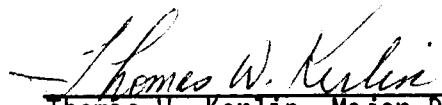
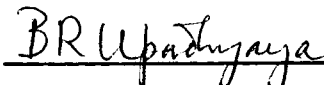
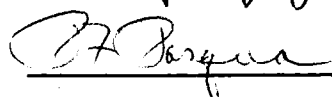
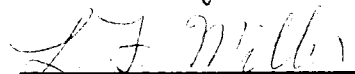


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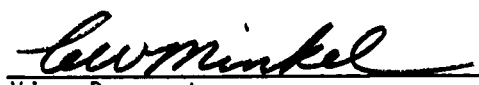
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MODELLING AND TESTING OF INDUSTRIAL
THERMOMETERS

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Majed N. Natour

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ABSTRACT

Thermocouples and Resistance Temperature Detectors are widely used in nuclear power plants and other industrial processes for temperature measurements. Such measurement is necessary for control and safety purposes. Thus temperature sensors and the various factors that may affect their speed of response are of great importance to manufacturers and users of temperature sensors.

This work is concerned with the numerical simulation of temperature sensors along with the experimental verification of theoretically developed correlations to characterize and better understand sensor behavior under laboratory and practical conditions.

A general purpose computer program for two-dimensional simulation of industrial temperature sensors has been developed. The program permits dynamic simulation of industrial sensors. Effects of porosity of the insulator, surface heat transfer coefficient, and physical properties of the sensor were examined to demonstrate the utility of the approach.

In addition, an extensive set of experiments was carried out to verify a theoretically developed correlation. This correlation indicates that the time constant varies inversely as the heat transfer coefficient. Experimentation involved different liquids, different velocities, different sensors, and different experimental facilities. The results have shown satisfactory validation of the theoretical relation.

As a continuation of the experimental program, an apparatus for

testing temperature sensors in dry and wet air has been constructed. Tests were carried out on four thermocouples. The results show that the response times increase dramatically in comparison with the response times in liquids.

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CHAPTER I

INTRODUCTION

Reliable temperature measurement is important in a number of process industries. It can potentially lead to better product quality and proper process performance. It also plays an important role in safeguarding power plants. For example, safety system actions may be taken based on temperature measurements in a nuclear power plant. Since temperature sensors, among other sensors, are so vital to the nuclear industry, a regulatory guide entitled, "Periodic Testing of Electric Power and Protection Systems" (1) was published by the Nuclear Regulatory Commission (NRC). This guide recommends the periodic testing of sensors.

Several methods and test techniques have been developed to study the response time characteristics of temperature sensors.

Mathematical models can also be used to simulate the behavior of temperature sensors. They can be used by engineers for performance evaluation or by manufacturers as a design tool.

Successful modelling of an actual sensor depends on the accuracy of the information known to the modeller. The necessary information includes the physical properties of the sensor's constituents and surrounding medium, dimensions, and location of the sensing elements. The physical properties include the thermal conductivity (k), specific heat capacity (C_p), and density (ρ) of the sensor's constituents. The necessary physical properties of the fluid are ρ , k , C_p ,

and the dynamic viscosity (μ). The velocity (u) of the fluid is also needed.

Two sensors of the same type and make may differ in their response to the same input because of the differences in their construction due to manufacturing tolerances. This indicates that modelling alone is not sufficient for evaluating sensor response. However, it helps in evaluating trade offs.

Testing of temperature sensors, however, is more reliable and widely used technique than modelling. Three test methods can be used. The first is the plunge test which is usually performed in laboratories. The second test method is the Loop Current Step Response (LCSR) test that can be performed in laboratory and in-situ. This is equivalent to introducing a step increase in heat generated inside the sensor. The third testing method is used to test for sensor degradation. It is referred to as the Self Heating test.

1.1 Previous Work

In 1975, the Nuclear Engineering Department, University of Tennessee, Knoxville, was involved in the theoretical development of LCSR testing. In 1976, the Electric Power Research Institute (EPRI) supported and funded a project to develop in-situ testing technology. In the period between 1976 and 1978, the LCSR method was confirmed in the Laboratory for RTDs. LCSR testing was also confirmed in three operating plants: Turkey Point, St. Lucie, and Oconee.

Test results showed that sensors can degrade and Kerlin(5) was involved in developing the Self Heating method to test for sensor

degradation. In 1978, a correction factor, for the LCSR method, which led to a reduction of the error from a maximum 20% to about 5% was developed. This factor accounts for the higher modes as will be described later.

Further analysis was performed recently to explore the effects of the medium around an immersion sensor on the sensor's response. A theoretical correlation that predicts the sensor response as a function of the film heat transfer coefficient was developed. Several sensors were tested in three testing facilities to verify this correlation.

1.2 Statement of Research Objectives

The purpose of this work is to explore dynamic modelling of industrial thermometers. Modelling is used to estimate the effects of fluid flow conditions on the sensor's response. Sensitivity analyses to study end effects, velocity and heat transfer coefficient effects, diameter effects, the physical properties effects, etc., are to be carried out.

Results of tests performed on five temperature sensors to verify the theoretical correlations, derived to predict the time constant as a function of the heat transfer coefficient and velocity, are presented. Discussion of the various problems encountered while testing and analyzing test results is also presented.

Finally, the physical properties of the most widely used sensor's constituents, fluids, and air are summarized in this thesis.

CHAPTER II

SYSTEM DESCRIPTION

2.1 Thermocouples

2.1.1 Introduction

In 1821, T. J. Seebeck discovered the thermoelectricity phenomenon.⁽²⁾ That is, if the ends of two dissimilar metals are joined together and each end is kept at different temperature, then a voltage is created. Thermocouples (TCs) were then developed by making use of this "effect."

In general, a thermocouple consists of two metallic wires joined mechanically by welding or brazing to form the measuring junction. These wires are typically .5 to 1.0 mm in diameter (some special TCs can be made smaller than human hair). These wires are usually insulated electrically and thermally by cement or a powder. The most commonly used powders are aluminum oxide (Al_2O_3) or magnesium oxide (MgO). The sensing wires and insulant are usually protected by a stainless steel sheath. The most commonly used stainless steels are types 304 or 446. Figure 2.1 shows a typical multilayered structure of a sheathed thermocouple.

In general TCs are characterized as being simple, inexpensive, versatile, durable, and accurate. They can cover ranges of temperatures between $-183^{\circ}C$ (cryogenic thermocouples) to $2500^{\circ}C$ (refractory thermocouples).⁽³⁾

2.1.2 Circuit Theory

As the measuring junction is placed in a temperature gradient,

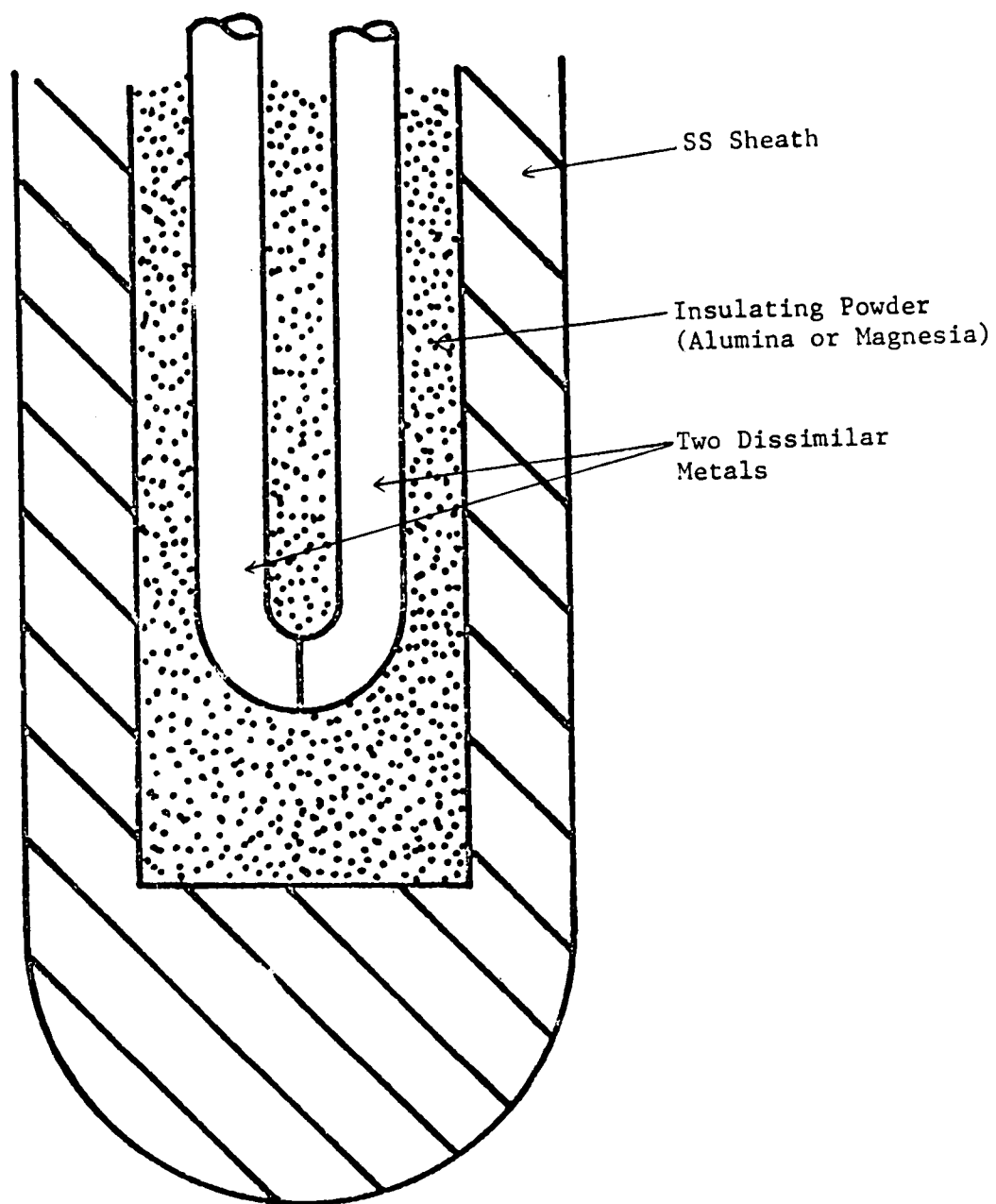


Figure 2.1 Typical Multilayered Structure of a Sheathed Thermocouple

(i.e. there exists a temperature difference between the measuring junction and the other end of the wires) an electromotive force (emf) is generated and measured at the other end. This emf is proportional to the temperature gradient. It can be calculated in terms of the difference in Seebeck coefficients, and the temperatures gradient as

$$V = (S_A - S_B)(T_M - T_R) \quad (2.1)$$

where all the variables are shown in Figure 2.2.

The Seebeck coefficient of a material is defined as the potential difference established along a wire of that material by the application of a unit temperature difference (3). This coefficient has the units of volts/°C and it is called the thermoelectric power of the material.

Two types of Seebeck coefficients are usually used. The first is the absolute Seebeck coefficient, which is a property of the material. It is denoted by S_A for material A. The second is the relative Seebeck coefficient (S_{AB}) defined as $S_{AB} = S_A - S_B$ where material B is normally pure platinum.

Making use of the relative and the absolute Seebeck coefficients, and the fact that the voltage at the open end of the thermocouple is equivalent to the sum of all voltages in the circuit, one can obtain several thermocouple circuits. A selected case is shown in Figure 2.3. In this case, the wires are assumed to pass through several temperature gradients. And the voltage is given by

$$V = S_a(T_1 - T_0) + S_a(T_2 - T_1) + S_a(T_H - T_2) + S_b(T_2 - T_H) + S_b(T_1 - T_2) + S_b(T_0 - T_1) \quad (2.2)$$

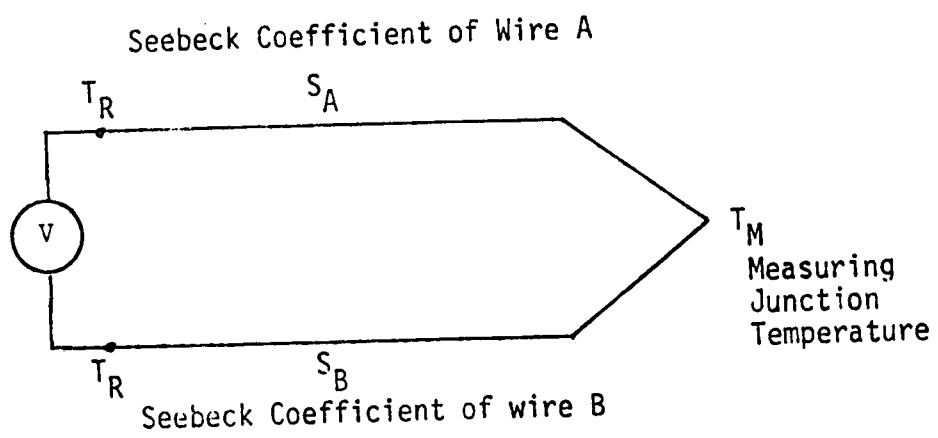


Figure 2.2 Basic Thermocouple Circuit

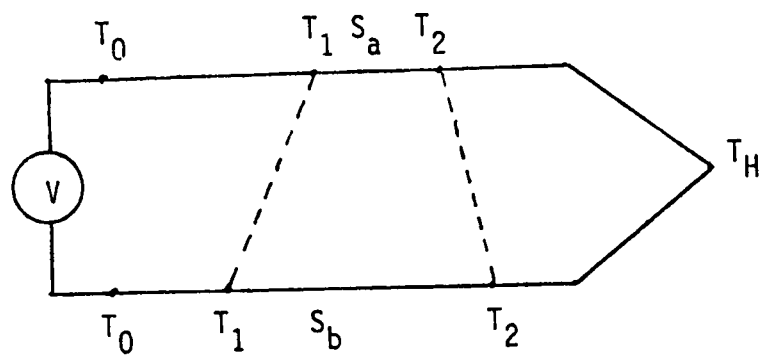


Figure 2.3 Selected TC Circuit

$$\begin{aligned}
 V &= S_a(T_H - T_0) - S_b(T_H - T_0) \\
 &= (T_H - T_0) (S_a - S_b) \\
 &= (T_H - T_0)S_{ab}.
 \end{aligned}
 \tag{2.3}$$

Equation 2.3 shows that the voltage for the circuit shown in Figure 2.3 is dependent on T_H , T_0 , and S_{ab} .

2.1.3 Types of Thermocouples

It is obvious that the properties of a thermocouple depend on the two metals forming the sensing element. Such metals are chosen on the basis of their stability, ductility, and sensitivity. TCs that have relatively large Seebeck coefficients and, hence, large emfs are preferred provided that this emf follows a linear relationship with temperature.

Thermocouples may be classified into three classes depending on the sensing metals (3):

1. Base-metal thermocouples for temperature ranges up to 1000 °C. Such TCs employ iron/constantan (copper-nickel alloy) (type J TC), copper/constantan (type T), Chromel/Alumel (type K), Chromel/constantan (type E), and other alloys of copper, nickel, iron, chromium, manganese, aluminum, and other elements.
2. Noble metal thermocouples that cover up to 2000 °C. These include platinum (Pt)-rhodium (type S, R, or B) and other alloys of Pt, iridium, ruthenium, niobium, and rhenium.

3. Refractory metal TCs that are useful up to 2600 °C. The principle elements are tungsten-rhenium. Some refractory TCs include tantalum TCs, molybdenum and their alloys TC. Types S, R and B employ the following sensing elements:

Type S: Pt-10% rhodium/Pt

Type R: Pt-13% rhodium/Pt

Type B: Pt-30% rhodium/Pt-6% rhodium.

Tables of temperature versus voltages (for the reference junction kept at 0°C), are established for all types of TCs. If the reference junction is kept at a temperature other than 0°C, one can still use those tables to find the temperature of the medium. This is done by simply finding the voltage that corresponds to the new reference temperature, using the table, and adding this voltage to the measured voltage. Then the temperature that corresponds to the total voltage is the medium's temperature.

The new advances in electronics led to the production of digital readout devices that indicate the process temperature directly. Such readout devices employ microprocessors and A/D convertors. The reference tables for a particular type are usually saved on special memory in the device. Some of these devices can be programmed to receive signals from any type of TC.

2.1.4 Thermocouple Styles

The most commonly manufactured TCs are those which employ one measuring junction and a reference junction. The measuring junction

can be insulated or grounded. Both cases are shown in Figures 2.1 and 2.4. The grounded sensors are usually faster than the insulated one.

Other thermocouple styles include the differential and thermopile thermocouples. Differential TCs are used to measure the difference in temperature between the measuring junctions as shown in Figure 2.5.

The thermopile TCs employ several measuring junctions that are all exposed to one temperature. They are connected in the manner shown in Figure 2.6. The advantage of such thermocouples is that they increase the output voltage by as many times as there are measuring junctions.

2.2 Resistance Temperature Detectors (RTDs)

2.2.1 Introduction

Sir Humphry Davy was probably the first scientist to notice that the conductivity of metals decreases with increasing temperature (4). In 1887, Hugh Callendar published the first paper on resistance thermometers based on an earlier outline of using platinum resistance thermometers by Sir William Siemens. In Callendar's paper, a description of a platinum resistance thermometer appeared. Moreover, he provided an equation that describes the variations in Pt resistance with temperature. This equation is still being used to measure temperature on the International Practical Temperature Scale from 0°C to 500°C (4).

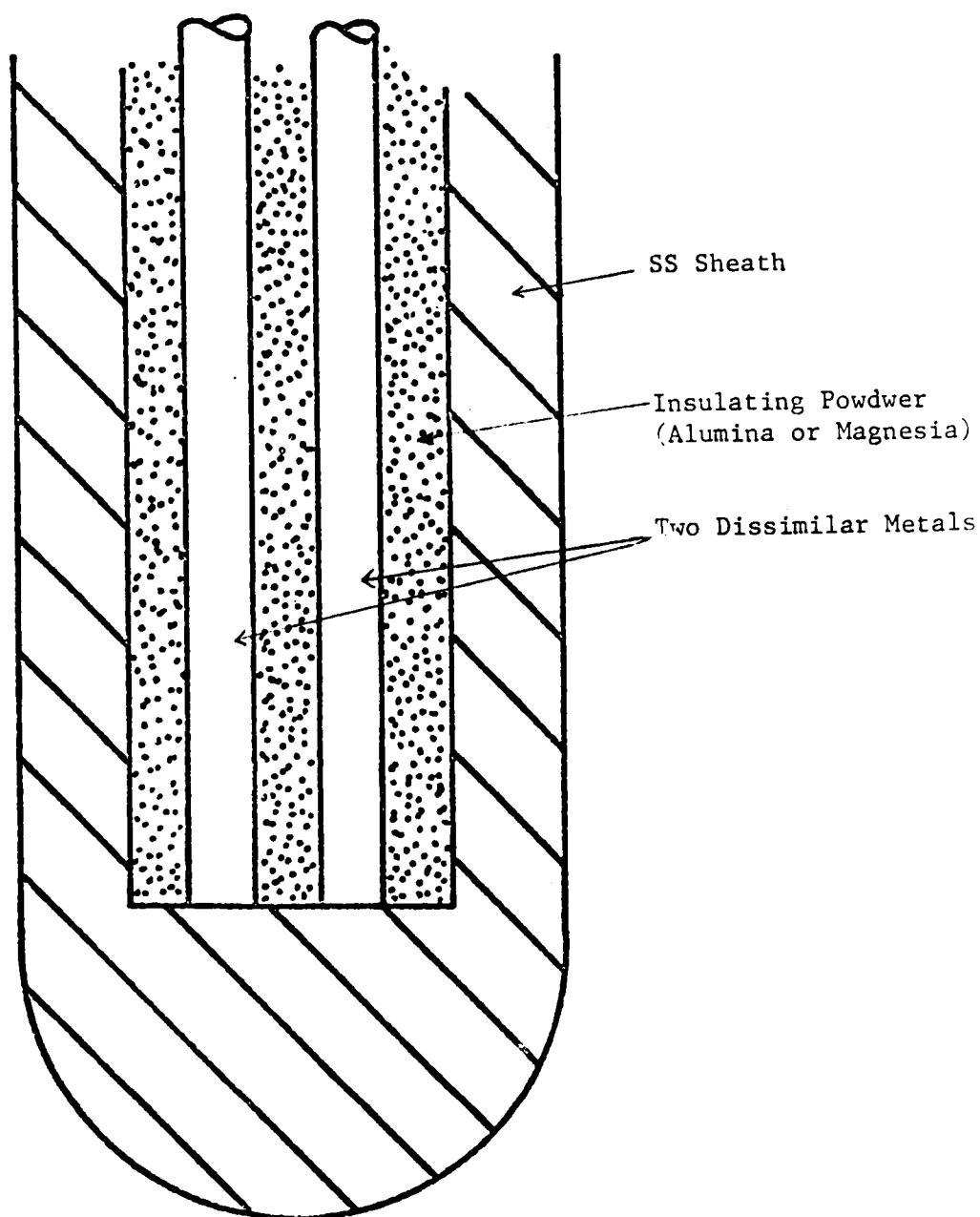


Figure 2.4 A Typical Grounded Junction Thermocouple

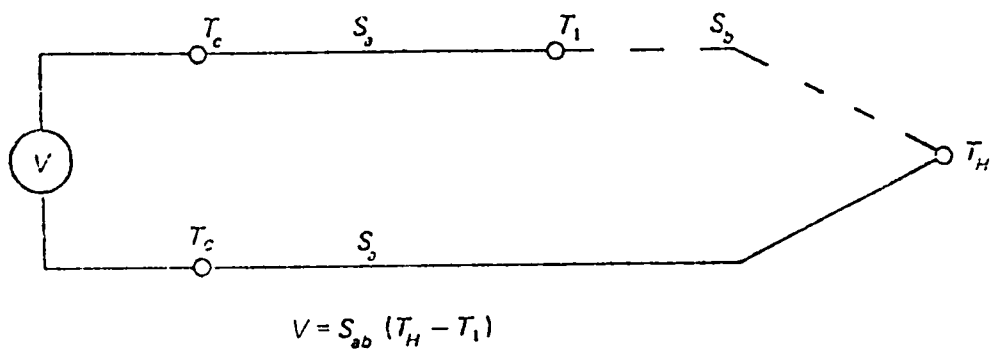


Figure 2.5 A Typical Differential Thermocouple

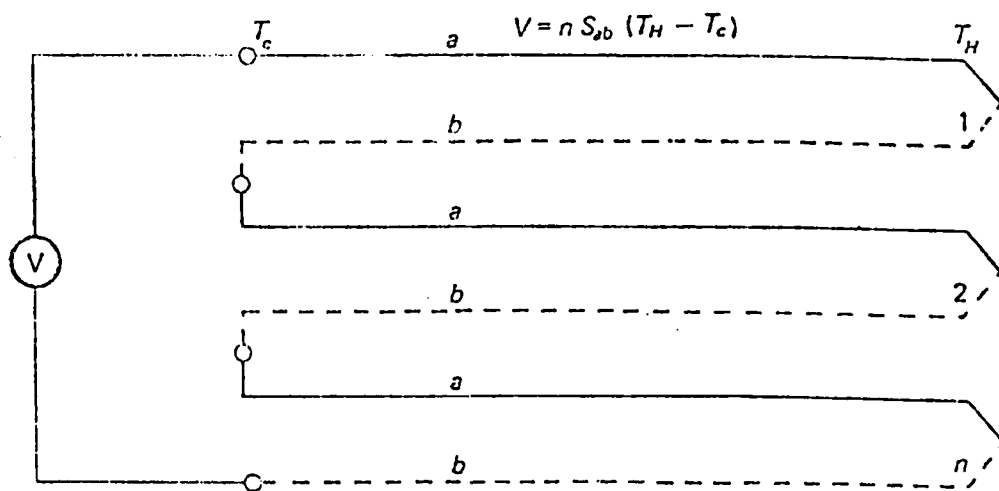


Figure 2.6 A Typical Thermopile Thermocouple

2.2.2 Principle of Operation

The governing principle of operation of RTD is that the metal resistance increases with increasing temperature. The electric resistance of the metal is expressed as (3)

$$R = \frac{r \times L}{A} \quad (2.4)$$

where

R = electric resistance (ohms)

r = resistivity (ohm.cm)

L = Length (cm)

A = cross sectional area (cm^2)

Metal selection for an RTD is usually based on practical resistivity, chemical inertness with respect to other constituents, large temperature coefficient (fractional change in resistance per $^{\circ}\text{C}$), and freedom from strain effects on resistance.

A standard equation that describes Pt resistance change with temperature is (4)

$$T = \frac{R_T - R_0}{R_{100} - R_0} \left[100 + \delta \left(\frac{T}{100} - 1 \right) \left(\frac{T}{100} \right) + \beta \left(\frac{T}{100} - 1 \right) \left(\frac{T}{100} \right)^3 \right] \quad (2.5)$$

where T = temperature, $^{\circ}\text{C}$

R_T = Pt resistance at temperature T

R_0 = Pt resistance at 0°C

R_{100} = Pt resistance at 100°C

δ = constant (characteristic of the particular thermometer)

β = constant (value is zero for $T > 0$)

Equation 2.5 was derived by M.S. Van Dusen as a modification to the initial version of Callender's equation. The new version (Equation 2.5) accounts for temperatures below the ice point.

2.2.3 Typical RTD's

There are several types and styles of RTDs depending on their uses and the method of the sensing wire support. A tradeoff between supporting the sensing wires and strain takes place. Unsupported elements are usually fragile and expensive.

One of the different types of the unsupported RTDs is shown in Figure 2.7. This RTD is called a bird-cage RTD where the sensing wires are allowed to move freely in a gas filled space. Partially supported elements designs are shown in Figures 2.8 and 2.9. Figures 2.10 and 2.11 show fully supported elements which proved to be rugged, inexpensive, and satisfactory for most industries.

Sensors that are directly exposed to a process are called immersion sensors. Such sensors consist of the sensing wires, insulators, (powders or cement), and a protective metallic sheath. Figure 2.12 shows a typical immersion sensor.

Sensors that are used to measure temperature of surfaces are called surface sensors. Those include clamped, bolted, or cemented sensors. Figure 2.13 shows two typical surface sensors.

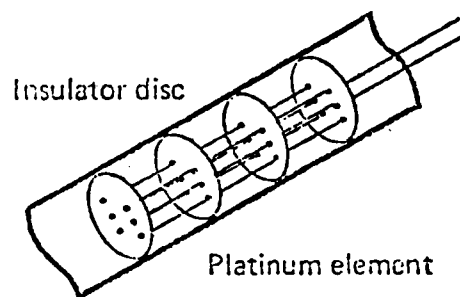


Figure 2.7 A Bird-Cage PRTD

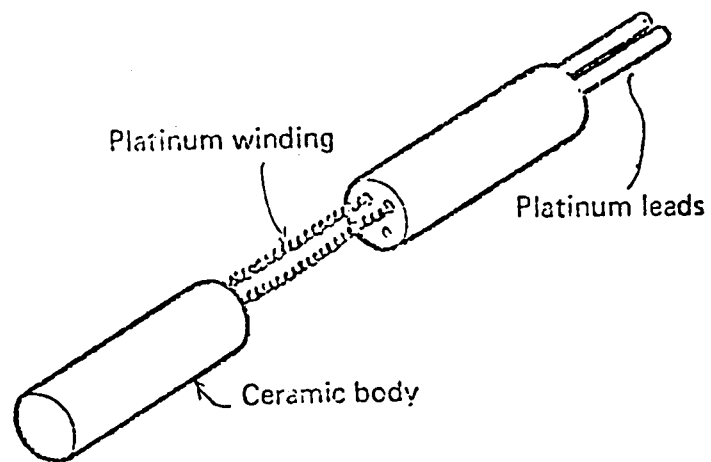


Figure 2.8 Partially Supported RTD

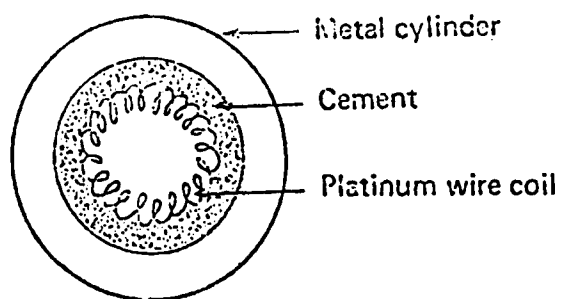


Figure 2.9 A Wall-Mount, Partially Supported RTD

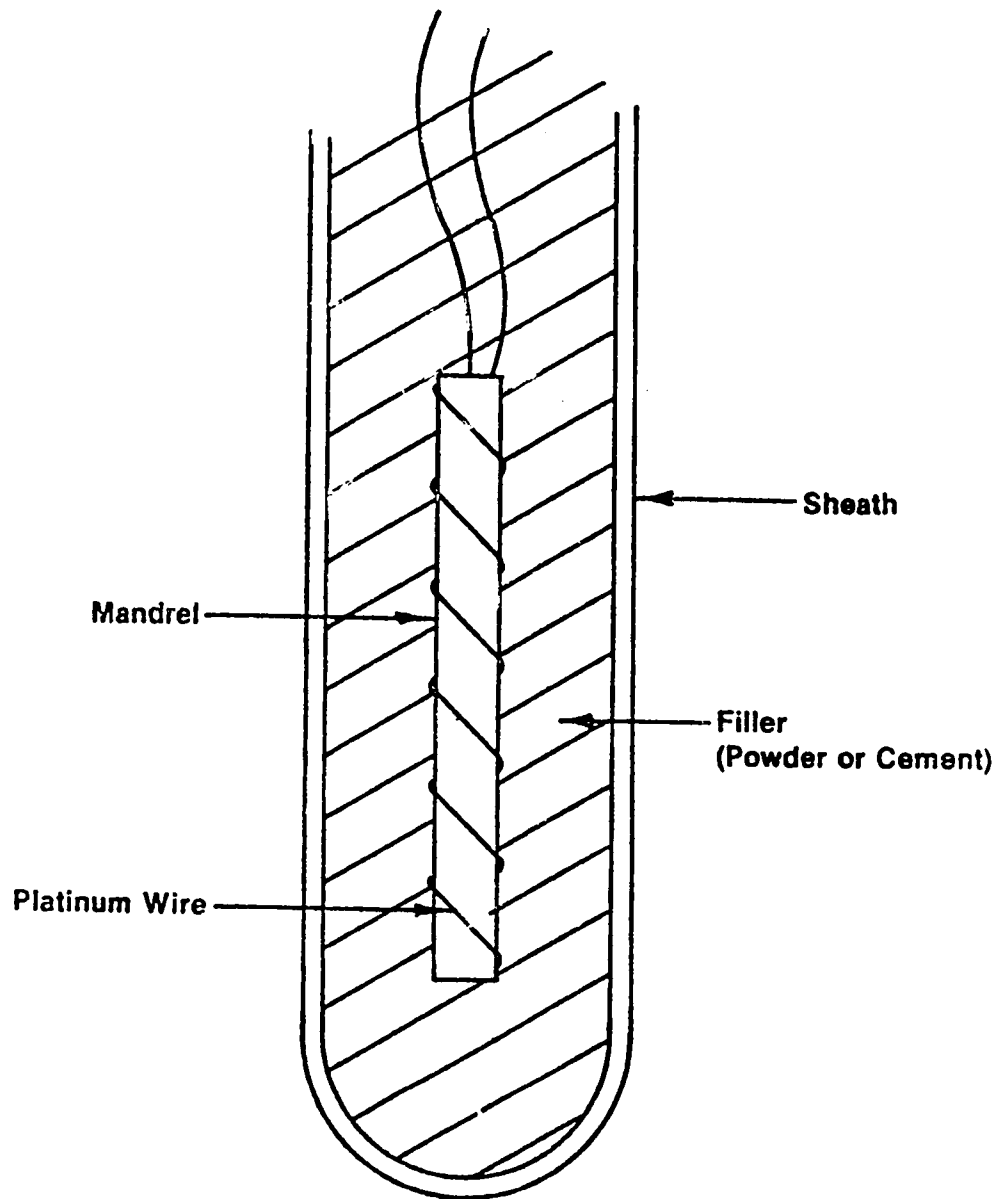


Figure 2.10 Fully Supported RTD
(Immersion Type Sensor)

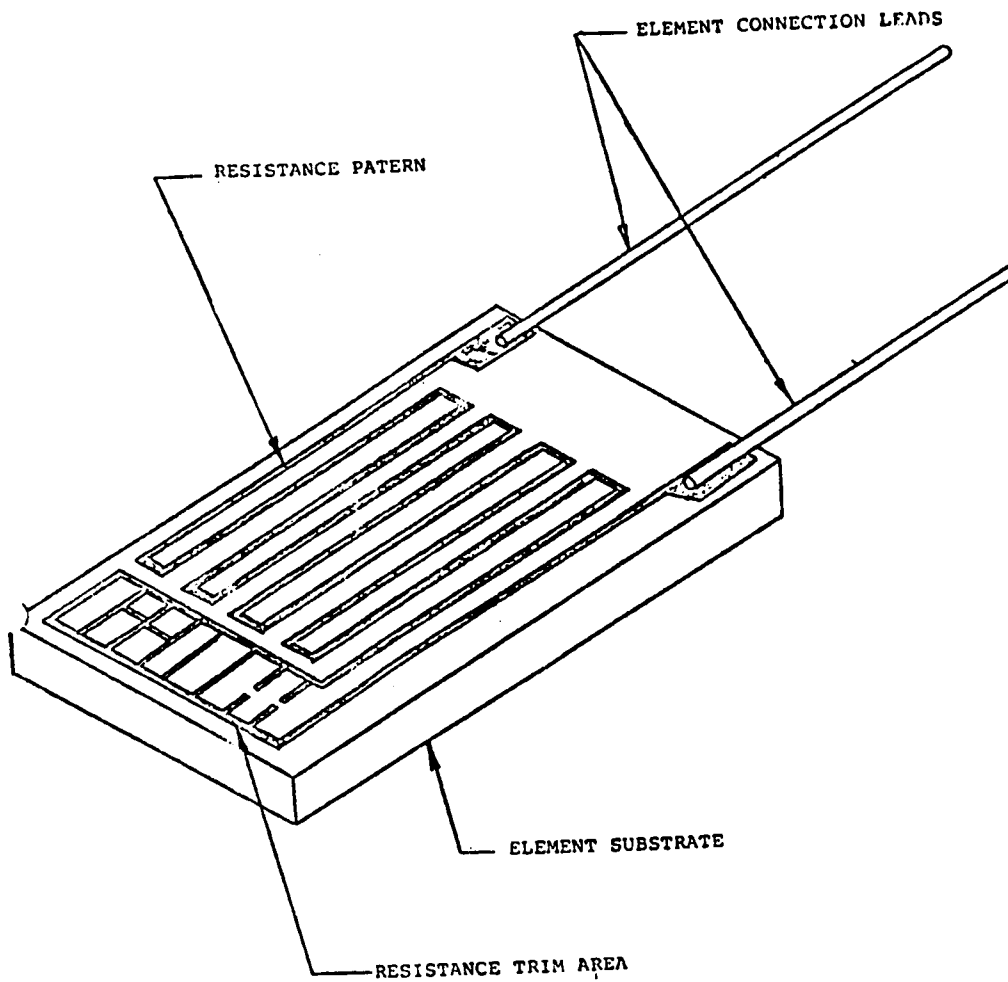


Figure 2.11 Fully Supported RTD

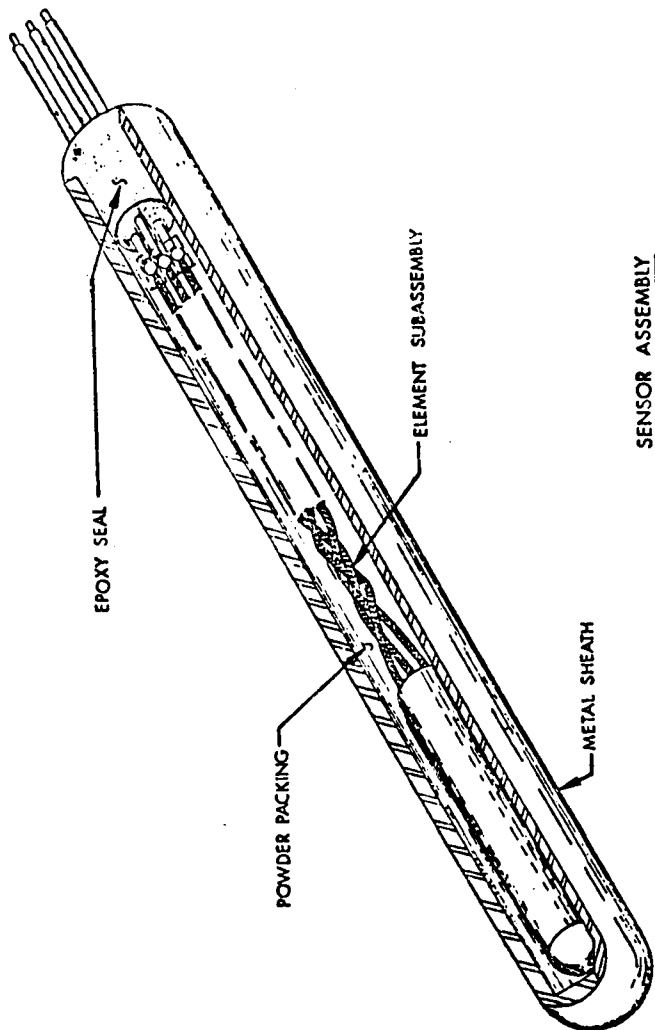


Figure 2.12 Typical Immersion Sensor

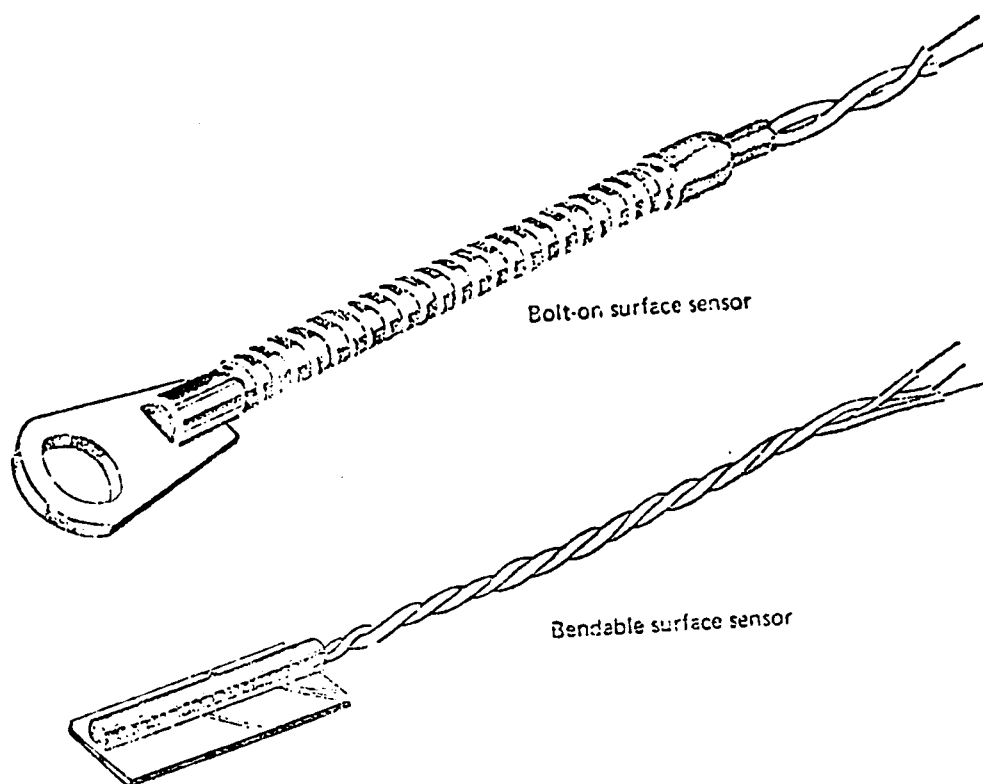


Figure 2.13 Typical Surface Sensors

2.2.4 Wheatstone Bridge

One way to measure the temperature of a medium using an RTD is by using a Wheatstone bridge, shown in Figure 2.14. This bridge provides a simple way to measure the RTD's resistance. R_{DB} can be varied until ΔV becomes equal to zero. Therefore

$$I R_{RTD} = I R_{DB}$$

where (2.6)

I = electrical current

R = electrical resistance

DB = decade box

also

$$I R_A = I R_B \quad (2.7)$$

Dividing Equation 2.6 by Equation 2.7 and rearranging, we get

$$R_{RTD} = \frac{R_A}{R_B} R_{DB} \quad (2.8)$$

and

$$R_{RTD} = R_{DB} \text{ if } R_A = R_B \quad (2.9)$$

Once the RTD's resistance is determined, reference tables can be used to determine the corresponding temperature.

To monitor the transient response of an RTD, one first initializes the voltage across the Wheatstone bridge. That is, make ΔV equal to zero by varying R_{DB} . Then, as the transient is started, one monitors ΔV which follows the relation (5)

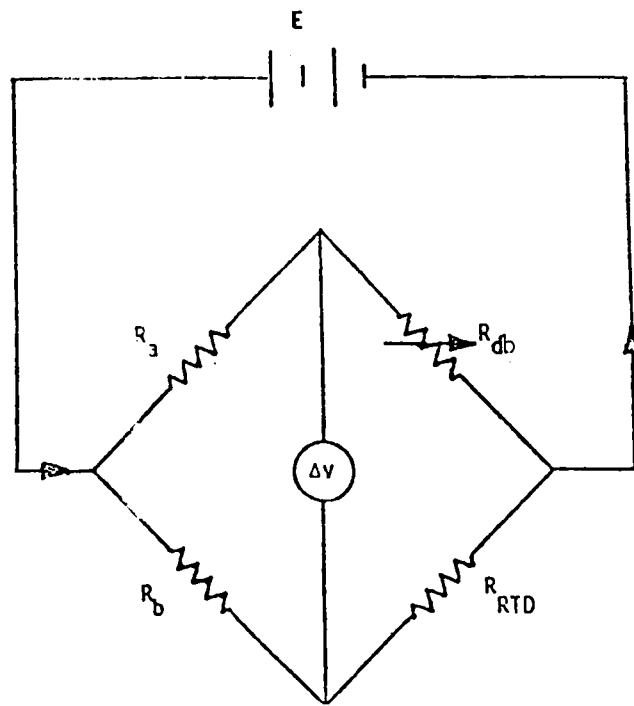


Figure 2.14 Wheatstone Bridge

$$\Delta V = E \left[\frac{R_R R_{RTD} - R_A R_{DB}}{(R_A + R_{RTD})(R_B + R_{DB})} \right]. \quad (2.10)$$

Equation (2.10) is nonlinear. However, the voltage drop across the bridge can be approximated by a linear function if R_B and R_{DB} are selected so that

$$\begin{aligned} &R_B > R_{RTD} \\ \text{and} \quad &R_{DB} \approx R_{RTD}. \end{aligned}$$

CHAPTER III

THEORY AND MATHEMATICAL DEVELOPMENT OF MODELS

3.1 Introduction

Development of modelling techniques based on numerical heat transfer analysis can be involved and complicated. A detailed three dimensional finite element approach is an example of such a complex technique. Therefore, it is essential to find a technique that is relatively inexpensive and adequate to provide useful information about the system under consideration. Such a technique is presented in this chapter. However, it is important to understand the theory behind modelling of industrial thermometers before describing the model.

3.2 Temperature Distribution Inside a Solid Homogeneous Cylinder

3.2.1 Introduction

Most industrial temperature sensors are cylindrically shaped. Usually, these sensors consist of several concentric layers of different materials. However, for illustrative purposes, the heat transfer process to or from a solid homogeneous cylinder is discussed.

Ideally, the temperature inside a sensor during steady state operation is uniform and equal to the monitored temperature. However, immersion sensors are often installed in a way that results in heat transfer to hotter or colder regions by conduction, convection or radiation heat transfer paths. This causes temperature gradients

inside the sensor. The internal temperature (where the sensing element is usually located) will differ from monitored temperatures.

When the monitored temperature undergoes a change, the sensor responds with an output that lags behind the monitored temperature. The lag is greater for positions farther from the surface of the sensor. Consequently, the sensor internal temperature varies with position and time following a change in monitored temperature. Figure 3.1 shows a calculated transient in a homogeneous solid cylinder following a step decrease in monitored temperature (3). Note that the temperature was uniform initially ($t=0$). After the change in monitored temperature the internal temperature starts to change, but the surface temperature changes faster than the center temperature. This gives a time dependent temperature profile that proceeds as follows:

flat - initially

skewed - during transient

flat - finally

3.2.2 Lumped Model

The solid homogeneous cylinder can be used as a crude representative of an actual cylindrical temperature sensor. In this representation, the sensor's internal constituents are represented by a single material with uniform properties. Moreover, a single temperature is assumed to describe the whole interior of the sensor. This is based on the assumption that the spatial variations are small inside the sensor.

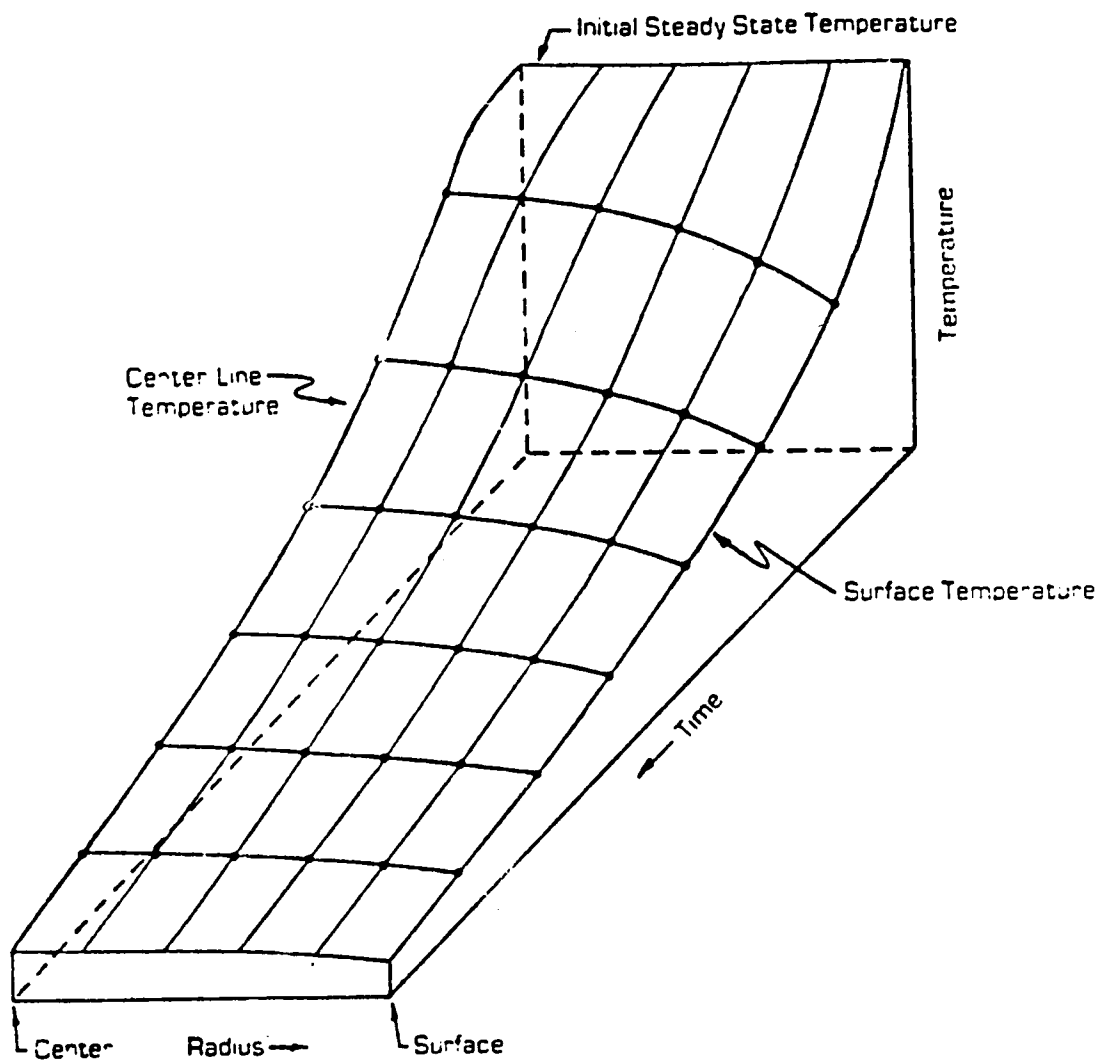


Figure 3.1 Temperature Within a Cylindrical Sensor Following a Step Decrease in Fluid Temperature

Such a simple representation is said to be a "lumped model" of the sensor.

The laws of heat conduction needed to develop a heat transfer model for a sensor are well known. The most exact model is the partial differential equation for heat conduction (6):

$$\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial t} - \dot{q}/k \quad (3.1)$$

where

T = temperature

t = time

$\alpha = k/\rho C$

k = thermal conductivity

ρ = density

C = specific heat capacity

$\dot{q}$ = volumetric heat generation

The following equation is an energy balance equation which is a simplification of Equation 3.1 for a lumped parameter model.

$$\frac{dE}{dt} = \dot{Q} - \frac{1}{R} (T - \theta)$$

or
$$\frac{dMC_p T}{dt} = \dot{Q} - \frac{1}{R} (T - \theta)$$

and
$$\frac{dT}{dt} = \frac{\dot{Q}}{MC_p} - \frac{1}{RMC_p} (T - \theta) \quad (3.2)$$

where

E = internal energy.

$\dot{Q}$ = volumetric heat generation.

M = mass of sensor.

C_p = specific heat capacity at constant pressure.

R = heat transfer resistance between the sensor
and the surroundings.

T = temperature of the sensor.

θ = medium temperature.

For no internal heat generation, we obtain

$$\frac{dT}{dt} = \frac{1}{RMC} (\theta - T) \quad (3.3)$$

The heat transfer resistance may be written,

$$R = \frac{1}{UA} \quad (3.4)$$

where

U = overall heat transfer coefficient
(internal and surface)

A = heat transfer area

Therefore,

$$\frac{dT}{dt} = \frac{UA}{MC} (\theta - T) \quad (3.5)$$

For a step change in surrounding temperature, this equation may be solved to obtain

$$\Delta T(t) = \Delta \theta [1 - e^{-(UA/MC)t}]. \quad (3.6)$$

The quantity, UA/MC , has the units of inverse time. It is usually expressed as a time constant as follows:

$$\tau = MC/UA. \quad (3.7)$$

Therefore, Equation 3.6 may be written as

$$\Delta T(t) = \Delta \theta (1 - e^{-t/\tau}) \quad (3.8)$$

where

$$\Delta T(\tau) = \Delta \theta (1 - e^{-1})$$

and

$$\Delta T(\tau) = \Delta \theta \times 0.632. \quad (3.9)$$

Equation 3.9 indicates that the response at $t = \tau$ after a step increase in monitored temperature is 63.2% of the total change. The step response for a lumped model is shown in Figure 3.2.

3.3 Multinode Model

3.3.1 Introduction

It is clear that a typical immersion TC or RTD has multilayers of different materials. For such geometry, solving Equation 3.1 to find the temperature distribution in the sensor becomes impractical. On the other hand, the lumped model is not good enough to describe the sensor response because the assumption that one temperature represents the whole interior of the sensor is not good for many practical situations.

An alternative may be numerical solutions of finite difference approximations to Equation 3.1. Such methods are capable of handling

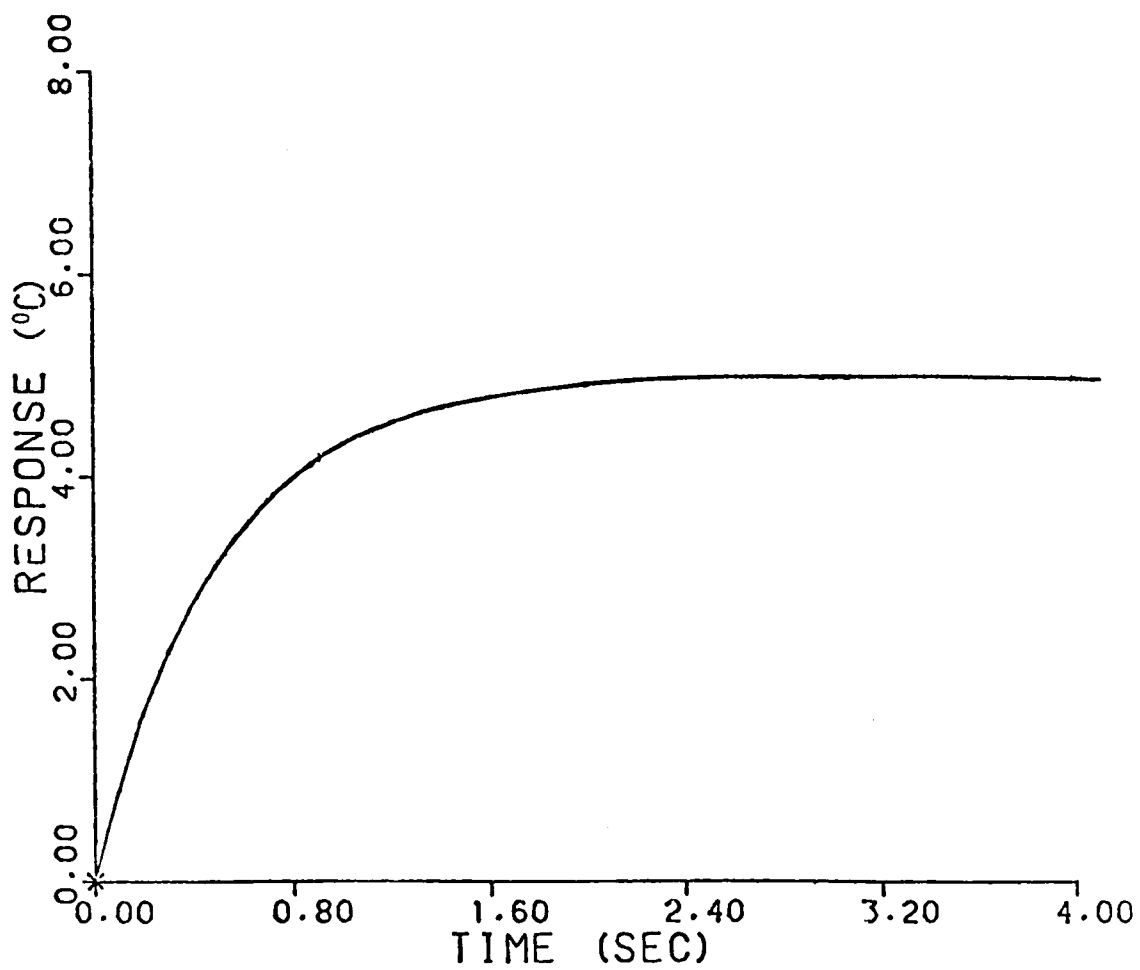


Figure 3.2 Typical Step Response Transient for a lumped Model

realistic geometries. This technique assumes that the sensor can be divided into many sections or "lumps." For each lump, one temperature is assumed to represent the whole lump. All lumps are interconnected via thermal resistances and have appropriate thermal capacitances.

3.3.2 Nodal Structure of a Cylindrical Sensor

The sensing element region of the sensor is usually the region of concern except for the case where stem loss analysis is concerned. Figure 3.3 shows a typical nodal structure of a sensor. The sensor is divided into radial and axial sections as shown in Figure 3.3. Each section is bounded by the dashed lines. This configuration is adequate if the sensor is symmetric around its axis. It is possible to have as many constituents as the number of nodes. Each node must consist entirely of one material. However, it is possible to have many nodes with the same material. The dimensions of the nodes are flexible. That is, the dimensions can be chosen to fit the modeller's requirements.

3.3.3 Energy Equations

To find the transient temperature history at the nodes of interest, dynamic energy balances are written for all the nodes. Every node is assumed to have a certain thermal capacitance and a certain thermal resistance between it and the adjacent nodes. An energy balance equation for a typical node i,j , shown in Figure 3.4, is of the form (7)

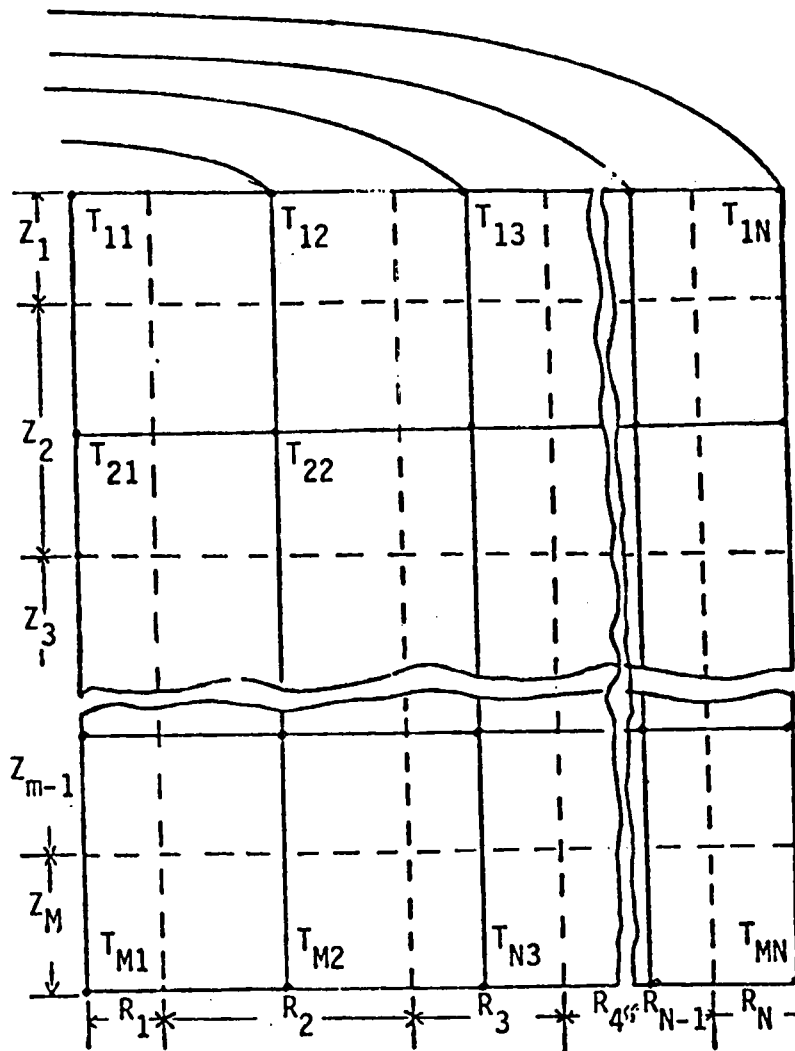


Figure 3.3 Schematic Diagram of a Multinodal Structure of a Cylindrical Sensor

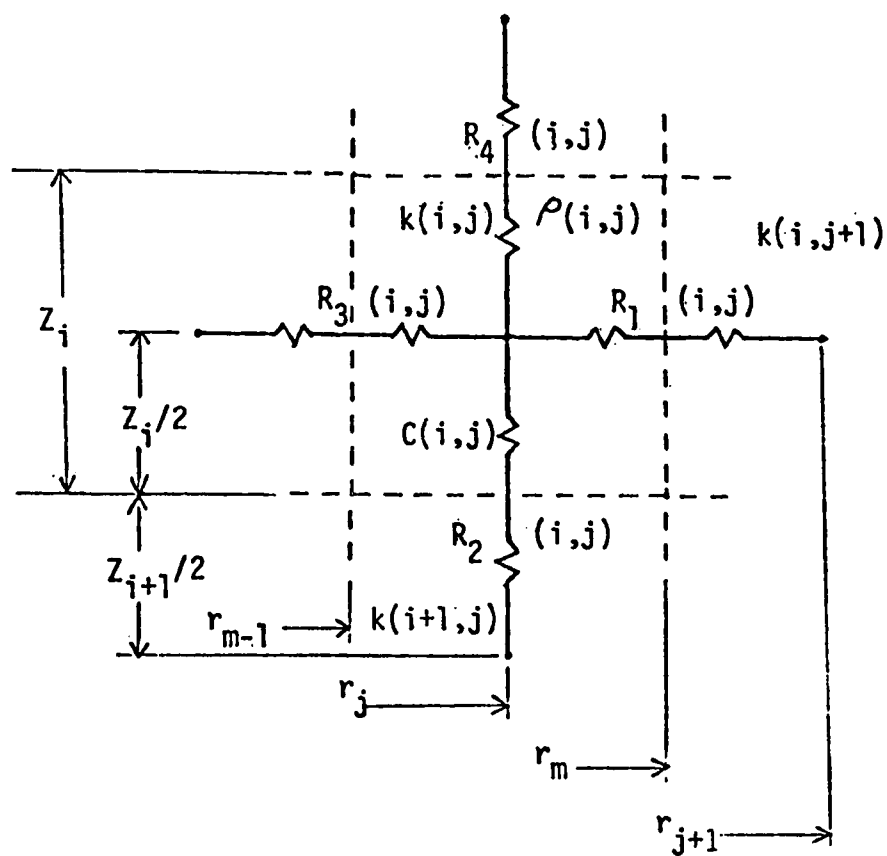


Figure 3.4 Thermal Resistances for an Internal Node

$$\left[\begin{array}{c} \text{rate of change} \\ \text{in stored energy} \end{array} \right] = \left[\begin{array}{c} \text{heat generated} \\ \text{in the node} \\ \text{energy out of} \\ \text{the node} \end{array} \right] + \left[\begin{array}{c} \text{energy into} \\ \text{the node} \end{array} \right] - \quad (3.10)$$

$$\frac{dE_{ij}}{dt} = \dot{Q} \times V + \left[\frac{T_{i,j+1} - T_{i,j}}{R_1(i,j)} + \frac{T_{i+1,j} - T_{i,j}}{R_2(i,j)} \right] - \left[\frac{T_{i,j} - T_{i,j-1}}{R_3(i,j)} + \frac{T_{i,i} - T_{i-1,j}}{R_4(i,j)} \right] \quad (3.11)$$

where

$E_{i,j} = MC_p T_{i,j}$ = energy stored in node i,j (J)

M = mass of material in node i,j (Kg)

C_p = specific heat constant (J/kg.°C)

$T_{i,j}$ = temperature of node i,j (°C)

$\dot{Q}$ = volumetric heat generation (W/m³)

V = volume (m³)

$R_1, R_2, R_3, R_4(i,j)$ = forward, downward, backward and upward thermal resistances (°C/W)

Assuming that the mass (m) and C_p are constants, and rearranging we get

$$\frac{dT_{i,j}}{dt} = \frac{1}{MC_p} \times \left\{ \left[\dot{Q} \times V + \frac{T_{i,j+1}}{R_1(i,j)} + \frac{T_{i,j-1}}{R_3(i,j)} + \frac{T_{i+1,j}}{R_2(i,j)} + \frac{T_{i-1,j}}{R_4(i,j)} \right] - \left[T_{i,j} \left(\frac{1}{R_1(i,j)} + \frac{1}{R_2(i,j)} + \frac{1}{R_3(i,j)} + \frac{1}{R_4(i,j)} \right) \right] \right\} \quad (3.12)$$

The term, MC_p is the thermal capacitance. The heat transfer resistance formulation is different for internal nodes and for surface nodes.

For internal nodes, the formulas are (6)

Forward Resistance (see Figure 3.4)

$$R_1(i,j) = \frac{(r_m - r_j)}{2\pi r_m Z_i k(i,j)} + \frac{(r_{j+1} - r_m)}{2\pi r_m Z_i k(i,j+1)} \quad (3.13)$$

If the material on both sides is the same, then

$$k(i,j) = k(i,j+1)$$

and

$$R_1(i,j) = \frac{(r_{j+1} - r_j)}{2\pi r_m Z_i k(i,j)} \quad (3.14)$$

where

$$k(i,j) = \text{thermal conductivity in node } i,j$$

Downward Resistance (see Figure 3.4)

$$R_2(i,j) = \frac{Z_i/2}{\pi(r_m^2 - r_j^2 - 1) k(i,j)} + \frac{Z_{i+1}/2}{\pi(r_m^2 - r_{j+1}^2 - 1) k(i+1,j)} \quad (3.15)$$

For surface node the formulas are

Side surface (see Figure 3.5)

$$R_1(i,j) = 1/h2\pi r_j Z_i \quad (3.16)$$

$$R_2(i,j) = \frac{Z_i/2}{\pi(r_j^2 - r_m - 1^2) k(i,j)} + \frac{Z_{i+1}/2}{\pi r_j^2 - r_m - 1^2) k(i+1,j)} \quad (3.17)$$

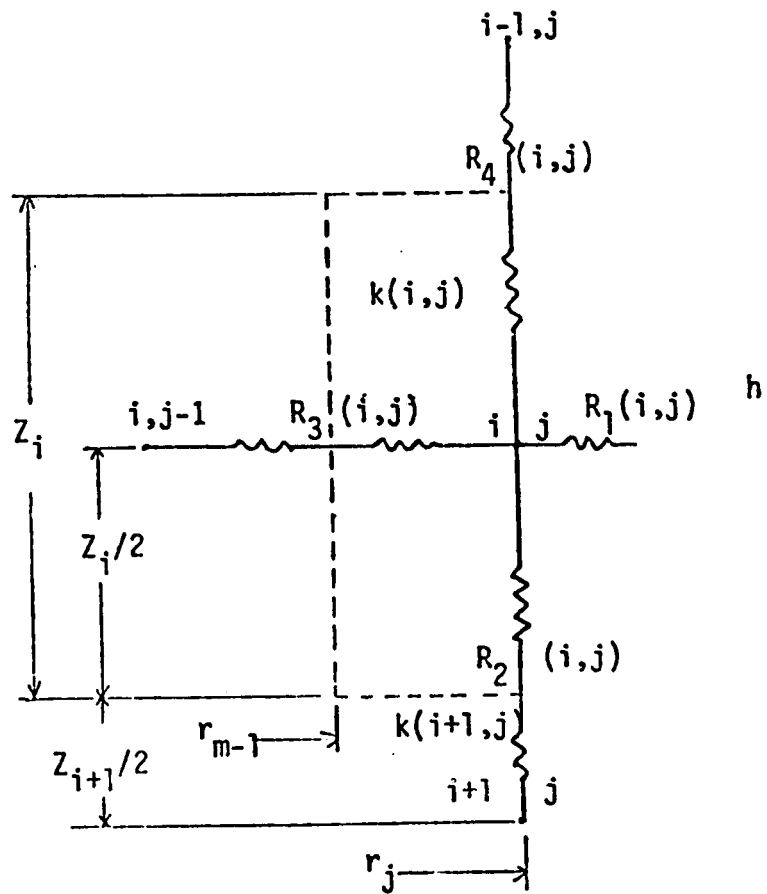


Figure 3.5 Thermal Resistances for Side Surface Node

Bottom Surface (see Figure 3.6)

$$R_1(i, j) = \frac{(r_m - r_j)}{2\pi r_m Z_i k(i, j)} + \frac{(r_{j+1} - r_m)}{2\pi r_m Z_i k(i, j+1)} \quad (3.18)$$

$$R_2(i, j) = 1/h\pi(r_m^2 - r_{m-1}^2) \quad (3.19)$$

Note

$$R_3(i, j) = R_1(i, j-1) \quad (3.20)$$

$$R_4(i, j) = R_2(i-1, j) \quad (3.21)$$

3.3.4 Forcing Function

Physically and mathematically, there are only two possible forcing functions. That is, there are two types of perturbation that one can apply to the sensor to cause a transient. The first can be introduced by changing the ambient temperature. A change in ambient temperature causes the sensor to depart from equilibrium and a transient can be detected on the sensor's output leads until it reaches a new steady state condition with the new thermal state.

The second perturbation can be introduced to the sensor by a variation in the heat generation inside the sensor. Heat can be generated at the sensor's sensing elements by simply heating those elements by passing a small electric current through the sensor's leads. This causes a transient in the sensor.

Both perturbations, heat generation change or ambient temperature change, can be arbitrarily selected functions of time. Examples of commonly used perturbations are a ramp, an impulse, or a step input.

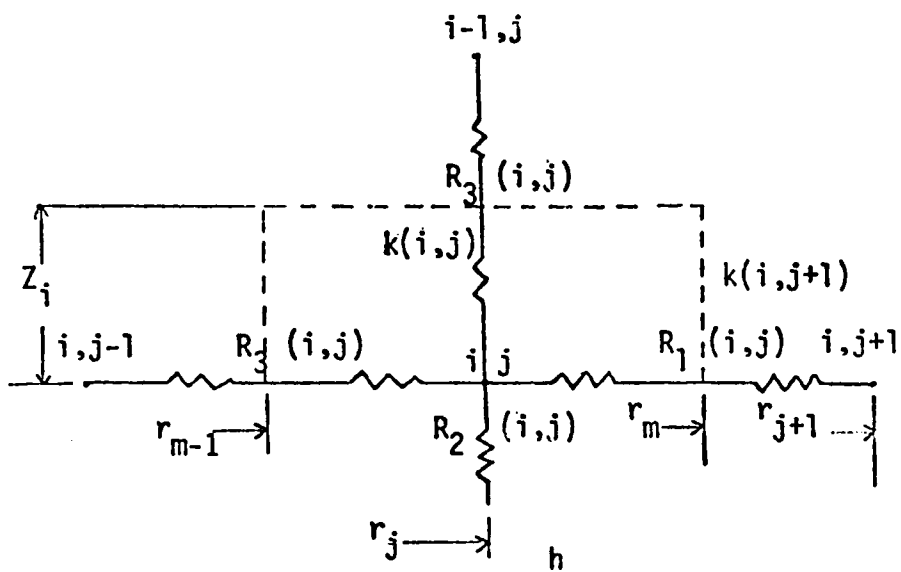


Figure 3.6 Thermal Resistances for an End Surface Node

However, the simplest and most commonly used test input is the step input. This can be accomplished experimentally if the sensor is moved rapidly from one medium to another at a different temperature to cause a step change in ambient temperature (called a "Plunge Test"), or a current is passed through the sensor leads then it is instantly changed to a different magnitude to cause a step change in heat generation rate (called the Loop Current Step Response Test (LCSR)).

3.3.5 Method of Solution

If the sensor is divided into M axial nodes and N radial nodes, then M x N coupled ordinary differential equations are needed to describe the sensor response. Once they are developed, they can be arranged in matrix form as (5)

$$\frac{dT}{dt} = \dot{T} = A T + F \quad (3.22)$$

where

$\dot{T}$ is (M x N) x 1, where each element contains the time derivative of the temperature.

A is (M x N) X (M x N) state matrix, contains five diagonal stripes. The non-zero elements contain the thermal resistances and capacitances.

T is (M x N) x 1 vector containing the temperatures at the center of all nodes.

F is (M x N) x 1 forcing vector, contains all the forcing terms.

Once the state variable matrix is developed and the forcing vector is generated, then the time response of the sensor may be predicted by solving the state variable equations for the temperature at

the center of each node as a function of time.

The solution for the system represented by Equation 3.22 is (8)

$$\underline{T}(t) = e^{At}T(0) + \int_0^t e^{A(t-\tau)}F(\tau)d\tau \quad (3.23)$$

This is the basis for the matrix exponential approach to solve a set of differential equations. If the forcing function ($F(\tau)$) is constant, then Equation 3.23 becomes (8)

$$\underline{T}(t) = e^{At}T(0) + e^{At}A^{-1}F - A^{-1}F \quad (3.24)$$

where

$$e^{At} = I + At + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \dots \quad (3.25)$$

3.3.6 Estimation of τ Using LCSR Test Results

In practice, the sensor response to variations in the ambient temperature is more important than its response to variations in heat generated inside the sensor. Hence, it is important to transform data obtained by imposing a step increase in the heat generated at the sensing element, as in the LCSR test, into data that would be obtained in a corresponding plunge test, which is obtained by causing the sensor to respond to a step change in ambient temperature. This is done to obtain an estimate of the time constant of the sensor when the LCSR test is performed. In a plunge test, the sensor response in the Laplace domain(s) is of the form (1)

$$T(s) = \frac{K}{s(s-P_1)(s-P_2)\dots} \quad (3.26)$$

Taking the inverse Laplace transform, we get

$$T(t) = A_0 + A_1 e^{p_1 t} + A_2 e^{p_2 t} + \dots \quad (3.27)$$

where p_i are the poles of the transfer function. They have negative values.

In the LCSR test, the sensor response in the s-domain is of the form

$$T(s) = \frac{K(s-Z_1)(s-Z_2)\dots}{s(s-P_1)(s-P_2)\dots} \quad (3.28)$$

In the time domain, the response becomes

$$T(t) = B_0 + B_1 e^{p_1 t} + B_2 e^{p_2 t} + \dots \quad (3.29)$$

It must be noted that the poles in both Equations 3.27 and 3.29 are the same. Hence if one can estimate the poles in Equation 3.28 then Equation 3.27 can be constructed easily.

There are two methods to determine the poles of LCSR test employed at The University of Tennessee. These are

1. computer optimization routines
2. graphical exponential stripping

In the first method, the sensor's signal obtained in the LCSR test is digitized and fed to microcomputer. The microcomputer is programmed to fit a function of the form

$$f(t) = a_0 + a_1 e^{p_1 t} + a_2 e^{p_2 t} \quad (3.30)$$

through the data points. This task requires a combined nonlinear functional minimization and least-squares fit techniques to be used to identify all the eigenvalues in Equation 3.30.

In the second method, the sensor response is approximated by (5)

$$O(t) = A_0 + A_1 e^{-p_1 t} + A_2 e^{-p_2 t} + w(t) \quad (3.31)$$

where

$$p_1 < p_2$$

A_0 = a constant = 0, if the test bridge is balanced at high current, and = the final value if the bridge is balanced at low current.

$w(t)$ = process noise

let

$$y(t) = O(t) - A_0 = A_1 e^{-p_1 t} + A_2 e^{-p_2 t} + w(t) \quad (3.32)$$

If the noise level is not significant, and the eigenvalues are widely separated, then for $t \gg 1/p_1$, the sensor response is dominated by $e^{-p_1 t}$ term, that is

$$y(t) \approx A_1 e^{-p_1 t} \text{ for } t \gg 1/p_1 \quad (3.33)$$

and

$$y(t) - A_1 e^{-p_1 t} = A_2 e^{-p_2 t} \text{ for } t \ll 1/p_1 \quad (3.34)$$

It follows from Equations 3.33 and 3.34 that a semilog plot of the sensor response versus time results in a skewed portion then a straight line portion. The straight line has a slope of p_1 . Subtracting the straight line (extrapolated to the initial time of the

transient) from the skewed portion results in new skewed and straight lines. The slope of the new straight line is equal to P_2 . Physically two poles are identifiable and adequate. Theoretically, one identifies n ($n = 1, 2, 3..$) poles.

Once at least two poles are identified, the residue theorem can be applied to Equation 3.26 in order to determine the constants (A_i 's). Then the plunge time constant can be determined by using Equation 3.27, or by using the correlation (5)

$$\pi_{PL} = \frac{1}{-P_1} \left[1 - \ln\left(1 - \frac{P_1}{P_2}\right) - \ln\left(1 - \frac{P_1}{P_3}\right) \cdots \right] \quad (3.35)$$

The limitation of not having the ability to identify more than two (or sometimes three) poles, and hence, some information about the sensor response is lost, led to the development of a correction factor (CF). It is defined as the factor which "an 'n' order fit result must be multiplied by to obtain the result from an infinite order fit "

(1). That is

$$CF = \frac{1 - \sum_{i=1}^2 \ln(1 - P_1/P_i)}{1 - \sum_{i=1}^n \ln(1 - P_1/P_i)}, \quad n=1,2,\dots,\infty \quad (3.36)$$

Extensive studies performed at UT concluded that the first two poles contain information about all the poles (1). A fifth order polynomial fitted through a curve that represented the theoretical CF versus pole spacing resulted in the following formula (1).

$$CF = 1.0043 + .05578(P_1/P_2) + 19.590 (P_1/P_2)^2 - 238.38(P_1/P_2)^3 + 1352.2(P_1/P_2)^4 - 2622.9(P_1/P_2)^5 \quad (3.37)$$

The correction factor as represented by Equation (3.37) appears to be valid for all cylindrical sensors where the LCSR method can be applied. Therefore, Equation 3.30 can be rewritten as

$$\tau = CF \left[\frac{1}{-P_1} \left[1 - \ln \left(1 - \frac{P_1}{P_2} \right) \right] \right] \quad (3.38)$$

where

CF is given by equation (3.37).

3.3.7 The SENSOR Program

The steady state or dynamic heat transfer analysis of a sensor should be possible for those interested in sensors or processes as well as those interested in heat transfer or simulation. This means that the computer software should be "user friendly." The analyst should be concerned with sensor performance, not heat transfer or computer programming or numerical analysis. A program was prepared with that objective as a design goal.

The program, called SENSOR, is implemented on the DEC-10 system. It is written in the BASIC computer language. This makes the program suitable for implementation on small microcomputers such as the Commodore 64. However, small modifications, such as the size of the dimension statement, may be required to accommodate the limited memory on such microcomputers.

SENSOR operates in two steps. In the first step, the A matrix of Equation 3.22 is generated with user-specified data about the sensor. The program is menu-driven, That is, the program asks the user for all necessary information. The coefficients in the matrix are then generated automatically. The data requested by the program are as follows (7):

1. Dimensions.
2. Physical properties of constituents.
3. Sensing element location.
4. Heat transfer coefficient or properties of surrounding medium.
5. Type of forcing function, either a step increase in ambient temperature or a step increase in the heat generation term.
6. The time step and number of time steps.

The second step in the analysis is to solve Equation 3.22 for the specified A matrix and the specified calculational time increment. DEMAT(8), a general purpose program that finds the response of a system with a constant forcing function using the matrix exponential approach, was used. The computer prints out the temperature at the center of each node and the average temperature of all nodes specified as the sensing element nodes.

A typical result is presented here for illustrative purposes. The sensor selected for simulation is a .236 inch outside diameter sensor with a .0197 inch thick 446 stainless steel sheeth and 0% porosity magnesium oxide insulator. The medium around the sensor is flowing water. The structure selected for the analysis is shown in

Figure 3.7. The actual input data in the order entered in the computer is presented in Table 3.1. The SENSOR output was used to find the time constant of the model due to a 5°C step increase in ambient temperature. The response is shown in Figure 3.8. The time constant was found to be 1.190 seconds.

The same sensor was simulated by a 6×6 order model. The time constant was found to be 1.198 seconds. Therefore, changing the model order from 4×4 to 6×6 has resulted in a small (.67%) increase in the time constant. Considering the computation cost involved and the small improvement in the time constant lead to the conclusion that 4×4 and 5×5 order models are adequate for simulating sensor response.

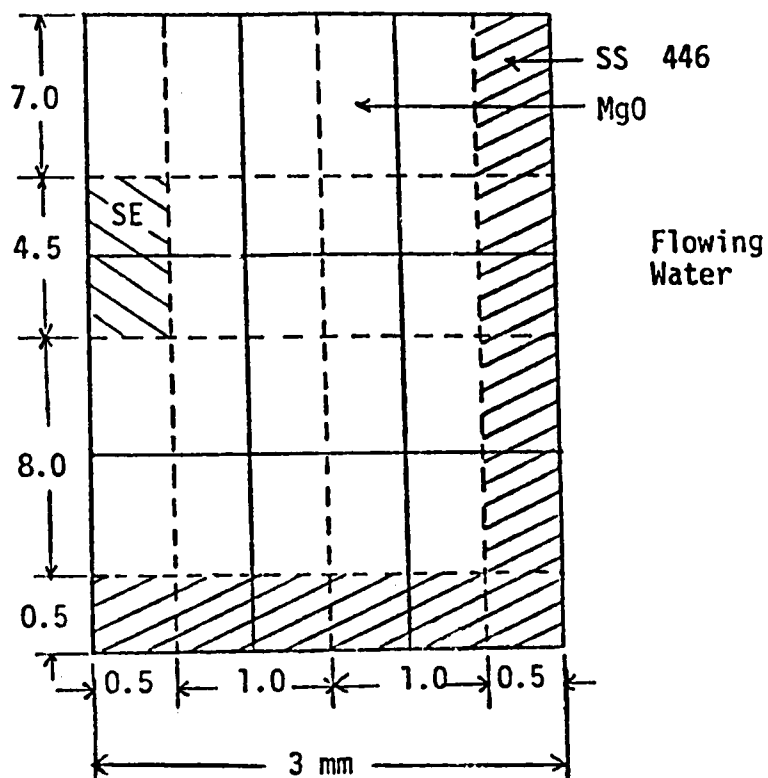


Figure 3.7 Nodal Arrangement for a Hypothetical Sensor

Table 3.1

Input Data for the Sensor Under Study

Input	Comments
N	The user does not want to see instructions
4	Number of radial sections
4	Number of axial sections
.05	Width of the first radial node (cm)
.1	Width of the second radial node (cm)
.05	Width of the fourth radial node (cm)
.7	Height of the first axial node (cm)
.45	Height of the second axial node (cm)
.8	Height of the third axial node (cm)
.05	Height of the fourth axial node (cm)
2	Number of the sensor constituents
38.51	Thermal conductivity of material 1 (MgO) (W/M.°C)
4000	Density of material 1 (kg/m ³)
786.97	Specific heat constant of material 1 (J/kg.°C)
1	Number of regions occupied by material 1
1,1	i and j to specify the top left node of region 1 material 1
3,3	i,j for the bottom right node of region 1 for material 1
20.32	Thermal conductivity of material 2 (SS 460) (W/M.°C)
7600	Density of material 2 (kg/m ³)
460.6	Specific heat constant of material 2 (J/kg.°C)
2	Number of regions occupied by material 2
1,4	i,j for the top left node of region 1 of material 2
4,4	i,j for the bottom right node of region 1 of material 2
4,1	i,j for the top left node of region 2 of material 2
4,3	i,j for the bottom right node of region 2 of material 2
2,1	i,j for the location of sensing element node
1	Indicates that the forcing function is a step increase in ambient temperature as opposed to 2 for internal heat perturbation
N	User doesn't wish to have the program calculate the heat transfer coefficient
5000	The heat transfer coefficient (W/M ² .°C)
N	No air gaps exist
.005	Time step (sec)
500	Number of time steps

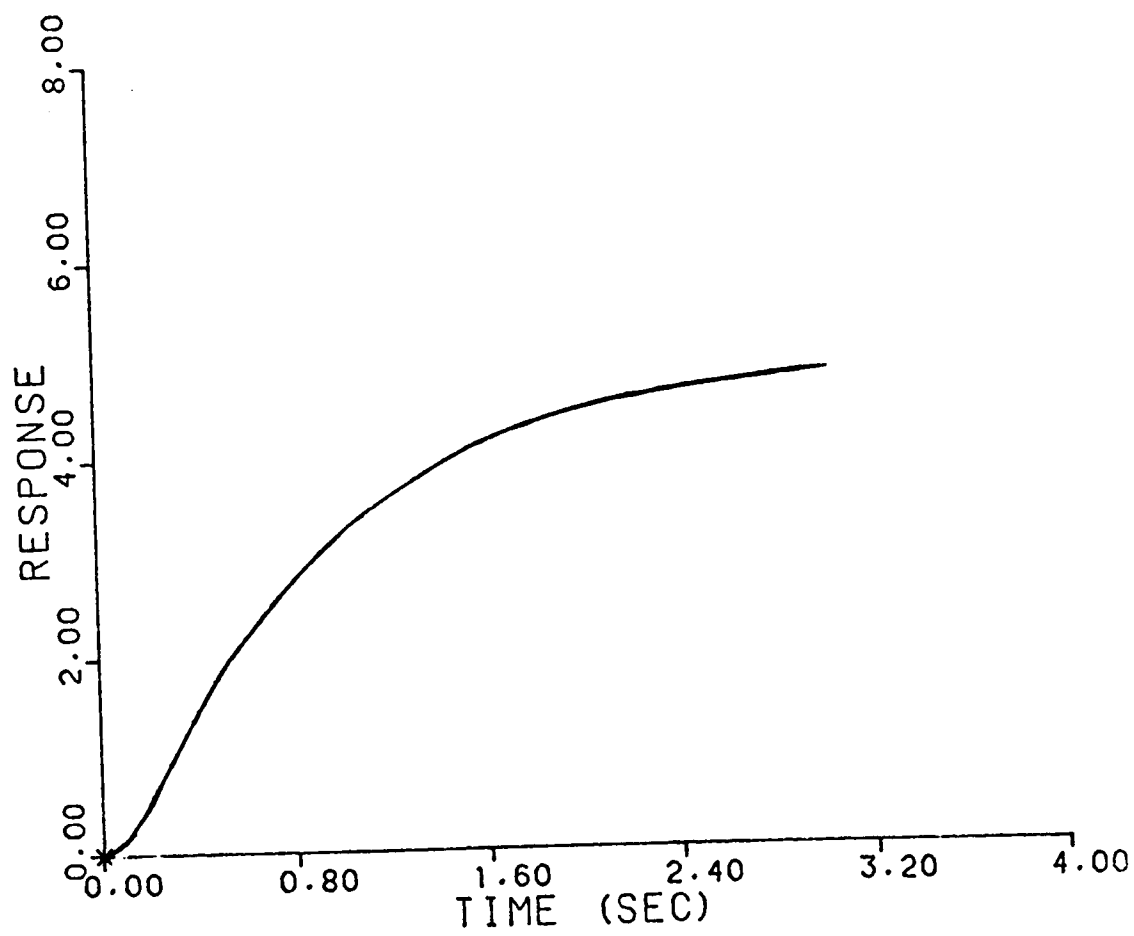


Figure 3.8 Typical Step Transient Response for the Hypothetical Sensor.

CHAPTER IV

THEORETICAL PREDICTION OF THE TIME CONSTANT

4.1 Introduction

The time constant of a temperature sensor may depend greatly on the ambient conditions. Thus, laboratory tests performed at standard conditions (i.e. water flowing at 1 m/s and at temperature of 20 or 80°C) may not be adequate to describe sensors installed in processes.

To overcome this problem, a theoretical correlation that indicates the time constant of the sensor as a function of the film heat transfer coefficient was developed. The correlation takes the form (3,9)

$$\tau = C_1 + C_2/h \quad (4.1)$$

where

τ = time constant of the sensor

C_1 = constant, internal component

C_2/h = external component

h = film heat transfer coefficient

C_2 = constant.

For a lumped parameter model, the constants C_1 and C_2 are

$$C_1 = \frac{\rho C r_o^2}{2k} \ln(r_o/r_i) \quad (4.2)$$

$$C_2 = \frac{\rho C r_o}{2} \quad (4.3)$$

For a distributed parameter model for a homogenous cylinder, C_1

and C_2 are given by

$$C_1 = .24 \frac{\rho C r_o^2}{k} \quad (4.4)$$

$$C_2 = \frac{\rho C r_o}{2} \quad (4.5)$$

where

ρ = density of the sensors material

C = specific heat capacity of the sensors material

k = thermal conductivity of the sensors material

r_o = outer radius of the sensor

r_i = inner radius at which the sensing elements are located.

The complete derivation of Equation 4.1 is shown in appendix A.

The assumptions involved in the derivation of Equation 4.1 are

1. the sensor is a cylinder
2. only radial heat transfer exists.

The constants C_1 and C_2 can be determined experimentally. This is done by measurements for a range of time constants (τ_i) and a corresponding range of calculated film heat transfer coefficients (h_i). The film heat transfer coefficient h can be changed by changing the fluid itself, or the velocity of the fluid. The constants C_1 and C_2 can then be obtained by finding the least-squares fit through the points ($1/h_i$, τ_i). Then Equation 4.1, with known C_1 and C_2 , can be used to predict the time constant of the sensor under actual process operating conditions.

It is obvious that C_1 and C_2 , as represented in Equations 4.2, 4.3, 4.4, and 4.5, depend on the physical properties of the sensor constituents which in turn depend on the temperature of the medium surrounding the sensor. Thus C_1 and C_2 cannot be used if the process temperature is different from the laboratory test temperature.

4.2 The Film Heat Transfer Coefficient

The validity of the prediction of the time constant, τ , using Equation 4.1 depends on using the best heat transfer correlation to calculate the film heat transfer coefficient, h . Therefore, a comprehensive search for heat transfer correlations that describe convection from cylinders in cross flow was pursued.

The following correlations meet the objective:

(a) Davis (1924) (10)

$$Nu = .56 Re^{.49} Pr^{.33}$$

for $18 < Re < 170$

(b) Perkins and Leppert (1962) (11)

$$Nu = .26 Re^{.5} Pr^{1/3}$$

for $40 < Re < 10^5$

(c) Fand (1965) (12)

$$Nu = .32 + 0.505 Re^{.52} Pr^{1/3}$$

for $1.1 \times 10^4 < Re < 6.3 \times 10^4$

Morgan (1975) has refitted Fand's data and obtained (10)

$$(c') \quad Nu = .455 Re^{0.53} Pr^{1/3}$$

(d) Fand and Keswani (1973) (13)

$$\text{Nu} = .231 + .633 \text{ Re}^{0.5} \text{ Pr}^{1/3}$$

for $4 \times 10^2 < \text{Re} < 3.7 \times 10^3$

(d') Morgan (1975) (10)

$$\text{Nu} = .665 \text{ Re}^{.494} \text{ Pr}^{1/3}$$

(e) Churchill and Bernstein (1977) (14)

$$\text{Nu} = .3 + .62 \text{ Re}^{.5} \text{ Pr}^{1/3} \left[1 + \left(\frac{\text{Re}}{282000} \right)^{5/8} \right]^{4/5}$$
$$/ [1 + (.4/\text{Pr})^{2/3}]^{1/4}$$

for $10^2 < \text{Re} < 10^7$

(f) Rohsenow and Choi (1961) (15)

$$\text{Nu} = 0.26 \text{ Re}^{0.6} \text{ Pr}^{0.3}$$

for $1000 < \text{Re} < 50000$

and

$$\text{Nu} = (.35 + .47 \text{ Re}^{.52}) \text{ Pr}^{0.3}$$

for $0.1 < \text{Re} < 1000$

(g) R. Helpert (1933) (16)

$$\text{Nu} = C \text{ Re}^n$$

where

C, n, and Re ranges are indicated in Table 4.1.

In all the correlations, the variables are

$$\text{Nu} = \text{Nusselt number} = dh/k$$

d = diameter of the cylinder

h = film heat transfer coefficient

k = thermal conductivity of the liquid

TABLE 4.1

n and c Values for the Helpert Correlation

Re	N	C	
		Gases	Liquids
1-4	0.330	0.891	$0.989(\text{Pr})^{1/3}$
4-40	0.385	0.821	$0.911(\text{Pr})^{1/3}$
40-4000	0.466	0.615	$0.683(\text{Pr})^{1/3}$
4000-40000	0.618	0.174	$0.193(\text{Pr})^{1/3}$
40000-250000	0.805	0.0239	$0.0266(\text{Pr})^{1/3}$

$$Re = \text{Reynolds number} = \frac{u d \rho}{\mu}$$

u = fluid velocity

ρ = fluid density

μ = fluid dynamic viscosity

$$Pr = \text{Prandtl number} = C \mu / k$$

C = specific heat capacity

In order to choose the correct correlation, one has to determine the Reynolds number for the process.

Calculating the Reynolds number depends on accurate knowledge of the physical properties of the fluid. A list of such physical properties at room temperature for five typical liquids is presented in Table 4.2. More detailed information on the thermophysical properties of those liquids is presented in Appendix B.

For illustrative purposes the film heat transfer coefficient was calculated for a 6.35 mm RTD tested in all five liquids with the physical properties presented in Table 4.2. The Churchill-Bernstein correlation was used to calculate h . The sensor was tested at different velocities in each liquid in a testing facility called "The Rotating Tank." A wide range of h values (from 300 to 5500 W/M².°C) has been obtained. In each test, the time constant of the sensor was obtained. (The details of the experimental facility and the testing procedures are discussed in the following chapter.) A plot of the time constant of the sensor versus $1/h$ is shown in Figure 4.1. It is obvious that τ is linearly related to $1/h$. The least square fit of a straight line through the points resulted in the

Table 4.2

Thermophysical Properties of Five Liquids at Room Temperature (6,17)

Liquid	Thermal Conductivity (k)	Specific Heat Constant (Cp)	Dynamic Viscosity (μ)	Density (ρ)
	W/m ⁰ C	J/kg./ ⁰ C	kg/m.s	kg/m ³
Water	.604	4179.0	.001	997.4
Engine Oil	.145	1880.0	.7994	888.23
Ethylene Glycol	.249	2382.0	.02 142	1116.65
Heptane	.125	2242.0	.0004	684.0
Carbon Tetrachlo- ride	.104	862.0	.000969	1585.0

H = (K/D) * (.3 + .62 * RE **.5 * PR **.33 * ...)
 SENSOR IS : 6.35 MM RTD

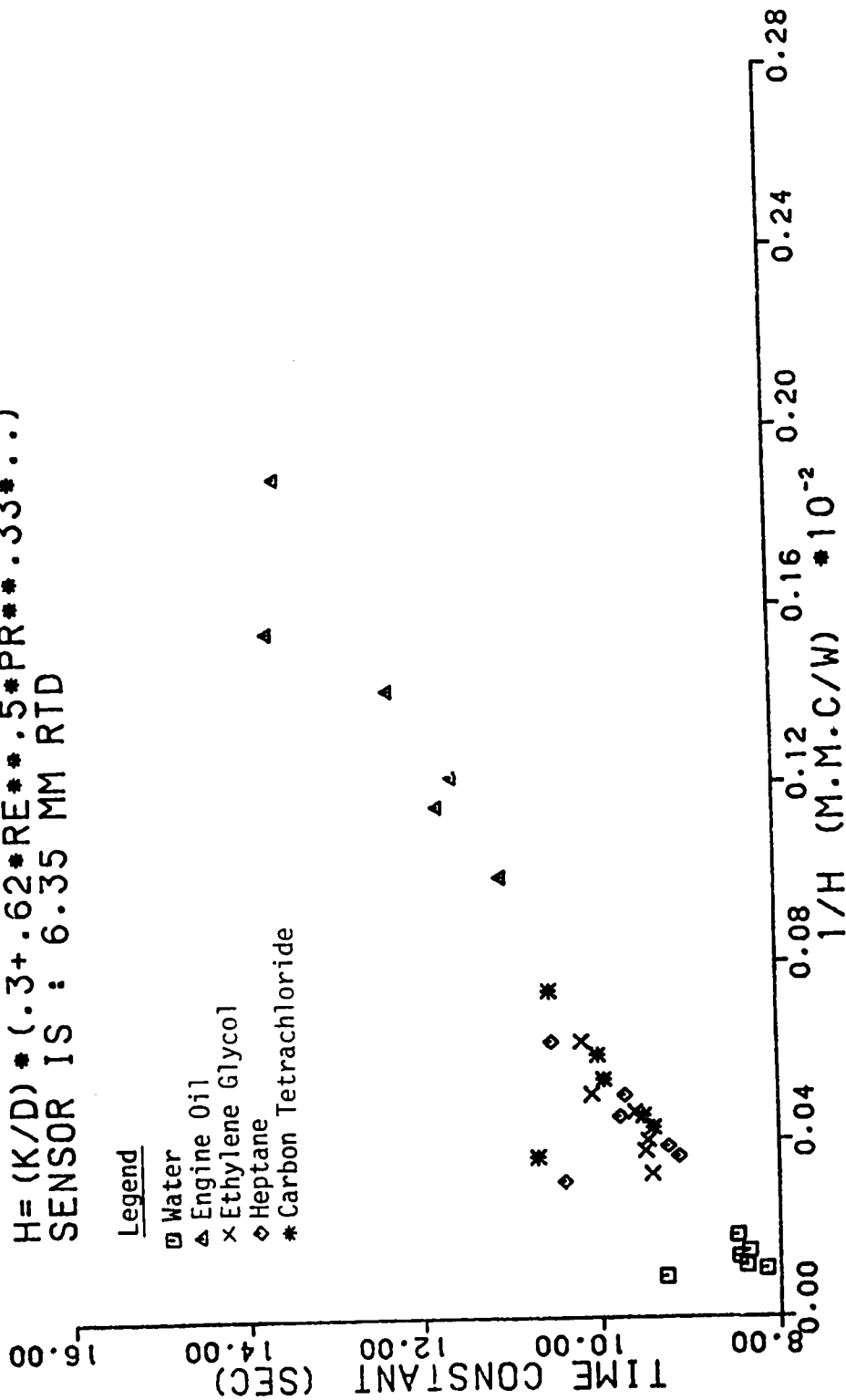


Figure 4.1 Relationship Between the Time Constant and 1/h for a 6.35 mm RTD

following equation

$$\tau = 2.10 + 3090/h \quad (4.6)$$

Therefore, for this sensor

$$C_1 = 2.1$$

$$C_2 = 3090$$

4.3. Fluid Velocity

It is possible to simplify Equation 4.1 to indicate the time constant as a function of the fluid velocity. This is done by substituting an expression for h into Equation 4.1. For example, if the Reynolds number for the process fluid lies within the range 40-4000, and Helpert's formula was found to be adequate to describe the convection process, then

$$Nu = .683 \left(\frac{d_o u \rho}{\mu} \right)^{.466} Pr^{1/3}$$

results in the following heat transfer coefficient

$$h = \frac{k}{d_o} Nu = \frac{k}{d_o} .683 \left(\frac{d_o u \rho}{\mu} \right)^{.466} Pr^{1/3}$$

or

$$h = .683 d_o^{-.534} \rho^{.466} c^{.333} k^{.667} \mu^{-.133} u^{.466}$$

which is of the form

$$h = c u^{.466} \quad (4.7)$$

where

$$c = \text{constant} = .683 d_0^{-.534} \rho^{.466} c^{.333} k^{.667} \mu^{-.133}$$

substituting Equation 4.7 into Equation 4.1 results in

$$\tau = C_1 + C_2 / C u^{.466}$$

or

$$\tau = C_1 + C_3 u^{-.466} \quad (4.8)$$

where

$$C_3 = C_2 / C$$

In general, it is possible to obtain a correlation of the form

$$\tau = C_1 + C_3 u^{-n} \quad (4.9)$$

where n = the power of Re number in the heat transfer correlation.

CHAPTER V

TESTING FACILITIES

A number of tests were performed on temperature sensors to check correlation 4.1 and obtain a relationship between τ and the flow velocity. Several problems were encountered in this process. Attempts to resolve those problems were made. In the following sections, the testing facilities and the types of tests performed are described. Discussion of the problems encountered are also presented in this chapter.

5.1 Introduction

Based on the nature of correlation 4.1, it was necessary to construct an experimental apparatus that allowed for obtaining a wide range of heat transfer coefficients (h). Such a wide range of values of h was achieved by varying

1. the velocity of the fluid
2. the fluid itself
3. the sensor

Three experimental facilities were constructed at the University of Tennessee, Knoxville. Several experimental procedures can be followed to test temperature sensors in each facility.

5.2 Types of Tests

There are two kinds of test procedures for response time testing of temperature sensors. The first involves a step change in the

ambient temperature. This is the basis for the "plunge" testing. The second is introduced to the sensor by causing a step change in the heat generated inside the sensor. This is called the Loop Current Step Response (LCSR) test.

5.2.1 Plunge Test

Plunge testing is a well known testing procedure. The standard plunge test is performed by the rapid movement of the sensor from an initial position in air to a position in flowing water at 20°C or 80°C and 1 m/s. There are other procedures to perform a plunge test. Those performed at the University of Tennessee (UT) include heating the sensor by warm air or hot water (depending on the facility used) then plunging it into the flowing fluid at room temperature. Another plunge testing procedure, which is also employed at UT, is performed by plunging the sensor from room air into hotter flowing air. The common factor among all these plunge testing procedures is that the sensor is exposed to a sudden (step) change in the ambient temperature. Then the sensor response during the transient is monitored, and the time constant (τ) can be determined.

5.2.2 Loop Current Step Response (LCSR) Test

This method was developed initially to measure the response time of thermocouples. It was then adapted for testing of RTDs (14).

The procedure for the LCSR testing of RTDs is simple. Initially, the RTD is allowed to reach a steady state condition with the medium

while a small current is passed through the sensor's head. Then the current is instantly increased. This action represents a step increase in Joule heat generated inside the RTD.

For a thermocouple, the LCSR procedure is slightly different. First, a high current is passed through the thermocouple's lead wires, and it is allowed to reach a steady state. Then the current is suddenly cut off to induce a step decrease in the heat generated inside the sensor. The sensor response is monitored during cooling.

The LCSR test has the advantage of allowing for in-situ testing of temperature sensors. In other words, since plunge tests cannot be performed on sensors installed in processes, it becomes necessary to use the LCSR testing method which does not require moving or removing the sensor from its place in the process. Furthermore, one must test in the process in order to properly account for process conditions.

The data obtained are used to determine the time constant of the sensor as has been mentioned in section 3.3.6.

A schematic of the LCSR test is shown in Figure 5.1. Figure 5.2 shows a typical response of a temperature sensor in a plunge test and an LCSR test. It must be noted that in the plunge test, the temperature gradient is first felt at the surface of the sensor. A time lag occurs before the heat conduction is felt at the sensing element. In the LCSR test, however, the transient is felt directly at the sensing element. Thus the response of the sensor in the LCSR test is always faster than its response in the plunge test.

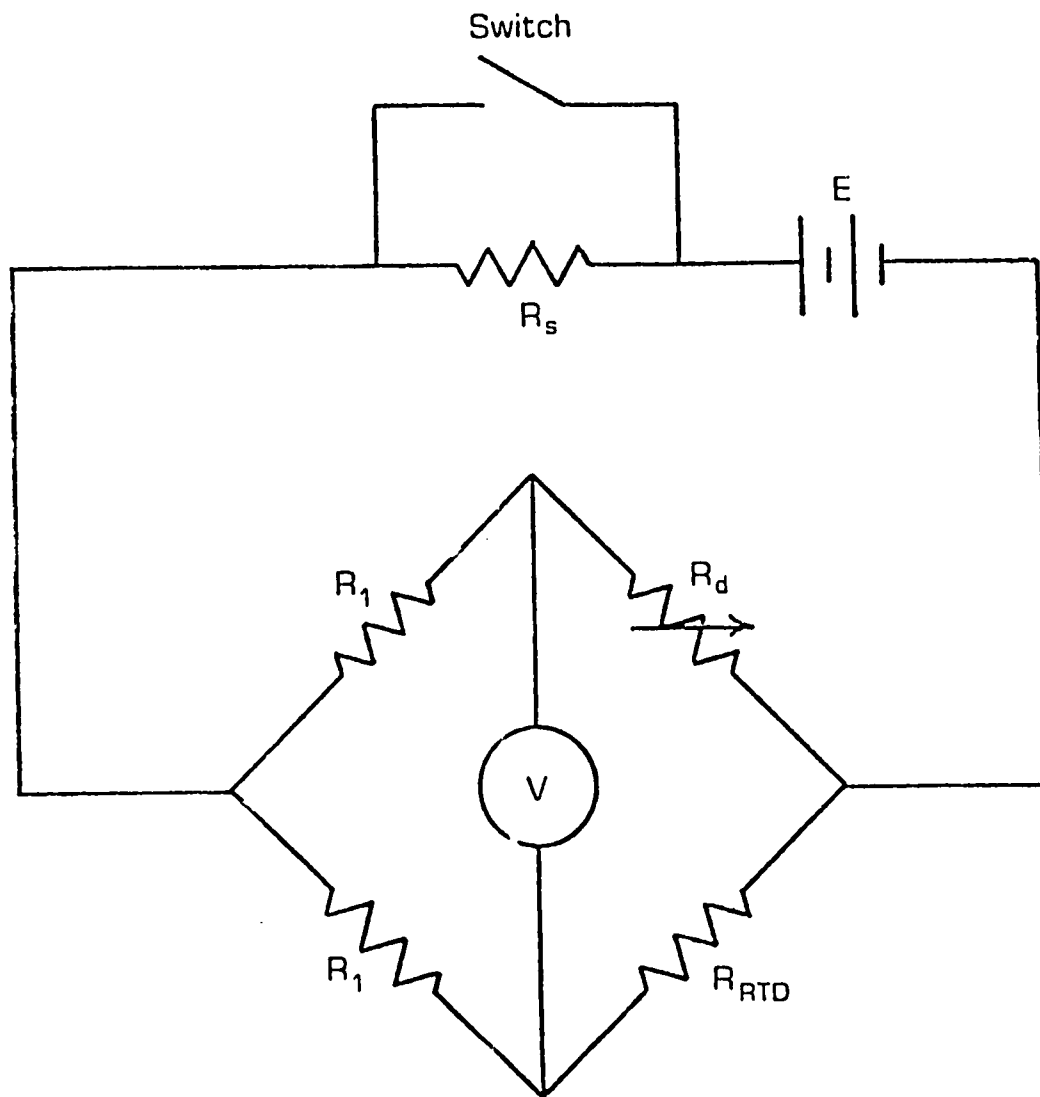


Figure 5.1 A Schematic of the LCSR Test

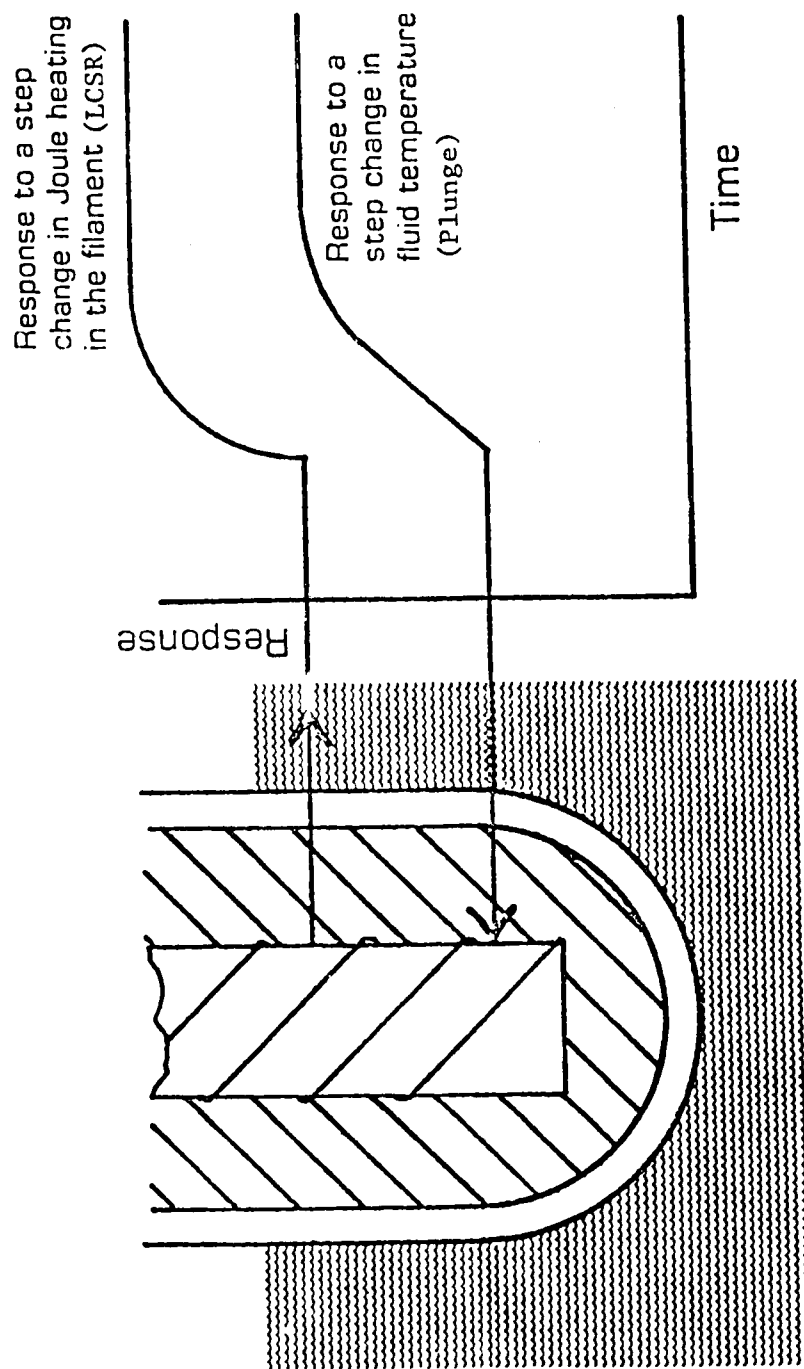


Figure 5.2 Typical Response of a Hypothetical Sensor in a Plunge and LCSR Tests

5.2.3. Self Heating Test

This test was not actually performed as part of this test program. However, it is discussed here since it may be simulated by using SENSOR.

The self heating tests may be performed on RTDs installed in processes. Their primary objective is to test for degradation of sensor heat transfer.

The self heating test is initiated by (13) increasing the current (I) passing through the sensor leads from 1-6 milliamperes to 20-60 milliamperes. The steady state resistance (R) is determined. The power (P) generated inside the sensor is calculated by:

$$P = I^2R \quad (5.1)$$

The current is increased again and a new R and P are obtained. The process is repeated several times to obtain a range of resistance values versus a range of power values. A plot of R versus P results in a straight line with a slope called the Self Heating Index (SHI). That is

$$SHI = \frac{\Delta R}{\Delta P} \frac{\text{ohms}}{\text{watts}} \quad (5.2)$$

The SHI is unique for each sensor. It must be determined experimentally. Any changes in SHI, under the same test condition, may be attributed to sensor degradation because of possible internal or surface changes.

5.3 Testing Facilities

5.3.1 The Loop Testing Facility

This facility consists of a tank, a pump, pipes, plunging mechanism, hot and cold water, and response monitoring equipment. The tank is half filled with water at room temperature. The pump is used to circulate the water through the pipes and back into the tank in the manner indicated in Figure 5.3. The velocity of the flowing water is regulated by a valve opening located downstream of the pump. The top horizontal pipe is a 9.1 cm in inner diameter and 1.3 m in length. A facility is provided for sensor insertion in the pipe. The maximum velocity obtainable in this facility is 1.71 m/s.

To perform a plunge test, the sensor is heated by using a hot water supply. Once the sensor reaches steady state at high temperature, it is plunged into the flowing colder water. Simultaneously, the hot water supply is cut off. The plunging is achieved by a two-way solenoid-actuated valve. As the sensor enters the colder region, an electric switch is turned on which closes an electric circuit, thus giving an electrical timing signal. This is detected on a strip chart recorder as a rapid jump. This indicates the time at which the transient starts.

Several problems are encountered while operating this facility. The main problem is caused by the constant raising of the cold water temperature by the hot water that is used to heat the sensor. Therefore, the tank must be drained and refilled with cold water.

The loop facility requires the use of about 150 gallons of fluid. Thus only water was used in this facility. This in turn limits the

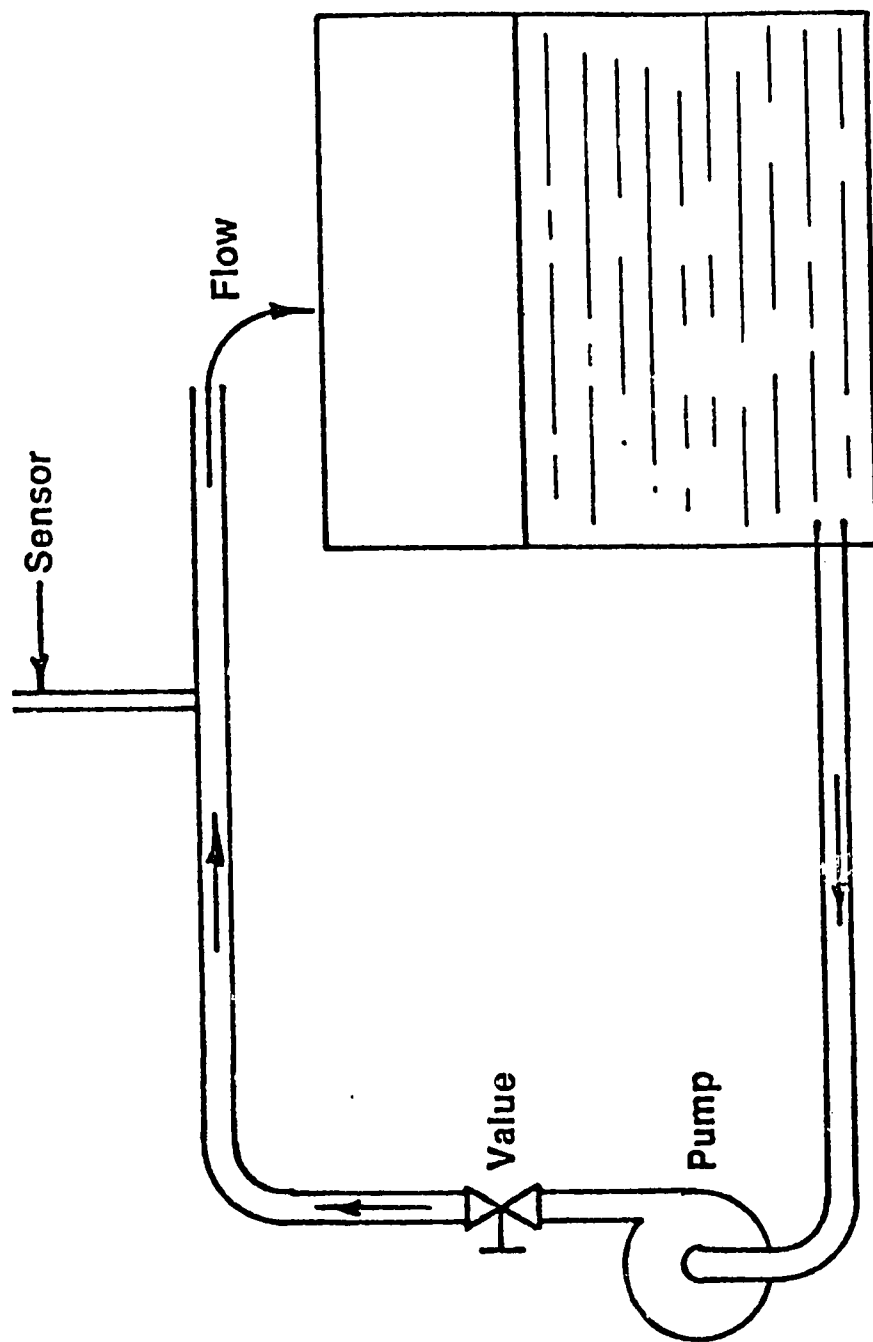


Figure 5.3 Schematic of the Loop Testing Facility

range of the film heat transfer coefficients (h) obtained. To overcome this problem the rotating tank facility, where smaller volumes of several liquids can be used, is used.

5.3.2 Rotating Tank Testing Facility

This facility was designed to accommodate testing in several fluids and at variable velocities. It consists of a cylindrical tank that is divided into four concentric cylinders. The tank is mounted on a shaft that is rotated using pulleys that are driven by an electric motor. The maximum obtainable fluid velocity is 1.38 m/s. On top of the tank, a hot air blower, timing switch, plunging mechanism, and the sensor are mounted. The sensor output signal is amplified and monitored on one channel of a strip chart recorder, while the timing signal is monitored on the other channel. A schematic of this set up is shown in Figure 5.4.

Approximately 20 gallons of fluid are needed to fill half of the tank. This makes it possible to use relatively expensive fluids.

The velocity of the fluid can be changed by changing the pulley setting which changes the rotational speed, and by shifting the sensor position radially.

To perform the plunge test on a temperature sensor, the hot air blower is used to heat the sensor until it reaches steady state. Then it is plunged into the flowing fluid which is kept at room temperature. The sensor response is monitored using the strip chart recorder.

Initially, a problem with sensor vibration was encountered. This would increase the uncertainties in heat transfer coefficient calcula-

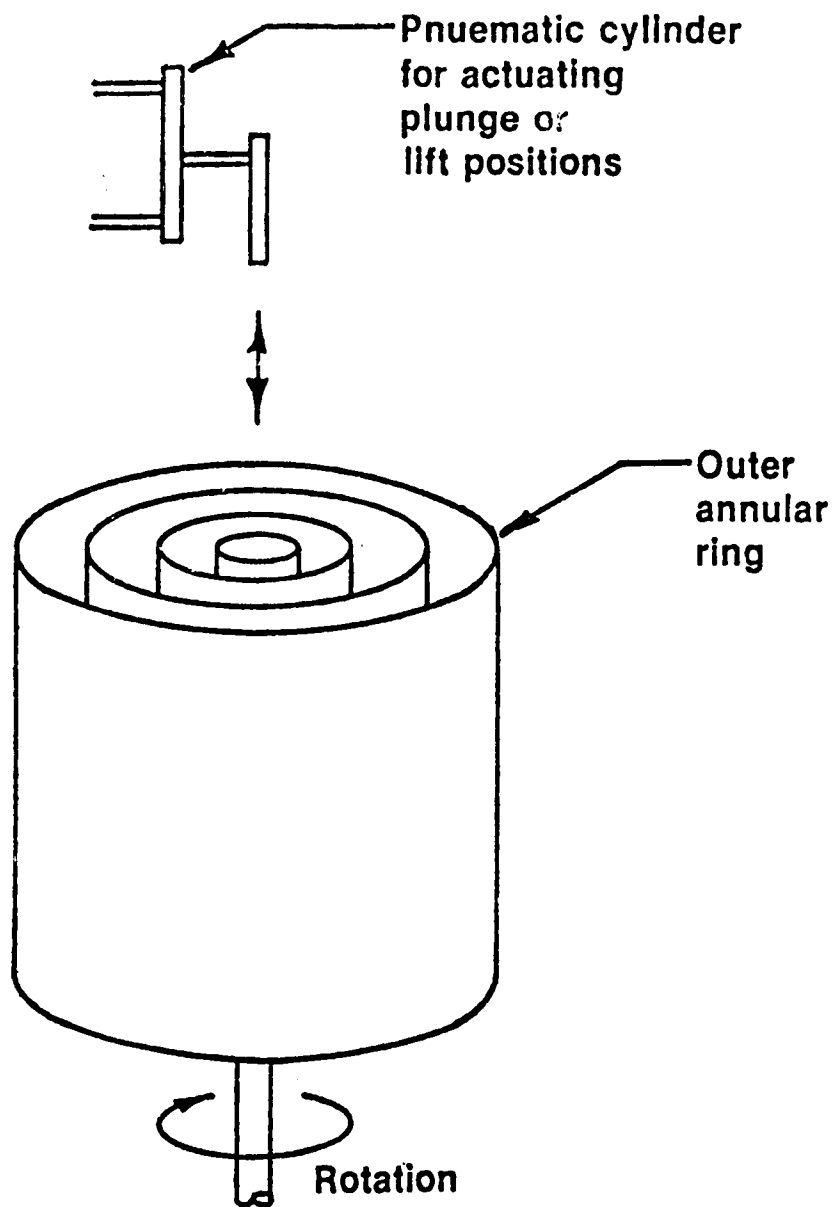


Figure 5.4 Schematic of the Rotating Tank Testing Facility

tions and quicken the sensor response. To overcome this problem, a weight was attached to the superstructure of the plunge mechanism. This reduced the vibration.

Agreement of results between the loop and the rotating tank facilities will be discussed in section 6.2.

5.3.3 Gas Loop Testing Facility

This facility has been designed primarily to test temperature sensors in flowing air. The humidity can be varied between the range of 0% to 100%. This facility permits characterizing and verifying correlation 4.1 at conditions far removed from conditions in flowing liquids.

The facility, shown in Figure 5.5, consists of a tank, three heaters, pipes, blowers, plunging mechanism, and monitoring equipment. The air inside the tank can be heated using any or all of the three heaters. A thermostat is used to control the heater output. The hot air is circulated through the pipes and back into the tank by using an air blower. The speed of rotation of the blower is controlled by using a variac. That is, the air velocity is controlled by changing the variac's setting. A maximum fluid velocity of 8.64 m/s is obtainable. The velocity is measured at the sensor location using a pitot tube. The temperature of the flowing air is monitored using a type K fast ($\tau < 0.5$ sec) thermocouple with a diameter of 1.5 mm.

To create high humidity inside the loop, a container filled with water, in which two heaters are immersed, is placed at the bottom of the tank. When the two heaters are turned on the water temperature

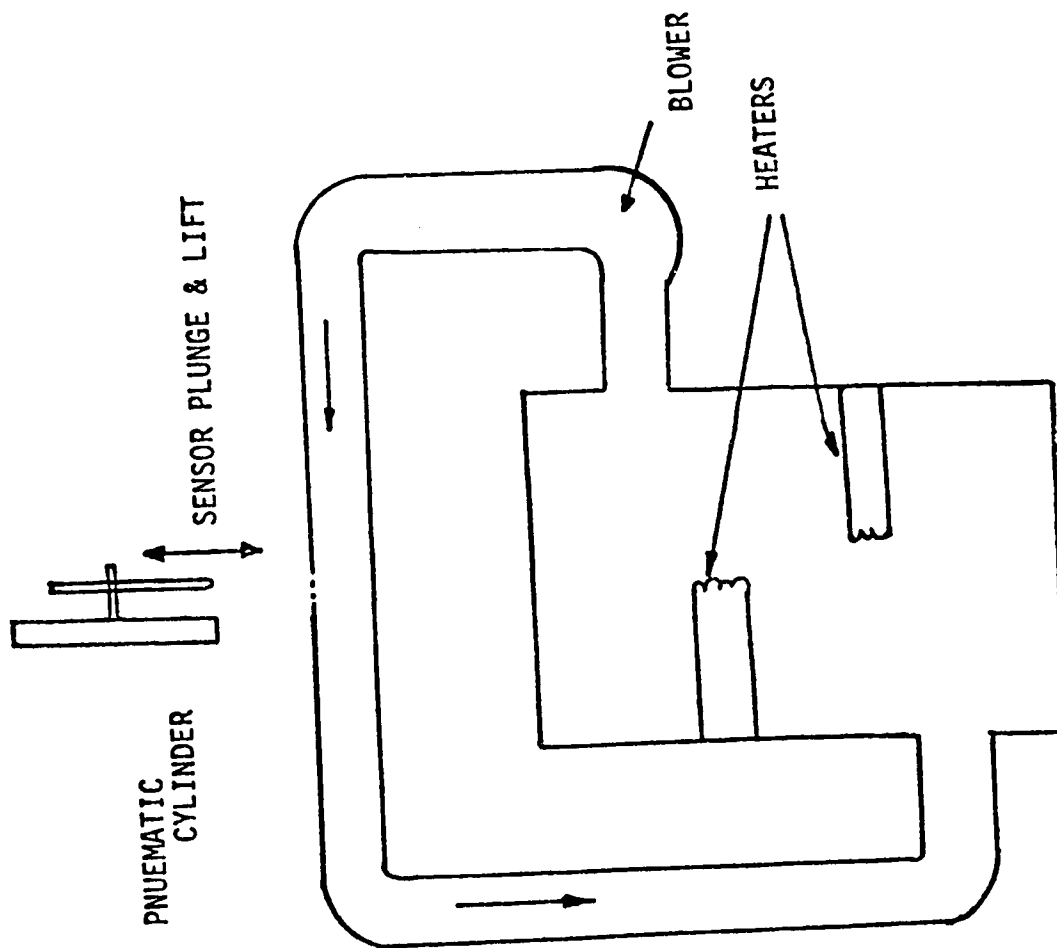


Figure 5.5 Schematic of the Gas Loop Testing Facility

rises to about 55°C. This increases the humidity. On the other hand, if low humidity (~ 0% humidity) is desired, then the water container is replaced with a moisture absorbing material such as DRIERITE which employs calcium sulfate (CaSO_4).

To perform the plunge test, the sensor is allowed to reach steady state at room temperature with the help of a blower. The blower quickens the return of the sensor to thermal equilibrium with the surroundings. Then it is plunged into the flowing air through an opening in the middle of the top horizontal pipe. The sensor response and the timing signals are monitored on a strip chart recorder.

CHAPTER VI

THEORETICAL AND EXPERIMENTAL RESULTS

This chapter is presented in two parts. In the first part, modelling results are presented. In the second part, experimental results are presented.

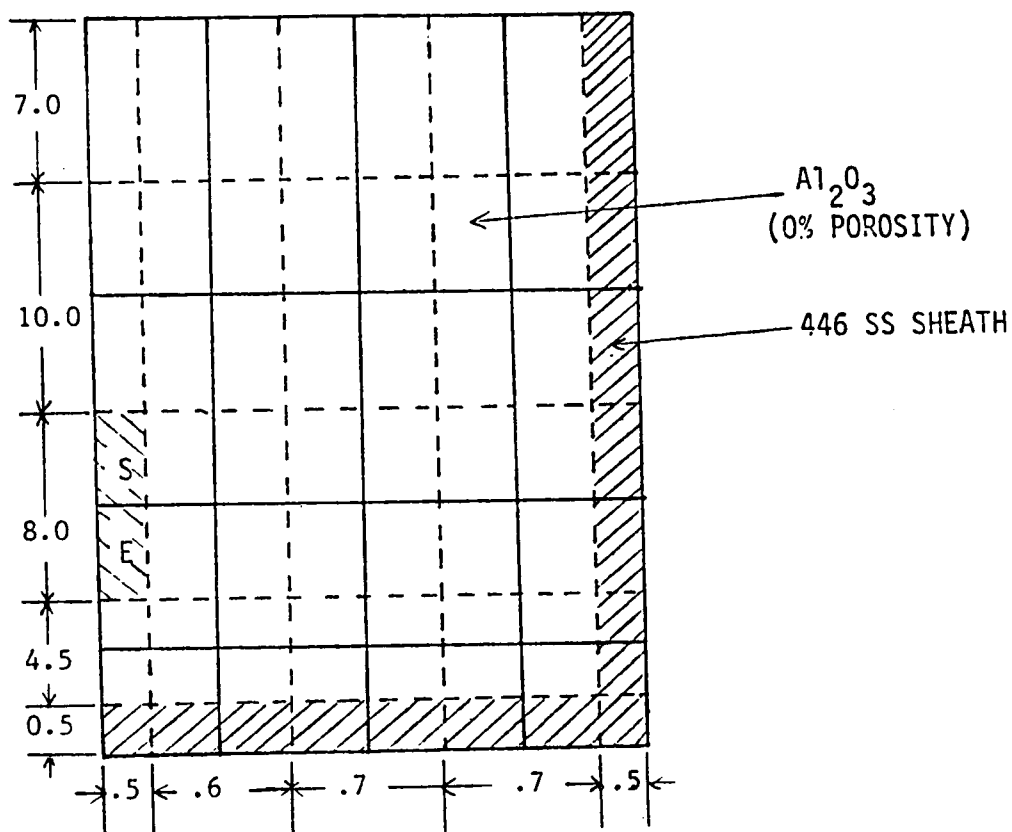
6.1 Modelling Results

Several hypothetical sensors were simulated to obtain results that would demonstrate the validity of the modelling approach as a designer's aid.

The hypothetical sensors used for simulation are shown in Figure 6.1. These sensors are considered as reference sensors. Their constituent's properties at the indicated temperature can be found in Appendix B. In many cases, however, it was necessary to vary the dimensions or the physical properties to suit the requirements of the specific analysis being performed. All the deviations from these reference cases will be discussed.

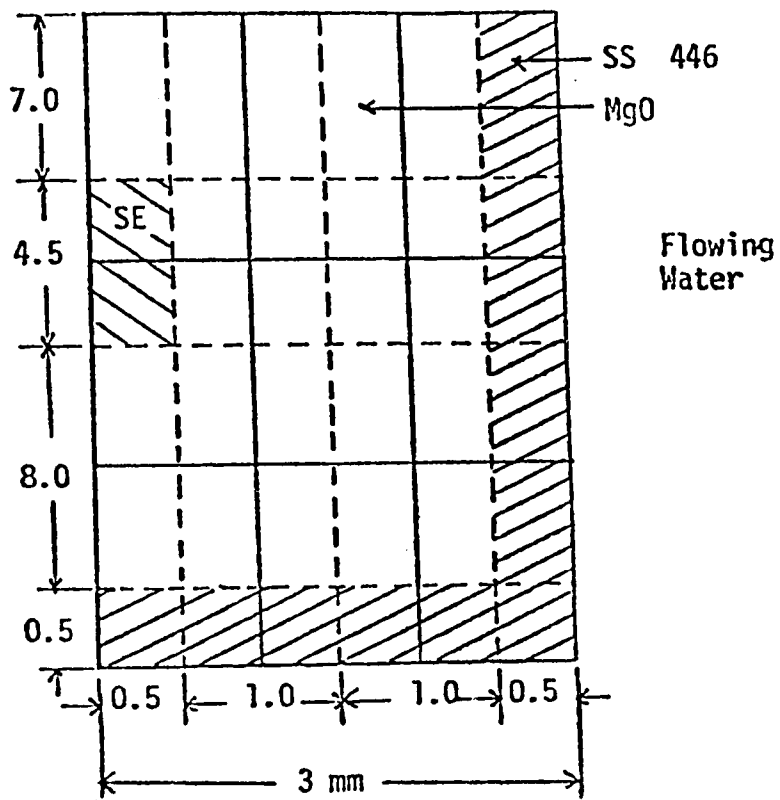
6.1.1 Effects of the Insulator Properties

The physical properties of insulators, such as aluminum oxide or magnesium oxide, are important factors in determining the speed of response of the sensor. The physical properties of interest are the thermal conductivity, k , the density, ρ , and the specific heat capacity, C_p . The sensor used in the study of the effects of k , ρ , and C_p on the sensor response is sensor A.



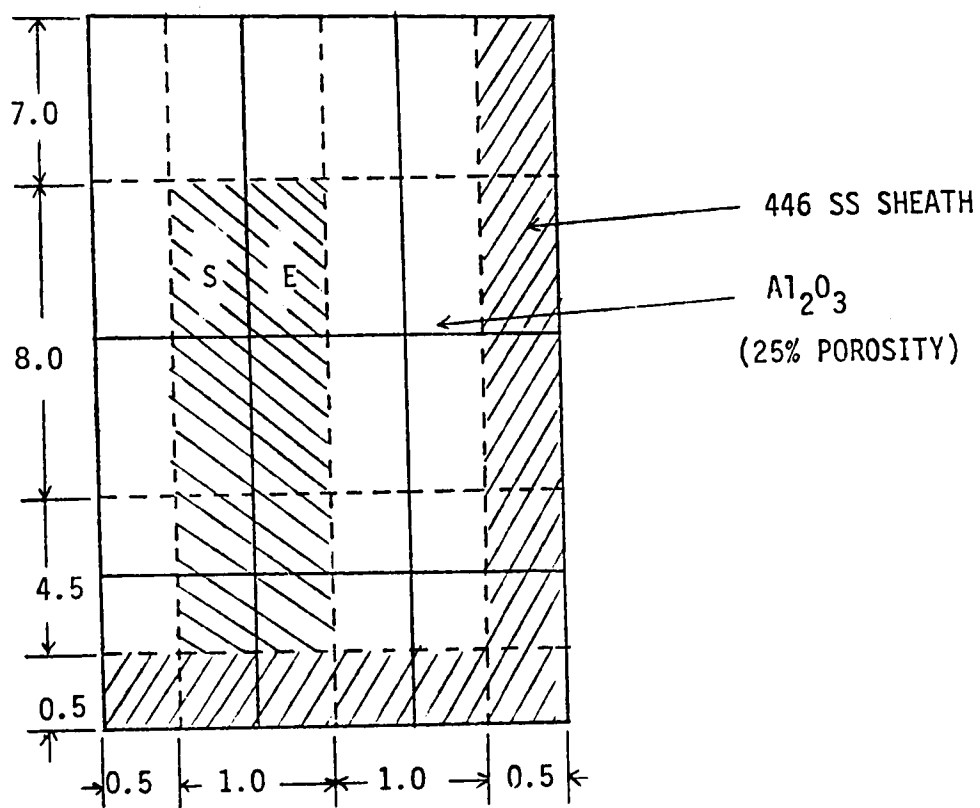
SENSOR A

Figure 6.1 Nodal Structure of the Hypothetical Sensors Used in the Simulations



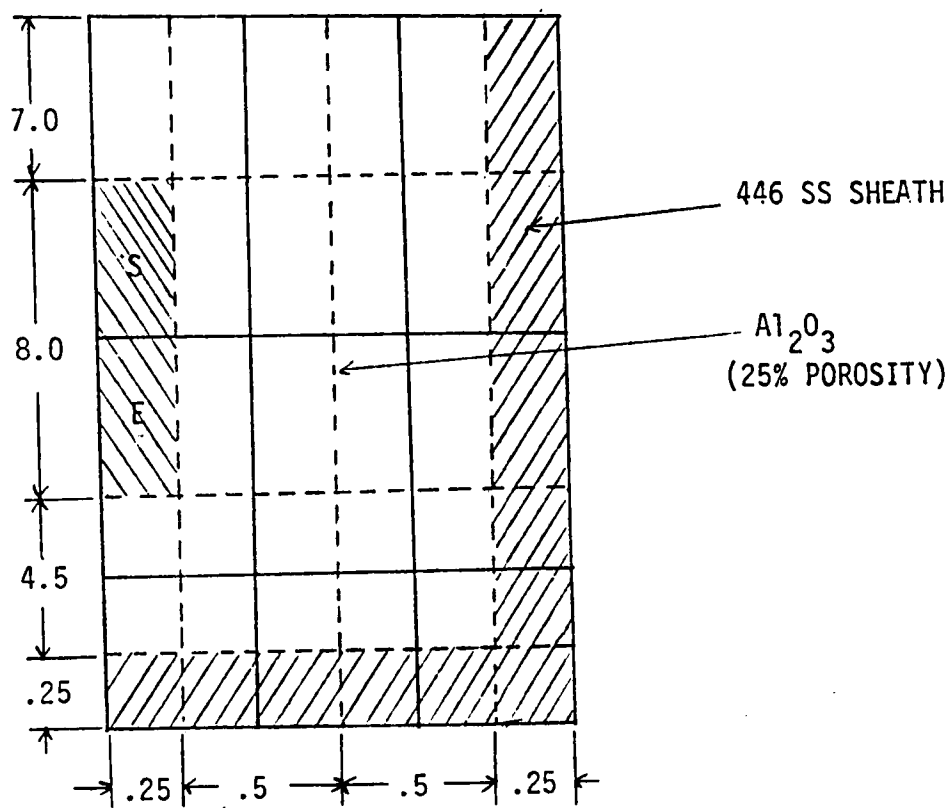
SENSOR B

Figure 6.1 (Continued)



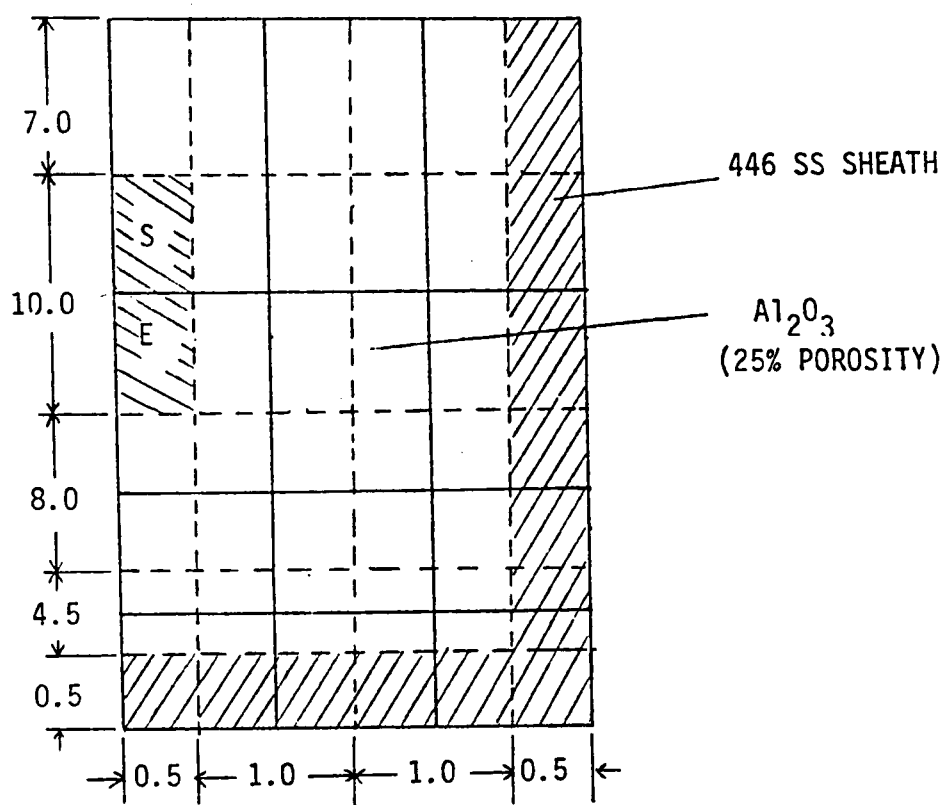
SENSOR C

Figure 6.1 (Continued)



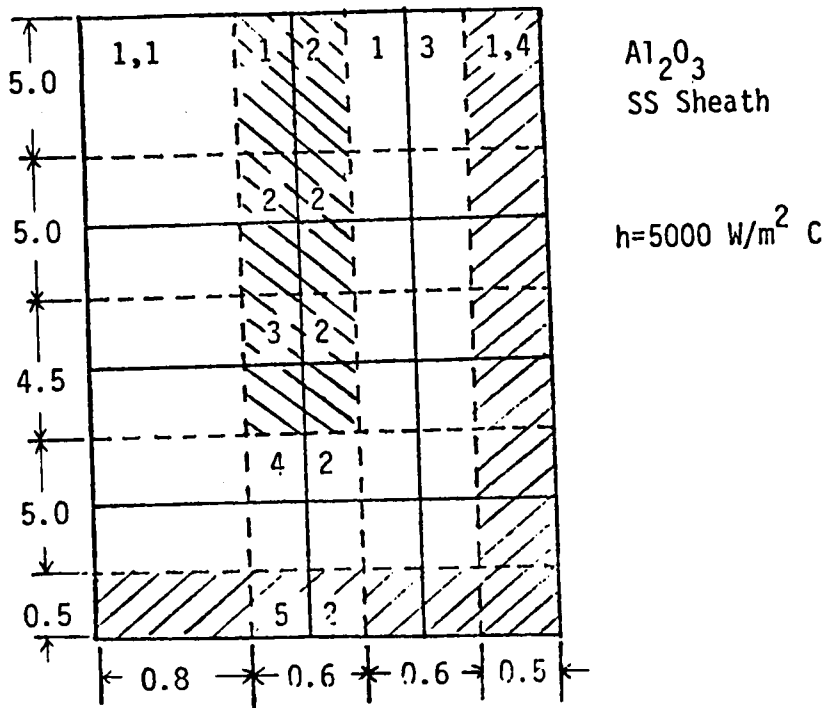
SENSOR D

Figure 6.1 (Continued)



SENSOR E

Figure 6.1 (Continued)



SENSOR F

Figure 6.1 (Continued)

First, the effect of variations in k on the sensor response was analyzed. This was accomplished by varying k from -30 to +20 percent of its reference value while maintaining ρ and C_p constant. Figure 6.2 shows the effects of k on the sensor response. It is clear that increasing k reduces the sensor response time.

A similar procedure was followed to study the effects of ρ and C_p , respectively, on the sensor response. Results are shown in Figures 6.3 and 6.4. In both cases, increasing ρ or C_p resulted in an increase in the time constant and, hence, a slower response.

These simulations are in agreement with the following approximate relation:

$$\tau = RMC \quad (6.1)$$

where

R is the thermal resistance = $\Delta x / Ak$
 M is the mass = ρv
 C is the specific heat capacity.

For illustrative purposes, Figures 6.2, 6.3, and 6.4 were combined to produce Figure 6.5. In terms of percent changes, Figure 6.5 indicates that C_p and ρ have greater effects on the sensor response than k does. This is as expected since k determines the internal heat transfer resistance which is only a part of the total resistance (which includes the resistance at the surface).

6.1.2 Effects of the Surface Conditions

Sensor B was used in the study of the influence of surface conditions on the sensor response. The physical properties and the flow

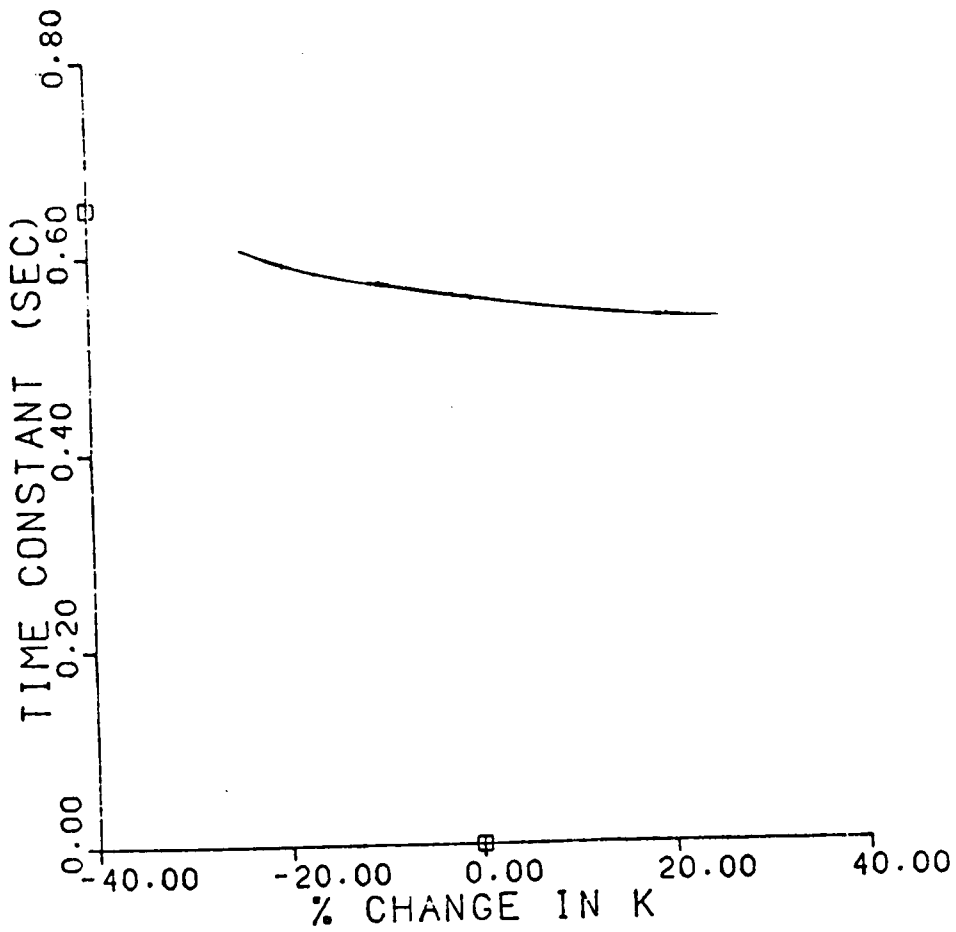


Figure 6.2 Effects of the Thermal Conductivity (k) of the Insulator of Sensor A

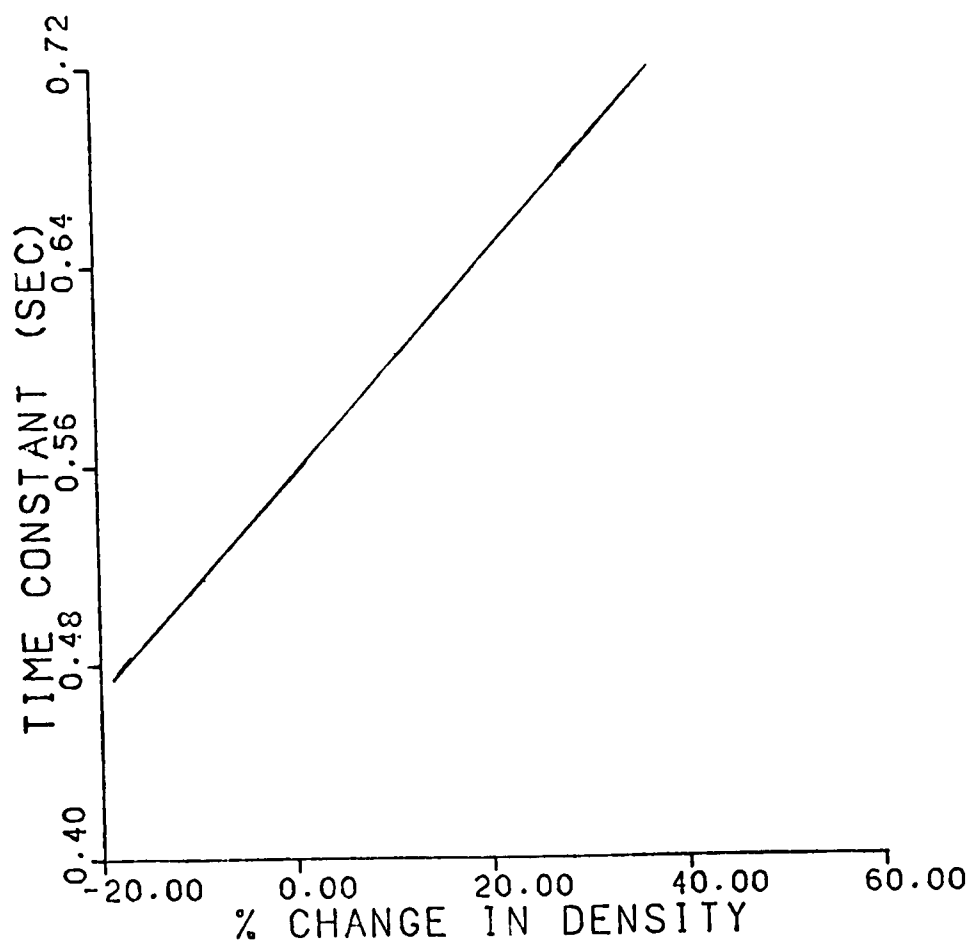


Figure 6.3 Effects of the Density of the Insulator of Sensor A

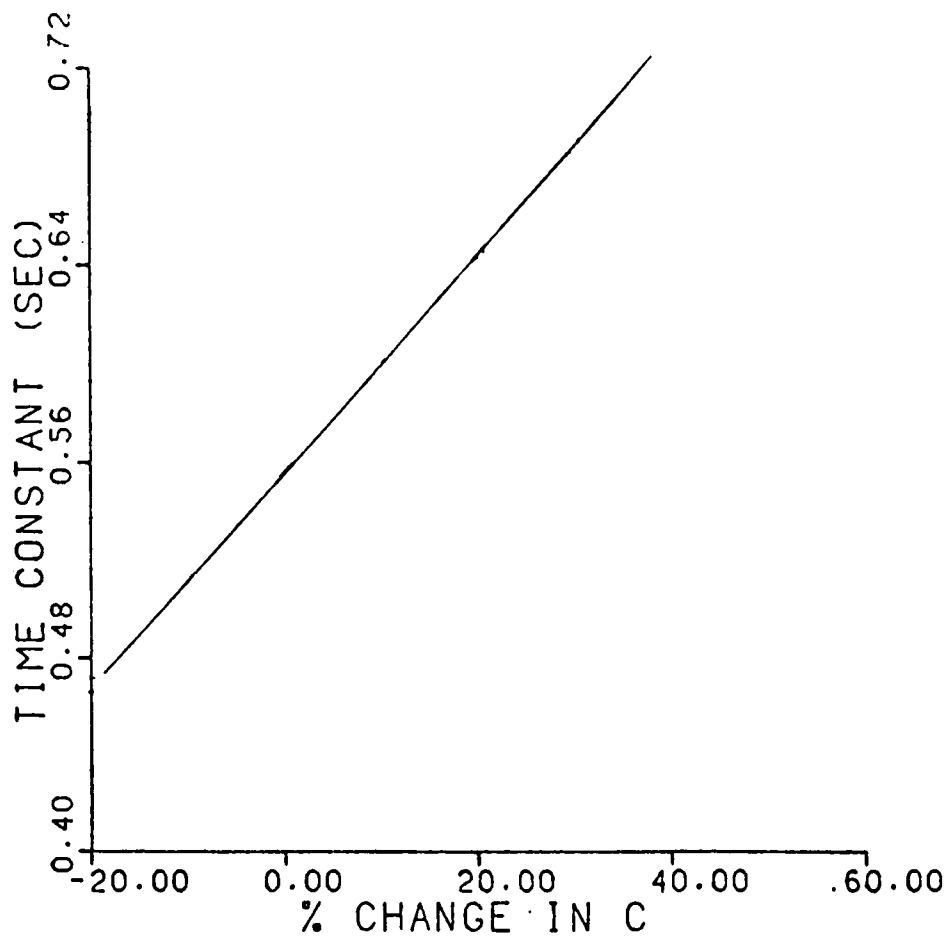


Figure 6.4 Effects of the Specific Heat Constant (C_p) of the Insulator of Sensor A

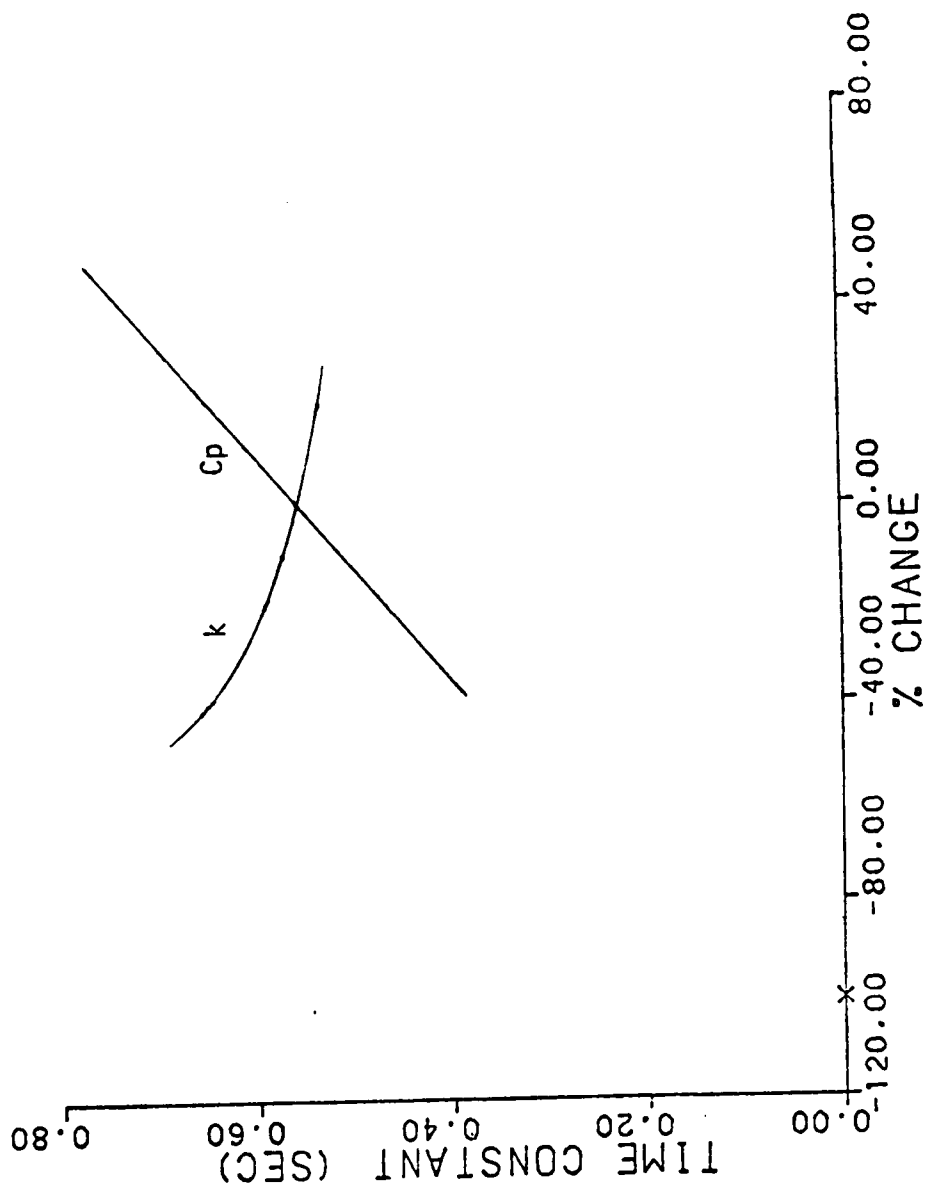


Figure 6.5 Effects of k , Density, and C_p of the Insulator of Sensor A

rate of the fluid flowing around the sensor determine the surface heat transfer coefficient. The response decreases for higher flows until the surface heat transfer resistance becomes negligible compared to internal heat transfer resistance inside the sensor. The dependence of the sensor's time constant on the fluid's velocity and surface heat transfer coefficient is shown in Figures 6.6 and 6.7 respectively.

Equation 4.1, repeated below, was derived using idealized models for temperature sensors.

$$\tau = C_1 + C_2/h \quad (4.1)$$

Equation 4.1 results when a lumped parameter model or a distributed parameter model for a homogeneous cylinder is used. Real sensors, however, are multilayered cylindrical regions composed of several materials. That is, real sensors behave as higher order systems and not as first order systems as assumed in deriving Equation (4.1). Thus additional validation of Equation 4.1 is needed for real sensors.

SENSOR, the new simulation package, can provide this validation.

Simulations were performed using sensors A, C, and D. The plots of the time constants versus the inverse of the heat transfer coefficients for sensors A, C, and D are shown in Figures 6.8, 6.9, and 6.10 respectively. These results clearly follow the correlation of Equation 4.1 and give further confidence in the general validity of Equation 4.1.

Next, sensor B was used to estimate the importance of axial heat transfer through the tip of the sensor on sensor response. The heat

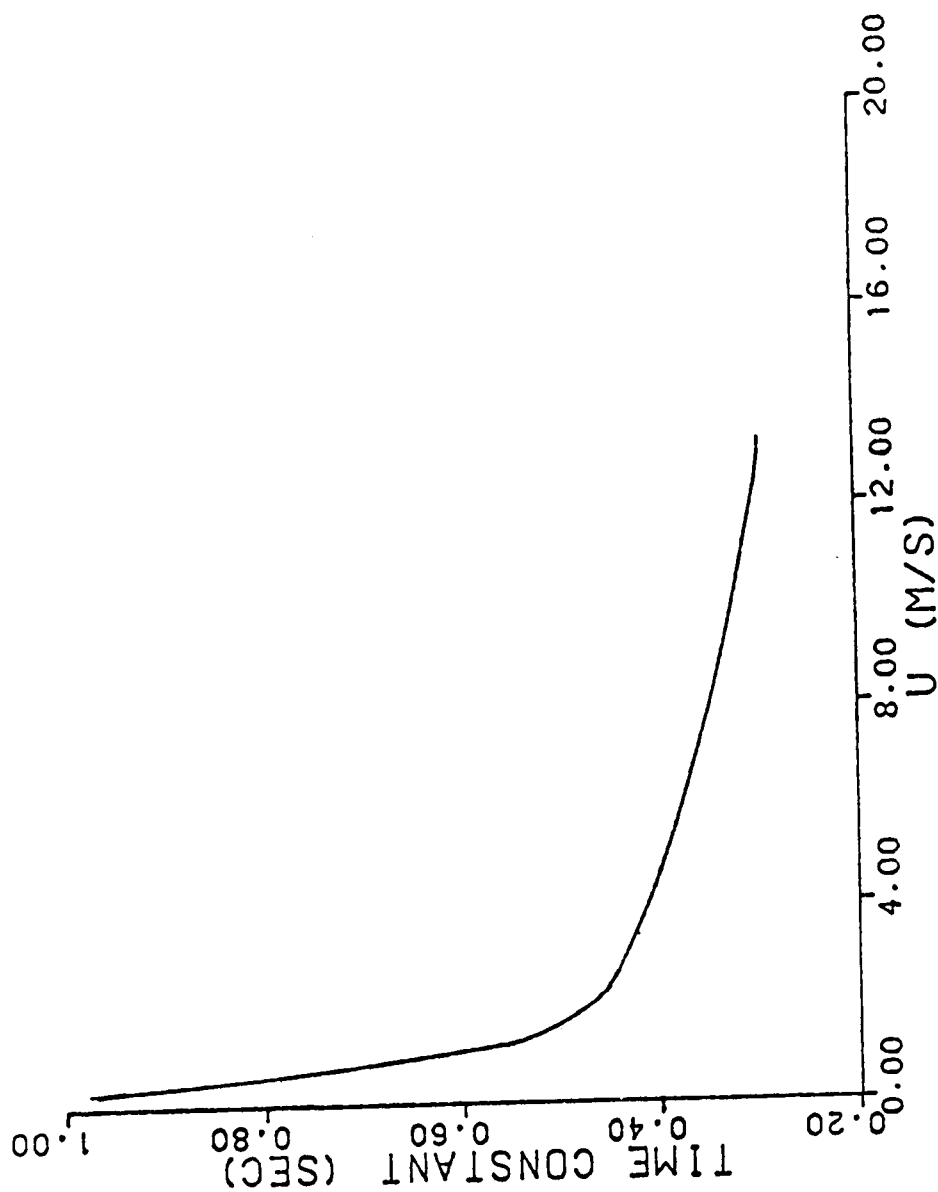


Figure 6.6 Dependence of the Time Constant of Sensor B on Fluid velocity

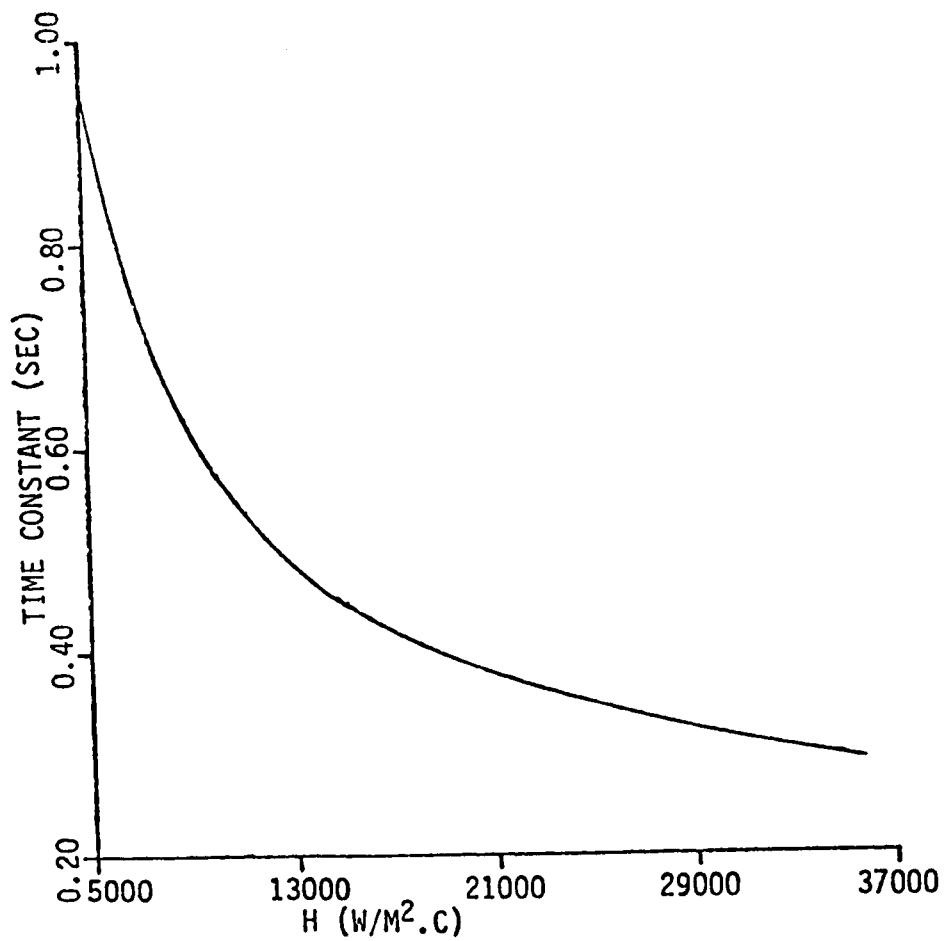


Figure 6.7 Dependence of the Time Constant of Sensor B on the Heat Transfer Coefficient

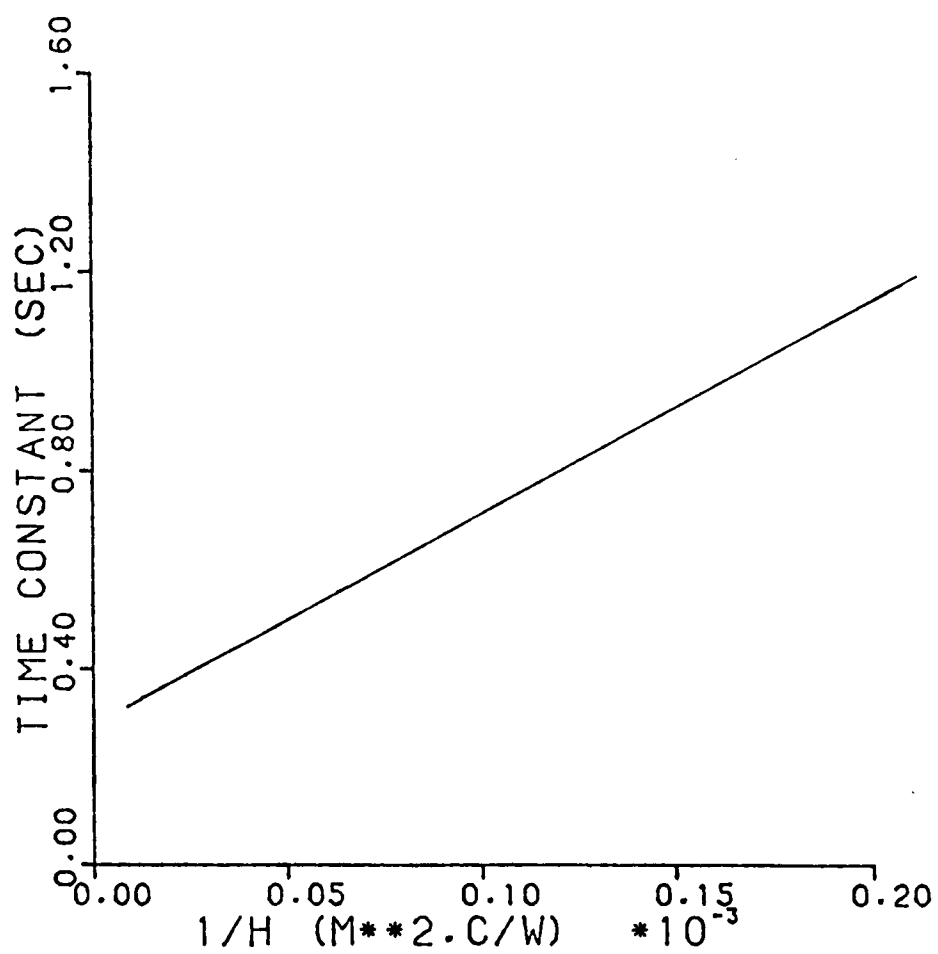


Figure 6.8 Relationship Between τ and $1/h$ for Sensor B

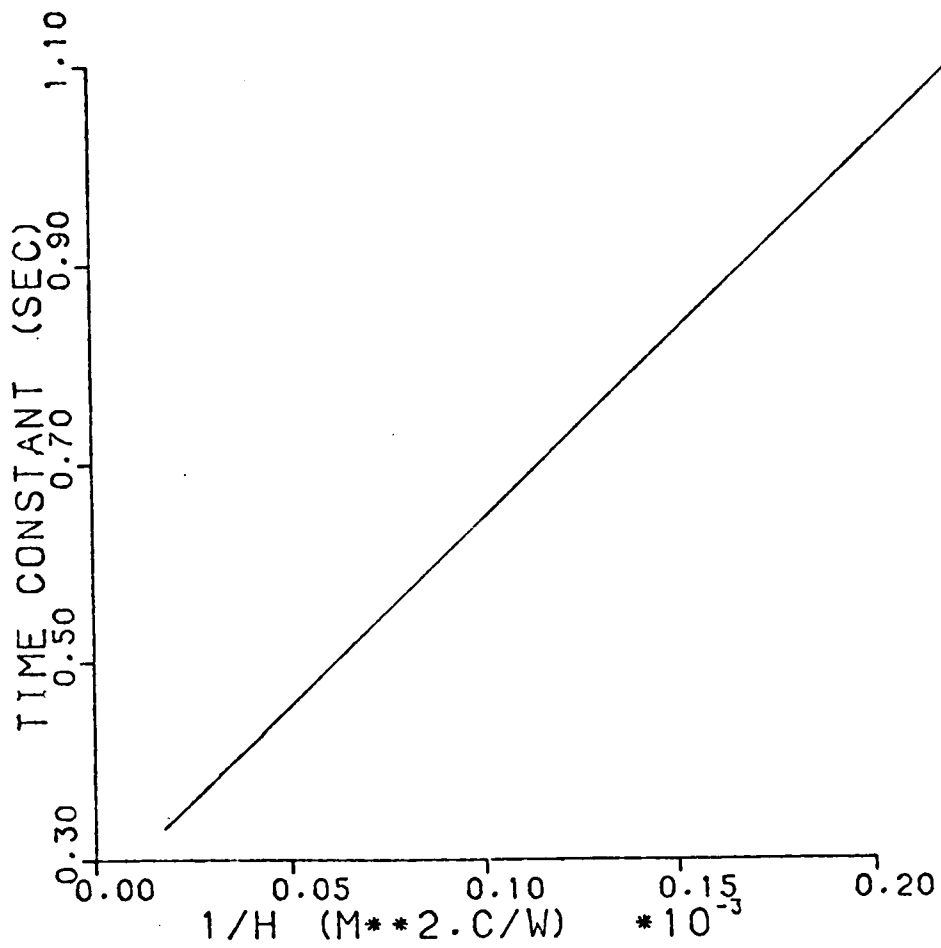


Figure 6.9 Relationship Between τ and $1/h$ for Sensor C (RTD)

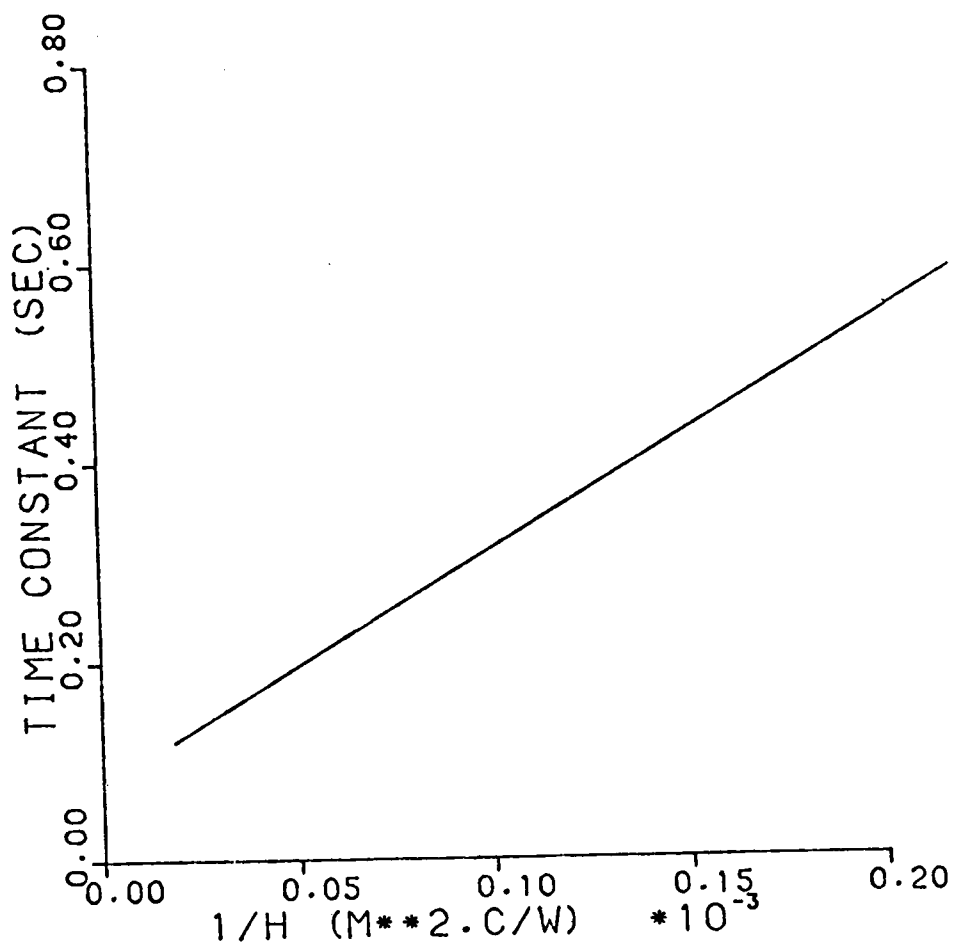


Figure 6.10 Relationship Between τ and $1/h$ for Sensor D

transfer coefficient for heat transfer through the tip of the sensor was varied and simulations were performed. The results are shown in Figure 6.11. This simulation shows that the time constant for the simulated sensor is weakly dependent on axial heat transfer through the sensor tip.

6.1.3. Sensing Element Location

In this section, a comparison between grounded junction sensors and insulated junction thermocouples is made. Experience shows that grounded TCs respond faster than insulated junction TCs. This is because the measuring junction in the grounded sensor is closer to the ambient fluid. The grounded junction is attached to the metal sheath. This leads to a faster response to changes in the ambient temperature.

The effects of sensing element location in the sensor study was not investigated independently. Instead, this study was incorporated as part of other studies such as the effects of surface conditions study and effect of porosity study.

Figure 6.12 shows the relation between $1/h$ and τ for grounded and insulated junction sensors. The sensors used were sensor D and sensor E. As this figure indicates, the fastest response occurred where sensor D with a grounded junction was used. The slowest response occurred when sensor E with an insulated junction was used. This is in agreement with the fact that grounded junction sensors respond faster to variations in ambient temperature than insulated junction sensors.

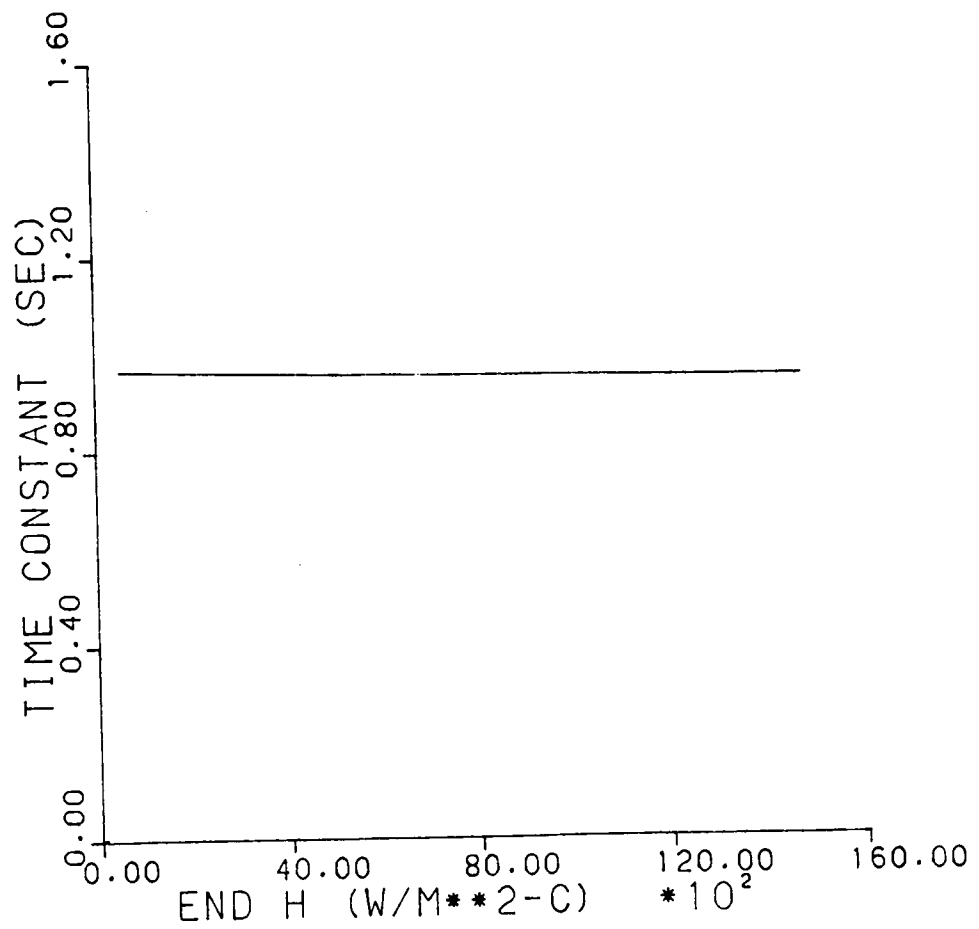


Figure 6.11 Time Constant Dependence on End Heat Transfer Coefficient

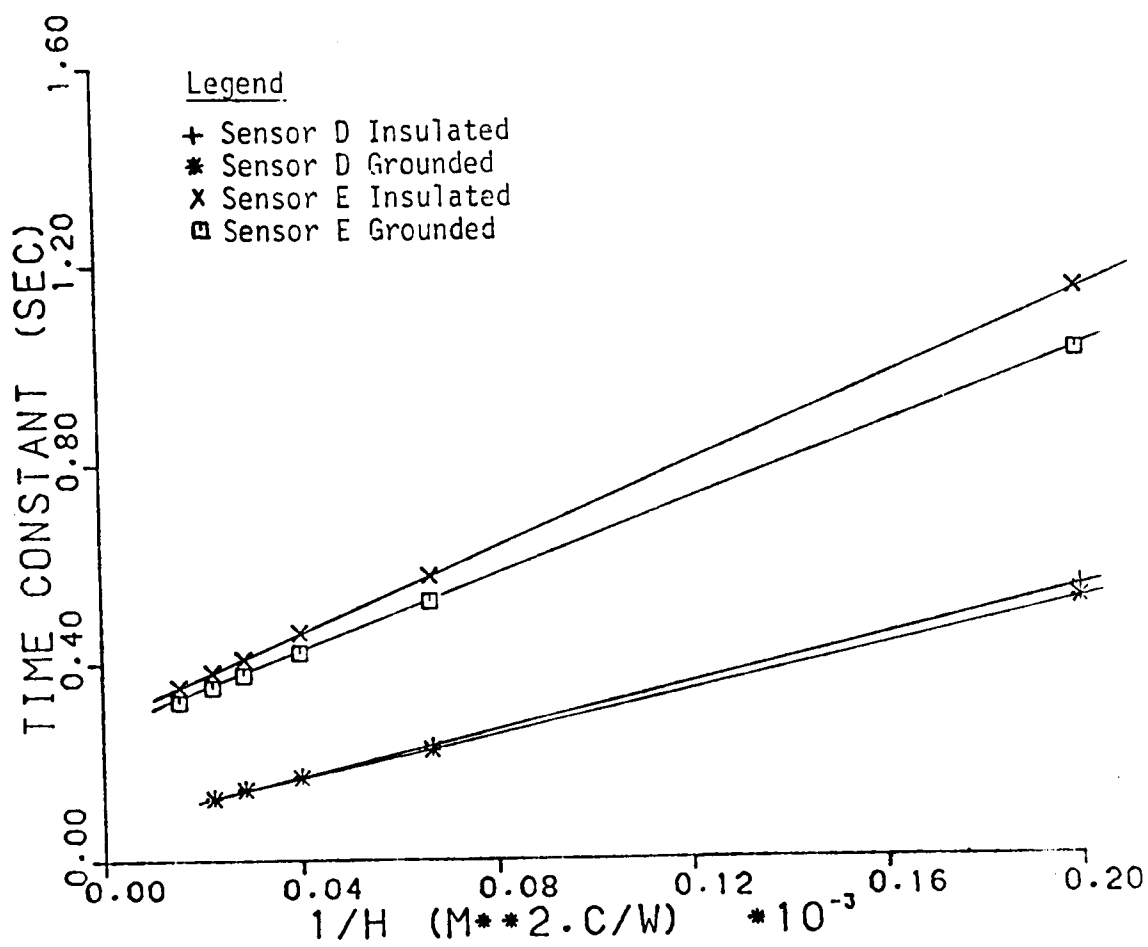


Figure 6.12 Relationship Between τ and $1/h$ for Sensor D and E

6.1.4 Effects of Diameter

The diameter of the sensor is one of the most important factors that determine the sensor response. Theoretical analysis based on first order approximation indicates that (3)

$$\tau_{int} = \frac{\rho C r^2 \ln(r/r_i)}{2k} \quad (6.2)$$

where

$$\begin{aligned} \tau_{int} &= \text{internal component of the time constant} \\ &\quad (\text{see appendix A}) \\ r &= \text{outer radius} \\ r_i &= \text{inner radius} \\ \rho &= \text{density} \\ C &= \text{specific heat constant} \\ k &= \text{thermal conductivity} \end{aligned}$$

This indicates that the sensor time constant is proportional to the square of the sensor outer diameter, i.e.

$$\tau \propto D^2 \quad (6.3)$$

The surface time constant, however, is given by

$$\tau_{surface} = \frac{\rho C r}{2h} \quad (6.4)$$

where

$$h = \text{film heat transfer coefficient.}$$

Substituting the heat transfer correlation f (see section 4.2) into Equation 6.4 results in the following relationship

$$\tau \propto D^{1.4} \quad (6.5)$$

Thus, large and slow sensors that are usually dominated by internal

heat transfer tend to follow the relationship (6.3). On the other hand, small and fast sensors, usually dominated by surface heat transfer resistances, have responses that follow relationship (6.5).

Experimental results (3) performed on sensors at standard conditions with varying the diameter seem to verify relationships (6.3) for large sensors and (6.5) for small sensors.

The new simulation package provided idealized conditions under which to study the effects of the diameter.

The diameter of sensor B was varied and simulations were performed at moderate film heat transfer coefficient (about $25000 \text{ W/m}^2 \cdot ^\circ\text{C}$). The results are shown in Figure 6.13. The time constant increases with the diameter as this figure indicates.

For a given sensor

$$\tau = \tau_{\text{int}} + \tau_{\text{surface}} \quad (6.6)$$

where τ_{int} and τ_{surface} are given by Equations 6.2 and 6.4. It follows that as the surface time constant becomes negligible, τ can be approximated by τ_{int} . This implies that the sensor response follows relationship (6.3) when τ_{surface} becomes insignificant. On the other hand, one expects the sensor to follow relationship (6.5) when the surface time constant is significant.

The effect of τ_{surface} on the total sensor response could be reduced simply by increasing the film heat transfer coefficient (h) which in turn reduces the surface thermal resistance. This fact prompted new simulations carried out on sensor B. In these simulations h was increased. Two cases were simulated at each h value. In

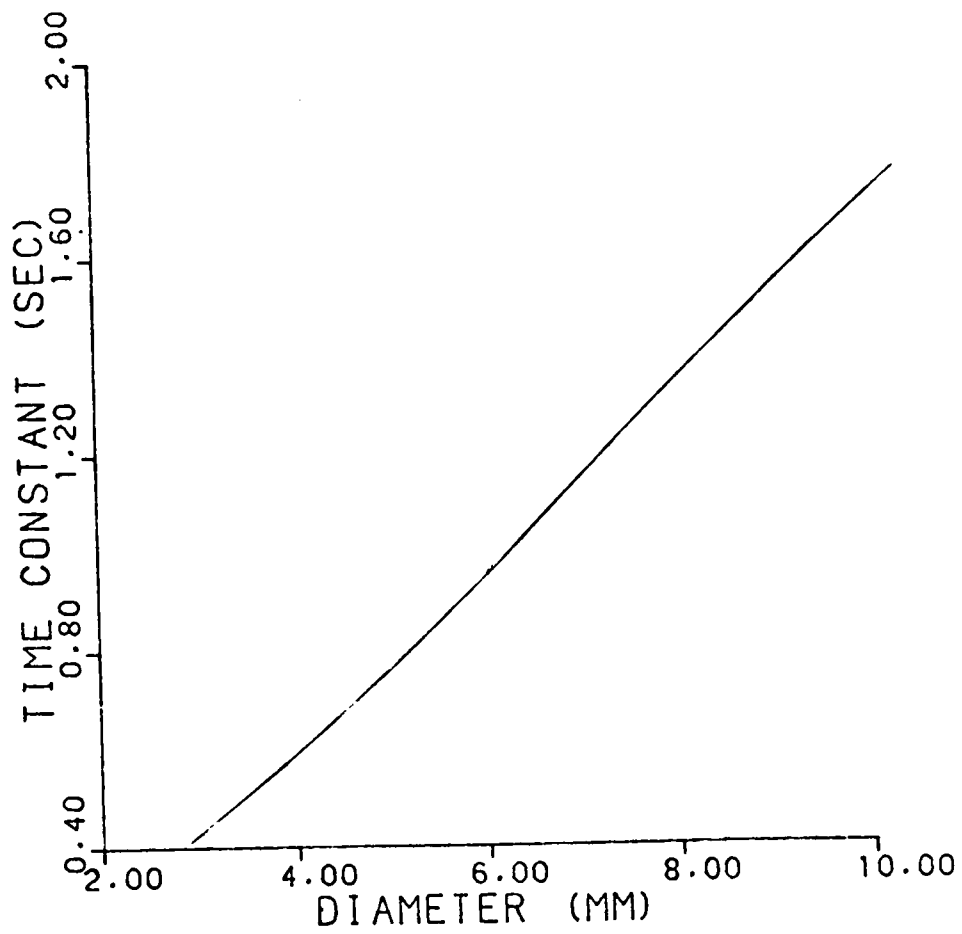


Figure 6.13 Effects of the Diameter on the Sensor Response

the first case the diameter of the sensor was taken as 6.0 mm. In the second case, the diameter was taken as 12.0 mm. The percent surface to total (internal plus surface) thermal resistance was calculated at each h value to be used as a measure of the importance of the surface time constant. Low percentages imply small surface time constant effect. The results of all the simulations and calculations are presented in Table 6.1. The results show that as the surface resistance becomes negligible, the sensor response seems to approach a relation that changes with the square of the diameter. This is in agreement with relationship (6.3). On the other hand, the sensor response seems to follow relationship (6.5) when the surface resistance is high.

6.1.5 Effects of Temperature

Sensor A was used to simulate the sensor response at low temperature (about 27°C) and at high temperature (about 527°C). The physical properties used in both cases are presented in Table 6.2. The response in both cases is shown in Figure 6.14. Higher temperature resulted in slower response. This was expected because the thermal conductivity of alumina decreases with increasing temperature. This is true within a temperature range between 300°k and 1150°k and a porosity range between 0% and 53% (see Appendix B).

6.1.6 Effects of Porosity

The porosity of insulators used in temperature sensors, mainly aluminum oxide or magnesium oxide, is difficult to predict. In general, two sensors of the same type, make, and size may not have the

Table 6.1

Importance of the Surface Resistance in Determining the Diameter
Effect on the Sensor Response

% Surface to Total Resistance	* Power (n)
78.2%	1.36
12.5%	1.70
3.5%	1.88
1.8%	1.90
0.7%	1.92

* The power (n) in the relation $\tau \propto D^n$

Table 6.2

Physical Properties of the Materials of Sensor A Used in the Study
of Temperature Effects on Sensor Response

Material	Temperature °C	Thermal Conductivity W/m.°C	Density kg/m ³	Specific Heat Constant J/kg.°C
% porosity Al ₂ O ₃	27	38.51	4000	786.97
	527	6.10	4000	1172.08
446 SS	27	20.32	7600	460.6
	527	24.51	7600	460.6

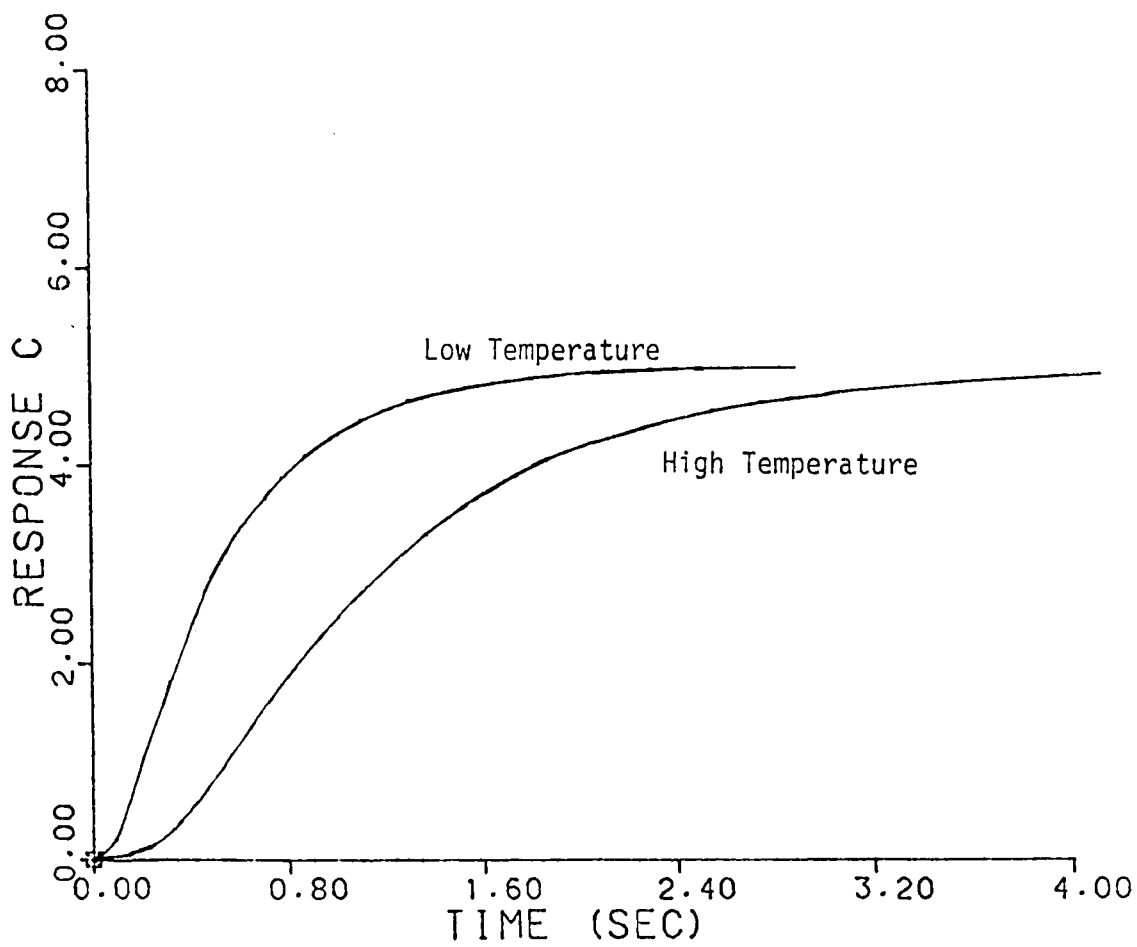


Figure 6.14 Temperature Effects

same porosity because of manufacturing tolerances. Further, the porosity is temperature dependant and also affected by vibrations. The porosity has a major effect on the heat transfer process. This is true because as the porosity increases, the thermal conductivity decreases and the specific heat capacity increases. Thus, a slower sensor response is expected as the porosity increases.

Figure 6.15 shows simulation results for sensor A (grounded junction) at low temperature (27°C) and high temperature (527°C). The physical properties of the sensor constituents are presented in Table 6.3. As expected the sensor became slower as the porosity increased. Figure 6.16 shows a similar set of results for an insulated junction (sensor A). The results are similar to those of the grounded junction TC.

6.1.7 Self Heating Effects

Self heating is a process whereby heat is generated inside the sensor due to the passing of a small current through the sensing wires (see section 5.2.3). The ratio of the steady state temperature to the amount of heat generated inside the sensor is called the Self Heating Index (SHI). The SHI is important in evaluating the sensor behavior. If the sensor internal structure or surface conditions degraded, then one expects to find a larger SHI value for the sensor.

Another commonly used index is the dissipation constant (DC). It is defined as the power (mW) generated in the sensor divided by the temperature rise (°C) (3). This constant also depends on the heat

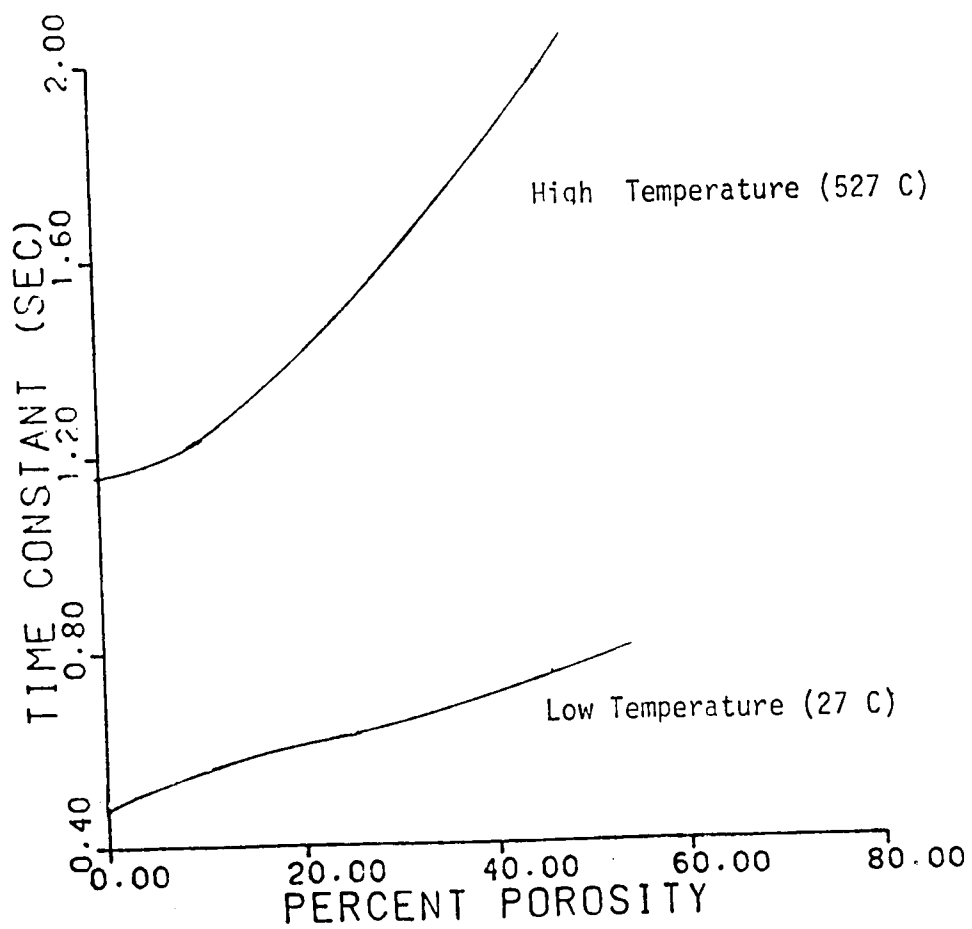


Figure 6.15 Porosity Effects on a Grounded Junction Version of Sensor A

Table 6.3

Physical Properties of the Materials of Sensor A Used in
the Study of the Porosity Effects on Sensor Response

Material	Temperature °C	Thermal Conductivity W/m.°C	Density kg/m ³	Specific Heat Constant J/kg.°C
% porosity	27	38.51	4000	786.97
Al ₂ O ₃	527	6.10	4000	1172.08
Al ₂ O ₃	27	26.16	3600	808.94
10% porosity	527	9.42	3600	1164.84
Al ₂ O ₃	27	20.93	3000	841.39
25% porosity	527	3.99	3000	1155.34
Al ₂ O ₃	27	15.7	2160	887.43
46% porosity	527	4.6	2160	1138.59
446 SS	27	20.32	7600	460.6
	527	24.51	7600	460.6

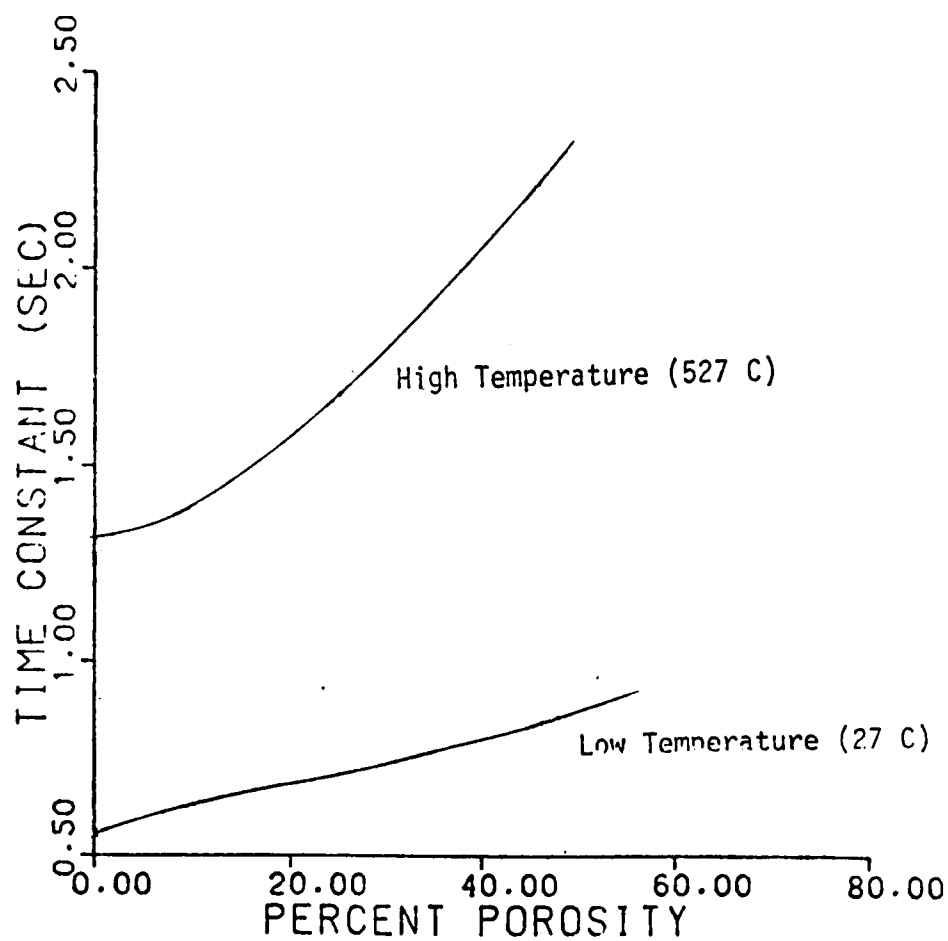


Figure 6.16 Porosity Effects on an Insulated Junction Version of Sensor A

transfer resistance created by the surrounding medium. It is used primarily to provide information on the electric current levels that can result in an accurate electric resistance measurements.

The SHI concept along with the new simulation package was used in making several studies. Those included the determination of the effects of the length of the sensing elements, the amount of heat generation, and the stem loss on sensor response. The relationship between the SHI and τ was also studied.

Three cases were necessary for this analysis. Those are

Case 1: Sensor F is used; and the sensing element is located in node (3, 2). (See Figure 6.1)

Case 2: Sensor F is used; and the sensing element is in nodes (3, 2) and (2, 2).

Case 3: Sensor F is used; and the sensing element is in nodes (3,2), (2, 2), and (1, 2).

Next, the effects of increasing the heat generation rate were determined using the previous simulations. The results show that increasing $\dot{Q}$ by a certain percentage, resulted in increasing the temperature by the same percentage.

Similar simulations were performed on the sensor after adding an extra layer of nodes on top of the original model. Figure 6.17 shows the modified model structure of sensor F. This is done to verify the existence and the extent of axial heat transfer as would occur if stem losses occur. If there is no axial heat transfer, then the sensor output would not change by adding a new row of nodes. The results

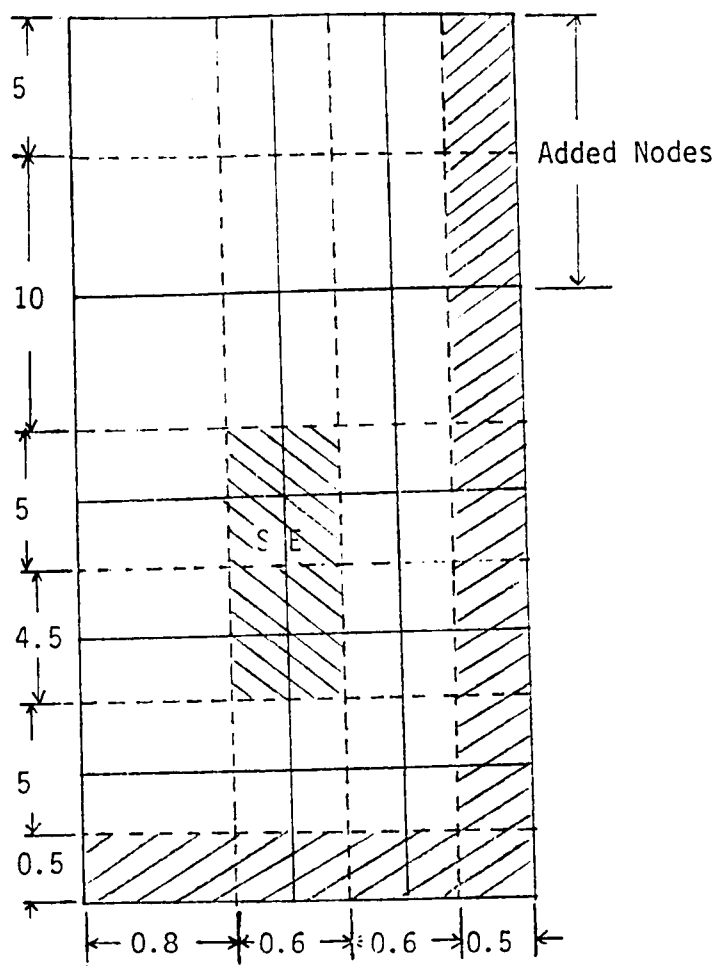


Figure 6.17 Nodal Structure of Sensor F with an Extra Layer of Material

show that the sensor output changes. The amount of change in output depends on the distance between the sensing element and the extra layer of nodes. The shorter the distance is the greater the effects. The time constant decreased by about 0.86%.

Figure 6.18 shows the results of the simulations performed in this section. As this figure indicates, the ratio of the temperature to Q decreases with adding extra nodes. This occurs because while Q remained constant, its effect was distributed over a larger volume. Thus the temperature difference is smaller and the ratio ($\Delta T/\Delta Q$) gets smaller.

Finally, simulations were performed on sensor F to investigate the relationship between the SHI and the time constant of the sensor. The radial surface thermal resistance was varied by varying h . For each value of h two simulations were performed. The first simulation produces the SHI (as in a self-heating test). The results are shown in Figure 6.19.a. These results confirm earlier experimental results obtained by other workers (5).

Theory (5), on the other hand, indicates that changing τ by a certain percentage should result in changes in the SHI by the same amount. To investigate this phenomenon, new simulations were performed on two modified versions of sensor B. The first modification included moving the sensing elements from node (2,1) to nodes (1,2) and (2,2) to simulate an RTD. The second modification included converting the model to 1×4 (i.e. the sensor is simulated by one row of nodes). The results of the simulations are presented in Figure 6.19.b. For

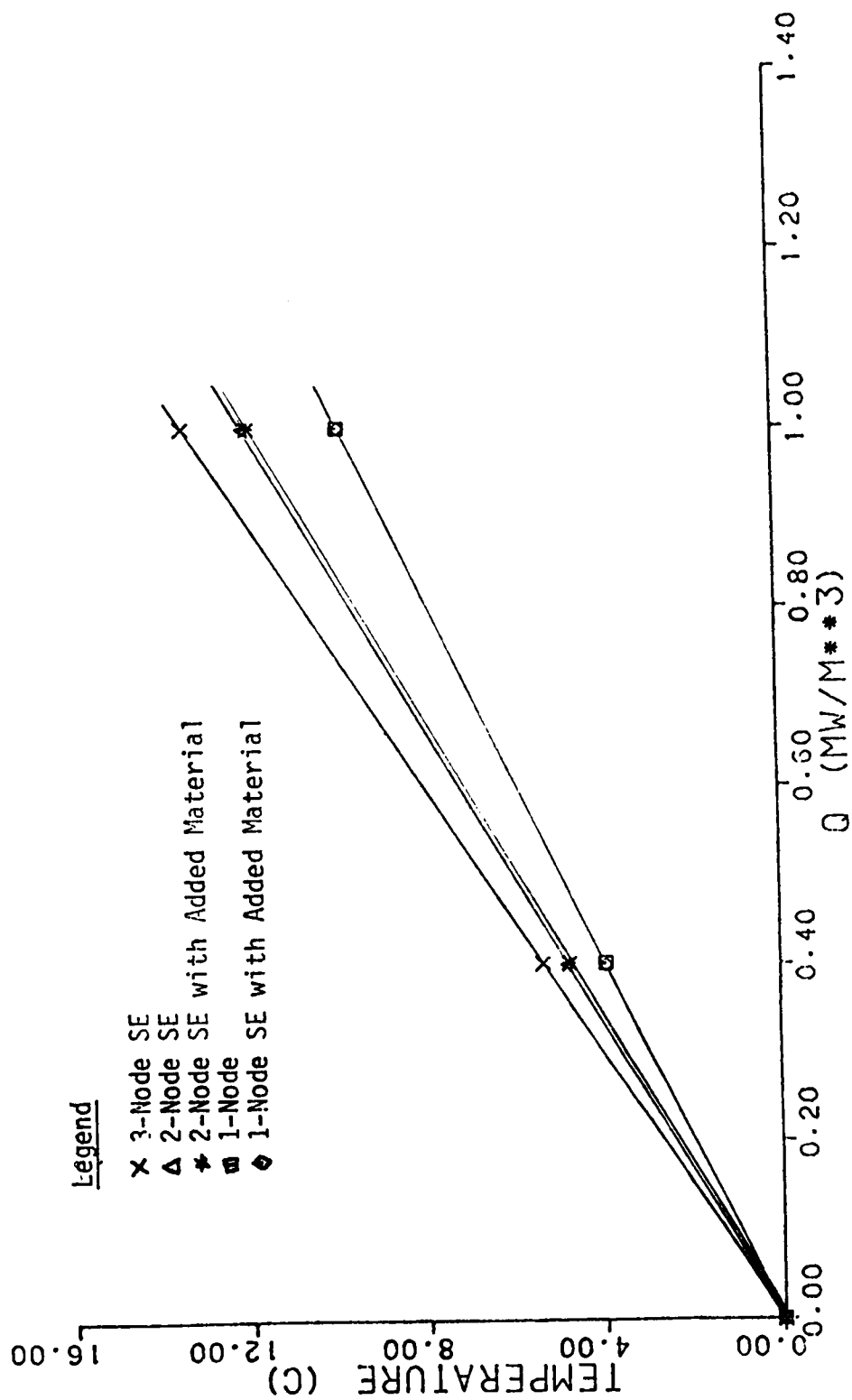
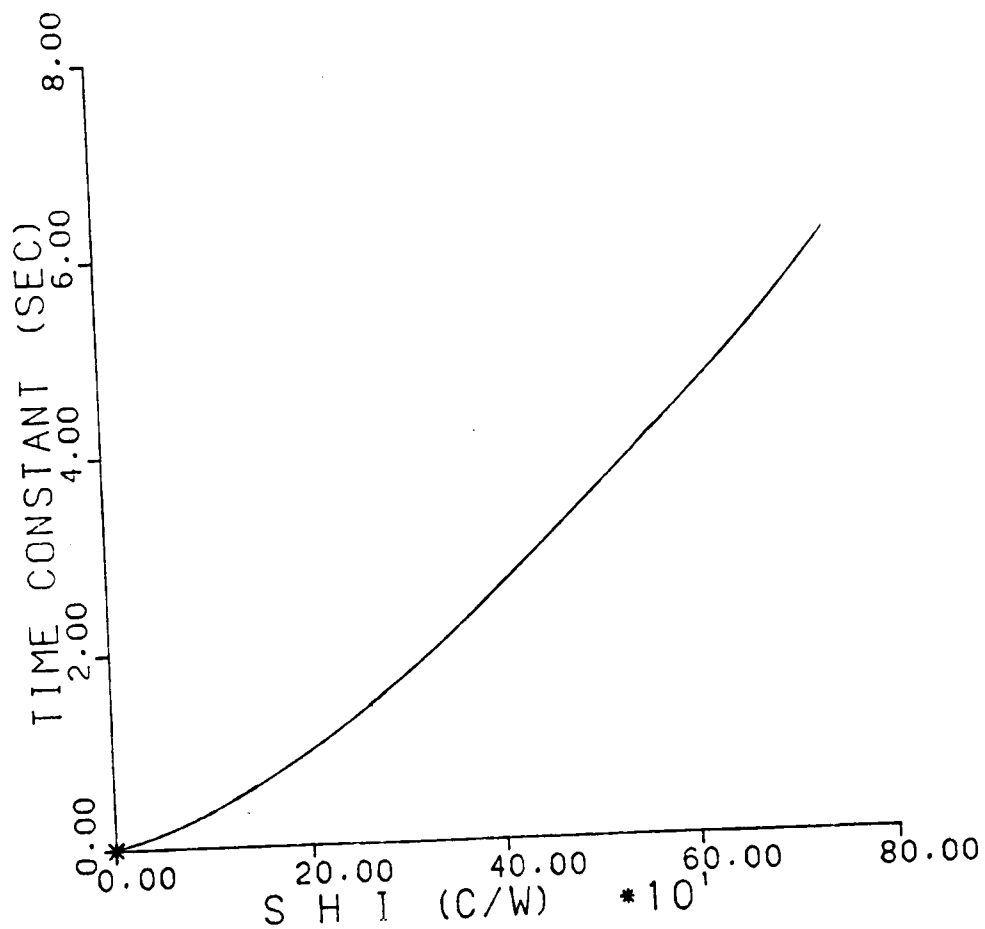
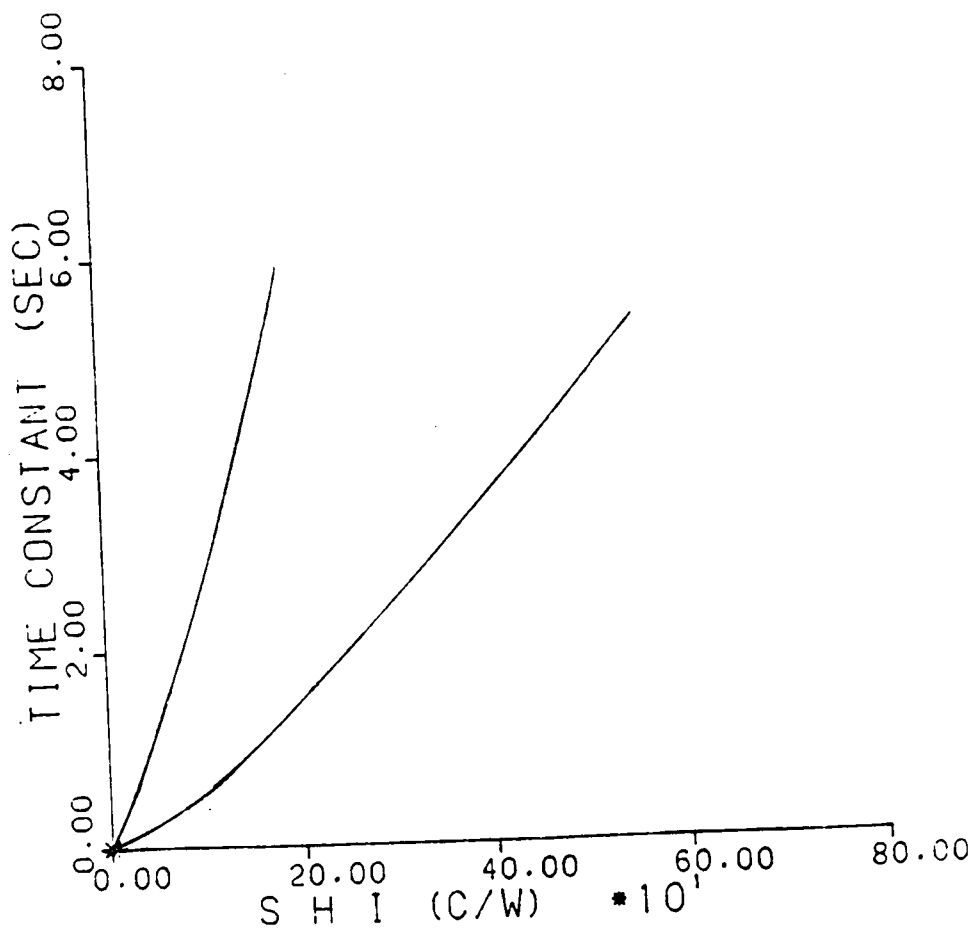


Figure 6.18 Effects of Sensing Element Length and Stem Loss



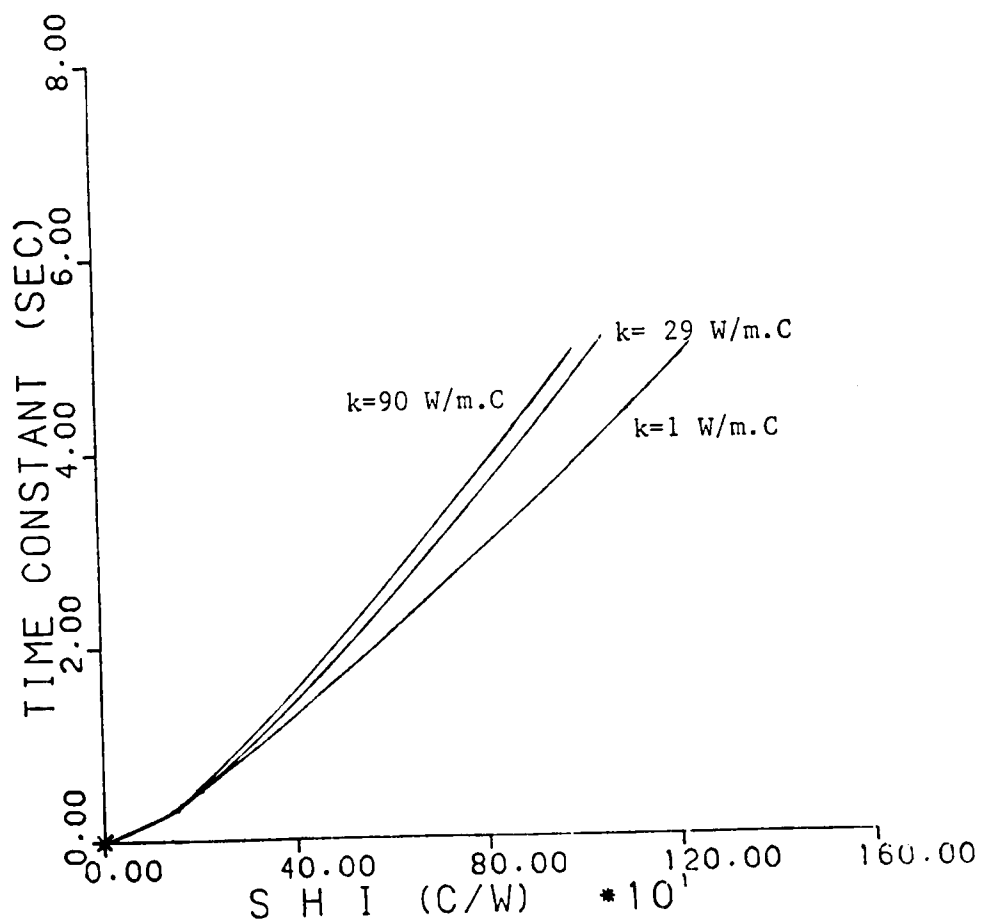
(a) The Relationship Between the Time Constant Obtained in an Ambient Temperature Transient and the Self Heating Index for Sensor B

Figure 6.19 The Relationship Between the Self-Heating Index and the Time Constant of the Sensor



(b) Self Heating Effects for 1-Dimensional and 2-dimensional Heat Transfer (for Sensor B)

Figure 6.19 (Continued)



(c) Effects of Changing the Thermal Conductivity of the Top Row of Nodes of Sensor B on τ vs. SHI Relationship

Figure 6.19 (Continued)

1-dimensional case, τ varies linearly with the SHI. This indicates that the sensor behaves in the manner indicated in Figures 6.19.a and 6.19.b (2-dimensional case) because of the presence of 2-dimensional heat transfer.

Finally, more simulations were performed on another modified version of sensor B. The new modification included homogenizing the sensor material. The thermal conductivity of the top row of nodes was varied to change axial thermal resistances. Three cases were considered. Those were at $k=1$ w/m. $^{\circ}$ C, $k=29$ w/m. $^{\circ}$ C (reference case), and $k=90$ w/m. $^{\circ}$ C. Eight simulations were performed for each case to determine the self-heating index and the corresponding plunge time constant of the sensor. The results were plotted as shown in Figure 6.19.c. The best straight line fit through the data points was found for each case by using the least-squares fit method. The deviations of the data points from the best fit lines were determined to find which case approached the linear relationship. The results showed that as the axial thermal resistances of the top row of nodes were increased by decreasing k , the relationship between τ and SHI approached linearity. This was expected since increasing the thermal resistances resulted in reduction of the axial heat transfer, which in turn led to mostly radial heat transfer. This caused the sensor to behave more like a first order system. Hence, the τ vs SHI curve approached linearity.

6.2 Experimental Results

6.2.1 Liquid Tests Results

This section focuses on the experimental results of the tests

performed in five different liquids at several velocities and on five sensors. The prime objective of these tests is to verify the correlation (4.1) repeated here for convenience.

$$\tau = C_1 + C_2/h \quad (4.1)$$

To check the postulate that τ changes linearly with $1/h$, the film heat transfer coefficient must be evaluated. To do that the following correlations were used: correlatons a, b, c, c', d, d', e in section 4.2. These correlations were investigated in an attempt to select a correlation that produces the best linear relation between τ and $1/h$.

Once a "good" heat transfer correlaton is selected, a new correlaton of the form

$$\tau = C_1 + C_3 U^{-n} \quad (4.9)$$

can be obtained, where n is the power of the Reynolds number in the selected correlation. All the liquids used in the tests along with their physical properties have been presented in Table 4.2.

The sensors used in the tests are listed in Table 6.4. The average time constant in water at room temperature and at 1 m/s, for these sensors ranged from .8 seconds to 9.5 seconds. This is a fairly wide range of response times.

The purpose of tests performed in the Loop Testing Facility was to show that at any velocity measurement, the time constants obtained were in agreement with those in the Rotating Tank Testing Facility. This implies that the heat transfer correlation applicable to the

Table 6.4
Sensors Used in the Liquid Tests

Sensor		Outer Diameter (mm)
Thermocouple	Omega: - TJ36-CASS-14U-12	6.25
Thermocouple	Omega: - TJ36-CASS-14U-12	6.20
Thermocouple	Omega: - TJ36-CASS-18U-12	3.05
Thermocouple	Omega: - TJ36-CASS-18U-12	3.07
RTD	RDF: - YHPOD1-18-3A	6.35

loop facility would be applicable to the rotating tank. If this is so then the rotating tank simulated the loop well. Results for three sensors are shown in Tables 6.5, 6.6 and 6.7.

The results show that, for larger thermometers (6.25 mm TC and 6.35 mm RTD), agreement is within 6% except at the highest velocity of 1.4 m/s. At the highest velocity, the fluid dynamic phenomena are peculiar to the rotating tank. A number of tests have shown that at the highest velocity, abnormally high time constants were obtained. The reasons for this are not yet known. Results for the smaller thermometer (3.05 mm O.D.) are not as good as for the larger thermometers but appear to be reasonable since there is relatively wide variability of data in the loop.

Results from the rotating tank will be presented in graphical form. Time constants are averages of three or more runs. Figures 6.20 through 6.24 show the results for each of the sensors used. Each sensor's results are those in the five liquids used. The abnormal behavior of response times for the highest velocity of 1.4 m/s are not presented for all the sensors. For oil, abnormal results were not observed, presumably because of the high viscosity. High viscosity leads to less turbulence around the sensor and a more stable flow pattern. It should be noted that the thermocouples of approximately the same diameter (6.25 and 6.20mm) produced different values of time constants. One gave 3.5 to 9.7 seconds and the other gave 2.0 to 7.7 seconds. This can be attributed to the internal distribution of materials. Presumably, the magnesium oxide in the second sensor was

Table 6.5

Correspondence of Rotating Tank and Straight Pipe
Response for 3.05 mm TC

Velocity m/s	Loop τ (sec)	Rotating Tank τ (sec)	%Difference
1.4	.838	.832	0.7%
1.0	.876	.704	19.6%
.91	.868	.784	9.6%
.67	.930	.824	11.4%
.57	.978	.836	14.5%
.37	1.01	.868	14.0%

Table 6.6

Correspondence of Rotating Tank and Straight Pipe
Response for 6.25 mm TC

Velocity m/s	Loop τ (sec)	Rotating Tank τ (sec)	%Difference
1.4	3.75	4.40	17.4%
1.0	3.79	3.64	3.9%
.91	3.79	3.66	3.4%
.67	3.82	3.82	0 %
.57	3.98	3.84	3.5%
.37	4.21	3.95	6.1%

Table 6.7

Correspondence of Rotating Tank and Straight Pipe
Response for 6.35 mm RTD

Velocity	Loop τ	Rotating Tank τ	%Difference
1.4	7.87	9.38	1.9%
1.0	7.94	8.15	2.6%
.91	8.09	8.4	3.8%
.67	8.3	8.45	1.8%
.57	8.16	8.34	2.2%
.37	8.48	8.50	.2%

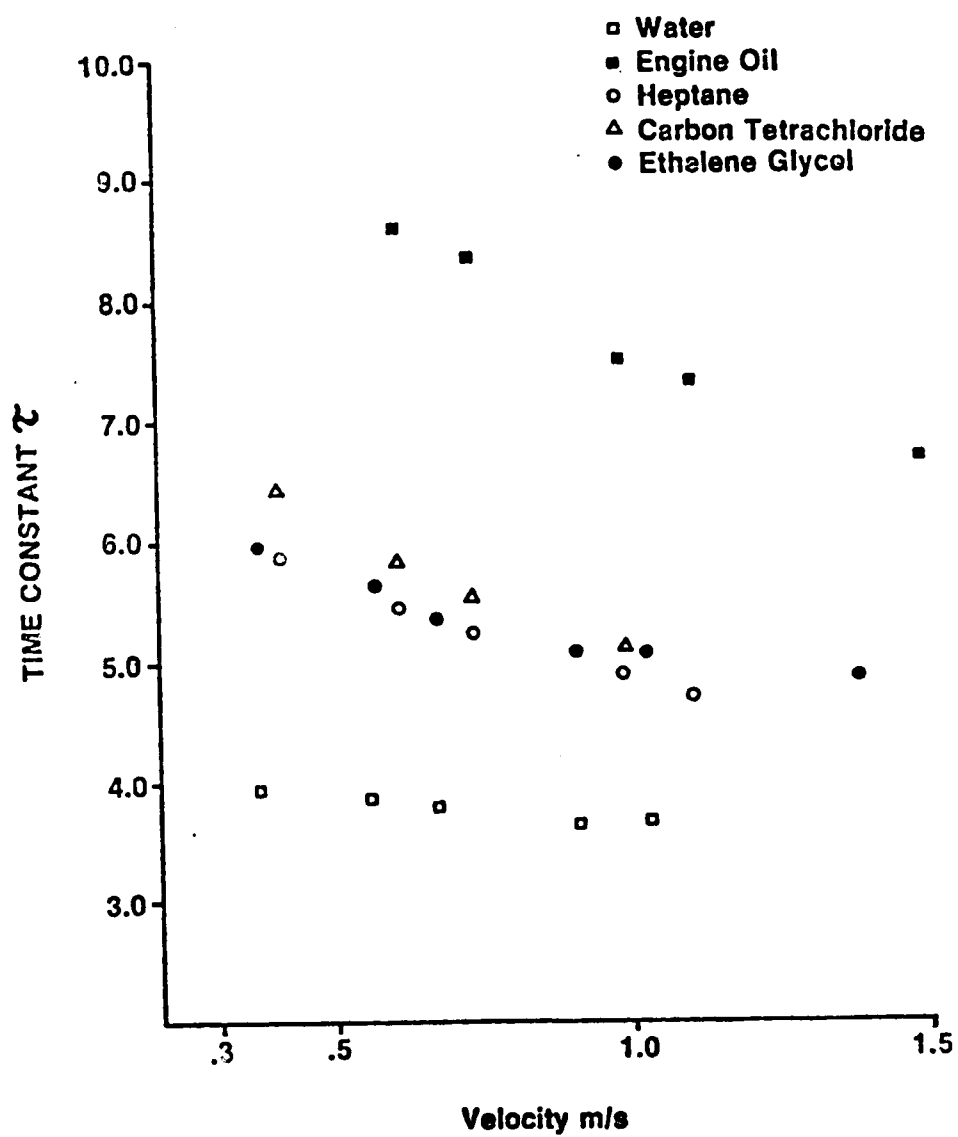


Figure 6.20 Results for 6.25 mm. Thermocouple

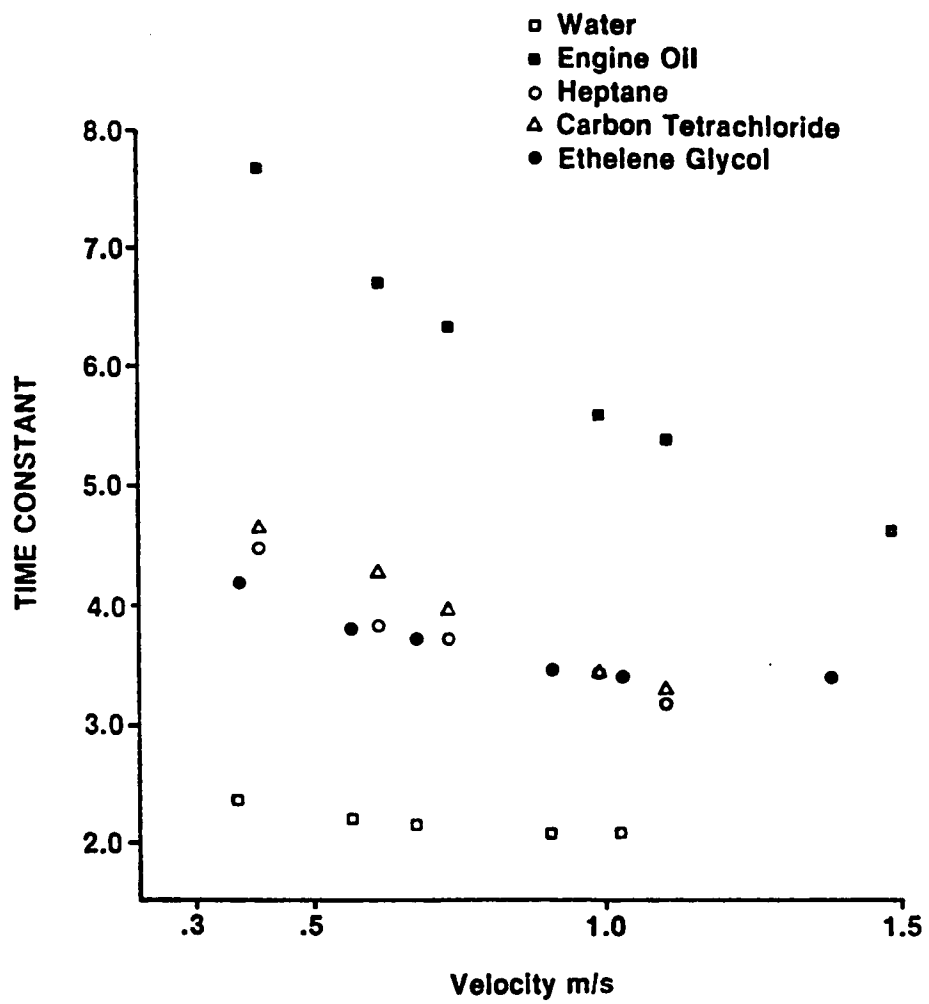


Figure 6.21 Results for 6.20 mm. Thermocouple

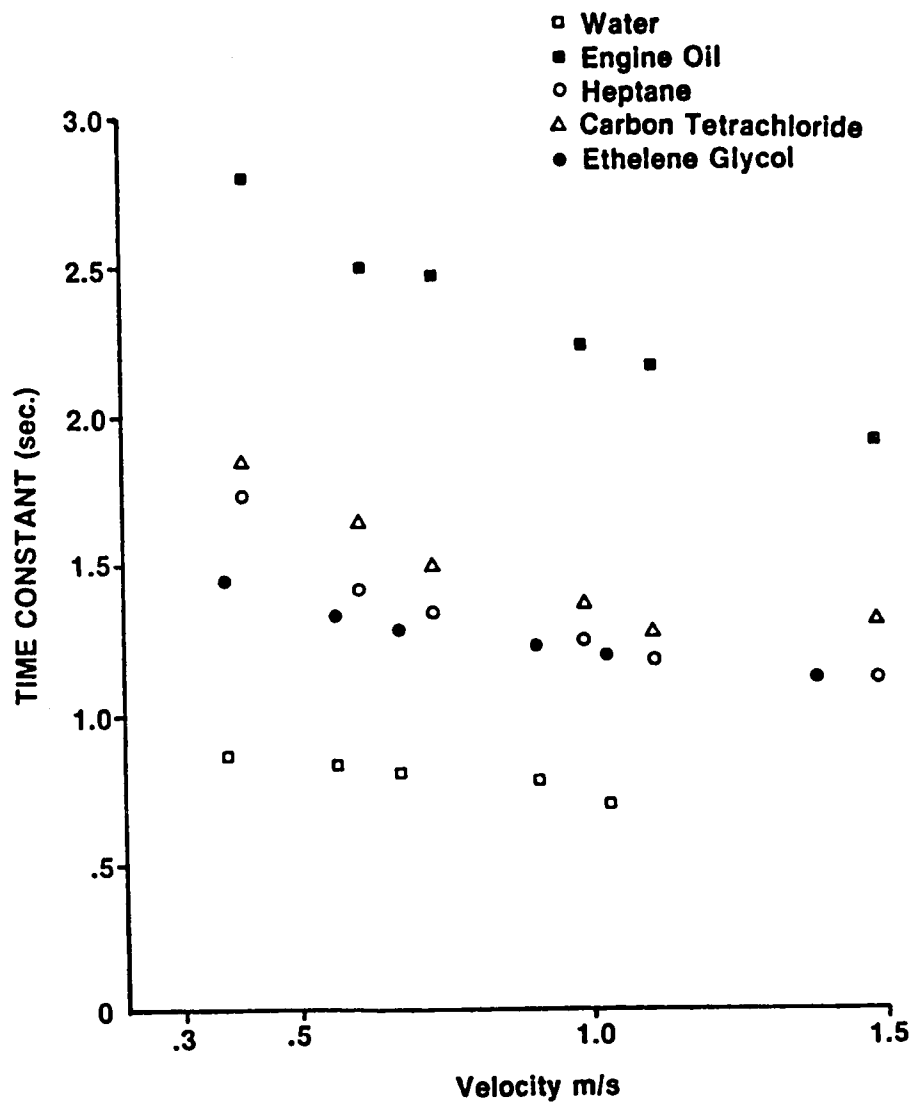


Figure 6.22 Results for 3.05 mm. Thermocouple

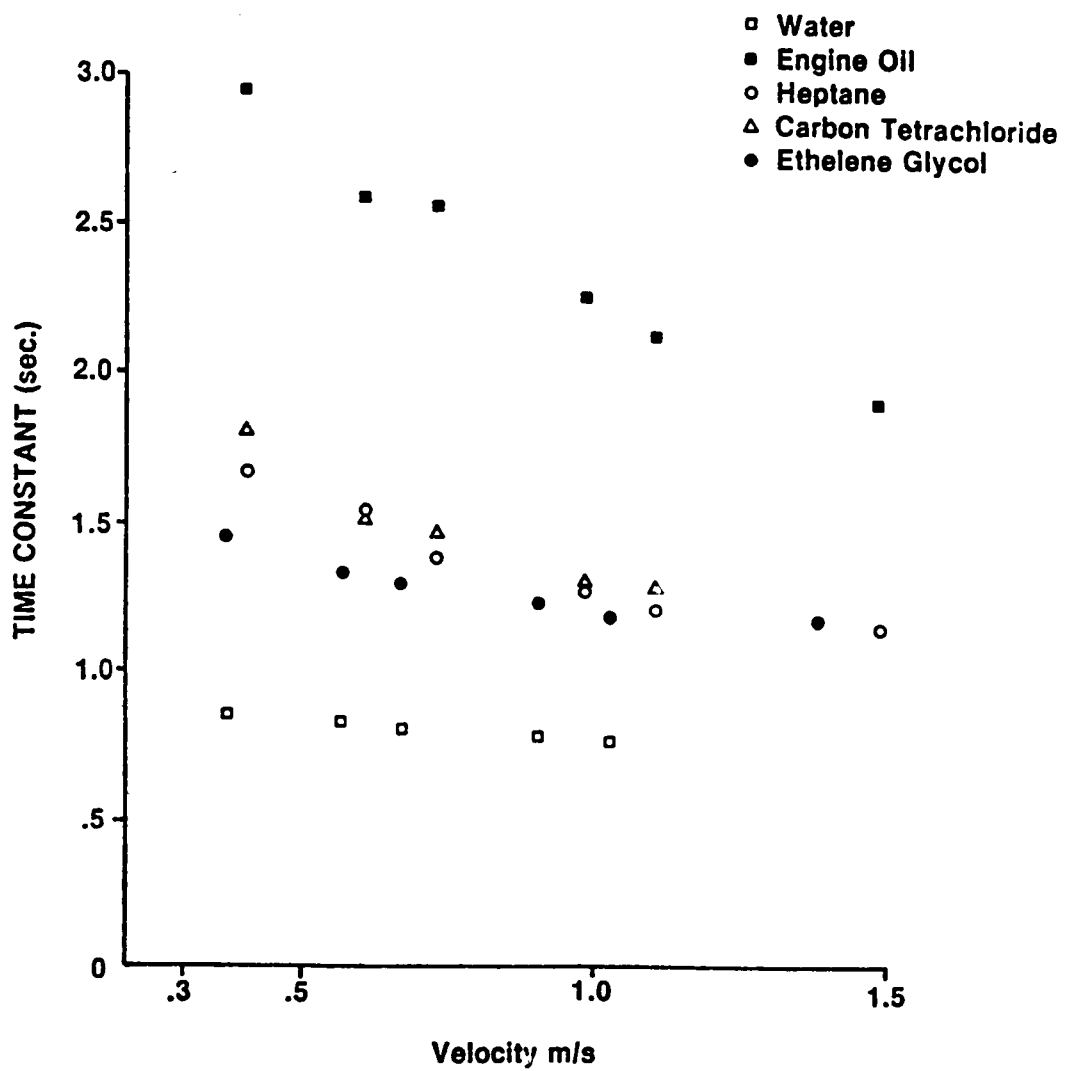


Figure 6.23 Results for 3.07 mm. Thermocouple

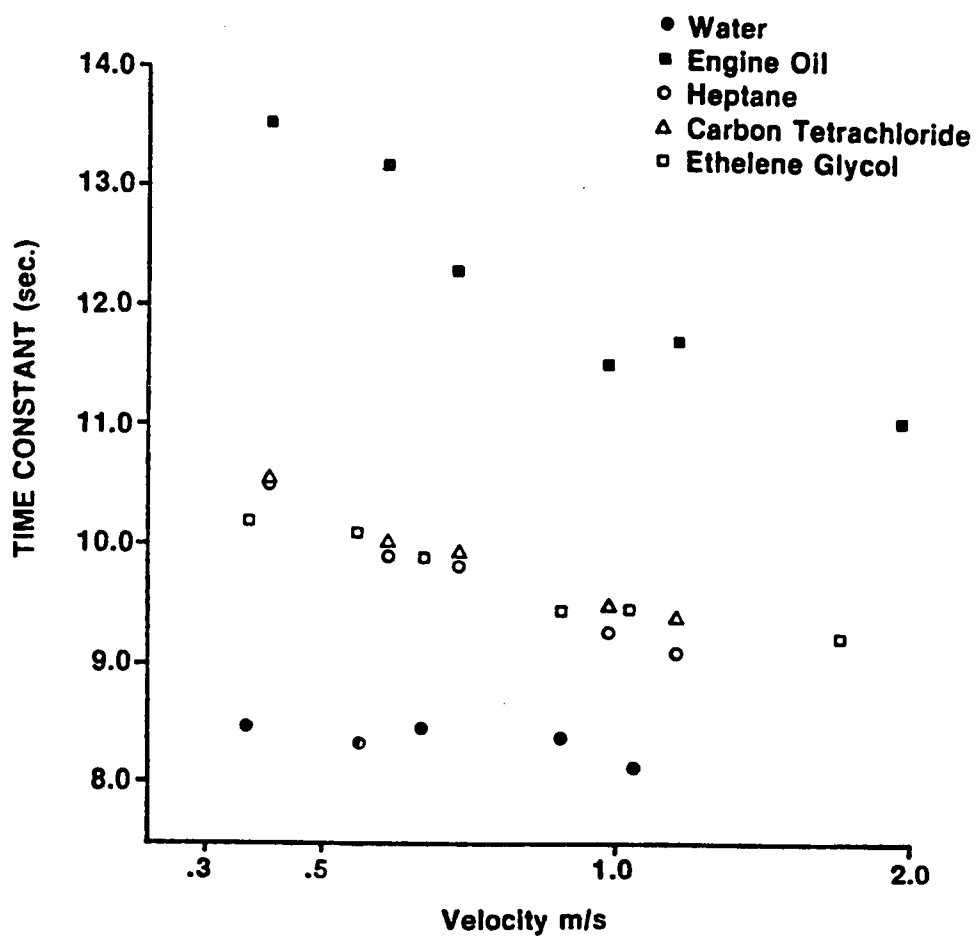


Figure 6.24 Results for 6.35 mm. RTD

more compacted giving a faster time constant range or the measuring junction was closer to the surface. In all cases the times of response in water increases two to three times by changing to engine oil for tests at low velocities. For all sensors, there is a wider variation of time constants (with respect to velocity variations) for engine oil than water.

The ranges of Reynolds numbers covered by each of the fluids for the sensors used are given in Table 6.8. The very low Reynolds numbers for engine oil lie outside the ranges of applicability of all the correlations presented above. Therefore, the oil test results were not considered in the analysis.

The inverse of the calculated values of h can be plotted against the time constants. Figures 6.25 through 6.32 present the results for all the correlations for the 6.25 mm sensor. The results show that correlation f and the experimental results exhibit the greatest deviation from linearity. The best linear relation has been observed when correlations such as the Churchill-Bernstein formula (Equation e) and the Perkins-Leppert formula (Equation b) are used. This phenomena has been observed for all the sensors tested. These correlations apply for a wide range of Reynolds number.

The results for these tests could also potentially provide a basis for deciding whether the time constant should vary as a velocity to the -0.5 power as in most of the correlations, or to -0.6 power (Equation f). Results for correlating data with Equation (4.2) and for $n = +0.5$ are shown in Figures 6.33 - 6.37. Similar results for $n =$

Table 6.8

Ranges of Reynolds Number Obtained in the Rotating Tank

Liquid	Reynolds Numbers
Water	$1 \times 10^3 < Re < 4.5 \times 10^3$
Ethylene Glycol	$50 < Re < 5 \times 10^2$
Heptane	$2 \times 10^3 < Re < 1.6 \times 10^4$
Carbon Tetrachloride	$1.6 \times 10^3 < Re < 1.5 \times 10^4$
Engine Oil	$1 < Re < 10$

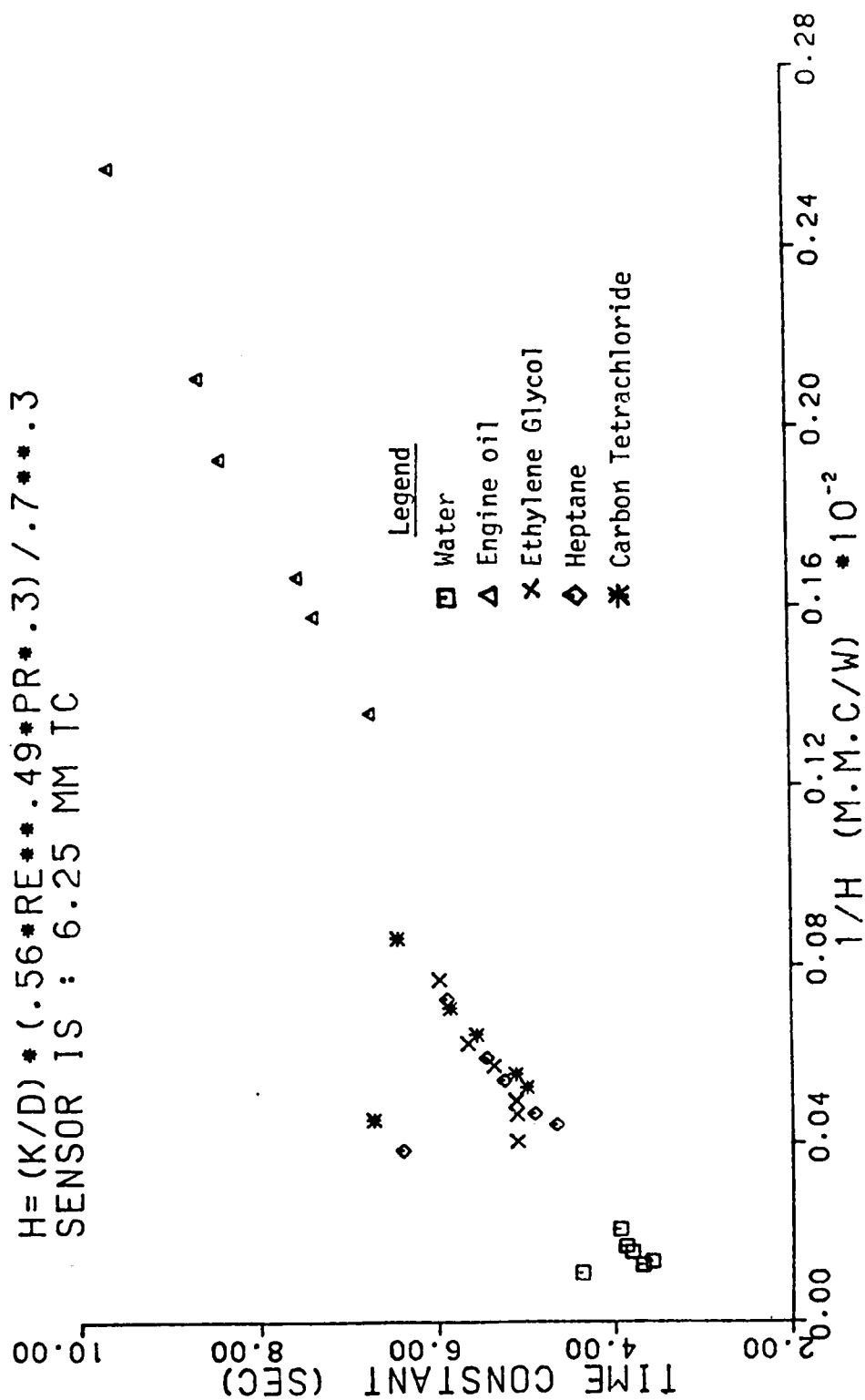


Figure 6.25 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation a

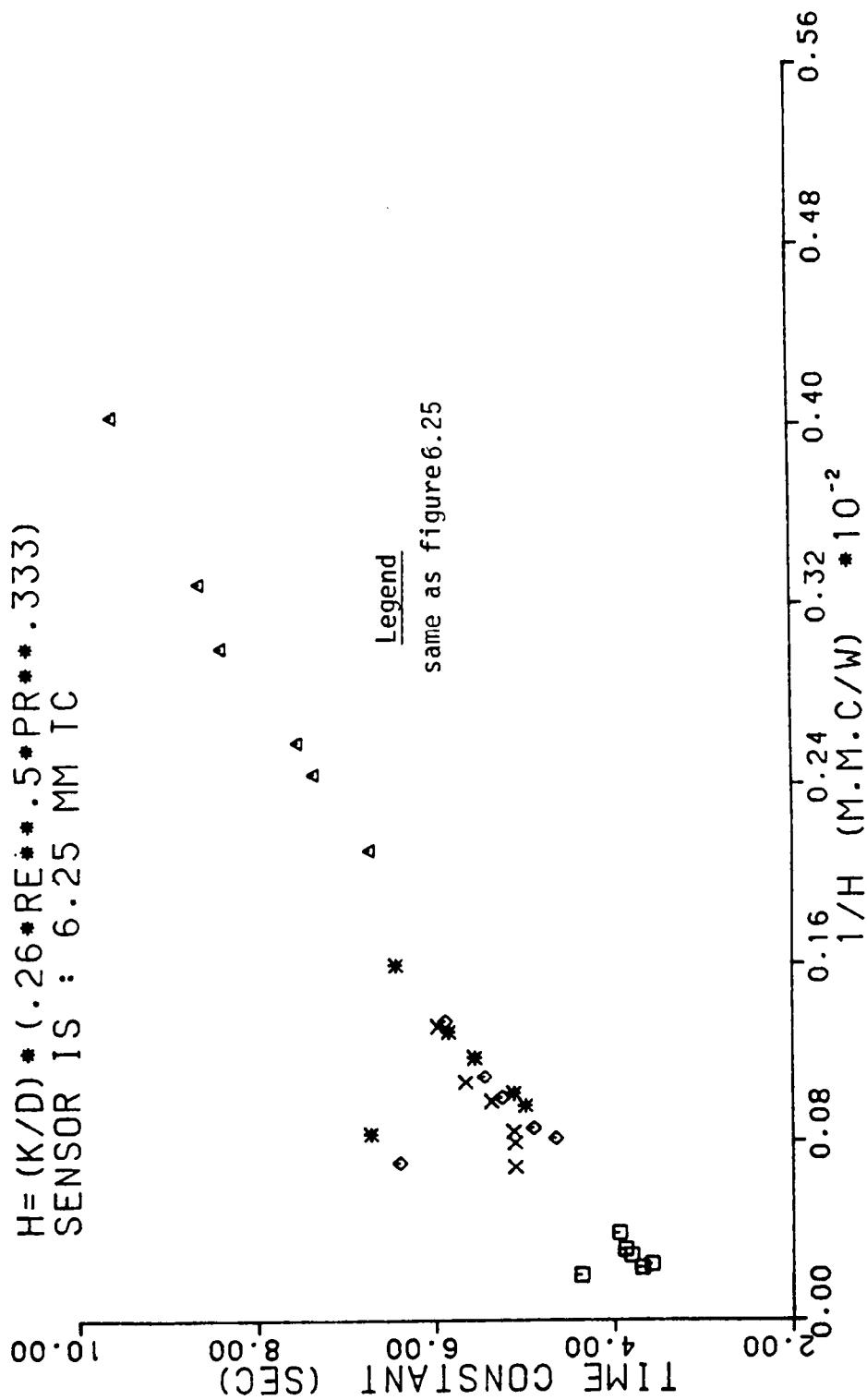


Figure 6.26 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation b

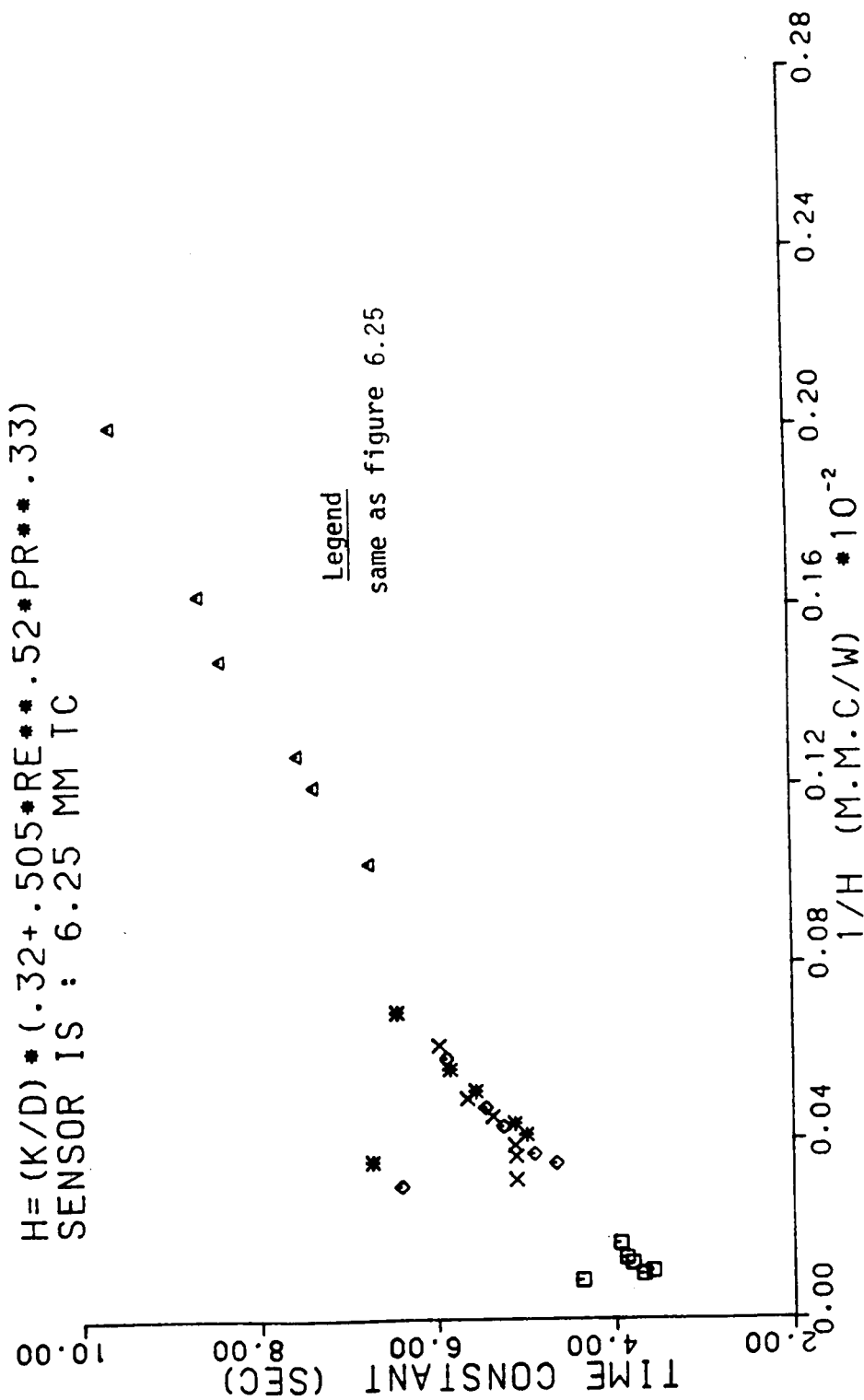


Figure 6.27 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation c

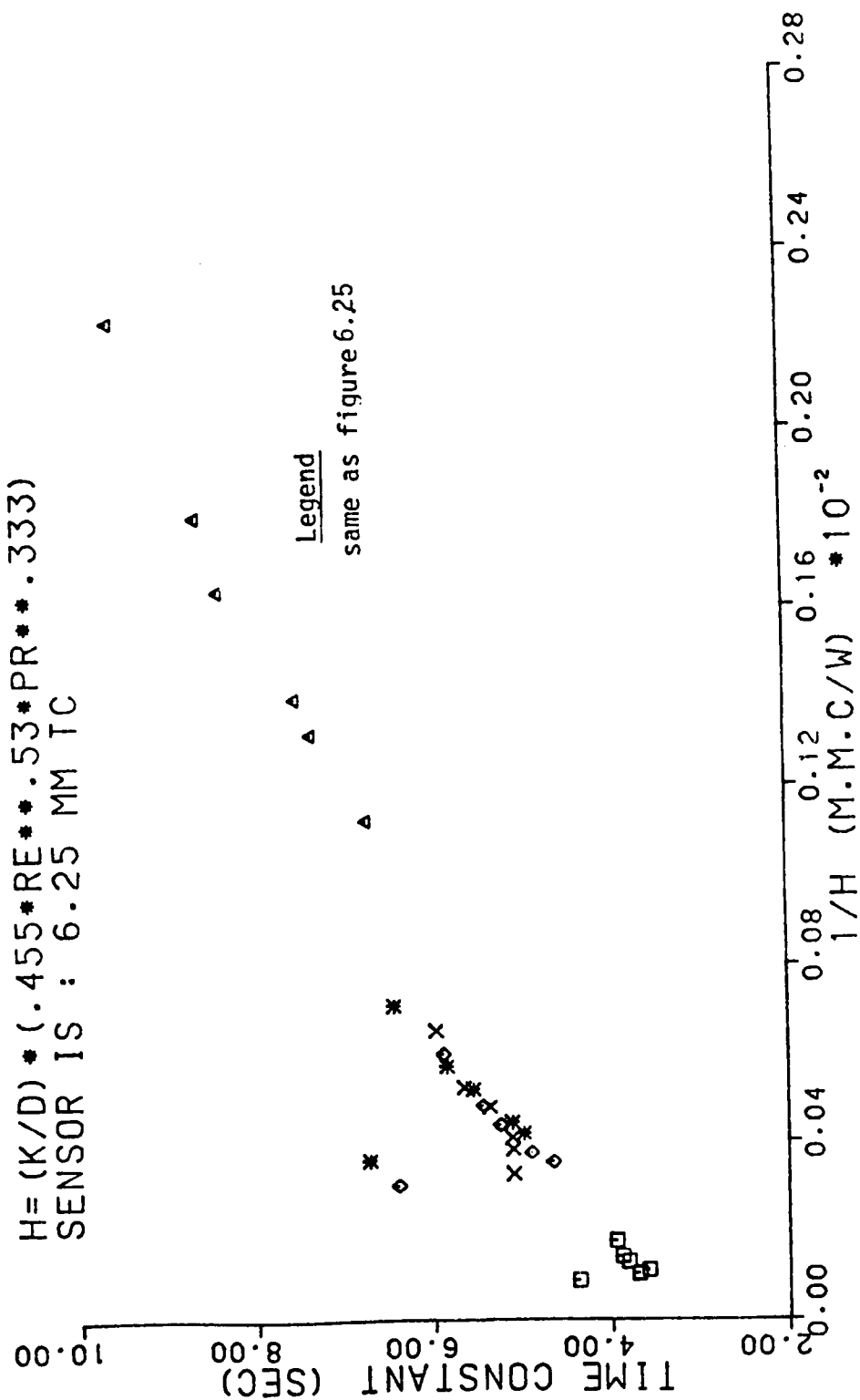


Figure 6.28 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation c'

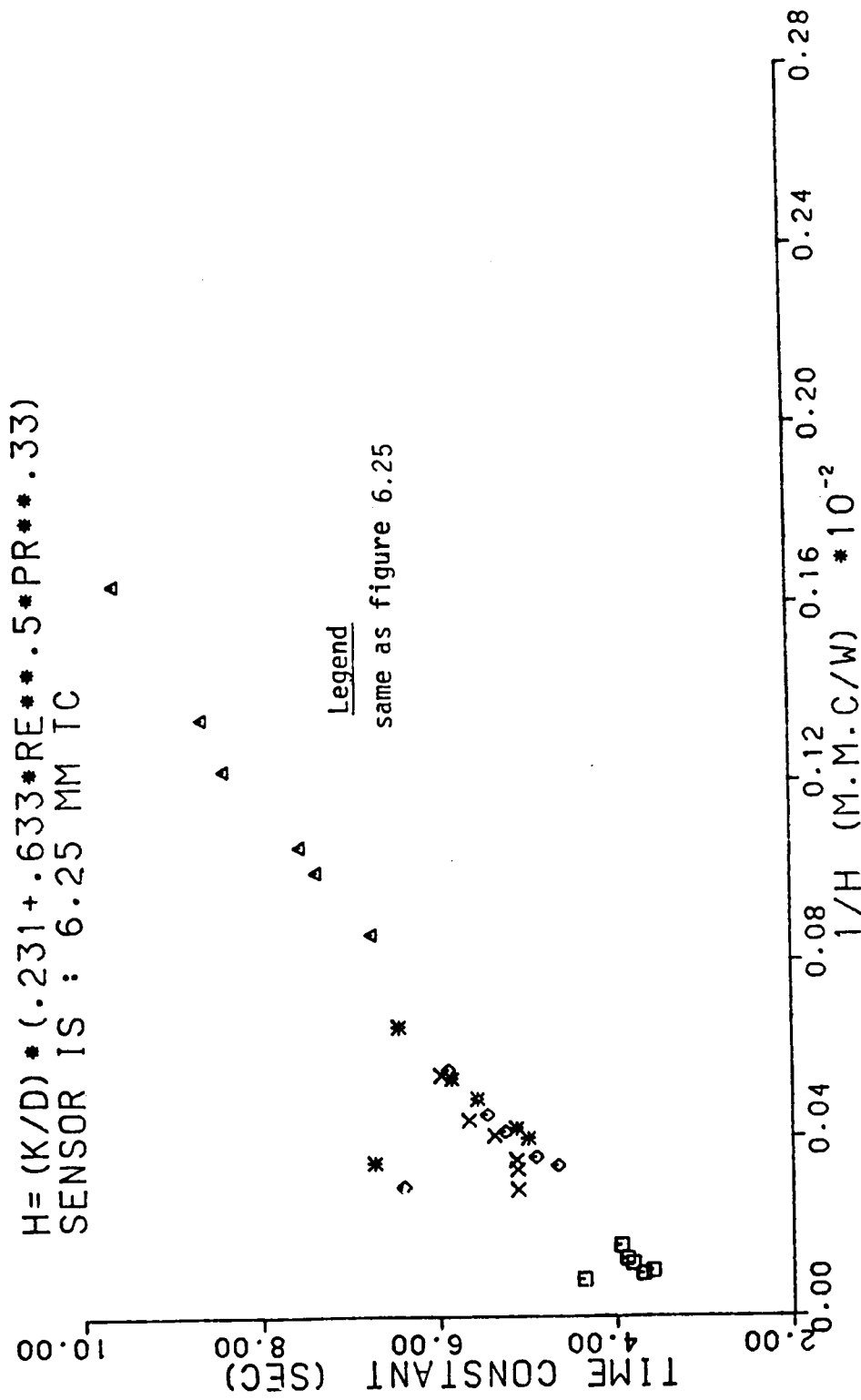


Figure 6.29 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation d

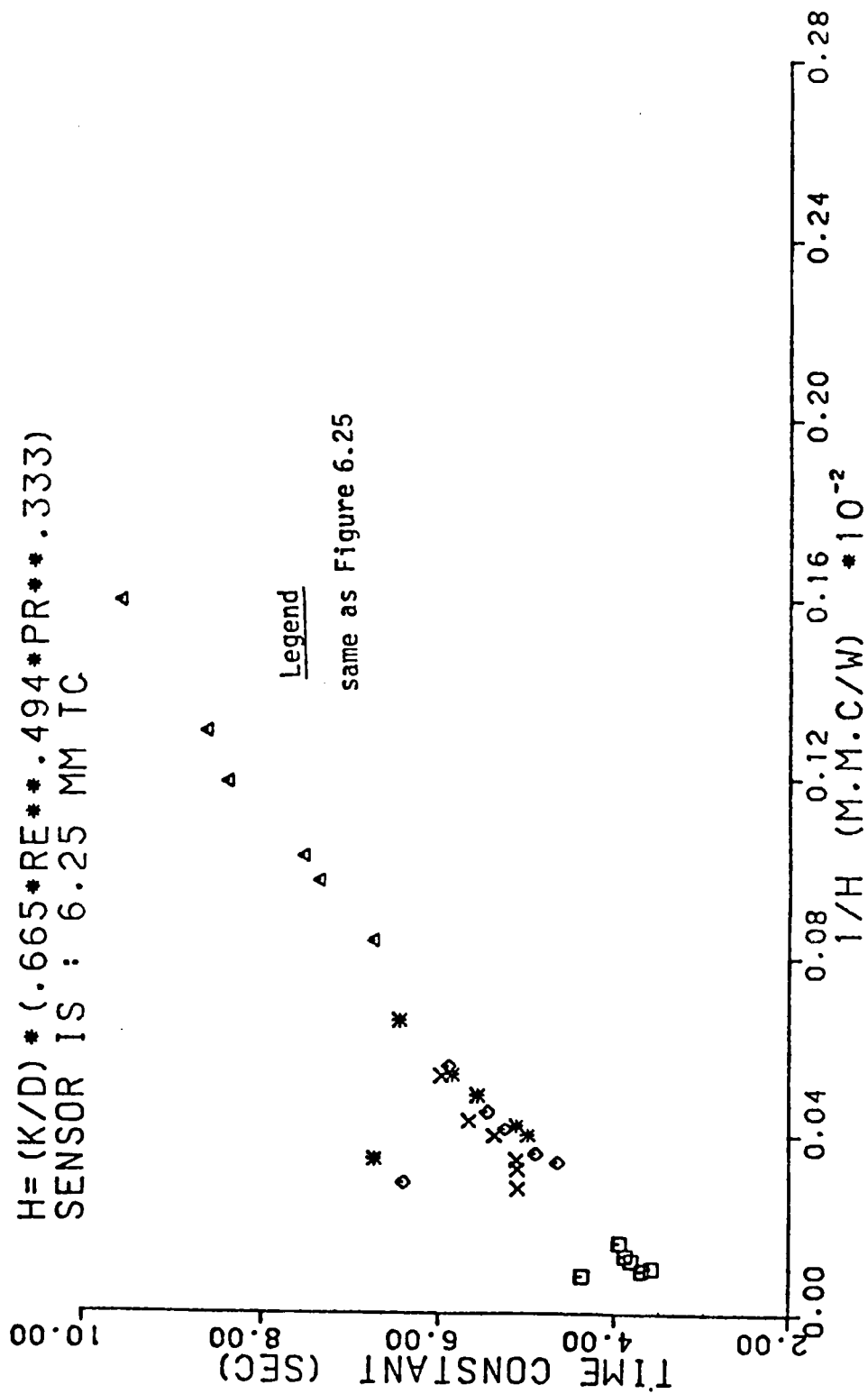


Figure 6.30 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation d'

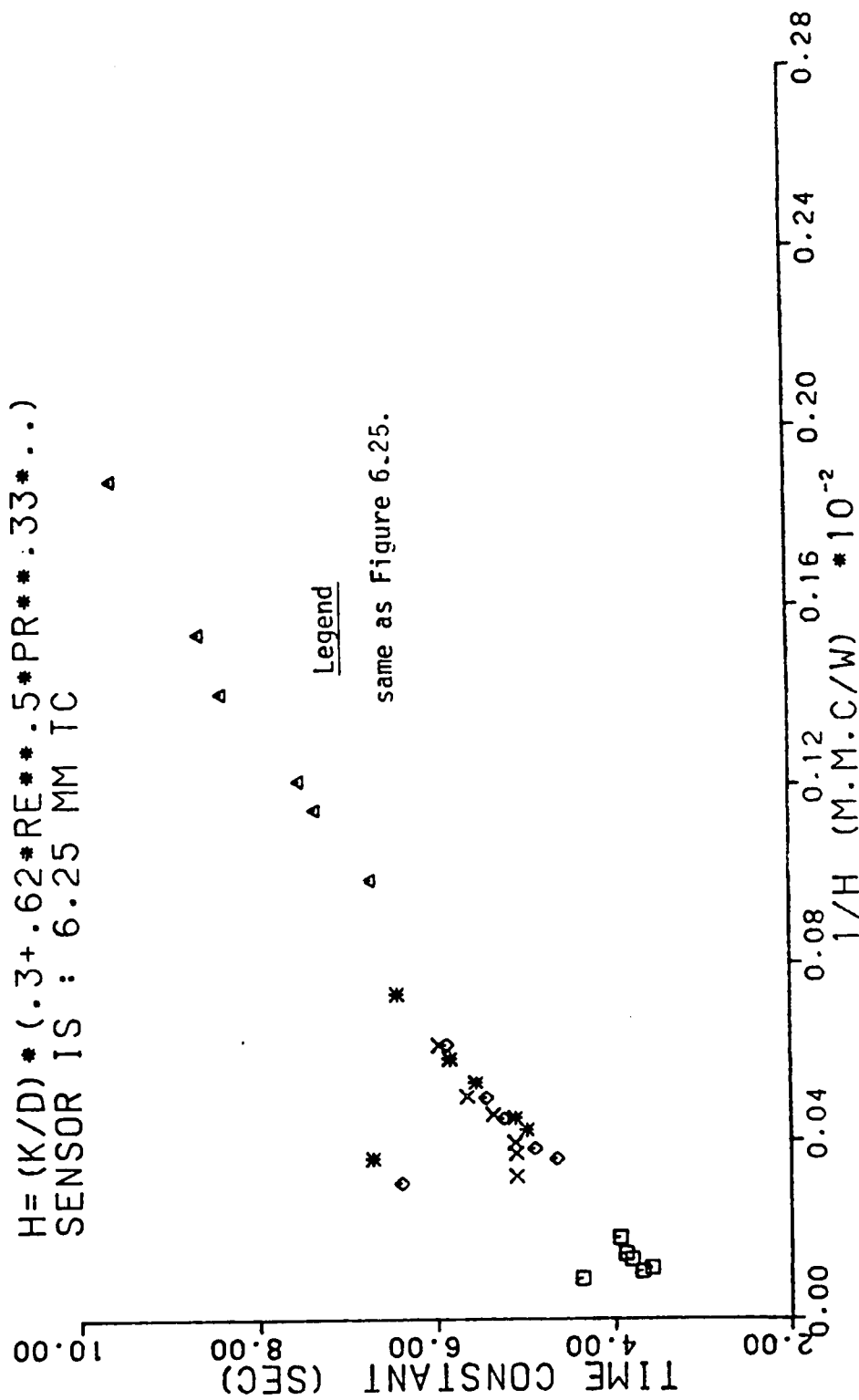


Figure 6.31 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation e

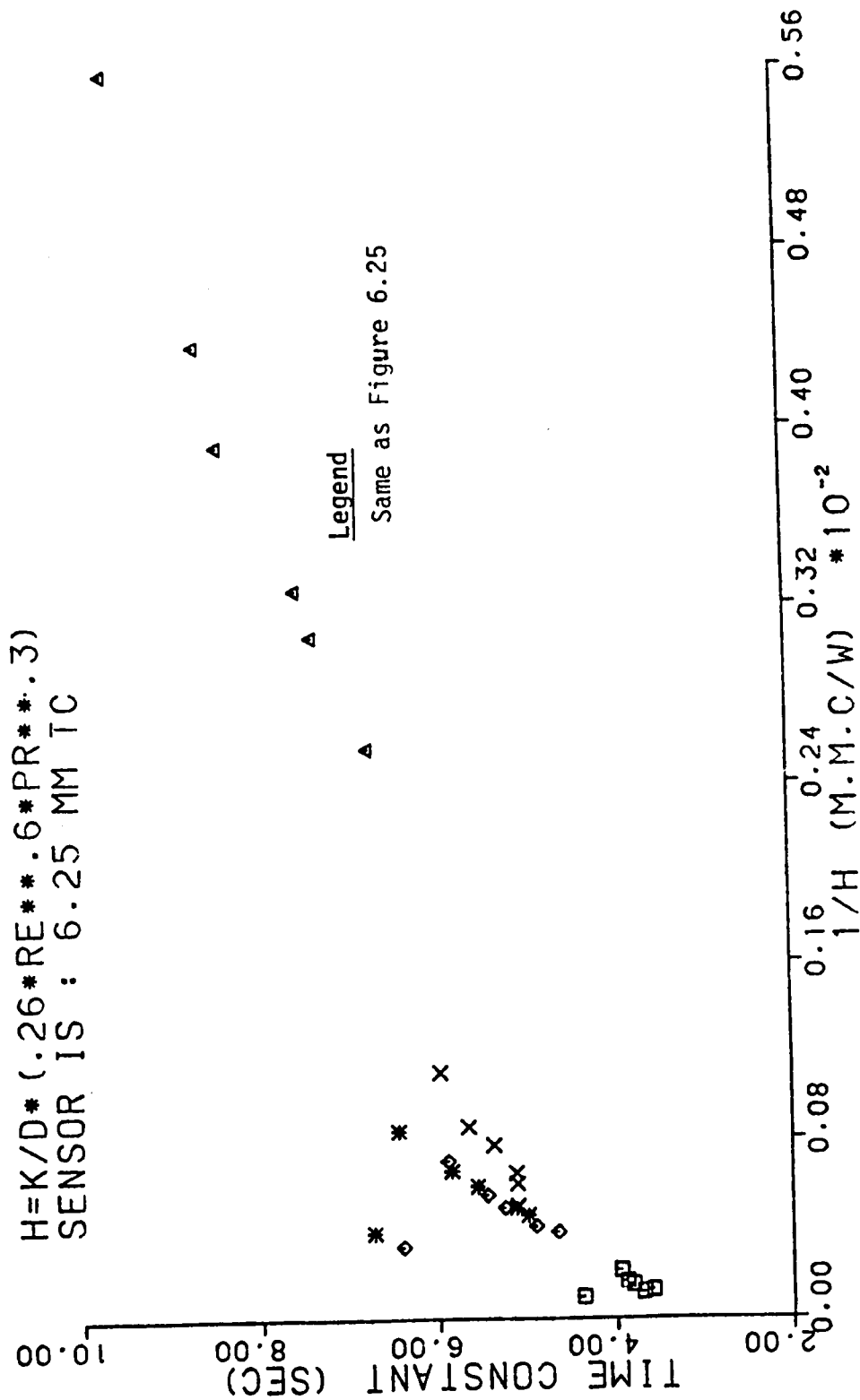


Figure 6.32 Application of Formula 4.1 for 6.25 mm. Thermocouple and Heat Transfer Correlation f

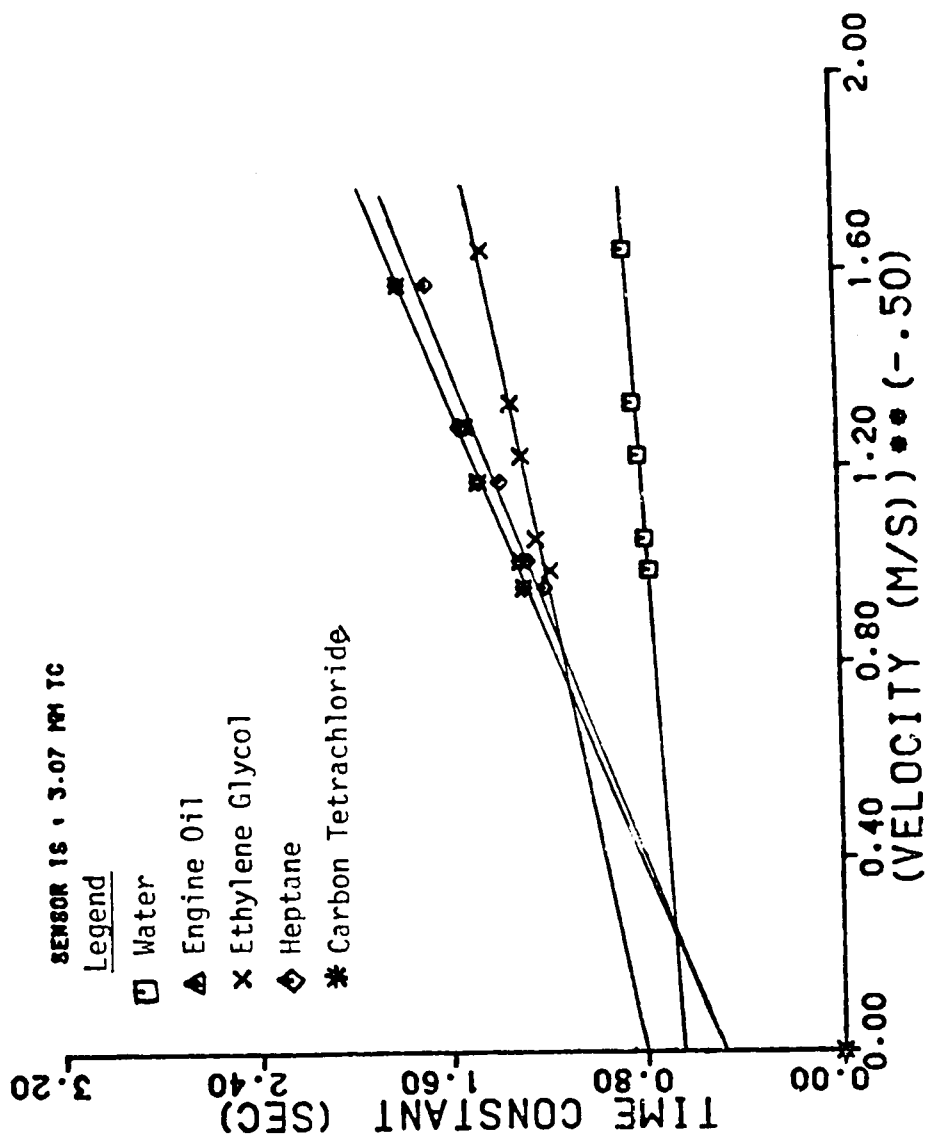


Figure 6.33 Application of Formula 4.9 for $n=0.5$ and a 3.07 mm Thermocouple

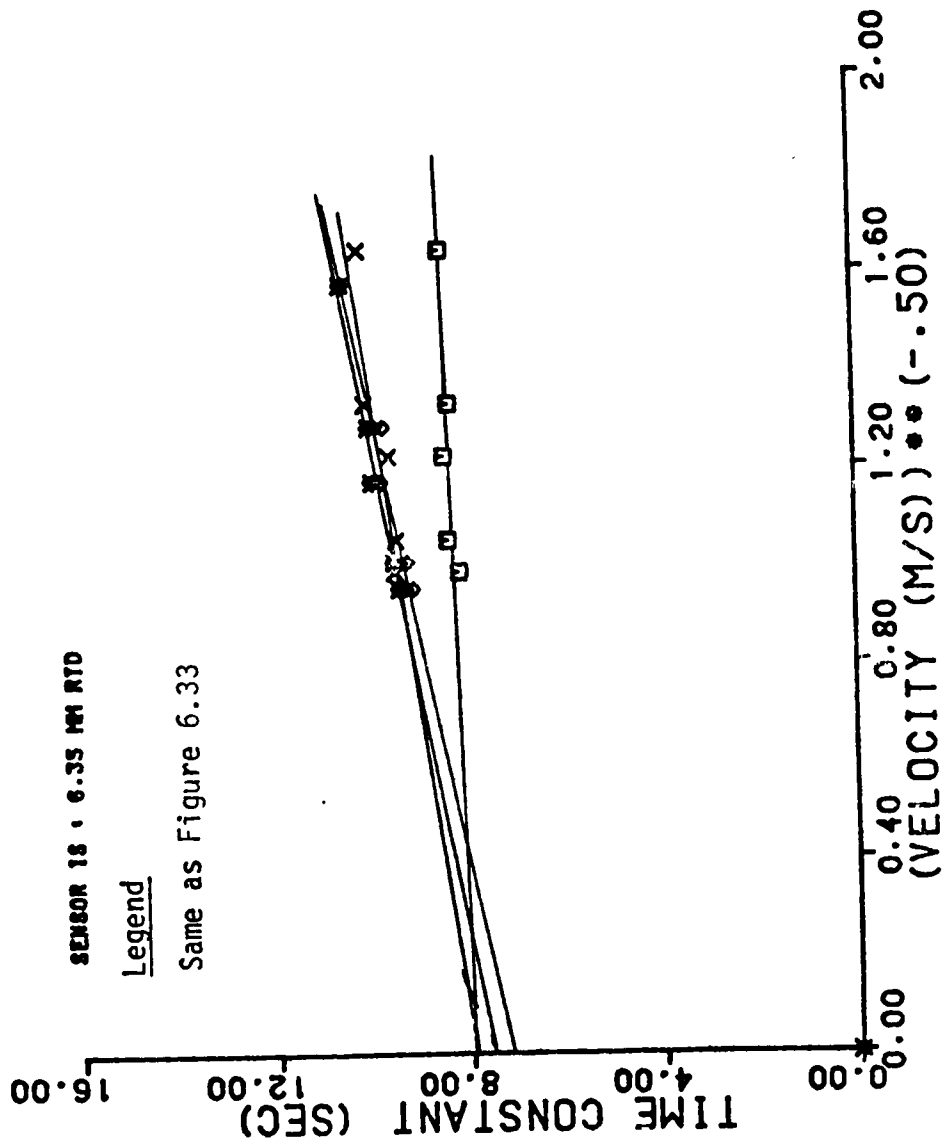


Figure 6.34 Application of Formula 4.9 for $n=0.5$ and a 6.35 mm. RTD

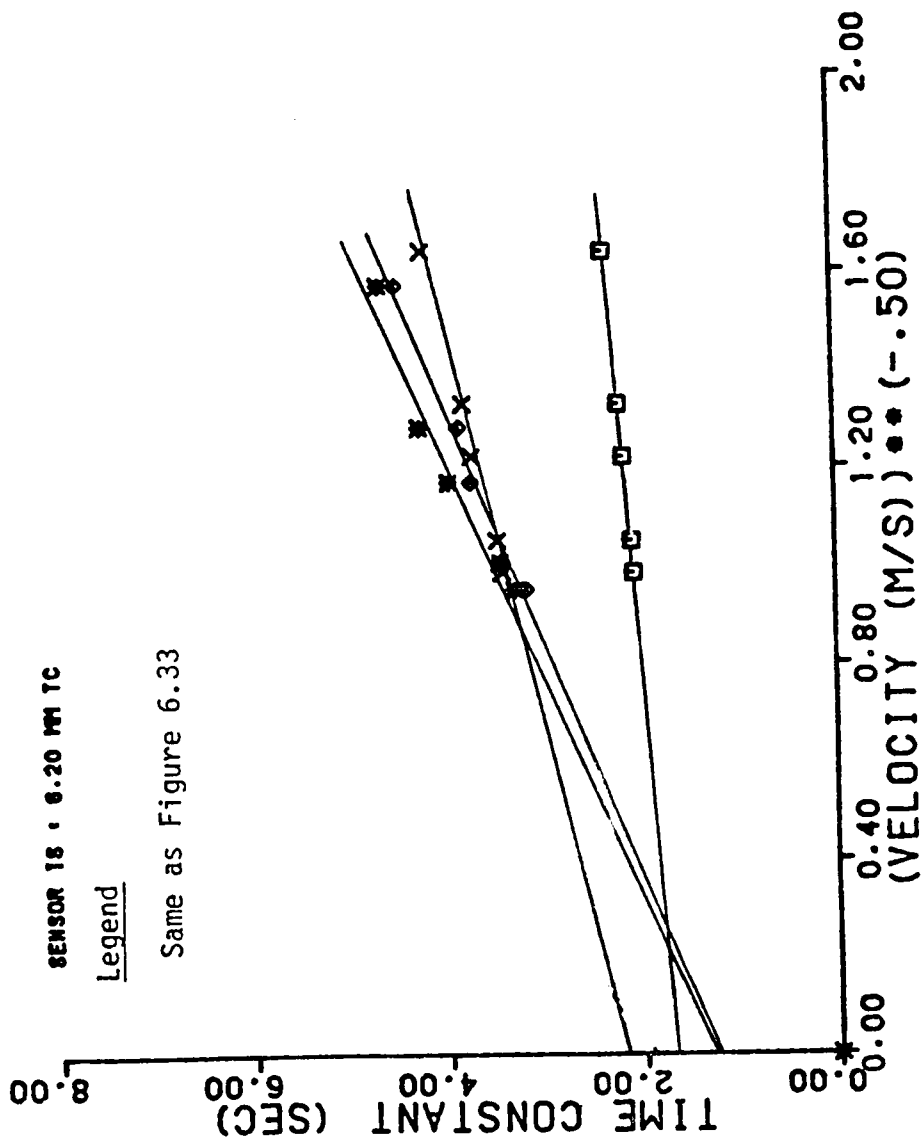


Figure 6.35 Application of Formula 4.9 for $n=0.5$ and a 6.20 mm. Thermocouple

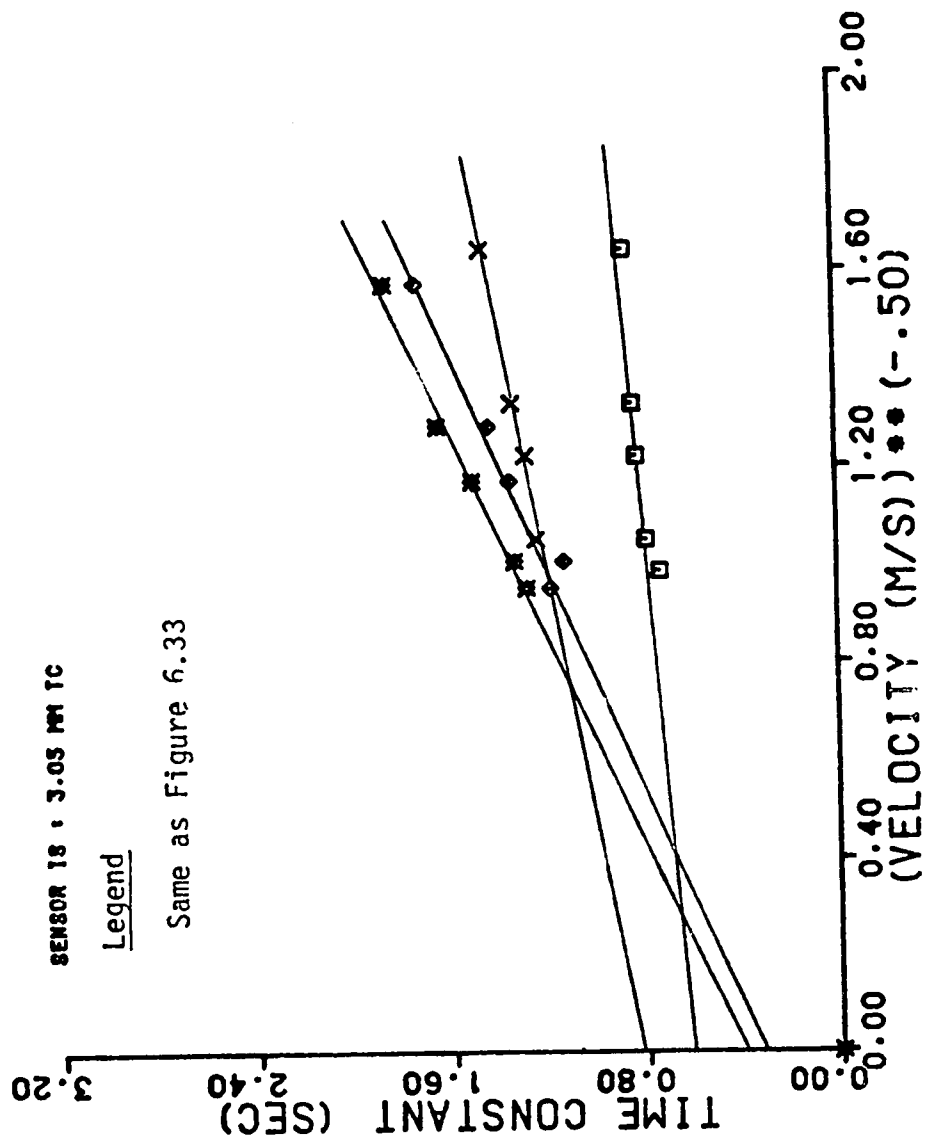


Figure 6.36 Application of Formula 4.9 for $n=0.5$ and a 3.05 mm. Thermocouple

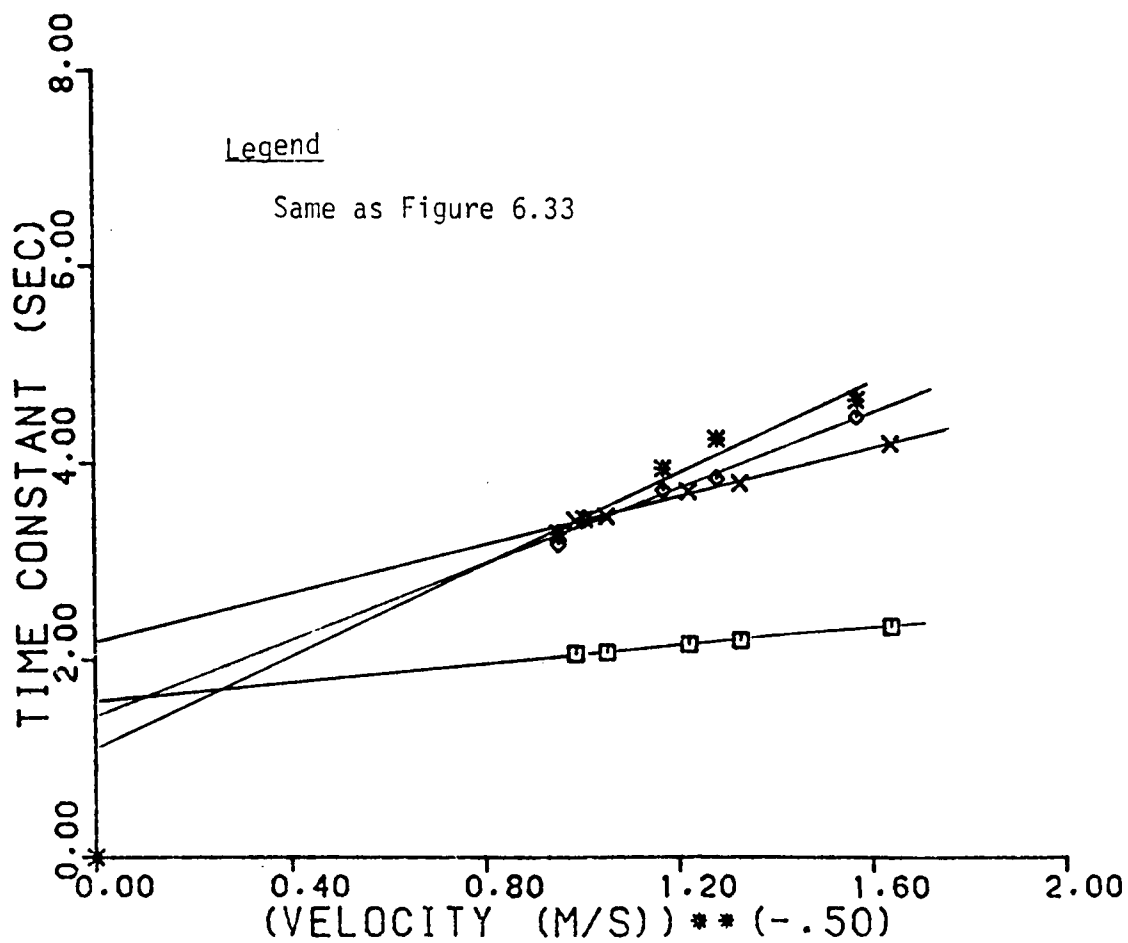


Figure 6.37 Application of Formula 4.9 for $n=0.5$ and a 6.25 mm Thermocouple.

0.6 are shown in Figures 6.38 - 6.42. The validity of Equation 4.9 is indicated both by the straightness and the equivalence of the intercepts (C_1) for tests made in different fluids. Both the exponents give straight line fits. However, there is variation of the intercepts for each of the sensors. This variation is pronounced for the faster responding sensors. Table 6.9 (P. 148) presents the maximum and minimum values for the intercept, C_1 , for all the sensors. The difference in the maximum and minimum values may suggest which exponent is more correct. This shows that -0.6 exponent gives a smaller overall difference than -0.5. However, least-squares fitting has shown that the C_1 is sensitive to errors in the data. Thus, for the number of data points and liquids used, Equation 4.9 cannot on its own prove or disprove the validity of the correctness of the exponent value.

6.2.2 Air Tests Results

Testing of temperature sensors in gaseous media is an extension of the tests performed on the same sensors described in Table 6.4 (p.117). Two sets of experiments were carried out at about 50°C and 100°C. The results are shown in Figures 6.43 and 6.44. These show that the responses of sensors of the same type and make are similar. Further, the responses at the higher temperature are slightly faster than those at the lower temperature. This is because the thermal conductivity of air increases with temperature. It is also important to notice the relatively much slower responses of sensors tested in

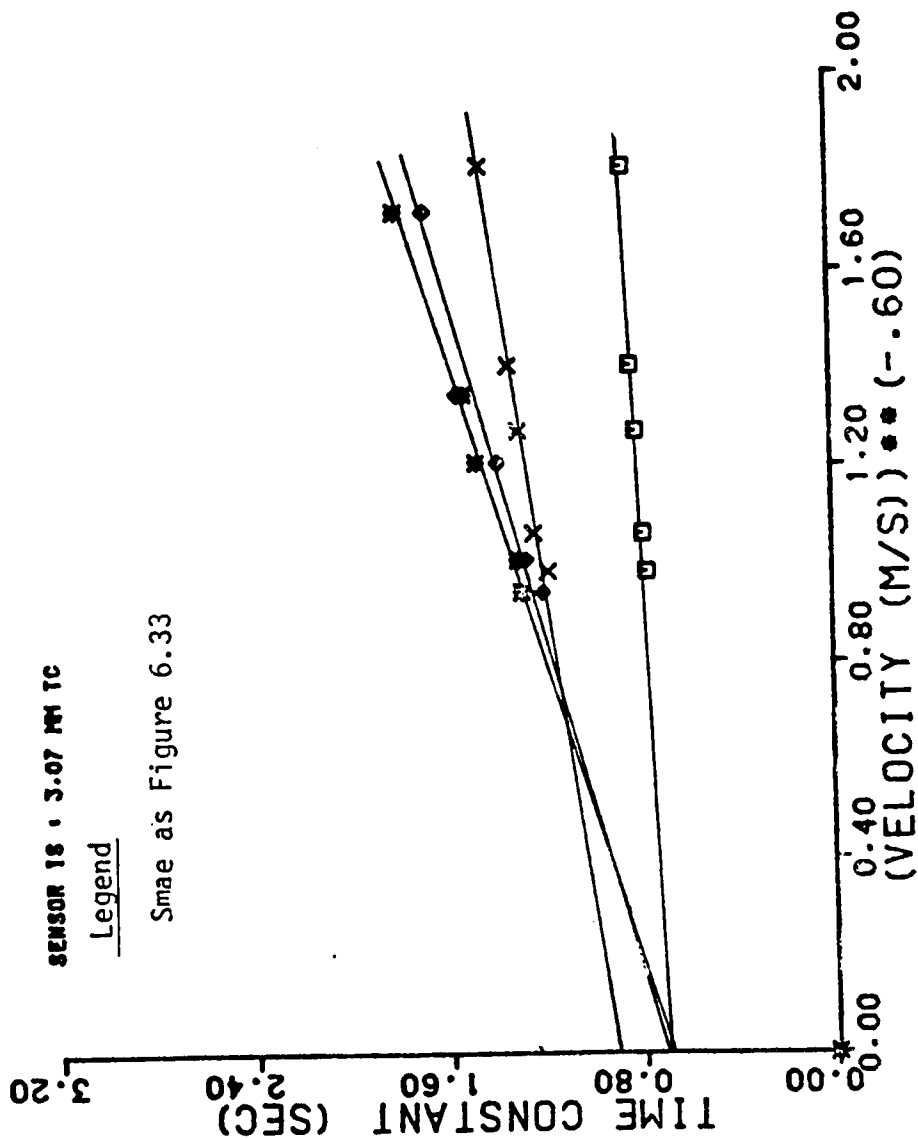


Figure 6.38 Application of Formula 4.9 for $n=0.6$ and a 3.07 mm. Thermocouple

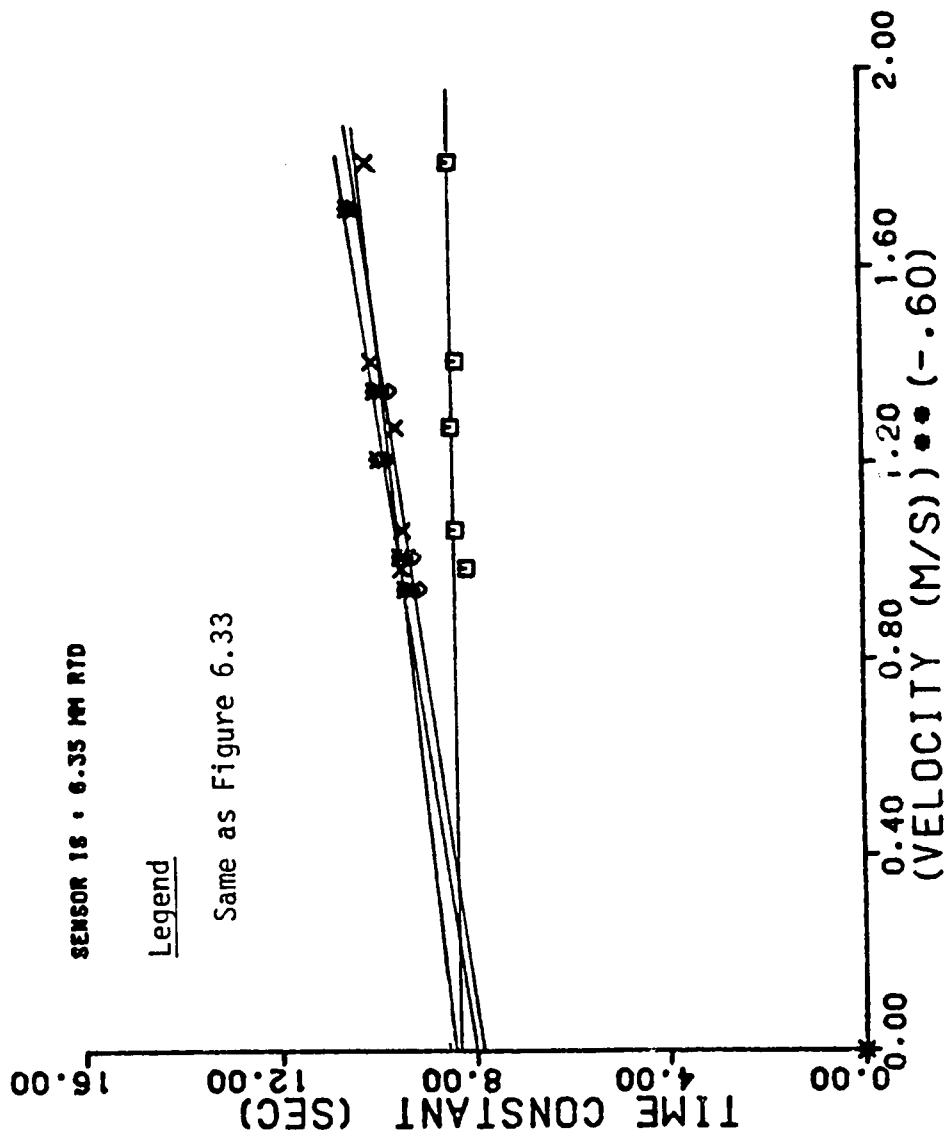


Figure 6.39 Application of Formula 4.9 for $n=0.6$ and a 6.35 RTD

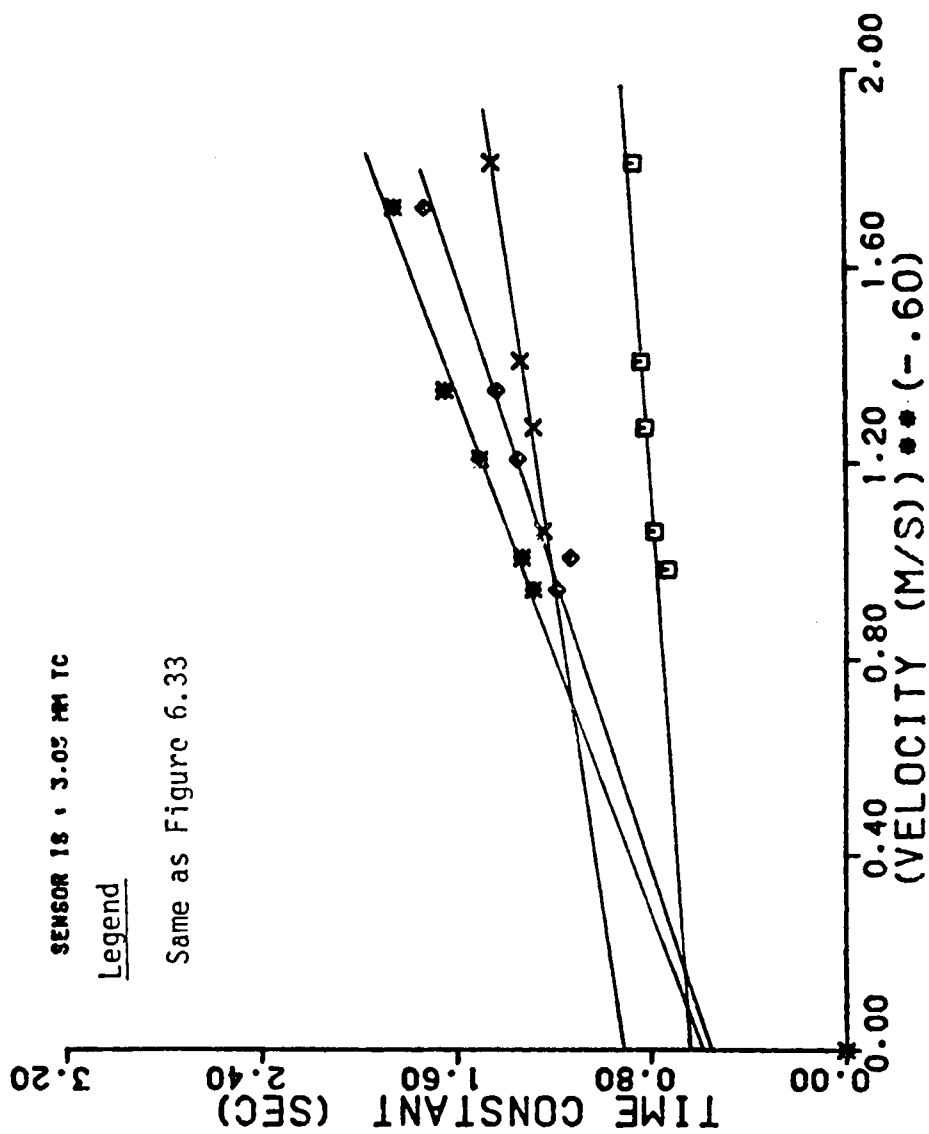


Figure 6.40 Application of Formula 4.9 for $n=0.6$ and a 3.05 mm. Thermocouple

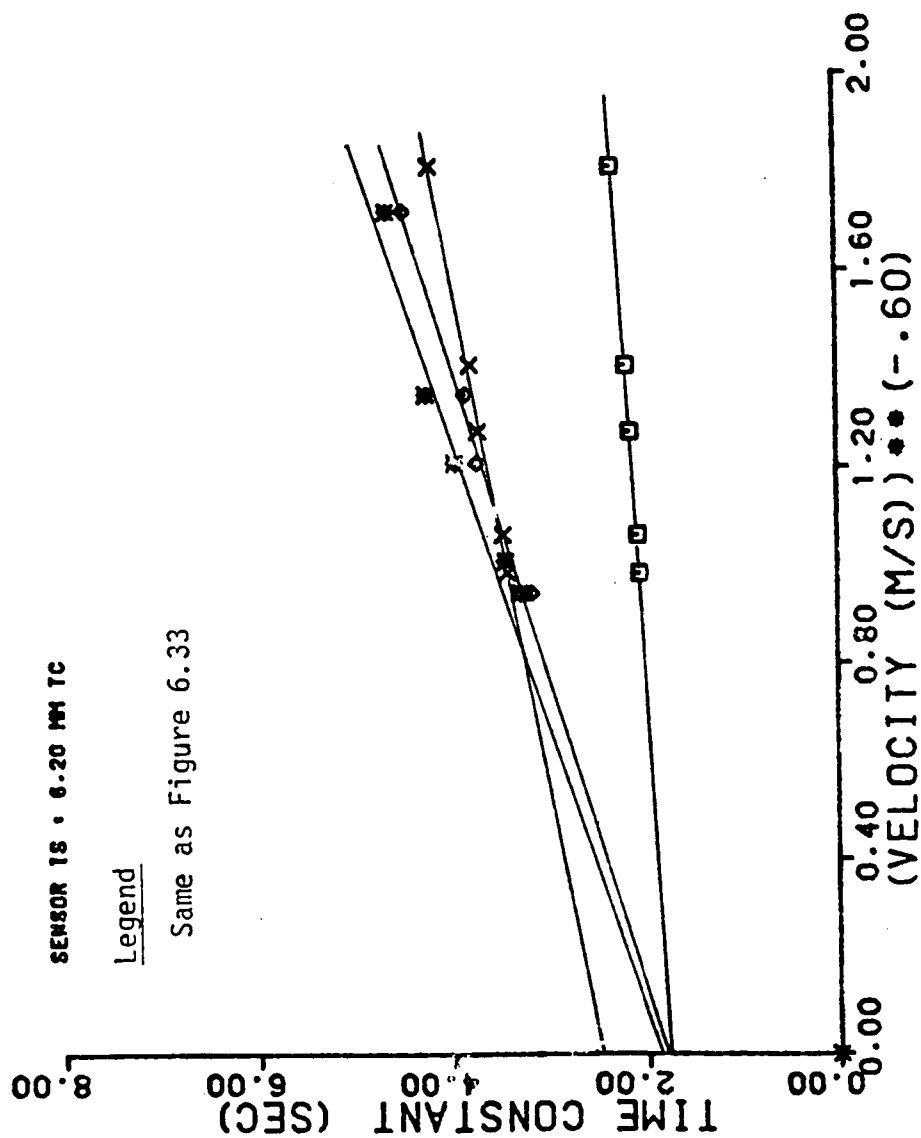


Figure 6.41 Application of Formula 4.9 for $n=0.6$ and a 6.20 mm. Thermocouple

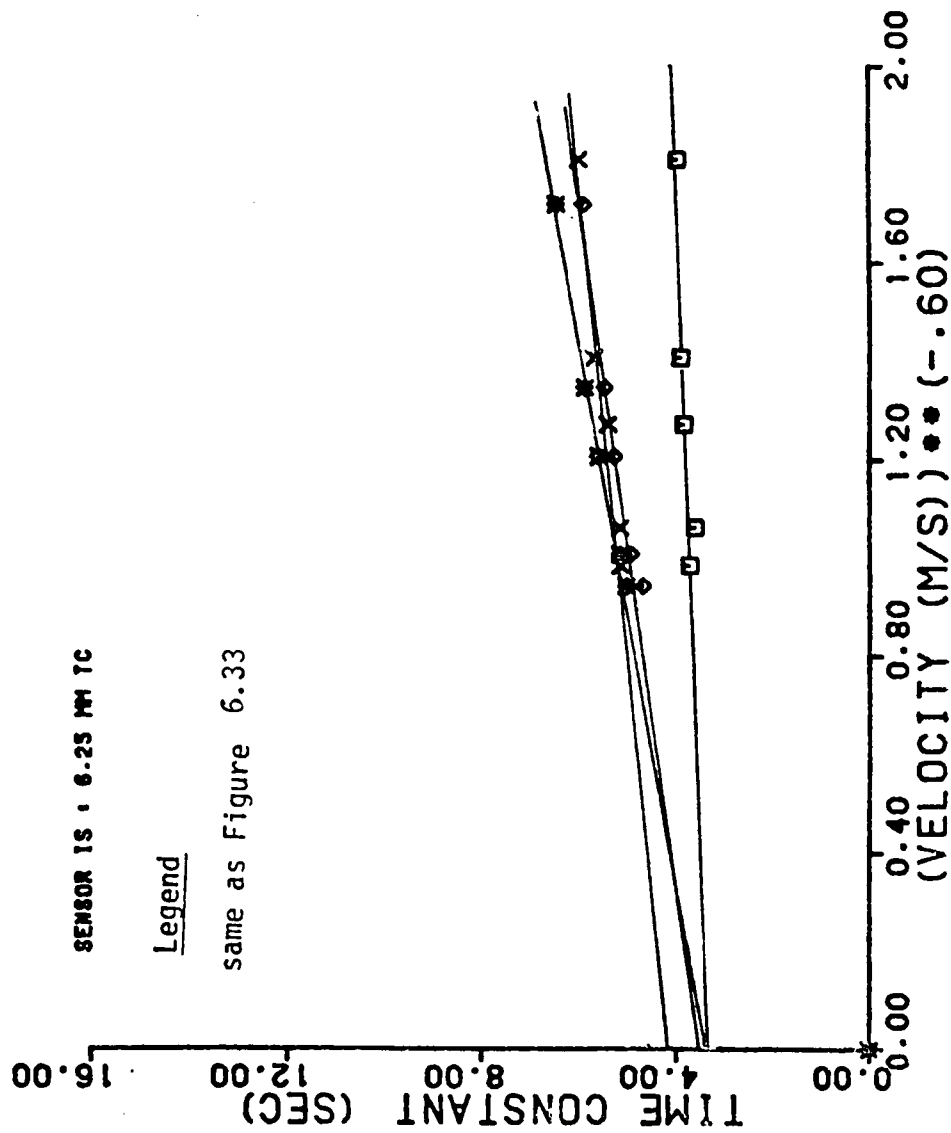


Figure 6.42 Application of Formula 4.9 for $n=0.6$ and a 6.25 mm. Thermocouple

Table 6.9

Maximum and Minimum Values of C_1 in Equation 4.9
When $n = 0.5$ and 0.6 with Engine Oil Results Neglected

Sensor	n	Max. C_1	Min. C_1	Diff.
3.07 mm TC	0.6	0.90	0.65	0.25
	0.5	0.82	0.48	0.34
3.05 mm TC	0.6	0.90	0.58	0.32
	0.5	0.573	0.23	0.34
6.20 mm TC	0.6	2.62	1.70	0.92
	0.5	2.21	1.23	0.98
6.25 mm TC	0.6	3.97	3.25	.72
	0.5	3.70	2.76	0.94
6.35 mm RTD	0.6	8.47	7.50	0.97
	0.5	8.24	7.03	1.21

□	SENSOR	IS :	6.25	MM	TC
△	SENSOR	IS :	6.20	MM	TC
◇	SENSOR	IS :	3.07	MM	TC
x	SENSOR	IS :	3.05	MM	TC

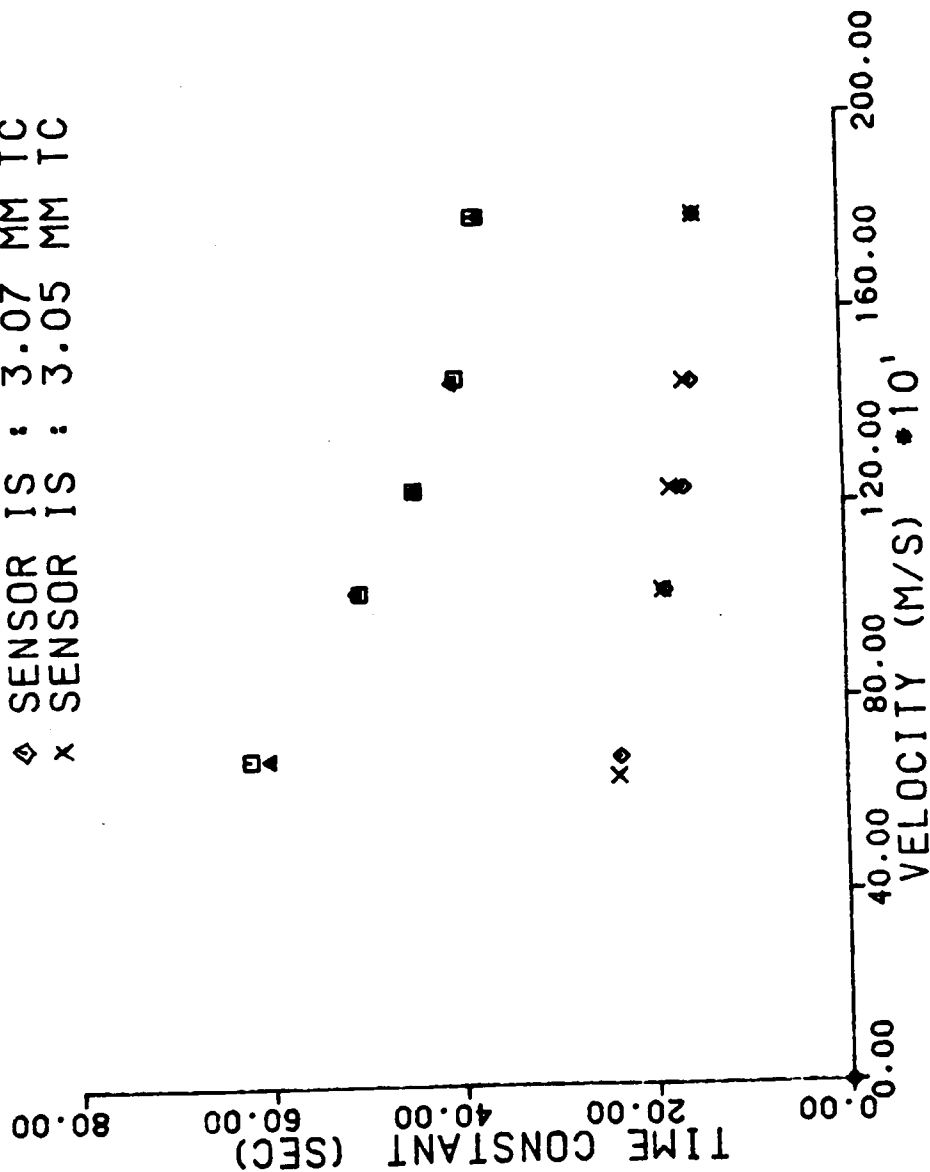


Figure 6.43 Results of Tests Performed on Four Thermocouples at about 50°C

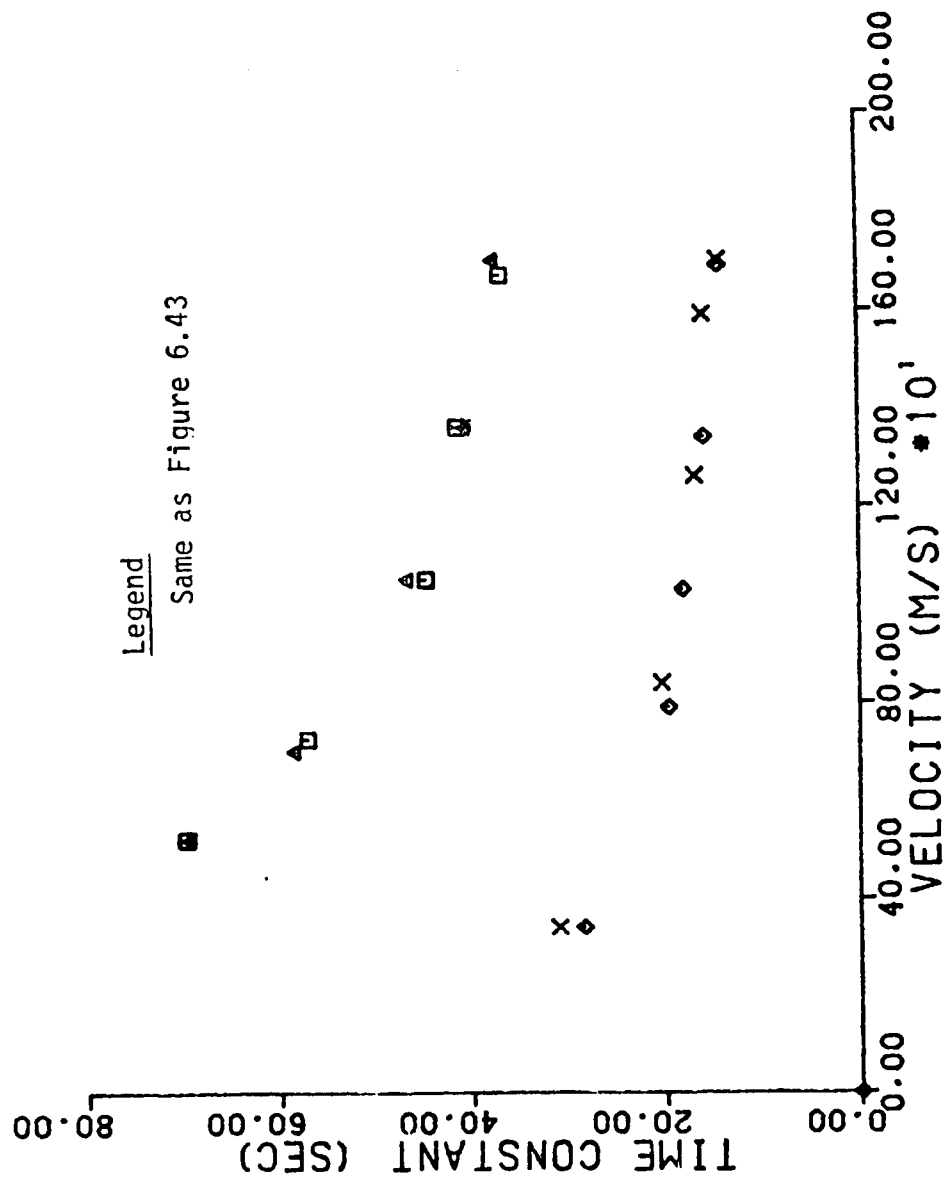


Figure 6.44 Results of Tests Performed on Four Thermocouples at about 100°C

air than their responses in liquids. This is due to the low surface heat transfer coefficients.

Earlier work (see section 6.2.1) suggested that the following may hold for liquids

$$\tau = C_1 + C_3 u^{-0.5} . \quad (6.7)$$

where C_1 and C_3 are constants and u is the fluid velocity. The results in Figures 6.43 and 6.44 are presented again as τ versus $u^{-0.5}$ in Figures 6.45 and 6.46. Though straight lines are suggested, this does not imply that relationship (6.6) is valid. Earlier experience showed that straight lines can also be fitted to τ versus $u^{-0.6}$. Figures 6.47 and 6.48 show the results as τ versus $u^{-0.6}$.

The least-squares fit method was used to determine the y-intercept (C_1) (see Equation 4.9) for the sensors tested in air. The results are presented in Table 6.10. For most sensors the C_1 values do not fall within the maximum and minimum C_1 values obtained in the liquid tests (see Table 6.9). These sensors, however, were tested at higher temperatures (50°C and 100°C) than that (room temperature) under which the liquid tests were performed. Furthermore, it appears that the C_1 for air results is quite sensitive to the exponent value (i.e. $-.5$ or $-.6$). C_1 value is also very sensitive to the velocity values. Thus small inaccuracies in determining the velocities may cause a big error in the C_1 value.

HIGH TEMPERATURE RESULTS

Legend

Same as Figure 6.43

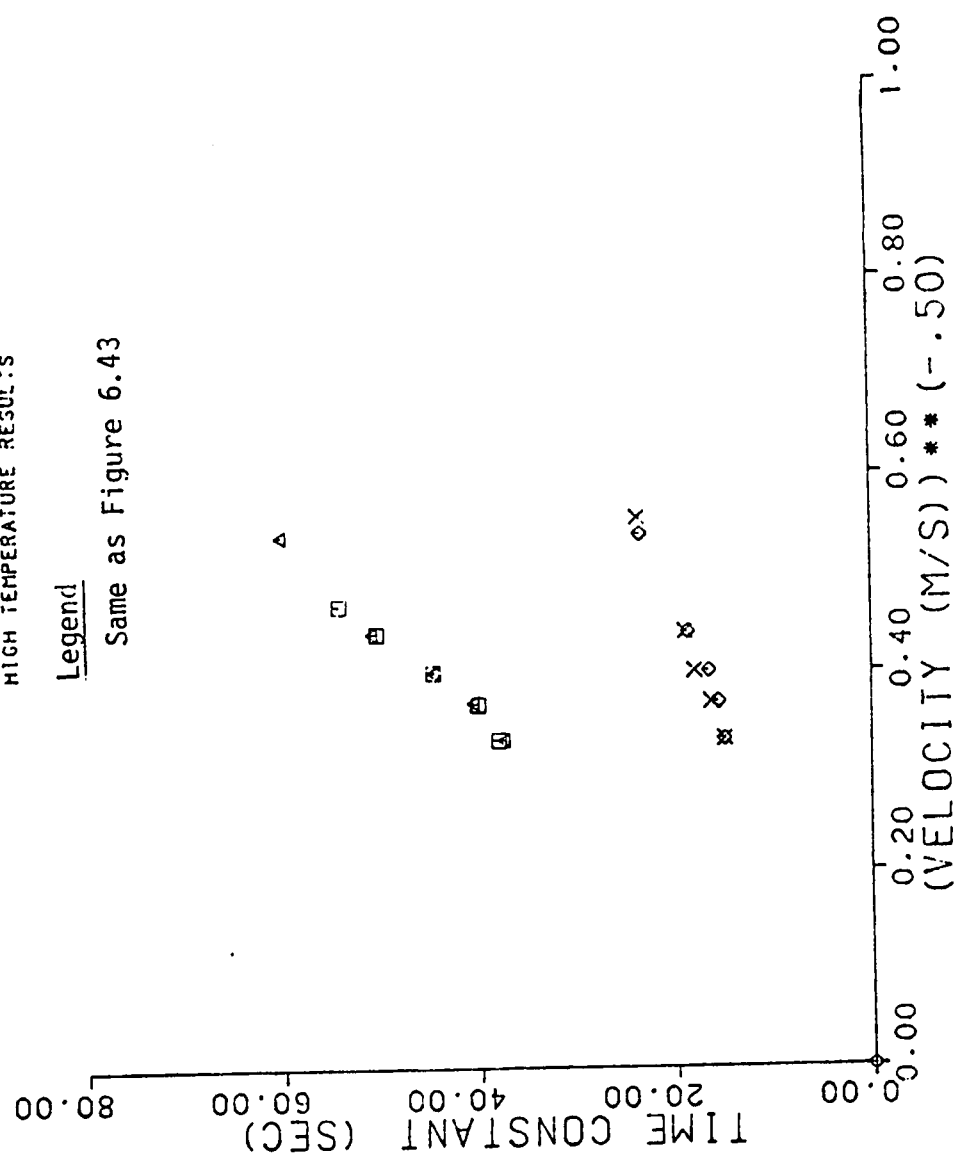


Figure 6.45 Application of Correlation 4.9 for $n=5$ and Four Thermocouples in Air

LOW TEMPERATURE RESULTS

Legend

Same as Figure 6.43

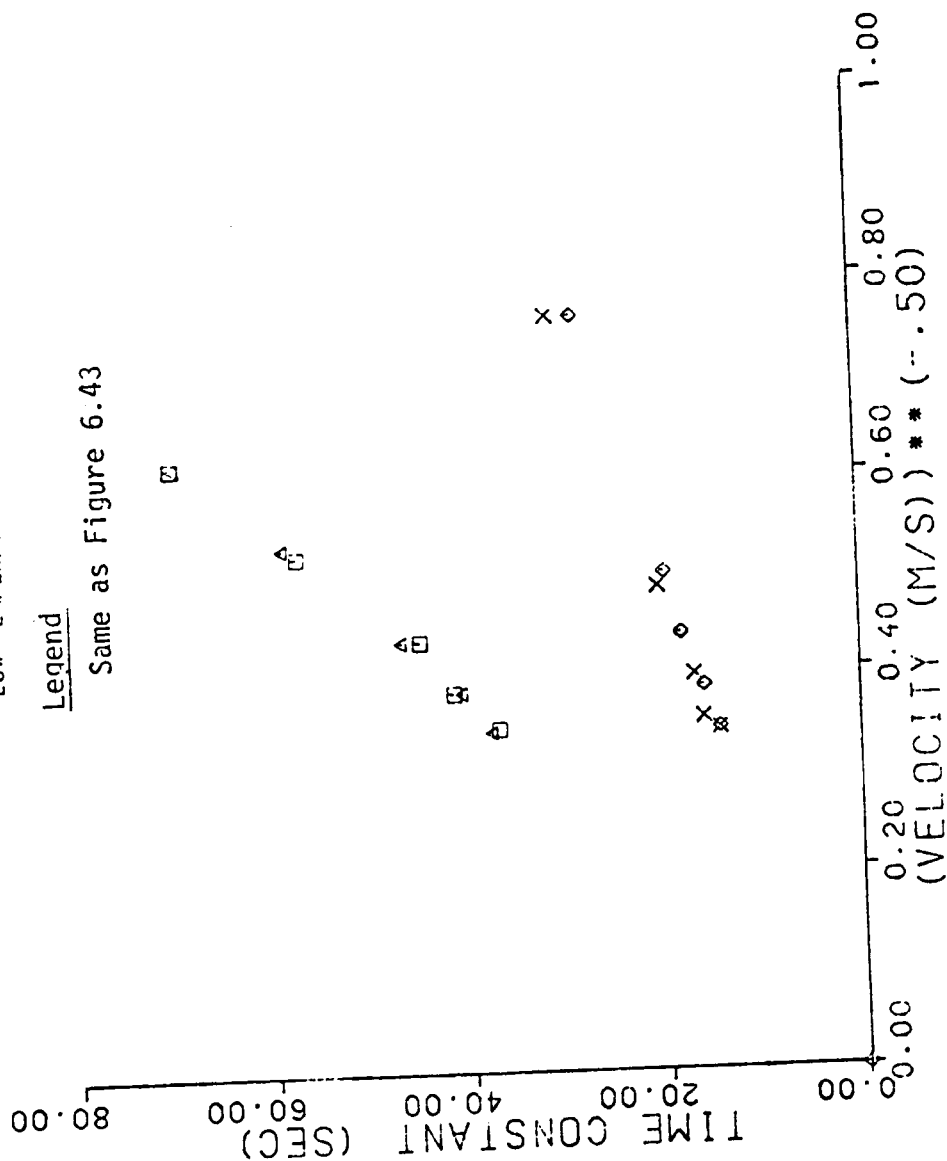


Figure 6.46 Application of Correlation 4.9 for $n=5$ and Four Thermocouples in Air

HIGH TEMPERATURE RESULTS

Legend

Same as Figure 6.43

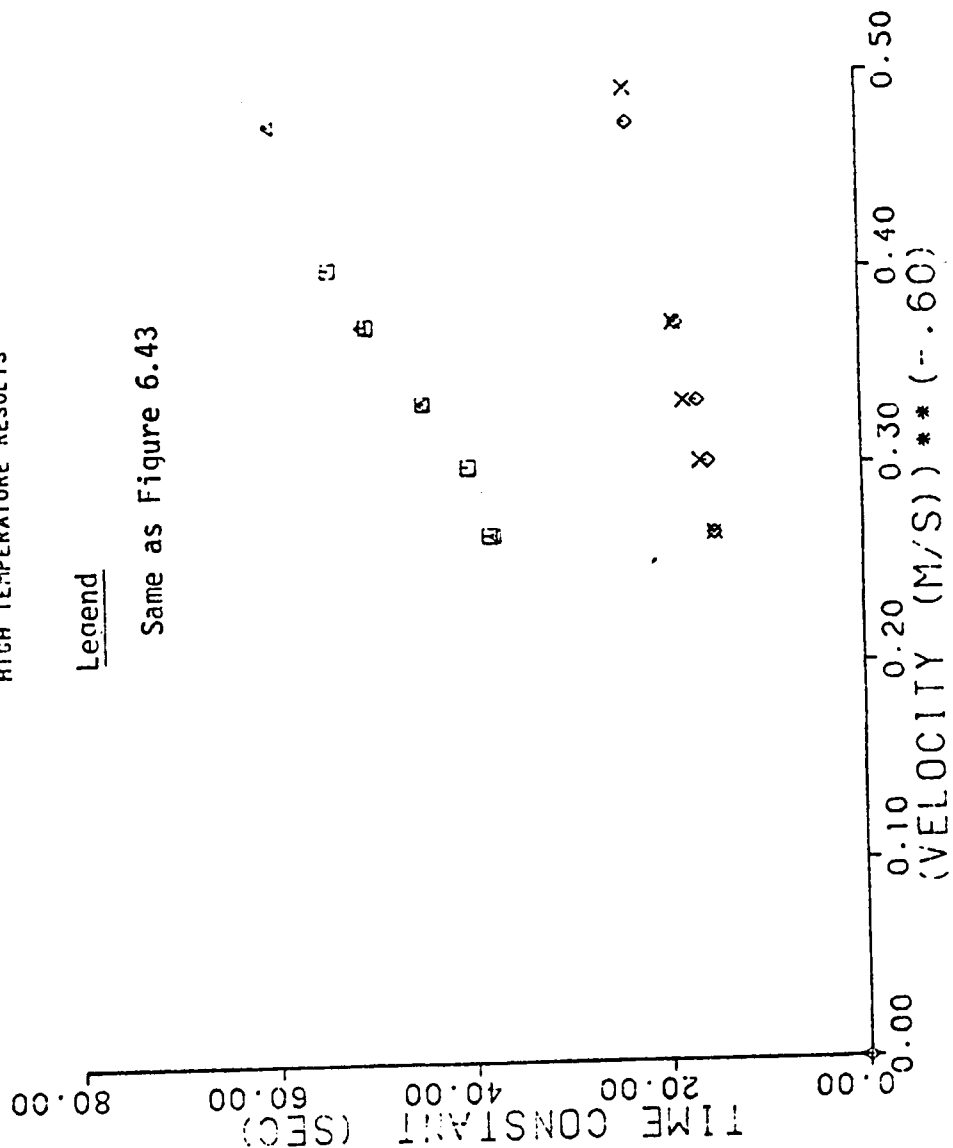


Figure 6.47 Application of Correlation 4.9 for n=.6 and Four Thermocouples

LOW TEMPERATURE RESULTS

Legend

Same as Figure 6.43

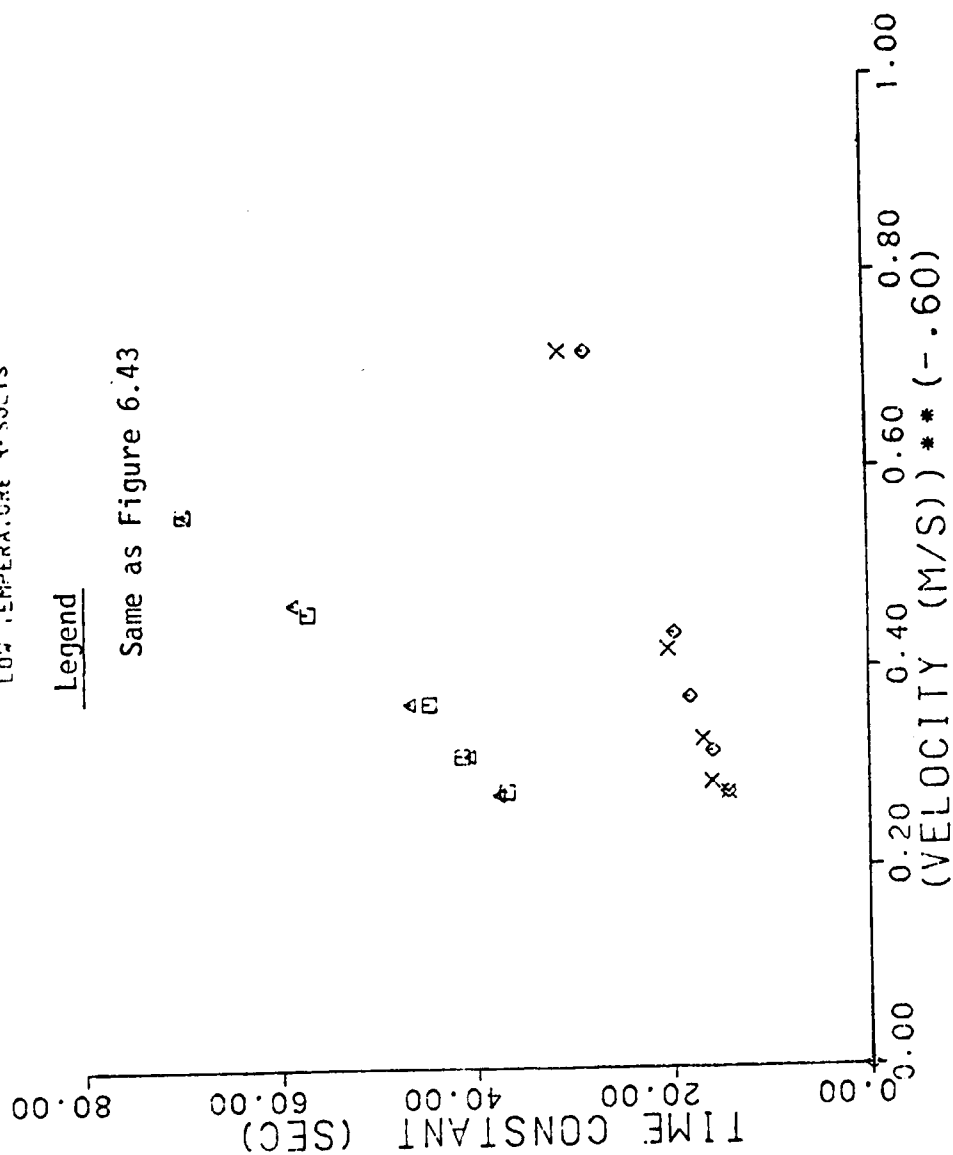


Figure 6.48 Application of Correlation 4.9 for $n=0.6$ and for Thermocouples in Air

Table 6.10

Values of C_1 in Equation 4.9 When $n=0.5$ and 0.6 for Air

Thermocouple Diameter (mm)	n	C_1	
		50°C	100°C
6.25	.5	-5.59	-4.16
	.6	3.68	3.94
6.20	.5	-3.84	-.41
	.6	5.19	7.69
3.07	.5	1.66	2.23
	.6	4.94	5.03
3.05	.5	3.09	.201
	.6	6.0	3.20

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Temperature sensor dynamic response depends on the properties and dimensions of the sensor constituents, and on the properties and velocity of the surrounding medium. Unfortunately, there are many uncertainties in the properties of the insulating powders. Additional uncertainties are due to manufacturing tolerances and manufacturing methods. Thus, sensors of the same make, type, and diameter have different responses. This raises questions about the importance of the simulation approach. It is obvious that accurate predictions of an actual sensor's time constant are impossible because of these uncertainties. However, the importance of simulation stems from the fact that it can provide insight on ways to permit meaningful interpolations and extrapolations, enable designers to evaluate design trade-offs, and to get rough estimates of response performance of new sensors.

A two-dimensional simulation package was developed with these benefits in mind. It was designed as a "user-friendly," flexible, and capable of adequate representation of real world sensors. The utility of this approach was demonstrated in analysis of problems such as the effects of insulator properties on sensor response, and extrapolating sensor response laboratory data; and in various sensitivity analyses.

In addition, dynamic response time testing of temperature sensors has been carried out in five different liquids. These results support

the theoretical conclusion that the time constant is inversely related to the heat transfer coefficient and the intercept and the slope are entirely dependent on the sensor. The formula for correlating time constant data with fluid conditions is

$$\tau = C_1 + C_2/h$$

where

τ = time constant

C_1, C_2 = constants which are dependent on the sensors

h = film heat transfer coefficient.

Both C_1 and C_2 are temperature dependant.

While the results show the validity of the above expression, the results also show that identification of the constants, C_1 and C_2 , requires very accurate test data.

Finally, an air loop for testing temperature sensors has been constructed and used for testing sensors. Typical results for thermocouples have been presented. These results provide data for low heat transfer coefficients. Liquid tests provide data for intermediate and high values of heat transfer coefficients.

Further work is suggested to complete the findings of this thesis. This recommended work is to be carried out in the areas of modelling and testing of industrial sensors.

First, the computer program SENSOR should be used to investigate some of the real-life problems in the sensor response behavior such as stem loss and others. It is recommended that SENSOR be implemented on a microcomputer with a large memory for easy and economical access.

Second, testing of industrial sensors in liquids should proceed with two objectives in mind. The first is to obtain statistically sound results, and the second is to investigate the temperature dependence of the coefficients C_1 and C_3 in Equation (4.9). To achieve these objectives, one should consider using a more accurate means of determining the velocity of the flowing liquid. Certainly, performing more tests, at least five tests, at a given velocity and temperature will give further confidence in the results. A more detailed error analysis should accompany these tests. On the other hand, new (unused) sensors should be used in the tests to eliminate the chances of having the sensor degradation and/or aging affecting the results.

The second objective, in particular, will require using a modified version of the rotating tub facility. The modification should provide for a controlled means to increase the temperature of the liquids. One should also consider using liquids other than carbontetrachloride and heptane since they become hazardous at higher temperatures.

Finally, tests in a modified version of the air loop should proceed. The modifications should include using a longer pipe (at least ten diameters long) in the test section to allow the air to have

a fully developed velocity profile. The sensor should be inserted at a location where the velocity profile is fully developed. Moreover, effects of humidity (0% to 100% Relative Humidity) on the sensor response should be studied.

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APPENDIXES

APPENDIX A

DEVELOPMENT OF EQUATION 4.1

For a Lumped parameter approach, the following assumptions are made

1. The thermometer is a first order system, that is

$$\tau = McR \quad (A.1)$$

where

τ = time constant of the sensor
 M = sensor's mass
 c = sensor's specific heat constant
 R = heat transfer resistance

Both M and C are average properties of the sensor.

2. Cylindrical sensor

$$M = (\pi r^2 L) \rho \quad (A.2)$$

where

r = sensor radius
 L = sensor length
 ρ = sensor material density

3. Total resistance is the sum of an internal and external (surface) heat transfer resistances, that is

$$R = R_{int} + R_{ext} \quad (A.3)$$

where

$$R_{int} = \frac{\ln (r/r_i)}{2\pi kL} \quad (A.4)$$

$$R_{ext} = 1/2\pi hLr \quad (A.5)$$

Therefore

$$R = \frac{\ln (r/r_i)}{2\pi kL} + 1/2\pi hLr \quad (A.6)$$

substituting A.6 and A.2 in A.1,

$$\tau = (\pi r^2 L \rho) c \ln(r/r_i) / 2\pi k L + 1/2 \pi r L h$$

rearranging

$$\tau = \frac{\rho c r^2 \ln(r/r_i)}{2k} + \frac{\rho c r}{2h} \quad (\text{A.7})$$

At a given temperature ρ , C , and k are constants. Then A.7 can be written in the in the form

$$\tau = C_1 + C_2/h$$

where

$$C_1 = \rho c r^2 \ln(r/r_i) / 2k$$

$$C_2 = \rho c r / 2.$$

APPENDIX B

PHYSICAL PROPERTIES TABLES

- B.1 Properties of Alumina
- B.2 Properties of Magnesia
- B.3 Properties of Stainless Steel
- B.4 Properties of Dry Air
- B.5 Properties of Liquid Water
- B.6 Properties of Engine Oil
- B.7 Properties of Ethylene Glycol
- B.8 Properties of Heptane
- B.9 Properties of Carbon Tetrachloride

Table B.1 Properties of Aluminum Oxide (18)

Material	Temperature		Thermal Conductivity		Specific Heat Constant		Density	
	°F	°C	B/h ft°F	W/m.°C	B/lb°F	J/kg.°C	lbm/ft ³	kg/m ³
0% Porosity (dense) Al ₂ O ₃	80.6	27	22.25	38.51	0.225	786.97	249.7	4000
	260.6	127	15.96	27.63	.225	941.85	"	"
	440.6	227	12.09	20.93	.250	1046.5	"	"
	620.6	327	9.19	15.91	.265	1109.29	"	"
	800.6	427	7.50	12.98	.275	1151.15	"	"
	980.6	527	6.10	10.55	.280	1172.08	"	"
	1160.6	627	5.12	8.87	.287	1201.38	"	"
	1340.6	727	4.59	7.95	.290	1213.94	"	"
	1520.6	827	4.11	7.12	.295	1234.87	"	"
	1700.6	927	3.87	6.70	.300	1255.8	"	"
							224.7	3600
10% Porosity Al ₂ O ₃	80.6	27	15.12	26.16	.193	808.94	"	"
	260.6	127	12.34	21.35	.227	949.13	"	"
	440.6	227	9.98	17.27	.249	1044.87	"	"
	620.6	327	7.98	13.81	.264	1103.56	"	"
	800.6	427	6.53	11.30	.273	1143.62	"	"
	980.6	527	5.44	9.42	.278	1164.84	"	"
	1160.6	627	4.64	8.04	.285	1193.43	"	"
	1340.6	727	4.11	7.12	.288	1206.70	"	"
	1520.6	827	3.75	6.49	.293	1227.33	"	"
	1700.6	927	3.36	5.86	.298	1247.68	"	"

Table B.1 (continued)

Material	Temperature		Thermal Conductivity		Specific Heat Constant		Density	
	°F	°C	B/h ft°F	W/m.°C	B/lb°F	J/kg.°C	lbm/ft ³	kg/m ³
20-30% Porosity Al ₂ O ₃	80.6	27	12.10	20.93	.201	841.39	187.3	3000
	260.6	127	9.79	16.95	.230	962.78	"	"
	440.6	227	7.86	13.60	.249	1042.31	"	"
	620.6	327	6.29	10.88	.255	1096.73	"	"
	800.6	427	4.96	8.58	.271	1134.41	"	"
	980.6	527	3.99	6.91	.276	1155.34	"	"
	1160.6	627	3.34	5.78	.282	1180.45	"	"
	1340.6	727	2.95	5.11	.286	1197.2	"	"
	1520.6	827	2.78	4.81	.291	1218.13	"	"
	1700.6	927	2.66	4.60	.295	1234.87	"	"
44-48% Porosity Al ₂ O ₃	80.6	27	9.07	15.7	.212	887.43	134.8	2160
	260.6	127	7.26	12.56	.233	975.34	"	"
	440.6	227	5.56	9.63	.248	1038.13	"	"
	620.6	327	4.35	7.53	.258	1079.99	"	"
	800.6	427	3.31	5.73	.267	1117.66	"	"
	980.6	527	2.66	4.6	.272	1138.59	"	"
	1160.6	627	2.18	3.77	.278	1163.71	"	"
	1340.6	727	2.06	3.56	.282	1180.45	"	"
	1520.6	827	1.94	3.35	.287	1201.38	"	"
	1700.6	927	1.88	3.26	.291	1218.13	"	"

Table B.1 (continued)

Material	Temperature		Thermal Conductivity		Specific Heat Constant		Density	
	°F	°C	B/h ft°F	W/m.°C	B/lb°F	J/kg.°C	lbm/ft ³	kg/m ³
53% Porosity Al ₂ O ₃	620.6	327	1.21	2.09	.258	1078.90	117.4	1880
	800.6	427	.99	1.72	.264	1111.21	"	"
	980.6	527	.907	1.57	.271	1133.69	"	"
	1160.6	627	.84	1.46	.277	1159.06	"	"
	1340.6	727	.77	1.34	.281	1175.43	"	"
	1520.6	827	.74	1.29	.285	1194.89	"	"
	1700.6	927	.73	1.26	.290	1212.77	"	"

Table B.2 Properties of Magnesium Oxide Powder(18)

Material	Temperature		Thermal Conductivity		Specific Heat Constant		Density	
	°F	°C	B/h ft°F	W/m.°C	B/lb°F	J/kg.°C	lbm/ft ³	kg/m ³
0% Porosity	40	4.44	24.5	42.40	.22	921.18	223.5	3580
	290	143.3	17.0	29.42	.248	1038.35	"	"
Mgo	540	282.2	12.5	21.63	.268	1122.09	"	"
(Poly-	790	421.11	9.5	16.44	.275	1151.40	"	"
Crystall-	1040	560	7.0	12.11	.30	1256.07	"	"
ine)	1290	698.8	5.5	9.52	.312	1306.31	"	"
	1540	837.7	4.5	7.78	.324	1356.56	"	"
	1640	893.3	4.25	7.36	.33	1381.68	"	"

Table B.3
Properties of Stainless Steel (15)

Type	Thermal Conductivity				Specific Heat Constant		Density	
	B/h ft°F		W/m.°C		B/lb°F		lbm/ft ³	kg/m ³
	212°F	932°F	100°C	500°C	(68-212)°C	(20-100)°C		
304	9.4	12.4	16.27	21.46	.12	502.4	493.2	7900
446	12.09	14.03	20.93	24.28	.11	460.6	474.5	7600

Table B.4
Properties of Dry Air (19)

Temperature	Density	Specific Heat	Thermal Conductivity	Dynamic Viscosity	Pr
$^{\circ}\text{K}$	kg/m^3	$\text{kJ/kg.}^{\circ}\text{K}$	W/m.k	kg/m.s	
200	1.769	1.004	1.810×10^{-2}	13.36×10^{-6}	.741
210	1.684	1.004	1.895	13.92	.738
220	1.607	1.004	1.980	14.47	.734
230	1.537	1.005	2.063	15.01	.731
240	1.473	1.005	2.145	15.54	.728
250	1.413	1.005	2.226	16.06	.725
260	1.359	1.005	2.305	16.57	.722
270	1.308	1.006	2.384	17.07	.720
280	1.261	1.006	2.461	17.57	.718
290	1.218	1.006	2.538	18.05	.715
300	1.177	1.006	2.614	18.53	.713
310	1.139	1.007	2.687	19.00	.712
320	1.103	1.007	2.759	19.46	.71
330	1.070	1.008	2.830	19.92	.71
340	1.038	1.008	2.900	20.37	.708
350	1.009	1.009	2.970	20.81	.707
360	.9805	1.01	3.039	21.25	.706
370	.9539	1.011	3.107	21.68	.705
380	.9288	1.012	3.173	22.11	.705
390	.9050	1.013	3.239	22.52	.704
400	.8822	1.014	3.305	22.94	.704
410	.8608	1.015	3.371	23.35	.703
420	.8402	1.017	3.437	23.75	.703
430	.8207	1.018	3.503	24.15	.702
440	.8021	1.020	3.568	24.54	.702
450	.7842	1.021	3.633	24.93	.701
460	.7672	1.023	3.697	25.32	.701
470	.7509	1.025	3.761	25.70	.700
480	.7351	1.026	3.825	26.07	.699
490	.7201	1.028	3.888	26.45	.699
500	.7057	1.03	3.951	26.82	.699
510	.6919	1.032	4.02	27.18	.698
520	.6786	1.034	4.08	27.54	.698
530	.6658	1.036	4.14	27.90	.698
540	.6535	1.038	4.2	28.25	.698
550	.6416	1.04	4.26	28.6	.698
560	.6300	1.042	4.32	28.95	.698
570	.6190	1.044	4.38	29.29	.698
580	.6084	1.047	4.44	29.69	.7
590	.5980	1.049	4.50	29.97	.699

Table B.4 (continued)

Temperature	Density	Specific Heat	Thermal Conductivity	Dynamic Viscosity	Pr
$^{\circ}\text{K}$	kg/m^3	$\text{kJ/kg.}^{\circ}\text{K}$	W/m.k	kg/m.s	
600	.5881	1.051	4.56×10^{-2}	30.30×10^{-6}	.698
610	.5785	1.054	4.62	30.63	.699
620	.5691	1.056	4.68	30.96	.699
630	.5601	1.059	4.73	31.28	.7
640	.5514	1.061	4.79	31.61	.7
650	.5428	1.063	4.84	31.93	.701

Table B.5
Properties of Water (Saturated Liquid) (6)

Temperature	Density	Specific Heat	Thermal Conductivity	Dynamic Viscosity	Pr
$^{\circ}\text{C}$	kg/m^3	$\text{kJ/kg}\cdot^{\circ}\text{C}$	$\text{W/m}\cdot^{\circ}\text{C}$	$\text{kg/m}\cdot\text{s}$	
0	999.8	4.225	0.566	1.79×10^{-3}	13.25
4.44	999.8	4.208	0.575	1.55	11.35
10	999.2	4.195	0.585	1.31	9.4
15.56	998.6	4.186	0.595	1.12	7.88
21.11	997.4	4.179	0.604	9.8×10^{-4}	6.78
26.67	995.8	4.179	0.615	8.6	5.85
32.22	994.9	4.174	0.623	7.65	5.12
37.78	993.0	4.174	0.630	6.82	4.53
43.33	990.6	4.174	0.637	6.16	4.04
48.89	988.8	4.174	0.644	5.62	3.64
54.44	985.7	4.179	0.649	5.13	3.30
60.0	983.3	4.179	0.654	4.71	3.01
65.55	980.3	4.183	0.659	4.3	2.73
71.11	977.3	4.186	0.665	4.01	2.53
76.67	973.7	4.191	0.668	3.72	2.33
82.22	970.2	4.195	0.673	3.47	2.16
87.78	966.7	4.199	0.675	3.27	2.03
93.33	963.2	4.204	0.678	3.06	1.90
104.4	955.1	4.216	0.684	2.67	1.66
115.6	946.7	4.229	0.685	2.44	1.51
126.7	937.2	4.250	0.685	2.19	1.36
137.8	928.1	4.271	0.685	1.98	1.24
148.9	918.0	4.296	0.684	1.86	1.17
176.7	890.4	4.371	0.677	1.57	1.02
204.4	859.4	4.467	0.665	1.36	1.00
232.2	825.7	4.585	0.646	1.20	.85
260.0	785.2	4.731	0.616	1.07	.83
287.7	735.5	5.024		9.51×10^{-5}	
315.6	678.7	5.703		8.68	

Table B.6
Properties of Engine Oil (6)

Temperature	Density	Specific Heat	Thermal Conductivity	Kinematic Viscosity	Pr
$^{\circ}\text{C}$	kg/m^3	$\text{kJ/kg.}^{\circ}\text{C}$	$\text{W/m.}^{\circ}\text{C}$	m^2/s	
0	899.12	1.796	0.147	0.00428	47100
20	888.23	1.880	0.145	0.00090	10400
40	876.05	1.964	0.144	0.00024	2870
60	864.04	2.047	0.140	0.0000839	1050
80	852.02	2.131	0.138	0.0000375	490
100	840.01	2.219	0.137	0.0000203	276
120	828.96	2.307	0.135	0.0000124	175
140	816.94	3.395	0.133	0.0000080	116
160	805.89	2.483	0.132	0.0000056	84

Table B.7
Properties of Ethylene Glycol (17)

Temperature	Density	Specific Heat	Thermal Conductivity	Kinematic Viscosity	Pr
$^{\circ}\text{C}$	kg/m^3	$\text{kJ/kg.}^{\circ}\text{C}$	$\text{W/m.}^{\circ}\text{C}$	m^2/s	
0	1130.75	2.294	0.242	57.53×10^{-6}	615
20	1116.65	2.382	0.249	19.18	204
40	1101.43	2.478	0.256	8.69	93
60	1087.66	2.562	0.260	4.75	51
80	1077.56	2.650	0.261	2.98	32.4
100	1058.50	2.742	0.263	2.03	22.4

Table B.8
Properties of Heptane (17)

Temperature	Density	Thermal Conductivity	Specific Heat Constant	Dynamic Viscosity
$^{\circ}\text{C}$	kg/m^3	$\text{W/m.}^{\circ}\text{C}$	$\text{kJ/kg.}^{\circ}\text{C}$	kg/m.s
0	700.5	0.134	2.16	0.000526
10	692.0	-	2.190	0.000466
20	683.6	0.129	2.226	0.000414
30	675.1	-	2.263	0.000373
40	666.5	0.123	2.301	0.000338
50	657.9	-	2.34	0.000308
60	649.1	0.118	2.39	0.000281
70	640.2	-	2.43	0.00026
80	631.1	0.113	2.47	0.000239
90	621.8	-	2.52	0.000217
100	612.4	0.0188	2.57	0.000198
110	602.7	-	2.62	0.0001809
120	592.6	0.0209	2.67	0.0001672
130	582.1	-	2.72	0.0001546
140	571.1	0.0232	2.78	0.0001427
150	559.8	-	2.83	0.0001318
160	548.1	0.0255	2.89	0.0001217
180	523.2	0.0278	3.01	0.0001028
200	495.2	0.0302	3.14	0.0000867
220	461.6	0.0326	3.30	0.0000722
240	417.7	0.0351	3.56	0.0000581
260	345.7	0.0377	-	0.0000447

Table B.9

Properties of Liquid Carbon Tetrachloride (17)

Temperature $^{\circ}\text{C}$	Density $109/\text{m}^3$	Thermal Conductivity $\text{W}/\text{m}^{\circ}\text{C}$	Dynamic Viscosity $\text{kg}/\text{m}^{\cdot}\text{s}$
-20	-	.113	-
-10	-	.1108	1.68×10^{-3}
0	1632.5	.1087	1.35
10	1613.2	.1065	1.13
20	1594.0	.1042	.97
30	1574.8	.1019	.84
40	1555.2	.0997	.74
50	1536.1	.0975	.65
60	1516.3	.0953	.59
70	1496.3	.0930	.52
80	1476.5	.0908	.472
90	1455.4	.0886	.426
100	1434.3	.0864	.387
110	1412.4	.0840	.352
120	1390.2	.0820	.323
130	1368.0	.0797	-
140	1345.0	.0775	.276
150	-	.0753	-
160	1298.2	.0731	.234
170	1273.4	.071	-
180	1247.0	.0688	.201
190	1219.2	.0666	-
200	1188.8	.0645	-

Table B.10

The Specific Heat Constant of Liquid Carbon Tetrachloride
as a Function of Temperature (17)

T $^{\circ}\text{K}$	260	270	280	290	300	310	320
Cp(kJ/kg.K)	.8386	.8482	.8545	.8600	.8637	.8654	.8667

APPENDIX C

LIST OF THE COMPUTER PROGRAM "SENSOR"

```

00010 RO=0
00020 FILES OUTPUT
00030 SCRATCH 71
00040 REM WRITTEN BY: MAJED HADJOK
00050 REM MAJ. ADVISOR: DR T. W. KERLIK
00060 REM FOR : FULFILLMENT OF IS IN ME
00070 REM DATE : JAN. 12/1984
00080 REM =====
00090 PRINT "DO YOU WANT TO SEE INSTRUCTIONS? (Y OR N)"
00100 INPUT K$
00110 IF K$="N" THEN 450
00120 PRINT "=====
00130 PRINT* THIS PROGRAM FINDS THE TRANSIENT TEMPERATURE DISTRIBUTION"
00140 PRINT* AT THE SENSING ELEMENTS OF RESISTANCE TEMPERATURE DETECTORS"
00150 PRINT* (RTD'S) AND THERMOCOUPLES (TC'S) IN ORDER TO ESTIMATE THE"
00160 PRINT* TIME CONSTANT OF THE SENSOR."
00170 PRINT* -----
00180 PRINT* "
00190 PRINT* TWO TEST CONDITIONS ARE SIMULATED:"
00200 PRINT* "
00210 PRINT* 1) PLUNGE TESTS:"
00220 PRINT* IMPLIES A STEP CHANGE IN AMBIENT TEMPERATURE."
00230 PRINT* "
00240 PRINT* 2) LOOP CURRENT STEP RESPONSE:"
00250 PRINT* IMPLIES A STEP CHANGE IN HEAT GENERATION AT "
00260 PRINT* THE SENSING ELEMENTS BY OHMIC HEATING WHILE "
00270 PRINT* AMBIENT TEMPERATURE REMAINS CONSTANT."
00280 PRINT* "
00290 PRINT* -----
00300 PRINT* "
00310 PRINT* USER MUST PROVIDE THE FOLLOWING INFORMATION:"
00320 PRINT* "
00330 PRINT* 1) THE PHYSICAL PROPERTIES AND THE DIMENSIONS OF THE SENS-
00340 PRINT* 2) THE LOCATION OF THE SENSING ELEMENTS"
00350 PRINT* 3) TYPE OF FORCING FUNCTION"
00360 PRINT* 4) THE HEAT TRANSFER COEFFICIENT"
00370 PRINT* 5) THE TIME STEP AND THE NUMBER OF TIME STEPS"
00380 PRINT* "
00390 PRINT* -----
00400 PRINT* "
00410 PRINT* THIS PROGRAM USES THE MATRIX EXPONENTIAL APPROACH TO"
00420 PRINT* SOLVE A SET OF COUPLED, LINEAR, ORDINARY, DIFFERENTIAL"
00430 PRINT* EQUATIONS WITH A CONSTANT FORCING FUNCTION."
00440 PRINT* "
00450 PRINT* =====
00460 DIM A(100,100),R(10),Z(10),K(10,10),D(10,10),C(10,10),T(100),Q(100)
00465 DIM A1(100,100),A4(100)
00470 DIM T1(100),C1(10,10),R1(10,10),R2(10,10),R3(10,10),R4(10,10),F(100)
00480 DIM Y(100),R5(10,10)
00490 DIM B(100,100),F1(100,100),G(100,100),C2(100,100)
00500 REM =====
00510 REM GENERATE THE NODES AND ASSIGN THE PHYSICAL PROPERTIES:
00520 REM
00530 PRINT "INPUT THE # OF RADIAL SECTIONS "
00540 INPUT N1
00550 PRINT#1," THE NUMBER OF RADIAL SECTIONS IS ="N1
00560 PRINT "INPUT THE # OF AXIAL SECTIONS"
00570 INPUT N2
00580 PRINT#1," THE NUMBER OF AXIAL SECTIONS IS ="N2
00590 RO=0

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00600 R=N1*N2
00610 FOR I=1 TO N1
00620 PRINT" INPUT DELTA RADIUS (CM) FOR NODE" I
00630 INPUT R
00640 R(I)=R/100
00650 PRINT#1,"R("I")="R(I)
00660 RO=RO+R(I)
00670 PRINT#1,"RO="RO
00680 NEXT I
00690 FOR I=1 TO N2
00700 PRINT" INPUT DELTA Z (CM) FOR AXIAL NODE " I
00710 INPUT Z
00720 Z(I)=Z/100
00730 PRINT#1,"Z("I")="Z(I)
00740 NEXT I
00750 REM
00760 PRINT"INPUT # OF MATERIALS THAT CONSTITUTES THE SENSOR"
00770 INPUT N3
00780 PRINT#1,"# OF SENSOR CONSTITUENTS="N3
00820 FOR L=1 TO N3
00830 PRINT"INPUT THE THERMAL CLNDUCTIVITY (W/M.C) FOR MATERIAL "L
00840 INPUT K
00850 PRINT" INPUT THE DENSITY (KG/M**3) FOR MATERIAL "L
00860 INPUT D
00870 PRINT"INPUT THE SPECIFIC HEAT CONSTANT (J/KG.C) R MATERIAL"L
00880 INPUT C
00890 PRINT" INPUT THE # OF ZONES THAT ARE MADE OF MATERIAL "L
00900 INPUT N4
00910 FOR P=1 TO N4
00920 PRINT" INPUT I,J FOR THE TOP LEFT NODE OF ZONE"P OF MATERIAL"L
00930 INPUT I1,J1
00940 PRINT"INPUT I,J OF THE BOTTOM RIGHT NCDE OF ZONE"P OF MATERIAL"L
00950 INPUT I2,J2
00960 FOR I=I1 TO I2
00970 FOR J=J1 TO J2
00980 K(I,J)=K
00990 D(I,J)=D
01000 C(I,J)=C
01010 NEXT J
01020 NEXT I
01030 NEXT P
01040 NEXT L
01050 FOR I=1 TO N2
01060 FOR J=1 TO N1
01070 PRINT#1,"K,D,C("I;J")="K(I,J);D(I,J);C(I,J)
01080 NEXT J
01090 NEXT I
01100 REM =====
01110 REM LCCATION OF SENSING ELEMENTS:
01120 PRINT" INPUT I,J OF THE TOP LEFT SENSING ELEMENT NODE"
01130 INPUT Y1,Y2
01140 PRINT"INPUT I,J OF THE BOTTOM RIGHT SENSING ELEMENT NODE"
01150 INPUT Y3,Y4
01160 FOR I=Y1 TO Y3
01170 FOR J=Y2 TO Y4
01180 PRINT#1," SE. AT#1, NODE("I;J")"
01190 NEXT J
01200 NEXT I
01210 REM =====
01220 REM INITIALIZE

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01230 PRINT"ASSUME THAT NO HEAT IS GENERATED AND THE TEMPERATURE OF THE "
01240 PRINT"SENSOR IS ZERO AT TIME ZERO"
01250 PRINT
01260 PRINT
01270 FOR I=1 TO N
01280 T(I)=0
01290 F(I)=0
01300 Q(I)=0
01310 NEXT I
01320 REM=====
01330 REM FORCING FUNCTION:
01340 PRINT" INPUT 1 IF FF IS A STEP CHANGE IN AMBIENT TEMPERATURE"
01350 PRINT" INPUT 2 IF FF IS A STEP CHANGE IN HEAT GENERATION"
01360 INPUT Y5
01370 IF Y5=2 THEN 1500
01380 REM FF IS A S.C. IN A.T.
01390 FOR I=N1 TO N STEP N1
01400 T(I)=5
01410 NEXT I
01420 REM FOR BOTTOM NODES:
01430 FOR I=N1*(N2-1)+1 TO N-1
01440 T(I)=5
01450 NEXT I
01460 FCRI=1TCN
01470 PRINT#1,"T1("I")="T1(I)
01480 NEXT I
01490 GOTC 1570
01500 REM FF IS A SC. IN HT. GR.
01510 Y6=(Y1*N1)-(N1-Y2)
01520 FOR I=Y6 TO N1*Y3 STEP N1
01530 Q(I)=1000000
01540 PRINT#1,"Q("I")="Q(I)
01550 NEXT I
01560 REM =====
01570 REM HEAT TRANSFER COEFFICIENT:
01580 PRINT"DO YOU WANT THE PROGRAM TO CALCULATE THE HEAT TRANSFER "
01590 PRINT"Coefficient (W/C.M**2) . (Y OR N)"
01600 INPUT W3
01610 IF W3="N" THEN 1810
01620 PRINT"INPUT THE VELOCITY OF THE FLUID. (M/S)"
01630 INPUT V1
01640 PRINT"INPUT K(W/M.C),D(KG/M**3),C(J/KG.C),MUE(KG/M.S) FOR THE LIQUID"
01650 PRINT"AT ABOUT 60 DEGREES CELCIOUS."
01660 INPUT K,D,C,M
01670 PRINT"K,D,C,M,VELOCITY OF FLUID="K;D;C;M;V1
01680 REM EVALUATE RENOLDS #'S
01690 E1=2*RO*V1*D/M
01700 PRINT"RENOLDS # ="E1
01710 IF E1>5000 THEN 1740
01720 IF E1>1000 THEN 1760
01730 IF E1>.1 THEN 1780
01740 PRINT"RENOLDS # IS TOO HIGH."
01750 GOTC 1810
01760 H=(K/(2*RO))*(.26*(E1**.6)*(C*M/K)**.3)
01770 GOTC 1830
01780 H=(K/(2*RO))*((.35+(.47*E1**.52))*((C*M/K)**.3))
01790 GOTC 1830
01800 PRINT"RENOLDS # IS TOO LOW."
01810 PRINT"USER MUST INPUT THE HEAT TRANSFER COEFFICIENT. (W/S.M**2)"
01820 INPUT H

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01830          PRINT#1,"I=";I
01840 REM =====
01850 REM CAPACITANCES:
01860 P6=0
01870 R7=0
01880 FOR I=1 TO N2
01890 FOR J=1 TO N1
01900 R6=R6
01910 R6=R(J)+R6
01920 C1(I,J)=3.14159*(R6**2-R7**2)*Z(I)*D(I,J)*C(I,J)
01930 PRINT#1,"C(I;J)=",C1(I,J)
01940 NEXT J
01950 R6=0
01960 R7=0
01970 NEXT I
01980 REM =====
01990 REM RESISTANCES:
02000 R6=0
02010 R7=0
02020 FOR I=1 TO N2
02030 FOR J=1 TO N1
02040 R6=R6
02050 R6=R(J)+R6
02060 R5(I,J)=2*3.14159*(R6**2-R7**2)*K(I,J)*K(I+1,J)
02070 IF I=N2 THEN 2160
02080 IF I=N2-1 THEN 2220
02090 IF I>1 THEN 2260
02100 REM I=1
02110 IF J=N1 THEN 2520
02120 IF J=N1-1 THEN 2610
02130 IF J>1 THEN 2400
02140 REM I=1, J=1
02150 GOTO 2430
02160 REM I=N2
02170 IF J=N1 THEN 2490
02180 IF J=N1-1 THEN 2640
02190 IF J>1 THEN 2550
02200 REM J=1, I=N2
02210 GOTO 2580
02220 REM I=N2-1
02230 IF J=N1 THEN 2670
02240 IF J=N1-1 THEN 2700
02250 IF J>1 THEN 2730
02260 REM J=1, I= N2-1
02270 GOTO 2760
02280 REM 1<I<N2-1
02290 IF J=N1 THEN 2460
02300 IF J=N1-1 THEN 2790
02310 IF J>1 THEN 2340
02320 REM J=1 & 1<I<N2-1
02330 GOTO 2370
02340 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J+1)*K(I,J)/((R(J)*K(I,J+1))+R(J+1)*K(I,J))
02350 R2(I,J)=R5(I,J)/(Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02360 GOTO 2810
02370 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(2*R(J)*K(I,J+1)+R(J+1)*K(I,J))
02380 R2(I,J)=R5(I,J)/(Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02390 GOTO 2810
02400 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(R(J)*K(I,J+1)+R(J+1)*K(I,J))
02410 R2(I,J)=R5(I,J)/(2*Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02420 GOTO 2810

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02430 R1(I,J)=4*3.14159*R(I,J)*R(I,J+1)*R6*Z(I)/(2*R(J)*K(I,J+1)+R(J+1)*K(I,J))
02440 R2(I,J)=R5(I,J)/(2*Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02450 GOTO 2810
02460 R1(I,J)=H*R6*2*3.14159*Z(I)
02470 R2(I,J)=K5(I,J)/(Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02480 GOTO 2810
02490 R1(I,J)=2*3.14159*R6*Z(I)*H
02500 R2(I,J)=H*(R6**2-R7**2)*3.14159
02510 GOTO 2810
02520 R1(I,J)=H*2*3.14159*Z(I)*R6
02530 R2(I,J)=R5(I,J)/(2*Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02540 GOTO 2810
02550 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(R(J)*K(I,J+1)+R(J+1)*K(I,J))
02560 R2(I,J)=H*(R6**2-R7**2)*3.14159
02570 GOTO 2810
02580 R1(I,J)=4*3.14159*R6*K(I,J)*K(I,J+1)*Z(I)/(2*R(J)*K(I,J+1)+R(J+1)*K(I,J))
02590 R2(I,J)=H*(R6**2-R7**2)*3.14159
02600 GOTO 2810
02610 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(R(J)*K(I,J+1)+2*R(J+1)*K(I,J))
02620 R2(I,J)=R5(I,J)/(2*Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02630 GOTO 2810
02640 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(R(J)*K(I,J+1)+2*R(J+1)*K(I,J))
02650 R2(I,J)=H*3.14159*(R6**2-R7**2)
02660 GOTO 2810
02670 R1(I,J)=H*3.14159*2*R6*Z(I)
02680 R2(I,J)=R5(I,J)/(Z(I)*K(I+1,J)+2*Z(I+1)*K(I,J))
02690 GOTO 2810
02700 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(R(J)*K(I,J+1)+2*R(J+1)*K(I,J))
02710 R2(I,J)=R5(I,J)/(Z(I)*K(I+1,J)+2*Z(I+1)*K(I,J))
02720 GOTO 2810
02730 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(R(J)*K(I,J+1)+R(J+1)*K(I,J))
02740 R2(I,J)=R5(I,J)/(Z(I)*K(I+1,J)+2*Z(I+1)*K(I,J))
02750 GOTO 2810
02760 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(2*R(J)*K(I,J+1)+R(J+1)*K(I,J))
02770 R2(I,J)=R5(I,J)/(Z(I)*K(I+1,J)+Z(I+1)*2*K(I,J))
02780 GOTO 2810
02790 R1(I,J)=4*3.14159*R6*Z(I)*K(I,J)*K(I,J+1)/(R(J)*K(I,J+1)+2*R(J+1)*K(I,J))
02800 R2(I,J)=R5(I,J)/(Z(I)*K(I+1,J)+Z(I+1)*K(I,J))
02810 R3(I,J)=R1(I,J-1)
02820 R4(I,J)=R2(I-1,J)
02830 PRINT#1,"R1,R2,R3,R4("I;J")="R1(I,J);R2(I,J);R3(I,J);R4(I,J)
02840 NEXT J
02850 R6=0
02860 NEXT I
02870 REM =====
02880 REM GENERATE THE A-MATRIX
02890 FOR I=1 TO N2
02900 FOR J=1 TO N1
02910 P=(I-1)*N1+J
02920 IF I>1 THEN 2980
02930 IF J>1 THEN 2960
02940 A(P,P+1)=R1(I,J)/C1(I,J)
02950 GOTO 3120
02960 IF J<N1 THEN 3100
02970 GOTO 3110
02980 IF I=N2 THEN 3030
02990 IF P<>I*N1 THEN 3090
03000 A(P,P+1)=0
03010 A(P,P-N1)=R4(I,J)/C1(I,J)
03020 GOTO 3110

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03030 I=N2
03040 IF J=N1 THEN 3000
03050 A(P,P+1)=R1(I,J)/C1(I,J)
03060 A(P,P-N1)=R4(I,J)/C1(I,J)
03070 A(P,P-1)=R3(I,J)/C1(I,J)
03080 GOTO 3130
03090 A(P,P-N1)=R4(I,J)/C1(I,J)
03100 A(P,P+1)=R1(I,J)/C1(I,J)
03110 A(P,P-1)=R3(I,J)/C1(I,J)
03120 A(P,P+N1)=R2(I,J)/C1(I,J)
03130 A(P,P)=- (R1(I,J)+R2(I,J)+R3(I,J)+R4(I,J))/C1(I,J)
03140 NEXT J
03150 NEXT I
03160 REM =====
03170 REM FF VECTOR
03180 Y9=0
03190 Y7=Y1*N1-(N1-Y2)
03200 Y8=Y3*N1-(N1-Y2)
03210 K1=0
03220 FOR J=1 TO Y2-1
03230 K1=R(J)+K1
03240 NEXT J
03250 FOR I=1 TO N2
03260 FOR J=1 TO N1
03270 P=(I-1)*N1+J
03280 IF Y5=2 THEN 3400
03290 REM THE ONLY FF IS A S.C. IN A.T.
03300 IF P<>I*N1 THEN 3320
03310 F(P)=T1(P)*R1(I,J)/C1(I,J)
03320 IF P<N1*(N2-1)+1 THEN 3470
03330 IF P=N THEN 3370
03340 F(P)=T1(P)*R2(I,J)/C1(I,J)
03350 PRINT#1,"F(P)"=F(P)
03360 GOTO 3470
03370 F(N)=(T1(N)*R1(N2,N1)+T1(N)*R2(N2,N1))/C1(N2,N1)
03380 PRINT#1,"F(N)"=F(N)
03390 GOTO 3490
03400 REM THE ONLY FF IS A S.C. IN HT. GK.
03410 Y6=Y7+Y9
03420 IF P<>Y6 THEN 3470
03430 F(P)=(Q(P)*3.14159*(((K1+R(J))*2)-K1*2))/C1(I,J)
03440 Y5=Y9+N1
03450 IF Y6>Y8 THEN 3490
03460 PRINT#1,"F(P)"=F(P)
03470 NEXT J
03480 NEXT I
03490 REM =====
03500 REM INITIAL CONDITIONS
03510 FOR I=1 TO N
03520 Y(I)=0
03530 NEXT I
03540 REM =====
03550 REM CALCULATE THE STEADY STATE TEMPERATURE:
03552 FOR I=1 TO N
03555 A4(I)=F(I)
03557 NEXT I
03560 FOR P=1 TO N-1
03570 FOR I=P+1 TO N
03580 A4(I)=A4(I)-A4(P)*A1(I,P)/A1(P,P)
03590 FOR J=N TO P STEP-1

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03600  A1(I,J)=A1(I,J)-A1(I,P)*A1(P,J)/A1(P,P)
03610  NEXT J
03620  NEXT I
03630  NEXT P
03640  FOR I=N TO 1 STEP -1
03650  Z=0
03660  FOR J=1 TO N
03661  Z=Z+A1(I,J)*T(J)
03662  NEXT J
03663  T(I)=(A4(I)-Z)/A1(I,I)
03664  NEXT I
03665  FOR I=1 TO N
03666  PRINT#1," THE STEADY STATE TEMPERATURE AT NODE "I" IS" T(I)
03667  PRINT" THE STEADY STATE TEMPERATURE AT NODE "I" IS" T(I)
03668  T(I)=0
03669  NEXT I
03670  REM =====
03680  REM DEMAT
03690  R5=0
03700  FOR I=1 TO N
03710  IF ABS(A(I,1))<R5 THEN 3730
03720  R5=ABS(A(I,1))
03730  NEXT I
03740  Q5=10
03750  PRINT"RECOMMENDED MAXIMUM VALUE TO USE FOR DELTA T="16.5/R5
03760  PRINT"INPUT DELTA T"
03770  INPUT T
03780  PRINT#1,"DELTA T="T
03790  PRINT"INPUT # OF TIME STEPS"
03800  INPUT N5
03810  Q=1
03820  Z=1
03900  PRINT"THESE ARE THE NONZERO FORCING VECTOR ELEMENTS:"
03910  FOR I=1 TO N
03920  IF F(I)=0 THEN 3940
03930  PRINT"F("I")="F(I)
03940  NEXT I
03950  FOR I=1 TO N
03960  FOR J=1 TO N
03970  B(I,J)=0
03980  F1(I,J)=0
03990  G(I,J)=0
04000  IF I<>J THEN 4020
04010  B(I,J)=1
04020  F1(I,J)=0
04030  G(I,J)=0
04040  NEXT J
04050  NEXT I
04060  P=T/Q
04070  FOR I=1 TO N
04080  FOR J=1 TO N
04090  F1(I,J)=F1(I,J)+B(I,J)
04100  C2(I,J)=B(I,J)*P
04110  G(I,J)=G(I,J)+C2(I,J)
04120  B(I,J)=0
04130  NEXT J
04140  NEXT I
04150  FOR I=1 TO N
04160  FOR J=1 TO N
04170  FOR L=1 TO N

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04180 B(I,J)=B(I,J)+C(I,L)*A(L,J)
04190 NEXT L
04200 NEXT J
04210 NEXT I
04220 Q=Q+1
04230 IF Q<=Q5 THEN 4060
04240 PRINT#1,"T=0"
04250 FOR I=1 TO N
04260 PRINT#1,"I("I")="T(I)
04270 NEXT I
04280 V9=.1
04290 IF ABS((T*Z)-V9)>.001 THEN 4310
04300 PRINT#1,"TIME IS="T*Z"SECONDS"
04310 FOR I=1 TO N
04320 FOR J=1 TO N
04330 T(I)=T(I)+F1(I,J)+Y(J)+G(I,J)*F(J)
04340 NEXT J
04350 IF ABS((T*Z)-V9)>.001 THEN 4370
04360 PRINT#1,"I("I")="T(I)
04370 NEXT I
04380 IF ABS((T*Z)-V9)>.001 THEN 4480
04390 Y6=(Y1*N1)-(N1-Y2)
04400 X=0
04410 K=0
04420 FOR I=Y6 TO N1*Y3 STEP N1
04430 K=K+1
04440 X=X+T(I)
04450 NEXT I
04460 PRINT#1,"THE AVERAGE TEMPERATURE OF ALL SENSING ELEMENTS IS"X/K
04470 V9=V9+.1
04480 FOR I=1 TO N
04490 Y(I)=T(I)
04500 T(I)=0
04510 NEXT I
04520 Z=Z+1
04530 IF Z<(N5+1) THEN 4290
04540 END

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VITA

Majed N. Natour was born December 17, 1958, in Amman, Jordan. He received his elementary, preparatory, and secondary education in the Jordanian public school system.

In November 1977, he came to the USA to pursue higher education. He attended the English Language Institute at Belmont, N.C. and obtained an English Diploma in December 1977. He joined Gaston College in Gastonia, N. C. in March 1978. He transferred to the University of Tennessee at Knoxville in June 1978. He obtained a Bachelor of Science Degree in Nuclear Engineering in June 1982. He received his Master of Science in Nuclear Engineering in March 1986. Since that time he has pursued the degree of Doctor of Philosophy in Nuclear Engineering.

He is a member of Tau Beta Pi, and the American Nuclear Society.