THE EFFECT OF UNBALANCED VOLTAGES
ON AN INDUCTION MOTOR

by

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A THESIS SUBMITTED FOR THE DEGREE OF

BACHELOR OF SCIENCE

ELECTRICAL ENGINEERING COURSE

The University of Tennessee

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I. - PREFACE.

In this paper it is hoped by the author that he will be able to present, through analysis of the results obtained, information which will be of some value, and which will add some little bit to the present knowledge of the action of induction motors when voltages which are not balanced are impressed upon the terminals of the motor. It is realized and freely acknowledged that the results obtained in the laboratory are not strictly comparable to those which would be obtained under operating conditions in the field. Therefore, it is not proposed to present the results contained herein as indicative of the exact conditions existing within a motor loaded as a machine would be in commercial practice, but it is proposed by analysis of the data obtained in this experiment to give some idea of the severity and abnormality of conditions which might exist within a commercial machine under conditions of voltage unbalance that might very easily exist on incoming power lines of even a well regulated power distribution system. The motor used was a small one, being only a 5 H.P. unit, so the data would not be altogether homologous to a unit of a very much larger capacity.

Acknowledgment is given to Prof. J. G. Tarboux for invaluable aid in planning and executing the experiment. It was due to his advice and help that the difficulties, of which there were a great many, were finally overcome and the project pursued to a successful conclusion. Acknowledgment is given to L. S. Dysart for much aid in the performance of the experimental work.

II. - SURVEY OF FIELD.

The problem under consideration, that of determining the effect of unbalanced currents upon induction motor operation, is one which is undoubtedly as old as the induction motor itself. In spite of the fact that modern power distribution systems are automatically regulated at strategic points to a very great extent, it is impossible to hold the voltages between lines at a constant value or to keep the line voltages at an equal value. Especially does unbalance occur where single phase loads, such as heavy lighting loads, are thrown upon the line. Unbalance may be caused by conditions within the customer's premises - conditions over which the power company has no control. In any case, unbalanced conditions are almost sure to exist to a greater or lesser degree, and it is of utmost importance that their effect upon induction motors, the most usual commercial method of converting electrical to mechanical energy, be studied and appreciated.

Much experimental work has undoubtedly been done upon the subject and several theories have been formulated. The theory of positive and negative sequences, which will be discussed at some length in another portion of this paper possesses the advantage of adaptability to motor theory and is of particular value in the study of motor operation. Without a doubt much experimental work has been done, but in the main the theory exists - still as theory - explained in terms which are hardly intelligible to the practical man or to the designer in the field. Practical results in the form of curve sheets should be made available to the designer and to the operating engineer so that the effect of probable values of unbalance could be studied and determined with some degree of accuracy.

III. - STATEMENT OF PURPOSE.

It was originally intended to obtain the experimental data and then to work out the positive and negative sequence currents for each condition. From these values the net torque and power for each phase could be computed and hence the net torque and power for the motor. This would have resulted in a vast amount of calculation, but it would have been a very interesting and instructive check upon the theory of positive and negative sequences. Interesting as the comparison of calculated and actual results would have been, this procedure would have added nothing to the practical value of the results. Since the primary object of the experiment was to determine the actual effect of unbalanced conditions upon the motor, the proof of the theory was abandoned as being of secondary interest, and stress was laid upon the interpretation of actual results in as practical a manner as possible.

In the light of the final decision, phase power, phase currents, and the power factor of the phase, for each one of the three phases were plotted against the same value, voltage unbalance. This makes values of current, power, and power factor available at the same conditions, so that at a glance the entire conditions of operation of the phase could be determined. For instance, for any specific load, the current and power being drawn by the phase at any specific condition could be determined from the curves. The heating effect in the phase under consideration could be calculated and the comparative heating around the circumference of the stator approximated.

IV. - GENERAL THEORY.

There are two general types of electrical machines, synchronous and asynchronous. Synchronous machines operate at synchronous speed determined by the number of poles, and require D. C. as well as A. C. excitation. Asynchronous machines vary somewhat in speed according to the load carried and do not require D. C. excitation. Induction motors fall in the latter class of machines.

The stator winding of a polyphase induction motor is similar to the armature winding of a polyphase alternator. The space relations of the windings for the different phases in electrical degrees must be the same as the time-phase relation between the currents they carry. In the motor used, which was wound for three phase, the phase windings were of course 120° apart. When a polyphase current is impressed upon a properly wound stator, a revolving field is produced, the fundamental of which revolves at synchronous speed with respect to the stator.

The rotor will have as many poles induced in it as there are poles on the stator. Currents will be present in it which are caused by induction. The currents will cause the rotor to revolve in the same direction as the magnetic field present in the stator. The current induced in the rotor is dependant upon the slip or difference in speed between the rotor and the stator revolving field. At no load the speed of the rotor is almost synchronous speed, while under load conditions just enough current will be induced in the rotor to produce the torque necessary to carry the load and overcome the rotational losses.

In the calculation of unbalanced voltages and currents, which in this experiment are impressed upon the motor under test, the solution can be obtained by application of Ohm's and Kirchoff's laws. When applied to rotating machines, the calculations may become somewhat lengthy and complicated. To offset this difficulty, a more convenient scheme known as the method of symmetrical co-ordinates has been devised. This method developed by C. L. Fortescue is dependent upon the fact that in any n-phase system, the actual sinusoidal unbalanced voltages and currents may, in general, be replaced by n-1 symmetrical n-phase systems of voltages and currents and one single phase system. If the circuit has an odd number of phases, the n-1 symmetrical systems will all be balanced n-phase systems.

When unbalanced currents are impressed upon an induction motor the currents which the motor takes will also be unbalanced. Since an ordinary commercial three phase motor is always delta-connected, or y-connected without a neutral, no single phase current can possibly flow. The unbalanced voltages and currents there form a closed triangle and may be broken up into two balanced systems, one of positive and one of negative sequence.

In considering the effect of unbalanced currents upon the motor, the effect of the balanced current systems of opposite phase sequence may be determined separately and the net result obtained by superimposing the results. Each balanced current system may be considered as producing a revolving field in the air gap. Reacting with the currents produced in the rotor, each of these systems produces a torque. Obviously the two torques produced will be in opposite directions, since they are produced by current systems of opposite sequence. The positive sequence torque is in the direction of rotation of the rotor, and the net torque when the motor is operating is the difference of the positive and negative torques.

When the rotor turns at synchronous speed, the positive sequence slip, or the actual slip, is zero while the negative sequence slip has its maximum value of 2. If the actual slip is S, the negative sequence slip is 2-S. By actually obtaining the slip torque curves from 200% to 0% negative slip, it will be seen that unbalance of the impressed voltages and hence the currents reduces the net torque for all values of slip. Hence to deliver a specified load the slip must increase when the voltages become unbalanced. The starting torque and the pull-out torque are lower than for balanced operation.

With unbalanced currents, the copper losses and heat produced in the three phases of the stator will be unequal. One phase or two phases will be heated considerably more than they would be for the same output under balanced conditions of operation. This can be readily seen by reference to the power factor curves obtained under actual test. The core loss will also increase somewhat when the motor is operated under unbalanced conditions. This increase is due for the most part to a double frequency core loss in the stator. Under balanced operation the rotor is almost without core loss due to the low frequency of the rotor flux. Thus the copper loss and the core loss both increase due to the unbalanced voltages. The variation in speed is not appreciable, so the rotational losses will remain sensibly constant. Due to the increase in the total losses, the motor will decrease in efficiency when operating under conditions of unbalance.

The theory involved in the actual performance of the experiment is rather simple. The input to the motor was measured by electrical instruments, a voltmeter, ammeter, and wattmeter in the individual phases. The mechanical output was absorbed by a dynamometer, consisting of an aluminum disc rotating in a strong magnetic field. Eddy currents set up in the disc oppose the rotation and so furnish the necessary resistance.

A scale was connected to the end of the arm bearing the D. C. coils which furnish the magnetic field. By applying the usual formula of the prony brake the output was easily calculated.

Regulation of the voltages was accomplished by single phase induction regulators, one in each of the three phases. The induction regulator is essentially a transformer with the primary across the line and the secondary in series with the line. The primary is excited at all times and the voltage induced in the secondary is dependent upon the position of the secondary coil. When the coils are parallel, maximum secondary voltage is obtained; when the coils are perpendicular, there is no voltage induced in the secondary. By turning the secondary coil, the voltages may be made to either add or subtract, and so by varying the position of the secondary coil, any voltage within the range of the device may be obtained. At the position when the secondary coil is perpendicular to the primary, no flux cuts the secondary and it would simply act as an impedance in series with the line. To prevent this condition, a tertiary or short-circuited coil is placed on the primary core with its axis perpendicular to that of the primary. When the primary and secondary coils are at right angles, the flux set up by the tertiary winding will serve to neutralize the reactance of the secondary winding. The single phase induction regulator results in algebraic addition or subtraction of voltages and does not change the angle between voltage and current of the phase regulated.

V. - METHOD AND APPARATUS.

The method used in the experiment has already been indicated in the discussion of theory. The input to the motor was determined electrically and the output determined by means of the dynamometer. This dynamometer was selected as the most accurate and feasible method of determining the output. The resistance to rotation or load on the motor was easily regulated by means of a rheostat in the D. C. Circuit of the dynamometer.

The apparatus used consisted of a 5 H.P. - 3 Phase 220 Volt-Westinghouse induction motor with a voltmeter, ammeter, and wattmeter in each phase. Due to the magnitude of the currents, a current transformer was used in each phase. An induction regulator was placed in each phase to regulate the voltages. The dynamometer was connected on the same shaft as the motor to measure output.

The sources of error are the following. The chief error was perhaps in reading the dynamometer scale. The divisions on this scale were in ounces which were rather large units. Friction in the bearings was a source of error so the calculated efficiency was that of the entire installation and not simply that of the motor. Some error was introduced in the use of current transformers and the use of the current transformer ratio multiplied the errors in reading the ammeters and wattmeters. The tachometer which measured speed was not absolutely accurate. The power plant from which the motor input was drawn is an independent unit operating without interconnection with any power system. Consequently, the speed regulation is very poor. The frequency varies within wide limits. These changes in frequency caused changes in the losses of the motor and so the results were not strictly comparative.

VI. - OBSERVATION AND PERFORMANCE.

Considerable difficulty was experienced in the satisfactory operation of the induction regulators. It was first attempted to connect the primaries of the regulators in delta but due to the internal connections of the regulators, this connection was not possible. A plan of connecting each phase of the motor separately was used in an attempt to secure separate and independent regulation, but due to magnetic coupling between the phases, such regulation could not be secured. Finally the regulators were connected in Y, as shown in the wiring diagram, and all attempts at independent regulation abandoned.

Since the phases were connected through the y-eonnection, the phases did not operate independently and all three phases had to be regulated simultaneously. After some practice the desired conditions could be obtained without so very much difficulty. At some loads, the unbalance required was obtained by drawing the load from the generator of a motor-generator set and thus adding the control in the field of the generator.

After actually beginning the load runs, readings were taken for runs at no load, one-quarter load, half load, three-quarters load, and full load. In each case the phase voltages, line voltages,phase currents, phase power, and the reading of the dynamometer scale were recorded. The same conditions of voltage unbalance were present in each case. These conditions were 2 phase voltages 10% low, one phase 10% low, 2 phases 5% low, one phase 5% low, all three voltages at normal phase voltage, one phase 5% high, two phases 5% high, one phase 10% high, and 2 phases 10% high. Taking all readings at the same conditions made possible the plotting of all values to the same scale of voltage unbalance. The phases not being regulated were held at normal phase voltage throughout the experiment.

VII. - CALCULATIONS.

The power factor of each phase was calculated by the use of the formula, $\cos \theta$ = power factor = $\frac{W}{E \times I}$, where W = phase power in watts, E = phase voltage, and I = phase current. The total input was determined by adding the power of the separate phases. The output was determined by the formula $P = \frac{2\pi nlf}{33,000} \times \frac{786}{1000}$, where n = R.P.M., l = length of the dynamometer arm, and f = the reading of the scale. The factor $\frac{786}{1000}$ changes the output to K. W. The motor efficiency was determined by the ratio of $\frac{\text{output}}{\text{input}} \times 100$ to express the ratio in percent.

VIII. - CONCLUSIONS.

From the observation of the curves of phase current, phase power, and phase power factor against voltage unbalance, it can be seen that these curves have a characteristic shape regardless of the condition of load. It would seem that the constants of the machine determine in a great measure the distribution of current and power in the individual phases. Even when the phases being regulated are changed, the curves still tend to hold the same general shape, though the tendencies are not so pronounced as before.

By reference to the power factor curves it can be seen that a high power factor in one phase is accompanied by an extremely low power factor in one or both of the other two phases. The degree of voltage and current unbalance would certainly not indicate such a great difference in power and hence in the power factor. Such a condition seems to indicate that power is actually transmitted from one phase to another by transformer action, thus causing the wide variation in the values of power factor.

The efficiency of the motor used was not very high, and also the exciting current was rather high. The relatively large value of exciting current drawn by the motor probably affected the results at the low loads. As has been said before, if a larger motor had been used in the test, the results would have been of more value.

DATA SHEET

Phase I					Phase II					Phase III										
Voltage	Current	Input	Power Factor	Line Voltage	Voltage	Current	Input	Power Factor	Line Voltage	Voltage	Current	Input	Power Factor	Line Voltage	Scale Reading	R.P.M.	Output	Input	Percent Efficiency	Condition
114	9.55	0.580	52.7	187	114.5	18.25	2.005	96.0	215	126	16.00	1.335	66.2	211	7 lb. 10½ oz.	1130	3.077	3.910	78.6	Full Load - 2- 10% Low
114	10.75	1.170	95.5	208	126.5	16.20	1.725	84.0	226.5	127	13.95	0.905	51.0	204		1130	3.077	3.800	81.0	" " 1- 10% "
120.5	10.60	0.875	68.5	205	121	14.65	1.605	90.5	219	127	14.05	1.285	72.0	216		1140	3.100	3.765	82.4	" " 2- 5% "
121	11.40	1.205	87.4	215.5	127	14.50	1.525	82.9	224	127	12.50	0.995	62.7	213		1140	3.100	3.725	83.1	" " 1- 5% "
127	12.50	1.225	77.1	222	127	12.30	1.205	77.1	221	127	12.50	1.255	79.0	222		1130	3.077	3.685	83.4	" " Balanced
133.5	13.65	1.205	66.1	227	127	10.40	0.875	66.3	217	127	14.15	1.650	92.0	232		1140	3.100	3.730	83.2	" " 1- 5% High
133	14.25	1.570	82.9	235	133	12.30	0.955	58.4	223.5	127	11.00	1.180	84.5	225		1140	3.100	3.705	83.5	" " 2- 5% "
140.5	15.90	1.345	60.2	236	127	8.60	0.375	34.3	210	127	17.00	2.105	97.5	242		1150	3.125	3.825	81.7	" " 1- 10% "
140	16.00	1.815	81.0	252	140	13.80	0.765	39.6	225	126	9.80	1.190	96.4	232		1150	3.125	3.770	83.0	" " 2- 10% "
114	7.90	0.300	33.3	187	114	16.40	1.795	97.0	217	127	14.60	1.120	60.4	213	5 lb. 12 oz.	1150	2.350	3.215	73.2	3/4 Load 2- 10% Low
114	10.00	1.100	96.5	211	127	13.95	1.375	77.6	227		12.60	0.660	41.2	203		1140	2.330	3.135	74.4	" " 1- 10% "
121	9.25	0.695	62.0	206	121	12.10	1.295	88.5	218		12.50	1.065	67.0	216		1150	2.350	3.065	76.6	" " 2- 5% "
121	9.85	1.010	84.7	216	127	12.35	1.220	77.7	224		14.00	0.850	47.8	214		1150	2.350	3.080	76.5	" " 1- 5% "
127	10.90	1.000	72.2	221	127	10.85	1.010	73.2	221		10.95	1.015	73.0	221		1140	2.330	3.025	77.0	" " Balanced
133	12.40	1.060	64.3	227	127	8.90	0.635	56.1	217	127.5	12.25	1.370	87.7	230		1150	2.350	3.065	76.7	" " 1- 5% High
133	12.60	1.285	76.6	236	133	11.15	0.765	51.5	224	127	9.45	0.980	81.7	227		1150	2.350	3.630	77.5	" " 2- 5% "
140	15.00	1.225	58.4	236	127	8.30	0.275	23.8	210		15.60	1.920	97.0	240		1150	2.350	3.420	68.8	" " 1- 10% "
140	14.30	1.545	77.2	251	140	12.35	0.620	35.9	228		8.05	0.945	92.5	233		1150	2.350	3.110	75.6	" " 2- 10% "
114	6.30	0.125	17.4	188	114	11.70	1.240	93.0	216	127	12.45	0.780	50.2	212	3 lb. 12 oz.	1150	1.534	2.145	71.5	1/2 Load 2- 10% Low
114	6.85	0.735	94.0	211	127	11.50	1.040	70.0	227		10.65	0.405	29.9	203		1150	1.534	2.180	70.5	" " 1- 10% "
121	7.50	0.425	46.8	206	121	9.50	0.965	84.0	218		10.65	0.785	58.0	216		1140	1.520	2.175	70.0	" " 2- 5% "
121	7.60	0.700	76.0	215	127	10.20	0.895	69.0	224		9.55	0.550	45.3	213		1140	1.520	2.145	70.9	" " 1- 5% "
127	8.95	0.705	62.0	220	127	9.00	0.720	63.0	221		9.05	0.725	63.0	221		1150	1.534	2.150	71.5	" " Balanced
133	10.95	0.790	56.8	228	127	7.55	0.370	38.6	206		10.00	1.050	82.7	230		1150	1.534	2.210	69.5	" " 1- 5% High
133	10.75	0.985	68.9	234	133	9.75	0.520	40.1	223		7.50	0.690	72.5	226		1150	1.534	2.195	70.0	" " 2- 5% "
140	12.80	0.865	48.3	236	127	7.50	0.025	2.6	209		12.40	1.440	91.5	239		1140	1.520	2.330	65.3	" " 1- 10% "
140	12.60	1.250	71.0	251	140	11.16	0.835	53.5	227		6.30	0.705	88.0	232		1150	1.534	2.790	54.3	" " 2- 10% "
127	10.00	0.475	37.4	213	114	5.85	-0.050	8.24	188	114	8.50	0.870	89.8	215	1 lb. 14 oz.	1170	0.780	1.295	60.2	1/4 Load 2- 10% Low
127	9.65	0.750	61.1	228	127	8.95	0.100	8.8	204	114	4.50	0.400	78.0	209		1170	0.780	1.250	62.3	" " 1- 10% "
127	8.75	0.455	41.0	216	121	6.25	0.130	17.2	205	121	7.50	0.645	71.0	216		1160	0.775	1.230	63.0	" " 2- 5% "
127	8.50	0.625	58.0	224	127	8.00	0.225	22.2	214	121	5.75	0.350	50.3	214		1170	0.780	1.200	65.0	" " 1- 5% "
127	7.35	0.425	45.5	220	127	7.55	0.400	41.7	221	127	7.25	0.375	40.7	220		1160	0.775	1.200	64.5	" " Balanced
133	9.05	0.450	37.4	227	127	6.65	0.125	14.8	216	127	7.40	0.625	66.5	228		1160	0.775	1.200	64.5	" " 1- 5% High
133	9.00	0.615	51.5	234	133	8.20	0.170	15.6	222	127	6.25	0.400	50.4	225		1160	0.775	1.185	65.4	" " 2- 5% "
140	10.65	0.490	32.8	239	127	7.25	-0.150	16.3	213	127	8.40	0.890	83.5	238		1150	0.766	1.230	62.3	" " 1- 10% "
140	10.10	0.785	55.5	250	140	9.80	0.085	6.2	228	127	4.75	0.335	55.5	232		1170	0.780	1.205	64.6	" " 2- 10% "
114	6.35	-0.275	38.0	186	114	5.85	0.480	22.8	214	127	8.65	0.215	19.6	214	0	1180	0	0.420	0	No Load 2- 10% Low
114	3.85	0.100	22.8	206	127	7.75	0.400	40.6	228		8.00	-0.125	12.3	204		1180		0.475		" " 1- 10% "
121	6.30	-0.035	4.6	204	121	6.20	0.280	37.4	218		6.95	0.150	17.0	218		1180		0.405		" " 2- 5% "
121	5.80	0.120	17.1	214	127	6.80	0.250	29.0	224		6.80	0.025	1.7	212		1180		0.395		" " 1- 5% "
127	6.65	0.125	14.7	220	127	6.55	0.125	14.7	220		6.55	0.125	14.7	220		1180		0.375		" " Balanced
133	5.80	0.120	13.0	227	127	6.80	0.250	0	216		6.80	0.025	32.9	228		1180		0.395		" " 1- 5% High
133	6.30	-0.035	27.6	234	133	6.20	0.280	0	223		6.95	0.150	16.4	224		1180		0.395		" " 2- 5% "
140	3.85	0.200	16.1	236	127	7.75	0.400	34.8	211		8.00	-0.125	6.7	237		1170		0.475		" " 1- 10% "
140	6.35	-0.275	38.0	251	140	5.85	0.480	5.9	228		8.65	0.215	14.9	228		1180		0.420		" " 2- 10% "

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-APPENDIX-

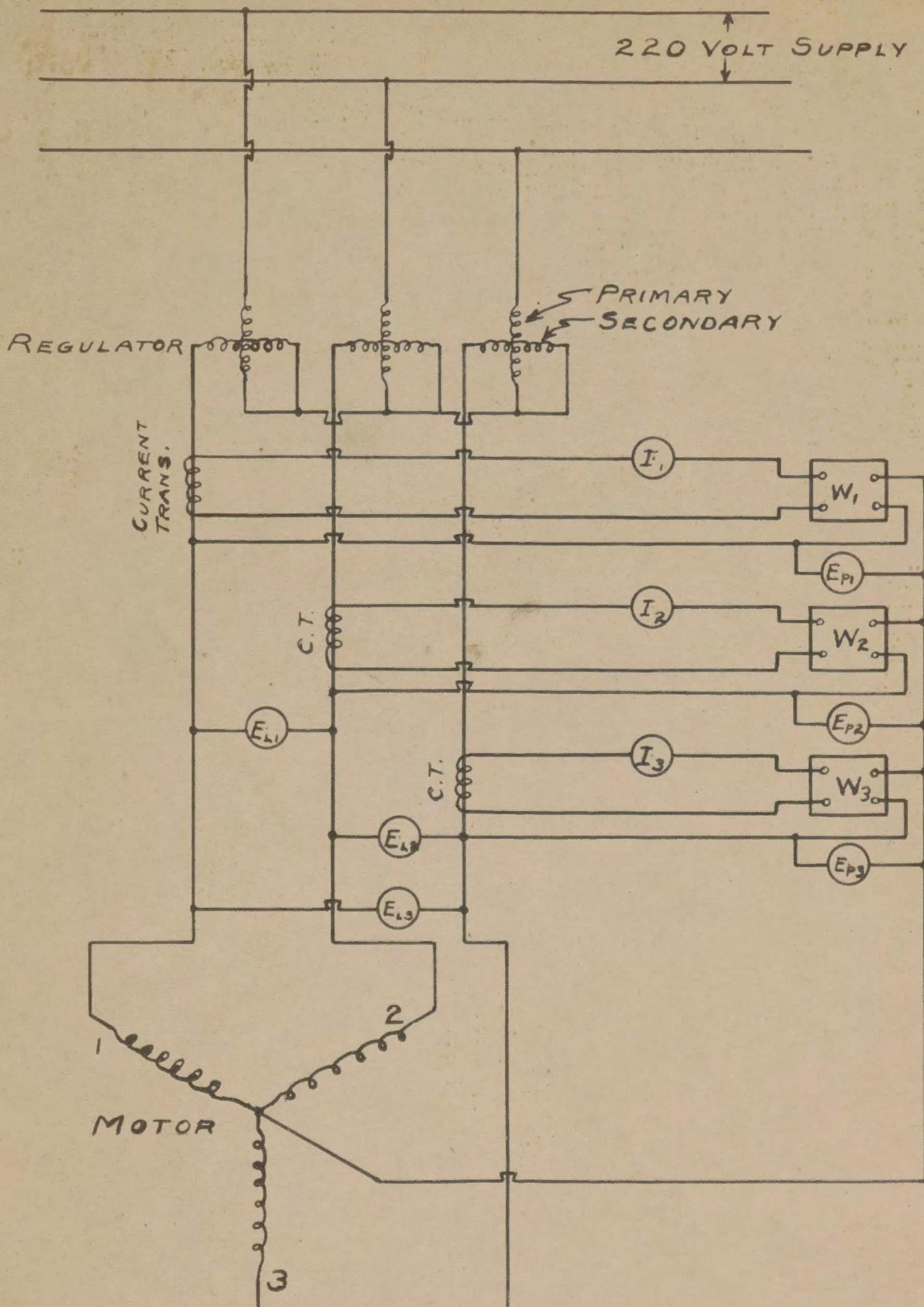


DIAGRAM OF CONNECTIONS
AND
LAYOUT OF APPARATUS

- C U R V E   S H E E T S -

In all of the curve sheets which follow, the following symbols are used for the obscissa:

a = 2 Phases 10% Low - 1 Phase normal.

b = 1 Phase 10% Low - 2 Phase normal.

c = 2 Phases 5% Low - 1 Phase normal.

d = 1 Phase 5% Low - 2 Phase normal.

e = All phases normal.

f = 1 Phase 5% High - 2 Phases normal.

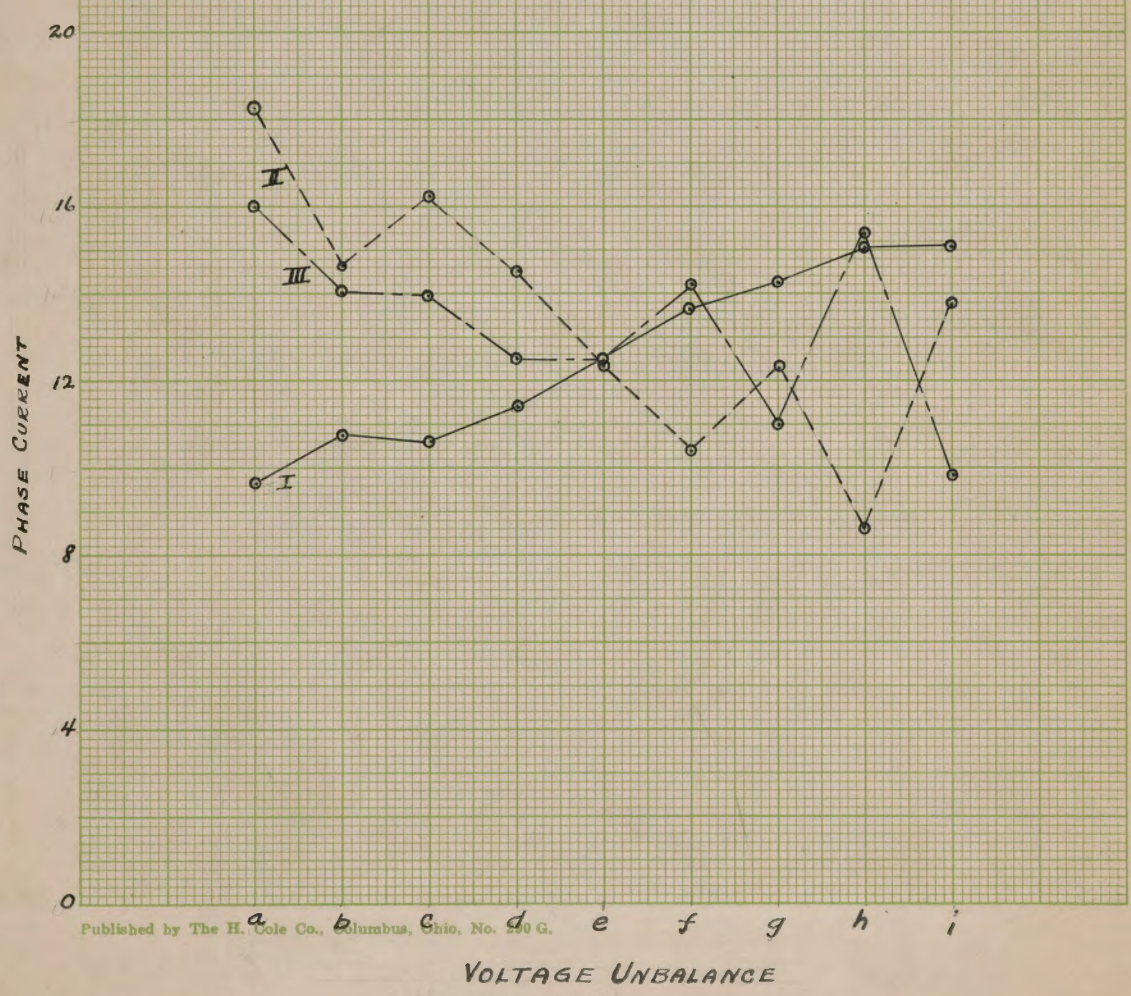
g = 2 Phases 5% High - 1 Phase normal.

h = 1 Phase 10% High - 2 Phases normal.

i = 2 Phases 10% High - 1 Phase normal.

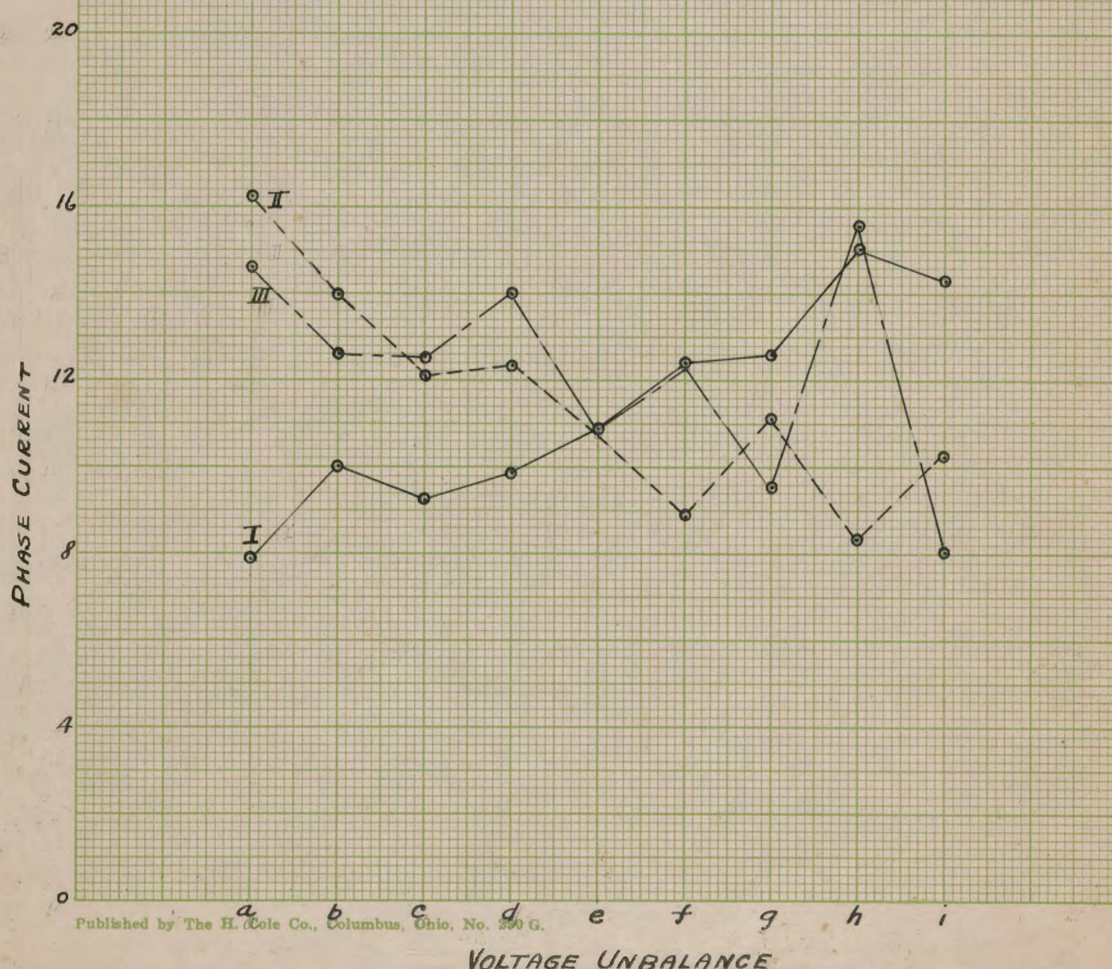
SHEET #1

FULL LOAD - PHASE CURRENTS



SHEET # 2

3/4 LOAD – PHASE CURRENTS



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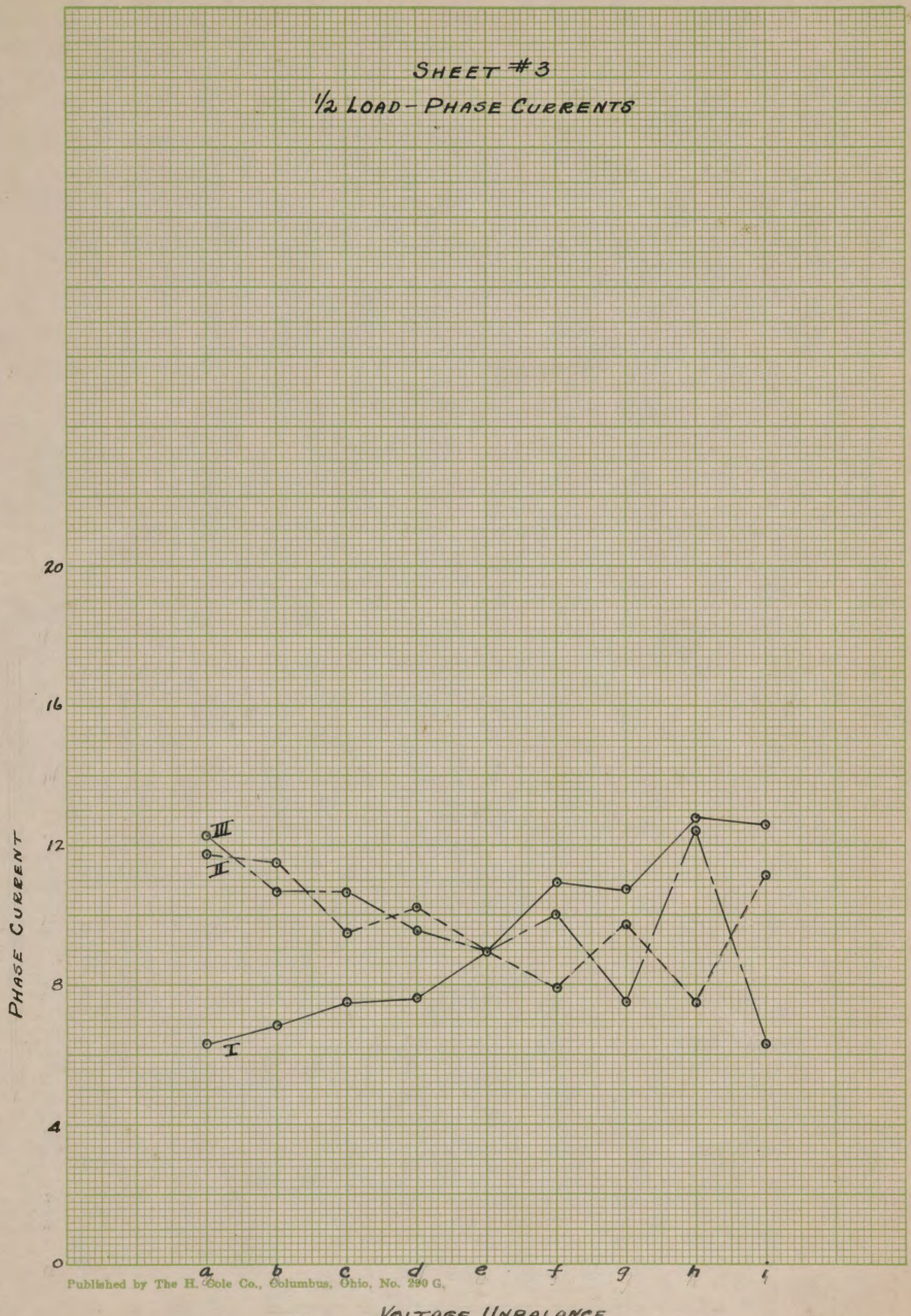
SHEET #3

½ LOAD – PHASE CURRENTS



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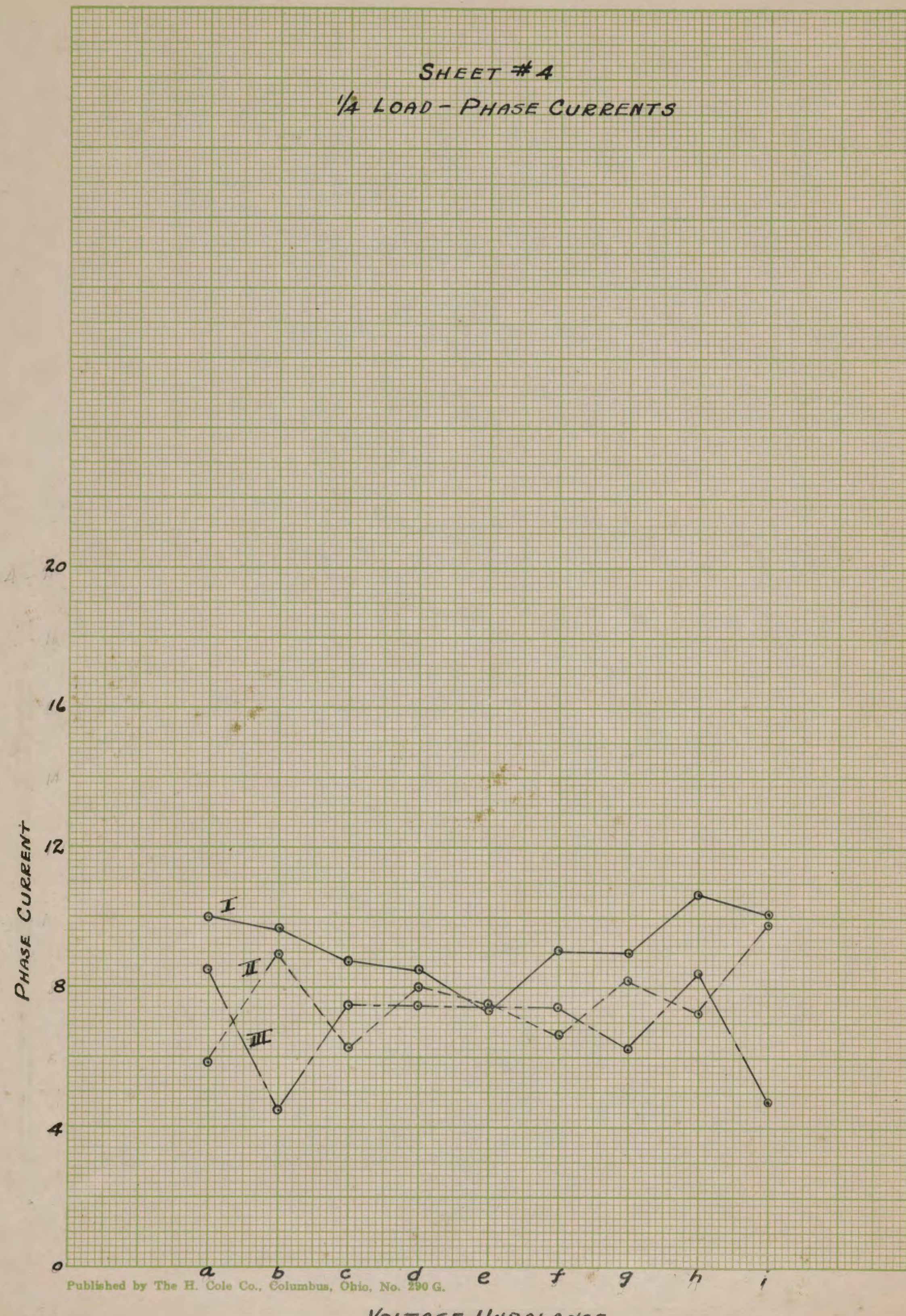

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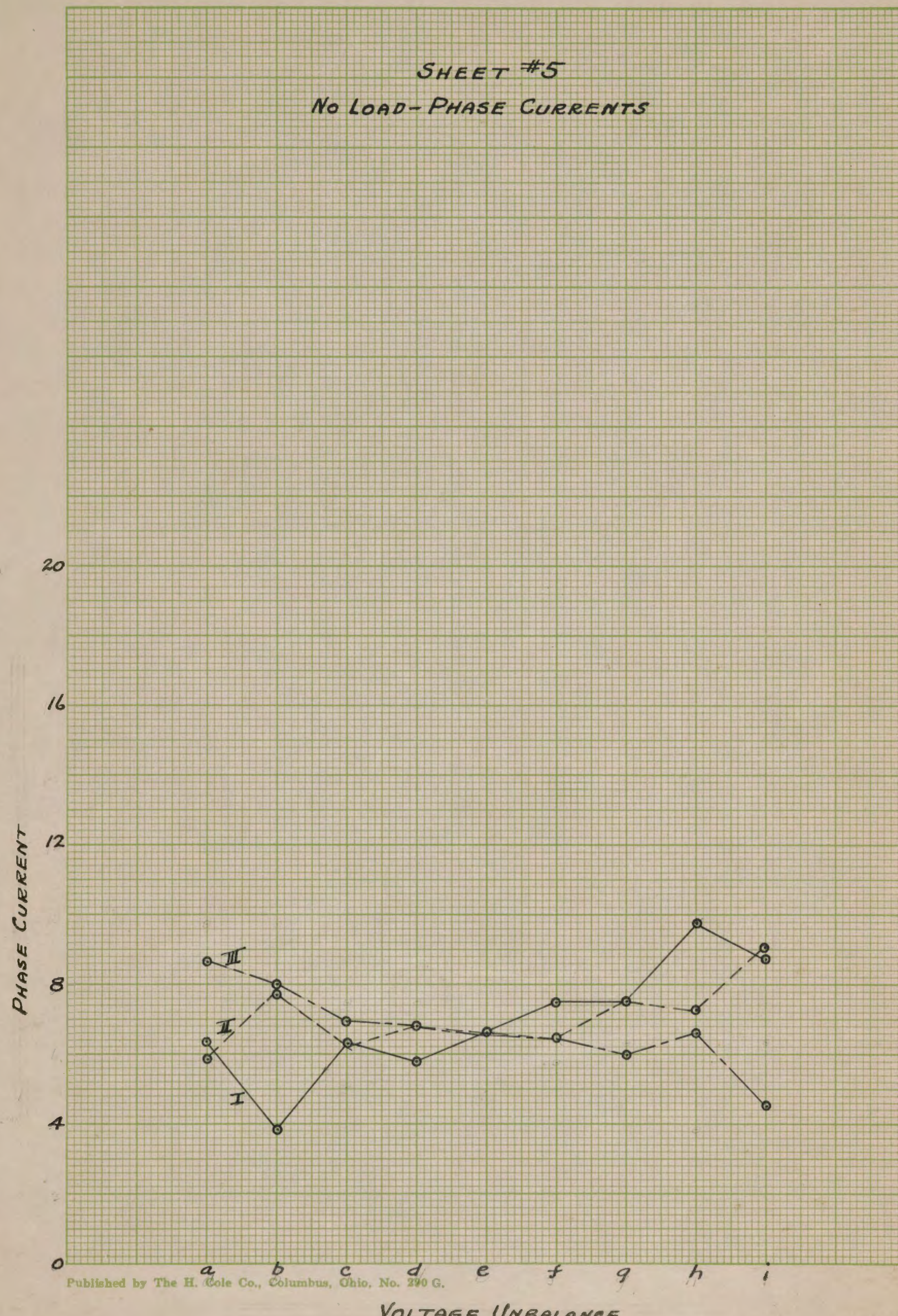

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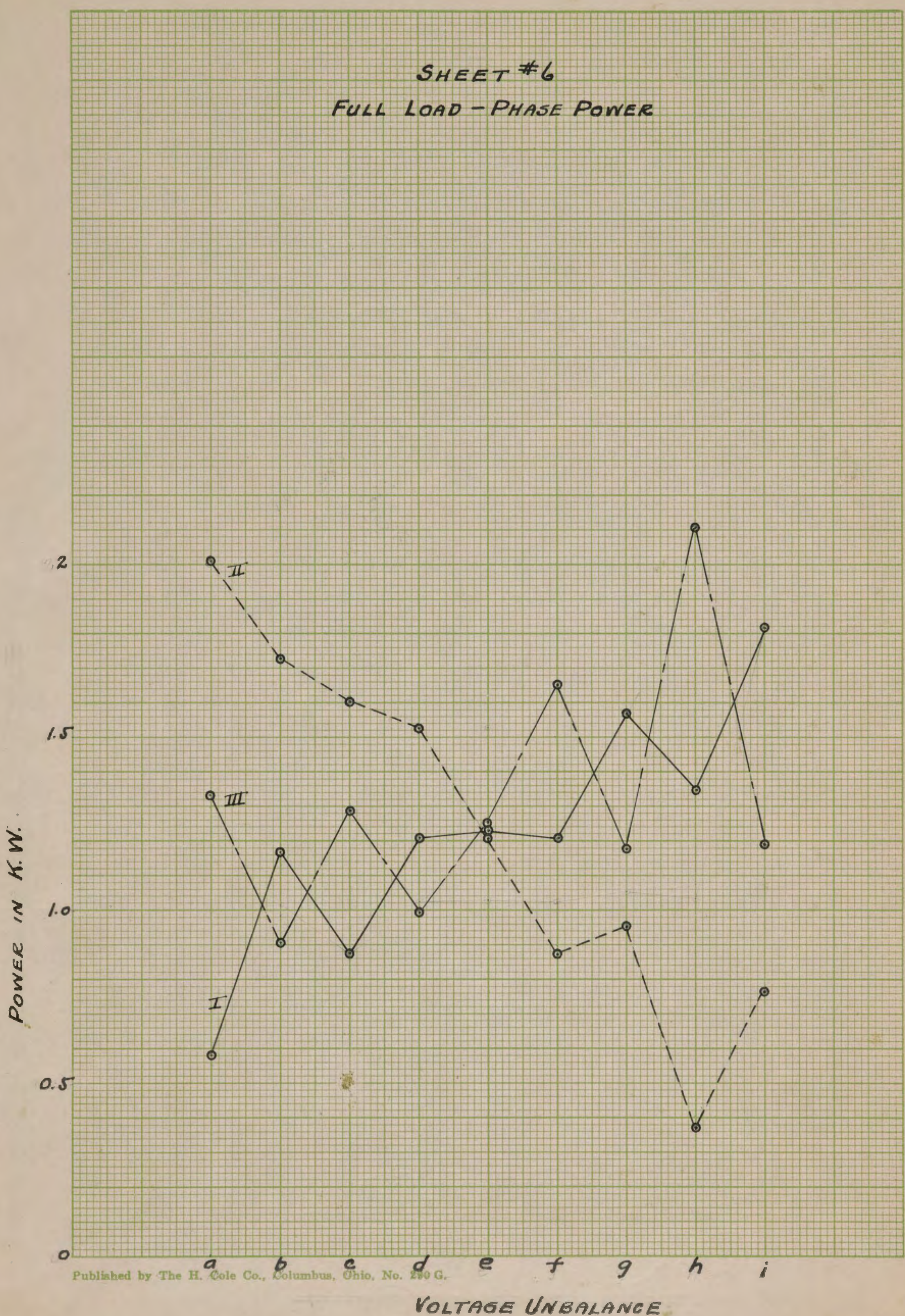
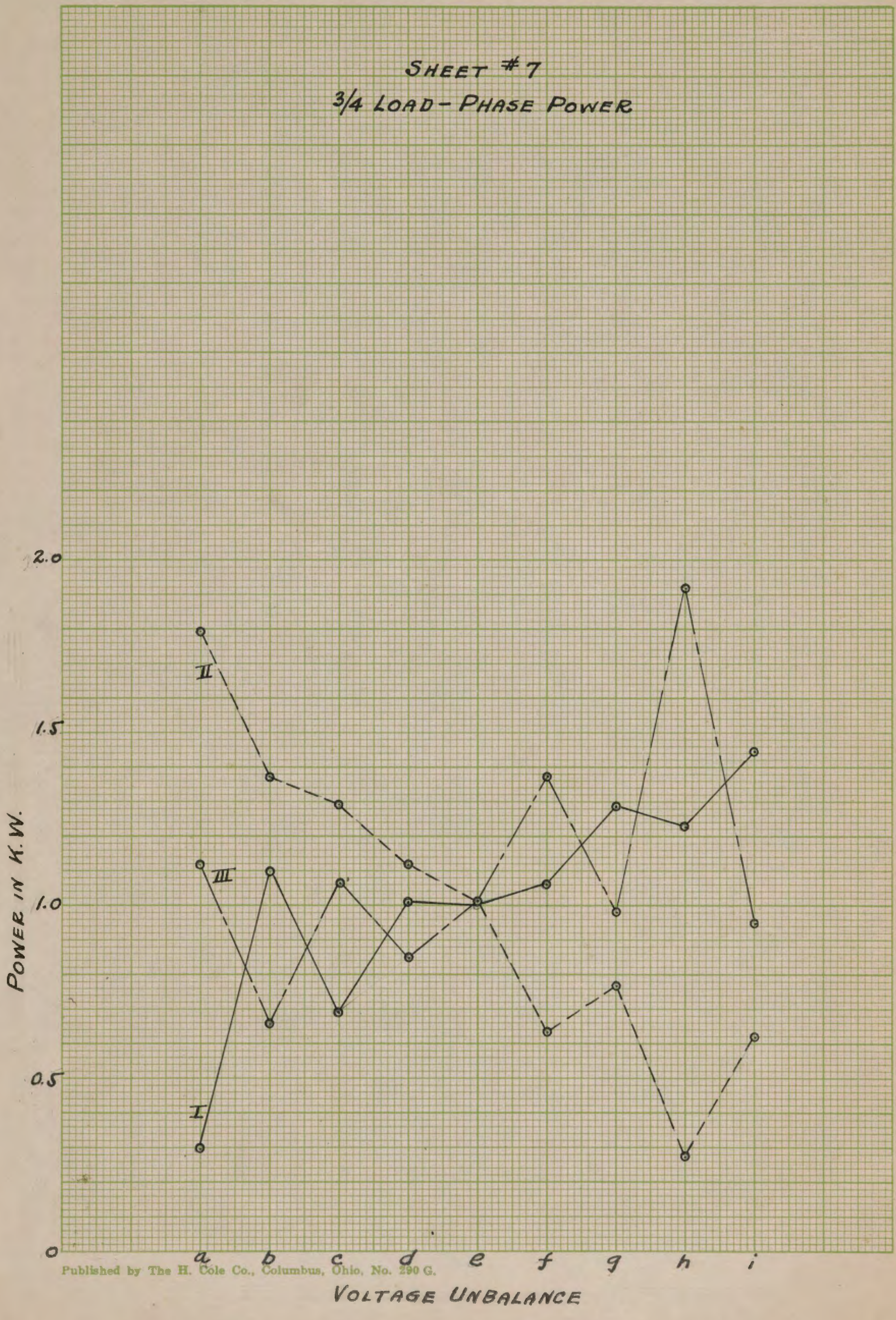

SHEET #6
FULL LOAD - PHASE POWER
POWER IN K.W.
VOLTAGE UNBALANCE
0
0.5
1.0
1.5
2
a
b
c
d
e
f
g
h
i
I
II
III



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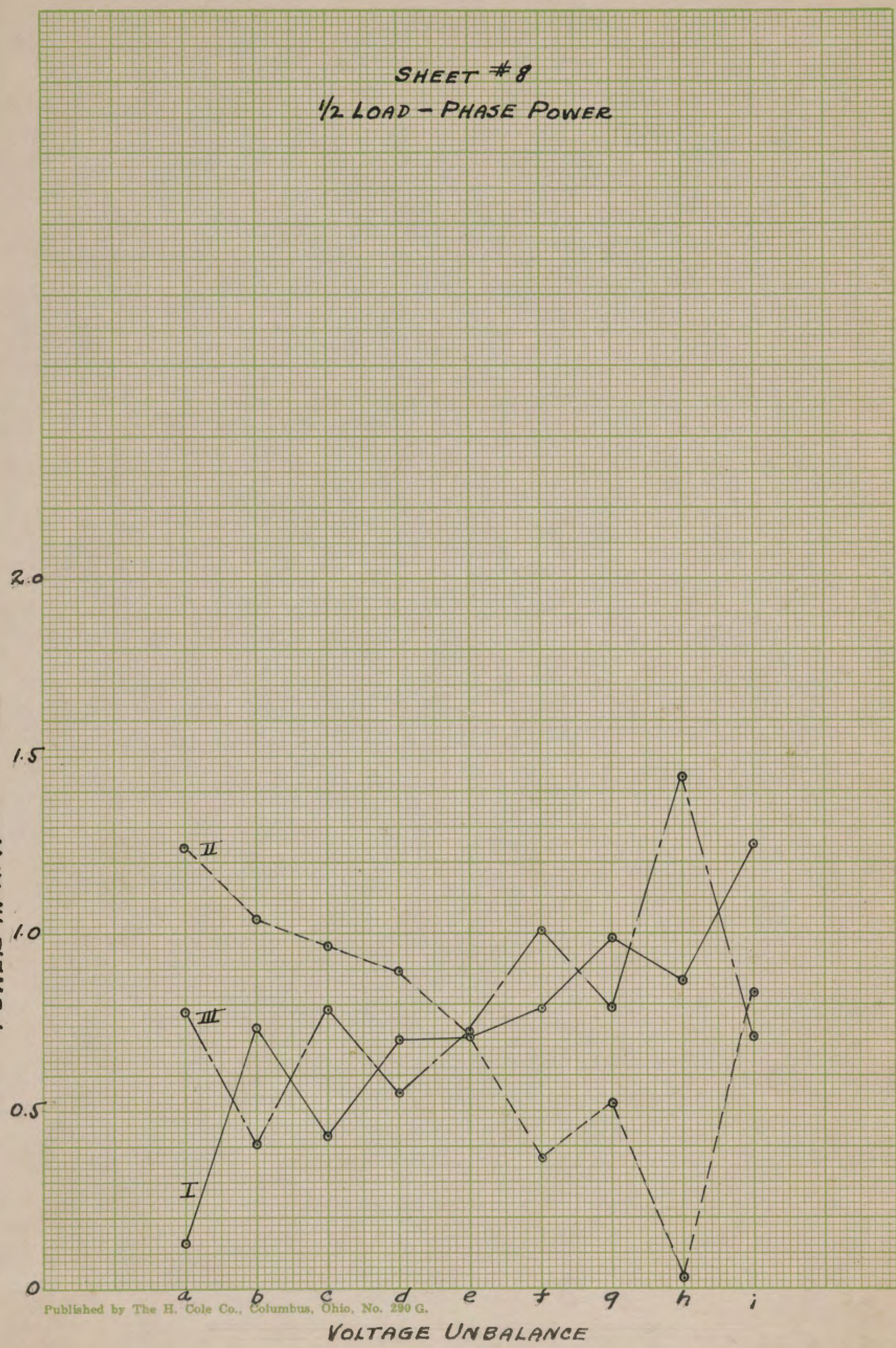
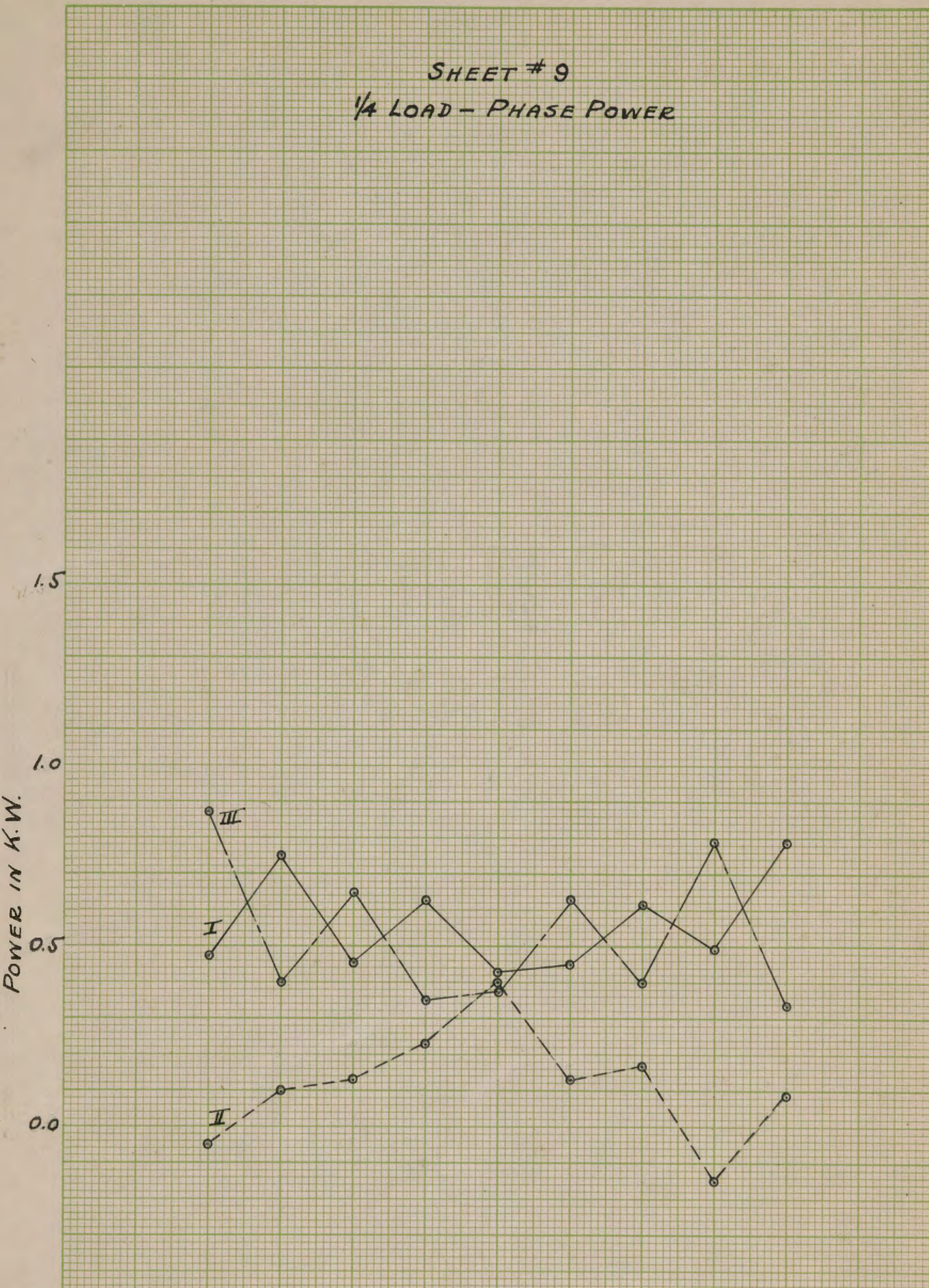


SHEET #8
1/2 LOAD - PHASE POWER
POWER IN K.W.
2.0
1.5
1.0
0.5
0
II
III
I
a
b
c
d
e
f
g
h
i
Published by The H. Cole Co., Columbus, Ohio, No. 290 G.
VOLTAGE UNBALANCE



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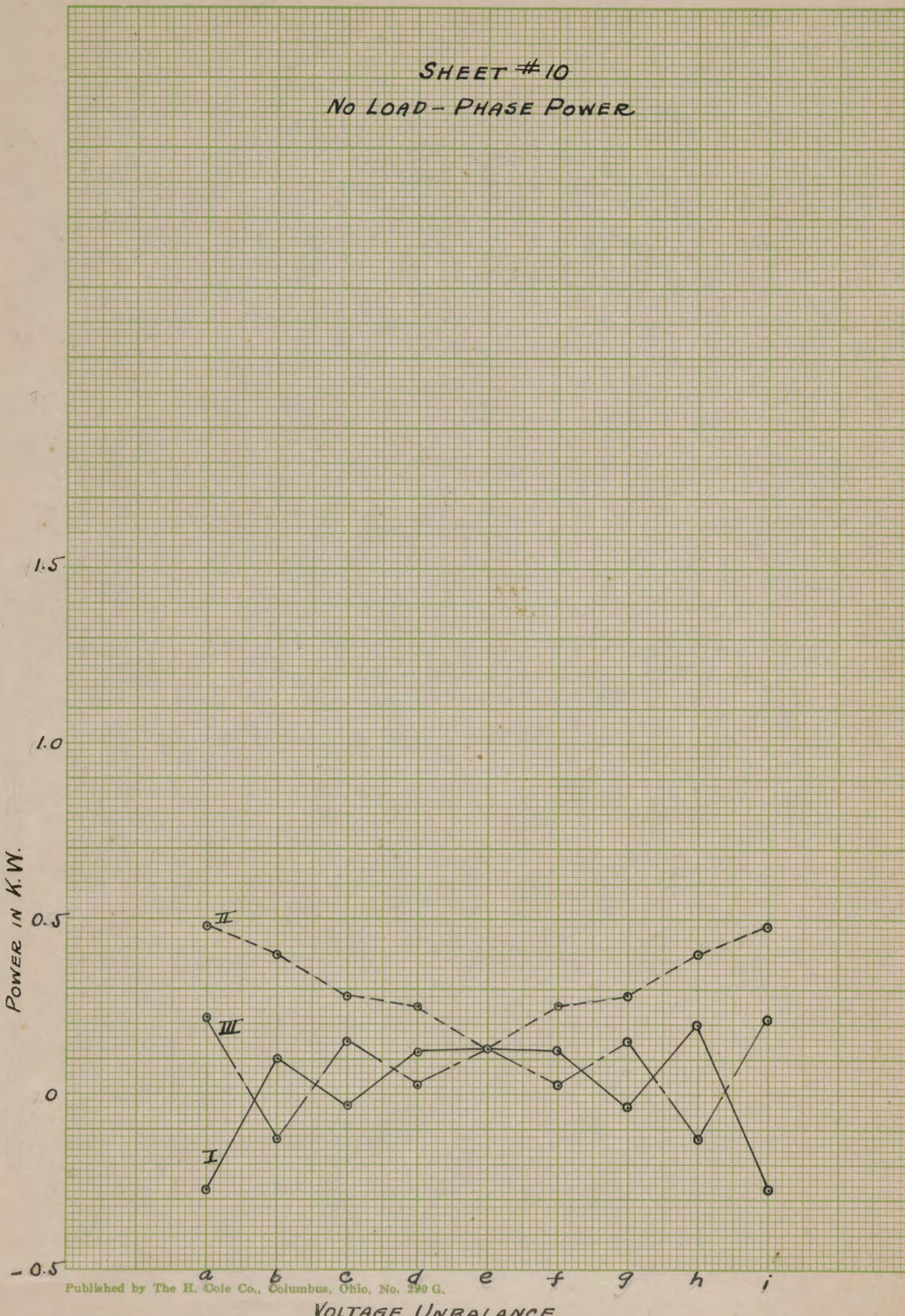
SHEET #10

NO LOAD – PHASE POWER



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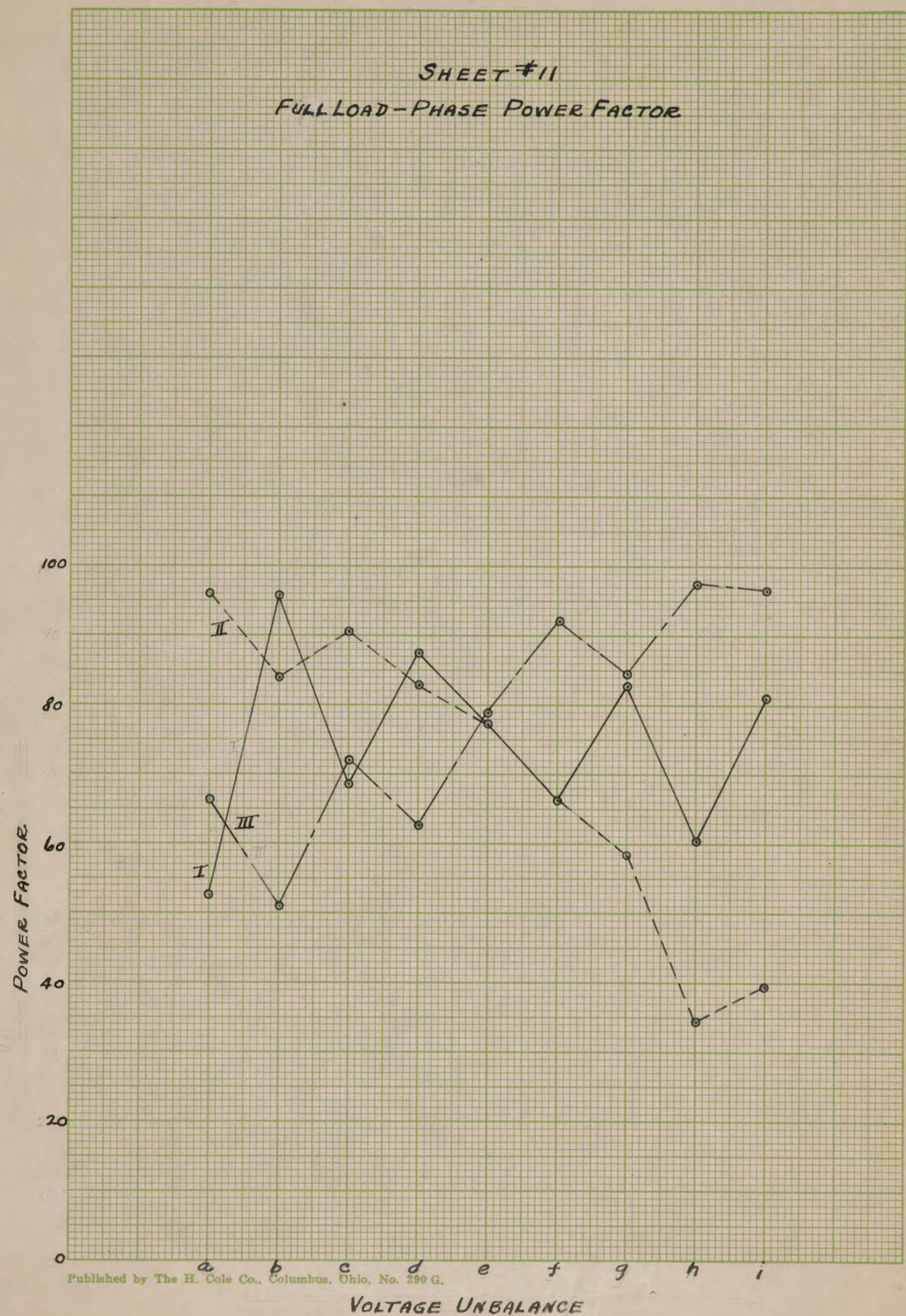

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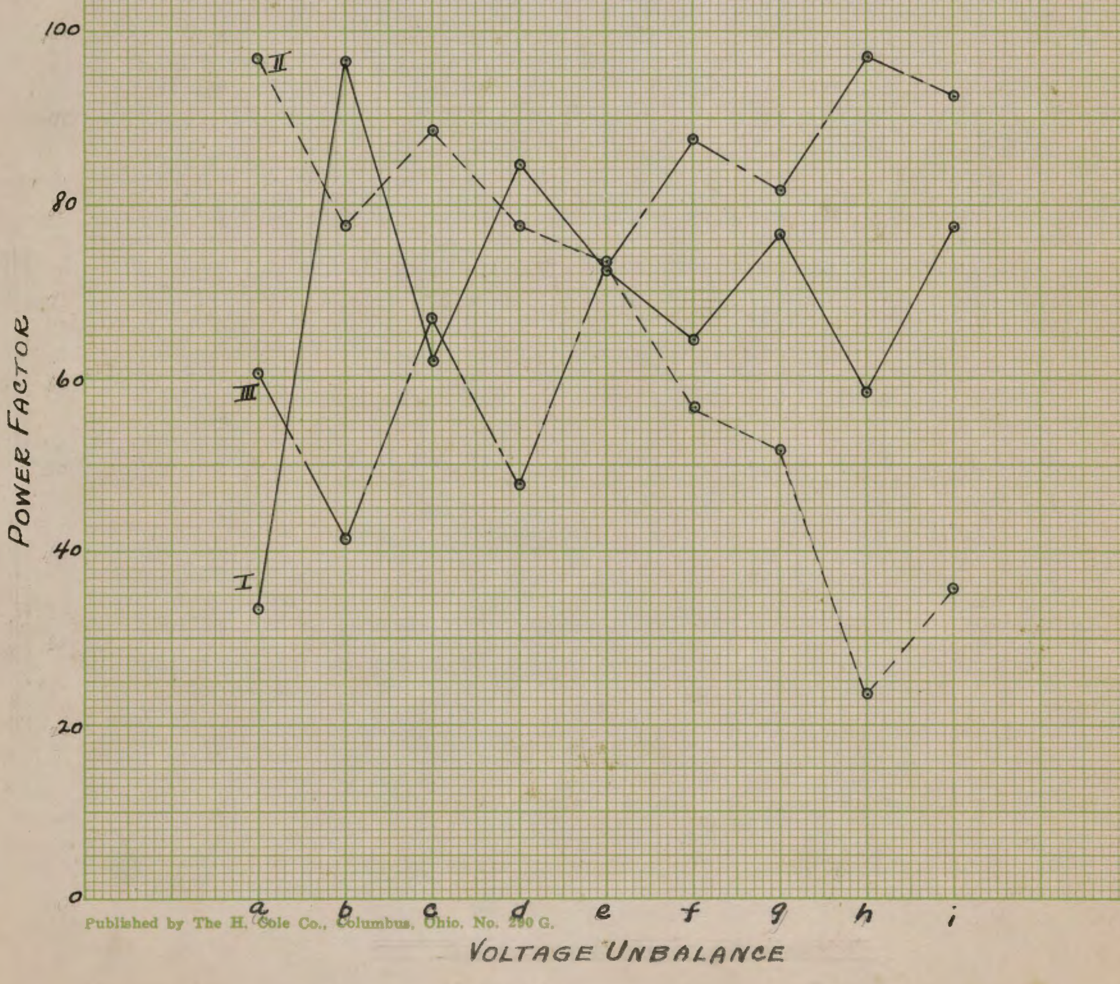
Sheet #12

3/4 Load - Phase Power Factor



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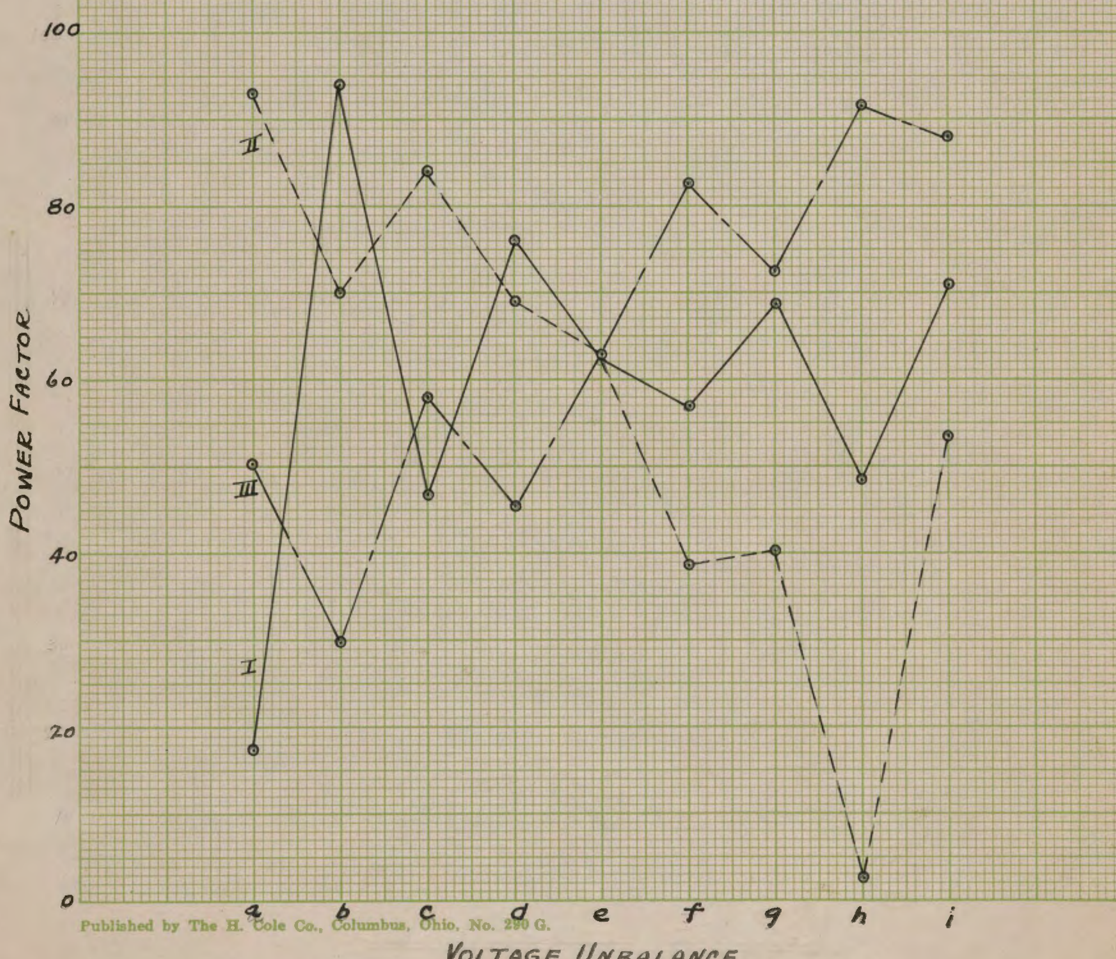
SHEET #13

½ LOAD – PHASE POWER FACTOR



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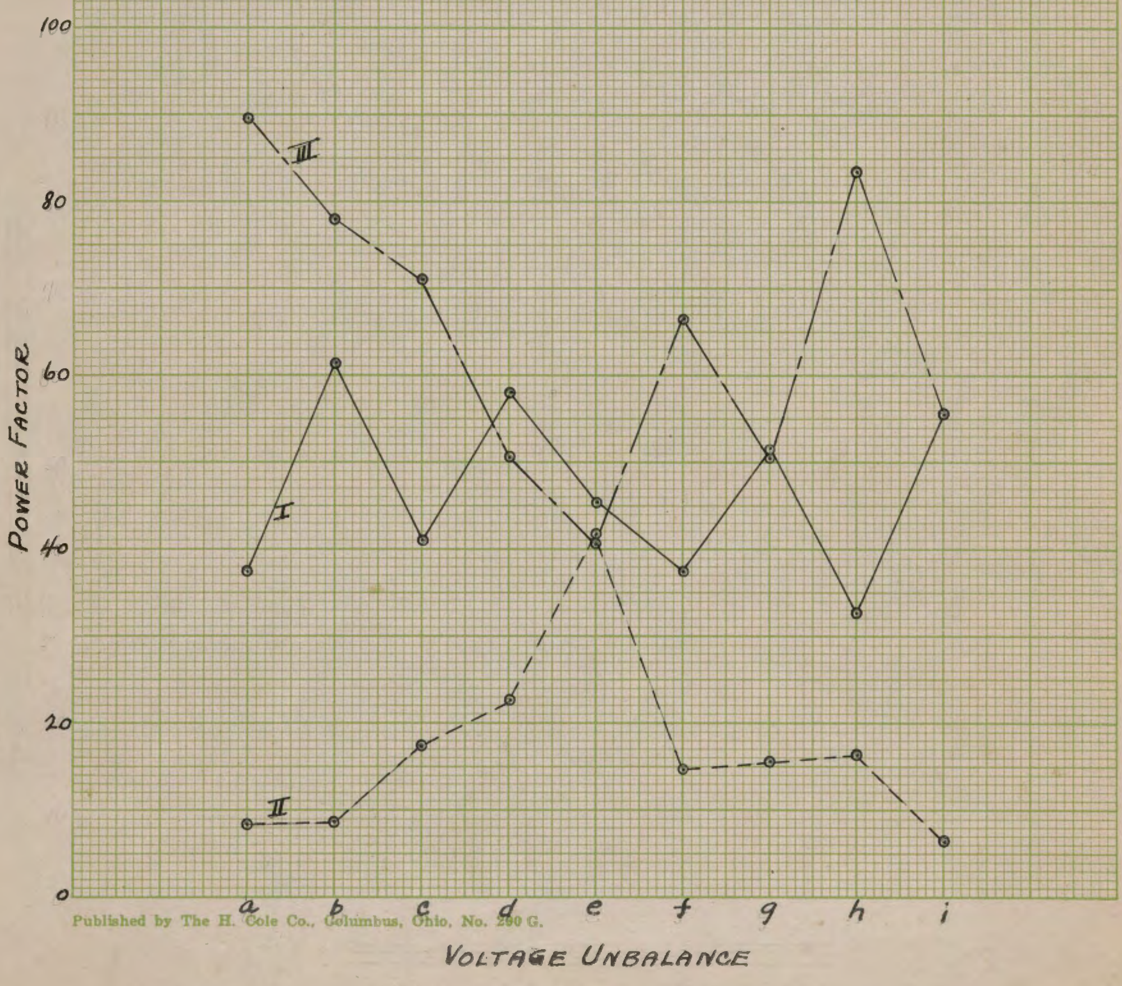
SHEET #14

1/4 LOAD - PHASE POWER FACTOR



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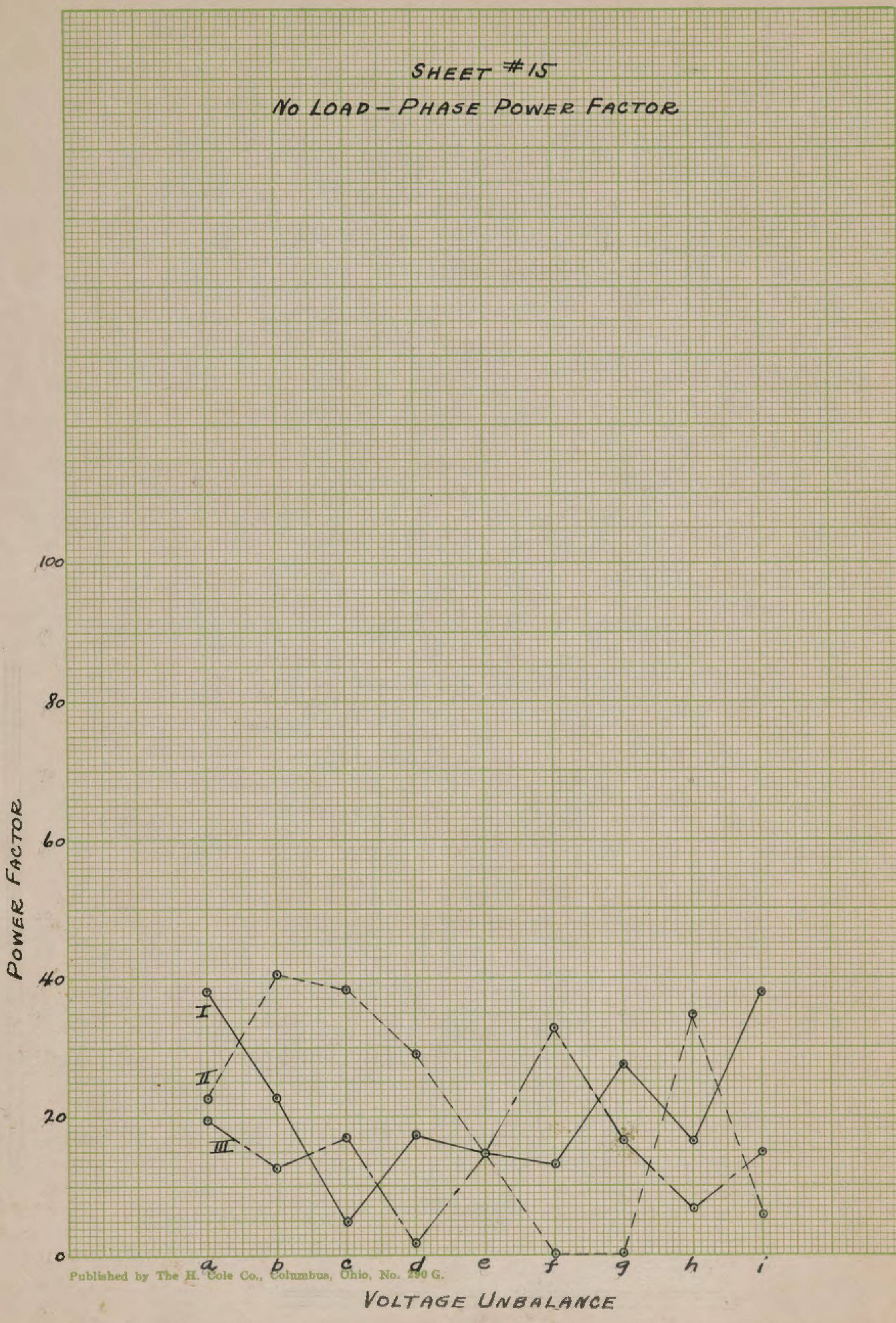

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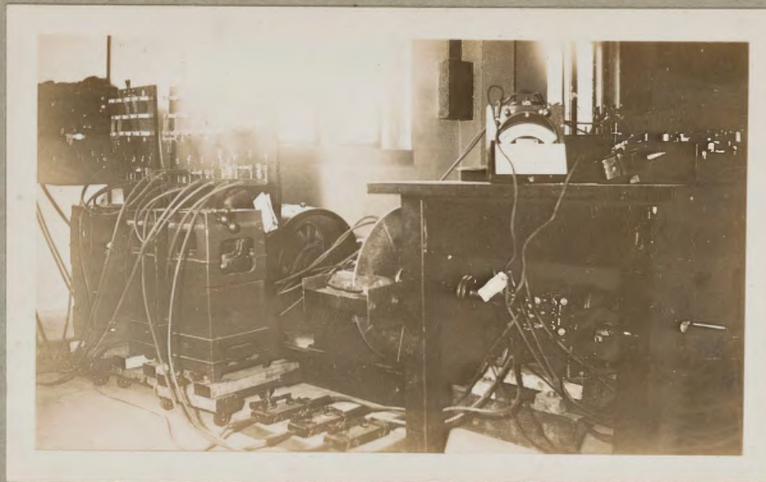

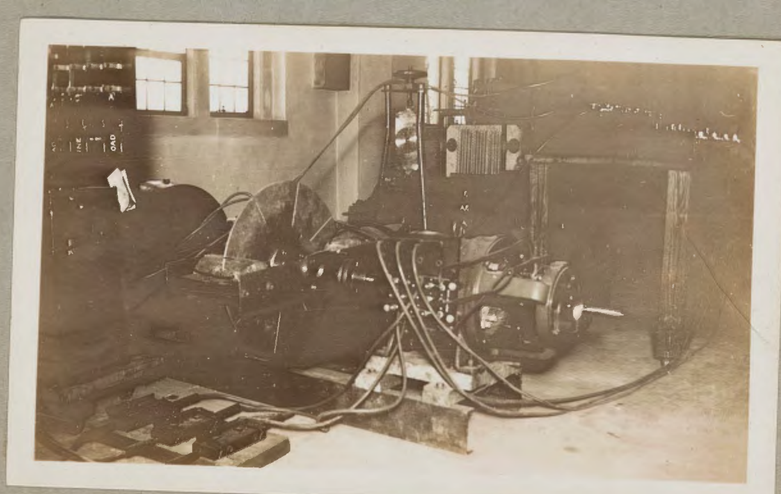

APPROVAL

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prerequisite to the degree for which it is submitted. It is understood that by this approval the undersigned does not necessarily endorse or approve any statement made, opinions expressed, or conclusions drawn therein, but approves the thesis only for the purpose for which it is submitted.

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