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To the Graduate Council:

I am submitting herewith a thesis written by Charles Lamar Brantley entitled "The Design and Analysis of a Vacuum Tube D. C. Voltage Regulator Having Specified Characteristics." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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THE DESIGN AND ANALYSIS OF A VACUUM TUBE D. C. VOLTAGE
REGULATOR HAVING SPECIFIED CHARACTERISTICS

A THESIS

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TABLE OF CONTENTS

	PAGE
INTRODUCTION	1
THEORETICAL CONSIDERATIONS	4
The General Regulator—D. C. Analysis	4
The General Regulator—A. C. Analysis	9
Impedance Considerations	13
Ripple Suppression Considerations	20
REGULATOR DESIGN USING SPECIFIED CHARACTERISTICS	21
D. C. Design	21
A. C. Design	29
SUMMARY AND CONCLUSIONS	48
REFERENCES	51
APPENDIXES	53
APPENDIX I	54
APPENDIX II	55
APPENDIX III	56
APPENDIX IV	59
APPENDIX V	64
APPENDIX VI	65

LIST OF FIGURES

FIGURE	PAGE
1. Basic Electronic Regulator	4
2. D. C. Regulator	9
3. Cathode Coupled Amplifier	11
4. Advanced Electronic Regulator	12
5. Non-unity Negative Feedback System	12
6. Basic Shunt Feedback Amplifier Block Diagram	14
7. Optimum AB Cutoff Characteristic	16
8. Graphical Analysis of Output Impedance for a Shunt Feedback Amplifier	17
9. Nyquist Plot for Three Stage Feedback Amplifier	18
10. Graphical Determination of $/(1 + AB)/$ at Gain Crossover	19
11. Noise in Feedback System	20
12. 6528 Plate Characteristics	22
13. Output Stage	23
14. Pentode D. C. Voltage Amplifier	25
15. Cathode Coupled Amplifier	26
16. Feedback Network	26
17. Regulator Design--D. C. Circuitry	28
18. Effect of Adding Pole to Z_p	30
19. $Z_p - /AB/$	31
20. Preliminary $/AB/$ Cutoff Design	33

FIGURE	PAGE
21. Feedback Network with Added Capacitor	34
22. A. C. Equivalent Circuit	35
23. Actual AB Transmission--No Corrective Networks	42
24. Amplifiers with Added Corrective Networks	41
25. Actual AB Transmission--Corrective Networks Added	42
26. Determination of Phase and Margin for Regulator	44
27. Final Design of Completed Regulator	47
28. D. C. Voltage Regulator Performance	50
29. $/AB/$ and $\frac{1}{1 + /AB/}$ Near Gain Crossover	58
30. Asymptotic Phase and Magnitude Plots	60
31. Combination of Simple Shapes	62

THE DESIGN AND ANALYSIS OF A VACUUM TUBE D. C. VOLTAGE REGULATOR HAVING SPECIFIED CHARACTERISTICS

INTRODUCTION

Advancing technology in the fields of Radar and Computers has placed increased emphasis on the performance of d. c. voltage regulators in the past few years, and there certainly is no reason to believe that this trend will subside in the future.

Vacuum tubes have been used for many years as a means of regulating output voltages of d. c. power supplies. The consequent development of a "cookbook" technique of design for such circuitry has led to many misconceptions and misunderstandings as to the characteristics available from the vacuum tube regulator and the methods of obtaining these characteristics. In many cases the design engineer, having a muddled concept of voltage regulators, specifies a regulated voltage having characteristics far exceeding his particular need. This overdesign and subsequent cost spiral is exceedingly wasteful. On the other hand, inadequate design leads to degradation of system performance.

Regulated power supplies are generally thought of as having at least three desirable characteristics. These are low output impedance, low output ripple voltage, and constancy of output voltage with changing output load and input voltage. The objective of this study was to present a method of analyzing and designing a typical voltage regulator from the standpoint of the above characteristics. To the writer's

knowledge there is no publication presently available in which the analysis of the vacuum tube d. c. voltage regulator is comprehensively treated. In concluding this study a design procedure for such circuitry will be formulated which, it is hoped, will be both useful and informative. The design and analysis techniques as advanced in this study are intended to be applicable in their basic concepts to almost any type of d. c. voltage regulator in which vacuum tubes are utilized.

Basically, there are two types of vacuum tube d. c. voltage regulators--the shunt regulator and the "series" regulator. Since the latter type apparently is the most misunderstood and abused of the two mentioned, it will serve as the pattern in this paper. The name "series" regulator is misleading, and the manner in which schematic diagrams of such circuitry is generally drawn only adds to the confusion. The "series regulator" is a direct coupled cathode follower power amplifier with negative feedback. Any analysis applying to amplifiers in this category may also be applied to power supply regulators without alteration.

There is a wealth of published information available on the general subject of d. c. regulators in which vacuum tubes are utilized. However, these publications individually tend to omit various concepts and analyses important to the thorough understanding of the subject. Foundations for d. c. analysis may be found in any good textbook on vacuum tube devices, while the basic concepts for the a. c. analysis may be found in textbooks concerned with feedback analysis and

stability criteria. The works of Bode, Black, Nyquist, and Terman are particularly enlightening on this general subject.

The best method for advancing a technique of design and analysis is probably by means of an example. To this end, a voltage regulator having the following characteristics served as the pattern in this study.

- a) Output voltage was to be + 300 volts from no load to full load over a line voltage variation of 10 per cent.
- b) ~~Maximum~~ load was to be a ~~minimum~~ of 300 milliamperes.
- c) Output impedance was to be less than one ohm from d. c. to 100 kilocycles.
- d) Output ripple was to be less than 5 millivolts peak to peak.

The input requirements were reasonable and were required to meet the following specifications.

- a) Input voltage was to be + 400 volts at 375 milliamperes. Variation was not to exceed ± 5 per cent over a line voltage variation of 10 per cent.
- b) Input ripple was not to exceed 4 volts peak to peak.

THEORETICAL CONSIDERATIONS

The General Regulator--D. C. Analysis

The general cathode follower type electronic d. c. regulator is made up of an output stage, a feedback network, a reference voltage, and a feedback amplifying device. In conformance with this the following circuit satisfies these requirements.

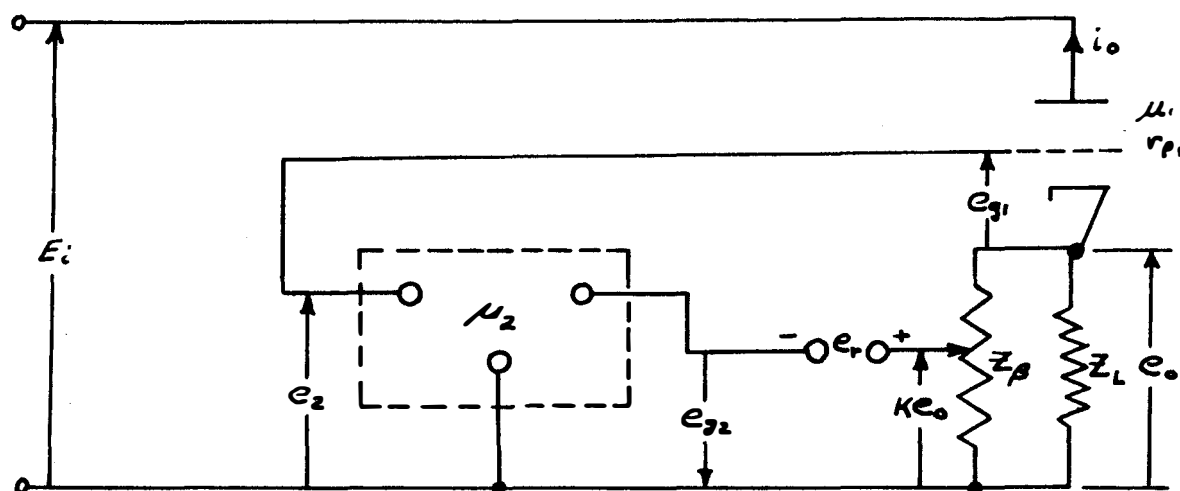


Figure 1. Basic electronic regulator

E_i - Input voltage from rectifier filter unit

V_1 - Output stage

Z_L - Load impedance

Z_b - Feedback network

Z_k - Z_L in parallel with Z_b

e_r - Reference voltage

K - Percentage of e_o that is fed back ($K \leq 1$)

u_2 - Means of amplifying feedback voltage

e_o - Output voltage

i_o - Load current

It is assumed that plate voltage for the u_2 device is independent of E_1 or e_o . Although this assumption is not valid in practice, it will have no adverse effect on the qualitative results to follow.

From Figure 1, d. c. equations may be written around various loops of the circuit.¹ Manipulation of these equations yields the following important relationships.

$$e_o = \frac{R_k(u_1 u_2 e_r + E_1)}{R_k(u_1 + 1 + u_1 u_2 K) + r_{pl}} \quad (1)$$

$$\frac{de_o}{de_r} = \frac{R_k}{R_k(u_1 + 1 + u_1 u_2 K) + r_{pl}} \quad \left| \begin{array}{l} e_r = C \\ R_k = C \end{array} \right. \quad (2)$$

$$\frac{de_o}{dE_1} = \frac{R_k u_1 u_2}{R_k(u_1 + 1 + u_1 u_2 K) + r_{pl}} \quad \left| \begin{array}{l} E_1 = C \\ R_k = C \end{array} \right. \quad (3)$$

$$\frac{de_o}{dR_k} = \frac{u_1 u_2 e_r + E_1}{R_k(u_1 + 1 + u_1 u_2 K) + r_{pl}} - \frac{R_k(u_1 u_2 e_r + E_1)}{[R_k(u_1 + 1 + u_1 u_2 K) + r_{pl}]^2} \quad \left| \begin{array}{l} E_1 = C \\ e_r = C \end{array} \right. \quad (4)$$

These four equations are derived in the Appendix. As written above, the equations are difficult to use in practice. However, if relative magnitudes are considered, one finds that $u_1 u_2 \gg (u_1 + 1)$, and $r_{pl}/R_k \ll u_1 u_2 K$. If these approximations are used, the equations above are reduced to the more easily applied forms:

$$e_o \approx e_r / K \quad (5)$$

$$\frac{de_o}{dE_1} \approx \frac{1}{u_1 u_2 K} \quad \left| \begin{array}{l} e_r = C \\ R_k = C \end{array} \right. \quad (6)$$

$$\frac{de_o}{de_r} \approx \frac{1}{K} \quad \left| \begin{array}{l} E_i = C \\ R_k = C \end{array} \right. \quad (7)$$

$$\frac{de_o}{dR_k} \approx \frac{e_o r_{pl}}{R_k^2 u_1 u_2 K} \quad \left| \begin{array}{l} E_i = C \\ e_r = C \end{array} \right. \quad (8)$$

In addition to the four relations given above a relationship between output voltage and output current must be derived in order to accomplish a complete analysis of the general regulator from the d. c. standpoint. By noting that $i_o = R_k/e_o$ and substituting this relationship for R_k into equation 1

$$e_o = \frac{u_1 u_2 e_r + E_i - i_o r_{pl}}{(u_1 + 1 + u_1 u_2 K)} \quad (9)$$

$$\frac{de_o}{di_o} = \frac{r_{pl}}{(u_1 + 1 + u_1 u_2 K)} \quad \left| \begin{array}{l} e_r = C \\ E_i = C \end{array} \right.$$

$$\frac{de_o}{di_o} \approx - \frac{1}{gm_1 u_2 K} \quad \left| \begin{array}{l} e_r = C \\ E_i = C \end{array} \right. \quad (10)$$

The Stabilization Ratio (Regulation Factor) may be recognized as de_o/dE_i and output impedance is given by de_o/di_o .

Thus it may be seen that equations 5, 6, 7, 8, and 10 contain all the information needed to predict the d. c. behavior of the general regulator. From equation 5, it is apparent that either e_r or K may be utilized as a means of controlling the magnitude of output voltage. Generally speaking, however, e_r is rather difficult to vary. Therefore

K is almost universally used for this application. From equation 6, it is seen that the stabilization ratio is primarily an inverse function of the u_2 device. ($u_1 < 1$, $K \leq 1$). Therefore, to insure good regulation the feedback amplifying device should have as much gain as possible. This conclusion is also borne out by equations 7, 8, and 10, since the ultimate desire is to make the denominator of the right hand side of these equations equal to infinity. It should also be noted that K appears as a factor in all of the figures of merit as given above. Consequently the device from which K is derived should be as stable and free from drift as possible. Generally a wirewound low temperature coefficient potentiometer is used in this application.

The reduction in any change in input voltage is clearly indicated by equation 6. This reduction also applies in modified form to any ripple voltage present in the regulator from the filter section. This analysis will be covered in the sections to follow.

The minus sign which appears in the definition of output impedance (equation 10) is extremely significant in that it proves that a change in load current in a positive sense is accompanied by a decrease in output impedance. Thus it is seen that the designer should initially use the minimum tolerable load as a design criterion for determining output impedance.

It should be noted that no particular care was taken in determining the polarity of voltages as shown in Figure 1. It takes but a moment of reflection to realize that the change in output from the u_2 device must be opposite to that of a change in input. This

usually limits the u_2 device to an odd number of stages where vacuum tubes are used except in the case where one grounded grid stage is used.

Examination of the equations and discussion given above leads to the following conclusions:

- a) The output stage should be characterized by a high current capacity, an extremely high transconductance, and a high plate power dissipation.
- b) The feedback network should be a high stability, high impedance device to reduce interaction.
- c) The reference voltage source should also be a high stability device.
- d) The feedback amplifying device should be characterized by a high gain. This generally means that pentode tubes should be used.

With these requirements in mind, the following type of regulator would probably suffice for perhaps 8 out of 10 d. c. applications. This regulator is capable of excellent performance. Maximum load current is determined only by the limits of the output tube V_1 . Output impedance is determined principally by the gain of u_2 , and with one stage of voltage amplification, this value should not be greater than 10 ohms. Stabilization ratios for this type vary between 0.05 and 0.005.

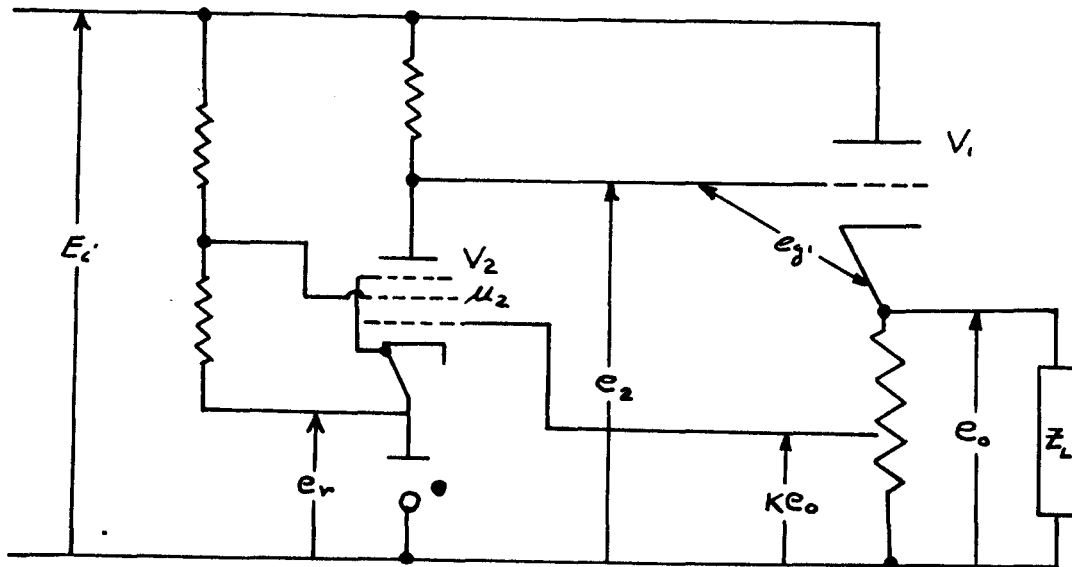


Figure 2. D. C. regulator

The General Regulator--A. C. Analysis

As was pointed out in the previous section, the use of only one stage of voltage amplification in the feedback amplifying device is generally sufficient to obtain adequate regulation and a relatively low output impedance. Under these conditions, if oscillations are present, they will occur in the region from 50 to 100 kilocycles. Applications in which the type of regulator shown in Figure 2 are used very rarely include a requirement that no high frequency oscillations be present. Thus little care is taken to prevent this occurrence.

In many applications, however, d. c. output impedance of a regulator system is required to be far less than that possible with the regulator of Figure 2. Such applications include uses with pulsed

radar and computer systems. When an electronic regulator is used in these cases, output impedance is generally specified over a fairly wide frequency range. In addition to the requirement for output impedance, there is also a requirement that no high frequency oscillations be present, since noise or any type of spurious signals may degrade the performance of the connected systems.

Addition of more amplification stages within the feedback amplifying device is of necessity accompanied by a stability problem, since maximum phase shift of the open loop system may exceed 180° . The fact that conductive coupling must be used throughout results in several problems of design and stability. One obvious design problem arises because plate potentials are considerably higher than grid potentials. It is therefore necessary either to have the cathode of a following stage at this higher potential from ground or to use a device to lower this potential the correct amount. It should be noted here that the gas regulator in u_2 of Figure 2 served a dual purpose. First it served to raise the d. c. voltage on the cathode of V_2 as required above and secondly it served as a reference source. The use of gas regulators or other low impedance devices to serve the former function results in a lower efficiency and a decrease in gain.

Since the entire regulator should operate from a single source of unregulated input voltage, the use of three or more stages of amplification in the u_2 system would generally result in failure, because d. c. electrode potentials could not be maintained for proper

biasing. Thus the designer is faced with the task of finding some device which will give amplification without phase inversion. A very convenient device for this application is the cathode coupled amplifier as shown below.²

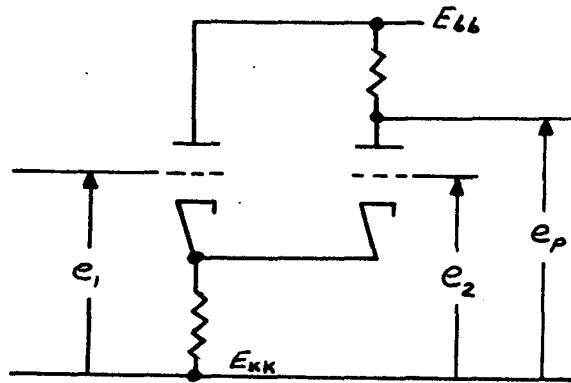


Figure 3. Cathode coupled amplifier

This circuit is also doubly useful, since e_p will be the differential between e_2 and e_1 amplified by a given amount if the voltage e_2 is held at a constant level. Thus e_2 becomes a convenient high impedance terminal for the required reference voltage.

Let us now use the circuit of Figure 3 and one stage of pentode voltage amplification in the u_2 device. Figure 1 will then become Figure 4.

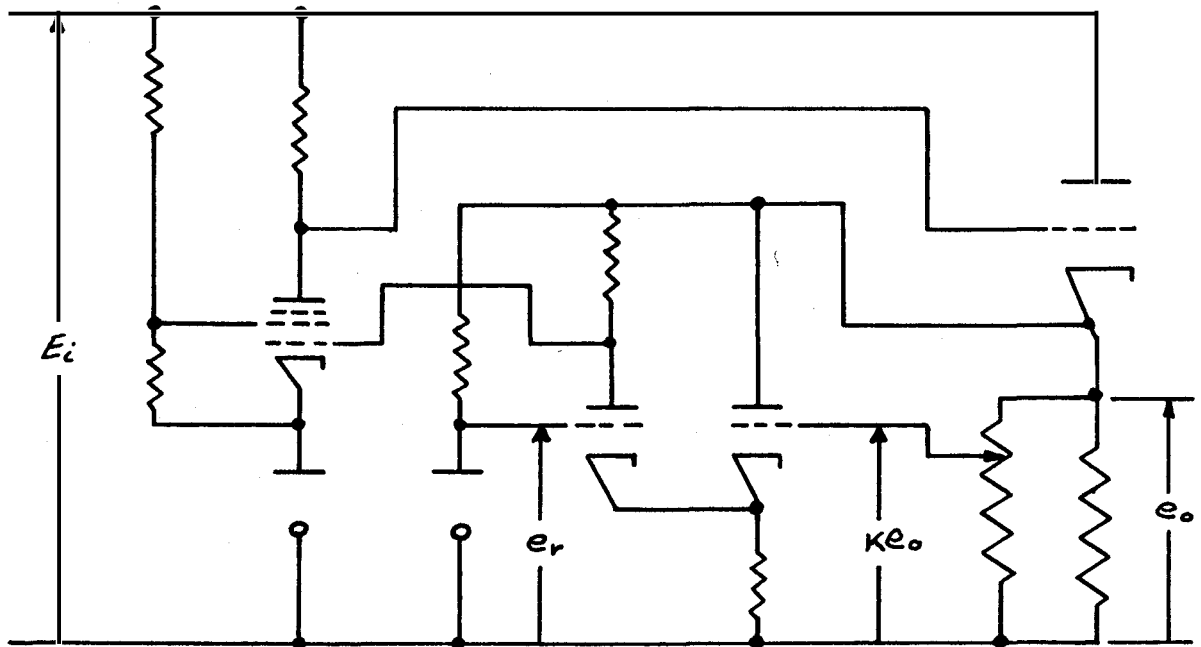


Figure 4. Advanced electronic regulator

From basic feedback principles³ it is known that if u is the amplification of a system similar to Figure 5 without feedback, the amplification with feedback is given by

$$G_{wf} = \frac{u}{1 + AB} \quad (11)$$

where AB is defined as uu_2k .

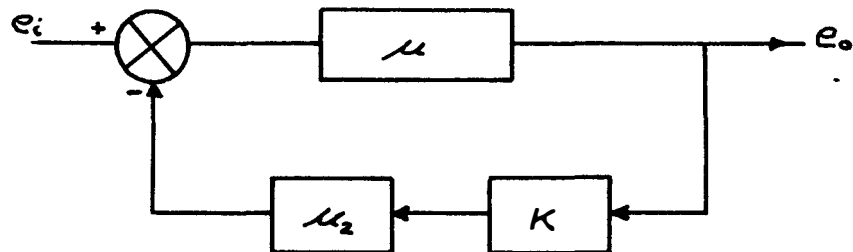


Figure 5. Non-unity negative feedback system

Again consider the basic regulator of Figure 1. In this case

$$u = \frac{u_1 Z_k}{r_{pl} + u_1 Z_k + Z_k} \quad (12)$$

and $(1 + AB) = 1 + \frac{Ku_1 u_2 Z_k}{r_{pl} + u_1 Z_k + Z_k} . \quad (13)$

Therefore $G_{wf} = \frac{\frac{u_1 Z_k}{r_{pl} + u_1 Z_k + Z_k}}{1 + \frac{Ku_1 u_2 Z_k}{r_{pl} + u_1 Z_k + Z_k}} = \frac{A_1}{1 + KA_1 A_2} . \quad (14)$

The parameters Z_k , A_1 , K_1 , and A_2 may now be functions of frequency. In the previous section frequency dependent functions were of no concern, since all calculations were made at d. c. Under these conditions the denominator of equation 14 must be examined for stability.⁵ For the moment, let us turn to a few remarks concerning impedance considerations in feedback amplifiers.

Impedance Considerations

Relations between impedance and feedback were derived in the early days of the development of feedback systems for a number of specific feedback amplifier configurations. In some cases these relations turned out to be very simple. For the most part, however, the relations were so complicated that they apparently defied reduction to a common form. This difficulty was overcome by R. B. Blackman⁶ who derived the equation

$$Z_f = \frac{Z[\bar{1} - (AB)_o]}{[\bar{1} - (AB)_\infty]} \quad (15)$$

where Z_f is the impedance between any two points of a feedback system, Z is the impedance without feedback, AB_o is the AB of the feedback system with the two points which define Z_f connected together, and AB_∞ is the AB of the feedback system with the short circuit removed.

Equation 15 provides a convenient tool for the analysis of the following shunt feedback system from the standpoint of impedance.

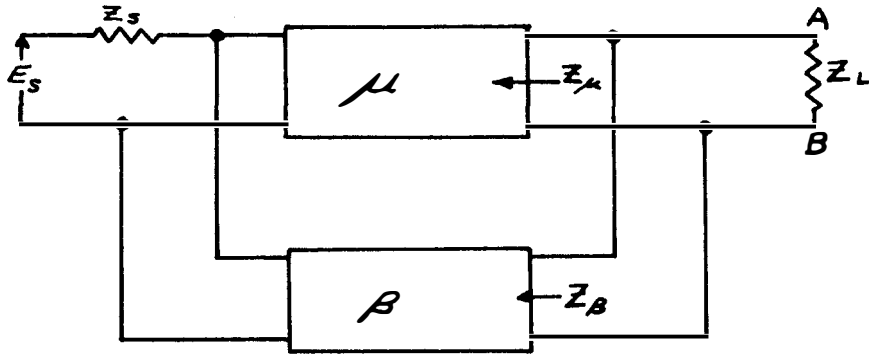


Figure 6. Basic shunt feedback amplifier block diagram

Consider the two points A and B. It is apparent that

$$\frac{1}{Z} = \frac{1}{Z_u} + \frac{1}{Z_B} + \frac{1}{Z_L} \equiv \frac{1}{Z_p}, \quad (AB)_o = 0, \quad (AB)_\infty = (AB). \quad \text{Consequently,}$$

equation 15 now reduces to the further simplified form

$$Z_{AB} = \frac{Z_p}{(1 - AB)}. \quad (16)$$

Consider now equation 16 and take the logarithm of both sides.

$$K \log_{10} Z_{AB} = K \log_{10} Z_p - K \log_{10} (1 - AB) \quad (17)$$

It may immediately be seen that the three terms making up equation 17 are of the general form

$$K \log_{10} \frac{A_1}{A_2}, \text{ where } A_2 = 1 \quad (18)$$

At this point A_1 may be a purely real or a complex quantity. Equation 18 is certainly indicative of the term "decibel above or below a quantity of one," where K would determine the units of voltage, current, or power. It is proposed here that the term "decibel" may also be applied to units of impedance as well as to voltage, current, and power without loss of clarity for the original definition of the decibel.

Design engineers have long been accustomed to expressing the gain of amplifiers in db, where it is generally understood to mean

$$G = 20 \log_{10} \frac{V_1}{V_2}, \text{ where } V_2 = 1 \text{ volt} \quad (19)$$

This term is used almost universally without regard to the fact that the impedance at which V_1 is measured may be radically different from that at which V_2 is measured. Much has been written in attempts to clarify the apparent lack of continuity between the original definition of the decibel and general usage of today. It is not the intent of this paper to either condone or condemn present usage—merely to extend it.

The term "decibel above or below one ohm" shall be defined as

$$\text{db}(\Omega) = 20 \log_{10} \frac{Z}{1 \Omega}. \quad (20)$$

If the various factors in equation 17 are functions of frequency, then

$$Z_{AB} [\text{db}(\Omega)] = 20 \log_{10} Z_p - 20 \log_{10} (1-AB). \quad (21)$$

For the moment let us assume that Z_p is purely resistive in nature. By the definition given above, Z_p may be expressed in db above or below one ohm.

$$Z_p [\text{db}(\Omega)] = 20 \log_{10} R_p \quad (22)$$

A plot of equation 22 on the Bode Diagram^{7,9} will be a straight line of magnitude $20 \log_{10} R_p$. There will be no associated phase shift. Now let us assume that the factor AB of equation 21 is characterized by the ideal AB high frequency cutoff function^{8,9} as shown below. A phase margin of 30° and a gain margin of 10 db will arbitrarily be used herein.

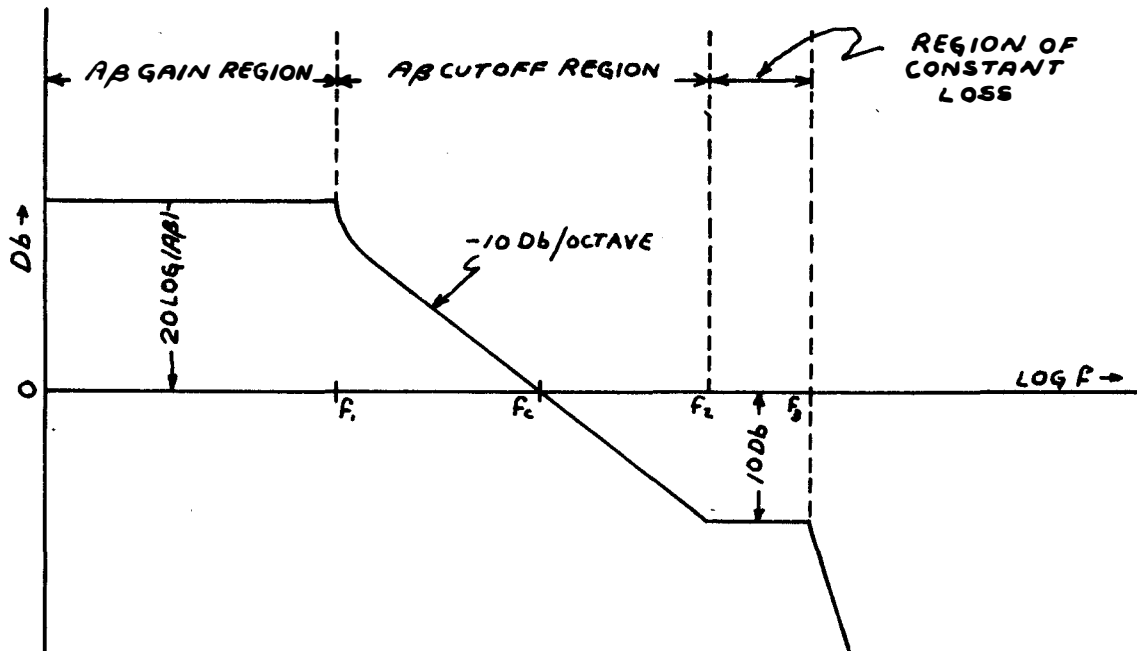


Figure 7. Optimum AB cutoff characteristic

In accordance with equation 21, the output impedance of the shunt feedback amplifier of Figure 6 may now be derived as a function of frequency by combining equation 22 with the definition of $|AB|$ in Figure 7.

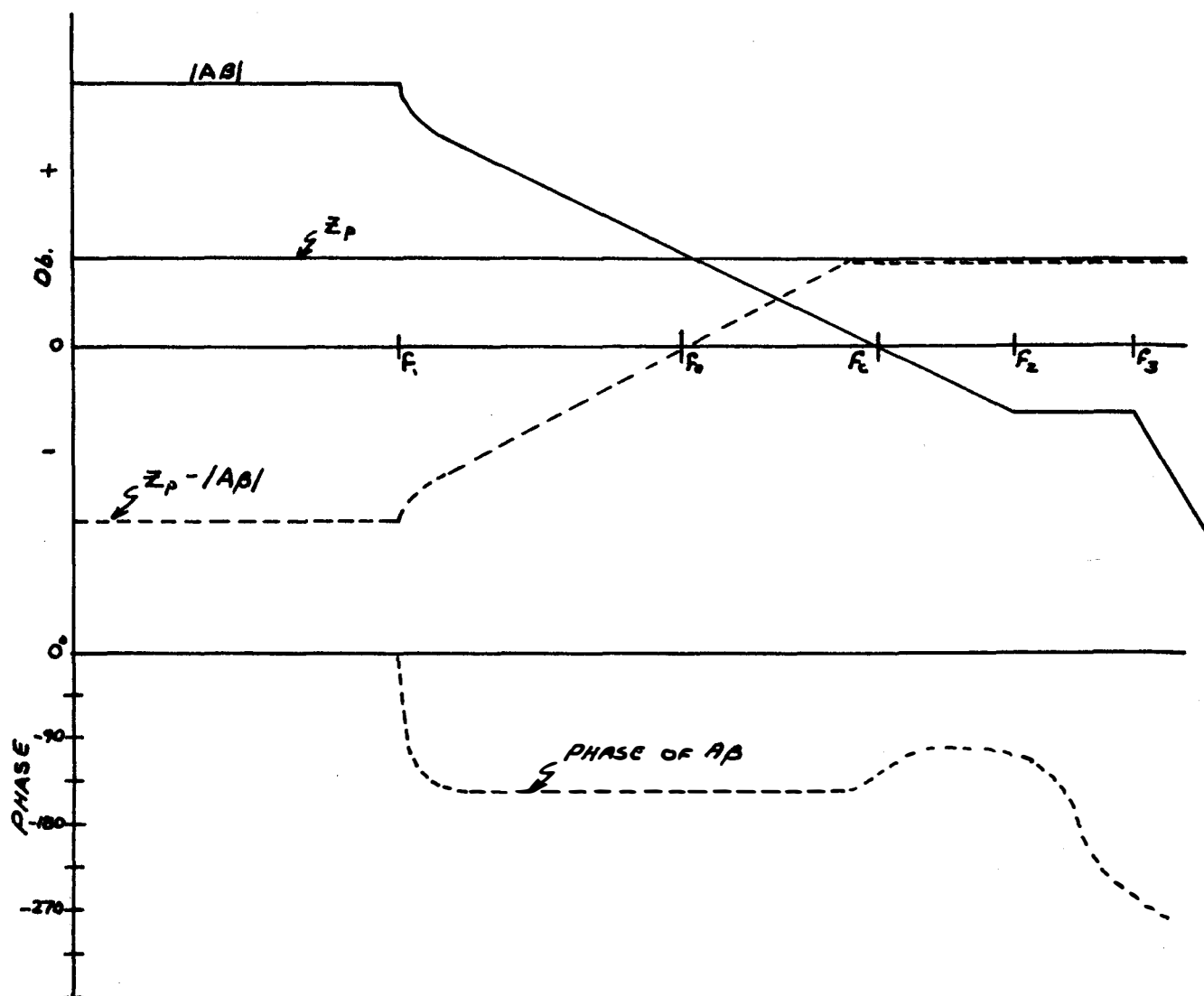


Figure 8. Graphical analysis of output impedance for a shunt feedback amplifier

A logical question to arise at this point is, "What happens near gain crossover when actual loop loss no longer approaches asymptotic loss?" Before this question is answered, one must realize that the output impedance asymptotes which appear in Figure 8 were derived under the assumption that $|1 + AB|$ is equal to $|AB|$. This certainly is a valid assumption as long as the factor $|AB|$ is much greater than one. This is equivalent to saying that as long as the manipulations similar to Figure 8 are carried out at frequencies remote from gain crossover, then, indeed, $|1 + AB|$ approaches $|AB|$. The answer to the question above may be readily answered by use of the Nyquist Diagram.⁵ Assuming that the shunt feedback amplifier of Figure 6 is made up of three stages, the Nyquist diagram would be similar to that shown below.

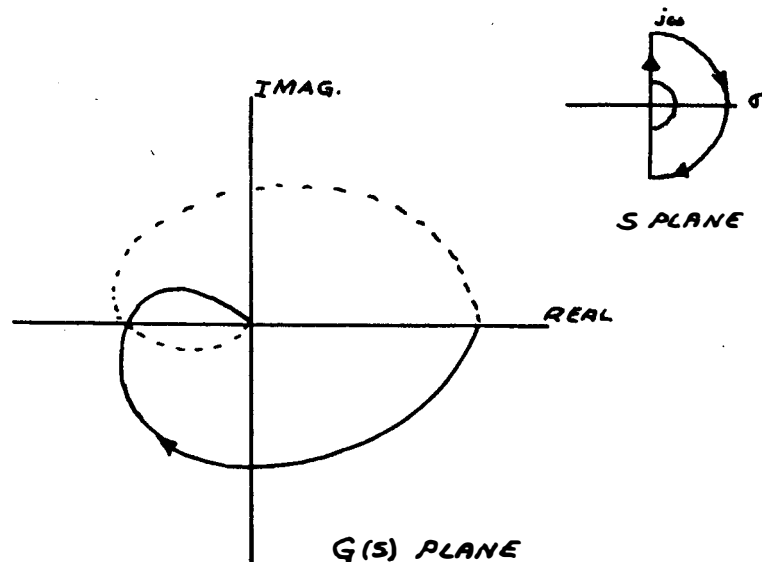


Figure 9. Nyquist plot for three stage feedback amplifier

It may easily be seen that the greatest deviation between $|AB|$ and $|1 + AB|$ occurs at gain crossover. The only method for determining the true error between actual values of $|1 + AB|$ and asymptotic values of $|AB|$ which the author knows is one of point calculation. The Nyquist plot is made for the system under consideration and vector methods are used to determine $|1 + AB|$. Further information on this point may be found in Appendix III.

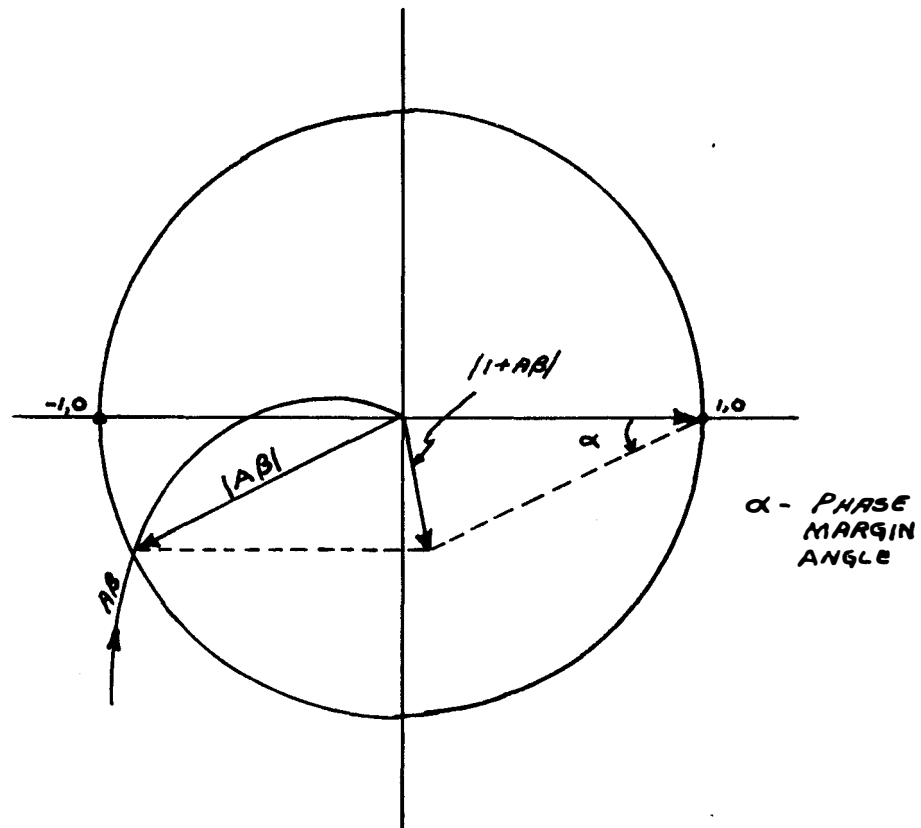


Figure 10. Graphical determination of $|1 + AB|$ at gain crossover

The basic cathode follower type electronic regulator is a shunt feedback amplifier and, consequently, may be analyzed as outlined above.

Ripple Suppression Considerations

Consider the feedback system of Figure 11. In this figure the noise voltage represents ripple voltage present in the system due to filtering action preceding the input. In the case of d. c. regulators this voltage may be considered as a fixed frequency determined by the type of rectifier filter section which is used. In any event, the noise voltage will be composed primarily of some multiple of line voltage.

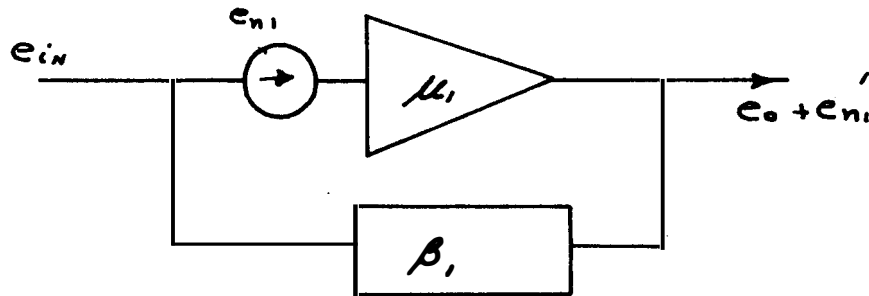


Figure 11. Noise in feedback system

$$E_{n1}' = \frac{u_1}{1 - u_1 \beta_1} E_{n1} \quad (23)$$

Any other noise voltage present in the system will be suppressed

by an amount $\frac{1}{(1 - u_1 \beta_1)}$, where $u_1 \beta_1$ is evaluated at the frequency of the noise voltage.

REGULATOR DESIGN USING SPECIFIED CHARACTERISTICS

D. C. Design

The output stage of the regulator must be capable of supplying ~~maximum~~ rated current of the regulator plus any other dependent current within the regulator itself. From the specified characteristics it is seen that this particular output stage must supply at least 300 milliamperes plus any other dependent current. Having in mind the comments given on page 8, the 6528 type twin triode appears to be a good selection for this application. From the tube plate characteristics, Figure 12, we find that we may operate this device at $E_b = 100$ volts, $I_p = 160$ milliamperes, per section. Under these conditions $\mu = 7.5$, $r_p = 375$ ohms, $g_m = 30,000$ umhos, and $E_{c1} = -5$ volts. Using two sections in parallel, we may imagine this a single tube having $g_m = 60,000$ umhos and $r_p = 187$ ohms. Thus total load may be 320 milliamperes at 300 volts when input voltage is 400 volts. These values were found at nominal input voltage for a constant load. Care must be taken to insure operation within tube limits for limit conditions, i.e., the limits of input voltage and output load. A current of 20 milliamperes internal to the regulator will be assumed.

Limit conditions may be "boxed" in as shown in Figure 12. It is seen that for a ΔE_{bb} of 40 volts centered at 400 volts at ~~maximum~~ load, the stage preceding the output must be capable of supplying a ΔE_{c1} of 5 volts centered around - 5 volts. At minimum load (no external load) ΔE_{c1} becomes 8 volts centered around - 12.5 volts.

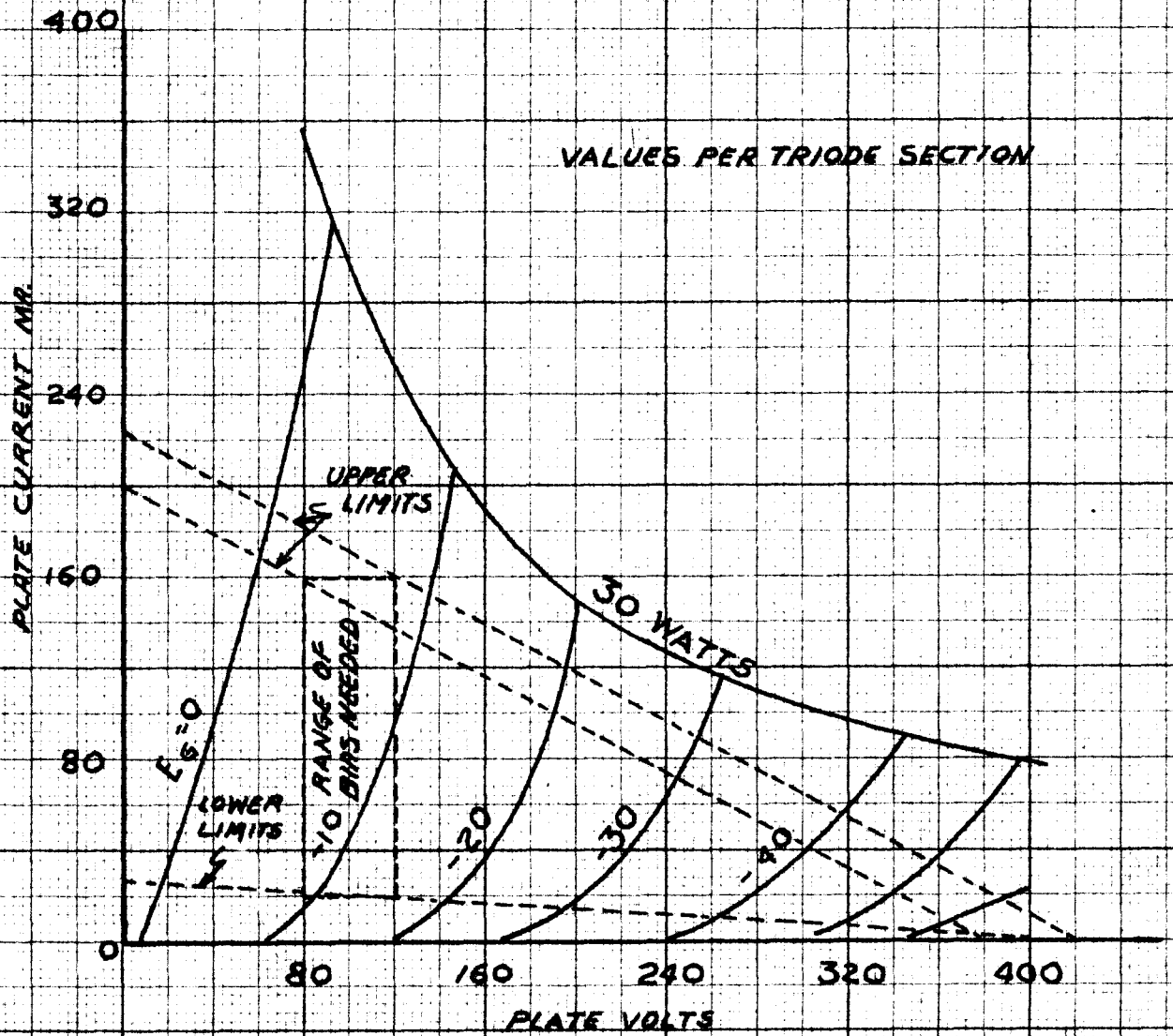


Figure 12. 6528 plate characteristics

Therefore, the stage preceding the output must be capable of supplying a bias range of - 2.5 to - 16.5 volts. Since the cathode of the output stage is to be maintained at + 300 volts, the range of output voltages from the previous stage becomes + 283.5 to + 297.5.

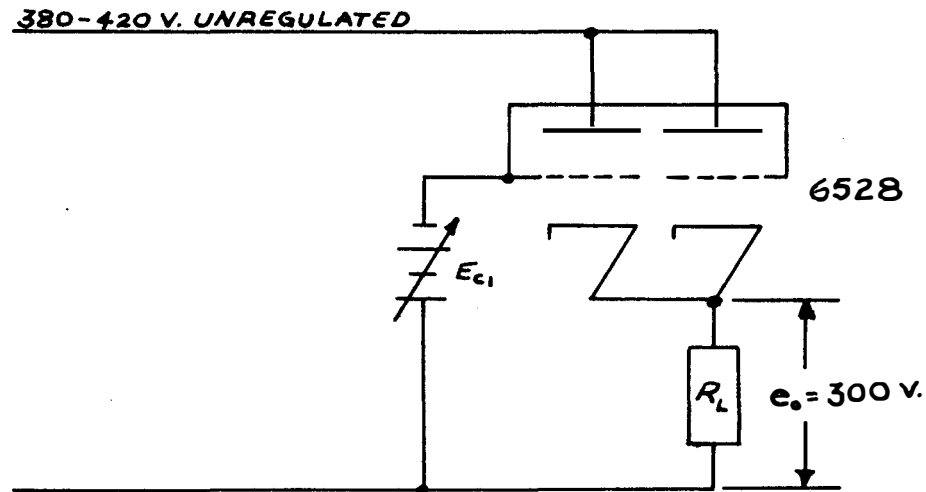


Figure 13. Output stage

Consider now the output resistance of this stage.

$$R_p = \frac{r_p R_k}{r_p + (u_1 + 1)R_k}$$

$$R_p \approx \frac{1}{g_m}$$

At minimum load (no external load) $g_m = 24,000$ umhos per section.

Therefore $R_p = 20$ ohms or 26.0 db above one ohm. It would appear that all that is needed in our feedback amplifying device is a gain greater than 26.0 db in order to reduce this output resistance below one ohm. This is quite true in the d. c. case, but, for reasons which

will become apparent later, we will choose the use of the differential amplifier of Figure 3 and one stage of pentode voltage amplification for our feedback amplifying device.

In the design to follow, pertinent comments will be offered. Basic design principles will not be covered. A more detailed survey of the design procedures used herein may be found in reference 2 of the list of references.

As stated above the pentode amplifier must be capable of delivering an output voltage range from + 283.5 to + 297.5 volts. Two choices are available as to plate supply voltage for this stage. These are the unregulated input voltage or the regulated output voltage. Choice of the latter would mean that the tube must be operated near cutoff to obtain proper output voltage. Such operation is inadvisable, and we are thus faced with using a plate supply voltage which generally exceeds the maximum rated plate supply voltage for most miniature pentodes. This problem is easily remedied by raising the d. c. voltage on the cathode of this stage until the plate voltage is within limits. This is generally accomplished by use of a low impedance gas regulator. The 6AU6 remote cutoff pentode is a good selection for this stage. The pentode amplification stage is shown below.

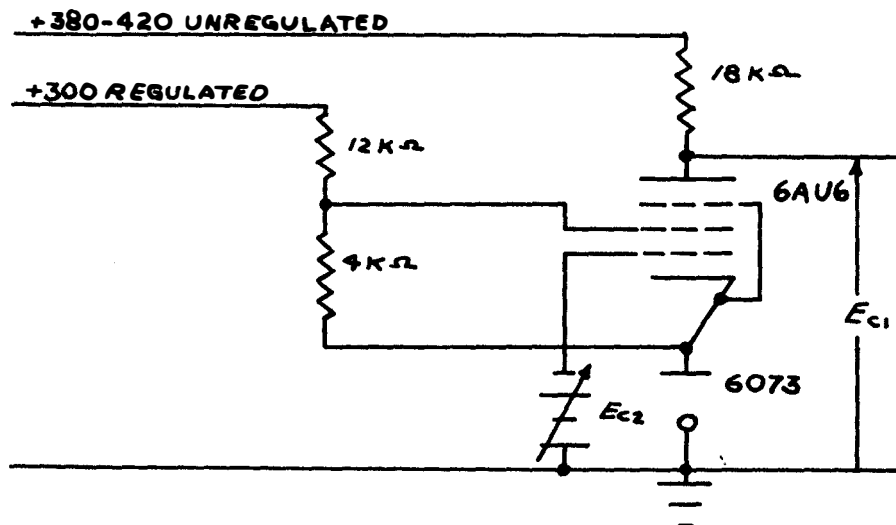


Figure 14. Pentode D. C. voltage amplifier

From the tube characteristic curves it may be seen that a voltage output swing from + 283.5 to + 297.5 may be obtained by allowing E_{c2} to swing from - 1.9 volts to - 2.1 volts. Since the cathode is now maintained at + 150 volts by the gas regulator, the bias to this stage must be from + 148.1 to + 147.9 volts.

The reference voltage source is to be connected to the cathode coupled amplifier as noted before. The 6141 type gas regulator is a good choice here, because this tube is characterized by extremely good stability. Ionization voltage is 100 volts with $I_{min} = 10$ ma. and $I_{max} = 20$ ma. Using this reference source with a 5751 twin triode, the cathode coupled amplifier is shown in Figure 15.

Operation within the range of the tubes has been assured by noting the various operating points on plate characteristics. As long as there is a differential between E_{c3} and e_r (corrected for bias conditions) the regulator will react to maintain a constant + 300 volt output. This statement also applies to limit conditions, and the regulator will function properly from no load to full load over a line voltage variation of ± 5 per cent. The complete regulator for d. c. operation is shown in Figure 17.

Let us now analyze the circuit of Figure 17 from the standpoint of output impedance, stabilization ratio, and load disturbances at minimum load.

<u>Circuit</u>	<u>D. C. Gain</u>	<u>DB</u>
Cathode coupled	48.6	33.7
Pentode	90.5	39.2
Feedback network	0.333	- 9.5
Output stage	0.750	- 2.5

From equation 16, the output resistance is given by $R_o = R_p / (1 + AB)$, where AB now becomes d. c. open loop gain.

$$R_o = \frac{20}{1 + (48.6)(90.5)(.33)(.85)} = 0.016 \text{ ohms}$$

Any d. c. change in input voltage will be diminished by the factor $\frac{1}{1240}$ or $S = 0.00086$. Changes in load current will be diminished by a factor similar to this determined by equation 8, where R_k is determined at minimum load.

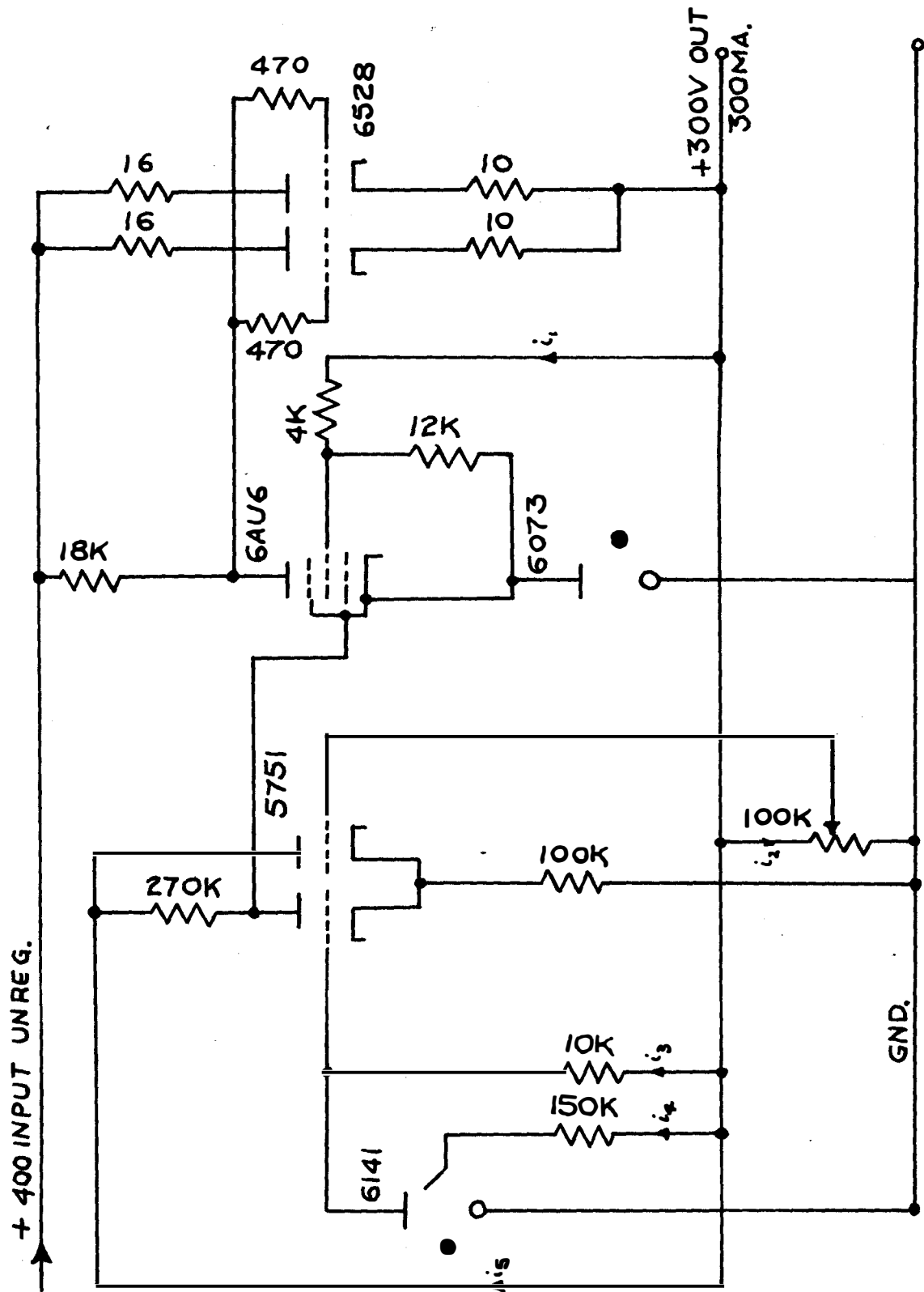
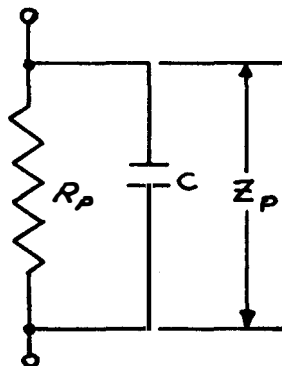


Figure 17. Regulator design--D. C. Circuitry

As far as d. c. operation is concerned, it is thus concluded that a satisfactory regulator has been designed which will meet all required d. c. specifications. However, since this is equivalent to a three stage feedback device, oscillations will be present at some high frequency which will be determined by parasitic and interelectrode capacitances. It only remains to stabilize the feedback loop to remove these oscillations. This must be done in such a manner as to meet the specification that output impedance be less than one ohm from d. c. to 100 kilocycles, and output ripple be less than 5 millivolts.

A. C. Design

Let us turn now to Figure 8, and, for the moment assume that our feedback loop has been designed to follow these asymptotic curves. It is easily seen that if Z_p were purely resistive f_o would have to be somewhat greater than 100 kilocycles to insure that specifications for this regulator were met. In any event, if Z_p were independent of frequency, eventually output impedance would rise above one ohm asymptotically to R_p . Suppose now that Z_p is made frequency sensitive in the following manner.



$$Z_p = \frac{R_p \cdot \frac{1}{j\omega C}}{R_p + \frac{1}{j\omega C}}$$

$$Z_p = \frac{R_p}{(\tau_1 s + 1)}$$

$$\text{where } \tau_1 = R_p C$$

Thus a pole has been added to Z_p at a frequency $1/\tau_1$.

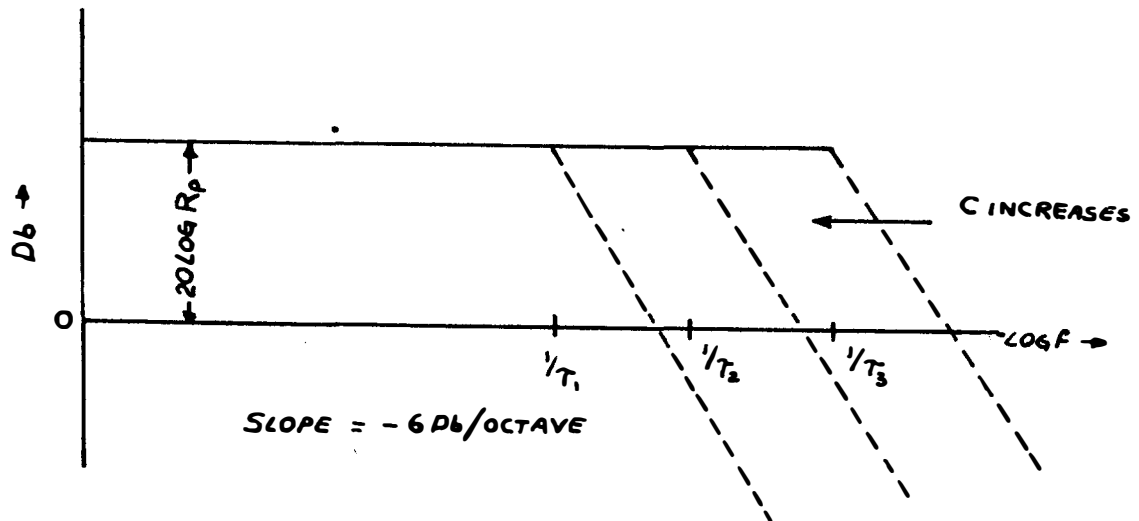


Figure 18. Effect of adding pole to Z_p

It may readily be seen that if the proper value for f_0 is chosen, the output impedance asymptotes may never reach one ohm. Generally speaking the capacitor C is always made as large as practical. This not only insures that the effect of Z_p is minimized at high frequencies but also insures good transient response from the overall system.⁷ A. C. design of the regulator circuitry then becomes a matter of combining the plot for Z_p with that of $/AB/$ to meet desired specifications. The best method of accomplishing this is to design the output circuit for desired in-band performance and then determine the asymptotic loop gain characteristic to meet the design objectives.

For this specific regulator, such an analysis is shown in Figure 19. From the d. c. section it is known that $/AB/$ at d. c. is 60.9 db above one ohm and R_p is 26 db above one ohm. Various

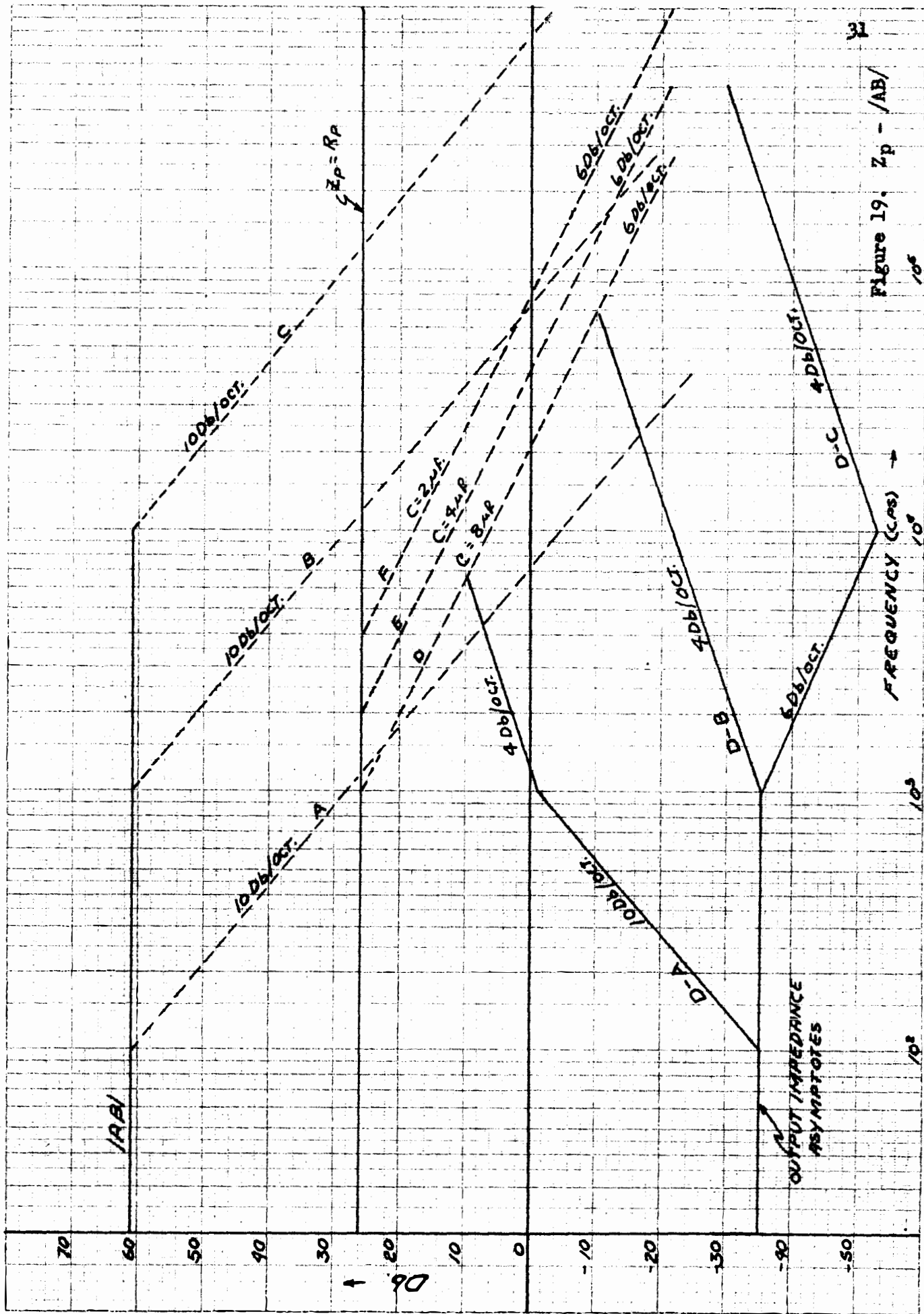


Figure 19. $Z_p - /AB$

combinations of ideal $/AB/$ cutoff and output capacities are shown in the figure. It appears that $/AB/$ cutoff starting in the region from 100 to 500 cycles per second in combination with a 4 mf. capacitor will suffice to give an overall output impedance less than one ohm from d. c. to 100 kilocycles.

Since circuit elements in this type of circuit will include only minimum phase networks, the ideal $/AB/$ cutoff must be approximated by use of multiples of 6 db per octave slopes. An approximation to the ideal is shown in Figure 20. The problem then becomes one of designing the loop cutoff to meet the approximation.

This may best be done by first determining loop transmission considering the d. c. circuit given in Figure 17 and the interelectrode and interstage wiring capacities. After this step is completed, it is relatively easy to determine corrective networks necessary to meet the approximation for $/AB/$ cutoff given in Figure 20.

Before the final $/AB/$ cutoff design is attempted, let us turn to ripple suppression again. From equation 6, it is seen that input voltage variations are suppressed by a value depending inversely on K . By using a frequency sensitive device (capacitor), we may make the value of K very nearly one at ripple frequency. Such a combination is shown in Figure 21.

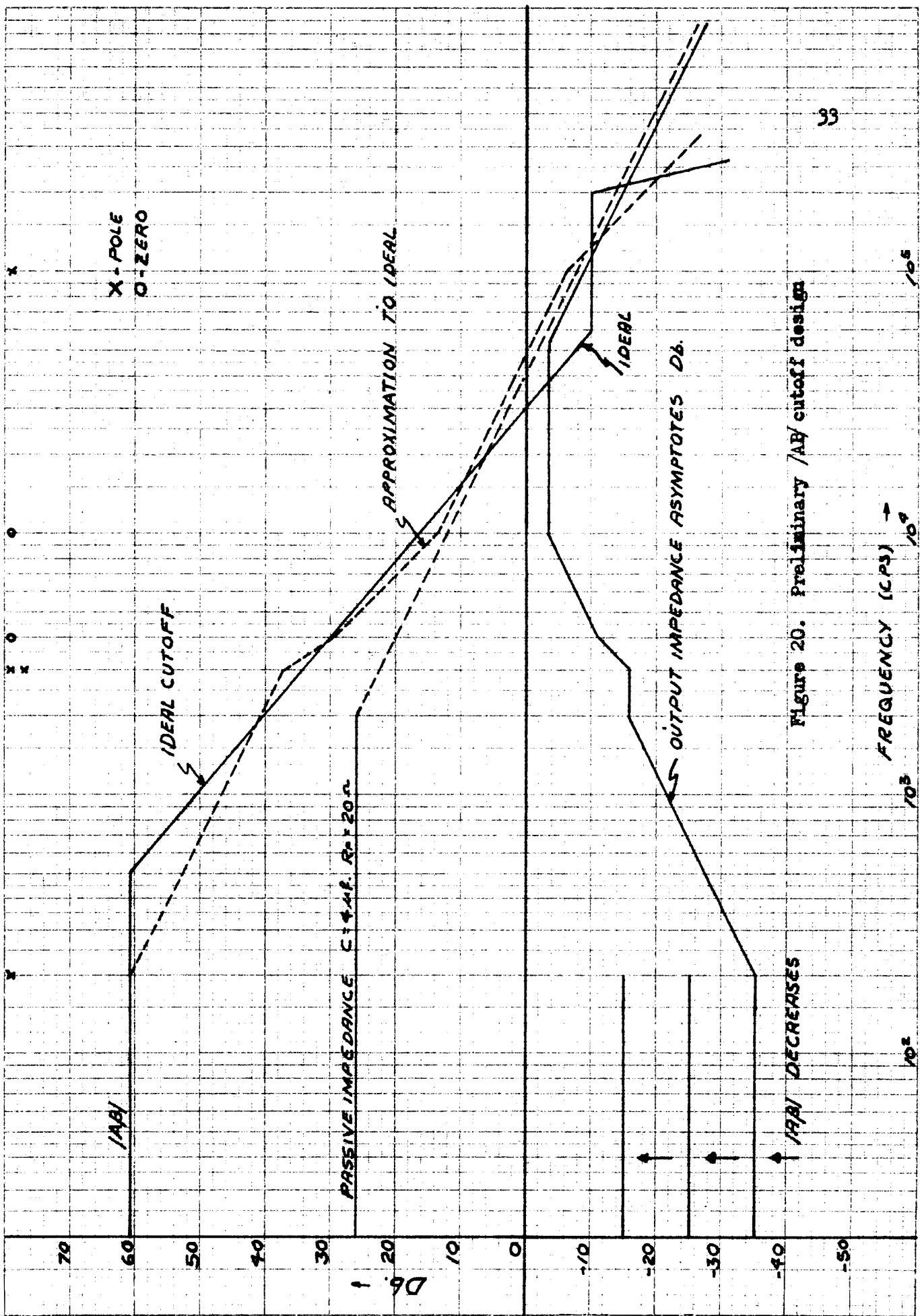


Figure 20. Preliminary $|AB|$ cutoff design

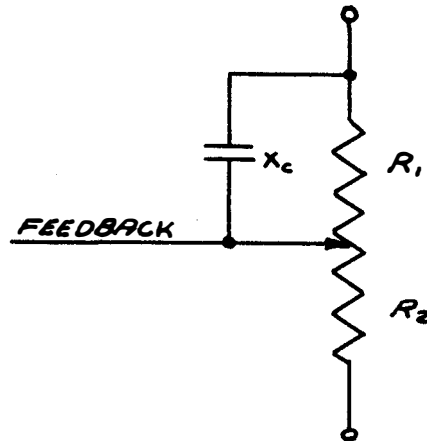


Figure 21. Feedback network with added capacitor

If X_c is made much smaller than R_1 at ripple frequency, it is thus seen that K for this particular frequency approximates a value of one. Actually this capacitor adds a zero preceding a pole to AB transmission in the frequency region of ripple voltage. Since $|AB|$ thus increases, then in accordance with equation 23 any ripple voltage will be further suppressed by the amount of this increase in $|AB|$.

Let us now proceed to determine actual AB transmission using only the d. c. circuitry and the two added capacitors as shown above. The equivalent circuit for the entire regulator is shown in Figure 22. A few comments concerning this circuit are in order. C_{p1} and C_{p2} as appear in the circuit represent output capacitance of the preceding stage plus input capacitance to the following stage plus any interstage wiring capacity. The values of these two elements may be computed from published data on the various tubes. However, a much better method is to actually measure their values by use of a capacitance bridge. The

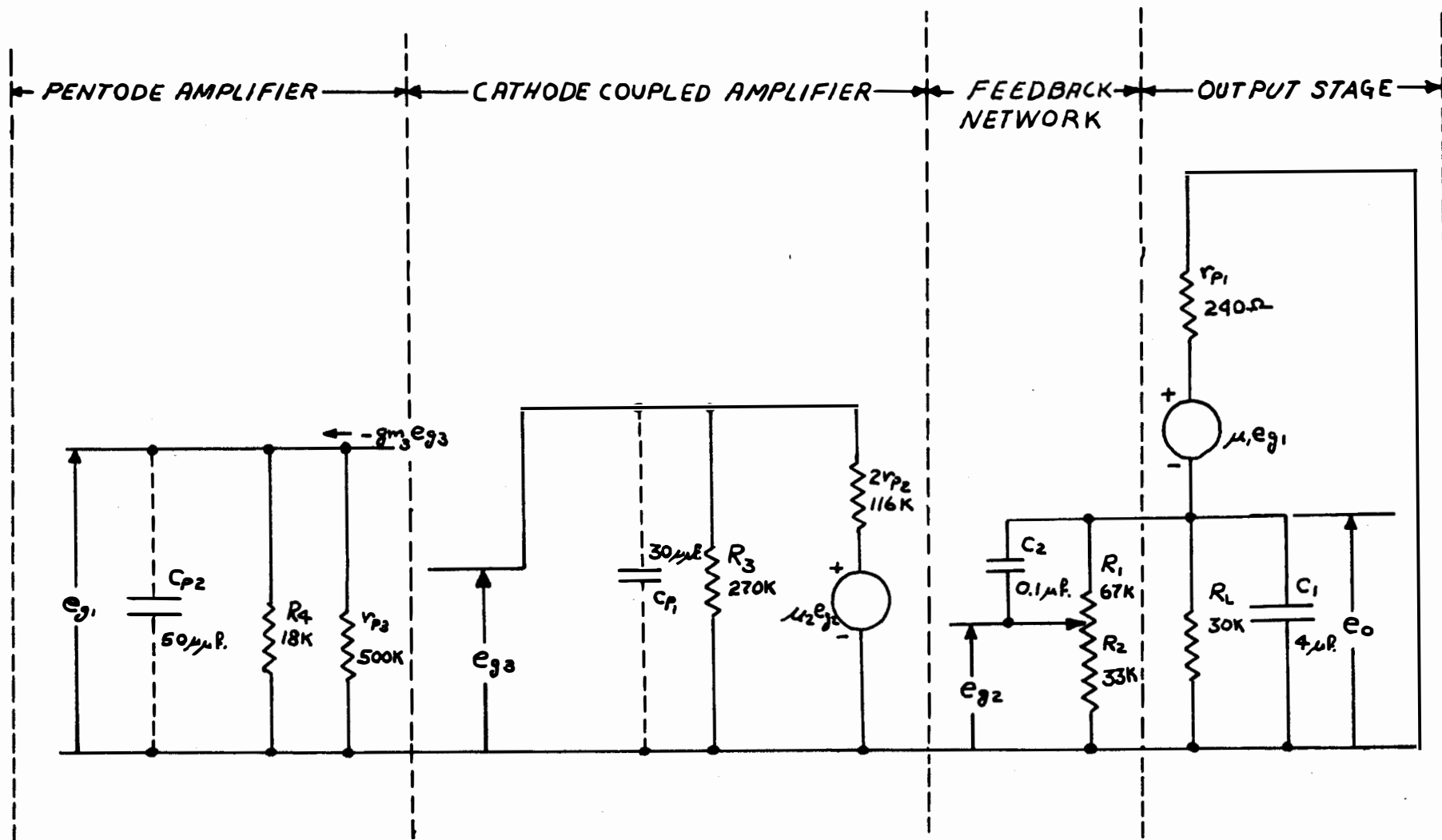


Figure 22. A. C. equivalent circuit

author has done this on a similar regulator using similar tubes. Values as shown in the figure were measured.

Since the regulator is a direct coupled device, and the individual stages have been designed to draw no current in grid circuits, the equivalent circuit may be broken into four sections and analyzed separately as shown. This statement may not appear to be true as applied to the output stage and feedback network since these two circuits share common elements (R_1 and C_1). However, since C_1 is much greater than C_2 , we may neglect their interacting effect.

The equivalent circuit for the cathode coupled amplifier may seem to be overly simplified, since capacitance of the cathode follower portion of the stage has been neglected. Reference to Valley and Wallman show the approximations to be correct.

A comparison of Figure 23 and the approximation to ideal AB cutoff reveals that corrective networks should be added to produce poles near 200 cycles per second and 2500 cycles per second, while zeros are needed near 4 kilocycles and 10 kilocycles. There is a multitude of various corrective networks that may be used¹¹ to accomplish this end, and much has been written on the subject of their application. For the sake of simplicity only a series combination of resistance and capacitance will be used herein. Adding such networks to both the pentode amplifier and the cathode coupled amplifier, the following equivalent circuits emerge.

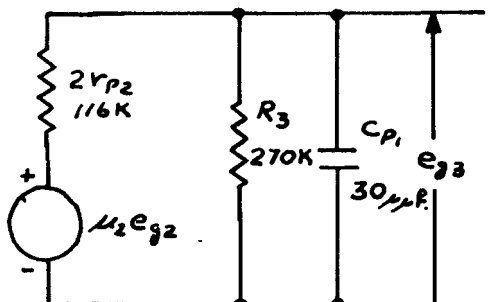
CIRCUIT

Cathode Coupled Amplifier

5751 Rated Characteristics:

$$r_p = 58 \text{ kohms}$$

$$\mu = 70$$



CALCULATIONS

$$e_{g3} = \mu_2 e_{g2} \times \frac{R_3 \frac{1}{j\omega C_{p1}}}{R_3 + \frac{1}{j\omega C_{p1}}} \cdot \frac{1}{\frac{R_3 \frac{1}{j\omega C_{p1}}}{R_3 + \frac{1}{j\omega C_{p1}}} + 2r_{p2}}$$

$$\frac{e_{g3}}{e_{g2}} = \frac{\mu_2 R_3}{R_2 + R_3} \cdot \frac{1}{(\tau_2 s + 1)}$$

$$\text{where } \tau_2 = \frac{(2r_{p2})(R_3) C_{p1}}{(2r_{p2} + R_3)}$$

Summary of Break Frequencies:

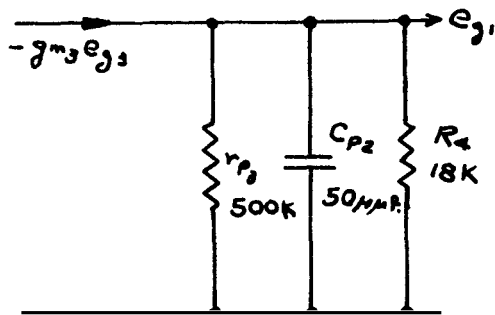
Components	Type	Freq.
$C_{p1} \parallel (R_3 \parallel 2r_{p2})$	X	65 KC

Pentode Voltage Amplifier

6AU6 Rated Characteristics:

$$r_p = 500 \text{ kohms}$$

$$g_m = 5200 \text{ umhos}$$



$$e_{g1} = -g_{m3} e_{g3} \times \left(\frac{1}{r_{p3}} + \frac{1}{R_4} + \frac{j\omega C_{p2}}{1} \right)$$

$$\frac{e_{g1}}{e_{g3}} = -g_{m3} \frac{(R_4) r_{p3}}{R_4 + r_{p3}} \cdot \frac{1}{(\tau_3 s + 1)}$$

$$\text{where } \tau_3 = \frac{R_4 r_{p3}}{R_4 + r_{p3}} C_{p2}$$

Summary of Break Frequencies:

Components	Type	Freq.
$C_{p2} \parallel r_{p3} \parallel R_4$	X	183 KC

OUTPUT STAGE

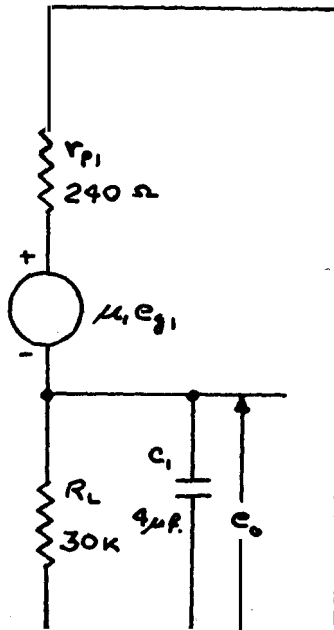
6528 Characteristics at
minimum load/section

$$u = 5$$

$$r_p = 240 \text{ ohms}$$

$$g_m = 24,000 \text{ umhos}$$

$$e_o = \frac{u_1 \frac{(R_L)(\frac{1}{j\omega C_1})}{R_L + \frac{1}{j\omega C_1}} e_{g1}}{r_{p1} + u_1 \left(\frac{(R_L)(\frac{1}{j\omega C_1})}{R_L + \frac{1}{j\omega C_1}} \right) + \left(\frac{R_L (\frac{1}{j\omega C_1})}{R_L + \frac{1}{j\omega C_1}} \right)}$$



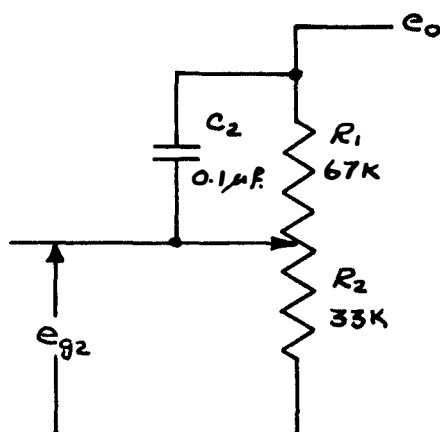
$$\frac{e_o}{e_{g1}} = \frac{u_1 R_2}{r_{p1} + (u_1 + 1)R_L} \cdot \frac{1}{(\gamma_4 S + 1)}$$

$$\text{where } \gamma_4 = \frac{r_{p1} R_L C_1}{r_p + (u_1 + 1)R_L}$$

Summary of Break Frequencies:

Components	Type	Frequency
$C_1 \& \frac{r_{p1} R_L}{r_{p1} + (u_1 + 1)R_L}$	X	2000 Cps

Feedback Network



$$e_{g2} = e_o \frac{R_2}{R_2 + (R_1) \left(\frac{1}{j\omega C_2} \right)} \cdot \frac{1}{R_1 + \frac{1}{j\omega C_2}}$$

$$\frac{e_{g2}}{e_o} = \frac{R_2}{R_1 + R_2} \cdot \frac{(\tau_5 S + 1)}{(\tau_6 S + 1)}$$

$$\text{where } \tau_5 = R_1 C_2$$

$$\tau_6 = \frac{R_1 R_2}{R_1 + R_2} C_2$$

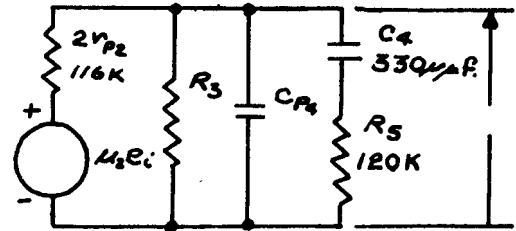
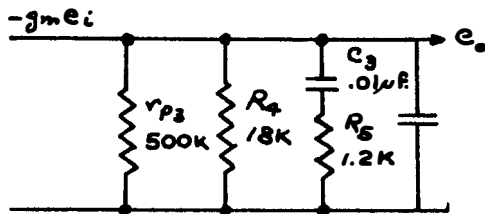
Summary of Break Frequencies:

Components	Type	Frequency
$C_2 \nless R_1$	0	24 Cps
$C_2 \nless R_1 \parallel R_2$	X	72 Cps

By combining all of the above relations

$$AB = - \frac{R_2}{R_1 + R_2} \cdot \frac{u_1 R_L}{r_{p1}(u_1 + 1)R_L} \cdot \frac{g_{m3} R_4 r_{p3}}{R_4 + r_{p3}} \cdot \frac{u_2 R_3}{R_2 + R_3} \cdot \frac{(\tau_5 S + 1)}{(\tau_2 S + 1)(\tau_3 S + 1)(\tau_4 S + 1)(\tau_6 S + 1)}$$

A plot of $-AB$ is shown in Figure 23.



$$e_o = -gm \frac{R_4 r_{p3}}{R_4 + R_{p3}} \frac{(\tau_1 s + 1)}{(\tau_2 s + 1)(\tau_3 s + 1)}$$

$$\text{if } C_3 \gg C_{p2}$$

$$\tau_1 = R_5 C_3$$

$$\tau_2 = \left(\frac{R_4 r_{p3}}{r_{p3} + R_4} + R_5 \right) C_2$$

$$\tau_3 = \left(\frac{1}{R_5} + \frac{1}{r_{p3}} + \frac{1}{R_4} \right) C_{p2}$$

$$\frac{e_o}{e_{in}} = \frac{u_2 R_3}{R_3 + 2r_{p2}} \frac{(\tau_4 s + 1)}{(\tau_5 s + 1)(\tau_6 s + 1)}$$

$$\text{if } C_4 \gg C_{p1}$$

$$\tau_4 = R_5 C_4$$

$$\tau_5 = \left(\frac{1}{2r_{p2}} + \frac{1}{R_3} + \frac{1}{R_5} \right) C_{p1}$$

$$\tau_6 = \left(\frac{2r_{p2} R_3}{2r_{p2} + R_3} + R_1 \right) C_4$$

Figure 24. Amplifiers with added corrective networks

In order to obtain zeros at the frequencies mentioned above, combinations of R and C are then chosen to place τ_1 & τ_4 at these frequencies. Care must be taken in the selection of these networks since their addition will alter the break frequencies already present in the circuit. From a Reactance Chart (see Appendix), it is seen that a combination of 330 uuf and 120 kohms will produce a zero at 4 k.c. and a combination of 0.01 uf and 1.2 kohms will produce a zero at 13.2 k.c. Calculations must then be made to ascertain the location of the poles

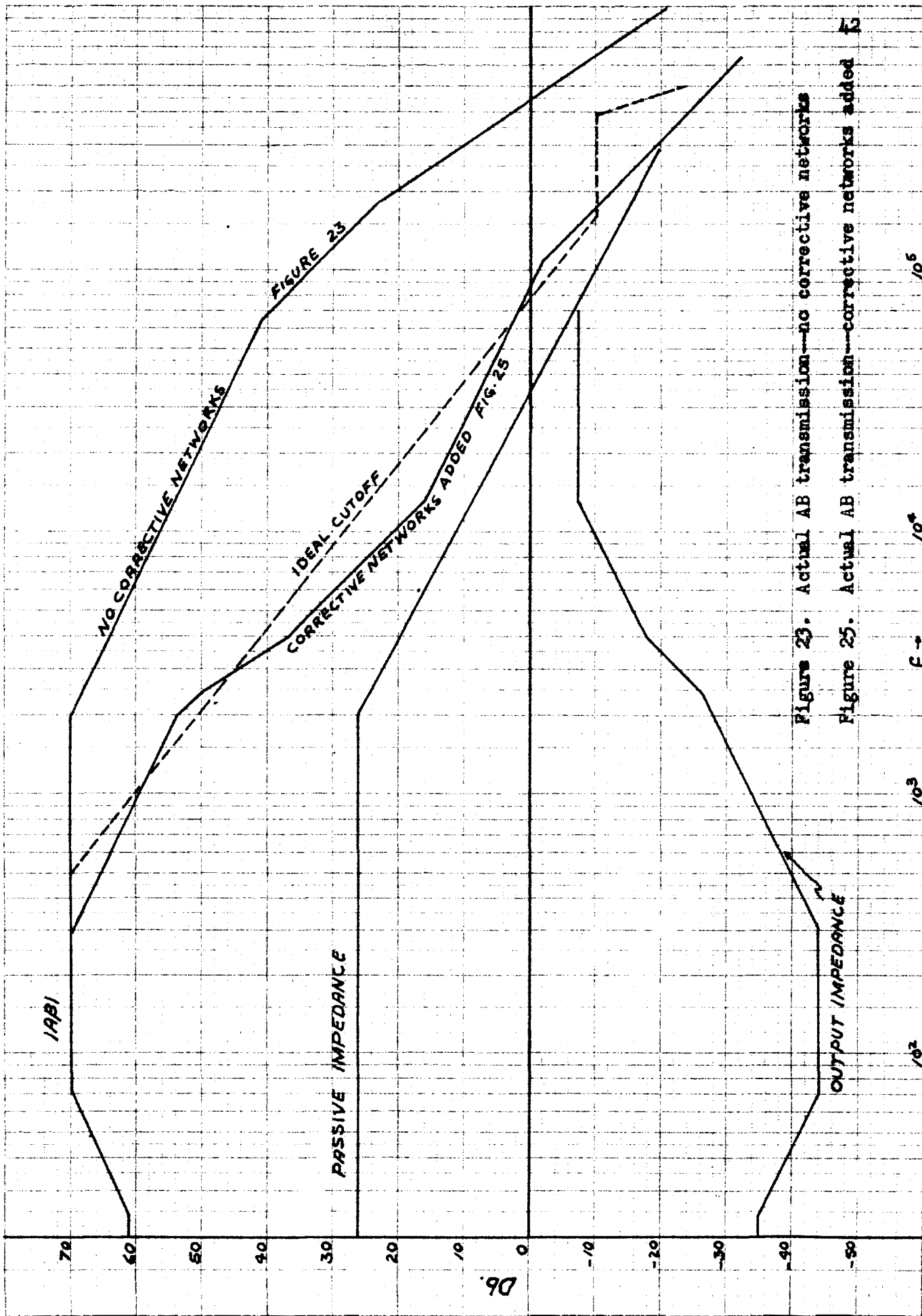


Figure 23. Actual AB transmission--no corrective networks

Figure 25. Actual AB transmission--corrective networks added

when these networks are added. These calculations are carried out below.

$$\begin{aligned}\tau_1 &= R_5 C_3 = (1.2)(10^3)(10^{-8}) & \tau_4 &= R_5 C_4 = (3.30)(10^{-12})(1.20)(10^3) \\ &= 1.2 \times 10^{-5} & &= 4 \times 10^{-5}\end{aligned}$$

$$f_1 = \frac{1}{\tau_1(2\pi)} = 13.2 \text{ KC} \qquad f_4 = \frac{1}{\tau_4(2\pi)} = 4 \text{ KC}$$

$$f_2 = 300 \text{ Cps}$$

$$f_5 = 110 \text{ KC}$$

$$f_3 = 2.7 \text{ MC}$$

$$f_6 = 2.4 \text{ KC}$$

It is thus seen that the pole due to C_{p2} has been shifted from 183 KC to 2.7 megacycles and the pole due to C_{p1} has been shifted from 65 kilocycles to 110 kilocycles.

A plot of /AB/ cutoff with corrective networks added must then be made and compared with the approximation to the ideal cutoff. Output impedance is then derived to insure that its value is less than one ohm from d. c. to 100 kilocycles. These computations are shown in Figure 25.

Throughout the previous discussion little care was taken to assure that phase and gain margin criteria were met. However, the ideal cutoff for AB as used herein assures a phase margin of 30° (Cutoff slope is 10 db per octave) and a gain margin of 10 db. Since actual AB cutoff is only an approximation to the ideal, computations must be made to assure that these criteria are met. A procedure somewhat similar to the asymptotic frequency response technique will be used here (see Appendix). These computations are shown in Figure 26.

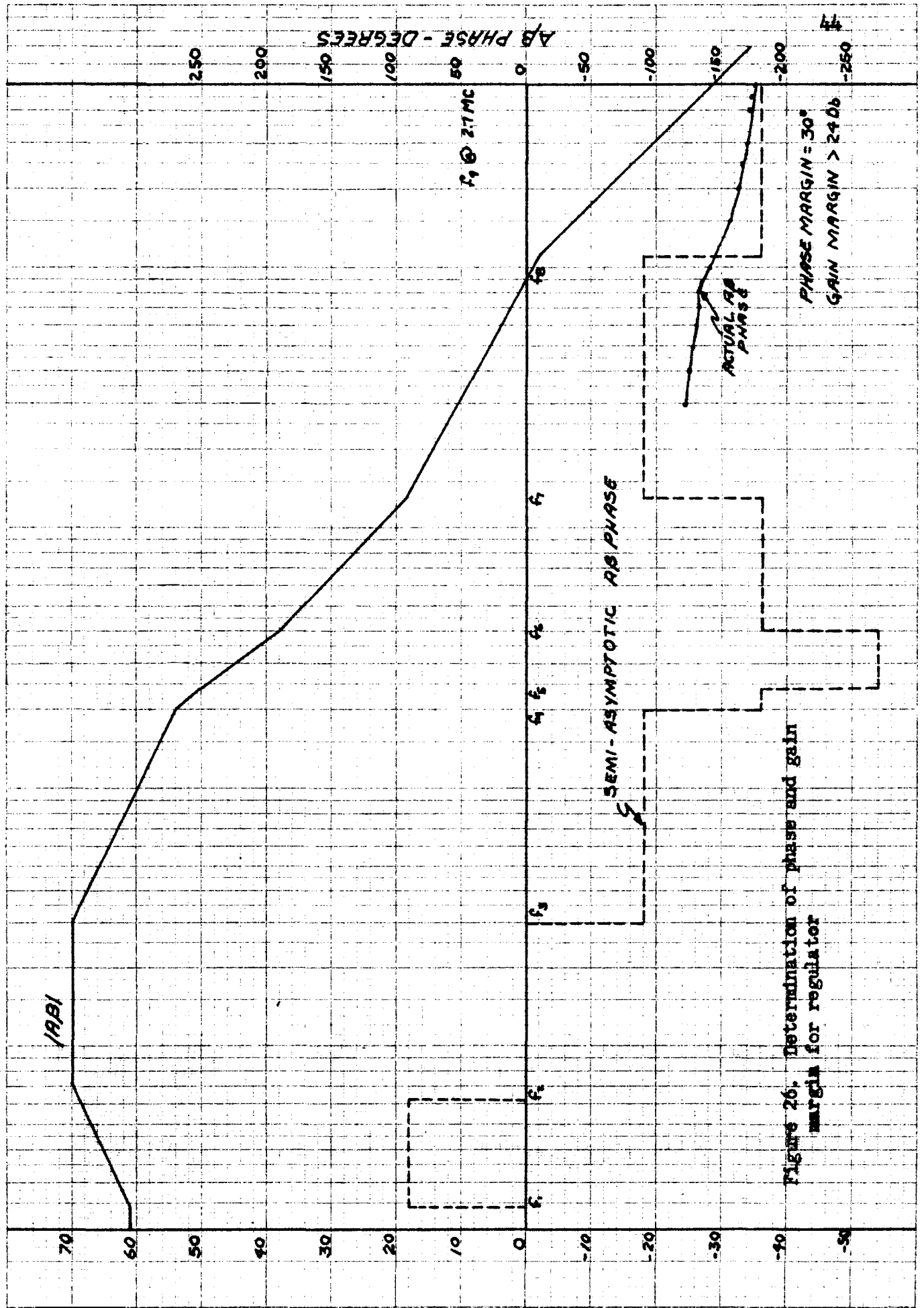


Figure 26. Determination of phase and gain margin for regulator

A final check must be made to assure that ripple voltage in the output is less than 5 millivolts. From equation 23 it was stated that any ripple voltage present in the system from the rectifier filter section would be suppressed by an amount $(\frac{1}{1 + AB})$ where AB is evaluated at ripple frequency. Assuming that the regulator circuitry is supplied from a 60 cycle full wave rectifier, ripple frequency will be 120 cycles per second. From Figure 25 it is seen that AB at 120 cycles per second is 69.5 db above one ohm. Therefore, since input ripple has been limited to 4 volts peak to peak, output ripple will be $\frac{4}{1 + 3000}$ or 1.33 mv peak to peak. The completed regulator is shown in Figure 27.

f_0	CONTRIBUTION IN DEGREES FROM									TOTAL
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	
20KC	+90	-90	-90	-84	-83	+77	+57	-10	0	-131
30KC	"	"	"	-86	-84	+82	+70	-15	0	-123
40KC	"	"	"	-88	-86	+84	+73	-20	0	-127
50KC	"	"	"	-89	-88	+86	+75	-25	0	-130
60KC	"	"	"	-90	-89	+88	+79	-28	0	-130
70KC	"	"	"	"	-90	+89	+81	-32	0	-132
80KC	"	"	"	"	"	+90	+82	-36	0	-134
90KC	"	"	"	"	"	"	+83	-39	-1	-137
100KC	"	"	"	"	"	"	+84	-42	-2	-141
200KC	"	"	"	"	"	"	+86	-61	-4	-161
300KC	"	"	"	"	"	"	+88	-70	-6	-170
400KC	"	"	"	"	"	"	+89	-75	-8	-173
500KC	"	"	"	"	"	"	+90	-77	-10	-178
600KC	"	"	"	"	"	"	"	-79	-13	-182
700KC	"	"	"	"	"	"	"	-81	-15	-186

Tabulation of AB phase between 20 kc and 700 kc

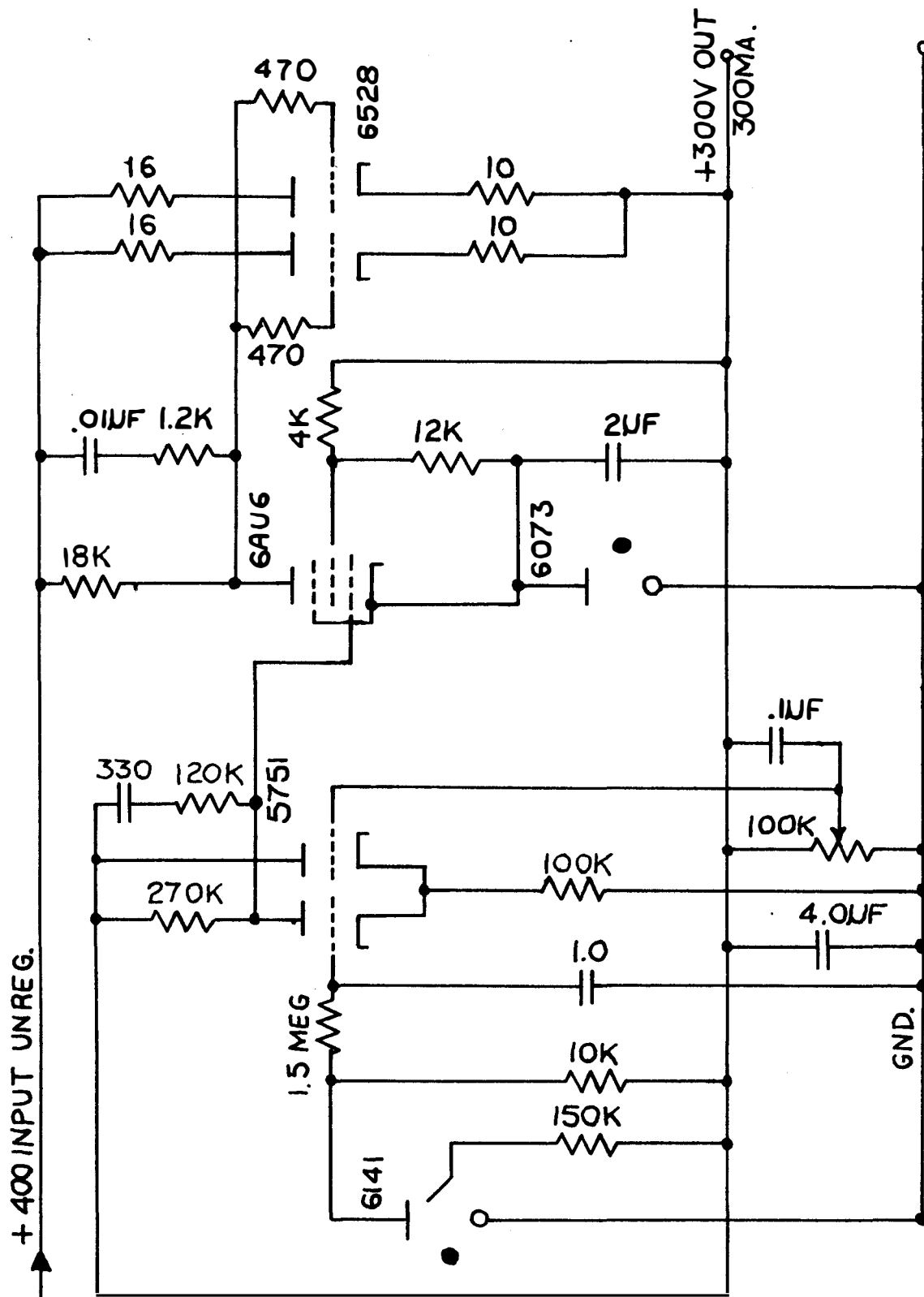


Figure 27. Final design of completed regulator

SUMMARY AND CONCLUSIONS

In summarizing, a sequence of design steps may be pointed out although the order of the sequence need not be followed rigorously.

- a) Choose output tube or tubes and reference source to meet the design objectives.
- b) From the design objectives of the regulator determine the amount of feedback needed. With this amount of feedback and desired phase and gain margins construct the required AB characteristics for the entire spectrum of interest.
- c) Design output circuit for the desired in-band performance.
- d) Determine required AB gain in the band.
- e) Make a check computation of the total in-band losses and determine how much gain will be required to meet the objectives stated in item b. This will also determine the number of stages required.
- f) Determine the asymptotic loop gain characteristics.
- g) Design interstage networks to realize the required in-band gain computed in item e.
- h) Determine the complete AB transmission for the components now specified. Plot this curve so that it can be compared with the design objectives.
- i) Build up the difference between the actual transmission and the objectives by corrective networks in the various parts of the loop. This can be done as the sum of simple shapes.
- j) For best results it will be often necessary to slightly modify some of the corrective networks first inserted.
- k) Make a final check to insure that all predetermined specifications are met by the final design.

It must be realized that the foregoing calculations and conclusions may, quite likely, be slightly in error when they are subjected to the rigors of the laboratory. Parameter variation, errors in

measurement and over-simplification are but a few of the pitfalls encountered when one attempts to verify theory with practice. The author had the opportunity while employed by Bell Telephone Laboratories to design and justify the type of regulator covered in this paper. The results of this design substantiate very well the theory presented herein. Because of proprietary rights, the actual circuitry cannot be shown in this paper. Permission has been granted, however, to include the calculated and measured performance of this regulator. It suffices to say that the same procedure outlined above was used on this regulator and the circuitry was quite similar. The performance data presented in Figure 28 was derived with an output voltage of + 300 volts and a total output current of 200 milliamperes. It may be seen from this figure that calculated and measured performance from the standpoint of output impedance agree very well within practical expectations.

An analysis of transient response for the type of regulator given in this paper is not covered herein. However, some pertinent comments on this general subject are given in Appendix VI.

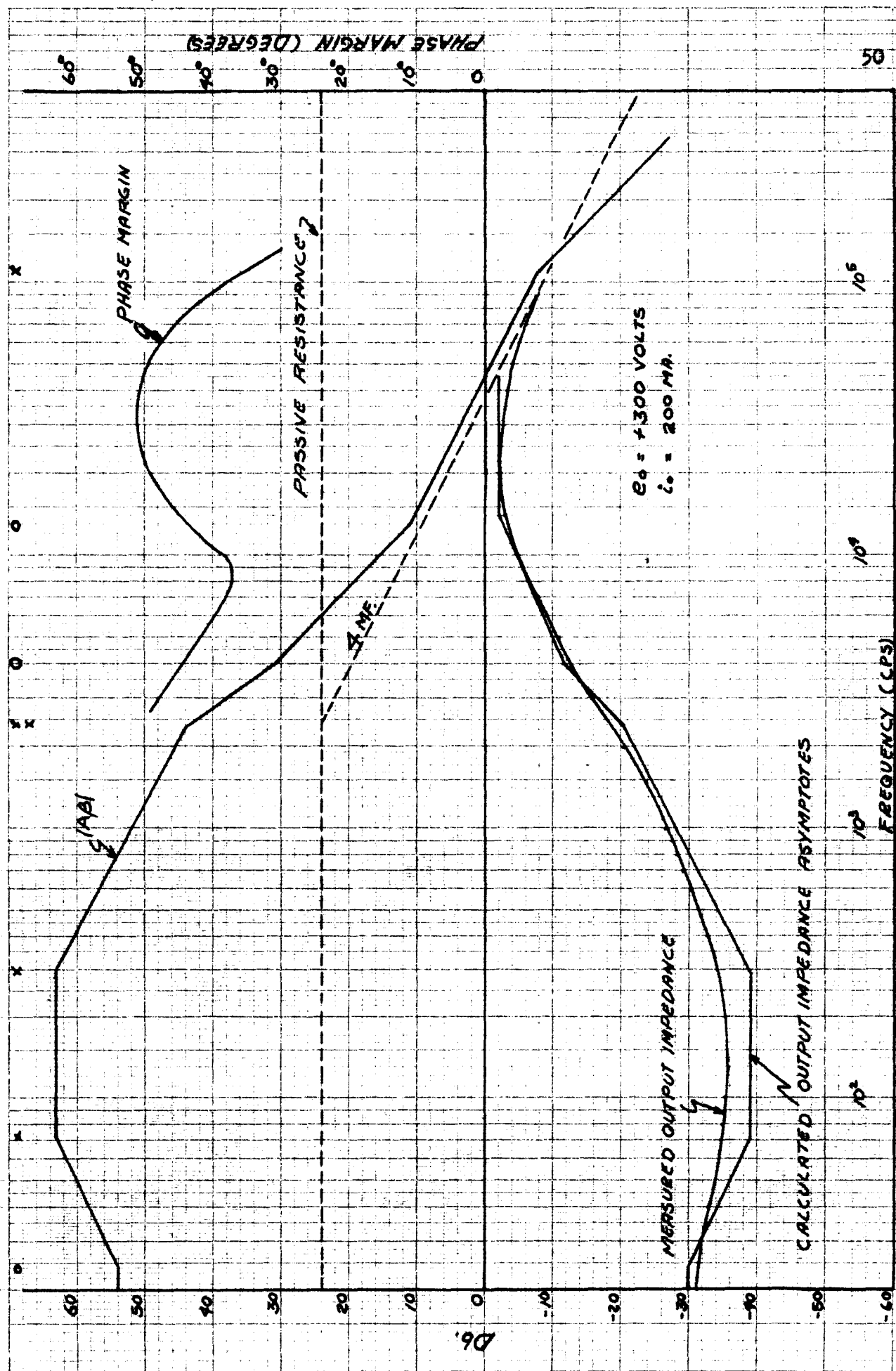


Figure 28. D. C. voltage regulator performance

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REFERENCES

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APPENDICES

APPENDIX I. DERIVATION OF EQUATION 1

Loop Equations from Figure 1:

$$-e_{g2} + e_r - Ke_o = 0 \quad (1)$$

$$e_2 = -u_2 (e_r - Ke_o) \quad (2)$$

$$e_2 - e_{g1} = e_o \quad (3)$$

$$i_o R_k = e_o \quad (4)$$

$$E_i - i_o r_{pl} + u_1 e_{g1} - e_o = 0 \quad (5)$$

From (4) and (5):

$$\begin{aligned} E_i - \frac{e_o}{R_k} r_{pl} + u_1 e_{g1} - e_o &= 0 \\ e_{g1} &= e_o \left[\frac{R_k + r_{pl}}{R_k u_1} \right] - \frac{E_i}{u_1} \end{aligned} \quad (6)$$

From (1), (2), and (3):

$$e_{g1} = -e_o - u_2 Ke_o + u_2 e_r \quad (7)$$

(6) = (7)

$$\begin{aligned} e_o \left[1 + \frac{R_k + r_{pl}}{R_k u_1} \right] - \frac{E_i}{u_1} &= u_2 Ke_o + u_2 e_r \\ e_o &= \frac{R_k u_1 u_2 e_r}{R_k(u_1 + u_1 u_2 K + 1) + r_{pl}} + \frac{R_k E_i}{R_k(u_1 + u_1 u_2 K + 1) + r_{pl}} \end{aligned} \quad (8)$$

Thus (8) becomes equation 1 of the body of the thesis.

APPENDIX II. CALCULATION OF D. C. OPEN LOOP GAIN FOR FIGURE 17

A) Cathode coupled amplifier

$$G_{dc} = \frac{uR_2}{2r_p + R_2 + r_p (r_p + R_2)/R_k(u + 1)}$$

$$= 48.6$$

$$= 33.706 \text{ Db.}$$

B) Pentode amplifier

$$G_{dc} = gm R_L$$

$$= (5200)(10^{-6})(17.4)(10^3)$$

$$= 90.5$$

$$= 39.206 \text{ Db}$$

C) Output stage

$$G_{dc} = \frac{uR_k}{r_p + uR_k + R_k}$$

$$= 0.750$$

$$= - 2.506 \text{ Db.}$$

D) Feedback network

$$G_{dc} = \frac{R_1}{R_2 + R_1}$$

$$= 0.333$$

$$= - 9.506 \text{ Db}$$

APPENDIX III. $(1 + AB)$ AND AB NEAR GAIN CROSSOVER

Consider the following function

$$\psi = \frac{Z_p}{1 + AB} \quad \begin{array}{l} Z_p > 0 \\ /AB/ > 0 \\ 180^\circ \leq \phi \leq 0 \end{array} \quad (1)$$

$$\psi \angle db = 20 \log_{10} Z_p - 20 \log_{10} (1 + AB \angle \phi) \quad (2)$$

When $/AB/ \gg 1$

$$\begin{aligned} \psi \angle db &\approx 20 \log_{10} Z_p - 20 \log_{10} (/AB/ \angle \phi) \\ &\approx 20 \log_{10} Z_p - (20 \log_{10} /AB/ + j\phi) \end{aligned} \quad (3)$$

When AB is in the vicinity of 1

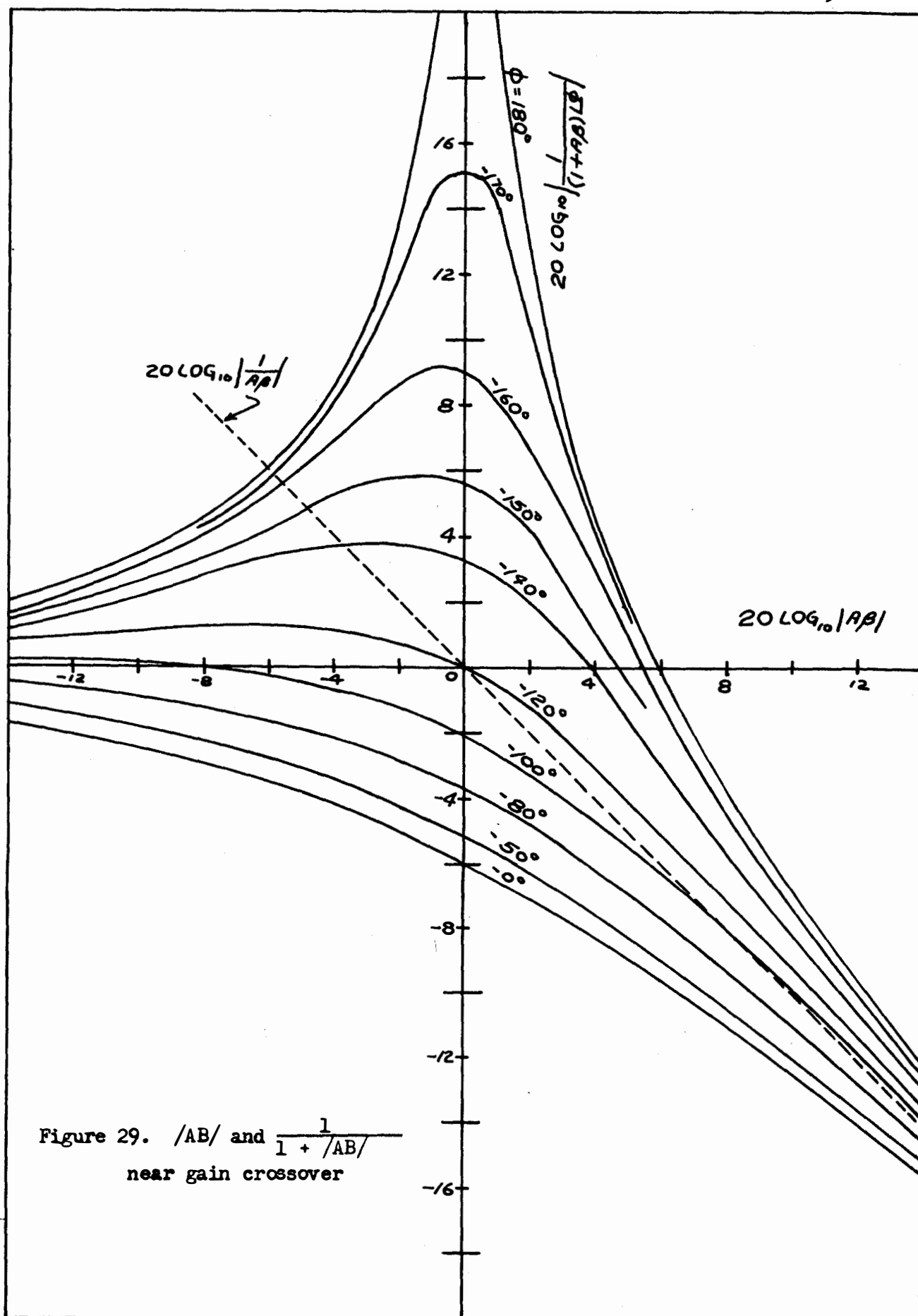
$$\begin{aligned} \psi \angle db &= 20 \log_{10} Z_p - \angle 20 \log_{10} (1 + AB) + j\Theta \\ \text{where } \Theta &= \tan^{-1} \frac{/AB/ \sin \phi}{1 + /AB/ \cos \phi} \end{aligned} \quad (4)$$

Reference to Figure 8 shows that this is actually an asymptotic plot of equation (3) above, where ϕ within the cutoff region is -150° and $/AB/$ is frequency dependent. The output impedance asymptotes were derived under the assumption that $/AB/ = /(1 + /AB/)/$. It is readily seen that this is not valid as $/AB/$ takes on values near unity. $/(1 + AB)/$ may be found in the vicinity of gain crossover ($/AB/ = 1$) by use of the curves given on the following page when open loop gain and phase of a given system are known. Either the Bode diagram or the Nyquist plot may be used to obtain this information. The curves were derived as follows.

Let $\phi = -140^\circ$.

$ AB $	$20\log AB $	$20\log \frac{1}{1+AB}$
0.251	-12	1.6
0.397	-8	2.8
0.63	-4	3.7
1.0	0	3.3
1.59	+4	-0.2
2.51	+8	-5.6
4.0	+12	-10.4

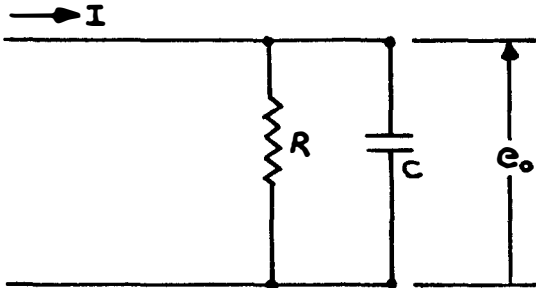
In a similar manner curves for $\phi = -170^\circ$, $\phi = -160^\circ$, etc. may be plotted. Thus if open loop gain and phase are known $/(1 + AB)/$ may be found. In feedback amplifiers, a relationship between $\frac{1}{(1 + AB)}$ and open loop gain is generally of more value. Hence these coordinates are used in the included graph. The restriction that $-180^\circ \leq \phi \leq 0$ was placed on this function in conformance with Nyquist's criterion for system stability.



APPENDIX IV. ASYMPTOTIC PHASE AND MAGNITUDE PLOTS

A very definite relationship exists between $\log_{10} |Z| = \text{Re}(\log Z)$ and $(\text{phase angle } Z) = \text{Im}(\log Z)$ as a consequence of the well known relationship between attenuation and phase in minimum phase networks.⁹ The knowledge of $(\text{angle } Z)$ determines $\log |Z|$ to within an additive constant, and the knowledge of $\log |Z|$ prescribes $(\text{angle } Z)$ exactly. Usually we will be concerned with obtaining the phase angle associated with a known $\log |Z|$.

Consider the following network. It is assumed that this network is driven by a constant current device.



$$e_o = \frac{R(\frac{1}{j\omega C}) I}{R + \frac{1}{j\omega C}} \quad (1)$$

$$e_o = \frac{RI}{1 + j f/f_o} \quad (2)$$

$$\text{where } 2\pi f_o = \frac{1}{RC}$$

Equation 2 is in normalized form and is the product of a level constant (IR) and a shaping function $(1 + j f/f_o)^{-1}$ which determines the shape of the $\log |Z|$ versus $\log f$ plot. The two straight lines representing their respective asymptotic structure intersect where $\frac{1}{\omega C} = R$. This defines the break frequency f_o .

An exact plot of the magnitude and phase for the shape function $(1 + j f/f_o)^{-1}$ is given in Figure 30. The actual magnitude and phase of other shaping functions such as $(1 + j f/f_o)$, $(1 - f_o/f)$, and

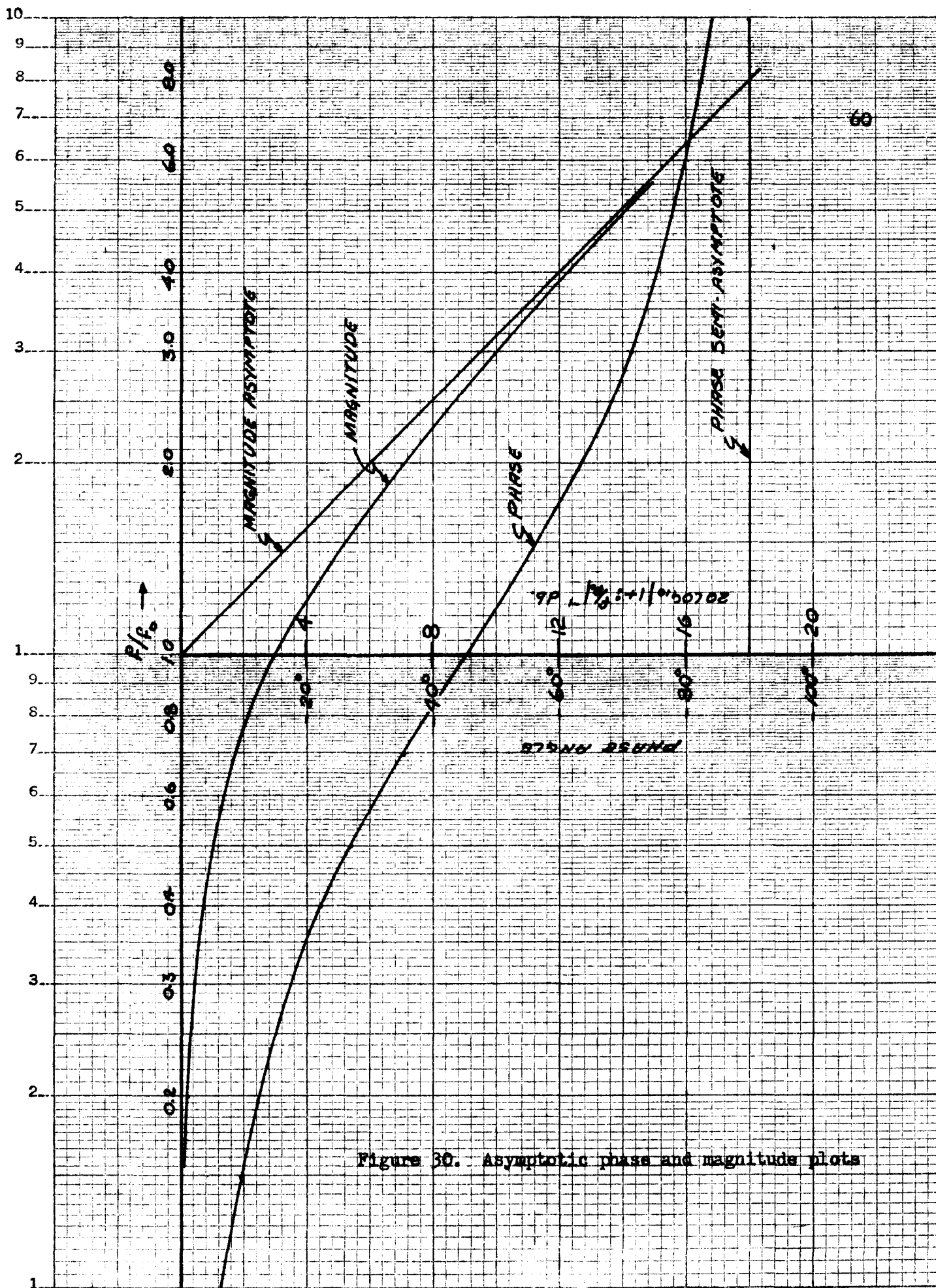
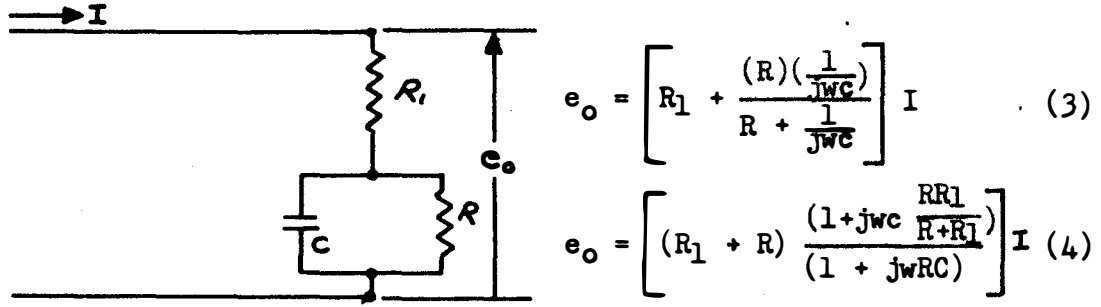


Figure 30. Asymptotic phase and magnitude plots

$(1 - f_o/f)^{-1}$ may also be obtained directly from the graph by taking advantage of their essential equivalence in character.

Now consider the following circuit.



It may be seen that there are two break frequencies defined by

$$\omega_1 = \frac{1}{RC} \quad (5)$$

$$\omega_2 = \left(\frac{RR_1}{R + R_1} \right) C \quad (6)$$

Rewriting equation 4

$$e_o = \left[(R + R_1) \left(1 + j \frac{\omega}{\omega_2} \right) \left(1 + j \frac{\omega}{\omega_1} \right)^{-1} \right] I \quad (7)$$

which exhibits two familiar shape functions. Since

$$\log_{10} e_o = \log_{10} (R + R_1) I + \log \left(1 + j \frac{\omega}{\omega_2} \right) - \log \left(1 + j \frac{\omega}{\omega_1} \right) \quad (8)$$

the asymptotic structure of $|e_o|$ is obtained by adding the asymptotic structures of the three factors on the right side of equation 8.

Similarly the semi-asymptotic structure of (angle e_o) is obtained by adding the semi-asymptotic structure of the three terms on the right of equation 8.

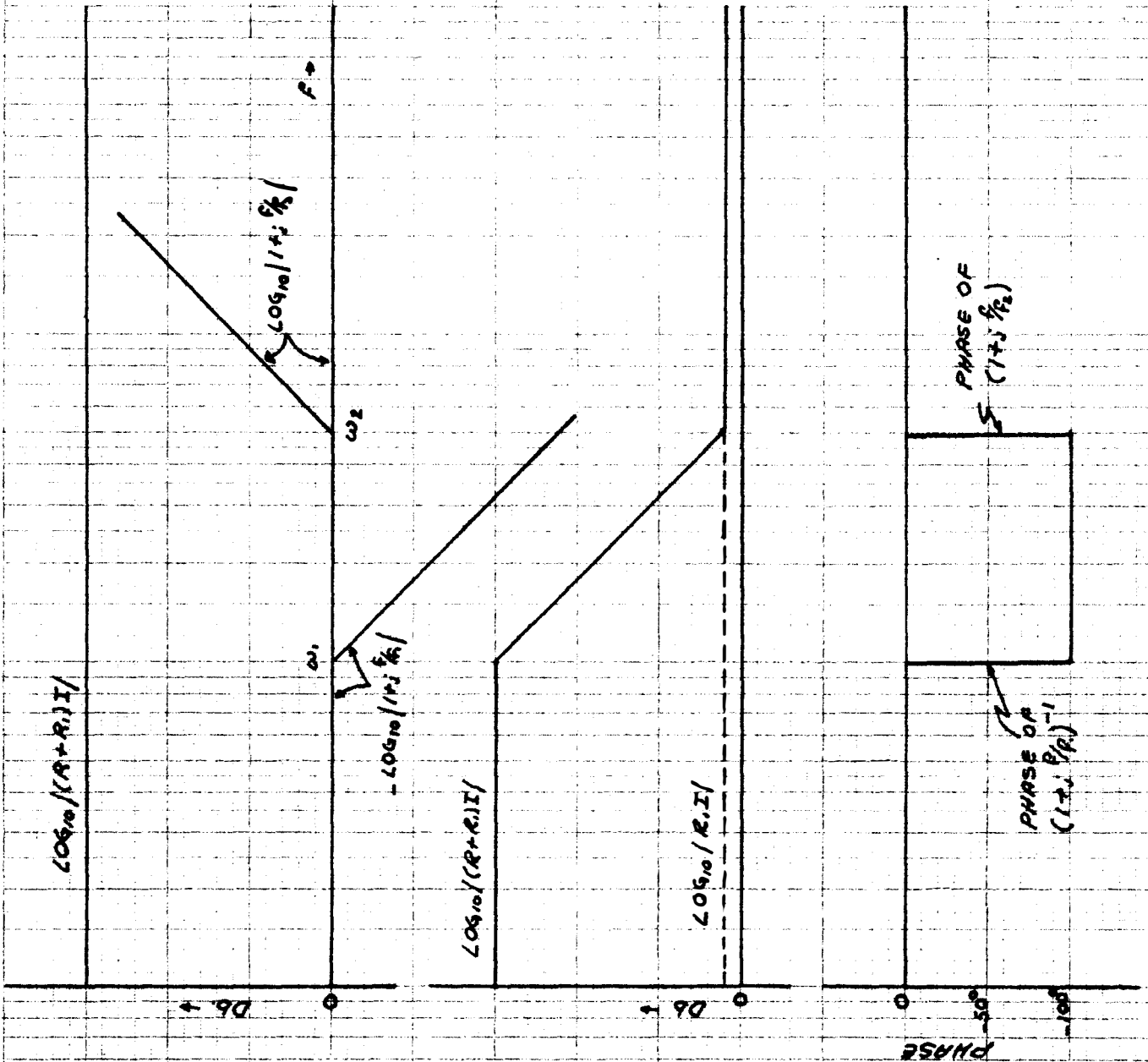
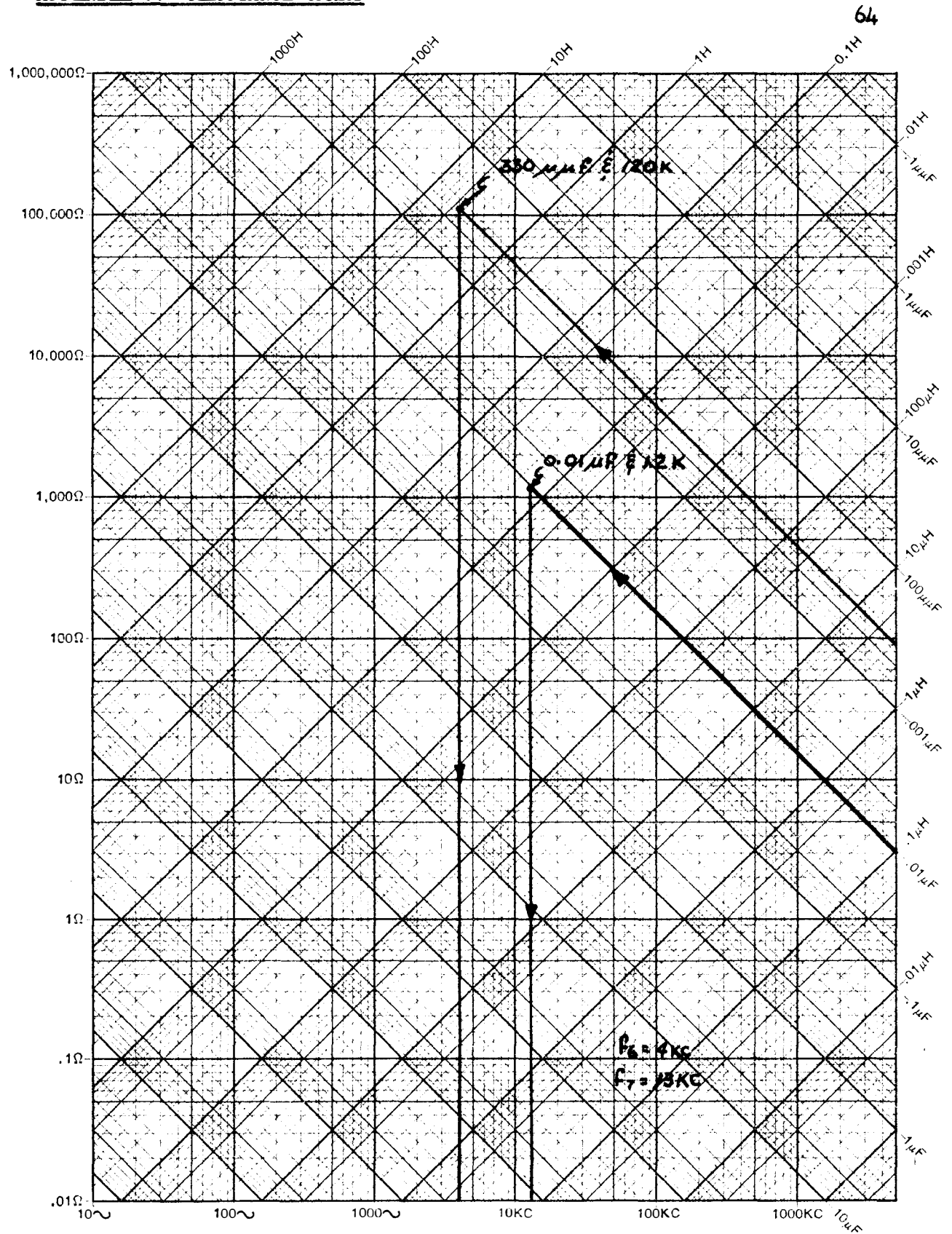


Figure 31. Combination of simple shapes

The actual magnitude and phase angle for e_o may be obtained by adding the actual magnitude and phase angle of the basic shapes of which e_o is composed. In a similar manner transfer functions composed of multiple break frequencies may be analyzed by using the sum of simple shapes.

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APPENDIX VI. TRANSIENT ANALYSIS

From equation 1 in the text of the thesis D. C. output voltage of the regulator is given by

$$e_o = \frac{u_1 u_2 R_k + R_k E_i}{R_k(u_1 + 1 + u_1 u_2 K) + r_{pl}} \quad (1)$$

In the S plane,

$$e_o(s) = \frac{u_1 u_2(s) Z_k(s) e_r + Z_k(s) E_i}{Z_k(s) (u_1 + 1 + u_1 u_2(s) K(s)) + r_{pl}} \quad (2)$$

where u_1 , e_r and E_i are considered as constants. Analysis given in the text revealed that

$$u_2(s) = \frac{K_1(\tau_1 s + 1)(\tau_4 s + 1)}{(\tau_2 s + 1)(\tau_3 s + 1)(\tau_5 s + 1)(\tau_6 s + 1)} \quad (3)$$

$$Z_k(s) = \frac{R_k}{(\tau_7 s + 1)} \quad (4)$$

$$K(s) = \frac{K(\tau_8 s + 1)}{(\tau_9 s + 1)} \quad (5)$$

where K , and K_1 are D. C. level constants. These constants along with the break frequencies $(\frac{1}{\tau_1}, \frac{1}{\tau_2}, \dots, \frac{1}{\tau_9})$ are defined in the text. The parameter R_k is a function of the load and is determined by the relationship $R_k = \frac{e_o}{i_{load}}$. Using equations 3, 4, and 5 in equation 2 and manipulating

$$e_o(s) = \frac{R_k \mu K_1 (\tau_1 s + 1)(\tau_4 s + 1)(\tau_7 s + 1) e_r + R_k (\tau_1 s + 1)(\tau_3 s + 1)(\tau_5 s + 1)(\tau_6 s + 1)(\tau_7 s + 1) E_i}{R_k (\mu + 1)(\tau_1 s + 1)(\tau_3 s + 1)(\tau_5 s + 1)(\tau_6 s + 1)(\tau_7 s + 1) + \mu K K_1 (\tau_1 s + 1)(\tau_4 s + 1)(\tau_8 s + 1) + r_{pl} (\tau_1 s + 1)(\tau_3 s + 1)(\tau_5 s + 1)(\tau_6 s + 1)(\tau_7 s + 1)} \quad (6)$$

It may easily be seen from equation 6 that any transient solution attempted will be extremely laborious and complicated.

$$e_o(t) = \frac{1}{2\pi j} \oint e_o(s) e^{-st} ds \quad (7)$$

Transient solutions involving simultaneous step changes in both e_r and E_i at a given fixed load may be treated in the standard inverse transform methods. The problem then becomes one of computing the roots of a sixth order algebraic equation.

Transient analysis involving changes in load become extremely complicated, since equation 6 now becomes non-linear. Transients within the system may be found for any given value of load (R_k), but these solutions do not give the reaction of the system in the act of a load change. The solution of non-linear differential equations of this nature is beyond the scope of this thesis.