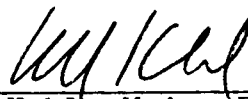


To the Graduate Council:

I am submitting herewith a dissertation written by Willard Wayne Montgomery entitled "Opposition Effect Experimentation and a Theoretical Investigation Involving Cylinders." I recommend that it be accepted in partial fulfillment of the requirement for the degree of Doctor of Philosophy, with a major in Physics.

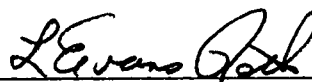


Ronald H. Kohl, Major Professor

We have read this dissertation
and recommend its acceptance:



Accepted for the Council:



Vice Chancellor
Graduate Studies and Research

OPPOSITION EFFECT EXPERIMENTATION
AND A THEORETICAL INVESTIGATION
INVOLVING CYLINDERS

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Willard Wayne Montgomery

August 1980

3045061

ABSTRACT

The existence, shape, and polarization characteristics of the backscattered peak due to the opposition effect are investigated using an optical system of greater resolution than has been used in previously reported studies. The samples studied, many previously unexamined, include some organic materials. Even with improved system resolution, opposition effect peaks produced by some samples are not fully resolved.

Three theoretical problems are investigated. Two of these address the scattering of plane electromagnetic waves by single infinite circular cylinders. In the first, the cylinder is embedded in a homogeneous half-space. In the second, the cylinder is adjacent to a homogeneous half-space. The third addresses the scattering produced by three identical, non-coplanar, dielectric cylinders. A back-scattered peak, comparable to that of the opposition effect, is obtained for cylinders with diameters comparable to the wavelength of the incident radiation.

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CHAPTER I

INTRODUCTION

Historical Background

The opposition, or retroreflection, effect is an increase in the observed brightness (Figure 1) of an illuminated surface as the direction of the observer approaches that of the illuminating source. This source-surface-observer angle is referred to as the "phase angle." This effect was first observed in studies of the lunar surface by Barabashev (1922) and by Markov (1924). These investigators determined that, when the moon was full, not only was the perceived brightness greater due to a maximum in illuminated area, but also each area element of the surface was brighter than at any other time during the lunar cycle. A similar effect had been observed (Seeliger, 1895) concerning the reflecting properties of Saturn's rings. Oetking (1966) reported the results of an investigation of the reflectance properties of a number of substances, primarily minerals. Most of the substances Oetking investigated displayed the pronounced backscatter of the opposition effect. Other studies (Seaney, 1966; Egan and Hilgeman, 1976; Egan and Hilgeman, 1977; Preston, 1967; Dunbar, 1931) have shown the opposition effect present in samples of some paints, barium sulfate, magnesium oxide

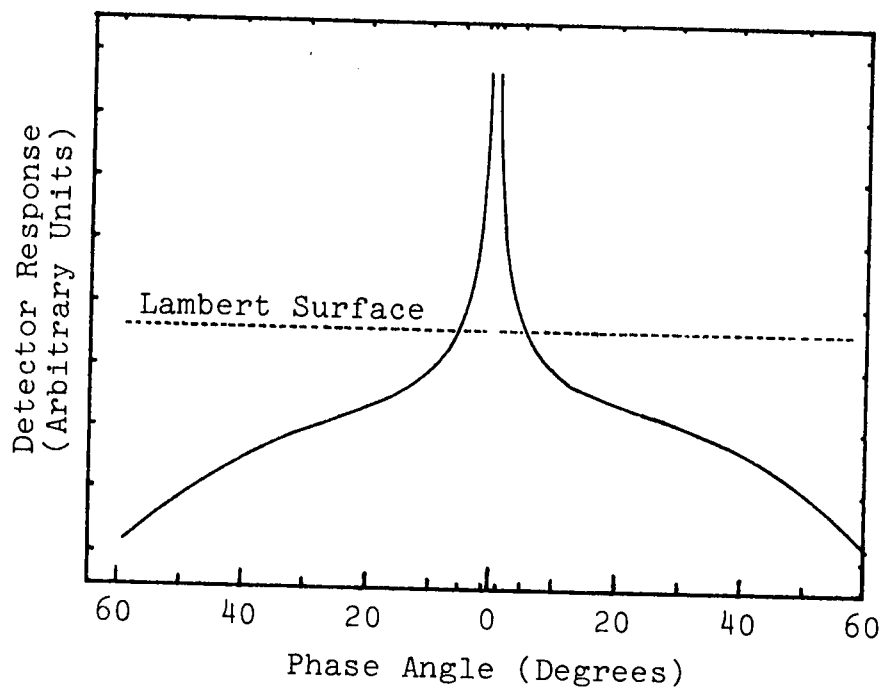


Figure 1. Smoothed Raw Data Curve for Smoked Magnesium Oxide Showing Opposition Effect, from Oetking (1966). The curve is discontinuous around zero degrees due to the detector obscuring the incident beam.

and other materials. Barium sulfate and magnesium oxide are used as reflectance standards (Grum and Becherer, 1979, pp. 320-322) because, except for the opposition effect, their reflectance properties approximate an ideal Lambertian surface.

The fact that the opposition effect was first observed in studies of extra-terrestrial surfaces reflects on the rather peculiar requirements for observing this effect. The peak itself is rather narrow, rising steeply as the phase angle approaches zero degrees. It is thus required that the detector field of view direction be allowed to approach the direction of illumination closely and that either the source direction or the detector direction be adjustable in order to demonstrate the change in received intensity versus phase angle. In most laboratory arrangements, the physical size of the source, detector, and associated optics, prevent the attainment of the required small phase angles. In the field, due to the larger scale, small phase angles are frequently encountered; but, in this case, the detector and source are usually fixed in position. As a result, while the opposition effect may play a role in the measurement being undertaken, it may not display itself in an obvious manner to the investigator. In both the laboratory and the field, phase angle averaging due to the finite size of the effective detector aperture

would tend to make the effect less dramatic and, thus, less noticeable.

Since electromagnetic radiation penetrates most substances to some extent, the quantitative features, width and relative magnitude, of the opposition effect should be related to the properties of the near surface of the material under investigation. There exists no complete theory, at present, which satisfactorily relates observation to material characteristics. Hapke (1963) utilized ray tracing and modeled the lunar surface layer as a cloud of point scatterers in which those rays scattered in the direction of incidence would have the greatest probability of avoiding further attenuation due to multiple scattering. In order for this model to fit the lunar surface reflectivity data, the surface layer of the moon would have to be composed of a cloud of dust which was 90 percent devoid of matter. This model led to the Thomas Gold-Bruce Hapke prediction that lunar landings would be difficult, if not impossible. Irvine (1966) refined Hapke's model for application to Saturn's rings. While these models were based upon the shadowing effect to produce enhanced back-scatter, Trowbridge (1978) added additional physical mechanisms which could produce the opposition effect. These included retroreflection by troughs and corners, and lens effects where spherical or cylindrical segments of

the surface focus onto sub-surface inclusions. The inclusions act as secondary sources for the lens segments.

The opposition effect thus depends on the phase angle of the measurement and the physical properties of the material under study. Its existence is independent of the angle of incidence of the illuminating source (Oetking, 1966). Since the proposed mechanisms producing this effect imply that the radiation making up the opposition effect peak is due to single or low-order scattering effects, it was expected that electromagnetic energy associated with this effect should not be depolarized. That is, with a polarized source, the detected peak should be due primarily to radiation of the same polarization as that of the source.

Problem Statement

The purpose of the experimental part of this investigation was to seek the existence of the opposition effect in some common materials not previously examined, including some organic materials. Polarization effects were considered. Further, the opposition effect peak was examined at a higher optical resolution than has been employed in previously reported investigations.

As the beginning of a search for a theoretical explanation of the opposition effect, a theoretical method for determining the scattering effects of an electromagnetic plane wave incident on a system of dissimilar objects consisting of a cylinder and a semi-infinite homogeneous

half-space was developed and applied to various configurations. With minor modifications, this procedure was then applied to the problem of scattering by a system of three parallel cylinders in order to investigate the shadowing theory for the opposition effect when multiple scattering effects may not be neglected. This procedure may be directly extended to include the case of a system composed of a large number of parallel cylinders; however, the increased complexity of the problem would result in a proportionately greater difficulty in obtaining numerical results.

CHAPTER II

EXPERIMENTAL OBSERVATIONS

Introduction

While previous studies have demonstrated the existence of the opposition effect in a variety of selected terrestrial materials, the purpose of this study was to determine the existence of such an effect in materials common to the outdoors environment. Thus, these materials are of interest to those who would optically monitor the earth's surface. The polarization content, relative to that of the source, of the backscattered radiation was examined for each material considered. The results of these examinations indicated that the enhanced backscattered radiation due to the opposition effect was composed of light which was polarized parallel to that of the illuminating source. This electromagnetic energy was in addition to a depolarized component which was constant over the range of phase angles considered for each material. While the resolution of the optical system utilized in this study was approximately 0.2 degrees, the peak was still not fully resolved in some cases.

Experimental Procedure

For this investigation a polarized helium-neon laser was employed as a source of illumination. The beam was

expanded to produce an illuminated spot on the sample surface 9.3 mm in diameter. The detector aperture was 11.0 mm in diameter. No collector optics were used. The spot size and detector aperture size combined to allow for sufficient speckle averaging that associated errors were reduced to an acceptable level.

The expanded beam was directed through a beam splitter, which was mounted on a frame constructed to allow orthogonal rotations, parallel and perpendicular to the plane of incidence, of the beam splitter. The orientation of the beam splitter was controlled by two micrometer heads which were spring loaded against the frame. The beam then proceeded to the sample surface, which was located at the beam waist of the expanded laser beam. The sample surface was mounted vertically and the surface normal was tilted approximately 15 degrees relative to the incident beam to avoid any specular reflection effects. The part of the beam split off by the beam splitter was dumped onto an optically black surface. The detector, a photomultiplier tube, was arranged to view the reflection of the illuminated surface in the beam splitter. The experimental layout is diagrammed in Figure 2.

Rotating the beam splitter changed the phase angle. In order to establish zero degrees phase angle, the sample was replaced by a plane mirror, oriented to reflect the incident beam back onto itself. The beam reflected by the

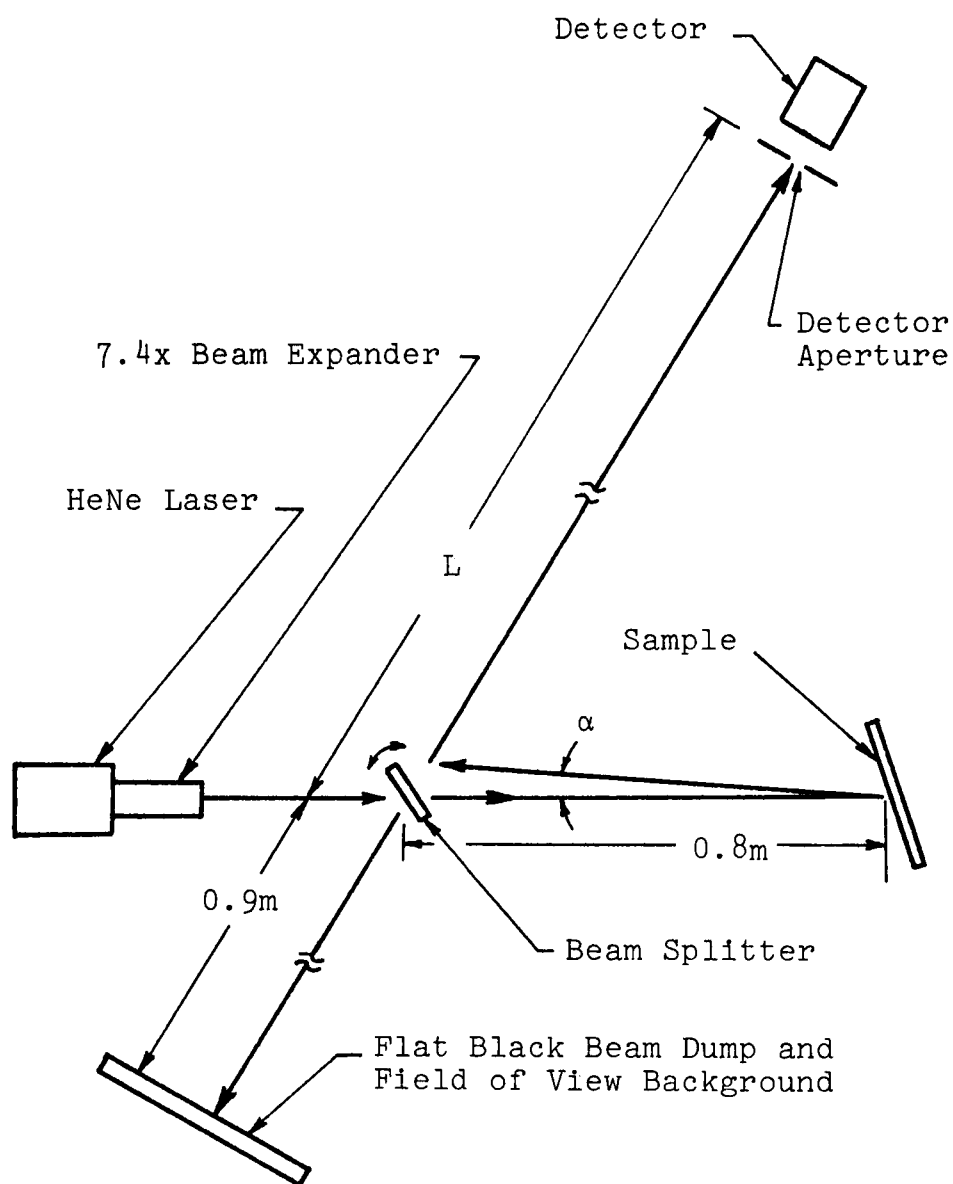


Figure 2. The Experimental Arrangement. The phase angle is indicated by α . The length, L , was 4.2m for the first five samples investigated and 2.7m for the remaining samples. These lengths resulted in optical system resolution of 0.17 degrees and 0.24 degrees respectively.

beam splitter then corresponded to zero degrees phase angle and the detector was located in this direction. Also, the micrometer settings on the beam splitter frame were recorded. Interference with the beam due to the frame limited the range of phase angles which could be attained. The calculated system angular resolution was obtained by taking the square root of the sum of the squares of the angle subtended by the detector aperture as viewed from the surface and the angle subtended by the illuminated spot as viewed from the detector aperture. The divergence angle of the expanded laser beam was small compared to these. Limitations due to the available laser power, sensitivity of the detector, beam size and detector aperture set the limit on angular resolution attainable with this system.

Detector output voltage was proportional to the intensity of the light on the photosensitive area of the detector and, therefore, with constant laser output proportional to the directional reflectivity of the sample material. The zero level detector output was determined by rotating the beam splitter approximately 15 degrees and thus preventing any radiation scattered from the illuminated sample reaching the detector aperture via the beam splitter. This zero level output, which was nulled, was insensitive to the beam splitter orientation. The detector output, as recorded, thus excluded the effects of photomultiplier dark current (10 mv) and the background light (up to 8 mv).

This last was due to room reflected stray radiation from the sample rather than scattered radiation from the beam splitter and beam dump. Data were taken by adjusting the micrometer on the beam splitter frame and thereby changing the phase angle by a known amount, and recording the photomultiplier tube output voltage as displayed on an oscilloscope. A phase angle scan with an ultraflat black surface in place of the sample gave a detector output of about 2 mv and showed there existed no untoward effects of the radiation scattered by the beam splitter and beam dump during such a scan.

One run required approximately 30 minutes to complete. At the end of each run, the micrometer was returned to its initial setting and the sample scan data rechecked to determine if the laser power had changed during the course of a run. Runs which showed evidence of laser output power drift were discarded. Results presented here are for the case of the polarization vector of the laser source oriented perpendicular to the plane of incidence. Investigation of the case of parallel orientation indicated that, within experimental error, source polarization relative to the plane of incidence does not influence the results. The polarization properties of the reflected light were investigated by mounting a linear polarizer in front of the detector aperture. Data could be taken with a bare detector or with the polarizer oriented either parallel or

perpendicular to the orientation of the laser polarization vector. Since the polarizer introduced some absorption, detector output voltage without the polarizer in place was not equal to the sum of the detector voltages obtained for the two polarizer orientations.

Experimental Results

Preliminary to considering the materials which were of primary interest, a barium sulfate sample was examined. Barium sulfate has been utilized as a reflectance standard for many years. This is due, in part, to the fact that its directional reflectivity is very nearly that of an ideal Lambertian surface. The fact that barium sulfate exhibits an opposition effect thus places a qualification on the use of such a material as an approximation of a Lambertian reflector.

The sample used in this investigation was prepared by applying the barium sulfate as a paint onto a plywood backing. Sufficient coats were applied so that the optical properties of the sample were solely dependent on the barium sulfate coating and independent of the supporting substrate. Figure 3 shows the results of measuring the reflectance properties of this sample. Here $\Delta\alpha$ represents the phase angle resolution of the optical system. The (vertical) error bars constitute estimated errors in the determination of the photomultiplier tube output voltage and were due

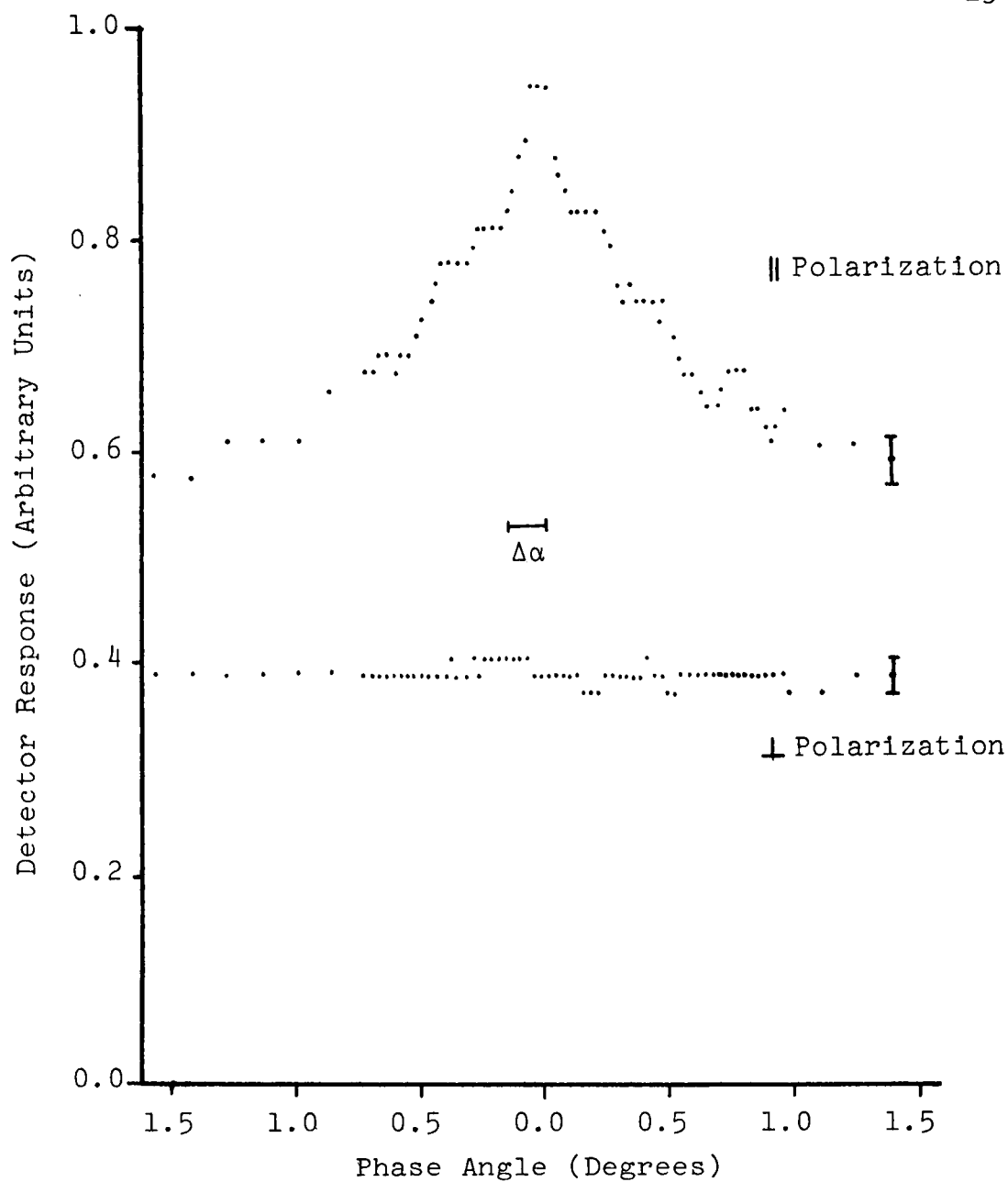


Figure 3. Measured Detector Response as a Function of Phase Angle for a Barium Sulfate Paint Sample. The calculated system resolution was $\Delta\alpha = 0.17^\circ$.

to system noise and difficulty in accurately reading the oscilloscope trace.

As demonstrated by the figure, the true height of the peak at zero degrees phase angle was not fully resolved due to phase angle averaging. Further, the perpendicular polarization component was not influenced by the opposition effect and, within experimental error, was constant for the range of phase angles investigated.

Figure 4 represents data collected for a yellow poplar leaf. Again, the opposition effect was confined to the parallel polarization component and the peak was instrument resolution limited. Because the perpendicular polarization component of reflectivity was constant, within experimental error, for this and all other samples investigated, data were collected for subsequent samples with a bare detector and then with the polarizer oriented perpendicular to the polarization vector of the laser source. A bare detector was preferred in order to enhance the signal-to-noise ratio, and the perpendicular polarizer was used to verify that this component was indeed constant for each sample over the phase angles considered.

Figure 5 represents data collected for a slightly weathered concrete block sample. Figure 6 is for a fir board sample while Figure 7 is for this same sample after it had been subjected to the effects of weathering for 6 weeks. Weathering had the influence of enhancing and

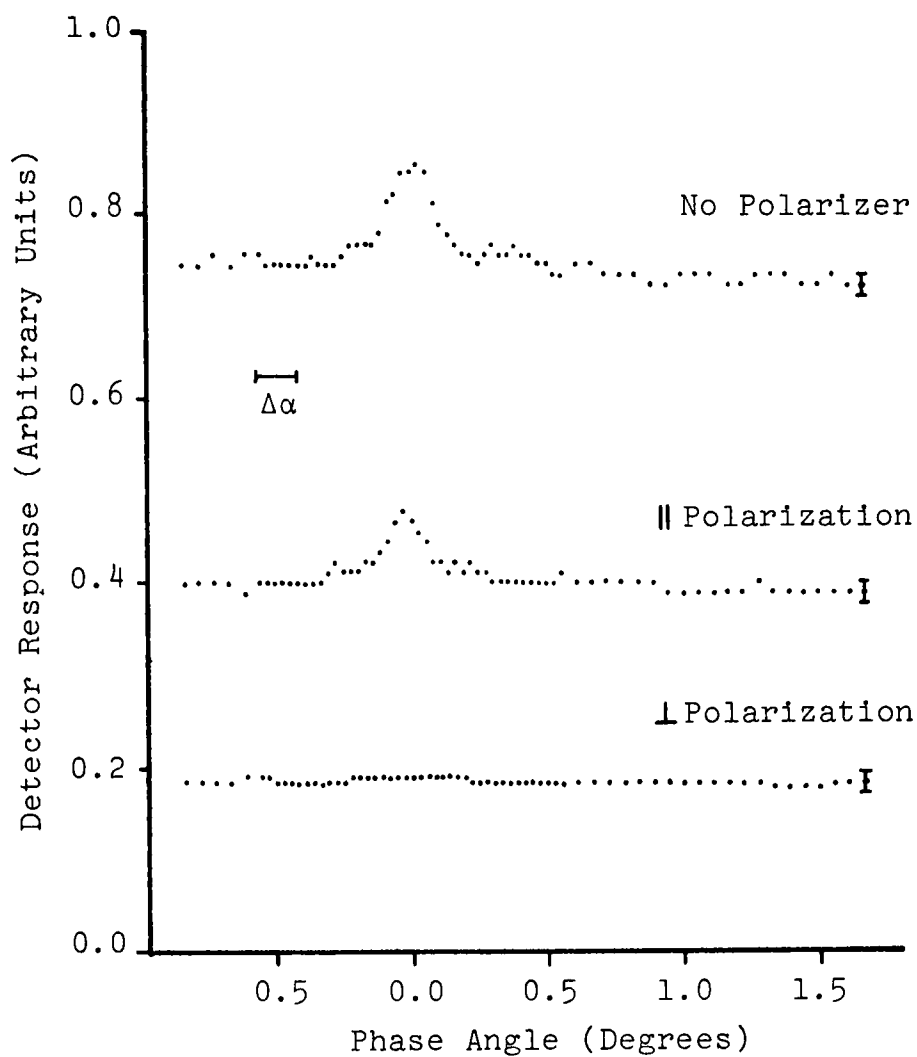


Figure 4. Measured Detector Response as a Function of Phase Angle for a Yellow Poplar Leaf. The calculated system resolution was $\Delta\alpha = 0.17^\circ$.

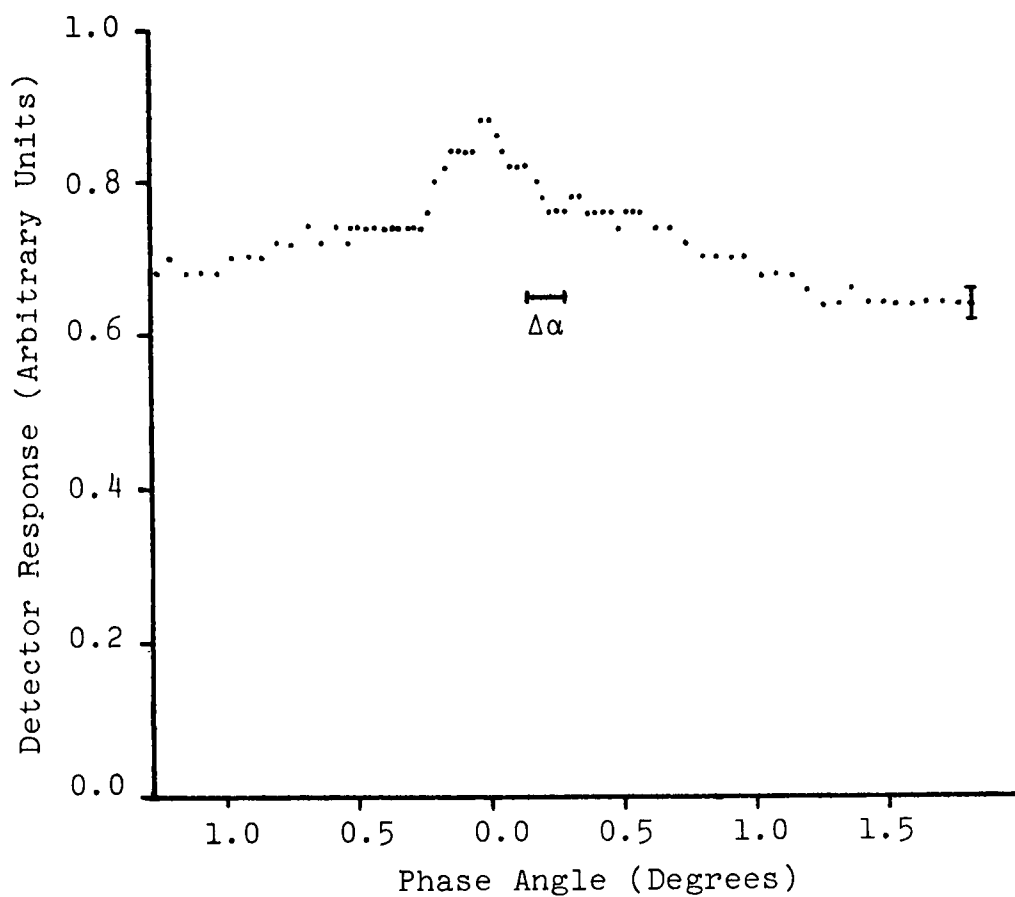


Figure 5. Measured Detector Response as a Function of Phase Angle for a Concrete Block Sample. The calculated system resolution was $\Delta\alpha = 0.17^\circ$.

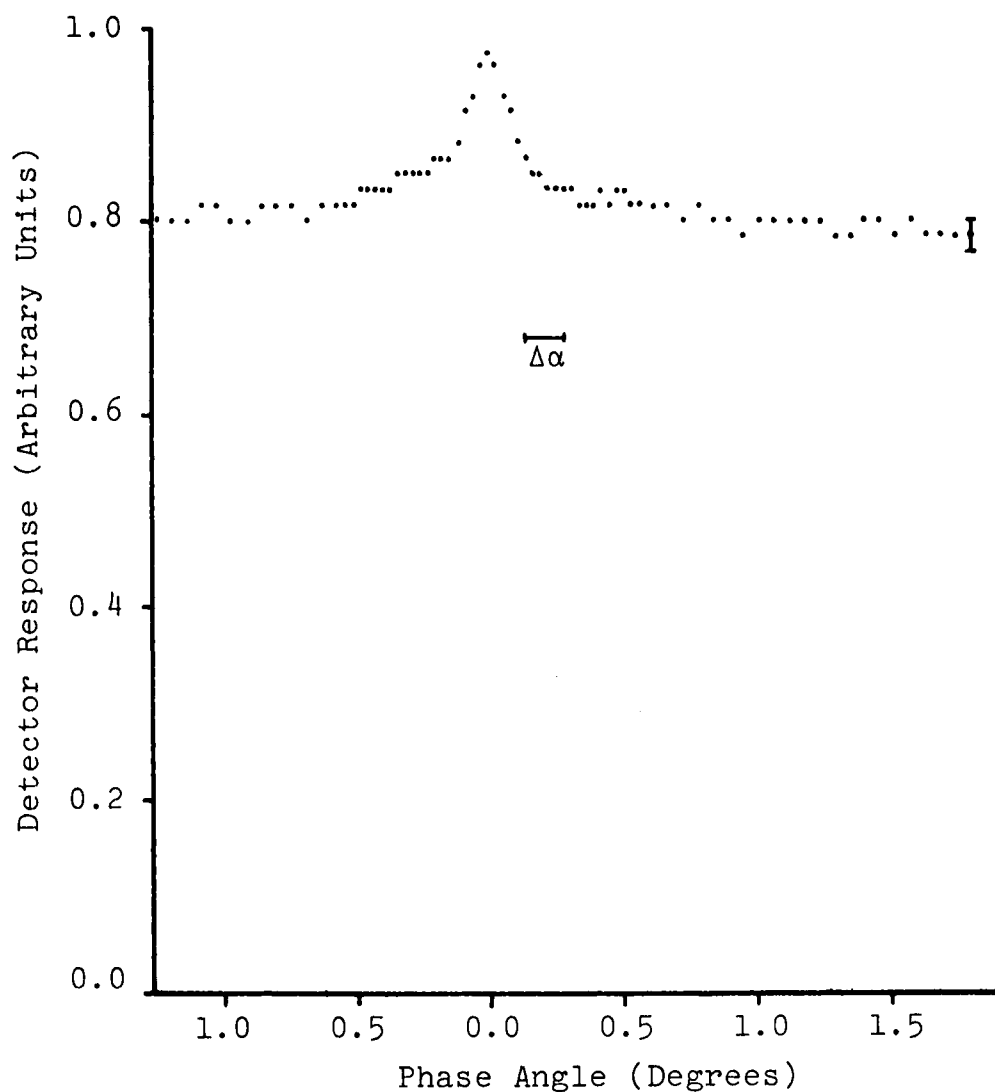


Figure 6. Measured Detector Response as a Function of Phase Angle for a Fir Board Sample. The calculated system resolution was $\Delta\alpha = 0.17^\circ$.

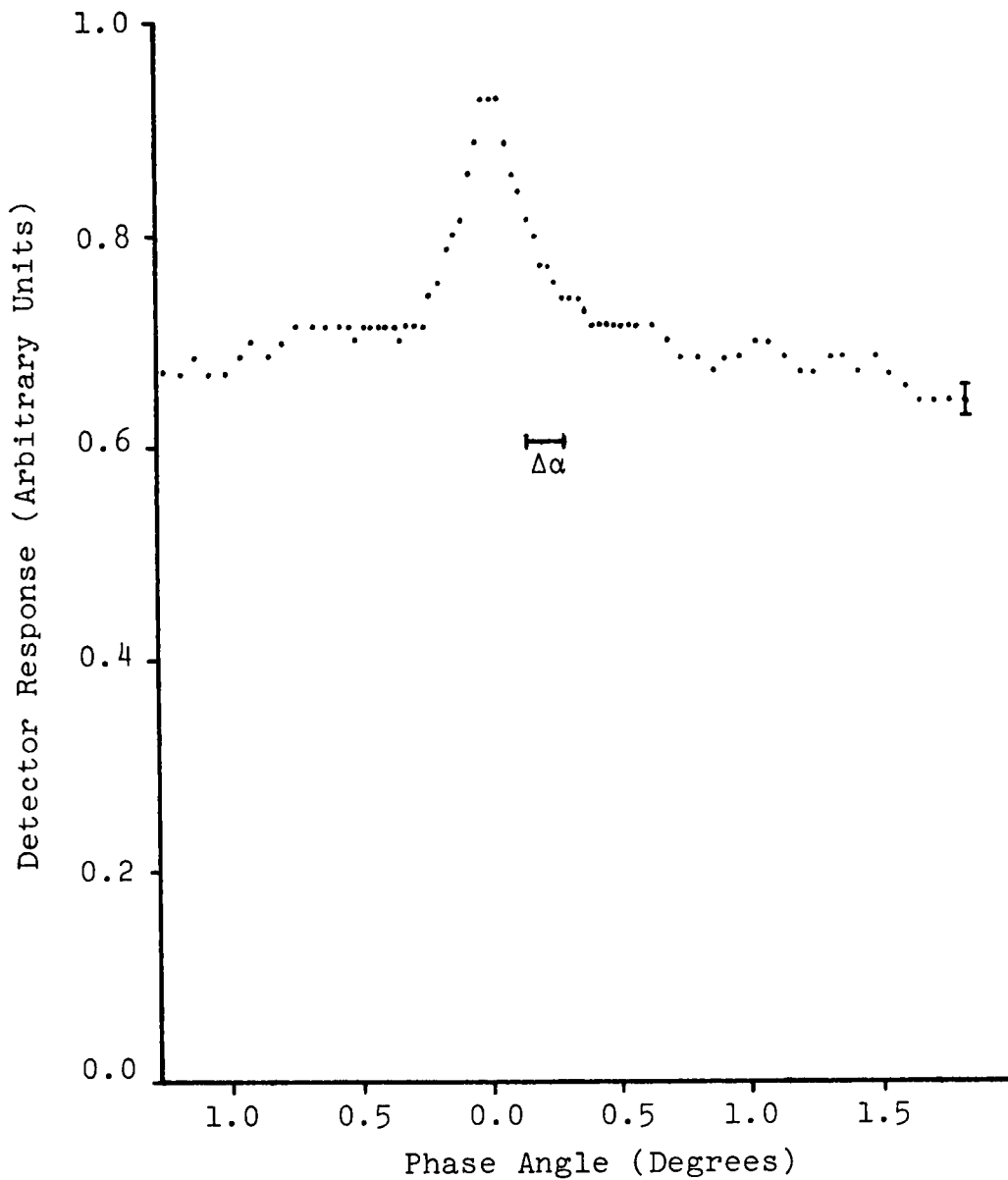


Figure 7. Measured Detector Response as a Function of Phase Angle for a Weathered Fir Board Sample. The calculated system resolution was $\Delta\alpha = 0.17^\circ$.

slightly widening the central peak. This was due to the creation of a rougher surface texture and separation of wood fibers near the board surface.

At this point in the data collection, the optical arrangement was modified by reducing the distance between the beam splitter and the detector. This was done to improve the perpendicular polarization signal-to-noise ratio in order to verify the constancy of this component of the reflected energy for the remaining samples. The result of this change was to make the system resolution more coarse, as is noted in the remaining figures. In order to investigate the influence of this alteration, data were again collected (Figure 8) with the same sample as was used for Figure 7. Comparison of Figures 7 and 8 demonstrates the broadening and reduction in relative peak height associated with the increased phase angle averaging for the lower resolution system.

A hickory bark sample produced the results displayed in Figure 9. This sample was very dark in color and had only a slight backscatter enhancement. Figure 10 is for a limestone rock sample which was of light color.

Because a single blade of grass was too narrow to fill the illuminating beam, a bundle of grass was mounted in a manner which simulated an optical system looking down on a grass covered area. Figure 11 shows the results of

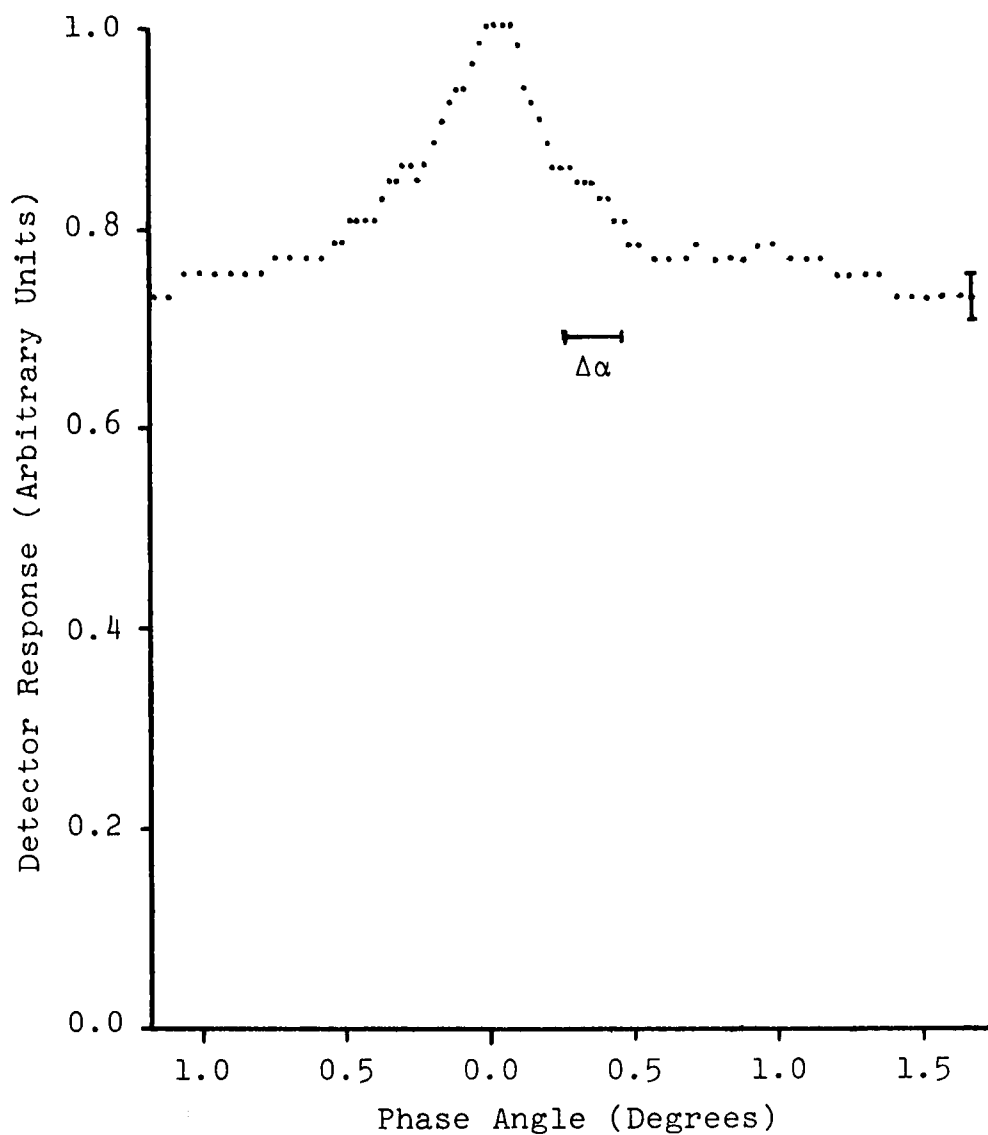


Figure 8. Demonstration of the Effect of Modification of System Resolution on the Results for the Same Sample Used for Figure 7. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

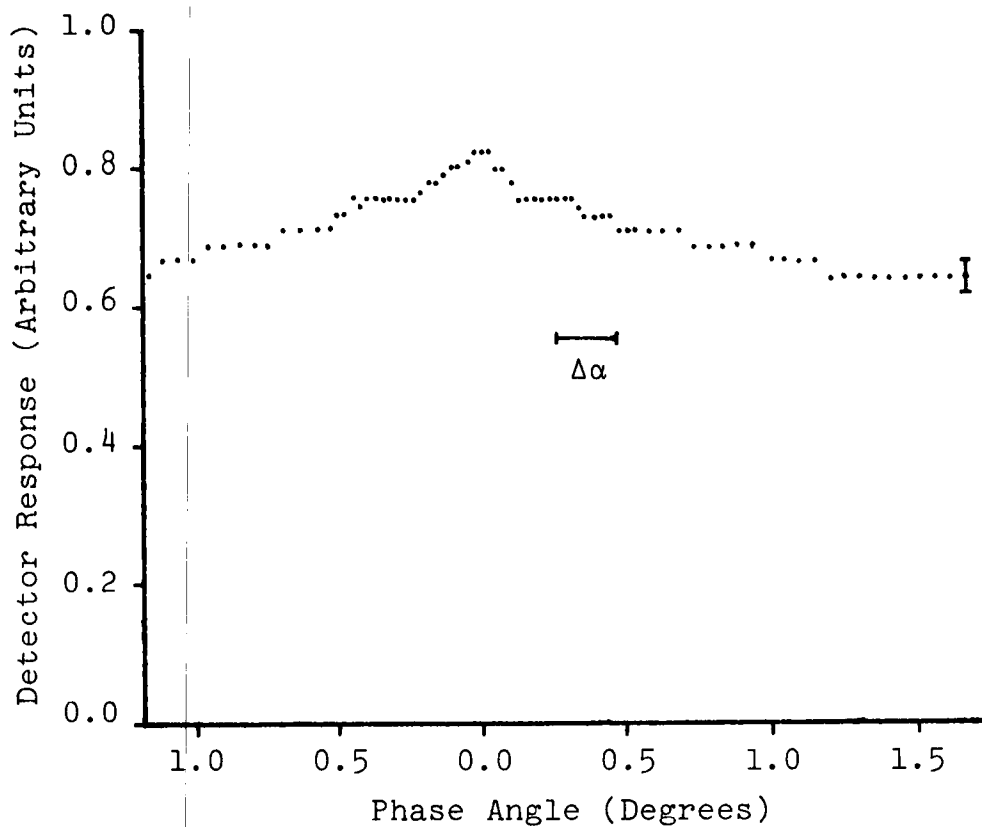


Figure 9. Measured Detector Response as a Function of Phase Angle for a Hickory Bark Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

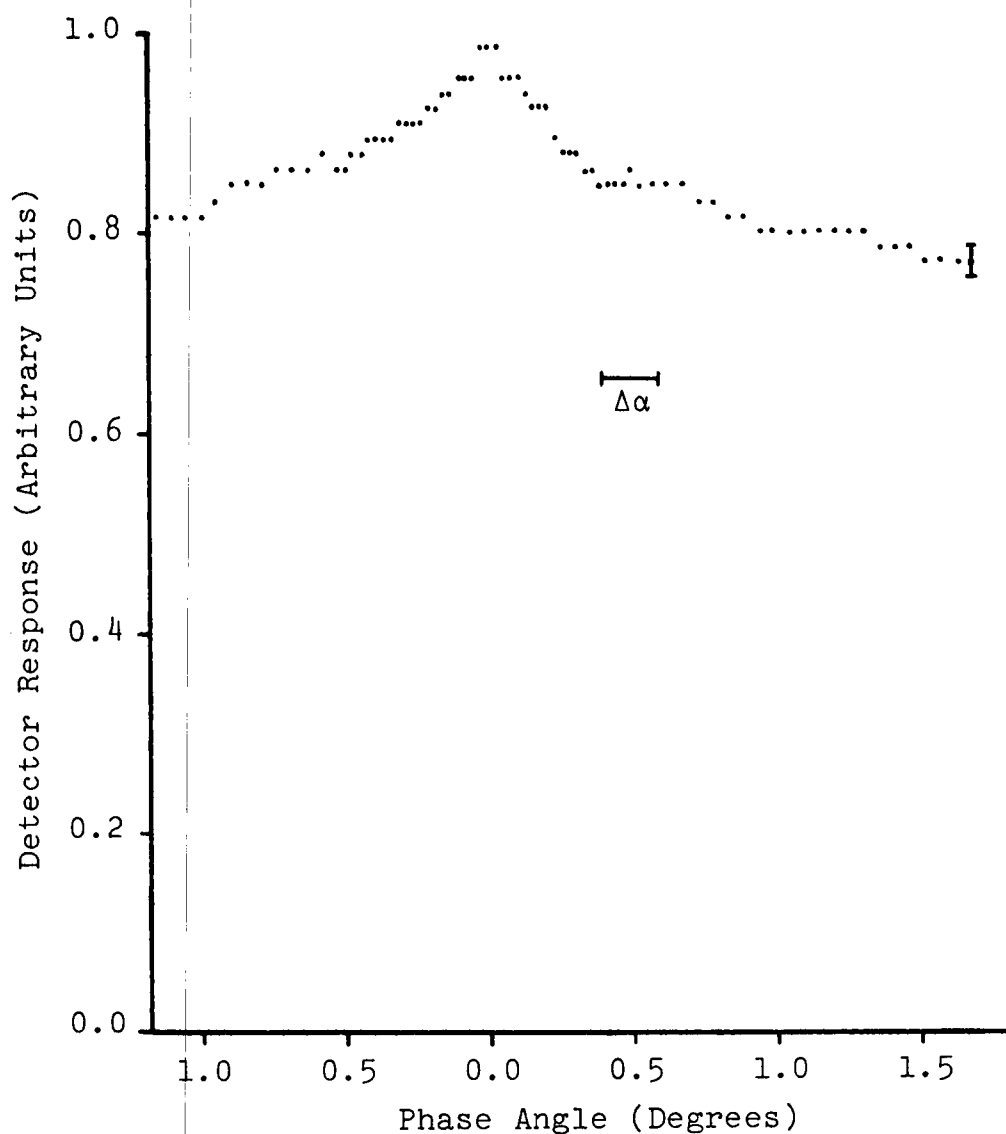


Figure 10. Measured Detector Response as a Function of Phase Angle for a Limestone Rock Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

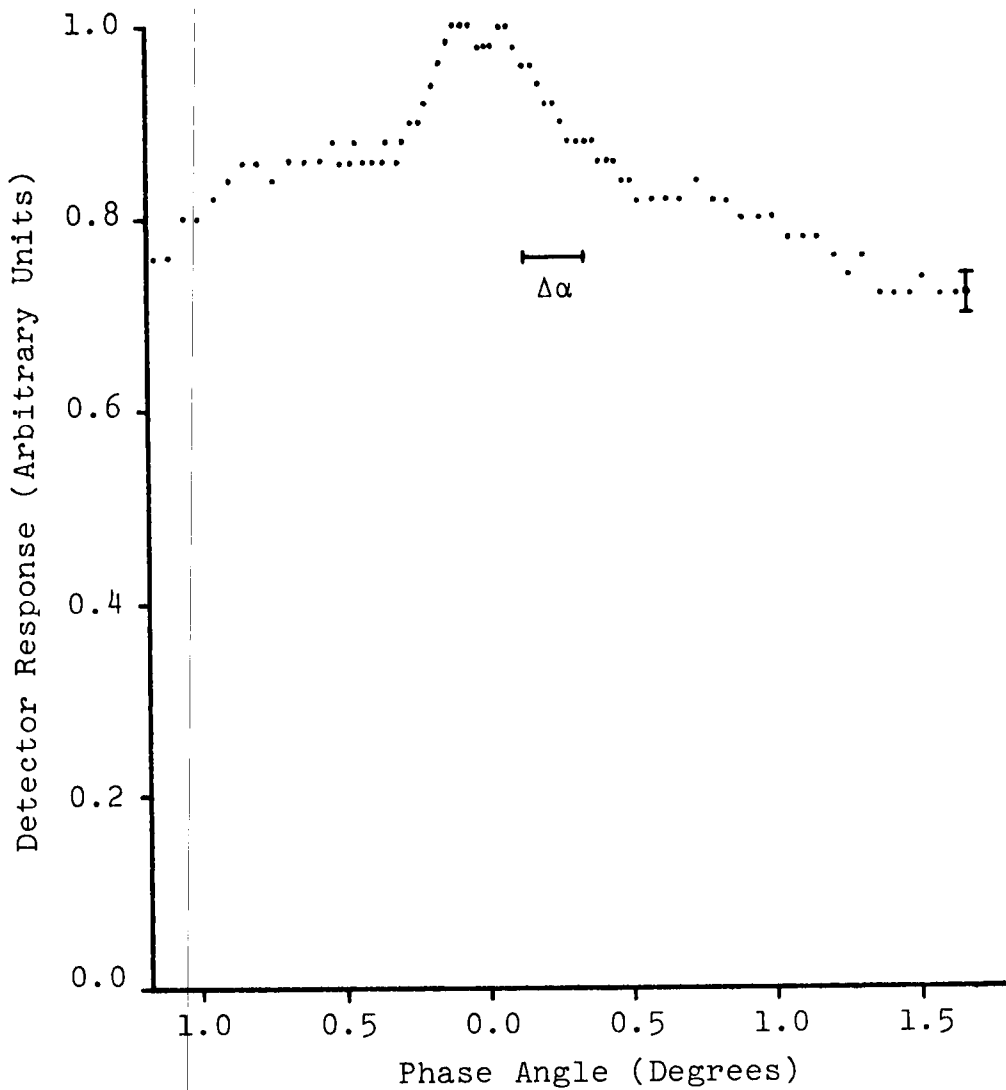


Figure 11. Measured Detector Response as a Function of Phase Angle for a Grass (*Digitaria Sanguinalis*, Crabgrass) Bundle. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

the examination of this grass bundle. This sample was light green in color.

Figure 12 shows the results of the examination of an unpainted, exterior grade, plywood sample. This sample possessed a more textured surface than did the weathered fir board sample which contributed to the broader retroreflection peak.

Figures 13 through 15 show the reflection properties of some paints. In spite of the difference in appearance of the two samples of flat white and glossy white paints, the measured reflection properties of these two are quite similar. This was due, in part, to the nonuniform texture of the glossy white paint sample, which displayed many small glossy areas separated by mat areas. The red paint was almost uniformly mat and resulted in a more narrow peak which appeared not to be fully resolved by the optical system.

Figure 16 and Figure 17 represent the results of measurements made on 2 brick samples. The first, Figure 16, was for a construction type brick sample while the second was a paving type brick sample. The second sample exhibited a slightly smoother surface but, within experimental error, these samples displayed similar reflection properties. This was probably due to the identical material content of the two samples.

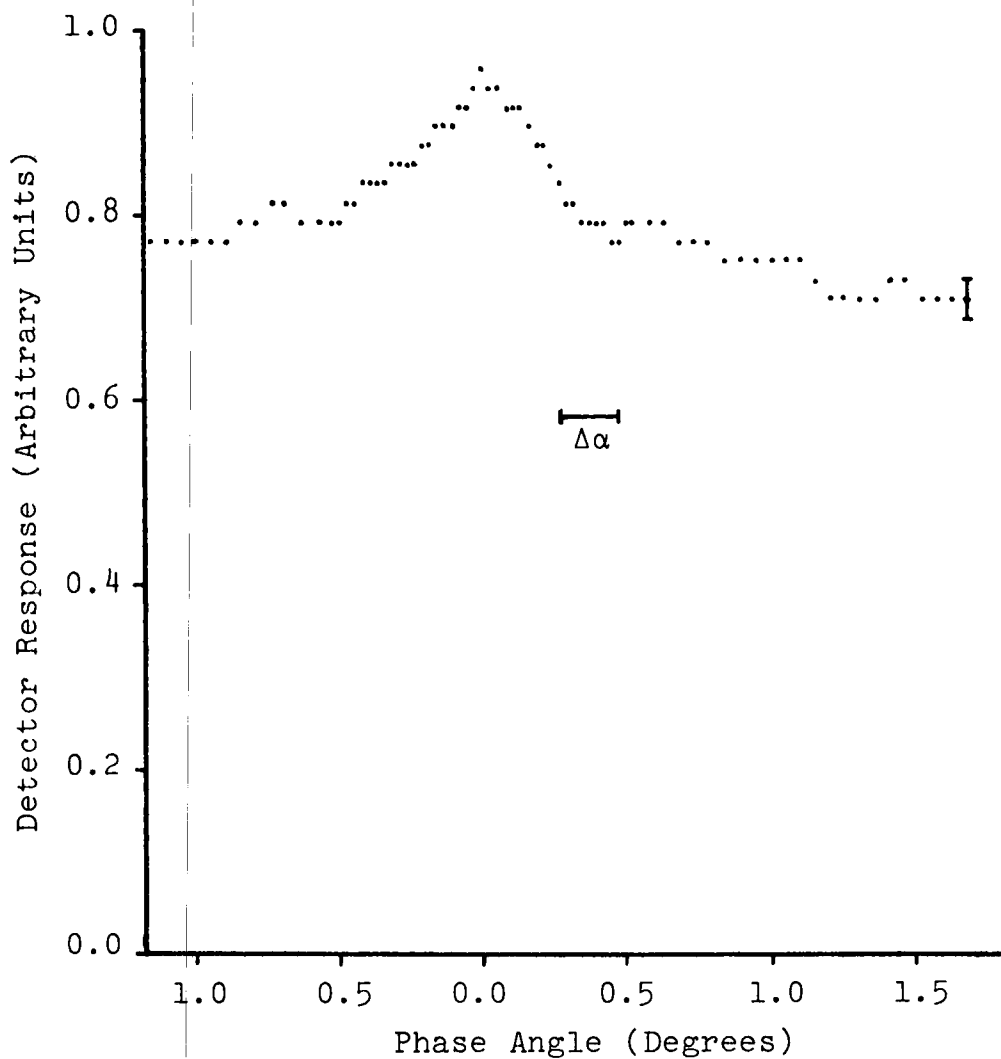


Figure 12. Measured Detector Response as a Function of Phase Angle for an American Plywood Association Exterior Type A-C, Pine Plywood Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

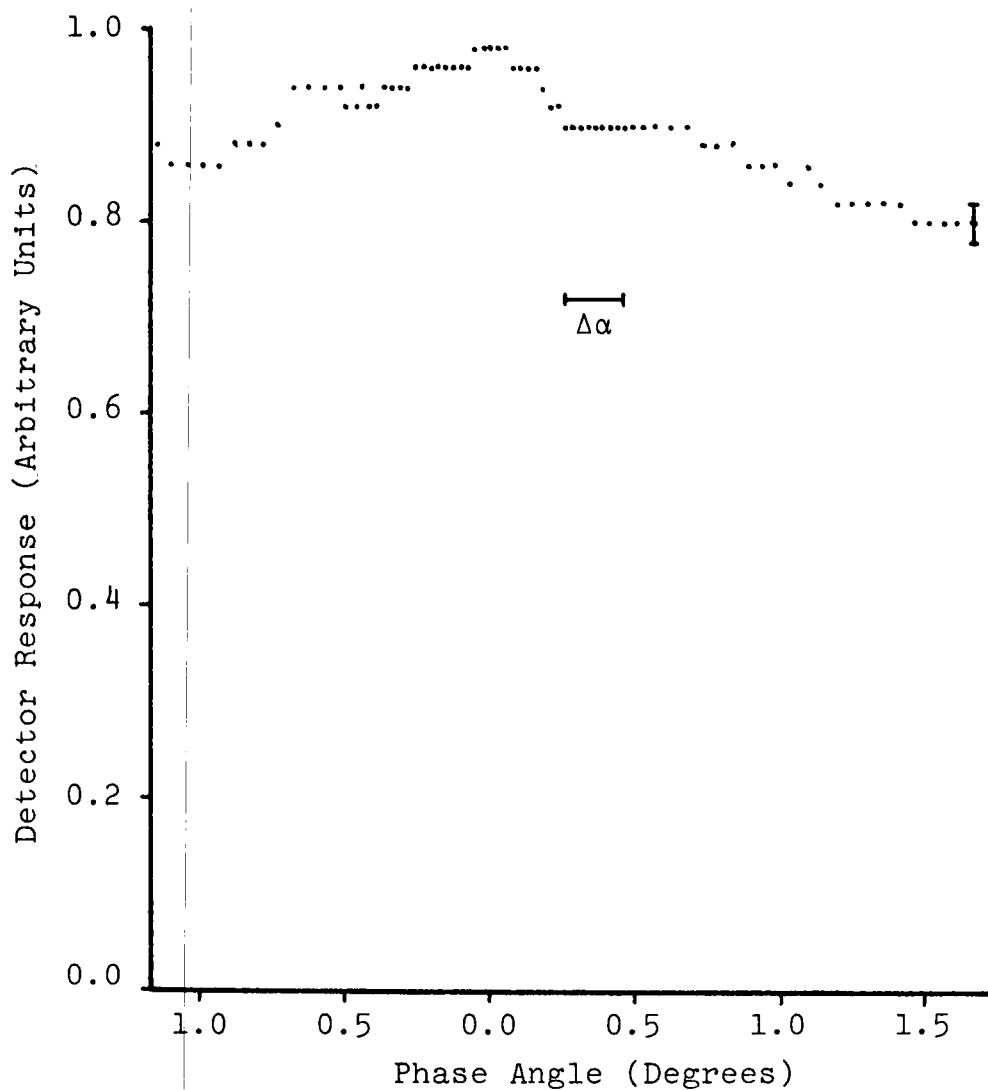


Figure 13. Measured Detector Response as a Function of Phase Angle for a KRYLON Flat White (1502) Enamel Paint Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

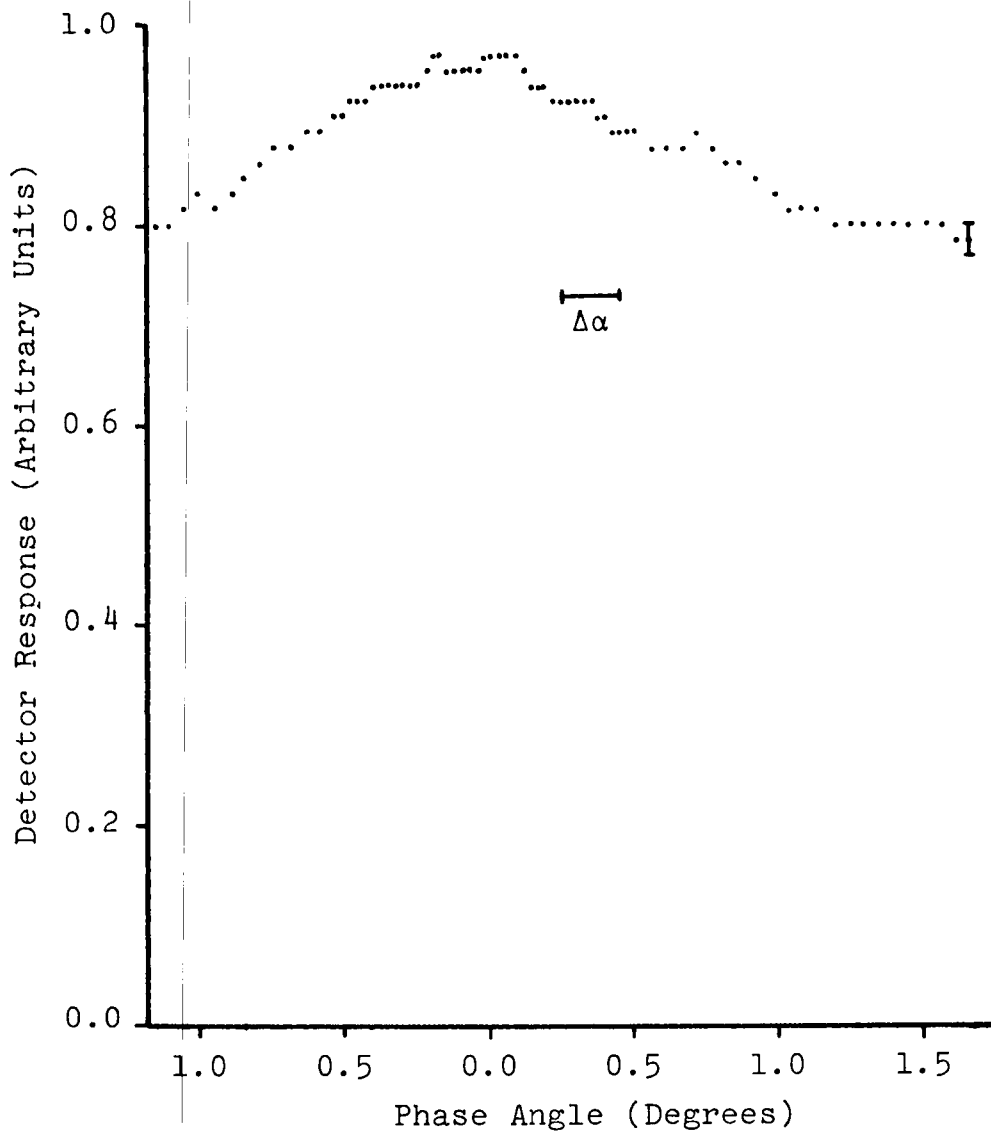


Figure 14. Measured Detector Response as a Function of Phase Angle for a KRYLON Glossy White (1501) Enamel Paint Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

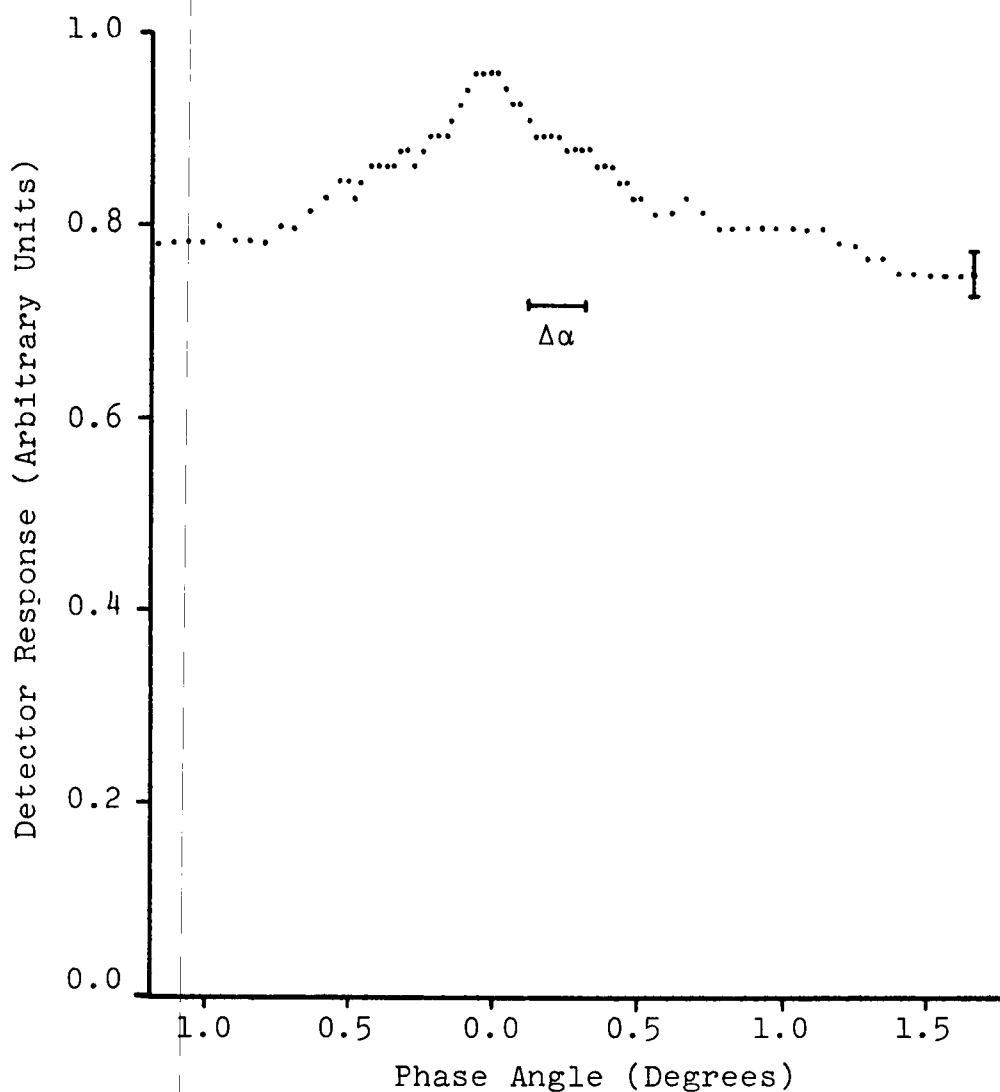


Figure 15. Measured Detector Response as a Function of Phase Angle for a KRYLON Cherry Red (2101) Enamel Paint Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

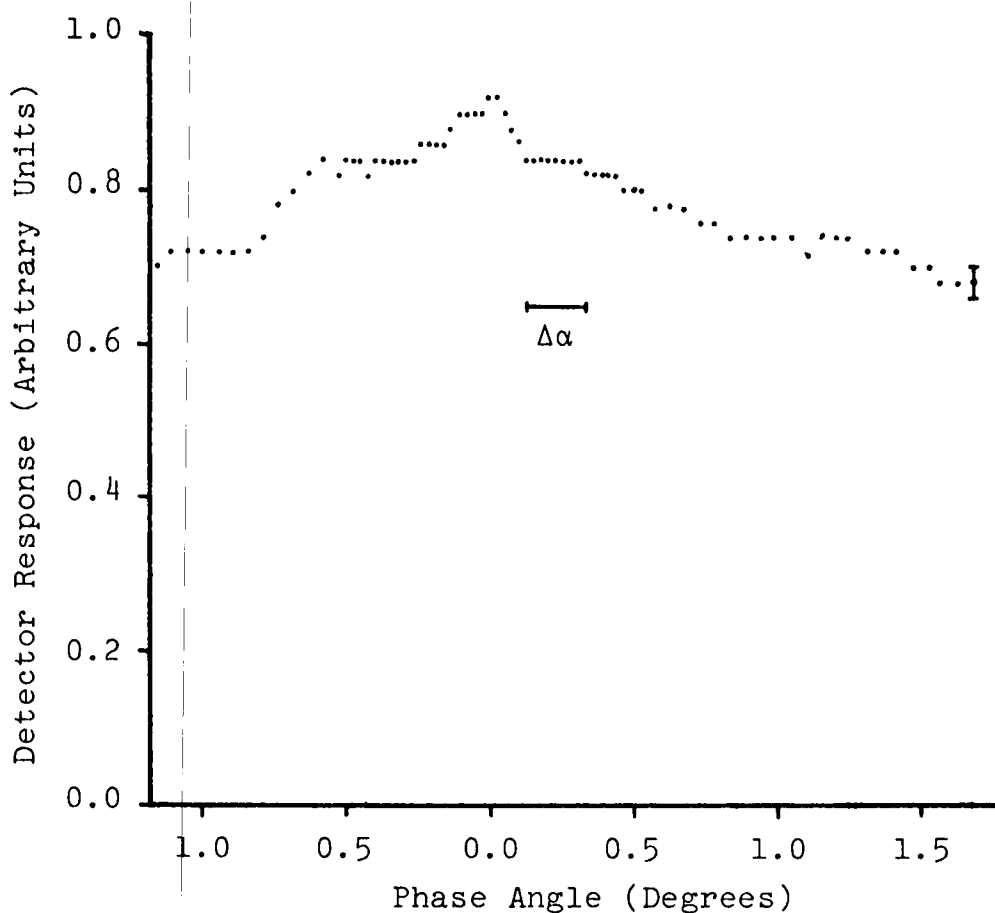


Figure 16. Measured Detector Response as a Function of Phase Angle for a General Shale Products Corporation, Red Range Barktex Construction Brick Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

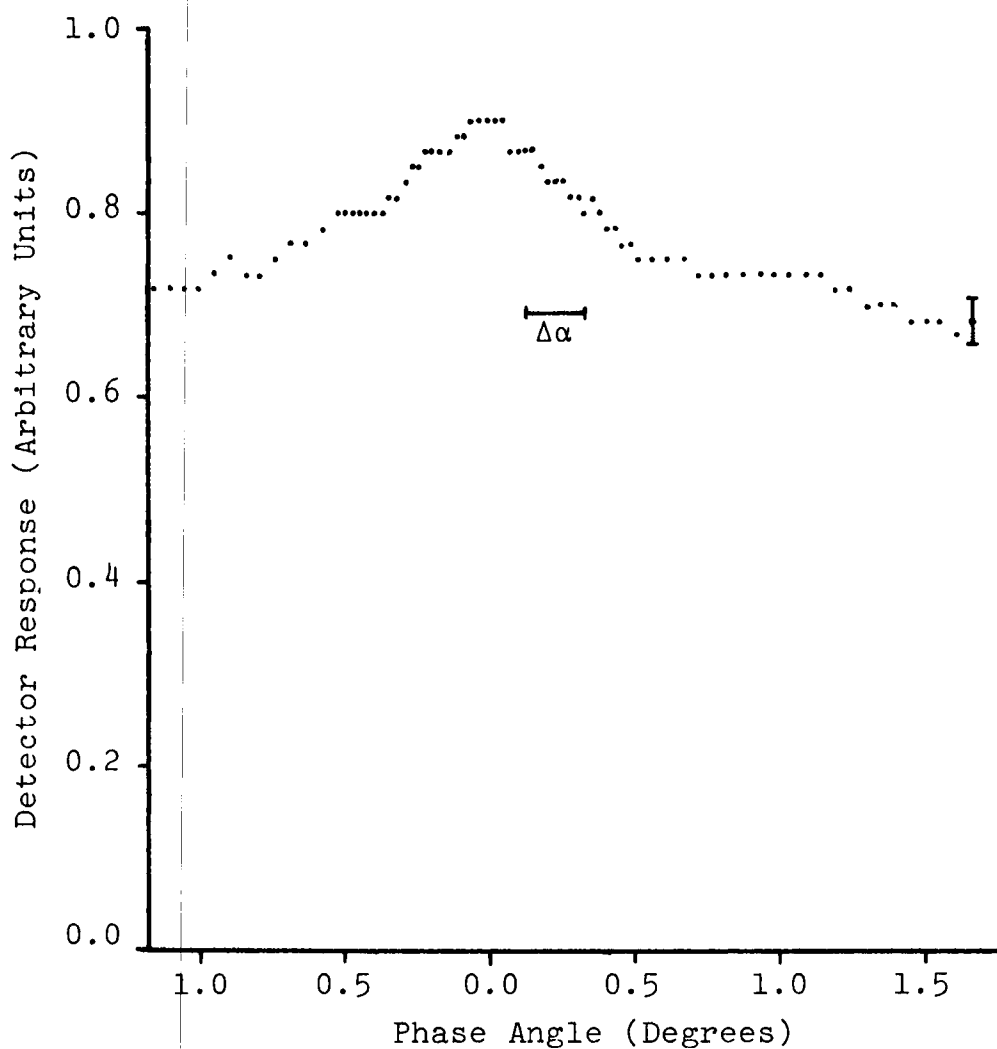


Figure 17. Measured Detector Response as a Function of Phase Angle for a General Shale Products Corporation, Patio Paver Brick Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

A section of used, road-paving asphalt was used to obtain the data shown in Figure 18. Because it had been under use, the sample surface contained a small quantity of embedded sand and dirt. The figure shows that this sample possessed a slight opposition effect enhancement. Figure 19 contains the results of measurements made on a sample of dry, red clay soil. The sample was made up primarily of small clay particles. The resultant reflection diagram shows a broad peak which appears to not be fully resolved at zero degrees phase angle.

A roofing shingle, composed of white, finely crushed stone affixed 3 or 4 layers deep onto a black, asphalt covered felt base, was used to obtain the measurements plotted in Figure 20. Comparison of these results with those in Figure 21, in which a sample made up of dry, light-colored, beach sand was used, shows that the reflection properties of these 2 samples are similar. The sizes of the individual grains making up the near surfaces of the two samples were comparable. However, the grains used in producing the surface of the roofing shingle were more jagged than were the grains of beach sand. The beach sand was also not as white as the roofing shingle surface.

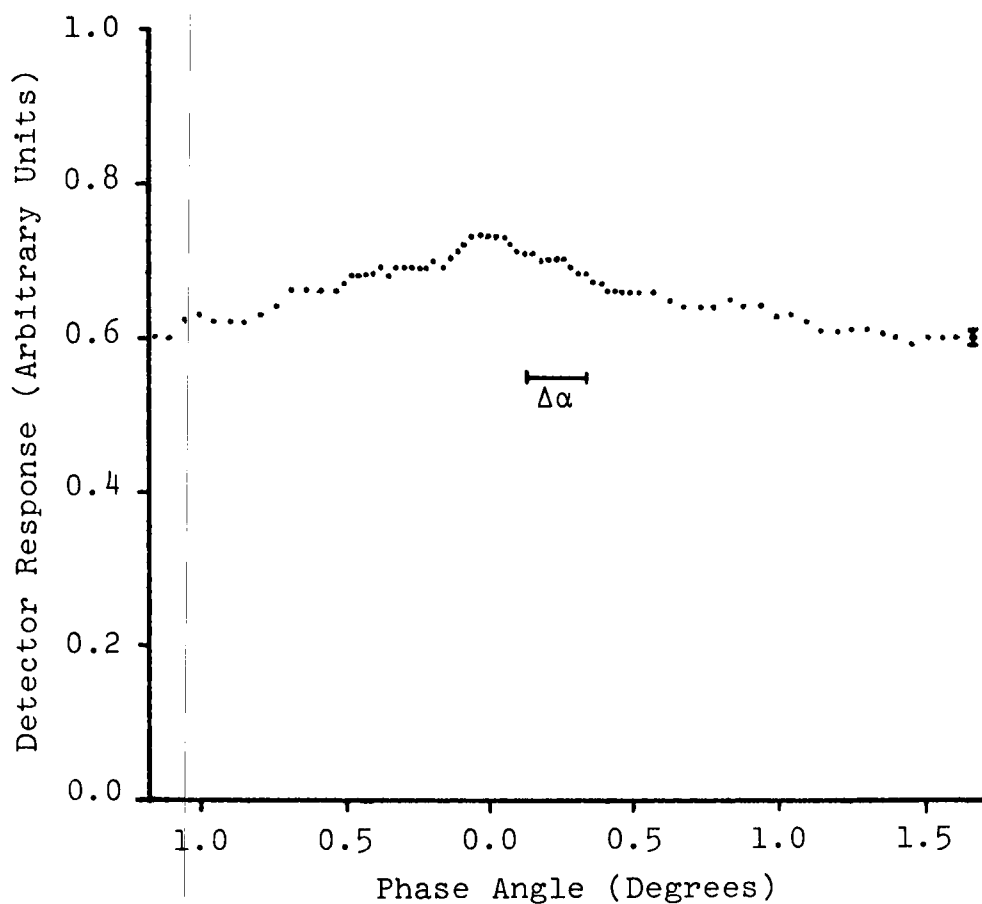


Figure 18. Measured Detector Response as a Function of Phase Angle for a Used, Paving Asphalt Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

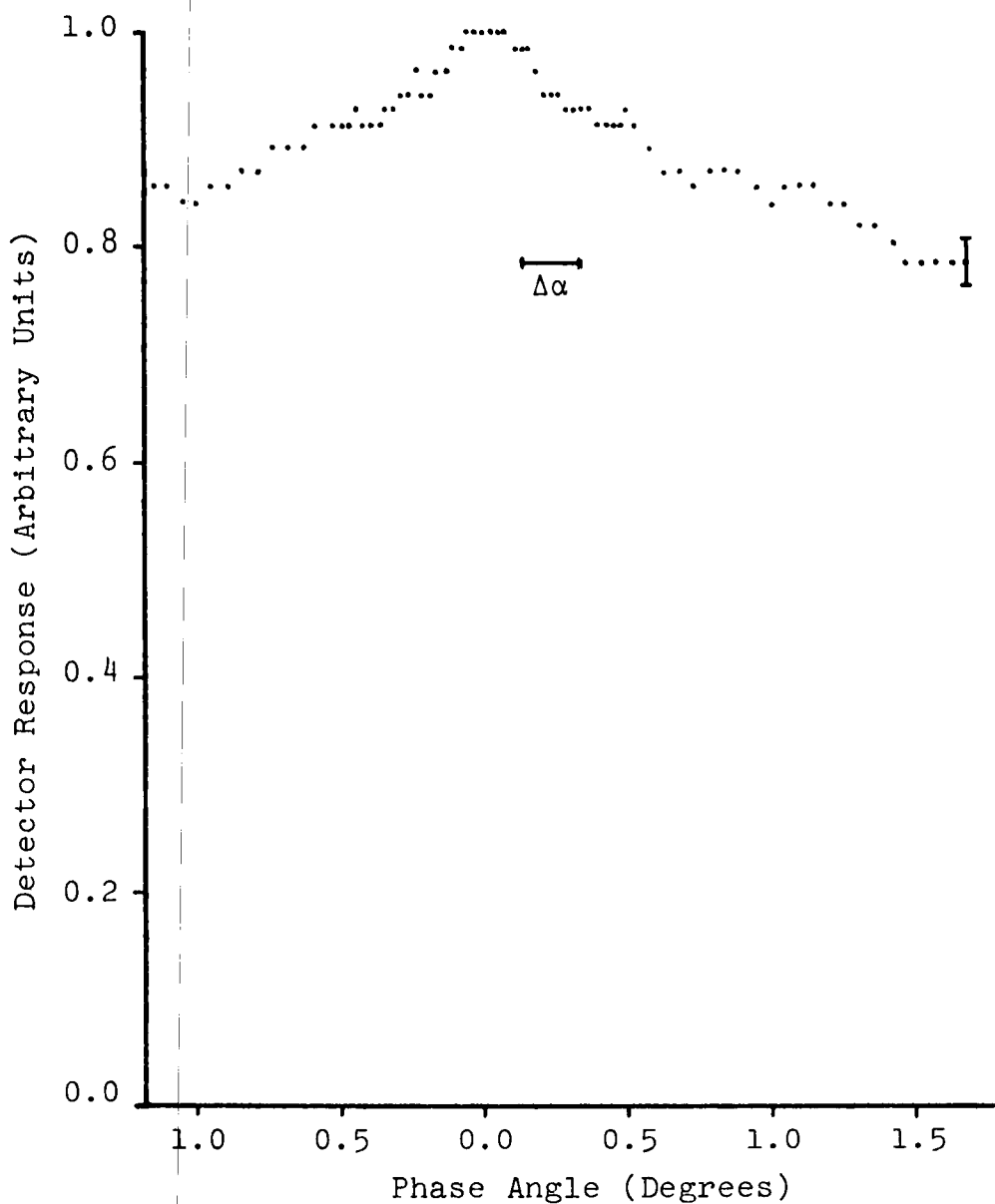


Figure 19. Measured Detector Response as a Function of Phase Angle for a Dry, Red Clay Soil Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

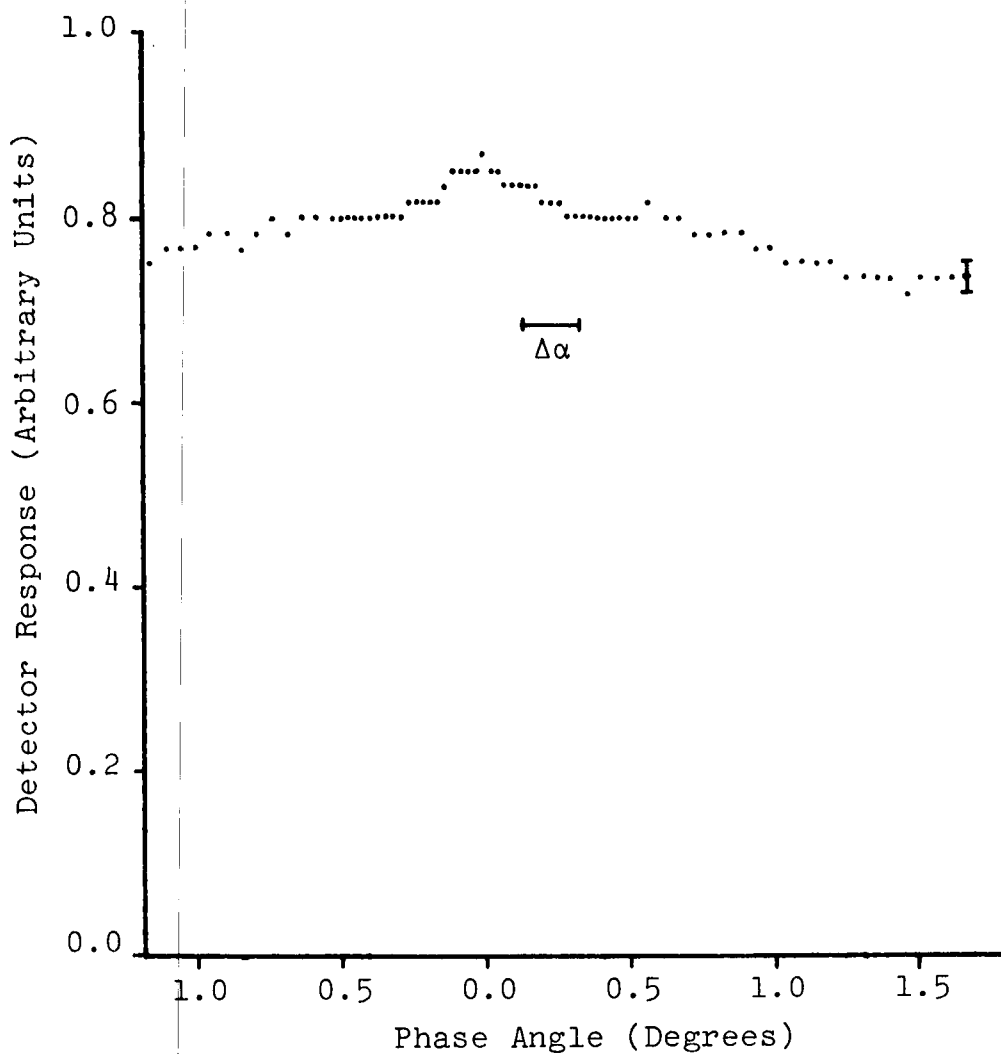


Figure 20. Measured Detector Response as a Function of Phase Angle for a CELOTEX, POLAR WHITE, Sol-Seal/15 Roofing Shingle Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

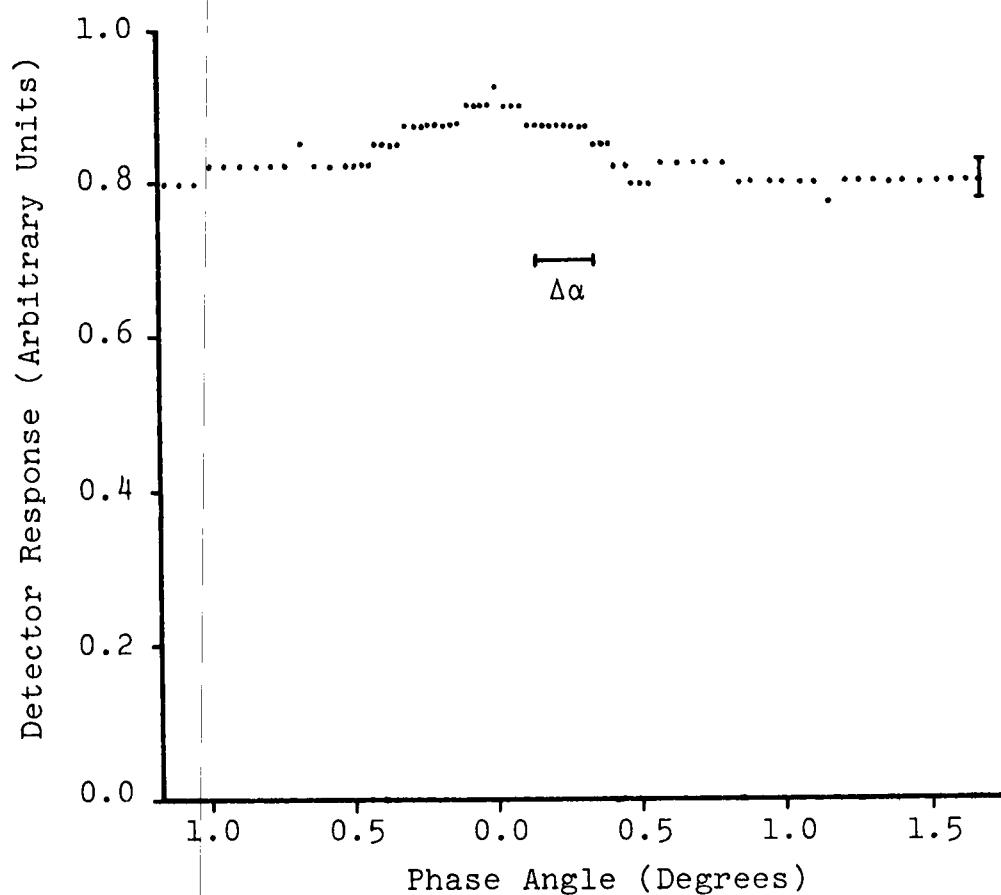


Figure 21. Measured Detector Response as a Function of Phase Angle for a Dry, Beach Sand Sample. The calculated system resolution was $\Delta\alpha = 0.24^\circ$.

Conclusions

The results of this study have shown the opposition effect to be present in the reflection properties, to some extent, of many common materials, including organic materials. While the resolution of the optical system employed in this study was better than in other reported investigations, the retroreflection peak was still not fully resolved for all samples considered. This study has shown that the energy associated with the opposition effect peak is polarized identically to the source polarization. Except for these, only statements concerning general trends may be made in this and other studies of the opposition effect due to a lack of structure characterization of the samples investigated. Unfortunately, it is very difficult to accurately characterize the structure of most materials. Because several mechanisms have been proposed (Trowbridge, 1978) which could contribute to the retroreflection enhancement, many material characteristics need to be measured and compared to opposition effect peak measurements before a general, quantitative theory may be advanced.

The trends noted in the samples investigated in this study are that, in most cases, color and near surface granularity influenced the measured reflection near zero degrees phase angle. Darker, more absorbing, samples displayed less enhancement than did the lighter colored samples. This was possibly due to absorption processes

reducing the depth of penetration of the incident beam into the sample and thus involving fewer material elements in the reflection process. Samples composed of larger structure elements produced a broader peak than those made up of smaller particles. The relative importance of large scale and small scale material elements, where the large scale elements were composed of collections of smaller elements, was unclear from this study.

Future investigations of the opposition effect should center on the determination of the relationship between material structure and the measured characteristics of the material opposition effect peak. As has been demonstrated by this study, optical systems of even higher resolution than was used here are required to fully resolve opposition effect peaks produced by some materials. This, too, should be a consideration in future efforts.

CHAPTER III

SCATTERING OF A PLANE ELECTROMAGNETIC WAVE BY A CIRCULAR CYLINDER EMBEDDED IN A HOMOGENEOUS HALF-SPACE

Introduction

While theoretical treatments of the scattering of electromagnetic radiation by obstacles possessing simple geometric shapes have been carried out for many years, scattering by these same objects in the presence of surfaces and regions of different geometric shape has received less attention. Scattering calculations for simple obstacles are generally based on the assumption that the source, detector and obstacle are all immersed in the same medium, while in many common situations, such as with the opposition effect, the scatterer may be located in a medium separate from that which contains the source and detector. If the scattering obstacle is located near the interface separating the two media, then adjustments of the theoretical predictions are not easily made.

Several investigators (Hohmann, 1971 ; Parry and Ward, 1971; Wait, 1971) have considered the problem of the scattering of radiation from a line source located on the plane interface separating 2 media by a buried cylinder. The source is oriented parallel to the scattering cylinder.

D'Yakonov (1959) approximated the problem of scattering by a circular cylinder contained in a half-space by considering scattering by 2 non-coaxial cylinders illuminated by a point source. Kelly and Schultz (1971) treated the scattering of radiation from a line source by a conducting cylinder located exterior to a dielectric half-space. Their treatment required relatively large values for the dielectric constant of the half-space in order to approximate the field at the cylinder.

The treatment presented in this chapter considers the scattering produced by a plane wave incident upon a circular cylinder located within a medium which is separated from the medium containing the source and detector by a plane interface. The following chapter will address the case of a cylinder located exterior to a half-space. One of the applications of this treatment is to the problem of predicting radar cross sections of objects near the surface of the earth.

Problem Approach

The problem is depicted in Figure 22. The propagation vector of the incident plane wave is orthogonal to the z -axis and, therefore, the incident plane wave is independent of the coordinate z . As the boundaries are also independent of the z -coordinate, the scattered fields will exhibit no z -coordinate dependence. The boundary conditions are time-independent so that all fields exhibit an identical

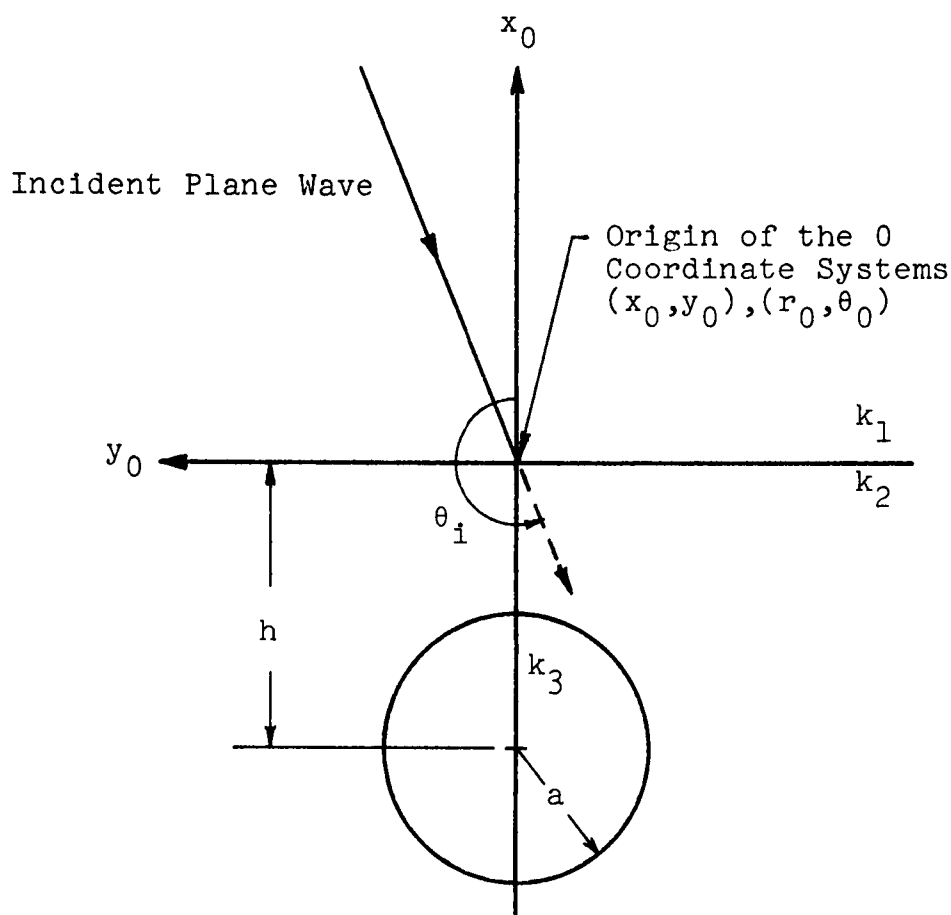


Figure 22. Diagram of the Problem of Scattering of a Plane Electromagnetic Wave by an Infinite Circular Cylinder Embedded in a Homogeneous Half-Space. The plane of incidence is normal to the cylinder axis.

dependence on time and these terms will be suppressed in the analysis. The total fields in each region are expressed as linear sums over field contributions due to the linearity of the wave equation and the boundary conditions.

Four cases are considered; namely, transverse magnetic fields ($E_z \neq 0$, $H_z = 0$) and transverse electric fields ($E_z = 0$, $H_z \neq 0$) with a metal (infinite conductivity) and with a dielectric (zero conductivity) cylinder embedded in a homogeneous half-space. The transverse magnetic and transverse electric field cases are treated separately because, due to the symmetry involved with this problem, boundary conditions may be satisfied in each case without requiring the introduction of fields of the other type. For the more general case of an incident wave being neither purely transverse magnetic nor transverse electric, results obtained here may be used by first resolving the incident wave into its transverse magnetic and transverse electric field components, then combining the weighted solutions as a final step.

Problem analysis may be simplified by utilizing scalar auxiliary functions from which the non-zero field components may be derived. These auxiliary functions are designated u , for those associated with transverse magnetic fields, and v , for those associated with transverse electric fields. Both u and v are solutions of the two-dimensional scalar Helmholtz equation:

$$\frac{1}{r} \left[\frac{\partial}{\partial r} r \frac{\partial(u,v)}{\partial r} + \frac{1}{r} \frac{\partial^2(u,v)}{\partial \theta^2} \right] + k^2(u,v) = 0 \quad (1)$$

The fields are related to the auxiliary functions (Ivanov, 1970) by

$$E_r = 0 \quad , \quad (2a)$$

$$E_\theta = 0 \quad , \quad (2b)$$

$$E_z = k^2 u \quad ; \quad (2c)$$

$$H_r = - \frac{i\omega k^2}{\mu c} \frac{1}{r} \frac{\partial u}{\partial \theta} \quad , \quad (3a)$$

$$H_\theta = \frac{i\omega k^2}{\mu c} \frac{\partial u}{\partial r} \quad , \quad (3b)$$

and

$$H_z = 0 \quad (3c)$$

for transverse magnetic fields, and

$$E_r = \frac{i\mu c}{\omega} \frac{1}{r} \frac{\partial v}{\partial \theta} \quad , \quad (4a)$$

$$E_\theta = - \frac{i\mu c}{\omega} \frac{\partial v}{\partial r} \quad , \quad (4b)$$

$$E_z = 0 \quad ; \quad (4c)$$

$$H_r = 0 \quad , \quad (5a)$$

$$H_\theta = 0 \quad , \quad (5b)$$

and

$$H_z = k^2 v \quad (5c)$$

for transverse electric fields; where (r, θ, z) are the coordinates of the cylindrical coordinate system, ω is the angular frequency of the electromagnetic wave, k is the propagation constant of the medium, and μ is

the magnetic permeability. For all cases considered here, the magnetic permeability is taken to be equal to one for all media.

The auxiliary functions are expanded in complete sets of orthogonal functions with unknown expansion coefficients. These expansion coefficients are determined by applying the boundary conditions over the interfaces and at infinity in each region. The boundary conditions at the interfaces require the continuity of the tangential components of the electric and magnetic fields across the boundaries while at points far removed from the boundaries, outgoing waves are required. The geometries of the interfaces separating the regions of differing propagation constants suggest that plane wave expansions and Fourier-Bessel expansions be used as the complete sets over which auxiliary functions are to be expanded.

A complete plane wave expansion (Clemmow, 1966) must include all homogeneous and inhomogeneous waves. For these, the angle between the propagation vector and the vector, $\bar{r}$, must be allowed to assume all real values from $-\pi$ to $+\pi$ and all imaginary values from $-i\infty$ to $+i\infty$. The terminal points of integration must be chosen such that the integral converges. A complete Fourier-Bessel expansion includes (Stratton, 1941) both Bessel functions, $J_n(\rho)$, and Hankel functions of the first kind, $H_n(\rho)$.

In the upper half-space, the total field includes contributions from the incident plane wave and scattered waves. The auxiliary function for the latter is represented by a plane wave expansion associated with the (r_0, θ_0) coordinate system (Figure 23). Within the homogeneous half-space, excluding the cylinder, the auxiliary function for the total field is taken to be the sum of a Fourier-Bessel expansion, associated with the (r_1, θ_1) coordinate system, and a plane wave expansion associated with the (r_2, θ_2) coordinate system. Note that this last coordinate system is rotated 180 degrees about the z-axis relative to the others. In the case of the field within the cylinder, the auxiliary function is expanded in a Fourier-Bessel expansion associated with the (r_1, θ_1) coordinate system.

The boundary condition of outgoing waves at infinity requires that the expansion coefficients of the $J_n(\rho)$ Bessel functions all be equal to zero for the auxiliary function within the homogeneous half-space excluding the cylinder. This also requires that the ranges of the 2 plane wave expansions be truncated so that now the angles over which the integration is performed are limited to real values from $-\pi/2$ to $+\pi/2$ and imaginary values from $-i\infty$ to $+i\infty$. Within the cylinder, the field must remain finite as r_1 approaches zero. Therefore, the expansion coefficients for the Hankel functions must all be set equal to zero. Further, note that the plane incident wave and the plane

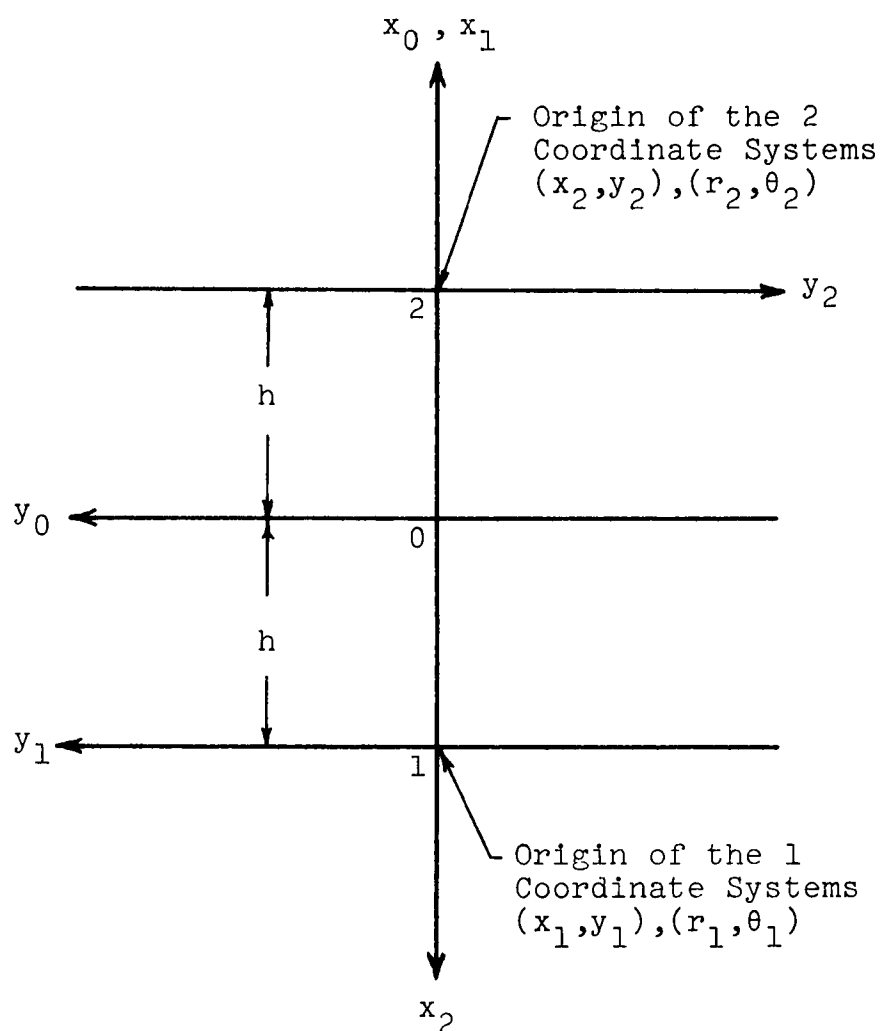


Figure 23. Coordinate Systems Used in Embedded Cylinder Scattering Problem.

interface separating the two half-spaces are both infinite in extent while the cylinder possesses a finite cross section. Clearly, at points infinitely distant from the cylinder there will be single plane propagating waves in the directions of simple reflection from the plane interface and simple refracted transmission through the plane interface. These will be modified by the existence of scattered fields due to the presence of the cylinder. For convenience, these plane waves are added to the total field expressions in their respective half-spaces so that the scattered fields need only account for the remaining contributions to the total fields.

Due to the linearity of the boundary condition equations, the auxiliary functions for the plane incident, reflected, and refracted waves may be required to satisfy the boundary conditions over the plane interface independent of the other scattered fields. This procedure meets the requirement that the total fields in each of the regions must satisfy the boundary conditions. The unique relationship between these waves and the other scattered waves will be determined when the boundary conditions over the surface of the cylinder are applied.

Within the cylinder, the propagation constant is represented by k_3 . In this region, the scalar auxiliary function is now expanded in the Fourier-Bessel expansion:

$$u_4 = \sum_n t_n J_n(k_3 r_1) \exp\{in\theta_1\} \quad , \quad (6)$$

where t_n is an unknown expansion coefficient (for a metal cylinder, all t_n are zero).

In the homogeneous half-space excluding the cylinder the total scalar auxiliary function is equal to $u_1 + u_2 + u^t$ with

$$u_1 = \sum_n p_n H_n(k_2 r_1) \exp\{in\theta_1\} \quad (7)$$

and

$$u_2 = \int_c q(\theta'') \exp\{ik_2 r_2 \cos(\theta_2 - \theta'')\} d\theta'' \quad , \quad (8)$$

where k_2 is the propagation constant for the homogeneous half-space, p_n is an unknown expansion coefficient, $q(\theta'')$ is an unknown weighting function and u^t is the auxiliary function for the refracted plane wave. The contour of integration, c , in equation (8) extends from $-\pi/2 + i\infty$ to $+\pi/2 - i\infty$.

In the medium exterior to the homogeneous half-space, the propagation constant is represented by k_1 . In this region, the auxiliary function for the total field is equal to $u^0 + u^r + u_3$ with u^0 and u^r representing the auxiliary functions for the plane incident wave and the plane refracted wave respectively, and

$$u_3 = \int_c s(\theta''') \exp\{ik_1 r_0 \cos(\theta_0 - \theta''')\} d\theta''' , \quad (9)$$

where $s(\theta''')$ is the unknown weighting function for this expansion.

Single modes of either the Fourier-Bessel expansions or the plane wave expansions are related to sums, either discrete or continuous, of the other (Ivanov, 1970; Stratton, 1941) by the following expressions:

$$\exp\{\pm ikr \sin\theta\} = \sum_n (\pm 1)^n J_n(kr) \exp\{in\theta\} , \quad (10)$$

$$J_n(kr) \exp\{in\theta\} = \frac{(-1)^n}{2\pi} \int_0^{2\pi} \exp\{ikr \cos(\theta - \theta') + in\theta'\} d\theta' , \quad (11)$$

and

$$H_n(kr) \exp\{\pm in\theta\} = \frac{(-1)^n}{\pi} \int_c \exp\{ikr \cos(\theta \mp \theta') + in\theta'\} d\theta' \quad (12)$$

for $|\theta| < \pi/2$, and the contour of integration, c , extends from $-\pi/2 + i\infty$ to $+\pi/2 - i\infty$. The integration variable has an imaginary component due to the inclusion of inhomogeneous waves in the expression.

While scalar auxiliary functions have been defined in this section for transverse magnetic fields only, definitions of the transverse electric auxiliary functions are similar.

Transverse Magnetic Fields--Metal Cylinder Case

In this section, the case of a transverse magnetic plane wave incident on a homogeneous half-space with a metal (infinite conductivity) cylinder embedded in it, is considered.

A plane polarized wave ($E_x^0 = E_y^0 = 0$) is incident on the half-space/cylinder system, traveling in the direction $\theta_0 = \theta_i$. Then

$$E_z^0 = E^0 \exp\{ik_1 r_0 \cos(\theta_0 - \theta_i)\} \quad , \quad (13)$$

where E^0 is the amplitude of the incident wave,

$$v^0 = 0 \quad , \quad (14)$$

and

$$u^0 = \frac{E^0}{k_1^2} \exp\{ik_1 r_0 \cos(\theta_0 - \theta_i)\} \quad . \quad (15)$$

The non-zero boundary conditions require that the radial component of the magnetic field (H_r) and the z component of the electric field (E_z) be continuous across the plane interface. The boundary conditions require the existence of two additional plane waves: one in the region containing the incident wave traveling in direction $\theta_0 = \theta_r$ and another in the homogeneous half-space traveling in direction $\theta_0 = \theta_t$. The appropriate field functions for these are

$$u^r = q^r \exp\{ik_1 r_0 \cos(\theta_0 - \theta_r)\} \quad , \quad (16)$$

and

$$u^t = s^t \exp\{ik_2 r_0 \cos(\theta_0 - \theta_t)\} \quad (17)$$

The boundary conditions require

$$k_1^2 \frac{\partial}{\partial \theta_0} (u^0 + u^r) = k_2^2 \frac{\partial}{\partial \theta_0} (u^t) \quad (18a)$$

and

$$k_1^2 (u^0 + u^r) = k_2^2 u^t \quad (18b)$$

be satisfied on the plane interface defined by

$$\theta_0 = \pm \frac{\pi}{2}.$$

The existence of boundary conditions on this interface, independent of a particular value of r_0 , imply

$$k_1 \cos(\pm \frac{\pi}{2} - \theta_i) = k_1 \cos(\pm \frac{\pi}{2} - \theta_r) = k_2 \cos(\pm \frac{\pi}{2} - \theta_t) \quad (19)$$

Then

$$\theta_r = \pi - \theta_i \quad (20a)$$

and

$$\theta_t = \sin^{-1} \left[\frac{k_1}{k_2} \sin \theta_i \right] \quad (20b)$$

Using these relationships, the boundary conditions become

$$k_1 E^0 \cos \theta_i - k_1^3 q^r \cos \theta_i = k_2^3 s^t \cos \theta_t, \quad (21)$$

where

$$\cos \theta_t = - \left[1 - \frac{k_1^2}{k_2^2} \sin^2 \theta_i \right]^{1/2} \quad (22)$$

and

$$E^0 + k_1^2 q^r = k_2^2 s^t . \quad (23)$$

Combining equations (21) and (23) leads to

$$q^r = \frac{E^0}{k_1^2} \frac{k_1 \cos \theta_i - k_2 \cos \theta_t}{k_1 \cos \theta_i + k_2 \cos \theta_t} \quad (24)$$

and

$$s^t = \frac{E^0}{k_2^2} \frac{2 k_1 \cos \theta_i}{k_1 \cos \theta_i + k_2 \cos \theta_t} . \quad (25)$$

Thus, the non-zero electric field components are

$$E_z^r = k_1^2 q^r = E^0 \frac{k_1 \cos \theta_i - k_2 \cos \theta_t}{k_1 \cos \theta_i + k_2 \cos \theta_t} \quad (26)$$

and

$$E_z^t = k_2^2 s^t = E^0 \frac{2 k_1 \cos \theta_i}{k_1 \cos \theta_i + k_2 \cos \theta_t} . \quad (27)$$

Next consider the boundary conditions required on the plane interface applied to the scattered fields.

The field auxiliary functions are

$$u_1 = \sum_n p_n H_n(k_2 r_1) \exp(in\theta_1) , \quad (28)$$

$$u_2 = \int_c q(\theta'') \exp\{ik_2 r_2 \cos(\theta_2 - \theta'')\} d\theta'' , \quad (29)$$

and

$$u_3 = \int_c s(\theta''') \exp\{ik_1 r_0 \cos(\theta_0 - \theta''')\} d\theta''' . \quad (30)$$

Using equation (12), equation (28) may be transformed:

$$u_1 = \sum_n p_n \int_c \frac{(-i)^n}{\pi} \exp\{ik_2 r_1 \cos(\theta_1 - \theta') + in\theta'\} d\theta' \quad . \quad (31)$$

The boundary conditions on the plane interface require

$$k_2^2(u_1 + u_2) = k_1^2 u_3 \quad (32)$$

and

$$k_2^2 \frac{\partial}{\partial \theta_0} (u_1 + u_2) = k_1^2 \frac{\partial}{\partial \theta_0} u_3 \quad , \quad (33)$$

where the equations are to be evaluated on the plane interface, which, in Cartesian coordinates, is defined by $x_0 = 0$. The equations that relate the various coordinate systems to the Cartesian coordinate system with its origin on the interface are

$$r_1 \cos \theta_1 = x_1 = x_0 + h \quad , \quad (34a)$$

$$r_1 \sin \theta_1 = y_1 = y_0 \quad , \quad (34b)$$

$$r_2 \cos \theta_2 = x_2 = -x_0 + h \quad , \quad (34c)$$

$$r_2 \sin \theta_2 = y_2 = -y_0 \quad , \quad (34d)$$

$$r_0 \cos \theta_0 = x_0 \quad , \quad (34e)$$

and

$$r_0 \sin \theta_0 = y_0 \quad . \quad (34f)$$

Placing the more stringent requirement on the field functions that not only must the full field functions satisfy the boundary conditions, but also the individual wavelets which make up the integral

representations of these field functions, results in the following equation for the boundary condition (32):

$$\begin{aligned}
 & k_2^2 \sum_n p_n \frac{(-1)^n}{\pi} \exp\{ik_2 h \cos \theta' + ik_2 y_0 \sin \theta' + in\theta'\} d\theta' \\
 & + k_2^2 q(\theta'') \exp\{ik_2 h \cos \theta'' - ik_2 y_0 \sin \theta''\} d\theta'' \\
 & = k_1^2 s(\theta''') \exp\{ik_1 y_0 \sin \theta'''\} d\theta''' \quad .
 \end{aligned} \tag{35}$$

Over the plane, the boundary conditions must be satisfied for each value of y_0 . This will be assured if each term in the equation exhibits an identical dependence on the variable y_0 . That is

$$k_2 \sin \theta' = -k_2 \sin \theta'' = k_1 \sin \theta''' \quad , \tag{36}$$

then

$$\theta'' = -\theta' \quad , \tag{37}$$

$$\theta''' = \sin^{-1} \left[\frac{k_2}{k_1} \sin \theta' \right] \quad , \tag{38}$$

and therefore

$$d\theta'' = -d\theta' \quad , \tag{39}$$

and

$$\begin{aligned}
 d\theta''' & = + \left[1 - \left(\frac{k_2}{k_1} \sin \theta' \right)^2 \right]^{-1/2} \frac{k_2}{k_1} \cos \theta' d\theta' \\
 & = \frac{k_2}{k_1} \frac{\cos \theta'}{\cos \theta'''} d\theta' \quad ,
 \end{aligned} \tag{40}$$

where the positive sign is given by the definition of

the integral contour c which is appropriate for u_3 , equation (9) with its coordinate system orientation.

The boundary condition equation (32) now requires

$$k_2 \exp\{ik_2 h \cos \theta'\} \left[\sum_n p_n \frac{(-i)^n}{\pi} e^{in\theta'} - q \right] = k_1 s \frac{\cos \theta'}{\cos \theta'''} . \quad (41)$$

Now consider the other boundary condition, equation (33), and note that

$$\frac{\partial}{\partial \theta_0} = x_0 \frac{\partial}{\partial y_0} - y_0 \frac{\partial}{\partial x_0} . \quad (42)$$

Then, proceeding as before, on the plane defined by $x_0 = 0$, this boundary condition requires

$$k_2^2 \exp\{ik_2 h \cos \theta'\} \left[\sum_n p_n \frac{(-i)^n}{\pi} e^{in\theta'} + q \right] = k_1^2 s . \quad (43)$$

Combining equations (41) and (43) results in

$$q(\theta'') = \sum_n p_n \frac{(-i)^n}{\pi} e^{-in\theta''} \left[\frac{k_1 \cos \theta''' - k_2 \cos \theta''}{k_1 \cos \theta''' + k_2 \cos \theta''} \right] \quad (44)$$

and

$$s(\theta''') = \sum_n \frac{k_2^2}{k_1^2} p_n \frac{(-i)^n}{\pi} \exp\{ik_2 h \cos \theta' + in\theta'\} \left[\frac{2 k_1 \cos \theta'''}{k_1 \cos \theta''' + k_2 \cos \theta'} \right] \quad (45)$$

where the relationships between θ' , θ'' and θ''' are given by equations (36) - (40). These equations, along with equations (29) and (30) provide expressions of the field functions u_2 and u_3 in terms of the expansion coefficients p_n . To determine these coefficients, the boundary conditions on the surface of the cylinder are used.

Up to this point, the derivation has been independent of the particular characteristics of the cylinder. Now, two cases are considered in turn; first, the case of a metal cylinder, then the case of a dielectric cylinder.

A metal cylinder of radius $r_1 = a$ and center located distance h below the plane interface separating the two half-spaces has, as a boundary condition,

$$u^t + u_1 + u_2 = 0 \quad (46)$$

over the surface of the cylinder, and where u^t is determined by equations (17) and (25), u_1 is defined by equation (28), and u_2 is determined by equations (29) and (44) in terms of the expansion coefficients, p_n . The (r_0, θ_0) and (r_2, θ_2) coordinates may be transformed to (r_1, θ_1) by the following:

$$r_0 \cos \theta_0 = r_1 \cos \theta_1 - h, \quad (46a)$$

$$r_0 \sin \theta_0 = r_1 \sin \theta_1, \quad (46b)$$

and

$$r_2 \cos(\theta_2 - \theta'') = -r_2 \cos(\theta_1 - \theta'') + 2h \cos \theta''. \quad (46c)$$

Using these equations and the identity

$$\exp\{ikr \cos(\theta - \theta')\} = \sum_n i^n J_n(kr) \exp\{in(\theta - \theta')\}, \quad (47)$$

the field functions may now be expressed

$$u_1 = \sum_n p_n H_n(k_2 r_1) e^{in\theta_1} \quad (48)$$

$$u_2 = \int_c q \exp\{2ik_2 h \cos \theta'' - ik_2 r_1 \cos(\theta_1 - \theta'')\} d\theta'' \quad , \quad (49)$$

and

$$u^t = s^t \exp\{-ik_2 h \cos \theta_t\} \sum_n i^n J_n(k_2 r_1) e^{in(\theta_1 - \theta_t)} \quad , \quad (50)$$

where

$$q(\theta'') = \sum_n p_n \frac{(-i)^n}{\pi} e^{-in\theta''} \frac{k_1 \cos \theta''' - k_2 \cos \theta''}{k_1 \cos \theta''' + k_2 \cos \theta''} \quad , \quad (51)$$

$$\cos \theta''' = \left[1 - \left(\frac{k_2}{k_1} \sin \theta'' \right)^2 \right]^{1/2} \quad , \quad (52)$$

and s^t is given by equation (25).

Next, substituting equations (48) - (50) into the boundary condition equation (46), multiplying through by $\exp\{-im\theta_1\} d\theta_1$, integrating from 0 to 2π , and noting that (Ivanov, 1970; Stratton, 1941)

$$\begin{aligned} \int_0^{2\pi} \exp\{-ik_2 a \cos(\theta_1 - \theta'') - im\theta_1\} d\theta_1 &= \sum_n (-i)^n J_n(k_2 a) e^{-im\theta''} \\ &\quad \int_0^{2\pi} e^{i(n-m)\theta_1} d\theta_1 \\ &= 2\pi (-i)^m J_m(k_2 a) e^{-im\theta''} \quad ; \end{aligned} \quad (53)$$

the boundary condition now requires

$$p_m = \frac{J_m(k_2 a)}{H_m(k_2 a)} \left[-s_t i^m \exp\{-ik_2 h \cos \theta_t - im\theta_t\} - (-i)^m \int_c q \exp\{2ik_2 h \cos \theta'' - im\theta''\} d\theta'' \right]. \quad (54)$$

Because q has been determined as a function of p_n in equation (51), equation (54) actually represents a system of simultaneous equations which remain to be solved. For numerical results, this solution may be accomplished by an iterative procedure which continues until convergence is attained. Truncation is required for such a procedure and this imposes a limit on the cases which may be considered. Finally, the integral in equation (54) must be evaluated. In general, the integral does not have a closed solution; however, the integral may be determined approximately for the cases when the quantity $2k_2 h$ is large.

Postponing further evaluation of equation (54) until the next section, the solution of this equation for all m does, in principle, provide all of the expansion coefficients, p_m . Thus, the field functions may now be written in terms of the p_m expansion coefficients and, from these, the total electric fields in the various regions may be determined. The field auxiliary functions are

$$u^0 = \frac{E^0}{k_1^2} \exp\{ik_1 r_0 \cos(\theta_0 - \theta_1)\} , \quad (55)$$

$$u^r = \frac{E^0}{k_1^2} \frac{k_1 \cos \theta_1 - k_2 \cos \theta_t}{k_1 \cos \theta_1 + k_2 \cos \theta_t} \exp\{ik_1 r_0 \cos(\theta_0 - \theta_r)\} , \quad (56)$$

$$u^t = \frac{E^0}{k_2^2} \frac{2k_1 \cos \theta_1}{k_1 \cos \theta_1 + k_2 \cos \theta_t} \exp\{ik_2 r_0 \cos(\theta_0 - \theta_t)\} , \quad (57)$$

$$u_1 = \sum_n p_n H_n(k_2 r_1) \exp(in\theta_1) , \quad (58)$$

$$u_2 = \int_c q(\theta'') \exp\{ik_2 r_2 \cos(\theta_2 - \theta'')\} d\theta'' , \quad (59)$$

$$u_3 = \int_c s(\theta''') \exp\{ik_1 r_0 \cos(\theta_0 - \theta''')\} d\theta''' , \quad (60)$$

where θ_1 is the angle of incidence,

$$\theta_r = \pi - \theta_1 , \quad (61)$$

$$\theta_t = \sin^{-1} \left(\frac{k_1}{k_2} \sin \theta_1 \right) , \quad (62)$$

$$q(\theta'') = \sum_n p_n \frac{(-1)^n}{\pi} e^{in\theta''} \frac{k_1 \cos \theta''' - k_2 \cos \theta''}{k_1 \cos \theta''' + k_2 \cos \theta''} , \quad (63)$$

$$s(\theta''') = \sum_n \frac{k_2^2}{k_1^2} p_n \frac{(-1)^n}{\pi} \frac{2k_1 \cos \theta'''}{k_1 \cos \theta''' + k_2 \cos \theta''} \exp\{ik_2 h \cos \theta' + in\theta'\} , \quad (64)$$

$$\cos \theta''' = \left[1 - \left(\frac{k_2}{k_1} \sin \theta'' \right)^2 \right]^{1/2} , \quad (65)$$

and

$$\theta' = \sin^{-1} \left[\frac{k_1}{k_2} \sin \theta''' \right] . \quad (66)$$

The electric field in the region exterior to the homogeneous half-space is given by

$$E_z = k_1^2 (u^0 + u^r + u_3) \quad (67)$$

and the electric field in the homogeneous half-space surrounding the cylinder is

$$E_z = k_2^2 (u^t + u_1 + u_2) . \quad (68)$$

Before proceeding to numerical results, it is instructive to examine two limiting cases of the solution previously obtained and compare these results with the known solutions. In the first case, the cylinder is mathematically removed by letting its radius approach zero; then in this limit, $J_m(k_2 a)/H_m(k_2 a)$ approaches zero and $p_m = 0$ for all m . With all $p_m = 0$, $q = 0$ and $s = 0$. The remaining field potentials, equations (56) and (57), are simply those of plane waves reflected and transmitted by the plane interface separating 2 homogeneous media (Born and Wolf, 1975). For the second case, the two half-spaces are mathematically combined into one by letting $k_2 = k_1$, thus the problem should become that of a plane, transverse magnetic wave incident upon a metal cylinder of radius a . If $k_2 = k_1$,

then $\theta_t = \theta_i$, $u^r = 0$ and $u^t = u^0$; thus, the reflected plane wave vanishes and the transmitted plane wave becomes identical to the incident wave. For the scattered fields, $\theta''' = \theta'$ so $u_2 = 0$ and, except for a phase term due to the translated coordinate system, u_3 is identical to u_1 . Thus, in this limiting case, this analysis yields as a result, a scattered electric field

$$E_s = -E^0 \sum_n i^n \frac{J_n(k_1 a)}{H_n(k_1 a)} H_n(k_1 r_1) \exp\{in(\theta_1 - \theta_i)\} \quad , \quad (69)$$

which is the correct solution (King and Wu, 1959).

Numerical Results--Metal Cylinder Case

It was desired to obtain the calculated far-field scattering diagram, in the quarter-space containing the source of the incident plane wave, for some metal cylinders, each embedded in a dielectric half-space. The differential cross section is defined as the limit, as r approaches infinity, of $2\pi r \langle S_s \rangle / \langle S_i \rangle$, where r is the radial coordinate of the observation point, $\langle S_s \rangle$ is the time-averaged Poynting vector of the scattered fields and $\langle S_i \rangle$ is the time-averaged Poynting vector of the incident field. This quantity was a function of observation angle. Further, the differential cross section was divided by 2π times the radius, a , of

the cylinder in order to non-dimensionalize the results.

Before the differential scattering cross section could be determined, equation (54) had to be solved for all p_m which contributed significantly to the results. Provided the quantity, $2k_2h$, was large, the term $(k_1 \cos \theta''' - k_2 \cos \theta'') / (k_1 \cos \theta''' + k_2 \cos \theta'')$ in the integral could be evaluated (Clemmow, 1966) at the saddle point and taken out of the integral. Once this was done, the remaining integral could be related to that of a Hankel function of argument, $2k_2h$, and order, $n+m$. In the analysis performed, the quantity, $2k_2h$, was restricted to values of 60 or greater.

With these approximations, equation (54) then became

$$p_m = - \frac{J_m(k_2 a)}{H_m(k_2 a)} \left[s^t i^m \exp\{-ik_2 h \cos \theta_t - im\theta_t\} + \left(\frac{k_1 - k_2}{k_1 + k_2} \right) \sum_n p_n H_{n+m}(2k_2 h) \right], \quad (70)$$

where s^t is given by equation (25). Equation (70) was solved in an iterative manner by computer. The number of equations was truncated by neglecting coefficients whose absolute values were less than that of the largest coefficient by eight orders of magnitude. The field auxiliary function of interest was

$$u_3 = \int_c s(\theta''') \exp\{ik_1 r_0 \cos(\theta_0 - \theta''')\} d\theta''' \quad (71)$$

where $s(\theta''')$ is given by equation (45). Because the field was to be evaluated in the far-field, r_0 was quite large and the integral in equation (71) could be approximated without difficulty by using the method of steepest descents (Born and Wolf, 1975).

Figure 24 shows the results of performing the calculation of the scattering diagram for a metal cylinder of radius equal to $20/2\pi$ times the wavelength, λ , in the upper medium; in the quarter-space containing the source of the plane, incident wave. The angle of incidence was set at 45 degrees from the interface normal. The center of the cylinder was separated from the plane interface by a length equal to $24\lambda/2\pi$. The index of refraction of the dielectric half-space was 1.33 times that of the upper half-space.

Figure 25 is included for comparison purposes and shows the scattering diagram for a metal cylinder, of the same radius, in a dielectric full-space ($n = 1.33$). Comparison of these two figures demonstrates that the presence of the dielectric half-space shifted and attenuated the peaks in the scattering diagram of the cylinder. The attenuation of the scattered energy increased as the observation angle approached 90 degrees from the normal to the plane interface. The dielectric

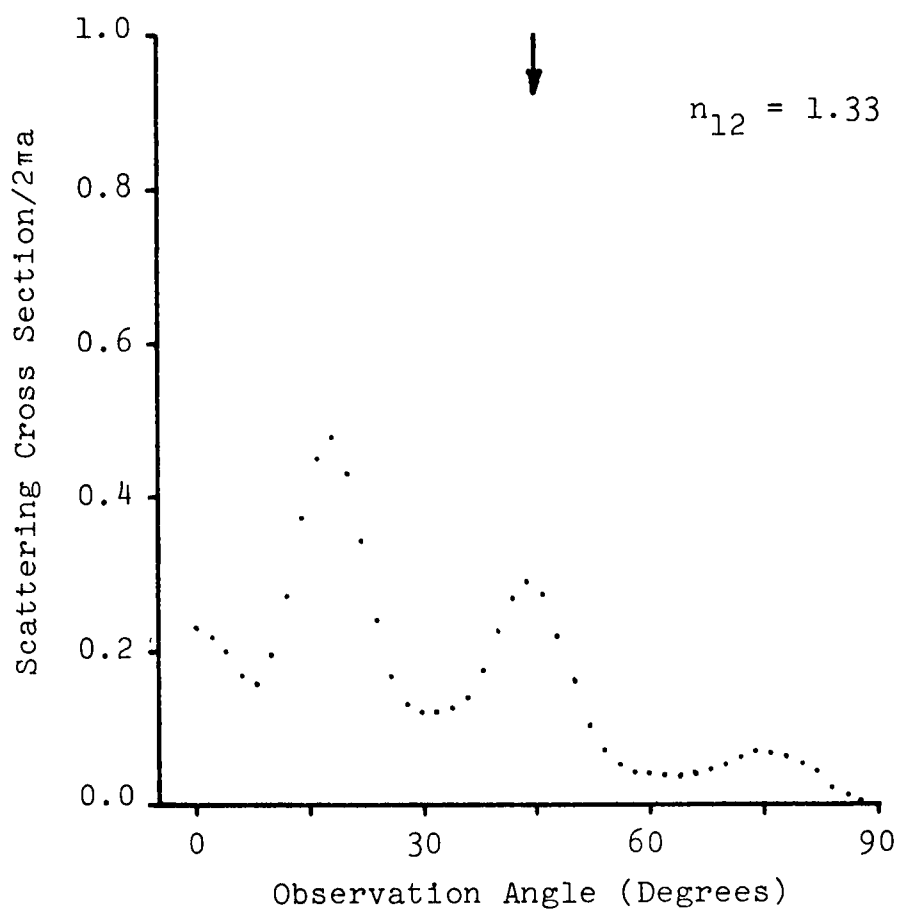


Figure 24. Calculated Results for an Embedded Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $24\lambda/2\pi$, Below the Plane Interface Separating the 2 Half-Spaces and a Plane Wave Incident at 45° .

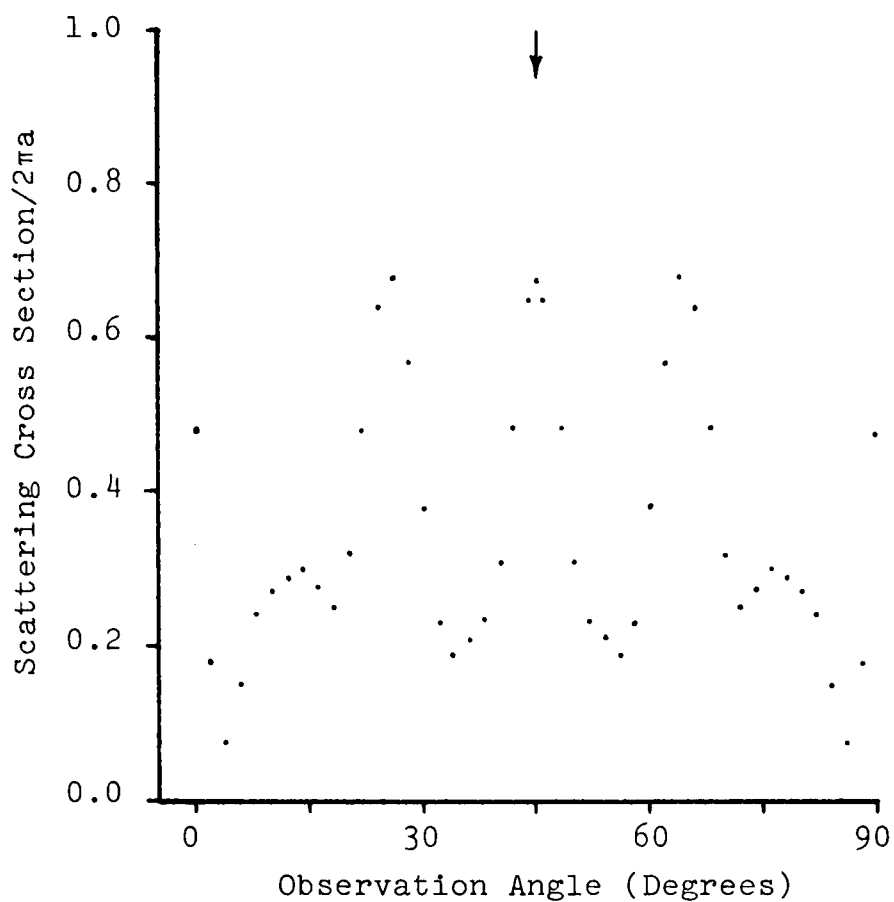


Figure 25. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) in a Homogeneous Dielectric and a Plane Wave Incident at 45° . The index of refraction of the dielectric full-space is 1.33 .

half-space had the effect of reducing the structure otherwise present in the scattering diagram of the cylinder.

Figure 26 is similar to Figure 24 except here the cylinder/interface separation was increased to $25\lambda/2\pi$. This slight change of $\lambda/2\pi$ in separation resulted in a decrease in the height of the first scattering peak located near 17 degrees while the second peak near 45 degrees was broadened and slightly enhanced. Figure 27 shows the effect of increasing the separation distance to $50\lambda/2\pi$.

The values of the parameters used to produce the results shown in Figure 28 were the same as those used in Figure 26 except that the angle of incidence was set at 5 degrees. For comparison, Figure 29 presents the calculated relative scattering cross section produced by the same cylinder but in a homogeneous dielectric.

Figure 30 graphically presents the calculated scattering associated with a cylinder of radius, $\lambda/2\pi$. As is shown by Figure 31, this small cylinder normally scattered uniformly in the back hemisphere.

In each case considered, in which the cylinder exhibited a highly structured scattering diagram when located in a dielectric full-space, the scattering diagram of this same cylinder in a dielectric half-space produced a peak near the backscatter direction. While the figures display scattering cross section values every two degrees, the scattering diagrams were actually calculated in

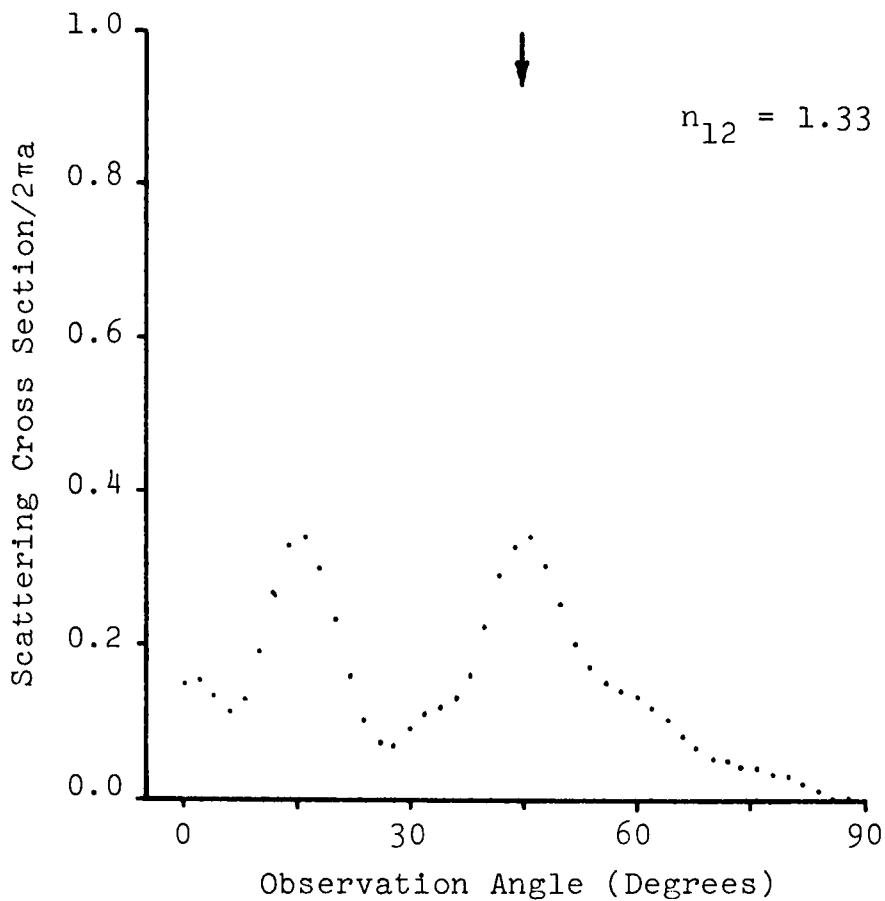


Figure 26. Calculated Results for an Embedded Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Below the Plane Interface and a Plane Wave Incident at 45° .

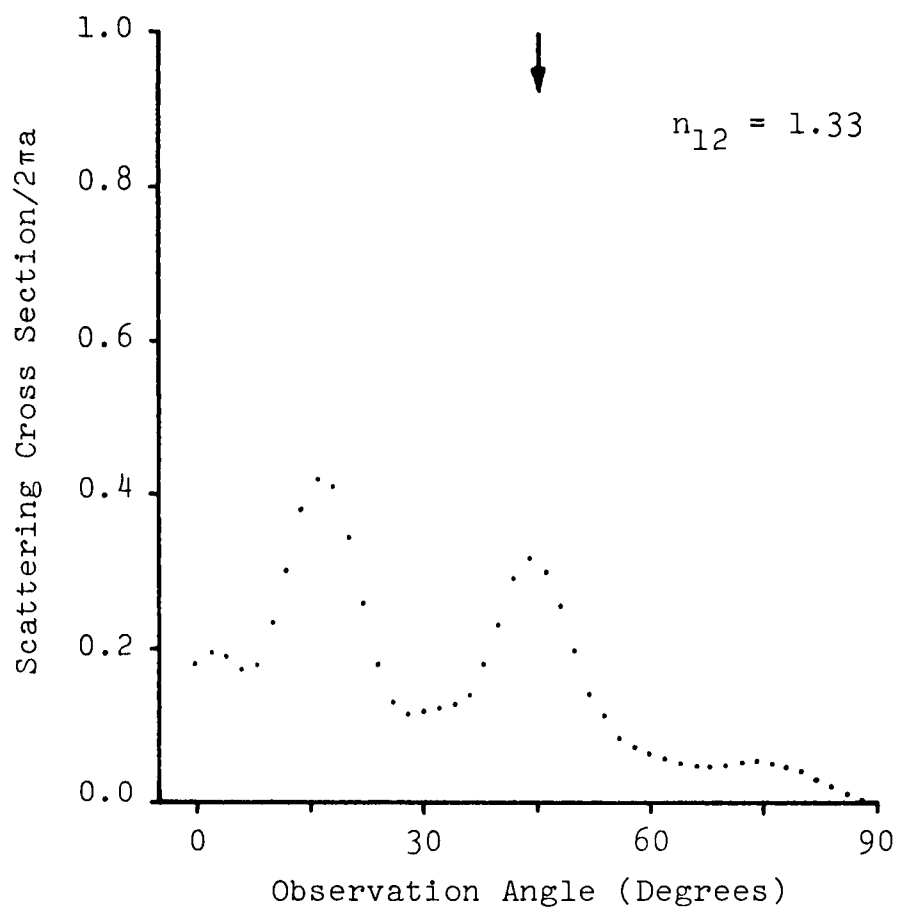


Figure 27. Calculated Results for an Embedded Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $50\lambda/2\pi$, Below the Plane Interface and a Plane Wave Incident at 45° .

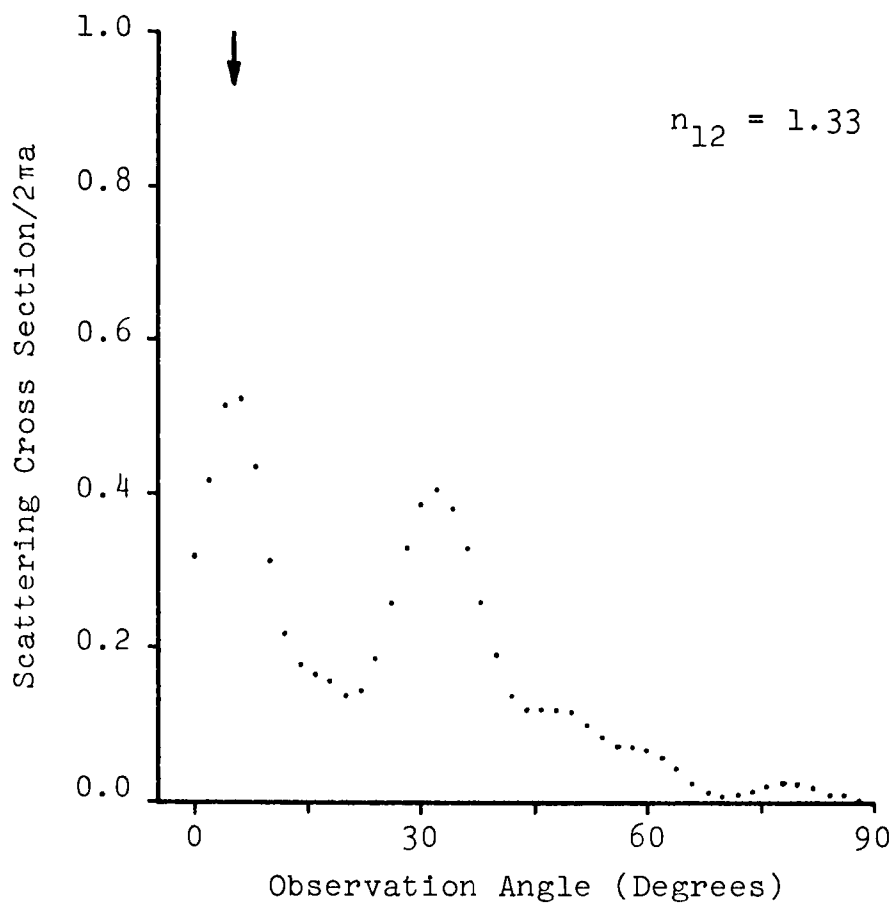


Figure 28. Calculated Results for an Embedded Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Below the Plane Interface and a Plane Wave Incident at 5° .

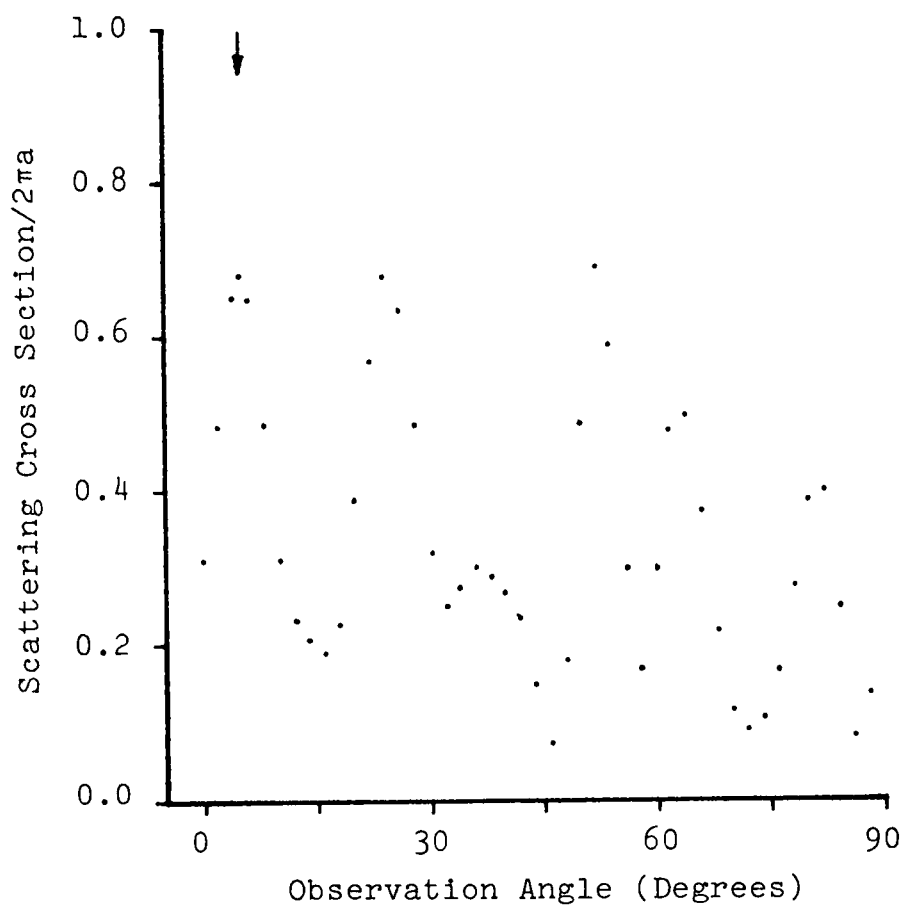


Figure 29. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) in a Homogeneous Dielectric and a Plane Wave Incident at 5° . The index of refraction of the dielectric full-space is 1.33 .

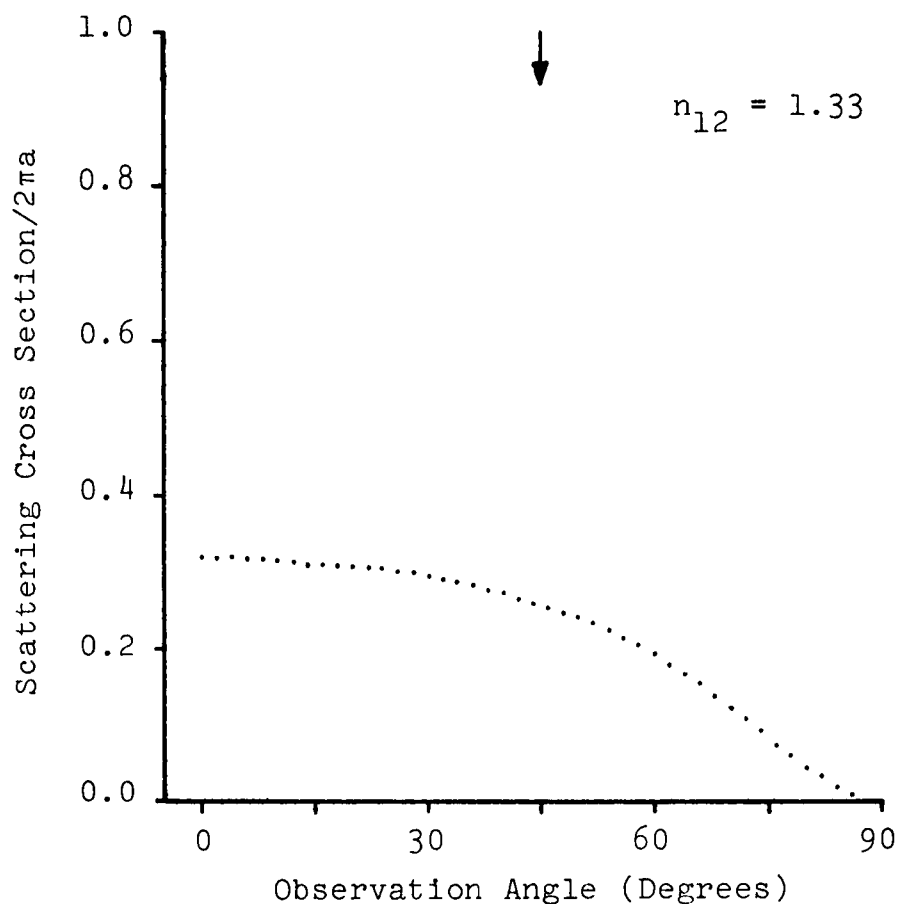


Figure 30. Calculated Results for an Embedded Metal Cylinder (Radius, $a = \lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Below the Plane Interface and a Plane Wave Incident at 45° .

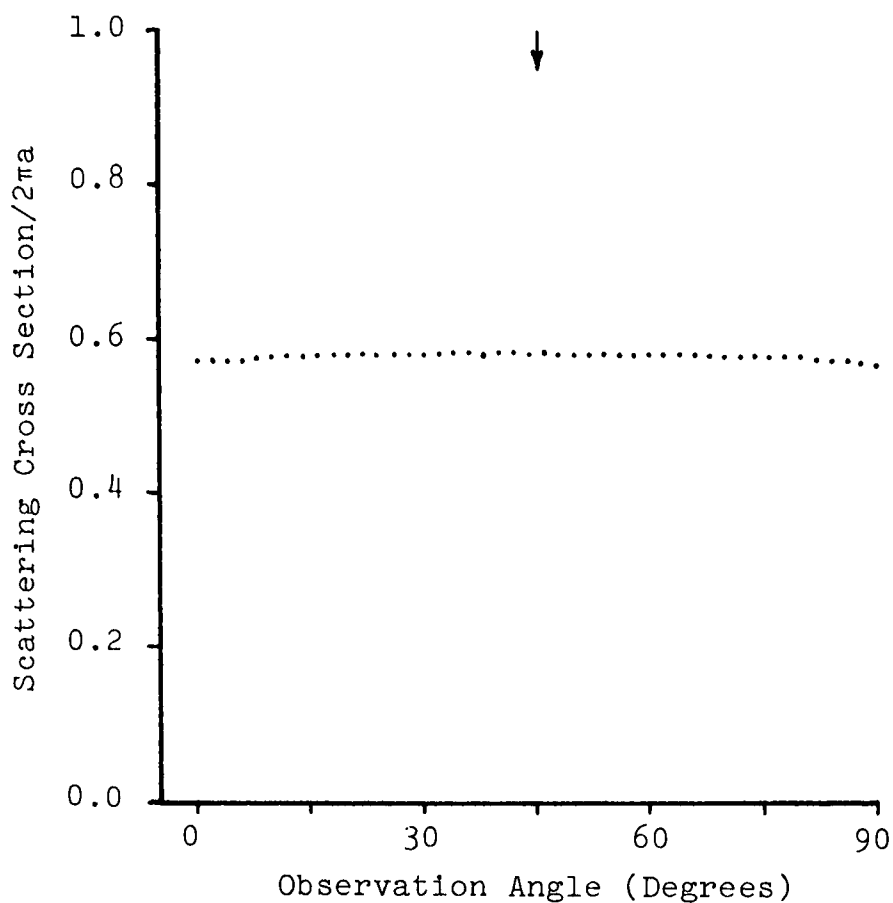


Figure 31. Calculated Results for a Metal Cylinder (Radius, $a = \lambda/2\pi$) in a Homogeneous Dielectric and a Plane Wave Incident at 45° . The index of refraction of the dielectric full-space is 1.33 .

one-quarter degree increments. This analysis demonstrated, for the cases considered, that the crest of this particular peak was usually shifted from the backscatter direction. The magnitude and sign of the shift depended upon the angle of incidence and the separation length of the cylinder from the plane interface.

In order to further explore this effect, the scattering diagrams shown in Figures 32, 33 and 34 were calculated. In Figure 32, the incidence angle is 30 degrees and the peak is shifted less than one-quarter of a degree. In Figures 33 and 34, where the angles of incidence are 60 and 85 degrees respectively, the nearest relative maximums in the scattering cross sections are toward zero degrees from the backscattered directions. Still, a relative maximum does occur near the backscattered direction although in Figure 34 its crest is located at 73 degrees. While this effect requires further study, its character does not suggest a conclusive application to the opposition effect based upon this study.

The results of this section have shown that the presence of a dielectric half-space reduces the structure normally present in the scattering diagram of a scattering cylinder. The dielectric half-space also reduces, with increasing observation angle, the observed value of the cylinder's scattering cross section.

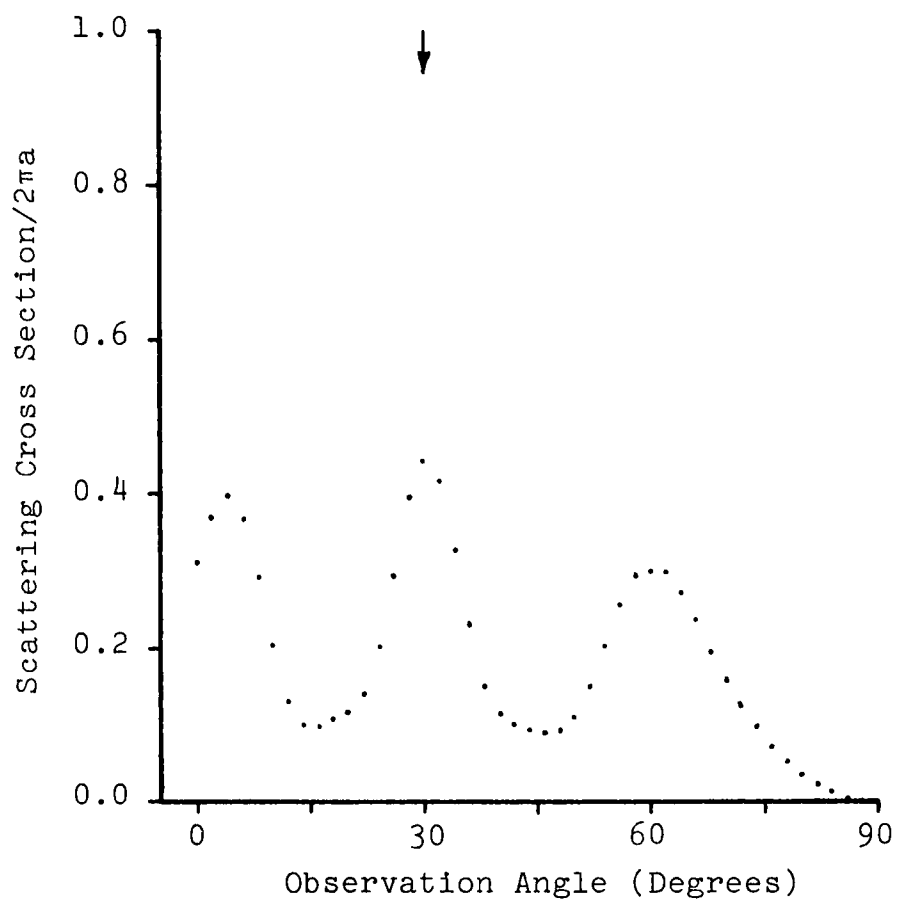


Figure 32. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Below the Plane Interface and a Plane Wave Incident at 30° .

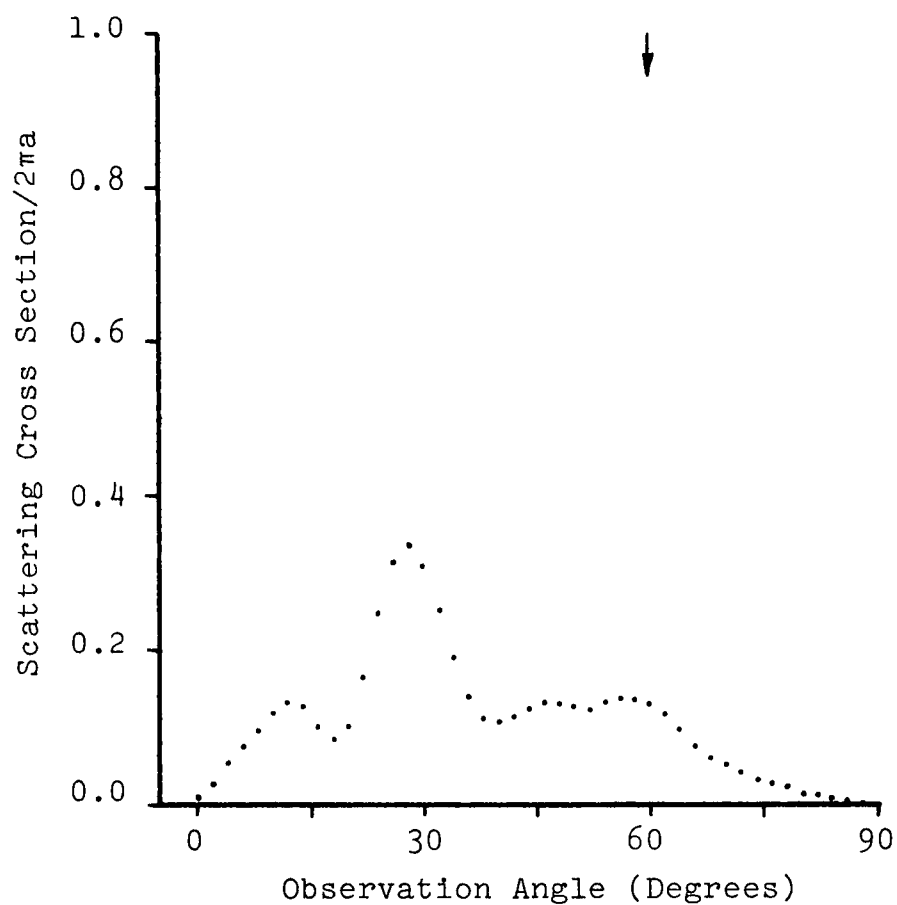


Figure 33. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Below the Plane Interface and a Plane Wave Incident at 60° .

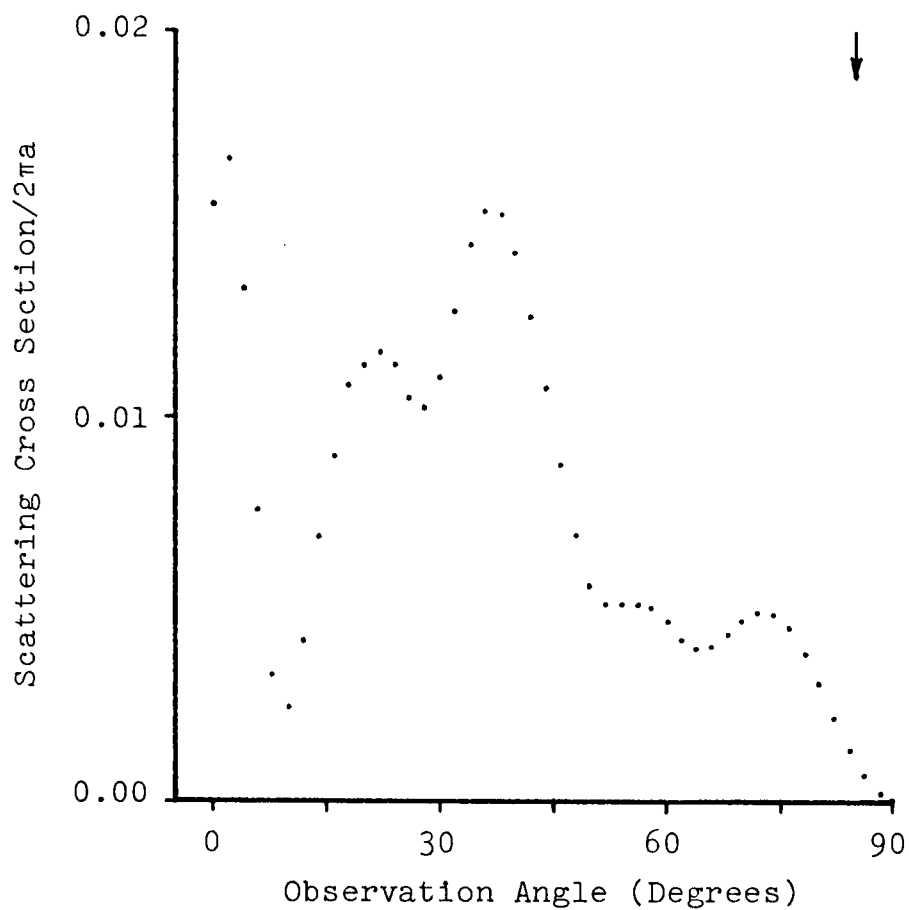


Figure 34. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Below the Plane Interface and a Plane Wave Incident at 85° .

Transverse Magnetic Fields--Dielectric Cylinder Case

This section addresses the problem of a plane, transverse magnetic wave incident upon a dielectric cylinder embedded in a homogeneous half-space.

As was previously indicated, the derivation up through equation (45) was independent of the particular characteristics of the embedded cylinder; therefore, the derivation here will commence at the point where the boundary conditions at the surface of the cylinder are to be applied.

A dielectric cylinder with propagation constant k_3 , radius a , and center located distance h below the plane interface separating the two half-spaces, has as boundary conditions: the z -components of the electric fields and the θ -components of the magnetic fields are continuous across the cylinder boundary defined by $r_1 = a$. Because the fields inside the cylinder must remain finite at the center of the cylinder, the field potential for the region made up of the interior of the cylinder is represented by

$$u_4 = \sum_n t_n J_n(k_3 r_1) \exp\{in\theta_1\} \quad . \quad (72)$$

In terms of the field auxiliary functions, the boundary conditions require, for $r_1 = a$,

$$k_2^2 (u_1 + u_2 + u^t) = k_3^2 u_4 \quad , \quad (73)$$

and

$$k_2^2 \frac{\partial}{\partial r_1} (u_1 + u_2 + u^t) = k_3^2 \frac{\partial}{\partial r_1} u_4, \quad (74)$$

where u^t , u_1 and u_2 are given by equations (57), (58) and (59) respectively. Next, all coordinates are transformed to the (r_1, θ_1) coordinate system, equations (73) and (74) are multiplied through by $\exp\{-im\theta_1\}d\theta_1$ and integrated over θ_1 from 0 to 2π . The resulting equations may be further simplified by using equation (53) in the first and the identity derived in the Appendix.

$$\begin{aligned} \int_0^{2\pi} \cos(\theta_1 - \theta'') \exp\{-ik_2 a \cos(\theta_1 - \theta'') - im\theta_1\} d\theta_1 \\ = 2\pi(-1)^m i e^{-im\theta''} J_m'(k_2 a) \end{aligned} \quad (75)$$

in the second, in which the general representation

$$R_n'(kr) = \frac{\partial}{\partial(kr)} R_n(kr) \quad (76)$$

where R_n represents either J_n or H_n , has been employed.

The two resulting equations may now be combined to yield

$$\begin{aligned} p_m = -\alpha_m(k_2, k_3, a) \left[s^t i^m \exp\{-ik_2 h \cos\theta_t - im\theta_t\} \right. \\ \left. + (-1)^m \int_c q(\theta'') \exp\{2ik_2 h \cos\theta'' - im\theta''\} d\theta'' \right] \end{aligned} \quad (77)$$

where

$$\alpha_m(k_2, k_3, a) = \frac{k_2 J_m(k_3 a) J'_m(k_2 a) - k_3 J'_m(k_3 a) J_m(k_2 a)}{k_2 J_m(k_3 a) H'_m(k_2 a) - k_3 J'_m(k_3 a) H_m(k_2 a)}, \quad (78)$$

s^t is given by equation (25) and $q(\theta'')$ by equation (51).

Again, equation (77) represents one of many equations which must be solved simultaneously in order that the p_m 's be numerically determined. Numerical evaluation of the scattering diagram proceeds identically to that of the previous section with the replacement of equation (54) by equation (77) for the expansion coefficients.

Transverse Electric Fields--Metal and Dielectric Cylinder Cases

Next, the case of transverse electric fields is considered for both a metal and a dielectric embedded cylinder. Since these derivations are so very similar to those of transverse magnetic fields, only essential points and results are presented.

The incident plane wave is represented by

$$H_z^0 = H^0 \exp\{i k_1 r_0 \cos(\theta_0 - \theta_1)\} \quad , \quad (79)$$

where H^0 is related to the electric field intensity by

$$H^0 = \frac{k_1 \omega}{\mu c} E^0 \quad , \quad (80)$$

with μ , the magnetic permeability, assumed to be equal to unity for all materials considered.

The scalar auxiliary function for the incident wave is

$$v^0 = \frac{\omega E^0}{k_1 c} \exp\{i k_1 r_0 \cos(\theta_0 - \theta_i)\} ; \quad (81)$$

those for reflected and transmitted waves are, respectively,

$$v^r = \frac{\omega}{c} Q^r \exp\{i k_1 r_0 \cos(\theta_0 - \theta_r)\} , \quad (82)$$

and

$$v^t = \frac{\omega}{c} S^t \exp\{i k_1 r_0 \cos(\theta_0 - \theta_t)\} . \quad (83)$$

The boundary conditions on the interface defined by $\theta_0 = \pm \pi/2$, require that the z-components of the magnetic fields and the radial components of the electric fields be continuous across the interface. Proceeding in the same manner as in the case of transverse magnetic fields yields

$$Q^r = \frac{E^0}{k_1} \frac{k_2 \cos\theta_i - k_1 \cos\theta_t}{k_2 \cos\theta_i + k_1 \cos\theta_t} \quad (84)$$

and

$$S^t = \frac{E^0}{k_2} \frac{2k_1 \cos\theta_i}{k_2 \cos\theta_i + k_1 \cos\theta_t} , \quad (85)$$

where

$$\theta_r = \pi - \theta_i \quad (86)$$

and

$$\theta_t = \sin^{-1} \left[\frac{k_1}{k_2} \sin\theta_i \right] . \quad (87)$$

The scattered field functions are represented by

$$v_1 = \sum_n \frac{\omega}{c} P_n H_n(k_2 r_1) e^{in\theta_1}, \quad (88)$$

$$v_2 = \frac{\omega}{c} \int_c Q(\theta'') \exp\{ik_2 r_2 \cos(\theta_2 - \theta'')\} d\theta'' \quad , \quad (89)$$

and

$$v_3 = \frac{\omega}{c} \int_c S(\theta''') \exp\{ik_1 r_0 \cos(\theta_0 - \theta''')\} d\theta''' \quad , \quad (90)$$

where P_n , Q , and S are to be determined by application of the boundary conditions. The field function, v_1 , may be further transformed by using equation (12) such that

$$v_1 = \frac{\omega}{c} \sum_n P_n \int_c \frac{(-1)^n}{\pi} \exp\{ik_2 r_1 \cos(\theta_1 - \theta') + in\theta'\} d\theta' \quad . \quad (91)$$

Applying the boundary conditions for the plane interface provides

$$Q = \sum_n P_n \frac{(-1)^n}{\pi k_2} e^{in\theta'} \frac{k_1 \cos\theta''' - k_2 \cos\theta'}{k_1 \cos\theta''' + k_2 \cos\theta'} \quad (92)$$

and

$$S = \sum_n P_n \frac{(-1)^n}{\pi k_1} e^{in\theta'} \frac{2k_2 \cos\theta'''}{k_1 \cos\theta''' + k_2 \cos\theta'} \quad (93)$$

where

$$\theta'' = -\theta' \quad (94a)$$

and

$$\theta''' = \sin^{-1} \left[\frac{k_2}{k_1} \sin \theta' \right] . \quad (94b)$$

In the case of an embedded metal cylinder, the boundary conditions require, on the cylinder surface defined by $r_1 = a$,

$$v_1 + v_2 + v^t = 0 . \quad (95)$$

This equation may be solved such that

$$\begin{aligned} P_m = & - \frac{J_m(k_2 a)}{H_m(k_2 a)} \left[S_1^{t,m} \exp\{-i(k_2 h \cos \theta_t + m \theta_t)\} \right. \\ & \left. + \sum_n P_n \frac{(-1)^{n+m}}{\pi} \int_c \left(\frac{k_1 \cos \theta''' - k_2 \cos \theta''}{k_1 \cos \theta''' + k_2 \cos \theta''} \right) \exp\{2ik_2 h \cos \theta'' - i(n+m)\theta''\} d\theta'' \right] \end{aligned} \quad (96)$$

where

$$\cos \theta''' = \left[1 - \left(\frac{k_2}{k_1} \sin \theta'' \right)^2 \right]^{1/2} . \quad (97)$$

In the case of an embedded dielectric cylinder, the boundary conditions over the surface of the cylinder require that the θ -component of the electric fields and the z -component of the magnetic field be continuous across the boundary. In terms of the field auxiliary functions, these require that

$$k_2^2 [v_1 + v_2 + v^t] = k_3^2 v_4 \quad (98)$$

and

$$\frac{\partial}{\partial r_1} [v_1 + v_2 + v^t] = \frac{\partial}{\partial r_1} v_4, \quad (99)$$

where the scalar auxiliary function for the cylinder interior is

$$v_4 = \sum_n \frac{\omega}{c} T_n J_n(k_3 r_1) e^{in\theta_1}, \quad (100)$$

where the T_n 's represent unknown expansion coefficients to be determined from the application of the boundary conditions. Using equations (98) and (99) yields

$$P_m = \beta_m(k_2, k_3, a) \left[i^m S^t \exp\{-ik_2 h \cos \theta_t - im\theta_t\} \right. \\ \left. + \sum_n P_n \frac{(-i)^{n+m}}{\pi} \int_c \left(\frac{k_1 \cos \theta''' - k_2 \cos \theta''}{k_1 \cos \theta''' + k_2 \cos \theta''} \right) \exp\{2ik_2 h \cos \theta'' - i(n+m)\theta''\} d\theta'' \right] \quad (101)$$

where

$$\beta_m(k_2, k_3, a) = \frac{k_2 J_m'(k_3 a) J_m(k_2 a) - k_3 J_m(k_3 a) J_m'(k_2 a)}{k_3 J_m(k_3 a) H_m'(k_2 a) - k_2 J_m'(k_3 a) H_m(k_2 a)} \quad (102)$$

and $\cos \theta'''$ is given by equation (97). Once the expansion coefficients, P_m , are determined for all m , using either equation (96) or equation (101), substitution of these into equations (93) and (90) determines v_3 . The scattering diagram may then be determined in a manner identical to that in the case of transverse magnetic fields.

CHAPTER IV

SCATTERING OF A PLANE ELECTROMAGNETIC WAVE BY A CIRCULAR CYLINDER NEAR A HOMOGENEOUS HALF-SPACE

Introduction

In this chapter, the case of a plane wave incident on an infinite circular cylinder positioned exterior and parallel to the plane surface of a homogeneous half-space is examined. As in Chapter III, here four cases are considered: transverse magnetic fields ($E_z \neq 0$, $H_z = 0$) and transverse electric fields ($E_z = 0$, $H_z \neq 0$) with a metal (infinite conductivity) cylinder and with a dielectric (zero conductivity) cylinder. The problem is diagrammed in Figure 35.

Problem Approach

The mathematical procedure employed is similar to that utilized in Chapter III. The coordinate system (r_1, θ_1) is centered on the cylinder, the system (r_2, θ_2) is centered in the homogeneous half-space at the mirror image of the cylinder, and the origin of the system (r_0, θ_0) is located on the plane interface separating the two half-spaces and between the origins of the other two coordinate systems. A plane wave is incident on the system composed of the cylinder and the two half-spaces.

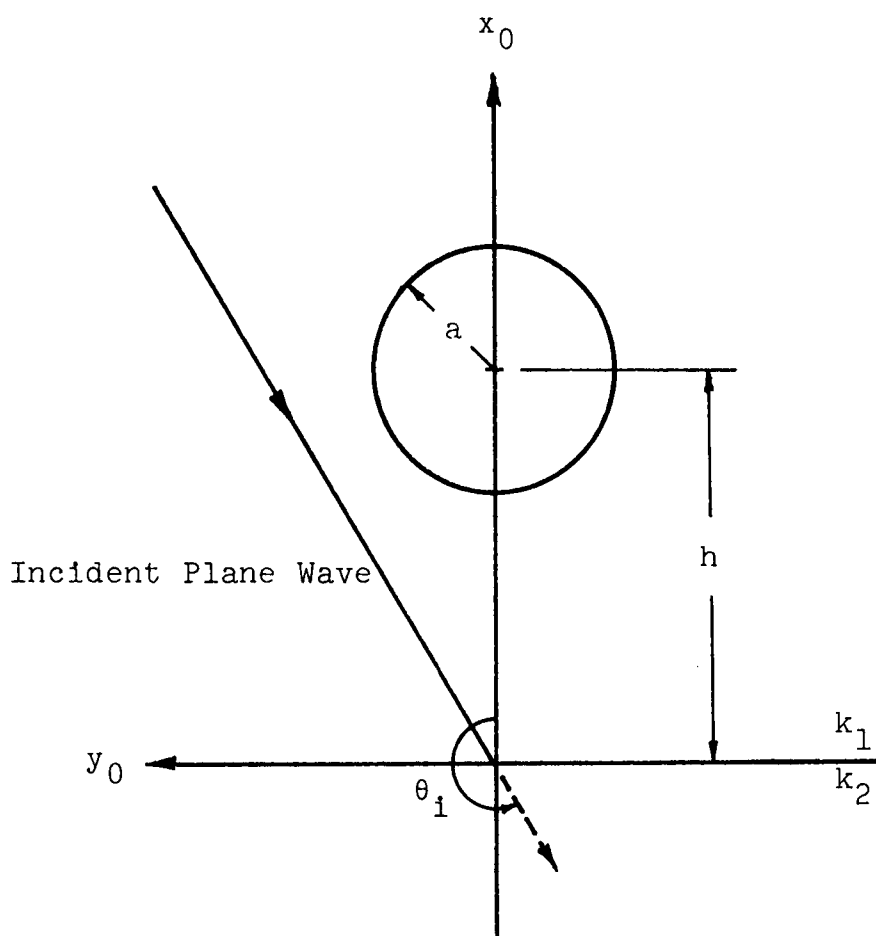


Figure 35. Diagram of the Problem of Scattering of a Plane Electromagnetic Wave by an Infinite Circular Cylinder Located Exterior to a Homogeneous Half-Space.

The source of the plane wave is taken to be located in the half-space containing the cylinder. The boundary conditions at the surfaces require the existence of scattered fields. Mathematically, the scalar functions representing the various fields in the problem are: $u_0(v_0)$ for the incident plane wave, $u_r(v_r)$ for the reflected, and $u_t(v_t)$ for the transmitted waves due only to the presence of the plane incident wave and the infinite plane boundary, $u_1(v_1)$ for scattered waves traveling away from the cylinder in the medium exterior to the homogeneous half-space, $u_2(v_2)$ for scattered waves reflected by the plane boundary, $u_3(v_3)$ for scattered waves transmitted through the plane interface, and, in the case of a dielectric cylinder, $u_4(v_4)$ for waves inside the cylinder. Keeping with the notation that was used in Chapter III, u_i represents a scalar field function corresponding to a transverse magnetic field and v_i corresponds to a transverse electric field auxiliary function.

The first cases to be considered are those of transverse magnetic plane waves incident on a metal, then dielectric, cylinder. Here, as was the case in Chapter III, due to the symmetry in the problem, the transverse magnetic and transverse electric cases may be treated independently. Also, time dependence will again be suppressed.

Transverse Magnetic Fields--Metal Cylinder Case

For the case of transverse magnetic waves, the boundary conditions require that the scalar functions satisfy the following equations on the plane defined by $x_0 = 0$:

$$k_1^2(u^0 + u^r) = k_2^2 u^t \quad , \quad (103)$$

$$k_1^2 \frac{\partial}{\partial \theta_0}(u^0 + u^r) = k_2^2 \frac{\partial}{\partial \theta_0} u_t \quad , \quad (104)$$

$$k_1^2(u_1 + u_2) = k_2^2 u_3 \quad , \quad (105)$$

and

$$k_1^2 \frac{\partial}{\partial \theta_0}(u_1 + u_2) = k_2^2 \frac{\partial}{\partial \theta_0} u_3 \quad . \quad (106)$$

The first two boundary condition equations are identical to similar equations required in Chapter III; thus, identical relations between u^0 , u^r , and u^t are obtained. The general expansions for the other field auxiliary functions are

$$u_1 = \sum_n p_n H_n(k_1 r_1) e^{in\theta_1} \quad , \quad (107)$$

$$u_2 = \int_c q(\theta'') \exp\{ik_1 r_2 \cos(\theta_2 - \theta'')\} d\theta'' \quad , \quad (108)$$

and

$$u_3 = \int_c s(\theta''') \exp\{ik_2 r_0 \cos(\theta_0 - \theta''')\} d\theta''' \quad . \quad (109)$$

Applying the boundary conditions for the plane interface leads to the following relationships between the expansion coefficients:

$$q(\theta'') = \sum_n p_n \frac{(-1)^n}{\pi} e^{-in\theta''} \left(\frac{k_2 \cos \theta''' - k_1 \cos \theta''}{k_2 \cos \theta''' + k_1 \cos \theta''} \right) \quad (110)$$

and

$$s(\theta''') = \sum_n p_n \frac{(-1)^n k_1^2}{\pi k_2^2} \exp\{ik_1 h \cos \theta' + in\theta'\} \left(\frac{2k_2 \cos \theta'''}{k_2 \cos \theta''' + k_1 \cos \theta'} \right), \quad (111)$$

where

$$\theta' = -\theta'' \quad (112a)$$

and

$$\cos \theta''' = \left[1 - \left(\frac{k_1}{k_2} \sin \theta' \right)^2 \right]^{1/2}. \quad (112b)$$

Then, in terms of the p_m expansion coefficients, the scalar functions for the other scattered fields may be written

$$u_2 = \sum_n p_n \frac{(-1)^n}{\pi} \int_c \left(\frac{k_2 \cos \theta''' - k_1 \cos \theta''}{k_2 \cos \theta''' + k_1 \cos \theta''} \right) \exp\{ik_1 r_2 \cos(\theta_2 - \theta'') - in\theta''\} d\theta'' \quad (113)$$

and

$$u_3 = \sum_n \frac{k_1^2}{k_2^2} p_n \frac{(-1)^n}{\pi} \int_c \left(\frac{2k_2 \cos \theta'''}{k_2 \cos \theta''' + k_1 \cos \theta'} \right) \exp\{ik_1 h \cos \theta' + ik_2 r_0 \cos(\theta_0 - \theta''') + in\theta'\} d\theta''' , \quad (114)$$

with

$$\theta' = \sin^{-1} \left[\frac{k_2}{k_1} \sin \theta''' \right] \quad (115a)$$

and

$$\cos \theta' = \left[1 - \left(\frac{k_2}{k_1} \sin \theta''' \right)^2 \right]^{1/2} . \quad (115b)$$

Next, the equations required by the boundary conditions for the cylinder surface are used.

The derivation up to this point has been independent of the particular characteristics of the cylinder.

In the case of a metal cylinder, the expansion coefficients must be chosen such that the following equation is satisfied on the surface defined by

$$r_1 = a :$$

$$u^0 + u_1 + u_2 + u^r = 0 . \quad (116)$$

Now

$$u^0 = \frac{E^0}{k_1^2} \exp\{ik_1 r_0 \cos(\theta_0 - \theta_1)\} \quad (117)$$

and

$$u^r = q^r \exp\{ik_1 r_0 \cos(\theta_0 + \theta_1 - \pi)\} , \quad (118)$$

where q^r is given by equation (24) of Chapter III.

Transforming coordinates, these may be written

$$u^0 = \frac{E^0}{k_1^2} \sum_n i^n J_n(k_1 r_1) \exp\{in(\theta_1 - \theta_1) - ik_1 h \cos \theta_1\} \quad (119)$$

and

$$u^r = q^r \sum_n (-i)^n J_n(k_1 r_1) \exp\{in(\theta_1 + \theta_1) + ik_1 h \cos\theta_1\} \quad . \quad (120)$$

Converting the coordinates in the other functions to the (r_1, θ_1) system, substituting the auxiliary function expansions into the boundary condition equation, multiplying through by $e^{-im\theta_1} d\theta_1$ and integrating both sides from 0 to 2π , provides the solution

$$p_m = -\frac{J_m(k_1 a)}{H_m(k_1 a)} \left[\frac{E^0}{k_1^2} i^m \exp\{-ik_1 h \cos\theta_1 - im\theta_1\} + (-i)^m q^r \exp\{ik_2 h \cos\theta_1 + im\theta_1\} + \int_c q(\theta'') (-i)^m \exp\{2ik_1 h \cos\theta'' - im\theta''\} d\theta'' \right] , \quad (121)$$

where $q(\theta'')$ is given by equation (110).

Solving equation (121) for all p_m and substituting these into equations (107), (113), and (114) completes the determination of the scalar functions. The total electric field in the region exterior to the homogeneous half-space and exterior to the cylinder is given by

$$E = k_1^2 (u^0 + u^r + u_1 + u_2) \quad . \quad (122)$$

The total electric field within the homogeneous half-space is

$$E = k_2^2 (u^t + u_3) \quad . \quad (123)$$

Numerical Results

The differential scattering cross section as a function of observation angle in the quarter-space containing the source of the incident plane wave was calculated for several cases. For these calculations metal cylinders, each located exterior to a dielectric half-space, were considered. The index of refraction of the dielectric half-space, relative to that of the upper half-space, was set equal to 1.33 in each case. Similar to what was done in Chapter III, the differential scattering cross section was divided by $2\pi a$, where a was the radius of the cylinder, to non-dimensionalize the results. The complex integrations were approximated in a manner identical to that which was employed in integrals of Chapter III, with identical restrictions. In particular, the quantity, $2k_1 h$, was restricted to values of 60 or greater.

Equation (121) may now be approximated by

$$\begin{aligned}
 p_m = & - \frac{J_m(k_1 a)}{H_m(k_1 a)} \left[\frac{E_0}{k_1^2} i^m \exp\{-ik_1 h \cos\theta_1 - im\theta_1\} \right. \\
 & + (-i)^m q^r \exp\{ik_2 h \cos\theta_1 + im\theta_1\} \\
 & \left. + \left(\frac{k_2 - k_1}{k_2 + k_1} \right) \sum_n p_n H_{n+m}(2k_1 h) \right] , \quad (124)
 \end{aligned}$$

where q^r is given by equation (24) of Chapter III.

The truncated set of equations was solved in an

iterative manner by computer. Truncation was accomplished by neglecting coefficients whose absolute values were less than that of the largest coefficient by eight orders of magnitude.

The field potentials of interest here were

$$u_1 = \sum_n p_n H_n(k_1 r_1) e^{in\theta_1} \quad (125)$$

and

$$u_2 = \sum_n p_n \frac{(-1)^n}{\pi} \int_c \left(\frac{k_2 \cos \theta''' - k_1 \cos \theta''}{k_2 \cos \theta''' + k_1 \cos \theta''} \right) \exp\{ik_1 r_2 \cos(\theta_2 - \theta'') - in\theta''\} d\theta'' \quad (126)$$

with

$$\theta''' = \sin^{-1} \left[-\frac{k_1}{k_2} \sin \theta'' \right] . \quad (127)$$

As the interest here was in the far-field, the integral in equation (126) could be evaluated by the method of steepest descents to yield

$$u_2 = \sum_n p_n (-1)^n \left[\frac{2}{\pi k_1 r_0} \right]^{1/2} \frac{\left[k_2^2 - k_1^2 \sin^2 \theta_0 \right]^{1/2} - k_1 \cos \theta_0}{\left[k_2^2 - k_1^2 \sin^2 \theta_0 \right]^{1/2} + k_1 \cos \theta_0} \exp\{ik_1 h \cos \theta_0 + ik_1 r_0 - in\theta_0 - i\frac{\pi}{4}\} . \quad (128)$$

In the limit of r_0 assuming large values, equation (125) became

$$u_1 = \sum_n p_n \left[\frac{2}{\pi k_1 r_0} \right]^{1/2} \exp\{-ik_1 h \cos \theta_0 + ik_1 r_0 + in(\theta_0 + \frac{\pi}{2}) - i\frac{\pi}{4}\} . \quad (129)$$

Using equation (128) and equation (129) in

$$E_s = k_1^2(u_1 + u_2) \quad , \quad (130)$$

the differential scattering cross section could be calculated directly.

Figure 36 shows the results of performing the calculation of the scattering diagram for a metal cylinder of radius equal to $20/2\pi$ times the wavelength, λ , in the upper medium; in the quarter-space containing the source of the plane, incident wave. The angle of incidence was set at 45 degrees from the interface normal. The center of the cylinder was separated from the plane interface by a length equal to $24\lambda/2\pi$. In Figure 37 the separation of the center of the cylinder and the plane interface was increased to $25\lambda/2\pi$. In Figure 38 this separation distance was increased to $50\lambda/2\pi$. Comparing these 3 figures shows that increasing the cylinder to interface separation distance reduces the first scattering cross section peak located near 10 degrees. All 3 figures show that, in the region between 30 degrees and 90 degrees, several resonances appeared which would not have been present in the absence of the dielectric half-space. Many of these resonances were approximately 15 degrees in width and a few order of magnitude changes in the calculated scattering cross section occurred in spans of less than 10 degrees.

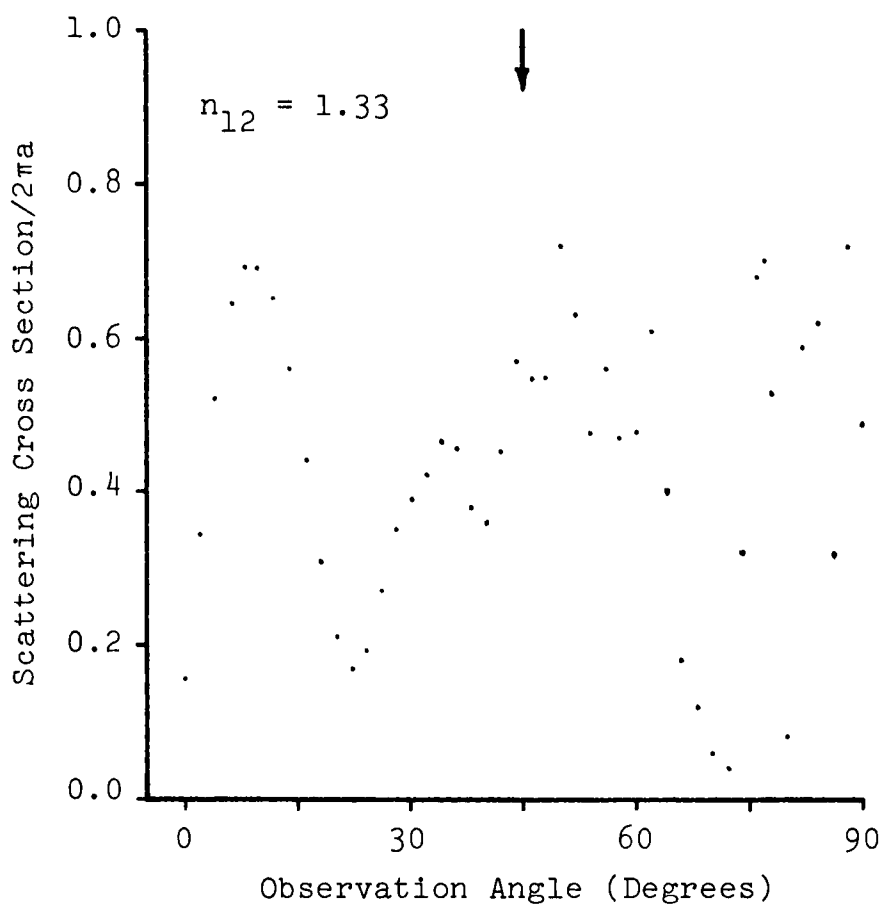


Figure 36. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $24\lambda/2\pi$, Exterior to a Dielectric Half-Space and a Plane Wave Incident at 45° .

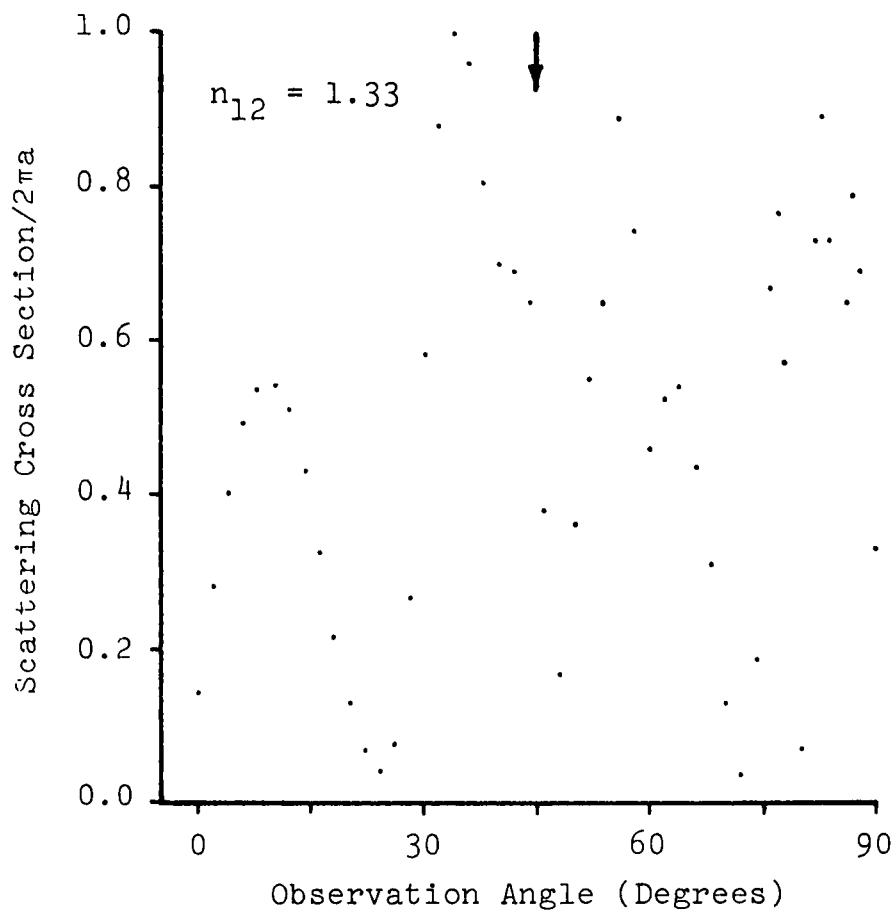


Figure 37. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Exterior to a Dielectric Half-Space and a Plane Wave Incident at 45° .

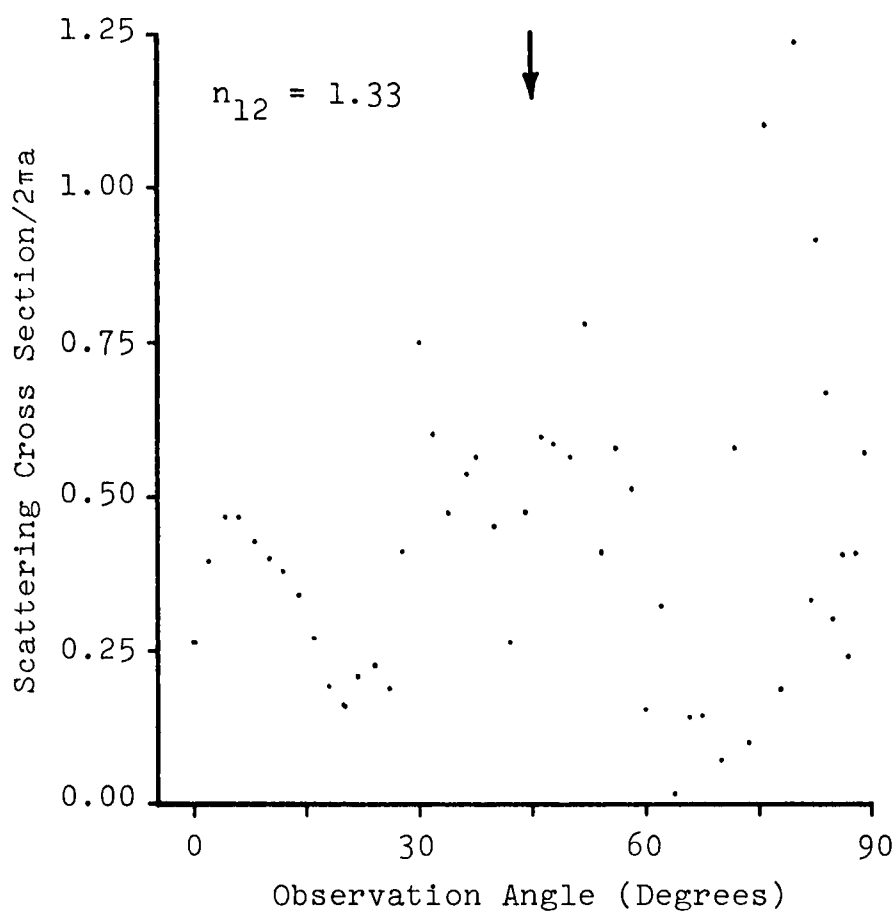


Figure 38. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $50\lambda/2\pi$, Exterior to a Dielectric Half-Space and a Plane Wave Incident at 45° .

Figure 39 is similar to Figure 37 except that, for the case depicted in Figure 39, the angle of incidence was changed to 5 degrees from normal. In this case, the more narrow resonances were shifted to larger observation angles. Even though virtually all other features of the calculated scattering diagram changed going from the conditions of Figure 37 to those of Figure 39, the backscattered cross section remained constant. Figure 40 is included for comparison.

The calculated results for a metal cylinder of radius equal to $\lambda/2\pi$ and separated from the plane interface by distance, $25\lambda/2\pi$, are shown in Figure 41. The angle of incidence of the plane wave was set at 45 degrees. The figure shows, beginning at 30 degrees and proceeding out to 84 degrees, a series of resonances which became more intense and more narrow as the observation angle increased. The angular separation of the resonant peaks decreased from 12 degrees to 7 degrees with increasing observation angle. The relative regularity of this scattering diagram compared to the others presented in this chapter was due to the smaller diameter of this cylinder which, in vacuum, exhibits a more uniform scattering diagram.

The results of this section demonstrate that dramatic changes may occur in scattering measurements when a dielectric half-space is introduced in the

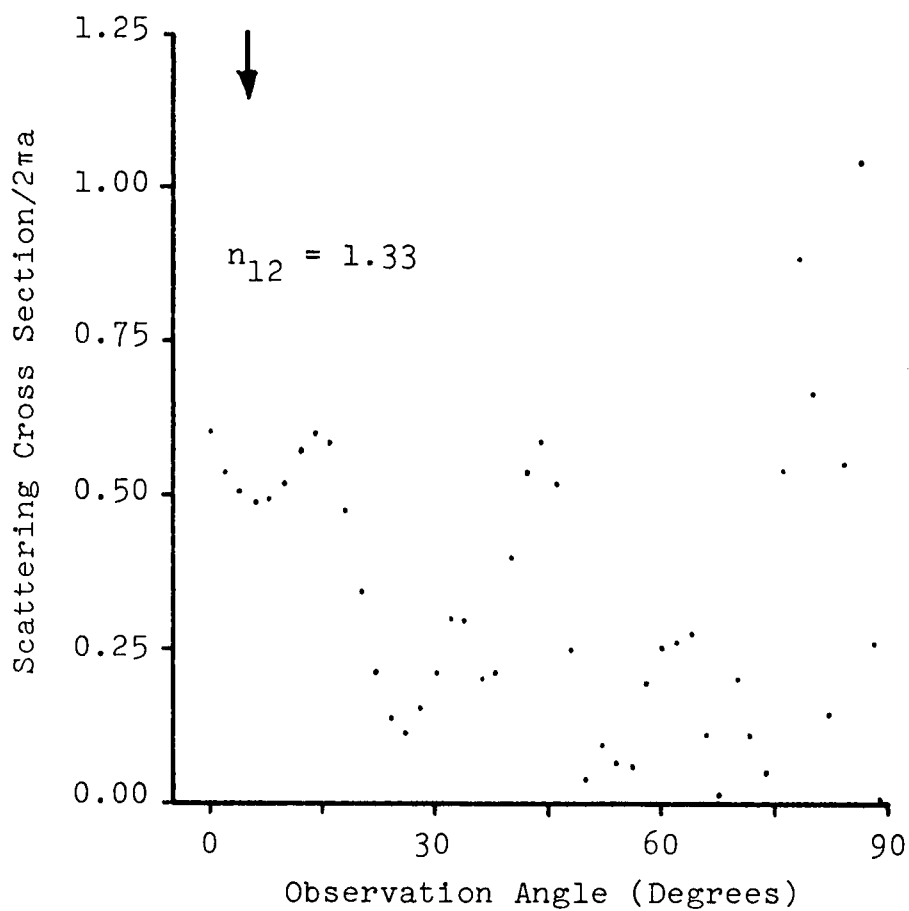


Figure 39. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Exterior to a Dielectric Half-Space and a Plane Wave Incident at 5° .

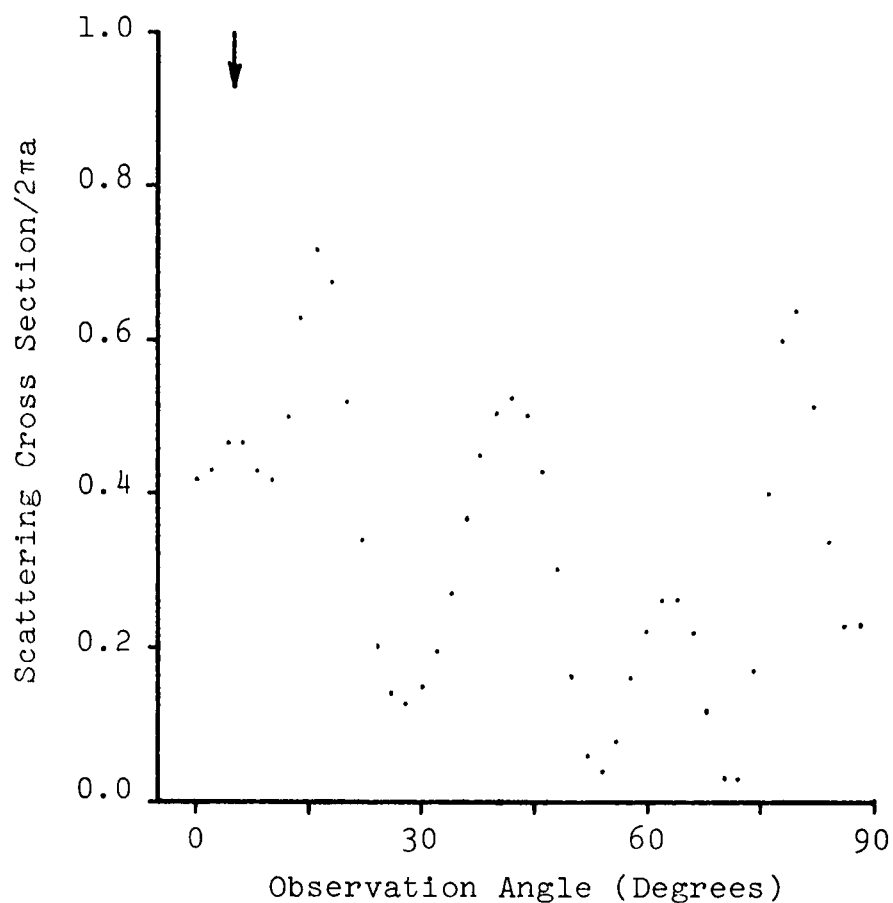


Figure 40. Calculated Results for a Metal Cylinder (Radius, $a = 20\lambda/2\pi$) in Vacuum and a Plane Wave Incident at 5° .

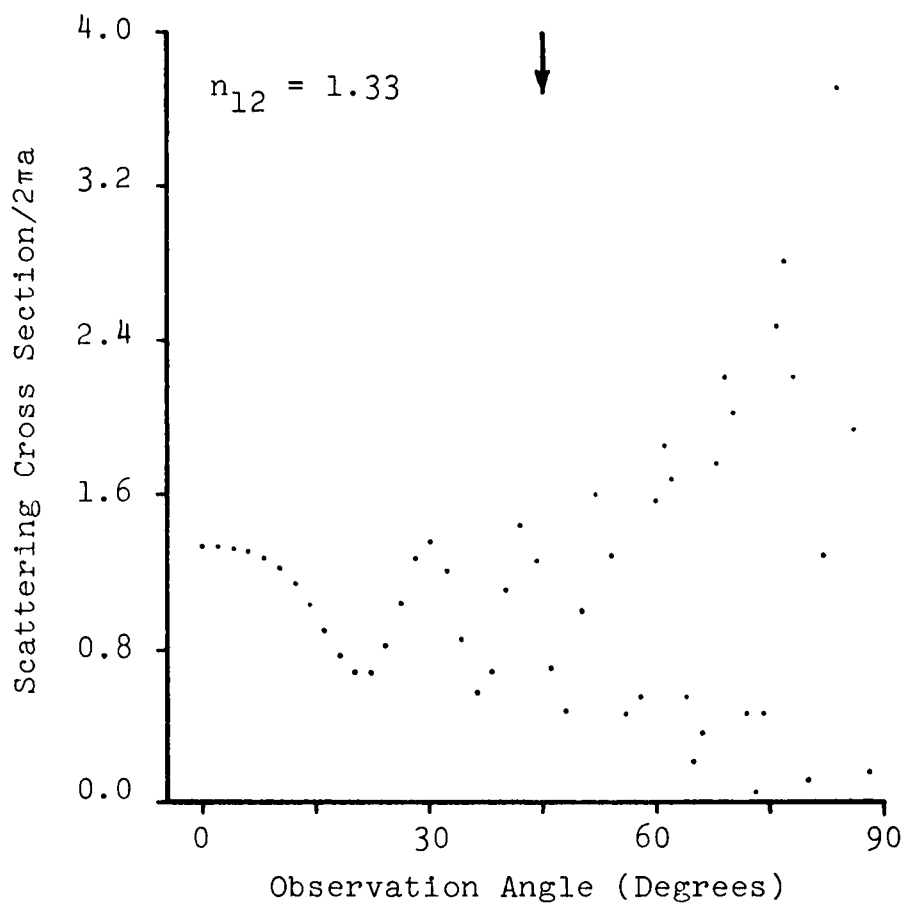


Figure 41. Calculated Results for a Metal Cylinder (Radius, $a = \lambda/2\pi$) Located Distance, $25\lambda/2\pi$, Exterior to a Dielectric Half-Space and a Plane Wave Incident at 45° .

neighborhood of other scatterers. In particular, an object near the surface of the earth may present a greatly modified scattering cross section to a remote detection system due to the interaction of the object under surveillance and planar surfaces in the object's vicinity.

Transverse Magnetic Fields--Dielectric Cylinder Case

For the case of a dielectric circular cylinder, 2 boundary conditions are required in addition to those included in equations (103)-(106). These 2 boundary conditions are applied at the cylinder surface defined by $r_1 = a$ and may be expressed, in terms of the scalar auxiliary functions,

$$k_1^2 \left[u^0 + u^r + u_1 + u_2 \right] = k_3^2 u_4 \quad (131)$$

and

$$k_1^2 \frac{\partial}{\partial r_1} \left[u^0 + u^r + u_1 + u_2 \right] = k_3^2 \frac{\partial}{\partial r_1} u_4, \quad (132)$$

where u_4 is the scalar function for the cylinder interior and is given by

$$u_4 = \sum_n t_n J_n(k_3 r_1) e^{in\theta_1}. \quad (133)$$

Converting all coordinates to the (r_1, θ_1) system, utilizing in equations (131) and (132) the expressions for the scalar functions, multiplying both equations by $e^{-im\theta_1}$ and integrating over θ_1 from 0 to 2π , t_m may be eliminated and the equation for p_m that

results is

$$\begin{aligned}
 p_m = & -\alpha_m(k_1, k_3, a) \left[(-i)^m \int_c q(\theta'') \exp\{2ik_1 h \cos\theta'' - im\theta''\} d\theta'' \right. \\
 & + (-i)^m q^r \exp\{ik_1 h \cos\theta_1 + im\theta_1\} \\
 & \left. + (i)^m \frac{E^0}{k_1^2} \exp\{-ik_1 h \cos\theta_t - im\theta_t\} \right], \quad (134)
 \end{aligned}$$

where

$$\alpha_m(k_1, k_3, a) = \frac{k_1 J_m(k_3 a) J'_m(k_1 a) - k_3 J'_m(k_3 a) J_m(k_1 a)}{k_1 J_m(k_3 a) H_m(k_1 a) - k_3 J'_m(k_3 a) H_m(k_1 a)}, \quad (135)$$

$q(\theta'')$ is given by equation (110) and q^r by equation (24) of Chapter III. The solutions of equation (134), for all m , may be substituted into equations (107), (113) and (114) to produce the solutions for the scalar functions u_1 , u_2 and u_3 respectively. These are inserted into equations (122) and (123), along with previous solutions for u^0 , u^r , u^t , to express the total electric field in the 2 half-spaces. Further, these values of p_m may be used directly in the equations of the previous section to determine the differential scattering cross section, within the limitations of the assumptions made in that section.

Transverse Electric Fields--Metal and Dielectric Cylinder Cases

The cases of transverse electric fields scattered by a metal or dielectric cylinder parallel and exterior to a homogeneous half-space may be treated by the same mathematical procedure as has been used in the cases of transverse magnetic fields and these same cylinders. Consequently, only essential steps and results are presented in this section.

The incident plane wave is represented by the expression given in equation (85) of Chapter III. The scalar potential functions, v^0 , v^r and v^t for plane waves incident upon, reflected by and transmitted by the plane interface are given by equations (81), (82) and (83), respectively, in Chapter III. The boundary conditions on the plane interface require the continuity of the z-components of the magnetic fields and the radial components of the electric fields across the interface. Applying the boundary conditions to these fields gives

$$Q^r = \frac{E^0}{k_1} \frac{k_2 \cos \theta_i - k_1 \cos \theta_t}{k_2 \cos \theta_i + k_1 \cos \theta_t} \quad (136)$$

and

$$S^t = \frac{E^0}{k_2} \frac{2k_1 \cos \theta_i}{k_2 \cos \theta_i + k_1 \cos \theta_t} \quad , \quad (137)$$

where

$$\theta_r = \pi - \theta_i \quad (138a)$$

and

$$\theta_t = \sin^{-1} \left[\frac{k_1}{k_2} \sin \theta_1 \right] . \quad (138b)$$

The scattered field functions are represented by

$$v_1 = \sum_n \frac{\omega}{c} P_n H_n(k_1 r_1) e^{in\theta_1} , \quad (139)$$

for waves diverging from the cylinder, this may be rewritten using equation (12) of Chapter III

$$v_1 = \frac{\omega}{c} \sum_n P_n \int_c \frac{(-i)^n}{\pi} \exp\{ik_1 r_1 \cos(\theta_1 - \theta') + in\theta'\} d\theta' ; \quad (140)$$

$$v_2 = \frac{\omega}{c} \int_c Q(\theta'') \exp\{ik_1 r_2 \cos(\theta_2 - \theta'')\} d\theta'' , \quad (141)$$

for scattered waves reflected by the plane interface and

$$v_3 = \frac{\omega}{c} \int_c S(\theta''') \exp\{ik_2 r_0 \cos(\theta_0 - \theta''')\} d\theta''' , \quad (142)$$

for waves transmitted into the homogeneous half-space.

Applying the plane interface boundary conditions to these scattered waves results in

$$Q(\theta'') = \sum_n P_n \frac{(-i)^n}{\pi k_1} e^{-in\theta''} \left[\frac{k_2 \cos \theta''' - k_1 \cos \theta''}{k_2 \cos \theta''' + k_1 \cos \theta''} \right] \quad (143)$$

where

$$\theta''' = \sin^{-1} \left[- \frac{k_1}{k_2} \sin \theta'' \right] \quad (144)$$

and

$$S(\theta''') = \sum P_n \frac{(-1)^n}{\pi k_2} e^{in\theta'} \left[\frac{2k_1 \cos \theta'''}{k_2 \cos \theta''' + k_1 \cos \theta'} \right] \quad (145)$$

where

$$\theta' = \sin^{-1} \left[\frac{k_2}{k_1} \sin \theta''' \right] . \quad (146)$$

Now, in the case of a metal cylinder, the boundary condition on the surface defined by $r_1 = a$ is

$$v_0 + v_r + v_1 + v_2 = 0 . \quad (147)$$

Substituting the previously determined expressions for these scalar field functions into this equation yields, after manipulation,

$$\begin{aligned} P_m = & -\frac{J_m(k_1 a)}{H_m(k_1 a)} \left[\frac{E_0}{k_1} \exp \left\{ i \left(\frac{m\pi}{2} - k_1 h \cos \theta_1 - m\theta_1 \right) \right\} \right. \\ & + Q^r \exp \left\{ i \left(k_2 h \cos \theta_1 + m\theta_1 - \frac{m\pi}{2} \right) \right\} \\ & \left. + \int_c Q(\theta'') \exp \left\{ i \left(2k_1 h \cos \theta'' - m\theta'' - \frac{m\pi}{2} \right) \right\} d\theta'' \right] . \quad (148) \end{aligned}$$

In the case of a dielectric cylinder, the boundary conditions on $r_1 = a$ require that the following equations be satisfied by the scalar auxiliary functions:

$$k_1^2 [v_0 + v_r + v_1 + v_2] = k_3^2 v_4 \quad (149)$$

and

$$\frac{\partial}{\partial r_1} [v^0 + v^r + v_1 + v_2] = \frac{\partial}{\partial r_1} v_4 \quad (150)$$

where k_3 is the propagation constant for the dielectric material of which the dielectric cylinder is composed and v_4 is the scalar function for the region interior to the cylinder defined

$$v_4 = \frac{\omega}{c} \sum_n T_n J_n(k_3 r_1) e^{in\theta_1} \quad (151)$$

Substituting the scalar function expressions into the boundary condition equations and eliminating T_m , results in the equation

$$\begin{aligned} P_m = & -\beta(k_1, k_3, a) \left[\frac{E^0}{k_1} \exp \left\{ i \left(m \frac{\pi}{2} - k_1 h \cos \theta_1 - m \theta_1 \right) \right\} \right. \\ & + Q^r \exp \left\{ i \left(k_2 h \cos \theta_1 + m \theta_1 - m \frac{\pi}{2} \right) \right\} \\ & \left. + \int_c Q(\theta'') \exp \left\{ i \left(2k_1 h \cos \theta'' - m \theta'' - m \frac{\pi}{2} \right) \right\} d\theta'' \right] , \quad (152) \end{aligned}$$

where

$$\beta(k_1, k_3, a) = \frac{k_3 J_m(k_3 a) J'_m(k_1 a) - k_1 J'_m(k_3 a) J_m(k_1 a)}{k_3 J_m(k_3 a) H'_m(k_1 a) - k_1 J'_m(k_3 a) H_m(k_1 a)} \quad (153)$$

Conclusions

The results of this chapter and of Chapter III have provided solutions to the problems of determining

the scattering effects of systems of cylinders and homogeneous half-spaces. The calculated results did not show an enhanced backscatter similar to that of the opposition effect. The calculated results did show that the presence of a dielectric half-space near or surrounding an object alters the differential scattering cross section, as a function of observation angle, of the object. The results presented in these two chapters may be directly applied to investigations of embedded cables and cylindrical objects near large plane surfaces.

CHAPTER V

SCATTERING BY THREE DIELECTRIC CYLINDERS

Introduction

This chapter deals with the theoretical aspects of scattering of electromagnetic waves by a system of three parallel, dielectric cylinders, which are not all coplanar. This problem is of interest because such a system of scatterers constitutes a simple model for a portion of the near surface of a material. Several mechanisms have been proposed to explain the existence of the opposition effect (Hapke, 1963; Irvine, 1966; Trowbridge, 1978). One mechanism advanced which results in an enhanced backscatter is shadowing. Shadowing occurs when scattering elements near the surface of a material block the escape of radiation scattered by illuminated elements further from the surface, except at or near the backscatter direction of the illuminating radiation. If the size of an individual scatterer is comparable with the wavelength of the illuminating radiation, a distinct shadow is not formed. In this case, diffraction effects are important as they modify the scattered flux.

The purpose of this study is to determine if the scattering produced by a system of three cylinders, arranged such that two of the cylinders partially shield the third

from the observation point, exhibits an enhanced backscatter. Only the cases of cylinders with diameters comparable with the wavelength of the incident radiation are considered although the theoretical results may be applied to other cases without restriction on cylinder size.

The investigation of this problem also serves the purpose of bridging the gap between the theoretical investigations of the preceding two chapters and the experimental investigation discussed in Chapter II. The application here is to the opposition effect while the theoretical treatment is based on the method developed for the cylinder/homogeneous half-space problems.

Previous investigations (Row, 1955; Olaofe, 1970) of the diffraction of electromagnetic waves by systems of cylinders, not restricted to widely separated cylinders, treated plane and cylindrical wave illumination of two cylinders. The analysis presented in this chapter is restricted to the case of the scattering of a plane, plane-polarized electromagnetic wave incident upon a system of three identical, parallel cylinders. Two of the cylinders are centered on the plane orthogonal to the direction of the incident wave. The plane of polarization of the incident wave is parallel to the cylinders.

Problem Approach

The cylinders are arranged as shown in Figure 42. The center of the third cylinder is located along a line bisecting and perpendicular to the line joining the first and second cylinders. The direction of the incident plane wave is also along this line. All three dielectric cylinders are taken to be of the same size and possess identical indices of refraction.

Four coordinate systems are utilized, one centered on each of the three cylinders and one with its origin located midway between the two paired cylinders. This last coordinate system is the one in which the final results are determined. The x-axes of all of the coordinate systems are parallel and oriented toward the direction from which the plane wave originates. Addressing only the case of a transverse magnetic wave, the field functions in the region exterior to the three cylinders may be written in the Fourier-Bessel expansions

$$u_1 = \sum_n p_n H_n(k_1 r_1) e^{in\theta_1}, \quad (154)$$

$$u_2 = \sum_n q_n H_n(k_1 r_2) e^{in\theta_2}, \quad (155)$$

and

$$u_3 = \sum_n s_n H_n(k_1 r_3) e^{in\theta_3}, \quad (156)$$

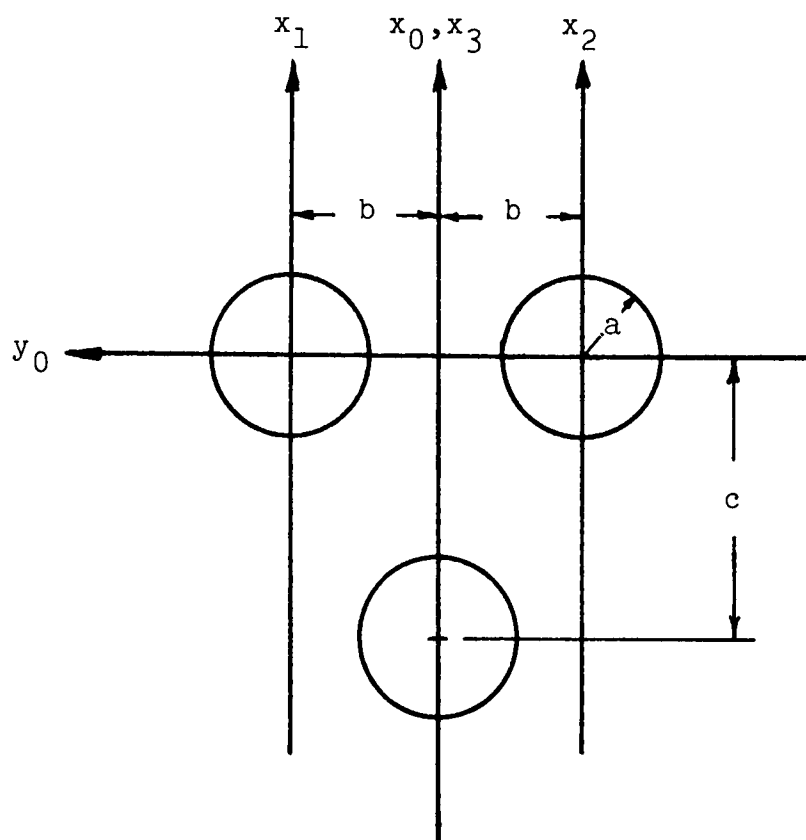


Figure 42. Diagram of the Problem of Scattering of a Plane Electromagnetic Wave by a System of Three Dielectric Cylinders.

where the subscripted coordinates refer to the three coordinate systems centered on each of the cylinders and k_1 is the propagation constant of the medium surrounding the cylinders.

The scalar auxiliary function for the incident wave, in the fourth coordinate system, is represented by

$$u^0 = \frac{E^0}{k_1^2} \exp\{-ik_1 r_0 \cos \theta_0\} \quad (157a)$$

$$= \frac{E^0}{k_1^2} \sum_n i^n J_n(k_1 r_0) \exp\{in(\theta_0 - \pi)\} \quad (157b)$$

The scalar auxiliary functions for the regions within the cylinders are represented by the general form

$$u_j = \sum_n t_n^{(j)} J_n(k_2 r_\ell) \exp\{ik_2 \theta_\ell\} \quad , \quad (158)$$

where

$$j = 4, 5, 6 \quad (159)$$

and

$$\ell = 1, 2, 3 \quad , \quad (160)$$

depending upon which cylinder is being considered.

The propagation constant, k_2 , for the dielectric material composing the three cylinders is assumed to have the same value for each of the cylinders. The cylinders are assumed to possess the same radius, a , and thus the same size parameter, $k_1 a$.

Boundary conditions are applied over the surface of each cylinder. These boundary conditions require the z-components of the electric fields and the θ -components of the magnetic fields to be continuous across each cylinder surface. In terms of the scalar auxiliary functions, the boundary conditions require

$$k_1^2(u_0 + u_1 + u_2 + u_3) = k_2^2 u_j \quad (161)$$

and

$$k_1^2 \frac{\partial}{\partial r_\ell} (u_0 + u_1 + u_2 + u_3) = k_2^2 \frac{\partial}{\partial r_\ell} u_j \quad (162)$$

for

$$r_\ell = a, \quad (163)$$

where

$$j = 4, 5, 6 \quad (164)$$

and

$$\ell = 1, 2, 3. \quad (165)$$

In order to solve the resultant equations for the expansion coefficients, all scalar auxiliary functions must be first re-written in the coordinate system for which equation (163) is satisfied. The Bessel and Hankel functions may be transformed by expanding each in a plane wave expansion, performing appropriate coordinate transformations, and forming sums over products of Bessel and Hankel functions. Equivalently,

translation theorems for Bessel and Hankel functions may be used (Stratton, 1941; Ivanov, 1970). Each equation is then multiplied by $\exp\{-im\theta_j\}d\theta_j$ and integrated over θ_j from 0 to 2π . The equations are combined in pairs to eliminate the expansion coefficients of the auxiliary functions for the cylinder interior regions. The resulting equations for the expansion coefficients are

$$p_m = -\alpha_m(k_1, k_2, a) \left[\exp\{-im \frac{\pi}{2}\} + \sum_n q_n H_{n-m}(2k_1 b) \exp\{i(n-m) \frac{\pi}{2}\} + \sum_n s_n H_{n-m}(k_1 \ell) \exp\{i(n-m) \tan^{-1}(\frac{b}{c})\} \right], \quad (166)$$

$$q_m = -\alpha_m(k_1, k_2, a) \left[\exp\{-im \frac{\pi}{2}\} + \sum_n p_n H_{n-m}(2k_1 b) \exp\{-i(n-m) \frac{\pi}{2}\} + \sum_n s_n H_{n-m}(k_1 \ell) \exp\{-i(n-m) \tan^{-1}(\frac{b}{c})\} \right] \quad (167)$$

and

$$s_m = -\alpha_m(k_1, k_2, a) \left[\exp\{i(k_1 c - m \frac{\pi}{2})\} + \sum_n p_m H_{n-m}(k_1 \ell) \exp\left\{i(n-m) \left(\pi + \tan^{-1}(\frac{b}{c})\right)\right\} + \sum_n q_n H_{n-m}(k_1 \ell) \exp\left\{i(n-m) \left(\pi - \tan^{-1}(\frac{b}{c})\right)\right\} \right]. \quad (168)$$

where

$$\alpha_m(k_1, k_2, a) = \frac{k_1 J_m(k_2 a) J'_m(k_1 a) - k_2 J'_m(k_2 a) J_m(k_1 a)}{k_1 J_m(k_2 a) H'_m(k_1 a) - k_2 J'_m(k_2 a) H_m(k_1 a)}; \quad (169)$$

b is one-half the distance separating the two paired cylinders; c is the perpendicular distance of the third cylinder from the plane defined by the centers of the two paired cylinders; and

$$l = (b^2 + c^2)^{1/2} . \quad (170)$$

These equations provide the relationships which the expansion coefficients must satisfy. The solutions to equations (166), (167) and (168) may be substituted into equations (154), (155) and (156) to provide the scalar auxiliary functions for the scattered fields. The associated, transverse magnetic, electric fields are given by

$$E_j = k_1^2 u_j , \quad j = 1, 2, 3 . \quad (171)$$

Numerical Results

In this section, the results of calculating the differential scattering cross sections of a few representative three cylinder systems are presented. Before the differential scattering cross section could be calculated, the expansion coefficients which were solutions to equations (166), (167) and (168) were determined. To accomplish this, a computer routine was developed which first estimated the value of the coefficients and then determined the truncation point at which only

coefficients of absolute value eight orders of magnitude less than the largest coefficient, or less, would be truncated. The computer program then determined the value of each retained coefficient by an iterative scheme.

Using the calculated expansion coefficients, the scalar auxiliary functions, electric fields and the differential scattering cross sections as functions of observation angle were determined. In order to non-dimensionalize the results, the resultant scattering cross sections were divided by $2\pi a$, where a was the radius of a single cylinder. In each case considered, the index of refraction of the material making up the cylinders, relative to that of the surrounding medium, was set at the value 1.5 .

The included figures show the calculated scattering diagrams of several three-cylinder systems for various cylinder size parameters and separation lengths. In each figure, the backscatter direction corresponded to 0° . Cylinder sizes and separation lengths were multiplied by the propagation constant, k_1 , of the medium surrounding the cylinders in order to non-dimensionalize the parameter values.

For cylinders of size parameter, $k_1 a$, equal to 7.0 , Figures 43, 44 and 45 demonstrate the effect of increasing the separation distance of the third cylinder from the forward two cylinders. In these figures the two paired

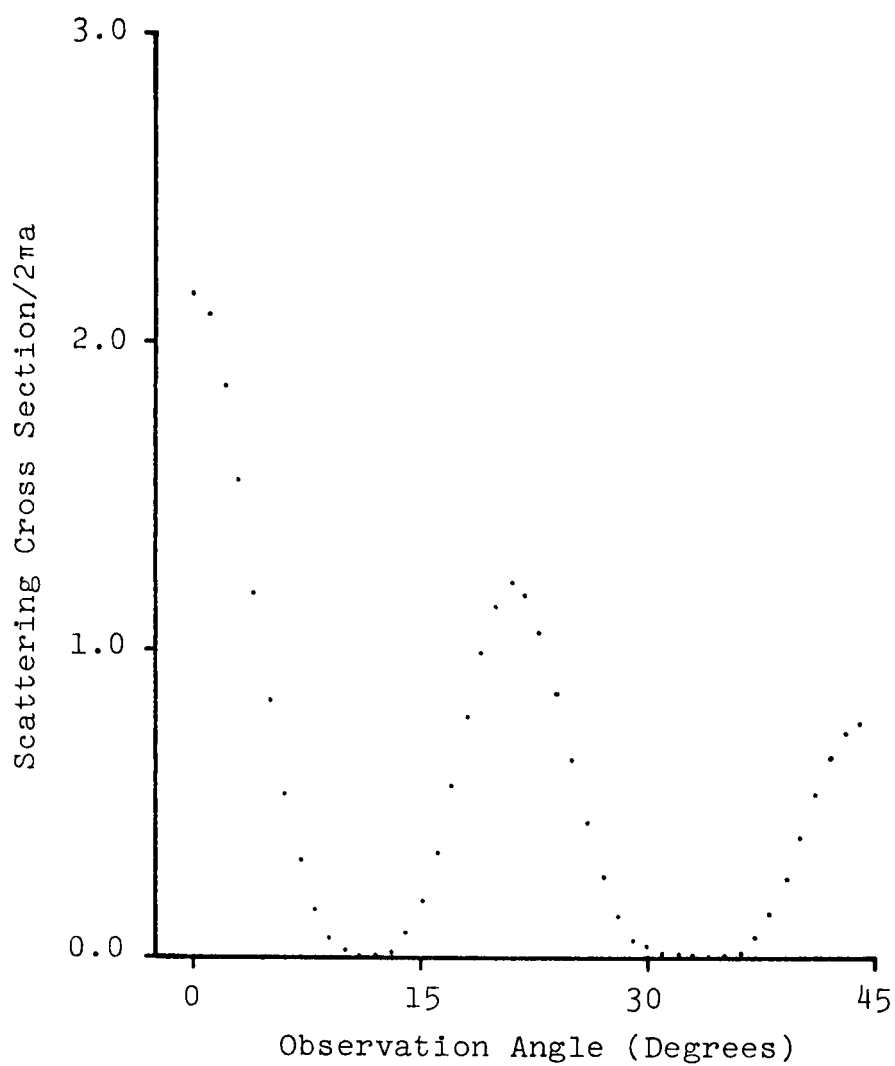


Figure 43. Calculated Results for a Three Cylinder System with $k_{1a} = 7.0$, $2k_{1b} = 20.0$, and $k_{1c} = 15.0$.

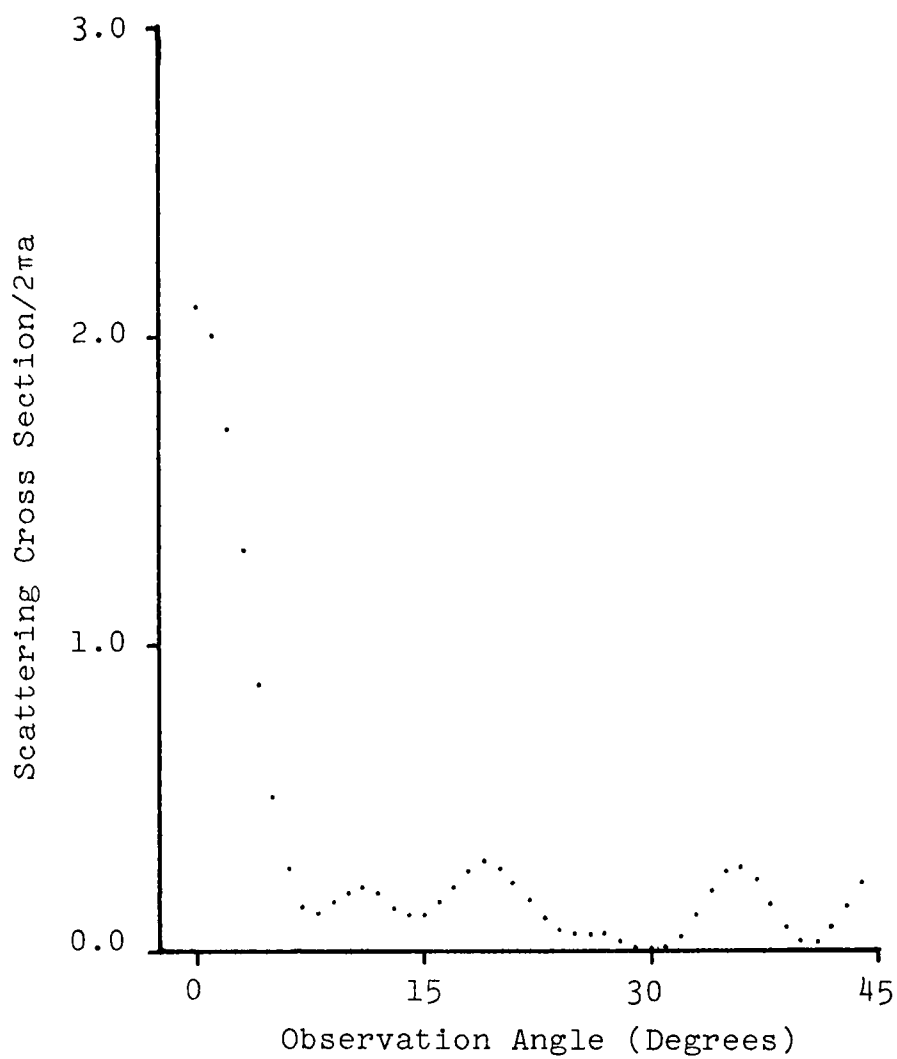


Figure 44. Calculated Results for a Three Cylinder System with $k_{1a} = 7.0$, $2k_{1b} = 20.0$, and $k_{1c} = 20.0$.

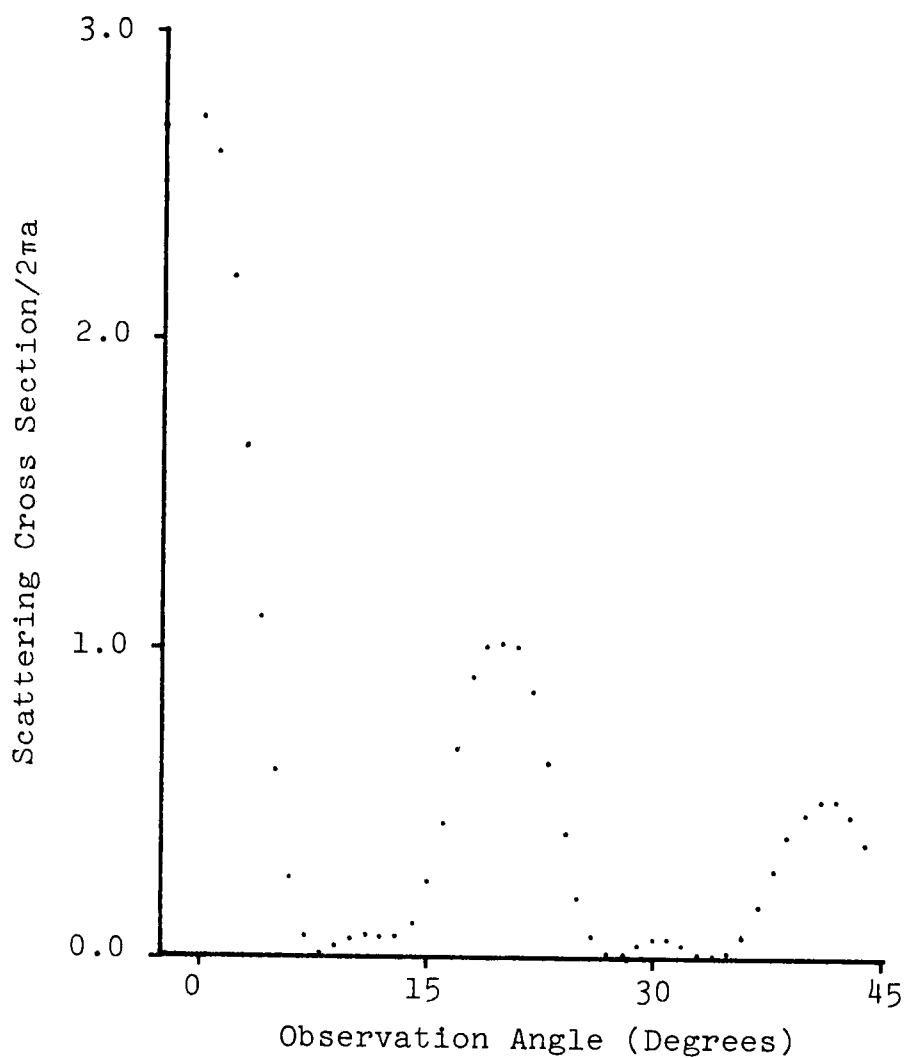


Figure 45. Calculated Results for a Three Cylinder System with $k_1a = 7.0$, $2k_1b = 20.0$, and $k_1c = 30.0$.

cylinders were held fixed. In each case, a pronounced backscattered peak was obtained. The width of this peak at one-half height did not change going from the conditions of Figure 44 to those of Figure 45. The magnitude of the backscattered peak first decreased, then increased, with increasing separation length. The magnitudes and angular locations of the other peaks in the displayed portions of the scattering diagrams did show a strong dependence on separation length. Comparison of Figure 46, which shows the scattering diagram of a single cylinder ($k_1 a = 7.0$), with Figures 43 and 45 demonstrates a tendency of systems of 3 cylinders to exhibit peaks in their scattering diagrams in the angular vicinity of peaks in the scattering diagram of a single cylinder when the diagonal separation of the cylinder centers is near an integer number of wavelengths. The fact that the widths of the backscattered peaks were not greatly dependent upon the separation lengths of the back cylinders from the sets of two forward cylinders indicated that the shadowing theory may not be extended to cases of scattering elements comparable to the wavelength of the incident radiation. However, even in cases in which diffraction effects predominate, a backscattered peak can occur and its existence does not critically depend on the exact separation distance of the front two cylinders from the third.

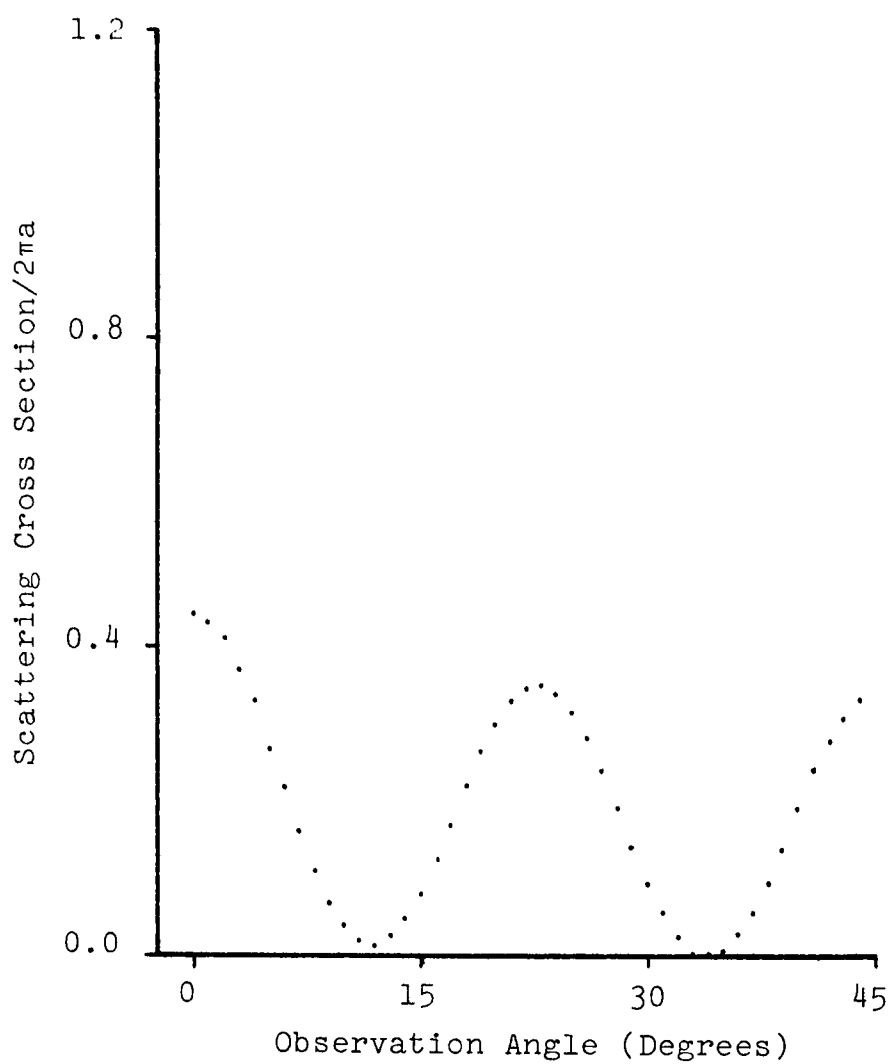


Figure 46. Calculated Results for a Single Cylinder with $k_1a = 7.0$.

Next, the influence of changing the separation distance between the two forward cylinders, while holding the third fixed, was investigated. Comparison of Figure 47 with Figure 48 demonstrated that increasing this separation distance twofold resulted in a decrease in both the magnitude and width of the backscattered peak while the angular locations and magnitudes of the secondary peaks were severely altered. Figure 49 shows that increasing the separation further changed the entire scattering diagram, greatly attenuating all features including the backscattered peak which was also broadened. These figures have shown that, with the back cylinder fixed in position, the magnitude of the backscattered peak decreases with increasing separation of the two forward cylinders. Secondary peak characteristics exhibited a more critical dependency upon this separation.

Figures 50, 51 and 52 show the influence of increasing cylinder diameter while holding separation lengths constant. Figures 53 and 54 provide the scattering diagrams of the individual cylinders making up the systems in Figures 50 and 52 respectively. The only common feature of the scattering diagrams of these three systems of cylinders was the existence of a backscattered peak. All other features were greatly dependent upon the size parameter of the individual cylinders, including the widths

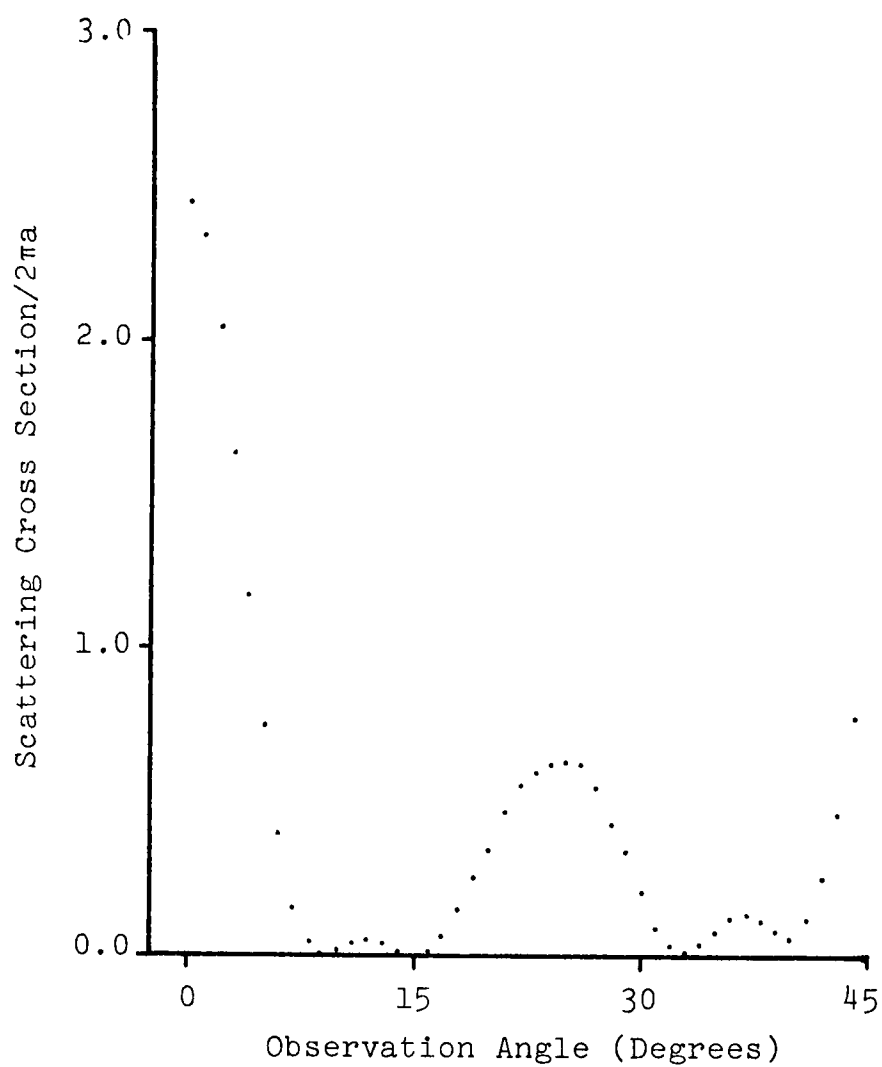


Figure 47. Calculated Results for a Three Cylinder System with $k_1a = 7.0$, $2k_1b = 16.0$, and $k_1c = 20.0$.

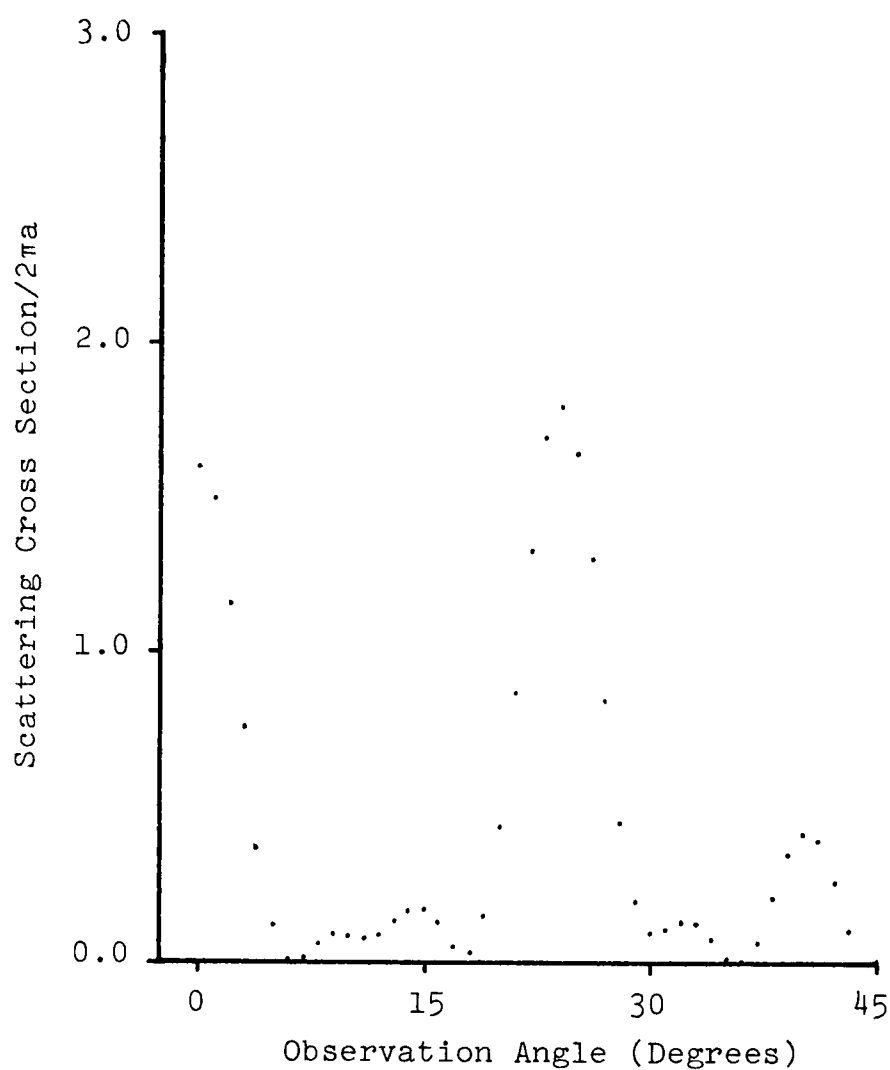


Figure 48. Calculated Results for a Three Cylinder System with $k_1a = 7.0$, $2k_1b = 32.0$, and $k_1c = 20.0$.

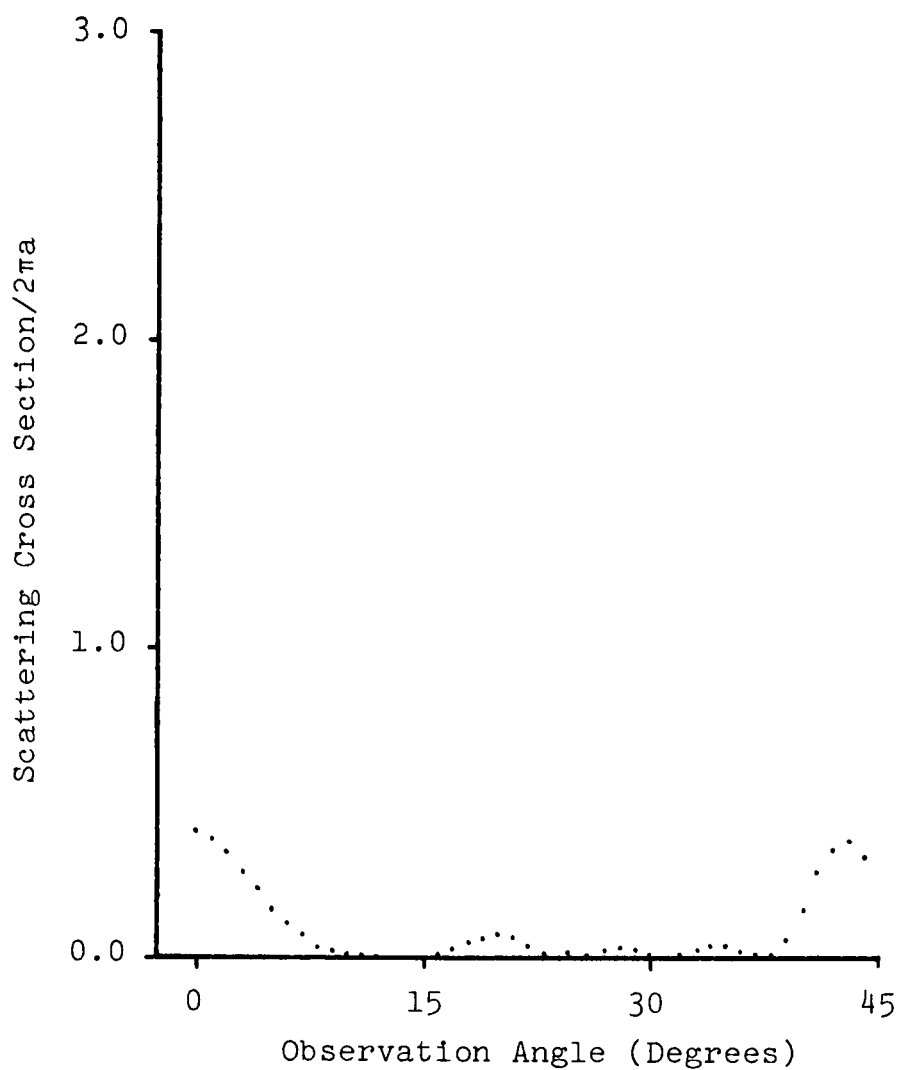


Figure 49. Calculated Results for a Three Cylinder System with $k_1a = 7.0$, $2k_1b = 40.0$, and $k_1c = 20.0$.

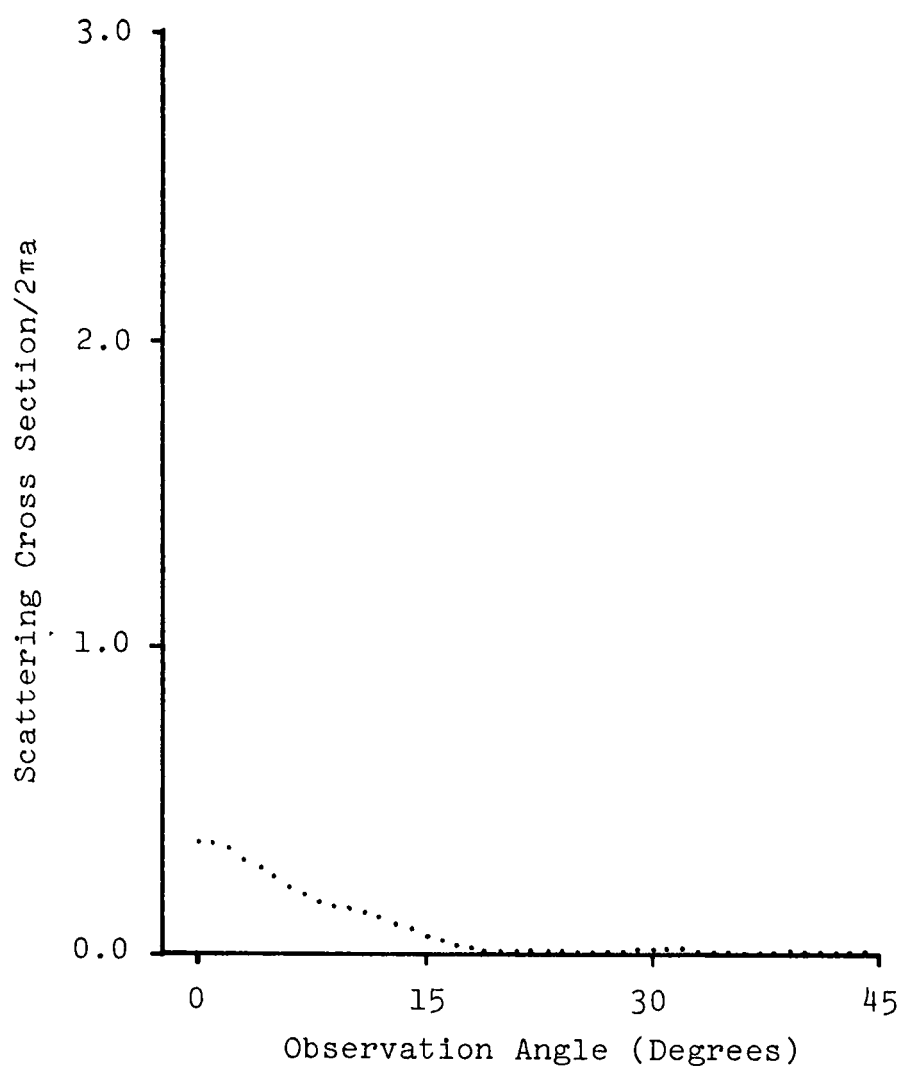


Figure 50. Calculated Results for a Three Cylinder System with $k_1a = 5.0$, $2k_1b = 60.0$, and $k_1c = 30.0$.

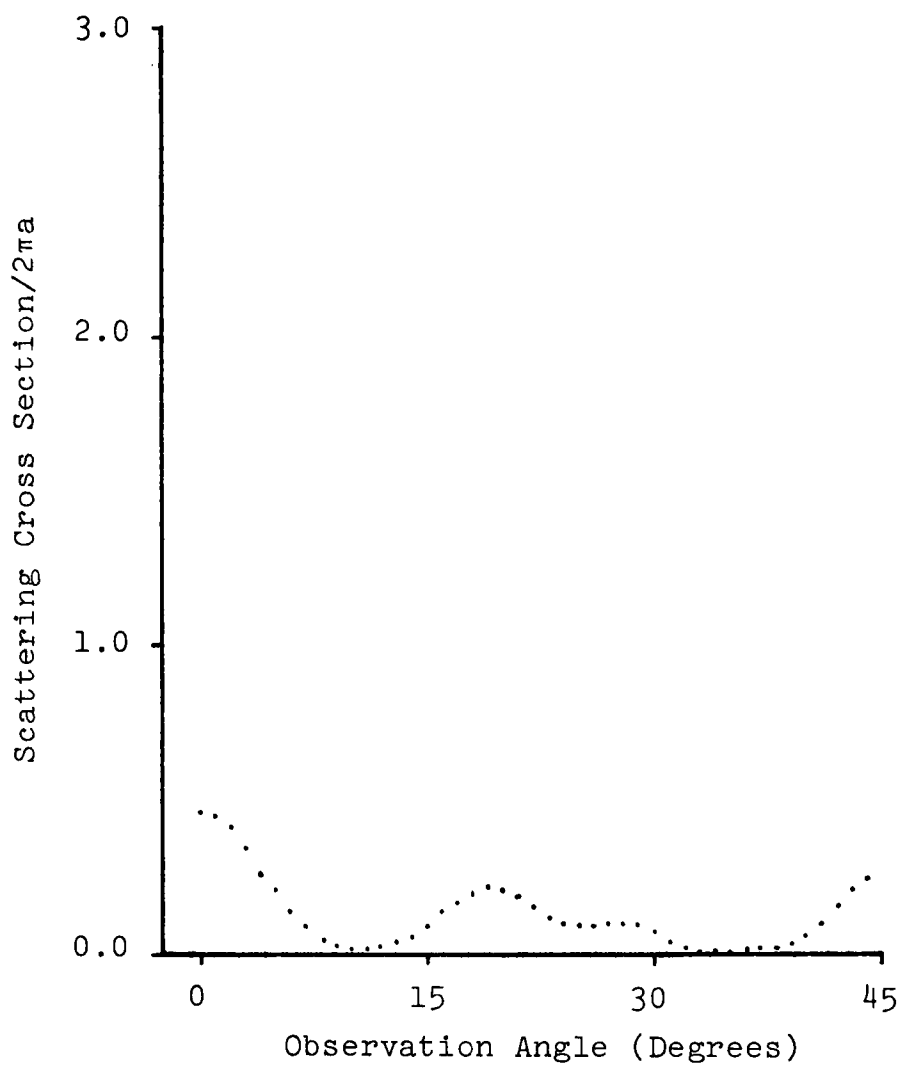


Figure 51. Calculated Results for a Three Cylinder System with $k_1a = 7.0$, $2k_1b = 60.0$, and $k_1c = 30.0$.

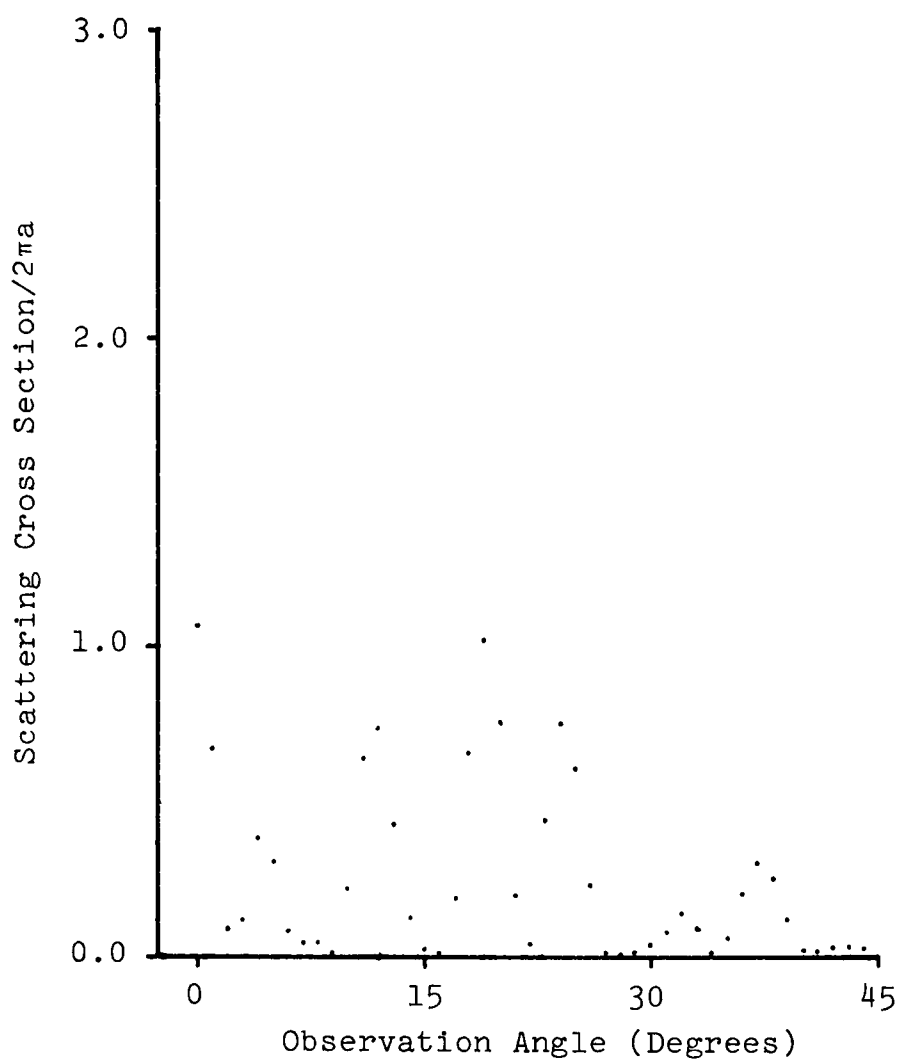


Figure 52. Calculated Results for a Three Cylinder System with $k_1a = 20.0$, $2k_1b = 60.0$, and $k_1c = 30.0$.

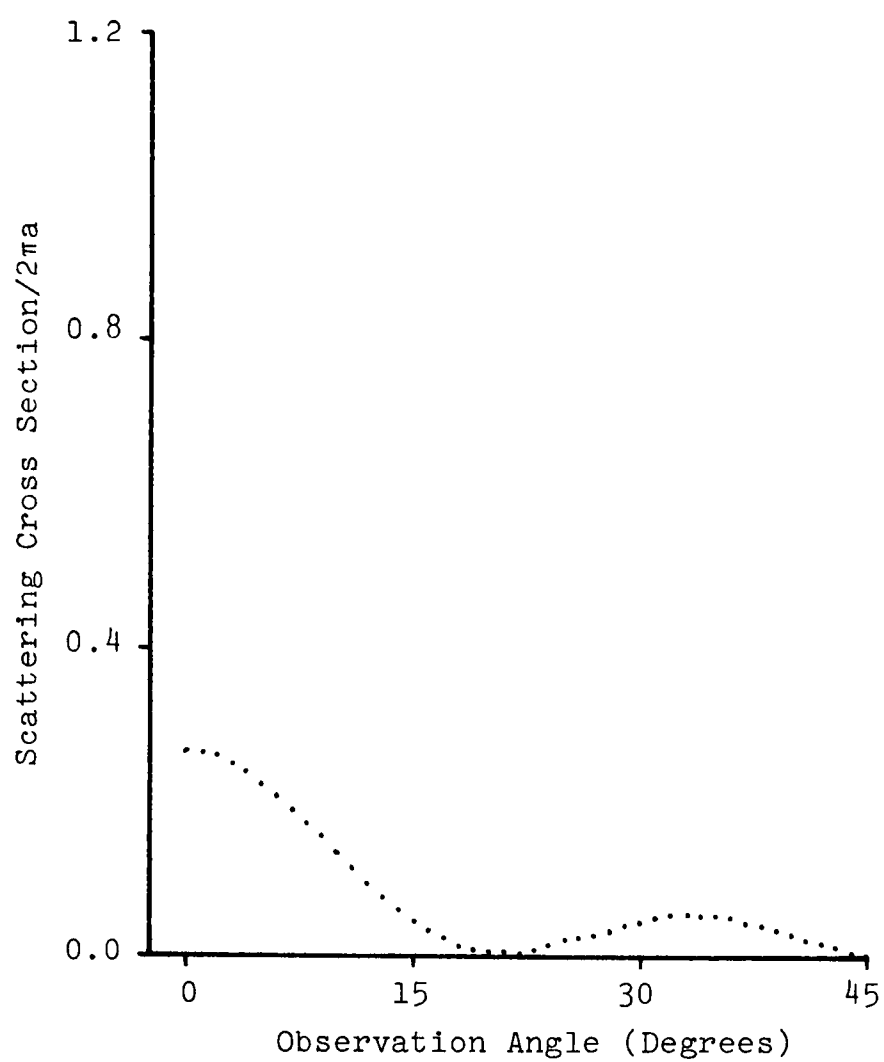


Figure 53. Calculated Results for a Single Cylinder with $k_1a = 5.0$.

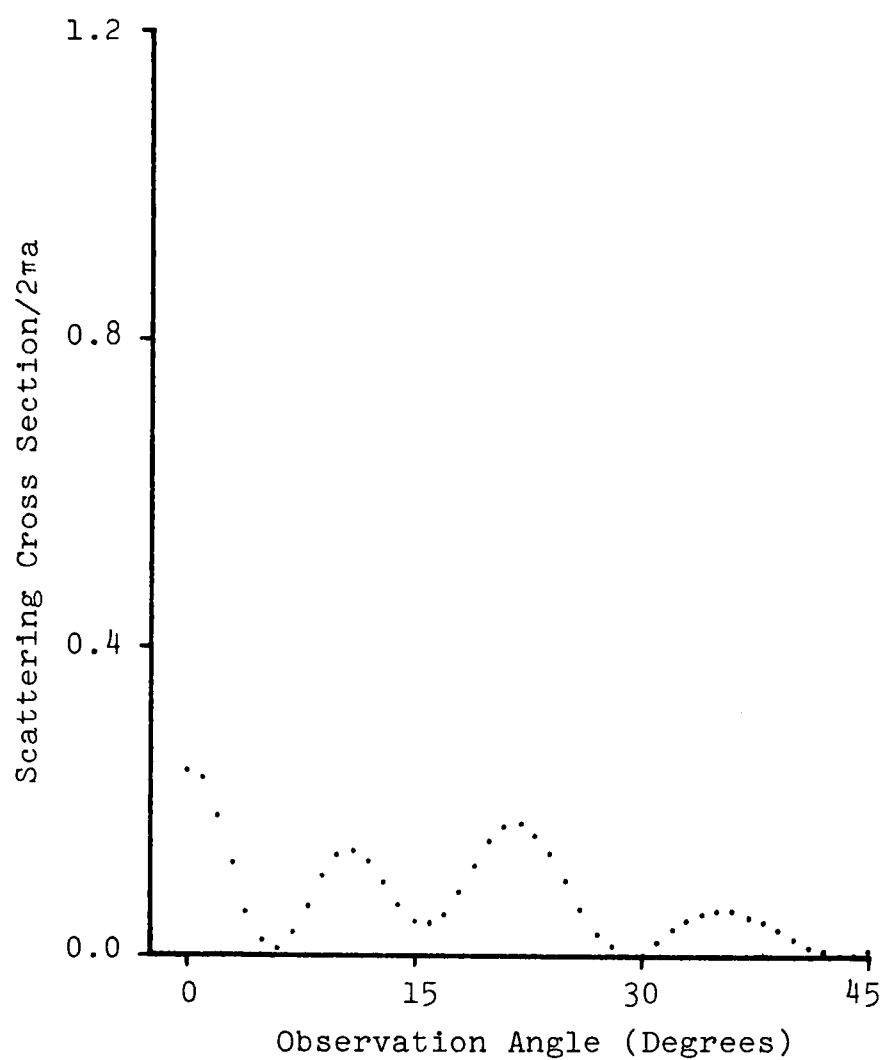


Figure 54. Calculated Results for a Single Cylinder with $k_1a = 20.0$.

of the backscattered peaks which decreased with increasing cylinder diameters.

Conclusions

Using systems of three non-coplanar dielectric cylinders as models for material surface elements, the results of this study have indicated that backscattered peaks comparable with those of the opposition effect could result when these systems of cylinders are illuminated by plane, electromagnetic waves. The magnitudes of these peaks in the calculated scattering cross sections were greatly influenced by the relative locations of the cylinders within a given system and, along with the full angular widths of the peaks, by the size parameters of the cylinders. The magnitudes and angular positions of other peaks in the scattering diagrams were found to be dependent upon both cylinder size and cylinder placement within a system.

In relation to the opposition effect, the primary result of this study was that, in the case of scattering cylinder diameters comparable with the wavelength of the incident radiation, pronounced backscattered peaks in the calculated scattering diagrams were shown to exist. The existence of these peaks required neither particular sized cylinders nor particular separation distances between cylinders within a system. Other peaks exist in particular

examples at other than zero degrees phase angle. These are sometimes of the size of the backscattered peak, but often are much smaller. From example to example these peaks occur in different observational directions.

While the directional distribution of scattered radiation from collections of such systems of cylinders should be calculated utilizing such collections, and the number of examples calculated here are limited, it certainly appears from the above that a surface model composed of a large collection of such systems, varying in size and separation parameters, would produce a peak similar to that of the opposition effect, even if the illumination were not monochromatic, the peaks at other angles averaging out over the collection. Thus, diffraction effects by groups of scatterers comparable with the wavelength of light must be added to the list of mechanisms which could contribute to the production of the opposition effect.

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APPENDIX

APPENDIX

DERIVATION OF EQUATION (75)

The integral

$$\int_0^{2\pi} \cos(\theta_1 - \theta'') \exp\{-ik_2 a \cos(\theta_1 - \theta'') - im\theta_1\} d\theta_1$$

may be evaluated using the identities

$$\exp\{-ik_2 a \cos(\theta_1 - \theta'')\} = \sum_n (-i)^n J_n(k_2 a) \exp\{in(\theta_1 - \theta'')\} \quad (A-1)$$

and

$$\cos(\theta_1 - \theta'') = \frac{1}{2} \left[\exp\{i(\theta_1 - \theta'')\} + \exp\{i(\theta'' - \theta_1)\} \right] \quad (A-2)$$

Substituting these into the integral results in

$$\begin{aligned} & \int_0^{2\pi} \cos(\theta_1 - \theta'') \exp\{-ik_2 a \cos(\theta_1 - \theta'') - im\theta_1\} d\theta_1 \\ &= \frac{1}{2} \sum_n (-i)^n J_n(k_2 a) e^{-in\theta''} \left[e^{-i\theta''} 2\pi \delta_{n-m, -1} + e^{i\theta''} 2\pi \delta_{n-m, +1} \right] \\ &= \pi \left[(-i)^{m-1} J_{m-1}(k_2 a) e^{-im\theta''} + (-i)^{m+1} J_{m+1}(k_2 a) e^{-im\theta''} \right] \\ &= i\pi (-i)^m e^{-im\theta''} \left[J_{m-1}(k_2 a) - J_{m+1}(k_2 a) \right] \\ &= 2\pi (-i)^m i e^{-im\theta''} J'_m(k_2 a) \quad (A-3) \end{aligned}$$

VITA

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