

**Over the Centerline Scaling and
Spanwise Characteristics of
Blunt-Fin- and Cylinder-Generated
Shock-Wave/Turbulent
Boundary-Layer Interactions**

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Doctor of Philosophy
Degree

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This work is dedicated to the advancement of supersonic flight; may it help in enabling new supersonic airliners and bring the world closer together.

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Abstract

An investigation of a shock-wave/turbulent boundary-layer interaction (SWBLI) generated by cylinders and blunt fins was performed. Emphasis was placed on using numerical methods, namely Reynolds-averaged Navier-Stokes (RANS) and large eddy simulations (LES), although companion wind tunnel experiments using high-speed schlieren imaging and oil-flow visualization were also performed. Scaling effects on the centerline separation distance (S) and the lambda-shock triple-point height (h_{tp}) as a function of the ratio of diameter to boundary-layer thickness (d/δ) were investigated; a new scaling law is proposed that normalizes these parameters as S/δ and h_{tp}/δ , which yielded highly-agreeable correlations for both linear and exponential best-fit curves, even when including data from the archival literature at vastly different flow conditions. Spanwise effects of the evolution of key flow features were also investigated, consisting of the separation line and bubble, pressure rise, and vortices. These flow features are most severe at the centerline and rapidly lose strength in the outboard direction. Separation was limited in the outboard extent to about $\pm 2d$ – $3d$, and the dynamic behavior of separation was identical to that on the centerline. The separation line persists as a disturbance in the pressure distribution even without flow separation, and its curvature linearized at around $\pm 6d$. Lastly, the advantages and limitations of RANS and LES as a method for gathering pertinent information on this SWBLI were described in the context of the generated data of interest to a vehicle designer.

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Nomenclature

α	Cylinder or blunt-fin angle of attack
β	Oblique shock angle
β_s	Clock-wise angle measured from the centerline to the point where the sonic line impacts on the surface
Δt	Time-step
δ	99 th -percentile boundary-layer thickness
δ^*	Incompressible displacement thickness
$\dot{q}$	Heat transfer rate
η	Cartesian-transformed wall-normal coordinate
γ	Intermittency of flow separation
γ	Ratio of specific heats
$\hat{n}$	Unit vector
κ	von Kármán constant
Λ	Cylinder or blunt-fin sweep-back angle
λ	(Centerline) Forward shock separation range
λ_1	Forward shock or separation shock

λ_2	Rearward shock or closure shock
$\mathcal{P}$	Probability function
GR	Exponential growth rate along a coordinate
Pr	Prandtl number
Re	Reynolds number
Re_τ	Friction Reynolds number
Sc	Schmidt number
St	Strouhal number
μ	Dynamic viscosity
$\overline{\phi}$	Time-averaged or mean component of the arbitrary variable ϕ
Φ	Dilatation of velocity
ϕ^*	Non-dimensional value of the arbitrary variable ϕ
ϕ^+	Non-dimensional length in mesh units of the arbitrary variable ϕ
ϕ'	Fluctuating component of the arbitrary variable ϕ
ϕ_0	Stagnation value of the arbitrary variable ϕ
ϕ_1	Value in the first cell off the wall of the arbitrary variable ϕ
ϕ_∞	Freestream value of the arbitrary variable ϕ
ϕ_c	Compressible form of the arbitrary variable ϕ
ϕ_e	Boundary-layer edge value of the arbitrary variable ϕ
ϕ_{ref}	Reference value of the arbitrary variable ϕ
ϕ_t	Turbulent form of the arbitrary variable ϕ

ϕ_w	Wall value of the arbitrary variable ϕ
ϕ_i	Einstein-indexed value of the arbitrary variable ϕ
ρ	Density
Σ	Slip-line location
σ	Standard deviation
τ	Pseudo-time term
τ	Shear stress
Θ	Azimuthal coordinate
θ	Flow deflection angle
θ	Incompressible momentum thickness
θ_s	Angle of the body surface tangent to the flow at the point where the sonic line impacts on the surface
$\tilde{\nu}$	Spalart–Allmaras turbulence working variable
$\vec{q}$	Heat flux density vector
$\vec{F}$	Inviscid flux vector
$\vec{G}$	Viscous flux vector
$\vec{Q}$	Dependent variable vector
ξ	Cartesian-transformed streamwise coordinate
ζ	Cartesian-transformed spanwise coordinate
ζ	Threshold value
A	(Centerline) Attachment location

a	Speed of sound
AL	Attachment line
B	(Centerline) Bow shock stand-off distance
C	Law of the wall log-law intercept
c_f	Coefficient of friction
c_P	Coefficient of pressure
d	Cylinder or blunt-fin diameter
e	Specific internal energy
F	Focal point or vortex core
f	Frequency
f_c	Separation shock zero-crossing frequency
G	Spectral density function
H	Incompressible shape factor
h	Cylinder or blunt-fin height
h_{tp}	Triple-point height
I	Cartesian streamwise coordinate
J	Cartesian wall-normal coordinate
J	Jacobian transformation matrix
K	Cartesian spanwise coordinate
k	Thermal conductivity
l	Blunt-fin length

L_i	Intermittent length of the forward shock separation range
M	Mach number
N	Node
N	Number of grid points
N'	Half-node
P	Pressure
q	Dynamic pressure
R	Specific gas constant
r	Radial coordinate
R^2	Coefficient of determination
S	(Centerline) Separation distance
S	Saddle
S	Sutherland constant
S'	Half-saddle
SL	Separation line
T	Separation-shock period
T	Temperature
t	Time
U	Contravariant velocity
u	Streamwise velocity
u_τ	Friction velocity

UI	Upstream influence location
v	Wall-normal velocity
w	Spanwise velocity
x	Streamwise coordinate
x_0	Streamwise distance needed to grow a flat plate boundary layer to a certain thickness
y	Wall-normal coordinate
Y_1	Skewness
Y_2	Kurtosis
z	Spanwise coordinate

Chapter 1

Introduction

1.1 Motivation

Sustained high-speed flight faces a variety of challenges that must be overcome to allow for wide-spread implementation. These range from designing efficient propulsion systems to reducing the sonic boom to developing durable, light-weight materials. The problems as a whole are without a doubt multi-disciplinary, and the aerodynamics of a high-speed vehicle strongly influence all other systems. Within aerodynamics, the remaining key challenges include laminar-turbulent transition, ablation, and shock-wave/boundary-layer interactions (SWBLI) [1]. Laminar-turbulent transition can be expected to occur on any flight vehicle, including those with natural laminar flow (NLF) designs. Ablation is generally present only in rocket nozzles and at very high Mach numbers, such as those systems defined for re-entry or for hypersonic DoD systems. As with transition, SWBLI are ubiquitous in high-speed flows, and have been selected as the focus of this study. This class of flow phenomena is geometry-dependent and thus of critical importance to vehicle design [1, 2]. The interactions generate high aerodynamic, thermal, acoustic, and structural loads that can significantly impair vehicle effectiveness or compromise re-usability and potentially cause component- or even vehicle failure [2–4]. An estimate of a generic hypersonic air-breathing vehicle predicted total failure to occur in 1 min or less due to these high loads [5]. For internal flow applications (e.g. high-speed inlets), these loads may result in flow degradation or even unstart [2, 3]. For external flow applications, control surfaces may be rendered ineffective or experience separation

may result in the package striking the vehicle. One technique to mitigate the danger of these loads is to reinforce the structure, but this results in added weight that impairs effectiveness and may push the vehicle out of its designed flight envelope. However, control surfaces are also subject to these loads, and structural reinforcements provide limited aid when the vehicle experiences loss of control. Consequently, high-speed vehicle designers are interested in *a priori* knowledge of the flowfield so that they can effectively account for the adverse effects caused by SWBLI, both internal and external. Thus, a fundamental understanding of the behavior and characteristics of these interactions is necessary.

SWBLI have been the focus of much research since the 1950s [3, 6]. A practical case that highlights their impact was the flight-test vehicle North American X-15 [7], pictured in Fig. 1.1 [8]. The hypersonic airplane conducted a total of 199 flights, and on Flight 188 it reached a top speed of Mach 6.7. During Flight 188, a dummy ramjet engine was being tested, which was mounted on the underside of the vertical tail in lieu of the lower ventral fin, as shown from a different flight in Fig. 1.2 [9]. The SWBLI generated by the dummy ramjet were so severe that they induced surface buckling, and a resulting shock/shock interaction damaged the structural integrity of the mounting pylon, which consisted of Inconel-X and was coated with a protective ablator, causing the dummy engine to be involuntarily jettisoned mid-flight [10]. A ground inspection of the pylon post-flight, shown in Fig. 1.3 [11], reveals the extent of the damage.

SWBLI are primarily classified by the geometry acting as the shock generator and the state of the incoming boundary layer (i.e., laminar, transitional, or turbulent). A number of canonical geometries have been used to study the behavior of SWBLI [3, 4, 6], representing various aspects of a high-speed vehicle. These geometries are classified as 2-D or 3-D, and closed-type or open-type [12]. For strongly-separated SWBLI, 2-D configurations are inherently closed-type in the sense that the flow recirculates in an upstream region. Example geometries for these closed-type, 2-D interactions include unswept impinging shock waves or unswept compression ramps. 3-D interactions are generally open-type in the sense that flow features are continuously swept downstream. Example geometries for open-type, 3-D interactions include swept compression ramps or sharp fins. The blunt fin and the cylinder in crossflow are both special geometries because they exhibit a closed-type, 3-D interaction



Figure 1.1: Boeing B-52 Stratofortress carrying the North American X-15 on its wing. Figure from NASA [8].



Figure 1.2: North American X-15 during a flight test. A dummy ramjet engine is carried on the underside of the vertical tail. Figure from NASA [9].



Figure 1.3: Pylon on the North American X-15 after Flight 188. The pylon carrying a dummy ramjet engine is severely damaged. Figure from Jones and Berry [11].

[12]. Such geometries are commonly found on vehicles in the form of wing-body junctions, blunted inlets, and other blunt protuberances. Moreover, these two geometries generate the same type of upstream interaction, allowing for a direct comparison and interchangeable usage [4, 13–15].

The geometries of interest in this study are both the blunt fin and the cylinder in crossflow. These geometries generate high loads on the centerline, particularly at the leading edge [4, 14, 16–18]. As a result, the vast majority of prior research has focused solely on the flow features at the centerline, while the development of features in the spanwise direction has received little attention [14]. However, in vehicle design, nearby structures will encounter the spanwise effects of this SWBLI due to its open-type nature. In order to determine the extent of the region of influence for these ubiquitous interactions, a thorough investigation of the spanwise flow characteristics is required.

1.2 Objectives of Current Work

The goal of this study is to characterize the structures and dynamic behavior of the spanwise flow features of SWBLI generated by semi-infinite blunt fins and cylinders. This also requires a thorough understanding of the flow behavior at the centerline, where the structures originate, and so a characterization of centerline scaling effects will also be performed. Computational fluid dynamics (CFD) excel at providing detailed information at any location in the flow field, and will thus be the primary method used to obtain the data. Both new, concurrently-obtained, and existing in-house-generated experimental data are available and will complement the CFD findings. The flow conditions are determined by the available wind tunnel facility, and are namely a Mach number of approximately 2 and a unit Reynolds number of approximately $3 \times 10^7 \text{ m}^{-1}$. The following objectives are set in the current work:

1. *Establish the scaling of pertinent flow features at the centerline.*

In order to quantify the spanwise characteristics of the flow features, knowledge of the scaling at their origin is necessary. A parametric study of independent flow variables will be performed to study their effects on the interaction scale.

Time-averaged simulations accompanied by newly-generated, simple experiments will be used to generate this large data set, alongside experiments from within the research group and from the archival literature to generate appropriate trends. These trends will be used as a baseline for time-accurate simulations and to determine the spanwise evolution of the flow features.

2. *Characterize the spanwise evolution of key flow features.*

The shock structure, separation bubble, and vortical structures will be inspected and traced from their origin at the centerline as they sweep downstream and outboard. An analysis will be performed to quantify characteristic frequencies, as well as determine where the flow linearizes outboard. Time-averaged and time-accurate simulations will be performed to study the behavior and strength of these features, with comparison to experimental data.

3. *Distinguish between implementation of time-averaged and time-accurate computational methods for recommended usage on these SWBLI.*

Since time-accurate simulations still require a large amount of computational resources, time-averaged simulations continue to remain the dominant simulation technique for high-Reynolds-number flows. This is particularly the case for industrial applications, where projects are far more time-sensitive than the research performed in academia. A quantitative comparison between time-averaged and time-accurate results can illustrate the respective effectiveness and could thus guide the selection for which method to use for a given variable of interest.

Chapter 2

Background and Literature Review

A general overview of the SWBLI structure is presented first, describing the key phenomena and the terminology used throughout this paper. This is followed by a more detailed inspection of the flow at the centerline, as well as prior attempts to scale the interaction based on the independent flow variables. The existing work over the off-centerline interaction is then discussed, along with a list of the research gaps.

2.1 General Interaction Components

A simple schematic of the interaction is provided in Fig. 2.1 for a cylinder, highlighting some of the most prominent features. The cylinder is represented by two geometric variables: the height, h , and the diameter, d . For a blunt fin, the diameter also represents the width, and the length, l , is a third geometric variable. Sweep-back or angle of attack, typically denoted by Λ and α , respectively, were not investigated here. For a fixed incoming boundary-layer state, the two geometric variables h [13, 14, 17–20] and d [4, 13–26] contribute the most to the scale of the interaction and are thus considered primary interaction variables. The freestream Mach number, M_∞ [13, 14, 18, 23, 26, 27], and the (99th-percentile) boundary-layer thickness, δ [14, 16, 18, 22–27], have secondary scaling effects on the interaction that contribute to the inviscid and viscous scales, respectively. These effects are influenced by the state of the incoming boundary layer and will be discussed in more detail later.

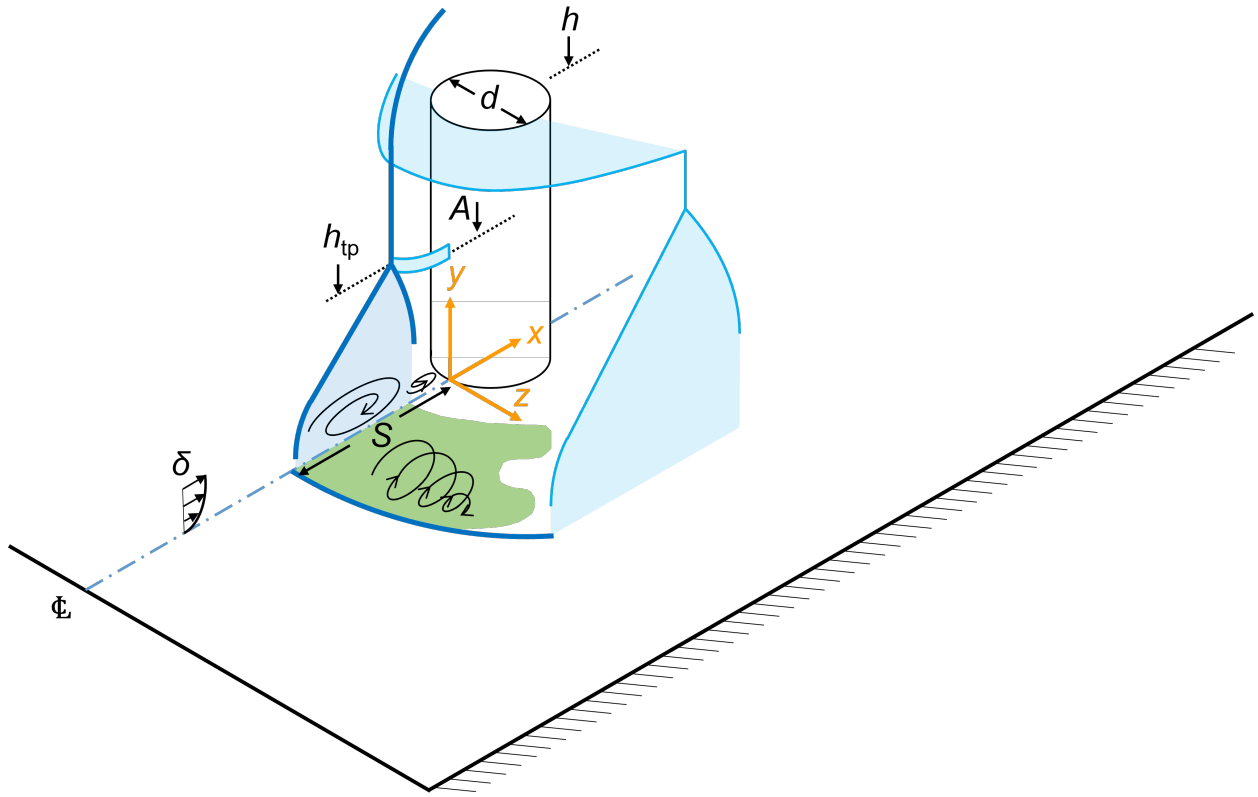


Figure 2.1: Simple schematic of a semi-infinite cylinder-generated SWBLI.

2.1.1 General Shock Structure

Figure 2.1 indicates the presence of a lambda-shock structure, whose details are depicted in Fig. 2.2. As the boundary layer approaches the shock generator, a strong adverse pressure gradient induces massive flow separation. The resulting flow deflection generates a shock wave known as the forward shock or separation shock, which is denoted by λ_1 ; the forward shock generally has some curvature near its foot due to viscous effects in the boundary layer. Although flow separation is an unsteady phenomenon, a time-averaged separation line can be found for laminar and turbulent interactions. Similar to a bow shock, the shortest distance from the separation line to the body exists at the centerline and is denoted by S . Moving off the centerline, the separation point is located further and further downstream as the adverse pressure gradient due to the shock generator is reduced, and the flow wraps around the cylinder or blunt fin in a spanwise, conical fashion (in the algebraic sense), as reflected by the hyperbolic-like separation line in Fig. 2.1. Additionally, the separated region, highlighted in green, approaches an outer bound. A line of coalescence following the original separation line persists, as does the lambda-shock structure due to the pressure disturbance. The inviscid bow shock experiences a similar pressure relief, both in the spanwise and wall-normal directions. Near the top of the shock generator, the bow shock begins to curve downstream, eventually wrapping over the top of the cylinder or blunt fin and continuing as a constant-angle oblique shock. If the shock generator is sufficiently tall, a normal bow shock can exist at the centerline; otherwise, some bow-shock curvature will be present throughout.

The intersection between the forward shock and the bow shock occurs at a triple point with height h_{tp} . The resultant structure is classified as an Edney type IV interaction, given a sufficiently high Mach number, or an Edney type III interaction if the Mach number is too low [16, 23, 24, 26, 28–31]. Additionally, the trailing shock, also known as the rearward shock or closure shock, is formed at this triple point, and is denoted by λ_2 . In the event of an Edney type IV interaction, a shock train is formed at the triple point, as illustrated in more detail by Fig. 2.3 [23]. For an Edney type III interaction, the shock train cannot form due to insufficient pressure, and the resulting structure instead consists of a slip line with a subsonic shear layer. In either case, the resulting flow downstream of the triple point impacts on the

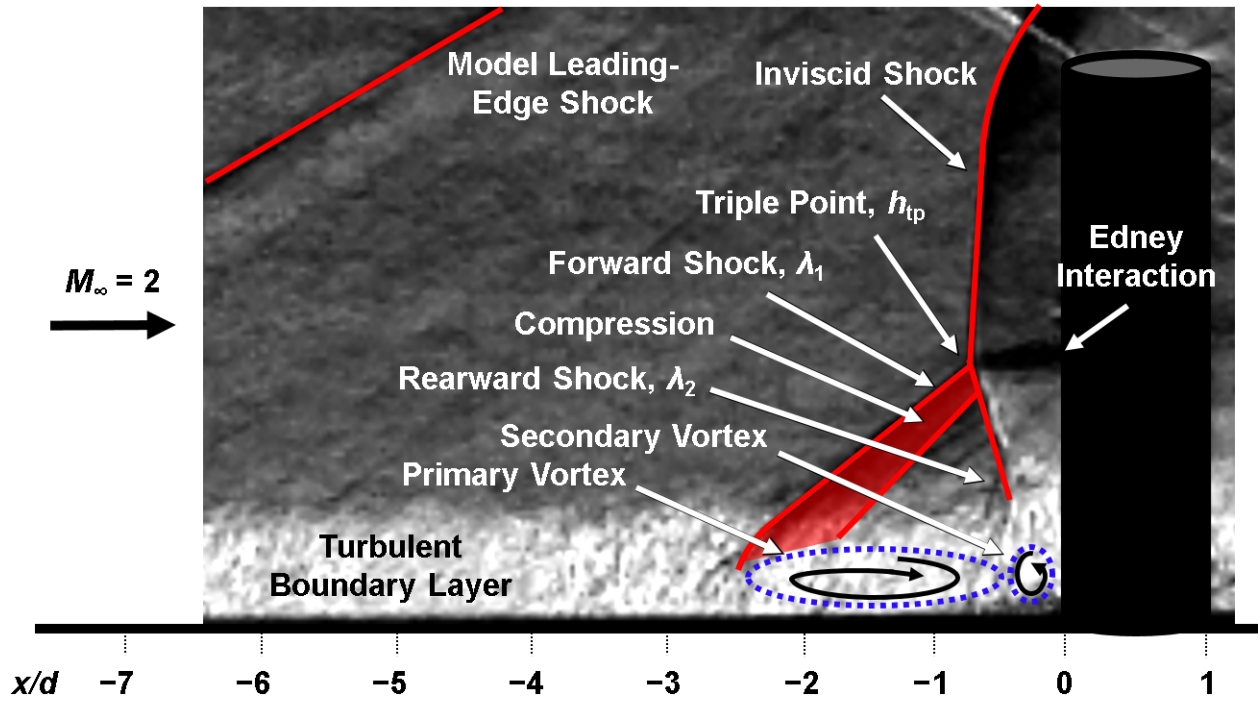


Figure 2.2: Instantaneous schlieren image of a semi-infinite cylinder-generated SWBLI with key flow features labeled. Flow is left to right.

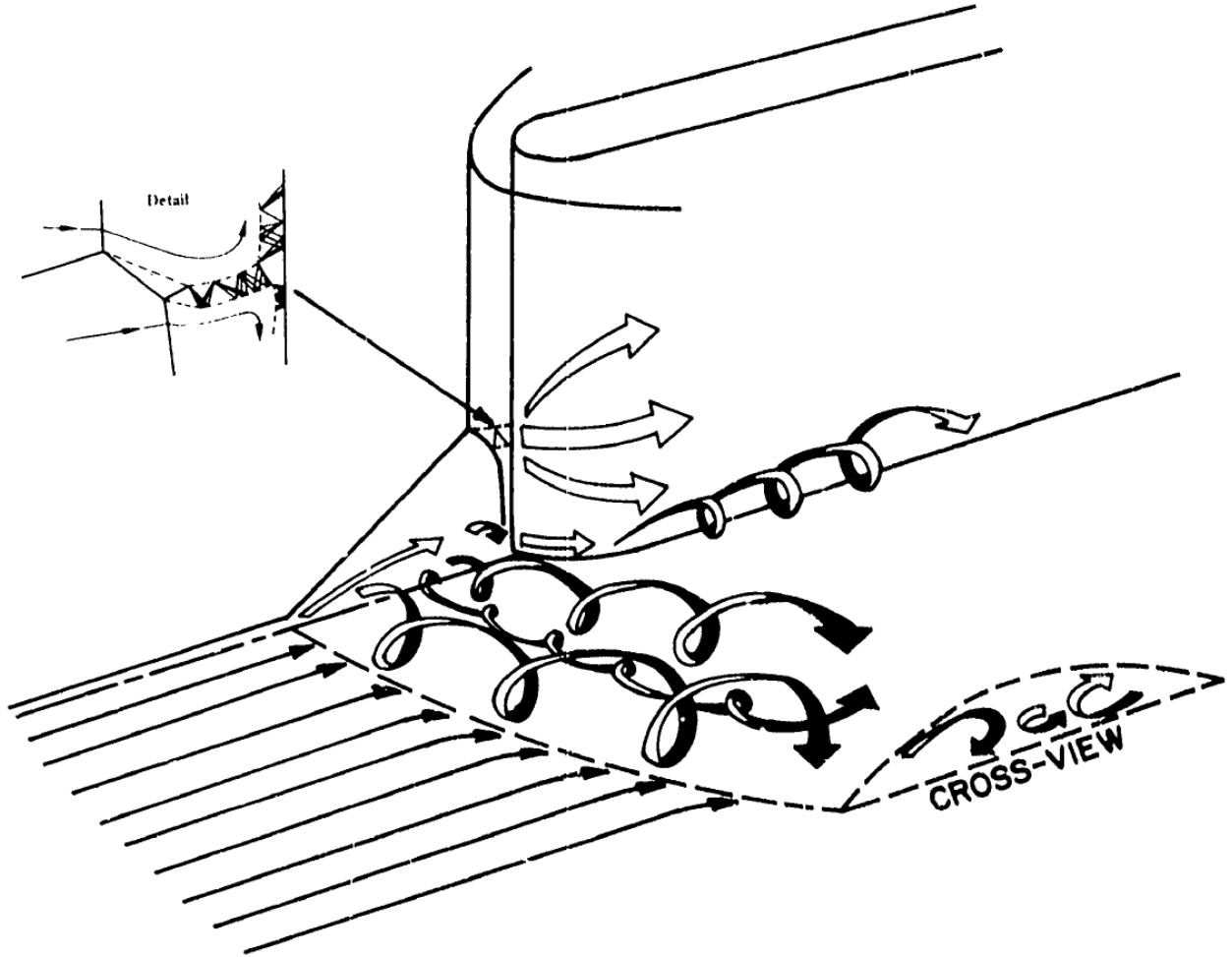


Figure 2.3: Schematic of a semi-infinite blunt-fin-generated SWBLI, indicating the Edney type IV interaction [28] and vortex structures in greater detail. Figure from Kaufman et al. [23].

surface at the leading edge, generating very high pressures and heat transfer rates. These loads can be reduced by sweeping the cylinder or blunt fin backwards [19, 23, 24, 27, 29], so that the bow shock changes from a normal shock to an oblique shock, regardless of the height of the shock generator. However, implementing a shock generator whose height is small enough to induce a triple point in the curved relief-shock region does not result in a reduced peak pressure or heat transfer [17, 25]. This study will focus exclusively on unswept (orthogonal) cylinders and blunt fins, despite the more commonly-used swept structures existing in vehicle designs, as the unswept geometries represent the maximum limits to the unsteady and potentially high-risk phenomena generated by this class of SWBLI.

In order to investigate a more uniform interaction, removing the effects of the shock generator height proves useful. Dolling and Bogdonoff [13] determined that an asymptotic limit, coined a “semi-infinite criterion”, exists when the blunt fin height is 2–3 times larger than the triple-point height ($h/h_{tp} > 2-3$), and approximated this criterion to be $h/d > 2.4$. When this criterion is met, changes in the blunt fin height no longer affect the interaction scale, and the presence of a normal bow shock is effectively guaranteed. As a result, all cylinders and blunt fins in this study meet the semi-infinite criterion, effectively removing h as an interaction variable.

2.1.2 General Vortex Structure

Both Fig. 2.1 and Fig. 2.3 [23] show the vortices generated by the interaction. Starting at the centerline, Fig. 2.1 displays a single large vortex under the lambda-shock structure, and a smaller counter-rotating vortex near the base of the cylinder. The large vortex is shown to spiral to a focal point, after which the flow spirals outboard with diminishing strength. Figure 2.3 [23], however, shows three large counter-rotating vortices spawning from the centerline that follow the same spiral pattern, as well as the smaller counter-rotating vortex at the base. Both schematics are possible, as the number of large vortices varies in the separation bubble, although Fig. 2.3 [23] does not take the reduced vortex strength into consideration. The large vortex structure is known as the primary vortex structure. Sedney and Kitchens [18] found that this consists of one, three, or five large counter-rotating vortices in a time-averaged sense. Özcan [32] achieved similar findings in that the number varied between one

or three. This variation is not well-defined, but is presumed to be a function of either the unit Reynolds number, Re_∞ , or the Reynolds number based on the diameter, Re_d [18, 25, 32]. In any case, Sedney and Kitchens [18] found that the Reynolds number exhibits the smallest, if any, changes to the turbulent interaction as a whole. The smaller secondary vortex at the base is always present, and as shown in Fig. 2.3 [23], its outboard flow pattern follows a path seemingly aligned with the blunt-fin wall, differing from the path of the primary vortex structure.

2.2 Detailed Time-Averaged Interaction Components

With a general understanding of the major components of the SWBLI defined, a closer look at each individual interaction component is presented. Prior work that has aimed at explaining these individual aspects is examined, as well as how they might affect other parts of the interaction and the overall flowfield. This section will examine the time-averaged (mean) components, and the time-accurate (unsteady) components will be discussed in Sec. 2.3.

2.2.1 Bow Shock

If a simple supersonic blunt body is considered, a detached bow shock exists in the shape of a nearly-conic section [33] (in the algebraic sense), as indicated by the schematic of a 2-D, inviscid, blunted-nose cone in Fig. 2.4a [34]. Between points a through c , a strong shock occurs, resulting in subsonic flow downstream. At point c' , the flow behind the shock becomes sonic. Further away from the centerline, for example at point d , the flow behind the shock remains supersonic. Naturally, points a through e can be found on a θ - β - M chart, shown in Fig. 2.4b [34]. Point a represents the strong shock limit as a normal shock. At the other end, point e and any point further outboard represents the weak shock limit, also known as a Mach wave, where no flow deflection occurs. Points a , c' , and e are of particular interest in this study.

At the centerline at point a , the bow shock has a stand-off distance, B , as indicated in Fig. 2.4a. For decades, researchers have attempted to find an analytical solution to B ,

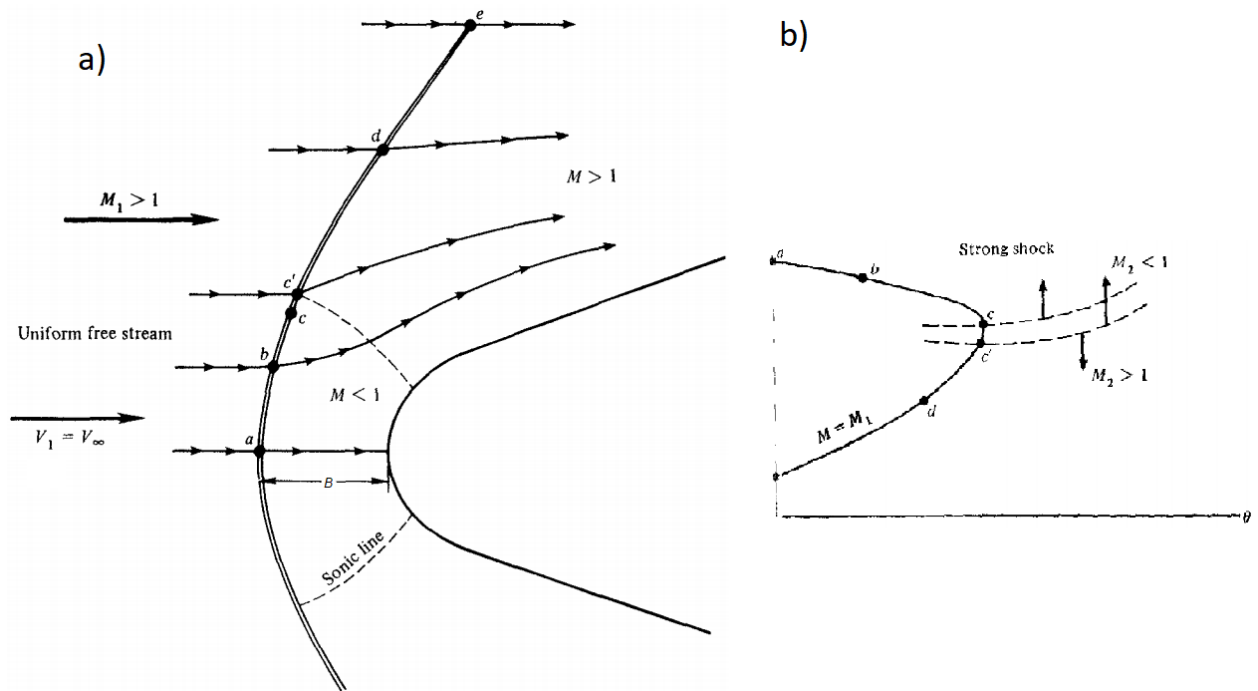


Figure 2.4: (a) Schematic of a blunted-nose cone with shock structure and (b) accompanying θ - β - M chart. Figure adapted from Anderson [34].

but this has so far eluded them. Approximations have been developed that are often based on Newtonian theory, but these tend to be valid only for air and deviate from the actual value at lower Mach numbers. However, a theoretically-based approximation was derived by Sinclair and Cui [35] that is valid over a wide range of Mach numbers and specific heat ratios, γ . This approximation is defined in Eq. 2.1 [35], and while it is typically normalized by the leading-edge radius, in this case it was rewritten for a normalization by the diameter:

$$B/d = \frac{\beta_s^2}{2\theta_s^2 \cos \beta_s} \sqrt{\frac{2 + (\gamma - 1) M_\infty^2}{2\gamma M_\infty^2 - \gamma + 1}}$$

where $\beta_s = \frac{\pi}{2} - \theta_s$

$$\theta_s = \arcsin \left(\sqrt{\frac{c_{P,s}}{c_{P,\max}}} \right)$$

$$c_{P,s} = \frac{2}{\gamma M_\infty^2} \left[\left(\frac{\gamma + 1}{2} \right)^{-\frac{\gamma}{\gamma-1}} \left(\frac{(\gamma + 1)^2 M_\infty^2}{4\gamma M_\infty^2 - 2(\gamma - 1)} \right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1 - \gamma + 2\gamma M_\infty^2}{\gamma + 1} \right) - 1 \right]$$

$$c_{P,\max} = \frac{2}{\gamma M_\infty^2} \left[\left(\frac{(\gamma + 1)^2 M_\infty^2}{4\gamma M_\infty^2 - 2(\gamma - 1)} \right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1 - \gamma + 2\gamma M_\infty^2}{\gamma + 1} \right) - 1 \right] \quad (2.1)$$

Here, β_s is the clock-wise angle measured from the centerline to the point where the sonic line impacts on the surface (see dashed line in Fig. 2.4a), θ_s is the angle of the body surface tangent to the flow at the point where the sonic line impacts, $c_{P,s}$ is the surface pressure coefficient at the point where the sonic line impacts, and $c_{P,\max}$ is the surface pressure coefficient at the (centerline) stagnation point.

A simpler approximation for air that remained relevant at the Mach number investigated here was developed by Amick [36]. This semi-empirical approximation is defined in Eq. 2.2 [36]:

$$B/d = 0.19 + \frac{1.2}{M_\infty^2 - 1} + \frac{0.7}{(M_\infty^2 - 1)^2} \quad (2.2)$$

As shown in Fig. 2.5, at a Mach number of 2 the deviation of Eq. 2.2 [36] from the approximation in Eq. 2.1 [35] is less than $0.03d$ ($< 4\%$). This small deviation indicates that

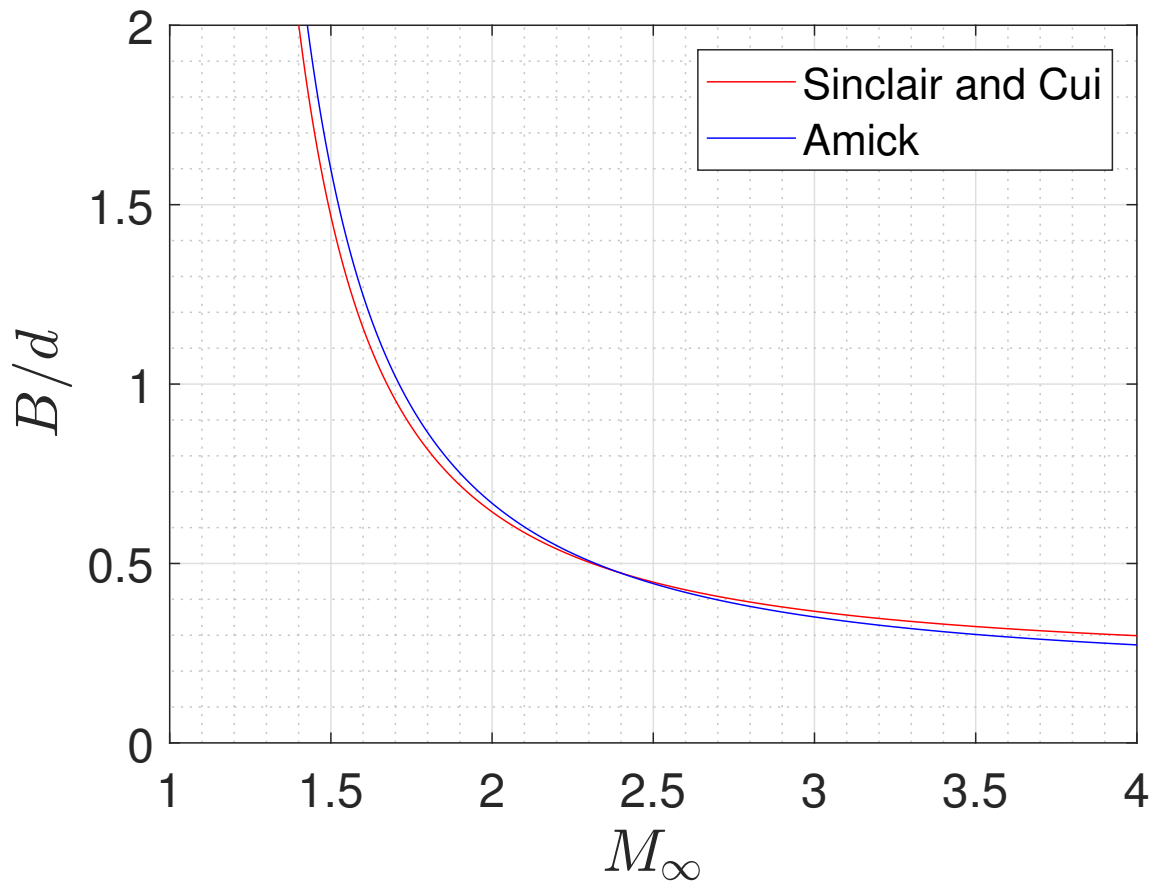


Figure 2.5: Comparison of bow shock standoff distance approximation for air with varying Mach number.

both approximations are in good agreement with one another for the current Mach number, and thus either approximation can be used for the present conditions.

2.2.2 State of the Incoming Boundary Layer

The state of the incoming boundary layer is the most critical determining factor regarding when flow separation occurs. The limiting cases are a completely laminar and a completely turbulent state. The “fullness” of the velocity profile provides a reasonable distinction regarding the state of the boundary layer [37], as indicated by an example Blasius [38] laminar boundary-layer profile in Fig. 2.6a and an example Musker [39] turbulent boundary-layer profile in Fig. 2.6b. A “fuller” velocity profile is less susceptible to flow separation, owing to greater momentum and energy near the wall [37]. As such, a laminar boundary layer will separate further upstream than a turbulent boundary layer under equal edge conditions.

Another method of distinguishing the state of the incoming boundary layer is by using the incompressible shape factor, H [31], defined in Eq. 2.3 [37]:

$$H(x) = \frac{\delta^*(x)}{\theta(x)}$$

$$\text{where } \delta^*(x) = \int_0^\delta \left(1 - \frac{u(x,y)}{u_e(x)}\right) dy \quad (2.3)$$

$$\theta(x) = \int_0^\delta \frac{u(x,y)}{u_e(x)} \left(1 - \frac{u(x,y)}{u_e(x)}\right) dy$$

Here, δ^* is the incompressible displacement thickness, θ is the incompressible momentum thickness, and u_e is the streamwise velocity at the edge of the boundary layer, i.e. at $y = \delta$. Usage of the incompressible version is preferred because the compressible shape factor is strongly dependent on the Mach number and therefore less universal [31]. Note that the velocity profile is effectively integrated in Eq. 2.3, allowing for the “fullness” of the boundary layer to be quantified. A “fuller” boundary-layer profile will result in a smaller shape factor, as is evident by typical values in the vicinity of $H = 1.3$ – 1.4 for a turbulent profile and an exact value of $H = 2.59$ for a Blasius profile [37]. Such values indicate that a boundary layer is “healthy,” and deviations quantitatively describe the “healthiness” of the boundary layer.

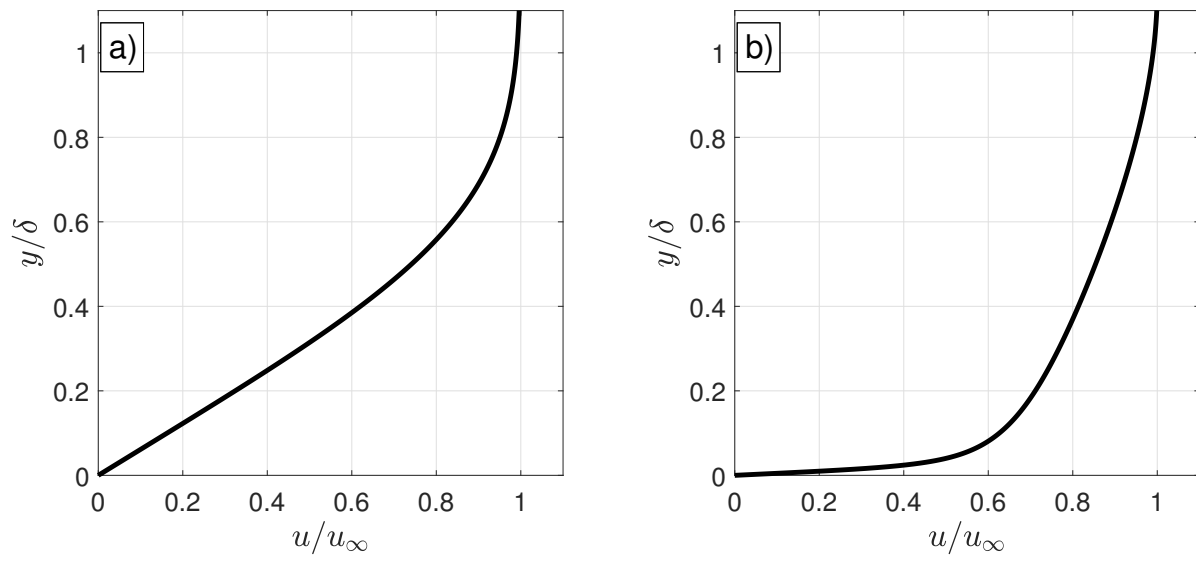


Figure 2.6: Normalized velocity profiles of a (a) Blasius laminar and (b) Musker turbulent boundary layer.

As a boundary layer continues to develop in the streamwise direction under a non-favorable pressure gradient, its thickness grows and the shape factor increases [37]. Natural flow separation of a turbulent boundary layer typically occurs around $H \approx 1.8$ [37]. For laminar boundary layers, natural flow separation is expected to occur at much higher shape factors exceeding $H > 4$ [40]; however, it is likely that laminar-turbulent transition happens before such a high shape factor is achieved. When the boundary layer undergoes transition, the shape factor drastically decreases from a laminar value to a turbulent value, as indicated by the example shown in Fig. 2.7 [41].

A comparison of the impact of the state of the incoming boundary layer on the flow separation in a semi-infinite cylinder-generated SWBLI is visually represented by Fig. 2.8 [42]. Separation occurs earliest for laminar boundary layers due to the aforementioned factors, and is shown in Fig. 2.8a [42]. The separation distance along the centerline, labeled S in Fig. 2.1, is typically around $6d$ – $9d$ for a laminar interaction and varies greatly with $Re_{x,sep}$ [22, 23, 32, 43–45]. As $Re_{x,sep}$ increases, transition is induced further upstream, and a transitional interaction can be observed, like the one shown in Fig. 2.8b [42]. The separation line is no longer symmetric for a transitional interaction and displays characteristics akin to the laminar separation line near the centerline, and akin to the turbulent separation line further outboard [23, 42, 43]. As the incoming boundary layer transitions further towards a turbulent state, the separation line shifts further downstream, eventually asymptoting towards that typical of a turbulent interaction. A turbulent interaction, shown in Fig. 2.8c [42], has a typical separation distance of around $2d$ – $3d$ [4, 13, 14, 17–19, 22–24, 43] and, as mentioned in Sec. 2.1.2, is largely insensitive to Re . With the focus of this study concentrating on a turbulent interaction, successive discussion will be limited to a turbulent boundary layer.

2.2.3 Separation Shock Curvature

As separation occurs, the flow is deflected upwards, causing compression that results in an oblique shock wave. Due to limitations with experimental diagnostic equipment, authors such as Sykes [21] or Westkaemper [17] assumed that the separation shock was at a constant angle. However, it has been shown that there is curvature of the separation shock in the

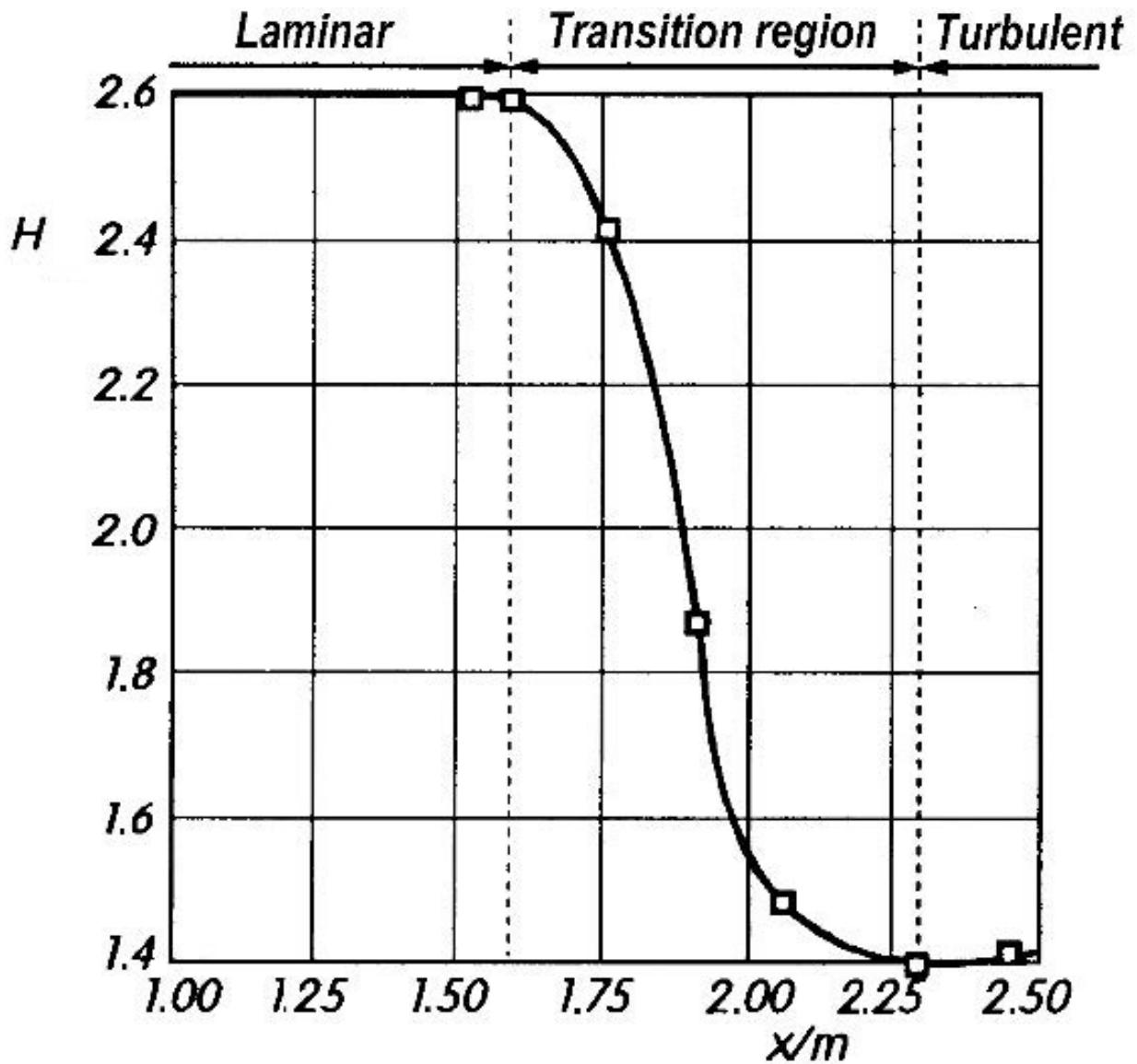


Figure 2.7: Example laminar-turbulent transition showing changes in shape factor. Figure adapted from Schlichting and Gersten [41]. Data and original figure by Persh [50], using data from Schubauer and Klebanoff [51].

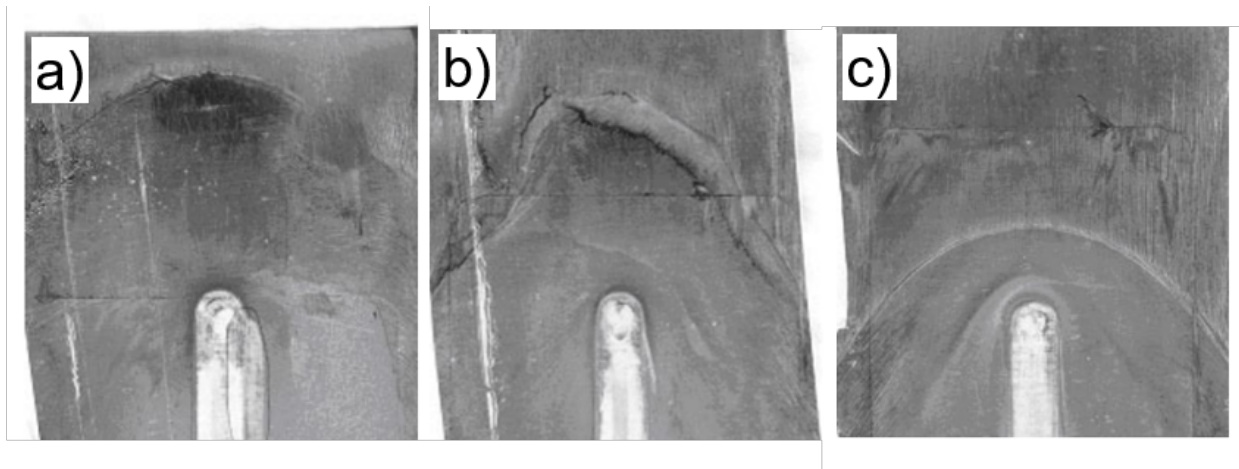


Figure 2.8: Kerosene-lampblack flow visualization of a cylinder-generated SWBLI, indicating separation line variation based on an incoming (a) laminar, (b) transitional, and (c) turbulent boundary layer. Flow is top to bottom. Figure adapted from Murphree [42].

boundary layer [18, 19, 23, 46]. This curvature can be explained using an analytical approach to the boundary layer known as the triple-deck terminology, which was developed by Lighthill [47] and expanded upon by Stewartson and Williams [48]. The triple-deck structure consists of three vertical tiers, as depicted in Fig. 2.9 [49]: the upper deck, where Eulerian or potential flow exists in the freestream; the middle deck, sometimes referred to as the main deck, where there is inviscid rotational flow in the boundary layer; and the inner deck, which is comprised of the near-wall regime where viscosity has an effect. In an attached, supersonic, turbulent boundary layer, the middle deck comprises over 90% of the boundary-layer thickness [31]. Moreover, the sonic point in a supersonic, turbulent boundary layer is quite close to the wall, as suggested by Fig. 2.6b; even for flows with moderate Mach numbers, the sonic point typically lies under $y/\delta < 0.01$ [31]. Shown in Fig. 2.10 [31] is a schematic of an oblique shock generated by a small-angle compression corner, such that the flow does not separate. In the outer deck, for $y/\delta > 1$, the oblique shock is shown to have a constant angle. In the middle deck, however, the curvature of the shock becomes evident. Enhancing the view on the inner deck indicates that compression shock waves emanate from the sonic line due to the disturbance of the boundary layer. These compression shocks then focus to create a single separation shock and induce its curvature. Hence, determining the separation distance via an extrapolation of the constant-angle region of the separation shock to the surface, perhaps through the use of side-view optical techniques, leads to a value that is upstream of the true separation location.

2.2.4 Upstream Surface Pressure

Chapman et al. [52] proposed the free-interaction theory, stating that the characteristics of the flow in the vicinity of separation are independent of the geometry of the shock generator and solely a function of the local flow conditions, particularly the pressure gradient. This theory has been verified by many authors for various geometries, including for the presently-concerning cylinder and blunt fin [22, 30–32, 53]. Since boundary layers are vulnerable to large pressure disturbances, and the shock generator imposes a strong adverse pressure gradient on the incoming boundary layer, it becomes imperative to analyze the upstream surface pressure profile. Shown in Fig. 2.11 is a schematic of the mean surface pressure

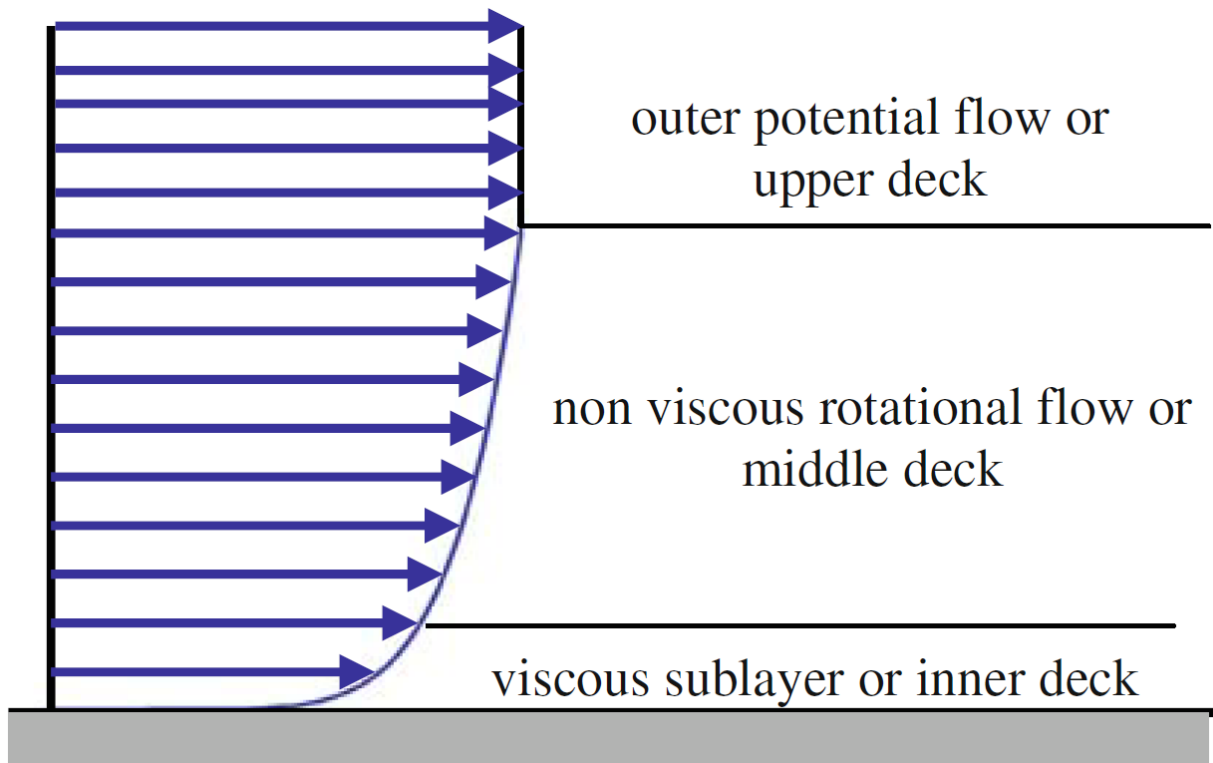


Figure 2.9: Schematic of a boundary layer broken up into the triple deck. Figure from Détery and Dussauge [49].

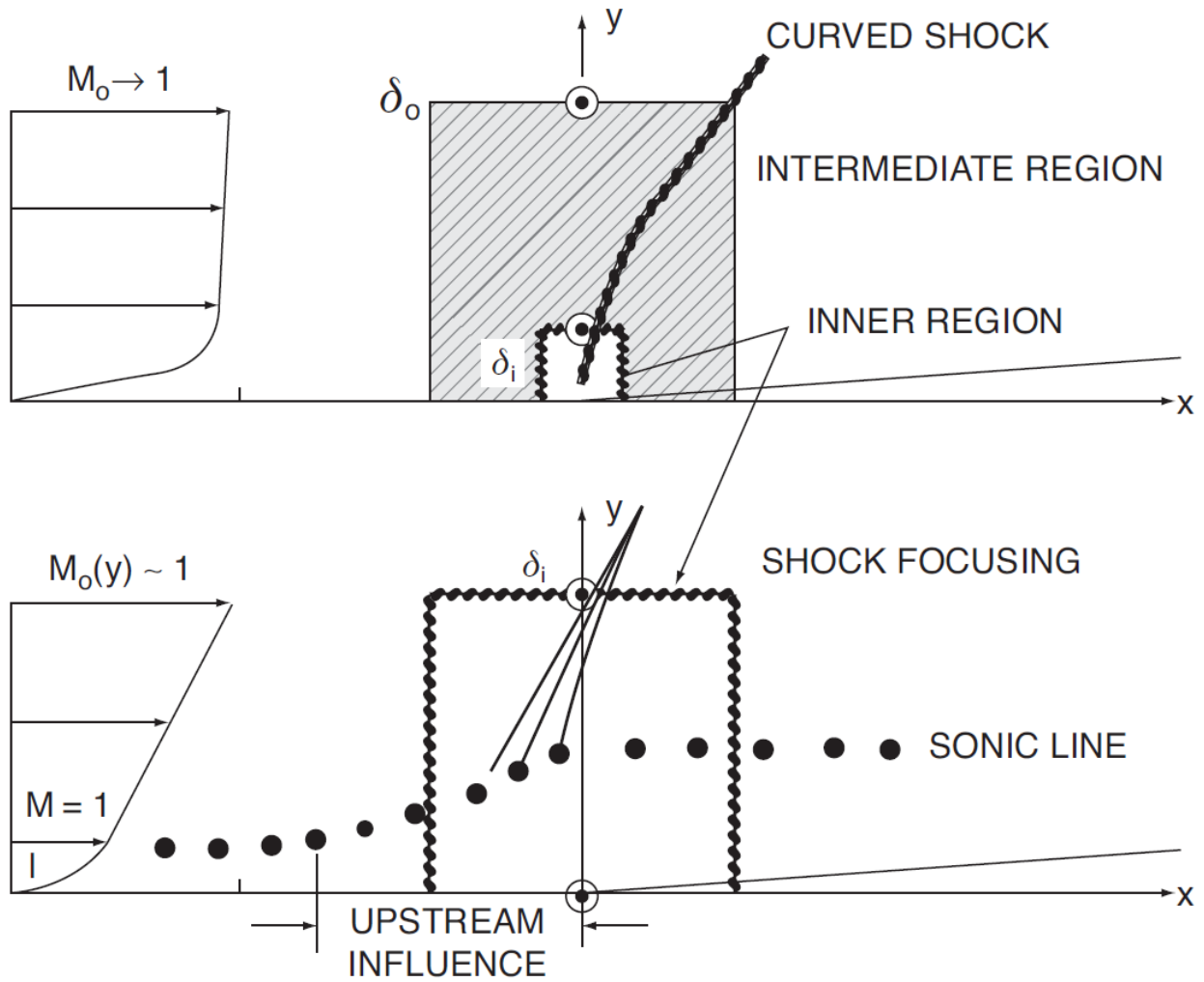


Figure 2.10: Schematic of the middle and inner deck, indicating curvature of an oblique shock within the boundary layer. Figure from Babinsky and Harvey [31].

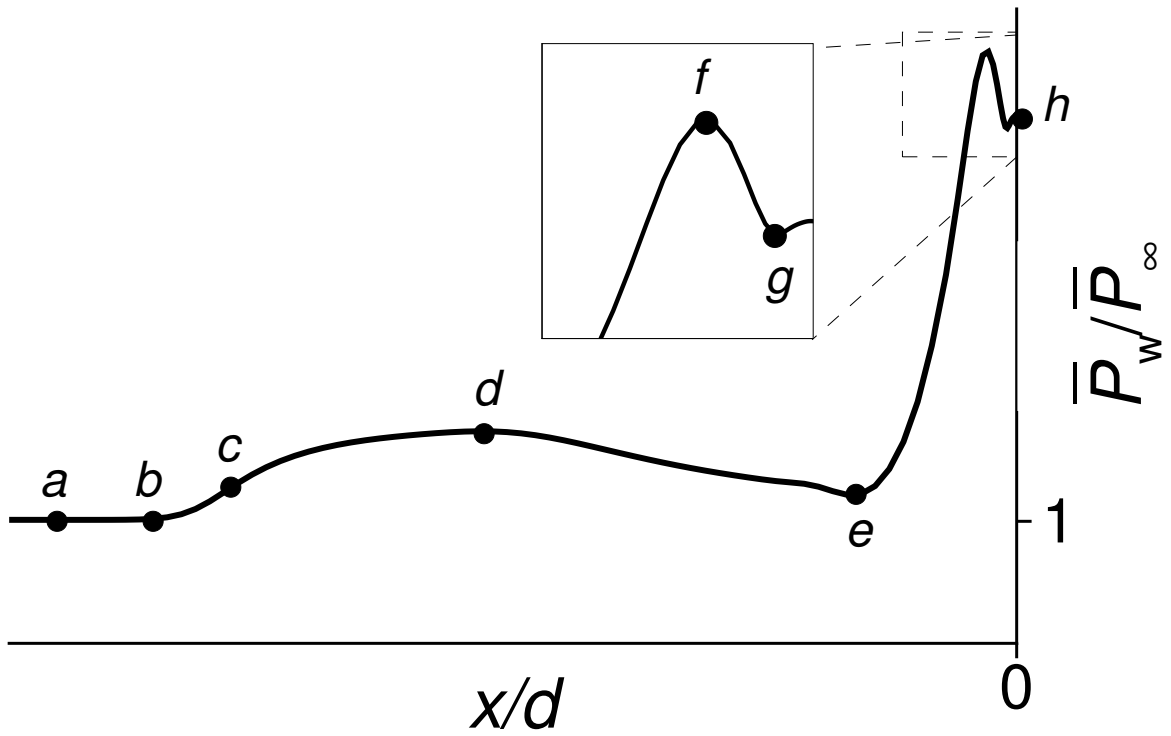


Figure 2.11: Schematic of the mean centerline surface pressure on the floor upstream of a cylinder- or blunt-fin-generated SWBLI.

profile at the centerline, normalized by the mean freestream pressure, where the sample mean is defined as in Eq. A.1. Although both separation and the SWBLI are unsteady phenomena, the schematic in Fig. 2.11 illustrates the expected pressure profile.

Upstream of point a , the flow remains undisturbed at a pressure of $\bar{P}_\infty$. The influence of the adverse pressure gradient imposed by the shock generator then becomes apparent in a pressure rise, beginning at point b in a mean sense. At point c , the boundary layer can no longer sustain the sudden disturbance and separates. Unfortunately, while an analytical understanding of the pressure rise needed to induce separation of a laminar boundary layer exists [31, 52, 54–56], this has not yet been discovered for a turbulent boundary layer with large-scale separation [54, 56, 57]. Instead, observations have approximated this rise to around $\bar{P}_w/\bar{P}_\infty \approx 1.2\text{--}1.3$ for a Mach number of $M_\infty = 2$ [22, 31, 52]. Following the initial mean pressure rise at point b , this rise yields a typical profile shape consisting of an initial plateau, a trough, and a final peak with relief, with magnitudes that are greatly dependent on the Mach number [26]. This shape generally remains valid when analyzing the instantaneous pressure profile, although the magnitudes will fluctuate over time. In a mean sense, the pressure plateau is denoted by point d in Fig. 2.11, and can be approximated for the cylinder or blunt fin. Such a semi-empirical approximation was developed by Truitt [58] for 2-D forward-facing steps and is shown in Eq. 2.4. Since the upstream centerline behavior of a cylinder and blunt fin is akin to that of a 2-D forward-facing step, this approximation holds [17, 22, 24].

$$\frac{\bar{P}_w}{\bar{P}_\infty} = 1 + \frac{\gamma M_\infty^2}{2} \frac{5.61}{(M_\infty \text{Re}_\infty)^{0.2}} \quad (2.4)$$

Between the mean plateau at point d and the trough at point e , an overall mean decrease in pressure is observed, though local extrema may be present due to fluctuations. The mean profile shape in Fig. 2.11 is retained even when the number of vortices in the primary vortex structure exceeds one. Downstream of point e , a sharp increase in pressure is observed whose (streamwise) spatial variation is minimal. The pressure increase peaks at point f in a mean sense, followed by a small pressure relief around point g , and finally reaching the base at the coordinate origin at point h .

2.2.5 Triple Point and Resulting Edney Interaction

The bow shock and separation shock intersect at a triple point. The result is a shock/shock interaction that causes redirection of the bow shock [16, 59], which creates the trailing shock and thus the well-known lambda-shock structure. Furthermore, either a subsonic shear layer or a supersonic jet with an enclosed compression/expansion shock train emanates from the triple point; which scenario exists depends on the freestream Mach number and the angle of incidence from the separation shock. Edney [28] classified a series of six different types of shock/shock interactions occurring ahead of blunt bodies. A shock/shock interaction ahead of the stagnation point is of type IV and is known to be the most severe regarding pressure and heat transfer rise.

A graphical representation of the Edney interaction [28] can be provided by a shock polar. Shown in Fig. 2.12a and c are example pressure-deflection diagrams, a type of shock polar, of an Edney type III and IV interaction [28], respectively, with a representation of the accompanying physical shock structure shown in Fig. 2.12b and d, respectively. The flow-deflection angle was arbitrarily set to $\theta = 10^\circ$ in both cases, but the Mach number was increased from 2 to 4, respectively. Across the forward shock (C_1), the flow remains supersonic at state 2; this change in flow conditions across the oblique shock is reflected in the shock polar by an increase in pressure ratio. Similarly, the flow changes states across the bow shock (C_2) but to a subsonic condition at state 3. For the Edney type III [28] interaction, the change in conditions across the trailing shock (C_3) to state 4 results in a matched static pressure but a mismatched velocity and stagnation pressure. This discontinuity is equalized in a shear layer centered about a slip line [31]. The size of the shear layer depends on the magnitude of the mismatch, which can be exacerbated if state 4 remains supersonic; yet the pressure ratio at the upper boundary of the second deflection (blue dashed line at $\bar{P}_w/\bar{P}_\infty = 4.5$) will always be insufficient to generate a third deflection. For the Edney type IV [28] interaction, the deflected shock C_3 causes state 4 to remain supersonic with a sufficiently high pressure ratio to induce yet another shock deflection (C_4). A series of alternating compression waves and expansion fans keeps the flow supersonic at states 6, 7,

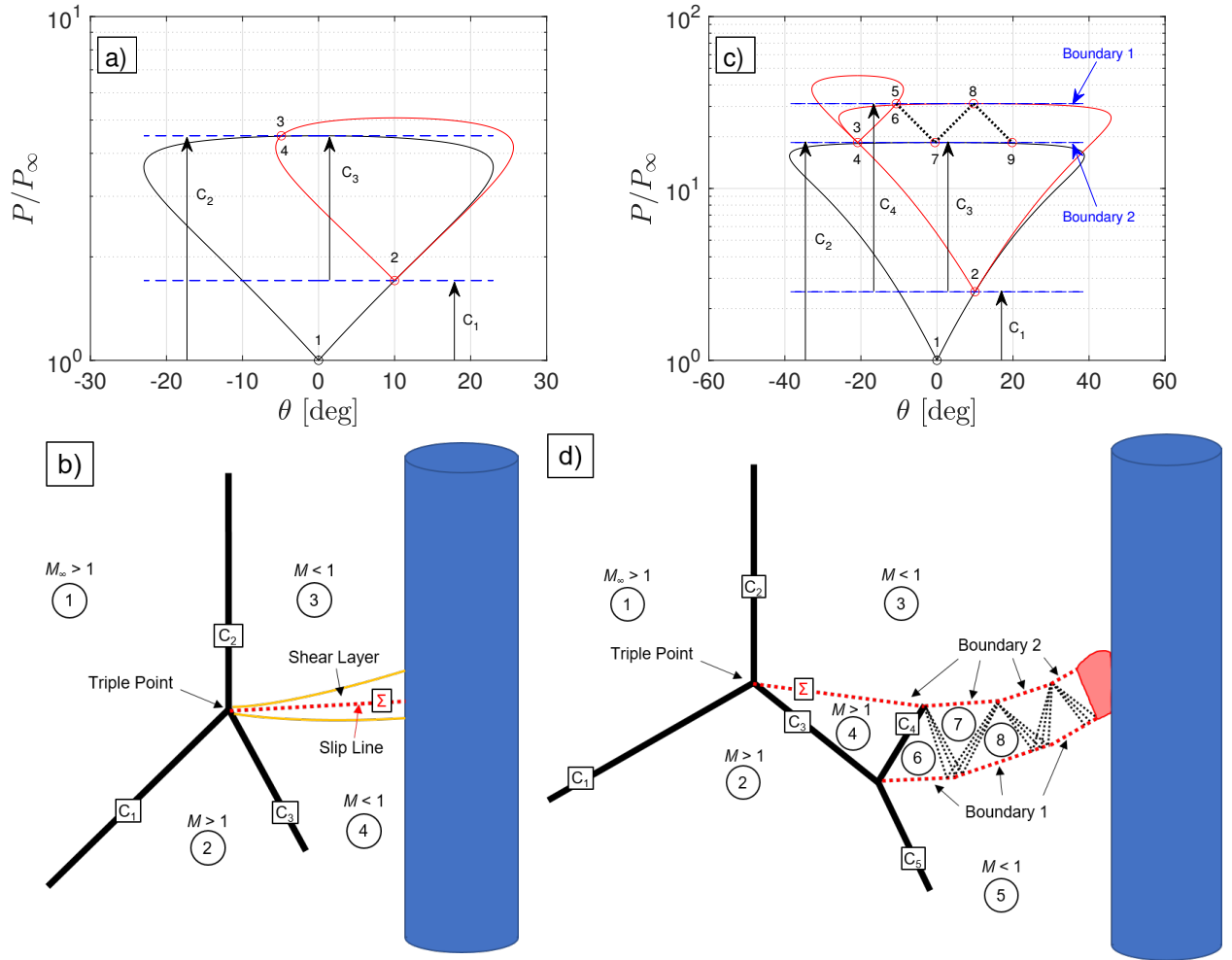


Figure 2.12: Pressure-deflection shock polar of an Edney type [28] (a) III and (c) IV interaction induced by a 10° deflection at a Mach number of 2 and 4, respectively. Physical plane representation with exaggerated length scales for the shock structure upstream of a cylinder with an Edney type [28] (b) III and (d) IV interaction.

8, etc., as the shock train moves downstream. This shock train is contained between two static pressure boundaries and leads all the way to the blunt body surface.

2.2.6 Cylinder/Blunt Fin Leading Edge Surface Pressure

Much research attention has been given to the floor pressure upstream, particularly at the centerline, due to the unsteadiness in the flow. Nevertheless, the pressure on the surface of the cylinder, especially at the centerline, far exceeds the pressure observed on the floor as a result of stagnation. This becomes apparent when comparing point h in both Fig. 2.11 and Fig. 2.13, which have identical magnitudes, and realizing that the mean peak of the surface pressure profile at the leading edge, point c in Fig. 2.13, is roughly twice as high, depending on the Mach number [23, 46].

As might be expected, point c is directly related to the impact of the Edney interaction [28] on the surface. In fact, the rise in mean pressure beyond that of the mean post-normal-shock stagnation pressure, $\bar{P}_{0,2}$, ranging from points b to c is overall caused by the Edney interaction [28], as evidenced by the relatively large impact area covered in Fig. 2.12d [21, 23]. The initial disturbance occurs at point b in the mean sense and fluctuates, possibly in accordance with the separation shock, over a range such that point a is considered to be the lowest wall-normal coordinate (with respect to the floor) at which the pressure remains uniform. That is, above point a , the pressure remains at the inviscid value. Closer to the surface and located under the triple point, the pressure drops significantly until it reaches a minimum mean value at point d . Instrumentation at the leading edge is difficult to install and thus data here have been experimentally collected in much fewer studies than on the floor upstream. However, computational results [60] have indicated that the mean pressure at points e , f , and g experiences variation as a result of a small tertiary vortex.

2.2.7 Centerline Vortex Topology

As mentioned in Sec. 1.1, a closed-type, 3-D interaction exists at the centerline for this geometry. Legendre [61] coined the term “wall streamlines” in his critical-point theory to describe the very-near-wall velocity field, and how these wall streamlines converge to

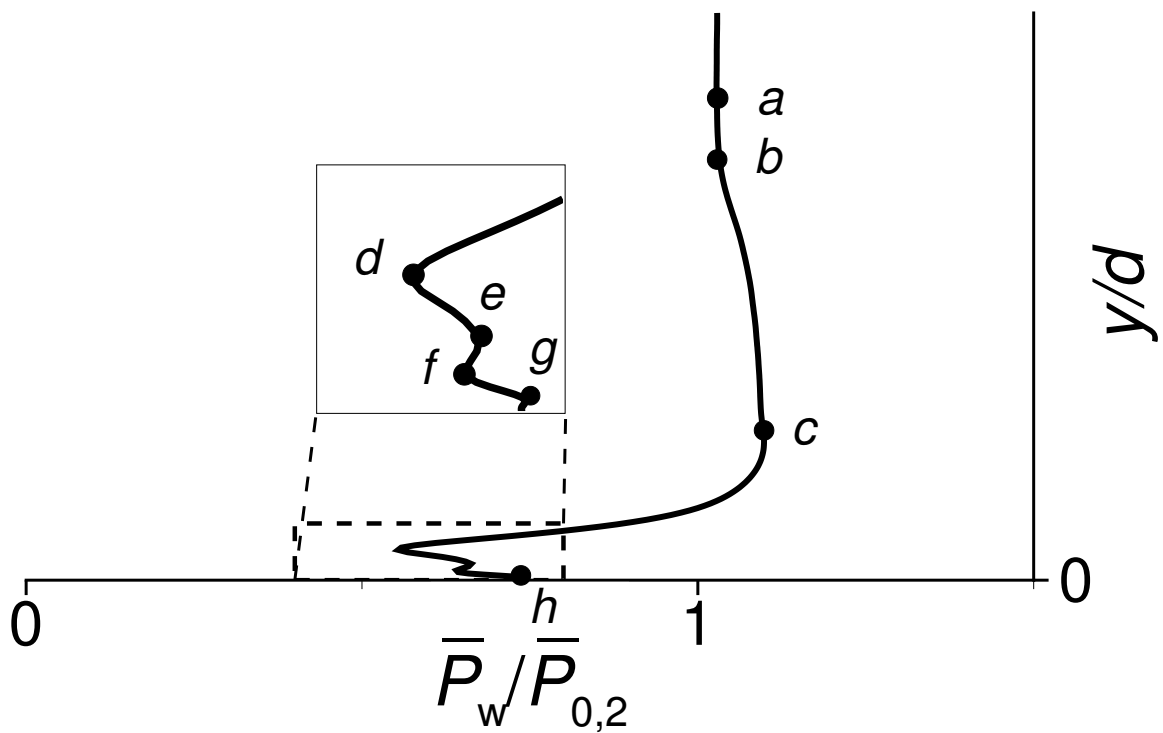


Figure 2.13: Schematic of the mean centerline surface pressure on the leading edge of a cylinder- or blunt-fin-generated SWBLI.

singularity separation lines. This work was extended by Lighthill [62] to use surface skin friction in viscous flows. Hunt et al. [63] then applied this surface method and extended it to the 3-D flowfield for 2-D planes. The singularities described by each of these authors are akin to those found in ordinary differential equations [63, 64], using nodes (N), half-nodes (N'), saddles (S), and half-saddles (S') to classify the flow features. Nodes and saddles occur in the flow, whereas half-nodes and half-saddles are located on a surface. Each node represents a vortex core, and each half-saddle represents either a separation line or an attachment line. The total number of nodes and half-nodes must equal the total number of saddles and half-saddles, as indicated by Eq. 2.5; note that two half-nodes and two half-saddles are equivalent to a single node and a single saddle, respectively.

$$\left(\sum N + \frac{1}{2} \sum N'\right) - \left(\sum S + \frac{1}{2} \sum S'\right) = 0 \quad (2.5)$$

As seen in Fig. 2.14, this method was applied to the upstream centerline flowfield of the cylinder and blunt fin SWBLI. To facilitate discussion, the mean flow was considered with a single primary vortex. At first glance, the topology in Fig. 2.14a indicates six limiting streamlines that result in two nodes and four half-saddles. Upon closer inspection via the enlarged view in Fig. 2.14b, there are, however, nine limiting streamlines consisting of three nodes and six half-saddles in total. The first node is located in the primary vortex structure and results in a focal point, $F1$, which creates the vortex core that extrudes in the spanwise direction. Though the primary vortex structure may result in multiple vortex cores, as discussed in Sec. 2.1.2, the discussion here will be limited to a single focal point. Accompanied by this single node are two half-saddles consisting of the primary separation line, $SL1$, whose distance is denoted by S in Fig. 2.1, and an attachment line, $AL1$. Note that although it is located in the general vicinity, $AL1$ does not correspond to the impact point of the Edney interaction [28] and is instead located below the impact point. This is evident by the upward flow deflection of the streamline in the freestream due to the forward separation shock. The secondary near-corner vortex is shown in greater detail in Fig. 2.14b and consists of another node, $F2$. This node is accompanied by another set of half-saddles consisting of a separation line ($SL2$) and attachment line ($AL2$). A tertiary near-corner

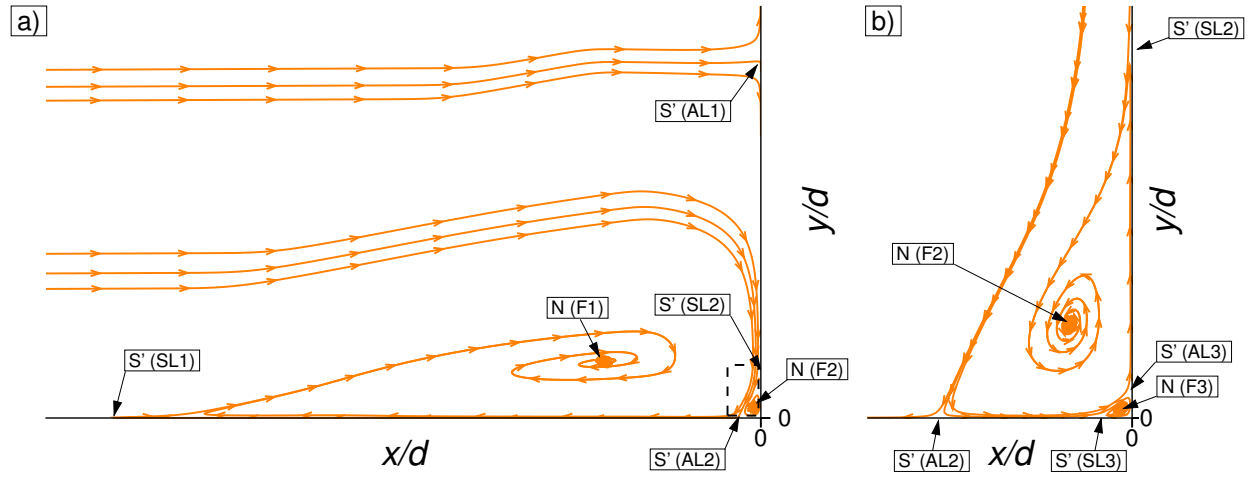


Figure 2.14: Schematic of the mean upstream centerline topology with labeled nodes and half-saddles. a) Global view. b) Enlarged view confined to the dashed lined box from a).

vortex with node $F3$ has also been found using computational means, but its small size has made it difficult to measure experimentally. The tertiary node is also accompanied by a set of half-saddles with separation line $SL3$ and attachment line $AL3$. All separation lines $SL1$, $SL2$, and $SL3$ are visible in Fig. 2.8c [42]. For any additional vortices in the primary vortex structure, the accompanying separation and attachment line pairs are confined to the floor where $y = 0$ and located between $SL1$ and $AL2$.

Coupling of the half-saddles and surface pressure profiles has also been determined. Referring back to Fig. 2.11 for the mean surface pressure profile along the upstream centerline, points f and g at the peak are of interest. As found in both computational [16] and experimental [24, 26] studies, the mean pressure peak at point f in Fig. 2.11 directly corresponds to the attachment line $AL2$ in Fig. 2.14. Moreover, the small pressure relief experienced downstream of the peak and terminating at point g is coupled to the tertiary vortex at the base of the cylinder or blunt fin, where point g corresponds to the separation line $SL3$ in Fig. 2.14b.

The tertiary node $F3$ appears to have a greater impact on the pressure profile at the leading edge, where it can be suspected to have some sort of coupling to points f and/or g in Fig. 2.13. Furthermore, it was found in a prior study that $SL2$ is loosely coupled to point d in Fig. 2.13, and that $AL1$ directly corresponds to point c [60].

2.3 Detailed Time-Accurate Interaction Components

The unsteadiness in the flow stems from flow separation and the subsequent movement of the forward shock. This behavior has experimentally been considered using analyses of surface pressure obtained via pressure transducers. Significant progress on this front was achieved by Dolling and co-authors in the 1980s and 1990s. This section mainly explores the various paths by which this unsteadiness has been characterized. Some comparisons to compression-ramp-generated SWBLI will be made, as Hou et al. [65] showed that the shock-foot unsteadiness is very similar to the cylinder- or blunt-fin-generated SWBLI, despite drastic differences in the mean behavior.

2.3.1 Flow Separation

Although mean flow separation over a flat plate is rather easy to determine by simply observing where the coefficient of skin friction, c_f , turns negative, instantaneous flow separation is a more difficult phenomenon to characterize. This is due to the constant fluctuations in the flow, resulting in small pockets of reverse flow that reattach a short distance downstream. Haller [66] derived several approaches that are applicable to various categories of flow problems. These approaches contain various criteria that ought to be met for an instantaneous flow to be considered truly separated. One of those approaches was taken and modified in the current work to better suit the data available, and it was confirmed with manual investigation of the flow that the criteria did not produce significant false positives (i.e. flow separating too far upstream) or negatives (i.e. flow separating too far downstream).

The modified approach consists of three criteria, of which only the third is not originally provided by Haller [66]. In the first criterion, the coefficient of skin friction at the wall must be negative, as shown in Eq. 2.6:

$$c_f \equiv \frac{\tau_w}{q_\infty} < 0 \quad (2.6)$$

where $\tau_w = \mu_w \left. \frac{du}{dy} \right|_{y=0}$

Here, τ_w is the shear stress at the wall, q_∞ is the freestream dynamic pressure, μ_w is the dynamic viscosity at the wall. Note that the only variable of interest here is the u -velocity, as all other variables are always non-negative and the criterion tracks the occurrence of a sign change. As such, the focus can be on the derivative term in Eq. 2.6, which, after eliminating the zero values at the wall due to the no-slip condition, reduces to simply observing when the u -velocity in the first cell off the wall becomes negative, as shown in Eq. 2.7.

$$u \Big|_{y_1} < 0 \quad (2.7)$$

The second criterion aims to track a departure of flow along the wall. For this to occur, the v -velocity gradient at the wall should be positive. Again, a simplification due to the

no-slip condition can be applied, resulting in the observation of when the v -velocity in the first cell off the wall becomes positive, as shown in Eq. 2.8.

$$v \Big|_{y_1} > 0 \quad (2.8)$$

These two criteria alone were insufficient in accurately predicting instantaneous flow separation. Several small pockets of reverse flow were eligible with these criteria, despite being well upstream of the actual separation location that caused the SWBLI. However, as one of the main drivers for flow separation is also the adverse pressure gradient due to the shock generator, a third criterion based on the mean pressure rise was created and implemented. This also ties back to the free-interaction theory [52] and the necessary pressure rise to induce separation. Shown in Eq. 2.9 is the third criterion, which is met when the local non-dimensionalized wall pressure exceeds half the mean non-dimensionalized wall pressure rise at the mean separation location.

$$\frac{P_w}{\bar{P}_\infty} > 1 + \frac{1}{2} \left(\frac{\bar{P}_w}{\bar{P}_\infty} \Big|_{x=\bar{S}} - 1 \right) \quad (2.9)$$

As described in Sec. 2.2.4, the mean wall pressure at separation is about $\bar{P}_w/\bar{P}_\infty \approx 1.20$ – 1.30 for the current flow conditions, which effectively converts this criterion to meet a local pressure rise by which $P_w/\bar{P}_\infty > 1.10$ – 1.15 .

2.3.2 Intermittency

Shown in Fig. 2.15 [67] is a schematic of three transient mean surface pressure profiles on the centerline upstream of a blunt fin. Although the pressure signal has been found to exhibit aperiodic behavior [14, 15, 53, 68–71], a snapshot sweep of transient mean pressure profiles is shown with time-stamps corresponding to an upstream motion of separation. Here, curve t_3 represents the upstream-most location of any transient mean pressure profile obtained in the interaction. Accordingly, curve t_3 contains the upstream-most point of any initial pressure rise in the interaction, which corresponds to point a in Fig. 2.11. Point a is also coined the upstream influence, UI , as the SWBLI has no impact on the flow upstream of this location.

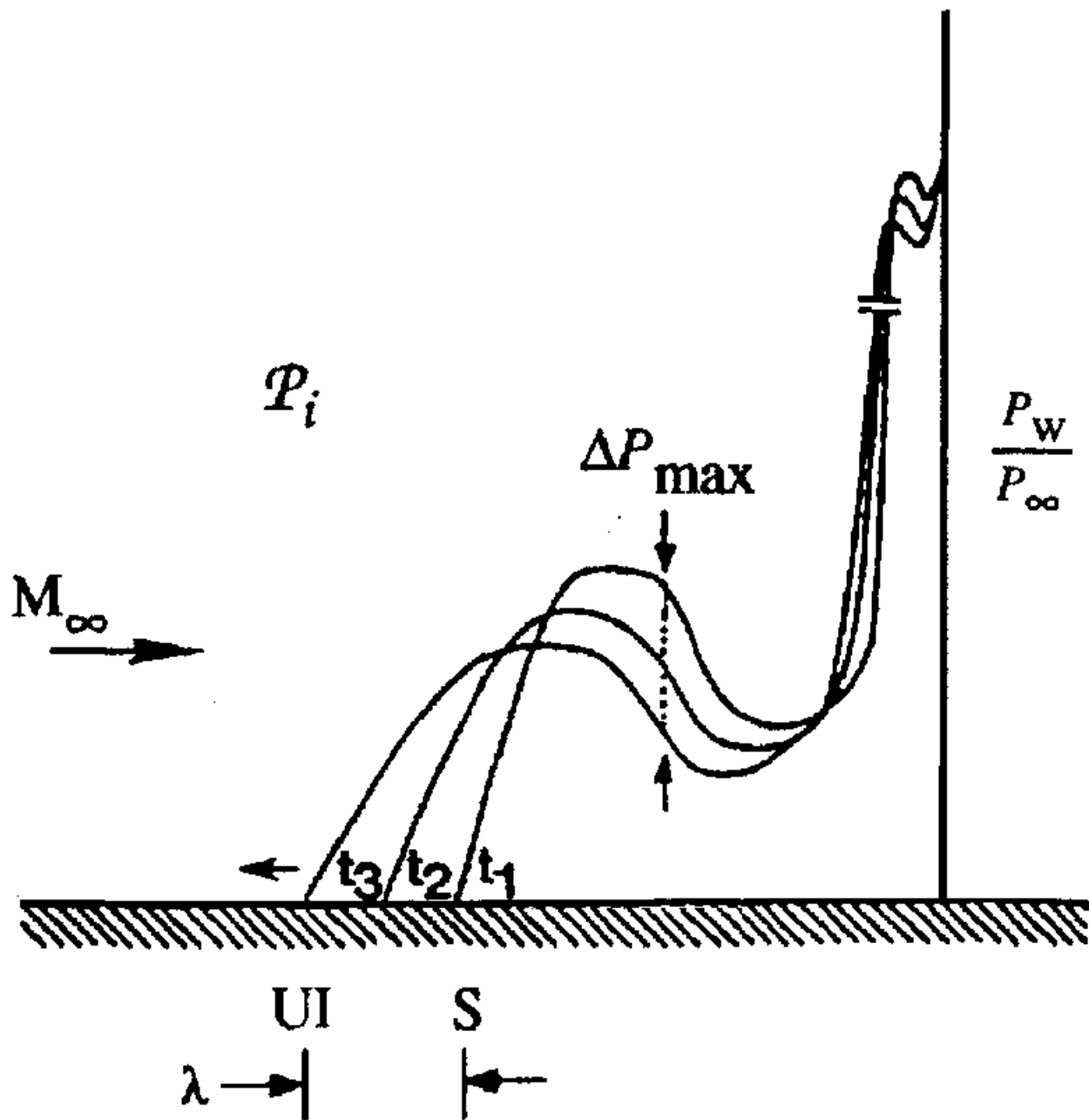


Figure 2.15: Schematic of the transient mean surface pressure on the centerline floor upstream of a cylinder- or blunt-fin-generated SWBLI. Figure adapted from Brusniak and Dolling [67].

Quantitatively, UI can be identified by transcribing the flow separation behavior in terms of an intermittency, γ , the fraction of time for which the flow is separated upstream of a particular location [69], defined in Eq. 2.10. As the flow is never separated upstream of UI , this location corresponds to a value of $\gamma = 0$.

$$\gamma(x) = \frac{t_{\text{sep}}(x)}{t_{\text{total}}} \quad (2.10)$$

Brusniak and Dolling [67] label the initial pressure rise of curve t_1 in Fig. 2.15 [67] as S , representing the downstream-most location for flow separation with $\gamma = 1$ and coinciding with the separation line obtained by surface flow visualization techniques as in Fig. 2.8c [42]. While it has been found that the separation line obtained from surface flow visualization does indeed correspond to the downstream-most location for flow separation with $\gamma = 1$ [69, 72], the instantaneous wall pressure at this location is that of the undisturbed boundary-layer value and thus not where S occurs. Note that careful interpretation of the results from prior studies is required, as the initial rise in pressure or heat transfer has sometimes been (erroneously) marked as the separation location. Additionally, side-view optical techniques have also shown to provide a value of S that is upstream of the true separation location [18, 19, 23, 46]. Sedney and Kitchens [18] suggested that oil-flow visualization should be the primary diagnostic technique used to measure S , as the separation line is clearly defined and unsteadiness in the flow is averaged out [52]. This proves useful for time-averaged analyses and subsequently, the experimental data from the archival literature that are used for comparison of S in a time-averaged sense were solely based on oil-flow measurements. When investigating time-accurate behavior, the notation used by Brusniak and Dolling [67] also proves useful, bearing in mind the aforementioned caveat.

The distance between UI and S in Fig. 2.15 is labeled as λ , which Brusniak and Dolling [67] intend to represent as the total distance over which flow separation will occur, i.e. the distance between the points where $\gamma = 0$ and $\gamma = 1$. The actual flow separation of curves t_3 and t_1 will be located at points just downstream of UI and S (where S is as defined in the figure), respectively, that exceed the necessary pressure rise ($P_w/\bar{P}_\infty \approx 1.2\text{--}1.3$ for current flow conditions). However, the offset between the initial pressure rise and the actual

separation location can be approximated to be equal for both curves, and thus λ is still approximately equal to the total distance over which flow separation occurs.

If curve t_2 in Fig. 2.15 [67] is taken to be the mean surface pressure profile, aligning with that in Fig. 2.11, then its intermittency would equal $\gamma = 0.5$, as separation in this interaction has been found to closely follow a Gaussian distribution [53, 67–70, 73–76]. Shown in Fig. 2.16 [75] is the intermittency in the separated region, starting at an upstream value of $\gamma = 0$ and ending at S with a value of $\gamma = 1$. The distribution is shown to exhibit a sigmoid shape, in particular one that aligns with the Gauss error function, $\text{erf}(x)$, indicating that a Gaussian distribution is present. Considering λ as the distance from UI to S , as in Fig. 2.15 [67], then γ is approximately equal to the Gauss error function as shown in Eq. 2.11.

$$\gamma(x) \cong \frac{1}{2} \left(\text{erf} \left(4 \frac{x - UI}{\lambda} + S \right) + 1 \right) \quad (2.11)$$

In the limit, the values of γ as an error function approach but never truly reach 0 and 1. Therefore, it becomes practical to bound this function and define a new variable, L_i , to represent the intermittent length over $\gamma = [0.05, 0.95]$ [76]. Dolling and Murphy [69] revisited a prior study by Dolling and Bogdonoff [68] to establish that L_i scales primarily with d for a blunt-fin interaction, yet Clemens and Narayanaswamy [12] note that L_i also increases with S , indicating an interdependence. Nevertheless, Dolling and Murphy [69] showed that a nondimensionalized version of L_i was located in the range $0.42 \leq L_i/d \leq 0.54$. Dolling and Smith [74] similarly revisited the study by Dolling and Bogdonoff [68] and conducted new tests on cylinders to find that L_i/d was in the range of 0.5–0.8.

For data analysis purposes, intermittency measured via pressure signals is defined by a conditional sampling algorithm. In this algorithm, the pressure signal must surpass a certain threshold value, after which the time is measured until the pressure signal drops below this threshold again, allowing for a boxcar transformation [77]. While early approaches used a “best guess” or “eyeing” method for determining the threshold value, Dolling and Murphy [69] provided a quantitative approach using a 3σ offset above the mean undisturbed pressure value, where σ is the standard deviation as defined in Eq. A.3. The approach by Dolling and Murphy [69] was coined the single threshold method. Improvements were made by Dolling

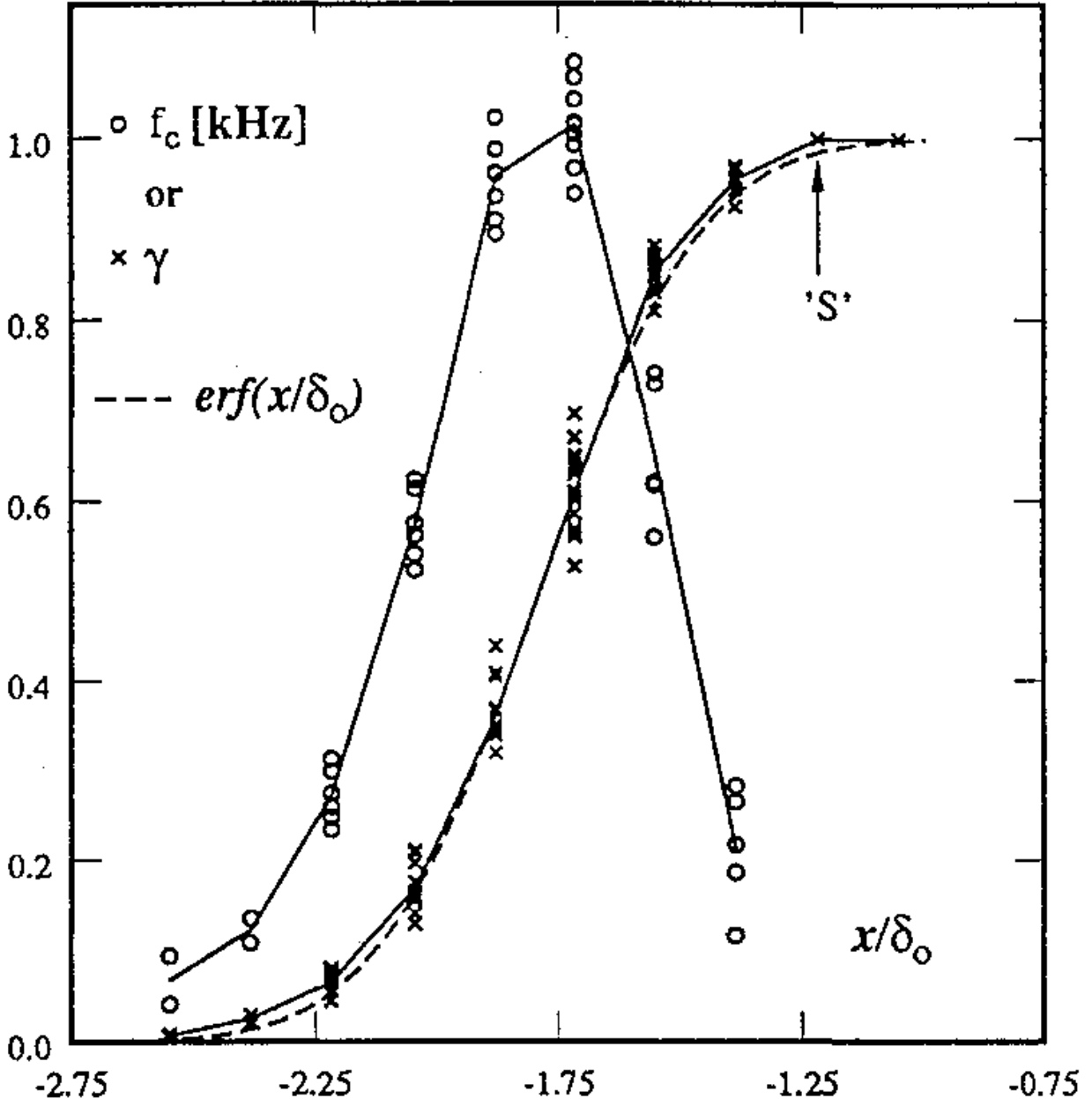


Figure 2.16: Shock zero-crossing frequency and intermittency in the separation region on the floor upstream of an unswept-compression-ramp-generated SWBLI. Figure adapted from Erenkil and Dolling [75].

and Narlo [70] to include a second threshold, and Brusniak [73] further refined this method to the version shown in Eq. 2.12.

$$\begin{aligned}\gamma(x) &= \frac{\sum_i (t_{\text{fall}}(x) - t_{\text{rise}}(x))_i}{t_{\text{total}}} \\ \text{where } t_{\text{fall}}(x) &\equiv P_w(x) < \zeta_1 \\ t_{\text{rise}}(x) &\equiv P_w(x) > \zeta_2 \\ \zeta_1 &= \bar{P}_\infty + 3\sigma_{P_\infty} \\ \zeta_2 &= \bar{P}_\infty + 6\sigma_{P_\infty}\end{aligned}\tag{2.12}$$

Here, t_{fall} is the time at which the pressure signal drops below the threshold value ζ_1 and t_{rise} is the time at which the pressure signal rises above the threshold value ζ_2 . Experimentally, the referenced authors used the wall pressure in the undisturbed boundary layer instead of the freestream pressure. An example two-threshold boxcar transformation is shown in Fig. 2.17 [73].

Applying this method to numerical results becomes difficult. As most LES are performed with a reduced Reynolds number to reduce the grid count, the pressure signal near the wall becomes dampened. It has been shown by Knight et al. [78], Wu and Martín [79], and Ringnette et al. [80] that the wall pressure signals obtained from LES or DNS do not always align with high-Reynolds number experimental counterparts. Ringnette et al. [80] determined that the low-Reynolds number flow, whether obtained via experiments or simulations, contains increased viscous effects that prevent the (separation) shock wave from penetrating as deeply into the boundary layer as a high-Reynolds-number-equivalent flow. The shock wave is instead spread out as a compression fan in the lower half of the boundary layer, and the characteristic return to an undisturbed pressure value as the shock wave passes downstream of a probe location, as seen in Fig. 2.17 [73], does not occur for the low-Reynolds number condition. This invalidates the aforementioned approach as it is fundamentally based on a return to an undisturbed pressure value [80]. Nevertheless, intermittency can still be measured numerically by tracking the separation shock-foot location and applying Eq. 2.10 to a coordinate location.

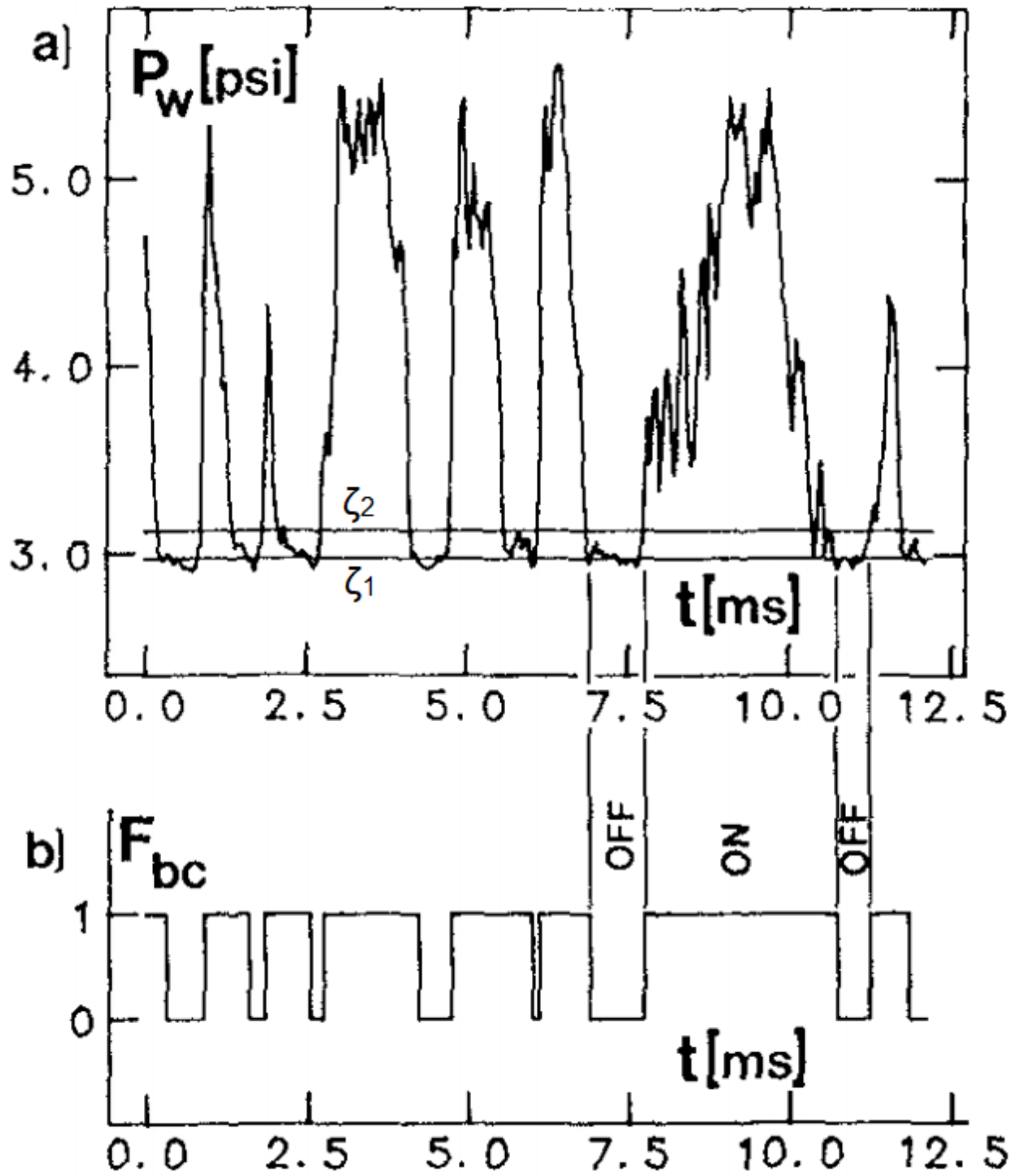


Figure 2.17: Sample boxcar transformation using a two-threshold method. a) Wall pressure signal with both thresholds indicated, b) resulting boxcar signal. Figure adapted from Brusniak [73].

Statistical moments regarding the intermittency were measured by Dolling and Smith [74] and Dolling and Or [53]. They showed that both skewness, Y_1 , and kurtosis, Y_2 , were solely functions of intermittency, and were not influenced by the geometry generating the interaction. Shown in Fig. 2.18 is the skewness of wall pressure in the separated flow region for two types of SWBLI: those generated by blunt fins, and those generated by unswept compression ramps. In both types of interactions, the strength was varied with the blunt-fin diameter and the ramp angle. Yet the results all collapse onto one curve and remain independent of the shock generator, owing to the free-interaction theory described in Sec. 2.2.4.

2.3.3 Shock Zero-Crossing Frequency

Also shown in Fig. 2.16 [75] is the separation shock zero-crossing frequency, f_c , which is defined as the number of shock crossings per second at a given location, as defined in Eq. 2.13.

$$\begin{aligned}
 f_c(x) &= 1/\overline{T_s}(x) \\
 &= 1 / \left(\frac{1}{N-1} \sum_{i=1}^N T_{s,i}(x) \right) \\
 &= 1 / \left(\frac{1}{N-1} \sum_{i=2}^N (t_{\text{rise},i}(x) - t_{\text{rise},i-1}(x)) \right)
 \end{aligned} \tag{2.13}$$

Here, $\overline{T_s}$ is the sample mean separation shock period, $T_{s,i}$ is the i^{th} separation shock period, represented in an experimental manner by the difference in subsequent threshold rising times from Eq. 2.12, and N is the total number of events in the sample. As can be inferred from the figure, the peak zero-crossing frequency is located at a value of $\gamma = 0.5$ [12, 70, 75, 81].

Dolling and Smith [74] determined that the shock zero-crossing frequency was dependent on both d and δ ; by holding δ constant and increasing d , a decrease in f_c was observed and conversely, by holding d constant and increasing δ , an increase in f_c was observed. Overall, the shock zero-crossing frequency remained in the vicinity of 1 kHz, which is in agreement with the results shown in Fig. 2.16 [75]. This behavior appears to affect several different

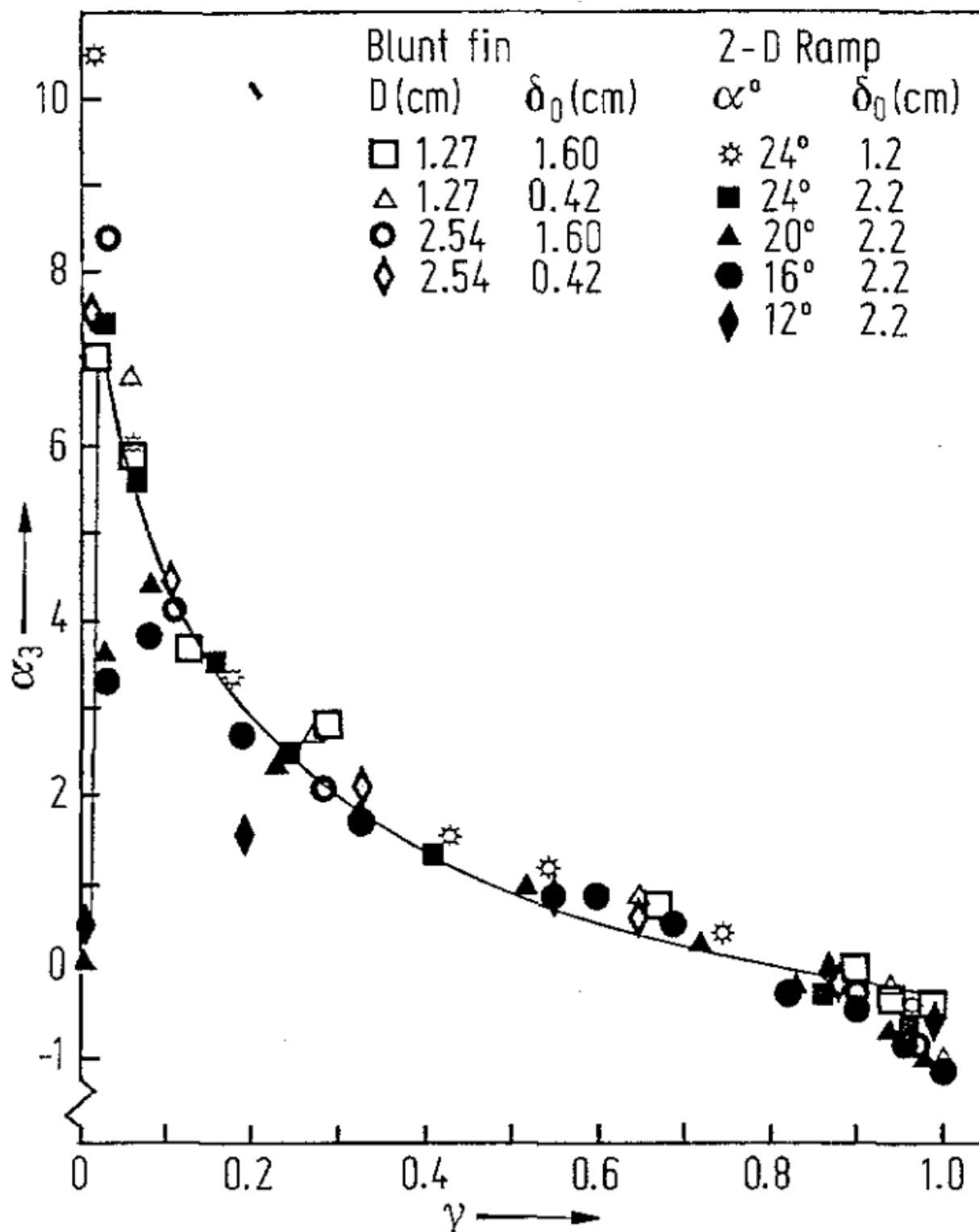


Figure 2.18: Skewness of wall pressure as a function of intermittency. Measurements taken in the separation region on the floor upstream of several blunt-fin-generated SWBLI of varying diameters and of several unswept-compression-ramp-generated SWBLI of varying angles. Figure from Dolling and Or [53].

geometries that generate SWBLI, not just the cylinder and blunt fin [6]. This was further confirmed by Gonsalez and Dolling [81] and Hou et al. [65].

2.3.4 Strouhal Number

Dolling and Narlo [70] showed that f_c is subject to scaling, primarily with d , and that using a Strouhal number, St , based on d and the mean freestream velocity, $\bar{u}_\infty$, yielded a more constant value ranging from $St_d = 0.024$ – 0.031 . Gonsalez and Dolling [81], however, found that a more appropriate scaling is performed when using L_i as the length scale, resulting in a nearly constant value of $St_{L_i} = 0.022$, and thus this equation is shown in Eq. 2.14.

$$St_{L_i} = \frac{f_{\max} L_i}{\bar{u}_\infty} \quad (2.14)$$

Further attempts to scale the Strouhal number to account for changes in Mach number and/or geometry of the shock generator were made by Piponniau et al. [82], proposing to use the length scale as L , the interaction length scale, and a frequency based on the breathing motion of the separation bubble. This bubble frequency is suggested to be a function of the ratio between the mass in the reverse flow and the rate of mass entrainment. A weighting factor was applied to account for the effects of velocity ratio, density ratio, and compressibility through the convective Mach number. Then they used a triangular approximation to define the separation bubble for an impinging shock and an unswept compression ramp. While this approximation is valid for these nominally 2-D cases, application to the 3-D interaction generated by the cylinder and blunt fin is not as straightforward. Piponniau et al. [82] did, however, apply an estimate to blunt fin data from Dolling and Brusniak [83], though the modified Strouhal number appeared to result in a greater variation for the blunt fin case. Consequently, the standard definition in Eq. 2.14 will be used in this study.

2.3.5 Shock-Foot Driving Mechanism

For decades, the driving mechanism has been a source of discussion among researchers. The main point of contention has been whether the unsteadiness in the forward shock is driven

be the inherent unsteadiness of the incoming boundary layer (upstream mechanism) or by the recirculation and breathing motion in the separation bubble (downstream mechanism) [3, 6, 12, 84]. Moreover, each of these views are in itself driven by a frequency range: the unsteadiness of the incoming boundary layer is driven by high-frequency ($\mathcal{O}(10 \text{ kHz} - 1 \text{ MHz})$) fluctuations of the turbulent structures, and the breathing motion in the separation bubble is driven by low-frequency ($\mathcal{O}(100 \text{ Hz} - 1 \text{ kHz})$) movement. Proponents of both sides have made key observations and deduced models that have often either conflicted with the literature or have later been disproven or found to be unrepeatable [6, 12, 82], making it difficult to establish a proper basis. Nevertheless, significant research on the low-frequency unsteadiness has provided a greater insight into the driving mechanism [12, 84].

One key finding was made by Ünalmsis and Dolling [85, 86], namely that Görtler vortices [87] were induced in the expansion section of the wind tunnel nozzle and subsequently affected the boundary layer in the test section, causing low-frequency pulsation in the separation shock foot. Similar findings of coherent Görtler-like structures [87] were also observed by Andreopoulos and Muck [88]. Additionally, Görtler-like structures [87] were found to be a driving mechanism in both the low- and high-frequency ranges in the simulations obtained by Priebe et al. [89] and Pasquariello et al. [90]. Dolling [6] had postulated that wind-tunnel-facility effects may play a factor in the driving mechanism, and that wind tunnel experiments might differ significantly from a true flight environment, but Poggie et al. [91] validated via flight-test experiment that neither wind-tunnel effects nor Reynolds-number effects, e.g. from either wind tunnel experiments or from CFD simulations, factored into the low-frequency dynamics.

Ünalmsis and Dolling [85] further proposed that the thickening/thinning motion of the boundary layer, induced by Görtler vortices [87], was causing the upstream/downstream sweep of the separation shock foot. Beresh et al. [92] replicated the experiment of Ünalmsis and Dolling [85] and determined that the thickening/thinning motion of the boundary layer was in fact not a driving mechanism; however, in the process, they established that the shock-foot unsteadiness was correlated to the u -velocity fluctuation, u' , in the upstream boundary layer. A positive fluctuation relative to the mean velocity corresponded to the upstream velocity profile becoming fuller and consequently moving the shock foot downstream, whereas

a negative fluctuation relative to the mean velocity corresponded to the velocity profile becoming more retarded and moving the shock foot upstream, as visualized in Fig. 2.19. Similar conclusions were drawn by Andreopoulos and Muck [88] and Hou et al. [93], who both quantified the separation shock velocity to be about $0.04\bar{u}_\infty$. This finding can be traced back and corroborated by the discussion in Sec. 2.2.2 concerning shape factor and the fullness/healthiness of the velocity profile. It becomes apparent that a fuller and thus more energized boundary layer will delay separation and subsequently force the separation shock further downstream, and vice versa. Moreover, Toubert and Sandham [94] used linear stability analysis to find a similar convective velocity of $0.05\bar{u}_\infty$ in the separation bubble, suggesting the global instability in the separation bubble may be a possible driving mechanism.

A model for the separation bubble mechanism was developed by Wu and Martín [95] for a compression-ramp-generated SWBLI, with similar findings obtained by Piponniau et al. [82] for an impinging-shock-generated SWBLI, in which the mass entrainment within the separation bubble was considered and consisted of a periodic mechanism. The (mass) recharge rate and the entrainment rate of the separated shear layer both led to a flapping motion that dictated low-frequency pulsation in the separation bubble. Additional support for these observations was provided by Priebe and Martín [96].

Brusniak and Dolling [15] described the overall unsteadiness as an entirely coupled mechanism in what they refer to as the “global flow field hypothesis”. They emphasized caution in placing too much weight on one sole mechanism and postulated that several driving mechanisms could factor into the unsteadiness in the interaction, a viewpoint that has been shared by successive authors [6, 12, 84]. However, Dussauge and Piponniau [97] and Clemens and Narayanaswamy [12] did conclude that the downstream mechanism plays a stronger role for strongly-separated SWBLI and that the upstream mechanism plays a stronger role for weakly-separated or incipient SWBLI. As the cylinder- and blunt-fin-generated SWBLI is considered a strongly-separated SWBLI, this study will pay particular attention to the downstream mechanism and the breathing motion of the separation bubble.

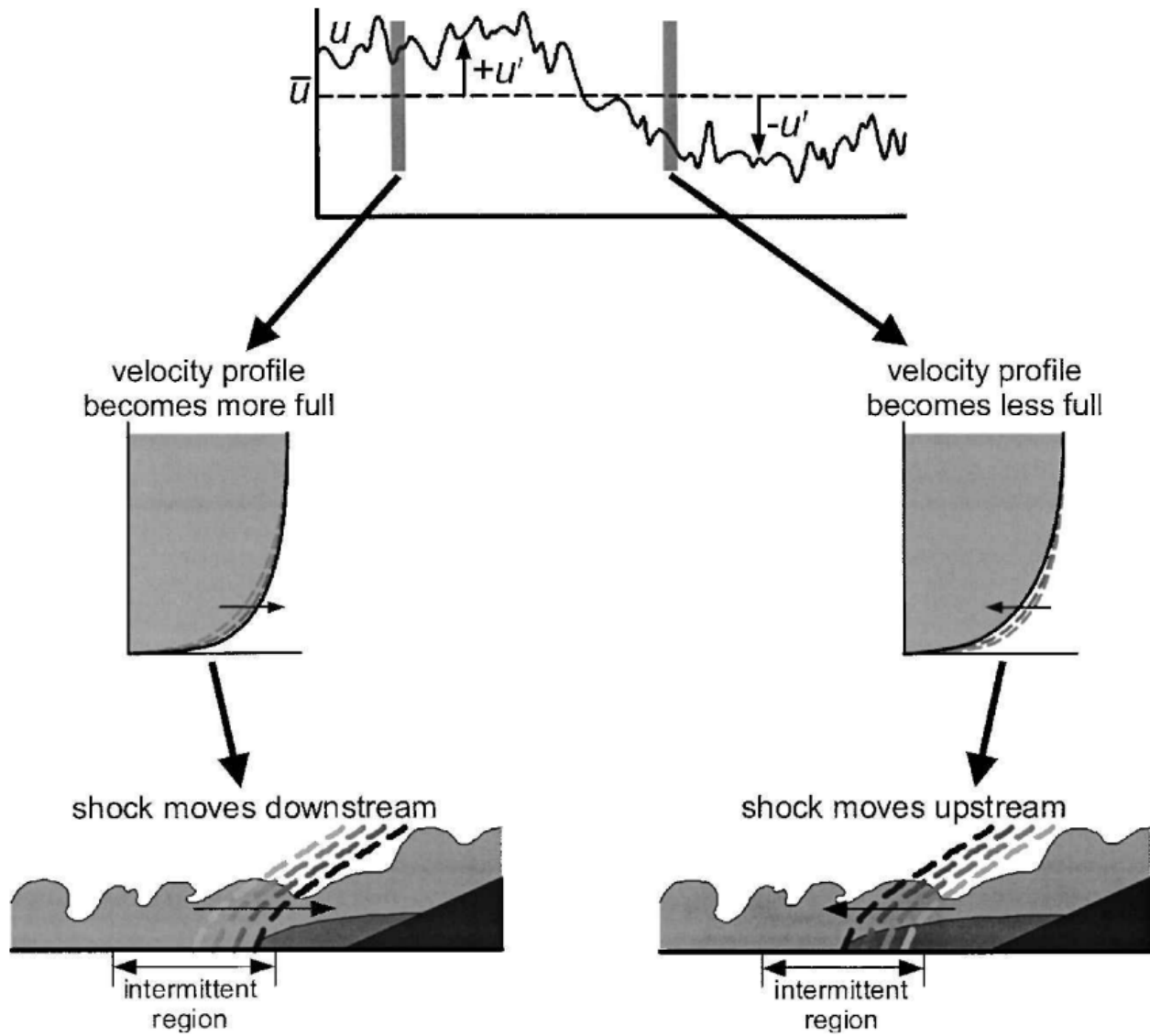


Figure 2.19: Relationship between the fluctuations in the incoming boundary layer and the shock-foot unsteadiness in an unswept compression ramp SWBLI. Figure from Beresh et al. [92].

2.4 Prior Centerline Interaction Scaling Attempts

A myriad of studies have been conducted in an attempt to attain *a priori* knowledge of the scales of the interaction. This provides high-speed vehicle designers with critical knowledge on how to shape blunt aspects of the vehicle. As mentioned in Sec. 2.1, the primary independent variable on the scales of the interaction is the cylinder or blunt-fin diameter, d , with the freestream Mach number, M_∞ , and boundary-layer thickness, δ , having secondary scaling effects, and the Reynolds number, Re , having a tertiary effect. The most common way for prior studies to measure the scale is through the separation distance, S , the peak pressure on the floor upstream, $(\bar{P}/\bar{P}_\infty)_{\max}$, and the peak heat transfer rate on the floor upstream, $(\bar{q})_{\max}$.

In 1962, Sykes [21] was among the first to discover that d is the primary variable affecting the interaction scale by using pressure orifices and schlieren imaging, and this has since been reinforced by several authors [13–20, 22–26, 30, 46]. The practice has been to normalize all other parameters by d . This works well for the inviscid phenomena, but not those where viscous effects are present. Unfortunately, the only truly inviscid phenomenon in the cylinder- and blunt-fin SWBLI is the bow-shock with centerline stand-off distance, B , which does scale very well with d and M_∞ , as described in Sec. 2.2.1. For all the other dependent variables in the flow field, variations are induced by differing values of δ , M_∞ , and Re . Consequently, no real correlation has been found thus far, despite many attempts to do so.

The major focus has been to establish a correlation for S/d , as the forward shock determines the length scale of the interaction along the centerline and could reveal a normalized behavior for other dependent variables. Westkaemper [17] concluded that $S/d \approx 2.65$ over a wide range of Mach numbers ($M_\infty = 2\text{--}20$) and that there was no clear effect due to δ . The constant separation distance was later on disproven by several authors, including Young et al. [22], Sedney and Kitchens [18], Dolling and Bogdonoff [13, 14], Fomison and Stollery [24], and Özcan and Yüceil [25, 98]. These authors conducted studies on scaling factors and found a correlation to exist with the factor d/δ . Young et al. [22] were among the first to mention this factor after testing a wide variety of blunt fins under differing flow conditions, concluding that the interaction scales with d when $d > \delta$.

Sedney and Kitchens [18] performed a systematic study of the independent variable effects on S/d for cylinders and found that this was a function of h/d , d/δ , Re_d , and M_∞ . A trend was found for d/δ , but a conclusion could not be drawn. Instead, it was determined that for large cylinders, S/d is solely a function of M_∞ . Dolling and Bogdonoff [13, 14] used blunt fins to conclude that although S/d is primarily dependent on d , a more consistent scaling of secondary effects is achieved by scaling with d/δ . They found that this trend holds true even when the lambda-shock structure is partially or fully immersed in the boundary layer. Fomison and Stollery [24] stated that the blunt-fin interaction is independent of M_∞ and Re_∞ , and also of d/δ for values of $d/\delta \geq 2$. Özcan and Yüceil [25, 98] corroborated some of the findings made by Sedney and Kitchens [18] and expanded on their proposed correlations.

Westkaemper [17] also concluded that the interaction was independent of M_∞ , a statement that was criticized by Kaufman et al. [23] and Sedney and Kitchens [18]. Dolling and Bogdonoff [13, 14] pointed out that the effect of M_∞ was unclear, particularly because M_∞ directly affects δ , such that the two variables may be coupled.

It quickly becomes evident that the conclusions found in the archival literature are not always consistent with one another. What seems to work well in establishing correlations for one set of collected data does not share universal agreement with data collected by others. One point of caution is that S is not always correctly identified in the archival literature. For example, Dolling and Bogdonoff [13] used a combination of oil-flow visualization, initial pressure disturbance, and initial heat transfer ratio disturbance to define the separation distance. However, as previously mentioned in Sec. 2.3.2, the initial pressure and heat transfer ratio disturbances occur upstream of actual flow separation. As a result, comparisons to archival literature data are solely performed against studies in which oil-flow visualization was used, as this remains the most effective experimental method in determining the separation line [18]. Table D.1 summarizes these data with key quantities of interest.

Few experimental studies have investigated the triple-point height, h_{tp} , as this requires an optical diagnostic technique. Westkaemper [17] derived a simple geometric formula based on S/d , B/d , and the angle of the forward shock. As it stands, this formula only works when the curvature of the forward shock is very small, as it does not take into account the near-wall curvature described in Sec. 2.2.3. Kaufman et al. [23] provided a closer analysis of the

flow surrounding the triple point by using shadowgraphy and intrusive Pitot pressure probes, providing an initial characterization of the jet, as shown in Fig. 2.3. However, no correlation to the remaining interaction scales was made. Notably, few computational studies that have been performed quantify the triple-point height, and instead usually only reference it with regard to leading-edge surface pressure peaks. It is often left to the reader to extrapolate h_{tp} from a centerline contour plot, and the accuracy by which this can be obtained is adequate at best.

Several studies have also tried to obtain scaling parameters for the surface pressure on the centerline upstream of the interaction and on the cylinder leading edge [14–16, 19, 20, 22, 24–26, 46]. Some of the findings were described in Sec. 2.2.4 and 2.2.6, including the consistent shapes of the pressure profiles normalized by freestream pressure or stagnation pressure. These remain primarily dependent on x/d and y/d for the surface and leading-edge pressure profiles, respectively. Variations in the magnitudes of these pressure ratios exist based on M_∞ , Re_∞ , and d/δ , with M_∞ having a stronger effect.

2.5 Prior Off-Centerline Research

Korkegi [4] stated in 1971 that off-centerline surface flow characteristics of the interaction were only known qualitatively. Continued research has since provided limited insight to these properties. Many authors have provided qualitative surface oil-flow images [20, 22, 23, 30, 46, 99], but not many have performed quantitative analyses. Dolling and Bogdonoff [14] reported in 1982 that the flow in the spanwise direction has received very little attention. Consequently, they performed a study to measure mean surface pressures off the centerline. They found that the peak pressure ratio decays rapidly over $z/d = 0$ to $z/d = 1$ from $\bar{P}_w/\bar{P}_\infty \approx 6$ to $\bar{P}_w/\bar{P}_\infty \approx 2$, and is barely detected at $z/d = 5$. At $z/d = 55$, the effect of the interaction is negligible, with a single peak at $\bar{P}_w/\bar{P}_\infty \approx 1.1$ [14], which could characterize the maximum spanwise extent of the interaction. Similarly, Price and Stallings [19], Kaufman et al. [23], Hung and Buning [16], and Özcan and Yüceil [25, 98] all reported mean surface pressure ratios and found that the maximum noticeable extent of the interaction was around $z/d \approx 5.5$. In addition, Hung and Buning [16] and Özcan and Yüceil [25, 98] provided surface

skin-friction lines, indicating separation and attachment lines. Burbank et al. [27], Young et al. [22], Winkelmann [29], Fomison and Stollery [24], Barnhart and Greber [26], and Yamamoto and Takasu [100] also reported mean surface pressure ratios off the centerline but did not make measurements far enough outboard to characterize the extent of the interaction.

Dolling and Bogdonoff [14] found that the surface pressure profiles along a constant z -ordinate collapse as a function of B/d , where B is the local streamwise location of the inviscid bow shock from Sec. 2.2.1 projected onto the floor. A depiction of this is shown in Fig. 2.20. The shift in x is thus a simple translation and not a scaling factor, which makes it convenient to study the decreasing strength of the surface pressure profiles in the outboard direction.

Dolling [101] determined that, similar to point e in the inviscid bow shock in Fig. 2.4, there is an outboard region upon which the blunt-fin-generated SWBLI mimics a sharp-fin-generated SWBLI. The interaction region of the blunt-fin-generated SWBLI was divided up into three regions: an “inner” region, where the bluntness and diameter primarily affect the scaling; an “outer” region, where the bluntness and diameter no longer affect the scaling; and a “boundary” region that serves as a transition layer between the inner and outer regions. In the inner region, the flow is said to be “leading-edge dominated”, as it is primarily dependent on d , and showed secondary scaling based on δ . However, in the outer region, the flow is primarily dependent on δ , as it would be for the entire flowfield around a sharp-fin-generated SWBLI. Dolling [101] was able to define the boundary extent for the outer region as a function of z/d when the angle of attack of the fins was $\alpha \geq 4^\circ$, but could not conclude a limit for lower angles of attack. Notably, at $\alpha = 0^\circ$, the sharp fin does not generate a SWBLI, and a direct comparison to the blunt-fin-generated SWBLI flowfield cannot be made.

Korkegi [4] also stated that the details of the flow field, including the horseshoe vortices, were poorly understood. Fomison and Stollery [24] provided some insight by generating qualitative sketches of the vortices originating from the separation bubble and their downstream and spanwise paths. Most other studies, including computational ones such as Hung and Buning [16] and Yamamoto and Takasu [100] only inspected the vortical structures at the centerline.

To date, the vast majority of prior research on the cylinder- and blunt-fin interaction has been conducted through experiments, particularly surface pressure measurements, oil-flow

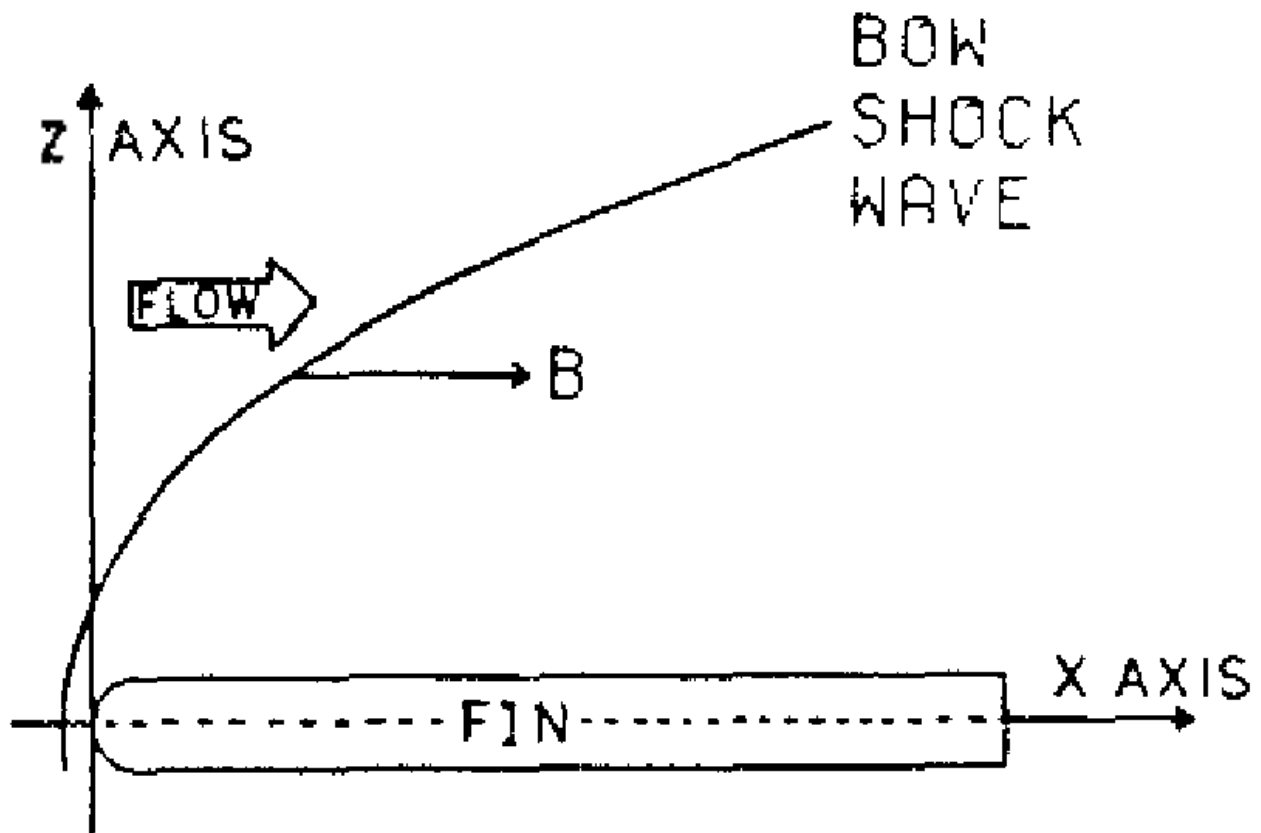


Figure 2.20: Coordinates used with streamwise origin of B in the spanwise direction, representing the inviscid bow shock projected onto the floor. Figure adapted from Dolling and Bogdonoff [68].

visualization, and schlieren imaging [6]. Planar, non-path-integrated diagnostic techniques have, to the author’s knowledge, not yet been applied to spanwise cross-sections of the flow. CFD simulations would provide the most effective method of quantifying the flow features along spanwise cross-sections. Hung and Buning [16] in 1985 used steady-state Reynolds-averaged Navier–Stokes (RANS) simulations with a Baldwin–Lomax algebraic turbulence model [102] to analyze the pressure and limiting streamlines of the floor surface near a blunt fin, as well as the surface pressure distribution along $z/d = 1, 2, 3$, and 4, and the fin surface pressure along the azimuthal direction. Yamamoto and Takasu [100] in 1998 used unsteady RANS (URANS) with the Baldwin–Lomax algebraic turbulence model [102] to investigate sweepback on blunt fins. Pressure distributions at $z/d = 0.5$ and $z/d = 1.5$ were analyzed and provided good agreement with experiments. More recently, in 2022, Kolesnik et al. [103] revisited the case from Hung and Buning [16] that referenced experimental conditions from Dolling and Bogdonoff [14] and used modern CFD methods to provide further detail to the fin surface and floor off-centerline pressure profiles. Although this provided additional resolution for a prior case, it did not draw any new conclusions regarding spanwise development of the flow.

Chapter 3

Methodology

Many studies have focused solely on investigating SWBLI experimentally or computationally. Often times, experimentalists do not report all of the necessary flow or boundary conditions necessary for a computationalist to truly replicate the experiment. Settles and Dodson [104] thus sought to create a database of experimental SWBLI studies that include sufficient data of the environment such that it can be easily reenacted by a computational study. Consequently, the importance of communication between experimentalists and computationalists becomes apparent, and the recommendation towards performing combined experimental and numerical studies instead of using such a database has been made [6, 104, 105]. As a result, both wind tunnel experiments and CFD simulations were performed to obtain data in this study, though the emphasis was strongly placed on the simulations. The experiments conducted used a relatively simple technique consisting of high-speed schlieren imaging, though more complex methods were applied to additional in-house experiments that are also shown; these consisted of surface oil-flow visualization, high-speed pressure transducers, and the use of fast-acting unsteady pressure-sensitive paint (uPSP). The CFD simulations consisted of both Reynolds-averaged Navier–Stokes (RANS) and implicit large-eddy simulations (ILES). A description of the assumptions made is provided first, followed by the setup and execution of the wind tunnel experiments, and then the setup and execution of the CFD simulations.

3.1 Flow Modeling Assumptions

This study assumed a continuum flow with a compressible, Newtonian fluid, namely air, that contains no impurities. A calorically-perfect gas assumption is made, simplifying the ideal gas equation of state from Eq. 3.1 to Eq. 3.2.

$$P = \rho RT \quad (3.1)$$

$$P = (\gamma - 1)\rho e \quad (3.2)$$

Here, ρ is the density, R is the specific gas constant, T is the temperature, γ is the ratio of specific heats, and e is the specific internal energy.

For the experiments, it was assumed that the stagnation conditions in the wind tunnel were constant during data collection, and that the freestream turbulence intensity was around 1% [106]. The experimental models were machined to have a tolerance of 0.005 in (127 μm). The flat plate model has a leading edge that was considered sharp at the time of machining, but due to repeated testing was instead assumed to have a blunted leading edge with a radius matching the machine tolerance. Crossflow effects due to a planar nozzle were not taken into account, and neither were variations in the freestream or span due to surface imperfections in the nozzle.

For the simulations, RANS simulations were assumed to generate the statistical mean flow in the interaction, and large-eddy-type simulations were assumed to provide a representative replication of the wind tunnel turbulent flow. The wind tunnel and the experimental models were replicated in the simulations with perfect machining precision; they were modeled without surface roughness, and the experimental models were assumed to have been perfectly installed into the wind tunnel. Moreover, the flow was considered adiabatic in the simulations, although it should be noted that an isothermal wall treatment would yield a more accurate comparison. Although the models used in the RANS simulations matched the experimental model height initially, this was extended to the height of the wind tunnel

test section to fully ensure a semi-infinite behavior; consequently, some of the experimental models were also created to span the length of the test section for direct comparison.

3.2 Wind Tunnel Facility

The High-Speed Wind Tunnel (HSWT) at the University of Tennessee Space Institute (UTSI) was used to perform all of the experiments listed here. It is a low-enthalpy blowdown facility with air as the working fluid, and is depicted by a computer-aided design (CAD) model in Fig. 3.1. Air is supplied by a 23.6 m³ bottle farm compressed to at most 20 MPa. The facility features a stilling chamber that maintains the stagnation pressure by a control valve and allows for run times up to 2 min, although the run times in this study were typically less than 30 s. Typical operating stagnation pressure values are around $\bar{P}_0 = 30\text{--}35$ psia (210–240 kPa). The flow is not pre-heated and the air is cooled by expansion through a mass-control valve, resulting in a mean stagnation temperature of $\bar{T}_0 = 287$ K. A planar nozzle accelerates the flow to a mean freestream Mach number of $\bar{M}_\infty = 2.01$ and a unit Reynolds number of around $\text{Re}_\infty = 3.0 \times 10^7 \text{ m}^{-1}$, depending on the stagnation pressure applied. The flow reaches a test section with cross-sectional dimensions of 8 in $\times$ 8 in (203 mm $\times$ 203 mm) and length 72 in (1.829 m). As inferred by the figure, the test section is divided into three segments. All experimental measurements were taken in the first third of the test section, where a turbulent boundary layer had already developed naturally on the floor. The mean boundary-layer thickness on the floor has been measured to be approximately $\delta = 11$ mm in the first third of the test section [107], with variation occurring depending on the stagnation pressure applied. Optical access to the experimental models is ensured through the use of BK7 glass windows on the wind tunnel sidewalls and an interchangeable window made of either BK7 glass or acrylic glass (poly(methyl methacrylate), PMMA) on the ceiling.

3.3 Test Models

A flat plate model was installed such that an experimental model was optically visible in the first third of the test section. The flat plate rested on a diamond airfoil sting, and both

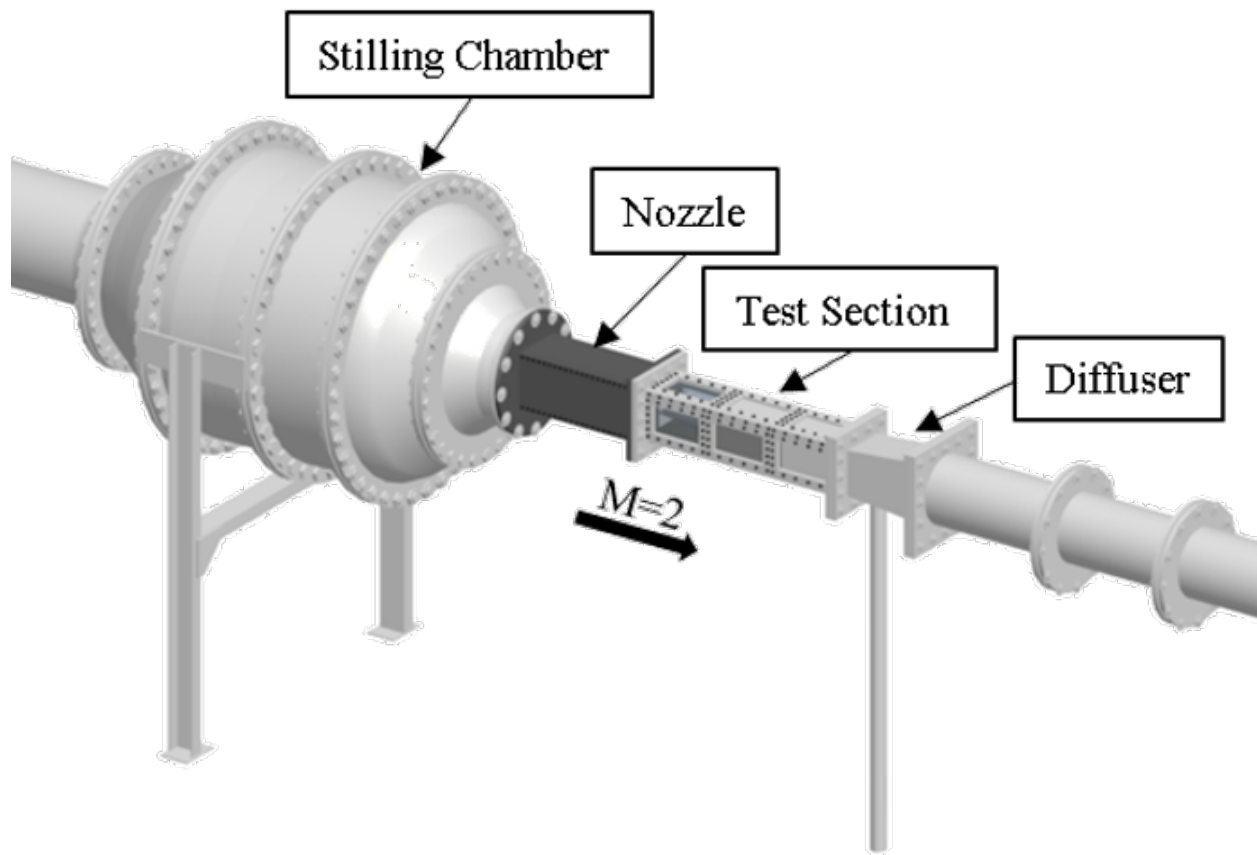


Figure 3.1: CAD model of the High-Speed Wind Tunnel at the University of Tennessee Space Institute. Flow is from left to right. Figure from Kocher et al. [107].

were manufactured using grade 303 stainless steel. A CAD model of the flat plate and sting is shown in Fig. 3.2 [108, 109]. The flat plate had a span of 8 in (203 mm), thus spanning the width of the wind tunnel test section, and a length of 182 mm. As mentioned in Sec. 3.1, although the flat plate was machined with a sharp leading edge, repeated testing had resulted in a blunted tip; the angle at which the leading edge was manufactured was 10° . The flat plate was installed at an angle of attack of -6.3° , such that the upper surface experienced a wedge-type flow with a favorable pressure gradient. Moreover, the oblique shock wave generated by the leading edge resulted in a different set of flow conditions, which are listed in Table 3.1 for two different stagnation pressures.

The wind tunnel test section floor was also used as a means of testing cylinders that have a larger diameter and are also subjected to a thicker boundary layer. A floor insert was created, visible in the CAD model of Fig. 3.2, upon which the cylinder is mounted. The floor insert was also made using grade 303 stainless steel and had a span of 2 in (50.4 mm), a length of 4 in (100.8 mm), and a thickness of 0.5 in (12.7 mm). A small gap initially existed with respect to the wind tunnel floor, which resulted in an oblique shock being generated. This gap was treated using vacuum grease and smoothed out using an exacto knife; this process is shown in Fig. 3.3. The grease treatment removed the oblique shock and had a negligible impact on the SWBLI, as validated by schlieren imaging.

Several cylinder models were implemented to create a test matrix that would encompass a wide range of flow conditions. The cylinders were made of stainless steel again, and were mounted orthogonally to the surface and thus perpendicular to the flow, whether this was on the flat plate or on the wind tunnel floor. The cylinders varied in diameter, resulting in the test matrix listed in Table 3.2. Both experimental and computational runs were performed to address the first objective from Sec. 1.2: establishing the scaling of flow features.

3.4 Performed Wind Tunnel Experiments

The experimental campaign was broken up into four sections. One of those sections was performed by the author, namely case E2 in Table 3.2, and the other three sections were performed in-house by others in the research group. Case E3 in Table 3.2 was not directly

Table 3.1: Mean flow conditions on flat plate model upper surface and in wind tunnel test section. Conditions shown for two different experimental stagnation pressures and one computational stagnation pressure.

	Flat Plate Edge			Test Section Freestream/Floor Edge	
P_0 [kPa]	239	209	79.7	240	210
ρ_0 [$\frac{\text{kg}}{\text{m}^3}$]	2.902	2.537	0.967	2.914	2.550
T_0 [K]	287	287	287	287	287
P_∞ [kPa]	42.6	37.2	14.2	30.2	26.4
ρ_∞ [$\frac{\text{kg}}{\text{m}^3}$]	0.847	0.739	0.282	0.663	0.579
T_∞ [K]	175.35	175.35	175.35	158.74	158.74
M_∞	1.784	1.784	1.784	2.01	2.01
u_∞ [$\frac{\text{m}}{\text{s}}$]	473.54	473.54	473.54	507.63	507.63
Re_∞ [m^{-1}]	33.745×10^6	29.527×10^6	11.248×10^6	31.026×10^6	27.148×10^6
δ [mm]	0.4	0.4	0.4	11.0	10.6

Table 3.2: Test matrix with run conditions for experiments (E) and computations (C).

Case	Model	Re_∞ [m^{-1}]	d [mm]	δ [mm]	d/δ	h [mm]	Trips
E1	Floor	3.1×10^7	9.525–19.05	11.0	0.87–1.73	76.2–101.6	No
E2	Floor	2.7×10^7	9.525–12.7	10.6	0.90–1.20	203	No
E3	Flat Plate	3.0×10^7	3.175	0.4	7.94	12.7	No
E4	Flat Plate	3.4×10^7	1.588–3.175	0.4–1.4	1.19–6.02	12.7	Yes
C1	Floor	3.1×10^7	3.175–25.4	1.32	2.40–19.17	203	-
C2	Floor	3.1×10^7	3.175–25.4	2.65	1.20–9.58	203	-
C3	Floor	3.1×10^7	3.175–25.4	5.30	0.60–4.79	203	-
C4	Floor	3.1×10^7	3.175–25.4	10.60	0.30–2.40	203	-
C5	Floor	3.1×10^7	3.175–25.4	21.20	0.15–1.20	203	-
C6	Flat Plate	1.1×10^7	1.588	1.22	1.30	40	Yes

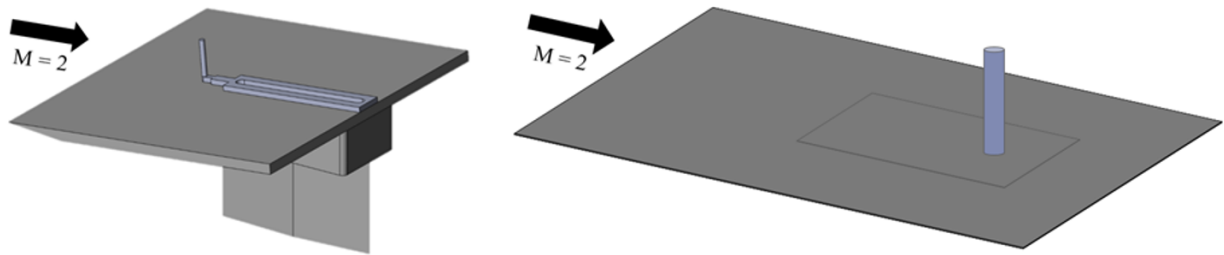


Figure 3.2: CAD model of the flat plate with diamond airfoil sting and CAD model of the floor insert. Both models show an installed cylinder. Figure with flat plate from Lash et al. [108], figure with floor insert from Combs et al. [109].



Figure 3.3: Photograph of wind tunnel test section with floor insert and mounted cylinder. Flow would be right to left. Vacuum grease is applied to the gap created by the floor insert. Excess grease is removed with an exacto knife.

part of this study, as data from it were previously collected and published [108, 110]; however, the data were relevant and available, and were thus included in the analysis section of this study. The goal of the experimental campaign was to provide sufficient data points to assess and determine a possible direct relationship between d and δ . As such, case E4 replicated parts of case E3 but included boundary-layer trips in the form of spanwise strips near the leading-edge of the flat plate upper surface. This enabled additional lower d/δ values for comparison on the flat plate. It should be noted, however, that the boundary-layer thickness for case E4 was estimated with significant error, owing to its small size and optical equipment limitations. A best-judgment approach based on the pixels collected from schlieren imaging, coupled with an approximation from Schlichting [37], thus yielded the value provided in the table.

High-speed schlieren imaging was employed to permit time-resolved shock position measurements. While this is a path-integrated technique, the density changes at (and in the near vicinity of) the centerline have a much greater magnitude than anywhere else in the flow. As a result, the primary structures captured by schlieren imaging are the centerline shock structure. A schematic of the Z-type schlieren system is shown in Fig. 3.4. A high-powered, pulsed light-emitting diode (LED) of the type CBT-140 by Luminus Devices, Inc., provided high-intensity, pulsed light at the selected acquisition rate with a 700 ns pulse duration; this duration was short enough to essentially freeze the motion of the shock structures in each image. This light source was developed based on the work of Willert et al. [111]. The sourced light encounters a schlieren mirror that collimates the light with focal length 2.67 m, passes it through the test section, until it is refocused with another schlieren mirror, parted with a horizontal knife edge, and subjected to a high-speed camera at its final focal point. The camera in use was a FASTCAM Mini UX100 from Photron USA, Inc., fitted with a 150 mm lens from SIGMA Corporation. Tens of thousands of images were collected during a single run. Those images were acquired at a resolution of 1024×72 pixels and a frame rate of 64 kHz; this frame rate had been shown to ensure the time-resolved measurement of the shock motion [108]. In addition, a larger field-of-view was used for insight of the start-up, nominal operating, and shut-down procedures; for these, a resolution of 896×488 pixels

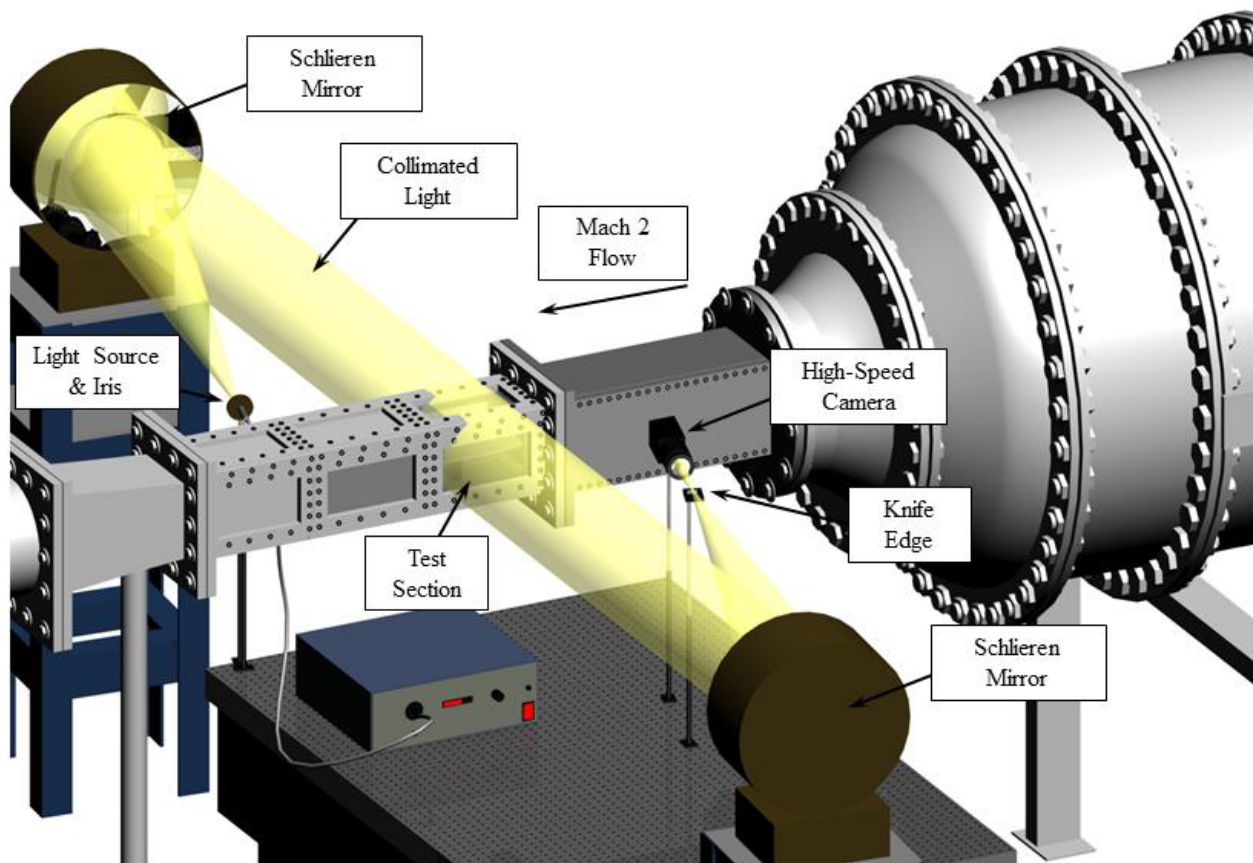


Figure 3.4: CAD model of the Z-type schlieren system employed. Flow is from right to left. Figure from Combs et al. [109].

and a frame rate of 50 Hz were selected. Further details to the schlieren imaging setup are provided by Combs et al. [112].

To ensure there were absolutely no effects of the cylinder height, h , factoring into the SWBLI, case E2 used cylinders that spanned the height of the test section. While the cylinders for both cases E1 and E2 were mounted using a floor insert, an additional mount on the ceiling was also required for the full-height cylinders. As such, a new ceiling piece was created out of grade 6061 aluminum that enabled mounting of the cylinders along with optical access using acrylic glass upstream of the mount. While attempting to mount a cylinder, it was discovered that the test section height was actually 202.7 mm. Subsequently, all of the cylinders were trimmed to match this length. In addition to the cylinder diameters listed in Table 3.2, two larger cylinders with $d = 19.05$ mm and $d = 25.4$ mm were tested but caused too much blockage to choke the flow. Moreover, two smaller cylinders with $d = 3.175$ mm and $d = 6.35$ mm were tested but encountered too much deformation to be considered orthogonal.

The campaign for case E2 consisted of three days of testing. During the first two days, the low-frequency acquisition rate of 50 Hz was selected to visualize the overall behavior. On the third and final day, the high-frequency acquisition rate of 64 kHz was selected. To avoid hysteresis, data were collected first with the smallest diameter, increasing to the largest diameter, then re-testing the largest diameter and reversing the order. No observable differences in the quantities of interest were detected. As a final detail, the ambient temperature in the laboratory containing the test section was around 295 K and the humidity was around 60%, as measured by a household thermometer and hygrometer.

3.5 Complementary Wind Tunnel Experiments

In-house experiments were also performed with high-speed schlieren imaging, both by itself and in combination with another method. In any event, the setup for the high-speed schlieren imaging was nearly identical to the one described in Sec. 3.4; the only difference was that additional images were acquired at a resolution of 1024×32 pixels and a frame rate of 100 kHz. The high-speed schlieren imaging was frequently combined with either high-speed

pressure transducers, surface oil-flow visualization, or uPSP, and sometimes these methods were also used by themselves for investigative purposes. Simultaneous data collection was enabled using two delay generators of the type DG535 from Stanford Research Systems.

Discrete surface pressure measurements were enabled using multiple pressure transducers of the type XCQ-062 from Kulite Semiconductor Products, Inc. The pressure transducers were installed on the flat plate [110, 113] and on the test section floor using the floor insert [108]. In both cases, the pressure transducers were located upstream of the cylinder and along the centerline, with the intent to capture the location and dynamic behavior of the forward shock. Data were collected simultaneously with the high-speed schlieren imaging at an acquisition rate of 100 kHz. Further details on this method are described by Lash et al. [110] and Combs et al. [113]. The use of pressure transducers mainly served to validate the high-speed schlieren imaging as a viable option to obtain the quantities of interest [113], and subsequent analyses were performed without pressure transducers.

Surface oil-flow visualization was performed to capture the mean separation location [108, 114]. Grade 5W-30 motor oil was mixed with titanium dioxide (TiO_2) in a 4 : 1 ratio by volume and applied to the surface of the flat plate or the test section floor. An approximately 2 mm thick coat was applied and covered the entire field-of-view. A FASTCAM Mini UX100 was mounted above the ceiling of the test section and collected images at a resolution of 1280×1024 pixels and a frame rate of 50 Hz. Capturing images during a wind tunnel run instead of after the run was necessary to avoid any distortion of the oil mixture that occurred during the wind tunnel shutdown procedure. Similar to the pressure transducers, surface oil-flow visualization was used to validate the high-speed schlieren imaging for the mean features [113]. A direct comparison of the two methods is shown in Fig. 3.5, where an extrapolated two-point line from the forward shock to the surface is shown to be just upstream of the actual separation line obtained from the oil mixture. This offset was consistently measured to be 2 mm upstream, for both the test cases on the test section floor and on the flat plate, thus allowing for a direct comparison. Note that the curvature of the forward shock near the surface is not taken into account in the extrapolation, yet the actual separation location does align well with the two methods. Because the high-speed schlieren images were able

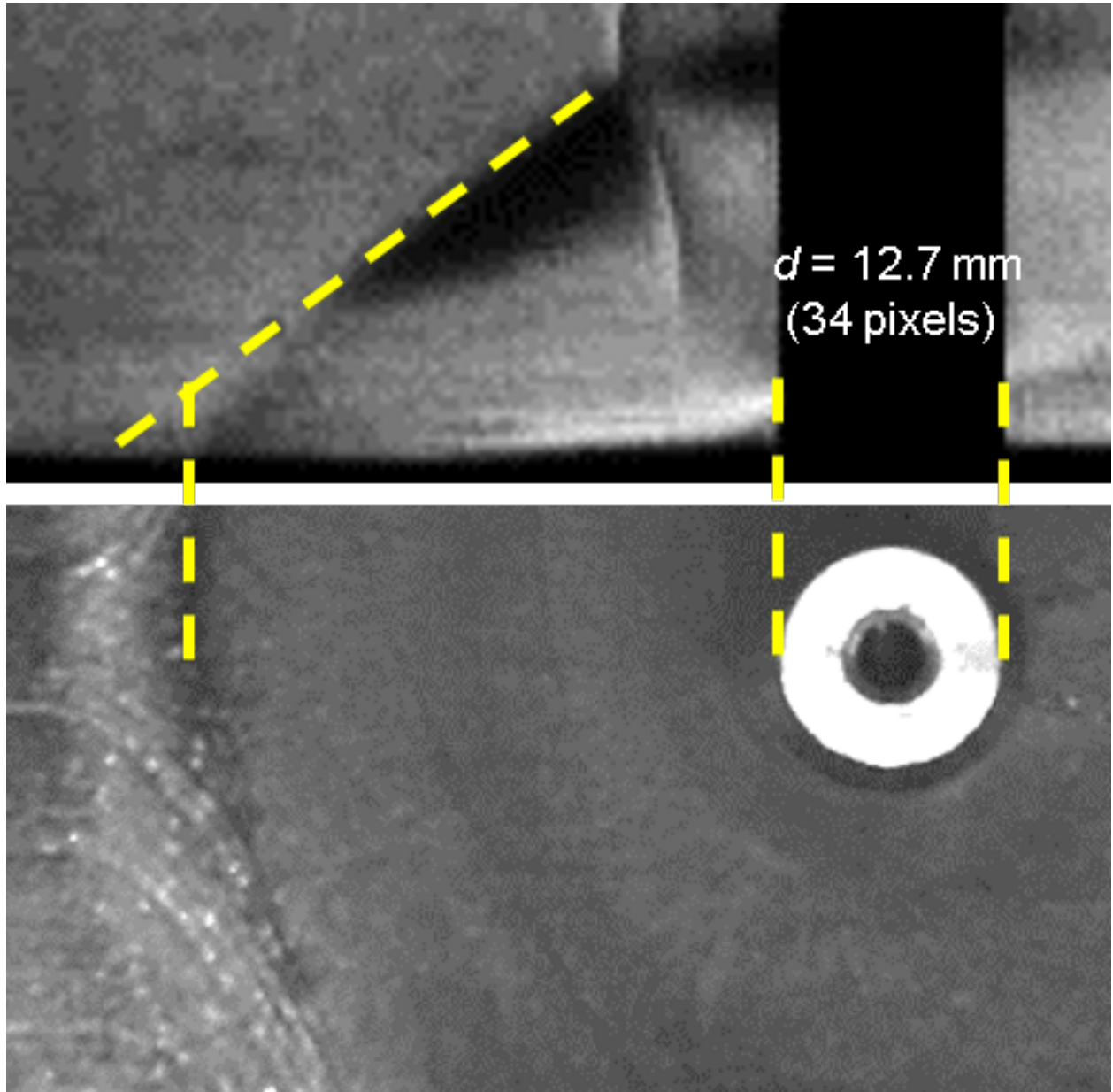


Figure 3.5: Comparison between time-averaged schlieren imaging and surface oil-flow visualization. Flow is from left to right.

to capture the mean separation location with the 2 mm offset applied, subsequent analyses were performed without surface oil-flow visualization.

Preliminary work using uPSP was performed using paint from Innovative Scientific Solutions, Inc. More details about the general methodology and how it differs from regular PSP are described by Crafton et al. [115]. However, uPSP has been shown to provide temporal resolution in excess of 10 kHz [116, 117], which is sufficient to resolve the dynamic behavior of the SWBLI. The paint consisted of platinum tetra(pentafluorophenyl) porphyrin (PtTFPP) in a porous polymer/ceramic binder that reached peak excitation at 400 nm. Two 2-in, water-cooled, ultraviolet LED arrays of the type LM2X-DM from Innovative Scientific Solutions, Inc., provided continuous illumination and excitation of the paint. A FASTCAM Mini AX200 from Photron USA, Inc., with a 60 mm lens captured images at a resolution of 640×480 pixels and a frame rate of 20 kHz. A schematic detailing this setup is provided in Fig. 3.6. Simultaneous data collection with high-speed schlieren imaging was also performed. Further details on this method are described by Lash et al. [108, 118].

3.6 Reynolds-averaged Navier–Stokes Simulations

The vast majority of the RANS simulations were conducted using CFD++ from Metacomp Technologies, Inc. Only a few RANS simulations were performed using Overflow from the NASA Langley Research Center to ensure consistent results were obtained. This section will describe the setup of CFD++ in detail, whereas Sec. 3.7 will cover the setup of Overflow.

The RANS simulations were run on a local cluster at UTSI that used the operating system Red Hat Enterprise Linux (RHEL) Server 7.2. The cluster contained 14 nodes with 16 cores each for a total of 224 cores. All of the cores were of the type Intel[®] Xeon[®] E5-2630 v3, clocking at 2.4 GHz and producing roughly 3.07×10^{11} floating point operations per second (307 GFLOPS). Each node had 64 GB of random access memory (RAM) available.

3.6.1 CFD++ Equation Set

As this research spanned several years, a number of versions of CFD++ were used, ranging from v15.1 to v18.1, though the majority of results shown here were obtained with CFD++

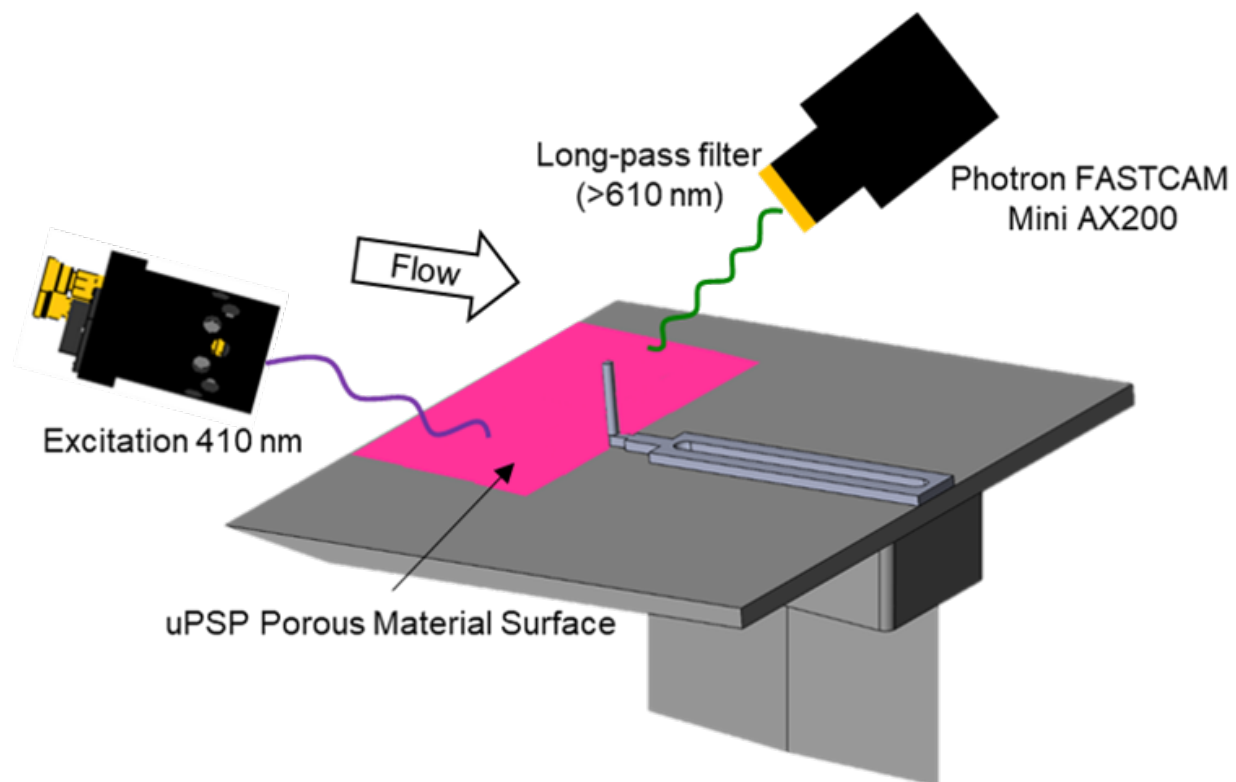


Figure 3.6: CAD model of the uPSP system employed. Figure from Lash et al. [118].

v16.1. The compressible, perfect-gas Navier–Stokes equations were solved in dimensional, Cartesian coordinates by a finite-volume method. The formulation used to solve these equations in CFD++ is in strong conservation form and is shown in Eq. 3.3–3.6.

$$\frac{\partial \vec{Q}}{\partial t} + \frac{\partial (\vec{F}_i + \vec{G}_i)}{\partial x_i} = 0 \quad (3.3)$$

$$\vec{Q} = \begin{bmatrix} e_0 & \rho & \rho u & \rho v & \rho w \end{bmatrix}^T \quad (3.4)$$

$$\vec{F}_i = \begin{bmatrix} (e_0 + P)u_i \\ \rho u_i \\ \rho w u_i + P \hat{n}_x \\ \rho v u_i + P \hat{n}_y \\ \rho w u_i + P \hat{n}_z \end{bmatrix} \quad (3.5)$$

$$\vec{G}_i = \begin{bmatrix} \vec{q}_i - u_j \tau_{ij} \\ 0 \\ -\tau_{ix} \\ -\tau_{iy} \\ -\tau_{iz} \end{bmatrix} \quad (3.6)$$

In Eq. 3.3, $\vec{Q}$ is the dependent variable vector, $\vec{F}_i$ are the inviscid flux vectors indexed by i in the x , y , and z directions, respectively, and $\vec{G}_i$ are the viscous flux vectors indexed by i . Velocities indexed by i (or j) represent u , v , and w , respectively. The total energy, e_0 , is solved using the equation of state shown in Eq. 3.7, which relates back to the calorically-perfect gas equation from Eq. 3.2.

$$e_0 = \frac{P}{\gamma - 1} + \frac{1}{2} \rho (u^2 + v^2 + w^2) \quad (3.7)$$

The term $\vec{q}_i$ in Eq. 3.6 is the heat flux density vector and is described using Fourier’s law of heat conduction, as shown in Eq. 3.8.

$$\vec{q}_i = -k \frac{\partial T}{\partial x_i} \quad (3.8)$$

The stresses and strains, respectively, for a Newtonian fluid are linearly related and are shown in Eq. 3.9.

$$\begin{aligned} \tau_{ii} &= 2\mu \frac{\partial u_i}{\partial x_i} - \frac{2}{3}\mu\Phi \\ \tau_{ij} &= \tau_{ji} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad i \neq j \\ \text{where } \Phi &= \frac{\partial u_k}{\partial x_k} \end{aligned} \quad (3.9)$$

Here, Φ is the dilatation of velocity. The molecular dynamic viscosity, μ , and the thermal conductivity, k , were computed using Sutherland's law, as shown in Eq. 3.10 and Eq. 3.11, respectively.

$$\frac{\mu}{\mu_{\text{ref}}} = \left(\frac{T}{T_{\text{ref}}} \right)^{3/2} \frac{T_{\text{ref}} + S_{\mu}}{T + S_{\mu}} \quad (3.10)$$

$$\frac{k}{k_{\text{ref}}} = \left(\frac{T}{T_{\text{ref}}} \right)^{3/2} \frac{T_{\text{ref}} + S_k}{T + S_k} \quad (3.11)$$

Here, μ_{ref} is the reference viscosity of $1.716 \times 10^{-5} \frac{\text{kg}}{\text{m s}}$, k_{ref} is the reference thermal conductivity of $2.410 \times 10^{-2} \frac{\text{W}}{\text{m K}}$, T_{ref} is the reference temperature of 273.11 K, S_{μ} is the Sutherland constant of 110.0 K for the viscosity equation, and S_k is the Sutherland constant of 194.0 K for the thermal conductivity equation.

Turbulence modeling was provided using a transport equation. The transport equation solved here was the 1-equation Spalart–Allmaras (SA) model [119] with both a rotation-curvature correction (RC) [120] and quadratic constitutive relations (QCR) [121], collectively referred to as SA-RC-QCR2013. This model solves for a (turbulence) working variable, $\tilde{\nu}$, which is also sometimes referred to as the undamped eddy viscosity. The transport equation for $\tilde{\nu}$ is shown and described in more detail in Appendix B.

3.6.2 CFD++ Discretization and Solver Set-Up

A compressible flow scheme provided second-order discretization in both space and time. A steady-state, point-implicit, backward (upwind) Euler method was applied together with multigrid acceleration to accelerate convergence. The multigrid method used a W-cycle with 4 cycles and 20 levels, and allowed for non-strong agglomerations that were recomputed every 5 steps until the agglomeration had one group in its level. The Courant–Friedrichs–Lewy (CFL) number was ramped linearly from 1 to 10 over the first 100 iterations. Time-step temporal smoothing with a factor of 0.5 was activated, as was time-step spatial smoothing with 4 smoothing passes and a maximum time-step growth factor of 1.5. All simulations were run for a total of 12,000 iterations, and all of the normalized L_2 residual norms for mass, momenta, energy, and turbulence were flat-lined below seven orders of magnitude, indicating convergence [122].

The Harten–Lax–van Leer contact (HLLC) approximate Riemann solver [123] was used together with the continuous van Leer flux limiter [124], which is total variation diminishing (TVD), to model the inviscid fluxes. This combination avoids spurious oscillations and maintains monotonicity near discontinuities. In addition, a pressure switch added extra dissipation in regions of strong pressure gradients, such as shock waves. The solution was run at first-order (in space) for the first 1000 iterations to wash out transients in the flow, during which the global residuals had typically dropped around four to five orders of magnitude from their initial value. A first-to-second-order blending function then linearly ramped the spatial discretization over 100 iterations, after which the solution remained at second-order for the duration of the simulation.

The SA-RC-QCR2013 turbulence model was activated on the 11th iteration, allowing for initial start-up transients to settle. A turbulence under-relaxation parameter of 0.75 was specified. Limits were enforced on the turbulence quantities, such that $\tilde{\nu}$ could not fall below 10^{-12} , nor could the maximum eddy viscosity ratio, μ_t/μ , exceed 10^{10} . The Prandtl number, Pr , was not directly specified; instead, a laminar-to-turbulent Prandtl number ratio, Pr/Pr_t , of 0.8 was maintained. Moreover, the turbulent Schmidt number, Sc_t , was specified at 0.7.

3.6.3 CFD++ Computational Domain

The streamwise, wall-normal, and spanwise extents of the computational domain were continuously reduced in a prior study [60]. Initial computational domain extents were the size of the wind tunnel test section, spanning the full height and width. After introducing the flat plate model and further reducing the domain size, no variations on the SWBLI were observed. The final domain extents were normalized by the cylinder diameter and resulted in an extent of $x/d = [-4.5, 10]$, $y = [0 \text{ mm}, 203 \text{ mm}]$, and $z/d = [0, 9.5]$. While testing variations in the boundary-layer thickness, the computational domain height came into question, as curvature in the bow shock began to form. The height of the domain was originally also non-dimensionalized to $y/d = [0, 10]$, but after setting the height to match the full height of the wind tunnel test section, the curvature disappeared. As stated in Sec. 2.5, prior studies have shown that the effective physical extents of the interaction reach out to around $z/d = 5.5$ [16, 19, 23, 25] in the span. However, extending the spanwise domain to $z/d = 9.5$ enabled the full capture of the interaction prior to reaching the outflow, which was found to be preferable in preventing pressure reflections off the spanwise boundary.

Boundary conditions for the domain were specified as follows: a symmetry plane was located at the centerline where $z/d = 0$; adiabatic, viscous (no-slip) walls were specified on the cylinder and floor; centroidal extrapolation was used at the farfield boundaries and at the outflow; the inflow used a Musker [39] turbulent boundary-layer profile, which generated the profile based on freestream variables and a user-defined 99.95% boundary-layer thickness. The classic definition of a 99% boundary-layer thickness was then obtained by measuring the profile directly at the inflow. The boundary conditions are visually identified in Fig. 3.7.

The initial condition consisted of a two-box method. The first box encompassed the entire domain and specified the primitive variables based on the freestream values. The second box was located downstream of the cylinder to the outflow, providing a rough outline of the wake; for this, the primitive variables were based on the stagnation values and a u -velocity of $1 \frac{\text{m}}{\text{s}}$.

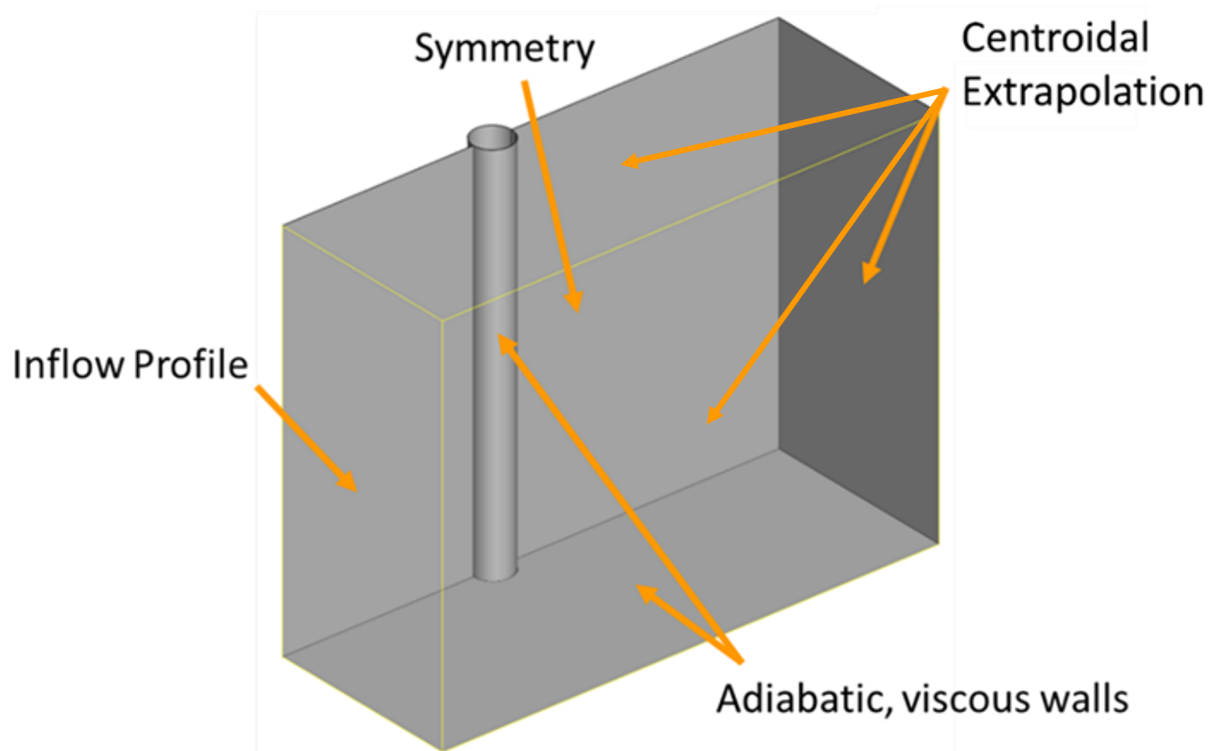


Figure 3.7: Computational domain for CFD++ cases with indicated boundary conditions.

3.6.4 CFD++ Mesh Generation

Mesh generation for CFD++ was performed in Pointwise v18.0R1 from Pointwise, Inc. (now Cadence Design Systems, Inc.). The foundation of the mesh consisted of a 2-D, structured, quadrilateral surface mesh in the x - z plane that included a radial-to-rectangular transition and an outboard transition to coarsen the mesh spacing. A sample mesh, with an enhanced view of both the radial and outboard transition regions, is shown in Fig. 3.8. The radial mesh extended $3d$ outwards. The transition region to the spanwise outboard section was located at $z/d = 5$ and implemented to cut the local cell count in half, thus saving approximately 20% in the total 3-D cell count.

The surface mesh was extruded in the wall-normal direction (y) to create a 3-D, hexahedral, structured, multi-block mesh. Mesh parameters were estimated *a priori* based on a non-dimensionalization of the wall-normal height, as described in Eq. 3.12.

$$\begin{aligned}
 y^+ &= \frac{\rho_\infty u_\tau y}{\mu_\infty} \\
 \text{where } u_\tau &= \sqrt{\frac{\tau_\infty}{\rho_w}} \\
 \tau_\infty &= \frac{\rho_\infty u_\infty^2 c_f}{2} \\
 c_f &= \frac{0.026}{\text{Re}_x^{1/7}}
 \end{aligned} \tag{3.12}$$

Here, the subscript w refers to values at the wall. Further, u_τ represents the (estimated) friction velocity, τ_∞ represents the shear stress estimated by freestream parameters, and c_f represents the skin friction coefficient. The skin friction coefficient was estimated using a simple approximation of flat plate turbulent flow related to the $1/7^{\text{th}}$ power law [125] and a reference length of 1 m.

The *a priori* spacing of the first cell in the wall-normal direction (y_1) was set to $y_1^+ = 1$, and successive cells were subjected to an exponential growth with a growth rate of $\text{GR}_y = 1.2$, until a maximum mesh spacing of $\Delta y_{\text{max}}^+ = 410$ was reached. Similar values were applied to the radial dimensions of the surface mesh, resulting in an initial spacing off the cylinder of $r_1^+ = 1$ with a growth rate of $\text{GR}_r = 1.2$ and maximum mesh spacing of $\Delta r_{\text{max}}^+ = 410$. The azimuthal spacing was held constant at a value of $\Delta \Theta^+ = 150$. Upon completion (and

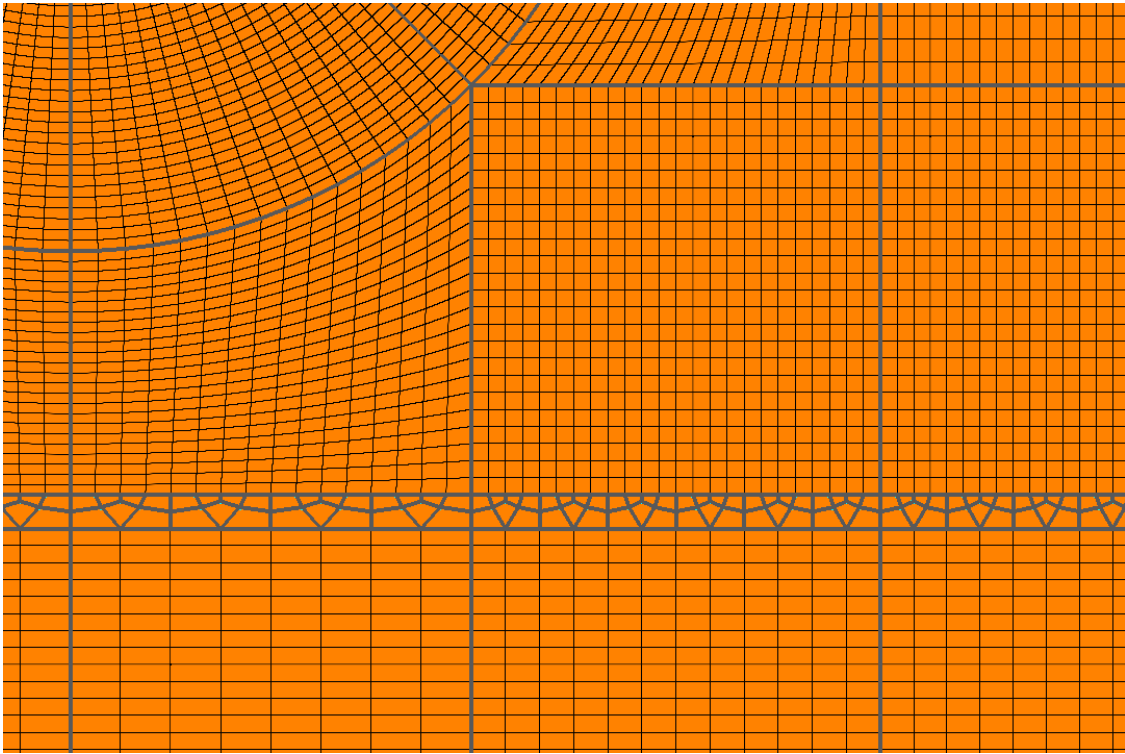
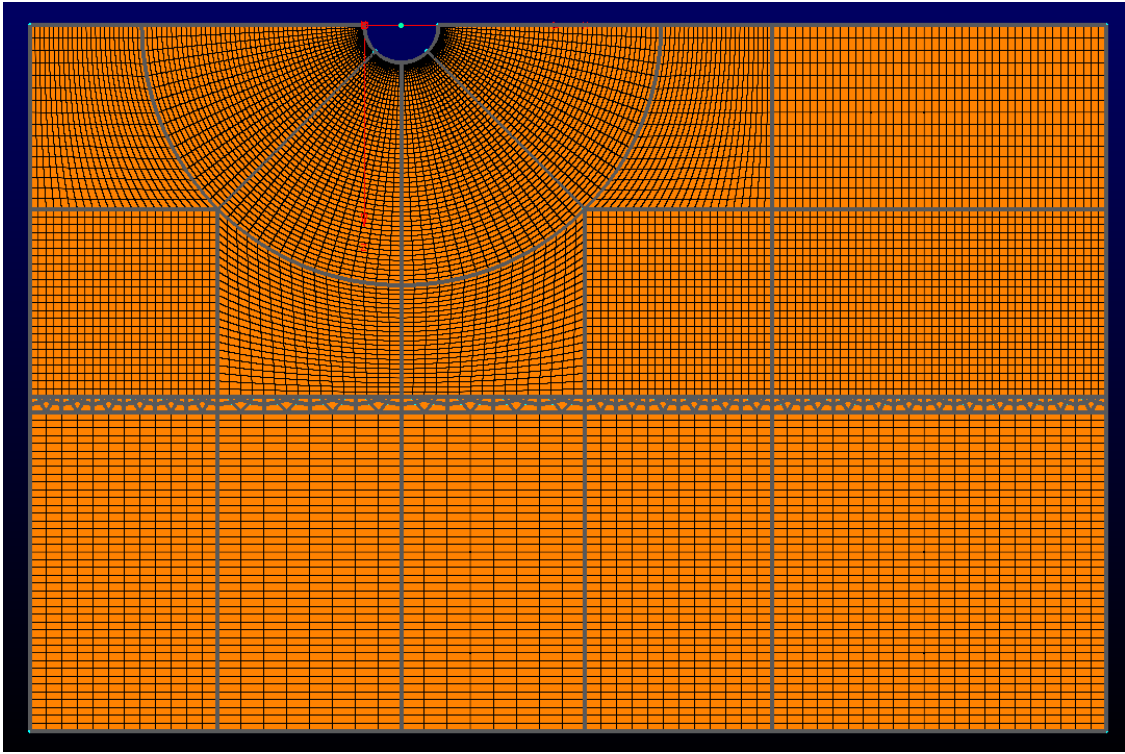


Figure 3.8: a) Top-down view of a sample surface mesh with b) enhanced view of the radial and outboard transition region.

convergence) of the simulation, the mesh parameters were computed *a posteriori* based on Eq. 3.13.

$$\begin{aligned}
y^+ &= \frac{\rho_w u_\tau y}{\mu_w} \\
\text{where } u_\tau &= \sqrt{\frac{\tau_w}{\rho_w}} \\
\tau_w &= \frac{\rho_w u_e^2 C_f}{2}
\end{aligned} \tag{3.13}$$

Here, τ_w is the computed shear stress at the wall obtained from Eq. 3.9.

As the shear stress and the skin friction coefficient varied throughout the simulation, the actual y^+ value for the first cell off the wall varied as well. It was found, however, to be approximately $y_1^+ \approx 0.2$ in the undisturbed region upstream of the SWBLI, and approximately $y_1^+ \approx 1.0$ and $r_1^+ \approx 1.0$ in the interaction region.

Since the domain extents were dependent on the cylinder diameter, as mentioned in Sec. 3.6.3, but the mesh spacing was dependent on y^+ , the total cell count varied significantly between the test cases C1–C5 in Table 3.2. As such, the smallest-diameter mesh contained approximately 150×10^3 cells and the largest-diameter mesh contained approximately 25×10^6 cells. Mesh independence was performed in a prior study [60]; the criteria used to determine mesh independence were the locations of the shock structures and the separation distance, which varied by less than 0.3% between successive iterations.

3.7 Implicit Large-Eddy Simulations

Initial attempts to perform large-eddy-type simulations using CFD++ were unsuccessful, likely owing to the built-in second-order bounded central-difference scheme being too dissipative. As Overflow has higher-order schemes available, and has found use in the archival literature for ILES, this code was chosen.

A majority of precursor and exploratory ILES set-up cases were run on the same local cluster at UTSI that was mentioned in Sec. 3.6. A small number of exploratory cases were also run on the DoD Supercomputing Resource Center (DSRC), a high-performance computing (HPC) facility hosted by various entities of the DoD. The DSRC cluster used

was Mustang, hosted by the U. S. Air Force Research Laboratory (AFRL), and is a Hewlett Packard Enterprise (HPE)[®] Silicon Graphics International (SGI)[®] 8600 system. The HPC cluster contained 1128 nodes with 48 cores each, but only one node was used when queueing in the background, and four nodes were used for the standard queue. All of the cores were of the type Intel[®] Xeon[®] Platinum 8168, clocking at 2.7 GHz and producing roughly 1459 GFLOPS. Each node had 180 GB of RAM available.

Yet another number of precursor and exploratory cases were run on the University of Tennessee, Knoxville (UTK), High Performance Scientific Computing (HPSC) center, particularly the Infrastructure for Scientific Applications and Advanced Computing (ISAAC) Next Generation (ISAAC-NG) cluster. The ISAAC-NG cluster contained 118 nodes with 48 cores each on the standard allocation, but only six nodes were available for a standard student queue for a total of 288 cores. All of the cores on the standard allocation were of the type Intel[®] Xeon[®] Gold 6248R, clocking at 3.0 GHz and producing roughly 768 GFLOPS. Each node had 192 GB of RAM available.

Lastly, a few precursor cases and all the cases with data collection were run on the UTK HPSC ISAAC Secure Enclave cluster. The ISAAC Secure Enclave cluster contained 26 nodes with 48 cores each on a private allocation, and all 1248 cores were run. The type of core and memory were the same per node as for the ISAAC-NG system.

The precursor and exploratory cases used smaller grids than those for final data collection, and a standard message passing interface (MPI) was sufficient to communicate between the nodes and cores. However, when Overflow performs domain decomposition to assign the decomposed grids to the cores, a three-point overlap is created in all directions to maintain a higher-order stencil. As a result, the grid count can rapidly inflate for a larger number of cores. To counteract this, the cases run on the ISAAC Secure Enclave cluster utilized a hybrid MPI/OpenMP structure to communicate, where a number of tasks are assigned to a node using communication through MPI, and a number of threads are assigned to each task using communication through OpenMP [126]. This enables fewer domains to be created by domain decomposition, with multiple cores running calculations on a single decomposed grid. It was found that assigning three MPI tasks per node (78 total MPI tasks) with 16 OpenMP threads per task provided the most optimal speed-up.

3.7.1 Overflow Equation Set

A couple of versions of Overflow were used, with 2.3a used on most all of the systems, but 2.2o used on Mustang. All of the results shown here were obtained with Overflow 2.3a. The non-dimensional, compressible, perfect-gas Navier–Stokes equations were solved in generalized, curvilinear coordinates by a finite difference method. The formulation used to solve these equations in Overflow is in strong conservation form and is shown in Eq. 3.14–3.17.

$$\frac{\partial \vec{Q}}{\partial \tau} + \frac{\partial (\vec{F}_i + \vec{G}_i)}{\partial \xi_i} = 0 \quad (3.14)$$

$$\vec{Q} = J^{-1} \begin{bmatrix} \rho & \rho u & \rho v & \rho w & e_0 \end{bmatrix}^T \quad (3.15)$$

$$\vec{F}_i = J^{-1} \begin{bmatrix} \rho U_i \\ \rho u U_i + \frac{\partial \xi_i}{\partial x} P \\ \rho v U_i + \frac{\partial \xi_i}{\partial y} P \\ \rho w U_i + \frac{\partial \xi_i}{\partial z} P \\ (e_0 + P) U_i - \frac{\partial \xi_i}{\partial t} P \end{bmatrix} \quad (3.16)$$

$$\vec{G}_i = -J^{-1} \begin{bmatrix} 0 \\ \frac{\partial \xi_i}{\partial x_j} \tau_{xj} \\ \frac{\partial \xi_i}{\partial x_j} \tau_{yj} \\ \frac{\partial \xi_i}{\partial x_j} \tau_{zj} \\ \frac{\partial \xi_i}{\partial x_j} \left(\frac{\gamma k}{\text{Pr}} \frac{\partial e}{\partial x_j} + u_k \tau_{jk} \right) \end{bmatrix} \quad (3.17)$$

In Eq. 3.14, $\vec{Q}$ is the dependent variable vector, τ is a pseudo-time term, $\vec{F}_i$ are the inviscid flux vectors indexed by i in the ξ , η , and ζ directions, respectively, and $\vec{G}_i$ are the viscous flux vectors indexed by i . All vectors are transformed from the curvilinear to the Cartesian coordinate system via the Jacobian transformation matrix, J , as shown in Eq. 3.18.

$$J = \frac{\partial x_i}{\partial \xi_j} \quad (3.18)$$

The u -velocities indexed by i (or j or k) represent u , v , and w , respectively, and U -velocities represent the contravariant velocities, which are defined in Eq. 3.19.

$$U_i = \frac{\partial \xi_i}{\partial t} + \frac{\partial \xi_i}{\partial x_j} u_j \quad (3.19)$$

Flow variables are non-dimensionalized as shown in Appendix C. The heat flux density vector and the stresses and strains are computed in the same manner as those shown in Eq. 3.8 and Eq. 3.9, respectively. The molecular dynamic viscosity and the thermal conductivity were computed using Sutherland's law, though the equation is provided in non-dimensionalized form as shown in Eq. 3.20 and Eq. 3.21, respectively.

$$\mu^* = \frac{\mu}{\mu_\infty} \approx \left(\frac{T}{T_\infty} \right)^{3/2} \frac{1 + S_\mu^*}{\frac{T}{T_\infty} + S_\mu^*} \quad (3.20)$$

$$k^* = \frac{k}{k_\infty} \approx \left(\frac{T}{T_\infty} \right)^{3/2} \frac{1 + S_k^*}{\frac{T}{T_\infty} + S_k^*} \quad (3.21)$$

Due to the non-dimensionalization, no reference values were needed. The Sutherland constant is altered slightly from the values in CFD++ to be $S_\mu = 199^\circ\text{R}$ (110.56 K) for the viscosity equation, and $S_k = 350^\circ\text{R}$ (194.44 K) for the thermal conductivity equation.

For the RANS simulation cases that were used to validate a code comparison between CFD++ and Overflow, the same SA-RC-QCR2013 model was used as described in Appendix B. For the LES cases, no turbulence modeling was provided. Moreover, no sub-grid scale (SGS) model was provided, as would otherwise be common in LES. Instead, the numerical algorithm is expected to supply the dissipation of turbulent energy that is lost due to the finest-scale structures not being resolved. This approach has been successfully employed by several authors as described in Georgiadis et al. [127].

3.7.2 Overflow LES Discretization and Solver Set-Up

The explicit, backward (upwind), classic Runge–Kutta (RK4) method was chosen to advance the solution, as it provides fourth-order temporal discretization. The scheme used for

iteratively solving the governing equations was the lower-upper symmetric-Gauss–Seidel (LU-SGS) method [128, 129]. This algorithm was selected for its efficiency, and proved to require the lowest wall-clock time per iteration when compared to other schemes. The physical time-step was set to $\Delta t^* = 1.42 \times 10^{-6}$, corresponding to $\Delta t \approx 3$ ns and $\Delta t^+ \approx 0.019$, where Δt^+ is defined in Eq. 3.22.

$$\Delta t^+ = \frac{\rho_w u_\tau^2 \Delta t}{\mu_w} \quad (3.22)$$

This time step easily satisfies the requirement for $\Delta t^+ \leq 1$ needed to resolve the viscous time-scale in the sublayer [130]. Additionally, as the selected time-integration method was explicit, the CFL number was also critical, but this was kept below 0.038 throughout. However, Yang and Griffin [131] found a stricter time-step requirement than the CFL criterion, which is shown in Eq. 3.23 for a wall-resolved large eddy simulation (WRLES), which is in the family of LES that an ILES belongs to.

$$\Delta t \leq \frac{74 \text{Re}_{x_0}^{1/7} \mu_\infty}{\rho_\infty u_\infty^2} \quad (3.23)$$

Here, $x_0 = 50.61$ mm is the equivalent distance over a flat plate based on a $1/7^{\text{th}}$ power-law estimate [125] of the boundary-layer thickness in the undisturbed, fully-turbulent region. Again, this criterion is satisfied as the upper bound yields a maximum time-step of 92 ns. The smaller time-step of 3 ns was selected based on guidelines from Adler et al. [132] and Adler and Gaitonde [133] to resolve 1000 time-steps for the freestream boundary layer to advect one boundary-layer thickness in the streamwise direction, as shown in Eq. 3.24.

$$\Delta t = \frac{\delta}{1000 u_\infty} \quad (3.24)$$

Precursor simulations were used to invoke turbulence and are described in Sec. 3.7.4; for these, the solution was run for a total of 800,000 iterations, with the latter half being used for time-averaging of the solution. Based on a qualitative inspection of the Q -criterion [134], initial turbulent structures had developed by about 30,000 iterations, and sufficient time was added as a buffer to allow a periodic/quasi-steady solution to settle. For the simulations for which data collection was performed, a total of 2,090,000 iterations were run over a

wall-clock time of about 60 days, and data was collected for 1,670,000 iterations (≈ 5 ms) to enable resolution of 400 Hz for two periods. One flow-through of the domain consisted of approximately 70,000 iterations, and six flow-throughs were granted to allow the flow and the lambda-shock structure to settle, which had developed initially around 50,000 iterations. The criterion used to determine the presence of quasi-periodic behavior was a running average of the forward-shock separation distance, S/d , which matched the experimental value within 1% after the sixth flow-through. For each time-step, the normalized L_2 residual norms for mass, momenta, and energy dropped below three orders of magnitude, which was deemed sufficient for a transient simulation.

The twice-improved Harten–Lax–van Leer–Einfeldt (HLLE++) approximate Riemann solver with a limiter invoked by the pressure gradient switch on the eigenvalues [135] was used together with the fifth-order, mapped, weighted, essentially non-oscillatory (WENO5M) flux limiter [136] to model the inviscid fluxes and provide fifth-order spatial discretization. Additional smoothing to the pressure/entropy switch was also provided in the simulations, consisting of a factor of 5, to mitigate slip lines in the vicinity of shock waves. This combination of algorithms allows for higher-order spatial discretization in flows with shock waves, in addition to providing significantly less artificial dissipation than a lower-order discretization; as a result, elongated, turbulent structures can be sustained over a significantly larger grid, which is critical for LES [137].

3.7.3 Overflow RANS Discretization and Solver Set-Up

A compressible flow scheme provided second-order discretization in space and first-order discretization in time. A steady-state, line-implicit, backward (upwind) Euler method was applied with a default “fudge factor” of 0.005. The CFL number was bounded to be between 0.005 and 5. The simulations were run for a total of 10,000 iterations, and all of the normalized L_2 residual norms for mass, momenta, energy, and turbulence were flat-lined below six orders of magnitude, indicating convergence [122].

The twice-improved HLLE++ approximate Riemann solver [135] was used together with the symmetric minmod flux limiter [138], which is total variation diminishing (TVD), to model the inviscid fluxes. This combination avoids spurious oscillations and maintains

monotonicity near discontinuities. Additional smoothing to the pressure/entropy switch was also provided with a factor of 5, as in Sec. 3.7.2. An symmetric successive over-relaxation (SSOR) scheme [139] was used to advance the solution. The SA-RC-QCR2013 turbulence model was used with a turbulence under-relaxation parameter of 0.75.

3.7.4 Overflow Inflow Turbulence

The specification of realistic turbulence at the inflow is a critical step in providing accurate LES [127, 140, 141]. Without it, the flow cannot be considered turbulent, as the defining features, such as eddies in both the boundary layer and the freestream, are missing. Dhamankar et al. [141] outline several methods that are commonly used to produce inflow turbulence, each with a set of advantages and disadvantages. One such method is that of recycling/rescaling [142–144], in which a precursor simulation is used to allow a turbulent flow field to develop over a relatively short computational domain by recycling the outflow (or an arbitrary streamwise plane near the outflow), rescaling the parameters to account for developmental growth over this domain, and imposing this rescaled flow as the new inflow. After several flow-throughs in time, the flow will resemble a desired outflow profile. Unfortunately, the periodic element associated with this method also introduces a signal that often interferes with SWBLI [94, 141, 145, 146]. Another method is that of digital-filter-based synthetic turbulence [94, 132, 147–149], which solely requires a precursor RANS simulation to specify zeroth-order statistics of the primitive variables. This method saves both computational cost and setup efforts compared to other inflow turbulence generation methods where precursor LES are required. However, no digital-filter-based synthetic turbulence method is currently available in Overflow. Moreover, Ngoh and Poggie [150] reported a departure in the unsteady dynamics of a blunt-fin SWBLI from the experiment when using this approach. As a result, a transition-inducing method [141, 145, 151] was selected, in which a laminar boundary layer is imposed at the inflow and tripped after a short distance downstream until an equilibrium turbulent boundary layer has developed.

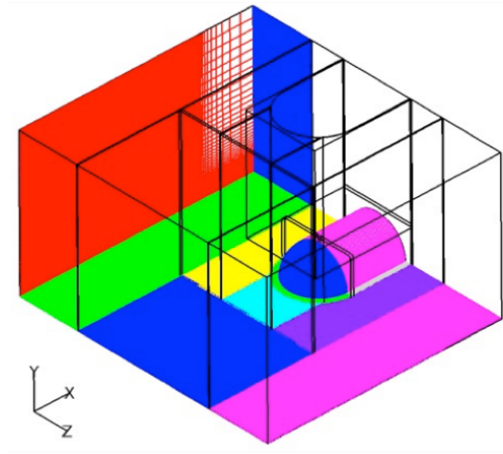
Mullenix et al. [145] found that a tripped, laminar boundary layer requires about 50 inflow-boundary-layer thicknesses, δ_{in} , to reach an equilibrium turbulent-boundary-layer state. For this study, as the spanwise development of the SWBLI is an objective focus, this

would result in a prohibitively large grid. Fortunately, Morgan and Visbal [152] developed a technique called Spanwise Periodically Repeating Inflow Turbulence (SPiRIT) that allows for a transition-inducing method without imposing a huge spanwise cell count. The idea is to establish an equilibrium turbulent boundary layer using the transition-inducing method on a slim grid (in the spanwise direction), then copy the outflow profile several times along the span to match the grid containing an SWBLI, as depicted in Fig. 3.9.

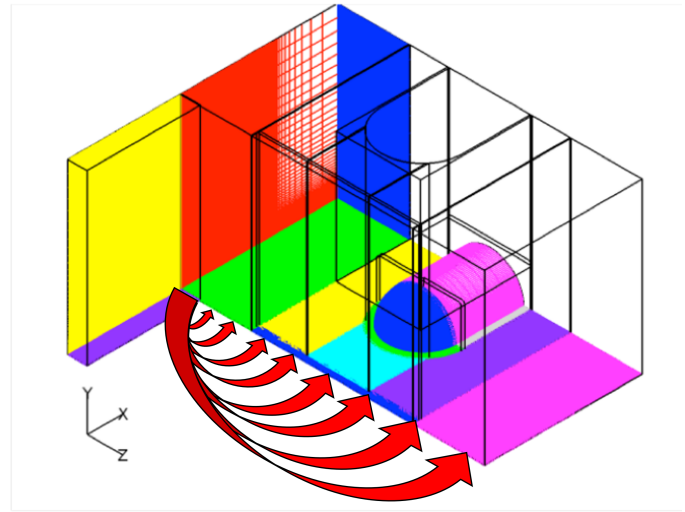
Using the SPiRIT technique, Morgan and Visbal [152] reduced their overall cell count by about a third. The streamwise length of their slim grid was set to $60\delta_{\text{in}}$, which was found to be sufficient to generate equilibrium turbulence. Moreover, the minimum width of the slim grid was also investigated by Morgan and Visbal [152], and this was determined to be approximately $5\delta_{\text{in}}$.

The tripping mechanism employed here leans on the approach of Sherer et al. [153, 154]. This approach consisted of a laminar inflow and a reverse flow at the wall a short distance downstream to trip the flow. A prescribed flow boundary condition with a velocity of $u = -0.5M_\infty$ was applied to two streamwise grid points at the wall, spanning the width of the domain and causing a disturbance (but not a shock wave) that effectively acts as a trip. Further advances to the mechanism of a reverse-flow trip were made by Schwartz and Garmann [149] by adding a sine wave in the span with amplitude of 10% the magnitude of a uniform trip. They found that this spanwise variation to the trip greatly accelerated the generation of an equilibrium turbulent boundary layer, cutting down on the necessary streamwise length from approximately $30\delta_{\text{in}}$ to approximately $15\delta_{\text{in}}$. The wave number did not seem to factor much into the effect of tripping length, as both 4 and 8 periods in the span were tested.

Three different strengths to the reverse flow tripping mechanism were tested in this study, with uniform values of $u = -0.3M_\infty$, $-0.5M_\infty$, and $-0.8M_\infty$. Unfortunately this data was lost, though results showed that the greater the disturbance, the more effective the trip became, and thus the value of $u = -0.8M_\infty$ was selected moving forward. Moreover, a sine wave with amplitude $0.05M_\infty$ and 5 periods across the span of the domain was added to the trip, which further increased its effectiveness. This wave is depicted in Fig. 3.10 and was applied over two streamwise grid points at two streamwise locations: at $10\delta_{\text{in}}$ downstream of



Original Mesh



SPiRiT Mesh

Figure 3.9: Sample grids of a hemispherically-generated SWBLI describing the SPiRiT approach of Morgan and Visbal [152]. Figure adapted from Morgan and Visbal [152].

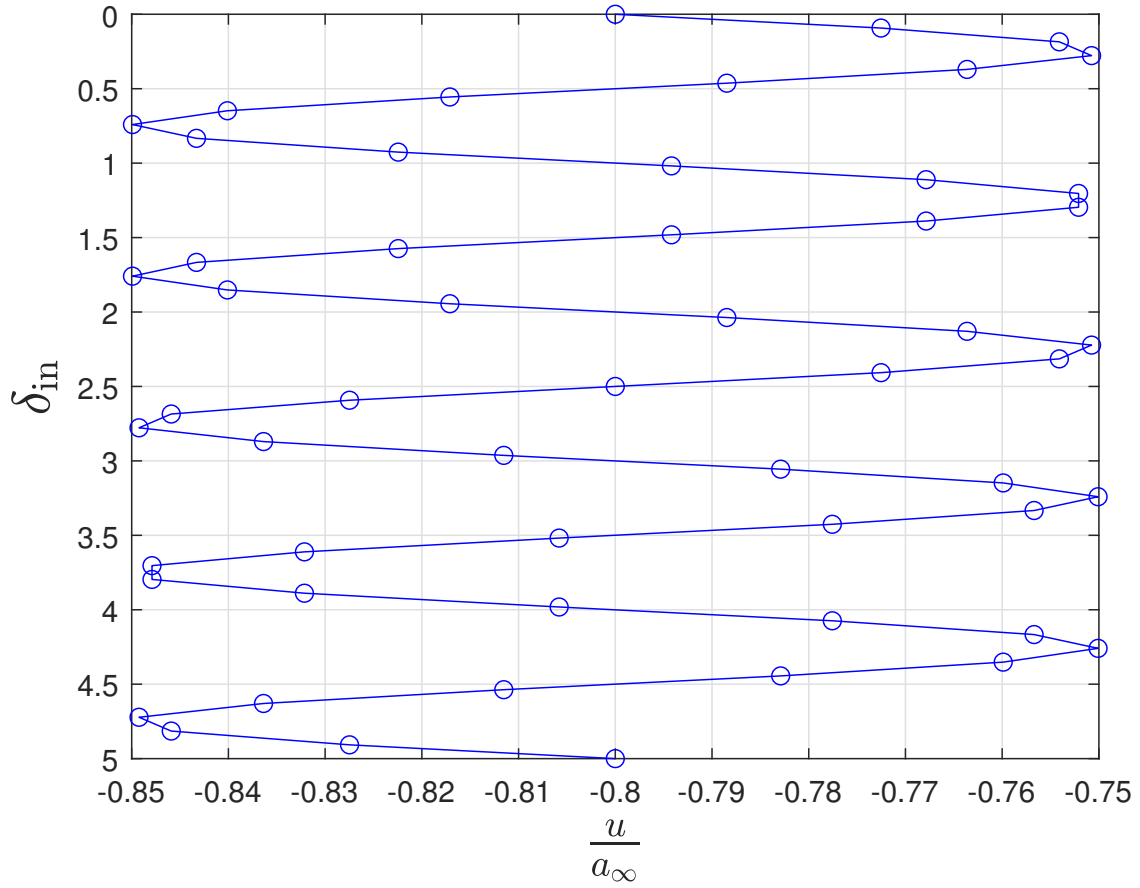


Figure 3.10: Tripping mechanism employed, showing the velocity value at each grid point across the span. Inflow boundary-layer thickness provided for reference.

the inflow and at $11\delta_{\text{in}}$. Two tripping locations were required, as the flow would occasionally re-laminarize when using just a single trip.

A Blasius laminar boundary-layer profile [38], which matched that in Fig. 2.6a, with $\delta_{\text{in}} = 0.4$ mm was prescribed at the inflow for all grid points in the span, which was $5\delta_{\text{in}}$ wide. A qualitative inspection of the instantaneous flow is shown in Fig. 3.11 via the Q -criterion [134] and in Fig. 3.12 via normalized temperature. Quantitative details are provided by Fig. 3.13a and b through the time- and spanwise-averaged values for the coefficient of friction, c_f , and the dimensionless wall distance for the first cell off the wall, y_1^+ , respectively, along the streamwise direction. After approximately $40\delta_{\text{in}}$, an equilibrium turbulent boundary layer had developed; for comparison, this was much shorter than the approximately $80\delta_{\text{in}}$ that were necessary when employing a uniform trip with a value of $u = -0.5M_\infty$. One reason for the increased length compared to other sources is the low Reynolds number employed here to minimize the cell count, which consequently delayed transition and the formation of turbulent structures.

Shown in Fig. 3.14 is the boundary-layer profile after $82.2\delta_{\text{in}}$, with a comparison to direct numerical simulation (DNS) data by Pirozzoli and Bernardini [155] at roughly similar flow conditions. The profiles align well with one another and closely resemble that of a turbulent profile. The data here were averaged in time and across the span, and then modified using the van Driest II transformation [156] to match an incompressible profile in the viscous sublayer [157, 158] and the logarithmic law of the wall [158, 159]. As the location shown was chosen to be downstream of all overset communication and spanwise copying, and a comparison to a profile at $60\delta_{\text{in}}$ yielded virtually the same profile, an equilibrium turbulent profile has been established just upstream of the SWBLI.

To enable replication of the results, all of the boundary layer variables and quantities are summarized in Table 3.3. These are averaged in both time and across the span at the same location of $82.2\delta_{\text{in}}$ downstream of the inflow or, alternatively, $x/d = -4.5$ upstream of the fin leading edge. The subscript e refers to boundary-layer edge values at $y = \delta$ and the subscript c refers to the compressible version of the shape factor variables, which are defined in Eq. 3.25 [37].

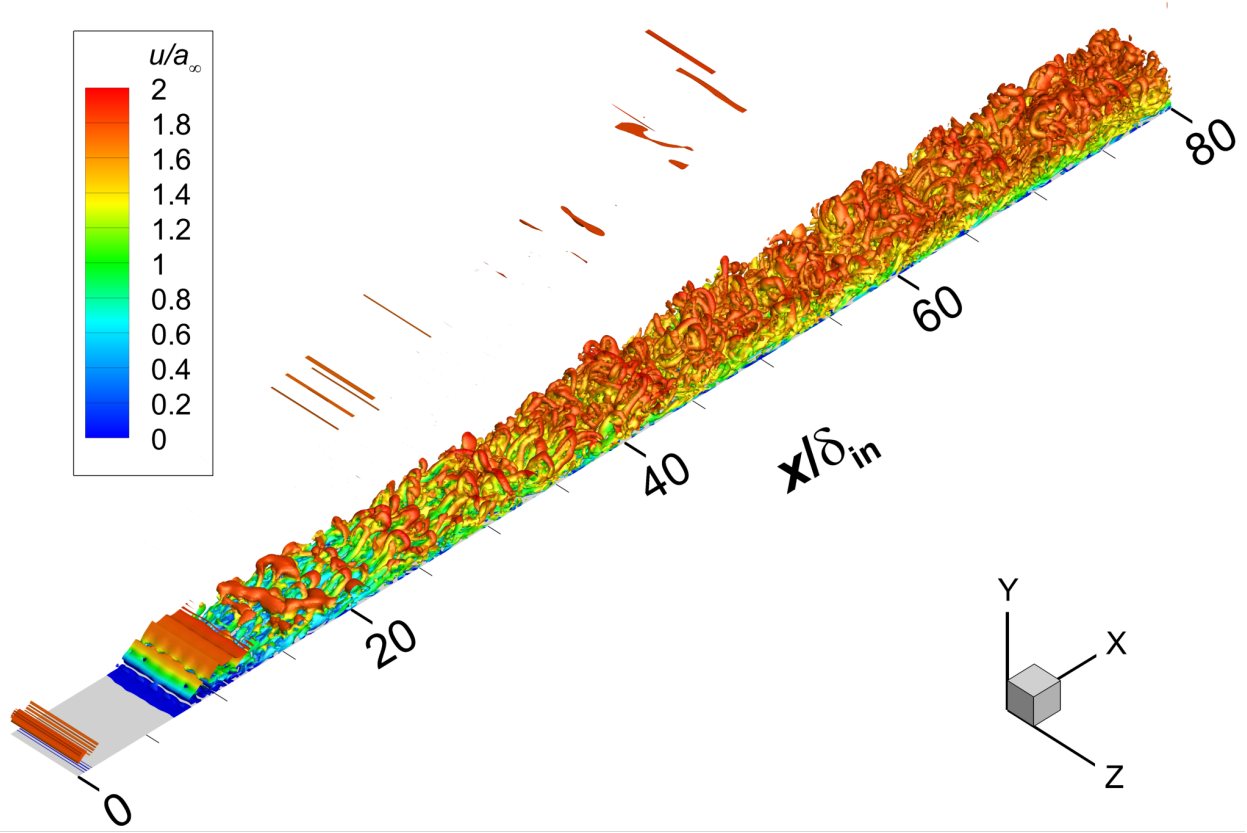


Figure 3.11: Qualitative inspection of instantaneous Q -criterion [134] with $Q = 1$, colored by u -velocity-equivalent Mach number.

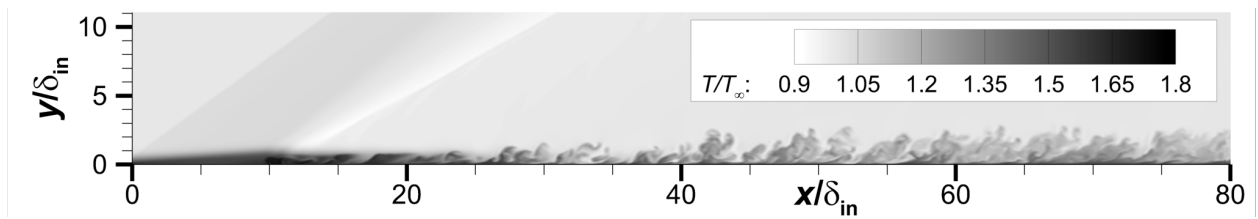


Figure 3.12: Instantaneous slice at $z/d = 0$ with normalized temperature contour.

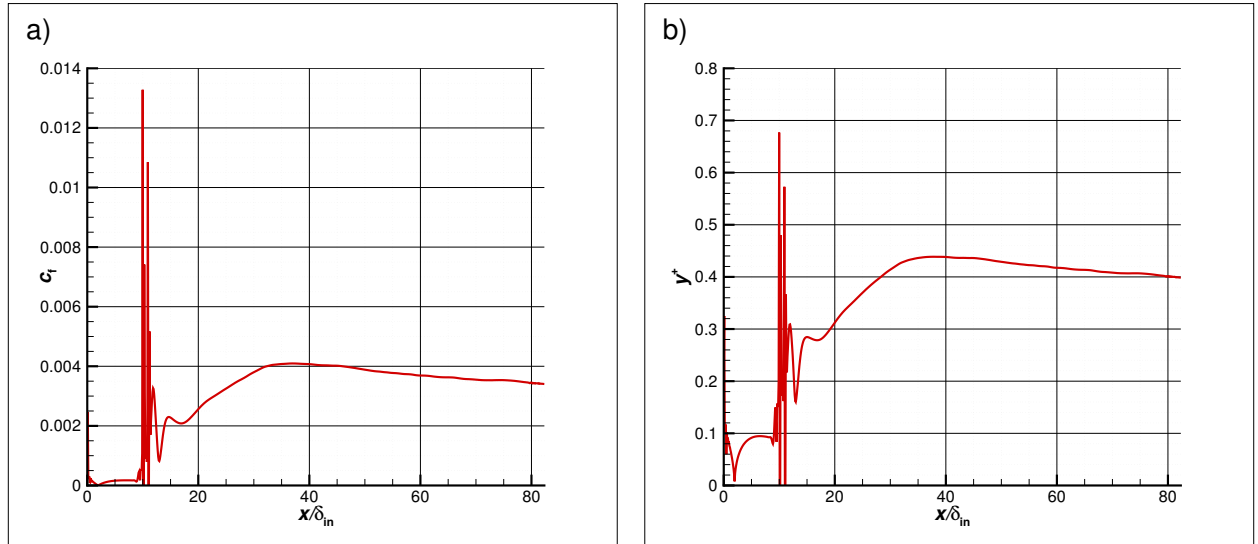


Figure 3.13: Temporally- and spanwise-averaged a) coefficient of friction and b) dimensionless wall distance in the slim grid.

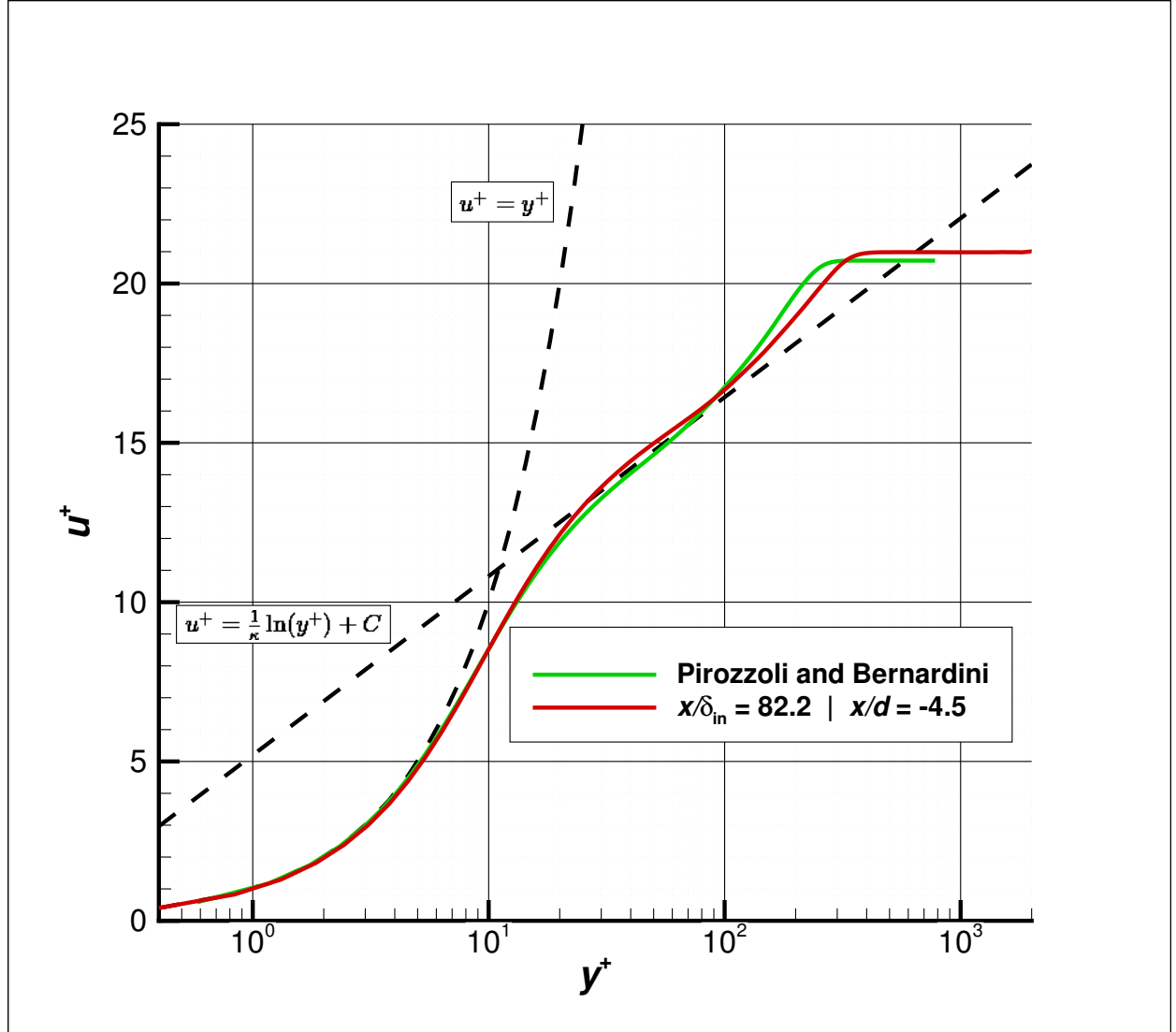


Figure 3.14: Temporally- and spanwise-averaged inner velocity profile with van Driest II transformation [156] at $x/\delta_{in} = 82.2$ or $x/d = -4.5$. Here, $\kappa = 0.41$ and $C = 5.2$. Data from Pirozzoli and Bernardini [155] at $M_\infty = 2$, $Re_\tau = 251$, $Re_\theta = 1122$, and $H = 1.51$.

Table 3.3: Boundary-layer conditions in the undisturbed turbulent region at $x/d = -4.5$, averaged in time and across the span.

Primitive Variables			BL Variables			Extra Variables		
P_e	14,514	Pa	δ_{in}	0.4	mm	τ_w	109.2	Pa
T_e	181.0	K	δ	1.219	mm	u_τ	23.98	$\frac{m}{s}$
ρ_e	0.2794	$\frac{kg}{m^3}$	δ^*	0.1816	mm	c_f	3.415×10^{-3}	
μ_e	1.1998×10^{-5}	$\frac{kg}{m \cdot s}$	θ	0.1249	mm	y_1^+	0.399	
u_e	468.7	$\frac{m}{s}$	H	1.45		x_0	50.61	mm
M_e	1.738		δ_c^*	0.2701	mm	Re_∞	11,248,333	$\frac{1}{m}$
P_w	14,572	Pa	θ_c	0.1696	mm	Re_θ	1405	
T_w	267.5	K	H_c	1.59		Re_τ	324	
ρ_w	0.1898	$\frac{kg}{m^3}$				Re_{x_0}	569,278	
μ_w	1.7115×10^{-5}	$\frac{kg}{m \cdot s}$						

$$\begin{aligned}
H_c(x) &= \frac{\delta_c^*(x)}{\theta_c(x)} \\
\text{where } \delta_c^*(x) &= \int_0^\delta \left(1 - \frac{\rho(x, y) u(x, y)}{\rho_e(x) u_e(x)} \right) dy \\
\theta_c(x) &= \int_0^\delta \frac{\rho(x, y) u(x, y)}{\rho_e(x) u_e(x)} \left(1 - \frac{\rho(x, y) u(x, y)}{\rho_e(x) u_e(x)} \right) dy
\end{aligned} \tag{3.25}$$

Additionally, a special note should be made about Re_τ , which is not computed using the freestream values, but rather as shown in Eq. 3.26 [155].

$$\text{Re}_\tau = \frac{\rho_w u_\tau \delta}{\mu_w} \tag{3.26}$$

3.7.5 Overflow Computational Domain

As the computational domain essentially consisted of two parts, one section for the trip to turbulence and another for the SWBLI, this discussion will focus on these parts separately. For the trip to turbulence, many details were already presented in Sec. 3.7.4, but are summarized again here for completeness. The domain extents were normalized by the boundary-layer thickness prescribed at the inflow, and consisted of values of $x/\delta_{\text{in}} = [0, 82]$, $y/\delta_{\text{in}} = [0, 100]$, and $z/\delta_{\text{in}} = [0, 5]$. As stated in Sec. 3.7.4, the streamwise and spanwise extents were sufficient to allow an equilibrium turbulent boundary layer to develop [145, 149, 152–154]. The height of the domain was chosen to prevent any reflection from the upper boundary back into the domain.

Boundary conditions for the trip-to-turbulence domain were specified as follows: periodic boundaries were enforced at $z/\delta_{\text{in}} = 0$ and $z/\delta_{\text{in}} = 5$; an adiabatic, viscous (no-slip) wall was specified on the floor; an inviscid (slip) wall was specified on the upper (farfield) boundary; the inflow prescribed a Blasius [38] laminar boundary-layer profile, which generated the profile based on a 99% boundary-layer thickness; the tripping mechanism consisted of a prescribed sinusoidal profile at two locations as described in Sec. 3.7.4; for testing purposes, the outflow used a centroidal extrapolation, but for the full simulation, an overset boundary existed at the downstream end. The boundary conditions for the trip-to-turbulence domain are visually identified in Fig. 3.15.

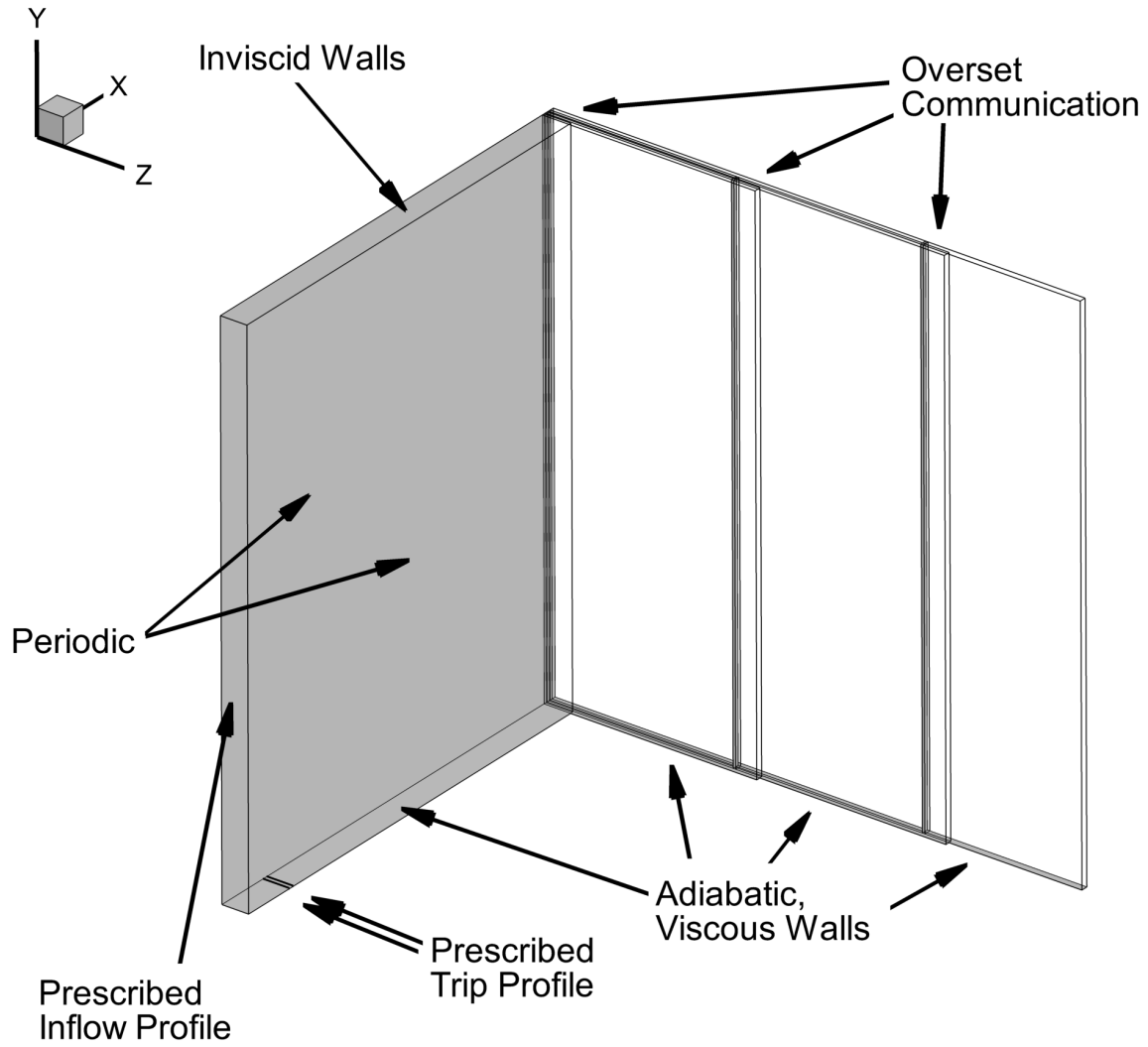


Figure 3.15: Computational domain for the trip-to-turbulence segment and downstream SPiRiT approach [152] of the LES for Overflow cases with indicated boundary conditions.

Of note is that several overset communication boundaries exist in the span, where the SPiRiT approach [152] was implemented. The reason for this is two-fold: first, Overflow only allowed 25 boundary conditions to be specified for each grid. The continuous copy/paste operation of the slim grid onto the wider one exceeded this limit, and would also greatly restrict any output from probe/slice boundaries. Second, there were grid size limitations due to domain decomposition, where a copy/paste boundary could not be decomposed, thus leaving large grids that prohibitively slowed down the simulation. The grid was thus broken up in a stair-stepping method to mitigate this limitation in domain decomposition. In the end, the CPUs computing decomposed grids were all assigned multiple decomposed grids, except for those handling the spanwise overset copy/paste boundaries.

The size of the computational domain required to resolve the SWBLI with sufficient spanwise extents leans heavily on that described in Sec. 3.6.3. Non-dimensionalization was performed using the fin diameter, resulting in extents of $x/d \approx [-4.76, 12.6]$, $y/d \approx [0, 25.2]$, and $z/d \approx [-12.6, 12.6]$. It was again validated that the upper boundary was sufficiently far away from the rest of the interaction to prevent any reflections back into the domain. Moreover, the spanwise extents were increased to properly study the spanwise evolution of the SWBLI, and to prevent reflections from the sidewall boundaries from affecting the interaction. The origin for this domain was shifted to be located at the leading edge of the base of the fin, such that negative z values exist. To provide a direct comparison to the trip-to-turbulence domain, one would align the periodic boundaries at the smallest dimensional z -location.

Boundary conditions for the SWBLI domain were specified as follows: periodic boundaries were enforced at $z/d = -12.6$ and $z/d = 12.6$; adiabatic, viscous (no-slip) walls were specified on the fin and floor; an inviscid (slip) wall was specified on the upper (farfield) boundary and aligned with the upper boundary from the trip-to-turbulence domain; centroidal extrapolation was used at the outflow; the inflow used the overset boundaries from the trip-to-turbulence domain when running the LES, or a prescribed Musker [39] turbulent boundary-layer profile when running a RANS simulation. The boundary conditions for the SWBLI domain are visually identified in Fig. 3.16.

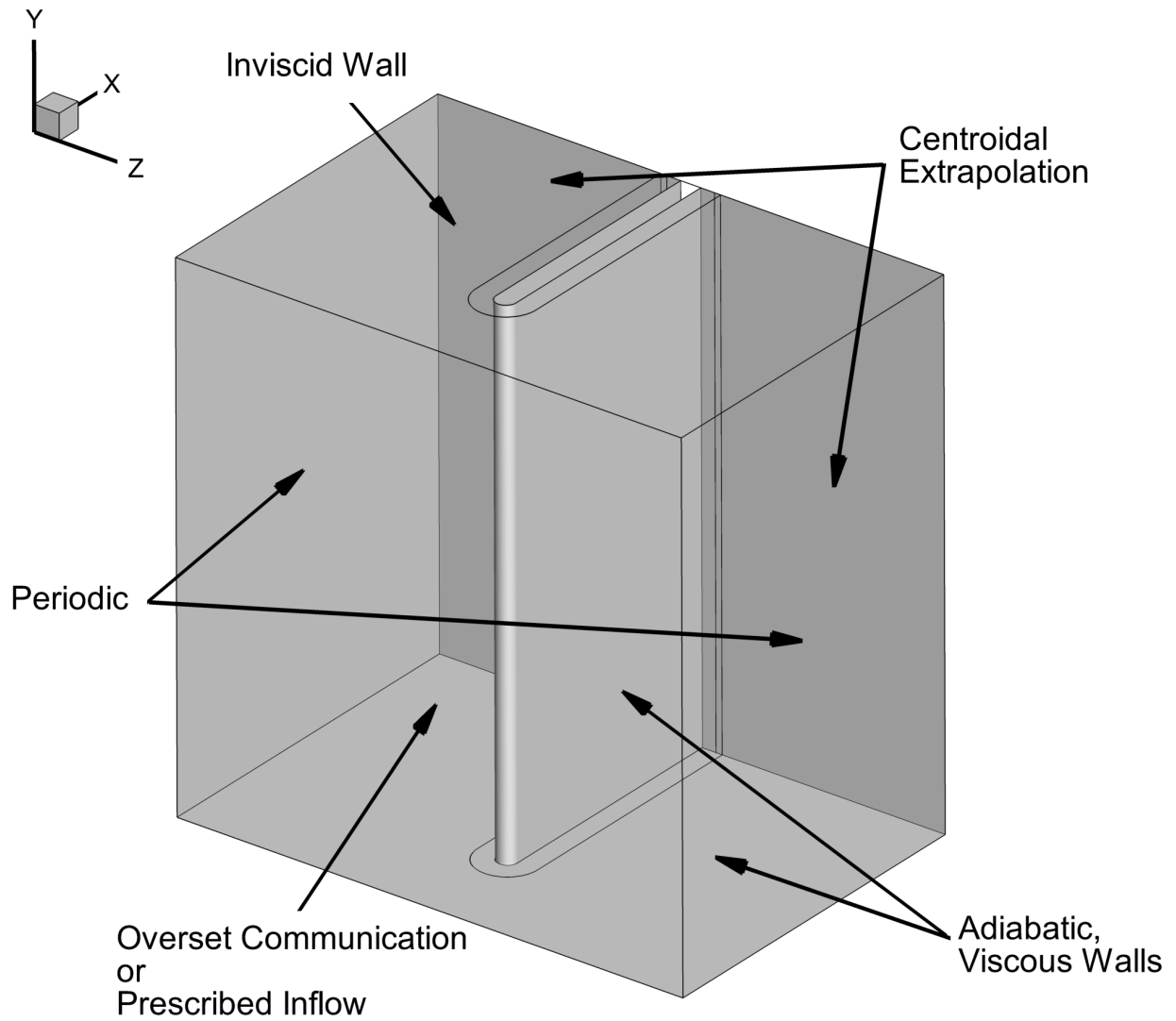


Figure 3.16: Computational domain for the main segment of the LES and the total domain for the RANS simulations for Overflow cases with indicated boundary conditions.

The initial condition for the entire computational domain for all Overflow simulations simply consisted of the freestream values at all locations.

3.7.6 Overflow Grid Generation

Grid generation for Overflow was performed in Pointwise v2022.1.1 and overset communication was established using Pegasus 5.2d from the NASA Langley Research Center [160]. A block-structured gridding approach was required for Overflow and was pursued such that the I , J , and K directions aligned with the x , y , and z coordinates, respectively. As shown in Fig. 3.17, the overall grid was broken up into eight smaller grids. As discussed in Sec. 3.7.5, grid 1 was used to trip the laminar inflow to turbulence, grids 2–6 were used for overset communication and implementing the SPiRiT approach [152], grid 7 was the main grid used to capture the SWBLI, and grid 8 provided near-wall conformity to the fin, as well as acting as a hole cutter to grid 7. In particular, grid 8 extended $1d$ in the radial direction and was aligned such that K extruded in the radial direction.

Akin to the RANS mesh described in Sec 3.6.4, the foundation of the Overflow grid consisted of a 2-D, block-structured, quadrilateral surface grid in the x - z plane. Spacing in both I (x) and K (z) was uniform for all grids, with the exception of the fin grid. The surface grid was extruded in the wall-normal direction (y) to create a 3-D, hexahedral, block-structured, multi-block grid. This extrusion followed an exponential growth with rate $GR_y = 1.1$ until a maximum grid spacing was reached that closely resembled isotropy. The extrusion in the radial direction about the fin followed a similar method. Above a height of $y/d > 3$, the SWBLI had only a marginal impact and the flow was effectively inviscid, so an additional exponential growth was specified to reduce the grid size. This growth was at a rate of $GR_y = 1.3$ and progressed continuously until the upper boundary. Additionally, to prevent reflections at the outflow from bouncing back into the domain, an exponential growth with rate $GR_x = 1.3$ was specified for $x/d > 10$.

Grid parameters were calculated *a posteriori* and are shown in Table 3.4 for the ILES and in Table 3.5 for the RANS simulation. Converging on the values for the ILES required a few iterations of adjustment to provide an equilibrium turbulent boundary layer with sufficient resolution. In particular, it was found that the grid-point requirements for LES provided

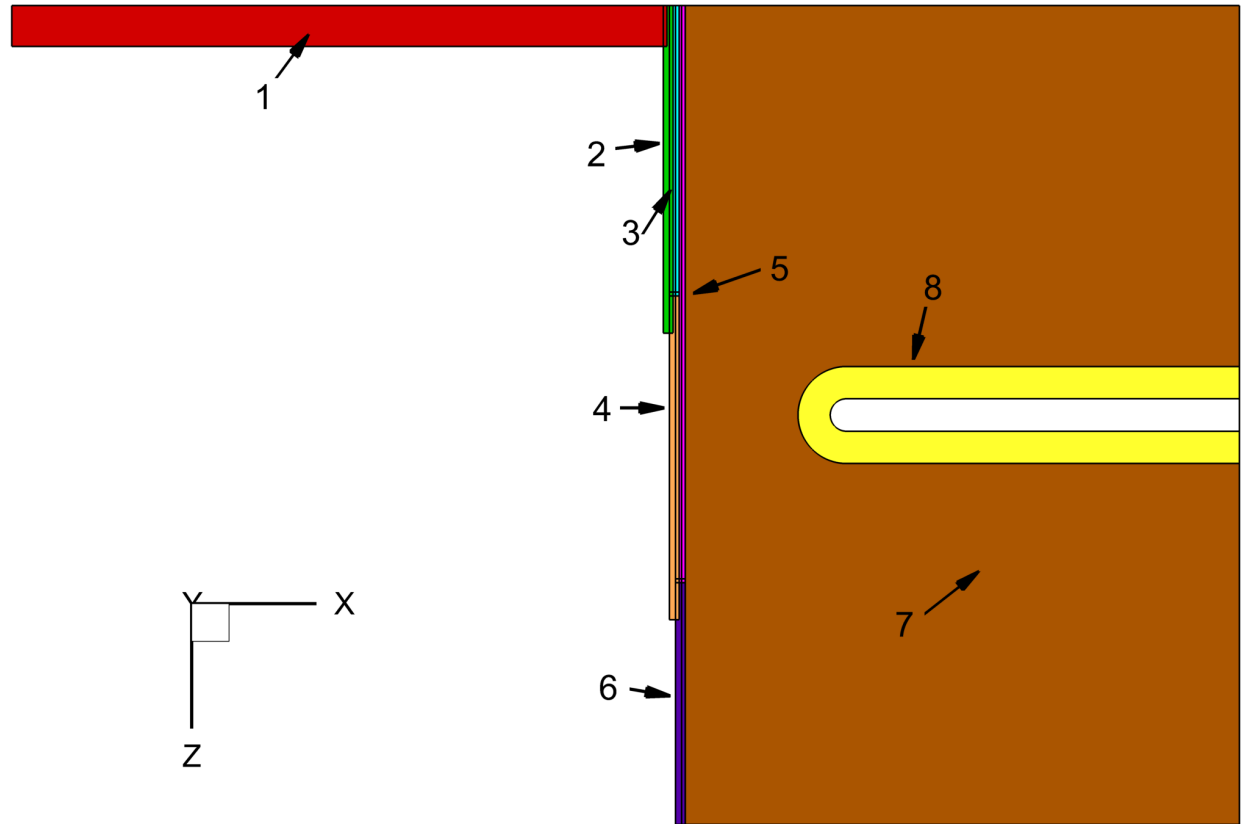


Figure 3.17: Top-down view of the grid employed for ILES. Grid 1 was used to trip the flow to turbulence. Grids 2–6 were used to employ the SPiRiT mechanism [152]. Grid 7 is the main domain and contains a hole cut-out of grid 8, which wraps around the fin.

Table 3.4: Grid parameters for each of the blocks used in the LES.

Grid	N_I	N_J	N_K	Δx^+	Δy_1^+	Δy_∞^+	Δz^+	Δr_1^+	Δr_∞^+	$\Delta\theta_{1,\min}^+$	$\Delta\theta_{1,\max}^+$	$\Delta\theta_\infty^+$
1	859	176	55	10	0.4	10	10					
2	14	176	433	10	0.4	10	10					
3	14	176	384	10	0.4	10	10					
4	14	176	433	10	0.4	10	10					
5	14	176	762	10	0.4	10	10					
6	14	176	325	10	0.4	10	10					
7	635	176	1081	10	0.4	10	10					
8	1037	176	68	10	0.4	10	10	0.4	10	4.3	10	10

Table 3.5: Grid parameters for each of the blocks used in the RANS simulation.

Grid	N_I	N_J	N_K	Δx^+	Δy_1^+	Δy_∞^+	Δz^+	Δr_1^+	Δr_∞^+	$\Delta\theta_{1,\min}^+$	$\Delta\theta_{1,\max}^+$	$\Delta\theta_\infty^+$
7	165	88	267	40	0.4	40	40					
8	271	88	50	40	0.4	40	40	0.4	40	6.5	40	40

by Piomelli and Balaras [127, 161] were too lax and generated a substantial overshoot of the log-layer in Fig. 3.14. This was also corroborated by Poggie et al. [162] and Schwartz [163]. The log-layer overshoot may be caused due to a lack of a SGS model in the ILES, but further investigation was not performed. It should be noted, however, that the limiting factor appeared to be isotropy, and that elongated grid spacing in x compared to the other directions seemed to be the culprit.

In total, the overall ILES grid was created with 147.3×10^6 points, of which 8.3×10^6 points were used to generate turbulence. However, due to domain decomposition creating an additional overlap of three cells in all directions with adjacent decomposed grids, the final grid count considered by Overflow was 175.5×10^6 points. The overall RANS grid was created with 5.1×10^6 points, and after domain decomposition the final grid count considered by Overflow was 9.1×10^6 points.

3.8 Post-Processing

All post-processing of results was performed on a standard desktop computer running at first Microsoft Windows® 7 Professional© 64-bit and later Microsoft Windows® 10 Pro© 64-bit. The desktop computer consisted of 4 cores of the type Intel® Core™ i7-4790, clocking at 3.6 GHz and producing roughly 216 GFLOPS. The desktop computer had 32 GB of RAM available.

Analysis of CFD data, obtained both from RANS simulations and ILES, was performed using Tecplot 360 EX from Tecplot, Inc., using versions 2015 R2 through 2018 R2. Data analysis from both CFD and experimental data was performed using MATLAB® from The MathWorks, Inc., using versions R2015b through R2019b. Additionally, image processing of experimental results was performed using Fiji, an open-source Java applet running ImageJ from the National Institutes of Health (NIH).

Quantitative data from the high-speed schlieren images were extracted using a custom MATLAB® script developed by Combs et al. [113] and validated using simultaneous high-speed surface pressure measurements using pressure transducers. First, a background subtraction to all images was performed using a time-averaged image under ambient

conditions to cancel out any noise. Then, the script non-dimensionalized all length scales by the cylinder diameter, as determined on-screen by the user. Finally, the script identified the forward and trailing lambda shocks via two-point lines in an automated, efficient, and accurate manner. An extrapolation of the forward shock to the surface reveals time-resolved data of the estimated separation location, something that cannot be measured using discretely-spaced pressure transducers alone. Moreover, the triple point was estimated based on the intersection of the forward and trailing shocks. Based on the optical setup described in Sec. 3.4 and 3.5, the estimated uncertainty ranged from ± 0.08 – $0.50d$, or ± 3.6 – 34.0% . This uncertainty was directly a function of the camera resolution and the size of the cylinder diameter. As a result, the uncertainty in the measurement increased as the cylinder diameter decreased.

Frequency spectra of the processed experimental data were computed using Welch’s method [164] with a Hamming window and a fast-Fourier transform (FFT) size of 1024 points with 50% overlap. This resulted in an average of 48.4 short-time Fourier transform segments with a frequency resolution of 98 Hz, equivalent to $St_{L_i} = 2.70 \times 10^{-4}$. For the ILES data, a Hann window was selected and the FFT-size was selected as 1392 points with a 75% overlap. This resulted in 9 short-time Fourier transform segments with a frequency resolution of 599 Hz, equivalent to $St_{L_i} = 1.65 \times 10^{-3}$.

Chapter 4

Scaling of Centerline Flow Features^{*}

Both qualitative and some quantitative effects of d/δ on the shock structure at the centerline are analyzed first in Sec. 4.1. A comparison between mean experimental and CFD RANS results is provided as well. Trends with lines of best fit are then presented for the mean separation distance and triple-point height in Sec. 4.2, which include a broad range of separation-distance data from the archival literature. Section 4.3 then describes unsteady scaling effects of d/δ on the interaction gathered from the experiment, and Sec. 4.4 analyzes the effects of surface pressure computed by the RANS simulations.

4.1 Shock-Structure Analysis

Qualitative schlieren imaging was first performed to visualize the primary flow features in the SWBLI. A sample comparison between a mean experimental schlieren image and a numerical-schlieren equivalent from RANS simulations is shown in Fig. 4.1. Rather than using a standard density-gradient magnitude, the numerical schlieren image was computed using $\sqrt{\sum_i \left(\frac{\partial \rho}{\partial x_i}\right)^2}$. Together with cutting off values below an arbitrary threshold, this approach filters out many extraneous flow features, yet leaves the shock structure visible. Several of the features called out in Fig. 2.2 are visualized, including the lambda-shock structure consisting

^{*}This section, in part, is a reprint of the material as it appears in *Shock Waves* **30**, pp. 395–407 titled “Scaling of Cylinder-Generated Shock-Wave/Turbulent Boundary-Layer Interactions” (2020). Authors: Stefen A. Lindörfer, Christopher S. Combs, Phillip A. Kreth, Ryan B. Bond, and John D. Schmisser. The dissertation author was the primary investigator and author of this paper. Copyright is held by **Stefen A. Lindörfer**, Christopher S. Combs, Phillip A. Kreth, Ryan B. Bond, and John D. Schmisser

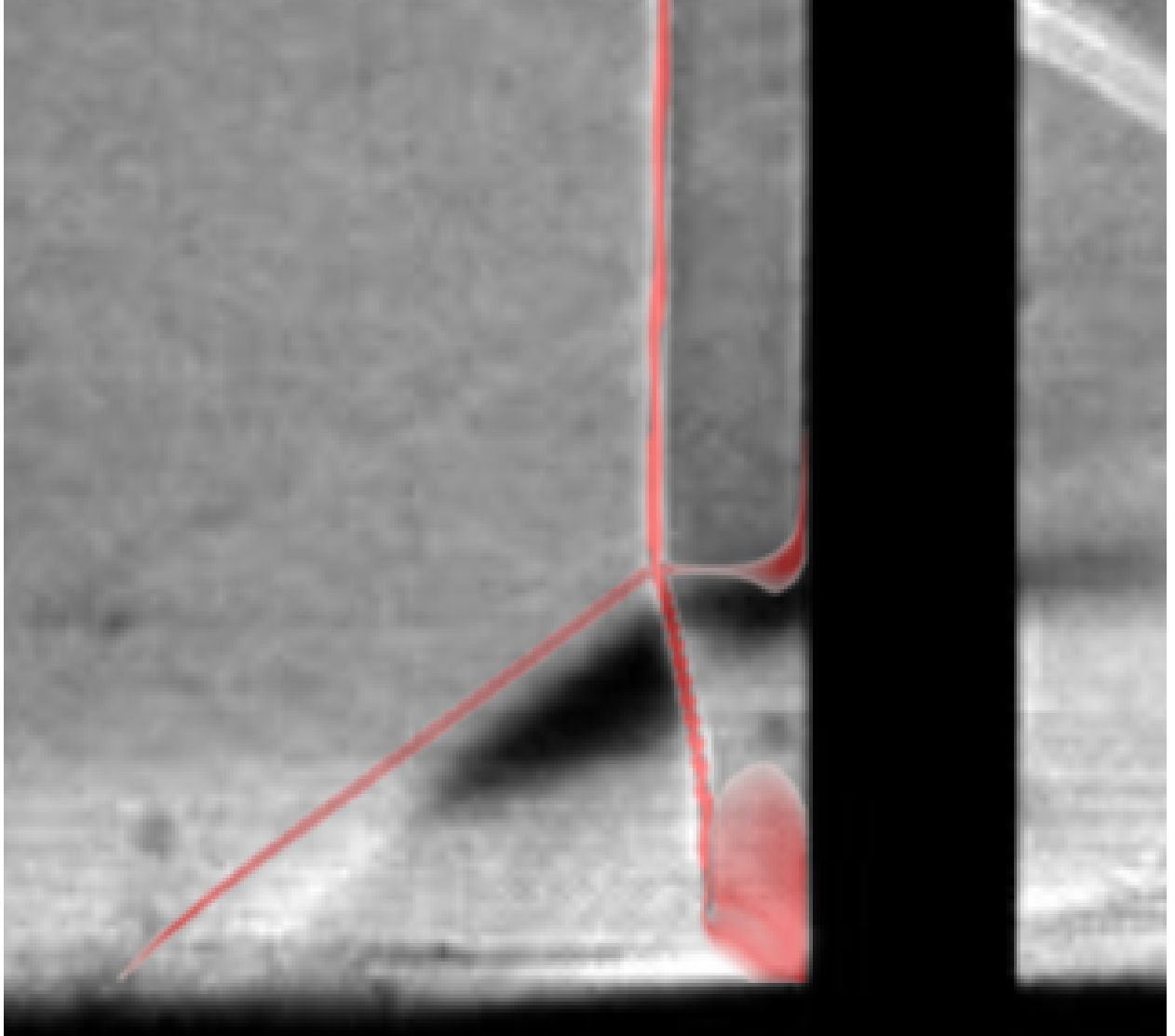


Figure 4.1: Superposition of numerical schlieren (red) to experimental mean schlieren image for same length scales of $d = 9.525$ mm and $\delta = 10.6$ mm.

of the bow shock in front of the cylinder, the forward separation shock, and the rearward closure shock. In the experimental schlieren image, the incoming, fully-turbulent boundary layer is also clearly visualized on the floor of the wind tunnel. The overlap between the red lambda-shock structure obtained by the RANS simulation and the experimental schlieren image is apparent. The calculated bow and rearward shocks are seen to qualitatively match the experiment well, but the forward shock appears to be further upstream. This is likely due to the conservative approach of the turbulence model when dealing with shallow flow separation. The shock-shock interaction at the triple point qualitatively matches as well, and a subsonic shear jet that emanates from the triple point and impinges on the cylinder leading edge is identifiable. This shear jet forms an Edney type III [28] interaction, as described in Sec. 2.2.5.

The same flow features can be found in the montage presented in Fig. 4.2. These schlieren images depict SWBLI generated with a variety of test configurations as defined in Table 3.2. The montage includes representative instantaneous (left) and corresponding mean (right) images for d/δ ranging from 0.87 to 7.94. Each image has been scaled so that the cylinder diameters are approximately equal in size and the leading edges of the cylinders have been aligned. Additionally, the montage contains a mixture of images obtained on the floor of the test section and on the flat-plate model. The incoming turbulent boundary layer, forward shock, and rearward shock were visualized in each image. The triple point was not visible in all images, however, due to field-of-view limitations imposed by the optical arrangement. Furthermore, the image resolution was restricted due to the high-speed CMOS sensor when a 100-kHz frame rate was employed. As a result, the image resolution of the wind-tunnel-floor test cases was significantly higher than that of the flat-plate test cases owing to magnification limitations with the optical setup. Nevertheless, it can be seen by the yellow dashed line in the figure that as d/δ decreases (from top to bottom), viscous effects become more relevant and the forward-shock position appears to move upstream. In addition, the forward shock becomes more rounded and behaves more akin to a detached oblique shock as the boundary layer encompasses more and more of the lambda-shock structure.

A similar montage for the numerical schlieren results is provided in Fig. 4.3 for lower values of d/δ and in Fig. 4.4 for higher values of d/δ . The results in Fig. 4.3 are taken at

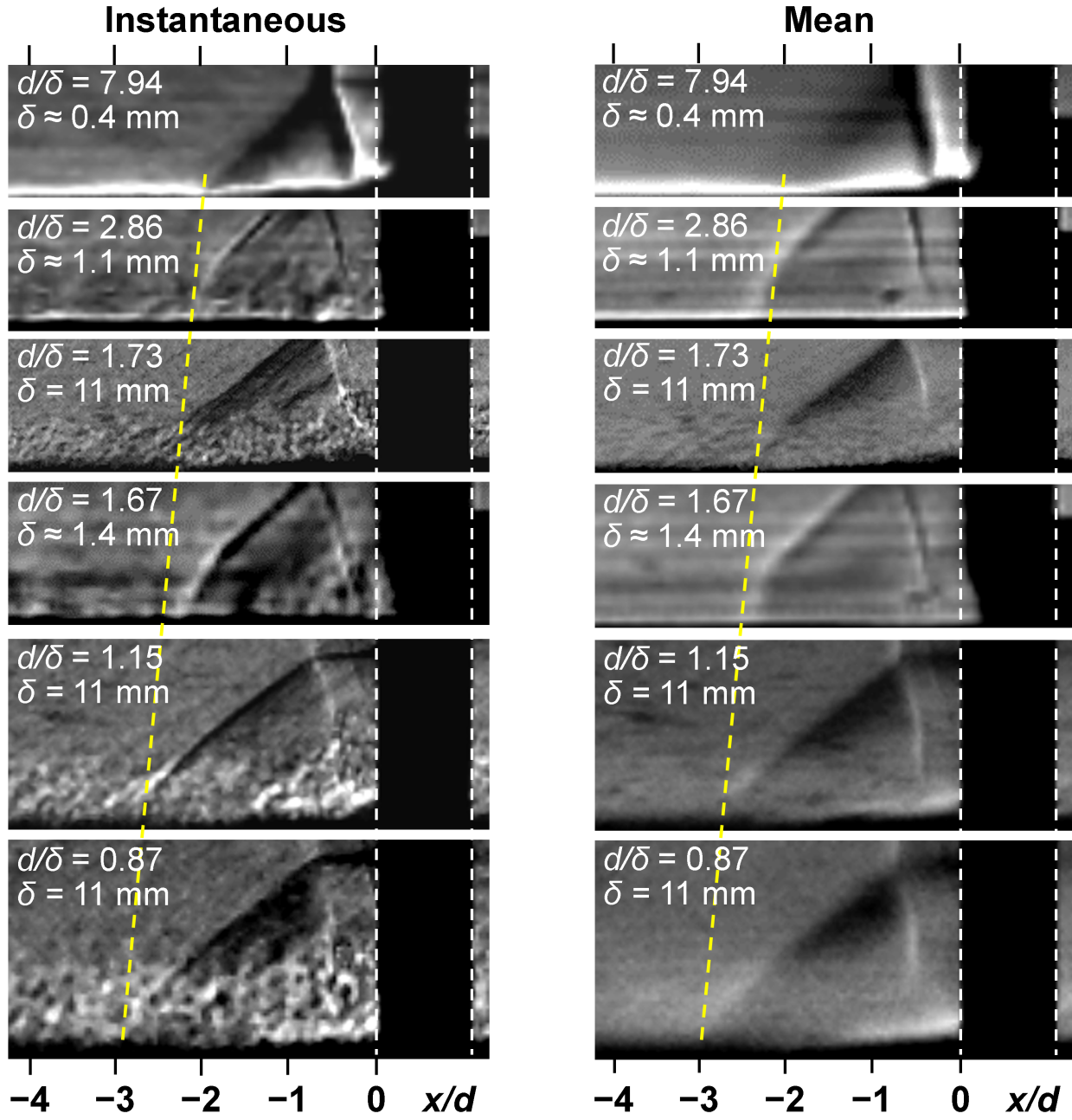


Figure 4.2: Schlieren image comparison of SWBLI generated over the full range of experimental test conditions investigated.

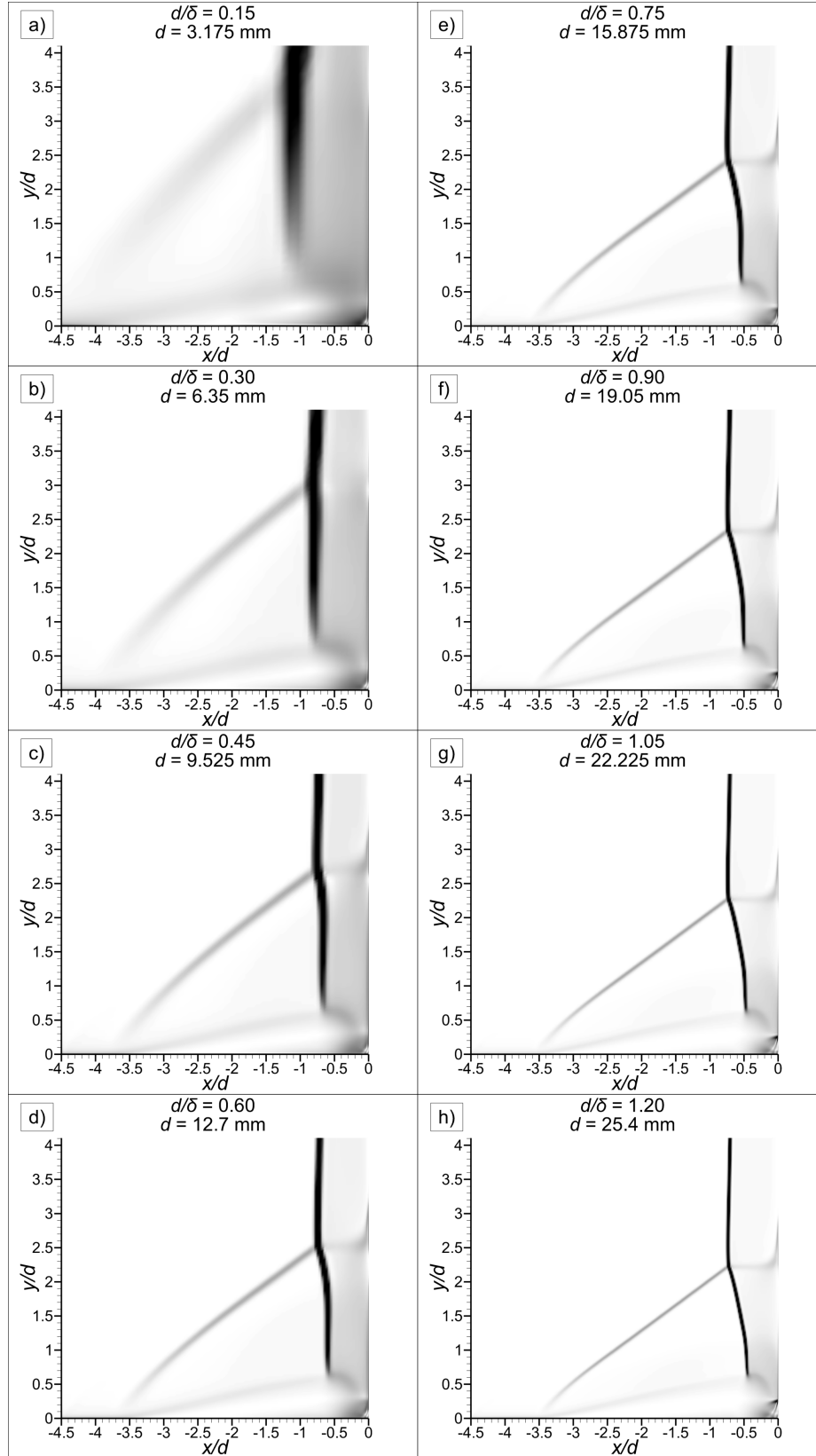


Figure 4.3: Numerical schlieren image comparison of SWBLI generated over low d/δ values of case C5 with $\delta = 21.2$ mm. All images taken on the centerline at $z/d = 0$.

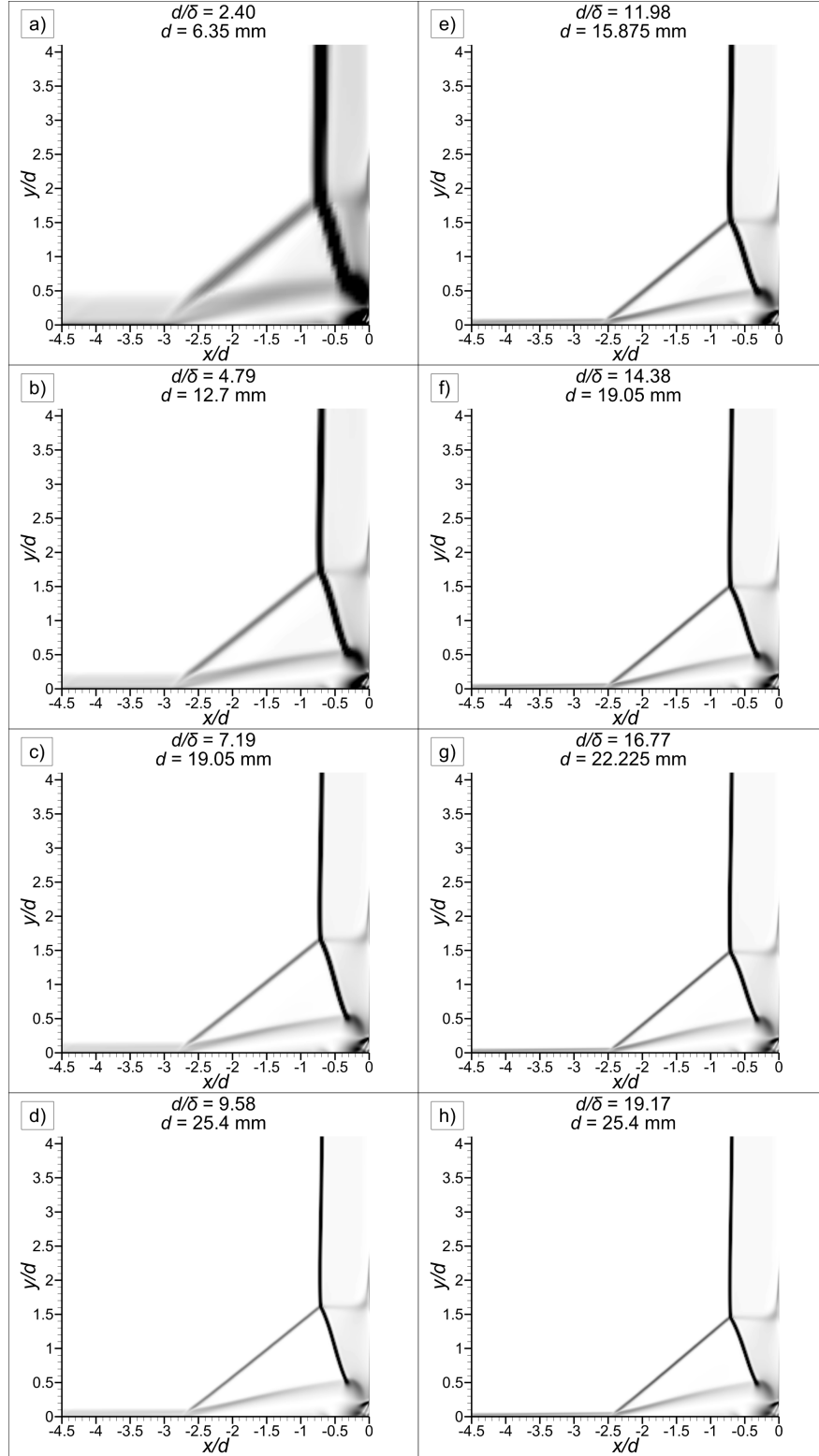


Figure 4.4: Numerical schlieren image comparison of SWBLI generated over medium d/δ values of case C2 with $\delta = 2.65$ mm in the left column and over high d/δ values of case C1 with $\delta = 1.32$ mm in the right column. All images taken on the centerline at $z/d = 0$.

the centerline for a constant boundary-layer thickness of $\delta = 21.2$ mm to illustrate the effect of varying just the diameter. Note that although the magnitudes in each contour plot are consistent, the smaller d/δ cases for each numerical case set appear thicker; this is due to the mesh being smaller, as the physical scales of the mesh were based on y^+ . For the smallest-diameter case in Fig. 4.3a, the forward shock is very weak and hardly redirects the rearward shock. As d/δ increases, the forward shock becomes stronger and less curved, indicating that the viscous effects on the interaction diminish. However, even for the value of $d/\delta = 4.79$ in Fig. 4.4b, there is still some curvature near the forward-shock foot. As indicated in Fig. 4.4, at higher d/δ values this curvature diminishes to the point where the shock foot can be approximated by a straight line; this appears to be the case when $d/\delta \geq 7.19$.

It is also clear that when properly scaled, these trends are present irrespective of the incoming boundary-layer thickness or model configuration, as showcased by the yellow dashed lines for the $d/\delta = 1.73$ and 1.67 images presented in Fig. 4.5. The image for $d/\delta = 1.73$ was collected on the floor of the wind tunnel test section with $d = 19.05$ mm and $\delta = 11.0$ mm, while the image for $d/\delta = 1.67$ was collected on the flat-plate model with $d = 2.39$ mm and $\delta = 1.4$ mm. Despite approximately an order of magnitude difference in scale, the structure and size of these interactions appear similar when properly scaled, since the x/d -locations of the forward shock and the triple point are seemingly identical.

A more quantitative comparison of the two experimental test cases from Fig. 4.5 with $d/\delta = 1.73$ and 1.67 is provided in Fig. 4.6, which presents probability density functions (PDFs) of the forward-shock position. The two PDFs are generally similar to peak values of $-2.87d$ and $-2.74d$, a difference of 4.5% that is within the measurement of uncertainty for the technique. Furthermore, the full-width at half-maximum (FWHM) values are $0.26d$ and $0.28d$ for the $d/\delta = 1.73$ and 1.67 cases, respectively, showing agreement within the bin size used to generate the PDFs.

4.2 Observation of Trends in the Scaled Data

As described in Sec. 2.4, several prior attempts were made to determine scaling effects for the maximum mean separation distance, S . One commonly proposed scaling recommendation

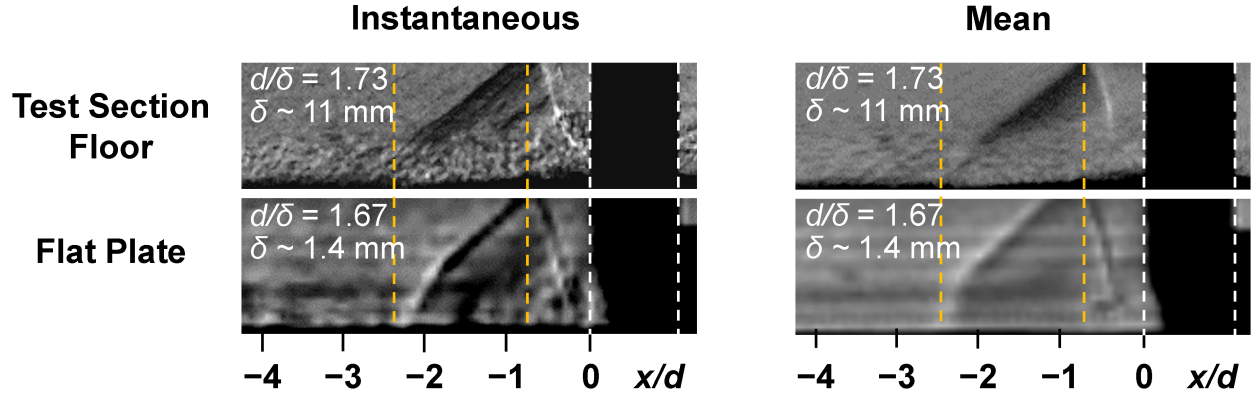


Figure 4.5: Schlieren image comparison indicating qualitative scaling similarity with d/δ at differing length scales.

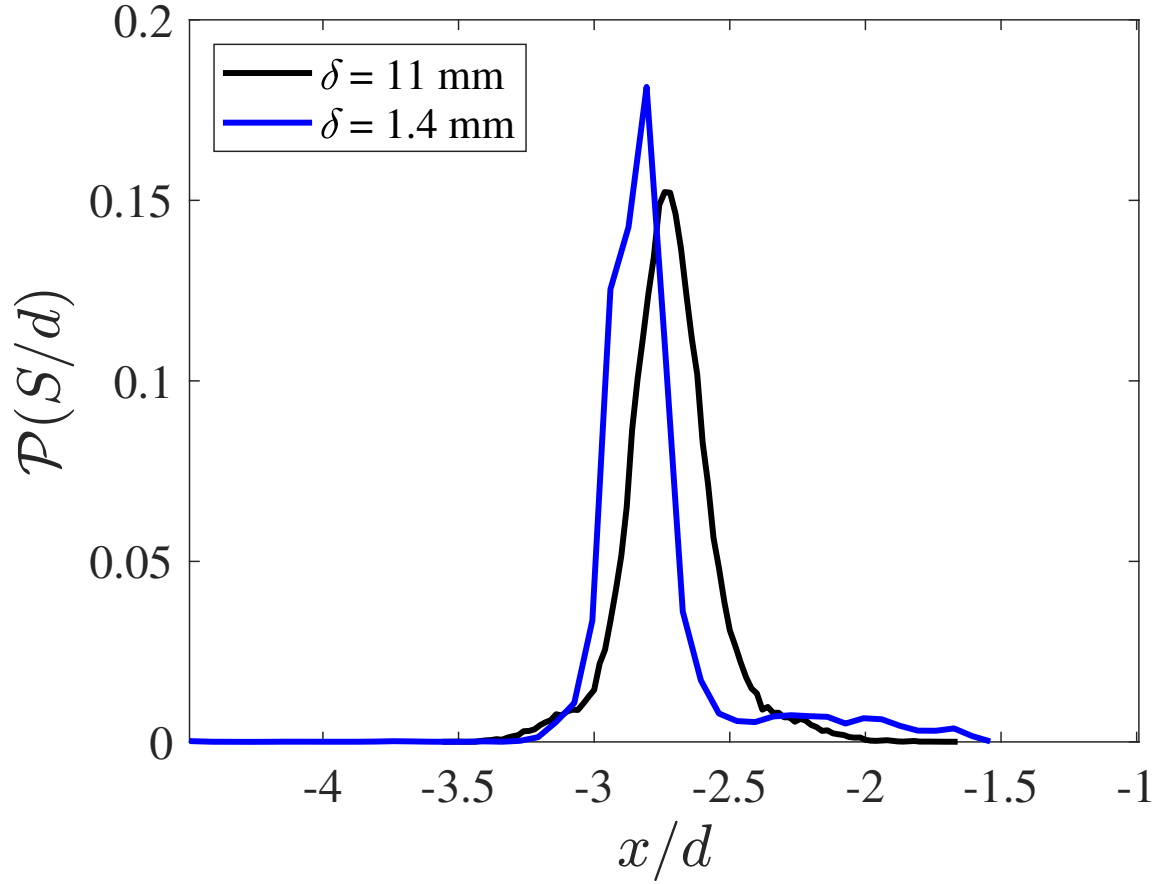


Figure 4.6: PDF comparison of the forward-shock foot location for SWBLI with qualitative scaling similarity with d/δ at differing δ .

for S is shown and applied to results gathered here and to a large variety of data from the archival literature. These data, summarized in Table D.1, cover a myriad of d , δ , d/δ , M_∞ , and Re values.

Following the scaling recommendations of Young et al. [22], Sedney and Kitchens [18], and Dolling and Bogdonoff [13, 14], the normalized term S/d is plotted against the parameter d/δ in Fig. 4.7 with a best-fit trendline. The parameter S/d lies mostly between 2 and 3, as described in Sec. 2.2.2. The CFD data are consistently predicting a larger separation distance from the experiments, as expected. A power-law trend provides the best fit, although the overall coefficient of determination value, R^2 , is only 0.37, indicating a weak correlation [165]. Power-law trend lines passing solely through the data gathered in this study yield high R^2 values of 0.90 and 0.84 for the CFD and experimental data, respectively. However, the results indicate that there are scaling effects not captured when including data from other sources.

As both Young et al. [22] and Sedney and Kitchens [18] found, S scales directly with d , and effects of other variables are secondary. However, the flow has also been found to scale with d/δ . Rather than normalizing the separation distance by d , a normalization by δ is proposed. This ensures that both the separation distance and the diameter are normalized by the same term (δ) to maintain the primary dependence of the two variables. This new relationship of S/δ as a function of d/δ is shown in Fig. 4.8, where it becomes evident that the boundary-layer thickness normalization for both terms seems to capture some of the secondary flow effects, as the spread at a given ordinate value is reduced. The overall R^2 values are 0.97 for a power-law trend and 0.96 for a linear trend, indicating a strong correlation for both methods [165]. Particularly for the data from the archival literature, where M_∞ and Re are not uniform as in the data gathered from this study, the low spread and high correlation would suggest that the proposed normalization fits the data well. The deviation between the power-law and linear trend lines occurs for values where the boundary-layer thickness is greater than the diameter. For these values of $d/\delta < 1$, the viscous effects become preminent and the power-law fit should be used. For the rest of the data set, the simpler linear trend line is equally appropriate, as the inertial forces dominate the interaction.

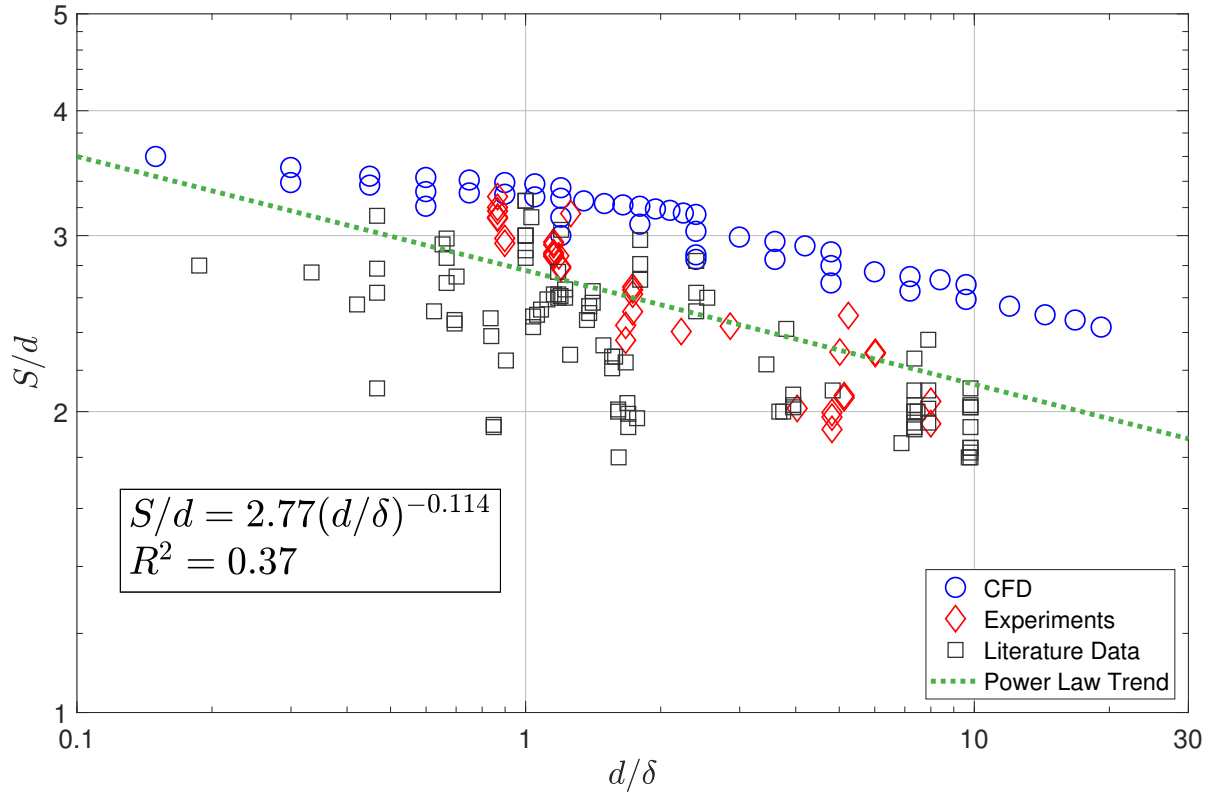


Figure 4.7: Separation distance scaled by cylinder diameter as a function of d/δ for CFD, experimental, and archival literature data, indicating a weak power-law correlation.

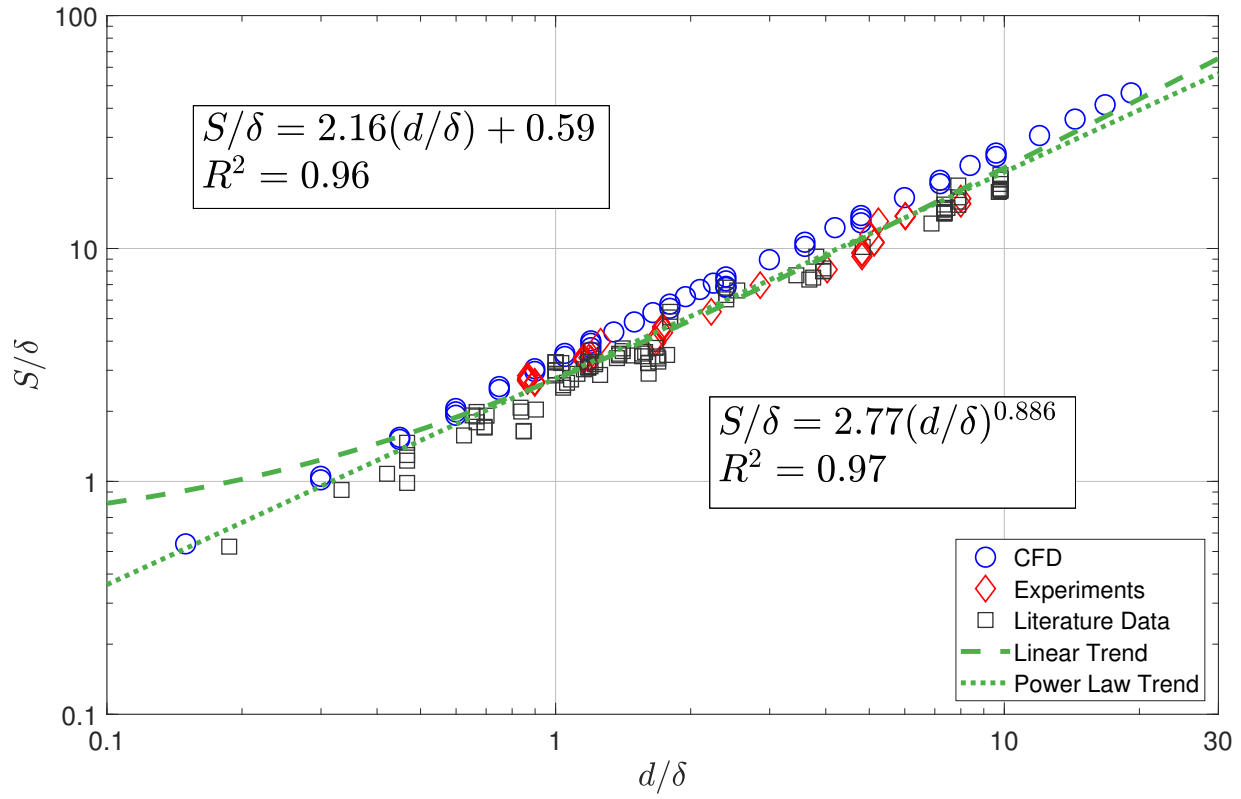


Figure 4.8: Separation distance scaled by boundary-layer thickness as a function of d/δ for CFD, experimental, and archival literature data, indicating strong linear and power-law correlations.

Further assessments of this scaling method were applied to the triple-point height, h_{tp} . This variable cannot be measured using surface measurements, but can be evaluated using optical diagnostic techniques. Moreover, due to the curvature of the forward shock near the boundary layer, as discussed in Sec. 2.2.3, a geometric formula from Westkaemper [17] based on the separation distance, the forward-shock angle, and the inviscid bow-shock standoff distance does not yield an accurate value for the triple-point height. In several instances where optical diagnostics had been used [21, 22, 24, 26], the schlieren image quality of the microfilm copies is often poor, and the triple point was generally used to assess the Edney interaction [28] on the leading edge or whether a semi-infinite interaction criterion [13] was met. However, a quantitative analysis might allow for a priori knowledge on whether a given cylinder or blunt-fin height can meet this $h/h_{\text{tp}} > 2\text{--}3$ criterion [13]. Shown in Fig. 4.9 is the measured triple-point height for the collected data, normalized by the boundary-layer thickness as in Fig. 4.8. As with the separation distance, the CFD results consistently predicted a higher value for the triple-point height, which was induced by the further upstream separation location. Similar best-fit conclusions can be drawn as from Fig. 4.8, although there were no quantitative data from the archival literature added for comparison. High R^2 values are observed with 0.97 for a linear trend and 0.94 for a power-law trend. The deviation of the two trend lines occurs around $d/\delta < 0.5$, which is a lower value than for the separation distance. Moreover, the CFD data for $d/\delta \leq 0.30$ both qualitatively and quantitatively indicated that the triple point was encompassed by and fully immersed within the boundary layer. The resulting viscous effects may thus dominate for such small d/δ values, resulting in the deviation between the linear and power-law trend lines. A second deviation occurs for higher values where $d/\delta > 10$, likely owing to the lack of experimental data in that region. Nevertheless, either the power-law trend line or the simpler linear trend line are appropriate for use when d/δ is between 0.5 and 10, and the power-law trend line should be used at the outer bounds.

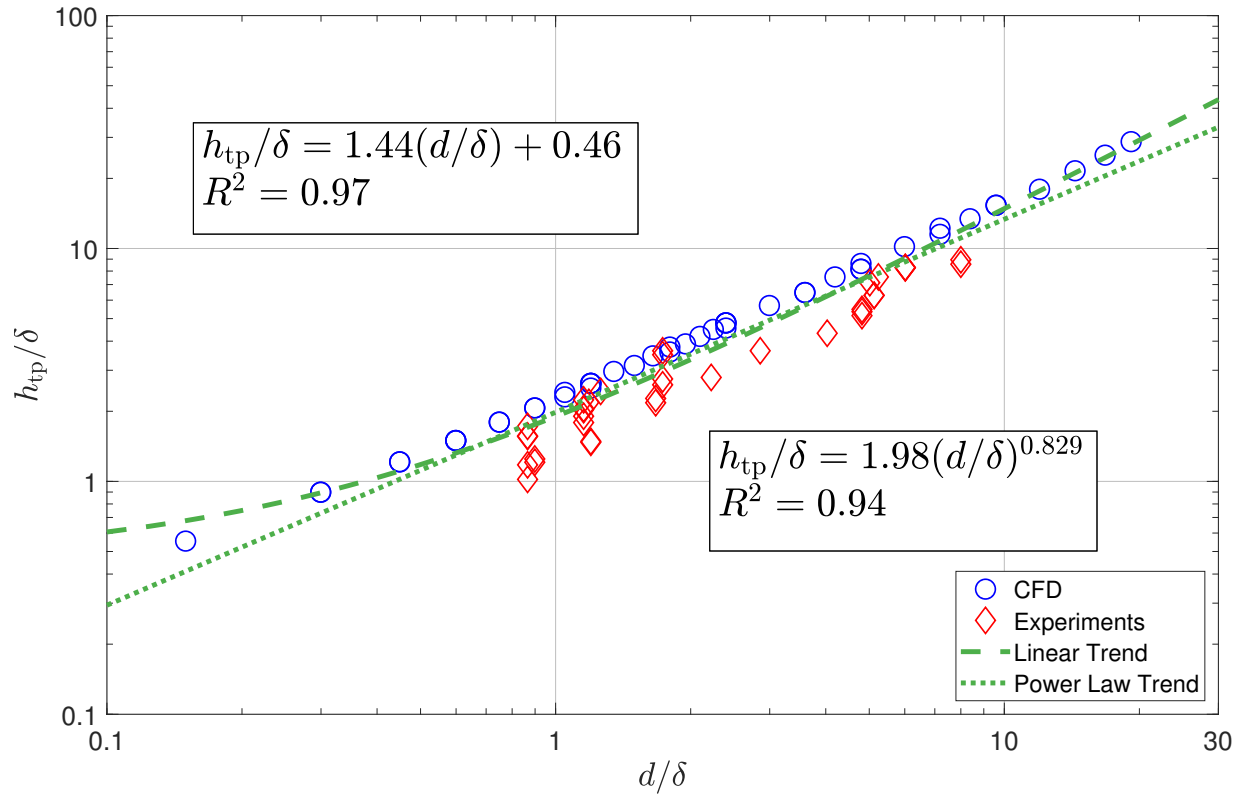


Figure 4.9: Triple-point height scaled by boundary-layer thickness as a function of d/δ for CFD, experimental, and archival literature data, indicating strong linear and power-law correlations.

4.3 Dynamic Analysis of Separation Distance

At the time the analysis in this chapter was completed, no LES had been performed. Consequently, the dynamic analysis shown here will focus solely on the experimental data that was gathered, as all simulations performed to this point were steady-state RANS simulations. Using the time-resolved experimental data, temporal scaling effects on the separation distance were investigated. A comparison of statistical moments as a function of d/δ is shown in Fig. 4.10. No discernible trend was observed for any of these quantities, as the values appear to be relatively constant with some experimental scatter. The standard deviation in Fig. 4.10a appears to have a fairly consistent value between 5–10% of the mean for values of $d/\delta < 5$, with slightly higher values approaching 20% of the mean for some of the large d/δ cases. Meanwhile, the skewness in Fig. 4.10b is primarily centered at zero, indicating distribution that are neither left- nor right-leaning. However, it seems to be a tendency for some cases to have slightly positive values, indicating distributions skewed downstream toward the cylinder. Lastly, the majority of test cases yielded distributions with a kurtosis of approximately 3, as evident in Fig. 4.10c, indicating a Gaussian distribution. A few cases demonstrate leptokurtic behavior (positive excess kurtosis), suggesting that the distributions have slightly sharper peaks (or longer tails) than a Gaussian distribution.

Normalized one-sided auto-spectral density functions, $G(f)$, were also calculated for time histories of S and are presented in Fig. 4.11 for a series of representative d/δ cases. It should be noted that the normalization has been performed using the resolution, df [Hz], of the spectra, rather than the frequency, f , at each point. This normalization was chosen to avoid artificially amplifying higher-frequency content and potentially producing misleading peaks in the presented spectra [112]. In comparing the spectra, it does not appear that there is a clear trend in the shapes or peaks of the spectra with varying d/δ . Each plot, particularly the $d/\delta = 1.15$ and 1.73 cases, appears to be dominated by broadband low-frequency ($f < 1$ kHz) content with little energy higher than 10 kHz, as has been observed in many previous studies of SWTBLI [12]. The spectra do not appear to demonstrate any dependence on d/δ , and the dynamic results thus suggest that while the position and structure of the SWBLI may

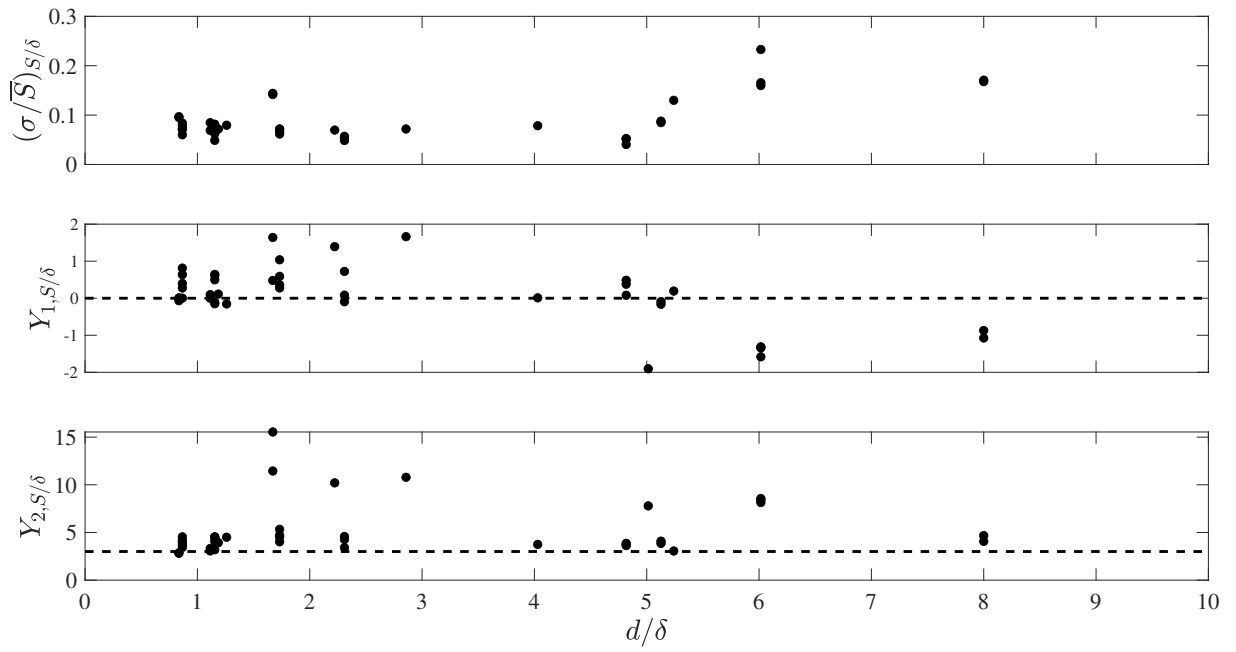


Figure 4.10: Statistical moments of a) standard deviation, b) skewness, and c) kurtosis for separation distance gathered from experimental data.

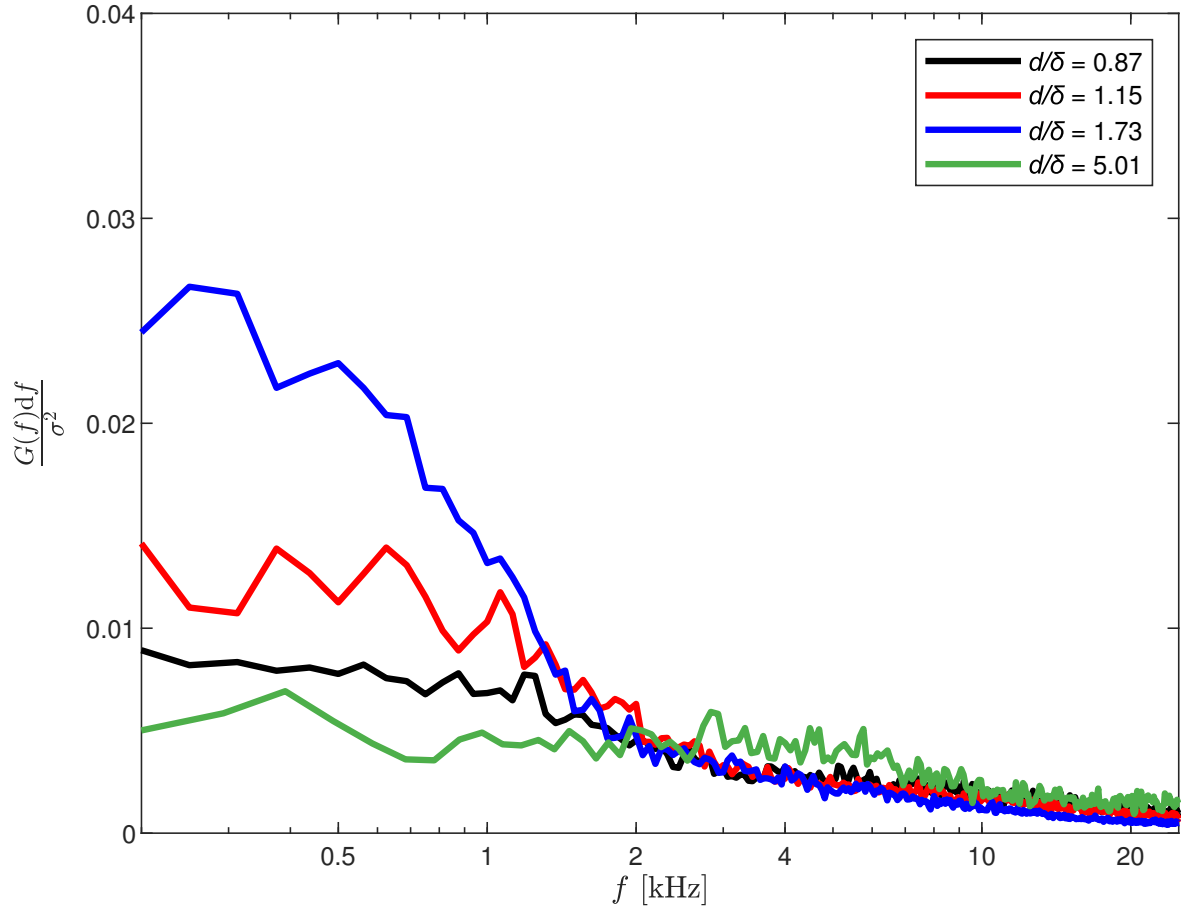


Figure 4.11: Comparison of one-sided auto-spectral density functions of separation distance for varying d/δ scaling.

change as a result of varied d/δ , scaling effects on the unsteady dynamics of the interaction are minimal.

4.4 Centerline Surface Pressure Analysis

Some of the SWBLI's most practically important features to the aerospace community are the centerline pressure profiles on the floor upstream of the cylinder and on the leading edge of the cylinder. Pressure profiles were extracted from the RANS simulations, but without experimental comparison. Related data have been reported by many authors, as described in Sec. 2.2.4 and Sec. 2.2.6, and current results indicate qualitative agreement. Shown in Fig. 4.12 are centerline surface pressure ratios on the floor just upstream for $d/\delta \leq 2.40$, normalized by the freestream pressure, $\bar{P}_\infty$. The coefficient of pressure, c_P , is provided for reference. The smaller d/δ values were chosen here to emphasize the nonlinear behavior that becomes qualitatively invisible for large d/δ values. The shape of the pressure profile is typical, as discussed in Sec. 2.2.4, sharing all the characteristics visible in Fig. 2.11, including the initial pressure rise with a plateau, followed by a pressure trough, and then a pressure peak with some relief. Based on a streamline comparison and separation analysis, the initial pressure rise occurs upstream of flow separation, as discussed in Sec. 2.2.4, and the pressure peak is a direct result of the attachment line for the secondary near-corner vortex, as discussed in Sec. 2.2.7. The free interaction theory [52] holds true for this geometry [22] and states that separation occurs for a pressure ratio of around $\bar{P}_w/\bar{P}_\infty = 1.2\text{--}1.3$ [22, 31, 52]; the pressure ratio at separation was between 1.27 and 1.30 for the cases in Fig. 4.12. The initial pressure rise follows the same trend as observed for S , where a smaller d/δ value corresponds to a further upstream location for the rise; this is likely due to the increased extent of viscous effects as the lambda-shock structure is immersed in the boundary layer.

The initial pressure rise plateaus at a pressure ratio of around 1.5; this value is similar to that calculated by the semi-empirical 2-D approximation from Eq. 2.4, which results in a pressure ratio of 1.43 for the current conditions. Closely tied to the initial pressure rise, the effects of a smaller d/δ also include a lower initial pressure plateau, a higher pressure trough, and a lower pressure peak. The added viscous effects are a significant contributor

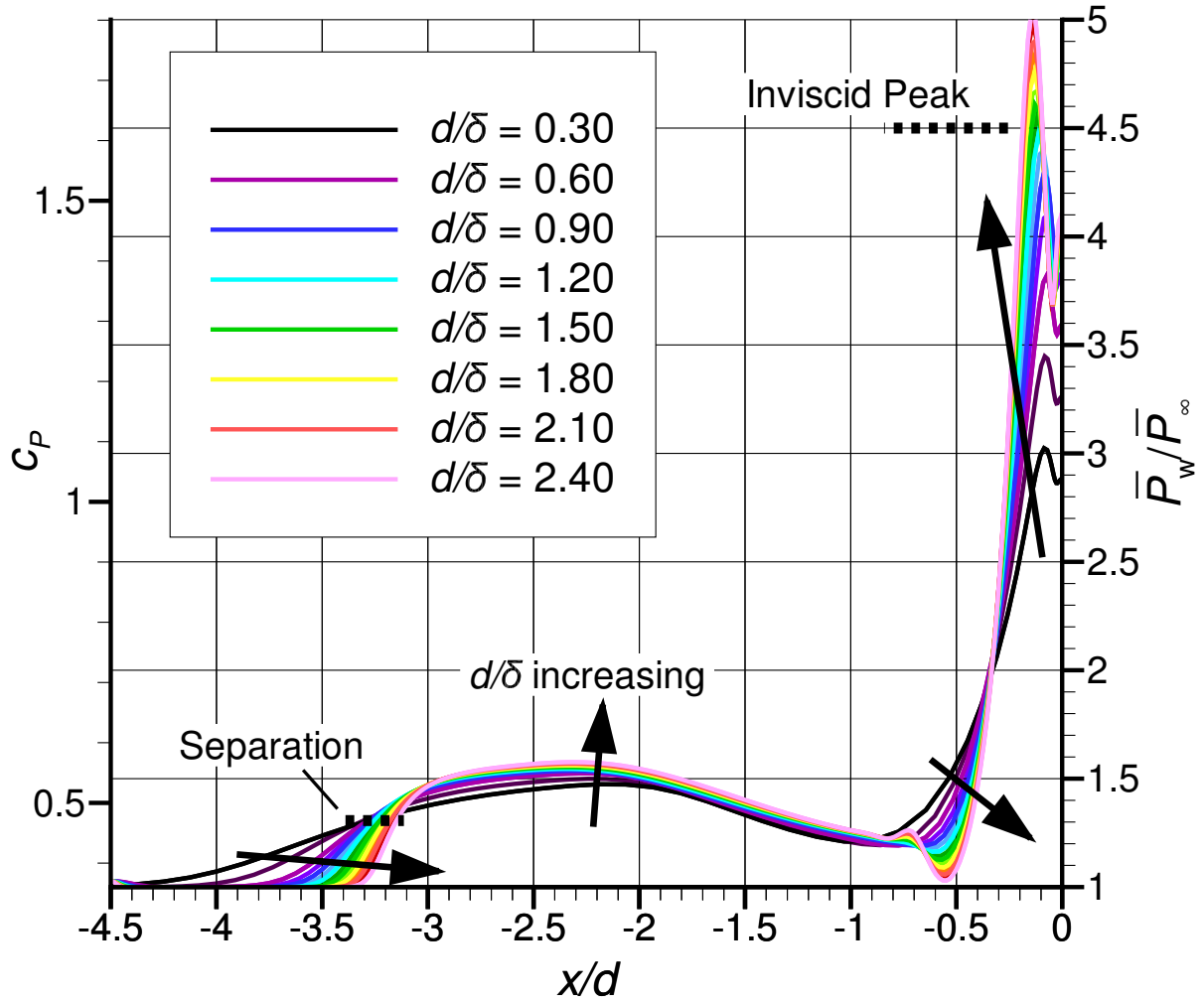


Figure 4.12: Surface pressure profiles for d/δ values of case C4 on the floor upstream of the cylinder at centerline $z/d = 0$. All cases have constant $\delta = 10.6$ mm.

to mitigating the severity of the interaction, changing the pressure peak from $\bar{P}_w/\bar{P}_\infty = 5$ for $d/\delta = 2.40$ to a value of only $\bar{P}_w/\bar{P}_\infty = 3$ for $d/\delta = 0.30$. Due to the power-law scaling of d/δ , this mitigation is not as effective for high d/δ , and this is reflected by the maximum pressure peak in this study being $\bar{P}_w/\bar{P}_\infty = 5.9$ for $d/\delta = 19.17$. For reference, the inviscid value of $\bar{P}_w/\bar{P}_\infty = 4.5$ obtained via a shock polar, as described in Sec. 2.2.5, is included in Fig. 4.12.

Surface pressure ratios on the cylinder leading edge for the same d/δ values are shown in Fig. 4.13, normalized by the stagnation pressure downstream of the inviscid bow shock $\bar{P}_{0,2} = 173$ kPa. Again, c_P is provided for reference. The shape of the pressure profile is typical for values of $d/\delta \geq 0.45$, and follows that described in Sec. 2.2.6. This consists of an increase in pressure from the inviscid value until the peak is reached, followed by a significant pressure relief within the interaction region, where $\bar{P}_w/\bar{P}_{0,2}$ falls under 1. A pressure trough is experienced, followed by a matching of the pressures at the base of the cylinder. the pressure peak is characterized by an Edney type III interaction [3, 23, 26, 28, 43], as described in Sec. 2.2.5, and its value remains consistent across $d/\delta \geq 0.60$, but smaller for the $d/\delta = 0.45$ case, and no pressure peak was observed for the $d/\delta = 0.30$ case. As indicated in Fig. 4.9, $h_{tp}/\delta < 1$ for the $d/\delta = 0.30$ and 0.45 cases, so the reduced peak value is due to the boundary layer encompassing the Edney interaction [28]. The pressure trough seems to indicate a nearly constant value of $\bar{P}_w/\bar{P}_{0,2} = 0.35$ at $y/d = 0.3$ for all cases investigated. Following another streamline analysis, this location corresponds to the separation line of the secondary near-corner vortex, labeled *SL2* in Fig. 2.14, and does not appear to be sensitive to viscous effects.

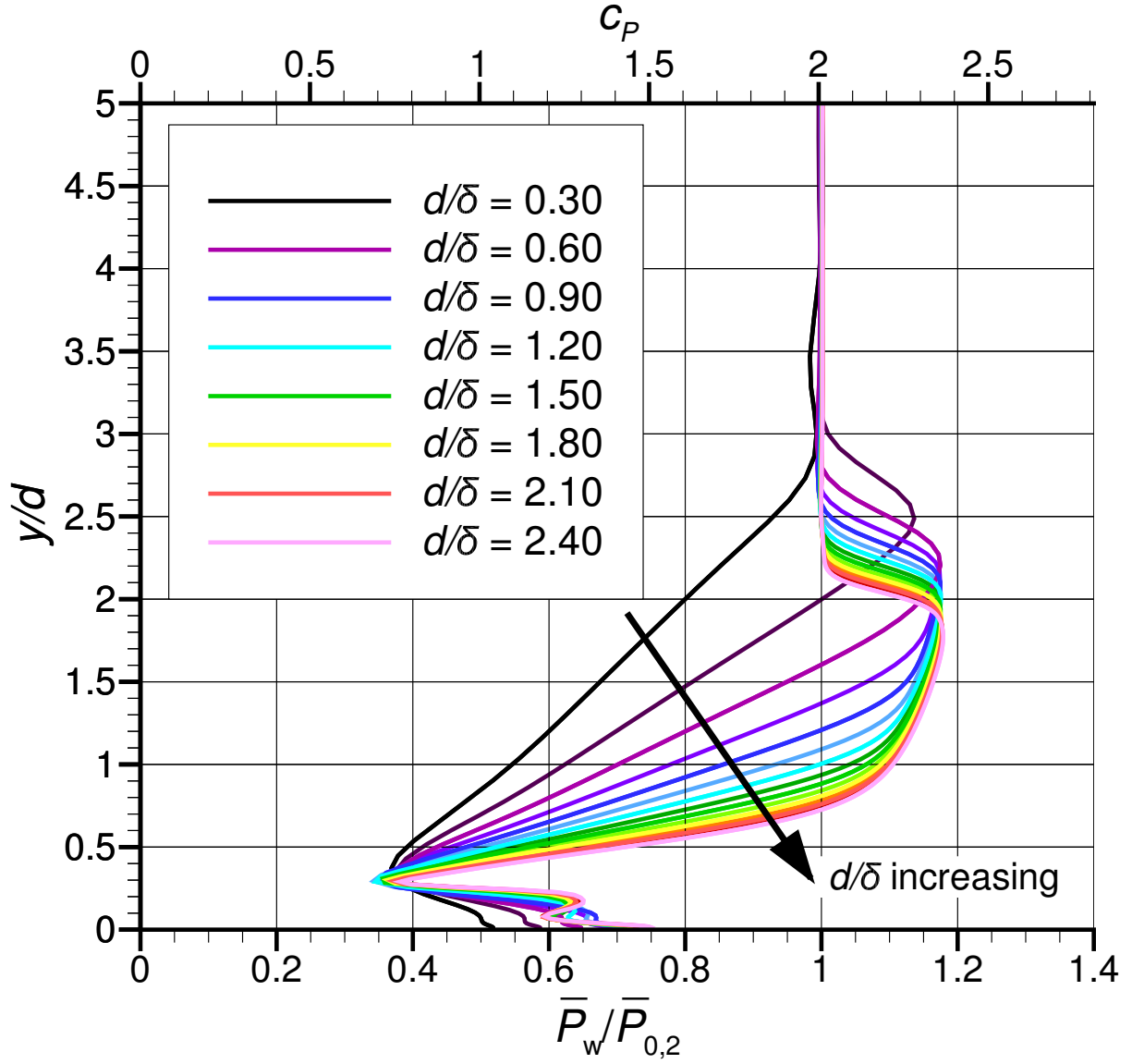


Figure 4.13: Surface pressure profiles for d/δ values of case C4 on the cylinder leading edge at centerline $z/d = 0$. All cases have constant $\delta = 10.6$ mm.

Chapter 5

Spanwise Evolution of Pertinent Flow Features

The spanwise effects are described with a great emphasis on case C6 in Table 3.2. Quantitative results of the time-averaged flow features are analyzed first in Sec. 5.1. A comparison between mean experimental and mean CFD results, both RANS and LES, is provided. This is followed by a comparison between time-accurate experimental and LES results in Sec. 5.2. As the pertinent flow features originate at the centerline before moving and evolving in the spanwise direction, emphasis is placed on matching experiments to the simulations at the centerline first. Moreover, this is also the location at which most of the experimental data were collected.

5.1 Time-Averaged Flow Features

The mean separation distance is taken into account first, and there are two approaches that can be followed to retrieve this: the time-accurate flow data could be time-averaged, and the mean value can be found by a simple investigation of the sign change of c_f ; or, the time-accurate data could be analyzed to provide dynamic separation distance, and the mean value can be found by time-averaging the dynamic results. These approaches were described in Sec. 2.3.1. In theory, both approaches should yield the same outcome, but in practice this can differ because the criteria may change with the approaches, generating errors. In

fact, for the data analyzed, these two approaches yielded approximately 5% difference. As the first approach of time-averaging the data and then finding the separation distance is universally accepted for both CFD data and through oil-flow visualization, this method is used moving forward. Similar approaches were used for analyzing the other flow features, such as the pressure on the floor and on the fin or cylinder.

5.1.1 Mean Separation on the Floor

Starting at the centerline, Fig. 5.1 shows the mean u -velocity-equivalent Mach number in the first cell off the wall, as a substitute for c_f . Three cases are presented, with the low-Reynolds number cases for ILES and RANS representing case C6 in Table 3.2, and the high-Reynolds number RANS case taken from case C4 in Table 3.2 at an approximately matching d/δ value of 1.3. Observing the upstream-most sign change, the mean separation distance for the ILES case was $S/d = 2.66$ or $S/\delta = 3.46$; using the trendline from Fig. 4.8, the linear trend predicts a value of $S/\delta = 3.40$ ($S/d = 2.62$), and the exponential trend predicts a value of $S/\delta = 3.50$ ($S/d = 2.69$), thus bounding the computed solution within $\pm 2\%$. It should be noted that the ILES result is more closely aligned with the experimental data in Fig. 4.8 than it is with the RANS simulation data, which resulted in a value of $S/d = 3.25$ and is nearly 23% higher. This further corroborates the trend that the RANS simulation data over-predicts separation due to the effects of the turbulence model; moreover, any effects due to the Reynolds number are eliminated by confirming that the RANS simulation results in Fig. 5.1 yield approximately the same separation distance at both low and high Re. Also found in Fig. 5.1 are the number of mean vortices in the primary vortex structure by observing the sign changes between the separation distance and the near-leading-edge behavior. For both of the low-Reynolds number cases, there were three zero-crossings, thus yielding three vortices; for the high-Reynolds number RANS case, the only sign change occurred at separation. This is in line with the observations described in Sec. 2.1.2.

Many of the characteristics observed at the centerline extend into the span, as shown in a top-down view of the mean ILES results in Fig. 5.2a. The streamlines reveal some of these characteristics, including the separation line, the reverse flow region, and the vortex extents. The separation line is marked by the upstream-most line of coalescence, and its

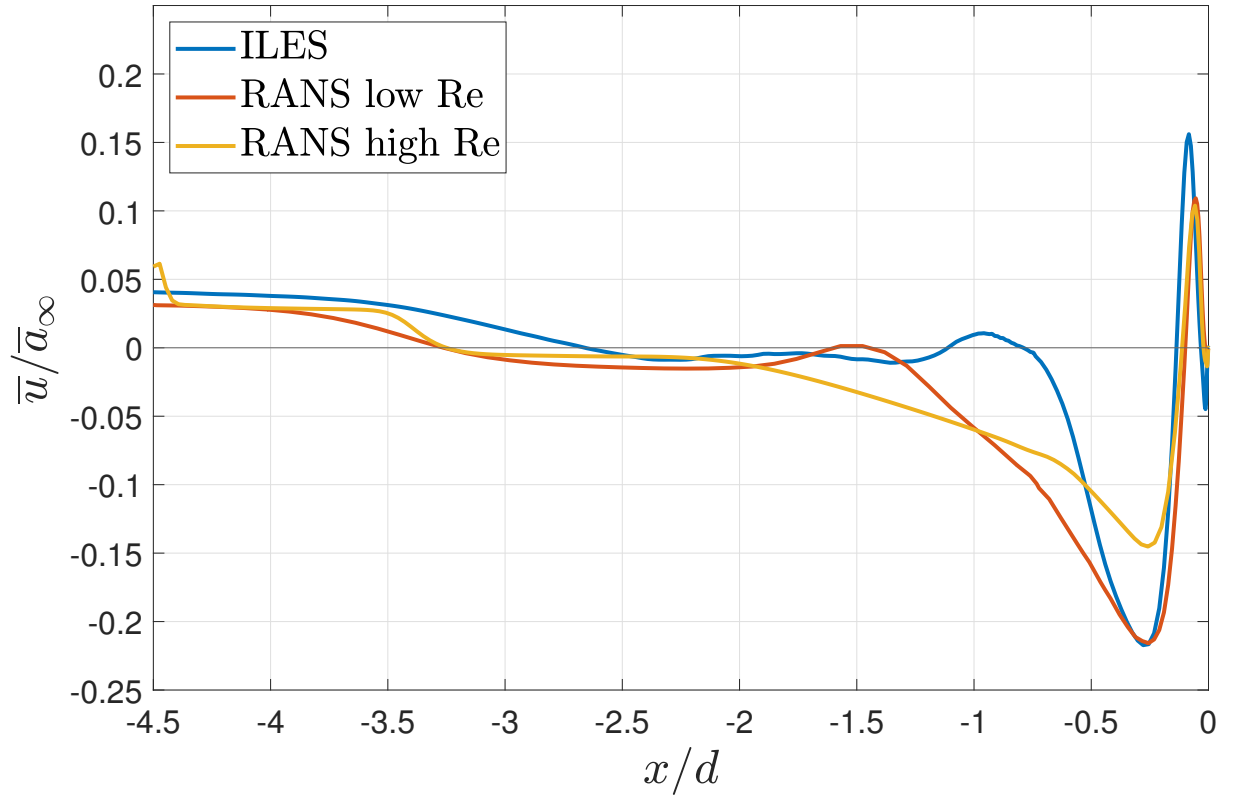


Figure 5.1: Mean u -velocity-equivalent Mach number in the first cell off the wall at the centerline on the floor.

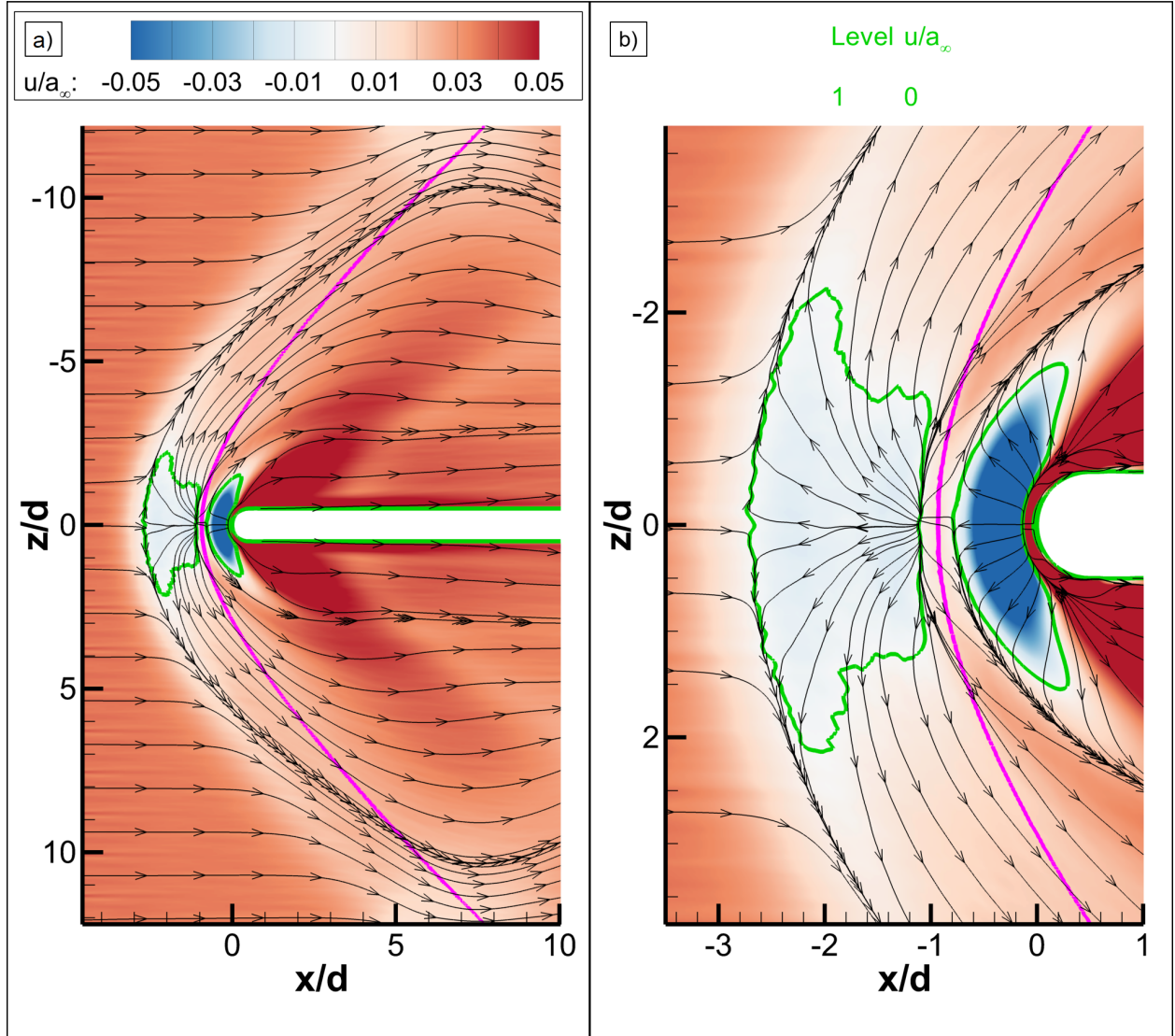


Figure 5.2: Contour of mean u -velocity-equivalent Mach number in the first cell off the wall with black streamlines. Flow is left to right. Green line is $u = 0$. Pink line is inviscid bow shock projected onto the floor for reference. a) Full view of main region of interest, b) enhanced view of leading-edge region.

furthest upstream extent is at the centerline with the aforementioned value of $S/d = 2.66$. This line curves in a hyperbolic manner similar to the inviscid bow shock, which is provided for reference in the figure. By around $z/d \gtrapprox \pm 6$, the separation line has linearized and its curvature has become negligible. Further outboard, the effects of the periodic boundary condition become evident for values of around $z/d \gtrapprox \pm 10$; for the purposes of this analysis, this outboard flow region can be ignored, although it may be of interest to someone who is keen on understanding the effect of placing blunt fins at equal spanwise distances apart (in this case, three fins located around $24d$ apart).

Additional lines of coalescence and divergence are more clearly displayed in the enhanced view in Fig. 5.2b. Near the fin leading edge, where the contour color changes from deep blue to deep red, a coalescing reattachment line forms at the centerline that evolves into a line of divergence at around $z/d \gtrapprox \pm 0.5$. This line is marked by the boundary of the secondary vortex described in Sec. 2.1.2 and labeled as *AL2* in Fig. 2.14. Further upstream, around $x/d \approx -1.1$ and $x/d \approx -0.8$ and near the centerline, a set of limiting streamlines are found, consisting of a line of divergence and a line of coalescence, respectively. These correspond to the bounds of the three vortices in the primary vortex structure, and their x -locations at the centerline are traceable to the zero-crossing locations in Fig. 5.1; for the high-Reynolds number RANS simulations, in which there is only one vortex in the primary vortex structure, these lines do not exist and the streamlines coalesce towards the separation line.

The green line in the contour marks the boundary of the reverse flow region in which the u -velocity is negative. The gap around $x/d \approx -1$ and near the centerline is due to the middle vortex in the primary vortex structure, as these are counter-rotating vortices; for the high-Reynolds number RANS simulations, there is no gap and a continuous green line is present. Interestingly, the maximum spanwise extent of the reverse flow region is around $z/d \lesssim \pm 2.2$ (or ± 2.7 for the high-Reynolds number RANS simulation results). This suggests that further outboard, the flow no longer separates (in the mean sense), even as it approaches the separation line. Supporting proof of this can be found in a streamline analysis shown in Fig. 5.3. Starting at the centerline at $z/d = 0$, the lambda-shock structure (without the bow shock) and the primary vortex structure, consisting of three vortices, are visible. The largest of the three vortices dominates in size and extends further away from the wall than

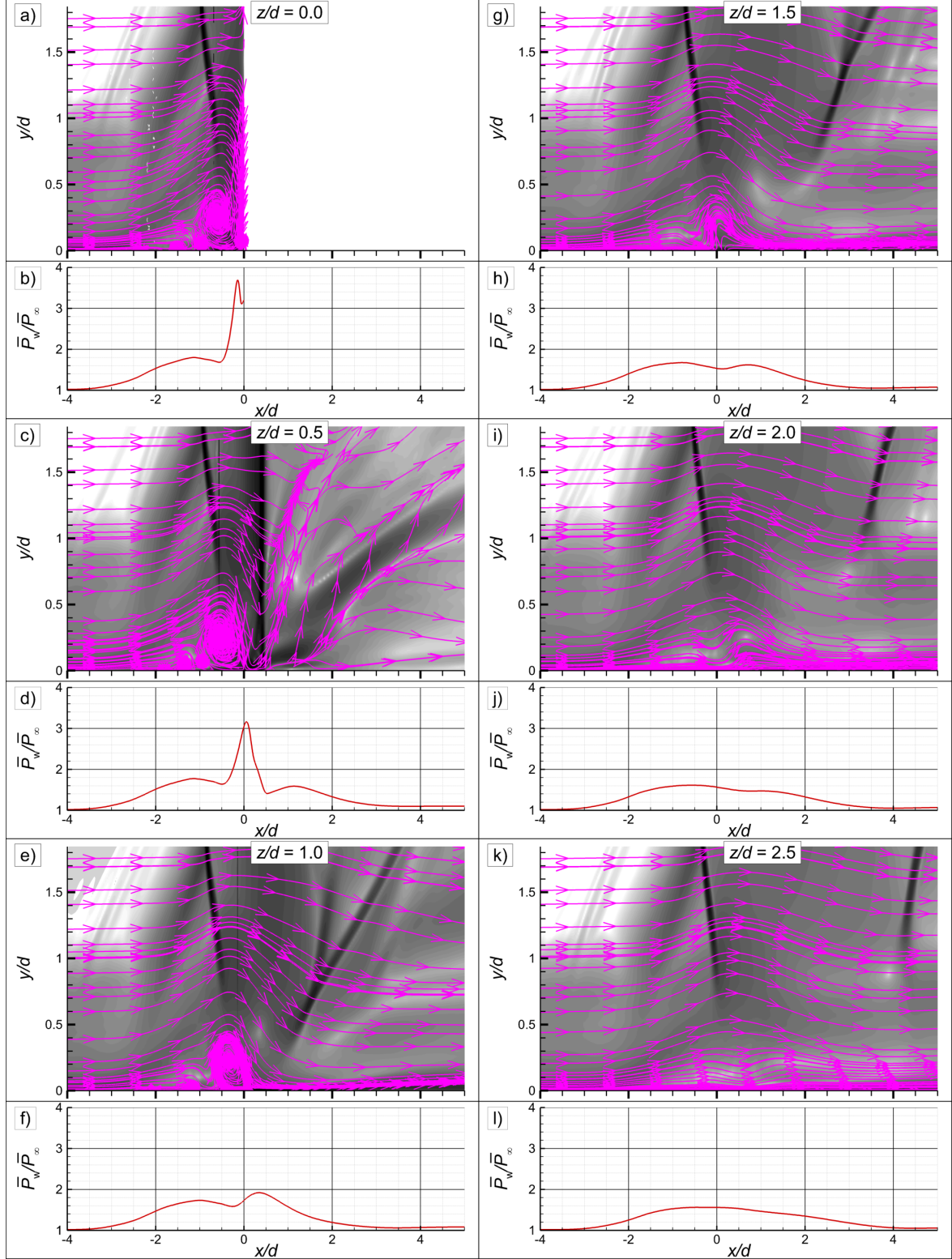


Figure 5.3: a), c), e), g), i), k) Contour slices of mean numerical schlieren and tangential streamlines with ordinate-to-abscissa ratio of 3:1. b), d), f), h), j), l) Mean normalized wall pressure on the floor at corresponding abscissa location. Slices at a), b) $z/d = 0$, c), d) $z/d = 0.5$, e), f) $z/d = 1.0$, g), h), $z/d = 1.5$, i), j), $z/d = 2.0$, k), l), $z/d = 2.5$.

the other two. The largest vortex is also situated at the terminating end of the rearward shock. Additionally, the secondary vortex in the near-corner region at the base of the fin is also visible. As the flow is observed to move outboard with increasing z/d slices, the primary vortex structure shrinks in size and diminishes in vorticity magnitude. At around $z/d \approx 1.5$, the smaller vortices in the primary vortex structure have disappeared entirely, and the swirling structure of the large vortex begins to straighten out. For $z/d \geq 2$, the vortex is qualitatively only marginally visible, and for $z/d \gtrsim 2.2$, no portion of the tangential flow moves upstream anymore. As a note, since the mean flow is symmetric about the centerline, this behavior is identical in the negative z -direction as well.

Given this limited separation flow extent, further analysis may reveal additional characteristic behavior. One such thought had been the subsonic flow feeding and energizing the separation bubble. However, a correlation between the extent of separation to the subsonic flow field induced by the separation shock and/or the bow shock could not be made, as these regions did not seem to align well with one another.

5.1.2 Mean Surface Pressure on the Floor

Many mean features regarding the surface pressure on the floor corroborate the findings of the mean separation behavior. However, caution should be taken when approaching the pressure analysis due to a lowered Reynolds number, as the two variables are coupled. The effects of this are evident in Fig. 5.4, which shows the mean wall pressure at the centerline for the same three cases as in Sec. 5.1.1. All three pressure profiles followed the generic behavior described in Fig. 2.11, though there are notable differences. For all three profiles, mean separation occurred at a mean pressure ratio of $\bar{P}_w/\bar{P}_\infty \approx 1.22$. The pressure rise is more gradual for the low-Re cases, and this viscous damping resembles that of the effects of lower d/δ seen in Fig. 4.12. However, the pressure plateau, corresponding to point d in Fig. 2.11, yields the opposite effect; the centerline plateau for the high-Re RANS case was around $\bar{P}_w/\bar{P}_\infty \approx 1.55$, which increased to ≈ 1.65 for the low-Re RANS case and ≈ 1.80 for the (low-Re) LES case. All of these are higher than the value of $\bar{P}_w/\bar{P}_\infty \approx 1.44$ predicted using the 2-D approximation from Eq. 2.4. This elevated pressure plateau for the low-Re cases also resulted in an elevated pressure trough and, more importantly, a lower

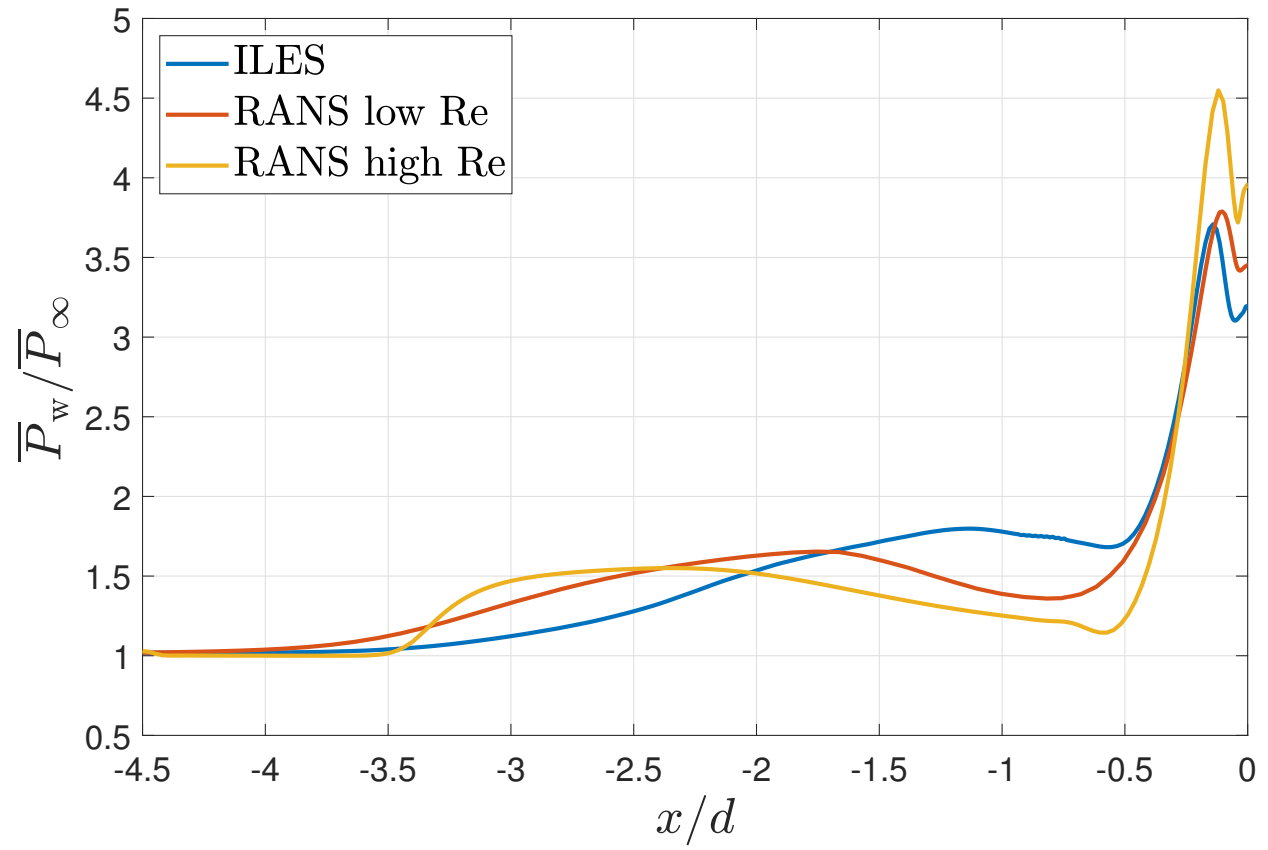


Figure 5.4: Mean normalized wall pressure at the centerline on the floor.

pressure peak near the base of the fin; whereas the peak for the high-Re RANS case is around $\bar{P}_w/\bar{P}_\infty \approx 4.55$, it is only around ≈ 3.79 for the low-Re RANS case and ≈ 3.71 for the LES case. The similarity and agreement of the two low-Re cases in computing the peak pressure may support agreement between RANS and LES results at a higher Re as well.

Extending the centerline observations from Fig. 5.4 to 2-D reveals additional details. A top-down view of the mean surface pressure from the ILES case is provided in Fig. 5.5a, with streamlines provided for reference. The separation line appears to follow a distinct pressure contour line of $\bar{P}_w/\bar{P}_\infty \approx 1.22$, the centerline pressure needed to induce flow separation, for values of around $z/d \approx \pm 5$. As the flow approaches the linear region the pressure slowly increases on this line of coalescence, but seems to remain at around $\bar{P}_w/\bar{P}_\infty \approx 1.3$ – 1.35 ; it becomes difficult to predict the behavior far outboard due to the effects of the periodic boundary conditions. As the flow moves past the fin, the pressure returns below $\bar{P}_w/\bar{P}_\infty < 1.1$ rather quickly, only $3d$ downstream of the leading edge in the vicinity of the fin surface. This contour line at $\bar{P}_w/\bar{P}_\infty = 1.1$ is angled at approximately the same value as the linear portion of the separation line and has a distinctly smaller (tighter) angle than the inviscid bow shock, which can also be identified in Fig. 5.2a.

A closer view at the mean surface pressure in the leading-edge region is shown in Fig. 5.5b. It can be seen that the high-pressure region is only found in the direct vicinity of the nose of the fin, and that these adverse effects rapidly deteriorate with increased distance away from the nose. A comparison to instantaneous experimental quantitative data using uPSP is included. A dark-blue spot with very low pressure is visible in the image, and is a result of rust developing on the stainless steel surface [118]. Nevertheless, several features are identifiable and corroborate the numerical findings. The mean experimental separation length is found to occur at about the same pressure rise value of $\bar{P}_w/\bar{P}_\infty \approx 1.22$. The separation line remains associated with this pressure contour line up to the spanwise extent of the experimental window. The location of the pressure plateau is very similar at around $x/d \approx -1.1$ – -1.2 , although the magnitude is lower in the experiment; however, since the experimental data are instantaneous, the mean value may have been in better agreement. Near the leading edge of the fin, the uPSP data indicate a drop below the mean freestream value, which is counter-intuitive and does not align with pressure transducer measurements

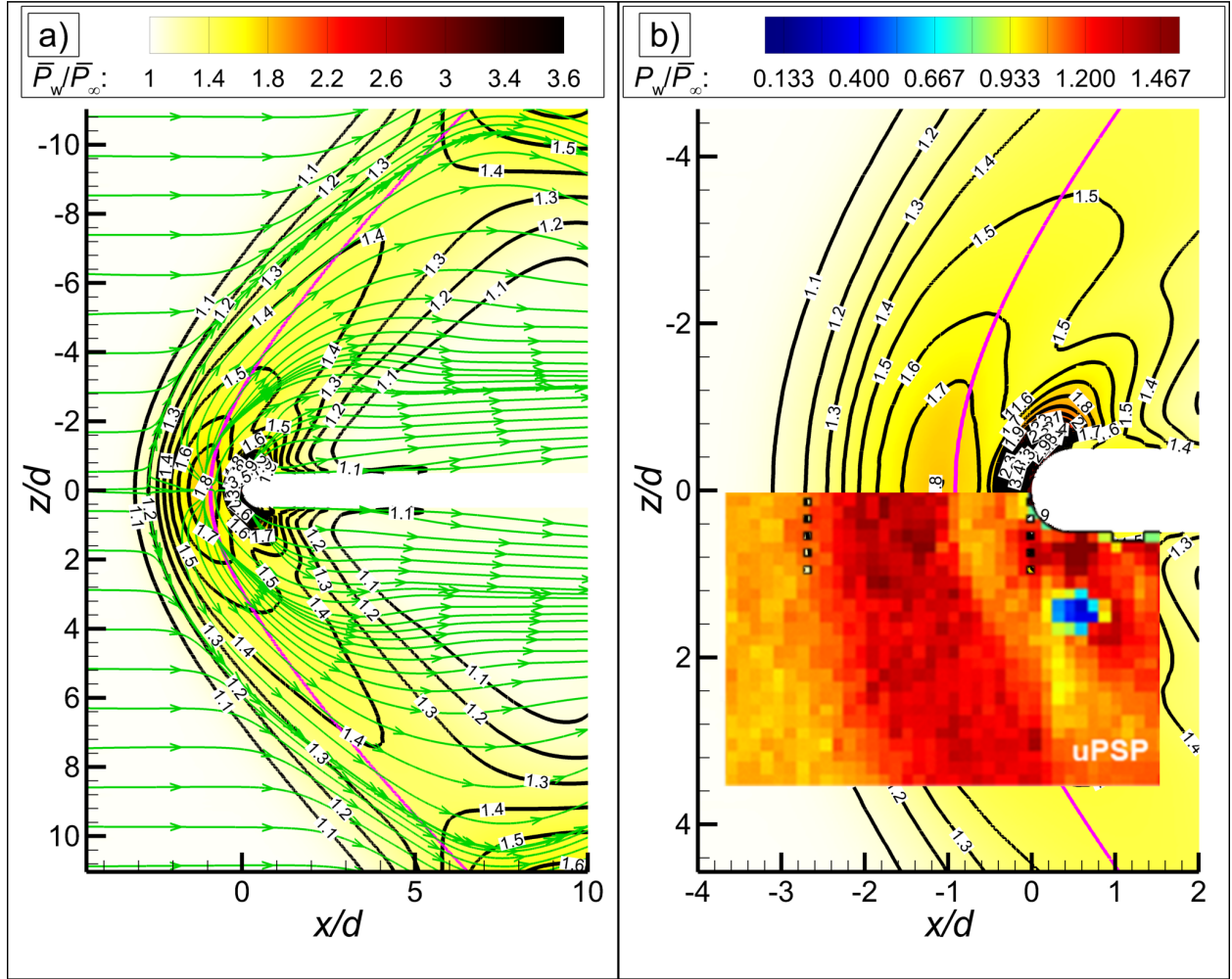


Figure 5.5: Contour of mean normalized wall pressure with labeled contour lines. Flow is left to right. Pink line is inviscid bow shock projected onto the floor for reference. a) Full view of main region of interest, with green streamlines from the first cell off the wall. b) Enhanced view of leading-edge region, with instantaneous normalized wall pressure obtained from uPSP inserted. Dashed lines represent centerline mean separation and blunt-fin leading edge in the experiment, respectively. Inserted figure adapted from Lash et al. [118].

or the CFD data. It is possible that the nearly-stagnant flow at the base of the fin could cause diagnostic issue, but the source of this discrepancy is unknown. Even so, the extent of the experimental high-pressure region near the fin surface appears to qualitatively align with the numerically computed one.

Returning to Fig. 5.3, the rapid decay of the high-pressure region becomes evident yet again. The peak pressure at the centerline reduced so much in magnitude, that by $z/d = 1.5$ it has become lower than the first pressure plateau. In fact, it appears that as the vortex loses strength and straightens out, the peak pressure diminishes. When the vortex vanishes and no more reverse flow is present at a given z -location, the local maximum of the pressure peak also disappears. This effect can be clearly seen at $z/d = 2.5$, where no vortex is present and the only local maximum is the pressure plateau. Contrary to the behavior of the pressure peak, the pressure plateau has a much slower decay at more outboard locations, only dropping from $\bar{P}_w/\bar{P}_\infty \approx 1.80$ at the centerline to $\bar{P}_w/\bar{P}_\infty \approx 1.56$ at $z/d = 2.5$. The behaviors and trends described here applied to the RANS simulation results as well, as shown in Fig. 5.6. For the RANS cases, the peak pressure from the centerline is still higher at the outboard location of $z/d = 1.5$ than the pressure plateau, but agreement to the trends observed in the LES data is evident. A notable difference is that the high-Re RANS case has a substantially lower trough pressure at around $x/d \approx 0$, and a much lower recovery pressure that dips below the freestream value. In addition, the pressure at separation for this outboard location was $\bar{P}_w/\bar{P}_\infty \approx 1.35$, which was noticeably higher than the value of $\bar{P}_w/\bar{P}_\infty \approx 1.23$ for the low-Re cases.

Also evident in Fig. 5.3 is that the local pressure maxima do not align at the same abscissa locations. A shift in the abscissa by the inviscid bow-shock x -location at every z -ordinate, described as B in Fig. 2.20, was applied, and the result is shown in Fig. 5.7. Overall, there is better alignment of the pressure rise, plateau, and trough. However, this method did not produce alignment across the three cases at a single z -location, and was only applicable to each case individually. For both of the low-Re cases, a more noticeable drop in the plateau pressure occurs than for the high-Re RANS case. The pressure trough seems roughly aligned with $(x - B)/d = 0$ for all three cases, with lower deviation from this value in both of the RANS cases, particularly the high-Re RANS case. For all cases,

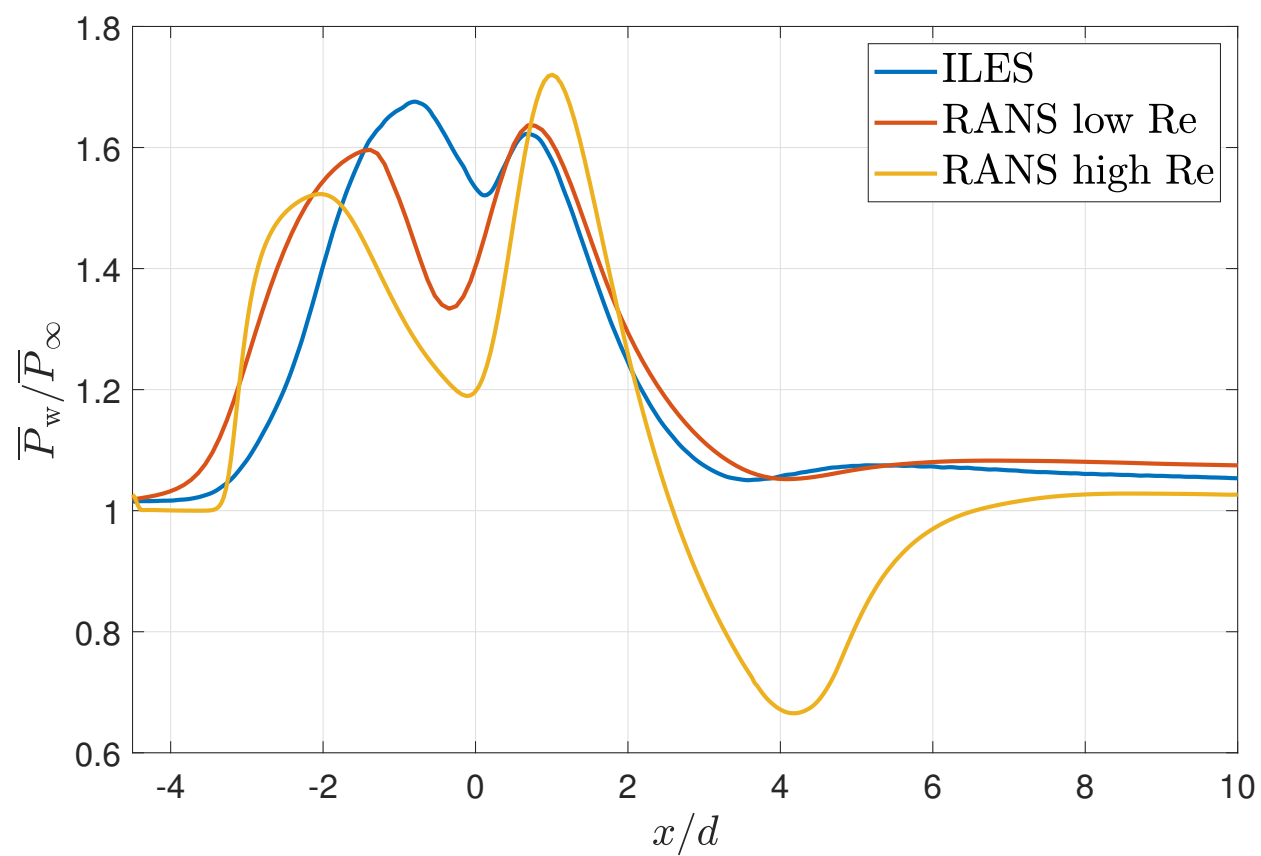


Figure 5.6: Mean normalized wall pressure at $z/d = 1.5$.

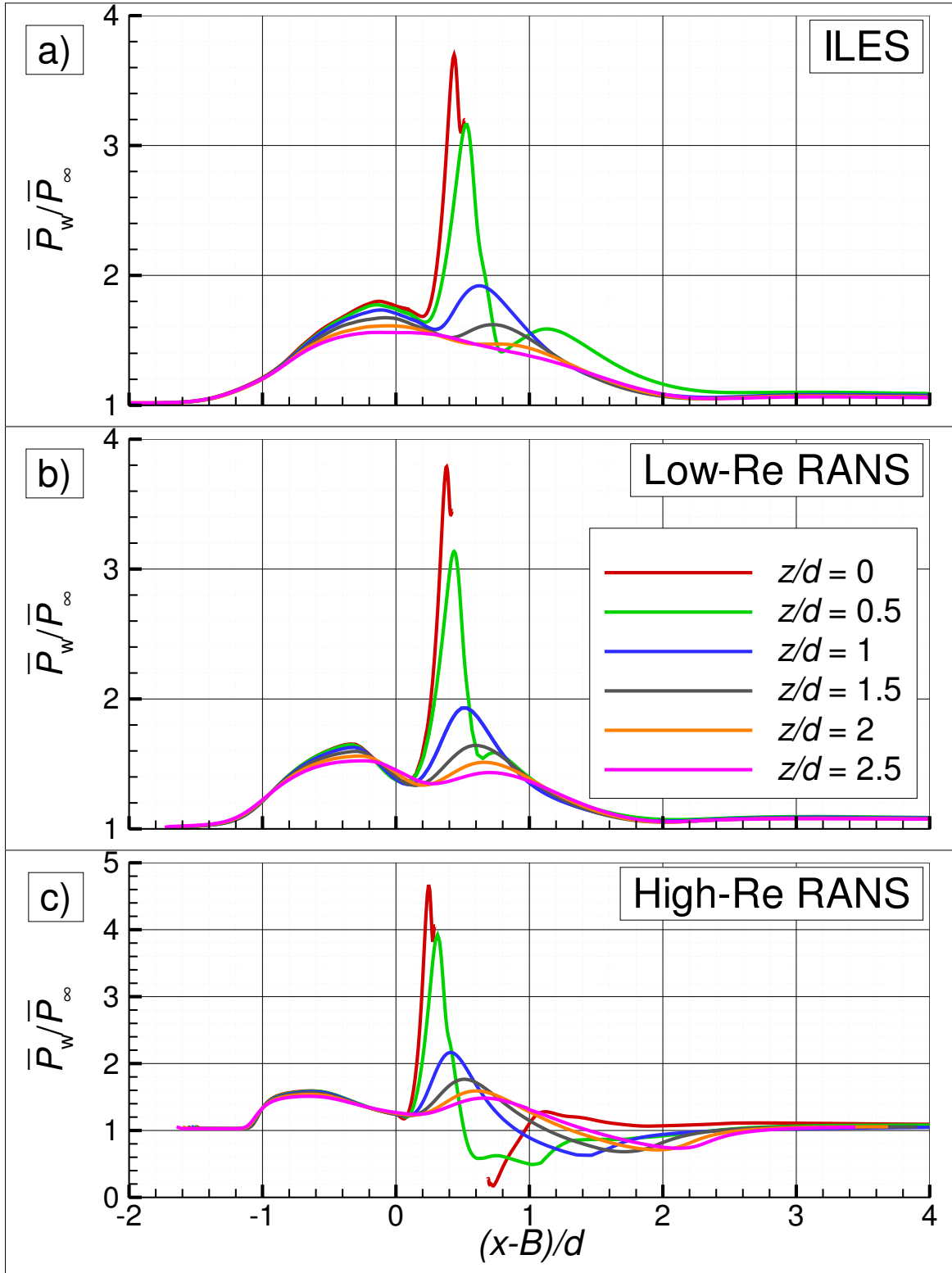


Figure 5.7: Mean normalized wall pressure at shifted x -values for various z -locations. a) ILES, b) low-Re RANS, c) high-Re RANS results.

the pressure peak does not align very well, but the rapid decay is easier to visualize than in Fig. 5.3. Note that since the high-Re RANS case used a cylinder as the shock generator, the pressure behind the cylinder is also included here; it is also important to note that the recovery pressure dips below the freestream value for all of the z -ordinate values shown, which was not the case for any of the low-Re cases.

5.1.3 Mean Attachment on the Fin

Referring back to the centerline streamline topology in Fig. 2.14a, the separation line on the floor ($SL1$) has a complementary attachment line impinging on the leading edge ($AL1$). This dividing streamline can be found through the zero-crossing of the coefficient of friction, c_f , or alternatively using the v -velocity in the first cell off the wall, as shown in Fig. 5.8. Note that for this and many successive figures relating to the attachment line, the abscissa and ordinate are swapped to provide a better physical representation with y “pointing up.” Also note that the ILES data should be handled with care above $y/d > 3$, as the grid spacing coarsens rapidly for increasing y -values; the flow not far above this threshold can be considered fully laminar. In the figure, at high y/d values, the flow is nearly stagnant in terms of v -velocity, marking the inviscid region where the flow simply wraps around the shock generator. Below this, a positive v -velocity followed by a zero-crossing indicates the region just above and at the attachment line. A negative v -velocity is then observed for all cases that corresponds to the recirculation of the primary vortex structure. A zero-crossing back to a positive v -velocity is observed near around $y/d \approx 0.3$, which relates to the secondary vortex with separation line $SL2$ in Fig. 2.14b. Near the base of the cylinder, an additional zero-crossing occurs that can be traced to the tertiary near-corner vortex and the attachment line $AL3$ in Fig. 2.14b. These trends are visible for all three cases. The point at which $AL1$ impinges, i.e. the zero-crossing location between the peak and trough v -velocity values, will henceforth be referred to as A , in a similar manner to the separation point on the floor. The mean value was found at $\bar{A}/d = 1.47$.

The structure within the boundary layer developing on the (side of the) fin can be visualized through the streamlines provided in Fig. 5.9. The figure simply provides the x -location at the abscissa as a projection, rather than accounting for the curvature of the

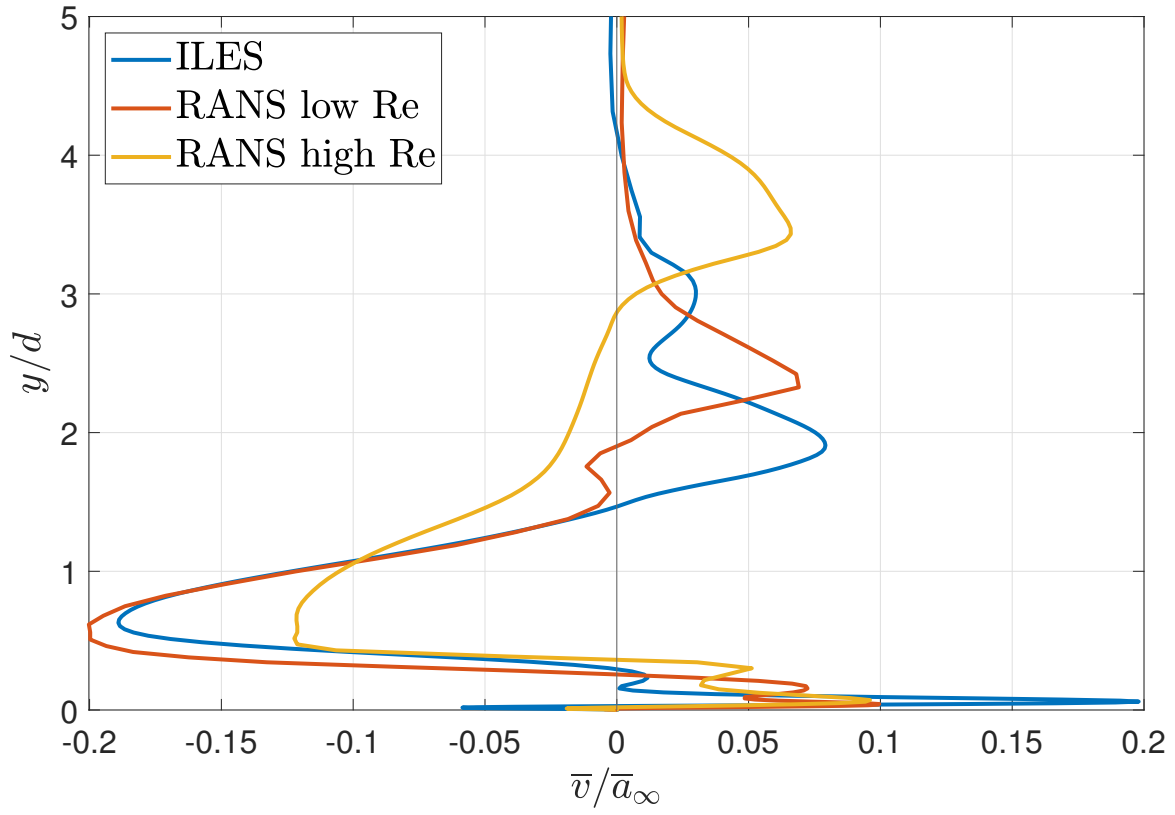


Figure 5.8: Mean v -velocity-equivalent Mach number in the first cell off the wall at the centerline on the leading edge.

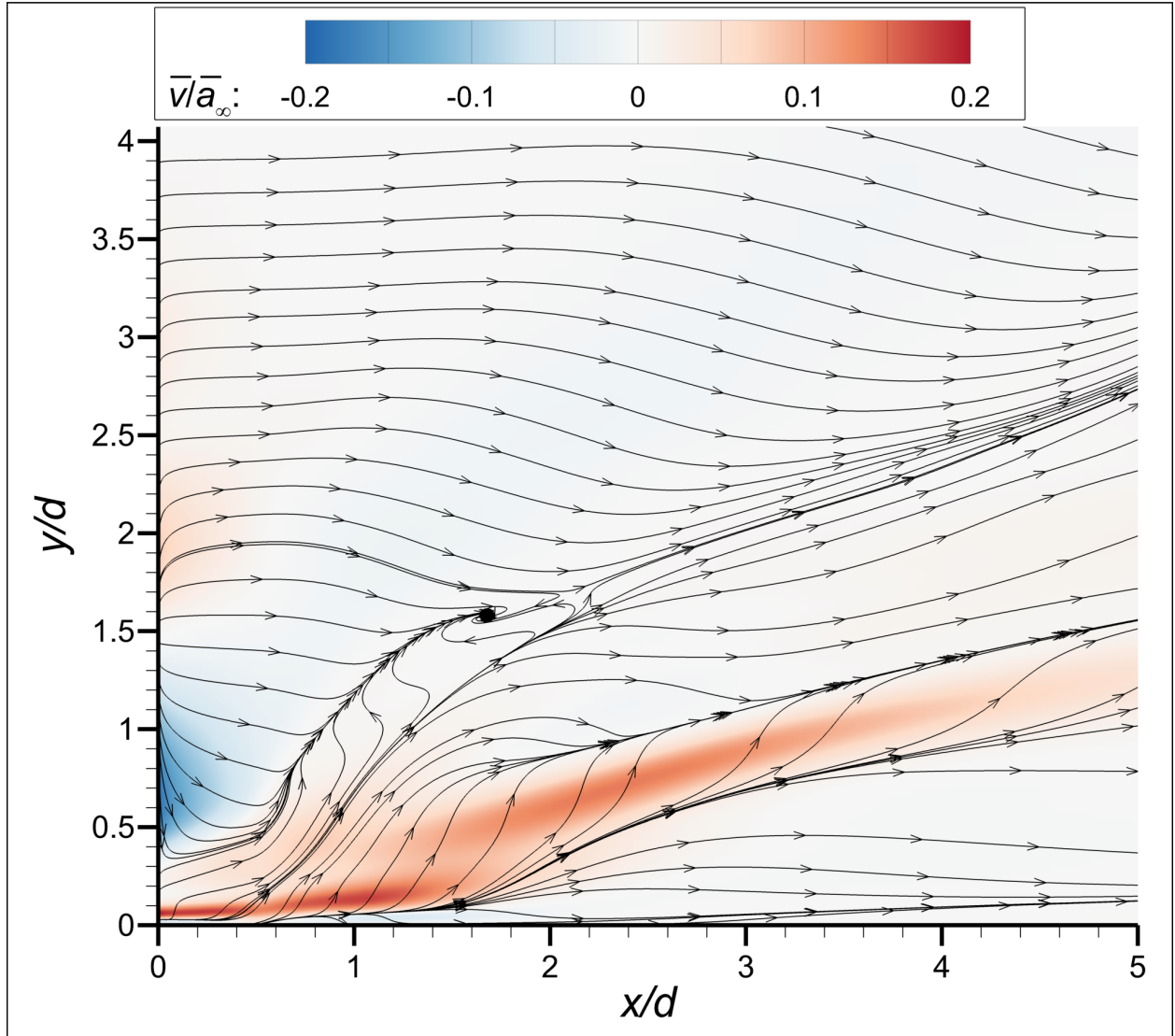


Figure 5.9: Contour of mean v -velocity-equivalent Mach number in the first cell off the wall with black streamlines. Flow is left to right.

nose. The same behavior observed at the centerline in Fig. 5.8 is seen at $x/d = 0$ in Fig. 5.9 regarding the positive and negative v -velocity regions. Several lines of coalescence and divergence are seen to form at the junction of the nose and side of the fin at $x/d = 0.5$ and under $y/d < 1$, all of which are associated with a positive v -velocity. The secondary vortex is the only leading-edge feature that extends past this junction and continues until about $x/d \approx 1.5$, after which the flow diverges. The tertiary vortex follows a similar path, moving up along the wall after it passes the nose/sidewall junction, and forming a single vortex together with the streamlines originating about the secondary vortex. This focus is observed on the side of the fin at $x/d \approx 1.5$ and $y/d \approx 1.5$. The streamlines observed at $x/d = 5$ continued in a mostly linear fashion until the downstream end of the domain, indicating that the behavior is mostly settled by around $x/d \gtrsim 3$.

5.1.4 Mean Surface Pressure on the Fin

Complementary to the velocity analysis in Sec. 5.1.3, the surface pressure on the leading edge of the fin is provided in Fig. 5.10. Recall that the normalization is performed through the mean stagnation pressure downstream of the inviscid bow shock, $\bar{P}_{0,2}$, rather than the mean freestream pressure, although this value is also provided for reference with regard to the ILES data. In the inviscid region, the expected value of $\bar{P}_w/\bar{P}_{0,2} = 1$ is observed. A pressure peak is found, corresponding to point c in Fig. 2.13, and loosely correlating with A . This is followed by a significant pressure relief in which the trough directly corresponds to $SL2$ in Fig. 2.14b. The pressure then oscillates again near the base of the fin in the vicinity of the tertiary vortex, and finally matches the pressure on the floor at the base. Note that the pressure on the leading edge exceeds the peak pressure found on the floor (or at the base of the fin), even in the inviscid region at this Mach number.

The pressure contours on the wall of the fin, normalized by the mean freestream value, are shown in Fig. 5.11. An upper limit was set to exclude the nose and provide a better understanding of the pressure on the side. Additionally, the same streamlines from Fig. 5.10 were retained for reference. Downstream of the nose section, i.e. $x/d > 0.5$, the pressure tends to sit right around or just below the mean freestream value of $\bar{P}_w/\bar{P}_\infty = 1$. The highest pressure is found near the base of the fin and around $x/d \approx 1$ and is traced to a line

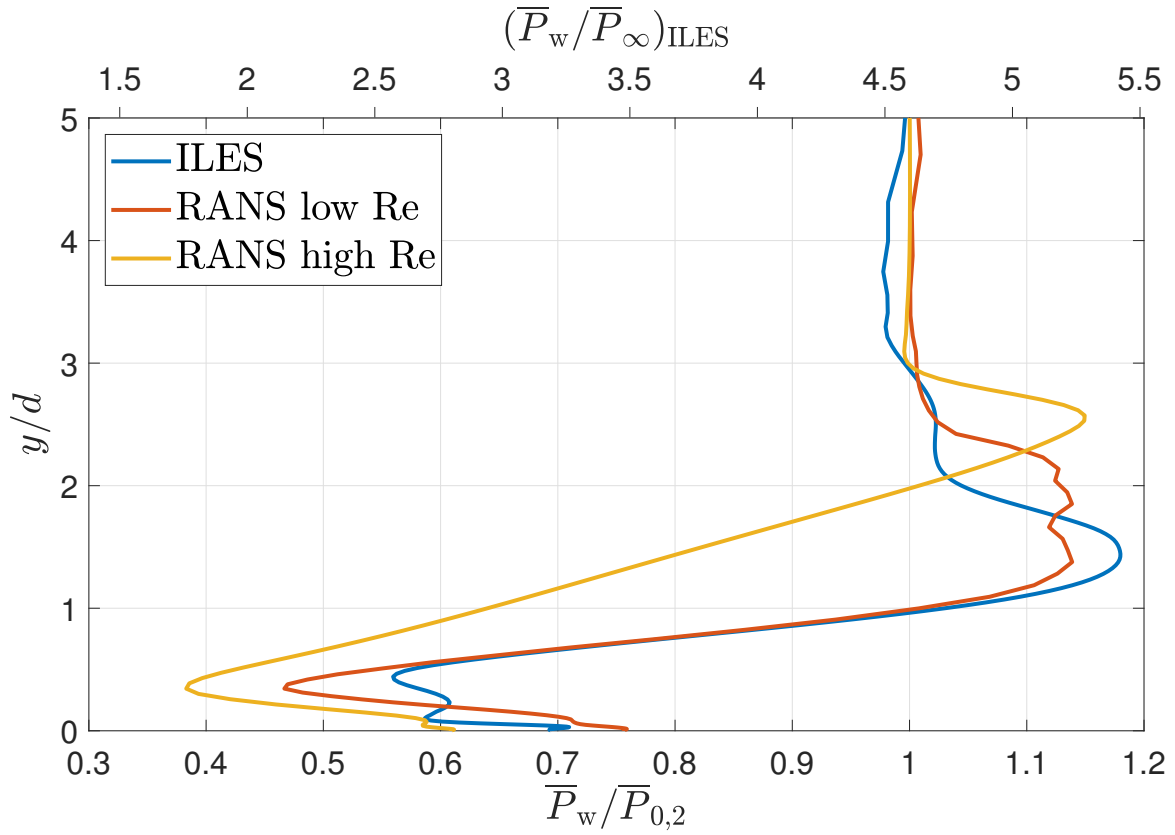


Figure 5.10: Mean wall pressure normalized by the post-normal shock stagnation pressure at the centerline on the leading edge. Normalization by the freestream pressure for the ILES data is provided for reference.

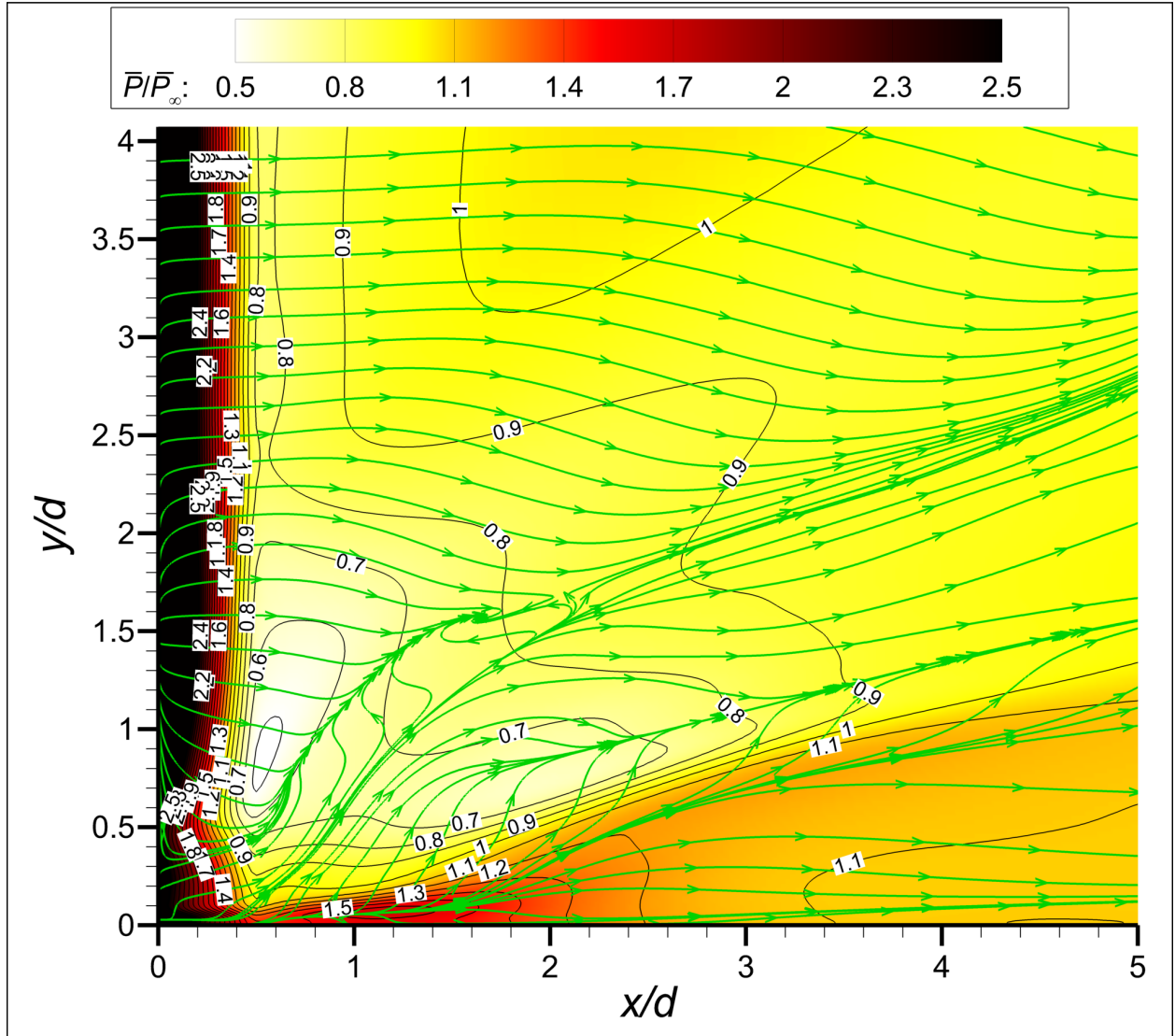


Figure 5.11: Contour of mean normalized pressure on the fin wall with black contour lines and green streamlines. Flow is left to right.

of coalescence that formed at the nose/sidewall junction. Overall, the sidewall pressure is mild compared to other surfaces, at least for zero angle of attack.

5.2 Time-Accurate Flow Features

The dynamic aspects of the ILES results are investigated in the same order as in Sec. 5.1. As the steady-state RANS simulations do not produce time-accurate results, the only comparisons drawn will be against experimental data. Many of the data analysis methods described in Sec. 2.3 are applied to the ILES data and presented here. To enable a more realistic comparison to the experimental data, some of the pressure data from the ILES results were low-pass filtered through an 8th-order Butterworth filter with a cut-off frequency of 20 kHz ($St_{L_i} \approx 0.057$). A sample view of the instantaneous flow field on the floor is provided in Fig. 5.12 for the u -velocity-equivalent Mach number and normalized pressure.

5.2.1 Dynamic Separation on the Floor

Using the method described in Sec. 2.3.1, Eq. 2.7–2.9 were used to obtain the instantaneous separation distance. The resulting time signal is shown in Fig. 5.13, along with the mean values obtained from first time-averaging the solution and using Eq. 2.7, and from time-averaging the instantaneous data, showing general agreement between the two.

Applying a PDF to the instantaneous separation distance and comparing it to a Gaussian curve, as shown in Fig. 5.14a, indicates that the separation distance follows a Gaussian distribution with a slightly positive skew. Moreover, the bins indicate that the separation distance on the centerline is bound by about $S/d \in [-3.5, -1.5]$, which is in line with the results of Fig. 4.6. Extending the centerline results in the spanwise direction, as shown in Fig. 5.14b, indicates that the separated region behaves very similarly, as the range of x/d values for a given z/d location remains relatively constant. The same streamwise translation by the local bow-shock projection as in Fig. 5.7 is applied, which appears to fit the data well in the outboard direction. The sudden blanked values in the PDF at $z/d \gtrsim 2.2$ occurs because the flow never separates in that region, which is in agreement with the mean results.

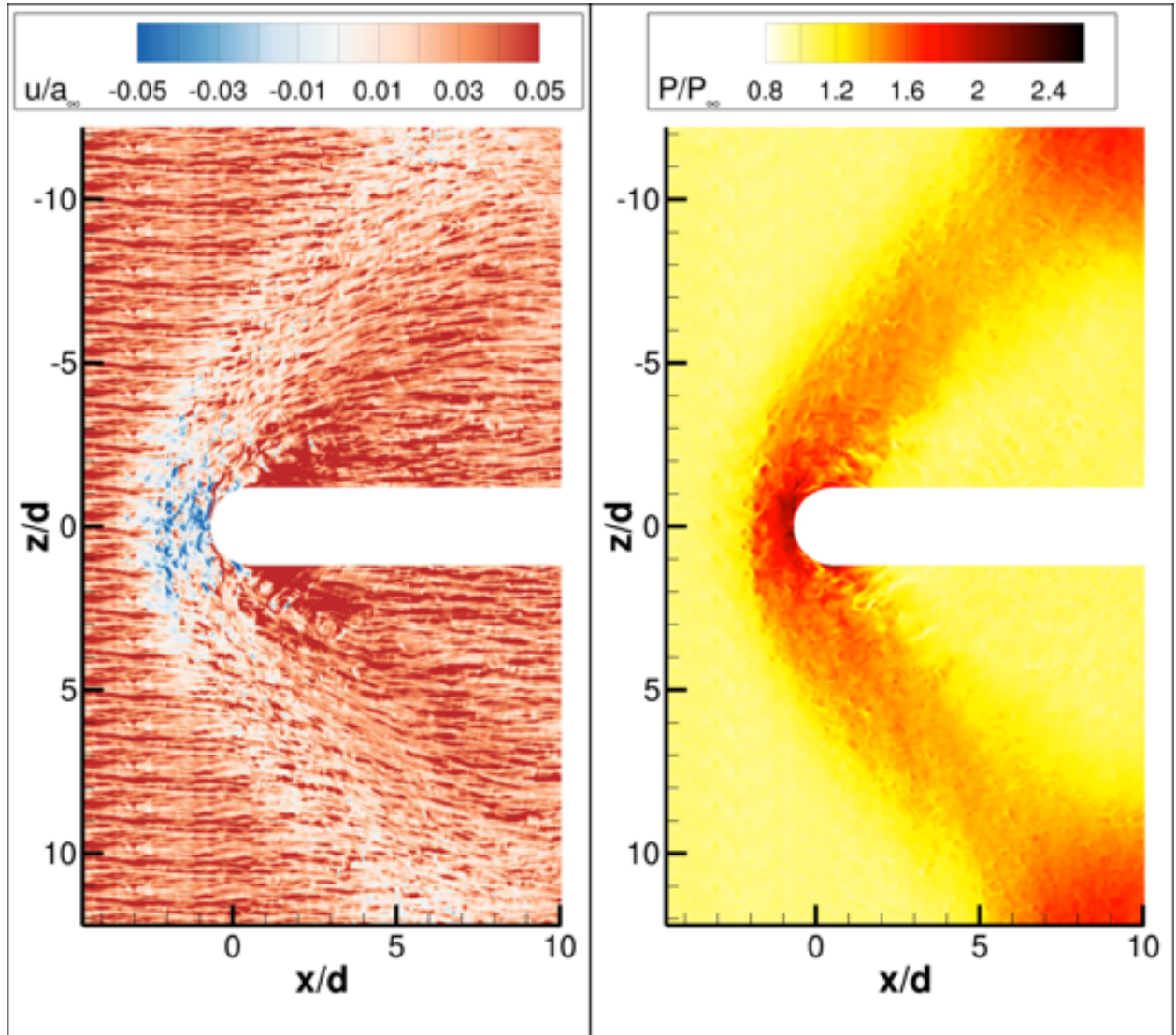


Figure 5.12: Instantaneous snapshot contour of a) u -velocity-equivalent Mach number in the first cell off the floor and b) normalized pressure on the floor.

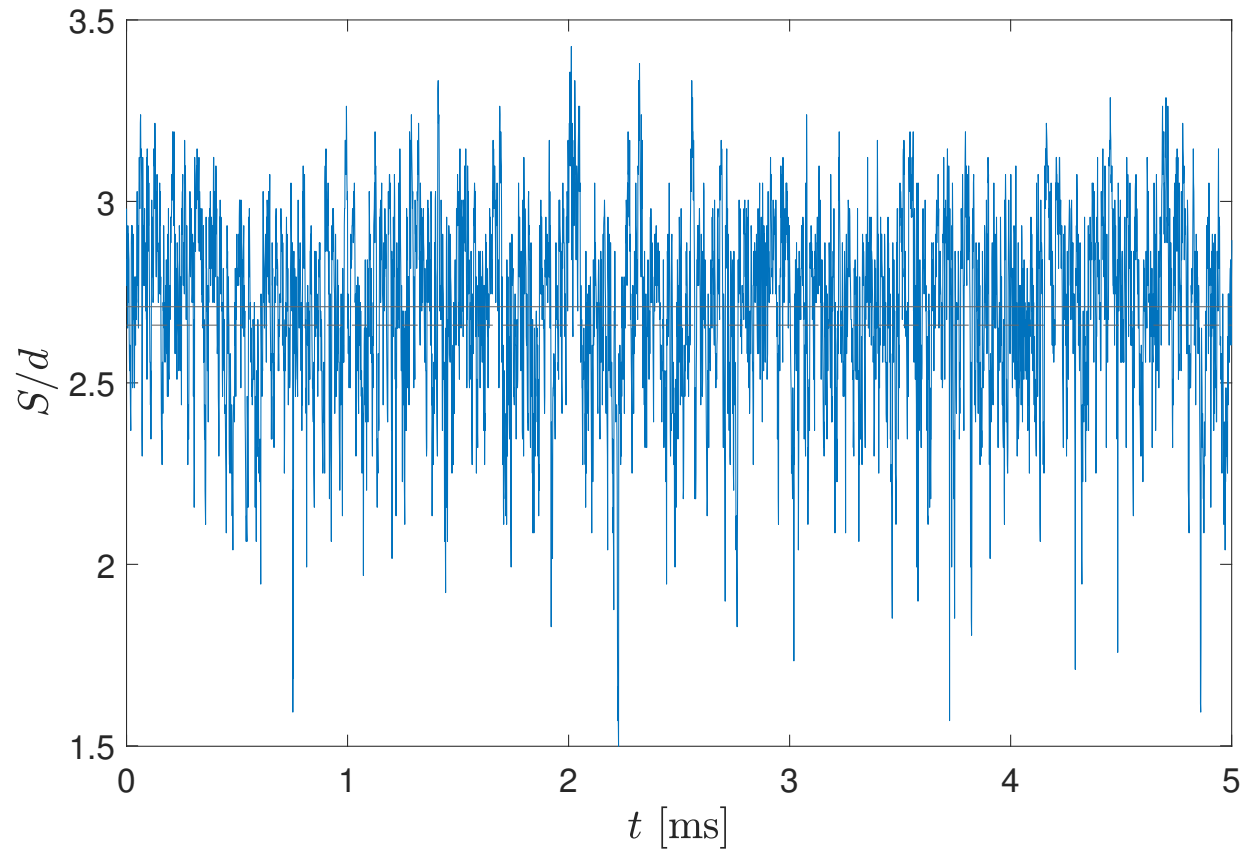


Figure 5.13: Instantaneous separation distance at the centerline on the floor. Solid line represents mean value of instantaneous separation data. Dashed line represents mean value of time-averaged separation data.

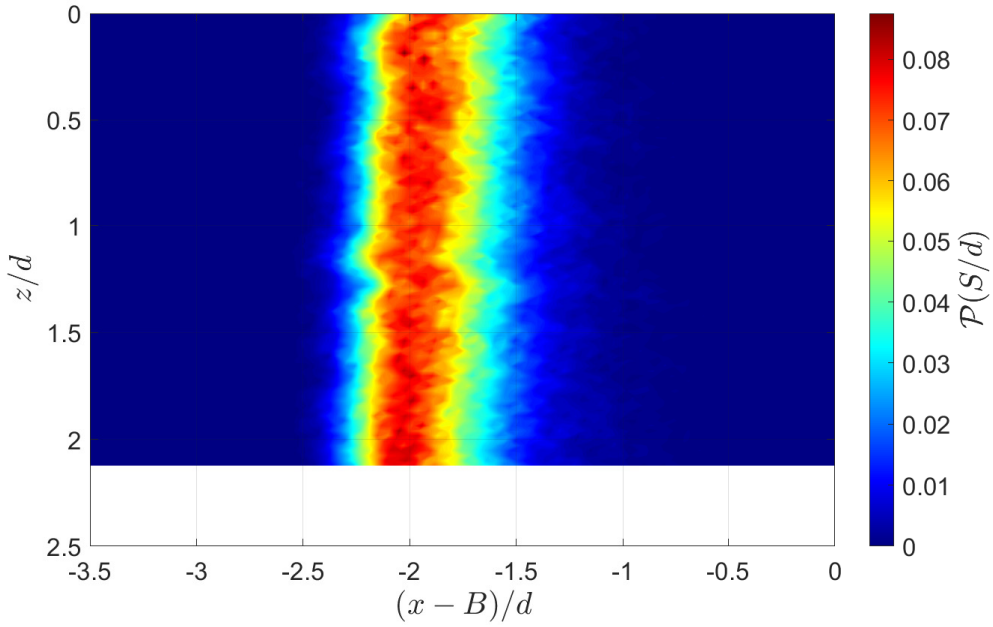
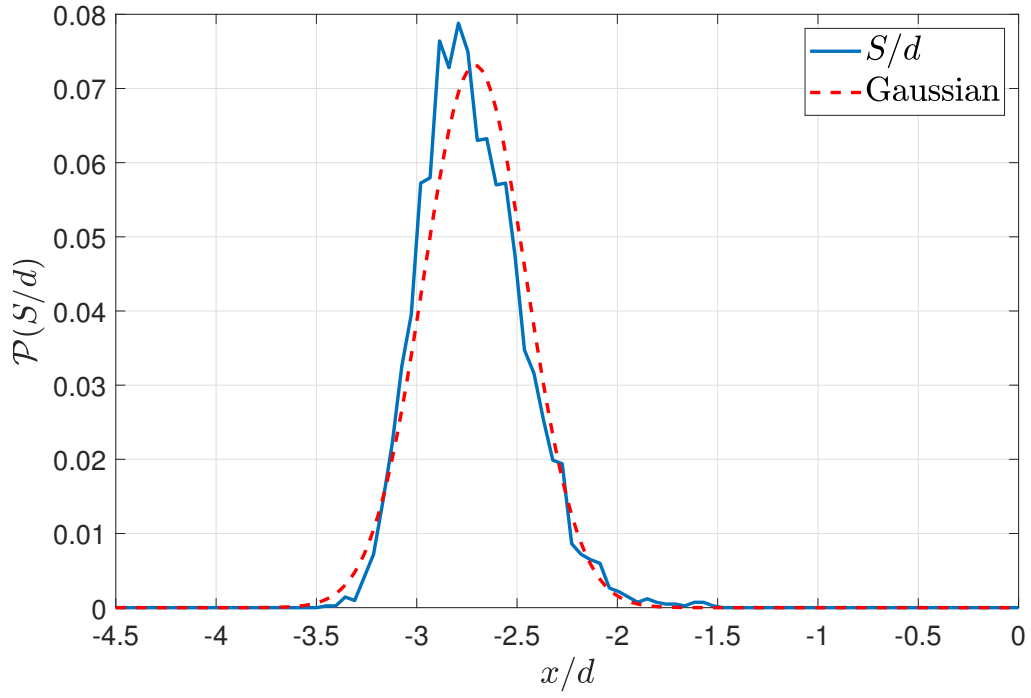


Figure 5.14: Probability density function of instantaneous separation distance a) at the centerline and b) as contour with shifted x -values on the floor. Dashed line in a) represents Gaussian-equivalent data.

Similar to the approach in Fig. 2.16, both the shock zero-crossing frequency and the intermittency, as described in Sec. 2.3.3 and Sec. 2.3.2, respectively, are plotted together in Fig. 5.15a. High-speed schlieren imaging and uPSP data for conditions similar to those in case E3 from Table 3.2 are added for comparison, as is the Gauss error function, $\text{erf}(x/d)$, described by Eq. 2.11. Starting with the zero-crossing frequency, there is good agreement with the abscissa location between the ILES and high-speed schlieren imaging results, though the frequencies differ by an order of magnitude. A similar offset is found when comparing to the data in Fig. 2.16, which are yet another order of magnitude below the schlieren data. The uPSP data exhibit a much narrower band than the other data sets, but its peak frequency is located approximately in between the values of the ILES and schlieren peak frequencies. Comparing the peak zero-crossing frequency to the intermittency of each data set, an alignment to $\gamma = 0.5$ is visible for both the ILES and schlieren data, but the uPSP peak frequency appears to miss this peak and instead aligns closer to a value of $\gamma \approx 0.01$. The intermittency results for the ILES and schlieren data are also mostly aligned, although a slight downstream offset exists between the two, which is likely due to the differing d/δ values. Nevertheless, a classic sigmoid shape is visible for all three data sets, and the ILES data aligns well with the Gauss error function, further indicating a Gaussian distribution. The spanwise distribution of intermittency is shown in Fig. 5.15b using the same shift in the abscissa. Similar to the PDF in Fig. 5.14b, there is little to no apparent deviation from the centerline behavior. The intermittency function abruptly ends at around $z/d \approx 2.2$ again, at the limit of the separation region. To get the intermittent length at the centerline, the abscissa locations for $\gamma = 0.05$ and 0.95 were examined. These yielded values of $x/d = -3.10$ and -2.27 , respectively, resulting in an intermittent length of $L_i/d = 0.82$. The intermittent length remains effectively constant for outboard locations, which suggests that the separation dynamics are not affected in the spanwise direction.

Another view that gauges the variability in the dynamic behavior of the SWBLI is presented in the time history of the u -velocity-equivalent Mach number contour in Fig. 5.16. The mean separation distance and intermittent length limits are added for reference, and visualize the extents of the forward shock. Further downstream, the effects of the primary and secondary vortex structure are also visible, and the abscissa locations of these effects

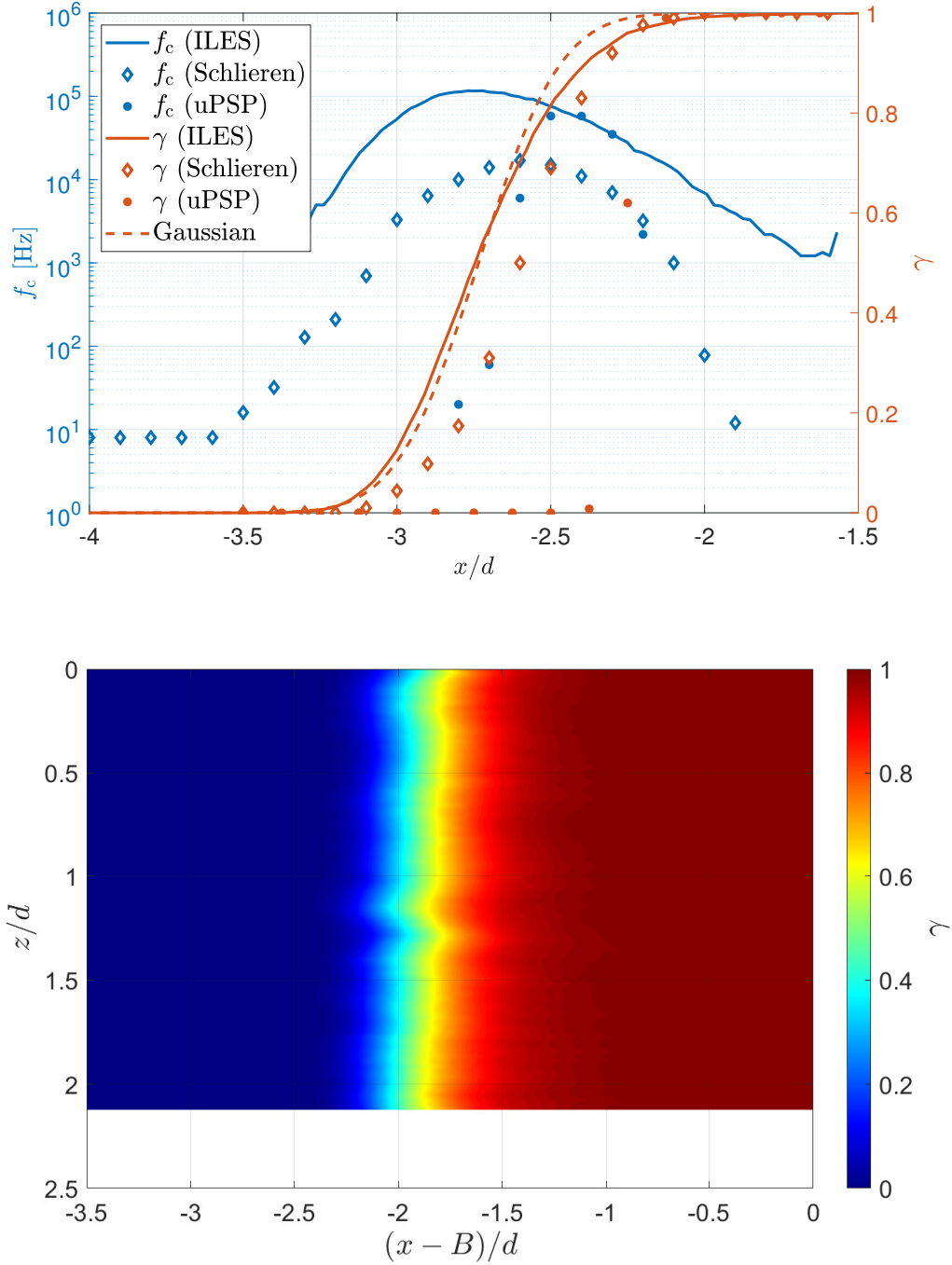


Figure 5.15: a) Zero-crossing frequency and intermittency of instantaneous separation distance along with experimental comparisons using high-speed schlieren imaging and uPSP. Intermittency chart includes Gaussian error function for reference. Experimental data from Lash et al. [108]. b) Contour of intermittency of instantaneous separation distance at shifted x -values on the floor.

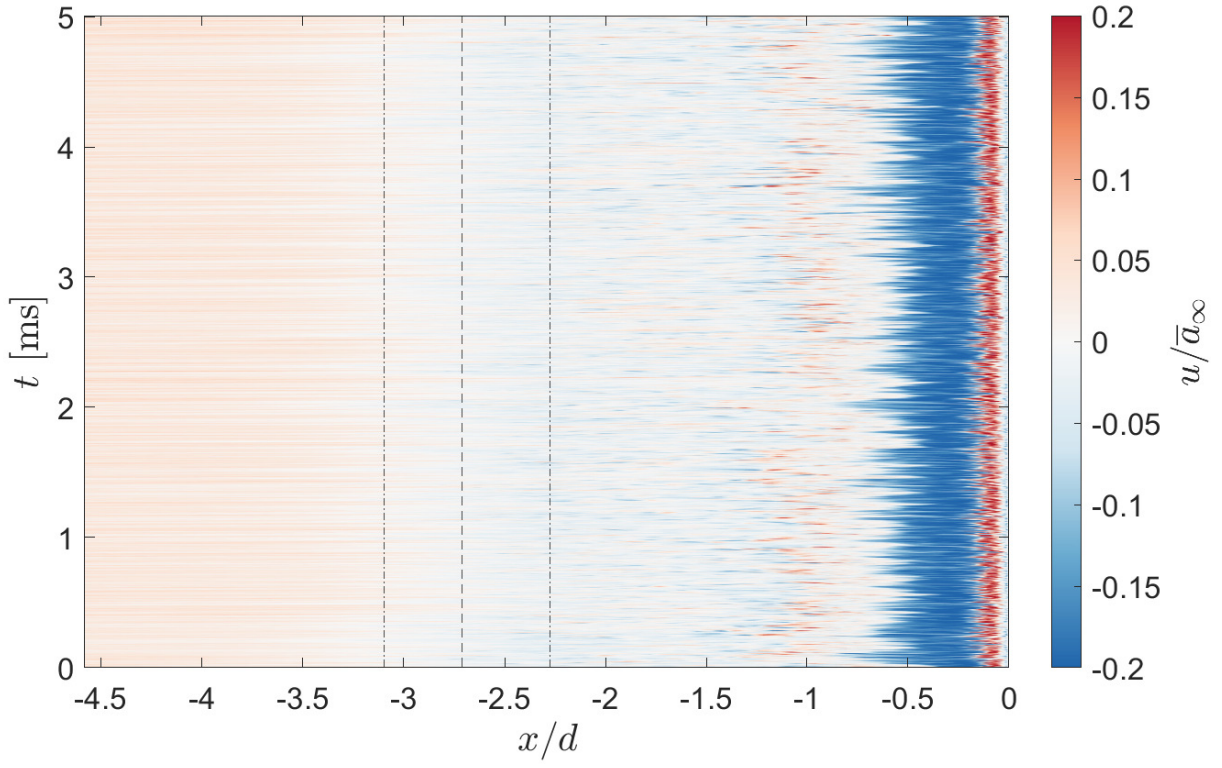


Figure 5.16: Instantaneous u -velocity contour at the centerline on the floor. Dashed line represents mean separation distance, dash-dotted lines represents intermittent length limits at $\gamma = 0.05$ and $\gamma = 0.95$.

can be traced back to Fig. 5.1; while there is a noticeable variation in the translation of the three vortices of the primary vortex structure, the secondary (and tertiary) vortex near the base of the fin remains mostly confined to its mean value.

With the intermittent length defined, the Strouhal number with respect to the intermittent length can be computed, as described in Sec. 2.3.4. Shown in Fig. 5.17 is the normalized power spectral density of the centerline separation distance as a function of the Strouhal number. A distinct peak is exhibited at $St_{L_i} = 0.0215$, which is in great agreement with the expected value of 0.022 from the archival literature. A secondary peak is seen at $St_{L_i} \approx 0.008$ as well. As the flow does not separate further outboard, a 2-D contour is provided at every spanwise grid point up to the outboard limit in Fig. 5.18. For nearly all spanwise locations, the dominant Strouhal number is observed at $St_{L_i} \approx 0.022$; the secondary peak also generally remains in the vicinity of $St_{L_i} \approx 0.008$, though with greater variability. This suggests that the dynamic behavior of the separation shock wave does not change from the centerline to the span. Similar tangential observations were made by Gonzalez and Dolling [81] regarding the centerline dynamics with respect to the blunt-fin sweepback angle; below a certain sweepback angle, the dynamic behavior remained unchanged.

5.2.2 Dynamic Surface Pressure on the Floor

The pressure signal was analyzed on the centerline first. A comparison to experimental pressure transducer measurements was performed, which were arranged as shown in Fig. 5.19a. The corresponding pressure signals are shown for a snapshot in time that is on the order of time for which the ILES data were collected. Transducer port P1 is located in the undisturbed region and measures only noise in the boundary layer. Transducer port P2, however, occasionally has a shock passing over it, causing a pressure spike, before returning back to an undisturbed pressure level. Transducer port P3 has an elevated pressure for most of the time and only returns to the undisturbed level infrequently. These behaviors align well with those described in Fig. 2.17 and can also be traced back to the intermittency in Fig. 5.15.

Recall from Sec. 2.3.2 that this approach does not yield the same desired output in a low-Reynolds number environment such as in this ILES. Evidence of this is shown in Fig.

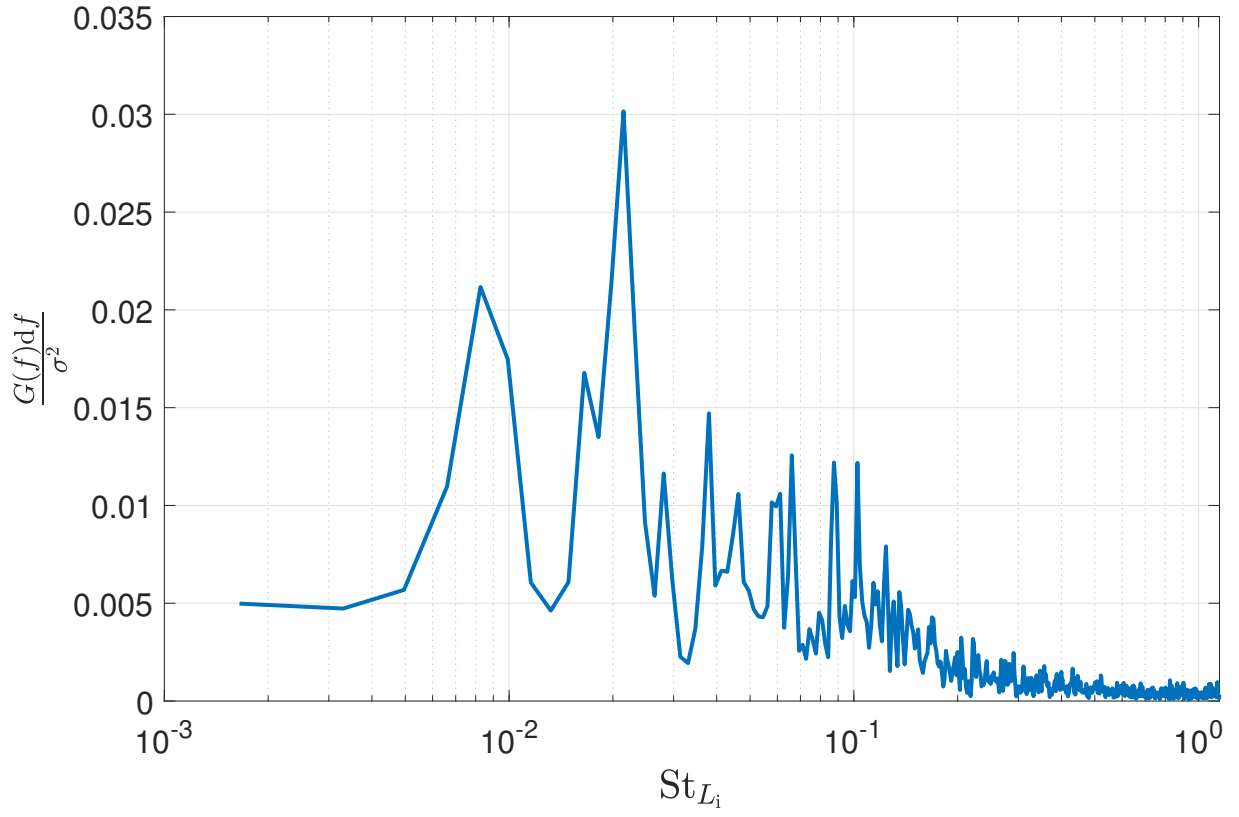


Figure 5.17: Normalized power spectral density of the separation distance related to the Strouhal number with respect to the intermittent length.

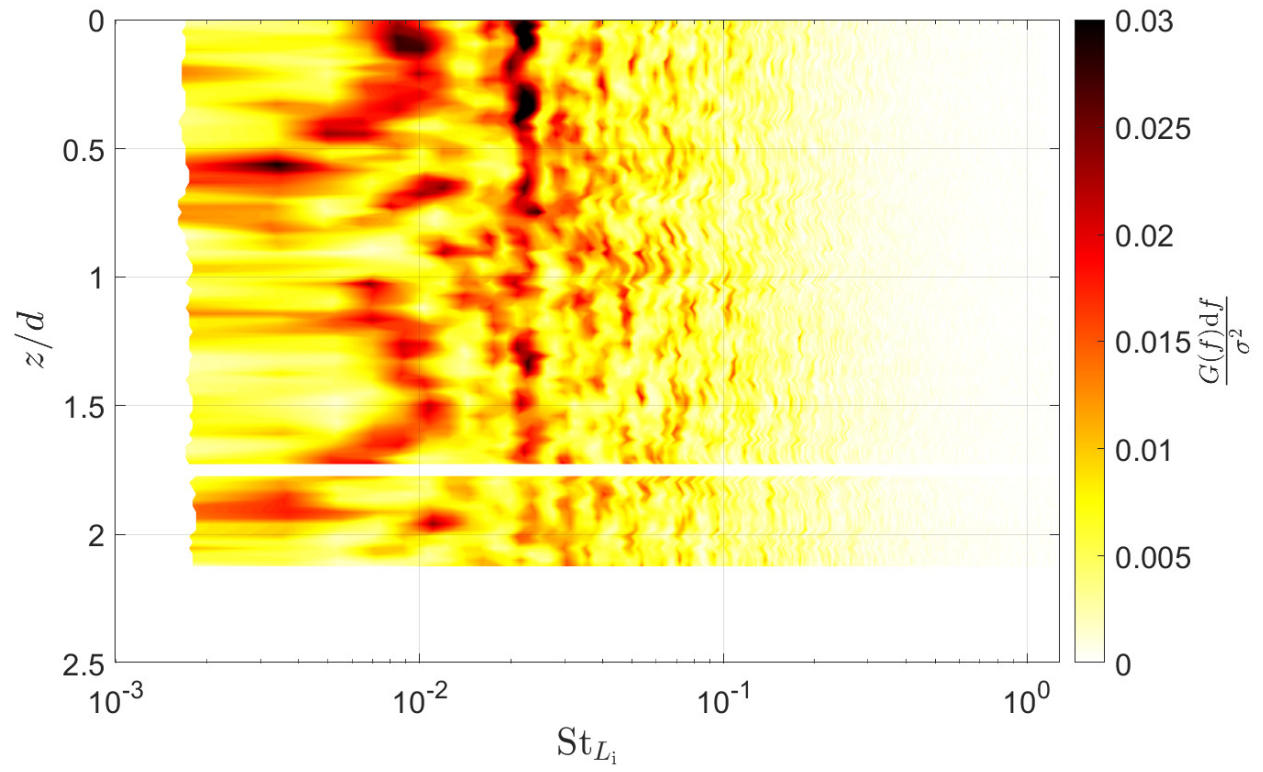


Figure 5.18: Contour of normalized power spectral density of the separation distance in the span on the floor related to the Strouhal number with respect to the intermittent length.

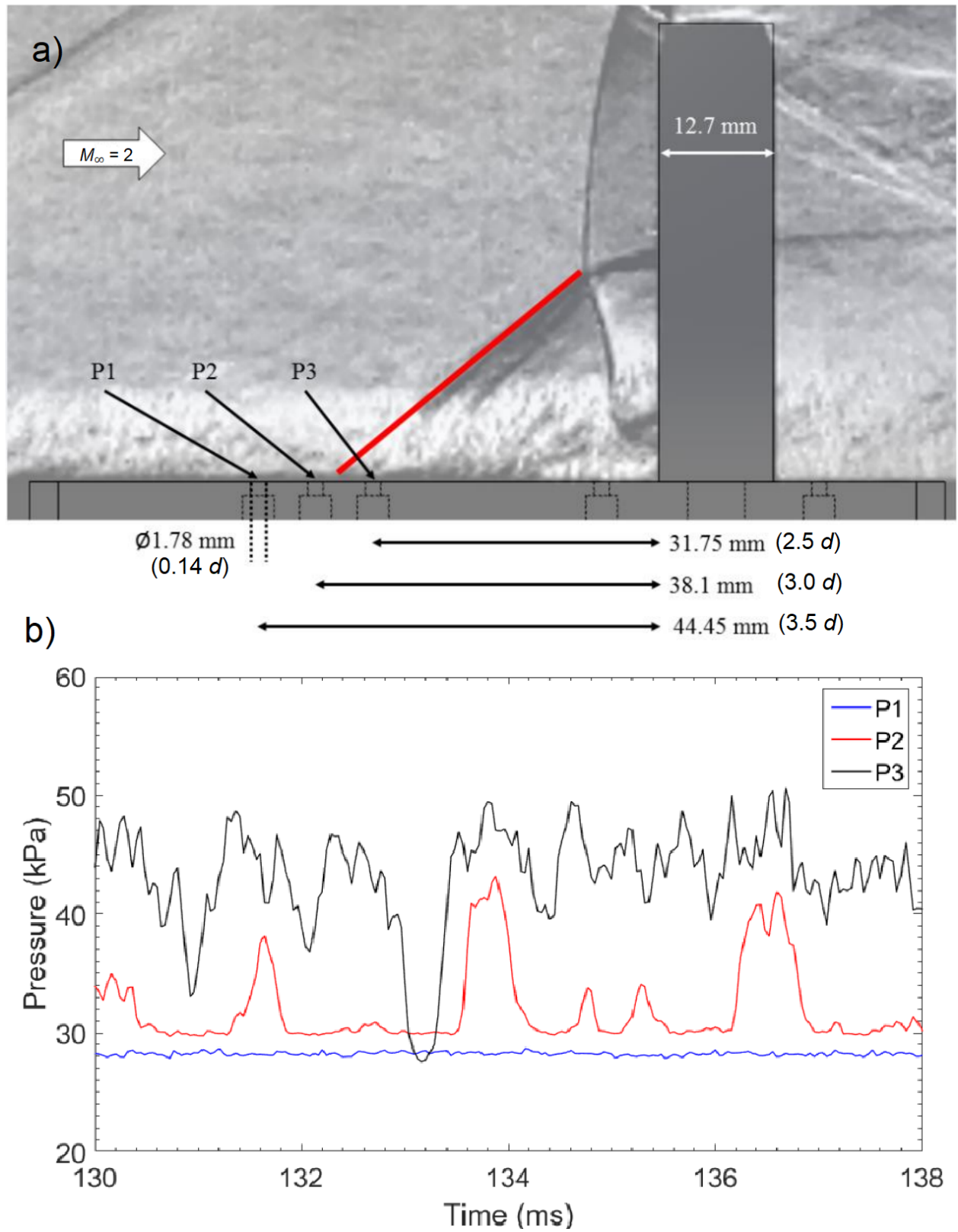


Figure 5.19: a) Experimental setup and b) corresponding pressure signals over time at centerline transducer locations. Figure adapted from Lash et al. [110].

5.20 which follows similar x/d -locations as the experiment; the undisturbed pressure port was shifted upstream to $x/d = -4.5$ to ensure an undisturbed environment, $x/d = -3$ and -2.5 match P2 and P3, respectively, and a downstream location of $x/d = -2$ was added. Notice that the disturbed pressure signals never return to an undisturbed value and remain elevated for the duration of data collection.

Despite the departure in raw signal behavior from the experiment, a PDF of these signals, shown in Fig. 5.21, reveals that the behavior is still Gaussian and is mildly skewed right. This gives confidence that a pressure analysis can still be performed on the ILES data, though some experimental methods and aspects may not be replicable. A closer look at the PDF at every grid location on the centerline is provided in Fig. 5.22. The contour depicts a Gaussian distribution in the undisturbed upstream region around $x/d \approx -4.5$. A positive skew becomes evident in the separation region from around $x/d \approx -3.5$ to $x/d \approx -2$. Then around $x/d \approx -1$, a negative skew is visible until the near-leading-edge flow becomes positively skewed again.

The pressure signal at every grid location on the centerline was analyzed for its statistical moments, similar to the separation shock in Fig. 4.10. These are shown in Fig. 5.23 for the standard deviation, skewness, and kurtosis, respectively, for both the unfiltered and low-pass-filtered ILES data. For standard deviation, the trends are similar but the magnitudes are higher for the unfiltered data. For skewness, the flow is observed to skew right in the separation region, as expected, although this begins further upstream in the filtered data set. A similar behavior exists for the kurtosis, in which a slight but persistent leptokurtic behavior for the unfiltered data, and a strongly leptokurtic in the upstream region for the filtered data that is followed by no significant kurtosis in the remainder of the separation region. Analyzing the rest of the floor using the unfiltered data, as shown in Fig. 5.24, similar trends are observed extending in the spanwise direction until about $z/d \approx 0.5$ for all three moments. The standard deviation then rapidly decays until it indicates only a minor disturbance for $z/d > 1$. Skewness and kurtosis are both abnormal near the separation region at the centerline and remain this way in the span along the separation line; this region continuously skews right and is leptokurtic. In the centerline region where the mean primary vortex structures contains three vortices, one of these appears to correlate to a reverse trend

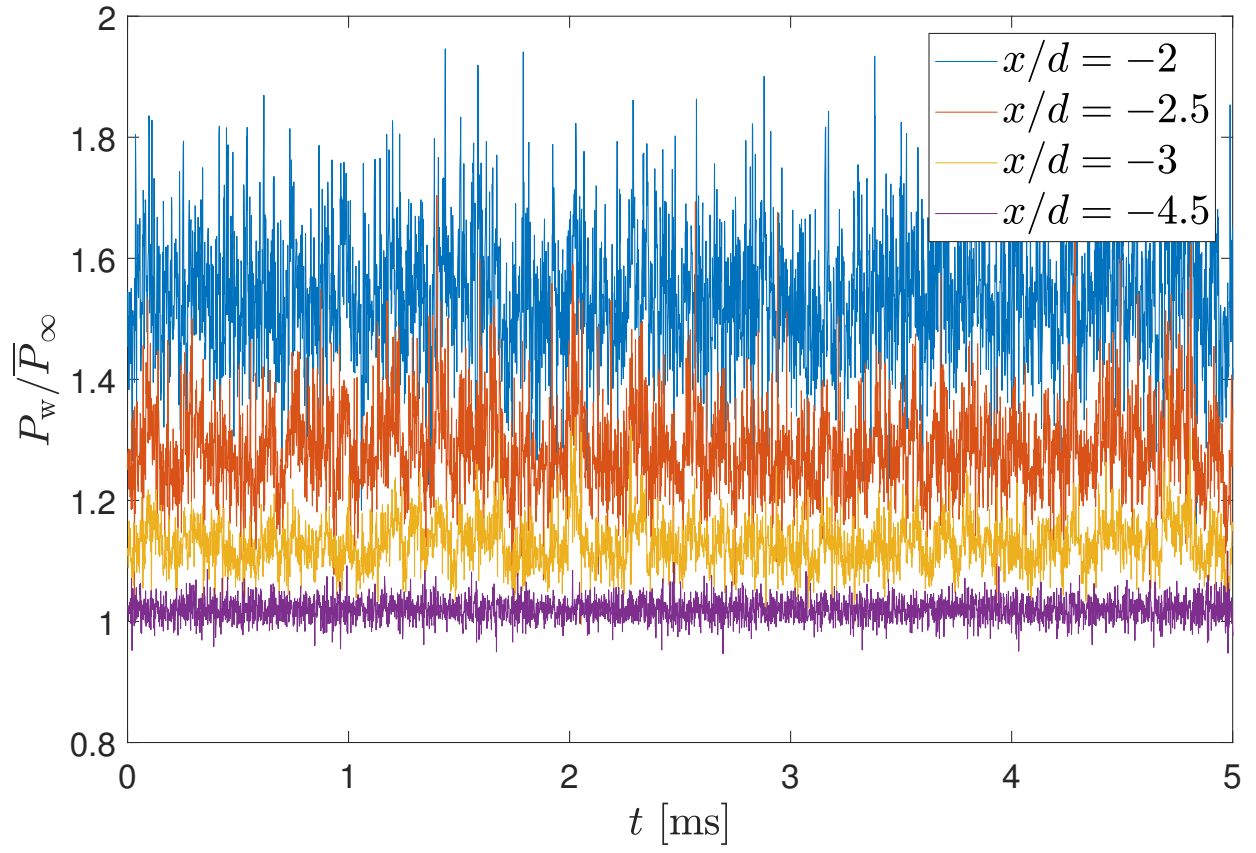


Figure 5.20: Instantaneous wall pressure at the centerline on the floor for various x/d -locations.

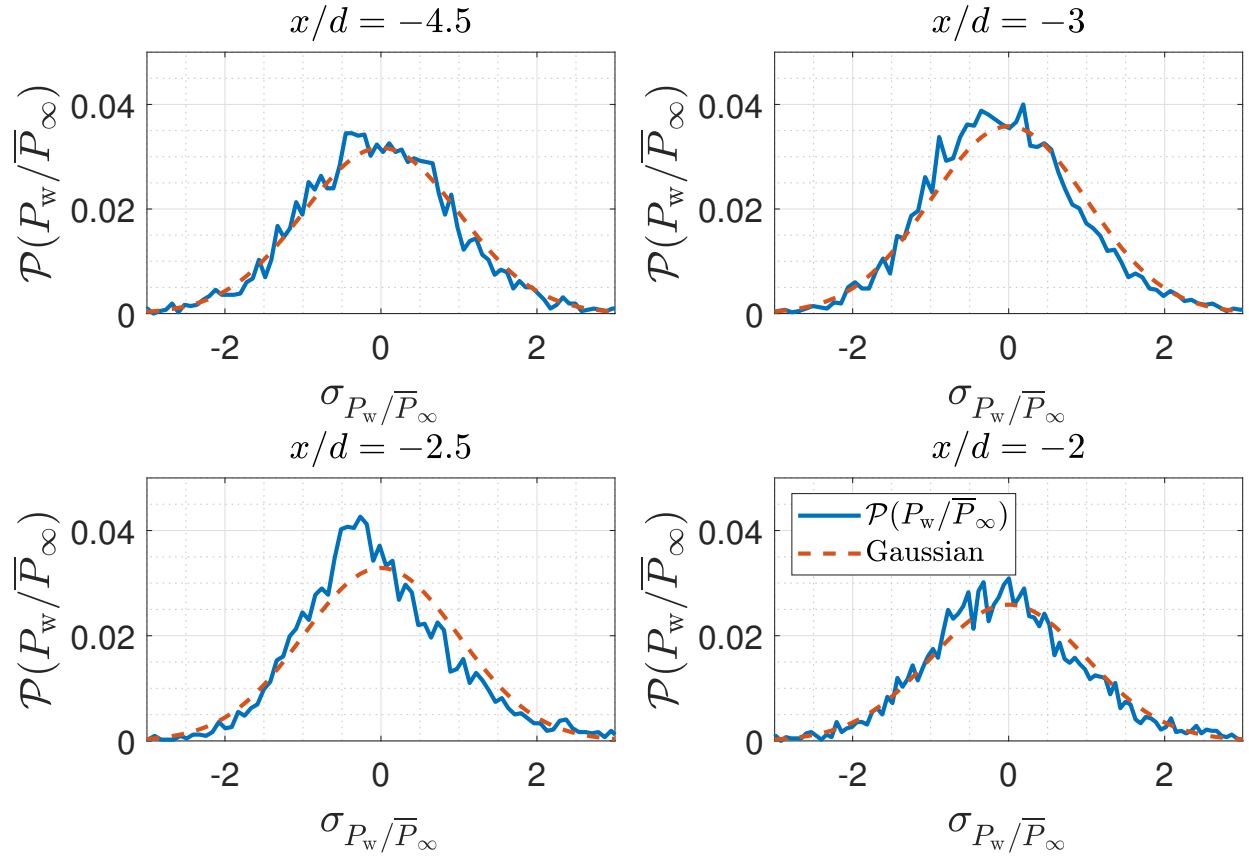


Figure 5.21: Probability density function of wall pressure at the centerline on the floor for various x/d -locations.

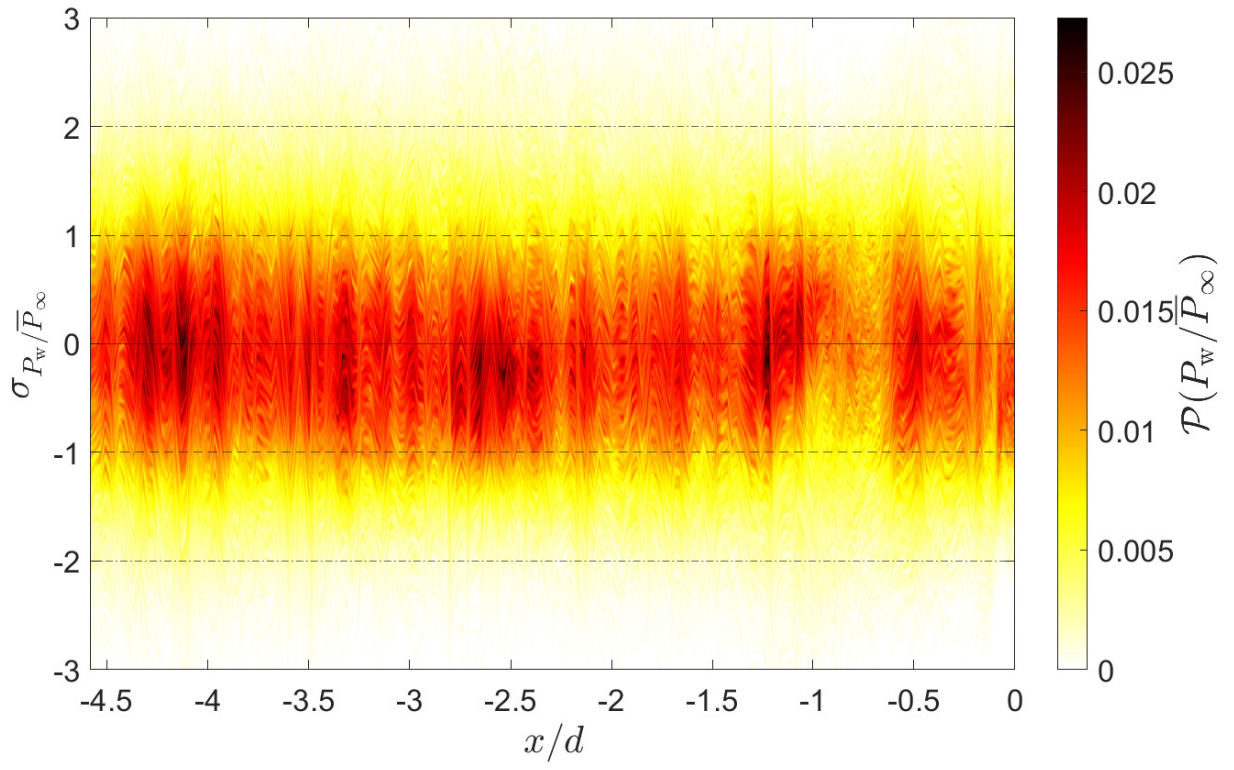


Figure 5.22: Contour of probability density function of wall pressure at the centerline on the floor.

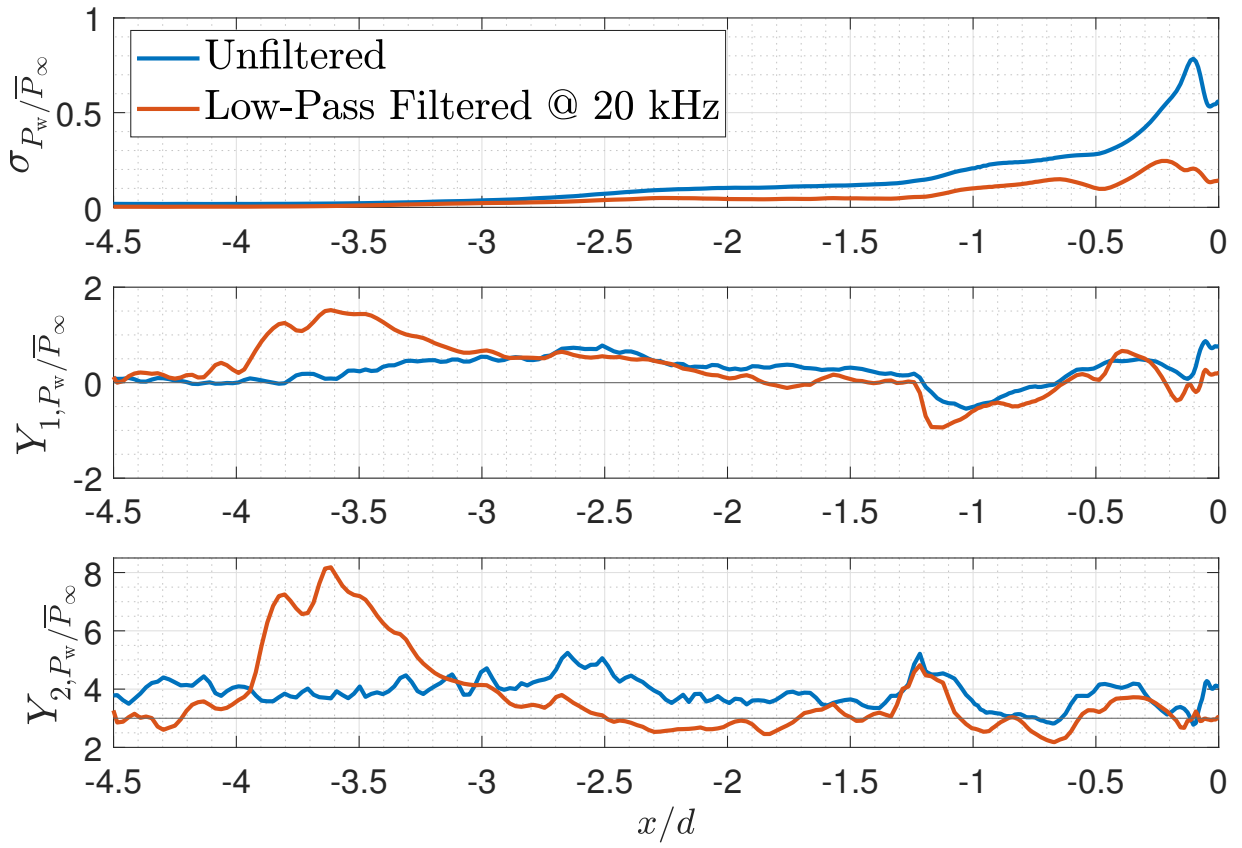


Figure 5.23: Statistical moments of a) standard deviation, b) skewness, and c) kurtosis for wall pressure at the centerline on the floor. Both unfiltered and low-pass filtered data are provided.

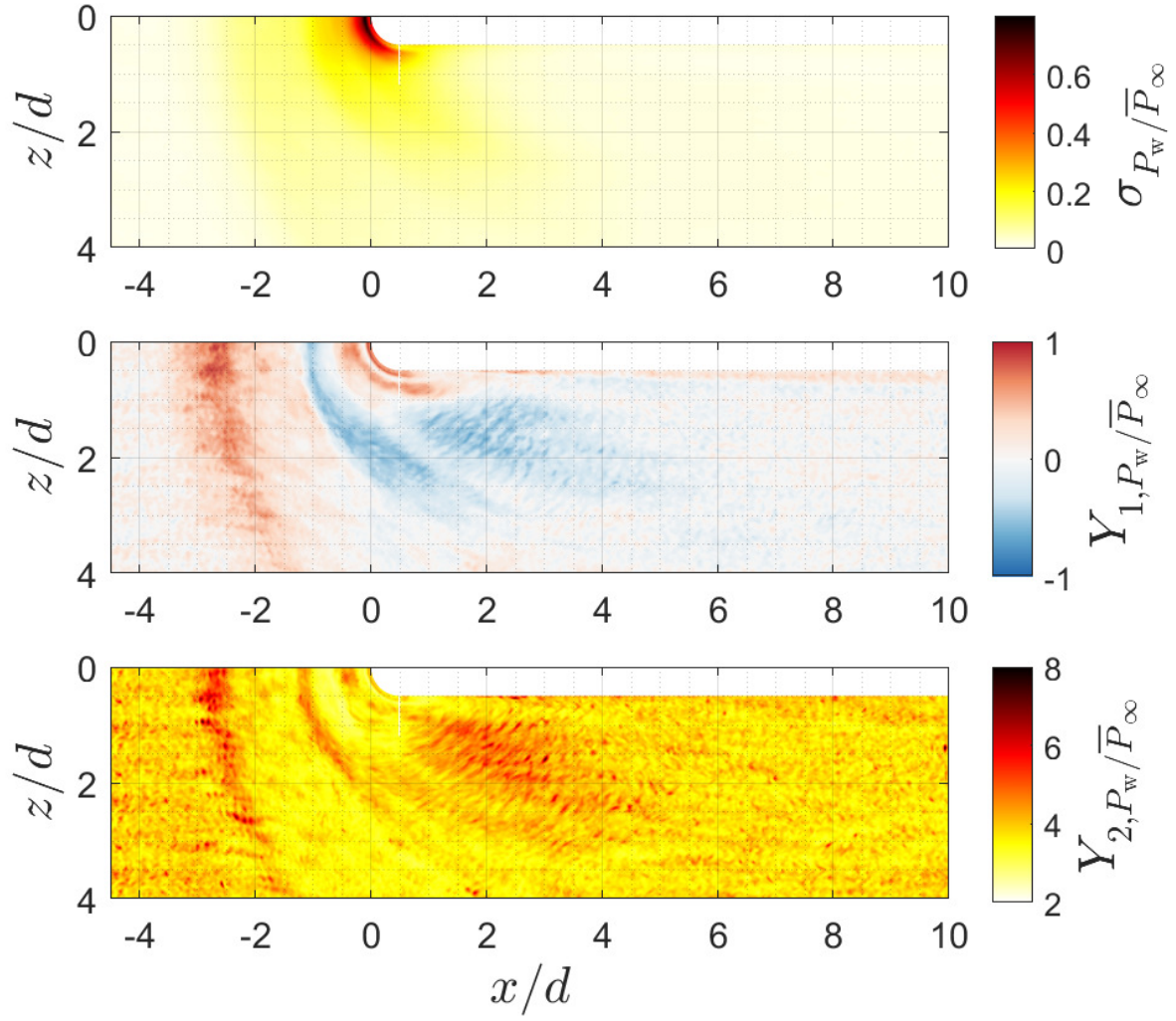


Figure 5.24: Contours of statistical moments of a) standard deviation, b) skewness, and c) kurtosis for wall pressure on the floor. Unfiltered data are provided.

in skewness, causing the flow to skew left. In addition, the flow becomes more leptokurtic in this region. This behavior seems to follow the outboard vortex path and grows in size as well, but also begins to dissipate by about $z/d \approx 3$, or by about $x/d \approx 5$. Since these additional vortices in the mean primary vortex structure are not present for high-Re flows, it is unclear if these patterns would reappear in a high-Re environment.

Similar to Fig. 5.17, the PSD was computed but using the wall pressure at the four centerline locations in some of the previous figures. These are shown in Fig. 5.25. In the undisturbed region at $x/d = -4.5$, no significant peaks are visible and the flow unsteadiness is primarily driven by the (undisturbed) turbulent boundary layer. For both of the locations in the separation region, at $x/d = -3$ and $x/d = -2.5$, the dominant Strouhal number is at $St_{L_i} = 0.0215$, matching the value in Fig. 5.17. A secondary peak of $St_{L_i} \approx 0.008$ appears again for the $x/d = -3$ location. Although the magnitude of the peak is lower, a similar Strouhal number of $St_{L_i} \approx 0.0231$ is exhibited for $x/d = -2$. This slight departure may be tied to the intermittency approaching a value of 1, and a contoured PSD at every grid point, shown in Fig. 5.26, confirms that the Strouhal number is dominant and constant at a value of ≈ 0.022 throughout the intermittent separation region. This is also the dominant Strouhal number nearer the leading edge at around $x/d \approx -1$ to -0.5 , which would suggest that the smaller vortices in the primary vortex structure oscillate at the same Strouhal number as the separation shock wave. The secondary peak of around $St_{L_i} \approx 0.008$ is also visible in the separation region for values of $\gamma \lesssim 0.5$.

Finally, a band power contour can be established by integrating the PSD over certain frequency bands to highlight where the dominant frequencies persist. This is shown for frequency bands of $St_{L_i} \in (0, 0.015]$, $St_{L_i} \in [0.015, 0.03]$, and $St_{L_i} \in [0.03, 0.045]$ on the centerline in Fig. 5.27a, b, and c, respectively, as well as on the floor in Fig. 5.28a, b, and c, respectively. The contours are normalized to reflect relative power. From these contours, it quickly becomes evident that the majority of unsteadiness is contained within the motion of the forward shock and the separation line, and that the dominant frequency band is between 0.015–0.03, as expected. The centerline contour reveals a roughly equal magnitude between the forward- and rearward-shock power, with very little power exhibited by the bow shock, even in the vicinity of the triple point. Moreover, a roughly equal magnitude of power

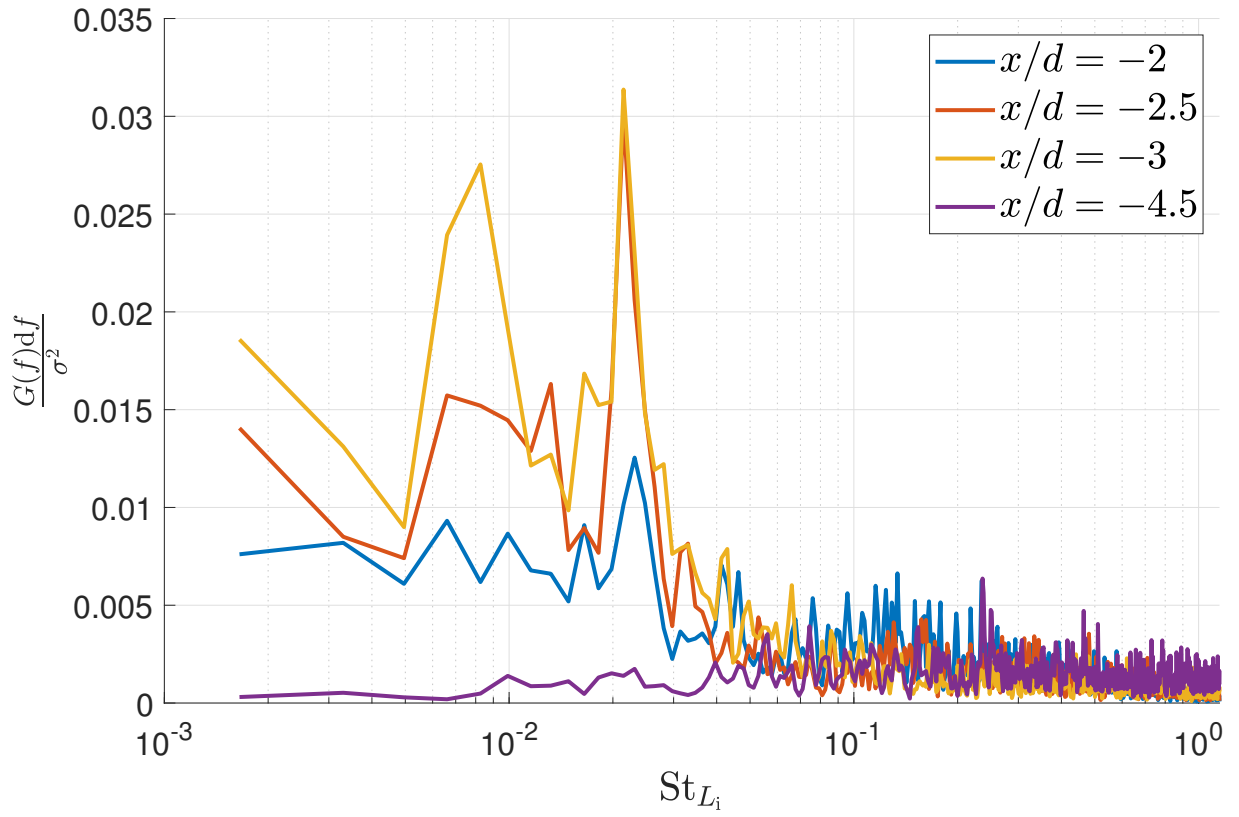


Figure 5.25: Normalized power spectral density of wall pressure at the centerline on the floor related to the Strouhal number with respect to the intermittent length.

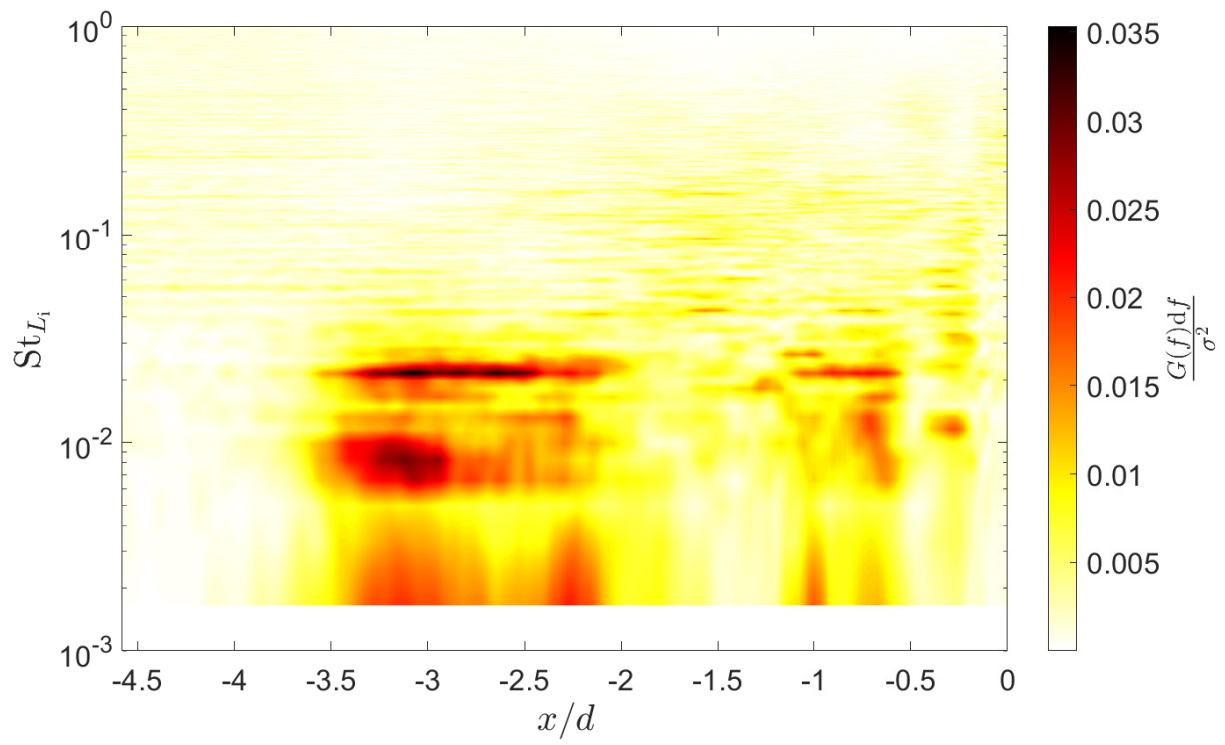


Figure 5.26: Contour of normalized power spectral density of wall pressure at the centerline on the floor related to the Strouhal number with respect to the intermittent length.

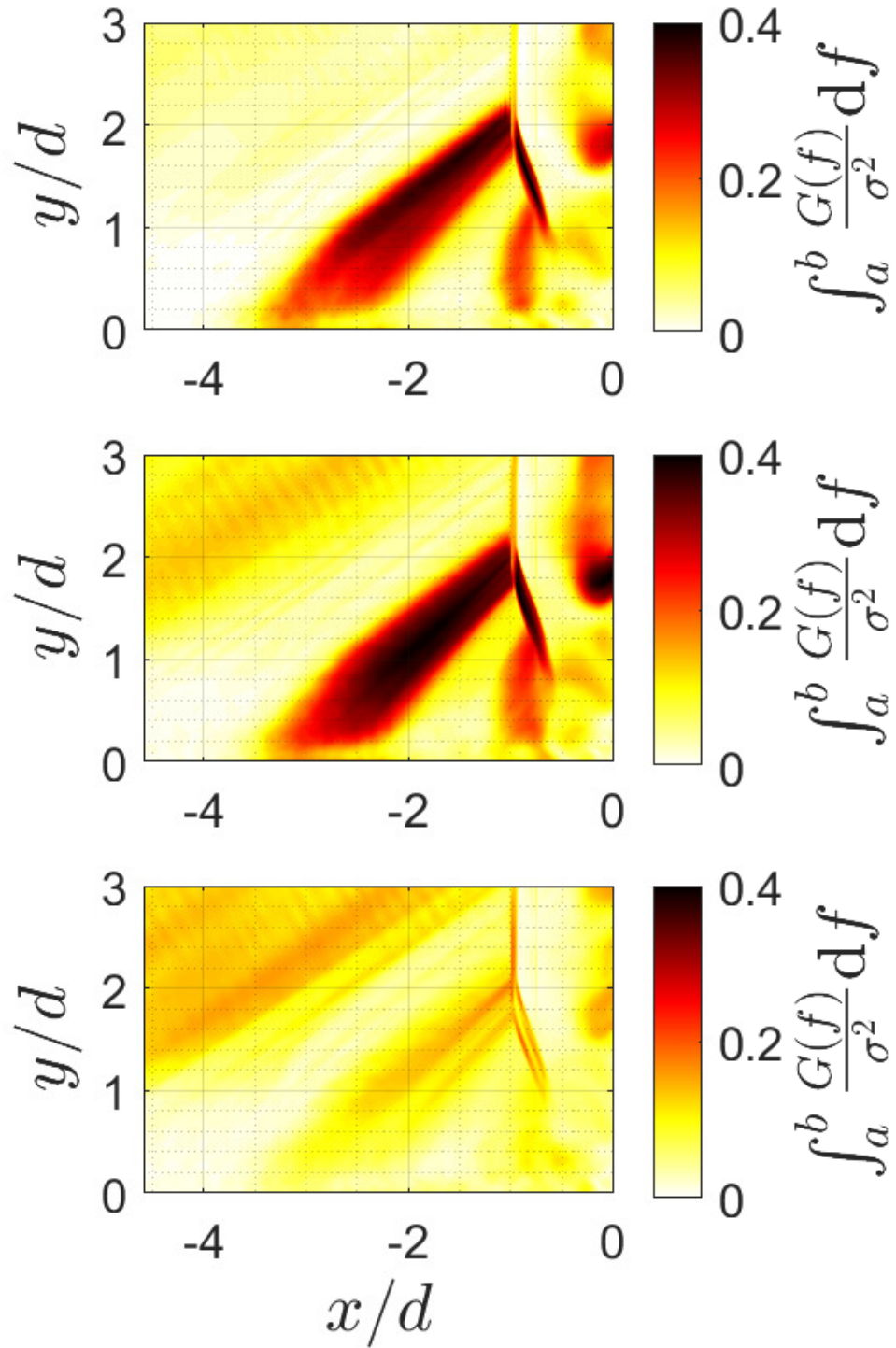


Figure 5.27: Contour of normalized band power of wall pressure at the centerline related to the Strouhal number with respect to the intermittent length. Frequency bands span St_{L_i} over a) $(0, 0.015]$, b) $[0.015, 0.03]$, c) $[0.03, 0.045]$.

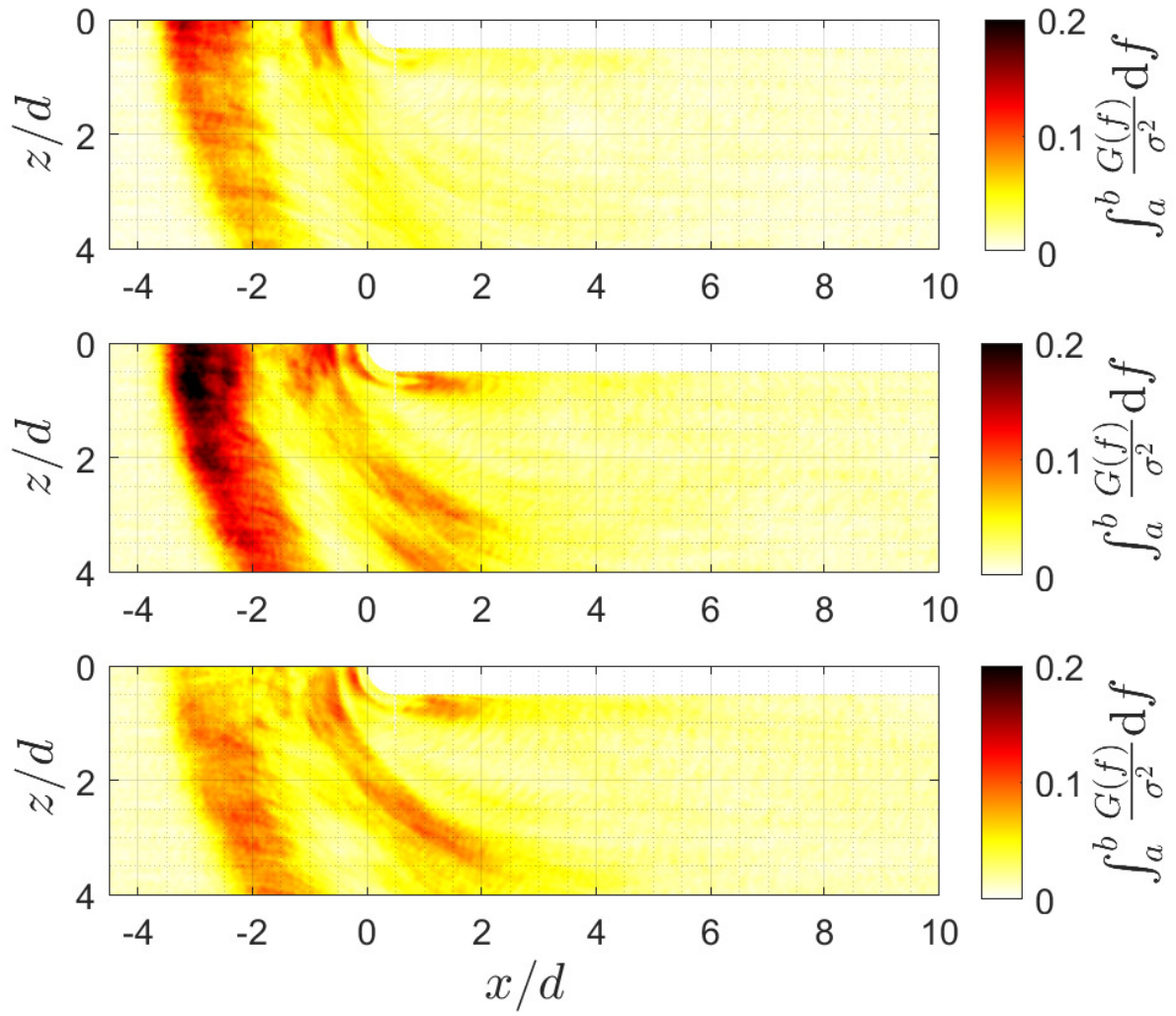


Figure 5.28: Contour of normalized band power of wall pressure on the floor related to the Strouhal number with respect to the intermittent length. Frequency bands span St_{L_i} over a) $(0, 0.015]$, b) $[0.015, 0.03]$, c) $[0.03, 0.045]$.

is also visible near the leading edge at about the same height as the triple point, which correlates closely to the impact location of the Edney interaction. On the floor, the drop in power magnitude along the spanwise direction is observed, but remains strongest along the separation line. Additionally, the secondary peak in Fig. 5.26 at $St_{L_i} \approx 0.008$ is particularly strong only near the centerline, but shares similar magnitude as the rest of the dynamic behavior does for the higher band of $St_{L_i} \in [0.03, 0.045]$. This could be the result of the closed-type vortex recirculation on the centerline causing a secondary dynamic influence.

5.2.3 Dynamic Attachment on the Fin Leading Edge

Just as the instantaneous separation distance on the flat plate floor was determined using a set of flow criteria, the attachment location on the fin leading edge can be subjected to similar criteria. Due to the orientation of the flow, rather than observing $u < 0$ in the first cell off the wall as a criterion, an equivalent $v < 0$ in the first cell off the wall was used as a criterion for y -locations between the mean pressure peak and trough (see Fig. 5.10). The resulting tracked attachment location over time is shown in Fig. 5.29, with the mean time-averaged value and the mean instantaneous value shown to agree rather well with one another. The attachment location is confined to lay within $A/d \in [1.2, 1.8]$, and the PDF shown in Fig. 5.30 confirms this. Moreover, the PDF reveals that the distribution is Gaussian.

The zero-crossing frequency and the intermittency of the attachment location are provided in Fig. 5.31. It is unclear what caused the zero-crossing frequency to spike around $y/d \approx 1.7$, but it is believed to be an artifact of the data analysis process and can likely be ignored. The peak frequency is at around ≈ 70 kHz, which is in the same ballpark as the peak frequency for the separation distance, and also corresponds to an intermittency value of $\gamma = 0.5$. The intermittency curve has a sigmoid shape and follows the Gaussian error function well, further confirming that the attachment location dynamically follows a Gaussian distribution. An intermittent length is again defined as the distance between $\gamma = 0.05$ and $\gamma = 0.95$, which were found to be at $y/d = 1.63$ and $y/d = 1.35$, respectively, and this yielded a length of $L_i/d = 0.28$.

Another view on the unsteadiness of the attachment location and the vortices on the leading edge is provided by the v -velocity-equivalent Mach number contour in Fig. 5.32. The

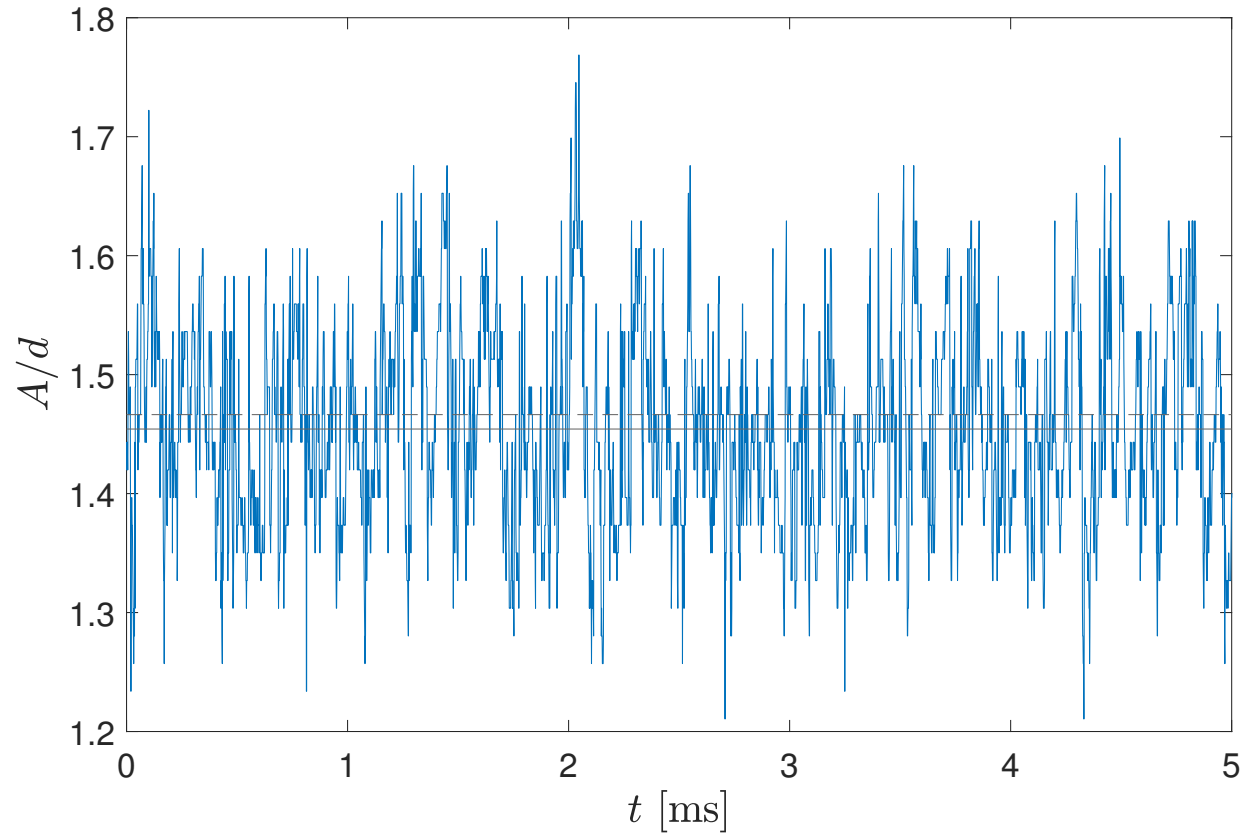


Figure 5.29: Instantaneous attachment location at the leading edge. Solid line represents mean value of instantaneous separation data. Dashed line represents mean value of time-averaged separation data.

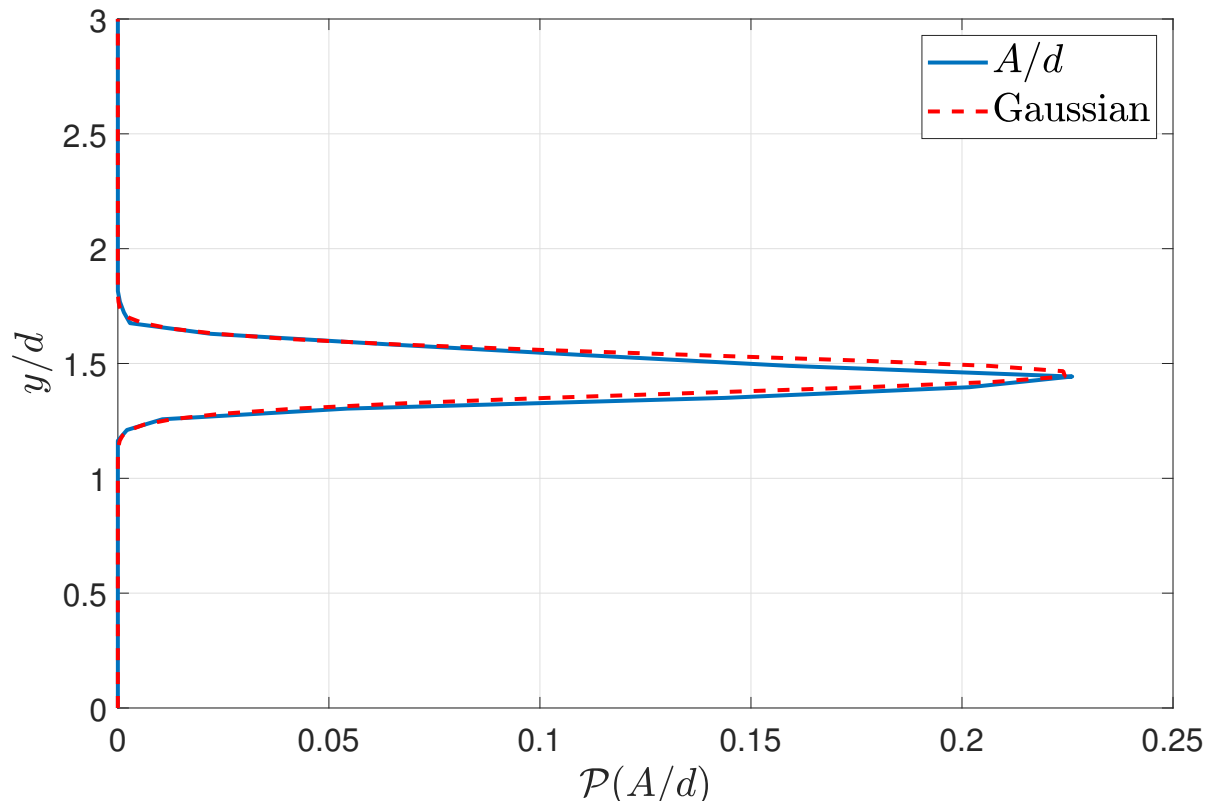


Figure 5.30: Probability density function of instantaneous attachment location on the leading edge. Dashed line represents Gaussian-equivalent data.

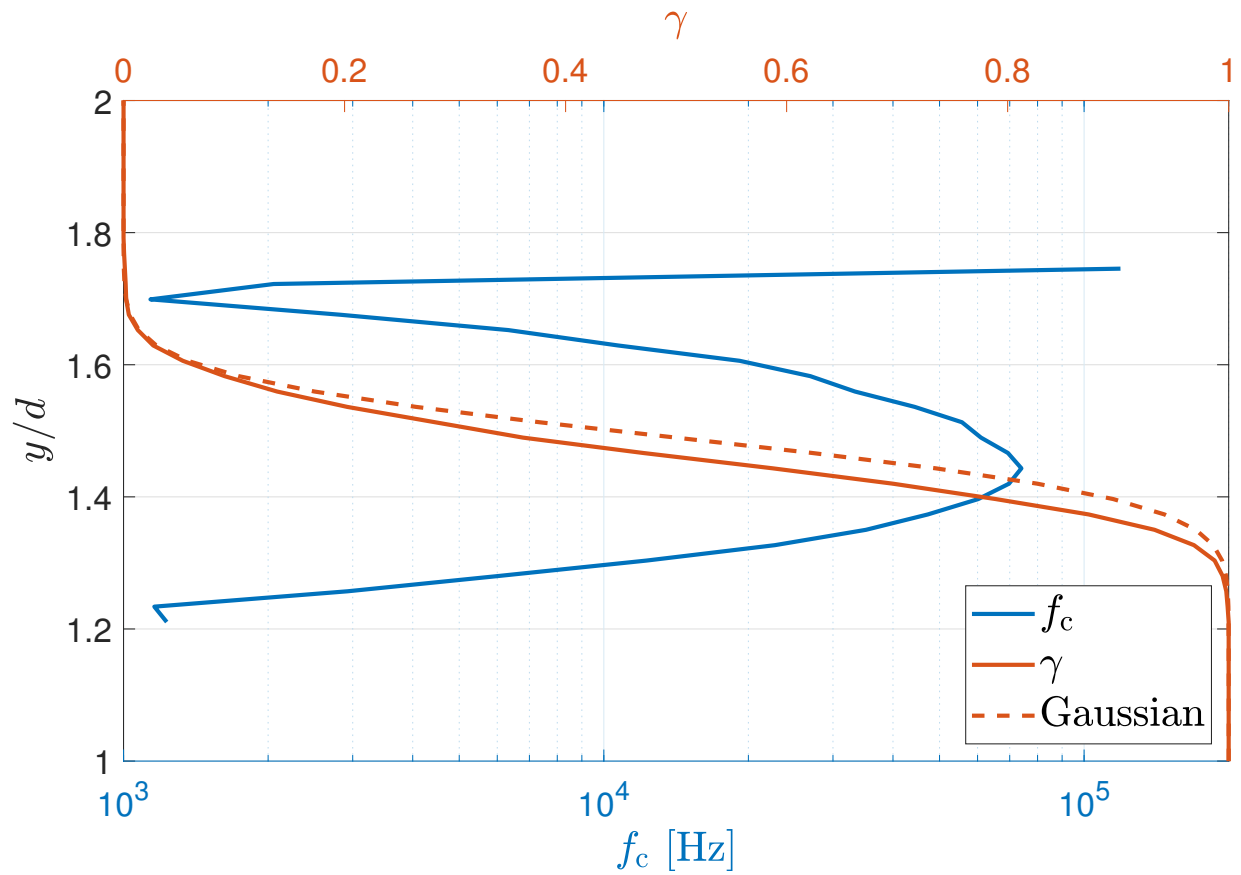


Figure 5.31: Zero-crossing frequency and intermittency of instantaneous attachment location. Intermittency chart includes Gaussian error function for reference.

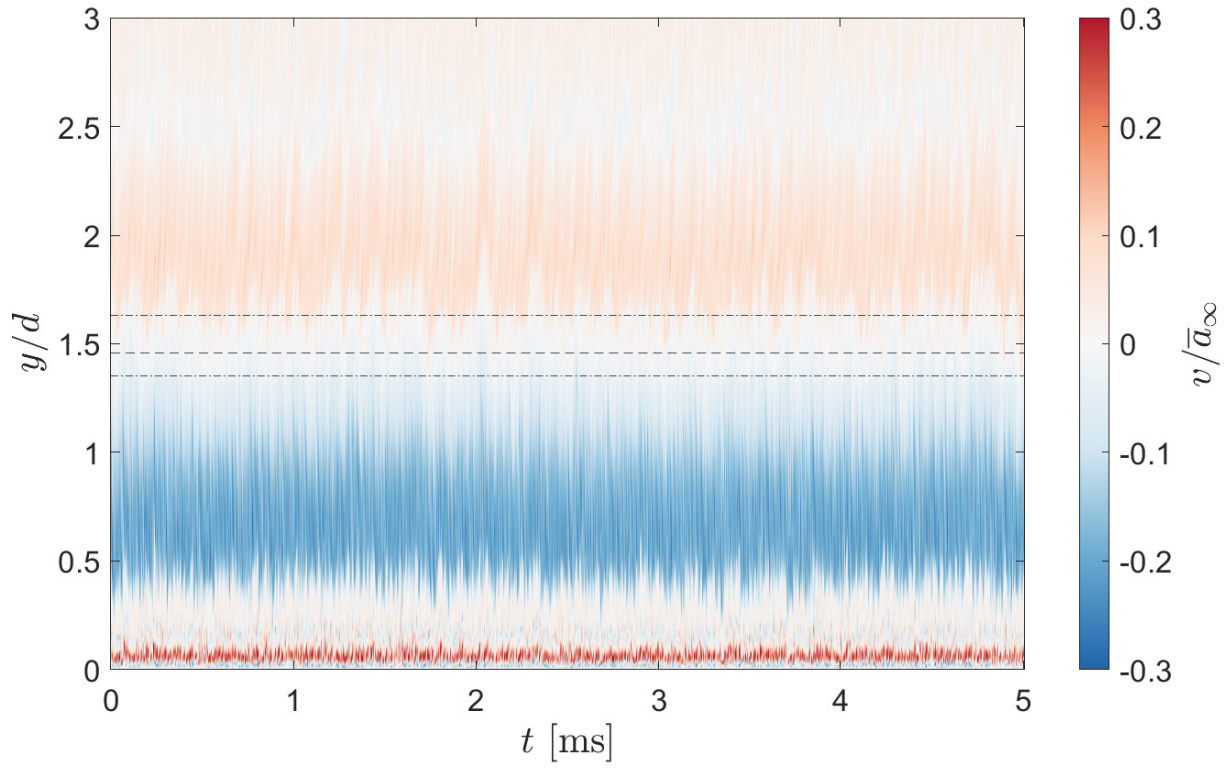


Figure 5.32: Instantaneous v -velocity contour at the leading edge. Solid line represents mean attachment location, dashed values represents intermittent length limits at $\gamma = 0.05$ and $\gamma = 0.95$.

attachment location is clearly shown as the divider, with the mean value and the intermittent limits shown for reference. The bands of strong positive and strong negative v -velocity are rather confined by their locations. Near the base, both the secondary vortex (red band, positive v -velocity) and the tertiary vortex (blue band, negative v -velocity) are also identified. There does seem to be some fluctuation in the overlap region around $y/d \approx 0.2$ – 0.3 , near where the secondary vortex separates around $SL2$ in Fig. 2.14b. At the top of the contour, a mildly positive v -velocity is visible, as expected from Fig. 5.8, as the inviscid region had not yet been reached.

Quantifying the characteristic frequencies of the attachment location is again done through the PSD as shown in Fig. 5.33. There are two distinct peaks visible at $St_{L_i} \approx 0.0028$ and ≈ 0.0073 . It is possible that the second peak correlates to the secondary peak in the separation distance PSD in Fig. 5.17. In addition to these two peaks, a substantial amount of energy content persists in the low-frequency range ($f < 600$ Hz) that was insufficiently resolved, which, though present, was not as pronounced for PSDs observed on the floor. Removing the normalization by an intermittent length allows for raw frequencies to be compared between the separation and attachment locations. This comparison is shown in Fig. 5.34 and reveals the direct correlation of the separation line, $SL1$, and the attachment line, $AL1$, with both values having variables sharing the exact same frequency peaks at $f = 3.0$ kHz and 7.8 kHz.

5.2.4 Dynamic Pressure on the Fin Leading Edge

For consistency, the wall pressure on the leading edge is normalized again by the stagnation pressure behind the inviscid bow shock. Looking at Fig. 5.35, the variability in the normalized wall pressure is shown. Around the attachment location, there is a moderate variability associated with the peak pressure location, but not with its magnitude. This type of pattern is visible elsewhere on the leading edge as well, where the locations may fluctuate over time, but the magnitudes are generally similar.

A quantitative inspection of the pressure variations is provided by the PDF contour shown in Fig. 5.36. Unlike the attachment location, the wall pressure does not produce a clearly distinguishing feature. The flow appears to be nearly Gaussian throughout the leading edge,

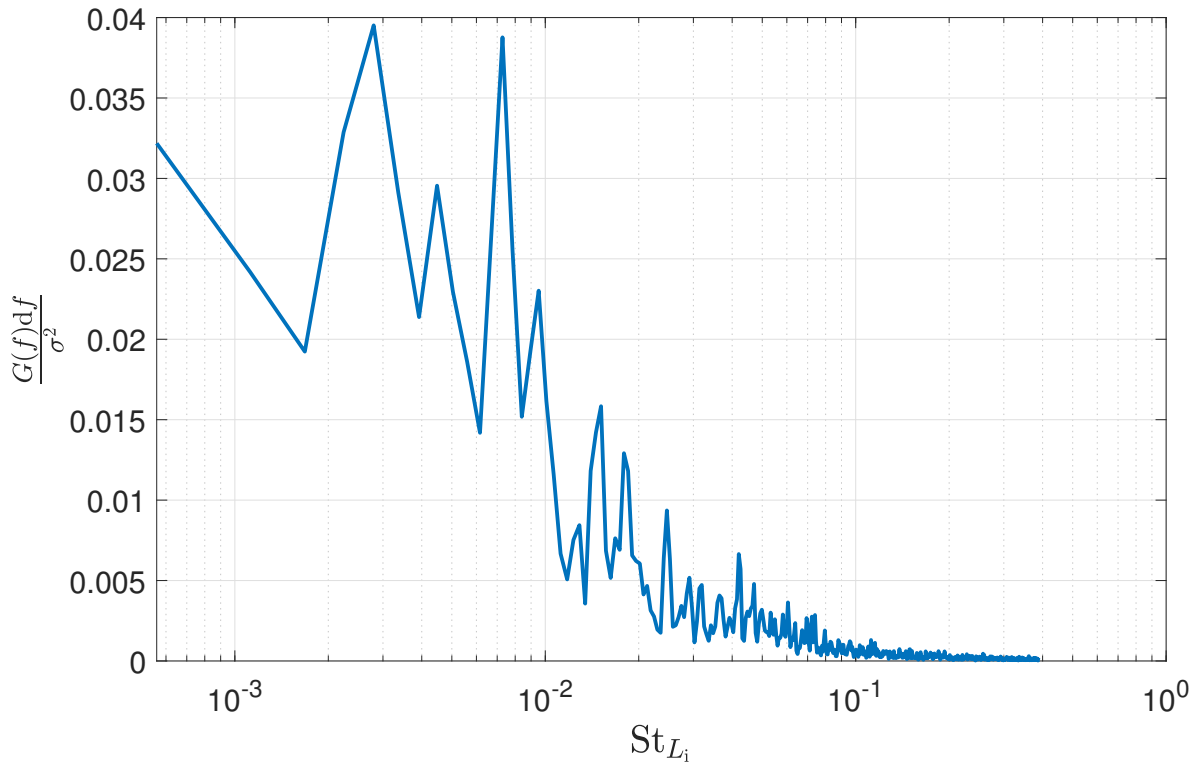


Figure 5.33: Normalized power spectral density of the attachment location related to the Strouhal number with respect to the intermittent length.

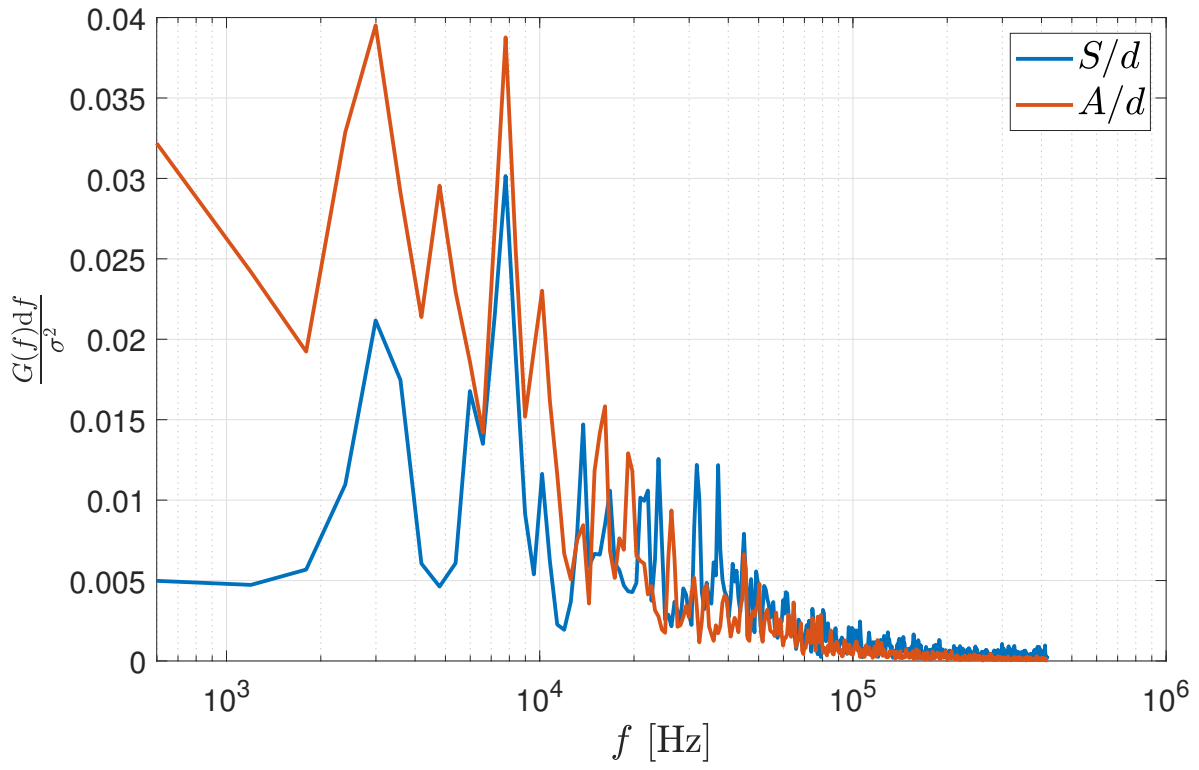


Figure 5.34: Normalized power spectral density of the separation and attachment locations related to frequency.

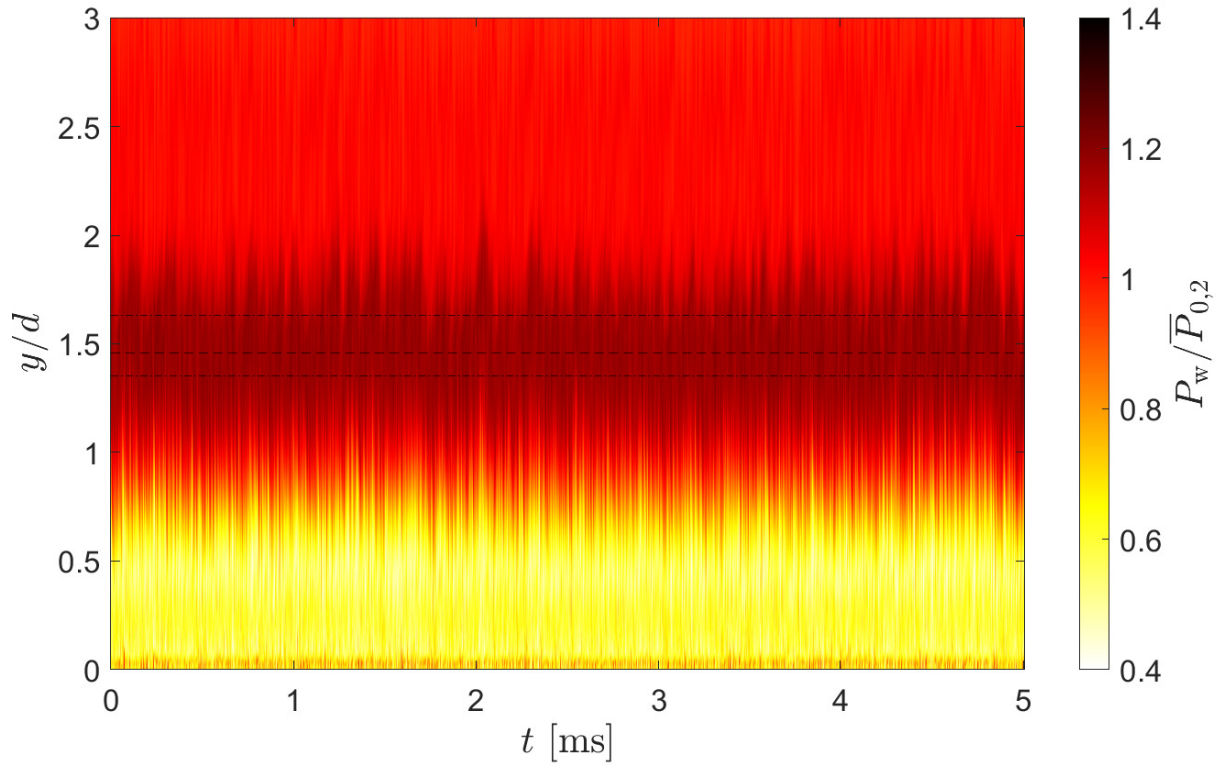


Figure 5.35: Instantaneous normalized wall pressure contour at the leading edge. Solid line represents mean attachment location, dashed values represents intermittent length limits at $\gamma = 0.05$ and $\gamma = 0.95$.

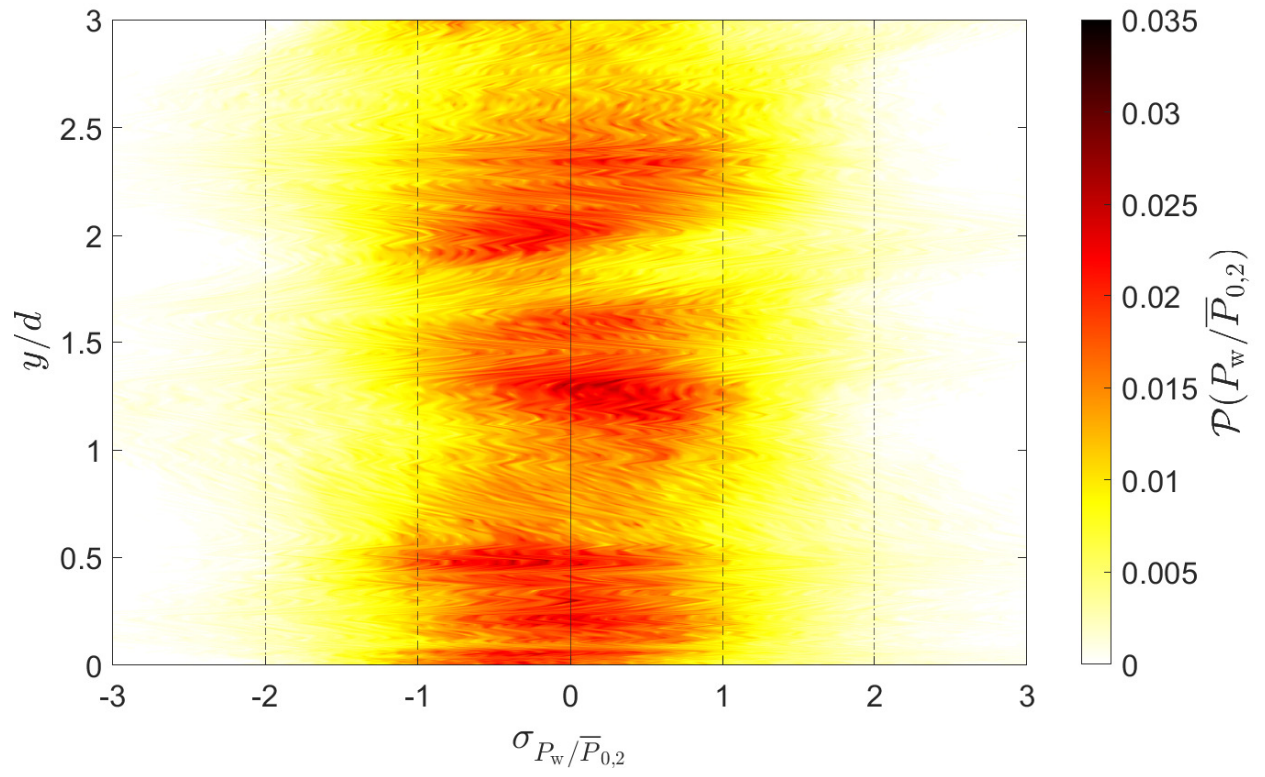


Figure 5.36: Contour of probability density function of normalized wall pressure at the leading edge.

being mostly confined to $\sigma_{P_w/\bar{P}_{0,2}} \lesssim \pm 1$. The skewness switches sides regularly, oscillating between left- and right-sided skewness. The strongest probability peaks are observed at $y/d \approx 1.3$, $y/d \approx 0.5$, and $y/d \approx 0.3$, which seem to correlate with $\gamma \approx 1$ and the trough and local maximum pressures associated with the secondary vortex, respectively (see Fig. 5.10). Also notable are regions that have a higher variance with content approaching $\sigma_{P_w/\bar{P}_{0,2}} \approx \pm 3$. These regions are found around $y/d \approx 2.5$ – 3 , near $y/d \approx 1.7$ – 1.8 , or near $y/d \approx 0.6$ – 1 . A further inspection of the statistical moments in Fig. 5.37a, using both the unfiltered and low-pass filtered data, confirms that the standard deviation at these locations is higher.

Also shown in Fig. 5.37b and c are the skewness and kurtosis, respectively. The skewness is seen to shift back and forth between left- and right skewness. The kurtosis is mildly to moderately leptokurtic for most of the flow region, although there are platykurtic sections, particularly for $y/d \gtrsim 2.5$. A peak is found at $y/d \approx 2$ for both skewness and kurtosis, which corresponds to the onset of the pressure rise. At the mean location for A/d , both skewness and kurtosis are at effectively Gaussian values. Several local maxima and minima are correlated between the higher moments, such as at $y/d \approx 1.6$, ≈ 1.3 , ≈ 0.5 , or ≈ 0.01 , where a local maximum or minimum in skewness also results in a local maximum or minimum in kurtosis.

Using the PSD to inspect the characteristic frequency on the leading edge, as shown in Fig. 5.38, indicates unexpected results. The dominant frequency is at around $St_{L_i} \approx 0.007$, but its location appears limited to $y/d \gtrsim 1.5$, with the greatest energy content found for $y/d \in [1.5, 2]$. At these locations, secondary peaks are found in the frequency band of around $St_{L_i} \approx 0.002$ – 0.005 and in the low-frequency range that is insufficiently resolved. Below the peak pressure and mean attachment location of $\bar{A}/d \approx 1.5$, broadband frequency content seems to be dominant over nearly an order of magnitude from $St_{L_i} \approx 0.002$ – 0.013 ; this broadband energy is found for $y/d \in [0.5, 1.35]$. For lower y -values, no distinguishable peaks are exhibited, indicating that the secondary vortex is characterized by random fluctuations rather than distinct frequencies.

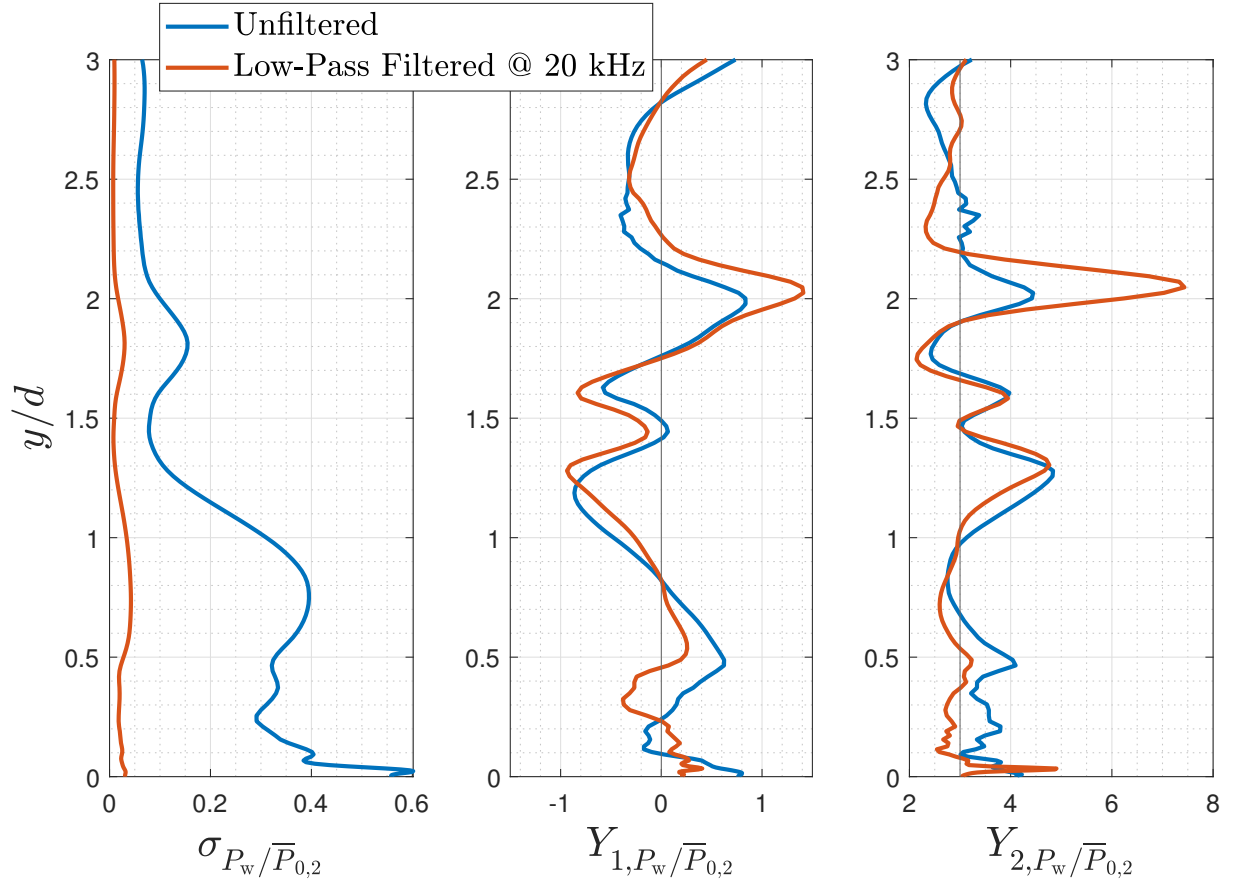


Figure 5.37: Statistical moments of a) standard deviation, b) skewness, and c) kurtosis for normalized wall pressure at the leading edge. Both unfiltered and low-pass filtered data are provided.

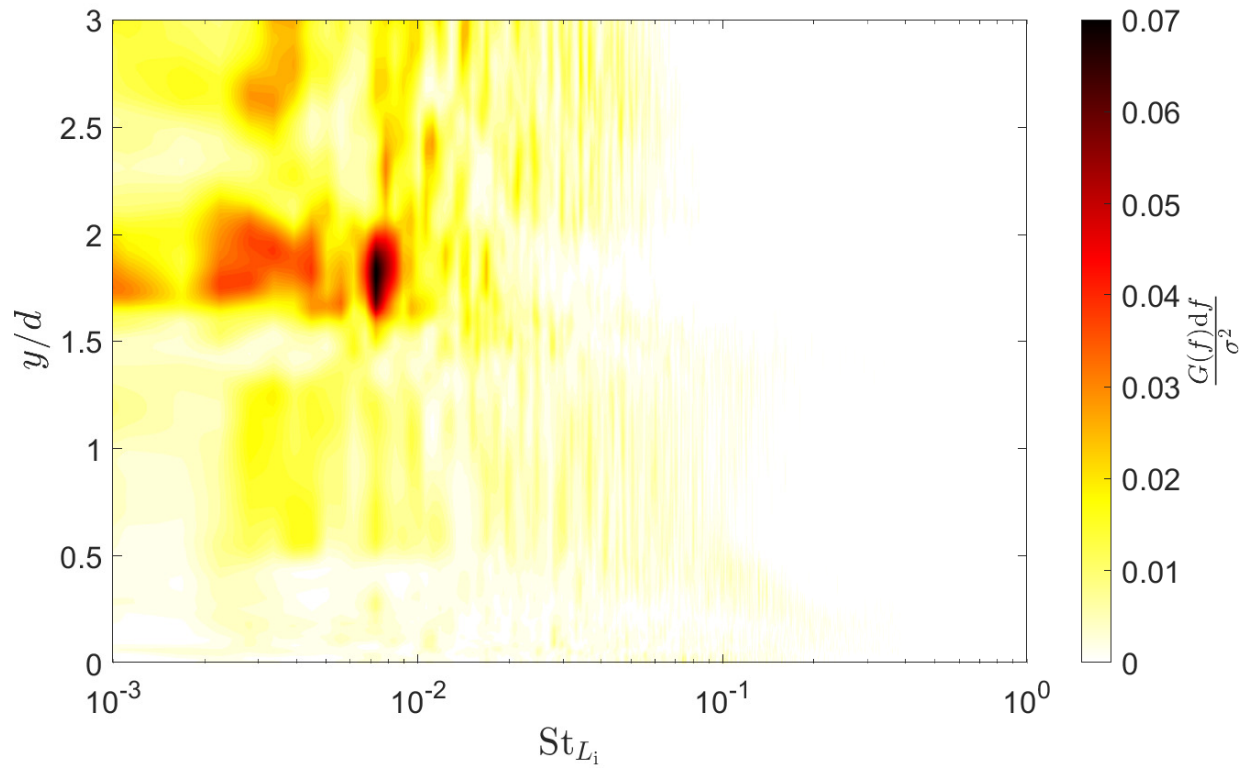


Figure 5.38: Contour of normalized power spectral density of normalized wall pressure at the leading edge related to the Strouhal number with respect to the intermittent length.

Chapter 6

Conclusions

The shock wave/turbulent boundary layer interaction generated by a semi-infinite, orthogonal cylinder and blunt fin at around Mach 2 was investigated. A combination of numerical and experimental methods was used to study the interaction scaling at the centerline, the spanwise evolution of key flow features, and establish best practices for the numerical methods.

Reynolds-averaged Navier–Stokes simulations were performed in conjunction with high-speed schlieren imaging and supplementary surface oil-flow visualization to investigate scaling effects of the parameter d/δ . This ratio between the cylinder or blunt-fin diameter, d , and the 99th-percentile boundary-layer thickness, δ , had historically provided an adequate scaling of the centerline separation distance, S , normalized by the diameter (S/d), though this was not universal for differing flow conditions. Instead, it was found that a normalization by the boundary-layer thickness, such that S/δ became a function of d/δ , yielded excellent agreement over a vast amount of flow conditions, including for data from the archival literature, when following either a linear or exponential trend. Moreover, the triple-point height, h_{tp} , showed similar agreement when normalized as h_{tp}/δ . The effects of d/δ were further found to provide viscous damping on the centerline pressure, both on the floor upstream and on the leading edge of the blunt body; a lower d/δ value provided more damping, and once the boundary layer encompassed the interaction, the peak pressure due to an Edney interaction was also reduced.

An implicit large-eddy simulation was performed with comparisons to supplementary high-speed schlieren imaging, high-speed pressure transducer, and fast-acting pressure sensitive paint measurements, to characterize the dynamics of the interaction and the spanwise evolution of flow features that originate at the centerline. The separation line was found to be a line of coalescence that does not contain flow separation beyond about $z/d > \pm 2-3$, depending on the Reynolds number employed. The primary vortex structure originating at the centerline diminishes and ultimately vanishes at the spanwise outboard location where flow separation ceases. The peak pressure on the floor upstream rapidly diminishes with increased spanwise distance away from the centerline and similarly ceases to be a local maximum at the point where separation ceases. A universal dynamic behavior was observed for both centerline and spanwise outboard locations, collectively indicating a universal intermittent length, L_i , and Strouhal number, St_{L_i} , in the regions where flow separation is present. Moreover, the centerline separation distance and leading-edge attachment location exactly shared dominant frequencies, indicating direct connectivity between the two. A near-Gaussian distribution was found to be omnipresent in the interaction, existing as such on the centerline on the floor, the spanwise outboard locations on the floor, and on the leading edge of the cylinder or fin. Further outboard, the (algebraically) conical nature of the separation line linearized, at around $z/d \gtrsim 6$.

The use of RANS simulations is deemed to be sufficient for mean features, as the time-averaged ILES generated very similar results. Differences observed between the two methods can generally be attributed to an early (upstream) flow separation, which are likely due to the conservative approach of the turbulence model. However, agreement on the separation location was found even for differing Reynolds numbers that used the same RANS approach. Early separation may alter the perception of overall scaling of the interaction, but still appears to grant insight into the strength and severity of the imposed surface pressures. The ILES was able to reproduce and align with experimental methods, albeit at a lower Reynolds number; the characteristic normalized frequencies (i.e. through St) were observed, but the reduced pressure in the simulation introduced viscous dampening. As such, it is recommended that RANS simulations are used to bound the mean impact of the SWBLI, bearing in mind the caveat of early separation that may also be considered part of an

engineering safety factor. Characteristic normalized frequencies can effectively be determined through (I)LES, and although these simulations are normally very expensive to produce, the cost can be reduced by shrinking the physical domain and simply maintaining the correct ratio d/δ and using a Reynolds number that sustains turbulent flow (based on Re_θ).

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Appendices

Appendix A

Statistical Analysis

Standard statistical analysis is applied to the results obtained, consisting not only of mean and standard deviation, but also the higher-order moments such as skewness and kurtosis. In all of these, ϕ is an arbitrary place-holder variable that can take on pressure, velocity, etc., and N is the sample size that is taken to be sufficiently large such that it generates a statistically-significant and repeatable result.

A.1 Sample Mean

The sample mean, or (arithmetic) average, is the first raw moment, defined as:

$$\bar{\phi} = \frac{1}{N} \sum_{i=1}^N \phi_i \quad (\text{A.1})$$

A.2 Standard Deviation

The biased standard deviation is defined as:

$$\sigma_{\phi,N} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi})^2} \quad (\text{A.2})$$

However, the biased version accounts for the entire population, which is not applicable in this study. When sampling large enough data sets, an unbiased estimation should be followed, and applying Bessel's correction and Jensen's inequality yields the unbiased sample standard deviation:

$$\sigma_{\phi} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (\phi_i - \bar{\phi})^2} \quad (\text{A.3})$$

If $\bar{\phi}$ is taken to be zero, then the biased sample standard deviation reduces to the root mean square (RMS). The RMS does not have a correction available.

$$\phi_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^N \phi_i^2} \quad (\text{A.4})$$

A.3 Variance

Squaring the biased sample standard deviation leads to the biased variance, another measure of the deviation from a random variable. The biased variance is the second central moment, and is defined as:

$$\sigma_{\phi,N}^2 = \frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi})^2 \quad (\text{A.5})$$

More generally, any n^{th} -order central moment can be described as follows:

$$m_n = \frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi})^n \quad (\text{A.6})$$

As with the standard deviation, the unbiased version should be obtained to account for the lack of information regarding the entire population. Squaring the unbiased sample standard deviation leads to the Bessel-corrected (unbiased) variance. This version fully takes into account sampling through Bessel's correction, as opposed to the estimated correction used in the standard deviation. The unbiased variance is the sample second central moment, and is defined as:

$$\sigma_{\phi}^2 = \frac{1}{N-1} \sum_{i=1}^N (\phi_i - \bar{\phi})^2 \quad (\text{A.7})$$

A.4 Sample Skewness

Skewness is a third-order moment that indicates the asymmetry of the probability distribution about the mean. A negative skew indicates that the tail is on the left and a positive skew indicates that the tail is on the right, while a symmetric (and Gaussian) distribution has a skewness of zero. The sample skewness is defined as:

$$\begin{aligned} Y_1 &= \frac{\sqrt{N(N-1)}}{N-2} \frac{m_3}{m_2^{3/2}} \\ &= \frac{\sqrt{N(N-1)}}{N-2} \frac{\frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi})^3}{\left[\frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi})^2 \right]^{3/2}} \end{aligned} \quad (\text{A.8})$$

A.5 Sample Kurtosis

Lastly, kurtosis is a fourth-order moment that measures the “tailedness” of a distribution. The higher the kurtosis, the greater the deviation of the data from the mean. A Gaussian distribution has a kurtosis of 3, and this value is sometimes subtracted to define excess kurtosis. This, in turn, leads to negative values which can be more difficult to conceptualize. By the standard definition, values with a kurtosis of less than 3 (or negative excess kurtosis) are said to be platykurtic, and values with a kurtosis greater than 3 (or positive excess kurtosis) are said to be leptokurtic. The sample kurtosis is defined as:

$$\begin{aligned}
Y_2 &= \frac{(N+1)(N-1)}{(N-2)(N-3)} \frac{m_4}{m_2^2} \\
&= \frac{(N+1)(N-1)}{(N-2)(N-3)} \frac{\frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi})^4}{\left[\frac{1}{N} \sum_{i=1}^N (\phi_i - \bar{\phi})^2 \right]^2}
\end{aligned} \tag{A.9}$$

Appendix B

Turbulence Modeling

Turbulence modeling is required to solve the transport equation in a RANS simulation. The model used in the RANS calculations was the 1-equation SA-RC-QCR2013 model. This model solves for $\tilde{\nu}$, a turbulence working variable that is also referred to as the undamped eddy viscosity. The overarching equation for $\tilde{\nu}$ is described by Eq. [B.1](#).

$$\begin{aligned} \frac{\partial u_j \tilde{\nu}}{\partial x_j} &= c_{b1} f_{r1} (1 - f_{t2}) \tilde{S} \tilde{\nu} + \frac{1}{\sigma} \{ \nabla \cdot [(\nu + \tilde{\nu}) \nabla \tilde{\nu}] + c_{b2} (\nabla \tilde{\nu})^2 \} - \left(c_{w1} f_w - \frac{c_{b1}}{\kappa^2} f_{t2} \right) \left(\frac{\tilde{\nu}}{d} \right)^2 \\ \text{where } \frac{\mu_t}{\rho} &= \nu_t = \tilde{\nu} f_{\nu 1} \\ f_{\nu 1} &= \frac{\chi^3}{\chi^3 + c_{\nu 1}^3} \\ \chi &= \frac{\tilde{\nu}}{\nu} \end{aligned} \tag{B.1}$$

Here, f_{r1} is the rotation function used in the RC model, and is defined in Eq. [B.2](#).

$$\begin{aligned}
f_{r1} &= (1 + c_{r1}) \frac{2r^*}{1 + r^*} [1 - c_{r3} \tan^{-1}(c_{r2}\tilde{r})] - c_{r1} \\
\text{where } r^* &= \frac{\hat{S}}{\omega} \\
\tilde{r} &= \frac{2\omega_{ik}\hat{S}_{jk}}{D^4} \left[\frac{D\hat{S}_{ij}}{Dt} + (\epsilon_{imn}\hat{S}_{jn} + \epsilon_{jmn}\hat{S}_{in})\Omega_m \right] \\
\frac{D\hat{S}_{ij}}{Dt} &= \frac{\partial\hat{S}_{ij}}{\partial t} + u_k \frac{\partial\hat{S}_{ij}}{\partial x_k} \\
\hat{S}_{ij} &= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \\
\omega_{ij} &= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) + \epsilon_{mji}\Omega_m \\
\hat{S}^2 &= 2\hat{S}_{ij}\hat{S}_{ij} \\
\omega^2 &= 2\omega_{ij}\omega_{ij} \\
D^2 &= \frac{1}{2} (\hat{S}^2 + \omega^2)
\end{aligned} \tag{B.2}$$

Further, f_{t2} is a sink term that allows for laminar-turbulent transition, and is defined in Eq. [B.3](#).

$$f_{t2} = c_{t3} \exp(-c_{t4}\chi^2) \tag{B.3}$$

Next, the modified vorticity rate, $\tilde{S}$, is defined in Eq. [B.4](#).

$$\begin{aligned}
\tilde{S} &= \omega + \frac{\tilde{\nu}}{(\kappa d)^2} f_{\nu 2} \\
\text{where } f_{\nu 2} &= 1 - \frac{\chi}{1 + \chi f_{\nu 1}}
\end{aligned} \tag{B.4}$$

The final sink term, f_w , is defined in Eq. [B.5](#).

$$\begin{aligned}
f_w &= g \left(\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right)^{1/6} \\
\text{where } g &= r [1 + c_{w2} (r^5 - 1)] \\
r &= \min \left(\frac{\tilde{\nu}}{\tilde{S}(\kappa d)^2}, 10 \right)
\end{aligned} \tag{B.5}$$

Instead of using the linear Boussinesq relation, a modified Reynolds stress term, $\tau_{ij,\text{QCR}}$, is added and used by the QCR model, and it is defined in Eq. B.6.

$$\tau_{ij,\text{QCR}} = \tau_{ij} - c_{cr1} (O_{ik}\tau_{jk} - O_{jk}\tau_{ik}) - c_{cr2}\nu_t \sqrt{\frac{S_{mn}S_{mn}}{2}}\delta_{ij}$$

$$\text{where } O_{ik} = \frac{\frac{\partial u_i}{\partial x_k} - \frac{\partial u_k}{\partial x_i}}{\sqrt{\frac{\partial u_m}{\partial x_n} \frac{\partial u_m}{\partial x_n}}}$$
(B.6)

The model constants are defined in Table B.1, where each row represents constants first found in Eq. B.1–B.6, respectively, with Eq. B.4 not requiring a new model constant.

In the above, ν is the molecular kinematic viscosity, κ is the von Kármán constant, d is the distance to the nearest wall, μ_t is the turbulent eddy viscosity, ν_t is the kinematic eddy viscosity, S_{ij} is the mean strain tensor, ω_{ij} is the mean vorticity magnitude, and Ω is the system rotation vector. As expected for Einstein notation, δ_{ij} and ϵ_{ijk} are the Kronecker delta and the Levi–Civita symbol, respectively. Additional variables that have not been named are used for intermediary calculations. No wall functions were used in the present simulations, although CFD++ does include models for this; hence, they are omitted from the discussion. Lastly, the boundary condition at the wall is defined in Eq. B.7.

$$\tilde{\nu} \Big|_{d=0} = 0$$
(B.7)

When the turbulence model is active, the viscosity and thermal conductivity used in Eq. 3.8 and Eq. 3.9 will be modified to include turbulent terms, as shown in Eq. B.8 and Eq. B.9, respectively.

$$\mu \Rightarrow \mu + \mu_t$$
(B.8)

$$k \Rightarrow k + k_t$$
(B.9)

Table B.1: Model constants for Spalart-Allmaras turbulence model with rotation-curvature correction and quadratic constitutive relations (2013).

Eq. B.1 :	$c_{\nu 1} = 7.1$	$c_{b1} = 0.1355$	$\sigma = \frac{2}{3}$	$c_{b2} = 0.622$	$\kappa = 0.41$	$c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1+c_{b2}}{\sigma}$
Eq. B.2 :	$c_{r1} = 1.0$	$c_{r2} = 12.0$	$c_{r3} = 1.0$			
Eq. B.3 :	$c_{t3} = 1.1$	$c_{t4} = 2$				
Eq. B.4 :	-					
Eq. B.5 :	$c_{w2} = 0.3$	$c_{w3} = 2$				
Eq. B.6 :	$c_{cr1} = 0.3$	$c_{cr2} = 2.5$				

Appendix C

Overflow Non-Dimensionalization

Flow and working variables in the NASA Langley Research Center code Overflow are non-dimensionalized, similar to the codes F3D and ARC3D. For an arbitrary variable ϕ , the non-dimensionalized variable is denoted by ϕ^* . In the present simulations, $L = 1$ m.

$$x^* = \frac{x}{L} \quad (\text{C.1})$$

$$\vec{V}^* = \frac{\vec{V}}{a_\infty} \quad (\text{C.2})$$

$$t^* = \frac{tu_\infty}{L} \quad (\text{C.3})$$

$$\rho^* = \frac{\rho}{\rho_\infty} \quad (\text{C.4})$$

$$P^* = \frac{P}{\rho_\infty a_\infty^2} = \frac{P}{\gamma_\infty P_\infty} \quad (\text{C.5})$$

$$T^* = \frac{TR_\infty}{a_\infty^2} = \frac{T}{\gamma_\infty T_\infty} \quad (\text{C.6})$$

$$R^* = \frac{R}{R_\infty} \quad (\text{C.7})$$

$$c_p^* = \frac{c_p}{R_\infty} \quad (\text{C.8})$$

$$c_v^* = \frac{c_v}{R_\infty} \quad (\text{C.9})$$

$$e^* = \frac{e}{a_\infty^2} \quad (\text{C.10})$$

$$\mu^* = \frac{\mu}{\mu_\infty} \quad (\text{C.11})$$

$$k^* = \frac{k}{k_\infty} \quad (\text{C.12})$$

$$S^* = \frac{S}{T_\infty} \quad (\text{C.13})$$

$$\nabla^* = L\nabla \quad (\text{C.14})$$

Appendix D

Literature Data Collection

The data collected from the archival literature, listed in Table [D.1](#), were selected based on two criteria: the cylinder or blunt fin must be sufficiently tall to meet the asymptotic height of $h/d \gtrsim 2.4$; and the separation distance must have been obtained using oil flow visualization, or in the case of in-house collected data, using high-speed schlieren imaging. Some data appear repeated but are separate runs.

Table D.1: Compilation of flow conditions and separation distance from the archival literature.

Author	Ref.	M_∞	$\text{Re}_\infty [\text{m}^{-1}]$	$d [\text{mm}]$	$\delta [\text{mm}]$	d/δ	h/d	S/d	S/δ
Burbank et al.	[27]	2.65	1.15×10^7	71.12	152.4	0.47	4.46	3.14	1.47
		3.51	1.15×10^7	71.12	152.4	0.47	4.46	2.11	0.98
		3.51	1.15×10^7	71.12	152.4	0.47	4.46	2.78	1.30
		4.44	1.15×10^7	71.12	152.4	0.47	4.46	2.63	1.12
Price and Stallings	[19]	3.71	9.80×10^6	50.8	152.4	0.33	5.00	2.76	0.92
Westkaemper	[166]	4.91	3.78×10^7	6.35	6.35	1.00	6.00	2.90	2.90
		4.91	3.78×10^7	6.35	6.35	1.00	4.00	2.85	2.85
		4.91	3.78×10^7	11.43	6.35	1.80	3.33	2.71	4.88
		4.91	3.78×10^7	11.43	6.35	1.80	3.33	2.81	5.06
		4.91	3.78×10^7	15.24	6.35	2.40	2.50	2.52	6.05
		4.91	3.78×10^7	15.24	6.35	2.40	2.50	2.63	6.31
Young et al.	[22]	3.02	3.24×10^7	19.05	1.96	9.72	4.00	1.80	17.49
		3.04	3.15×10^7	9.525	1.97	4.84	8.00	2.10	10.15
		3.02	3.15×10^7	3.175	1.97	1.61	24.00	1.80	2.90
		3.01	3.20×10^7	19.05	2.55	7.47	4.00	2.00	14.94
		3.02	3.26×10^7	9.525	2.54	3.75	8.00	2.00	7.50
Mashburn	[20]	4.81	4.23×10^7	6.35	6.35	1.00	3.00	3.00	3.00
		4.81	4.23×10^7	6.35	6.35	1.00	4.00	3.00	3.00
		4.81	4.23×10^7	6.35	6.35	1.00	4.00	3.25	3.25

Table D.1: Continued

Author	Ref.	M_∞	$\text{Re}_\infty [\text{m}^{-1}]$	$d [\text{mm}]$	$\delta [\text{mm}]$	d/δ	h/d	S/d	S/δ
Lucas	[46]	4.81	4.23×10^7	6.35	6.35	1.00	5.00	3.25	3.25
		4.81	4.23×10^7	6.35	6.35	1.00	6.00	3.00	3.00
		4.81	4.23×10^7	6.35	6.35	1.00	6.00	3.25	3.25
		4.81	4.23×10^7	7.62	6.35	1.20	6.00	3.04	3.65
		4.81	4.23×10^7	11.43	6.35	1.80	6.00	2.97	5.35
		4.81	4.23×10^7	15.24	6.35	2.40	6.00	2.83	6.79
		2.5	3.12×10^7	19.1	2.41	7.89	4.00	2.01	15.89
		2.99	3.17×10^7	25.4	2.59	9.80	2.50	1.84	18.04
		2.99	3.17×10^7	25.4	2.59	9.80	2.50	1.84	18.04
		3.0	3.17×10^7	25.4	2.59	9.80	2.50	1.80	17.65
Kaufman et al.	[23]	2.5	3.12×10^7	9.525	2.41	3.95	8.00	2.08	8.21
		3.0	3.17×10^7	9.525	2.59	3.68	8.00	2.00	7.35
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	2.00	14.71
		2.5	3.12×10^7	19.05	2.41	7.89	4.00	1.95	15.39
		3.0	3.17×10^7	25.4	2.59	9.80	2.50	1.82	17.84
		4.0	1.87×10^7	19.05	2.77	6.88	4.00	1.86	12.80
		2.5	3.12×10^7	9.525	2.41	3.95	8.00	2.02	7.97
		2.5	3.12×10^7	9.525	2.41	3.95	8.00	2.03	8.01
		2.5	3.12×10^7	19.05	2.41	7.89	4.00	2.10	16.58

Table D.1: Continued

Author	Ref.	M_∞	$\text{Re}_\infty [\text{m}^{-1}]$	$d [\text{mm}]$	$\delta [\text{mm}]$	d/δ	h/d	S/d	S/δ
		2.5	3.12×10^7	19.05	2.41	7.89	4.00	2.36	18.63
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	1.95	14.34
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	2.10	15.44
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	1.93	14.19
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	2.03	14.93
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	1.93	14.19
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	2.00	14.71
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	1.92	14.12
		3.0	3.17×10^7	19.05	2.59	7.35	4.00	2.26	16.62
		3.0	3.17×10^7	25.4	2.59	9.80	2.50	2.02	19.80
		3.0	3.17×10^7	25.4	2.59	9.80	2.50	1.93	18.92
		3.0	3.17×10^7	25.4	2.59	9.80	2.50	2.02	19.80
		3.0	3.17×10^7	25.4	2.59	9.80	2.50	2.03	19.90
		3.0	3.17×10^7	25.4	2.59	9.80	2.50	2.11	20.69
		4.0	1.87×10^7	9.525	2.77	3.44	8.00	2.23	7.67
Winkelmann	[29]	4.93	2.43×10^7	12.37	66.04	0.19	12.30	2.80	0.52
Sedney and Kitchens	[18]	2.5	9.80×10^6	19.1	22.50	0.85	5.32	1.93	1.64
		2.5	9.80×10^6	38.1	22.50	1.69	2.67	1.93	3.27
		3.5	9.80×10^6	19.1	27.50	0.69	5.32	2.47	1.72

Table D.1: Continued

Author	Ref.	M_∞	$\text{Re}_\infty \text{ [m}^{-1}\text{]}$	$d \text{ [mm]}$	$\delta \text{ [mm]}$	d/δ	h/d	S/d	S/δ
		3.5	9.80×10^6	38.1	27.50	1.39	2.67	2.55	3.53
		2.5	9.80×10^6	19.1	22.50	0.85	5.32	1.94	1.65
		3.5	9.80×10^6	19.1	27.50	0.69	5.32	2.45	1.70
		2.5	9.80×10^6	38.1	22.50	1.69	2.67	1.99	3.37
		3.5	9.80×10^6	38.1	27.50	1.39	2.67	2.51	3.48
		4.0	9.80×10^6	38.1	32.25	1.18	2.67	2.76	3.26
		4.5	9.80×10^6	38.1	37.00	1.03	2.67	3.13	3.22
		2.5	2.97×10^6	38.1	24.44	1.56	2.67	2.27	3.54
		2.5	3.10×10^6	38.1	24.44	1.56	2.67	2.21	3.45
		2.5	3.60×10^6	38.1	24.07	1.58	2.67	2.27	3.59
		2.5	4.91×10^6	38.1	23.70	1.61	2.67	2.01	3.23
		2.5	6.06×10^6	38.1	23.70	1.61	2.67	2.00	3.22
		2.5	9.27×10^6	38.1	22.60	1.69	2.67	2.04	3.44
		2.5	1.96×10^7	38.1	21.50	1.77	2.67	1.97	3.49
		3.5	2.06×10^6	38.1	36.67	1.04	2.67	2.43	2.52
		3.5	2.19×10^6	38.1	36.67	1.04	2.67	2.49	2.59
		3.5	2.83×10^6	38.1	35.93	1.06	2.67	2.50	2.65
		3.5	3.60×10^6	38.1	35.20	1.08	2.67	2.53	2.74
		3.5	4.65×10^6	38.1	34.07	1.12	2.67	2.59	2.90

Table D.1: Continued

Author	Ref.	M_∞	$\text{Re}_\infty [\text{m}^{-1}]$	$d [\text{mm}]$	$\delta [\text{mm}]$	d/δ	h/d	S/d	S/δ
		3.5	5.67×10^6	38.1	32.96	1.16	2.67	2.62	3.03
		3.5	6.06×10^6	38.1	32.22	1.18	2.67	2.60	3.07
		3.5	6.19×10^6	38.1	32.22	1.18	2.67	2.62	3.10
		3.5	6.43×10^6	38.1	31.85	1.20	2.67	2.61	3.12
		3.5	7.09×10^6	38.1	31.11	1.22	2.67	2.60	3.18
		3.5	7.09×10^6	38.1	31.11	1.22	2.67	2.65	3.25
		3.5	1.10×10^7	38.1	27.00	1.41	2.67	2.57	3.63
		3.5	1.10×10^7	38.1	27.00	1.41	2.67	2.64	3.73
		3.5	1.98×10^7	38.1	25.56	1.49	2.67	2.33	3.47
Saida	[30]	2.48	4.00×10^7	10.0	7.30	1.37	3.50	2.47	3.38
Dolling and Narlo	[70]	4.96	5.50×10^6	12.7	5.00	2.54	4.00	2.60	6.60
		4.96	5.50×10^6	19.05	5.00	3.81	4.00	2.42	9.22
Fomison and Stollery	[24]	2.4	2.60×10^6	6.4	15.20	0.42	23.81	2.56	1.08
		2.4	2.60×10^6	9.5	15.20	0.63	13.37	2.52	1.58
		2.4	2.60×10^6	12.7	15.20	0.84	9.00	2.48	2.07
		2.4	2.60×10^6	19.1	15.20	1.26	4.65	2.28	2.87
		2.4	2.60×10^6	25.4	15.20	1.67	3.50	2.24	3.74
Barnhart and Greber	[26]	1.97	1.70×10^7	25.4	28.10	0.90	5.98	2.25	2.03
		2.94	1.50×10^7	25.4	30.30	0.84	5.98	2.38	2.00

Table D.1: Continued

Author	Ref.	M_∞	$\text{Re}_\infty \text{ [m}^{-1}\text{]}$	$d \text{ [mm]}$	$\delta \text{ [mm]}$	d/δ	h/d	S/d	S/δ
Stephen et al.	[167]	3.94	1.41×10^7	25.4	36.20	0.70	5.98	2.73	1.92
		4.99	1.39×10^7	25.4	38.90	0.65	5.98	2.94	1.92
		3.0	3.50×10^6	2.0	3.00	0.67	3.00	2.69	1.79
		3.0	3.50×10^6	2.0	3.00	0.67	3.00	2.85	1.90
		3.0	3.50×10^6	2.0	3.00	0.67	3.00	2.98	1.99
Lash et al.	[114]	1.91	2.87×10^7	3.175	1.15	2.76	14.56	2.45	6.76
Lash et al.	[108]	1.80	2.87×10^7	3.175	1.15	2.76	14.56	2.50	6.90

Vita

Stefen A. Lindörfer was born in El Paso, TX, to the parents of Dr. med. Hans W. Lindörfer and Maria D. Dellaplain. He was raised in Gütersloh, Germany, where he attended the Evangelisch Stiftisches Gymnasium Gütersloh. During high school, he moved to Plano, TX, where he attended and graduated from Plano East Senior High School. Seeking out a challenging and inspiring field, Stefen went on to study Aeronautical and Astronautical Engineering at Purdue University. After four years, he obtained his Bachelor of Science degree with a minor in Mathematics in May 2014 from Purdue. He accepted a graduate research assistant position at the University of Tennessee Space Institute in Aerospace Engineering, providing computational support to the group of Dr. John D. Schmisser. After two years, Stefen graduated with his Master of Science degree in May 2017. He stayed on as a full-time doctoral student until October 2018, after which he began working as a CFD specialist at National Aerospace Solutions, LLC. at the Arnold Engineering Development Complex at Arnold Air Force Base. He remained a part-time student until graduation, having additionally worked for Calspan Corporation and Stratolaunch, LLC.

Stefen is a third-generation doctor. He also traces his academic genealogy as follows:

Gottfried W. Leibniz (Universität Leipzig, 1666)

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Jacob Bernoulli (Universität Basel, 1684)

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Johann Bernoulli (Universität Basel, 1694)

↓

Leonhard Euler (Universität Basel, 1726)

↓

Joseph-Louis Lagrange (Università di Torino, 1754)

↓

J.-B. Joseph Fourier (École Normale Supérieure, 1795)

↓

Gustav P. L. Dirichlet (Rheinische Friedrich-Wilhelms-Universität Bonn, 1827)

↓

Rudolf O. S. Lipschitz (Universität Berlin, 1853)

↓

Felix C. Klein (Rheinische Friedrich-Wilhelms-Universität Bonn, 1868)

↓

C. L. Ferdinand von Lindemann (Friedrich-Alexander-Universität Erlangen-Nürnberg, 1873)

↓

Emil Hilb (Ludwig-Maximilians-Universität München, 1903)

↓

Richard Bär (Bayerische Julius-Maximilians-Universität Würzburg, 1916)

↓

Hans W. Liepmann (Universität Zürich, 1938)

↓

Steven P. Schneider (California Institute of Technology, 1989)

↓

John D. Schmisser (Purdue University, 1997)

↓

Stefen A. Lindörfer (University of Tennessee, 2024)

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