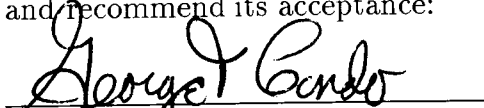


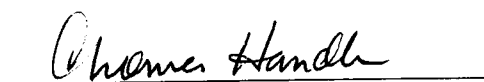
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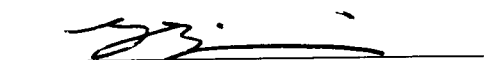
I am submitting herewithin a dissertation written by Chehyon Shio, entitled "YFS Monte Carlo Approach to Higher Order Radiative Corrections to LHC/SSC Physics Processes." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy with a major in Physics.


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YFS MONTE CARLO APPROACH TO HIGHER ORDER
RADIATIVE CORRECTIONS TO LHC/SSC PHYSICS
PROCESSES

A DISSERTATION
PRESENTED FOR THE
DOCTOR OF PHILOSOPHY
DEGREE
THE UNIVERSITY OF TENNESSEE, KNOXVILLE

BY
CHEHYON SHIO
MAY 1994

Dedication

This work is dedicated to my parents.

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Abstract

We extend the methods of Yennie, Frautschi and Suura (YFS) to compute, via Monte Carlo methods, the effects of multiple gluon emission in the processes $q + (\bar{q})' \rightarrow q'' + (\bar{q})''' + n(G)$, where G is a soft gluon. We show explicitly that the infrared singularities in the respective simulations are canceled to all orders in α_s . Some discussion of this result from the standpoint of confinement is given. More importantly, we present, for the first time ever, sample numerical Monte Carlo data on multiple soft gluon emission in the rigorously extended YFS framework. We find that such soft gluon effects must be taken into account for precise SSC/LHC physics simulations.

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Chapter 1

Introduction

While the SSC has been cancelled and is being re-evaluated, the LHC planning stages continue to gain momentum, so that it still becomes more and more necessary to prepare for the physics exploration which the LHC, at the least, will provide. The primary issue in this exploration is the comparison between predictions from the standard $SU_{2L} \otimes U_1 \otimes SU_3^C$ theory and its possible extensions, and what will be observed by the GEM and SDC collaborations, with the discovery of the Higgs particle or whatever it represents as of course a primary goal of such comparisons. Thus, it is important to know the theoretical predictions as accurately as is necessary to exploit the expected detector performances of GEM and SDC to the fullest extent. In particular, higher order radiative corrections to these predictions (due to multiple photon and multiple gluon effects) are in fact essential to obtain the proper precision on the signal and background processes in the respective SSC/LHC environment. In Refs. [1], we have developed Yennie–Frautschi–Suura [2] (YFS) multiphoton Monte Carlo event generators for calculating $n\gamma$ effects in SSC/LHC processes. In what follows, we now develop the extension of our own YFS Monte Carlo–based higher order radiative correction methods to multiple gluon effects in SSC/LHC processes. Such a calculation of $n(G)$ effects has not appeared elsewhere, where we use G to denote a gluon.

More precisely, we want to use the fact that, by the uncertainty principle,

infinite-wavelength gluons should not affect the motion of quarks and gluons inside a proton, whose radius is ~ 1 fm. Thus, we have recently realized [3] this physical requirement in perturbation theory by showing that in our prototypical processes $q + (\bar{q})' \rightarrow q'' + (\bar{q})''' + (G)$, the infrared singularities cancel at $O(\alpha_s)$, just as they cancel in the analogous QED process $f + f' \rightarrow f + f' + \gamma$. We then are able to define the QCD analogues of the famous YFS infrared functions [2] B and $\tilde{B}$ which describe the virtual and real infrared singularities in QED processes to all orders in α . These functions, B_{QCD} and $\tilde{B}_{\text{QCD}}$, then describe the infrared singularities in QCD in the processes $q + (\bar{q})' \rightarrow q'' + (\bar{q})''' + n(G)$ to all orders in α_s . The sum $B_{\text{QCD}} + \tilde{B}_{\text{QCD}}$ is infrared finite and allows us to extend our YFS Monte Carlo methods, widely used in SLC/LEP physics for higher order QED radiative effects, to the analogous Monte Carlo methods in QCD processes for SSC/LHC physics objectives. We call the resulting programs SSCYFSG and SSCBHLG, for they are the QCD extensions of the QED $n\gamma$ Monte Carlos SSCYFS2 and SSCBHL. Here we recall that SSCYFS2 was already described in Ref. [1] and SSCBHL is the analogous SSC extension of the YFS Monte Carlo BHLUMI 2.01 [4] for the SLC/LEP luminosity process $e^+e^- \rightarrow e^+e^- + n\gamma$.

In what follows, we shall present sample Monte Carlo data for the process $q + (\bar{q})' \rightarrow q'' + (\bar{q})''' + n(G)$ from the event generator SSCBHLG. Analogous results from SSCYFSG will appear elsewhere [5]. The latter Monte Carlo only simulates initial-state gluon radiation, whereas SSCBHLG simulates initial-state and final-state n -gluon radiation, as well as the respective initial-final-state interference effects. Further, we emphasize that (unlike BHLUMI 2.01) SSCBHLG is not restricted to small scattering angles. Note finally that SSCYFSG is an initial-state restriction of SSCBHLG and hence is useful for cross-checking our work.

This work is organized as follows. In the next Chapter, we review the YFS theory [2] and the Renormalization-Group-Improved YFS theory [8]. In Chapter 3, we renew the YFS exponentiation to n -gluon emission and the attendant extension of SSCYFS2 and SSCBHL. In Chapter 4, we present sample Monte Carlo data for

n -gluon emission for the SDC and GEM acceptances. Chapter 5 contains some concluding remarks.

Chapter 2

Review of Theory

2.1 YFS Theory

The cancellation of infrared divergences to all orders in QED, was obtained by Yennie, Frautschi and Suura, by factorizing the IR divergent contributions of virtual and real parts of matrix elements. Here, we follow the development in Ref. 2 in the Bibliography.

2.1.1 Virtual Photon Radiative Corrections

Consider a process where n virtual photons are attached to the initial or final electron leg, which contributes to the infrared divergence when they are on the mass shell. The matrix element is expanded

$$M(p, p') = \sum_{n=0}^{\infty} M_n(p, p') \quad (1)$$

The M_n has the structure.

$$\begin{aligned} M_0 &= m_0 \\ M_1 &= m_0 \alpha B + m_1 \\ M_2 &= m_0 \frac{(\alpha B)^2}{2} + m_1 \alpha B + m_2 \end{aligned}$$

$$M_n = \sum_{r=0}^n m_{n-r} \frac{(\alpha B)^r}{r!} \quad (2)$$

where m_j is an infrared finite function of order α^j relative to M_0 . Then each factor αB , which contains the IR contribution from one virtual photon, is factorized by

$$M = \exp(\alpha B) \sum_{n=0}^{\infty} m_n \quad (3)$$

In order to prove (2) we begin with the definition

$$M_n = \frac{1}{n!} \int \cdots \int \prod_{i=1}^n \frac{d^4 k_i}{k_i^2 - \lambda^2} \rho_n(k_1 \cdots k_n) \quad (4)$$

where λ is a photon mass for infrared cutoff. The $n!$ is introduced into (4) by symmetrization of the n virtual photons in ρ_n . This symmetrization will play an essential role in obtaining the symmetric form (2).

In the Feynman amplitude, using the anticommutation relation of Dirac γ -matrices, ρ_n is separated to an infrared finite piece and remaining one by exchanging the ordering of k_n and p , and by working on infrared limit $k_n \rightarrow 0$ for remaining matrix element.

$$\rho_n(k_1 \cdots k_n) = S(k_n) \rho_{n-1}(k_1 \cdots k_{n-1}) + \beta_n^{(1)}(k_1 \cdots k_{n-1}; k_n) \quad (5)$$

Therefore $S(k_n)$ contains the k_n infrared contribution and the remaining part β has no infrared divergence in k_n . We may extract next infrared finite piece by iteration of (5).

$$- S(k_{n-1}) \beta_{n-1}^{(1)}(k_n \cdots k_{n-2}; k_n) + \beta_n^{(1)}(k_1 \cdots k_{n-1}; k_n) = \beta_n^{(2)}(k_1 \cdots k_{n-2}; k_{n-1}, k_n) \quad (6)$$

The symmetry of ρ_n in k_{n-1} and k_n implies invariance of this quantity under interchange of k_{n-1} and k_n . Eq. (6) is noninfrared in k_n for all values of the other k_i . That means $\beta_n^{(2)}$ is infrared finite to both k_n and k_{n-1} virtual photons. By repeating it to all n photons, we have

$$\rho_n(k_1 \cdots k_n) = \sum_{perm} \sum_{r=0}^n \frac{1}{r!(n-r)!} \prod_{i=1}^r S(k_i) \beta_{n-r}(k_{r+1} \cdots k_n) \quad (7)$$

The β_{n-r} are noninfrared. Insertion of (7) into (4) yields

$$M_n = \sum_{r=0}^n \frac{1}{r!(n-r)!} \left(\int \frac{d^4 k S(k)}{k^2 - \lambda^2} \right)^r \int \prod_{i=1}^{n-r} \frac{d^4 k_i}{k_i^2} \beta_{n-r}(k_1 \cdots k_{n-r}) \quad (8)$$

Therefore (8) becomes the conjecture (2) with the following identification

$$\alpha B[p, p'(\epsilon)] \equiv \int \frac{d^4 k S(k)}{k^2 - \lambda^2} \quad (9)$$

and

$$m_r[p, p'(\epsilon)] \equiv \frac{1}{r!} \int \prod_{i=1}^r \frac{d^4 k_i}{k_i^2} \beta_r(k_1 \cdots k_r) \quad (10)$$

The famous infrared function B can be represented, for example, by the gauge invariant expression for the process which has two external legs in the same electron line.

$$B = \frac{i}{(2\pi)^3} \int \frac{d^4 k}{k^2 - \lambda^2} \left(\frac{2p'_\mu - k_\mu}{2p' \cdot k - k^2} - \frac{2p_\mu - k_\mu}{2p \cdot k - k^2} \right)^2 \quad (11)$$

2.1.2 Real Photon Radiative Corrections

The differential cross section for emission of n undetected real photons with total energy ϵ has the form

$$\frac{d\sigma_n}{d\epsilon} = \exp(2\alpha B) \frac{1}{n!} \int \prod_{m=1}^n \frac{d^3 k_m}{(k_m^2 + \lambda^2)^{1/2}} \delta(\epsilon - \sum_{i=1}^n k_i) \tilde{\rho}_n(p, p', k_1, \cdots k_n) \quad (12)$$

with the infrared factor $\exp(2\alpha B)$ from virtual part. The complete differential cross section is given by

$$\frac{d\sigma}{d\epsilon} = \lim_{\lambda \rightarrow 0} \sum_{n=0}^{\infty} \frac{d\sigma_n}{d\epsilon} \quad (13)$$

The quantity $\tilde{\rho}_n$ has the similar structure in the virtual case, ρ_n ,

$$\tilde{\rho}_n(k_1 \cdots k_n) = \tilde{S}(k_1) \cdots \tilde{S}(k_n) \tilde{\beta}_0 + \sum_{i=1}^n \tilde{S}(k_1) \cdots \tilde{S}(k_{i-1}) \tilde{S}(k_{i+1}) \cdots \tilde{\beta}_1(k_i)$$

$$+ \cdots + \sum_{i=1}^n \tilde{S}(k_i) \tilde{\beta}_{n-1}(k_1 \cdots k_{i-1} k_{i+1} \cdots k_n) + \tilde{\beta}_n(k_1 \cdots k_n) \quad (14)$$

In this real case, photons must terminate exclusively on external charge lines in order to contribute infrared terms. That means the δ function ensures that $\tilde{\rho}_n$ contribute to the cross section only when the energy conservation $E' = E - \epsilon$ holds. Thus $\tilde{\beta}(k_m)$ is defined on the energy shell $E' = E - \sum k_m$, and $\tilde{\beta}_0$ only at $E' = E$.

The energy-conserving δ function is conveniently represented by

$$\delta(\epsilon - \sum_{m=1}^n k_m) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dy \exp \left[iy(\epsilon - \sum_{m=1}^n k_m) \right] \quad (15)$$

Substitution of (12), (14), and (15) into (13) yields

$$\begin{aligned} \frac{d\sigma}{d\epsilon} = \lim_{\lambda \rightarrow 0} \exp(2\alpha B) \frac{1}{2\pi} \int_{-\infty}^{\infty} dy e^{iy\epsilon} \exp \left[\int^{k \leq \epsilon} \frac{d^3 k}{(k^2 + \lambda^2)^{1/2}} \tilde{S}(k, p, p'(\epsilon)) e^{-iyk} \right] \\ \left(\tilde{\beta}_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{m=1}^n \frac{d^3 k_m}{k_m} e^{-iyk_m} \tilde{\beta}_n(p, p', k_1 \cdots k_n) \right) \end{aligned} \quad (16)$$

The real infrared factor in the third exponential of (16), which is still kinematically connected with the other real photons by the factor e^{-iyk} which insures that $\sum k = \epsilon$, can be separated by

$$\int^{k \leq \epsilon} \frac{d^3 k}{(k^2 + \lambda^2)^{1/2}} \tilde{S} e^{-iyk} = 2\alpha \tilde{B} + D \quad (17)$$

where

$$2\alpha \tilde{B}(p, p'(\epsilon)) \equiv \int^{k \leq \epsilon} \frac{d^3 k}{(k^2 + \lambda^2)^{1/2}} \tilde{S} \quad (18)$$

and

$$D(p, p'(\epsilon), y) \equiv \int^{k \leq \epsilon} \frac{d^3 k}{k} \tilde{S} (e^{-iyk} - 1) \quad (19)$$

The term $2\alpha \tilde{B}$ is independent of y , while the integrand of D is well behaved as $k \rightarrow 0$. The $\exp(2\alpha \tilde{B})$ violates the condition $\sum k = \epsilon$, but $\exp(D)$ just restore this condition and thereby maintains energy conservation.

Therefore we show the famous YFS cross section, which holds in the infrared limit $k \rightarrow 0$,

$$\frac{d\sigma}{d\epsilon} = e^{2\alpha(B+\tilde{B})} \frac{d\hat{\sigma}}{d\epsilon} \quad (20)$$

with the infrared finite part in the same limit

$$\frac{d\hat{\sigma}}{d\epsilon} \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} dy e^{iy\epsilon+D} \left(\tilde{\beta}_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{m=1}^n \frac{d^3 k_m}{k_m} e^{-iyk_m} \tilde{\beta}_n \right) \quad (21)$$

As in the case of virtual infrared function B , the real infrared function, $\tilde{B}$, can be represented by the gauge invariant expression

$$\tilde{B} = \frac{-1}{8\pi^2} \int_0^\epsilon \frac{d^3 k}{(k^2 + \lambda^2)^{1/2}} \left(\frac{p'_\mu}{k \cdot p'} - \frac{p_\mu}{k \cdot p} \right)^2 \quad (22)$$

B and $\tilde{B}$ represent known infrared contributions from the lowest order radiative correction. Thus the cancellation of infrared terms to all orders is assured by the cancellation in lowest order since $d\hat{\sigma}/d\epsilon$ is noninfrared as $k \rightarrow 0$.

The result can be generalized to the case of energy-momentum conservation, which is related to angle dependence of detector cutoff $K_m = K_m(\Omega)$ by using

$$\delta_4(K - \sum k_i) = \frac{1}{(2\pi)^4} \int \cdots \int d^4 x e^{ix \cdot (K - \sum k_i)} \quad (23)$$

Then the cross section becomes

$$\sigma = \lim_{\lambda \rightarrow 0} \exp(2\alpha B) \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} d^4 x e^{ix \cdot K} \exp \left[\int \frac{d^3 k}{(k^2 + \lambda^2)^{1/2}} \tilde{S}(q, k) e^{-ix \cdot k} \right] \left(\tilde{\beta}_0(q) + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{m=1}^n \frac{d^3 k_m}{k_m} e^{-ix \cdot k_m} \tilde{\beta}_n(q, k_1 \cdots k_n) \right) \quad (24)$$

where q stands for the set of all momenta of the observed particles and photons. With $K_m(\Omega) \gg \lambda$ in all directions we define D by

$$D(q, x, [K_m]) \equiv \int \frac{d^3 k}{k} \tilde{S} \left(e^{-ix \cdot k} - \theta[K_m(\Omega) - k] \right) \quad (25)$$

Then we have the desired form

$$\sigma = \exp[2\alpha(\text{Re } B + \tilde{B})] \frac{1}{(2\pi)^4} \int d^4x e^{iK \cdot x + D} \left(\tilde{\beta}_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{m=1}^n \frac{d^3k_m}{k_m} e^{-ik_m \cdot x} \tilde{\beta}_n \right) \quad (26)$$

with the definitions

$$\begin{aligned} \tilde{B} &= \frac{1}{8\pi^2} \int_0^{k < K_m} \frac{d^3k}{(k^2 + \lambda^2)^{1/2}} \sum_{i < j} Z_i \theta_i Z_j \theta_j \left(\frac{p_{i\mu}}{k \cdot p_i} - \frac{p_{j\mu}}{k \cdot p_j} \right)^2 \\ &= \sum_{i < j} \tilde{B}_{ij} \end{aligned} \quad (27)$$

and

$$\begin{aligned} B &= \frac{-i}{8\pi^3} \int \frac{d^4k}{k^2 - \lambda^2} \sum_{i < j} Z_i \theta_i Z_j \theta_j \left(\frac{(2p_i \theta_i - k)_\mu}{k^2 - 2k \cdot p_i \theta_i} + \frac{(2p_j \theta_j + k)_\mu}{k^2 + 2k \cdot p_j \theta_j} \right)^2 \\ &= \sum_{i < j} B_{i < j} \end{aligned} \quad (28)$$

where Z_i has the sign of the i th charge and $\theta_i = +(-)$ if i is outgoing (incoming). The contribution to B from the photon propagator pole, where the virtual photon lies on the mass shell, cancels the infrared part of $\tilde{B}$. If particles i and j in the initial or final states are identical, B and $\tilde{B}$ will clearly be symmetric in the variables of these particles. Thus infrared factors do not change the overall symmetry of the matrix elements they multiply.

2.2 Renormalization-Group-Improved YFS Theory

In order to address the physical consequences of the large UV effects in (24), we will appeal to the renormalization-group equation of Weinberg [15]. We follow the development in Ref. 8 in the Bibliography.

The basic idea of Weinberg is that multiplicatively renormalized Green's functions Γ of a theory may be subtracted with the massless limits of the subtraction

constants for the theory at a Euclidean scale μ . The fact that the unrenormalized theory is independent of μ then implies the equation

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_R) \frac{\partial}{\partial g_R} - \gamma_\Theta(g_R) m_R \frac{\partial}{\partial m_R} - \gamma_\Gamma(g_R) \right) \Gamma = 0 \quad (29)$$

where, for simplicity, we imagine we have one renormalized coupling g_R and one renormalized mass m_R . In the $SU(2)_L \times U(1)$ theory, we would have two couplings, e_R and g_{WR} , where g_{WR} is the $SU(2)_L$ coupling and e_R is the electric charge of the positron, we would have renormalized mass parameters for the fermions in the presumed three families of quarks and leptons, we would have the mass parameter of the W^+ , W^- , and Z_0 bosons, and the mass parameter of the physical Higgs particle (or the quadratic coupling of the physical Higgs particle), as a minimal set of masses and couplings. The physics beyond the standard model would enlarge this set. The coefficient functions β , γ_Θ , and γ_Γ are computable in renormalized perturbation theory. Specializing the (29) to QED [note that (29) tacitly presumes the gauge of Landau], we can write ($Q_i e_R$ is the electric charge of fermion i)

$$\begin{aligned} \beta(e_R) &= \frac{1}{12\pi^2} \sum_i Q_i^2 e_R^3 + \dots, \\ \gamma_{\Theta_i} &= \frac{3}{8\pi^2} Q_i^2 e_R^2 + \dots, \\ \gamma_\Gamma &= n_\gamma \beta(e_R) / e_R, \end{aligned} \quad (30)$$

where n_γ is the number of external photon lines in Γ . We note that, if we write the respective amplitudes $\mathcal{M}^{(n)}$ as (we suppress the amputated photon labels)

$$\mathcal{M}^{(n)} = (\bar{u}_f)_{\alpha_1} (v_{\bar{f}})_{\alpha_2} (u_e)_{\nu_1} (\bar{v}_{\bar{e}})_{\nu_2} \mathcal{M}_{\alpha_1 \alpha_2}^{(n) \nu_2 \nu_1} \quad (31)$$

then $\mathcal{M}_{\alpha_1 \alpha_2}^{(n) \nu_2 \nu_1}$ is an amputated connected *multiplicatively renormalized* (on-shell) Green's function. Thus, it satisfies the appropriate version of (29):

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_R) \frac{\partial}{\partial g_R} - \sum_i \gamma_{\Theta_i}(g_R) m_{iR} \frac{\partial}{\partial m_{iR}} - \gamma_{\mathcal{M}^{(n)}}(g_R) \right) \mathcal{M}_{\alpha_1 \alpha_2}^{(n) \nu_2 \nu_1} = 0 \quad (32)$$

Further, it is convenient to write

$$\begin{aligned} p_e &\equiv (\lambda\sqrt{s_0}/2, (\lambda^2 s_0/4 - m_e^2)^{\frac{1}{2}} \hat{z}) \\ p_{\bar{e}} &\equiv (\lambda\sqrt{s_0}/2, -(\lambda^2 s_0/4 - m_e^2)^{\frac{1}{2}} \hat{z}) \end{aligned} \quad (33)$$

and, in $\mathcal{M}^{(n)}$,

$$\begin{aligned} p_f^0 + p_{\bar{f}}^0 &= \lambda[\sqrt{s_0} - \sum_{i=1}^n k_{0i}^0] \quad k_i \equiv \lambda k_{0i}, \\ p_f + p_{\bar{f}} &= -\lambda \sum_{i=1}^n k_{0i}. \end{aligned} \quad (34)$$

We can always do this in the physical region provided that $\lambda\sqrt{s_0} > 2m_f$ and $\lambda\sqrt{s_0} > 2m_e$. We will always imagine, that $\sqrt{s_0} > 2m_e$ and that $\lambda > 1$. Then, since $\gamma_{\mathcal{M}}^{(n)} = n\beta(e_R)/e_R$,

$$\left(-\lambda \frac{\partial}{\partial \lambda} + \beta(e_R) \frac{\partial}{\partial e_R} - \sum_i (1 + \gamma_{\Theta_i}(e_R)) m_{iR} \frac{\partial}{\partial m_{iR}} - \frac{n\beta(e_R)}{e_R} + \overline{D}_{\mathcal{M}^{(n)}} \right) \mathcal{M}_{\alpha_1 \alpha_2}^{(n) \nu_2 \nu_1} = 0 \quad (35)$$

so that, using

$$\mathcal{M}_{\alpha_1 \alpha_2}^{(n) \nu_2 \nu_1} = \exp(\alpha B) \sum_{n'=0}^{\infty} m_{n' \alpha_1 \alpha_2}^{(n) \nu_2 \nu_1} \quad (36)$$

we find the solution

$$\begin{aligned} \mathcal{M}_{\alpha_1 \alpha_2}^{(n) \nu_2 \nu_1} &= \exp[\alpha(1)B(s_0, m_{iR}(\lambda))] \sum_{n'=0}^{\infty} m_{n' \alpha_1 \alpha_2}^{(n) \nu_2 \nu_1}(p_i(1), m_{iR}(\lambda), \alpha(\lambda), \mu) \lambda^{\overline{D}_{\mathcal{M}}^{(n)}} \\ &\quad \times \exp\left(-\int_1^\lambda \gamma_{\mathcal{M}^{(n)}}(e_R(\lambda')) d\lambda'/\lambda'\right) \\ &= \exp[\alpha(1)B(s_0, m_{iR}(\lambda))] \sum_{n'=0}^{\infty} m_{n' \alpha_1 \alpha_2}^{(n) \nu_2 \nu_1}(p_i(1), m_{iR}(\lambda), \alpha(\lambda), \mu) \lambda^{\overline{D}_{\mathcal{M}}^{(n)}} [e_R(\lambda)/e_R(1)]^{-n} \end{aligned} \quad (37)$$

where $\overline{D}_{\mathcal{M}^{(n)}}$ is the engineering dimension of $\mathcal{M}^{(n)}$ and is given by

$$\overline{D}_{\mathcal{M}^{(n)}} = -2 - n \quad (38)$$

$$\lambda \frac{d}{d\lambda} e_R = \beta(e_R(\lambda))$$

$$\lambda \frac{d}{d\lambda} m_{iR} = -[1 + \gamma_{\Theta_i}(e_R(\lambda))] m_{iR}(\lambda) \quad (39)$$

and

$$\alpha(\lambda) \equiv e_R^2(\lambda)/4\pi \quad (40)$$

We emphasize that (37) is a rigorous consequence of renormalization-group equation.

The running masses m_{iR} are

$$m_{iR}(\lambda) = [m_{iR}(1)/\lambda] \exp \left(- \int_1^\lambda \gamma_{\Theta_i}(e_R(\lambda')) d\lambda' / \lambda' \right)$$

$$\simeq [m_{iR}(1)/\lambda] [e_R(\lambda)/e_R(1)]^{-c_{\Theta_i}^0/b_0} \quad (41)$$

where

$$\beta(e_R) \equiv b_0 e_R^3 + \dots, \quad \gamma_{\Theta_i}(e_R) \equiv c_{\Theta_i}^0 e_R^2 + \dots \quad (42)$$

Note that, even though $f\bar{f}$ may be $e\bar{e}$, the error on the approximation in (41) is a factor of order

$$\exp[-\alpha(\lambda) + \alpha(1)] \simeq 1 - 6.4 \times 10^{-4} \quad (43)$$

Thus, one-loop coefficient functions should be adequate for our purposes, provided that n is not too big.

The important point to note about (37) is that it has the same algebraic structure as the original amplitude $\mathcal{M}$ in (3). Thus, the YFS program can be applied to (37). In this way we arrive at

$$d\sigma = \exp \left(2\alpha(1) [ReB(p_i(1), m_{iR}(\lambda)) + \tilde{B}(p_i(1), m_{iR}(\lambda), K_{max}/\lambda)] \right) \frac{1}{(2\pi)^4}$$

$$\int d^4y \ e^{iy \cdot (p_e + p_{\bar{e}} - P_{X'}) + D} \left(\tilde{\beta}_0(q_0) + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{l'=1}^n \frac{d^3 k_{l'}}{k_{l'}} e^{-iy \cdot k_{l'}} \tilde{\beta}_n(q_n) \right) dE_{X'} d^3 P_{X'} \quad (44)$$

where

$$\tilde{\beta}_n(q_n) \equiv \lambda^{2\overline{D}_{\mathcal{M}}^{(n)}} \overline{\beta}_n(p_i(1), k_{0j}, m_{iR}(\lambda), \alpha(\lambda), \mu) [e_R(\lambda)/e_R(1)]^{-2n} \quad (45)$$

and

$$D \equiv D(p_i(1), m_{iR}(\lambda), \alpha(\lambda), K_{max}/\lambda) \quad (46)$$

The result (44) is the basic result of this chapter.

The physical interpretation of (44) in comparison with (24) is the following: The exponentiation of the infrared effects occurs now with the running coupling $\alpha(1)$. The β_n are scaled by $[e_R(\lambda)/e_R(1)]^{-2n} \lambda^{2\overline{D}_{\mathcal{M}}^{(n)}}$. It is interesting that the invariance of the physical masses $m_{i,phys}(m_{iR}, e_R, \mu)$ under the renormalization-group operator means that, if the amplitudes $\mathcal{M}$ are on shell, the net effect of the running masses $m_{iR}(\lambda)$ is to keep the renormalization-group-improved amplitudes on shell. We emphasize that (44) is a rigorous consequence of the renormalization-group equation.

In the practical applications, we have in mind to work to the value $n = 2$ in (44). The error on this real two-photon emission term due to the factor of $[e_R(\lambda)/e_R(1)]^{-4}$ in (44), which we approximate to the leading order in $\beta(e_R(\lambda))$ in (39) and (42) is $\sim 0.02\%$ and this hard term is itself $\sim 0.85 \times 10^{-2}$ relative to the Born cross section in $\tilde{\beta}$. Thus, we see that the one-loop values of the coefficient functions are indeed adequate. Thus, the result (44), taken together with the one-loop expressions (41) and

$$e_R^2(\lambda) \simeq e_R^2(1)/[1 - 2b_0 e_R^2(1) \ln \lambda] \quad (47)$$

provides a basis for the Monte Carlo simulation of $e^+e^- \rightarrow f\bar{f}, f\bar{f} + \gamma, f\bar{f} + \gamma_1\gamma_2$ where γ, γ_1 , and γ_2 are hard photons and arbitrary numbers of soft photons are understood, and where $\sqrt{s} \cong M_{Z_0}$, to the level of $\sim 0.3\%$ of the basic Born cross section in $\tilde{\beta}$.

Chapter 3

Extension of YFS Theory to QCD Process

3.1 Introduction

Calculation of any QCD processes needs to treat $SU(3)$ color matrix element by definition of theory itself. This additional terms to the QED integrals completely alter the transition probability amplitude of each diagram including both the virtual and real diagrams. For example, each of diagrams has different additional weight which is combinations of C_F and C_A . Virtual part of the diagrams still have both UV and IR divergences, except box diagrams which have only IR divergences. Fortunately real part of the theory contains only IR divergences by the definition of real diagrams. And with the unknown strong coupling constant and quark mass in the limit $Q^2 \rightarrow m_q^2$ the transition probabilities can not be simply obtained if the On-shell scheme as in QED is used to calculate such QCD processes [17]. Therefore it is possible to set up an arbitrary mass parameter, μ , and subtract the UV and IR divergences in the single parameter. $\overline{MS}$ scheme is independent to the fermion mass to avoid infrared terms in coupling constant [18]. The On-shell

scheme is completely mass dependent which make possible the infrared contribution. Especially the YFS method needs the mass terms in the form factors by the definition of theory itself [2]. Therefore the two entirely independent schemes are hard to combine. To see the infrared contribution mainly, with the help of that physical quantities are independent on subtraction schemes, first thing is to look at the final results of Feynman amplitude with both $\overline{MS}$ and On-shell schemes, which means to keep all mass terms in the denominator of the Feynman amplitude, before doing any subtraction to the probability. Then by comparing with the fixed contribution of real part to the result of virtual parts, we may decide the suitable subtraction which satisfy the both ideas of mass independent and mass dependent-ness. As we will show the key is separate the single parameter ϵ to ϵ_{uv} and ϵ_{ir} . Then which one contribute to mass dependent and which one or not, and the independence is absorbed to coupling constant renormalization and the dependence goes to the YFS method via gluon mass. In this way the YFS method may get in the QCD process even though there is alteration by the color contribution, which is completely absorbed to the new definition of YFS theory. Therefore looking at the lowest order process and extracting YFS form factors in this process, the result can be easily extended to all orders by through YFS theory and the renormalization-group-improved YFS theory as shown the Chapter 2. With the guide of real part of weight change, we carefully examine the virtual part weight change, then make arrangement to cancel out the both the UV and IR divergences. Then we extract IR contribution which assure the application of YFS method to QCD process [16, 20]. Therefor first thing to do is to examine the real part diagrams as in following section. In there, the completely fixed results will be derived just as in QED case with additional weight change due to color matrix. This result will be reference to fix the UV and IR divergences in the virtual part of theory. In the next virtual part, we follow the usual dimensional regularization method with fermion and gluon mass term in the denominator. The result, before any subtraction, will be compared to the results of real part to get finite result even though

there is alteration of weight, which completely satisfies the both schemes. To see how to extend the YFS program to QCD process [21, 22, 23], we mainly concern on the lowest contribution i.e., $O(\alpha_s)$ t-channel $qq \rightarrow qqg$ process. In here, the infrared function will be extracted. Then as a next step, the result will be incorporated to higher order terms by the YFS theory and it's renormalized-group-improved frame work.

3.2 Real Corrections

The t-channel $O(\alpha_s)$ QCD process consists of five diagrams in its real gluon emission part as in Fig. 1. Four of them, diagrams in Fig. 1(c-i)-(c-iv), are basically the same type to the same order of QED process. The remaining one in Fig. 1(c-v), the contribution from the triple gluon vertex, is a new feature to the corresponding QED process. The result of calculation to these five diagrams is well-known in the literature [28, 29, 27]. Therefore basic strategy is simply to follow the well-established ways in this real part calculation, as natural. Furthermore, as far as we concern the soft gluon contribution to this $O(\alpha_s)$ process, the calculation can be greatly simplified by working in the limit $k \rightarrow 0$, where k is a momentum of respected gluon [2, 29].

Throughout this chapter, we follow the standard Bjorken-Drell notations [19], with metric $diag \ g_{\mu\nu} = (1, -1, -1, -1)$. And p_1 and p_2 subtend for the initial incoming quark momentum for upper line and lower line, respectively. The final outgoing momenta of quarks for upper line and lower line are represented by p_3 and p_4 .

The Feynman amplitudes for each diagrams in Fig. 2, are given by

$$M_i = (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4 - k) \sqrt{\frac{m}{E_1}} \sqrt{\frac{m}{E_2}} \sqrt{\frac{m}{E_3}} \sqrt{\frac{m}{E_4}} \left(\frac{1}{(2\pi)^{3/2}} \right)^4 \frac{1}{\sqrt{(2\pi)^3 2\omega_k}} \mathcal{M}_i \quad (48)$$

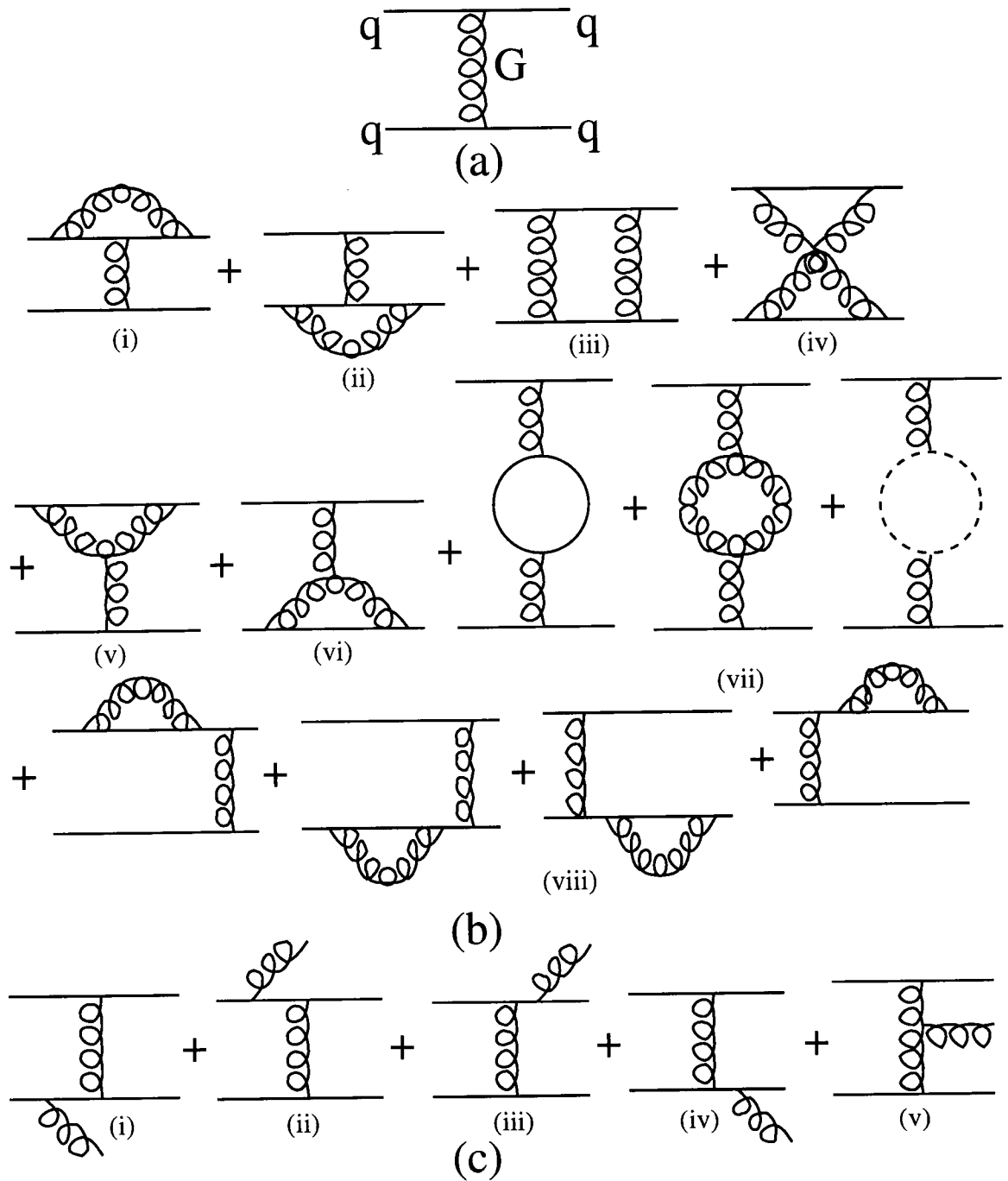


Figure 1: The process $q + \bar{q}' \rightarrow q + \bar{q}' + (G)$ to $O(\alpha_s)$: (a) Born approximation; (b) $O(\alpha_s)$ virtual correction; (c) $O(\alpha_s)$ bremsstrahlung process.

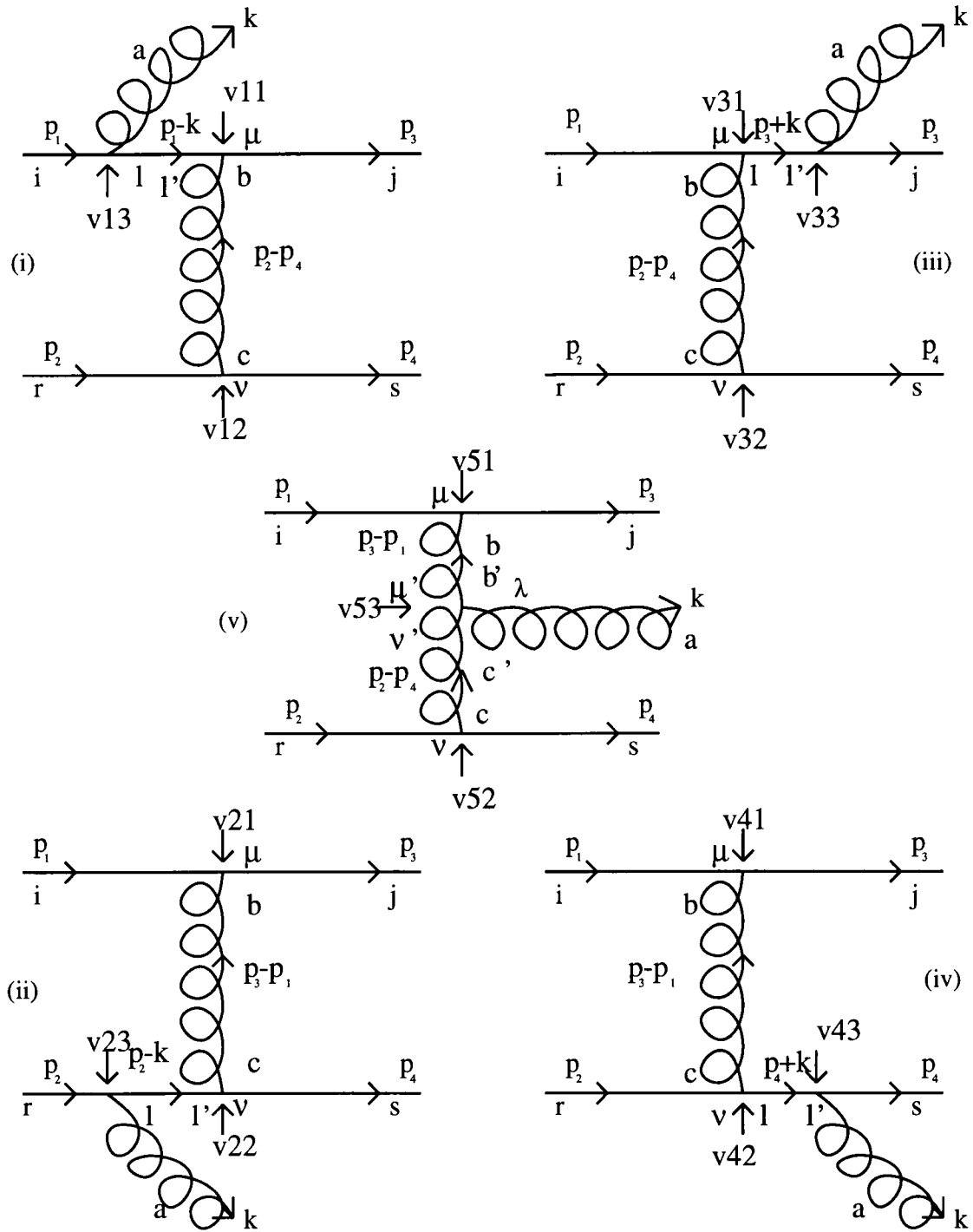


Figure 2: $O(\alpha_s)$ corrections to the real gluon bremsstrahlung process where $v11$ represents $-ig_s T_{ij}^b \gamma^\mu$, $v12$ for $-ig_s T_{rs}^c \gamma^\nu$, $v13$ for $-ig_s T_{il}^a / \epsilon$, $v21$ for $-ig_s T_{ij}^b \gamma^\mu$, $v22$ for $-ig_s T_{rs}^c \gamma^\nu$, $v23$ for $-ig_s T_{il}^a / \epsilon$, $v31$ for $-ig_s T_{il}^b \gamma^\mu$, $v32$ for $-ig_s T_{rs}^c \gamma^\nu$, $v33$ for $-ig_s T_{il}^a / \epsilon$, $v41$ for $-ig_s T_{ij}^b \gamma^\mu$, $v42$ for $-ig_s T_{rs}^c \gamma^\nu$, $v43$ for $-ig_s T_{il}^a / \epsilon$, $v51$ for $-ig_s T_{ij}^b \gamma^\mu$, $v52$ for $-ig_s T_{rs}^c \gamma^\nu$, and $v53$ for $g_s f^{ab'c'} F^{\lambda\mu'\nu'} \epsilon_\lambda$.

where $i=1,2,3,4,5$ correspond to the respective diagrams in Fig. 2(i),(ii),(iii),(iv) and (v), with its matrix element, $\mathcal{M}_i$

$$\mathcal{M}_1 = \bar{u}(p_3)(-ig_s T_{l'j}^b \gamma^\mu) \frac{i\delta_{ll'}}{\not{p}_1 - \not{k} - m + i\epsilon} (-ig_s T_{il}^a \not{\epsilon}) u(p_1) \frac{-ig_{\mu\nu}\delta_{bc}}{(p_2 - p_4)^2 + i\epsilon} \bar{u}(p_4)(-ig_s T_{rs}^c \gamma^\nu) u(p_2) \quad (49)$$

$$\mathcal{M}_2 = \bar{u}(p_3)(-ig_s T_{ij}^b \gamma^\mu) u(p_1) \frac{-ig_{\mu\nu}\delta_{bc}}{(p_3 - p_1)^2 + i\epsilon} \bar{u}(p_4)(-ig_s T_{l's}^c \gamma^\nu) \frac{i\delta_{ll'}}{\not{p}_2 - \not{k} - m + i\epsilon} (-ig_s T_{rl}^a \not{\epsilon}) u(p_2) \quad (50)$$

$$\mathcal{M}_3 = \bar{u}(p_3)(-ig_s T_{l'j}^a \not{\epsilon}) \frac{i\delta_{ll'}}{\not{p}_3 + \not{k} - m + i\epsilon} (-ig_s T_{il}^b \gamma^\mu) u(p_1) \frac{-ig_{\mu\nu}\delta_{bc}}{(p_2 - p_4)^2 + i\epsilon} \bar{u}(p_4)(-ig_s T_{rs}^c \gamma^\nu) u(p_2) \quad (51)$$

$$\mathcal{M}_4 = \bar{u}(p_3)(-ig_s T_{ij}^b \gamma^\mu) u(p_1) \frac{-ig_{\mu\nu}\delta_{bc}}{(p_3 - p_1)^2 + i\epsilon} \bar{u}(p_4)(-ig_s T_{l's}^a \not{\epsilon}) \frac{i\delta_{ll'}}{\not{p}_4 + \not{k} - m + i\epsilon} (-ig_s T_{rl}^c \gamma^\nu) u(p_2) \quad (52)$$

$$\mathcal{M}_5 = \bar{u}(p_3)(-ig_s T_{ij}^b \gamma^\mu) u(p_1) \frac{-ig_{\mu\mu'}\delta_{bb'}}{(p_3 - p_1)^2 + i\epsilon} (g_s f^{ab'c'} F^{\lambda\mu'\nu'} \epsilon_\lambda) \frac{-ig_{\nu\nu'}\delta_{cc'}}{(p_2 - p_4)^2} \bar{u}(p_4)(-ig_s T_{rs}^c \gamma^\nu) u(p_2) \quad (53)$$

with

$$F^{\lambda\mu'\nu'} = (-(p_3 - p_1) - (p_2 - p_4))^\lambda g^{\mu'\nu'} + ((p_2 - p_4) + k)^{\mu'} g^{\nu'\lambda} + (-k + (p_3 - p_1))^{\nu'} g^{\lambda\mu'} \quad (54)$$

We need to use Dirac equation and the soft gluon approximation to extract simple form of infrared contributions from these matrix elements. For example, in the first denominator of (49), after squaring, limit $k^2 \rightarrow 0$ and in the numerator, using anticommutation relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, *limit* $/k \rightarrow 0$ with $(\not{p}_1 - \not{k} + m)u(p_1) = (2\epsilon \cdot p_1 - \not{k}(\not{p}_1 - m))u(p_1) = 2\epsilon \cdot p_1 u(p_1)$. Then the matrix elements become to more readable forms

$$\mathcal{M}_1 = g_s(T_{lj}^b T_{il}^a T_{rs}^b) \left(\frac{\epsilon \cdot p_1}{-k \cdot p_1} \right) \mathcal{M}_B \quad (55)$$

$$\mathcal{M}_2 = g_s(T_{ij}^b T_{ls}^b T_{rl}^a) \left(\frac{\epsilon \cdot p_2}{-k \cdot p_2} \right) \mathcal{M}_B \quad (56)$$

$$\mathcal{M}_3 = g_s(T_{lj}^a T_{il}^b T_{rs}^b) \left(\frac{\epsilon \cdot p_3}{k \cdot p_3} \right) \mathcal{M}_B \quad (57)$$

$$\mathcal{M}_4 = g_s(T_{ij}^b T_{ls}^a T_{rl}^b) \left(\frac{\epsilon \cdot p_4}{k \cdot p_4} \right) \mathcal{M}_B \quad (58)$$

$$\mathcal{M}_5 = (-ig_s)(f^{abc} T_{ij}^b T_{rs}^c) \left(\frac{2\epsilon \cdot (p_1 - p_3)}{(p_3 - p_1)^2} \right) \mathcal{M}_B \quad (59)$$

where the the Born term, $\mathcal{M}_B$, has the usual expression

$$\mathcal{M}_B = \bar{u}(p_3)(-ig_s \gamma^\mu)u(p_1) \frac{-ig_{\mu\nu}}{(p_1 - p_2)^2} \bar{u}(p_4)(-ig_s \gamma^\nu)u(p_2) \quad (60)$$

Therefore sum of the each contribution shows factorization of infrared terms from the Born amplitude

$$\mathcal{M}_{tot} = g_s \epsilon \cdot \left(C_1 \frac{p_1}{-k \cdot p_1} + C_2 \frac{p_2}{-k \cdot p_2} + C_3 \frac{p_3}{k \cdot p_3} + C_4 \frac{p_4}{k \cdot p_4} + C_5 \frac{2(p_1 - p_3)}{(p_1 - p_3)^2} \right) \mathcal{M}_B \quad (61)$$

with the color matrix C_i

$$C_1 = T_{lj}^b T_{il}^a T_{rs}^b, C_2 = T_{ij}^b T_{ls}^b T_{rl}^a, C_3 = T_{lj}^a T_{il}^b T_{rs}^b, C_4 = T_{ij}^b T_{ls}^a T_{rl}^b, C_5 = -if^{abc} T_{ij}^b T_{rs}^c \quad (62)$$

Total transition probability can be obtained by sum of polarization, spin and color of the quark and gluon.

$$|\overline{\mathcal{M}}_{tot}|^2 = \frac{1}{9} \sum_{color} \frac{1}{4} \sum_{spin} \sum_{pol} |\mathcal{M}_{tot}|^2 \quad (63)$$

Following known tables of γ matrix and color matrix [18], and using relation $\sum_{pol} \epsilon \cdot a \epsilon \cdot b = -a \cdot b$ [19], the color terms becomes

$$\begin{aligned} C_1 C_1^* &= C_2 C_2^* = C_3 C_3^* = C_4 C_4^* = C_F \frac{N^2 - 1}{4} \\ C_1 C_2^* &= C_3 C_4^* = C_4^* C_3 = C_2^* C_1 = (2C_F - C_A) \frac{N^2 - 1}{4} \\ C_1 C_3^* &= C_2 C_4^* = C_4^* C_2 = C_3^* C_1 = (C_F - \frac{1}{2}C_A) \frac{N^2 - 4}{4} \\ C_1 C_4^* &= C_2 C_3^* = C_2^* C_3 = C_4^* C_1 = (2C_F - \frac{1}{2}C_A) \frac{N^2 - 1}{4} \\ C_1 C_5^* &= -C_2 C_5^* = -C_3 C_5^* = C_4 C_5^* = \frac{1}{2}C_A \frac{N^2 - 1}{4} \\ C_5 C_5^* &= C_A \frac{N^2 - 1}{4} \end{aligned} \quad (64)$$

with the Casimir operators $C_F = \frac{N^2 - 1}{2N}$, $C_A = N$, $N = 3$ for SU(3) color matrices. Then the differential cross section for the real soft gluon emission is

$$\left(\frac{d\sigma}{d\Omega} \right)_{real} = \left(\frac{d\sigma}{d\Omega} \right)_{Born} \delta_{real} \quad (65)$$

with

$$\left(\frac{d\sigma}{d\Omega} \right)_{Born} = \frac{g_s^4}{64\pi^2 s} \frac{N^2 - 1}{4N^2} 2 \frac{s^2 + u^2}{t^2} \quad (66)$$

The real correction can be obtained with the cut-off, the detector resolution, $K_m \equiv \frac{\epsilon\sqrt{s}}{2}$,

$$\begin{aligned} \delta_{real} &= N_C \frac{-g_s^2}{(2\pi)^3} \int_0^{K_m} \frac{d^3 k}{2w_k} \\ &\left| C_1 \frac{p_1}{-k \cdot p_1} + C_2 \frac{p_2}{-k \cdot p_2} + C_3 \frac{p_3}{k \cdot p_3} + C_4 \frac{p_4}{k \cdot p_4} + C_5 \frac{2(p_1 - p_3)}{(p_3 - p_1)^2} \right|^2 \end{aligned} \quad (67)$$

where $\omega_k = \sqrt{\vec{k}^2 + \lambda^2}$, λ = a fictitious gluon mass. The normalization factor $N_C = \frac{4}{N^2-1}$ is due to separation of color contribution to Born and infrared correction term. The expressions in (64) always exist in both real and virtual part expression. Thus it is probable to define them as another symbols which represent momentum part contribution as they stand.

With invariants $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$,

$$C_t \equiv C_F - \frac{1}{2}C_A, \quad C_s \equiv 2C_F - C_A, \quad C_u \equiv 2C_F - \frac{1}{2}C_A \quad (68)$$

Then the correction term δ_{real} might be arranged to the following form to see the color contribution easily,

$$\begin{aligned} \delta_{real} = & C_F(I_{11} + I_{33} + I_{22} + I_{44}) + C_t(I_{13} + I_{24}) + C_s(I_{12} + I_{34}) + C_u(I_{14} + I_{23}) \\ & + C_A(I_{55} + I_{53} + I_{51} + I_{54} + I_{52}) \end{aligned} \quad (69)$$

where subscripts 1-5 correspond to the momentum contribution of each processes in Fig. 2(i)-(v). Therefore the momentum distribution of corresponding QED process is altered by the color contribution with different weights, C_F for self-interactions, C_s for initial up and initial down interference contributions, C_t for initial up and final up interference, C_u for initial up and final down interference, and C_A for interference emission from triple gluon vertex and that of quark lines. The momentum integration for corresponding interaction part represented by, for the QED analogous part with $i,j=1,2,3$ and 4,

$$I_{ij} = -\frac{\alpha_s}{4\pi^2} \int_0^{K_m} \frac{d^3k}{\omega_k} \frac{np_i \cdot p_j}{k \cdot p_i k \cdot p_j} \quad (70)$$

with $n = +1$ for C_F part, $n = -2$ for C_t and C_u part, $n = +2$ for C_s part,

The pure QCD contribution to momentum integration are

$$I_{55} = -\frac{\alpha_s}{4\pi^2} \int_0^{K_m} \frac{d^3k}{\omega_k} \frac{4}{(p_3 - p_1)^2} \quad (71)$$

$$I_{5j} = -\frac{\alpha_s}{4\pi^2} \int_0^{K_m} \frac{d^3k}{\omega_k} \frac{n}{k \cdot p_i} \quad (72)$$

where $n = +1$ for $j = 3, 4$, $n = -1$ for $j = 1, 2$.

Next step is to do the k-space integrations. The integration of QED analogous part in (70) is exactly same as those of QED calculations. In the infinitesimal qq CMS product system of t-channel, we define $p_1 = (\frac{\sqrt{s}}{2}, \vec{P})$, and $p_3 = (\frac{\sqrt{s}}{2}, -\vec{P})$. Then in this frame the results of each integrations can be summarized by

$$\frac{2\alpha_s}{\pi} I_F \equiv I_{11} + I_{33} + I_{22} + I_{44} = \frac{2\alpha_s}{\pi} \left(-\log \frac{4K_m^2}{\lambda^2} + \log \frac{t}{m^2} \right) \quad (73)$$

$$\begin{aligned} \frac{2\alpha_s}{\pi} I_t &\equiv I_{13} + I_{24}, \quad \frac{2\alpha_s}{\pi} I_s \equiv -I_{12} - I_{34}, \quad \frac{2\alpha_s}{\pi} I_u \equiv I_{14} + I_{31}, \\ \frac{2\alpha_s}{\pi} I_a &= \frac{2\alpha_s}{\pi} \left(\log \frac{4K_m^2}{\lambda^2} \log \frac{a}{m^2} - \frac{1}{2} \log^2 \frac{a}{m^2} + \frac{\pi^2}{6} \right) \end{aligned} \quad (74)$$

with $a = t, s, u > 0$ and the additional $i\pi^2$ is included due to their analytic continuation. This result is exactly same as well-known one in the literature [25, 26]. The remaining pure QCD contributions are

$$I_{55} = \frac{2\alpha_s}{\pi} \frac{K_m^2}{-t} \cong 0, \quad I_{51} + I_{53} = I_{52} + I_{54} = 0 \quad (75)$$

It turns out these pure QCD contributions are safely neglected relative to the QED analogous contributions. In fact, we already saw it in (59) when we factorize IR contributions from matrix elements.

Therefore, in the soft gluon limit, the real gluon emission correction to the cross section for $O(\alpha_s)$ t-channel $qq \rightarrow qqg$ process has basically same structure of QED case with additional weight to each terms such as

$$\delta_{real}^{QCD} = \frac{2\alpha_s}{\pi} \left(C_F I_F^{QED} + C_t I_t^{QED} - C_s I_s^{QED} + C_u I_u^{QED} \right) \quad (76)$$

where C_F, C_s, C_t , and C_u are defined in (64) and (68), and I_F, I_t, I_s , and I_u are given in (73) and (74). When we set $C_F = C_t = C_s = C_u = 1$ and $\alpha_s \rightarrow \alpha_e$, (3.29) becomes exactly the corresponding QED result. The fact that color interactions alters the QED transition probabilities by introducing different additional weights

to each $c, s, u,$ and F contributions means we may not simply use the same subtraction scheme as in QED to get IR finite result and to extend YFS program to QCD process. But the result, (3.29), serves as a reference to our purpose because it is UV finite.

3.3 Virtual Corrections

As indicated in section 3.1, if we use the $\overline{MS}$ (Modified Minimal Subtraction) scheme [17], which is independent on fermion mass, say neglecting all fermion mass in the amplitude and using arbitrary mass point, then we get both UV and IR finite results. In which both effect is mixed in a single parameter ϵ . Therefore it is hard to see separately the IR contributions in a collision process. Furthermore if one wants to use YFS method to QCD process, one should have fermion mass because YFS theory requires fermion mass terms and gluon mass terms in the infrared functions B and $\tilde{B}$ by the definition of theory itself to factorize IR contribution. On the other hand, if we use On-shell scheme solely to QCD process, one cannot get finite result due to the fact virtual and real part have different weights (color contributions) to each process, as shown in (76), unlikely QED case which has same weight 1 to both parts. Therefore it seems we have two opposite cases, one is to use the mass independent scheme for high energy effect and the other is to use mass dependent scheme to isolate low energy effect (if we allow the substitution of the word infrared to low energy relative to high energy to simplify logic). We might get a possible solution by defining clearly our purpose, the extension of YFS method to QCD process. That means we need to have both YFS theory with fermion mass terms and $\overline{MS}$ scheme with arbitrary mass point. Therefore following both requirement, first thing is to keep all mass terms as need and second thing is to see the final results of unsubtracted contributions with the incorporated arbitrary mass point. Then third thing is to decide subtraction methods which gives finite results and mass terms for YFS method. As we will see the $\overline{MS}$ scheme is precisely

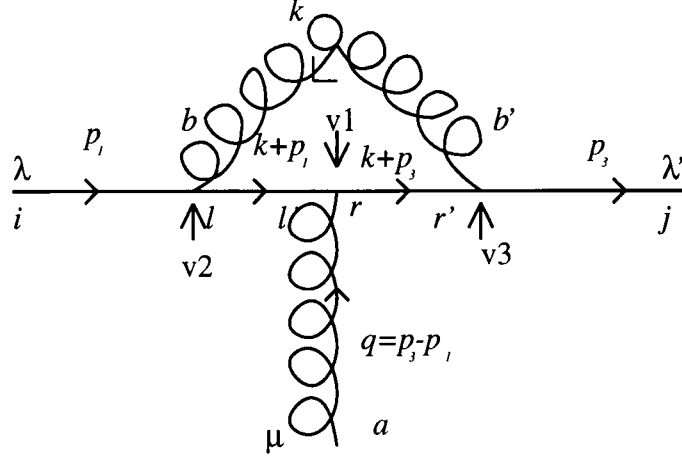


Figure 3: $O(\alpha_s)$ correction to the qqq-g vertex, where v1 represents $-ig_s T_{l'r}^a \gamma^\mu$, v2 for $-ig_s T_{il}^b \gamma^\lambda$, and v3 for $-ig_s T_{r'j}^{b'} \gamma^{\lambda'}$

the required one and therefore we automatically allow to use the YFS theory in QCD process with $\alpha_s/\pi \simeq .263$ [11] in the limit $k^2 \rightarrow 0$.

3.3.1 Quark-Quark-Gluon Vertex Correction

The matrix element, following the Feynman rules, for the diagram in Fig. 3 is

$$\begin{aligned}
 \mathcal{M}_{qqg} &= \bar{u}(p_3) \int \frac{d^4 k}{(2\pi)^4} (-ig_s T_{r'j}^{b'} \gamma^{\lambda'}) \frac{i\delta_{rr'}}{k + \not{p}_3 - m + i\epsilon} (-ig_s T_{l'r}^a \gamma^\mu) \\
 &\quad \frac{i\delta_{ll'}}{k + \not{p}_1 - m + i\epsilon} (-ig_s T_{il}^b \gamma^\lambda) \frac{-i\delta_{\lambda\lambda'} \delta_{bb'}}{k^2 - \lambda^2 + i\epsilon} u(p_1) \\
 &= \bar{u}(p_3) (-ig_s T_{ij}^a \Gamma_{qqg}^\mu) u(p_1)
 \end{aligned} \tag{77}$$

$$\begin{aligned}
 -ig_s T_{ij}^a \Gamma_{qqg}^\mu &= -g_s^3 (T_{il}^b T_{lr}^a T_{rj}^b) \\
 &\quad \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma_\lambda (\not{k} + \not{p}_3 + m) \gamma^\mu (\not{k} + \not{p}_1 + m) \gamma^\lambda}{((k + p_3)^2 - m^2 + i\epsilon)((k + p_1)^2 - m^2 + i\epsilon)(k^2 - \lambda^2 + i\epsilon)}
 \end{aligned} \tag{78}$$

The color sum reduces to

$$(T^b T^a T^b)_{ij} = (T^a T^b T^b)_{ij} + ([T^b, T^a] T^b)_{ij} = T_{ij}^a (C_F - \frac{1}{2} C_A)$$

Then

$$\Gamma_{qqg}^\mu = -ig_s^2(C_F - \frac{1}{2}C_A) \int \frac{d^4k}{(2\pi)^4} \frac{N_{qqg}^\mu}{D_{qqg}} \quad (79)$$

The denominator D_{qqg} can be written as

$$D_{qqg} = (k^2 - \lambda^2 + i\epsilon)(k^2 + 2k \cdot p_3 + i\epsilon)(k^2 + 2k \cdot p_1 + i\epsilon) \quad (80)$$

Using the two Feynman-Schwinger integrals sequentially,

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2} \quad (81)$$

$$\frac{1}{A^2B} = \int_0^1 dx \int_0^1 d\alpha \frac{1}{[xA + (1-x)B]^3} \quad (82)$$

The inverse of denominator becomes

$$\begin{aligned} \frac{1}{D_{qqg}} &= \frac{1}{(k^2 - \lambda^2 + i\epsilon)} \int_0^1 d\alpha_1 \frac{1}{[k^2 + 2k \cdot (\alpha_1 p_3 + (1 - \alpha_1)p_1) + i\epsilon]^2} \\ &= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \frac{1}{[k^2 + 2k \cdot \alpha_2 Q - (1 - \alpha_2)\lambda^2 + i\epsilon]^3} \\ &= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \frac{1}{(k'^2 - d_{qqg} + i\epsilon)^3} \end{aligned} \quad (83)$$

with

$$\begin{aligned} Q &= \alpha_1 p_3 + (1 - \alpha_1)p_1, & k' &= k + \alpha_2 Q, \\ d_{qqg} &= \alpha_2^2 Q^2 + (1 - \alpha_2)\lambda^2, & Q^2 &= m^2 + \alpha_1(1 - \alpha_1)(-q^2) \end{aligned} \quad (84)$$

where k is shifted to k' to simplify the numerator, and we use space-like vector $-q^2 = (p_3 - p_1)^2 > 0$ to avoid any unnecessary singularities during integrations by requiring $Q^2 > 0$ and $d_{qqg} > 0$. After done integrations (79) will be analytically continued to the region $q^2 > 0$ by the simple relation $\log(-q^2) = \log q^2 + i\pi$.

The numerator N_{qqg}^μ can be simplified by neglecting mass terms. But we still keep mass terms in the denominator as in (84). Using the 't Hooft-Veltman dimensional regularization [17].

$$N_{qqg}^\mu = \gamma_\nu (\not{p}_3 + \not{k} + m) \gamma^\mu (\not{p}_1 + \not{k} + m) \gamma^\nu$$

$$\begin{aligned}
&= \gamma_\nu (\not{k}' - \alpha_2 \not{Q} + \not{p}_3) \gamma^\mu (\not{k}' - \alpha_2 \not{Q} + \not{p}_1) \gamma^\nu \\
&= \gamma_\nu \not{k}' \gamma^\mu \not{k}' \gamma^\nu + \gamma_\nu (\not{p}_3 - \alpha_2 \not{Q}) \gamma^\mu (\not{p}_1 - \alpha_2 \not{Q}) \gamma^\nu \\
&= \left(\frac{(2-n)^2}{n} k'^2 + (n-2) \alpha_2^2 Q^2 + 2(1-\alpha_2)(-q^2) \right) \gamma^\mu \quad (85)
\end{aligned}$$

Therefore with this dimensional regularization the vertex function Γ_{qqg} is extracted from Born terms by through (3.31)

$$\Gamma_{qqg} = (-ig_s^2)(C_F - \frac{1}{2}C_A) \int \frac{d^n k'}{(2\pi)^n} \frac{N_{qqg}}{D_{qqg}} \quad (86)$$

Using Feynman-Schwinger integration method, the momentum integration part becomes

$$\begin{aligned}
\int \frac{d^n k'}{(2\pi)^n} \frac{N_{qqg}}{D_{qqg}} &= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \frac{-i}{16\pi^2} \left(\frac{4\pi\mu^2}{d_{qqg} - i\epsilon} \right)^{2-n/2} \frac{\Gamma(2-n/2)}{\Gamma(3)} \\
&\quad \left(\frac{-n(2-n)^2}{2n} + \frac{2-n/2}{d_{qqg} - i\epsilon} [(n-2)\alpha_2^2 Q^2 + 2(1-\alpha_2)(-q^2)] \right) \quad (87)
\end{aligned}$$

Now we expand the dimensional parameter in the limit $n \rightarrow 4$,

$$\Gamma(2-n/2) \rightarrow \frac{1}{2-n/2} - \gamma_E + \dots \quad (88)$$

$$\left(\frac{4\pi\mu^2}{d_{qqg} - i\epsilon} \right)^{2-n/2} \rightarrow 1 + (2-n/2) \log \frac{4\pi\mu^2}{d_{qqg} - i\epsilon} + \dots \quad (89)$$

$$-\frac{1}{2}(2-n)^2 \rightarrow -2 + 4(2-n/2) - 2(2-n/2)^2 \quad (90)$$

$$n-2 \rightarrow 2 - 2(2-n/2) \quad (91)$$

In this limit, (86) becomes

$$\Gamma_{qqg} = (C_F - \frac{1}{2}C_A) \frac{\alpha_s}{4\pi} \left[N_\epsilon - 2 - \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \alpha_2 \left(2 \log \frac{d_{qqg}}{\mu^2} + \frac{R_{qqg}}{d_{qqg}} \right) \right] \quad (92)$$

where

$$N_\epsilon \equiv \frac{1}{2-n/2} - \gamma_E + \log 4\pi \quad (93)$$

and $d_{qqg} = \alpha_2^2 Q^2 + (1 - \alpha_2)\lambda^2$ is from (84). $Q^2 = m^2 + \alpha_1(1 - \alpha_1)(-q^2)$. μ is an arbitrary mass due to the dimensionality of mass in n -dimension and R_{qqg} represents

$$R_{qqg} = 2\alpha_2^2 Q^2 + 2(1 - \alpha_2)(-q^2) \quad (94)$$

By direct calculating the Feynman parameter integrations, we get

$$\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \alpha_2 \log \frac{d_{qqg}}{\mu^2} = \frac{1}{2} \log \frac{-q^2}{\mu^2} - \frac{3}{2} \quad (95)$$

$$\begin{aligned} \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \alpha_2 \frac{R_{qqg}}{d_{qqg}} &= \int_0^1 d\alpha_1 \left(1 - 2 \frac{-q^2}{Q^2} + \frac{-q^2}{Q^2} \log \frac{Q^2}{\lambda^2} \right) \\ &= 1 - 2[2 \log \frac{-q^2}{m^2}] + [2 \log \frac{-q^2}{m^2} \log \frac{-q^2}{\lambda^2} - \log^2 \frac{-q^2}{m^2} - \frac{\pi^2}{3} + 2(i\pi)^2 - (i\pi)^2] \end{aligned} \quad (96)$$

where last two $(i\pi)$ terms are due to analytic continuation of $-q^2$. Therefore we have final result with (94), (95) and (96),

$$\begin{aligned} \Gamma_{qqg}(q^2) &= (C_F - \frac{1}{2}C_A) \frac{\alpha_s}{4\pi} \\ &\quad \left((N_\epsilon - \log \frac{q^2}{\mu^2}) + 4 \log \frac{q^2}{m^2} - (2 \log \frac{q^2}{m^2} \log \frac{q^2}{\lambda^2} - \log^2 \frac{q^2}{m^2} - \frac{4\pi^2}{3}) \right) \end{aligned} \quad (97)$$

The renormalization factor Z_{1qqg} is defined by

$$Z_{1qqg}^{-1} - 1 \equiv (C_F - \frac{1}{2}C_A) \frac{\alpha_s}{4\pi} (N_\epsilon - \log \frac{q^2}{\mu^2}) \quad (98)$$

Then the qqg -vertex correction with counting both upper and lower lines becomes

$$\delta_{qqg} = (C_F - \frac{1}{2}C_A) \frac{2\alpha_s}{\pi} \left(2 \log \frac{q^2}{m^2} - (\log \frac{q^2}{m^2} \log \frac{q^2}{\lambda^2} - \frac{1}{2} \log^2 \frac{q^2}{m^2} - \frac{2\pi^2}{3}) \right) \quad (99)$$

For $q^2 = 0$ case, by following same way, we get

$$\Gamma_{qqg}(q^2 = 0) = (C_F - \frac{1}{2}C_A) \frac{\alpha_s}{4\pi} \left(N_\epsilon - \log \frac{m^2}{\mu^2} + 4 - 2 \log \frac{m^2}{\lambda^2} \right) \quad (100)$$

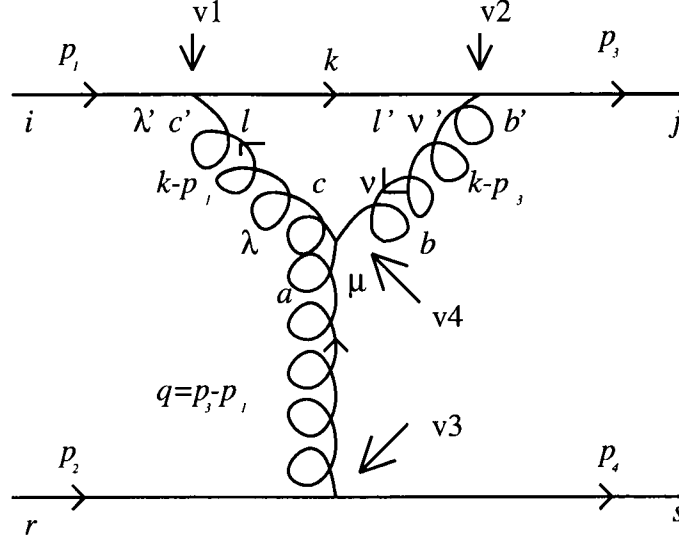


Figure 4: $O(\alpha_s)$ correction to the ggg-vertex, where v1 represents $-ig_s T_{il}^{c'} \gamma^{\lambda'}$, v2 for $-ig_s T_{lj}^{b'} \gamma^{\nu'}$, v3 for $-ig_s T_{rs}^{a'} \gamma^{\mu'}$, and v4 for $-g_s f^{abc} F^{\nu\mu\lambda}$

3.3.2 Gluon-Gluon-Gluon Vertex Correction

The matrix element to the ggg-coupling diagram, which comes from Non-Abelian structure of theory, on Fig. 4 is

$$\begin{aligned}
 \mathcal{M}_{ggg} &= \int \frac{d^4 k}{(2\pi)^4} \bar{u}(p_3) \frac{i}{k - m + i\epsilon} (-ig_s T_{il}^{c'} \gamma^{\lambda'}) u(p_1) \frac{-ig_{\lambda'\lambda} \delta_{c'e}}{(k - p_1)^2 - \lambda^2 + i\epsilon} \\
 &\quad (g_s f^{abc} F^{\nu\mu\lambda}) \frac{-ig_{\mu\mu'} \delta_{aa'}}{(p_3 - p_1)^2 + i\epsilon} \frac{-ig_{\nu\nu'} \delta_{bb'}}{(k - p_3)^2 - \lambda^2 + i\epsilon} \bar{u}(p_4) (-ig_s T_{rs}^{a'} \gamma^{\mu'}) u(p_2) \\
 &= \bar{u}(p_3) (-ig_s T_{ij}^a \Gamma_{ggg}^\mu) u(p_1) \frac{-ig_{\mu\mu'} \delta_{aa'}}{(p_3 - p_1)^2 + i\epsilon} \bar{u}(p_4) (-ig_s T_{rs}^{a'} \gamma^{\mu'}) u(p_2) \quad (101)
 \end{aligned}$$

with

$$F^{\nu\mu\lambda} = ((p_3 - p_1) + (k - p))^\nu g^{\mu\lambda} + (-(k - p_1) - (k - p_3))^\mu g^{\lambda\nu} + ((k - p_3) - q)^\lambda g^{\nu\mu} \quad (102)$$

$$\begin{aligned}
 -ig_s T_{ij}^a \Gamma_{ggg}^\mu &= (-ig_s) (-g_s^2) (f^{abc} T_{lj}^b T_{il}^c) \\
 &\quad \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma_\nu (k + m) \gamma_\lambda F^{\nu\mu\lambda}}{(k^2 - m^2 + i\epsilon)((k - p_3)^2 - \lambda^2 + i\epsilon)((k - p_1)^2 - \lambda^2 + i\epsilon)} \quad (103)
 \end{aligned}$$

Born contribution to the color sum can be separated out

$$f^{abc}T_{ij}^b T_{il}^c = f^{abc}\frac{1}{2}[\frac{1}{N}\delta^{cb} + (d^{cbd} + if^{cbd})T^d]_{ij} = -\frac{i}{2}f^{abc}f^{dbc}T_{ij}^d = -\frac{i}{2}C_A T_{ij}^a \quad (104)$$

where $C_A = N = 3$. Then

$$\Gamma_{ggg}^\mu = ig_s^2 \frac{C_A}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{N_{ggg}^\mu}{D_{ggg}} \quad (105)$$

$$D_{ggg} = (k^2 - m^2 + i\epsilon)(k^2 - 2k \cdot p_3 + m^2 - \lambda^2 + i\epsilon)(k^2 - 2k \cdot p_1 + m^2 - \lambda^2 + i\epsilon) \quad (106)$$

$$\begin{aligned} \frac{1}{D_{ggg}} &= \frac{1}{(k^2 - m^2 + i\epsilon)} \int_0^1 d\alpha_1 \frac{1}{[k^2 - 2k \cdot (\alpha_1 p_3 + (1 - \alpha_1)p_1) + m^2 - \lambda^2 + i\epsilon]^2} \\ &= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \frac{1}{[k^2 - 2k \cdot \alpha_2 Q + \alpha_2(m^2 - \lambda^2) - (1 - \alpha_2)m^2 + i\epsilon]^3} \\ &= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \frac{1}{(k'^2 - d_{ggg} + i\epsilon)^3} \end{aligned} \quad (107)$$

with

$$\begin{aligned} Q &= \alpha_1 p_3 + (1 - \alpha_1)p_1, & Q^2 &= m^2 + \alpha_1(1 - \alpha_1)(-q^2) \\ k' &= k - \alpha_2 Q, & d_{ggg} &= \alpha_2^2 Q^2 + \alpha_2(\lambda^2 - 2m^2) + m^2 \end{aligned} \quad (108)$$

where k is shifted to k' to simplify the numerator, and we use space-like vector $-q^2 = (p_3 - p_1)^2 > 0$.

$$\begin{aligned} N_{ggg}^\mu &= \gamma_\nu (\not{k} + m) \gamma_\lambda F^{\nu\mu\lambda} \\ &= \gamma_\nu \not{k} \gamma_\lambda [(k - p_1 + q)^\nu g^{\mu\lambda} + (p_3 + p_1 - 2k)^\mu g^{\lambda\nu} + (k - p_3 - q)^\lambda g^{\nu\mu}] \\ &= (\not{k}' + \alpha_2 \not{Q} + 2 \not{q})(\not{k}' + \alpha_2 \not{Q}) \gamma^\mu + \gamma^\mu (\not{k}' + \alpha_2 \not{Q})(\not{k}' + \alpha_2 \not{Q} - 2 \not{q}) \\ &\quad + (2 - n)(\not{k}' + \alpha_2 \not{Q})(p_3 + p_1 - 2k' - 2\alpha_2 Q)^\mu \\ &= 2(k'^2 \gamma^\mu - (2 - n) \not{k}' k'^\mu) + (\alpha_2 \not{Q} + 2 \not{q}) \alpha_2 \not{Q} \gamma^\mu \\ &\quad + \gamma^\mu \alpha_2 \not{Q} (\alpha_2 \not{Q} - 2 \not{q}) - (2 - n) 2\alpha_2^2 \not{Q} Q^\mu \\ &= \left(k'^2 (-4) \frac{1 - n}{n} + (n - 1) 2\alpha_2^2 Q^2 - 2\alpha_2 (-q^2) \right) \gamma^\mu \end{aligned} \quad (109)$$

where $\int d^n k' k'^\alpha k'^\beta f(k'^2) = \frac{g^{\alpha\beta}}{n} \int d^n k' k'^2 f(k'^2)$ is used. The vertex function is extracted from Born contribution of the spin and color in (104) by the separation processes (105) and (110).

$$\Gamma_{ggg} = ig_s^2 \frac{1}{2} C_A \int \frac{d^n k'}{(2\pi)^n} \frac{N_{ggg}}{D_{ggg}} \quad (110)$$

Using Feynman-Schwinger integration method, the momentum integration part becomes

$$\begin{aligned} \int \frac{d^n k'}{(2\pi)^n} \frac{N_{ggg}}{D_{ggg}} &= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \frac{-i}{16\pi^2} \left(\frac{4\pi\mu^2}{d_{ggg} - i\epsilon} \right)^{2-n/2} \frac{\Gamma(2-n/2)}{\Gamma(3)} \\ &\quad \left(2(1-n) + \frac{2-n/2}{d_{ggg} - i\epsilon} [(n-1)2\alpha_2^2 Q^2 - 2\alpha_2(-q^2)] \right) \end{aligned} \quad (111)$$

With the same expansion as in quark-quark-gluon vertex part

$$\Gamma_{ggg} = \frac{1}{2} C_A \frac{\alpha_s}{4\pi} \left[3N_\epsilon - 2 - \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \left(6\log \frac{d_{ggg}}{\mu^2} + \frac{R_{ggg}}{d_{ggg}} \right) \right] \quad (112)$$

where N_ϵ is given in (93) and the remaining part

$$R_{ggg} = 6\alpha_2^2 Q^2 - 2\alpha_2(-q^2) \quad (113)$$

By direct calculating the Feynman parameter integrations, we get

$$\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \log \frac{d_{ggg}}{\mu^2} = \frac{1}{2} \log \frac{-q^2}{\mu^2} - \frac{3}{2} \quad (114)$$

$$\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \frac{R_{ggg}}{d_{ggg}} = 3 - 4 \log \frac{-q^2}{m^2} \quad (115)$$

where last two ($i\pi$) terms are due to analytic continuation of $-q^2$. Therefore we have final result with (112), (114) and (115),

$$\Gamma_{ggg}(q^2) = \frac{1}{2} C_A \frac{\alpha_s}{4\pi} \left(3(N_\epsilon - \log \frac{q^2}{\mu^2}) + 4(1 + \log \frac{q^2}{m^2}) \right) \quad (116)$$

The renormalization factor Z_{1ggg} is defined by

$$Z_{1ggg}^{-1} - 1 = \frac{1}{2} C_A \frac{\alpha_s}{4\pi} 3(N_\epsilon - \log \frac{q^2}{\mu^2}) \quad (117)$$

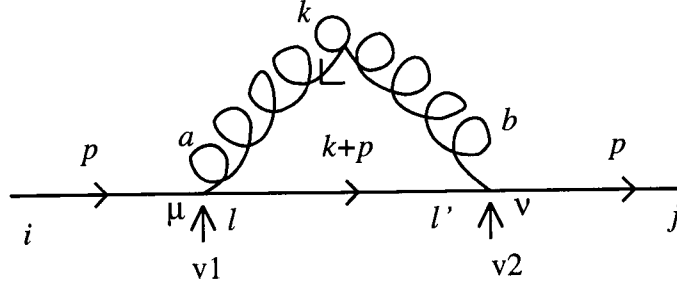


Figure 5: $O(\alpha_s)$ correction to the quark self-energy, where v1 represents $-ig_s T_{il}^a \gamma^\mu$ and v2 for $-ig_s T_{l'j}^b \gamma^\nu$

The ggg-vertex correction becomes

$$\delta_{ggg} = \frac{1}{2} C_A \frac{2\alpha_s}{\pi} \left(2 + 2 \log \frac{q^2}{m^2} \right) \quad (118)$$

For $q^2 = 0$ case, by following same way, we get

$$\Gamma_{ggg}(q^2 = 0) = \frac{1}{2} C_A \frac{\alpha_s}{4\pi} \left(3(N_\epsilon - \log \frac{m^2}{\mu^2}) + 4 \right) \quad (119)$$

3.3.3 Quark Self Energy Correction

The matrix element for the typical quark self-energy as in Fig. 5, is given by

$$\begin{aligned} -i\delta_{ij}\Sigma(p) &= \int \frac{d^4k}{(2\pi)^4} (-ig_s T_{l'j}^b \gamma^\nu) \frac{i\delta_{ll'}}{k + \not{p} - m + i\epsilon} (-ig_s T_{il}^a \gamma^\mu) \frac{-ig_{\mu\nu}\delta_{ab}}{k^2 - \lambda^2 + i\epsilon} \\ &= (ig_s)^2 (T_{il}^a T_{l'j}^a) \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu (\not{k} + \not{p} + m) \gamma_\mu}{(k^2 - \lambda^2 + i\epsilon)((k+p)^2 - m^2 + i\epsilon)} \end{aligned} \quad (120)$$

Born contribution to the color matrix can be separated out

$$T_{il}^a T_{l'j}^a = \frac{\delta^{aa}}{2N} \delta_{ij} = \frac{N^2 - 1}{2N} \delta_{ij} = C_F \delta_{ij} \quad (121)$$

Then the Green function can be written as

$$\Sigma = -ig_s^2 C_F \int \frac{d^4k}{(2\pi)^4} \frac{N_{se}}{D_{se}} \quad (122)$$

with the definition

$$D_{se} = (k^2 - \lambda^2 + i\epsilon)(k^2 + 2k \cdot p + p^2 - m^2 + i\epsilon) \quad (123)$$

$$\begin{aligned} \frac{1}{D_{se}} &= \int_0^1 d\alpha_1 \frac{1}{[k^2 + 2k \cdot \alpha_1 p + \alpha_1 p^2 - \alpha_1 m^2 - (1 - \alpha_1)\lambda^2 + i\epsilon]^2} \\ &= \int_0^1 d\alpha_1 \frac{1}{[k'^2 - d_{se} + i\epsilon]^2} \end{aligned} \quad (124)$$

where

$$k' = k + \alpha_1 p, \quad d_{se} = \alpha_1(\alpha_1 - 1)p^2 + \alpha_1 m^2 + (1 - \alpha_1)\lambda^2 \quad (125)$$

The numerator can be reduced to

$$\begin{aligned} N_{se} &= \gamma^\mu (\not{k} + \not{p} + m) \gamma_\mu = \gamma^\mu (\not{k}' + (1 - \alpha_1) \not{p} + m) \gamma_\mu \\ &= (2 - n)(1 - \alpha_1) \not{p} + nm \end{aligned} \quad (126)$$

In this n -dimensional space-time,

$$\Sigma = -ig_s^2 C_F \int \frac{d^n k'}{(2\pi)^n} \frac{N_{se}}{D_{se}} \quad (127)$$

Using Feynman-Schwinger integration method, the momentum integration part becomes

$$\int \frac{d^n k'}{(2\pi)^n} \frac{N_{se}}{D_{se}} = \int_0^1 d\alpha_1 \frac{-i}{16\pi^2} \left(\frac{4\pi\mu^2}{d_{se} - i\epsilon} \right)^{2-n/2} \Gamma(2 - n/2) [(2 - n)(1 - \alpha_1) \not{p} + nm] \quad (128)$$

With the same expansion as in quark-quark-gluon vertex part

$$\Sigma = C_F \frac{\alpha_s}{4\pi} \left[N_\epsilon (\not{p} - 4m) - (\not{p} - 2m) - \int_0^1 d\alpha_1 (2(1 - \alpha_1) \not{p} - 4m) \log \frac{d_{se}}{\mu^2} \right] \quad (129)$$

By direct calculating the Feynman parameter integrations, we get

$$\int_0^1 d\alpha_1 \log \frac{d_{se}}{\mu^2} = \log \frac{p^2}{\mu^2} + \log \frac{p^2}{\lambda^2} - 2 \quad (130)$$

$$\int_0^1 d\alpha_1 \alpha_1 \log \frac{d_{se}}{\mu^2} = \log \frac{p^2}{\mu^2} + \frac{1}{2} (\log \frac{p^2}{\lambda^2} - 2) \quad (131)$$

Therefore we have final result with (129), (130) and (131),

$$\begin{aligned} \Sigma(p^2) = & C_F \frac{\alpha_s}{4\pi} \left(m[3(N_\epsilon - \log \frac{p^2}{\mu^2}) + (5 - 3\log \frac{p^2}{\lambda^2})] \right. \\ & \left. - (\not{p} - m)[(N_\epsilon - \log \frac{p^2}{\mu^2}) + (1 - \log \frac{p^2}{\lambda^2})] \right) \end{aligned} \quad (132)$$

The mass counter term is given by

$$\delta_m = C_F \frac{\alpha_s}{4\pi} \left(m[3(N_\epsilon - \log \frac{p^2}{\mu^2}) + (5 - 3\log \frac{p^2}{\lambda^2})] \right) \quad (133)$$

And the renormalization factor is defined as

$$Z_2^{-1} - 1 = C_F \frac{\alpha_s}{4\pi} \left(N_\epsilon - \log \frac{p^2}{\mu^2} \right) \quad (134)$$

Thus the quark self energy correction to Born cross section is

$$\delta_{se} = \frac{2\alpha_s}{\pi} C_F \log \frac{q^2}{\lambda^2} \quad (135)$$

3.3.4 Quark+Gluon+Ghost Loop Corrections

The matrix element to the one quark-loop shown in Fig. 6(i) is

$$\begin{aligned} -ig_s^2 \Pi_{quark}^{\mu\nu,ab}(q) = & - \int \frac{d^4 k}{(2\pi)^4} \\ & tr \left[(-ig_s T_{ij}^a \gamma^\mu) \frac{i\delta^{ii'}}{\not{k} - m + i\epsilon} (-ig_s T_{j'i'}^b \gamma^\nu) \frac{i\delta^{j'j}}{\not{k} + \not{q} - m + i\epsilon} \right] \\ = & -g_s^2 T_{ij}^a T_{ji}^b \int \frac{d^4 k}{(2\pi)^4} \frac{tr[\gamma^\mu(\not{k} + m)\gamma^\nu(\not{k} + \not{q} + m)]}{[(k+q)^2 - m^2 + i\epsilon][k^2 - m^2 + i\epsilon]} \end{aligned} \quad (136)$$

$$T_{ij}^a T_{ji}^b = n_f T_F \delta^{ab} = C_F \delta^{ab} \quad (137)$$

where n_f = number of quark flavors, and $T_F = 1/2$. Then

$$g_s^2 \Pi_{quark}^{\mu\nu} = -ig_s^2 C_F \int \frac{d^4 k}{(2\pi)^4} \frac{N_{quark}^{\mu\nu}}{D_{quark}} \quad (138)$$

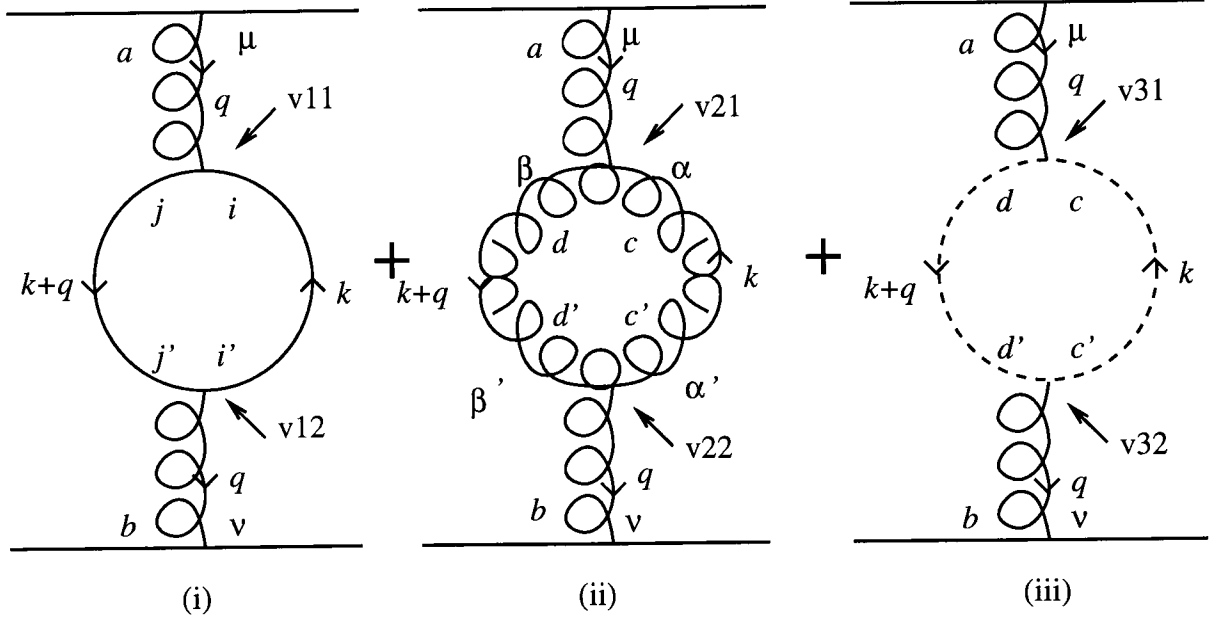


Figure 6: $O(\alpha_s)$ corrections to the gluon propagator, where v_{11} represents $-ig_s T_{ij}^a \gamma^\mu$, v_{12} for $-ig_s T_{j'i}^b \gamma^\nu$, v_{21} for $g_s f^{adc} F^{\mu\beta\alpha}$, v_{22} for $g_s f^{d'c'b} F^{\beta'\alpha'\nu}$, v_{31} for $g_s f^{adc} (k+q)^\mu$, and v_{32} for $g_s f^{bd'c'} (-k)^\nu$

The denominator D_{quark} can be written as

$$D_{quark} = (k^2 - m^2 + i\epsilon)(k^2 + 2k \cdot q + q^2 - m^2 + i\epsilon) \quad (139)$$

Using Feynman-Schwinger integrals, The inverse of denominator becomes

$$\begin{aligned} \frac{1}{D_{quark}} &= \int_0^1 d\alpha \frac{1}{[(k + \alpha q)^2 - ((\alpha^2 - \alpha)q^2 + m^2) + i\epsilon]^2} \\ &= \int_0^1 d\alpha \frac{1}{(k'^2 - d_{quark} + i\epsilon)^2} \end{aligned} \quad (140)$$

with

$$k' = k + \alpha q, \quad d_{quark} = \alpha(\alpha - 1)q^2 + m^2 \quad (141)$$

The numerator N_{quark}^μ can be simplified by neglecting mass terms. Note that we still keep mass terms in the denominator as in (141). Working in the n -dimensional space-time,

$$\begin{aligned} N_{quark}^{\mu\nu} &= tr[\gamma^\mu(\not{k} + m)\gamma^\nu(\not{k} + \not{q} + m)] \\ &= tr[\gamma^\mu(\not{k}' - \alpha \not{q})\gamma^\nu(\not{k}' + (1 - \alpha)\not{q})] \\ &= tr[\gamma^\mu \not{k}' \gamma^\nu \not{k}'] + (\alpha^2 - \alpha)tr[\gamma^\mu \not{q} \gamma^\nu \not{q}] \\ &= 2^{n/2} \left(g^{\mu\nu} [k'^2 \frac{2-n}{n} - \alpha(\alpha - 1)q^2 + m^2] + 2\alpha(\alpha - 1)q^\mu q^\nu \right) \end{aligned} \quad (142)$$

Therefore with this dimensional regularization method (138) becomes

$$g_s^2 \Pi_{quark}^{\mu\nu} = (-ig_s^2) C_f \int \frac{d^n k'}{(2\pi)^n} \frac{N_{quark}^{\mu\nu}}{D_{quark}} \quad (143)$$

Using Feynman-Schwinger integration method, the momentum integration part becomes

$$\begin{aligned} \int \frac{d^n k'}{(2\pi)^n} \frac{N_{quark}^{\mu\nu}}{D_{quark}} &= \int_0^1 d\alpha \frac{-i}{16\pi^2} \left(\frac{4\pi\mu^2}{d_{quark} - i\epsilon} \right)^{2-n/2} \Gamma(2 - n/2) \\ &\quad 2\alpha(\alpha - 1)4(-q^2 g^{\mu\nu} + q^\mu q^\nu) \end{aligned} \quad (144)$$

Therefore

$$-ig_s^2 \Pi_{quark}^{\mu\nu, ab}(q) = -ig_s^2 \delta^{ab} (-q^2 g^{\mu\nu} + q^\mu q^\nu) \Pi_{quark}(q^2) \quad (145)$$

In this limit, (3.145) becomes

$$g_s^2 \Pi_{quark}(q^2) = -\frac{4}{3} C_f \frac{\alpha_s}{4\pi} \left(N_\epsilon + \int_0^1 d\alpha 6\alpha(\alpha-1) \log \frac{d_{quark}}{\mu^2} \right) \quad (146)$$

where

$$N_\epsilon \equiv \frac{1}{2 - n/2} - \gamma_E + \log 4\pi \quad (147)$$

and $d_{quark} = m^2 + \alpha(1 - \alpha)(-q^2)$ is from (141). μ is an arbitrary mass due to the dimensionality of mass in n -dimension. By direct calculating the Feynman parameter integrations, we get

$$\int_0^1 d\alpha \alpha(\alpha-1) \log \frac{d_{quark}}{\mu^2} = \frac{5}{18} - \frac{1}{6} \log \frac{-q^2}{\mu^2} \quad (148)$$

Thus quark contribution to vacuum polarization becomes

$$\Pi_{quark}(q^2) = -\frac{4}{3} C_f \frac{\alpha_s}{4\pi} \left((N_\epsilon - \log \frac{q^2}{\mu^2}) + \frac{5}{3} \right) \quad (149)$$

For $q^2 = 0$ case, by following same way, we get

$$\Pi_{quark}(q^2 = 0) = -\frac{4}{3} C_f \frac{\alpha_s}{4\pi} \left(N_\epsilon - \log \frac{m^2}{\mu^2} \right) \quad (150)$$

The gluon-loop correction, second part of vacuum polarization, through the diagram on Fig. 6(ii) is

$$\begin{aligned} -ig_s^2 \Pi_{gluon}^{\mu\nu, ab}(q) &= \frac{1}{2!} \int \frac{d^4 k}{(2\pi)^4} (g_s f^{adc} F^{\mu\beta\alpha}) \frac{-ig_{\alpha\alpha'} \delta_{cc'}}{k^2 + i\epsilon} \\ &\quad (g_s f^{d'c'b} F^{\beta'\alpha'\nu}) \frac{-ig_{\beta\beta'} \delta_{d'd}}{(k+q)^2 + i\epsilon} \\ &= -\frac{1}{2!} g_s^2 f^{adc} f^{dcb} \int \frac{d^4 k}{(2\pi)^4} \frac{F^{\mu\beta\alpha} F_{\beta\alpha}^\nu}{((k+q)^2 + i\epsilon)(k^2 + i\epsilon)} \end{aligned} \quad (151)$$

with

$$g_s f^{adc} F^{\mu\beta\alpha} = g_s f^{adc} \left((-2k - q)^\mu g^{\beta\alpha} + (k - q)^\beta g^{\alpha\mu} + (2q + k)^\alpha g^{\mu\beta} \right) \quad (152)$$

$$g_s f^{d'c'b} F^{\beta'\alpha'\nu} = g_s f^{d'c'b} \left((q-k)^{\beta'} g^{\alpha'\nu} + (-2q-k)^{\alpha'} g^{\nu\beta'} + (2k+q)^\nu g^{\beta'\alpha'} \right) \quad (153)$$

Color term becomes

$$f^{adc} f^{dcb} = C_A \delta^{ab} \quad (154)$$

with $C_A = N$. Then

$$g_s^2 \Pi_{gluon}^{\mu\nu} = -i g_s^2 \frac{1}{2} C_A \int \frac{d^4 k}{(2\pi)^4} \frac{N_{gluon}^{\mu\nu}}{D_{gluon}} \quad (155)$$

The denominator D_{gluon} can be written as

$$D_{gluon} = (k^2 + i\epsilon)(k^2 + 2k \cdot q + q^2 + i\epsilon) \quad (156)$$

and

$$\begin{aligned} \frac{1}{D_{gluon}} &= \int_0^1 d\alpha \frac{1}{[(k + \alpha q)^2 - ((\alpha^2 - \alpha)q^2) + i\epsilon]^2} \\ &= \int_0^1 d\alpha \frac{1}{(k'^2 - d_{gluon} + i\epsilon)^2} \end{aligned} \quad (157)$$

with

$$k' = k + \alpha q, \quad d_{gluon} = \alpha(\alpha - 1)q^2 \quad (158)$$

The numerator N_{gluon}^μ can be simplified by neglecting mass terms. With (157) the numerator yields

$$\begin{aligned} N_{gluon}^{\mu\nu} &= F^{\mu\beta\alpha} F_{\beta\alpha}^\nu \\ &= - \left(g^{\mu\nu} [k'^2 6 \frac{n-1}{n} + (5 + 2(\alpha^2 - \alpha))q^2] \right. \\ &\quad \left. + q^\mu q^\nu [(n-6) + 2(\alpha^2 - \alpha)(2n-3)] \right) \end{aligned} \quad (159)$$

Therefore, in this dimension, (160) becomes

$$g_s^2 \Pi_{gluon}^{\mu\nu} = (-i g_s^2) C_f \int \frac{d^n k'}{(2\pi)^n} \frac{N_{gluon}^{\mu\nu}}{D_{gluon}} \quad (160)$$

Using Feynman-Schwinger integration method, the momentum integration part becomes

$$\int \frac{d^n k'}{(2\pi)^n} \frac{N_{gluon}^{\mu\nu}}{D_{gluon}} = \int_0^1 d\alpha \frac{-i}{16\pi^2} \left(\frac{4\pi\mu^2}{d_{gluon} - i\epsilon} \right)^{2-n/2} \Gamma(2-n/2) \left(-q^2 g^{\mu\nu} \left[\frac{5}{2} + (\alpha^2 - \alpha)(1 + 3\frac{1-n}{2-n}) \right] + q^\mu q^\nu [1 + (2-n/2) + (\alpha^2 - \alpha)(-5 + 4(2-n/2))] \right) \quad (161)$$

Now we expand the dimensional parameter in the limit $n \rightarrow 4$, In this limit, (3.110) becomes

$$g_s^2 \Pi_{gluon}^{\mu\nu}(q) = C_A \frac{\alpha_s}{4\pi} \left(-q^2 g^{\mu\nu} \left[\frac{19}{12} N_\epsilon - \frac{1}{4} - \int_0^1 d\alpha \left(\frac{5}{2} + \frac{11}{2}(\alpha^2 - \alpha) \right) \log \frac{d_{gluon}}{\mu^2} \right] + q^\mu q^\nu \left[\frac{11}{6} N_\epsilon + \frac{1}{3} - \int_0^1 d\alpha (1 - 5(\alpha^2 - \alpha)) \log \frac{d_{gluon}}{\mu^2} \right] \right) \quad (162)$$

where

$$N_\epsilon \equiv \frac{1}{2-n/2} - \gamma_E + \log 4\pi \quad (163)$$

μ is an arbitrary mass due to the dimensionality of mass in n -dimension. By direct calculating the Feynman parameter integrations, we get

$$\int_0^1 d\alpha \log \frac{d_{gluon}}{\mu^2} = -2 + \log \frac{q^2}{\mu^2} \quad (164)$$

$$\int_0^1 d\alpha (\alpha^2 - \alpha) \log \frac{d_{gluon}}{\mu^2} = \frac{5}{18} - \frac{1}{6} \log \frac{q^2}{\mu^2} \quad (165)$$

This result explicitly shows current is not conserved.

$$g_s^2 \Pi_{gluon}^{\mu\nu}(q) = C_A \frac{\alpha_s}{4\pi} \left(-q^2 g^{\mu\nu} \left[\frac{19}{12} (N_\epsilon - \log \frac{q^2}{\mu^2}) + \frac{29}{9} \right] + q^\mu q^\nu \left[\frac{11}{6} (N_\epsilon - \log \frac{q^2}{\mu^2}) + \frac{67}{18} \right] \right) \quad (166)$$

The third contribution to the vacuum polarization part by the ghost loop as shown in Fig. 6(iii) can be written as

$$-i g_s^2 \Pi_{ghost}^{\mu\nu, ab}(q) = (-) \int \frac{d^4 k}{(2\pi)^4} (g_s f^{adc} (k+q)^\mu) \frac{-i \delta_{cc'}}{(k+q)^2 + i\epsilon}$$

$$\begin{aligned}
& (g_s f^{bd'c'} (-k)^\nu) \frac{-i\delta_{dd'}}{k^2 + i\epsilon} \\
&= -g_s^2 f^{adc} f^{bdc} \int \frac{d^4 k}{(2\pi)^4} \frac{(k+q)^\mu k^\nu}{((k+q)^2 + i\epsilon)(k^2 + i\epsilon)} \quad (167)
\end{aligned}$$

with color term

$$f^{adc} f^{bdc} = C_A \delta^{ab} \quad (168)$$

Then as usual expression,

$$g_s^2 \Pi_{ghost}^{\mu\nu} = -ig_s^2 C_A \int \frac{d^4 k}{(2\pi)^4} \frac{N_{ghost}^{\mu\nu}}{D_{ghost}} \quad (169)$$

The denominator D_{gluon} can be written as

$$D_{ghost} = (k^2 + i\epsilon)(k^2 + 2k \cdot q + q^2 + i\epsilon) \quad (170)$$

The inverse of denominator becomes

$$\begin{aligned}
\frac{1}{D_{ghost}} &= \int_0^1 d\alpha \frac{1}{[(k + \alpha q)^2 - ((\alpha^2 - \alpha)q^2) + i\epsilon]^2} \\
&= \int_0^1 d\alpha \frac{1}{(k'^2 - d_{ghost} + i\epsilon)^2} \quad (171)
\end{aligned}$$

with

$$k' = k + \alpha q, \quad d_{ghost} = \alpha(\alpha - 1)q^2 \quad (172)$$

The numerator N_{ghost}^μ can be simplified by neglecting mass terms. Using (167) and (162), the numerator yields, with $k \rightarrow k'$,

$$\begin{aligned}
N_{ghost}^{\mu\nu} &= (k+q)^\mu k^\nu = (k' - \alpha q + q)^\mu (k' - \alpha q)^\nu \\
&= \left(g^{\mu\nu} k'^2 \frac{1}{n} + q^\mu q^\nu (\alpha^2 - \alpha) \right) \quad (173)
\end{aligned}$$

and

$$g_s^2 \Pi_{ghost}^{\mu\nu} = (-ig_s^2) C_A \int \frac{d^n k'}{(2\pi)^n} \frac{N_{ghost}^{\mu\nu}}{D_{ghost}} \quad (174)$$

Using Feynman-Schwinger integration, the momentum integration part becomes

$$\int \frac{d^n k'}{(2\pi)^n} \frac{N_{ghost}^{\mu\nu}}{D_{ghost}} = \int_0^1 d\alpha \frac{-i}{16\pi^2} \left(\frac{4\pi\mu^2}{d_{ghost} - i\epsilon} \right)^{2-n/2} \Gamma(2 - n/2)$$

$$(\alpha^2 - \alpha) \left(-q^2 g^{\mu\nu} \left[-\frac{1}{2} - \frac{1}{2}(2 - n/2) \right] + q^\mu q^\nu \right) \quad (175)$$

Now we expand the dimensional parameter in the limit $n \rightarrow 4$, In this limit, (174) becomes

$$g_s^2 \Pi_{ghost}^{\mu\nu}(q) = C_A \frac{\alpha_s}{4\pi} \left(-q^2 g^{\mu\nu} \left[\frac{1}{12} N_\epsilon + \frac{1}{12} + \int_0^1 d\alpha \frac{1}{2} (\alpha^2 - \alpha) \log \frac{d_{ghost}}{\mu^2} \right] + q^\mu q^\nu \left[\frac{-1}{6} N_\epsilon - \int_0^1 d\alpha (\alpha^2 - \alpha) \log \frac{d_{ghost}}{\mu^2} \right] \right) \quad (176)$$

where

$$N_\epsilon \equiv \frac{1}{2 - n/2} - \gamma_E + \log 4\pi \quad (177)$$

By direct calculating the Feynman parameter integrations, we get

$$\int_0^1 d\alpha (\alpha^2 - \alpha) \log \frac{d_{ghost}}{\mu^2} = \frac{5}{18} - \frac{1}{6} \log \frac{q^2}{\mu^2} \quad (178)$$

Therefore we have final result from (176) and (178)

$$g_s^2 \Pi_{ghost}^{\mu\nu}(q) = C_A \frac{\alpha_s}{4\pi} \left(-q^2 g^{\mu\nu} \left[\frac{1}{12} (N_\epsilon - \log \frac{q^2}{\mu^2}) + \frac{2}{9} \right] + q^\mu q^\nu \left[\frac{-1}{6} (N_\epsilon - \log \frac{q^2}{\mu^2}) - \frac{5}{18} \right] \right) \quad (179)$$

Therefore sum of gluon and ghost contributions, the current conserved quantity, becomes

$$g_s^2 \Pi_{gluon+ghost}^{\mu\nu}(q) = (-q^2 g^{\mu\nu} + q^\mu q^\nu) \frac{5}{3} C_A \frac{\alpha_s}{4\pi} \left((N_\epsilon - \log \frac{-q^2}{\mu^2}) + \frac{31}{15} \right) \quad (180)$$

Sum of quark, gluon and ghost contribution to vacuum polarization is

$$g_s^2 \Pi_{vp}(q^2) = \frac{\alpha_s}{4\pi} \left(\left(\frac{5}{3} C_A - \frac{4}{3} C_f \right) (N_\epsilon - \log \frac{-q^2}{\mu^2}) + \left(\frac{31}{9} C_A - \frac{20}{9} C_f \right) \right) \quad (181)$$

The renormalization factor Z_3 is defined by

$$Z_3 - 1 \equiv \frac{\alpha_s}{4\pi} \left(\frac{5}{3} C_A - \frac{4}{3} C_f \right) (N_\epsilon - \log \frac{-q^2}{\mu^2}) \quad (182)$$

The correction of vacuum polarization part becomes

$$\delta_{vac} = \frac{2\alpha_s}{\pi} \left(\frac{31}{36} C_A - \frac{5}{9} C_f \right) \quad (183)$$

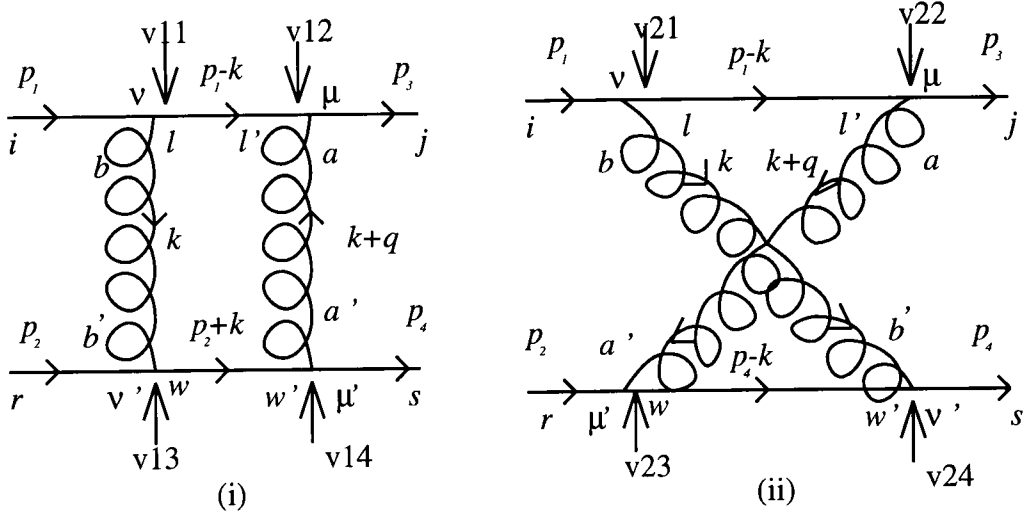


Figure 7: $O(\alpha_s)$ corrections to the two gluons exchange processes with v11 for $-ig_s T_{il}^b \gamma^\nu$, v12 for $-ig_s T_{l'j}^a \gamma^\mu$, v13 for $-ig_s T_{rw}^{b'} \gamma^{\nu'}$, v14 for $-ig_s T_{w's}^{a'} \gamma^{\mu'}$, v21 for $-ig_s T_{il}^b \gamma^\nu$, v22 for $-ig_s T_{l'j}^a \gamma^\mu$, v23 for $-ig_s T_{rw}^{b'} \gamma^{\mu'}$, and v24 for $-ig_s T_{w's}^{a'} \gamma^{\nu'}$

3.3.5 Box Diagrams

The two gluons exchange process is given in Fig. 7(i) and (ii)

$$\begin{aligned}
 \mathcal{M}_{box} &= \int \frac{d^4 k}{(2\pi)^4} \bar{u}(p_3) (-ig_s T_{l'j}^a \gamma^\mu) \frac{i\delta_{ll'}}{(\not{p}_1 - \not{k} - m + i\epsilon)} (-ig_s T_{il}^b \gamma^\nu) u(p_1) \frac{-ig_{\nu\nu'} \delta_{bb'}}{k^2 - \lambda^2 + i\epsilon} \\
 &\quad \frac{-ig_{\mu\mu'} \delta_{aa'}}{(q+k)^2 - \lambda^2 + i\epsilon} \bar{u}(p_4) (-ig_s T_{w's}^{a'} \gamma^{\mu'}) \frac{i\delta_{ww'}}{(\not{p}_2 + \not{k} - m + i\epsilon)} (-ig_s T_{rw}^{b'} \gamma^{\nu'}) u(p_2) \\
 &= g_s^4 (T_{lj}^a T_{il}^b T_{ws}^a T_{rw}^b) \int \frac{d^4 k}{(2\pi)^4} \\
 &\quad \frac{\bar{u}(p_3) \gamma^\mu (\not{p}_1 - \not{k} + m) \gamma^\nu u(p_1) \bar{u}(p_4) \gamma_\mu (\not{p}_2 + \not{k} + m) \gamma_\nu u(p_2)}{(k^2 - \lambda^2 + i\epsilon)(k^2 + 2k \cdot q + q^2 - \lambda^2 + i\epsilon)(k^2 - 2k \cdot p_1 + i\epsilon)(k^2 + 2k \cdot p_2 + i\epsilon)}
 \end{aligned} \tag{184}$$

The color sum reduces to

$$T_{lj}^a T_{il}^b T_{ws}^a T_{rw}^b = (T^b T^a)_{ij} (T^b T^a)_{rs} = (T_{ij}^c T_{rs}^c) \left(\frac{-1}{N} \right) = C_{Born} (2C_F - C_A)$$

Then

$$\mathcal{M}_{box} = g_s^2 (2C_F - C_A) C_{Born} \int \frac{d^4 k}{(2\pi)^4} \frac{N_{box}^\mu}{D_{box}} \tag{185}$$

The inverse of denominator becomes

$$\begin{aligned}
\frac{1}{D_{box}} &= \frac{1}{(k^2 - \lambda^2 + i\epsilon)(k^2 + 2k \cdot q + q^2 - \lambda^2 + i\epsilon)} \int_0^1 d\alpha_1 \frac{1}{[k^2 - 2k \cdot (\alpha_1 p_1 + (1 - \alpha_1)p_2) + i\epsilon]^2} \\
&= \frac{1}{k^2 - \lambda^2 + i\epsilon} \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \frac{2\alpha_2}{[k^2 - 2k \cdot (\alpha_2 Q_1 - (1 - \alpha_2)q) + (1 - \alpha_2)(q^2 - \lambda^2) + i\epsilon]^3} \\
&= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \int_0^1 d\alpha_3 \frac{6\alpha_2 \alpha_3^2}{[k^2 - 2k \cdot \alpha_3 Q_2 + \alpha_3(1 - \alpha_2)(q^2 - \lambda^2) - (1 - \alpha_3)\lambda^2 + i\epsilon]^4} \\
&= \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \int_0^1 d\alpha_3 3\alpha^2 \frac{1}{(k'^2 - d_{box} + i\epsilon)^4} \quad (186)
\end{aligned}$$

with

$$\begin{aligned}
Q_1 &= \alpha_1 p_1 + (1 - \alpha_1)p_2, & k' &= k - \alpha_3 Q_2, \\
Q_2 &= \alpha_2 Q_1 - (1 - \alpha_2)q, & q &= p_3 - p_1, \\
d_{box} &= \alpha_3^2 Q_2^2 - \alpha_3(1 - \alpha_2)(q^2 - \lambda^2) + (1 - \alpha_3)\lambda^2, \quad (187)
\end{aligned}$$

The numerator N_{box}^μ can be simplified by neglecting mass terms. But we still keep mass terms in the denominator as in (187). we use the n -dimensional γ matrix to use the t' Hoot-Veltman n -dimensional regularization method.

$$\begin{aligned}
N_{box} &= \bar{u}(p_4)\gamma_\mu(\not{p}_2 + \not{k} + m)\gamma_\nu u(p_2) \\
&= -k'^2 \frac{1}{n} N_1(n) + N_2(n) \quad (188)
\end{aligned}$$

with

$$\begin{aligned}
N_1(n) &= \bar{u}(p_3)\gamma_\mu \gamma^\alpha \gamma^\mu u(p_1) \bar{u}(p_2)\gamma_\mu \gamma_\alpha \gamma_\nu u(p_2) \\
&= a_0 + (2 - n/2)a_1 + (2 - n/2)^2 a_2 + \dots \\
N_2(n) &= \bar{u}(p_3)\gamma^\mu (\not{p}_1 + m - \alpha_3 \not{Q})\gamma^\nu u(p_1) \bar{u}(p_2)\gamma_\mu (\not{p}_2 + m + \alpha_3 \not{Q}_2)\gamma_\nu u(p_2) \\
&= b_0 + (2 - n/2)b_1 + (2 - n/2)^2 b_2 + \dots \quad (189)
\end{aligned}$$

With this expansion in mind, $\mathcal{M}_{box}$ can be written as terms by through (3.148) Then

$$\mathcal{M}_{box} = g_s^2 (2C_F - C_A) C_{Born} \int \frac{d^n k'}{(2\pi)^n} \frac{N_{box}^\mu}{D_{box}} \quad (190)$$

Using Feynman-Schwinger integration method, the momentum integration part becomes

$$\int \frac{d^n k'}{(2\pi)^n} \frac{N_{box}}{D_{box}} = \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 2\alpha_2 \int_0^1 d\alpha_3 3\alpha_3^2 \frac{-i}{16\pi^2} \left(\frac{4\pi\mu^2}{d_{box} - i\epsilon} \right)^{2-n/2} \frac{\Gamma(2-n/2)}{\Gamma(4)} \\ \left(-\frac{(2-n/2)}{2} \frac{N_1(n)}{d_{box}} + (3-n/2)(2-n/2) \frac{N_2(n)}{d_{box}^2} \right) \quad (191)$$

Now we expand the dimensional parameter in the limit $n \rightarrow 4$, In this limit, (190) becomes UV finite expression

$$\mathcal{M}_{box} = g_s^2 (2C_F - C_A) C_{Born} \frac{1}{16\pi^2} \quad (192)$$

$$\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \alpha_2 \int_0^1 d\alpha_3 \alpha_3^2 \left(\frac{1}{2} \frac{N_1(n=4)}{d_{box}} + \frac{N_2(n=4)}{d_{box}^2} \right) \quad (193)$$

Since the box diagram is UV finite, we might refer well-known results [26, 31]. The result is

$$\delta_{box1} = (2C_F - C_A) \frac{2\alpha_s}{\pi} \left(I(s, m^2, s, \lambda^2) - \frac{t^2}{s^2 + u^2} A_{box} \right) \\ A_{box} = \frac{s-u}{t} \frac{1}{4} \log^2 \frac{s}{t} + \frac{u}{t} \frac{1}{2} \log \frac{s}{t} \quad (194)$$

Similarly, for the crossed two gluon exchange diagram, the correction term is

$$\delta_{box2} = (2C_F - \frac{1}{2}C_A) \frac{2\alpha_s}{\pi} (-) \left(I(u, m^2, u, \lambda^2) - \frac{t^2}{s^2 + u^2} B_{box} \right) \\ B_{box} = \frac{u-s}{t} \frac{1}{4} \log^2 \frac{s}{t} + \frac{s}{t} \frac{1}{2} \log \frac{u}{t} \quad (195)$$

And

$$I(a, m^2, b, \lambda^2) \equiv \log \frac{a}{m^2} \log \frac{b}{\lambda^2} - \frac{1}{2} \log^2 \frac{a}{m^2} - \frac{2\pi^2}{3} \quad (196)$$

We are now to collect all virtual corrections, with $t = q^2 = (p_3 - p_1)^2 > 0$,

$$\delta_{virtual} = \frac{2\alpha_s}{\pi} \left(C_t \left[2 \log \frac{t}{m^2} - I(t, m^2, t, \lambda^2) \right] + C_s \left[I(s, m^2, s, \lambda^2) - \frac{t^2}{s^2 + u^2} A_{box} \right] \right. \\ \left. + C_u (-) \left[I(u, m^2, u, \lambda^2) - \frac{t^2}{u^2 + s^2} B_{box} \right] + C_F \left[\log \frac{t}{\lambda^2} \right] \right)$$

$$+\frac{1}{2}C_A[2+2\log\frac{t}{m^2}]+(\frac{31}{36}C_A-\frac{5}{9}C_f)) \quad (197)$$

Therefore sum of the real correction, eq.(29), and the virtual correction, eq.(154), shows infrared finite correction to Born cross section, using $\overline{MS}$ scheme,

$$\delta_{real} + \delta_{virtual} = \frac{2\alpha_s}{\pi} \left(C_t \log \frac{4K_m^2}{t} \log \frac{t}{m^2} - C_s \log \frac{4K_m^2}{s} \log \frac{s}{m^2} + C_u \log \frac{4K_m^2}{u} \log \frac{u}{m^2} \right. \\ \left. C_F (\log \frac{t}{4K_m^2} + 3 \log \frac{t}{m^2}) + (C_u B'_{box} - C_s A'_{box}) + C_A + C_F \frac{5\pi^2}{6} + (\frac{31}{36}C_A - \frac{5}{9}C_f) \right) \quad (198)$$

with $A'_{box} = \frac{t^2}{s^2+u^2} A_{box}$ and $B'_{box} = \frac{t^2}{u^2+s^2} B_{box}$. Therefore UV term is absorbed to coupling constant as usual. We can now use this result to get β_0 in QCD version of YFS theory.

3.4 YFS Infrared Functions in QCD

In this Section, we derive the analogue of the YFS infrared functions B and $\tilde{B}$ for QCD, B_{QCD} and $\tilde{B}_{QCD}$, for the process $q + (\bar{q})' \rightarrow q'' + (\bar{q})''' + (G)$. We will focus on the case of most interest to us, in which a gluon is exchanged in the t -channel and we require that the exchanged gluon carries large momentum transfer so that the outgoing quark (anti-quark) is in the $|\eta| \leq 2.8$ acceptance of the SDC (and GEM). This situation is depicted in Figs. 1. We remark that if the relevant exchange is a color singlet exchange (such as W or Z^0 exchange), we can obtain the corresponding formulas for B_{QCD} and $\tilde{B}_{QCD}$ by dropping the terms in the color-exchange case which arise from non-commutativity of the respective QCD λ^a matrices in the relevant quark representation.

Specifically, following the kinematics in Fig. 1a, we note that, for the SDC and GEM acceptance, we always have $-Q^2 = -(q_1 - q_2)^2 \gg \Lambda_{QCD}^2$, so that perturbative QCD methods are applicable. What we wish to do is to extract the infrared-singular part of the $O(\alpha_s)$ amplitude in Fig. 1b, in complete analogy with the YFS [2] extraction of the infrared-singular portion of the analogous amplitude in QED

via the famous YFS virtual infrared function B. To this end, we first note that, by explicit calculation, the graphs in (v)-(vii) are not infrared-divergent (we compute the gluon vacuum polarization to order α_s as well), so that the infrared-singular part of Fig. 1b is given by just the same graphs as the infrared-singular part of the analogous process in QED. We follow the YFS methods in Ref. [2] and find the infrared-singular part of Fig. 1b to be given by

$$\mathcal{M}_{v,IR} = \alpha_s B_{\text{QCD}} \mathcal{M}_{8B} + \alpha_s \bar{B}_{\text{QCD}} \mathcal{M}_{0B} \quad , \quad (199)$$

where $\mathcal{M}_{8B}$ is the respective Born amplitude in Fig. 1a and where

$$\begin{aligned} B_{\text{QCD}} = & \frac{i}{(8\pi^3)} \int \frac{d^4 k}{(k^2 - \lambda^2 + i\epsilon)} \left[C_F \left(\frac{2p_1 + k}{k^2 + 2k \cdot p_1 + i\epsilon} + \frac{2p_2 - k}{k^2 - 2k \cdot p_2 + i\epsilon} \right)^2 \right. \\ & + \Delta C_s \frac{2(2p_1 + k) \cdot (2p_2 - k)}{(k^2 + 2k \cdot p_1 + i\epsilon)(k^2 - 2k \cdot p_2 + i\epsilon)} + C_F \left(\frac{2q_1 + k}{k^2 + 2k \cdot q_1 + i\epsilon} - \frac{2q_2 - k}{k^2 - 2k \cdot q_2 + i\epsilon} \right)^2 \\ & + \Delta C_s \frac{2(2q_1 + k) \cdot (2q_2 - k)}{(k^2 + 2k \cdot q_1 + i\epsilon)(k^2 - 2k \cdot q_2 + i\epsilon)} + C_F \left(\frac{2p_2 + k}{k^2 + 2k \cdot p_2 + i\epsilon} - \frac{2q_2 + k}{k^2 + 2k \cdot q_2 + i\epsilon} \right)^2 \\ & + \Delta C_t \frac{2(2q_2 + k) \cdot (2p_2 + k)}{(k^2 + 2k \cdot q_2 + i\epsilon)(k^2 + 2k \cdot p_2 + i\epsilon)} + C_F \left(\frac{2p_1 + k}{k^2 + 2k \cdot p_1 + i\epsilon} - \frac{2q_1 + k}{k^2 + 2k \cdot q_1 + i\epsilon} \right)^2 \\ & + \Delta C_t \frac{2(2q_1 + k) \cdot (2p_1 + k)}{(k^2 + 2k \cdot q_1 + i\epsilon)(k^2 + 2k \cdot p_1 + i\epsilon)} - C_F \left(\frac{2p_1 + k}{k^2 + 2k \cdot p_1 + i\epsilon} - \frac{2q_2 + k}{k^2 + 2k \cdot q_2 + i\epsilon} \right)^2 \\ & + \Delta C_u \frac{2(2p_1 + k) \cdot (2q_2 + k)}{(k^2 + 2k \cdot p_1 + i\epsilon)(k^2 + 2k \cdot q_2 + i\epsilon)} - C_F \left(\frac{2q_1 + k}{k^2 + 2k \cdot q_1 + i\epsilon} - \frac{2p_2 + k}{k^2 + 2k \cdot p_2 + i\epsilon} \right)^2 \\ & \left. + \Delta C_u \frac{2(2q_1 + k) \cdot (2p_2 + k)}{(k^2 + 2k \cdot q_1 + i\epsilon)(k^2 + 2k \cdot p_2 + i\epsilon)} \right] \quad (200) \end{aligned}$$

with $C_F = 4/3$ =quadratic Casimir invariant of the quark color representation, λ equal to the standard infrared regulator mass (Ref. [6]) and

$$\begin{aligned} \Delta C_s &= \begin{cases} -1, & qq' \text{ incoming} \\ -1/6, & q\bar{q}' \text{ incoming} \end{cases}, \quad \Delta C_t = -3/2, \text{ and} \\ \Delta C_u &= \begin{cases} -5/2, & qq' \text{ incoming} \\ -5/3, & q\bar{q}' \text{ incoming} \end{cases}. \end{aligned} \quad (201)$$

Thus, we see that the non-Abelian nature of QCD causes $\Delta C_j \neq 0$.

The additional term in (199), which arises from color algebra, consists of what we call the “singlet” exchange Born amplitude which is obtained from Fig. 1a by substituting λ^0 for the color matrices λ^a , where $\lambda^0 \equiv I/\sqrt{6}$. The corresponding value of $\tilde{B}_{\text{QCD}}$ in (199) is obtained from (200) by setting $C_F \equiv 0$ and $\Delta C_t = 0$ in (200) and by setting

$$\Delta C_u = \Delta C_s = \begin{cases} -C_F, & qq' \text{ incoming} \\ C_F, & q\bar{q}' \text{ incoming} \end{cases} \quad (202)$$

in (200). We note that, to order α_s , the “singlet” amplitude does not contribute to our cross section for the process in Fig. 1.

Turning now to the soft real emission process in Fig. 1b, we again follow the methods of YFS in Ref. [2] and extract the infrared-divergent part of the real gluon bremsstrahlung as

$$d\sigma_{\text{soft}}^{B1} = d\sigma_0(2\alpha_s \tilde{B}_{\text{QCD}}) \quad (203)$$

where the real QCD infrared function is

$$2\alpha_s \tilde{B}_{\text{QCD}} = \int^{k \leq K_{\text{max}}} \frac{d^3 k}{k_0} \tilde{S}_{\text{QCD}}(k) \quad (204)$$

with

$$\begin{aligned} \tilde{S}_{\text{QCD}}(k) = & -\frac{\alpha_s}{4\pi} \left\{ C_F \left(\frac{p_1}{p_1 \cdot k} - \frac{q_1}{q_1 \cdot k} \right)^2 + \Delta C_t \frac{2p_1 \cdot q_1}{k \cdot p_1 k \cdot q_1} + C_F \left(\frac{p_2}{p_2 \cdot k} - \frac{q_2}{q_2 \cdot k} \right)^2 \right. \\ & + \Delta C_t \frac{2p_2 \cdot q_2}{k \cdot p_2 k \cdot q_2} \\ & + C_F \left(\frac{p_1}{p_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right)^2 - \Delta C_s \frac{2p_1 \cdot p_2}{k \cdot p_1 k \cdot p_2} + C_F \left(\frac{q_1}{q_1 \cdot k} - \frac{q_2}{q_2 \cdot k} \right)^2 - \Delta C_s \frac{2q_1 \cdot q_2}{k \cdot q_1 k \cdot q_2} \\ & \left. - C_F \left(\frac{q_1}{q_1 \cdot k} - \frac{p_2}{p_2 \cdot k} \right)^2 + \Delta C_u \frac{2q_1 \cdot p_2}{k \cdot q_1 k \cdot p_2} - C_F \left(\frac{q_2}{q_2 \cdot k} - \frac{p_1}{p_1 \cdot k} \right)^2 + \Delta C_u \frac{2q_2 \cdot p_1}{k \cdot q_2 k \cdot p_1} \right\}, \end{aligned} \quad (205)$$

where K_{max} corresponds to the relevant gluon jet detector resolution energy (~ 3 GeV at the SSC/LHC) and $k_0 \equiv \sqrt{\vec{k}^2 + \lambda^2}$ so that we regulate the infrared singularities in (205) in complete analogy with our gluon mass regulator in (200). (Note that graph (v) in Fig. 1c is not infrared-divergent.) Here, $d\sigma_0$ is the respective Born differential cross section. The results (199) to (205) represent the complete infrared (IR) structure of QCD for the cross section for $q + (\bar{q})' \rightarrow q'' + (\bar{q})''' + (G)$ to order α_s . The result (205) is consistent with the pioneering work of Berends and Giele in Ref. [7]. Following the YFS theory, we see that the criterion for the cancellation of the $O(\alpha_s)$ and IR singularities in the cross section for our scattering process in Fig. 1 is that

$$\text{SUM}_{\text{IR}}(\text{QCD}) \equiv 2\alpha_s \tilde{B}_{\text{QCD}} + 2\alpha_s \text{Re} B_{\text{QCD}} \quad (206)$$

should be independent of λ . Using the well-known results [2],[8] for the integrals in (200) and (205) (notice that these are just the same integrals that arise in the YFS analysis of QED with different color-based weights) we see that, indeed, λ does cancel out of (206). We are left with the fundamental result (for, e.g., $m_q = m_{q'} = m$)

$$\text{SUM}_{\text{IR}}(\text{QCD}) = \frac{\alpha_s}{\pi} \sum_{A=\{s,t,u,s',t',u'\}} (-1)^{\rho(A)} (C_F B_{\text{tot}}(A) + \Delta C_A B'_{\text{tot}}(A)) \quad (207)$$

where

$$B_{\text{tot}}(A) = \log(2K_{max}/\sqrt{|A|})^2 (\ln(|A|/m^2) - 1) + \frac{1}{2} \ln(|A|/m^2) - 1 - \pi^2/6 + \theta(A)\pi^2/2 \quad ,$$

$$B'_{\text{tot}}(A) = \log(2K_{max}/\sqrt{|A|})^2 + \frac{1}{2} \ln(|A|/m^2) - \pi^2/6 + \theta(A)\pi^2/2 \quad (208)$$

and

$$\rho(A) = \begin{cases} 0, & A = s, s', t, t' \\ 1, & A = u, u' \end{cases} \quad (209)$$

The results (207) and (208) are then the fundamental results of our analysis.

Indeed, repeating the arguments in Ref. [2] and/or using the factorization results in Ref. [7], we see that by virtue of (207), we may compute the hard gluon radiation residuals at $O(\alpha_s)$ as

$$\bar{\beta}_0 = d\sigma^{1-loop} - 2\alpha_s \text{Re} B_{\text{QCD}} d\sigma_0 \quad (210)$$

and

$$\bar{\beta}_1 = d\sigma^{B1} - \tilde{S}_{\text{QCD}} d\sigma_0 \quad (211)$$

in the exponentiated formula

$$d\sigma_{exp} = \exp[\text{SUM}_{\text{IR}}(\text{QCD})] \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^n \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{+iy(p_1+p_2-q_1-q_2-\sum_j k_j)+D} \bar{\beta}_n(k_1, \dots, k_n) \frac{d^3 q_1 d^3 q_2}{q_1^0 q_2^0} \quad (212)$$

where

$$D = \int \frac{d^3 k}{k_0} \tilde{S}_{\text{QCD}} \left[e^{-iy \cdot k} - \theta(K_{max} - |\vec{k}|) \right] \quad (213)$$

Here, we use the virtual 1-loop correction

$$d\sigma^{1-loop} = d\sigma_0 \delta_{vir}, \quad (214)$$

where

$$\begin{aligned} \delta_{vir} = \frac{2\alpha_s}{\pi} & \left[(C_F - \frac{1}{2}C_A) \left\{ \ln \frac{|t|}{m^2} \left(2 - \ln \frac{|t|}{\lambda^2} + \frac{1}{2} \ln \frac{|t|}{m^2} \right) \right. \right. \\ & + \ln \frac{s}{m^2} \left(2 \ln \frac{s}{\lambda^2} - \ln \frac{s}{m^2} \right) + \frac{tu}{s^2+u^2} \ln \frac{s}{|t|} \left(\frac{1}{2}(s/u - 1) \ln \frac{s}{|t|} - 1 \right) \Big\} \\ & - (C_F - \frac{1}{4}C_A) \left\{ \ln \frac{|u|}{m^2} \left(2 \ln \frac{|u|}{\lambda^2} - \ln \frac{|u|}{m^2} \right) + \frac{st}{s^2+u^2} \ln \frac{u}{t} \left(\frac{1}{2}(1 - u/s) \ln \frac{u}{t} - 1 \right) \right\} \\ & \left. + C_F \ln \frac{|t|}{\lambda^2} + C_A \left(\ln \frac{|t|}{m^2} + 1 \right) + \frac{31}{36}C_A - \frac{5}{9}C_F + \frac{\pi^2}{6}C_F \right] \quad (215) \end{aligned}$$

The respective soft real gluon bremsstrahlung cross is given by (203) with

$$\begin{aligned}
2\alpha_s \tilde{B}_{QCD} = & \frac{2\alpha_s}{\pi} \left[(C_F - \frac{1}{2}C_A) \left\{ \ln \frac{4K_{max}^2}{\lambda^2} + \ln \frac{|t|}{m^2} \left(1 + \ln \frac{4K_{max}^2}{\lambda^2} - \frac{1}{2} \ln \frac{|t|}{m^2} \right) \right. \right. \\
& \left. \left. - 2 \ln \frac{s}{m^2} \left(1 + \ln \frac{4K_{max}^2}{\lambda^2} - \frac{1}{2} \ln \frac{s}{m^2} \right) \right\} \right. \\
& \left. + (C_F - \frac{1}{4}C_A) \left\{ -2 \ln \frac{4K_{max}^2}{\lambda^2} + 2 \ln \frac{|u|}{m^2} \left(1 + \ln \frac{4K_{max}^2}{\lambda^2} - \frac{1}{2} \ln \frac{|u|}{m^2} \right) - \frac{\pi^2}{3} \right\} \right]
\end{aligned} \tag{216}$$

. It allows $\bar{\beta}_0$ to be identified as $d\sigma_{soft}^{O(\alpha_s)} - 2\alpha_s(\tilde{B}_{QCD} + B_{QCD})d\sigma_0$, where $d\sigma_{soft}^{O(\alpha_s)} = d\sigma^{1-loop} + d\sigma_{soft}^{B1}$, for example. The above expressions are for incoming $q\bar{q}$ and need to be modified accordingly for qq interactions. The hard gluon residual $\bar{\beta}_1$ makes a vanishing contribution in the soft gluon limit so that it will be presented elsewhere [5]; for, here we focus on the soft gluon effects in (212) so that we will not need $\bar{\beta}_1$ in the current discussion.

The results (210) and (214) – (216) are now realized in (212) via Monte Carlo methods by replacing the QED forms of $\bar{\beta}_n$, SUM_{IR} as described in Refs. [1] for our SSCYFS2 and SSCBHL event generators by the respective QCD forms. The resulting event generators are SSCYFSG and SSCBHLG, where in the former, only only gluon radiation from the initial state is treated. Sample Monte Carlo data from SSCBHLG are illustrated in the next section. Similar data for SSCYFSG will appear elsewhere [5].

We should stress that two limits of the QCD coupling constant are needed in (210), (211) and (212). One is the perturbative $-q^2 \gg \Lambda_{QCD}^2$ regime in the hard gluon residuals $\bar{\beta}_n$ in (212). This regime is well-known and we use the standard type of formula [9]

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2n_f) \ln(\mu^2/(\Lambda_{n_f}^{\overline{MS}})^2)} \tag{217}$$

with n_f = number of open flavors and $\Lambda_n^{\overline{MS}}$ the respective value of Λ_{QCD} : we take $\Lambda_4^{\overline{MS}} \simeq .238$ GeV for definiteness.

The second regime occurs in SUM_{IR} in (207), where the $k^2 \rightarrow 0$ limit is relevant for “on-shell” gluons of 4-momentum k . (We follow Tarrah [10] and use the

concept of on-shell quarks and gluons in perturbation theory at large momentum transfer.) Recently [11], it has been pointed out by Mattingly and Stevenson that this limit of α_s/π exists as an infrared fixed point and its limiting value is $\simeq .263$. We use this value of (α_s/π) in SUM_{IR} in (207) and (211). Because of the fixed point behavior, the value one uses for the $k^2 \rightarrow 0$ coupling of gluons is not sensitive to the precise value $\Lambda_4^{\overline{MS}}$, for example. Further, since we only work to $O(\alpha_s)$ in the hard interactions, there is no contradiction between our use of $\Lambda^{\overline{MS}}$ values in (217) and the renormalization scheme of Mattingly and Stevenson in Ref. [11].

We should also note that our light quark masses are always those determined by Leutwyler and Gaisser in Ref. [12]: $m_u(1 \text{ GeV}) \simeq 5.1 \times 10^{-3} \text{ GeV}$, $m_d(1 \text{ GeV}) \simeq 8.9 \times 10^{-3} \text{ GeV}$, $m_s(1 \text{ GeV}) \simeq .175 \text{ GeV}$, $m_c(1 \text{ GeV}) \simeq 1.3 \text{ GeV}$, $m_b(m_b) \simeq 4.5 \text{ GeV}$; for m_t , we take $m_t(m_t) \simeq 164 \text{ GeV}$. The running of these masses is readily incorporated into our calculations [12]; we ignore this running in the current paper, since it does not affect our results at the level of accuracy of interest to us here.

With these explanatory remarks in mind, we now turn to explicit Monte Carlo data illustrations for multiple gluon effects in $q + (\bar{q})' \rightarrow q'' + (\bar{q})''' + n(G)$ at SSC/LHC energies in the ATLAS-CMS(SDC-GEM) acceptances.

Chapter 4

Results

In this Chapter, we illustrate our multiple-gluon Monte Carlo event generators in the sample case of $u + u \rightarrow u + u + n(G)$, where we require the out-going u quarks to satisfy $|\eta| \leq 2.8$ and $E > 58$ GeV, for definiteness, at $\sqrt{s} = 15.4$ TeV. (We comment about the SSC case $\sqrt{s} = 40$ TeV as well.) For completeness, we will show explicit Monte Carlo data for SSCBHLG, since in that generator initial, initial-final interference, and final state $n(G)$ radiation is simulated.

Our results are shown in Figs. 8, 9 and 10, where we show respectively the gluon multiplicity, the distribution of $v = (s - s')/s$, where s' is the squared invariant mass of the final quark-quark system, and the total gluon transverse momentum in TeV units. What we see is that there is a pronounced effect from the multi-gluon radiation at $\sqrt{s} = 15.4$ TeV: the average value of the number of radiated gluons with energy $E_G > 3$ GeV is, e.g., at $\sqrt{s} = 15.4$ TeV,

$$\langle n_G \rangle = 28.5 \pm 5.5. \quad (218)$$

The average value of v is, for $\sqrt{s} = 15.4$ TeV,

$$\langle v \rangle = 0.92 \pm 0.14. \quad (219)$$

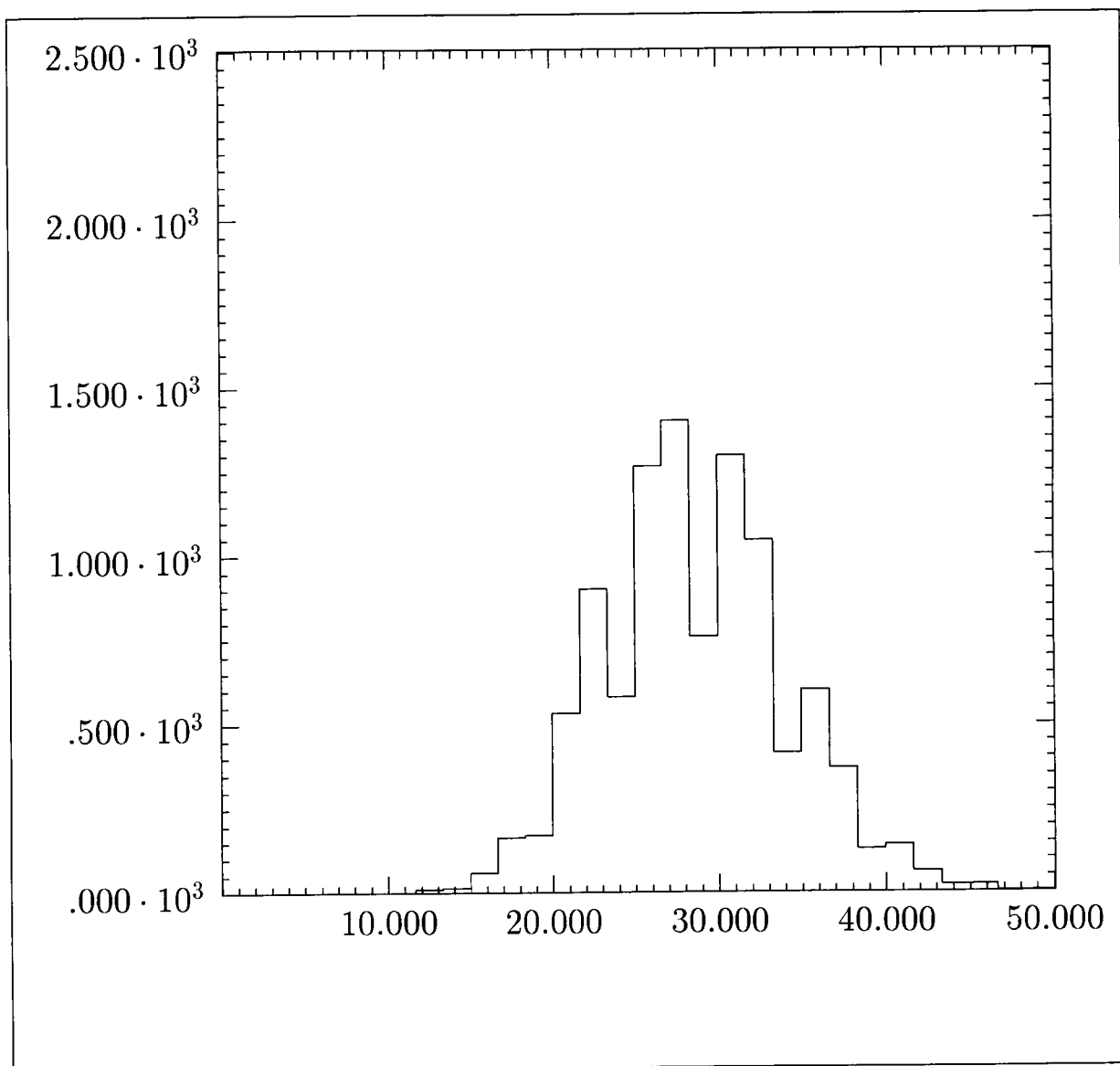


Figure 8: Gluon multiplicity distribution in $u+u \rightarrow u+u+n(G)$ at $\sqrt{s} = 15.4$ TeV.

The average value of the total transverse momentum in gluons is

$$\langle p_{\perp, tot} \rangle \equiv \langle |\sum_{i=1}^n \vec{k}_{i\perp}| \rangle = (0.19 \pm 0.23) \text{ TeV}. \quad (220)$$

This means that a substantial amount of incoming and outgoing energy is radiated into gluons (due to the usual collinearity of radiation with its source, most of the final state radiation would be a part of the typical jet associated with its parent quark). Entirely similar results hold for the SSC $\sqrt{s} = 40$ TeV case. Evidently, any realistic treatment of QCD corrections to LHC/SSC processes must analyze the full event-by-event n -gluon effects as they interact with the detector efficiencies and cuts. Our SSCBHLG event generator provides the first amplitude-based exponentiated Monte Carlo realization of such n -gluon effects and we hope to participate in a study of their interplay with detector effects elsewhere [5].

An important consequence of our work is a prediction for the effect on the overall normalization of an LHC/SSC physics process due to multiple-gluon radiation, in an unambiguous way, since all IR-singular effects have canceled out of our calculation. For example, if we simply compute the $O(\alpha_s)$ soft cross section, $E_G < 3$ GeV, we get (for $u + u \rightarrow u + u + (G)$, $\sqrt{s} = 40$ TeV) that

$$\sigma/\sigma_{Born} = 1 + \delta_{vir} + \delta_{real} = -22.1, \quad (221)$$

whereas if we compute the cross section for $u + u \rightarrow u + u + n(G)$ and require $v < 3$ GeV/7.7 TeV = .00039, we get

$$\sigma/\sigma_{Born} = 1.2 \times 10^{-11}. \quad (222)$$

Hence, we see that multi-gluon radiation is crucial to getting the proper normalization of the cross section.

We emphasize that the results for $\sqrt{s} = 40$ TeV give similar conclusions [5] to those in Figs. 8-10. Entirely analogous results hold for the pure initial-state multigluon event generator SSCYFSG, except only two lines radiate, so that $\langle n_G \rangle$ is reduced by a factor ~ 2 , with the appropriate corresponding reductions in $\langle v \rangle$

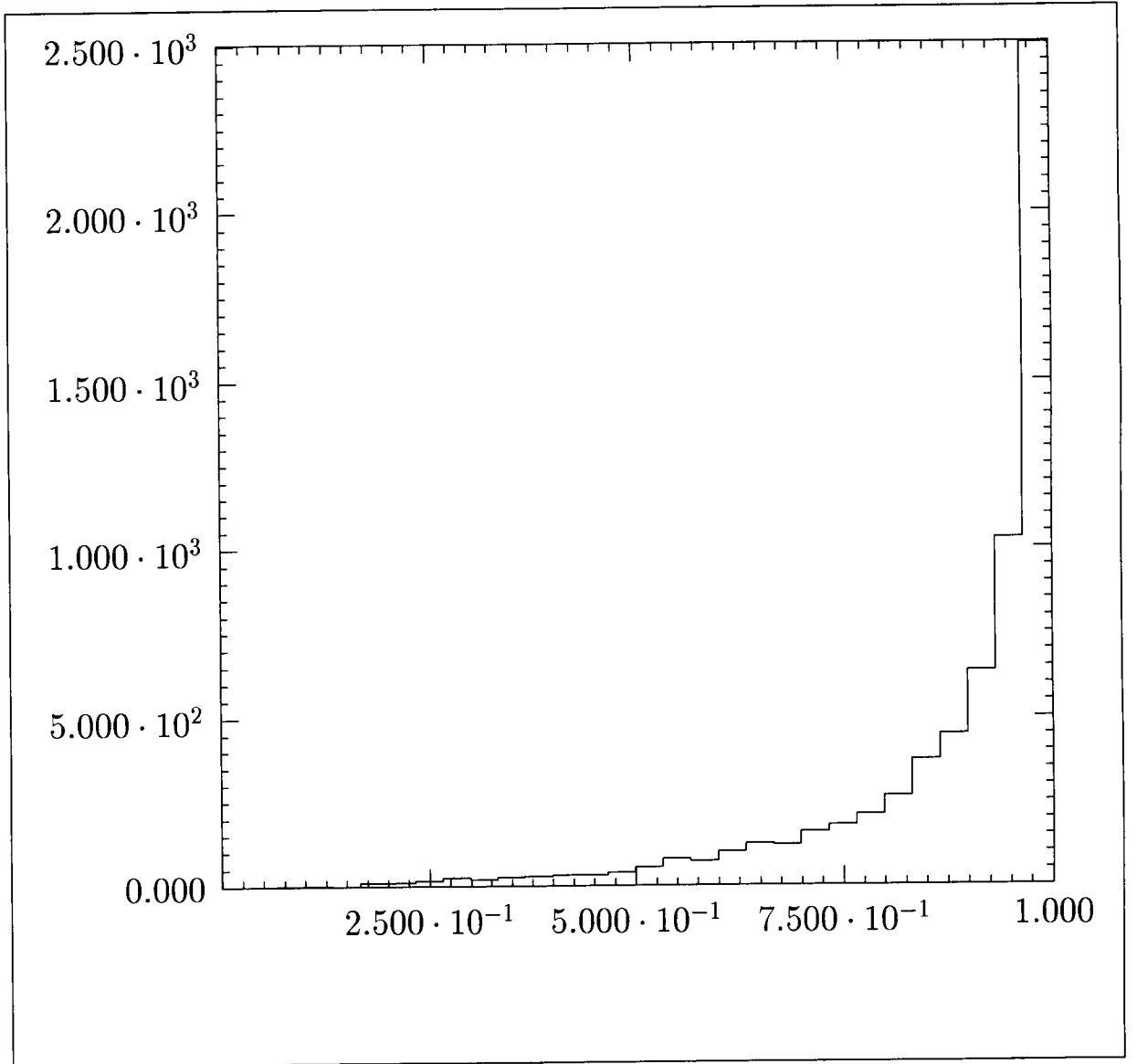


Figure 9: v -distribution in $u + u \rightarrow u + u + n(G)$ at $\sqrt{s} = 15.4$ TeV.

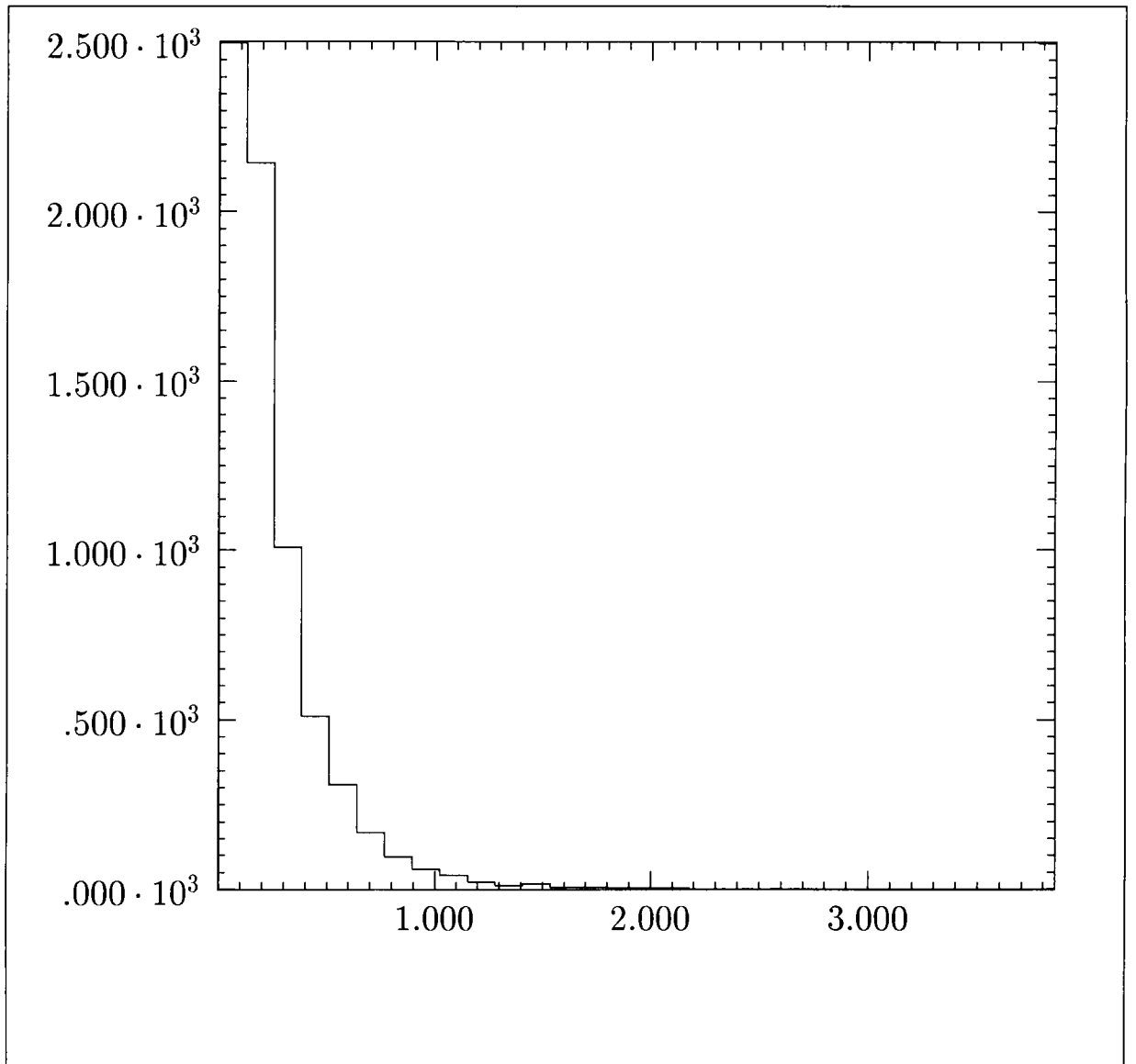


Figure 10: Total transverse momentum of gluons in $u + u \rightarrow u + u + n(G)$ at $\sqrt{s} = 15.4$ TeV. The units of the horizontal axis are TeV.

and $\langle p_{\perp, tot} \rangle$. Such results are effectively an integration over the events used in Figs. 8–10, wherein one integrates over all gluons within some cones close to the outgoing fermions. We do not show a separate set of plots for SSCYFSG here but we have presented a more detailed discussion of the respective initial-state n -gluon radiation elsewhere [5]. Here, we have focused on the complete gluon radiation problem.

Chapter 5

Conclusions

In this work, we have shown that the IR singularities in fermion-(anti-)fermion scattering at LHC/SSC energies in QCD cancel at order α_s , permitting an immediate extension of YFS QED exponentiation methods to such processes. The YFS Monte Carlo methods invented by S. Jadach and B.F.L. Ward [32] for QED radiation in Z^0 physics can then be extended to multi-gluon Monte Carlo event generators with well-defined IR behavior to all orders in α_s . Such a Monte Carlo realization of amplitude-based n -gluon effects opens the way to a new era in higher-order QCD corrections to hard processes such as those of interest to the LHC/SSC physics program.

More specifically, we have realized our exponentiated, n -gluon effects via the Monte Carlo event generators SSCBHLG and SSCYFSG. Since the latter amounts to an initial-state restriction of the former, we have presented results from the former generator in this work. We find, for example, that the average number of radiated gluons in $u + u \rightarrow u + u + n(G)$ at $\sqrt{s} = 15.4$ TeV is 28.5 ± 5.5 and that $\langle v \rangle = 0.92 \pm 0.14$. Further, the cross-section normalization is strongly affected by our n -gluon effects. Hence, only by taking these effects into account, as they interact with detector simulation, for example, can a truly detailed picture of the SSC/LHC physics platform be obtained. We hope to participate in such an investigation elsewhere.

In summary, we have presented, for the first time, the realistic event-by-event Monte Carlo realization of multi-gluon effects in hard QCD processes at SSC/LHC energies in which IR singularities are canceled to all orders in α_s . A new approach to LHC/SSC physics has thus been created.

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Vita

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