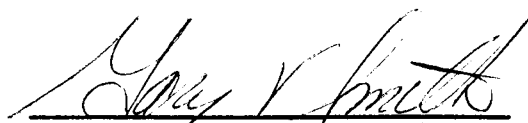
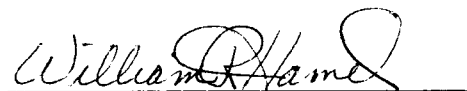


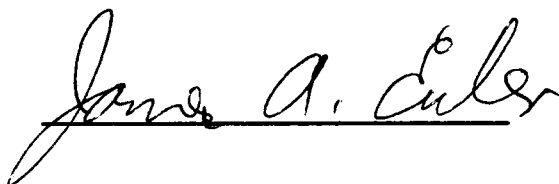
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DEVELOPMENT AND ANALYSIS OF A GAS CANTILEVER CORIOLIS
MASS FLOWMETER CONCEPT

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Andrew Gregory Denny

August 1987

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ever have been completed. Words alone are insufficient to fully express my love and appreciation for her, but if ever a woman could come close to being a perfect helpmate, I think she would find Janice had beaten her to it. My sincerest hope is that in the time to come, I can repay to her some of the debt of love which I owe her, and I will continually pray that God shower her with many rich blessings.

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ABSTRACT

A concept of a cantilever Coriolis mass flowmeter for gas flow measurement has been developed and tested. This work draws from original work performed by W. R. Hamel, who developed the cantilever Coriolis flowmeter concept. The concept has been developed as an in-line device utilizing a null-type configuration for the use with gas flows where the Mach number remains less than approximately 0.3. This constraint was placed on the flowmeter to ensure that compressibility effects were minimized, so that the approximate, single-mode, analytical solution to the governing equation, which Hamel derived, could be used, since the governing equation was derived assuming incompressible flow.

The governing equation is a fourth-order, non-self-adjoint, homogeneous, partial differential equation; and the approximate solution mentioned above relates the unforced cantilever vibration characteristics to the density and mean velocity of the flow. The results given by the solution have been utilized along with a framework laid out by Hamel to establish a design methodology for systems of flows whose compositions are constant. This synthesis procedure centers on maximizing the sensitivity of the tube to the dissipative Coriolis forces with respect to the various tube parameters.

The analytical solution to the governing equation was also used to develop measurement and calibration algorithms for use in proof-of-principle testing. A bench-size prototype flowmeter was constructed and tested using an air circulation loop with a flow rate range of 0 - 0.3 SCMM (Standard Cubic Meters per Minute), where the maximum air velocity reached approximately 100 m/s (300 ft/s). Three different sets of tubes

were tested, comprising two materials and sizes. A laminar flow element was used to provide a comparison measurement. The results were in general promising, demonstrating measurable and repeatable trends in the comparison of the mass flow measurements, although a fair amount of scatter was present in the results. Future work should include the determination of ways to eliminate the scatter and improve the precision and rangeability of such a flowmeter.

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LIST OF SYMBOLS

A_i	constant
A_f	flow area of the tube
a_{fx}	fluid acceleration in x-direction
a_{fy}	fluid acceleration in y-direction
A_t	material cross-sectional area of tube
a_{tx}	tube acceleration in x-direction
a_{ty}	tube acceleration in y-direction
a_1	calibration constant
a_2	calibration constant
a_3	calibration constant
a_3'	calibration constant
B_i	constant
c	damping coefficient per unit length
$\bar{c}$	nondimensionalized damping coefficient per unit length
C_i	constant
d_i	inner diameter of tube
D_i	constant
d_o	outer diameter of tube
E	modulus of elasticity for tube
$f(t)$	nonzero offset for damped sinusoid
F	Coriolis force
$g(t)$	upper envelope for damped sinusoid with $f(t) \neq 0$
G	shear modulus of elasticity for tube
$h(t)$	lower envelope for damped sinusoid with $f(t) \neq 0$
I	area moment of inertia for tube

j	$\sqrt{-1}$
K_x	orthogonality constant
K'_x	orthogonality constant
K''_x	orthogonality constant
L	tube length
Ma	Mach number
M	moment
$\dot{m}$	mass flow rate
m_f	mass per unit length of fluid
m_t	mass per unit length of tube
P	pressure
q	fluid shear
Q	transverse shear in tube
R_{c1}	measured value of decay constant for the reference tube
R_{c2}	measured value of frequency, squared, for the reference tube
R_d	diameter ratio
R_f	density ratio
R_ω	ratio of sampling frequency to empty tube natural frequency
R_1	measured value of decay constant for the active tube
R_2	measured value of frequency, squared, for the active tube
s	displacement along centerline of tube
S_m	sensitivity
$\hat{t}$	nondimensional time variable
T	tension force in tube element
$T_i(t)$	assumed time function
t_R	relaxation time

$T_{\frac{1}{2}}$	average half-period of waveform
u	fluid velocity
$\hat{u}$	nondimensional fluid velocity
u_m	maximum flow velocity
w_x	uncertainty in any quantity x
$\hat{x}$	nondimensional spatial variable
$X_i(x)$	eigenfunction of Bernoulli-Euler beam equation
$\tilde{y}$	approximate solution in Galerkin's method
$z(t)$	envelope for damped sinusoid

Greek Letters

α_i	nondimensional eigenvalue
β	mass ratio parameter
β_0	mass ratio parameter at zero flow
γ	nondimensional damping coefficient
δ	ratio of specific heats, c_p/c_v
ϵ	residual error from approximate solution
η_0	speed of sound at stagnation point
θ	general variable for angular displacements
κ_i	constant of virtue in Galerkin solution
λ	root to characteristic equation
μ	coefficient of viscosity
ν	eigenvalue for Bernoulli-Euler beam equation
ρ, ρ_f	fluid density
ρ_t	tube density
$\sigma_{\max}$	maximum allowable stress

τ	time constant
τ_u	velocity sensitivity
τ_p	density sensitivity
τ^{-1}	decay constant for tube
τ_e^{-1}	empty tube decay constant
τ_{diff}^{-1}	difference of τ^{-1} and $(1 - \beta) \tau_e^{-1}$
$\phi_j(x)$	function in Galerkin solution
Ψ	internal perimeter of tube
ω	angular velocity
$\bar{\omega}$	nondimensional angular velocity (or frequency)
ω_e	empty tube fundamental natural frequency
ω_s	sampling frequency

CHAPTER I

INTRODUCTION

Background

All Coriolis mass flowmeters are based on the principle of the Coriolis acceleration. This acceleration, which is the familiar $(2\vec{\omega} \times \vec{u})$ quantity from classical dynamics, arises whenever a body moving with some linear velocity $\vec{u}$ is exposed to an angular velocity field $\vec{\omega}$ and the vectors $\vec{u}$ and $\vec{\omega}$ are not parallel. An example which may serve to illustrate this concept is the merry-go-round, as found in numerous playgrounds. If a passenger on the merry-go-round attempted to walk from the center of the spinning wheel to the outside rim (or vice-versa), he would experience a force pushing him sideways. This force would represent the product of his mass and a Coriolis acceleration arising from the interaction of the man's velocity, which would be parallel to the ground, and the wheel's angular velocity, and would be directed perpendicular to the ground. A force equal in magnitude but opposite in direction would be applied to the wheel of the merry-go-round.

It is not a long jump from this point to see how this principle can be applied to mass flow measurement. One note should be made first: It is not necessary that the angular velocity field be purely rotational, as was that of the merry-go-round example. The Coriolis acceleration will also arise when the angular velocity field is the result of harmonic vibration.

Consider a fluid element dm moving with some velocity u through an element ds of a piping network, as shown in Figure 1. If the element ds were given some angular velocity vector ω perpendicular to u , the situation would be analagous to that of the man on the merry-go-round. An interactive force would develop between the fluid element and tube element due to a Coriolis acceleration component. This force would be equal to $2\omega u \rho A ds$ and the total force over the length L of piping undergoing some angular velocity would be

$$F = \int_L 2\omega u \rho A ds.$$

A principle idea in any Coriolis flowmeter is that if this fluid bearing element is "properly" designed, this force will result in a measurable effect which is directly related to the product of the velocity and density of the fluid in the tube provided the other quantities in the integral are known.

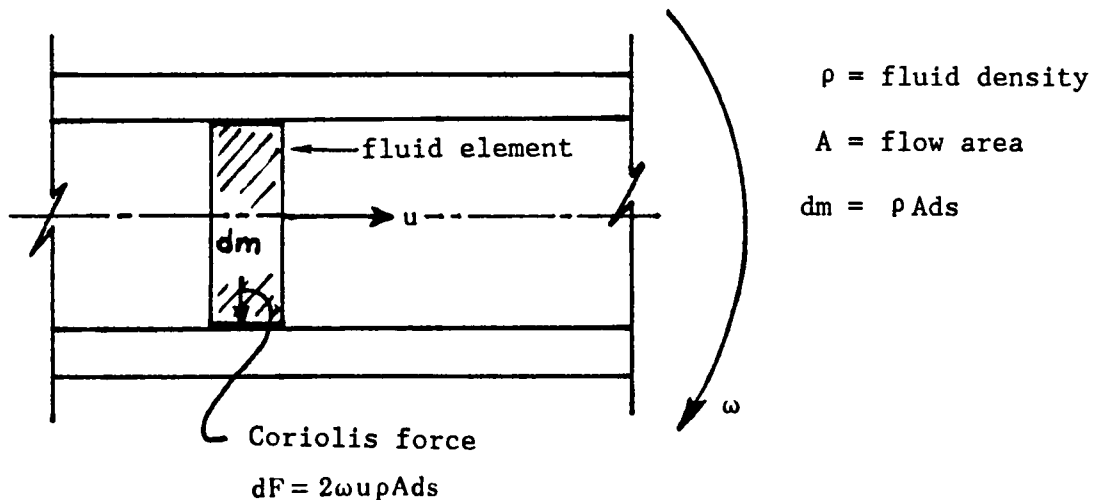


Figure 1. Illustration of Coriolis Effect.

The concept of using the Coriolis principle to measure mass flow rate first appeared in the literature in the early 1950's. The earliest studies concentrated on rotational configurations, that is, using pure rotation to generate the necessary angular velocity vector. Li [1]* developed a simple theory for the "T- shaped" geometry shown in Figure 2. The fluid flows in the legs of the "T", where the angular velocity and fluid velocity vectors are perpendicular. The Coriolis accelerations which are produced are mutually perpendicular to both the

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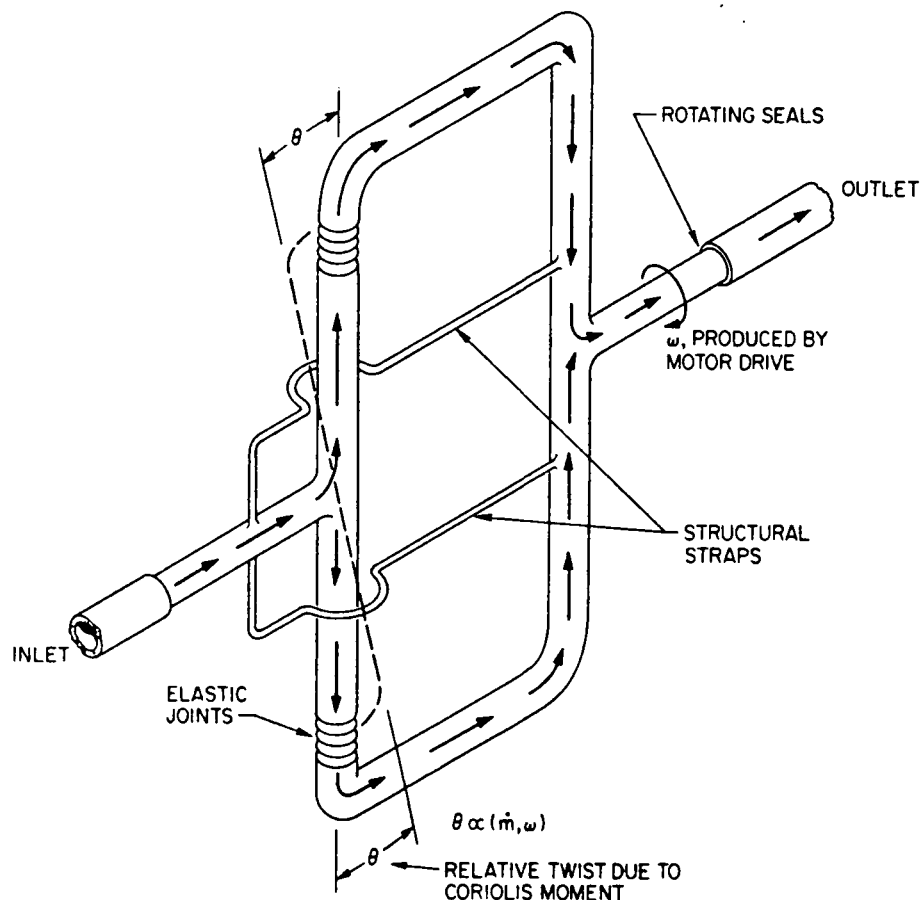


Figure 2. Rotating "T-Shape" Configuration Developed by Li [1].

* Numbers in [] refer to references in the Bibliography.

angular velocity vector and fluid velocity vector, and produce forces which are equal in magnitude but opposite in direction in the different halves of the "T". In other words, a twisting moment is generated relative to the axis of rotation which is proportional to the mass flow rate. This moment is then measured with torsion-sensing strain gages. This design suffers from several problems, among which are the necessity for rotating seals and slip-ring commutation of low-level strain gage signals.

Domestic literature of the period 1950-1960 reveals no other dynamic approaches to flow measurement with the exception of a flowmeter based on angular momentum exchange, as noted by Hamel [2]. (As a side note, angular momentum flowmeters have been demonstrated to perform quite adequately for the measurement of mass flow rate of gases at high flow rates, as noted by Reinecke [3].) However, the Russians were quite active in the development of Coriolis flow measurement techniques during this time. Much of their work consisted of extending Li's earlier work and developing variations of his basic "T-shape" to reduce sealing and torque measurement problems. One variation, noted by Hamel [2], consisted of using harmonic vibration with the "T-shape", which eliminated the need for rotating seals and commutators for the strain gage signals.

In the mid-1960's, several patents ([4], [5], [6], and [7]) were issued in this country for flowmeters based on the Coriolis principle. Although the theory and mathematical modeling behind these attempts are fairly simple, these patents provide an interesting perspective to the history of Coriolis flow technology.

Due to the complexity of the Coriolis flow measurement problem, both in terms of hardware and theory, it was passed over in favor of simpler flow measurement techniques. However, with the tremendous advances in electronics that have been made in the last thirty years, interest in Coriolis flow measurement devices has been renewed. At present there are at least two companies marketing mass flowmeters based on the Coriolis effect. One of these companies, Micromotion, Inc., began marketing a Coriolis flowmeter in 1976 which utilized a U-tube sensing element, similar to that in Figure 3, which undergoes harmonic oscillation. As can be seen, the fluid enters one leg of the sense element and exits the other, while the U-tube vibrates harmonically. This gives rise to a force couple (due to the Coriolis accelerations which are produced) which tends to twist the tube about an axis of

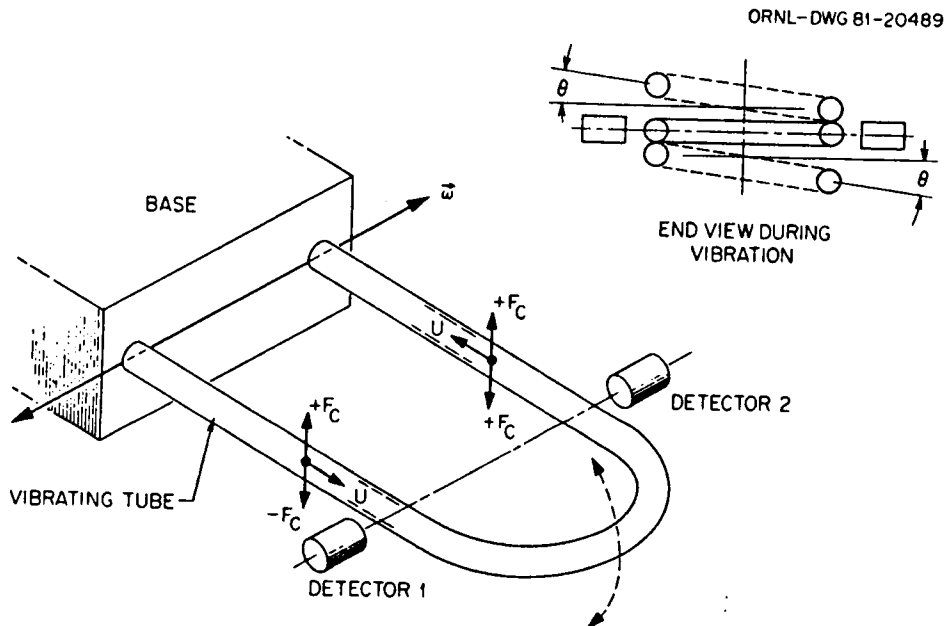


Figure 3. U-Tube Sense Element.

symmetry. The torsional deflection caused by this twisting moment is then measured and related to the mass flow rate. More recently, the EXAC Corporation has marketed a Coriolis flowmeter utilizing a similar principle with a double helical sensing element. Both of these flowmeters, while representing innovations in the area of commercial mass flowmeters, seem to be the result of the trial and error design method rather than any detailed analysis of the phenomenon.

In 1981, W. R. Hamel [2] performed a detailed theoretical analysis of the Coriolis principle as applied to a sense element in the form of a cantilever tube and developed an analytical modeling methodology for designing a cantilever Coriolis mass flowmeter. He applied the results of his analytical modeling in proof-of-principle testing using water as the working fluid. The results were very positive, yielding measurements of mass flow rates within $\pm 10\%$ of reading compared to a industrial grade thermal flowmeter and density measurements within $\pm 1\%$ to $\pm 4\%$ of reading for the known loop water density, all for very simple mechanical flow sensor arrangements. One other interesting note about his work is that he used transient vibration of the cantilever tube as the angular velocity field. In this setup, the tubes' vibration characteristics of frequency and decay rate are affected by the Coriolis force created from the interaction of the beam and the fluid. The amount that these characteristics are affected is then related to the velocity and density of the flowing fluid, from which the mass flow rate can be determined.

No information has been found concerning the use of a Coriolis flowmeter to measure the mass flow rates of gases, although in a recent

edition of Intech the statement was made that "application to gases is more limited in that the density of low pressure gases is often insufficient to accurately operate the units" [6]. On that note this study begins.

Scope

This study has two objectives. These are:

1. Building on Hamel's work on the cantilever Coriolis mass flowmeter, design and construct a prototype flowmeter to measure the mass flow rates of gas flows, as an in-line device, in the range of incompressible flow, typically $Ma < 0.3$. For Mach numbers in this range, compressibility effects can usually be ignored. This follows from an assumption of isentropic flow, which should be a fair approximation for the flow in this study. Since for isentropic flow

$$\left(\frac{\rho_0}{\rho}\right)^{\frac{\delta-1}{\delta}} = 1 + \frac{1}{2}(\delta-1)Ma^2,$$

it follows that for low Mach numbers, $\rho_0 \approx \rho$.

2. Assess the performance, practicality, and potential of such a gas mass flowmeter.

Very little will be added to Hamel's original work in the way of theory, since this is by and large an experimental study. For this study, air will be the working fluid due to ease of access in the laboratory. The incompressible constraint restricts the maximum flow rate to less than 1.4 SCMM (50 SCFM) for bench-size prototypes. This follows from the fact that the flow rate Q will be limited by both the maximum velocity and the size of the tube practical for a bench size

prototype. This is very low in terms of the flows which would be encountered in industrial processes, where flows may range over 1000 SCFM, but this range should be satisfactory for the purposes of proof of principle testing in this study.

Method of Attack

This study has been divided into three distinct phases. The first of these consists of the design and analysis of a prototype sense element according to the methodology presented by Hamel [2]. In this phase, much of the work involved finding materials suitable for the sensing tube. Also to be considered is the requirement of constructing an in-line device based on this cantilever sensing element.

The second phase of this study is the actual construction of the prototype, as well as the development of the necessary electronics and computer-based measurement system.

The third phase of this work is the testing and evaluation of the prototype. The prototype will be tested in a flow loop with air as the working fluid and the results will be compared with those of a laminar flowmeter.

CHAPTER II

THEORETICAL WORK

Development of Governing Equation

The development of the governing equation for the hydroelastic cantilever beam follows that given by Hamel [2], which he referenced from Gregory and Paidoussis [8]. Figure 4 shows the x-axis is coincident with the centerline of the undeflected tube. All vibrations are assumed to occur in the plane formed by the x- and y-axes. The fluid enters the tube through the fixed end with a steady mean stream velocity u . The beam itself is a tube with structural rigidity EI and possessing a mass per unit length of m_t . The fluid is assumed to be incompressible with a mass density of ρ and a mass per unit tube length of m_f . External damping is represented by constant damping coefficient per unit length of tube, c . It is assumed that c has the proper units for whatever damping model is used.

An element of the beam structure is composed of two parts: the fluid element and the tube element. Freebody diagrams of both elements are shown in Figure 5, where A_f is the flow area of the tube and Ψ is

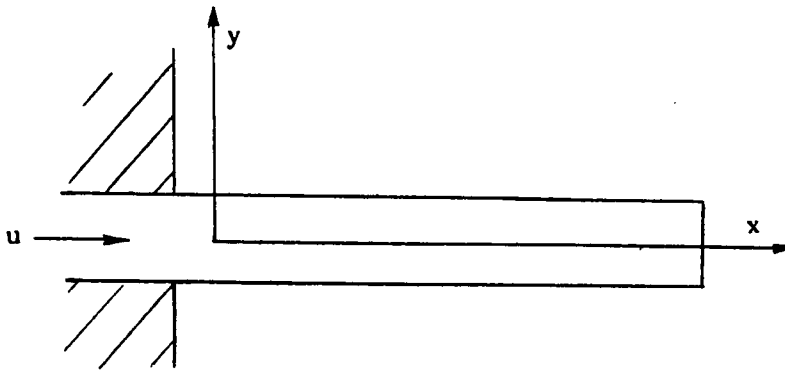


Figure 4. Cantilever Tube Geometry

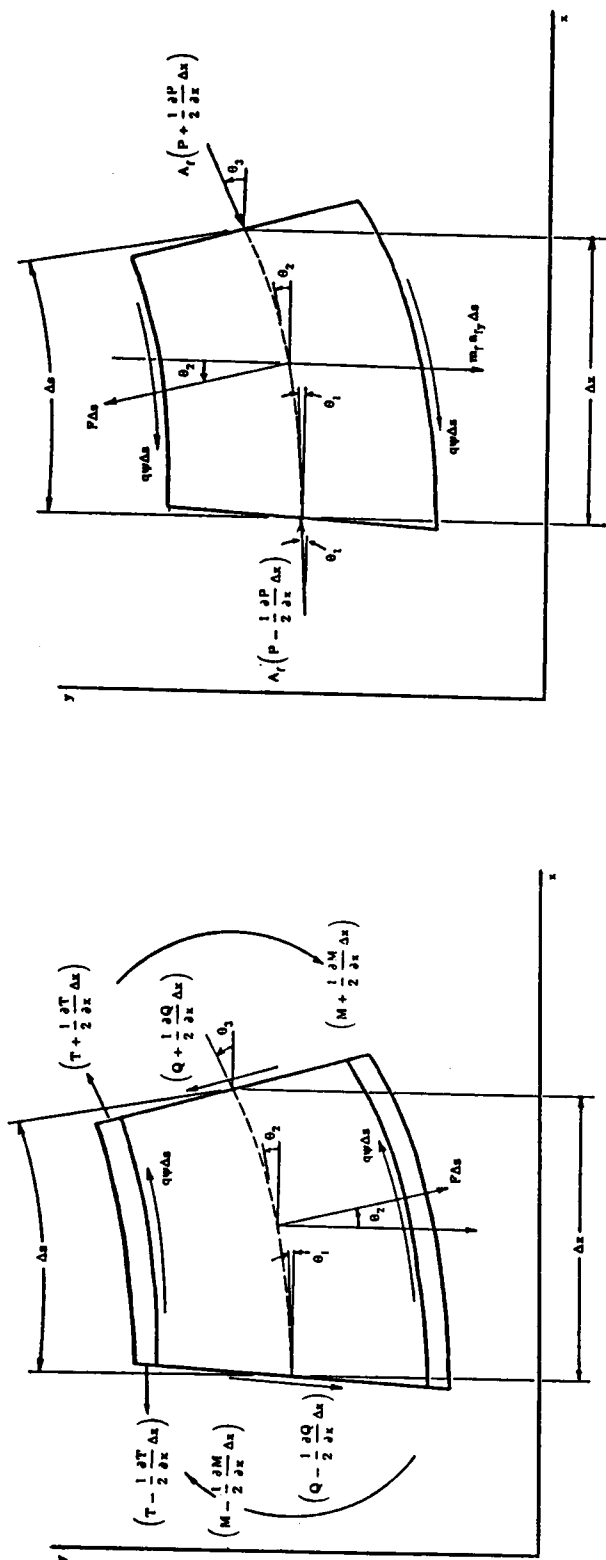


Figure 5. Freebody Diagrams.

(Note: All forces are centered about midpoints of elements.)

the internal perimeter of the tube. The x-axis coincides with the centerline of the tube in the static deflection position. The s-coordinate refers to the displacement along the arc formed by the neutral axis of the tube in the deflected position; hence, the element's length is Δs , not Δx . However, as will be shown, $\Delta s = \Delta x$ for the small angles involved. According to Niordson [9], if the cross sectional dimensions of the tube are small compared with the wavelength of any disturbances to its centerline during vibration, which should be true in this study, the accelerations of fluid particles along the x- and y-axes are

$$(1) \quad a_{fx} = 0$$

and

$$(2) \quad a_{fy} = \left[\left(\frac{\partial}{\partial t} \right) + u \left(\frac{\partial}{\partial x} \right) \right]^2 y$$

to the first order in small deflections, y. Beginning with the fluid element, the equations of motion are

$$(3) \quad \sum^+ F_{fx} = 0$$

and

$$(4) \quad +1 \sum F_{fy} = m_f \Delta s a_{fy}.$$

Summing forces in the x-direction yields

$$(5) \quad \sum^+ F_{fx} = A_f P \Big|_{s - \frac{\Delta s}{2}} \cos \theta_1 - A_f P \Big|_{s + \frac{\Delta s}{2}} \cos \theta_3 - F \Big|_s \Delta s \sin \theta_2 - q \Big|_s \Psi \Delta s \cos \theta_2 = 0$$

where P represents the pressure acting on the fluid element, q is the fluid shear per unit length, and F represents the force of interaction

between the fluid element and the tube element per unit length. Since the fluid is assumed to be incompressible, it is further assumed that the interaction force is similar to that of one body sliding over another and acts perpendicular to the contact surface. This model for the interaction force was used by Hamel [2], with good results. Others, such as Gregory and Paidoussis [8], have also used this assumption with equally good results.

Summing forces in the y-direction gives

$$(6) \quad +1 \sum F_{fy} = \Lambda_f P \Big|_{s + \frac{\Delta s}{2}}^{\sin \theta_1} - \Lambda_f P \Big|_{s + \frac{\Delta s}{2}}^{\sin \theta_3} + F \Big|_s^{\Delta s \cos \theta_2 - q} \Big|_s^{\psi \Delta s \sin \theta_2} \\ = m_f \Delta s a_{fy}.$$

To simplify equations (5) and (6), make note of the following:

$$(7) \quad \cos \theta_2 = \frac{\Delta x/2}{\Delta s/2} \Big|_s = \frac{\Delta x}{\Delta s} \Big|_s.$$

For small angles, $\cos \theta_1 \approx \cos \theta_2 \approx \cos \theta_3 \approx 1$, which, when coupled with (7), gives

$$(8) \quad 1 = \frac{\Delta x}{\Delta s} \Big|_s$$

or

$$(9) \quad \Delta x = \Delta s$$

which states that $\Delta x = \Delta s$ for small deflections and implies that Δx goes to zero as fast as Δs goes to zero, since Δx and Δs are arbitrary. It can be shown that using $\cos \theta_1$ or $\cos \theta_3$ will produce the same result. It is also necessary to find an expression for the various sine terms in

equation (6). Remembering the trigonometric relationship

$$(10) \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

which, for small angles θ , reduces to

$$(11) \quad \tan \theta = \sin \theta$$

results in

$$(12) \quad \begin{aligned} \sin \theta_1 &= \tan \theta_1 \\ \sin \theta_2 &= \tan \theta_2 \\ \sin \theta_3 &= \tan \theta_3 \end{aligned}$$

Since the tangent of the angle is equal to the slope of the curve at that point, where the slope is given by $\frac{\partial y}{\partial x}$, equation (12) becomes

$$(13) \quad \begin{aligned} \sin \theta_1 &= \left. \frac{\partial y}{\partial x} \right|_{x - \frac{\Delta x}{2}} \\ \sin \theta_2 &= \left. \frac{\partial y}{\partial x} \right|_x \\ \sin \theta_3 &= \left. \frac{\partial y}{\partial x} \right|_{x + \frac{\Delta x}{2}} \end{aligned}$$

By applying Taylor's Formula to (13), the desired relations may be obtained as

$$(14) \quad \begin{aligned} \sin \theta_1 &= \left. \frac{\partial y}{\partial x} \right|_x - \frac{1}{2} \left. \frac{\partial^2 y}{\partial x^2} \right|_x \Delta x \\ \sin \theta_2 &= \left. \frac{\partial y}{\partial x} \right|_x \\ \sin \theta_3 &= \left. \frac{\partial y}{\partial x} \right|_x + \frac{1}{2} \left. \frac{\partial^2 y}{\partial x^2} \right|_x \Delta x \end{aligned}$$

It is also necessary to apply Taylor's Formula to the quantities $\left(P\right|_{s-\frac{\Delta s}{2}})$ and $\left(P\right|_{s+\frac{\Delta s}{2}})$. This yields

$$(15) \quad \begin{aligned} P\left|_{s+\frac{\Delta s}{2}} &= P\left|_s + \frac{1}{2} \frac{\partial P}{\partial s}\right|_s \Delta s \\ P\left|_{s-\frac{\Delta s}{2}} &= P\left|_s - \frac{1}{2} \frac{\partial P}{\partial s}\right|_s \Delta s \end{aligned}$$

where higher order terms have been dropped. Assume a function, $f(s)$, is defined along the s -coordinate with the derivative

$$(16) \quad \frac{\partial f}{\partial s} = \lim_{\Delta s \rightarrow 0} \frac{\Delta f}{\Delta s}.$$

Since $\Delta s = \Delta x$ and Δs goes to zero as fast as Δx , as shown previously, then (16) can be rewritten as

$$(17) \quad \frac{\partial f}{\partial s} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x} = \frac{\partial f}{\partial x}.$$

Applying this to (15) shows that

$$(18) \quad \frac{\partial P}{\partial s}\Big|_s = \frac{\partial P}{\partial x}\Big|_x.$$

The equations of motion for the fluid element can thus be rewritten as

$$(19) \quad \begin{aligned} 0 &= A_f \left[P\Big|_x - \frac{1}{2} \frac{\partial P}{\partial x}\Big|_x \Delta x \right] - A_f \left[P\Big|_x + \frac{1}{2} \frac{\partial P}{\partial x}\Big|_x \Delta x \right] \\ &\quad - F\Big|_x \Delta x \frac{\partial y}{\partial x}\Big|_x - q\Big|_x \Delta x \psi \end{aligned}$$

and

$$\begin{aligned}
 m_f \Delta x a_{fy} &= A_f \left[P \Big|_x - \frac{1}{2} \frac{\partial P}{\partial x} \Big|_x \Delta x \right] \left[\frac{\partial y}{\partial x} \Big|_x - \frac{1}{2} \frac{\partial^2 y}{\partial x^2} \Big|_x \Delta x \right] \\
 (20) \quad &- A_f \left[P \Big|_x + \frac{1}{2} \frac{\partial P}{\partial x} \Big|_x \Delta x \right] \left[\frac{\partial y}{\partial x} \Big|_x + \frac{1}{2} \frac{\partial^2 y}{\partial x^2} \Big|_x \Delta x \right] \\
 &+ F \Big|_x \Delta x - q \Big|_x \Delta x \Psi \frac{\partial y}{\partial x} \Big|_x.
 \end{aligned}$$

Making the necessary algebraic simplifications yields

$$(21) \quad 0 = -A_f \frac{\partial P}{\partial x} \Big|_x \Delta x - F \Big|_x \frac{\partial y}{\partial x} \Big|_x \Delta x - q \Big|_x \Psi \Delta x$$

$$(22) \quad m_f \Delta x a_{fy} = -A_f \Delta x \frac{\partial}{\partial x} \left[P \Big|_x \frac{\partial y}{\partial x} \Big|_x \right] + F \Big|_x \Delta x - q \Big|_x \Delta x \Psi \frac{\partial y}{\partial x} \Big|_x$$

and after dividing by Δx the result is

$$(23) \quad 0 = A_f \frac{\partial P}{\partial x} + F \frac{\partial y}{\partial x} + q \Psi$$

and

$$(24) \quad m_f a_{fy} = -A_f \frac{\partial}{\partial x} \left(P \frac{\partial y}{\partial x} \right) + F - q \Psi \frac{\partial y}{\partial x}$$

which is the final form of the equations of motion for the fluid element.

The next step is to derive the equations of motion for the tube element. As can be seen from Figure 5, page 10, the element has several forces and moments acting on it. The interactive forces between the tube and fluid are the fluid shear q and normal force $F\Delta s$, where F is the force per limit length. The moment, M , and shear force, Q , are present due to the bending of the tube. The tensile force, T , is

present due to the presence of fluid shear. The quantity $cf\left(\frac{\partial y}{\partial t}\right)$ represents the external damping in its general form. The function $f\left(\frac{\partial y}{\partial t}\right)$ will assume a particular form for the damping model which will be chosen. The acceleration of the tube element is given by

$$(25) \quad \begin{aligned} a_{tx} &= 0 \\ a_{ty} &= \frac{\partial^2 y}{\partial t^2} \end{aligned}$$

for small deflections.

Summation of forces in the x-direction, yields

$$(26) \quad \begin{aligned} \sum^+ F_{tx} = 0 &= -T \Big|_{s - \frac{\Delta s}{2}} \cos \theta_1 + T \Big|_{s + \frac{\Delta s}{2}} \cos \theta_3 + F \Big|_s \Delta s \sin \theta_2 \\ &+ Q \Big|_{s - \frac{\Delta s}{2}} \sin \theta_1 - Q \Big|_{s + \frac{\Delta s}{2}} \sin \theta_3 + q \Big|_s \Psi \Delta s \cos \theta_2 . \end{aligned}$$

The tensile and shear forces may be expressed via Taylor's formula as

$$(27) \quad \begin{aligned} T \Big|_{s + \frac{\Delta s}{2}} &= T \Big|_s + \frac{1}{2} \frac{\partial T}{\partial s} \Big|_s \Delta s \\ T \Big|_{s - \frac{\Delta s}{2}} &= T \Big|_s - \frac{1}{2} \frac{\partial T}{\partial s} \Big|_s \Delta s \end{aligned}$$

and

$$(28) \quad \begin{aligned} Q \Big|_{s + \frac{\Delta s}{2}} &= Q \Big|_s + \frac{1}{2} \frac{\partial Q}{\partial s} \Big|_s \Delta s \\ Q \Big|_{s - \frac{\Delta s}{2}} &= Q \Big|_s - \frac{1}{2} \frac{\partial Q}{\partial s} \Big|_s \Delta s \end{aligned}$$

where higher order terms have been dropped. Substituting these

expressions into (26) and making use of the small angle assumption as well as equations (14), (16), and (17) gives

$$\begin{aligned}
 0 = & - \left[T \Big|_x - \frac{1}{2} \frac{\partial T}{\partial x} \Big|_x \Delta x \right] + \left[T \Big|_x + \frac{1}{2} \frac{\partial T}{\partial x} \Big|_x \Delta x \right] \\
 & + \left[Q \Big|_x - \frac{1}{2} \frac{\partial Q}{\partial x} \Big|_x \Delta x \right] \frac{\partial y}{\partial x} \Big|_x - \frac{1}{2} \left[Q \Big|_x - \frac{1}{2} \frac{\partial Q}{\partial x} \Big|_x \Delta x \right] \frac{\partial^2 y}{\partial x^2} \Big|_x \Delta x \\
 (29) \quad & - \left[Q \Big|_x + \frac{1}{2} \frac{\partial Q}{\partial x} \Big|_x \Delta x \right] \frac{\partial y}{\partial x} \Big|_x - \frac{1}{2} \left[Q \Big|_x + \frac{1}{2} \frac{\partial Q}{\partial x} \Big|_x \Delta x \right] \frac{\partial^2 y}{\partial x^2} \Big|_x \Delta x \\
 & + F \Big|_x \Delta x \frac{\partial y}{\partial x} \Big|_x + q \Big|_x \Psi \Delta x.
 \end{aligned}$$

Making the necessary simplifications and dividing by Δx yields

$$(30) \quad 0 = \frac{\partial T}{\partial x} - \frac{\partial}{\partial x} \left(Q \frac{\partial y}{\partial x} \right) + F \frac{\partial y}{\partial x} + q \Psi.$$

This is the equation of motion in the x-direction for the tube element.

To obtain the equation of motion in the y-direction, sum forces in the y-direction:

$$\begin{aligned}
 + \sum F_{ty} = m_t \Delta s a_{ty} = & - T \Big|_{s - \frac{\Delta s}{2}} \sin \theta_1 - Q \Big|_{s - \frac{\Delta s}{2}} \cos \theta_1 - F \Big|_s \Delta s \cos \theta_2 \\
 (31) \quad & + Q \Big|_{s + \frac{\Delta s}{2}} \cos \theta_3 + T \Big|_{s + \frac{\Delta s}{2}} \sin \theta_3 + q \Big|_s \Psi \Delta s \sin \theta_2 \\
 & + c f \left(\frac{\partial y}{\partial t} \right) \Big|_s \Delta s.
 \end{aligned}$$

Making use of the previously stated assumptions and formulations for T and Q yields

$$\begin{aligned}
 (32) \quad m_t a_{ty} \Delta x = & - \left[T \Big|_x - \frac{1}{2} \frac{\partial T}{\partial x} \Big|_x \Delta x \right] \left[\frac{\partial y}{\partial x} \Big|_x - \frac{1}{2} \frac{\partial^2 y}{\partial x^2} \Big|_x \Delta x \right] \\
 & - \left[Q \Big|_x - \frac{1}{2} \frac{\partial Q}{\partial x} \Big|_x \Delta x \right] - F \Big|_x \Delta x + \left[Q \Big|_x + \frac{1}{2} \frac{\partial Q}{\partial x} \Big|_x \Delta x \right] \\
 & + \left[T \Big|_x + \frac{1}{2} \frac{\partial T}{\partial x} \Big|_x \Delta x \right] \left[\frac{\partial y}{\partial x} \Big|_x + \frac{1}{2} \frac{\partial^2 y}{\partial x^2} \Big|_x \Delta x \right] \\
 & + q \Big|_x \Psi \Delta x \frac{\partial y}{\partial x} \Big|_x - cf \left(\frac{\partial y}{\partial t} \right) \Big|_x \Delta x
 \end{aligned}$$

which reduces to

$$(33) \quad m_t a_{ty} = \frac{\partial}{\partial x} \left(T \frac{\partial y}{\partial x} \right) + \frac{\partial Q}{\partial x} - F + q \Psi \frac{\partial y}{\partial x} - cf \left(\frac{\partial y}{\partial t} \right)$$

after dividing by Δx . This equation is the equation of motion for the tube element in the y-direction.

For a tube with negligible rotary inertia, the relationship between the bending moment and transverse shear is

$$(34) \quad \frac{\partial M}{\partial x} = -Q$$

where the bending moment M is given by

$$(35) \quad M = EI \frac{\partial^2 y}{\partial x^2} .$$

For constant rigidity EI, therefore

$$(36) \quad Q = -EI \frac{\partial^3 y}{\partial x^3}$$

and thus

$$(37) \quad \frac{\partial Q}{\partial x} = -EI \frac{\partial^4 y}{\partial x^4}.$$

Substituting equation (37) into equation (30) gives

$$(38) \quad 0 = \frac{\partial T}{\partial x} + F \frac{\partial y}{\partial x} + q \Psi - \frac{\partial}{\partial x} \left[EI \frac{\partial^3 y}{\partial x^3} \cdot \frac{\partial y}{\partial x} \right]$$

and neglecting higher order terms as very small yields

$$(39) \quad 0 = \frac{\partial T}{\partial x} + F \frac{\partial y}{\partial x} + q \Psi.$$

Subtracting equation (23) from (39) gives

$$(40) \quad A_f \frac{\partial P}{\partial x} - \frac{\partial T}{\partial x} = 0$$

or

$$(41) \quad \frac{\partial}{\partial x} \left[PA_f - T \right] = 0$$

for constant flow area.

Equation (41) indicates that the quantity $(PA_f - T)$ is a constant over all x . Since this quantity is zero at the free end of a cantilever, it is then zero everywhere.

Now, using equations (24) and (33), the governing equations can be obtained as

$$(24) \quad m_f a_{fy} = -A_f \frac{\partial}{\partial x} \left(P \frac{\partial y}{\partial x} \right) + F - q \Psi \frac{\partial y}{\partial x}$$

and

$$(33) \quad m_t a_{ty} = \frac{\partial}{\partial x} \left(T \frac{\partial y}{\partial x} \right) + \frac{\partial Q}{\partial x} - F + q \Psi \frac{\partial y}{\partial x} - cf \left(\frac{\partial y}{\partial t} \right).$$

Adding these two equations together gives

$$(42) \quad m_f a_{fy} + m_t a_{ty} = -\frac{\partial}{\partial x} \left[(PA_f - T) \frac{\partial y}{\partial x} \right] + \frac{\partial Q}{\partial x} - cf \left(\frac{\partial y}{\partial t} \right).$$

Recalling that $(PA_f - T)$ is zero for the cantilever tube, substitution of equations (2), (25), and (37) into equation (42) yields

$$(43) \quad 0 = EI \frac{\partial^4 y}{\partial x^4} + m_f u^2 \frac{\partial^2 y}{\partial x^2} + 2m_f u \frac{\partial^2 y}{\partial t \partial x} + (m_f + m_t) \frac{\partial^2 y}{\partial t^2} + cf \left(\frac{\partial y}{\partial t} \right).$$

For the sake of simplicity, a linear damping model is chosen [2], so $f \left(\frac{\partial y}{\partial t} \right)$ can be replaced by $\frac{\partial y}{\partial t}$ and the equation becomes

$$(44) \quad 0 = EI \frac{\partial^4 y}{\partial x^4} + m_f u^2 \frac{\partial^2 y}{\partial x^2} + 2m_f u \frac{\partial^2 y}{\partial t \partial x} + (m_f + m_t) \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t}.$$

For steady, incompressible flow, equation (44) is a linear, constant-coefficient partial differential equation which is fourth-order in space and second-order in time. The boundary conditions that go with equation (44) are

$$(45) \quad \begin{aligned} y(0, t) &= 0 \\ y_x(0, t) &= 0 \\ y_{xx}(L, t) &= 0 \\ y_{xxx}(L, t) &= 0. \end{aligned}$$

The initial conditions are chosen based on the type of beam excitation used. Hamel [2] ascertained that static deflection through the application of a point load would be the simplest form of excitation to use, so the initial conditions become

$$(46) \quad \begin{aligned} y(x, 0) &= X(x) \\ y_t(x, 0) &= 0 \end{aligned}$$

where $X(x)$ is the initial static deflection curve.

Equation (46) may be nondimensionalized through the use of appropriate scaling factors. By defining the scaled variables as

$$(47) \quad \hat{x} = \frac{x}{L}$$

and

$$(48) \quad \hat{t} = \frac{t}{\sqrt{\frac{m_f + m_t}{EI}}}$$

and defining the following non-dimensional parameters

$$(49) \quad \hat{u} = \sqrt{\frac{m_f L^2}{EI}} u \quad \text{dimensionless velocity}$$

$$(50) \quad \gamma = \frac{cL}{m_f u} \quad \text{dimensionless damping coefficient}$$

$$(51) \quad \beta = \frac{m_f}{m_f + m_t} \quad \text{mass density ratio}$$

the governing equation becomes

$$(52) \quad y_{\hat{x}\hat{x}\hat{x}\hat{x}} + \hat{u}^2 y_{\hat{x}\hat{x}} + 2\sqrt{\beta} u y_{\hat{x}\hat{t}} + \sqrt{\beta} \hat{u} \gamma y_{\hat{t}} + y_{\hat{t}\hat{t}} = 0.$$

The coefficients and variables of this equation are now nondimensional.

This is the final form of the governing equation. The boundary conditions and initial conditions are

$$(53) \quad \begin{aligned} y(0, \hat{t}) &= 0 \\ y_{\hat{x}}(0, \hat{t}) &= 0 \\ y_{\hat{x}\hat{x}}(1, \hat{t}) &= 0 \\ y_{\hat{x}\hat{x}\hat{x}}(1, \hat{t}) &= 0 \end{aligned} \quad \begin{aligned} y(\hat{x}, 0) &= X(\hat{x}) \\ y_{\hat{t}}(\hat{x}, 0) &= 0 \end{aligned}$$

where $X(\hat{x})$ is the nondimensional static deflection curve of the beam.

Assumptions

A model is only as good as the assumptions that go into it. Table 1 lists all of the assumptions which have been made in the development of this model, either explicitly or implicitly.

One of the major assumptions listed in Table 1 is that the flow is incompressible, since equation (52) is meant to describe a tube with a gas flowing in it for the case of a gas flowmeter. It might behoove this study to examine this assumption and its validity. Shapiro [10] presents an expression which he labels as a correction to the incompressible Pitot-tube formula. This expression, including terms up to $(u/\eta_0)^4$, is

$$(54) \quad \frac{P_o - P}{\frac{1}{2} \rho_o u^2} = 1 - \frac{1}{4} \left(\frac{u}{\eta_0} \right)^2 + \frac{1}{16} \left(\frac{u}{\eta_0} \right)^4 - \dots$$

where the subscript "o" refers to quantities at the stagnation point and η_0 is the speed of sound in the fluid at the stagnation state. This equation reduces to Bernoulli's Equation for incompressible flow when (u/η_0) becomes zero. Departure from unity on the right hand side of equation (54) can thus be taken as a measure of the error incurred in ignoring compressibility effects, assuming the flow is ideal in all other respects. Dropping the higher order terms [above $(u/\eta_0)^4$], the error can be plotted as shown in Figure 6. The ordinate represents the difference $\left(\frac{P_o - P}{\frac{1}{2} \rho_o u^2} - 1 \right)$ expressed in percent, and the abscissa is the quantity (u/η_0) . For the purposes of this study, incompressibility will be approximated by limiting the error to three percent. This gives a $(u/\eta_0) \approx 0.3$ and for air at 21°C (70°F), the maximum velocity is about

Table 1. Simplicity Assumptions In Derivation of
Governing Equation

Phenomena	Assumptions
beam bending	1. thin wall, negligible radial stresses 2. uniform cross-section 3. negligible rotary inertia 4. simple elastic bending relative to neutral axis 5. small deflections 6. material and geometric properties remain constant
fluid flow	1. steady and incompressible 2. fully developed 3. completely filling tube 4. $L \gg d_i$ such that secondary flow effects are negligible 5. hydrostatic pressure gradients are negligible 6. $Ma < 0.3$
beam vibration	1. small deflections 2. linear external viscous damping 3. internal material damping is negligible or can be lumped with linear external damping model chosen 4. longitudinal and rotary dynamics are negligible 5. gravitational effects are negligible
fluid/beam interaction	1. tube vibration effects on fluid pressure and velocity with respect to the tube are insignificant 2. fluid transmits inertial force to the tube directly through reaction forces at tube wall interface 3. tube diameter is small in comparison to radius of curvature in deflected state

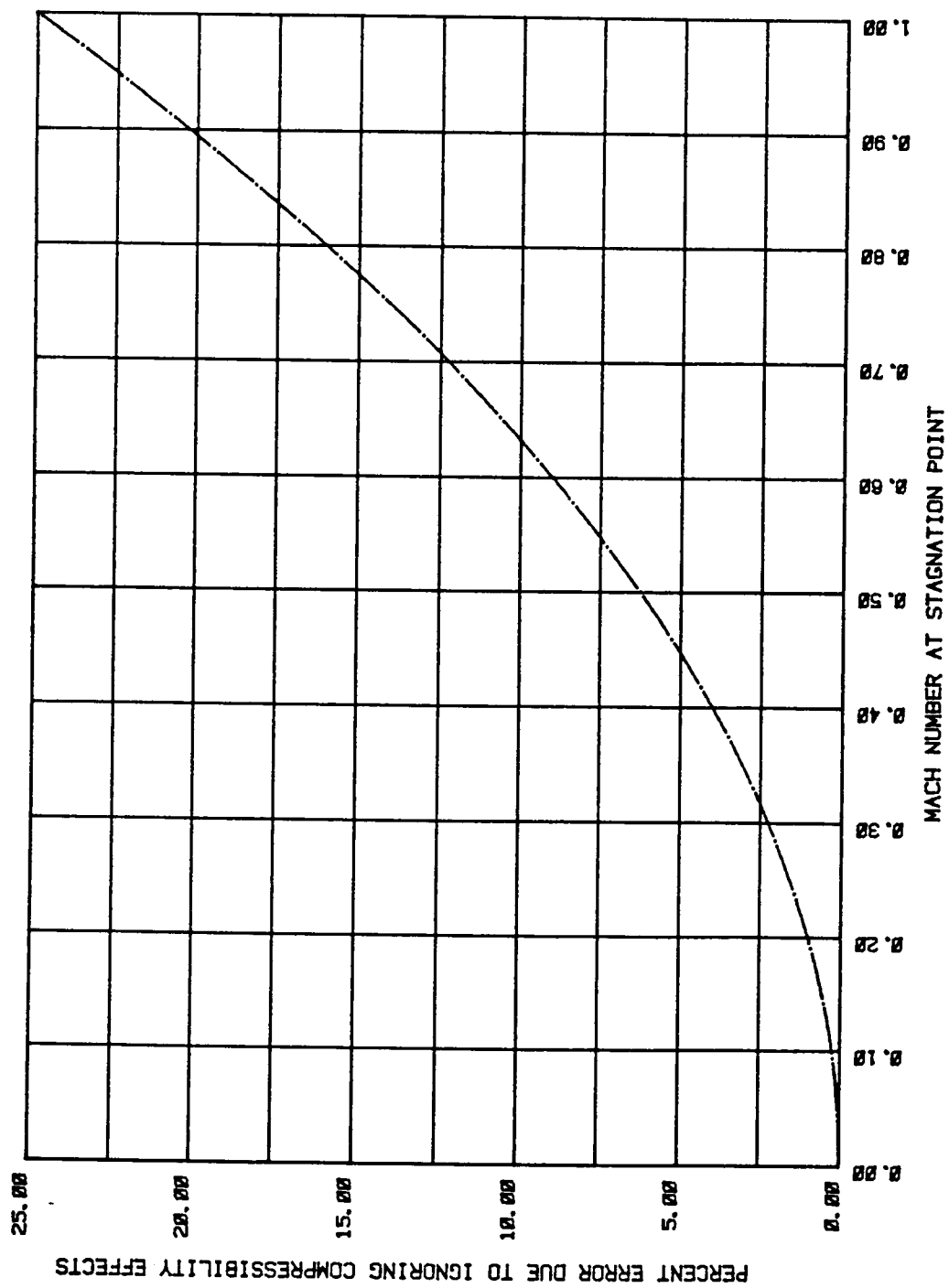


Figure 6. Error Introduced Through Incompressibility Assumption.

100 m/s (340 ft/s). This will be the limiting velocity for the tests described in this report to ensure that the incompressibility assumption retains some validity. This velocity, when coupled with the relatively small flow areas expected to be encountered in bench-size flowmeters, should limit the flow rates to less than 50 SCFM. As stated before, this is low in terms of some industrial processes; but for the purposes of testing, it should be adequate.

Solution to the Governing Equation

It is necessary to find a "simple" analytic solution to the governing equation for the purpose of putting together a flow measurement algorithm. Hamel [2] initially attempted to obtain an approximate solution via a two-timing perturbation technique. This resulted in a fairly complex solution (algebraically), even after discarding all but the fundamental mode in the perturbation solution. However, the type of manipulation involved in obtaining the perturbation solution, where spatial dependence was replaced by constants of virtue by making use of mode shape orthogonality, prompted Hamel to consider a solution via a Galerkin-type approach. The solution obtained in this fashion, which he termed the "serendipity solution", was shown to be algebraically equivalent to that obtained by the perturbation technique where the error incurred in the approximation is of order u^2 , $O(u^2)$, and that both solutions were in good agreement with an exact numerical solution. This "serendipity solution" yielded a somewhat simpler form for the purpose of developing a flow measurement algorithm, and given the equivalence of it with a solution of a known (small) amount of

error, the "serendipity solution" will be presented here, to be used in the development of a flowmeter.

The Galerkin method assumes an approximate solution to the equation

$$(55) \quad f(D, y, x) = 0$$

where f is a (perhaps nonlinear) function of the derivative operator ($D = d/dx$), of the dependent variable y , and of the independent variable x . This approximate solution is assumed to be valid only over some finite range, $a \leq x \leq b$.

This assumed solution takes the form

$$(56) \quad \tilde{y}(x) = \sum_{i=1}^n \kappa_i \phi_i(x)$$

where the $\sim$ signifies that this is an approximate solution.

Substituting (56) into (55) gives

$$(57) \quad f(D, \tilde{y}, x) = \epsilon$$

where ϵ is some small remainder due to the fact that $\tilde{y}$ is an approximate solution. If the ϕ_i 's possess orthogonality, over the interval $a \leq x \leq b$, such that

$$(58) \quad \int_a^b \phi_i \phi_j dx \quad \begin{cases} = 0 & \text{for } i \neq j \\ \neq 0 & \text{for } i = j \end{cases},$$

then the constants κ_i can easily be determined from

$$(59) \quad \int_a^b f(D, \tilde{y}, x) \phi_j(x) dx = 0 \quad j = 1, 2, \dots$$

Hamel was prompted to use a modified form of this approach, where the $\phi_i(x)$'s were the eigenfunctions of a nonhydroelastic beam and the

constants κ_i 's were replaced with functions of time $T_i(t)$'s. By making use of the known orthogonality of the eigenfunctions, the resulting equations from (59) could be uncoupled and the time functions could be easily determined. The first step in this process is to assume a solution of the form

$$(60) \quad y(\hat{x}, \hat{t}) = \sum_{i=1}^n X_i(\hat{x}) T_i(\hat{t}) \quad 0 \leq \hat{x} \leq 1$$

where it should be remembered that the variables are still in scaled space and time, and $X(\hat{x})$ are the n eigenfunctions to be used in this solution. These eigenfunctions are the mode shapes of a nonhydroelastic beam, as determined from the solution of the Bernoulli-Euler beam equation,

$$(61) \quad EI \frac{\partial^4 y}{\partial x^4} + m_t \frac{\partial^2 y}{\partial t^2} = 0.$$

To obtain the eigenfunctions (mode shapes), a solution of the form

$$(62) \quad y(x, t) = X_i(x)(\cos \omega t + \sin \omega t)$$

is assumed where $X_i(x)$ are the desired mode shapes. On substituting (62) into (61), the differential equation reduces to

$$(63) \quad \frac{\partial^4 y}{\partial x^4} - v^4 y = 0$$

$$\text{where } v = \frac{m_t \omega^2}{EI}.$$

This fourth-order differential equation can be solved easily by assuming a solution of the form

$$(64) \quad y(x) = X(x) = e^{ax}$$

which satisfies (63) when

$$(65) \quad a = \pm jv \quad \text{and} \quad a = \pm v \quad \text{where } j = \sqrt{-1}.$$

From this, it follows that the general form of $X(x)$ can be written as

$$(66) \quad X_i(x) = A_i \cos v_i x + B_i \sin v_i x + C_i \cosh v_i x + D_i \sinh v_i x$$

where the subscript i refers to the i^{th} mode shape corresponding to the i^{th} natural frequency of the beam. The natural frequencies of the beam are given by

$$(67) \quad \omega_i = v_i^2 \sqrt{\frac{EI}{m_t}} = (v_i L)^2 \sqrt{\frac{EI}{m_t L^4}}.$$

Since the governing equation is in terms of a nondimensional length $\hat{x}$, it is more convenient to also nondimensionalize the eigenfunctions with respect to $\hat{x}$, where $\hat{x} = x/L$. Making this substitution yields

$$(68) \quad X_i(\hat{x}) = A_i \cos(v_i L) \hat{x} + B_i \sin(v_i L) \hat{x} + C_i \cosh(v_i L) \hat{x} + D_i \sinh(v_i L) \hat{x}.$$

By letting $(v_i L) = a_i$, the nondimensional eigenfunctions can be rewritten as

$$(69) \quad \begin{aligned} X_i(x) = K_i [& (\sin \sqrt{a_i} + \sinh \sqrt{a_i}) (\cos \sqrt{a_i} x - \cosh \sqrt{a_i} x) \\ & - (\cos \sqrt{a_i} + \cosh \sqrt{a_i}) (\sin \sqrt{a_i} x - \sinh \sqrt{a_i} x)] \end{aligned}$$

where K_i is a normalization constant. The a_i 's represent the spatial eigenvalues, and for the case of a cantilever beam, are defined by the transcendental equation

$$(70) \quad \cos \sqrt{a_i} \cosh \sqrt{a_i} + 1 = 0.$$

The governing equation (52) may be rewritten as

$$(71) \quad y_{xxxx} + \hat{u}^2 y_{xx} + 2\sqrt{\beta} \hat{u} y_{xt} + \sqrt{1-\beta} \frac{cl^2}{\sqrt{m_l EI}} y_t + y_{tt} = 0$$

where the external damping has been redimensionalized. Hamel used only the fundamental mode shape, so that equation (60) becomes

$$(72) \quad y(\hat{x}, \hat{t}) = X(\hat{x})T(\hat{t})$$

where it is understood now that $X(\hat{x})$ is only the fundamental eigenfunction, given by

$$(73) \quad X(\hat{x}) = K_1 [(\sin \sqrt{a} + \sinh \sqrt{a}) (\cos \sqrt{a} \hat{x} + \cosh \sqrt{a} \hat{x}) - (\cos \sqrt{a} + \cosh \sqrt{a}) (\sin \sqrt{a} \hat{x} + \sinh \sqrt{a} \hat{x})],$$

To ensure the validity of this assumption, only the first mode should be excited according to the initial conditions. The static deflection curve should therefore be identical to the fundamental mode shape. Hamel determined that for a cantilever beam, this would occur for a point load placed at $x = 3/4 L$ along the beam, yielding a difference between the static deflection curve and the mode shape of less than 1%. Furthermore, Hamel also demonstrated that a software filter to filter out the unwanted higher frequencies in the actual flow

measurement algorithm would serve to make the model work even in the presence of higher modes in the beam response.

Substituting equation (72) the governing equation yields

$$(74) \quad \frac{d^4 X(\hat{x})}{d\hat{x}^4} T(\hat{t}) + \hat{u}^2 \frac{d^2 X(\hat{x})}{d\hat{x}^2} T(\hat{t}) + 2\sqrt{\beta} \hat{u} \frac{dX(\hat{x})}{d\hat{x}} \cdot \frac{dT(\hat{t})}{d\hat{t}} \\ + \sqrt{1-\beta} \bar{c} X(\hat{x}) \frac{dT(\hat{t})}{d\hat{t}} + X(\hat{x}) \frac{d^2 T(\hat{t})}{d\hat{t}^2} = 0$$

$$\text{where } \bar{c} = \frac{cL^2}{\sqrt{m_t EI}}.$$

The spatial variable $\hat{x}$ can be removed via the procedure described above, using equation (59). This gives

$$(75) \quad \int_0^1 \left[\frac{d^4 X(\hat{x})}{d\hat{x}^4} T(\hat{t}) + \hat{u}^2 \frac{d^2 X(\hat{x})}{d\hat{x}^2} T(\hat{t}) + 2\sqrt{\beta} \hat{u} \frac{dX(\hat{x})}{d\hat{x}} \cdot \frac{dT(\hat{t})}{d\hat{t}} \right. \\ \left. + \sqrt{1-\beta} \bar{c} X(\hat{x}) \frac{dT(\hat{t})}{d\hat{t}} + X(\hat{x}) \frac{d^2 T(\hat{t})}{d\hat{t}^2} \right] X(\hat{x}) d\hat{x} = 0.$$

It can be shown, from equations (61) and (69), that

$$(76) \quad \frac{d^4 X(\hat{x})}{d\hat{x}^4} = a^2 X(\hat{x}).$$

With the following definitions

$$(77) \quad K_x = \int_0^1 X^2(\hat{x}) d\hat{x} \\ K_x' = \int_0^1 \frac{dX(\hat{x})}{d\hat{x}} X(\hat{x}) d\hat{x} \\ K_x'' = \int_0^1 \frac{d^2 X(\hat{x})}{d\hat{x}^2} X(\hat{x}) d\hat{x}$$

the integral in equation (75) reduces to the ordinary differential equation

$$(78) \quad \frac{d^2 T}{d\hat{t}^2} + \left(2\sqrt{\beta} \hat{u} \frac{K_x'}{K_x} + \sqrt{1-\beta} \bar{c} \right) \frac{dT}{d\hat{t}} + \left(u^2 \frac{K_x''}{K_x} + a^2 \right) T = 0.$$

The characteristic equation for this ODE is

$$(79) \quad \lambda^2 + \left(2\sqrt{\beta} \hat{u} \frac{K_x'}{K_x} + \sqrt{1-\beta} \bar{c} \right) \lambda + \left(\hat{u}^2 \frac{K_x''}{K_x} + a^2 \right) = 0$$

for which the two distinct roots are

$$(80) \quad \lambda_{1,2} = - \left(\sqrt{\beta} u \frac{K_x'}{K_x} + \sqrt{1-\beta} \frac{\bar{c}}{2} \right) \pm j \left[\left(u^2 \frac{K_x''}{K_x} + a^2 \right) - \left(\sqrt{\beta} u \frac{K_x'}{K_x} + \sqrt{1-\beta} \frac{\bar{c}}{2} \right)^2 \right]^{\frac{1}{2}}$$

where the real and imaginary parts are the decay constant and frequency respectively. The solution to (78) can be written as

$$(81) \quad T(\hat{t}) = e^{-\hat{\lambda}_r \hat{t}} [B_1 \cos \bar{\omega} \hat{t} + B_2 \sin \bar{\omega} \hat{t}]$$

where the constants A and B can be determined from the initial conditions, equation (53), so that the solution becomes

$$(82) \quad T(\hat{t}) = T(0) e^{-\hat{\lambda}_r \hat{t}} \left[\cos \bar{\omega} \hat{t} + \frac{1}{\bar{\omega} \tau} \sin \bar{\omega} \hat{t} \right].$$

The total solution can therefore be written as

$$(83) \quad y(\hat{x}, \hat{t}) = X(\hat{x}) T(0) e^{-\hat{\lambda}_r \hat{t}} \left[\cos \bar{\omega} \hat{t} + \frac{1}{\bar{\omega} \tau} \sin \bar{\omega} \hat{t} \right]$$

where it should be remembered that $X(\hat{x})$ represents the spatial eigenfunction describing the fundamental normal mode vibration for a nonhydroelastic beam.

Since $X(\hat{x})T(0) = y(\hat{x}, 0)$, the solution becomes

$$(84) \quad y(\hat{x}, \hat{t}) = y(\hat{x}, 0) e^{-\hat{\lambda}_r \hat{t}} \left[\cos \bar{\omega} \hat{t} + \frac{1}{\bar{\omega} \tau} \sin \bar{\omega} \hat{t} \right].$$

As stated previously, Hamel showed that this solution was algebraically identical to the solution obtained through a two-timing perturbation method, where the error was of the order of u^2 . He used this "serendipity solution" in the development of his flow measurement algorithm, and that procedure will be followed here.

CHAPTER III

PROTOTYPE DEVELOPMENT

Derivation of Measurement Algorithms

Equation (80) represents the roots of the solution governing the transient vibration of a cantilever tube in scaled time. For real time measurements these roots need to be unscaled. The equations which come from equation (78) are

$$(85) \quad \iota^{-1} = \sqrt{\beta} \hat{u} \frac{K_x'}{K_x} + \sqrt{1-\beta} \frac{\bar{c}}{2}$$

and

$$(86) \quad \bar{\omega}^2 = \hat{u} \frac{K_x''}{K_x} + a^2 - \iota^{-2}.$$

Making use of the time-scaling factor and the previous definitions for $\hat{u}$ and β , the equations become

$$(87) \quad \left(\iota \frac{\bar{t}}{\hat{t}} \right)^{-1} = \beta \frac{K_x'}{K_x} \frac{u}{L} + (1-\beta) \frac{\bar{c}}{2m_t}$$

and

$$(88) \quad \left(\frac{\bar{\omega}}{\omega} \iota \frac{\bar{t}}{\hat{t}} \right)^2 = (1-\beta) \frac{a^2 EI}{m_t L^4} + \beta \frac{K_x''}{K_x} \frac{u^2}{L} - \left| \left(\iota \frac{\bar{t}}{\hat{t}} \right)^{-1} \right|^2.$$

The quantity $\bar{c}/2m_t$ can be defined as ι_e^{-1} , which represents the amount of damping present for no flow through the tube. The first quantity on the right side of equation (86) is the fundamental natural frequency of an empty tube, that is

$$(89) \quad \omega_e^2 = \frac{a^2 EI}{m_t L^4}$$

so the equations become

$$(90) \quad \tau^{-1} = \beta \frac{K_x'}{K_x} \frac{u}{L} + (1-\beta) \tau_e^{-1}$$

and

$$(91) \quad \omega^2 = (1-\beta) \omega_e^2 + \beta \frac{K_x''}{K_x} \frac{u^2}{L} - \tau^{-2}.$$

These equations relate the vibratory response of a cantilever tube to the physical parameters of the tube and the flow properties ρ_f and u , in real time. If the physical parameters of the tube are known, then fluid density and velocity should be determinable from these equations. The determination of the decay rate and frequency of a signal is no problem for a computer-based measurement system such as the one to be used in this study.

The basic approach to this determination may be discussed with the use of Figure 7, which shows some sinusoidal waveform of decaying amplitude, where the top half of the envelope is equal to the bottom half of the envelope. The envelope is defined by the peaks and is generally given as

$$(92) \quad z(t) = Ae^{-t/\tau}.$$

This corresponds to a tube vibrating about a zero reference. Let P_i be the i^{th} peak and P_{i-1} be the $(i-1)^{\text{th}}$ peak. The time between these peaks is equal to one-half period of the waveform. The average half-period of the waveform is given by

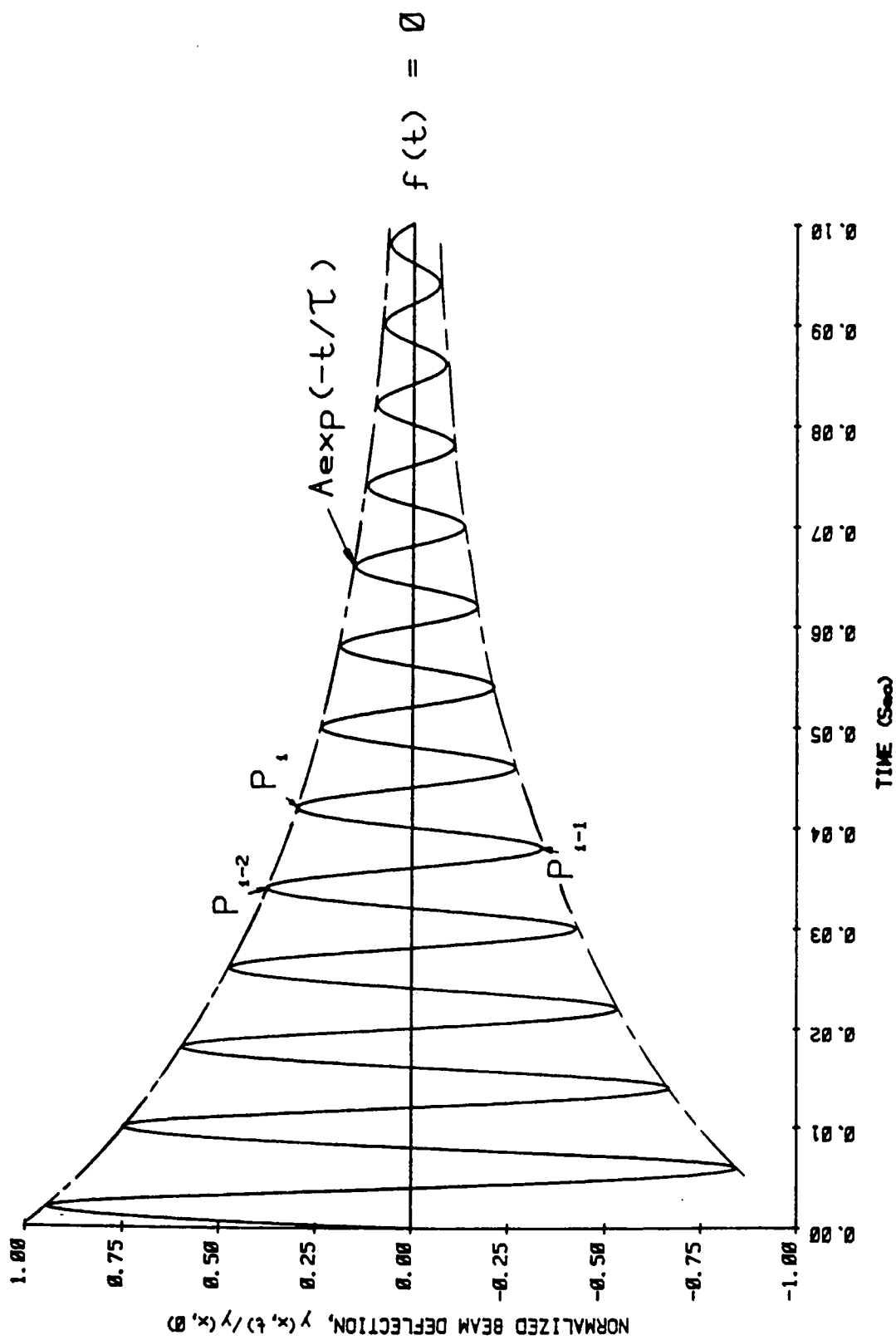


Figure 7. Damped Sinusoid Vibrating About Zero.

$$(93) \quad T_{\frac{1}{2}} = \sum_{i=1}^n \frac{t_i - t_{i-1}}{n}.$$

The frequency is equal to the reciprocal of the period, or

$$(94) \quad f = \frac{1}{2T_{\frac{1}{2}}},$$

and the circular frequency ω is given by

$$(95) \quad \omega = 2\pi f.$$

The decay constant for a simple damped sinusoid can be found by using the logarithmic decrement,

$$(96) \quad \tau^{-1} = f \left| \ln \left(\frac{P_{i-2}}{P_i} \right) \right|.$$

However, preliminary results indicated that a problem existed which precluded the use of the simple algorithm discussed above. Figure 8 is the time history of one of the tubes which was later tested. As can be seen, the overall vibration of the tube has occurred about a time-varying offset, and the exponential envelope mentioned previously has degraded into two unequal functions; that is, the function that describes the curve formed by the upper peaks of the waveform differs from the curve described by the lower peaks not only in sign, but also in magnitude. The cause of this behavior lies in the time-dependent stress-strain characteristics of the tube materials which were used, and a more detailed discussion of this phenomenon and the implications that lie therein will be made shortly. For the present, it is enough to state that a program based on the aforementioned algorithm for finding

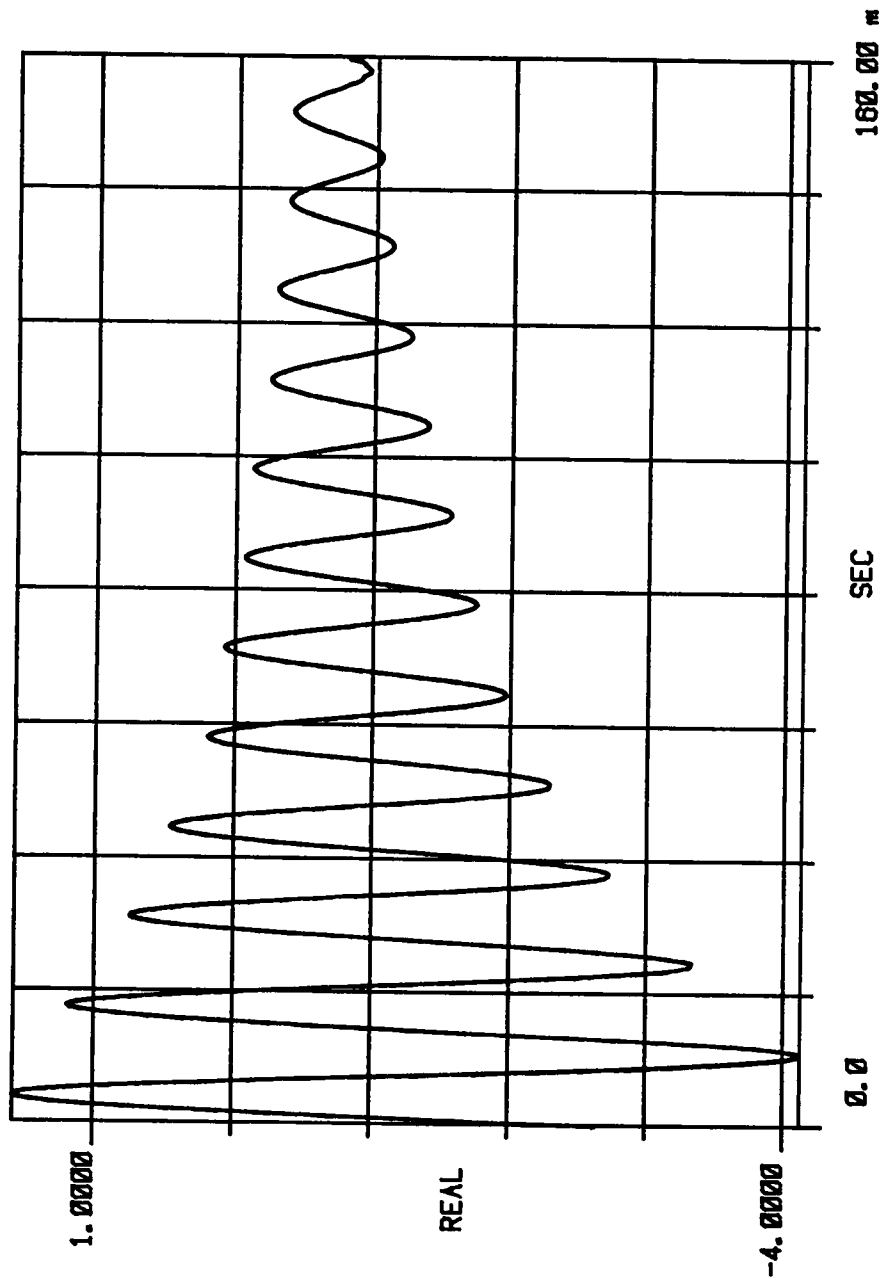


Figure 8. Time History of 1.27 cm OD x 0.15875 cm
(0.5 inch OD x 0.0625 inch) Plexiglas Tube.

frequency and decay constant failed miserably for the simple reason that it was based on finding the zeroes of a waveform, and this waveform possessed a distribution of zeroes that was not palatable to the code. Even finding the peaks was only enough to give frequency. Because of this, it became necessary to formulate a new algorithm. The development of this new algorithm follows.

The first assumption that was made was that the offset was due solely to material effects and that the effects of the tube-fluid interaction which were sought (i.e. decay rate and frequency) could be separated out in their entirety. In other words, the two effects were treated as being a linear combination of two independent phenomenon, where the offset induced by the material effects could be subtracted out without affecting the signal change due to the flow, much as one might subtract a DC offset from an AC voltage without distorting the information contained in the AC waveform. The idea, then, was to subtract out the offset and shift the curve up to a zero offset position, where the curve would take on the appearance of a simply damped sinusoid as mentioned before. The derivation which follows is based on Figure 9; this figure shows the type of curve under study. The function $f(t)$ represents the time-varying reference position for the beam vibration. (Note that for $f(t) = 0$, the case degenerates into the simpler case discussed previously.) The functions $g(t)$ and $h(t)$ are defined by the upper and lower peaks, respectively. Let $z(t)$ (not pictured) represent the envelope of the shifted curve, i.e. after the time-varying offset has been subtracted out. This $z(t)$ is identical to that already mentioned and contains all of the information concerning

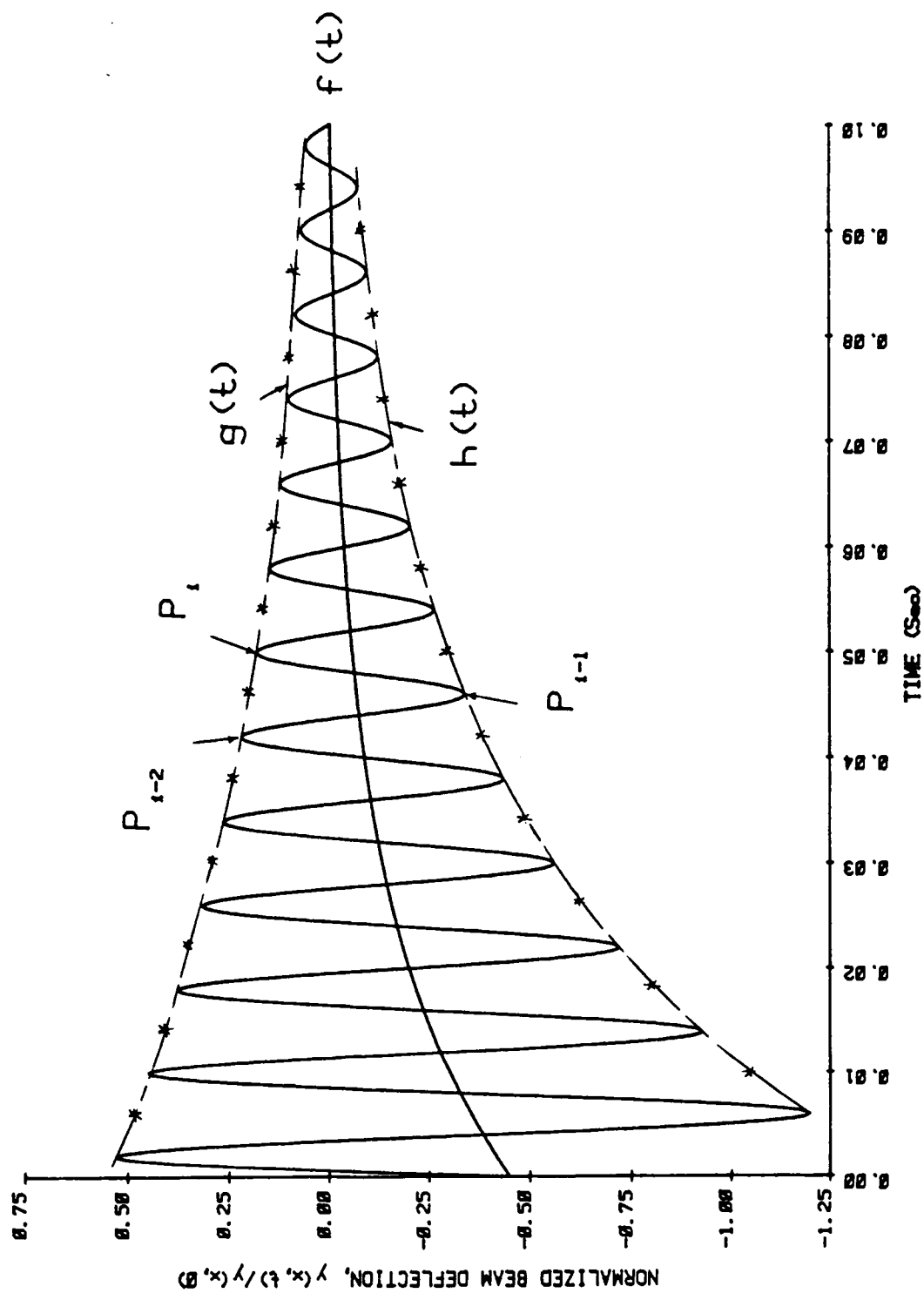


Figure 9. Damped Sinusoid With Non-Zero Offset.

τ^{-1} , the beam decay constant of interest. The shifted position of any point in the waveform is given by the difference of its unshifted position and $f(t)$. One aside: this "shifting" is a completely static process. Only the relative amplitudes of the various curves are changed. This is in line with the assumption that the material effects and any flow effects are independent and the total beam response is a linear combination of the two. Thus, the frequency of the signal will be unchanged. Since this shifting can be applied to every point in the waveform, it can particularly be applied to the peaks, yielding

$$(97) \quad z(t) = g(t) - f(t)$$

and

$$(98) \quad -z(t) = h(t) - f(t)$$

where $z(t)$ represents the desired "shifted" envelope given by equation (92). These equations only state that $g(t)$ and $h(t)$ will be transformed into a simple exponential envelope by subtracting out the offset $f(t)$.

Equations (97) and (98) may be combined to give

$$(99) \quad z(t) = \frac{g(t)}{2} - \frac{h(t)}{2}.$$

This equation defines $z(t)$, the desired envelope, in terms of the existing ones. Thus, to compute $z(t)$ (and hence find τ^{-1}) the problem reduces to finding $g(t)$ and $h(t)$, and $f(t)$ never enters the picture. One way of accomplishing this is to locate the peaks of the waveform and apply least-square curve fits to obtain approximations of $g(t)$ and $h(t)$. Unfortunately, to apply a curve fit routine properly requires some idea

of the form of the analytic equation sought, and since $g(t)$ and $h(t)$ are probably sums of exponentials, the algebraic task of developing the necessary formulas for the various quantities appears daunting at best, particularly since a better way exists. This better way involves computing $z(t)$ at certain points and fitting a curve to this discrete version of $z(t)$. Since $z(t)$ is assumed to be a simple exponential, the amount of algebra is reduced manifold and the decay constant τ^{-1} of the shifted waveform $z(t)$ (which is the quantity of interest) is computed in a more straightforward way. One problem, however, is immediately apparent. Since $g(t)$ and $h(t)$ are only known at opposite peaks, which differ by $T/2$ for any one period T , it is impossible to calculate $z(t)$ without knowing the intermediate points, represented by *'s in Figure 9. These points can be determined through an interpolation scheme. It was decided to use the simplest interpolation scheme which would work adequately. Since the frequency of the signals encountered in this study were on the order of 100 Hz, the time between a peak and the following "*" was on the order of 0.05 milliseconds. This is a small interval, and it was felt that a linear interpolation scheme would suffice. So, upon capturing the waveform on computer, the peaks were located through a peak search routine and intermediate values were determined via linear interpolation. All of these points were used to compute $z(t)$ according to equation (99). An exponential least-squares curve fit was then applied to the points representing $z(t)$. The two constants which are determined by such a curve fit are the amplitude A and the time constant τ . The time constant τ holds the most interest in this study, since by virtue of the assumptions previously made, it is

the inverse of the decay constant which is sought. The formula for can be derived easily from such a least-squares curve fit, and is

$$(100) \quad \tau^{-1} = \left[\frac{\frac{\sum t_i \sum t_i}{n} - \sum t_i^2}{\sum t_i \sum \ln c(t_i) - \frac{1}{n} \sum \ln c(t_i) \sum t_i} \right]$$

where $z(t_i)$ are the calculated values of $z(t)$ at time t_i , and n is the number of these points. The summations are understood to go from i equals one to n . The decay constant is then given by

$$(101) \quad \tau^{-1} = \frac{1}{\tau}.$$

These equations are the ones which were used in the measurement algorithm to determine decay constant. The frequency was determined as before, since $f(t)$ did not affect it.

Once τ^{-1} and ω have been determined, equations (90) and (91) can be used to solve for the density and velocity of the fluid. The prototype used a two-tube, null-type setup where one tube remained empty during the measurement process. In this way, the empty tube response characteristics were determined at the same time that the response characteristics of the flowing tube were measured. These measured values are denoted as

R_{c1} = measured decay constant for $u = 0$

R_{c2} = measured square of the frequency for $u = 0$

R_1 = measured decay constant for $u = u(t)$

R_2 = square of measured frequency for $u = u(t)$

Equations (90) and (91) yield four equations upon substitution of the parameters. The equations are

$$(102) \quad R_{c1} = (1 - \beta_0) \iota_e^{-1}$$

$$(103) \quad R_{c2} + R_{c1}^2 = (1 - \beta_0) \omega_e^2$$

$$(104) \quad R_1 = \beta \frac{K_x'}{K_x} \frac{u}{L} + (1 - \beta) \iota_e^{-1}$$

and

$$(105) \quad R_2 + R_1^2 = (1 - \beta) \omega_e^2 + \beta \frac{K_x''}{K_x} \frac{u^2}{L}$$

where β_0 is the value of β at zero velocity (for the empty tube) and β is the value at some velocity u (for the active tube). Another equation is the defining relation for β , which is

$$(106) \quad \beta = \left[\frac{\rho_f}{\frac{\rho_t A_t}{A_f} + \rho_f} \right].$$

By defining the following constants,

$$(107) \quad \begin{aligned} a_1 &= \frac{\rho_t A_t}{A_f} \\ a_2 &= \frac{K_x'}{K_x L} \\ a_3 &= \frac{K_x''}{K_x L} \end{aligned}$$

equations (102), (103), (104), (105), and (106) then become

$$(108) \quad R_{c1} = (1 - \beta_0) \tau_e^{-1}$$

$$(109) \quad R_{c2} + R_{c1}^2 = (1 - \beta_0) \omega_e^2$$

$$(110) \quad R_1 = a_2 \beta u + (1 - \beta) \tau_e^2$$

$$(111) \quad R_2 + R_1^2 = (1 - \beta_0) \omega_e^2 + a_3 \beta u^2$$

and

$$(112) \quad \beta = \frac{\rho_f}{a_1 + \rho_f}.$$

These equations can be combined to give

$$(113) \quad R_1 = a_2 \beta u + \frac{(1 - \beta)}{(1 - \beta_0)} R_{c1}$$

$$(114) \quad R_2 + R_1^2 = \left(\frac{(1 - \beta)}{(1 - \beta_0)} (R_{c2} + R_{c1}^2) + a_3 \beta u^2 \right)$$

and

$$(115) \quad \beta = \frac{\rho_f}{a_1 + \rho_f}.$$

At this point, there are two paths which may be taken. Strictly speaking, β_0 is not equal to β , since the fluid that will be used is not incompressible, but will undergo some density change due to compressibility effects as the velocity increases. The reference tube will "see" a fluid density that is representative of the bulk density of the fluid surrounding the tubes, where the velocity is much lower than that in the active tube. Furthermore, this density could be time

dependent (particularly in the case of multi-constituent flows with time-varying compositions), so that the densities, and hence the β 's, may never be equal. Thus, a more precise way to proceed would involve both β and β_0 as variables to be determined through measurement.

However, it can be shown that

$$(116) \quad \frac{d\beta}{\beta} = \left(\frac{a_1}{\rho_f + a_1} \right) \frac{d\rho_f}{\rho_f}$$

or

$$(117) \quad \frac{d\beta}{\beta} \approx \frac{d\rho_f}{\rho_f}$$

since $a_1 = \frac{\rho_t A_t}{A_f}$, which is several orders of magnitude larger than ρ_f for typical gas flows such as those dealt with in this study. Changes in β are therefore about the same as changes in the density. Referring back to the Mach number constraint, it is anticipated that the density will undergo small variations so that $\beta \approx \beta_0$ and $(1 - \beta_0) \approx (1 - \beta)$. For single constituent flows or flows where the composition remains fixed, such as the case with the experimental work in this study. Note that this is only true in light of the Mach number constraint, and that in general, for a compressible substance, $\beta \neq \beta_0$. This approach leads to the following equations:

$$(118) \quad R_1 - R_{c1} = a_2 \beta u$$

$$(119) \quad (R_2 + R_1^2) - (R_{c2} + R_{c1}^2) = a_3 \beta u^2$$

and

$$(120) \quad \beta = \frac{\rho_f}{\rho_f + a_1}.$$

If a_1 , a_2 , and a_3 are known, this set of equations represent three equations in the unknowns β , ρ_f , and u . The constants a_1 , a_2 , and a_3 could be determined by careful measurements of the tubes and calculations of the various quantities such as K_x'' , K_x' , and K_x . A better, and more traceable way, though, would be to calibrate the flowmeter, i.e. determine a_1 , a_2 , and a_3 for various known values of ρ_f and u . The calibration equations are found by solving the previous equations for a_1 , a_2 , and a_3 . Thus,

$$(121) \quad a_1 = \rho_f \left(\frac{1-\beta}{\beta} \right)$$

$$(122) \quad a_2 = \frac{R_1 - R_{c1}}{\beta u}$$

$$(123) \quad a_3 = \frac{(R_2 + R_1^2) - (R_{c2} + R_{c1}^2)}{\beta u^2}.$$

By operating the flowmeter in conjunction with a secondary standard and measuring R_1 , R_2 , R_{c1} , R_{c2} , ρ_f , and u over a suitable range, the calibration constants can be determined.

Early results pointed out a major problem in this approach. The constant a_3 varied widely over the 10- to 15-point calibration range used, having variances equal to over twice the mean in some cases. Hamel [2] also ran into this problem and interpreted it as the result of negligible frequency shift due to flow. He therefore set a_3 equal to zero and re-derived his measurement algorithms. In this case, however, setting a_3 to zero reduces equation (123) to a simple identity, leaving only two equations and three unknowns. The reason for this is that in Hamel's work, he was able to neglect β_0 completely, since β was defined

for water while β_0 was defined for air. Thus, $\beta \gg \beta_0$ for any flow rate. In this case the form of the equations was such that setting a to zero did not cause the frequency equation to reduce to an identity, unlike the present situation.

This is not a good situation for a measuring device. The answer to this problem came through the re-introduction of the $(1 - \beta_0)$ term into the equations. This is equivalent to taking the approach that $\beta_0 \neq \beta$ and using the equations without any simplifications. For the conditions present in this study, it was believed that β_0 would be constant over the flow ranges to be encountered, and in fact results showed that the variance of the quantity $1/(1 - \beta_0)$ was on the order of 10^{-4} for all tests, with means on the order of 1.00. Thus, the $(1 - \beta_0)$ quantity was treated as another calibration constant, and was determined in the same manner as a_1 and a_2 . The new calibration equations are

$$(124) \quad a_1 = \rho_f \left(\frac{1 - \beta}{\beta} \right)$$

$$(125) \quad a_2 = \frac{R_1 - a_3' (1 - \beta) R_{c1}}{\beta u}$$

$$(126) \quad a_3 = \frac{1}{a_3'} = \frac{R_2 + R_1^2}{(1 - \beta)(R_{c2} + R_{c1}^2)}.$$

where $a_3' = (1 - \beta_0)$. Observations made over the entire flow range for each set of tubes tested showed that this quantity varied less than 1% at any time.

With this set of calibration constants, the flow measurement algorithms can be written as

$$(127) \quad (1-\beta) = \frac{R_2 + R_1^2}{a_3(R_{c2} + R_{c1}^2)} \quad ; \quad a_3 = \frac{1}{a_3'}$$

$$(128) \quad \beta = 1 - (1-\beta)$$

$$(129) \quad \rho_f = \frac{a_1 \beta}{1 - \beta}$$

$$(130) \quad u = \frac{R_1 - a_3(1-\beta)R_{c1}}{a_2 \beta}$$

and

$$(131) \quad \dot{m} = \rho_f A_f u .$$

These are the equations which were used in the FORTRAN computer programs that performed the actual calibration and measurement procedures.

Design of a Gas Cantilever Coriolis Mass Flowmeter

The design of an in-line Coriolis flowmeter is a two-step process. First, the sensing element, which is the cantilever tube, must be selected. Then the housing which contains the sensing element and allows the flowmeter to be an in-line device can be designed. The first step, selecting a tube to be used as the sensing element, should be done with regard to maximizing the effect that changes in flow rate will have on the vibration characteristics of the tube. In this light, a sensitivity analysis of how various tube parameters (physical dimensions, mass, etc.) affect the roots given by equation (80) is of primary importance.

The decay constant and frequency of vibration were given in equations (90) and (91) are reproduced here for convenience:

$$(90) \quad \tau^{-1} = \beta \frac{K_x'}{K_x} \frac{u}{L} + (1-\beta) \tau_e^{-1}$$

and

$$(91) \quad \omega^2 = (1-\beta) \omega_e^2 + \beta \frac{K_x''}{K_x} \frac{u^2}{L} - \tau^{-2}.$$

These equations represent the real-time vibration characteristics as determined by Hamel [2]. Figure 10 is a plot of theoretical response curves for a set of typical plastic tube parameters and flowrates. As can be seen, the amplitudes of the responses are definitely affected by the amount of flow present, indicating that the decay constant is a relatively strong function of flow compared to the frequency, which from all appearances in the figure changes negligibly with flow. In actual testing, as noted previously, this was the case. Therefore, a parameter sensitivity analysis was performed only for the decay constant, and aside from the constraint that the tube frequency should remain in a "reasonable" range, the frequency parameter was not considered in the design sequence.

From equation (90), it is apparent that the decay constant is a function of both flow-related variables and material parameters. The flow-related variables are the fluid velocity u and the mass-ratio term β , which includes fluid density implicitly. Note that the density and velocity of the flow are independent variables. Because the expression for the decay constant can be rewritten as

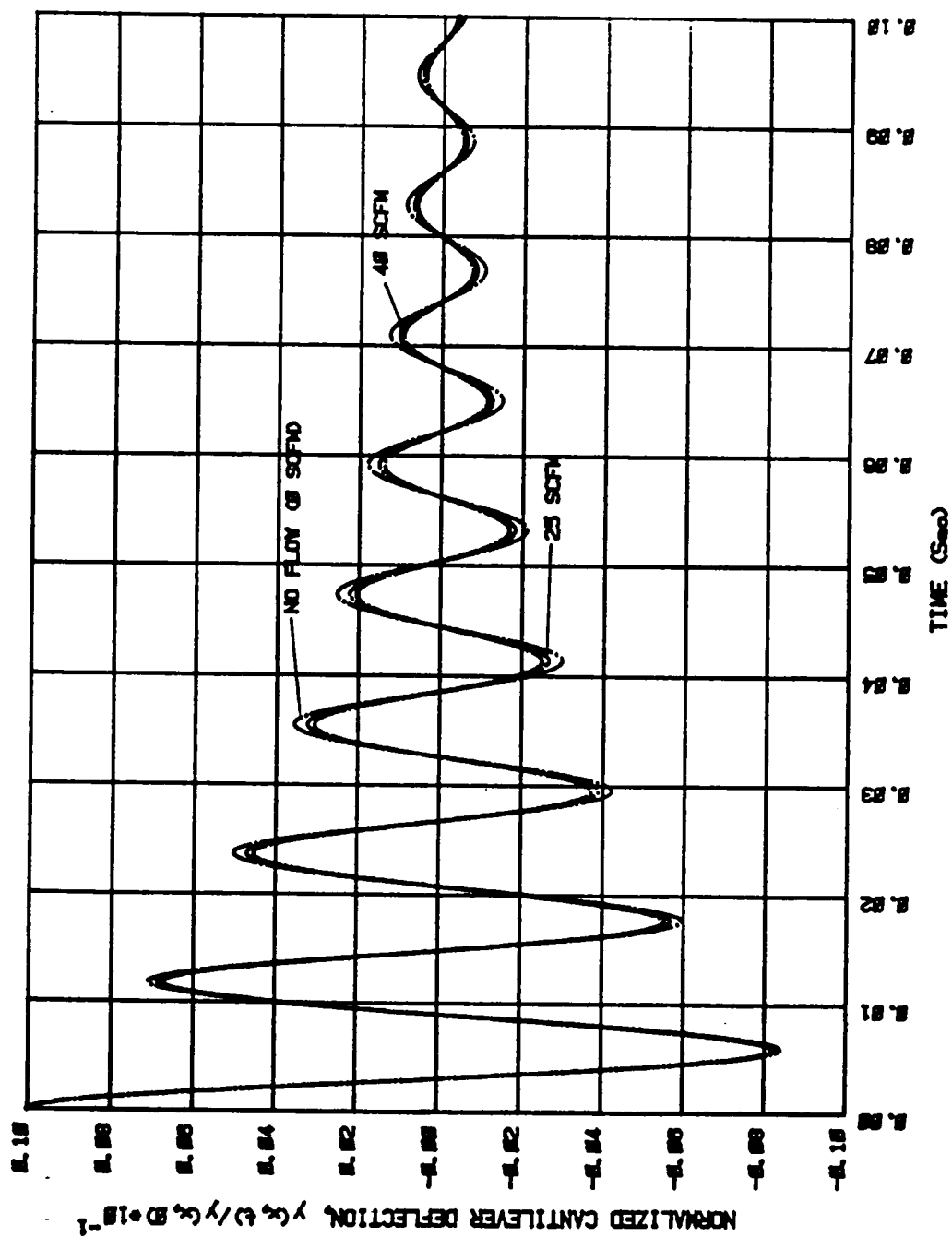


Figure 10. Theoretical Results for ABS Cantilever Tube.

$$(132) \quad \tau^{-1} = \frac{\rho_f A_f}{\rho_t A_t + \rho_f A_f} \cdot \frac{K_x'}{K_x} \frac{u}{L} + \frac{\rho_t A_t}{\rho_t A_t + \rho_f A_f} \tau_e^{-1}$$

it is not possible to express the decay rate as an explicit single-variable function of the mass flow rate. Mathematically this means that there is no single sensitivity of the form

$$(133) \quad S_{\dot{m}} = \frac{d\tau^{-1}}{d\dot{m}}$$

since τ^{-1} is multi-variable function. In fact, it is possible to vary and u such that $\dot{m}$ would not change, and yet τ^{-1} would vary according to equation (132). This does not mean that the flowmeter would not accurately determine the mass flow, for it would. It simply means that a parameter sensitivity analysis for τ^{-1} cannot be based on mass flowrate.

A better (and more meaningful) approach consists of looking at sensitivities based on the fluid velocity u and fluid density ρ_f . The differential of τ^{-1} can be expressed as

$$(134) \quad d\tau^{-1} = \left(\frac{\partial \tau^{-1}}{\partial u} \right) \bigg|_{\rho_f} du + \left(\frac{\partial \tau^{-1}}{\partial \rho_f} \right) \bigg|_u d\rho_f.$$

This equation states that an incremental change in the decay constant is a weighted sum of incremental changes in the velocity and density. The coefficients of du and $d\rho_f$ in equation (134) are the sensitivities that are sought, that is,

$$(135) \quad \tau_{\rho_f} = \left(\frac{\partial \tau^{-1}}{\partial \rho_f} \right) \Big|_u \quad \text{"density sensitivity"}$$

and

$$(136) \quad \tau_u = \left(\frac{\partial \tau^{-1}}{\partial u} \right) \Big|_{\rho_f} \quad \text{"velocity sensitivity"}$$

From equation (90), these sensitivities can be shown to be

$$(137) \quad \tau_{\rho_f} = \frac{A_f}{\rho_t A_t} (1-\beta)^2 \left| \frac{K_x'}{K_x} \frac{u}{L} - \tau_e^{-1} \right|$$

and

$$(138) \quad \tau_u = \frac{\rho_f A_f}{\rho_t A_t} (1-\beta) \frac{K_x'}{K_x}.$$

Note that the sensitivities are functions of the fluid density, mean fluid velocity, tube density and geometry, and in the case of the density sensitivity, the external damping. In general, for a flow consisting of multiple phases and/or constituents where the relationships between phases between constituents are not fixed, density of the flow is a function of masses of the various constituents and phases (and may be quite complicated as well); and time dependency of the density and velocity may enter into the picture. These factors alone make the general case most forbidding, and the added presence of the external damping serves to make the picture more dismal for utilizing these sensitivities in a general flowmeter design algorithm. However, as engineers have always done when faced with difficult problems, simplifications were sought.

One such simplification occurs if the flow is incompressible with a fixed, static relationship between the constituents which make up the flow. An example would be a flow of a water-alcohol mixture where the ratio of alcohol mass to water mass is constant. Obviously, single-constituent, incompressible flows also fall into this category. In this case, ρ_f is constant, so that $d\rho_f$ is zero, and the change in τ^{-1} is simply

$$(139) \quad d\tau^{-1} = \left(\frac{\partial \tau^{-1}}{\partial u} \right) \bigg|_{\rho_f} du$$

and the velocity sensitivity becomes a function only of the tube parameters. A similar situation occurs if the flow from the first case is allowed to be compressible, but if the changes in density due to compressibility effects are small enough to ignore. Given the Mach number assumption, changes in fluid density should be small enough compared to velocity changes that again,

$$(139) \quad d\tau^{-1} = \left(\frac{\partial \tau^{-1}}{\partial u} \right) \bigg|_{\rho_f} du$$

and the velocity sensitivity becomes a function solely of the tube parameters. Since this is the case for this study, the only sensitivity which need be maximized in the design is the velocity sensitivity, and the more complex case will not be covered here. This presents no loss anyway, since the solution to the governing equation, from which equation (90) is obtained, is based on the assumption of incompressible flow in the measurement tube. Thus, only the case of varying composition would need to be looked at, unless one wishes to solve a

modified governing equation which takes into account unsteady mass in the tube.

A study of the sensitivities reveals some interesting and useful facts about what maximizes these functions. Take the velocity sensitivity,

$$(138) \quad \tau_u = \frac{\partial \tau_{diff}^{-1}}{\partial u} = \frac{\rho_f A_f (1 - \beta)}{\rho_t A_t L} \frac{K_x'}{K_x}$$

which can be rewritten as

$$(140) \quad \tau_u = \left(\frac{1}{R_\rho [R_d^2 - 1] + 1} \right) \frac{K_x'}{K_x L}$$

$$R_d^2 = \left(\frac{d_o}{d_i} \right)^2$$

$$R_\rho = \frac{\rho_t}{\rho_f}.$$

This equation shows that the velocity sensitivity is a function of the two dimensionless variables, R_d and R_ρ , and the tube length L . To maximize τ_u , R_ρ should be as small as possible, which corresponds to a low density tube material for gas flows; and R_d should be very close to 1, which corresponds to a tube with thin walls. These conclusions are probably intuitive. It would appear the the length L should also be small, although this may not be so. It is felt that there is probably some intermediate value of L where τ_u would be maximized. The reason for this belief stems from the knowledge that while a very long tube would be more flexible and yield more to the applied Coriolis force, and

would have a larger area for the force to act on, the drop in frequency would cause the Coriolis force to suffer a reduction in magnitude. In contrast, a short tube would vibrate at a much higher frequency, and thus yield a larger Coriolis force; however, the stiffness of the shorter tube would tend to resist the damping effects of the Coriolis force, thus making it (possibly) less sensitive. More qualitative work on this idea is probably in order.

Hamel [2] laid out a design procedure for cantilever sensing tubes consisting of three equations, based on the following constraints:

- (1) maximum allowable stress in the tube due to static deflection must remain well within the elastic region
- (2) allowable pressure drop through the tube must be less than or equal to maximum pressure drop allowed for measurement
- (3) maximum allowable tube frequency must remain low enough for adequate reconstruction of the digitized signal by the measurement system

Constraints (1) and (2) are valid for either gas or liquid flows. The second constraint is not very useful for gas flows, however, since the maximum pressure drop that is likely to be encountered for the size of tubes used in this study will be less than one psi for a wide range of tubes. Additionally, the desire to optimize performance by maximizing the sensitivities has some merit, since for gas flows there is a definite need to ensure that the Coriolis force generated by a gas flow will be strong enough to influence the tube. For liquids, this need is not nearly so great, since the densities of liquids are several orders of magnitude larger than those for gases, and the Coriolis forces

will be correspondingly larger. Therefore, a design procedure will be presented utilizing constraints (1) and (3) and the optimization of τ_u as decided in the previous section.

The first consideration in the design of a flow tube is that the maximum stress in the tube must remain well within the elastic range, not only to avoid failure due to tension but also to ensure as long a fatigue life as possible. The maximum stress in a cantilever tube with a point load applied at $3/4 L$ is given by

$$(141) \quad \sigma_{\max} = \frac{16E}{9} \frac{d_o}{L^2} y(L,0).$$

The second consideration is that the frequency of the vibrating tube must be low enough to allow adequate reconstruction of the sampled signal by the measurement system. For good reconstruction, the sampling frequency should be at least 10 times larger than the frequency of the beam deflection transient, which will always be less than or equal to the empty tube frequency ω_e given by equation (89). This can be expressed mathematically as

$$(142) \quad \frac{\omega_s^2}{R_\omega^2 \alpha^2 g_c} = \left(\frac{E}{\rho_t} \right) \left(\frac{d_o^2 + d_i^2}{16L^4} \right)$$

where $R_\omega = \frac{\omega_s}{\omega_e}$ and is always greater than or equal to 10.

The third condition is the maximization of the velocity sensitivity τ_u , where τ_u is given by

$$(138) \quad \tau_u = \frac{\rho_f A_f (1 - \beta)}{\rho_t A_t L} \frac{K_x'}{K_x}$$

which can be expressed alternately as

$$(143) \quad v_u = \left| \frac{\rho_f}{\rho_l \left(\frac{d_o^2 - d_i^2}{d_i^2} \right) + \rho_f} \right| \frac{K_x'}{K_x}$$

where ρ_f is some nominal value of the expected fluid density. For air at 21°C (70°F) and 30 mm Hg (14.7 psi), this value is 1.22 kg/m³ (0.076 lbm/ft³).

An additional provision can be introduced from the initial assumptions that went into the model. To ensure that secondary flow effects are negligible L/d_i must be greater than or equal to 10. For values of L/d_i greater than 10, Katys [12] found that secondary flow effects were negligible. Thus,

$$(144) \quad \left(\frac{L}{d_i} \right) \geq 10.$$

The design problem reduces to the following statement:

Maximize

$$(143) \quad v_u = \left| \frac{\rho_f}{\rho_l \left(\frac{d_o^2 - d_i^2}{d_i^2} \right) + \rho_f} \right| \frac{K_x'}{K_x L}$$

subject to

$$(141) \quad \sigma_{\max} = \frac{16E}{9} \frac{d_o}{L^2} y(L, 0)$$

and

$$(140) \quad \frac{\omega_s^2}{R_\omega^2 a^2 g_c} = \left(\frac{E}{\rho_l} \right) \left(\frac{d_o^2 + d_i^2}{16L^4} \right)$$

with the provision that

$$(144) \quad \left(\frac{L}{d_i}\right) \geq 10.$$

Equations (141), (142), and (143) reduce to three equations in the unknowns τ_u , d_o , d_i , and L when the parameters ω_s , σ_{max} , $y(L,0)$, E , and ρ_i are known, with equation (144) serving as a sort of final check.

These parameters represent information that comes from knowing the type of measurement system used and what material to make the flow tube from. If this information is known, the design problem becomes a simple optimization that could be done easily on a computer, utilizing perhaps a steepest ascent routine or some similar program.

When this information is not known, and the design question includes the need to find a material which will give the best response, this design routine must be combined with trial-and-error to locate the optimum material and the best system characteristics. This study falls into that category. In fact, one of the most time-consuming segments of this project concerned the search for materials.

At first, work centered on using Hamel's original design procedure, with the pressure drop constraint. Early calculations showed that the mass of the tube was the primary variable of interest, and that it needed to be as small as possible. Further computations showed that the two secondary parameters of density and wall thickness (which directly affect the mass) were the variables which most affected the theoretical response of a given tube. In what was primarily a trial-and-error routine (although the term "iterative search" sounds more professional) a simple BASIC computer program was used to solve the three equations in

Hamel's synthesis procedure for length and inner and outer diameters when a host of input data was entered. This input data consisted of material properties, allowable stress, allowable pressure drop, fluid density, and other bits of information. Materials which were checked out in this manner were chosen primarily for low density, and included magnesium alloys and several plastics. Although the program worked, it became apparent that the geometric values which were being returned had no relationship to the actual ability of the tube to measure flow. That is, perfectly reasonable values of length and diameters, with respect to what would be a good size for a bench top prototype, would yield tubes whose sensitivities were so low as to be useless.

At this point, it was decided that in some way the maximization of the decay constant needed to be included in the overall search. The pressure drop constraint was removed from a position of primary concern, and the quantity

$$(145) \quad \tau^{-1} - \tau_e^{-1}(1 - \beta) = \beta \frac{K'_x}{K_x} \frac{u}{L}$$

was included, with the condition that $\beta \frac{K'_x}{K_x} \frac{u}{L}$ (hereafter referred to as τ_{diff}^{-1}) would be maximized for some u (equivalent to maximizing τ_u for a constant density process). This u was chosen to be $u = u_m \approx 300$ ft/s, and with the assumption of constant density, τ_{diff}^{-1} becomes a function only of the tube parameters density, length, and inner and outer diameter. By this time, the material search had narrowed to plastics, due to the relatively low densities of such materials. Magnesium, the lightest structural metal, had shown very little promise and so had been dropped from consideration.

The BASIC computer program was modified to allow as an input a desired value of decay constant. The output again consisted of the geometric parameters for the tube, which were visually scanned for promising combinations. As can be imagined, this process got rather tedious, and a different approach was taken. It was noted that

could be manipulated as

$$(146) \quad L \tau_{diff}^{-1} = \frac{(K_x'/K_x)u}{R_p(R_d^2 - 1) + 1}$$

where R_d and R_p are defined as before in equation (140). For a fixed u ($= u_m$) and given material the quantity $L \tau_{diff}^{-1}$ can be plotted as a function of diameter ratio R_d for air as the flowing fluid. Figure 11 shows plots for several materials. This graph greatly reduced the amount of time required in running through material checks. Once the material curve was plotted, a diameter ratio was selected, producing a $(L \tau_{diff}^{-1})$ quantity. The length L was selected as seemed reasonable (usually between one and two feet), and τ_{diff}^{-1} was quickly determined. The inner diameter was then determined by ensuring that $L/d_i > 10$ (and that d maintained a reasonable value for a bench top prototype); and then the outer diameter was computed. The procedure was admittedly rough around the edges, but worked quite well in speeding through material checks. Once the geometric quantities were obtained, other quantities such as natural frequency were computed. The stress requirement could be met by adjusting the initial deflection, if necessary. This type of procedure could be used effectively as a first step in a general design sequence, such as that outlined previously, to narrow a wide selection of materials down to a handful. In the case of

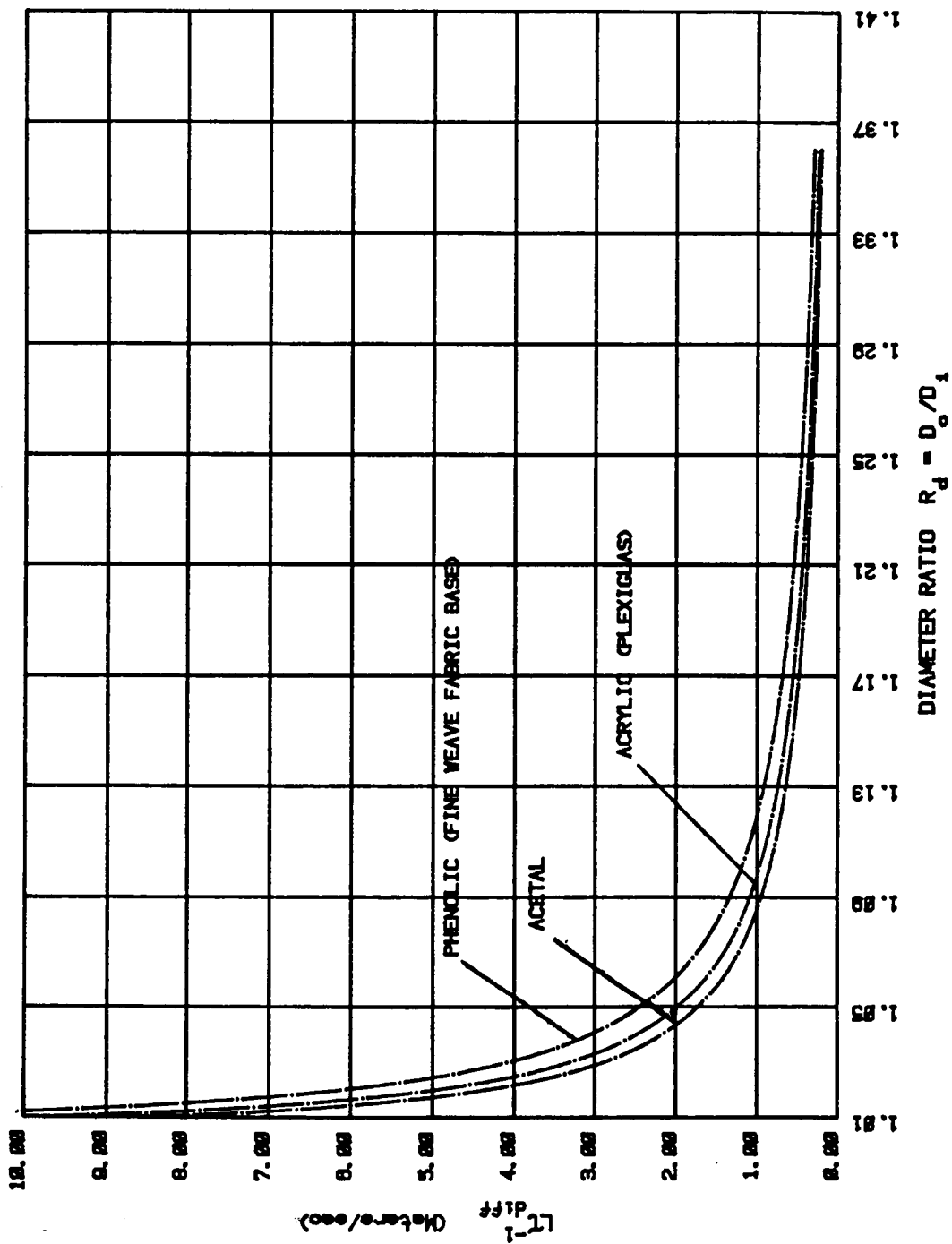


Figure 11. Comparison of Material Properties.

this project, considerations of economy and availability quickly narrowed the field to three sets of tubes: 1.27 cm x 0.15875 cm (1/2 inch x 0.0625 inch) plexiglas, 1.5875 cm x 0.15875 cm (5/8 inch x 0.0625 inch) fabric-filled phenolic, and 1.5875 cm x 0.15875 cm (5/8 inch x 0.0625 inch) plexiglas. These are the tubes which were used in the tests to be discussed in the following chapter.

The choice of plastics as the tube materials to be tested has some bearing on measurement ability. Although the materials which were selected are easily available and have low densities, plastics in general possess a non-linear stress-strain relationship where strain is not only dependent on the load but also on the duration of the load application. Thus, the classical Hookean engineering formulas, which work so well for metals, cannot be used when working with plastics [11]. Brostow [13] separates the strain in plastic materials into two parts: an elastic strain similar to that in metals, and a viscous strain resulting from the fact that plastics will tend to flow (much like a fluid) in the presence of a load. It is this viscous strain which is time dependent, and grows larger as the load duration is increased. In fact, if a plastic structure were subjected to a static load long enough, the deformation could be permanent, even if the load did not cause the elastic strain to exceed the limit necessary to cause failure. This behavior shows up in the vibration characteristics of the cantilever tubes, as mentioned before in the development of the measurement algorithm, and thus can affect the ability of the measurement system. Because the static deflection load is applied for a finite length of time, there is some buildup of viscous strain in the

tube, as well as an instantaneous formation of elastic strain in the tube. Once the load is removed, the elastic strain vanishes instantly, much as in the case of metals, but there is a finite time necessary for the dissipation of the viscous strain, and thus the vibratory waveform of the tube appears as an oscillation occurring about a non-zero, time-varying offset that eventually approaches zero. This is not the case with metals, where all of the strain vanishes instantly upon removal of the load (assuming deflections have remained within the elastic range) and oscillation thus occurs about the zero (or static) equilibrium position [14].

The effects of the viscous strain can be minimized by applying and removing the load very quickly. Brostow [13] defines a relaxation time as the time required for the viscous (time-dependent) strain to equal the elastic strain as

$$(147) \quad t_R = \frac{\mu}{G}$$

where μ is the coefficient of viscosity and G is the elastic shear modulus. For load durations which are much less than the relaxation time, material behaviour approaches that describes by Hooke's law. Thus, in establishing the static deflection curve for the tubes, it will be desirable to use as little time as possible in order to minimize the effects of the non-linearities on the measurement system (i.e. minimize the $f(t)$ offset described previously in the section on measurement algorithms).

The relaxation time parameter can also be helpful in choosing materials. It is desirable to have t_R approach infinity, since that

would imply zero viscous strain. Thus, a desirable material would have a very large viscosity and a relatively small shear modulus. The exact values of these parameters would need to be balanced against the other design constraints.

CHAPTER IV

EXPERIMENTAL SETUP

To carry out the proof-of-principle testing as proposed in this study, it became necessary to design a housing to hold the tubes, the selection of which was discussed in the previous chapter, and to put together an experimental setup capable of adequately performing the functions of data acquisition and control.

Design of Flowmeter Housing

The successful adaptation of the flowmeter into an in-line device hinges upon the ability to convey the fluid leaving the end of the sensing tube back into the line. This is the function of the flowmeter housing. Figure 12 shows what the flowmeter housing looked like for the prototype tested in this study. As can be seen, it consisted of a laminated construction of plexiglas plates with a chamber area, in which the tubes were housed, leading into a receiver/diffuser segment, which converted some of the dynamic pressure of the flow back into static pressure in an effort to minimize pressure losses across the flowmeter. The flow was then necked back down to the exiting line size and excited back into the line.

A second feature of the housing was that it must hold the necessary hardware required to deflect the tubes. In this case, deflection was provided by a solenoid sealed in a plexiglas container which was mounted to the top of the flowmeter housing.

The laminated assembly was connected at either end to PVC flanges,

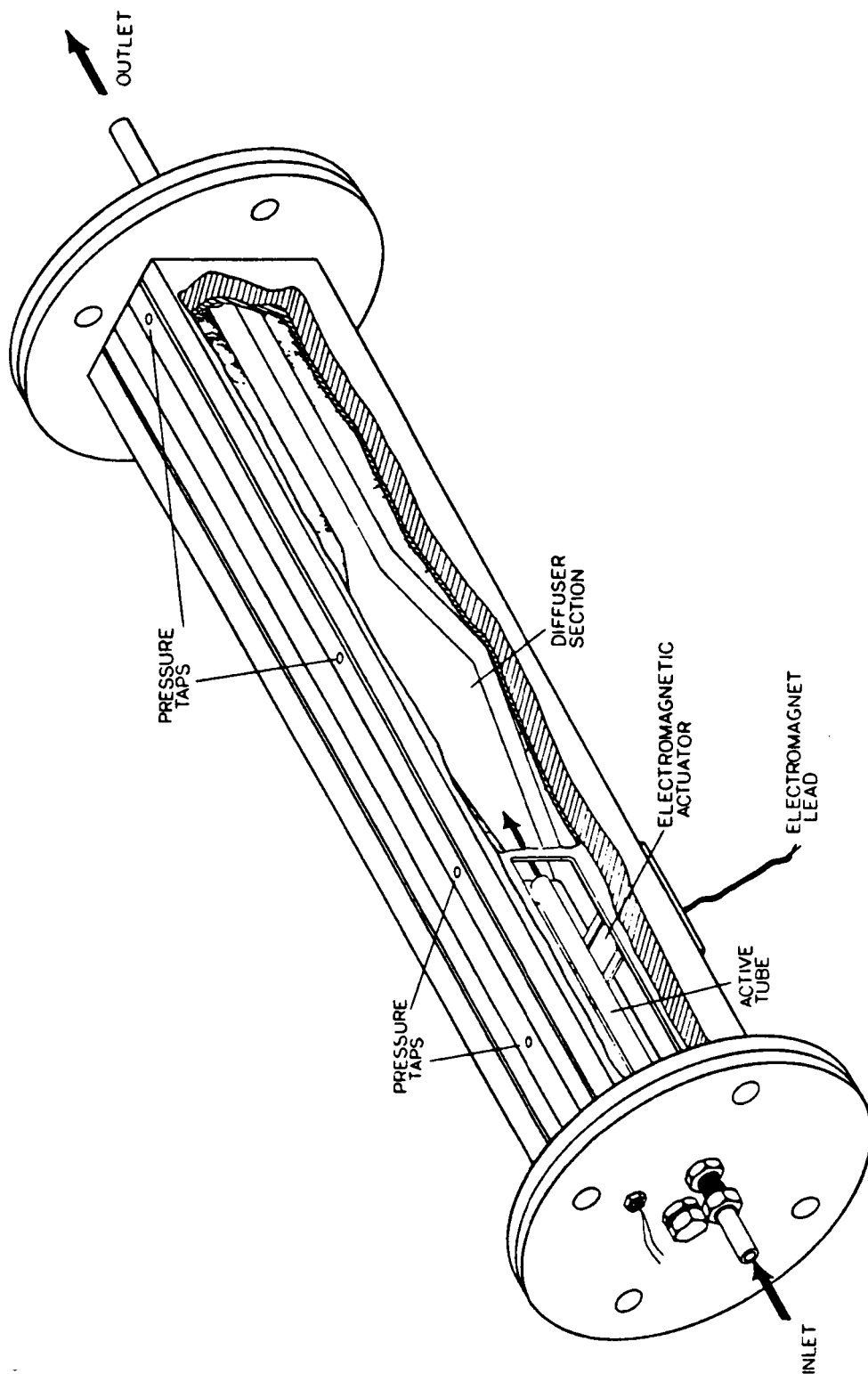


Figure 12. Prototype Housing.

and the incoming and outgoing sides of the line could be flanged to the housing. One advantage was that the tubes themselves were mounted in the supply line flange, and the whole assembly (tubes and flange) could be removed simply by unscrewing the eight flange bolts. This allowed easy access to the tubes.

No special provisions were made in the design other than an attempt to make it small enough to fit easily into the laboratory.

Basic Experimental Approach

One of the main points of interest in this work is the determination of usefulness and effectiveness of the cantilever Coriolis concept in measuring flow rates of gas flow. To this end, it was necessary to compare the performance of the Coriolis flowmeter against the performance of another flowmeter. In this way, a qualitative idea of the Coriolis flowmeter's performance relative to this secondary flowmeter could be formulated and used as a means of ascertaining the ability of the Coriolis concept to measure the mass flow rate.

In order to simplify the test setup as much as possible, it was decided to use available laboratory compressed air as the working fluid and set up the flow circuit as an open loop, with the air exhausting to the atmosphere. As mentioned in Chapter III, the Coriolis flowmeter was calibrated relative to a flow element of known accuracy (henceforth referred to as the secondary flow element), and all the data were reduced to the form of plots of the Coriolis' output versus the secondary flow element's output. The basis of comparison thus became the linearity with respect to a line and repeatability about that line, or how well

the two meters tracked together as the flow rate varied. This procedure was first used by Hamel [2]. "Good results are characterized by 'tight' clustering of the data around straight lines when the flowmeter outputs are plotted against each other." The selection of a secondary flow element was based on availability and economics. It was decided to use a flow measurement system based around a laminar flow element, which was available in the Metrology Laboratory of the Oak Ridge National Laboratory (ORNL) Instrumentation and Controls Division. The laminar flow element is basically a tube bundle which works by reducing the Reynold's number of the flow to the point where the flow is laminar. In this type of flow, the pressure loss due to frictional effects is a linear function of velocity, so the output of the laminar flow element is in the form of a pressure drop which can be converted easily to flow velocity. With independent measurements of the temperature and pressure of the flow to determine the density of the flow, the mass flow rate may thus be calculated. The particular system used in these tests consisted of a Meriam Laminar Flow Element, model number 50MC2-2, rated at a maximum capacity of 100.00 standard cubic feet per minute (SCFM), a Diametric Barocel differential pressure sensor, a thermocouple and System Teknik digital temperature readout for temperature measurement, and a Hewlett-Packard 9825B calculator to turn the measured pressure drop and temperature into a volume flow through the circuit at standard temperature (20°C) and pressure (1 atmosphere). A Wallace and Tiernan pressure gauge, with a resolution of 0.1 inches of water, was used to measure the static pressure of the flow at the inlet to the Coriolis meter, and this together with the volume flow through the circuit at STP

established the mass flow through the circuit. A complete list of the equipment which was used is provided in Appendix B.

Manufacturer's specifications on the laminar flow element state that "Meriam Instrument certifies that the completed LFE unit has been calibrated and correlated at several points of flow rate to within $\pm 1/2\%$ of Meriam Flow Standards, which are controlled per the calibration system requirements of MIL-STD-45662 and traceable to the National Bureau of Standards." A copy of the calibration curve is presented in Figure 13. As can be seen, the flow element's response is, to all appearances, very linear to over 100 ACFM (actual cubic feet per minute) at 70°F and 29.92 inches Hg absolute. In addition, it was observed that this laminar flow element was used as a calibration standard for a rotameter, so it is felt that sufficient accuracy was provided by it.

The flow circuit consisted of three meters connected in series in the following order: a rotameter, the Coriolis meter, and then the laminar flow element. The rotameter was used in conjunction with a pressure regulator and valve combination located at the air supply tap to quickly establish (i.e. eyeball) a flow measurement point. Fine tuning was provided by the laminar element. The control afforded by such an arrangement was adequate to allow reasonable repeatability of set points. Figures 14.a. and 14.b. are photographs of the flow circuit. The panel-mounted pressure gauges shown on top of the Coriolis flowmeter were used to measure the pressure drops across various segments of the meter, with the maximum pressure drop across the entire flowmeter on the order of 0.15 to 0.22 mm Hg (two or three inches of water) at a maximum flow of 0.7 SCMM (25 SCFM).

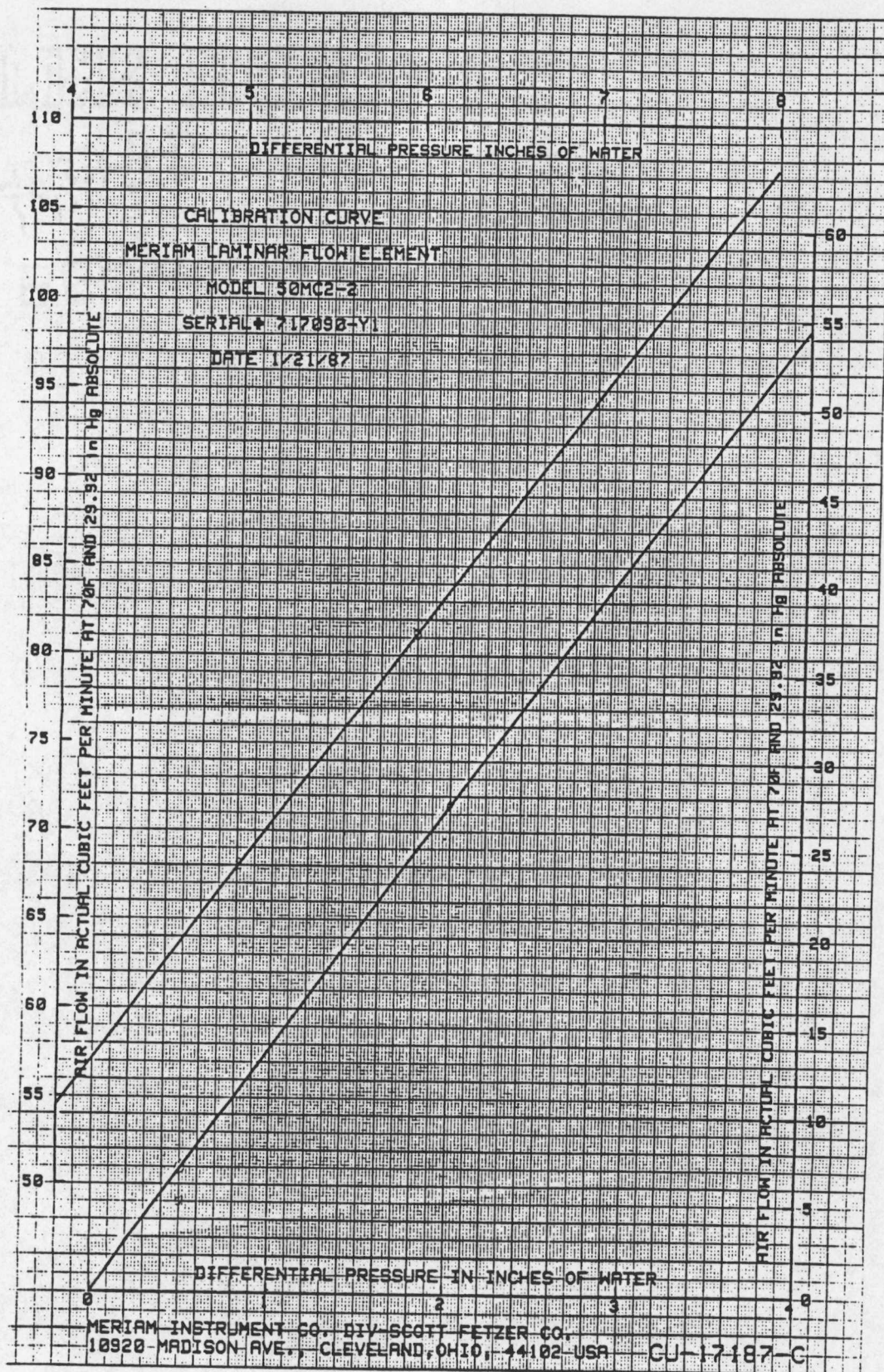


Figure 13. Laminar Flow Element Calibration Curve.

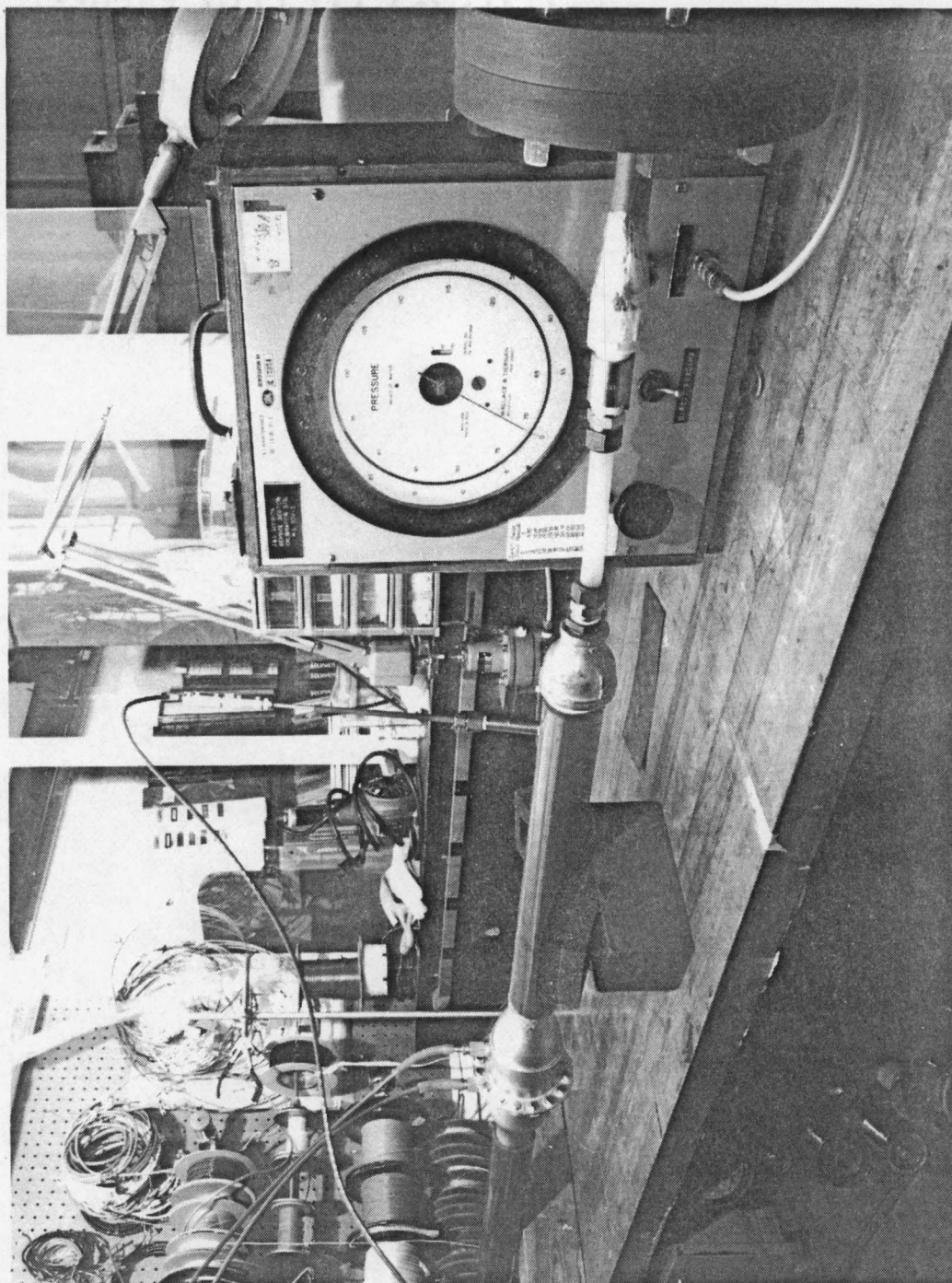


Figure 14. Flow Circuit.

(a) Laminar flow element.

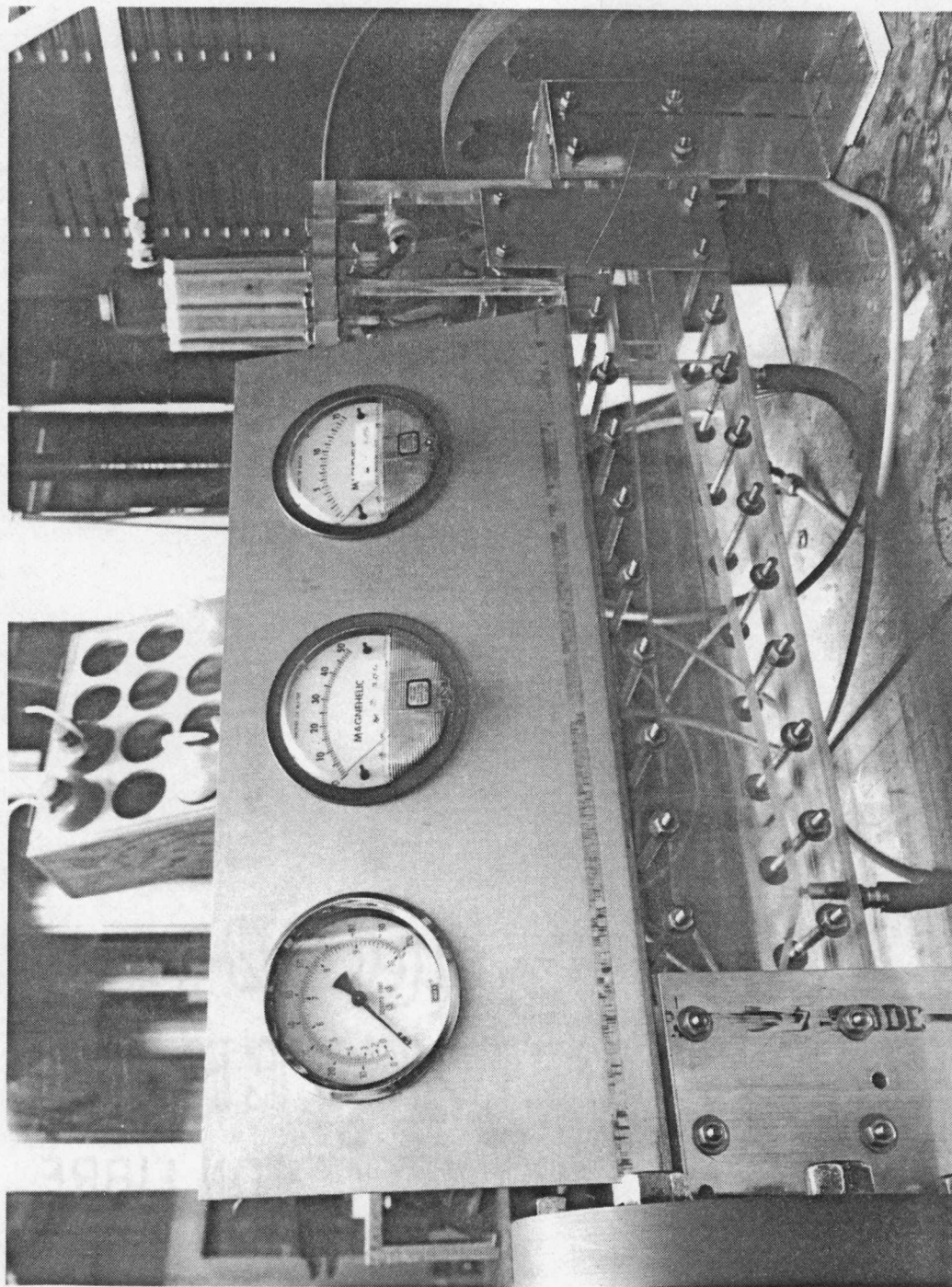


Figure 14. (continued)

(b) Coriolis flow meter.

Experimental Measurement and Control System

The measurement and control system must serve three basic purposes. These are: (1) control the operation of the flowmeter, specifically the deflection mechanism, (2) record and store the beam deflection data, and (3) perform the necessary flow measurement and calibration algorithm computations. Additionally, the system was used to plot the analytical and theoretical data. The secondary flowmeter was tied into a separate HP 9825B calculator via an IEEE 488 bus and the data from the flowmeter was transferred to the Coriolis measurement system through the keyboard of the VT100 monitor used for the Coriolis system.

Figure 15 is a schematic of the experimental measurement and control system. The centerpiece of the system is a Digital Equipment Corporation (DEC) LSI-11/23 16-bit microprocessor. Data storage was provided by a ten megabyte hard disk drive and three 512K floppy disk drives. Operation of the system occurred through a monochrome terminal connected to the microprocessor through an RS-232C serial interface. A Hewlett-Packard digital plotter (model 7225A) was also connected via the serial interface. The cantilever deflection data was read through an eight-channel, twelve-bit analog to digital converter (Data Translation model DT2782-DI). Precise sampling intervals were provided by a programmable real-time clock (Data Translation model DT2769) operating at a one megahertz clock rate. The beam excitation actuator, an AC solenoid, was controlled by using one output bit of a 16-bit parallel input/output interface (DEC model DRV11).

The DEC RT-11 realtime operating system was used to operate the measurement and control system with all programming written in DEC

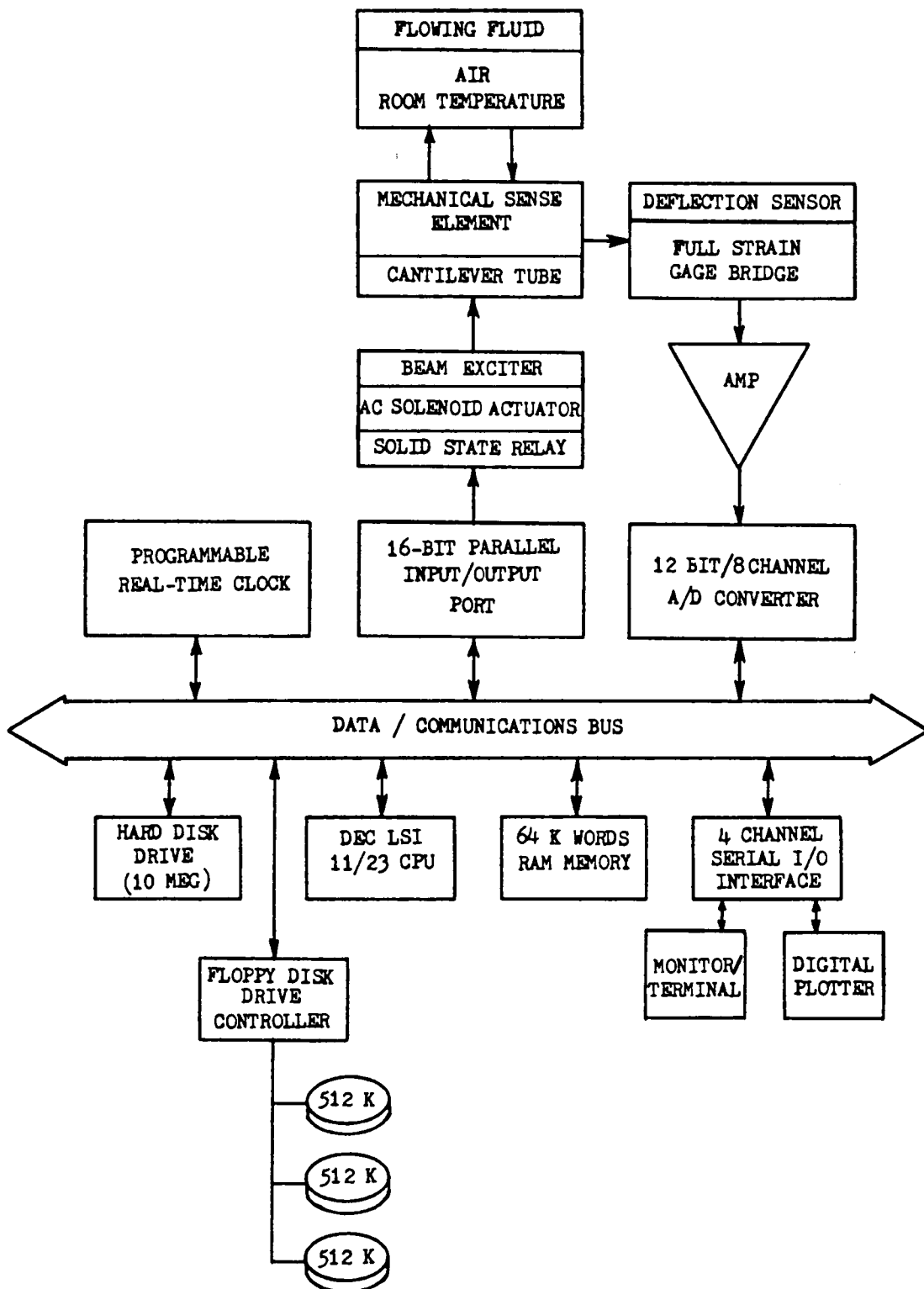


Figure 15. Block Diagram of Coriolis Flowmeter Experimental Measurement and Control System Based on Idea from Hamel [2].

FORTTRAN IV. The exception to this was the library of machine language programs provided by Data Translation, which were used to control the realtime clock, A/D converter, and the parallel I/O interface. Listings of the FORTRAN codes which were used are presented in Appendix A.

In operation, the two beam channels were sampled at intervals of 0.2 to 0.25 ms for approximately 500 samples. This is a throughput rate on the order of 8,000 to 10,000 channels/second, a rate which was deemed satisfactory for these tests. Figures 16 and 17 show the PDP 11/23 computer used for the measurement and control system, and the electronics associated with the laminar flow element.

Flowmeter Setup and Flow Sensors

The mechanical configuration of the Coriolis flowmeter was described in Chapter III. Figure 18 illustrates the basic layout. The end flange on the inlet side holds the two cantilever tubes in bulkhead fittings. One tube remained empty during the testing to provide the empty tube vibration characteristics. This was the reference tube. The other tube, the active tube, was positioned so that its exhaust entered into the diffuser segment. See Figure 19 for this detail.

Early design work focused on using an electromagnetic actuator to deflect the tubes. This approach was unsatisfactory from the standpoint that the actuator could not develop the necessary force required to deflect the tubes. The flowmeter housing was then modified to incorporate an AC solenoid actuator which was used to create the cantilever static deflection. The maximum deflection thus created was estimated to be approximately one centimeter. The solenoid was

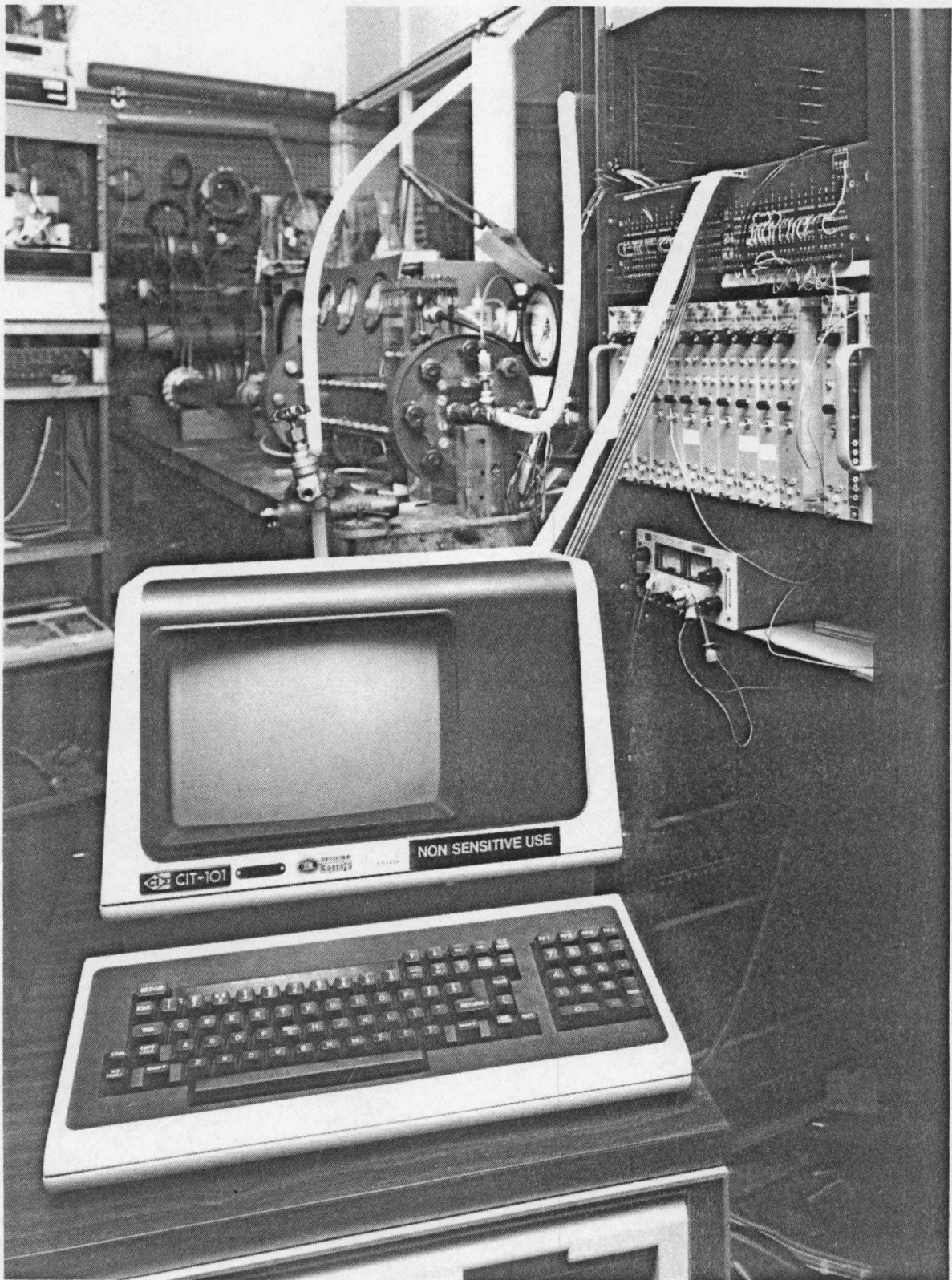


Figure 16. PDP 11/23 Computer and Strain Gage Amplifiers.

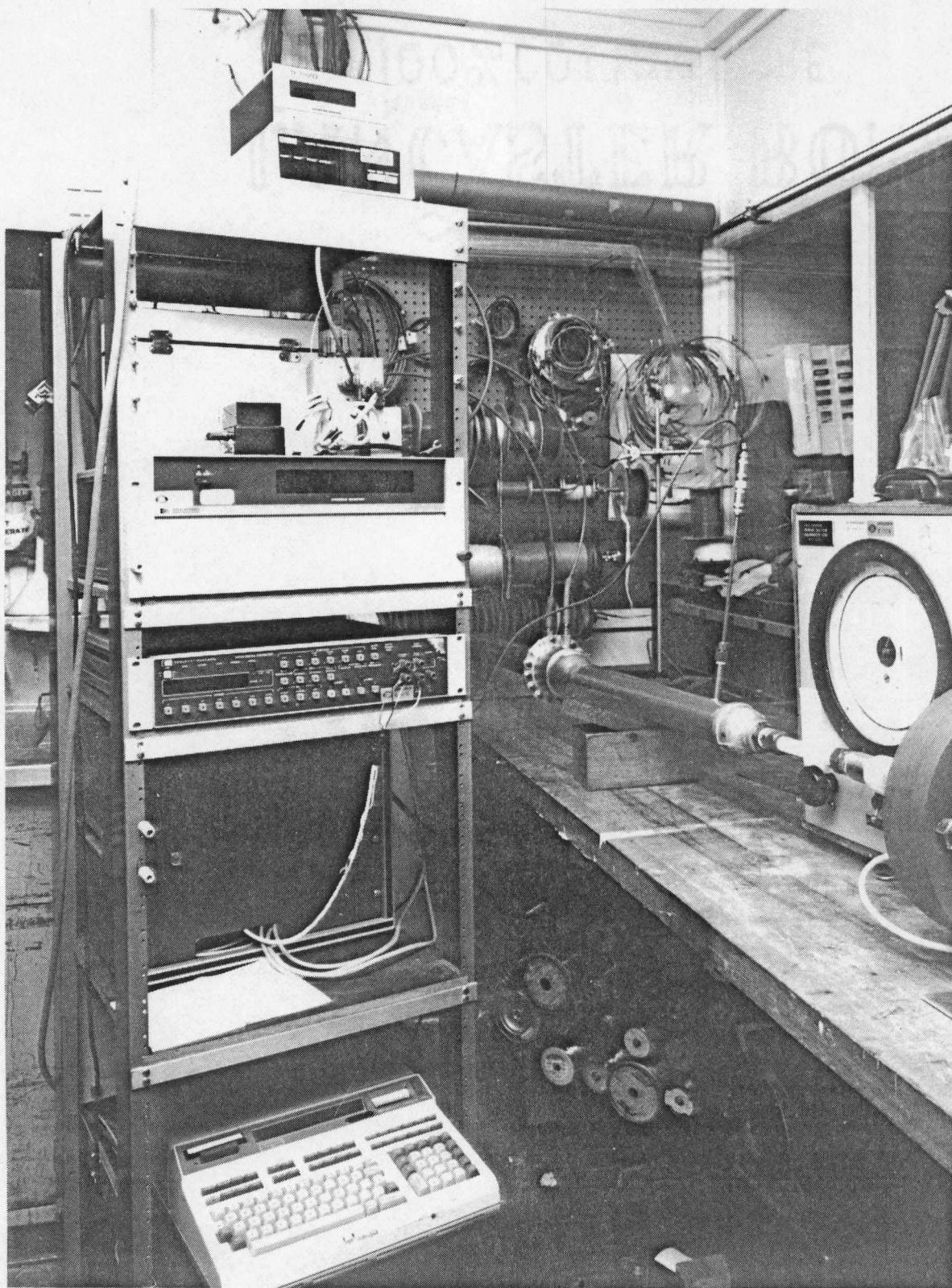


Figure 17. Electronics for LFE.

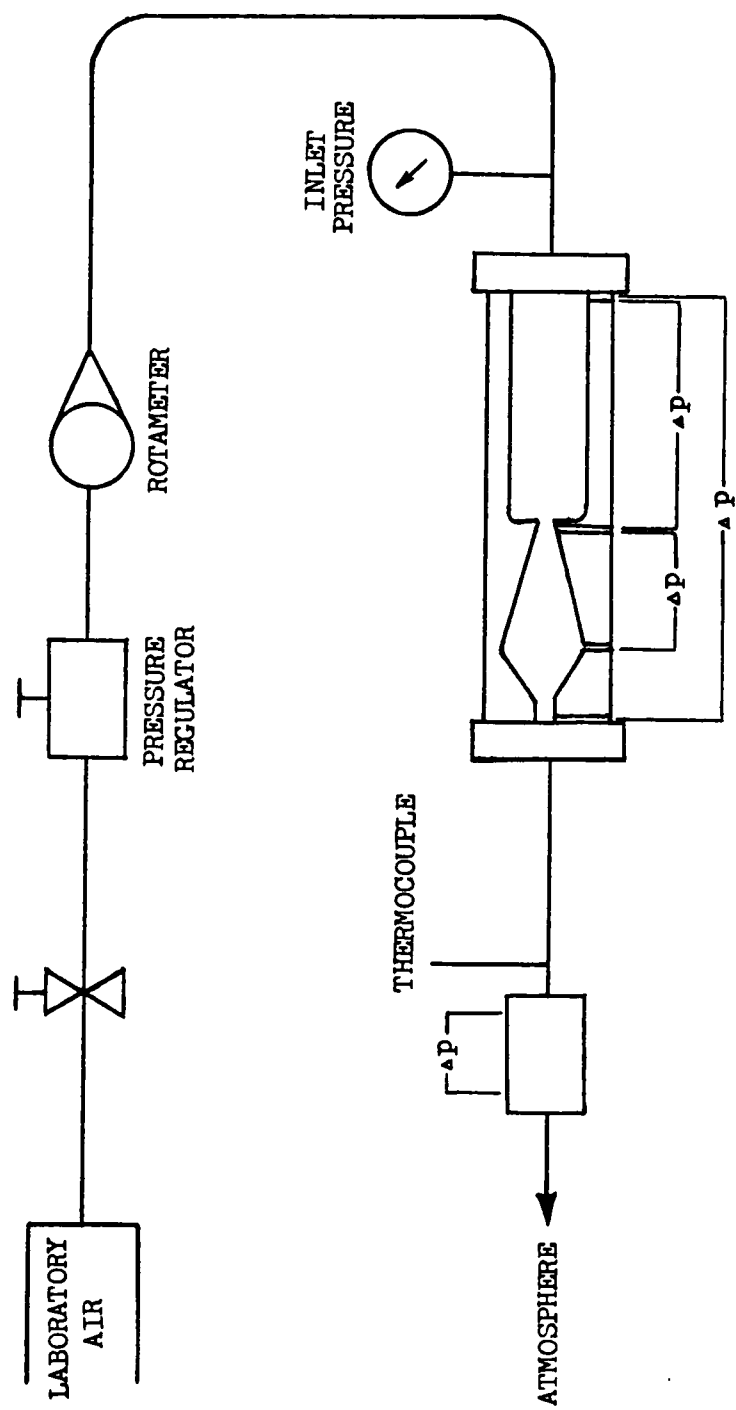


Figure 18. Schematic of Flow Circuit.

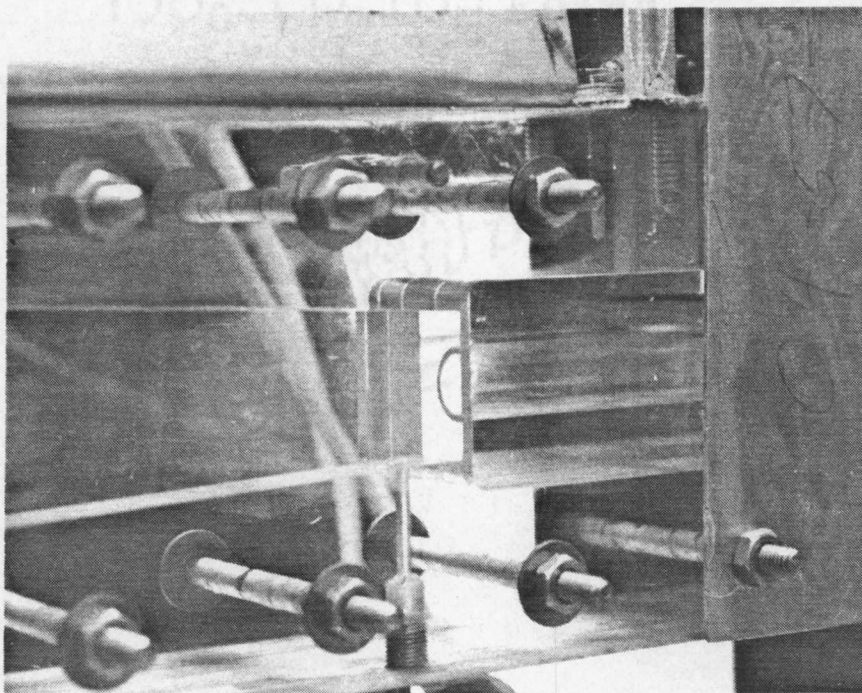


Figure 19. Close-up of the Cantilever Exhaust.

controlled via the DRV11 parallel I/O port through a solid-state relay, using machine language routines provided by Data Translation. The LSI 11/23 was programmed to pause for (approximately) one second upon energizing the solenoid to ensure that any vibrations would be damped out and the deflection curve would be truly static. The amount of pause obtained from the control program fluctuated somewhat due to the fact that the single-job monitor version of RT-11 had to be used with the measurement/control and calibration routines because of memory requirements. The single-job monitor version did not support the timer features which would allow more precise control of the pause time, so the computer was induced to pause momentarily by means of a loop.

The deflection transient from each tube was measured by means of a strain gage bridge composed of 350 Ω strain gages (Micromasurement type EA-06-125PC-350) attached near the fixed ends of the cantilever tubes. The strain gage lead wires were passed out through an airtight fitting in the inlet flange and connected to strain gage amplifiers (designed at ORNL). These amplifiers provided approximately 70db of amplification to bring the low-level strain gage signals up to the ± 10 volt input range of the A/D converter.

The bridge excitation voltage used with the gage was limited to between 2.0 and 2.5 volts, of approximately ± 4 volts peak to peak (maximum). The reason for this is related to the poor thermal conductivity of the plastic tubes. Too large a bridge excitation voltage would yield large amounts of drift from the gages, so the appropriate excitation voltages was chosen using trial-and-error approach to get as large an output as possible without introducing

unwanted large drift effects. This came at a price, of course.

Reducing the excitation voltage, and hence the bridge output, effectively reduced the signal-to-noise ratio and could have possibly had adverse effects on the results which are presented in the following chapter.

CHAPTER V

DISCUSSION OF RESULTS

With the development of the necessary measurement and calibration algorithms, as discussed in Chapter III, and the construction of a flowmeter prototype, as described in Chapters III and IV, the next step in the development of a cantilever Coriolis mass flowmeter for gases is testing the prototype and evaluating its performance against some standard. The testing discussed here is a proof-of-principle demonstration and the results illustrate qualitative trends and relationships rather than just quantitative results. Nevertheless, information gained from such testing should not be lightly esteemed because of the lack of "hard evidence". Results gleaned from proof-of-principle tests can demonstrate the accuracy of a model, provide insight into the actual physical relationships present in a system, verify (or negate) assumptions, and indicate whether more in-depth work should take place on a project.

Experimental Limitations

The results to be presented in this chapter are, like any experimental data, subject to certain limitations. The various parameter ranges which are relevant to these tests are shown in Table 2. Note that in all three cases the maximum flow rate is relatively low. This is due to the combination of using tubes with small cross-sectional areas and maintaining the maximum flow velocity at no more than 100 m/s (340 ft/s), as discussed previously.

Table 2. Experimental Parameter Ranges (Active Tubes)

Parameter	Tube Configuration		
	1.27 x 0.15875 cm (1/2 x 0.0625 inch) plexiglas	1.5875 x 0.15875 cm (5/8 x 0.0625 inch) phenolic	1.5875 x 0.15875 cm (5/8 x 0.0625 inch) plexiglas
Outside Diameter (mm)	12.7 (0.500 in.)	15.88 (0.625 in.)	15.88 (0.625 in.)
Inside Diameter (mm)	9.25 (0.375 in.)	12.7 (0.500 in.)	12.7 (0.500 in.)
Length (cm)	24.13 (9.50 in.)	23.50 (9.25 in.)	23.50 (9.25 in.)
Empty Natural Frequency (Hz)	74.37	94.16	87.36
External Damping Coefficient [(N-s)/m]	0.67 (0.0459 lbf-s/ft)	0.71 (0.0488 lbf-s/ft)	1.09 (0.0746 lbf-s/ft)
Nondimensional Parameters:			
$\hat{u} = \sqrt{\frac{m_f l_f^2}{E_f}} u$	0.0013	0.0021	0.0018
$\beta = \frac{m_f}{m_c + m_t}$	0.017	0.015	0.015
Maximum Flow Conditions:			
Volumetric Flow Rate (SCMH)	0.40	0.70	0.70
Mean Velocity (m/s)	100.0	100.0	100.0

As can be seen, the excessive amount of damping present at zero flow is indicative that material damping was far from negligible in these tests. In fact, it is possible that the majority of the damping present was due to material damping. The assumption that material damping was negligible is obviously a bad assumption. This could be dealt with in two ways: (1) re-derive an entirely new governing equation taking into account the material effects, or (2) ignore or try to subtract out the effects in the measurement. Since the actual figure of merit sought was a change in damping caused by the addition of the hydroelastic interaction, it was felt that a large amount of material damping could still be ignored if its effect was essentially independent of the hydroelastic interaction. Measurements of the empty tube responses for both tubes (reference and active) showed that the amount of damping was relatively constant, with deviations from the mean values on the order of one percent, even with some variation in the amount of time that the tubes were deflected. Additionally, since the temperature did not change greatly during testing (which could possibly have had a strong effect on the material viscosity and hence the material damping) it was felt that material effects, such as the damping and those discussed in Chapter III, could be separated out from the desired signal, as discussed in the development of the measurement algorithms, or ignored. This is not to say that these phenomena may not have adversely affected the results. In this study, though, they were dealt with in the manner and for the reasons already discussed.

Also, it is apparent from Table 2 that there was not an overwhelmingly wide range of tube materials used in these tests. As

pointed out in Chapter III, the tubes which were tested were selected with economics, availability, and potential usefulness all playing an equal role. It should be stated here for the record that the underlying goal of these tests was not to prove the cantilever Coriolis flowmeter theory; that has already been done by Hamel [2]. This study, and the selection of materials used in the testing, had the goal of demonstrating the possible usefulness and ability of a cantilever Coriolis mass flowmeter to measure the mass flow rates of gas flows.

The velocity constraint, which limited the maximum attainable flow rate, and the simplicity of the equipment used, which limited the minimum measurable flow rate, combined to limit the maximum rangeability in these tests to less than six.

A further limitation in this testing was that only air was used as the flowing medium. The choice of air was based on availability and the simplicity of the hardware required, but it made it impossible to judge the flowmeter's capabilities over a very wide density range.

Experimental Results

In order to get some idea of the overall performance of the cantilever Coriolis gas flowmeter, proof of principle tests for the three tube selections were performed with the intent of judging the quality of the Coriolis flowmeter relative to the laminar flow element. The basic approach, which was identical for all three sets of tubes, was to first verify that the unimodal assumption discussed in Chapter II was valid by performing a frequency spectrum analysis on both tubes of each set. After assuring that the vibration was unimodal, zero flow measurements

were taken for both tubes using the main flow measurement routine given in Appendix A. This was done to help insure that the material effects could be ignored while still obtaining useful results, and also so that (in later testing) the averaged empty tube parameters could be used, as will be discussed shortly. With the knowledge that the tube vibrations were satisfactory, the calibration routine given in Appendix A was executed to obtain the calibration constants from equations (124), (125), and (126). The laminar flow element was used as a secondary standard to obtain values for the flow rate and temperature. The density was determined by making a separate measurement of the pressure at the inlet to the cantilever tube. The maximum flow rate range was divided into approximately ten equal ranges and calibration constants were determined at each flow rate. Statistical averages and standard deviations were then computed and the average values were used in equations (127), (128), (129), (130), and (131) in the main flow measurement routine to obtain results from the cantilever Coriolis flowmeter. The flow range was again divided into 10 to 12 flow rates and data was obtained at each point. This data and the data from the laminar flow element were then stored and, upon completion of the data acquisition, plotted against each other for comparison. A linear regression was also performed to characterize the degree of linearity between the two outputs. The Coriolis density measurement was likewise compared with a computed density value based on measured temperature and pressure. In the following sections, the results are discussed for each set of tubes.

Results for the 1.27 x 0.15875 cm (1/2 x 0.0625 inch)

Plexiglas Tubes

The first set of tubes to be tested were the two 1.27 x 0.15875 cm (1/2 x 0.0625 inch) plexiglas tubes. Figures 20, 21, 22, and 23 show the frequency spectra and time histories of the active tube and reference tube, as taken by a digital signal analyzer (Hewlett-Packard model 5420). The frequency spectrum for each tube is quite nicely unimodal, with the natural frequencies occurring at approximately 72 Hz and 74 Hz for the reference tube and active tube, respectively. The time histories are also very clean and show signs of degradation only as the amplitudes get very small. The offset in the vibration reference point, the material effect mentioned in Chapter III, can also be noted. It should be pointed out that this slow stress relief appears in the frequency spectra as a large zero hertz content. The differences in frequencies and decay constants raise some implications concerning the measurement ability of the null-type configuration used in these tests, where a reference tube and an active tube are deflected together and the resulting signals are used to get the vibration parameters ω and τ^{-1} at zero flow and the current flow rate in the same measurement cycle. Preliminary testing using the algorithms presented in Chapter III yielded results that were highly inconsistent. The inconsistencies were traced directly to two sources: (1) difficulties in relating the empty tube parameters to the flowing tube parameters in the two tube approach, and (2) using too long a cable, as mentioned in the manual provided with the converter. As discussed in the manual, long cables could tend to adversely affect the solid-state multiplexer due to stray capacitive

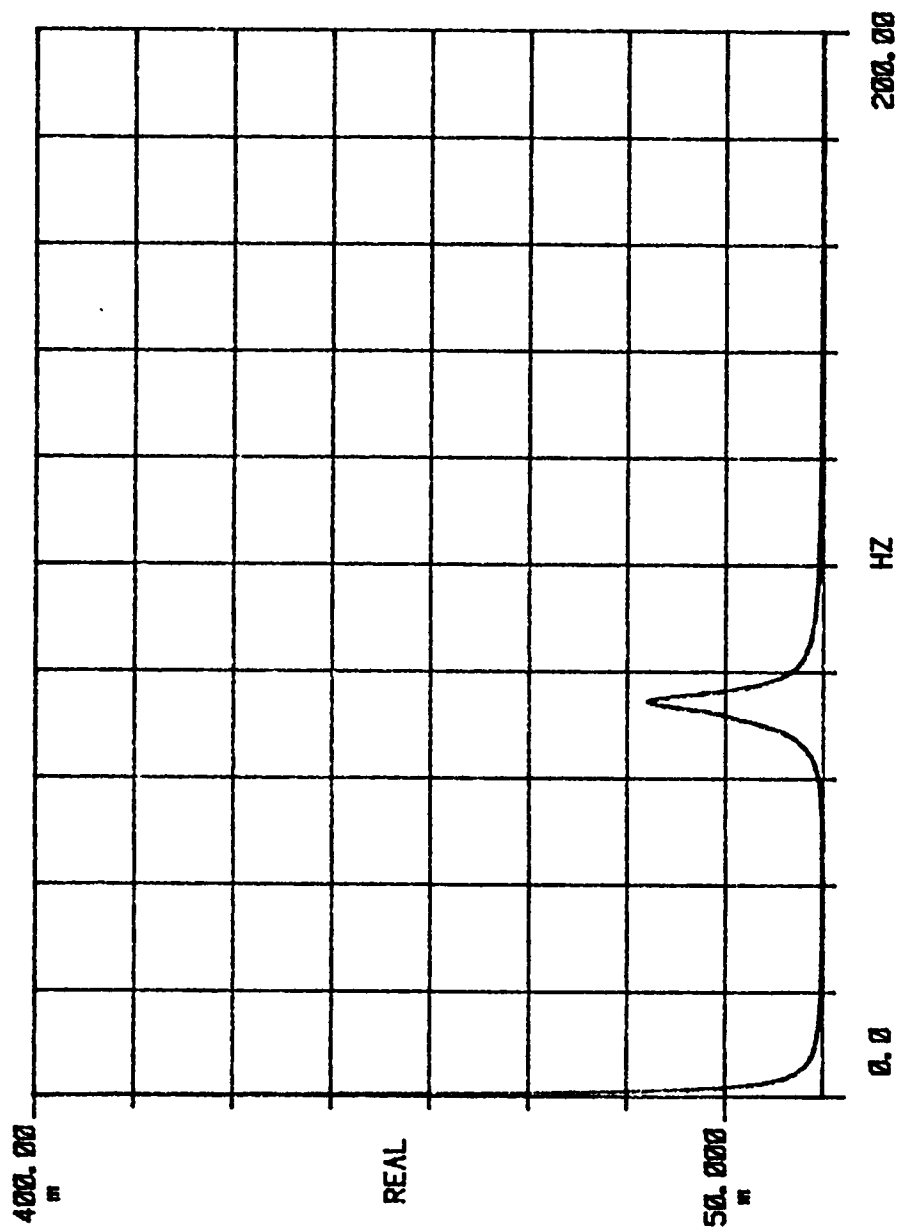


Figure 20. Frequency Spectrum of Active Tube for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

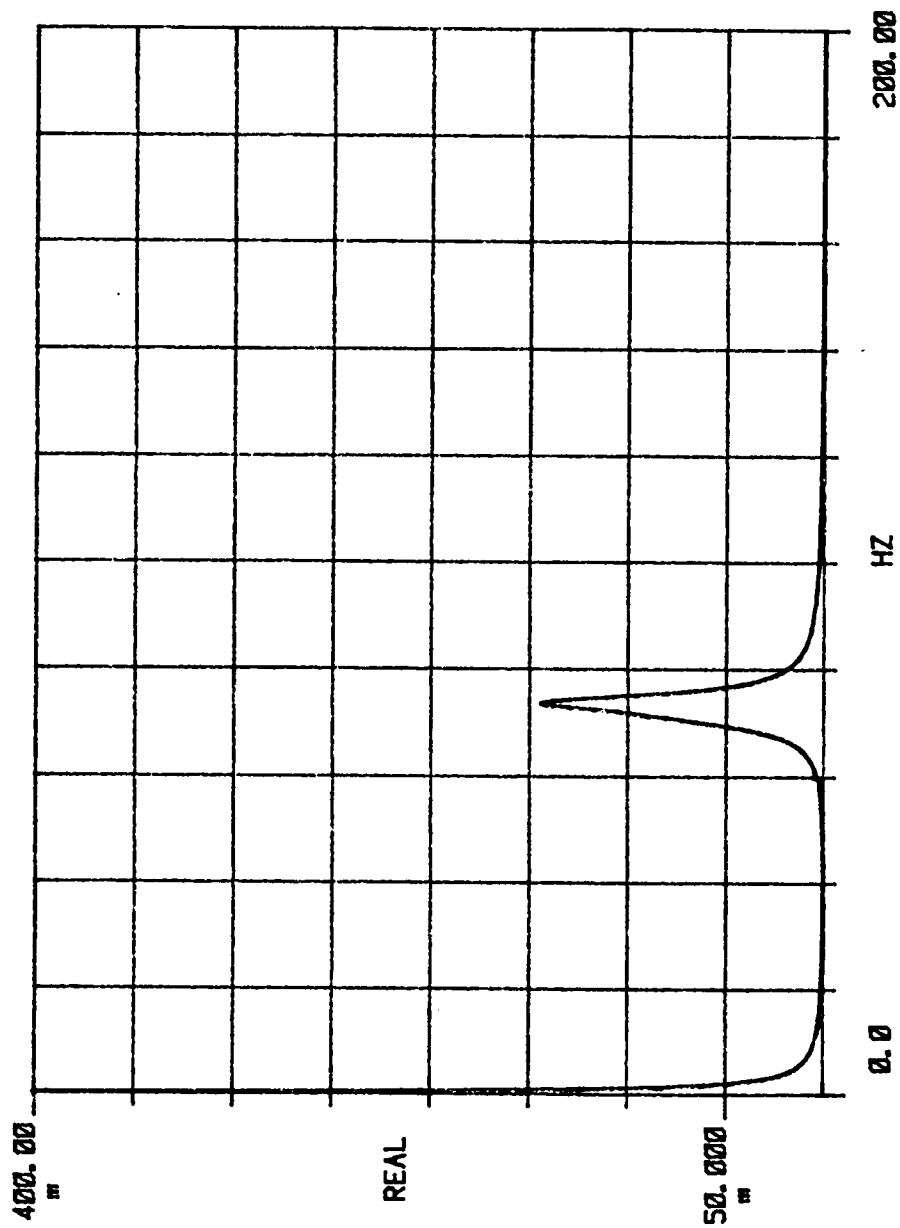


Figure 21. Frequency Spectrum of Reference Tube for 1.27 x 0.15875 cm
(1/2 x 0.0625 inch) Plexiglas Tubes.

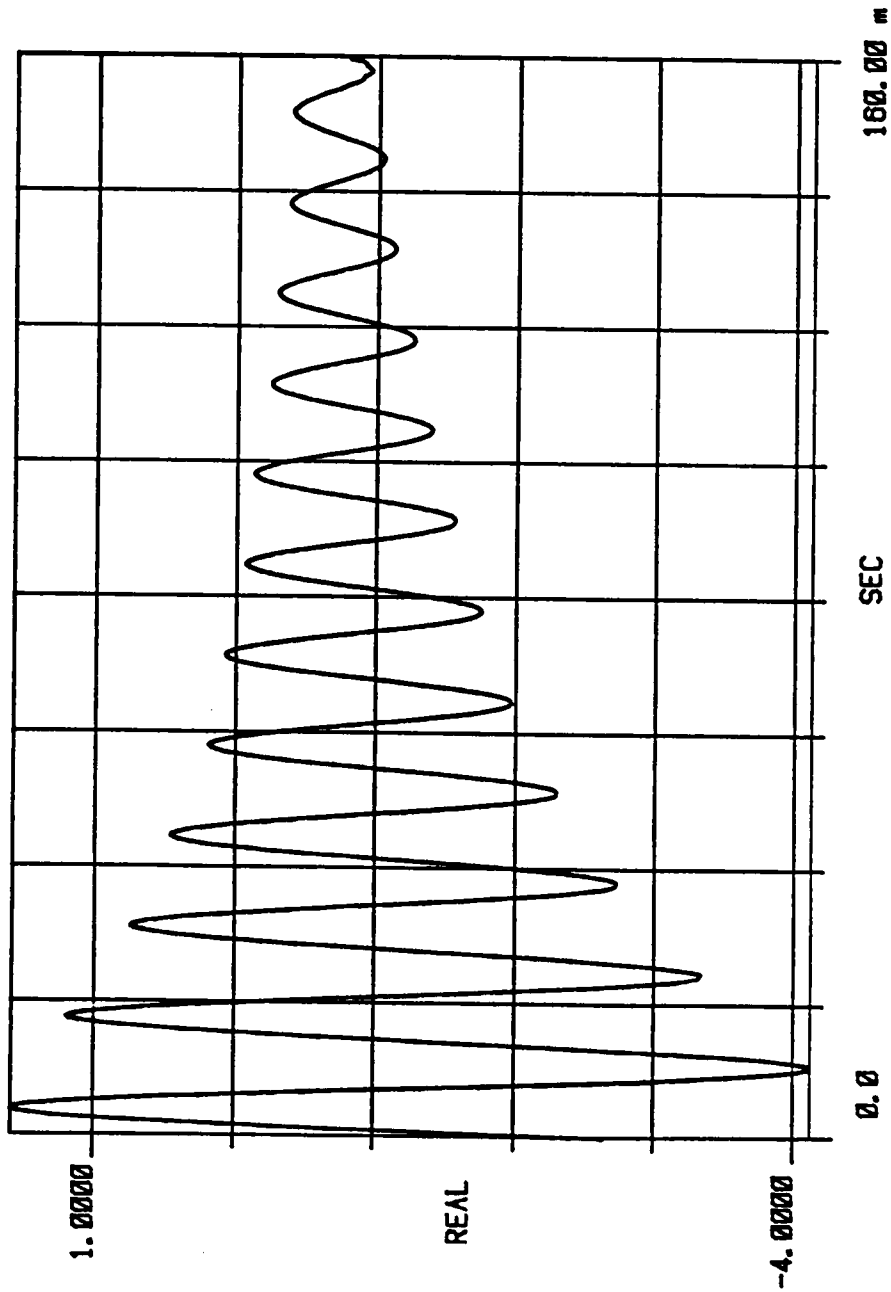


Figure 22. Time History of Active Tube for 1.27 x 0.15875 cm
(1/2 x 0.0625 inch) Plexiglas Tubes.

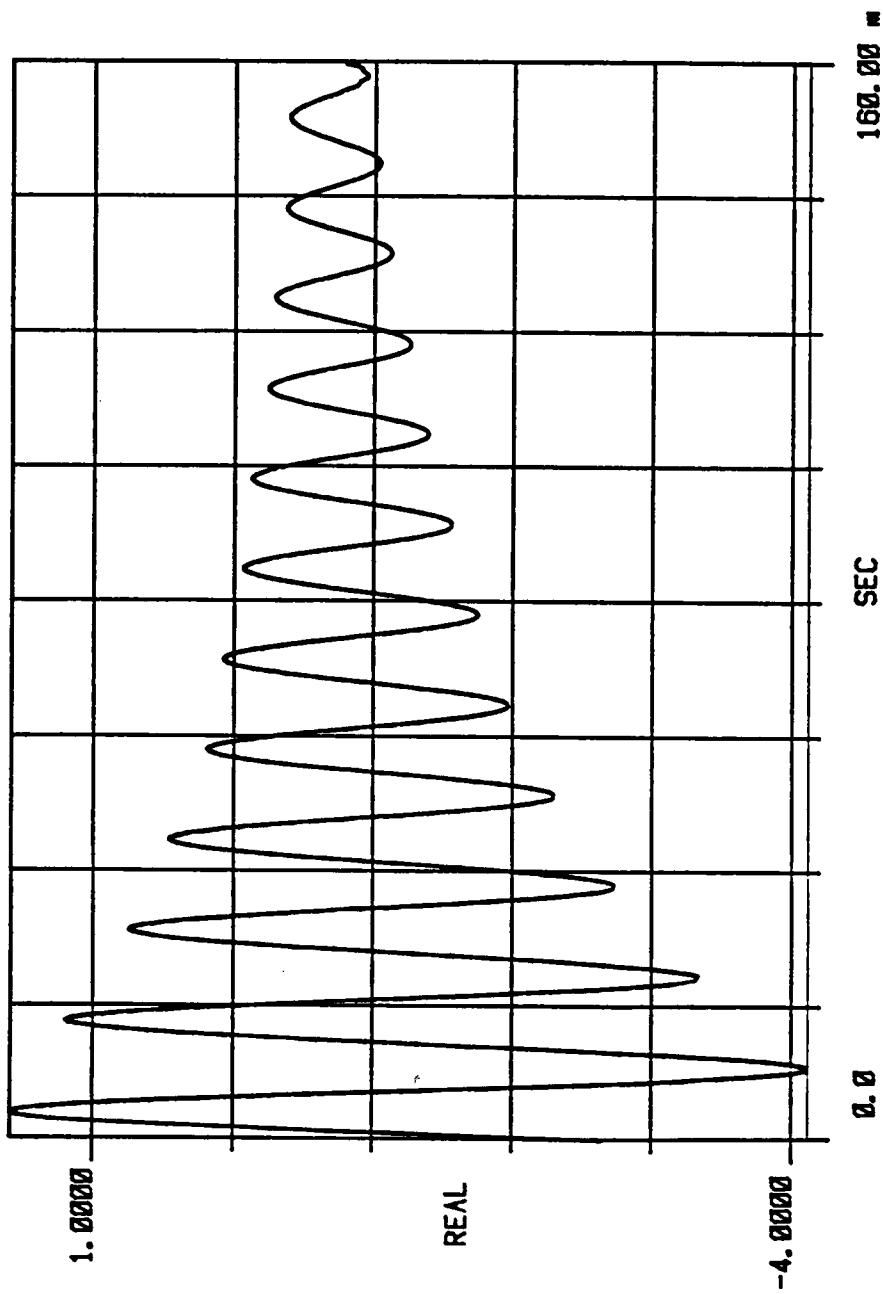


Figure 23. Time History of Reference Tube for 1.27 x 0.15875 cm
(1/2 x 0.0625 inch) Plexiglas Tubes.

buildup [15]. To connect the analog inputs to the A/D converter. The second difficulty was remedied easily by simply shortening the cable. The first difficulty was satisfied in two ways. First, since all evidence showed that frequency shift due to flow would be negligible, the empty tube frequency R_{c2} was set equal to the active tube frequency in equation (127), so that now

$$(127)' \quad (1 - \beta) = \frac{R_2 + R_1^2}{a_3(R_2 + R_{c1}^2)}$$

This equation was used in the calibration and measurement algorithms for the 1.27 x 0.15875 cm (1/2 x 0.0625 inch) plexiglas tubes. Table 3 shows values of frequency and decay constant for ten

Table 3. Empty Tube Vibration Characteristics For 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes

Active Tube		Reference Tube	
ω	τ^{-1}	ω	τ^{-1}
(rad/s)	(s ⁻¹)	(rad/s)	(s ⁻¹)
468.259	16.581	474.202	15.434
466.993	16.754	474.401	15.646
466.111	16.883	473.947	15.559
470.298	16.620	469.673	15.780
465.156	16.608	473.607	15.445
466.993	16.893	470.650	15.588
466.573	16.989	473.947	15.739
467.731	16.627	468.894	15.677
466.573	16.500	470.298	15.478
468.259	16.390	473.131	15.327
mean	467.295	472.275	15.567
std. dev.	1.425	2.136	0.146
std. dev.			
(% of mean)	0.3%	0.4%	0.9%

runs of the main flow measurement program at zero flow. As can be seen, the numbers are reasonably stable, and are in good agreement with values which can be determined from the time histories in Figures 22 and 23.

Figures 24 and 25 show the actual dynamic response of the active tube and reference tube. The voltage scales for the plots are one-tenth the actual values in order to make the change in amplitude due to flow noticeable. This should be expected, since the amount of Coriolis force from low density gases is not enough to create the forces which would be necessary to cause large amplitude changes. It is interesting to compare the experimental results to theoretical results, and Figure 26 is a plot of the nondimensional deflection versus time as predicted by the solution of the governing equation for a tube with plexiglas properties. As can be seen, the only significant difference between the plots is the presence of the offset reference for the vibration in the experimental plot, as discussed in Chapter III. The amount of amplitude change due to the hydroelastic interaction is seen to be fairly small in the analytical plot as well as the experimental results, and tends to bear out the belief that the real development of a Coriolis flowmeter for gases will hinge more on the ability to measure the small signal changes which occur rather than any major theoretical considerations.

With relative assurance that the tubes were vibrating satisfactorily, the calibration program given in Appendix A was run, with the results as shown in Table 4. Coefficients a_1 and a_2 are quite constant, showing zero variations over the flow range. Coefficient a_3 was not nearly as invariant, with a deviation of nearly 25% over the flow range of zero to ten SCFM. This is typical of all three

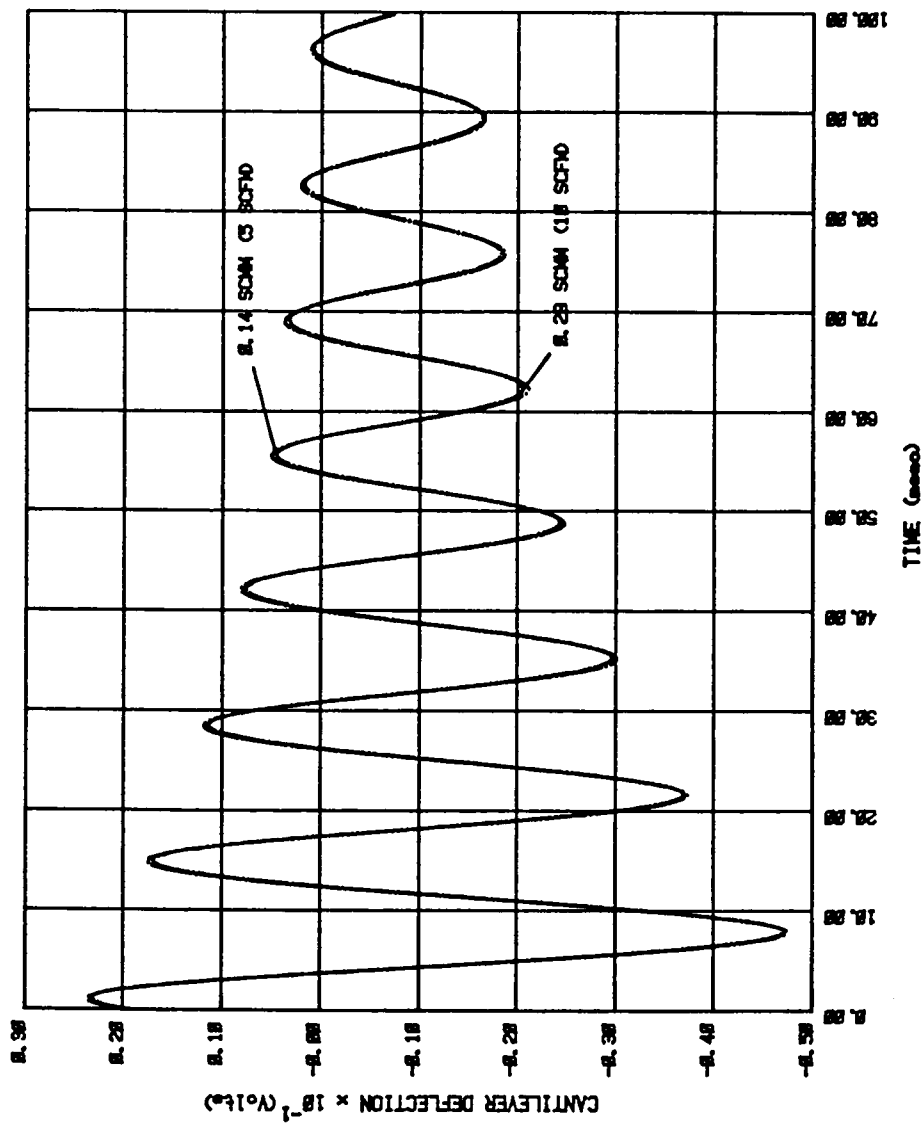


Figure 24. Typical Dynamic Response of Active Tube for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

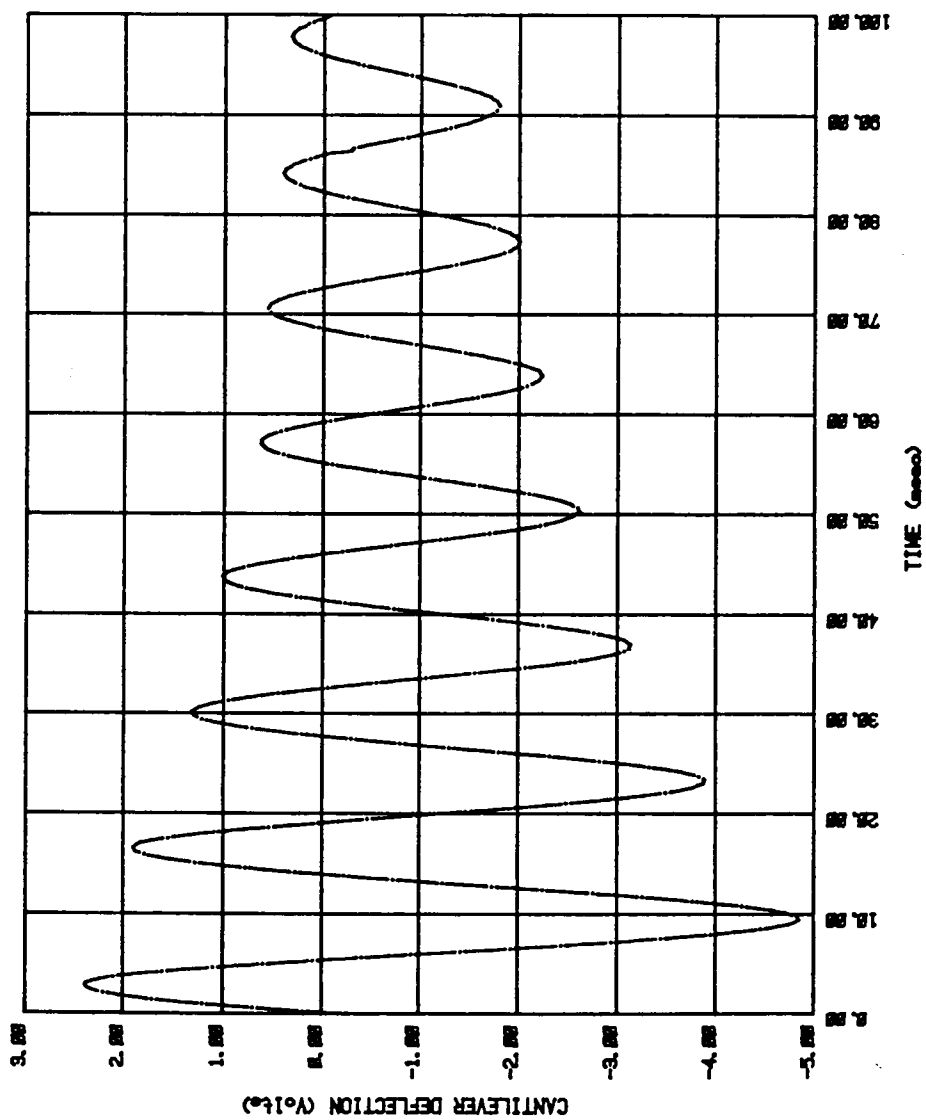


Figure 25. Typical Dynamic Response of Reference Tube for
1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

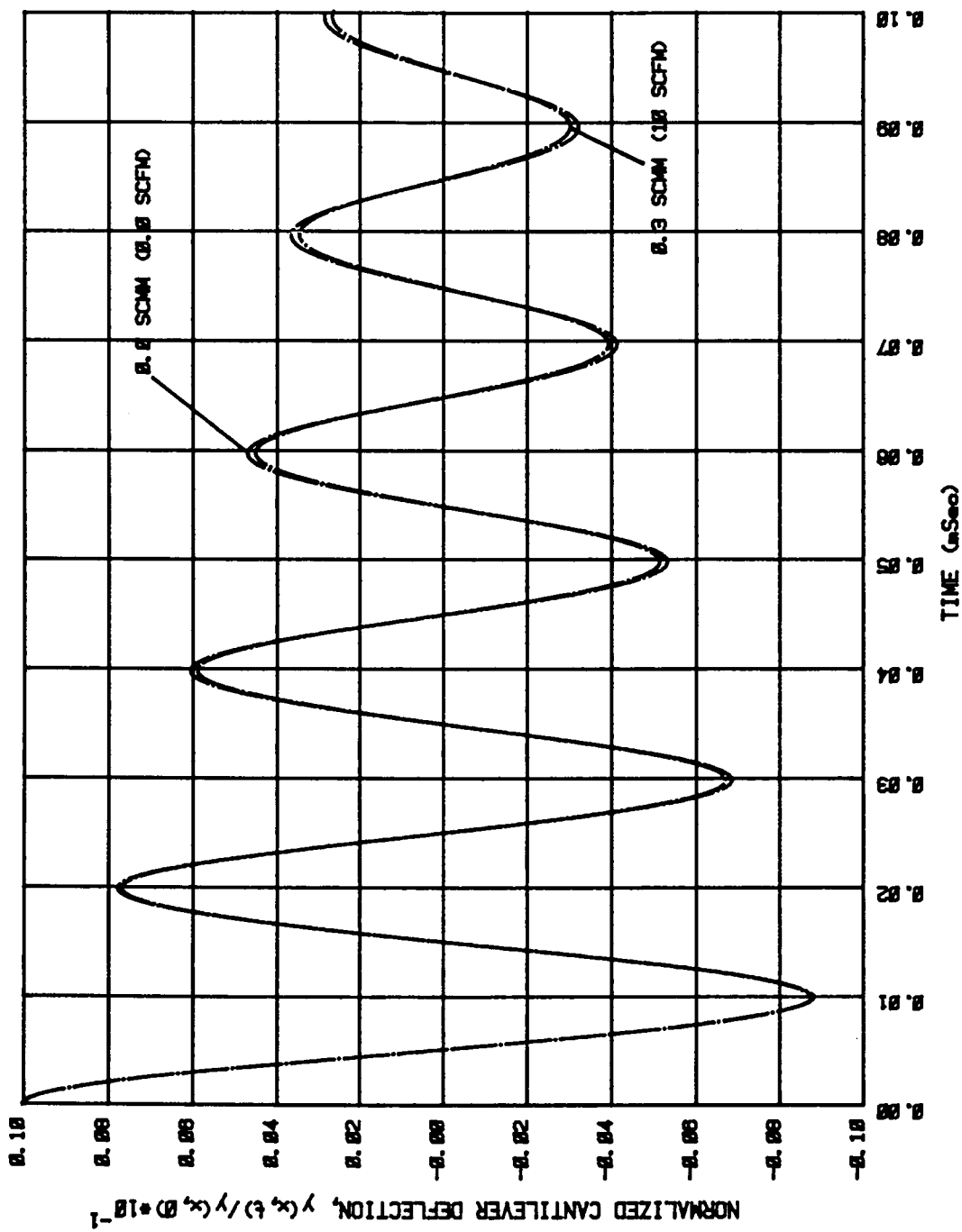


Figure 26. Nondimensional Analytical Plot of Dynamic Response for
1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

Table 4. Calibration Constants For 1.27 x 0.15875 cm
(1/2 x 0.0625 inch) Plexiglas Tubes

Calibration Constant	Mean	Standard Deviation	Std. Dev. (% of Mean)
a_1	56.394	$0(10^{-5})$	$0(10^{-6})$
a_2	8.774	2.402	23.3
a_3	1.0013	$0(10^{-4})$	$0(10^{-4})$

configurations that were tested. The constants a_1 and a_3 are dependent on density and material properties and showed little variation with flow, while a_2 is mass flow related and showed much larger variations. This large a variation in what should be a constant parameter is not ordinarily acceptable. However, measurements were made in hopes of obtaining useful information.

Actual mass flow measurements are shown in Figure 27, with the Coriolis flowmeter output plotted against the laminar flow element output. The dotted line is the linear regression which indicates the degree of linearity between the two signals. Ideally, the line should be at 45°. The shallow slope is an indication of relative insensitivity on the part of the plexiglas tube. Multiple results are shown in Figure 28, and it can be seen that the basic trend of the previous figure is repeated. In Figure 29, the results are plotted as percent difference or variation from the regression line. Obviously there is a great deal of scatter present in the results. Referring back to Figure 28, it can be seen that the two regions of greatest divergence from the ideal 45° line are at the low end of the flow range and the high end of the flow

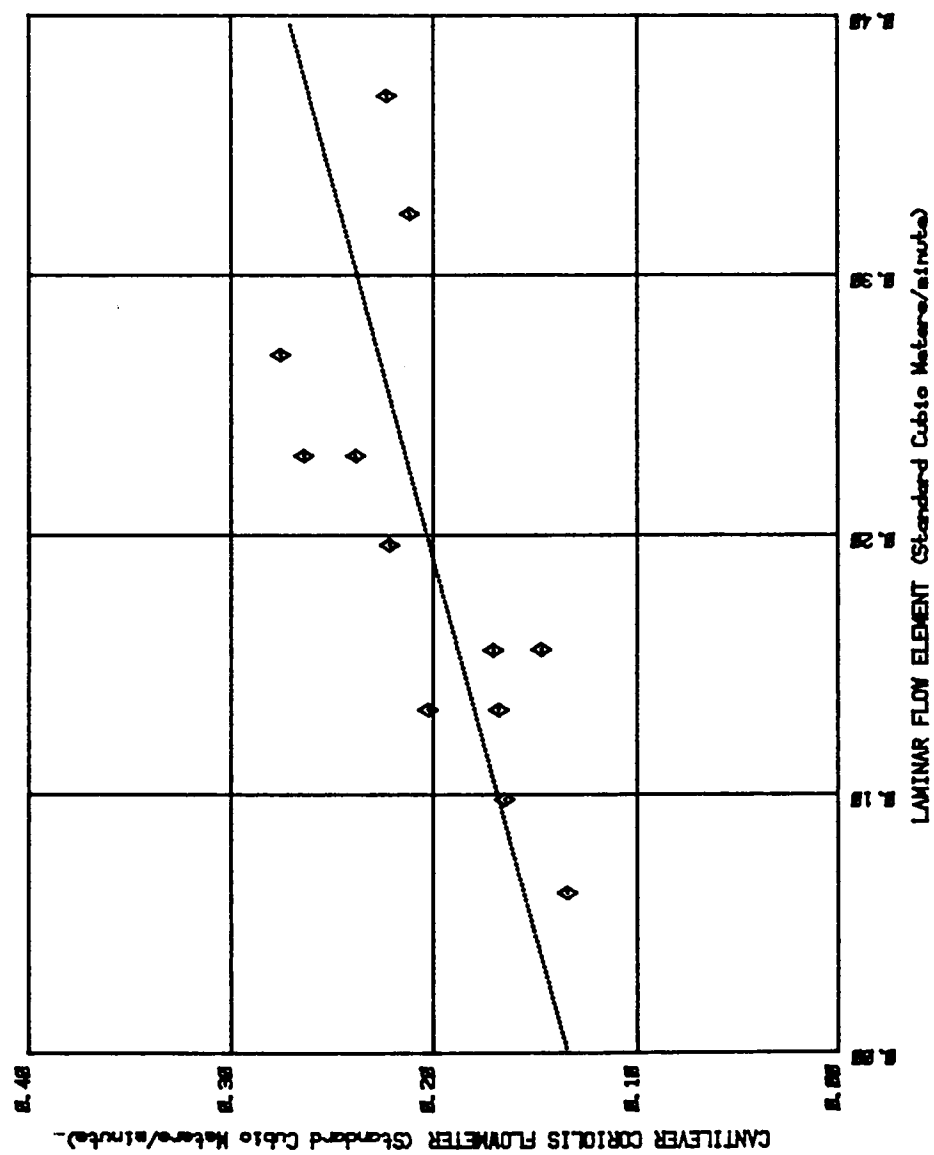


Figure 27. Typical Result and Regression for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

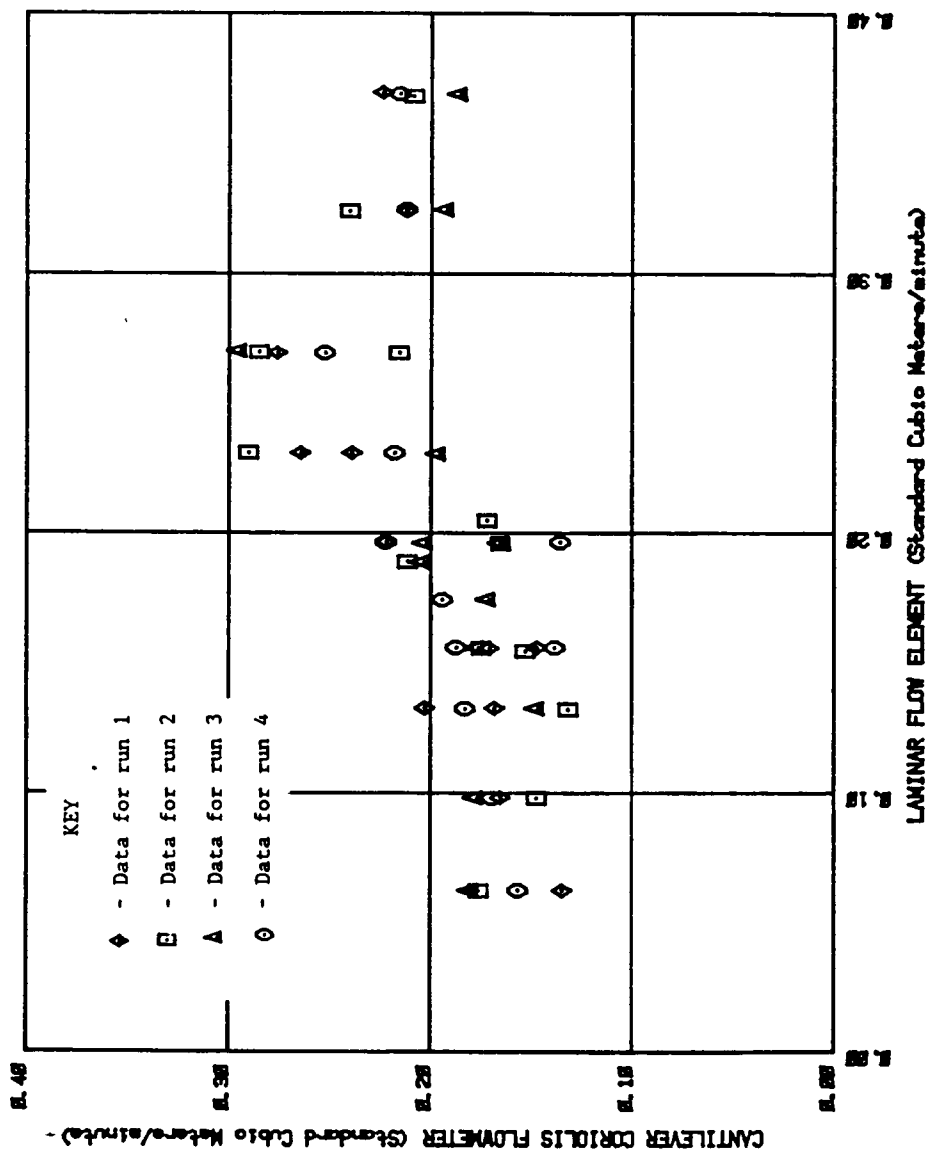


Figure 28. Multiple Results for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

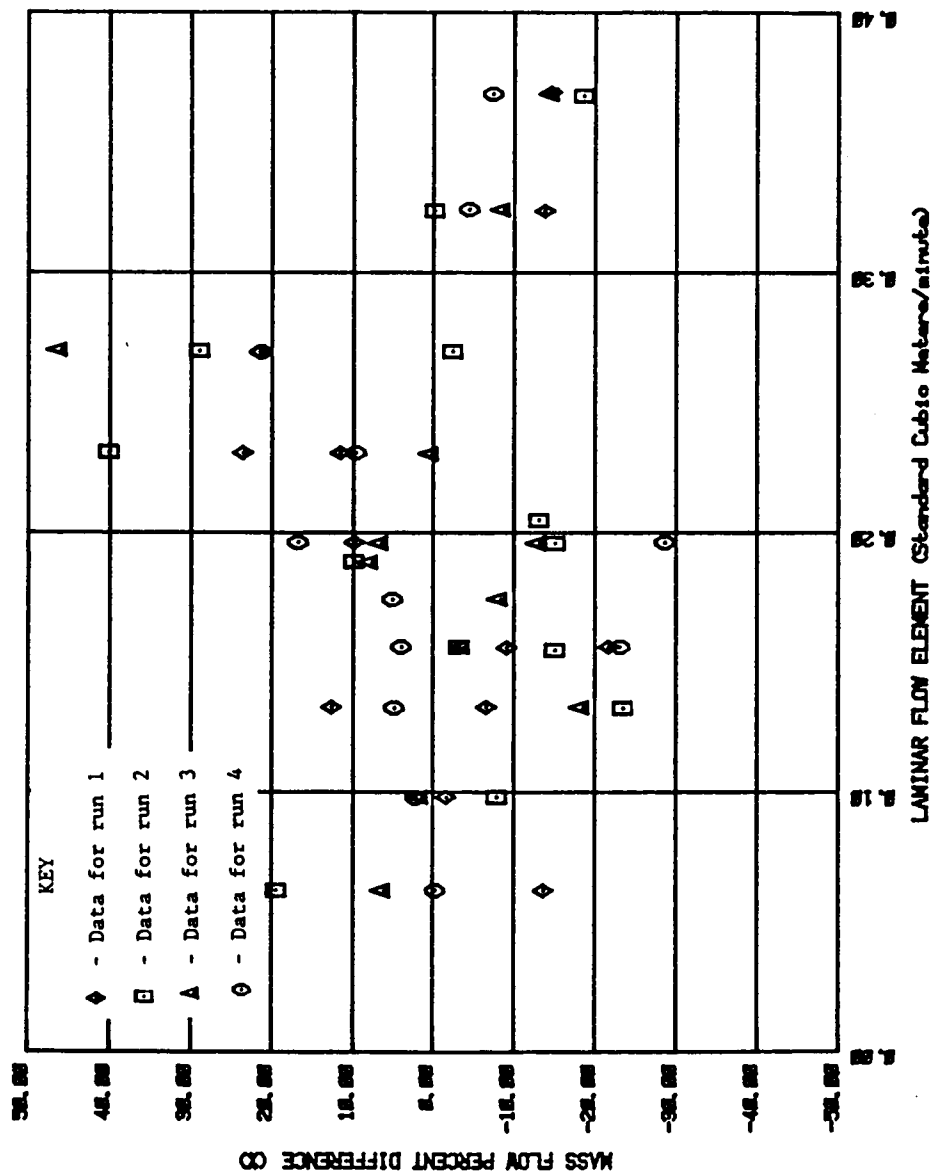


Figure 29. Mass Flow Regression Comparison for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

range. Divergence at the lower end of the flow range can be explained by noting that at low flows the Coriolis force produced by the hydroelastic interaction is correspondingly low and thus the meter is more insensitive. The divergence at the high end of the scale is an unexpected phenomenon which was observed in all three configurations to some extent, and it could not be determined why it occurred in this study. Some possible reasons for this behavior are: (1) breakdown of the incompressible flow assumption, (2) strange flow patterns induced at the tube exit due to the flow housing geometry, or (3) flow-induced instability not related to the geometry of the housing but rather due to the interaction force between the fluid and the tube. This third possibility has been the subject of much research in the area of hydroelasticity (such as Gregory and Paidoussis [8]), and it is a well-established fact that increases in fluid velocity can affect the eigenvalues of the fluid-bearing structure, which in turn affects the stability of the structure. The flowmeter is such a structure. Recalling the governing equation for the flowmeter, equation (52), one may note that the total solution was written as

$$(72) \quad y(\hat{x}, \hat{t}) = X(\hat{x}) T(\hat{t})$$

taking only the fundamental mode into account, where $T(\hat{t})$ was of the form $Be^{(\sigma + j\omega)t}$ and $(\sigma + j\omega)$ represented the eigenvalue for the first mode. For positive σ ($\sigma > 0$), the time solution is unstable. Hamel [2] began his work by going through a slightly more detailed analysis and provided root locus plots of the eigenvalues for the first two modes for the cantilever configuration. Since the eigenvalues are

functions of β and $\hat{u}$ (non-dimensional velocity), each plot is actually a family of curves. Using the plot for the fundamental mode which Hamel provided, and with $\hat{u} \approx 0.15$ (from the velocity at which the instability first appeared) and $\beta \approx 0.002$, it is entirely possible that this phenomenon could explain the instability, although it is impossible to say for sure. One interpretation of the behavior is that at some flow rate energy begins to be added to the system and this energy addition shows up as a drop in the decay rate, which in turn yields a lower reading for the mass flow. For this set of tubes, this behavior became apparent at approximately 0.3 SCMM (10 SCFM). Since this behavior is anomalous, it might be constructive to delete the mass flow points beyond the point at which the instability began. Figure 30 shows what the modified mass flow response plot looks like. As can be seen, both the linearity and sensitivity are much better with the elimination of the high velocity data. Figure 31 is a plot of the percent difference of the Coriolis value and the linear regression. It is apparent that the agreement is improved over that of Figure 29, with the mass flows falling within roughly $\pm 20\%$ of the regression line, and for the most part within $\pm 10\%$. By eliminating the high flow points, which were of questionable reliability for reasons unknown but believed unrelated to the flowmeter's ability at lower flow rates, reasonably good mass flow results were obtained using the 1.27 x 0.15875 cm (1/2 x 0.0623 inch) plexiglas tubes.

The density results are shown in Figure 32, which is a plot of the percent difference of the Coriolis value from a calculated value using measured temperature and pressure. The amount of divergence varies

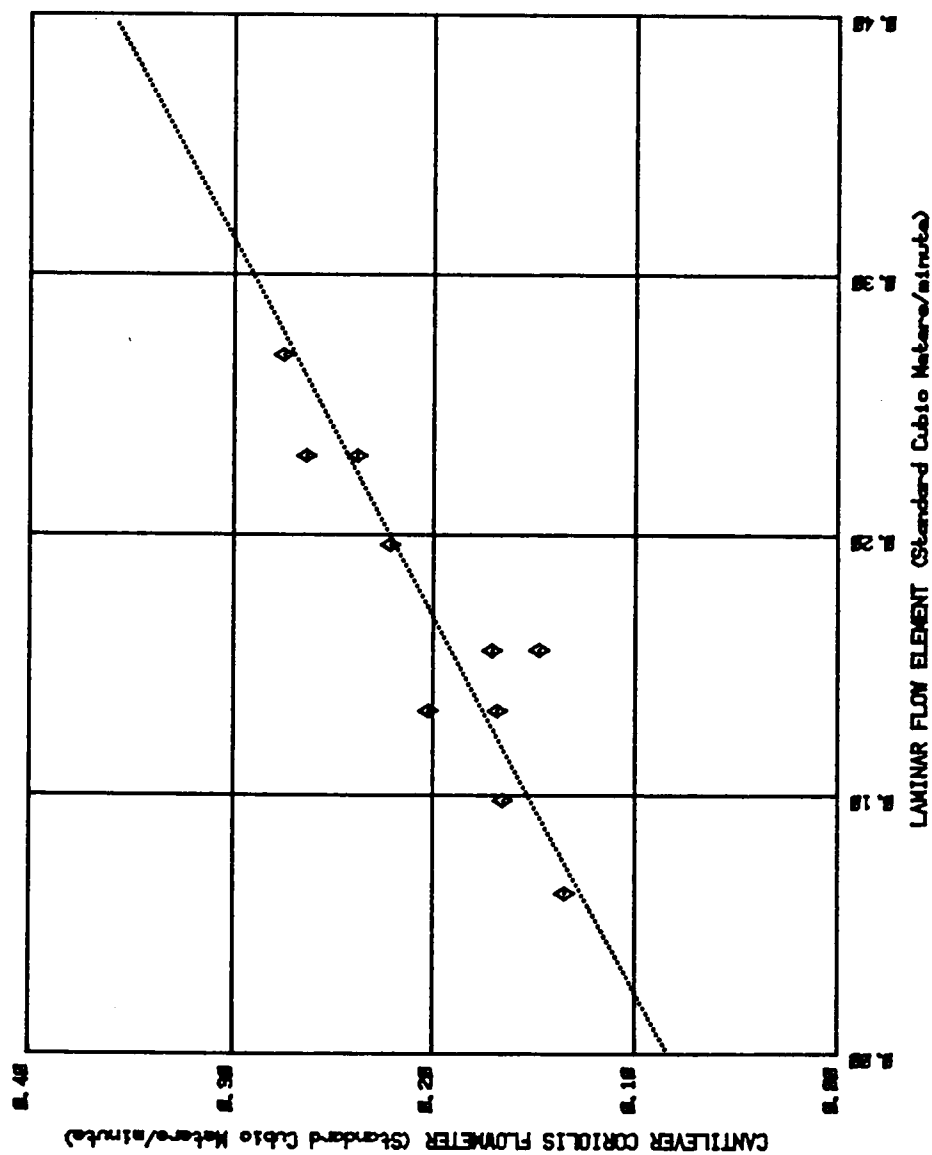


Figure 30. Result and Regression With High Flow Rate Points Removed for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

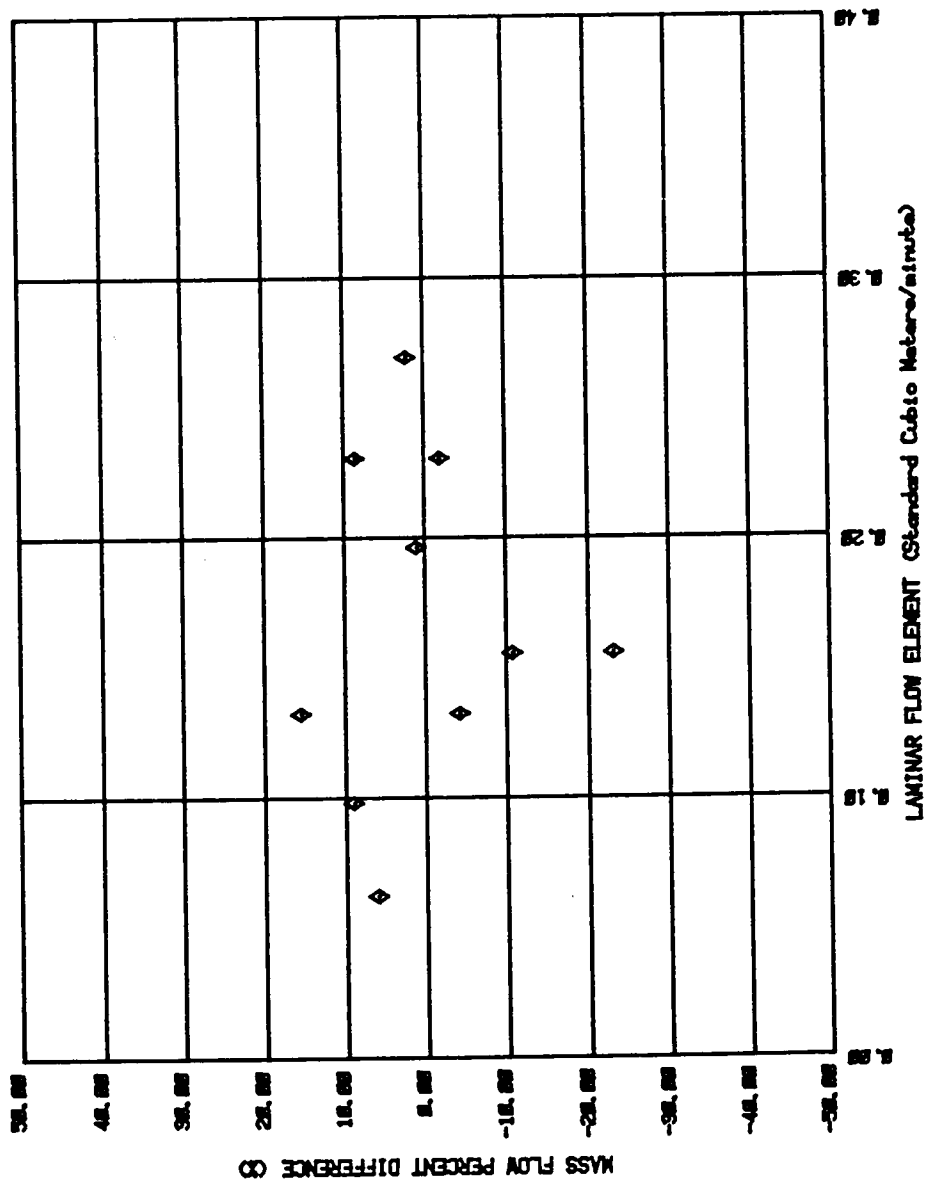


Figure 31. Mass Flow Regression Comparison With High Flow Rate Points Removed for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

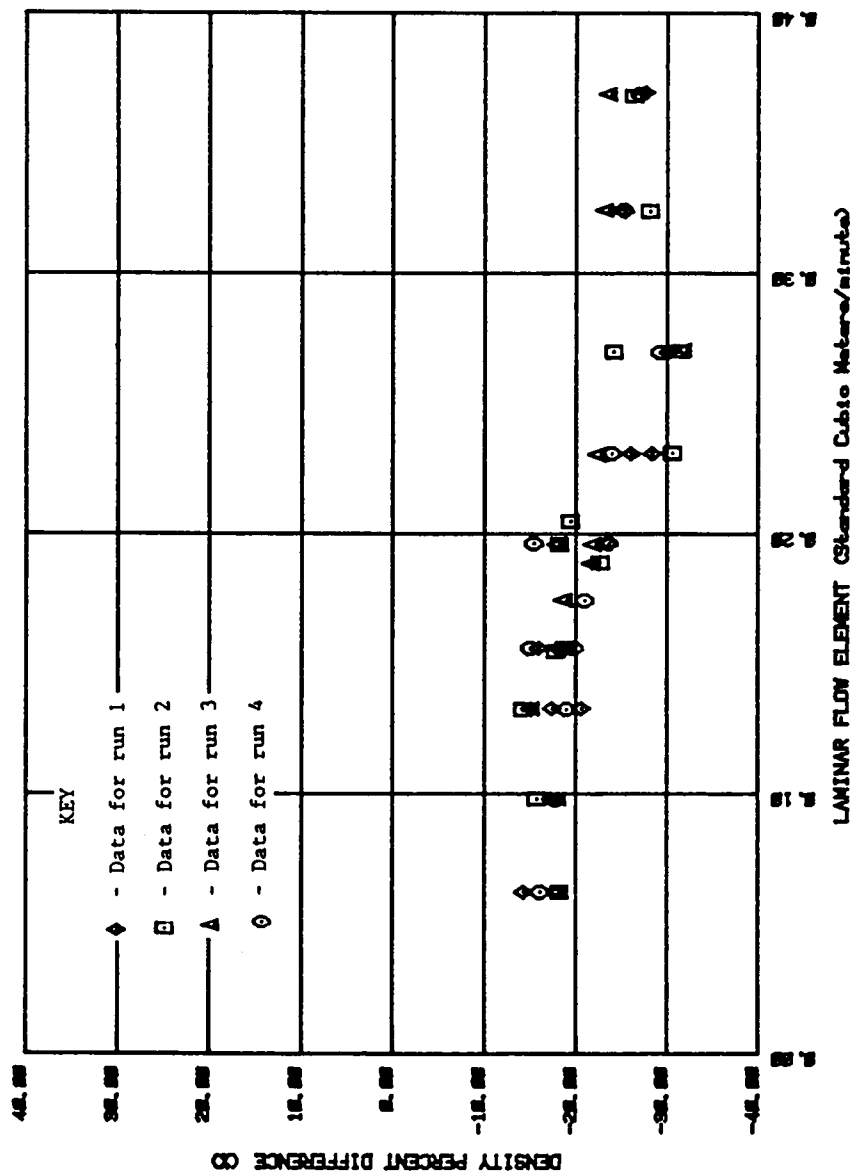


Figure 32. Density Regression Comparison for 1.27 x 0.15875 cm (1/2 x 0.0625 inch) Plexiglas Tubes.

(Percent difference is taken with respect to actual density values.)

somewhat, being relatively constant at about -20% over half the flow range and then dropping down to -30% before creeping back up to -25% at the upper end of the flow range. The reason for such a large overall divergence can be traced back to differences between the decay constants of the two tubes at zero flow. Since the reference tube signal was meant to provide the empty tube parameters for the measurement algorithm, ideally the two empty tube values should have been identical. However, as was seen before, the values exhibit some difference. This difference is relatively constant, and would result in a large offset in the computation of the density from equation (129). The measurement and calibration algorithms were modified to account for this in the last two tests by multiplying the value of R_{c1} for each measurement by the ratio of the mean values of the empty tube decay constants, $(\tau_e^{-1})_{act} / (\tau_e^{-1})_{ref}$, and it will be seen that the density results improved as a result.

The dip in Figure 32, where the percent difference increases negatively, is not as easily explained. One possible reason could be that the computed "standard" value of density may not have been as "standard" as it should have. The reason for this lies in how the pressure and temperature which were used to calculate the density were measured. A static pressure gage at the inlet to the Coriolis flow element measured the pressure, while the temperature was taken by a thermocouple located downstream of the Coriolis flowmeter. The assumption was made that the actual temperature values at any point in the circuit would not vary much from this value. Discrepancies in either of these values could show up as an additional error in the density plot by making the computed "standard" density value be too high

or low. The dip in Figure 32 implies that this calculated value of density was too high, which would indicate that the measured pressure was too high or the measured temperature was too low, if this were the reason for the increase. This effect, however, would not suffice to explain away the entire problem, and in fact, the raw density data actually showed a slight dip in the Coriolis density as the flow rate increased. This phenomenon also was repeated in the other tube configurations and is as yet unexplained. The slight decrease in the divergence at the higher flow rates reflects the lower sensitivity of the density computations to the decreasing decay constants in the region of apparent instability.

Results for the 1.5785 x 0.15875 cm (5/8 x 0.0625 inch)

Phenolic Tubes

The second set of tubes which were tested were the 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) phenolic tubes. The phenolic resin had been filled with a fine weave fabric, and exhibited promising characteristics from the start. Its smaller diameter ratio made it less structurally massive than the plexiglas tubes and its lower density increased hopes that the results obtained from testing would be the best yet.

The procedure followed did not vary from that previously used except for the modification in the measurement and calibration routines which was discussed in the last section. Figures 33, 34, 35, and 36 show the frequency spectra and time histories for both phenolic tubes. Notice how smooth and clean the vibrations are. Also note that the material effect, which caused the vibration reference offset and was so

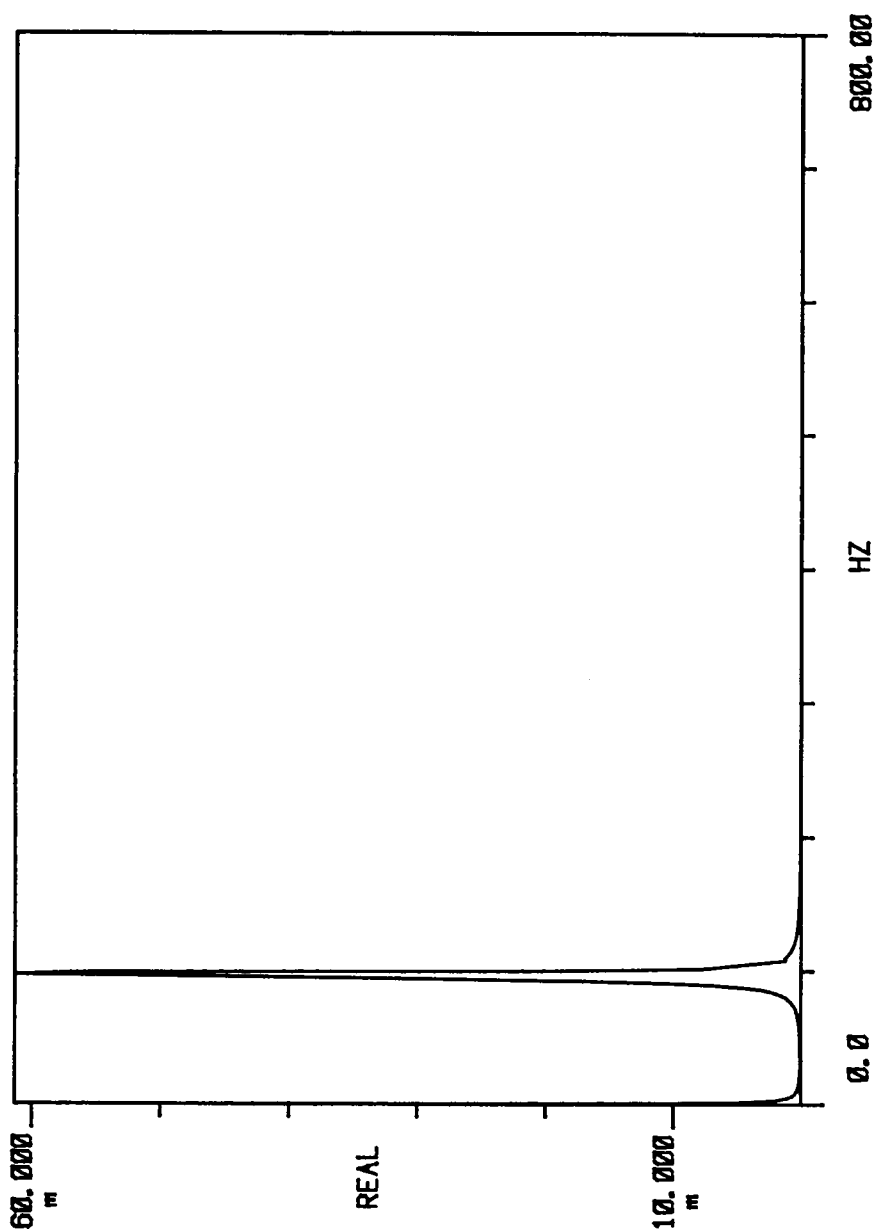


Figure 33. Frequency Spectrum of Active Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

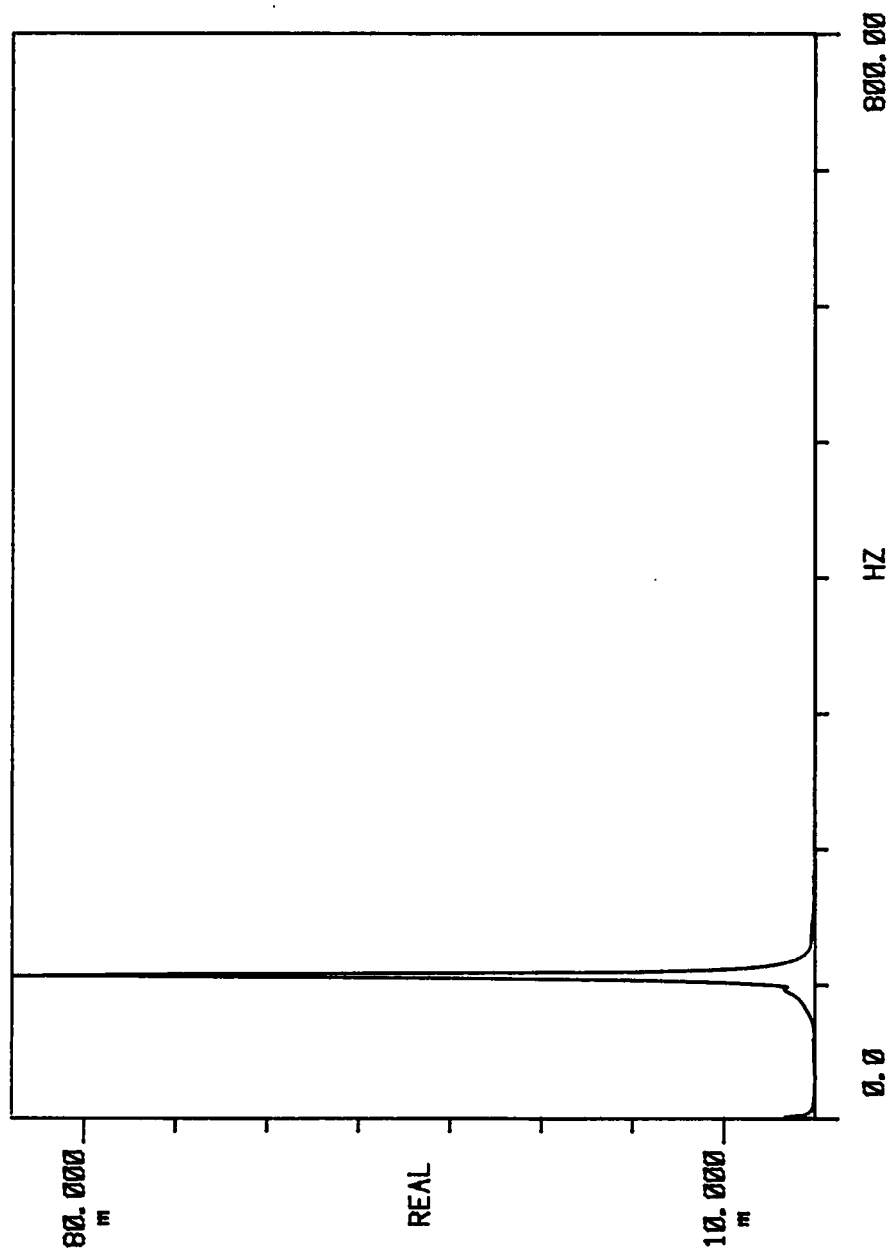


Figure 34. Frequency Spectrum of Reference Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

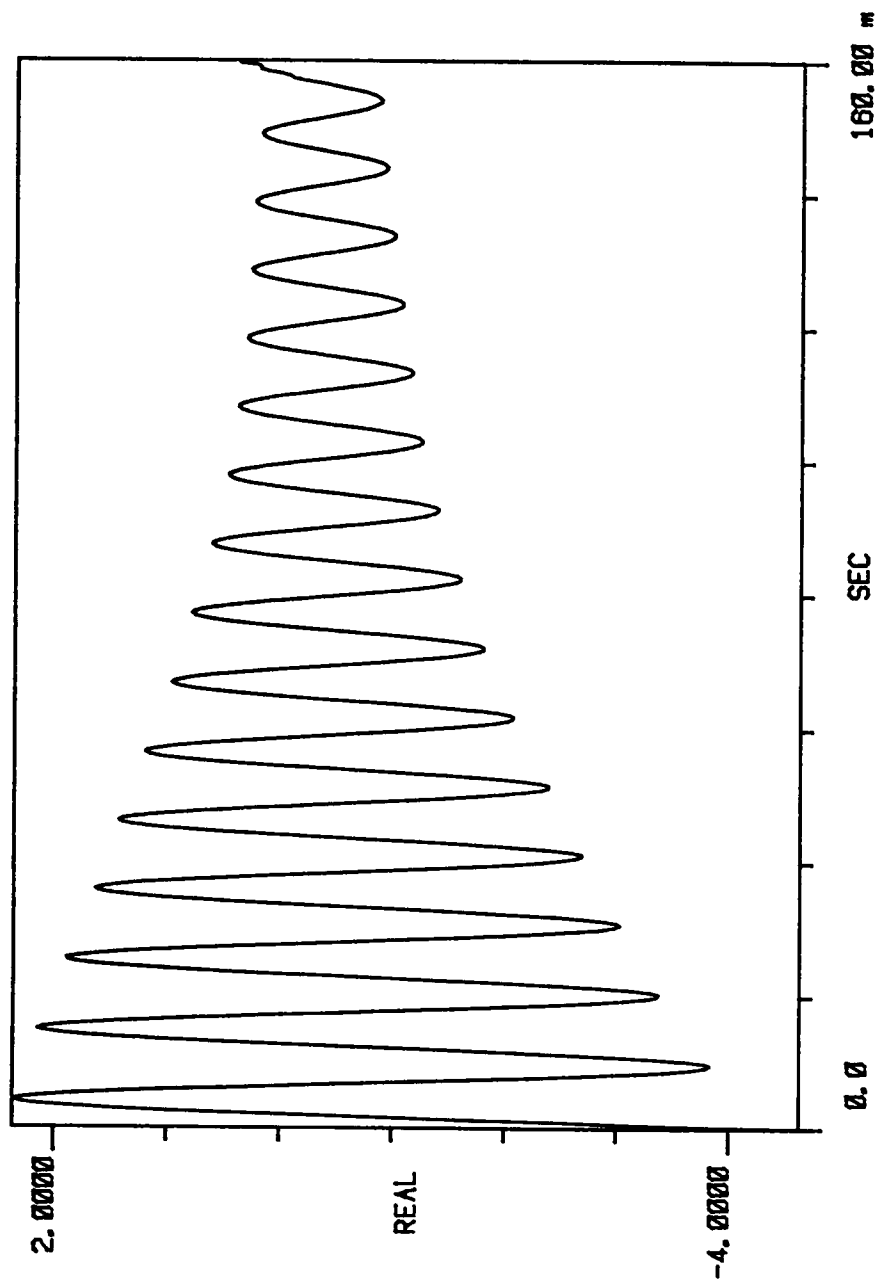


Figure 35. Time History of Active Tube for 1.5875 x 0.15875 cm
(5/8 x 0.0625 inch) Phenolic Tubes.

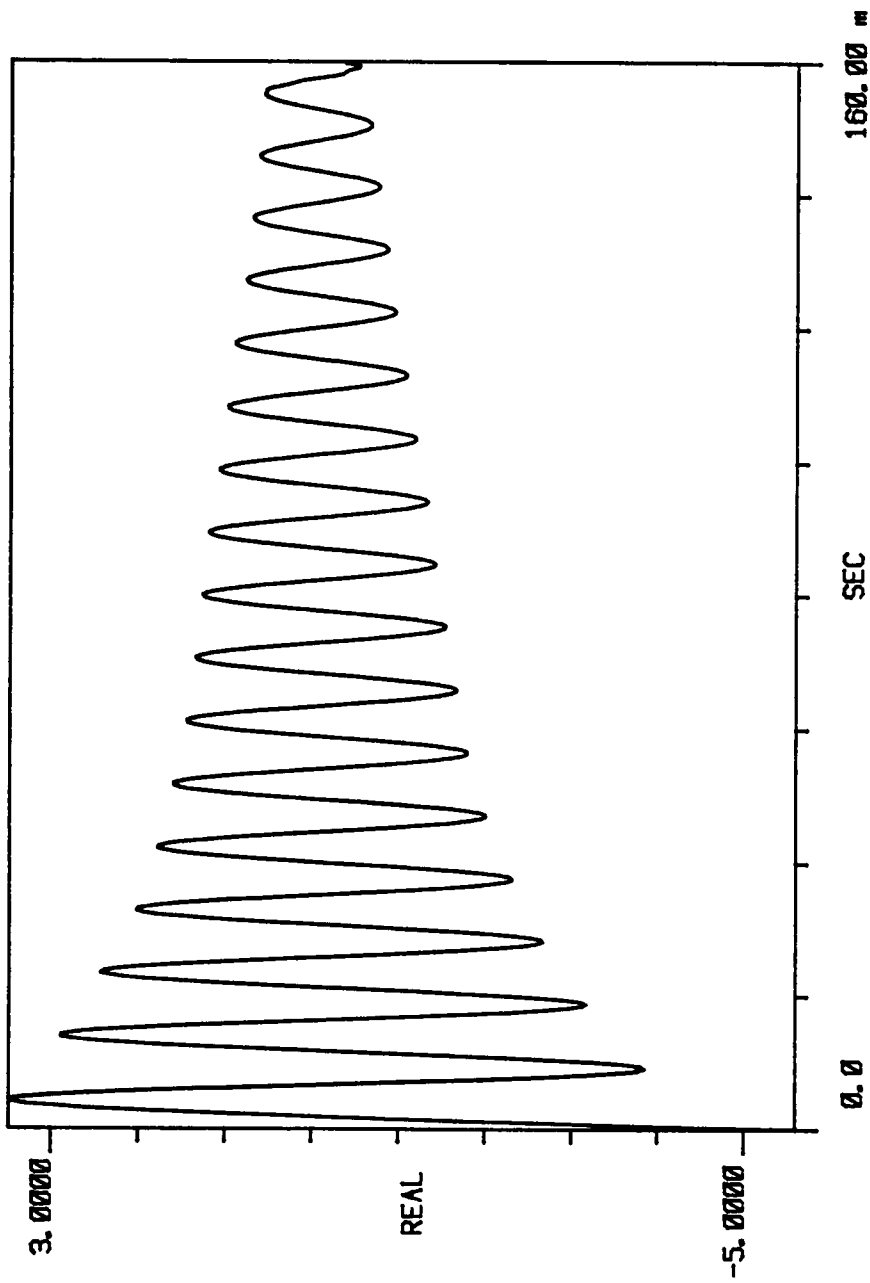


Figure 36. Time History of Reference Tube for 1.5857 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

prominent in the 1.5875 x 0.15875 cm (1/2 x 0.0625 inch) plexiglas tubes, is hardly noticable in the phenolic tubes. This could be one reason why the average empty tube vibration characteristics were very repeatable, as shown in Table 5. In fact, the variations in these characteristics were the lowest of all three sets tested. The dynamic responses of the tubes as measured by the measurement system are shown in Figures 37 and 38, and the theoretical nondimensionalized response is given in Figure 39. Again the ordinates have been scaled by a factor of ten to make it easier to see the amplitude changes caused by the fluid flow. The theoretical and experimental plots are in agreement. The calibration results for this configuration are given in Table 6. As

Table 5. Empty Tube Vibration Characteristics For 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes

Active Tube		Reference Tube	
ω	τ^{-1}	ω	τ^{-1}
(rad/s)	(s ⁻¹)	(rad/s)	(s ⁻¹)
593.499	15.643	654.498	14.868
591.358	15.796	654.498	14.922
590.183	15.669	670.206	14.876
592.753	15.950	670.206	14.666
590.183	15.635	654.498	14.800
591.159	15.863	654.498	14.822
591.358	15.807	654.498	14.625
591.159	15.695	654.498	14.538
591.358	15.722	654.498	14.762
593.499	15.893	654.498	14.754
mean	591.651	657.640	14.763
std. dev.	1.206	6.623	0.122
std. dev. (% of mean)	0.20%	1.01%	0.82%

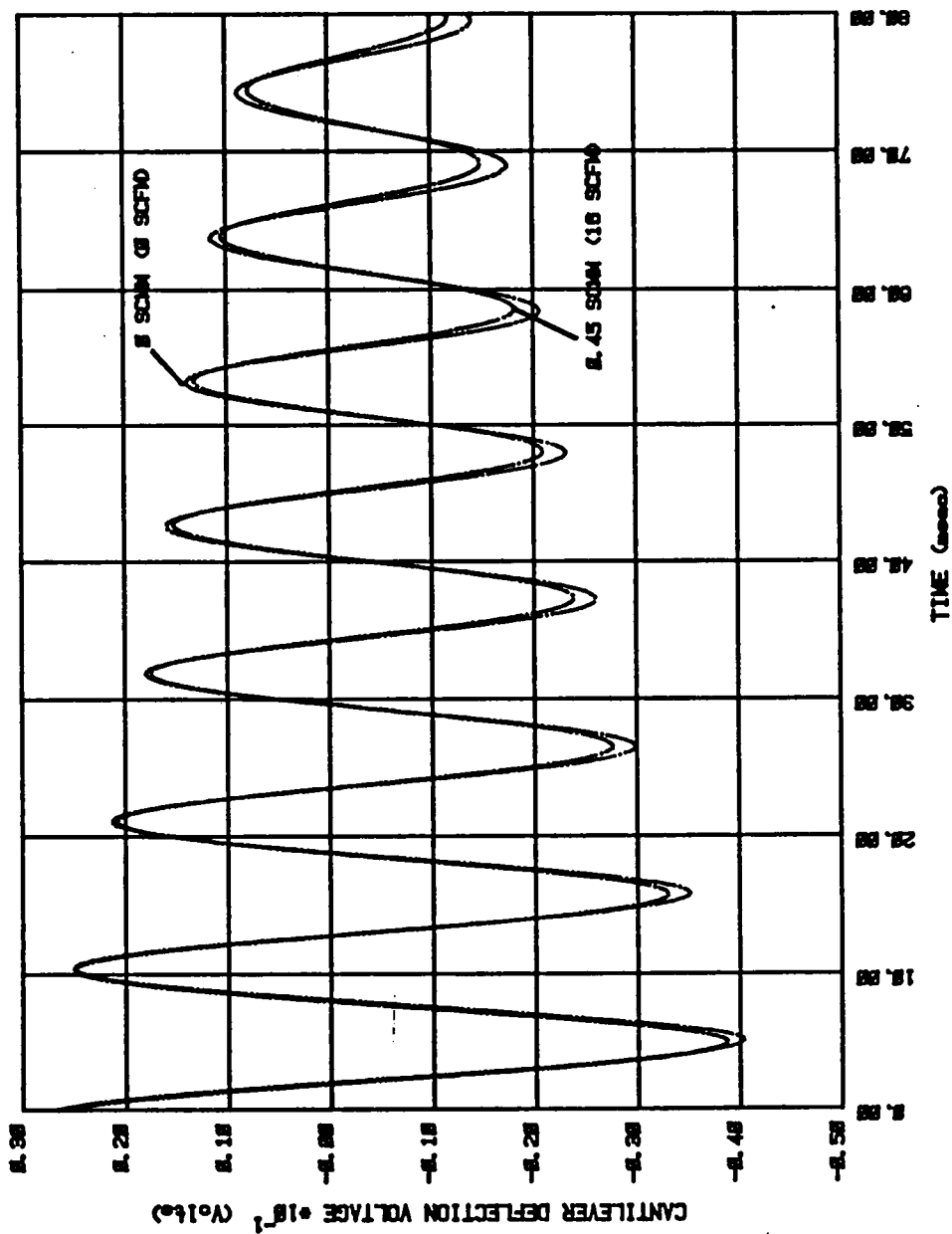


Figure 37. Typical Dynamic Response of Active Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

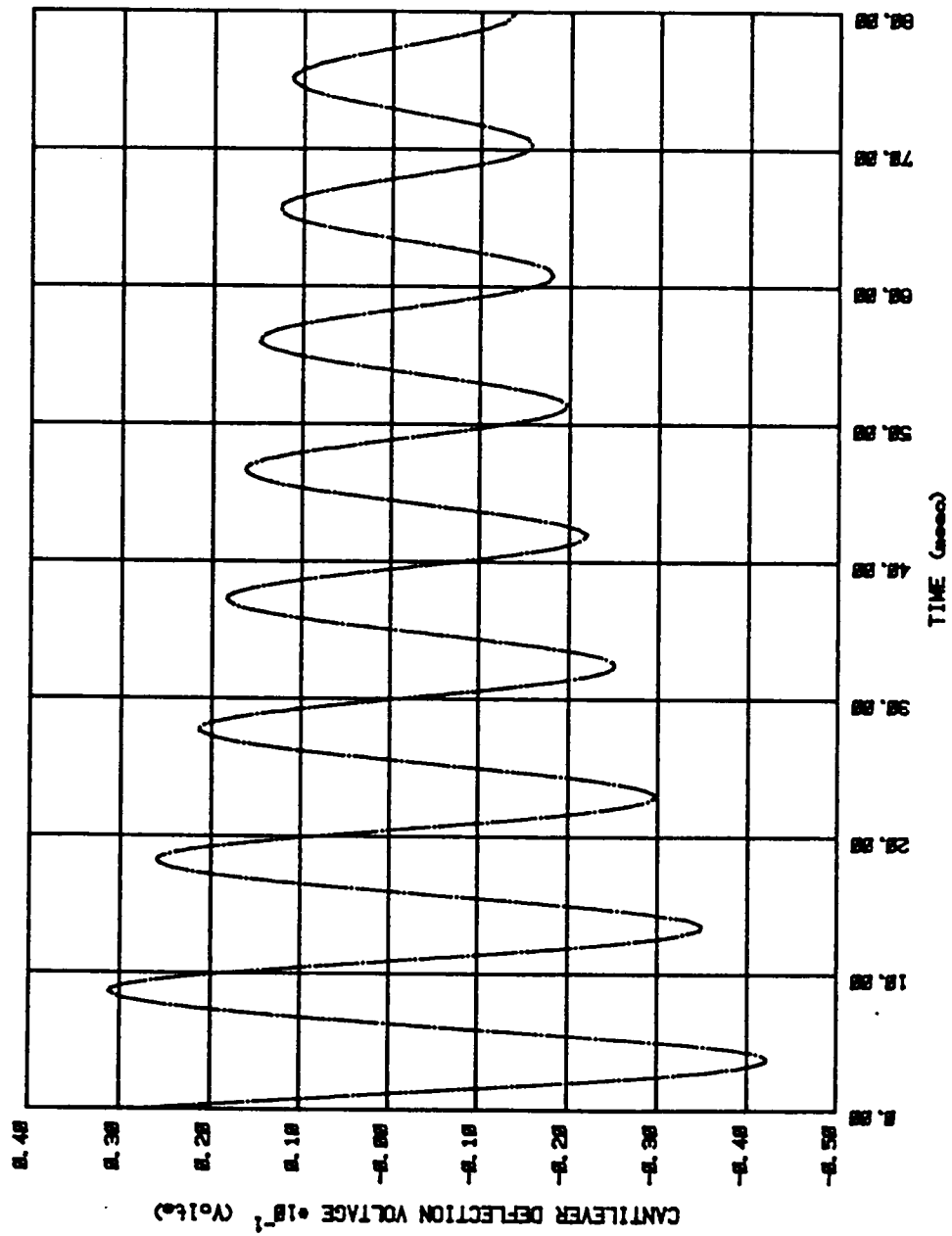


Figure 38. Typical Dynamic Response of Reference Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

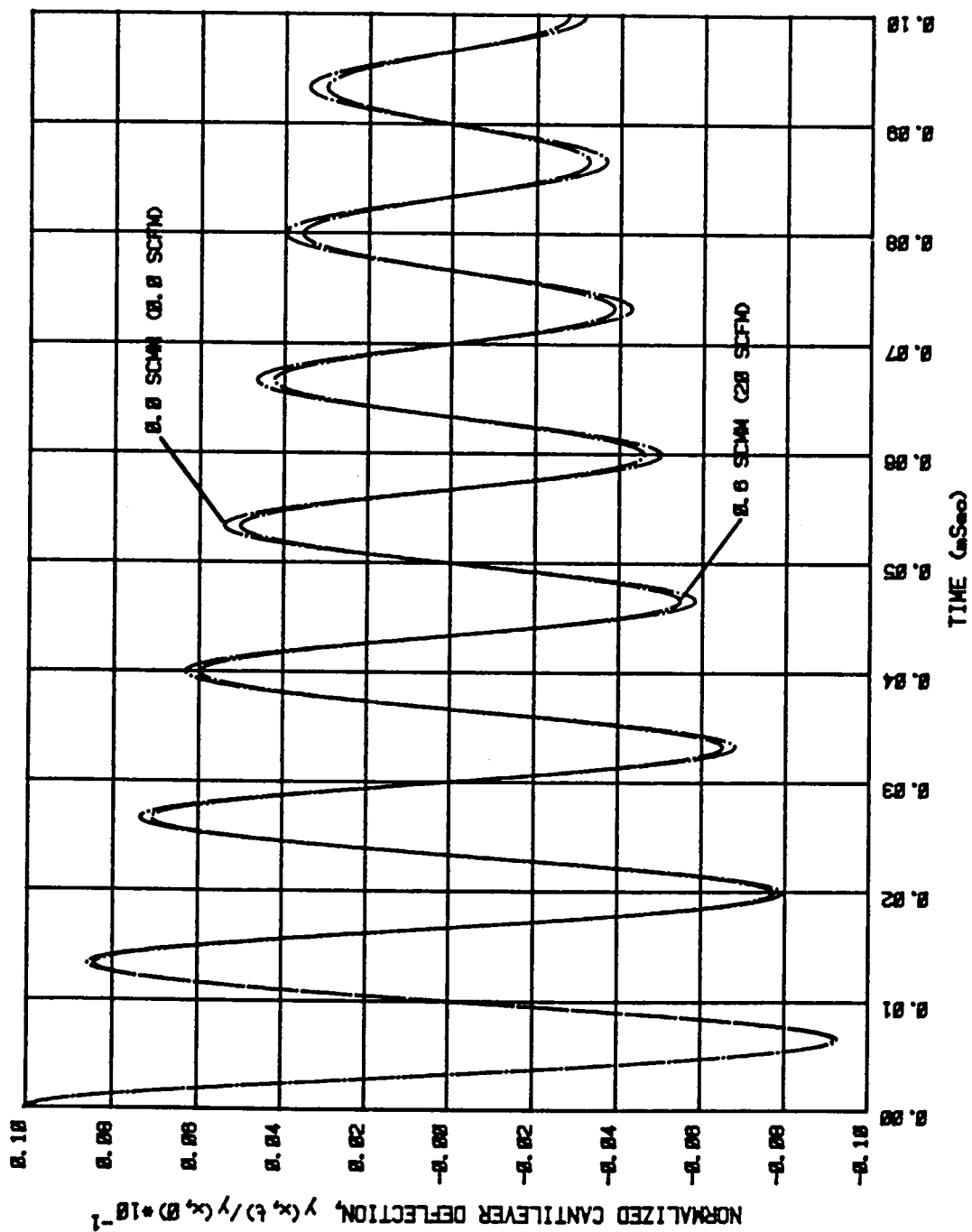


Figure 39. Nondimensional Analytical Plot of Dynamic Response for 1.5857 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

Table 6. Calibration Constants For 1.5875 x 0.15875 cm
(5/8 x 0.0625 inch) Phenolic Tubes

Calibration Constant	Mean	Standard Deviation	Std. Dev. (% of Mean)
a_1	34.162	$0(10^{-5})$	$0(10^{-6})$
a_2	3.078	0.939	30.5
a_3	1.0022	$0(10^{-4})$	$0(10^{-4})$

before, a_1 and a_3 are solid with negligible variation, while the variation in a_2 is rather large. Again, in spite of this uncertainty (and because no other tubes were available) tests were made using these tubes.

Mass flow results are shown in Figures 40, 41, and 42. Figure 40 is a typical result and regression for this configuration. Like before, note the drop off in the Coriolis values as flow rate increases past a certain point, about 0.6 SCMM (20 SCFM). The velocity at this flow rate for these tubes is very nearly the same as the velocity at which the first set of plexiglas tubes began to show problems, which could indicate that the problem is related to the fluid velocity. Figure 43 is a plot of decay constant versus flow for both the phenolic tubes and the previous set of plexiglas tubes, and shows the correlation of this "drop-off" velocity at $u \approx 225$ ft/s (the amount of scatter present in the plexiglas data is due to the lower sensitivity compared to the phenolic tubes). A plot with the highest flow rate data removed is not shown, since the effect is not as extreme as before and it is easier to visualize what the net change would be.

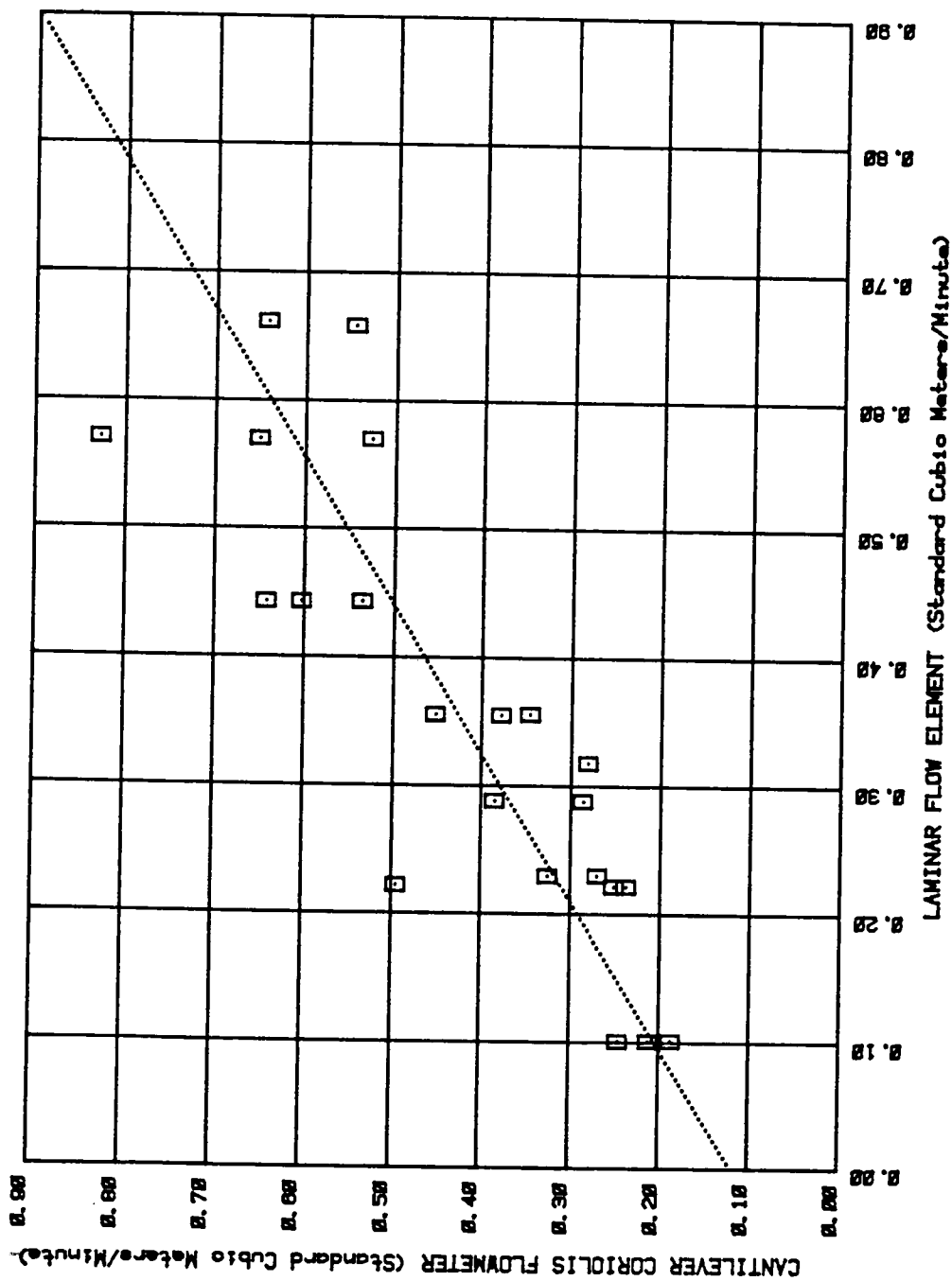


Figure 40. Typical Result and Regression for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

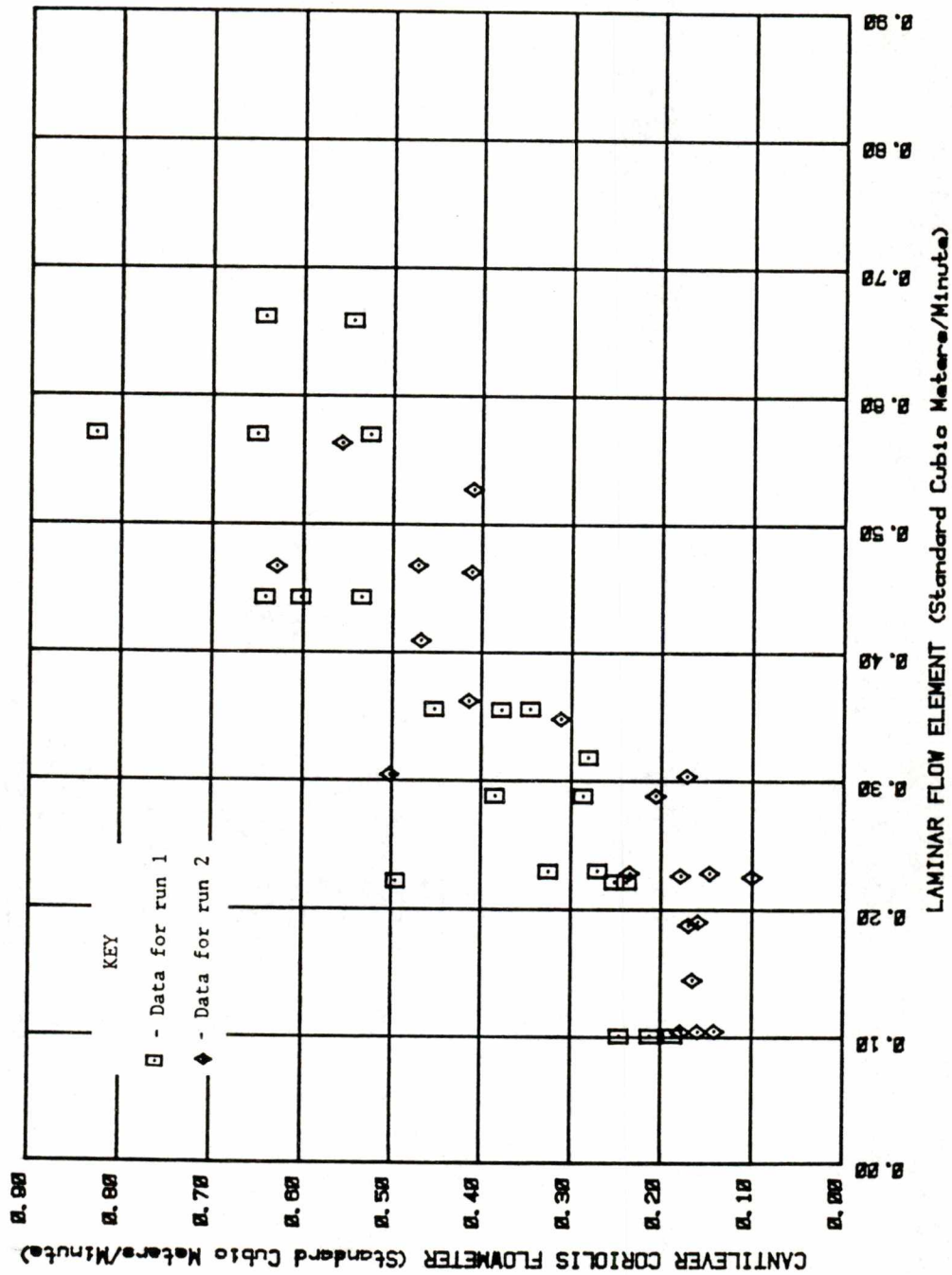


Figure 41. Multiple Results for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

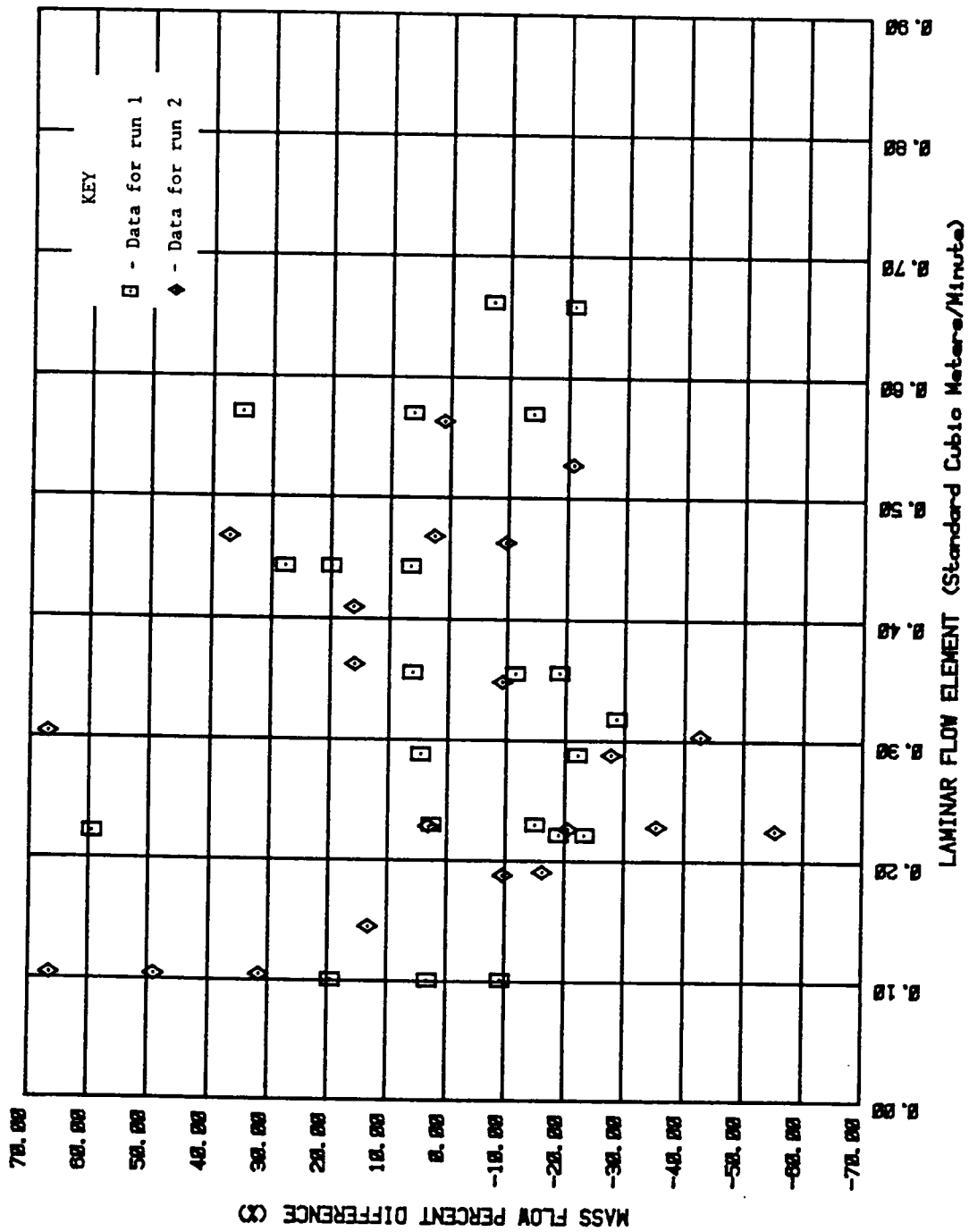


Figure 42. Mass Flow Regression Comparison for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

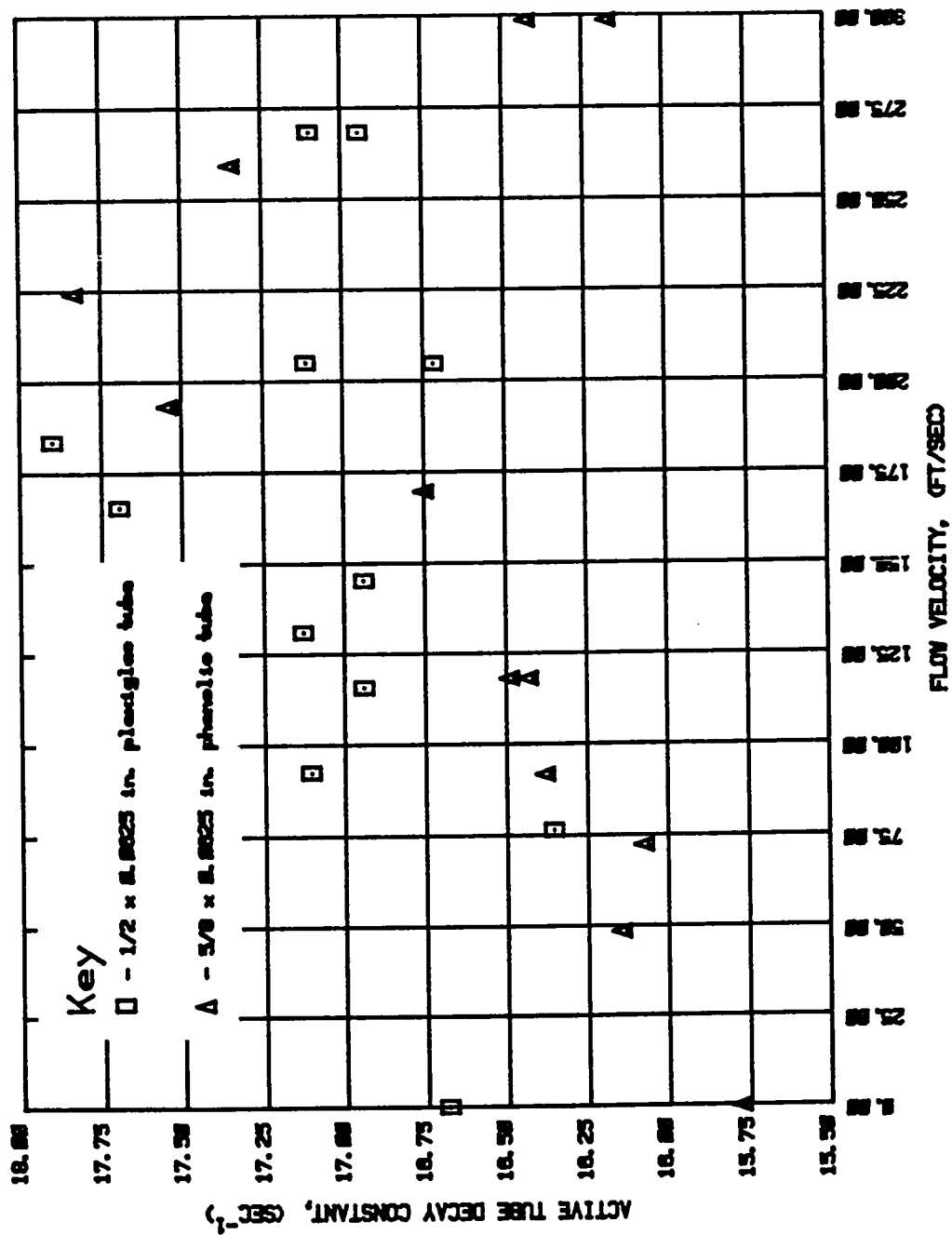


Figure 43. Illustration of Decay Constant Behavior.

The results are also similar to those of the previous configuration in that there is a noticeable amount of scatter (see Figure 28, page 99). However, the general trend is apparent and tends to bear out the suggestion that in spite of large calibration uncertainties and the relative insensitivities of the tubes chosen, the results are positive and further work on this idea is in order.

Even with the highest flow data, it is apparent that these tubes are more sensitive than the last set tested, yielding a steeper regression line and showing better clustering about the regression line at low flow rates. Since the phenolic tubes have a better sensitivity, as predicted by the theory of Chapter III, better clustering at lower flows should be expected.

Figure 40 shows the regression comparison for multiple results. Some scatter is present, but the results generally fall within $\pm 30\%$, with a large percentage of the readings falling within $\pm 20\%$ of the regression line.

The density results are shown in Figure 44. It can be seen that the modification in the algorithms had a great impact on density measurement. The trend is similar to that seen in the previous configuration, although the actual values are lower, ranging from a low of zero percent difference to a high of -15% difference.

Results for the 1.5875 x 0.15875 cm (5/8 x 0.0625 inch)

Plexiglas Tubes

The 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) plexiglas tubes have the same diameter ratio as the phenolic tubes which were tested, and

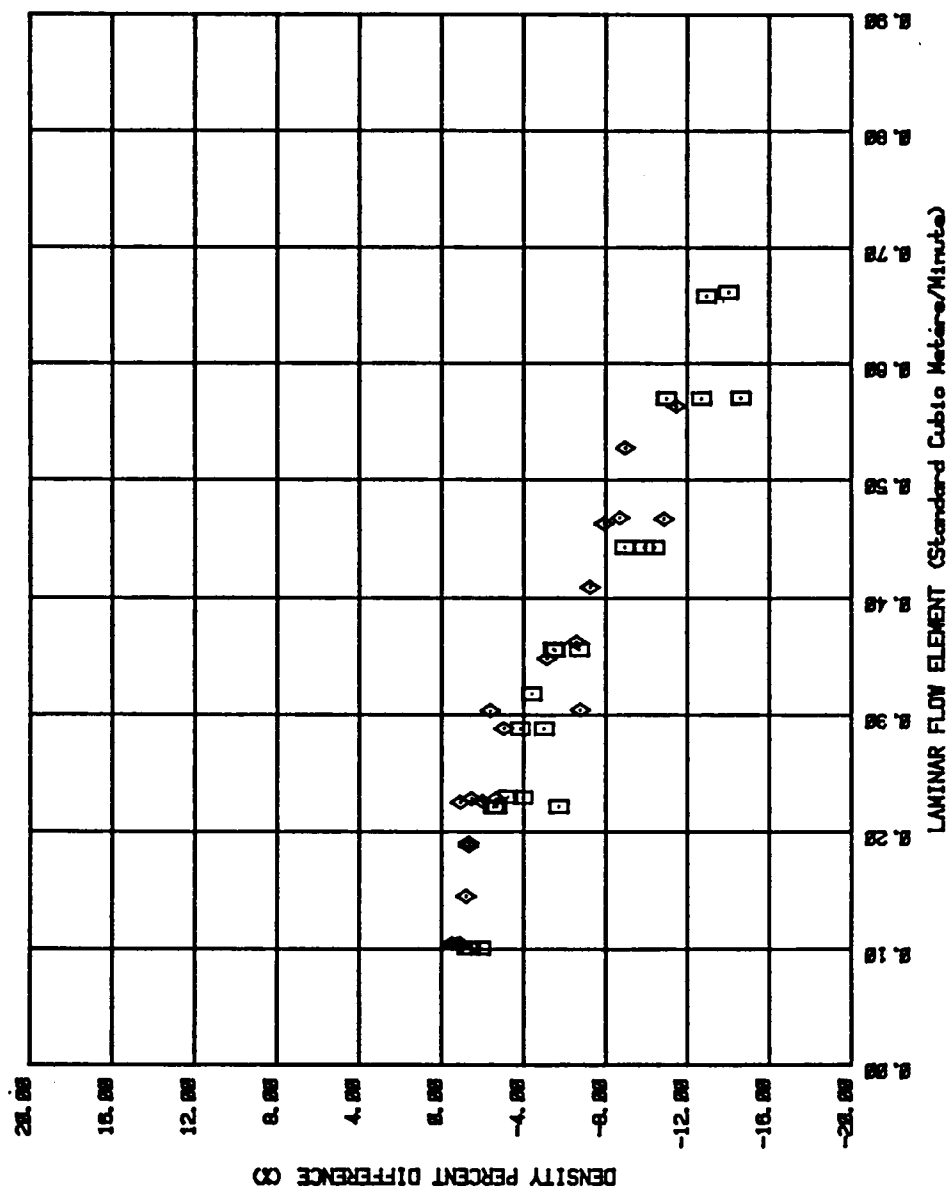


Figure 44. Density Regression Comparison for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Phenolic Tubes.

(Percent difference is taken with respect to actual density values.)

have a slightly higher density. So the results should show a sensitivity lying between those of the 1.27×0.15875 cm ($1/2 \times 0.0625$ inch) plexiglas configuration and the 1.5875×0.15875 cm ($5/8 \times 0.0625$ inch) phenolic tubes.

The frequency spectra and time histories are shown in Figures 45, 46, 47, and 48. As before, the tubes' vibrations are very nicely unimodal and free from distortion, with the natural frequencies occurring at about 92 Hz and 95 Hz for the active tube and reference tube respectively. Like the previous set of plexiglas tubes there is a noticeable imbalance in the time histories due to a large, finite stress release time. Table 7 shows the empty tube vibration parameters, which are nicely steady. The calibration results for this set of tubes are shown in Table 8 and reflect the trends which have already been discussed. The dynamic responses of the tubes as obtained by the measurement system are shown in Figures 49 and 50. The theoretical response is shown in Figure 51 for comparison.

At this point, everything was proceeding as before, and hopes were high of obtaining results which were at least as positive as those of the previous configurations. However, for reasons still unknown this particular configuration was very difficult to work with, and yielded results which are probably unreliable to some degree. As stated before, the cause of these difficulties was never determined, as time ran out for testing and the test setup was broken down. Suffice it to say that tests run over a period of several days all yielded some discrepancies which showed up in the plotted results as excessive scatter and a distinct lack of repeatability. One possible explanation: the larger

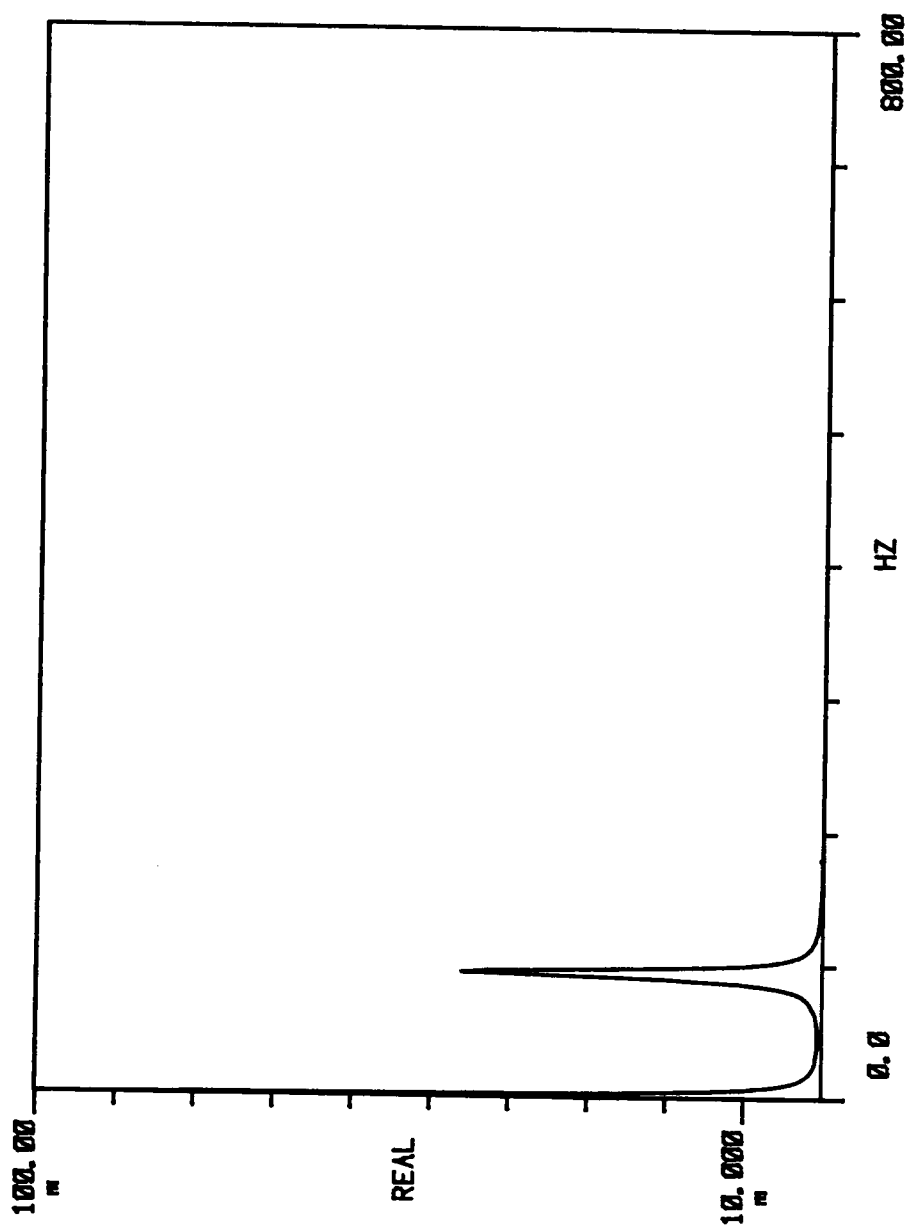


Figure 45. Frequency Spectrum of Active Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

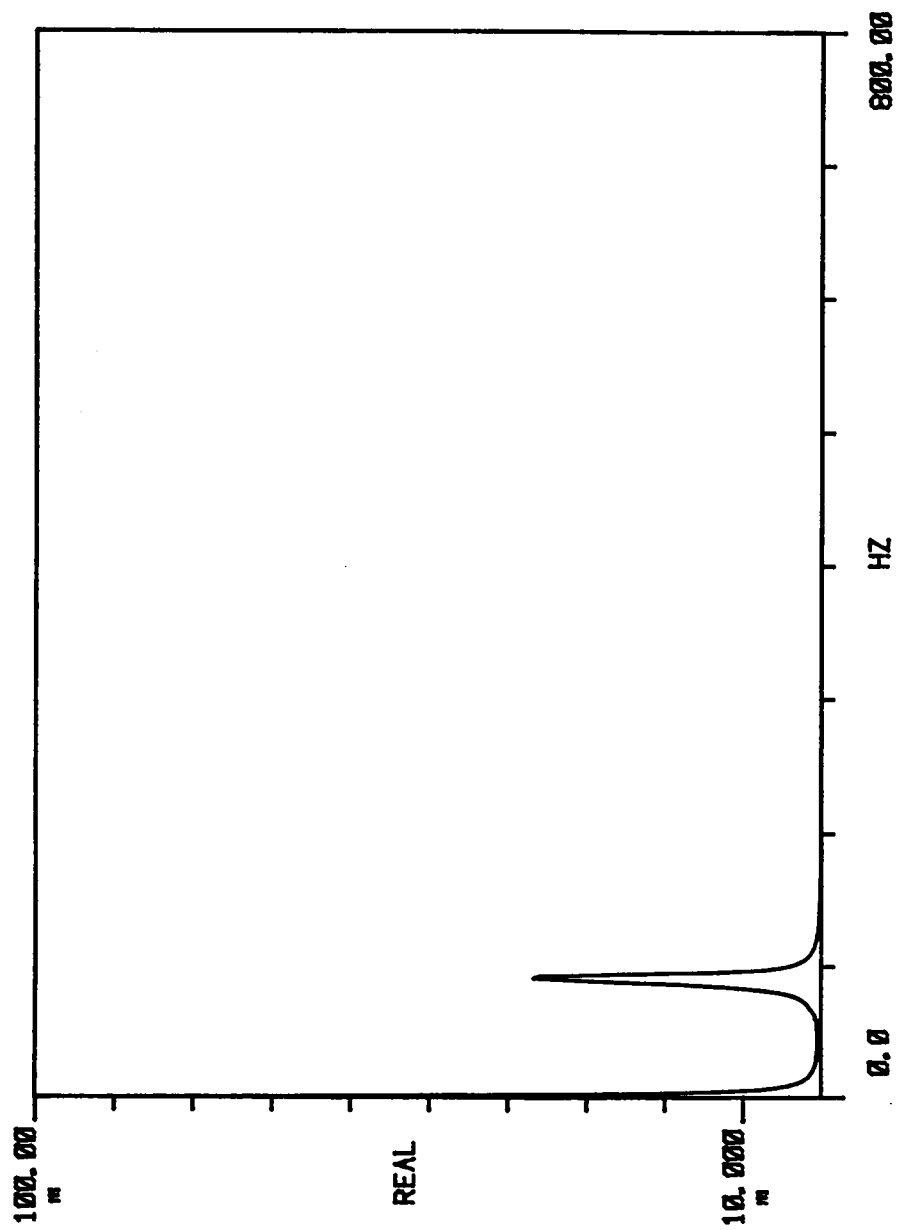


Figure 46. Frequency Spectrum of Reference Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

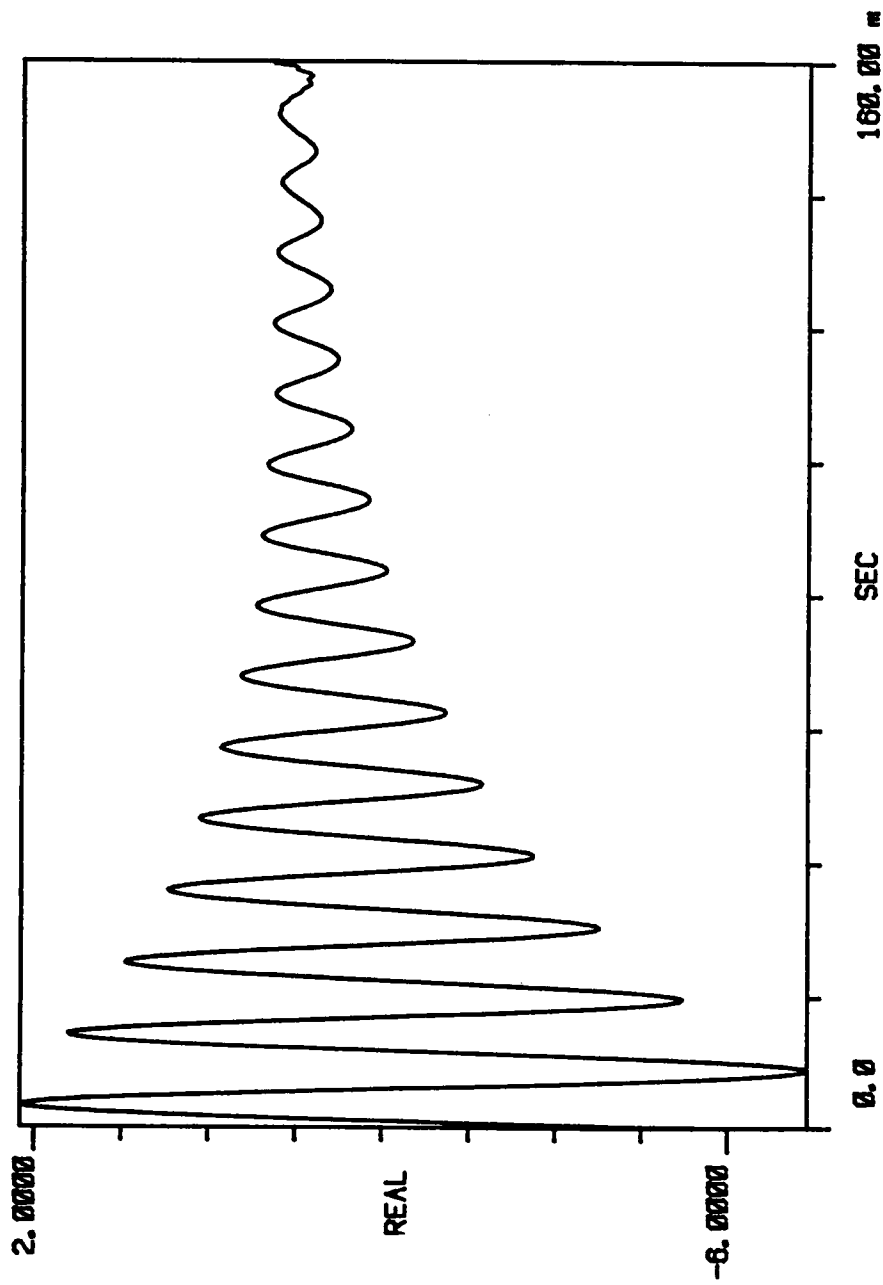


Figure 47. Time History of Active Tube for 1.5875 x 0.15875 cm
(5/8 x 0.0625 inch) Plexiglas Tubes.

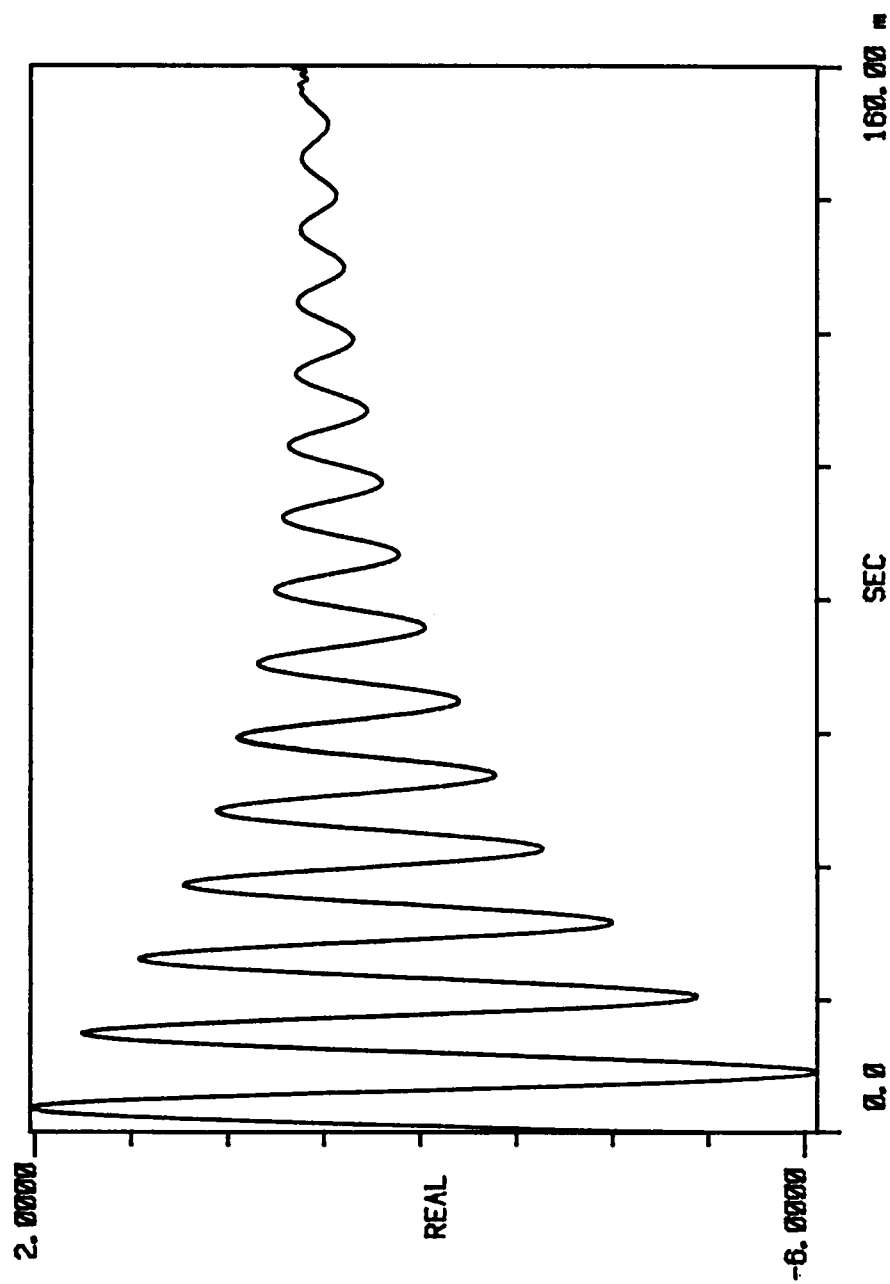


Figure 48. Time History of Reference Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

Table 7. Empty Tube Vibration Characteristics For 1.5875 x 0.15875 cm
(5/8 x 0.0625 inch) Plexiglas Tubes

Active Tube		Reference Tube	
ω	τ^{-1}	ω	τ^{-1}
(rad/s)	(s ⁻¹)	(rad/s)	(s ⁻¹)
551.156	22.039	572.686	20.746
550.512	22.010	572.686	21.263
548.934	22.260	574.681	21.081
545.998	22.168	572.686	21.270
551.156	22.305	581.776	21.298
548.934	22.137	581.776	21.459
548.934	22.321	581.776	22.093
551.156	22.403	581.776	22.068
550.413	22.164	581.776	22.388
549.229	22.418	581.776	21.353
mean	549.642	578.340	21.502
std. dev.	1.605	4.474	0.514
std. dev. (% of mean)	0.3%	0.8%	2.4%

Table 8. Calibration Constants For 1.5875 x 0.15875 cm
(5/8 x 0.0625 inch) Plexiglas Tubes

Calibration Constant	Mean	Standard Deviation	Std. Dev. (% of Mean)
a_1	41.220	0(10 ⁻⁵)	0(10 ⁻⁶)
a_2	3.543	0.9687	27.3
a_3	1.0018	0(10 ⁻⁴)	0(10 ⁻⁴)

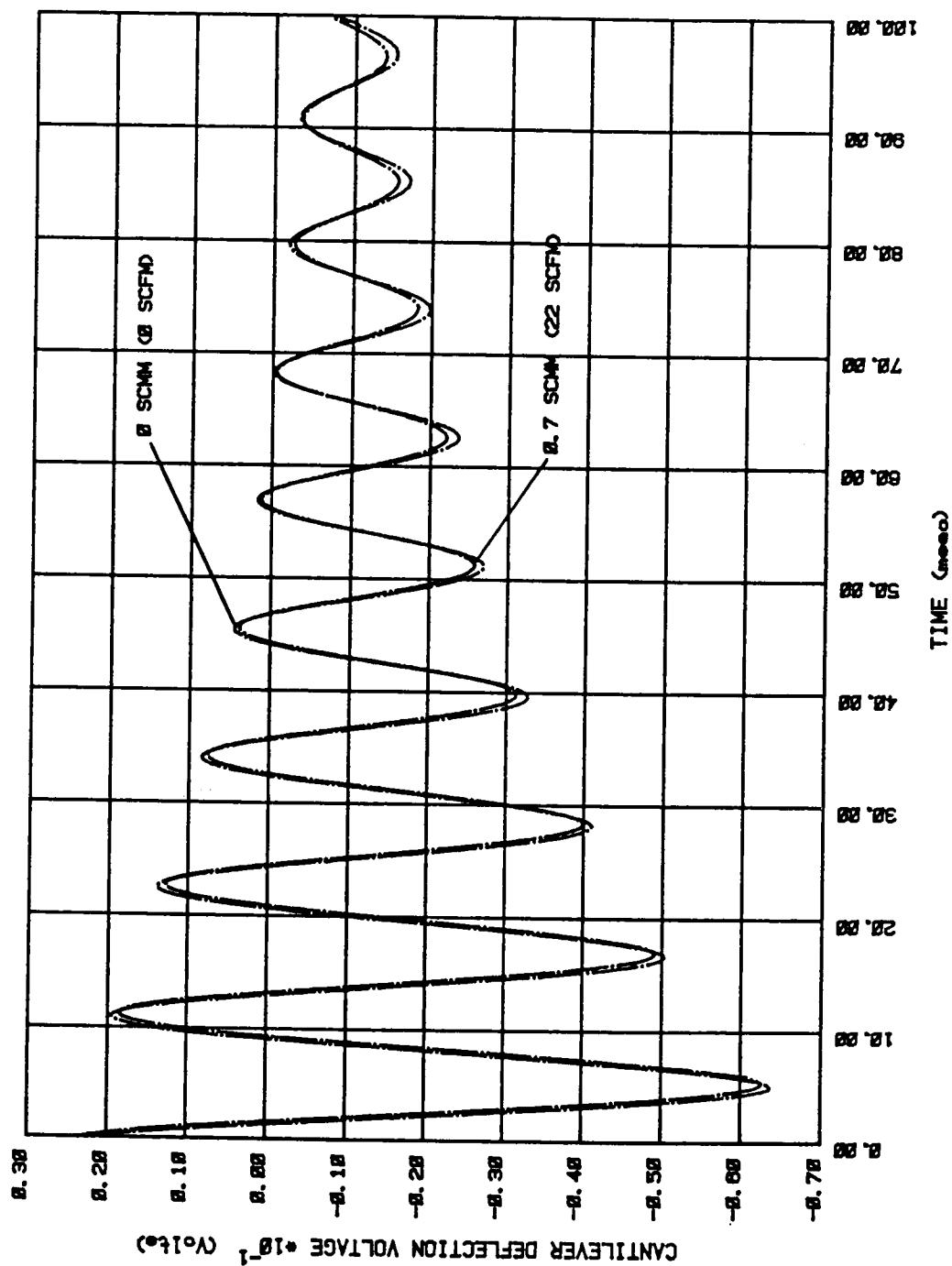


Figure 49. Typical Dynamic Response of Active Tube for 1.5875 x 0.15875 (5/8 x 0.0625 inch) Plexiglas Tubes.

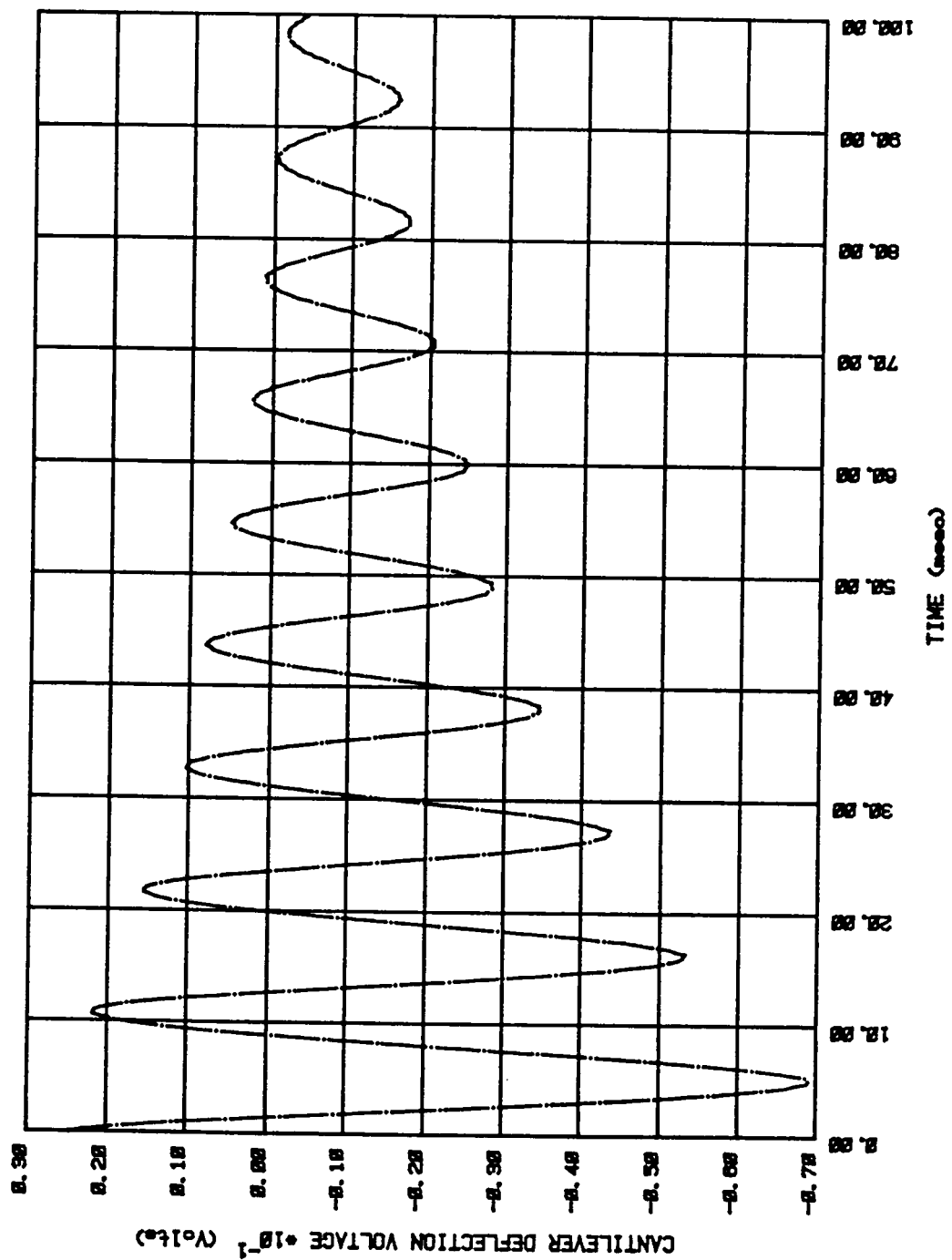


Figure 50. Typical Dynamic Response of Reference Tube for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

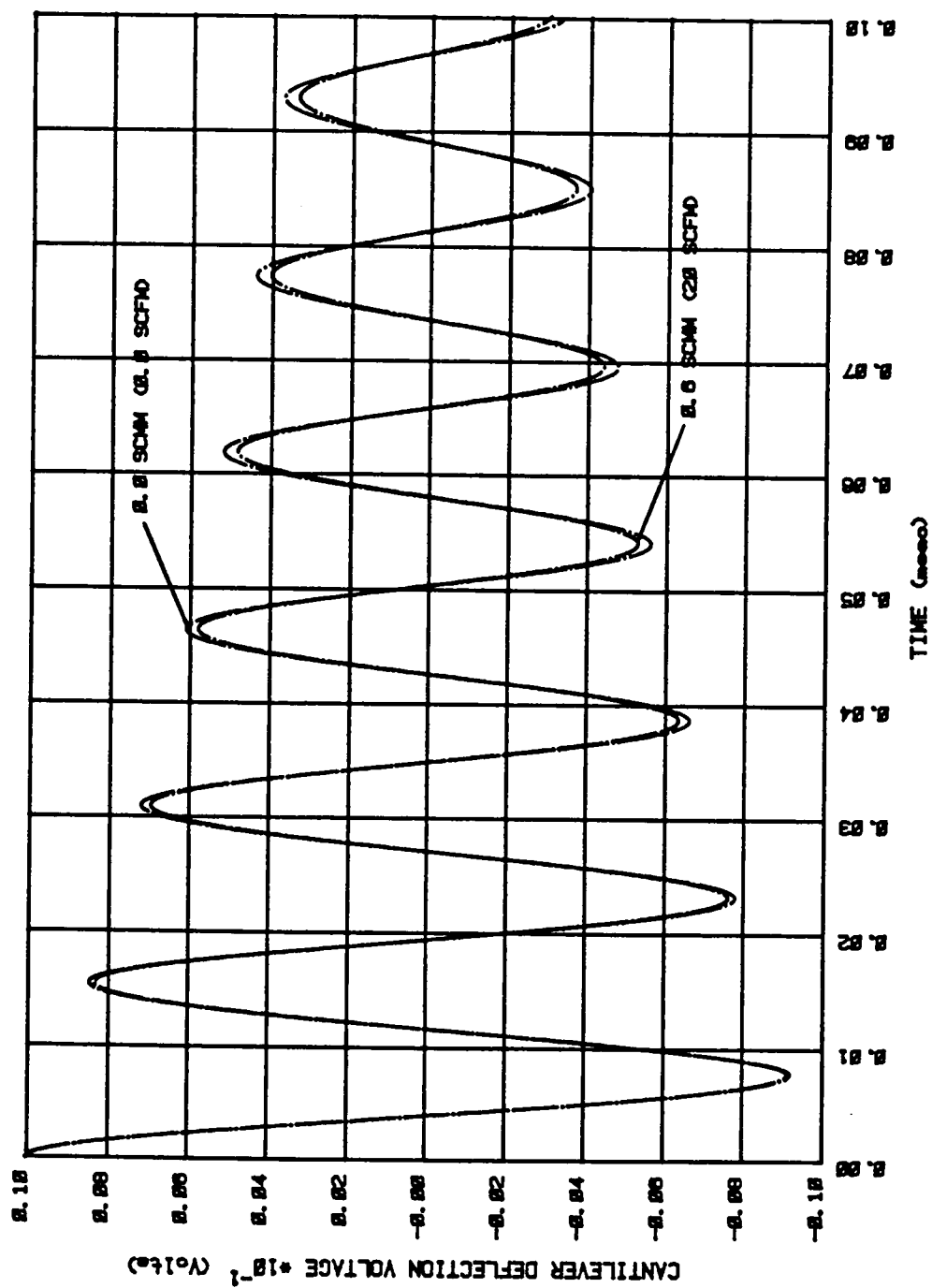


Figure 51. Nondimensional Analytical Plot of Dynamic Response for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

cross-section contained enough anisotropies as to yield unpredictable vibrational patterns.

The actual mass flow results for the 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) configuration are shown in the following figures. Figures 52, 53, and 54 are plots of typical values for three separate runs. As can be seen, the first of these plots shows results which are quite acceptable, illustrating a good trend and showing reasonable uniform behavior. In fact, even the problem of a drop off in the decay constant at high flow rates has seemed to disappear. However, in the next plot the drop-off is back, and the linear regression exhibits a marked decrease in slope as well as a negative intercept. This negative intercept characteristic is reflected in the third plot, Figure 54, to a larger degree, and it is unclear whether the high flow drop off occurred. Figure 55 shows the multiple results for this tube set and indicates a good deal of scatter. Overall, the flow measurement quality of this setup was the worst of the three tested, as indicated in Figure 56, the mass regression comparison. In general, the amounts varied within 40% of the regression line.

The density measurements were actually better than might be expected, and are shown in Figure 57. The (by now) familiar trend of increasing divergence with flow rate is apparent, and the measurements fall between 0 % and -24% of the "actual" density value.

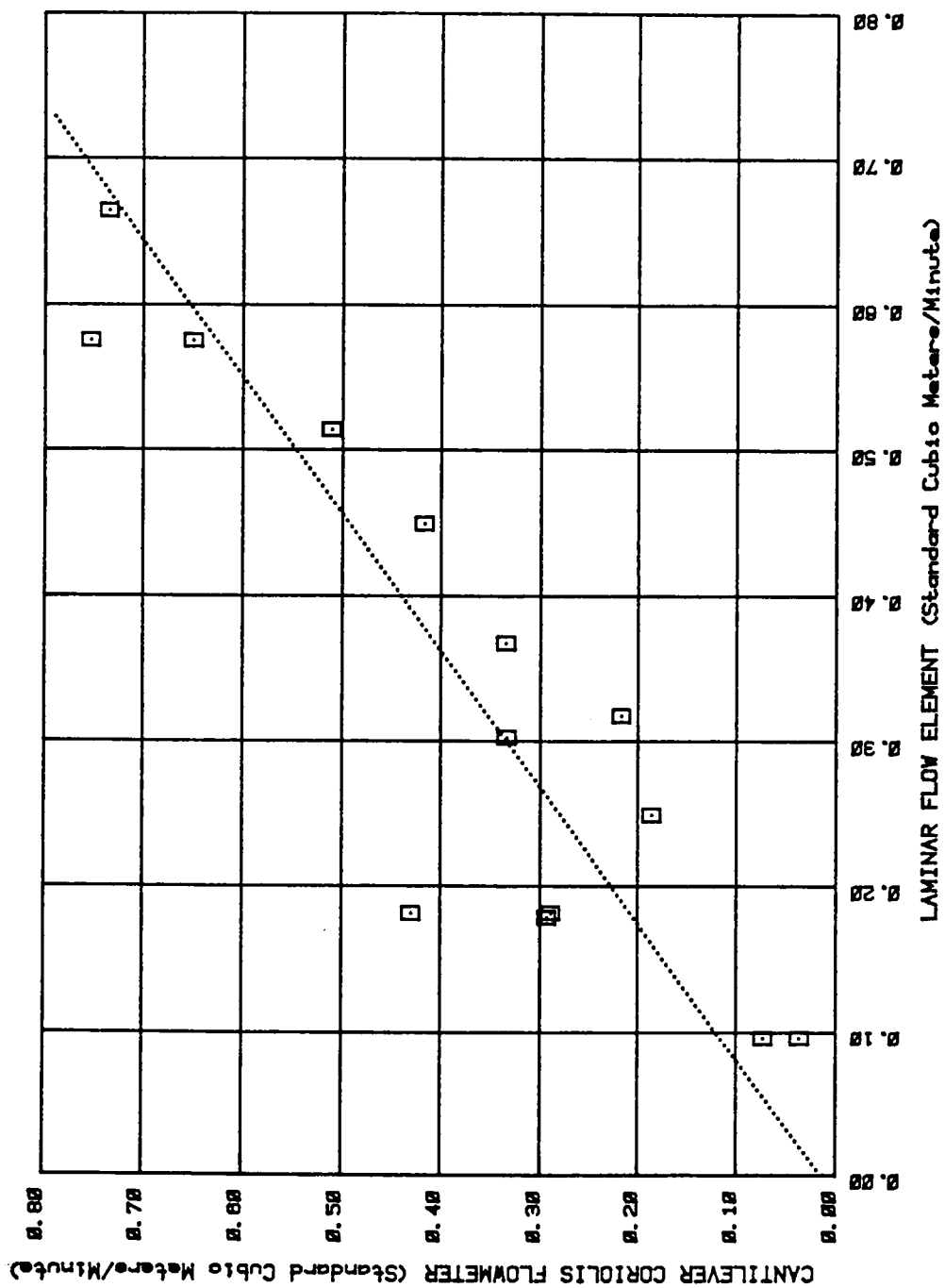


Figure 52. Typical Result and Regression for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes. (Run 1)

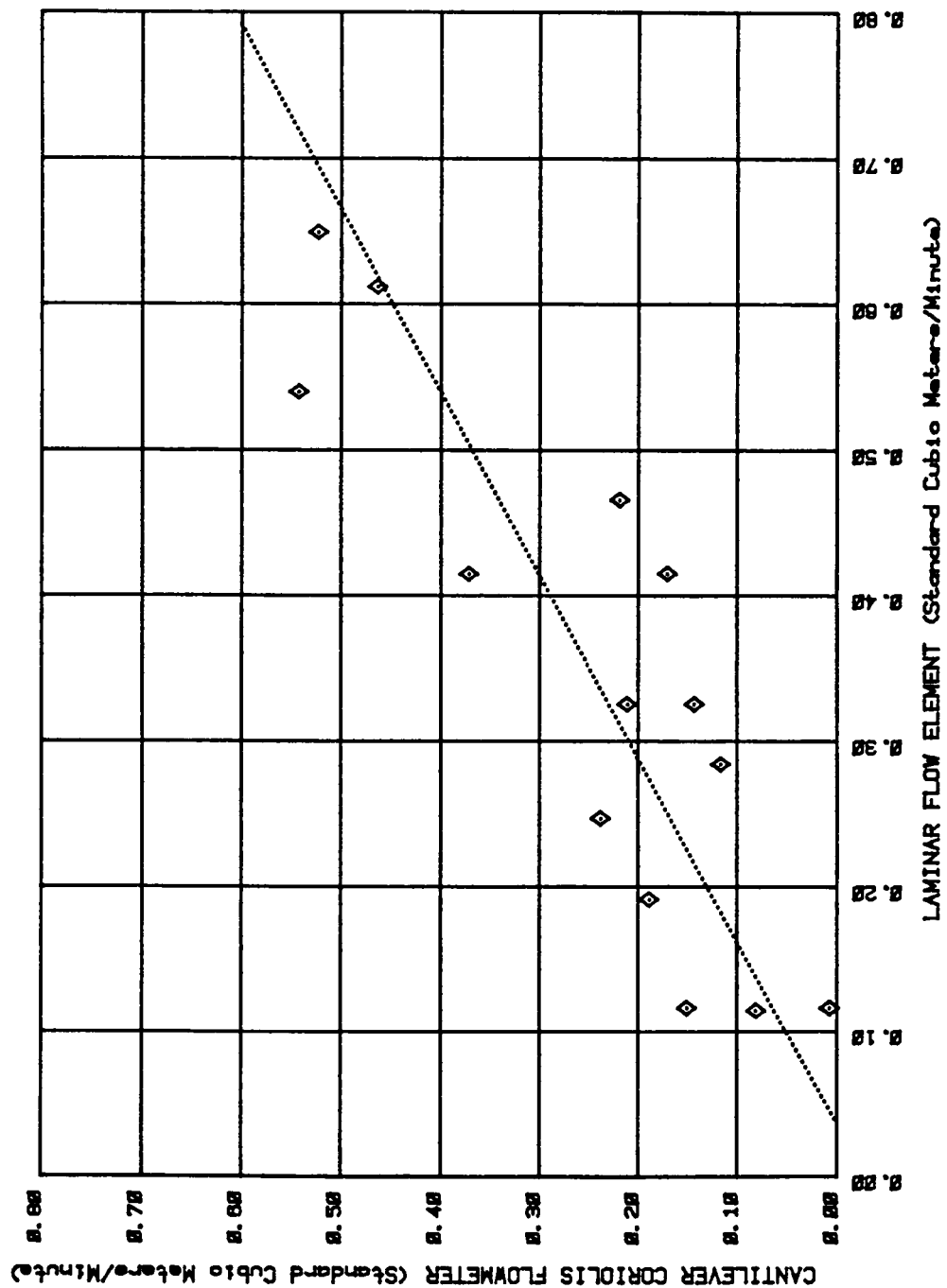


Figure 53. Typical Result and Regression for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes. (Run 2)

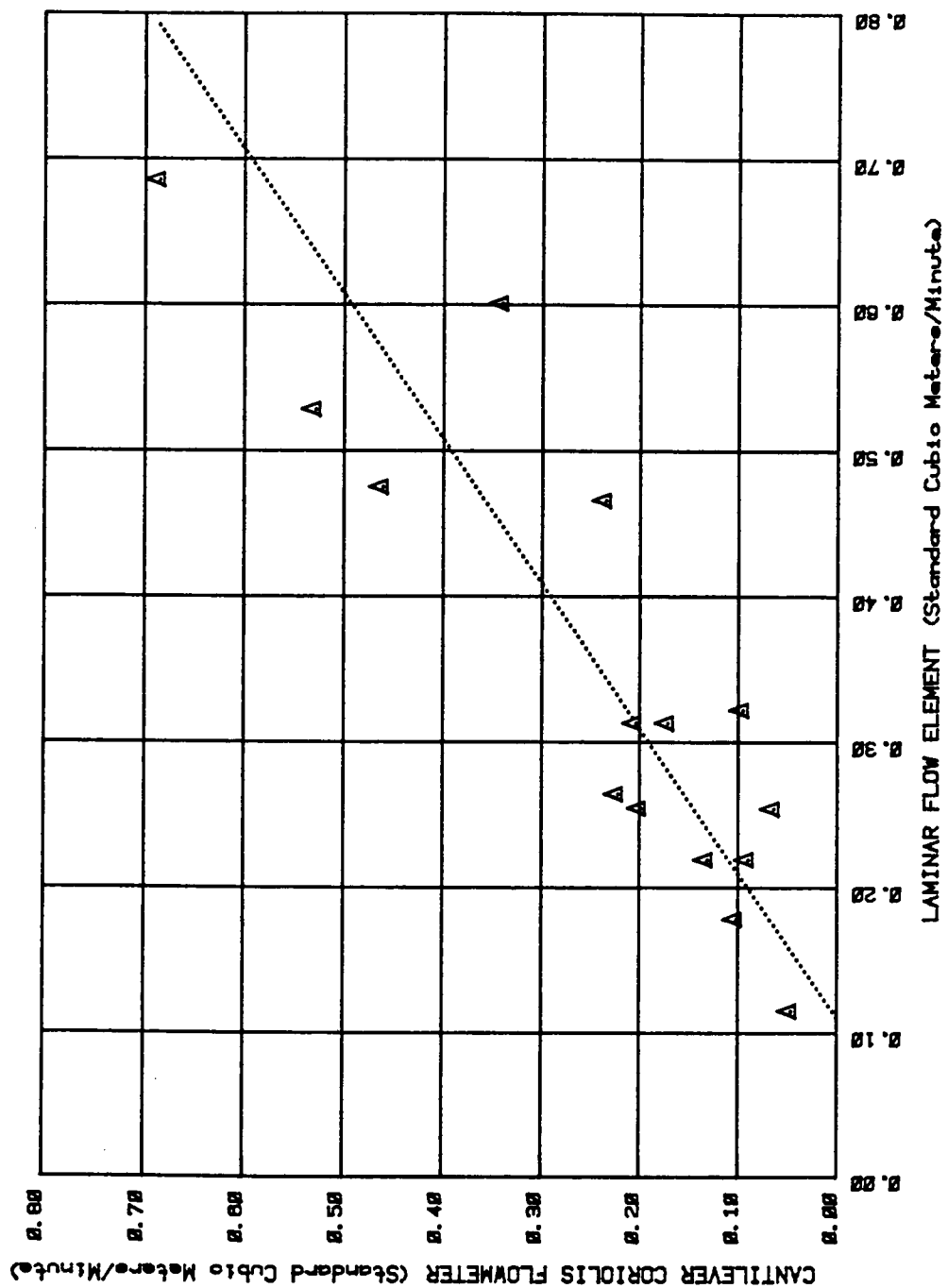


Figure 54. Typical Result and Regression for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes. (Run 3)

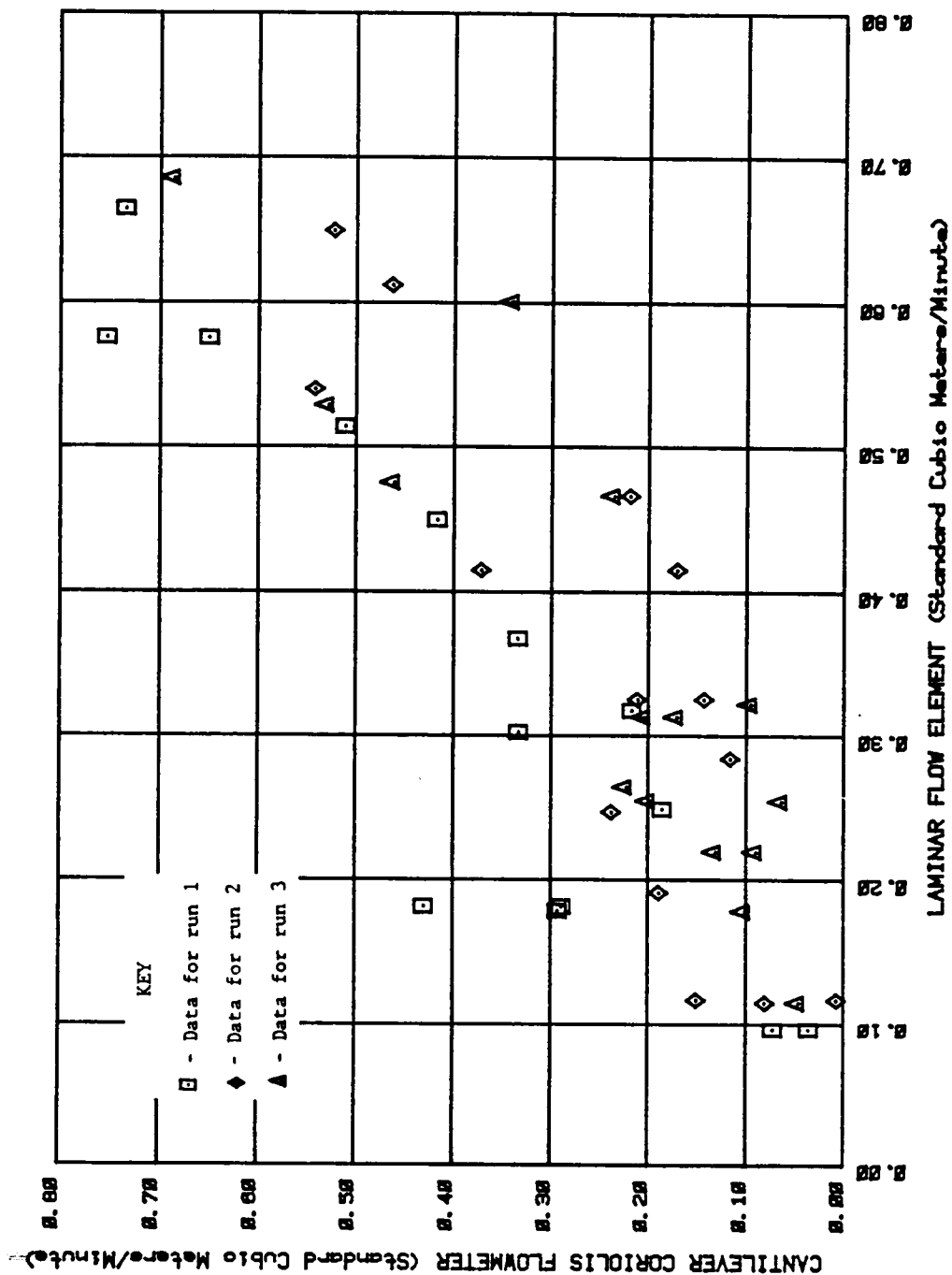


Figure 55. Multiple Results for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

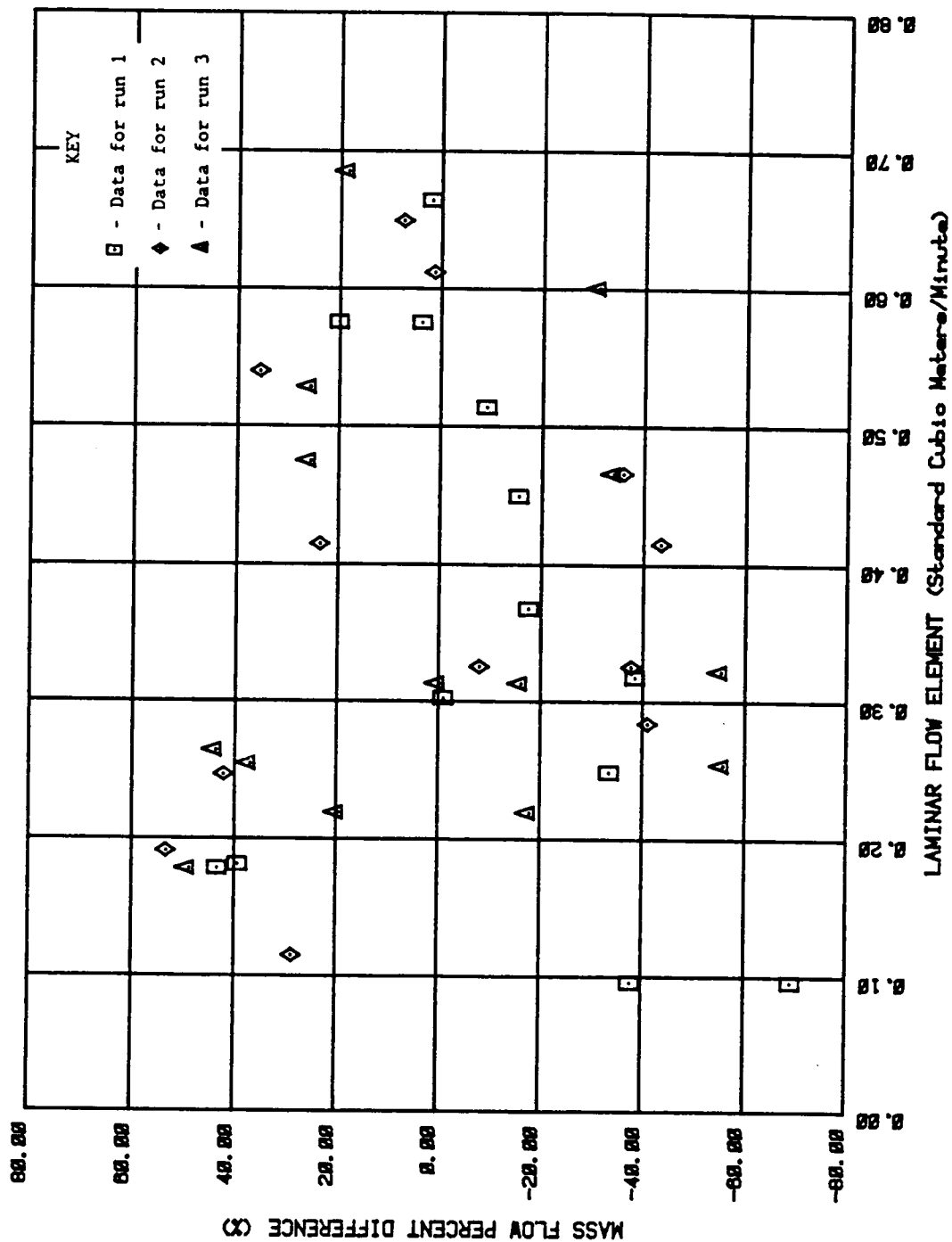


Figure 56. Mass Flow Regression Comparison for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

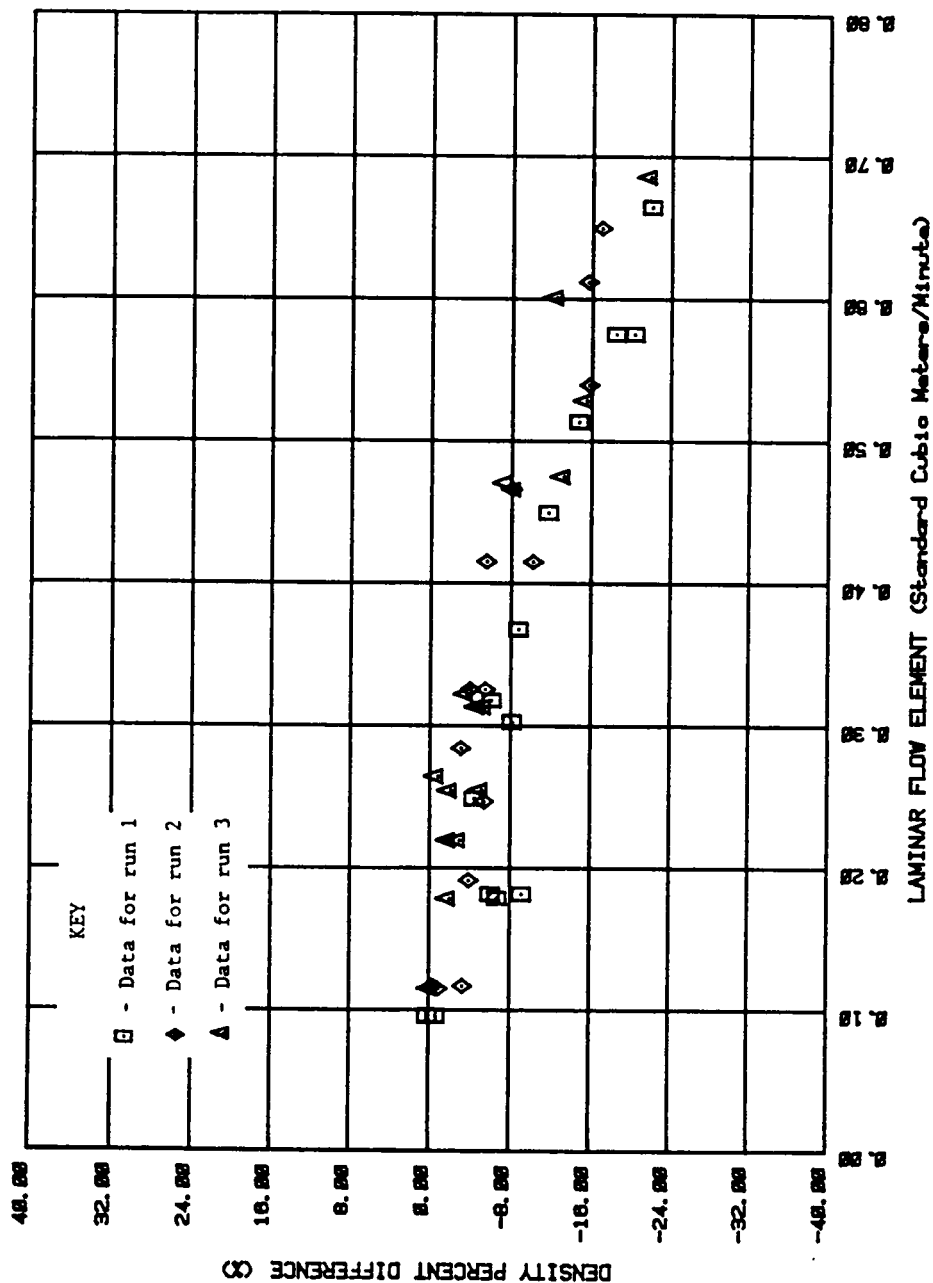


Figure 57. Density Regression Comparison for 1.5875 x 0.15875 cm (5/8 x 0.0625 inch) Plexiglas Tubes.

(Percent difference is taken with respect to actual density value.)

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The objectives of this study were to design and build a prototype cantilever Coriolis gas mass flowmeter using theory developed by W. R. Hamel [2], and evaluate the effectiveness of such a Coriolis gas flowmeter. The actual development and construction of such a meter was limited somewhat by constraints of economy and material availability; however, results which were obtained through proof-of-principle testing for three different flowmeter configurations showed clearly the potential of such a meter. The trends which were established in the results were reasonably well-defined and repeatable, at least for the 1.27×0.15875 cm ($1/2 \times 0.0625$ inch) plexiglas and 1.5875×0.15875 cm ($5/8 \times 0.0625$ inch) phenolic configurations; and the results were in good agreement with those which would be predicted from theory presented in Chapters II and III, even with the addition of time-dependent strain characteristics due to the use of plastics. The simple adjustments made to the overall model to deal with this additional material interaction seemed to work well, within certain limits, and the null-type configuration which was developed showed definite promise, although problems with "matching" empty tube parameters may have contributed to the scatter which pervaded all of the results.

This scatter was present in all of the results presented here, and represents the most disappointing aspect of the entire project; for while the results illustrated reasonable trends the repeatability (and hence precision) of the meter must be greatly improved if it is ever to

have commercial appeal. Therefore, future work should center on reducing scatter and improving the precision of the flowmeter. Listed below are several recommendations for accomplishing this.

1. Locate and obtain materials with properties that are optimally suited for use in a gas Coriolis flowmeter. That is, find tube materials from which tubes with high sensitivity to small Coriolis forces could be produced, and which possess properties which would be suitable for a null-type configuration. Such properties would include material property stability with respect to load and temperature, the capability of being formed into a uniform cross section, and the relationship between stress, strain, and time. In this study, the choice of materials was rather limited, and although efforts were made to determine which materials would have the best characteristics, the final choice was a compromise between economy, time, and material availability. A case could be made that a study on the selection of promising materials might constitute an entire thesis itself, and since a basis for selection has already been laid and a test stand is now available that would allow material samples to be tested, this approach definitely has some merit. The tube is the heart of the flowmeter, and finding tube materials with the optimal blend of characteristics (i.e., density, vibrational properties, and stress-strain relationship) would probably yield the greatest performance improvement over other possible areas of work. It is entirely possible that the greatest amount of the scatter present in these results came

from a combination of low tube sensitivity, mismatch of tube characteristics, buildup of viscous strain (reference Chapter III), and irregularities in the tube cross-section (i.e., thickness, material property variations) which could have produced unusual (and irregular) bending vibrations. All of these problems are related directly to the material properties of the tube. Future work should seek to determine what role each of these effects plays in the ability of the flowmeter and find materials which will give the "best" meter.

2. Optimize measurement system sensitivity. If the tube is the heart of the flowmeter, the digital measurement and control system is the brain necessary for taking in data, analyzing the tube response, and calculating mass flow, as well as controlling the measurement sequence. In the proof-of-principle tests discussed in this study, the average change in amplitude of the transient beam deflection voltage due to a 50% change in mass flow was on the order of tens of millivolts. The resolution of the A/D converter used was at best ± 2.4 mV due to quantization error for a 12-bit converter. This is quite a good resolution unless the signal change which is to be measured is of the same order of magnitude. Future work should include a determination of whether some of the scatter in the results of this study can be traced back to a (relatively) poor measurement system resolution, and if so, seek ways to improve the resolution of the measurement system. Two ways such an improvement could be had are the use of an A/D converter with more bits and more

amplification of the strain gage signals (and hence more amplification of the change in signals). Future studies should address the effectiveness of such changes in improving the performance of the flowmeter.

A third area of future work which has not been previously mentioned should center on the study of the instability noted in these tests, determine the cause of such instabilities, and seek to incorporate into the design methodology whatever is necessary to ensure that tube stability is not a factor in the construction and use of a gas flowmeter. This is particularly important for gases and vapors (like steam), since velocities are so much higher in general for these flows than for liquids, and it appears from the results that the instability is velocity-dependent. Results from this kind of a study would not necessarily lead to performance improvements such as those previously discussed, but could help determine how practical such a flowmeter really is. Obviously, it doesn't matter how sensitive a meter is if it cannot be used for fear of going unstable.

In summary, the results which were obtained in this study were very promising, and it is felt that the areas of work listed above hold the most hope for producing improvements in the performance of the gas cantilever Coriolis mass flowmeter.

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A

PROGRAMS USED FOR CALIBRATION AND MEASUREMENT

The calibration and measurement programs used in this study are listed. Data reduction was accomplished through the use of programs from W. R. Hamel's dissertation [2].

CALIBRATION ROUTINE

```

C*****C
C
C PROGRAM CFLMD.FOR: THIS PROGRAM IS DESIGNED TO BE THE MAIN CAL- C
C IBRATION PROGRAM FOR THE GAS CANTILEVER CORIOLIS MASS FLOWMETER. C
C MUCH OF THE PROGRAMMING IS TAKEN FROM DR. W.R. HAMEL'S DOCTORAL C
C DISSERTATION. C
C
C GREG DENNY DECEMBER 10, 1986 C
C
C*****C
C
C DECLARE VARIABLES
C
C COMMON IFLAG
C COMMON/PEST/BIAS0,BIAS1
C COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
C COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
C COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA
C 1,RADIAN(2),TAU(2),KCROSS,OMEGA
C COMMON/CALI/A1A(10),A2A(10),A3A(10),QS,PP,PP2,TF,A1,A2,A3
C REAL*4 YES,NO,DEC
C DATA YES,NO/'YES','NO'/
C IFLAG = 0
C
C ZERO OUT RELAY CONTROLLING ACTUATOR. USE IDOR ROUTINE TO SET OUTPUT
C BIT OF INTERFACE TO HIGH (+5V).
C
50 MASK = '1
IWORD = '1
CALL IDOR(0,0,MASK,IWORD)
C
C TURN REALTIME CLOCK OFF TO AVOID ERRORS FROM SETTING OPERATING
C PARAMETERS.
C
CALL SETR(-1,,0,0,0)
C
C GET DATA-SAMPLING PARAMETERS FROM TERMINAL (OPERATOR).
C
TYPE 100
FORMAT(' TYPE IN SAMPLING PERIOD IN MILLISECONDS. <CR>')
ACCEPT *,PRESET
PRESET = PRESET*1000
TYPE 200
FORMAT(' HOW MANY SAMPLES? <CR> (MAX 1000)')
ACCEPT *,NREAD
C
C GET VALUES OF T1E,T2E MEANS
C
TYPE 210
FORMAT(' TYPE IN MEAN VALUES OF T1E,T2E')
ACCEPT *,T1E,T2E
RATIO = T1E/T2E
C
C REMEMBER, WE HAVE TWO INPUT CHANNELS, STARTING AT CHAN 1.
C
NCCH = 2
ISTCHN = 1
ISIZ = NCCH*NREAD
C
C SET UP REALTIME CLOCK FOR SAMPLING PURPOSES.
C
IRATE = 1
MODE1 = 9
ICMF1 = 0
CALL SETR(IRATE,,MODE1,PRESET,ICMF1)
C
C THIS SETS THE CLOCK OSCILLATOR AT 1 MHZ. THE CLOCK WILL FEED
C PULSES TO THE A/D CONVERTER AT THE RATE OF 1 MHZ/PRESET. EACH
C PULSE WILL TRIGGER AN A/D CONVERSION.
C
C BEGIN FLOW CALIBRATION LOOP

```

```

        PAUSE ' SET FLOW RATE FOR FIRST CALIBRATION POINT'
        DO 1000 L=1,10
C
250      CALL VBIAS1
        CONTINUE
C
C THIS LOOP IS PROVIDED TO ENSURE BIASES ARE CONSTANT BEFORE TAKING
C A MEASUREMENT.
C
        TYPE 260
260      FORMAT(' TAKE NEW BIAS READING? (YES/NO)')
        ACCEPT 270,DEC
270      FORMAT(A4)
        IF (DEC.EQ.YES) GO TO 250
C
C
C NOW WE WANT DATA FROM VIBRATING BEAM. GO TO FIRE CONTROL ROUTINE.
C
        CALL FIRE1
        CONTINUE
        IF (IFLAG.EQ.1) GO TO 800
C
C SHOULD NOW HAVE DATA TO CONVERT INTO FREQUENCY AND DECAY RATE.
C NOW GO TO ALGOR ROUTINE.
C
        CALL ALGOR
        CONTINUE
        TYPE 280,BIAS0,BIAS1
280      FORMAT(' BIAS ON CHAN1 IS: ',F8.5,' VOLTS',/,
1' BIAS ON CHAN2 IS: ',F8.5,' VOLTS',/)
C NOW INPUT DATA FROM STANDARD THROUGH KEYBOARD
        TYPE 300
300      FORMAT(' PLEASE INPUT THE FOLLOWING DATA:',/,
1' 1. GAUGE PRESSURE AT PROTOTYPE (IN. H2O)',/,
2' 2. TEMPERATURE OF FLOW (DEG C)',/,
3' 3. FLOW RATE IN SCFM',/,
4' 4. GAUGE PRESSURE AT REFERENCE TUBE (PSI)',/)
        ACCEPT *,PP,TF,QS,PP2
C CALCULATE A1,A2,A3
        CALL FCAL1
        A1A(L) = A1
        A2A(L) = A2
        A3A(L) = A3
        IF(L.EQ.10) GO TO 1000
        PAUSE ' SET NEXT FLOW RATE'
1000     CONTINUE
C PERFORM STATISTICAL ANALYSIS ON CALIBRATION CONSTANTS
        CALL CALAVE
800      CONTINUE
        STOP
        END
        SUBROUTINE VBIAS1
C
C*****C
C SUBROUTINE BIAS1: THIS ROUTINE USES DTLIB CONTROL AND MEASURE- C
C MENT ROUTINES TO MEASURE THE VOLTAGE BIASES OF THE MEASURING C
C TUBES. THESE VOLTAGES ARE THEN OUTPUTTED TO THE SCREEN THROUGH C
C THIS ROUTINE. C
C C
C GREG DENNY C
C OCTOBER 23, 1985 C
C C
C*****C
C DECLARE VARIABLES C
C
        COMMON/PEST/BIAS0,BIAS1
        COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
        COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
C
C DEFINE BIT WEIGHTINGS
C
        ANLSBV = 20.0/4096

```

```

C
C SET PARAMETERS
C
      NBUF = 1
      IGAIN = 1
      MODE = 2
      ICMF = 0
      IREF = 0

C
C SET VARIABLES TO ZERO
C
      DO 50 I = 1,2000
      IBUF(I) = 0
50      CONTINUE
      DO 75 I = 1,1000
      VOLT(I) = 0.0
      VOLT1(I) = 0.0
      VOLT2(I) = 0.0
75      CONTINUE
C MEASURE EMPTY TUBE BIASES
C
      CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE,ICMF,IREF)
      IVAR = LWAIT(ICMF,0)

C
C CONVERT DATA TO ENGINEERING UNITS
C
      BIAS0 = 0.0
      BIAS1 = 0.0
      DO 100 J=1,NREAD
      BIAS0 = BIAS0 + ANLSBV*FLOAT(IBUF(2*J-1))
      BIAS1 = BIAS1 + ANLSBV*FLOAT(IBUF(2*J))
100     CONTINUE
C
C AVERAGE READINGS
C
      BIAS0 = BIAS0/NREAD
      BIAS1 = BIAS1/NREAD

C
C OUTPUT DATA
C
      TYPE 300,BIAS0,BIAS1
300     FORMAT(// ' CHANNEL 1 BIAS =',F15.7,'VOLTS',/,
1         ' CHANNEL 2 BIAS =',F15.7,'VOLTS',/)
      RETURN
      END
      SUBROUTINE FIRE1

C
C*****C
C SUBROUTINE FIRE1: THIS ROUTINE CONTROLS THE DEFLECTION OF THE C
C TUBES BY ACTIVATING THE ELECTROMAGNETIC ACTUATOR VIA A PARALLEL C
C I/O PORT CONTROLLED BY THE DTLIB ROUTINE IDOR. C
C C
C GREG DENNY OCTOBER 27,1986 C
C C
C*****C
C
C DECLARE VARIABLES
C
      COMMON IFLAG
      COMMON/PEST/BIAS0,BIAS1
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      REAL*4 YES,NO,DECIDE
      INTEGER*4 ITIME
      DATA YES,NO/'YES','NO'/

C
C AGAIN, ZERO OUT RELAY USING IDOR
C
      MASK = '1
      IWORD = '1
      CALL IDOR(0,0,MASK,IWORD)

```

```

C
C QUERY USER FOR MEASUREMENT ATTEMPT
C
      TYPE 100
100    FORMAT(' INITIATE MEASUREMENT? [YES/NO]')
      ACCEPT 200,DECIDE
200    FORMAT(A4)
      IF (DECIDE.EQ.YES) GO TO 300
      IFLAG = 1
      GO TO 400

C
C DEFLECT TUBES
C
300    PAUSE ' <CR> TO INITIATE MEASUREMENT'
      CALL IDOR(0,0,MASK,'0')

C
C HOLD TUBES DOWN FOR 1 SECOND
C
      CALL GTIM(ETIME)
      CALL CUTTIM(ETIME,IHRS,IMIN,ISEC,ITCK)
      ISECR = ISEC + 1
350    CALL GTIM(ETIME)
      CALL CUTTIM(ETIME,IHRS,IMIN,ISEC,ITCK)
      IF (ISEC.LT.ISECR) GO TO 350

C
C RELEASE TUBES
C
      CALL IDOR(0,0,MASK,IWORD)

C
C GET DATA
C
      CALL GETDAT
400    CONTINUE
      RETURN
      END
      SUBROUTINE GETDAT

C
C*****C
C SUBROUTINE GETDAT: THIS ROUTINE GATHERS THE DATA TO BE CONVERTED C
C INTO FREQUENCY AND DECAY RATE. THIS DATA IS PASSED TO ALGOR IN C
C TWO VECTORS: VOLT1 AND VOLT2. C
C C
C GREG DENNY OCTOBER 27,1986 C
C C
C*****C
C DECLARE VARIABLES
C
      COMMON/PEST/BIAS0,BIAS1
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)

C
C DECLARE BIT WEIGHTINGS
C
      ANLSBV = 20.0/4096

C
C DEFINE SET-UP PARAMETERS FOR A/D CONVERSION
C
      NBUF = 1
      IGAIN = 1
      MODE = 2
      ICMF = 0
      IBEF = 0

C
C COLLECT DATA
C
      CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,
1IGAIN,MODE,ICMF,IBEF)
      IVAR = LWAIT(ICMF,0)

C
C CONVERT DATA TO ENGINEERING UNITS AND REMOVE BIASES
C

```

```

      DO 100 J = 1,NREAD
      VOLT1(J) = ANLSBV*FLOAT(IBUF(2*J-1)) - BIAS0
      VOLT2(J) = ANLSBV*FLOAT(IBUF(2*J)) - BIAS1
100   CONTINUE
      RETURN
      END

      SUBROUTINE ALGOR

C
C*****C
C
C  SUBROUTINE ALGOR: THIS SUBROUTINE FINDS THE PEAKS
C  FROM THE RAW DATA AND CONVERTS THESE INTO THE FREQUENCY OF VIBRATION
C  AND DECAY RATE. IT CALLS SEVERAL OTHER SUBROUTINES, WHICH ARE ALL
C  GROUPED UNDER THIS TITLE BLOCK.
C
C
C          GREG DENNY                      OCTOBER 27, 1986
C
C*****C
C
C  DECLARE VARIABLES
C
      COMMON/MESS/NFLAG
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
      1TAU(2),KCROSS,OMEGA
      COMMON PEAK(50),KPEAK
      COMMON/XYZ/JL,JR
      REAL*4 MINUM,MAXUM

C
C  INITIALIZE FLAG
C
      NFLAG = 1

C
C  READ DATA INTO VOLT ARRAY
C
50    CONTINUE
      IF (NFLAG.EQ.2) GO TO 200
      IF (NFLAG.GT.2) GO TO 800
      DO 100 J = 1,NREAD
      VOLT(J) = VOLT1(J)
100   CONTINUE
      GO TO 325
200   CONTINUE
      DO 250 J = 1,NREAD
      VOLT(J) = 0.0
250   CONTINUE
      DO 300 J = 1,NREAD
      VOLT(J) = VOLT2(J)
300   CONTINUE
C
C  INITIALIZE STORAGE ARRAYS
C
325   DO 350 I=1,50
      TIME(I) = 0.0
      PKODD(I) = 0.0
      PKEVEN(I) = 0.0
      PEAK(I) = 0.0
      JTIME(I) = 0
350   CONTINUE
C
C  NOW HAVE DATA IN VOLT ARRAY; FIND PEAKS, DECAY RATE, AND FREQUENCY
C  FOR THIS PARTICULAR TUBE
C
400   J = 0
      K = 1

C
C  BEGIN
C
      IF (VOLT(1).LT.VOLT(2)) GO TO 600
500   CONTINUE
C

```

```

C LOOK FOR MINIMUM PEAK
  CALL MINIM(J)
C
C CHECK AND SEE IF END OF DATA HAS OCCURRED
C
  IF (J.GE.NREAD) GO TO 700

C
C STORE VALUES FOR THIS PEAK
C
  CALL TRAP(J,K)
C
C LOOK FOR A MAXIMUM PEAK
C
600  CALL MAXIM(J)
     IF (J.GE.NREAD) GO TO 700
     CALL TRAP(J,K)
C
C KEEP GOING UNTIL DATA USED UP
C
  GO TO 500
C
C START NEW SECTION; ALL PEAKS HAVE BEEN LOCATED
C
700  K = K-1
C
C KCROSS EQUALS NUMBER OF PEAKS MINUS ONE
C
  KCROSS = K
  KPEAK = K
C
C CALCULATE WAVEFORM FREQUENCY
C
  CALL PERIOD
  CONTINUE
C
C FIND WAVEFORM PEAKS AND DECAY RATES
C
  CALL PEAKER
  CONTINUE
C
C CHECK FLAG; GO AGAIN IF NECESSARY
C
  NFLAG = NFLAG + 1
  IF (NFLAG.LE.2) GO TO 50
800  RETURN
  END
C
C SUBROUTINE MAXIM(J)
C
  SUBROUTINE MAXIM(J)
C
C DECLARE VARIABLES
C
  COMMON/MESS/NFLAG
  COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
  COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
  COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
  COMMON/XYZ/JL,JR
  REAL*4 MINUM,MAXUM,PK
  INTEGER*2 JMAX
C
  IF (K.GE.3) GO TO 300
100  J = J+1
C
C CHECK AND SEE IF DATA FILE AT END
C
  IF (J.GE.NREAD) GO TO 200
C
C LOOK FOR A MAXIMUM
C

```

```

      MAXUM = AMAX1(VOLT(J),VOLT(J+1))
      IF (MAXUM.EQ.VOLT(J)) GO TO 200
      GO TO 100
200    CONTINUE
      GO TO 400
300    CONTINUE
      IF (JR.GE.NREAD) J=JR
      IF (J.EQ.JR) GO TO 400
      PK = -100.00
      JMAX = 0
      DO 325 J=JL,JR
      PK = AMAX1(PK,VOLT(J))
325    CONTINUE
      DO 350 J=JL,JR
      IF (PK.EQ.VOLT(J)) JMAX=J
350    CONTINUE
      J = JMAX
400    CONTINUE
      RETURN
      END

C
C SUBROUTINE MINIM(J)
C
      SUBROUTINE MINIM(J)
C
C DECLARE VARIABLES
C
      COMMON/MESS/NFLAG
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
      COMMON/XYZ/JL,JR
      REAL*4 MINUM,MAXUM,PK
      INTEGER*2 JMIN

C
      IF (K.GE.3) GO TO 300
100    J = J+1
C
C CHECK AND SEE IF DATA FILE IS AT END
C
      IF (J.GE.NREAD) GO TO 200
C
C LOOK FOR MINIMUM
C
      MINUM = AMIN1(VOLT(J),VOLT(J+1))
      IF (MINUM.EQ.VOLT(J)) GO TO 200
      GO TO 100
200    CONTINUE
      GO TO 400
300    CONTINUE
      IF (JR.GE.NREAD) J=JR
      IF (J.EQ.JR) GO TO 400
      PK = 0.0
      JMIN = 0
      DO 325 J=JL,JR
      PK = AMIN1(PK,VOLT(J))
325    CONTINUE
      DO 350 J=JL,JR
      IF (PK.EQ.VOLT(J)) JMIN=J
350    CONTINUE
      J = JMIN
400    CONTINUE
      RETURN
      END

C
C SUBROUTINE TRAP(J,K)
C
      SUBROUTINE TRAP(J,K)
C
C DECLARE VARIABLES
C

```

```

COMMON/MESS/NFLAG
COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
COMMON PEAK(50),KPEAK
COMMON/XYZ/JL,JR
REAL*4 MINUM,MAXUM

C
C STORE VALUES OF PEAKS;
C
      PEAK(K) = VOLT(J)
C
C CONVERT TIMING TO REAL TIME
C
      TIME(K) = J*PRESET*1.0E-6
      JTIME(K) = J
      IF (K.LE.1) J = J + 10
      IF (K.LE.1) GO TO 100
C
C WANT FLAG TO KICK OUT
C
      IF (NREAD-J.LE.5) GO TO 200
C
C IF AT LEAST SECOND PEAK HAS BEEN LOCATED, ENOUGH INFORMATION EXISTS
C TO STEP FORWARD MORE THAN ONE J-UNIT TO THE REGION OF THE NEXT PEAK.
C
      T = TIME(K) - TIME(K-1)
      JL = J + JTIME(K) - JTIME(K-1) - 5
      JR = JL + 10
      J = JL
100    CONTINUE
C
C K = # OF PEAKS
C
      K = K+1
200    CONTINUE
      RETURN
      END
C
C SUBROUTINE PERIOD
C
      SUBROUTINE PERIOD
C
C DECLARE VARIABLES
C
      COMMON/MESS/NFLAG
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
      REAL SIGTA,PERCEN,SUM,DIFF(50)
C
C DETERMINE PERIOD (ACTUALLY ONE-HALF PERIOD)
C
      SUM = 0.0
      DO 200 J=1,KCROSS-2
      SUM = SUM + TIME(J+1) - TIME(J)
200    CONTINUE
      TA = SUM/(KCROSS-2)
C DO STATISTICAL ANALYSIS OF WAVEFORM NOW RATHER THAN IN
C MAIN ROUTINE.
      DO 250 J=1,KCROSS-2
      DIFF(J) = 0.0
250    CONTINUE
      DO 300 J=1,KCROSS-2
      DIFF(J) = TIME(J+1) - TIME(J)
300    CONTINUE
C COMPUTE STANDARD DEVIATION
      SIGTA = 0.0
      DO 400 J=1,KCROSS-2
      SIGTA = SIGTA + (DIFF(J)-TA)**2
400    CONTINUE

```

```

      SIGTA = SQRT(SIGTA/(KCROSS-2))
C DETERMINE WHICH POINTS TO TOSS
      DO 500 J=1,KCROSS-2
      PERCEN = ABS((TA-DIFF(J))/DIFF(J))
      IF (PERCEN.GE.0.03) DIFF(J)=0.0
500    CONTINUE
C CALCULATE NEW PERIOD
      SUM = 0.0
      K = 1
      DO 600 J=1,KCROSS-2
      SUM = SUM + DIFF(J)
      IF (DIFF(J).EQ.0.0) K=K-1
      K = K + 1
600    CONTINUE
      TA = SUM/(K-1)
C
C DETERMINE FREQUENCY
C
      OMEGA = 0.5/TA
      RADIAN(NFLAG) = OMEGA*6.28318
      RETURN
      END

C SUBROUTINE PEAKER
C
      SUBROUTINE PEAKER
C
C DECLARE VARIABLES
C
      DIMENSION C(50),PKMIN(50),PKMAX(50)
      COMMON/MESS/NFLAG
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
      COMMON PEAK(50),KPEAK
      DO 10 I=1,50
      PKMIN(I) = 0.0
      PKMAX(I) = 0.0
      C(I) = 0.0
10    CONTINUE
C
C PARTITION PEAKS
C
      DO 20 J=1,KPEAK/2
      PKEVEN(2*J) = PEAK(2*J)
      PKODD(2*J-1) = PEAK(2*J-1)
20    CONTINUE
C
C PERFORM INTERPOLATION
C
      DO 50 J=2,KPEAK/2
      PKODD(2*J-2) = (PKODD(2*J-3) + PKODD(2*J-1))/2.0
      PKEVEN(2*J-1) = (PKEVEN(2*J-2) + PKEVEN(2*J))/2.0
50    CONTINUE
C
C GET DATA INTO MAX AND MIN VECTORS
C
      KPEAK = 2*INT(KPEAK/2.0)
      IF (PKODD(1).GT.PKEVEN(2)) GO TO 55
      DO 30 J=1,KPEAK
      PKMIN(J) = PKODD(J)
      PKMAX(J) = PKEVEN(J)
30    CONTINUE
      GO TO 100
55    DO 60 J=1,KPEAK
      PKMIN(J) = PKEVEN(J)
      PKMAX(J) = PKODD(J)
60    CONTINUE
100   CONTINUE

```

```

DO 400 I=1,KPEAK
TYPE 450,I,PEAK(I)
450   FORMAT(' PEAK',I3,' =',F8.4,' VOLTS.')
400   CONTINUE
C
C COMPUTE C(K) AND GET DECAY CONSTANT
C
DO 500 J=2,KPEAK-1
C(J) = (PKMAX(J) - PKMIN(J))/2.0
500   CONTINUE
SUMT = 0.0
SUMTS = 0.0
SLNC = 0.0
STLNC = 0.0
DO 600 J=2,KPEAK-1
SUMT = SUMT + TIME(J)
SUMTS = SUMTS + TIME(J)**2
SLNC = SLNC + ALOG(C(J))
STLNC = STLNC + TIME(J)*ALOG(C(J))
600   CONTINUE
N = KPEAK-2
TAU(NFLAG) = (STLNC - SLNC*SUMT/N)/(SUMT**2/N - SUMTS)
RETURN
END

SUBROUTINE FCAL1
C
C SUBROUTINE FCAL1 COMPUTES VALUES FOR THE CALIBRATION CONSTANTS A1,
C A2, AND A3.
C
C DECLARE VARIABLES
C
COMMON/CALI/A1A(10),A2A(10),A3A(10),QS,PP,PP2,TF,A1,A2,A3
COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
REAL*4 MDOTS,NUM
C
C ASSIGN VALUES TO MEASURED QUANTITIES
C
R1 = TAU(1)
RC1 = TAU(2)*RATIO
R2 = RADIAN(1)**2
RC2 = RADIAN(2)**2
C
C COMPUTE DENSITY AND VELOCITY OF FLOW
C
PP = PP*0.036111 + 14.7
PP2 = PP2 + 14.7
TF = 9.0*TF/5.0 + 32.0 + 460.0
R = 53.34
AREA1 = 0.001363538
AREA2 = 0.000766990
U = (QS/AREA1)*(14.7/PP)*(TF/530.0)/60.0
RHOF = PP/(R*TF)*144.0
RHOF0 = PP2/(R*TF)*144.0
RHOT = 73.2796
NUM = RHOF*AREA1
DEN = NUM + RHOT*AREA2
BETA = NUM/DEN
BETAM1 = 1.0 - BETA
MDOTS = QS*14.7/(R*530)*144.0/60.0
C
C CALCULATE CALIBRATION CONSTANTS
C
A1 = RHOF*BETAM1/BETA
A3 = 1 + RHOF0/A1
A2 = (R1 - A3*BETAM1*RC1)/(BETA*U)

```

```

      TYPE 10,A1,A2,A3,RC1,RC2,R1,R2,BETA,MDOTS,U,RHOF
10    FORMAT(//,
1' CALIBRATION RESULTS:  A1 =',E15.7,/,
2'                      A2 =',E15.7,/,
3'                      A3 =',E15.7,/,
4'                      RC1 =',E15.7,/,
5'                      RC2 =',E15.7,/,
6'                      R1 =',E15.7,/,
7'                      R2 =',E15.7,/,
8'                      BETA =',E15.7,/,
9'                      MDOTS =',E15.7,/,
1'                      U =',E15.7,/,
2'                      RHOF =',E15.7,/)
      RETURN
      END
      SUBROUTINE CALAVE
C THIS SUBROUTINE AVERAGES THE READINGS FOR THE CALIBRATION
C CONSTANTS AND COMPUTES THE MEANS AND STANDARD DEVIATIONS OF
C THE CONSTANTS.  THESE CONSTANTS ARE STORED FOR USE BY THE FLOW
C CALCULATION PROGRAM.
C
C DECLARE VARIABLES
C
      COMMON/CALI/A1A(10),A2A(10),A3A(10),QS,PP,PP2,TF,A1,A2,A3
C
C AVERAGE A1,A2,A3; OBTAIN MEANS
C
      A1 = 0.0
      A2 = 0.0
      A3 = 0.0
      DO 10 J=1,10
      A1 = A1 + A1A(J)
      A2 = A2 + A2A(J)
      A3 = A3 + A3A(J)
10    CONTINUE
      A1 = A1/10.0
      A2 = A2/10.0
      A3 = A3/10.0
C
C COMPUTE STANDARD DEVIATIONS
C
      SIGA1 = 0.0
      SIGA2 = 0.0
      SIGA3 = 0.0
      DO 20 J=1,10
      SIGA1 = SIGA1 + (A1A(J)-A1)**2
      SIGA2 = SIGA2 + (A2A(J)-A2)**2
      SIGA3 = SIGA3 + (A3A(J)-A3)**2
20    CONTINUE
      SIGA1 = SQRT(SIGA1/9.0)
      SIGA2 = SQRT(SIGA2/9.0)
      SIGA3 = SQRT(SIGA3/9.0)
C
C STORE RESULTS
C
      OPEN(UNIT=2,NAME='STAT.CAL',ACCESS='SEQUENTIAL',TYPE='NEW',
1FORM='UNFORMATTED')
      WRITE(2) A1,A1,A3,SIGA1,SIGA2,SIGA3
      CLOSE(UNIT=2)
C
C OUTPUT RESULTS
C
      TYPE 30,A1,SIGA1,A2,SIGA2,A3,SIGA3
30    FORMAT(' CALIBRATION CONSTANT          STANDARD DEVIATION',//,
1' A1 =',E15.7,'          SIGA1 =',E15.7,/,
2' A2 =',E15.7,'          SIGA2 =',E15.7,/,
3' A3 =',E15.7,'          SIGA3 =',E15.7,/)
      RETURN
      END

```

MEASUREMENT ROUTINE

```

C*****C
C
C PROGRAM FLMD.FOR: THIS PROGRAM IS DESIGNED TO BE THE MAIN MEAS- C
C UREMENT AND CONTROL PROGRAM FOR THE GAS CANTILEVER CORIOLIS MASS C
C FLOWMETER. MUCH OF THE PROGRAMMING IS TAKEN FROM DR. W.R. HAMEL'S C
C DOCTORAL DISSERTATION. C
C
C GREG DENNY OCTOBER 23, 1986 C
C
C*****C
C DECLARE VARIABLES
C
C COMMON IFLAG
C COMMON/PEST/BIAS0,BIAS1
C COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
C COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
C COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA
C 1,RADIAN(2),TAU(2),KCROSS,OMEGA
C LOGICAL*1 FILE(15)
C REAL*4 YES,NO,DEC,STOR,RITE
C DATA YES,NO/'YES','NO'/
C IFLAG = 0
C ICHECK = 0
C
C ZERO OUT RELAY CONTROLLING ACTUATOR. USE IDOR ROUTINE TO SET OUTPUT
C BIT OF INTERFACE TO HIGH (+5V).
C
50 MASK = *1
IWORD = *1
CALL IDOR(0,0,MASK,IWORD)
C
C TURN REALTIME CLOCK OFF TO AVOID ERRORS FROM SETTING OPERATING
C PARAMETERS.
C
CALL SETR(-1,,0,0,0)
C
C CLOSE FILE IF ALREADY OPEN
C
IF (ICHECK.EQ.1) CLOSE(UNIT=8)
C
C GET NAME OF DATA STORAGE FILE
C
TYPE 60
FORMAT(' TYPE IN NAME OF STORAGE FILE...DYN:MFL0XX.DAT')
ACCEPT 70,FILE
70 FORMAT(14A1)
FILE(15)=0
OPEN(UNIT=8,NAME=FILE,ACCESS='SEQUENTIAL',TYPE='NEW',
1FORM='UNFORMATTED')
C
C GET DATA-SAMPLING PARAMETERS FROM TERMINAL (OPERATOR).
C
TYPE 100
FORMAT(' TYPE IN SAMPLING PERIOD IN MILLISECONDS. <CR>')
ACCEPT *,PRESET
PRESET = PRESET*1000
TYPE 200
FORMAT(' HOW MANY SAMPLES? <CR> (MAX 1000)')
ACCEPT *,NREAD
TYPE 225
FORMAT(' TYPE IN MEAN EMPTY TUBE DECAY CONSTANTS, T1E,T2E')
ACCEPT *,T1E,T2E
RATIO = T1E/T2E
C
C REMEMBER, WE HAVE TWO INPUT CHANNELS, STARTING AT CHAN 1.
C
250 CONTINUE
NCCH = 2
ISTCHN = 1
ISIZ = NCCH*NREAD

```

```

C
C SET UP REALTIME CLOCK FOR SAMPLING PURPOSES.
C
      IRATE = 1
      MODE1 = 9
      ICMF1 = 0
      CALL SETR(-1,,0,0,0)
      CALL SETR(IRATE,,MODE1,PRESET,ICMF1)
C
C THIS SETS THE CLOCK OSCILLATOR AT 1 MHZ. THE CLOCK WILL FEED
C PULSES TO THE A/D CONVERTER AT THE RATE OF 1 MHZ/PRESET. EACH
C PULSE WILL TRIGGER AN A/D CONVERSION.
C
C
C NOW, MEASURE VOLTAGE BIAS (VOLTAGE FROM UNDEFLECTED BEAM).
C
C
C THIS LOOP IS PROVIDED TO ENSURE BIASES HAVE STEADIED OUT.
C
49      CALL VBIAS1
      CONTINUE
      TYPE 51
51      FORMAT(' DO YOU WISH TO CHECK BIAS AGAIN? (YES/NO)')
      ACCEPT 52,DEC
52      FORMAT(A4)
      IF (DEC.EQ.YES) GO TO 49
C
C
C NOW WE WANT DATA FROM VIBRATING BEAM. GO TO FIRE CONTROL ROUTINE.
C
      CALL FIRE1
      CONTINUE
      IF (IFLAG.EQ.1) GO TO 800
C
C SHOULD NOW HAVE DATA TO CONVERT INTO FREQUENCY AND DECAY RATE.
C NOW GO TO ALGOR ROUTINE.
C
      CALL ALGOR
      CONTINUE
C
C OUTPUT FREQUENCY.
C
      FREQ1 = RADIAN(1)
      FREQ2 = RADIAN(2)
      TYPE 600,FREQ1,FREQ2
600     FORMAT(' FREQUENCY OF TUBE ON CHAN1 IS W1 =',F9.3,'RAD/SEC.',/,
1' FREQUENCY OF TUBE ON CHAN2 IS W2 =',F9.3,'RAD/SEC.')
      TAU1 = TAU(1)
      TAU2 = TAU(2)
      TYPE 700,TAU1,TAU2
700     FORMAT(' DECAY CONSTANT, CHAN1 T1 =',F9.3,'SEC-1',/,
1' DECAY CONSTANT, CHAN2 T2 =',F9.3,'SEC-1')
      TYPE 805,BIAS0,BIAS1
805     FORMAT(' BIAS ON CHAN1 IS: ',F8.5,' VOLTS',/,
1' BIAS ON CHAN2 IS: ',F8.5,' VOLTS',/)
      TYPE 810
810     FORMAT(' CALCULATE MASS FLOW RATE?')
      ACCEPT 815,DEC
815     FORMAT(A4)
      IF(DEC.EQ.NO) GO TO 820
      CALL FLOW
C
C CHECK AND SEE IF RAW DATA IS TO BE STORED
C
820     TYPE 821
821     FORMAT(' DO YOU WISH TO STORE RAW DATA? (YES/NO)')
      ACCEPT 822,STOR
822     FORMAT(A4)
      IF (STOR.EQ.NO) GO TO 823
      CALL STORIT
823     CONTINUE

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```

C
C SEE IF DATA PRINT IS DESIRED
C
      TYPE 824
824      FORMAT(' DO YOU WISH TO PRINT DATA ON TERMINAL? (YES/NO)')
      ACCEPT 826,RITE
826      FORMAT(A4)
      IF(RITE,EQ,NO) GO TO 827
      CALL RITER
827      CONTINUE
      TYPE 825
825      FORMAT(' ANOTHER MEASUREMENT?(YES/NO)')
      ACCEPT 850,DEC
850      FORMAT(A4)
      IF(DEC,EQ,NO) GO TO 900
      IFLAG = 0
      TYPE 875
875      FORMAT(' DO YOU WISH TO CHANGE SETUP PARAMETERS?(YES/NO)')
      ACCEPT 880,DEC
880      FORMAT(A4)
C
C SET FLAG TO SAY FILE IS OPEN
C
      ICHECK = 1
      IF(DEC,EQ,YES) GO TO 50
      GO TO 250
800      CONTINUE
      CLOSE(UNIT=8)
900      STOP
      END
      SUBROUTINE VBIAS1
C
C*****C
C SUBROUTINE BIAS1: THIS ROUTINE USES DTLIB CONTROL AND MEASURE- C
C MENT ROUTINES TO MEASURE THE VOLTAGE BIASES OF THE MEASURING C
C TURES. THESE VOLTAGES ARE THEN OUTPUTTED TO THE SCREEN THROUGH C
C THIS ROUTINE. C
C GREG DENNY OCTOBER 23, 1985 C
C*****C
C DECLARE VARIABLES
C
      COMMON/PEST/BIAS0,BIAS1
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
C
C DEFINE BIT WEIGHTINGS
C
      ANLSRV = 20.0/4096.0
C
C SET PARAMETERS
C
      NBUF = 1
      IGAIN = 1
      MODE = 2
      ICMF = 0
      IBEF = 0
C
C SET VARIABLES TO ZERO
C
      DO 50 I = 1,2000
      IBUF(I) = 0
50      CONTINUE
      DO 75 I = 1,1000
      VOLT(I) = 0.0
      VOLT1(I) = 0.0
      VOLT2(I) = 0.0
75      CONTINUE
C MEASURE TURE BIASES
C

```

```

      CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE,ICMF,IBEF)
      IVAR = LWAIT(ICMF,0)
C
C CONVERT DATA TO ENGINEERING UNITS
C
      BIAS0 = 0.0
      BIAS1 = 0.0
      DO 100 J=1,NREAD
      BIAS0 = BIAS0 + ANLSRV*FLOAT(IBUF(2*J-1))
      BIAS1 = BIAS1 + ANLSRV*FLOAT(IBUF(2*J))
100    CONTINUE
C
C AVERAGE READINGS
C
      BIAS0 = BIAS0/NREAD
      BIAS1 = BIAS1/NREAD
C
C OUTPUT DATA
C
      TYPE 300,BIAS0,BIAS1
300    FORMAT(// ' CHANNEL 1 BIAS =',F15.7,'VOLTS',/,
1       ' CHANNEL 2 BIAS =',F15.7,'VOLTS',/)
      RETURN
      END
      SUBROUTINE FIRE1
C
C*****C
C SUBROUTINE FIRE1: THIS ROUTINE CONTROLS THE DEFLECTION OF THE C
C TUBES BY ACTIVATING THE ELECTROMAGNETIC ACTUATOR VIA A PARALLEL C
C I/O PORT CONTROLLED BY THE DTLIB ROUTINE IDOR. C
C C
C GREG DENNY OCTOBER 27,1986 C
C C
C*****C
C DECLARE VARIABLES
C
      COMMON IFLAG
      COMMON/PEST/BIAS0,BIAS1
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      REAL*4 YES,NO,DECIDE
      INTEGER*4 ITIME
      DATA YES,NO/'YES','NO'/
C
C AGAIN, ZERO OUT RELAY USING IDOR
C
      MASK = *1
      IWORD = *1
      CALL IDOR(0,0,MASK,IWORD)
C
C QUERY USER FOR MEASUREMENT ATTEMPT
C
      TYPE 100
100    FORMAT(' INITIATE MEASUREMENT? (YES/NO)')
      ACCEPT 200,DECIDE
200    FORMAT(A4)
      IF (DECIDE.EQ.YES) GO TO 300
      IFLAG = 1
      GO TO 400
C
C DEFLECT TUBES
C
300    PAUSE ' <CR> TO INITIATE MEASUREMENT'
      CALL IDOR(0,0,MASK,'0')
C
C HOLD TUBES DOWN FOR 1 SECOND
C
      CALL GTIM(ITIME)
      CALL CVTIM(ITIME,IHRS,IMIN,ISEC,ITCK)
      ISECR = ISEC + 1

```

```

350      CALL GTIM(ITIME)
          CALL CVTTIM(ITIME,IHRS,IMIN,ISEC,ITCK)
          IF (ISEC.LT.ISECR) GO TO 350
C
C  RELEASE TUBES
C
          CALL IDOR(0,0,MASK,IWORD)
C
C  GET DATA
C
          CALL GETDAT
400      CONTINUE
          RETURN
          END
          SUBROUTINE GETDAT
C
C*****C
C  SUBROUTINE GETDAT: THIS ROUTINE GATHERS THE DATA TO BE CONVERTED
C  INTO FREQUENCY AND DECAY RATE. THIS DATA IS PASSED TO ALGOR IN
C  TWO VECTORS: VOLT1 AND VOLT2.
C
C          GREG DENNY                      OCTOBER 27,1986
C
C*****C
C  DECLARE VARIABLES
C
          COMMON/PEST/BIAS0,BIAS1
          COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
          COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
C
C  DECLARE BIT WEIGHTINGS
C
          ANLSBV = 20.0/4096.0
C
C  DEFINE SET-UP PARAMETERS FOR A/D CONVERSION
C
          NBUF = 1
          IGAIN = 1
          MODE = 2
          ICMF = 0
          IBEF = 0
C
C  COLLECT DATA
C
          CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,
1          IGAIN,MODE,ICMF,IBEF)
          IVAR = LWAIT(ICMF,0)
C
C  CONVERT DATA TO ENGINEERING UNITS AND REMOVE BIASES
C
          DO 100 J = 1,NREAD
              VOLT1(J) = ANLSBV*FLOAT(IBUF(2*J-1)) - BIAS0
              VOLT2(J) = ANLSBV*FLOAT(IBUF(2*J)) - BIAS1
100      CONTINUE
          RETURN
          END
          SUBROUTINE ALGOR
C
C*****C
C  SUBROUTINE ALGOR: THIS SUBROUTINE FINDS THE PEAKS
C  FROM THE RAW DATA AND CONVERTS THESE INTO THE FREQUENCY OF VIBRATION
C  AND DECAY RATE. IT CALLS SEVERAL OTHER SUBROUTINES, WHICH ARE ALL
C  GROUPED UNDER THIS TITLE BLOCK.
C
C          GREG DENNY                      OCTOBER 27, 1986
C
C*****C
C  DECLARE VARIABLES
C

```

```

COMMON/MESS/NFLAG
COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
COMMON PEAK(50),KPEAK
COMMON/XYZ/JL,JR

C
C INITIALIZE FLAG
C
      NFLAG = 1

C
C READ DATA INTO VOLT ARRAY
C
50      CONTINUE
      IF (NFLAG.EQ.2) GO TO 200
      IF (NFLAG.GT.2) GO TO 800

      DO 100 J = 1,NREAD
      VOLT(J) = VOLT1(J)
100     CONTINUE
      GO TO 325
200     CONTINUE
      DO 250 J = 1,NREAD
      VOLT(J) = 0.0
250     CONTINUE
      DO 300 J = 1,NREAD
      VOLT(J) = VOLT2(J)
300     CONTINUE
C
C INITIALIZE STORAGE ARRAYS
C
325     DO 350 I=1,50
      TIME(I) = 0.0
      PKODD(I) = 0.0
      PKEVEN(I) = 0.0
      PEAK(I) = 0.0
      JTIME(I) = 0
350     CONTINUE
C
C NOW HAVE DATA IN VOLT ARRAY; FIND PEAKS, DECAY RATE, AND FREQUENCY
C FOR THIS PARTICULAR TUBE
C
400     J = 0
      K = 1

C
C BEGIN
C
      IF (VOLT(1).LT.VOLT(2)) GO TO 600
500     CONTINUE
C
C LOOK FOR MINIMUM PEAK
      CALL MINIM(J)
C
C CHECK AND SEE IF END OF DATA HAS OCCURRED
C
      IF (J.GE.NREAD) GO TO 700
C
C STORE MINIMUM VALUE
C
      CALL TRAP(J,K)
C
C LOOK FOR A MAXIMUM PEAK
C
600     CALL MAXIM(J)
      IF (J.GE.NREAD) GO TO 700
      CALL TRAP(J,K)
C
C KEEP GOING UNTIL DATA USED UP
C
      GO TO 500

```

```

C
C START NEW SECTION; ALL PEAKS HAVE BEEN LOCATED
C
700      K = K-1
C
C KCROSS EQUALS NUMBER OF PEAKS MINUS ONE
C
      KCROSS = K
      KPEAK = K
C
C CALCULATE WAVEFORM FREQUENCY
C
      CALL PERIOD
      CONTINUE
C
C INTERPOLATE AND FIND BEST CURVE; GET DECAY RATE
C
      CALL PEAKER
      CONTINUE
C
C CHECK FLAG; GO AGAIN IF NECESSARY
C
      NFLAG = NFLAG + 1
      IF (NFLAG.LE.2) GO TO 50
800      RETURN
      END
C
C SUBROUTINE MAXIM(J)
C
      SUBROUTINE MAXIM(J)
C
C DECLARE VARIABLES
C
      COMMON/MESS/NFLAG
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
      COMMON/XYZ/JL,JR
      REAL*4 MINUM,MAXUM,PK
      INTEGER*2 JMAX
C
C
      IF (K.GE.3) GO TO 300
100      J = J+1
C
C CHECK AND SEE IF DATA FILE AT END
C
      IF (J.GE.NREAD) GO TO 200
C
C LOOK FOR A MAXIMUM
C
      MAXUM = AMAX1(VOLT(J),VOLT(J+1))
      IF (MAXUM.EQ.VOLT(J)) GO TO 200
      GO TO 100
200      CONTINUE
      GO TO 400
300      CONTINUE
      IF (JR.GE.NREAD) J=JR
      IF (J.EQ.JR) GO TO 400
C
C SET PK LOW INITIALLY
C
      PK = -100.00
      JMAX = 0
      DO 325 J=JL,JR
      PK = AMAX1(PK,VOLT(J))
325      CONTINUE
      DO 350 J=JL,JR
      IF (PK.EQ.VOLT(J)) JMAX=J
350      CONTINUE
      J = JMAX
400      CONTINUE

```

```

        RETURN
        END
C
C SUBROUTINE MINIM(J)
C
        SUBROUTINE MINIM(J)
C
C DECLARE VARIABLES
C
        COMMON/MESS/NFLAG
        COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
        COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
        COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
        1TAU(2),KCROSS,OMEGA
        COMMON/XYZ/JL,JR
        REAL*4 MINUM,MAXUM,PK
        INTEGER*2 JMIN
C
        IF (K.GE.3) GO TO 300
100      J = J+1
C
C CHECK AND SEE IF DATA FILE IS AT END
C
        IF (J.GE.NREAD) GO TO 200
C
C LOOK FOR A MINIMUM PEAK
C
        MINUM = AMIN1(VOLT(J),VOLT(J+1))
        IF (MINUM.EQ.VOLT(J)) GO TO 200
        GO TO 100
200      CONTINUE
        GO TO 400
300      CONTINUE
        IF (JR.GE.NREAD) J=JR
        IF (J.EQ.JR) GO TO 400
C
C SET PK HIGH INITIALLY
C
        PK = 0.0
        JMIN = 0
        DO 325 J=JL,JR
        PK = AMIN1(PK,VOLT(J))
325      CONTINUE
        DO 350 J=JL,JR
        IF (PK.EQ.VOLT(J)) JMIN=J
350      CONTINUE
        J = JMIN
400      CONTINUE
        RETURN
        END
C
C SUBROUTINE TRAP(J,K)
C
        SUBROUTINE TRAP(J,K)
C
C DECLARE VARIABLES
C
        COMMON/MESS/NFLAG
        COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
        COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
        COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
        1TAU(2),KCROSS,OMEGA
        COMMON/PEAK(50),KPEAK
        COMMON/XYZ/JL,JR
C
C STORE VALUES
C
        PEAK(K) = VOLT(J)
C
C CONVERT TIMING TO REAL TIME
C
20      TIME(K) = J*PRESET*1.0E-6
        JTIME(K) = J

```

```

      IF (K.LE.1) J = J + 10
      IF (K.LE.1) GO TO 100
C
C WANT FLAG TO KICK OUT
C
      IF (NREAD-J.LE.5) GO TO 200
C
C IF AT LEAST SECOND PEAK HAS BEEN LOCATED, ENOUGH INFORMATION EXISTS
C TO STEP FORWARD MORE THAN ONE J-UNIT TO THE REGION OF THE NEXT PEAK.
C
      T = TIME(K) - TIME(K-1)
      JL = J + JTIME(K) - JTIME(K-1) - 5
      JR = JL + 10
      J = JL
100  CONTINUE
C
C K = # OF PEAKS
C
      K = K+1
200  CONTINUE
      RETURN
      END
C
C SUBROUTINE PERIOD
C
      SUBROUTINE PERIOD
C
C DECLARE VARIABLES
C
      COMMON/MESS/NFLAG
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIAN(2),
1TAU(2),KCROSS,OMEGA
      REAL*4 SIGTA,PERCEN,SUM,DIFF(50)
C
C DETERMINE PERIOD (ACTUALLY ONE-HALF PERIOD)
C
      SUM = 0.0
      DO 200 J=1,KCROSS-2
      SUM = SUM + TIME(J+1) - TIME(J)
200  CONTINUE
      TA = SUM/(KCROSS-2)
C DO STATISTICAL ANALYSIS OF WAVEFORM NOW RATHER THAN IN
C MAIN ROUTINE.
      DO 250 J=1,KCROSS-2
      DIFF(J) = 0.0
250  CONTINUE
      DO 300 J=1,KCROSS-2
      DIFF(J) = TIME(J+1) - TIME(J)
300  CONTINUE
C COMPUTE STANDARD DEVIATION
      SIGTA = 0.0
      DO 400 J=1,KCROSS-2
      SIGTA = SIGTA + (DIFF(J)-TA)**2
400  CONTINUE
      SIGTA = SQRT(SIGTA/(KCROSS-2))
C DETERMINE WHICH POINTS TO TOSS
      DO 500 J=1,KCROSS-2
      PERCEN = ABS((TA-DIFF(J))/DIFF(J))
      IF (PERCEN.GE.0.03) DIFF(J)=0.0
500  CONTINUE
C CALCULATE NEW PERIOD
      SUM = 0.0
      K = 1
      DO 600 J=1,KCROSS-2
      SUM = SUM + DIFF(J)
      IF (DIFF(J).EQ.0.0) K=K-1
      K = K + 1
600  CONTINUE
      TA = SUM/(K-1)

```

```

C
C DETERMINE FREQUENCY
C
      OMEGA = 0.5/TA
      RADIANT(NFLAG) = OMEGA*6.28318
      RETURN
      END

C
C SUBROUTINE PEAKER
C
      SUBROUTINE PEAKER
C
C DECLARE VARIABLES
C
      DIMENSION C(50),PKMAX(50),PKMIN(50)
      COMMON/MESS/NFLAG
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
      COMMON/DAT1/IRUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,RADIANT(2),
      1TAU(2),KCROSS,OMEGA
      COMMON PEAK(50),KPEAK
      DO 50 I=1,50
      PKMAX(I) = 0.0
      PKMIN(I) = 0.0
      C(I) = 0.0
50    CONTINUE
C
C SPLIT PEAKS INTO TWO VECTORS FOR INTERPOLATION. NOTE THAT ONE PEAK
C MAY BE LOST IF KPEAK IS ODD.
C
      DO 100 J=1,KPEAK/2
      PKEVEN(2*J) = PEAK(2*J)
      PKODD(2*J-1) = PEAK(2*J-1)
100   CONTINUE
C
C PERFORM INTERPOLATION
C
      DO 200 J=2,KPEAK/2
      PKODD(2*J-2) = (PKODD(2*J-3) + PKODD(2*J-1))/2.0
      PKEVEN(2*J-1) = (PKEVEN(2*J-2) + PKEVEN(2*J))/2.0
200   CONTINUE
C
C NOW GET DATA INTO MAX AND MIN VECTORS
C
      KPEAK = 2*INT(KPEAK/2.0)
      IF (PKODD(1).GT.PKEVEN(2)) GO TO 300
      DO 250 J=1,KPEAK
      PKMIN(J) = PKODD(J)
      PKMAX(J) = PKEVEN(J)
250   CONTINUE
      GO TO 350
300   DO 325 J=1,KPEAK
      PKMIN(J) = PKEVEN(J)
      PKMAX(J) = PKODD(J)
325   CONTINUE
350   CONTINUE
C
C COMPUTE C(K). REMEMBER, CAN'T USE FIRST OR LAST POINTS.
C
      DO 375 I=2,KPEAK-1
      C(I) = (PKMAX(I) - PKMIN(I))/2.0
375   CONTINUE
      DO 400 I=1,KPEAK
      TYPE 450,I,PEAK(I)
450   FORMAT(' PEAK',I3,' =',F8.4,' VOLTS.')
400   CONTINUE
C
C FIT BEST EXPONENTIAL TO C(K) AND OBTAIN TIME CONSTANT
C
      SUMT = 0.0
      SUMTS = 0.0
      SLNC = 0.0
      STLNC = 0.0

```

```

DO 500 J = 2,KPEAK-1
SUMT = SUMT + TIME(J)
SUMTS = SUMTS + TIME(J)**2
SLNC = SLNC + ALOG(C(J))
STLNC = STLNC + TIME(J)*ALOG(C(J))
500 CONTINUE
N = KPEAK-2
TAU(NFLAG) = (STLNC-SLNC*SUMT/N)/(SUMT**2/N-SUMTS)
RETURN
END
SUBROUTINE FLOW

C
C THIS SUBROUTINE CALCULATES THE MASS FLOW RATE THROUGH THE
C PROTOTYPE AND THE MASS FLOW RATE THROUGH THE SECONDARY STANDARD
C FOR COMPARISON. IT ALSO COMPUTES THE DENSITIES AND VELOCITIES
C FOR COMPARISON.
C
C DECLARE VARIABLES
C
REAL*4 MDOT,MDOTS,R1,R2,RC1,RC2,A1,A2,A3,QS,PS,PP,TF
COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ,RATIO
COMMON/DAT2/PKEVEN(50),PKODD(50),TIME(50),JTIME(50),TA,
1RDIAN(2),TAU(2),KRCROSS,OMEGA
COMMON/HSTOR/FILE(15)
REAL YES,NO,REC
DATA YES,NO/'YES','NO'/

C
C DEFINE MEASURED CONSTANTS
C
R1 = TAU(1)

C
C REMEMBER, RATIO IS OUR FUDGE FACTOR FOR RC1
C
RC1 = TAU(2)*RATIO
R2 = RDIAN(1)**2

C
C SINCE FREQUENCY SHIFT IS NEGLIGIBLE, USE R2 FOR RC2.
C
RC2 = RDIAN(1)**2

C
C AREAS AND DENSITY OF TUBE ARE OBTAINED FROM LAB MEASUREMENTS
C
AREA1 = 0.001363538
AREA2 = 0.000766990
RHOT = 73.2796

C
A1 = 41.220
A2 = 3.5427
A3 = 1.0018

C
C GET STANDARD VALUES FROM TERMINAL
C
TYPE 10
FORMAT(' INPUT THE FOLLOWING DATA: VOLUMETRIC FLOW RATE AS',/,
1' MEASURED BY SECONDARY STANDARD (SCFM), MEASURED TEMPERATURE',/,
2' (CELSIUS) AT STANDARD, AND GAGE PRESSURE (IN. H2O) AT CORIOLIS',/,
3' FLOWMETER INLET.')
```

```

TYPE 20
FORMAT(' THIS WILL ALLOW CALCULATION OF REFERENCE DENSITY AND
4 MASS FLOW RATE.')
```

```

ACCEPT *,QS,TF,PP
PP = PP*0.0361111 + 14.7
TF = 9.0*TF/5.0 + 492.0

C
C DEFINE GAS CONSTANT VALUE
C
R = 53.34

C
C COMPUTE REFERENCE VALUES
C
MDOTS = ((QS*14.7)/(R*530.0))*(144.0/60.0)
RHOF5 = (PP*144.0)/(R*TF)
US = ((QS*14.7*TF)/(AREA1*PP*530.0))/60.0

```

```

C
C COMPUTE CORIOLIS VALUES
C
      BETAM1 = (R2 + R1**2)/(A3*(RC2 + RC1**2))
      BETA = 1.0 - BETAM1
      RHOF = A1*BETA/BETAM1
      U = (R1 - A3*BETAM1*RC1)/(A2*BETA)
      MDOT = RHOF*AREA1*U
C
C SEE IF DATA STORAGE IS DESIRED
C
      TYPE 36
36      FORMAT(' STORE MEASUREMENT RESULTS? (YES/NO)')
      ACCEPT 37,REC
37      FORMAT(A4)
      IF (REC.EQ.NO) GO TO 100
C
C READ VALUES INTO STORAGE
C
      WRITE(8) MDOT,MDOTS,RHOF,RHOF5,U,US
100      TYPE 40, MDOT,MDOTS,RHOF,RHOF5,U,US
40      FORMAT(' FLOW MEASUREMENT RESULTS',//,
1' MASS FLOW RATE:',/,
2' MDOT =',E15.7,' MDOTS =',E15.7,/,
3' DENSITY:',/,
4' RHOF =',E15.7,' RHOF5 =',E15.7,/,
5' VELOCITY:',/,
6' U =',E15.7,' US =',E15.7,/)
      TYPE 50,TF
50      FORMAT(' ABSOLUTE TEMPERATURE IS ',F9.4,' R')
      PAUSE ' THIS HALT IS TO PROVIDE TIME TO RECORD DATA, IF
7 NECESSARY.'
      RETURN
      END
      SUBROUTINE STORIT
C
C*****C
C SUBROUTINE STORIT: THIS ROUTINE STORES THE RAW DATA ON FLOPPY C
C DISK. THE USER SPECIFIES THE FILE NAME. THIS PROGRAM IS BASED C
C LARGELY ON WORK DONE BY W.R. HAMEL. C
C GREG DENNY MARCH 12, 1987 C
C*****C
C
C DECLARE VARIABLES
C
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      LOGICAL*1 FILE(15)
C
C GET FILE NAME FROM TERMINAL
C
      TYPE 10
10      FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
      ACCEPT 20,FILE
20      FORMAT(14A1)
      FILE(15) = 0
C
C OPEN DISK FILE DY1:NAME.DAT FOR STORAGE
C
      OPEN(UNIT=2,NAME=FILE,ACCESS='SEQUENTIAL',TYPE='NEW',
1FORM='UNFORMATTED')
C
C WRITE DATA SAMPLING PARAMETERS
C
      WRITE(2) PRESET,NREAD,NCCH
C
C NOW WRITE DEFLECTION DATA FOR BOTH TUBES INTO SEQUENTIAL FILE
C DY1:NAME.DAT
C
      WRITE(2)(VOLT1(J),VOLT2(J),J=1,NREAD)

```

```

C
C CLOSE DISK FILE
C
      CLOSE(UNIT=2)
C
C SEND DATA STORED MESSAGE TO TERMINAL
C
      TYPE 30,(FILE(KK),KK=1,15)
30    FORMAT(' DATA STORED ON ',15A1)
      RETURN
      END
C
C
      SUBROUTINE RITER
C
C*****C
C THIS SUBROUTINE ALLOWS THE OPERATOR TO PRINT OUT SELECTED C
C DATA TO THE TERMINAL DEVICE. IT IS TAKEN FROM WORK DONE BY C
C W.R. HAMEL. C
C*****C
C DECLARE VARIABLES
C
      COMMON/PAR/PRESET,NCCH,NREAD,ISTCHN,ISIZ
      COMMON/DAT1/IBUF(2000),VOLT1(1000),VOLT2(1000),VOLT(1000)
      REAL*4 YES,NO
      DATA YES,NO/'YES','NO'/
      DIMENSION T(1000)
10    CONTINUE
      TYPE 20
20    FORMAT(' SPECIFY FIRST AND LAST [ARRAY INDICES] TO BE
      1 PRINTED. '//, ' FIRST > 1 AND LAST < NREAD')
C
C GET FIRST AND LAST OF DATA PRINTOUT
C
      ACCEPT *,IFIRST,ILAST
      DO 30, J=IFIRST,ILAST
      T(J) = (J-1)*PRESET*1.0E-3
30    CONTINUE
      TYPE 40
40    FORMAT(//,
      1' TIME(MS)      CHAN 0 DEFLECTION(VOLTS)      CHAN 1 DEFLECTION
      2(VOLTS)')
      TYPE 50
50    FORMAT(' -----')
      1-----)
      TYPE 60,(T(J),VOLT1(J),VOLT2(J),J=IFIRST,ILAST)
      FORMAT(1X,F8.4,9X,F9.5,20X,F9.5)
      TYPE 70
70    FORMAT(' IS THAT ALL THE PRINTING YOU WISH?(YES/NO)')
80    CONTINUE
      ACCEPT 90,RIT
90    FORMAT(A4)
      IF(RIT.EQ.NO) GO TO 10
      RETURN
      END

```

APPENDIX B

EQUIPMENT

Specific information on the equipment used in this study is documented below.

Meriam Model #50MC2-2 Laminar Flow Element
serial number: 717090-41
flow standard number: 67-8657-5

Fairchild Model 4880 Instrument Coupler
ORNL number: 038365

System Teknik Digital Temperature Readout
ORNL number: 038340

Analog Devices Modular Power Converter
number: MPC15D200T

Diametric Barocel Pressure Sensor
type: 590D-10W-2Q8-V1-4D
serial number: 810-6353
ORNL number: IC 24197
range: 10 inches of water

Power Supply
type: 1134
serial number: 810-219

Hewlett-Packard 3495A Scanner
ORNL number: IC039191
serial number: 1428A08332
IC maintenance number: M101149

Hewlett-Packard 3455A Digital Voltmeter
ORNL number: IC24139
serial number: 1637A00322

Hewlett-Packard 9825B Calculator
ORNL number: IC037613

Wallace and Tiernan Pressure Gage
ORNL number: IC17354
IC maintenance number: M104346
range: 0 - 120 inches of water
IC Division Service Section calibration standard
number: S-100-1

Foxboro DP Cell

serial number: 4071710
range: 0 - 100 inches of water

Dwyer Magnehelic Gages

range: 0 - 15 inches of water
least count: 0.50 inches of water

Wika Pressure Gage

output: square root percentage
least count: 0.1%

U. S. Gauge Dial

number: 33001
range: 0 - 15 psi
least count: 0.1 psi

SGA20 Amplifiers

designed at ORNL

Hewlett-Packard Dual DC Power Supply

number: 6205B
range: 0 - 40 V at 3 A
0 - 20 V at 6 A
ORNL number: IC032323
IC maintenance number: M101270

CIT-101 VT100 Terminal

ORNL number: IC041975
US DOE number: 137181

DEC LSI 11/23 Computer

F & P Company Rotameter

number: FP-1-35-G-10/77
range: 0 - 100%
0 - 33.3 LPM of water
0 - 1019 SLPM of air

Data Translation A/D Converter

model: DT2782-DI
range: +10 V
least count: + 1/2 LSB or 4.88 mV

Data Translation Realtime Clock

model: DT2769

APPENDIX C

ERROR ANALYSIS

The uncertainty in the overall measurement of mass flow rate enters the final computation through two sources. First, error (or uncertainty) is introduced in the calibration of the flowmeter. This error shows up in the calibration constants a_1 , a_2 , and a_3 , and comes from uncertainties in the values of pressure, temperature, flow rate, etc. obtained by the measurement system of the secondary standard; and uncertainties in decay constant and frequency as determined by the Coriolis flow measurement system during the calibration routine. The second source of error comes from uncertainties in the decay constant and frequency which are computed from the actual flow measurement. This analysis will be a brief study of those errors, and an attempt will be made to bound the error in these processes. The Kline-McKlintock method [16] will be used, in which the total error associated with a variable f , which is a function of several variables x , y , z , . . . , is given by

$$w_f = \left[\left(\frac{\partial f}{\partial x} w_x \right)^2 + \left(\frac{\partial f}{\partial y} w_y \right)^2 + \left(\frac{\partial f}{\partial z} w_z \right)^2 + \dots \right]^{\frac{1}{2}}$$

The equations relating mass flow to the vibrational characteristics of a cantilever tube are

$$\dot{m} = \rho_f A_f u$$

$$\rho_f = \frac{a_1 \beta}{1 - \beta}$$

$$(1-\beta) = \frac{R_2 + R_1^2}{a_3(R_{c2} + R_{c1}^2)}$$

$$\beta = 1 - (1-\beta)$$

$$u = \frac{R_1 - a_3(1-\beta)R_{c1}}{a_2\beta}$$

The uncertainty in a calculation for mass flow is given by

$$w_m = \left[\left(\frac{\partial m}{\partial \rho_f} w_{\rho_f} \right)^2 + \left(\frac{\partial m}{\partial A_f} w_{A_f} \right)^2 + \left(\frac{\partial m}{\partial u} w_u \right)^2 \right]^{\frac{1}{2}}$$

or

$$w_m = \left[(A_f u w_{\rho_f})^2 + (\rho_f u w_{A_f})^2 + (\rho_f A_f w_u)^2 \right]^{\frac{1}{2}}$$

The quantities w_{ρ_f} , w_{A_f} , and w_u represent the uncertainties in the density, flow area, and velocity respectively.

The uncertainty in the area comes from uncertainty in the diameter measurement. The density and velocity are also functions of the measured values of R_{c1} , R_1 , R_{c2} , and R_2 , and the calibration constants a_1 , a_2 , and a_3 . The equations which were used to compute the uncertainties of the various quantities are listed below.

$$w_u = \left[\left(\frac{1}{a_2\beta} w_{R_1} \right)^2 + \left(\frac{(1-\beta)R_{c1}}{a_2\beta} w_{a_3} \right)^2 + \left(\frac{a_3(1-\beta)}{a_2\beta} w_{R_{c1}} \right)^2 + \left(\frac{R_1 - a_3(1-\beta)R_{c1}}{a_2^2\beta} w_{a_2} \right)^2 + \left(\frac{R_1 - a_3(1-\beta)R_{c1}}{a_2\beta^2} w_{\beta} \right)^2 \right]^{\frac{1}{2}}$$

$$w_{\rho_f} = \left[\left(\frac{\beta}{1-\beta} w_{a_1} \right)^2 + \left(\frac{a_1}{1-\beta} w_{\beta} \right)^2 + \left(\frac{a_1\beta}{(1-\beta)^2} w_{1-\beta} \right)^2 \right]^{\frac{1}{2}}$$

$$w_{1-\beta} = \left[\left(\frac{R_2 w_{R_2}}{a_3 (R_{c2} + R_{c1}^2)} \right)^2 + \left(\frac{2R_1}{a_3 (R_{c2} + R_{c1}^2)} w_{R_1} \right)^2 + \left(\frac{R_2 + R_1^2}{a_3^2 (R_{c2} + R_{c1}^2)} w_{a_3} \right)^2 \right. \\ \left. + \left(\frac{R_{c2} (R_2 + R_1^2)}{a_3 (R_{c2} + R_{c1}^2)^2} w_{R_{c2}} \right)^2 + \left(\frac{2R_{c1} (R_2 + R_1^2)}{a_3 (R_{c2} + R_{c1}^2)^2} w_{R_{c1}} \right)^2 \right]^{\frac{1}{2}}$$

$$w_{\beta} = w_{1-\beta}$$

where the partials are taken from the preceding equations. As can be seen, uncertainties in the calibration constants and in frequencies (R_2 , R_{c2}) and decay constants (R_1 , R_{c1}) for both the active tube and reference tube influence the overall uncertainty in mass flow through the density and velocity computations. The frequency and decay constant are given by

$$R_{c2} = \frac{n\pi}{\sum (t_i - t_{i-1})}$$

$$R_2 = \frac{n\pi}{\sum (t_i - t_{i-1})}$$

$$R_1 = \left[\frac{\frac{\sum t_i \sum t_i}{n} - \sum t_i^2}{\sum t_i \ln z(t_i) - \frac{1}{n} \sum \ln z(t_i) \sum t_i} \right]^{-1}$$

$$R_{c1} = \left[\frac{\frac{\sum t_i \sum t_i}{n} - \sum t_i^2}{\sum t_i \ln z(t_i) - \frac{1}{n} \sum \ln z(t_i) \sum t_i} \right]^{-1}$$

where

$$z(t_i) = \frac{g(t_i) - h(t_i)}{2}$$

For the sake of computation ease, the decay constant is represented by a modified logarithmic decrement

$$R_1 = \frac{f}{n} \ln \left| \frac{z_1}{z_n} \right|$$

$$R_{c1} = \frac{f}{n} \ln \left| \frac{z_1}{z_n} \right|$$

where z_1 and z_n are the modified peaks at t_1 and t_n respectively. The assumption behind this approximation is that if $z(t_i)$ (the shifted waveform) is a true exponential (or very close to one), as was assumed in making use of the curve fit (discussed in Chapter III), then the amount of error introduced through the use of the curve-fit is negligible compared to the uncertainties in other quantities, and it should be possible to use the modified logarithmic decrement to represent the decay constant. At worst, this should tend to give a value for the uncertainty that is less than the maximum uncertainty but probably closer to a more typical value. (In fact, this version of the logarithmic decrement was used in actual testing, with no apparent difference from the more "complicated" curve-fit expression. The curve-fit was used since no gain was seen in one or the other and all the programming had been constructed around the curve-fit.)

The uncertainties in frequency and decay constant can be shown to be

$$w_{R_2} = \frac{4R_2^2}{n\pi^2} w_{t_i}$$

$$w_{R_{c2}} = \frac{4R_2^2}{nn^2} w_{t_i}$$

$$w_{R_1} = \frac{1}{nn} \left[\left(w_{R_2} \ln \frac{z_1}{z_n} \right)^2 + \left(\frac{R_2}{z_1} w_{z_1} \right)^2 + \left(\frac{R_2}{z_n} w_{z_n} \right)^2 \right]^{\frac{1}{2}}$$

$$w_{R_{c1}} = \frac{1}{nn} \left[\left(w_{R_{c2}} \ln \frac{z_1}{z_n} \right)^2 + \left(\frac{R_{c2}}{z_1} w_{z_1} \right)^2 + \left(\frac{R_{c2}}{z_n} w_{z_n} \right)^2 \right]^{\frac{1}{2}}$$

Where it should be noted that z_1 and z_n are those values corresponding to each particular tube. The quantity w_{t_i} is the uncertainty in the measured time of any peak and is given by

$$w_{t_i} = \pm 1/2 T_s$$

where T_s is the sampling period of the A/D converter. This uncertainty arises from the fact that the sampled signal's "peak" may not be the peak of the continuous signal.

The quantities w_{z_1} and w_{z_n} are obtained from using the defining equation for z .

$$z_1 = \frac{g_1 - h_1}{2}$$

$$z_n = \frac{g_n - h_n}{2}$$

It is apparent that the uncertainties in g and h will be needed. In general, a peak of the waveform is surrounded by a "box" of uncertainty,

defined along the time axis by $2w_{t_i}$ and along the voltage axis by a voltage uncertainty $2w_v$. The maximum uncertainty is thus defined as one-half the diagonal of the "box" (the true value of the peak being centered in the box), or something similar to

$$w_p = \left[w_{t_i}^2 + w_v^2 \right]^{\frac{1}{2}}$$

This expression is not dimensionally consistent, but should serve to illustrate the basic idea. The quantities $g(t_i)$ and $h(t_i)$ are these peaks whose uncertainties are a combination of w_{t_i} and w_v . For simplification, it will be assumed that the "box" discussed above is long in the voltage direction and narrow in the time direction (i.e. uncertainty in voltage is much larger than uncertainty in time) such that

$$w_p \approx w_v$$

This should at least give a minimum bound on the total uncertainty in the peaks. The quantity w_v is a combination of quantization error from the A/D converter and error introduced by the amplifiers. On the advice of Dr. Hamel, it was assumed that the amplifier error was 0.1% of full scale (+10V) or about $\pm 10\text{mV}$. The quantization error of the 12-bit A/D converter is $\pm 2.44\text{mV}$. Thus the maximum overall uncertainty is

$$w_p = w_v = \pm 12.44 \text{ mV}$$

This is the quantity which is used for the uncertainty in the peak values. For the $g(t_i)$ and $h(t_i)$ which are obtained via linear interpolation, the uncertainty is

$$w_{g,h} = 1/2 w_p$$

so that the uncertainty in z after simplification may be expressed as

$$w_{z_1} = w_{z_n} = \frac{\sqrt{5}}{4} w_p$$

Thus, the uncertainties in decay constants and frequencies can be determined.

These uncertainties in R_1 and R_{c1} also play a part in the computation of the uncertainties of the calibration constants, since

$$a_1 = \rho_f \left(\frac{1-\beta}{\beta} \right)$$

$$a_2 = \frac{R_1 - a_3' (1-\beta) R_{c1}}{\beta u}$$

$$a_3 = 1 + \frac{\rho_{f0}}{a_1}$$

Using the equations for the calibration constants, the uncertainties in the calibration constants are

$$w_{a_1} = \left[\left(\frac{1-\beta}{\beta} w_{\rho_f}' \right)^2 + \left(\frac{\rho_f}{\beta} w_{1-\beta}' \right)^2 + \left(\frac{\rho_f (1-\beta)}{\beta^2} w_{\beta}' \right)^2 \right]^{1/2}$$

$$w_{a_2} = \left[\left(\frac{w_{\rho_{fu}}'}{a_1} \right)^2 + \left(\frac{\rho_{f0} w_{a_1}}{a_1^2} \right)^2 \right]^{1/2}$$

$$w_{a_3} = \left[\left(\frac{w_{R_1}}{\beta_u} \right)^2 + \left(\frac{(1-\beta) R_{c1}}{\beta_u} w_{a_3} \right)^2 + \left(\frac{a_3 R_{c1}}{\beta_u} w'_{1-\beta} \right)^2 \right. \\ \left. + \left(\frac{a_3 (1-\beta)}{\beta_u} w_{R_{c1}} \right)^2 + \left(\frac{(R_1 - a_3) (1-\beta) R_{c1}}{\rho_u^3} w'_u \right)^2 \right. \\ \left. + \left(\frac{(R_1 - a_3) (1-\beta) R_{c1}}{\beta^2 u} w'_\beta \right)^2 \right]^{\frac{1}{2}}$$

where the prime (') indicates that these quantities are not the same as those in the first set of equations in this section. These quantities are evaluated using the equations in the calibration routine, and arise from uncertainties in measured values of pressure, temperature, flow rate, tube mass, etc. in the calibration routine. They are found from the following equations.

flow velocity:

$$u = (Q_s / A_f) \left(\frac{14.7}{P_a} \right) \left(\frac{T_f}{530.0} \right) 160.0 \\ w'_u = 4.62 \times 10^{-4} \left[\left(\frac{T_f}{P_a A_f} w_{Q_s} \right)^2 + \left(\frac{Q_s T_f}{P_a A_f^2} w_{A_f} \right)^2 \right. \\ \left. + \left(\frac{Q_s}{P_a A_f} w_{T_f} \right)^2 + \left(\frac{Q_s}{P_a^2 A_f} w_{P_a} \right)^2 \right]^{\frac{1}{2}}$$

density of fluid in active tube:

$$\rho_f = \frac{P_a}{RT_f}$$

$$w'_{\rho_f} = \left[\left(\frac{w_{P_a}}{RT_f} \right)^2 + \left(\frac{P_a w_{T_f}}{RT_f^2} \right)^2 \right]^{\frac{1}{2}}$$

density of fluid in reference tube:

$$\rho_{f_0} = \frac{P_b}{RT_f}$$

$$w'_{\rho_{f_0}} = \left[\left(\frac{w_{P_b}}{RT_f} \right)^2 + \left(\frac{P_b w_{T_f}}{RT_f^2} \right)^2 \right]^{\frac{1}{2}}$$

density of tube:

$$\rho_t = \frac{m_t}{A_t L}$$

$$w_{\rho_t} = \left[\left(\frac{w_{m_t}}{A_t L} \right)^2 + \left(\frac{m_t w_{A_t}}{A_t^2 L} \right)^2 + \left(\frac{m_t w_L}{A_t L^2} \right)^2 \right]^{\frac{1}{2}}$$

flow area:

$$A_f = \frac{\pi d_i^2}{4}$$

$$w_{A_f} = \frac{\pi}{2} d_i$$

tube area:

$$A_t = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$w_{A_t} = \frac{\pi}{2} \left[(d_o w_{d_o})^2 + (d_i w_{d_i})^2 \right]^{\frac{1}{2}}$$

mass ratio, β :

$$\beta = \frac{\rho_f A_f}{\rho_t A_f + \rho_f A_f}$$

$$w'_\beta = \beta \left[\left(\frac{w_{\rho_t}}{\rho_t + \frac{\rho_f A_f}{A_t}} \right)^2 + \left(\frac{w_{A_t}}{A_t + \frac{\rho_f A_f}{\rho_t}} \right)^2 + \left(\frac{w_{\rho_f}}{\rho_f \left(1 + \frac{\rho_f A_f}{\rho_t A_t} \right)} \right)^2 + \left(\frac{w_{A_f}}{A_f \left(1 + \frac{\rho_f A_f}{\rho_t A_t} \right)} \right)^2 \right]^{\frac{1}{2}}$$

one minus mass ratio, $(1 - \beta)$:

$$(1 - \beta) = 1 - \beta$$

$$w'_{(1-\beta)} = -w'_\beta$$

where Q_s = volumetric flow rate measured by laminar flow element (SCFM)

P_a = pressure measured in active tube (psi)

P_b = pressure measured in reference tube (psi)

T_f = temperature of flow (R)

Additionally, the following uncertainties are known:

$w_{Q_s} = 0.005 Q_s$ (from manufacturer's specifications)

$w_{P_a} = 0.2$ inches of water (minimum resolution of pressure gage)

$w_{P_b} = 0.05$ psi (minimum resolution of pressure gage)

$w_{T_f} = 0.01^\circ\text{C}$ (least count on digital readout)

$w_L = 0.1$ inch (least count on scale)

$w_{d_i} = w_{d_o} = 0.001$ inch (least count on vernier caliper)

$w_{m_t} = 0.1$ gram (least count on balance)

Obviously, unit conversions will be necessary. However, these equations, together with those for the uncertainties in R_1 and R_{c1} , can be used to compute the uncertainties in the calibration constants. Once these have been determined, they may be used in the first set of equations in this section, along with the uncertainties computed for R_1 , R_{c1} , R_2 , and R_{c2} , to determine the actual measurement uncertainties in mass flow, density, and velocity as found from the Coriolis flowmeter. For the 1.27×0.15875 cm ($1/2 \times 0.0625$ inch) set of plexiglas tubes, this procedure has been performed at one point, with the results given below.

Calibration Uncertainty

data: $Q_s = 9.15 \text{ SCFM} \pm 0.05 \text{ SCFM}$

$T_f = 79.50^\circ\text{F} \pm 0.02^\circ\text{F}$

$P_a = 15.310 \text{ psi} \pm 0.007 \text{ psi}$
 $P_b = 14.7 \text{ psi} \pm 0.05 \text{ psi}$
 $L = 9.5 \text{ inches} \pm 0.1 \text{ inches}$
 $d_i = 0.375 \text{ inches} \pm 0.001 \text{ inches}$
 $d_o = 0.500 \text{ inches} \pm 0.001 \text{ inches}$
 $m_t = 0.035 \text{ lbm} \pm 0.002 \text{ lbm}$
 $T_s = 0.2 \text{ msec}$
 flow area: $A_f = 0.000767 \text{ ft}^2$
 $w_{A_f} = 0.000049 \text{ ft}^2 = 6.3\%$
 tube area: $A_t = 0.000596 \text{ ft}^2$
 $w_{A_t} = 0.000082 \text{ ft}^2 = 13.7\%$
 tube density: $\rho_t = 74.26 \text{ lbm/ft}^3$
 $w_{\rho_t} = 11.08 \text{ lbm/ft}^3 = 14.9\%$
 density of fluid in active tube: $\rho_f = 0.0766 \text{ lbm/ft}^3$
 $w_{\rho_f} = 0.000035 \text{ lbm/ft}^3 = 0.04\%$
 density of fluid in reference tube: $\rho_{f_0} = 0.0749 \text{ lbm/ft}^3$
 $w_{\rho_{f_0}} = 0.00025 \text{ lbm/ft}^3 = 0.3\%$
 mass ratio, β : $\beta = 0.00136$
 $w_\beta = 0.00029 = 21.3\%$
 one minus mass ratio, $(1 - \beta)$: $(1 - \beta) = 0.99864$
 $w_{(1-\beta)} = -0.00029 = -0.03\%$
 flow velocity: $u = 183.542 \text{ ft/sec}$
 $w_u = 12.45 \text{ ft/sec} = 6.8\%$
 active tube decay constant: $R_1 = 17.901 \text{ sec}^{-1}$
 $w_{R_1} = 0.1047 \text{ sec}^{-1} = 0.5\%$

empty tube decay constant: $R_{c1} = 15.591 \text{ sec}^{-1}$

$$w_{R_{c1}} = 0.078 \text{ sec}^{-1} = 0.5\%$$

calibration constants:

$$a_1 = 56.394 \text{ lbm/ft}^3$$

$$w_{a_1} = 12.0 \text{ lbm/ft}^3 = 21.3\%$$

$$a_2 = 9.286 \text{ ft}^{-1}$$

$$w_{a_2} = 2.12 \text{ ft}^{-1} = 22.8\%$$

$$a_3 = 1.0013$$

$$w_{a_3} = 0.00028 = 0.03\%$$

Substituting these values into the first set of equations in this section to get the measurement uncertainty in the Coriolis values of density, velocity, and mass flow rate yields the following uncertainty values.

Measurement Uncertainty

frequency of active tube: $R_2 = 466.11 \text{ rad/sec}$

$$w_{R_2} = 0.677 \text{ rad/sec} = 0.15\%$$

frequency of empty tube: $R_{c2} = 466.11 \text{ rad/sec}$

$$w_{R_{c2}} = 0.677 \text{ rad/sec} = 0.15\%$$

active tube decay constant: $R_1 = 17.833 \text{ sec}^{-1}$

$$w_{R_1} = 0.085 \text{ sec}^{-1} = 0.5\%$$

empty tube decay constant: $R_{c1} = 15.424 \text{ sec}^{-1}$

$$w_{R_{c1}} = 0.078 \text{ sec}^{-1} = 0.5\%$$

one minus mass ratio, $(1 - \beta)$: $(1 - \beta) = 0.9990696$

$$w_{(1-\beta)} = 0.0041 = 0.02\%$$

$$\begin{aligned}
\text{mass ratio, } \beta: \quad \beta &= 0.0093 \\
w_{\beta} &= -0.0041 = 427\% \\
\text{fluid density*}: \quad \rho_f &= 0.0525 \text{ lbm/ft}^3 \\
w_{\rho_f} &= 0.2317 \text{ lbm/ft}^3 = 440\% \\
\text{velocity*}: \quad u &= 294.385 \text{ ft/sec} \\
w_u &= 1153.58 \text{ ft/sec} = 392\% \\
\text{mass flow rate*}: \quad \dot{m} &= 0.012564 \text{ lbm/sec} \\
w_{\dot{m}} &= 0.06997 \text{ lbm/sec} = 560\%
\end{aligned}$$

As can be seen, the uncertainty in mass flow is incredibly large, topping the numbers for velocity, density, and the mass ratio. It can be seen (by going back to the equations) that the real problem lies in the computation of $(1 - \beta)$, from which β and the uncertainty in β both come. The quantity $(1 - \beta)$ is a strong function of frequency (as can be noted by studying the equation for it) and even small changes (or uncertainties) in the frequencies, even 0.15%, can cause noticeable changes in $(1 - \beta)$. This is no problem until one attempts to compute β . The uncertainties in β and $(1 - \beta)$ are the same (on an absolute scale), being 0.0041 in this case. However, what is a 0.4% uncertainty in $(1 - \beta)$ becomes a whopping 427% uncertainty in β . The other terms in the expansion for $(1 - \beta)$ contribute at most an additional 30% to the uncertainty in β , indicating that future work may focus on ways to improve the frequency-measuring ability of the setup. Work might also focus on ways to avoid the very large-very small numerical problems (with β being very small relative to other parameters of interest) which were encountered in this study.

* As measured by Coriolis flowmeter.

VITA

Andrew Gregory Denny was born and raised in Knoxville, Tennessee. After attending both elementary school and high school in the Knoxville area (finishing with some degree of success), he began his college career at the University of Tennessee, Knoxville, in September, 1981. Greg (as he prefers to be called) graduated in June, 1985, with a Bachelor of Science degree in Mechanical Engineering after successfully completing the prescribed course of study. Commencement was promptly followed by graduate school and the pursuit of a Master of Science degree in the same field. It was also followed by his marriage to Janice Fay Conner, a wonderful lady, who has continually provided special blessings and great comfort to him. He received his Master of Science degree in August, 1987, and is currently employed by Calspan Corporation at the Arnold Engineering Development Center in Tullahoma, Tennessee.