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COMPUTATION OF ISOTHERMAL LAMINAR SWIRLING
FLOW IN A MODEL ARC HEATER

A Thesis
Presented for the
Master of Science
Degree
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ABSTRACT

The electric arc heater was used to provide required enthalpy-pressure environments for testing the reentry thermal protection system and for the development and evaluation of missiles nosetip material. However, the arc within the arc heater often generate a "kink" force driving the arc toward the wall. Therefore, swirling flow was introduced into the arc heater flowfield to stabilize the "kink" force, and analytical modeling of arc heater was greatly needed to determine an improved configuration for a swirl-stabilized arc heater.

A water tunnel study was initiated by Arvin-Calspan/AEDC to study the complex flowfield that exists within the arc heater. It was the objective of the present work to prepare and apply a numerical computational fluid dynamics model of the mean steady flow in the water tunnel model for a given geometry. An existing full Navier-Stokes computer code for axisymmetry internal flows was modified for this purpose.

The numerical model examined the variation of various parameters that were controlling the flowfield. The pressure variation across various cross sections in the arc heater were analysed to see the stability of the arc. Streamlines and vorticity were determined, and separation points and reattachment points inside the arc heater were located. The numerical code, however, could not address any unsteadiness of the core vortex flow.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
II. CODE DEVELOPMENT	4
A. THEORETICAL DEVELOPMENT	4
B. BACKGROUND OF THE COMPUTER CODE	6
C. GEOMETRIC DESCRIPTION OF NUMERICAL MODEL AND SOLUTION GRID	10
D. MASS INFLOW MODELING	12
III. CODE IMPLEMENTATION	14
A. BOUNDARY CONDITION	14
B. STATIC PRESSURE COMPUTATION	19
C. PARAMETERS OF THE INVESTIGATION	20
IV. NUMERICAL RESULTS	23
A. GENERAL DESCRIPTION OF THE FLOW	23
B. FLOW WITH DIFFERENT SWIRL ANGLES AND CONSTANT MASS FLUX	24
C. FLOWS WITH CONSTANT SWIRL ANGLES FOR DIFFERENT MASS FLUXES	30
V. CONCLUSIONS AND RECOMMENDATIONS	34
BIBLIOGRAPHY	37
APPENDIX	39
VITA	82

LIST OF FIGURES

FIGURE	PAGE
1.1. Water Tunnel H-1 Arc Heater Model AEDC/UTSI.	40
2.1. Physical Geometry Showing Finite Difference Method Used.	41
2.2. Flow Chart of the Computer Code.	42
2.3. Physical Geometry and Grid Lines.	43
3.1. Definition of Swirl Angle, β	44
4.1. Streamlines and Tangential Velocity Profiles- Case B.	45
4.2. Vorticity Contours and Axial Velocity Profiles- Case B.	46
4.3. Pressure Coefficient along the Wall-Case B.	47
4.4. Swirl Number-Case B.	47
4.5. Pressure Coefficient at Various Cross Sections- Case B.	48
4.6. Streamlines and Tangential Velocity Profiles- Case A.	50
4.7. Vorticity Contours and Axial Velocity Profiles- Case A.	51
4.8. Pressure Coefficient along the Wall-Case A.	52
4.9. Swirl Number-Case A.	52
4.10. Pressure Coefficient at Various Cross Sections- Case A.	53
4.11. Streamlines and Tangential Velocity Profiles- Case C.	55

4.12. Vorticity Contours and Axial Velocity Profiles-	
Case C.	56
4.13. Pressure Coefficient along the Wall-Case C.	57
4.14. Swirl Number-Case C.	57
4.15. Pressure Coefficient at Various Cross Sections-	
Case C.	58
4.16. Streamlines and Tangential Velocity Profiles-	
Case D.	60
4.17. Vorticity Contours and Axial Velocity Profiles-	
Case D.	61
4.18. Pressure Coefficient along the Wall-Case D.	62
4.19. Swirl Number-Case D.	62
4.20. Pressure Coefficient at Various Cross Sections-	
Case D.	63
4.21. Streamlines and Tangential Velocity Profiles-	
Case F.	65
4.22. Vorticity Contours and Axial Velocity Profiles-	
Case F.	66
4.23. Pressure Coefficient along the Wall-Case F.	67
4.24. Swirl Number-Case F.	67
4.25. Pressure Coefficient at Various Cross Sections-	
Case F.	68
4.26. Streamlines and Tangential Velocity Profiles-	
Case E.	70
4.27. Vorticity Contours and Axial Velocity Profiles-	
Case E.	71

4.28. Pressure Coefficient along the Wall-Case E.	72
4.29. Swirl Number-Case E.	72
4.30. Pressure Coefficient at Various Cross Sections- Case E.	73
4.31. Streamlines and Tangential Velocity Profiles- Case G.	75
4.32. Vorticity Contours and Axial Velocity Profiles- Case G.	76
4.33. Pressure Coefficient along the Wall-Case G.	77
4.34. Swirl Number-Case G.	77
4.35. Pressure Coefficient at Various Cross Sections- Case G.	78

LIST OF SYMBOLS

A_c	Cross sectional area
C_p	Specific heat at constant pressure
D	Diameter of circular duct
G	Decay function
J	Jacobian
IN	Total number of grid points in the ξ -direction
JN	Total number of grid points in the η -direction
P	Static Pressure
q	Mass flow rate
Re_D	Reynolds number based on diameter
S	Swirl number
U	Mean velocity
U_{ref}	Maximum Velocity
u	Axial velocity
$\vec{V}$	Velocity vector
v	Radial velocity
w	Tangential velocity
(x, r, θ)	Cylindrical coordinates system
β	Swirl angle
ϕ	Any variable
τ	Shear stress
ξ	Axial coordinate in computational plane
η	Radial coordinate in computational plane
μ	Dynamic viscosity coefficient

ν	Kinematic viscosity coefficient
ρ	Density
ω_x	Vorticity component in the streamline direction
$\vec{\omega}$	Vorticity vector
ψ	Streamfunction

CHAPTER I

INTRODUCTION

The electric arc heater was used to provide an environment for testing reentry thermal protection systems and for the development and evaluation of missiles nosetip materials. For the electric arc to efficiently heat the fluid in the test section, it is important that the arc remain in the center of the flow and not touch the wall. However, if the arc develops a bend then it will generate a "kink" force which will further distort the arc and drive it toward the wall. Therefore a stabilizing force which will appose this "kink" force is necessary to stabilize the arc and one way of providing this force is to introduce a swirl to the gas flow.

Approximate analytical models have been developed which have limited utility in the basic heater design. No complete computer code has ever been developed to simulate the arc heater fluid dynamics incorporating both the Navier-Stokes equations and Maxwell's equations. However, J.F. Shaeffer [1] produced a code to solve the swirling, turbulent and compressible arc heater flowfield, but he restricted the flow to be quasi-cylindrical. This assumed that the radial velocities in the tube were much smaller than the axial or circumferential velocities, $v \ll u \sim w$, the gradients in the radial direction were much stronger than those in the axial direction, $\frac{\partial}{\partial r} \gg \frac{\partial}{\partial x}$, and that the flow had axial symmetry, $\frac{\partial}{\partial \theta} = 0$. Neither the analytical models or this computer code can address the stability of the arc within the swirling flow.

To increase understanding of the complex flowfield that exists within the arc heater, a water tunnel study was initiated by Arvin-Calspan Corporation/AEDC Division, [2] (Figure 1.1) *. The complexities of the flowfield in the model were

* all figures and tables may be found in the appendix

examined by photographing movies of fluorescent dye streaks introduced into the model flowfield at various locations. The variation of the stability of the central vortex, without the electromagnetic forces, as influenced by modest geometric and flow changes, was observed and documented. It was the objective of present work to prepare and apply a numerical computational fluid dynamics model of the mean steady flow in the water tunnel model for a given geometry. An existing full Navier-Stokes computer code for axisymmetric internal flows was modified for this purpose. The numerical simulation of the water tunnel model was based on continuous distributed wall injection of mass inflow, whereas discrete injection drilled at different locations was used in the water tunnel model. The numerical model, being a steady flow model could not address any unsteadiness of the core vortex flow observed in the water tunnel model.

The numerical model allowed an examination of the variation of various parameters that were suggested by Arvin-Calspan Corporation for use during future experiments in the water tunnel. With this code one can:

1. compute the flow rate, swirl number, Reynolds number, magnitude of the velocity components, pressure coefficient, vorticity and streamfunction at any location;
2. vary the swirl strength to see its effect on the flowfield;
3. vary the swirl angle to modify the swirl strength and see its effect on the flowfield;
4. vary the influx mass flow rate to change the Reynolds number and swirl strength and see their effects on the flowfield;
5. vary the nozzle throat diameter;
6. change the dimensions of any section of the arc heater to see the effect on the flowfield.

The last two features were not examined in the present work but are recommended for future examination.

Chapter II introduces the history of the development of the computer code, and the geometry and solution grid used for the numerical water tunnel model. Chapter III describes the fluid dynamic boundary conditions that were applied, and introduces the flow controlling parameters to be investigated. Chapter IV describes the results of the numerical investigations and, finally, the conclusions are summarized in Chapter V.

CHAPTER II

CODE DEVELOPMENT

A. THEORETICAL DEVELOPMENT

The objective of this research was to study the effect of swirling flow upon the mean flow in the water tunnel model of the arc heater facility. The Arvin-Calspan arc heater model was tested in a water tunnel at The University of Tennessee Space Institute. The flow rates used in the water tunnel model justified testing the flow theoretical as steady, isothermal, laminar, incompressible and axisymmetric swirling flow. To create swirl in the flow, continuous tangential mass injection along the wall was assumed. Gravitational forces and heat transfer were neglected in the study. The equations governing the flow were the full, elliptic Navier-Stokes equations and the total mass continuity equation.

The steady flow Navier-Stokes equations in vector (coordinate system free) form is

$$\rho (\vec{V} \cdot \nabla) \vec{V} = -\nabla P - (\nabla \cdot \vec{\tau}) \quad (2.1)$$

where $\vec{\tau}$ is tensor. The total steady mass continuity equation is

$$\nabla \cdot (\rho \vec{V}) = 0 \quad (2.2)$$

Since the flow was incompressible, the energy equation was not required to solve the flowfield.

In many types of axisymmetry flows without swirl, the tangential velocity component and its gradient are neglected compared to the axial component. However, in swirling flows the flow is usually characterized by a vortical motion which is superimposed on the axial flow. Therefore, its tangential velocity component

has a magnitude comparable to that of axial velocity component and cannot be neglected.

Performing the vector and tensor operations on the Navier-Stokes equations above for a cylindrical coordinate system, the axial, radial and tangential or swirl momentum components were obtained, which for axisymmetric flows (neglecting θ variation) were

axial component

$$\rho \left(v \frac{\partial u}{\partial r} - u \frac{\partial u}{\partial x} \right) = - \frac{\partial P}{\partial x} - \mu \left[\frac{\partial^2 u}{\partial r^2} - \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2} \right] \quad (2.3)$$

radial component

$$\rho \left(v \frac{\partial v}{\partial r} - \frac{w^2}{r} - u \frac{\partial v}{\partial x} \right) = - \frac{\partial P}{\partial r} - \mu \left[\frac{\partial^2 v}{\partial r^2} - \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial x^2} \right] \quad (2.4)$$

tangential or swirl component

$$\rho \left(v \frac{\partial w}{\partial r} - \frac{vw}{r} - u \frac{\partial w}{\partial x} \right) = \mu \left[\frac{\partial^2 w}{\partial r^2} - \frac{1}{r} \frac{\partial w}{\partial r} - \frac{w}{r^2} + \frac{\partial^2 w}{\partial x^2} \right] \quad (2.5)$$

The equations formed a set of non-linear, elliptic partial differential equations. In order to reduce the number of dependent variables to be obtained by numerical solution, this set of equations was cast into a set of equations called the vorticity-streamfunction equations. The vorticity-streamfunction approach is one of the most popular methods for solving the two-dimensional planar and axisymmetric incompressible Navier-Stokes equations [3] because the continuity equation will be automatically satisfied, since a change of variables from velocity components to vorticity and streamfunction (independent of the pressure term) was attainable by combining the continuity and momentum equations. The vorticity transport and streamfunction equations have been derived previously by many investigators, including J.J. Lee who developed a code for his Ph.D. dissertation [4] to solve the Navier-Stokes equations in planar and axisymmetric coordinate systems. This present research was done using his computer code to handle the complex geometry

and unusual boundary conditions that the arc heater model defined. A geometric configuration identical to the water tunnel model of the H-1 arc heater facility was formulated in a finite-difference, non-orthogonal computational mesh upon which the vorticity-streamfunction equations were solved. In addition, Lee's code is capable of computing turbulent flows with compressibility effect but these features were not utilized for this work.

B. BACKGROUND OF THE COMPUTER CODE

The streamfunction and vorticity equations can be written in a general non-linear elliptic form as

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial r^2} - a \frac{\partial \phi}{\partial x} - b \frac{\partial \phi}{\partial r} - c = 0 \quad (2.6)$$

where a, b, c are functions of x and r .

Since the above partial differential equation is quasi-linear, numerical techniques must be used to solve it. The irregular physical plane (x, r) is transformed into a rectangular computational plane (ξ, η) , where finite-difference methods can be applied more easily, through the relationship

$$\xi = f(x, r), \quad (2.7)$$

$$\eta = g(x, r). \quad (2.8)$$

By chain rule differentiation, a derivative of the variable ϕ in the physical plane can be transformed into variations in the computational plane as follows

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x}, \quad (2.9)$$

$$\frac{\partial \phi}{\partial r} = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial r} + \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial r}. \quad (2.10)$$

Higher derivatives can be obtained by repeated application of the chain rule application. $\frac{\partial \phi}{\partial \xi}$ and $\frac{\partial \phi}{\partial \eta}$ can be represented by using a second order accurate finite-difference formulation. For example, at interior points, a central differences representation of these quantities is

$$\frac{\partial \phi_{i,j}}{\partial \xi} = \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\Delta \xi}, \quad (2.11)$$

$$\frac{\partial \phi_{i,j}}{\partial \eta} = \frac{\phi_{i,j+1} - \phi_{i,j-1}}{2\Delta \eta}, \quad (2.12)$$

where i is the index for variation in the ξ -direction and j is the index for variation in the η -direction, which is orthogonal to the i -direction.

The use of central differences, forward differences or backward differences depends on the point to be evaluated, that is, whether it is an interior point or at the boundary (see Figure 2.1 and Table 2.1), and upon whether or not "upward" difference need be applied.

The quantities $\frac{\partial \xi}{\partial x}$, $\frac{\partial \eta}{\partial x}$, $\frac{\partial \xi}{\partial r}$, $\frac{\partial \eta}{\partial r}$, which are not representable by finite-difference, can be transformed into $\frac{\partial x}{\partial \xi}$, $\frac{\partial x}{\partial \eta}$, $\frac{\partial r}{\partial \xi}$, and $\frac{\partial r}{\partial \eta}$, which can be approximated by finite-difference, by using the well known Jacobian determinant

$$J = \frac{\partial(x, r)}{\partial(\xi, \eta)} = \frac{\partial x}{\partial \xi} \frac{\partial r}{\partial \eta} - \frac{\partial r}{\partial \xi} \frac{\partial x}{\partial \eta} \neq 0, \quad (2.13)$$

$$\frac{\partial \xi}{\partial x} = \frac{1}{J} \frac{\partial r}{\partial \eta}, \quad (2.14)$$

$$\frac{\partial \xi}{\partial r} = -\frac{1}{J} \frac{\partial x}{\partial \eta}, \quad (2.15)$$

$$\frac{\partial \eta}{\partial x} = -\frac{1}{J} \frac{\partial r}{\partial \xi}, \quad (2.16)$$

$$\frac{\partial \eta}{\partial r} = \frac{1}{J} \frac{\partial x}{\partial \xi}. \quad (2.17)$$

After transforming the equations and some algebraic work, the quasi-linear elliptic partial differential equation (2.6) can be transformed into an equation of the same general form in the computational plane, namely

$$A_{1\phi} \frac{\partial^2 \phi}{\partial \xi^2} + A_{2\phi} \frac{\partial^2 \phi}{\partial \eta^2} - B_{1\phi} \frac{\partial \phi}{\partial \xi} - B_{2\phi} \frac{\partial \phi}{\partial \eta} + D_\phi = 0. \quad (2.18)$$

Equation (2.18) can then be represented by various finite-difference schemes. The scheme employed by Lee was the "decay function" approach, developed by Chien [5.6] to solve the finite-difference form of equation (2.18) and to make the finite-difference equation stable, speed convergence, and improve accuracy. The approximation in the method occurs when the axial and radial solutions are uncoupled except through source-like terms that arise in the transformation from differential equations to finite-difference equations. Lee demonstrated that the use of the decay function sped convergence of the solution for complex flow with high mesh Reynolds or Peclet number.

Applying the decay function, $G_{\phi\xi}$, to equation (2.18), one gets

$$\frac{A_{1\phi}}{G_{\phi\xi}} \frac{\partial^2 \phi}{\partial \xi^2} - \frac{A_{2\phi}}{G_{\phi\eta}} \frac{\partial^2 \phi}{\partial \eta^2} - B_{1\phi} \frac{\partial \phi}{\partial \xi} - B_{2\phi} \frac{\partial \phi}{\partial \eta} + D_\phi = 0 \quad (2.19)$$

The decay functions, $G_{\phi\xi}$ and $G_{\phi\eta}$, are exponential functions of the grid Reynolds number and their approximations and the criteria for numerical stability of the equation will be discussed in the next section.

After the second-order accurate finite-difference representation of all terms, including the decay function, have been determined at all interior points, one has the following implicit finite-difference representation of ϕ

$$\phi_{i,j} = K_A \phi_{i+1,j} - K_B \phi_{i-1,j} - K_C \phi_{i,j-1} - K_D \phi_{i,j+1} + D_P, \quad (2.20)$$

where the coefficients are defined by

$$K_A = \left(\frac{A_{1\phi}}{G_{\phi\xi} \Delta \xi^2} - \frac{B_{1\phi}}{2 \Delta \xi} \right) / E_P.$$

$$\begin{aligned}
K_B &= \left(\frac{A_{1\phi}}{G_{\phi\xi}\Delta\xi^2} + \frac{B_{1\phi}}{2\Delta\xi} \right) / E_P, \\
K_C &= \left(\frac{A_{2\phi}}{G_{\phi\eta}\Delta\eta^2} - \frac{B_{2\phi}}{2\Delta\eta} \right) / E_P, \\
K_D &= \left(\frac{A_{2\phi}}{G_{\phi\eta}\Delta\eta^2} + \frac{B_{2\phi}}{2\Delta\eta} \right) / E_P, \\
D_P &= D_\phi / E_P,
\end{aligned}$$

and

$$E_P = 2 \left(\frac{A_{1\phi}}{G_{\phi\xi}\Delta\xi^2} + \frac{A_{2\phi}}{G_{\phi\eta}\Delta\eta^2} \right).$$

For details of the derivation of this equation see [4].

Applying these transformations over a finite-difference mesh of dimensions $IN \times JN$, yields $(IN-2)(JN-2)$ coupled algebraic equations which must be solved. IN and JN are the maximum number of grid points in the ξ and η direction, respectively. There are numerous ways of solving this set of equations and an iteration method was employed in this code, in the following fashion. The code uses Gauss-Seidel, point by point iterative solution procedure with over and under-relaxation parameters. For each value of $\phi_{i,j}$ computed at a point is given by

$$\phi_{i,j} = (\phi_{i,j})_{old} - R_p \left[(\phi_{i,j})_{new} - (\phi_{i,j})_{old} \right] \quad (2.21)$$

where R_p is the relaxation parameter used to under/over relax the solution for convergence. R_p lies between 0 and 2 and the appropriate factor which causes rapid convergence must be determined by experience for each type of flowfield.

A solution is said to converge when a residual, defined by,

$$err = (\phi_{i,j} - (\phi_{i,j})_{old}) / (\phi_{i,j})_{old}$$

is less than 10^{-4} everywhere in the flowfield. A flow chart of the code with or without swirling flow is summarized in Figure (2.2).

C. GEOMETRIC DESCRIPTION OF NUMERICAL MODEL AND SOLUTION GRID

The decay functions, as developed by Chien, for the solution of the finite-difference formulation of the partial differential equations of two-dimensional and axisymmetric fluid dynamics can be approximated by the following algebraic functions

$$G_{\phi\xi} = 1 - 0.0625R_{\phi\xi}^2 \quad \text{for} \quad |R_{\phi\xi}| \leq 2,$$

$$G_{\phi\xi} = \frac{2}{|R_{\phi\xi}|} - \frac{1}{(R_{\phi\xi})^2} \quad \text{for} \quad |R_{\phi\xi}| > 2,$$

$$G_{\phi\eta} = 1 - 0.0625R_{\phi\eta}^2 \quad \text{for} \quad |R_{\phi\eta}| \leq 2,$$

and

$$G_{\phi\eta} = \frac{2}{R_{\phi\eta}} - \frac{1}{(R_{\phi\eta})^2} \quad \text{for} \quad |R_{\phi\eta}| > 2.$$

where the grid Reynolds number are defined as

$$R_{\phi\xi} = \frac{B_{1\phi}}{A_{1\phi}} \Delta\xi,$$

and

$$R_{\phi\eta} = \frac{B_{2\phi}}{A_{2\phi}} \Delta\eta.$$

A fine grid with small grid Reynolds number is desired in most cases to obtain a stable, accurate solution. Therefore, the physical grid points were closely spaced in those regions where it was suspected that rapid flow changes were taking place. Because of the long length of the arc heater, the numerical model configuration was divided into four sections, as shown in Figure 2.3. The total number of grid points used in the computation was 140 by 21 ($i < j$).

1. Anode section

The Anode section had axial grid points from $I = 1$ to $I = I2$. It consisted of a hemispherically-shaped cap at the left end wall with a 6 inch diameter straight section followed by a conical converging section that ended in a 4 inch diameter tube. The total length of this section was about 19.72 inches. The cross-stream grids were packed closer together axially where significant changes of the flow were suspected to occur and, therefore, grid lines were pushed closer together at the hemispherically shaped end wall and at the intersection of the straight 6 inch diameter section with the convergent cone. Similarly, in the convergent section, the grid lines were pushed toward both ends where the flow patterns were expected to change rapidly. The convergent conical section had a half angle of 15.7 degrees.

2. Constrictor section

The constrictor section consisted of a circular tube 4 inches in diameter and 114 inches long. From a water tunnel flow visualization test of the heater model, separated flow was seen at the point where the constrictor section meets the cathode section. Therefore, the grid lines were forced closer together at both ends of the constrictor, while they were more widely spaced in the middle. The constrictor section has axial grid points from $I = I2$ to $I = I3$.

3. Cathode and Nozzle section

The cathode and nozzle section consisted of a divergent conical section, a straight circular section of 6 inch diameter and a 1.8 inch diameter exit nozzle. Its total length was about 14.1 inches. Grid lines were again crowded toward both ends at every portion where the flow pattern was expected to change rapidly. The

cathode grids extended from $I = I3$ to $I = I5$ and the nozzle from $I = I5$ to $I = 129$.

D. MASS INFLOW MODELING

The water tunnel model had tangential holes drilled through the walls from the anode to the cathode section. This resulted in tangential mass injection at discrete points along the model. The computational model, on the other hand, assumed that the mass was injected continuously along the same length ($I = 1$ to $I = I5$). The nozzle section had a solid wall and the normal no-slip boundary condition was applied at the wall.

The grid lines were constructed as follows. The axial lines were equally spaced in the radial direction from the centerline to the wall, the centerline being $J = 1$ and the outer wall being $J = JN = 21$. The points of intersection of the $I =$ constant lines with the wall were identified, as described in the above sections, and then the $I =$ constant lines were algebraically constructed internally down to the axis. This was accomplished by reading in the boundary grid locations and interpolating to find the interior intersections of the tranverse with the axial grid lines.

The anode sections $I =$ constant grid lines were constructed so that they had curvature which was originally equal to that of the hemispherically-shaped end-cap and then slowly decreased to form straight lines at the downstream of the constrictor section. The interpolating formula used for the anode section was

$$x_{i,j} = x_{i-1,j} + \frac{x_{I2,j} - x_{1,j}}{x_{I2,JN} - x_{1,JN}} * (x_{i,JN} - x_{i-1,JN})$$

All remaining $I =$ constant grid lines were perpendicular to the axis. The final grid lines in the physical plane are shown in Figure. 2.3.

A separate program was constructed to determine the boundary grids location and then the physical plane was transformed into a rectangular computational grid (ξ, η) . The transformation was designed to be one-to-one, and was checked by ensuring that the Jacobian was always non-zero. This generated the values of ξ_i, η_j in the computational plane, and the values of the Jacobian at each computational point. Moreover, the solution could then be transformed back to the physical plane after the computation converges.

CHAPTER III

CODE IMPLEMENTATION

A. BOUNDARY CONDITION

The correct wall boundary conditions were required to determine the characteristics of the flow. They also characterized the finite-difference form to be used. The boundaries were divided into five parts.

1. Left End Wall (Spherical Cap)

Since there is no flow across the wall, the streamfunction must remain constant along the wall. For velocities, the no-slip condition was imposed at the wall.

From the continuity equation, one can define the streamfunction (for axisymmetric flow) as

$$\frac{\partial \psi}{\partial r} = \rho u r \quad \text{and} \quad \frac{\partial \psi}{\partial x} = -\rho v r, \quad (3.1)$$

$$d\psi = \frac{\partial \psi}{\partial r} dr - \frac{\partial \psi}{\partial x} dx. \quad (3.2)$$

Integrating from some reference point

$$\int d\psi = \int \left(\frac{\partial \psi}{\partial r} dr + \frac{\partial \psi}{\partial x} dx \right), \quad (3.3)$$

$$\psi = \psi_{ref} + \int \left(\frac{\partial \psi}{\partial r} dr + \frac{\partial \psi}{\partial x} dx \right). \quad (3.4)$$

ψ_{ref} is usually chosen at grid (1, 1) to have a value of zero. From equation (3.1), with $u, v, = 0$, the streamfunction along the entire left end wall is zero.

Vorticity plays an important role in the development of viscous flow. With the no-slip condition, the velocity gradient is significant at the wall, resulting in stress generation and hence the production of vorticity. This vorticity is then transported

and diffused downstream in the flow. Vorticity is defined by the relation

$$\vec{\omega} = \nabla \times \vec{V}.$$

In axisymmetric flow, with $\frac{\partial}{\partial \theta} = 0$, the only component of vorticity that is important is ω_θ , where

$$\omega_\theta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial r}. \quad (3.5)$$

In axially symmetric flows the tangential velocity does not contribute to the formation of vorticity at the wall. After transformation, one gets

$$\omega_\theta = \frac{\partial v}{\partial \xi} \frac{\partial \xi}{\partial x} - \frac{\partial v}{\partial \eta} \frac{\partial \eta}{\partial x} - \frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial r} - \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial r}. \quad (3.6)$$

Equation (3.6) was approximated by finite-differences. Care has to be taken when computing the vorticity at grid (1, JN). This consists of the point of intersection between the hemisphere and the anode tube. In the present case the surface of the hemisphere is the line $I = 1$, with $J = 1$ on the center line and $J = JN$ at the intersection. This becomes a rectangular corner in the computational plane. Usually there is no injection at point (1, JN) so there is no velocity gradient. Also, because of the way the present computational plane was constructed this is also a point of zero vorticity. However, in this investigation, there was injection at the point (1, JN) and so the vorticity at that point cannot be zero. The vorticity was computed using equation (3.6) except a forward-difference in the ξ direction and a backward-difference in the η direction were used.

2. Injection Wall

Mass is continuously injected through the wall into the arc heater with both radial inflow and a tangential velocity component. The magnitude of the radial

component and swirl component depends on the defined flow rate and is determined as follows

$$\text{mass flow rate} = (\text{velocity component}) \times (\text{area normal to velocity component}).$$

Thus given the total inflow mass flow rate, Q ,

$$Q = v\rho A_c \quad (lbm/sec),$$

hence the radial inflow velocity is

$$v = \frac{Q}{\rho A_c} \quad (ft/sec). \quad (3.7)$$

Defining β , the swirl angle (see Figure 3.1), as the angle with the wall tangent, then

$$w = \frac{v}{\tan \beta}. \quad (3.8)$$

Hence, u , v and w are specified along the boundaries and can be used to compute the streamlines and vorticity. Notice that if Q is fixed while β is varied, then the kinetic energy introduced to the flow is also changed. Thus, the kinetic energy at lower β is much greater than at higher β , for fixed Q .

From equations (3.1) and (3.3)

$$\psi = \psi_{ref} + \int (\rho u r dr - \rho v r dx).$$

Since u is equal to zero along the wall, the equation reduces to

$$\psi = \psi_{ref} - \int_0^x \rho v r dx \quad (3.9)$$

where the integral is evaluated along the wall. For incompressible flow and v constant along the model length,

$$\psi = \psi_{ref} - \rho v \int_0^x r dx. \quad (3.10)$$

Equation (3.10) depends on r only, and, from the geometry, one can see that r was discontinued and thus, this equation had to be integrated from section to section.

Vorticity is computed using equation (3.6). The velocity gradients at the wall are needed to compute vorticity, while only the velocity components themselves are used to compute the streamfunction distribution along the wall. At the boundaries, the velocity gradient in finite-difference form at a point requires information at the two neighboring points. For example, using a backward difference form for the derivative:

$$\frac{\partial u_{i,j}}{\partial \eta} = \frac{3u_{i,j} - 4u_{i,j-1} - u_{i,j-2}}{2\Delta\eta}.$$

No information is available for $u_{i,j-1}$ and $u_{i,j-2}$ at the beginning of the calculation. Therefore, the value of the vorticity at the boundaries has to be found implicitly in the course of the solution by iterating over the interior grids while the streamfunction can be computed immediately once the velocity and $r(x)$ are specified along the boundaries. The streamfunction is no longer a constant along the wall when mass is added through the wall.

3. Nozzle

Since this is a solid wall nozzle where no more mass is added, the wall velocity components are zero. With this condition, the streamfunction along the nozzle wall will be constant. Although this is the last section of the flow that is of interest, the exit flow condition that was previously used by Lee and was originally in the code was not imposed here. Lee assumed that the longitudinal dissipation was negligible, that is, $\frac{\partial^2 \phi}{\partial x^2} = 0$, where ϕ is any variable, which implies that $\frac{\partial \phi}{\partial x} = \text{constant}$. By expanding in a Taylor series, he obtained $\phi_{IN,J}$ in terms of $\phi_{IN-1,J}$, $\phi_{IN-2,J}$ and $\phi_{IN-3,J}$. But in a contracting nozzle $\frac{\partial^2 \phi}{\partial x^2} = 0$, does not approximate

the exit condition too well, and therefore, the exit condition which will be discussed next was artificially imposed.

4) Exit Condition

In a closed water tunnel, the water is continuously recirculated by a pump. The exit nozzle therefore was followed by a long length of straight tube through which the water recirculates to return to the holding tank. Although the flow inside this tube is not of interest, a straight section of ducting is added to the numerical model to permit a proper exit boundary condition to be applied downstream.

Swirling motion when established, is most persistent. Talbot [7] and A. J. Ward-Smith [8] have shown that swirl will decay exponentially downstream while the angular momentum possesses by the swirling flow will typically decrease by only a half in about twenty diameters. Therefore, the condition that $\frac{\partial^2 \phi}{\partial x^2} = 0$ when applied at about twenty diameters downstream of the nozzle will approximate the criteria used by Lee. Hence a straight circular pipe of about one and a half feet long was added to the exit nozzle to permit the exit boundary condition that was originally in the code (see Figure 2.3).

5) Axis Boundary Condition.

Since the flow is symmetrical about the axis,

$$\frac{\partial \phi}{\partial r} = 0 \quad \text{at} \quad r = 0,$$

for all variables except w , v , ψ and ω_θ , which were assumed to be zero on the axis. These boundary conditions were also assumed by Lee and he modeled $\frac{\partial \phi}{\partial r}$ by a backward differences approximation obtaining $\phi_{i,1}$ in terms of $\phi_{i,3}$ and $\phi_{i,2}$, using a Taylor's series expansion.

B. STATIC PRESSURE COMPUTATION

A good feature about the vorticity-streamfunction approach for incompressible flow is that, ψ , ω_θ , u , v and w can be solved independently of the static pressure field. Therefore, after a converged solution of all the above variables has been obtained, the static pressure can be computed. Lee computed the static pressure by two methods, 1) from the Poisson pressure equation and 2) from an integration method. The integration method was used in this investigation.

From equations (2.3) and (2.4) one can write

$$\begin{aligned}\frac{\partial P}{\partial x} &= \mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial x^2} \right) - \rho \left(v \frac{\partial u}{\partial r} - u \frac{\partial u}{\partial x} \right), \\ \frac{\partial P}{\partial r} &= \mu \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} - \frac{\partial^2 v}{\partial x^2} \right) - \rho \left(v \frac{\partial v}{\partial r} - \frac{w^2}{r} - u \frac{\partial v}{\partial x} \right), \\ dP &= \frac{\partial P}{\partial x} dx - \frac{\partial P}{\partial r} dr, \\ \int dP &= \int \left(\frac{\partial P}{\partial x} dx - \frac{\partial P}{\partial r} dr \right), \\ P &= P_{ref} - \int \left(\frac{\partial P}{\partial x} dx - \frac{\partial P}{\partial r} dr \right).\end{aligned}$$

The integral was computed in the computational plane by replacing the partial derivatives by finite-differences. The results were transferred back to the physical plane by Jacobian transformation.

Since the pressure distribution across a section determines the stability of the arc, the criteria for swirl stabilization will be discussed here. One minimum requirement for the stability of the arc would be that $\frac{\partial P}{\partial r} > 0$ everywhere across each cross section. If viscous diffusion is neglected, equation (2.4) becomes

$$-\rho v \frac{\partial v}{\partial r} - \rho \frac{w^2}{r} - \rho u \frac{\partial v}{\partial x} \approx \frac{\partial P}{\partial r}.$$

Assuming u , v and w are of equal magnitude, and that $\frac{\partial v}{\partial r} \approx \frac{\partial v}{\partial x} \ll u, v, w$, then

$$\frac{\partial P}{\partial r} = \rho \frac{w^2}{r}.$$

Therefore, no matter how the flow rotates, the swirl stabilizes the flow. As will be discussed later, the swirl velocity can be developed into a mixture of a forced and a free vortex region, and, therefore the possibility for stability for each type of vortex motion must be examined. For solid body rotation,

$$w = r\Omega,$$

$$w^2 = r^2\Omega^2,$$

and

$$\frac{\partial P}{\partial r} = \rho r\Omega^2.$$

Thus, it is seen that $\frac{\partial P}{\partial r}$ increases with r . For a potential vortex,

$$w = \frac{k}{r},$$

$$w^2 = \frac{k^2}{r^2},$$

and

$$\frac{\partial P}{\partial r} = \rho \frac{k^2}{r^3}.$$

In this case, $\frac{\partial P}{\partial r}$ decreases rapidly as r increases. Therefore, it can be concluded that, the swirl velocity should increase as the radius is increased, and, hence, rigid body flow rotation is a minimum requirement for arc stability.

C. PARAMETERS OF THE INVESTIGATION

Before stating all the parameters involved in the investigation, a commonly known swirl number is introduced. The swirl number is defined as the ratio of the angular momentum flux, $M_{\alpha n_j}$, to the axial momentum flux, $M_{\alpha x}$, at a given

cross section. [7] For incompressible flow, neglecting the pressure term in the axial momentum [9]

$$S = \frac{M_{ang}}{RM_{ax}} = \frac{\int_0^R uwr^2 dr}{R \int_0^R u^2 r dr}. \quad (3.11)$$

It can be seen that the swirl number can be increased by increasing the tangential velocity component which can be achieved by either increasing the flow rate or decreasing the swirl angle, β . Continuous tangential injection along the wall, which produces continuous swirl has the advantage of possibly keeping the electromagnetic arc source term stable at the axis for a particular constrictor length, depending on the intensity of swirl. This is possible because the tangential velocity component, which is comparable in magnitude to its axial component, will produce a radial force of magnitude approximately equal to $\frac{\rho w^2}{r}$. Usually in an arc heater, this force is used to stabilize the "kink" force produced by the arc, to keep the arc from touching the wall and causing a short circuit. Therefore, the main objective of this work is to investigate the development of the swirl velocity as the flow proceeds from upstream to downstream through the various model sections, assuming laminar flow.

Table 3.1 show the list of parameters that were tested. Cases A, B, and C investigated the variation of swirl angle for constant mass flow rate. This was then changed to a fixed swirl angle for different mass flow rates. With a continuous addition of mass, the flow pattern changed downstream; hence, swirl number, Reynolds number and velocity components changed continuously as the flow proceeded downstream. Swirl number, Reynolds number and mean axial velocity were computed at different cross sectional locations along the model from the equations

$$S = \frac{\int_0^R uwr^2 dr}{R \int_0^R u^2 r dr},$$

$$U = \frac{2}{R^2} \int_0^R ur dr, \quad (3.12)$$

$$Re_D = \frac{\rho U D}{\mu}. \quad (3.13)$$

Special care was taken at the anode section where the grid lines were constructed based on the geometry of the hemispherical cap and was not perpendicular to the axis. Since only information on the grid points was known, values at a certain cross section were interpolated to calculate the above parameters.

Finally, the mass flow rate at the exit is computed to check for continuity

$$Q = 2\pi \int_0^R r u dr. \quad (3.14)$$

Simpson's composite integration rule was used in the code to compute the integrals. More information on this method can be found in ref [10]. The computations were performed on a VAX 711-85. With a computational mesh of 140×21 , a typical solution required 650 iterations for convergence, which took 19 minutes of CPU time.

CHAPTER IV

NUMERICAL RESULTS

A demonstration of the utility of solving a streamfunction-vorticity formulation of the Navier-Stokes equations for elliptic flows by a finite-difference method which incorporated the special decay function to provide numerical solution stability and control of truncation errors can be found in ref [4].

The following assumptions were made for all calculations: steady, laminar, incompressible, and constant temperature flow with the no-slip wall boundary condition.

A. GENERAL DESCRIPTION OF THE FLOW

In general, water was injected through the walls in a plane perpendicular to the axis at an angle β to the wall tangent. Since no flow was coming in axially through the hemispherical end, the water coming in tangentially was being forced to attach to the walls due to the no-slip condition and to flow axially at the axis. This creates a region of reverse flow near the hemispherical end. The flow then sped up as it passed through the convergent section upstream of the constrictor section. In the long constrictor section, the flow began to develop and the profiles became invariant with x as it moved further downstream. This was due to the constant mass injection and the high length to the diameter ratio in the constrictor section. This could best be seen in the axial velocity profile since the profile was similar at different downstream sections. As the flow entered the divergent section where the pressure increased, the flow slowed down, separated and reverse flow occurred. The flow reattached itself in the downstream end of the cathode section

when enough mass flow was injected, creating enough momentum to carry it downstream.

The contours shown on all contour plots were those that left specific grid points, the points being the same for all plots. This had the advantage of showing, for example, the path lines of dye released from these points and allowed comparison between different cases. The disadvantage was that the contours had different values on different plots and several contours could have approximately the same value on a given plot.

B. FLOW WITH DIFFERENT SWIRL ANGLES AND CONSTANT MASS FLUX

As mentioned previously, swirl intensities could be varied in two ways. The effect of varying the swirl angle for constant mass flow was investigated first.

1. Case B : $Q = 0.1$ (lbm/sec), $\beta = 45$ degree.

a. Streamline Contours

Case B was picked as a reference case to which other results were compared. Figure 4.1 was a plot of the streamlines and tangential velocity profiles. The tangential profiles were turned 90 degrees so as to make them visible in a longitudinal plot. On this and all following plots the constrictor section was not shown. It could be seen that the initial streamlines attached themselves to the wall until they reached the axis where they flow axially. The streamlines from the third grid point showed weak reverse flow near the wall but it was quickly turned back to flow axially downstream. After this grid point no streamlines were parallel to the wall because mass was coming through the wall, hence ψ had a non-constant

value. The wall with injection became the source of emanating streamlines. As the flow moved into the constrictor section, the streamlines converged toward the axis, showing an increase of axial velocity.

In the divergent cone, the streamlines did not follow the wall contour, i.e., they did not diverge, because the injection of mass along the divergent cone and cathode walls built up an artificial wall which carried the former streamlines downstream. At the boundary of the divergent section and cathode walls, there was a lack of axial momentum, compared to the tangential and radial momentum, caused by the adverse axial pressure gradient, and the flow was reversed. At points further away from the wall, axial momentum was large enough to overcome the axial pressure gradient, and therefore, the flow moved axially downstream. Near the boundary, mass was continuously injected which increased the axial momentum downstream and hence the flow reattached to the wall. Starting at the nozzle, there was no more mass injection at the wall and the streamlines turned and followed the contour of the wall while the flow velocity increased.

b. Tangential Velocity Profiles

The generation of swirl in the flow was caused by the impartation of a tangential velocity component to the flow and since the profiles of the velocity injected tangentially determined the dynamics of the flow downstream in the arc heater, it was important to examine the shape of such profiles.

The tangential velocity was zero at the axis, from the axial symmetry boundary condition, and attained a constant specified value at the wall boundary where injection occurred. For walls where no injection occurred, the tangential velocity was zero (no-slip). In references [7] and [11], the tangential velocity profiles of a swirling flow were observed and predicted to result from a combination of a forced

and a free (potential) vortex. If this was the case, at some radial location, a maximum value of w was achieved. Further increase in r would decrease w inversely with r (free-vortex region) and a decrease in r toward zero would decrease w linearly (forced-vortex region). From Figure 4.1, one can see that in the anode region a forced-vortex dominated most of the tangential velocity profiles. Only a small free-vortex region with a slight tangential velocity decrease was observed. In the converging section, a steep gradient in the tangential velocity profile was noticed in the free-vortex region near the walls. This meant that the maximum tangential velocity had increased. As the flow moved downstream into the constrictor region the tangential velocity profiles developed and became invariant with further increase in x . When the flow moved into the divergent and cathode sections, the tangential velocity profiles did not change significantly. This was interesting, because in the cathode section, the axial velocity profiles showed that reversed flow occurred. This constancy of the tangential velocity must be caused by the injection of additional swirl flow into the cathode section through the cathodic walls. In fact, two different values of the tangential velocity gradient were seen and the maximum tangential velocity occurred at the wall. When the flow moved into the nozzle section, the viscous effects of the wall dominated the profile and the tangential velocity was rapidly diminished to zero at the wall from the no-slip condition.

c. Axial Velocity Profile

The axial velocity was very small in the initial, upstream portion of the model. Near the hemispherical cap the flow slowly crept upstream, attaching itself to the end wall. The axial velocity profiles were shown in Figure 4.2 where they had been expanded by a factor of six in the anode section to magnify the result. It

could be seen that only in the convergent section did the axial velocity increase and then it rapidly developed into an invariant form in the constrictor section. The axial speed continued to increase as the mass flow increased downstream in the constrictor section. As the flow moved into the divergent section, the sudden increase in area created an adverse pressure gradient that slowed down the flow near the wall until $\frac{\partial u}{\partial r} = 0$, and reverse flow occurred near the wall. Separation occurred between grid points 80–81. Separation took place because the axial momentum near the wall could not overcome the axial pressure gradient, and hence the fluid near the walls could not be carried downstream. The flow began to reattach to the wall again once it moved downstream to the cathode section where the flow had gained enough mass flow and speed. Reattachment took place between grid points 101–102.

d. Vorticity Contours

A zero vorticity contour was established at the wall which separated the negative vorticity produced on the solid wall at the end-cap from the positive vorticity which originates from the injecting wall. A reverse flow was thus established with such a vorticity gradient.

Vorticity could neither be created nor destroyed within a viscous incompressible flow but could be created on the walls or convected through the walls from a source outside the walls or through the nozzle. The vorticity intensity increased as the flow moved through the convergent section and decreased through the divergent section due to a change in the effective moment of inertia (ignoring injection effect). Therefore, one could see that the vorticity contours were more closely spaced in the convergent and nozzle sections. In the cathode section, the area increased resulting in decreased velocity and an adverse pressure gradient. This

resulted in the existence of another surface of zero vorticity and separation with reversed flow near the wall. The spacing between the vorticity lines was very small within this separated region near the wall.

e. Pressure Distribution

Pressure distribution along the wall was plotted in Figure 4.3. Notice the slight increase in pressure at the cathode section which was followed by a great drop as it moved through the nozzle. The great drop in pressure corresponded to a significant axial speed increase. As mentioned before, swirling flow produced a force that was approximately equal to $\frac{\partial P}{\partial r}$ which could be utilized to stabilize any undesirable radial force; therefore, radial plots of pressure distribution at various cross sections were of interest. Figure 4.5 shows plots of the pressure coefficients at cross sections corresponding to grid points at $I = 4$ (anode), $I = 51$ (constrictor) and $I = 91$ (cathode). The pressure coefficient was computed based on a reference pressure at a point which was chosen to be (1, JN),

$$C_p = \frac{P - P_{ref}}{0.5\rho U_{ref}^2}. \quad (4.3)$$

From the previous discussion it was indicated that arc stability required that $\frac{\partial P}{\partial r} > 0$. From Figures 4.5a and 4.5b, in the anode and constrictor sections, it could be seen that $\frac{\partial P}{\partial r} < 0$, showing that the stability condition was not satisfied. However, from Figure 4.5b, $\frac{\partial P}{\partial r} > 0$ for $\frac{r}{R} > 0.6$. By contrast $\frac{\partial P}{\partial r} > 0$ in the central part of the cathode region. Figure 4.5c, and the stability criteria were satisfied for this portion of the flow.

2. Case A : $Q = 0.1$ (lbm/sec), $\beta = 75$ degrees

Figures 4.6 to 4.10 showed the results of Case A where the swirl velocity was decreased by increasing the swirl angle β . Since v and u were still the same at the boundaries, decreasing the swirl angle from $\beta = 45$ degrees to $\beta = 75$ degrees had only a slight effect on the flowfield. The largest change occurred in the swirl number, Figure 4.9. The maximum swirl number dropped from 0.3 to 0.083. This concluded that flow was not sensitive to decreasing percentage change in swirl velocity. Therefore, for the rest of the investigation with a higher mass flow rate, $\beta = 75$ degrees would be used to simulate flow with a low swirl number and that would be compared to flow with a high swirl number. $\beta = 5$ degrees.

3. Case C : $Q = 0.1$ (lbm/sec), $\beta = 5$ degrees

Figures 4.11 to 4.15 represented the results of Case C. From the swirl number plot, Figure 4.14, the maximum swirl number increased to 1.13. The streamline contour plot, Figure 4.11, indicated that there was a great change in the flowfield pattern in the anode section, whereas not much change was observed in the constrictor and cathode sections. With a small swirl angle, high swirl velocity and kinetic energy were introduced and the streamlines tended to be pushed further back to the wall near the hemispherical end-cap. The flow injected through the wall did not have a tendency to move downstream until after grid point eleven. The zero vorticity contour originating from the wall was pushed further upstream to the end wall and all negative vorticities produced by the wall were squeezed together, producing a dense spacing of the vortex lines which indicated a high vorticity gradient at this location. In the reverse flow region, the streamlines turned to flow axially closer to the axis than for Cases A and B. Once the flow moved

axially, the streamlines began to resume the patterns of the previous cases. Looking at the vorticity contour plot and equation 2.4, one could tell that an adverse pressure gradient was established near the wall over a longer distance than for Cases A and B. In the cathode region, the only difference was that the reverse flow had a slightly weaker strength in the separation zone (the difference being very small). The flow separated between grids 80-81 and reattached to the wall between grids 101-102. Not much change was observed in the tangential and axial velocity profiles. In the vorticity contour plot, Figure 4.12, loops of constant vorticity of higher intensity than the entering vorticity were observed to occur.

It could be seen that for a higher swirl flow (small β), the pressure distribution across a section satisfied the condition for arc stability. Figures 4.15a,b and c all showed conditions where $\frac{\partial P}{\partial r} > 0$ and clearly it was a case that was better for arc stability than the previous case which had a larger swirl angle.

C. FLOWS WITH CONSTANT SWIRL ANGLES FOR DIFFERENT MASS FLUXES

1. $\beta = 75$ degrees

The swirl velocity could also be increased by increasing the mass flow rate while keeping the swirl angle, β , constant, as mentioned before. Cases D and F both illustrated this effect by using a higher mass flow rate while keeping β at 75 degrees. The flowfield patterns of these cases were compared to that of Case A where $Q = 0.1$ and $\beta = 75$ degrees. Figures 4.16 to 4.20 showed the result for Case D and Figures 4.21 to 4.25 showed the results for Case F. Looking at the swirl number graphs of Figures 4.19 and 4.24, of Cases D and F, respectively, it was seen that the maximum swirl number increased from 0.083 (case A) to 0.1

and 0.125, for Cases D and F, respectively. This indicated that increasing the flow rate did not increase the swirl number significantly. From the streamline contour plots of Figures 4.16 and 4.21, one could see that there wasn't much change taking place in the flow pattern in the anode section. However, the streamline contours in the separation zone of the divergent and cathode sections did vary between the cases. An axial momentum increase due to a higher flow rate would enhance reattachment of the flow. This could be seen from the streamline contour plots at the same grid point by looking at the amount of backflow of the three cases. This was true because increasing the mass flow rate increased the axial speed which in turn increased the ratio of the axial momentum to the swirl momentum. It could be seen from the vorticity contours that reverse flow occurred between grids 79–80 and between grids 78–79 for Case D and Case F, respectively. Reattachment occurred between grids 101–102 and between grids 98–99 for Case D and Case F, respectively. The most noticeable changes, however, occurred in the tangential velocity profile. In Cases D and F, the tangential velocity profile quickly developed into a mixture of free and forced-vorticity. However, Case F had a greater decrease in w in the free-vortex region, meaning that the free-vortex flow was more dominant at the mass flow rate. As the flow moved into the divergent section, the tangential velocity profile in Case D developed into a saddle-like velocity profile with two points of maximum tangential velocity which finally flattened out near the nozzle. In Case F, the tangential velocity profile also had two peaks but the peak near the axis was still more dominant than the peak near the wall. In fact, the forced-vortex region did not change much throughout the flow for all these cases: it was the profile of the free-vortex region that changed at different sections. The axial velocity profiles, Figures 4.17 and 4.22, remained similar in

profile for the three cases except for the fact that the velocity was higher with the higher mass flow rates.

Examination of the pressure distribution graph (Figures 4.20 and 4.25) revealed that the pressure profiles for Cases D and F, respectively, had almost the same shape except for changes in the magnitude of the slopes.

2. $\beta = 5$ degrees

Figures 4.14, 4.29, and 4.34 for Cases C, E and G, respectively, showed surprisingly that if β is 5 degrees, then increasing the mass flow rate decreased the maximum swirl number from 1.13 to 0.95 to 0.9. Comparing Cases C, E and G was almost the same as comparing Cases A, D and F. The streamlines (Figures 4.26 and 4.31) moved much further back toward the end wall with increasing mass flow rate. The zero vorticity line moved to the wall for the higher mass flow rate thus creating a very high vorticity gradient (see Figures 4.27 and 4.32). These vorticity gradients determined the strength of the reverse flow, as was illustrated by Case G which had a stronger reverse flow velocity than the other cases. Vorticities of higher intensities formed closed loops which were also pushed back to the end wall with increased mass flow rate. These loops were formed before vorticity of the same magnitude was generated downstream and carried in from sources outside the boundary. These loops also formed high vorticity gradients as the flow turned and moved axially closer to the axis. The flow downstream in the divergent and cathode sections had the same trend as the low swirl flows with increasing mass, that was, improved reattachment. The flow in Cases E and G separated between grids 79-80 and between grids 78-79, respectively, and it reattached between grids 101-102 and between grids 98-99, respectively.

Cases E and G have pressure distribution profiles across a section which were similar in form to Case C (Figures 4.30 and 4.35), except for the difference in magnitude of the slopes, and behaved similarly as in the low swirl flows with increasing mass flow rate.

Cases D and E and Cases F and G can be similarly compared to show the effects of swirl velocity increase by changing swirl angle at a higher mass flow rate. The same trends of flow property variations appeared in these cases as with Cases A and C except now the tangential velocity profile developed into a mixture of free and forced-vortex flow.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

A computer code was developed to compute the flowfield of a swirl-stabilized arc heater, using as a flow model incompressible laminar steady flow without electromagnetic forces. The boundary conditions and geometry grid were carefully modelled to match those of a water tunnel model of the arc heater. The computations were performed to show the effect of swirl angle and mass flow rate on the flowfield. Although this is a very simplified model of the actual flow encountered in a swirl-stabilized arc heater, the laminar flowfield computations can be considered to be the first step toward computing the turbulence and compressibility effects which are expected to dominate the real problem. A summary of the predictions obtained from the investigation are given below. They indicate how and in which model section a particular parameter will affect the flowfield. Many of these predictions should also apply to the arc heater flow.

- a. Increasing swirl intensities by decreasing the swirl angle at constant mass flow rate

The most prominent flow changes occur in the anode section where the streamlines and vorticity contours moved upstream toward the end wall as the swirl intensity was increased. The swirl intensity was increased as the swirl angle β decreased. Reverse flow was stronger in the anode section and weaker in the cathode section for the higher swirl intensities. The pressure distribution across the model sections in the anode, constrictor and cathode are completely different as a result of different flow behavior in the three sections.

b. Increasing mass flow rate at $\beta = 75$ degree

The swirl number was increased as the mass flow rate was increased at a constant swirl angle of 75 degrees. Not much change was observed at the anode section except that increasing the mass flow rate changed the tangential velocity profile into that characteristic of an inner region of a forced vortex and outer region of a free vortex more quickly than for the cases summarized in A. The axial velocity was increased as the mass flow rate was increased. In the separated region within the cathode section, reattachment was enhanced with higher mass flow rate. The pressure profiles maintained the same shape for all mass flow rates at each section except for the magnitude in slopes.

c. Increasing mass flow rate at $\beta = 5$ degree

The swirl number was decreased as the mass flow rate was increased at a constant swirl angle of 5 degrees. Both the anode and cathode section experienced flowfield changes. The streamlines and zero vorticity contours moved upstream toward the end wall in the anode section as the mass flow rate was increased, encouraging a strong reverse flow region. Increased mass flow rate also improved reattachment of the flow in the separation zone of the cathode section. Increasing the mass flow rate also developed the tangential velocity profile into that characteristic of a free and forced vortex as in the case above.

d. Radial Pressure Gradient

Only the cases computed for $\beta = 5$ degrees (high swirl velocity) led to a positive radial pressure gradient at all cross sections in the arc heater. Therefore,

it can be concluded that high swirl velocities, and consequently high kinetic energy input, is a requirement for arc stability.

e. Swirl Number

The value of the swirl number is not by itself a good indication of the usefulness of a vortical flow for an arc heater. It does not indicate whether the pressure gradient is positive nor does it give any indication of the mix free and forced vorticity that might occur at any cross section.

f. Future Work

The present work is preliminary and additional work should be performed, using the same computer code. The following tasks are suggested for future investigation.

1. Examine discrete, rather than continuous mass injection.
2. More correctly model the rough wall of the actual arc heater.
3. Vary the geometry.
4. Add turbulence to the computations.
5. Examine the effect of density variations on the flow.
6. Perform a stability analysis on the mean flows computed. This analysis could not use the present code.

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BIBLIOGRAPHY

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APPENDIX

Note: All Dimensions in Inches

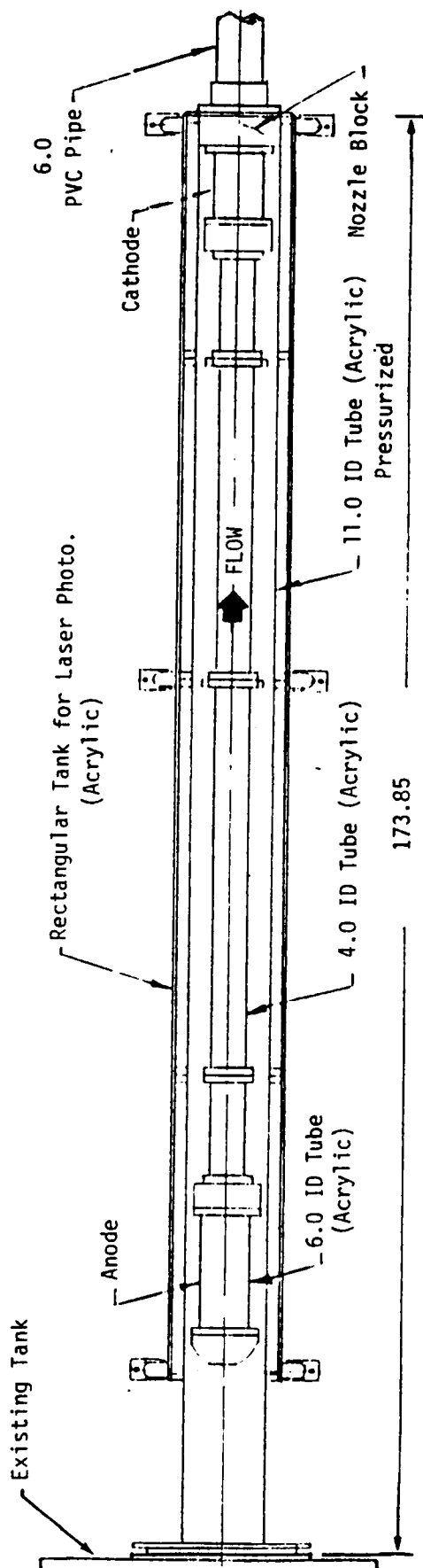


Figure 1.1. Water Tunnel H-1 Arc Heater Model AEDC/UTSI.

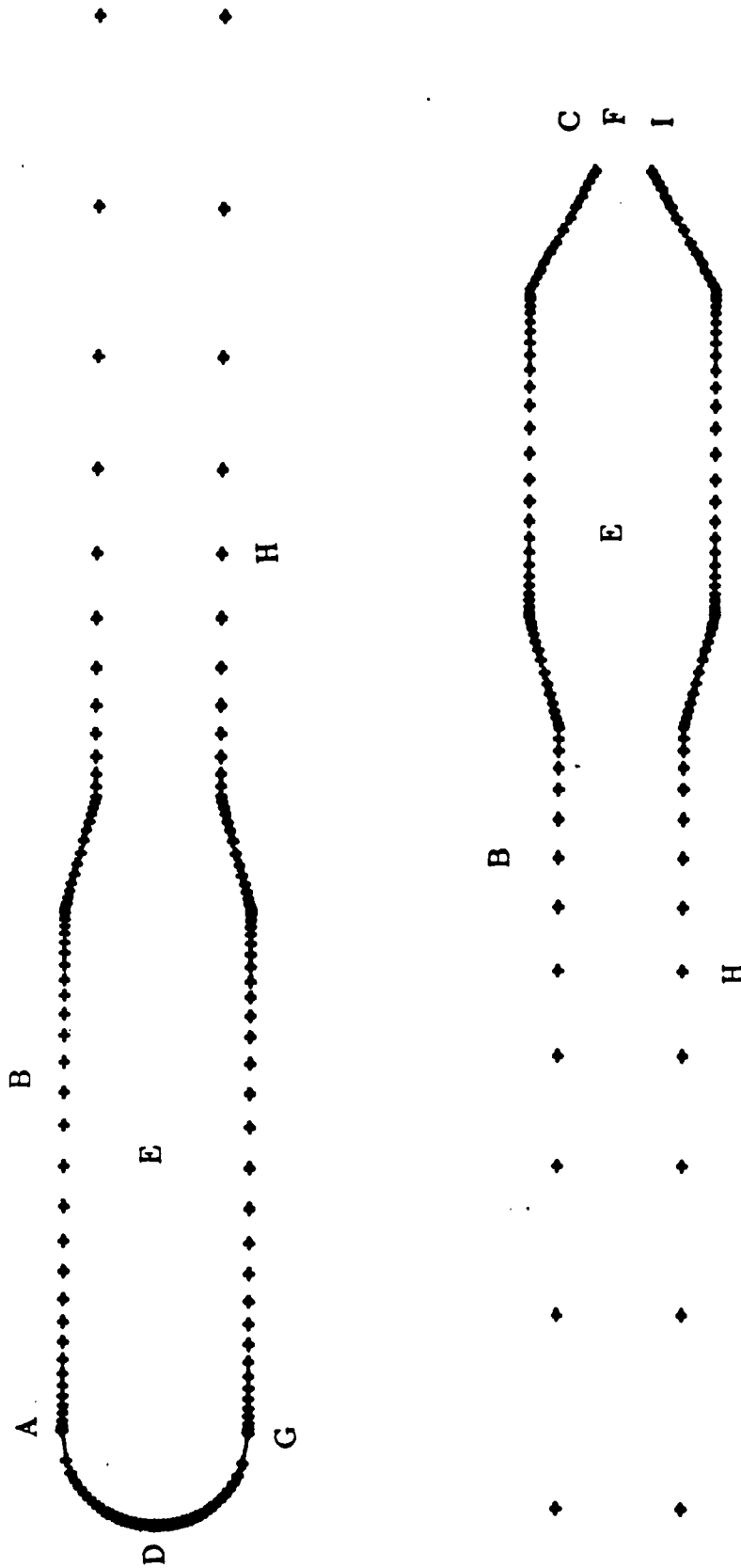


Figure 2.1. Physical Geometry Showing Finite Difference Method Used.

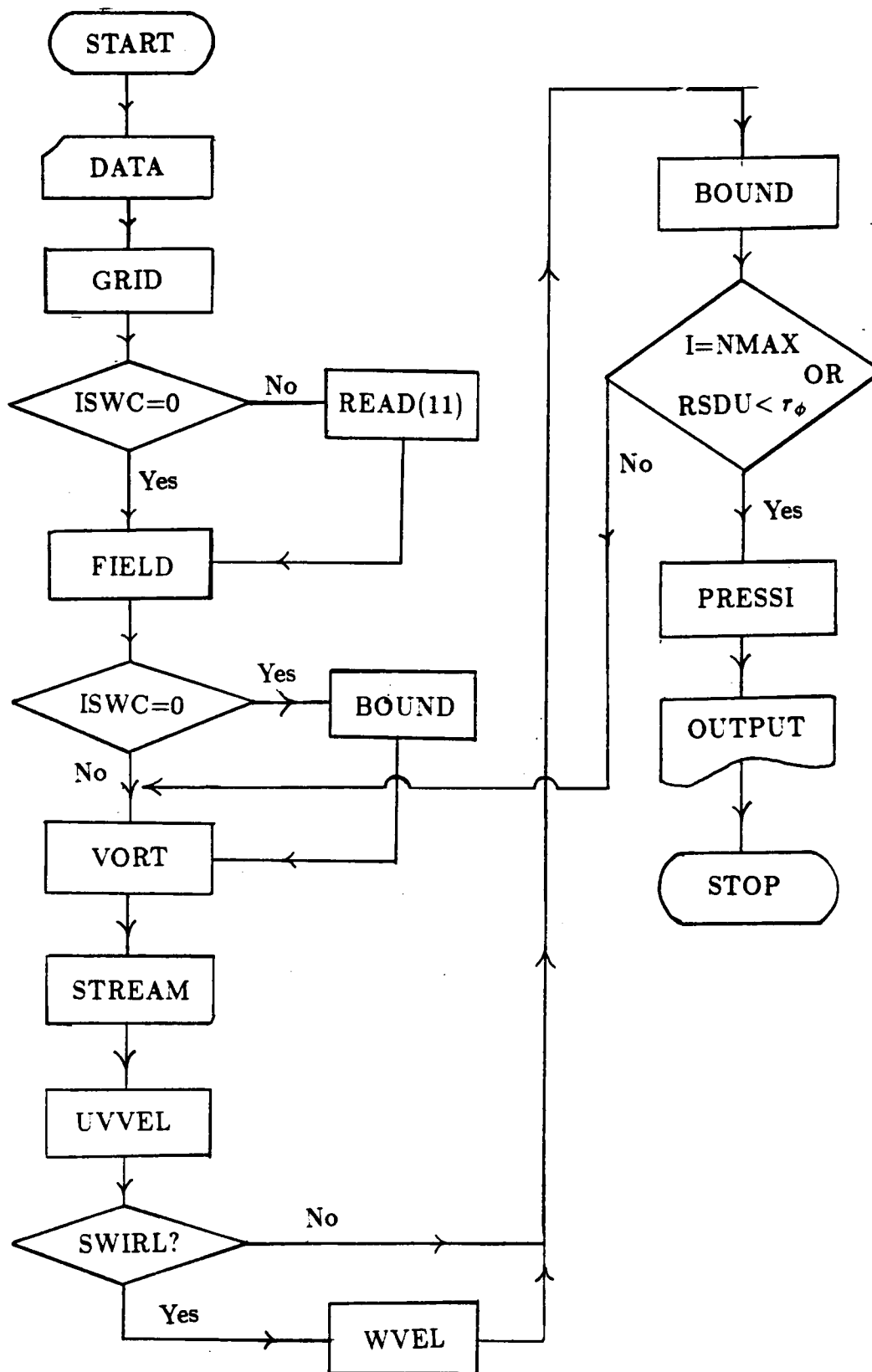


Figure 2.2. Flow Chart of the Computer Code.

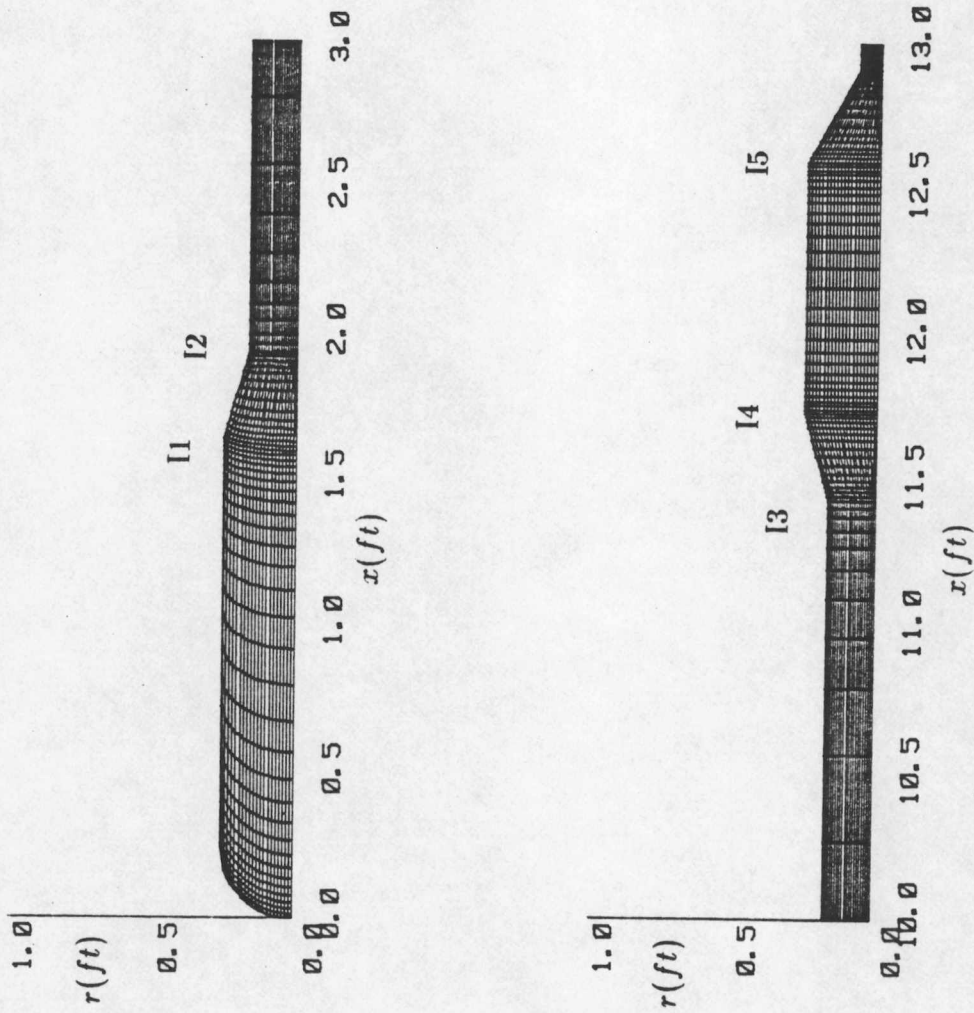


Figure 2.3. Physical Geometry and Grid Lines.
(note absence of central portion of constriction)

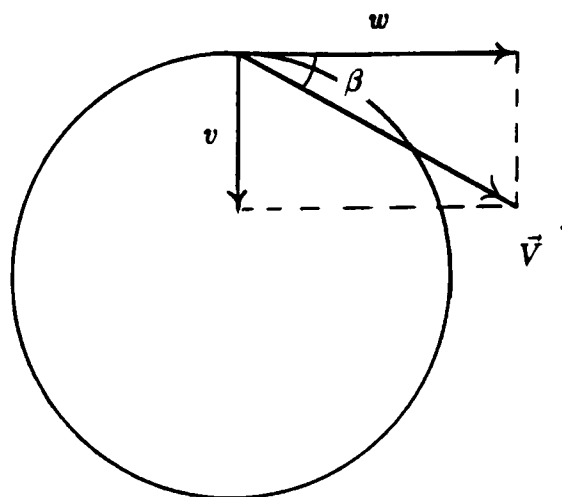


Figure 3.1. Definition of Swirl Angle, β .

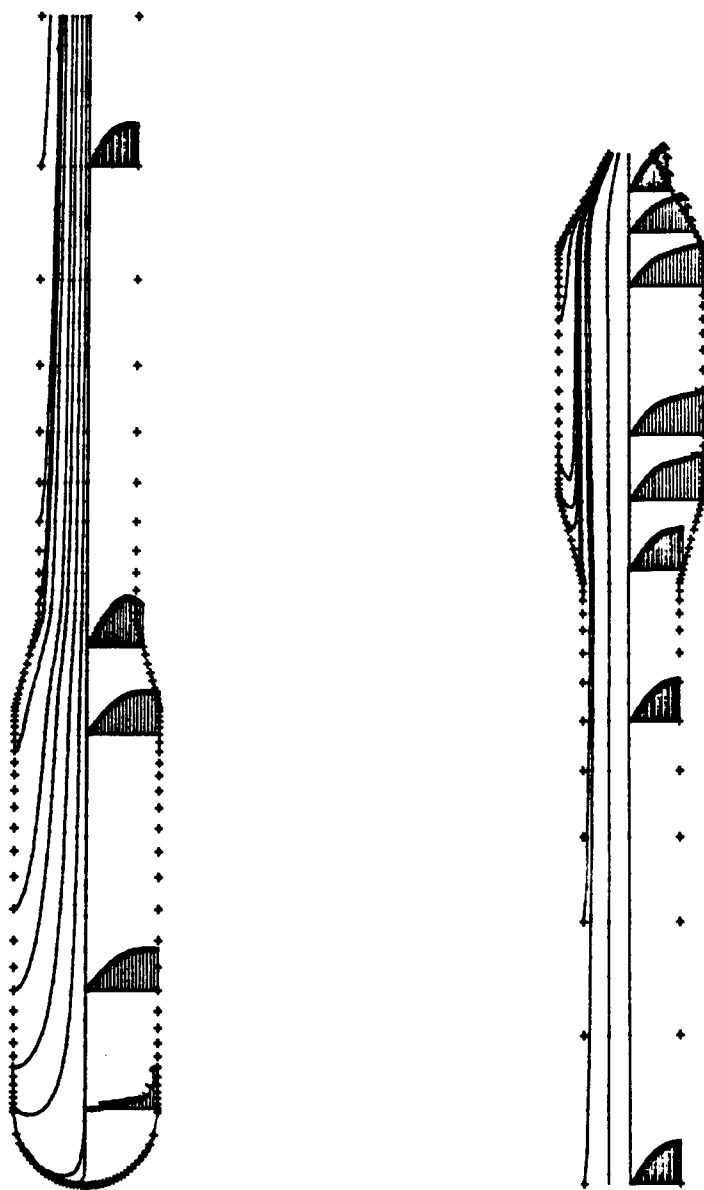


Figure 4.1. Streamlines and Tangential Velocity Profiles-Case B. $Re_{exit} = 1132$

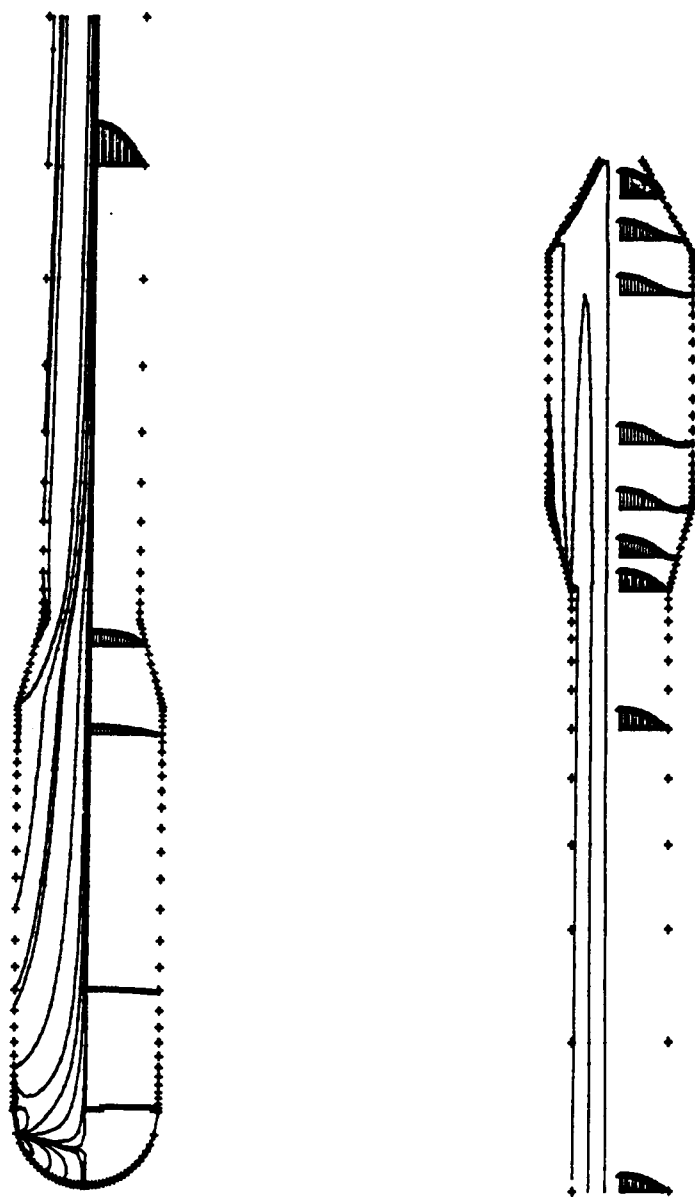


Figure 4.2. Vorticity Contours and Axial Velocity Profiles—Case B. $Re_{exit} = 1132$

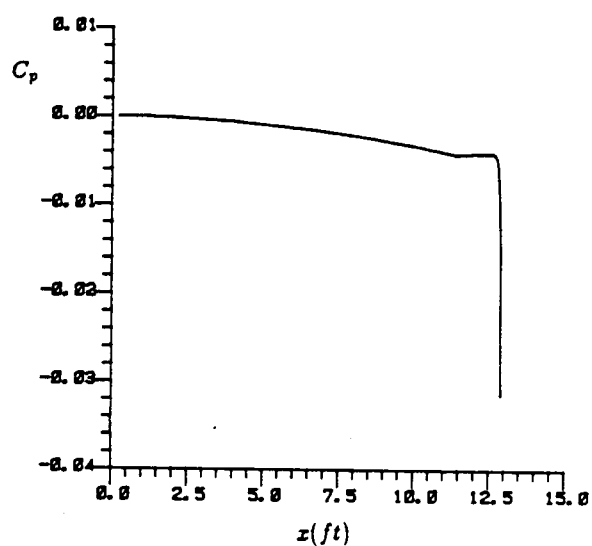


Figure 4.3. Pressure Coefficient along the Wall-Case B.

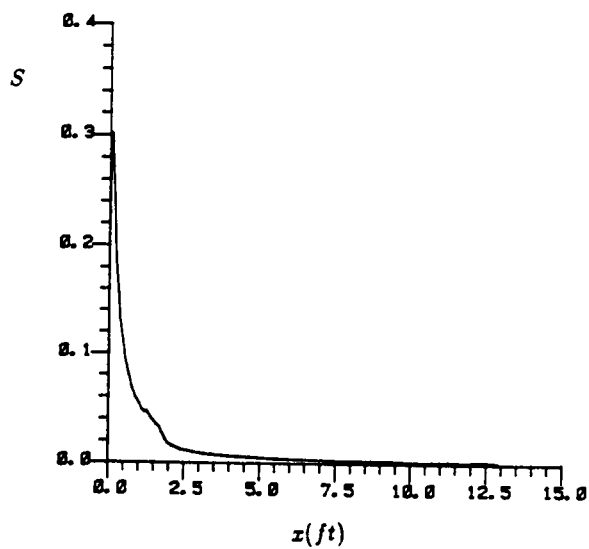


Figure 4.4. Swirl Number-Case B.

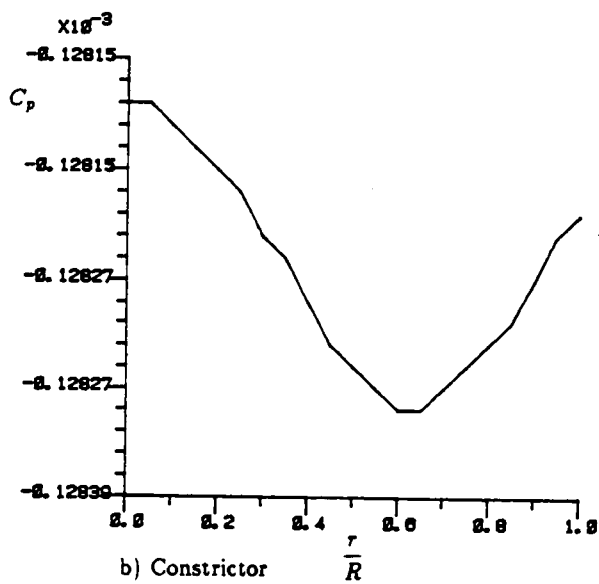
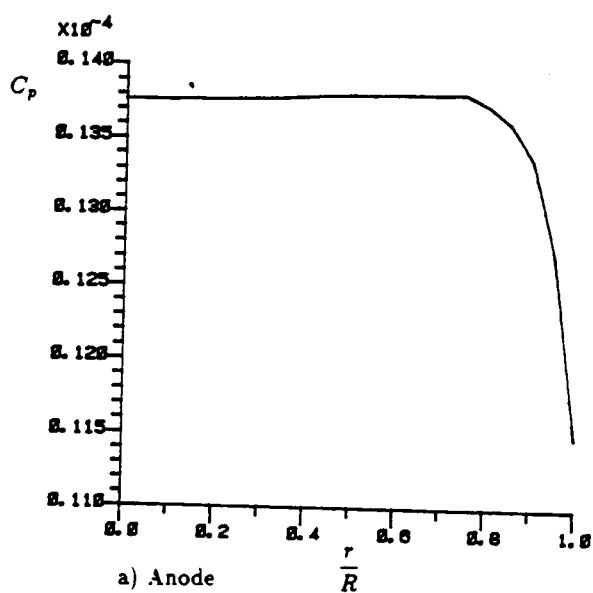


Figure 4.5. Pressure Coefficient at Various Cross Sections—Case B.

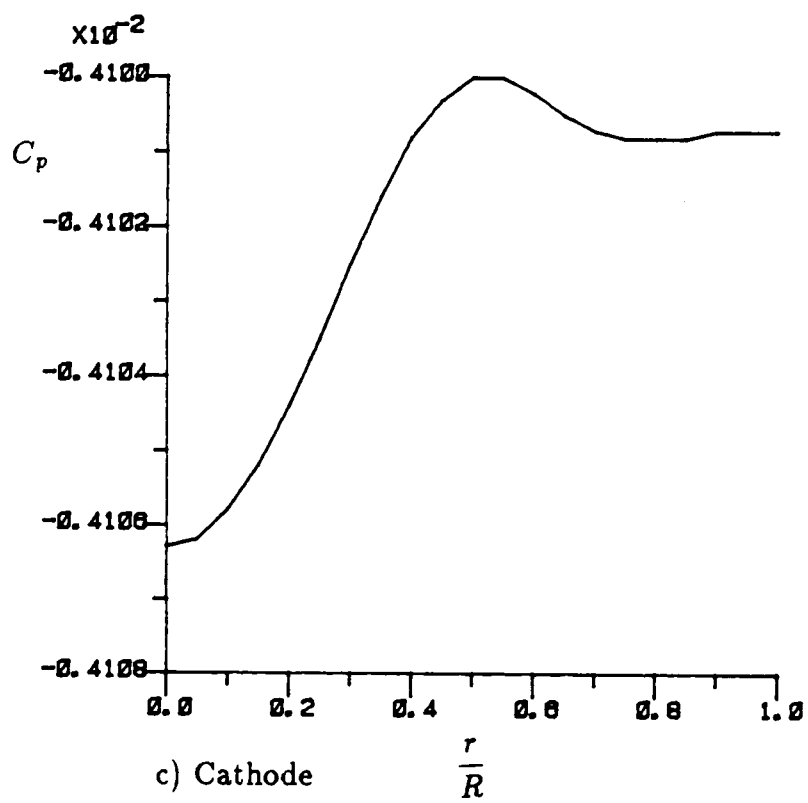


Figure 4.5. (continued)

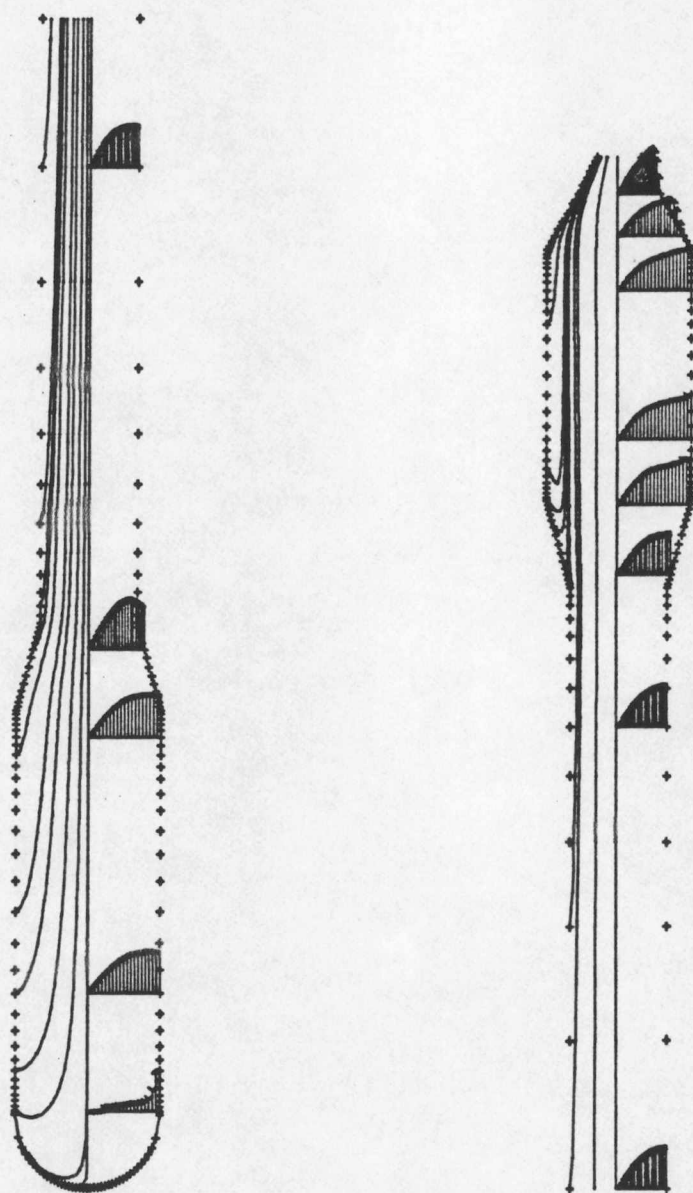


Figure 4.6. Streamlines and Tangential Velocity Profiles-Case A. $Re_{exit} = 1132$

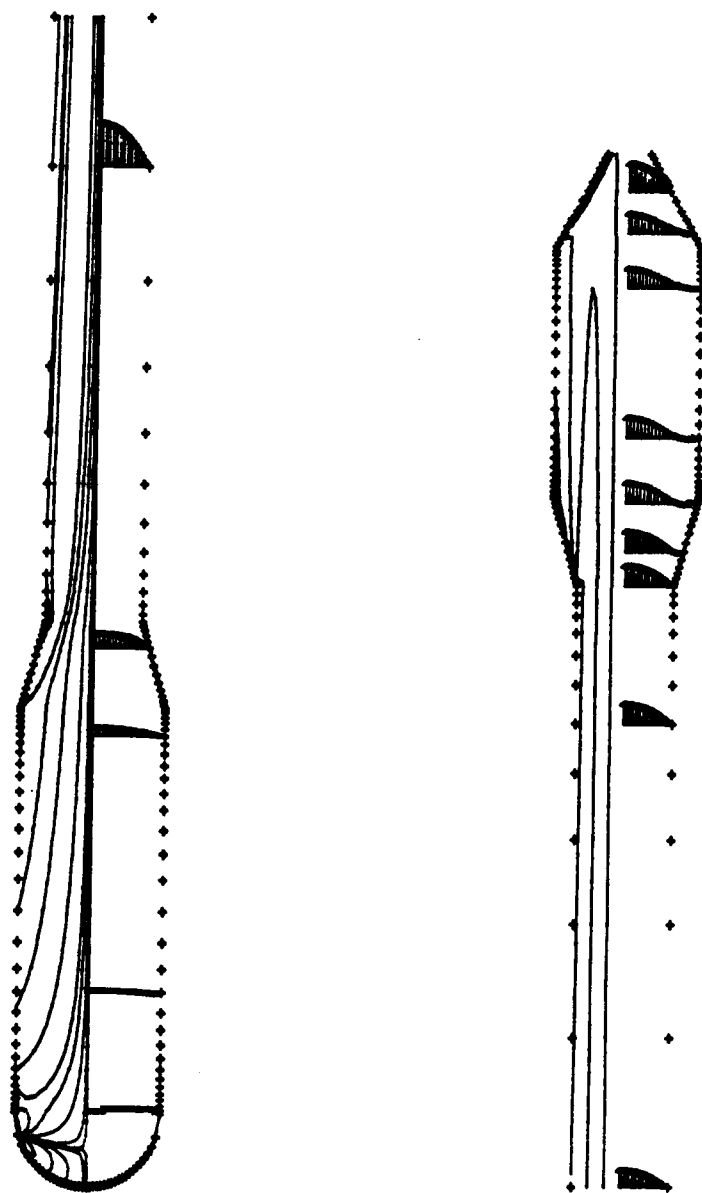


Figure 4.7. Vorticity Contours and Axial Velocity Profiles-Case A. $Re_{exit} = 1132$

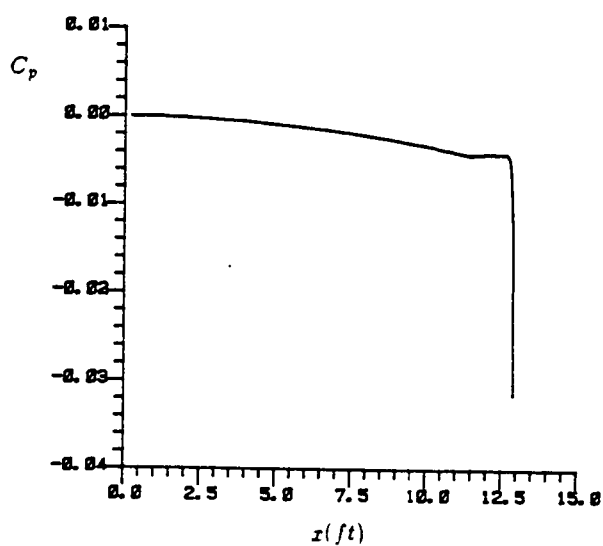


Figure 4.8. Pressure Coefficient along the Wall-Case A.

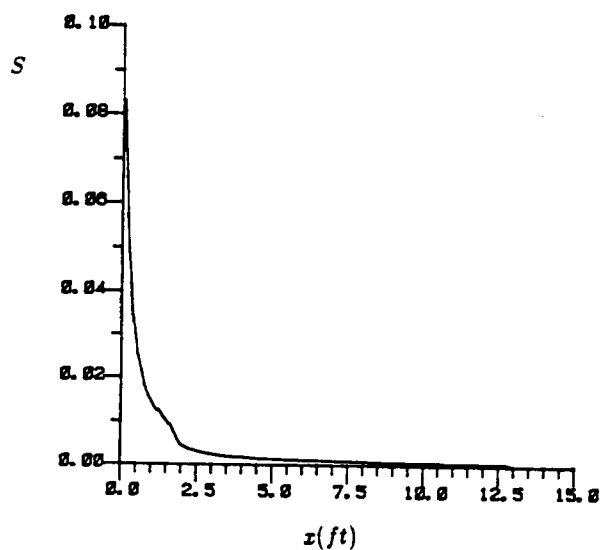


Figure 4.9. Swirl Number-Case A.

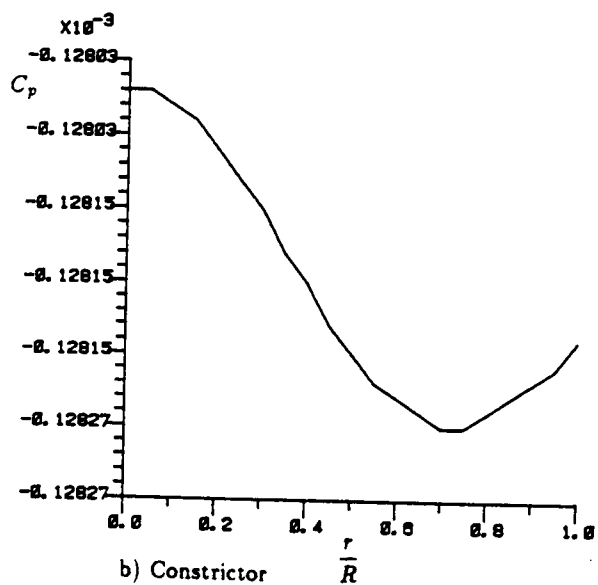
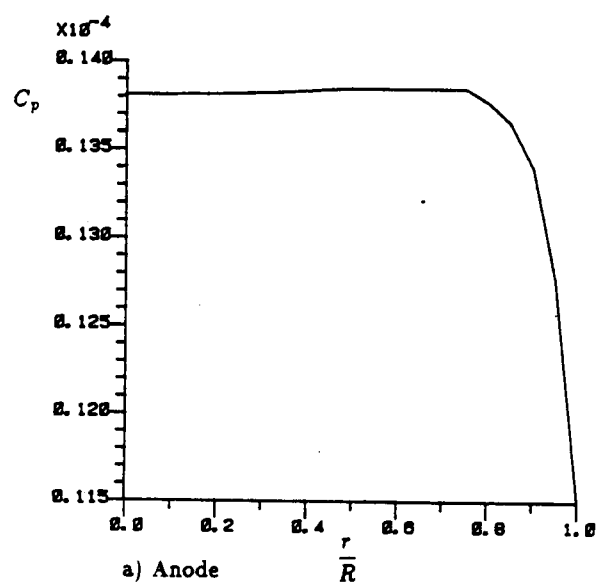


Figure 4.10. Pressure Coefficient at Various Cross Sections—Case A.

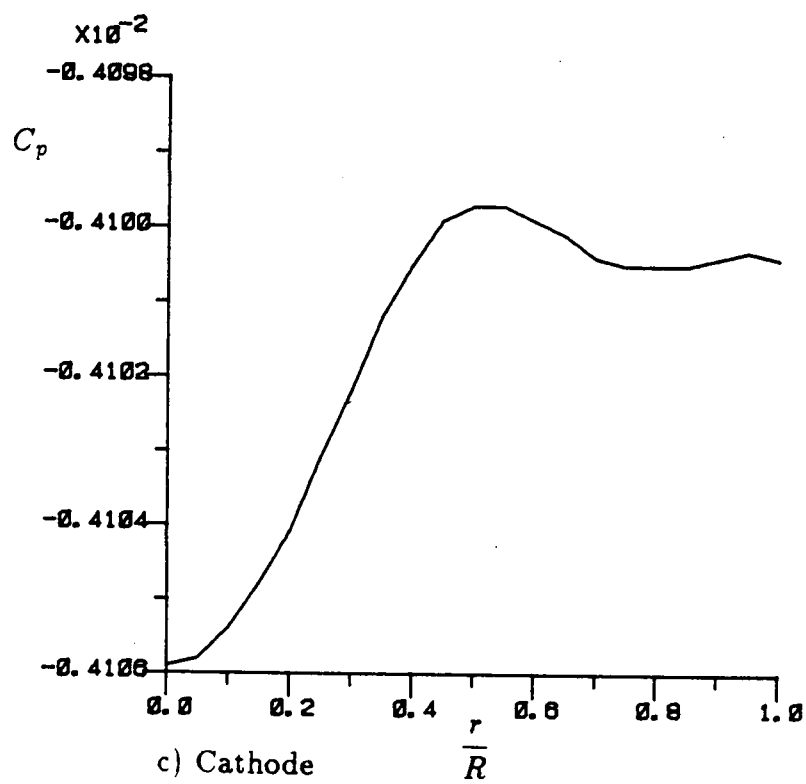


Figure 4.10. (continued)

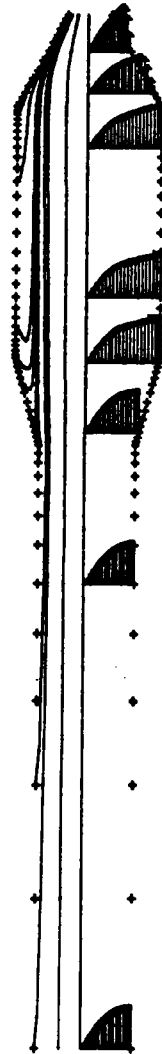
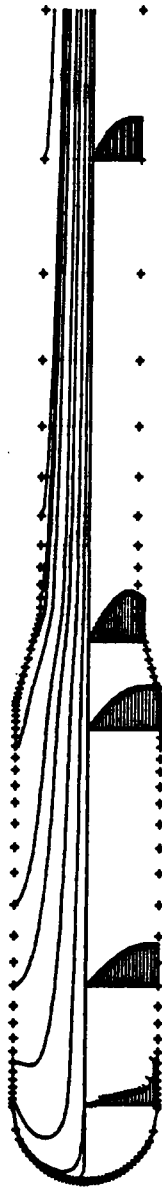


Figure 4.11. Streamlines and Tangential Velocity Profiles-Case C. $Re_{exit} \approx 1132$

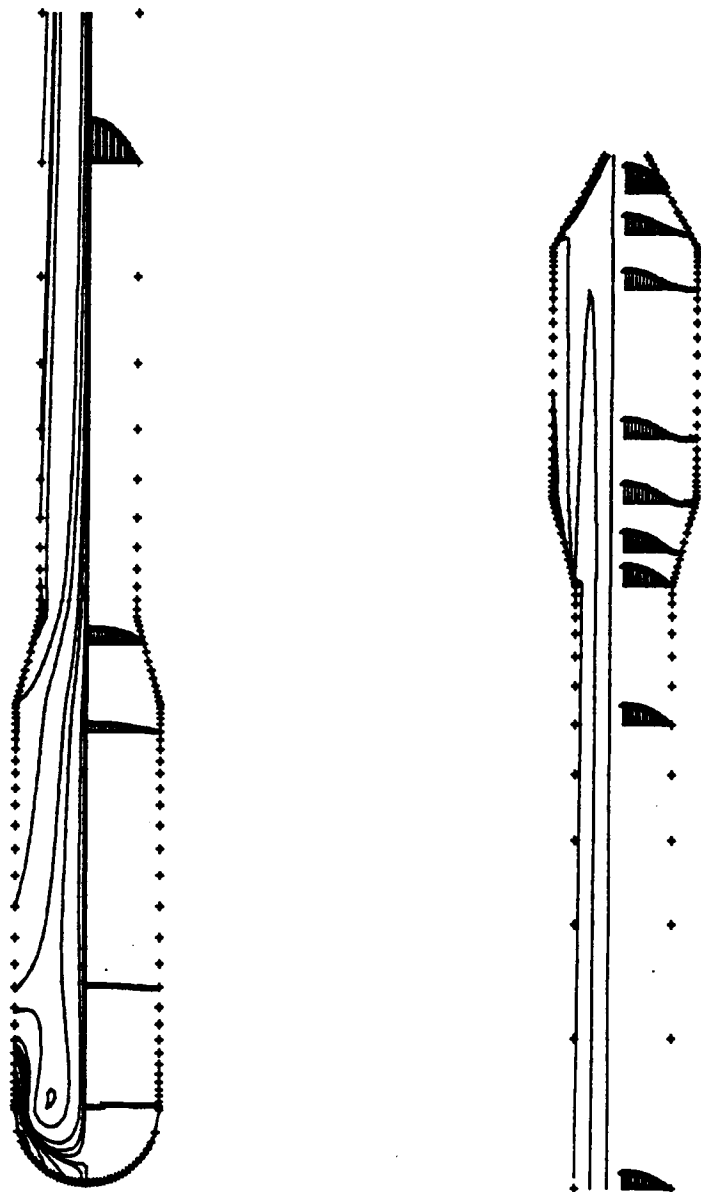


Figure 4.12. Vorticity Contours and Axial Velocity Profiles—Case C. $Re_{crit} = 1132$

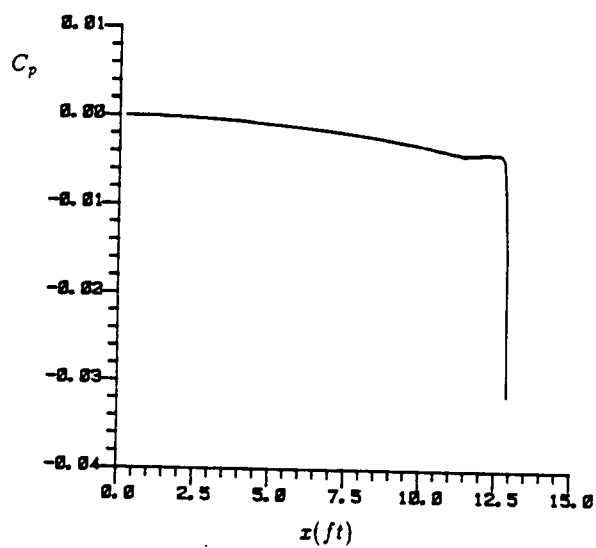


Figure 4.13. Pressure Coefficient along the Wall-Case C.

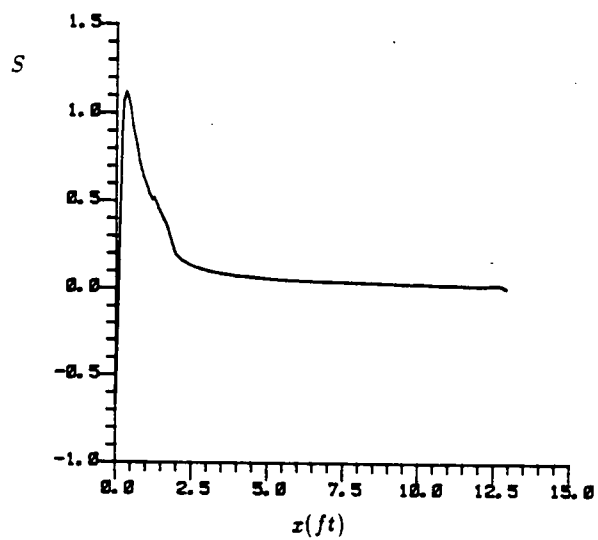


Figure 4.14. Swirl Number-Case C.

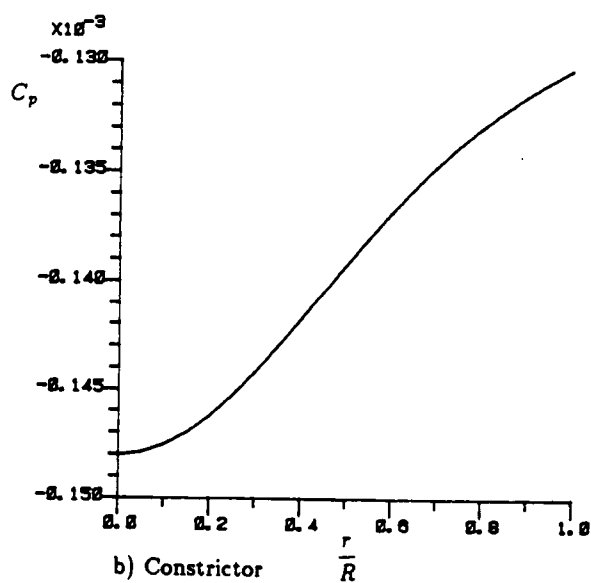
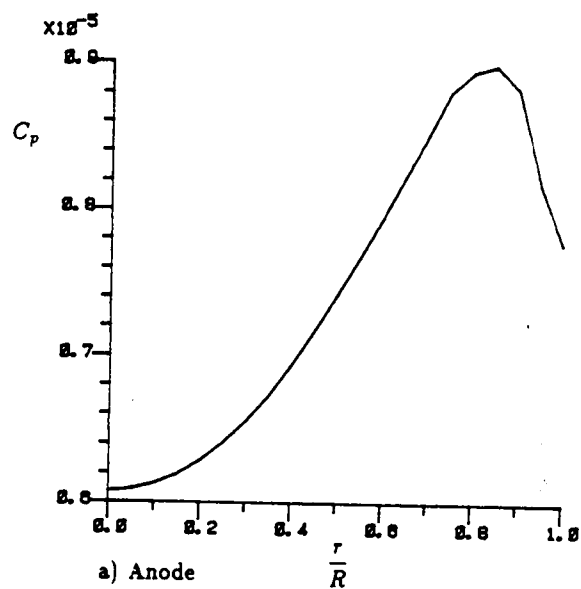


Figure 4.15. Pressure Coefficient at Various Cross Sections—Case C.

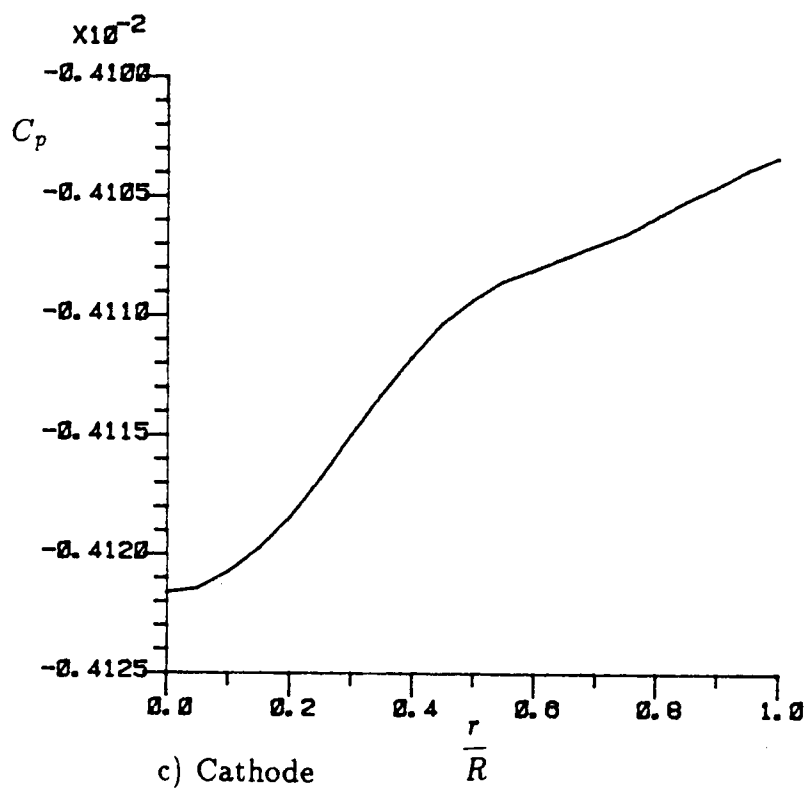


Figure 4.15. (continued)

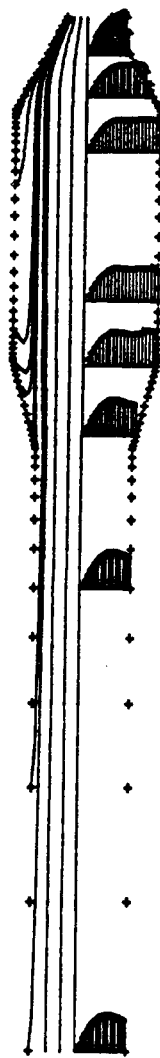
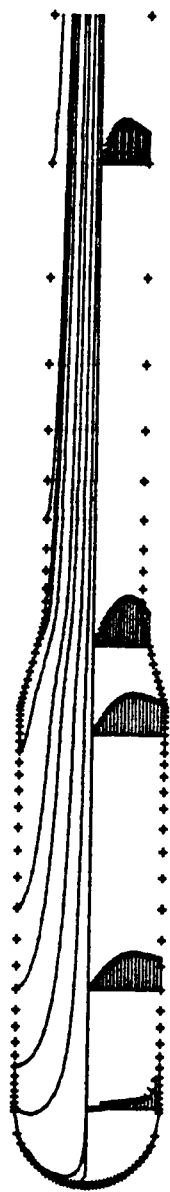


Figure 4.16. Streamlines and Tangential Velocity Profiles-Case D. $Re_{exit} = 2235$

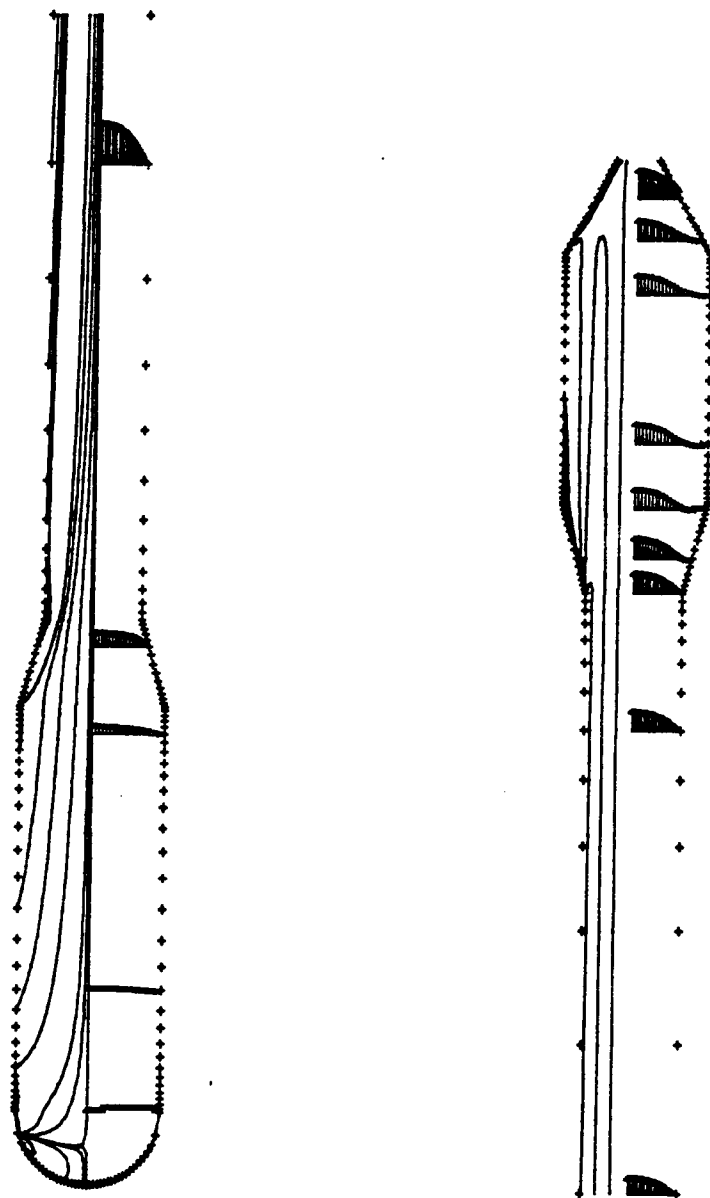


Figure 4.17. Vorticity Contours and Axial Velocity Profiles-Case D. $Re_{exit} = 2235$

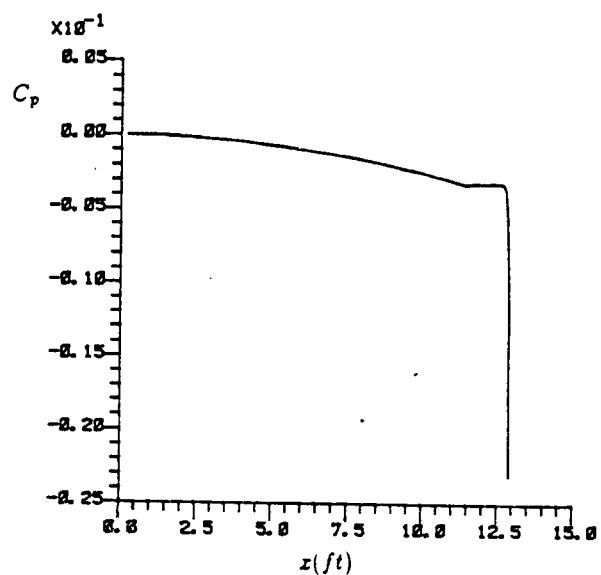


Figure 4.18. Pressure Coefficient along the Wall-Case D.

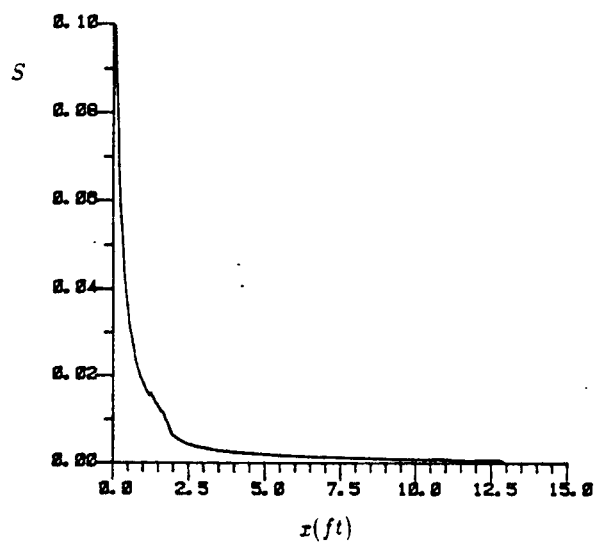


Figure 4.19. Swirl Number-Case D.

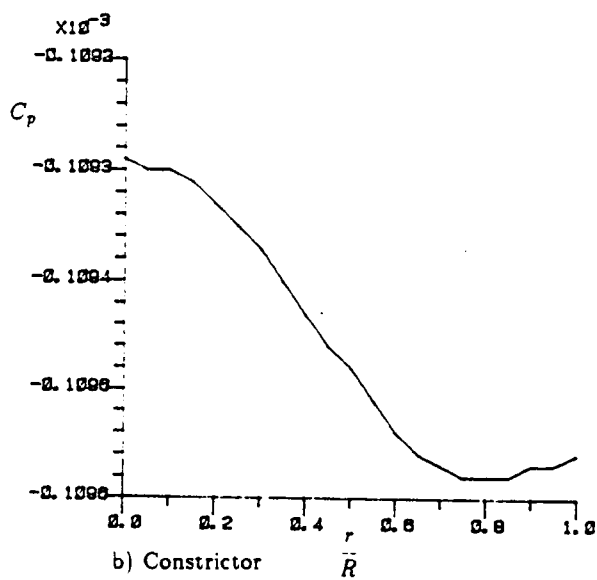
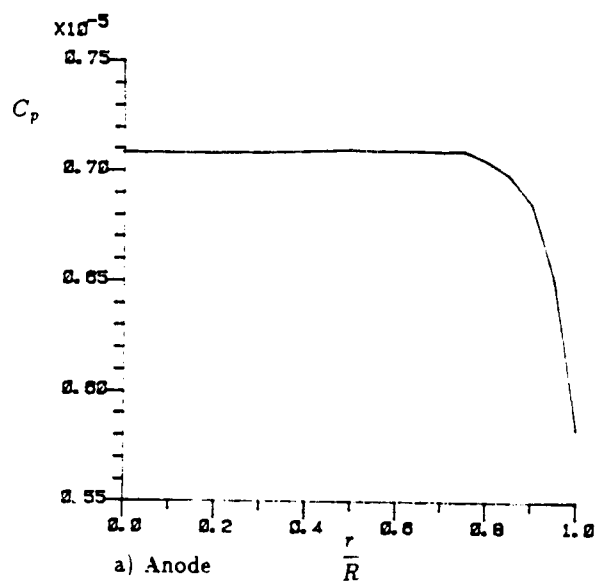


Figure 4.20. Pressure Coefficient at Various Cross Sections—Case D.

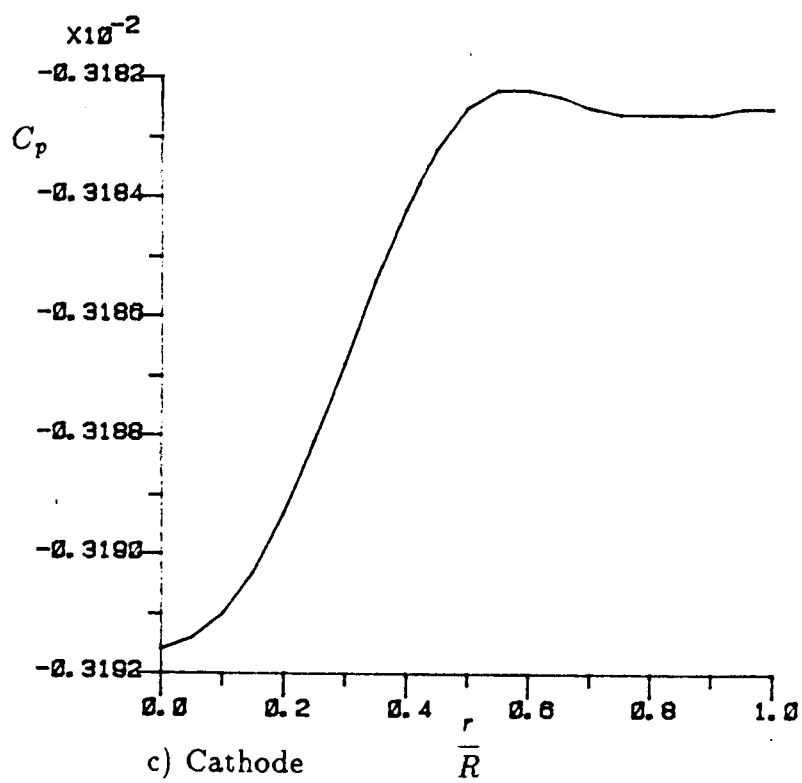


Figure 4.20. (continued)

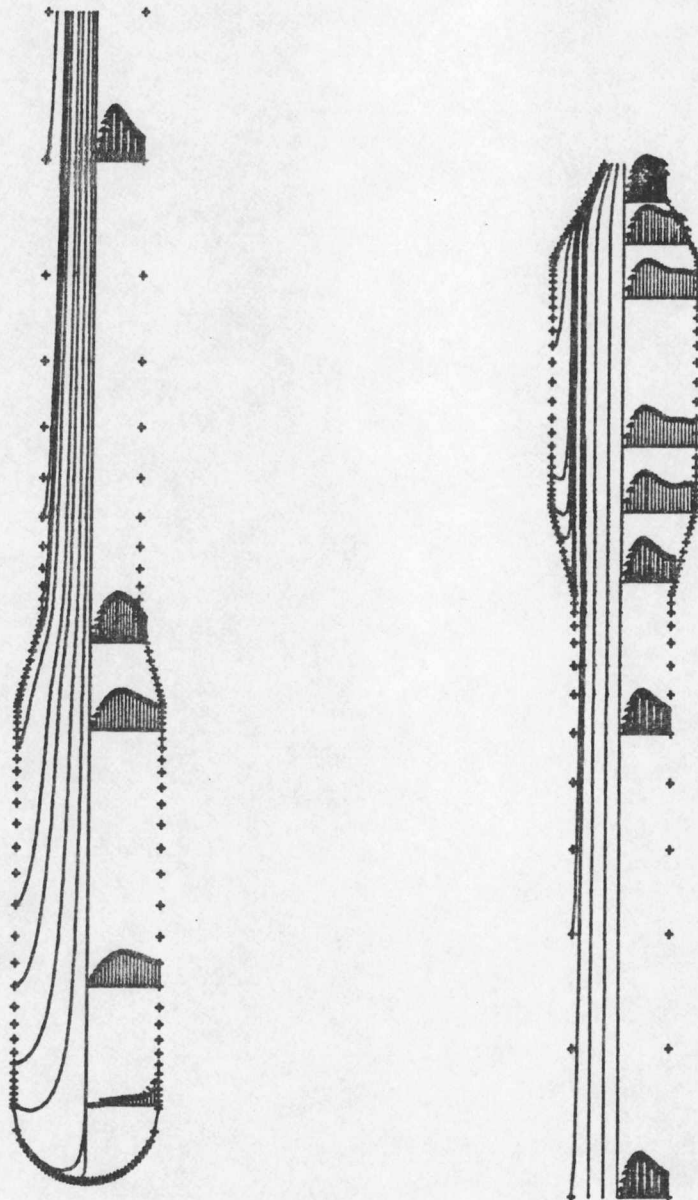


Figure 4.21. Streamlines and Tangential Velocity Profiles-Case F. $Re_{exit} = 4440$

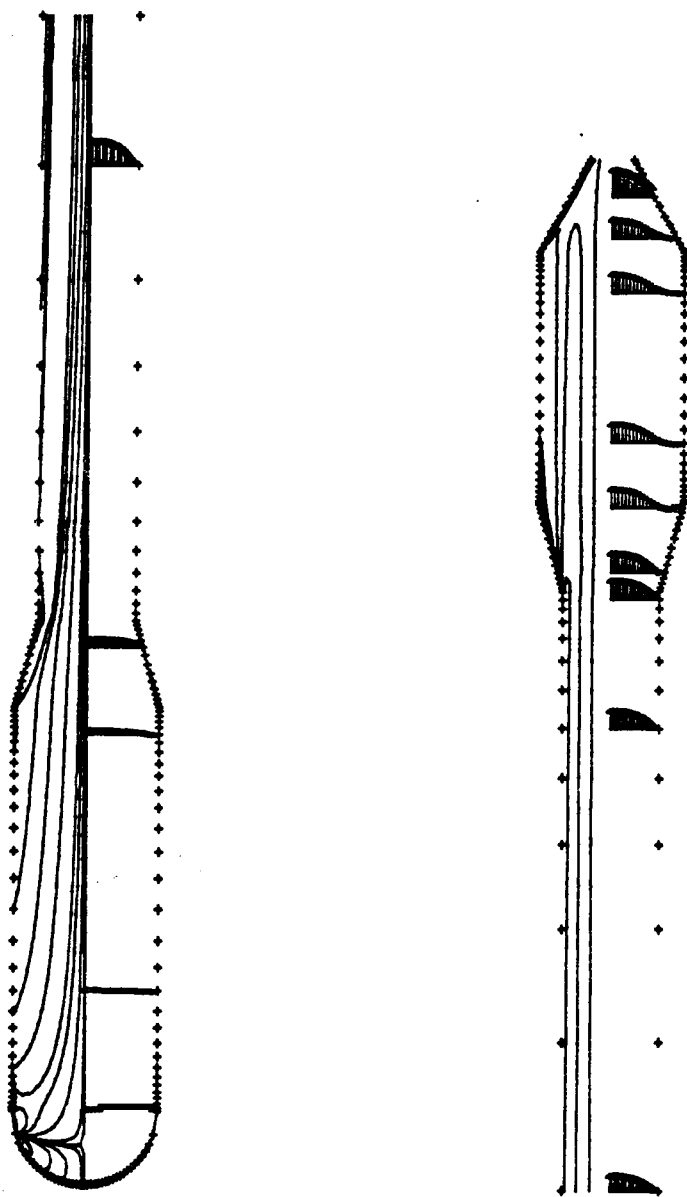


Figure 4.22. Vorticity Contours and Axial Velocity Profiles—Case F. $Re_{exit} = 4440$

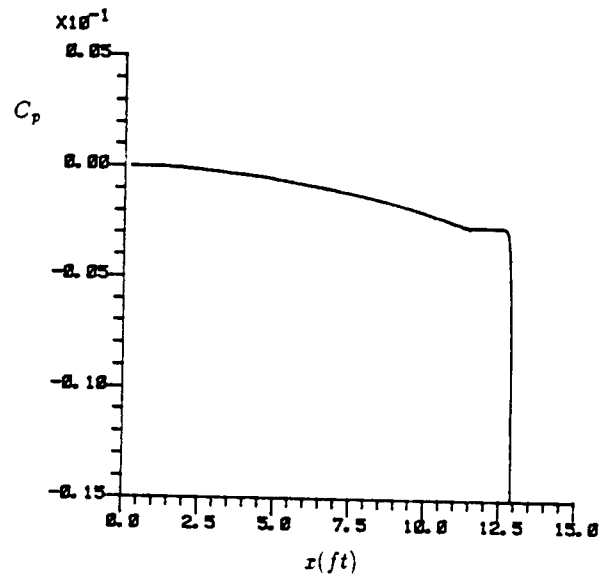


Figure 4.23. Pressure Coefficient along the Wall-Case F.

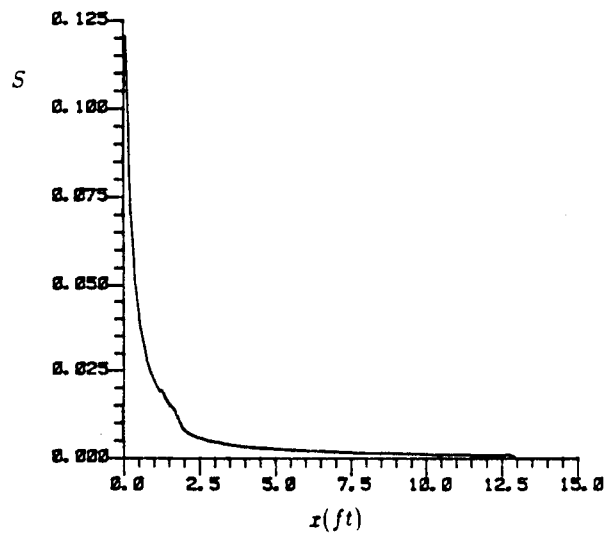


Figure 4.24. Swirl Number-Case F.

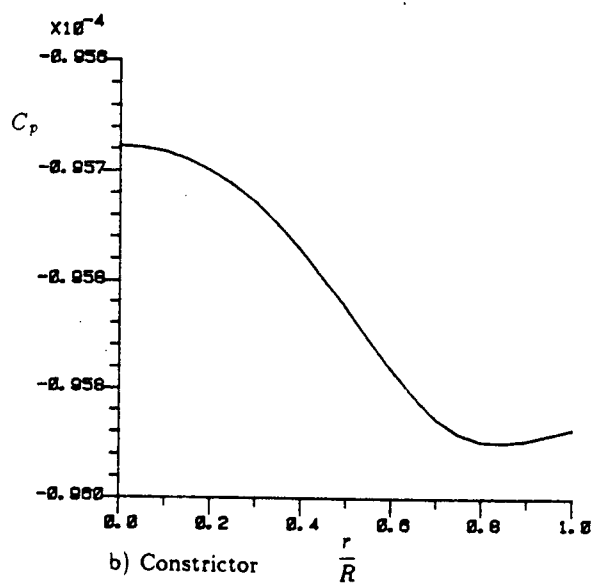
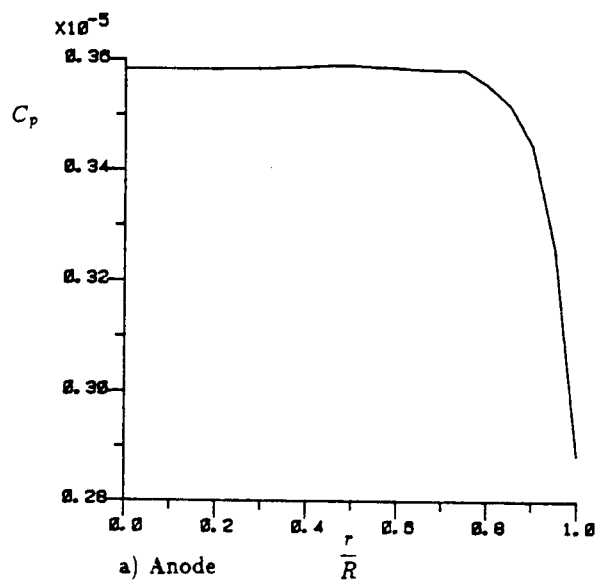


Figure 4.25. Pressure Coefficient at Various Cross Sections—Case F.

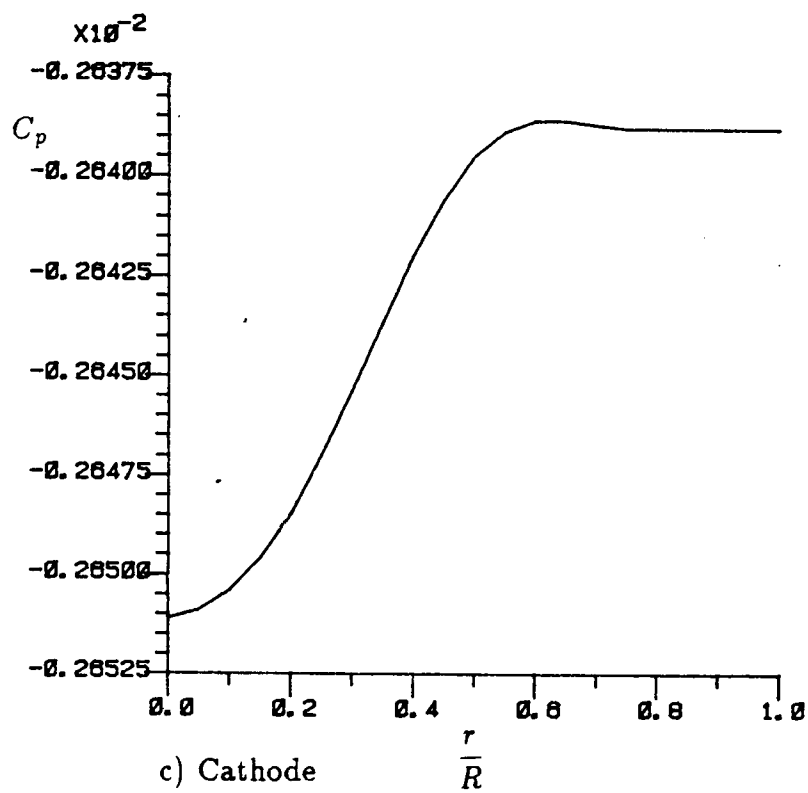


Figure 4.25. (continued)

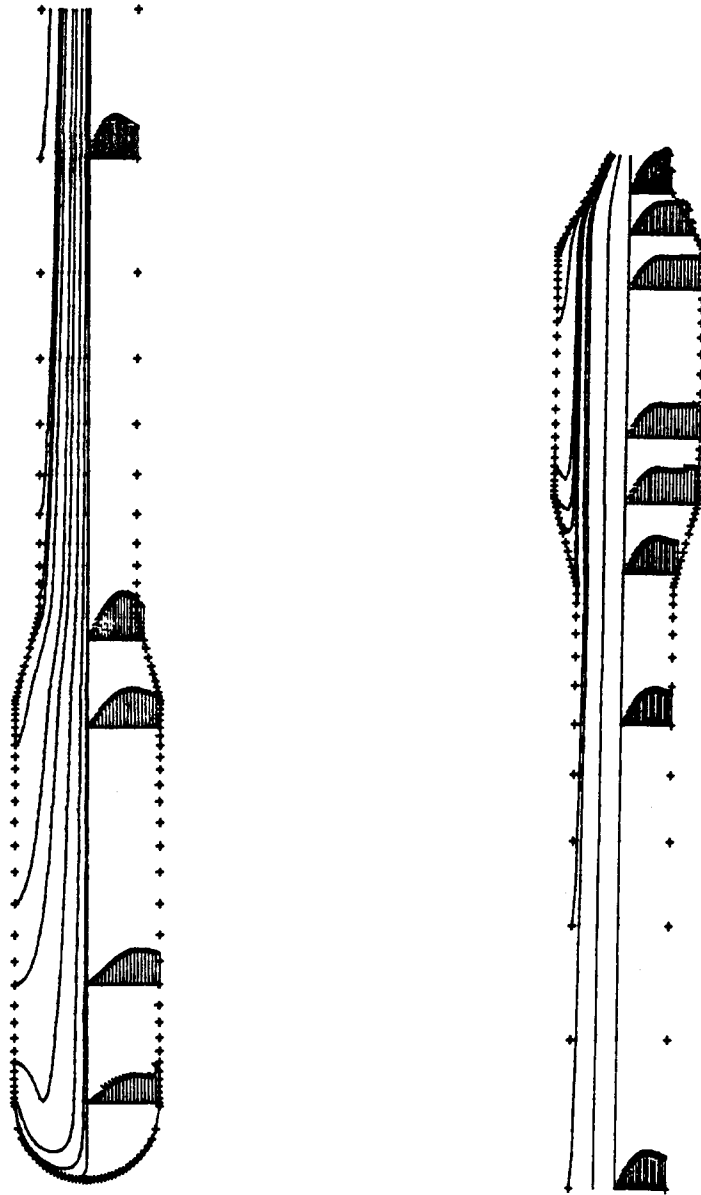


Figure 4.26. Streamlines and Tangential Velocity Profiles-Case E. $Re_{exit} = 2235$

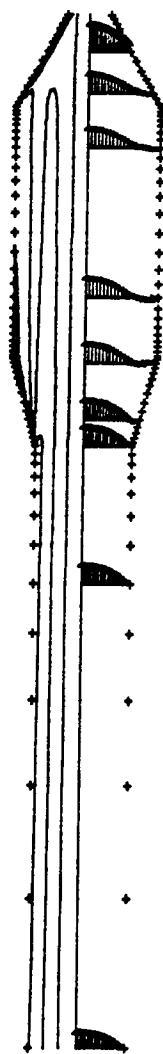
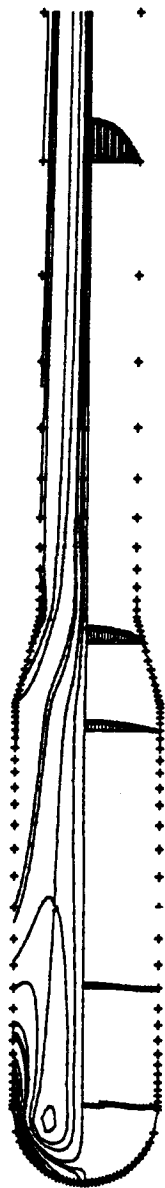


Figure 4.27. Vorticity Contours and Axial Velocity Profiles-Case E. $Re_{exit} = 2235$

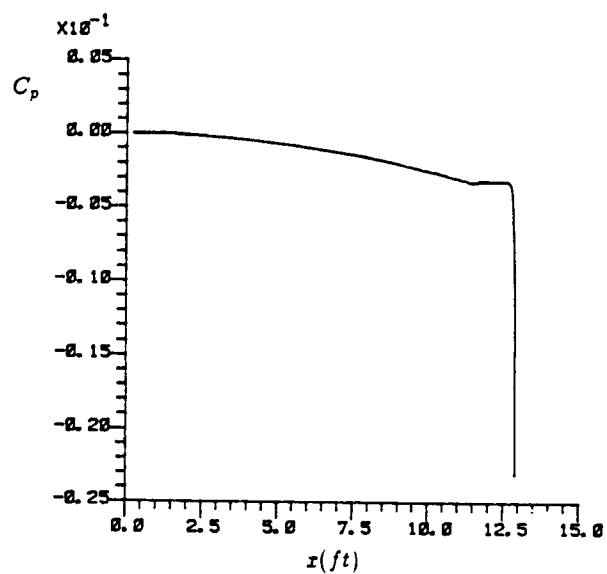


Figure 4.28. Pressure Coefficient along the Wall-Case E.

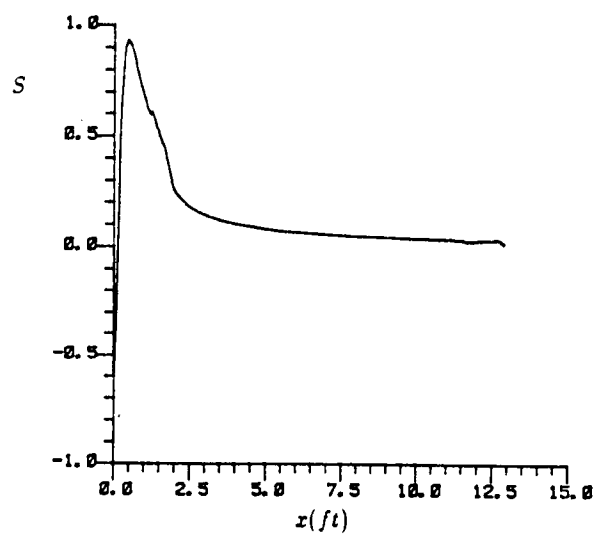


Figure 4.29. Swirl Number-Case E.

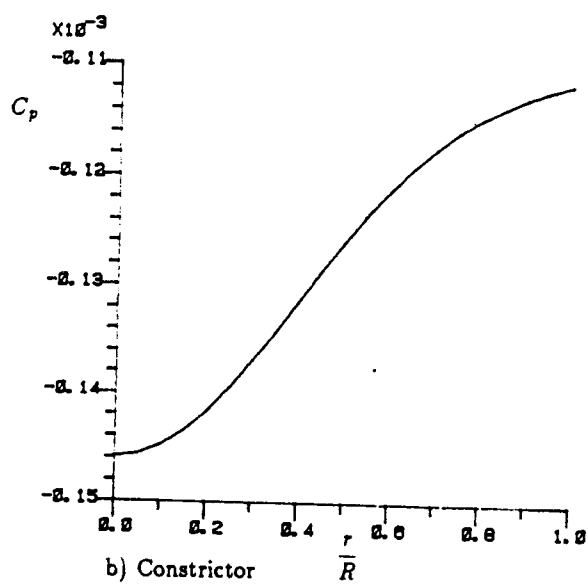
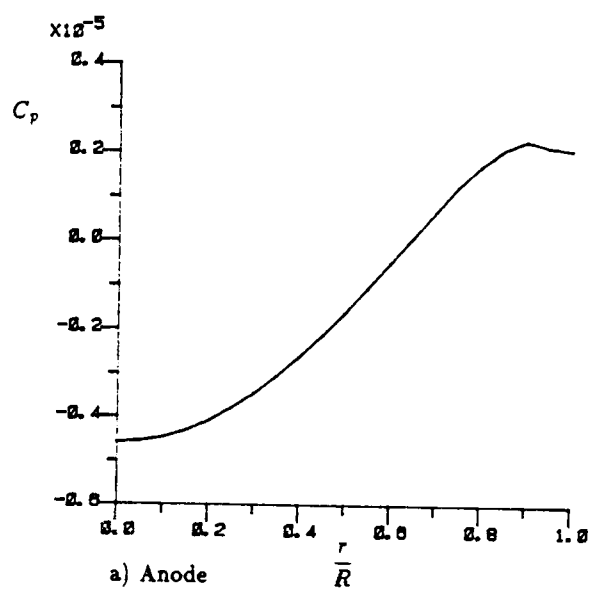


Figure 4.30. Pressure Coefficient at Various Cross Sections-Case E.

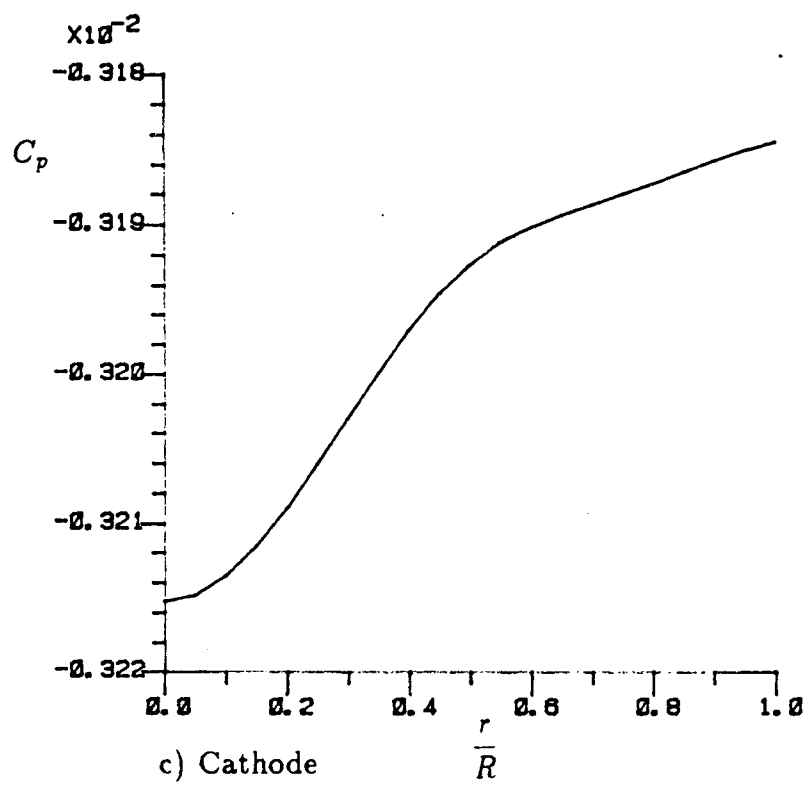


Figure 4.30. (continued)

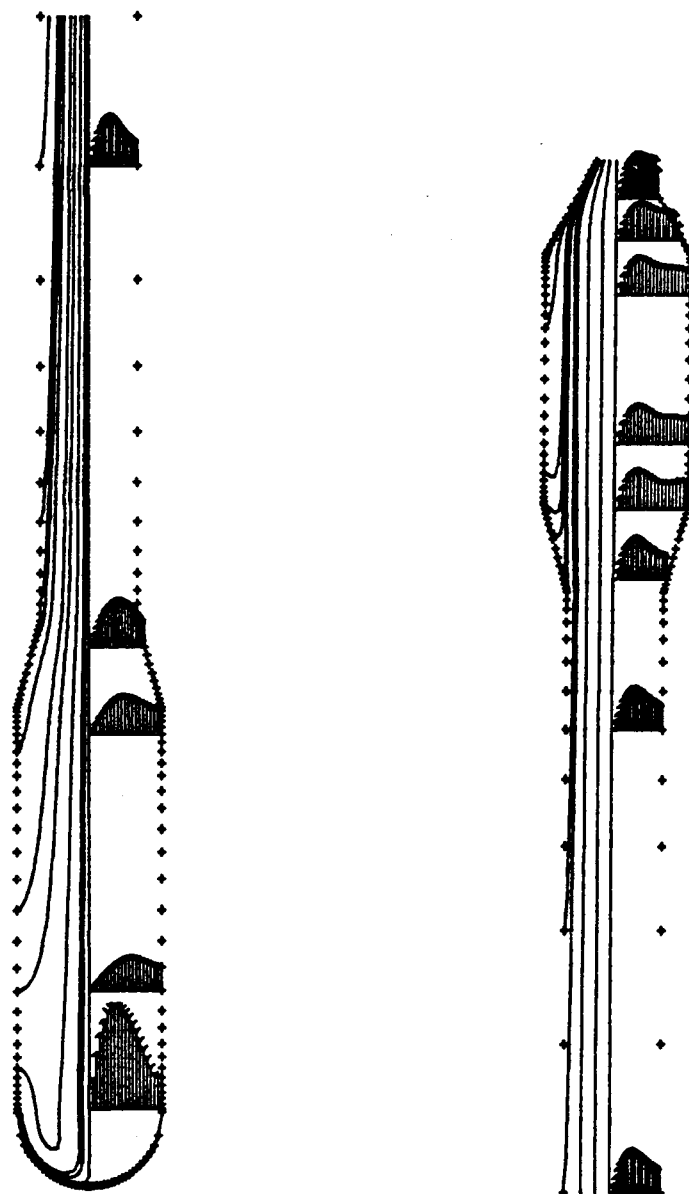


Figure 4.31. Streamlines and Tangential Velocity Profiles-Case G. $Re_{exit} = 4439$

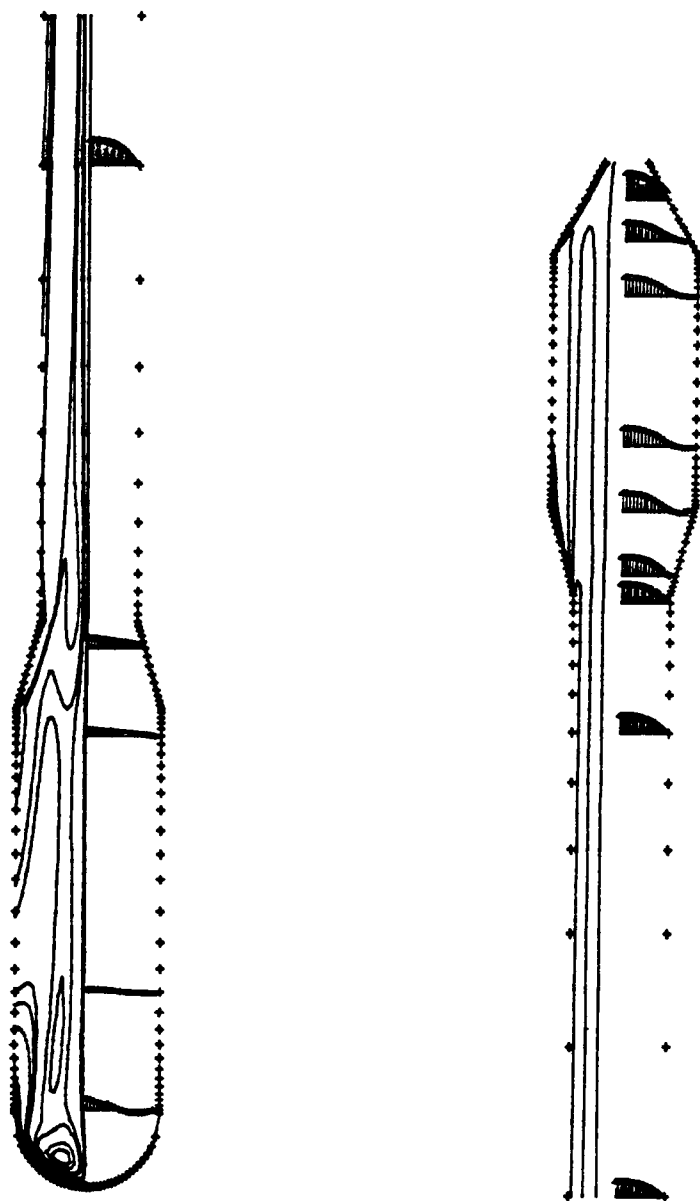


Figure 4.32. Vorticity Contours and Axial Velocity Profiles—Case G. $Re_{exit} = 4439$

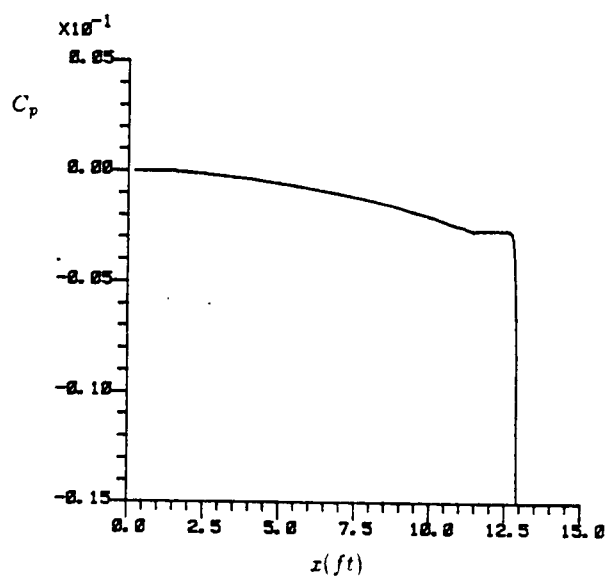


Figure 4.33. Pressure Coefficient along the Wall-Case G.

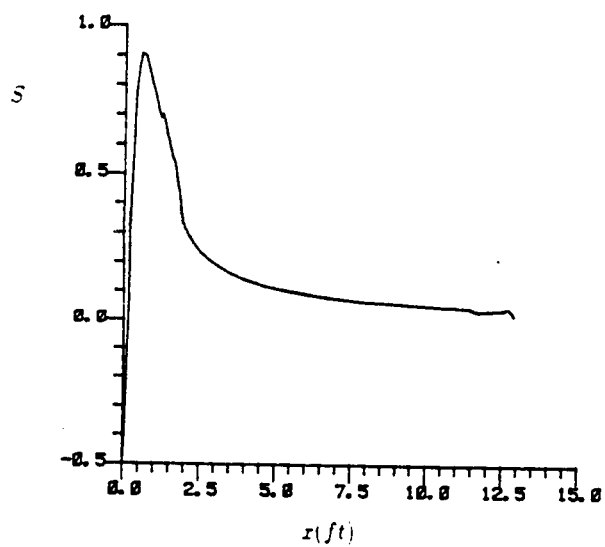


Figure 4.34. Swirl Number-Case G.

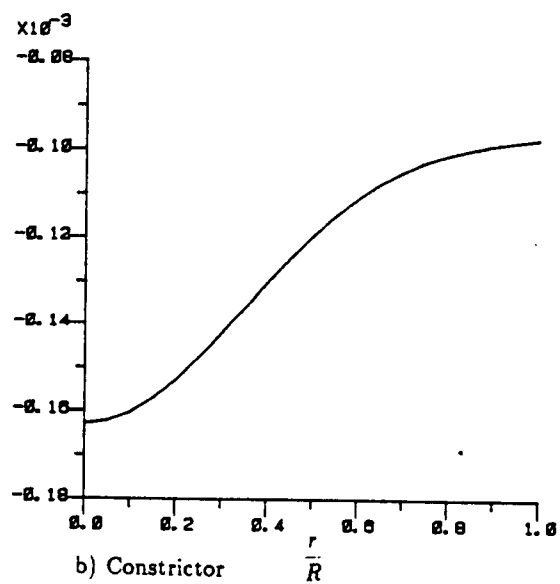
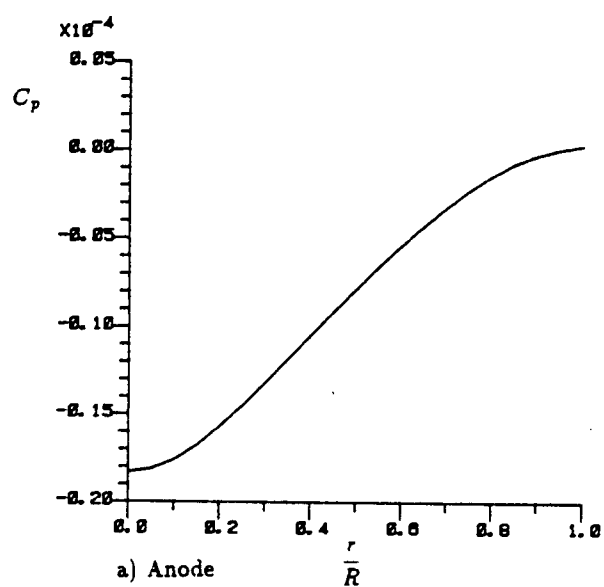


Figure 4.35. Pressure Coefficient at Various Cross Sections—Case G.

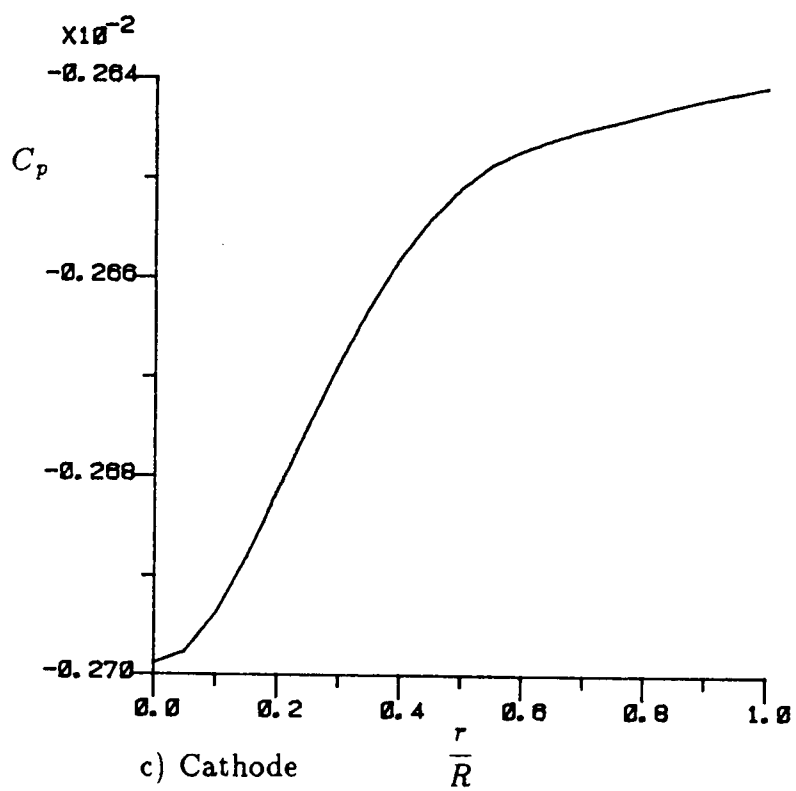


Figure 4.35. (continued)

Table 2.1 Finite Difference Method, used in Regions Identified in Figure 2.1.

- | | |
|---|--|
| A | : Forward difference in ξ and backward difference in η |
| B | : Central difference in ξ and backward difference in η |
| C | : Backward difference in ξ and backward difference in η |
| D | : Forward difference in ξ and central difference in η |
| E | : Central difference in ξ and central difference in η |
| F | : Backward difference in ξ and central difference in η |
| G | : Forward difference in ξ and forward difference in η |
| H | : Central difference in ξ and forward difference in η |
| I | : Backward difference in ξ and forward difference in η |

Table 3.1 Summary of Variables of the Arc Heater Model.

Cases	Q (lbm/sec)	β (Degrees)	v (ft/sec)	w (ft/sec)	Re_{exit}
A	0.1	75	-1.125×10^{-4}	3.013×10^{-5}	1132
B	0.1	45	-1.125×10^{-4}	1.125×10^{-4}	1132
C	0.1	5	-1.125×10^{-4}	1.285×10^{-3}	1132
D	0.2	75	-2.249×10^{-4}	6.026×10^{-5}	2235
E	0.2	5	-2.249×10^{-4}	2.571×10^{-3}	2235
F	0.4	75	-4.498×10^{-4}	1.205×10^{-4}	4440
G	0.4	5	-4.498×10^{-4}	5.141×10^{-3}	4439

VITA

Andrew S.T. Chiang was born in Kuching, Sarawak, Malaysia on July 28, 1961. He grew up and attended schools in Sarikei, Sarawak, Malaysia. He graduated from St' Anthony Secondary School in December 1978, and went to London, England to continue his A-level course in Willesden College. He graduated from Willesden College in 1980, and was admitted to Queen Mary college of London University majoring in Aeronautical Engineering. He studied there for one trimester and was transferred to Tamkang University in Taiwan majoring in Aerospace Engineering. After one year in Taiwan, he was transferred to Parks College of St' Louis University in the United States where he obtained his Bachelor Degree in Aerospace Engineering in June, 1985.

Since that time, he has been accepted as a Graduate Research Assistant at The University of Tennessee Space Institute, where he is studying for a Master of Science degree in Aerospace Engineering.