

**Appendix D - UNIVERSITY HONORS PROGRAM
SENIOR PROJECT - APPROVAL**

Name: Pamela Murray

College: Engineering Department: Mechanical Engineering

Faculty Mentor: Dr. William Hamel

PROJECT TITLE: Empirical Comparisons of Multi-Robot
Cooperation Strategies

I have reviewed this completed senior honors thesis with this student and certify that it is a project commensurate with honors level undergraduate research in this field.

Signed: William Hamel, Faculty Mentor

Date: 11/30/99

Comments (Optional):

- 1) Very well written report.
- 2) Experimental work is thorough and disciplined.
- 3) Technical topic is central to current research in multi-agent autonomous robotics
- 4). Superb job!

University Honors Senior Project

Empirical Comparisons of Multi-Robot Cooperation Strategies: Application to Cooperative Multi-Robot Observation of Multiple Moving Targets*

Pamela Murray

Dr. William R. Hamel, Faculty Advisor

December 8, 1999

Prepared in partial fulfillment of the requirements of the DOE Energy Research Undergraduate Laboratory Fellowship Program (ERULF) under the direction of research advisor Dr. Lynne Parker in the Computer Science and Mathematics Division of Oak Ridge National Laboratory.

*Research sponsored by the U.S. Department of Energy at ORNL under contract DE-AC05-96OR22464 with Lockheed Martin Energy Research Corporation.

By acceptance of this article, the publisher or recipient acknowledges the U.S. Government's right to retain a nonexclusive, royalty-free license in and to any copyright covering this article.

Empirical Comparisons of Multi-Robot Cooperation Strategies:
Application to Cooperative Multi-Robot Observation of Multiple Moving Targets

Pamela Murray

Abstract

This project explored the performance of physical robot teams executing the cooperative observation of multiple moving targets. In previous research at Oak Ridge National Laboratory (ORNL), two algorithms that can accomplish this type of observation were developed and then tested and compared by running computer simulations of robots using these algorithms. In this project, extensive experimentation and data collection was conducted on a physical robot team. The effectiveness of the algorithms with a physical robot team was determined by varying the number of obstacles and the number of tracker and target robots. Then, the statistical significance of the resulting data was studied. The results of this project will contribute to the successful use of robot teams in critical surveillance, reconnaissance, and security applications.

1. Introduction

From high risk activities like security, surveillance, and reconnaissance to repetitive tasks like tracking items in a warehouse or monitoring automated assembly processes in a factory and even to cutting edge technological applications like visually following tissue during medical procedures, it would be desirable to have a machine instead of a human presence. Autonomous robots programmed to carry out these observational tasks would be a solution to this problem. These robots would have the advantage of movement without the need of a human operator and could track moving targets unlike a stationary monitoring system. Past work has focused on

predicting the trajectories of moving targets to determine a tracker's path or has only assumed one tracker or target or a fixed and known area.

Previous research at ORNL has focused on multiple trackers and targets in an unfamiliar environment as might be encountered in an actual reconnaissance mission. Then, these concepts were incorporated into cooperation strategies for the tracker robots that maximized the collective observation of targets and were developed into two algorithms called (1) Local and (2) Cooperative Multi-Robot Observation of Multiple Moving Targets (CMOMMT). The algorithms were then tested and compared on computer simulated robots.

In current work, the tracker algorithms were modified to achieve optimal target observation and were implemented on a physical team of four autonomous Nomad robots that served as trackers and targets. This project empirically compared the performance of the algorithms on this physical team to determine which one optimized target observation. Performance differences caused by changes in the numbers of trackers, targets, and obstacles were studied to determine the algorithms' effectiveness under varying conditions. Then, all the resulting data was tested for statistical significance and compared.

In the following sections, the methods used to collect and analyze the data are addressed, the data is presented, the results are described, the findings are discussed, and the project is summarized.

2. Explanation of Methods

2.1 Algorithms

During experimentation, Local, CMOMMT, Local 2.0, and CMOMMT 2.0 algorithms were used to calculate the homing points of the tracker robots based on the positions of the other

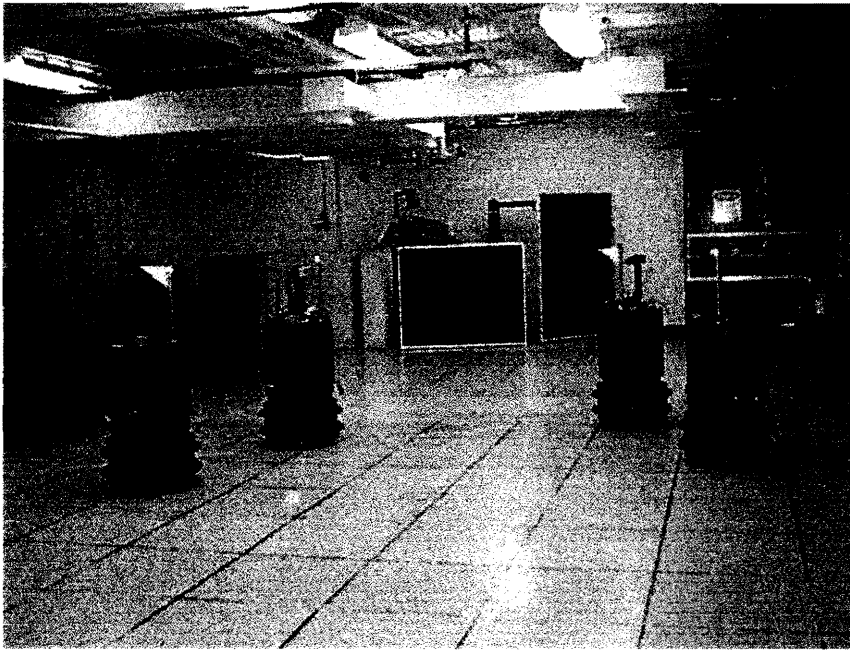
trackers and targets. A Fixed configuration for the tracker robots was also used. In every experiment configuration, target robots moved within the bounded area without regard to the location of trackers or other targets. Fixed is the most basic algorithm and keeps the trackers in fixed locations. In Local, the trackers are attracted to targets and repelled by other trackers. The CMOMMT algorithm is a modified version of the Local algorithm in which the trackers are additionally repelled by targets already under observation from another tracker. Local 2.0 and CMOMMT 2.0 are modified versions of the Local and CMOMMT algorithms that allow the trackers to circumnavigate obstacles in their paths.

2.2 Positioning

Tracker and target robots were given random initial positions. The algorithms employed produce non-random movements based on the environment, so changing the initial team configuration provided the difference necessary to lead to different team movements. During all experiments, positioning was accomplished by dead reckoning. Dead reckoning makes use of the odometric sensors on the Nomad robots for movement calculations.

The obstacles were randomly positioned according to values produced by a random number generator, and the obstacle configuration was different for every run.

2.3 Bounded Area



The bounded experimentation area is 20 feet by 42-feet. The robots are each two feet in diameter. Two sizes of obstacles, 9 in. x 15 in. x 34 in. and 17 in. x 25 in. x 33 in., were used.

2.4 Statistical Significance

An experiment run was considered valid and kept in the data set if no robot's reported and actual ending x or y position differed by more than one hundred inches. For each ten-minute experiment run, the average distance from each target to its closest tracker was calculated and recorded. The resulting data was tested for statistical significance using the student's t-test.

2.5 Experiment sets

Since the physical team consists of only four robots, the following configurations contain all possible combinations of trackers and targets. Ten valid ten-minute runs of each of the following experiment configurations were conducted:

No obstacles:

Fixed, Local

One tracker, one target

One tracker, two targets

One tracker, three targets

Fixed, Local, and CMOMMT

Two trackers, one target

Two trackers, two targets

Three trackers, one target

With 4, 8, and 12 obstacles:

Fixed, Local, CMOMMT, Local 2.0, and CMOMMT 2.0

Two trackers, two targets

2.6 Purposes of experiment configurations

The Fixed experiments provide baseline performance data. This data allows for a comparison of a fixed set of observers to a mobile set. The Local experiments demonstrate the simplest form of cooperative tracking. The CMOMMT experiments demonstrate a more

advanced and complex form of tracking. The Fixed, Local, and CMOMMT sets can be compared to determine if the Local and CMOMMT algorithms provide statistically significant changes in tracking performance within various robot team configurations. Additionally, comparisons can be made between the performance of an algorithm with different team configurations.

After these experiments were conducted in a bounded area free of obstacles, four, eight, and finally twelve obstacles were then added to the same bounded area. These experiments were only conducted with one robot team configuration: two trackers and two targets. The new data could then be compared to the previously obtained data for the same configuration without obstacles to determine the degradation of performance that occurred with the introduction of obstacles. Also, these results can be compared to those produced by Local 2.0 and CMOMMT 2.0 to determine the effect of obstacle avoidance on the local and CMOMMT algorithms.

2.7 Data Analysis

The data was analyzed by calculating the percentage of improvement or degradation of the average distance between each target and its closest tracker when algorithms, robot team configurations, or the number of obstacles was changed.

3. Results

The data was checked for statistical significance using the student's t-test. The student's t-test determines whether two groups of data come from populations with the same mean. The student's t-test used assumed unequal variances and a two tailed distribution was used. The resulting value (see tables in Appendix) is the probability that the two samples come from

populations with the same mean. The confidence level was set at .98. Therefore, all probabilities less than .02 ($2E-2$) were considered statistically significant. The statistically significant data was then analyzed by comparing the percent improvements between different configurations, algorithms, and numbers of obstacles.

Figure 1

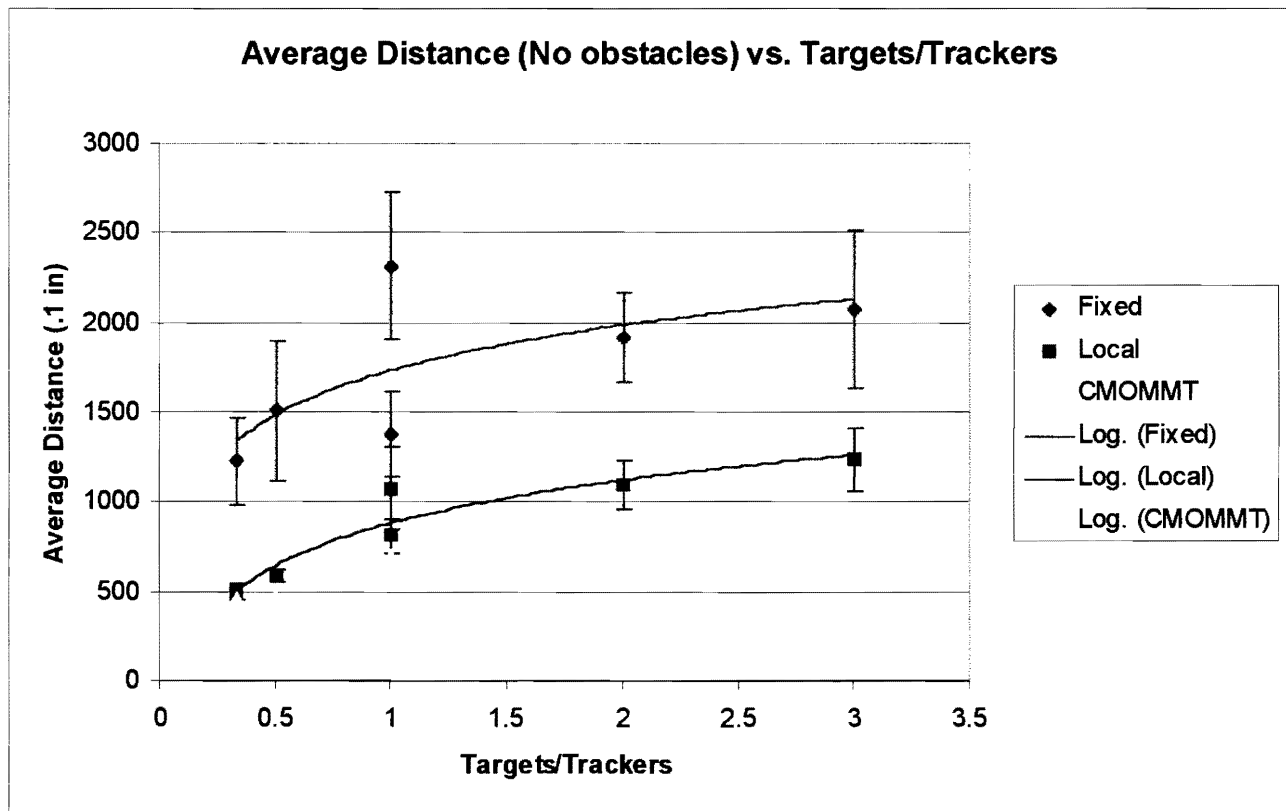


Figure 1: A graphical representation of the data presented in Tables 1 and 2.

A targets to trackers ratio is used for the dependent variable, x . Using this method, configurations with fewer trackers than targets have $x > 1$ and greater average distances than configurations with more trackers than targets which corresponds to $x < 1$. Two configurations - one tracker, one target and two trackers, two targets -- have a ratio of one. Therefore, two points are present at $x=1$ for Fixed and Local. CMOMMT has only one point at $x=1$ because CMOMMT is not possible with only one tracker. The graph shows that as the ratio of targets to

trackers increases the average distance from each target to its closest tracker increases logarithmically.

In the following tables, gray cells denote data that is not statistically significant.

Table 1

Tracker, Target	Percent improvement between (no obstacles)		
	Fixed--Local	Fixed--CMOMMT	Local--CMOMMT
1,1	65.03%		
1,2	42.94%		
1,3	40.35%		
2,1	61.25%	73.71%	32.15%
2,2	21.68%	49.02%	34.91%
3,1	59.40%	62.19%	

Table 1:

This table shows the percent improvements produced through the use of different algorithms. The Fixed to Local comparisons show that Local is always statistically better than Fixed by 21.68% to 65.03%. The Fixed to CMOMMT comparisons show that CMOMMT is always better than Fixed by 49.02% to 73.71%. Local shows the greatest improvement over Fixed when there is only one target (59-65% versus 21-42%) because these configurations have multiple trackers following a single target. This is not true for two trackers, two targets because neither target is closely followed by a tracker, but both trackers stay between the targets. CMOMMT is statistically better than Local two-thirds of the time. This algorithm is not significantly better than Local in the three trackers, one target configuration because the 3:1 tracker to target ratio is so high already. This data proves that at some point, having many trackers is as good as having communication (CMOMMT). CMOMMT has the greatest percent improvement over Local in the two trackers, two targets configuration. This result is expected because CMOMMT increases the probability that one tracker will follow each target.

CMOMMT shows the least improvement over Fixed in the two trackers, two targets configuration because CMOMMT is not required at all times in this configuration because sometimes each tracker is with a different target already, and, therefore, CMOMMT is not needed. This requirement is in opposition to the two trackers, one target and three trackers, one target configurations that require communication between trackers at all times.

Table 2

Tracker, Target--Tracker, Target	Percent improvement between (no obstacles)		
	Fixed	Local	CMOMMT
1,1--2,1	34.91%	27.88%	51.07%
2,1--3,1		12.69%	
1,2--2,2	28.42%		36.05%

Table 2

This table shows the percent improvements between different robot team configurations using the same algorithms. This data shows that in Fixed, as expected, when the number of trackers is doubled, the change is statistically significant and about equal for the two cases. When the ratio is only increased by fifty percent, the improvement is not enough to be significant. The same reasoning holds within Local and CMOMMT. Also, for CMOMMT, from one tracker, one target to two trackers, one target sees the most improvement because this configuration change requires a change from never needing CMOMMT to always needing it. An important note is that in this data for one tracker, one target and one tracker, two targets the Local algorithm is equivalent to the CMOMMT algorithm. The reason that the data for one target, two trackers to two trackers, two targets in Local is not significant is because for enough of the time the trackers are between the targets in two trackers, two targets, and the targets are far enough apart that one tracker, two targets is approximately equal to two trackers, two targets.

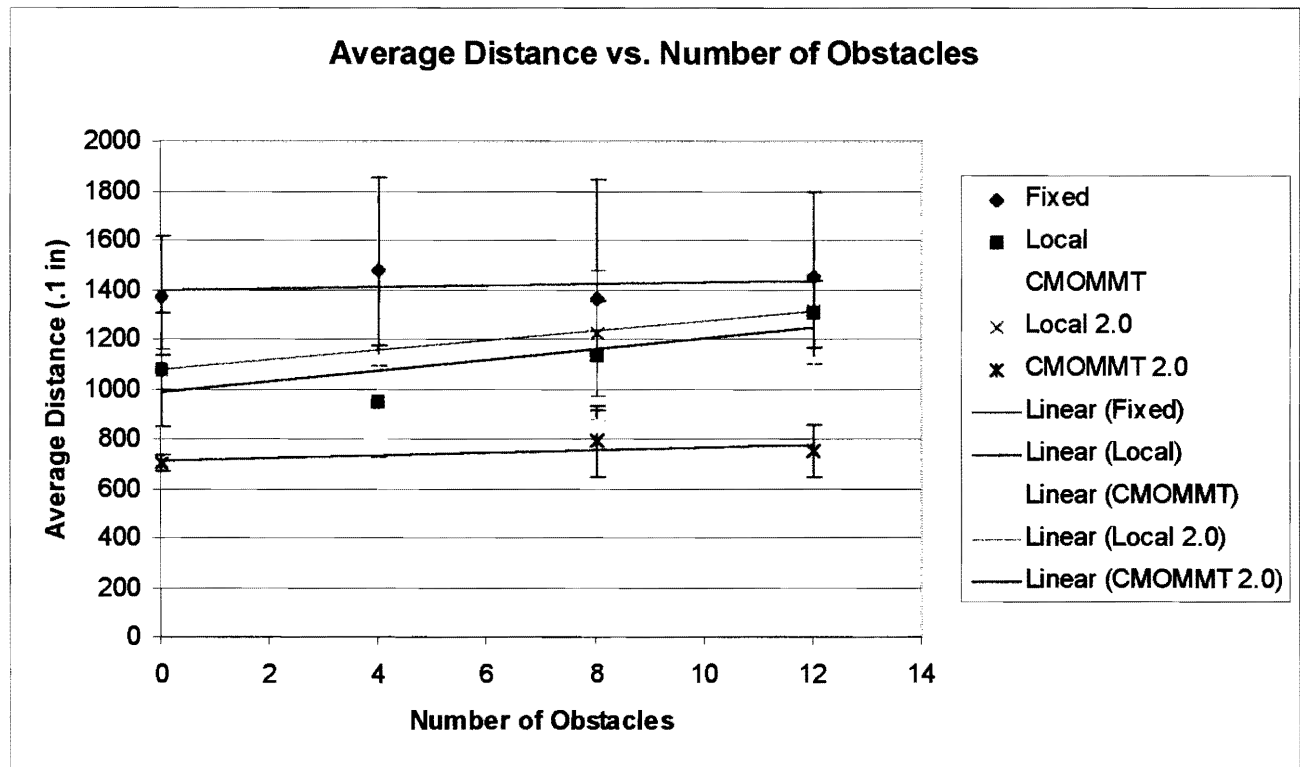
Figure 2

Figure 2: A graphical representation of the data presented in Tables 3-6.

The graph shows that as the number of obstacles increases the average distance from each target to its closest tracker increases linearly. The exceptions are the Fixed and CMOMMT 2.0 algorithms that stay approximately constant regardless of the number of obstacles present.

Table 3

Percent improvement between: (2 trackers, 2 targets-With obstacles)			
Number of Obstacles	Fixed--Local	Fixed--CMOMMT	Local--CMOMMT
0	21.68%	49.02%	34.91%
4	35.61%	41.89%	
8		37.82%	25.31%
12		34.46%	26.67%

Table 3

This table shows the improvement produced by changing the algorithm with a set number of obstacles. This data shows that Local is most improved over Fixed when zero or four obstacles are present. This result is due to the few obstacles hindering the tracker less than many obstacles. Having eight or twelve obstacles prevents enough tracker following to make the improvement over Fixed not be statistically significant. Local could hurt tracking by pulling both trackers between the targets. Therefore, the trackers would never be close to a target as could happen in Fixed. CMOMMT is always better than Fixed because the above problem is avoided. With CMOMMT, one tracker is often with each target as opposed to Fixed in which tracker position is not influenced by target position. The improvement decreases with increasing obstacles because the trackers will become trapped behind the obstacles thus degrading tracking ability and resulting performance. Fixed to CMOMMT always shows the greatest percent improvement because the CMOMMT algorithm increases the amount of time that one tracker follows each target. The percent improvement is greater than that of Local to CMOMMT because within Local this may be happening at times already.

Table 4

Percent improvement between (2 trackers, 2 targets--With obstacles)					
# of obstacles--# of obstacles	Fixed	Local	CMOMMT	Local 2.0	CMOMMT 2.0
0--4					
0--8			-20.94%		
0--12		-21.10%	-35.78%		
4--8					
4--12		-37.32%			
8--12					

Table 4

Table four shows the percent degradations resulting from changing the number of obstacles in the bounded area while using the same robot configuration. No statistical significance exists in changing the number of obstacles in Fixed. This result is expected because the targets are able to circumnavigate the obstacles, and thus, obstacles have no true impact on performance. Since there is no statistical significance between Local for zero to four obstacles, it would be expected that since zero to twelve obstacles produces significant data that four to twelve obstacles would also be significant, and it is. These are the only changes that cause a significant decrease in performance. Since zero to eight obstacles is not significant as well, it would be expected that eight to twelve obstacles would produce significant results in Local. This result is almost realized as can be seen by the 94.7% chance (Appendix — t-test with obstacles between 8—12 in Local) that it is significant which falls just below our accepted limit of 98% probability. In CMOMMT, eight obstacles must be added to the area before performance degradation is noticed. Then, adding more obstacles produces additional degradation. In the change from zero to twelve obstacles, CMOMMT performance degrades more than Local because in CMOMMT each tracker often follows one target and if one tracker becomes trapped behind an obstacle, performance is dramatically affected. In contrast, Local performance is not

affected as much because the trackers stay between the targets and compensate for each other when one becomes trapped and provide similar data.

Table 5

Percent improvement between (2 trackers, 2 targets--With obstacles)			
Number of Obstacles	Fixed--Local 2.0	Fixed--CMOMMT 2.0	Local 2.0--CMOMMT 2.0
4			
8		42.07%	35.60%
12		48.33%	43.12%

Table 5

This table shows equivalent data to Table 3 except that Table 5 makes comparisons to Local 2.0 and CMOMMT 2.0. Like Fixed to Local, changing from Fixed to Local 2.0 did not produce statistically significant results. In this case, the lack of significance is due to Local 2.0 keeping the two trackers together at the center of mass of the two targets as intended. Thus, the trackers are often far away from the targets as in Fixed. As with Fixed to CMOMMT, the difference between the average distance resulting from Fixed to CMOMMT 2.0 is significant. CMOMMT 2.0 offers the best mechanism to allow each tracker to follow one target. The amount of improvement is approximately the same with increasing obstacles because the average distance for Fixed stays approximately constant regardless of the number of obstacles as does CMOMMT 2.0 as can be seen by the differences being statistically insignificant (Appendix). The same reasoning holds for the change between Local 2.0 and CMOMMT 2.0. A parallel conclusion can be drawn for the differences between Fixed and CMOMMT 2.0 and those of Fixed to CMOMMT and Local to CMOMMT for eight and twelve obstacles. In both instances, the differences were greater for Fixed to the CMOMMT version than between the Local version and the equivalent CMOMMT version. This result is due to Local keeping one tracker with each

target for some of the time as CMOMMT would as opposed to Fixed which rarely brought a target near a tracker.

Table 6

Percent improvement (2 trackers, 2 targets--With obstacles)		
Number of Obstacles	Local--Local 2.0	CMOMMT--CMOMMT 2.0
4		
8		
12		21.15%

Table 6

This table shows comparisons of the original Local and CMOMMT versions to the 2.0 versions that include obstacle circumnavigation. The only time that the CMOMMT 2.0 was significantly better than CMOMMT was with twelve obstacles. With eight obstacles, the improvement was not large enough to be statistically significant. Although the difference between eight and twelve obstacles for both CMOMMT and CMOMMT 2.0 are not significant, enough of a difference exists to make the difference between the two for twelve obstacles be significant. CMOMMT 2.0 is significantly better than CMOMMT when many obstacles are present because of the obstacle circumnavigation modification in CMOMMT 2.0 that allows each tracker to stay with one target, whereas, in CMOMMT, the target and tracker would often be separated when the tracker became trapped behind an obstacle. The performance changes caused by Local 2.0 were not statistically significant when compared to the performance of Local. By allowing obstacle circumnavigation, the trackers were better able to achieve their goal of remaining together at the center of mass of the targets. Therefore, the average distances were much more consistent as can be seen by the lower standard deviation (see Appendix). In Local,

one tracker would sometimes become trapped near a target thus lowering the average distance.

As a result, Local 2.0 did not lower the average distance.

4. Conclusion

From these observations it can be concluded that when CMOMMT is used as intended with multiple trackers and multiple targets which was observed in the case of two trackers, two targets, this algorithm is an improvement over the Local and Fixed algorithms. The only exception to this result was seen when the number of trackers was three times that of the number of targets proving that a high tracker to target ratio produces equivalent results to communication among the trackers. These results are in agreement with the results of the earlier computer simulations. However, CMOMMT performance, like Local performance, is noticeably decreased when obstacles are added. Local 2.0 and CMOMMT 2.0 were developed in an attempt to improve the performance in the presence of obstacles, but Local 2.0 yielded greater average distances and thus a worse performance than Local did. However, CMOMMT performance did improve from the use of obstacle circumnavigation when twelve obstacles were in the area. Therefore, CMOMMT 2.0 would be the best algorithm to use in most applications with multiple moving trackers and targets and many obstacles.

Bibliography

1. L. Parker. Distributed Algorithms for Multi-Robot Observation of Multiple Moving Targets.
Autonomous Robots, to appear, 1999.

Appendix

No obstacles

a1

Fixed--1 tracker, 1 target

Run	Distance
1	2352.53
2	1752.29
3	1997.88
4	2467.61
5	2668.94
6	3053.59
7	1866.38
8	2050.22
9	2639.04
10	2325.31

avg dist: 2317.379
Variance 165457.1819
Std dev 406.764283

a2

Local--1 tracker, 1 target

Run	Distance
1	789.398
2	803.232
3	761.375
4	803.778
5	726.181
6	768.711
7	814.225
8	795.423
9	770.646
10	1071.54

avg dist: 810.4509
Variance 9087.1082
Std dev 95.326325

b1

Fixed--1 tracker, 2 targets

Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1	2377.73	1841.9	2109.815
2	2549.73	1959.82	2254.775
3	1235.25	2117.69	1676.47
4	2030.92	2527.96	2279.44
5	1567.48	1773.13	1670.305
6	1887.61	1866.81	1877.21
7	1998.44	2092.2	2045.32
8	1772.74	1434.57	1603.655
9	2234.77	1584.82	1909.795
10	1775.42	1757.4	1766.41

avg dist: 1919.3195
Variance 60028.3608
Std dev 245.0068587

b2

Local--1 tracker, 2 targets

Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1	1343.22	1181.7	1262.46
2	854.194	874.717	864.4555
3	1140.83	779.283	960.0565
4	1324.87	1219.73	1272.3
5	1040.77	1070.94	1055.855
6	1413.42	1076.65	1245.035
7	1363.14	950.442	1156.791
8	1050.45	1118.42	1084.435
9	1156.74	896.249	1026.4945
10	1047.94	1001.2	1024.57

avg dist: 1095.24525
Variance 18718.2247
Std dev 136.814563

c1 Fixed--1 tracker, 3 targets

Run	Target #1 Distance	Target #2 Distance	Target #3 Distance	Avg Target Avg Distance
1	1537.25	1696.96	1755.77	1663.3267
2	1620.49	1406.08	1659.65	1562.0733
3	1771.31	2027.07	1805.43	1867.9367
4	1891.92	2265.44	2351.41	2169.59
5	1829.55	1706.73	1820.03	1785.4367
6	2574.48	2506.36	3266.78	2782.54
7	2042.13	2177.55	2208.18	2142.62
8	1314.72	2063.08	1772.69	1716.83
9	2648.49	1839.59	2383.65	2290.5767
10	2722.62	2566.34	3008.86	2765.94

avg dist: 2074.687
Variance 191731.06
Std dev 437.87106

c2 Local--1 tracker, 3 targets

Run	Target #1 Distance	Target #2 Distance	Target #3 Distance	Avg Target Avg Distance
1	2072.99	1361.86	1200.93	1545.26
2	1051.18	932.424	1582.2	1188.601333
3	804.626	949.685	1039.95	931.4203333
4	1567.43	1402.63	1190.51	1386.856667
5	1856.96	995.002	1347.88	1399.947333
6	1181.34	1219.31	1365.02	1255.223333
7	1019.11	1141.09	1283.44	1147.88
8	1535.37	1137.31	1088.1	1253.593333
9	1151.08	1038.36	1010.82	1066.753333
10	1020.41	1215.64	1361.88	1199.31

avg dist: 1237.484567
Variance 30967.37375
Std dev 175.9754919

d1

Fixed--2 trackers, 1 target

Run	Distance
1	1133.88
2	1373.48
3	2033.24
4	1806.23
5	1246.94
6	1149.81
7	2119.02
8	1004.34
9	1694.42
10	1523.06

avg dist: 1508.442
Variance 153480.8997
Std dev 391.7663841

d2

Local--2 trackers, 1 target

Run	Distance
1	591.871
2	526.047
3	586.236
4	578.829
5	536.862
6	638.653
7	633.704
8	599.231
9	564.109
10	589.343

avg dist: 584.489
Variance 1309.5
Std dev 36.187

d3

CMOMMT--2 trackers, 1 target

Run	Distance
1	350.674
2	434.573
3	395.858
4	453.559
5	300.051
6	488.604
7	282.911
8	431.891
9	344.097
10	483.273

avg dist: 396.5491
Variance 5446.87348
Std dev 73.8029368

e1 Fixed--2 trackers, 2 targets			
Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1	1311.38	1056.93	1184.155
2	959.236	1372.11	1165.673
3	2067.41	1701.64	1884.525
4	1460.34	1617.44	1538.89
5	1542	1600.28	1571.14
6	1157.03	1332.14	1244.585
7	1205.36	964.34	1084.85
8	1375.5	1185.54	1280.52
9	1198.63	1630.5	1414.565
10	1314.91	1425.8	1370.355
avg dist:			1373.9258
Variance			57431.46173
Std dev			239.6486214

e2 Local--2 trackers, 2 targets			
Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1	1319.01	1305.62	1312.315
2	1127.77	1209.26	1168.515
3	965.337	1524.34	1244.8385
4	821.868	859.221	840.5445
5	1162.86	1431.08	1296.97
6	765.18	742.3	753.74
7	939.444	1266.49	1102.967
8	1038.03	1337.51	1187.77
9	953.156	1388.53	1170.843
10	703.406	661.591	682.4985
avg dist:			1076.10015
Variance			53120.7846
Std dev			230.479467

e3 CMOMMT--2 trackers, 2 targets			
Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1	683.532	666.997	675.2645
2	643.11	676.479	659.7945
3	781.886	680.437	731.1615
4	708.781	620.775	664.778
5	749.145	701.594	725.3695
6	838.787	649.133	743.96
7	619.005	736.972	677.9885
8	684.284	680.173	682.2285
9	786.851	695.609	741.23
10	759.927	644.404	702.1655
avg dist:			700.39405
Variance			1057.714508
Std dev			32.52252308

f1 Fixed--3 trackers, 1 target	
Run	
1	1175.88
2	991.833
3	1372.91
4	1229.12
5	805.398
6	1214.4
7	1055.7
8	1262.08
9	1544.61
10	1595.17
avg dist:	1224.7101
Variance	58229.45802
Std dev	241.3078076

f2 Local--3 trackers, 1 target	
Run	
1	520.206
2	534.956
3	557.178
4	444.52
5	435.481
6	418.887
7	510.319
8	498.316
9	532.051
10	520.761
avg dist:	497.2675
Variance	2245.8352
Std dev	47.390243

f3 CMOMMT--3 trackers, 1 target	
Run	
1	372.99
2	530.891
3	426.041
4	560.33
5	540.532
6	397.552
7	491.391
8	397.441
9	421.434
10	492.251
avg dist:	463.0853
Variance	4614.672
Std dev	67.931377

4 Obstacles

g1 Fixed--2 trackers, 2 targets				g2 Local--2 trackers, 2 targets				g3 CMOMMT--2 trackers, 2 targets			
Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance	Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance	Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1	1917.18	1296.85	1607.015	1	1161.66	1045.71	1103.685	1	725.03	777.524	751.278
2	1639.04	901.318	1270.179	2	1060	1032.75	1046.375	2	1329.5	1877.42	1603.46
3	1165.09	1152.53	1158.81	3	894.881	711.423	803.152	3	896.9	771.645	834.271
4	2497.4	2036.99	2267.195	4	873.562	756.822	815.192	4	682.19	682.874	682.5305
5	1402.79	1067.96	1235.375	5	958.159	981.74	969.9495	5	793.3	893.451	843.375
6	2815.8	1182.6	1999.2	6	987.745	772.858	880.3015	6	708.2	732.414	720.3075
7	1174	1399.19	1286.595	7	1043.55	1114.31	1078.93	7	795.31	1062.87	929.088
8	1370.45	1118.86	1244.655	8	1109.08	1462.43	1285.755	8	640.45	611.193	625.8235
9	1387.42	946.803	1167.1115	9	1036.49	1058.77	1047.63	9	787.32	732.629	759.9745
10	1673.48	1326.99	1500.235	10	446.142	470.643	458.3925	10	903.35	722.384	812.865
			avg dist: 1473.63705				avg dist: 948.93625				avg dist: 856.2973
			Variance 144789.2486				Variance 50939.2462				Variance 76466.95
			Std dev 380.5118245				Std dev 225.697244				Std dev 276.5266

g4 Local 2.0--2 trackers, 2 targets				g5 CMOMMT 2.0--2 trackers, 2 targets			
Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance	Run	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1			#DIV/0!	1			#DIV/0!
2			#DIV/0!	2			#DIV/0!
3			#DIV/0!	3			#DIV/0!
4			#DIV/0!	4			#DIV/0!
5			#DIV/0!	5			#DIV/0!
6			#DIV/0!	6			#DIV/0!
7			#DIV/0!	7			#DIV/0!
8			#DIV/0!	8			#DIV/0!
9			#DIV/0!	9			#DIV/0!
10			#DIV/0!	10			#DIV/0!
			avg dist: #DIV/0!				avg dist: #DIV/0!
			Variance #DIV/0!				Variance #DIV/0!
			Std dev #DIV/0!				Std dev #DIV/0!

8 obstacles

h1 Fixed--2 trackers, 2 targets				h2 Local--2 trackers, 2 targets				h3 CMOMMT--2 trackers, 2 targets			
Run	Target #1	Target #2	Avg Target	Run	Target #1	Target #2	Avg Target	Run	Target #1	Target #2	Avg Target
	Distance	Distance	Avg Distance		Distance	Distance	Avg Distance		Distance	Distance	Avg Distance
1	1313.93	1638.63	1476.28	1	1041.08	1024.02	1032.55	1	793.44	1017.02	905.2315
2	1210.77	1072.17	1141.47	2	1739.15	1504.29	1621.72	2	1111.8	1035.47	1073.615
3	1432.99	904.892	1168.941	3	1189.22	1283.84	1236.53	3	717.55	789.504	753.5265
4	941.34	1172.46	1056.9	4	1095.76	1405.32	1250.54	4	937.1	812.361	874.7325
5	1072.5	1215.91	1144.205	5	910.909	931.492	921.2005	5	769.67	645.825	707.745
6	993.053	1375.77	1184.4115	6	958.117	773.676	865.8965	6	765.11	738.681	751.894
7	1022.56	1236.11	1129.335	7	1396.47	1121.43	1258.95	7	964.23	1039.66	1001.945
8	1010.38	1408.25	1209.315	8	1045.96	1023.3	1034.63	8	1022.7	752.888	887.809
9	1489.27	1351.48	1420.375	9	1207.93	911.333	1059.6315	9	732.1	786.556	759.3295
10	2327.06	3056.8	2691.93	10	1213.14	905.654	1059.397	10	769.51	739.335	754.421
avg dist: 1362.31625				avg dist: 1134.10455				avg dist: 847.0249			
Variance 235621.0757				Variance 47333.6732				Variance 14957.67			
Std dev 485.4081537				Std dev 217.563033				Std dev 122.3016			

h4 Local 2.0--2 trackers, 2 targets				h5 CMOMMT 2.0--2 trackers, 2 targets			
Run	Target #1	Target #2	Avg Target	Run	Target #1	Target #2	Avg Target
	Distance	Distance	Avg Distance		Distance	Distance	Avg Distance
1	1347.8	951.634	1149.717	1	591.503	763.888	677.6955
2	1220.58	1071.77	1146.175	2	1160.27	913.016	1036.643
3	1376.84	1020.37	1198.605	3	652.715	613.334	633.0245
4	1427.38	1201.37	1314.375	4	750.845	792.467	771.656
5	1348.35	1186	1267.175	5	682.863	696.475	689.669
6	1267.4	1015.71	1141.555	6	604.941	671.972	638.4565
7	1545.72	958.49	1252.105	7	1035.97	924.557	980.2635
8	1604.73	1091.01	1347.87	8	791.814	843.804	817.809
9	1338.49	930.895	1134.6925	9	804.935	680.17	742.5525
10	1096.45	1507.62	1302.035	10	818.205	989.665	903.935
avg dist: 1225.43045				avg dist: 789.17045			
Variance 6580.777428				Variance 20405.97			
Std dev 81.12199102				Std dev 142.849466			

Fixed—2 trackers, 2 targets			
	Target #1	Target #2	Avg Target
Run	Distance	Distance	Avg Distance
1	947.767	996.259	972.013
2	1509.73	1212.53	1361.13
3	1046.71	1169.6	1108.155
4	1099.19	1242.34	1170.765
5	1538.44	1404.01	1471.225
6	984.911	1490.49	1237.7005
7	1406.77	1735.63	1571.2
8	1575.72	2154.72	1865.22
9	1205.67	2844.51	2025.09
10	1037.39	2419.68	1728.535
		avg dist:	1451.10335
		Variance	119124.0428
		Std dev	345.1435105

Run	Local 2.0-2 trackers, 2 targets		
	Target #1	Target #2	Avg Target
	Distance	Distance	Avg Distance
1	1644.97	1013.91	1329.44
2	1444.46	1465	1454.73
3	2324.83	768.146	1546.488
4	969.833	1495.36	1232.5965
5	1242.57	1354.44	1298.505
6	987.341	1214.37	1100.8555
7	1244.73	1148.46	1196.595
8	3211.56	543.917	1877.7385
9	1244.94	779.271	1012.1055
10	1312.06	957.035	1134.5475
	avg dist:		1318.36015
	Variance		64569.95662
	Std dev		254.1061916

Run	Local--2 trackers, 2 targets		
	Target #1 Distance	Target #2 Distance	Avg Target Avg Distance
1	1375.18	1122.99	1249.085
2	1529.09	1160.31	1344.7
3	1504.42	1014.87	1259.645
4	1539.85	1128.6	1334.225
5	1045.81	1363.82	1204.815
6	833.95	1765.62	1299.785
7	1465.49	1279.22	1372.355
8	1143.77	1207.05	1175.41
9	1956.17	1287.65	1621.91
10	1188.12	1150.24	1169.18
avg dist:			1303.111
Variance			17539.0651
Std dev			132.435135

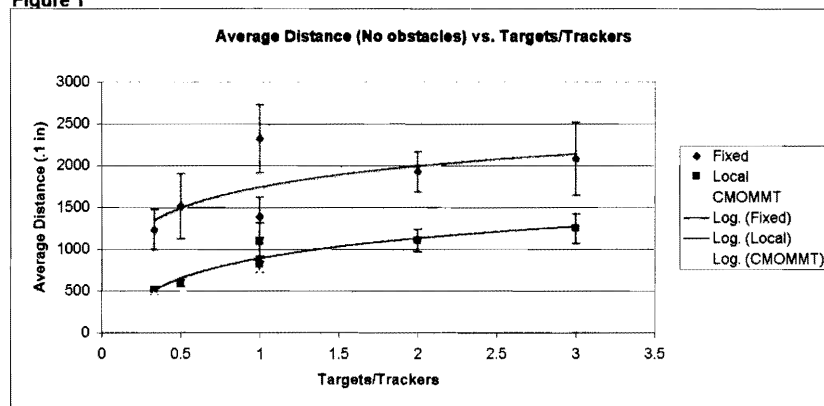
Run	CMOMMT 2.0--2 trackers, 2 targets		
	Target #1	Target #2	Avg Target
	Distance	Distance	Avg Distance
1	602.302	751.866	677.084
2	861.038	729.545	795.2915
3	872.926	989.291	931.1085
4	800.721	744.212	772.4665
5	674.688	791.381	733.0345
6	842.656	1002.11	922.383
7	650.635	607.809	629.222
8	749.797	731.829	740.813
9	557.535	722.595	640.065
10	682.581	631.058	656.8195
	avg dist:	749.82875	
	Variance	11790.7295	
	Std dev	108.585125	

CMOMMT--2 trackers, 2 targets			
	Target #1	Target #2	Avg Target
Run	Distance	Distance	Avg Distance
1	767.54	678.883	723.211
2	774.06	908.612	841.336
3	1084	937.351	1010.676
4	1030.3	910.507	970.3935
5	1367.2	1387.14	1377.15
6	1165.8	939.948	1052.849
7	756.89	990.586	873.739
8	950.48	797.472	873.9745
9	802.48	1056.19	929.335
10	962.58	751.988	857.282
	avg dist:	950.9946	
	Variance	31220.33	
	Std dev	176.6928	

Average distance (No obstacles)				
Trackers	Targets/trackers	Fixed	Local	CMOMMT
1,1	1	2317.379	810.4509	
1,2	2	1919.3195	1095.24525	
1,3	3	2074.687	1237.484567	
2,1	0.5	1508.442	584.4885	396.5491
2,2	1	1373.9258	1076.10015	700.39405
3,1	0.3333	1224.7101	497.2675	463.0853

Average distance (No obstacles)							
Trackers	Targets/trackers	F-Std Dev	Fixed	L-Std Dev	Local	C-Std Dev	CMOMMT
1,1	1	406.764283	2317.379	95.32632479	810.4509		
1,2	2	245.0088587	1919.3195	136.8145632	1095.24525		
1,3	3	437.8710595	2074.687	175.9754919	1237.484567		
2,1	0.5	391.7863841	1508.442	36.18696423	584.4885	73.80293678	396.5491
2,2	1	239.6486214	1373.9258	230.4794668	1076.10015	32.52252308	700.39405
3,1	0.333333333	241.3078076	1224.7101	47.39024321	497.2675	67.93137734	463.0853

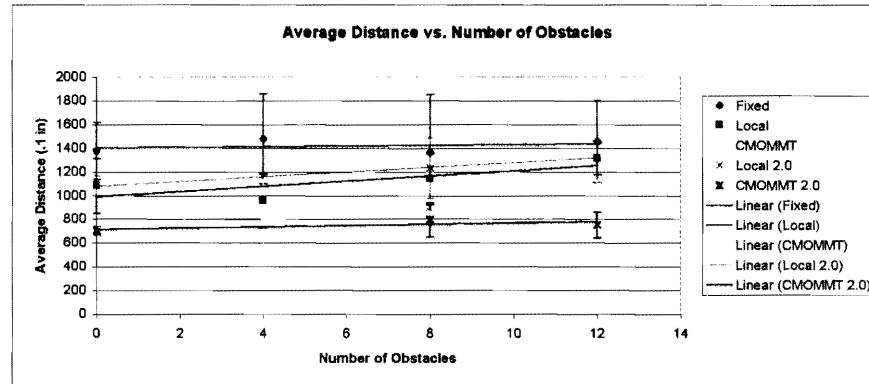
Figure 1



Average distance (.1 in)					
2 trackers, 2 targets--With Obstacles					
Algorithm					
Number of Obstacles	Fixed	Local	CMOMMT	Local 2.0	CMOMMT 2.0
0	1373.9258	1076.10015	700.39405		
4	1473.63705	948.93625	856.2973		
8	1362.31625	1134.10455	847.0249	1225.43045	789.17045
12	1451.10335	1303.111	950.99455	1318.36015	749.82875

Average distance (.1 in)										
2 trackers, 2 targets--With Obstacles										
Algorithm										
Number of Obstacles	F-Std Dev	Fixed	L-Std Dev	Local	C-Std Dev	CMOMMT	L2-Std Dev	Local 2.0	C2-Std Dev	CMOMMT 2.0
0	239.65	1373.93	230.48	1076.10015	32.52	700.39405	230.48	1076.10015	32.52	700.39405
4	380.51	1473.64	225.7	948.93625	276.53	856.2973				
8	485.41	1362.32	217.56	1134.10455	122.3	847.0249	81.12	1225.43045	142.85	789.17045
12	345.14	1451.1	132.44	1303.111	176.69	950.99455	254.11	1318.36015	108.59	749.82875

Figure 2



T-test between (no obstacles)			
Tracker, Target	Fixed--Local	Fixed--CMOMMT	Local--CMOMMT
1,1	4.763E-07		
1,2	2.168E-07		
1,3	1.204E-04		
2,1	3.643E-05	6.376E-06	6.388E-06
2,2	1.105E-02	8.022E-06	5.677E-04
3,1	3.664E-06	1.679E-06	2.102E-01

T-test between (no obstacles)			
Tracker, Target--Tracker, Target	Fixed	Local	CMOMMT
1,1--2,1	2.603E-04	1.745E-05	4.762E-09
2,1--3,1	7.016E-02	2.474E-04	5.043E-02
1,2--2,2	8.670E-05	7.606E-01	4.628E-06

Grey cells denote data that is not statistically significant.

T-test between (2 trackers, 2 targets--With obstacles)			
Number of Obstacles	Fixed-Local	Fixed-CMOMMT	Local-CMOMMT
0	1.105E-02	8.022E-06	5.677E-04
4	2.005E-03	7.149E-04	
8		8.493E-03	2.643E-03
12		1.226E-03	1.059E-04

T-test between (2 trackers, 2 targets--With obstacles)				
# of obstacles--# of obstacles	Fixed	Local	CMOMMT	Local 2.0
0-4				
0-8			4.174E-03	
0-12		1.694E-02	1.445E-03	
4-8				
4-12		7.029E-04		
8-12				

T-test between (2 trackers, 2 targets--With obstacles)			
Number of Obstacles	Fixed-Local 2.0	Fixed-CMOMMT 2.0	Local 2.0-CMOMMT 2.0
4			
8		4.281E-03	2.125E-05
12		8.145E-05	2.710E-05

T-test between (2 trackers, 2 targets--With obstacles)		
Number of Obstacles	Local-Local 2.0	CMOMMT-CMOMMT 2.0
4		
8		
12		7.847E-03

Grey cells denote data that is not statistically significant.