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AN ANALYTIC STUDY OF MICROLAYER EVAPORATION
AND RELATED BUBBLE GROWTH EFFECTS
IN NUCLEATE BOILING

A Dissertation
Presented to
The Graduate Council of
The University of Tennessee

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

by
Gerald Steven Dzakowic
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NOMENCLATURE

a	Heater thickness
A	Surface area
B	Evaporation heat flux constant
C	Specific heat capacity (with subscripts s or l)
C	Constant
d	Dimensionless transport property ratio
f	Denotes a function
F	The non-integral contributions of Equation 121 in Appendix A
g	Gravitational acceleration $\left(g_c = 32.17 \frac{\text{lb}_m \text{ft}}{\text{lb}_f \text{sec}^2}\right)$
G	The integrand of Equation 121 in Appendix A
h_{fg}	Latent heat of vaporization
H	Time constant defined by $a/\alpha^{1/2}$
i	Denotes higher order complimentary error functions
j	Summation index
J	Laplace transformed evaporative boundary condition (defined in Appendix A)
Ja	Jakob number
k	Thermal conductivity
K	Boltzmann's constant
l	Liquid slab thickness

$\mathcal{L}$	Laplace transform operator
m	Mass per molecule
$\dot{m}$	Mass flow rate
$\dot{m}/dA$	Mass flow rate per unit area
n	Index for time step numerical calculations
p	Laplace transformed variable
P	Pressure
Pr	Prandtl number
q	Heat transfer rate
q/A	Heat transfer rate per unit area
$\hat{q}$	$(1 - d)/(1 + d)$ as defined in Equation 68
r	Radial measurement in spherical coordinates
r^*	Dimensionless radial coordinate $= r/r_c$
r_c	Nucleating cavity radius or initial bubble radius
R	Specific gas constant
R_b	Bubble radius
Re	Reynold's number
t	Time
T	Temperature ($\Delta T_{sat} = T_w - T_{sat}$, superheat temperature difference; $\Delta T_{sub} = T_{sat} - T_{bulk}$, subcooled temperature difference)
v_{fg}	Specific volume change associated with vaporization
V_r	Radial component of liquid velocity
V_θ	Azmuthal component of the liquid velocity
x	One-dimensional coordinate

x	Dimensionless x-coordinate
x_l^*	$(Pr^2 r_c)/a$, characteristic nucleating cavity dimension
Y	$(T_w/T_{sat})^{1/2}$ as defined in Appendix A
α	Thermal diffusivity = $k/\rho C$
β_{wm}	Dimensionless initial temperature gradient in the heated solid at the end of microlayer evaporation
γ	Dimensionless evaporation parameter
Γ	Dimensionless constant defined in Equation 71
δ	Microlayer thickness
ε	Transmission or accommodation coefficient
θ	Azmuthal or polar angle
Θ	Dimensionless temperature (based upon saturation temperature)
λ	Dummy integration variable
μ	Transformed azmuthal angle ($\mu = \cos \theta$)
μ_l	Liquid dynamic viscosity
ν or ν'	Bubble growth constants
ξ	Dimensionless one-half power of the liquid-vapor interface temperature
ρ	Density
σ	Liquid-vapor surface tension
τ	Dimensionless time
u	Dimensionless internal heat generation (based upon the bulk liquid temperature)

ϑ or ϕ	Dimensionless temperature (based upon the bulk liquid temperature)
Φ	Dimensionless internal heat generation (based upon the saturation temperature)
Ψ	Dimensionless interface evaporation parameter
ζ	Total volume

Subscripts

b	Bubble interface
bi	Initial bubble interface
bulk	Bulk liquid
c	Initial bubble
evap	Evaporation
fg	Change in property value associated with vaporization
i	Initial
inc	Incipient nucleate boiling condition
l	Liquid
m	Microlayer interface
max	Maximum value
n	nth time step calculation
s	Solid or internal heat generating material
sat	Saturated liquid
v	Vapor
w	Heater surface

wm Condition determined at the end of microlayer
 evaporation

Superscripts

- ' Denotes a dummy integration variable
- " Denotes the independent variable is $(\tau - \lambda)$
- * Signifies a dimensionless quantity
- Laplace transformed quantity

CHAPTER I

INTRODUCTION

Nucleate boiling heat transfer is of technical interest since it allows a high heat flux with only a relatively small temperature difference. The demand for a better understanding of the behavior of a liquid as it exceeds its saturation temperature and changes to vapor has been the result of design problems in liquid cooled rocket nozzles and nuclear reactor cores. Moreover, to improve the performance of these high heat flux systems an understanding and control of the phenomena of boiling must be achieved. This dissertation is an attempt to establish the criteria for the heat and mass transfer processes which take place during bubble formation in nucleate boiling heat transfer.

I. BACKGROUND

Figures 1 and 2 illustrate the boiling heat transfer "regimes". The heat transfer characteristics are best described by comparing average surface heat flux, (q/A) , to superheat temperature differences, $\Delta T_{\text{sat}} = T_w - T_{\text{sat}}$, as shown in Figure 1. Corresponding to Figure 1, Figure 2 presents what one observes for successive superheat

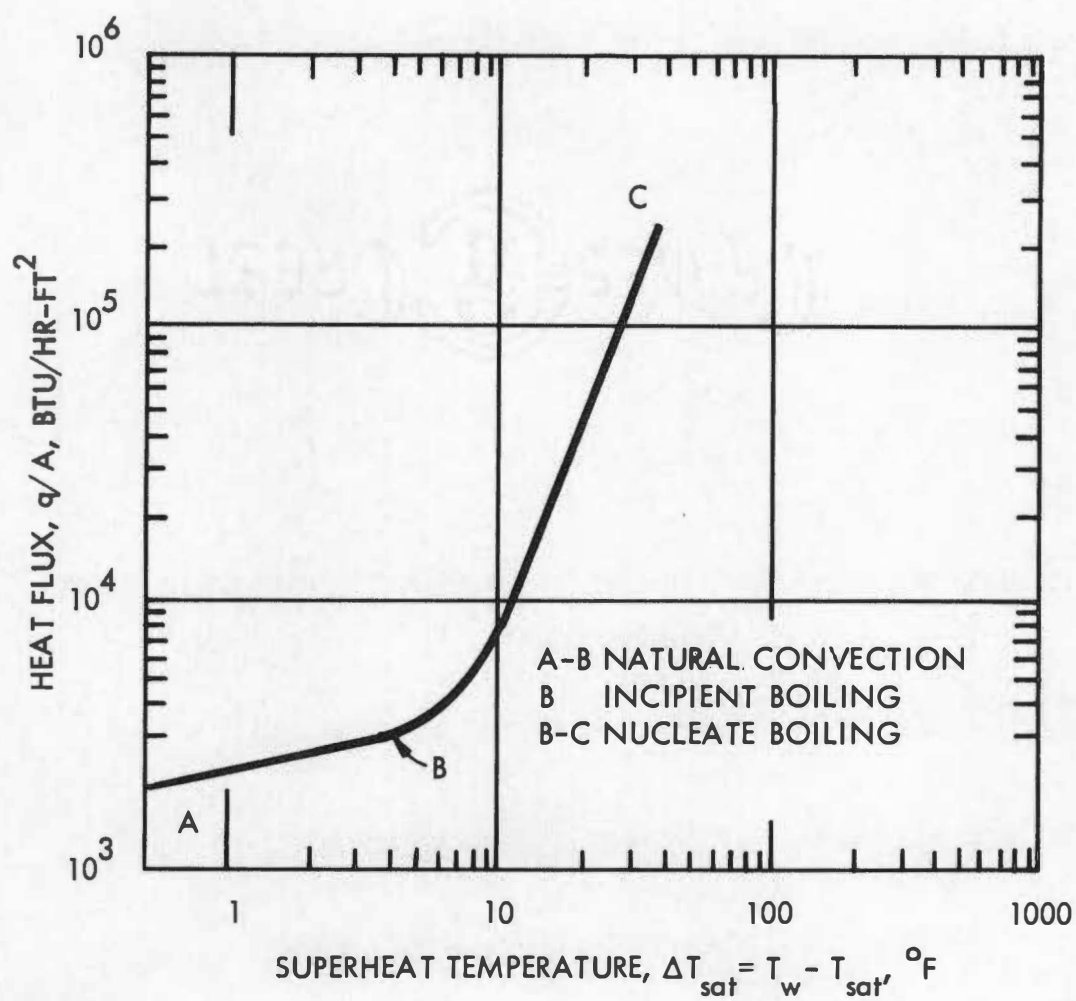
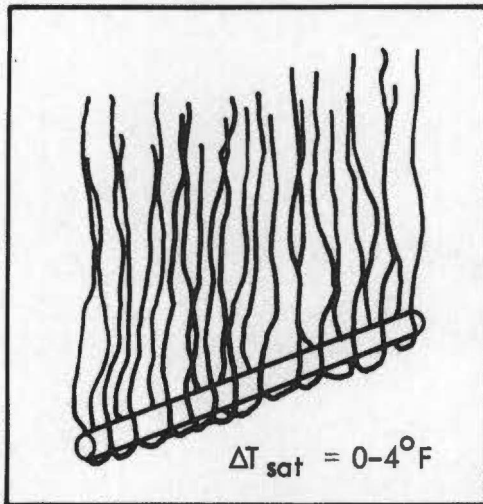
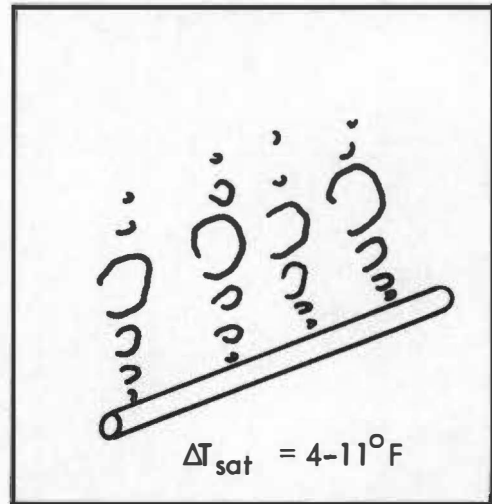


Figure 1. Pool boiling curve for water at one atmosphere.

NATURAL CONVECTION



INCIPIENT BOILING



NUCLEATE BOILING

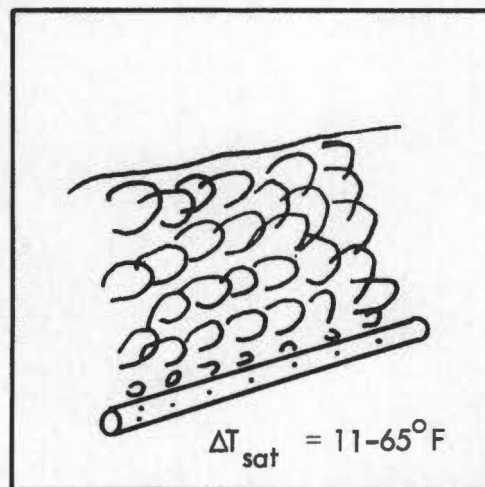


Figure 2. Observed nucleate boiling from Reference (1).

temperature differences. The heat transfer regimes are categorized as follows:

1. Natural or Forced Convection to the Liquid Phase:
As a heated surface exceeds the equilibrium saturation temperature of the liquid, a metastable convection persists in the liquid.
2. Incipient Boiling: Once the liquid is sufficiently superheated, the amount of vapor which is entrapped in microscopic cavities in the heated surface increases and produces the first visible ebullition (Point B, Figure 1, page 2).
3. Nucleate Boiling: In this regime discrete bubbles form on the heated surface and grow into the liquid phase. As illustrated in Figure 1 by the line BC, a small increase in the heater surface temperature will significantly increase the average heat flux. Thus, for efficient cooling, nucleate boiling is the desirable operating regime for most high heat flux systems.

As one can see from Figure 2, page 3, the random bubble formations associated with nucleate boiling are not amenable to an analytic description of the heat transfer processes surrounding them. Consequently, this analysis will consider an individual vapor bubble at the heater surface.

II. STATEMENT OF THE PROBLEM

The present investigation is concerned with establishing a model for bubble growth during nucleate boiling heat transfer. The investigation will focus on bubbles in the liquid which are also in contact with the heated surface. Interest in vapor bubbles at the heated surface has been stirred by the recent experimental findings of Moore and Mesler (2)¹ and Sharp (3) who have revealed the presence of a microlayer of liquid separating a growing vapor bubble from the heated surface (see Figure 3). Their qualitative analyses suggest that an extremely high heat flux occurs from the evaporation of the liquid microlayer and momentarily cools the heated surface locally. This evaporation and cooling is a major consideration in the theoretical bubble behavior proposed here. To the author's knowledge, no bubble growth analysis has yet considered liquid microlayer evaporation.

In summary, the problem is to analyze the transport processes associated with the growth of a vapor bubble which is partly separated from a heated surface by a liquid microlayer evaporating into the vapor cavity.

¹Numbers in parentheses refer to similarly numbered references in the bibliography.

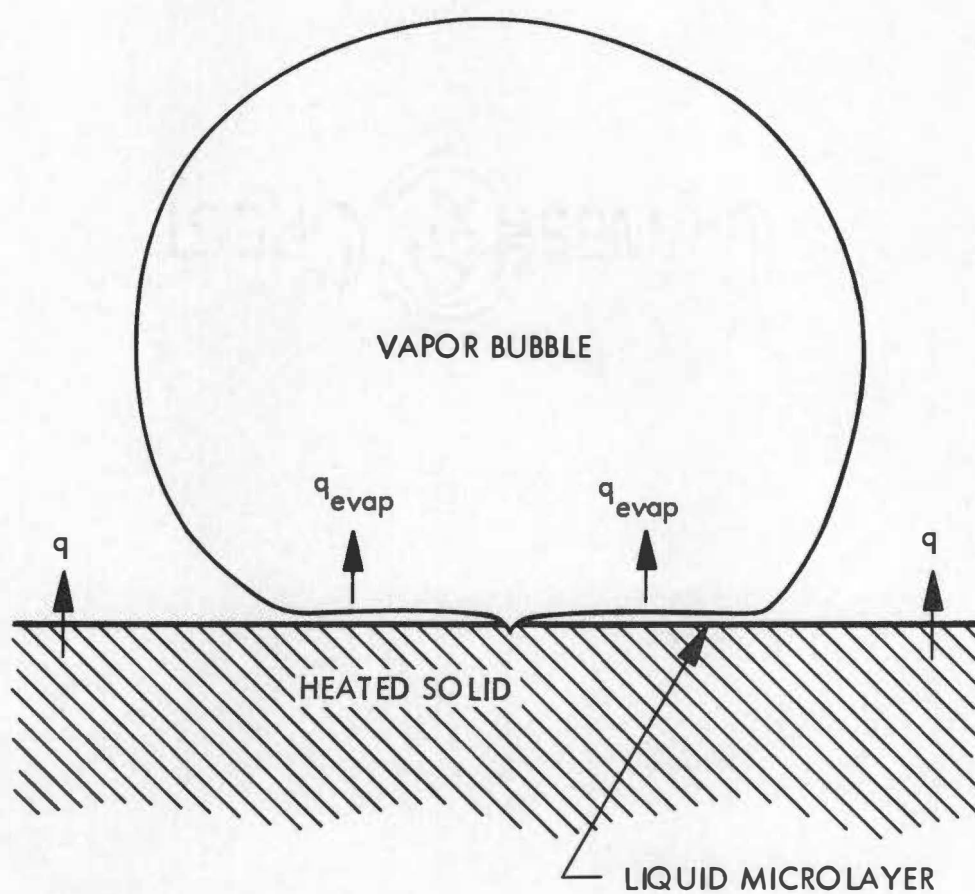


Figure 3. A newly formed vapor bubble illustrating an evaporating liquid microlayer.

The dissertation is divided into six chapters according to the problem under consideration. Chapter II is a review section which presents the governing equations of nucleate boiling heat transfer. In Chapters III, IV, and V the bubble growth problem is analyzed by examining the effects of microlayer evaporation, the waiting time, and the transport processes at the bubble interface respectively. A discussion of microlayer evaporation in Chapter VI concludes the analysis.

CHAPTER II

LITERATURE SURVEY

The intention of this chapter is to bring the reader up to the present "state-of-the-art" in the theoretical analysis of the heat transfer phenomena pertinent to bubble growth in nucleate boiling. This presentation is by no means intended to be all inclusive; the interested reader can find more complete bibliography lists in References (4), (5), and (6).

I. INCIPIENT BOILING

A mathematical prediction of the inception of bubble nucleation on a heated surface, denoting the transition from single-phase to boiling heat transfer, is often required in designs where heat transfer is of importance.

The analytical requirements for a stable nucleating cavity on a heated surface were first specified by Bankoff (7). He determined that microscopic pits or scratches which contained dissolved gas and/or vapor are the sites from which bubbles begin to grow. Subsequent experimental verifications of his analysis by References (8), (9), (10), and (11) have proven Bankoff's analysis. Thus, the equilibrium equations for predicting incipient boiling assume

the presence of a small nucleating bubble on a heated surface.

A force balance for a hemispherical vapor bubble of radius, r_c , which represents the characteristic dimension of a nucleating cavity, gives the Hemholtz relation,

$$P_v - P_l = \frac{2\sigma}{r_c} . \quad (1)$$

For pure vapor in the nucleating cavity P_l is less than P_v and $T_v = T_l$, hence, the liquid temperature must be superheated with respect to the liquid pressure. The vapor temperature in the bubble, T_v , is then found by integrating the Clausius-Clapeyron equation with the perfect gas approximation for the vapor to yield

$$\ln(P_v/P_l) = \frac{h_{fg}(T_v - T_{sat})}{T_v T_{sat} R} . \quad (2)$$

Equation 2 combined with Equation 1 becomes

$$T_v = T_{sat} + \frac{T_v T_{sat} R}{h_{fg}} \ln \left(1 + \frac{2\sigma}{r_c P_l} \right) . \quad (3)$$

The superheated liquid temperatures for incipient boiling predicted by Equation 3 are much lower than those which were observed by Griffith and Wallis (10). The exact

modification required so that the theoretical and experimental work agree remains unknown. Some progress, however, has been made by Bergles and Rohsenow (12), who postulated the following theory.

A heated surface is assumed to contain a wide range of cavities from which hemispherical vapor bubbles protrude. If a bubble protruding from the heated surface is to grow, the thermal layer surrounding it must attain a temperature such that there is net heat transfer from the liquid to the bubble. In the laminar region near a heated wall, the liquid temperature profile is nearly linear and may be expressed as:

$$T_{\ell} = T_w - (q/A) \frac{x_{\ell}}{k_{\ell}} . \quad (4)$$

Reference 12 simply postulates that the bubble nucleus of hemispherical shape with radius r_c will grow if the liquid temperature at a distance $x_{\ell} = r_c$ from the wall is greater than the critical vapor temperature defined by Equation 3. In terms of Equations 3 and 4, this is the minimum wall temperature for which T_{ℓ} and T_v have a common value. Figure 4 shows this condition to be satisfied when the curves of T_{ℓ} and T_v are tangent. Thus, the criterion for incipient boiling is:

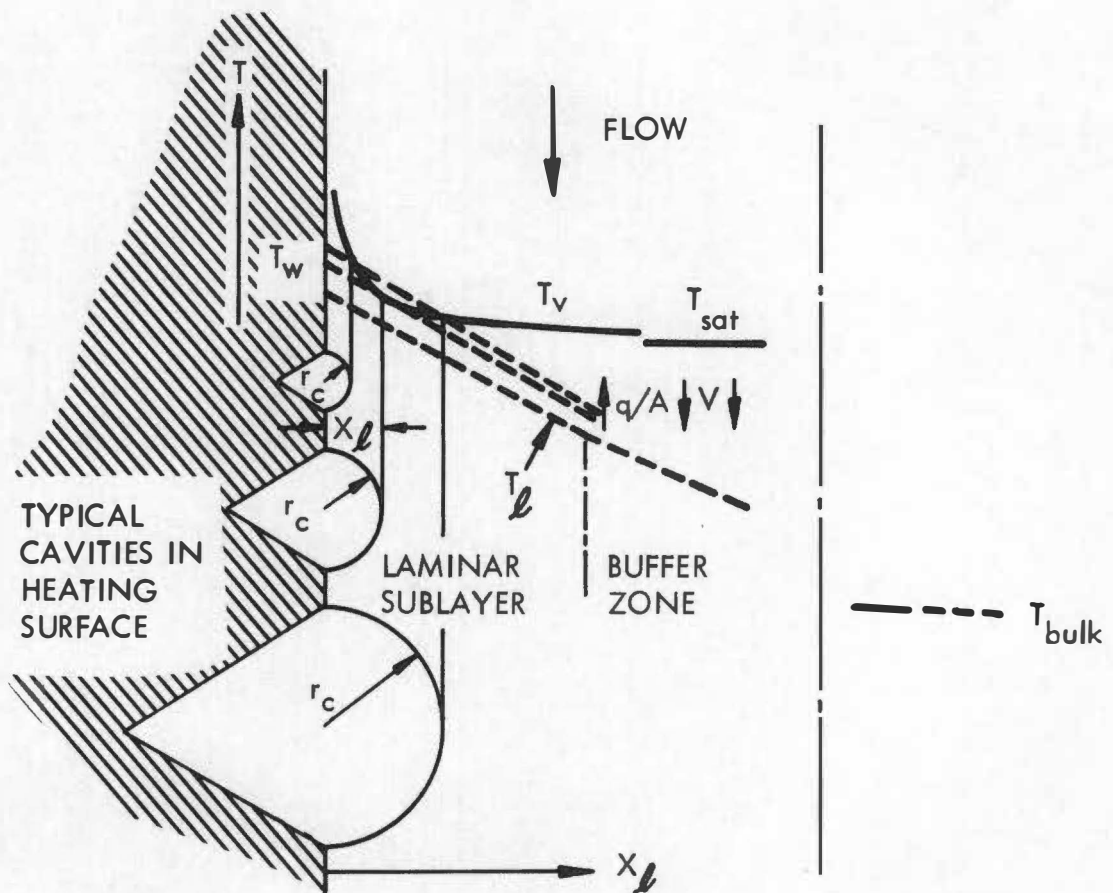


Figure 4. Conditions for incipient bubble formation according to Reference (12).

$$T_l = T_v \quad \text{and} \quad \frac{dT_l}{dx_l} = \frac{dT_v}{dr_c} \quad \text{at} \quad x_l = r_c. \quad (5)$$

This solution, which is completed by a graphical or trial-and-error procedure, has predicted incipient boiling for water on commercially finished surfaces. However, its predictions for other liquids exhibits considerable disagreement with experimental data (13).

Several refinements of this analysis (14), (15), and (16) have had limited success in correlating incipient boiling data for water. These will not be discussed. However, Reference (13) eliminates the aforementioned shortcomings of Bergles and Rohsenow's analysis by altering two assumptions.

1. The Clausius-Clapeyron equation is integrated assuming the slope of the vapor pressure curve $h_{fg}/v_{fg}T_{sat}$ is constant, rather than assuming the vapor is an ideal gas.
2. The incipient boiling criterion is changed to read $x_l = Pr_l^2 r_c$. An extensive comparison with experimental data showed this was necessary in order to obtain satisfactory correlation of incipient boiling data for fluids other than water.

Subject to these alterations the boiling conditions are given by

$$T_v = T_{sat} + \frac{2\sigma_{vf}qT_{sat}}{h_{fg}r_c}, \quad (6)$$

$$T_l = T_w - (q/A) \frac{x_l}{k_l}, \quad (7)$$

and

$$T_l = T_v; \quad \frac{dT_v}{dr_c} = \frac{dT_l}{dx_l} \quad \text{at} \quad x_l = Pr_l^2 r_c. \quad (8)$$

Under the conditions of Equations 6, 7, and 8, the inception of boiling is given by

$$\frac{\Delta T_{sat}}{Pr_l (q/A)^{1/2}} = 2 \left[\frac{2\sigma_{vf}qT_{sat}}{h_{fg}k_l} \right]^{1/2}, \quad (9)$$

where ΔT_{sat} and (q/A) can be related by the appropriate single-phase heat convection correlation (17). Of interest to the present study is the expression for the nucleating cavity size at the point of bubble inception (13)

$$r_c = \left[\frac{2\sigma_{fg} T_{sat} k_l}{h_{fg} Pr_l^2 (q/A)_{inc}} \right]^{1/2} . \quad (10)$$

This value will later be used as an initial condition for the analytic solution.

II. NUCLEATE BOILING

The analyses of nucleate boiling have followed two fundamental approaches:

1. Those leading to mathematical expressions which describe the overall heat transfer rate and can be directly applied to design, and
2. Analytic attempts at a basic understanding of the heat transfer mechanisms associated with boiling.

Although this dissertation will follow the latter approach, parts of its development will draw upon the design equations; hence a review of both techniques is presented.

Nucleate Boiling Heat Transfer Correlations

The most successful correlations of boiling heat transfer data have resulted from dimensional analysis.

Rohsenow (18) assumed that the convective motion in the liquid, which is induced by the bubble growth, governs the

heat transfer process and based his analysis upon the dimensionless variables analogous to convective heat transfer

$$Nu_b = f(Re_b, Pr_\ell) , \quad (11)$$

to obtain

$$\frac{C_\ell \Delta T_{sat}}{h_{fg}} = C_{sf} \left[\frac{q/A}{\mu_\ell h_{fg}} \left\{ \frac{g_c \sigma}{g(\rho_\ell - \rho_v)} \right\}^{\frac{1}{2}} \right]^r \left[\frac{C_\ell \mu_\ell}{k_\ell} \right]^s , \quad (12)$$

where r and s are empirical constants and C_{sf} is a constant which is related to the surface-fluid combination. No restriction on the condition of the bulk liquid (subcooled or saturated) was made. However, if forced flow is to be analyzed, the appropriate single phase heat flux predicted by a standard forced convection correlation must be added to Equation 12 to obtain the total heat transfer rate with surface boiling.

A second approach by Forster and Grief (19) assumed that the controlling mechanism of boiling heat transfer is "pumping" of hot liquid from the heated surface by the vapor bubbles. A dimensional analysis similar to the aforementioned approach yields the result

$$q/A = 0.0012 \left[\frac{\alpha_l C_l \rho_l T_w}{h_{fg} \rho_v \sigma^{1/2}} \right] \left[\frac{C_l T_w \alpha_l^{1/2}}{(h_{fg} \rho_v)^2} \right]^{1/4} \left[\frac{\rho_l}{\mu_l} \right]^{5/8} Pr_l^{1/3} (\Delta P)^2 \quad (13)$$

where T_w is the average wall temperature and ΔP is the pressure difference across the liquid-vapor interface, $(P_v - P_l)$.

Both Equations 12 and 13 have been moderately successful in correlating experimental nucleate boiling heat transfer data of liquids which wet (adhere to) the heated surface. However, they shed no light on the physics of the boiling process and thus provide limited information regarding optimization of design techniques. For this a more fundamental study is required. The basis of such a study logically begins by analyzing the heat transfer mechanisms which occur around an individual vapor bubble.

Bubble Growth in Nucleate Boiling

A qualitative description of the events which take place in the vicinity of a nucleating bubble site during subcooled nucleate boiling is presented in Figure 5, which is taken from Ellion's (20) dissertation. A portion of his description of Figure 5 is as follows: (Note: The T_{sat}

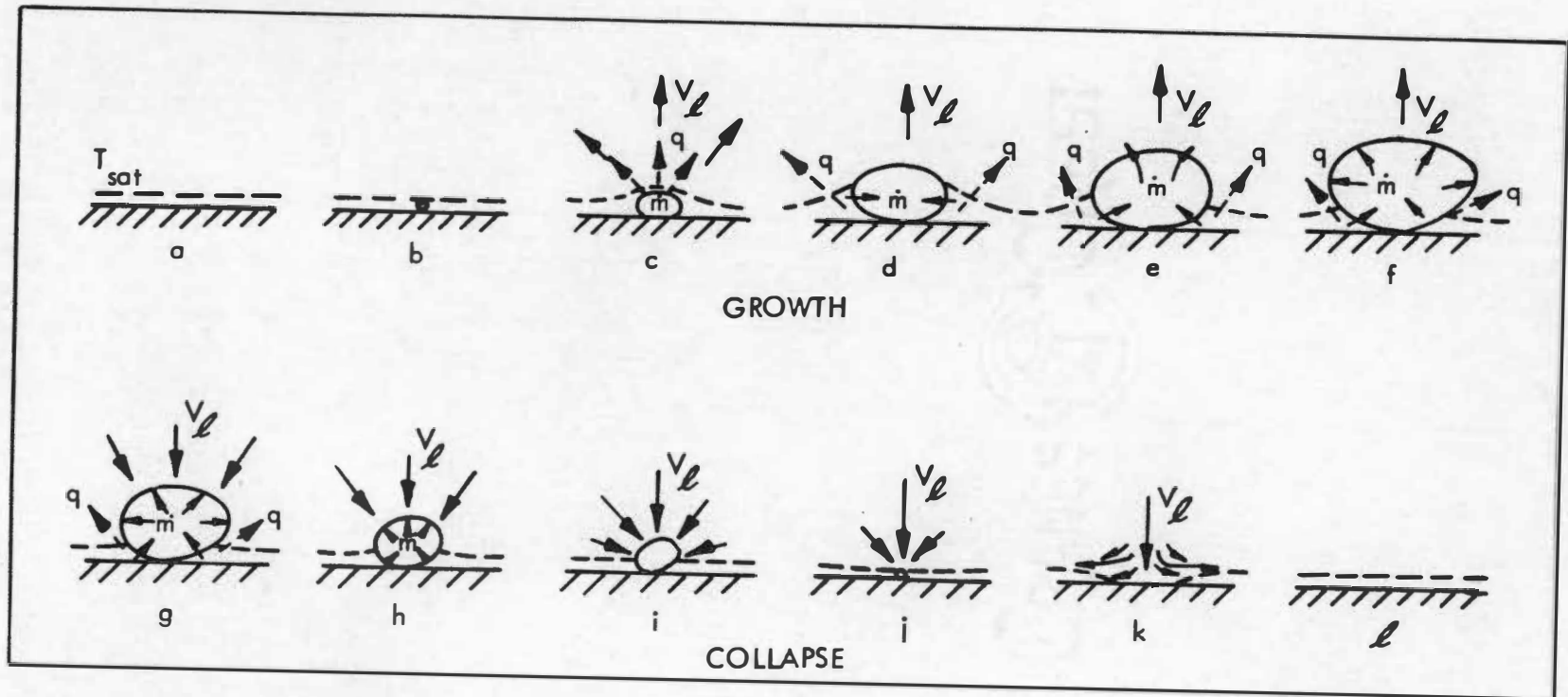


Figure 5. Qualitative description of bubble growth-collapse in nucleate boiling from Reference (20).

isotherm is the dashed line in each sequence of the figure.)

- a. The temperature of the wall and the adjacent liquid film are above the liquid saturation temperature.
- b. A gas-vapor nucleus has commenced bubble growth.
- c. The resistance of the wall to the spherical growth of the bubble results in approximately a hemispherical shape. The layer of superheated water that was above the nucleation volume is pushed away from the wall by the bubble. This water film is stretched over the top of the bubble with the result that the temperature gradient within the film is increased. Heat is removed from the displaced film by conduction and convection to the cooler surrounding liquid (dotted arrows) and by evaporation of liquid into the bubble (solid arrows). The liquid surrounding the bubble has been set in motion by the motion of the bubble during growth (large solid arrows).
- d. At the time corresponding to this sketch the liquid film at the top of the bubble has been cooled to the vapor saturation temperature, and evaporation ceases from this region. The bubble continues to grow although slower than previously because of the lower rate of evaporation per unit bubble volume.
- e. The water surrounding the top region of the bubble is now well below the saturation temperature and some of the vapor within the bubble condenses at the pole. The base of the bubble is still surrounded by superheated liquid. As a result, there is a flow of vapor from the hot base of the bubble to the cool pole.
- f. In this figure the inertia force of the water has been dissipated and the excess pressure within the bubble changed from positive to negative. The bubble has been deformed and the volume increased by the outward flow of the surrounding liquid. The evaporation from the base of the bubble decreases and the condensation from the pole increases.
- g. Condensation will occur over a larger area and the bubble will continue to collapse.
- h. The cooler surrounding liquid now has an appreciable inward radial velocity.
- j. The intrushing cool water has collapsed the bubble back to the critical size.
- k. The hot wall is splashed with the cool liquid and the local wall temperature decreases.
- l. The liquid near the wall has been once again superheated, and the bubble cycle may repeat.

For a saturated liquid the bubble collapse sequence of Figure 5, page 17, would be replaced by continued growth and, eventually, a bubble departure from the heated surface. Much scientific effort has been expended in analyzing the cycle of bubble growth, collapse or departure, and the subsequent waiting time between successive vapor bubbles. Since only the growth phase of the cycle is of interest in the proposed investigation, no further discussion of the collapse or departure will be made. A review of the governing equations that have been applied to bubble growth follows.

The early studies of Jakob and co-workers (21), (22), and (23) considered the latent heat of vaporization and conduction in the liquid surrounding the bubble as that which controls the motion of the liquid-vapor interface. For example, Fritz and Ende (23) solved the conduction equation applicable to the liquid surrounding the vapor cavity. Their governing equation is

$$\frac{\partial T}{\partial t} = \alpha_l \frac{\partial^2 T}{\partial x^2} ; \quad x \geq 0 ; \quad t \geq 0 , \quad (14)$$

where x is the coordinate into the liquid measured from the liquid-vapor interface. This analysis neglects bubble curvature, surface tension, liquid inertia, and motion of

the liquid. Although their solution seems to be an obvious over simplification of bubble growth, its agreement with their experimental investigations (21), (22), and (23) seems to indicate that conduction at the bubble interface does influence bubble growth and consequently the theory of nucleate boiling.

More recent analytical formulations of bubble growth are those of Plesset and Zwick (24), (25), and (26), Bankoff (27), (28), and Forster and Zuber (29). These theories consider a combination of the liquid inertia, surface tension and heat conduction in a uniform temperature field surrounding a vapor bubble.

The equation of motion for the liquid which surrounds the growing vapor bubble is a modified form of Rayleigh's equation developed by Plesset and Zwick (24). It is

$$R_b \frac{d^2 R_b}{dt^2} + \frac{3}{2} \left[\frac{dR_b}{dt} \right]^2 + \frac{2\sigma}{\rho_l R_b} = \frac{P_v - P_l}{\rho_l} . \quad (15)$$

The energy equation of the liquid for a spherically symmetric vapor bubble which has radial fluid motion is given by

$$\alpha_l \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) \right]$$

$$= \frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial r} , \quad (16)$$

and the coupling boundary condition at the liquid-vapor interface is

$$\frac{dR_b}{dt} = - \frac{k_l}{\rho_v h_{fg}} \int \frac{\partial T}{\partial r} \Big|_{r=R_b} \sin \theta \, d\theta , \quad (17)$$

where Equation 17 is obtained from an energy balance at the liquid-vapor interface.

The analysis of Plesset and Zwick (24), (25) suggested that bubble growth could be simplified by subdividing the bubble motion as illustrated in Figure 6. The initially high growth rate of region I is believed to be controlled by the dynamics of the liquid motion. Plesset and Zwick (24) found that during "asymptotic bubble growth" the effects of liquid inertia and surface tension are not important. Hence, shortly after a vapor cavity is visible, the asymptotic growth is believed to be governed by the heat transfer process in the liquid. Only theoretical results for asymptotic growth are presented here.

Bubble growth equations of References (24), (27), (29), (30), and (43) can be put into a meaningful form by expressing the bubble radius-time relationship as suggested by Reference (30)

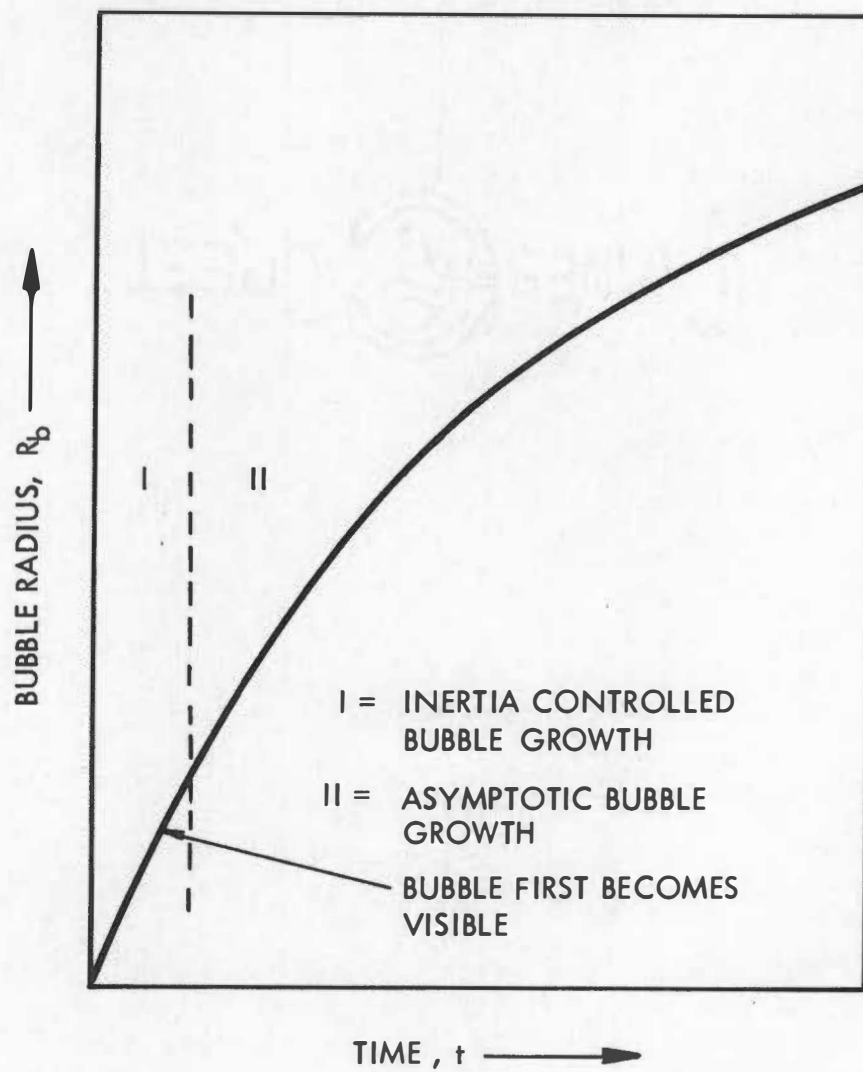


Figure 6. Subdividing of bubble growth.

$$R_b = Cv(\alpha_l t)^{1/2} . \quad (18)$$

C is a constant which accounts for interface curvature and v , called the "growth constant," is a function of the liquid superheat and properties of the liquid. Equation 18 which expresses the one-half power relationship of time to bubble size, is strictly applicable for the case of uniformly superheated liquid.

Zuber (31) extended the earlier theories of Bosnjakovic and Jakob to include the non-uniform liquid temperature distribution which occurs when the vapor bubble forms at a heated surface. By superposition of the wall heat flux into the growth rate equation, he obtained

$$R_b = C \left[v(\alpha_l t)^{1/2} + v' \alpha_l t \right] , \quad (19)$$

where v corresponds to the growth constant of Equation 18 and the modification term, v' , is proportional to the surface heat flux. Equation 19 was proposed for either saturated or subcooled nucleate boiling bubble behavior. Equation 19 predicts bubble collapse as well as growth where as Equation 18 does not. However, Zuber noted the collapse predicted by Equation 19 disagrees with experimental data. He reasoned that, unlike bubble growth, collapse occurs with such turbulent motion that isothermal conditions prevail.

Other analyses (32), (33) have extended Zuber's approach by employing more rigorous mathematical methods of determining v and v' . These refinements, however, have not resulted in substantial improvement over Zuber's correlation of measured results.

Cole and Shuleman (34) have recently compared bubble growths predicted by the aforementioned methods with experimentally measured growth curves taken over a wide range of Jakob numbers.¹ Reasonable agreement was obtained only for Jakob numbers less than 100. These authors conclude that serious shortcomings exist in current bubble growth theories particularly at high Jakob numbers.

In view of this apparent weakness in existing analyses and in light of the recent observations of bubbles growing on an evaporating microlayer of liquid beneath which severe temperature drops in the heated surface occur (2), (3), (34); the present author was prompted to re-examine many of the assumptions underlying the current prediction techniques. Those assumptions which bear examination are:

1. Uniform temperature in the wall and in the liquid during bubble growth,

¹The Jakob number as defined in Reference (31) is

$$\frac{C_l \rho_l \Delta T}{\rho_v h_{fg}} .$$

2. Negligible influence of the thermal properties of the heated surface and also of internal generation therein, and
3. Omission of a liquid microlayer separating the vapor cavity from the heated wall.

The analysis proposed here will investigate bubble growth without recourse to these questionable assumptions.

The Waiting Time

The initial phase of any bubble growth prediction must begin by analyzing the liquid heating period between successive bubble formations. This establishes the non-uniform temperature field within which the bubble develops. The early observations of Jakob (21) noted that the waiting time between bubbles was approximately equal to the bubble growth period. Further experimental work (32) suggested that Jakob's observation may be the exception rather than the general case with waiting time being longer than bubble growth time. A recent approach by Hsu and Graham (32) to ascertain the waiting time and thus the temperature distribution in the liquid at the beginning of bubble growth considered the transient heating of a stationary liquid slab adjacent to the heated surface of thickness, ℓ . The transient one dimensional heat conduction equation was then solved for this region with the boundary conditions:

1. A constant heat flux, at the liquid-solid interface, and
2. A constant surface and initial temperature.

The criterion for incipient boiling as cited earlier, Equation 5, was introduced to denote the end of the waiting period. Hsu and Graham's expression for calculating the waiting time, t_{wait} , is

$$\frac{2(q/A)}{k_l} (\alpha_l t_{\text{wait}})^{1/2} \sum_{j=0}^{\infty} (-1)^j \left[\text{ierfc} \frac{(2j+1)l - x_c}{2(\alpha_l t_{\text{wait}})^{1/2}} - \text{ierfc} \frac{(2j+1)l + x_c}{2(\alpha_l t_{\text{wait}})^{1/2}} \right] = \Delta T_{\text{sub}} + \frac{2\sigma T_{\text{sat}}}{1.25 r_c \rho_v h_{fg}}, \quad (20)$$

where $x_c = l - 2r_c$. Numerical values of the liquid slab thickness, l , are not assigned in Reference (32); hence, these must be determined by a separate analysis or experiment.

An analysis similar to the above mentioned one was performed by Han and Griffith (14); however, their criterion for spontaneous bubble growth was modified so as to be in accord with their incipient boiling theory. Their solution for the waiting time is

$$t_{\text{wait}} = \frac{9}{4\pi\alpha_l} \left\{ \frac{(T_w - T_{\text{bulk}})r_c}{T_w - T_{\text{sat}} \left[1 + \frac{2\sigma}{r_c \rho_v h_{fg}} \right]} \right\}^2. \quad (21)$$

Both expressions for the waiting time are a function of the nucleating cavity size, r_c . A heating surface (commercially finished) contains a wide variety of possible nucleating cavities, and consequently, a distribution of waiting times should be anticipated. The temperature distribution present in the liquid at bubble inception is found by the substitution of these waiting times into the solution of the transient conduction equation.

Microlayer Experimental Observations

Before commencing the proposed analysis of bubble behavior, it may be beneficial to discuss in more detail the recent literature concerning a liquid microlayer on a heated surface (see Figure 3, page 6).

The existence of a liquid microlayer under a vapor bubble formed during boiling heat transfer was first hypothesized by Snyder (35). Qualitative verification followed in the experimental work of Moore and Mesler (2). They observed a local surface temperature fluctuation during boiling by recording the output of a high response

thermocouple in conjunction with an oscilloscope. A typical oscilloscope trace of surface temperature versus time is shown in Figure 7. The severe temperature drops suggest a six-fold increase in the local surface heat flux, followed by a relatively long period of surface temperature recovery. The high instantaneous heat flux could only be attributed to evaporation of a liquid microlayer beneath the vapor bubbles on the heated surface.

Further qualitative evidence of a liquid microlayer was given by Hendricks and Sharp (34). These authors coordinated oscilloscope traces of surface temperature response with high speed motion pictures and established that the sudden decrease in surface temperature occurred simultaneously with the formation of a vapor cavity on the surface thermocouple.

The actual presence of the microlayer was finally ascertained by Sharp (3). Employing interferometer techniques coupled with high speed photography, Sharp observed the microlayer and measured its thickness as a function of time and position. His plot of microlayer thickness on a horizontal flint-glass surface is reproduced as Figure 8, page 30. Unfortunately, no measurements of surface temperature were made.

Inasmuch as an analytic model for microlayer evaporation has not been formulated to date, none of the

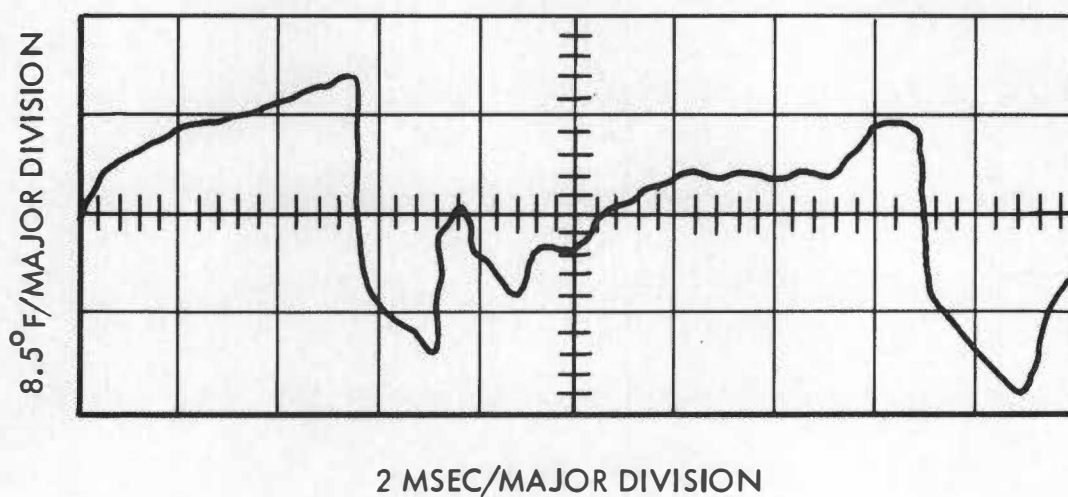


Figure 7. Typical photograph of the oscilloscope face showing surface temperature behavior, Reference (2).

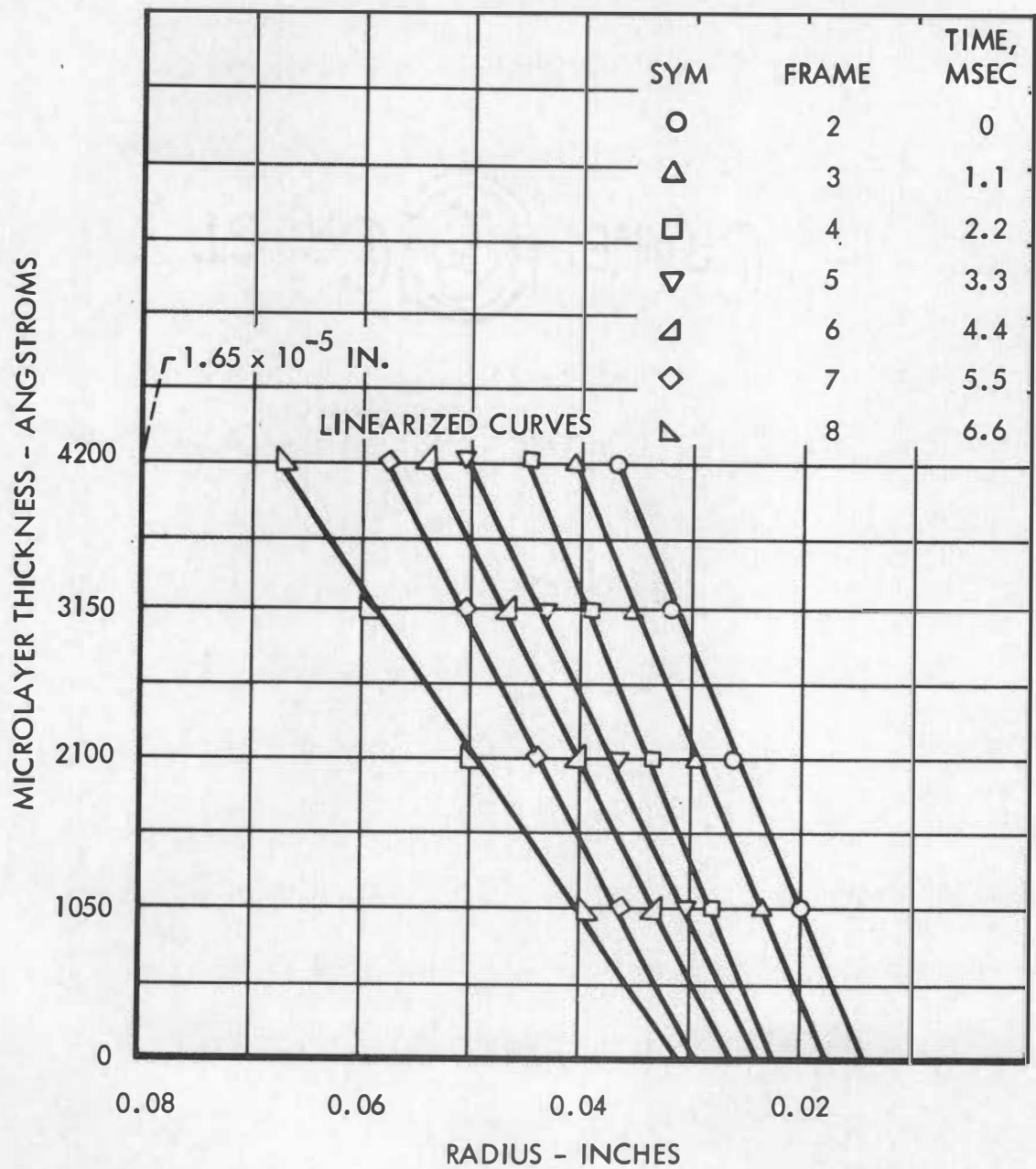


Figure 8. Microlayer thickness as a function of bubble base radius and time, Reference (3).

aforementioned bubble growth and waiting time analyses consider this effect. Hence, before proceeding with the present theoretical development of bubble growth, a model for microlayer evaporation will be formulated, and its effect on the waiting time will be examined.

CHAPTER III

ANALYSIS OF LIQUID MICROLAYER EVAPORATION

The experimental results of References (2) and (3) have shown the existence of a thin layer of liquid (approximate thickness 10^{-5} inches) which separates part of a newly formed vapor cavity from the heated surface. The intention of this chapter is to present an analytic model for the transient behavior of the microlayer as it evaporates into the vapor bubble. By a comparison with reported experimental results, verification of the model is obtained.

I. PROPOSED MODEL FOR MICROLAYER EVAPORATION

Figure 9 illustrates the model of the liquid microlayer upon which the following analysis is based. Only the sudden initial temperature drop is to be analyzed in this chapter and the prediction of the temperature recovery period (see Figure 7, page 29) is given in Chapter IV.

The following assumptions are embodied in the proposed model:

1. The bulk of vapor molecules is in equilibrium at a constant temperature, T_{sat} ,
2. The liquid microlayer has a constant initial thickness,

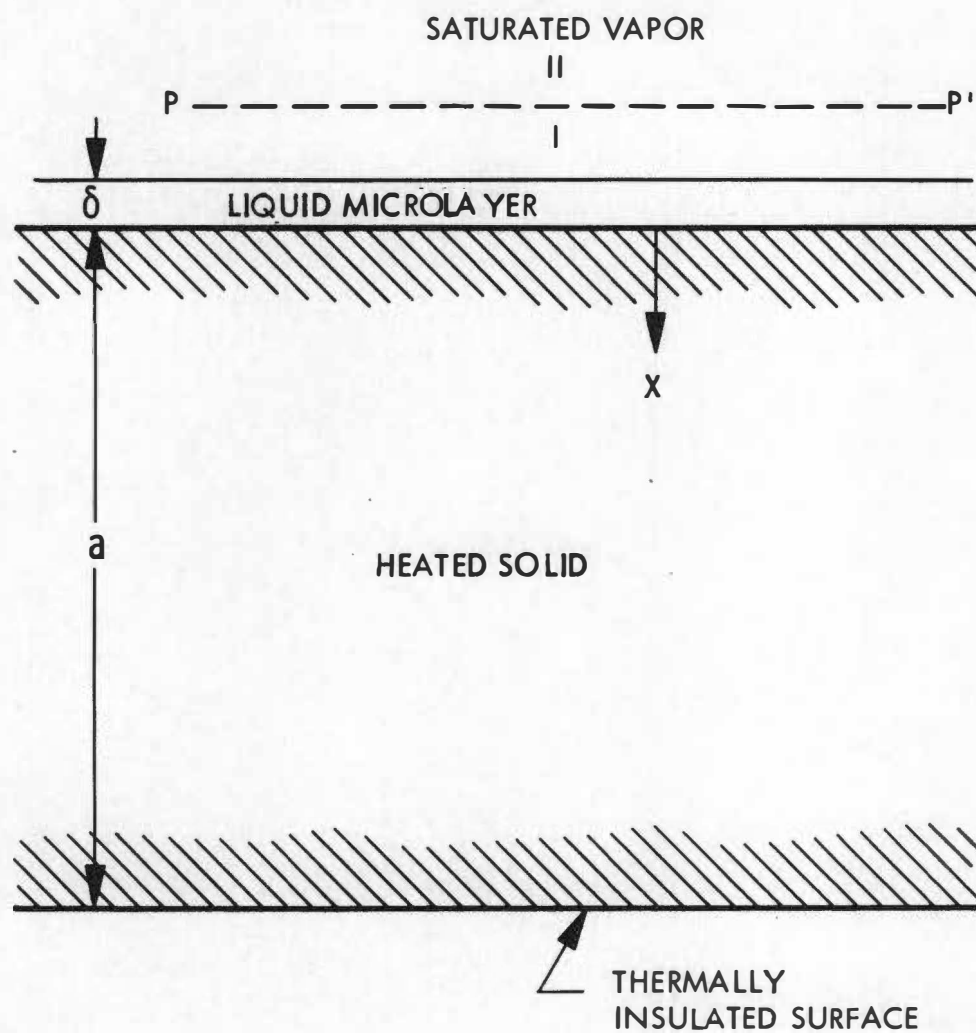


Figure 9. Model for an evaporating liquid microlayer.

3. The liquid microlayer temperature is always uniform and equal to the heater surface temperature,
4. Heat is generated at a constant, uniform rate in the heated solid,
5. Heat conduction from the solid is one-dimensional and normal to the heater surface, and
6. Microlayer evaporation can be determined from kinetic theory where the vapor phase is treated as ideal gas.

The assumption of constant vapor temperature follows from the view that the vapor within the bubble is highly turbulent. Thus only in a thin layer near the liquid-vapor interface is vapor temperature expected to depart appreciably from its saturation value.

Figure 8, page 30, from the experimental work of Reference (3) shows that the microlayer forms with an approximately constant initial thickness (in the particular case of Figure 8, this thickness was reported as 1.65×10^{-5} inches).

A uniformity of temperature in the microlayer is substantiated in Reference (3) where it is noted that 95 per cent of any heater surface temperature change is felt through a 10,000 Angstroms water microlayer in approximately 0.64 microseconds--see also Reference (22). Moreover, the

microlayer will acquire the heater surface temperature since the total heat capacity of the liquid microlayer is much less than the heated solid.

Inasmuch as the significant heat transfer is normal to the heated surface, one-dimensional heat conduction is justified. In actual microlayer evaporation beneath a vapor bubble there is a radial temperature distribution which is neglected in the present analysis.

II. THE HEAT FLUX BOUNDARY CONDITION FOR AN EVAPORATING INTERFACE

To determine the boundary condition at the liquid microlayer interface, two regions in the vapor phase are considered as separated by the fictitious plane PP' in Figure 9. Kennard (38) concludes that Region I represents a layer of vapor molecules with a Maxwellian velocity distribution characterized by temperature, T_l . Region I is composed of vapor molecules which have recently evaporated from the liquid microlayer and vapor molecules which have migrated from Region II with subsequent adjustment of their energies to those characterized by T_l . Region II represents vapor molecules relatively far from any interface with a Maxwellian velocity distribution characterized by temperature, T_v . Approximating the motion of all molecules by ideal gas behavior, the rate at which vapor molecules cross

PP' per unit area in the +x direction is

$$\left. \frac{dn}{dt} \right|_{+x} = N \left(\frac{K}{2\pi m} \right)^{1/2} (T_v)^{1/2}, \quad (22)$$

and for vapor molecules crossing PP' in the -x direction

$$\left. \frac{dn}{dt} \right|_{-x} = N \left(\frac{K}{2\pi m} \right)^{1/2} (T_\ell)^{1/2}. \quad (23)$$

The net rate of molecules crossing PP' is then

$$\left. \frac{dn}{dt} \right|_{\text{net}} = N \left(\frac{K}{2\pi m} \right)^{1/2} \left[(T_\ell)^{1/2} - (T_v)^{1/2} \right]. \quad (24)$$

If $T_\ell = T_v$ the net transport of molecules is zero as it must be for equilibrium.

For an evaporating liquid microlayer ($T_\ell > T_v$), the net molecular transport yields the approximate heat flux boundary condition

$$(q/A)_{\text{evap}} = \left. \frac{dn}{dt} \right|_{\text{net}} m h_{fg}. \quad (25)$$

Combining Equations 24 and 25 and converting microscopic properties to their macroscopic counterparts, one obtains

$$(q/A)_{\text{evap}} = \frac{\rho_v h_{fg}}{4} \left(\frac{8R}{\pi} \right)^{1/2} \left[(T_\ell)^{1/2} - (T_v)^{1/2} \right]. \quad (26)$$

III. ANALYSIS

The one-dimensional heat conduction in an electrically heated solid of thickness "a" can be expressed by the equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + G \quad (27)$$

where G is given by $(q/A)/apc$.

The boundary conditions are

$$k \left. \frac{\partial T}{\partial x} \right|_{x=0} = B \left[(T_\ell)^{1/2} - (T_{\text{sat}})^{1/2} \right] \quad (28)$$

$$\left. \frac{\partial T}{\partial x} \right|_{x=a} = 0 \quad (29)$$

where B is constant for a given saturation temperature and is evaluated from Equation 26

$$B = \frac{\rho_v h_{fg}}{4} \left(\frac{8R}{\pi} \right)^{1/2}. \quad (30)$$

The initial condition is based on the steady state temperature distribution which would exist in a slab with uniform internal heat generation, a prescribed surface temperature T_{wi} at $x = 0$, and an adiabatic surface $dT/dx = 0$ at $x = a$. It has the form

$$T \Big|_{t=0} = T_{wi} + \frac{G}{2\alpha} (2ax - x^2) . \quad (31)$$

The temperature T_l refers to the liquid microlayer and, as stated, is assumed equal to T at $x = 0$. T_{sat} refers to the bulk vapor temperature.

The dimensionless parameters of this set of equations are

$$\begin{aligned} \Theta &= \frac{T - T_{sat}}{T_{wi} - T_{sat}} , \\ X &= \frac{x}{a} , \\ \tau &= \frac{\alpha t}{a^2} , \\ \Phi &= \frac{(q/A)a}{k(T_{wi} - T_{sat})} , \\ \Psi &= \frac{Ba [(T_l)^{1/2} - (T_{sat})^{1/2}]}{k(T_{wi} - T_{sat})} . \end{aligned} \quad (32)$$

In terms of these parameters, the dimensionless form of Equations 27, 28, 29, and 31 is

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial X^2} + \Phi, \quad (33)$$

$$\left. \frac{\partial \Theta}{\partial X} \right|_{X=0} = \Psi, \quad (34)$$

$$\left. \frac{\partial \Theta}{\partial X} \right|_{X=1} = 0, \quad (35)$$

$$\Theta \Big|_{\tau=0} = 1 + \Phi \left(X - \frac{X^2}{2} \right). \quad (36)$$

The solution for the surface temperature of the heated solid (or the liquid microlayer) is outlined in Appendix A with the result

$$\begin{aligned} \Theta \Big|_{X=0} = \Theta_w = 1 - \frac{2}{\pi^{1/2}} \int_0^\tau \Psi'' \left[\frac{1}{2} + \sum_{j=1}^{\infty} \exp\left(\frac{-j^2}{\lambda}\right) \right] \lambda^{-1/2} d\lambda \\ + 4\Phi \left(\frac{\tau^{1/2}}{\pi^{1/2}} \left[\frac{1}{2} + \sum_{j=1}^{\infty} \exp\left(\frac{-j^2}{\tau}\right) \right] - \sum_{j=1}^{\infty} j \operatorname{erfc}\left(\frac{j}{\tau^{1/2}}\right) \right) \end{aligned} \quad (37)$$

where λ is a dummy variable of integration and Ψ'' is a function of $(\tau - \lambda)$.

IV. NUMERICAL SOLUTION AND RESULTS

Solution of Equation 37 was obtained numerically by successive calculations of Θ_w with increasing time increments. Since any error introduced by the numerical integration would propagate into the surface temperature response of successive time increments, a constant time step, which allowed a maximum change of less than one degree Rankine per increment, was used. Hence, the error in the temperature response always remained within acceptable limits.

The dimensionless solution of Equation 37 with $\Phi = 0$ is plotted in Figure 10. The parameter, γ , is obtained by factoring

$$\int_0^\tau \Psi'' d\lambda$$

into

$$\gamma = \frac{Ba}{k(T_{wi} - T_{sat})^{1/2}} \quad (38)$$

and

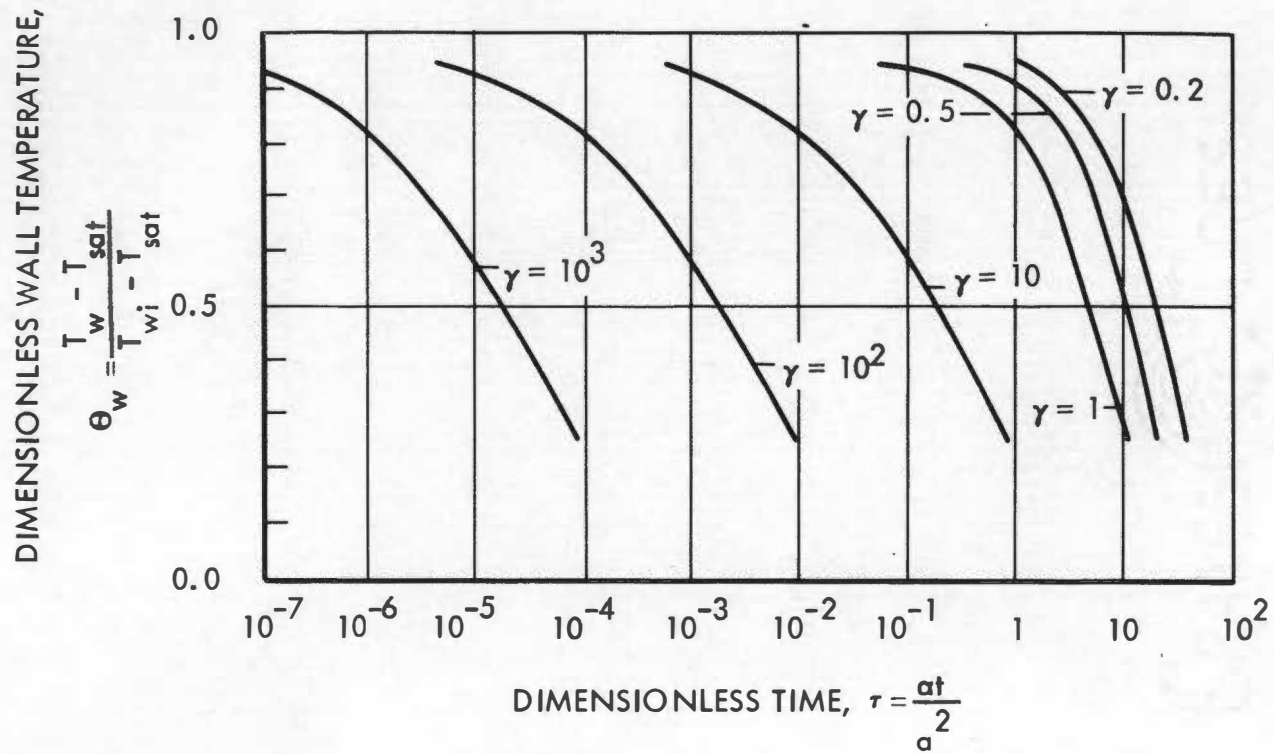


Figure 10. Dimensionless temperature-time response beneath an evaporating microlayer (heat generation parameter, $\Phi = 0$).

$$\Omega = \int_0^{\tau} \frac{(T_w'')^{1/2} - (T_{sat})^{1/2}}{(T_{wi} - T_{sat})^{1/2}} d\lambda, \quad (39)$$

and for convenience, γ is used as an approximation of

$\int_0^{\tau} \Psi'' d\lambda$. If the influence of Ω were included on Figure 10,

page 41, a family of curves indistinguishably clustered around an individual γ curve would result. This remains true if $T_w^{1/2} - T_{sat}^{1/2} \ll (T_{wi} - T_{sat})^{1/2}$, as is the case in the present analysis.

Values of γ less than ten characterize what is effectively the influence of a thin heater. This influence which is due to limited thermal capacity of the heater is illustrated by the steepening of the Θ_w versus τ curves with decreasing γ (Figure 10). On the other hand, values of γ greater than one hundred characterize thick heaters, for which the solution can be approximated by that of a semi-infinite solid. The semi-infinite solid approximation of heater surface temperature is governed by the following set of equations:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, \quad (40)$$

$$k \left. \frac{\partial T}{\partial x} \right|_{x=0} = B \left[(T_l)^{1/2} - (T_{sat})^{1/2} \right] , \quad (41)$$

$$T \Big|_{x \rightarrow \infty} \rightarrow T_{wi} , \quad (42)$$

$$T \Big|_{t=0} = T_{wi} . \quad (43)$$

The surface temperature, obtained by the technique of Appendix A, is

$$\Theta_w = 1 - \frac{1}{\pi^{1/2}} \int_0^\tau \Psi'' \lambda^{-1/2} d\lambda , \quad (44)$$

and the numerical solution is shown in Figure 11.

The introduction of the internal heat generation parameter, Φ , into the foregoing discussion results in the temperature response shown in Figure 12, page 45. Solutions for $\Phi = 0$ along with those for $\Phi \neq 0$ are presented for various values of γ . It is apparent that as γ becomes large the influence of Φ diminishes. Consequently, for large γ and normal values of Φ , the temperature response is equivalent to that of the semi-infinite solid shown in Figure 11.

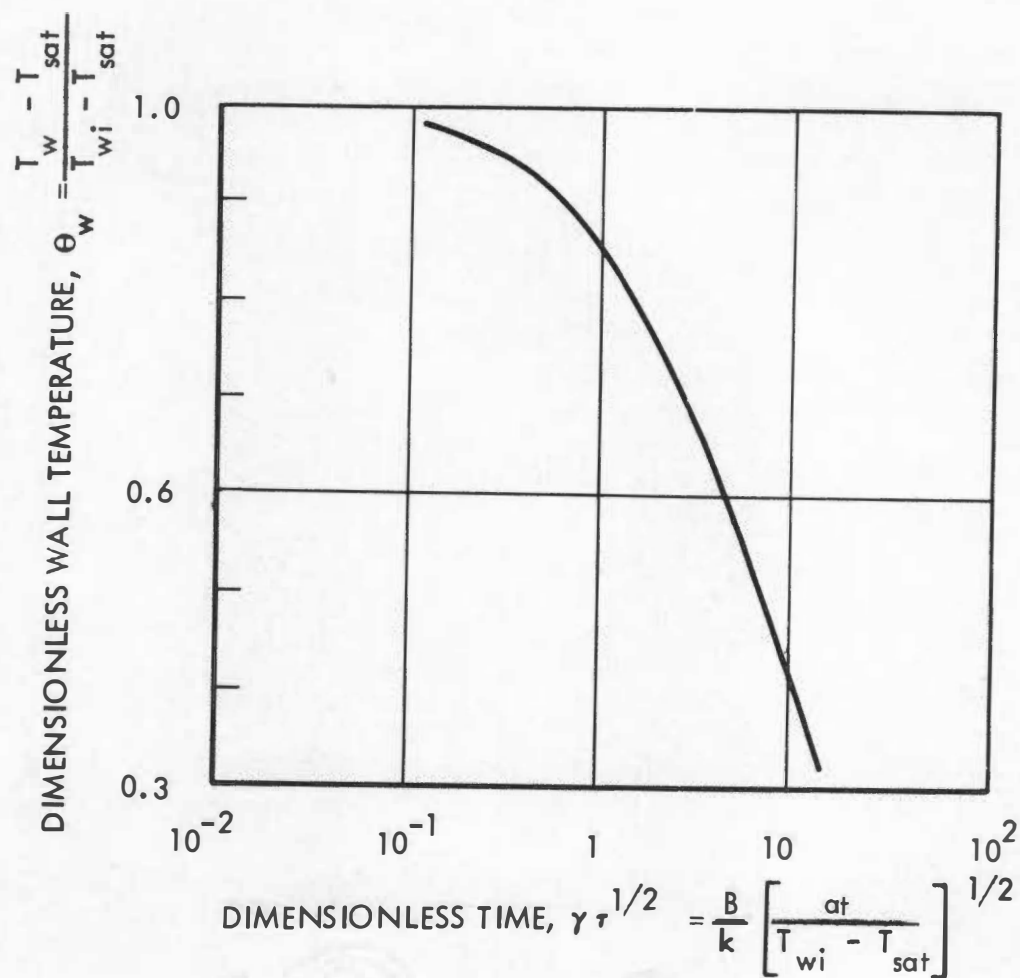


Figure 11. Dimensionless temperature-time response with a microlayer evaporating on the surface of a semi-infinite solid.

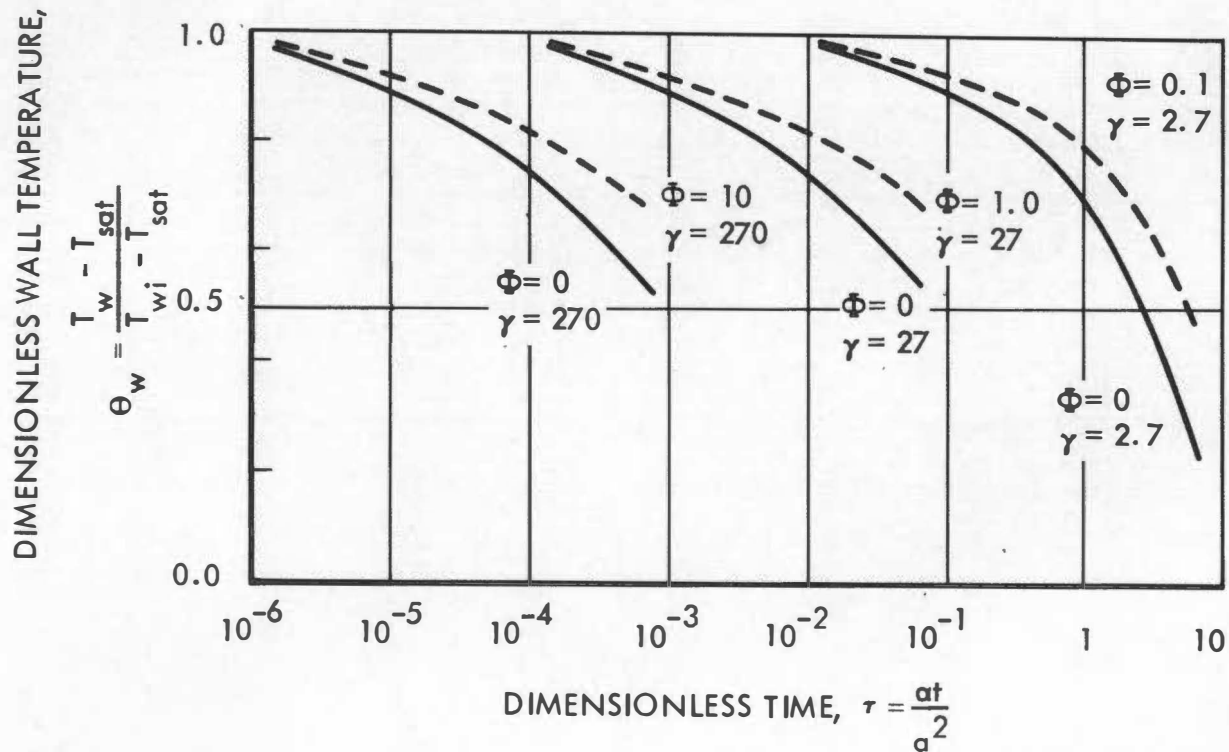


Figure 12. Dimensionless temperature-time response beneath an evaporating microlayer (effect of heat generation parameter, Φ).

V. COMPARISON WITH EXPERIMENTAL DATA

This section compares the foregoing analysis with the results of experimental studies (2), (3), and (34) of liquid microlayer evaporation beneath vapor bubbles formed during nucleate boiling.

In Figure 13 the measured surface temperature response beneath an evaporating microlayer (2) and (34) is compared with that predicted analytically. In general the agreement is good; particularly for the well-defined data of Hendricks and Sharp (34). Unfortunately, Reference (2) reported only temperature fluctuation about an unknown datum; and in order to make a quantitative comparison with the analytical model, it was necessary to assume some magnitude for the initial wall temperature. The value assumed was the overall average wall temperature derivable from the quoted superheat of 23°F. However, this introduces sizeable uncertainty in the computed temperature response (see Figure 7, page 29).

Attention is drawn to the comparison, also made in Figure 13, of the solution for a semi-infinite solid with the data of Hendricks and Sharp (3). Since these authors employed a 0.001 inch heater in their investigation, this comparison emphasizes the thin heater effect pointed out previously in the discussion of the analytical solution.

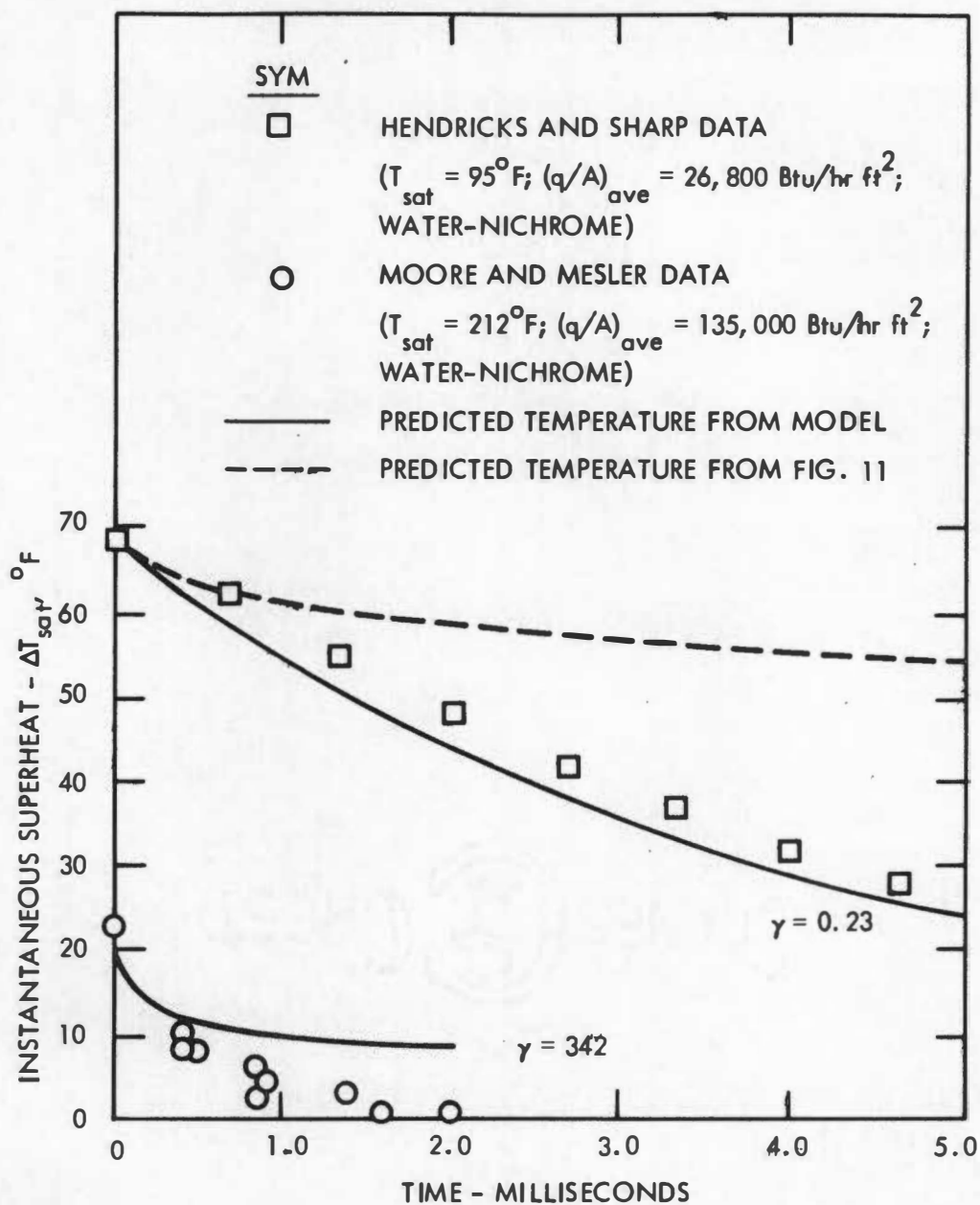


Figure 13. Comparison of model with reported experimental results.

Comparison can also be made with the reported evaporative heat fluxes (2) and (34) as follows:

1. The average evaporative heat flux for the temperature drops shown in Figure 7, page 29, is 790,000 Btu/hr-ft² (2); whereas the similarly averaged heat flux using Equation 26 is 750,000 Btu/hr-ft².
2. The maximum time averaged evaporative heat flux between data points for Reference (34) was reported as 500,000 Btu/hr-ft²; a similar analytic heat flux is 250,000 Btu/hr-ft².

The factor of two discrepancies in the latter data is felt to result from neglecting the heat capacity of the thermocouple which is believed to be considered when the reported heat flux was determined.

As a final comparison the heat fluxes of Reference (3) are listed with predicted values in Table I. Since no surface temperature measurements were made, an initial wall superheat of 18°F, predicted from incipient boiling theory (12), was used for the analytic computation. The large value of γ computed for Sharp's data ($\gamma = 1780$) permitted using the semi-infinite solid solution in calculating the predicted heat fluxes. The agreement which is demonstrated by this approximation is felt to substantiate its applicability to cases with large values of γ .

TABLE I
COMPARISON OF ANALYTIC SOLUTION WITH DATA FROM REFERENCE (3)

Surface Averaged Values Quoted in Reference (3)		Local Values Calculated from Figure 8 @ $r = 0.04$ Inches		Local Values Predicted with Model $(\Delta T_{\text{sat}})_i = 18^\circ\text{F}$	
Frame	Heat Flux (Btu/hr-ft ²)	Mass Evap. (lb _m /ft ²) $\times 10^5$	Heat Flux (Btu/hr-ft ²)	Mass Evap. (lb _m /ft ²) $\times 10^5$	Heat Flux (Btu/hr-ft ²)
2	42,000				
3	50,000	1.75	60,400	1.89	63,500
4	36,300	1.40	47,900	1.36	45,500
5	29,000	0.85	29,200	1.13	38,900
6	32,000	1.07	36,700	0.98	35,400
7	41,000	1.14	38,900	0.96	32,900
8		—		—	
		Total 6.21		Total 6.37	

In summary, the data have agreed with the proposed microlayer model. The wide range of γ which these data cover, justify its use in predicting the behavior of low pressure water microlayer evaporation. Further experimental verification of this model for other wetting fluids is required before a general validity is established.

CHAPTER IV

WAITING TIME ANALYSIS

The calculation of Chapter III illustrates that a significant temperature change will take place at the heated surface during a vapor bubble's formation. This chapter considers the influence of these surface temperature fluctuations on the waiting time which occurs between successive vapor bubbles formed at a given nucleating cavity. Determination of the heating of the liquid before the inception of a new vapor bubble is necessary to establish the frequency of bubble formation, as well as to ascribe the initial temperature distribution in the liquid at bubble growth. The approaches to this problem which are outlined in Chapter II assume either a constant heater surface temperature (14) or a constant surface heat flux (32). Internal heat generation is neglected in all cases. The experimental evidence of surface temperature fluctuation and the preceding microlayer evaporation analysis suggest, however, that such assumptions may introduce considerable uncertainty into the waiting time analysis. These assumptions are not necessary in the following model which is proposed to predict the temperature build-up in the liquid and the condition determining the waiting time.

I. PROPOSED MODEL FOR THE WAITING TIME

The coordinate system and the assumed model for the heat transfer to the liquid are illustrated in Figure 14. As can be seen in this figure only liquid is in contact with the heated surface. The information required to predict the waiting time between the formation of successive vapor bubbles is:

1. A solution for the transient superheat temperature distribution in the liquid near the heated surface, and
2. A knowledge of the defining temperature distribution for which the inception of spontaneous bubble growth will occur.

A summary of the general assumptions to be embodied in the proposed model is:

1. The liquid which replaces the space initially occupied by a departing vapor bubble, has a uniform temperature, T_{bulk} , (i.e., the liquid temperature far from the heated surface),
2. The energy transferred from the heated surface to the liquid can be approximated as one-dimensional heat conduction, and
3. The influence of neighboring bubbles is neglected.

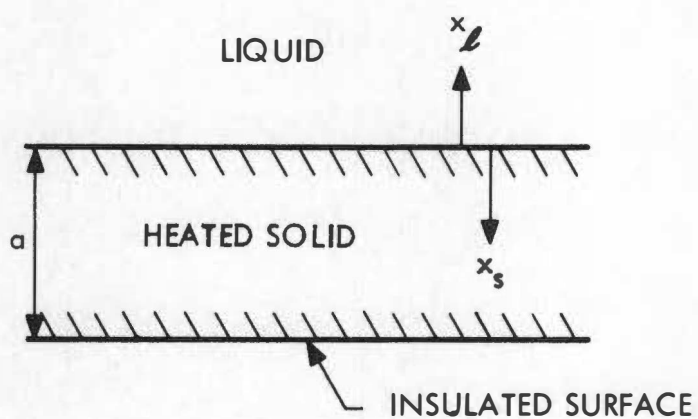


Figure 14. Model for waiting time analysis.

The observations of Hsu and Graham (32), who utilized schlieren techniques to examine the thermal boundary layer which surrounds a vapor bubble, lead to the conclusion that the boundary layer is torn from the heated surface as a bubble departs. Similar observations (20) have been made of vapor bubbles collapsing on a heated surface. This destruction of the boundary layer has been attributed to strong convective motion of the liquid at the end of the collapse process. Hence, the assumption of uniform bulk liquid temperature at the surface prior to the build-up of the superheated layer seems justified.

The approximation of one-dimensional heat transfer to the liquid is founded on the reported concept of a "bubble region of influence" (32). The dimensions of this region are of the order of the maximum diameter of the bubble, (10^{-1} inches), whereas, the diameter of the nucleating cavity is of the order 10^{-3} or 10^{-4} inches. Hence, in the vicinity of the nucleating site, the breakdown of the thermal boundary layer will appear to be complete. In turn the nucleating vapor bubble will be in the laminar sublayer during the waiting time, where conduction may be safely assumed the dominant mode of heat transfer.

The assumption of no influence from neighboring vapor bubbles is introduced for mathematical simplicity. The determination of when this condition is satisfied represents

a formidable problem in itself and the interested reader is referred to (7), (11), and (37). Fortunately in many practical cases a nucleating cavity will fulfill assumption 3 (11); this analysis will consider only such a cavity.

II. ANALYSIS FOR WAITING TIME TEMPERATURE DISTRIBUTION

Subject to the introduced assumptions, the temperature in the heated solid is governed by the equation (Figure 14, page 53),

$$\frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T_s}{\partial x_s^2} + G , \quad (45)$$

with the boundary conditions,

$$T_s \Big|_{t=0} = f_1(x_s) , \quad (46)$$

$$T_s \Big|_{x_s=0} = g_1(t) , \quad (47)$$

and

$$\frac{\partial T_s}{\partial x_s} \Big|_{x_s=a} = 0 . \quad (48)$$

The function $f_1(x_s)$ is the temperature distribution in the solid at the end of microlayer evaporation and the function $g_1(t)$ represents the transient surface temperature response. Equation 48 is the assumption of an insulated back surface.

The heat conduction problem for the liquid is given by

$$\frac{\partial T_l}{\partial t} = \alpha_l \frac{\partial^2 T_l}{\partial x_l^2} \quad (49)$$

with its boundary conditions,

$$\begin{aligned} T_l \Big|_{t=0} &= T_{\text{bulk}} \\ T_l \Big|_{x_l \rightarrow \infty} &\rightarrow T_{\text{bulk}} . \end{aligned} \quad (50)$$

The coupling boundary conditions at $x_s = x_l = 0$ require that

$$T_l \Big|_{x_l=0} = T_s \Big|_{x_s=0} = g_1(t) , \quad (51)$$

and

$$-k_l \frac{\partial T_l}{\partial x_l} \Big|_{x_l=0} = k_s \frac{\partial T_s}{\partial x_s} \Big|_{x_s=0} , \quad (52)$$

where Equations 51 and 52 impose continuity of temperature and heat flux at the liquid-solid interface. By introducing

into Equations 45 through 52, the dimensionless variables,

$$\varphi_s = \frac{T_s - T_{bulk}}{T_{wi} - T_{bulk}} ,$$

$$\varphi_l = \frac{T_l - T_{bulk}}{T_{wi} - T_{bulk}} ,$$

$$\tau_s = \frac{\alpha_s t}{a^2} ,$$

$$\tau_l = \frac{\alpha_l t}{a^2} , \quad (53)$$

$$X_s = \frac{x_s}{a} ,$$

$$X_l = \frac{x_l}{a} ,$$

and

$$v = \frac{Ga^2}{\alpha_s (T_{wi} - T_{bulk})} = \frac{(q/A)a}{k_s (T_{wi} - T_{bulk})} ,$$

the resulting equations are

$$\frac{\partial \varphi_s}{\partial \tau_s} = \frac{\partial^2 \varphi_s}{\partial X_s^2} + v , \quad (54)$$

$$\varphi_s \Big|_{\tau_s=0} = f(X_s) , \quad (55)$$

$$\varphi_s \Big|_{X_s=0} = g(t) , \quad (56)$$

$$\frac{\partial \varphi_s}{\partial X_s} \Big|_{X_s=1} = 0 , \quad (57)$$

where $f(X_s)$ and $g(t)$ are the dimensionless counterparts of $f_1(x_s)$ and $g_1(t)$ respectively. The equations for the liquid become

$$\frac{\partial \varphi_l}{\partial \tau_l} = \frac{\partial^2 \varphi_l}{\partial X_l^2} , \quad (58)$$

$$\varphi_l \Big|_{\tau_l=0} = 0 , \quad (59)$$

$$\varphi_l \Big|_{X_l \rightarrow \infty} \rightarrow 0 , \quad (60)$$

and

$$\varphi_l \Big|_{X_l=0} = \varphi_s \Big|_{X_s=0} = g(t) , \quad (61)$$

$$- \frac{\partial \varphi_l}{\partial X_l} \Big|_{X_l=0} = \frac{k_s}{k_l} \frac{\partial \varphi_s}{\partial X_s} \Big|_{X_s=0} . \quad (62)$$

To specify the functional form of $f(X_s)$, a polynomial approximation is introduced. This assumption is justified in view of the computational effort which must be expended if the microlayer solution of Chapter III is to be completed for the entire solid. The coefficients of the polynomial are evaluated from the known conditions

$$T_s \Big|_{x_s=0} = T_{wm} ,$$

$$\frac{\partial T_s}{\partial x_s} \Big|_{x_s=0} = \frac{B}{k_s} \left(T_{wm}^{1/2} - T_{sat}^{1/2} \right) ,$$
(63)

and

$$\frac{\partial T}{\partial x_s} \Big|_{x_s=a} = 0 ,$$

where T_{wm} represent the heater surface temperature at the conclusion of microlayer evaporation. The following quadratic temperature distribution results,

$$T_s \Big|_{t=0} = f_1(x_s) = T_{wm} + \frac{B}{k_s} \left(T_{wm}^{1/2} - T_{sat}^{1/2} \right) \left(x_s - \frac{x_s^2}{2a} \right) ,$$
(64)

which in dimensionless terms becomes

$$\varphi_s \Big|_{\tau_s=0} = \varphi_{wm} + \beta_{wm} \left(x_s - \frac{x_s^2}{2} \right), \quad (65)$$

where

$$\varphi_{wm} = \frac{T_{wm} - T_{bulk}}{T_{wi} - T_{bulk}},$$

and

(66)

$$\beta_{wm} = \frac{Ba (T_{wm}^{1/2} - T_{sat}^{1/2})}{k_s (T_{wi} - T_{bulk})}.$$

The mathematical details of the heat conduction solution are contained in Appendix B. The final expression for the temperature in the liquid as a function of time and distance from the heated plate is

$$\begin{aligned} \varphi_l = & \frac{\varphi_{wm}}{1+d} \left\{ \text{erfc}(E) \right. \\ & + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) \text{erfc} \left[E + \tau_s^{-1/2} (j+1) \right] \Bigg\} \\ & + \frac{2\beta_{wm}\tau_s^{1/2}}{1+d} \left\{ \text{ierfc}(E) \right. \end{aligned}$$

$$\begin{aligned}
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} + \hat{q}^j) \operatorname{ierfc} \left[E + \tau_s^{-1/2}(j+1) \right] \Bigg\} \\
& + \frac{4(v - \beta_{wm})\tau_s}{1+d} \left\{ i^2 \operatorname{erfc}(E) \right. \\
& \left. + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) i^2 \operatorname{erfc} \left[E + \tau_s^{-1/2}(j+1) \right] \right\}, \quad (67)
\end{aligned}$$

where the standard notation for the higher order complementary error functions--see Reference (40)--has been introduced, along with the dimensionless expressions

$$\begin{aligned}
E &= \frac{x_\ell \tau_\ell^{-1/2}}{2}, \\
\hat{q} &= \frac{1-d}{1+d},
\end{aligned} \quad (68)$$

and

$$d = \frac{k_\ell}{k_s} \left[\frac{\alpha_s}{\alpha_\ell} \right]^{1/2}.$$

III. CRITERION FOR BUBBLE INCEPTION

Equation 67 presents the temperature increase of the liquid near the heated plate as a function of time; time being measured from the instant the previous vapor bubble left the heated surface. The waiting period can be evaluated as soon as the conditions under which the vacated nucleation site will generate a second bubble are established.

These conditions are identical to those discussed in Reference (13) and outlined in Chapter II on incipient boiling with the exception that the steady state temperature

distribution $T_\ell = T_{wi} - \frac{(q/A)x_\ell}{k_\ell}$ is now replaced by

Equation 67.

The temperature of the vapor inside the nucleating cavities of radius, r_c , is given in Equation 6,

$$T_v = T_{sat} + \frac{2\sigma_{fg}T_{sat}}{h_{fg}r_c}, \quad (69)$$

which in dimensionless form becomes,

$$\Phi_v = \Phi_{sat} + \frac{\Gamma}{x_\ell}, \quad (70)$$

where

$$\varphi_v = \frac{T_v - T_{bulk}}{T_{wi} - T_{bulk}},$$

$$\varphi_{sat} = \frac{T_{sat} - T_{bulk}}{T_{wi} - T_{bulk}}, \quad (71)$$

and

$$\Gamma = \frac{2\sigma v_{fg} T_{sat} Pr_\ell^2}{h_{fg} a (T_{wi} - T_{bulk})},$$

and the Prandtl number has been introduced in Equation 70 to relate the dimension into the liquid to the cavity size.

Dimensionless distance evaluated at r_c is given the symbol x_ℓ^*

$$x_\ell^* = Pr_\ell^2 \frac{r_c}{a}. \quad (72)$$

The criterion for incipient bubble growth as stated in Reference (13) is

$$\varphi_v \Big|_{x_\ell = x_\ell^*} = \varphi_\ell \Big|_{x_\ell = x_\ell^*},$$

and

$$\frac{d\varphi_\ell}{dx_\ell} \Big|_{x_\ell = x_\ell^*} = \frac{d\varphi_v}{dx_\ell} \Big|_{x_\ell = x_\ell^*}. \quad (73)$$

Equating Equation 70 to φ_ℓ given by Equation 67 according to the requirements of Equation 73 gives

$$\varphi_{\text{sat}} + \frac{\Gamma}{X_\ell} \bigg|_{X_\ell = X_\ell^*} = \varphi_\ell \bigg|_{X_\ell = X_\ell^*}, \quad (74)$$

and

$$- \frac{\Gamma}{X_\ell^2} \bigg|_{X_\ell = X_\ell^*} = \frac{d\varphi_\ell}{dX_\ell} \bigg|_{X_\ell = X_\ell^*}, \quad (75)$$

which can be combined to yield,

$$\varphi_{\text{sat}} = \left[\varphi_\ell + X_\ell \frac{d\varphi_\ell}{dX_\ell} \right]_{X_\ell = X_\ell^*}, \quad (76)$$

or

$$\begin{aligned} \varphi_{\text{sat}} = & \frac{\varphi_{\text{wm}}}{1 + d} \left| \text{erfc}(E^*) \right. \\ & + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) \text{erfc} \left[E^* + \tau_{\text{s,wait}}^{-1/2} (j + 1) \right] \bigg| \\ & + \frac{2\beta_{\text{wm}} \tau_{\text{s,wait}}^{1/2}}{1 + d} \left| \text{ierfc}(E^*) \right| \end{aligned}$$

$$\begin{aligned}
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} + \hat{q}^j) \operatorname{ierfc} \left[E^* + \tau_{s, \text{wait}}^{-1/2} (j+1) \right] \Bigg| \\
& + \frac{4(v - \beta_{wm}) \tau_{s, \text{wait}}}{1+d} \left| i^2 \operatorname{erfc}(E^*) \right. \\
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) i^2 \operatorname{erfc} \left[E^* + \tau_{s, \text{wait}}^{-1/2} (j+1) \right] \Bigg| \\
& - \frac{2E^* [\varphi_{wm} + \tau_{s, \text{wait}} (v - \beta_{wm})]}{(1+d) \pi^{1/2}} \left\{ \exp(-E^{*2}) \right. \\
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) \exp \left[- [E^* + \tau_{s, \text{wait}}^{-1/2} (j+1)]^2 \right] \Bigg| \\
& - \frac{2E^* \beta_{wm} \tau_{s, \text{wait}}^{1/2}}{1+d} \left\{ \operatorname{erfc}(E^*) \right. \\
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} + \hat{q}^j) \operatorname{erfc} \left[E^* + \tau_{s, \text{wait}}^{-1/2} (j+1) \right] \Bigg\}, \quad (77)
\end{aligned}$$

where

$$E^* = \frac{X_l^* \tau_{l,wait}^{-1/2}}{2} ,$$

and

$$\tau_{s,wait} = \frac{\alpha_s}{\alpha_l} \tau_{l,wait} .$$

For a given r_c the time which satisfies the equality given by Equation 77 is the required waiting time.

IV. RESULTS AND DISCUSSION

Equation 77 is a functional relationship between the characteristic cavity dimension, X_l^* , and the waiting time, $\tau_{l,wait}$. The dimensionless waiting time solutions which will be discussed in this section assume X_l^* to be the independent variable where Equation 72 is recommended to relate X_l^* and the physical cavity dimension r_c .

Figures 15, 16, 17, and 18 present the dimensionless solution of Equation 77 in graphical form. The waiting period, $\tau_{l,wait}$, versus characteristic nucleating cavity dimension, X_l^* , is illustrated with the parameters φ_{sat} and φ_{wm} , the dimensionless saturation and initial heater surface temperatures.

The dimensionless saturation temperature can be interpreted as a measure of the subcooling which exists in

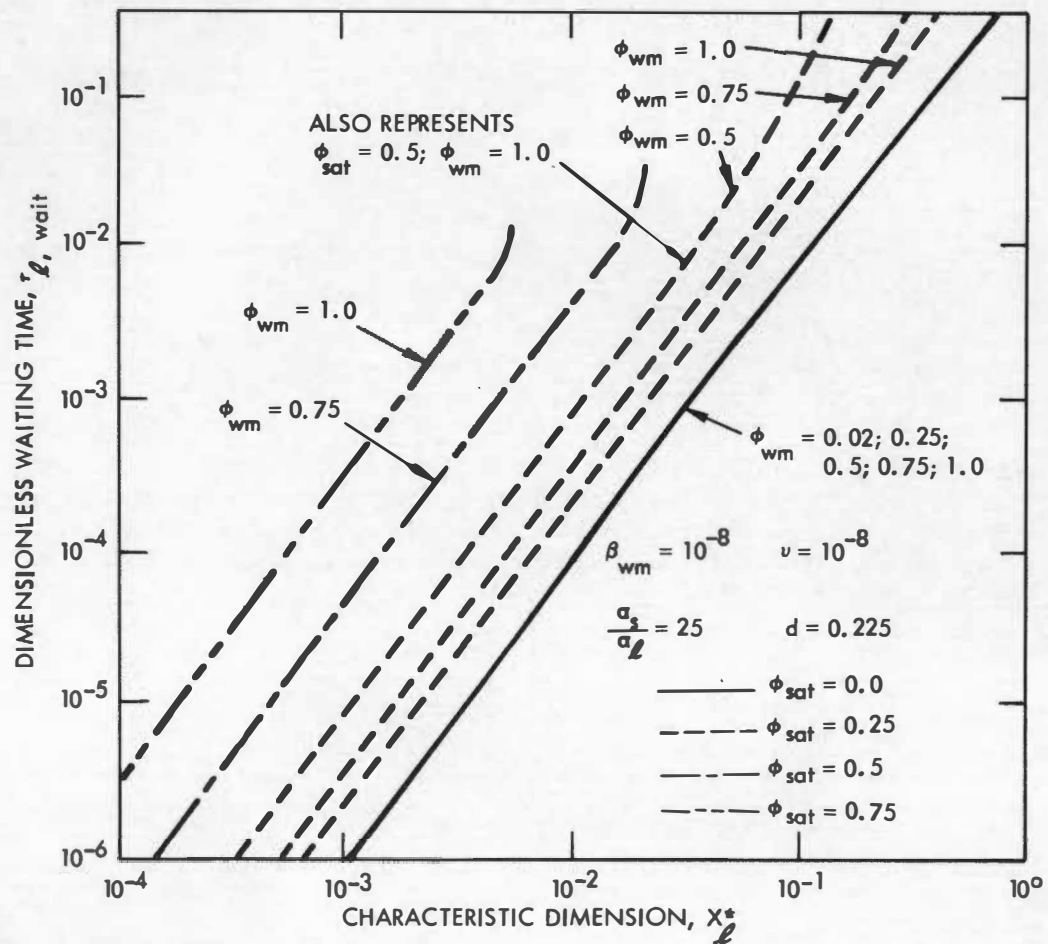


Figure 15. Dimensionless waiting time for variable initial heater surface temperature.

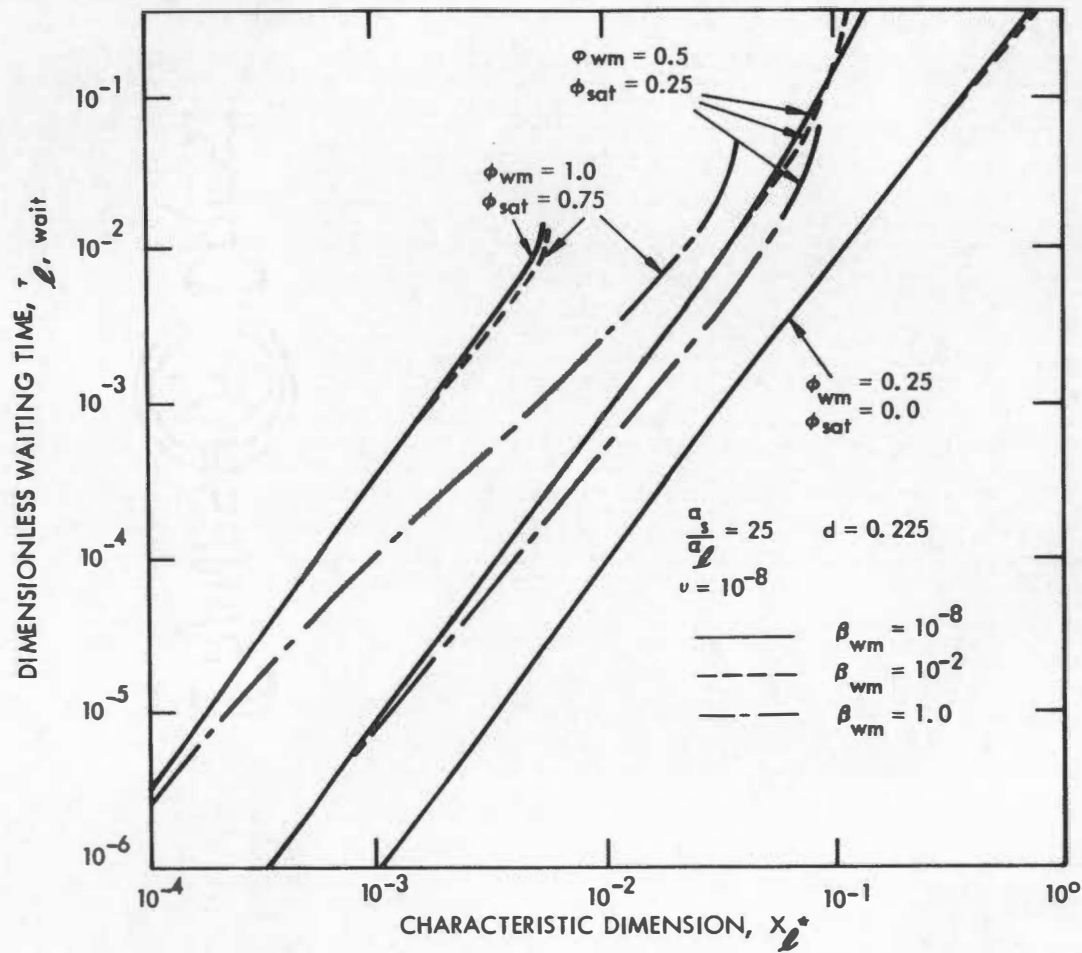


Figure 16. Dimensionless waiting time for variable initial temperature gradient in the heated plate.

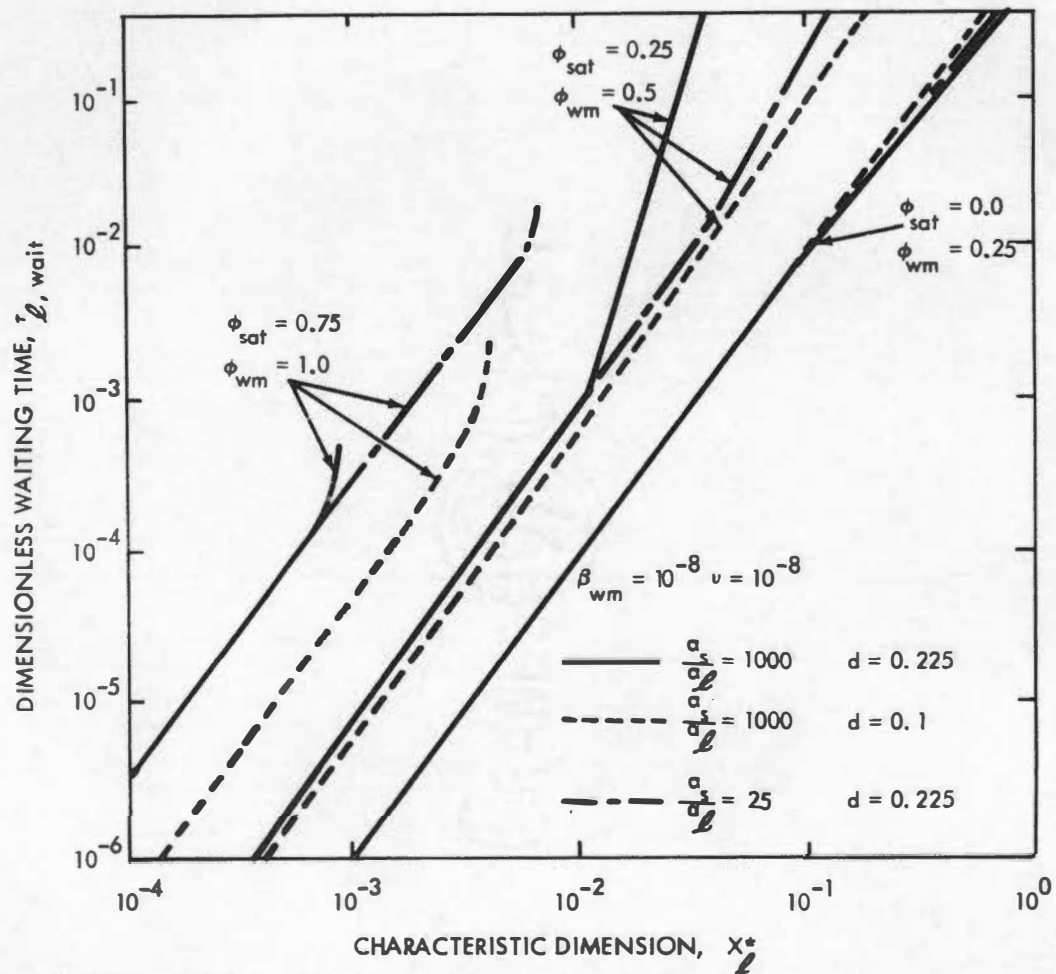


Figure 17. Dimensionless waiting time-effect of thermal transport properties.

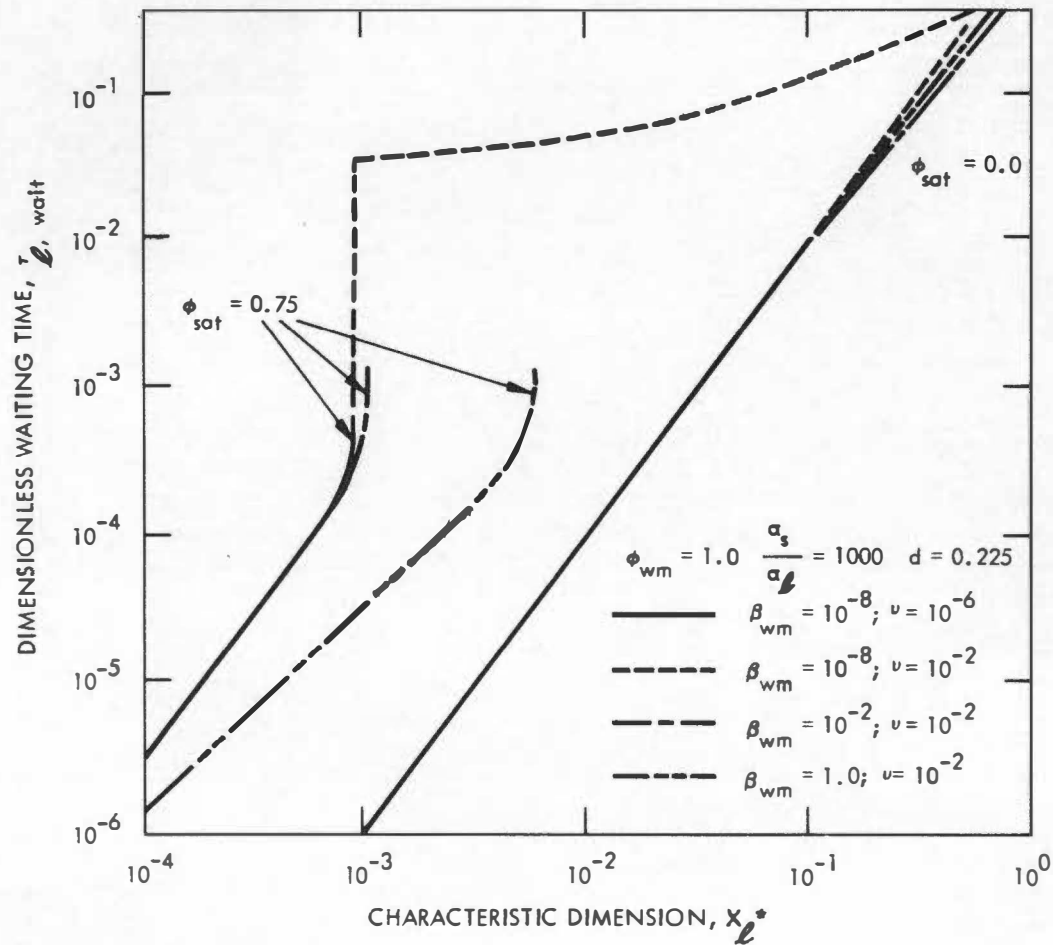


Figure 18. Dimensionless waiting time for variable initial temperature gradients in the heated plate and internal heat generation.

the bulk liquid. Hence, if ϕ_{sat} is zero the bulk liquid is saturated. Figure 15, page 67, illustrates the solution of Equation 77 for very small values of the parameters β_{wm} and v , which characterize the magnitudes of the initial temperature gradient and the internal heat generation, respectively. With ϕ_{sat} equal to zero, Figure 15 shows that all curves for values of ϕ_{wm} from 0.02 to 1.0 coincide with each other indicating that initial wall temperature exerts no influence on the waiting period in a saturated liquid. However, as the bulk liquid temperature becomes subcooled (i.e., $\phi_{\text{sat}} > 0$), the reduced initial wall temperatures increase the waiting time. To observe this in Figure 15, consider a fixed value of X_{ℓ}^* , and note the range of $\tau_{\ell, \text{wait}}$ for a particular value of ϕ_{sat} . The larger the ϕ_{sat} selected the more pronounced is the effect of ϕ_{wm} on $\tau_{\ell, \text{wait}}$. Thus a surface which is highly cooled by microlayer evaporation during bubble growth may have pronounced differences in the waiting period.

For conditions of β_{wm} and v effectively zero the initial heater temperature distribution throughout the solid is essentially uniform and the solution presented in Figure 15 represents the waiting periods subject to these assumptions. Insight into the physical effects of heat diffusion and the heater capacity may be gained from this figure.

The relatively high heat diffusion in the solid compared with the liquid maintains a nearly constant liquid-solid interface temperature and thus the straight line portions of the curves shown in Figure 15, page 67, represent waiting time solutions for a semi-infinite liquid exposed to a fixed, step change in the surface temperature. Since heat diffusion in the heater material is limited by the physical dimension of the solid, a , the stored thermal energy in the heater may be exhausted and this is shown in Figure 15 by the curving of the straight lines. If the stored energy is exhausted and the internal generation is zero ($u = 0$) the waiting period tends to infinity.

Figure 16, page 68, illustrates the influence of the initial temperature gradient in the heated surface, β_{wm} , which produces almost no effect on waiting period until this parameter is approximately 10^{-2} . For higher β_{wm} values one observes a shift in the subcooled waiting period curves toward lower values as shown by the dashed line in Figure 16. Thus, higher values of β_{wm} tend to reduce the spread of waiting times which result from a variable heater surface temperature, ϕ_{wm} . Physically, an initial temperature profile in the heated solid results in more energy stored near the heater surface. Thus the curving of the τ_{wm} lines, previously shown to be due to a limited heat capacity of the solid, occurs at lower values of $\tau_{l,wait}$. This is

reasonable since temperatures in the liquid near the heater increase faster with the larger supply of energy in the heated solid.

The same general conclusions about the influence of σ_{wm} and β_{wm} on the dimensionless waiting times apply when the ratio of thermal diffusivities is changed. Figure 17, page 69, presents the waiting period solution when α_s/α_l is changed from twenty-five to one thousand. This span of α_s/α_l encompasses the behavior of most metallic heater-to-liquid combinations of interest to the present bubble growth study. It should be emphasized, however, that no restriction on the magnitude of the ratio α_s/α_l is necessary in Equation 77. Figure 17 shows that a variation in the ratio α_s/α_l maintains the same relative magnitude of the dimensionless waiting periods. However, the dimensional waiting time, t_{wait} , may change appreciably since $\tau_{l,wait}$ is defined in terms of α_l .

Also illustrated in Figure 17 is the dependence of waiting time on the parameter d . The effect of decreasing the value of d is to reduce the influence of σ_{wm} and β_{wm} on the waiting period. This is what one would expect from a physical consideration of the ratio of thermal transport properties. For small values of d the relative mobility of the thermal energy present in the heated surface compensates for the influence of the variable surface temperature and

the initial temperature gradient in the heater material, and thus tends to overshadow the effects of the parameters φ_{wm} and β_{wm} on the waiting time. In summary, both the liquid and the solid heater transport properties influence the waiting period. By increasing the magnitude of the thermal transport properties of the solid with respect to the liquid the significance of initial wall temperature and initial temperature gradient can be reduced.

The remaining parameter which affects the waiting period is internal heat generation, v . Its influence on the waiting period for several values of β_{wm} is illustrated in Figure 18, page 70, for curves of φ_{wm} equal to unity, and $\varphi_{sat} = 0.00$ and $\varphi_{sat} = 0.75$ respectively. The two solid lines are duplicated from Figure 17, page 69, to serve as convenient reference positions on the diagram. While changes in the waiting period resulting from variations of β_{wm} and v for φ_{sat} equal to zero remain very small, their effect becomes more pronounced for a subcooled bulk liquid ($\varphi_{sat} > 0$) as is shown in Figure 18 with $\varphi_{sat} = 0.75$.

The three combinations of β_{wm} with fixed values of v equal to 10^{-2} illustrate that variations in the magnitudes of the initial temperature gradient produce a pronounced influence on the waiting period. Notable is that presented in Figure 18 at the value of X_p^* where the constant φ_{sat} curves become discontinuous. For X_p^* less than this value

the waiting period is approximately 10^{-3} whereas for greater values the waiting period is approximately 10^{-1} . For waiting periods of the order 10^{-3} the criterion for bubble inception is satisfied by liquid heating from the stored energy in the heated solid. The discontinuity occurs when this stored energy is exhausted. For waiting periods of the order 10^{-1} the criterion for bubble inception is satisfied by energy derived from internal heat generation. Large values of v also produce a shift of the waiting periods toward the $\phi_{\text{sat}} = 0$ line of Figure 18, page 69, and hence tend to reduce the ϕ_{wm} influence on waiting time.

In summary one can conclude that all of the dimensionless parameters appearing in Equation 77 significantly influence the waiting period between the formation of individual bubbles at a given nucleation site on a boiling surface.

V. COMPARISON OF THE PROPOSED MODEL WITH EXPERIMENTALLY REPORTED WAITING PERIODS

The waiting period analysis presents the influence of those parameters, ϕ_{wm} , β_{wm} , and X_p^* , which have heretofore received little analytical or experimental consideration. Thus, the quantitative experimental measurements required for an accurate evaluation of the present theory are scarce.

Recently, Hatton and Hall (39) measured the waiting period between successive vapor bubbles nucleated from artificial sites on the heated surface. Their measurements of the waiting period were obtained during nucleate boiling of saturated water at $P_{\text{sat}} = 2.69$ psia and their data are compared with the present theory in Figure 19. The nucleating cavities were mechanically drilled into the stainless steel heating surface and thus the nucleating cavity radius, r_c , was accurately known. The waiting periods ranged from 0.1 to 3.0 seconds. The extremely good agreement shown in Figure 19 of the analytical results with the measured waiting times for the low pressure boiling conditions of Reference (39) lends confidence to the quantitative predictions of the present theory.

Interestingly, Hatton and Hall also reported that waiting times at $P_{\text{sat}} = 9.6$ psia and 14.7 psia were immeasurably short. This effect on the waiting period is also predicted in the waiting model, since near atmospheric pressure the Prandtl number for water approaches unity. With this pressure change X_l^* becomes approximately one-tenth for the cavity sizes of Reference (39), and the predicted waiting period is reduced by three orders of magnitude (Figure 19). Thus the presence of the Prandtl number in Equation 72 as suggested by Reference (13)

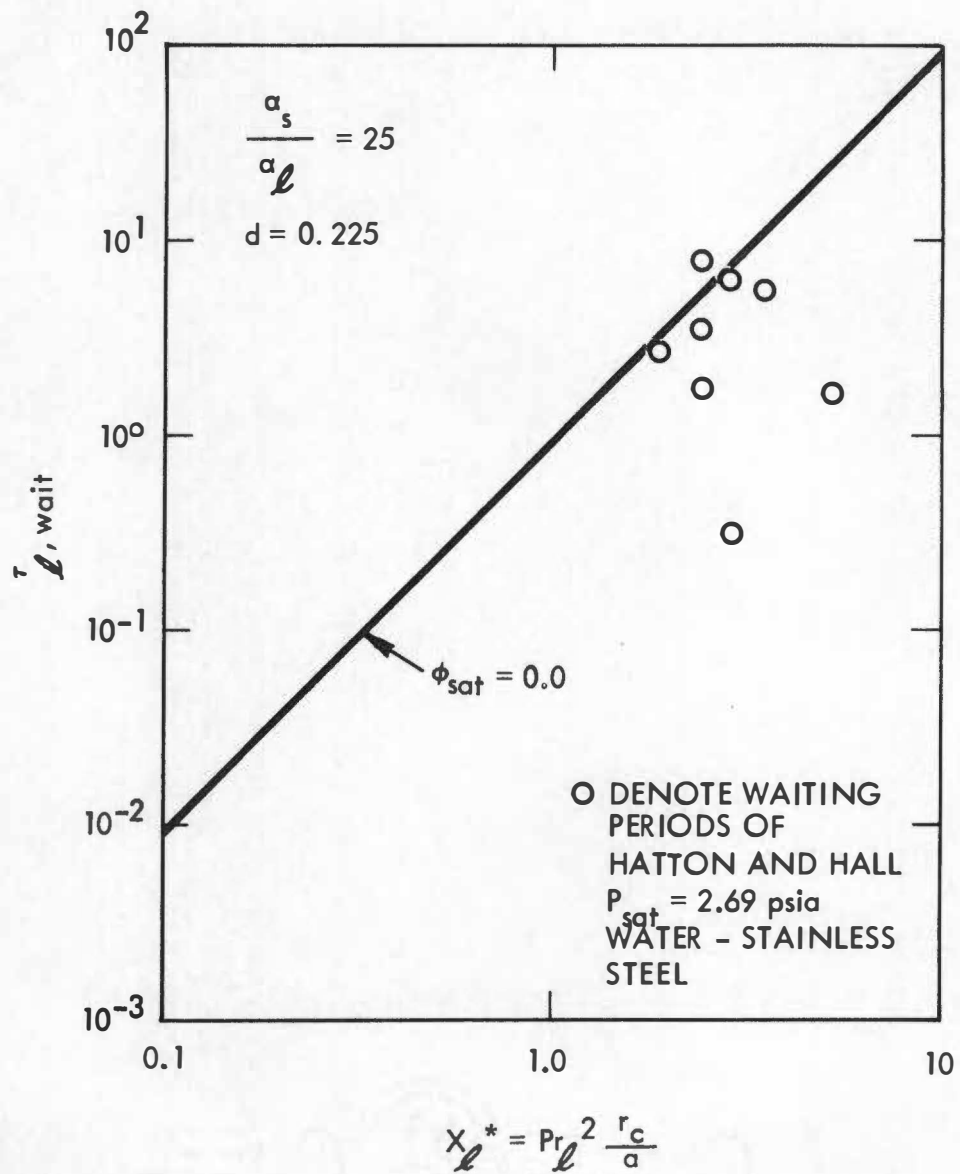


Figure 19. Comparison of waiting time model with reported experimental measurements of Reference (39).

produces the necessary qualitative correction required by experiment.

Further measurements of the waiting period have been reported by References (14), (32), and (40); however, since no information concerning nucleating cavity sizes is given, these observations can only be considered from a qualitative viewpoint. References (14) and (32) report relatively long waiting periods when compared with the data of Reference (40).

<u>Reference</u>	<u>t_{wait}</u>
Hsu and Graham (32)	0.0 through 0.1935 seconds
Han and Griffith (14)	0.0216 through 0.1640 seconds
Howell and Siegel (40)	0.0 seconds

The waiting periods of References (14) and (32) were obtained during nucleate boiling in subcooled liquid ($\phi_{\text{sat}} > 0$), whereas the measurements of Reference (40) were with a saturated liquid ($\phi_{\text{sat}} = 0$). Qualitatively, the present theory predicts longer waiting periods for subcooled boiling in agreement with the reported values.

From the experimental waiting time data which have been presented for comparison with the present theory the significance of the nucleating cavity size, the Prandtl number, and the bulk liquid subcooling can be qualitatively established. Although this comparison of data does not

produce complete verification of the present waiting time solution, it indicates analytically that widely different waiting times can result from temperature fluctuations in the heater due to the microlayer evaporation which occurs prior to commencement of the waiting time solution. The demonstrated influence of the parameters, σ_{wm} , β_{wm} , ν , α_s/α_l , d , and Pr_l , which were neglected in the waiting time analyses of References (14) and (32) suggests that their solutions represent an oversimplification of the problem.

CHAPTER V

BUBBLE GROWTH ANALYSIS

The purpose of this chapter is twofold. First, it will formulate equations which describe the motion of a bubble interface resulting from volumetric expansion due to liquid vaporization. Secondly, it will present the results of a numerical solution for the equations which describe bubble growth.

I. GOVERNING EQUATIONS

Continuity of the Vapor in the Bubble

Figure 20 depicts the bubble model to be analyzed. The bubble is initiated at a nucleation site and grows outward forming beneath it, as it progresses, a microlayer of liquid. The microlayer forms with a constant initial thickness, δ_i , Reference (3), and immediately commences evaporating. Thus, the microlayer thickness varies radially on the heated surface since portions near the center (nucleation site) have undergone longer periods of evaporation. As time progresses, the center portion of the microlayer will completely evaporate leaving a "dry spot" of increasing diameter on the heated surface.

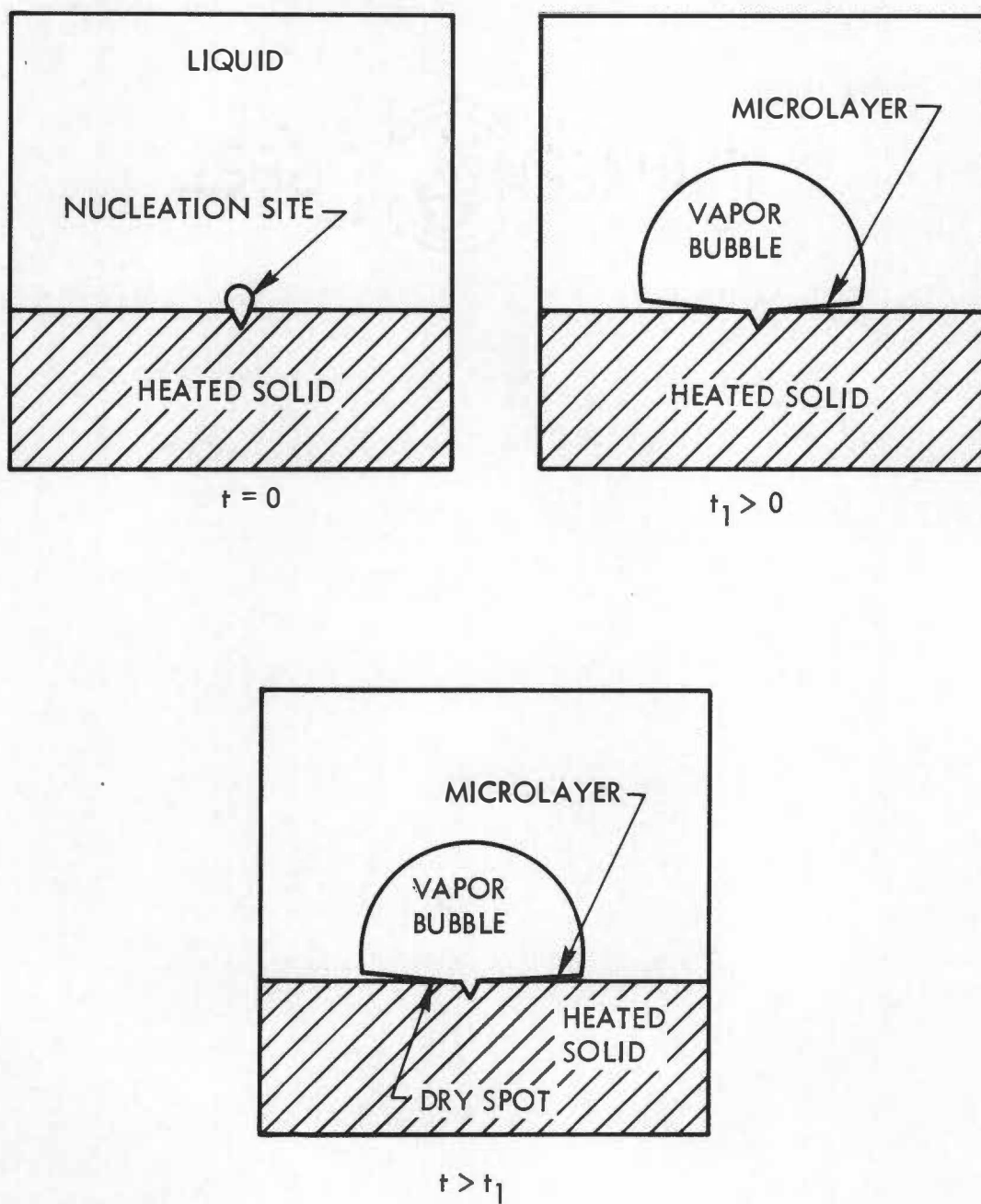


Figure 20. Schematic bubble growth with microlayer evaporation.

For convenience of analysis, the liquid-vapor interface is separated into two parts:

1. The microlayer liquid-vapor interface, henceforth called the microlayer and denoted by the subscript m , and
2. The curved bubble interface, henceforth called the bubble interface and denoted by the subscript b .

The net rate at which mass enters the bubble is obtained by integrating over the appropriate interfacial area and may be expressed as

$$\dot{m}|_{\text{net}} = \int_{A_m} \left(\frac{d\dot{m}}{dA} \right) dA_m + \int_{A_b} \left(\frac{d\dot{m}}{dA} \right) dA_b . \quad (78)$$

The solution of Equation 78 is dependent upon bubble geometry. Since photographic observations of growing bubbles indicate that in many cases the initial geometry of a bubble is approximately hemispherical, such a geometry was assumed throughout this analysis.

The instantaneous mass flux from the microlayer at any radial position measured from the center of the bubble base is given by

$$\frac{d\dot{m}}{dA} = \frac{B}{h_{fg}} \left[T_m^{1/2}(r', t) - T_{sat}^{1/2} \right], \quad (79)$$

where $T_m(r', t)$ denotes the instantaneous temperature of the microlayer.

Consistent with the kinetic theory analysis of the microlayer interface, the mass flux through the bubble interface is specified by

$$\frac{d\dot{m}}{dA} = \frac{B}{h_{fg}} \left[T_b^{1/2}(\theta, t) - T_{sat}^{1/2} \right], \quad (80)$$

where $T_b(\theta, t)$ denotes the instantaneous temperature of the bubble interface.

Combining Equations 78, 79, and 80, the net rate at which mass enters a hemispherical growing vapor bubble becomes

$$\dot{m}|_{net} = \frac{2\pi B}{h_{fg}} \left\{ \int_{r_c}^{R_b(t)} \left[T_m^{1/2}(r', t) - T_{sat}^{1/2} \right] r' dr' + R_b^2(t) \int_0^{\pi/2} \left[T_b^{1/2}(\theta, t) - T_{sat}^{1/2} \right] \sin \theta d\theta \right\}. \quad (81)$$

From a consideration of the volumetric expansion which the vaporizing liquid undergoes upon entering the hemispherical bubble, the net mass rate is related to bubble radius by

$$\dot{m}|_{\text{net}} = \rho_v \frac{d\zeta}{dt} = 2\pi\rho_v R_b^2(t) \dot{R}_b(t), \quad (82)$$

where $\dot{R}_b(t)$ denotes the time derivative of the bubble radius. Equation 82 neglects the volume of the liquid in comparison with the vapor. Equating Equations 81 and 82 gives an expression for the bubble growth rate, $\dot{R}_b(t)$

$$R_b^2(t) \dot{R}_b(t) = \frac{B}{h_{fg}\rho_v} \left\{ \int_{r_c}^{R_b(t)} \left[T_m^{1/2}(r',t) - T_{\text{sat}}^{1/2} \right] r' dr' + R_b^2(t) \int_0^{\pi/2} \left[T_b^{1/2}(\theta,t) - T_{\text{sat}}^{1/2} \right] \sin \theta d\theta \right\}. \quad (83)$$

Bubble radius as a function of time is thus given by

$$\frac{R_b^3(t) - r_c^3}{3} = \int_0^t R_b^2(t) \dot{R}_b(t) dt =$$

$$\frac{B}{h_{fg} \rho_v} \int_0^t \left\{ \int_{r_c}^{R_b(t)} \left[T_m^{1/2}(r', t) - T_{sat}^{1/2} \right] r' dr' + R_b^2(t) \int_0^{\pi/2} \left[T_b^{1/2}(\theta, t) - T_{sat}^{1/2} \right] \sin \theta d\theta \right\} dt . \quad (84)$$

Subsequently, a time stepping procedure for the solution of Equation 84 will be given. First, however, since this procedure requires knowledge of $T_b(\theta, t)$ as a function of angular position and time, the governing equations for the transient temperature distribution in the liquid surrounding the bubble will be developed.

Motion of the Liquid

When the vapor pressure in the nucleating cavity exceeds the ambient pressure of the liquid, the bubble grows. During initial growth, the excess vapor pressure, the inertia of the surrounding liquid, and the interfacial forces contribute to the motion of the bubble interface. Consider the hemispherical bubble (Figure 21) surrounded by an incompressible inviscid liquid, the equation of motion in the liquid domain is

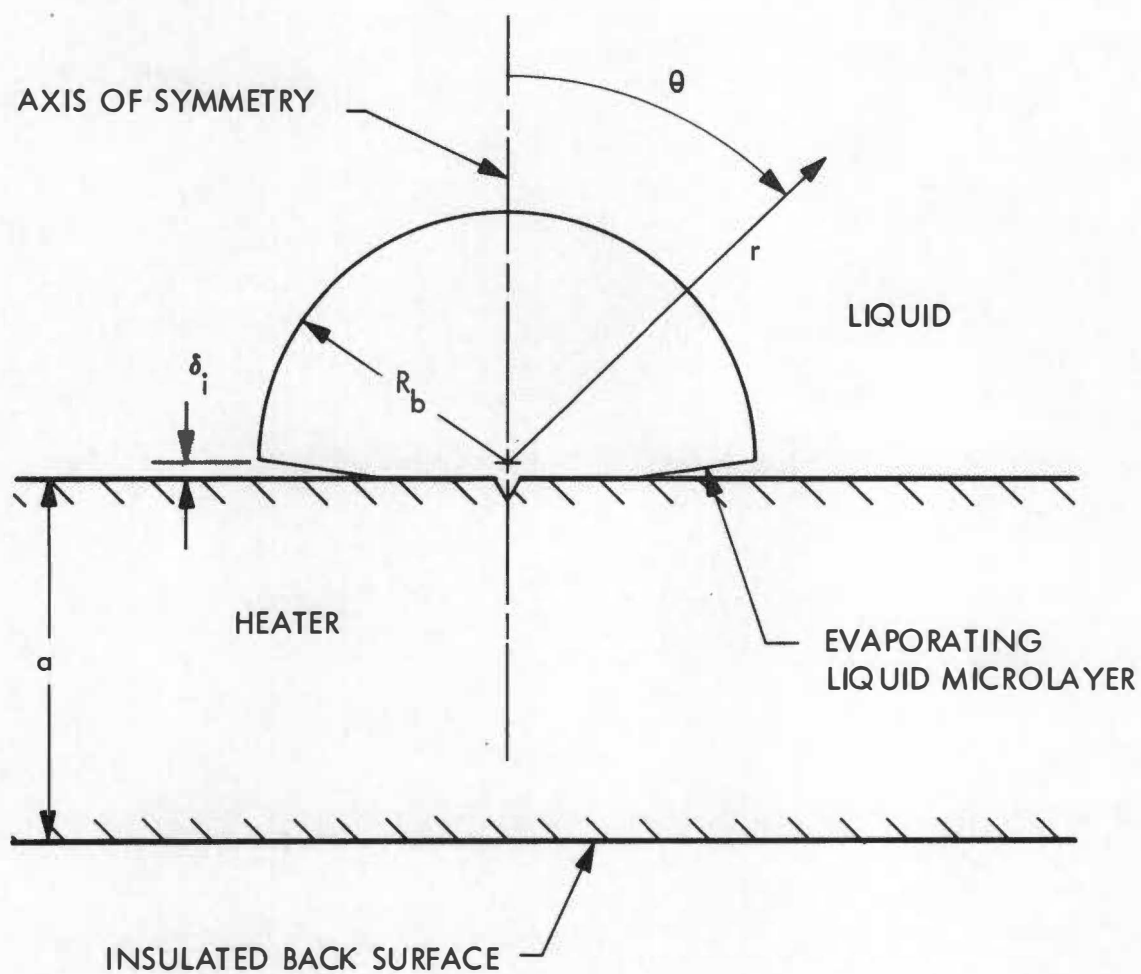


Figure 21. Bubble growth model.

$$\rho_l \left[\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} \right] = - \frac{\partial P_l}{\partial r} . \quad (85)$$

Integrating the equation of continuity

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_r) = 0 \quad (86)$$

from the bubble interface, R_b , to a position, r , in the liquid, gives the radial liquid velocity as a function of the bubble growth rate, $\dot{R}_b(t)$,

$$V_r = \frac{R_b^2(t) \dot{R}_b(t)}{r^2} . \quad (87)$$

Equation 87 substituted into Equation 85 gives

$$- \frac{\partial P_l}{\partial r} = \rho_l \left[\frac{2R_b(t) \dot{R}_b^2(t) + R_b^2(t) \ddot{R}_b(t)}{r^2} - \frac{2\dot{R}_b(t) R_b^4(t)}{r^5} \right] , \quad (88)$$

which upon integration from the bubble interface to infinity, where the ambient liquid pressure, P_{bulk} , is assumed constant, yields

$$P_l \Big|_{r=R_b} - P_{bulk} = \rho_l \left[R_b(t) \ddot{R}_b(t) + \frac{3}{2} \dot{R}_b^2(t) \right] . \quad (89)$$

The liquid pressure at the bubble interface can be related to the vapor pressure inside the bubble by Equation 1, to yield the modified Rayleigh's equation,

$$\frac{P_v - P_{bulk}}{\rho_l} = R_b(t) \ddot{R}_b(t) + \frac{3}{2} \dot{R}_b^2(t) + \frac{2\sigma}{\rho_l R_b(t)} . \quad (90)$$

Energy Equation for the Liquid

The temperature distribution in the liquid is governed by the energy equation, which in spherical coordinates and considering radial liquid motion only, becomes

$$\begin{aligned} \frac{\partial T}{\partial t} = & \alpha_l \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right. \\ & \left. + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) \right] - \frac{R_b^2(t) \dot{R}_b(t)}{r^2} \frac{\partial T}{\partial r} . \end{aligned} \quad (91)$$

Equation 87 has been introduced for V_r in Equation 91.

II. PROCEDURE FOR SOLVING THE BUBBLE GROWTH EQUATIONS

Equations 84, 87, 90, and 91 represent the governing equations which determine the motion of the bubble interface. Inspection of Equation 91 shows that the unknown bubble radius Equation 84 is actually a boundary condition for the energy equation. Furthermore, this boundary condition is also dependent upon the equation of motion (Equation 90) through the Clausius-Clapeyron relationship between T_{sat} and P_v . Hence Equations 91 and 90 are coupled.

Some simplification is introduced into the analysis by noting that solutions for isothermal bubble growth have shown the pressure difference, $(P_v - P_{\text{bulk}})$ to be very small for the major portion of the bubble's life. It thus appears permissible as a first approach to assume $P_v = P_{\text{bulk}}$. This permits two simplifications to the bubble growth analysis:

1. The vapor temperature in the bubble is constant at the equilibrium saturation temperature, T_{sat} , corresponding to the liquid pressure, P_{bulk} , and
2. Equations 84 and 91 become independent of the equation of motion.

Having once obtained $R_b(t)$ from Equations 84 and 91, one may

return to Equation 90 to check the validity of the assumption $P_v - P_{bulk} \approx 0$. This is done in the present work and it will subsequently be shown that the assumption $P_v = P_{bulk}$ is justified.

Analysis of the Energy Equation

Attention is now drawn to the energy equation (Equation 91) which is a non-linear partial differential equation subjected to the non-linear, non-homogeneous boundary condition, Equation 84. The solution of Equations 84 and 91 for the liquid temperature distribution and for the bubble radius $R_b(t)$ was attempted, but the only feasible method involved computer time out of proportion to the required precision of the results. However, as noted in the discussion of Equation 84, all that is required for calculation of the bubble radius as a function of time is the transient temperature distribution at the bubble interface. Thus, in lieu of solving Equations 84 and 91 directly, two approximate solutions for bubble interface temperature were investigated. With these solutions upper and lower bounds on the bubble growth curves can be obtained.

The first approach attributes the sole heat transfer in the liquid to convection which amounts to neglecting the conduction terms of Equation 91. This represents the most severe case of cooling at the bubble interface since

convection carries energy from the interface at the same time evaporation removes the remaining energy stored in a fictitious thin layer of liquid surrounding the bubble.

An alternate way of visualizing the convective effect is presented by Florschuetz and Chao (41) who point out that the convective term in the energy equation (Equation 91) assumes a negative value during bubble growth and thus acts as a heat sink which cools the bubble interface. Hence in the present analysis the convection term decreases the bubble growth rate by cooling the bubble interface to T_{sat} almost immediately upon the commencement of growth. Consequently, almost the entire vapor mass in the bubble is generated from evaporation of the microlayer; thus, the first approach is expected to represent the lower limit of the bubble growth curve.

The second approach neglects convection but includes the effect of conduction. Inasmuch as the temperature gradient at the interface of a growing bubble is positive, the energy flows from the liquid to the bubble interface. Under these conditions evaporation occurs both from the microlayer and from the liquid periphery for the major fraction of the bubble's life. From the above considerations one expects this second approach to give the upper limit of the bubble growth curve.

Case I: Liquid Convection Approximation

By neglecting the conduction terms of Equation 91, the energy equation for the bubble growth reduces to

$$\frac{\partial T}{\partial t} + \frac{R_b^2(t) \dot{R}_b(t)}{r^2} \frac{\partial T}{\partial r} = 0 . \quad (92)$$

The boundary condition to $R_b(t)$ for Equation 92 must specify a heat flux which is consistent with the mass flux of Equation 80

$$(q/A) \Big|_{r=R_b(t)} = h_{fg} \left(\frac{dm}{dA} \right)_b = B \left[T_b^{1/2}(\theta, t) - T_{sat}^{1/2} \right] \quad (93)$$

which becomes

$$k_l \frac{\partial T}{\partial r} \Big|_{r=R_b(t)} = B \left[T_b^{1/2}(\theta, t) - T_{sat}^{1/2} \right] . \quad (94)$$

The initial condition for Equation 92 is the temperature distribution obtained from Equation 67 evaluated at the end of the waiting period

$$T \Big|_{t=0} = f(X_l, \tau) \Big|_{\tau=\tau_{wait}} = f(r_c \cos \theta) . \quad (95)$$

A new set of dimensionless parameters for Equations 92, 94, and 95 are:

$$r_b^* = \frac{R_b}{r_c} ,$$

$$r^* = \frac{r}{r_c} ,$$

$$\mu = \cos \theta ,$$

$$\tau = \frac{\alpha_l t}{r_c^2} , \quad (96)$$

$$\Theta_l = \frac{T - T_{bulk}}{T_{wi} - T_{bulk}} ,$$

and

$$\Psi_l = \frac{Br_c}{k_l} \left[\frac{T_b^{1/2} - T_{sat}^{1/2}}{T_{wi} - T_{bulk}} \right] .$$

Introducing Equation 96 into Equations 92, 94, and 95 gives

$$\frac{\partial \Theta_l}{\partial \tau} + \frac{r_b^{*2} \dot{r}_b^*}{r^{*2}} \frac{\partial \Theta_l}{\partial r^*} = 0 , \quad (97)$$

$$\left. \frac{\partial \Theta_l}{\partial r^*} \right|_{r^*=r_b^*} = \Psi_l , \quad (98)$$

and

$$\Theta_l \Big|_{\tau=0} = f(\mu) , \quad (99)$$

where $\dot{r}_b^*$ now denotes the τ derivative of r_b^* . In spite of the simplifications of the energy equation that have been introduced by the convection approximation, Equation 97 remains non-linear because of the velocity coupling of Equations 84 and 92. However, a numerical procedure for calculating the bubble radius permits a linearization of Equation 97 by approximating the non-linear bubble radius, r_b^* , and bubble growth rate, $\dot{r}_b^*$, with a constant for each time step. The constant, $r_b^{*2} \dot{r}_b^*$, is determined from the previous time step solution of Equation 84 and it is then introduced into Equation 97 to specify the subsequent temperature distributions at the bubble interface. The details of the numerical solution of Equation 97 with $r_b^{*2} \dot{r}_b^*$ as a constant are contained in Appendix C. The result is

$$\begin{aligned} \Theta_{l,n} \Big|_{r^*=r_{b,n}^*} &= \Theta_{l,n-1} \Big|_{r^*=r_{b,n-1}^*} \\ &- \Psi_{l,n-1} \dot{r}_{b,n-1}^* (\tau_n - \tau_{n-1}) , \end{aligned} \quad (100)$$

where the numerical evaluation of the right hand side of Equation 100 is accomplished from the initial conditions ($n = 1$) or the previous time step calculations.

Equation 100 correctly approximates the bubble interface temperature if $r_{b,n-1}^*$ is approximately equal to $r_{b,n}^*$ (42). The solution of Equation 84 showed that this approximation is valid for 10^{-6} seconds after the bubble interface commences its growth. The same order of time (10^{-6} seconds) was required by Equation 100 to cool the bubble interface to a uniform temperature, T_{sat} ; after which, the bubble interface was set equal to the uniform interface temperature, T_{sat} (i.e., effectively insulated). Subsequent calculations of $r_{b,n}^*$ considered evaporation from the liquid microlayer only.

Case II: Heat Conduction Approximation

An approximation of the bubble interface temperature when conduction in the liquid is large can be gained by dividing the bubble interface into a number of nodes and by applying the semi-infinite heat conduction solution for surface temperature presented in Chapter III to each node. Letting x represent the distance into the liquid measured from the bubble interface, the liquid temperature is governed by

$$\frac{\partial T}{\partial t} = \alpha_l \frac{\partial^2 T}{\partial x^2}, \quad (101)$$

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = \frac{B}{k_l} \left[T_b^{1/2} - T_{sat}^{1/2} \right] , \quad (102)$$

$$T \Big|_{x \rightarrow \infty} \rightarrow T_{bi}(r_c \cos \theta) , \quad (103)$$

and

$$T \Big|_{t=0} = T_{bi}(r_c \cos \theta) , \quad (104)$$

where Equation 102 insures that the temperature gradient in the liquid at the bubble interface is consistent with the bubble interface evaporative heat flux. Equations 103 and 104 are specified at each node on the bubble interface evaluated at the end of the waiting period. The bubble interface temperature ($x = 0$) which is obtained from the solution of Equations 101, 102, 103, and 104 is presented in Figure 11, page 44, of Chapter III. This figure presents the dimensionless surface temperature solution of the transient one-dimensional heat conduction equation for a semi-infinite media which is suddenly subjected to the evaporative heat flux boundary condition (i.e., Equation 102). The initial bubble interface temperature, T_{wi} , which appears in the solution shown in Figure 11 was found from Equation 104.

The bubble interface temperatures evaluated by this heat conduction approximation neglect heat diffusion in the θ direction, bubble curvature, and liquid convection. Heat transfer in the polar direction can be neglected for the following reasons:

1. The evaporative heat flux condition (Equation 102) produces a much larger heat flux normal to the bubble interface than can possibly occur by thermal conduction in the polar direction, and
2. Polar heat conduction is the only energy transport mechanism in the θ direction since $V_\theta = 0$ (Equation 91).

The significance of bubble curvature is to produce a larger bubble as noted by Reference (31). To correct for bubble curvature, Reference (31) multiplies the bubble growth by $\pi/2$. However, this factor is omitted since a conservative estimate of the upper limit for bubble growth is produced by neglecting liquid convection. In summary, determination of the bubble interface temperature by assuming semi-infinite heat conduction represents a more severe case of energy transport to the bubble interface than is physically possible. The only limitation on the evaporation at the vapor-liquid interface is that imposed by the rate of energy diffusion.

Summary of the Limitations for the Bubble Growth Solution

The two approximate solutions for the bubble interface temperature from the energy equation for the liquid domain neglect condensation at the bubble interface (i.e., T_b less than T_{sat}). Thus the present bubble growth study is limited to a saturated bulk fluid temperature to eliminate the possibility of condensation at the bubble interface. A summary of the foregoing limitations to the bubble growth model is as follows:

1. A saturated bulk liquid temperature,
2. A uniform and constant vapor temperature, T_{sat} ,
3. A constant initial microlayer thickness and
initial temperature for microlayer evaporation,
4. A hemispherical vapor bubble,
5. The vapor approximates an ideal gas and has
negligible density when compared with the
liquid,
6. Heat diffusion in the polar direction of the
liquid domain is neglected, and
7. From Chapter III, the radial heat diffusion in
the heat generating material beneath the liquid
microlayer is neglected.

Summary of the Numerical Procedure for Bubble
Growth Calculation

Having indicated the upper and lower limits of the transient bubble interface temperature, the numerical solution of Equation 84 can be described. Equation 84 in its dimensionless form is

$$\begin{aligned} \frac{r_b^{*3} - 1}{3} = Ja \, \gamma_\ell \int_0^\tau \left[\int_1^{r_b^*} \xi_m r' dr' \right. \\ \left. + r_b^{*2} \int_0^1 \xi_b d\mu \right] d\tau , \end{aligned} \quad (105)$$

where, in addition to the parameters presented in Equation 96, the following dimensionless parameters have been introduced:

$$\begin{aligned} Ja &= \frac{\rho_\ell C_\ell (T_{wi} - T_{sat})}{\rho_v h_{fg}} , \\ \gamma_\ell &= \frac{Br_c}{k_\ell (T_{wi} - T_{sat})^{1/2}} , \\ \xi_m &= \frac{T_m^{1/2} - T_{sat}^{1/2}}{(T_{wi} - T_{sat})^{1/2}} , \end{aligned} \quad (106)$$

and

$$\xi_b = \frac{T_b^{1/2} - T_{sat}^{1/2}}{(T_{wi} - T_{sat})^{1/2}} . \quad (106)$$

One notes that the dimensionless group $Ja \, \gamma_l$ contains the dimensionless Jakob number which relates the present results to the isothermal bubble growth solutions discussed in Chapter II.

A description of the numerical procedure to determine bubble radius as a function of time from Equation 105 is contained in Appendix D. Briefly, it is as follows:

1. For given initial parameters the liquid micro-layer temperature was calculated as a function of evaporation time according to the procedure outlined in Chapter III,
2. The initial temperature distribution at the nucleating bubble interface, T_b , (Equation 67 evaluated at t_{wait}) was determined as outlined in Chapter IV,
3. Successive numerical values of the bubble radius were evaluated from the difference equations of Appendix C with the liquid microlayer forming as the bubble grew and evaporating into the vapor bubble in accordance with the solution obtained in step 1, and

4. Once the bubble interface cooled to T_{sat} , successive time steps assumed no mass contribution from the bubble interface and continued bubble growth resulted from microlayer evaporation only.

III. RESULTS OF THE NUMERICAL SOLUTION

The dimensionless parameters, ξ_m and ξ_b , which appear in Equation 105 are not suitable for a dimensionless presentation of the solution, since they are variable functions of time and position. However, these parameters must implicitly contain the parameters which were introduced in Chapters III and IV. To establish which of the aforementioned parameters are significant to the bubble growth solution, a preliminary study was made. Table II summarizes the parameters which were found to be significant or insignificant to the present bubble growth models. A description of the procedure used in ascertaining the insignificant parameters is contained in Appendix E. The resulting dimensionless parameters which characterize the visible lifetime of the vapor bubble are:

$$\delta_i * \tau^{-1/2} = \frac{\delta_i}{(\alpha_l t)^{1/2}},$$

TABLE II
SUMMARY OF PRELIMINARY STUDY OF BUBBLE GROWTH

Parameter Symbol	Description	Asymptotic Bubble Growth ¹		Dimensionless Parameters
		Convection Approx.	Conduction Approx.	
δ_i	Initial Microlayer Thickness	Significant	Significant	$\delta_i^* = \frac{\delta_i}{r_c}$
r_c	Initial Bubble Size	Insignificant	Insignificant	$r_b^*; \tau; \gamma_l; \delta_i^*$
T_{wi}	Initial Heater Surface Temperature	Significant	Significant	$Ja; \xi_b; \xi_m$
α_s and k_s	Thermal Transport Properties of the Heater Material	Significant	Significant	$d = \frac{k_l}{k_s} \frac{\alpha_s}{\alpha_l}^{1/2}$
a	Heater Thickness	Possibly Significant	Possibly Significant	$\gamma = \frac{Ba}{k_s (T_{wi} - T_{sat})^{1/2}}$
$P_v - P_{bulk}$	Excess Pressure Inside the Vapor Bubble	Insignificant	Insignificant	Contained in Modified Rayleigh Equation 90
$\frac{(q/A)}{a}$	Internal Heat Generation in the Solid	Insignificant	Insignificant	ψ or ϕ
$T_{ci}(\theta)$	Non-Uniform Initial Temperature Distribution Surrounding Vapor Bubble	Insignificant	Insignificant	$\theta_{l,b}$

¹As defined in Figure 6, page 22.

$$r_b^* \tau^{-1/2} = \frac{R_b}{(\alpha_l t)^{1/2}},$$

$$\gamma_l \tau^{1/2} = \frac{B(\alpha_l t)^{1/2}}{k_l (T_{wi} - T_{sat})^{1/2}},$$

$$d = \frac{k_l}{k_s} \left[\frac{\alpha_s}{\alpha_l} \right]^{1/2},$$

and

$$\gamma = \frac{Ba}{k_s (T_{wi} - T_{sat})^{1/2}},$$

if $\gamma \leq 100$. The above dimensionless groups have been written with $(\alpha_l t)^{1/2}$ as the characteristic length, rather than r_c as used previously. Appendix E notes that the characteristic dimension, r_c , is significant to bubble size during the initial growth, but becomes insignificant during the latter stages or visible lifetime of its development and thus is not a characteristic dimension.

Figures 22 through 24 present the dimensionless convection approximation for asymptotic bubble growth which represents the growth of a hemispherical vapor bubble resulting solely from evaporation of a liquid microlayer since the bubble interface is effectively insulated during

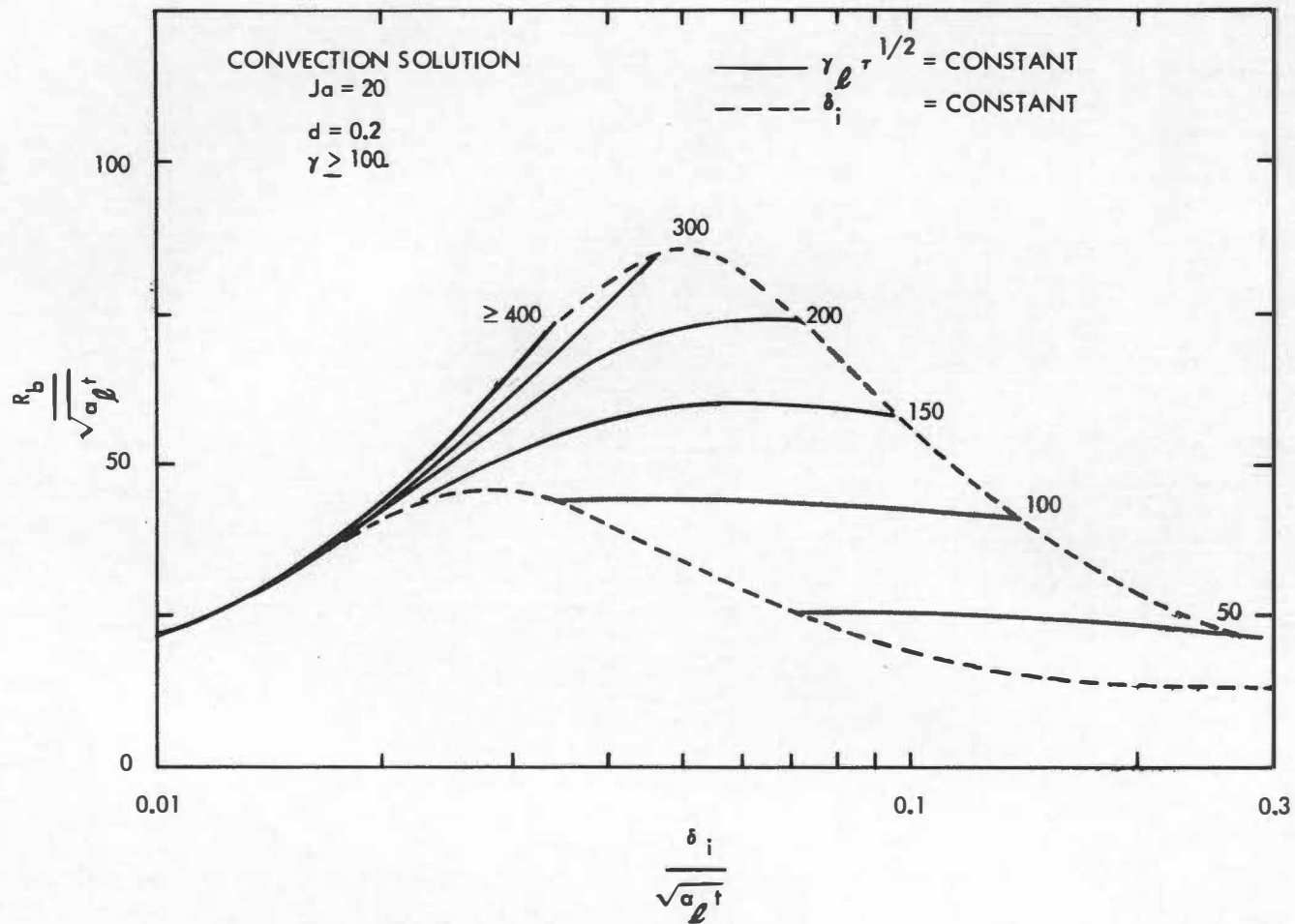


Figure 22. Dimensionless bubble growth solution (convection approximation--Jakob number equals twenty).

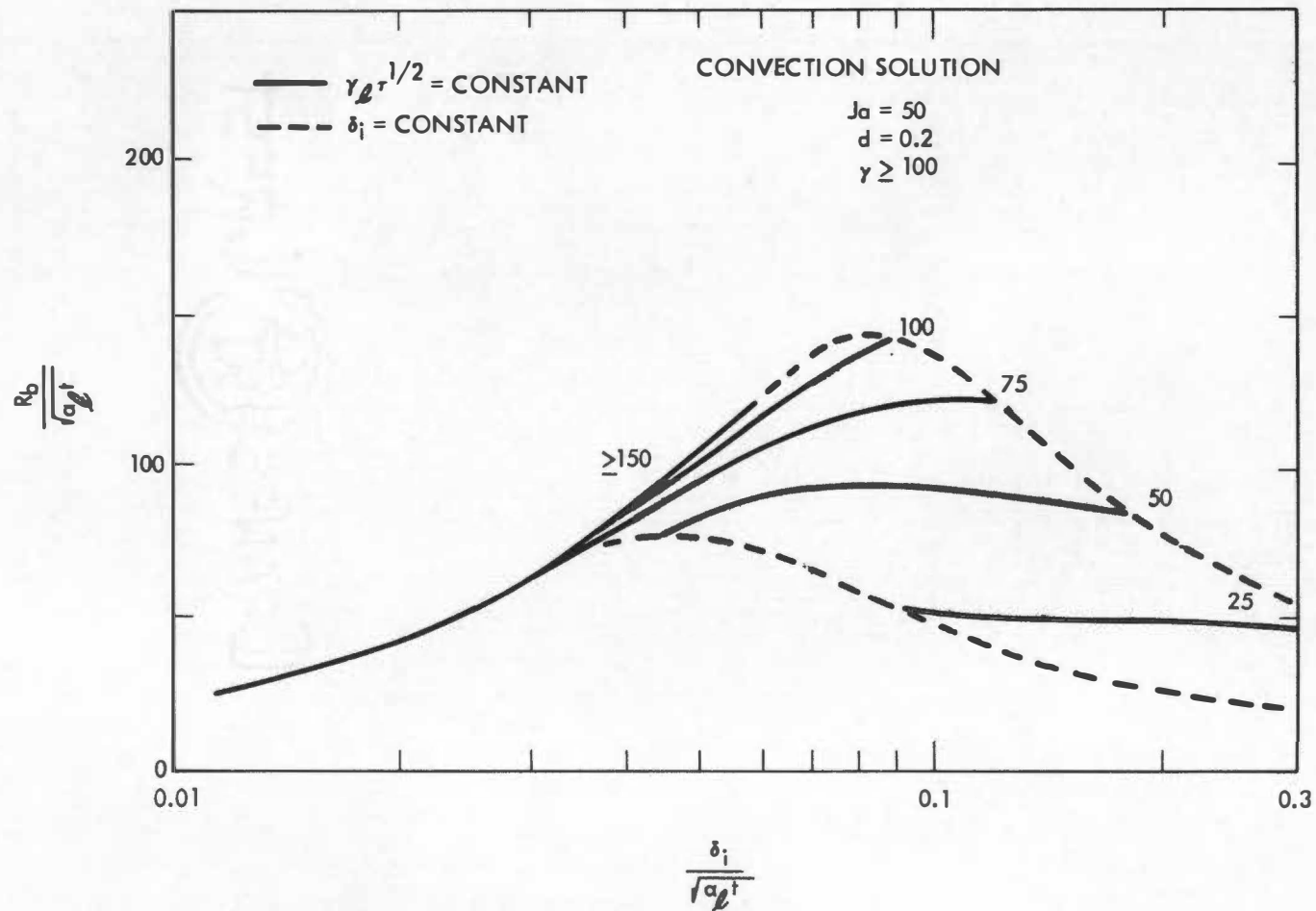


Figure 23. Dimensionless bubble growth solution (convection approximation--Jakob number equals fifty).

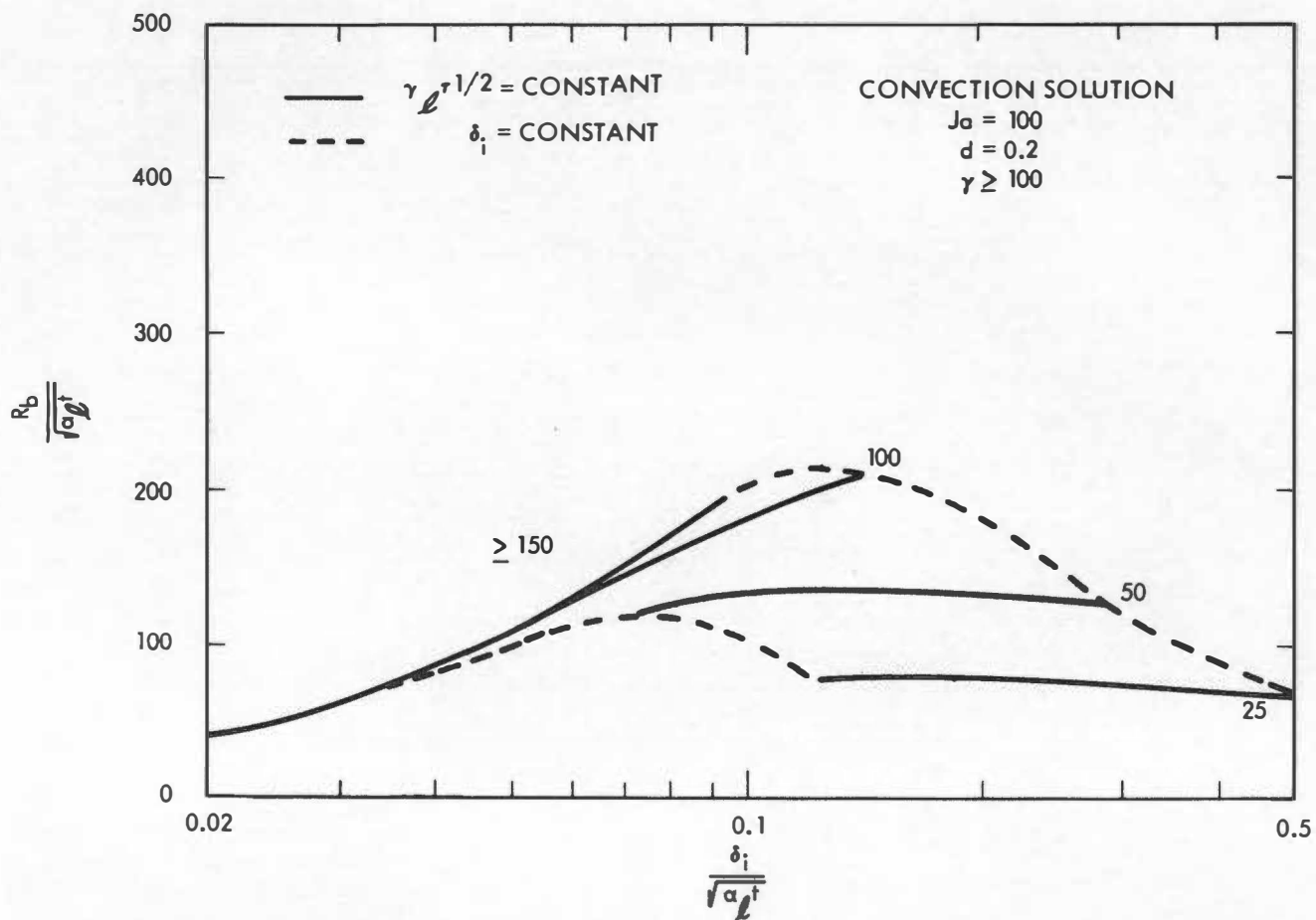


Figure 24. Dimensionless bubble growth solution (convection approximation--Jakob number equals one hundred).

asymptotic bubble lifetime in the convection solution. Figures 25 through 27 present the dimensionless conduction approximation for bubble growth which results from evaporation on both the bubble interface and on the microlayer interface during the major portion of the growth period. Shown in these figures are $r_b * \tau^{-1/2}$ versus $\delta_i * \tau^{-1/2}$ as a function of the parameter $\gamma_\ell \tau^{1/2}$ with Jakob number and d equal to a constant in each figure. The remaining significant parameter, γ , will only alter the bubble growth results for small values and will be discussed later in this section. The above mentioned figures thus present an implicit relationship between bubble size R_b , and visible bubble lifetime, t .

The dashed lines in each figure represent the bubble growth for a constant δ_i . The magnitude of $\delta_i * \tau^{-1/2}$ will decrease as bubble lifetime is increased. The figures do not represent a particular dimensionless bubble growth; rather these figures afford a means for rapid determination of bubble size, R_b , for a specific value of time, which is achieved by the following procedure.

For selected values of time, fluid, and heater material, the parameters $\delta_i / (\alpha_\ell t)^{1/2}$ and $B(\alpha_\ell t)^{1/2} / [k_\ell (T_{wi} - T_{sat})^{1/2}]$ are evaluated and $R_b / (\alpha_\ell t)^{1/2}$ is determined from the appropriate figure corresponding to the specified

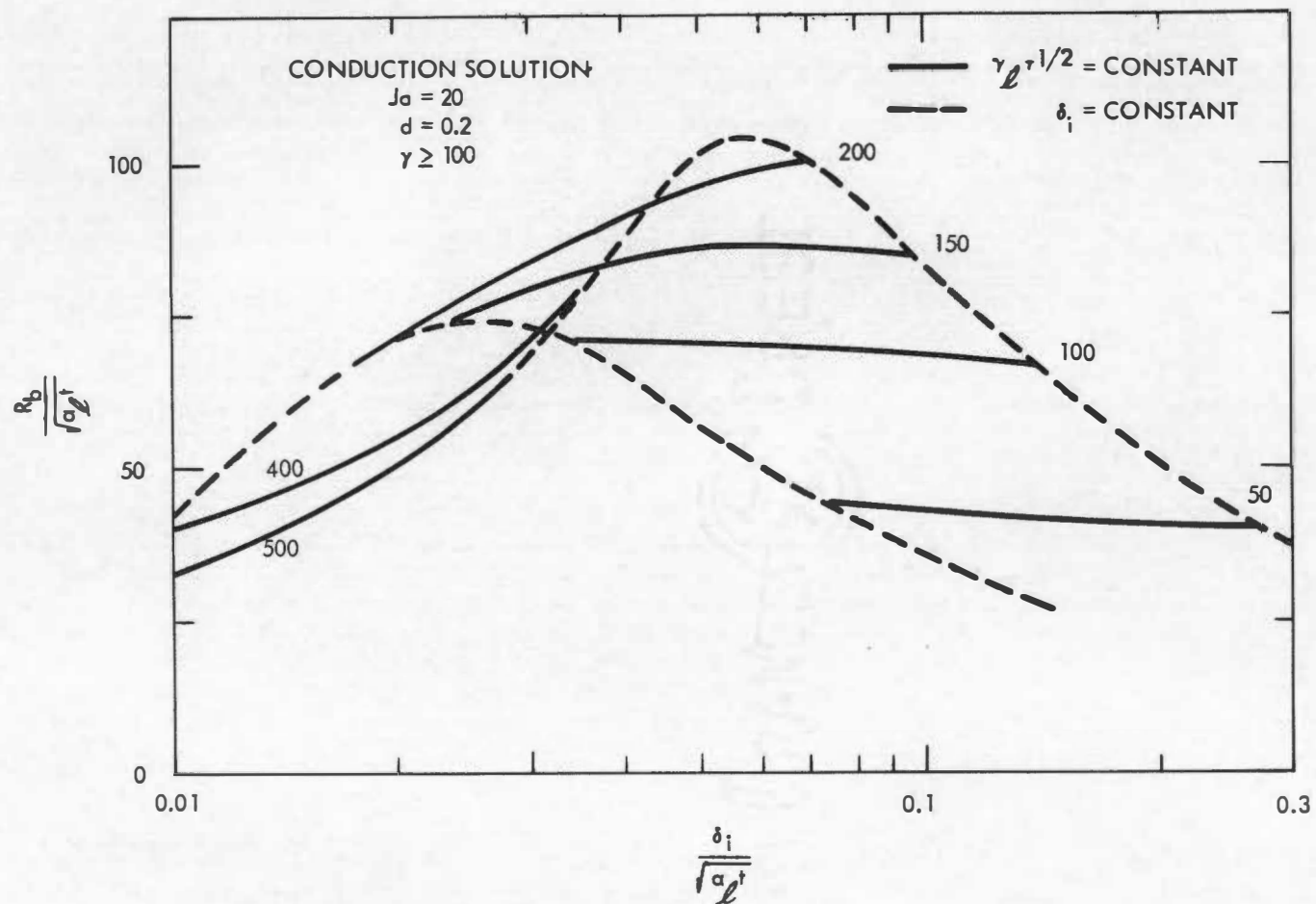


Figure 25. Dimensionless bubble growth solution (conduction approximation--Jakob number equals twenty).

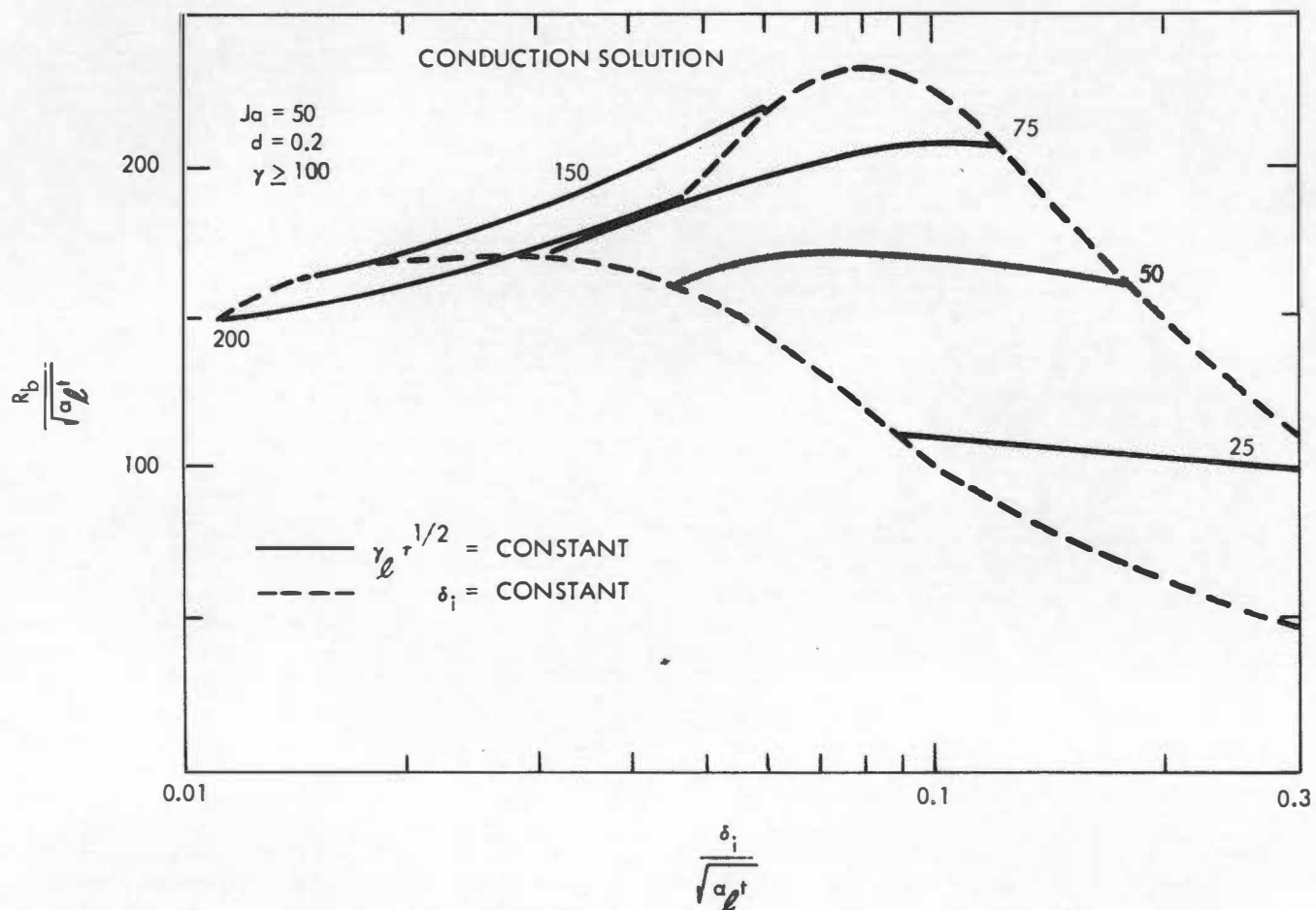


Figure 26. Dimensionless bubble growth solution (conduction approximation--Jakob number equals fifty).

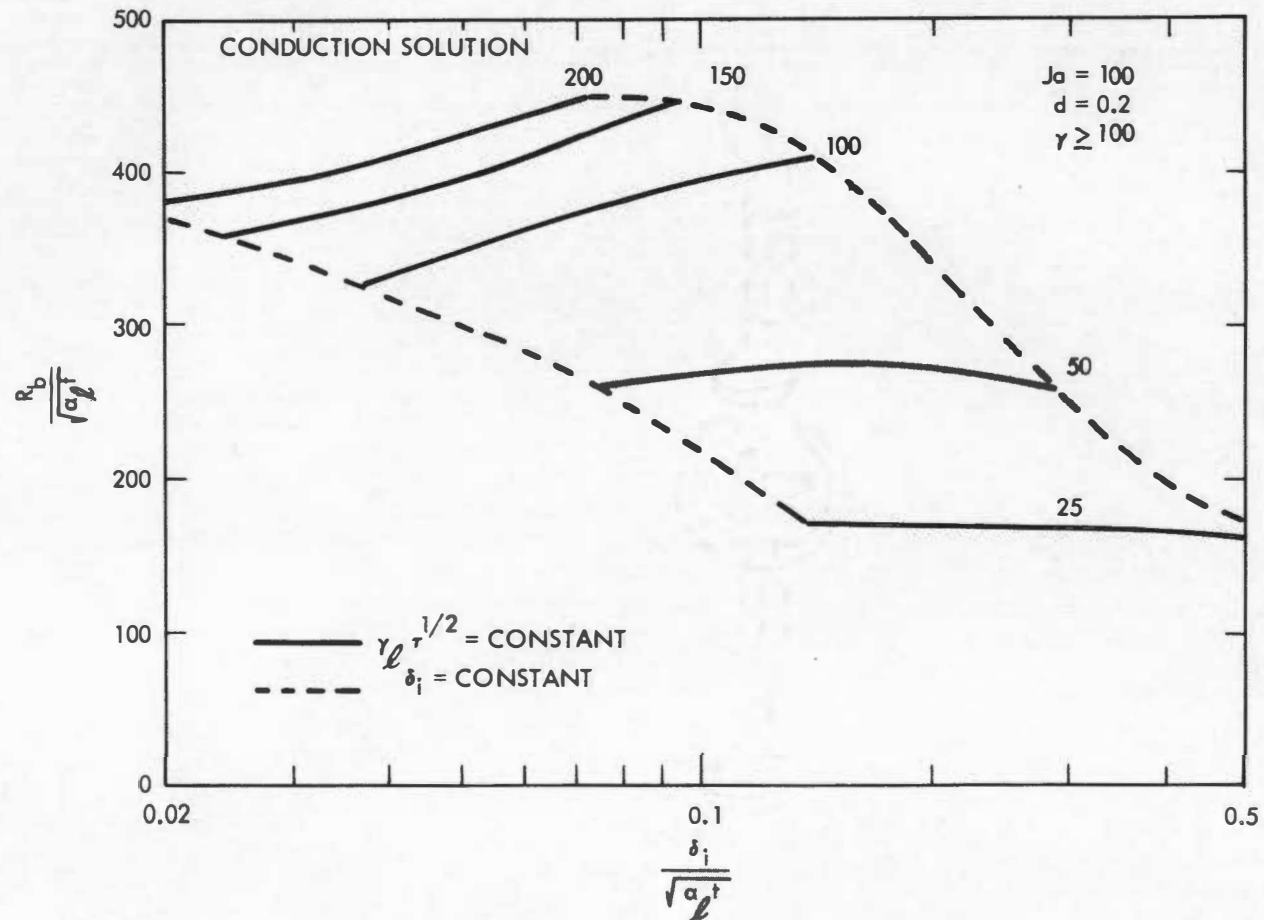


Figure 27. Dimensionless bubble growth solution (conduction approximation--Jakob number equals one hundred).

values of Ja and d . From the predetermined value of time the bubble size can be computed from $R_b/(\alpha_l t)^{1/2}$.

However, without calculating a numerical value of R_b , one can observe the effect of microlayer thickness from the two dashed lines of constant δ_i in each of these figures. These lines represent microlayer thicknesses differing by a factor of four with the larger value of δ_i equal a constant producing larger values of $R_b/(\alpha_l t)^{1/2}$.

The presented form of the dimensionless solution was selected since it affords a direct comparison of the present solution with previously reported bubble growth analyses. The discussion of bubble growth in Chapter II noted that isothermal bubble growth solutions report $R_b/(\alpha_l t)^{1/2}$ as a constant. However, from the present dimensionless bubble growth solution one observes that a maximum value of $R_b/(\alpha_l t)^{1/2}$ is attained for each constant value of δ_i . If the isothermal solution of $R_b/(\alpha_l t)^{1/2} = C_v$ were included in these dimensionless figures a single point would result along the ordinate for in these solutions microlayer evaporation was not considered ($\delta_i = 0$). If the microlayer evaporation bubble growth solution would have yielded $R_b/(\alpha_l t)^{1/2}$ as a constant; a horizontal, straight dashed line would have resulted. By using $\delta_i/(\alpha_l t)^{1/2}$ as the abscissa for the present solution, the lines of δ_i equal constant show that the present bubble growth solution is of the form

$$\frac{R_b}{(\alpha_l t)^{1/2}} = f(t) \neq C, \quad (107)$$

where $f(t)$ in Equation 107 is a function of time which achieves a maximum value as shown by the lines of δ_i equal a constant in Figures 22 through 24, pages 104 through 106, and Figures 25 through 27, pages 108 through 110. Since it will be instructive to examine the present bubble growth in light of reported isothermal bubble growth solutions as well as to fully examine the parameters Ja and δ_i , it is necessary to select a representative value of $R_b/(\alpha_l t)^{1/2}$. The most logical choice for this value is the maximum $R_b/(\alpha_l t)^{1/2}$ given by the dashed lines in Figures 22 through 27.

Figures 28 through 31 illustrate the maximum value of $R_b/(\alpha_l t)^{1/2}$ as a function of Ja for constant values of δ_i . These figures show an increase in $[R_b/(\alpha_l t)^{1/2}]_{\max}$ with increasing values of Ja number and with increasing values of δ_i . The significance of the ratio of heater to liquid thermal transport properties, d , is also shown in these figures. One observes, by comparing Figure 28 to Figure 29 and Figure 30 to Figure 31, that a change of the parameter, d , from 0.2 to 0.05 increases the slope of the curves for $[R_b/(\alpha_l t)^{1/2}]_{\max}$ versus Ja number such that larger maxima occur at low values of Ja . Physically a reduction in d increases the rate of vaporization from the liquid

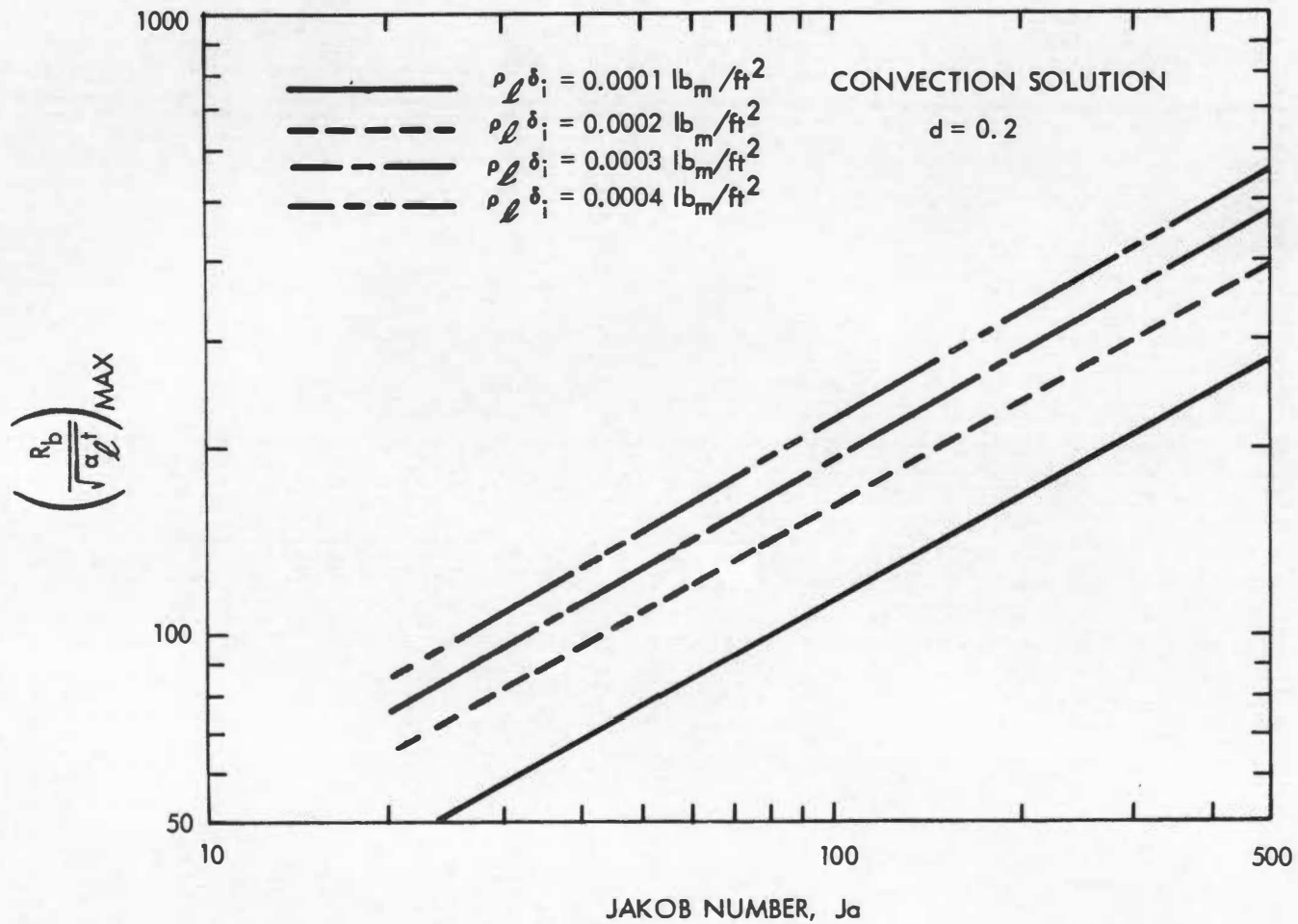


Figure 28. $\left[R_b / (\alpha_l t)^{1/2} \right]_{\text{max}}$ versus Jakob number (convection solution--d = 0.2).

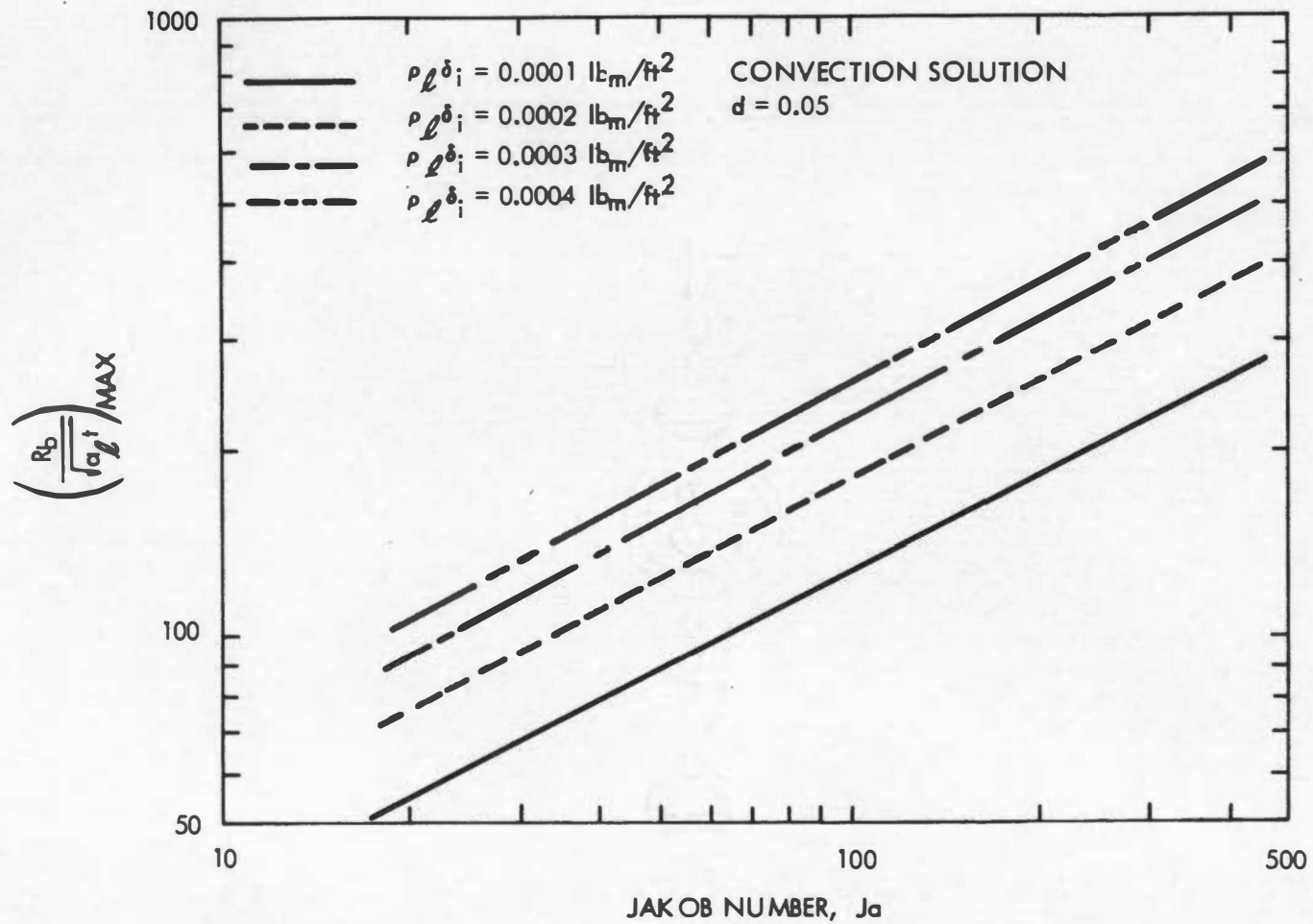


Figure 29. $[R_b / (\alpha_l t)^{1/2}]_{\text{max}}$ versus Jakob number (convection solution-- $d = 0.05$).

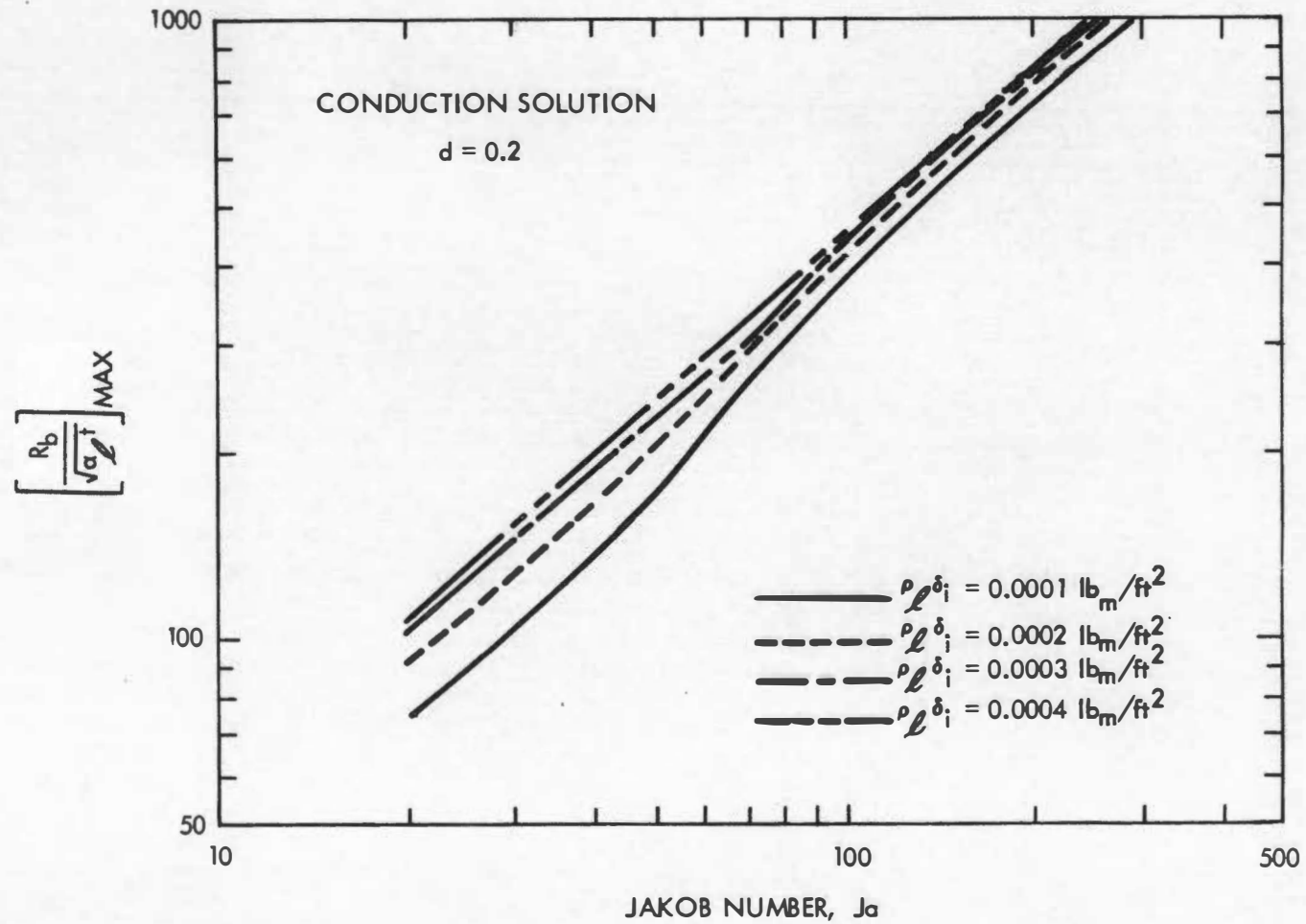


Figure 30. $\left[R_b / (\alpha_l t)^{1/2} \right]_{MAX}$ versus Jakob number (conduction solution--d = 0.2).

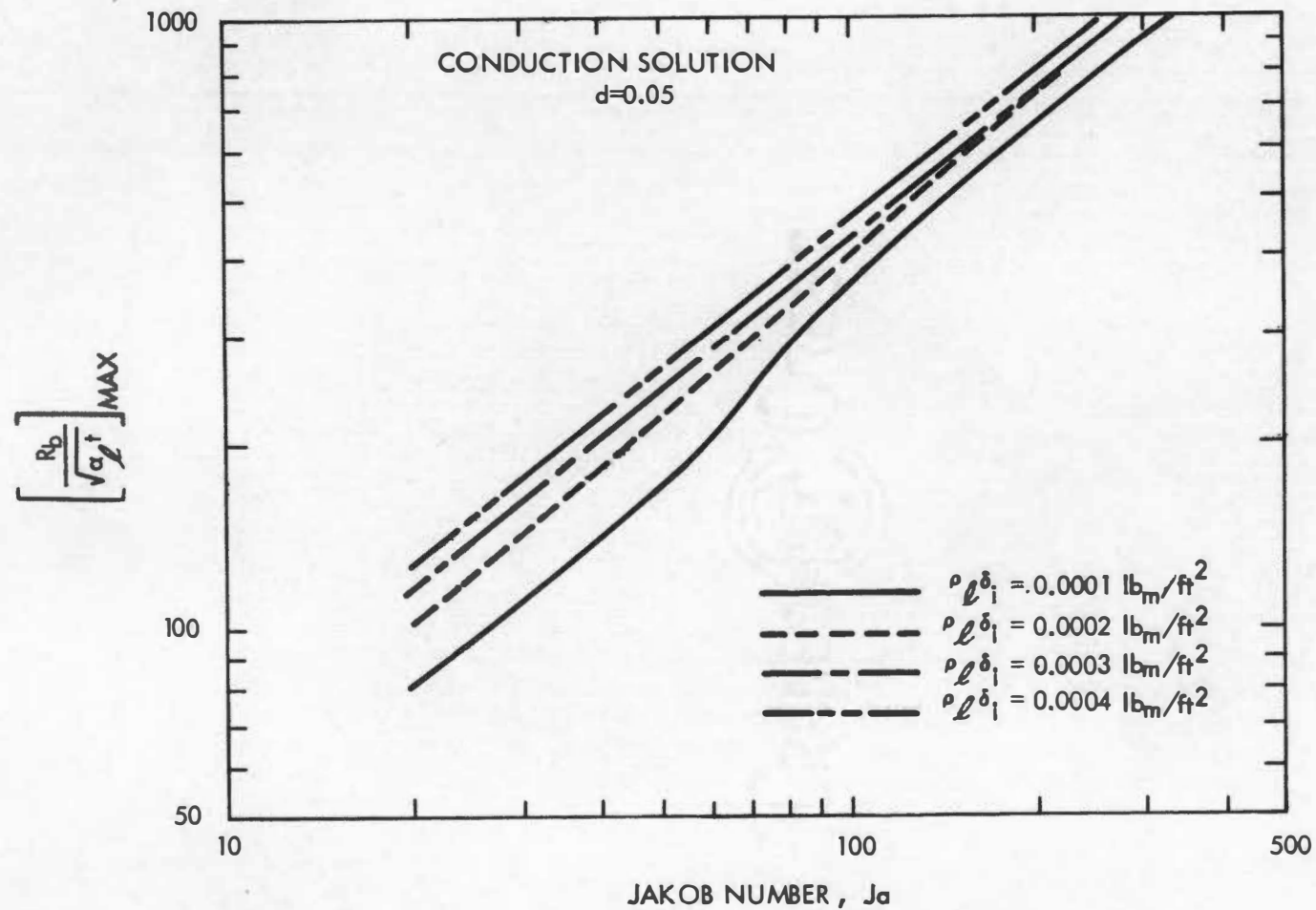


Figure 31. $\left[R_b / (\alpha_l t)^{1/2} \right]_{max}$ versus Jakob number (conduction solution--d = 0.05).

microlayer and thus produces a larger value of $[R_b/(\alpha_l t)^{1/2}]_{\max}$. The importance of the parameter, d , diminishes at high value of Ja number since bubble growth conditions for these cases are typically of subatmospheric pressure boiling where the rate of vaporization from the liquid microlayer becomes relatively slow. It should thus be anticipated that thermal diffusion in the liquid and in the solid will produce almost no effect on microlayer evaporation and consequently bubble growth.

In Figure 32 the variation of $[R_b/(\alpha_l t)^{1/2}]_{\max}$ with Jakob number is compared with the variation of analytically computed values, i.e., C_v in Equation 18, (24), (29), and (23), and with empirically determined values (33). The comparison shown in this figure is of a qualitative nature since no physical significance can be ascribed to $[R_b/(\alpha_l t)^{1/2}]_{\max}$ and is intended to illustrate the order of magnitude results of the present study by comparing them with the well established bubble growth analyses in the literature.

The slope of $[R_b/(\alpha_l t)^{1/2}]_{\max}$ versus Ja for the conduction approximation agrees with those predicted by References (23), (24), and (29). The added contribution to bubble size which is made from the evaporation of the liquid microlayer tends to produce maxima for $R_b/(\alpha_l t)^{1/2}$ which are larger than the value of C_v suggested in References (23), (24), and (29).

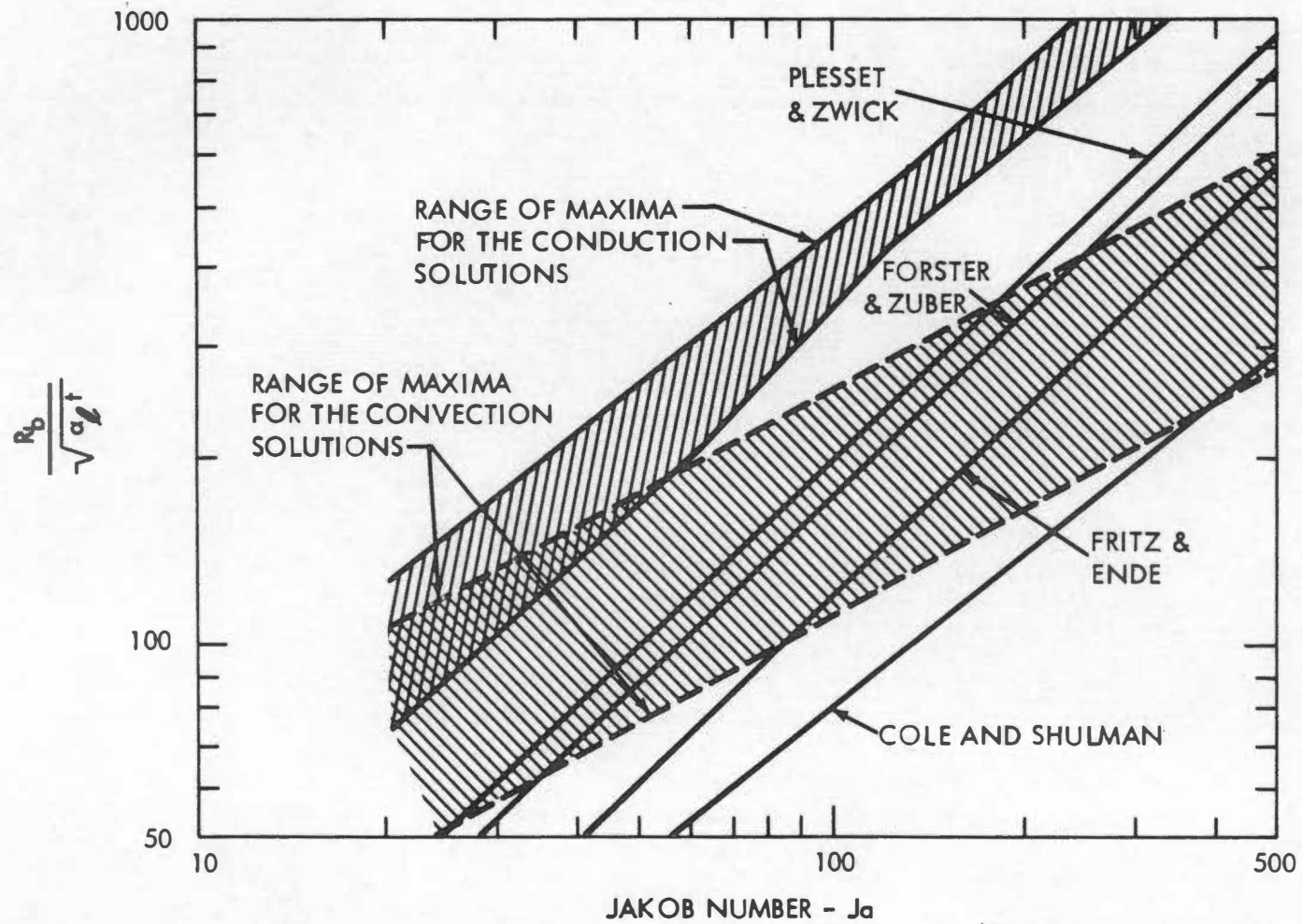


Figure 32. Qualitative comparison of $[R_b / (\alpha_l t)^{1/2}]_{\max}$ with existing bubble growth constants.

The bubble growth predictions of Cole and Shulman (33) are the result of experimental measurements of bubble size versus time for Ja numbers generally greater than one hundred and for several different hydrocarbon fluids. As seen in Figure 32, page 118, the convection approximation approaches the predictions of Reference (33) for Ja greater than one hundred.

Comparing the conduction and convection maxima of $R_b/(\alpha_l t)^{1/2}$ for the present bubble growth solutions and remembering that the convection maxima predominantly represent the microlayer evaporation contributions to bubble size, one will note in Figure 32 that the percentage of bubble size resulting from evaporation at the bubble interface is most significant at large values of Ja, and diminishes as Ja is decreased.

Further visualization of microlayer evaporation influence upon bubble growth is presented in Figures 33 and 34 with the dimensional plots of R_b versus $(\alpha_l t)^{1/2}$. The families of curves for each Ja number are the asymptotic bubble growth solutions for different values of δ_i equal to a constant. In both the conduction and convection approximations larger values of δ_i result in an overall larger bubble size. The combinations of Ja numbers and $B/[k_l(T_{wi} - T_{sat})^{1/2}]$ were arbitrarily selected to examine the influence of T_{wi} while property values remained constant.

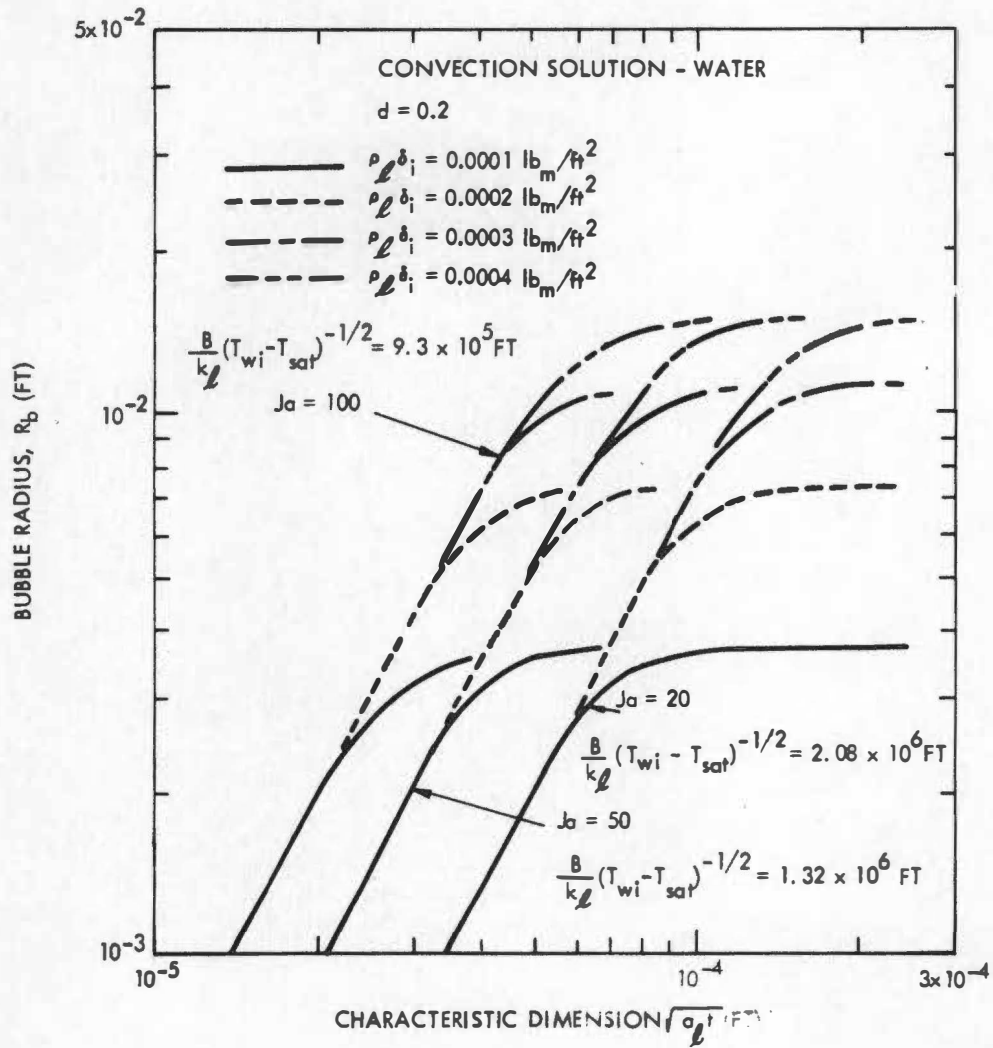


Figure 33. Dimensional bubble growth (convection approximation).

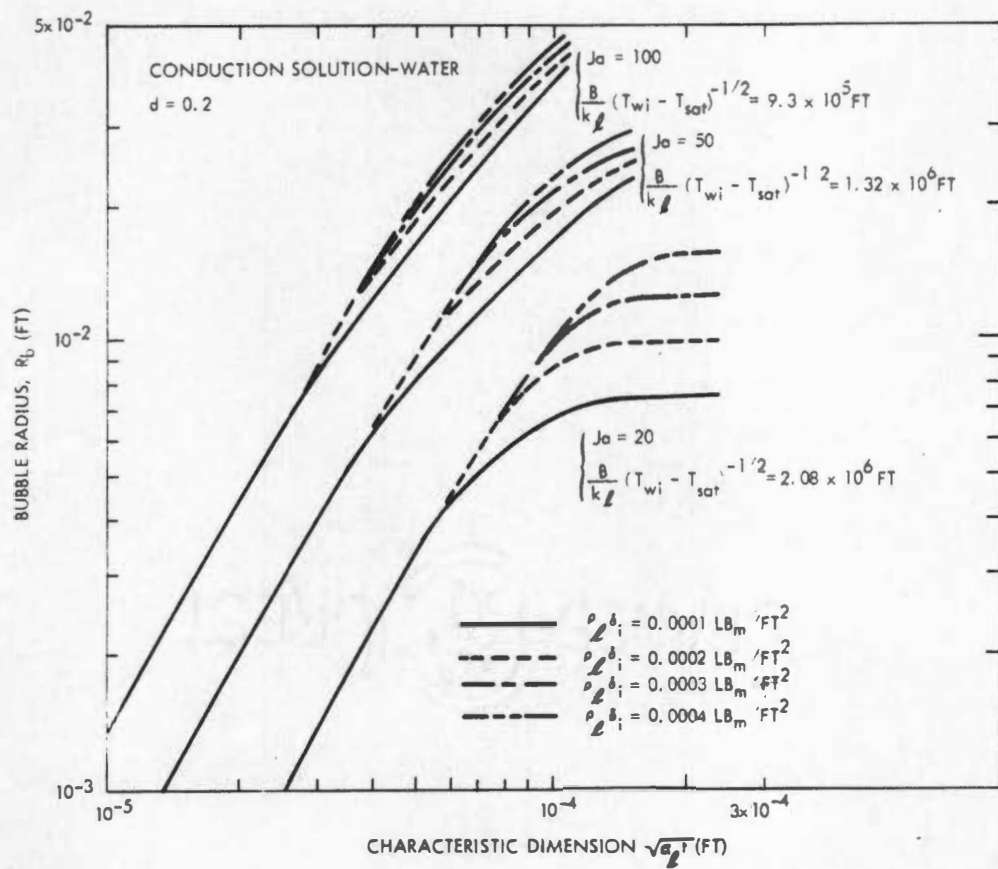


Figure 34. Dimensional bubble growth (conduction approximation).

The dimensional group, $B/[k_l(T_{wi} - T_{sat})^{1/2}]$ is derived from $r_l \tau^{1/2}$ by factoring out the characteristic dimension $(\alpha_l t)^{1/2}$. In Figure 33, page 120, one observes in the convection solution that the initial wall temperature exerts no influence on the maximum bubble size when all remaining properties are constant. This should be anticipated since the bubble grows solely from microlayer evaporation and the initial wall temperature, T_{wi} , will only alter the time required to reach the maximum size. For constant fluid properties the maximum bubble size is dictated by the amount of liquid contained in the microlayer as is clearly shown by the figure.

The dimensional form of the conduction approximation is presented in Figure 34, page 121, and illustrates that as T_{wi} is increased the heat conduction at the bubble interface increases resulting in larger bubble sizes. This is apparent from a comparison of Figures 33 and 34 for a given value of Ja. At a Jakob number of twenty the growth solutions for the two approximations are nearly identical. Thus conduction at the bubble interface for values of Ja less than twenty is small and the vapor in the bubble can be attributed to evaporation from the liquid microlayer. Contrastingly at Ja equal to one hundred the conduction solution yields a bubble size which is considerably larger than the convection approximation. Thus, for Jakob numbers of

one hundred or greater the results suggest that vaporization at the bubble interface contributes an appreciable portion of the total vapor volume.

The final parameter, γ , considered in the bubble growth solutions illustrates the "thin" heater effect as was noted in Chapter III for γ less than one hundred. Figures 35 and 36 illustrate the influence of small values of γ on the bubble growth solutions for two values of Ja number. For both the conduction and convection solutions the thin heater effect is to reduce the size of the vapor bubble for a given growth time. For the convection approximation the thin heater effect disappears once the maximum bubble size is achieved. In the conduction approximation, bubble growth upon a thin heater always lags the corresponding bubble growth that would occur upon a thick heater surface (i.e., $\gamma > 100$).

The significant effects upon bubble growth which result from inclusion of evaporation for a liquid microlayer are summarized as follows:

1. Microlayer evaporation represents most of the vapor contained in a bubble when the Jakob number is less than twenty. These values of Jakob number are such that the bubble interface must cool almost instantaneously to the

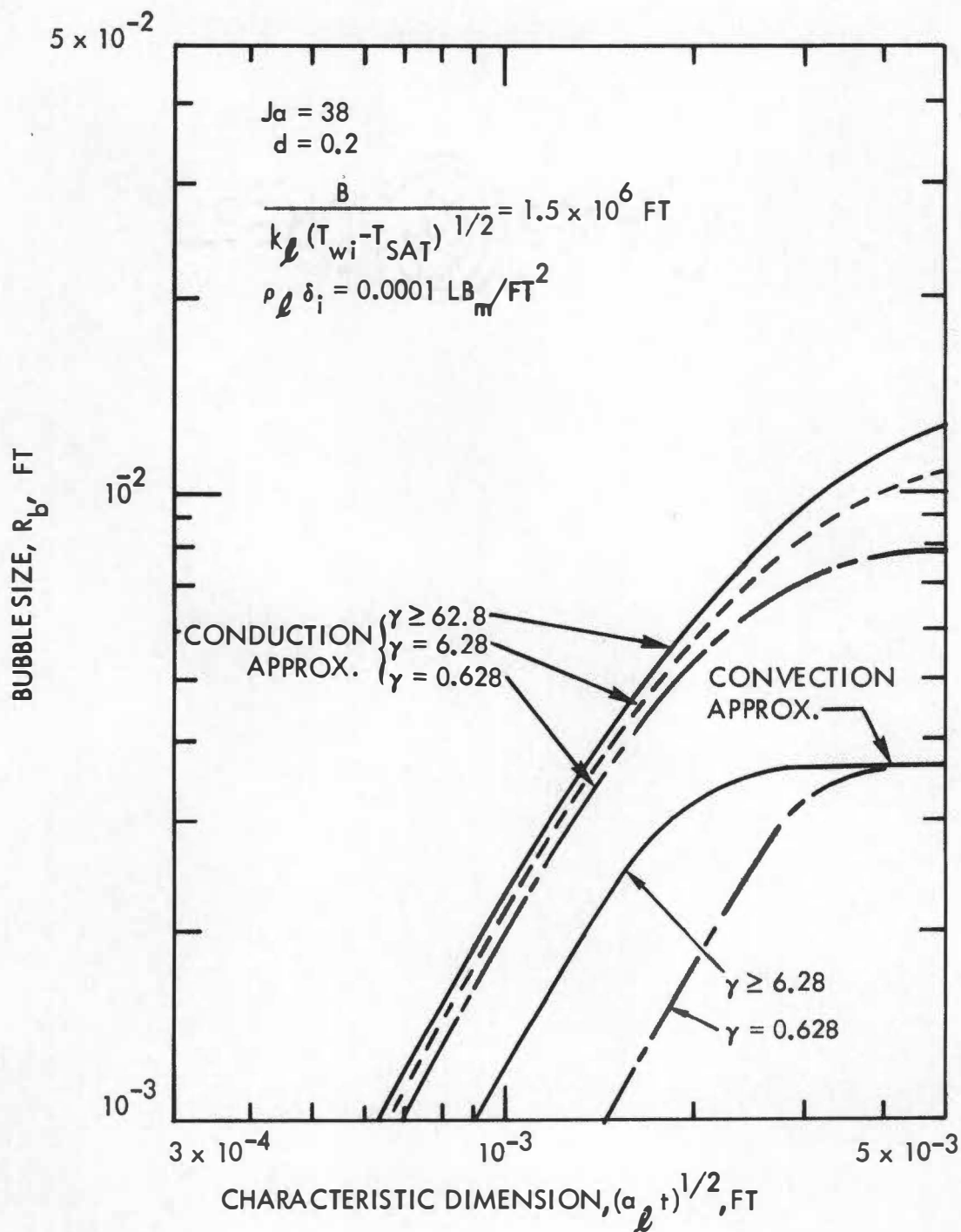


Figure 35. Bubble growth--effect of thin heaters--Jakob number equals thirty-eight.

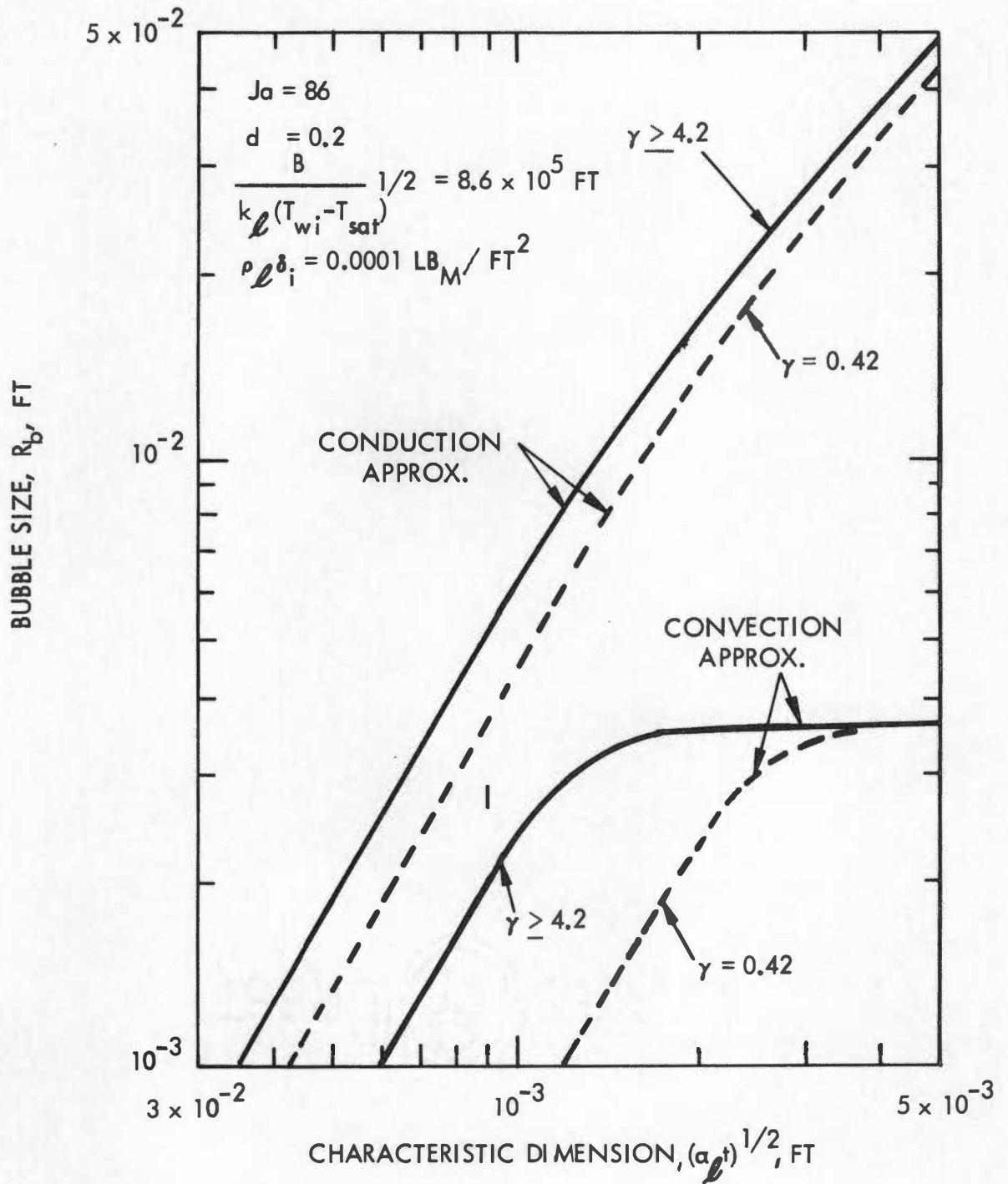


Figure 36. Bubble growth--effect of thin heaters--Jakob number equals eighty-six.

equilibrium saturation temperature and effectively insulate the bubble interface.

2. For values of Jakob number greater than one hundred, microlayer evaporation rates diminish relative to the heat conduction in the liquid toward the bubble interface. This effect results in vaporization occurring at both the bubble interface and the liquid microlayer during the major fraction of the bubble lifetime.
3. The thermal transport property ratio, d , can alter bubble growth such that a larger bubble size at a given time results from improving the thermal transport properties in the heater material relative to the liquid.
4. Thin heaters, as defined in the criterion of Chapter III, will produce a slower bubble growth than thick heaters.

IV. COMPARISON OF THE BUBBLE GROWTH MODELS WITH EXPERIMENTAL DATA

A direct comparison of the present bubble growth solutions with the experimentally measured data of References (32), (14), (33), and (44) is made in Figures 37 through 39. The shaded area in each figure represents the

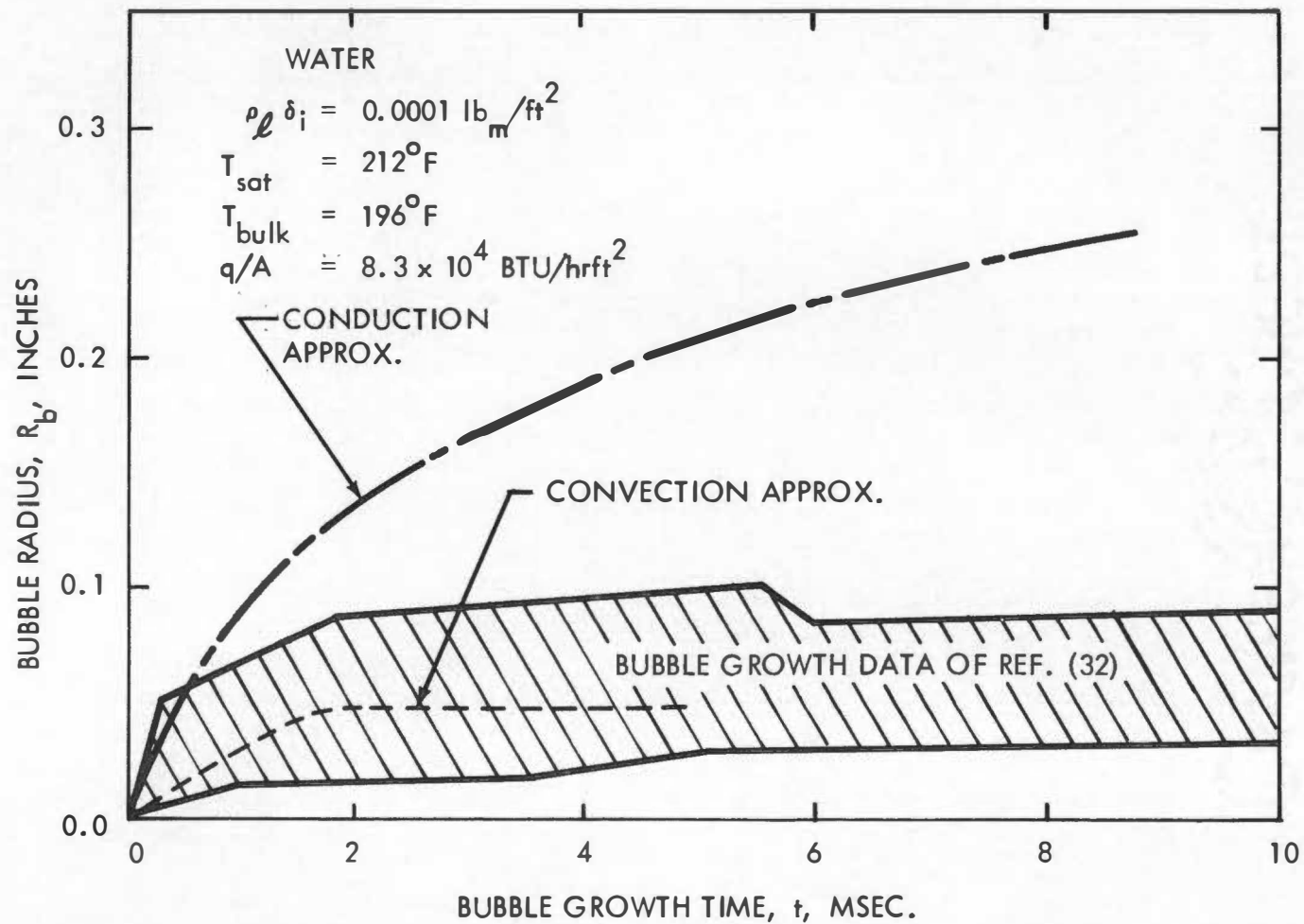


Figure 37. Comparison of bubble growth model with reported experimental results of Reference (32).

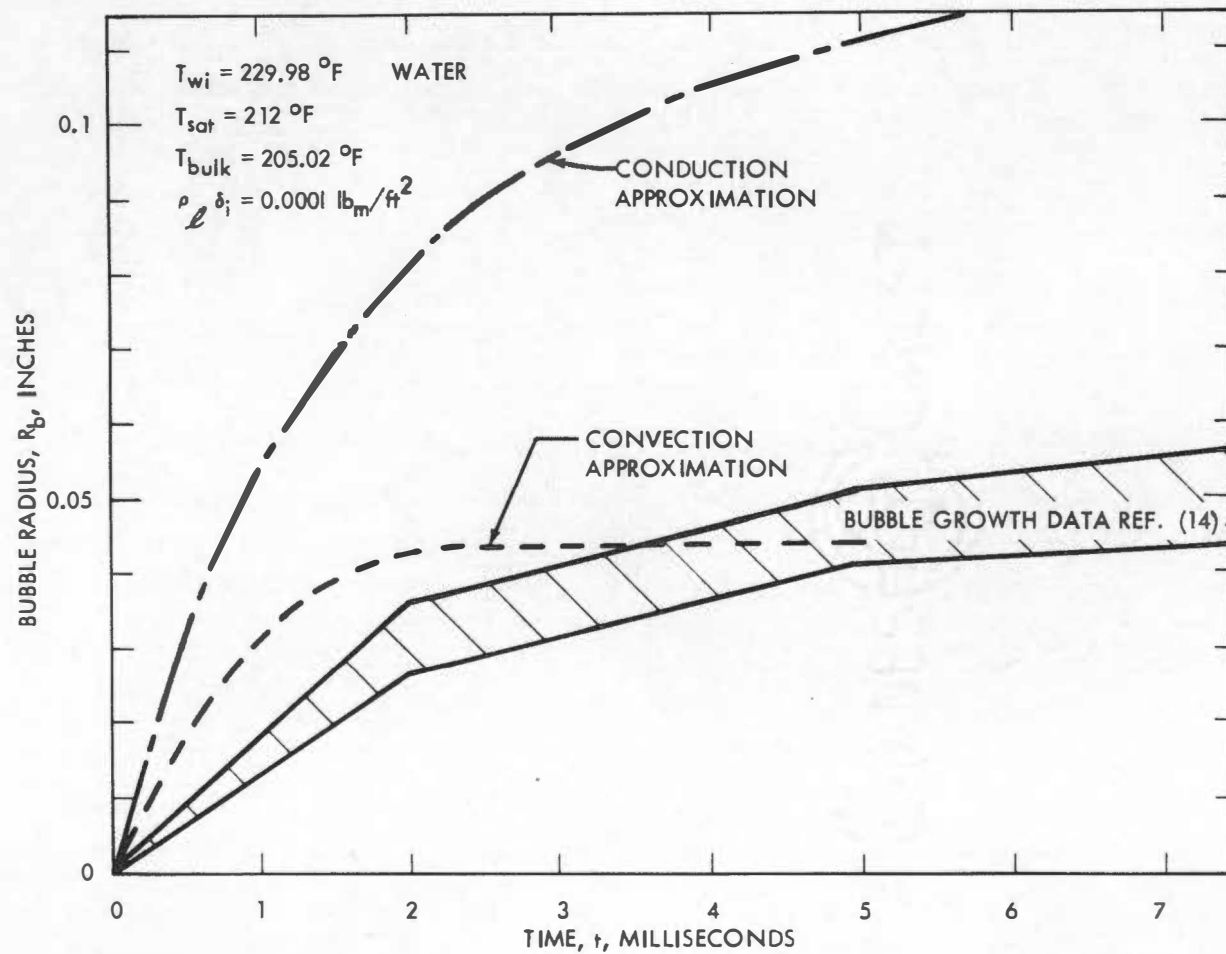


Figure 38. Comparison of bubble growth model with reported experimental results of Reference (14).

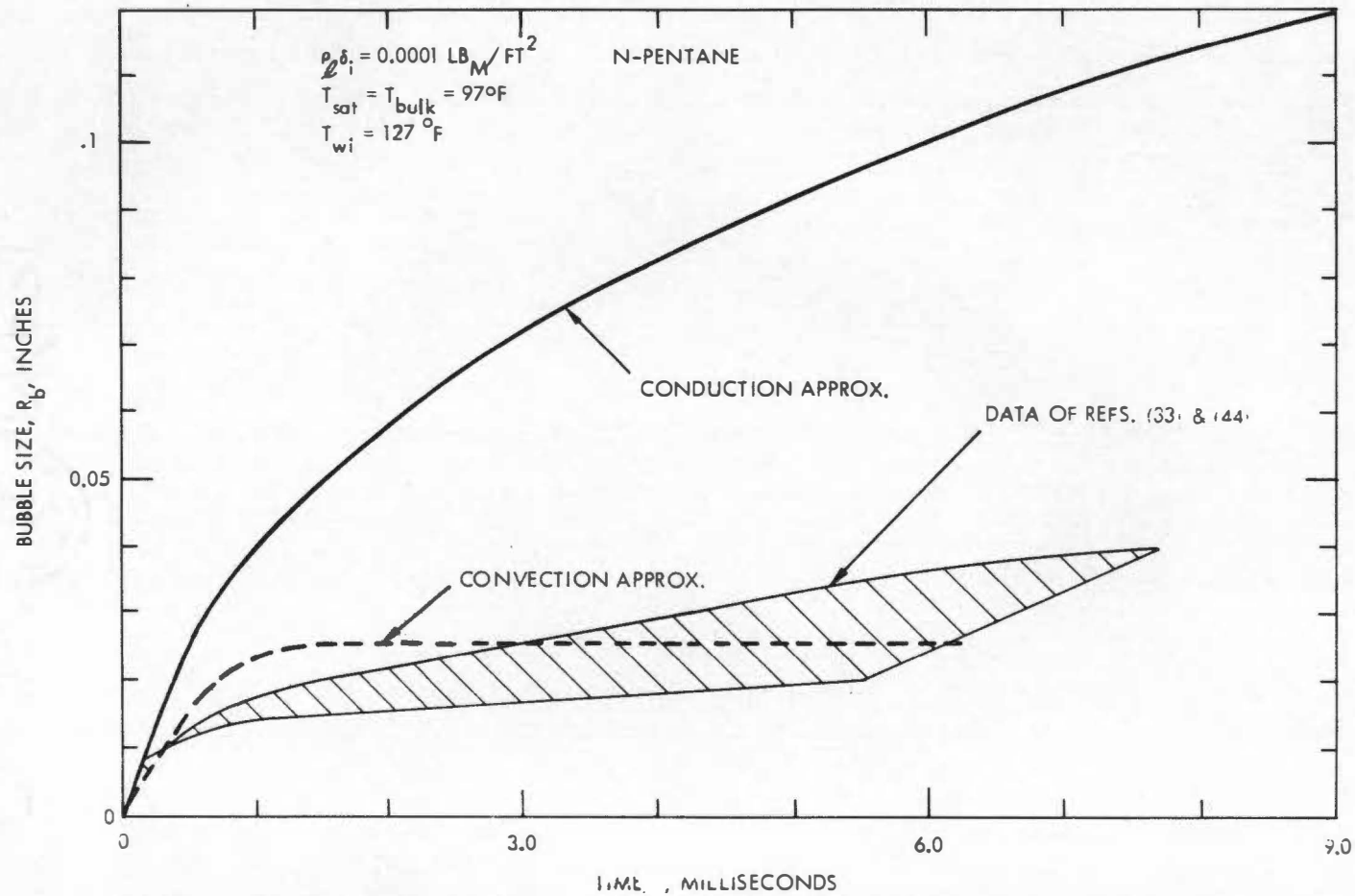


Figure 39. Comparison of bubble growth model with reported experimental results of References (33) and (44).

range of observed bubble sizes measured during steady state heat transfer with nucleate boiling. The curves represent analytical solutions of the present model. The initial wall temperature was taken to be identical with the reported average superheat temperature difference, and heater thicknesses were assumed to be such that γ remained greater than one hundred throughout this comparison.

Figures 37 through 39, pages 127 through 129, in general, suggest that the convection approximation for bubble interface temperature produces good agreement between theoretical analysis and experimental data. The applicability of the convection approximation suggests that the vapor contained in a bubble is essentially the result of evaporation from the liquid microlayer. Additional qualitative support for this view has already been given in Figure 32, page 118, for values of Jakob number greater than one hundred by the comparison of the convection solution with the empirical correlation of Cole and Shulman (33).

A quantitative assertion of this conclusion can not be made at the present time, since the present models state that the assumed values of microlayer thickness, δ_i , will appreciably alter the resulting bubble sizes. Further restriction to the present comparisons must be noted in the limitation imposed by bubble geometry. If bubble shape is appreciably different from the assumed hemi-spherical

geometry, microlayer evaporation interface area will be altered and may result in a significantly different bubble growth. Further comment upon these uncertainties will be made in Chapter VI.

In summary, the experimental data of References (14), (32), (33), and (44) suggest that the vapor contained in a bubble generated during nucleate boiling is mostly the result of evaporation from the liquid microlayer.

CHAPTER VI

DISCUSSION OF THE PROPOSED MODEL

A model of bubble growth which includes the effect of liquid microlayer evaporation has been developed. Results predicted by this model are subject to assumptions regarding the magnitude of certain parameters which influence the bubble growth. The purpose of this chapter is to discuss these parameters and assumptions. Additionally, some observed nucleate boiling phenomena are interpreted in terms of the microlayer evaporation model.

I. THE KINETIC THEORY BOUNDARY CONDITION FOR A VAPOR BUBBLE

The non-equilibrium kinetic theory boundary conditions for the evaporating interfaces represent a fundamental assumption of the theoretical development.

Previous analyses (20), (45), and (46) have applied gas kinetic theory to obtain evaporation rates from a liquid. In the particular case of dipole molecules (for example water), these analyses introduce a transmission or accommodation coefficient, ϵ . This multiplier is most often determined by comparing observed evaporation rates with those predicted by ideal gas kinetic theory. Thus, the

transmission coefficient can be assigned positive values less than or equal to unity, and according to Reference (45) the transmission coefficient is a correction for internal molecular non-equilibriums which result from phase change. However, the correct values of ϵ which should be assigned to a particular type of molecule remain unknown, especially for the non-equilibrium interface boundary conditions used in the present investigation. In particular, the transmission coefficient for water has been reported to be of the order of 0.04. This value of ϵ was not used in the present study of microlayer evaporation since a gross disagreement with experiment would occur. Moreover, the analysis of Kennard (36) which was noted in Chapter III suggests that the above value of the transmission coefficient may be in appreciable error if the liquid temperature at the interface is appreciably different from the temperature of the vapor.

Graham and Hendricks (46) recently reviewed their earlier experimental observations of liquid microlayer evaporation and analyzed the existing theory for obtaining transmission coefficients. They noted that the experimental measurements of evaporation rates used to ascertain transmission coefficients are highly idealized and represent the case of a steady state evaporation into a vacuum reservoir. These test conditions are not representative of the evaporation process during nucleate boiling.

An alternate view of evaporation has been presented by Wyllie (47) who considers an analytical model for evaporation composed of three layers, vapor, mobile liquid, and liquid substrate. The liquid substrate is defined as liquid molecules which are in ordered layers near the evaporating interface. Separating the liquid substrate from the vapor molecules is a disordered mobile layer of liquid. Wyllie envisioned the controlling rate process for evaporation as that which occurs between the liquid substrate and the mobile liquid layer, since this region would be most influenced by the intermolecular forces of the molecules. If this model of evaporation were applied to the liquid microlayer presented in Chapter III, it is reasonable to envision the liquid microlayer as composed mainly of the mobile liquid layer (due to the limited thickness of the microlayer). Hence, one would anticipate the transmission coefficient from the mobile liquid layer to the vapor to be nearly unity, and thus Equation 24 presents the correct evaporation rate from the liquid microlayer.

However, the transmission coefficient for the bubble interface could be significantly different from unity since this interface possesses the liquid substrate envisioned by Wyllie. To account for the possibility of the bubble interface's transmission coefficient departing from unity, Figure 40 illustrates the bubble growth data for water when the

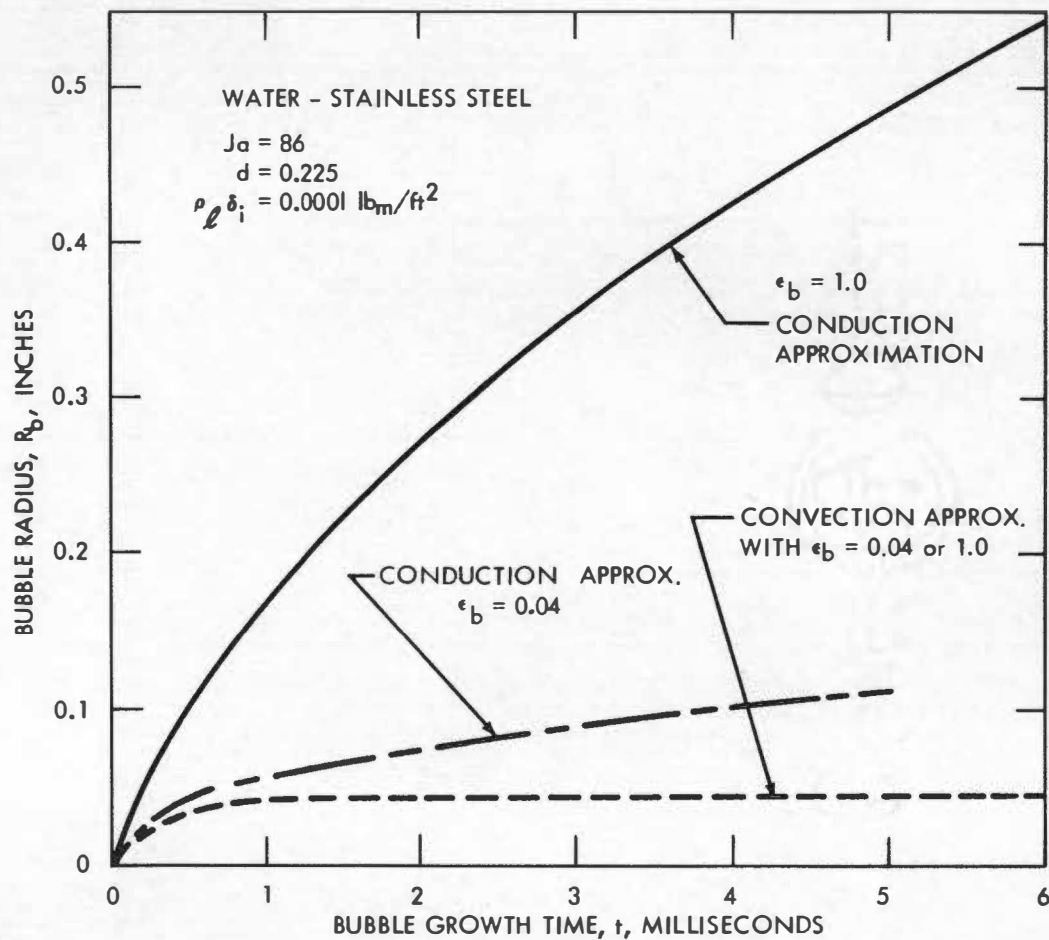


Figure 40. Bubble growth for dipole molecules with variable transmission coefficient on the bubble interface--Jakob number equals eighty-six.

bubble interface transmission coefficient assumes values of 1.0 and 0.04. This figure presents the two extremes of the reported values of the transmission coefficient at the bubble interface. (In Figure 40, page 135, the transmission coefficient for the liquid microlayer is unity.) As would be expected the transmission coefficient has no appreciable effect on the lower limit to bubble size as specified in Chapter V (the convection solution for bubble interface temperature); whereas, an assumed transmission coefficient of 0.04 substantially reduces bubble size in the upper limit solution. However, as was discussed in Chapter V, the correct bubble size can also be significantly affected by the assumed bubble geometry, by the assumed microlayer thickness, and by the initial temperature distribution surrounding the bubble. Therefore, the present analysis was carried out with a transmission coefficient of unity since sufficient justification for an alternate value is not available.

II. EVAPORATION PARAMETERS AFFECTING BUBBLE GROWTH

Aside from the parameters which directly alter the evaporation rate of the microlayer discussed in Chapter III, the indirect effects of bubble geometry (i.e., interface surface area) and mass of liquid in the microlayer must also

be considered during bubble growth. Thus, a brief discussion of these parameters follows.

Bubble Geometry

Johnson, de La Pena, and Mesler (48) have photographed bubble behavior for several different shapes as they appeared on the heated surface. Figure 41 which is taken from Reference (48) illustrates the observed growth curves for various bubble shapes. Reference (48) implies that a hemispherical vapor bubble will grow larger than a spherical bubble as a result of the different liquid microlayer evaporation areas. The fact that the spherical bubble growth is much less than the hemispherical bubble growth is consistent with the present model in view of the reduced microlayer evaporation area for the spherical bubble. Hence the values reported in Chapter V may be expected to depart appreciably from those observed, if the bubble geometry is altered from hemispherical.

Bubble Interface Temperature

Another effect which may change bubble growth is the true rate at which the bubble interface temperature approaches T_{sat} . In Chapter V this temperature behavior was approximated for two limiting cases of convection and of conduction. The combined effect of convection and conduction in the energy equation would result in a more exact

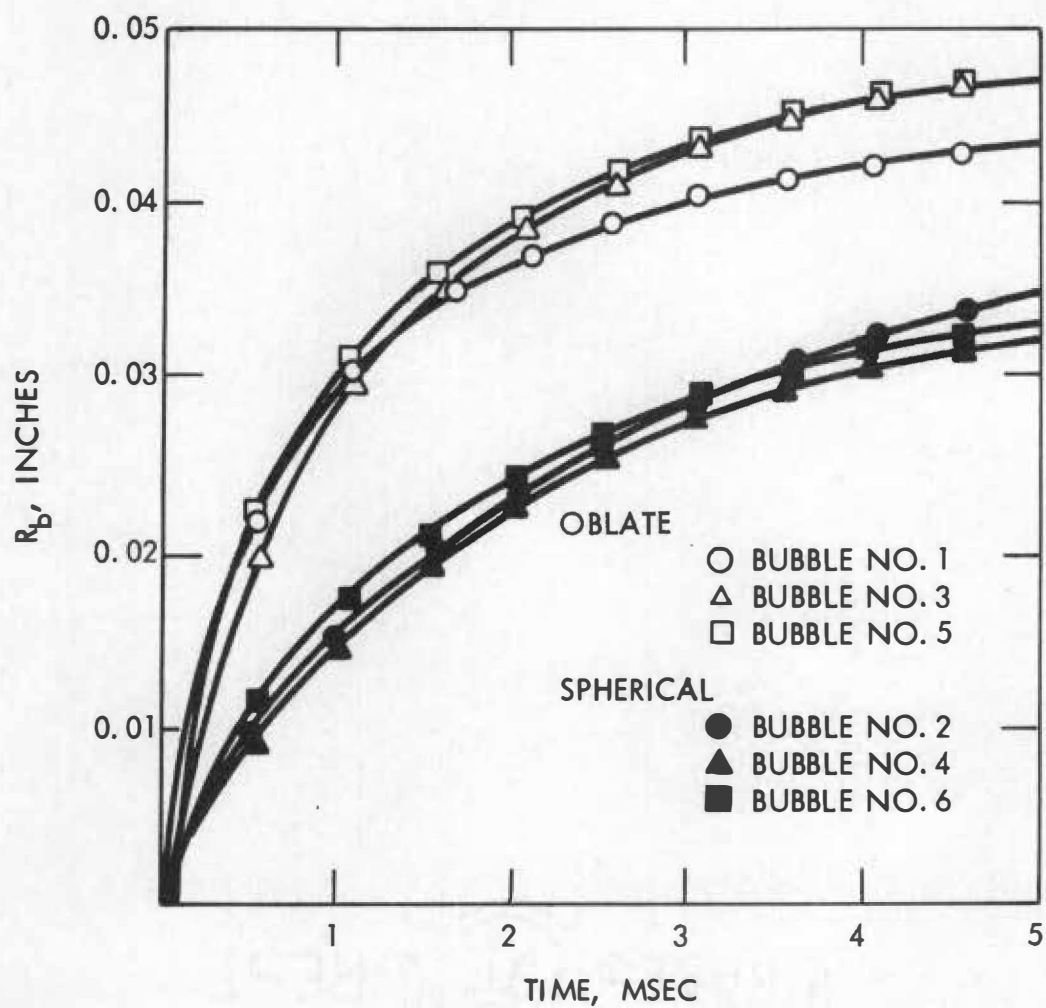


Figure 41. Effect of bubble geometry on bubble growth from Reference (48).

expression for the bubble interface temperature. As was discussed earlier this required a multidimensional numerical solution of considerable computational difficulty.

Liquid Microlayer Thickness

The reported experimental measurements of microlayer thickness (2), (3), (46), and (49) range from 1.65×10^{-5} inches through 10.2×10^{-5} inches. One might expect microlayer thickness to be a function of the adhesive qualities of the liquid-surface combinations. However, Reference (49) reports a wide range of microlayer thickness formed under identical liquid-surface combination. It is apparent that further experiments are required before precise determination of the variables which influence microlayer thickness will be known.

Thus, bubble growth behavior when microlayer evaporation is considered may be significantly influenced by microlayer thickness, bubble shape, and bubble interface temperature response. The present analysis is expected to give more precise predictions when further knowledge of these important bubble growth parameters is available.

Initial Temperature of the Evaporating Microlayer

Throughout the analysis, the initial temperature for the evaporating microlayer was assumed to be constant and equal to the average heater surface temperature. This is an

idealization. In actual fact the initial wall temperature which is dependent on the waiting time, will be higher than the average value, and this temperature will vary radially as the microlayer is formed on the heated surface. The models of Chapters III and IV provide an iterative procedure with which an improved approximation of the unknown initial microlayer temperature can be obtained.

The initial wall temperature, T_{wi} , can be determined from the waiting time model by evaluating Equation 67 at $X_l = 0$ and $\tau_l = \tau_{l,wait}$. However, the waiting time solution is dependent upon the final temperature distribution throughout the solid which results from the microlayer evaporation due to the previous bubble growth. To improve the assumed value of the initial wall temperature, T_{wi} , it is necessary to iterate between the wall temperature solutions of Chapters III and IV with a known initial microlayer thickness. This was not attempted in the present solution for bubble growth since the microlayer thickness was an assumed value. As a result of not completing this iteration for the initial microlayer temperature, T_{wi} , a temperature discontinuity appeared in the bubble growth solution at the junction of the liquid microlayer and the bubble interfaces. The newly formed liquid microlayer was assigned the assumed initial temperature, T_{wi} , whereas the initial bubble interface temperature at the heated surface was assigned the

calculated value from the waiting time solution. However, the bubble interface temperature cools so rapidly, particularly for the convection solution, that a variation in the interface temperature at this junction will have negligible effect on the mass transfer.

A further complication to successfully predicting the initial wall temperature occurs as bubble growth commences and forms the liquid microlayer on the heated surface. The advancing bubble interface at the heated surface tends to cool the initial wall temperature the further it advances from the nucleation site due to the cooling which occurs on the bubble interface. From the experimental evidence (50) it is anticipated that this effect is small. Thus, although T_{wi} depends on the waiting period, microlayer thickness, and radial position measured from the nucleation site, the assumption of constant T_{wi} is believed to represent a very good first approximation for the initial microlayer temperature distribution.

III. QUALITATIVE VIEW OF BUBBLE GROWTH WITH MICROLAYER EVAPORATION

It is instructive to observe the qualitative influence of microlayer evaporation on bubble growth which is apparent when the bubble growth rate, $\dot{R}_b$, is examined as a function of time. This is illustrated in Figure 42 for the

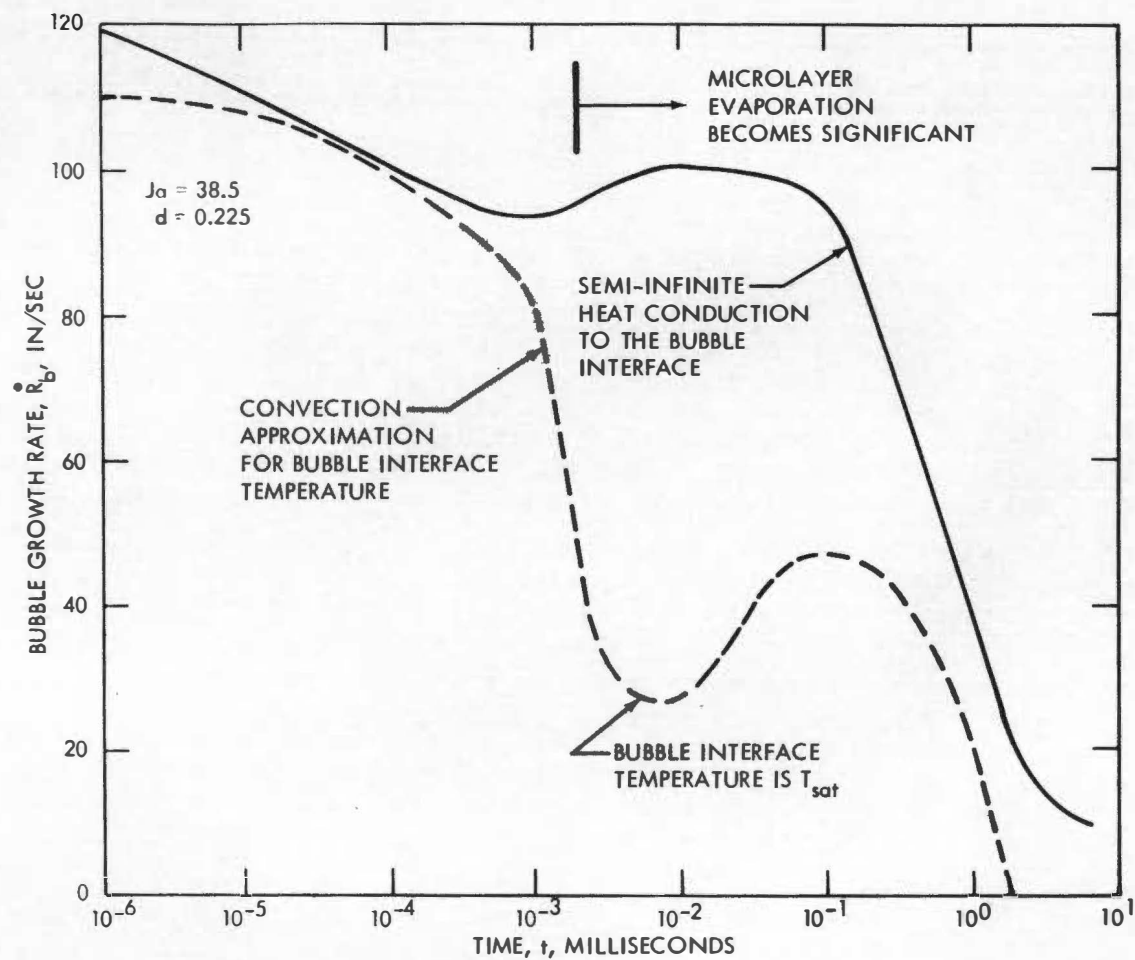


Figure 42. Typical bubble growth rates for the approximate solutions with a liquid microlayer.

two approximate solutions for bubble interface temperature. The convective approximation (dashed curve) illustrates that the interface velocity shows a rapid decline at approximately 10^{-3} milliseconds corresponding to the time at which the bubble interface temperature effectively reaches T_{sat} . However, at the end of this time the area of the microlayer has grown sufficiently so that the resulting evaporation increases the growth rate until a maximum occurs at $t = 10^{-1}$ milliseconds. The subsequent decline in bubble growth rate results because the microlayer surface growth is proportional to R_b^2 whereas the bubble volume growth is proportional to R_b^3 . The maximum bubble size occurs for $\dot{R}_b = 0$.

As an upper limit for bubble growth rate, the conduction approximation to bubble interface temperature, as illustrated in Figure 42, page 142, declines at approximately one-tenth of a millisecond after bubble growth has commenced. The contribution from microlayer evaporation after 10^{-3} milliseconds, again causes a small increase in bubble growth rate. In addition, the sustained evaporation at the bubble interface permits a bubble to grow indefinitely, that is, it does not produce a maximum bubble size ($\dot{R}_b = 0$).

IV. NUCLEATE BOILING PHENOMENA ATTRIBUTABLE TO MICROLAYER EVAPORATION

During the study of microlayer evaporation, the saturation properties of four fluids, water, nitrogen, normal pentane, and ammonia were investigated in conjunction with the thermal transport properties of two heater materials, type 302 stainless steel and copper. These properties were selected since they can be combined to yield extremely different liquid and heater surface combinations. The microlayer temperature-time responses are presented in Figure 43 for the above named fluid-heater material combinations and for internal heat generation values that are typical of nucleate boiling heat transfer ($P_{\text{sat}} \approx 1 \text{ atm.}$). The superheat temperature difference, $T_{\text{wi}} - T_{\text{sat}}$, for microlayer evaporation was arbitrarily selected to be 25°F . From this figure, one observes the following:

1. Liquid microlayer evaporation cools the stainless steel surface (low thermal property values) more rapidly than the copper surface (high thermal property values), and
2. For a given heater material, the rate of surface cooling for the different fluids increases in the order, n-pentane, water, ammonia, nitrogen.

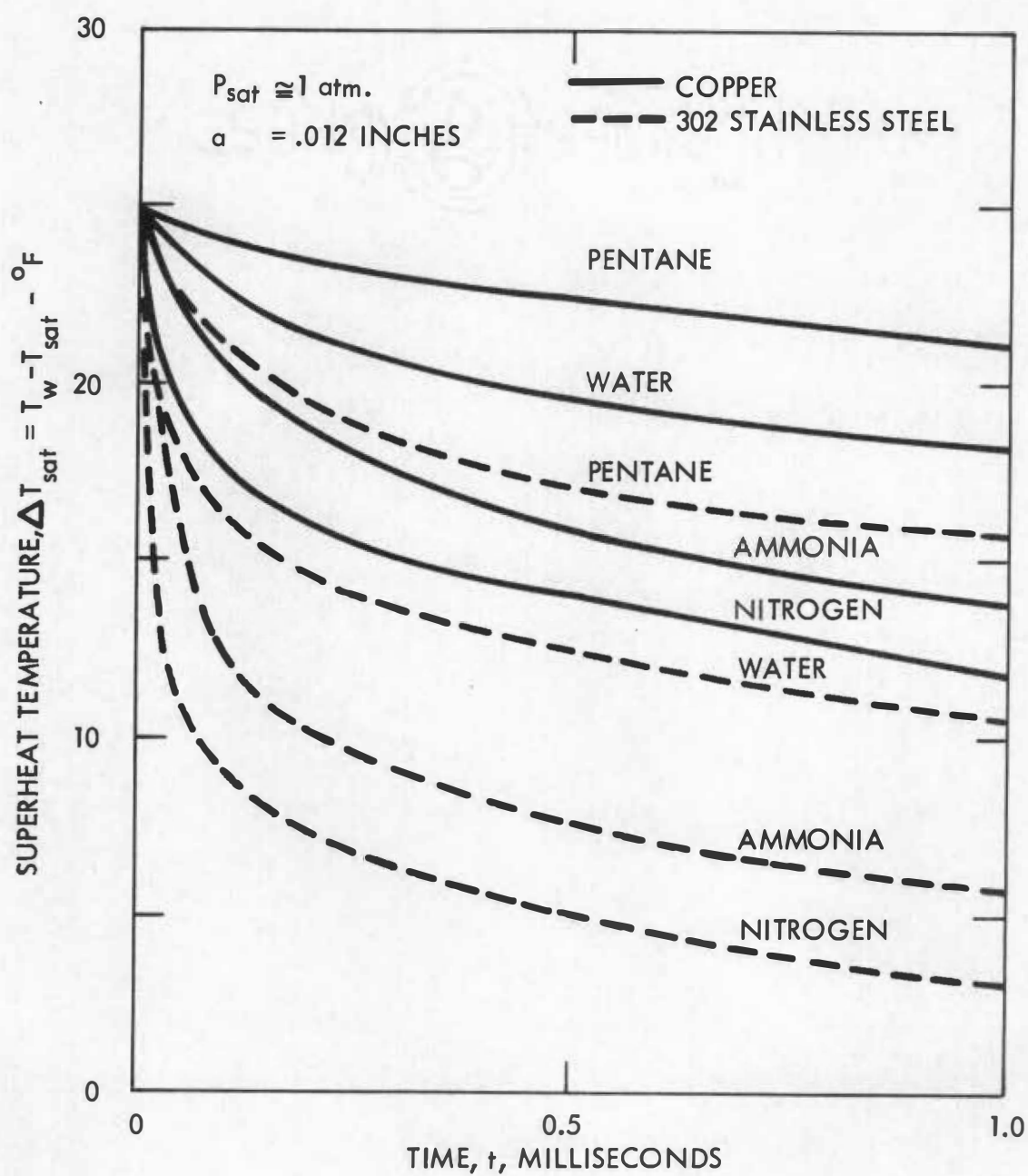


Figure 43. Microlayer temperature-time response for different liquid-heater combinations.

These observations can be qualitatively related to macroscopically observed nucleate boiling behavior. For example, a qualitative description of maximum heat flux phenomena is as follows.

When the magnitude of the nucleate boiling heat flux is increased, the rate of vapor bubble generation becomes so great that bubbles no longer can occur from discrete nucleation sites on the heated surface. The generated vapor "blankets" portions of the heated surface and acts as insulation to the heat transfer process. This produces a maximum nucleate boiling heat flux which is commonly called burnout.

In view of the temperature recovery which must occur at the completion of microlayer evaporation, temperature fluctuations in stainless steel surfaces are greater than in copper surfaces. Carne (51) has observed that the maximum heat flux increases with magnification of the thermal transport properties in the heated surface. Thus fluctuations around the average surface temperature being larger in the stainless steel permit excursions into the transitional boiling region at a lower heat flux.

Consistent also with this discussion is the observed lowering of the maximum heat flux with decreased heater thickness (51) and (52). As illustrated in Figure 13, page 47, thin heaters result in increased surface temperature

fluctuations in a similar manner as reduced thermal transport properties.

Secondly the different microlayer evaporating rates for the different fluids shown in Figure 43, page 145, are ordered coincident with the ordering of the superheat temperatures required for typical nucleate boiling. Figure 44 presents the observed nucleate boiling curves of average surface heat flux versus average wall superheat temperature for the four fluids considered as calculated by the Forster-Grief correlation (Equation 13). One observes that for given superheat temperature differences the average nucleate boiling heat flux is larger for nitrogen, ammonia, water, and n-pentane, in that order. This follows the view that larger surface temperature drops must be associated with higher rates of microlayer evaporation or higher local heat flux; and hence, a higher average heat flux.

Furthermore nucleate boiling superheat temperature differences decrease as saturation pressure is increased (1). Correspondingly microlayer evaporation rates increase with saturation pressure since more molecules will be leaving the evaporating microlayer as saturation pressure is raised. Thus, at the same heat flux, surface cooling would be greater due to increased evaporation.

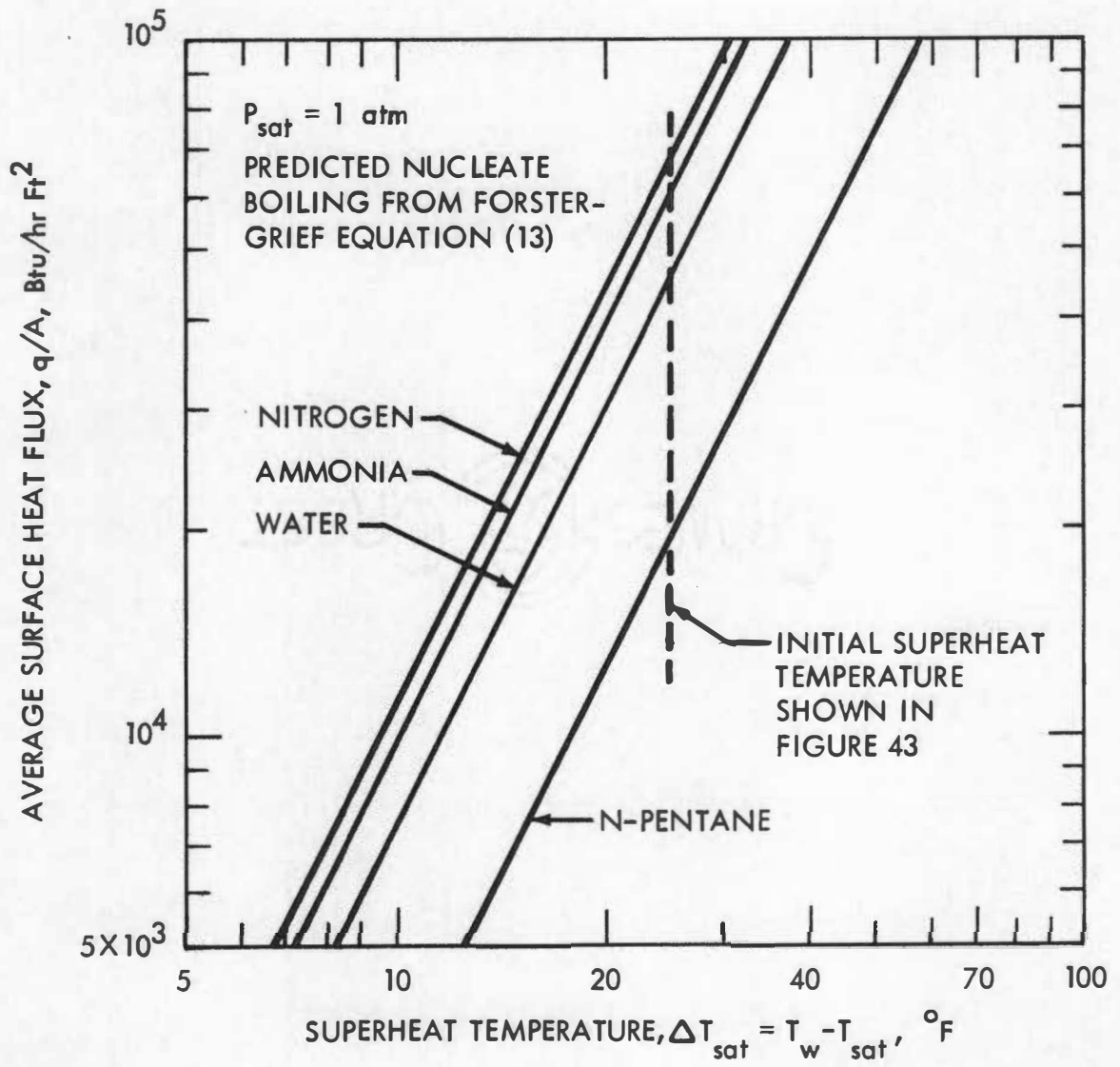


Figure 44. Typical nucleate boiling curves for different liquids.

CHAPTER VII

CONCLUSIONS

Based on the analytical models for the growth of a vapor bubble on heated surface with a liquid microlayer evaporating into the bubble and reported herein, it is concluded that:

1. Evaporation of a liquid microlayer is mathematically characterized by a non-equilibrium evaporation which is derivable from kinetic theory. A transient, dissipating temperature discontinuity appearing at the liquid-vapor interface is the driving potential for the evaporation process.
2. The rate at which the liquid-vapor interface cools to the equilibrium saturation temperature is dictated by the thermal transport properties of the heater material; and the net rate at which thermal energy is removed from the heated surface due to the vaporization of the liquid microlayer.
3. The fact that local temperature excursions occur in the heated material when a vapor bubble forms and grows at a heated surface will

appreciably increase the magnitude of time required for liquid heating so that a succeeding vapor bubble may commence growth from a particular nucleation site on the heated surface.

4. The time period which is required for heating the liquid between successive vapor bubble formations from an active nucleating cavity is a function of the extent of temperature disturbance from steady state which exists in the heater temperature profile as the waiting period commences, the bulk liquid temperature, the thermal transport properties of the liquid and heater materials, and the rate of internal heat generation in the heated solid.
5. Modification of the continuity equation which describes the mass contained in a growing vapor bubble so as to include evaporation of a liquid microlayer, results in larger bubble sizes which are directly related to the thickness of the liquid microlayer formed on the heated surface.
6. The growth of a hemispherical vapor bubble upon a heated surface as analytically established by an upper and lower limit for the bubble growth

demonstrates the following qualitative behavior for a fixed value of bubble lifetime.

- a. Larger bubble sizes occur with thicker liquid microlayers.
- b. Larger vapor bubbles occur with larger values of Jakob number.
- c. Larger vapor bubbles occur with a thick heater than a relatively thin heater.
- d. Most of the vapor contained in the bubble is the result of liquid microlayer evaporation; particularly for values of Jakob number less than twenty where the kinetic theory boundary condition for the bubble interface quickly cools this interface to an equilibrium condition, and thereby stops the net flow of vapor into the bubble. For this case, continued bubble growth can only occur as a result of evaporation from the liquid microlayer.

7. A qualitative comparison of the present microlayer evaporation analysis with experimentally observed nucleate boiling heat transfer suggests that the peak nucleate boiling heat flux, and the average superheat temperature

difference which is required to transmit a fixed average heat flux is directly compatible with the analytical solutions for evaporation rates from the liquid microlayer.

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APPENDIXES

APPENDIX A

SOLUTION OF THE EQUATIONS FOR MICROLAYER EVAPORATION

I. MATHEMATICAL SOLUTION FOR HEATER SURFACE TEMPERATURE DURING MICROLAYER EVAPORATION

The solution of Equation 33 with its associated boundary conditions is obtained by the technique of Laplace transformations, defined by

$$\mathcal{L}\{\Theta\} = \bar{\Theta} = \int_0^{\infty} e^{-pt} \Theta \, dt . \quad (108)$$

The dimensionless equations are

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial X^2} + \Phi , \quad (109)$$

$$\left. \frac{\partial \Theta}{\partial X} \right|_{X=0} = \Psi , \quad (110)$$

$$\left. \frac{\partial \Theta}{\partial X} \right|_{X=1} = 0 , \quad (111)$$

and

$$\left. \Theta \right|_{\tau=0} = 1 + \Phi \left(X - \frac{X^2}{2} \right) . \quad (112)$$

Applying the Laplace transformation to Equation 109 and using Equation 112 gives

$$\frac{d^2 \bar{\Theta}}{dX^2} - p\bar{\Theta} = - \left[1 + \Phi \left(X - \frac{X^2}{2} \right) + \frac{\Phi}{p} \right]. \quad (113)$$

The solution of this equation can be written

$$\bar{\Theta} = C_1 e^{-p^{1/2} X} + C_2 e^{p^{1/2} X} + \frac{1}{p} + \frac{\Phi}{p} X - \frac{\Phi}{2p} X^2, \quad (114)$$

with the transformed boundary conditions,

$$\left. \frac{\partial \bar{\Theta}}{\partial X} \right|_{X=0} = J, \quad (115)$$

and

$$\left. \frac{\partial \bar{\Theta}}{\partial X} \right|_{X=1} = 0, \quad (116)$$

where J is

$$J = \int_0^\infty \psi e^{-pt} dt. \quad (117)$$

The constants C_1 and C_2 in Equation 114 are evaluated from the transformed boundary conditions as

$$C_1 = \frac{(\Phi/p - J)}{p^{1/2}(1 - e^{-2p^{1/2}})}, \quad (113)$$

and

$$C_2 = \frac{(\Phi/p - J)e^{-2p^{1/2}}}{p^{1/2}(1 - e^{-2p^{1/2}})}. \quad (119)$$

Rather than obtain the solution for the temperature distribution in the entire solid, some simplification is achieved if the solution is completed for the wall temperature only ($X = 0$). Equation 114 then reduces to

$$\bar{\Theta}_w = \frac{(\Phi/p - J)}{p^{1/2}} \left[\frac{1 + e^{-2p^{1/2}}}{1 - e^{-2p^{1/2}}} \right] + \frac{1}{p}. \quad (120)$$

By expanding the exponential terms in a series of $e^{-jp^{1/2}}$ and introducing the convolution theorem to complete the inversion of J , the surface temperature solution can be written as (38)

$$\begin{aligned} \Theta_w = 1 - \frac{2}{\pi^{1/2}} \int_0^\tau \Psi'' \left[\frac{1}{2} + \sum_{j=1}^{\infty} \exp\left(-\frac{j^2}{\lambda}\right) \right] \lambda^{-1/2} d\lambda \\ + 4\Phi \left\{ \frac{\tau^{1/2}}{\pi^{1/2}} \left[\frac{1}{2} + \sum_{j=1}^{\infty} \exp\left(-\frac{j^2}{\tau}\right) \right] - \sum_{j=1}^{\infty} j \operatorname{erfc}\left(\frac{j}{\tau^{1/2}}\right) \right\}. \end{aligned} \quad (121)$$

II. NUMERICAL PROCEDURE USED IN THE COMPUTER PROGRAM

The numerical procedure used to calculate successive values of the heater surface temperature from Equation 121 is outlined as follows:

A general form of this equation for the n th time step is

$$Y^2(t_n) = F(t_n) + \int_0^{t_n} Y(t_n - \lambda) G(\lambda) d\lambda, \quad (122)$$

where $Y = \left(\frac{T_w}{T_{sat}} \right)^{1/2}$.

Because of the singular point in $G(\lambda)$ which occurs at $\lambda = 0$, a separate treatment of the first time step was introduced

$$\begin{aligned} Y^2(t_n) = & F(t_n) + \int_0^{t_1} Y(t_n - \lambda) G(\lambda) d\lambda \\ & + \int_{t_1}^{t_n} Y(t_n - \lambda) G(\lambda) d\lambda \end{aligned} \quad (123)$$

by letting $z^2 = \lambda$ in the first integral of Equation 123. Inserting this transformation gives

$$\begin{aligned}
Y^2(t_n) = & F(t_n) + \int_0^{t_1^{1/2}} Y(t_n - z^2) T G(z^2) dz \\
& + \int_{t_1}^{t_n} Y(t_n - \lambda) G(\lambda) d\lambda, \quad (124)
\end{aligned}$$

where $TG(z^2) = 2zG(z^2)$, and the singularity has been removed. Expressing Equation 124 in difference form using the trapezoidal approximation of the integrals, then gives

$$\begin{aligned}
Y^2(t_n) = & F(t_n) + \left[Y(t_n - t_1)TG(t_1) + Y(t_n)TG(0) \right] \frac{t_1^{1/2}}{2} \\
& + \left[Y(t_n - t_1)G(t_1) + Y(t_n - t_2)G(t_2) \right] \frac{(t_2 - t_1)}{2} \\
& + \left[Y(t_n - t_2)G(t_2) + Y(t_n - t_3)G(t_3) \right] \frac{(t_3 - t_2)}{2} \\
& + \dots + \left[Y(t_n - t_{n-1})G(t_{n-1}) \right. \\
& \left. + Y(t_n - t_n)G(t_n) \right] \frac{(t_n - t_{n-1})}{2}. \quad (125)
\end{aligned}$$

Since all values of Y which are evaluated at times less than t_n are assumed known, the above equation represents a

quadratic equation in the unknown $Y(t_n)$. By introducing a constant time step such that $t_n - t_{n-1} = t_{n-1} - t_{n-2} = \dots = t_1 - 0 = \Delta t$, the value of $Y(t_n)$ can be immediately expressed in terms of the calculated values of Y

$$\begin{aligned}
 Y(t_n) = & TG(0) \frac{(\Delta t)^{\frac{1}{2}}}{4} \pm \frac{1}{2} \left\{ TG(0)^2 \frac{\Delta t}{4} + 4 \left[F(t_n) \right. \right. \\
 & + \left(Y(t_{n-1})TG(\Delta t) \right) \frac{(\Delta t)^{\frac{1}{2}}}{2} + \left(Y(t_{n-1})G(t_1) \right. \\
 & + \left. Y(t_{n-2})G(t_2) \right) \frac{\Delta t}{2} + \dots + \left(Y(t_1)G(t_{n-1}) \right. \\
 & \left. \left. + Y(0)G(t_n) \right) \frac{\Delta t}{2} \right] \Bigg\}^{\frac{1}{2}} .
 \end{aligned} \tag{126}$$

The negative root in the above equation was discarded, since it represents a negative absolute temperature, and hence, this could be assumed as physically impossible.

APPENDIX B

SOLUTION OF THE CONDUCTION EQUATIONS FROM THE WAITING TIME ANALYSIS

The governing equations are

$$\frac{\partial \varphi_s}{\partial \tau_s} = \frac{\partial^2 \varphi_s}{\partial X_s^2} + v, \quad (\text{for the solid}) \quad (127)$$

and

$$\frac{\partial \varphi_l}{\partial \tau_l} = \frac{\partial^2 \varphi_l}{\partial X_l^2}, \quad (\text{for the liquid}) \quad (128)$$

with the boundary and initial conditions

$$\varphi_s \Big|_{\tau_s=0} = \varphi_{wm} + \beta_{wm} X_s - \frac{\beta_{wm}}{2} X_s^2, \quad (129)$$

$$\frac{\partial \varphi_s}{\partial X_s} \Big|_{X_s=1} = 0, \quad (130)$$

$$\varphi_s \Big|_{X_s=0} = g(t), \quad (131)$$

$$\varphi_l \Big|_{\tau_l=0} = 0 , \quad (132)$$

$$\varphi_l \Big|_{x_l \rightarrow \infty} \rightarrow 0 , \quad (133)$$

$$\varphi_l \Big|_{x_l=0} = \varphi_s \Big|_{x_s=0} = g(t) , \quad (134)$$

and

$$- \frac{\partial \varphi_l}{\partial x_l} \Big|_{x_l=0} = \frac{k_s}{k_l} \frac{\partial \varphi_s}{\partial x_s} \Big|_{x_s=0} . \quad (135)$$

The solution of Equation 127 is accomplished by taking the Laplace transformation with respect to time,

$$\frac{d^2 \bar{\varphi}_s}{dx_s^2} - H_s^2 p \bar{\varphi}_s = -H_s^2 \left[\varphi_{wm} + \beta_{wm} \left(x_s - \frac{x_s^2}{2} \right) \right] - \frac{v}{p} , \quad (136)$$

where H_s^2 is the time constant in the heated material and the transformed boundary conditions are

$$\bar{\varphi}_s \Big|_{x_s=0} = \bar{g}(p) , \quad (137)$$

and

$$\left. \frac{d\bar{\varphi}_s}{dX_s} \right|_{X_s=1} = 0 . \quad (138)$$

The solution of Equation 136 is then

$$\bar{\varphi}_s = C_1 e^{-H_s p^{1/2} X_s} + C_2 e^{H_s p^{1/2} X_s} + \bar{\varphi}_{\text{part}} , \quad (139)$$

where $\bar{\varphi}_{\text{part}}$ is the particular solution for the non-homogeneous terms on the right hand side of Equation 136.

The particular solution is

$$\bar{\varphi}_{\text{part}} = \frac{v - \beta_{wm}}{H_s^2 p^2} + \frac{\varphi_{wm}}{p} + \frac{\beta_{wm}}{p} \left(X_s - \frac{X_s^2}{2} \right) . \quad (140)$$

Inserting the two transformed boundary conditions permit the evaluation of the constants C_1 and C_2 in terms of $\bar{g}(p)$, the unknown transformed temperature of the heated surface

$$C_1 = \frac{\bar{g}(p) - \left[\frac{(v - \beta_{wm})}{H_s^2 p^2} + \frac{\varphi_{wm}}{p} \right]}{1 + e^{-2H_s p^{1/2}}} , \quad (141)$$

and

$$C_2 = \frac{\bar{g}(p) - \left[\frac{v - \beta_{wm}}{H_s^2 p^2} + \frac{\varphi_{wm}}{p} \right]}{1 + e^{2H_s p^{1/2}}} . \quad (142)$$

On the other hand, the solution for the liquid can be completed analogous to the method outlined for the heated solid. This gives

$$\frac{d^2 \varphi_l}{dx_l^2} - H_l^2 p \bar{\varphi}_l = 0 , \quad (143)$$

or

$$\bar{\varphi} = C_3 e^{-H_l p^{1/2} x_l} , \quad (144)$$

where the boundary condition (Equation 133) has been introduced in Equation 144 to eliminate the second term of the solution. C_3 and $\bar{g}(p)$ can now be obtained by applying the transformed coupling boundary conditions which correspond to Equations 134 and 135

$$C_3 = \bar{g}(p) ,$$

$$\bar{g}(p) = \frac{\left[\frac{v - \beta_{wm}}{H_s^2 p^2} + \frac{\varphi_{wm}}{p} \right] + \frac{\beta_{wm}}{H_s p^{3/2}} \operatorname{ctnh}(H_s p^{1/2})}{1 + d \operatorname{ctnh}(H_s p^{1/2})} ,$$

and

$$d = \frac{H_\ell k_\ell}{H_s k_s} = \frac{k_\ell}{k_s} \left(\frac{\alpha_s}{\alpha_\ell} \right)^{1/2}. \quad (145)$$

Considering just the denominator of $\bar{g}(p)$; it can be expanded as follows:

$$\begin{aligned} \frac{1}{1 + d \operatorname{ctnh}(H_s p^{1/2})} &= \frac{\sinh(H_s p^{1/2})}{d \cosh(H_s p^{1/2}) + \sinh(H_s p^{1/2})} \\ &= \frac{e^{H_s p^{1/2}} - e^{-H_s p^{1/2}}}{(1 + d)e^{H_s p^{1/2}} - (1 - d)e^{-H_s p^{1/2}}} \\ &= \frac{1}{1 + d} \left[\frac{1 - e^{-2H_s p^{1/2}}}{1 - \hat{q} e^{-2H_s p^{1/2}}} \right] \\ &= \frac{1}{1 + d} \left[1 + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) e^{-2(j+1)H_s p^{1/2}} \right], \end{aligned}$$

where $\hat{q} = (1 - d)/(1 + d)$. Similarly,

$$\frac{\operatorname{ctnh}(H_s p^{1/2})}{1 + d \operatorname{ctnh}(H_s p^{1/2})} = \frac{1}{1 + d} \left[1 + \sum_{j=0}^{\infty} (\hat{q}^{j+1} + \hat{q}^j) e^{-2(j+1)H_s p^{1/2}} \right],$$

and hence the solution for $\bar{\varphi}_\ell$ becomes,

$$\begin{aligned} \bar{\varphi}_\ell = \frac{e^{-H_\ell p^{1/2} \chi_\ell}}{1 + d} & \left\{ \left[\frac{v - \beta_{wm}}{H_s^2 p^2} + \frac{\varphi_{wm}}{p} \right] \right. \\ & \left[1 + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) e^{-2(j+1)H_s p^{1/2}} \right] \\ & \left. + \frac{\beta_{wm}}{H_s p^{3/2}} \left[1 + \sum_{j=0}^{\infty} (\hat{q}^{j+1} + \hat{q}^j) e^{-2(j+1)H_s p^{1/2}} \right] \right\}. \end{aligned} \quad (146)$$

The inverse transformation follows immediately from Reference (38):

$$\varphi_\ell = \frac{\varphi_{wm}}{1 + d} \left\{ \operatorname{erfc}(E) \right.$$

$$\begin{aligned}
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) \operatorname{erfc} \left[E + \tau_s^{-1/2}(j+1) \right] \Big\} \\
& + \frac{2\beta_{wm}\tau_s^{1/2}}{1+d} \left\{ i \operatorname{erfc}(E) \right. \\
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} + \hat{q}^j) i \operatorname{erfc} \left[E + \tau_s^{-1/2}(j+1) \right] \Big\} \\
& + \frac{4(v - \beta_{wm})\tau_s}{1+d} \left\{ i^2 \operatorname{erfc}(E) \right. \\
& + \sum_{j=0}^{\infty} (\hat{q}^{j+1} - \hat{q}^j) i^2 \operatorname{erfc} \left[E + \tau_s^{-1/2}(j+1) \right] \Big\} , \quad (147)
\end{aligned}$$

where

$$E = \frac{\tau_l^{-1/2} x_l}{2} . \quad (148)$$

APPENDIX C

SOLUTION OF THE BUBBLE GROWTH EQUATION

I. THE ENERGY EQUATION IN DIFFERENCE FORM (CONVECTION APPROXIMATION)

The dimensionless energy and continuity equation for the liquid surrounding a growing vapor bubble is approximated by

$$\frac{\partial \Theta_l}{\partial \tau} + \frac{r_b^{*2} \dot{r}_b^*}{r^{*2}} \frac{\partial \Theta_l}{\partial r^*} = 0 , \quad (149)$$

with the boundary conditions,

$$\Theta_{l,b} \Big|_{\tau=0} = f(\mu) ,$$

and (150)

$$\frac{\partial \Theta_l}{\partial r^*} \Big|_{r^*=r_b^*} = \Psi_l .$$

Considering the n th time step of Θ_l and assuming $\dot{r}_b^* r_b^{*2}$ has a constant numerical value which is known from the $(n - 1)$ th time step, the difference form of Equation 149 becomes

$$\frac{\Theta_{l,n} - \Theta_{l,n-1}}{\tau_n - \tau_{n-1}} + \frac{r_{b,n-1}^{*2} \dot{i}_{b,n-1}^*}{r^{*2}} \frac{\partial \Theta_{l,n}}{\partial r^*} = 0, \quad (151)$$

where in Equation 151 all $n - 1$ terms are assumed to be known. Solving for $\Theta_{l,n}$ gives

$$\Theta_{l,n} = C \exp \left[\frac{-r^{*3}}{3(\tau_n - \tau_{n-1}) r_{b,n-1}^{*2} \dot{i}_{b,n-1}^*} \right] + \Theta_{l,n-1}. \quad (152)$$

Evaluation of C from the boundary condition (Equation 150) gives

$$C = -\Psi_{l,n-1} \left\{ \frac{\exp \left[\frac{-r_{b,n-1}^{*3}}{3(\tau_n - \tau_{n-1})} \right]}{\dot{i}_{b,n-1}^* (\tau_n - \tau_{n-1})} \right\}^{-1}. \quad (153)$$

By replacing C in Equation 152 with its value from Equation 153, and by evaluating $\Theta_{l,n}$ at the bubble interface, $r^* = r_{b,n-1}^*$, the following equation results:

$$\begin{aligned} \Theta_{l,n} \Big|_{r^*=r_{b,n}^*} &= \Theta_{l,n-1} \Big|_{r^*=r_{b,n-1}^*} \\ &- \Psi_{l,n-1} \dot{i}_{b,n-1}^* (\tau_n - \tau_{n-1}). \end{aligned} \quad (154)$$

The initial condition (Equation 150) assigns the values of $\Theta_{\ell,0} \Big|_{r^*=r_{b,0}^*}$ which must be known before successive values of $\Theta_{\ell,n} \Big|_{r^*=r_{b,n-1}^*}$ can be calculated.

II. CONTINUITY EQUATION FOR THE VAPOR BUBBLE IN DIFFERENCE FORM

The dimensionless continuity equation for the vapor bubble is

$$r_b^{*2} \dot{r}_b^* = Ja \gamma_{\ell} \left[\int_1^{r_b^*} \xi_m r_b^{*'} dr_b^{*'} + r_b^{*2} \int_0^1 \xi_b d\mu \right] . \quad (155)$$

If this equation is integrated from $\tau = 0$ to the n th time step, the result is (note: $r_b^* = 1$ at $\tau = 0$)

$$\frac{r_{b,n}^{*3} - 1}{3} = Ja \gamma_{\ell} \int_0^{\tau_n} \left[\int_1^{r_b^*} \xi_m r_b^{*'} dr_b^{*'} + r_b^{*2} \int_0^1 \xi_b d\mu \right] d\tau . \quad (156)$$

By considering the integrands of the τ integration and by denoting successive integrands as I_n , the following difference equation results:

$$\begin{aligned}
\frac{r_{b,n}^{*3} - 1}{3} = J_a \gamma_\ell \left[(I_0 + I_1) \frac{(\tau_1 - \tau_0)}{2} \right. \\
+ (I_1 + I_2) \frac{(\tau_2 - \tau_1)}{2} + \dots \\
\left. + (I_{n-1} + I_n) \frac{(\tau_n - \tau_{n-1})}{2} \right]. \quad (157)
\end{aligned}$$

Equation 157 can be rearranged to give

$$\begin{aligned}
\frac{r_{b,n}^{*3} - 1}{3} = J_a \gamma_\ell \left[I_0 \frac{(\tau_1 - \tau_0)}{2} \right. \\
+ I_1 \frac{(\tau_2 - \tau_0)}{2} + I_2 \frac{(\tau_3 - \tau_1)}{2} + \dots \\
\left. + I_{n-1} \frac{(\tau_n - \tau_{n-2})}{2} + I_n \frac{(\tau_n - \tau_{n-1})}{2} \right], \quad (158)
\end{aligned}$$

where particular τ integrands are

$$I_0 = \int_1^{r_{b,0}^*} \xi_m r_b^{*'} dr_b^{*'} + r_{b,0}^{*2} \int_0^1 \xi_b d\mu.$$

Since $r_{b,0}^* = 1$ the first integral is zero, then

$$I_0 = \int_0^1 \xi_b d\mu = \sum_{j=1}^{20} [\xi_{b,j} + \xi_{b,j-1}] \frac{\Delta\mu}{2} ,$$

$$I_1 = \int_1^{r_{b,1}^*} \xi_m r_{b,1}^* dr_{b,1}^* + r_{b,1}^{*2} \int_0^1 \xi_b d\mu ,$$

$$I_1 = \left[\xi_m \right]_{t_{\text{evap}}=0}^{r_{b,1}^*} + \xi_m \left[\right]_{t_{\text{evap}}=t_1-t_0}^{r_{b,0}^*} \left[\frac{r_{b,1}^* - r_{b,0}^*}{2} + r_{b,1}^{*2} \int_0^1 \xi_b d\mu , \right. \\ \left. \begin{array}{c} \vdots \\ \vdots \end{array} \right]$$

$$I_n = \left[\xi_m \right]_{t_{\text{evap}}=t_n-t_0}^{r_{b,0}^*} + \xi_m \left[\right]_{t_{\text{evap}}=t_n-t_1}^{r_{b,1}^*} \left[\frac{r_{b,1}^* - r_{b,0}^*}{2} \right. \\ \left. + \left[\xi_m \right]_{t_{\text{evap}}=t_n-t_1}^{r_{b,1}^*} + \left[\xi_m \right]_{t_{\text{evap}}=t_n-t_2}^{r_{b,2}^*} \left[\frac{r_{b,2}^* - r_{b,1}^*}{2} + \dots \right] \right]$$

$$\begin{aligned}
& + \left[\xi_m \Big|_{t_{\text{evap}}=t_n-t_{n-1}} r_{b,n-1}^* \right. \\
& + \left. \xi_m \Big|_{t_{\text{evap}}=t_n-t_n} r_{b,n}^* \right] \frac{r_{b,n}^* - r_{b,n-1}^*}{2} \\
& + r_{b,n}^{*2} \sum_{j=1}^{20} \left[\xi_{b,j} + \xi_{b,j-1} \right] \frac{\Delta \mu}{2} . \quad (159)
\end{aligned}$$

To solve for $r_{b,n}^*$ all values on the right hand side of Equation 158 are known except for the $r_{b,n}^*$ terms in I_n , thus considering I_n further gives

$$\begin{aligned}
I_n = & \sum_{j=0}^{n-2} \left\{ \left[\xi_m \Big|_{t_{\text{evap}}=t_n-t_j} r_{b,j}^* \right. \right. \\
& + \left. \left. \xi_m \Big|_{t_{\text{evap}}=t_n-t_{j+1}} r_{b,j+1}^* \right] \frac{(r_{b,j+1}^* - r_{b,j}^*)}{2} \right\} \\
& + \xi_m \Big|_{t_{\text{evap}}=t_n-t_{n-1}} \frac{r_{b,n-1}^* r_{b,n}^*}{2} \\
& + \xi_m \Big|_{t_{\text{evap}}=0} \frac{r_{b,n}^{*2}}{2} - \xi_m \Big|_{t_{\text{evap}}=t_n-t_{n-1}} \frac{r_{b,n-1}^{*2}}{2}
\end{aligned}$$

$$\begin{aligned}
& - \xi_m \Big|_{t_{\text{evap}}=0} \frac{r_{b,n}^* r_{b,n-1}^*}{2} \\
& + r_{b,n}^{*2} \sum_{j=1}^{20} [\xi_{b,j} + \xi_{b,j-1}] \frac{\Delta\mu}{2} . \quad (160)
\end{aligned}$$

Inserting Equations 159 and 160 into Equation 158 gives the cubic equation for $r_{b,n}^*$

$$\begin{aligned}
& - \frac{1}{3} r_{b,n}^{*3} + \frac{Ja \gamma_\ell (\tau_n - \tau_{n-1})}{2} \left[\frac{1}{2} \xi_m \Big|_{t_{\text{evap}}=0} \right. \\
& + \sum_{j=1}^{20} [\xi_{b,j} + \xi_{b,j-1}] \frac{\Delta\mu}{2} \Big] r_{b,n}^{*2} \\
& + \frac{Ja \gamma_\ell (\tau_n - \tau_{n-1})}{2} \frac{r_{b,n-1}^*}{2} \left[\xi_m \Big|_{t_{\text{evap}}=t_n-t_{n-1}} \right. \\
& \left. - \xi_m \Big|_{t_{\text{evap}}=0} \right] r_{b,n}^* + \frac{1}{3} \\
& - \frac{Ja \gamma_\ell (\tau_n - \tau_{n-1})}{2} \frac{r_{b,n-1}^{*2}}{2} \xi_m \Big|_{t_{\text{evap}}=t_n-t_{n-1}}
\end{aligned}$$

$$\begin{aligned}
& + \sum_{j=0}^{n-2} \left\{ \left[\xi_m \right]_{t_{\text{evap}}=t_n-t_j} r_{b,j}^* \right. \\
& \left. + \xi_m \right|_{t_{\text{evap}}=t_n-t_{j+1}} r_{b,j+1}^* \left. \right] \frac{(r_{b,j+1}^* - r_{b,j}^*)}{2} \times \\
& \frac{Ja \gamma_\ell (\tau_n - \tau_{n-1})}{2} \left\{ + Ja \gamma_\ell I_0 \frac{(\tau_1 - \tau_0)}{2} \right. \\
& + Ja \gamma_\ell I_1 \frac{(\tau_2 - \tau_0)}{2} + \dots \\
& \left. + Ja \gamma_\ell I_{n-1} \frac{(\tau_n - \tau_{n-2})}{2} = 0 \right. . \tag{161}
\end{aligned}$$

Equation 161 represents the difference equation for $r_{b,n}^*$ as a function of the previous $n - 1$ values of r_b^* . The roots of the cubic equation were easily obtained by standard techniques--see Reference (55)--and in all cases only one real root occurred from the numerical evaluation of Equation 161.

III. DIFFERENCE FORM FOR RAYLEIGH'S EQUATION

With $r_{b,n}^*$ known from continuity of the vapor bubble consideration, the determination of the pressure inside the vapor bubble is readily known from the modified Rayleigh's Equation,

$$r_{b,n}^* \ddot{r}_{b,n}^* + \frac{3}{2} \dot{r}_{b,n}^{*2} + \frac{\Lambda_I}{r_{b,n}^*} = \Lambda_{II} , \quad (162)$$

where

$$\Lambda_I = \frac{2\sigma r_c}{a_l^2 \rho_l} ,$$

and

$$\Lambda_{II} = \frac{(P_v - P_{sat}) r_c^2}{a_l^2 \rho_l} . \quad (163)$$

The unknown pressure inside the vapor bubble is contained in Λ_{II} . With the calculation for $n = 1$, enough information concerning bubble growth has been obtained from energy and continuity considerations to numerically evaluate the left-hand side of Equation 162. The equation for the first time derivative of $r_{b,n}^*$ was obtained from Equation 155

$$\dot{r}_{b,n}^* = \frac{Ja \gamma_\ell}{r_{b,n}^{*2}} \int_1^{r_{b,n}^*} \xi_m r_b^{*'} dr_b^{*'} + Ja \gamma_\ell \int_0^1 \xi_b d\mu ,$$

(164)

and the second time derivative is given by

$$\ddot{r}_{b,n}^* = \frac{\dot{r}_{b,n}^* - \dot{r}_{b,n-1}^*}{\tau_n - \tau_{n-1}} .$$

(165)

APPENDIX D

DESCRIPTION OF THE NUMERICAL PROCEDURE FOR CALCULATING THE BUBBLE RADIUS

Having obtained the upper and lower limits of the transient bubble interface temperature, T_b , the numerical solution of Equation 105 can now be described:

$$\begin{aligned} \frac{r_b^{*3} - 1}{3} = & Ja \gamma_l \int_0^\tau \left[\int_1^{r_b^*} \xi_m r^{*'} dr^{*'} \right. \\ & \left. + r_b^{*2} \int_0^1 \xi_b d\mu \right] d\tau . \end{aligned} \quad (105)$$

The right hand side of Equation 105 suggests that a numerical time stepping procedure be utilized to evaluate the time and surface area integrals. This procedure was carried out by using a trapezoidal approximation. The liquid-vapor area integration in Equation 105 was determined by dividing the bubble interface into 20 segments, and by considering 21 temperature nodes equally spaced in the μ domain by increments of $\Delta\mu = 0.05$. The bubble interface temperature at each node could be determined from Equation 100 or Figure 11, page 44, of Chapter III depending upon

which solution of the energy equation described earlier is used to evaluate T_b .

Unlike the nodes for the bubble interface, the nodes for the area integrations of the microlayer were not known a priori, but rather were specified as the bubble interface advanced on the heated surface. The nodes for the numerical integration of the evaporating microlayer were assigned positions, r^* , at each evaluation of the bubble interface, r_b^* . Figure 45 presents a schematic illustration of the microlayer profile as a function of bubble interface position. Starting with the initial bubble size, r_c , the first advance of the bubble interface (i.e., first time step) will create liquid microlayer beneath the vapor bubble from r_c to $R_{b,1}$ (Figure 45). During the first time step, the newly formed microlayer will evaporate a mass which corresponds to the area I in Figure 45. With microlayer nodes at r_c and $R_{b,1}$ the microlayer mass contribution to the bubble growth during this time step is obtained by time averaging the instantaneous mass fluxes from the microlayer at the beginning and at the end of this step. The initial mass flux is zero; and the final mass flux must be averaged over the surface area of the microlayer. By Equation 79 the instantaneous mass flux from the microlayer is related to the temperature distribution. In turn, temperature distribution, $T_m(r',t)$ can be evaluated by shifting the evaporation

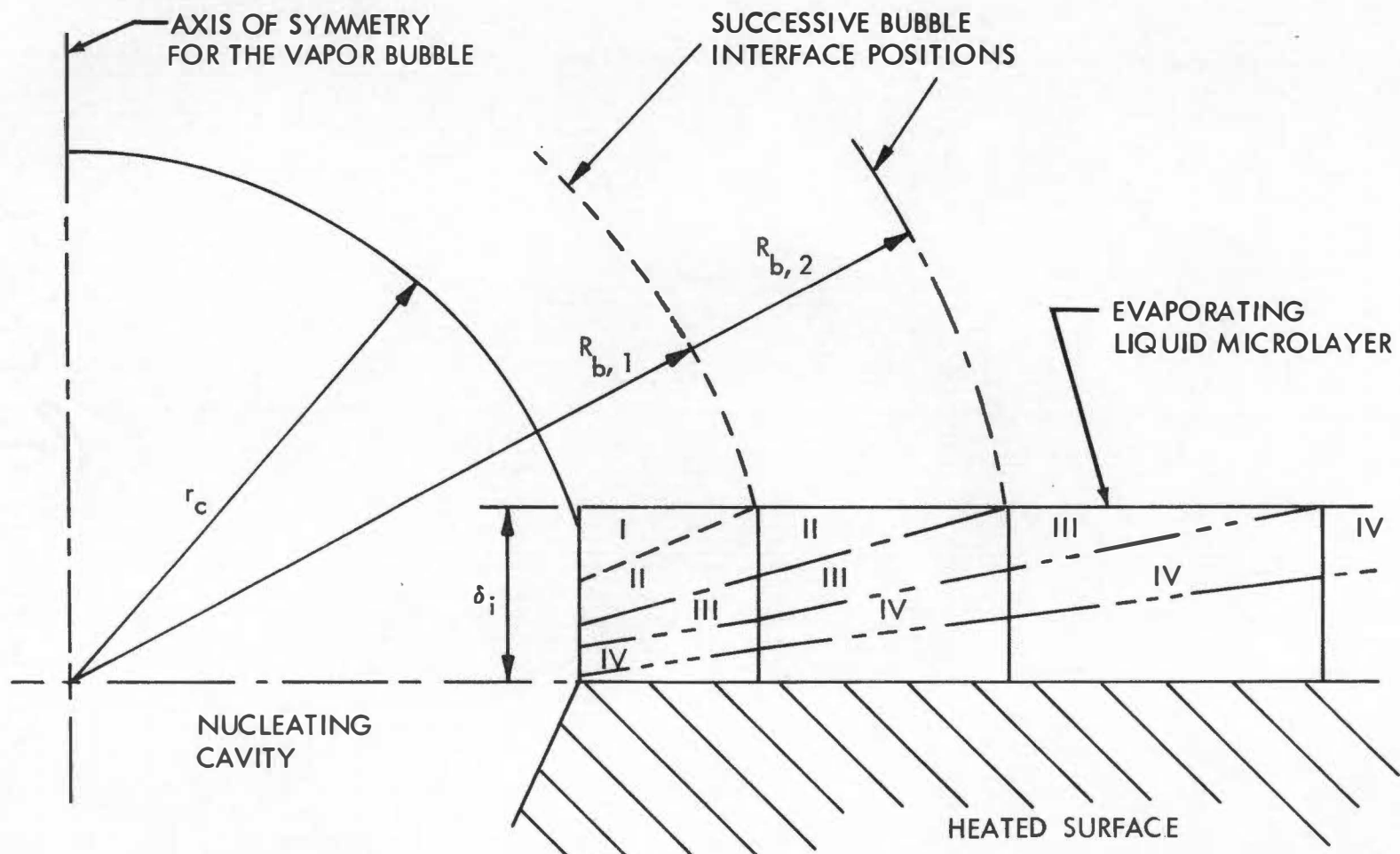


Figure 45. Schematic integration procedure for the liquid microlayer evaporation during bubble growth.

time scale used, in the Chapter III temperature solution. For example, considering the microlayer at a time, $t = t_1$, measured from the beginning of bubble growth, the microlayer evaporation at node, r_c , has occurred for a time $t_{\text{evap}} = t_1$; whereas evaporation from node, $R_{b,1}$, has occurred for $t_{\text{evap}} = 0$. Then, by referring to Figure 12, page 45, of Chapter III and introducing the evaporation times for each microlayer node, the microlayer temperature distribution is ascertained. Thus, the mass flux contributions from the liquid microlayer at an instant of bubble growth time can be area averaged to complete the first time step integration. Successive time steps require microlayer integrations as labeled II, III, and IV in Figure 45, page 186. The mathematical development of the difference equations for the n th numerical time integration is outlined in more detail in Appendix C.

The model for liquid microlayer evaporation during bubble growth requires knowledge of the amount of liquid, δ_i , which is contained in the microlayer to ascertain when a dry spot will appear on the heated surface. The few experimental measurements of microlayer thicknesses reported in the literature are insufficient to estimate how microlayer thickness varies from system to system. Hence, for purposes of this analysis the total mass of liquid evaporated from the microlayer was assigned values between 0.0001 lbm/ft^2 and 0.0004 lbm/ft^2 . These microlayer masses of liquid

correspond to the reported values of References (3) and (2) respectively.

Based on these assigned values the lifetime of the liquid microlayer at any location or node can be determined from the results of Chapter III. Once this lifetime is exceeded in the time stepping procedure, no further contribution from that node to the microlayer area integration of Equation 105 was considered.

Successive evaluations of the surface area integrations in Equation 105 complete the numerical integration. The pertinent difference equations for the time integration of Equation 105 are developed in Appendix C. The one real root which resulted from the cubic difference equation for r_b^* at the n th time step (Equation 161 in Appendix C) was used in succeeding calculations of r_b^* .

Magnitude of Time Steps

Because the evaporation rate at the liquid-vapor interface could initially be very rapid when $T_l - T_{sat}$ is large, it was necessary to employ a variable time step to conserve computer time and storage. This also insured that the temperature changes at the liquid-vapor interface remained within acceptable limits during the initial time steps, yet afforded a more rapid calculation as these transients dissipated. The time steps started at a small

value (about 10^{-9} seconds) and successive time increments were increased by an arbitrary factor of 1.4 until a time increment of about 10^{-4} seconds was reached; after which the time increments were kept constant. Figure 46 illustrates the effect of the maximum time step used in the bubble growth calculation. The maximum size of any time step must not exceed the time required to completely evaporate the liquid microlayer (for the conditions of Figure 46 this limit is 0.5 milliseconds). A maximum time step size of 60 steps per millisecond allowed thirty integration steps in the liquid microlayer before a dry spot appeared.

To utilize the microlayer evaporation temperature response of Chapter III in the variable time stepping procedure the temperature solution of Figure 11, page 44, was fitted to a polynomial by standard curve fitting techniques--see Reference (56). Values of T_m for use in Equation 105 corresponding to the appropriate microlayer evaporation times were then calculated with the polynomial curve fit.

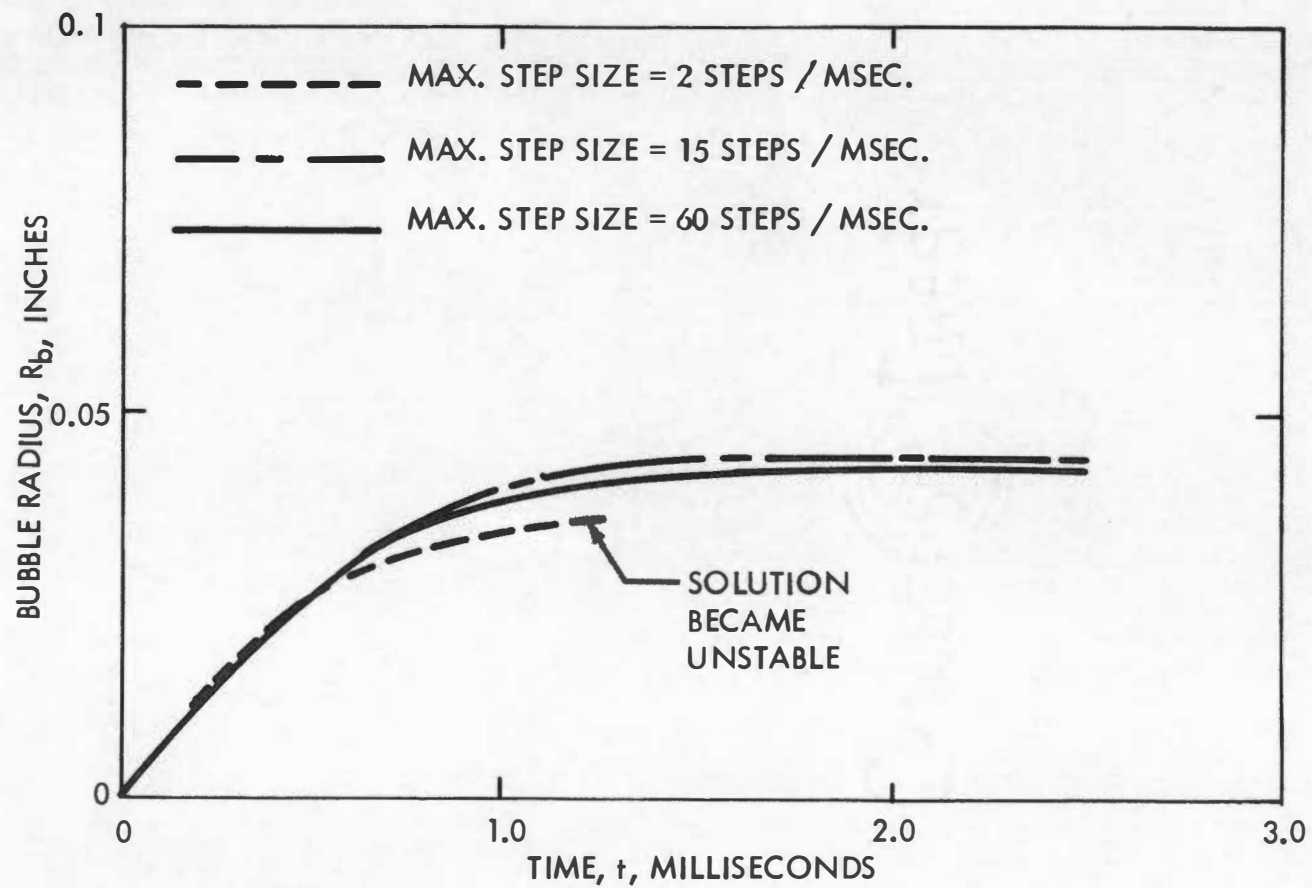


Figure 46. Effect of maximum time steps on the analytical bubble growth solution.

APPENDIX E

PRELIMINARY INVESTIGATION OF THE BUBBLE GROWTH SOLUTION

A preliminary investigation of the numerical bubble growth solution was conducted to ascertain the significance of the following bubble growth parameters:

1. Nucleating cavity size, i.e., initial bubble size, r_c ,
2. The vapor pressure departure from $P_v = P_{bulk}$,
 $P_v - P_{bulk}$,
3. The non-uniform initial temperature distribution specified at the nodes of the bubble interface, i.e., the effect of waiting period, $T_{bi}(\mu)$,
4. Internal heat generation in the heated solid,
 Φ or u ,
5. Heater thickness, a ,
6. Thermal transport properties of the heated solid and the liquid, d ,
7. Initial thickness of the liquid microlayer, δ_i ,
and
8. The initial microlayer temperature, T_{wi} .

Each of the above named parameters was individually varied for selected combinations of the remaining parameters.

The parameters were selected so as to duplicate the nucleate boiling heat transfer environments which are applicable to the present model. This procedure was necessary since an extensive variation of each of the above parameters requires prohibitive computation time. The results of the preliminary study justified neglecting the following parameters.

Initial Bubble Radius

Since a given heated surface will have a random distribution of active cavity sizes during nucleate boiling, it is necessary to define representative values of r_c which may be employed during the evaluation of Equation 105. This is accomplished with the aid of Equation 10 of Chapter II:

$$r_c = \left[\frac{2\sigma v_{fg} T_{sat} k_l}{h_{fg} Pr_l^2 (q/A)} \right]^{1/2}, \quad (10)$$

where $(q/A)_{inc}$ has been replaced with the average surface heat flux (q/A) . The range of active nucleating cavity sizes for bubble growth was approximated by alternately introducing the incipient heat flux and the average heat flux into Equation 10 to obtain upper and lower limits of r_c and then calculate for each value of r_c the resulting bubble growth. Figure 47 illustrates the bubble radius versus time solution of Equations 105 and 100 for the

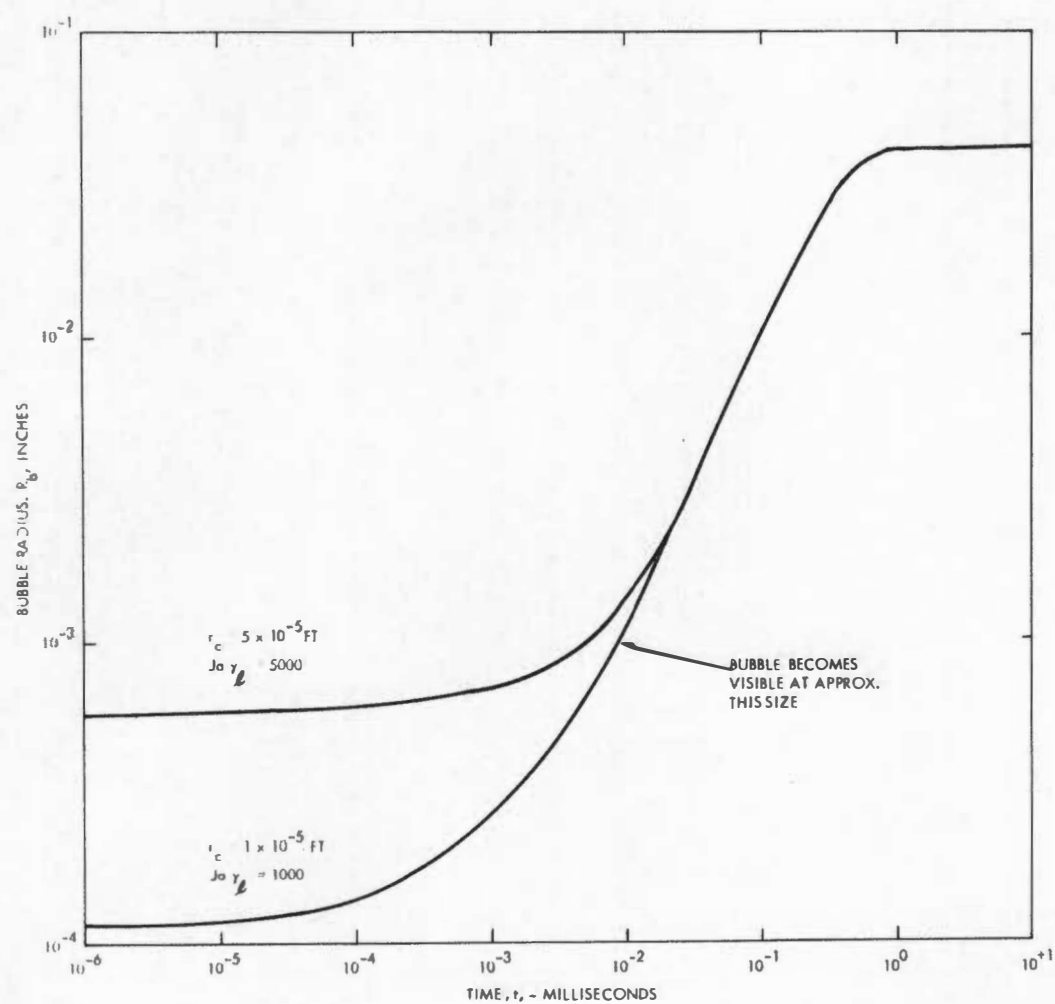


Figure 47. Dimensional comparison of the bubble growth solution with different values of r_c .

abovementioned limits on the nucleating cavity size. It is apparent that r_c is only significant during the very early stages of the bubble growth and after approximately 10^{-2} milliseconds the vapor bubble growth curves can no longer be distinguished by nucleating cavity size. Thus, during the bubble's visible lifetime this parameter can be neglected.

Excess Pressure in the Vapor Bubble

From the governing equation of motion for bubble growth the excess pressure $P_v - P_{bulk}$ inside the vapor bubble can be related to the dynamic motion of the bubble interface. This procedure was outlined earlier with the result that the pressure excess is given by the modified Rayleigh's equation (Equation 90). During the bubble growth the time derivatives of the bubble interface position were approximate by the finite difference procedure outlined in Appendix C. Figure 48 presents the average dimensional excess pressure $P_v - P_{bulk}$ (in psia) versus bubble growth time for several parameters of the bubble growth solutions of Equations 100 and 105. All of the presented bubble growth solutions monitored the excess pressure requirements of Equation 90 and conclusively demonstrated that the relatively small excess pressures required during bubble growth justified the earlier assumption of $P_v = P_{bulk}$, i.e.,

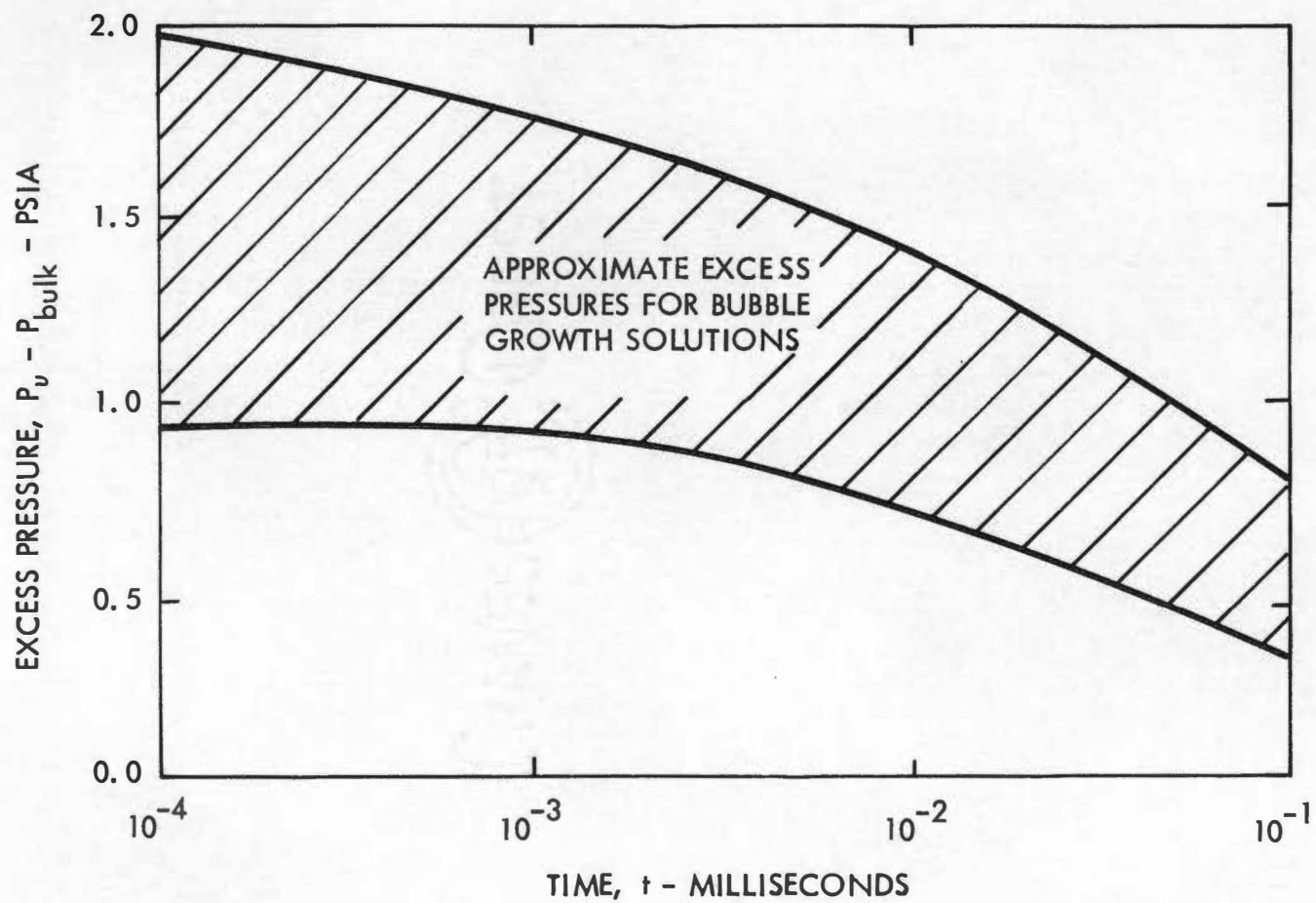


Figure 48. Excess vapor pressure during bubble growth.

the saturation temperature of the vapor, T_{sat} , can be assumed constant.

Non-Uniform Temperatures Surrounding the Vapor Bubble

The initial bubble interface temperatures were calculated from Equation 67 of Chapter IV evaluated at the end of the waiting period. Thus, various initial bubble interface distributions can be assigned to the bubble growth by varying the waiting period. As bubble growth commenced, the bubble interface cooled toward a uniform saturation temperature. The rate at which this cooling occurred for the two approximate solutions of the energy equation for the liquid (Equation 100 or Figure 11, page 44) was quite different. However, in the present cases the boundary condition (Equation 94) requires that the bubble interface cools more rapidly at nodes which were assigned higher initial superheat temperatures. For the convection approximation, the dimensionless temperature distribution on the bubble interface at successive values of time is shown in Figure 49. The physical time which corresponds to $\tau = 100 \times 10^{-4}$ at which $\Theta_{l,b} = 0$ is $t = 10^{-6}$ seconds, which is too short of a period for the non-uniform temperature distribution to have any influence on the visible growth of the bubble. In the case of the conduction solution the temperature does not rapidly approach zero; however, the non-uniformity of the

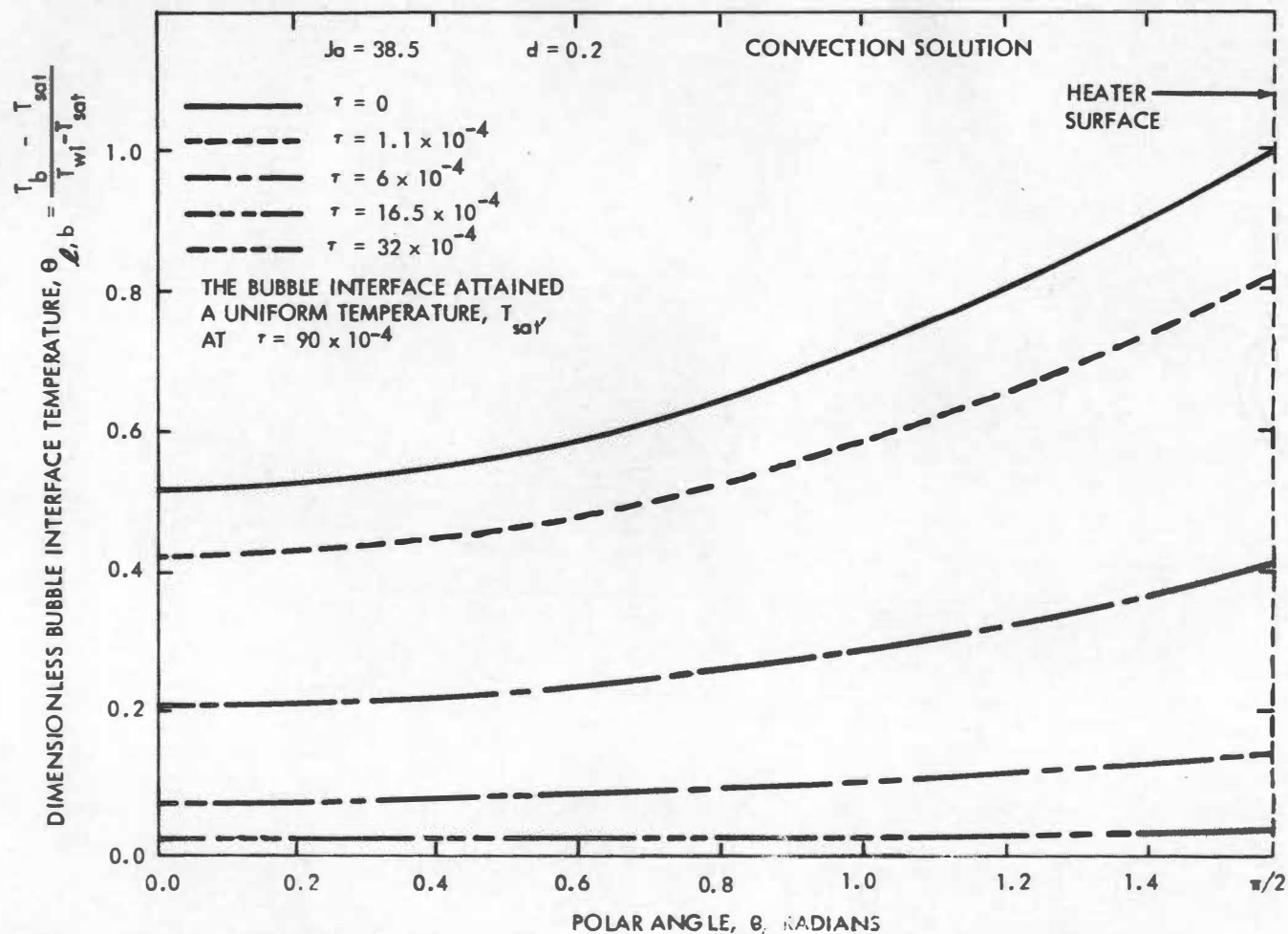


Figure 49. Dimensionless bubble interface temperature profiles surrounding the growing vapor bubble for the convection approximation.

profile is dissipated before the bubble is visible. Thus, the bubble interface temperature can be characterized by T_{wi} which serves as a representative liquid superheat temperature.

Internal Heat Generation

The rate at which heat is generated in the solid was found to have no appreciable influence on the present bubble growth models. The procedure used to ascertain this was to compute bubble growth with internal heat generation ranging from zero to a maximum consistent with nucleate boiling (51) while maintaining the remaining parameters constant. All curves were found to coincide. Among the parameters held constant was the initial heater surface temperature, T_{wi} , which, physically, will vary according to the steady state rate at which energy is generated in the heated solid. Inasmuch as the initial wall temperature was considered to be an independent parameter in this analysis, the internal heat generation exerts no influence on bubble growth.

The remaining bubble growth parameters listed at the beginning of this section must be considered in the bubble growth analysis; whereas, the above discussion of the preliminary results suggested that no further parameterization be made for nucleating cavity size, excess vapor pressure

during bubble growth, non-uniform temperature distribution, and internal heat generation.