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**GEOMECHANICAL MODELLING USING A BOUNDARY LOADED BASE
FRICTION MACHINE**

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Steven Craig Smith

December 1994

DEDICATION PAGE

This thesis is dedicated to my parents

Allen and Marilyn Smith

who have given me unconditional support and love

in all of my endeavors throughout life. I also dedicate

this thesis to my fiance

Lisa Mize

who has supported me through the entire process and never given up or lost patience.

ACKNOWLEDGMENTS

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ABSTRACT

This research deals with the design and demonstration of a modified base friction machine. The Base Friction Method, or BFM for short, was first conceived in the late 1960's by Dr. R.E. Goodman and has been in use since 1973. The premise behind the BFM permits the replacement of gravity in the plane of a two dimensional physical model by drag forces acting along its base. The machine built for this thesis utilizes a stationary base and a moving model, which is a variation of the standard continuous belt machines that have previously been used. This new machine uses air pistons to apply vertical and horizontal boundary stresses to the model, where as most of the previous machines applied a surface pressure to the model in order to increase the unit weight of the material. In order to demonstrate the machine, two case studies consisting of a tunnel sequence with varying vertical and horizontal boundary stresses and a slope sequence are performed.

The machine uses scaled material models in order to simulate movement due to gravity over a time period of a few minutes, as opposed to several years for full scale studies. The models consist of a geologic modelling material where the properties of the material can be scaled to match the properties of the prototype rock in question. The material used in this thesis is a mixture of flour, vegetable oil and Ottawa Sand.

The machine demonstrates with considerable accuracy how tunnels and slopes deform under gravity. By applying the boundary stresses, the tunnel sequence demonstrates how changing the confining stress on a tunnel effects its stability.

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CHAPTER 1

INTRODUCTION

1.1 History of Base Friction Technique

Physical modelling of rock masses has been used extensively for the past few decades in parallel with the considerable advancement in numerical techniques. Egger (1979) states that the capacity of numerical solutions to simulate realistically complex geotechnical situations such as the interaction of several joint sets or the opening of joints due to block rotation, is still unsatisfying. Even with the advancement in computer technology over the last decade, the behavior of rock can not be predicted accurately by numerical techniques alone. Depending on the type of rock, depth of interest, and the stress state of the rock, the rock mass can behave in several different ways.

A single rock mass can have regions that exhibit elastic behavior and regions that exhibit plastic behavior, such as the case of a tunnel in hard rock. The hard rock can generally be characterized by an elastic analysis, while the region around the tunnel may be characterized by plastic behavior. In order to model this case, a complicated nonlinear finite element model could be used which would require a considerable amount of time formulating and refining, or a scaled physical model could be used to study the behavior of the rock mass with less time involved in preparation. Either option could be used to study rock behavior, but Erguvanli and Goodman (1972) suggest that both options be used in a way that they complement each other. The finite element method

can be used to examine the effect of changing properties from one rock layer to another, since several problems can be run from a single program set-up. The physical models can be used to identify the most probable modes of failure which can be helpful in setting up the more expensive computer models (Erguvanli and Goodman 1972).

In the past, there has been considerable time and money invested in the physical modelling of discontinuities such as joints and faults in order to construct realistic models. According to Stimpson (1979), the absence of a simple method of simulating discontinuities had restricted the use and application of physical models. In the area of gravity simulation and other two dimensional model simulations, this need for simple modelling methods led to the development of the Base Friction Method.

The Base Friction Method, or BFM for short, has been in use since 1973, but the idea was first conceived in the late 1960's by Dr. R.E. Goodman. The premise behind the BFM permits the replacement of gravity in the plane of a two dimensional physical model by drag forces acting along its base. These forces are applied by drawing a continuous belt under a stationary model or by pushing the model over a flat surface (Bray and Goodman 1981). Egger (1979) states that this technique is particularly well suited for near-surface problems where the stresses change considerably over the height of the model.

In 1969, Goodman reported the results of experiments with stacked sugar cubes pushed over a flat surface, at a lecture at Imperial College, which suggested that gravity effects could be achieved in models in this way (Bray and Goodman 1981; Hoek 1971). Later on in 1971, Hoek (1971) demonstrated the principle of this model at his inaugural

lecture by pulling a sheet of paper out from beneath a pattern of blocks, where the frictional force on the base of the blocks simulates gravitational loading of the rock mass. He went on to discuss the effectiveness of this simple model for studying complex movement patterns of rock. The first base friction machine was constructed in 1973 at Imperial College. Some of the reasons the first machine was built in London are; Professor Hoek's interest in the technique, the ease of model preparation and testing, and the apparent correctness of results as compared with tilt frame models. (Bray and Goodman 1981).

1.2 Previously developed machines

Since 1973, several other machines have been built and used for qualitative and quantitative experiments. The following sections will discuss the differences of the most popular machines.

1.2.1 Stationary table vs. moving tables

The machines can be broken down into two major categories: stationary table or moving table. The main difference in the two types of tables is in the way the drag force is applied. The drag force for the stationary table is applied by running a continuous belt under the model, while the moving table applies the force by moving the model over a stationary flat surface. When Goodman developed the base friction method he used the moving table concept, but when Hoek demonstrated the method during his inaugural lecture he used the stationary table concept (Goodman 1976; Hoek 1971).

The stationary table concept caught on and has been the basis for the design of

all of the well known machines. Table 1.1 summarizes the most well-known machines. The table shows that the basic design concept behind all of the machines is a stationary model with the friction force, or drag force, being applied by a continuous belt of sand paper. The only exceptions are the machines using the stiff plate and the vinyl belt (Kameda et al. 1992; Shimada 1992). The reason for these deviations will be discussed in a later section. There were no moving table concepts found in the literature except for the models used by Goodman when developing the base friction method. The machine built for this thesis is a moving table machine and the reasons for choosing this method will be discussed in a later section also.

1.2.2 Friction Surfaces

The majority of the machines built use a continuous belt of sand paper due to the availability of long continuous belts for sanding equipment. This belt works most of the time, but under a large base friction force, the continuous belt has been found to stretch. This led Kameda et al. (1987) to use a very stiff metal plate coated with urethane rubber instead of the continuous belt. By using the continuous belt there is also the possibility that the frictional properties along the belt will change as the model material erodes away and material deposits build up on the belt. Bray and Goodman (1981) acknowledge this possibility and suggest using a smooth belt in place of a rough belt to reduce model degradation and to reduce the amount of particle build up along the belt.

1.2.3 Side pressure vs. Top pressure

Before Egger built his machine in 1979, the base friction machine was only used for demonstration purposes. In 1979 Egger was trying to develop a way to overcome

Table 1-1: Summary of Base Friction Machines

Location	Drag Force	Reference
Imperial College	continuous belt of sand paper	Bray and Goodman 1981
University of California, Berkeley	continuous belt of sand paper	Goodman 1976
Swiss Federal Institute of Technology, Lausanne	continuous belt of sand paper	Egger 1979; Karaca 1992
University of Tokyo	vinyl belt	Shimada 1992
Department of Earth Science, University of Leeds	continuous belt of sand paper	Al-Harthi and Hencher 1992
Research Institute of Industrial Science, Kyushu University	stiff urethane coated metal plate	Kameda et al. 1987

large scale factor values. The only way to lower the scale factor is to increase the weight of the material or increase the friction force acting along the base of the model. Egger determined that the easiest and most efficient way to do this was to apply a controllable air pressure across the top of the entire model. This in effect, increased the unit weight of the model material (Egger 1979). According to Bray and Goodman (1981), this idea was an important innovation in base friction modelling because Egger greatly improved the chances of attaining stress and strength similitude over a range of length and density scales. By being able to maintain similitude, Egger was able to record accurate data on stress and strains by attaching various pieces of measuring equipment such as hydraulic pressure cells and optical equipment (Egger 1979). Due to this advancement in base friction modelling, the base friction method became a tool capable of accurate quantitative predictions (Bray and Goodman 1981).

Kameda et al. (1987) took this idea one step further and added a lateral loading system which consisted of loading jacks on two sides of the model and load cells on one end of the model. By including lateral pressure, this allowed horizontal stresses to be incorporated into the model, along with the pressure over the entire model.

Shimada (1992) also modified the base friction model to incorporate horizontal stresses. Shimada developed the Modified Base Friction Method which uses wooden dowels and springs to apply the horizontal stresses. In this model the dowels and the springs apply the horizontal stress on one side of the model only. Shimada's model does not allow for a controlled pressure to be applied to the top surface of the model.

As Egger stated, by applying the air pressure to the top of the model it is easier

to establish the similitude requirements necessary for accurate geological modelling. With the addition of horizontal stress capabilities by Kameda et al. and Shimada, the stress state for many different situations can be more closely approximated. This addition also gives a wider range of geometric and strength scaling factors for the modelling materials being used. The machine built for this thesis incorporates a slightly different external loading mechanism which will be explained in a later section.

1.3 UT Base Friction Machine

The base friction machine built for this thesis is a combination of the concepts used in the machines that were previously built. The machine uses a moving table design, and uses air pistons to apply pressure to the sides of the sample instead of the air pressure being applied to the top of the model. This machine is an adaptation of the Esaki's (1992) idea of a rigid, slow moving table. Instead of a slow moving table, a rigid, slow moving mold is used, which produces the same result as will be shown later. The side pressure is applied in the plane of the model to two sides of the model simultaneously, where the surface pressure previously used is applied out of plane or perpendicular to top surface of the model in order to increase the unit weight of the sample. Figure 1-1 defines the top and sides of the model.

The sample mold has the capability of applying air pressure to all four sides of the model, but pressure can only be applied to three sides at one time. The reason for the air pistons on the sides of the model is to more closely approximate similitude. Since the pistons can be individually controlled, the pressure applied to the side and top of the

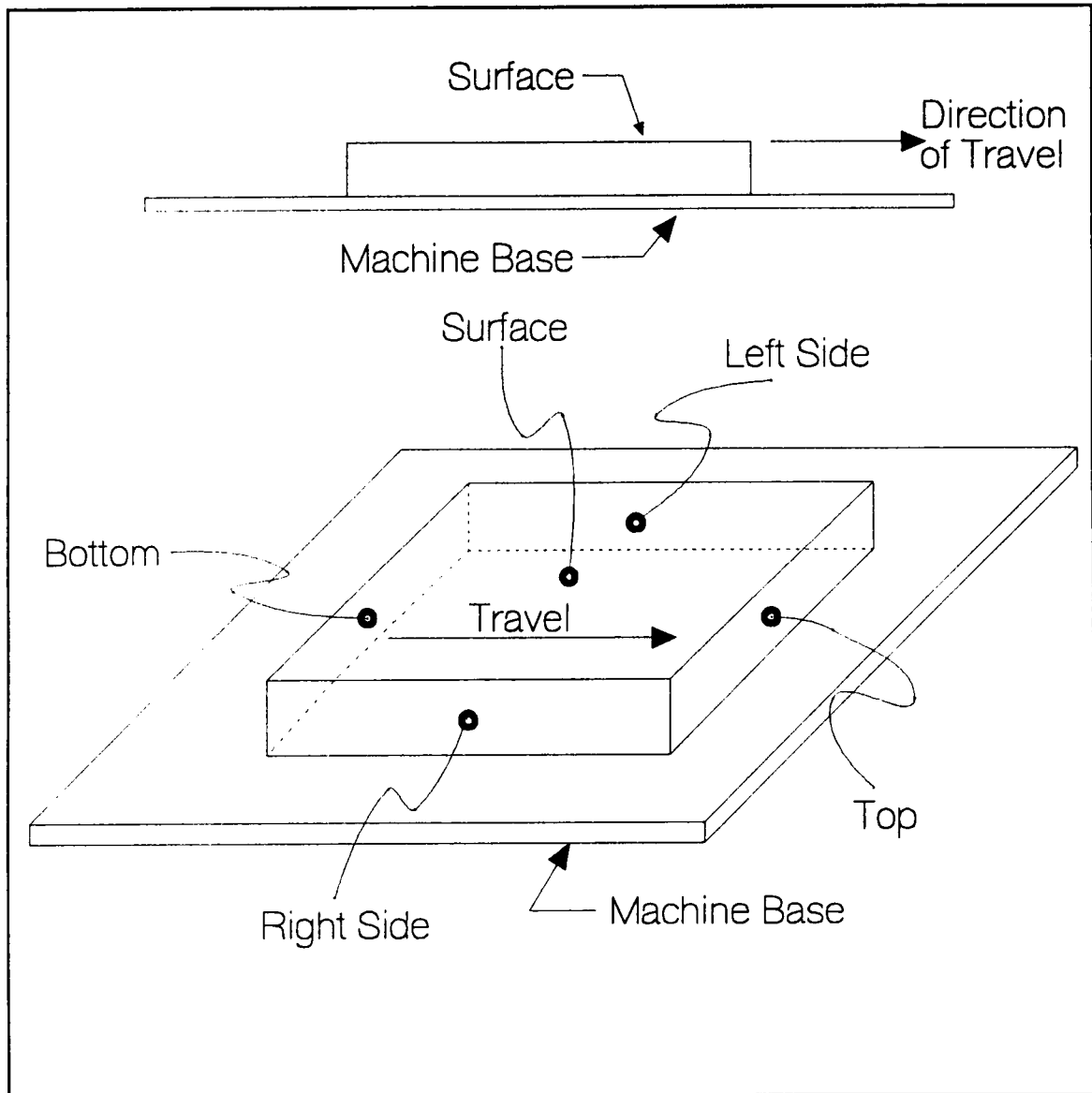


Figure 1-1: Definition of base friction model's surfaces

model can be considerably different. The variability of the air pressure allows the sample to demonstrate behavior at depth with different side pressures or horizontal stresses.

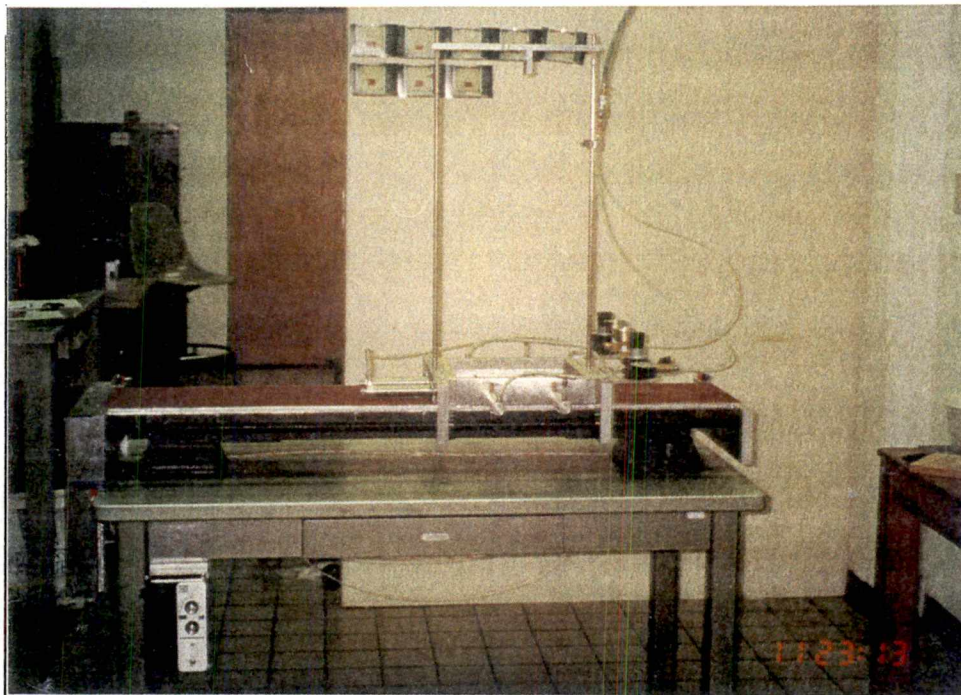
Unlike the machines built previously, a choice of frictional surfaces may be used. The frame travels over an aluminum plate with a frictional surface taped to it with double stick tape. This allows for any surface such as different grades of sand paper or rubber mats to be used. By making the surface interchangeable, different modelling materials as well as different frictional surfaces may be used to get the desired results. This type of flexibility gives a broader testing range than the machines built before this machine.

The actual model mold size is 12 in. wide x 12 in. long x 2.5 in. deep (30.48 cm wide x 30.48 cm long x 6.35 cm deep). This size was chosen due to the size of available frictional surfaces. The mold travels along a 13 in. (33.02 cm) wide aluminum plate. The frictional material is cut to be 12 in. (30.48 cm), leaving the extra space for the wheels on the sample mold. The length of travel for the sample is 44 in. (111.76 cm). The mold height above the frictional surface is adjustable. The frame can adjust from resting on the surface, up to a height of 1 in. (2.54 cm). The mold is driven by two threaded rods, one on each side of the machine.

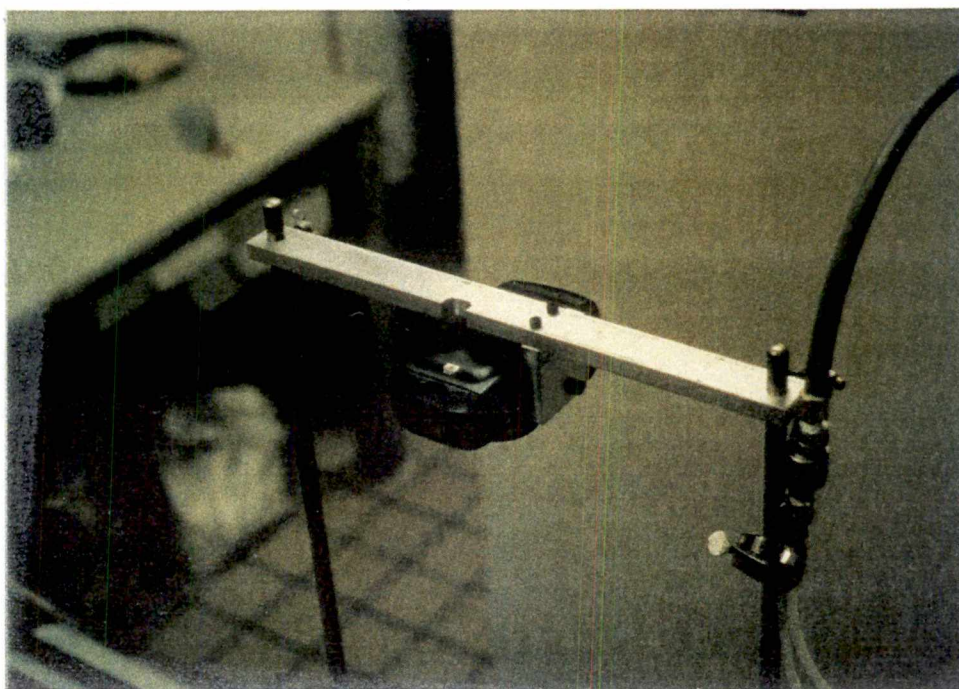
The machine is driven by a Bodine DC motor, and is controlled by a Bodine adjustable speed, reversible controller. The motor is a 1/8 Hp motor with an output torque rating of 310 ft-lbs (35 N-m). The drive unit is a 1:1 chain drive system. The velocity range of the machine is 0.08 in/min to 0.86 in/min (2.03 mm/min - 21.84 mm/min). If more or less speed is desired, different sized sprockets can be used.

The machine is connected to a central air pressure system capable of 150 psi (1034 kPa), but the air pressure into the machine has been regulated to 60 psi (414 kPa). The main line splits into two regulators; where one regulator operates the top or bottom pistons and the other regulator operates the pistons that supply the pressure on the left and right sides. Each regulator has two pressure gauges in order to read high and low pressures accurately. The high pressure gauge has a range of 0 psi to 60 psi (0 kPa to 414 kPa), and the low pressure gauge has a range of 0 psi to 30 psi (0 kPa to 207 kPa). The air pistons used to apply the pressure to the sample are four Bimba air pistons, model 094-DP, capable of applying a maximum pressure of 54 psi (372 kPa) each. The pistons have a stroke length of 4 in. (10.16 cm).

A bracket is attached to the mold in order to attach a still photo camera, or a video camera for recording the tests. A Pentax WR90 35 mm camera is used in this thesis for documenting the test. Plates I-A and I-B show photographs of the base friction machine and various components. For further detail, appendix B contains a schematic of the machine.

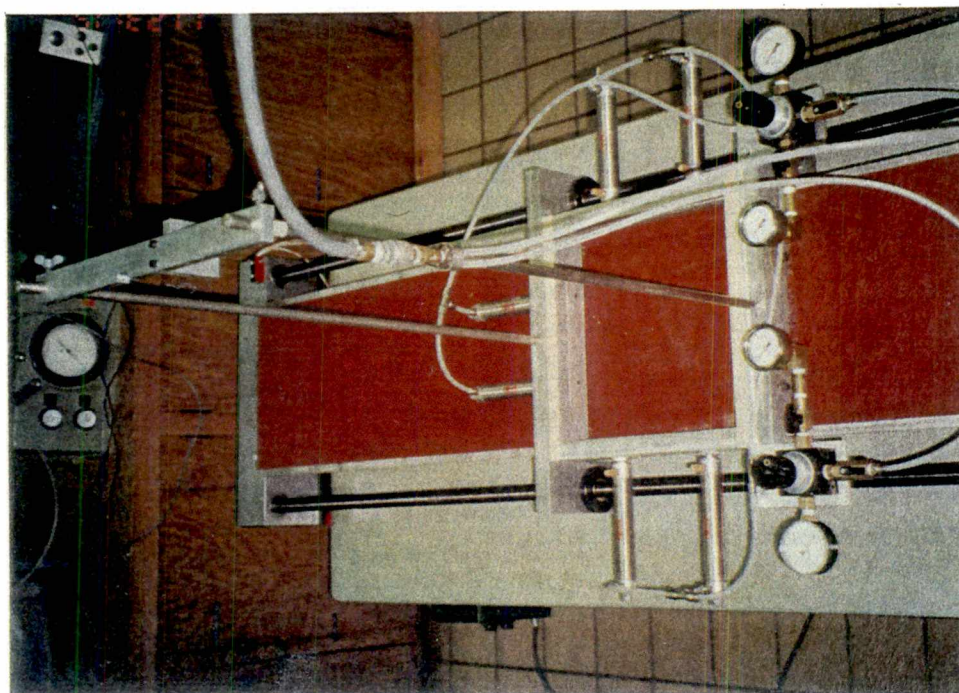


(a)

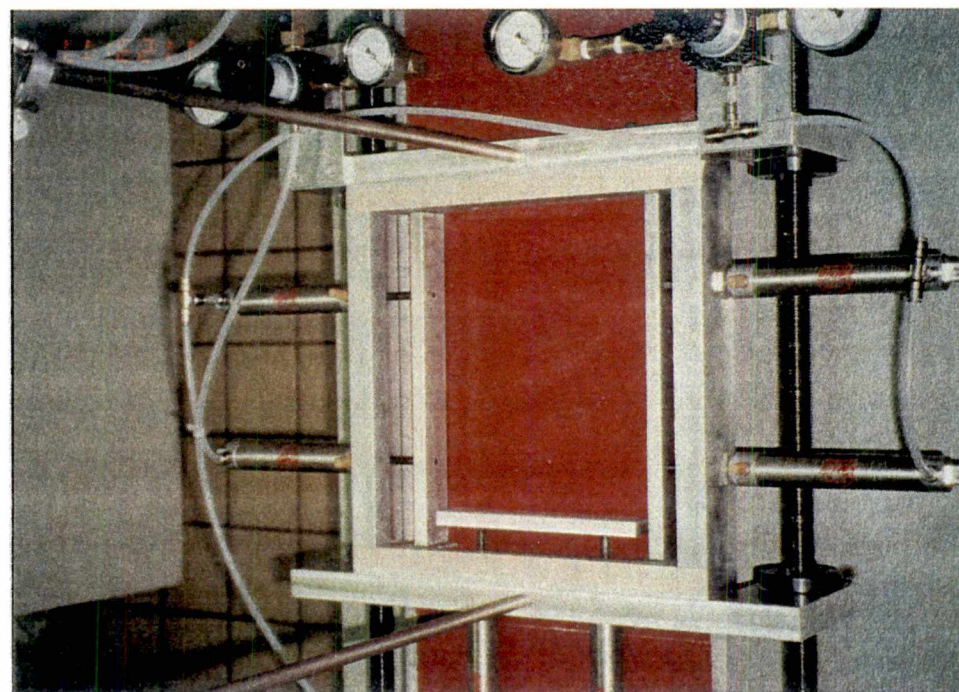


(b)

Plate I-1: (a) Base Friction Machine; (b) Close-up of camera mount with camera



(a)



(b)

Plate I-2: (a) Model Frame Assembly; (b) Mold with pistons extended

CHAPTER 2

BASE FRICTION THEORY

2.1 Gravity Simulation

Forces due to gravity are a special case and have to be considered differently from other external body forces. The main difference between gravity loads and surface forces, or contact forces, is the fact that the force due to gravity never changes as the model translates or changes position. As a body moves, the magnitude and direction of the gravitational force is constant. The same can not be said for other forces such as the reactions induced by adjacent rock. As the rock mass in question rotates or translates, the in-situ stresses are released or changed in some manner, and as the mass continues to reposition, the stresses will change back to a state of equilibrium.

Bray and Goodman (1981) discuss this in relation to the difficulties that arise from movements of blocks isolated by intersecting joint surfaces in shallow or surficial excavations. They compare the rock movements due to stress concentrations and resulting rock fracturing to the movement that results from being driven by a stiff testing machine, because the block or rock mass will resist the load until it fractures and then upon fracturing, the load finds a new reaction in contiguous unfractured ground. This load transference is not the case for gravity or a block moving under its own weight, because the pull of gravity is not lessened by any amount of fall. Misleading results are likely to be drawn from models of excavations that rely exclusively on external pressures to supply load to the model (Bray and Goodman 1981). There are three types of models

in which gravity can be simulated (Bray and Goodman 1981):

1. Tilt Table: The model is built while it lies flat or on an inclined plane and then tilted to increase gravity up to 1g.
2. Centrifuge model: The model is spun in a centrifuge; and
3. Base Friction model: The model can be subjected to base friction.

Tilting models have been used for a long time but they are difficult to control. Other problems associated with this type of experiment consist of unwanted vibrations which prematurely loosen key blocks, and no easy way to observe failure modes without "turning off" gravity or by using high speed cameras (Bray and Goodman 1981).

Centrifuge models have also been used for a long time but they are even more difficult and more expensive to perform than the tilting models. These models have a size limitation imposed on them due to the testing apparatus. This size limitation and the fact that gravity is controlled by the rate of rotation requires that excessive angular velocities must be used for rock masses. It is possible to fail the model material due to the excessive velocities achieved, which leads to distortions in the model material (Bray and Goodman 1981).

Base friction models, on the other hand, are cheap and easy to perform. They use readily available materials and the failure modes are easy to record. "Gravity" can also be turned on and off as needed in order to record and photograph any data or distortions that may be observed. Even though these models are cheap and easy, they are not without their limitations.

2.2 BFM Limitations

As mentioned previously, the principle behind the base friction method is the replacement of gravity by drag forces acting along the base of the model. The BFM became very popular due to the ease of model preparation and the accuracy of the results. Even though the BFM is a very simple and widely acclaimed modelling technique, it is not without limitations.

The limitations involved with this method are due to the fact that this technique uses 2-dimensional models. Therefore, the BFM can only be used when the problem can be scaled down from three dimensions to two dimensions. The 2-dimensional limitation also changes the type of stress situation that is modelled. By using a 2-dimensional model, a plane-stress situation is approximated rather than the plane-strain situations which occur in nature or at depth in 3-dimensional models (Al-Harthi and Hencher 1992). Another major limitation sighted by Al-Harthi and Hencher (1992) is buckling and crushing. Unrealistic situations such as buckling and crushing which consist of plastic movement normal to the frictional surface may occur. This situation can be prevented by choosing the appropriate model material, and by paying close attention to the model dimensions and thickness.

This method is also restricted to kinematic modelling due to incorrect simulation of momentum. In situations where the rock mass attains significant horizontal momentum and then impacts another surface or loses contact with adjacent surfaces, the method is deficient in this type of model simulation (Stimpson 1979).

Even with these limitations, the BFM is still a very effective and tool which is easy and cheap to use.

2.2.1 Advantages and Disadvantages of Method

The major advantage to the method is the ease with which the test can be performed. The model materials that can be used are cheap and readily available. The test is also very easy to setup and run, therefore there is not much time lost due to model preparation and testing. Another advantage is that the results that can be obtained compare very well with more sophisticated and more complicated test procedures such as tilt tests or centrifuge test.

A disadvantage to the test is the limitations placed on it by the 2-dimensional model being used. This disadvantage only allows the modelling of situations where gravity is the only disturbing force being applied, and the method is only valid when the 3-dimensional problem can be scaled down accurately to a 2-dimensional plane stress model. Stimpson (1979) discusses another disadvantage involved with the stresses induced on the model. Since the BFM induces low stresses on the model, it is difficult to model compressive rock fracture and crushing. He states that a very weak model material can fail in a means similar to a thrust fault, which would pass from the base of the model to the surface of the model, invalidating the model.

The machine built for this thesis will handle some of these limitations and disadvantages through the careful choice of model material for strength. The low stress disadvantage will also be dealt with through the application of the variable side pressures.

2.3 Derivation of Base Friction Force

The BFM replaces the gravitational force with a frictional force that develops between the contact of the model material and the machine surface. Bray and Goodman (1981) give a detailed discussion of the derivation of the base friction force in their paper; therefore this section will just outline the derivation of the force.

Before deriving the base friction force, the chosen sign convention and axis orientation will be established. Compressive stresses will be positive and the orientation of the model axis are according to figure 2-1, which is taken from Bray and Goodman (1981). Static equilibrium conditions will be the only conditions discussed in this thesis. Limiting equilibrium and dynamic conditions due to toppling and sliding are outside the scope of this section and will not be discussed. If the reader wishes to study these conditions, Bray and Goodman's paper gives a detailed discussion.

Under static conditions consider a model that is restrained by one or more fixed barriers, which are assumed frictionless, in order to prevent movement. Static conditions are assumed after the initial disturbance on the model from starting the machine and separating the model from the frictional surface. The resistance to the drag along the base of the model is called the base friction force. The base friction force, F_b , that develops is in the plane of the model and opposite the drag force on the model. The magnitude of this force is dependent upon the weight of the material and the coefficient of friction of the base surface.

The drag force on the model can be described by considering a differential slice of thickness, dz , of the model under its own weight. The incremental drag force (dT)

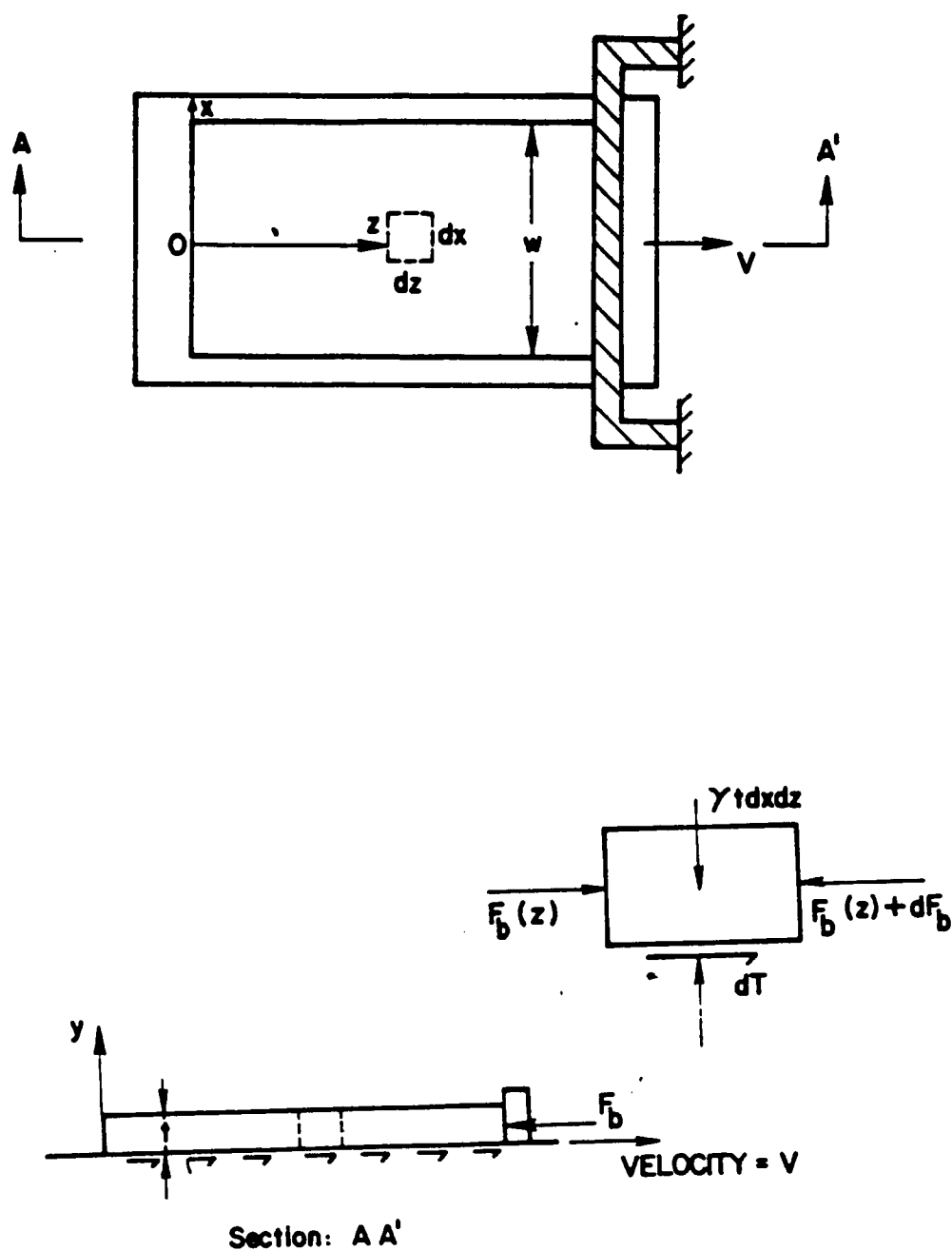


Figure 2-1: Base Friction Force Derivation (Bray and Goodman 1981)

can then be described by:

$$dT = \mu_b \gamma t dx dz \quad (2-1)$$

where: μ_b is the coefficient of friction between the model and the frictional surface; γ is the unit weight of the model material; and t is the thickness or height of the model. The equation for describing true gravity in the prototype is very similar to equation (2-1) for the model. The incremental body forces due to "gravity" (dG) act on each element of the prototype and yield the following equation:

$$dG = \gamma t dx dz \quad (2-2)$$

The only difference between equations (2-1) and (2-2) is the coefficient of friction for the base.

Since the base friction force is equal in magnitude to the drag force on the model, the base friction force of each element of the model (dF_b) can be obtained by assuming a constant width w , and integrating equation (2-1) over the length, L , of the model.

$$dF_b = \int dT = \int_{z=0}^L \mu_b \gamma t dx dz \quad (2-3)$$

By assuming a model of uniform width, w , the base friction force at any depth z is:

$$F_b(z) = \mu_b (\gamma t) wz \quad (2-4)$$

The normal stress, σ_z , parallel to the direction of model can be obtained by assuming a uniform width, w , and dividing equation (2-4) by the uniform cross-sectional area, wz . This yields the following equation, which is equivalent to vertical stress at depth.

$$\sigma_z = \mu_b \gamma z \quad (2-5)$$

Equation (2-5) shows the relationship between vertical stress at depth z due to true gravity, which is γz , and the equivalent to this vertical stress in the base friction model. This relationship holds true, as long as frictionless lateral restraints are assumed in the z direction.

2.4 Model Scaling and Similitude Laws

Scale modelling allows an experimenter to study large or complicated problems that would otherwise be too large to study. By constructing a scaled model of the problem, the experimenter can study the long term behavior of the full scale problem in a short amount of time. The whole purpose of a scale model is to scale down the physical size of the actual problem, and the time needed to analyze the problem, into some realistic model which is similar to the actual problem in size and behavior. Scale models are intended to duplicate structural deformations in miniature (Erguvanli and Goodman 1972).

For example, the base friction model is used to examine the affects of gravity in rock structures. The experimenter can use the model to produce similar tunnel deformations and slope failures in a few minutes, which could take from days to years to happen in real life. The experimenter can also vary aspects of the model in order to study different situations in a short amount of time.

In order for the models to produce accurate results they must be scaled in a way that the model will accurately portray the behavior of the full scale problem. To ensure that the model will behave appropriately, the experimenter must follow certain rules or laws governing similitude. By following these laws, distortion of important model parameters is prevented and a good representation of the problem can be developed.

In geologic structures, it is known that as the size of a body is changed its various other physical properties change, but not necessarily at the same rate the size is changed (Hubbert 1937). This principle holds true for many situations in every day occurrences.

An easy way to illustrate this principle is by using Hubbert's cube of lead example. Figure 2-2 illustrates how the dimensional properties change as the size of the cube is changed. By considering a single cube of lead as having a width of one and maintaining constant density, the change in the length, area, volume, mass, and pressure can be illustrated as the width, n , increases from 1 to 3.

All of the quantities associated with the single cube will be considered to have the unit dimensions, or unity (i.e. unit length, unit area, unit volume, unit mass, etc.). When n is increased to 2, the length of any side will be 2, the area will be 4, the

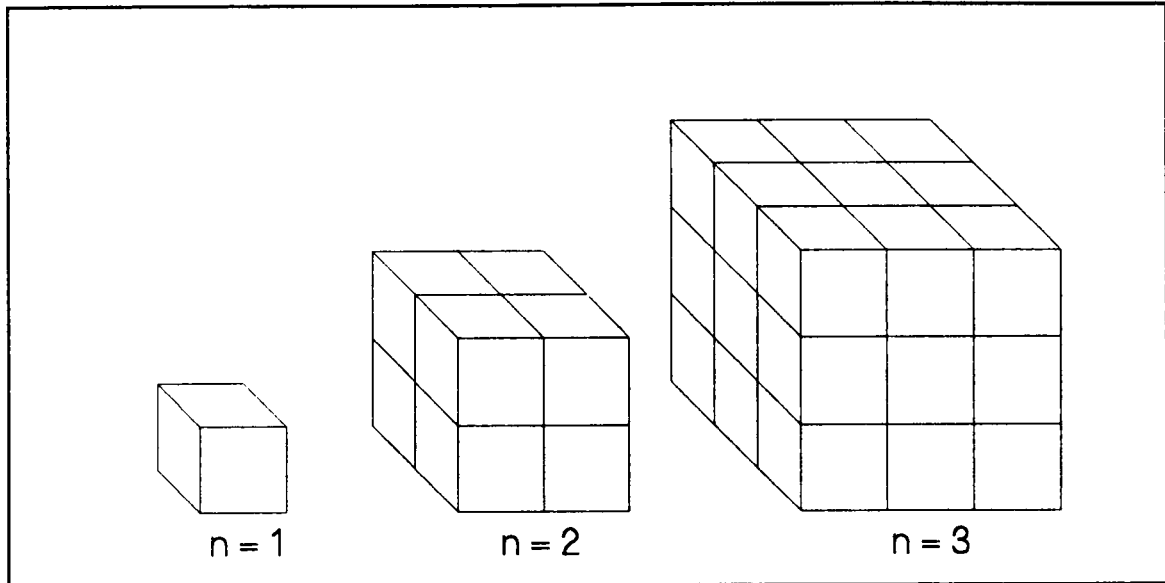


Figure 2-2: Cube example illustrating change in size principle
 (Source: Hubbert, M. K. 1937).

volume is 8, and the mass is also 8. The pressure is weight divided by area, and therefore becomes 2. For $n = 3$, the side is 3, the area is 9, the volume, mass, and weight is 27, and the pressure becomes 3.

From this comparison, it can be shown that area increases as the square of the linear dimension; the volume, mass and weight increases as the cube of the linear dimension; and the pressure is equivalent to the increase in width, n .

This example illustrates that scaling is not a linear process. For this reason, laws of similitude have been established in order to maintain the similarity between the model and the prototype, or full scale problem. As mentioned above, the main consideration is to prevent distortion of the important model parameters, such as Young's modulus, strength parameters, and dimensional properties.

Hubbert (1937) and Fumagalli (1968) both give good discussions on scale models and the theory associated with similitude, and it would be advantageous for any experimenter to read these papers before attempting any scale modelling. This section will present the ideas of both authors, but will concentrate mainly on Hubbert's paper, since the ideas presented by both authors are similar. Hubbert (1937) gives a very detailed discussion of the similitude laws and the theory behind similitude. For the purpose of this thesis, this discussion will be limited to the discussion of scaling two dimensional models in the elastic range.

2.4.1 Types of Similarity

When discussing scaling between the model and the prototype, the similarity of the model and the prototype must be discussed. Similarity, or similitude, describes the correlation between the dimensions of the model and the prototype. In order to get the model to behave in the same manner as the prototype, the model must be "similar" to the prototype.

When trying to maintain similarity, there are three types of similarity that must be considered and dealt with. The three types or degrees of similarity are defined as follows (Hubbert 1937):

1. Geometric Similarity: Two bodies are geometrical similar when all corresponding lengths are proportional and all of the corresponding angles of the two bodies are equal.
2. Kinematic Similarity: When two geometrically similar bodies undergo similar changes in shape or position, the two are said to be

kinematically similar, provided the time required for the change in both bodies is similar.

3. Dynamic Similarity: Requires that the model has a mass distribution similar to the prototype.

2.4.1(a) Geometric Similarity

The definition for geometric similarity states that all linear dimensions of the two bodies must be proportional. With this in mind, Hubbert (1937) defines the fundamental ratio of length as:

$$\lambda = \frac{l_p}{l_m} \quad (2-6)$$

where: λ = the constant of proportionality of length, or the model ratio of length

l_p = length in the prototype

l_m = length in the model

The ratio, λ , can either be greater than one or less than one depending on whether the scale model is an enlargement or a reduction. Since the length, l , can be any linear dimension in the model and the prototype, this ratio can be set at the beginning of testing. This ratio is normally governed by the size of the sample mold.

Since the area of a body is equal to the square of its linear dimensions and the lengths of the two bodies are proportional by definition, then an equation for the ratio of the areas can be obtained from the model ratio of length.

$$\lambda^2 = \frac{A_p}{A_m} \quad (2-7)$$

where A_p and A_m are the areas of the prototype and the model respectively. Then substituting equation (2-6) into equation (2-7), equation (2-8) is developed:

$$\lambda^2 = \frac{A_p}{A_m} = \frac{l_p^2}{l_m^2} \quad (2-8)$$

An expression for volume can also be derived based on the definition of geometric similarity. With volume being the cube of the length dimension, then a fundamental expression for the volume ratio of geometrically similar bodies is:

$$\lambda^3 = \frac{V_p}{V_m} \quad (2-9)$$

where V_p and V_m are the volumes of the prototype and the model respectively. Then again by substituting equation (2-6) in the above expression, the equation for the ratio of corresponding volumes is:

$$\lambda^3 = \frac{V_p}{V_m} = \frac{l_p^3}{l_m^3} \quad (2-10)$$

2.4.1(b) Kinematic Similarity

Time is another fundamental dimension that must be scaled. When dealing with gravity, as is this base friction machine, velocity and acceleration become a factor in similitude. Even though time is a factor, the dimension of time eventually drops out of the scaling equations, as will be shown later. The fundamental ratio of time is derived as follows:

$$\tau = \frac{t_p}{t_m} \quad (2-11)$$

where t_p and t_m are the times required for the prototype and model to undergo the given transformation, and τ is the model ratio of time.

If two bodies are kinematically similar, then by definition the time required for the respective bodies to undergo changes must be proportional. With this in mind, the same can be said for velocities and accelerations. The fundamental units of velocity and acceleration is length and time, therefore the velocities and accelerations for kinematically similar bodies should be proportional.

By definition, velocity is length over time. With this in mind, the equation for the ratio of velocities is:

$$\beta = \frac{v_p}{v_m} = \frac{l_p / t_p}{l_m / t_m} = \frac{l_p}{l_m} \cdot \frac{t_m}{t_p} = \frac{\lambda}{\tau} \quad (2-12)$$

where v_p and v_m are the respective velocities of the prototype and the model, and β is the model ratio of corresponding velocities. A more simplified version of the above equation is given as equation (2-13):

$$\beta = \lambda \tau^{-1} \quad (2-13)$$

The same fundamental relationship can be developed for accelerations since the basic units of acceleration are length over the square of time. The relationship is as follows:

$$\alpha = \frac{a_p}{a_m} = \frac{l_p / t_p^2}{l_m / t_m^2} = \frac{\lambda}{\tau^2} = \lambda \tau^{-2} \quad (2-14)$$

where a_p and a_m are the accelerations for the prototype and the model respectively, and α is the model ratio of acceleration (Hubbert 1937).

2.4.1(c) Dynamic Similarity

The similarity between linear dimensions and motion has been explained above, but when gravity is to be considered, the forces due to gravity and the inertial reactions of mass must also be similar (Hubbert 1937). The gravitational forces and the inertial forces that act on bodies are governed by the mass of the body. This requires that the mass distribution in the model must be similar to that of the prototype (Hubbert 1937).

The fundamental ratio of mass can be discussed by looking at the mass of a

differential volume of any differential element in a body. By studying any arbitrary element, the model ratio of mass can be developed as:

$$\zeta = \frac{dm_p}{dm_m} \quad (2-15)$$

where dm is the differential element of mass for the prototype and the model respectively, and ζ is the model ratio of mass (Hubbert 1937).

Now that the ratio for mass has been defined, the ratio for density can be defined since density is, by definition, mass over volume. The ratio for volume is obtained directly from equation (2-10). Therefore, a density relationship is established as:

$$\frac{\rho_p}{\rho_m} = \frac{dm_p / dV_p}{dm_m / dV_m} = \frac{dm_p}{dm_m} \cdot \frac{dV_m}{dV_p} = \zeta \lambda^{-3} \quad (2-16)$$

where ρ is the density of the prototype and the model respectively.

The forces that act upon this arbitrary element can be divided into two types of forces; surface forces and body forces. Hubbert (1937) defines body forces as the forces originating inside the body of the volume. The body forces can be broken down even further; into forces consisting of the forces due to gravity, f_g , and the forces due to inertia, f_a . These elementary equations are:

$$f_g = mg \quad (2-17)$$

where g is the force per unit mass due to gravity, and by Newton's Second Law of Motion the equation for the inertial forces is:

$$f_a = ma \quad (2-18)$$

where a is the local acceleration of mass. Equations (2-17) and (2-18) represent the entire body, and when a differential element of volume is considered the equations become:

$$df_g = dm \cdot g \quad (2-19)$$

and

$$df_a = dm \cdot a \quad (2-20)$$

Since the base friction machine moves very slowly, the acceleration is considered negligible making equation (2-20) equal to zero.

Surface forces are external forces whereas the above body forces are internal forces. The surface forces act only on the external surface of the element of volume being considered. Since the forces act on the outside of the element, they are proportional to the area they act upon. Surface forces are commonly referred to as stresses and are equal to the force over the area acted upon, or

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \quad (2-21)$$

Equation (2-21) is the general equation for stress. Some examples or special cases of stress are pressure and tension.

Pressure is stress that is directed toward and normal to the surface being acted upon, and tension is force or stress that is acting away from the surface being acted upon and normal to it. The most common type of stress is oriented obliquely to the surface in question. With the stress being oriented obliquely, it can be broken into its component parts. These stress components are referred to as the normal component and the tangential components of stress. In the case of elastic solids, surface forces result from both the normal and tangential components of the stress related to elastic strain.

In general, each element of mass is in a state of dynamic equilibrium among as many forces as are acting simultaneously, therefore, it can be said that the vector sum of the body forces is equal to the vector sum of the surface forces (Hubbert 1937). The initial definition of dynamic similarity states that there must be a similarity between the mass of the model and the mass of the prototype. Now forces can be incorporated into the definition. Since the forces are in equilibrium, it can be said that the forces acting on each element of mass occupying each element of volume of the model must be oriented in a way that the motion is geometrically and kinematically similar to the corresponding motion of the corresponding element of mass in the original (Hubbert 1937). In other words, for each force acting on the model there should be a corresponding force with the same orientation acting on the prototype. Since the forces

are equal in orientation and should cause the same motion, a ratio of the magnitude of the forces can be developed, with the general equation being:

$$\phi = \frac{F_p}{F_m} \quad (2-22)$$

where F_p and F_m are the forces in the prototype and the model, and ϕ is the model ratio of force. This ratio is assumed constant throughout the model and the prototype. When equation (2-22) is broken down into specific forces the ratio becomes

$$\phi = \frac{F_{gp}}{F_{gm}} = \frac{F_{pp}}{F_{pm}} = \frac{F_{ep}}{F_{em}} = \frac{F_{ip}}{F_{im}} \quad (2-23)$$

where F_g , F_p , F_e , and F_i are the forces due to gravity, pressure, elasticity, and inertia, respectively, for the prototype and the model.

2.4.2 Common Forces

Forces in general have been defined above, but the different types of forces that can potentially act upon the elements in the two bodies need to be discussed in relation to maintaining the scale ratio ϕ . The scale ratios for all forces can now be defined in terms of the fundamental model ratios for length, time and mass (λ , τ , and γ) as all of the previous scale ratios have been.

2.4.2(a) Body Forces

The fundamental body force equations have been defined in equations (2-19) and (2-20). When these two equations are substituted into equation (2-23) the scale ratio for body forces can be developed. Initially,

$$\phi = \frac{F_{gp}}{F_{gm}} = \frac{dm_p g_p}{dm_m g_m} = \frac{F_{ip}}{F_{im}} = \frac{dm_p a_p}{dm_m a_m} \quad (2-24)$$

and recalling equations (2-14) and (2-15) for the fundamental ratios of μ and γ , equation (2-24) then becomes:

$$\phi = \frac{F_{gp}}{F_{gm}} = \frac{F_{ip}}{F_{im}} = \zeta \lambda \tau^{-2} \quad (2-25)$$

Equation (2-25) states that when both gravitational and inertial forces exist, they must conform to the same scaling ratio, where the fundamental terms can be chosen arbitrarily. Here the inertial forces are being made to fit the ratio of the gravitational forces, which reduces the arbitrariness that should exist (Hubbert 1937).

As mentioned before, the base friction machine moves at such a slow, constant rate, the inertial forces are very small. From equation (2-24) it can be said that the ratio of gravity equals the ratio of acceleration. The ratio of gravity can be assumed to be unity since both bodies are on the earth's surface and the gravitational constant for the model is the same as that for the prototype. This can be illustrated by:

$$\frac{g_p}{g_m} = 1 = \frac{a_p}{a_m} = \alpha = \lambda \tau^{-2} \quad (2-26)$$

therefore;

$$\tau = \lambda^{1/2} \quad (2-27)$$

Now λ and τ are no longer mutually independent and when either ratio is chosen arbitrarily the other can be found directly. By substituting either equation (2-26) or (2-27) into equation (2-25), the force ratio, ϕ , becomes:

$$\phi = \frac{F_{gp}}{F_{gm}} = \zeta \quad (2-28)$$

2.4.2(b) Surface Forces

As mentioned before, the surface forces acting on a small element of mass must be equal and opposite to the body forces. The body forces are proportional to the mass and the volume, but this is not the same for the surface forces. Since the surface forces act on the surface or outside of the element, they are proportional to the stress on the element and the area of the element. Since length is a fundamental characteristic of similitude, the body and surface forces can be related since they are both proportional to properties which are illustrated using length or linear dimensions. Using l as a characteristic length, the ratio of body and surface forces is (Hubbert 1937):

$$\frac{\text{Body Forces}}{\text{Surface Forces}} = k \cdot \frac{l^3}{l^2} \quad (2-29)$$

where k is some arbitrary constant. If the size of the element that is being studied is reduced to the point that l approaches zero, then equation (2-29) will approach the limit zero. This shows that the body forces diminish with size more quickly than the surface forces because the length proportionality for the body forces is the cube of the length and the surface forces are proportional to the square of the length. Therefore, if a small enough element of volume is used the body forces can be ignored, which would require the surface forces to be in equilibrium with themselves (Hubbert 1937).

With this being the case, then the only forces that have to be modelled on the base friction machine are the surface forces or the stresses being applied to the model. This assumption leads to the discussion about dynamic similarity and the state of stress at any point in a body, which states that at corresponding points within the two bodies the state of stress must be similar at corresponding times.

2.4.2(c) Stress state at any point

By assuming an infinitesimal element of volume with no body forces, it is understood that the surface forces are in equilibrium with themselves, and that there is no resultant force present that will accelerate or rotate the bodies.

The incremental length of each side of the element of volume can be taken as dx , dy , and dz . The stresses can now be resolved into normal and shearing components acting on each face of the element of volume or cube. The stresses acting on this infinitesimal cube form the 3-D stress cube. Keep in mind that while this element is

three dimensional, the base friction model is only a two dimensional model. Therefore, only two dimensions will be discussed. Figure 2-3 illustrates the normal stress orientation for the two dimensional case. For this discussion, the positive normal stress is considered to be in compression. Figure 2-3 also maintains the axis orientation from figure 2-1.

By eliminating the third dimension, the number of stress components is reduced from six to four. This can be reduced even further by assuming that the element does not rotate about either axis. This means that $\tau_{zx} = \tau_{xz}$. This leaves three fundamental components of stress (Hubbert 1937):

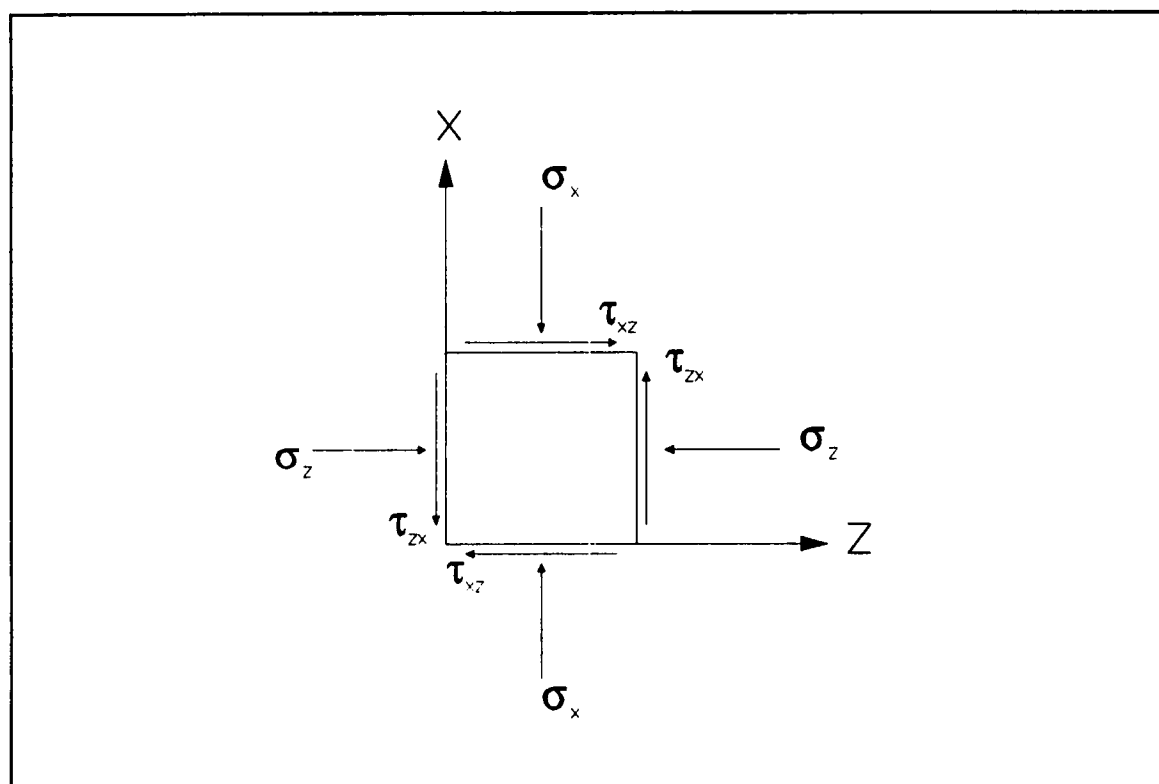


Figure 2-3: Force orientation for 2-D Base Friction Model

σ_x, σ_z = normal stresses

τ_{zx} = shear stress

These normal stresses become principal stresses when the X and Z axes are oriented such that the shear stresses disappear or are equal to zero. This is important because once the principal stresses are known the shear and normal components can be found for any plane acting through that particular point in the element.

2.4.2(d) Strain and Elasticity

When studying the stresses in an elastic body, the strains produced by these stresses must also be considered. For linear elastic materials, stress and strain can be related by Hooke's Law in terms of some constant of elasticity:

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant} \quad (2-30)$$

where the *constant* is a modulus value.

If a homogeneous isotropic material is assumed, stress and strain in the Z-direction can be related by the generalized form of Hooke's Law for isotropic materials (Popov 1968):

$$\epsilon_z = \frac{\sigma_z}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \quad (2-31)$$

where: ϵ = the length strain parallel to the axes

σ = normal stress

E = Young's modulus of elasticity

ν = Poisson's ratio

and for shear strain, the generalized equation is:

$$\theta_{xz} = \frac{\tau_{xz}}{G} \quad (2-32)$$

where: θ_{xz} = the angle of shear

τ_{xz} = the shear stress parallel to the Z-X plane

G = Shear modulus of elasticity

The reader should note that the above equations are for the Z-X plane because this plane is the horizontal plane in the base friction model. This notation is used in order to maintain the same axis orientations and directions as figure 2-3.

The above equations indicate that there are three elastic constants E , ν , and G , but in the case of isotropic materials, E and ν are coupled in the equation and only two elastic constants exist. The two elastic constants G and E are related by (Popov 1968):

$$G = \frac{E}{2(1 + \nu)} \quad (2-33)$$

Under the condition of uniaxial stress, when all normal stresses except for one are zero, the modulus E can be determined by the following equation:

$$E = \frac{\sigma}{\epsilon} \quad (2-34)$$

which leads to the discussion of the scaling of the modulus values.

As with stress, the moduli must also be scaled between the prototype and the model. The moduli for the prototype must also be dynamically similar to the model.

A ratio for similarity can now be developed for the both of the moduli:

$$\frac{E_p}{E_m} = \frac{\sigma_p / \epsilon_p}{\sigma_m / \epsilon_m} = \frac{\sigma_p}{\sigma_m} \cdot \frac{\epsilon_m}{\epsilon_p} \quad (2-35)$$

and for the shear modulus G :

$$\frac{G_p}{G_m} = \frac{\tau_p / \theta_p}{\tau_m / \theta_m} = \frac{\tau_p}{\tau_m} \cdot \frac{\theta_m}{\theta_p} \quad (2-36)$$

since both equations have the same general form, a more general equation can be written for all moduli ratios (Hubbert 1937):

$$\frac{(Modulus)_p}{(Modulus)_m} = \frac{(Stress)_p / (Strain)_p}{(Stress)_m / (Strain)_m} = \frac{(Stress)_p}{(Stress)_m} \cdot \frac{(Strain)_m}{(Strain)_p} \quad (2-37)$$

As mentioned before, the units for strain are some geometrical dimension over the same geometrical dimension, such as length/length. Recalling the definition for geometric similarity, requires that all lengths be proportional and all angles be equal in the model and the prototype. Therefore, the ratios of ϵ_m/ϵ_p and θ_m/θ_p from equations (2-35) and (2-36) are equal to one. In other words the model ratio of strain is unity, which is also the case for other quantities such as Poisson's ratio and the friction angle

(Fumagalli 1968). Consequently this makes the ratio for the elastic moduli equal to the ratio for stress, thus changing equation (2-37) to:

$$\frac{(Modulus)_p}{(Modulus)_m} = \frac{(Stress)_p}{(Stress)_m} = \xi \quad (2-38)$$

where ξ is the model ratio of stress. Knowing that stress is equal to force over the area that the force acts upon, the model ratio of stress can be derived in terms of the fundamental ratios of λ , ζ , and τ . Therefore ξ becomes:

$$\xi = \frac{f_p / A_p}{f_m / A_m} = \frac{f_p}{f_m} \cdot \frac{A_m}{A_p} = \zeta \lambda \tau^{-2} (\lambda^{-2}) \quad (2-39)$$

then breaking the equation down further reveals:

$$\xi = \zeta \lambda^{-1} \tau^{-2} \quad (2-40)$$

and finally remembering equation (2-27) and substituting it into equation (2-40) yields:

$$\xi = \zeta \lambda^{-2} \quad (2-41)$$

where ξ is the model ratio of stress and the elastic moduli.

Even though the modulus ratio is equivalent to the stress ratio, maintaining a specific modulus ratio is not considered for this thesis.

2.4.2(e) Strength

Strength of an elastic body is defined as the particular stress combination at which a given material fails by fracture or flows plastically (Hubbert 1937). For the purpose of the base friction model and this discussion, strength will be measured in terms of stress. Therefore, if the prototype fails in a certain manner at a certain stress, then the model should fail in a similar at a correspondingly similar stress. With this in mind, it can be said that the ratio of strength must be the same as the ratio of stress. Therefore, incorporating equation (2-41) yields the ratio for strength:

$$\frac{(Strength)_p}{(Strength)_m} = \frac{(Stress)_p}{(Stress)_m} = \xi = \zeta \lambda^{-2} \quad (2-42)$$

2.5 Similarity that the Base Friction Method models

The previous sections on scaling and similarity discuss these issues in a general sense. This section will discuss the particular types of similarity that the machine built for this thesis models. In general the three types of similarity are geometric, kinematic and dynamic. Geometric similarity deals with scaling linear dimensions, kinematic deals with the time required for similar changes in shape or position to occur, and dynamic deals with the mass distribution of the model and prototype. According to these definitions, the machine for this thesis deals with geometric and dynamic similarities only.

Geometric scaling can be handled because the scaling factor, λ , can be arbitrarily chosen based on the size of the prototype and the fixed dimensions of the model mold. The factor λ is then used to obtain the other scale factors.

Since dynamic similarity deals with all mass dimensioned properties, it is not achieved in the base friction models because the density ratio is not considered in any of the scaling parameters for the base friction model. For example, the density ratio in equation (2-16) is not satisfied.

Kinematic similarity is not dealt with because the dimension of time is not a factor in the base friction model. The rate at which things happen is not studied.

2.6 Scaling and Similitude Laws applied to the Base Friction Model

The previous sections discuss the derivation of the base friction concept and the concept of model scaling separately. This section will pull both concepts together and discuss the equations necessary for the scaling of base friction models.

The cube example in figure 2-2 shows that stress is proportional to the weight divided by the area. In this instance, this is true because the weight is proportional to the cube of the length, and the area is proportional to the square of the length. This can be shown algebraically by;

$$\sigma \propto \frac{L^3}{L^2} = L \quad (2-43)$$

where, σ is the stress exerted by the cube and L is any linear dimension, or the length of the cube. Equation (2-43) can now be used to describe the scaling of stresses between models and prototype for a general case. In the cube example, equation (2-43) shows that the stresses are proportional to the length, therefore a ratio between the model and prototype stresses can be developed in the following equation;

$$\frac{\sigma_p}{\sigma_m} = \frac{L_p}{L_m} = \lambda \quad (2-44)$$

where the subscripts p and m refer to the prototype and model respectively. The term λ is called the fundamental ratio of length, as defined previously.

Equation (2-44) is a basic relationship and it only holds true when the model material has the same density as the prototype material, and when the model and prototype are under equivalent loading and boundary conditions. This is not the case in the base friction model because the modelling material is not the same as the prototype material, and the base friction model simulates gravity in a 2-D plane where the prototype is actually undergoing gravity in a 3-D field. Since this is the case, the relationships have to be modified for the base friction model.

Since the model and prototype materials are different, a specific weight ratio η is defined in order to relate the unit weight of the model material to the prototype material.

$$\eta = \frac{\gamma_p}{\gamma_m} \quad (2-45)$$

where η is the specific weight ratio and γ is the unit weight of the materials. Equation (2-42) is a generic equation for the ratio of stress and with equation (2-45) it can be adapted to the base friction model.

To do this, consider the vertical stress at depth z_m in the model. From equation (2-44) the equivalent depth in the prototype, z_p , is equal to;

$$z_p = \lambda z_m \quad (2-46)$$

Stress at depth is equal to the unit weight of the material multiplied by the depth of interest, therefore the vertical stress, σ_z , in the prototype is equal to:

$$(\sigma_z)_p = z_p \gamma_p = \lambda z_m \gamma_p \quad (2-47)$$

Equation (2-5) gives the equivalent vertical stress in the base friction model. In the base friction machine built for this thesis, the coefficient of friction, μ_b , in equation (2-5) is equal to $\tan \delta$ where δ is the friction angle between the model material and the machine surface. Equation (2-5) now becomes:

$$(\sigma_z)_m = z_m \gamma_m \tan \delta \quad (2-48)$$

The stress ratio ξ from equation (2-42) can now be related to the base friction model by the following equation:

$$\xi = \frac{(\sigma_z)_p}{(\sigma_z)_m} = \frac{\lambda z_m \gamma_p}{z_m \gamma_m \tan \delta} = \frac{\lambda \eta}{\tan \delta} \quad (2-49)$$

Equations (2-44) and (2-49) can now be used to scale any stresses or lengths in the base friction model, or they can be used to determine what type of prototype conditions the model is modelling.

2.6.1 Application of Stress Scaling: Cantilever Beam Example

A simple cantilever beam example can be used to illustrate stress scaling by finding the tensile stress ratio for the prototype beam and the base friction model of the beam. Figure 2-4 shows the beam and model for this example.

By considering the prototype first, and by summing the moments around the middle of the fixed end, the following expression is obtained:

$$\frac{1}{2}WL_p = \frac{2}{3}Th_p \quad (2-50)$$

where T is the tensile force in the upper part of the beam. Taking $W = \gamma_p L_p h_p t_p$ and substituting this expression into the above equation, and solving for T , yields the

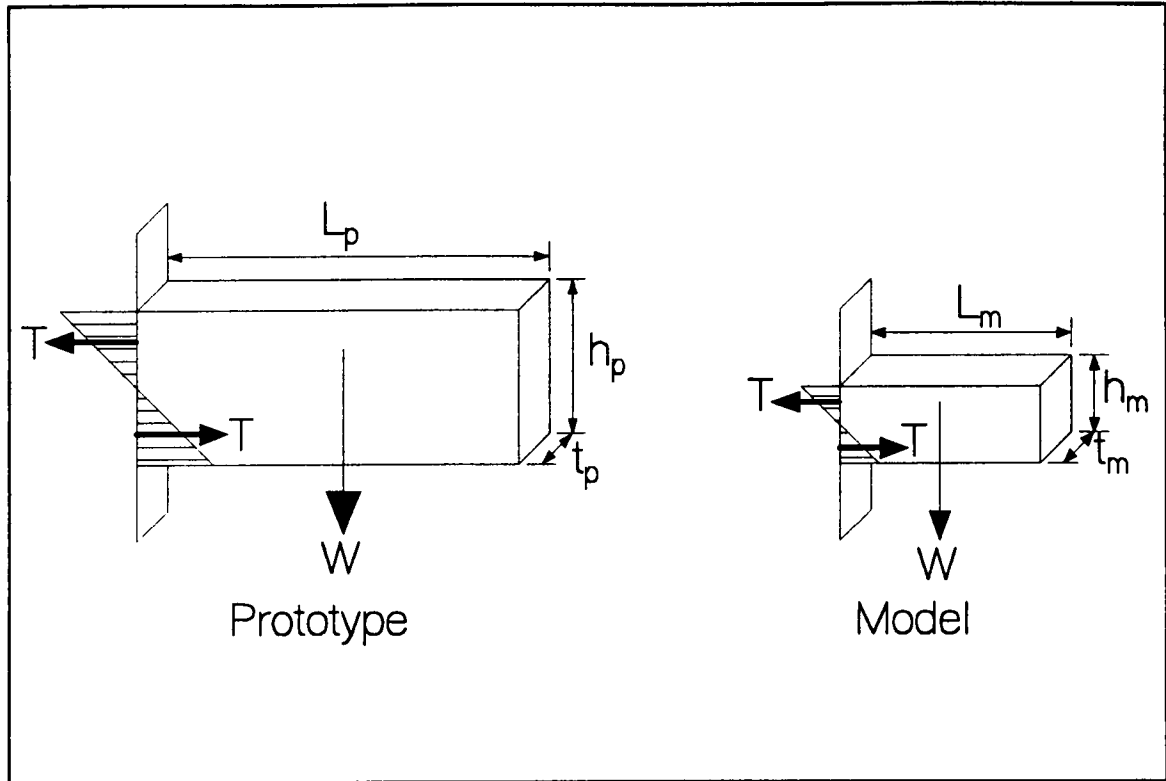


Figure 2-4: Cantilever Beam example for Body Forces

following equation:

$$T = \frac{3}{4} \gamma_p L_p^2 t_p \quad (2-51)$$

where T is the tensile force of the prototype, which is equal to:

$$T = \frac{1}{2} \sigma_{t_{\max}} t_p \frac{h_p}{2} \quad (2-52)$$

Letting $h_p = \psi L_p$, where ψ is a constant, and combining equations (2-51) and (2-52), an equation is obtained for the maximum tensile stress in the prototype.

$$(\sigma_t)_p = \frac{3\gamma_p L_p}{\psi} \quad (2-53)$$

The derivation of the tensile stress in the base friction model is slightly different than that for the prototype. In figure 2-4, the model and prototype are under their own weight, but when the model is on the base friction machine, the weight is replaced with the base friction force, F_b , which is the premise behind the base friction method. Therefore, when the base friction model is used, the forces have to be modelled differently than in the prototype. Figure 2-5 shows the model as it would be on the base friction machine.

Since the model is laying on the table, the gravitational force is not acting in the plane of the model, therefore the gravitational force or weight is simulated using the base friction force. Looking at the model in plan view, if the moment is summed about the center of the model an equation similar to equation (2-49) is obtained where F_b is the base friction force.

$$\frac{1}{2} F_b L_m = \frac{2}{3} T h_m \quad (2-54)$$

By taking $F_b = \gamma_m L_m h_m t_m \tan \delta$, which is equivalent to equation (2-4), and substituting the expression into equation (2-54) and solving for the tensile force, T , yields the following equation;

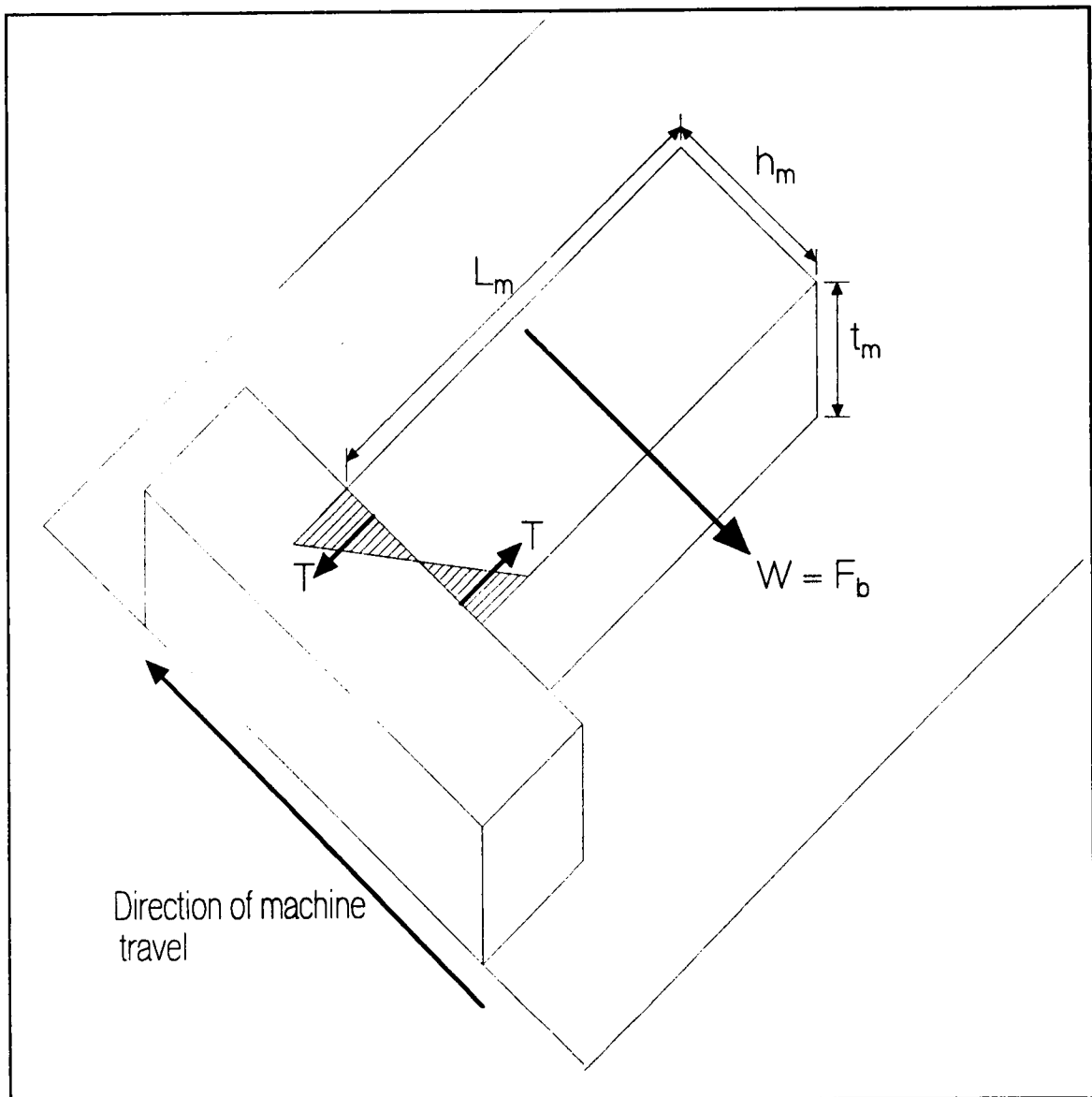


Figure 2-5: Body Force components of base friction model

$$T = \frac{3}{4} \gamma_m L_m^2 t_m \tan \delta \quad (2-55)$$

Recalling equation (2-4), the term μ_b , which is the coefficient of friction between the base and the model, is equal to the $\tan \delta$ term in equation (2-55). The tensile force, T , can be expressed in terms of the maximum tensile stress by an equation similar to equation (2-52):

$$T = \frac{1}{2} \sigma_{t_{\max}} t_m \frac{h_m}{2} \quad (2-56)$$

Once again setting equation (2-55) and (2-56) equal to each other and solving for the tensile stress assuming $h_m = \psi L_m$ yields the following equation:

$$(\sigma_t)_m = \frac{3L_m \gamma_m \tan \delta}{\psi} \quad (2-57)$$

From equations (2-53) and (2-54), the stress ratio ξ becomes:

$$\xi = \frac{(3 \gamma_p L_p) / \psi}{(3 \gamma_m L_m \tan \delta) / \psi} = \frac{\gamma_p L_p}{\gamma_m L_m \tan \delta} \quad (2-58)$$

and finally substituting λ and η into the equation, equation (2-58) becomes:

$$\xi = \frac{\eta \lambda}{\tan \delta} \quad (2-59)$$

which is the strength ratio for the body forces, and is identical to equation (2-49). This confirms the stress ratio with a specific example.

2.6.2 Stress Scaling in relation to the Surface Forces

Surface forces are different from body forces because of the way they act. The surface forces act on the outside of the body while body forces act internally. Therefore, it can be said that the stress caused by the surface forces is proportional to the weight of the body divided by the cross-sectional area that the force acts upon. This is illustrated by looking at another simple example.

Figure 2-6 shows a typical model and prototype of a tunnel with overburden. In the prototype, the stress at any depth is determined by the classical soil mechanics equation:

$$\sigma_z = \gamma_p z_p \quad (2-60)$$

In order to model the overburden condition in the model, the overburden in the prototype must be broken into two components for the model. The first component of the prototype is the depth of overburden, z_p , which is modelled by the actual depth of overburden, z_m , in the model itself; and the second component is the amount of stress needed from the air pistons to model the additional depth, z_p' , in the prototype which is not accounted for in the model itself.

The additional overburden is modelled in the base friction model by adjusting the air pressure in the two pistons, which have been installed in the top of the mold frame. In order to derive the stress which is applied by the air pistons, consider a cross-

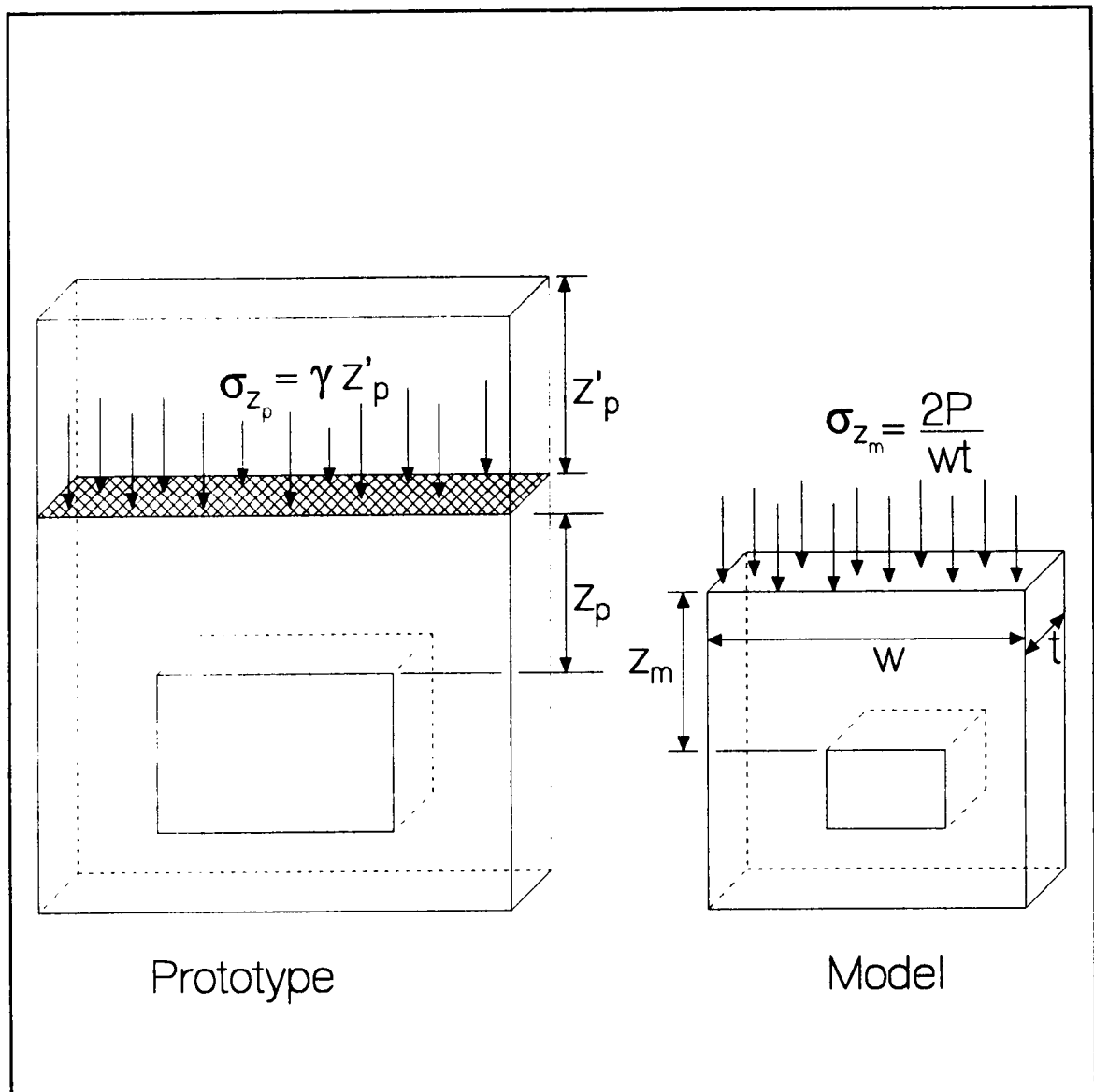


Figure 2-6: Stress Scaling example for Surface Forces

sectional area, wt , which is the top surface of the model. If the force applied by the pistons is called P , then the additional overburden is equivalent to;

$$\sigma_{z_1} = \frac{2 P}{wt} \quad (2-61)$$

In addition to the overburden applied by the pistons, the overburden in the model has to be accounted for. Recalling equation (2-5), the stress at depth in the model itself is

$$\sigma_{z_2} = \mu_b \gamma_m z_m = \gamma_m z_m \tan \delta \quad (2-62)$$

Therefore, the total amount of overburden being modelled is the sum of the two components of overburden or,

$$\sigma_{z_m} = \sigma_{z_1} + \sigma_{z_2} = \frac{2 P}{wt} + \gamma_m z_m \tan \delta \quad (2-63)$$

The stress ratio for the surface forces can now be developed. Knowing that the ratio of the stresses is equal to ξ yields the following equation:

$$\frac{\sigma_{z_p}}{\sigma_{z_m}} = \frac{\gamma_p (z_p + z_p')}{(2P/wt + \gamma_m z_m \tan \delta)} = \xi = \frac{\lambda \eta}{\tan \delta} \quad (2-64)$$

and solving the equation for z_p' yields the following equation:

$$\gamma_p (z_p + z_p') = \left[\frac{2 P}{w t} + \gamma_m z_m \tan \delta \right] \xi \quad (2-64a)$$

and remembering $z_p = \lambda z_m$,

$$z_p' = \left[\frac{2 P}{w t \gamma_p} + \frac{\gamma_m z_m \tan \delta}{\gamma_p} \right] \xi - \lambda z_m \quad (2-64b)$$

$$z_p' = \left[\frac{2 P}{w t \gamma_p} + \frac{z_m \tan \delta}{\eta} \right] \frac{\lambda \eta}{\tan \delta} - \lambda z_m \quad (2-64c)$$

and breaking the equation down further yields:

$$z_p' = \left[\frac{2 P \lambda \eta}{w t \gamma_p \tan \delta} + z_m \lambda \right] - z_m \lambda \quad (2-64d)$$

remembering equation (2-46) yields the final expression for z_p' :

$$z_p' = \frac{2 P \lambda}{w t \gamma_m \tan \delta} \quad (2-65)$$

This equation allows for the determination of the additional depth the air pistons are modelling directly. Equation (2-65) can be rearranged and used to find the piston force, P , required to model a particular depth in the prototype. By reworking equation (2-65), the expression for P becomes:

$$P = \frac{w t \gamma_m z_p' \tan \delta}{2 \lambda} \quad (2-66)$$

Knowledge of the piston area permits selection of the line pressure needed to obtain the desired P . This is discussed later in the thesis.

2.7 Determination of Modelling Material Tensile Strength

The tensile strength of the model material is determined by performing some simple clamped beam experiments with a base friction model. Figure 2-7 shows the two clamped beam tests that were run in order to get the tensile strength of the material. Both tests maintained a uniform thickness while either the span was lengthened or the height or depth of the beam was reduced until a tensile crack formed.

Figure 2-7(a) shows the test where the span length was increased. In this test, the depth of the beam was maintained at 4 cm (1.57 in.) and the initial span length was 12 cm (4.72 in.). The test was performed by running the machine and lengthening the span length until a tensile crack started forming in the top of the beam at the supports. The span was lengthened by stopping the machine and cutting away material from the top and bottom of the beam at the supports. The x - dimension in figure 2-7(a) shows the direction of the material removal. In this test, the tensile cracked formed when the span was increased to 18 cm (7.10 in.) in length.

Figure 2-7(b) shows the second test that was used. In this test, the span was maintained at 12 cm (4.72 in.) and the beam depth was reduced. This experiment was

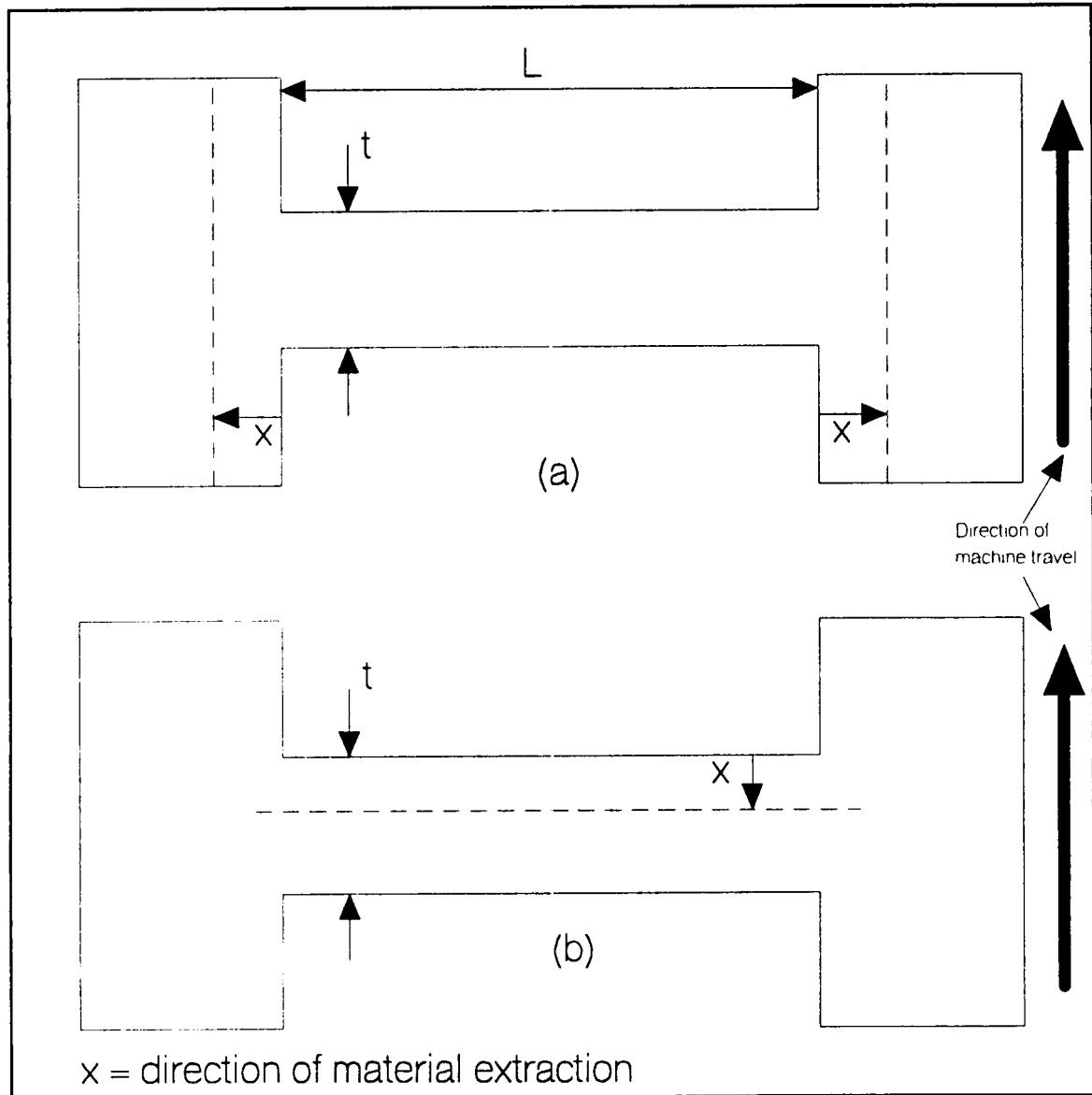


Figure 2-7: Plan view of clamped beam test: (a) beam span is lengthened, (b) beam thickness is reduced

performed the same way as the previous test, except that the material is cut away from the top of the beam. Again, the x - dimension in (b) shows the direction that the material was taken from. The span depth started at 4 cm (1.57 in.) and the tensile crack formed when the depth was reduced to 2 cm (0.787 in.). The air line pressure for both tests was held constant at 4 psi (27.6 kPa) to maintain a constant horizontal stress.

The tensile strength of the model is calculated by using a classical beam theory equation for a clamped beam where (Goodman 1989):

$$\sigma_{\max} = \frac{\gamma L^2}{2 t} - \sigma_h \quad (2-67)$$

where: $\sigma_{\max}$ is the maximum tensile stress at the top of the beam near the ends,

γ is the unit weight of the base friction model

L is the length of the free span of the beam

t is the thickness or depth of the beam

σ_h is the horizontal stress applied to the model

In the above equation, γ is the apparent unit weight in the plane of the base friction model and not the unit weight of the material. The unit weight of the model material, γ_m , is 93.3 lb/ft³ (14.66 kN/m³), but when it is converted to the base friction model the unit weight becomes a body force per unit volume. The unit weight, γ'_m , is derived by dividing the base friction force, F_b , by the volume. The equation for the unit weight is:

$$\gamma_m' = \frac{F_b}{\text{volume}} = \frac{\gamma_m L_m t_m h_m \tan \delta}{L_m t_m h_m} \quad (2-68)$$

Equation (2-68) simplifies to:

$$\gamma_m' = \gamma_m \tan \delta \quad (2-69)$$

where δ is the friction angle between the surface and the model, which is 31° for the surface used in these experiments. For a δ of 31° , equation (2-69) gives,

$$\gamma_m' = \left(93.3 \frac{\text{lb}}{\text{ft}^3} \right) \tan 31^\circ = 56.06 \frac{\text{lb}}{\text{ft}^3} \rightarrow 0.032 \frac{\text{lb}}{\text{in}^3} \rightarrow 8.80 \frac{\text{kN}}{\text{m}^3}$$

The friction angle δ was determined by small was determined by a tilt test. A small model was placed on a piece of the sandpaper that was attached to a piece of wood. The wood and the model were tilted up until the model began to slide. This angle of inclination was used as the friction angle δ . The internal angle of friction ϕ was determined by performing a simple slope sequence on the base friction machine. The angle of the bedding planes was increased until two adjacent planes started sliding. The angle at which the planes started to slide was considered to be the friction angle of the material. The internal friction angle was determined to be 55° to 60° .

2.7.1 Application of horizontal stress

The horizontal stress, σ_h , in equation (2-67) is the horizontal stress applied by the machine. This stress is equivalent to the force of the piston, P , divided by the cross-sectional area of the model contact surface. The piston force, P , is equal to a push factor times the air line pressure. The push factor is supplied by the manufacturer, and it is equal to the cross-sectional area of the piston bore. The cross-sectional area of the piston bore used in these pistons is 0.9 in² (5.8 cm²); therefore the push factor is equal to 0.9 when the air line pressure is in units of lb/in² and the piston area is in units of in²; and 0.00058 when the line pressure is in units of kPa and the piston area in cm². Therefore the equation for the force exerted by each piston is:

$$P = 0.9 p_A \quad (\text{English Units}) \quad (2-70a)$$

$$P = 0.00058 p_A \quad (\text{S.I. Units}) \quad (2-70b)$$

where p_A is the air line pressure and P is the force exerted by one piston.

The horizontal pressure applied by each side of the mold is equal to the force exerted by the two pistons divided by the cross-sectional area of the contact surface. The cross-sectional area of the model contact surface is equal to the model thickness times the length of the model. The thickness of the model is 0.75 in. (1.9 cm) and the length is 12 in. (30.4 cm), and both values are always constant. The equation for the applied horizontal stress is:

$$\sigma_h = \frac{2 P \text{ (lb)}}{(0.75 \text{ in.}) (12 \text{ in.})} = \frac{1.8 p_A \text{ (lb)}}{9 \text{ (in}^2\text{)}} = 0.2 p_A \frac{\text{lb}}{\text{in}^2} \quad (2-71a)$$

or,

$$\sigma_h = \frac{2 P \text{ (kN)}}{(0.019 \text{ m}) (0.305 \text{ m})} = \frac{0.0012 p_A \text{ (kN)}}{0.00581 \text{ (m}^2\text{)}} = 0.207 p_A \text{ kPa} \quad (2-71b)$$

Now equation (2-67) can be used to estimate the tensile strength of the modelling material. Using the above equations, equation (2-67) becomes:

$$\sigma_{t_{\max}} = \frac{\gamma L^2}{2 t} - \sigma_h = \frac{\gamma'_m L^2}{2 t} - 0.2 p_A \quad (2-72)$$

Using an air line pressure of 4 lb/in² (27.6 kPa) and by taking an average of the experimental values from pages 55 and 56, the maximum tensile strength of the model material is determined to be:

$$\sigma_{t_{\max}} = \frac{\left(0.032 \frac{\text{lb}}{\text{in}^3} (4.72 \text{ in})^2\right)}{2 (0.787 \text{ in})} - 0.2 \left(4 \frac{\text{lb}}{\text{in}^2}\right) = -0.35 \frac{\text{lb}}{\text{in}^2} \rightarrow -2.4 \text{ kPa}$$

Table 2-1 will summarize the material properties.

Table 2-1: Material Property summary of modelling material

Material Properties	English Units	S.I. Units
Unit Weight, γ_m	93.3 lb/ft ³ (0.054 lb/in ³)	14.66 kN/m ³
Base Friction Unit Weight, γ'_m	56.1 lb/ft ³ (0.032 lb/in ³)	8.80 kN/m ³
Internal angle of friction, ϕ	55° - 60°	55° - 60°
Base Friction coefficient of friction, δ	31°	31°
Tensile strength, σ_t	50.4 lb/ft ² (0.35 lb/in ²)	2.4 kPa

2.7.2 Model material tensile strength comparison to various rock types

The idea behind stress scaling is the ability to compare the model results or behavior to the prototype's behavior. With the scaling equations formulated and the material parameters determined, it is now possible to compare the tensile strength of the modelling material to the tensile strength of various rock types. Recalling equation (2-49), the tensile strength of the prototype is:

$$(\sigma_t)_p = \xi \cdot (\sigma_t)_m = \frac{\lambda \eta}{\tan \delta} \cdot (\sigma_t)_m \quad (2-73)$$

since δ and $(\sigma_t)_m$ are known, equation (2-73) becomes:

$$(\sigma_t)_p = \frac{\lambda \eta}{\tan 31^\circ} \left(0.35 \frac{lb}{in^2} \right) = 0.583 \lambda \eta \left(\frac{lb}{in^2} \right) \quad (2-74)$$

Table 2-2 gives a range of prototype tensile strengths calculated from a range of λ and η using equation (2-74). Based on the tensile strength that is desired, table 2-2 can be used to get the appropriate stress scaling factors, or the tensile strength can be determined by choosing the desired scaling factors. The metric equivalent (in kPa) to the tensile strengths in table 2-2 can be obtained by multiplying the tensile strengths by 6.9 kPa, or by using table 2-3.

In order to illustrate how to use table 2-2, the appropriate length scale will be found in order to model an underground cavern or tunnel in Berea Sandstone. Table 3.1 from Goodman (1989), gives the tensile strength of the sandstone as 170 lb/in². The unit weight of Berea Sandstone is assumed to be 163 lb/ft³. Then for the model material, the specific weight ratio η can be determined from equation (2-46) as follows:

$$\eta = \frac{\gamma_p}{\gamma_m} = \frac{163 \frac{lb}{ft^3}}{93.3 \frac{lb}{ft^3}} = 1.75$$

With the specific weight ratio, the experimental results for the tensile strength of the model material, and the prototype tensile strength being known, the length ratio λ can be determined by entering table 2-2 and finding the intersection of $\eta = 1.75$ and λ where the tensile strength is 170 lb/in². Table 2-2 shows that the intersection is between a λ of 160 and 180. After interpolating between the two ratios, λ is determined to be 167. Based on this information, Berea Sandstone can be modelled by using a η of 1.75 and a λ of 167.

In practice, the model parameters which are most amenable to being controlled are the model tensile strength and the base friction angle δ . Now suppose that the specific weight ratio is fixed at $\eta = 1.75$, and the prototype tensile strength is taken as $(\sigma_t)_p = 170 \text{ lb/in}^2$ for Berea Sandstone. Then equation (2-73) can be rearranged to solve for λ in terms of the model tensile strength $(\sigma_t)_m$ or;

Table 2-2: Prototype Tensile Strength (English units)

λ												
η	100	120	140	160	180	200	220	240	260	280	300	
1	58.3	69.9	81.6	93.2	104.9	116.5	128.2	139.8	151.5	163.1	174.8	
1.25	72.8	87.4	101.9	116.5	131.1	145.6	160.2	174.8	189.3	203.9	218.4	
1.5	87.4	104.9	122.3	139.8	157.3	174.8	192.2	209.7	227.2	244.7	262.1	
1.75	101.9	122.3	142.7	163.1	183.5	203.9	224.3	244.7	265.0	285.4	305.8	
2	116.5	139.8	163.1	186.4	209.7	233.0	256.3	279.6	302.9	326.2	349.5	
2.25	131.9	157.3	183.5	209.7	235.9	262.1	288.3	314.6	340.8	366.9	393.2	
2.5	145.6	174.8	203.9	233.0	262.1	291.3	320.4	349.5	378.6	407.8	436.9	
2.75	160.2	192.2	224.3	256.3	288.3	320.4	352.4	384.5	416.5	448.5	480.6	
3	174.8	209.7	244.7	279.6	314.6	349.5	384.4	419.4	454.4	489.3	524.3	
3.25	189.3	227.2	265.0	302.9	340.8	378.6	416.5	454.4	492.2	530.1	567.9	
3.5	203.9	244.7	285.4	326.2	366.9	407.8	448.5	489.3	530.1	570.9	611.6	
3.75	218.4	262.1	305.8	349.5	393.2	436.9	480.6	524.3	567.9	611.6	655.3	
4	233.0	279.6	326.2	372.8	419.4	466.0	512.6	559.2	605.8	652.4	699.0	

NOTE: The tensile strength units are lb/in²

Table 2-3: Prototype Tensile Strength (S.I. units)

λ												
η	100	120	140	160	180	200	220	240	260	280	300	
1	401.9	482.3	562.7	643.1	723.5	803.9	884.2	964.6	1045.0	1125.4	1205.8	
1.25	502.4	602.9	703.4	803.8	904.3	1004.8	1105.3	1205.8	1306.3	1406.7	1507.2	
1.5	602.9	723.5	844.0	964.6	1085.2	1205.8	1326.3	1446.9	1567.5	1688.1	1808.7	
1.75	703.4	844.0	984.7	1125.4	1266.1	1406.7	1547.4	1688.1	1828.8	1969.4	2110.1	
2	803.9	964.6	1125.4	1286.2	1466.9	1607.7	1768.5	1929.2	2090.0	2250.8	2411.6	
2.25	904.3	1085.2	1266.0	1446.9	1627.8	1808.7	1989.5	2170.4	2351.3	2532.1	2713.0	
2.5	1004.8	1205.8	1406.7	1607.7	1808.7	2009.6	2210.6	2411.6	2612.5	2813.5	3014.4	
2.75	1105.3	1326.4	1547.4	1768.5	1989.5	2210.6	2431.6	2652.7	2873.8	3094.8	3315.9	
3	1205.8	1446.9	1688.1	1929.2	2170.4	2411.6	2652.7	2893.9	3135.0	3376.2	3617.3	
3.25	1306.3	1567.5	1828.8	2090.0	2351.3	2612.5	2873.8	3135.0	3396.3	3657.5	3918.8	
3.5	1406.7	1688.1	1969.4	2250.8	2532.1	2813.5	3094.8	3376.2	3657.5	3938.9	4220.2	
3.75	1507.2	1808.7	2110.1	2411.5	2713.0	3014.4	3315.9	3617.3	3918.8	4220.2	4521.7	
4	1607.7	1929.2	2250.8	2572.3	2893.9	3215.4	3536.9	3858.5	4180.0	4501.6	4823.1	

NOTE: The tensile strength units are kPa

$$\lambda = \frac{170 \text{ psi}}{(\sigma_v)_m} \cdot \frac{\tan \delta}{1.75} = \frac{97 \tan \delta}{(\sigma_v)_m}$$

where $(\sigma_v)_p$ is in units of lb/in² or kPa. This relationship can then be presented in tabular form as tables 2-4 for english units and 2-5 for metric units. Table 2-3 shows how the length ratio λ can be changed by adjusting the friction angle δ or the model tensile strength.

2.8 Similitude Scaling Example

In order to illustrate how the scaling laws apply, the Berea Sandstone example from above will be used. The values for λ , δ , and γ_{rock} determined from above will be used in this example. Figure 2-8 gives the dimensions of the model along with the side and top pressure being applied to the model. The goal of this problem is to determine what physical conditions the base friction model simulates.

Using equation (2-44) the equivalent linear dimensions of the prototype can be related to the corresponding dimensions in the model. From this expression, the tunnel can be sized. Using equation (2-44) and a λ of 167, the prototype tunnel size is:

$$\text{Height} = l_m \lambda = (8 \text{ cm})(167) = 1336 \text{ cm} \rightarrow 13.3 \text{ m} \rightarrow 44 \text{ ft}$$

$$\text{Width} = (15 \text{ cm})(167) = 2505 \text{ cm} \rightarrow 25.1 \text{ m} \rightarrow 82.2 \text{ ft}$$

Table 2-4: Effects of model tensile strength and δ on the length ratio λ (English units)

	$\tan \delta$					
$(\sigma_t)_p$ (psi)	0	0.2	0.4	0.6	0.8	1.0
0		∞	∞	∞	∞	∞
0.2	0	97.0	194.0	291.0	388.0	485.0
0.4	0	48.5	97.0	145.5	194.0	242.5
0.6	0	32.3	64.7	97.0	129.3	161.2
0.8	0	24.3	48.5	72.8	97.0	121.3
1.0	0	19.4	38.8	58.2	77.6	97.0

Table 2-5: Effects of model tensile strength and δ on length ratio λ (S.I. units)

	$\tan \delta$					
$(\sigma_t)_p$ (kPa)	0	0.2	0.4	0.6	0.8	1.0
0		∞	∞	∞	∞	∞
0.2	0	1173	2346	3519	4692	5865
0.4	0	587	1173	1760	2346	2933
0.6	0	391	782	1173	1564	1955
0.8	0	293	587	880	1173	1466
1.0	0	235	469	704	938	1173

The thickness of the bedding layers in the prototype is also determined the same way.

$$\text{Layer thickness} = (2 \text{ cm})(167) = 334 \text{ cm} \rightarrow 3.3 \text{ m} \rightarrow 11 \text{ ft}$$

The depth of overburden that the model is simulating can be obtained directly from previous equations. The depth of overburden in the prototype, z_p , is obtained directly from the amount of overburden in the model. In figure 2-8, the amount of model overburden z_m is taken as the number of layers from the top of the model to down to the top of the tunnel. There are 8 layers at 2 cm (0.787 in.) per layer, therefore the depth overburden is determined to be 16 cm (6.3 in.). Using the same equation as above, the amount of overburden simulated by the model is:

$$z_p = \lambda z_m = (167) (16 \text{ cm}) = 2672 \text{ cm} \rightarrow 26.7 \text{ m} \rightarrow 87.7 \text{ ft}$$

Remembering that the model only accounts for part of the total overburden in the prototype, the air pistons account for the additional overburden z_p' in the prototype. This additional overburden is determined from equation (2-65). Figure 2-8 shows that the vertical stress being applied is 0.4 lb/in² (2.8 kPa). This pressure is equivalent to a piston force P of 2 lb (8.9 N), which is needed for equation (2-66). The cross-sectional area, wr , is equivalent to the typical 3/4 in x 12 in (1.9 cm x 30.48 cm) size of the contact surface between the model and the piston surface. Therefore z_p' is calculated as:

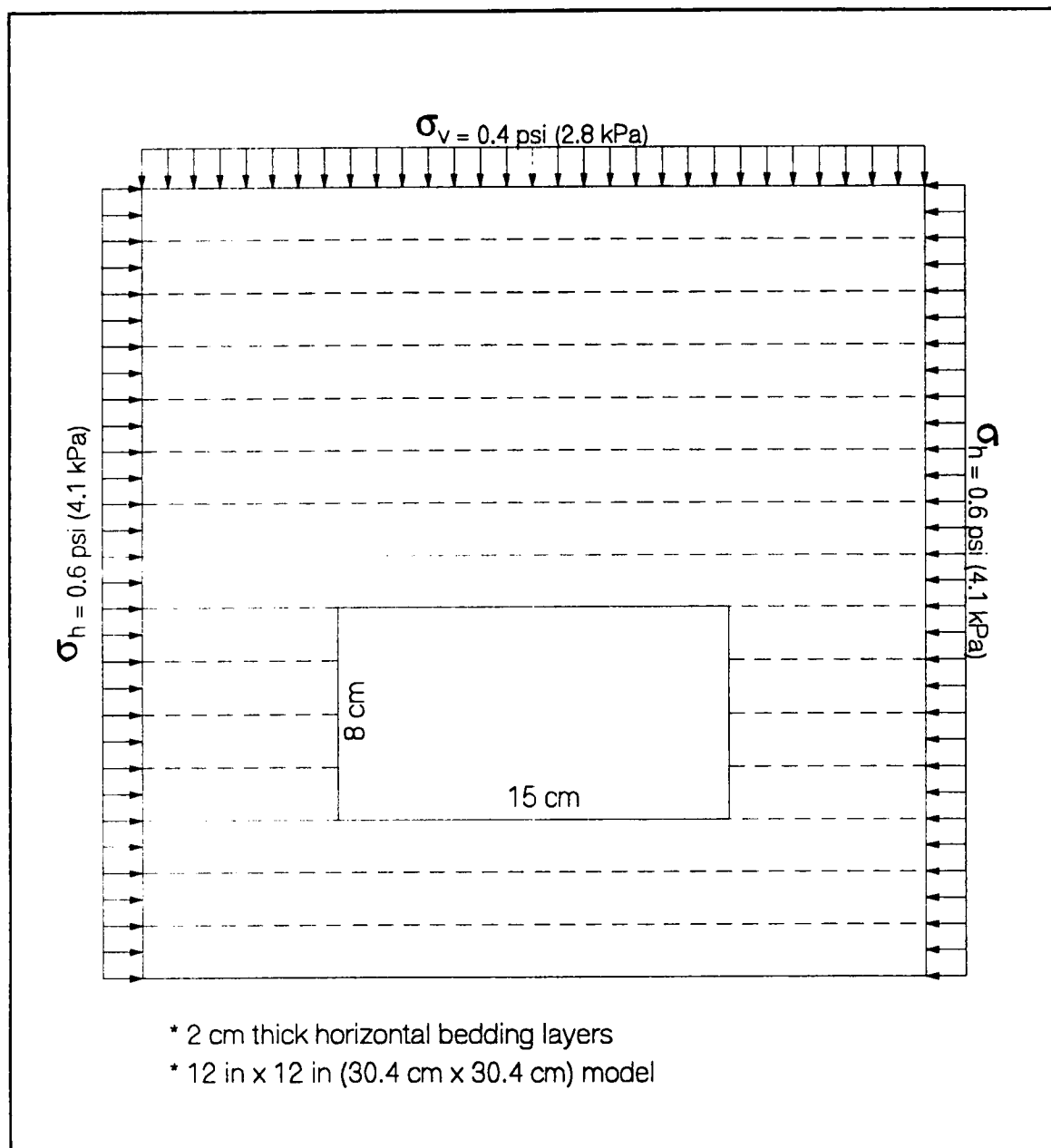


Figure 2-8: Scaling and similitude example

$$z_p' = \frac{2 P \lambda}{w t \gamma_m \tan \delta} = \frac{2 (2 \text{ lb}) (167)}{(0.75 \text{ in}) (12 \text{ in}) \left(0.054 \frac{\text{lb}}{\text{in}^3}\right) \tan 31^\circ}$$

$$\therefore z_p' = 2287 \text{ in.} \rightarrow 190 \text{ ft} \rightarrow 58 \text{ m}$$

The total depth of overburden being modelled is the sum of z_p and z_p' from above. Therefore, this particular model is modelling a tunnel at a depth of 278 ft (85 m).

In summary, this particular model represents a tunnel that is 25.1 m wide x 13.3 m high (82 ft x 44 ft) that is built in Berea sandstone with an assumed unit weight of 163 lb/ft³ (25.6 kN/m³). The prototype has horizontal bedding layers which are 3.3 m (11 ft) thick. The tunnel is at a depth of 85 m (278 ft), and the tensile strength of the rock is 170 psi (1173 kPa). All of these dimensions are realistic except for the bedding layer thickness. The 3 m thickness is unrealistic and could be improved upon by adjusting the assumed unit weight and or adjusting the length ratio λ . A more realistic thickness would be about half this amount or 1.5 m (5 ft).

CHAPTER 3

BASE FRICTION MODELLING MATERIALS

3.1 Introduction to Modelling Materials

Choosing the appropriate modelling material for the modelling situation is a major consideration in physical models. The modelling material must be able to deform and exhibit characteristics similar to the real material that is being modelled. When choosing the scaling factors to satisfy the similitude requirements, the model material must be able to meet the appropriate requirements, e.g. strength. Stimpson (1970) for reasons of simplicity, defines a suitable model material as a material where one material property only satisfies the similitude requirements. In the example from section 2.8, the similitude requirements for tensile strength have been satisfied, but the requirements for compressive strength, elastic modulus, etc. have not been satisfied or considered.

The modelling material plays a significant role in satisfying similitude requirements. The model material must behave properly within the limits set by the scaling requirements. A key factor in choosing a material is the strength characteristics that the material displays. In order to obtain accurate scaling and model behavior, the strength curves for the model material should resemble the curves for actual material within the appropriate scale (Fumagalli 1968; Erguvanli and Goodman 1972).

As mentioned above, a modelling material can be any material which satisfies similitude under given conditions. The only thing that limits the type of material used

is the limits or criteria set forth by the experimenter. The modelling materials chosen for this thesis were chosen based on seven major criteria:

1. Cheap and easy to make;
2. Readily available and easy to obtain;
3. Must be easily reusable;
4. Must be easy to work with;
5. Must be inert, or non-reactive;
6. Little or no changes in the material properties over time, once testing;

has started.

7. General use and cleanup should not be messy.

With these criteria in mind, the mix needed for this thesis needed to be relatively deformable, but brittle enough to represent brittle rock.

For a more comprehensive study or discussion of available materials for modelling, the reader should consult Stimpson's (1970) paper on material models. His paper gives a comprehensive discussion of different material models which includes their respective strength characteristics, and their behavior characteristics. The paper is also a very good source for locating other references on any material models that may be of interest.

3.2 Testing of Possible Modelling Material

Several different mixtures were tested in order to find a low strength mix that was brittle with a small amount of plasticity. The mix should act like a low strength rock,

which deforms before breaking. Therefore, the mix should exhibit some cohesive properties, instead of crumbling when handled. Ideally, the mix should behave similar to "play dough".

3.2.1 Experimental Mix Testing Procedure

In order to find a suitable mix, several mix designs previously used by other people were tested. The testing procedure consisted of mixing up a small batch of the mix and pushing several small scale models by hand. Samples were pushed over both a rough and smooth frictional surface. The test procedure was as follows:

1. Approximately 1000g to 1500g of the proposed mix was prepared.
2. The mix was compacted into 4 in. x 4 in. x 1 in. (10.16 cm x 10.16 cm x 2.54 cm) test molds. Several different samples were made with the thickness varying from 0.25 in. to 1 in. (0.64 cm to 2.54 cm). After they were compacted, discontinuities were cut into the sample. The test cases were a rock slope or a tunnel opening with bedding planes at various angles.
3. Once the samples had cured, if needed, they were pushed by hand along one of two possible frictional surfaces. The frictional surfaces being used were a coarse sandpaper and a smooth rubber mat. A fine grit sandpaper was also used in later test runs, but it was discarded because it did not work. Both frictional surfaces were tested with each mix.
4. The performance of each mix was recorded after each test push. Each mix was tested several times in order to determine if it could be used.

3.3 Modelling Materials Tested

A vast number of materials have been tested in order to find a mix that behaved liked "play dough". Several published materials, that other authors had used, were tested in order to get a reference as to what kind of material they were working with. Using these mixes as a reference, new mixes were developed which were variations of these published mixes and some were entirely new ideas. The mixes that were tested, consisted of various combinations of flour, oil, shellac, barite powder, plaster-of-paris, and sand. A sample of cookie dough was also tested. Play dough was not tested because it dries out over time, and it would reduce the life of the sandpaper.

3.4 Tested Materials that were unusable

Many of the materials tested were unusable. Most of the materials were good modelling materials, they just did not behave in the manner that was required for this thesis. Some problems were also encountered when mixing certain materials. Depending on the order in which the ingredients were mixed, the consistency of the material changed. Determining the correct mixing order was a problem because most of the references did not give their instructions about how these materials were mixed together. This led to problems when trying to determine if the material mixed was the same as the material in the references. All of the materials that were tested are outlined throughout the rest of this section.

3.4.1 Plaster-of-Paris

Plaster-of-Paris was the first material tested in the base friction machine. The plaster was mixed with water until it turned into a paste. The mix was then placed in the machine, and the discontinuities were cut into the model. A tunnel was used for the simulation with horizontal bedding layers approximately 3/4 in. (1.91 cm) wide. The plaster was allowed to set completely, approximately 3 hours, until it had hardened. Once the plaster had hardened, it was assumed that the mix would behave as brittle rock. With plaster being brittle, it was thought that the model would show good crack propagation once deformation and failure started. This was not the case. The plaster was much stronger than anticipated, and the model took horizontal pressures in excess of 24 psi (165.5 kPa).

Further testing of this mix was abandoned due to strength of the material and the time required to prepare the sample. Plaster-of-paris could prove to be a good mix if it is mixed with sand to weaken the material. Stimpson (1970) discusses several cases where plaster-of-paris was used in conjunction with other materials. These mixes were not tried due to the unavailability of the other ingredients required.

3.4.2 Sand and Shellac mixture

Several mixtures of sand and shellac were tested using the experimental procedure mentioned above. There were two experimental mixes used, consisting of various types of sand with set proportions of shellac. The mixes were categorized by the percentage by weight of shellac in each mix. The two shellac percentages were 1.5% and 3.0%. These percentages were taken from a study done by Stimpson (1979). The two types of

sand used were ASTM T-190 Ottawa Sand and Quickrete® all-purpose sand. Table 3-1 gives the corresponding sand and shellac mixtures along with comments about the mix.

The mixes were prepared by mixing 2.5 lbs (1134 g) of sand to 1.5% and 3% (by weight) of shellac respectively. Once the samples were compacted, they were allowed to dry for approximately 30 minutes to 1 hour. Several batches were made with different drying times. The ASTM T-190 Ottawa sand is 20-30 size sand particles. This is coarser than the +40 to -30 size that Stimpson (1979) used in his tests. The coarser sand was used because it was the closest size available at the time of testing. The all-purpose sand is a fairly well graded sand with a smaller grain size than the Ottawa sand used.

The results are summarized in table 3-1. The table shows that mixes 1 and 4 behaved in the same manner. These mixes were very strong and behaved fairly well. The only problem with them was their brittleness. The models were very brittle and broke suddenly. The brittleness allowed for very little deformation before failure.

Mix no. 2 was too strong. This mix was considerably stronger than the other three and it was also more brittle. The sample did not produce any useful results due to the strength required to break it.

Mix no. 4 was the worst mix of the four. The low shellac percentage and the smaller grain size of the all-purpose sand produced a very dry mix. There was not enough shellac to cover all of the sand particles, therefore the mix had little to no cohesion. This mix was very weak and fell apart as soon as it was pushed.

The sand-shellac mixture was eventually abandoned due to several reasons. The shellac wore out the sandpaper much faster than any of the other mixes tested. The

Table 3-1: Summary of Sand and Shellac mixtures

MIX #	SHELLAC %	SAND	COMMENT
1	1.5%	Ottawa Sand	good mix
2	3.0%	Ottawa Sand	too strong
3	1.5%	all-purpose sand	no cohesion
4	3.0%	all-purpose sand	good mix

mix also wore out the rubber mat that was being tested. In effect, the sand and shellac mix was coarse enough to "sand" off the top surface of the mat making it an extremely smooth surface. The drying or curing time of the mix also presented problems. The amount of time the samples dried, dictated how the mix behaved. The longer the samples dried, the more brittle they became. As the shellac evaporated the cohesion of the mix increased. Since the samples had to dry for at least 30 minutes, this reduced the amount of testing that could be done in a day. The mix is also non-reusable because the shellac changes the frictional properties of the sand. When the mix is broken back up, the shellac is still on the sand particles and there is a noticeable difference in the properties of the used sand versus the sand that had not been mixed yet. There was also a problem with fumes from the shellac during the mixing process. The mix had to be

prepared in a well ventilated area, otherwise the fumes from the shellac would overcome the person mixing up the mix.

This mix may be very good for certain applications, but it goes against some of the criteria for choosing a mix; such as a mix that is reusable and easy to make.

3.4.3 Flour-powder-sand and oil mixture

This mixture was used by Al-Harthi and Hencher (1992) at the University of Leeds. A variation of this mix was also tested. The actual mix consisted of the following: 40% by weight of barite (barium sulfate powder), 25% fine sand, 24% flour, and 11% vegetable oil. The variation that was used substituted plaster-of-paris for the barite powder. The variation was used because the barite powder is not a readily available material, and the powder had to be ordered from a chemical supply house.

According to Al-Harthi and Hencher (1992), the model material is of low strength and high deformability. Both the actual mix and the variation of the mix behaved as a material of low strength and high deformability. There was no noticeable difference between the barite powder and the plaster. The only problem with each mix was the low strength. The deformation was acceptable, but the strength of the material was too low for this particular machine. The material was too weak to withstand the hand pushing method of testing. This was an indication that it would be too weak for the base friction machine.

The weakness may be attributed to the way the material was mixed. Al-Harthi and Hencher do not give any indication as to how they mixed the material, therefore the procedure used to mix the material could be considerably different than their procedure.

The mixing procedure consisted of mixing all dry materials together, and then mixing the oil into the dry mix. This procedure produced a low strength, semi-cohesive material that could withstand moderate deformation. This mix could still possibly be used if the cohesive properties of the mix could be improved. A possible way of doing this would be to increase the oil content until a satisfactory mix is found.

3.4.4 Plaster, sand and oil mixture

Another attempt was made at using plaster-of-paris in order to get the strength needed. This time, fine Ottawa sand (-30 to +40 particle size) and vegetable oil were mixed with the plaster in order to reduce the strength of the mix.

The first sample mix just used the Ottawa sand with 5% plaster mixed with water added to it. This mix was allowed to set for 15 minutes and then tested. The result was a material that had lower strength, but it was not plastic enough. The second mix had 2.5% vegetable oil added to the mix and then allowed to set for 15 minutes. This mix also was not plastic enough.

After these two attempts, the use of plaster-of-paris was abandoned because the deformations needed could not be achieved, and the mix has to harden before it is used.

3.4.5 Cookie Dough

The idea for using cookie dough came about because of the type of behavior that was being modelled. The desired characteristics were deformability with some brittleness, which is consistent with the behavior of childrens play dough or cookie dough. Cookie dough was chosen because it was easier and cheaper to obtain in the quantities needed. The "Pillsbury Sugar Cookies" dough, which is premade, was used

because it just had to be rolled out into the sample mold. Sugar cookie dough was chosen because it was plain dough with nothing else in it.

The dough worked very well for the first few times. It deformed and broke better than expected. The only problem was that the dough started to get soft and "melt" after about 15 minutes of working with it. This was the only problem encountered with the dough. Even though the cookie dough did not work, it did provide a very good idea of the type of behavior that was wanted.

3.4.6 Sand, oil and flour mixture

The cookie dough idea led to experimenting with a mixture of fine sand, vegetable oil, and flour, which was also used by Goodman (1976). The problem was finding the right percentages to produce a mix with the same consistency as cookie dough.

In order to determine the best percentages, several test mixes were made using a set amount of all-purpose flour, fine Ottawa sand, and then adding small amounts of vegetable oil until it became doughy. Fine Ottawa sand was added to the mix to make the mixture more brittle and to enhance the cohesive strength of the material.

The percentage of vegetable oil ranged from 20% to 38% by weight, and the percentage of sand ranged from 30% to 80% by weight. The percentages for the vegetable oil were determined from trial and error, while an idea about the sand percentages was obtained through correspondence with Karaca (1994).

The results from the model tests were very promising. The various models had close to the consistency and the brittleness that was desired, but most of the models were too greasy. The models were very messy and left a considerable amount of material on

the sandpaper. Because of this, the sandpaper had to be replaced before it was worn out. This material produced the best results out of all of the materials tested, with the exception of the cookie dough. Even with the problems that were encountered, this mixture led to the model material that is being used in the base friction machine.

3.5 Acceptable Model Material

Further testing was done on the flour, sand, and oil mixture in order to find a consistency that was usable. The main goal of the second round of testing was to find the optimum working consistency. In order to find the optimum consistency, Dr. R. E. Goodman (1994) was consulted about the mix that he had reported using.

In his letter, Goodman (1994) gave several suggestions that were helpful in finding the best mix. He suggests mixing a few percent oil to the flour and then add an equal amount of sand. Then he suggests performing a simple toppling slope analysis on each mix in order to judge the behavior of the material. He went on to say that, "the mix should not be oily to the touch or greasy in appearance". The complete letter is included in appendix C.

Using Goodman's suggestions, the best flour and oil mixture was determined before the Ottawa sand was added. The sand was mixed fifty/fifty with the flour and oil mixture. The optimum percentage of oil was determined to be 30% by weight of flour.

The oil percentage of 30% was determined to be optimum; because a mix with 28% oil was a little dry, while a mix at 32% was too oily. Further testing of the mix with 30% oil revealed a mix with that performed very well, and it could withstand the

disturbance of breaking the adhesion at the base of the model. The mix also was not messy and it did not destroy the sandpaper.

The final mix design consists of a mix that is 50% of flour and oil, and 50% fine Ottawa sand. The oil content is 30% by weight of flour. The exact ingredients being used are as follows:

1. Flour: Bakers and Chefs all-purpose bleached flour.
2. Oil: Wesson vegetable oil
3. Sand: Flintshot 4.0 Ottawa sand (-30 to +40 size) from U.S. SILICA.

There were four main observations made during the development and testing of this mixture. These observations proved to affect the outcome of the results. These observations are now used as a general rule of thumb:

1. The results were easier to obtain when using a name brand flour, rather than using a generic brand. For whatever reason, the name brand flours tend to mix up better with the oil. The generic flour and oil tended to form large clumps, where as the name brand flour did not clump up as bad.
2. Choose one brand of flour and stick with it because there are variations among the different brands. No two brands of flour give the same results with the same mix percentages. When changing brands of flour, the oil percentage may have to be altered 1 to 2%.
3. Mix the material up in small portions. Depending on the size of the mixing bowl, large quantities of flour and oil in the bowls did not mix very well. The material used for this testing was generally mixed in 1300g portions.
4. Mix only enough material needed. This mix was only good for approximately 2 days with repeated use. It was finally determined that the mix would only be used 1 day and then thrown out. After two days, the mix properties tend to change slightly. The material tends to be slightly drier and more brittle, therefore fresh material was always used.

By following these general guidelines, the material properties and behavior remain fairly uniform and consistent each time a new batch is mixed.

CHAPTER 4

TESTING PROCEDURE

4.1 Overview of Testing Procedure

The base friction test procedure used in these experiments was developed through a trial and error process. The procedure is not modelled after any previously used procedure. The testing procedure consists of four main steps: 1. the model material preparation, 2. the machine setup, 3. the placement of the model material, and 4. actual testing.

4.2 Model Preparation

The model material being used is a flour, oil, and sand mixture. As mentioned in section 3.5, the model material is 50% sand and oil, and 50% ottawa sand.

4.2.1 Mixing Procedure

Approximately 1500g of model material is needed to get the desired model thickness. The material is mixed in a two step process:

1. The flour is mixed with 30% vegetable oil by weight.
2. The flour and oil mixture is then mixed with an equal amount of fine Ottawa sand. The mixture is then mixed very well making sure that there are no clumps of flour and oil with no sand. Special care should taken to ensure that all of the clumps are broken up, and that the mix is mixed very well.

4.2.2 Storage

The mix should be stored in an air tight container when it is not used. Any mix that is left over after compaction should be returned to the air tight container to prevent it from drying out. A new batch should be mixed daily, because the mix tends to last only approximately 24 hours. After 24 hours and several test runs, the mix tends to be more brittle and does not behave very well. This is due most likely to some of the moisture in the oil evaporating.

4.3 Machine Setup

Before the model is placed in the mold, the machine must be setup properly. The mold should be moved to either end of the machine, in order to give enough room to run a test, and the friction surface should be cleaned or replaced.

The threaded rods should be oiled at the beginning of each day and after every fourth or fifth model. The reason for oiling the rods is to keep them lubricated and to prevent the machine from binding. If the rods are not oiled properly, the grit and sand particles that get in the threads can lock-up or jam the sample mold. The oil also helps keep the mold moving smoothly which helps prevent the development of vibrations that could blur the photographs that are being taken during the testing. The oil that is used to oil the rods is Amoco Ultimate Gold SAE 10W-30 motor oil.

4.3.1 Mold Placement

When a test is being run using the top pressure cylinders, the mold should be started at the end of the machine that allows the mold to travel in the direction were the

pressure cylinders are at the top. When the top cylinders are being used, there is only one direction of travel. When the top cylinders are not being used, the machine may be started at either end of the table.

4.3.2 Friction Surface

The two types of frictional surfaces being used are a rubber mat and sandpaper. The type of modelling material being used will dictate which surface can be used. The sandpaper can be used for all types of modelling materials, but the rubber mat is best when it used with a dry material. The sand-oil-flour mix that is being used did not do very well on the rubber mat because the oil in the mix made the mat slick, therefore the sandpaper was the surface that was used on the base friction machine.

Both frictional surfaces are attached to the aluminum base with double stick tape. The tape enables the surfaces to be replaced easily and quickly. The tape is not a very strong sticking tape, therefore it does not leave much glue on the base surface when the sandpaper is removed.

The tape is placed on the back of the sandpaper in intervals which prevent the paper from moving or bowing up as the model moves across it. A strip is placed on each side of the sandpaper, with a third strip in the middle of the paper. Another strip is placed on the end of the paper where the two sheets of sandpaper meet together. This extra strip makes sure that there is a tight seam between the two sheets of paper, and that no gap forms between the paper with constant testing.

The sandpaper that is being used, Klingspor PS23OCP80 1302, is good for approximately 15 to 20 separate samples. Before each test, the sandpaper is vacuumed

to remove any material from previous tests. By vacuuming off the paper after each sample, it is possible to maintain consistent results with consecutive trials of the same case, because removing all the excess material helps keep a consistent coefficient of friction.

On average, one test uses no more than half the length of the base surface. Therefore, at least two to three separate tests can be run before the mold has to be returned to one end of the table and before the paper has to be vacuumed. This is done by advancing the mold to a new starting position just in front of the used paper. Then, once the mold is at the other end the paper can be cleaned off.

4.4 Model Placement

The third step in preparing for a test is the placement of the model. Model placement involves compacting the material into the mold, breaking the adhesion, cutting the discontinuities, and finally cutting out the tunnel or slope.

4.4.1 Compaction of model

As mentioned above, approximately 1500g of material is required to achieve an appropriate model thickness. If the model is too thick, there is a chance of the model failing along a weak plane in the middle of the sample, or if the model is compacted well it can be stronger than desired and the model may not deform as wanted. If the model is too thin, then it will not withstand the disturbance due to cutting the joints, and breaking the adhesion. A thin model tends to be very weak and it will fail quickly. Therefore, good compaction and model thickness is essential. The best model thickness

for this machine is approximately 1/2 in. to 3/4 in. (1.27 cm. to 1.91 cm.) in thickness.

The compaction procedure uses a block of wood and a hammer to achieve the desired compaction. Since the mold is 12 in. x 12 in. (30.48 cm. x 30.48 cm.), a 6 in. x 6 in. (15.24 cm. x 15.24 cm.) block of wood is used to compact the material. The hammer is a typical 10 oz. carpenter's hammer.

The compaction process begins by placing the 1500g of material into the mold and then evenly spreading out the material. When the material is spread out, the pre-compacted thickness should be approximately 1 in. (2.54 cm.). The material should compact to almost half of the pre-compacted thickness. Once the material is spread out, the next step is to place the square block of wood into any corner and start compacting.

The actual compaction begins in any corner and moves around the mold. In each corner of the mold, the sample is compacted by 20 blows with the hammer. The blows are delivered in a circular motion around the square block, working from the outside into the center of the block. This blow sequence is repeated at each corner of the mold. This leaves a uniformly compacted sample that is approximately 1/2 in. (1.27 cm.) thick. The next step is to break the adhesion.

4.4.2 Breaking the adhesion

Once the material has been compacted, the next step is to break the adhesion between the frictional surface and the base of the model. This is done in order to allow the model to move without immediate failure. If top and side pressure is going to be used, then it is applied at this time in order to compress the model slightly and to give the pistons some working room.

The first step is to apply the side and top pressure, if it is going to be used. Air pressure is applied until the pistons begin to push on the model and compresses it about 1/16 in. (2 mm.), then the air pressure is held constant at 2 psi (13.79 kPa) while the adhesion is being broken. The 2 psi is chosen because it is a low pressure and a good pressure to start the tests.

Once the pressure is applied, the adhesion is broken by turning on the machine. If external pressure is not used then the machine is just turned on until the model begins to move. The speed setting is very low, 20% on the speed indicator dial, in order to prevent from disturbing the sample as much as possible. Once the sample begins to move the machine is turned off and the pressure on the pistons is released. The sample is now ready to have the appropriate features added.

4.4.3 Adding the modelling features

The first step in adding the features is to add any joints or discontinuities that want to be modelled. These features are added by carefully cutting the model with a knife. Bedding layers are added one of two ways.

If the horizontal bedding planes are desired, they are cut in with a pre-sized template. Templates which are 1 cm, 2 cm, and 4 cm in width are used to cut the horizontal planes, depending on the thickness desired. Layers of constant thickness, and thick over thin layers, or vice versa, can be modelled. Angled bedding planes are cut by following markings on the mold. Masking tape is placed on the edge of the mold and the thickness of the desired layers is marked off on the tape. A straight edge is then used

to cut the angled layers into the model. Any other discontinuities, such as joints, are measured with a ruler and cut into the model with a knife.

The final step is to place the tunnel or cut out the desired slope. The size of the tunnel or slope is marked with a ruler and then cut with the knife. The unwanted material is then carefully removed with a scoop, trying not to disturb the rest of the model. Once the unwanted material is removed, the cutout section is then vacuumed out to remove any left over material and to clean the model up for a picture. The final step before running a test is to focus the mounted camera onto the model.

4.4.5 Preparing the camera

Once the model is prepared and ready to test, the camera is mounted on the mounting bracket above the model. The mounting bracket can then be moved up or down to get the desired field of view. Once the bracket is set, the camera is focused and zoomed in on the model. Pictures can now be taken by using the camera's timer, or by using a wireless remote control shutter.

4.5 Actual Testing

Before the testing begins, two pictures are taken. The first picture is of the sample with a label telling the test number and the side and top pressures. The second picture is of the sample prior to testing. Once the pictures are taken, the machine is turned on, slowly at first, and then to the desired speed. Pictures are taken as failure or deformation occurs.

When a tunnel is being modelled, the machine can be stopped at any time to increase the pressure, widen the tunnel, or remove blocks that have dropped out, etc. When the tunnel is widened or the pressure is changed, this would indicate a new test and would require another picture with a new label showing the changes.

When roof collapses start to occur the machine can be stopped and the collapse can be replaced. The side or top pressure can be changed to show that the mode of collapse changes as the pressure is changed. The same can be done for the tunnel being widened. When the tunnel is widened, a different collapse pattern should appear.

When slopes are modelled, the different types of collapses can be illustrated by stopping the machine and changing the model periodically.

Once the test is over and the model has collapsed, the model material can be scooped out of the mold and a different model can be compacted into the mold and the testing procedure is started over from the beginning.

CHAPTER 5

CASE STUDIES

5.1 Overview

In order to demonstrate the base friction method, two case studies will be discussed. The two case studies are a slope sequence and a tunnel sequence. These two conditions were studied because their failure mechanisms are very different. The slope sequence demonstrates a toppling mode of failure, while the tunnel demonstrates various degrees of roof collapse.

The model material that is used in the geologic model can simulate many different rock types by adjusting the scaling ratios, as mentioned in chapter 2. Since the model does not simulate just one rock type, it is possible to run a series of test and correlate the model behavior with the expected behavior of several different rock types. As mentioned in chapter 2, the factor linking the model to the prototype is the relationship between the tensile strength of the materials.

Since the tensile strength of the material is a constant, the type of rock being modelled can be changed by varying the strength ratio ξ , which is based on the specific weight ratio η and the length ratio λ . These scaling factors can be obtained by using table 2-2 if the tensile strength of the rock prototype is known, or by entering the table with known scaling factors to get the tensile strength being modelled.

Each case study is documented with pictures which are located in appendix A-1 and A-2. The slope sequences are in appendix A-1 and the tunnel sequences are in

appendix A-2. Each test sequence is considered a Plate and each individual picture is labelled by Stages.

5.2 Slope Sequence

The slope sequence used for this thesis is taken from an actual slope failure that occurred in a rock quarry (Ogden 1993). The walls of the rock quarry has bedding planes that are dipping into the wall and a haul road is cut into the side of the quarry. A section of the quarry wall hanging over the road failed over time closing the haul road. This cantilevered rock configuration is what the base friction model simulates.

The test was performed in two parts using two different road widths. The initial cut out was 5 cm (1.97 in.) wide by 9 cm (3.54 in.) high. The base of the cut out is widened to 9 cm when the 5 cm wide cut stabilizes. The cut outs simulate the haul road being cut into the quarry wall, and then eventually being widened. The model consists of 2 cm (0.787 in.) thick layers that are dipping 45° into the slope. No external pressure is being applied to the model since it is a slope.

The plate sequences in appendix A-1 show the progression of both tests and demonstrate how the real slope may have failed. Plate 1 shows the failure mode for the 5 cm cut out test, and Plate 2 show the failure sequence for the 9 cm cut out.

For plate 1, stages 1 to 5 illustrate the test sequence from the start of the test to the end of the test. Stage 1 shows the model just before gravity is turned on, while stage 2 illustrates how the bedding layers begin to bend and flex over time. Stage 2 also shows a tension crack forming in the first overhang layer of the cut-out. Stage 3 shows

the first overhang layer breaking away from the wall, and it shows the formation of tension cracks further back in the slope. The tension cracks are forming in the third and forth layers. Stage 4 illustrates the tension cracks getting bigger, and it also shows that the second layer is still intact and starting to take the weight of the third and forth layers as they tend to break away too.

The first layer has also broken completely away from the wall and is falling down into the cut. The bottom layer has been removed since it was free falling in stages 5 and 6. In these pictures, a tension crack is starting to form in the second layer due to gravity and due to the weight of the third and forth layers. A tension crack is also starting to form in the fifth layer.

Stages 5 and 6 show that the tension cracks have formed along a straight line projecting back into the slope at approximately the same angle as the slope. This line represents the point of critical overburden for this slope angle. Critical overburden is the minimum amount of overburden above each bedding layer needed to cause toppling. As the slope angle is decreased the amount of overburden above each layer is decreased, which reduces the horizontal stresses in the slope to some minimum value and causes a toppling failure. The line created by the tension cracks in stages 5 and 6 represents this line of minimum horizontal stress. If the test was continued, the stages would show that the slope is stable except for the blocks that are already toppling.

By applying the scale factors to this slope and relating this model to a full size prototype, a safe and stable slope configuration can be obtained. This test shows that if

the full size slope in question would be laid back to this line of critical horizontal stress, then the slope would be stable.

Plate 2 illustrates the same sequence as plate 1, except that the base of the cut out has been widened to 9 cm (3.54 in.). The model in this test sequence is the same model from the first test with the base widened. The toppling blocks from the first test have also been put back in place to illustrate what happens if the blocks are not removed but the base is widened.

Stage 1 for this sequence is taken as the test is in progress. Stages 1 and 2 show that the first and second layers are cracking in a different pattern than in the previous test. The bottom layer is not toppling at the tension crack formed in the last test, but at the face of the wall. This is most likely due to the fact that this section of the bedding layer is acting as a cantilevered beam, and the beam is cracking at the fixed end due to the weight of the toppling blocks above it. The second block is also breaking in a different manner than before. The top part of the block is breaking along the preformed tension crack, but the bottom part of the block is breaking deeper into the bedding layer; at the same point the cantilevered beam section is separating. The third and forth layers are toppling along the tension crack surface, as they did in the previous test sequence.

Stages 3 through 4 illustrate how the toppling progresses. The blocks topple along the line of critical horizontal stress as in the previous test. The main difference in the toppling regions of the two test sequences is the location of the tension zones where toppling occurred. In the first test sequence, the tension zone progresses back into the slope at the same angle as the slope, but in the second test sequence the tension zone

is steeper than the slope angle. The steepness of the angle in the second test sequence is due to the bottom layer breaking at the face of the wall.

The failure pattern that the second test sequence produced is different than the expected pattern. The expected pattern would be the same as the first test sequence where the toppling zone progresses back into the slope at the same angle as the slope. The failure pattern is different because the base of the cut is deeper into the slope, therefore the amount of overburden over each layer is greater. This would cause the tension zone to be steeper because the horizontal stresses are greater. Since the angle of the tension zone is steeper, this would cause the zone of failure to propagate further up into the slope or embankment. This is the case since stage 6 shows a block in the fifth layer starting to topple, where there were only four layers that toppled in stage 6 of the first test sequence.

5.3 Tunnel Sequence

The second case study is a series of test using a tunnel model to illustrate how changing horizontal and vertical stresses will affect the volume of material that falls out of the tunnel roof. Two types of testing sequences were used. The first sequence involves maintaining a constant stress ratio K , where K is equal to $\frac{1}{2}$. The second testing sequence holds the vertical stress constant at 2 lb/in² (13.8 kPa) as K increases.

The same model configuration is used for both testing sequences. The joint pattern being used is a standard brick pattern, where each individual brick overlaps the

brick beneath it halfway. After the model material is compacted, the model is cut into a brick pattern, where each brick is 3 cm long x 2 cm high x 1.9 cm thick (1.18 in x 0.79 in x 0.75 in). Since all the bricks are the same dimension, the volume of one brick is taken to be a unit volume or 11.43 cm³ (0.698 in³). The tunnel in each test is 8 cm high x 15 cm wide (3.15 in. x 5.91 in.).

By using a unit volume, the total volume of collapse for the prototype can be determined by knowing the length ratio λ . Equation 2-9, shows that the ratio of the prototype volume and model volume is equal to the cube of λ . With this model configuration, figure 5-1 and 5-2 can be used to determine the total volume of collapse in the prototype by using the value of λ obtained from table 2-2. Figure 5-1 is in metric units while 5-2 is the same graph in english units.

5.3.1 Tunnel sequence with constant K

The first testing sequence demonstrates how horizontal stress affects the number of blocks that drop out of the roof. For this test sequence, K is held constant at a value of 1/2 as the horizontal and vertical stresses are increased. The photographs in Plate 3 of appendix B-2 document the results of the test.

The first test series, illustrated in Plate 3 stages 1 through 6, uses fairly low pressures to model low stress conditions. The top and lateral air line pressures used in the test are 1 lb/in² (6.9 kPa) and 0.5 lb/in² (3.45 kPa) respectively, which correspond to an applied vertical stress of 0.2 lb/in² (1.38 kPa) and a horizontal stress of 0.1 lb/in² (0.69 kPa) being applied to the sample. Using the Berea Sandstone ratios for η and λ

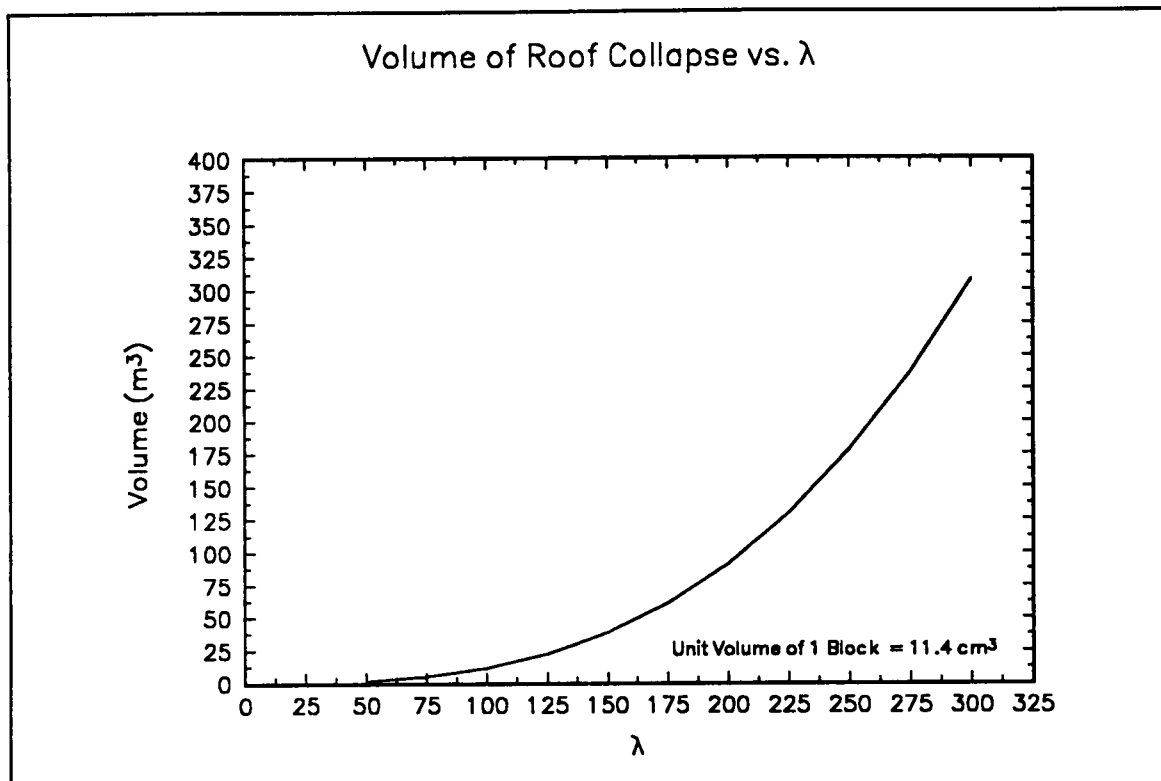


Figure 5-1: Estimation of Volume of Collapse in Prototype (metric units)

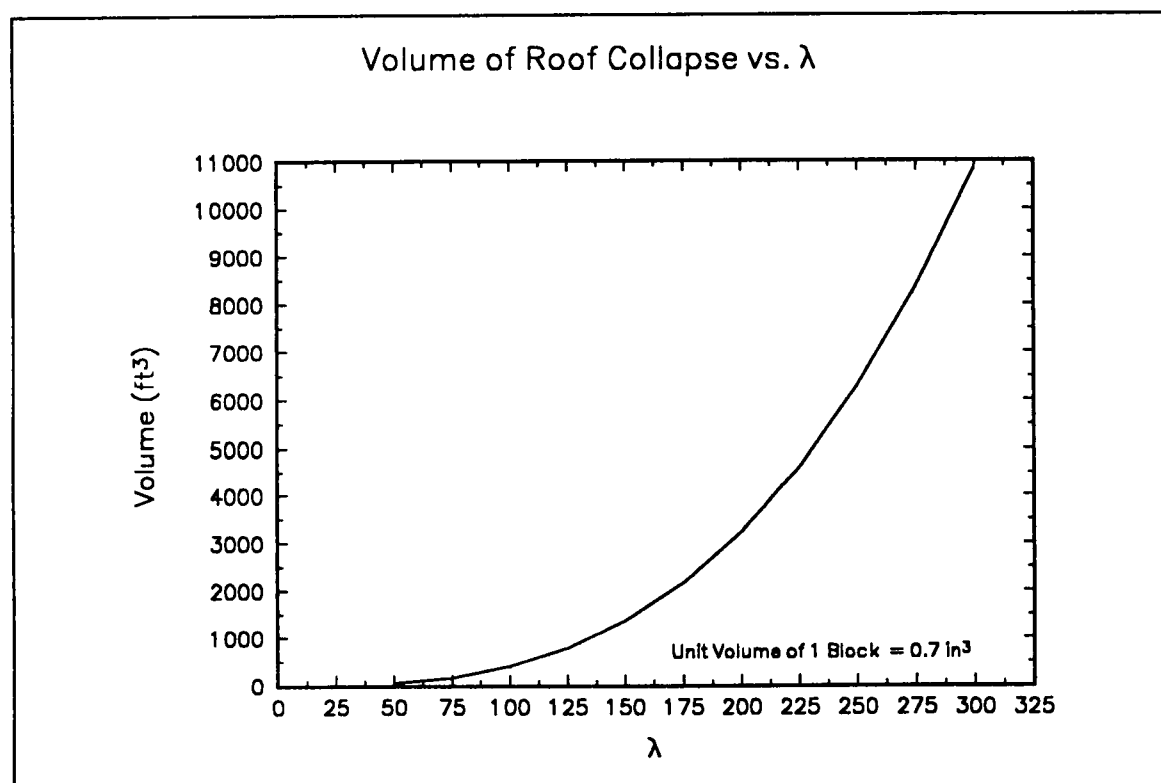


Figure 5-2: Estimation of Volume of Collapse for Prototype (english units)

from chapter 2 and the relationship in equation (2-49), these stresses can be equated to actual field stresses for this tunnel condition in Berea Sandstone. Using a η of 1.75 and λ of 167, a stress ratio ξ equal to 486 is obtained, so that the prototype vertical stress is 97.2 lb/in² (671 kPa) and the horizontal stress is equal to 48.6 lb/in² (335 kPa). The photographs illustrate a classic failure due to low overburden, and they show how the width of the collapse progresses up into the overburden material in a step-like fashion until a stable span width is reached.

In this test, 18 blocks of the roof collapse, which is a majority of the overburden. In the cases of low overburden material and K being assumed constant, low overburden means low horizontal stresses or confining pressure. Large roof collapses can be attributed to low overburden or low horizontal stresses.

The second test, Plate 3 stages 1 to 5, uses top and lateral air line pressures 2 lb/in² (13.8 kPa) and 1 lb/in² (6.9 kPa) respectively; creating applied vertical and horizontal stresses of 0.4 lb/in² (2.76 kPa) and 0.2 lb/in² (1.38 kPa) respectively. Again these stresses equate to prototype stresses of 194.4 lb/in² (1341 kPa) for vertical stress and a horizontal stress of 97.2 lb/in² (671 kPa).

The amount roof collapse in this test is only 16 blocks as compared to 18 blocks in the first test. The reduction in material is due to the increase in horizontal stress, which is due to the increase in the simulated overburden by the applied model vertical stress. The step-like failure surface, shown in stages 4 and 5 of this test is similar to the failure surface in the first test.

The third test sequence, shown in Plate 3 stages 1 through 5, illustrates the affect of a high confining stress. The top and lateral air line pressures in this test are 2.5 lb/in² (17.25 kPa) and 1.25 lb/in² (8.63 kPa) respectively, giving applied vertical and horizontal stresses of 0.5 lb/in² (3.45 kPa) and 0.25 lb/in² (1.73 kPa). These stresses are equivalent to a prototype vertical and horizontal stress of 243 lb/in² (1677 kPa) and 122 lb/in² (838 kPa). These values may seem low, but the compressive strength of the material is around 0.8 lb/in² (5.52 kPa). The higher confining stress is evident in the pictures because the roof collapse happens later in the test; starting at stage 3, where it began in stage 2 of each of the previous two tests. Also, the amount or volume of collapse drops from 16 blocks to 13 blocks.

The effects of high confining stress are shown very well in stages 4 and 5. These photographs show that the collapse mechanism is no longer purely sliding or dropping out. The blocks in these pictures are exhibiting beam bending and flexure failures; with compressive failure in the bottom layer of both pictures. In these later stages of this first test sequence, it is evident that the brick pattern has been lost due to compression of the material. The cause of this will be explained in a later section.

The fourth test is the last test in this sequence. This test is illustrated in Plate 3 stages 1 through 4. In this test, the vertical and lateral air line pressures are 3 lb/in² (20.7 kPa) and 1.5 lb/in² (10.35 kPa), producing applied vertical and horizontal stresses of 0.6 lb/in² (4.14 kPa) and 0.3 lb/in² (2.07 kPa) respectively. In the field, these stresses equate to a vertical stress of 292 lb/in² (2012 kPa) and a horizontal stress of 146 lb/in²

(1006 kPa). These pictures show that the horizontal stress is great enough to prevent roof collapse. The model eventually fails during this test sequence.

A thrust fault or failure can be seen in the lower left hand corner of stages 3 and 4. The model is considered to have failed when the model starts thrusting, because this means there is movement in the third dimension. Recalling section 2.2, movement in the third dimension invalidates the base friction model.

The effect of confining stress can be illustrated graphically by plotting the volume of collapse versus the horizontal stress. Figures 5-3 and 5-4 show that the amount of material that drops out of the roof decreases as the applied horizontal stress increases, which is what is expected. The amount of material that falls out is determined by multiplying the number of blocks that fall out by the volume of 1 block or 11.43 cm (4.5 in.).

5.3.2 Tunnel sequence with increasing K

This series of tunnel test studies the affect of increasing K has on the volume of roof collapse. In this series, the vertical stress is held constant at 0.4 lb/in² (1.38 kPa) while the horizontal stress is increased in order to increase K . The results produced in this test series are similar to the previous series of test where K is constant. Plate 4 illustrates the results of this test series.

The first test in stages 1 through 7 is conducted at low pressures just as the first test in the previous test series is. The vertical and lateral air line pressures are 1 lb/in² this is equal to is 97.2 lb/in² (671 kPa) and 48.6 lb/in² (335 kPa) respectively.

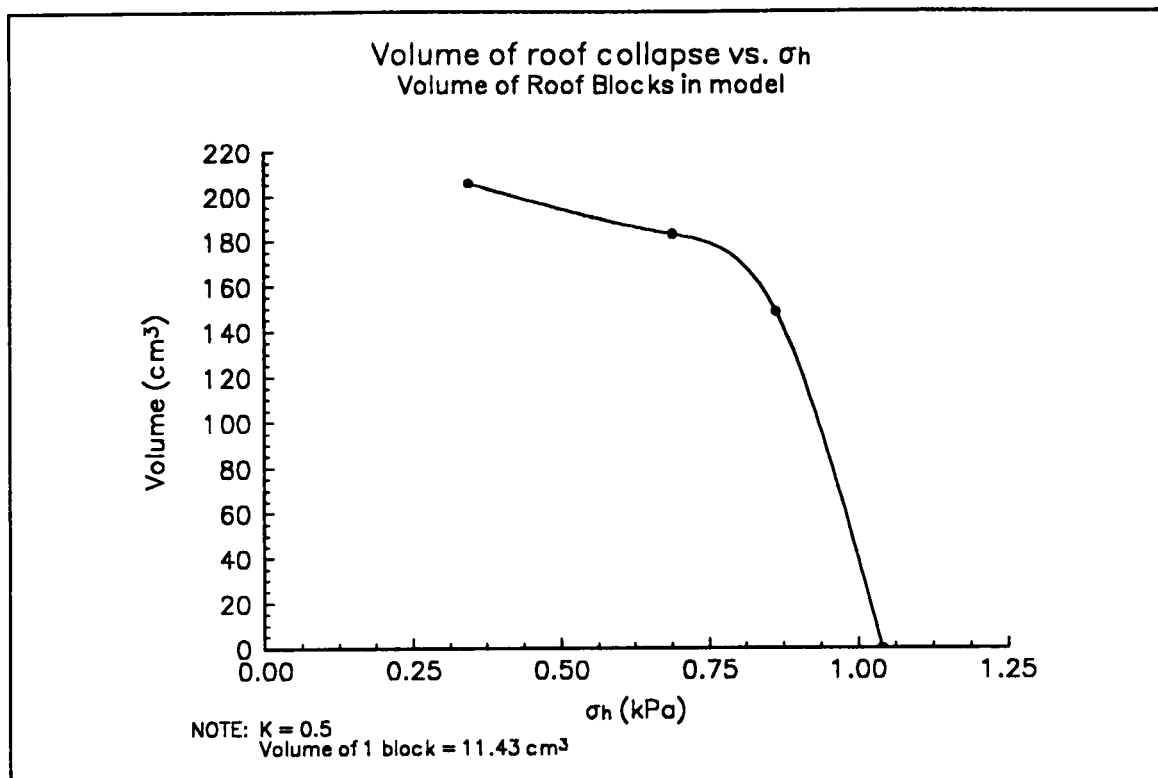


Figure 5-3: Effects of confining stress on roof collapse $K = 0.5$ (metric units)

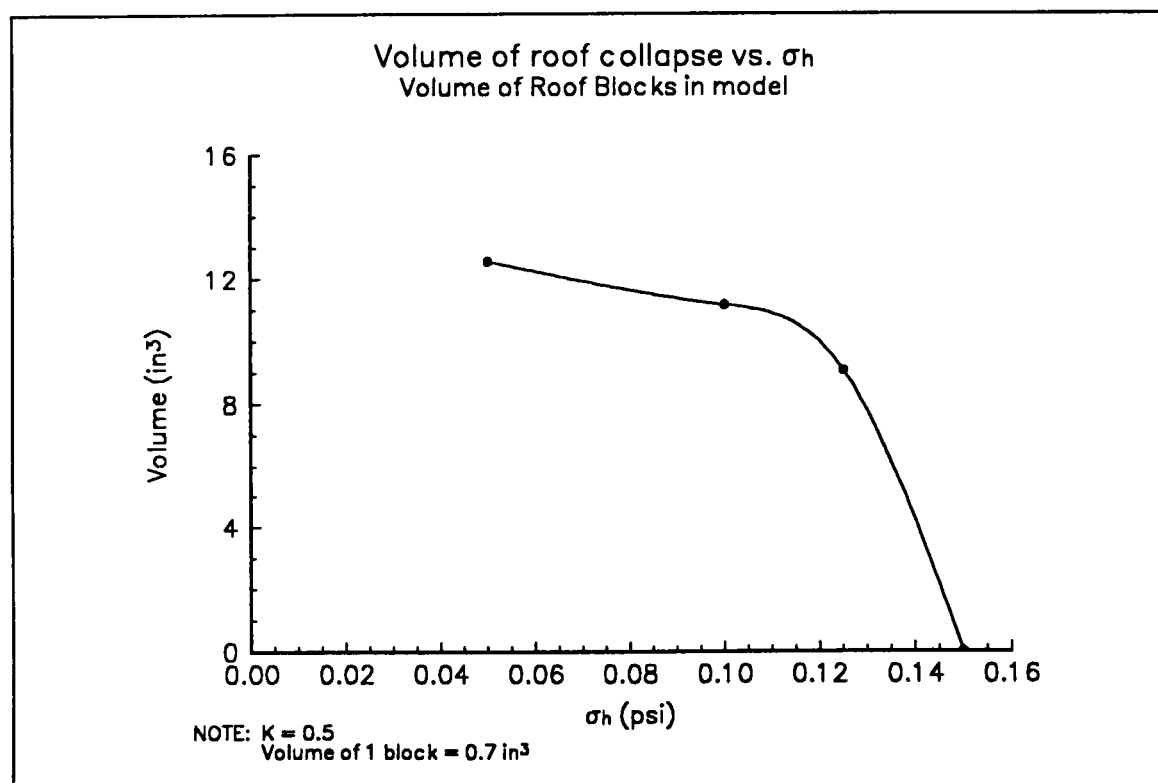


Figure 5-4: Effects of confining stress on roof collapse $K = 0.5$ (english units)

As with the first test in the previous series, the roof collapse is a classic step-like collapse where the span length of the collapse decreases until a stable span length is reached. In the case of this test and the first test of the previous series, the stable span length is 6 cm (2.4 in.). As with the first test of the previous series, 20 blocks collapsed out of this roof also. This is expected since the vertical and lateral pressures are the same. This test also shows that low confining pressure can lead to a large volume of material falling out of the roof.

The second test, shown in stages 1 through 9, maintains both the vertical and horizontal air line pressures at 2 lb/in² (13.8 kPa) giving a $K = 1$. This test also demonstrates a large roof collapse. This collapse produces the same step-like collapse that has been present in the previous tests. This test produced the same amount of collapse as the first test, 20 blocks. This is due to the fact that the stresses are still in the lower stress range. Stage 7 shows that some of the blocks are falling out due to bending and tensile failure. The reason for these types of failures is due to the slight increase in confining stress.

Test three in stages 1 to 4 uses a K of 1.25 with the vertical and horizontal air line pressures at 2 lb/in² (13.8 kPa) and 2.5 lb/in² (17.25 kPa), respectively, producing applied vertical and horizontal stresses of 0.4 lb/in² (2.76 kPa) and 0.5 lb/in² (3.45 kPa). This produces prototype vertical and horizontal stresses of 194 lb/in² (1339 kPa) and 243 lb/in² (1677 kPa). With this increase in stress, the roof blocks failed as a single mass instead of individual blocks as before. The roof blocks slide along a failure surface created in the previous test due to the tensile failure of certain blocks. As mentioned in

the first series of test, the increase in pressure is beginning to compress the blocks into one mass, causing them to lose the block characteristics. Even though these blocks fell as one unit, only 18 blocks fell during this test, which is expected with increased stresses.

Stages 1 through 7 illustrate the fourth test. This test uses vertical and horizontal air line pressures of 2 lb/in² (13.8 kPa) and 3 lb/in² (20.7 kPa) producing a K of 1.5. This produces applied vertical and horizontal stresses of 0.4 lb/in² (2.76 kPa) and 0.6 lb/in² (4.14 kPa) respectively, yielding prototype vertical and horizontal stresses of 194 lb/in² (1339 kPa) and 292 lb/in² (2012 kPa). This test is in the high range of stresses and the typical step-like roof collapse is not seen. This is due to the fact that the high stress range has compressed the material into one solid mass. The failure surface that formed in the previous two test also contributes to the failure mechanism in this test. Even though the material is acting like a solid mass, the stress increase only allowed 11 blocks to drop out of the roof.

The final test, illustrated in stages 1 through 5, is near the compressive strength of the material. The air line pressures are 2 lb/in² (13.8 kPa) vertically and 3.5 lb/in² (24.15 kPa) horizontally, making $K = 1.75$. This produced applied vertical and horizontal stresses of 0.4 lb/in² (2.76 kPa) and 0.7 lb/in² (4.83 kPa), which produces prototype stresses of 194 lb/in² (1339 kPa) and 340 lb/in² (2347 kPa). As shown in the all stages, the blocks do not fall out until the blocks start failing in compression. The blocks begin to fail in stage 3, while stages 4 and 5 show the layers have all failed in compression. Stage 5 also shows that the sample is starting to buckle in the bottom left

corner of the sample. As mentioned previously, this signifies the sample material has failed. Failure is expected in this stress range since the horizontal stress is 0.35 lb/in^2 and the compressive strength of the material is around 0.4 lb/in^2 (2.76 kPa). Before the sample failed, only 8.5 blocks collapsed out of the roof.

As in the previous test, the stress-volume relationship can be shown graphically. As K increases, the volume of collapse decreases which is the expected behavior. Figures 5-5 and 5-6 illustrates this relationship.

5.4 Conclusions and Recommendations

A base friction machine which allows the application of external surface loads on the side and top of the model has been designed. The addition of the external pressure cylinders has allowed more freedom to fine tune the scale models to the condition being modelled. The vertical cylinders allow the experimenter to model any depth, while the horizontal cylinders work well for modelling any horizontal stress. Since both sets of cylinders work independently of each other, any stress condition or K -value can be modelled. The results produced from this machine are representative of the situations being modelled and the behavior of the models compare well with the actual field behavior.

The graphs of the volume of roof collapse versus K and horizontal stress show the expected results. Figure 5-3, volume versus confining stress, shows that the volume of collapse decreases as the confining stress increases, which is expected since stability increases as horizontal stress increases. The graph of volume versus K , figure 5-5 and

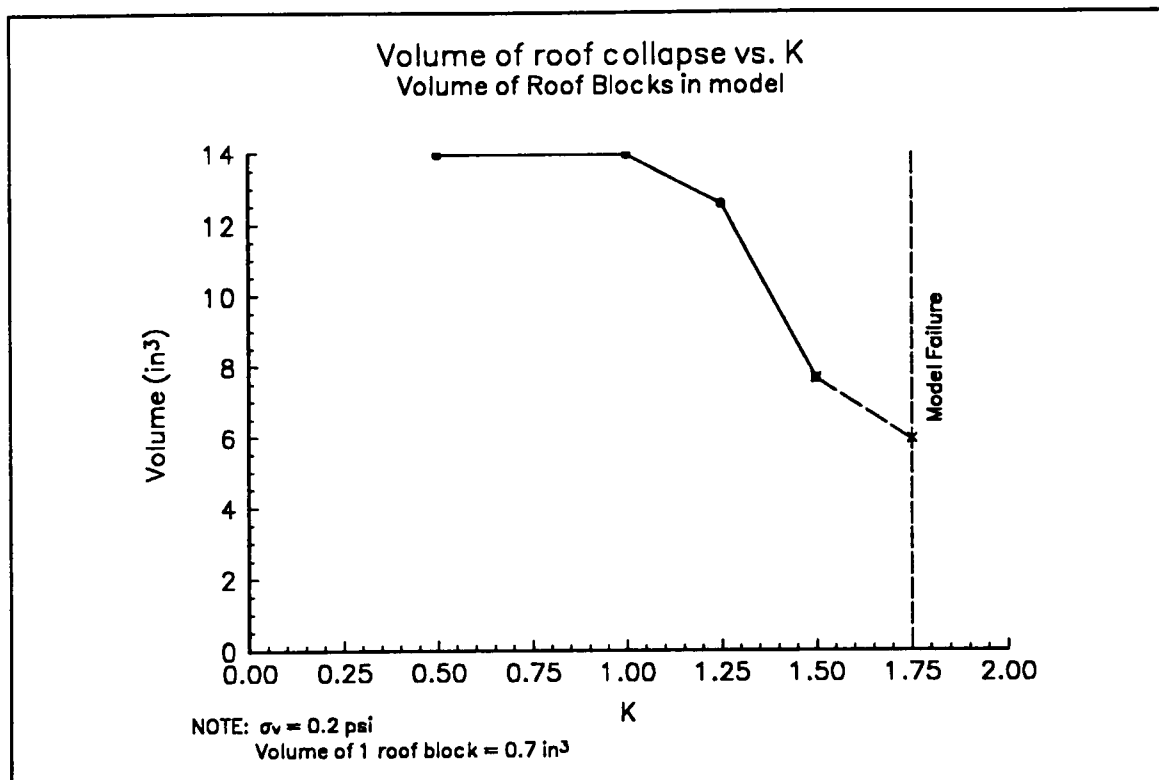


Figure 5-5: Effect of increasing K on roof collapse (metric units)

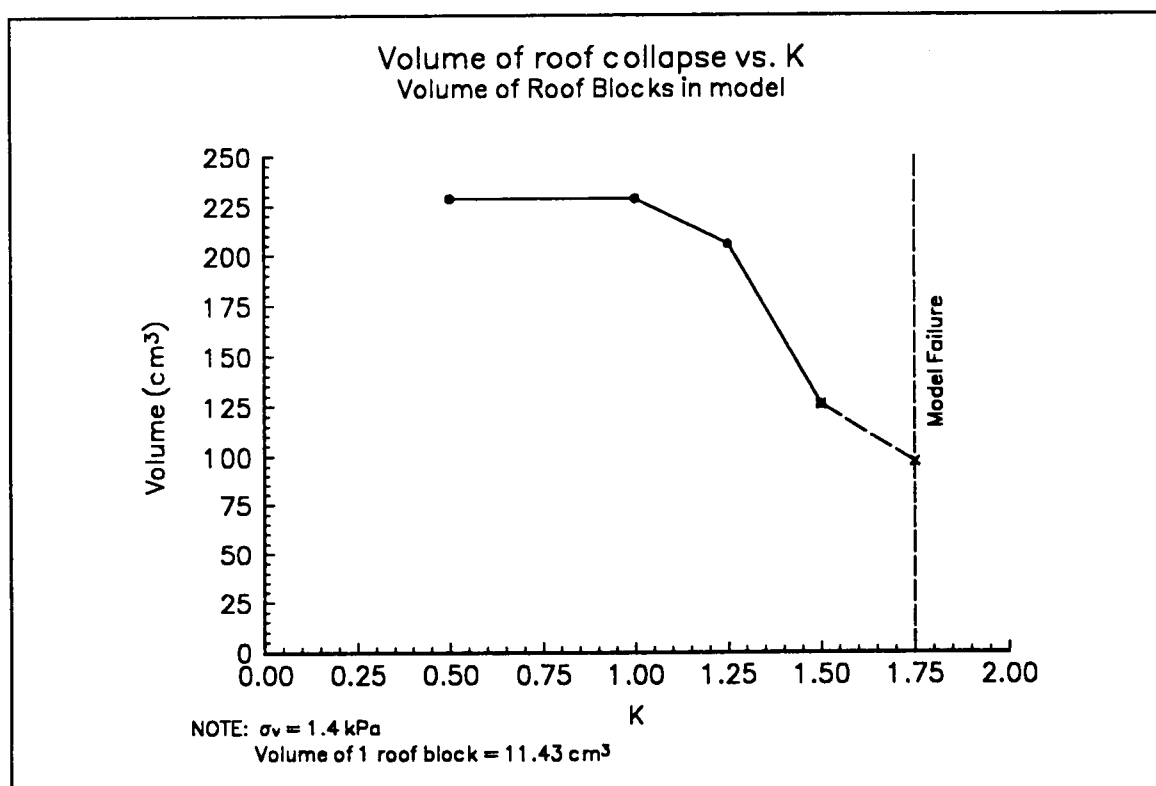


Figure 5-6: Effect of increasing K on Roof Collapse (english units)

5-6, also illustrates this decrease in volume. This is true because as the vertical stress is held constant and K is increased, the horizontal stress increases; which increases stability.

The model material being used is versatile and has proven to be useful for many situations. The material is a inexpensive mix to make and it can be used several times over again. The main disadvantage to the mix is that the properties change after approximately 24 hours. Therefore all of the testing with a particular batch of mix needs to be performed in a 24 hour period.

The model material works well for all of the conditions that were tested for this thesis, except for the higher stress ranges in the tunnel testing. When the applied stresses started getting close to the compressive strength of the material, the material started to compress and remold into a solid mass instead of the brick pattern that was being modelled. The material then started to behave as a single mass and the possibility of individual blocks dropping out was no longer modelled.

There are a few recommendations that can be made to improve these tests and this machine. The first recommendation is to perform the tunnel tests with a different material. The new material should not be as plastic or deformable as the mix used in this thesis. The mix or material should be fairly rigid and brittle in order for the bricks to hold their shape. The author suggests trying sugar cubes or balsa wood blocks and use these results as a comparison to develop a reusable mix. Sugar cubes are very brittle and should have about the same compressive strength as the flour and oil mixture used in this thesis. The cubes have been used successfully by several of the authors

referenced in this thesis. Balsa wood is also a brittle material. It is not as brittle as sugar cubes, but it is very light and has a fairly low strength. There have been several experiments with balsa wood so data for comparison is available.

The second suggestion is to incorporate some type of data acquisition system to see if the stresses in the model are close to the ones being calculated. If they are close, then the base friction method can start to be used as a more rigorous research tool.

BIBLIOGRAPHY

Aydan, Ö., Shimizu, Y., and Ichikawa, Y. (1989). "The Effective Failure Modes and Stability of Slopes in Rock Mass with Two Discontinuity Sets," *Rock Mechanics and Rock Engineering*, Vol. 22, pp. 163-188.

Aydan, Ö., and Kawamoto, T. (1992). "The Stability of Slopes and Underground Openings Against Flexural Toppling and Their Stabilisation," *Rock Mechanics and Rock Engineering*, Vol. 25, No. 3, pp. 143-165.

Al-Harthi, A., and Hencher, S. (1992). "Physical and Numerical Modelling of Underground Excavations in Dilational Rock Masses." *Fractured and Jointed Rock Masses Conference*, Lake Tahoe, California, June 3 - June 5, pp. 608 - 615.

Beck, B. F., and Chen, J. (1989). "Qualitative modelling of the cover-collapse process." *Proceedings of the Third Multidisciplinary Conference on Sinkholes and the Engineering and Environmental Impacts of Karst*, St. Petersburg Beach, Florida, October 2 - October 4, pp. 89 - 95.

Bray, J. D., Seed, R. B., and Seed, H. B. (1993). "1 g Small-Scale Modelling of Saturated Cohesive Soils," *Geotechnical Testing Journal*, GTJODJ, Vol. 16, No. 1, March, pp. 46-53.

Bray, J. W. and Goodman, R. E. (1981). "The Theory of Base Friction Models," *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, Vol. 18, No. 6, December, pp. 453-468.

Egger, P. (1979). "A New Development in the Base-friction Technique," *International Colloquium on Physical Geomechanical Models*, Bergamo, Italy, March 29 - March 30, pp. 67-81.

Erguvanli, K. A. and Goodman, R. E. (1972). "Applications of Models to Engineering Geology for Rock Excavations," *Bulletin of the Association of Engineering Geologists* Vol. IX, No. 2, pp. 89-103.

Esaki, T. (1992). Personal communication to Dr. Matthew Mauldon.

Fumagalli, E. (1968). "Model Simulation of Rock Mechanics Problems, in *Rock Mechanics in Engineering Practice*." Zienkiewicz and Stagg, ed., J. Wiley & Sons, New York.

Goodman, R. E. (1994). E-mail correspondence through Dr. Matthew Mauldon

Goodman, R. E. (1976). *Methods of Geological Engineering in Discontinuous Rocks*, West Publishing Company pp. 277-299.

Goodman, R. E. (1989). *Introduction to Rock Mechanics*, Second Edition, John Wiley & Sons pp. 61, 236.

Hoek, E. (1971). "Rock Engineering," Inaugural lecture, Imperial College, London.

Hubbert, M. K. (1937). "Theory of Scale Models as Applied to the Study of Geologic Structures." *Bulletin of the Geological Society of America*, Vol. 48, pp. 1459 - 1520.

Kameda, N., Esaki, T., Nishida, T., and Kimura, T. (1987). "Prediction of the behaviour of roof cavities in discontinuous bedded rock using the new base friction technique," *28th US Symposium of Rock Mechanics*, Tucson, Arizona, June 29- July 1, pp. 789-796.

Karaca, M. (1994). Mail correspondence and communication through Dr. Matthew Mauldon.

Karaca, M. (1992). "The Effects of Discontinuities on the Stability of Shallow Tunnels in Weak Rock," *Fractured and Jointed Rock Masses Conference*, Lake Tahoe, California, June 3 - June 5, pp. 759 - 764.

Ogden Environmental and Energy Services (1983). *Stability Assessment Southern Highwall Union Valley Road Quarry Oak Ridge, Tennessee*, Stability Analysis Report for Rogers Group, Inc., October 1993.

Shimada, S. (1992). "Observation of Roof Fall Mechanism Modelling Jointed Rocks by Modified Base Friction Method (MBFM)," *Journal of the Mining and Materials Processing Institute of Japan*, Vol. 108, No. 6, pp. 449-454.

Stimpson, B. (1970). "Modelling Materials for Engineering Rock Mechanics," *International Journal of Rock Mechanics and Mining Sciences*, Vol. 7, No. 1, January, pp. 77-121.

Stimpson, B. (1979). "A New Approach to Simulating Rock Joints in Physical Models," *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, Vol. 16, No. 3, June, pp. 215-216.

Stimpson, B. (1979). "A Simple Physical Modelling Technique for the Demonstration of Interaction between Underground Openings," *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, Vol. 16, No. 3, June, pp. 217-219.

APPENDICES

Appendix A
Photographs

Appendix A-1: Slope Sequence

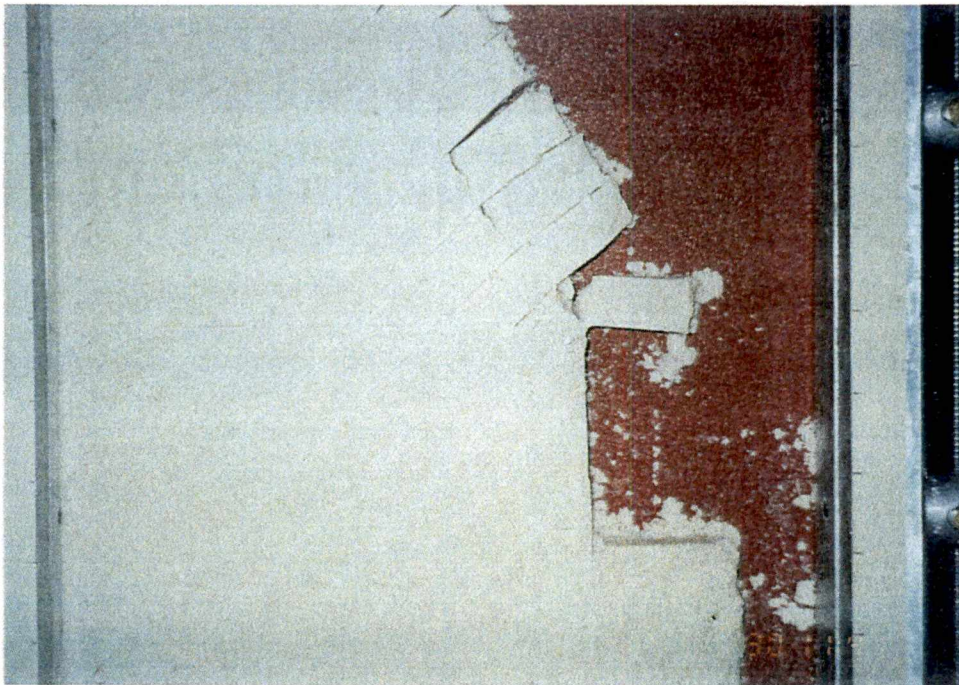
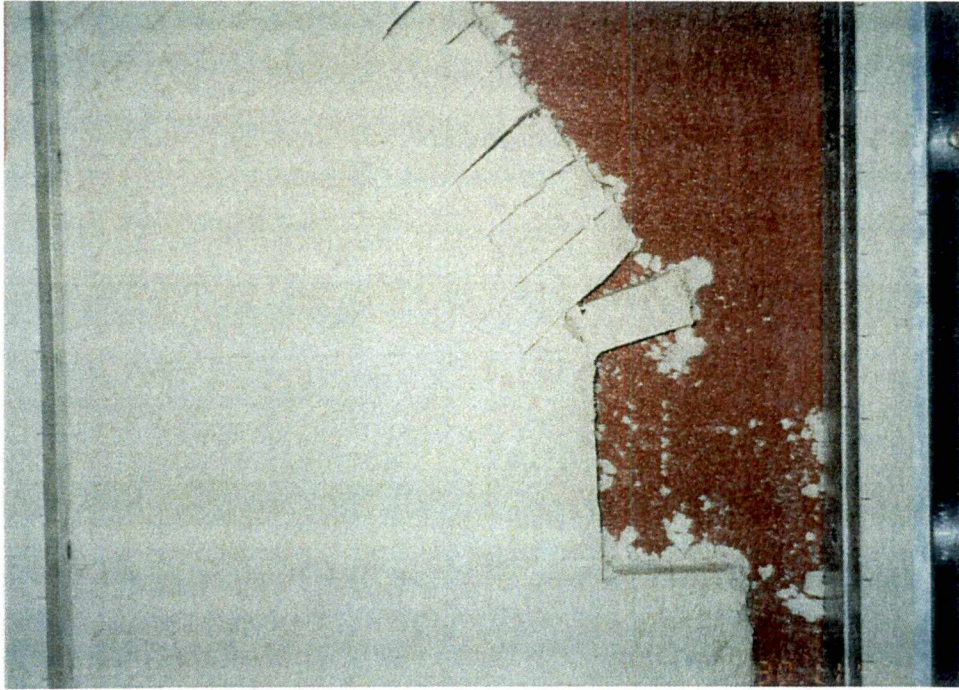


Plate 1: (continued): Stage 3 shows third and fourth layers starting to breaking away. Stage 4 shows the failure plane beginning to form at the same angle as the slope.

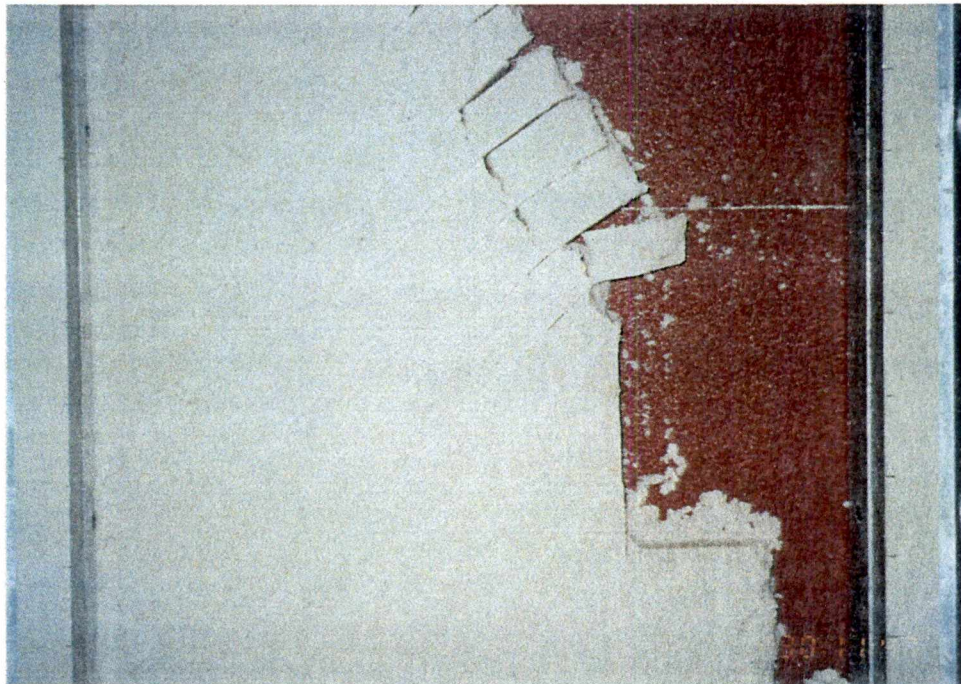
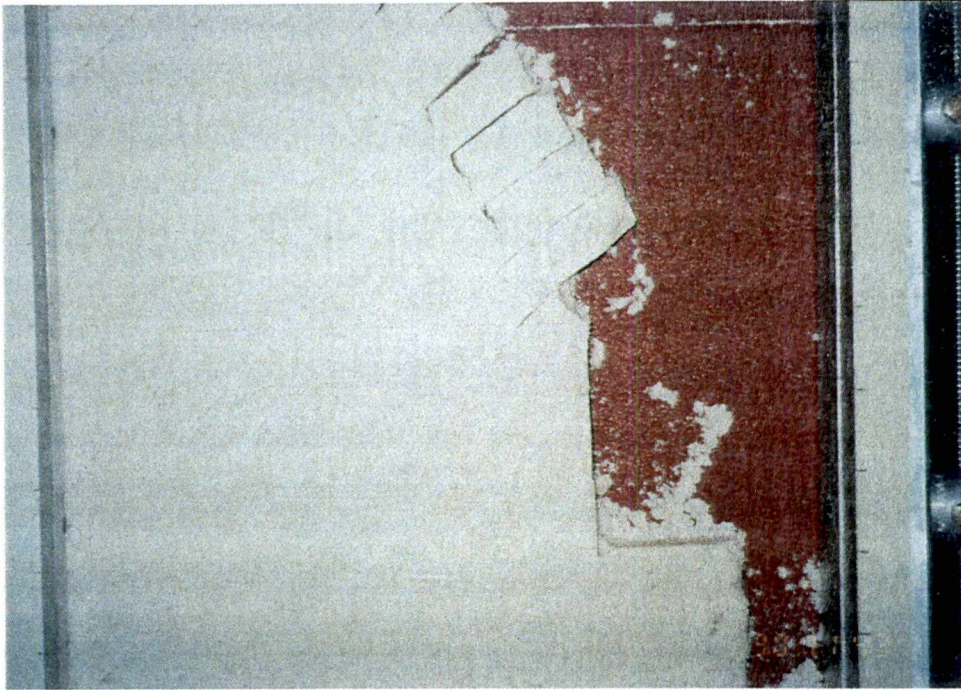


Plate 1: (continued): Stage 5 continues to show formation of the failure plane. Stage 6 shows that the slope has reached equilibrium and only the bottom five layers appear to be toppling.

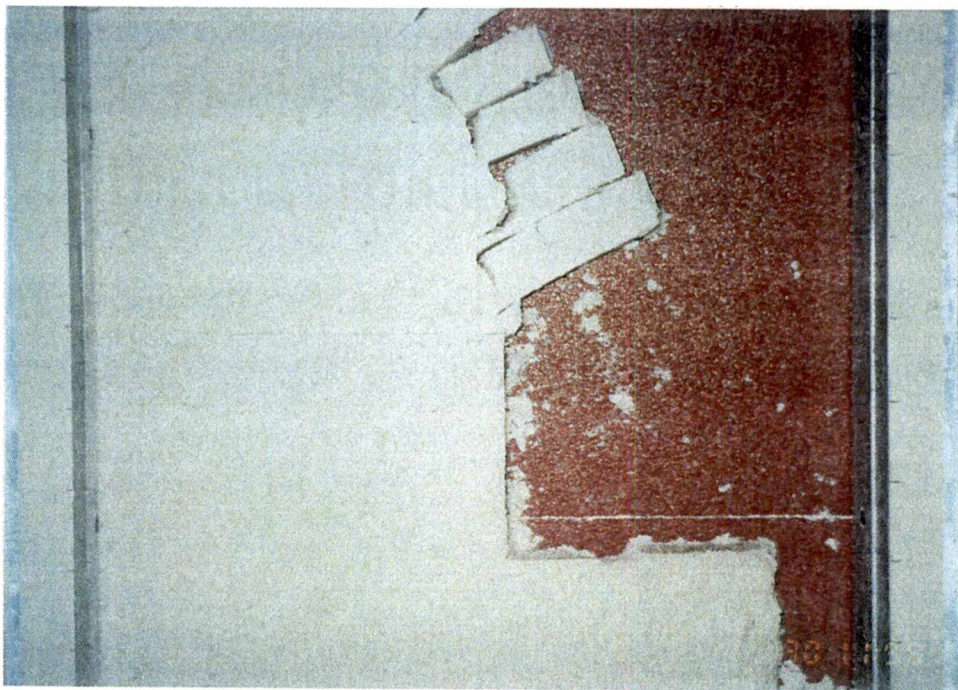
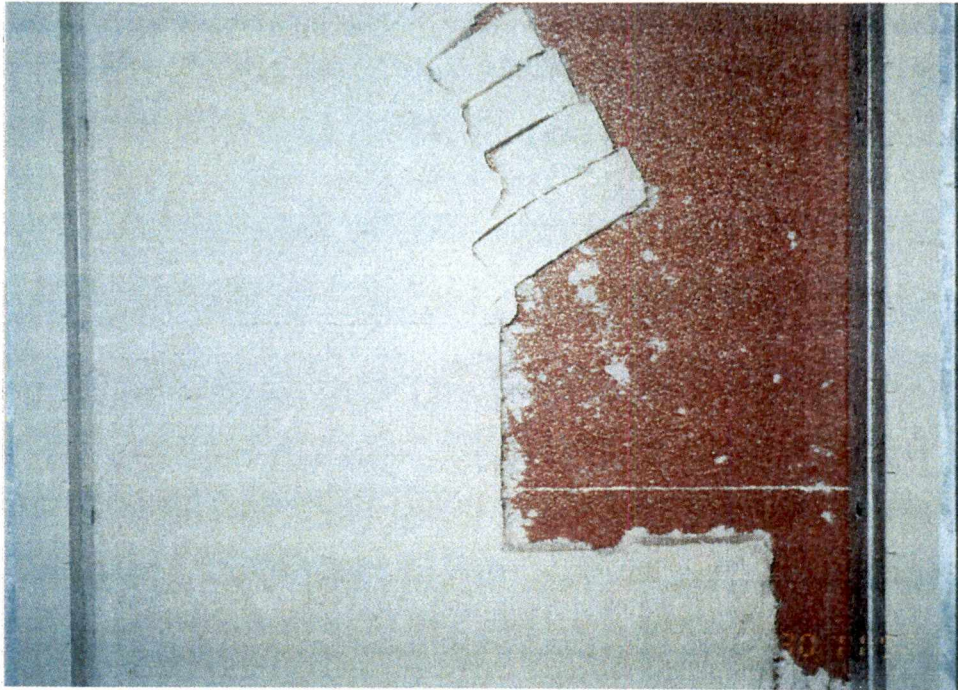


Plate 2: Slope: 9 cm base. Same model as plate 1 except with wider base. Stage 1 shows the bottom layer breaking at the face of the cutout and the other layers toppling at the preformed failure plane. Stage 2 shows all of four bottom layers toppling.

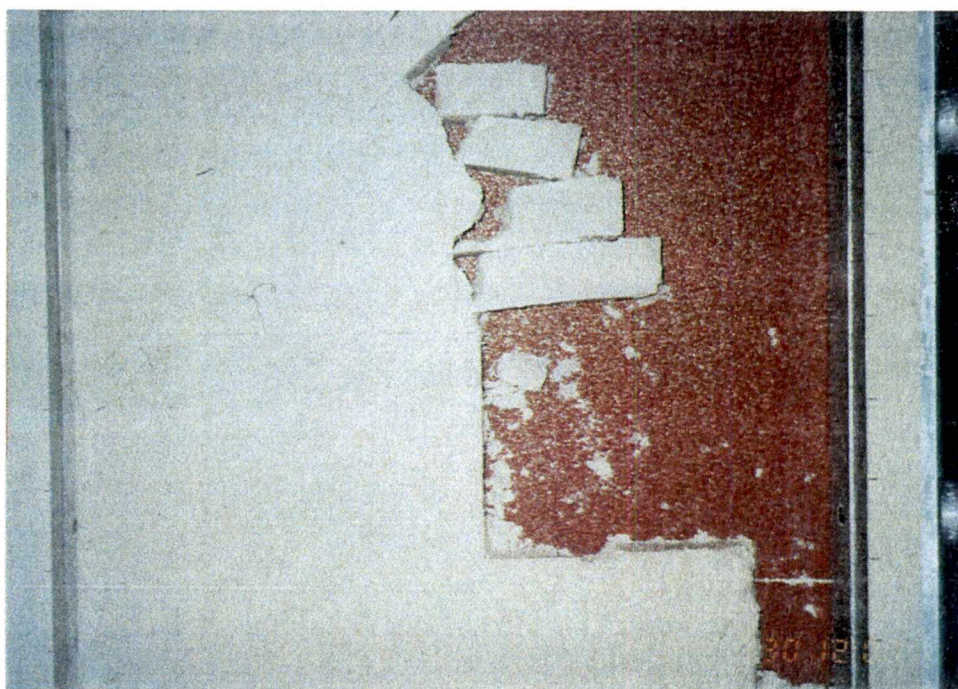
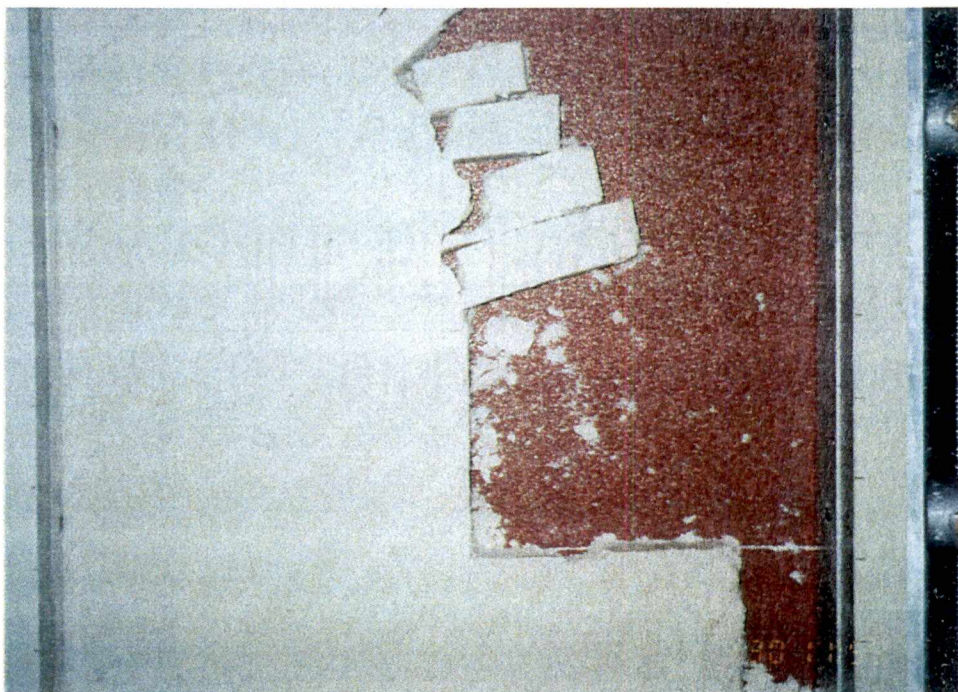


Plate 2: (continued): Stage 3 shows the same layers toppling as in stage 6 of plate 1. Stage 4 shows the slope in equilibrium. Notice that the failure surface is not like plate 1. The bottom layer broke deeper into the slope than it did in plate 1.

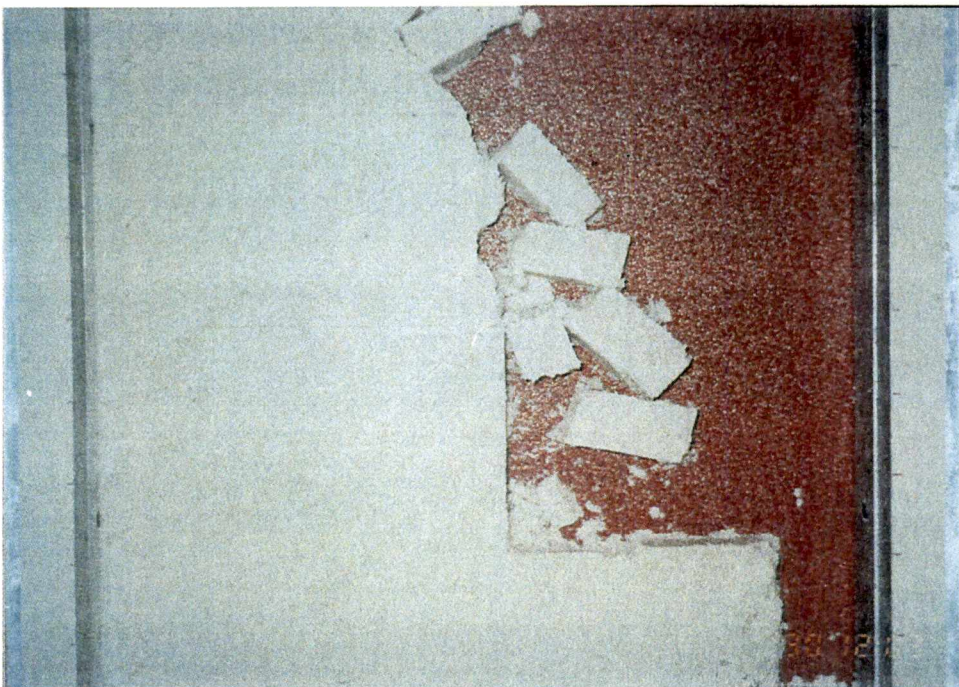
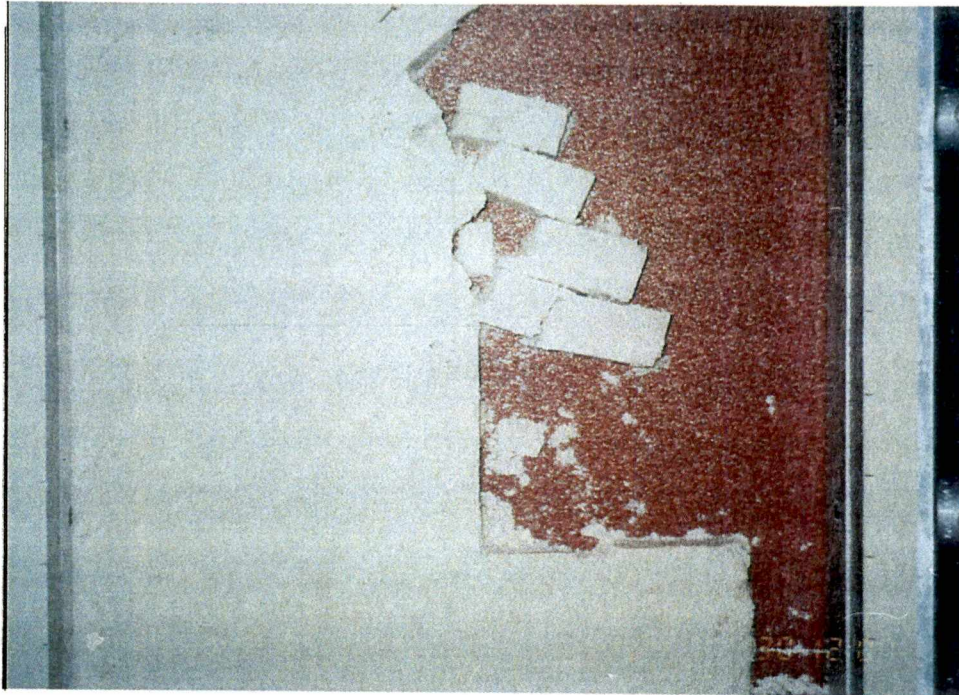


Plate 2: (continued): Stage 5 shows the bottom four layers are the only ones that toppled. Stage 6 shows the end of the test. The failure surface in this series is much steeper than in plate 1.

Appendix A-2: Tunnel Sequences

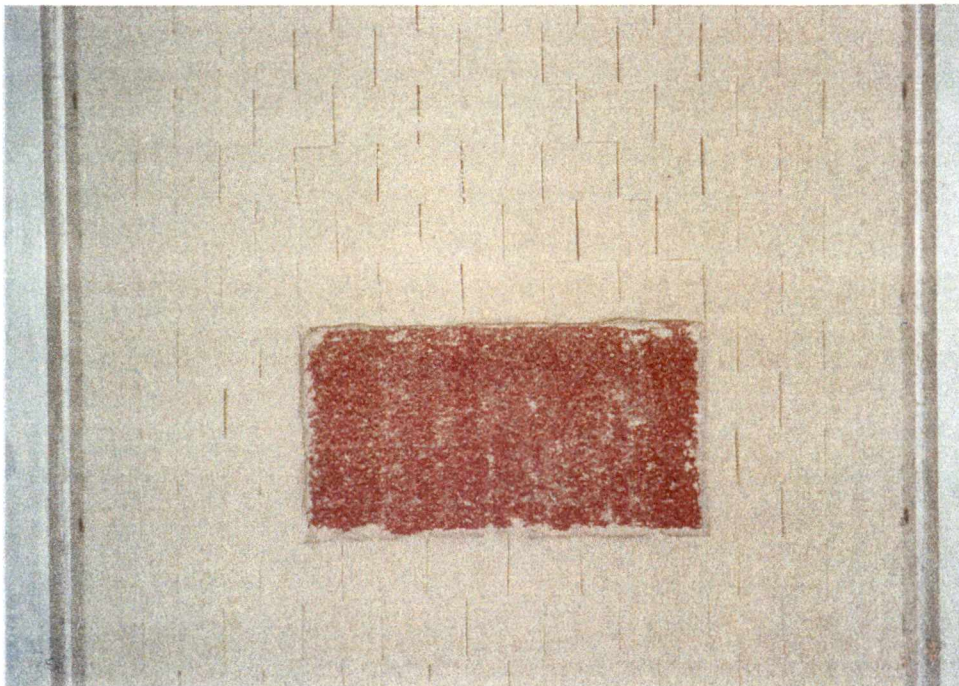
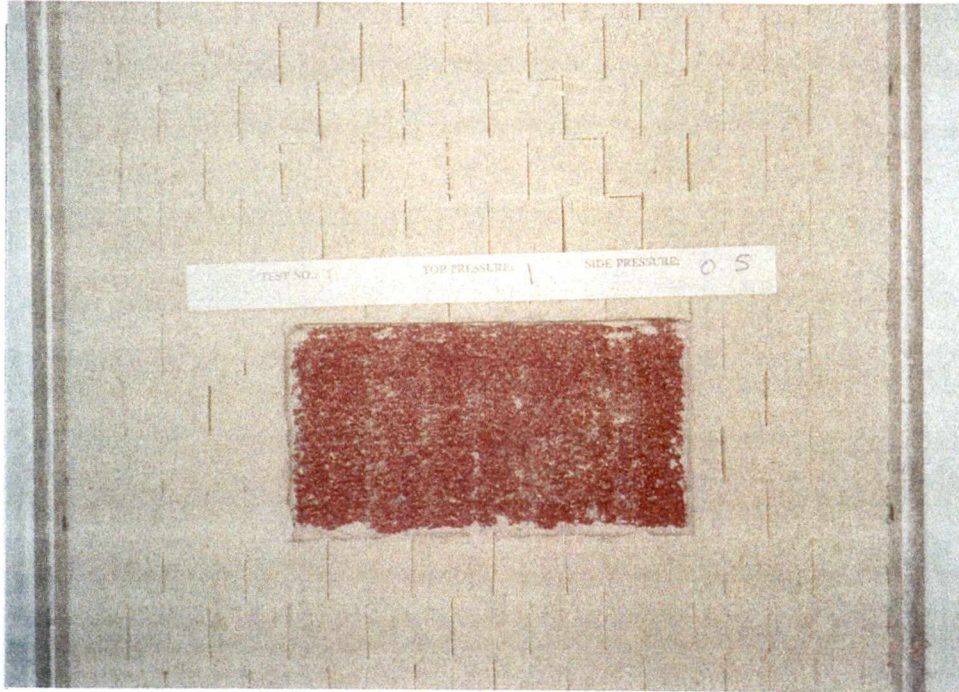


Plate 3: Tunnel Test 1: $K = 0.5$ and $\sigma_v = 0.2$ psi (1.4 kPa). Stage 1 of 6 shows the model with the test information. Stage 2 of 6 is a picture just as the test is starting.

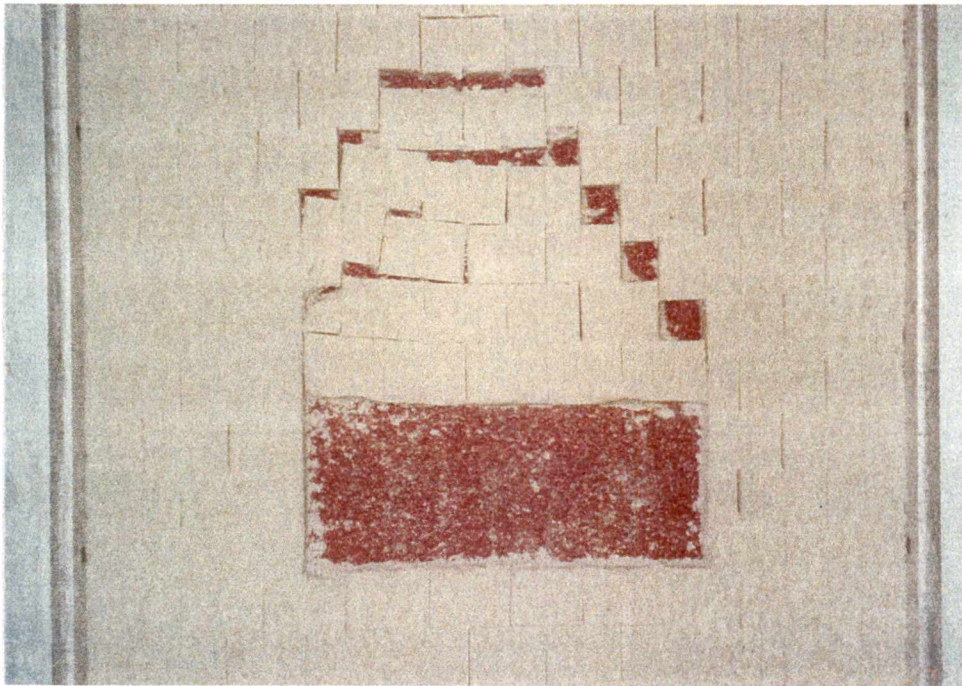
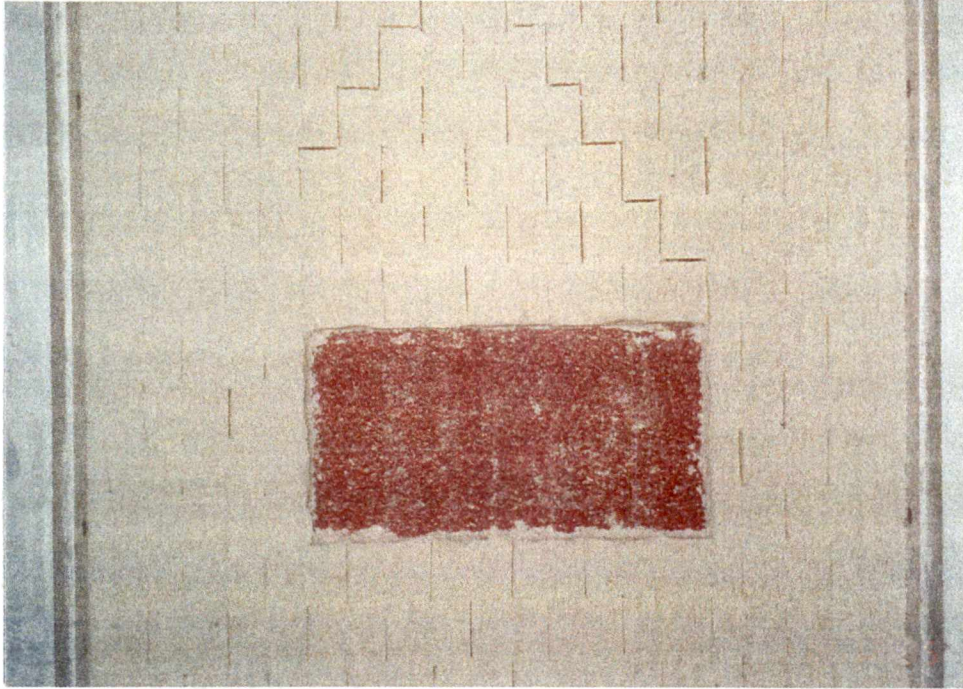


Plate 3: Tunnel Test 1 (continued): Stage 3 of 6 shows the start of roof collapse with the stairstep failure pattern. Stage 4 of 6 shows the majority of the roof dropping out.

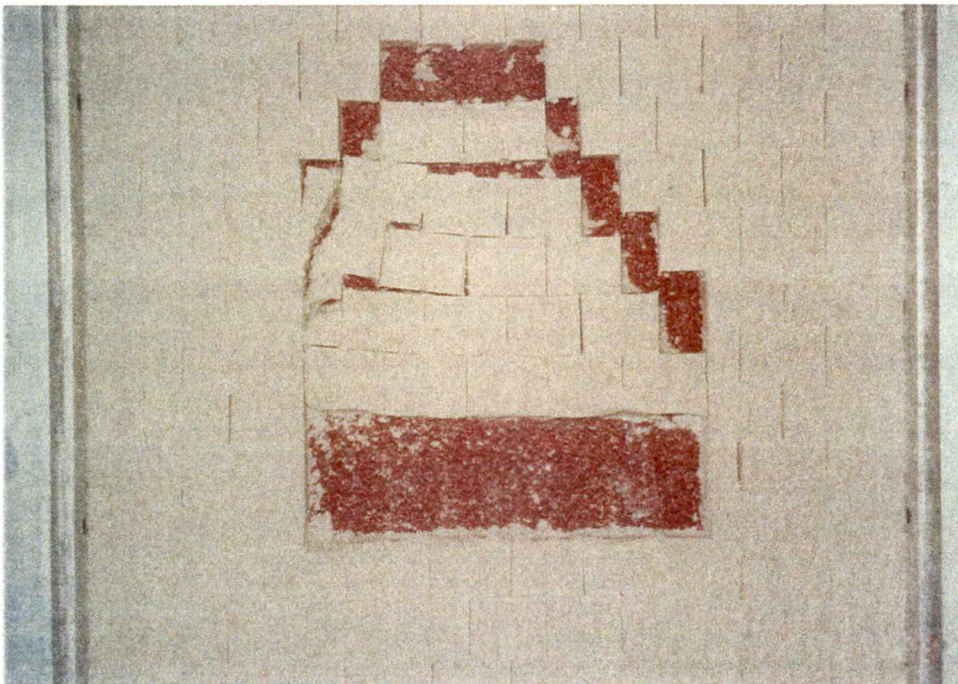
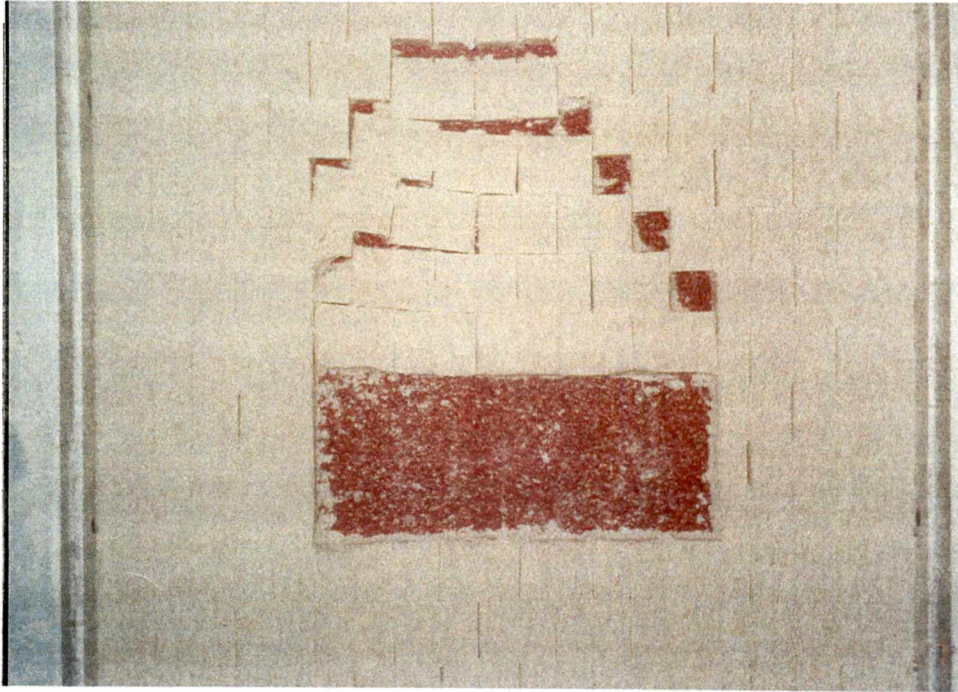


Plate 3: Tunnel Test 1 (continued): Stage 5 of 6 continues to show the roof collapsing. Stage 6 of 6 shows the that the tunnel is back at equilibrium and only 18 blocks fell in this test series.

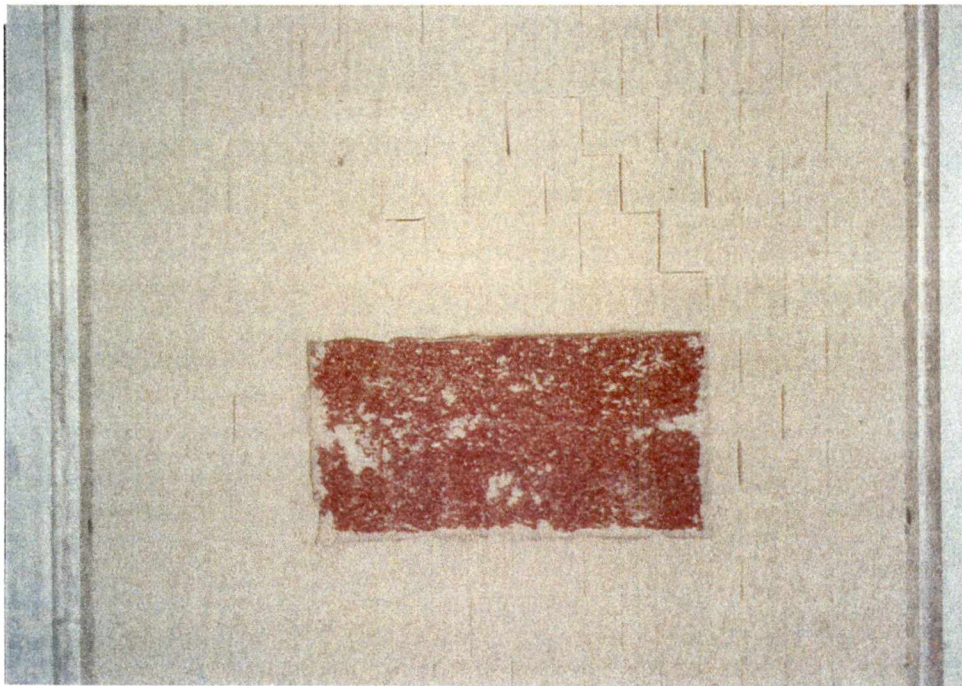
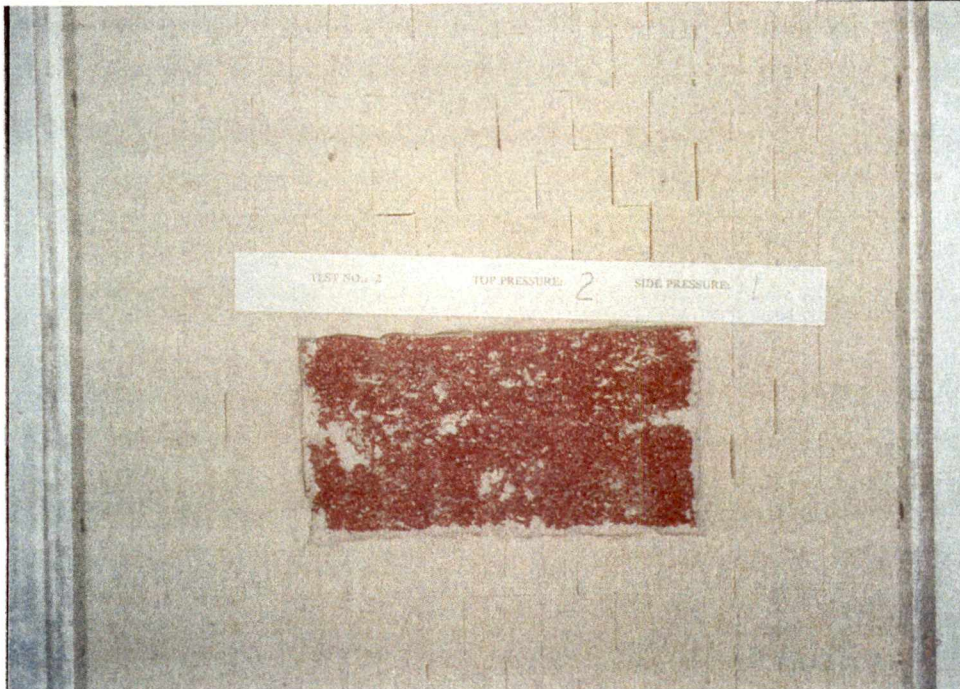


Plate 3: Tunnel Test 2: $K = 0.5$ and $\sigma_v = 0.4$ psi (2.8 kPa). Stage 1 of 5 shows the information for a new test. Stage 2 of 5 is the first picture of the test sequence, before test is started.

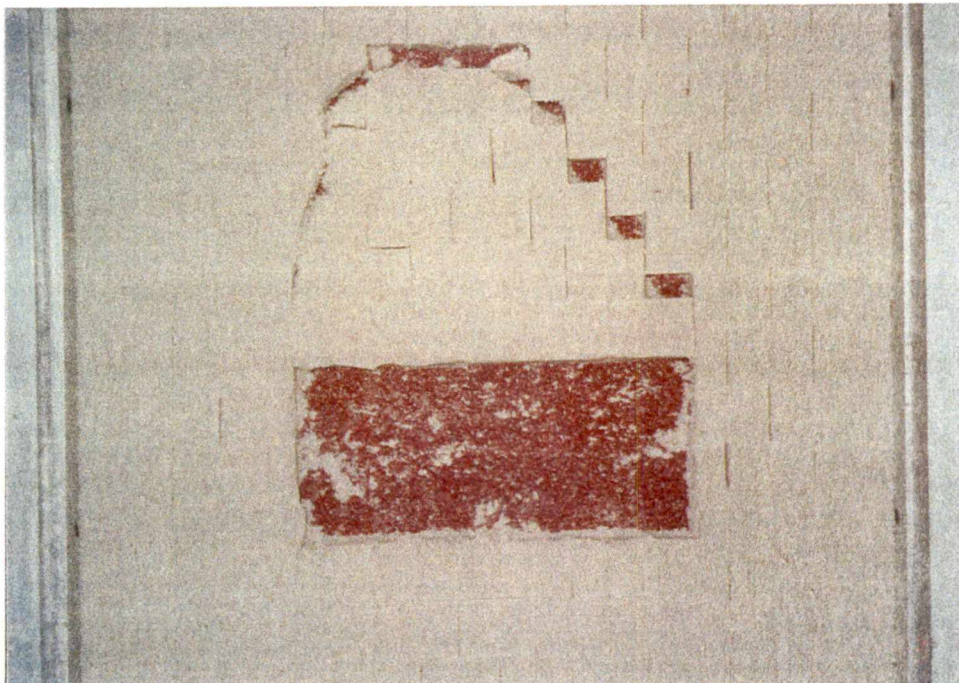
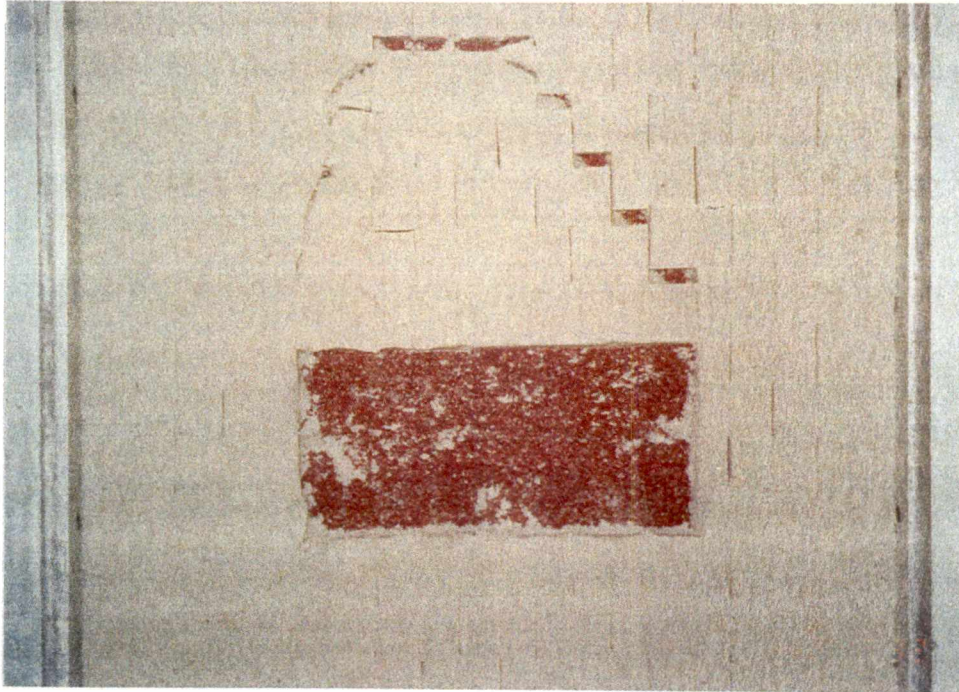


Plate 3: Tunnel Test 2 (continued): Stage 3 of 5 shows that the roof begins to drop out at the start of the test. Stage 4 of 5 shows the same stair step failure as the first test sequence.

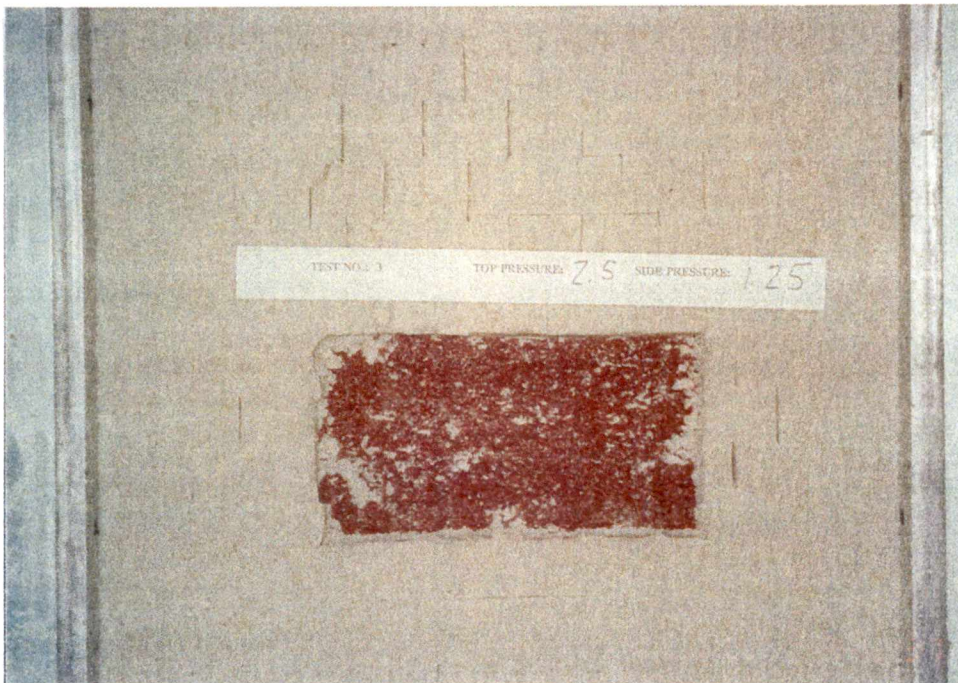
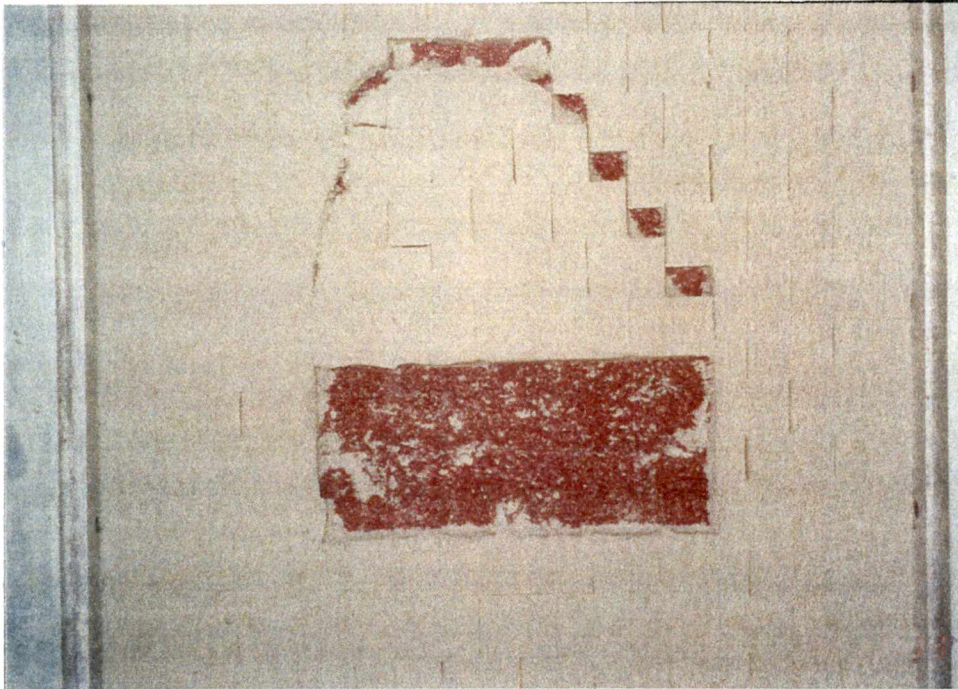


Plate 3: Tunnel Test 2 (continued): Stage 5 of 5 shows the end of the test. The roof is in equilibrium again with only 16 blocks dropping out in this test. Bottom picture is stage 1 of Test 3 where $K = 0.5$ and $\sigma_v = 0.5$ psi (3.5 kPa).

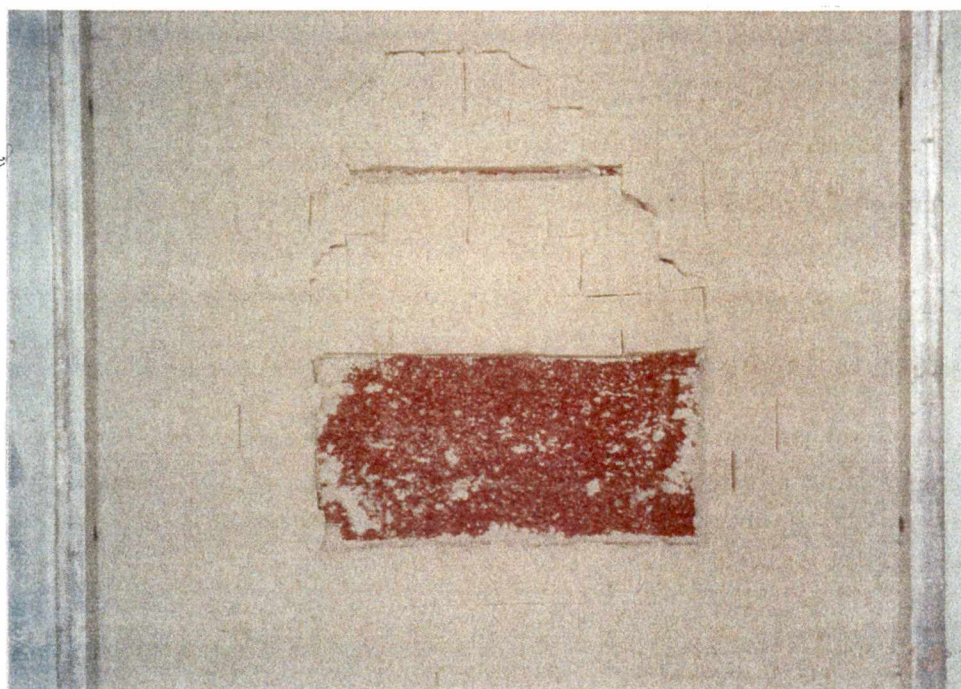
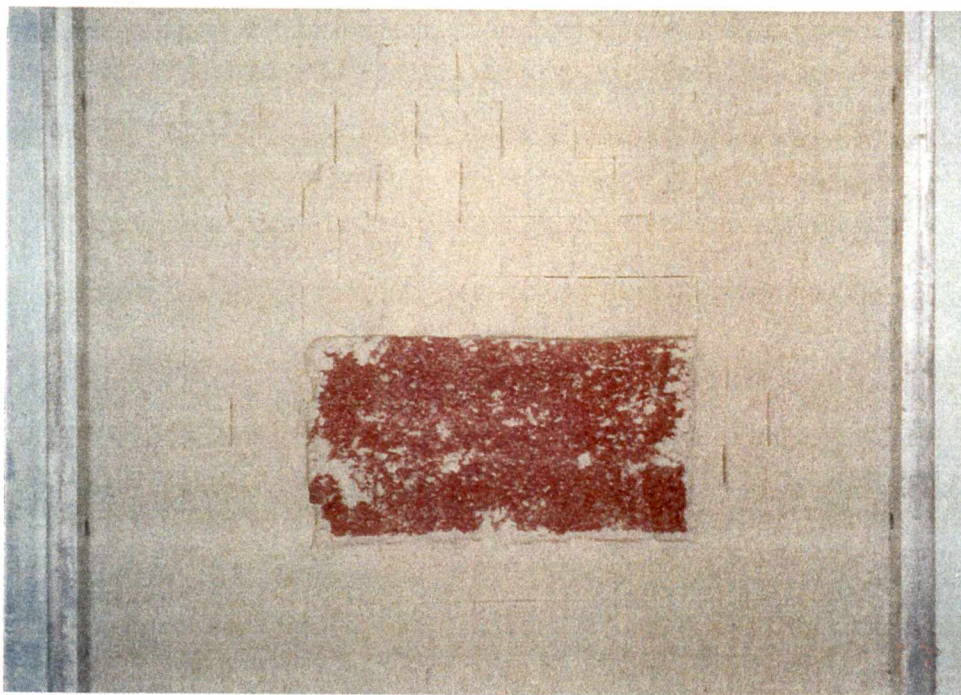


Plate 3: Tunnel Test 3: $K = 0.5$ and $\sigma_v = 0.5$ psi (3.5 kPa) (continued): Stage 2 of 5 is the start of the third test sequence. Stage 3 of 5 shows a different roof failure than in the previous two test. Notice the separation in the layers.

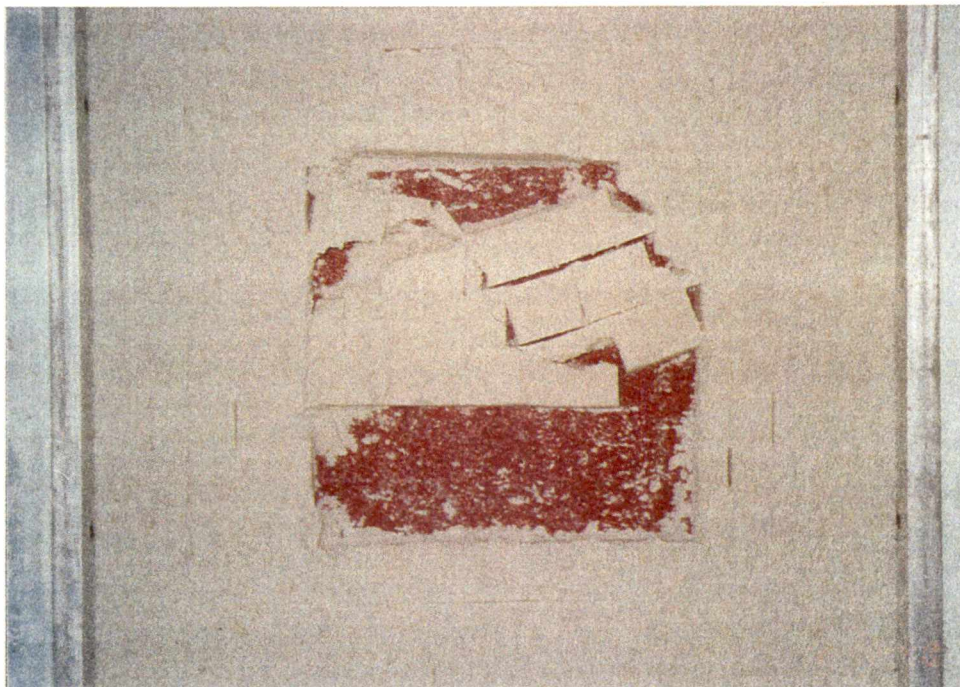
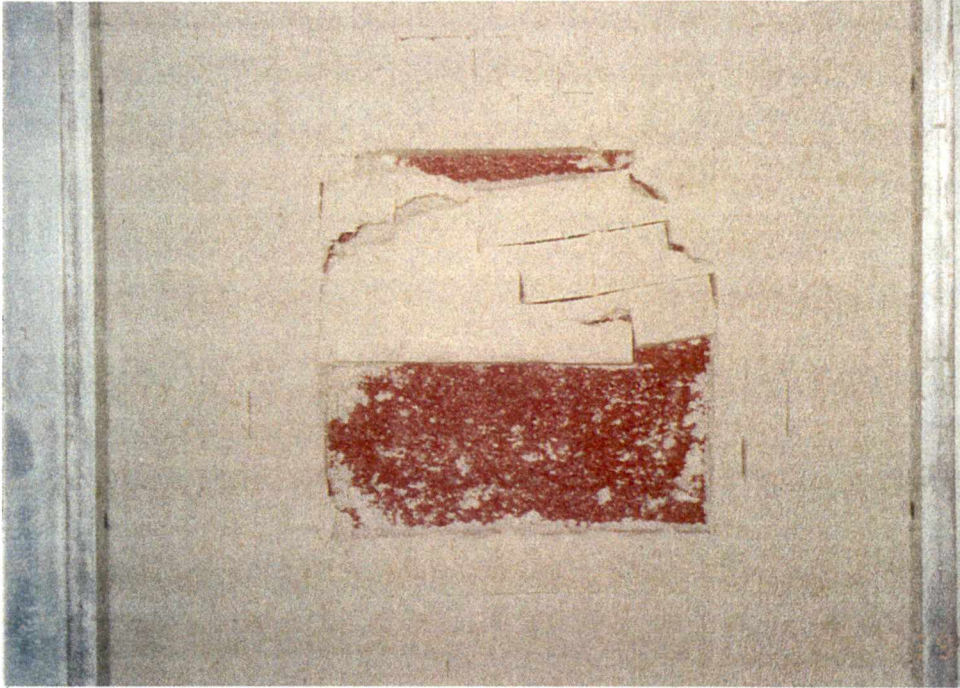


Plate 3: Tunnel Test 3 (continued): Stage 4 of 5 shows the layers starting to fail due to shear and bending. Stage 5 of 5 is the end of the sequence and only 13 blocks fail here.

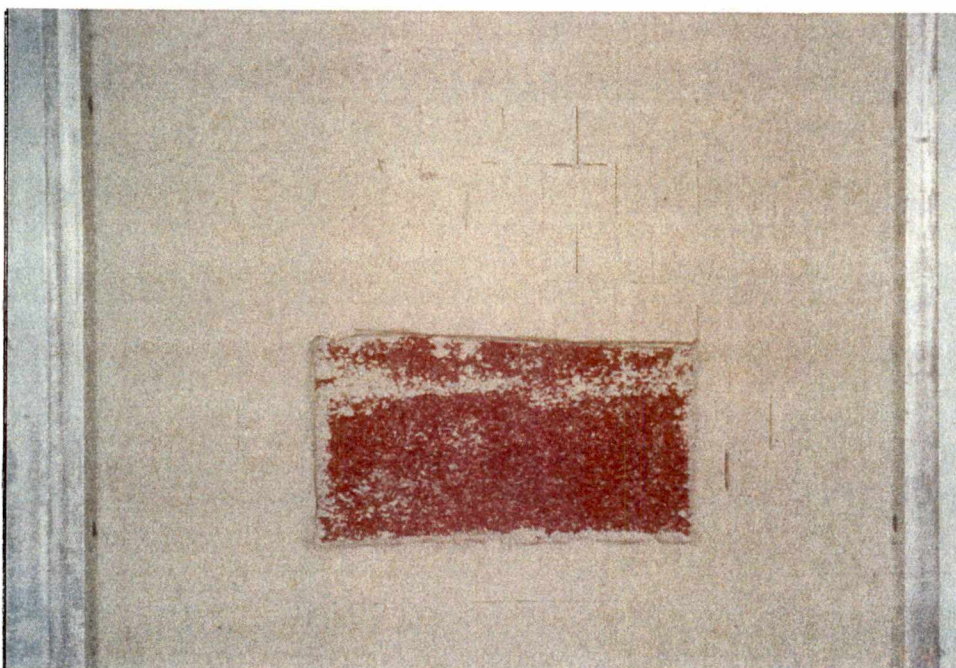
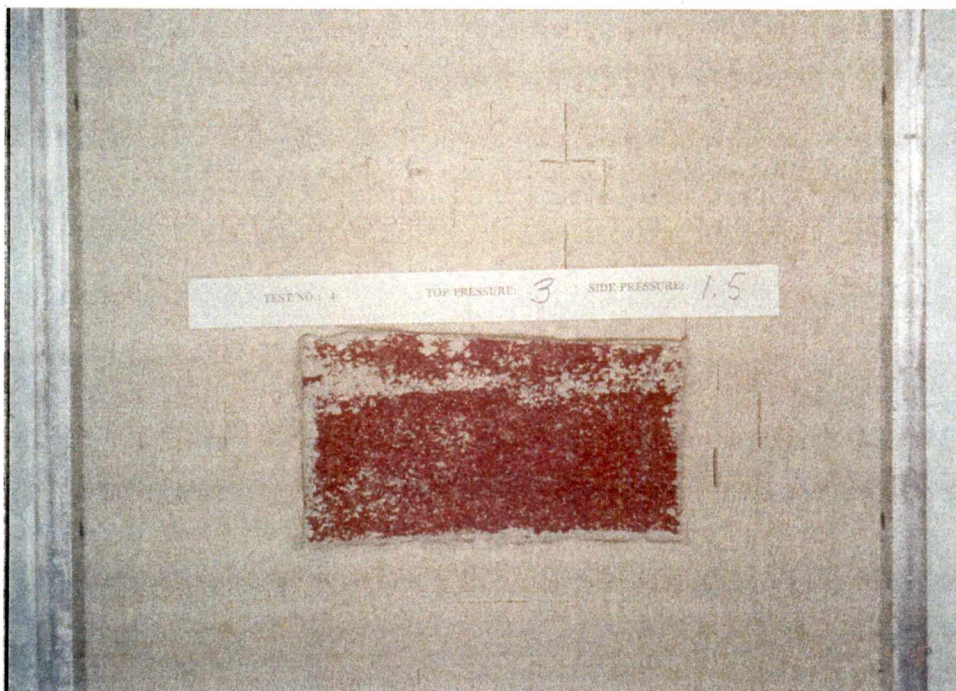


Plate 3: Tunnel Test 4: $K = 0.5$ and $\sigma_v = 0.6$ psi (4.1 kPa). Stage 1 of 4 starts the fourth and final test. Stage 2 of 4 is the first picture of the test sequence.

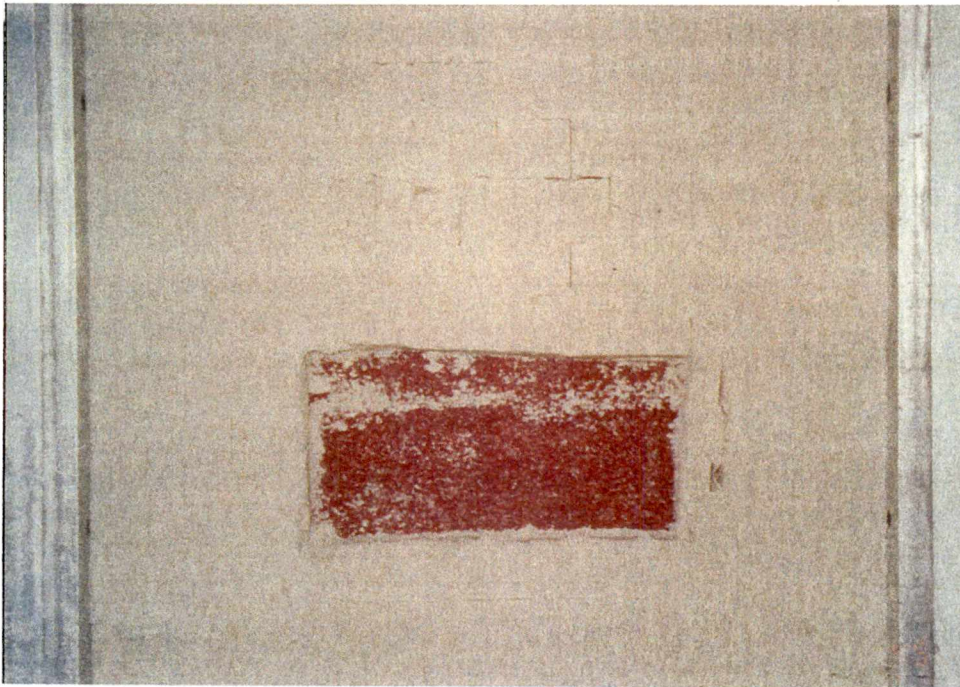
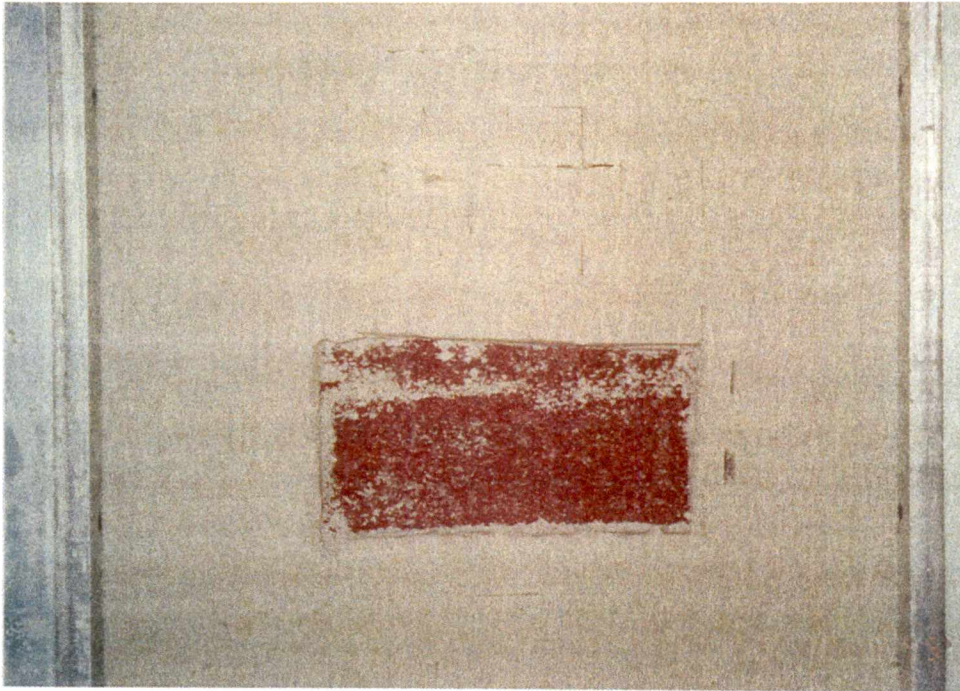


Plate 3: Tunnel Test 4 (continued): Stage 3 of 4 is in the middle of the test and no roof collapse has taken place. Stage 4 of 4 shows a shear failure (thrust fault) of the model in the bottom left of the picture.

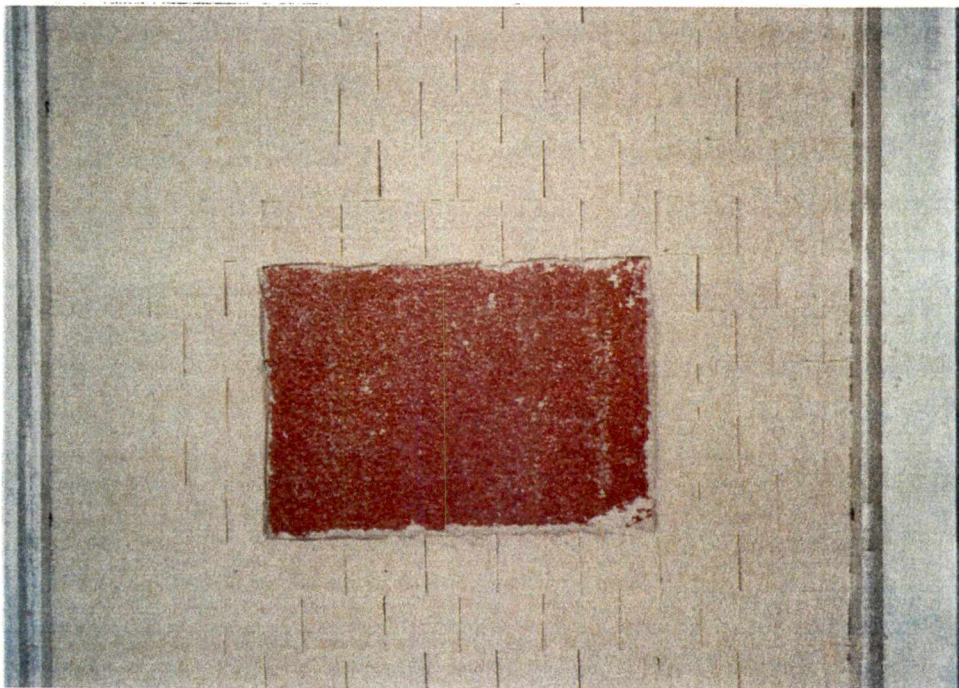
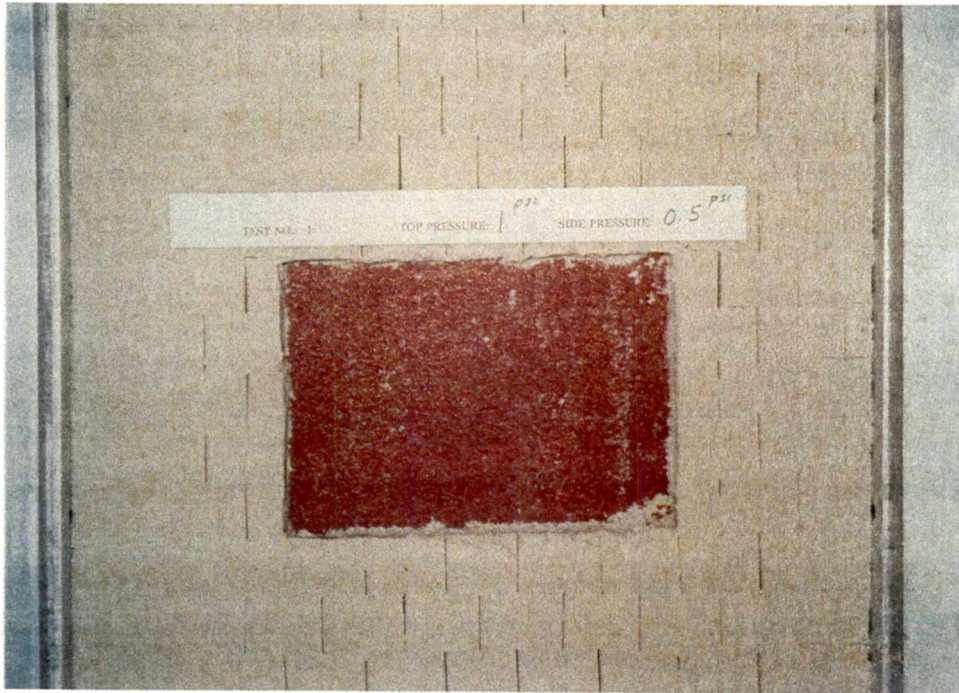


Plate 4: Tunnel Test 1: $K = 0.5$ and σ_v constant at 0.2 psi (1.4 kPa). Stage 1 of 7 is the first sequence with a $K = 0.5$. Stage 2 of 7 is the first picture of the test showing no movement.

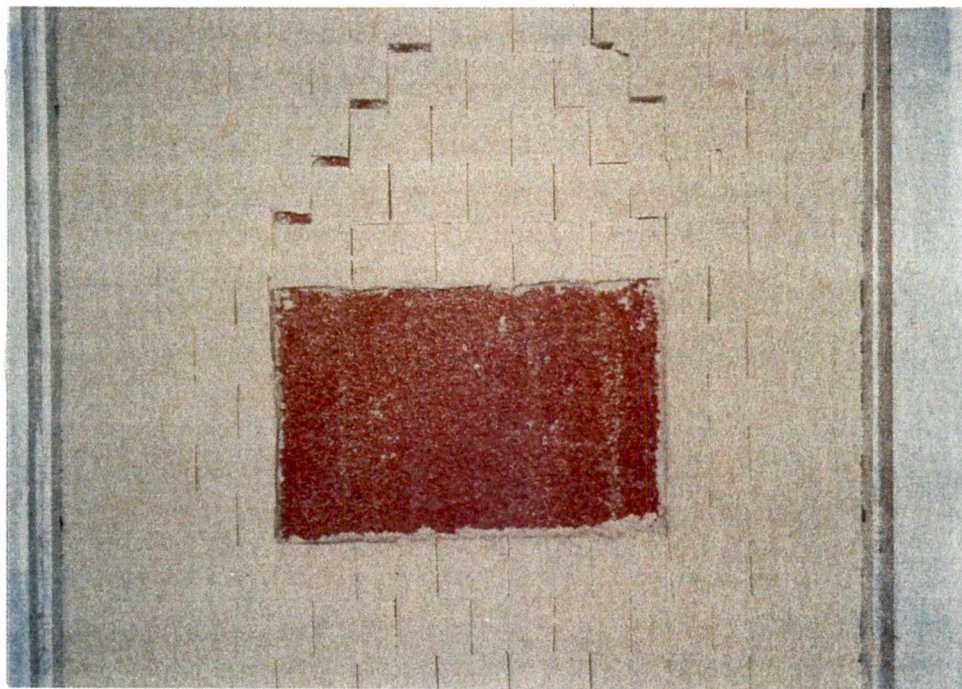
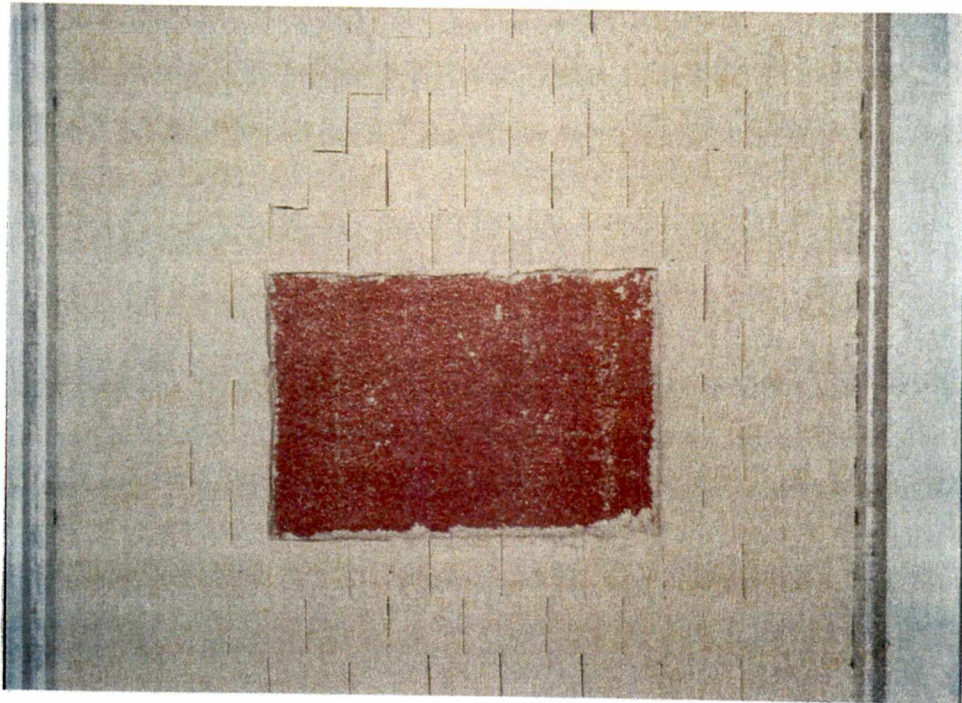


Plate 4: Tunnel Test 1 (continued): Stage 3 of 7 shows the roof blocks starting to separate. Stage 4 of 7 shows a definite roof collapse starting.

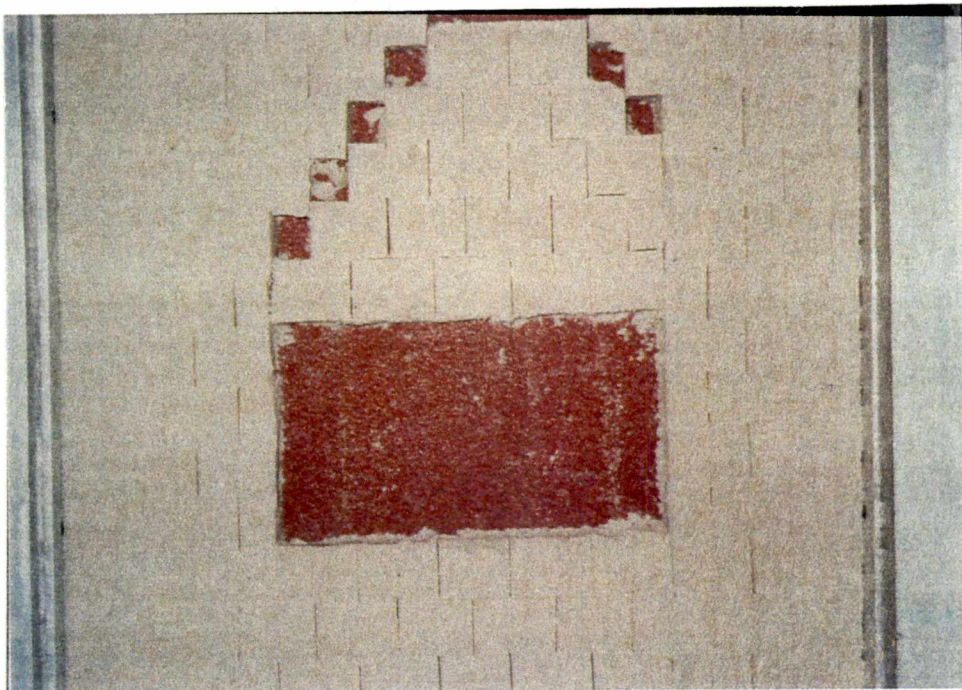
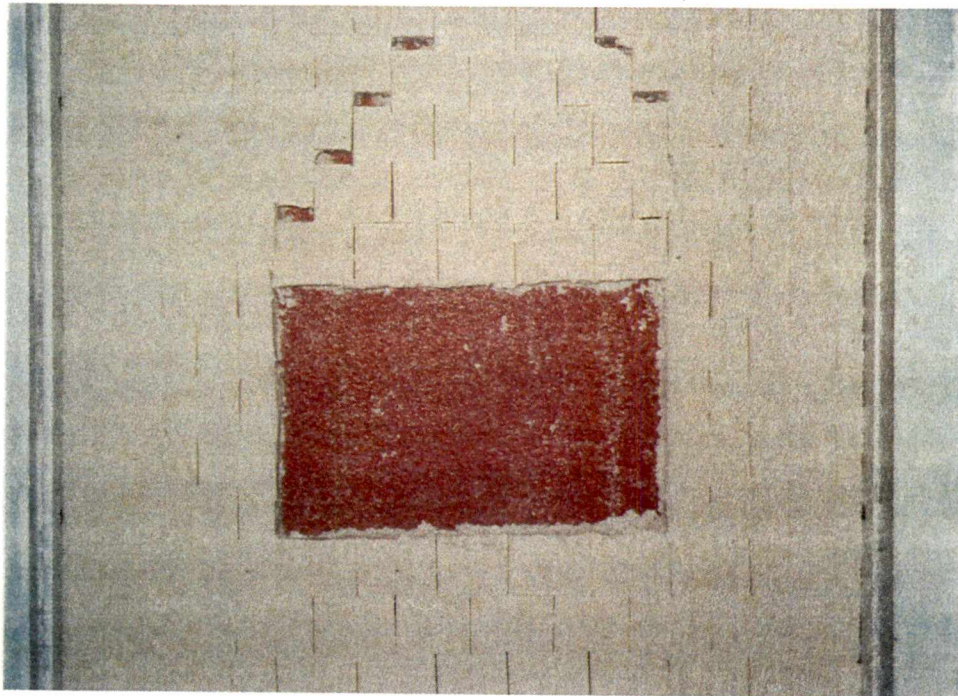


Plate 4: Tunnel Test 1: (continued): Stage 5 of 7 shows the roof blocks starting to slide or drop. Stage 6 of 7 shows the final amount of roof material starting to collapse.

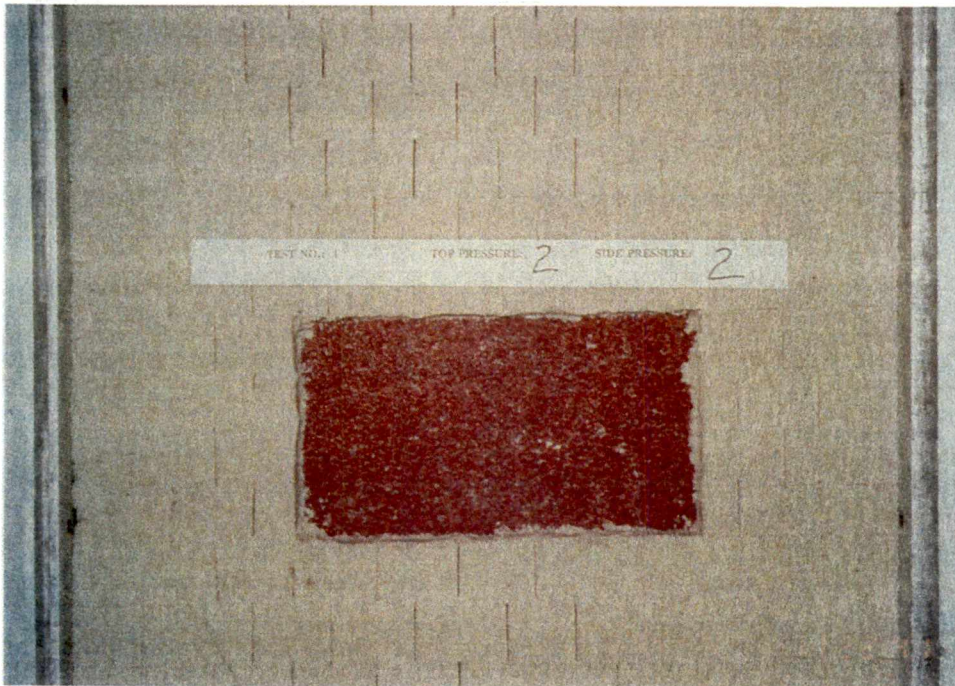
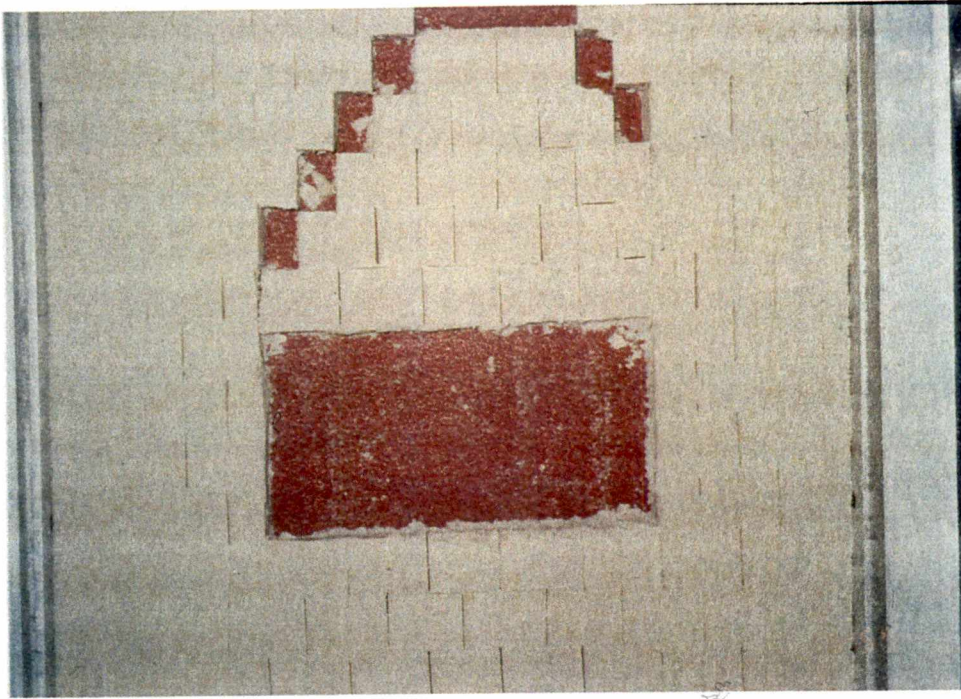


Plate 4: Tunnel Test 1 (continued): Stage 7 of 7 shows the tunnel back in equilibrium and 20 blocks sliding. Bottom picture is Stage 1 of 9 of a new test, with a $K = 1$.

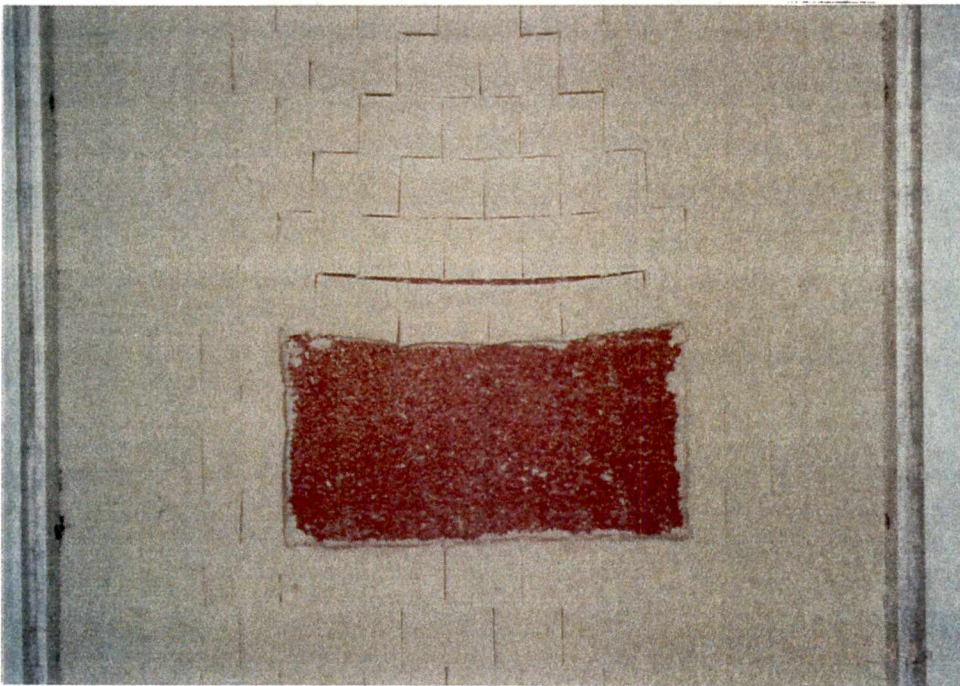
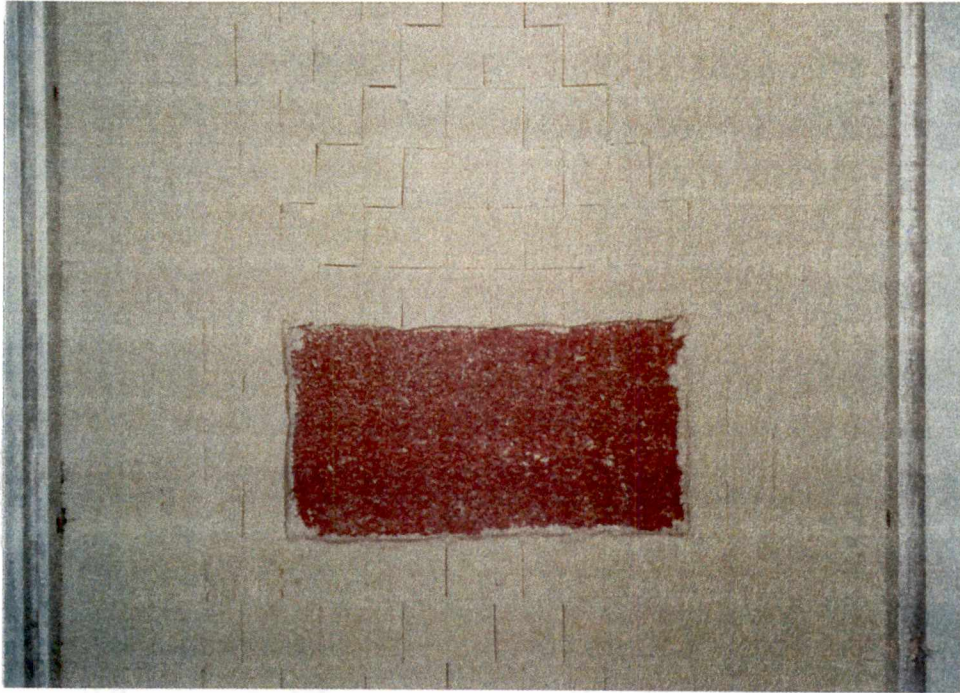


Plate 4: Tunnel Test 2: $K = 1.0$ and σ_v constant at 0.4 psi (2.8 kPa). Stage 2 of 9 shows the same stair step pattern forming. Stage 3 of 9 shows the bottom level starting to break away from the rest of the roof mass.

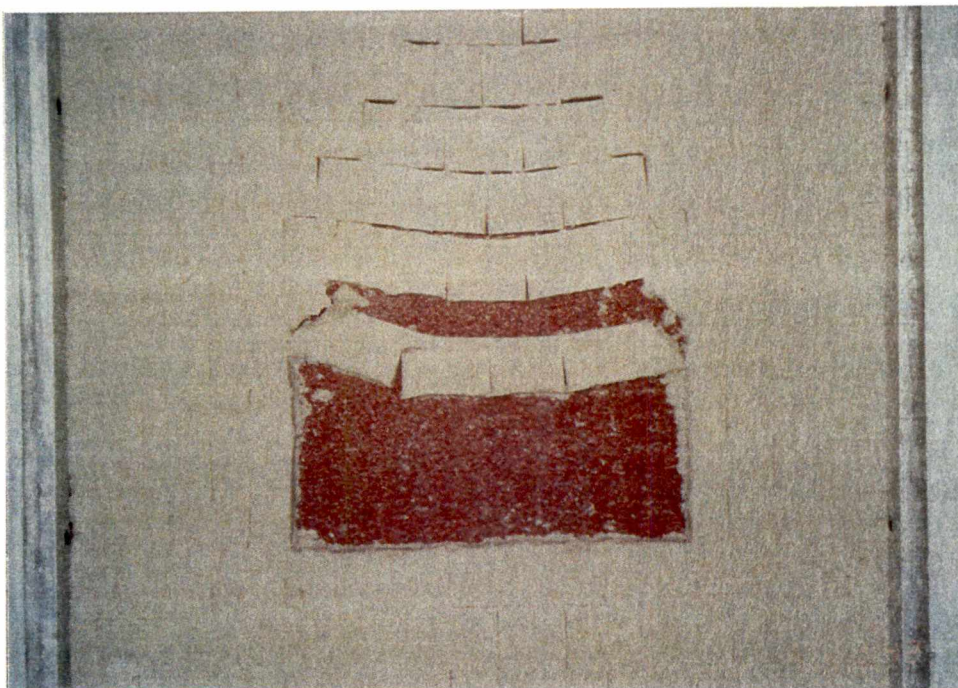
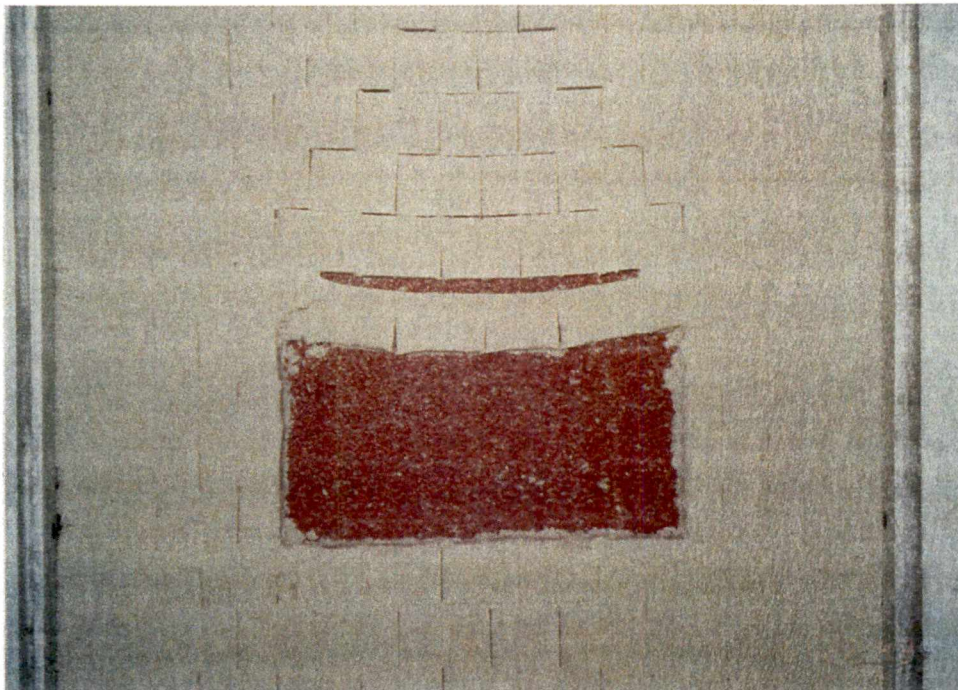


Plate 4: Tunnel Test 2 (continued): Stage 4 of 9 shows the bottom layer starting to bend instead of dropout. Stage 5 of 9 shows the rest of the mass starting to bend instead of slide.

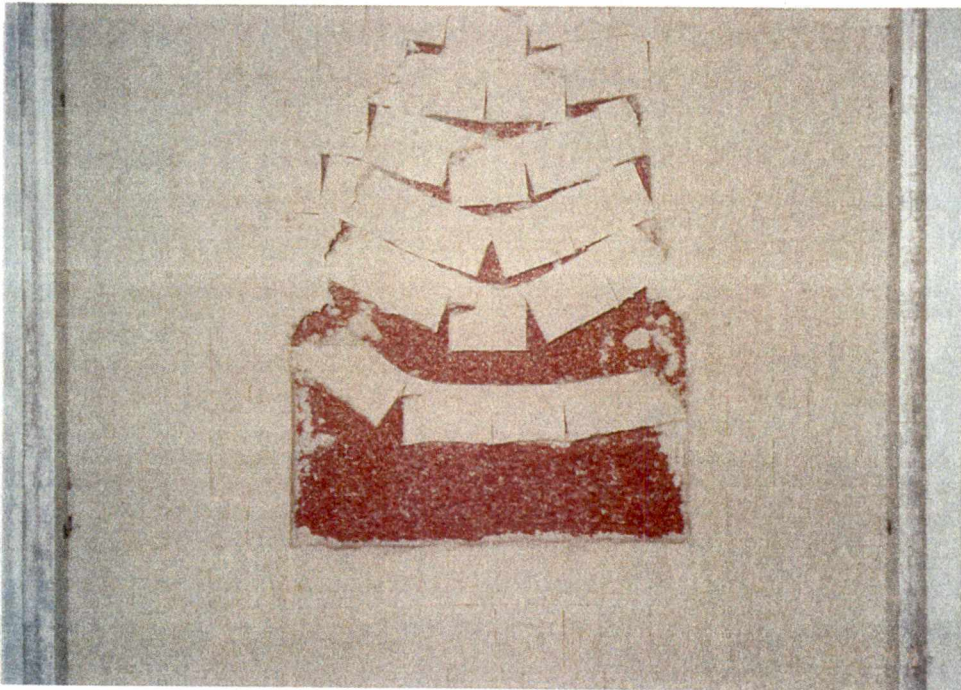
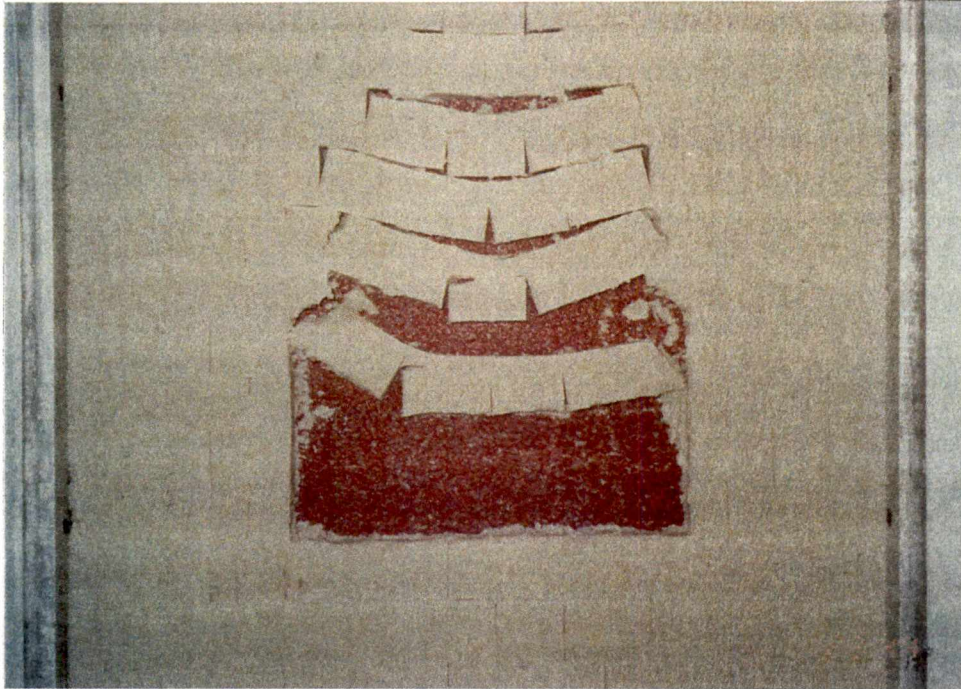


Plate 4: Tunnel Test 2 (continued): Stage 6 of 9 shows the blocks collapsing due to bending failure. Stage 7 of 9 shows that all of the blocks have dropped out due to shear and bending.

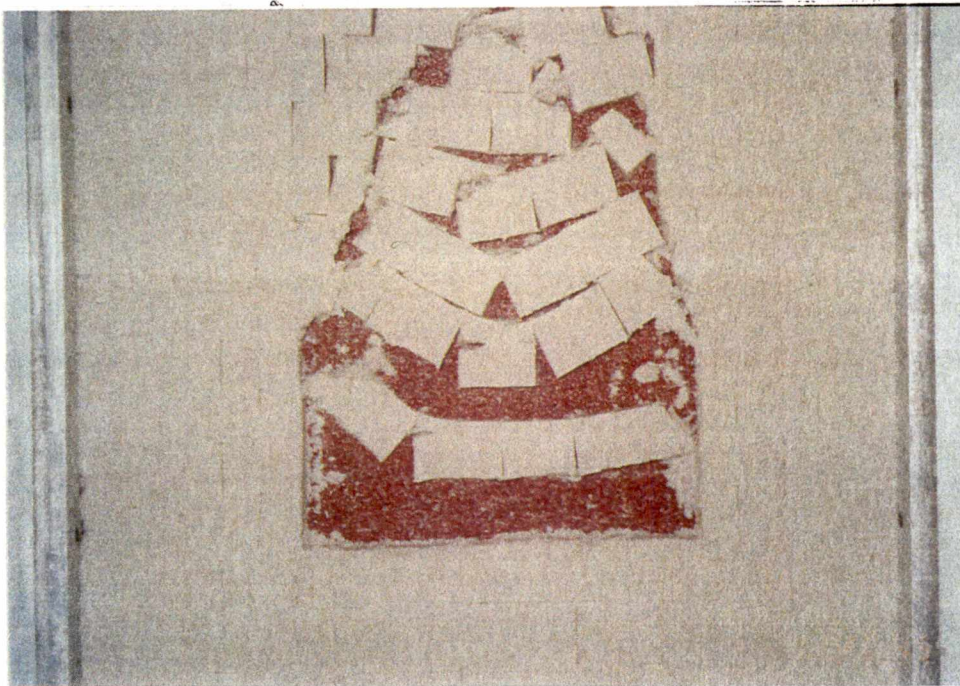
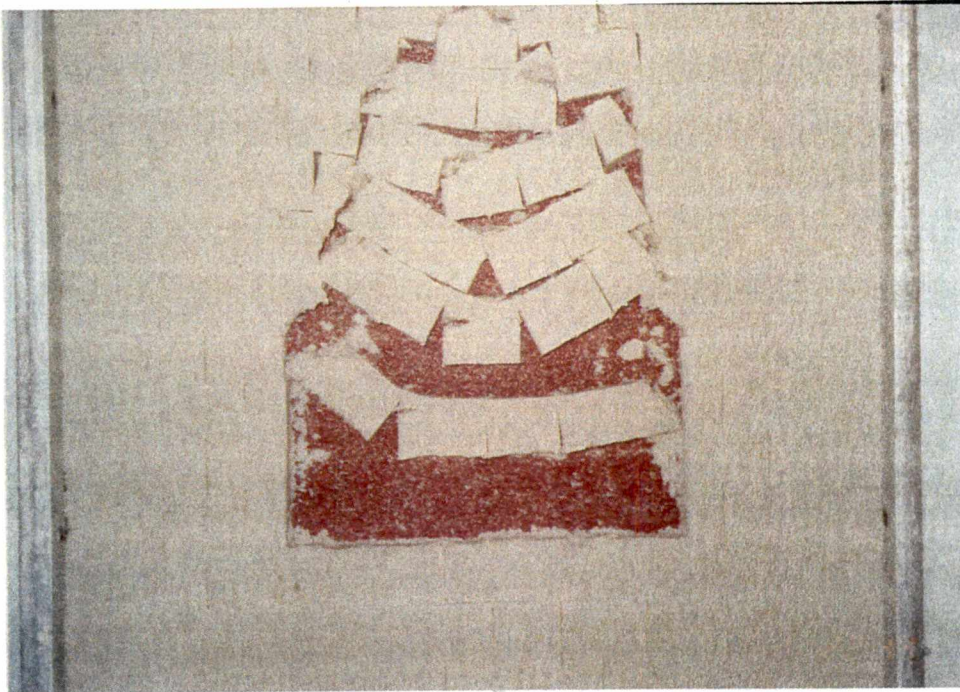


Plate 4: Tunnel Test 2 (continued): Stage 8 of 9 shows that the step like failure pattern is not seen here. Stage 9 of 9 shows the tunnel has reached equilibrium with 20 blocks bending.

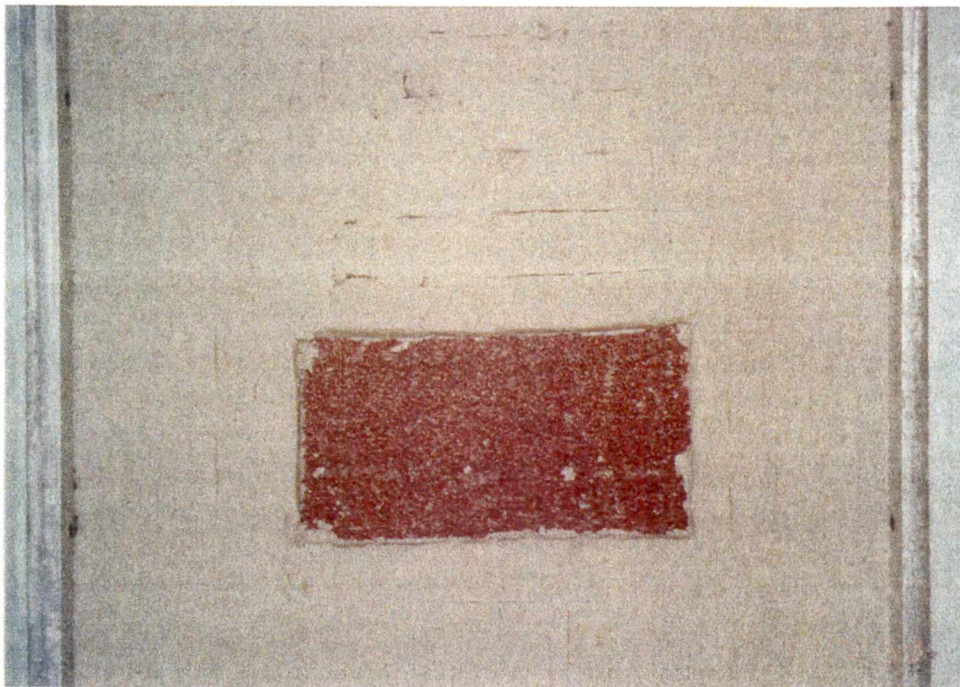
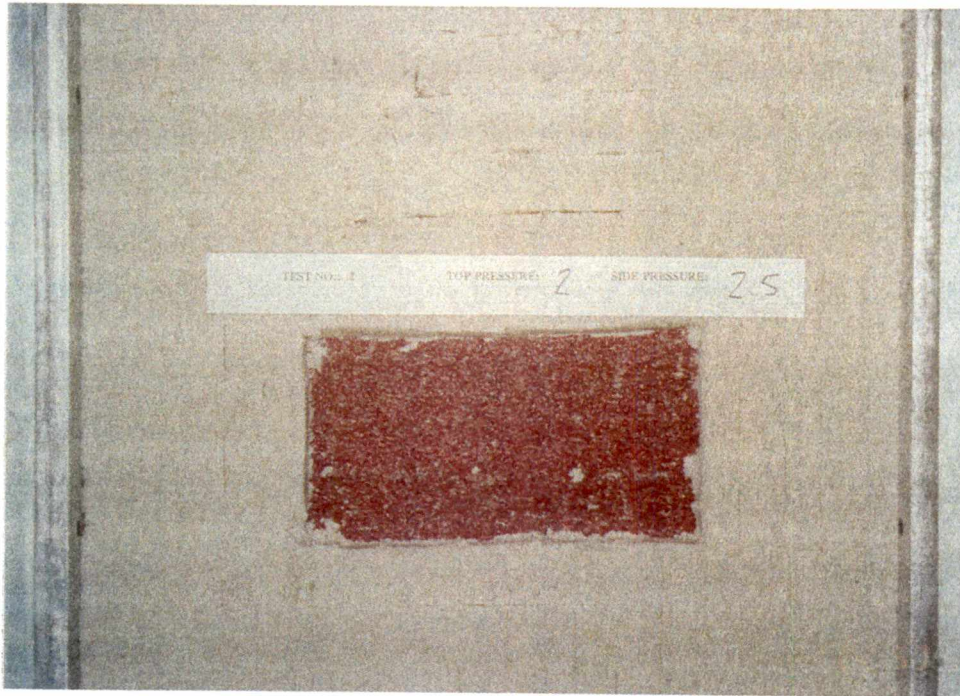


Plate 4: Tunnel Test 3: $K = 1.25$ and σ_v constant at 0.4 psi (2.8 kPa). Stage 1 of 4 is the start of a new test. Stage 2 of 4 is the start of the test.

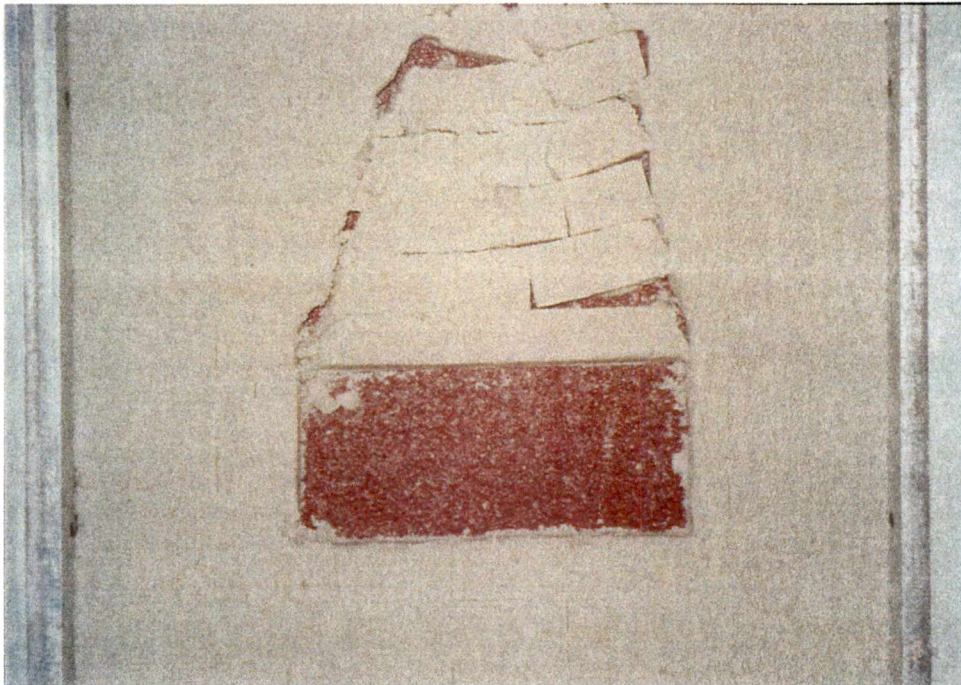
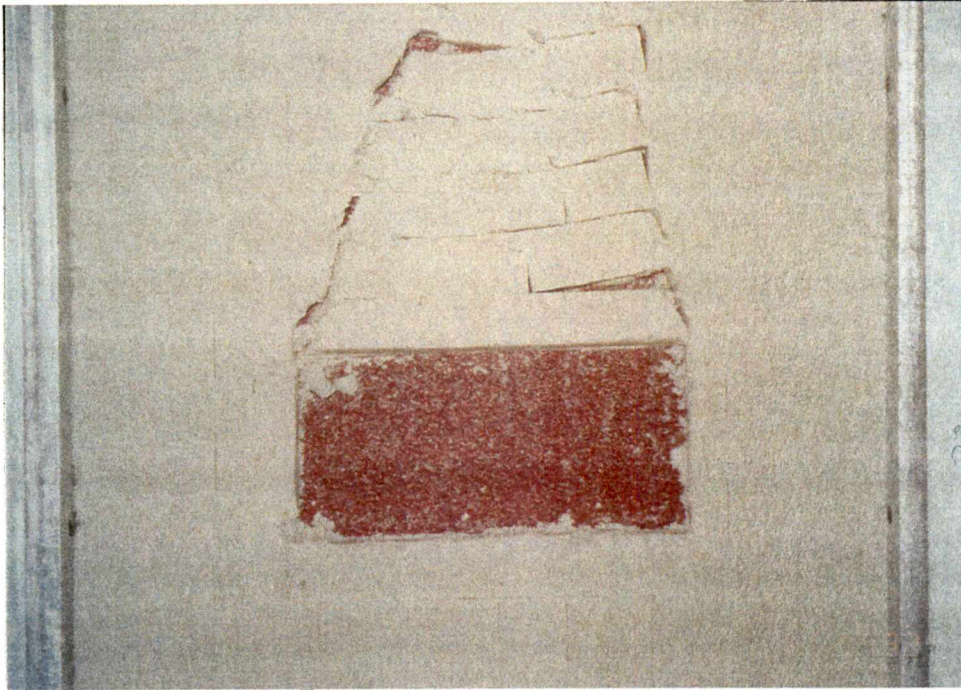


Plate 4: Tunnel Test 3 (continued): Stage 3 of 4 shows the roof is sliding as one mass. Stage 4 of 4 shows the tunnel back in equilibrium with 18 blocks dropping out.

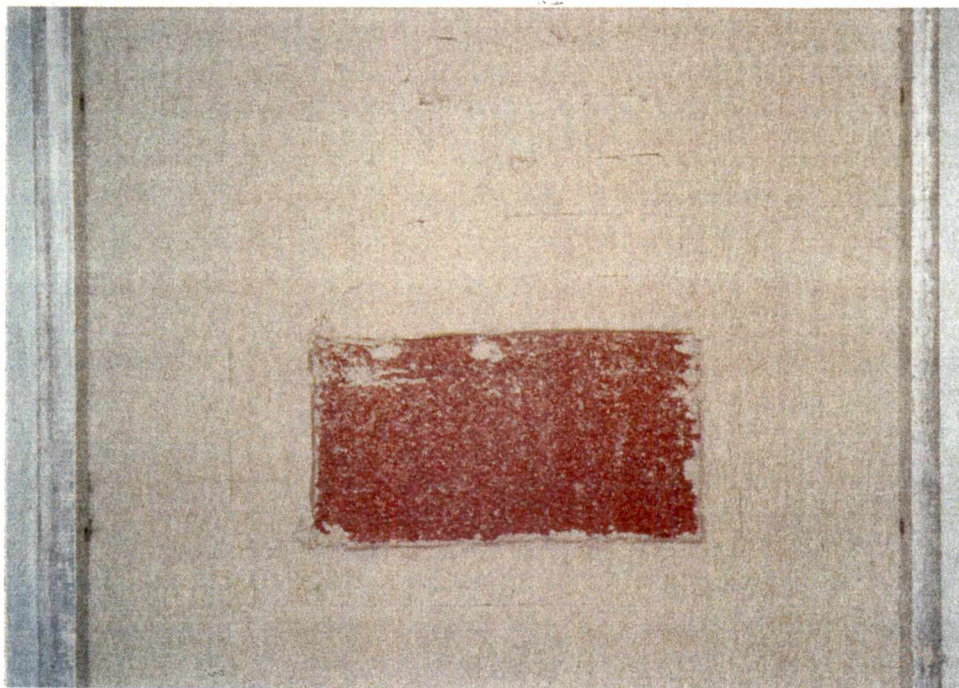
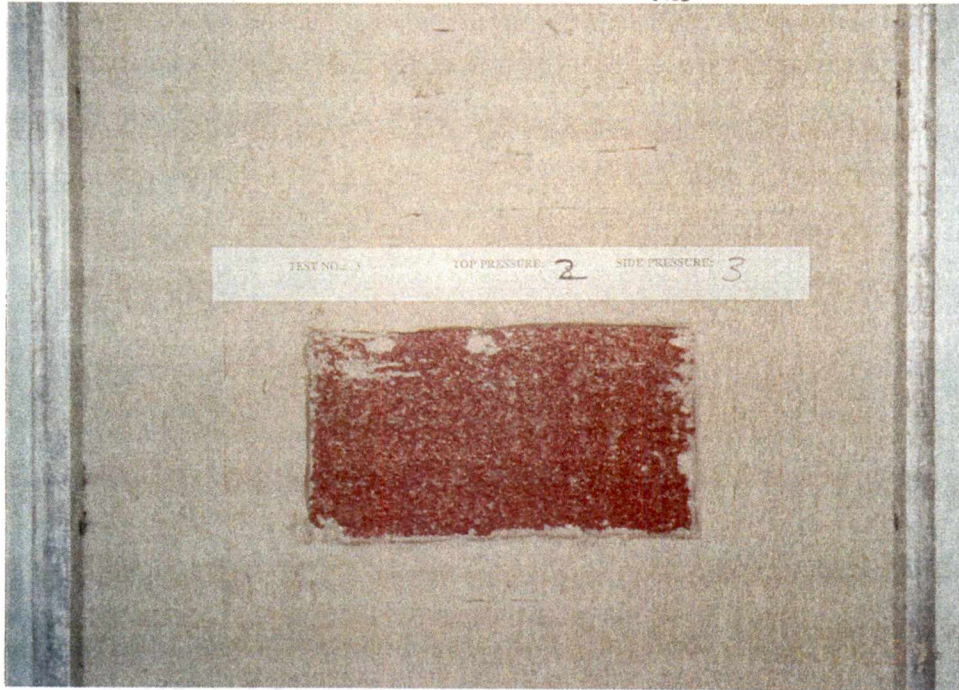


Plate 4: Tunnel Test 4: $K = 1.5$ and σ_v constant at 0.4 psi (2.8 kPa). Stage 1 of 7 is the start of the fourth test series. Stage 2 of 7 is the start of the test sequence.

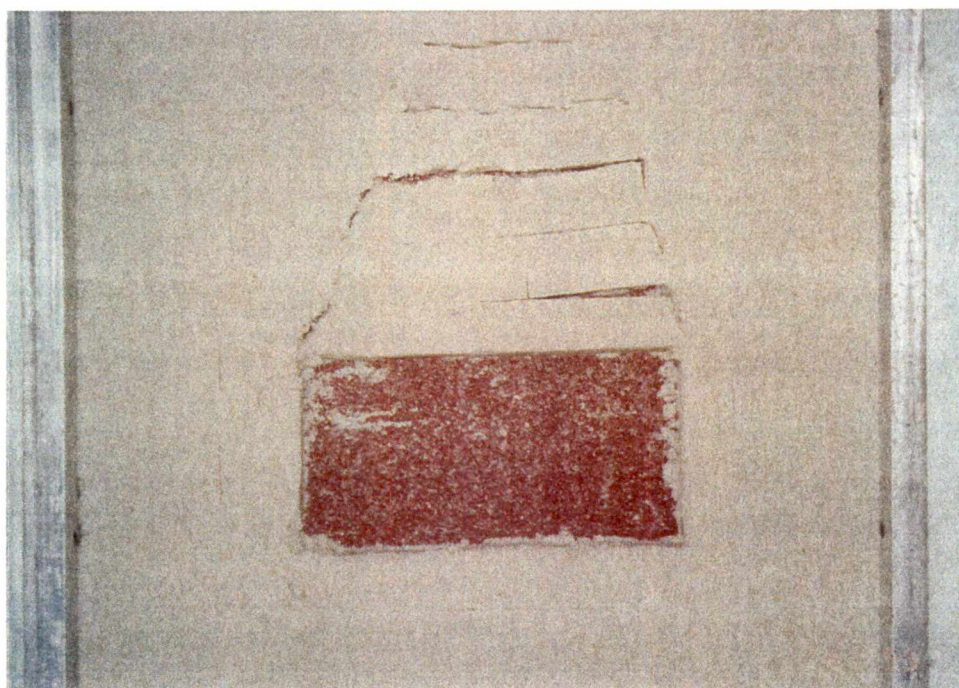
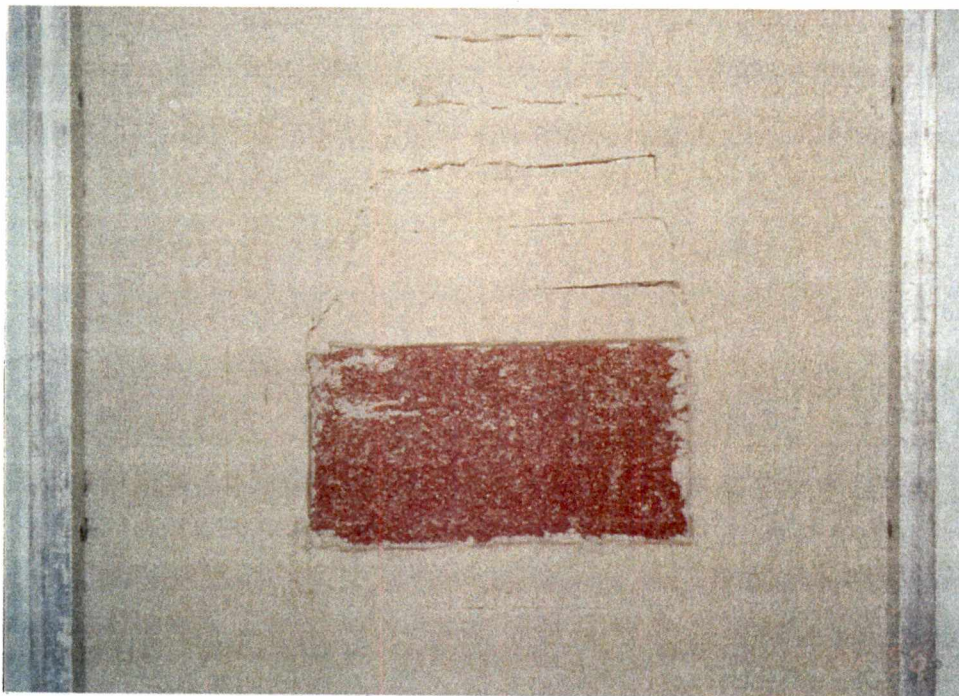


Plate 4: Tunnel Test 4 (continued): Stage 3 of 7 shows the same arch failure that has start in the previous tests. Stage 4 of 7 shows the bottom 3 layers sliding out of the roof.

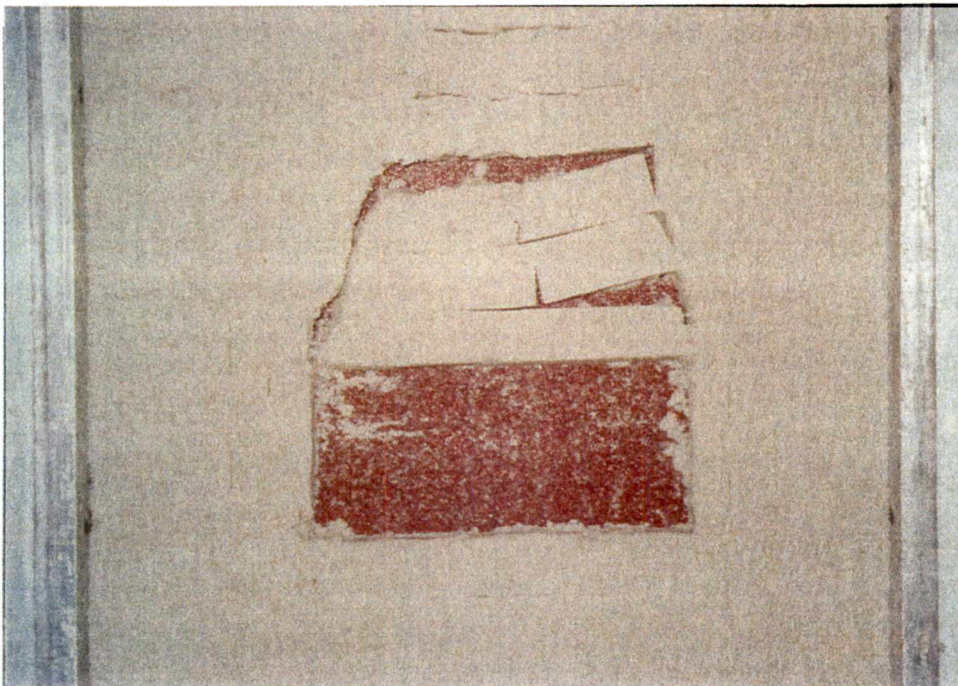
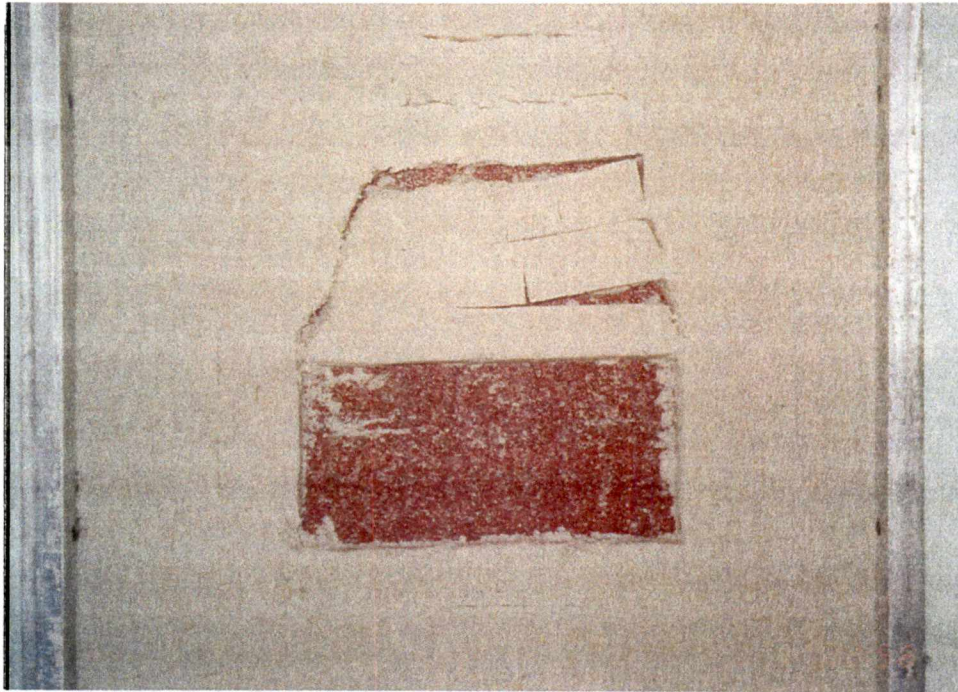


Plate 4: Tunnel Test 4 (continued): Stage 5 of 7 continues to shows the three layers sliding. Stage 6 of 7 shows the roof back in equilibrium.

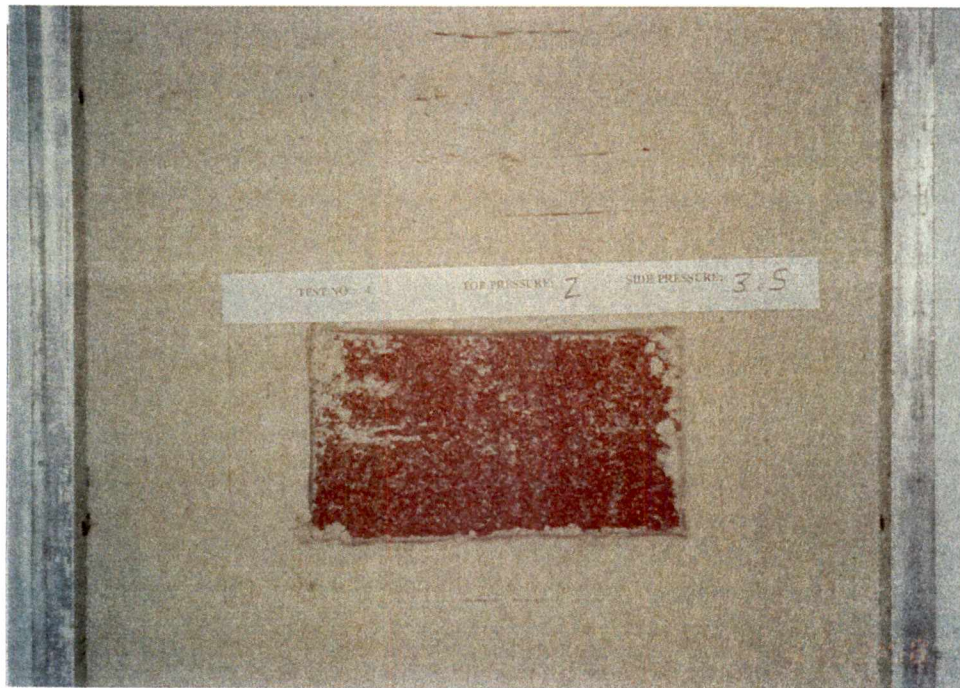
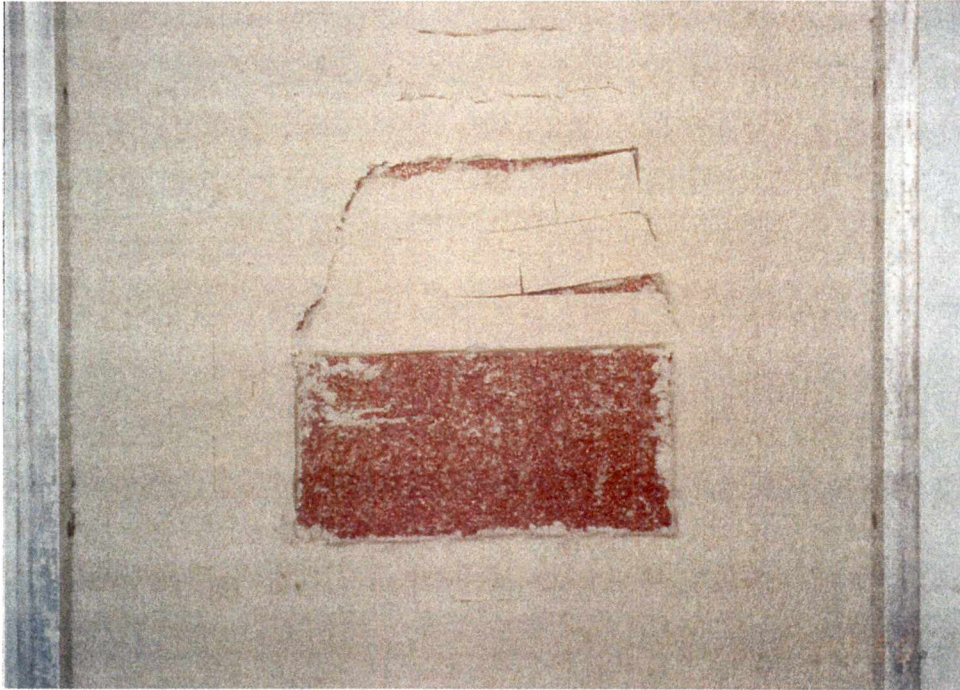


Plate 4: Tunnel Test 4 (continued): Stage 7 of 7 shows the roof is in equilibrium and only 11 blocks dropping out. Bottom picture is stage 1 of 5 of a new test with $K = 1.75$.

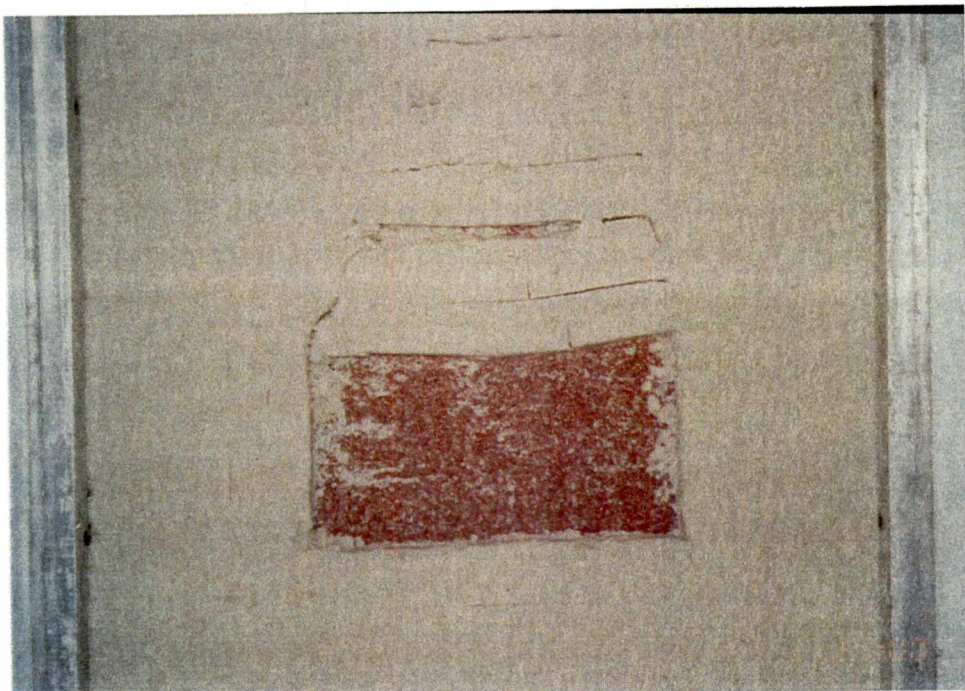
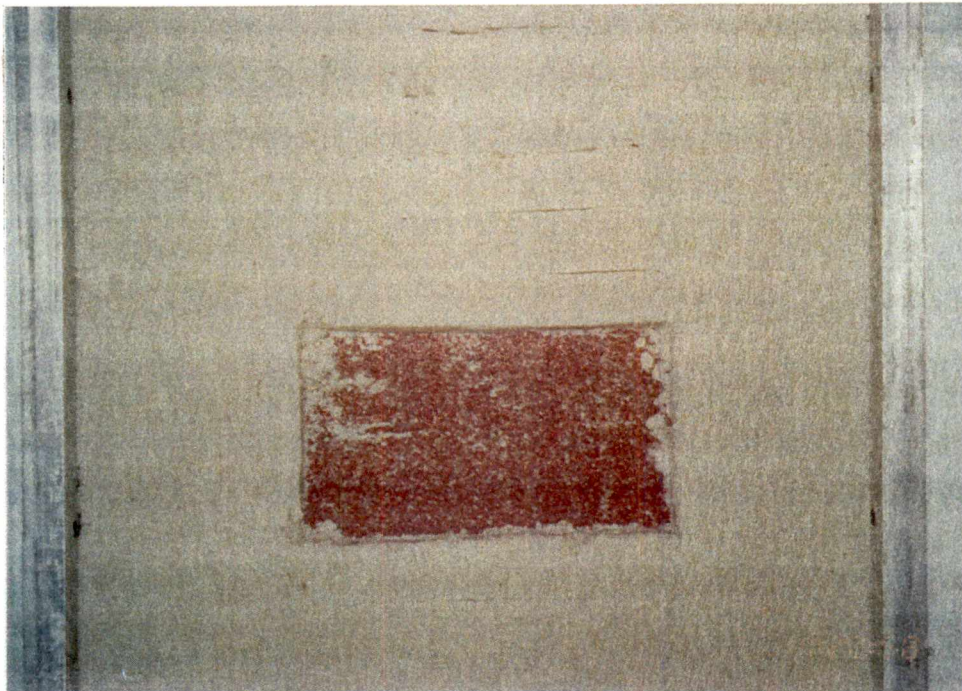


Plate 4: Tunnel Test 5: $K = 1.75$ and σ_v constant at 0.4 psi (2.8 kPa). Stage 2 of 5 is the start of the final test series. Stage 3 of 5 shows only the bottom two layers starting to slide out immediately.

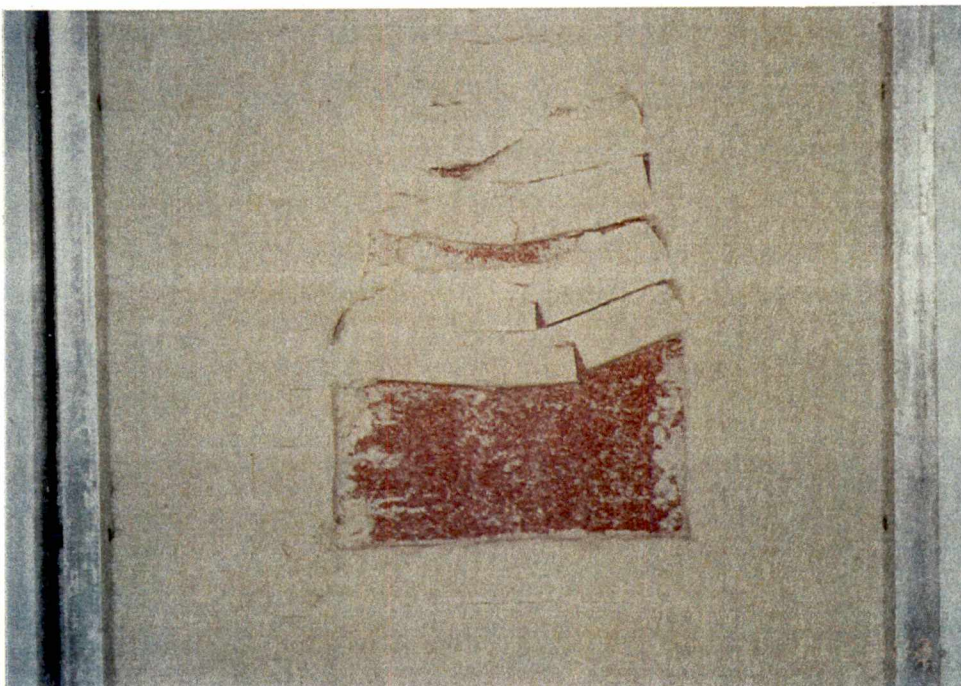
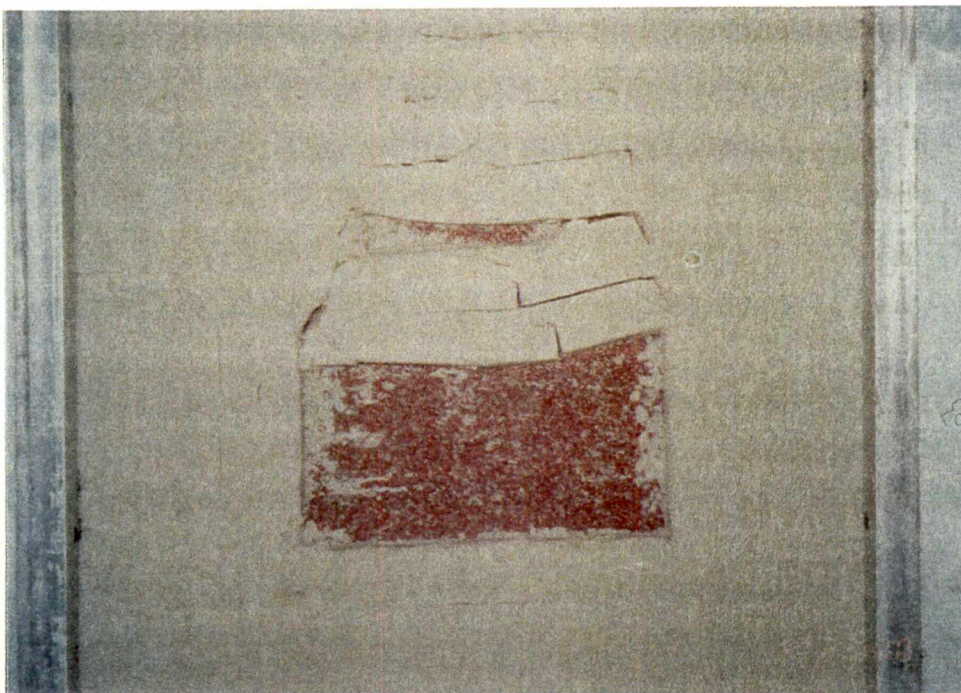
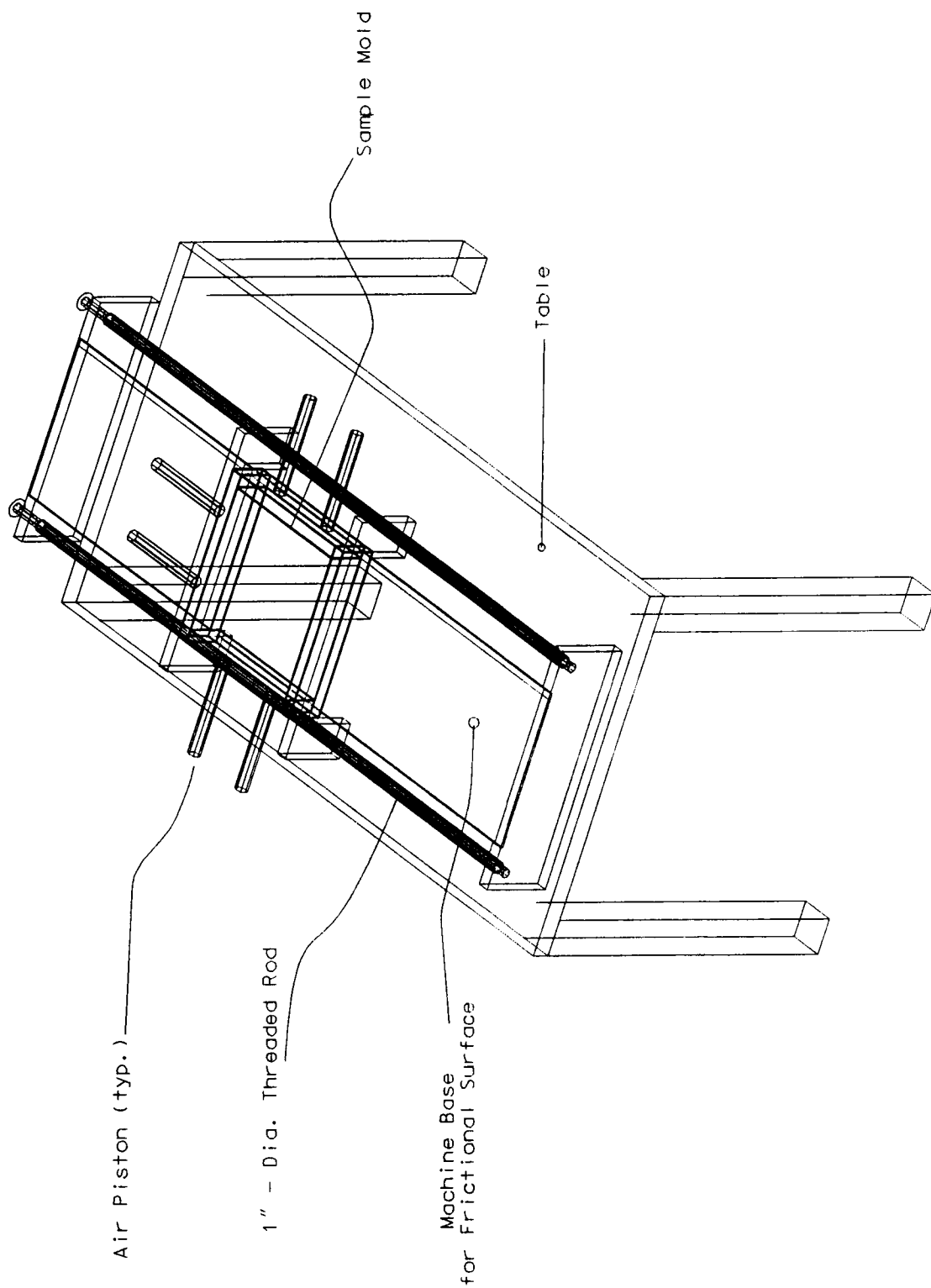


Plate 4: Tunnel Test 5 (continued): Stage 4 of 5 shows the same layers dropping as well as the sample starting to buckle. Stage 5 of 5 shows the model failing after 8.5 blocks drop out.

Appendix B

Base Friction machine schematic



Appendix C

Dr. R.E. Goodman's Letter on model mix dated February 2, 1994

Matt,

I had a student do a paper on materials for base friction modelling but I will have to do a search of the library to find it. In the meantime, you want to know that the purpose of the flour/oil is to bind to sand and the amount is dependent on the properties you want. A sand mix with very little flour/oil will be brittle and easily destroyed at low stresses. A flour/oil mix with very little sand will be plastic and capable of large deformations without much cracking. I typically mix just a few percent of oil to flour and then mix that about fifty/fifty with the sand. Then test the mix by making a simple toppling slope case to see how ready the material is to crack. The material should not be oily to the touch or greasy in appearance; also it should survive handling and cutting with a knife without going to pieces. Remember to break the adhesion at the base of the model sheet before beginning to cut details into the model; you break the adhesion by turning the belt on for a spell after laying the model sheet but before starting to put joints and layers and openings in.

Regards,

Dick Goodman

VITA

Steven Craig Smith was born at Fort Gordon Army Hospital in Augusta, Georgia on February 19, 1969. He attended elementary school at Bacon County Elementary School in Alma, Georgia until the fourth grade, when he moved to LaGrange, Georgia and attended schools in the LaGrange City School District. He graduated from LaGrange High School in June 1987. The following August he attended the University of Tennessee, Knoxville and in August, 1992 he received the degree of Bachelor of Science in Civil Engineering. He reentered the University of Tennessee, Knoxville in August, 1992 and in December, 1994 received a Master of Science degree in Civil Engineering, with an emphasis in Geotechnical Engineering.

He is presently employed as a staff Geotechnical Engineer with Ogden Environmental and Energy Services Company in Oak Ridge, Tennessee.