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I am submitting herewith a dissertation written by Mahmoud PourArsalan entitled “Secondary Light Particle Data Base Development Using a Thermodynamic Coalescence Model”. I have examined the final paper copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Nuclear Engineering.

---

Lawrence W. Townsend, Major Professor

We have read this dissertation and  
recommend its acceptance:

---

Dr. Ivan Maldonado

---

Dr. Laurence F. Miller

---

Dr. Thomas Handler

Acceptance for the Council:

---

Carolyn R. Hodges  
Vice Provost and Dean of Graduate Studies

(Original signatures are on file with official student records.)

**Secondary Light Particle Data Base Development Using a  
Thermodynamic Coalescence Model**

**A Dissertation**

**Presented for partial fulfillment of the**

**Doctor of Philosophy  
Degree**

**The University of Tennessee, Knoxville**

**Mahmoud PourArsalan**

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## **ABSTRACT**

As heavy ions are transported through shielding and interact with shielding materials accurate values of total, elastic scattering, reactions cross sections and angular distributions of the emitted nucleons, light high energy particles such as deuteron, triton, helion, alpha particles and other heavy ions are required in order to design appropriate and adequate shielding to protect the human crews and instruments from ionizing radiations during long duration space missions. Double-differential (energy and angle) light energetic particle production cross sections must be known for ion energies from tens of MeV/nucleon to tens of GeV/nucleon for all emitted light energetic particles for space radiation protection studies. A coalescence thermodynamics model is used to estimate the coalescence radii and emitting source radii for both symmetric and asymmetric systems of heavy ion collisions in order to evaluate emitted light particle double differential cross sections.

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# 1- INTRODUCTION

The NUCFRG series of nuclear fragmentation codes [1-4] were developed at NASA Langley Research Center in the mid 1980's in order to generate nuclear fragmentation cross section data bases for Galactic Cosmic Ray (GCR) transport codes and identify and design adequate shielding to protect human crews and instruments. Such cross section databases are essential for space radiation protection studies for future long duration missions to Mars and beyond. NUCFRG codes use an abrasion-ablation model of heavy ion fragmentation [5] to give reasonable estimates of isotopic production cross sections comparable to the experimentally measured isotopic production cross sections. High energy GCR can penetrate deep into thick shields and, when undergoing head on collisions with shielding materials, can emit high energy light particles, mainly hydrogen and helium isotopes, gamma rays and neutrons. The secondary high-energy light particles penetrate deeper into the shielding over most of the forward hemisphere and may significantly contribute to radiation exposure. Incorporating high energy light particle spectral and angular distribution cross section databases, in the transport codes enable them to transport nearly any radiation field, in three dimensions, that humans and instruments might be exposed to in space, near accelerators or during charged particle radiotherapy.

NUCFRG2 [2] uses some assumed parameters, such as a coalescence radius in momentum space and the multiplicities of the emitted light energetic particles emitted, that require further study. The coalescence model results need to be compared and validated with published experimental results, but there are a limited number of light ion production data sets

available from published experiments, especially spectral and angular distributions for emitted high energy secondary light particles in relativistic heavy ion collisions, with which to do these comparisons.

In the abrasion-ablation model of heavy ion fragmentation (see Figure 1), the participant region is roughly spherical, extremely hot and compressed, expands violently outwardly and in the process cools off. It then becomes less dense and emits high energy light particles (deuterons, tritons, helions, and alpha particles) in the spherical regions where the relative momentum of the nucleons is less than the coalescence radius. A slight perturbation will be sufficient to have them coalesce into high-energy light particles. Such a perturbation can arise, for example, from the average potential generated by other particles in their vicinity. High-energy light particles, which are emitted from participant regions, are detected at wider angles (over most of the forward hemisphere). High-energy light particles from the excited projectile spectator are emitted more in the forward direction. Low energy light particles from the target spectator are emitted isotropically. Compound nucleus emission cannot explain such trends and suggest emission from a source, which consists of comparable distributions from both projectile and target. The expansion, cooling and high-energy secondary light particle emissions continue until the system reaches the freeze out radius where nucleons fly outward freely and coalescence ceases. Most of the light energetic particles are emitted as the hot and dense source region forms and starts expanding. Very few light ions are produced near freeze out. Alpha particles may be emitted at the early stages of expansion because alphas have a high binding energy (28.3 MeV), while the deuterons with much lower binding energy (2.2 MeV) may be emitted in all stages of

the expansion. Deuterons emitted in the early stage of the expansion, may be transmuted into heavier particles or break up into nucleons [7].

Figure 2 shows the creation of the participant region right after the collision (region with radius A) to a thermodynamic model radius (region with radius B) where chemical and thermal equilibrium are maintained, and finally the freeze out radius (region with radius C) where coalescence ceases and nucleons fly freely outwardly.

The energy, projectile and target dependent probability of the high energy secondary light particle emissions and the radius of the region emitting these secondary light particles can be related with a coalescence model [7-9]. The coalescence model is a pure phase space approach and makes no assumptions about the dynamics of the emitting source region. It involves a single free parameter namely the coalescence radius  $P_0$ , which is the radius of the sphere in momentum space within which nucleons that are spatially close together coalesce to form high-energy secondary light particles. After the participant region reaches the freeze out radius, nucleons will not coalesce anymore and nucleons then freely fly outward.

The thermodynamic coalescence model [7, 9] is based on the assumption that any two nucleons with relative momentum less than the coalescence radius, coalesce to form a deuteron, any three nucleons within a different coalescence radius in momentum space form a triton and so forth. In the thermodynamic model, the invariant phase space density of the formed light particles  $d^3N_A/d^3p_A$  (number of nuclei per event per unit element of phase space) and the phase space proton density  $d^3N_p/d^3p$  (number of protons per event per unit element of phase space)

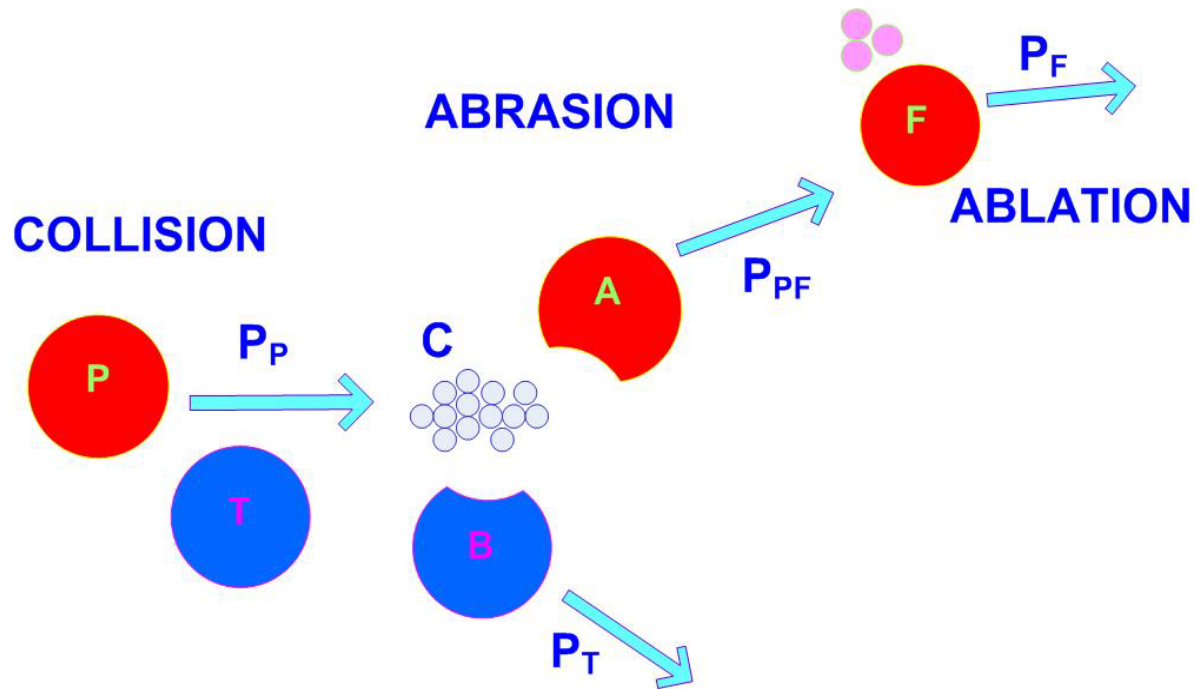


**Figure 1** Schematic of abrasion-ablation model and the corresponding momentum distributions at different steps. The two-step model is shown as projectile **P** colliding with target **T** in the first step. Consequently, the system partitions into projectile prefragment (**A**), target spectator (**B**) and participants (**C**). The pre-fragment then decays to the ground state by emitting secondary particles and gamma rays. The figure is adapted from Giacomelli et. al [6]

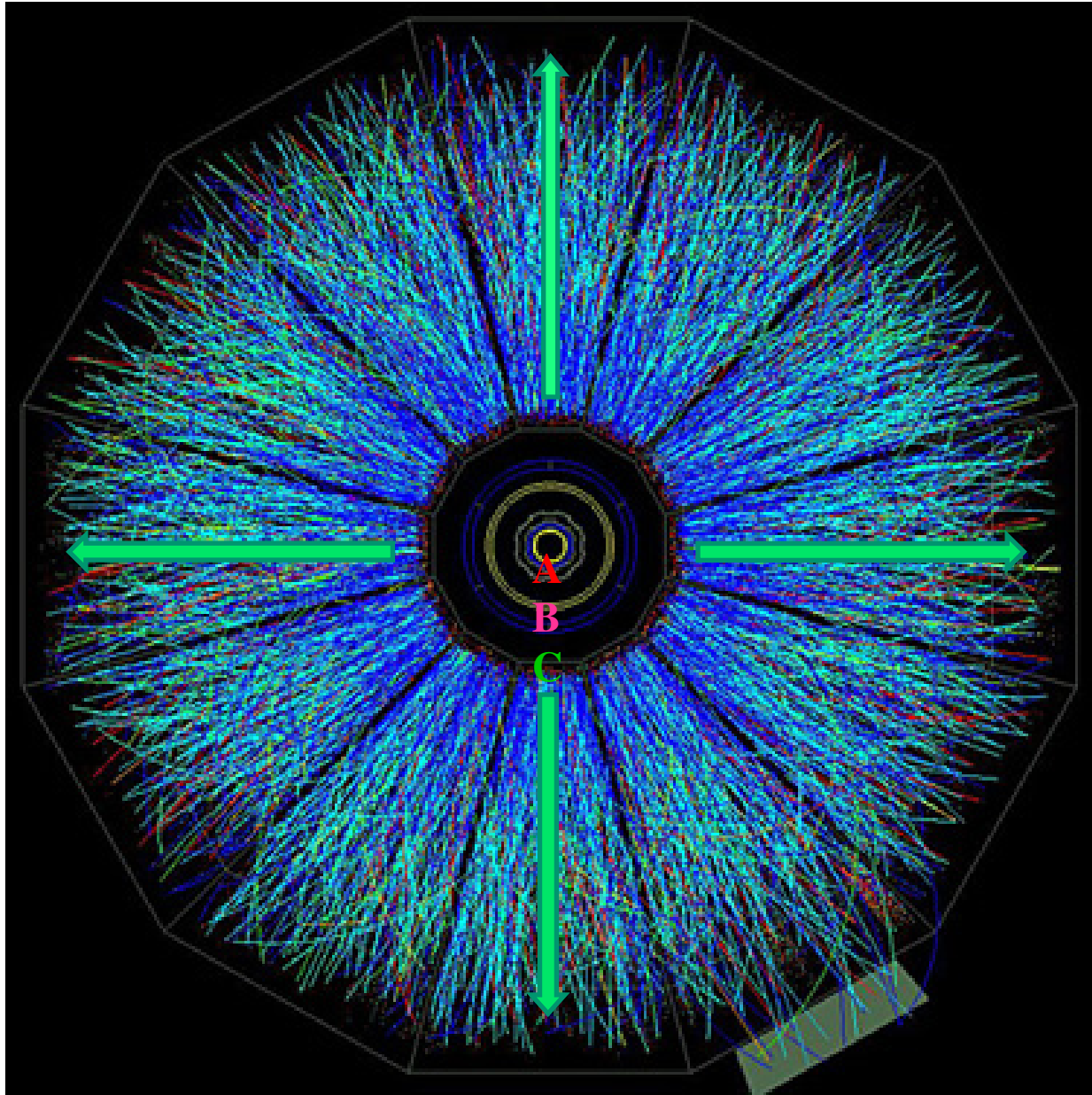


**Figure 2** This figure, adapted from the Brookhaven Laboratory web site (<http://www.bnl.gov/rhic/images.asp>), shows an end view of a collision of two 30-billion electron-volt gold beams in the STAR detector at the Relativistic Heavy Ion Collider at Brookhaven National Laboratory. The beams travel in opposite directions at nearly the speed of light before colliding. The different radii are not to scale. They only illustrate the **radius for right after the collision (A)**, the **largest average radius where thermal and chemical equilibrium is maintained (B)**, and the **freeze out radius (C)**. Beyond the freeze out radius, nucleons fly freely outwardly without interacting.

can be related to the volume  $V$  of the sphere with average radius  $R$ , where chemical and thermal equilibrium of the emitting source region is maintained. The volume,  $V$ , is inversely related to the coalescence radius  $P_0$ . Beginning in the early 1970's, thermodynamic coalescence models were used to estimate the emitting source size. These emitting source radius estimates were reasonable and much simpler to calculate than using other methods, such as the Hanbury, Brown and Twist method (HBT method) [10-14].

A power law formula connects the secondary light particle emission cross section with the proton cross section raised to the  $A^{\text{th}}$  power where  $A$  is the mass number of the secondary light particle [7, 9, 15-17]. The power law for the thermodynamic coalescence model [7-9] is presented in section 4.3.

The thermodynamic coalescence model has been used more frequently than other models because its calculated results are more often in agreement with the published experimental data. Despite its simplicity the model appears to be capable of predicting a wide spectrum of experimental data.

Are chemical and thermal equilibrium of the emitting source region reasonable assumptions? A characteristic time scale for the collisions is the mean time it takes a nucleon to cross the nucleus, which is on the order of  $3 \pm 2 \times 10^{-23}$  s. We compare the various reaction rates for the production of light particles to the emitting source region expansion rate, which is the reciprocal of this transit time, i.e.,  $\sim 10^{22}$ /s. Consequently, an equilibrium concentration is

achieved in the initial stages of the expansion when almost all of the high energy secondary light particles are emitted. This conclusion is also confirmed in the detailed calculation by Kapusta et. al [18]. The same arguments can be made for the formation of other light particles and it will be found that, they are also in chemical and thermal equilibrium [7]. Volume in the coalescence thermodynamic model,  $V$ , reflects the largest volume where chemical and thermal equilibrium is maintained and is different for different projectile-target systems. Volume in the thermodynamic model is larger than the pre-emitting volume (emitting source region right after collision) and smaller than the freeze-out volume.

If the emitted, high-energy secondary light particle inclusive spectra and the proton inclusive spectra are measured experimentally for any heavy ion collision system, the coalescence radius can be calculated using a simple coalescence model (See section 4). The coalescence radius can also be estimated analytically using a thermodynamic coalescence model as described in section 4.

The volume of the emitting light particle is inversely related to the cube of the coalescence radius. The source size of the nearly spherical region emitting the high-energy secondary light ions can be estimated. However only limited number of experimentally measured multiplicities for the emitted light particles for heavy ion collision systems is available. Therefore, there exists a very limited number of coalescence radius estimates.

**We are attacking the problem from the opposite direction. Rather than using experimental cross sections to extract  $P_0$  using a coalescence model to estimate the**

coalescence radius, we develop a method for estimating the emission source radius for a wide range of relativistic heavy ion collision systems. We then use them to estimate the coalescence radius, which in turn is used to calculate spectral and angular cross sections for the emitted high energy secondary light particles in the relativistic heavy ion collision systems for both symmetric and asymmetric systems from tens of MeV/nucleon to tens of GeV/nucleon.

The emitting source radius increases with increasing system size. This system size dependence is expected from the larger volume due to a larger number of participants in the central source region. With increasing beam energy, the source radius does not change appreciably, as expected.

In the coalescence model, high-energy secondary light particles are emitted into the forward hemisphere mainly from the participant region. Therefore, we will estimate the average participant mass contribution from the projectile and target [19, 20]. Then we can use the average participant mass to estimate the emitted secondary light particle source radius.

Therefore, the main objective of our study is to develop a methodology that can be used to calculate spectral and angular cross section estimates of emitted high energy secondary light particles using an analytical thermodynamic coalescence model. Emitting source radii are used to estimate coalescence radii for light energetic particles, from many sets of heavy-ion systems, using the model for both asymmetric and symmetric systems for kinetic energies from tens of MeV/nucleon to tens of GeV/nucleon.

The outline of the paper is as follows: In section 2, justification and originality of this work is presented. The abrasion-ablation model, and NUCFRG models and codes are explained in section 3. A complete historical overview and the different coalescence models are discussed in section 4. Then, we present the models used to estimate the coalescence radius using a thermodynamic coalescence model and the source radius in section 5. In section 6, the “sourceRadius” and “CoalescenceRadius” subroutines are explained. These FORTRAN subroutines are listed in Appendices I.2 and I.3 respectively. Our source radii estimates and coalescence radii are presented and discussed in section 7 and are also compared with results from other analyses. We also estimate some total light ion production cross sections to show the capabilities of the methodology. In section 8, a summary of this work and suggested future work are presented. Appendices are listed in section 9 and finally section 10 has list of the reference.

## 2- JUSTIFICATION AND ORIGINALITY

In space radiation protection studies, as heavy ion radiation is transported through the shielding materials, accurate values of the total, elastic scattering and reaction cross sections must be known. The primary (incident) heavy ions are transported mainly in the extreme forward direction through the shielding materials. Therefore total cross sections are used in the heavy ion transport calculations (1-D problem). Some heavy ions collide head on with shielding materials as they move through a shield and generate neutrons, high energy secondary light particles (mainly hydrogen and helium isotopes) that are emitted at broader forward angles. In the abrasion-ablation model of heavy ion fragmentation, high-energy secondary light particles ( $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$  and  $^4\text{He}$ ) are emitted from a nearly spherical participant region in the forward hemisphere. Therefore angular cross sections for those light particles are required in the heavy ion transport calculations (3-D problem). There are limited numbers of experimentally measured light particle cross section and multiplicities available in the published literature. In order to transport these secondary light particles through shielding materials in three spatial dimensions we need to theoretically estimate angular distribution cross sections for emitted high-energy secondary light particles for a wide range of collision systems. Such angular cross sections enable us to perform secondary, light particle, high-energy transport calculations that human and instruments might be exposed to in low earth orbit or deep space during long duration space flights.

The new and innovative feature of this work is that we analytically estimate the coalescence radius for a wide range of heavy ion collision systems, for both symmetric and asymmetric collision systems, using a thermodynamic coalescence model and program it as a FORTRAN

subroutine. Such a FORTRAN subroutine can then be integrated into the recently released NUCFRG3 code [21] used by NASA to generate nuclear interaction databases for use in space radiation protection studies.

### **3- ABRASION-ABLATION MODEL AND NUCFRG MODELS AND CODES**

#### **3.1 Abrasion-Ablation Model**

The abrasion-ablation model describes a two stage process that involves the collision between two relativistic heavy ions. In the first stage, the abrasion process, a relativistic projectile P, moving at an initial momentum,  $p$ , with respect to the stationary target T, collides with the target, and their overlapping volumes shear off. The sheared off volume has nucleon contributions from both projectile P and target T. It is called the participant region (C). In the second stage, the non-overlap piece of the projectile P, the projectile spectator, is in an excited state and continues in the direction of the pre-collision projectile at the essentially the same velocity as before the collision. The excited projectile and target spectators decay by emitting gamma rays and/or disintegrate into lighter particles and nucleons. The second stage is known as the ablation process. The abrasion-ablation process is schematically illustrated in Figure 1. Detailed literature search and detailed theoretical basis for the Abrasion, Ablation model are listed in [21].

#### **3.2 NUCFRG Models and Codes**

The NUCFRG series of nuclear fragmentation [3, 22] codes were developed at NASA Langley Research Center in the mid 1980's in order to generate nuclear fragmentation cross section data bases for Galactic Cosmic Ray (GCR) transport codes and identify and design adequate shielding to protect human crews and instruments. Such cross section databases are

essential for future long duration missions to Mars and beyond for space radiation protection studies. NUCFRG codes use an abrasion-ablation model [5] of heavy ion fragmentation and estimate isotopic production cross sections in reasonable agreement with the experimentally measured isotopic production cross section databases. High energy GCR can penetrate deep into thick shields. When head on collisions with nuclei in shielding materials occur, emit high energy secondary light particles, mainly hydrogen and helium isotopes and neutrons. The emitted secondary energetic light particles penetrate deeper into the shield over most of the forward hemisphere and may significantly contribute to radiation exposures. With a reliable high-energy, secondary, light particle spectral and angular distribution cross section database, transport codes are able to transport nearly any radiation field, in three spatial dimensions, that humans and instruments might be exposed to in space, near accelerators or during charged particle radiotherapy.

A coalescence model has been used to predict secondary light ion production in heavy ion collision systems for nearly fifty years. A simple version of the coalescence model that can be used to estimate light ion ( $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$  and  $^4\text{He}$ ) total yields (total cross section) was incorporated in NUCFRG2. The NUCFRG2 model [2, 3, 23] was based on the classical geometric overlapping approach presented in the abrasion-ablation model by Bowman et. al [5]. Several major improvements were included in the abrasion model used in NUCFRG over these earlier LBNL versions. An average transmission factor for the projectile and target nuclei was used at a given impact parameter to account for the finite mean free path in nuclear matter. Also, for use in the ablation stage of the reaction, a second order correction was implemented in the surface energy term used to estimate the prefragment excitation energies. A semi-empirical

higher-order correction for the surface deformation energy of the abrasion products was added. Electromagnetic dissociation effects in interaction of relativistic heavy ions, which are important for heavier systems and at high energies were added. The distributions of energy transfer across an interaction boundary and finally the Rudstam et. al [24] formalism for approximating the final charge distribution of the fragmentation products were also incorporated. The NUCFRG2 version by Wilson et. al [2] included additional improvements, such as a maximum transmission factor in the overlap region, pre-equilibrium emission of spectator nucleons in abraded fragments and finally a semi-empirical correction for distortion in the surface energy for light projectiles. The cross section predictions by Townsend and Cucinotta et. al [1] showed that using NUCFRG2 yielded general agreement within 25% with published experimental isotope production cross sections. There were also better agreements between predictions and measurements for elemental production cross-sections comparisons. Although, the NUCFRG2 version accurately accounts for all fragment yields with mass number  $A > 4$ , it poorly predicted the nuclear yields for light ion fragments [4]. There is a need to incorporate a correction for predicting production cross sections from the fragmentation of nuclei for mass numbers 2, 3 and 4 into the current model. Secondary high-energy light particles with mass numbers 2, 3 and 4 are emitted from both the projectile spectator in the forward direction and from participant region in the forward hemisphere by coalescence of the nucleons. This work estimates production by coalescence of high-energy secondary light particles emitted from the participant region. Since the NUCFRG2 code is based on a semi-empirical formulation, the resulting predictions are in the form of total yields and production cross sections. The model does not include the calculations of spectral or angular distributions for the secondary fragments. For future long duration space applications using three dimensional transport models, the contributions of secondary high-

energy heavy-ion particles and light ion particles at all angles are essential and a means of incorporating differential energy and angular production cross sections into the model is needed. Once our coalescence model is incorporated into the newly-released NUCFRG3 code [22], an improved method for predicting secondary light energetic particles emitted from participant region by coalescing nucleons will be fully accounted for.

## 4- COALESCENCE RADIUS ESTIMATES

### 4.1 Coalescence Formalism Overview

The concept of secondary light particle production by coalescence in high energy heavy ion collisions was proposed to account for the emission of deuterons in reactions induced with protons at energies of 25-30 GeV [25, 26]. Butler and Pearson et. al [25] have related the deuteron density in the momentum space to the square of the proton density in the momentum space. Schwartzschild and Zupancic et. al [26] extended that to secondary light particle emissions in nucleon-nucleus collisions. The concept was then extended to high-energy heavy ion collisions [15, 16, 27]. Gosset et. al [16] formulated their model for nucleus-nucleus collisions. Based on the theoretical formalism presented by both studies, Awes and others [27-30] have applied the coalescence model to the emission of high-energy secondary light particles in high-energy heavy ion collisions. The basic physical assumption of the coalescence model is that, during high-energy heavy ion collisions, secondary light particles, mainly hydrogen and helium isotopes, are produced by the coalescence of nucleons which happen to share the same spatial and momentum space volume. Such nucleons are produced in the abrasion-ablation model of heavy ion collision systems in the participant region (see Figure 1). The coalescence model is based in the idea that any two nucleons whose relative momenta are within the coalescence radius,  $P_0$ , of each other, coalesce to form a deuteron, any three nucleons within a different coalescence radius  $P_0$  form a triton and so forth. The coalescence model relates secondary light particle spectra to the  $A^{\text{th}}$  power of the proton spectrum where  $A$  is the secondary light particle mass number (see equation 4.2.1.1 or 4.2.2.1).

Awes et. al [30] introduced the coalescence model using a Poisson distribution to calculate the composition of the secondary light ions. In their work, the probability  $P$  for finding one primary nucleon in the coalescence volume centered at a momentum per nucleon  $\mathbf{p}$  is given by the product of this volume with the single nucleon momentum density

$$P = \frac{4\pi}{3} P_0^3 \frac{\gamma}{\bar{m}} \frac{d^3 N}{d p^3} \quad (4.1.1)$$

where  $d^3 N/d p^3$  denotes the differential nucleon multiplicity,  $P_0$  is the coalescence radius in (MeV/c),  $\bar{m}$  is the average nucleon multiplicity, and  $\gamma = 1+T/m$ . Assuming an independent probability for the observation of neutrons and protons, the average probability for finding  $N$  neutrons and  $Z$  protons in the coalescence volume is

$$\langle P(N, Z) \rangle = \frac{(\bar{m}_Z P_Z)^Z}{Z!} \frac{(\bar{m}_N P_N)^N}{N!}. \quad (4.1.2)$$

The parameter  $P(N, Z)$  in eq. (4.1.2) represents the probability of forming composite particles in the coalescence volume. The neutron distribution is calculated by multiplying the neutron to proton ratio  $(N_p + N_t)/(Z_p + Z_t)$  of the collision system by the proton distribution

$$\frac{d^3 N(0,1)}{d p^3} = \frac{N_p + N_t}{Z_p + Z_t} \frac{d^3 N(1,0)}{d p^3} \quad (4.1.3)$$

where  $N_p$  and  $N_t$  are the projectile and target neutron numbers, and  $Z_p$  and  $Z_t$  are the projectile and target charge numbers. Substituting eqs. (4.1.1) and (4.1.3) into eq. (4.1.2) and dividing by the coalescence volume, a power law for simple coalescence is given as

$$\frac{d^3 N_A(Z, n)}{d p_A^3} = \frac{1}{N! Z!} \left( \frac{N_p + N_t}{Z_p + Z_t} \right)^N \left( \frac{4\pi \gamma}{3} P_0^3 \right)^{A-1} \left[ \frac{d^3 N_p(1,0)}{d p^3} \right]^A \quad (4.1.4)$$

where  $Z$ ,  $N$ , and  $A$  are the secondary light particle charge, neutron number and mass, respectively.  $P_0$  is the radius of a sphere in momentum space where nucleons will form secondary light particles when their momentum lies within this sphere. Note that  $d^3N_p/dp^3$  and  $d^3N_A/dp^3$  are the differential nucleon multiplicity per event and light particle multiplicity per event respectively and are related to the nucleon momentum distribution and light particle momentum distribution by

$$\begin{aligned} \frac{d^3N_p}{dp^3} &= \frac{1}{\sigma_0} \frac{d^3\sigma}{dp^3} = \frac{1}{\sigma_0} \frac{d^3\sigma}{p^2 dp d\Omega} && \text{for protons and} \\ \frac{d^3N_A}{dp_A^3} &= \frac{1}{\sigma_0} \frac{d^3\sigma}{dp_A^3} = \frac{1}{\sigma_0} \frac{d^3\sigma}{p_A^2 dp_A d\Omega} && \text{for secondary light particles} \end{aligned} \quad (4.1.5)$$

where  $\sigma_0$  is the projectile-target total reaction cross section. It is also important to note that the evaluation of the neutron differential cross section and its momentum distribution occurs in the rest frame of the projectile. However, all differential cross sections are required to be evaluated in the laboratory rest frame or the center of mass frame. The Lorentz invariant differential cross section is related to the double differential cross section as

$$E_n \frac{d^3\sigma}{dp^3} = \frac{1}{p_n} \frac{d^2\sigma}{dE_n d\Omega_n} \quad (4.1.6)$$

where  $E_n$ ,  $p_n$ ,  $\Omega_n$  are kinetic energy, momentum, and direction of nucleon  $n$  in the laboratory rest frame. To transform the composite spectrum into the lab rest frame using the Lorentz invariant technique, eq. (4.1.5) is substituted into eq. (4.1.4), and then converted to the Lorentz invariant cross section of the composite system using eq. (4.1.6). The double differential momentum distribution is expressed as

$$\frac{d^2\sigma(Z, n)}{dE_n d\Omega} = R(Z, n) \frac{d^2\sigma(n)}{dE_n d\Omega} \quad (4.1.7)$$

where

$$R(Z, n) = \frac{A^3}{N!Z!} \left( \frac{N_p + N_T}{Z_p + Z_T} \right)^N \left( \frac{4\pi p_0^3}{3\sigma_0 p_n m} \right)^{A-1} \quad (4.1.8)$$

## 4.2 Coalescence Models

All commonly used coalescence models can be grouped into four different types in order to predict secondary light particle emission cross sections in the heavy ion collision systems [31].

A brief description of each group with their assumptions is written in the next 4 sub-sections.

### 4.2.1 The Empirical Nucleon Coalescence Model

The empirical nucleon coalescence model [7, 9] assumes that any two nucleons whose relative momentum is less than a coalescence radius (a constant value in momentum space),  $P_0$ , coalesce to form a deuteron while any three nucleons whose relative momentum is less than a different coalescence radius,  $P_0$ , coalesce to form triton and so on. This simple model implicitly assumes infinite source temperature, which makes this model unrealistic whenever the temperature of the participant source region that emits the high energy secondary light particles is not significantly more than the binding energy of the coalesced secondary light particles that are emitted. The multiplicity distribution of light ions with mass number  $A$  is given in terms of the nucleon multiplicity, which is scaled to the proton multiplicity, as

$$\frac{d^3 N_A}{dp^3} = \left( \frac{N_P + N_T}{Z_P + Z_T} \right)^N A^3 \left( \frac{4\pi}{3} P_0^3 \right)^{A-1} \left( \frac{d^3 N_P}{d^3 p} \right)^A \quad (4.2.1.1)$$

Equation 4.2.1.1 can be solved for the coalescence radius,  $P_0$ , to get equation 4.2.1.2 below. Such equation can be used to estimate the coalescence radius when experimentally measured secondary light particle and proton multiplicities are available.

$$P_0 = \left( \frac{3}{4\pi} \right)^{1/3} (A)^{\frac{1}{(A-1)}} \left( \frac{\frac{d^3 N_A}{dp^3}}{\left( \frac{N_P + N_T}{Z_P + Z_T} \right)^N \left( \frac{d^3 N_P}{d^3 p} \right)^A} \right)^{\frac{1}{3(A-1)}} \quad (4.2.1.2)$$

Where

$$\frac{d^3 N_A}{dp_A^3} = \text{Emitted light particle multiplicity;}$$

$$\frac{d^3 N_P}{d^3 p} = \text{Proton multiplicity}$$

$A$  = emitted light energetic particle mass number and

$$p_A = Ap$$

## 4.2.2 The Thermodynamic Nucleon Coalescence Model

The thermodynamic nucleon coalescence model [7-9], allows for a finite value of the source region temperature. After a particular stage in the evolution of the collision volume, the model assumes an abrupt transition from a strongly interacting system to a free-streaming one. In the thermodynamic coalescence model, the invariant phase space density of the formed light particles  $d^3 N_A / d^3 p_A$  (number of nuclei per event per unit element of phase space) and the phase

space proton density  $d^3N_p / d^3p$  (number of protons per event per unit element of phase space) can be related to the average volume  $V$  of the sphere with average radius  $R$ , where chemical and thermal equilibrium of the emitting source region are maintained. The volume,  $V$ , is inversely related to the cube of the coalescence radius  $P_0$ . It is assumed that light energetic particles are formed in their ground state. A power law formula connects the high-energy secondary light particle emission cross section with the  $A^{\text{th}}$  power of the proton cross section where  $A$  is the light particle mass number. In typical applications, it is also assumed that nucleons fill the volume  $V$  uniformly and freeze out happens suddenly (coalescence ceases at freeze out). It is also assumed that neutron and proton spectra have similar distributions and differ by a factor of the neutron to proton ratio of the system.

$$\frac{d^3N_A}{d^3p} = \left( \frac{N_p + N_T}{Z_p + Z_T} \right)^N A^3 \frac{2s_A + 1}{2^A} \left( \frac{\gamma h^3}{V} \right)^{A-1} \frac{e^{\frac{BE_A}{\theta_{ave}}}}{N!Z!} \left( \frac{d^3N_p}{d^3p} \right)^A \quad (4.2.1.2)$$

Therefore

$$P_0 = (A)^{\frac{1}{(A-1)}} \left( \frac{2s_A + 1}{2^A} \right)^{\frac{1}{3(A-1)}} \left( e^{\frac{BE_A}{\theta_{ave}}} Z!N! \right)^{\frac{1}{3(A-1)}} \left( \frac{9\gamma}{2\pi^2} \right)^{\frac{1}{3}} \frac{h}{2R} \quad (4.2.2.2)$$

where

$BE_A$  = binding energy of the light particle emitted MeV

$S_A$  = spin of the light particle emitted in ground state

$N$  = neutron number of the light particle emitted

$Z$  = charge number of the light particle emitted

$A$  = mass number of the light particle emitted

$\theta_{ave}$  = emitting source average temperature

$V$  = largest volume of emitting source region while equilibrium is maintained

$\tilde{R}$  = largest average radius of emitting source region while equilibrium is maintained

$$\langle r^2 \rangle = \text{root mean squar radius} = \left( \frac{3}{5} \right) \tilde{R}^2$$

$P_0$  = coalescence radius MeV/c

$N_p$  = projectile neutron number

$N_T$  = target neutron number

$$p_A = A \times p$$

$p_A$  = momentum of the light energetic particle emitted

$p$  = momentum of the proton

### 4.2.3 The Density Matrix Nucleon Coalescence Model

In the density matrix coalescence model of Sato and Yazaki et. al [32] it is assumed that the emitted high-energy secondary light particles can be represented as density matrices. Chemical and thermal equilibrium are not assumed and temperature is finite. Gaussian forms are assumed for the wave functions of the nucleons and their spatial distribution of these nucleons in the excited participant region where coalescence occurs.

### 4.2.4 The Dynamic Nucleon Coalescence Model

Recent coalescence prescriptions have been applied in event generating models to describe the dynamical evolution of the reaction. The dynamical calculation predicts the momenta and spatial positions of all of the nucleons as a function of time. Two nucleons are assumed to coalesce to form a deuteron if their relative momentum  $\Delta p = |\vec{p}_1 - \vec{p}_2|$  and relative position  $\Delta r = |\vec{r}_1 - \vec{r}_2|$  are less than a certain value for the known properties [33-35]. Sudden

Freeze-out is not assumed. However such an approach may not be valid for intermediate energy collisions, where large-scale fluctuations are present due to liquid-gas phase separation. If the projectile energy exceeds 100 MeV/nucleon, phase separation is not important.

### 4.3 A Thermodynamic Coalescence Model for Heavy Ion Collision

#### System

The power law for a simple coalescence model equation (4.1.4) can be modified to derive a thermodynamic coalescence model equation. Notice that,  $P_0$  contains other implicit factors such as a spin alignment factor and phase factor [7]. We need to account for the spin alignment factor in order to give the correct spin of the secondary light particle to be formed. For secondary light particles this factor is  $(2s+2)/2^A$  where  $A$  is light particle mass number and  $s$  is secondary light particle spin in the ground state. The second factor to be accounted for is the phase space factor,  $A^3$ , which comes from  $p_A = Ap$ , thus we have  $d^3N_A/d^3p_A = A^3 d^3N_A/d^3p$ . We explicitly remove the spin alignment factor and phase space factor by absorbing them into a new  $\tilde{P}_0$  as follows

$$(P_0^3)^{A-1} = A^3 \frac{2s+1}{2^A} (\tilde{P}_0^3)^{A-1}$$

equation 4.1.4 then becomes

$$\frac{d^3N_A(Z, n)}{d^3p^3} = \frac{1}{N!Z!} \left( \frac{N_p + N_T}{Z_p + Z_T} \right)^N A^3 \frac{2s+1}{2^A} \left( \frac{4\pi\gamma}{3} \tilde{P}_0^3 \right)^{A-1} \left[ \frac{d^3N_p(1,0)}{d^3p^3} \right]^A \quad (4.3.1)$$

Comparing equation (4.3.1) with the thermodynamic model from Mekjian et. al [7] we have

$$\left( \frac{4\pi}{3} \tilde{P}_0^3 \right)^{A-1} = \left( \frac{\gamma h^3}{V} \right)^{A-1} e^{\frac{BE_A}{\theta_{ave}}} Z!N! \quad (4.3.2)$$

Hence the power law for a thermodynamic coalescence model is

$$\frac{d^3 N_A}{d^3 p} = A^3 \left( \frac{N_P + N_T}{Z_P + Z_T} \right)^N \frac{2s_A + 1}{2^A} \left( \frac{\gamma h^3}{V} \right)^{A-1} \frac{e^{\frac{BE_A}{\theta_{Ave}}}}{N!Z!} \left( \frac{d^3 N_P}{d^3 p} \right)^A \quad (4.3.3)$$

Equation (4.3.3) can then be used to estimate the coalescence radius used in this work

$$\left( \frac{4\pi}{3} P_0^3 \right) = \left( A^3 \frac{2s_A + 1}{2^A} e^{\frac{BE_A}{\theta_{ave}}} Z!N! \right)^{\frac{1}{(A-1)}} \left( \frac{\gamma h^3}{V} \right) \quad (4.3.4)$$

Yielding

$$P_0 = (A)^{\frac{1}{(A-1)}} \left( \frac{2s_A + 1}{2^A} \right)^{\frac{1}{3(A-1)}} \left( e^{\frac{BE_A}{\theta_{ave}}} Z!N! \right)^{\frac{1}{3(A-1)}} \left( \frac{9\gamma}{2\pi^2} \right)^{\frac{1}{3}} \frac{h}{2R} \quad (4.3.5)$$

#### 4.4 A modified Thermodynamic Coalescence Model for Low Energy Heavy Ion Collision Systems (<25 MeV/nucleon)

In the thermodynamic model derivation in section 4.3, equation (4.3.3), the effects of Coulomb repulsion felt by protons and other secondary light particles were not considered. At high beam energies, Coulomb repulsion does not affect our thermodynamic coalescence model estimates. However, at energies below 25 MeV/nucleon, Coulomb repulsion must be considered [27, 30]. Detail derivation of the differential coalescence power law was performed by Datta et. al [27]. We start with the differential cross section power law, equation (4.4.1), for beam energies less than 25 MeV/nucleon, with Coulomb repulsion as derived by Datta et. al [27] and derive the equivalent differential thermodynamic cross section power law that includes Coulomb repulsion.

$$\left( \frac{d^2\sigma}{dE_A d\Omega} \right)_A = C \left( \frac{d^2\sigma}{dE_p d\Omega} \right)_{proton}^A \quad \text{where} \quad (4.4.1)$$

$$C = \frac{1}{N!Z!} \left( \frac{N_B + N_T}{Z_B + Z_T} \right)^N \left( \frac{4\pi}{3\sigma_{tot}} P_0^3 \right)^{A-1} \left( \frac{m_A}{(m_p)^A} \right) \left( \frac{A}{(p^2 - 2m_p f_p)^{\left(\frac{A-1}{2}\right)}} \right)$$

$$f_p = \frac{1.44(Z_B + Z_T - 1)}{r_0(A_B + A_T - 1)^{1/3}}; \quad r_0 = 1.29 \text{ fm}$$

The differential nucleon multiplicity per event can be related to the differential cross section by

$$\frac{d^3N}{dp^3} = \frac{1}{\sigma_{tot}} \frac{d^3\sigma}{dp^3} = \frac{1}{\sigma_{tot}} \frac{d^3\sigma}{p^2 dp d\Omega} \quad (4.4.2)$$

where  $\sigma_{tot}$  is the total reaction cross section.

We define a new variable  $\tilde{P}_0$  which explicitly removes the spin alignment factor, phase factor and Coulomb repulsion factor from  $P_0$  and compare with the thermodynamic coalescence model [8]. This improved thermodynamic coalescence model has additional terms which are due to Coulomb repulsion effects for beam energies less than 25 MeV/nucleon.

$$\left( \frac{4}{3} \pi P_0^3 \right)^{A-1} \equiv \frac{2s_A + 1}{2^A} \left( \frac{4\pi \tilde{P}_0^3}{3\sigma_{tot}} \right)^{A-1} \left( \frac{m_A}{(m_p)^A} \right) \left( \frac{A}{(p^2 - 2m_p f_p)^{\left(\frac{A-1}{2}\right)}} \right) \quad (4.4.3)$$

Now let

$$\frac{\left( \frac{4}{3} \pi \tilde{P}_0^3 \right)^{A-1}}{Z!N!} = \exp^{\frac{BE_A}{\theta_{Ave}}} \left( \frac{h^3}{V} \right)^{A-1}$$

and write the coalescence volume in terms of secondary light particle spin, source volume and Coulomb repulsion factors as follows

$$\left(\frac{4\pi}{3}P_0^3\right) = \left(\frac{2s_A+1}{2^A} e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{(A-1)}} \left(\frac{\gamma h^3}{V}\right) \left(\frac{1}{\sigma_{tot}}\right) \left(\frac{m_A}{(m_p)^A}\right)^{\frac{1}{(A-1)}} \left(\frac{A}{(p^2 - 2m_p f_p)^{\frac{(A-1)}{2}}}\right)^{\frac{1}{(A-1)}} \quad (4.4.4)$$

Now we solve equation (4.4.4) for coalescence radius,  $P_0$ ,

$$P_0 = \left(\frac{2s_A+1}{2^A}\right)^{\frac{1}{3(A-1)}} \left(e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{3(A-1)}} \left(\frac{1}{\sigma_{tot}}\right) \left(\frac{m_A}{(m_p)^A}\right)^{\frac{1}{(A-1)}} \left(\frac{A}{(p^2 - 2m_p f_p)^{\frac{(A-1)}{2}}}\right)^{\frac{1}{3(A-1)}} \left(\frac{9\gamma}{2\pi^2}\right)^{\frac{1}{3}} \frac{h}{2R} \quad (4.4.5)$$

Finally, we can simplify equation (4.4.5), rearrange and use equation (4.4.6) to estimate the coalescence radius in MeV/c using an improved thermodynamic coalescence model. Notice that the first 4 terms are exactly like  $P_0$  estimate of equation (4.3.5). The last three terms are as a result of including Coulomb repulsion in our coalescence radius estimates.

$$P_0 = \left(\frac{2s_A+1}{2^A}\right)^{\frac{1}{3(A-1)}} \left(e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{3(A-1)}} \left(\frac{9\gamma}{2\pi^2}\right)^{\frac{1}{3}} \left(\frac{h}{2R}\right) \left(\frac{1}{\sigma_{tot}}\right) \left(\frac{m_A}{(m_p)^A}\right)^{\frac{1}{(A-1)}} \left(\frac{A^{\frac{1}{3(A-1)}}}{(p^2 - 2m_p f_p)^{1/6}}\right) \quad (4.4.6)$$

## 5- COALESCENCE RADIUS AND EMITTING SOURCE RADIUS ESTIMATES USING A THERMODYNAMIC COALESCENCE MODEL

The coalescence radius is inversely related to the emitting secondary particle source radius. Therefore we will first estimate the source radius analytically.

### 5.1 Emitting Source Radius Estimates

The emitting source radius in this work is defined as the average radius corresponding to the largest participant volume where chemical and thermal equilibrium is maintained. Figure 2 shows the participant region stages: average radius (A) right after collision, average radius (B) where chemical and thermal equilibrium are maintained, and C, the freeze out radius. Beyond the freeze out radius, nucleons fly freely outward without interacting. The emitting source radius increases with increasing system size. This system size dependence is expected from the larger volume due to a larger number of participants in the central source region.

The emitting source radius, in fm, for any system for emitted light particles can be evaluated using Mekjian et. al [7].

$$\begin{aligned} R(fm) &= 0.24 \left( A_p^{1/3} + A_T^{1/3} \right) + 2.0 \quad \text{for } {}^2H \\ R(fm) &= 0.24 \left( A_p^{1/3} + A_T^{1/3} \right) + 1.6 \quad \text{for } {}^3H; {}^3He \quad \text{and} \\ R(fm) &= 0.24 \left( A_p^{1/3} + A_T^{1/3} \right) + 1.0 \quad \text{for } {}^4He \end{aligned} \tag{5.1.1}$$

$A_P$  and  $A_T$  are projectile and target mass numbers respectively. Although equations (5.1.1) have given reasonable source radii estimates for the emitted high-energy secondary light particles for

different heavy ion collision systems, one expression does not fit all collision systems. The coefficient, 0.24, for example, needs to be modified slightly for different systems.

In the coalescence model, secondary light particles are emitted in the forward hemisphere mainly from the participant region. Therefore, we will estimate the average participant mass contribution from the projectile and target [19, 20] and use the average participant mass, equation (5.1.2), to estimate the emitted light particle source radius.

$$\begin{aligned} \langle A^{Participant} \rangle = & \left( \langle Z_{Projectile}^{Participant} \rangle + \langle Z_{Target}^{Participant} \rangle \right) + \\ & \left( \langle N_{Projectile}^{Participant} \rangle + \langle N_{Target}^{Participant} \rangle \right) \end{aligned} \quad (5.1.2)$$

where

$\langle A^{Participant} \rangle$  is an average participant mass number,  $\langle Z_{Projectile}^{Participant} \rangle$  and  $\langle Z_{Target}^{Participant} \rangle$  are average proton numbers in the participant region from projectile and target respectively, and  $\langle N_{Projectile}^{Participant} \rangle$  and  $\langle N_{Target}^{Participant} \rangle$  are average neutron numbers in the participant region from the projectile and target respectively.

The average number of participant protons, which come from the beam nucleus (projectile), is proportional to the ratio of the target cross sectional area to the total cross section area. The average number of participant protons, which come from the target nucleus is proportional to the ratio of the projectile cross sectional area to the total cross sectional area and the average number of target protons.

$$\langle Z_{Projectile}^{Participant} \rangle = Z_P \times (target \text{ cross section}) / \sigma_G = \frac{Z_P A_T^{2/3}}{(A_P^{1/3} + A_T^{1/3})^2} \quad (5.1.3)$$

$$\langle Z_{Target}^{Participant} \rangle = \frac{Z_T A_P^{2/3}}{(A_P^{1/3} + A_T^{1/3})^2} \quad (5.1.4)$$

where

$$\sigma_G = \text{Geometrical cross section} = \pi r_0^2 (A_P^{1/3} + A_T^{1/3})^2;$$

The average neutron numbers of the participants from the projectile and target nuclei are estimated by multiplying the corresponding average charge numbers with the neutron to proton ratio of the heavy ion collision system. Neutrons and protons are assumed to have the same spectral shapes.

$$\langle N_{Projectile}^{Participant} \rangle = \langle Z_{Projectile}^{Participant} \rangle \times \left( \frac{N_P + N_T}{Z_P + Z_T} \right) \quad (5.1.5)$$

$$\langle N_{Target}^{Participant} \rangle = \langle Z_{Target}^{Participant} \rangle \times \left( \frac{N_P + N_T}{Z_P + Z_T} \right) \quad (5.1.6)$$

The source radius can be estimated using the cube root of the average participant mass (equations 5.1.7). The coefficients a, c, e, and g are independent of beam energy, but are slightly different for different group collision systems. They are also slightly different for different emitted secondary light particles. The groups of collision systems are formed using the collision system average mass (csam), incremented in units of 30 mass numbers.

$$\begin{aligned} R(\text{fm}) &= a \langle A^{Participant} \rangle^{1/3} + b \quad \text{for } {}^2\text{H} \\ R(\text{fm}) &= c \langle A^{Participant} \rangle^{1/3} + d \quad \text{for } {}^3\text{H} \\ R(\text{fm}) &= e \langle A^{Participant} \rangle^{1/3} + f \quad \text{for } {}^3\text{He} \\ R(\text{fm}) &= g \langle A^{Participant} \rangle^{1/3} + h \quad \text{for } {}^4\text{He} \end{aligned} \quad (5.1.7)$$

where a, c, e, and g are constants, but are slightly different for different group collision systems.

$\langle A^{Participant} \rangle$  is the participant average mass number estimated by equation (5.1.2). The constant are chosen such that our coalescence radius estimates are comparable to other analyses estimates for the same system.

The source radius can also be estimated using the cube root of average participant from beam plus the cube root of average participant mass from the target (equations 5.1.8) and get similar results as equations (5.1.7) with slightly different coefficients a, b, c, and d.

$$\begin{aligned}
 R(\text{fm}) &= a \left( \langle A_{\text{Projectile}}^{Participant} \rangle^{1/3} + \langle A_{\text{Target}}^{Participant} \rangle^{1/3} \right) + 2.0 \quad \text{for } {}^2\text{H} \\
 R(\text{fm}) &= b \left( \langle A_{\text{Projectile}}^{Participant} \rangle^{1/3} + \langle A_{\text{Target}}^{Participant} \rangle^{1/3} \right) + 1.6 \quad \text{for } {}^3\text{H} \\
 R(\text{fm}) &= c \left( \langle A_{\text{Projectile}}^{Participant} \rangle^{1/3} + \langle A_{\text{Target}}^{Participant} \rangle^{1/3} \right) + 1.55 \quad \text{for } {}^3\text{He} \quad \text{and} \\
 R(\text{fm}) &= d \left( \langle A_{\text{Projectile}}^{Participant} \rangle^{1/3} + \langle A_{\text{Target}}^{Participant} \rangle^{1/3} \right) + 1.0 \quad \text{for } {}^4\text{He}
 \end{aligned} \tag{5.1.8}$$

Notice that, the average participant mass is the same if we exchange the beam and the target. Therefore the estimated source radius will be the same if we switch the projectile and the target.

## 5.2 Coalescence Radius Estimates

### 5.2.1 Coalescence Radius Estimates Using the Thermodynamic Coalescence Model.

Equation (4.3.3) can be used to estimate the coalescence radius. The coalescence volume can be related to the inverse of the source radius. All of the assumptions used to derive the thermodynamic model are assumed here also.

$$\left(\frac{4\pi}{3}P_0^3\right) = \left(A^3 \frac{2s_A + 1}{2^A} e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{(A-1)}} \left(\frac{\gamma h^3}{V}\right) \quad (5.2.1.1)$$

Now we solve equation (5.2.1.1) for the coalescence radius  $P_0$ . Equations (5.2.1.2) are used in our estimation of the coalescence radii.

$$P_0 = (A)^{\frac{1}{(A-1)}} \left(\frac{2s_A + 1}{2^A}\right)^{\frac{1}{3(A-1)}} \left(e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{3(A-1)}} \left(\frac{9\gamma}{2\pi^2}\right)^{\frac{1}{3}} \frac{h}{2R} \quad (5.2.1.2)$$

## 5.2.2 Coalescence Radius Estimates Using the Improved Thermodynamic Coalescence Model.

In section 4.4 power law is in double differential form [equation (4.4.1)] that can be used to estimate the coalescence radii, however we used secondary light particle and proton multiplicities in our analyses rather than double differential form. In order to be consistent in this work we will start with equation (4.4.1) and use equation (4.1.6) and equation (4.1.5) and transform double differential cross section power law [equation (4.4.1) in terms of multiplicities. as follow

$$\frac{d^2\sigma}{dE_n d\Omega} = E_n p_n \frac{d^3\sigma}{dp^3} \quad \text{and} \quad \frac{d^2\sigma}{dE_A d\Omega} = E_A p_A \frac{d^3\sigma}{dp^3} \quad (5.2.2.1)$$

Substitute equations (5.2.2.1) into equation (4.4.1) to get

$$\begin{aligned}
E_A p_A \frac{d^3 \sigma}{dp_A^3} &= C \left( \frac{1}{\sigma_{tot}} \right)^{A-1} \left( E_n p_n \frac{d^3 \sigma}{dp^3} \right)^A & \text{where} \\
C &= \frac{1}{N!Z!} \left( \frac{N_B + N_T}{Z_B + Z_T} \right)^N \left( \frac{4\pi}{3} P_0^3 \right)^{A-1} \\
&\quad \left( \frac{m_A}{(m_p)^A} \right) \left( \frac{A}{(p^2 - 2m_p f_p)^{\left(\frac{A-1}{2}\right)}} \right)
\end{aligned} \tag{5.2.2.2}$$

Simplify equation (5.2.2.2), substitute equation (5.2.2.3) and  $p_A = Ap$  to derive the power law in terms of multiplicities, equation (5.2.2.4).

$$\frac{d^3 N_p}{dp^3} = \frac{1}{\sigma_{tot}} \frac{d^3 \sigma}{dp^3} \quad \text{and} \quad \frac{d^3 N_A}{dp_A^3} = \frac{1}{\sigma_{tot}} \frac{d^3 \sigma}{dp_A^3} \tag{5.2.2.3}$$

$$\begin{aligned}
\frac{d^3 N_A}{dp^3} &= C \left( \frac{d^3 N_p}{dp^3} \right)^A & \text{where} \\
C &= \frac{A^3}{N!Z!} \left( \frac{N_B + N_T}{Z_B + Z_T} \right)^N \left( \frac{4\pi}{3} P_0^3 \right)^{A-1} \\
&\quad \left( \frac{E_n p_n}{m_p} \right)^A \left( \frac{m_A}{E_A p_A} \right) \left( \frac{A}{(p^2 - 2m_p f_p)^{\left(\frac{A-1}{2}\right)}} \right)
\end{aligned} \tag{5.2.2.4}$$

then use equation (5.2.2.4) and [7, 9] to write the coalescence volume in terms of the source volume, secondary light particle spin, and Coulomb repulsion term as follows

$$\left(\frac{4\pi}{3}P_0^3\right) = A^{\frac{3}{(A-1)}} \left(\frac{2s_A+1}{2^A}\right)^{\frac{1}{(A-1)}} \left(e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{(A-1)}} \left(\frac{\gamma h^3}{V}\right) \left(\left(\frac{E_n p_n}{m_p}\right)^A \left(\frac{m_A}{E_A p_A}\right)\right)^{\frac{1}{(A-1)}} \left(\frac{A}{(p^2 - 2m_p f_p)^{\left(\frac{A-1}{2}\right)}}\right)^{\frac{1}{(A-1)}} \quad (5.2.2.5)$$

Yielding a coalescence radius,  $P_0$ , estimate of

$$P_0 = (A)^{\frac{1}{(A-1)}} \left(\frac{2s_A+1}{2^A}\right)^{\frac{1}{3(A-1)}} \left(e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{3(A-1)}} \left(\frac{9\gamma}{2\pi^2}\right)^{\frac{1}{3}} \frac{h}{2R} \left(\left(\frac{E_n p_n}{m_p}\right)^A \left(\frac{m_A}{E_A p_A}\right)\right)^{\frac{1}{3(A-1)}} \left(\frac{A}{(p^2 - 2m_p f_p)^{\left(\frac{A-1}{2}\right)}}\right)^{\frac{1}{3(A-1)}} \quad (5.2.2.6)$$

Finally, we can simplify equation (5.2.2.6) and use equation (5.2.2.7) to estimate the coalescence radius using an improved thermodynamic coalescence model for beam energies less than 25 MeV/nucleon.

$$P_0 = (A)^{\frac{4}{3(A-1)}} \left(\frac{2s_A+1}{2^A}\right)^{\frac{1}{3(A-1)}} \left(e^{\frac{BE_A}{\theta_{ave}}} Z!N!\right)^{\frac{1}{3(A-1)}} \left(\frac{9\gamma}{2\pi^2}\right)^{\frac{1}{3}} \frac{h}{2R} \left(\left(\frac{E_n p_n}{m_p}\right)^A \left(\frac{m_A}{E_A p_A}\right) A\right)^{\frac{1}{3(A-1)}} \left(\frac{1}{(p^2 - 2m_p f_p)^{1/6}}\right) \quad (5.2.2.7)$$

Where secondary light particle energy and proton energy [27] are related as follow

$$E_A = A^2 \frac{m_p}{m_A} (E_p - f_p) + f_A \quad (5.2.2.8)$$

## **6- SUBROUTINES “SOURCERADIUS” AND “COALESCENCERADIUS”**

### **6.1 Subroutine "SourceRadius"**

The “SourceRadius” subroutine requires beam energy in MeV/nucleon, projectile proton and mass numbers, and target proton and mass numbers. The output consists of estimates of source radii for  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$ , and  $^4\text{He}$  secondary light particles in units of fm for any heavy ion collision system. The FORTRAN subroutine “SourceRadius” is listed in the appendix I.2

### **6.2 Subroutine "CoalescenceRadius"**

The “CoalescenceRadius” subroutine requires source radii for the secondary light particles and uses the thermodynamic coalescence model to estimate coalescence radii for  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$ , and  $^4\text{He}$  secondary light particles in MeV/c for any heavy ion collision system. FORTRAN Subroutine CoalescenceRadius used to estimate coalescence radii is listed in appendix I.3.

## 7- REPRESENTATIVE SOURCE AND COALESCENCE RADII ESTIMATES

### 7.1 Source Radii Estimates and Discussion of Results

Our model (equations (5.1.7)) is used to estimate source radii for emitted secondary hydrogen and helium isotopes for a wide range of heavy ion collision systems from alpha-alpha to Pb+U collisions for both symmetric and asymmetric systems. The source radii do not depend on beam energy. The beam energy per nucleon investigated ranged from 100 Mev/nucleon to 4 GeV/nucleon for Ne+NaF and Ne+U heavy ion collision systems. Source radius estimates increase as system size (projectile and target mass numbers) increase and are different for different emitted secondary light particles. Source radii are larger for symmetric systems compare to asymmetric systems with the same collision system average mass. The rise and fall of source radii for  $csam < 150$  are directly related to the symmetry of the collision system. Collision systems are divided into seven groups, incremented by 30 mass units, according to the collision system average mass ( $csam$ ), which is calculated as  $csam = \frac{(\text{projectile mass} + \text{target mass})}{2}$ . For group 1  $csam \leq 30$ , for group 2,  $30 < csam \leq 60$ , and so forth. The coefficients a, c, e, and g are slightly larger for heavier collision systems and they are also different for different emitted secondary light particles. The source radii output data are listed in appendix I.5. The source radii of emitted secondary light particles are plotted as a function of collision system average mass ( $csam$ ) in figure 3. As expected, source radii are the largest for  $^2\text{H}$  and the smallest for alpha particles for any heavy ion collision system. Alpha

particles have the highest binding energy and deuterons have the lowest binding energy. Source radii increase as system size increases as expected.

The plots in figure 3 rise and fall for systems of nearly equal size. In heavy ion collision systems, symmetric systems have the largest overlap of the beam and target nuclei therefore the source radii are larger. On the other hand, in asymmetric systems, overlap is limited by the smaller nuclear size. Therefore, assuming the same collision system average mass, we expect to get smaller source radii for asymmetric systems relative to symmetric systems. The sharp rise and fall of the source radii estimates depend on whether it is a symmetric system or an asymmetric system. Table 1 and Table 2 compare source radii for three collision systems with average masses of 60.5 and 150.0 respectively. Notice that the more symmetric the system the higher the source radius at a given average system mass. The main reason that the plots are smoother when collision system average mass,  $csam$ ,  $> 150$  is because collision systems are nearly symmetric there. Our estimated source radii are compared with other analyses for  $^{20}\text{Ne}+\text{NaF}$ ,  $^{12}\text{C}+^{12}\text{C}$ , and  $^{20}\text{Ne} + ^{238}\text{U}$  in table 3, table 4, and table 5 respectively.

Even though coalescence radii estimates from other analyses were used to validate our coalescence radii estimates, our source radii estimates are within 20% of other analyses. The source radii in our calculations are the largest average source radii where chemical and thermal equilibrium are maintained. Other analyses [17] use the source radius right after collision, as it starts expanding. Thus source radii in this work are larger than those. On the other hand, still other analyses [7] use the freeze out radius so the source radii in this work are smaller than those.

Our estimated source radius for  $^{20}\text{Ne}+\text{NaF}$  (see table 3) for  $^2\text{H}$  is 8% higher, for  $^3\text{H}$  15% lower and for  $^3\text{He}$  18% lower than Lemaire et. al [17] estimates.

The estimated source radius from this work for  $^{12}\text{C}+^{12}\text{C}$  (see table 4) for  $^2\text{H}$  is 10% larger, for  $^3\text{H}$  it is 4% higher, and for  $^3\text{He}$  it is the same as values from Lemaire et. al [17].

The estimated source radius from this work for  $^{20}\text{Ne}+^{238}\text{U}$  (see table 5) for  $^2\text{H}$  is 6% higher, for  $^3\text{H}$  it is 5% lower, for  $^3\text{He}$  it is 8% lower, and for  $^4\text{He}$  it is 10% lower compared to estimates from Mekjian et. al [7].

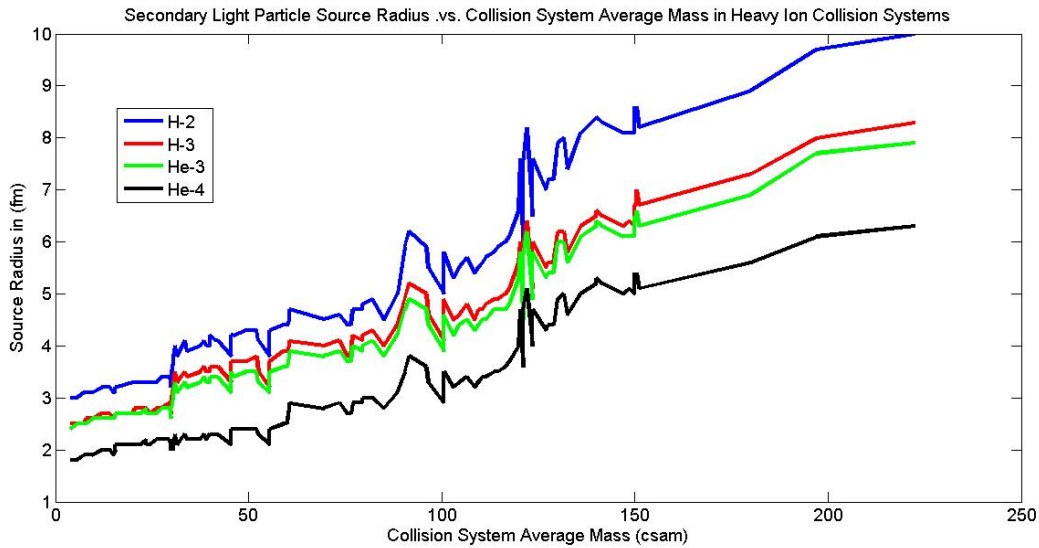


**Figure 3** Source radius vs. collision system average mass of secondary light particles in heavy ion collision systems. Equations (5.1.7) were used to estimate source radii. Data are listed in Appendix I.5.

**Table 1** Three collision systems having the same collision system average mass of 60.5 and the same equations were used to estimate source radii. Source radii are higher for symmetric systems than asymmetric systems of the same average mass.

| collision System                  | csam | Source Radius<br>$^2\text{H}$ (fm) | Source Radius<br>$^3\text{H}$ (fm) | Source Radius<br>$^3\text{He}$ (fm) | Source Radius<br>$^4\text{He}$ (fm) |
|-----------------------------------|------|------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|
| $^{24}\text{Mg} + ^{97}\text{Tc}$ | 60.5 | 4.9                                | 4.0                                | 3.8                                 | 2.8                                 |
| $^{35}\text{Cl} + ^{86}\text{Rb}$ | 60.5 | 5.0                                | 4.1                                | 3.9                                 | 2.9                                 |
| $^{56}\text{Fe} + ^{65}\text{Zn}$ | 60.5 | 5.1                                | 4.1                                | 3.9                                 | 2.9                                 |

**Table 2** Three collision systems having the same collision system average mass of 150 and equations (5.1.7) were used to estimate source radii. Source radii are higher for symmetric systems than asymmetric systems of the same average mass.

| Collision System                    | csam  | Source Radius<br>$^2\text{H}$ (fm) | Source Radius<br>$^3\text{H}$ (fm) | Source Radius<br>$^3\text{He}$ (fm) | Source Radius<br>$^4\text{He}$ (fm) |
|-------------------------------------|-------|------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|
| $^{55}\text{Mn} + ^{245}\text{Am}$  | 150.0 | 8.5                                | 6.3                                | 6.1                                 | 5.0                                 |
| $^{122}\text{Sb} + ^{178}\text{Hf}$ | 150.0 | 8.9                                | 6.7                                | 6.4                                 | 5.4                                 |
| $^{150}\text{Sm} + ^{150}\text{Sm}$ | 150.0 | 9.0                                | 6.7                                | 6.5                                 | 5.4                                 |

**Table 3** Comparison of source radii estimates from this work with estimates from Lemaire et. al [17] for the  $^{20}\text{Ne} + \text{NaF}$  system at beam energy 400 MeV/nucleon.

|               | Source Radii $^{20}\text{Ne} + \text{NaF}$ |                                  | % Diff |
|---------------|--------------------------------------------|----------------------------------|--------|
|               | This Model<br>Source Radius<br>(fm)        | Lemaire<br>Source Radius<br>(fm) |        |
| $^2\text{H}$  | 3.7                                        | 3.4                              | +8%    |
| $^3\text{H}$  | 2.8                                        | 3.3                              | -15%   |
| $^3\text{He}$ | 2.7                                        | 3.3                              | -18%   |
| $^4\text{He}$ | 2.1                                        | Not Available                    |        |

**Table 4** Comparison of source radii estimates from this work with estimates from Lemaire et. al [17] for the  $^{12}\text{C}+^{12}\text{C}$  system at beam energy 800 MeV/nucleon.

|               | Source Radii $^{12}\text{C}+^{12}\text{C}$ |                                  | % Diff |
|---------------|--------------------------------------------|----------------------------------|--------|
|               | This Model<br>Source Radius<br>(fm)        | Lemaire<br>Source Radius<br>(fm) |        |
| $^2\text{H}$  | 3.2                                        | 2.9                              | +10%   |
| $^3\text{H}$  | 2.7                                        | 2.6                              | +4%    |
| $^3\text{He}$ | 2.6                                        | 2.6                              | 0%     |
| $^4\text{He}$ | 2.0                                        | Not Available                    |        |

**Table 5** Comparisons of source radii estimates with estimates from Mekjian et. al [7] for the  $^{20}\text{Ne} + ^{238}\text{U}$  system at beam energy 400 MeV/nucleon.

|               | Source Radii $^{20}\text{Ne} + ^{238}\text{U}$ |                                  | % Diff |
|---------------|------------------------------------------------|----------------------------------|--------|
|               | This Model<br>Source Radius<br>(fm)            | Mekjian<br>Source Radius<br>(fm) |        |
| $^2\text{H}$  | 7.2                                            | 6.8                              | +6%    |
| $^3\text{H}$  | 5.6                                            | 5.9                              | -5%    |
| $^3\text{He}$ | 5.4                                            | 5.9                              | -8%    |
| $^4\text{He}$ | 4.4                                            | 4.9                              | -10%   |

## 7.2 Coalescence Radius Results and Discussion

The thermodynamic coalescence model (equations (5.2.1.2)) was used to estimate coalescence radii for emitted secondary light particles for a wide range of heavy ion collision systems from alpha+alpha to Pb+U for both symmetric and asymmetric systems. The coalescence radius increases less than 6% as beam energy increases from 100 MeV/nucleon to 4 GeV/nucleon for Ne+NaF and Ne+U heavy ion collision systems. Thus, the coalescence radius is nearly independent of beam energy. However, the coalescence radius decreases as system size (projectile and target mass number) increases since the coalescence radius is inversely related to the source radius. Coalescence radii are generally lowest for emitted deuterons and highest for

emitted alpha particles. Coalescence radii were estimated for a wide range of symmetric and asymmetric collision systems. The results are shown in appendix I.6. Coalescence radii are also plotted versus collision system average mass (csam) in figure 4. Note that the coalescence radius increases as source radius decreases and vice versa. Coalescence radius estimates are lower for a symmetric collision system compared to an asymmetric system with the same collision system average mass. Tables 6 and 7 compare coalescence radii for three collision systems with average masses of 60.5 and 150.0 respectively. Note that the more symmetric the system, the larger the source radii whereas the coalescence radii are smaller for a specific average system mass. The thermodynamic coalescence model estimates of coalescence radii for  $^{12}\text{C}+^{12}\text{C}$ ,  $^{20}\text{Ne}+\text{NaF}$ ,  $^{20}\text{Ne}+^{207}\text{Pb}$ ,  $^{16}\text{O}+^{238}\text{U}$  and  $^{20}\text{Ne}+^{238}\text{U}$  are compared in Tables 8 through Table 12 with other analyses. Results from this work are comparable to those from other works [15, 17, 30].

The estimated coalescence radius from this work for  $^{12}\text{C}+^{12}\text{C}$  (see table 8) for  $^2\text{H}$  is 8% smaller, for  $^3\text{H}$  it is 1% larger and for  $^3\text{He}$  it is 3% higher than estimates from Lemaire et. al [17].

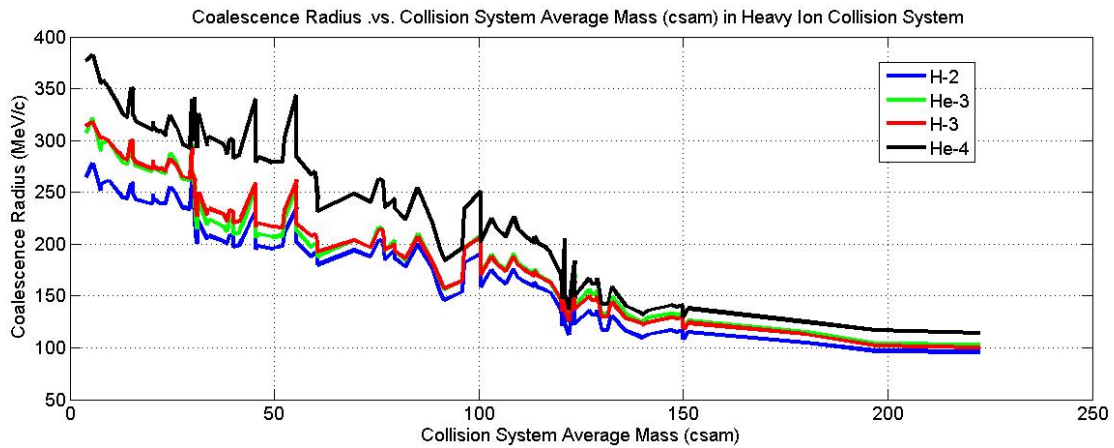


**Figure 4** Coalescence radius vs. collision system average mass of secondary light particles in heavy ion collision systems. Equations (5.2.1.2) were used to estimate the coalescence radii.

**Table 6** Three collision systems having the same collision system average mass number of 60.5. Equation (5.2.1.2) were used to estimate the coalescence radii. Coalescence radii are smaller for symmetric systems than asymmetric systems with the same average mass.

| collision System                  | csam | Coalescence<br>Radius<br>$^2\text{H}$ (MeV/c) | Coalescence<br>Radius<br>$^3\text{H}$ (MeV/c) | Coalescence<br>Radius<br>$^3\text{He}$ (MeV/c) | Coalescence<br>Radius<br>$^4\text{He}$ (MeV/c) |
|-----------------------------------|------|-----------------------------------------------|-----------------------------------------------|------------------------------------------------|------------------------------------------------|
| $^{24}\text{Mg} + ^{97}\text{Tc}$ | 60.5 | 205                                           | 198                                           | 201                                            | 244                                            |
| $^{35}\text{Cl} + ^{86}\text{Rb}$ | 60.5 | 202                                           | 195                                           | 197                                            | 239                                            |
| $^{56}\text{Fe} + ^{65}\text{Zn}$ | 60.5 | 195                                           | 188                                           | 192                                            | 232                                            |

**Table 7** Three collision systems having the same collision system average mass number of 150. Equation (5.2.1.2) was used to estimate coalescence radii. Coalescence radii are smaller for a symmetric system than for asymmetric systems of the same average mass number.

| Collision<br>System                 | csam  | Coalescence<br>Radius<br>$^2\text{H}$ (MeV/c) | Coalescence<br>Radius<br>$^3\text{H}$ (MeV/c) | Coalescence<br>Radius<br>$^3\text{He}$ (MeV/c) | Coalescence<br>Radius<br>$^4\text{He}$ (MeV/c) |
|-------------------------------------|-------|-----------------------------------------------|-----------------------------------------------|------------------------------------------------|------------------------------------------------|
| $^{55}\text{Mn} + ^{245}\text{Am}$  | 150.0 | 123                                           | 133                                           | 129                                            | 141                                            |
| $^{122}\text{Sb} + ^{178}\text{Hf}$ | 150.0 | 114                                           | 124                                           | 121                                            | 131                                            |
| $^{150}\text{Sm} + ^{150}\text{Sm}$ | 150.0 | 113                                           | 123                                           | 120                                            | 130                                            |

**Table 8** Comparisons of the thermodynamic coalescence model estimate with estimates from Lemaire et. al [17] for the  $^{12}\text{C}+^{12}\text{C}$  system at beam energy 800 MeV/nucleon.

|               | Coalescence Radii $^{12}\text{C}+^{12}\text{C}$ |                    | % Diff |
|---------------|-------------------------------------------------|--------------------|--------|
|               | Thermodynamic<br>Model<br>(MeV/c)               | Lemaire<br>(MeV/c) |        |
| $^2\text{H}$  | 280                                             | 304                | -8%    |
| $^3\text{H}$  | 284                                             | 280                | +1%    |
| $^3\text{He}$ | 288                                             | 280                | +3%    |
| $^4\text{He}$ | 333                                             | Not Available      |        |

The estimated coalescence radius from this work for  $^{20}\text{Ne}+\text{NaF}$  (see table 9) for  $^2\text{H}$  is 8% larger, for  $^3\text{H}$  it is 5% larger and for  $^3\text{He}$  it is 6% larger than estimates from Lamaire et. al [17].

The estimated coalescence radius from this work for Radii  $^{20}\text{Ne}+^{207}\text{Pb}$  (see table 10) for  $^2\text{H}$  is 17% smaller, for  $^3\text{H}$  it is 14% smaller and for  $^3\text{He}$  it is 9% smaller than estimates from Lamaire et. al [17].

The estimated coalescence radius from this work for  $^{16}\text{O}+^{238}\text{U}$  (see table 11) for  $^2\text{H}$  is 15% smaller, for  $^3\text{H}$  it is 27% smaller and for  $^3\text{He}$  it is 38% smaller than estimates from Awes et. al [30]. Awes et. al [28] uses source radii right after collision which is smaller than the thermodynamic model source radii. As a result the estimated coalescence radii from this model are much smaller than Awes et. al [30] estimates.

The estimated coalescence radius from this work for  $^2\text{H}$  is 8% larger, for  $^3\text{H}$  is 18% larger, for  $^3\text{He}$  it is 13% larger, and for  $^4\text{He}$  it is 14% larger than estimates from Gutbrod et. al [15].

For collision systems with mass numbers larger than 150, coalescence radii estimates are within 20% of other analyses except for  $^{16}\text{O}+^{238}\text{U}$ . Coalescence Radii estimates are listed in Appendix I.6.

### 7.3 Total Cross Section Estimates

This work estimates coalescence radii for collision systems  $^{12}\text{C}+^{12}\text{C}$ ,  $^{16}\text{O}+^{12}\text{C}$ , and  $^{12}\text{C} + ^{64}\text{Cu}$  is used with a simplified version of recently released NUCFRG3 in order to estimate the total cross sections in mb. The estimated total cross sections are listed in table 13 and table 14. This work estimated total cross sections are also compared with available experimental results.

**Table 9** Comparisons of the thermodynamic coalescence model estimates with estimates from Lemaire et. al [17] for  $^{20}\text{Ne}+\text{NaF}$  system at beam energy 800 MeV/nucleon.

| Coalescence Radii $^{20}\text{Ne}+\text{NaF}$ |                             |                 |        |
|-----------------------------------------------|-----------------------------|-----------------|--------|
|                                               | Thermodynamic Model (MeV/c) | Lemaire (MeV/c) | % Diff |
| $^2\text{H}$                                  | 272                         | 259             | +5%    |
| $^3\text{H}$                                  | 274                         | 260             | +5%    |
| $^3\text{He}$                                 | 275                         | 260             | +6%    |
| $^4\text{He}$                                 | 312                         | Not Available   |        |

**Table 10** Comparisons of the thermodynamic coalescence model coalescence radius estimates with Lemaire et. al [17] for  $^{20}\text{Ne}+^{207}\text{Pb}$  at beam energy 800 MeV/nucleon.

| Coalescence Radii $^{20}\text{Ne}+^{207}\text{Pb}$ |                             |                 |        |
|----------------------------------------------------|-----------------------------|-----------------|--------|
|                                                    | Thermodynamic Model (MeV/c) | Lemaire (MeV/c) | % Diff |
| $^2\text{H}$                                       | 170                         | 205             | -17%   |
| $^3\text{H}$                                       | 172                         | 199             | -14%   |
| $^3\text{He}$                                      | 171                         | 189             | -9%    |
| $^4\text{He}$                                      | 202                         | Not Available   |        |

**Table 11** Comparisons of the thermodynamic coalescence model coalescence radius estimates with Awes et. al [30] for  $^{16}\text{O}+^{238}\text{U}$  system at beam energy 315 MeV/nucleon.

|               | Coalescence Radii $^{16}\text{O}+^{238}\text{U}$ |               |        |
|---------------|--------------------------------------------------|---------------|--------|
|               | Thermodynamic Model (MeV/c)                      | Awes (MeV/c)  | % Diff |
| $^2\text{H}$  | 144                                              | 170           | -15%   |
| $^3\text{H}$  | 156                                              | 215           | -27%   |
| $^3\text{He}$ | 150                                              | Not Available |        |
| $^4\text{He}$ | 167                                              | 270           | -38%   |

**Table 12** Comparisons of the thermodynamic coalescence model coalescence radius estimates with estimates from Gutbrod et. al [15] for  $^{20}\text{Ne}+^{238}\text{U}$  system at beam energy 400 MeV/nucleon.

|               | Coalescence Radii $^{20}\text{Ne}+^{238}\text{U}$ |                 |        |
|---------------|---------------------------------------------------|-----------------|--------|
|               | Thermodynamic Model (MeV/c)                       | Gutbrod (MeV/c) | % Diff |
| $^2\text{H}$  | 140                                               | 129             | +8%    |
| $^3\text{H}$  | 152                                               | 129             | +18%   |
| $^3\text{He}$ | 146                                               | 129             | +13%   |
| $^4\text{He}$ | 162                                               | 142             | +14%   |

**Table 13**  $P_0$  used for  $^{12}\text{C}+^{12}\text{C}$  are 280, 284, 288, 333 and for  $^{16}\text{O}+^{12}\text{C}$  are 273, 277, 281, and 322 for  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$ , and  $^4\text{He}$  respectively

| Emitted nuclei | Total Cross Section (mb)<br>$^{12}\text{C} + ^{12}\text{C}$ |                     | Reference     | Total Cross Section (mb)<br>$^{16}\text{O}+ ^{12}\text{C}$ |                     | Reference     |
|----------------|-------------------------------------------------------------|---------------------|---------------|------------------------------------------------------------|---------------------|---------------|
|                | This work NUCFRG3                                           | Experimental Result |               | This work NUCFRG3                                          | Experimental Result |               |
| $^2\text{H}$   | 188.9                                                       | $292 \pm 26$        | Lindstrom[36] | 258.1                                                      | $406 \pm 36$        | Lindstrom[36] |
| $^3\text{H}$   | 74.7                                                        | $118 \pm 7$         | Lindstrom[36] | 108.1                                                      | $151 \pm 11$        | Lindstrom[36] |
| $^3\text{He}$  | 81.2                                                        | $135 \pm 8$         | Lindstrom[36] | 117.8                                                      | $136 \pm 8$         | Lindstrom[36] |
| $^4\text{He}$  | 99.3                                                        | $404 \pm 36$        | Lindstrom[36] | 140.8                                                      | $474 \pm 42$        | Lindstrom[36] |

**Table 14** Value  $P_0$  used for  $^{12}\text{C} + ^{64}\text{Cu}$  237, 231, 240, 312 for  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$ , and  $^4\text{He}$  respectively.

| Emitted nuclei | Total Cross Section (mb)<br>$^{12}\text{C} + ^{64}\text{Cu}$ |                        | Reference     |
|----------------|--------------------------------------------------------------|------------------------|---------------|
|                | This work<br>NUCFRG3                                         | Experimental<br>Result |               |
| $^2\text{H}$   | 78.6                                                         | $485 \pm 52$           | Lindstrom[36] |
| $^3\text{H}$   | 16.2                                                         | $219 \pm 19$           | Lindstrom[36] |
| $^3\text{He}$  | 20.4                                                         | $209 \pm 14$           | Lindstrom[36] |
| $^4\text{He}$  | 45.11                                                        | $568 \pm 61$           | Lindstrom[36] |

## **8- CONCLUSIONS AND FUTURE WORK**

### **8.1 Source Radius Estimates**

Average participant mass can be used to estimate source radius analytically for a wide range of heavy ion collision systems, both symmetric and asymmetric, from 25 MeV per nucleon to several GeV per nucleon beam energy and yield reasonable estimates within 20% of other methods. The estimated source radii in this work are the largest average source radius where chemical and thermal equilibrium are maintained. Source radii are independent of the beam energy, but increase as system average mass number increases. Source radii also depend on the symmetry of the system. Symmetric collision systems have larger source radii than an asymmetric collision system of the same total mass number.

### **8.2 Coalescence Radius Estimates**

The thermodynamic coalescence model can be used to estimate coalescence radii analytically for both symmetric and asymmetric systems from tens of MeV per nucleon to several GeV per nucleon. The thermodynamic coalescence model estimates coalescence radii within 10% or less of other methods. The thermodynamic coalescence radii increase by only 6% for beam energies from 100 MeV per nucleon to 4 GeV per nucleon. The coalescence radii decrease as system mass number increases. The coalescence radii are smaller for a symmetric system compared to an asymmetric system of equal size (collision system average mass the same). We can now estimate coalescence radii for any collision system analytically by using the limited number of available experimental data to validate the estimates. Coalescence radii

estimates are very sensitive to the estimated source radii. Therefore better estimates of source radii, such as using the Hanbury, Brown and Twist method (HBT method), may improve coalescence radii estimates.

### **8.3 Total Cross Section Estimates**

Total cross sections estimates do not quite compare with the experimental results but needed improvements are unclear due to the lack of sufficient experimental measurements. More experimental results are needed in order to get better coalescence radii estimates and better estimates for the total cross sections calculations using the NUCFRG3 code.

### **8.4 Suggested Future Work**

The method of two-particle intensity interferometry, originally developed by Hanbury Brown and Twiss (HBT) [10, 11] to measure angular distances of stars and other stellar objects, exploits quantum statistical correlations between particle intensities in two-particle coincidence measurements to access spatial information on the emitting source region [14]. Hanbury Brown/Twiss (HBT) particle interferometry methods for relativistic heavy-ion collisions has been used successfully to exploit identical two-particle correlations to predict space-time geometry and dynamics of the fireball created after relativistic heavy-ion collisions. The connection between the measured correlations in momentum space and the phase-space structure of the particle emitter has been established [12, 37]. Coalescence radii are inversely related to source radii, therefore better estimates of source radii in relativistic heavy-ion collisions,

obtained using HBT methods, may improve our coalescence radii estimates using the thermodynamic coalescence model.

More experimentally measured cross sections data would improve the estimates from this work.

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## **10- APPENDICES**

## Appendix I: Main FORTRAN Program, SourceRadius Subroutine, CoalescenceRadius Subroutine, Input, and Output Data Files

The FORTRAN program including subroutines, and input data files which were used to estimate source radii and coalescence radii are listed in Appendix I.1 through Appendix I.4. This FORTRAN program was run on a PC with Compaq Visual FORTRAN 6.1.

## Appendix I.1: Main FORTRAN Program Listing

```
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$  
  
c Program POESTIMATE, estimates the emitting source radius in fm and  
c coalescence radius in MeV/c for emitted secondary light particles H-2, c H-  
c 3, He-3 and He-4 for any high energy heavy ion collision system. The c  
c required inputs are:  
C  
C      1- projectile and target charge and mass numbers  
c      2- projectile energy per nucleon in MeV per nucleon  
c      3- Emitted light particle charge, mass numbers, binding energy and  
c          spin  
  
c M. PourArsalan June 2012  
  
c $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$  
  
program POESTIMATE  
IMPLICIT NONE  
  
real*8 zp,np,Ap,zt,nt,At,csm  
real*8 ave_pA,Epn,Tave,Tave_MeV  
real*8 RD2_MP, RT3_MP,RHe3_MP,RHe4_MP  
real*8 RD2_Mpty, RT3_Mpty, RHe3_Mpty, RHe4_Mpty  
real*8 RD2_Mek,RT3_Mek,RHe3_Mek,RHe4_Mek  
real*8 P0D2_MP,P0T3_MP,P0He3_MP,P0He4_MP  
real*8 P0D2_Mpty,P0T3_Mpty,P0He3_Mpty,P0He4_Mpty  
real*8 P0D2_Mek,P0T3_Mek,P0He3_Mek,P0He4_Mek  
real*8 temp1_D2,temp2_D2,temp3_D2,temp4_D2,temp5_D2  
character(len=15) temp2  
c      character*14 temp2  
integer i
```

```

C *****
C Open and write output data
C *****
      open(20,file='SourceRadius_MP.dat',status='old')
      open(25,file='SourceRadius_Mek.dat',status='old')
      open(40,file='SourceRadius_MPty.dat',status='old')
      open(60,file='SourceRadius_MP_plot.dat',status='old')
C
      write(20,*),'*****'
      write(20,*),'*** Our Model estimates Average Participant Mass ***'
      write(20,*),'*****'
      write(20,*),'  System      Energy/n  csam  H-2    H-3    He-3    He-4'
      write(20,*),'Proj<->Target  (MeV)          (fm) (fm)  (fm)  (fm)'
      write(20,*),'*****          *****          *****          *****          *****'
C
      write(40,*),'*****'
      write(40,*),'*** Our Model estimates Total Participant Yield ***'
      write(40,*),'*****'
      write(40,*),'  System      Energy/n  csam  H-2    H-3    He-3    He-4'
      write(40,*),'Proj<->Target  (MeV)          (fm) (fm)  (fm)  (fm)'
      write(40,*),'*****          *****          *****          *****          *****'
C
      write(25,*),'*****'
      write(25,*),'* Mekjian estimates Projectile and Target Mass ****'
      write(25,*),'*****'
      write(25,*),'  System      Energy/n  csam  H-2    H-3    He-3    He-4'
      write(25,*),'Proj<->Target  (MeV)          (fm) (fm)  (fm)  (fm)'
      write(25,*),'*****          *****          *****          *****          *****'
C
      open(30,file='CoalscenceRadius_MP.dat',status='old')
      open(35,file='CoalscenceRadius_Mek.dat',status='old')
      open(45,file='CoalscenceRadius_MPty.dat',status='old')
      open(80,file='CoalscenceRadius_MP_plot.dat',status='old')
C
      write(30,*),'*****'
      write(30,*),'*** Our Model estimates Average Participant Mass ****'
      write(30,*),'*****'
      write(30,*),'  System      Energy csam  H-2    H-3    He-3    He-4'
      write(30,*),'Proj<->Target  (MeV)          MeV/c (MeV/c) (MeV/c) (MeV/c)'
      write(30,*),'*****          *****          *****          *****          *****'
C
      write(35,*),'*****'
      write(35,*),'** Mekjian estimates Projectile and Target Mass **'
      write(35,*),'*****'
      write(35,*),'  System      Energy/n  H-2    H-3    He-3    He-4'
      write(35,*),'Proj<->Target  (MeV)    (MeV/c) (MeV/c) (MeV/c) (MeV/c)'
      write(35,*),'*****          *****          *****          *****          *****'

```

```

      write(45,*), '*****'
      write(45,*), '*** Our Model estimates Total Participant Yield ***'
      write(45,*), '*****'
      write(45,*), '  System      Energy/n   H-2      H-3      He-3   He-4'
      write(45,*), 'Proj<->Target (MeV)    (MeV/c) (MeV/c) (MeV/c) (MeV/c)'
      write(45,*), '*****'

c *****

c Open and read Heavy Ion Collision System input data.

c SYSTEM INPUT DATA FORMAT.  system data must be entered exactly in this
order

c Projectile<->Target System maximum 15 characters, Projectile Energy per
nucleon
c in MeV/n, Projectile charge number, Projectile mass number, Target charge
number,
c and Target mass number.

c *****

      open(unit=10, file='CollissionSystemInputData.txt', status='old')

      do i=1,121

c      read(10,*) temp2,EpN,zp,np,zt,nt
      read(10,*) temp2,EpN,zp,Ap,zt,At

      CALL SourceRadius(EpN,zp,Ap,zt,At,ave_pA,
&      csam, RD2_MP, RT3_MP, RHe3_MP, RHe4_MP,
&      RD2_Mpty, RT3_Mpty, RHe3_Mpty, RHe4_Mpty,
&      RD2_Mek, RT3_Mek, RHe3_Mek, RHe4_Mek)

      CALL CoalescenceRadius(EpN,zp,Ap,zt,At,ave_pA,
&      RD2_MP, RT3_MP, RHe3_MP, RHe4_MP,
&      RD2_Mpty, RT3_Mpty, RHe3_Mpty, RHe4_Mpty,
&      P0D2_MP, P0T3_MP, P0He3_MP, P0He4_MP,
&      RD2_Mek, RT3_Mek, RHe3_Mek, RHe4_Mek,
&      P0D2_Mek, P0T3_Mek, P0He3_Mek, P0He4_Mek,
&      P0D2_Mpty, P0T3_Mpty, P0He3_Mpty, P0He4_Mpty)

c *****

c Write output data.  Emitting Source Radius and Coalescence Radius

c *****

      write(20,100) temp2,EpN,csam,RD2_MP,RT3_MP,RHe3_MP,RHe4_MP
      write(25,100) temp2,EpN,csam,RD2_Mek,RT3_Mek,RHe3_Mek,RHe4_Mek

```

**Appendix I.1 Main FORTRAN Program Listing****Continued.....**

```
      write(40,100) temp2,EpN,csam,RD2_MPTy,RT3_MPTy,RHe3_MPTy,RHe4_MPTy
      write(60,600) csam,RD2_MP,RT3_MP,RHe3_MP,RHe4_MP
      write(30,300) temp2,EpN,csam,int(P0D2_MP),int(P0T3_MP),
&                  int(P0He3_MP),int(P0He4_MP)

      write(35,300) temp2,EpN,csam,int(P0D2_Mek),int(P0T3_Mek),
&                  int(P0He3_Mek),int(P0He4_Mek)

      write(45,300) temp2,EpN,csam,int(P0D2_MPTy),int(P0T3_MPTy),
&                  int(P0He3_MPTy),int(P0He4_MPTy)
      write(80,800) csam,int(P0D2_MP),int(P0T3_MP),
&                  int(P0He3_MP),int(P0He4_MP)

      enddo

100      format(1x,A,F6.1,2x,F5.1,2x,4(F4.1,2x))
600      format(2x,5(F6.1,1x))
300      format(1x,A,F6.1,1x,F5.1,2x,4(1x,I3,2x))
800      format(1x,F5.1,1x,4(1x,I3,2x))
c
70      close(10)
      close(20)
      close(25)
      close(30)
      close(35)
      close(40)
      close(45)

      stop
      end
```

## Appendix I.2: SourceRadius Subroutine Listing

```

C *****

c Subroutine SourceRadius
c This is a preliminary subroutine for calculating Source Radius for any
heavy ion collision system.

C *****

      subroutine SourceRadius(EpN,zp,Ap,zt,At,ave_pA,
&          csam,RD2_MP,RT3_MP,RHe3_MP,RHe4_MP,
&          RD2_MPTy,RT3_MPTy,RHe3_MPTy,RHe4_MPTy,
&          RD2_Mek,RT3_Mek,RHe3_Mek,RHe4_Mek)

      real*8 ave_zfp,ave_nfp, ave_zft,ave_nft, ave_pA
      real*8 EpN,zp,np,  zt, nt,  Ap, At,sma,gcs,r0
      real*8 nzRatio,pi, csam, coef1, coef2,coef3, coef4
      real*8 coef5, coef6, coef7, coef8
      real*8 RD2_MP, RT3_MP, RHe3_MP, RHe4_MP
      real*8 RD2_MPTy, RT3_MPTy, RHe3_MPTy, RHe4_MPTy
      real*8 RD2_Mek,RT3_Mek,RHe3_Mek,RHe4_Mek

c      Ap = zp + np
c      At = zt + nt
c Projectile neutron number
      np = Ap - zp
c Target neutron number
      nt = At - zt
c Collision System Mass Average
      csam = (Ap + At)/2.0
c System (Projectile + Target) neutron to proton ratio.

      nzRatio = (np + nt)/(zp + zt)

C *****

c Calculation of average participant mass for any heavy ion collision system
C *****

      pi = 4.0 * atan(1.0)

c r0 in (fm) from Nagamiya 1981

      r0 = 1.2

c Average number of participant charges from Projectile only

      ave_pfp = (zp*At**(2./3.))/(Ap**(1./3.)+At**(1./3.))**2

c Average number of participant Neutrons from Projectile only. assume

```

## Appendix I.2 SourceRadius Subroutine Listing      Continued.....

c Neutrons have the same spectrum as protons and multiply average proton  
c number by neutron to proton ratio of the projectile.

```
ave_nfp = ave_pfp * ((np+nt)/(zp+zt))
```

c Average number of participant charges from Target only

```
ave_pft = (zt*Ap**(2./3.))/(Ap**(1./3.)+At**(1./3.))**2
```

c Average number of participant Neutrons from Target only. assume  
c Neutrons have same spectrum as protons and multiply average proton  
c number by neutron to proton ratio of the target.

```
ave_nft = ave_pft * ((np+nt)/(zp+zt))
```

c The geometrical cross section in heavy ion collisions is estimated as  
(Nagamiya 1981)

```
gcs = (pi*r0**2)*(Ap**(1./3.)+At**(1./3.))**2
```

c Average participant mass for any heavy ion collision system.  
c from Projectile, Target, and participant region.

```
ave_Afp = ave_pfp+ave_nfp
ave_Aft = ave_pft+ave_nft
ave_pA = ((ave_pfp+ave_nfp)+(ave_pft+ave_nft))
```

c Average Total Participating Yield from projectile and target

```
ave_totalyield_pA = ave_pA * gcs
```

c Check Collision System Average Mass (csam) and set coefficient constant  
values

```
coef2 = 2.0
coef4 = 1.6
coef6 = 1.55
coef8 = 1.0
```

c Set coef1, coef3, coef5, and coef7 for group 1 (0<csam<=30)  
c group 1 was validated with C-12 C-12 (Lamaire 1979)

```
if(csam .le. 30.0) then
    coef1 = 0.25
    coef3 = 0.25
    coef5 = 0.26
    coef7 = 0.3
```

c Set coef1, coef3, coef5, and coef7 for group 2 (30<csam<=60)  
c group 2 was not validated because no estimates are available

```
elseif((csam .gt. 30.0) .and. (csam .le. 60.0)) then
```

## Appendix I.2 SourceRadius Subroutine Listing

Continued.....

```
        coef1 = 0.45
        coef3 = 0.45
        coef5 = 0.4
        coef7 = 0.3

c Set coef1, coef3, coef5,and coef7 for group 3 (60<csam<=90)
c group 3 was not validated because no estimates are available

        elseif((csam .gt. 60.0) .and. (csam .le. 90.0)) then
            coef1 = 0.52
            coef3 = 0.51
            coef5 = 0.48
            coef7 = 0.4

c Set coef1, coef3, coef5,and coef7 for group 4 (90<csam<=120)
c group 4 was validated with C-12 Pb-207 from (Lamaire 1979)

        elseif((csam .gt. 90) .and. (csam .le. 120)) then
            coef1 = 0.8
            coef3 = 0.7
            coef5 = 0.65
            coef7 = 0.55

c Set coef1, coef3, coef5,and coef7 for group 5 (120<csam<=150)
c group 5 was validated with Ne-20 U-238 from (Gutbrod 1976)

        elseif((csam .gt. 120) .and. (csam .le. 150)) then
            coef1 = 1.1
            coef3 = 0.85
            coef5 = 0.82
            coef7 = 0.75

c Set coef1, coef3, coef5,and coef7 for group 6 (150<csam<=180)
c group 6 was not validated because no estimates are available

        elseif((csam .gt. 150) .and. (csam .le. 180)) then
            coef1 = 1.1
            coef3 = 0.9
            coef5 = 0.85
            coef7 = 0.75

c Set coef1, coef3, coef5,and coef7 for group 7 (180<csam)
c group 7 was not validated because no estimates are available

        elseif(csam .gt. 180.0)then
            coef1 = 1.2
            coef3 = 1.0
            coef5 = 0.95
            coef7 = 0.8

    endif
```

## Appendix I.2 SourceRadius Subroutine Listing      Continued.....

c Source Radius of any Heavy Ion Collision System for H-2, H-3, He-3 and He-4  
c emitted secondary light particles.

c Source Radius in (fm) for Deuteron

c Source Radius in (fm) for Deuteron using projectile and target masses  
[Mekjian 1978]

$$Rd2\_Mek = 0.24 * (Ap^{1./3.} + At^{1./3.}) + 2.0$$

c Source Radius in (fm) for Deuteron using average participant masses [Our  
Model]

c         $Rd2\_MP = 0.2 * (ave\_Afp^{1./3.} + ave\_Aft^{1./3.}) + 2.0$   
       $Rd2\_MP = coef1 * ave\_pA^{1./3.} + coef2$

c Source Radius in (fm) for H-2 using ave total participant yield

$$Rd2\_MPty = 0.045 * ave\_totalyield\_pA^{1./3.} + 2.0$$

c root mean squared radius

$$\begin{aligned} Rd2\_Mek &= Rd2\_Mek * \sqrt{5./3.} \\ Rd2\_MP &= Rd2\_MP * \sqrt{5./3.} \\ Rd2\_MPty &= Rd2\_MPty * \sqrt{5./3.} \end{aligned}$$

c Source Radius(fm) for Triton

c Source Radius(fm) for H-3 [Mekjian 1978]

$$RT3\_Mek = 0.24 * (Ap^{1./3.} + At^{1./3.}) + 1.6$$

c Source Radius(fm) for H-3 [Our Model]

c         $RT3\_MP = 0.2 * (ave\_Afp^{1./3.} + ave\_Aft^{1./3.}) + 1.6$   
       $RT3\_MP = coef3 * ave\_pA^{1./3.} + coef4$   
       $RT3\_MPty = 0.09 * ave\_totalyield\_pA^{1./3.} + 1.6$

c root mean squared Radius

$$\begin{aligned} RT3\_MP &= RT3\_MP * \sqrt{5./3.} \\ RT3\_MPty &= RT3\_MPty * \sqrt{5./3.} \\ RT3\_Mek &= RT3\_Mek * \sqrt{5./3.} \end{aligned}$$

c Source Radius in (fm) for He-3

c Source Radius in (fm) for He-3 [Mekjian 1978]

$$RHe3\_Mek = 0.24 * (Ap^{1./3.} + At^{1./3.}) + 1.6$$

c Source Radius(fm) for He-3 [Our Models]

## Appendix I.2 SourceRadius Subroutine Listing      Continued.....

```
c      RHe3_MP = 0.2 * (ave_Afp**(1./3.)+ave_Aft**(1./3.)) +1.4
      RHe3_MP = coef5 * ave_pA**(1./3.) + coef6
      RHe3_Mpty = 0.09 *ave_totalyield_pA**(1./3.) + 1.4

c root mean squared radius

      RHe3_MP = RHe3_MP * sqrt(5./3.)
      RHe3_Mpty = RHe3_Mpty * sqrt(5./3.)
      RHe3_Mek = RHe3_Mek * sqrt(5./3.)

c Source Radius in (fm) for He-4 (Alpha particle)

c Source Radius in (fm) for He-4 [Mekjian 1978]

      RHe4_Mek = 0.24 * (Ap**(1./3.)+At**(1./3.)) +1.0

c Source Radius in (fm) for He-4 [Our Model]

c      RHe4_MP = 0.2 * (ave_Afp**(1./3.)+ave_Aft**(1./3.)) +1.0
      RHe4_MP = coef7 * ave_pA**(1./3.) + coef8
      RHe4_Mpty = 0.08 * ave_totalyield_pA**(1./3.) + 1.0

c root mean squared radius

      RHe4_MP = RHe4_MP * sqrt(5./3.)
      RHe4_Mpty = RHe4_Mpty * sqrt(5./3.)
      RHe4_Mek = RHe4_Mek * sqrt(5./3.)

      return
      End
```

## Appendix I.3: CoalescenceRadius Subroutine Listing

```
C *****
c Subroutine CoalescenceRadius

c This is a preliminary subroutine for calculating Coalescence Radius of c
any heavy ion collision system.
```

```
C *****
```

```
subroutine CoalescenceRadius(EpN,zp,Ap,zt,At,ave_pA,
&      RD2_MP,RT3_MP,RHe3_MP,RHe4_MP,
&      RD2_MPTy,RT3_MPTy,RHe3_MPTy,RHe4_MPTy,
&      P0D2_MP,P0T3_MP,P0He3_MP,P0He4_MP,
&      RD2_Mek,RT3_Mek,RHe3_Mek,RHe4_Mek,
&      P0D2_Mek,P0T3_Mek,P0He3_Mek,P0He4_Mek,
&      P0D2_MPTy,P0T3_MPTy,P0He3_MPTy,P0He4_MPTy)

real*8 ave_zfp,ave_nfp, ave_zft,ave_nft, ave_pA
real*8 EpN,zp,np,  zt, nt, Ap, At,gcs,r0
real*8 RD2_MP, RT3_MP, RHe3_MP, RHe4_MP
real*8 RD2_MPTy, RT3_MPTy, RHe3_MPTy, RHe4_MPTy
real*8 RD2_Mek,RT3_Mek,RHe3_Mek,RHe4_Mek
real*8 light,k,h,gama,pi,Tave,Tave_MeV
real*8 D2A,nzFD2,D2n,BeD2,SD2
real*8 T3A,nzFT3,T3n,BeT3,ST3
real*8 He3A,nzFHe3,He3n,BeHe3,SHe3
real*8 He4A,nzFHe4,He4n,BeHe4,SHe4
real*8 temp1_D2,temp2_D2,temp3_D2,temp4_D2,temp5_D2,temp6_D2
real*8 temp5_D2ty,temp5_T3ty,temp5_He3ty,temp5_He4ty
real*8 temp1_T3,temp2_T3,temp3_T3,temp4_T3,temp5_T3,temp6_T3
real*8 temp1_He3,temp2_He3,temp3_He3,temp4_He3,temp5_He3
real*8 temp6_He3,temp6_He4
real*8 temp1_He4,temp2_He4,temp3_He4,temp4_He4,temp5_He4
real*8 nzRatio,ave_Afp,ave_Aft
real*8 P0D2_MP,P0T3_MP,P0He3_MP,P0He4_MP
real*8 P0D2_MPTy,P0T3_MPTy,P0He3_MPTy,P0He4_MPTy
real*8 P0D2_Mek,P0T3_Mek,P0He3_Mek,P0He4_Mek

      np = Ap - zp
      nt = At - zt
```

```
c System (Projectile + Target) neutron to proton ratio.
```

```
      nzRatio = (np + nt)/(zp + zt)
```

```
C *****
```

```
c Calculation of average participant mass for any heavy ion collisssion system
```

## Appendix I.3 CoalescenceRadius Subroutine Listing      Continued.....

```

C *****

      pi = 4.0 * atan(1.0)

C r0 in (fm) from Nagamiya 1981

      r0 = 1.2

C Light speed in m/s

      light = 299792458.0

C Boltzman constant in MeV/k

      k = 8.617343E-11

C Plank Constant in MeV s

      h = 4.13566732E-21

C
      Ap = zp + np
      gama =1.0 + EpN /(Ap*931.)

C average source temperature in degrees k

      Tave = (2. * EpN) / (3. * k)

C average source temperature in MeV

      Tave_MeV = Tave * k

C Deuteron mass; neutron number; n!z!; binding energy in MeV and spin

      D2A = 2.
      D2n = 1.
C n!z!
      nzFD2 = 1.

      BeD2 = 2.22452
      SD2 = 1.

C Coalescence Radius P0 estimates in MeV/c for H-2 using MP Source Radius
estimates

      temp1_D2 = D2A**(1./(D2A-1.))
      temp2_D2 = ((2.*SD2+1.)/(2.*D2A))**(1./(3.*(D2A-1.)))
      temp3_D2 = (exp((BeD2/D2A)/Tave)*nzFD2)**(1./(3.*(D2A-1.)))
      temp4_D2 = (4.5 * Gama/pi**2)**(1./3.)

C Largest volume where chemical and thermal equilibrium is maintained
C root mean squared Radius used

```

## Appendix I.3 CoalescenceRadius Subroutine Listing      Continued.....

```

temp5_D2 = (h*light)/(2.*RD2_MP*(10.**-15))

temp6_D2 = nzRatio**(D2n/(3.*(D2A-1.)))
P0D2_MP = temp1_D2*temp2_D2*temp3_D2*temp4_D2*temp5_D2*temp6_D2

c Coalescence Radius P0 estimates in MeV/c for D-2 using MPty Source Radius
estimates (Total Participant Yield)

temp5_D2ty = (h*light)/(2.*RD2_MPty*(10.**-15))
P0D2_MPty = temp1_D2*temp2_D2*temp3_D2*temp4_D2*temp5_D2ty*
& temp6_D2

c Coalescence Radius P0 estimates in MeV/c for H-2 using Mekjian Source
Radius estimates

temp5_D2 = (h*light)/(2.*RD2_Mek*(10.**-15))
P0D2_Mek = temp1_D2*temp2_D2*temp3_D2*temp4_D2*temp5_D2*temp6_D2

c Triton mass number; Triton neutron number; n!z!; binding energy in MeV and
spin

T3A = 3.
T3n = 2.
nzFT3 = 2.
BeT3 = 4.481821
ST3 = 0.5

c Coalescence Radius P0 estimates in MeV/c for H-3 using MP Source Radius
estimates

temp1_T3 = T3A**(1./(T3A-1.))
temp2_T3 = ((2*ST3+1.)/(2.**T3A))**(1./(3.*(T3A-1.)))
temp3_T3 = (exp((BeT3/T3A)/Tave)*nzFT3)**(1./(3.*(T3A-1.)))
temp4_T3 = (4.5 * Gama/pi**2)**(1./3.)

c Largest volume where chemical and thermal equilibrium is maintained
c root mean squared Radius used

temp5_T3 = (h*light)/(2.*RT3_MP*(10.**-15))
temp6_T3 = nzRatio**(T3n/(3.*(T3A-1.)))
P0T3_MP = temp1_T3*temp2_T3*temp3_T3*temp4_T3*temp5_T3*temp6_T3

c CoalescenceRadius P0 estimates in MeV/c for H-3 using MP Source Radius
estimates (Total Participant Yield)

temp5_T3ty = (h*light)/(2.*RT3_MPty*(10.**-15))
P0T3_MPty=temp1_T3*temp2_T3*temp3_T3*temp4_T3*temp5_T3ty*
& temp6_T3

temp5_T3 = (h*light)/(2.*RT3_Mek*(10.**-15))
P0T3_Mek = temp1_T3*temp2_T3*temp3_T3*temp4_T3*temp5_T3*temp6_T3
c He-3 mass; n!z!; binding energy in MeV and spin

```

## Appendix I.3 CoalescenceRadius Subroutine Listing      Continued.....

```

      He3A = 3.
      He3n = 1.
      nzFHe3 = 2.
      BeHe3 = 7.718043
      SHe3 = 0.5

c Coalescence Radius P0 estimates in MeV/c for He-3 using MP Source Radius
estimates
c Largest volume where chemical and thermal equilibrium is maintained
c root mean squared Radius is used

      temp1_He3 = He3A**(1. / (He3A-1.))
      temp2_He3 = ((2.*SHe3+1.) / (2.**He3A))**(1. / (3.*(He3A-1.)))
      temp3_He3 = (exp((BeHe3/He3A)/Tave)*nzFHe3)**(1. / (3.*(He3A-1.)))
      temp4_He3 = (4.5 * Gama/pi**2)**(1./3.)

      temp5_He3 = (h*light)/(2.*RHe3_MP*(10.**-15))
      temp6_He3 = nzRatio**(He3n/(3.*(He3A-1.)))

      P0He3_MP = temp1_He3*temp2_He3*temp3_He3*
&               temp4_He3*temp5_He3*temp6_He3

c Coalescence Radius P0 estimates in MeV/c for He-3 using MPty Source Radius
estimates (Total Participant Yield)

      temp5_He3 = (h*light)/(2.*RHe3_MPty*(10.**-15))
      P0He3_MPty = temp1_He3*temp2_He3*temp3_He3*
&               temp4_He3*temp5_He3*temp6_He3

c Coalescence Radius P0 estimates in MeV/c for He-3 using Mek Source Radius
estimates (Total Participant Yield)

      temp5_He3 = (h*light)/(2.*RHe3_Mek*(10.**-15))
      P0He3_Mek = temp1_He3*temp2_He3*temp3_He3*
&               temp4_He3*temp5_He3*temp6_He3

c He-4 mass; n!z!; binding energy in MeV and spin

      He4A = 4.
      He4n = 2.
      nzFHe4 = 4.
      BeHe4 = 28.3007
      SHe4 = 0.

c Coalescence Radius P0 estimates in MeV/c for He-4 using MP Source Radius
estimate
c Largest volume where chemical and thermal equilibrium is maintained
c root mean squared radius is used.

```

## Appendix I.3 CoalescenceRadius Subroutine Listing      Continued.....

```

temp1_He4 = He4A**(1. / (He4A-1.))
temp2_He4 = ((2.*SHe4+1.) / (2.**He4A))**(1. / (3.*(He4A-1.)))
temp3_He4 = (exp((BeHe4/He4A)/Tave)*nzFHe4)**(1. / (3.*(He4A-1.)))
temp4_He4 = (4.5 * Gama/pi**2)**(1./3.)

temp5_He4 = (h*light)/(2.*RHe4_MP*(10.**-15))
temp6_He4 = nzRatio**(He4n/(3.*(He4A-1.)))

c CoalescenceRadius P0 estimates in MeV/c for He-4 using MP Source Radius
estimate

P0He4_MP = temp1_He4*temp2_He4*temp3_He4*
&          temp4_He4*temp5_He4*temp6_He4

c Coalescence Radius P0 estimates in MeV/c for He-4 using MPty Source Radius
estimates (Total Participant Yield)

temp5_He4 = (h*light)/(2.*RHe4_MPty*(10.**-15))
P0He4_MPty = temp1_He3*temp2_He3*temp3_He3*
&          temp4_He3*temp5_He3*temp6_He3

c Coalescence Radius P0 estimates in MeV/c for He-4 using Mek Source Radius
estimate

temp5_He4 = (h*light)/(2.*RHe4_Mek*(10.**-15))
P0He4_Mek = temp1_He4*temp2_He4*temp3_He4*
&          temp4_He4*temp5_He4*temp6_He4

return
End

```

## Appendix I.4: Heavy Ion Collision System input data text file and format

This is the listing of the heavy ion collision systems input data text file used to estimate source radii and coalescence radii. If you add data for an addition collision system, you need to update the number of the rows with system data in the main program manually. SYSTEM DATA INPUT FORMAT must be followed as written below

*Projectile-mass number <->Target-mass number maximum 15 characters,  
Projectile Energy in MeV/Nucleon,  
Projectile proton number, Projectile mass number,  
Target proton number and Target mass number.*

```
He-4<->He-4,400.0,2.0,4.0,2.0,4.0
He-4<->Li-7,400.0,2.0,4.0,3.0,7.0
He-3<->C-12,400.0,2.0,3.0,6.0,12.0
He-4<->C-12,400.0,2.0,4.0,6.0,12.0
Li-7<->C-12,400.0,3.0,7.0,6.0,12.0
C-12<->C-12,800.0,6.0,12.0,6.0,12.0
N-14<->C-12,400.0,7.0,14.0,6.0,12.0
O-16<->C-12,400.0,8.0,16.0,6.0,12.0
He-3<->Al-27,400.0,2.0,3.0,13.0,27.0
He-4<->Al-27,400.0,2.0,4.0,13.0,27.0
Li-7<->Mg-24,400.0,3.0,7.0,12.0,24.0
C-12<->F-19,400.0,6.0,12.0,9.0,19.0
C-12<->Ne-20,400.0,6.0,12.0,10.0,20.0
Si-28<->C-12,400.0,14.0,28.0,6.0,12.0
N-14<->Al-27,400.0,7.0,14.0,13.0,27.0
Ne-20<->NaF-42,400.0,10.0,20.0,10.0,21.0
Ne-20<->NaF-42,800.0,10.0,20.0,10.0,21.0
Ne-20<->NaF-42,2100.0,10.0,20.0,10.0,21.0
N-14<->Al-27,400.0,7.0,14.0,13.0,27.0
O-16<->Al-27,400.0,8.0,16.0,13.0,27.0
F-19<->Mg-24,400.0,9.0,19.0,12.0,24.0
C-12<->S-32,400.0,6.0,12.0,16.0,32.0
Ne-20<->Al-27,400.0,10.0,20.0,13.0,27.0
Cl-35<->C-12,400.0,17.0,35.0,6.0,12.0
Ar-40<->Be-9,400.0,18.0,40.0,4.0,9.0
Ar-40<->C-12,400.0,18.0,40.0,6.0,12.0
Si-28<->Al-27,400.0,14.0,28.0,13.0,27.0
Al-27<->S-32,400.0,13.0,27.0,16.0,32.0
He-4<->Fe-56,400.0,2.0,4.0,26.0,56.0
Na-23<->Cl-37,400.0,11.0,23.0,17.0,37.0
He-4<->Fe-57,400.0,2.0,4.0,26.0,57.0
Li-7<->Mn-54,400.0,3.0,7.0,25.0,54.0
Cl-35<->Al-27,400.0,17.0,35.0,13.0,27.0
Li-7<->Fe-56,400.0,3.0,7.0,26.0,56.0
Ar-40<->Al-27,400.0,18.0,40.0,13.0,27.0
Fe-56<->C-12,400.0,26.0,56.0,6.0,12.0
O-16<->Ni-59,400.0,8.0,16.0,28.0,59.0
Ar-40<->KCl-74,92.0,18.0,40.0,18.0,37.0
Ar-40<->KCl-74,800.0,18.0,40.0,18.0,37.0
N-14<->Cu-64,400.0,7.0,14.0,29.0,64.0
O-16<->Cu-64,400.0,8.0,16.0,29.0,64.0
```

**Appendix I.4 Heavy Ion Collision System input data****Continued.....**

Ar-40<->Ca-40, 92.0, 18.0, 40.0, 20.0, 40.0  
Ar-40<->Ca-40, 137.0, 18.0, 40.0, 20.0, 40.0  
Fe-56<->Al-27, 400.0, 26.0, 56.0, 13.0, 27.0  
Ne-20<->Cu-64, 400.0, 10.0, 20.0, 29.0, 64.0  
He-4<->Sr-87, 400.0, 2.0, 4.0, 38.0, 87.0  
Na-23<->Zn-68, 400.0, 11.0, 23.0, 30.0, 68.0  
Ca-40<->V-51, 400.0, 20.0, 40.0, 23.0, 51.0  
Sc-45<->Ti-46, 400.0, 21.0, 45.0, 22.0, 46.0  
Si-28<->Cu-64, 400.0, 14.0, 28.0, 29.0, 64.0  
Cl-35<->Cu-64, 400.0, 17.0, 35.0, 29.0, 64.0  
Ar-40<->Cu-64, 92.0, 18.0, 40.0, 29.0, 64.0  
Ar-40<->Cu-64, 137.0, 18.0, 40.0, 29.0, 64.0  
C-12<->Nb-93, 400.0, 6.0, 12.0, 41.0, 93.0  
He-3<->Ag-108, 400.0, 2.0, 3.0, 47.0, 108.0  
Kr-84<->Al-27, 400.0, 36.0, 84.0, 13.0, 27.0  
Ni-58<->Ni-60, 400.0, 28.0, 58.0, 28.0, 60.0  
Fe-56<->Cu-64, 400.0, 26.0, 56.0, 29.0, 64.0  
Mg-24<->Tc-97, 400.0, 12.0, 24.0, 43.0, 97.0  
Cl-35<->Rb-86, 400.0, 17.0, 35.0, 37.0, 86.0  
Fe-56<->Zn-65, 400.0, 26.0, 56.0, 30.0, 65.0  
Ne-20<->Sn-119, 400.0, 10.0, 20.0, 50.0, 119.0  
Si-28<->Sn-119, 400.0, 14.0, 28.0, 50.0, 119.0  
C-12<->La-139, 400.0, 6.0, 12.0, 57.0, 139.0  
C-12<->Pr-141, 400.0, 6.0, 12.0, 59.0, 141.0  
Cl-35<->Sn-119, 400.0, 17.0, 35.0, 50.0, 119.0  
Xe-132<->Al-27, 400.0, 51.0, 132.0, 13.0, 27.0  
Ar-40<->Sn-119, 400.0, 18.0, 40.0, 50.0, 119.0  
Fe-56<->Ag-108, 400.0, 26.0, 56.0, 47.0, 108.0  
O-16<->Sm-154, 400.0, 8.0, 16.0, 62.0, 154.0  
Kr-84<->Nb-93, 400.0, 36.0, 84.0, 41.0, 93.0  
Al-27<->Gd-154, 400.0, 13.0, 27.0, 64.0, 154.0  
Fe-56<->Te-127, 400.0, 26.0, 56.0, 52.0, 127.0  
Ho-165<->Al-27, 400.0, 67.0, 165.0, 13.0, 27.0  
C-12<->Ta-181, 3400.0, 6.0, 12.0, 73.0, 181.0  
He-4<->Au-197, 400.0, 2.0, 4.0, 79.0, 197.0  
Ne-20<->Ta-181, 400.0, 10.0, 20.0, 73.0, 181.0  
Be-9<->Au-197, 400.0, 4.0, 9.0, 79.0, 197.0  
C-12<->Au-197, 400.0, 6.0, 12.0, 79.0, 197.0  
O-16<->Au-197, 400.0, 8.0, 16.0, 79.0, 197.0  
Be-9<->Pb-208, 400.0, 4.0, 9.0, 82.0, 208.0  
C-12<->Pb-207, 400.0, 6.0, 12.0, 82.0, 207.0  
N-14<->Pb-207, 400.0, 7.0, 14.0, 82.0, 207.0  
O-16<->Pb-207, 400.0, 8.0, 16.0, 82.0, 207.0  
Ne-20<->Pb-207, 400.0, 10.0, 20.0, 82.0, 207.0  
Ne-20<->Pb-207, 800.0, 10.0, 20.0, 82.0, 207.0  
Ne-20<->Pb-207, 2100.0, 10.0, 20.0, 82.0, 207.0  
Ne-20<->Bi-209, 400.0, 10.0, 20.0, 83.0, 209.0  
Mg-24<->Bi-209, 400.0, 12.0, 24.0, 83.0, 209.0  
Si-28<->Pb-207, 400.0, 14.0, 28.0, 82.0, 207.0  
Zr-91<->Sm-149, 400.0, 40.0, 91.0, 62.0, 149.0  
K-39<->Ti-202, 400.0, 19.0, 39.0, 81.0, 202.0

**Appendix I.4 Heavy Ion Collision System input data****Continued.....**

He-4<->U-238,250.0,2.0,4.0,92.0,238.0  
Cl-35<->Pb-207,400.0,17.0,35.0,82.0,207.0  
Sb-122<->Sb-122,400.0,51.0,122.0,51.0,122.0  
Be-9<->U-238,400.0,4.0,9.0,92.0,238.0  
Ar-40<->Pb-207,92.0,18.0,40.0,82.0,207.0  
Ar-40<->Pb-207,800.0,18.0,40.0,82.0,207.0  
O-16<->U-238,250.0,8.0,16.0,92.0,238.0  
Ne-20<->U-235,400.0,10.0,20.0,92.0,235.0  
Ne-20<->U-238,250.0,10.0,20.0,92.0,238.0  
Ne-20<->U-238,400.0,10.0,20.0,92.0,238.0  
Ne-20<->U-238,2100.0,10.0,20.0,92.0,238.0  
Fe-56<->Ti-204,400.0,26.0,56.0,81.0,204.0  
Fe-56<->Pb-207,400.0,26.0,56.0,82.0,207.0  
Al-27<->U-238,2100.0,13.0,27.0,92.0,238.0  
Pb-208<->Cu-64,400.0,82.0,208.0,29.0,64.0  
Pd-106<->Lu-174,400.0,46.0,106.0,71.0,174.0  
Ba-137<->Nd-143,400.0,56.0,137.0,60.0,143.0  
Kr-86<->Au-197,400.0,36.0,86.0,79.0,197.0  
Fe-56<->U-238,400.0,26.0,56.0,92.0,238.0  
Ni-59<->U-238,400.0,28.0,59.0,92.0,238.0  
Mn-55<->Am-245,400.0,25.0,55.0,95.0,245.0  
Sb-122<->Hf-178,400.0,51.0,122.0,72.0,178.0  
Sm-150<->Sm-150,400.0,62.0,150.0,62.0,150.0  
Zr-91<->At-210,400.0,40.0,91.0,85.0,210.0  
Ba-137<->Ho-164,400.0,56.0,137.0,67.0,164.0  
Cu-64<->U-238,400.0,29.0,64.0,92.0,238.0  
Ta-180<->-180,400.0,73.0,180.0,73.0,180.0  
Au-197<->Au-197,400.0,79.0,197.0,79.0,197.0  
Pb-207<->U-238,400.0,82.0,207.0,92.0,238.0

120 data sets

## Appendix I.5: Source Radii Output Listing

```

*****
*** Our Model estimates Average Participant Mass ***
*****

```

| System<br>Proj<->Target | Energy/n<br>(MeV) | csam  | H-2<br>(fm) | H-3<br>(fm) | He-3<br>(fm) | He-4<br>(fm) |
|-------------------------|-------------------|-------|-------------|-------------|--------------|--------------|
| *****                   | *****             | ***** | *****       | *****       | *****        | *****        |
| He-4<->He-4             | 400.0             | 4.0   | 3.0         | 2.5         | 2.4          | 1.8          |
| He-4<->Li-7             | 400.0             | 5.5   | 3.0         | 2.5         | 2.5          | 1.8          |
| He-3<->C-12             | 400.0             | 7.5   | 3.1         | 2.5         | 2.5          | 1.9          |
| He-4<->C-12             | 400.0             | 8.0   | 3.1         | 2.6         | 2.5          | 1.9          |
| Li-7<->C-12             | 400.0             | 9.5   | 3.1         | 2.6         | 2.6          | 1.9          |
| C-12<->C-12             | 800.0             | 12.0  | 3.2         | 2.7         | 2.6          | 2.0          |
| N-14<->C-12             | 400.0             | 13.0  | 3.2         | 2.7         | 2.6          | 2.0          |
| O-16<->C-12             | 400.0             | 14.0  | 3.2         | 2.7         | 2.6          | 2.0          |
| He-3<->Al-27            | 400.0             | 15.0  | 3.1         | 2.6         | 2.6          | 1.9          |
| He-4<->Al-27            | 400.0             | 15.5  | 3.1         | 2.6         | 2.6          | 2.0          |
| Li-7<->Mg-24            | 400.0             | 15.5  | 3.2         | 2.7         | 2.6          | 2.0          |
| C-12<->F-19             | 400.0             | 15.5  | 3.2         | 2.7         | 2.7          | 2.1          |
| C-12<->Ne-20            | 400.0             | 16.0  | 3.2         | 2.7         | 2.7          | 2.1          |
| Si-28<->C-12            | 400.0             | 20.0  | 3.3         | 2.7         | 2.7          | 2.1          |
| N-14<->Al-27            | 400.0             | 20.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| Ne-20<->NaF-42          | 400.0             | 20.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| Ne-20<->NaF-42          | 800.0             | 20.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| Ne-20<->NaF-42          | 2100.0            | 20.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| N-14<->Al-27            | 400.0             | 20.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| O-16<->Al-27            | 400.0             | 21.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| F-19<->Mg-24            | 400.0             | 21.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| C-12<->S-32             | 400.0             | 22.0  | 3.3         | 2.8         | 2.7          | 2.1          |
| Ne-20<->Al-27           | 400.0             | 23.5  | 3.3         | 2.8         | 2.8          | 2.2          |
| Cl-35<->C-12            | 400.0             | 23.5  | 3.3         | 2.8         | 2.7          | 2.1          |
| Ar-40<->Be-9            | 400.0             | 24.5  | 3.3         | 2.7         | 2.7          | 2.1          |
| Ar-40<->C-12            | 400.0             | 26.0  | 3.3         | 2.8         | 2.7          | 2.2          |
| Si-28<->Al-27           | 400.0             | 27.5  | 3.4         | 2.8         | 2.8          | 2.2          |
| Al-27<->S-32            | 400.0             | 29.5  | 3.4         | 2.9         | 2.8          | 2.2          |
| He-4<->Fe-56            | 400.0             | 30.0  | 3.2         | 2.7         | 2.6          | 2.0          |
| Na-23<->Cl-37           | 400.0             | 30.0  | 3.4         | 2.9         | 2.8          | 2.2          |
| He-4<->Fe-57            | 400.0             | 30.5  | 3.7         | 3.2         | 3.0          | 2.0          |
| Li-7<->Mn-54            | 400.0             | 30.5  | 3.8         | 3.3         | 3.1          | 2.1          |
| Cl-35<->Al-27           | 400.0             | 31.0  | 4.0         | 3.5         | 3.3          | 2.3          |
| Li-7<->Fe-56            | 400.0             | 31.5  | 3.8         | 3.3         | 3.1          | 2.1          |
| Ar-40<->Al-27           | 400.0             | 33.5  | 4.1         | 3.5         | 3.3          | 2.3          |
| Fe-56<->C-12            | 400.0             | 34.0  | 3.9         | 3.4         | 3.2          | 2.2          |
| O-16<->Ni-59            | 400.0             | 37.5  | 4.0         | 3.5         | 3.3          | 2.2          |
| Ar-40<->KCl-74          | 92.0              | 38.5  | 4.1         | 3.6         | 3.4          | 2.3          |
| Ar-40<->KCl-74          | 800.0             | 38.5  | 4.1         | 3.6         | 3.4          | 2.3          |
| N-14<->Cu-64            | 400.0             | 39.0  | 4.0         | 3.5         | 3.3          | 2.2          |
| O-16<->Cu-64            | 400.0             | 40.0  | 4.0         | 3.5         | 3.3          | 2.3          |
| Ar-40<->Ca-40           | 92.0              | 40.0  | 4.2         | 3.6         | 3.4          | 2.3          |
| Ar-40<->Ca-40           | 137.0             | 40.0  | 4.2         | 3.6         | 3.4          | 2.3          |
| Fe-56<->Al-27           | 400.0             | 41.5  | 4.1         | 3.6         | 3.4          | 2.3          |
| Ne-20<->Cu-64           | 400.0             | 42.0  | 4.1         | 3.6         | 3.4          | 2.3          |

## Appendix I.5 Source Radii Output Listing      Continued.....

| *****          |          |       |      |       |       |       |
|----------------|----------|-------|------|-------|-------|-------|
| System         | Energy/n | csam  | H-2  | H-3   | He-3  | He-4  |
| Proj<->Target  | (MeV)    |       | (fm) | (fm)  | (fm)  | (fm)  |
| *****          | *****    | ***** | **** | ***** | ***** | ***** |
| He-4<->Sr-87   | 400.0    | 45.5  | 3.8  | 3.3   | 3.1   | 2.1   |
| Na-23<->Zn-68  | 400.0    | 45.5  | 4.2  | 3.6   | 3.4   | 2.3   |
| Ca-40<->V-51   | 400.0    | 45.5  | 4.2  | 3.7   | 3.5   | 2.4   |
| Sc-45<->Ti-46  | 400.0    | 45.5  | 4.2  | 3.7   | 3.5   | 2.4   |
| Si-28<->Cu-64  | 400.0    | 46.0  | 4.2  | 3.7   | 3.4   | 2.4   |
| Cl-35<->Cu-64  | 400.0    | 49.5  | 4.3  | 3.7   | 3.5   | 2.4   |
| Ar-40<->Cu-64  | 92.0     | 52.0  | 4.3  | 3.8   | 3.5   | 2.4   |
| Ar-40<->Cu-64  | 137.0    | 52.0  | 4.3  | 3.8   | 3.5   | 2.4   |
| C-12<->Nb-93   | 400.0    | 52.5  | 4.1  | 3.5   | 3.3   | 2.3   |
| He-3<->Ag-108  | 400.0    | 55.5  | 3.8  | 3.2   | 3.1   | 2.1   |
| Kr-84<->Al-27  | 400.0    | 55.5  | 4.3  | 3.7   | 3.5   | 2.4   |
| Ni-58<->Ni-60  | 400.0    | 59.0  | 4.4  | 3.9   | 3.6   | 2.5   |
| Fe-56<->Cu-64  | 400.0    | 60.0  | 4.4  | 3.9   | 3.6   | 2.5   |
| Mg-24<->Tc-97  | 400.0    | 60.5  | 4.5  | 4.0   | 3.8   | 2.8   |
| Cl-35<->Rb-86  | 400.0    | 60.5  | 4.6  | 4.1   | 3.9   | 2.9   |
| Fe-56<->Zn-65  | 400.0    | 60.5  | 4.7  | 4.1   | 3.9   | 2.9   |
| Ne-20<->Sn-119 | 400.0    | 69.5  | 4.5  | 4.0   | 3.8   | 2.8   |
| Si-28<->Sn-119 | 400.0    | 73.5  | 4.6  | 4.1   | 3.9   | 2.9   |
| C-12<->La-139  | 400.0    | 75.5  | 4.4  | 3.8   | 3.7   | 2.7   |
| C-12<->Pr-141  | 400.0    | 76.5  | 4.4  | 3.8   | 3.7   | 2.7   |
| Cl-35<->Sn-119 | 400.0    | 77.0  | 4.7  | 4.2   | 4.0   | 2.9   |
| Xe-132<->Al-27 | 400.0    | 79.5  | 4.7  | 4.1   | 3.9   | 2.9   |
| Ar-40<->Sn-119 | 400.0    | 79.5  | 4.8  | 4.2   | 4.0   | 3.0   |
| Fe-56<->Ag-108 | 400.0    | 82.0  | 4.9  | 4.3   | 4.1   | 3.0   |
| O-16<->Sm-154  | 400.0    | 85.0  | 4.5  | 4.0   | 3.8   | 2.8   |
| Kr-84<->Nb-93  | 400.0    | 88.5  | 5.0  | 4.4   | 4.2   | 3.1   |
| Al-27<->Gd-154 | 400.0    | 90.5  | 5.9  | 4.9   | 4.7   | 3.5   |
| Fe-56<->Te-127 | 400.0    | 91.5  | 6.2  | 5.2   | 4.9   | 3.8   |
| Ho-165<->Al-27 | 400.0    | 96.0  | 5.9  | 5.0   | 4.7   | 3.6   |
| C-12<->Ta-181  | 3400.0   | 96.5  | 5.5  | 4.6   | 4.4   | 3.3   |
| He-4<->Au-197  | 400.0    | 100.5 | 5.0  | 4.1   | 3.9   | 2.9   |
| Ne-20<->Ta-181 | 400.0    | 100.5 | 5.8  | 4.9   | 4.6   | 3.5   |
| Be-9<->Au-197  | 400.0    | 103.0 | 5.3  | 4.5   | 4.2   | 3.2   |
| C-12<->Au-197  | 400.0    | 104.5 | 5.5  | 4.6   | 4.4   | 3.3   |
| O-16<->Au-197  | 400.0    | 106.5 | 5.7  | 4.8   | 4.5   | 3.4   |
| Be-9<->Pb-208  | 400.0    | 108.5 | 5.4  | 4.5   | 4.3   | 3.2   |
| C-12<->Pb-207  | 400.0    | 109.5 | 5.5  | 4.7   | 4.4   | 3.3   |
| N-14<->Pb-207  | 400.0    | 110.5 | 5.6  | 4.7   | 4.5   | 3.4   |
| O-16<->Pb-207  | 400.0    | 111.5 | 5.7  | 4.8   | 4.5   | 3.4   |
| Ne-20<->Pb-207 | 400.0    | 113.5 | 5.8  | 4.9   | 4.7   | 3.5   |
| Ne-20<->Pb-207 | 800.0    | 113.5 | 5.8  | 4.9   | 4.7   | 3.5   |
| Ne-20<->Pb-207 | 2100.0   | 113.5 | 5.8  | 4.9   | 4.7   | 3.5   |
| Ne-20<->Bi-209 | 400.0    | 114.5 | 5.9  | 4.9   | 4.7   | 3.5   |
| Mg-24<->Bi-209 | 400.0    | 116.5 | 6.0  | 5.0   | 4.7   | 3.6   |
| Si-28<->Pb-207 | 400.0    | 117.5 | 6.1  | 5.1   | 4.8   | 3.7   |
| Zr-91<->Sm-149 | 400.0    | 120.0 | 6.6  | 5.6   | 5.3   | 4.0   |
| K-39<->Ti-202  | 400.0    | 120.5 | 7.6  | 6.0   | 5.8   | 4.7   |
| He-4<->U-238   | 250.0    | 121.0 | 5.9  | 4.7   | 4.5   | 3.6   |

## Appendix I.5 Source Radii Output Listing      Continued.....

| *****           |          |       |       |       |       |       |
|-----------------|----------|-------|-------|-------|-------|-------|
| System          | Energy/n | csam  | H-2   | H-3   | He-3  | He-4  |
| Proj<->Target   | (MeV)    |       | (fm)  | (fm)  | (fm)  | (fm)  |
| *****           | *****    | ***** | ***** | ***** | ***** | ***** |
| Cl-35<->Pb-207  | 400.0    | 121.0 | 7.5   | 5.9   | 5.7   | 4.7   |
| Sb-122<->Sb-122 | 400.0    | 122.0 | 8.2   | 6.4   | 6.2   | 5.1   |
| Be-9<->U-238    | 400.0    | 123.5 | 6.5   | 5.1   | 4.9   | 4.0   |
| Ar-40<->Pb-207  | 92.0     | 123.5 | 7.6   | 6.0   | 5.8   | 4.7   |
| Ar-40<->Pb-207  | 800.0    | 123.5 | 7.6   | 6.0   | 5.8   | 4.7   |
| O-16<->U-238    | 250.0    | 127.0 | 7.0   | 5.5   | 5.3   | 4.3   |
| Ne-20<->U-235   | 400.0    | 127.5 | 7.2   | 5.6   | 5.4   | 4.4   |
| Ne-20<->U-238   | 250.0    | 129.0 | 7.2   | 5.6   | 5.4   | 4.4   |
| Ne-20<->U-238   | 400.0    | 129.0 | 7.2   | 5.6   | 5.4   | 4.4   |
| Ne-20<->U-238   | 2100.0   | 129.0 | 7.2   | 5.6   | 5.4   | 4.4   |
| Fe-56<->Ti-204  | 400.0    | 130.0 | 7.9   | 6.2   | 6.0   | 4.9   |
| Fe-56<->Pb-207  | 400.0    | 131.5 | 8.0   | 6.2   | 6.0   | 5.0   |
| Al-27<->U-238   | 2100.0   | 132.5 | 7.4   | 5.8   | 5.6   | 4.6   |
| Pb-208<->Cu-64  | 400.0    | 136.0 | 8.1   | 6.3   | 6.1   | 5.0   |
| Pd-106<->Lu-174 | 400.0    | 140.0 | 8.4   | 6.5   | 6.3   | 5.2   |
| Ba-137<->Nd-143 | 400.0    | 140.0 | 8.4   | 6.6   | 6.4   | 5.3   |
| Kr-86<->Au-197  | 400.0    | 141.5 | 8.3   | 6.5   | 6.3   | 5.2   |
| Fe-56<->U-238   | 400.0    | 147.0 | 8.1   | 6.3   | 6.1   | 5.0   |
| Ni-59<->U-238   | 400.0    | 148.5 | 8.1   | 6.4   | 6.1   | 5.1   |
| Mn-55<->Am-245  | 400.0    | 150.0 | 8.1   | 6.3   | 6.1   | 5.0   |
| Sb-122<->Hf-178 | 400.0    | 150.0 | 8.5   | 6.7   | 6.4   | 5.4   |
| Sm-150<->Sm-150 | 400.0    | 150.0 | 8.6   | 6.7   | 6.5   | 5.4   |
| Zr-91<->At-210  | 400.0    | 150.5 | 8.4   | 6.8   | 6.5   | 5.3   |
| Ba-137<->Ho-164 | 400.0    | 150.5 | 8.6   | 7.0   | 6.6   | 5.4   |
| Cu-64<->U-238   | 400.0    | 151.0 | 8.2   | 6.7   | 6.3   | 5.1   |
| Ta-180<->-180   | 400.0    | 180.0 | 8.9   | 7.3   | 6.9   | 5.6   |
| Au-197<->Au-197 | 400.0    | 197.0 | 9.7   | 8.0   | 7.7   | 6.1   |
| Pb-207<->U-238  | 400.0    | 222.5 | 10.0  | 8.3   | 7.9   | 6.3   |

## Appendix I.6: Coalescence Radii Output Listing

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*****
*** Our Model estimates Average Participant Mass ***
*****
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| System         | Energy | csam  | H-2     | H-3     | He-3    | He-4    |
|----------------|--------|-------|---------|---------|---------|---------|
| Proj<->Target  | (MeV)  |       | (MeV/c) | (MeV/c) | (MeV/c) | (MeV/c) |
| *****          | *****  | ***** | *****   | *****   | *****   | *****   |
| He-4<->He-4    | 400.0  | 4.0   | 300     | 308     | 314     | 377     |
| He-4<->Li-7    | 400.0  | 5.5   | 314     | 322     | 318     | 382     |
| He-3<->C-12    | 400.0  | 7.5   | 284     | 290     | 302     | 355     |
| He-4<->C-12    | 400.0  | 8.0   | 292     | 298     | 303     | 358     |
| Li-7<->C-12    | 400.0  | 9.5   | 294     | 299     | 299     | 351     |
| C-12<->C-12    | 800.0  | 12.0  | 280     | 284     | 288     | 333     |
| N-14<->C-12    | 400.0  | 13.0  | 275     | 278     | 283     | 325     |
| O-16<->C-12    | 400.0  | 14.0  | 273     | 277     | 281     | 322     |
| He-3<->Al-27   | 400.0  | 15.0  | 290     | 295     | 300     | 351     |
| He-4<->Al-27   | 400.0  | 15.5  | 292     | 297     | 299     | 348     |
| Li-7<->Mg-24   | 400.0  | 15.5  | 284     | 288     | 290     | 335     |
| C-12<->F-19    | 400.0  | 15.5  | 278     | 282     | 283     | 324     |
| C-12<->Ne-20   | 400.0  | 16.0  | 272     | 275     | 279     | 319     |
| Si-28<->C-12   | 400.0  | 20.0  | 267     | 270     | 273     | 310     |
| N-14<->Al-27   | 400.0  | 20.5  | 272     | 274     | 275     | 313     |
| Ne-20<->NaF-42 | 400.0  | 20.5  | 270     | 272     | 273     | 310     |
| Ne-20<->NaF-42 | 800.0  | 20.5  | 272     | 274     | 275     | 312     |
| Ne-20<->NaF-42 | 2100.0 | 20.5  | 278     | 280     | 281     | 318     |
| N-14<->Al-27   | 400.0  | 20.5  | 272     | 274     | 275     | 313     |
| O-16<->Al-27   | 400.0  | 21.5  | 270     | 272     | 273     | 309     |
| F-19<->Mg-24   | 400.0  | 21.5  | 269     | 271     | 272     | 308     |
| C-12<->S-32    | 400.0  | 22.0  | 268     | 270     | 274     | 310     |
| Ne-20<->Al-27  | 400.0  | 23.5  | 267     | 268     | 270     | 304     |
| Cl-35<->C-12   | 400.0  | 23.5  | 269     | 271     | 272     | 309     |
| Ar-40<->Be-9   | 400.0  | 24.5  | 285     | 288     | 282     | 324     |
| Ar-40<->C-12   | 400.0  | 26.0  | 277     | 279     | 276     | 313     |
| Si-28<->Al-27  | 400.0  | 27.5  | 262     | 263     | 265     | 296     |
| Al-27<->S-32   | 400.0  | 29.5  | 261     | 262     | 263     | 293     |
| He-4<->Fe-56   | 400.0  | 30.0  | 293     | 297     | 294     | 340     |
| Na-23<->Cl-37  | 400.0  | 30.0  | 270     | 271     | 268     | 300     |
| He-4<->Fe-57   | 400.0  | 30.5  | 256     | 253     | 261     | 342     |
| Li-7<->Mn-54   | 400.0  | 30.5  | 246     | 242     | 250     | 327     |
| Cl-35<->Al-27  | 400.0  | 31.0  | 220     | 215     | 227     | 293     |
| Li-7<->Fe-56   | 400.0  | 31.5  | 245     | 241     | 250     | 326     |
| Ar-40<->Al-27  | 400.0  | 33.5  | 225     | 219     | 228     | 295     |
| Fe-56<->C-12   | 400.0  | 34.0  | 229     | 224     | 234     | 304     |
| O-16<->Ni-59   | 400.0  | 37.5  | 223     | 217     | 229     | 296     |
| Ar-40<->KCl-74 | 92.0   | 38.5  | 218     | 212     | 222     | 287     |
| Ar-40<->KCl-74 | 800.0  | 38.5  | 220     | 213     | 223     | 288     |
| N-14<->Cu-64   | 400.0  | 39.0  | 230     | 224     | 233     | 302     |
| O-16<->Cu-64   | 400.0  | 40.0  | 227     | 221     | 230     | 299     |
| Ar-40<->Ca-40  | 92.0   | 40.0  | 215     | 209     | 220     | 283     |
| Ar-40<->Ca-40  | 137.0  | 40.0  | 215     | 209     | 220     | 283     |
| Fe-56<->Al-27  | 400.0  | 41.5  | 218     | 211     | 222     | 286     |
| Ne-20<->Cu-64  | 400.0  | 42.0  | 223     | 217     | 226     | 293     |

## Appendix I.6 Coalescence Radii Output Listing Continued.....

| *****          |        |       |         |         |         |         |
|----------------|--------|-------|---------|---------|---------|---------|
| System         | Energy | csam  | H-2     | H-3     | He-3    | He-4    |
| Proj<->Target  | (MeV)  |       | (MeV/c) | (MeV/c) | (MeV/c) | (MeV/c) |
| *****          | *****  | ****  | *****   | *****   | *****   | *****   |
| He-4<->Sr-87   | 400.0  | 45.5  | 258     | 254     | 259     | 340     |
| Na-23<->Zn-68  | 400.0  | 45.5  | 224     | 217     | 225     | 292     |
| Ca-40<->V-51   | 400.0  | 45.5  | 213     | 206     | 217     | 279     |
| Sc-45<->Ti-46  | 400.0  | 45.5  | 213     | 206     | 217     | 279     |
| Si-28<->Cu-64  | 400.0  | 46.0  | 217     | 210     | 220     | 284     |
| Cl-35<->Cu-64  | 400.0  | 49.5  | 214     | 207     | 217     | 279     |
| Ar-40<->Cu-64  | 92.0   | 52.0  | 215     | 208     | 216     | 279     |
| Ar-40<->Cu-64  | 137.0  | 52.0  | 216     | 208     | 216     | 279     |
| C-12<->Nb-93   | 400.0  | 52.5  | 232     | 225     | 233     | 303     |
| He-3<->Ag-108  | 400.0  | 55.5  | 260     | 256     | 262     | 344     |
| Kr-84<->Al-27  | 400.0  | 55.5  | 220     | 213     | 219     | 284     |
| Ni-58<->Ni-60  | 400.0  | 59.0  | 205     | 197     | 208     | 267     |
| Fe-56<->Cu-64  | 400.0  | 60.0  | 209     | 201     | 210     | 270     |
| Mg-24<->Tc-97  | 400.0  | 60.5  | 205     | 198     | 201     | 244     |
| Cl-35<->Rb-86  | 400.0  | 60.5  | 202     | 195     | 197     | 239     |
| Fe-56<->Zn-65  | 400.0  | 60.5  | 195     | 188     | 192     | 232     |
| Ne-20<->Sn-119 | 400.0  | 69.5  | 211     | 204     | 204     | 249     |
| Si-28<->Sn-119 | 400.0  | 73.5  | 204     | 197     | 197     | 240     |
| C-12<->La-139  | 400.0  | 75.5  | 223     | 216     | 214     | 263     |
| C-12<->Pr-141  | 400.0  | 76.5  | 220     | 214     | 213     | 261     |
| Cl-35<->Sn-119 | 400.0  | 77.0  | 201     | 193     | 194     | 235     |
| Xe-132<->Al-27 | 400.0  | 79.5  | 211     | 204     | 200     | 244     |
| Ar-40<->Sn-119 | 400.0  | 79.5  | 201     | 193     | 193     | 234     |
| Fe-56<->Ag-108 | 400.0  | 82.0  | 192     | 184     | 186     | 224     |
| O-16<->Sm-154  | 400.0  | 85.0  | 217     | 210     | 207     | 254     |
| Kr-84<->Nb-93  | 400.0  | 88.5  | 191     | 183     | 183     | 221     |
| Al-27<->Gd-154 | 400.0  | 90.5  | 164     | 165     | 166     | 196     |
| Fe-56<->Te-127 | 400.0  | 91.5  | 155     | 156     | 157     | 184     |
| Ho-165<->Al-27 | 400.0  | 96.0  | 164     | 165     | 165     | 196     |
| C-12<->Ta-181  | 3400.0 | 96.5  | 195     | 197     | 196     | 234     |
| He-4<->Au-197  | 400.0  | 100.5 | 206     | 209     | 207     | 251     |
| Ne-20<->Ta-181 | 400.0  | 100.5 | 169     | 171     | 170     | 202     |
| Be-9<->Au-197  | 400.0  | 103.0 | 188     | 190     | 188     | 225     |
| C-12<->Au-197  | 400.0  | 104.5 | 180     | 182     | 180     | 215     |
| O-16<->Au-197  | 400.0  | 106.5 | 173     | 175     | 174     | 207     |
| Be-9<->Pb-208  | 400.0  | 108.5 | 188     | 191     | 188     | 226     |
| C-12<->Pb-207  | 400.0  | 109.5 | 180     | 182     | 180     | 215     |
| N-14<->Pb-207  | 400.0  | 110.5 | 177     | 179     | 177     | 211     |
| O-16<->Pb-207  | 400.0  | 111.5 | 174     | 176     | 174     | 207     |
| Ne-20<->Pb-207 | 400.0  | 113.5 | 169     | 171     | 169     | 201     |
| Ne-20<->Pb-207 | 800.0  | 113.5 | 170     | 172     | 171     | 202     |
| Ne-20<->Pb-207 | 2100.0 | 113.5 | 174     | 176     | 174     | 207     |
| Ne-20<->Bi-209 | 400.0  | 114.5 | 169     | 170     | 169     | 201     |
| Mg-24<->Bi-209 | 400.0  | 116.5 | 165     | 166     | 165     | 196     |
| Si-28<->Pb-207 | 400.0  | 117.5 | 162     | 163     | 163     | 192     |
| Zr-91<->Sm-149 | 400.0  | 120.0 | 145     | 146     | 147     | 171     |
| K-39<->Ti-202  | 400.0  | 120.5 | 128     | 139     | 135     | 148     |
| He-4<->U-238   | 250.0  | 121.0 | 173     | 188     | 180     | 205     |

## Appendix I.6 Coalescence Radii Output Listing Continued.....

| *****           |        |       |         |         |         |         |
|-----------------|--------|-------|---------|---------|---------|---------|
| System          | Energy | csam  | H-2     | H-3     | He-3    | He-4    |
| Proj<->Target   | (MeV)  |       | (MeV/c) | (MeV/c) | (MeV/c) | (MeV/c) |
| *****           | *****  | ****  | *****   | *****   | *****   | *****   |
| Cl-35<->Pb-207  | 400.0  | 121.0 | 130     | 141     | 137     | 151     |
| Sb-122<->Sb-122 | 400.0  | 122.0 | 118     | 128     | 126     | 137     |
| Be-9<->U-238    | 400.0  | 123.5 | 157     | 171     | 164     | 184     |
| Ar-40<->Pb-207  | 92.0   | 123.5 | 129     | 140     | 136     | 149     |
| Ar-40<->Pb-207  | 800.0  | 123.5 | 130     | 141     | 137     | 150     |
| O-16<->U-238    | 250.0  | 127.0 | 144     | 156     | 150     | 167     |
| Ne-20<->U-235   | 400.0  | 127.5 | 139     | 151     | 146     | 162     |
| Ne-20<->U-238   | 250.0  | 129.0 | 139     | 151     | 146     | 162     |
| Ne-20<->U-238   | 400.0  | 129.0 | 140     | 152     | 146     | 162     |
| Ne-20<->U-238   | 2100.0 | 129.0 | 144     | 156     | 150     | 167     |
| Fe-56<->Ti-204  | 400.0  | 130.0 | 123     | 133     | 130     | 142     |
| Fe-56<->Pb-207  | 400.0  | 131.5 | 123     | 133     | 130     | 142     |
| Al-27<->U-238   | 2100.0 | 132.5 | 138     | 149     | 144     | 159     |
| Pb-208<->Cu-64  | 400.0  | 136.0 | 121     | 132     | 128     | 140     |
| Pd-106<->Lu-174 | 400.0  | 140.0 | 115     | 125     | 123     | 133     |
| Ba-137<->Nd-143 | 400.0  | 140.0 | 115     | 125     | 122     | 132     |
| Kr-86<->Au-197  | 400.0  | 141.5 | 118     | 129     | 125     | 136     |
| Fe-56<->U-238   | 400.0  | 147.0 | 122     | 133     | 129     | 141     |
| Ni-59<->U-238   | 400.0  | 148.5 | 121     | 132     | 128     | 139     |
| Mn-55<->Am-245  | 400.0  | 150.0 | 123     | 133     | 129     | 141     |
| Sb-122<->Hf-178 | 400.0  | 150.0 | 114     | 124     | 121     | 131     |
| Sm-150<->Sm-150 | 400.0  | 150.0 | 113     | 123     | 120     | 130     |
| Zr-91<->At-210  | 400.0  | 150.5 | 115     | 120     | 119     | 133     |
| Ba-137<->Ho-164 | 400.0  | 150.5 | 114     | 119     | 118     | 131     |
| Cu-64<->U-238   | 400.0  | 151.0 | 121     | 126     | 124     | 138     |
| Ta-180<->-180   | 400.0  | 180.0 | 110     | 115     | 113     | 125     |
| Au-197<->Au-197 | 400.0  | 197.0 | 101     | 104     | 102     | 117     |
| Pb-207<->U-238  | 400.0  | 222.5 | 100     | 103     | 100     | 114     |

## Appendix II: Estimated Source Radii and Coalescence Radii From Other Analyses

**Appendix II.1:** Estimated source radii from other analyses for different systems and projectile energies per nucleon for the  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$  and  $^4\text{He}$  emitted light particles.

|    | Collision System<br>Proj $\rightarrow$ Target | csam  | Energy/n<br>(MeV/n) | $^2\text{H}$<br>(fm) | $^3\text{H}$<br>(fm) | $^3\text{He}$<br>(fm) | $^4\text{He}$<br>(fm) | [Reference]  |
|----|-----------------------------------------------|-------|---------------------|----------------------|----------------------|-----------------------|-----------------------|--------------|
| 1  | C-12 $\rightarrow$ C-12                       | 12.0  | 800.0               | 2.9                  | 2.6                  | 2.6                   |                       | Lemaire [17] |
| 2  | Ne-20 $\rightarrow$ NaF                       | 20.5  | 400.0               | 3.4                  | 3.3                  | 3.3                   |                       | Lemaire [17] |
| 3  | Ne-20 $\rightarrow$ NaF                       | 20.5  | 800.0               | 3.4                  | 2.8                  | 2.8                   |                       | Lemaire [17] |
| 4  | Ne-20 $\rightarrow$ NaF                       | 20.5  | 2100.0              | 3.4                  | 3.5                  | 3.5                   |                       | Lemaire [17] |
| 5  | Ar-40 $\rightarrow$ KCl                       | 38.5  | 800                 | 3.5                  | 3.0                  | 3.0                   |                       | Lemaire [17] |
| 6  | Ar-40 $\rightarrow$ Ca-40                     | 40.0  | 92                  | 5.3                  | 4.5                  | 4.8                   | 3.7                   | Jacak [37]   |
| 7  | Ar-40 $\rightarrow$ Ca-40                     | 40.0  | 137                 | 5.2                  | 4.2                  | 4.7                   | 3.8                   | Jacak [37]   |
| 8  | Ar-40 $\rightarrow$ Cu-64                     | 52.0  | 92                  | 5.4                  | 4.6                  | 5.2                   | 4.0                   | Jacak [37]   |
| 9  | Ar-40 $\rightarrow$ Cu-64                     | 52.0  | 137                 | 5.4                  | 4.6                  | 5.2                   | 4.0                   | Jacak [37]   |
| 10 | C-12 $\rightarrow$ Pb-207                     | 109.5 | 800.0               | 3.9                  | 3.4                  | 3.3                   |                       | Lemaire [17] |
| 11 | Ne-20 $\rightarrow$ Pb-207                    | 113.5 | 400.0               | 4.2                  | 3.6                  | 3.7                   |                       | Lemaire [17] |
| 12 | Ne-20 $\rightarrow$ Pb-207                    | 113.5 | 800.0               | 4.2                  | 3.7                  | 3.9                   |                       | Lemaire [17] |
| 13 | Ne-20 $\rightarrow$ Pb-207                    | 113.5 | 2100.0              | 5.0                  | 3.9                  |                       | 3.7                   | Lemaire [17] |
| 14 | Ar-40 $\rightarrow$ Pb-207                    | 123.5 | 800.0               | 4.1                  | 3.6                  | 2.6                   |                       | Lemaire [17] |
| 15 | O-16 $\rightarrow$ U-238                      | 127.0 | 315.0               | 5.6                  | 4.2                  | 3.7                   |                       | Awes [28]    |
| 16 | Ne-20 $\rightarrow$ U-235                     | 127.5 | 400.0               | 3.5                  | 3.0                  | 3.0                   |                       | Lemaire [17] |
| 17 | Ne-20 $\rightarrow$ U-238                     | 129.0 | 400.0               | 6.8                  | 5.9                  | 5.9                   | 4.9                   | Mekjian [7]  |

No estimates are available, if boxes are blank.

**Appendix II.2:** Estimated coalescence radii from other analyses for different systems and projectile energies per nucleon for the  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$  and  $^4\text{He}$  emitted secondary light particles.

|    | System<br>Proj $\rightarrow$ Target | Csam  | Energy/n<br>(MeV/n) | $^2\text{H}$<br>(MeV/c) | $^3\text{H}$<br>(MeV/c) | $^3\text{He}$<br>(MeV/c) | $^4\text{He}$<br>(MeV/c) | [Reference]  |
|----|-------------------------------------|-------|---------------------|-------------------------|-------------------------|--------------------------|--------------------------|--------------|
| 1  | C-12 $\rightarrow$ C-12             | 12.0  | 800.0               | 304                     | 280                     | 280                      |                          | Lemaire [17] |
| 2  | Ne-20 $\rightarrow$ NaF             | 20.5  | 400.0               | 259                     | 223                     | 223                      |                          | Lemaire [17] |
| 3  | Ne-20 $\rightarrow$ NaF             | 20.5  | 800.0               | 259                     | 260                     | 260                      |                          | Lemaire [17] |
| 4  | Ne-20 $\rightarrow$ NaF             | 20.5  | 2100.0              | 259                     | 212                     | 212                      |                          | Lemaire [17] |
| 5  | Ar-40 $\rightarrow$ KCl             | 38.5  | 800.0               | 248                     | 179                     | 179                      |                          | Lemaire [17] |
| 6  | C-12 $\rightarrow$ Pb-207           | 109.5 | 800.0               | 221                     | 219                     | 226                      |                          | Lemaire [17] |
| 7  | Ne-20 $\rightarrow$ Pb-207          | 113.5 | 400.0               | 205                     | 207                     | 198                      |                          | Lemaire [17] |
| 8  | Ne-20 $\rightarrow$ Pb-207          | 113.5 | 800.0               | 205                     | 199                     | 189                      |                          | Lemaire [17] |
| 9  | Ne-20 $\rightarrow$ Pb-207          | 113.5 | 2100.0              | 173                     | 190                     | 198                      |                          | Lemaire [17] |
| 10 | He-4 $\rightarrow$ U-238            | 121.0 | 400.0               | 126                     | 127                     | 127                      | 132                      | Gutbrod [15] |
| 11 | Ar-40 $\rightarrow$ Pb-207          | 123.5 | 800.0               | 213                     | 205                     | 208                      |                          | Lemaire [17] |
| 12 | O-16 $\rightarrow$ U-238            | 127.0 | 315.0               | 170                     | 215                     |                          | 270                      | Awes [28]    |
| 13 | Ne-20 $\rightarrow$ U-238           | 129.0 | 250.0               | 126                     | 140                     | 135                      | 147                      | Gutbrod [15] |
| 14 | Ne-20 $\rightarrow$ U-238           | 129.0 | 400.0               | 129                     | 129                     | 129                      | 142                      | Gutbrod [15] |
| 15 | Ne-20 $\rightarrow$ U-238           | 129.0 | 2100.0              | 106                     | 116                     | 106                      | 118                      | Gutbrod [15] |

No estimates are available, if boxes are blank.

## **11- VITA**

Mahmoud PourArsalan was born in Isfahan, Iran in June 1955. He grew up in Isfahan and, graduated from Saremieh High School in Isfahan in May 1974. Upon completion of compulsory military service he entered Millersville State College (Currently Millersville University of Pennsylvania) in Millersville, Pennsylvania and received a Bachelor of Arts degree in Physics in May 1983. He entered the graduate school at the University of New Mexico, Albuquerque, New Mexico, and in May of 1986, he received a Master of Science degree in Nuclear Engineering. In August 1986 he began working towards the Doctor of Philosophy degree in Nuclear Engineering at the University of Arizona in Tucson, Arizona, which he completed all required course work and passed qualifying exam. He was a graduate student intern at Palo Verde Nuclear Generating Station in Phoenix, AZ during summer of 1989. He then, took off from school and worked for S3 technology until March 1992 and supported designing, testing software for nuclear power plant simulators for different nuclear power plants such as Palo Verde, Cooper, Quad City, and PP&I. He taught math at Pima Community college in Tucson, Arizona for several years. He earned a graduate Criticality Safety Certificate from the University of Tennessee and a second Master in nuclear engineering with concentration in Radiological Engineering in 2009. His dissertation research was sponsored by NASA and he earned a PhD in Nuclear Engineering at the University of Tennessee in August 2012.