

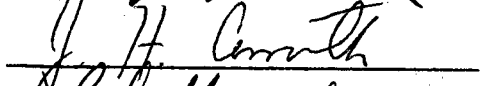
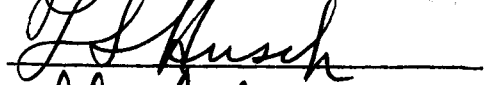
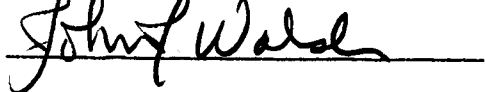
To the Graduate Council:

I am submitting herewith a dissertation written by Donald Kriss Preston entitled "A Study of Product Decompositions of Topological Manifolds." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Mathematics.



R. J. Daverman, Major Professor

We have read this dissertation
and recommend its acceptance:

Accepted for the Council:



Vice Chancellor
Graduate Studies and Research

Thesis
796
.P748
cop. 2

A STUDY OF PRODUCT DECOMPOSITIONS
OF TOPOLOGICAL MANIFOLDS

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Donald Kriss Preston

August 1979

1398235

DEDICATION

This dissertation is dedicated to the author's wife, Jean, without whose patience and support it could never have been completed.

ACKNOWLEDGMENTS

I would like to express my greatitide to Professor R.J. Daverman for his guidance and remarkable patience. His professionalism and and his friendship have shown me that he is an educator with a genuine concern for his students.

ABSTRACT

This work is concerned with upper semi-continuous, cell-like decompositions of topological manifolds. The first major result is a theorem which states that a sliced decomposition of $E^n \times E^1$ is shrinkable if each slice yields an $(n + 1)$ -manifold factor. The second major theorem states that the product of two cell-like, upper semi-continuous decompositions is shrinkable provided each yields an ANR and at least one is of codimension one in the source manifold.

The first result is proved using a spreading and shrinking in stages technique which is similar to methods used previously by E. Woodruff. The second result is proved via R.D. Edwards' theorem on approximating cell-like maps by homeomorphisms when the image possesses the Disjoint Discs Property.

TABLE OF CONTENTS

CHAPTER	PAGE
I. PRELIMINARIES	1
II. SLICED DECOMPOSITIONS	4
III. PRODUCTS OF DECOMPOSITION SPACES	18
BIBLIOGRAPHY	29
VITA	32

PRELIMINARIES

It is assumed that the reader has a familiarity with basic concepts concerning cell-like, upper semi-continuous decompositions of topological manifolds. A good source for this background material is Daverman [7].

Whenever G is a decomposition of a topological space X , we use the symbol H_G to denote the collection of those elements of G which contain more than one point. The union of the elements of H_G is denoted by N_G and is called the nondegeneracy set of G . If, in addition, we have a space Y with a decomposition K , the decomposition $G \times K$ is defined to be the collection of all subsets of $X \times Y$ of the form $g \times k$ where $g \in G$ and $k \in K$. When we refer to the decomposition X of a space X , we mean that decomposition of X so that $N_X = \emptyset$. For a subset C of X and a decomposition $G(C)$ of C we define the trivial extension of $G(C)$ to X as that decomposition G of X with $H_{G(C)} = H_G$.

The earliest substantial results concerning upper semi-continuous (USC) decompositions of manifolds are attributed to R.L. Moore. Moore [18] showed that subject to certain elementary connectedness requirements (cell-likeness, for example) on the elements of a USC decomposition of E^2 , the decomposition space is homeomorphic to E^2 .

In 1957, R.H. Bing [3] set forth an example of a decomposition of E^3 which showed that even if H_G consists of a relatively sparse collection of tame arcs, E^3/G need not be homeomorphic to E^3 . The decomposition space of Bing's example, known as the Dogbone Space, is not a manifold. However, Bing [4] did prove later that the product of his example with E^1 does yield E^4 . Eaton [12] generalized Bing's Dogbone Decomposition to higher dimensional Euclidean spaces to produce nonmanifold decomposition spaces of these dimensions. Bing [2] is also to be credited for isolating a criterion under which we may be assured that a decomposition space is homeomorphic to its source manifold. His notion of shrinkability can be stated in its most general form as follows:

G is a shrinkable decomposition of a manifold M if, for any open cover $\mathcal{V}$ of M and any G -saturated open cover $\mathcal{U}$ of M , there is a homeomorphism h of M to itself so that for each $g \in G$, there exists some $V \in \mathcal{V}$ so that $h(g) \subset V$ and for each $x \in M$, there exists some $U \in \mathcal{U}$ so that both x and $h(x)$ lie in U . In the manifold setting, G shrinkable implies that the decomposition space is homeomorphic to the source. A shrinkable decomposition is said to be strongly shrinkable if the shrinking homeomorphism h can be chosen to be the identity outside any predetermined neighborhood of N_G . Using the Partial Straightening Lemma of Edwards and Kirby [14], one can show that a USC decomposition of a manifold is shrinkable if and only if it is strongly shrinkable. For a proof of this see Daverman [7, Chap. XIV, Theorem 1]. If G is a shrinkable decomposition of a manifold M , then it is also well known that G is realized by a pseudo-isotopy.

That is, there is a homotopy ψ_t of M to itself so that $\psi_0 = 1$, ψ_t is a homeomorphism for $0 \leq t < 1$, and ψ_1 is a closed map of M to itself so that $G = \{\psi_1^{-1}(x) | x \in M\}$.

In the early to mid 1970's, the emphasis of decomposition theory shifted. For a number of years previous to then, the study of decompositions had focussed on properties of the nondegenerate elements and their relative positions in the source manifold. The new emphasis was placed on properties of the decomposition space itself which would assure shrinkability. J.W. Cannon [5] first isolated a property called the Disjoint Discs Property (DDP) which he showed in many cases implies shrinkability. The DDP requires that any two maps of the 2-disc B^2 into the decomposition space be approximable by maps with disjoint images. R.D. Edwards [13], in a work now being prepared for publication, proved that, for $n \geq 5$, a finite dimensional, cell-like, USC decomposition of an n -manifold is shrinkable if the decomposition space satisfies the DDP. Both Edwards' theorem and his techniques have been used to generate a flurry of new results in decomposition and manifold theory.

In particular, R.J. Daverman [8] has shown that if G is a cell-like, USC decomposition of an n -manifold M where $n \geq 3$ and M/G is finite dimensional, then $M/G \times E^2$ is an $(n + 2)$ -manifold. The major unanswered question in this area is whether cell-like, USC decompositions of n -manifolds yield $(n + 1)$ -manifold factors. In Chapter II of this work, we examine a generalization of product decompositions which we call sliced decompositions and in Chapter III we prove a theorem about product decompositions where each factor is assumed to have high dimension.

CHAPTER II

SLICED DECOMPOSITIONS

A decomposition G of $X \times Y$ is said to be sliced with respect to Y if each decomposition element g lies in some slice $X \times \{y_g\}$ where $y_g \in Y$. Product decompositions of the form $G \times Y$ make up a natural and important class of sliced decompositions. Moreover, there are powerful results concerning the shrinkability of cell-like product decompositions of $E^n \times E^1$. The major result of this chapter develops a connection, based on elementary techniques, between the shrinkability of such product decompositions and that of analogous sliced decompositions. More specifically, we prove that a sliced decomposition of $E^n \times E^1$ is shrinkable if the decomposition space induced in each slice $E^n \times \{s\}$ is an $(n + 1)$ -manifold factor.

From this point on, we shall compactify to consider sliced decompositions of $S^n \times S^1$. In this setting, shrinkability can be reduced to epsilonic controls and our work is considerably simplified. Techniques like ours will apply to sliced decompositions of $E^n \times E^1$ to give the results alluded to above.

With any sliced decomposition G of $S^n \times S^1$, we can associate two new decompositions for each slice $S^n \times \{s\}$. The decomposition G^s defined by

$$G^s = \{g \subset S^n \mid g \times \{s\} \in G\}$$

is a decomposition of S^n and $G(s)$ defined by

$$G(s) = \{g \in G \mid g \subset S^n \times \{s\}\} \cup [S^n \times (S^1 - \{s\})]$$

is a decomposition of $S^n \times S^1$. The latter is the trivial extension of $G^S \times \{s\}$ to all of $S^n \times S^1$. Observe that if $G^S \times E^1$ is a shrinkable decomposition of $S^n \times E^1$, then so is $G^S \times \{s\}$ trivially extended to $S^n \times S^1$ [7, Chap. XIV, Theorem 2].

The following theorem is the main result of this chapter.

Theorem 2.1. Suppose G is a sliced, cell-like USC decomposition of $S^n \times S^1$ satisfying

- (1) For each $s \in S^1$, the decomposition $G(s)$ of $S^n \times S^1$ is shrinkable

and

- (2) S^1 contains a countable, dense set $D = \{d(i)\}$ for which each of the decompositions $G^{d(i)}$ of S^n yields an $(n+1)$ -manifold factor (that is, $G^{d(i)} \times E^1$ is shrinkable).

Then G is shrinkable.

Before proving Theorem 2.1, we state and prove several lemmas.

Lemma 2.2. Suppose that G is a sliced, USC decomposition of $S^n \times S^1$ and that C is a compact, 0-dimensional subset of S^1 satisfying

$$(1) \quad N_G \subset S^n \times C$$

and

(2) for each $c \in C$, the decomposition $G(c)$ is shrinkable.

Then G is shrinkable.

Proof. Fix a positive number ε and a metric ρ on $S^n \times S^1/G$.

We shall produce a homeomorphism h of $S^n \times S^1$ to itself so that

(1) $\rho(\Pi h, \Pi) < \varepsilon$ where $\Pi: S^n \times S^1 \rightarrow S^n \times S^1/G$ is the decomposition map

and

(2) the diameter of $h(g)$ is less than ε for all $g \in G$.

Since Π is uniformly continuous, we may choose $\delta \in (0, \varepsilon/3)$ so that if F is a homeomorphism of $S^n \times S^1$ to itself moving points less than δ then $\rho(\Pi, \Pi F) < \varepsilon/3$. According to hypothesis (2) and an earlier remark, each decomposition $G(c)$ is strongly shrinkable. Let h_c be a homeomorphism of $S^n \times S^1$ to itself shrinking each $g \in G(c)$ to diameter less than $\varepsilon/3$, satisfying $\rho(\Pi, \Pi h_c) < \varepsilon/3$ and fixing points outside $S^n \times U_c$ where U_c denotes the $\delta/2$ -neighborhood of c in S^1 .

Since $h_c(G)$ is a USC decomposition for each $c \in C$, there is an open interval J_c such that $c \in J_c \subset U_c$, $C \cap (\text{Bdry } J_c) = \emptyset$, and for each $g \in G$ lying in $S^n \times J_c$, the diameter of $h_c(g)$ is less than $\varepsilon/3$. We extract a subcover $\{J_{c(i)} \mid i = 1, 2, \dots, N\}$ of C and cut back each $J_{c(i)}$ if necessary to make $J_{c(i)} \cap J_{c(j)} = \emptyset$ for $i \neq j$.

For $i = 1, 2, \dots, N$, let f_i be a homeomorphism of $C \times U_{C(i)}$ onto $C \times J_{C(i)}$ fixing $J_{C(i)} \cap C$ pointwise. Let $F_i = 1 \times f_i: S^n \times S^1 \rightarrow S^n \times S^1$ and let h be defined by

$$h = \begin{cases} F_i h_{C(i)} F_i^{-1} & \text{on } S^n \times J_{C(i)} \\ 1 & \text{elsewhere} \end{cases}$$

Clearly h satisfies the conditions required for our shrinking homeomorphism.

Lemma 2.3. Let G be a decomposition of S^n such that $G \times S^1$ is a shrinkable decomposition of $S^n \times S^1$. Also let C be a compact subset of S^1 and θ a map of $S^n \times S^1$ to itself realizing $G \times C$ trivially extended to $S^n \times S^1$. Then for each point c of C , $\theta(S^n \times \{c\})$ is bicollared in $S^n \times S^1$.

Proof. Let $\Pi: S^n \times S^1 \rightarrow S^n \times S^1/G \times S^1 = (S^n/G) \times S^1$ denote the decomposition map. By [7, Chap. XIV, Corollary 2A], the decomposition $\theta(G \times S^1)$ is strongly shrinkable. Thus there is a map θ' of $S^n \times S^1$ to itself fixing $\theta(S^n \times C)$ and realizing $\theta(G \times S^1)$. The map $\theta' \theta \Pi^{-1}: S^n \times S^1/G \times S^1 \rightarrow S^n \times S^1$ is a homeomorphism and takes $S^n/G \times \{c\}$ to the set

$$\theta' \theta \Pi^{-1}(S^n/G \times \{c\}) = \theta' \theta(S^n \times \{c\}) = \theta(S^n \times \{c\}).$$

Since $S^n/G \times \{c\}$ is bicollared in $S^n/G \times S^1$, its image $\theta(S^n \times \{c\})$ is also bicollared in $S^n \times S^1$.

Lemma 2.4. Let G be a shrinkable, sliced decomposition of $S^n \times S^1$, θ a map of $S^n \times S^1$ to itself realizing G , and $s \in S^1$ so that $G^s \times S^1$ is shrinkable. Then $\theta(S^n \times \{s\})$ is bicollared in $S^n \times S^1$.

Proof. Since $G^s \times S^1$ is shrinkable, we know that $G(s)$ is a shrinkable decomposition of $S^n \times S^1$. Hence, there exists a map f of $S^n \times S^1$ to itself realizing $G(s)$. As in the proof of Lemma 2.3, $f(G)$ is a strongly shrinkable decomposition of $S^n \times S^1$. Thus there is a map f' of $S^n \times S^1$ to itself fixing $f(S^n \times \{s\})$ and realizing $f(G)$. The map $f'f\theta^{-1}$ is a homeomorphism. By Lemma 2.3, the set $f'f(S^n \times \{s\}) = f(S^n \times \{s\})$ is bicollared in $S^n \times S^1$ so that $(f'f\theta^{-1})^{-1}(f'f(S^n \times \{s\})) = \theta(S^n \times \{s\})$ is bicollared in $S^n \times S^1$.

Lemma 2.5. Suppose that for $j = 1, 2, \dots$, there exists an embedding h_j of a compact space $X_j \times [-1, 1]$ in $S^n \times S^1$ satisfying

(1) $h_j(X_j \times \{\pm 1\})$ is bicollared

(2) $h_j(X_j \times [-1, 1]) \cap h_k(X_k \times [-1, 1]) = \emptyset$ for $j \neq k$

and

(3) $\lambda_j = \text{Max}\{\text{diameter of } h_j(p \times [-1, 1]) \mid p \in X_j\}$ approaches 0 as j approaches ∞ .

Then the decomposition G of $S^n \times S^1$ into points and the arcs $h_j(p \times [-1, 1])$ where $p \in X_j$ and $j = 1, 2, \dots$ is USC and shrinkable.

Proof. G is USC since N_G is the disjoint union of nondegeneracy sets of countably many USC decompositions with maximal diameters going to zero. The proof that G is shrinkable is modelled on the proof given by R.J. Bean [1] that a null sequence, star-like equivalent decomposition of E^3 is shrinkable.

Fix $\epsilon > 0$ and U a neighborhood of N_G . The fact that λ_i approaches zero means that we may assume that only $h_1(X_1 \times [-1, 1])$ has any fiber arcs of diameter at least ϵ . We need only find a homeomorphism h of $S^n \times S^1$ to itself which is the identity outside U and shrinks the arcs of $h_1([X_1 \times [-1, 1]])$ to size less than ϵ without stretching any other nondegenerate elements bigger than ϵ .

By hypothesis (1), we may assume that h_1 is actually an embedding of $X_1 \times [-2, 2]$ in $S^n \times S^1$. Let V be a neighborhood of $h_1(X_1 \times [-1, 1])$ which is G -saturated and contained in U and in the image of h_1 . Since h_1 is uniformly continuous, there exists $\delta > 0$ so that if $d(x, y) < \delta$, then $d(h_1(x), h_1(y)) < \epsilon$ for x and y in $X_1 \times [-2, 2]$. Choose a positive integer K so that $K\delta/16 > 1 + \delta/8$ and numbers $\beta_i > 1$ for $i = 1, 2, \dots, K$ so that

$$(1) \quad X_1 \times [-\beta_K, \beta_K] \subset h_1^{-1}(V) \cap (X_1 \times (-1 - \delta/16, 1 + \delta/16))$$

$$(2) \quad \beta_i < \beta_{i+1} \quad \text{for } i < K$$

and

$$(3) \quad \text{if } g' \in G \text{ and } g' \cap h_1(X_1 \times [-1, 1]) = \emptyset \text{ but}$$

$$g' \cap h_1(X_1 \times [-\beta_i, \beta_i]) \neq \emptyset \text{ then the diameter of } h_1^{-1}(g')$$

$$\text{is less than } \delta/16 \text{ and if } i < K, \text{ then } g' \subset h_1(X_1 \times [-\beta_{i+1}, \beta_{i+1}]).$$

Now define $h': X_1 \times [-2, 2] \rightarrow X_1 \times [-2, 2]$ by

$$\begin{aligned} h'(x, -\beta_i) &= (x, -i\delta/16) \quad \text{if } \beta_i \geq i\delta/16 \\ &= (x, -\beta_i) \quad \text{if not} \end{aligned}$$

and

$$\begin{aligned} h'(x, \beta_i) &= (x, i\delta/16) \quad \text{if } \beta_i \geq i\delta/16 \\ &= (x, \beta_i) \quad \text{if not .} \end{aligned}$$

The map h' can be extended to all of $X_1 \times [-2, 2]$ in a linear fashion on $X_1 \times ([-\beta_i, -\beta_{i-1}] \cup [\beta_{i-1}, \beta_i])$ and as the identity outside $X_1 \times [-\beta_K, \beta_K]$. Then

$$\begin{aligned} h &= h_1 h' h_1^{-1} \quad \text{on } h_1(X_1 \times [-2, 2]) \\ &= 1 \quad \text{elsewhere} \end{aligned}$$

shrinks the fibers of $h_1(X \times [-1, 1])$ to ε -size and stretches no other nondegenerate elements bigger than ε -size.

Proof of Theorem 2.1.

Step 1: Stretching the original decomposition in the S^1 direction.

Let $f: S^1 \rightarrow S^1$ denote a monotone map with the property that

$N_f = \{f^{-1}(d(i)) \mid i = 1, 2, \dots\}$. Such an f is easy to construct using

the fact that any two countable dense subsets of S^1 are equivalently embedded there. We denote the product map $1 \times f: S^n \times S^1 \rightarrow S^n \times S^1$ by F and the decomposition induced by F we call G_F . G_F resembles G except that elements of G from the $d(i)$ levels have been stretched in the S^1 direction.

We show that G_F is shrinkable. Before proceeding with that, we point out how to establish the shrinkability of G assuming G_F is shrinkable.

Consider the diagram:

$$\begin{array}{ccc}
 S^n \times S^1 & \xrightarrow{F} & S^n \times S^1 \\
 \downarrow \Pi_F & \searrow F^* = \Pi_F F^{-1} & \downarrow \Pi_G \\
 S^n \times S^1 / G_F & \xrightarrow{H} & S^n \times S^1 / G
 \end{array}$$

$H = \Pi_G F \Pi_F^{-1}$ is clearly a homeomorphism. Assuming G_F is shrinkable, we also know that F and Π_F are approximable by homeomorphisms. We shall show that this implies $F^* = \Pi_F F^{-1}$ is approximable by homeomorphism so that by uniform continuity $H F^* = \Pi_G$ is also approximable by homeomorphisms.

So let $\epsilon > 0$ be given and choose a homeomorphism $h_1: S^n \times S^1 \rightarrow S^n \times S^1 / G_F$ which is $\epsilon/4$ close to Π_F . Since $F \Pi_F^{-1}(U)$ is an open cover of $S^n \times S^1$, we can find a homeomorphism h_2 of $S^n \times S^1$ to itself which is $\epsilon/4$ close to $F \Pi_F^{-1}(U)$. If $x \in S^n \times S^1$, then $h_2 h_1^{-1}(x) = x$ and $F h_2^{-1}(x)$ belong to

some common element of $F \Pi_F^{-1}(U)$ which we shall call $F \Pi_1^{-1}(U)$. Thus $F^{-1}(x)$ and $F^{-1} F h_2^{-1}(x)$ are each subsets of $F^{-1} F \Pi_F^{-1}(U) = \Pi_F^{-1}(U)$. This implies that $\Pi_F F^{-1}(x) \in U$ and $\Pi_F F^{-1} F h_2^{-1}(x) \in U$. But since h_1 is U -close to Π_F , there is some $V \in U$ so that $h_1 h_2^{-1}(x)$ and $\Pi_F h_2^{-1}(x) = \Pi_F F^{-1} F h_2^{-1}(x)$ both lie in V . Therefore, $\Pi_F F^{-1}(x)$ and $h_1 h_2^{-1}(x)$ both lie in $U \cup V$ which has diameter less than ϵ . So $\rho(\Pi_F F^{-1}, h_1 h_2^{-1}) < \epsilon$.

We now proceed to show that G_F is shrinkable.

Step 2: Slicing the stretched decomposition.

We will shrink G_F in stages by shrinking several related decompositions. The first of these is a sliced decomposition which is a kind of stratification of G_F , defined by

$$G_1 = \{g_F \cap (S^n \times \{s\}) \mid g_F \in G_F \text{ and } s \in S^1\}.$$

Step 3: Opening the stratified version of G_F .

The second related decomposition is a fenestration of G_1 obtained by eliminating nondegenerate elements from a certain dense and open collection of levels. Specifically, if we let N_f denote the nondegenerate point inverses of $f: S^1 \rightarrow S^1$, G_2 consists of singletons from $S^n \times \text{Int } N_f$ together with $\{g_F \cap (S^n \times \{s\}) \mid g_F \in G_F \text{ and } s \in \text{Cl}(S^1 - N_f)\}$. In other words, the nondegenerate elements of G_2 are precisely those nondegenerate elements of G_1 from levels not interior to any elements of N_f .

Step 4: Shrinking G_F .

According to Lemma 2.2, the fenestrated decomposition G_2 is shrinkable. Hence, there exists a map θ_2 of $S^n \times S^1$ to itself realizing G_2 . That is, $G_2 = \{\theta_2^{-1}(x) | x \in S^n \times S^1\}$. We may assume that θ_2 is the end of a pseudo-isotopy ψ_t^2 defined on $S^n \times S^1$ where ψ_t^2 is controlled by any G_2 -saturated open cover that we choose in advance. By selecting the cover $\{\pi_F^{-1}(N_{\epsilon/3}(x)) | x \in S^n \times S^1/G_F\}$ we can insure that $\rho(\pi_F, \pi_F \psi_t^2) < \epsilon/3$.

Naturally, we turn our attention to the decomposition $\theta_2(G_1)$ whose nondegenerate elements are partitioned into countably many (curvilinear) product decompositions. Explicitly, for each inverse set $A_i = f^{-1}(d(i))$ in S^1 , $\theta_2(G_1 \cap (S^n \times \text{Int } A_i))$ is a decomposition which is equivalent to $G^{d(i)} \times E^1$ which by hypothesis is shrinkable. Hence, there is a homeomorphism of $S^n \times S^1$ to itself, fixed outside $\theta_2(S^n \times A_i)$, which shrinks $\theta_2(G_1 \cap (S^n \times A_i))$ to any preassigned degree of smallness. Only a finite number of indices i give rise to a $\theta_2(G_1)$ -nondegenerate element of diameter greater than any preassigned positive number since the only nondegenerate elements of $\theta_2(G_1)$ occur in the images under θ_2 of levels interior to some A_i . Thus $\theta_2(G_1)$ is shrinkable and can be realized by a map θ_1 of $S^n \times S^1$ to itself which is the end of a pseudo-isotopy ψ_t^1 such that $\rho(\pi_F, \pi_F \psi_t^1) < \epsilon/3$.

Finally, we turn to the decomposition $\theta_1\theta_2(G_F)$ from which all of the strata have been crushed to points, leaving only products of an arc with $S^n/G^{d(i)}$ associated with a dense set $\{d(i) | i = 1, 2, \dots\}$ of levels. These fiber arcs can be shrunk using Lemmas 2.4 and 2.5, so that we get a pseudo-isotopy ψ_t^F with $\theta_F = \psi_1^F$ so that θ_F realizes

$$\theta_1 \theta_2(G_F) \text{ and } \rho(\Pi_F, \Pi_F \psi_t^F) < \varepsilon/3 .$$

By choosing $\gamma \in [0, 1)$ close enough to 1, we get a homeomorphism $\psi_\gamma^F \psi_\gamma^1 \psi_\gamma^2$ of $S^n \times S^1$ to itself which shrinks G_F to small size and

$$\rho(\Pi_F, \Pi_F \psi_\gamma^F \psi_\gamma^1 \psi_\gamma^2) \leq \rho(\Pi_F, \Pi_F \psi_\gamma^2) + \rho(\Pi_F, \Pi_F \psi_\gamma^1) + \rho(\Pi_F, \Pi_F \psi_\gamma^F) < \varepsilon .$$

Therefore, G_F is shrinkable.

Corollary 2.6. If G is a sliced, USC decomposition of E^{n+1} such that, for each $s \in E^1$, $G^s \times E^1$ is shrinkable, then G is shrinkable.

Proof. As stated earlier, $G^s \times E^1$ shrinkable implies $G(s)$ shrinkable.

Corollary 2.7. If G is a sliced, USC decomposition of E^{n+1} such that, for each $s \in E^1$, G^s is a shrinkable decomposition of E^n , then G is shrinkable.

Corollary 2.7 is a special case of a result proved by Sher [9] and Walsh [20]. Using Corollary 2.7 and results of Moore [18], we can offer a proof of a result by Dyer and Hamstrom [11].

Corollary 2.8. Every sliced, cell-like decomposition of E^3 is shrinkable.

Corollary 2.9. If G is a sliced, cell-like, USC decomposition of E^{n+1} and E^{n+1}/G is an ANR, then $G \times E^1$ is shrinkable.

Proof. The important observation is that $G \times E^1$ can be viewed as a sliced decomposition of E^{n+2} where each slice is a product decomposition of $E^n \times E^1$. By Daverman [8], each slice is an $(n+2)$ -manifold factor so that the hypotheses of Theorem 2.1 are satisfied.

In support of our conviction that sliced decomposition theory must parallel product decomposition theory, we offer the following generalization of Daverman's result concerning products of cell-like decompositions with E^2 .

Corollary 2.10. If G is a sliced, cell-like decomposition of $E^n \times E^2$ in the sense that for each $g \in G$, there exists $s_g \in E^2$ so that $g \subset E^n \times \{s_g\}$, then G is shrinkable.

Proof. This follows directly from Corollary 2.9 and Theorem 2.1.

Corollary 2.11. If G is a 0-dimensional, cell-like decomposition of E^n considered as $E^n \times \{0\} \subset E^n \times E^1$, then the trivial extension of G to all of E^{n+1} is shrinkable.

Proof. Using methods similar to those in [15, Chapter V], we can get a map $f: E^n/G \rightarrow E^1$ so that $f|_{\Pi(N_G)}$ is an embedding. Let θ denote the E^n -coordinate preserving homeomorphism of E^{n+1} to itself defined by $\theta(\langle x, s \rangle) = \langle x, s + f\Pi(x) \rangle$. Then $\theta(G)$ is a sliced, cell-like decomposition of E^{n+1} and in each slice there is at most a single nondegenerate element. Therefore, each slice is an $(n+1)$ -manifold factor. This requires the use of Cannon [5] and Edwards [13]. Thus, by Corollary 2.6, $\theta(G)$ is shrinkable and G must be also.

Corollary 2.12. Suppose that G is a sliced, cell-like USC decomposition of E^{n+1} ($n \geq 4$) such that E^{n+1}/G is finite dimensional and E^1 contains a dense subset D for which the decompositions G^d yield E^{n+1} factors. Then G is shrinkable.

Proof. It follows from Daverman [8] that, for each $s \in E^1$, $G(s)$ is shrinkable.

Corollary 2.13. Suppose G is a sliced, cell-like USC decomposition of E^{n+1} ($n \geq 4$) such that, for each $s \in E^1$, $G(s)$ is a closed, $(n - 2)$ -dimensional decomposition in the sense that $Cl(\pi(N_{G(s)}))$ is $(n - 2)$ -dimensional. Then G is shrinkable.

Proof. Cannon [5] and Edwards [13] show that $G^S \times E^1$ is shrinkable.

Corollary 2.14. If G is a sliced, cell-like, USC decomposition of E^4 such that, for each $s \in E^1$, $G(s)$ is 0-dimensional, then G is shrinkable.

Proof. By [10], $G^S \times E^1$ is shrinkable and by Corollary 2.11, $G(s)$ is shrinkable.

Corollary 2.15. If G is a 1-dimensional, cell-like, USC decomposition of $E^3 \times \{0\} \subset E^3 \times E^1$, then the trivial extension of G to E^4 is shrinkable.

Proof. Much as in the proof of Corollary 2.11, we can spread G to be 0-dimensional on each slice and then apply Corollary 2.14.

Corollary 2.16. If G is a sliced, cell-like, USC decomposition of E^4 such that, for each $s \in E^1$, $N_{G(s)}$ has embedding dimension less than two then G is shrinkable.

Proof. Since $N_{G(s)}$ is of embedding dimension one, it has dimension one. Then Kozłowski [16] shows that its image in $E^4/G(s)$ is also 1-dimensional. By Corollary 2.15, $G(s)$ is shrinkable. By [9], $G^S \times E^1$ is shrinkable as well.

Corollary 2.17. If G is a sliced, cell-like, USC decomposition of E^{n+1} ($n \geq 4$) so that each G^S is $(n - 3)$ -dimensional, then G is shrinkable.

Proof. Daverman [8] shows that $G^S \times E^1$ is shrinkable.

CHAPTER III

PRODUCTS OF DECOMPOSITION SPACES

As we observed in Chapter I, one of the major open questions in decomposition theory concerns products of decomposition spaces with E^n . For $n = 2$, Daverman [8] has completely resolved the issue. In this chapter, we study products of decomposition spaces with other decomposition spaces. Daverman's work coupled with that of Moore [18] covers the question of shrinkability when one factor has dimension at least three and the other has dimension two. Thus, our work can be restricted to product decompositions of manifolds $M^m \times N^n$ where $m, n \geq 3$. We should point out that if G and G^* are decompositions of M^m and N^n respectively, then $M/G \times N/G^*$ is homeomorphic to $M \times N/G \times G^*$. Our main result is the following theorem where we again restrict our attention to decompositions of spheres in order to simplify the epsilonics.

Theorem 3.1. Let G be a cell-like, USC decomposition of S^n that is $(n - 1)$ -dimensional in the following sense: If $\pi: S^n \rightarrow S^n/G$ is the decomposition map, then $\dim \pi(N_G) \leq n - 1$. Also, let G' be a cell-like, USC decomposition of S^m which yields a finite dimensional space or equivalently, an ANR. Then if $n, m \geq 3$, $S^n/G \times S^m/G'$ is homeomorphic to $S^n \times S^m$ and $G \times G'$ is shrinkable.

Before presenting the proof, we offer several lemmas which are needed for it.

Lemma 3.2. If f and g are maps of B^2 into S^n/G where G is cell-like, USC, and $(n-1)$ -dimensional and $n \geq 3$, then f and g can be approximated by maps f_* and g_* where

$$\dim f_*^{-1}(f_*(B^2) \cap g_*(B^2)) \leq 1 \quad \text{and}$$

$$\dim g_*^{-1}(f_*(B^2) \cap g_*(B^2)) \leq 1 .$$

Proof. The proof relies on the fact that $\Pi(N_G)$ contains no open subset of S^n/G which in turn depends upon the fact that a cell-like map of a manifold without boundary to an ANR cannot lower dimension [see 7, Chap. 12].

We shall obtain f_* and g_* as limits of maps f_i and g_i . Let $f_0 = f$ and $g_0 = g$ and let $Q = \{r_1, r_2, \dots\}$ be a countable, dense subset of B^2 . Fix $\epsilon > 0$. We intend to choose f_i and g_i so that

$$(1) \quad \rho(f_i, f_{i-1}) < \epsilon/2^{i+1} ,$$

$$(2) \quad \rho(g_i, g_{i-1}) < \epsilon/2^{i+1} ,$$

$$(3) \quad \{r_1, r_2, \dots, r_i\} \subset B^2 - [f_i^{-1}(f_i(B^2) \cap g_i(B^2)) \cup g_i^{-1}(f_i(B^2) \cap g_i(B^2))] ,$$

and property (3) is preserved in the limit. These properties will

clearly imply the desired conclusion.

To choose f_1 and g_1 , consider $f_0(r_1)$. Since S^n/G is locally path connected, we can choose a neighborhood U of $f_0(r_1)$ so that any point in U can be connected to $f_0(r_1)$ by a path in $N_{\epsilon/8}(f_0(r_1))$. Because G is $(n-1)$ -dimensional, there is a point r'_1 of $U - \Pi(N_G)$. Let f'_1 be a map of B^2 in S^n/G which agrees with f_0 outside $N_\delta(r_1)$ where δ is so small that $N_\delta(r_1) \subset f_0^{-1}(U)$, sends $N_\delta(r_1) - N_{\delta/2}(r_1)$ to $f_0(N_\delta(r_1) - r_1)$ in a "linear" fashion, and sends $N_{\delta/2}(r_1)$ along the path between $f_0(r_1)$ and r'_1 so that $f'_1(r_1) = r'_1$. Note that $\rho(f_0, f'_1) < \epsilon/4$ and since $\Pi^{-1}(r'_1)$ is a point in S^n , we can use an approximate lift of g_0 to S^n and general position is to get a map g'_1 of B^2 in S^n/G which is $\epsilon/4$ close to g_0 with $r'_1 \notin g'_1(B^2)$.

Now consider $g'_1(r_1)$. As with f_0 , we can find a map g_1 of B^2 in S^n/G agreeing with g'_1 outside a small neighborhood of r_1 and sending r_1 to a point r''_1 which is not in $\Pi(N_G)$. By a careful choice of the neighborhood mentioned above, we can insure that $\rho(g_0, g_1) < \epsilon/2$ and that $f'_1(r_1) \notin g_1(B^2)$. As before, we can also find a general position map f_1 of B^2 in S^n/G so that $g_1(r_1) \notin f_1(B^2)$ and so close to f'_1 that $\rho(f_0, f_1) < \epsilon/2$ and $f_1(r_1) \notin g_1(B^2)$. The maps f_1 and g_1 have the desired properties.

Now suppose that f_i and g_i have been chosen so that

$$(1) \quad \rho(f_j, f_{j+1}) < \epsilon/2^j \quad \text{for } j = 0, 1, 2, \dots, i-1$$

$$(2) \quad \rho(g_j, g_{j+1}) < \epsilon/2^j \quad \text{for } j = 0, 1, 2, \dots, i-1$$

$$(3) \quad \{r_1, r_2, \dots, r_i\} \subset B^2 - [f_i^{-1}(f_i(B^2) \cap g_i(B^2)) \cup g_i^{-1}(f_i(B^2) \cap g_i(B^2))]$$

$$(4) \quad \rho(f_i(r_j), g_i(B^2)) \geq \left(\frac{1}{2} + \frac{1}{i+2}\right) \rho(f_j(r_j), g_j(B^2)) > 0$$

for $j = 1, 2, \dots, i-1$

and

$$(5) \quad \rho(g_i(r_j), f_i(B^2)) \geq \left(\frac{1}{2} + \frac{1}{i+2}\right) \rho(g_j(r_j), f_j(B^2)) > 0$$

for $j = 1, 2, \dots, i-1$.

The last two conditions preserve condition (3) in the limit. To construct f_{i+1} and g_{i+1} , consider $f_i(r_{i+1})$. If $f_i(r_{i+1}) = f_i(r_j)$ for some $j \leq i$, then we do not need to adjust $g_i(B^2)$ off of $f_i(r_{i+1})$. Otherwise, choose $\gamma > 0$ so small that

$$\rho(f_i(r_{i+1}), f_i(r_j)) \geq \gamma \quad \text{and} \quad \rho(f_i(r_{i+1}), g_i(r_j)) \geq \gamma \quad \text{for } j \leq i \quad \text{and}$$

so that any point of $N_\gamma(f_i(r_{i+1}))$ can be joined to $f_i(r_{i+1})$ by a path in $N_{\epsilon/2^{i+1}}(f_i(r_{i+1}))$. As in the $i = 0$ case, find a point

r'_{i+1} of $N_\gamma(f_i(r_{i+1})) - \Pi(N_G)$ and let f'_{i+1} be a map of B^2 in S^n/G which sends a small neighborhood of r_{i+1} which misses

$\{r_1, r_2, \dots, r_i\}$ to the path mentioned above and which sends r_{i+1}

to r'_{i+1} . We can require $\rho(f_i, f'_{i+1}) < \epsilon/2^{i+2}$ and for $j = 1, 2, \dots, i$

$$\rho(f'_{i+1}(B^2), g_i(r_j)) \geq \left(\frac{1}{2} + \frac{3}{4(i+3)} + \frac{1}{4(i+2)}\right) \rho(f_j(B^2), g_j(r_j)) .$$

Now let g'_{i+1} be a map satisfying

$$(1) \quad \rho(g_i, g'_{i+1}) < \varepsilon/2^{i+2}$$

$$(2) \quad f'_{i+1}(r_{i+1}) \notin g'_{i+1}(B^2)$$

$$(3) \quad \rho(f'_{i+1}(B^2), g'_{i+1}(r_j)) \geq \left(\frac{1}{2} + \frac{1}{2(i+3)} + \frac{1}{2(i+2)}\right) \rho(f_j(B^2), g_j(r_j))$$

for $j = 1, 2, \dots, i$

and

$$(4) \quad \rho(g'_{i+1}(B^2), f'_{i+1}(r_j)) \geq \left(\frac{1}{2} + \frac{1}{2(i+3)} + \frac{1}{2(i+2)}\right) \rho(g_j(B^2), f_j(r_j))$$

for $j = 1, 2, \dots, i$.

If $g'_{i+1}(r_{i+1}) \notin f'_{i+1}(B^2)$, we can let $f_{i+1} = f'_{i+1}$ and $g_{i+1} = g'_{i+1}$. Otherwise, consider $g'_{i+1}(r_{i+1})$. Exercising controls similar to those above, we can move $g'_{i+1}(r_{i+1})$ to a point nearby off which we can push $f'_{i+1}(B^2)$ to get maps f_{i+1} and g_{i+1} subject to the controls we need.

The study of decomposition spaces has been greatly advanced in recent years by the examination of general position properties which they possess. The DDP is known to imply shrinkability of decompositions provided that the decomposition space is an ANR. The Disjoint Arcs Property (DAP) requires that any two maps of the unit interval into

M/G be approximable by maps with disjoint images. The following lemma asserts that all decomposition spaces that we shall study possess the DAP.

Lemma 3.3. If G is a cell-like, USC decomposition of an orientable n -manifold M where $n \geq 3$, then M/G has the DAP.

Proof. Let f and g be maps of $I = [0, 1]$ into M/G and ϵ a positive number. Our first step is to find a map F which is ϵ -close to f so that $F(I)$ is 0-LCC in M/G . To this end, let $0 = t_0 < t_1 < \dots < t_k = 1$ be a partition of I of such small mesh that $f([t_i, t_{i+1}])$ has diameter less than ϵ for $i = 0, 1, 2, \dots, k-1$. Since $f([t_i, t_{i+1}])$ is a Peano continuum and hence arcwise connected, we can find a map f_i of $[t_i, t_{i+1}]$ into $f([t_i, t_{i+1}])$ so that $f_i([t_i, t_{i+1}])$ is an arc running from $f(t_i)$ to $f(t_{i+1})$ or is a point if $f(t_i) = f(t_{i+1})$. Let F be the union of the maps $\{f_i | i = 0, 1, \dots, k-1\}$.

Observe that $A_i = F([t_i, t_{i+1}])$ is 0-LCC in M/G for $i = 0, 1, 2, \dots, k-1$. For if U is a neighborhood of $a \in A_i$, then there is a connected neighborhood V of a , contained in U , so that $V \cap A_i$ is connected. Then $V \cap A_i$ is either a point or homeomorphic to E^1 or $[0, \infty)$. A simple duality argument will show that $H_0(V - (V \cap A_i)) = 0$ so that A_i is 0-LCC.

Now, since A_i is 0-LCC for $i = 0, 1, 2, \dots, k-1$, $F(I) = \bigcup_{i=0}^{k-1} A_i$ is 0-LCC. For this, it suffices to show that if $a \in A_i \cap A_j$ and U is a neighborhood of a , then there is a neighborhood

V of a so that pairs of points in $V - (A_i \cup A_j)$ bound singular arcs in $U - (A_i \cup A_j)$. So let V be a neighborhood of a satisfying 0-LCC conditions for A_i and let x and y be points of $V - (A_i \cap A_j)$. We get a map α of $[0, 1]$ into $U - A_i$ so that $\alpha(0) = x$ and $\alpha(1) = y$. Since A_j is 0-LCC, $A_j \cap \alpha([0, 1])$ is compact, and M/G is locally path connected, we can find a partition $0 = S_0 < S_1 < \dots < S_m = 1$ of $[0, 1]$ and a map α' of $[0, 1]$ into U which agrees with α on $\{0, 1\}$ and so that either $\alpha'([S_k, S_{k+1}]) \cap A_j = \emptyset$ or $\alpha'(S_k)$ and $\alpha'(S_{k+1})$ miss A_j and are so close together that they bound a singular arc in $U - (A_i \cup A_j)$. This last condition will give us the property needed for 0-LCC.

Finally, to see the DAP, we need only approximate g by a map G so that $G(I) \cap F(I) = \emptyset$. This is accomplished in a manner similar to that used above to show that $\alpha(I)$ can be pushed off $A_i \cup A_j$.

One can easily prove that any map of a 1-complex into a reasonable space with the DAP can be approximated by an embedding. The next lemma generalizes this result.

Lemma 3.4. If Q is a compact, locally contractible, separable, metric ANR with the DAP, then any map of a 1-dimensional compactum into Q can be approximated by an embedding.

Proof. Let $M_\epsilon(X, Q)$ denote the space of ϵ -mappings of X into Q and $M(X, Q)$ the space of all mappings of X into Q . Clearly $M_\epsilon(X, Q)$ is an open subset of $M(X, Q)$ if X is compact. We shall show that if X is also 1-dimensional and Q has DAP, then $M_\epsilon(X, Q)$ is also dense in $M(X, Q)$. By Baire's Theorem, this will suffice to prove the lemma.

Let f be a map of X into Q and ϵ and δ be positive numbers. We shall find an ϵ -map f' of X into Q that is δ -close to f . Since Q is locally contractible and compact, there is a positive number β so that any β -subset of Q contracts in an $(\epsilon/5)$ -subset of Q . Let γ be a positive number so that, for any γ -subset A of X , $f(A)$ has diameter less than β and $\delta/4$. Let $\mathcal{U} = \{U_1, U_2, \dots, U_r\}$ be an order 1 cover of X by open sets of diameter less than $\gamma/2$ and $\epsilon/2$. Without loss, we assume that we can also pick a point x_i of $U_i - \bigcup_{j \neq i} U_j$ for each $i = 1, 2, \dots, r$.

Denote by N the nerve of the covering $\mathcal{U}$ and let y_i be the vertex of N corresponding to U_i . By $[y_i, y_j]$ we shall mean the 1-simplex of N joining y_i and y_j whenever $U_i \cap U_j \neq \emptyset$. Let g denote the $\mathcal{U}$ -map of X onto N promised by Hurewicz and Wallman [15, Theorem V9]. Note that $g(U_i - \bigcup_{j \neq i} U_j) = y_i$ for $i = 1, 2, \dots, r$.

We have a map h of the vertices of N into Q defined by $h(y_i) = f(x_i)$. Since $\rho(x_i, x_j) < \gamma$ when $U_i \cap U_j \neq \emptyset$, $\rho(h(y_i), h(y_j)) < \beta$ whenever $[y_i, y_j]$ is a 1-simplex of N . Thus, we can find an extension of h to all of N which sends $[y_i, y_j]$ into a subset of Q of diameter less than $\epsilon/5$. Let h' be an embedding of N into Q which is so close to h that $h'([y_i, y_j])$ is contained in an $(\delta/4)$ subset of Q and $\rho(h'(y_i), f(x_i)) < \delta/4$ for all choices of i and j for which the above make sense. Finally, let $f' = h'g$. Since h' is an embedding and g is an ϵ -map, f' is an ϵ -map. Also $\rho(f, f') < \delta$ since if $x \in U_i$

$$\rho(f(x), f'(x)) \leq \rho(f(x), f(x_i)) + \rho(f(x_i), f'(x_i)) + \rho(f'(x_i), f'(x)) < \delta/4 + \delta/4 + \delta/4 < \delta.$$

Proof of Theorem 3.1. We shall show that $S^n/G \times S^m/G'$ has the DDP. Let f and g be maps of B^2 into $S^n/G \times S^m/G'$ and let p_1 and p_2 respectively represent the cartesian projection maps of $S^n/G \times S^m/G'$ to S^n/G and S^m/G' . By Lemma 3.2, we can approximate $p_1 f$ and $p_1 g$ by maps f_* and g_* so that if $X = f_*(B^2) \cap g_*(B^2)$, then $f_*^{-1}(X)$ and $g_*^{-1}(X)$ are 1-dimensional subsets of B^2 .

Now let Y represent the disjoint union of the sets $f_*^{-1}(X)$ and $g_*^{-1}(X)$. We have a map $p_2 f \cup p_2 g$ of Y into S^m/G' . Lemmas 3.3 and 3.4 now give us maps $f_\#$ and $g_\#$ of $f_*^{-1}(X)$ and $g_*^{-1}(X)$ respectively which approximate $p_2 f$ and $p_2 g$ on the sets over which they are defined and which have disjoint images. By [7, Chap. 12], we can extend $f_\#$ and $g_\#$ to maps of B^2 into S^m/G' which are close to $p_2 f$ and $p_2 g$.

Let $f' = f_* \times f_\#$ and $g' = g_* \times g_\#$. Clearly f' and g' have disjoint image and could have been chosen to approximate f and g as closely as desired. Thus, by Edwards [13], we have that $G \times G'$ is shrinkable and so $S^n/G \times S^m/G'$ is homeomorphic to $S^n \times S^m$.

We shall list several corollaries of Theorem 3.1 but first we will need the following lemma which we feel is worth stating for its own sake.

Lemma 3.5. If Π is a cellular, USC map of an m -manifold M onto an ANR Q , then Π can be approximated by a cell-like, USC map f with $f(N_f)$ having dimensions less than m .

Proof. First observe that a subset of Q is m -dimensional if and only if it contains an open subset of Q . This follows since π cannot lower dimension on manifolds without boundary or raise dimension when it maps a locally compact ANR onto an ANR. We shall find f close to π so that $f(N_f)$ misses some countable, dense subset of Q .

Let $R = \{r_i | i = 1, 2, \dots\}$ be dense in Q . The map f will be defined as a limit of maps f_i where $f_i(N_{f_i}) \cap \{r_1, r_2, \dots, r_i\} = \emptyset$ and f_i can be chosen as close as desired to f_{i-1} . We show only how to construct f_1 from π .

Consider $\pi^{-1}(r_1) = X$ and assume X is not a point. Then X is cellular in M so the decomposition $\{X\} \cup \{\text{singletons of } M - X\}$ is shrinkable. Let θ be a map of M to M fixed outside a very small neighborhood of X and realizing the decomposition described above. Then $f_1 = \pi \theta^{-1}$ is one to one over $\{r_1\}$ as we desired.

Corollary 3.6. If G is a cellular, USC decomposition of an m -manifold M ($m \geq 3$) such that M/G is an ANR and G' is a cell-like decomposition of an n -manifold N ($n \geq 3$) such that N/G' is an ANR, then $G \times G'$ is shrinkable.

Proof. This follows easily from Theorem 3.1 and Lemma 3.5.

Theorem 3.1 gives us a simple proof of a special case of a result by Curtis and McMillan [6] concerning cellularity of products of cell-like sets. This result is given as the next corollary and Corollary 3.8 makes use of that result.

Corollary 3.7. If G_1 and G_2 are cell-like, USC decompositions of manifolds N^n and M^m ($n \geq 2, m \geq 3$) so that N/G_1 and M/G_2

are ANRs then $G_1 \times G_2$ is a cellular decomposition of $N \times M$.

Proof. Let X and Y be elements of G_1 and G_2 respectively. We need to show that $X \times Y$ is cellular in $N \times M$. For this we need only show that the decomposition K of $N \times M$ defined as the product of the decompositions $\{X\} \cup \{\text{singletons of } N - X\}$ and $\{Y\} \cup \{\text{singletons of } M - Y\}$ is shrinkable. Since each of these is a codimension one decomposition, Theorem 3.1 gives the desired result. In case $n = 2$, Daverman [8] gives the conclusion.

Corollary 3.8. If G_1, G_2 , and G_3 are cell-like, USC decompositions of spheres S^m, S^n , and S^q respectively so that each decomposition space is an ANR, then $G_1 \times G_2 \times G_3$ is shrinkable.

Proof. If $m, n, q \geq 3$, then $G_1 \times G_2$ is cellular so Corollary 3.6 applies. If any one of m, n , and q is two, or if any two of m, n , and q sum to two, then Daverman [8] and Moore [18] give the desired result. If $m = 1$ and $n > 3$, then $G_1 \times G_2$ is cellular by McMillan [17] so that Corollary 3.7 applies. Finally, if $m = 1, n = 3$, and $q = 3$, then McMillan [17] shows that $G_1 \times G_2$ is cellular since handbodies in S^3 can be pushed into 3-cells nearby when we cross with S^1 .

I wish to point out to the reader that shortly after completion of this paper, Charles Bass announced that he has proved that the product of any two cell-like decompositions is shrinkable provided that each factor is an ANR. His result is being prepared for publication and should appear soon.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Bean, R.J., Decompositions of E^3 with a null sequence of star-like equivalent non-degenerate elements are E^3 , Illinois J. of Math. Vol. 11, No. 1(1967), 21 - 23.
2. Bing, R.H., Upper semicontinuous decompositions of E^3 , Ann. of Math. 65(1957), 363 - 374.
3. _____, A decomposition of E^3 into points and tame arcs such that the decomposition space is topologically different from E^3 , Ann. of Math. 65(1957), 484 - 500.
4. _____, The cartesian product of a certain nonmanifold and a line is E^4 , Ann. of Math. 70(1959), 399 - 412.
5. Cannon, J.W., Shrinking cell-like decompositions of manifolds, Codimension three, Ann. of Math., to appear.
6. Curtis, M.L. and McMillan, D.R., Cellularity of sets in products, Michigan Math. J. 9(1962), 299 - 302.
7. Daverman, R.J., Decomposition theory class notes, The University of Tennessee, Knoxville, (1979).
8. _____, Detecting the disjoint discs property, in preparation.
9. Daverman, R.J. and Preston, D.K., Cell-like 1-dimensional decompositions of S^3 are 4-manifold factors, to appear.
10. Daverman, R.J. and Row, W.H., Cell-like 0-dimensional decompositions of S^3 are 4-manifold factors, Trans. Amer. Math. Soc., to appear.
11. Dyer, E. and Hamstrom, M.E., Completely regular mappings. Fund. Math. 45(1958), 103 - 118.
12. Eaton, W.T., A generalization of the dog-bone space to E^n , Proc. Amer. Math. Soc. 39(1973), 379 - 387.
13. Edwards, R.D., Approximating certain cell-like maps by homeomorphisms, Notices Amer. Math. Soc. 24(1977) A649. Abstract #751-G5.
14. Edwards, R.D. and Kirby, R.C., Deformations of spaces of embeddings, Ann. of Math. 93(1971), 63 - 88.
15. Hurewicz, W. and Wallman, H., Dimension Theory, Princeton: Princeton University Press, 1948.

16. Kozłowski, G., Images of ANR's, Trans. Amer. Math. Soc., to appear.
17. McMillan, D.R., Compact, acyclic subsets of three-manifolds, Michigan Math J. 16(1969), 129 - 136.
18. Moore, R.L., Concerning upper semi-continuous collections of continua, Trans. Amer. Math. Soc. 27(1925), 416 - 428.
19. Sher, R., Realize cell-like maps in Euclidean space, Gen. Top. and Appl. 2(1972), 75 - 89.
20. Walsh, J.J., Fiber preserving cellular decompositions, Bull. Amer. Math. Soc. 78(1972), 746 - 748.
21. Woodruff, E., Decomposition spaces having arbitrarily small neighborhoods with z -sphere boundaries, Trans. Amer. Math. Soc. 232(1977), 195 - 204.

VITA

Donald Kriss Preston was born in Laurens, South Carolina on December 24, 1951. He graduated from Hillcrest High School near Greenville, South Carolina in 1970. The following September he entered Georgia Institute of Technology where he was enrolled for two years before transferring to Furman University. He graduated there in 1974 with a Bachelor of Science degree in Mathematics. In the fall of 1974, he began graduate studies at The University of Tennessee, Knoxville, where he was awarded the Doctor of Philosophy degree in Mathematics in August 1979.

The author is a member of Phi Beta Kappa. He will be employed by Hartwick College in Oneonta, New York after graduation.

He is married to the former Jean Patton of Mauldin, South Carolina, and they have a daughter, Katy, born October 1, 1977.