

**The Relationship Between Teachers' Subject Matter Knowledge for Teaching and
Student Achievement in Struggling and Non-Struggling Students**

A Dissertation Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Jami Terese Garner
May 2017

Copyright © 2017 by Jami Garner.
All rights reserved.

Acknowledgements

“If you were successful, somebody along the line gave you some help.”

-President Barack Obama

I have been fortunate in life and in my academic pursuits to have received help from innumerable somebodies, and it is because of that help that I was able to complete what often seemed an insurmountable task. It would be impossible to acknowledge all of the people who have inspired and encouraged me in this limited space. So, I will limit myself to those who have influenced this project directly, but please know that I am eternally grateful to all who have helped me grow as a person and a scholar.

Dr. Jo Ann Cady, thank you for your mentorship and encouragement. Nearly 12 years ago, you began the work of helping a new teacher discover a passion for math education and throughout the years, you have continued to nurture that passion with your knowledge and example. Throughout the arduous process of conquering this dissertation beast, your advice and guidance on writing and the numerous hours you spent helping me through the editing process were invaluable.

Dr. Jennifer Morrow, thank you for helping me discover a love for data and statistics. In each of the research, evaluation, and analysis classes you taught, my interest and knowledge of research methods grew, and, if given the chance, I would do it all over again. Thank you also for your time and dedication to helping me determine the appropriate analyses to use for this project and to helping me clearly and correctly communicate the results of those analyses. After all, if I learned nothing else, I know that regardless of what the numbers say, “it depends.”

Dr. Stergois Botzakis and Dr. Yujeong Park, thank you for serving on my committee and for your time, expertise, and recommendations throughout this process. I truly appreciate your time and advice.

To my fellow doctoral colleagues, Ann, Sondra, Journey, Ashley, Sarah, and Alicia, your commiseration, friendship, and support made this process much more bearable. A special thanks to Jessica for the time you spent as a second coder for this project, for your writing advice (I believe I have used every ounce of that 5-gallon bucket of glue) and for the innumerable texts and phone calls of support.

To my friends and work colleagues, Julie, Tracey, Kelly, Lynn, Sherrie and numerous others, thank you for your support and encouragement, without which, there were days I surely would have given up.

To my family, although you don’t always understand my ambition, your love and support have been my rock and comfort in every endeavor.

Most of all, thank you to my beautiful wife, April, who bore the brunt of the psychosis that ensues when one is going through the dissertation process. Your support, encouragement and undying confidence carried me through. I love you.

Abstract

The purpose of this mixed-methods study was to investigate the relationship between teachers' mathematical knowledge for teaching (MKT) and student growth on the Aimsweb Mathematical Computation (M-COMP) screening assessment. District participation in a structured Response to Intervention (RtI) program, provided the opportunity to fill a gap in the research by placing an intentional focus on the relationship between both classroom teachers' and interventionists' MKT and students who struggle. Students were classified as struggling if they scored below a certain level on the M-COMP, qualifying them to receive interventions as part of the RtI program. This study included 20 classroom teachers, 4 interventionists and their 485 students in a moderate sized district in Tennessee. Student inclusion in the study was dependent on their teacher's completion of the subject matter MKT survey.

To determine the relationship between teachers' and interventionists' MKT and student growth, several quantitative analyses were used. The analysis revealed a significant negative correlation between classroom teachers' MKT and student growth ($p < .01$), when looking at all students, indicating that higher classroom teacher MKT may result in less student growth. However, the strength of that relationship was small, rendering it insignificant for practical purposes. Additionally, struggling students' growth was also negatively correlated ($p < .01$) with classroom teachers' MKT, but there was no significant relationship between interventionists' MKT and struggling students' growth.

Classroom observations of teachers at different levels of subject matter MKT and interventionists with different levels of pedagogical training provided insight into the relationship between MKT and teachers' or interventionists' instructional practice. An analysis of questions asked by teachers during those observations indicated that teachers with higher levels of MKT asked more questions and gave their students more opportunities to reason. However, one teacher who scored low on the subject matter MKT but had more focused training in math pedagogical content knowledge, was able to compensate for the deficiency in subject matter knowledge. Conversely, the uncertified interventionists who had high levels of subject matter MKT were not able to compensate for their lack of pedagogical training.

Table of Contents

Chapter One Introduction	1
Problem Statement	5
Purpose of the Study	7
Need for the Study	8
Limitations	9
Delimitations	10
Assumptions	11
Definition of Terms	12
Chapter Two Review of Literature	18
Teacher Knowledge	18
Mathematical Knowledge for Teaching	27
Response to Intervention	53
Chapter Three Method	59
Research Design	59
Research Questions	60
Context of the Study	60
Participants	61
Data Collection and Instruments	62
Data Analysis	69
<i>Research Question 1: Is there a relationship between classroom teachers' MKT and student's achievement as measured by the Aimsweb M-Comp growth scores?</i>	<i>72</i>
<i>Research Question 2: How is teachers' MKT related to achievement in struggling students or non-struggling students?</i>	<i>73</i>
<i>Research Question 3: Do the instructional practices of teachers correspond to their MKT scores?</i>	<i>75</i>
Chapter Four Results and Discussion	80
Results	83
<i>Relationship Between MKT and Student M-COMP Growth</i>	<i>83</i>
<i>Relationship of MKT and Achievement in Struggling and Non-Struggling Students</i>	<i>84</i>
<i>Instructional Practices and MKT</i>	<i>88</i>
Discussion	105
<i>MKT and Student Achievement</i>	<i>105</i>
<i>MKT and Instructional Practice of Classroom Teachers</i>	<i>111</i>
<i>Certified Status, MKT, and the Instructional Practice of Interventionists</i>	<i>114</i>
Chapter Five Conclusion and Recommendations	119
Relationship Between Classroom Teachers' MKT and Student M-COMP Growth ..	120
Relationship of MKT with Achievement in Struggling and Non-Struggling Students	121
Instructional Practices and MKT	122
Implications	123
Future Research	124

References.....	126
Appendix.....	139
APPENDIX A	140
APPENDIX B	141
APPENDIX C	147
APPENDIX D	152
APPENDIX E	166
Vita	178

List of Tables

Table 2.1. Selected Studies Investigating Teachers' Subject Matter Knowledge	29
Table 2.2. Mathematics Content Areas and Domains Represented.....	40
Table 2.3. Promax Rotated Factor Loadings, Form A.....	43
Table 2.4. Reliabilities of KCS in Number and Operations	44
Table 2.5. Select Studies Investigating Mathematical Knowledge for Teaching Relationships.....	52
Table 2.6. Minimum Time Requirements for Math RtI.....	58
Table 3.1. Student and Classroom Teacher Participants.....	63
Table 3.2. Classroom Teacher MKT Scores and Class Mean of Students' AIMSweb Growth.....	63
Table 3.3. Content Measured by M-COMP, by Grade	66
Table 3.4. Criterion Validity of M-COMP Scores with Respect to G-MADE Scores.....	67
Table 3.5. Classroom Teachers Selected for Observation	68
Table 3.6. Interventionists Selected for Observation.....	68
Table 3.7. Abbreviated Analysis Plan.....	71
Table 3.8. Pearson Correlation Strength of Association Guidelines	73
Table 4.1. Student and Classroom Teacher Participants.....	82
Table 4.2. Classroom Teacher MKT Scores and Class Mean of Students' AIMSweb Growth.....	82
Table 4.3. Correlation Table for Classroom Teachers' Subject Matter MKT and M-COMP Growth	84
Table 4.4. Hierarchical (Moderator) Multiple Regression Predicting Aimsweb Growth from Teachers' Subject Matter MKT and Struggling Student Status.....	86
Table 4.5. Correlation Table for Teachers' and Interventionists' Subject Matter MKT and M-COMP Growth in Struggling Students.....	87
Table 4.6. Multiple Regression Predicting Struggling Students' Aimsweb Growth From Classroom Teachers' and Interventionists' Subject Matter MKT	88
Table 4.7. Classroom Teachers Selected for Observation	89
Table 4.8. Interventionists Selected for Observation.....	89
Table 4.9. Types of Questions Asked by Classroom Teachers.....	96
Table 4.10. Types and Number of questions asked, scaled for a 10-minute lesson	97
Table 4.11. Time Spent on Activity Types	99
Table 4.12. Types of Questions Asked by Interventionists	104
Table 4.13. Classroom Teacher MKT Scores by School and Grade	108

List of Figures

Figure 1.1. Intertwined Strands of Proficiency and descriptions of each strand	5
Figure 2.1. Shulman’s Content Knowledge for Teaching domain	23
Figure 2.2. Ball’s Mathematical Knowledge for Teaching.....	33
Figure 2.3. Item measuring common content knowledge.....	37
Figure 2.4. Item measuring specialized content knowledge	38
Figure 2.5. Item measuring knowledge of content and students.....	39
Figure 2.6. The three tiered RTI model	54
Figure 2.7. Tennessee’s Response to Intervention Framework	56
Figure 2.8. Tennessee’s Response to Intervention Decision-Making Process	57
Figure 3.1. Domain map for mathematical knowledge for teaching	64
Figure 3.2. Framework for question categories	77
Figure 3.3. Bloom’s Taxonomy and Revised Bloom’s Taxonomy	79
Figure 4.1. Framework for question categories	91
Figure 4.2. Excerpt of Core Fluency Practice Set B	92
Figure 4.3. Proportion of question types asked by classroom teachers in different grade levels and with differing levels of MK	98
Figure 4.4. Proportion of question types asked by certified and non-certified interventionists.....	103
Figure 4.5. Number of students associated with each teacher.	106

Chapter One

Introduction

More than 30 years ago, the National Council of Teachers of Mathematics (NCTM) introduced its first set of standards (NCTM, 1989), which included a call for drastic changes in the way mathematics was taught in American public schools as well as the guiding principles for carrying out those changes. The standards were meant to “create a coherent vision of what it means to be mathematically literate,” and “to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision,” (NCTM, 1989, p. 1). Although one goal of the standards was to provide guidance as to *what* mathematics should be taught at each grade level, the more pressing goal was to provide guidance for changing *how* mathematics should be taught. Toward meeting this goal, NCTM suggested the use of ways of thinking about mathematics, such as Problem Solving, Communication, Reasoning, and Mathematical Connections at each grade band. In the elementary grades, these standards moved the emphasis from a rote use of symbols and operations to increase conceptually oriented attention on number sense, estimation, and reasoning through problem solving and sense-making (NCTM, 1989). Even with National Science Foundation (NSF) and other funding for a variety of curriculum and teacher enhancement projects, systematic change to the understanding of how mathematics should be taught proved to be a much more daunting endeavor (Romberg, 1993). NCTM’s, Principles and Standards for School Mathematics (2000), elaborated on their Curriculum and Evaluation Standards document by suggesting a more explicit change to instruction with a set of Process Standards. These Process Standards,

include (a) Problem Solving, (b) Reasoning and Proof, (c) Communication, (d) Connections, and (e) Representations and “function as the simultaneous goals for student learning, activities, habits, and processes through which mathematical content is learned,” (Bosse, Lee, Swinson, & Faulconer, 2010, p. 263). In addition to setting forth these goals, Principles and Standards for School Mathematics elaborated on how each of these goals should be met. In brief, students should (a) solve real world mathematics problems in a variety of contexts using a variety of strategies; (b) make and investigate mathematical conjectures, as well as develop mathematical arguments, justifications, and proofs; (c) clearly communicate their mathematical thinking in addition to analyzing and evaluating the mathematical thinking of others; (d) make connections among mathematical topics and in contexts outside of mathematics; and (e) use multiple representations to solve problems and communicate mathematical ideas, (NCTM, 2000). However, the implementation of these processes required a paradigm shift for most teachers about how mathematics should be taught and learned. Additionally, this shift would require a more robust understanding of mathematics for many elementary teachers than the understandings they had gained through their own mathematics experiences. For example, American in-service and pre-service teachers alike struggle when providing explanations about adding, subtracting, multiplying, and dividing whole numbers. Teachers are often capable of performing a procedural algorithm to solve whole number operations. However, both pre-service and in-service teachers tend to rely on the algorithmic rules when asked to explain the math operations because they lack the knowledge necessary for discussing the operations conceptually with place value

reasoning, (Ball, 1988a; Ma, 1999; Thanheiser, 2009). Between this deficiency in teachers' mathematical content knowledge, and the remaining emphasis on mathematical skills as opposed to problem solving and conceptual understanding in many state standards and associated assessments, the impetus for paradigm shift to occur on a systematic level was small, resulting in only modest changes to existing practices. (Larson, Fennell, Adams, Dixon, Kobett, & Wray, 2012).

The Common Core State Standards for Mathematics (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) elaborated once again on the process standards and introduced eight mathematical practices in which students should engage in the mathematics classroom to enhance learning. In addition, assessments reflecting the ideals of the mathematical practices were introduced and subsequently adopted by many states, leading teachers and districts to begin to systematically embrace the changes in the teaching of mathematics recommended by the NCTM Process Standards and included in the Common Core State Standards for Mathematics (CCSSM) as an integral part of the Standards for Mathematical Practice, (Larson et al., 2012; NGA & CCSSO, 2010).

The Standards for Mathematical Practice, with foundations in the NCTM Process Standards (i.e. problem solving, reasoning and proof, communication, representation, and connections), as well as in the five intertwined strands for mathematical proficiency included in the National Research Council's (2001) report *Adding It Up* (see *Figure 1.1*), describe the skills necessary for students to have a deep conceptual understanding of

mathematics and the ability to connect ideas and approach problems in a thoughtful way (Pilgrim, 2014). These mathematically empowering standards:

1. Make sense of problems and persevere in solving.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning, (NGA & CCSSO, 2010, pp.6-8)

require a level of mathematical understanding that goes beyond the procedural fluency and rote calculation skills that were once valued and espoused as the epitome of mathematical proficiency. Instead, students must be able to make sense of problems, then persevere through struggles to solve them. In doing so, they must make sense of mathematics by reasoning about quantities and their meanings either abstractly ($2 + 4$) or supported by context then flexibly move between multiple representations (i.e. numerical symbols, diagrams, tables, graphs, and formulas) to represent the mathematics in a coherent way. Students must be accurate with their calculations and determine the level of precision required by the context of the problem, then explain and justify their thinking as they communicate with classmates and teachers. Finally, students must recognize the

structure of mathematics (e.g. in place value, the place to the left of a digit has a value that is ten times greater than the digit) and use that structure within mathematics to solve problems and generalize about mathematics (NGA & CCSSO, 2010; Larson et al., 2012). It follows, that in order for students to achieve this level of proficiency, teachers must also know and understand mathematics in the same way.

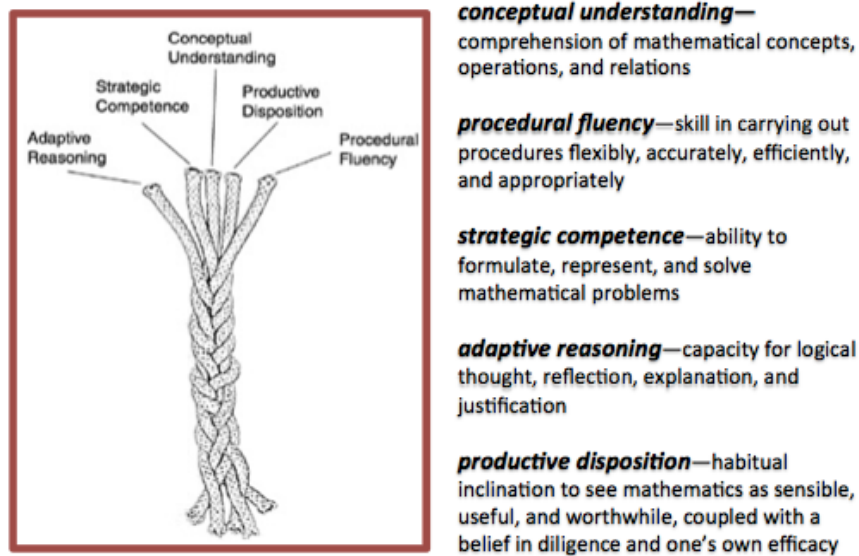


Figure 1.1. Intertwined Strands of Proficiency and descriptions of each strand (National Research Council, 2001 p. 116-117)

Problem Statement

The paradigm shift needed for incorporating the mathematical practices into the mathematics classroom, requires students to be more active learners and the teacher's role changes to that of facilitator rather than the sage on the stage. For example, rather

than having students memorize rules and procedures, they are now expected to know why these rules work and make connections among mathematical concepts. However, many elementary teachers may lack the knowledge of mathematics that allows them to develop this depth of understanding in their students (Ball, 1988a; Ma, 1999). Although teachers may be competent in calculation and other mathematical tasks, they may be missing some key components such as the ability to explain why an algorithm works or to create real world scenarios for calculation with fractions (Ball, 1988; Ma, 1999). With the increase in the rigor of mathematics standards in the elementary grades and the intentional focus placed on the mastery of mathematical content, including the conceptual understanding required to apply the mathematics and solve problems, we must consider the effect that teachers' mathematical knowledge, or lack thereof, has on student achievement, especially when it comes to those students who struggle the most.

For decades, researchers have been attempting to identify exactly what knowledge elementary teachers need in order to do the work of teaching mathematics (Ball, 1988b; Ball & Bass, 2003; Ball, Thames, & Phelps, 2000; Ma, 1999; Stein, Baxter, & Leinhardt, 1990) and how that knowledge influences student mathematical achievement (Hill, Rowan, & Ball, 2005). More specifically, these researchers have contributed greatly to the field by introducing a theoretical framework describing the types of knowledge teachers use when doing the work of teaching mathematics, and a tool for measuring that knowledge. This study attempts to contribute to that literature by investigating if teachers' knowledge, as measured by the Mathematical Knowledge for Teaching survey (Hill, Schilling, & Ball, 2004; Hill, Ball, & Schilling, 2008) influences

student achievement. It goes one step further by also investigating if the mathematical knowledge of interventionists influences the achievement of struggling learners receiving interventions as part of a response to intervention program.

Purpose of the Study

The purpose of this study is to investigate the relationship between teachers' mathematical knowledge for teaching (MKT) and student achievement as measured by the Aimsweb Mathematical Computation (M-COMP) assessment growth scores. In addition, this study will address the gaps in the current literature with regards to the MKT of teachers providing interventions to students identified as struggling through a response to intervention (RtI) program. Finally, this study seeks to find a relationship among the instructional strategies used by classroom teachers and interventionists and their levels on the MKT scale. Specifically, this study attempts to answer the following research questions:

1. Is there a relationship between classroom teachers' MKT and students' achievement as measured by Aimsweb M-Comp growth scores?
2. How are teachers' MKT related to achievement in struggling students or non-struggling students?
3. Do the instructional practices of teachers correspond to their MKT scores?

Based on previous findings (Hill, Rowan, & Ball, 2005), a positively correlated relationship is expected between MKT and student achievement. Additionally, it is expected that teachers with higher MKT scores will utilize instructional strategies that

foster quality mathematical understanding (Charalambous & Hill, 2012; Hill, Umland, Litke, & Kapitula, 2012; Hill & Charalambous, 2012).

Need for the Study

It has been said that achievement in high school mathematics courses is the gatekeeper to college success (Bryk & Treisman, 2010) and that mathematics success in the middle grades heavily influences courses taken and mathematics achievement in high school (Wang & Goldschmidt, 2003). It follows that achievement in elementary mathematics courses has a significant effect on that outcome. It is for this reason that we must make mathematics instruction a priority in the elementary grades. According to Adding it Up, a 2001 report by the National Research Council, the main goal of elementary mathematics is to develop proficiency with number and number operations. Fortunately, there is a wide body of research on how students learn these concepts and the best practices for teaching them (Carpenter, Fenema, Franke, Levi, & Empson, 1999; Kamii, 2000; National Research Council, 2005; Van De Walle, 2007; Lamon, 2012; Richardson, 2012). However, the research is lacking when it comes to understanding the knowledge required to effectively implement these practices.

Ball's Mathematical Knowledge for Teaching (MKT)(2005) provides a framework for explaining the complexity of the knowledge required for the work of teaching, and the development of a valid and reliable instrument to measure that knowledge has provided the means for researchers to empirically establish the relationship between teachers' MKT and student achievement (Ball & Bass, 2003; Ball, Hill, & Bass, 2005; Hill, Schilling, & Ball, 2004; Hill, Schilling, & Ball, 2008). Despite

this, since the development of the MKT survey in 2002, there have been a limited number of studies looking at this relationship (Hill, Rowan, and Ball, 2005). Additionally, there has been no research looking at the relationship between teachers' MKT and the achievement of students who struggle.

With the high level of importance placed on mathematics achievement, and the increase in attention being paid to providing appropriate interventions to students who struggle, more research is needed to better explain the relationship between teachers' mathematical knowledge for teaching and student achievement. This research is needed in order to better inform colleges of education on the importance of including mathematics and mathematics methods courses that increase both common math knowledge and the math knowledge specific to the work of teaching in their elementary education programs. Likewise, it will inform districts as they look to hire the most qualified teachers and interventionists, as well as giving direction to districts' planning of professional development that is most effective for helping teachers improve mathematics learning and achievement in their students.

Limitations

The findings from this study are limited by a few factors. First, this study is limited by the teachers' willingness to complete the mathematical knowledge for teaching survey. Teachers who are more confident in their mathematical ability may be more willing to complete the survey, leading to an underrepresentation of teachers with low MKT scores. Another limitation of this study is the use of the AimsWeb Mathematical Computation(M-COMP) assessment for student achievement growth scores. M-COMP

is a brief test of math operations; while it is valid and reliable for providing a snapshot of students' performance levels (NC Pearson, 2012), it does not measure student understanding of standards-based mathematical concepts. This limitation is significant because the extent of teachers' knowledge is more likely to be related to students' conceptual understanding than their computational skill (Begle, 1979; Hunkler, 1968). M-COMP is being used in this study because it is administered to all students in elementary grades three times per year in the district where this study will be carried out. The final limitation in this study is the inconsistency in the number of students assigned to an interventionist among the different schools. Interventionists with larger groups of students may be limited in their ability to target needed interventions to individual students, resulting in lower student achievement influenced by the group size rather than the teacher's MKT.

Delimitations

First, this study is delimited by the population being sampled. The sample will be drawn from one school district and will target only grades 1 through 5. The decision to sample from one school district was for manageability of the study. However, utilizing only one school district in the study could cause results to be confounded by characteristics particular to that school district and limits the generalizability of the findings. The targeting of first through fifth grades is due to the use of a common assessment and the established protocol for providing interventions to struggling students, making it possible to utilize both classroom teacher and interventionist MKT scores when looking at the relationship between student achievement and MKT. Another

delimitation in this study is the measurement of only two strands of the subject matter knowledge subdomain from the larger mathematical knowledge for teaching domain. The decision for the exclusion of the knowledge strands belonging to the pedagogical content knowledge subdomain is based on the reliability of the survey instrument for measuring the subject matter knowledge strands, and the inability of the instrument to discriminate between the knowledge of content and students (KCS), belonging to the pedagogical content knowledge subdomain, and specialized content knowledge (SCK), belonging to the subject matter knowledge subdomain (see literature review). Finally, this study is delimited by the teachers selected for observation. The criteria used for selecting teachers for observation was meant to provide instructional representations of teachers in different grade bands (1-2 or 3-5), different settings (whole group or intervention), and different MKT levels.

Assumptions

This study will take into account a few assumptions. First, it is assumed that teachers who choose to complete the Mathematical Knowledge for Teaching survey do so independently without help, providing an accurate measure of their mathematical content knowledge in the area of numbers and operations. Similarly, it is assumed that students provided their best effort when taking the M-COMP assessment, and that the administration of the assessment was done with fidelity at each school within the school district. It is also assumed that the instructional practices observed during the observation phase of the study reflect practices that are typical to the teacher being observed. Finally, it is assumed that all data provided by the school district is correct.

Definition of Terms

In this section, terms are defined in order to give the reader clarity in regards to how these terms are being used in this study.

Classroom Teacher- In this study, the classroom teacher is defined as the certified teacher providing Tier 1 core math instruction.

Common Content Knowledge (CCK)- Common Content Knowledge is the knowledge of mathematics that is in common with how the content is used outside the profession of teaching (Hill, Ball & Shilling, 2008).

Content Knowledge For Teaching- Content Knowledge for Teaching is the name Lee Shulman (1986) gave to the knowledge domain used in the work of teaching which includes subject matter knowledge, pedagogical content knowledge and curricular knowledge. The categories within this domain, particularly the interaction between subject matter knowledge and pedagogical content knowledge formed the basis for Ball's Mathematical Knowledge for Teaching.

Interventionist-In this study, an interventionist is defined as any teacher, certified or uncertified who is providing interventions to students as a part of the response to intervention program.

Knowledge of Content-Knowledge of Content is a sub-category within the subject matter category of the Content Knowledge for Teaching domain. It refers to knowledge of factual information and central concepts—the ability to identify, define, and discuss the concepts (Grossman, Wilson & Shulman, 1989).

Mathematics Computation Assessment (M-COMP)-M-COMP is the assessment used

by the district as a universal screener. It is a quick assessment of computational skill and is used for identifying students in need of interventions (NC Pearson, 2012).

Mathematical Knowledge for Teaching (MKT)- Mathematical Knowledge for Teaching is Ball and colleagues' reconceptualization of Schulman's Content Knowledge for Teaching domain that maps the knowledge used for the work of teaching math. The two larger domains, Subject Matter Knowledge and Pedagogical Content Knowledge, are each subdivided into three categories of knowledge.

MKT Survey-MKT survey refers to the survey being used for this study that measures teachers' CCK and Specialized Content Knowledge.

MKT Score- MKT score refers to the score teachers received on the MKT survey. Scores range from -3 to +3 with 0 being the score received by a teacher with average CCK and SCK (Learning Mathematics for Teaching, 2014)

Pedagogical Content Knowledge-Pedagogical content knowledge is the "special kind of teacher knowledge the links content and pedagogy" (Ball & Bass, 2003, p. 4). In other words, it is the subject matter for teaching. It's the knowledge of a subject and how to represent it in a way that makes it understandable to others (Shulman, 1986).

Response to Intervention (RtI)-RtI is the three tiered system used by the school district for providing intervention to struggling students. Within the tiered system, Tier 1 refers to the core classroom, Tiers 2 and 3 refer to interventions some students receive in addition to Tier 1 instruction. A universal screener is used by the to identify students who need Tier 2 or 3 interventions. Ideally, 80% of the students' needs are met through Tier 1 instruction with the bottom 20% receiving additional support in

either Tier 2 (15%) or the most intense interventions in Tier 3 (5%) (Riccomini & Witzel, 2010)

Specialized Content Knowledge (SCK)- Specialized Content knowledge is the knowledge of mathematics that is specific to teaching. It includes knowledge of how to represent mathematical ideas, provide mathematical explanations for rules and procedures, and understand unusual solution methods to problems (Ball, Hill, & Bass, 2005).

Subject matter knowledge-Subject matter knowledge is a category of the Content Knowledge for Teaching domain. It refers to the understanding of the subject in the mind of the teacher including knowledge of the content as well as understanding of how the subject matter relates to and is represented in different contexts.

Struggling student- In this study, a struggling student is defined as any student receiving interventions as part of the three tiered RtI program.

Teacher-In this study, a teacher is any individual, certified or uncertified who is instructing students. This includes core classroom teachers and interventionists.

Tier 1 Instruction-Tier 1 Instruction refers to the core classroom instruction that is given to all students.

Tier 2 and Tier 3 Interventions-Tier 2 and Tier 3 interventions are the interventions given to students who struggle. Students receiving these interventions are identified based on their performance on the Universal Screener given to all students. The lowest 20% of students receive interventions with the lowest 5% receiving the most intensive Tier 3 interventions and the next 15% receiving Tier 2 interventions. The

more intense the intervention, the smaller the group size, with the recommendation for Tier 2 being no more than 6 students and the recommendation for Tier 3 being no more than 3 students (Buffum, Mattos, & Weber, 2012).

Theoretical Framework

The purpose of a theoretical perspective is to place the current study into perspective among other relevant studies (Simon, 2011). It explains how and why sets of constructs (or variables) are related, providing a lens through which to view the problem (Creswell, 2009). In this study, the relationship between teacher knowledge and student achievement are being looked at through the lens of Ball's mathematical knowledge for teaching (MKT) domain, which she defines as "the mathematical knowledge needed to carry out the work of teaching mathematics," (Ball, Thames, & Phelps 2008, p.395).

The idea that teachers with more mathematical content knowledge are more likely to be competent math instructors is not new. However, the relationship between teachers' mathematical content knowledge and student achievement has been difficult to establish empirically. Perhaps this difficulty can be attributed to the misunderstanding of the kinds of mathematical knowledge needed for teaching. For example, measures such as the number of upper level mathematics courses or achievement tests have been utilized as measures of mathematical content knowledge as researchers look to establish a relationship between that knowledge and student achievement (Begle, 1979). These measures, looking at teachers' abilities to apply mathematical concepts in their highly compressed form simply don't align with the way teachers of mathematics must utilize

their knowledge of the concepts, in their decompressed form (Ball, Hill, & Bass, 2005).

Thus, through an in depth analysis of the work of teaching and all that it requires, Ball and her colleagues developed a “practice-based theory of the mathematical knowledge for teaching,” (Ball & Bass, 2002, p.5) to include a structural framework of that knowledge in addition to a tool for measuring it. Beginning with the framework of Shulman’s (1986) Content Knowledge for Teaching domain, they were able to give structure to the multidimensionality of teachers’ MKT. Shulman’s Subject Matter Knowledge and Pedagogical Content Knowledge categories became the two domain headings, each with three subdomains specific to the teaching of mathematics. Under the Subject Matter Knowledge domain are types of knowledge that are purely mathematical in nature: Common Content Knowledge (CCK), mathematical knowledge common to those outside the teaching profession; Specialized Content Knowledge (SCK), mathematical knowledge specific to the work of teaching; and Knowledge at the Mathematical Horizon, knowledge of how the mathematical topics are related across the curriculum (Ball, Thames, & Phelps, 2008). The Pedagogical Content Knowledge domain is where the convergence of the knowledge about teaching practice and pure content knowledge takes place (Shulman, 1986). It is highly dependent on subject matter knowledge, but no less important to high quality math instruction. Included in this domain are: Knowledge of Content and Students (KCS), knowledge about students and their thinking about mathematics; Knowledge of Content and Teaching (KCT), knowledge about the design of instruction and how to best sequence and implement that instruction; and Knowledge of Content and Curriculum, knowledge of curriculum

resources, materials, and programs for teaching the content (Ball et al., 2008).

The development of this framework and a reliable tool for measuring teachers' MKT opened the door for researchers to empirically establish relationships between teachers' mathematical knowledge and the many aspects of the work of teaching. Thus, this study will build on the existing research base by using the content knowledge aspects of the MKT domain as a lens for studying the relationship between teachers' mathematical knowledge and student achievement. Of particular interest in this study is the relationship between teachers' MKT and student achievement in students who struggle.

Chapter Two

Review of Literature

A review of literature summarizes the existing research on a topic in order to establish the need for a study. It establishes a framework for situating the study within the larger, ongoing dialogue in the literature, filling gaps and extending prior studies (Creswell, 2008/2009). The purpose of this study is to investigate the relationship between teachers' mathematical knowledge for teaching (MKT) and student achievement of both on-level and struggling learners. Thus, this review of literature situates the current research within the general category of teacher knowledge and more specifically adds to the body of research addressing MKT. It seeks to examine the MKT domain and its constructs as well as the development of the survey used in this study, which was designed to measure teachers' MKT. Prior research utilizing the MKT survey as well as the existing research addressing math instructional strategies and teacher effects on student achievement are also discussed. Finally, this review identifies gaps in the research being addressed by the current study.

Teacher Knowledge

The work of teaching is varied and complex, as are the variables contributing to or detracting from a student's achievement. Through the years, researchers have attempted to determine which factors contribute the most to students' achievement in school and have looked to teachers for the answer by investigating the relationship between a number of teacher variables and student achievement. Among these variables, are those meant to be measures of either teachers' subject matter knowledge or

pedagogical knowledge. Subject matter knowledge includes knowledge of the content of a subject area, such as major concepts and relationships among concepts (Grossman & Richert, 1988). Pedagogical knowledge—knowledge about the practice of teaching—includes theories of learning, principles of instruction, understanding of the various philosophies of education, knowledge of learners, and techniques of classroom management (Grossman & Richert, 1988). The studies trying to establish these relationships fall into two main categories of research, presage-product and process-product. Whereby, presage-product studies look to establish a relationship between teacher characteristics such as demographic data or knowledge of the subject and student achievement, while the process-product studies sought to describe the relationship between teacher behaviors—what teachers did in their classrooms—and student achievement (Doyle, 1977). Thus, the comparison can be made that while a number of the presage-product studies placed a heavy emphasis on the importance of teachers' subject matter knowledge, the process-product studies placed a heavy emphasis on the importance of teachers' pedagogical knowledge to student achievement.

Presage-Product Studies

Historically, those studies that investigated the relationship between subject matter knowledge and student achievement have used the number of college math courses a teacher had taken or scores on subject specific standardized tests as the measure of teachers' knowledge, then looked for relationships with some measure of student achievement (Gage & Needles, 1989). These studies were meant to identify and support the need for teachers to have a robust knowledge of their subject. Begle (1972,1979), one

of the most well-known researchers in this area, was unable to find any educationally significant correlation between teacher subject matter knowledge and student performance. Although somewhat surprising, this finding was in line with other studies on the same topic (Copeland & Doyle, 1973; Hunkler, 1968). Begle's study differed from these studies in that instead of investigating the relationship between teacher subject matter knowledge and student achievement in elementary grades, his study centered on ninth grade Algebra I teachers and students. The measures used in his study included skills tests created by Begle using questions from the School Mathematics Study Group question bank. In his study, teachers were given two separate tests, one to measure understanding of algebra of the real number system (the content included in ninth grade algebra), as well as a second test to measure higher-level abstract algebraic concepts. Meanwhile, students were given a series of pre and post-tests to measure mathematical calculation and algebraic understanding. Although there was a statistically significant relationship between teachers' understanding of algebra of the real number system and student achievement in understanding ninth grade algebra, "it was so small as to be educationally insignificant," (Begle, 1972, p.8). As he tried to reconcile the negative results with the belief that teachers' subject matter knowledge is an important influence the quality of instruction, and thus student achievement, Begle concluded that there must be a "lower bound" of teacher knowledge in order for the relationship to exist. In other words, he postulated that all teachers in this study, who were participants in a National Science Foundation Institute, must have high enough levels of math knowledge, that even

those on the lower end in this study were not low enough to adversely affect student achievement.

Because of the selection bias involved in Begle's study, Eisenberg (1977) duplicated the study with the hypothesis that when teachers were sampled from a standard population a positive correlation between teacher knowledge and student performance would appear. In addition to teacher test scores, Eisenberg also looked to establish a relationship between teachers' years of experience, number of post-calculus math courses, or collegiate mathematics grade point averages and student performance. Despite the standard population, Eisenberg's results mirrored Begle's, leading him to suppose that if in fact there is a lower bound to teacher knowledge in order for the correlation between teacher knowledge and student achievement to appear, it must be so low that "it is not worth worrying about," (Eisenberg, 1977, p.221). In addition, Eisenberg found no correlation between any of the other teacher variables and student achievement. Eisenberg's findings were in line with Begle's (1979) later analysis of the relationship between the number of courses teachers had taken past calculus and student performance which resulted in positive effects in only 10% of the cases with negative effects in 8% of the cases.

These disheartening results and those of the studies cited earlier, led researchers to believe that there must be some other factor more important to student learning than the teacher's subject matter knowledge. This belief led to a more pronounced focus on a paradigm for research known as process-product, in which there is a search for relations

between classroom processes and products, in other words between how teachers teach and what students learn (Gage & Needels, 1989).

Process-Product Studies

The goal of the process-product research was to identify the behaviors that discriminate successful from unsuccessful teachers so that the instructional methods of successful teachers could be emphasized in colleges of education as well as in teacher professional development programs (Brophy & Everston, 2010). Relationships between certain teaching practices (behaviors) and student achievement outcomes such as, the quality of feedback, lesson pacing and questioning (Good & Grouws, 1977; Gage & Needels, 1989) have been identified by these studies. Despite the finding of these relationships, Doyle (1977) points out that few of the studies explain why the variable (the teacher behavior) relates to student achievement; rather, the studies are often focused on discrete classroom events without considering the larger picture of other influences that could potentially affect student achievement. Regardless, the results of these studies have informed many policy decisions regarding teacher certification and evaluation. The emphasis is on the capacity to teach (pedagogical knowledge), and is justified as being research based due to the growing body of process-product research identifying the patterns of teacher behavior that have contributed to improved academic performance (Shulman, 1986). As Shulman (1986) and his colleagues with the Knowledge Growth in Teaching project at Stanford University began a closer investigation of the knowledge required for the work of teaching. The realization that the subject matter was often absent from studies on teacher effectiveness inspired their intentional focus on the interaction of

subject matter knowledge and pedagogical knowledge for the transformation from “knowledge of the teacher to the content of instruction,” (Shulman, 1986, p. 6).

Pedagogical Content Knowledge

With an intentional focus placed on the interaction between teachers’ subject matter knowledge and pedagogical knowledge, a new domain, content knowledge for teaching, was conceptualized. The three categories included in the domain were subject matter knowledge, pedagogical content knowledge, and curricular knowledge (see Figure 2.1). Among these, the researchers concentrated on subject matter knowledge and the role it plays in teaching, as well as what they termed “the missing paradigm,” or pedagogical content knowledge (Shulman, 1986; Wilson, Shulman, & Richert, 1987).

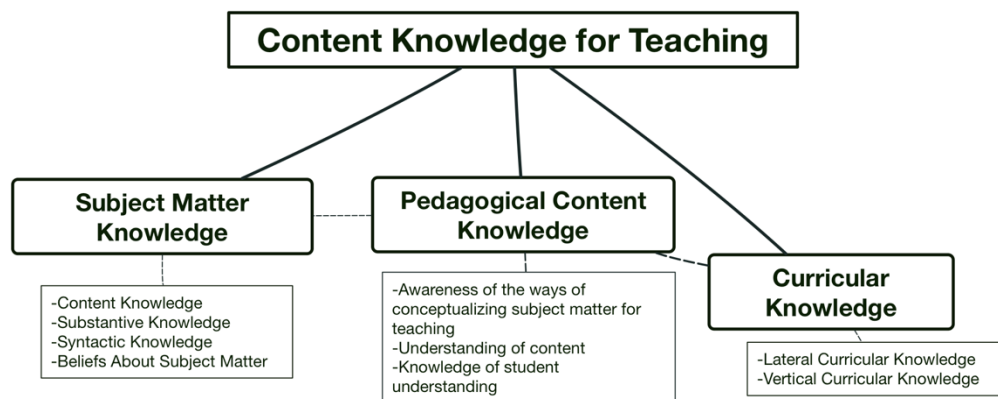


Figure 2.1. Shulman’s Content Knowledge for Teaching domain

The researchers were able to identify four dimensions of subject matter knowledge that influence teachers’ teaching: knowledge of content, substantive knowledge, syntactic knowledge, and beliefs about subject matter (Wilson et. al, 1987). Knowledge of content is quite simply the “stuff” of a discipline. It involves knowledge

of factual information and central concepts—the ability to identify, define, and discuss the concepts individually and how they relate to other concepts within the field as well as those concepts external to the discipline (Grossman, Wilson & Shulman, 1989). Both substantive and syntactic knowledge are ways of understanding the structure of the subject matter. The substantive structures of the discipline include the explanatory frameworks utilized to guide inquiry and make sense of the content (Schwab, 1978). A teacher's tacit or explicit knowledge of substantive structures "...has important implications for how and what teachers choose to teach," (Grossman et. al, 1989 p. 29). Knowledge of syntactic structures is an understanding of the ways in which new knowledge is brought into the field—the ways in which truth or falsehood, validity or invalidity are established (Schwab 1978). "Teachers who lack knowledge of the syntactic structures of the subject matter fail to incorporate that aspect of the discipline in their curriculum," (Grossman et. al, 1989, p. 30). Thus, teachers run the risk of misrepresenting the subject matter and may struggle to adequately teach that subject matter to their students. Finally, subject matter knowledge is also greatly influenced by teacher beliefs such as what they choose to teach and how they orient or represent that instruction (Grossman et. al, 1989). For example, in mathematics, do teachers believe more importance should be placed on rote memorization of facts and procedures, or on conceptual understanding.

With a structure for teachers' subject matter knowledge established, the researchers approached the question of "...how the subject matter was transformed from the knowledge of the teacher into the content of instruction," (Shulman, 1986, p.6). This

formed the basis for further research into what they termed “the missing paradigm” in the study of teacher knowledge. This missing paradigm became what is now known as pedagogical content knowledge (PCK). PCK is subject matter knowledge for teaching. It goes beyond the content knowledge of a subject matter expert to include knowledge of, “the most useful forms of representation, the most powerful analogies, illustrations, examples, explanations and demonstrations—in a word, the ways of representing and formulation the subject that makes it comprehensible to others,” (Shulman, 1986, p.9).

PCK includes three broad components. The first of these is an understanding of the content which includes all of the elements of subject matter knowledge for the specific content to be taught. Additionally, “The teacher need not only understand *that* something is so; the teacher must further understand *why* it is so” and why it is worth knowing (Shulman, 1986, p. 9). Without a solid understanding of the content to be taught, a teacher would surely be deficient in the other two components, including an awareness of the ways of conceptualizing subject matter for teaching and knowledge of student understanding (Grossman & Richert, 1988). Conceptualizing subject matter for teaching includes knowledge of the most useful forms of representations, analogies, illustrations, examples, explanations, and demonstrations for making the subject comprehensible to others. The knowledge of student understanding includes knowledge of students’ conceptions or misconceptions, their background with the content and interests within the field. In addition, it includes knowledge of the strategies most likely to enhance the understanding of learners who come to the teacher with misconceptions. The ability to use, and knowledge of when to use, certain representations is very closely

tied to the knowledge of student understanding (Grossman & Richert, 1988; Shulman, 1986).

The final category in the domain content knowledge for teaching is curricular knowledge. Curricular knowledge includes knowledge of resources available for teaching particular subject matter, such as textbooks, instructional programs, digital media, and other curriculum materials (Grossman & Richert, 1988; Shulman, 1986). In addition to curricular knowledge of the subject matter being taught, this category also includes lateral and vertical curricular knowledge. Lateral curriculum knowledge involves the understanding of and ability to relate the content to other topics being taught simultaneously. Vertical curriculum knowledge is quite simply knowledge of what has been taught in earlier school years and what will be taught in later years (Shulman, 1986).

Although these three categories have been defined separately under the domain heading “Content Knowledge for Teaching,” it is important to realize that they are intertwined and overlapping. However, subject matter knowledge can and does stand on its own outside the profession of teaching. For example, a scientist might possess all four dimensions of subject matter knowledge for his discipline, without possessing the kinds of knowledge necessary for transferring that knowledge to others. The kind of knowledge necessary for that transference to take place is PCK. Because PCK relies heavily on the level of subject matter knowledge, with curricular knowledge also playing an important role in the level of PCK in the mind of a teacher, the work of teaching requires all three forms of knowledge. In other words, all three forms of knowledge are highly related, and robust pedagogical content knowledge relies on also having subject

matter knowledge and curricular knowledge. As Shulman (1987) points out, the quality of the content being taught is highly dependent on teachers' knowledge structures including pedagogical reasoning, which places emphasis on intellectual understandings rather than on behavior alone for teacher performance. However, the body of research on content knowledge in the minds of teachers and its transference to the content of instruction is limited (Shulman, 1986). Yet, this research is necessary for the reform of colleges of education to include the permeation of pedagogical content knowledge into the teacher preparation curriculum (Shulman, 1987).

Mathematical Knowledge for Teaching

The conception of pedagogical content knowledge (PCK) by Shulman and his colleagues reframed the study of teacher knowledge to place more emphasis on the role of content in teaching. They represented PCK as a “special kind of technical knowledge key to the profession of teaching,” (Ball, Thames & Phelps, 2008, p.390), suggesting that what matters for teaching is not a separate knowledge of pedagogy and knowledge of content, but an amalgam of the two. Thus, distinguishing the knowledge of the content specialist from that of the teacher (Shulman, 1987; Ball, Thames & Phelps, 2008).

Ball followed Shulman's call for attention to content knowledge in her doctoral studies by investigating prospective teachers' knowledge of and attitudes towards mathematics. The mathematical pedagogy framework she used as a lens for her study emphasized the need for students to gain “...understandings about mathematics: what it means to ‘do’ mathematics and how one establishes the validity of answers,” (Ball, 1988a, p.2) as well as knowledge of mathematics. In other words, as important as

learning mathematical concepts and procedures, is learning what mathematics is and how to engage with it. Likewise, “teachers must themselves be grounded in knowledge about mathematics,” (Ball, 1988a, p.191). Through interviews conducted for her study, Ball realized that the pre-service teachers had a very limited view of mathematics in general and were correspondingly limited in their conceptual understanding of mathematical connections and procedures. She found this to be true of both elementary and secondary trained teachers.

In the years following her doctoral studies, Ball joined other math education researchers (Leinhardt & Smith, 1985; Stein, Baxter, & Leinhardt, 1990; Simon, 1993; Putnam, 1992; Cohen, 1990; Borko, Eisenhart, Brown, Undersell, Jones, & Award, 1992) in the quest to better understand the role of subject matter knowledge in mathematics instruction and learning (see Table 2.1). Perhaps one of the most well-known studies looking at teachers’ subject matter knowledge was done by Ma, a Chinese graduate student at Michigan State University, who worked as a graduate assistant with the Teacher Education and Learning to Teach Study (TELT) coding interview transcripts collected as part of Ball’s work. As a result of coding transcripts, and a subsequent visit to an American elementary math classroom, Ma began to realize that the expectations she held about elementary teachers’ knowledge of mathematics in China, did not seem to hold in the United States. She found that, “even expert teachers, experienced teachers who were mathematically confident, and teachers who actively participated in current mathematics teaching reform did not seem to have a thorough knowledge of the mathematics taught in elementary school,” (Ma, 1999, p.xix).

Table 2.1

Selected Studies Investigating Teachers' Subject Matter Knowledge

Author(s)	Context of the Study	Findings
Leinhardt t & Smith, 1985	Interviews and Observations Expert and Novice Fourth Grade Teachers Equivalent Fractions	More knowledgeable teachers presented more conceptual information, emphasized more powerful aspects of equivalence, and used representations more appropriately.
Stein, Baxter, & Leinhardt, 1990	Case Study Experienced Fifth Grade Teacher Functions and Graphing	The teacher was missing knowledge of key concepts in regards to functions and graphing, and the knowledge he did have was not structured in a way that he was able to transform for accessibility for learners.
Cohen, 1990	Case Study Second Grade Veteran Teacher Teaching within a new pedagogical framework (Teaching math for Understanding)	Concrete materials were utilized in an effort to more effectively teach for understanding, but in actuality, the lack of mathematical knowledge of the teacher resulted in the materials being used to convey a traditional message of concepts lacking connections to related concepts or discussion about mathematics in the classroom. Also, due to her own lack of understanding, the teacher was unable to recognize when problems were presented poorly and often identified poor lessons as great ones.
Putnam, 1992	Case Study Fifth Grade Veteran Teacher Teaching within a new pedagogical framework (Teaching math for Understanding)	Although the teacher tries to shift her pedagogical style and is using a problem-based curriculum, there is still an emphasis on procedures and the view that problem solving is a means for applying algorithms to practical situations. In addition, the teacher's mathematical knowledge was not developed enough for her to understand the mathematical complexities of the problem necessary for meaning making.

Table 2.1 Continued

Author(s)	Context of the Study	Findings
Borko, Eisenhart, Brown, Underhill, Jones & Agard, 1992	Case Study Sixth Grade Pre-Service Teacher Division of Fractions	The teacher held a belief of making math meaningful for students, however her level of subject matter knowledge was such that she was unable to provide illustrations of the math content that made it meaningful. She was able to increase some of her conceptual knowledge as a result of methods courses, but the increase was not extensive enough and she still had difficulty assimilating what was learned in the methods course into her own knowledge structure. This resulted in a stronger belief in the value of practice and the authority of the textbook for answers.
Simon, 1993	Survey and Interviews Elementary Pre-Service Teachers Division	Conceptual knowledge was weak, including: conceptual understanding of the algorithm, the relationship between real world problems and symbolic division, identification of quantities included in division problems, and the difference between measurement and partitive division.

It seemed to make sense that this lack of mathematical knowledge among teachers might be a better explanation of American students being outperformed on international math achievement tests than the commonly held notion that cultural differences might be to blame. However, it was puzzling that American teachers, who are more formally educated than Chinese teachers, would not have as much or more knowledge of mathematics. She wondered if perhaps the mathematical knowledge of elementary

teachers in the two countries was structured differently— that instead of the subject matter knowledge common to anyone who studies mathematics, the Chinese teachers possessed a knowledge of mathematics that was structured in a way that allowed for more comprehensible representations to students, in other words, that they had a better grasp of what Shulman called pedagogical content knowledge, (Ma, 1999; Shulman, 1986). These realizations provided the impetus for Ma’s doctoral study, in which she compared Chinese and U.S. teachers’ understandings of elementary mathematics topics. Her findings were in line with Ball and others in the field in that none of the above average U.S. Teachers demonstrated a profound understanding of elementary mathematics. “In fact, the knowledge gap between U.S. and Chinese teachers parallels the learning gap between U.S. and Chinese students,” (Ma, 1999, p. 144). Thus, in order for U.S. students’ learning in mathematics to improve, teachers’ knowledge of school mathematics must also improve.

As a result of these findings, researchers were convinced that teachers’ mathematical knowledge does play a role in the quality of their math instruction. However, the kind of mathematical knowledge required for high quality math instruction transcends the tacit mathematical understanding that characterizes much personal knowledge (Polanyi, 1958). Tacit knowledge of mathematics can be such that it allows mathematicians to unravel complex problems, yet be unable to articulate the intricacies of the mathematics. This articulation, needed for the work of teaching, requires explicit knowledge that goes beyond surface level representation to allow discussion of the meaning and reasons for relationships or procedures as well as connections among

mathematical topics (Ball, 1988b). Even with this understanding about the type of knowledge needed for the transference of mathematical knowledge, questions specific to the work of teaching remained (Ball & Bass, 2003, p.5).

- What mathematical knowledge is entailed by the work of teaching mathematics?
- Where and how is mathematical knowledge used in teaching mathematics?
- How is mathematical knowledge intertwined with other knowledge and sensibilities in the course of that work?

Finding the answers to these questions required Ball and her colleagues to place a heavy focus on the mathematical demands of the tasks involved in teaching (Ball, Thames, & Phelps, 2008). This focus resulted in the development of a practice based theory of mathematical knowledge for teaching (Ball & Bass, 2003).

The practiced based theory of the mathematical knowledge needed for teaching was not developed through examination of the curriculum. Rather, it was developed based on a thorough examination of the practice itself (Ball & Bass, 2000). This examination involved the usage of a National Science Foundation funded database which documented a year of mathematics instruction in a third grade public school classroom. Documents included in this database are; videotapes, audiotapes, transcripts, students' classwork, homework and quizzes, as well as the teacher's plans, notes, and reflections (Ball & Bass, 2000; Ball & Bass 2003; Ball et al. 2008). As a result of these analyses and

building from Shulman's conception of PCK, Ball and her colleagues were able to give structure and descriptive definition to the mathematical knowledge required for teaching.

Mathematical Knowledge for Teaching (MKT) is represented by two domains, Subject Matter Knowledge and Pedagogical Content Knowledge. Each of these larger domains is made up of three subdomains. Those belonging to the Subject Matter Knowledge domain are Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Knowledge at the Mathematical Horizon, with Knowledge of Content and Students (KCS), Knowledge of Content and Teachers (KCT), and Knowledge of Content and Curriculum belonging to the Pedagogical Content Knowledge Domain (see Figure 2.2). Ball's subject matter domain is quite simply a reimagining of what Shulman (1986) called content knowledge. More specifically, Shulman was most likely referring to what Ball and her colleagues call CCK. In mathematics, CCK is the knowledge that allows a person to understand and solve a variety of mathematics problems. It's the knowledge that is common to anyone who has studied mathematics.

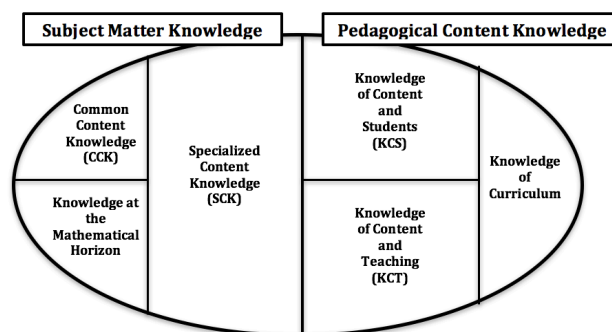


Figure 2.2. Ball's Mathematical Knowledge for Teaching by Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.

This mathematical knowledge, that allows precise calculation and problem solving, is compressed in the brain allowing it to be recalled and used as chunks of related information. While this compression is necessary and useful for solving complex math problems, it is not conducive to providing conceptual instruction in chunks manageable for learners (Ball & Bass, 2003). Teaching requires a specialized form of subject matter knowledge, knowledge that is purely mathematical, not mixed with knowledge of students or pedagogy (Ball et al., 2008). Thus, the largest subdomain in the subject matter domain is SCK. This type of knowledge is what Ma (1999) was referring to when she argued that teachers must be able to reorganize what they know—to deconstruct their own mathematical knowledge into its less polished form—what Ball and Bass (2000) refer to as decompression. This decompressed knowledge is what allows teachers to engage in the tasks required for teaching, such as accurately representing mathematical ideas, providing mathematical explanations for common rules and procedures, as well as examining and understanding unusual solution methods to problems (Ball, Hill, & Bass, 2005). Finally, the last subdomain of Subject Matter Knowledge is Horizon Content Knowledge, which refers to an awareness of how mathematical topics are related throughout the mathematics curriculum (Ball, 1993). Each of these subdomains is purely mathematical requiring no knowledge of students, teaching, or pedagogy. However, the subdomains included in the Pedagogical Content Knowledge (PCK) domain of the MKT construct are highly dependent on a well-developed subject matter knowledge.

Included in the PCK domain of MKT are KCS, KCT, and Knowledge of Content and Curriculum. KCS is knowledge at the intersection of content and students—what Shulman (1986) referred to as knowing “the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons,” (p. 9). It’s what allows teachers to interpret students’ solutions and explanations to understand their thinking, anticipate what students may find easy or difficult, identify misconceptions, and to predict what will be motivating or interesting for students (Ball et al., 2008). Whereas, KCT is knowledge at the intersection of content and teaching—what Shulman (1986) meant when he said teachers must know “the ways of representing and formulating the subject that makes it comprehensible to others,” (p.9). It’s teachers’ knowledge of instructional design and sequence—knowing which examples and representations to use as well as when to “pause for clarification” or “use a student’s remark to make a mathematical point,” (Ball et al., 2008, p. 401). Finally, Knowledge of Content and Curriculum is quite simply knowledge of the curricular materials and resources available for teaching the content.

Although “a teacher might have a strong knowledge of the content itself but weak knowledge of how students learn the content and vice versa,” (Hill et al. 2008), the four main subdomains of MKT (CCK, SCK, KCS, and KCT) are equally important and complimentary when it comes to teachers’ ability to provide high quality math instruction. For example, when teaching students about ordering decimals, CCK is used when the teacher orders decimals him or herself, SCK is used when generating a list to be ordered that may reveal key mathematical issues, KCS is used when recognizing which

decimals may cause students the most difficulty, and KCT is used when deciding what to do about student difficulties (Ball et al., 2008).

With this theoretical structure for Mathematical Knowledge of Teaching established, the question of its validity as a knowledge construct and its relationship to student learning remained. Thus, Ball and her colleagues engaged in the work of developing an instrument that would measure MKT. They hoped to empirically prove the existence of each domain of the theoretical structure and to provide a tool that could be used in research to empirically link teachers' MKT with student learning.

Development of the MKT Survey

In an effort to validate the theorized construct, mathematical knowledge for teaching, Ball and her colleagues constructed, administered and analyzed a set of “survey-based teaching problems thought to represent components of the knowledge of mathematics needed for teaching,” (Hill, Schilling, & Ball, 2004, p.12). As a result of this initial analysis, they were able to statistically establish the multidimensionality of teachers' knowledge for teaching elementary mathematics and the reliability of the new instrument for measuring such knowledge (Hill et al. 2004). The development of the instrument began with the construction and collection of a large number of survey items meant to address teachers' CCK, SCK and KCS. In addition to addressing the aforementioned knowledge domains, researchers selected the mathematical domain of number and operations due to its prominence in the elementary curriculum, and the less researched domain because of its newness to the elementary curriculum, patterns, functions, and algebra.

1. Mr. Allen found himself a bit confused one morning as he prepared to teach. Realizing that ten to the second power equals one hundred ($10^2 = 100$), he puzzled about what power of 10 equals 1. He asked Ms. Berry, next door. What should she tell him? (Mark (X) ONE answer.)

a) 0

b) 1

c) Ten cannot be raised to any power such that ten to that power equals 1.

d) -1

e) I'm not sure.

Figure 2.3. Item measuring common content knowledge. Adapted from “Developing Measures of Teachers’ Mathematical Knowledge for Teaching,” by Hill, Schilling, and Ball, 2004, *The Elementary School Journal*, 105e(1), p.28.

The items, produced by a range of experts including math educators, mathematicians, professional developers, classroom teachers and researchers with the Study of Instructional Improvement, were situated within the context of teachers’ daily work, (Ball, Hill, & Bass, 2005; Hill et al., 2004). Because these items were to be included on a survey and administered to a very large number of teachers, the items were all multiple choice to facilitate efficient scoring and analysis, (Ball et al. 2005). Items meant to measure CCK of mathematics (see Figure 2.3) were set in the context of a teaching scenario, yet only required a mathematical understanding common to those within as well as outside of the teaching profession. Similarly, items meant to measure SCK of mathematics (see Figure 2.4) required no knowledge of students and were purely mathematical. However, these items were designed to measure knowledge

of mathematics that is specialized for the teaching profession, such as common representations, error analysis, explanations for common rules and procedures, or analysis of alternate strategies for solving problems (Ball et al., 2005).

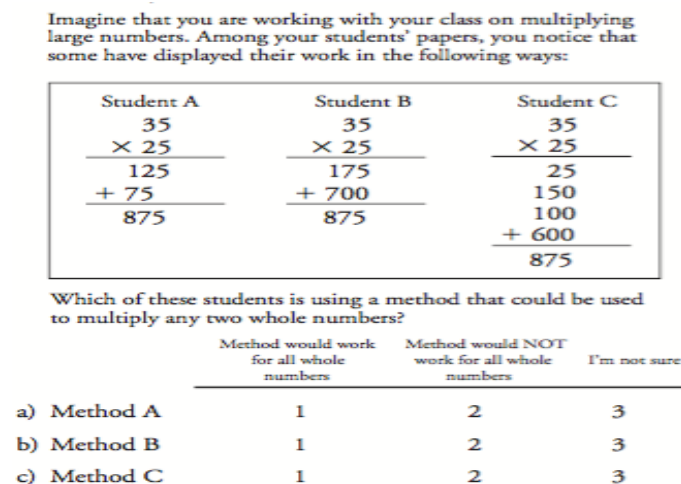


Figure 2.4. Item measuring specialized content knowledge. From “Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide?” by Ball, Hill, and Bass, 2005, *American Educator*, 29, p.43.

The distinction between CCK and SCK can be seen by comparing the sample items. In Figure 2.3, the item measuring CCK, the question is set in a teaching situation, but does not require knowledge specialized to the profession of teaching. Understanding that $10^1=0$ is common to anyone who has studied mathematics. Conversely, the survey item in Figure 2.4 involves the analysis of multiple strategies for solving a two digit by two-digit multiplication problem. Although the knowledge needed to answer this question is purely mathematical, it is not knowledge people outside of the profession of teaching would need or use making it specialized for teachers. The final knowledge

category included on the survey was KCS. These items (see Figure 2.5) were designed so that an amalgam of subject matter knowledge and knowledge of students should be used to answer the question. Researchers hoped to provide distinction between SCK and KCS in the survey items by designing the KCS items such that “in order to analyze an item and arrive at their answer, respondents should use knowledge of students’ thinking around particular mathematics topics, rather than purely their own mathematical knowledge, test taking skills, or other processes,” (Hill, Ball, & Schilling, 2008, p.378).

- 1. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)**
- a. Bonny doesn’t know how large 23 is.**
 - b. Bonny thinks that 2 and 20 are the same.**
 - c. Bonny doesn’t understand the meaning of the places in the numeral 23.**
 - d. All of the above.**

Figure 2.5. Item measuring knowledge of content and students. Adapted from “Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers’ topic-specific knowledge of students,” by Hill, Ball and Schilling, 2008, Journal for research in

The item in Figure 2.5 is asking respondents to make a judgement about student understanding based on a scenario about a student error, thus requiring pedagogical knowledge as well as content knowledge.

Once created, test items were divided among three test forms (A, B, and C) such that each knowledge and content domain was nearly equally represented (see Table 2.2),

then administered to teachers participating in California's Mathematics Professional Development Institutes (MPDIs) on teaching numbers and operations in the elementary school. The surveys were administered as pre and post-tests, resulting in 640 cases for Form A, 535 for Form B, and 377 for Form C. Using the data from these surveys, researchers were able to analyze them using factor analysis (FA), to statistically show the multidimensionality of MKT, use item response theory (IRT) to determine the reliability of survey items, and to measure growth of institute participants' MKT, (Hill et al., 2004; Hill et al., 2008).

Table 2.2
Mathematics Content Areas and Domains Represented

Content Area	Domain	
	Knowledge of Content (includes CCK and SCK)	Knowledge of Students and Content
Number Concepts	11-15 items	11-15 items
Operations	11-15 items	11-15 items
Patterns, Functions, Algebra	11-15 items	<i>*no items due to limited amount of research for use during item writing.</i>

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, 105e(1), p. 16.

Validity and Reliability of the Survey

Once items had been written and administered, the researchers attempted verification of their intended constructs (CCK, SCK, and KCS in number concepts, operations, and patterns and functions in algebra) through the use of exploratory factor analysis (FA). This type of FA provides researchers with a tool for consolidating

variables into a smaller number of categories, or factors, based on the underlying characteristics of the observed variables (Tabachnick & Fidell, 2001). Because of the complexity involved in identifying the knowledge types being measured by the MKT survey, FA was run in a number of different ways. The first of these was an exploratory FA run on each of the three test forms. On all three forms, content knowledge questions about number concepts and operations loaded most heavily on the same factor (Factor 1), leading researchers to believe that the same knowledge base was utilized to answer these questions. Likewise, the questions intended to measure content knowledge about patterns, functions, and algebra loaded most heavily on one factor (Factor 3) for all three test forms. However, the loadings for questions meant to measure KCS in number concepts and operations were not so clear with loadings split between the same factor as the content knowledge questions (Factor 1) and a third factor (Factor 2) on Form A (see Table 2.3) and loadings split between all three factors on the other two forms (Hill et al., 2004). What was clear from these findings is that among the questions intended to measure content knowledge, the combined mathematical domains of number concepts and operations formed one underlying construct while patterns, functions, and algebra formed another. What wasn't clear is whether or not KCS could be separated and recognized as its own construct. Thus, researchers decided to remove the questions meant to measure patterns, functions, and algebra content knowledge so as not to confound the results of the FA. In other words, they felt it was possible the additional mathematical content domain was obscuring the potential discrimination between content knowledge and pedagogical knowledge (KCS) (Hill et al., 2004). With those questions

removed, FA models of increasing complexity were fit to the data, including both two and three factor models. Once those analyses had been run with results similar to the first analyses, the researchers accepted the original three-factor model indicating that at least three knowledge types were being measured by the survey. These included: knowledge of content in numbers and operations; knowledge of content in patterns, functions and algebra; and knowledge of content and students in elementary number and operations (Hill et al., 2004).

Questions still remained, especially considering that the intent had been to measure two different types of content knowledge, common and specialized, for each mathematical domain. Thus, a bi-factor analysis, which intentionally allows items to load on more than one factor, was run. It was in this analysis that they were able to determine the existence of a fourth “general” factor that explained between 72% and 77% of the overall variation in teachers’ responses to the items (Hill et al., 2004), and a fifth factor for SCK. The appearance of this fifth factor offered some explanation for the multidimensionality of the items meant to measure knowledge of content.

After completion of the analyses to determine what was being measured, researchers set about the task of determining the reliability, or utility of the instrument for measuring the intended knowledge types as well as discriminating between knowledgeable and less knowledgeable respondents (Hill et al. 2008). Item response theory (IRT) was used to determine these reliabilities. Simply put, IRT utilizes probabilities to determine the likelihood of respondents at differing knowledge levels answering a question correctly and the coherence of similar item types to assign

Table 2.3
Promax Rotated Factor Loadings, Form A

Item	Factor 1	Factor 2	Factor 3
Knowledge of Content:			
Number Concepts:			
1	.512	.138	.063
2	.473	.113	-.059
3	.260	-.132	.165
4	.219	.062	-.180
5	.444	.158	-.133
6	.228	.097	.086
7	.139	-.039	.295
Operations:			
1	.732	.068	-.203
2	.246	.084	.136
3	.637	-.210	.101
4	.643	.042	.036
5	.704	-.292	-.023
6	.511	-.143	-.052
Patterns, functions, and algebra:			
1	-.290	-.111	.773
2	.259	-.038	.403
3	.015	-.016	.675
4	.156	.175	.314
5	.337	-.047	.419
6	.039	-.113	.639
Knowledge of Content and Students:			
Number concepts:			
1	.061	.275	.121
2	.263	.327	.158
3	.311	.149	-.021
4	.002	.365	.010
5	-.109	.248	.212
6	.180	.386	.047
7	.352	-.041	.019
Operations:			
1	-.055	.946	-.098
2	.466	.181	-.058
3	.018	.249	.130
4	.493	-.022	.017
5	-.125	.699	-.124
6	-.012	.417	-.022
7	-.015	.149	.151

Note –Boldface indicates highest loadings for each item

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, 105(1), 11-30.

correlation coefficients for use in reporting (Weiss & Yoes, 1991). The first round of analyses reported very strong reliabilities for the identified content domains as well as the combined KCS in numbers and operations. These reliabilities ranged from .709 to .839 with overall reliabilities ranging from .907 to .931, which are quite strong according to industry standards (Hill et al., 2008; Tabachnick & Fidell, 2001). However, there were some weaknesses in the determination of these reliabilities, which was addressed by later analyses (Hill et al., 2008).

Table 2.4
Reliabilities of KCS in Number and Operations

Scale	Number Stems	Number Items	Reliability	Test Information Curve Max
Form A				
Pretest	14	20	.60	-1.5
Posttest	14	20	.67	-1.12
Form B				
Pretest	14	19	.68	-1.0
Posttest	14	19	.65	-1.25
Form C				
Pretest	15	21	.58	-1.37
Posttest	15	21	.69	-.0875

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education*, 372-400.

As noted above, respondents at the CMPDI's were given both pre and post-tests. The 2004 analysis lumps both the pre and post tests for each form together creating a sample size that allowed an IRT model that would "overweight highly discriminating items in the estimation of reliability," (Hill et al. 2008, p.386). Because the focus of the second analysis was specifically aimed at describing the measurement of the KCS

domain, the researchers only amended the reporting of those reliabilities. They did so by using IRT on the pretest and posttest of each form individually (see Table 2.4). This reduced the sample size and did not allow the IRT to overweight the reliability of high performing items, reducing the reliability scores from the .71 for form A, .73 for form B, and .78 for form C on the combined analyses to scores ranging from .58 to .69 on the separate pre and post-test analysis. The newly calculated scores were “lower than industry standards suggest are sufficient for use in research and evaluation projects,” (Hill et al. 2008, p. 386). Thus, the survey cannot reliably discriminate among teachers of differing levels of KCS as well as the researchers had hoped. This was not surprising given the complexity of establishing KCS as its own construct in the factor analysis, and it only makes sense that respondents would draw on content knowledge as well as their pedagogical knowledge of students, that isolating this knowledge of students from the content knowledge on a multiple-choice test would be difficult if not impossible.

Although IRT showed a deficiency in the ability of the survey to discriminate between teachers of varying knowledge levels, test information curves generated as part of the IRT analysis showed the survey to be most accurate for teachers falling between one and two standard deviations below the mean. Training modules provided by the Learning Mathematics for Teaching Project out of the University of Michigan for researchers interested in using the instrument reinforce that this is the case, in addition to reporting updated reliabilities ranging from .71 to .92 in the content areas, from .51 to .78 for KCS in number and operations, and .64 in the newly researched knowledge of content and teaching in the number and operations domain (Learning Mathematics for Teaching,

2014d). Thus, the instrument is most reliable when used for measuring teachers' content knowledge resulting in higher instances of studies relating those aspects of the MKT construct to the teaching and learning of mathematics.

MKT Relationships in Research

When the MKT survey was conceptualized and designed, researchers were not only attempting to determine the structure of the knowledge teachers use when teaching math, but to design a tool that could be used in research to reliably answer questions about the effects of this knowledge on instructional practices and student achievement, as well as being a measure of teachers' growth as a result of professional development. In this section, I will address two studies in which the MKT was used to determine what, if any correlation exists between the level of either student achievement or instructional practices while implementing a standards based curriculum.

The first study, whose timeline coincided with the collection of data for the piloting of the MKT survey instrument at the MPDI's in California, was centered on determining if and how mathematical knowledge for teaching contributes to student gains (Hill, Rowan, & Ball, 2005). In this study, data was collected on both first and third grade students and teachers in 115 elementary schools. Student achievement data was based on student scores on the Terra Nova battery of tests given each spring and fall. Researchers used the raw scores of these assessments given to 8 students per classroom, randomly selected in the first year of the study, to calculate gain scores for use in analysis. In addition, data about students' rate of absence, gender, and minority status was collected from school staff, while data about students' academic history was

collected via phone interviews with a parent or guardian. Teacher level data included detailed mathematics teaching logs, completed by teachers up to 60 times per year, which consisted of a highly structured self-report instrument including information about the time spent on math instruction, mathematical content being covered, and instructional practices used to teach the content. These logs had previously been subject to extensive piloting, development, and validation work, making them an acceptable method for collecting these types of data (Hill et al. 2005). In addition to the logs, teachers answered a questionnaire each year that included items about experience, certification, and related coursework (content and methods courses) as well as select questions meant to measure teachers' mathematical knowledge for teaching. Although the intent was to include both content knowledge and knowledge of student and content questions on the assessment, researchers chose to only include questions in the content domain when the pilot studies showed that the KCS items did not meet the criteria for inclusion in a study such as this.

The data in this study was analyzed using linear mixed models. These models are predictive in nature, meant to estimate the influence of student, teacher, and school characteristics on a student's achievement gain (Hill et al. 2005). The key problems for researchers in conducting these analyses, were student attrition rates and missing variables. Missing data was dealt with in a variety of ways depending on the type and amount of data missing. For example, teachers who did not return questionnaires containing background information were excluded from the analysis entirely, but for those who simply did not answer enough of the MKT questions, researchers used mean imputation to adjust their scores. While this is an acceptable way of dealing with missing

data, it is not ideal. The researchers acknowledge the loss of covariance between variables because of this.

Although student level variables, such as SES and student scores on the first test, were the strongest predictors of student gain scores, there were several relationships between teacher variables and student gains. Most notable among these was the relationship between the MKT questions and student achievement. Analyses of these two variables showed that “teachers’ content knowledge for teaching mathematics was a significant predictor of student gains at both grade levels,” (Hill et al. 2005, p. 396). However, the most significance was found when comparing the lowest 20% of teachers in first grade and the lowest 30% in third grade to those in the top four deciles. This is not surprising given that the pilot study found a failure of the measure to discriminate among those at the higher knowledge levels (Hill et al. 2008). So, while it suggests that knowledgeable teachers are more effective math teachers, it also suggests that there may be a threshold at which more teacher content knowledge doesn’t necessarily equate to better student achievement. This finding was in line with Begle’s (1979) findings when using the number of math classes taken as a measure of teachers’ math knowledge. Accurate measures of teacher knowledge in the KCS domain might help discriminate among teachers with very high levels of content and specialized content knowledge for teaching mathematics. The trouble comes in finding a reliable way to isolate and measure the construct.

The other study of interest here was purely qualitative in nature. The study by Charalambous and Hill, was a series of case studies looking at the contributions made by

both curriculum materials and teacher knowledge to the quality of mathematics instruction (Charalambous & Hill, 2012). In doing this, they adopted a theoretical framework that “derives from the work on curriculum enactment, that looks at instructional quality as shaped by the interactions between teachers and curriculum resources,” (Charalambous & Hill, 2012, p. 450). In this series of four case studies, researchers videotaped lessons, collected post lesson interview responses from teachers, conducted interviews with each teacher meant to capture orientation toward materials, beliefs about teaching and other information about teaching practices, analyzed curriculum materials, and administered the MKT survey. The MKT survey administered consisted of questions addressing common and specialized content knowledge on numbers and operations, algebra, and proportional reasoning. Researchers looked at the results of the MKT survey in two ways. First they looked at teachers’ global MKT by assigning teachers percentile rankings as compared to a nationally representative sample of middle school math teachers. Then they looked at what they called the teachers’ local MKT, by isolating just those questions that related to the lessons being observed.

A grounded theory approach to analysis allowed the researcher to make hypotheses from the data, which researchers were then able to test with further analysis of the data. The findings of this research were in line with those of the quantitative study looking at student achievement and with other quantitative studies utilizing the MKT, in that they conclude that “MKT can legitimately be considered a key contributor to the mathematical quality of instruction,” (Hill & Charalambous, 2012, p. 560). Despite the

lack of quantitative analysis, the descriptive elements of the study are useful for illustrating how MKT affects math instruction in the classroom, not just that it does.

In addition to looking at teachers' content knowledge for teaching, as measured by the MKT survey, and its relationship to instruction, researchers were particularly interested in the ability of teachers at different knowledge levels to effectively implement a standards based curriculum that is both rigorous for students and more demanding for the teachers that teach it. Teachers in this study were all middle school teachers using the *Connected Mathematics Project* (CMP) curriculum materials. Observations of two or three teachers of varying MKT levels were done for each of four different lessons from the CMP materials. Teachers at all levels varied as to how closely they followed the curriculum materials, which in turn affected the mathematical quality of instruction, but more so for teachers in the lower ranges of MKT (Hill & Charalambous, 2012). For example, when comparing the lessons Rebecca (low MKT), Bonita (low MKT), and Mercedes (high MKT) each taught from the curriculum on adding and subtracting integers, researchers found that Mercedes facilitated a lesson that was mathematically rich, precise, and cohesive, as would be expected of a teacher scoring in the high range on the MKT survey. Rebecca was able to utilize the supports within the curriculum to compensate for her weaknesses in order to provide students with a more productive math lesson, but the lesson lost some of its coherence when she attempted to supplement from outside the curriculum. Bonita, on the other hand, opted not to utilize the supports built into the curriculum materials for teachers, resulting in a lesson that lacked precision and coherence (Charalambous, Hill, & Mitchell, 2012).

Overall, teachers scoring high on the MKT, were able to use and improve upon the CMP materials for quality math instruction. This was true for all high scoring teachers except for one who was not positively oriented toward the curriculum and in turn taught lessons that were mathematically precise, but not necessarily productive for helping students gain conceptual understanding of the content (Hill & Charalambous, 2012).

In the form used, the MKT survey was a measurement of teachers' common and specialized content knowledge. Teachers who score high in these areas, but are still instructionally weak, more than likely fall short when it comes to the unmeasured pedagogical portions of the MKT structure including knowledge of content and students, and knowledge of content and teaching. Additionally, teachers who score low in these areas are able to provide adequate to strong instruction, but only when they are able to closely follow instructional materials, and when those materials sufficiently support the enactment of the lesson.

Since the conceptualization of the MKT domain and the creation of the instrument for measuring that knowledge, a number of other studies utilizing the survey and domain descriptions have been carried out (see Table 2.5). However, despite establishing an empirical relationship between teachers' MKT and student achievement, as well as between teachers' MKT and the quality of mathematics instruction, none of these studies has looked specifically at the relationship between teachers' MKT and the achievement

Table 2.5

Select Studies Investigating Mathematical Knowledge for Teaching Relationships

Author(s)	The Study	Findings
Hill, Umland, Litke, & Kapitula, 2012	-Elementary and middle school classrooms -MKT survey to measure teacher knowledge -Mathematical Quality of Instruction (MQI) observation tool -Value-add scores for measure of student achievement	-Poor MKT performance predicted poor classroom performance (instruction and student achievement) -Strong MKT performance predicted strong classroom performance (instruction and student achievement) -Classroom performance of teachers in the middle on MKT performance varied greatly
Hill, Rowan, & Ball, 2005	-First and third grade classrooms; schools engaged instructional improvement with overrepresentation of high-poverty -MKT survey to measure teacher knowledge -Student Assessments drawn from the Terra Nova Battery for student achievement -Parent interviews to control for outside variables	Teachers' mathematical knowledge was significantly related to student achievement, even after controlling for co-variables.
Delaney, 2012	-MKT pilot in Ireland -Videotaped math lessons -How do teachers' MKT scores relate to classroom instruction in Ireland	Only a weak association was found between teachers' MKT scores and the mathematical quality of instruction.
Marshall & Sorto, 2012	-3rd Grade classrooms in rural Guatemala -MKT survey to measure teacher knowledge (separated into CCK and SCK for analysis) -Two years of achievement test scores	-Effective teachers have different kinds of mathematical knowledge -Higher levels of mathematical knowledge are linked with higher levels of student achievement
Bell, Wilson, Higgins & McCoach, 2010	-Teachers completing 2 modules of the Developing Mathematical Ideas (DMI) program were compared to teachers who did not complete any modules -MKT survey to measure teacher knowledge	-The DMI participants outperformed the comparison group on both assessments by a statistically significant margin
Charalambous, 2010	-2 elementary teachers at differing MKT levels compared while teaching tasks -MKT survey to measure teacher knowledge -Mathematical Tasks Framework (MTF) for analyzing videotaped lessons	-Quantitative and qualitative data provide evidence of a positive relationship between teachers' MKT and the cognitive level of task presentation in the classroom.
Copur-Gencturk, 2015	-Longitudinal study of 21 K-8 teachers to study the change in mathematical knowledge and teaching practice over 3 years -Interviews, observations and MKT survey	-Changes in teachers' mathematical knowledge predicted changes in the quality of their lesson design, their mathematical agenda, and classroom climate.

of students who struggle. This research seeks to fill the gap by investigating that relationship.

Response to Intervention

One of the most talked about trends in the field of education today is the implementation of a tiered approach to identifying and providing support to struggling students in both reading and mathematics. The reauthorization of the Individuals with Disabilities Education Improvement Act (IDEA) in 2004 provided much of the impetus for this movement. The document described and encouraged the use of a new, then untested, method for the identification of students with learning disabilities called response to intervention (Douglas Fuchs, Fuchs, & Compton, 2012). This new model depends on ongoing evaluation and documentation of a student's responsiveness to targeted interventions, as opposed to the commonly used discrepancy model, which uses IQ and other assessments to determine a discrepancy between potential and achievement, for identification of learning disabilities (Iii, Martens, Bamett, Witt, & Olson, 2007).

Despite its use as an identification tool for students with learning disabilities, response to intervention (RTI) provides a framework that can be used to meet the needs of all students (Larson et al., 2012). The RTI framework has two key instructional pieces—core instruction and intervention—both of which must be research and evidence based (Lembke, Hampton, & Beyers, 2012a). These two keys are further divided into three tiers, universal (Tier 1), strategic intervention (Tier 2), and intensive intervention (Tier 3) (Fuchs & Fuchs, 2006).

Student placement and the support they receive as a part of the tiered framework is dependent on the use of assessments, making those assessments crucial to the RTI process (Riccomini & Smith, 2011). The first assessment practice, used for identifying students who may be at risk, is the administration of a universal screener, a brief assessment administered three times per year. Students who fall below the benchmark levels on the screener are considered for placement in Tier 2 for strategic interventions on critical content. In addition to curriculum embedded measures for formative assessment use, a progress monitoring tool is used to show a student's growth as the result of intervention (Clarke, Lembke, Hampton, & Hendricker, 2011). Students in Tier 2 are progress monitored one to three times per month. Those not responding to Tier 2 interventions may be administered a more in-depth diagnostic assessment, with the purpose of giving educators a better understanding of the student's mathematical strengths and weaknesses (Riccomini & Witzel, 2010).

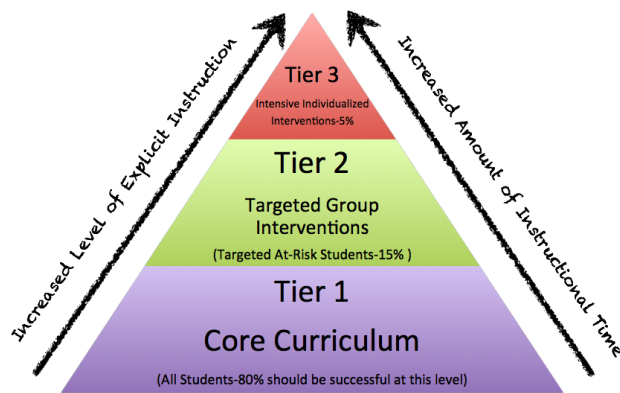


Figure 2.6. The three tiered RTI model (adapted from: Riccomini & Witzel, 2010 pg 10)

Students who fail to show progress at Tier 2 level are then considered for Tier 3 where the student receives more intensive interventions (see *Figure 2.6*). If the student continues to struggle and is not progressing, the student is referred for special education services and the presence of a learning disability is considered.

State and Local RtI Frameworks

The state of Tennessee is one of many states that have adopted this practice for addressing the needs of struggling students and identifying students in need of special education services. The Tennessee Department of Education (2015) describes its RTI² framework as an integral part of the education system:

In Tennessee, the education system will be built around a tiered intervention model that spans from general education to special education. Tiered interventions in the areas of reading, math, and/or writing occur in general education depending on the needs of the student. If a student fails to respond to intensive interventions and is suspected of having a Specific Learning Disability, then the student may require special education interventions (i.e. the most intensive interventions and services). (Tennessee Department of Education, 2015)

The model of this framework, published in the state implementation guide is very similar to the model represented in the research (see *Figure 2.7*), with Tier 1 for all students and Tiers 2 and 3 designated for students who struggle. According to the implementation guide, the local education agency (LEA) must administer a “nationally normed, skills-based universal screener,” to all students. This universal screener is used to determine

whether or not students are performing at a level consistent with other students in their grade level. This data can then be used for placement of students in appropriate interventions (see *Figure 2.8*).

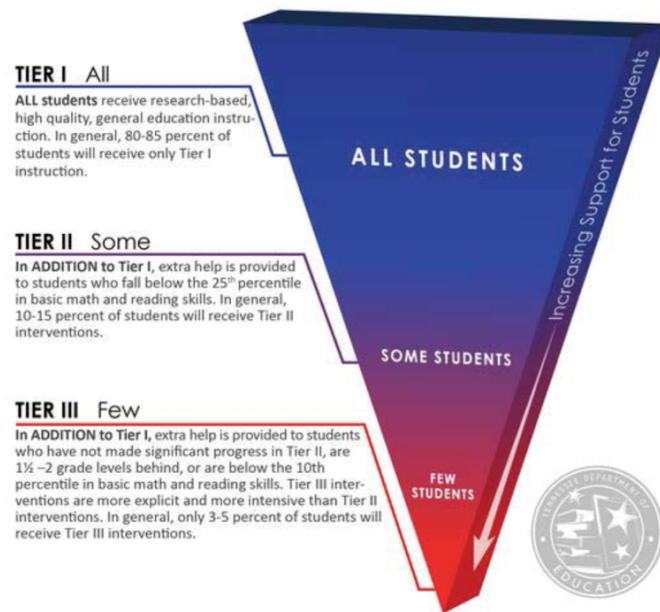


Figure 2.7. Tennessee’s Response to Intervention Framework (Tennessee Department of Education, 2015, p. 14).

According to district policy (2015), this district meets the requirement by utilizing the Aimsweb battery of assessments in kindergarten through fifth grade. In addition, the state recommends that students who fall below the 25th percentile on the universal screener should be considered at “at-risk,” with more at-risk students receiving priority for interventions. The school district represented in this study makes this statement more explicit by stating “If the number of students in the lowest 25th percentile is greater than 15-20% of the total school population, the percentile (for placement) can be lowered,” (district policy, 2015). Thus, students receiving interventions at Tier 2 or 3 can be

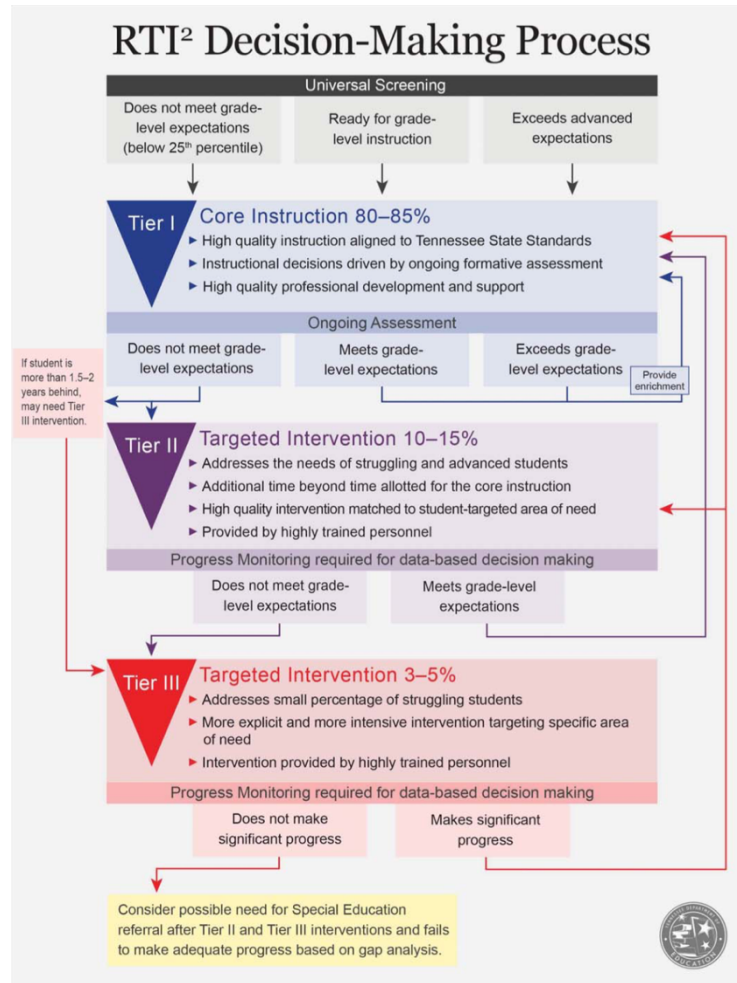


Figure 2.8. Tennessee’s Response to Intervention Decision-Making Process (Tennessee Department of Education, 2015, p. 14).

considered struggling students for the school population.

This research is particularly timely, because as more and more states set rigid time requirements (see Table 2.6) for the structured interventions of students who struggle, districts often have difficulty meeting those requirements with classroom teachers who have been trained in math instruction. Because of this, they are becoming dependent on teachers and paraprofessionals who may not be trained in math instruction to provide those interventions. A positive relationship between teachers' MKT and the achievement of students who struggle may indicate less than favorable results when allowing less qualified teachers to provide interventions.

Table 2.6
Minimum Time Requirements for Math RtI

Grade	Tier 1	Tier 2	Tier 3
K	60	20	40
1	60	20	40
2	75	30	45
3	90	30	45
4	90	30	45
5	90	30	45

Chapter Three

Method

With the increase in attention paid to student achievement in mathematics, and the increased rigor of mathematics standards, researchers have been attempting to isolate the factors contributing to students' mathematical achievement. Some of these factors include, but aren't limited to, the student's socioeconomic status (Apple, 1992; Berliner & Glass 2014), previous math experiences (Wang & Goldschmidt, 2003), instructional practices of the teacher (Van De Wall, 2007; Richardson, 2012; Kamii, 2000; Lamon, 2012; Carpenter, Fennema, Franke, Levi, and Empson, 1999), the classroom teacher's mathematical knowledge for teaching (MKT), and in some cases, the interventionist's MKT (Ma, 1999; Van De Wall, 2007; Hill, Rowan, & Ball, 2005). Because of the complexity involved in measuring students' previous math experience and the abundance of existing research on the relationship between socioeconomic status and student achievement, this study has chosen to focus on the relationship between classroom teachers' MKT and interventionists' MKT and student achievement. Specifically, this study used the common content knowledge(CCK) and specialized content knowledge(SCK), domains of the MKT survey to explore the relationship between teachers' subject matter MKT and student achievement.

Research Design

An explanatory mixed methods design was selected for this study (Creswell, 2008). In this approach, the researcher placed a priority on the quantitative data, which was collected first. In the quantitative portion of this study, the researcher was looking

for a simple association between two or more variables; in other words, that they co-varied (Creswell, 2008). Of specific interest here was the covariance between classroom teachers' and interventionists' MKT and student achievement. Following the collection and initial analysis of the quantitative data, teachers' MKT and students' growth scores, qualitative data was collected in order to look at the relationship between MKT and instructional practices, which might explain the covariance. Classroom observations of classroom teachers and interventionists at varying levels of the MKT scale were conducted to further explore the relationship between MKT and instructional practices.

Research Questions

The study addresses the following research questions:

1. Is there a relationship between the classroom teachers' subject matter MKT and students' achievement as measured by M-Comp growth scores?
2. How are classroom teachers' and interventionists' MKT related to achievement in struggling students or non-struggling students?
3. Do the instructional practices of teachers correspond to their MKT scores?

Context of the Study

The variables of interest in this study (classroom teachers' subject matter MKT, interventionists' subject matter MKT, and student M-COMP growth scores) required the collection of data about both students and their teachers. As with all districts in the state of Tennessee, Crystal Lake School District is required to adhere to a three-tiered response to intervention (RtI) model. Because of this requirement, the system administers a

universal screening assessment three times per year in order to identify 15-20 percent of its lowest-achieving students, who are then provided interventions through an RtI model. This provided an opportunity to also investigate the influence of interventionists' mathematical knowledge for teaching on student achievement.

Crystal Lake School District, a mid-sized school district in East Tennessee, was selected for this study because the researcher has an ongoing relationship with the district and teachers. This established relationship allowed for easy access to a large number of participants. In addition, rapport with teachers had already been established by the researcher, potentially increasing participation in the survey necessary for this study.

Participants

The population included all students in first through fifth grade within the Crystal Lake School District. In these grade levels, the district has 1,667 students, with 61% qualifying for free or reduced lunch. The population is 69% White, with 17.8% Black, 8.2% Hispanic, and 4.1% Asian. Students were identified as struggling and in need of intervention by their score on the universal screener. Based on the assumption that 20% of students were placed in tiered interventions, the subpopulation of struggling students consists of approximately 300 students. A convenience sample, with participants selected because they are willing and available to be studied (Creswell, 2008), was taken from the larger population of students. In this case, inclusion of the student in the study was dependent on the completion of the MKT survey by their math teacher.

All teachers and paraprofessionals who provide math instruction or math intervention to students in first through fifth grades were emailed a request to complete

the MKT survey and an access code. The email contained a brief description of the study as well as a statement informing teachers of the steps taken to ensure that results of the survey are confidential. The testing window remained open for two weeks, with reminder emails sent at the end of the first week and middle of the second week. Thirty-two teachers and interventionists completed the survey.

All students of teachers who voluntarily completed the MKT survey were included in the study. Unfortunately, it was found that some first-grade students were administered the Aimsweb Number Identification assessment rather than the Aimsweb Math Computation (M-COMP) assessment in the fall. Therefore, due to the differing assessments, it was impossible to calculate growth scores for first grade students. Those students who ended up with no corresponding teacher MKT score were eliminated from the study. Additionally, the failure of a student to make progress during tiered interventions is used as one of the indicators that the student has a learning disability and should be referred for special education services. Thus, it was decided that only students in second through fifth grade not receiving special education services should be included in the study. This resulted in the inclusion of 20 classroom teachers and 4 interventionists (a few classroom teachers provided intervention as well) and 485 of these classroom teachers' and interventionists' students, 67 of these students were participating in tiered interventions and were classified as struggling students (see Tables 3.1 and 3.2).

Data Collection and Instruments

The instrument used to collect data on the teachers' mathematical knowledge for teaching is the Mathematical Knowledge for Teaching (MKT) survey

Table 3.1

Student and Classroom Teacher Participants

	Grade 2	Grade 3	Grade 4	Grade 5	Total
Non-struggling Students	101	26	137	154	418
Struggling Students	22	6	18	21	67
Total Students	123	32	155	175	485
Classroom Teachers	9	2	6	3	20

Note. In addition to the 4 interventionists who served students at multiple grade levels, 6 classroom teachers acted as both classroom teacher and interventionist. However, the students who were provided interventions by these classroom teachers may or may not have received Tier 1 instruction from the same teacher.

Table 3.2

Classroom Teacher MKT Scores and Class Mean of Students' AIMSweb Growth Scores

Classroom Teacher	MKT Scores	Mean AIMSweb Growth
Sidney	-0.746	19.86
Glen	-0.550	4.12
Fred	-0.403	25.64
Pamela	-0.384	19.29
Jack	-0.318	25.13
Laurie	-0.282	26.46
Kristen	-0.209	20.81
Gale	-0.207	22.31
Meg	-0.187	31.06
Marge	0.044	17.62
Michael	0.226	18.38
Tatum	0.227	15.62
Casey	0.293	22.73
Alex	0.437	18.06
Marie	0.478	21
Nancy	0.820	25.66
Jason	1.015	15.58
Judy	1.066	15.54
Russ	1.346	11.60
Alice	1.526	8

(Appendix B), developed by the University of Michigan’s Learning Mathematics for Teaching Project. At its inception, the intent of the researchers was to develop an instrument that could reliably measure the subject matter knowledge and pedagogical content knowledge (PCK) subsections of the MKT domain (see figure 3.1 and refer to the literature review).

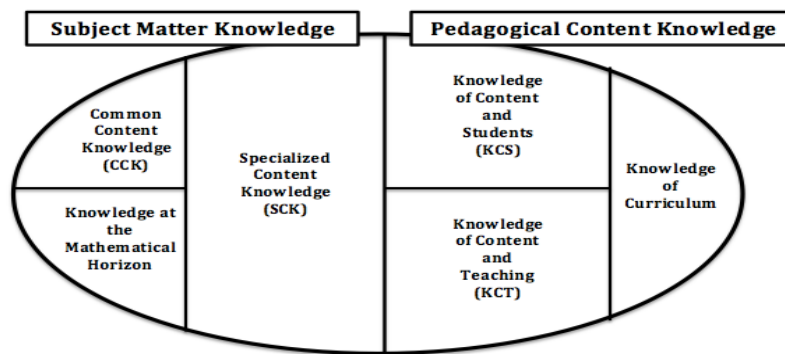


Figure 3.1. Domain map for mathematical knowledge for teaching. (Hill, Ball, & Schilling, 2008, p.377)

However, factor analysis of the test items indicated that those items intended to measure the PCK construct, Knowledge of Content and Students (KCS), were loading on more than one factor, and those loadings were inconsistent among different trials. This led researchers to believe that teachers “might use KCS, mathematical reasoning, or both” to get the answers. In other words, “even if KCS is not present (in participants), mathematical reasoning and knowledge can compensate,” (Hill, Ball, & Schilling, 2008, p. 390). In addition, the inability of items intended to measure KCS to discriminate between KCS and the content knowledge constructs, resulted in a low reliability of those items (0.51-0.78). Those items intended to measure only the content knowledge areas—

Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK)—had reliabilities that were much more acceptable (0.71-0.89) (Learning Mathematics for Teaching (LMT) Training Module 5). This study utilizes only the more reliable content knowledge assessment, aimed at measuring teachers' subject matter knowledge (CCK and SCK) in the area of Number Concepts and Operations.

The Teacher Knowledge Assessment System (TKAS), an online test administration system, developed by Geoffrey Phelps (PI) and Steve Schilling with grant funding from the National Science Foundation (LMT Training Module 1), was utilized for administration of the MKT survey. The survey was designed to measure a range of teachers' MKT reliably with easy, medium, and difficult items. Therefore, the average teacher should be able to answer 50% of the items correctly (Hill, Schilling & Ball, 2005). The version of the online assessment utilized for this study is adaptive, meaning that each participant received a selection of items that were tailored to their ability, greatly reducing the time it took to complete the survey. For example, if an item was answered incorrectly, the participant received an easier question. Similarly, if an item was answered correctly, the participant received a more difficult question. Once the survey was completed, Item Response Theory (IRT) was utilized by the system to convert the raw scores into standardized IRT scores ranging from -3 to +3. Zero is the mean score, indicating a teacher of average MKT ability, with the majority of teachers typically ranging from -2 to +2 (LMT Training Module 4).

All of the student data that was utilized in this study was provided by the school district, including demographic information and the Aimsweb M-COMP universal

screening results. Because this data was already in existence, no additional consent was required from parents. The demographic information that was provided by the school system included the students' school, grade level, socioeconomic status, race, and teacher.

The school system utilizes the M-COMP (Appendix A), an 8-minute test of math operations with national norms including 30 different test forms at each grade level, for universal screening and as progress monitoring probes (NC Pearson, 2012). According to the Aimsweb Technical Manual, a content blueprint for the assessment items (see Table 3.3) was developed based on guidance from experts in the field. The reliability and validity of the assessment was checked during a field testing correlation study.

Table 3.3
Content Measured by M-COMP, by Grade

Domain	1	2	Grade 3	4	5
Column Addition	✓	✓	✓		
Basic Facts	✓	✓	✓	✓	✓
Complex Computation	✓	✓	✓	✓	✓
Decimals				✓	✓
Fractions				✓	✓
Conversions					✓
Percentages					✓

NCS Pearson. (2012). aimsweb: Technical Manual. Bloomington, MN.
Retrieved from <http://www.aimsweb.com/wp-content/uploads/aimsweb-Technical-Manual.pdf>.

The *Group Mathematics Assessment and Diagnostic Evaluation* (G-MADE), an established diagnostic test distributed by Pearson, was utilized to determine how strongly the results correlated with the M-COMP. The M-COMP was found to be highly correlated with the G-MADE (see Table 3.4). Reliability of the assessments was also checked in grades 1-8 during field testing with acceptable reliability, ranging from .74 to .82 (NC Pearson, 2012). The M-COMP is administered to all students three times per year as a universal screener. The district provided student scores for all three assessments, but only the fall and spring scores were utilized to calculate an overall growth score to be used as the student achievement variable.

Table 3.4

Criterion Validity of M-COMP Scores with Respect to G-MADE Scores

Grade	N	M-COMP		G-MADE		r
		Mean	SD	Mean	SD	
1	98	36.0	12.2	17.9	5.6	.84
3	98	45.5	14.2	15.9	5.7	.73
8	54	28.4	13.6	13.4	3.8	.76

NCS Pearson. (2012). *aimsweb: Technical Manual*. Bloomington, MN. Retrieved from <http://www.aimsweb.com/wp-content/uploads/aimsweb-Technical-Manual.pdf>.

The final piece of data collected for this study was qualitative in nature, meant to give further explanation of the instructional differences that might be seen from teachers at different levels of the MKT scale during both classroom instruction and interventions. Once the survey had been administered, 8 teachers were selected for observation. Among the 8 teachers, the researcher attempted to select 4 classroom teachers and 4 interventionists from different grade bands (1-2 and 3-5), with 2 of each scoring at the high and 2 of each scoring at the low end of the MKT scale. The researcher was able to

adhere to those parameters when doing observations of the classroom teachers. However, due to the performance level of the interventionists included in the study, the shortage of volunteers for observation, and staff turnover, two non-certified interventionists, one certified interventionist, who carries the title of math specialist, and one certified classroom teacher administering an intervention were observed (see Tables 3.5 and 3.6). Each teacher was observed one time for 30 minutes. During the observation, the researcher scripted the lesson including notes about instructional activities that were utilized and questions that were asked.

Table 3.5
Classroom Teachers Selected for Observation

MKT	Classroom Teacher	MKT Score
High	Jason– Grade 2	1.02
	Alice- Grade 5	1.53
Low	Pamela- Grade 2	-0.38
	Jack- Grade 4	-0.32

Table 3.6
Interventionists Selected for Observation

Status	Interventionist	MKT Score
Certified	Tommy-Grade 4	2.24
	Brenda- Grade 1	0.61
Non-Certified	Marcie-Grade 4	1.52
	Paul-Grade 3	1.52

Data Analysis

Once survey data had been collected and demographic data had been provided by the school system, teacher MKT scores were matched with their students' M-COMP growth scores, and students with no teacher MKT were eliminated from the study. After all teacher data had been matched to the correct students, students' and teachers' identifying information was eliminated. With all data appropriately recorded, the process of cleaning and analyzing the data began. The twelve steps of data cleaning recommended by Morrow and Skolits (2015) was used to guide the data cleaning process. The first two steps included the creation of a codebook (that lists detailed information about each variable included in the study) and a data analysis plan (Appendix C). The analysis plan will serve as a detailed guide to how analysis for the project was carried out, including step-by-step descriptions of the analyses as well as the syntax used to carry out each analysis. Because of the detail involved, the analysis plan was started during the data cleaning stage, then added to as analysis was carried out using the SPSS software package (IBM, 2015). Performing initial frequencies is the third step of the process. Running frequencies on all variables allowed for a quick look at the data and led directly into the fourth step, identification of initial errors including coding errors and missing data. Once errors in the data had been identified and corrected, step five in the process was to create or modify variables. This is the stage in the process where variables such as socioeconomic status and students' struggling or non-struggling status were recoded for later analysis, teachers' and interventionists' MKT were recoded as categorical (either high or low), and student growth scores were calculated by subtracting the score of each

student's fall Aimsweb M-COMP score from the spring M-COMP score. With variables recoded and new variables created, steps six and seven required running frequencies on all variables, new and old, and obtaining descriptive statistics on all continuous variables, which allowed outliers in the data to be identified and subsequently dealt with. Steps eight, nine, and ten involved final checks to make sure the data was ready for analysis. Because a common assumption of many inferential statistics is that the data be normally distributed, step eight in the data cleaning process was a check for normality. Normality was not met for either the classroom teacher MKT scores or the interventionist MKT scores. In this stage, data can be manipulated to meet the assumption or the decision to utilize non-parametric statistics can be made. However, due to the relatively small number of teachers represented in the study (20) and the robust nature of the analyses to be utilized, the decision to proceed with the data in its current form was made (Tabachnick and Fidell, 2011). In addition to checking for normality, missing data was dealt with, and sample size is checked. With the data in its final stage, frequencies and descriptive statistics were run one last time so that figures could be generated to display characteristics of the sample. Finally, the last step in the data cleaning process, checking assumptions, was done for each analysis to be run (Morrow & Skolits, 2015). With the exception of the normality assumption which has already been discussed, all assumptions were met.

There are three research questions associated with this study. Each question was approached using different set of analyses (see Table 3.7). These analyses will be looked

Table 3.7
Abbreviated Analysis Plan

Research Question	Variables Utilized to Address the Question	Proposed Analysis and Figures
Is there a relationship between the instructors' MKT and students' achievement as measured by Aimsweb M-Comp growth scores?	• Classroom teacher MKT score • Student Aimsweb growth score	Bivariate correlation • Correlations Table
How is teachers' MKT related to achievement in struggling and non-struggling students	• Intervention (yes or no) • Classroom teacher MKT score • Interventionist MKT • Student Aimsweb growth score • Categorical classification of students as struggling or non-struggling	Moderator Multiple Regression • Descriptives Table • Frequency chart • Regression Results Table Bivariate correlation • Correlations Table Multiple Regression • Regression Results Table
Do the instructional practices of teachers correspond to their MKT scores?	Classroom observation data coded for instructional practices and questioning.	Coding for the identification of patterns among types of questions asked by teachers with similar MKT scores. Comparison analysis of instructional activities utilized during interventions.

at both individually and comprehensively in an effort to better understand the relationship between MKT and student achievement.

Research Question 1: Is there a relationship between classroom teachers' MKT and student's achievement as measured by the Aimsweb M-Comp growth scores?

This question involved all subjects in the sample and was analyzed using bivariate correlation. The bivariate correlation is a statistical analysis used to determine the direction and strength of the relationship between two variables (Tabachnick & Fidell, 2001). In this case, the independent variable (IV) was the classroom teachers' MKT score in the form of a z-score calculated by the TKAS assessment administration system, and the dependent variable (DV) was students' growth scores. Both of these variables were continuous and paired, thus meeting the first two assumptions of this analysis. There must be a linear relationship between the variables in addition to the absence of outliers and an approximate normal distribution. These assumptions were checked as part of the data cleaning process. The bivariate correlation analysis generated a correlation coefficient known as Pearson's r . This coefficient has a value between -1.00 and 1.00, with stronger correlations falling closer to either 1.00 or -1.00. The direction of the association is indicated by the sign on the coefficient with positive correlations showing that as the IV goes up (or down), the DV also goes up (or down). The inverse is true of negative correlations, as the IV goes up, the DV goes down or vice versa (Tabachnick and Fidell, 2001). Rules about how to interpret the strength of the relationship based on the value of Pearson's r vary somewhat. However, for this study recommendations made by Cohen (1988) were utilized (see Table 3.8).

Table 3.8
Pearson Correlation Strength of Association Guidelines

Coefficient Value	Strength of Association
$0.1 < r < 0.3$	small correlation
$0.3 < r < 0.5$	medium/moderate correlation
$ r > 0.5$	large/strong correlation

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). New York, NY: Psychology Press.

Research Question 2: How is teachers' MKT related to achievement in struggling students or non-struggling students?

This question required the division of the population into two sub-populations, struggling and non-struggling students, and utilized three different analyses to more fully illustrate the nature of the relationship between teacher subject matter MKT and student achievement. Both sub-populations were included in the first analysis in which moderator multiple regression was utilized to determine if there was a stronger relationship between teachers' subject matter MKT and growth in struggling or non-struggling students.

Regression analysis is "a set of statistical techniques that allow one to assess the relationship between one DV and one or more IV," (Tabachnick & Fidell, 2001, pg. 111), and requires that a number of assumptions have been met. At this point, the assumptions of normality and absence of outliers had been addressed by the data cleaning. In addition to those assumptions, there must be little or no multicollinearity among the IV's, meaning they should not be too highly correlated. Therefore, anything that was correlated at the 0.8 level or higher could be problematic and the variables should not be used together in the same regression analysis (Tabachnick & Fidell, 2001). As with the assumptions for bivariate correlations, there should be a linear relationship between each IV and the DV.

Finally, the assumption of homoscedasticity must be met. This simply means that there is a constant variance of the errors, in addition to independence of errors, meaning that the distribution of the errors is random, not influenced or correlated with each other (Tabachnick & Fidell, 2001; Morrow, 2015a). The data met all assumptions.

Moderator multiple regression is a type of hierarchical regression used to find out if one IV has a stronger relationship with the DV of one group of subjects than another (Tabachnick & Fidell, 2001; Morrow, 2015b). In this analysis, the DV was student achievement as measured by Aimsweb growth scores. One of the IV's was classroom teachers' subject matter MKT score, the other IV was a categorical variable indicating whether a student is struggling or non-struggling based whether or not they had received tiered interventions, and a third IV is an interaction variable created by multiplying the first two IV's together. The hierarchical multiple regression allowed the IV's to be entered into regression analysis in a particular order, which made any change in variance apparent. Thus, this analysis indicated whether classroom teachers' MKT had a stronger relationship with struggling or non-struggling students.

Following this analysis focus was placed solely on students identified as receiving interventions as part of a tiered intervention program and thus, identified as struggling. Because an interventionist often administers these interventions, the subject matter MKT of the interventionist must also be considered in addition to considering the subject matter MKT of the classroom teacher. Two analyses were utilized to investigate the relationship between teachers' subject matter MKT and achievement in struggling students. The first of these analyses was bivariate correlation, the same analysis used to

address the first research question. In this case, the independent variable (IV) is the interventionist MKT score, and the dependent variable (DV) is struggling students' growth scores. Both of these variables are continuous and paired, thus meeting the first two assumptions of this analysis. There must be a linear relationship between the variables in addition to the absence of outliers and an approximate normal distribution. These assumptions were checked as part of the data cleaning process.

While a simple comparison of correlation coefficients from the two analyses might address concerns about the influence or lack of influence in regards to classroom teachers' or interventionists' MKT on student growth scores, a more robust analysis to explain levels of variance among the variables is multiple regression (Tabachnick & Fidell, 2001). In this standard multiple regression, classroom teachers' subject matter MKT scores and interventionists' subject matter MKT scores were IVs and student growth was the DV. The multiple regression analysis generated Pearson's r for the model, in addition to generating an R^2 value and an adjusted R^2 value. The adjusted R^2 was used to explain the proportion of variance in student growth scores attributed to classroom teachers' and interventionists' MKT scores.

Research Question 3: Do the instructional practices of teachers correspond to their MKT scores?

This question involved the naturalistic observation of teachers at varying levels of the MKT scale (see Tables 3.4 and 3.5) followed by analysis of the scripted observational notes (Appendix D), so that further information about whether the observed instructional differences of these teachers might be explained by their MKT level. The method used

for analysis of this observational data was initial coding followed by axial coding. The initial coding is quite simply the breaking down of the qualitative data into discrete parts, then comparing them for similarities and differences (Saldana, 2015). Initial coding is a first cycle coding method intended as a starting point. Once this has been done, the researcher can engage in axial coding in order to reassemble the data into categories and subcategories (Saldana, 2015). Of particular interest during this coding process were the methods of instruction and types of questions being asked. Thus, coding focused on the categorization of questions and activities based on rigor and cognitive demand.

The first stage of analysis, initial coding, for both the classroom and intervention observations involved the listing of all questions asked during the thirty-minute observations. The questions were then coded by two separate researchers using the framework conceptualized by Barnes (1969) for his studies on the effects of teacher behavior and language on student learning (See *Figure 3.2*). After the first pass coding questions, the two researchers were in agreement on 30% of the questions. Then, the researchers collaborated to clarify interpretation of the definitions for each coding category and discussed each question to ensure agreement in regards to the code assigned as well as to ensure that all similar questions were assigned the same code.

The framework for coding the questions consisted of four main categories along with six subcategories. The first main category, Factual, are questions that ask “What?” and correspond to the first level of Bloom’s Taxonomy (see *Figure 3.3*), which require students to recall factual knowledge from memory. The “Factual” category is divided

into two subcategories, the first of which, Information, includes questions that require students to respond with some fact, but require no reasoning or application of that knowledge. In other words, these are questions that could be asked in any environment and an answer could be provided. The second subcategory, Naming, requires students to name some phenomenon, such as the number of objects on a page or the answer to a calculation problem, but requires no reasoning about its use (Barnes, 1969).

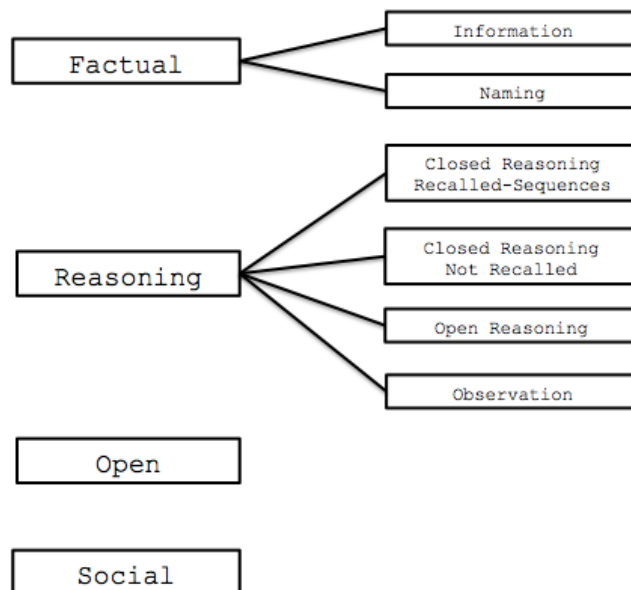


Figure 3.2. Framework for question categories. (Barnes, 1969, p.17)

The second main category of questions, “Reasoning”, are questions that ask “How?” or “Why?” and could correspond to either the second, third, fourth, or fifth levels of Bloom’s Taxonomy. Reasoning is divided into four subcategories. The first two ask for “closed reasoning,” meaning that the questions require reasoning, but only have one acceptable answer. Questions that are “Closed Reasoning with Recalled Sequences” are

those in which a student uses reasoning about a recalled sequence, such as a particular solution strategy, to arrive at the correct answer. Those that are “Closed Reasoning-Not Recalled” are those in which there is only one acceptable answer, but the reasoning used to arrive at the answer is not recalled from a sequence previously taught.

“Open Reasoning” questions are those that require students to reason about an answer, but any number of different answers might be acceptable. One example of an “Open Reasoning” question is one in which a student has to defend or explain his or her answer. The last subcategory, “Reasoning-Observation,” is open in the sense that these questions could result in a number of acceptable answers, but they require students to interpret what they perceive such as from a word problem or model. The final two main categories are “Open,” questions with a number of acceptable answers that do not require students to reason, and “Social,” questions that are asked for clarification or for other social reasons (Barnes, 1969). Once questions asked by all classroom teachers and interventionists had been coded, the number and types of questions asked were tallied for comparison among classroom teachers with varying levels of subject matter MKT and interventionists with varying levels of subject matter MKT and formal pedagogical training.

Next, the focus was placed on the types of activities used during the lesson observations. Because each of the classroom teachers observed were teaching from the district mathematics program, analysis of their lessons was only focused on the questions asked during implementation. On the other hand, interventionists had more flexibility in the activities they chose to utilize during interventions, so in addition to a comparison of

the questions asked, the activities used were categorized as either fluency, concept building, or problem solving and compared among interventionists.

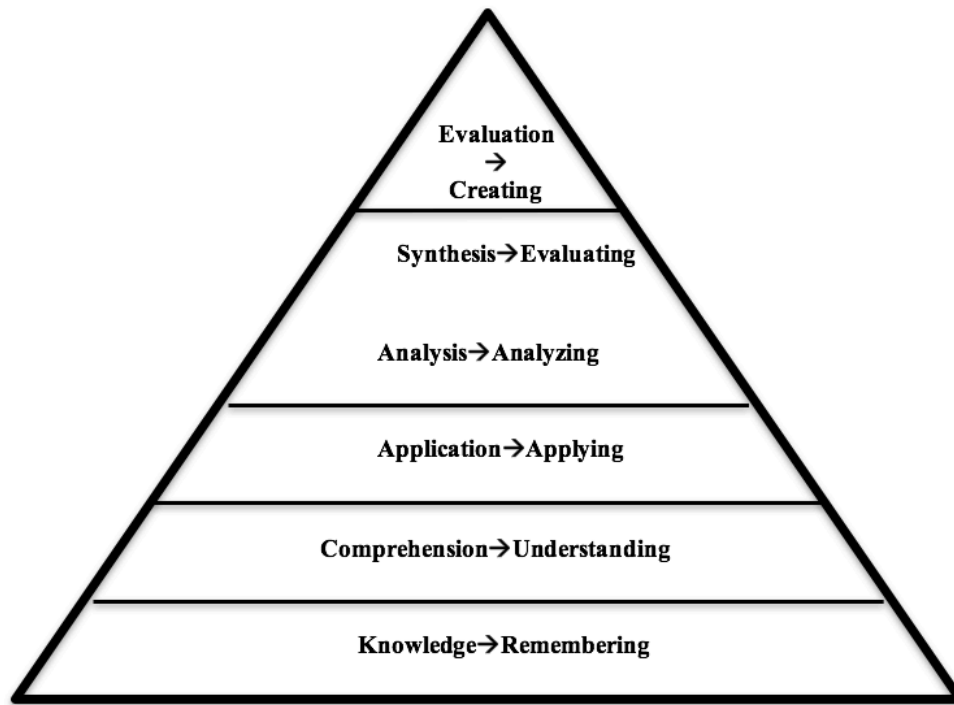


Figure 3.3. Bloom's Taxonomy and Revised Bloom's Taxonomy. (Forehand, 2010)

Chapter Four

Results and Discussion

The purpose of this mixed methods study was to investigate the relationship between teachers' and interventionists' mathematical knowledge for teaching (MKT) and struggling and non-struggling students' Aimsweb Mathematical Computation (M-COMP) growth scores. Specifically, this study attempted to answer the following research questions:

1. Is there a relationship between the classroom teachers' subject matter MKT and students' achievement as measured by M-Comp growth scores?
2. How is teachers' MKT related to achievement in struggling students or non-struggling students?
3. Do the instructional practices of teachers correspond to their MKT scores?

As explained in chapter three, teacher data was collected from teachers through the voluntary completion of the subject matter MKT survey, which includes assessment of teachers' common content knowledge (CCK) and specialized content knowledge (SCK). The district provided fall and spring M-COMP scores of students corresponding to teacher participants. These scores were used to calculate student growth by subtracting the fall score from the spring score. In addition, qualitative data regarding instructional practices was collected during classroom observations of teachers who had completed the MKT. This chapter presents the findings of this research.

Data Cleaning

To evaluate the relationship between teachers' subject matter MKT and student achievement, a twelve-step data cleaning process (Morrow & Skolits, 2015) was used to

identify any issues with the data such as outliers or missing data as well as to describe the data and to check the assumptions for each statistical analysis being used. There were no outliers and missing data was dealt with in the manner described in chapter three. All assumptions were met except for the assumption of normality for both Teacher MKT and Interventionist MKT as assessed by Shapiro-Wilk's test ($p < .05$) (Tabachnick & Fidell, 2001). However, despite Teacher MKT being positively skewed (.309) with a negative kurtosis (-1.464) and Interventionist MKT being negatively skewed (-1.138) with a positive kurtosis (1.421), it was determined that the small sample size, 485, allowed analyses to be carried out despite the violation. With this determination made, the appropriate analyses were carried out to answer each research question. These analyses included bivariate correlation and multiple regression.

Sample Description

The student sample for this study was comprised of 485 students representing four grade levels (see Tables 4.1 and 4.2). As indicated by the table, the student sample was subdivided into two subgroups, Struggling (SS) and Non-struggling students (NS). Student inclusion in the Struggling Students subgroup was dependent on whether they received either Tier 2 or Tier 3 interventions as part of a response to intervention program. Additionally, because this study was investigating the relationship between teachers' MKT and students' M-COMP growth scores, the 485 students correspond to one of the 20 classroom teachers represented in the study, with the 67 struggling students also corresponding with one of the ten teachers providing interventions. Of the ten teachers providing interventions, four were dedicated interventionists who provided Tier

Table 4.1
Student and Classroom Teacher Participants

	Grade 2	Grade 3	Grade 4	Grade 5	Total
Non-struggling Students	101	26	137	154	418
Struggling Students	22	6	18	21	67
Total Students	123	32	155	175	485
Classroom Teachers	9	2	6	3	20

Note. In addition to the 4 interventionists who served students at multiple grade levels, 6 classroom teachers acted as both classroom teacher and interventionist. However, the students who were provided interventions by these classroom teachers may or may not have received Tier 1 instruction from the same teacher.

Table 4.2
Classroom Teacher MKT Scores and Class Mean of Students' AIMSweb Growth

Classroom Teacher	MKT Scores	Mean AIMSweb Growth
Sidney	-0.746	19.86
Glen	-0.550	4.12
Fred	-0.403	25.64
Pamela	-0.384	19.29
Jack	-0.318	25.13
Laurie	-0.282	26.46
Kristen	-0.209	20.81
Gale	-0.207	22.31
Meg	-0.187	31.06
Marge	0.044	17.62
Michael	0.226	18.38
Tatum	0.227	15.62
Casey	0.293	22.73
Alex	0.437	18.06
Marie	0.478	21
Nancy	0.820	25.66
Jason	1.015	15.58
Judy	1.066	15.54
Russ	1.346	11.60
Alice	1.526	8

2 or Tier 3 interventions to students at multiple grade levels throughout the day. The remaining six teachers providing interventions were classroom teachers who also provided either Tier 2 or Tier 3 interventions, but not necessarily to students in their own Tier 1 class.

Results

Relationship Between MKT and Student M-COMP Growth

Pearson's bivariate correlation was conducted to determine the direction and strength of the relationship between student achievement and classroom teachers' subject matter MKT. Both struggling and non-struggling students ($N=485$) were included in the analysis. The mean for the corresponding MKT scores was .335 ($SD=.786$), and the mean M-COMP growth score was 17.86 ($SD=14.245$). There was a significant correlation between teachers' subject matter MKT and student growth, $r(483) = -.269, p < .001$. However, according to Cohen (1988) the strength of this correlation is small. Additionally, when the data is disaggregated by grade level and the same analysis is run, the small negative correlation with MKT and student achievement only holds for grade 5, $r(173) = -.280, p < .001$. In all other cases, there was no significant correlation (see Table 4.3).

Table 4.3
Correlation Table for Classroom Teachers' Subject Matter MKT and M-COMP Growth

Classroom Teachers' MKT	N	Aimsweb M-COMP Growth	
		Correlation Coefficient	Significance (2-tailed)
All Grade Levels	485	-.269**	.000
Grade 2	123	.106	.245
Grade 3	32	-.131	.474
Grade 4	155	.094	.245
Grade 5	175	-.280**	.000

** Correlation is significant at the .01 level (2-tailed)

Relationship of MKT and Achievement in Struggling and Non-Struggling Students

In order to fully examine the relationship between teachers' subject matter MKT and student achievement in both struggling and non-struggling students, three analyses were run. The first analysis used was moderator multiple regression, a type of hierarchical regression used to find out if one independent variable (IV) has a stronger relationship with the dependent variable (DV) of one group of subjects than another (Tabachnick & Fidell, 2001; Morrow, 2015b). In this analysis, the DV was student achievement as measured by Aimsweb growth scores. One of the IV's was classroom teachers' subject matter MKT score, another IV was a categorical variable indicating whether a student is struggling or non-struggling based on whether or not they had received tiered interventions, and a third IV is an interaction variable created by multiplying the first two IV's together. The hierarchical multiple regression allows the IV's to be entered into regression analysis in a particular order, which makes any change in variance apparent. In this case, teachers' MKT and student status as struggling was

entered first (Model 1) followed by the interaction variable (Model 2). Thus, this analysis indicated whether the effect of classroom teachers' subject matter MKT on student growth depended on whether a student was classified as struggling or non-struggling.

The overall regression results indicated that classroom teachers' subject matter MKT and whether or not the student was classified as struggling significantly predicted student achievement, $F(2,482)=18.872$, $p < .001$; adjusted $R^2 = .069$. However, that does not mean that each individual variable significantly contributed to that relationship. Further investigation of the individual variables indicated that while there was a significant association between MKT ($b = -4.339$, $SE_b = .836$, $\beta = -.249$, $p < .001$) and student growth, there was no significant association between students' status as struggling ($b = -.112$, $SE_b = 1.606$, $\beta = -.003$, $p = .945$) and student growth or the interaction between MKT and student status as struggling ($b = -2.379$, $SE_b = 2.121$, $\beta = -.054$, $p = .263$) (see Table 4.4). In other words, the effect of teachers' subject matter MKT on student achievement was not dependent on students' status (SS or NS). Additionally, although there was a significant relationship between MKT and student achievement, the strength of the relationship was small with only 5% of the variance in student growth scores explained by teachers' subject matter MKT ($sr^2 = R_{change}^2 = (-.228)^2 = .052$) (Cohen, 1988).

Table 4.4
Hierarchical (Moderator) Multiple Regression Predicting Aimsweb Growth from
Teachers' Subject Matter MKT and Struggling Student Status

Variable	Model 1		Model 2	
	B	β	B	β
Constant	18.714		18.648	
N_TMKT				
(Teachers' MKT	-4.709	-.270**	-4.339	-.249**
centered for analysis)				
Struggling Student	-.156	-.004	-.112	-.003
Interaction				
(N_TMKT *			-2.379	-.054
Struggling Student)				

Note. $N=485$. ** $p < .001$

The final two analyses focus on the relationship between both classroom teachers' and interventionists' subject matter MKT and struggling students' M-COMP growth scores. To begin, Pearson's bivariate correlations were run to determine the direction and strength of the relationship between struggling students' M-COMP growth scores and classroom teachers' subject matter MKT as well interventionists' subject matter MKT. When including just struggling students, the mean for the corresponding classroom teachers' MKT scores was .324 (SD=.767), for corresponding interventionists' MKT scores was 1.53 (SD=.684) and the mean M-COMP growth score was 18.612 (SD=14.41). Among the students classified as struggling, there was a moderate negative correlation between teachers' subject matter MKT and student achievement, $r(67) = -.358$, $p < .01$, but there was no statistically significant correlation between interventionists' subject matter MKT and student achievement, $r(63) = .116$, $p = .366$ (see Table 4.5) (Cohen, 1988).

Table 4.5

Correlation Table for Teachers' and Interventionists' Subject Matter MKT and M-COMP Growth in Struggling Students

	Aimsweb M-COMP Growth		
	N	Correlation Coefficient	Significance (2-tailed)
Teachers' MKT	67	-.358	.003
Interventionists' MKT	63	.116	.366

In other words, although the interventionists' subject matter MKT was not related to the achievement of their SS, the classroom teachers' subject matter MKT was negatively correlated, meaning that higher subject matter MKT in classroom teachers may result in lower achievement for SS. Multiple regression was used to determine if classroom teachers' and interventionists' subject matter MKT could be used to predict struggling students' M-COMP growth scores as well as to explain the proportion of variance in struggling students' growth scores that can be attributed to the MKT of their teachers (see Table 4.6). Classroom teachers' and interventionists' subject matter MKT significantly predicted growth at the .05 level, $F(2,60)=4.078$, $p < .05$; adjusted $R^2 = .09$. However, the strength of this relationship was small (Cohen, 1988; Tabachnick & Fidell, 2001). In other words, although the ability of classroom teachers' and interventionists' subject matter MKT to predict growth in SS is statistically significant, the practicality of using these measures to predict growth in SS is not ideal.

Table 4.6

Multiple Regression Predicting Struggling Students' Aimsweb Growth From Classroom Teachers' and Interventionists' Subject Matter MKT

Variable	B	β
Constant	18.43	
Teachers' MKT	-6.045	-.333
Interventionists' MKT	.952	.046
R^2	.12	
F	4.078*	

Note. $N=62$. * $p<.05$

Instructional Practices and MKT

In order to illustrate differences in instructional practices by classroom teachers and interventionists with differing levels of subject matter MKT and professional training, one 30-minute lesson observation on each of 4 classroom teachers and 4 interventionists was conducted and analyzed. Classroom teachers were selected based on their MKT level and grade level taught. Two teachers in each grade band (1-3 and 4-5) were selected with one at each grade band scoring above average on the MKT and one at each grade band scoring below average (see Table 4.7). This variation in MKT levels allowed for comparison of instructional practices based on MKT score. Because Item Response Theory analysis showed the survey to be most accurate for teachers falling between one and two standard deviations below the mean, it would have been ideal to observe two teachers falling in that range, and two teachers scoring above the mean (Learning Mathematics for Teaching, 2014d). However, no teachers scored in that range, so teachers classified as “low” were chosen from among all teachers willing to be observed who scored below zero. Teachers classified as scoring “high” were chosen from among

all teachers who were willing to be observed and scored above one. Because the majority of interventionists scored above average on the survey, and not all teachers were willing to be observed during interventions, the opportunity to select interventionists for observation in the same manner as the selection of classroom teachers was limited.

Table 4.7
Classroom Teachers Selected for Observation

MKT	Classroom Teacher	MKT Score
High	Jason– Grade 2	1.02
	Alice- Grade 5	1.53
Low	Pamela- Grade 2	-0.38
	Jack- Grade 4	-0.32

Thus, I chose to observe two interventionists who are certified teachers and two who are not for comparison (see Table 4.8). In doing this, I was hoping to explore whether or not the interventionists with higher MKT scores were able to use that knowledge to compensate for their lack of formal pedagogical training. Likewise, I wondered if the certified interventionist with lower MKT would be able to compensate for that because of her formal pedagogical training.

Table 4.8
Interventionists Selected for Observation

Status	Interventionist	MKT Score
Certified	Tommy-Grade 2	2.24
	Brenda- Grade 1	0.61
Non-Certified	Marcie-Grade 4	1.52
	Paul-Grade 3	1.52

The use of a common curriculum with pre-determined instructional components by all of the classroom teachers made the analysis of the type tasks teachers use during instruction impractical. Thus, a significant amount of attention was paid to the number and type of questions asked during each lesson. The impact questioning has on student engagement and thinking makes it a critical component of instruction. Additionally, it is a component of instruction that has the potential to encourage reasoning about the mathematics despite the cognitive demand of the instructional task. The more students reason about the mathematics, the more likely they are to demonstrate understanding of those concepts in other contexts and at other times. Thus, analyzing questions based on the type of answer required of students provided insight into the development of conceptual understanding in the minds of students being facilitated by the teacher. The framework used for analyzing these questions was adapted from the framework used by Barnes (1969) for his research looking at the effects of language on student learning. The framework consists of four main categories with six subcategories divided among two of the main categories. A question was classified as belonging to a category based on the type of thinking required to answer the question. For example, Factual questions were those that asked “What?” and required students to simply recall factual knowledge from memory, while Reasoning questions asked “How?” or “Why?” requiring students to reason or think more critically to produce an answer (See *Figure 4.1* and refer to Chapter 3 for more detailed descriptions of the categories).

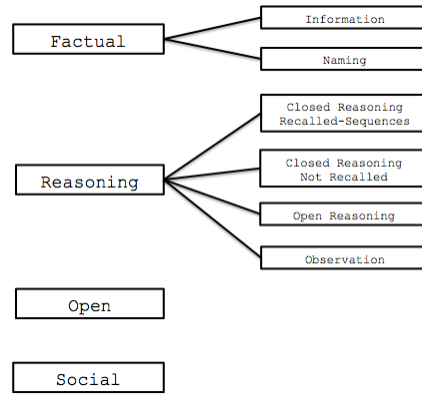


Figure 4.1. Framework for question categories. (Barnes, 1969, p.17).

Classroom teacher observations.

Each of the classroom teachers taught a lesson from the district math curriculum, *Eureka Math*. Each lesson in the curriculum includes fluency work, an application problem, concept development, problem set, and an exit ticket. Most of the classroom observations took place during the first 30 minutes of the class period. Lessons were scripted then later analyzed (see Appendix D).

Pamela

Pamela is a second-grade teacher scoring below average on the MKT (-0.38). The observation of her lesson took place a few weeks before the end of the school year. During the first 10 minutes of her lesson, students did fluency work. For this lesson, the fluency work consisted of leveled core fluency worksheets, each with 40 addition and subtraction problems to 20 at varying levels of difficulty (see Figure 4.2). The students in the class all knew which level they were on (a-e) and seemed excited to be moving toward the next level. After completing the fluency work, students moved to the carpet

area. Pamela started the discussion by asking a number of questions to determine what students remembered about a unit on measurement they had completed earlier in the year.

1.	$10 + 6 =$	21.	$3 + 8 =$
2.	$10 + 9 =$	22.	$9 + 4 =$
3.	$7 + 10 =$	23.	$\underline{\hspace{1cm}} + 6 = 11$
4.	$3 + 10 =$	24.	$\underline{\hspace{1cm}} + 9 = 13$
5.	$5 + 11 =$	25.	$8 + \underline{\hspace{1cm}} = 14$
6.	$12 + 8 =$	26.	$7 + \underline{\hspace{1cm}} = 15$

Figure 4.2. Excerpt of Core Fluency Practice Set B. (Great Minds, 2015, p. 21)

Then, she presented students with the application problem and gave them time to discuss the problem and solution with each other.

Frances is moving the furniture in her bedroom. She wants to move the bookcase to the space between her bed and the wall, but she is not sure it will fit. What could Frances use as a measurement tool if she doesn't have a ruler? How could she use it? (Great Minds, 2015, pg. 199)

After two minutes, Pamela led a whole class discussion about this problem, then went directly into the concept development portion of the lesson in which she modeled and discussed measurement with an inch tile. The observation ended as students were beginning to work on the problem sets that required them to measure several objects at their desk using an inch tile.

Jason

Jason is a second-grade teacher scoring above average on the MKT (1.02). The observation of his lesson took place about a week after Pamela's. His lesson began with a brief discussion about the lesson objective of telling time followed by the fluency work portion of the lesson. The fluency work for this lesson did not include the use of a Sprint, or fluency worksheet. Rather, it centered on renaming for a smaller unit and subtraction with renaming. Students participated in this fluency work by working problems on their marker boards then engaging in discussion about their solution strategies. The fluency portion of the lesson ended a little more than twenty minutes into the observation. Following the fluency work, Jason engaged in a discussion with students to review their knowledge about clocks and telling time, after which students were instructed to return to their desks where they began constructing their own clocks out of card stock.

Jack

Jack is a fourth-grade teacher scoring below average on the MKT (-0.32). The observation of his lesson took place in the same week as Pamela's lesson. At the school where Jack teaches, students are homogeneously grouped for math and the observed class was the "high" math group. The lesson objective was for students to use a right angle to determine if other angles were greater than, less than, or equal to a right angle, then use the right angle to generate their own obtuse, acute and right angles. Like the second-grade lessons, Jack's lesson began with fluency work. This fluency work was split into three parts, the first was using partial products to solve a three-digit by one-digit multiplication problem. Students solved the problem on individual marker boards, then

very briefly discussed the answer as a whole class. The next part of the fluency involved Jack drawing various two dimensional geometric figures on the board for students to identify followed by Jack and the class modeling these figures using body movements. Following the fluency work, Jack chose to skip over the application problem of the day in favor of immediately starting work on the concept development. For concept development, students cut out a circle, then folded the circle to form a right angle. While cutting and folding the circles, Jack led a short discussion about the fractional parts and attributes of the circle. When all the circles were cut out, he had students move around the room with a partner identifying all objects with a right angle. After 5 minutes, students returned to their desks to compare their list of objects with others at their table. The observation ended as students were beginning to draw angles on their individual marker boards.

Alice

Alice is a fifth-grade teacher scoring above average on the MKT (1.53). Crystal Lake School District is different than most districts in that fifth grade is included at the middle school level, rather than at the elementary level. Because of this, and the nature of the middle school schedule, Alice's class period is approximately half as long as her elementary school colleagues resulting in the need to alter the recommended lesson structure of *Eureka Math*. Thus, the portion of the lesson observed included students working and discussing several word problems from a problem set with the objective of multiplying fractions by non-unit fraction multipliers. During the entire 30-minute observation students worked in small groups and discussed as a whole class how to draw

the model and the equation used to find a solution to two different word problems involving the multiplication of a fraction by a fraction.

Analysis of questioning during classroom observations.

The number of questions asked by classroom teachers during each 30-minute observation ranged from 20 to 55, with the opportunity for students to reason ranging from 10 to 28 questions (see Table 4.9). By the numbers, the two teachers scoring above average on the MKT asked the most questions and gave students the most opportunities to reason about the math when answering those questions. These results may have been confounded by the fact that Pamela had less whole group instruction time because of the individual time required for implementation of the core fluency practice worksheets included with the lesson she was teaching. Thus, it is important to note the number of questions per minute of instructional time as well as the percentage of each question type asked during the lesson (see *Figure 4.3*). In order to more fully illustrate teachers' questioning with equity in whole group instruction time, the number of questions each teacher asked per minute was scaled to a ten-minute lesson. Teachers with above average MKT still asked more total questions and provided students with more opportunities to reason (see Table 4.10).

Table 4.9
Types of Questions Asked by Classroom Teachers

	Pamela-2 nd (MKT= -0.38)	Jack-4 th (MKT= -0.32)	Jason-2 nd (MKT= 1.02)	Alice-5 th (MKT= 1.53)
Factual- Information	1	2	7	1
Factual-Naming	1	1	11	1
Reasoning- Closed-Recalled Sequence	5	7	15	8
Reasoning- Closed-Non- recalled	2	0	1	0
Reasoning- Observation	1	1	2	17
Reasoning-Open	5	2	8	3
Total Reasoning Questions	13 (65%)	10 (42%)	26 (47%)	28 (85%)
Open	2	0	5	0
Shared	3	1	6	3
Total	20	24	55	33
Questions per Minutes of Whole Group Instruction	1	0.8	1.8	1.1

Table 4.10

Types and Number of questions asked, scaled for a 10-minute lesson

	Jack-4th (MKT=-0.32)	Alice-5 th (MKT=1.53)	Pamela-2 nd (MKT=-0.38)	Jason-2 nd (MKT=1.02)
Factual	4.3	0.7	1	5.9
Reasoning	3.4	9.4	6.5	8.5
Open	0	0	1	1.6
Shared	.3	1	1.5	2
Total Questions	8	11	10	18
Questions per Minutes of Whole Group Instruction	0.8	1.1	1	1.8

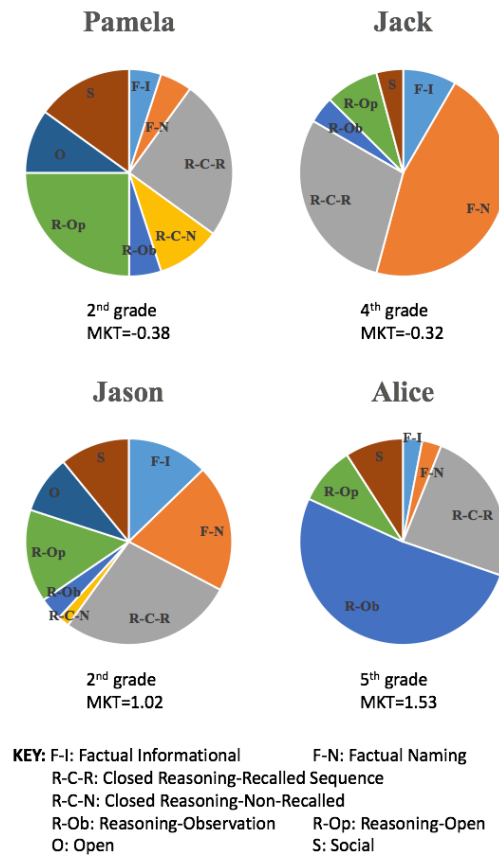


Figure 4.3. Proportion of question types asked by classroom teachers in different grade levels and with differing levels of MKT.

Interventionist observations.

Intervention time is meant to fill gaps in the skills needed for students to access grade level standards during Tier 1 (core classroom) instruction. Because of the range of needs of struggling students, and the inability of any one curricular program to meet those needs, interventionists have quite a bit of flexibility in planning the activities they will use during intervention. Despite this flexibility, the activities used for instruction can be divided into three categories, fluency work, concept building, or problem solving.

The amount of time spent on each type of activity varied among the four interventionists (see Table 4.11).

Table 4.11

Time Spent on Activity Types				
	Brenda-1 st Certified (MKT=0.61)	Tommy-4 th Certified (MKT=2.24)	Marcie-4 th Non-Cert (MKT=1.52)	Paul-3 rd Non-Cert (MKT=1.52)
Fluency	25	3	25	8
Concept Building	5	2	5	22
Problem Solving	0	25	0	0

Brenda

Brenda is a certified first grade teacher who scored just above average on the MKT survey. Although her subject matter MKT is above average, her score was quite a bit lower than the other interventionists being observed. During the observation, Brenda was providing Tier 3 (most intensive) intervention to one student. As indicated above, the majority of the intervention session was spent on fluency work. Brenda began the session with number identification flash cards followed by having the student count the number of objects on another set of cards then write the number on a marker board. The number identification and counting flash cards were followed by a timed fluency worksheet with “+2” facts. The student was given one minute to complete as many problems as he could, then Brenda and the student graphed the number correct to show that he had improved by one problem from the day before. Afterwards, the student practiced on fact sheets of “+0” and “+1.” After completing the practice worksheets, Brenda pulled out a tablet computer for the student to have more practice adding two in

the digital environment. After a couple of minutes practicing with the tablet, Brenda pulled out the number identification flash cards again followed by practice writing numbers to twenty, then extending to see if the student could write past twenty to thirty-three. Finally, the student used the tablet to work on placing numbers in the hundreds chart where he was beginning to recognize the patterns of numbers in the chart.

Tommy

Tommy is a certified teacher who scored higher than any other teacher in the district. He serves as a math specialist for one of the district's highest need schools and provides all of the Tier 3 interventions to students with the highest need. The observed intervention session included five fourth grade students who spent the majority of the thirty-minute session making sense of and solving the word problem;

To climb Mount Everest, a person climbs 66,000 feet. When the climber reached the first camp, 42,369 feet remained to climb. How far was it to the first camp? As the lesson progressed, students were asked to model the problem by drawing their own picture, then by representing it on an open number line. As students struggled to understand 42,369 feet as the space between the first camp and the top rather than the "mile marker" at the first camp, Tommy helped students understand the meaning by having them stand up and walk a number of steps, stop, then walk another three steps to get to the "top" in order to build the concept of value representing the space left to go. At another point during the lesson when it seemed that students were losing focus, he had them practice their fluency with the 4's multiplication fact by having them count by 4's

as they jumped to each number on a Math in Movement[®] counting mat. Beyond these two “breaks,” students spent the entire intervention period on the problem.

Marcie

Marcie is a non-certified para-professional who scored well above average on the subject matter MKT and serves in the role of math interventionist. In her role, she provides tiered interventions to students at different grade levels throughout the day. The lesson observed included three fourth grade students. Two of the students received tier 3 interventions, while the other received tier 2. Upon entering, the tier 2 student was asked to work on DreamBox[®], an online, adaptive computer software program for building and practicing math concepts and skills. He worked on DreamBox[®] for 20 minutes while the tier 3 students took turns listing number facts to ten on the board then building the listed fact on a ten frame with two color counters. After all facts had been listed and built, the students played a card game called “7-Up” in which the object was to collect the most pairs of cards that added up to make ten. After playing one round of the game with the tier 3 students, Marcie called all students to a giant Math in Movement[®] hundreds chart where she had students jump to the sums as they added up the points they each earned on previous assignments. Finally, the last five minutes of the observation, the tier 3 students returned to practicing numbers to ten by playing the “7-Up” card game, and Marcie worked with the tier 2 student individually on adding and subtracting double digit numbers.

Paul

Like Marcie, Paul is a non-certified para-professional who scored well above average on the subject matter MKT and serves in the role of math interventionist. In his role, he provides tiered interventions at different grade levels throughout the day. The observed tier 2 intervention served four third grade students. During the first 8 minutes of the intervention session, students worked on fluency individually using turnover cards, a matching game of sorts in which cards with answers on one side and problems on the other side are dealt with the answers face up. Instead of an answer, one card says “Start.” The reverse of the start card has the first problem. The student finds the answer to that problem then turns the card over to find the next problem. The reverse side of the last correct answer says “End.” If the end card is the last to be turned over, students know they have answered the problems correctly, if not, they can turn all of the cards over and try again. Paul supervised and provided support as students solved the turnover cards with addition facts to ten, doubles addition facts, and multiples of 2. After completing the turnover cards, the remaining 22 minutes was spent working on the concept of multiplication as students drew arrays, modeled them on an array card, and wrote repeated addition equations to model problems such as, “...five plants are planted in each of five rows. How many plants are there?”

Analysis of questioning during intervention observations.

The number of questions asked by interventionists during each 30-minute observation ranged from 15 to 47, with the opportunity for students to reason ranging from 9 to 32 questions (see Table 4.12). Because each teacher was observed for the same

amount of time and had the same opportunity to question students during the lesson, it is not necessary to scale the lessons for comparison. However, the percentage of each question type asked may provide insight into what questioning from that teacher might look like if the number of questions asked were to increase or decrease in subsequent lessons (see *Figure 4.4*).

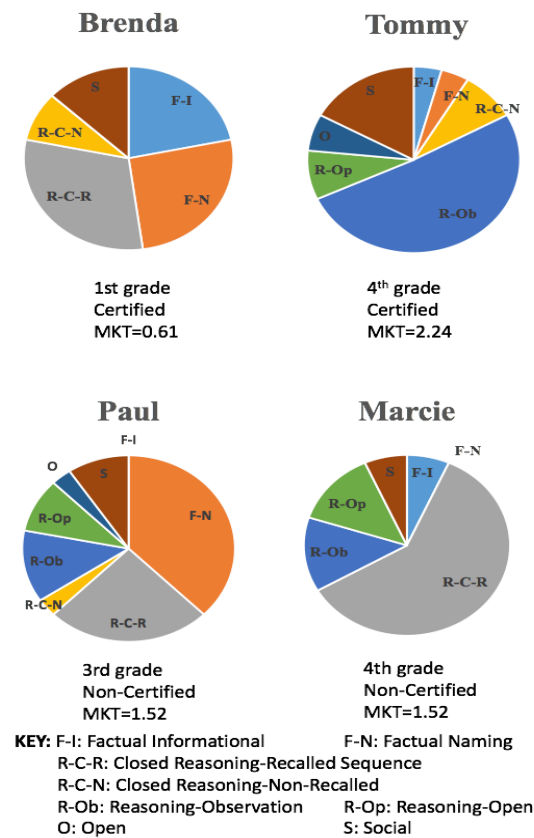


Figure 4.4. Proportion of question types asked by certified and non-certified interventionists

Table 4.12
Types of Questions Asked by Interventionists

	Brenda-1 st Certified (MKT=0.61)	Tommy-4 th Certified (MKT=2.24)	Marcie-4 th Non-Cert (MKT=1.52)	Paul-3 rd Non-Cert (MKT=1.52)
Factual- Information	5	2	1	0
Factual- Naming	6	2	0	12
Reasoning- Closed- Recalled Sequence	7	0	9	8
Reasoning- Closed-Non- recalled	2	4	0	1
Reasoning- Observation	0	24	2	4
Reasoning- Open	0	4	2	3
Open	0	3	0	1
Shared	3	8	1	3
Total	23	47	15	32
Questions per Minutes of Whole Group Instruction	0.8	1.6	0.5	1.1

Discussion

MKT and Student Achievement

When looking at the relationship between teachers' subject matter MKT and student achievement, the expected outcome was a positive correlation between the two, meaning that teachers with higher subject matter MKT were expected to have students with higher growth scores. However, the results of this study did not support that hypothesis. In fact, when a relationship was found, the correlation was negative, meaning that teachers with higher subject matter MKT generally had students with lower growth scores. Although the strength of the negative correlation was small when including all students in the analysis and moderate when restricting the analysis to only students who struggle, the results are perplexing and warrant further explanation as to what may have contributed to the outcome being different than expected.

This deviation might be best explained with a more thorough investigation of the teachers and students included in the study. The small sample size of 20 teachers and 485 related students could provide some explanation as to why the relationship between the variables (teachers' subject matter MKT and student achievement) in the study was different than expected. Among the 20 teachers who were included in the study, there was not much variance in teachers' subject matter MKT (see Table 4.2). A larger sample of teachers may have resulted in a greater variance in these scores, in addition to providing a larger sample of corresponding students. The larger sample size and greater variance would have made for a stronger statistical analysis. In addition to the small sample size, there was inequality in the number of students connected to each teacher, creating a situation

where students were disproportionately assigned to teachers scoring high on the MKT scale (See *Figure 4.5*). Part of the disproportionality stems from the fact that Crystal Lake School District is different from other districts in that 5th grade is included at the middle school level rather than at the elementary level. With only two middle schools in the district and only two math teachers at each grade level, each 5th grade math teacher is assigned a number of students than teachers at the elementary level. Thus, the inclusion in the study of three of the four 5th grade math teachers in the district resulted in the inclusion of three fourths of fifth graders in the district (not including students receiving special education services). Additionally, these three 5th grade teachers have the three highest subject matter MKT scores. Despite the high MKT scores of these teachers, there was quite a bit of variance in the growth scores of 5th grade students in the study, making it unlikely that teacher subject matter MKT adequately explains the differences in student growth scores at this grade level, thus skewing the overall results.

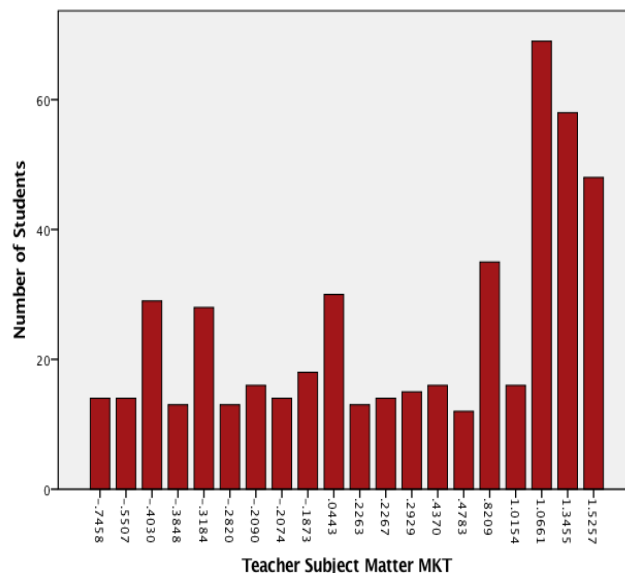


Figure 4.5. Number of students associated with each

Other potentially confounding factors present at the fourth and fifth grade levels must also be taken into consideration. One of these factors lies in the difference in the socio-economic make-up of the two middle schools. 63.61% of the students at Higgins Haven Middle School qualify for free or reduced lunch (one indicator of low socio-economic status) while only 46.26% of the students at Pinehurst Middle School qualify. Because socio-economic status is often related to student achievement (Berliner & Glass, 2014), it is expected that students at Higgins Haven Middle would struggle to achieve the same level of growth as students at Pinehurst Middle, thus, providing one explanation for the higher mean growth at Pinehurst, despite the lower subject matter MKT of that 5th grade teacher when compared with the two teachers at Higgins Haven (see Table 4.13). Another factor that may confound results when looking at the relationship between teachers' subject matter MKT and student achievement across all grade levels is the amount of time dedicated to mathematics instruction in each grade level. All students in grades 2 through 4 received ninety minutes of math instruction per day, while 5th grade students only received 45 minutes of math instruction per day. Therefore, the teachers with the highest subject matter MKT provided a much lower amount of instruction to each student, thus, providing one possible explanation for the negative correlation between MKT and student achievement. Finally, students of Forest Green Elementary were homogeneously grouped for math instruction. This was unlike the heterogeneous grouping of students for math instruction at the other three elementary schools, where each teacher was responsible for teaching students achieving at all levels. Jack, a fourth-grade teacher at Forest Green, scored well below the mean on the MKT survey, but was

responsible for teaching students who achieved in the upper 50% of students in fourth grade at Forest Green, meaning that he taught only non-struggling students who were likely to be successful regardless of the teacher. Thus, the disparity in student characteristics among teachers may have skewed the results of analyses looking at the relationship between teacher subject matter MKT and student achievement.

Table 4.13
Classroom Teacher MKT Scores by School and Grade

School	% Low SES	Grade	Classroom Teacher MKT Scores	Mean AIMSweb Growth
Jarvis Elementary	55.22	2	0.478	21
			1.015	15.58
		3	0.437	18.06
		4	0.044 0.820	17.62 25.66
Vorhees Elementary	77.89*	2	-0.550	4.12
			-0.384	19.29
			-0.282	26.46
			0.226	18.38
		4	-0.403 0.293	25.64 22.73
Forest Green Elementary	49.28	4	-0.318	25.13
Ginny Field Elementary	68.75*	2	-0.746	19.86
			-0.207	22.31
			0.227	15.62
		3	-0.209	20.81
		4	-0.187	31.06
Pinehurst Middle	46.26	5	1.066	15.54
Higgins Haven Middle	63.61	5	1.346	11.60
			1.526	8

*These schools qualify for the Community Eligibility Provision of the Healthy, Hunger Free Kids Act, which means that all students at the school receive breakfast and lunch for free. Thus, percentages could actually be higher due to parents not being required to submit forms to receive benefits. (School Meals –Community Eligibility Provision, (2016, 9/6). Retrieved from <http://www.fns.usda.gov/school-meals/community-eligibility-provision.>)

In addition to considering differences in the characteristics of students assigned to each teacher, it would be prudent to also consider what characteristics of teachers in the study would allow them to score below average on the MKT survey, but have students with above average growth scores or vice versa. One rationalization for this phenomenon might come from an examination of exactly what subject matter knowledge was assessed by the survey. The survey, which measures teachers' subject matter knowledge of the numbers and operation strand of mathematics for kindergarten through fifth grade, assesses teacher understanding of early number concepts, place value, addition and subtraction, and fraction concepts. In order for teachers to do well on the survey, they needed in depth knowledge and understanding in each of these areas. However, it could be argued that second grade teachers don't need an in depth understanding of fractions or fraction operations because it is not something they teach. What is required in second grade is an in depth understanding of early number concepts and place value in addition to an understanding of how those topics are utilized in teaching students to fluently add and subtract. An examination of second grade teachers in this study showed that of the nine second grade teachers, seven scored below the mean (.335). However, four of those seven had students with a mean Aimsweb growth score above the average of 19.22 (see Table 4.13). Thus, teachers' MKT scores might be misleading in regards to how much knowledge they have about their grade level content, as well as their ability to increase student understanding of that content. The argument concerning the appropriateness of the content of the MKT was beyond what second grade teachers might need doesn't hold when discussing third, fourth, and fifth grade teachers who are responsible for knowing

and teaching fraction concepts. Therefore, there must be some other explanation for the small to moderate negative correlation between classroom teachers' MKT and student growth.

The portion of the MKT survey used for this study measured common content knowledge and specialized content knowledge, the two categories of teachers' subject matter knowledge. Yet, the importance of pedagogical content knowledge (PCK) cannot be overstated, and its absence must be considered. In considering the importance of PCK, it's worth noting that Crystal Lake School District places an emphasis on providing its teachers with regular, high quality professional development in all content areas; thus, providing teachers the opportunity to continually grow and develop their PCK. Although having robust subject matter knowledge is fundamental to PCK, it is possible that the intentional focus on training and development of pedagogy specific to teaching grade level math content provided to teachers in this study may have increased their understanding of pedagogy for math instruction and allowed them to compensate for any subject matter knowledge they may lack.

Finally, in looking at these results, we must consider the limitations of using an assessment such as Aimsweb as a measure of student growth. As described in Chapter 3, Aimsweb is simply a test of math operations meant to place students into nationally normed percentile groups. It provides no assessment of conceptual understanding of grade level content in the minds of the students. Conversely, a high score by teachers on the subject matter MKT would indicate an in depth conceptual understanding of mathematics, the very thing necessary to facilitating the development of conceptual

understanding in the minds of students. Thus, the use of a different assessment for the measurement of student growth might provide a quite different view of the relationship between teachers' subject matter MKT and student growth. Unfortunately, the state of Tennessee has been in period of transition, and the standardized test that would have been a better measure of conceptual understanding was not administered during the period of this study.

MKT and Instructional Practice of Classroom Teachers

With less than conclusive results in regards to the relationship between teachers' subject matter MKT and student achievement, teacher observations were done in an effort to describe how teachers' MKT might be related to their instructional practices. For this study, analysis of instructional practice involved a close examination of the question types used during instruction. It was expected that teachers who scored higher on the MKT survey would ask questions that have a greater potential for facilitating the development of mathematical conceptual understanding in the minds of their students. Although the four observations completed for this study were too few for determining any kind of statistical relationship, the qualitative analysis suggests that teachers with higher MKT provided more opportunities for students to reason about mathematics. However, the similarities and differences present among the teachers at different grade levels and different MKT levels are interesting and warrant further discussion.

The quantity of reasoning questions asked by teachers with a high subject matter MKT was higher when compared to the quantity of reasoning questions asked by teachers with a low subject matter MKT at both the 1-3 and the 4-5 grade band (see Table

4.9). However, when looking at the percentage of the reasoning questions asked by each, the difference between Jack and Alice (teachers at the 4-5 grade band) is much more pronounced than the difference between Jason and Pamela (teachers at the 1-3 grade band). In fact, when comparing the percentage of reasoning questions of the two second grade teachers who were observed, Pamela, the teacher with the low subject matter MKT, asked more reasoning questions. Even so, it is worth noting that the percentage of reasoning questions Pamela asked (65%) is out of a much lower total (13 out of 20 total questions) when compared with Jason (26 out of 55 total questions). So, even though the percentage of reasoning questions Jason asked (47%) was lower, he did provide students with more reasoning questions and thus more opportunities to reason about the mathematics. In any case, the number of questions asked and the number of opportunities Pamela gave for her students to reason was more closely related to Jason and Alice, teachers with high subject matter MKT, than to Jack, who also had low subject matter MKT (see Table 4.7).

Pamela's case provides a perfect illustration of the supposition that with intentional focus on developing a deeper understanding of pedagogy for math instruction, teachers could compensate for subject matter knowledge that may be lacking. Pamela is known in the district as a model teacher. She participates in all required and optional professional development (PD) opportunities offered by the district and engages in self-study about math instruction. In addition to participating in PD and self-study, she is reflective in a way that allows her to alter her instruction based on what she has learned. Throughout the observation, it was evident that despite the limitations of the lesson

content, Pamela wanted to make sure students were thinking critically about the mathematics. They were engaged in such a way that indicated a classroom culture where student thinking and ideas are valued.

This was not unlike the cultures of both Alice's and Jason's classrooms where there was an expectation that students were responsible for providing the answers as well as the reasoning and explanation behind those answers. In these classrooms, the teachers' job was to ask questions in order to facilitate discussion and promote thinking about the underlying mathematical concepts. Additionally, the difference in the number of questions asked by each is more a function of the difference in grade levels and the complexity of grade level content than it is a function of a difference between the two teachers. Conversely, although the student disposition towards mathematics and engagement in the lesson was positive, it was clear that students in Jack's class were used to being asked questions that required them to respond with rote answers rather than explanations of their thinking about the mathematics. Like Pamela, Jack is well thought of in the district. However, although he participates in all required and some optional PD offered by the district, he is not as likely to engage in self-study or to alter his instructional practice as a result of that training. Additionally, the more advanced mathematical content that is taught in fourth grade makes it less likely that he could entirely compensate for low subject matter MKT with increased pedagogical understanding of math instruction.

Certified Status, MKT, and the Instructional Practice of Interventionists

Because schools are required to provide 30-45 minutes of intervention instruction per day to students who struggle, many schools have had to hire non-certified para-professionals to provide these interventions. Thus, when designing this study, it seemed relevant to look at interventionists as well as classroom teachers when investigating the relationship between subject matter MKT and student achievement (M-COMP growth) in struggling students. It was expected that the inclusion of specialized content knowledge on the subject matter MKT assessment would prevent those with no formal education training from scoring well. However, the opposite proved to be true, with the mean subject matter MKT score among those teachers in the study serving only as interventionists (1.53) far surpassing the mean subject matter MKT score of classroom teachers (0.335). So, in addition to attempting to describe the relationship between subject matter MKT and instructional practice during intervention sessions, the certified status of the interventionist was also considered. Of the four interventionists selected for observation, two were certified (Brenda and Tommy) and two were not (Marcie and Paul). Additionally, Brenda scored just above average on the subject matter MKT survey, while the other three scored well above average (see Table 4.13).

The two certified teachers, Brenda and Tommy, were very different from each other in both the activities they selected for the intervention (see Table 4.11) and in their questioning of students during intervention (see Table 4.12). Tommy spent the majority of the intervention period working with students on solving one word problem. Like Alice and Jason, he did none of the telling, meaning that he was able to guide students

through the process of interpreting the problem, modeling the problem, and finally, doing the calculation necessary to solve the problem through questioning. The 47 questions he asked, 32 of which gave students the opportunity to reason, made it clear that his main objective was to help students think critically and make sense of the mathematics. In addition to helping students think about the math, it was very clear that he wanted to understand how the students were thinking about the mathematics. On the other hand, Brenda spent the majority of her intervention time working with one student on a variety of fluency activities. She asked a total of 23 questions with 9 opportunities for her student to reason about the mathematics. It was evident throughout that the activities she used were very intentional and aimed at helping the student become proficient at identifying and writing numbers as well as helping him memorize math facts. However, there weren't many opportunities for the student to think conceptually about the mathematics either with the use of concrete materials or by providing a context. The differences in the two certified teachers might be explained in a variety of ways. The most obvious explanation might be that the needs of students in fourth grade—making sense of, modeling, and solving a word problem—naturally allow the teacher more opportunities to ask reasoning questions when compared with the needs of the first-grade student to develop number sense and fluency. Although this may be true, it doesn't explain the absence of visual or concrete representations that are often used as students work to develop a strong sense of number. This absence would be better explained by the level of PCK in the mind of the teacher. Although Brenda is a dedicated teacher who participates in both required and optional professional development offered by the

district, her focus as a first-grade teacher is often on increasing student literacy. Thus, although her subject matter MKT is at a sufficient level, she may be lacking some of the strategies for teaching early number concepts that come with increased PCK. On the other hand, Tommy is a certified teacher who has worked for many years as a math specialist. He has completed a number of courses aimed at increasing PCK for math instruction in addition to participating in district PD and engaging in self-study about strategies and philosophies for teaching mathematics, which has likely contributed to a high level of PCK for mathematics instruction. Thus, in combination with his very high subject matter MKT, his deep understanding of how students learn and make sense of mathematics allows him to plan activities that give opportunities for students to reason and make sense of mathematics using both concrete and abstract representations.

Likewise, the two uncertified interventionists, Marcie and Paul, were different from each other in both the activities they chose for the intervention and in their questioning of students (see Tables 4.11 and 4.12). Marcie's intervention was with three students, two of whom were at the tier 3 level and one who was at the tier 2 level. So, she split her time between the two groups. Despite this, the methods she used with each group were very similar. At the beginning of the observation, she stated that she was using a program called "Do the Math," which was written by Marilyn Burns and purchased by the district. Because the program is research based, the methods Marcie utilized during the intervention were pedagogically sound. They included concrete modeling of addition facts to 10 with counters and a ten frame for the tier 3 students, and using a diagram to model double digit addition for the tier 2 student. However, even

though the methods and activities were sound, the implementation was lacking. Much of Marcie's instruction involved her telling students what they should know about the math with very little opportunity for them to reason and make sense of it. Throughout the 30-minute observation, Marcie asked fewer questions than anyone (15), so even though the percentage of the total questions that were reasoning questions was high (87%), the quantity was only 13. Thus, unlike Pamela, who was able to compensate for her low subject matter MKT with her extensive pedagogical knowledge, Marcie was unable to compensate for her pedagogical knowledge deficit with her high subject matter MKT. On the other hand, despite the fact that their backgrounds were similar, Paul demonstrated a bit more understanding of pedagogy for math instruction. In addition to utilizing a fluency card game for students to practice combinations to ten, doubles facts, and multiples of two, Paul also used activities from the "Do the Math" program. However, during his lesson, he asked more than twice as many questions (32) when compared to Marcie and provided more opportunities to reason (16). It is unclear why there was such a difference between the two, but perhaps Paul's use of an activity from a resource other than "Do the Math," indicates that he has spent some time in self-study trying to understand how children learn math.

When comparing the certified interventionists to the non-certified interventionists, the importance of PCK in addition to subject matter MKT is glaringly apparent. This is most obvious when comparing Tommy's instruction to the other three. In providing numerous opportunities for students to reason about the mathematics, he kept them engaged and held them accountable by asking them to explain their thinking or

interpretation of the mathematics. The other three interventionists asked fewer questions than Tommy, and each provided students the opportunity to reason. However, the reasoning questions they asked were mostly closed, in which students were expected to reason about the math to give one correct answer. On the other hand, the majority of Tommy's questions were open or observation questions, in which students had the freedom to explain their own thinking or interpretation of the math. Tommy's increased pedagogical understanding is what made him aware of the types of questions he should ask in order for his students to think most critically about the mathematics, but his subject matter knowledge allowed him to continually probe deeper with regards to the mathematical content. Thus, if either pedagogical knowledge or subject matter knowledge were lacking, Tommy might have struggled to maintain productive struggle in his students as they worked to make sense of the mathematics.

Chapter Five

Conclusion and Recommendations

The National Council for Teachers of Mathematics' (NCTM), Principles and Standards for School Mathematics (2000) and the National Research Council's 2001 report Adding It Up made the complexities of students' mathematical proficiency more explicit and the challenges of helping students achieve that proficiency all the more apparent. The acknowledgement of these depictions of what it means to know and do mathematics accelerated the movement for systematic change in mathematics instructional practice. Although the movement was slow to take hold, the recent adoption of the Common Core State Standards for Mathematics (CCSSM)—including The Standards for Mathematical Practice—and the associated assessments has prioritized the need for systematic change and the urgency for a shift in paradigms about mathematics instruction among teachers and districts (Council of Chief State School Officers [CCSSO], 2010; Larson, Fennell, Adams, Dixon, Kobett, & Wray, 2012). Yet, studies by Ball (1988a) and Ma (1999) suggest that elementary teachers may need support in developing the mathematical knowledge necessary to fully realize this shift in paradigm or in developing the conceptual understanding in students required for them to become mathematically proficient. Mindful of this, the current study investigated the relationship between teachers' subject matter mathematical knowledge for teaching (MKT) and student achievement, as measured by the Aimsweb Mathematical Computation (M-COMP) assessment growth scores. In addition, this study addresses gaps in the current literature with regards to the subject matter MKT of teachers providing both classroom instruction and interventions to students identified as struggling through a response to intervention

(RtI) program. Finally, this study investigated the relationship among the instructional strategies used by classroom teachers and interventionists and their subject matter MKT.

Relationship Between Classroom Teachers' MKT and Student M-COMP Growth

The findings of the present study indicated that classroom teachers with higher levels of subject matter MKT may have lower levels of student growth. However, the statistical significance of this relationship between MKT and student growth was small, limiting its practicality. At first glance, this finding appears to be in conflict with the findings of Hill, Rowan, and Ball (2005) who found that teachers' mathematical knowledge was a significant predictor of student gains. However, Hill et al. found that this relationship was most significant when comparing the achievement of students whose teachers who scored in the lowest 20 to 30 percent of teachers taking the MKT survey to those in the top 40%. In contrast, this study had little variation in the teachers' MKT scores, as only four teachers scored high (more than one standard deviation above the mean) on the subject matter MKT survey, and no teachers scored particularly low (2 to 3 standard deviations below the mean). Additionally, Hill, Umland, Litke and Kapitula (2012) also found considerable variance in the performance of students whose teachers scored in the middle on the MKT survey. Because most of the teachers in this study also scored in the middle on the MKT survey, the findings of this study support the findings of Hill et al. (2012).

Relationship of MKT with Achievement in Struggling and Non-Struggling Students

The analysis for the second research question sought to fill a gap in the research by looking specifically at struggling students. In this study, students were identified as struggling if they were receiving interventions as part of a tiered intervention program. Although the findings indicated that while there was a small significant relationship between teachers' subject matter MKT and student growth, this relationship was not dependent on whether or not a student was classified as struggling. However, the strength of that relationship was moderate when only looking at students who struggle, meaning that it's possible that teachers with higher levels of subject matter MKT may have difficulties showing growth among their students who struggle. Although none of the previous research has looked specifically at struggling students, the finding of small to moderate correlations between MKT and student growth is in accordance with previous studies that included all students (Hill et al., 2005; Hill et al. 2012). However, even more interesting is the similarity between these findings and those of Begle (1972) and Eisenberg (1977) who also found some negative correlations between teachers' subject matter knowledge and student achievement, albeit with a different measure of subject matter knowledge. Additionally, they supposed that there must be some lower bound to teachers' subject matter knowledge, above which the significance of the relationship between teachers' subject matter knowledge and student achievement is insignificant for practical applications. Thus, the inconsequential quantitative results of this study may be explained by the existence of a lower bound to teachers' subject matter MKT, as there were no teachers that scored particularly low.

Instructional Practices and MKT

To illustrate differences in instructional practices by classroom teachers and interventionists with differing levels of subject matter MKT and professional training, the types of questions asked during one 30-minute lesson observation were analyzed. The findings of this study indicated that classroom teachers with above average subject matter MKT will likely ask more questions and provide their students with more opportunities to reason than teachers with below average subject matter MKT. This was in accordance with the findings of Hill and Charalambous (2012), that “MKT can legitimately be considered a key contributor to the mathematical quality of instruction,” (p. 560). Additionally, Charalambous, Hill and Mitchell (2012) found that one of the teachers in their study with low MKT was able to utilize curricular supports within the curriculum to compensate for her weaknesses. Likewise, this study found that one of the teachers with low subject matter MKT was able to compensate by drawing on the pedagogical content knowledge (PCK) she had gained through her participation in district provided, grade level specific professional development in math instruction as well as through her own self-study of best practices for teaching mathematics. However, although it appears that teachers may be able to compensate for lower subject matter MKT with content specific pedagogical training, the uncertified interventionists, without any formal pedagogical training, were not able to compensate for that discrepancy with their high subject matter MKT. This finding is particularly important for schools and districts as they look to hire teachers, certified or non-certified, to serve their most needy students. If districts are to hire non-certified para-professionals, they should be aware that simply knowing math is

not enough and be prepared to provide ongoing training in pedagogical practices for math instruction to help these individuals reach their students.

Implications

Although the quantitative results of this study are somewhat inconclusive, the qualitative results suggest that districts and colleges of education should place an intentional focus on increasing teachers' PCK in mathematics (Knowledge of Content and Students [KCS] and Knowledge of Content and Teaching [KCT]) rather than on increasing pure content knowledge. This is not to say that the subject matter subdomains (Common Content Knowledge [CCK] and Specialized Content Knowledge [SCK]) are unimportant, PCK relies heavily on the teacher's level of subject matter knowledge (Shulman, 1987). However, findings by Monk (1994) suggest that although the subject matter knowledge of teachers (as measured by the number of undergraduate math courses a teacher had taken) with a modest level of teacher training had positive effects on the achievement of high school juniors and seniors. The number of mathematics education courses taken by the teachers had an even greater effect on student achievement. Additionally, a recent two-year longitudinal study of K-8 teachers taking math education and science education courses indicated that as a result of these courses, teachers were able to increase their MKT, and as their MKT increased, so did the quality of their instruction, (Copur-Gencturk, 2015). It's not surprising that courses in math education allowed teachers to increase their quality of instruction, which usually leads to increased growth in their students. Unlike pure math courses that place a priority on calculations and finding solutions to complex math problems, math education courses place a priority

on how students learn math including which types of instructional activities are most effective, and what mathematical representations are most likely to help students develop conceptual understanding. In short, math education courses are focused on developing teachers' PCK, and are designed to help teachers transform their knowledge of mathematics into the content of instruction, (Shulman, 1986). Thus, by providing preservice and in-service teachers training intentionally focused on increasing teachers' PCK for mathematics instruction, districts and colleges of education could help increase the quality of mathematics instruction provided by these teachers. Likewise, providing training intentionally focused on PCK for math instruction to both certified and non-certified interventionists could increase the quality of instruction during interventions provided to struggling students as part of a response to intervention (RtI) program.

Future Research

The body of research on MKT and its effects on student achievement and instructional practice has been growing steadily since the development of the MKT construct and the tool for measuring that construct. However, this is the first study that placed an emphasis on looking at the relationship in regards to students who struggle. However, this study was limited in its scope by the low number of teacher and interventionist respondents, which contributed to the less than desirable variance in MKT scores and a low quantity of struggling students for inclusion in the study. A larger study, with an intentional focus on including a sufficient number of struggling students as well as teachers and interventionists scoring on both the low and high ends of the MKT scale would certainly shed more light on the relationship between classroom teachers'

and interventionists' MKT and student growth. Additionally, using a different assessment, such as, a standards based diagnostic assessment to more reliably show conceptual growth with at least 3 administrations, would add to the validity of the study.

Yet, the current finding that teachers may be able to compensate for a subject matter MKT deficiency with increased PCK, developed through intentionally focused professional development (PD) and training programs, underscores the importance of building on the body of research for increasing MKT and thus, the quality of instruction as a result of such programs. Studies have focused on classroom teachers' MKT growth and instructional quality as a result of PD or math education courses (Bell, Wilson, Higgins & McCoach, 2010; Copur-Gencturk, 2015), but there has been no exploration of the effects that training might have on non-certified interventionists' quality of instruction. A study of this nature is particularly timely because of the growing requirement for school systems across the country to implement response to intervention (RtI) programs, without the extra funding to do so. The requirement to implement RtI programs generally comes with intervention time recommendations that most schools have great difficulty meeting with their current staff. As a result, many are hiring non-certified para-professionals to provide these interventions. A priority must be placed on developing and testing intentionally focused training programs that will help these individuals gain the knowledge necessary to mathematically empower students who struggle.

References

- Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education*, 412-431.
- Ball, D. L. (1988a). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education* (Doctoral dissertation, Michigan State University).
- Ball, D. L. (1988b). Research on Teaching Mathematics: Making Subject Matter Knowledge Part of the Equation. East Lansing, Michigan: National Center for Research on Teacher Education.
- Ball, D.L., & Bass, H., (2000). Interweaving content and pedagogy in teaching and learning to teach: Kowing and using mathematics. In J. Boaler (ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp83f-104). Westport, CT: Ablex.
- Ball, D.L. & Bass, H. (2003) Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.). *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (p 3-14). Edmonton, Alberta, Canada: Canadian Mathematics Education Study Group.
- Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, 29, 14-22.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.

- Barnes, D. (1969). Language in the secondary classroom. *Language, the learner, and the school*, 11-76.
- Begle, E. G. (1979). Critical variables in mathematics education. *Mathematical Association of American and National Council of Teachers of Mathematics, Washington, DC*.
- Berliner, D. C., & Glass, G. V. (Eds.). (2014). *50 myths and lies that threaten America's public schools: The real crisis in education*. Teachers College Press.
- Bossé, M. J., Lee, T. D., Swinson, M., & Faulconer, J. (2010). The NCTM process standards and the five Es of science: Connecting math and science. *School Science and Mathematics*, 110(5), 262-276.
- Broke, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*. 23(3), 194-222.
- Buffum, A., Mattos, M., Weber C. (2012). *Simplifying Response to Intervention: Four essential guiding principles*. Solution Tree Press.
- Bryk, A. S., & Treisman, U. (2010). Make math a gateway, not a gatekeeper. *Chronicle of Higher Education*, 56(32), B19-B20.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. (1999). *Children's Mathematics Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.
- Charalambous, C. Y., & Hill, H. C. (2012). Teacher knowledge, curriculum materials, and quality of instruction: Unpacking a complex relationship. *Journal of*

Curriculum Studies, 44(4), 443-466.

- Charalambous, C. Y., Hill, H. C. & Mitchell, R. (2012). Two negatives don't always make a positive: Exploring how limitations in teacher knowledge and the curriculum contribute to instructional quality. *Journal of Curriculum Studies*, 44(4), 489-513.
- Clarke, B., Lembke, E. S., Hampton, D. D., & Hendricker, E. (2011). Understanding the R in RTI. In R. Gersten & R. Newman-Gonchar (Eds.), *Understanding RTI in Mathematics* (pp. 35–47). Baltimore, MA: Paul H. Brookes.
- Copeland, W. D., & Doyle, W. (1973). Laboratory skill training and student teacher classroom performance. *The Journal of Experimental Education*, 42(1), 16-21.
- Copur-Gencturk, Yasemin (2015). The effects of changes in Mathematical Knowledge on Teaching: A Longitudinal Study of Teachers' Knowledge and Instruction. *Journal for Research in Mathematics Education*, 46(3), 280-330.
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). New York, NY: Psychology Press.
- Creswell, John W. (2008). *Educational Research: Planning, conducting, and evaluating quantitative and qualitative research*. Upper Saddle River, NJ: Pearson.
- Creswell, John W. (2009). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. Thousand Oaks, CA: Sage Publications.
- Cohen, David K. (1990). A revolution in one classroom: the case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, 12(3), 311-329.
- District (2015). District RTI² Handbook
- Doyle, Walter. (1977). Paradigms for Research on Teacher Effectiveness. *Review of*

- Research in Education*, Vol. 5, 163-198.
- Forehand, M. (2010). Bloom's taxonomy. *Emerging perspectives on learning, teaching, and technology*, 41-47.
- Fuchs, L. S., & Fuchs, D. (2006). A Framework for Building Capacity for Responsiveness to Intervention. *School Psychology Review*, 35(4), 621–626.
- Fuchs, D., Fuchs, L. S., & Compton, D. L. (2012). Smart RTI: A Next-Generation Approach to Multilevel Prevention. *Exceptional Children*, 78(3), 263–279
- Gage, N. L., & Needels, M. C. (1989). Process-product research on teaching: A review of criticisms. *The elementary school journal*, 253-300.
- Good, T. L., & Grouws, D. A. (1977). Teaching Effects: A Process-Product Study in Fourth Grade Mathematics Classrooms. *Journal of Teacher Education*, 28(3), 49-54.
- Grossman, P. L., Wilson, S. M., & Shulman, L. S. (1989). Teachers of substance: Subject matter knowledge for teaching. In M. Reynolds (Ed.), *The knowledge base for beginning teachers* (pp. 23–36). New York: Pergamon.
- Great Minds. (2015a). *Eureka Math: A Story of Units. Grade 2. Module 7, Problem Solving with Length, Money and Data*. Great Minds. USA
- Grossman, Pamela L., & Richert, Anna E. (1988). Unacknowledged knowledge growth: A re-examination of the effects of teacher education. *Teaching & Teacher Education*. 4(1), 53-62.
- Hiebert, J. (2003). Signposts for teaching mathematics through problem solving. *Teaching Mathematics Through Problem Solving Prekindergarten-Grade 6*.

- Hill, H. C., Ball, D.L., & Schilling, S.G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education*, 39(4), 372-400.
- Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, 105e(1), 11-30.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American educational research journal*, 42(2), 371-406.
- Hill, H.C., & Charalambous, C.Y. (2012). Teacher knowledge, curriculum materials, and quality of instruction: Lessons learned and open issues. *Journal of Curriculum Studies*, 44(4), 443-466.
- Hill, H.C., & Charalambous, C.Y. (2012). Teaching (un)Connected Mathematis: Two teachers' enactment of the pizza problem. *Journal of Curriculum Studies*, 44(4), 467-487.
- Hill, H., Umland, K., Litke, E. & Kapitula, L. (2012) Teacher quality and quality teaching: Examining the relationship of a teacher assessment to practice. *American Journal of Education*, 118(4), 489-519
- Hunkler, R.F. (1968). Achievement of sixth grade pupils in modern mathematics as related to their teachers' math preparation. Unpublished doctoral dissertation, Texas A & M University, College Station.

IBM Corp. Released 2015. IBM SPSS Statistics for Mac, Version 23. [Computer

- software]. Arming, NY: IBM Corp.
- Iii, E. J. D., Martens, B. K., Bamett, D., Witt, J. C., & Olson, S. C. (2007). Varying Intervention Delivery in Response to Intervention : Confronting and Resolving Challenges With Measurement , Instruction , and Intensity. *School Psychology Review*, 36(4), 562–581.
- Kamii, C. (2000). *Young Children Reinvent Arithmetic: Implications of Piaget's Theory. Early Childhood Education Series*. Teachers College Press, PO Box 20, Williston, VT 05495-0020.
- Kersaint, Gladis. (2015) Orchestrating mathematical discourse to enhance student learning: Creating deeper learning by engaging students in math discussion. *District Administration*, 51(12), 2
- Laerd Statistics (2015). Statistical Tutorials and Software Guides. Retrieved from <https://statistics.laerd.com>.
- Lamon, S. J. (2012). *Teaching Fractions and Rations for Understanding; Essential Content Knowledge and Instructional Strategies for Teachers* (Third Edition ed.). New York, New York: Routledge.
- Larson, M. R., Fennell, F. (Skip), Adams, T. L., Dixon, J. K., Kobett, B. M., & Wray, J. A. (2012). *Common Core Mathematics in a PLC at Work: Grades 3 - 5*. (T. Kanold, Ed.). Bloomington, IN: Solution Tree.
- Learning Mathematics for Teaching (2014a). Mathematical Knowledge for Teaching (MKT) online training Module 1. Ann Arbor, MI: University of Michigan.
- Learning Mathematics for Teaching (2014b). Mathematical Knowledge for Teaching

- (MKT) online training Module 2. Ann Arbor, MI: University of Michigan.
- Learning Mathematics for Teaching (2014c). Mathematical Knowledge for Teaching
- (MKT) online training Module 3. Ann Arbor, MI: University of Michigan.
- Learning Mathematics for Teaching (2014d). Mathematical Knowledge for Teaching
- (MKT) online training Module 4. Ann Arbor, MI: University of Michigan.
- Learning Mathematics for Teaching (2014e). Mathematical Knowledge for Teaching
- (MKT) online training Module 5. Ann Arbor, MI: University of Michigan.
- Lembke, E. S., Hampton, D., & Beyers, S. J. (2012a). Response to intervention in mathematics: Critical elements. *Psychology in the Schools*, 49(3), 257–272.
doi:10.1002/pits.21596
- Lewis, J.M. & Blunk, M.L. (2012). Reading between the lines: Teaching linear algebra. *Journal of Curriculum Studies*, 44(4), 443-466.
- Leinardt, Gaea & Smith, Donald A. (1985). Expertise in mathematics instruction: subject matter knowledge. *Journal of Educational Psychology*, 77(3), 247-271.
- Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understandings of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of education review*, 13(2), 125-145.
- Morrow, J.A. (2015a). Lecture 3: Advanced Regression—Part 1. University of Tennessee EDPY 678, Statistics in Applied Fields III.

- Morrow, J.A. (2015b). Lecture 3: Multiple Regression—Part 2. University of Tennessee
EDPY 678, Statistics in Applied Fields III.
- Morrow, J.A. & Skolits, G. (2011). *Twelve Steps of Data Cleaning: Strategies for
Dealing with Dirty Evaluation Data*. Knoxville, TN.
- Norusis, Marija, J. (2011). PASW Statistics 19: Statistical Procedures Companion. Upper
Saddle River, NJ: Prentice Hall
- National Council of Teachers of Mathematics. (1980). *An agenda for action:
recommendations for school mathematics of the 1980s*. NCTM.
- National Council of Teachers of Mathematics. Commission on Standards for School
Mathematics. (1989). *Curriculum and evaluation standards for school
mathematics*. NCTM.
- National Council of Teachers of Mathematics (Ed.). (2000). *Principles and standards for
school mathematics* (Vol. 1). National Council of Teachers of.
- National Council of Teachers of Mathematics. (2006, September). Curriculum focal
points for prekindergarten through grade 8 mathematics: A quest for coherence.
NCTM.
- National Governors Association Center for Best Practices & Council of Chief State
School Officers. (2010). *Common Core State Standards for Mathematics*.
Washington, DC: Authors.
- National Research Council. (2001). *Adding It Up: Helping Children Learn Mathematics*.
(J. Kilpatrick, J. Swafford, & B. Findell, Eds.). Washington, D.C.: National
Academy Press.

- National Research Council. (2005). *How Students Learn: Mathematics in the Classroom*. Committee on *How People Learn*, A Targeted Report for Teachers, M.S. Donovan and J.D. Bransford, Editors. Division of Behavioral and Social Sciences and Education. Washington, D.C: The National Academies Press.
- NCS Pearson. (2012). aimsweb: Technical Manual. Bloomington, MN. Retrieved from <http://www.aimsweb.com/wp-content/uploads/aimsweb-Technical-Manual.pdf>.
- NCS Pearson. (2012). aims web: Mathematics Computation Administration and Scoring Guide. Bloomington, MN. Retrieved from http://www.aimsweb.com/wp-content/uploads/MCOMP_Admin_Scoring-Guide_2.01.pdf.
- Pearson. (n.d.). aims web FAQs. Retrieved from <http://www.aimsweb.com/about/faqs>.
- Pilgrim, M. E. (2014). Addressing the standards for mathematical practice in a calculus class. *Mathematics Teacher*, 108(1), 52-57
- Putnam, Ralph T. (1992). Teaching the “hows” of mathematics for everyday life: a case study of a fifth-grade teacher, *The Elementary School Journal*, 93(2), 163-177.
- Riccomini, P. J., & Smith, G. W. (2011). Introduction of Response to Intervention in Mathematics. In R. Gersten & R. Newman-Gonchar (Eds.), *Understanding RTI in Mathematics* (pp. 1–16). Baltimore, MA: Paul H. Brookes.
- Riccomini, P. J., & Witzel, B. S. (2010). *Response to Intervention in Math*. Thousand Oaks, CA: Corwin Press.
- Richardson, Kathy. (2012). *How Children Learn Number Concepts: A Guide to the Critical Learning Phases*. Math Perspectives.
- Romberg, Thomas A. (1993). NCTM Standards: A rallying flag for mathematics teachers.

- Educational Leadership*, 50(5), 36-41.
- Saldaña, J. (2015). *The coding manual for qualitative researchers*. Sage.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-22.
- Simon, M. A. (1993). Elementary teachers' knowledge of division. *Journal for Research in Mathematics Education*, 24(3), 233-254.
- Simon, M. K. (2011). *Dissertation and scholarly research: Recipes for success*. Dissertation Success, LLC.
- Sleep, L. & Eskelson, S. L. (2012). MKT and curriculum materials are only part of the story: Insights from a lesson on fractions. *Journal of Curriculum Studies*, 44(4), 537-558.
- Smith, M. S., & Stein, M. K. (2011). 5 Practices for Orchestrating Productive Math Discussions. Reston, VA: National Council of Teachers of Mathematics.
- Stein, M. K., Baxter, J. A., & Leinhardt, G. (1990). Subject-matter knowledge and elementary instruction: a case from functions and graphing. *American Educational Research Journal*, 27(4), 639-663.
- Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American educational research journal*, 33(2), 455-488.

- Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, 2(1), 50-80.
- Tabachnick, B.G. & Fidell, L.S. (2001). *Using Multivariate Statistics* (4ed). Needham Heights, MA: Allyn & Bacon.
- Tennessee Department of Education. (2015). RTI² Framework: Response to Instruction & Intervention Framework. Nashville, TN. Retrieved from http://tncore.org/sites/www/Uploads/RTI_templates/RTI2_Manual_revision_1_15%20final.pdf.
- Thanheiser, E. (2009). Preservice elementary school teachers' conceptions of multidigit whole numbers. *Journal for Research in mathematics Education*, 251-281.
- Vacc, N. N. (1993). Teaching and learning mathematics through classroom discussion. *Arithmetic Teacher*, 41(4), 225-230
- Vacc, N. N. (1993). Implementing the 'professional standards for teaching mathematics': questioning in the mathematics classroom. *Arithmetic Teacher*, 41(2), 88-92.
- Van De Walle, J. A. (2007). *Elementary and Middle School Mathematics: Teaching Developmentally* 6 ed. Boston, MA: Pearson.
- Wang, J., & Goldschmidt, P. (2003). Importance of middle school mathematics on high school students' mathematics achievement. *The Journal of Educational Research*, 97(1), 3-17.
- Weiss, D. J., & Yoes, M. E. (1991). Item response theory. In *Advances in educational and*

psychological testing: Theory and applications (pp. 69-95). Springer Netherlands.

Wilson, S., Shulman, L., & Richert, A. (1987). 150 different ways of knowing:

Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring teachers' thinking* (pp. 104-123).

Appendix

APPENDIX A

Student:	Teacher:	Date:	
1 $15 + 16 =$ 	2 $18 - 14 =$ 	3 $\begin{array}{r} 5 \\ \times 3 \\ \hline \end{array}$	4 $\begin{array}{r} 576 \\ + 72 \\ \hline \end{array}$
5 $\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$	6 $\begin{array}{r} 41 \\ \times 5 \\ \hline \end{array}$	7 $\begin{array}{r} 343 \\ + 50 \\ \hline \end{array}$	8 $\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$
9 $\begin{array}{r} 696 \\ - 339 \\ \hline \end{array}$	10 $\begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$	11 $\begin{array}{r} 346 \\ + 132 \\ \hline \end{array}$	12 $\begin{array}{r} 62 \\ \times 3 \\ \hline \end{array}$
13 $\begin{array}{r} 346 \\ - 36 \\ \hline \end{array}$	14 $\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$	15 $\begin{array}{r} 340 \\ - 200 \\ \hline \end{array}$	16 $\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$
17 $3 \overline{)27}$	18 $\begin{array}{r} 9.8 \\ + 4.5 \\ \hline \end{array}$	19 $\begin{array}{r} 60 \\ \times 2 \\ \hline \end{array}$	20 $\frac{5}{9} + \frac{2}{9} =$

APPENDIX B

Study of Instructional Improvement/Learning Mathematics for Teaching Content Knowledge for Teaching Mathematics Measures (MKT measures) Released Items, 2008

ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

2. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

- a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
- b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
- c) Check to see whether 371 is divisible by any prime number less than 20.
- d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ +600 \\ \hline 875 \end{array}$

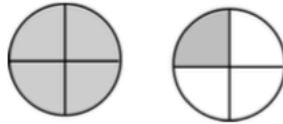
Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

4. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

- a) Four is an even number, and odd numbers are not divisible by even numbers.
- b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
- c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.
- d) It only works when the sum of the last two digits is an even number.

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



- a) $\frac{5}{4}$
- b) $\frac{5}{3}$
- c) $\frac{5}{8}$
- d) $\frac{1}{4}$

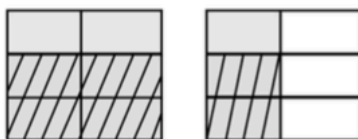
6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

A)



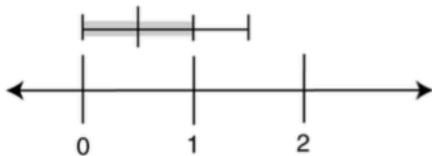
B)



C)



D)



7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have \$1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

$$\begin{array}{r} 983 \\ \times 6 \\ \hline 488 \\ + 5410 \\ \hline 5898 \end{array}$$

What is Todd doing here? (Mark ONE answer.)

- a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.
- b) Todd is using the traditional multiplication algorithm but working from left to right.
- c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.
- d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

9. Ms. James' class was investigating patterns in whole-number addition. Her students noticed that whenever they added an even number and an odd number the sum was an odd number. Ms. James asked her students to explain why this claim is true for all whole numbers.

After giving the class time to work, she asked Susan to present her explanation:

I can split the even number into two equal groups, and I can split the odd number into two equal groups with one left over. When I add them together I get an odd number, which means I can split the sum into two equal groups with one left over.

Which of the following best characterizes Susan's explanation? (Circle ONE answer.)

- a) It provides a general and efficient basis for the claim.
- b) It is correct, but it would be more efficient to examine the units digit of the sum to see if it is 1, 3, 5, 7, or 9.
- c) It only shows that the claim is true for one example, rather than establishing that it is true in general.
- d) It assumes what it is trying to show, rather than establishing why the sum is odd.

APPENDIX C

ANALYSIS PLAN

Question:	What does the data look like?
Variables:	All
Analysis:	Frequencies
Graphics:	None
Syntax:	FREQUENCIES VARIABLES=ALL /BARCHART /ORDER=ANALYSIS.

Question:	Recode Socioeconomic Status
Variables:	Socio
Analysis:	Recode
Syntax:	RECODE socio ('A'=0) ('B'=1) INTO economic. Execute.

Question:	New Variable-Struggling Student based on screener
Variables:	BOY_NP
Analysis:	recode
Syntax:	RECODE BOY_NP (Lowest THRU 24 =1) (25 THRU HIGHEST=0) INTO atrisk. EXECUTE.

Question:	Calculate Growth Score
Variables:	BOY_QC, EOY_QC
Analysis:	Compute
Syntax:	COMPUTE growth=EOY_QC-BOY_QC. VARIABLE LABELS growth 'Aimsweb Growth Score'. EXECUTE.

Question:	Calculate NP Growth
Variables:	BOY_NP, EOY_NP
Analysis:	Compute
Syntax:	COMPUTE NP_growth=EOY_NP-BOY_NP. VARIABLE LABELS NP_growth 'National Percentile Growth'. EXECUTE.

Question:	Recode categorical MKT for Teachers and Interventionists
Variables:	TeacherMKT, InterventionistMKT
Analysis:	Recode
Syntax:	RECODE TeacherMKT (0 thru HIGHEST=1) (ELSE = 0) into TMKT_Cat. EXECUTE. RECODE InterventionistMKT (0 thru HIGHEST=1) (-3 THRU - 0.99999 = 0) into IMKT_Cat. EXECUTE.

Question:	Data Cleaning step 6: Frequencies and descriptives round 2
Variables:	grade strugglingstudent boy_QC boy_NP Eoy_QC Eoy_NP TeacherMKT InterventionistMKT economic atrisk growth NP_growth TMKT_Cat IMKT_Cat
Analysis:	Frequencies, Descriptives
Syntax:	FREQUENCIES Variables=grade strugglingstudent boy_QC boy_NP Eoy_QC Eoy_NP TeacherMKT InterventionistMKT economic atrisk growth NP_growth TMKT_Cat IMKT_Cat /BARCHART /Order=ANALYSIS. DESCRIPTIVES variables=boy_QC boy_NP Eoy_QC Eoy_NP TeacherMKT InterventionistMKT growth NP_growth /save /STATISTICS=MEAN STDDEV min MAX KURTOSIS SKEWNESS.

Question:	Assess for Normality
Variables:	TeacherMKT interventionistMKT growth np_growth
Analysis:	Examine Variables
Syntax:	EXAMINE VARIABLES= TeacherMKT InterventionistMKT growth NP_Growth /PLOT BOXPLOT STEMLEAF NPLOT /COMPARE GROUPS /STATISTICS DESCRIPTIVES /CINTERVAL 95 /MISSING LISTWISE /NOTOTAL.

Question:	RQ1: Is there a relationship between classroom teachers' MKT and students' achievement as measured by the Aimsweb M-Comp growth scores?
Variables:	TeacherMKT, growth
Analysis:	Correlations, Nonpar Corr
Syntax:	CORRELATIONS /VARIABLES=TeacherMKT growth /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE. NONPAR CORR /VARIABLES=TeacherMKT growth /PRINT=SPEARMAN TWOTAIL NOSIG /MISSING=PAIRWISE.

Question:	RQ1: Is there a relationship between classroom teachers' MKT and students' achievement as measured by the Aimsweb M-Comp National percentile growth scores?
Variables:	TeacherMKT, NP_growth
Analysis:	Correlations, Nonpar Corr
Syntax:	CORRELATIONS /VARIABLES=TeacherMKT NP_growth /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE. NONPAR CORR /VARIABLES=TeacherMKT NP_growth /PRINT=SPEARMAN TWOTAIL NOSIG /MISSING=PAIRWISE.

Question:	RQ2: How is teachers' MKT related to achievement in struggling students and non-struggling students?-Checking assumptions
Variables:	Growth, TeacherMKT, Struggling Student
Analysis:	Regression
Syntax:	REGRESSION /DESCRIPTIVES MEAN STDDEV CORR SIG N /MISSING LISTWISE /STATISTICS COEFF OUTS CI(95) R ANOVA COLLIN TOL CHANGE ZPP /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT growth /METHOD=ENTER TeacherMKT StrugglingStudent /PARTIALPLOT ALL /RESIDUALS DURBIN HISTOGRAM(ZRESID) NORMPROB(ZRESID)

	/CASEWISE PLOT(ZRESID) OUTLIERS(3) /SAVE PRED COOK LEVER SRESID SDRESID.
--	---

Question:	RQ2: How is teachers' MKT related to achievement in struggling students and non-struggling students?: Moderator Regression
Variables:	Growth, TeacherMKT, StrugglingStudent, N_TMKT, int
Analysis:	Compute, regression
Syntax:	Compute N_TMKT=teacherMKT-.335224. EXECUTE. Compute int=N_TMKT * strugglingstudent. EXECUTE. REGRESSION /DESCRIPTIVES MEAN STDDEV CORR SIG N /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA COLLIN TOL CHANGE ZPP /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT GROWTH /METHOD=ENTER N_TMKT STRUGGLINGSTUDENT /METHOD=ENTER int /RESIDUALS DURBIN HISTOGRAM(ZRESID) /CASEWISE PLOT(ZRESID) OUTLIERS(3) /SAVE RESID PRED ZPRED.

Question:	RQ2: How is teachers' MKT related to achievement in struggling students and non-struggling students?: Correlations-Struggling Students Only
Variables:	InterventionistMKT, TeacherMKT, growth, NP_growth
Analysis:	Correlations
Syntax:	CORRELATIONS /VARIABLES=InterventionistMKT growth /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE. NONPAR CORR /VARIABLES=InterventionistMKT growth /PRINT=SPEARMAN TWOTAIL NOSIG /MISSING=PAIRWISE. CORRELATIONS /VARIABLES=InterventionistMKT NP_growth

	<pre> /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE. NONPAR CORR /VARIABLES=InterventionistMKT NP_growth /PRINT=SPEARMAN TWOTAIL NOSIG /MISSING=PAIRWISE. CORRELATIONS /VARIABLES=TeacherMKT growth /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE. NONPAR CORR /VARIABLES=TeacherMKT growth /PRINT=SPEARMAN TWOTAIL NOSIG /MISSING=PAIRWISE. CORRELATIONS /VARIABLES=TeacherMKT NP_growth /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE. NONPAR CORR /VARIABLES=TeacherMKT NP_growth /PRINT=SPEARMAN TWOTAIL NOSIG /MISSING=PAIRWISE. </pre>
--	---

Question:	RQ2: How is teachers' MKT related to achievement in struggling students and non-struggling students?: Regression-Struggling Students Only
Variables:	InterventionistMKT, TeacherMKT, growth, NP_growth
Analysis:	Regression
Syntax:	<pre> REGRESSION /DESCRIPTIVES MEAN STDDEV CORR SIG N /MISSING LISTWISE /STATISTICS COEFF OUTS CI(95) R ANOVA COLLIN TOL CHANGE ZPP /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT growth /METHOD=ENTER TeacherMKT InterventionistMKT /PARTIALPLOT ALL /RESIDUALS DURBIN HISTOGRAM(ZRESID) NORMPROB(ZRESID) /CASEWISE PLOT(ZRESID) OUTLIERS(3) /SAVE PRED COOK LEVER SRESID SDRESID. </pre>

APPENDIX D

CLASSROOM TEACHER LESSON TRANSCRIPTS

UL-MKT=-0.32

T-322 times 7...I'm sorry, my projector is not working, so I'm going to have to improvise.

(Students working the problem independently)

T- Give a thumbs up when you are done. O.k. so using partial product we multiply 7 times 2 is (student choral response 14) and then we have 7 times...(20) is (140) and then 7 times (300 is 2,100) and that gave us a product of (2,254) ok very good. Alright, clear your white boards.

T- Today we are going to spend some time reviewing yesterday's lesson, identifying line segments, umm line, line segments, rays and uh, angles. So what did I have...what is this? (line segment, point) What am I pointing to (point) Point what? (point A). OK point A, what do I have here? (point B) Point B together, if I point to the entire thing, the entire thing is referenced as...(line segment AB) Ok, line segment AB. Now on your white board, I want you to draw for me line segment CD. CD, good job *T walks around room checking students' work on white boards*

T-Ok , everyone has drawn line segment CD. This time I want you to draw me line EF. What does line EF look like? *T walks around room checking students' work on white boards* What was this one called? What was CD? (line segment) I said this time draw me line EF. Ok good job. Line EF. What are we missing from here? This is the line segment? We want a line here. What do we know about a line segment. Turn and talk to your neighbor, tell them the difference between a line and a line segment.

T-Clap-Class-(Yes) Ok Daniel, can you tell us the difference between a line segment and a line (a line segment has two fixed end points) alright, does everyone agree? (yes) and a (line goes on forever and doesn't stop) and what do we have to do to indicate that? (arrows) ok, we have arrows to indicate that they go one and on indefinitely in both directions.

T- This time I want you to draw me ray GH, ray GH. *T walks around room checking students' work on white boards*. O.k, you guys did an awesome job on ray GH. This time we are going to draw angle JKL, angle JKL. What is the K going to be? (the middle) The middle, which is called our (vertex) vertex ok. OK so our angle JKL, I have this or something similar. OK, so we agree that our middle point K would be (middle) OK the middle which is called the vertex ok. So, what do you notice I have up here? What does this look like from here to here? (a ray) A ray, o.k. and this looks like a (a ray) ok, so in order to form our angle we are putting two what together(rays) Ok, we are putting two rays together.

T-Alright let's stand up. Go ahead and put your top on your markers. Attention, I need everyone to do this. What do you notice about my fists? (They are balled up) So what do they represent? (points) O.k. so what am I representing? (A line segment) O.k, Let me do this. What do I still have over here? (a point) And I'm going infinitely in this direction so

I have? (A ray) If I do this, do I still have a ray? (yes) So what happened to my ray? What changed? (The direction) Ben tell us what changed (it changed what side the point is on) Ok, my point changed and my arrow is going in a different direction. Now I am going to have both of my hands extended out (a line). So we have what here? How would we represent an angle? O.k. let's go...shout it out (teacher forms points, line segments, lines, rays and angles with body and students say what the shape is).

T- So our objective today is to use right angles to determine if angles are the same larger than or smaller than a right angle. So I'm going to give you circle and you guys need some scissors... go ahead and cut out your circle. Once you get your circles cut out give me a thumbs up. Anyone have any idea what we're going to do with the circle? (fold it) alright give me five. I'm going to show you how to use the circle to measure angles, okay? We're going to take this to form a right angle. So you're going to take your circle and fold once. (it's like a taco, it's 180 degrees). How do you know it's one hundred eighty degrees? So how many degrees is the circle? You are going to fold in half again, so what do we have? So what do we know about 90° angles? (right angles) So what I want you to do is, space I want you to take your fingers cross the top where it meets when vertex and then down again. We have what type of corner there? (right angle- square corner). What I'm going to let you do, with your elbow partner and get a note card out of the basket. She said we can tie it in with fractions. This is a half of a circle and this is (a whole circle) good. We have five minutes for this next activity, you're going to walk around the room with your shoulder partner and identify everything that has a right angle.

T-Return to your seats. Why Don't you to do is take a quick look at your list, compare your list of the person in front of you. What you get out your whiteboard and draw what you see here. (Teacher draws point a, b, and c) Use a straight edge, I want you to connect this many one segments if you can't with this. How many lines segments can you connect with those three points?

UH-MKT= 1.53

Students are working on word problems in groups...the teacher is moving around the room answering questions and guiding groups

T-Thumbs up when you are looking at problem 2...we are looking at problem 2...eyes on me...we are struggling with the back side, so look at the back. When we are doing a word problem, how much of it are we going to try and do at one time? (one sentence) Looking at problem 2, Andrea read the first sentence...So, what do we know from that sentence? ($\frac{5}{8}$) Can we make a model of that? (yes) So what do I need to do? (draw a box) When you don't know what math to do, you need to model what you do know. I have a question, Is that box all I know from that first sentence? What do I need to do? (put 7 lines) Why do I need 7 lines? (so you have 8 parts) Why do I need 8 parts? What about $\frac{5}{8}$ told me I need 8 parts? O.K, so I have 8 parts is that all I know from this first sentence? (no, shade 5) Shade 5 of them, why? (cuz 5 is in the $\frac{5}{8}$) I'm done right? (no, you have to read the other sentence) Make sure you label it. What does the $\frac{5}{8}$ represent? (hip hop songs) Before we go on, can we figure anything else out about this model?

(there's still $\frac{3}{8}$ of the music player empty) So we still have $\frac{3}{8}$ remaining. Put it on your model. Have we done anything more than the first sentence? (no) So we already have lots of information from our first sentence, looking at our model, is there anything else?

T- Second sentence ($\frac{1}{3}$ of the remaining songs...) $\frac{1}{3}$ of the remaining. Are we saying that $\frac{1}{3}$ of the hip hop songs? (no, of the $\frac{3}{8}$ because that's the remaining). Oh, so I need $\frac{1}{3}$ of this. So what math do I need to do? $\frac{1}{3} \times \frac{3}{8}$. OK, do that and stand when you have an answer, I need your answer in simplest form. When everyone at your group has an answer, check to make sure everyone has an answer in simplest form. Jada, you all have said $\frac{1}{3} \times \frac{3}{8}$ is that what you did? I had several people who wrote this down. Jada what did you do to get to an answer? (1 times 3 is 3 and 3×8 is 24). Did anyone write it down differently before multiplying? ($\frac{3}{8} \times \frac{1}{3}$) Do you remember the commutative property? If I change that order, will it change my answer? Will that change my answer? (No, you can do it either way) Drake, what did you do differently? (simplified before multiplying-student explained) What is 1 copy of $\frac{1}{8}$? Instead of rewriting, he saw a 3 on top and a 3 on bottom and he knew that it was 1 whole.

T-Look at the next one, a sentence at a time, Darcy read # 3 just the first sentence ($\frac{3}{5}$ of the students in the room are girls). Draw a model, go. Give me a model for that first sentence. Can anybody tell me something that's on their model that you know, but it didn't say it in sentence 1? Something you just know, but it didn't tell you? ($\frac{2}{5}$ are boys) How did you know that? Do you have $\frac{3}{5}$ labeled as girls and $\frac{2}{5}$ labeled as boys?

T-Read just the second sentence ($\frac{1}{3}$ of the girls have blonde hair) Ok, $\frac{1}{3}$ of the girls have blonde hair. What would I do? A says, what fraction of all the students are girls with blonde hair. Does it want a number or a fraction? Do I know what fraction of the girls have blonde hair? What else do I know to try to figure out what fraction of the girls in the class have blonde hair? Dean, show your math on the board. Stand up if $\frac{3}{15}$ is a correct answer? Marcus what do we need to do? (simplifies) Dray-show what you did different.

T-Look at problem B...same model ($\frac{1}{2}$ of the boys have brown hair). Put your pencils down, I want us to think without our pencils for a minute. William can you read that again? Who can tell me which fraction of the boys do not have brown hair? Stand when you can tell me...no pencils.

So the whole class is boys? (no, $\frac{2}{5}$) So I need $\frac{1}{2}$ of $\frac{2}{5}$. Do the math. Stand when you have it.

Do I need half of this or half of this?

Jill, I haven't heard from you today, show us the math. (student puts the math on the board) Ok, she got $\frac{2}{10}$ and simplified to get $\frac{1}{5}$.

O.k. Look at problem 4. I want you to read just the first sentence. So when you draw your model, how many lawns do you need to draw? Do we know anything else from this sentence? Anna, read the next sentence (Dad tells Cody to mow $\frac{1}{4}$ of the yard) Fix your model, go. Be sure you label. Next sentence. (He tells Sam to mow $\frac{1}{3}$ of the remainder

of the yard) So Sam is going to go out and mow the yard. Is he going to mow the same section that Cody is?

LL-MKT=-.038

T-We are going to do a quick Go Noodle to get the wiggles out then we are going to do math. You have 2 minute for Core Fluency, remember for these you have two minutes. If you are on Core Fluency E....teacher passes out leveled fluency pages. Students know what level they are on and seem excited about it.

Teacher sets timer for 2 minutes, students work on problems at their desks. Teacher collects work, students excited about their progress.

Put your pencils and privacy screens away and meet me on the carpet. Alright, go to the carpet please. Alright, we are starting a new, we are going back to measurement. Who can remember when we did measurement at the beginning of the year, and we used centimeters. What did we learn about measuring? What were some of the rules of measuring that you remember? 1 foot is 12 inches. Did you remember that from using these rulers? What do you mean lined up? You have to line the 0 up with the starting point, and then what? How do I know when to stop? What if it's not exactly on a number? So I have to decide if it's $\frac{1}{2}$ or more than $\frac{1}{2}$ or less than $\frac{1}{2}$ and I can use the word "about". Let's look at the application problem and I want you to think about this problem. This is Francis's bedroom down here. She wants to move the bookcase to the space bed and the wall, but she doesn't have a ruler. What could she use to figure this out? What could she use instead and how would she do it? Now, I want you to very quietly tell the person next to you and then we are going to take a collection of ideas. What were you and Jacob talking about? Can you explain, if you were using a book what would she do to measure? So, she would use a book to measure the shelf and then she would use the same book to measure the wall? **Multiple students share ideas. Could she use, she's got a lot of different books. Could she just use a group of books? Why wouldn't that work? So, if I had laid those books down and they weren't the same size, I wouldn't have a good measurement? Now, we are going to be using inches. We are going to use a tile like this. Now we are going to use the mark and move with this one. So let's look at what happens when we measure these lines with an inch tile. Models where to start measuring line on board with tile, then models the mark and move method. As long as I am lined up with that line, am I okay? Will one more fit? What happened? It's a little bit too big, so what would we say? We have to decide about this tile, is it closer to 4 or closer to 5? What would we call it? It's closest to 5 because it is a little bit more than the half way mark. Models again.

Here's your job. I'm going to give you a tile and you are going to measure these items. I'm going to give you a piece of paper so that if you want to mark and move you can. You should have most of these objects at your seat. Students go to desks and work individually to measure objects listed on worksheet. It's very important to always include the measurement label.

LH-MKT=1.02

T-We're going to start this morning with our I can, remember last week we were learning about shapes and dividing shapes into equal parts and this week we are going to take what we learned about that to learn about telling (time) time! So let's start with our I can statement....Some big words. Tell me one that you don't know. What do those words mean? (analog) Analog, what does analog mean? (that type of clock) Thumbs up if you agree that analog means that type of clock...hmm, we're not so sure about that (no, that's a digital clock) Are you sure? You are right, that's a digital clock and the one on your computer or cell phone is a digital clock. On my computer, what does it say? (8:47 A.M.) It tells you the time in digital, just numbers. After the time it says A.M. (that means it's morning) and what does P.M. mean? (afternoon) (At morning and past morning) Is that how you remember it? O.K. Let's read our I can statement about dividing circles into equal parts. (I can divide circles and rectangles into two, three, or four equal parts, describe the shares using the words using the words half, thirds, fourths) So, today we are going to look specifically at the circle and we are going to learn a new way to talk about fourths.

T-O.k. on your boards, rename for the smaller unit, if I have One hundred and One Ten write down how many tens that is...eyes on your own board. If I have one hundred and one ten, how many tens is that? How many tens are in this number? Let's first decide how many 10's are in 100 (10). So how many 10's are in one hundred and one ten? (11) Alright, erase your boards. You are renaming for the smaller unit. I'm going to give you 2 hundreds, and that is the same as one hundred and how many tens? Write it. **student checking work and asking students to try again and reinforces question if they make a mistake** How many tens? (10) one hundred plus ten tens is, what is ten tens the same as? (100) right, so one hundred plus another hundred is two hundred. Alright, two hundred. What if I have 1 hundred, 9 tens, write how many ones I need. Alright, good job, 10 ones gives us another ten, 10 tens makes another hundred, a hundred and a hundred makes...(200). 300 equals 2 hundreds and how many tens? 3 hundreds equals 2 hundreds, 9 tens and how many ones? 4 hundreds equals 3 hundreds, how many 10's? Erase, last one 4 hundreds equals 3 hundreds, 9 tens and how many ones?

T-O.k, we are using our subtraction strategies to rename to make those friendly numbers easy for our brains to think about. Start with $132 - 118$, write that at the top of your board please. Underneath that, write it in a way that makes it easier for you to solve. Think about what you remember from when we did the tape diagram keeping those like differences. Alright, let's talk about it, if we have 132 **Models with place value disks on chart on board** and we want to take away 118, who can tell me what they did for their strategy? (I made it a friendly number, so I did $130 - 120$) Ok, how did you do that? (I took away the two from 132 and I gave it to the 118 so I could have 130 and 120) My friends who have hands up, what are you thinking? Is his strategy going to work? Why? (he didn't add the same number) He didn't keep the difference the same. When we add, we can borrow from one number and give it to the other to make a friendly number. Like $27 + 19$, instead of adding 19, we can do $26 + 20$. We just have to make one number friendly. When we do our subtraction method, if we are going to do $27 - 19$, remember the

bars, and we can put 27 in one and 19 in one. What are we going to do to 19 to make it friendly? (add one) And what are we going to do to 27? (add one) So what's our new equation? (28-20) We had to do the same thing to both numbers. Can you see that mistake on yours? Sir? (I did 132-100 to get to 32) So you took part by part, you took out place value steps, so you took out 100, then you took out a 10, then I saw that you broke up 8 into 2 and 6 so you took out 2 then you took out 6 more. Reese? (I knew that 18 was close to 20, so I added 2 to both of them so I could do 134-120). Alright, so we are going to add 2 to 132 so now we have 134, now can I take out 120? What am I left with? (14) Thumbs up if you got 14. O.k. let's try another one, 183-129. If you need to solve it another way, do that. Chip method if you need to. Look at your answer, does that make sense? Alright, let's hear some of the answers you got. (54, 54,54) Who can tell me their strategy? What was the equation that they used? (184-130) So you changed it to 184-130? Does that make it an easier problem to solve? 278-159. Thumbs up when you've got it. **to one student--When you added 2, did that make this an easy number to subtract? What could you add to make this an easy number?** O.k. what's the answer (129) thumbs up if you agree. 438-239. Look at your numbers carefully. Be smart mathematicians. Alright, look at what Jaden did. Lots of you got the correct answer, but he got it fastest. He didn't use the same strategy he's been using over and over again, he looked at the numbers. What strategy did you use? Talk to us about it, what did you notice? (I knew that if I did minus 1 to that I would have 200 and I have one left over and 200-1 is 199) O.k, so you changed your equation by making it a friendly number. He saw that 239 is really close to 238 and do we know what 38 -38 is (0). Right, so we can look at our numbers and not just use the same strategy we always use, we can think about numbers and use the best strategy for working with those numbers. 438-238 is what? (200). Did we take enough away? (No) How many do we still need to take away? (1) Do you know what 200-1 is? Can you almost do that one in your head?

T-Okay, clear off your boards, we are going to have a quick review of clocks and time. I don't really know what you did in 1st grade. So here's our clock. What are things you know or might see on a clock? (hour hand) Do you know which one is the hour hand, the big hand or the little hand? (little) The little hand is the hour hand. (Second hand) Second hand, does my clock have a second hand? (No) Does our school clock have a second hand? (Yes) What color is the second hand?(red) What else do we know about the second hand? (it's skinny and it's fast) What else do we know about clocks? (minute hand) That's right, the minute hand it's the (longest) that's right the minute hand is the longest hand. Paige, what do you know? (That if the minute hand is pointing at the 12 it's o'clock) Okay, if the minute hand is pointing at the 12, we say the words o'clock. Becca? (sometimes it goes by 5's) What goes by 5? (the time) So we can count by 5, each of the numbers is an interval of 5. What do the little lines go by? (seconds, ones) We have a little difference of opinion. When you say seconds, what do you mean? How many seconds? 9(it means one) Okay, so the little lines mean one because in between we have 1,2,3,4, (5), 6,7,8,9,(10). Anything else we know? (When the minute hand goes all the way around the clock, it's been one hour). O.k. so when the minute hand travels all the way around the clock, it's been one hour. Does anyone know what the front of the

clock it's called? (the face) You said when the minute hand travels all the way around the clock it's been one hour, does anyone know how many minutes that is? (60) 60 minutes in an hour.

T-We're going to go make a clock. Leave your marker and board, go to your desk and get out scissors. Wait for directions before your start to cut please. You'll see that our clocks are missing some important information, we are going to fix that today. Do clocks have to have numbers? (No) Some fancy watches don't. Some of them have lines, some of them just have a dot at the top and you have to learn where on the clock those numbers might be. Okay, your first job is take your piece of cardstock and cut carefully right on the dotted line. There will only be one scrap. Go carefully, cut on the dotted line.

INTERVENTIONIST LESSON TRANSCRIPTS

3rd-Interventionist-MKT=1.52 (Not certified)

As students enter, they are given a set of turnover cards to practice addition with numbers to 10. When the first set has been completed, they are given a different set of cards with doubles addition facts, then a third set with multiples of 2.

T-What cards do you have left? Which one could it be? That one was correct, but you did something else wrong because you had several cards left. Let's try again to see what you are doing. How did you do?

T-So, before we get started, I want you to skip count by 2's (2,4,6,8...). Let's do it one more time. Today we are going to finish working on Amanda Bean, I have some worksheets that we are going to do. These worksheets are working on grouping and showing repeated addition and how repeated addition is related to addition. We are going to do two worksheets today. The first one we are going to do together and then the next one I want you to do by yourself. Since we have been talking about arrays, we are going to show those problems using arrays because it's helpful for you to see that. Let's look at this first problem, write your name on it please. On the first problem it says how many plants and it already shows you an array. How many rows do we have in that? (5) We have 5 rows. How many columns do we have? (6) So our array is going to have 5 rows and 6 columns **teacher models using a multiplication array card with clear strip that can be moved to show columns and rows** So we can look at this in two different ways. We can look at it as 1, 2,3, 4, 5, 6 groups with 5 in each group or we can look at it as 5 groups with 6 in each group. So, you can write your addition problem any way you want to. I want you to write an addition problem, repeated addition problem for this array. I want to see what you do. Go ahead and write a repeated addition for this problem. (students work independently) Solve it. **to students as they finish writing the addition sentence** All of you except Emanuel chose to use 6 groups of 5. E chose to use 5 groups of 6. Why did you choose to use 6 groups of 5? (it's easier to count) Yes, it's easier to count

by 5's. I want you to write the multiplication problem that goes with that underneath. Good job.

T-Let's look at the next one. It's not laid out as an array, so let's see if we can make an array with our next problem. It says how many knitting needles. How are we going to do that? (I don't know) Well, think about that for just a minute, how could we figure out how many knitting needles. (we can see there's two...) What were you going to say? (you can make an array with all the knitting needles) Okay, how would I do that? (you would put them in array with how many columns and rows) But that would mean I would have to count them all in the first place, I don't want to count them all. What do you think Emanuel, what could I do? (you can't count by 2's) Okay, I can count by 2's. How many people are there? (7) Okay, if I have 7 people and they each have two knitting needles, can I make an array that looks like that? ****models on array card**** Yes, because I have 1,2...7 people, and each of those 7 people has 2 knitting needles, which means I have 7 groups of (2) or 2 groups of (7). Right, I want you to write a repeated addition problem for this. After you have written the problem, you can solve it, but you have to write it first. Okay, everyone chose to do two's, so in that case, you were adding two how many times? And what was your answer? (14) So, we had 7 groups of 2 right, or, we could have done 2 groups of 7 right? Would this have been easier or would this have been easier? $(7+7)$ $7+7$ because we have been working on our doubles and you should know $7+7$ by now and it equals (14). So, I want you to write a multiplication problem for your repeated addition and answer it. How many sets of two did you have? How many sets of 7 did you have? We are going to do another sheet. This time I want you can do on your own. I'm going to give each of you an array helper so that you can see your arrays ****passes out the array cards and worksheets**** Read your first question and see if you can make an array for it. I don't want you to count the sheep eyes, I want you tell me how you make an array for that problem. So you've got 2, each sheep has 2. How many sheep? (5) So, what's your array going to look like? (5 and 5) Why 5 and 5? You have 5 sheep, but do they have 5 eyes? (No) How many eyes do they have? (2) So your array is going be 5 and (2). Make me an array with 5 and 2, I want to see you do it. Does that make sense? We have 5 sheep and each sheep has what (2) 2eyes, so does that make sense? Good, let me see the repeated addition and then multiplication. Okay, do the next one. Now write your multiplication part. How would you do that? There were 8 in all, but were there 8 in each row? (no) So does that work? How many were in each row? What did you do? How many rows do I have? How many columns do I have? So what is the repeated addition problem? How many groups of 2 do I have? How many groups of 4 do I have? So how do I write the repeated addition problem? You can write it two ways.

4th-Intervention-MKT=1.52- (not certified)

T-Hey Diego, did you see the girls coming this way? Why don't you get on Dreambox, then we'll come together in a little while. I'm going to have you use the ten grid and some counters to model some equations. Last time we were working on our worksheets and I just wanted to point out Audrey that I know you were in a little bit of a hurry, but

these all had just two addends right, we just invited 3 clowns and 4 mice. So you did great on those, but this one you actually had a third addend. If you would go ahead and correct that I would appreciate it. Katelyn, you did fine on that. I would like to remind you about equations with addends that equal 10. Katelyn, could you go to the board and write one equation with two addends that equal 10. I'm going to have you swap off and do that. So, any equation. Perfect. Then, I want you to go back and do it on your ten frame. Now Audrey, you do one. That's a good one. Now go ahead and model that on your ten frame please. Alright, Katelynn, it looks like you've got that one can you write another one on the board please. Good now go model that. (This exchange continues until all equations to ten had been written and modeled. Do you remember how we came up with our complete list of addends that equal ten? (we started with $1+9$, then...) Did we start with 1? If we put these in order, we start with $1+9$. What about this one (0). That one comes before this one doesn't it. $0 + 10 = 10$. So what was the pattern? (start from 0 and count up) So you start from 0 and you count up to 9? (10) 10 because you can always flip this one around. And the second column you start at 10 and go down to...(0). Think about that when you are trying to think of other equations. Is that different from this one right here? It is different, you started with 2 instead of 8. Go ahead and model that one. (Can you do $5+5$ twice) Well, I guess we could, but we don't. Can you think of a new one? (Can we do subtraction?) Not in this case because we are working with addends. (you could do $11-1$) Yes, there's an awful lot of them that you could do, and there's a reason we work with these numbers a lot. We want to know these pairs of numbers because it's extremely important. If we can remember that 7 and 3 will always equal 10 then we will also know that if we subtract 3 from 10, how many will we have? If we can remember these pairs of numbers so that they are just right there at the top of our head, then when we do subtraction, addition, whatever, then we can be much faster in our calculations right? Alright, can you model this one. Just like I was saying, by knowing these addends that equal 10 it makes us much quicker with that game 7-up that I showed you. So we are going to do that **play 7-up** then we are going to go back and do the number grid, then you girls are going to play 7-up while I work with Diego. So, come sit over here. I really want you two to check each other, so when your neighbor takes the 7 and 2 to make 10, what would you say? (you could do 3) Yeah, you would want to say wait a minute, 7 and 2 don't make 10. So you two want to check each other. You want to make sure that your neighbor didn't make a mistake. I'm going to put 7 up. So what are we trying to do here? (we are trying to make 10 with these 7 numbers) Right, so if you see a pair of numbers that makes 10, go ahead and take them. Now, we have 0's in this deck, so you can't just take 10, you have to wait for the zero. Good she got that one 9 and 1. Now when I put the next layer of cards down, I'm going to put them on top of the ones that are still here. I'm not just going to fill in the blanks. Okay, but you have to leave them on top. You can't yank them until I have put them all out. I don't see any more do you? Wait, remember, this is the 9. The bottom is where the line is. Good job. Alright, are you ready? You can only do two addends. So the better you know those pairs of numbers, the quicker you can get those matches. When you come back to do this together, remember to check each other.

T-We're going to go and do the grid. Diego, are you at a stopping point? Okay, let's go do the grid and this cards will be ready for you when you get back. Okay, everyone starts at zero. Zero is not on the grid, but you stand over there. Now, what I am going to do is call out the points you got on the math relay, so you can add your points. So for Diego I might say +3, so Diego ($0+3$ equals 3) Do you see what he did? He said 0 plus 3 equals 3. For every number I tell you, you start with the number you left plus the number I told you and you should end up with the number you landed on. If you want to actually make that equation first to figure out where to go, that's fine too. These numbers are pretty small. (what do we get for points?) Well, at the end of the semester, just before Christmas, we will, I'll hand out prizes. Audrey, +1, great, plus 2 ($1+2=3$) plus 2 ($3+2=5$) ***continues for various points each student has earned***

T- I need you girls to sit down at the table and play the 7-up game. Practice making 10. Diego, we've got something else to talk about. Alright, what we are going to be working on today is splitting numbers to add numbers. We are going to start with the problem $57 + 23$. Now, if you had a math sheet, this would be in the first box and they would say split that second number into 10's and ones. So 23 in tens and ones is what? So I have 2 tens which is the same as (20) and (3 ones) So what we need to do is start with our number line, alright, so 57 and first we are going to add the 10's with the hop, so where are we going so how big is our hop? We are going to add 20, so where do we end up? (77) Good, so now we are going to add our ones... $77 + (3)$ (80). If we split the number we are adding or subtracting, it makes it that much easier. Now, let's do another one. Alright on this one, $28 - 12$, where do we start (28). Start at 28, and where we end up? (student drew jump from 28-18 then from 18-16). Now in this case, you didn't have to split the ones column since it was just 2. Here's a different example. We have $36-18$. Where do we start in this problem?(36) What do we do first?(-10) And where do we end up? (26) It's easy to subtract tens right? So now we need to split the 8 into what? (6 and 2) So what do we subtract next?

1st- Intervention-MKT=0.61 (certified teacher)

T-Alright, we're going to see if you remember that number **number flash cards, student practices number id** Alright, let's try it again... **goes through flash cards again** Struggles with a few numbers. Alright, now count those for me. **objects on card** I want you to write the number for me. How many were on the card? (10) Okay, write a 10 for me. Now do this one, count them out loud... **student misses number** ...wait just a minute, what comes after this one 13,(14,...).How would you write 19? Good. Let's count out loud for just a minute. **counts to 20 together*** Good. We don't want to forget any of the numbers! Count that one for me. (11) Okay, now write that number. Can you count that one for me? Awesome. Alright, I want you to go slow and steady on this one...what comes after 14? Right, you don't want to forget 15! Try that one for me! (I already know what it is) What is it? If you already know, what is it? (10) It's not 10, try again. (counts,8) Did you think these were rows of 5? They were rows of 4. $4+4$ is 8. Great, we're going to put these away for a minute and we're going to see how fast you

can go with your plus 1's today. Oops, not +1's, +2's (cuz I moved up) Yes, you moved up. Now, yesterday you got 15. See if you can go a little bit faster today. Tell me when you are ready (ready) Go. $6+2$ is what? (8) What did you write? (6) Guess what? You got all the rest of these correct, so you got 16. ****graphs result in student folder**** So your line went up today! So tomorrow, you are going to try to get more than 16. Alright, now we are going to see what you remember on your +0's. ****student works practice sheet with plus zeros, fast**** Okay, now we are going to try the +1's. I'm going to see if you can get them in 30 seconds. Tell me when you are ready and I will set the timer. (ready) Go. Oh! You did it in 17 seconds, but look what happened right here, what is $1+9$? What is that one (10). What is $1+4$ (5). Sometimes you get in a rush and you forget that, so I want you to slow down just a little bit because you still had 12 seconds left to go. Alright, now we're going to practice your plus 2's. I just want you to do this row and this row. Good. You caught yourself there. Okay, now we are going to try some on the tablet ****digital game practicing +2, student says answer out loud before typing it**** What's that number? What's your answer? (12) What's $7+2$? (9) Remember to put the highest number in your brain? What's this number (5) plus 2 more (9) Look, ****shows fingers**** 5, 6, 7, 8, 9 so 5 and how many more would give me 9? (4) Yes, 5 and 4 give me 9 so what's five and two more? (7) right. 4 plus 2 more would be (6) Good. What's $2+2$? (4) Okay, you've got 3 more. What's $5+2$ (9, 7) Right, remember 5 and 4 more is 9. 7 plus 2 more is (9). Last one. Alright, we are going to go back to our numbers ****number recognition flash cards**** What's the trick I showed you on this one? (Nine...teen) This is the tricky one (13). Remember, look at the number in the ones place. Now, I want you write your numbers from 1-20, go slow and steady and say them while you are writing them. I'm going to put a finger space because if we put them too close together, what does that look like. (5) Now that looks like an s. Think about how to make that 5. (13) Oh you got it! You didn't let him trick you that time! What's last...how would you write 20? (0) What number do you have to write first? (2) Now I'm going to challenge you today. Let's see if you can go to 29. (that's easy) Then what comes after 29? 10, 20 (30) How would you write thirty? (three and a zero) How would you write 31? (3 and 1) How would you write 32? (3 and 2) How would you write 33 (3 and 3). Awesome! Very good job. It's almost time for my next groups. Can you erase this board for me please. Let's try one more thing. Remember I have this game on here that lets you see if you can figure out what number comes next on the number chart. We've had a little trouble with that because when we first started doing it..what number is this (26) Where would it go on the hundreds chart. (after 27) After 27? Show me.. ****touches correct spot**** Good, so you're going to touch where it goes. Good, what number is this and where does it go (in the 6 line) Look at the number up her (12) What's that number? Look at the number? Look at the number that's in the tens place (23) so it goes in the 3 row. What's this number (13) Not 13, it's got a 3 in the 10's place, let's read it (30), so where would 30 go ****T counts up from 21 pointing at numbers**** (It goes here...in the 10's) That's right, it goes in the 10's column. (15) 15, (so it goes in the 5 column). (1, so it goes in the 1 column) Awesome, what number is that...etc. There you go! Good job, I want you to go right here and get on DreamBox.

4th - Intervention-MKT=2.24

Does anyone remember what we talked about before the break? That's where we are going to pick up today, so let me erase what we talked about in the last class, because I left what we did before the break up here. (Mount Everest) You are right buddy! Grab a pencil. Here was you thinking to start with ****passes out papers**** Okay, we were looking at Mt. Everest. And where we were struggling with this was being able to make a map about what was happening in the story. Because here's what happened. Remember how I talked to you, you just pulled numbers out and started doing stuff with numbers, but you didn't quite get what the numbers stood for, did you? So, we started talking about making a map, okay. So here's what we are going to do today. We're going to go back, we're going to read the story...we're going to read out loud so I can make sure that we all catch all the words. So we want to kind of review where we were. Everybody ready? To climb...read with me...****To climb Mt. Everest, a person climbs 66,000 feet. When the climber reached the first camp, 42,369 feet remained to climb. How far was it to the first camp.**** Now, look at your buddy and tell your buddy what was happening with the Mount Everest situation. What were we trying to discover? What do you think was happening in this situation. It didn't tell how far it was to the first camp. Is that what you're saying? Do you agree with him? So, you've got your thought. Girls, what did you say over there? And when they got those more feet, where would they be?

Do you remember this picture we made? What did the dot stand for? Turn over on the back for a minute then make this visual representation about this story we were reading about climbing Mt. Everest. Remember, that's something a lot of people aspire to do and they train for a long time. They have to stay dedicated to it for a long time to reach it. So put a picture on your paper somewhere. It doesn't have to look exactly like mine, that's just what we came up with before break. This is our initial drawing right? So, look at your situation about the people climbing Mt. Everest. We had some numbers happen in this story, didn't we? Alright we had 66,000 and 42,369 and those both talk about feet. Where in the picture could we put those numbers. Where in this picture could those numbers go? What part of the picture did they match? Don't show anybody else, just put in your picture what part of the picture you think those numbers represent. ****T** moves from child to child asking questions about their thinking****** That's a good thought. That's really helping your brain create an image. Is this the top of the mountain? And here's the person climbing the mountain. Where in your picture does that amount of feet happen? Okay, now talk to your buddy about what you ended up drawing, what your numbers stood for in the picture and whether or not you are in agreement. What do you think?

Emma did something interesting. All of your pictures look very similar, but Emma did something different, so I want you to look at her representation about what was happening. Emma, can you tell them what you did? I'll hold it for you, I'll be your Vanna White. (First I put 42,369 here and 66,000 here, but then I switched them) Why did this one have to be at the top of the mountain? (more steps) Yes, because it was more steps, so it had to be at the top. Do you see how she used her picture to help herself with

the story? That really helped her brain to synthesize the information. Now, go back and read just the last sentence on this story.

Okay, so here's my question. Here's what I like that you did today that you did not do on Friday. Remember this dot right here? This dot right here, you said represented 42,369. But today, you said 42,369 was represented here. So, by going away and coming back, it made more sense to you. Now, here's the question. I agree you have it placed in the right spot, but does it represent the space from the start to the base camp or does it represent the distance from the base camp to the top. Think about it. Which one do you think? Which distance does it represent? Tell your buddy about it. **T is one student's buddy...Do you think it measures from here to here? Why do you say it measures from here to here. So what we don't know is this distance.** Let's do a show of hands. Raise your hand if you think it represents the distance from the start to the first camp. Raise your hand if you think it represents the distance from here to here.

When we looked at this last week, you all had no clear idea what it represented. When we looked at it today, instantly you all knew what it represented. What do you think caused that difference to happen that you now have a clear picture in your brain? Do you know what happened? Lots of times, ideas come when you go away and come back even if it's two weeks. Do you have a clear picture in your brain of what's happening now?

Okay, go get a marker and an eraser. You don't need anything else just a marker and eraser. Alright, in a minute, we are going to work on what the question is asking us, but right now, we are going to take this picture and we are going to try to represent it on this tool. Who remembers what this tool is called? What kind of number line is it? Open, because it doesn't have any start or end numbers and you can make them up, whatever you want them to be. Now, can it go this way? What about this way? Yes, you can slant it if that helps you think about the story. So, you know at the top of the mountain what distance. 66,000 yards? Miles? (feet) We know that the first camp was where? (42,369) The distance from here to here is 42,369. So that's not the dot, but this distance right? Now, can you represent it on this number line? Be sure you label what these numbers stand for. I see some great thinking going on. Okay, great. Alright, I want you to stop and look at everyone's work. Hold up your number line so other people can see your work. Look around. Okay, so, here's my first question. Do you see anybody's who looks like yours? Does anybody match Emma's? I think so. Put your marker down for a minute. How many of you don't have anything at the bottom yet? How many of you put 42,369 at the spot of the 1st camp. Now, does yours represent the spot of the first camp...is that what yours represents? What does that mean when he put this line from the first camp to the top then wrote the number underneath? So does the number represent the spot or this space? How many of you wrote the number at the spot of the first camp. So, Jordan, did you put your 42,369 at the spot or does it represent that distance? (spot) Which one is it really? Is 42,369 the spot of the first camp or the distance from the first camp to the top? What did it say in the story? (student reads that part of the problem) Janaya, Do you think it means this spot or this distance? Go back and read from the beginning. Okay, stop right there, so, if we climb Mt. Everest, all this

distance is how many feet? (66,000) 66,000 feet covers all this space, doesn't it? Okay, read me the next spot. (when the climber reached the first camp) Okay, put your finger on the first camp. The climber went .;aj;oj;oj and got to the first camp. (42,369 remained to climb). Okay, what does that mean? They've got left to climb, so is it this amount of space or this spot. Okay, so you agree that it's the amount of space. So if they reach this base camp and they have 42,369 feet to go...Let's do this, stand up right here, now, walk four steps toward something. Now, you to get to the top, you have 3 more steps to go. Show me those steps. Alright, did you move further when you took those 3 steps? Did those three steps represent the distance ahead of you or the spot you were standing? Alright, so we know then...that you agree that this 42,369 is this part of the journey. So, does anybody want to change their number line that it represents this space, not this spot on the number line? Did you want this quantity to represent this spot on the number line or this space? Good, but where did this distance start and end? So you want this humpy line to show that. Now, that was hard work wasn't it.

Everybody stand up. Before we go on to the next step, we are going to take a brain break and practice our skip counts to four. ***gets out math and movement mat, students practice jumping the multiples of four. Students say the numbers as they jump on them. T asks them to go back if they make a mistake on their jumping.

Now, let me hear you skip count 4's with your eyes closed. Good. Now, go back to Mt. Everest and read me the very first sentence that begins with how. What distance are you trying to find? (how much the distance is) From where to where? (from the first camp to the top) Look at your picture. Do you have a number there? (yes) Now, go back and read me the question Cameron. So are you trying to find the distance to the top? (No, from the bottom to the camp) Do you agree with him? What do you think? Do we know the distance to the first camp to the top. We know the distance from the first camp to the top. So, what letter could I use to represent that unknown distance? (m for mountain...ect.) Choose a letter that works with your brain to represent that distance. I'm going to use z, he used h, and she used m. If that's the distance from the first camp to the top, is that distance you are trying to find.

Alright, I'm going to give you a fresh sheet of paper. Now, think about how you would write this as a number sentence.

Does anyone have the same thinking as yours?
How is his thinking different than yours?

APPENDIX E

CODED QUESTIONS

Interventionists

Teacher	MKT	Question	Code 1	Code 2	Code 3
Brenda	0.61	What comes after 14?	F-I	F-N	F-I
Brenda	0.61	What's that number?	F-N	F-I	F-I
Brenda	0.61	What number do you have to write first?	F-I	F-I	F-I
Brenda	0.61	What's that number?	F-N	F-I	F-I
Brenda	0.61	What's this number?	F-N	F-I	F-I
Brenda	0.61	How many were on the card?	F-N	F-I	F-N
Brenda	0.61	6+2 is what?	F-N	F-I	F-N
Brenda	0.61	What is 1 + 9?	F-N	F-I	F-N
Brenda	0.61	What's 7 + 2?	F-N	F-I	F-N
Brenda	0.61	What's 5 and 2 more?	F-N	F-I	F-N
Brenda	0.61	What's 2 + 2?	F-N	F-I	F-N
Brenda	0.61	So, 5 and how many more would give me 9?	R-C-R	F-I	R-C-N
Brenda	0.61	Where would it go on the hundreds chart?	R-C-N	F-I	R-C-N
Brenda	0.61	What's the trick I showed you on this one?	S	F-I	R-C-R
Brenda	0.61	How would you write 20?	R-C-R	F-I	R-C-R
Brenda	0.61	Then what comes after 29?	R-C-R	F-I	R-C-R
Brenda	0.61	How would you write 30?	R-C-R	F-I	R-C-R
Brenda	0.61	How would you write 31?	R-C-R	F-I	R-C-R
Brenda	0.61	How would you write 32?	R-C-R	F-I	R-C-R
Brenda	0.61	How would you write 33	R-C-R	F-I	R-C-R
Brenda	0.61	If you already know, what is it?	O		S
Brenda	0.61	Did you think these were rows of 5?	S	S	S
Brenda	0.61	What did you write?	F-I	S	S
Paul	1.52	How many rows do we have in that?	F-N	F-I	F-N
Paul	1.52	How many columns do we have?	F-N	F-I	F-N
Paul	1.52	How many people are there?	F-N	F-I	F-N

Paul	1.52	How many sets of two did you have?	F-N	F-N	F-N
Paul	1.52	How many sets of 7 did you have?	F-N	F-N	F-N
Paul	1.52	How many sheep?	F-N	F-N	F-N
Paul	1.52	How many were in each row?	F-N		F-N
Paul	1.52	How many rows do I have?	F-N	F-I	F-N
Paul	1.52	How many rows do I have?	F-N	F-I	F-N
Paul	1.52	How many columns do I have?	F-N	F-I	F-N
Paul	1.52	How many groups of 2 do I have?	F-N	F-I	F-N
Paul	1.52	How many groups of 4 do I have?	F-N	F-I	F-N
Paul	1.52	What did you do?	O	R-C-R	O
Paul	1.52	Why 5 and 5?	R-C-N	R-C-N	R-C-N
Paul	1.52	What cards do you have left, which one could it be?	R-C-R		R-C-R
Paul	1.52	How would I do that?	R-Op	R-C-R	R-C-R
Paul	1.52	Okay, everyone chose to do two's, so in that case, you were adding how many times?	R-C-R	F-I	R-C-R
Paul	1.52	So, we had 7 groups of 2 right, could we have done 2 groups of 7?	R-C-N	R	R-C-R
Paul	1.52	What's your array going to look like?	R-C-R	F	R-C-R
Paul	1.52	How would you do that?	R-Op	R-C-R	R-C-R
Paul	1.52	So, what is the repeated addition problem?	R-C-R	R-Ob	R-C-R
Paul	1.52	How do I write the repeated addition problem?	R-C-R	R-C-R	R-C-R
Paul	1.52	Okay, if I have 7 people and they each have two knitting needles, can I make an array that looks like that?	R-Ob	F-I	R-Ob
Paul	1.52	You have 5 sheep, but do they have 5 eyes?	R-Ob	R-C-R	R-Ob
Paul	1.52	How many eyes do they have?	F-N	R-Ob	R-Ob
Paul	1.52	There were 8 in all, but were there 8 in each row?	R-Ob	R-Ob	R-Ob
Paul	1.52	Why did you choose to use 6 groups of 5?	R-Op	R-Op	R-Op
Paul	1.52	How are we going to do that? (make an array using info from a problem)	R-Op	R-Op	R-Op
Paul	1.52	Does that work?	R-C-N	R-Op	R-Op
Paul	1.52	What were you going to say?	S	S	S

Paul	1.52	What do you think Emanuel, what could I do?	R-Op	S	S
Paul	1.52	What was your answer?	S	S	S
Marcie	1.52	Did we start with 1?	F-I	F-I	F-I
Marcie	1.52	Do you remember how we came up with our complete list of addends that equal 10?	R-Op	R-C-R	R-C-R
Marcie	1.52	If we can remember that 7 and 3 will always equal 10, then we will also know that if we subtract 3 from 10 how many will we have?	F-N	R-C-R	R-C-R
Marcie	1.52	So, 23 in tens and ones is what?	R-C-R	F-I	R-C-R
Marcie	1.52	Alright, on this one, 28-12, where do we start?	R-C-R	R-C-R	R-C-R
Marcie	1.52	We have 36-18, where do we start on this problem?	R-C-R	R-C-R	R-C-R
Marcie	1.52	What do we do first?	R-C-R	F-N	R-C-R
Marcie	1.52	Where do we end up?	R-C-R		R-C-R
Marcie	1.52	So, now we need to split the 8 into what?	R-C-R	R-C-R	R-C-R
Marcie	1.52	What do we subtract next?	R-C-R	R-C-R	R-C-R
Marcie	1.52	Is that different from this one right here?	R-Ob	R-Ob	R-Ob
Marcie	1.52	I don't see any more do you?	R-Ob	F-I	R-Ob
Marcie	1.52	Can you think of a new one?	O	R-Op	R-Op
Marcie	1.52	When your neighbor takes 7 and 2 to make 10, what would you say?	R-Op	R-Ob	R-Op
Marcie	1.52	Alright, are you ready?	S	S	S
Tommy	2.24	Can it go this way?	F-I	F-I	F-I
Tommy	2.24	What about this way?	F-I	F-I	F-I
Tommy	2.24	Who remembers what this tool is called?	F-N	F-N	F-N
Tommy	2.24	What kind of number line is it?	F-N	F-N	F-N
Tommy	2.24	What do you think caused that difference to happen that you now have a clear picture in your brain?	O	R-Op	O
Tommy	2.24	Do you agree with him, what do you think?	O	S	O
Tommy	2.24	So, what letter could I use to represent the unknown distance?	O		O

Tommy	2.24	So that's not the dot but the distance right?	R-C-N	F-I	R-C-N
Tommy	2.24	Can you represent it on the number line?	S	R-C-N	R-C-N
Tommy	2.24	Alright, did you move further when you took those 3 steps?	R-C-N	R-Ob	R-C-N
Tommy	2.24	Did those three steps represent the distance ahead of you or the spot you were standing?	R-C-N	R-Ob	R-C-N
Tommy	2.24	What were we trying to discover?	R-Ob	O	R-Ob
Tommy	2.24	What do you think was happening in this situation?	R-Ob	R-C-R	R-Ob
Tommy	2.24	When they got those more feet, where would they be?	R-Ob	R-C-R	R-Ob
Tommy	2.24	Do you remember this picture we made, what did the dot stand for?	R-Ob	R-C-R	R-Ob
Tommy	2.24	We had some numbers happen in this story. Where in the picture could we put those numbers?	R-Ob	R-Ob	R-Ob
Tommy	2.24	What part of the picture did they match?	R-Ob		R-Ob
Tommy	2.24	Where in your picture does that amount of feet happen?	R-Ob	R-Ob	R-Ob
Tommy	2.24	Does it represent the space from the start to the base camp, or does it represent the distance from the start to the first camp?	R-Ob	R-C-N	R-Ob
Tommy	2.24	Which one do you think?	R-Ob	R-C-N	R-Ob
Tommy	2.24	Do you think it measures from here to here?	R-Ob	R-C-N	R-Ob
Tommy	2.24	We know that the first camp was where?	R-Ob	F-I	R-Ob
Tommy	2.24	Does yours represent the spot of the first camp...is that what yours represents?	S	R-Ob	R-Ob
Tommy	2.24	What does that mean when he put this line from the first camp to the top then wrote the number underneath?	R-Ob	R-C-R	R-Ob
Tommy	2.24	So, does that number represent the spot or the space?	R-Ob	R-C-R	R-Ob
Tommy	2.24	Jordan, did you put your 42,369 at the spot or does it represent that distance?	R-Ob	R-Ob	R-Ob
Tommy	2.24	Which one is it really, is 42,369 the spot of the first camp or the distance from the first camp to the top?	R-Ob	R-Ob	R-Ob
Tommy	2.24	What did it say in the story?	R-Ob	F-I	R-Ob

Tommy	2.24	Janaya, do you think it means the spot or the distance?	R-Ob	R-Ob	R-Ob
Tommy	2.24	So, if we climb Mt Everest, all this distance is how many feet?	R-Ob	R-Ob	R-Ob
Tommy	2.24	Okay, what does that mean?	R-Ob	R-Ob	R-Ob
Tommy	2.24	Where did this distance start and end?	R-Ob	F-I	R-Ob
Tommy	2.24	What distance are you trying to find?	R-Ob	F-I	R-Ob
Tommy	2.24	From where to where?	R-Ob	F-I	R-Ob
Tommy	2.24	So, are you trying to find the distance to the top?	R-Ob	R-Ob	R-Ob
Tommy	2.24	What do you think?	O	R-Op	R-Op
Tommy	2.24	Why did this one have to be at the top of the mountain?	R-Ob	R-C-R	R-Op
Tommy	2.24	Do you see how she used her picture to help herself with the story?	S	O	R-Op
Tommy	2.24	How is his thinking different than yours?	O	R-Op	R-Op
Tommy	2.24	Does anyone remember what we talked about before the break?	S	S	S
Tommy	2.24	Girls, what did you say over there?	S	S	S
Tommy	2.24	Do you have a clear picture in your brain of what's happening now?	S	R-Op	S
Tommy	2.24	Do you see anybody's who looks like yours?	S	S	S
Tommy	2.24	Does anybody match Emma's?	S	S	S
Tommy	2.24	How many of you don't have anything at the bottom yet?	S	S	S
Tommy	2.24	Did you want this quantity to represent this spot on the number line or this space?	S	R-C-N	S
Tommy	2.24	Does anyone have the same thinking as yours?	O		S

Classroom Teachers

Teacher	MKT	Question	Code 1	Code 2	Code 3
Jack	-0.32	How many degrees in the circle?	F-N	F-I	F-I

Jack	-0.32	What do we know about 90° angles?	O	F-I	F-I
Jack	-0.32	What do I have here?	F-N	F-I	F-N
Jack	-0.32	What was this one called...what was CD?	F-N	F-I	F-N
Jack	-0.32	This time we are going to draw angle JKL. What is K going to be?	F-N	F-I	F-N
Jack	-0.32	What do you notice I have up here? What does this look like from here to here?	F-N	F-I	F-N
Jack	-0.32	What do you notice about my fists...what do they represent?	F-N	F-I	F-N
Jack	-0.32	What do I still have over here, and I am going infinitely in this direction, so I have?	F-N	F-I	F-N
Jack	-0.32	What do we have here?	F-N	F-I	F-N
Jack	-0.32	We have what type of corner there?	F-N	F-I	F-N
Jack	-0.32	So what did I have...what is this?	F-N	F-N	F-N
Jack	-0.32	What am I pointing to?	F-N	F-N	F-N
Jack	-0.32	Point What?	F-N	F-N	F-N
Jack	-0.32	How would we represent an angle?	R-C-N	R-C-R	R-C-R
Jack	-0.32	How do you know it's one hundred eighty degrees?	R-Op	R-C-R	R-C-R
Alice	1.53	Do you remember the commutative property, if I change that order, will it change my answer?	F-I		F-I
Alice	1.53	What is 1 copy of $1/8$?	F-N	F-I	F-N
Alice	1.53	When we are doing a word problem, how much of it are we going to try and do at one time?	R-C-R	S	R-C-R
Alice	1.53	Can we make a model of that...what do I need to do?	R-C-N	R-C-R	R-C-R
Alice	1.53	What math do I need to do?	R-C-R	R-Ob	R-C-R
Alice	1.53	What would I do?	R-Ob	R-C-R	R-C-R
Alice	1.53	Why do I need 7 lines?	R-C-R	R-C-R	R-C-R
Alice	1.53	Why do I need 8 parts?	R-C-R	R-C-R	R-C-R

Alice	1.53	What about $\frac{5}{8}$ told me I need 8 parts?	R-C-R	R-C-R	R-C-R
Alice	1.53	Shade five of them why?	R-C-R	R-C-R	R-C-R
Alice	1.53	What do we know from the first sentence?	R-Ob	F-I	R-Ob
Alice	1.53	Is that box all I know from the first sentence?	R-Ob	F-I	R-Ob
Alice	1.53	Okay, so I have 8 parts, is that all I know from this first sentence?	R-Ob	F-I	R-Ob
Alice	1.53	Can we figure anything else out about this model?	R-Ob	O	R-Ob
Alice	1.53	Is there anything else we can do with the first sentence?	R-Ob	O	R-Ob
Alice	1.53	Are we saying $\frac{1}{3}$ of the hip hop songs?	R-Ob		R-Ob
Alice	1.53	Does it want a number or a fraction?	R-Ob	R-C-R	R-Ob
Alice	1.53	Do I know what fraction of the girls have blonde hair?	R-C-N	R-C-R	R-Ob
Alice	1.53	What else do I know to try to figure out what fraction of the girls in the class have blonde hair?	R-Ob	R-C-R	R-Ob
Alice	1.53	Who can tell me which fraction of the boys do not have brown hair?	R-C-N	S	R-Ob
Alice	1.53	Is the whole class boys?	R-C-N	F-I	R-Ob
Alice	1.53	Do I need half of this or half of this?	R-C-N	F-I	R-Ob
Alice	1.53	How many lawns do you need to draw?	R-Ob	R-C-R	R-Ob
Alice	1.53	Do we know anything else from this sentence?	R-Ob	F-I	R-Ob
Alice	1.53	Is Sam going to mow the same section that Cody is?	R-C-N		R-Ob
Pamela	-0.38	As long as I am lined up with that line am I okay?	F-I	R-Ob	F-I
Pamela	-0.38	What would we call it?	R-C-N	F-N	F-N
Pamela	-0.38	What did we learn about measuring?	O	F-I	O

Pamela	-0.38	What are some of the rules of measuring that you remember?	O	F-I	O
Pamela	-0.38	What if it's not exactly on a number?	R-C-N	R-C-N	R-C-N
Pamela	-0.38	We have to decide about this tile, is it closer to 4 or closer to 5?	R-C-N	R-C-N	R-C-N
Pamela	-0.38	Can you explain, if you were using a book, what would she do to measure?	R-Op	R-C-R	R-C-R
Pamela	-0.38	Could she just use a group of books?	R-Op	R-Ob	R-C-R
Pamela	-0.38	It's a little too big, so what would we say?	R-C-N	R-C-R	R-C-R
Pamela	-0.38	You have to line the 0 up with the starting point, then what?	R-C-R	R-C-R	R-C-R
Pamela	-0.38	How do I know when to stop?	R-C-R	R-C-R	R-C-R
Pamela	-0.38	Will one more fit?	R-C-N	R-Ob	R-Ob
Pamela	-0.38	What do you mean lined up?	R-Op	R-C-R	R-Op
Pamela	-0.38	What could she use to figure this out?	R-Ob	R-C-N	R-Op
Jason	1.02	Analog, what does analog mean?	F-N	R-C-R	F-I
Jason	1.02	After the time it says A.M., what does that mean?	F-N	R-C-R	F-I
Jason	1.02	And what does P.M. mean?	F-N	R-C-R	F-I
Jason	1.02	You said when the minute hand travels all the way around the clock it's been one hour, does anyone know how many minutes that is?	F-N	S	F-I
Jason	1.02	Do you know which one is the hour hand, the big hand or the little hand?	F-I	F-I	F-I
Jason	1.02	Second hand, does my clock have a second hand?	F-I	F-I	F-I
Jason	1.02	Does anyone know what the front of the clock is called?	F-N	F-N	F-I
Jason	1.02	On my computer, what does it say?	F-N	F-I	F-N
Jason	1.02	What is the answer to 132-118?		F-I	F-N
Jason	1.02	What is 183-129?		F-I	F-N

Jason	1.02	What is 278-159?		F-I	F-N
Jason	1.02	What is 438-239?		F-I	F-N
Jason	1.02	438-238 is what?		F-I	F-N
Jason	1.02	Do you know what 200-1 is?	F-N	F-I	F-N
Jason	1.02	What color is the second hand?	F-N	F-I	F-N
Jason	1.02	What goes by 5?	F-N	F-I	F-N
Jason	1.02	What do the little lines go by?	F-N	F-I	F-N
Jason	1.02	When you say seconds, what do you mean? How many seconds?	F-N	F-I	F-N
Jason	1.02	What do those words mean?	O	R-C-R	O
Jason	1.02	What are things you know or might see on a clock?	O	F-N	O
Jason	1.02	What else do we know about the second hand?	O	F-I	O
Jason	1.02	What else do we know about clocks?	O	F-I	O
Jason	1.02	Do we know anything else?	O	O	O
Jason	1.02	How many do we still need to take away?	R-C-R	F-I	R-C-N
Jason	1.02	What are we going to do to 19 to make it friendly?	R-C-R	F-I	R-C-R
Jason	1.02	What are we going to do to 27?	R-C-R	F-I	R-C-R
Jason	1.02	So, what's our new equation?	F-N	F-I	R-C-R
Jason	1.02	We are going to add 2 to 132 so now we have 134, now can I take out 120	F-I	R-C-R	R-C-R
Jason	1.02	What could you add to make this an easy number?	R-C-R	F-I	R-C-R
Jason	1.02	If you have one hundred and one ten, how many tens is that, how many tens are in this number?	R-C-R	R-C-R	R-C-R
Jason	1.02	So, how many 10's are in one hundred and one ten?	R-C-R	R-C-R	R-C-R
Jason	1.02	I'm going to give you 2 hundreds, and that is the same as one hundred and how many tens?	R-C-R	R-C-R	R-C-R
Jason	1.02	How many tens?	R-C-R	R-C-R	R-C-R

Jason	1.02	One hundred plus ten tens is, what is ten tens the same as?	R-C-R	R-C-R	R-C-R
Jason	1.02	Alright, 200, what if I have 1 hundred, 9 tens, how many ones do I need?	R-C-R	R-C-R	R-C-R
Jason	1.02	300 equals 2 hundreds and how many tens?	R-C-R	R-C-R	R-C-R
Jason	1.02	3 hundreds equals 2 hundreds, 9 tens and how many ones?	R-C-R	R-C-R	R-C-R
Jason	1.02	4 hundreds equals 3 hundreds and how many tens?	R-C-R	R-C-R	R-C-R
Jason	1.02	4 hundreds equals 3 hundreds, 9 tens, and how many ones?	R-C-R	R-C-R	R-C-R
Jack	-0.32	What does line EF look like?	R-C-R	R-C-R	R-C-R
Jack	-0.32	What are we missing from here?	R-C-R	R-C-R	R-C-R
Jack	-0.32	Can you tell us the difference between a line segment and a line?	R-C-R	R-C-R	R-C-R
Jack	-0.32	If I do this, do I still have a ray?	R-C-R	R-C-R	R-C-R
Jack	-0.32	What happened to my ray, what changed?	R-C-R	R-C-R	R-C-R
Jack	-0.32	You are going to fold in half again, so what do we have?	Op	F-I	R-Ob
Jack	-0.32	Does anyone have any idea what we're going to do with the circle?	R-Op	O	R-Op
Jack	-0.32	How many line segments can you connect with those three points?	R-Op	O	R-Op
Jack	-0.32	Does everyone agree?	S	S	S
Alice	1.53	What does the $\frac{5}{8}$ represent	R-Ob	R-Ob	R-Ob
Alice	1.53	Can anybody tell me something that's on their model that you know but it didn't say in the problem?	R-Ob	R-Ob	R-Ob
Alice	1.53	What did you do to get an answer?	R-C-R	R-Op	R-Op
Alice	1.53	Drake, what did you do differently?	R-Op	R-C-R	R-Op
Alice	1.53	How do you know that?	R-Op	R-C-N	R-Op

Alice	1.53	Did anyone write it down differently before multiplying?	O	R-Ob	S
Alice	1.53	You all have said $1/3 \times 3/8$, is that what you did?	S	S	S
Alice	1.53	Do you have $3/5$ labeled as girls and $2/5$ labeled as boys?	S	S	S
Pamela	-0.38	What could she use instead, and how would she do it?	R-Ob	R-C-N	R-Op
Pamela	-0.38	So, if I laid those books down and they weren't the same size, would I have a good measurement?	R-C-N	R-Ob	R-Op
Pamela	-0.38	Why wouldn't that work?	R-Op	R-Op	R-Op
Pamela	-0.38	Did you remember that from using these rulers?	S	F-I	S
Pamela	-0.38	So she would use a book to measure the shelf, and then use the same book to measure the wall?	S	R-Ob	S
Pamela	-0.38	What were you and Jacob talking about?	S	S	S
Jason	1.02	Look at your answer, does that make sense?	R-Ob	O	R-Ob
Jason	1.02	Did we take enough away?	R-C-N	F-I	R-Ob
Jason	1.02	What strategy did you use?	O	R-Op	R-Op
Jason	1.02	How did you do that?	O	R-C-R	R-Op
Jason	1.02	My friends who have hands up, what are you thinking, is his strategy going to work?	R-Op	O	R-Op
Jason	1.02	Why?	R-Op	R-C-N	R-Op
Jason	1.02	Who can tell me their strategy, what equation that you used?	R-Ob	S	R-Op
Jason	1.02	What strategy did you use?	O	R-Op	R-Op
Jason	1.02	Talk to us about it, what did you notice?	O	R-Op	R-Op
Jason	1.02	Do clocks have to have numbers?	R-Op	R-Ob	R-Op
Jason	1.02	Are you sure?	R-Ob	S	S
Jason	1.02	Can you see that mistake on yours?	S	O	S

Jason	1.02	So you changed it to 184-130, does that make it an easier problem to solve?	S	F-I	S
Jason	1.02	When you added 2, did that make this an easy number to subtract?	S	F-I	S
Jason	1.02	Is that how you remember it?	S	S	S
Jason	1.02	Can you almost do that one in your head?	S	S	S

Vita

Jami Garner was born and grew up in Arlington, Texas. She moved to Knoxville, Tennessee with her family after graduating high school and now lives in nearby Oak Ridge with her wife and an assortment of pets. She began her career in education as a middle school math teacher after receiving her BS in Multidisciplinary Studies from Tennessee Technological University. She later earned her MS in Math Education, PhD in Teacher Education, and an Evaluation, Statistics, and Measurement certificate from The University of Tennessee. After leaving the classroom, Jami has served as a school based math coach, a math consultant for the Tennessee Statewide Personnel Development project, a consultant for the Tennessee Academic Consultants, and is currently serving as the math coordinator for Oak Ridge Schools.