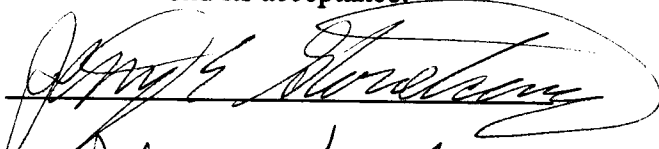


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Date 19 November 1992

A Large Volume Strain Plasticity Constitutive Model for Impact Limiter Materials

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Kevin Herbert Brown
December 1992

DEDICATION

This thesis is dedicated to my parents
Mr. Harry K. Brown and Mrs. Annette F. Brown
who have given me invaluable opportunities.

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ABSTRACT

In medical as well as military applications it is necessary to transport modest amounts of nuclear material. The statistically safest way to transport these materials is by airplane. The nuclear material must be transported in a container that will not expose the public to any radiation even if the airplane crashes. The tests to verify the survivability of this type of package are very difficult and expensive. Because they are so difficult and expensive, it is useful, perhaps even necessary, to supplement the tests with numerical simulations.

The finite element method is well suited to perform this type of analysis if the proper material description or constitutive model is available. This thesis develops a material model that yields both volumetrically and deviatorically like the packaging materials used in air transportable containers for nuclear materials. The model is tailored to a new design under consideration that uses wrapped screen wire and Kevlar cloth. As an alternative material to the wrapped screen wire and Kevlar, perforated plate is also considered.

Material tests are performed on screen wire and Kevlar cloth to determine the necessary material properties. The material properties of perforated plate are determined computationally using the finite element method. These properties are then fit to the constitutive model developed in this work.

The constitutive model consists of two parts. The first is an isotropic crush model which consists of a yield cone capped by an ellipsoidal end described in the pressure- $\sqrt{J_2}$ stress space. The elastic material constants are variable and depend on how much plastic volume crushing has taken place locally in the material. The isotropic crush model yields both volumetrically and deviatorically. Non-associative flow rule are used on both surfaces, although both flow rules are associative in the limited deviatoric stress space.

The second part of the constitutive model simulates the in-plane tensile behavior of the screen wire and Kevlar fibers. The in-plane tensile model is treated independently of the isotropic crush model. The in-plane tension model is subjected to the same deformations as the isotropic part of the model and the resulting stress fields from the two parts are superimposed giving an anisotropic material description. The constitutive model is implemented and verified as a stand alone model first and then introduced into a non-linear large strain finite element code.

It was concluded that the constitutive model developed in this work could be applied to a wide class of volumetrically yielding materials. It was also concluded that the test procedures developed for testing the material properties of screen wire and Kevlar could also be applied to other similar materials. Finally, future areas of study were recommended.

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LIST OF SYMBOLS

a_0, a_1	Constants to Determine the Shape of the Conical Yield Surface
C	Ratio of Major to Minor Axis on the Ellipsoidal Cap
ε	Logarithmic Strain
d_{ij}	Stretching Tensor
d_{ij}^e	Elastic Part of the Stretching Tensor
d_{ij}^p	Plastic Part of the Stretching Tensor
δ_{ij}	Croneker Delta
ε_{vp}	Logarithmic Plastic Volume Strain
e_v^*	Logarithmic Volume Strain Since Tensile Fracture
E	Young's Modulus
E_{eff}	Effective Young's Modulus
E_{ijkl}	Matrix of Elastic Constants
$[F]$	Deformation Gradient
f_c	Yield Function for the Cone
f_{eps}	Yield Function for the Ellipsoidal Cap
G	Elastic Shear Modulus
J_2	Second Invariant of the Deviatoric Stress Tensor
K	Elastic Bulk Modulus
$\lambda, \hat{\lambda}$	Constant used in Calculation of the Final Stress State During Yielding
ν	Poisson's Ratio
σ	Cauchy Stress
p	Pressure
p_c	Pressure Coordinate of the Center of Ellipsoidal Cap

p_{frac}	Tensile Fracture Pressure
$[R]$	Rotation Tensor in the Polar Decomposition of the Deformation Gradient
S_{ij}	Deviatoric Cauchy Stress
t	Time
$[V]$	Symmetric Part of the Deformation Gradient
V_{ij}	Direction of the Plastic Stretching Tensor

CHAPTER 1

General Introduction and Overview

Introduction

In medical, as well as defense applications, it is necessary to transport small amounts of nuclear materials. These materials are sometimes placed inside a containment or cask and transported by airplanes since this is statistically safer than other forms of transportation. To ensure the public's safety, the casks must be able to survive the most severe plane crash without leaking.

The tests to demonstrate the survivability of a package of this type are extremely expensive and difficult to perform. Because of these facts, it is desirable to be able to analyze a package design with a numerical method as a supplement to physical tests. The finite element method is well suited for this type of analysis. The theory and applicability of this method are well documented in codes such as PRONTO2D (Taylor and Flanagan, 1987), SPECTROM331 (Key, 1987), JAC2D (Biffle, 1981), DYNA3D (Hallquist, 1981), and many other commercial codes.

The work presented in this thesis concentrates on developing a constitutive model to describe materials used for these packages. As part of this development, material tests are performed on specific materials and fitted to this model. This work was funded by Sandia National Laboratories, Albuquerque, New Mexico. Much of the work presented here appeared in the final report to Sandia (Krieg and Brown, 1992).

Because these packages are transported by airplane, a crash could subject these packages to a severe environment. This environment is represented by a set of tests to

which a package is subjected to in succession. The first is a perpendicular impact onto a flat unyielding target at a velocity of 422 feet per second with the package in the most severe orientation. This requirement is meant to represent the initial impact of the crash. The next test a package must survive is a crush test which is meant to represent the weight of the aircraft hull applied to the package after the package has come to rest. This test is represented by applying a static load of seventy thousand pounds transmitted through a two inch wide steel bar. To represent the possible puncture from part of the aircraft or part of its cargo, a steel probe weighing five hundred pounds is dropped onto the package from a height of ten feet. If the package weighs less than five hundred pounds, this criteria is met by dropping the package on the probe. To represent part of the aircraft or its contents tearing the package, the package must survive a steel angle dropped one hundred fifty feet onto the package twice with the package tilted at an angle of forty-five degrees. Because it is possible that the aircraft will burst into flames upon impact, the package must then be subjected to a pool fire of aviation fuel for a period of one hour and left to self-extinguish or be extinguished after three hours, whichever is worse. As a final test, the package must be submerged under three feet of water for eight hours.

Any external loads cause stresses and strains in a body. Upon removal of these loads the body may or may not return to its original configuration. If all deformations are recoverable, the process is called elastic. If upon removal of all loads, all deformations are not recovered, the unrecovered deformations are called plastic deformations. Yielding is defined as the region where plastic deformations occur. The yield criterion is defined as the limit of elastic deformations expressed by a combination of states of stress (Desai, 1984). Because of the application, most materials used for this type of package yield volumetrically. A new design currently being explored consists of a titanium

containment surrounded by wrapped screen wire and Kevlar®* cloth and placed inside a steel drum (Pierce, 1992). The screen wire is intended to absorb the majority of the energy of the impact through plastic work. The Kevlar is added to provide additional tensile strength to prevent the package from fracturing during the impact and to help in fire retardation.

Volumetric yielding presents a difficulty in performing analyses for this type of package. The majority of constitutive models in finite element codes today decompose the stress state into volumetric and deviatoric components. The volumetric component, as its name implies, is the component associated with volume change. The deviatoric component is the component associated with a constant volume distortion of shape as in a shearing mode. The volumetric component in conventional metal plasticity is treated as elastic while only the deviatoric component is treated as elastic-plastic. The lack of volumetric yielding make constitutive models of this type inappropriate for modeling materials used for impact limiters. Most constitutive models that do incorporate volumetric yielding, are used to simulate soils, concretes, or other geologic materials. In general, these models are also inappropriate for performing analysis on impact limiters.

Another complication in performing analyses on impact limiters, is that the materials used in these limiters are usually not isotropic. A design in use today, uses grain oriented wood with steel outer casings. The properties of wood depend on the direction of the grain, therefore wood is anisotropic in behavior. A new design, proposed by Pierce, uses layered screen wire and Kevlar which is also anisotropic. The screen wire can not carry any tension out of the plane of the weave and in the plane of the weave it only carries tension in the two orthogonal directions of the weave and its shear strength is

* Kevlar is a registered trademark of DuPont Corporation and is considered to be such throughout the remainder of this manuscript.

only carries tension in the two orthogonal directions of the weave and its shear strength is virtually zero. In addition, the Kevlar can carry tensile loads in its two orthogonal in-plane direction but can not carry any tensile load in the out-of-plane direction. The majority of constitutive models in use today are for isotropic materials and are unable to model this type of orthogonal behavior.

History of Transport Packages

The first certified package for the air transport of nuclear materials is PAT-1 (Plutonium Air Transport), which was developed in the late 1970's (Andersen et. al., 1978). PAT-1 was designed to carry 3.15 kilograms of plutonium dioxide but was only certified to carry 2 kilograms. The regulation concerning packages of this type is NUREG-0360 (U.S. Nuclear Regulatory Commission, 1978).

PAT-1 weighs five hundred pounds including the payload of plutonium with the containment and sample representing approximately six percent of the total weight. PAT-1 was made from a steel inner tube surrounded by redwood with load spreaders further surrounded by grain oriented redwood encased in two steel drums. Because of the use of wood as an impact limiter, it is necessary to know the properties of wood under impact loading. Johnson (1986) has a paper discussing impact on several different types of wood.

PAT-2 (Andersen et. al., 1981), which was developed in the early 1980's, was designed to carry only 15 grams of plutonium-239, which is significantly less than the 3.15 kilograms that PAT-1 carries. PAT-2 weighs between seventy and seventy-four pounds depending on the exact properties of the materials used in its construction. PAT-2 is made from a steel inner package surrounded by grain oriented maple with load

spreaders, further surrounded by grain oriented redwood encased in a double wall of steel.

PAT-1 and PAT-2 are very expensive to build. In addition, it has been found that these packages are not easily scaled to accommodate different size payloads. Because of these facts, a new design was needed that is cheaper to manufacture and can be scaled to accommodate different size payloads. A new package currently under design is made from wrapped screen wire and Kevlar cloth (Pierce, 1992). Patents are currently pending on this design. This package is capable of carrying up to 8 kilograms of plutonium dioxide. All tests have been performed on a one quarter scale package which is thirty inches in length and 14.5 inches in diameter. A series of initial impact tests, run at speeds up to 636 feet per second, demonstrated that this design is feasible (Pierce, 1992).

The environment that PAT-1 and PAT-2 were certified under is a 422 feet per second impact into an unyielding target in the worst configuration. Additional requirements included puncture, fire resistance, slash, and submersion in water. This environment has recently been questioned due to the occurrence of plane crashes at even higher speeds. Because of this uncertainty, a 650 feet per second impact into an unyielding target is being used to test the current design by Pierce.

Background of Plasticity and Constitutive Models

A general historical review of plasticity can be found in Hill (1950), Mendelson (1968), Kachanov (1971), and Slater (1979). There are two major aspects that constitute plastic behavior (Desai, 1984). The first is the yield condition and the second is the post yield behavior. Both aspects must be addressed to develop a constitutive model in plasticity.

The majority of early constitutive relations used to describe metals assume that the volumetric behavior is elastic and is uncoupled from the deviatoric behavior. Many constitutive models have been developed using this assumption. In fact, the majority of constitutive models implemented in finite element codes today use an elastic volumetric assumption.

Volumetrically yielding materials are used in many everyday applications, especially as impact limiters. For example, wooden crates are used to transport heavy packages. Polyurethane and Styrofoam are used every day for packaging of fragile items. Honeycomb has traditionally been used to mitigate impact loads. Another indirect application of a volumetrically yielding package is a suitcase on an airplane. Under impact loadings, the suitcase is usually dented but the contents are undamaged.

The majority of work concerning volumetrically yielding materials has been concentrated in one of three general areas. The first, and probably most studied area, is that of geologic materials such as concrete, soils, and clays. A little work has been devoted in recent years to the study of open and closed cell foams as well as honeycomb which was designed to mitigate impact loads. A third area which has received considerable attention is the characterization of voids in metals.

Geologic Materials

Considerable work has been devoted to this class of materials because of their use as foundations as well as many other reasons. Recent works in this area include Shreyer and Babcock (1985), Shreyer and Bean (1985), and Kavvadas (1991). A wide range of papers can be traced from these references. Desai (1984) has a book on constitutive modeling which emphasizes this class of materials. He includes several cap models to

describe volumetric yielding as well as the general plasticity models. A brief description of these particular models are presented below.

Drucker-Prager Model

The original plasticity descriptions applied to metals used only the second invariant of the deviatoric stress and did not take into account the pressure in the description of the yield surface. This was not an adequate model to describe all materials, particularly geologic materials. Drucker and Prager (1952) suggested a model that accounts for the effects of all principal stresses by using the first and second invariants of the stress. The yield function for this model is

$$f = \sqrt{J_{2D}} - \alpha J_1 - k \quad (1.1)$$

where α and k are positive material parameters, J_1 is the first invariant of the stress tensor and is defined as

$$J_1 = \sigma_{kk} \quad (1.2)$$

and J_{2D} is the second invariant of the deviatoric stress tensor which is defined as

$$J_{2D} = \frac{1}{6} [(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2)] \quad (1.3)$$

An associative flow rule is used in this model so that the direction of the plastic increment is

$$d\epsilon^p = \lambda \frac{\partial f}{\partial \sigma} \quad (1.4)$$

where $d\epsilon^p$ is the plastic strain increment and λ is a positive constant selected such that the final stress state is on the yield surface.

A Third-Invariant Plasticity Theory for Low-Strength Concrete

Third invariant models are used because they allow for different behavior in tension than in compression which is often seen in geologic materials. A third invariant plasticity theory for low-strength concrete (Shreyer and Babcock, 1985) uses the first and third invariants of the stress rather than the traditional first and second invariants. The first invariant is the traditional trace of the stress tensor or the pressure. The other invariant used is a particular combination of the three stress invariants, which is also invariant, and is defined by

$$L = - \left[(\sigma_1 - \sigma_s)(\sigma_2 - \sigma_s)(\sigma_3 - \sigma_s) - \sigma_s^3 \right]^{1/3} \quad (1.5)$$

where σ_s is defined as the cohesion. Because an associative flow rule provided too much dilatation, the potential function, which is slightly different from the flow function, is defined as

$$\phi^* = \gamma^* P - L - \sigma_s \quad (1.6)$$

where P is the pressure and γ^* is the strain hardening function plus a material constant.

The non associative flow rule implies that the inelastic strain increment is given by:

$$d\epsilon^i = d\lambda \frac{\partial \phi^*}{\partial \sigma} \quad (1.7)$$

where λ is a positive parameter. The authors implement this theory in its full generality for a low-strength concrete and demonstrate the merit of this model. This model does not incorporate the tensile behavior, strain softening, or strain rate effects.

Third-Invariant Model for Rate-Dependent Soils

The third-invariant model for rate-dependent soils (Shreyer and Bean, 1985) is very similar to the model described above. The main difference is that this model incorporates rate effects where the model above does not. This model also uses the same invariants of the stress as the model above. Again, a non associative flow rule is used and the potential function is given by Eq. (1.6) while the flow rule is given by Eq. (1.7).

The strain rate effect is incorporated into the model using the relation

$$\sigma_R = \sigma_0 \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^m \quad (1.8)$$

where σ_R is additional cohesion due to strain rate, σ_0 and m are material properties, $\dot{\epsilon}_0$ is a reference strain rate, usually one per second, and $\dot{\epsilon}$ is the current strain rate. The authors use this model and compare the results with experimental results using two sandy materials.

Foams

Krieg Model

The simplest model is one developed for lightweight concrete but has been used as a soils and crushable foams model (Krieg, 1978). This model serves as the basis for many of the volumetric models. This model decomposes the stress state into a deviatoric and a volumetric component. These components are treated separately with an explicit one-way coupling occurring in the yield surface for the deviatoric part. The yield surface consists of cone with a flat cap on the end (See Figure 1.1*).

In this model, the pressure is assumed to be a function of the volume strain on loading as

$$p = f(\epsilon_v) \quad (1.9)$$

If the stress state is unloading, the pressure is calculated as

$$p = \text{MAX} \left[p_{frac}, \left(p_o - 3K \dot{\epsilon}_v \Delta t \right) \right] \quad (1.10)$$

where p_{frac} is the fracture pressure, p_o is the pressure at the start of the time increment, K is the elastic bulk modulus, $\dot{\epsilon}_v$ is the rate of change of the volume strain, and Δt is the time increment. A tensile fracture is included in this model because most materials of this type fracture under tensile loading.

* All figures are located in Appendix I

The deviatoric behavior is taken to be independent of the volumetric behavior except in the yield condition which is

$$f_s = J_2 - (a_0 + a_1 p + a_2 p^2) = 0 \quad (1.11)$$

where J_2 is the second invariant of the deviatoric stress, p is the pressure, and a_0 , a_1 , and a_2 are constants. The deviatoric behavior is treated as elastic-perfect plastic. A non-associative flow rule is used to solve the plasticity equations for the final stress state.

This model has been the basis for much of the work in volumetric plasticity. This model is very good for hydrostatic loading. It is pretty good if the loading is uniaxial such that the flat end of the loading surface is contacted. The model is not bad deviatorically for materials where the yield surface is circular in cross section. The model is not good if the loading is near the intersection of the two yield surfaces.

Open-Cell Foams

The Krieg model described above generally applies to closed cell foams. Closed cell foams generally have much higher densities than do open cell foams. Open and closed cell foams are used for different applications. Warren and Kraynik (1988) looked at the linear elastic properties of open-cell foams.

The authors begin by considering a unit tetrahedral unit cell with four half-struts of length L . After determining the response to the unit for pure shear and uniaxial extension, they consider a representative volume of cellular material to contain tetrahedral elements of all possible orientations with equal probability. The elastic properties of this representative cell must be isotropic because the results are averaged over all possible orientations and there can be no preferred direction within the material

which exhibits a different elastic response. This allows the effective elastic constants to be determined in terms of L , the half length of the struts, M , an axial compliance, and N , a transverse of bending compliance. The authors use this method to determine the effective elastic constants for a couple of different open-cell foams.

Low Density Polyurethane Foam Model

Neilsen, Morgan, and Krieg (1987) developed a phenomenological constitutive model for low density Polyurethane foams. The authors decompose the stress components σ_{ij} into skeleton stress components and a component to represent the air contribution to the normal stress components as:

$$\sigma_{ij} = \sigma_{ij}^{sk} + \sigma^{air} \delta_{ij} \quad (1.12)$$

where σ_{ij}^{sk} are the skeleton stress components and σ^{air} represents the air contribution to the normal stress components. By considering the ideal gas law the air contribution to the normal stress is found to be

$$\sigma^{air} = \frac{p_0^{air} \left[\gamma + (1-\phi) \left(1 - \frac{T_1}{T_0} \right) \right]}{\gamma + 1 - \phi} \quad (1.13)$$

where γ is the volume strain, ϕ is the volume fraction of solid material, p_0^{air} is the internal air pressure when no load is applied, T_1 is the absolute temperature, and T_0 is the absolute temperature when no load is applied. The skeleton stress response is now determined from experimental data by substituting Eq. (1.13) into Eq. (1.12) to give

$$\sigma_{ij}^{sk} = \sigma_{ij} - \sigma^{air} \delta_{ij} \quad (1.14)$$

The yield stress in each principal direction is expressed as

$$g = A \langle II' \rangle + B (1 + C \gamma) \quad (1.15)$$

where g is the yield stress, II' is the second invariant of the deviatoric strain $\langle \rangle$ is the heavyside step function, and A , B , and C are constants. The constants A , B , and C are determined from experimental data.

At the start of a strain step, the initial skeleton stresses are calculated using Eq. (1.14). Trial skeleton stresses are then calculated using the assumption that the entire step is elastic. Then the principal skeleton stresses and directions are calculated and then each principal stress component is compared to the yield stress in that direction calculated using Eq. (1.15). If the trial stress in the principal direction is greater than the yield stress, then the stress component is set equal to the yield stress. Otherwise, the trial stress is the final stress. The skeleton stresses are then rotated back to the global coordinate system. New normal stresses for the air enclosed is then calculated using Eq. (1.13) and added to the final skeleton stresses to give the final stress state of the material.

The authors fit the constants needed in the constitutive model to experimental data to test the model. The authors then implement the model into a quasi-static and a dynamic finite element code and compare the results obtained to the experimental data. All comparisons were made to static test data. Very good correlation was obtained between the experimental data and the results predicted by the model.

Voids in Metals

Voids in metals has been studied extensively by people in the fracture and failure areas. The growth of the voids and possible fracture or failure have been studied to try and predict the behavior in real structures or machines. This work is of particular importance because it is not possible at this time to manufacture any metal without having some voids or defects in it. Because the voids or defects are unavoidable, it is necessary to know how these voids grow and lead to a failure.

The work done in this area has ranged from looking at a single void in an infinite medium (Glennie, 1972, and Ortiz and Molinari, 1992) to a general porous metal (Duszczyk, 1991). Considerable work has been done on the interaction and growth of voids in a metal, the interested reader is referred to Berg, 1969, Kahlow and Avitzur, 1969, Needleman, 1972, Rice and Tracey, 1969, and Gurson, 1975. Several of these works are outlined below.

Growth of Voids

The Strain Hardening and Rate Sensitive Model for the Growth of a Void (Ortiz and Molinari, 1992) model described here looks at the effect of strain hardening and rate sensitivity for the growth of a spherical void in an unbounded solid under the action of remote hydrostatic tension. The matrix surrounding the void responds to monotonic stressing as a rigid-viscoplastic solid with flow rule

$$d_{ij} = \dot{\epsilon}_e \frac{3S_{ij}}{2\sigma_e} = \frac{\partial \phi(s)}{\partial S_{ij}} \quad (1.16)$$

where d_{ij} is the rate of deformation tensor, S_{ij} is the deviatoric stress tensor, and

$$\dot{\epsilon}_e = \dot{\epsilon}_0 \left(\frac{\sigma_e}{\sigma_0} \right)^m \quad (1.17)$$

$$\phi(s) = \frac{\sigma_0 \dot{\epsilon}_0}{m+1} \left(\frac{\sigma_e}{\sigma_0} \right)^m \quad (1.18)$$

where σ_e and ϵ_e are the effective Von Mises stress and strain respectively and σ_0 , $\dot{\epsilon}_0$, and m are material constants. The flow stress is assumed to obey the hardening law

$$\sigma_0 = \sigma_y \left(\frac{\epsilon_e}{\epsilon_y} \right)^{\frac{1}{n}} \quad (1.19)$$

where σ_y , ϵ_y , and n are material constants.

In this model it is assumed that the void remains spherical throughout the deformation. The authors go on to look at the short and long term behavior since the full equations can not be solved analytically in closed form. They also explore the response to impact loading.

Perforated Plate

Perforated plate of various thicknesses and hole spacing has been looked at because of the relatively high strength to weight ratio, which is one reason it is considered as a possible alternative for an impact limiter. General studies on the stress distribution in perforated plate can be found by Koiter (1960), Slot (1967), and Meijers (1967).

Slot and O'Donnell (1971) also looked at the effective elastic constants for thick (generalized plane strain) and thin (plane stress) perforated plate. Using elasticity theory and the specific geometry of the plates analyzed the authors are able to determine

effective elastic constants for perforated plates with square and triangular penetration patterns. The method used by Slot and O'Donnell is similar to that used in this thesis to determine the effective elastic constants except that instead of purely theoretical basis used by Slot and O'Donnell, computational data is used in this thesis to determine the effective elastic constants. The work presented here also determines the large strain plasticity behavior which was not considered by Slot and O'Donnell.

Thesis Outline

The work presented in this thesis develops a volumetrically yielding constitutive model including the determination of the necessary material properties. Wrapped screen wire and Kevlar is the main design under consideration. As a possible alternative, perforated plate is also considered. In the process of determining the material properties, test methods are developed for testing compressive properties of layered screen wire and Kevlar and for testing the tensile properties of screen wire, Kevlar, and S2-Glass. The design being analyzed uses approximately fifteen layers of screen wire for every one layer of Kevlar. S2-Glass is included in the tensile tests because it is a cheaper alternative to Kevlar. It was not included in the compressive tests because the cloths have a minimal role in that type of test.

The fundamental ideas behind the constitutive model are similar to other cap models developed for volumetrically yielding materials. This work differs from previous constitutive models in that the elastic constants are functions of the plastic volume crush. In addition, the behavior of the model is complicated by including orthotropic rather than the usual isotropic behavior.

The constitutive model developed in this work is intended to be used in conjunction with a large-strain non-linear dynamic finite element code although it will have other applications. The finite element method used in such codes integrates the equations of motion forward in time using an explicit time integration operator. Given the accelerations, the integrator integrates them forward in time to obtain the velocities and displacement at the next time step. The new velocities and displacements are used to calculate new shape functions and a new gradient operator, analogous to the strain-displacement matrix in linear finite elements. The strain increment is found from the strain-displacement matrix and the displacements. The constitutive model is then used to relate the strain increment to a stress increment for each gauss point in each element. Having the new stress, the gradient operator is again used to calculate the nodal loads caused by the internal stresses. Using the nodal loads caused from internal stresses and adding any external loads the global load vector is obtained. Now having the nodal loads, new accelerations are calculated and the iterative process is repeated until the problem is finished.

A fundamental choice in the finite element method described above is the choice of stress and strain measures to use. For a long time, the Cauchy stress, the stretching tensor, and the co-rotational stress flux were used in codes such as HONDOII (Key, et al., 1978). It was later discovered that this stress flux has problems when large shear strains or rotations are encountered. In more modern codes, the unrotated Cauchy stress, the unrotated stretching tensor, and the Green-Naghdi-Dienes (Dienes, 1979) stress flux are used for constitutive modeling and the Cauchy stress is used in the equations of motion. The polar decomposition theorem (Malvern, 1969) is used to decompose the deformation gradient into the product of an orthogonal tensor and a symmetric positive definite tensor. The orthogonal matrix is simply a rotation tensor. The orthogonal tensor $[R]$ is used to unrotate the needed quantities back to the original orientation. By

unrotating the needed quantities, $[R]$ allows the orthotropic behavior of the screen wire and Kevlar to be modeled by always rotating the strain and stress increments back to the material axes.

Chapter 2 of this thesis discusses material testing of screen wire and Kevlar. Methods to perform normal-to-the-plane compressive tests and in-plane tensile tests are developed. For the normal-to-the-plane compressive tests, a fixture is designed to allow the sample to be held in compression until the time the sample is tested but does not interfere with the actual test. For these tests, both axial and lateral strain are measured. For the in-plane tension tests, a method is developed that allows fabric type samples to be held in hydraulic grips without causing failure at the grips.

Chapter 3 is devoted to determining the material properties of stacked perforated plate. Because neither the samples necessary or the testing equipment were available to perform actual experiments, the properties were determined computationally using the finite element method. Both normal-to-the-plane compressive tests and in-plane tensile and compressive tests are modeled. The analysis is performed on an infinite number of stacked plates which are infinite in length and width. Therefore, the results are independent of boundary conditions. This independence is needed in order to construct a constitutive model. Data reduction is performed on the results from the finite element method to determine the parameters necessary for a constitutive model.

In chapter 4, a theoretical constitutive models is developed for wrapped screen wire and Kevlar and for stacked perforated plate. The model for the screen wire and Kevlar is a volumetrically yielding isotropic crush model superimposed on an orthotropic model for the tensile behavior of the screen wire and Kevlar, resulting in a transversely isotropic model. The same model is used for the stacked perforated plate by fitting the constants of the model to the data for the perforated plate.

Chapter 5 details the data fitting of the experimental data to the constants needed for the constitutive model. This is done both for the screen wire and Kevlar but also for the perforated plate. This chapter also shows how other materials of this type can be fit to this model.

Chapter 6 details the numerical implementation of the screen wire and Kevlar constitutive model. This model includes the orthotropic behavior of the screen wire and Kevlar fibers. Also, included in this chapter is a FORTRAN listing of the constitutive model.

In chapter 7, the screen wire and Kevlar constitutive model is implemented in a stand alone environment with a FORTRAN driver and in the finite element code PRONTO2D (Flanagan and Taylor, 1987). The stand alone driver is used to demonstrate the constitutive model and show the model produces realistic results for some very simple test problems under simple loadings. The model is then implemented in PRONTO2D to show the capability of the model in the finite element environment.

Chapter 8 contains the summary and conclusions. The applicability of the constitutive model and the testing procedures are discussed for other materials. In addition, recommendations are made for further areas of study and possible improvements to the constitutive model itself.

CHAPTER 2

Material Testing of Screen Wire, Kevlar, and S2-Glass

Introduction

The impact limiter being considered is made from a composite of screen wire and Kevlar. To analyze the limiter with a finite element model, the material properties were needed so that a constitutive model could be constructed. Both the in-plane tensile properties as well as the compressive properties normal-to-the-plane were needed for the composite. Because S2-Glass was a possible substitute material for the Kevlar, the in-plane behavior for the S2-Glass was also needed. Compression tests normal-to-the-plane were not performed with S2-Glass because the screen wire dominates this behavior. In-plane tension tests were only performed in the direction of the fibers or wires because layers of screen wire, Kevlar and S2-Glass have virtually no in-plane shear strength. Since the Kevlar sheets are always between screen wire in their final application, all specimens were built to represent this fact. In addition, the ratio of screen wire to Kevlar in the final application is fifteen to one so all tests were performed using this ratio.

Compression Normal-to-the-Plane

Introduction

For the tests with compression normal-to-the-plane, screen wire and Kevlar were layered together in a ratio of fifteen layers of screen wire to one layer of Kevlar. This is the ratio that is used in building the final impact limiter, so this is the ratio that needs to be characterized.

Due to the application of this material, the tests must be performed to very large strains. If possible, it would be desired to compress the specimen to densities approaching that of solid aluminum. Due to the very large stresses that would be required to achieve this, the specimens must have relatively small cross sectional areas.

Several problems were encountered while building and testing the composite of layered screen wire and Kevlar. The first problem concerned specimen coherence. Once the specimen was layered together, a compressive stress was required to prevent the specimen from falling apart. The second problem was how to layer the specimens so a random orientation of the holes in the screen wire is obtained. This was necessary because the limiter is built by wrapping the layers around a cylinder. Due to the changing diameter, the holes line up in a random way. Moreover, the edges must be smooth to allow for the lateral expansion to be measured (i.e. Poisson's effect). Finally, the problem of lubricating the specimen so the top and bottom were not restricted in lateral movement by friction was addressed.

Holding the Specimen

A test fixture was designed to keep the specimen in compression at all times. The fixture consists of two steel blocks and four bolts to keep the specimen in compression (See Figure 2.1). Once the specimen is built, it is placed between the two blocks and the bolts are tightened to keep the specimen in compression. When the specimen is tested, the testing machine takes up the compression carried in the bolts and after this point the bolts slide freely through the bolt holes in the fixture plates and no longer play any role in the test.

Constructing the Rough Specimens

Building the specimens takes place in several steps. First the specimens are rough cut to the approximate dimensions needed and placed in the test fixture. The bolts are then tightened to keep the specimen in compression. Finally whatever process is necessary to smooth the edges will be performed.

The first step was to rough cut the screen wire to 1.5 by 1.5 inch squares. The screen wire was always cut parallel to a direction in which the wires ran. Using leather shears, the Kevlar was hand cut into squares of approximately the same size. This produced rough edges and some raveling at the edges occurred. Once the screen wire and Kevlar had been rough cut, they were layered together in the fifteen to one ratio (See Figure 2.2).

Finishing the Edges

The edges needed to be smooth to allow the lateral expansion to be measured and to allow the cross sectional area to be more accurately determined. Due to the difficulty in layering the screen wire and Kevlar after they have been cut to the final dimensions, a method was needed which trims both materials to their final dimensions after they have been layered together and placed in the test fixture. To accomplish this, the specimen was layered and put into the fixture immediately after being rough cut. Several methods to finish the edges were attempted, each method exhibited a varying degree of success.

As a first attempt to finish the edges, the specimens were cut using a band saw with different blades, including cutting with a flat blade with no sharp edges. Regardless of the blade used, the Kevlar always pulled and raveled leaving a very rough edge. Since the screen wire and Kevlar must be layered together before the edges are finished, this approach did not represent an acceptable method.

As a second alternative to finish the edges, the specimens were cut with a milling machine. This presented a problem because the specimen deformed under the pressure of the tool and the Kevlar wrapped around the cutting tool and unraveled. To add strength to the edges during machining, Universal's "UNI-WAX #40" was applied to stiffen the material. This wax has a softening point of 71-77 degrees Celsius. The intended use for the wax was to hold materials, especially metals, during machining processes. To impregnate the specimens with the wax, the specimen and the fixture were heated to the softening temperature of the wax. When the specimen was not heated, the wax hardened before it could penetrate into the specimen, thereby defeating the purpose of the wax. The wax was then heated until it was in a liquid state and poured over the edges making sure an even layer was applied. This was done to all four sides and then the specimen was allowed to cool.

The fixture was then placed in a clamp, to keep the specimen in compression, and two of the bolts removed to allow the edge to be machined. During the machining process, water was sprayed over the specimen to keep the edges from heating up and melting the wax. This process yielded a fairly smooth edge. Because the wax did not penetrate into the specimen very far, it was reimpregnated between machining operations.

One problem remained in this method, which was removing the excess wax from the specimen. First the specimen was reheated to allow as much wax as possible to drain out of the edges. The manufacturer recommended using acetone to dissolve the wax. This was tried but some wax still remained in the Kevlar and screen wire as verified by weighing a specimen, impregnating it with wax, placing it in acetone and weighing the specimen upon removal from the acetone. Because no material was removed, if all the wax was removed, the specimen should have had an identical weight to that of the specimen before impregnation, but this was not the case. In addition, there was some question what effects the acetone had on the screen wire and the Kevlar. The acetone could have been corroding the aluminum, being absorbed into the aluminum, or possibly chemically attacking the Kevlar. Therefore this procedure also proved to be inadequate for preparing the specimens.

As another alternative, grinding the edges was attempted. A diamond tip cutting wheel of one quarter inch width was used. The specimen was machined by placing the fixture in a clamp and removing two bolts allowing the grinding wheel to machine one edge at a time. After one edge was machined, the bolts were replaced and the specimen was rotated with each edge machined in turn. This produced an extremely smooth edge with no impregnation necessary and no raveling of the Kevlar. One specimen was taken apart after this process to examine the effects on the edges of the material. The edges were compressed to solid aluminum but this effect was very localized to approximately the

diameter of one wire. The method of grinding the edges was by far the method of choice and was used to construct all subsequent specimens.

Lubricating the Ends

Some kind of lubrication was necessary on the ends where the specimen contacts the fixture to prevent the friction at the contact surface from hindering the lateral expansion. Due to the extremely high stresses desired, a solid lubricant was used. Both graphite and very thin Teflon sheets were tried as possible lubricants. Both were applied to the face of the fixture before the specimen was placed in the fixture and machined. These test specimens were then loaded in compression to determine which would provide better lubrication. The graphite was pushed out of the way where the screen wire met with the fixture, therefore the graphite was not performing the desired task and was not used. The Teflon sheet was indented where the screen wire was against it but it did not cut through and appeared to still be providing the necessary lubrication. On the basis of these results, the Teflon was judged to be the best lubricant and was used for all subsequent tests.

Initial Testing of the Compression Specimens

A specimen of one inch tall and 1.5 by 1.5 inch cross section was tested from the specimen that was built by impregnating the specimen with wax and machining the edges. The one inch corresponds to sixty layers of screen wire and four layers of Kevlar. The process of clamping the material and grinding it reduces the height of the specimen to 0.708 inches. This occurred because very small loads caused large elastic and plastic strains when the material is first layered together. Each time the specimen was clamped, a little more compression was applied to the specimen. Because the bolts had to be removed, a

machining C-clamp was used to keep the specimen in compression. To ensure the specimen did not fall apart when the bolts were removed, the C-clamp was tightened until the bolts just started to become loose.

The specimen was tested using a sixty kip testing machine with only a load gauge. The displacement was measured by setting up displacement dial indicators on the cross heads to measure their relative displacement thereby indicating the strain of the specimen. The lateral displacement was measured by placing blocks of known dimensions against the sides and using a micrometer to measure the lateral dimension of the specimen. The blocks were used to prevent the need of placing hands between the crossheads during the test, and to get an average value of the lateral dimension as opposed to a local value since the location of the measurement could change during the test. The lateral displacement was then calculated by subtracting the initial lateral dimension of the specimen and the width of the blocks from the measurements taken during the test.

This specimen was tested to get an idea of how the material would behave. The specimen was crushed 0.212 inches corresponding to a logarithmic strain of -0.357. This displacement and strain are dependent on where the zero state is defined. The zero deformation state was defined to be when the test began even though the specimen had undergone quite a bit of strain due to compression of the fixture. At this strain level the load was only 3400 pounds which corresponds to a stress of only 1522 pounds per square inch. Up to this point in the test, no measurable lateral strain had occurred. At the end of the test, the ratio of the height to the width was approximately 0.333.

With an aspect ratio of 0.333, the end effects can dominate the results of the test. Because the experimental data is to be used to construct a constitutive model, the data needs to be independent of the boundary conditions. Because of these facts, it was determined taller specimens would be needed, where the ratio of the initial height of the specimen to the width of the specimen would be much greater. Another specimen was constructed using

the grinding technique that was 2.073 inches tall, 0.666 inches wide, and 0.696 inches deep. This specimen buckled almost immediately preventing any useful data acquisition. Other specimens showed this buckling behavior when the aspect ratio was larger than about 1.0.

Final Testing of the Compression Specimens

From the previous testing, it was determined that a new method was needed which would allow larger strains to be obtained without buckling. The procedure for rough cutting the screen wire and Kevlar, layering the composite, and grinding the edges smooth for the final specimen worked well and would not be changed. A new test procedure was devised to allow larger strains to be obtained.

Since no lateral expansion occurred at the early strain values, the load was very small, and good reproducible small strain data had been obtained, it was decided to construct a final specimen by compressing several of these initial specimens from one inch to 0.33 inches and then stacking these specimens to make the final precompressed specimen. This precompression allowed a much larger strain to be obtained without buckling and with better aspect ratios.

This procedure was accomplished in several steps. First, layers of screen wire and Kevlar were rough cut to dimensions of approximately 1.25 by 1.25 inches. These layers were stacked to give three specimens of 60 layers of screen wire and 4 layers of Kevlar each. These three specimens were then compressed down to a height of approximately 0.33 inches. These specimens were then stacked up one on top of the other and clamped in the test fixture. The final specimen was then ground to a cross section of 1.014 by 1.015 inches with a height of 1.080 inches. This specimen was then compressed from the zero state to a total logarithmic strain of -1.172. The zero state of deformation was taken to be

3.000 inches. This was the theoretical height of the layers with no compression. The testing machine was accidentally unloaded at this strain level which caused the specimen to slip. This slippage initiated buckling when the specimen was reloaded.

Results for Compression Normal-to-the-Plane

Figure 2.3 shows the stress-strain curves for the initial three specimens as well as superposition of the three to get the early portion of the stress-strain curve for the total specimen (i.e. the three specimens stacked together). This superposition was obtained by averaging the stresses at a given displacement. A logarithmic strain and a Cauchy stress measure were used.

When the initial specimens were loaded and compressed to approximately 0.30 inches and unloaded, the specimens regained their elastic displacement, which was only a small fraction of the total deformation. The specimens had undergone enough plastic deformation to hold them together which allowed them to be handled without any compression. After these specimens were stacked together and machined, their height had changed due to the elastic recovery and the clamping procedure. In addition, the exact initial height of the initial specimens and the point from which displacement was measured were unknown. This caused a discontinuity in the stress-strain curves.

To eliminate this discontinuity, the initial part of the test data needed to be shifted to align with the second part of the test data. From previous tests and the data just collected, this material exhibited an exponential behavior, so the form for the stress-strain behavior was assumed to be:

$$\sigma = A (e^{\beta \epsilon} - 1) \quad (2.1)$$

In Eq. (2.1), A and β are constants that must be determined. Eq. (2.1) is a non-linear equation that must be solved for the constants. A least squares technique for non-linear functions was employed to solve for the constants (Chappra and Canale, 1988). In this method an initial guess for the constants is used and then iterations are performed to get more accurate approximations. The method was broken down into two steps. First, the second part of the data was fit to Eq. (2.1) using the non-linear least squares fit to get the constants A and β . The displacement of the initial data points was shifted to correspond with the second part of the data. This necessary shift was on the displacement, therefore, the initial data was shifted using an equation of the form:

$$\sigma = A (e^{\beta \ln(c + e^e)} - 1) \quad (2.2)$$

The desired shift for the initial data is the constant c . The constants A and β are the same constants fitted from Eq. (2.1). A least squares method was employed to find c . Now all the data, including the initial data, is refit using Eq. (2.1). A new value of c is then determined by performing a least squares fit on Eq. (2.2). This process is repeated until the constants converge to a constant value. Figure 2.4 shows the curve fit and the shifted data.

There was an appreciable amount of lateral expansion in the second part of the test. This data was plotted and it was apparent that some lateral expansion had occurred during the initial portion of the test. The lateral expansion data suggested using a quadratic function to extrapolate the lateral expansion back to an axial strain of zero. The minimum shift possible was desired. In addition, the curve must be concave upward at all times. For small shifts, the linear term was negative which caused the curve to have a minima. As the shift was increased, the linear coefficient went to zero and then turned positive. Since the minimum shift possible was desired, the linear term was not used; only a constant and a

quadratic term were used for the fitting process. This data was fit with a least squares method. The data as taken as well as the curve fit to this data are shown in Figure 2.5. This analysis showed that approximately one sixteenth of an inch of lateral expansion occurred in the initial portion of the test. Figure 2.6 shows the data and the fit after they have been shifted.

In-Plane Tension

Introduction

In-plane tension tests were performed on screen wire, Kevlar and S2-Glass. Unlike the normal-to-the-plane compression tests, the goal was to test until failure or fracture occurred. These tests presented several major difficulties. First, how were the specimens to be held in order not to cause stress concentrations that would corrupt the test? Another consideration was how to cut the Kevlar and the S2-Glass.

The desired result from these tests was to determine the behavior of the composite of screen wire and Kevlar in the fifteen-to-one ratio. It was decided to perform single layer tests first to devise a method for building and testing a multi-layer specimen. Once this method had been perfected, the multi-layer specimen would be built and tested.

Testing Equipment

An Interlocken twenty-two kip machine was used for all the in-plane tension tests. The machine was equipped with hydraulic grips to hold the specimens. The grips could exert up to forty thousand pounds per square inch of pressure on the specimens. Load and

crosshead displacement were measured by the testing machine and output as an analog signal to a plotter and digitally to a data file.

Holding the Specimens

Because hydraulic grips were used, a method to hold the specimen that would not initiate failure at the grips was needed. If the material itself was simply placed between the grips, the grip pressure would have damaged the fibers and would have always initiated tensile failure at the grips. To eliminate this problem, it was decided to use epoxy to bond the materials to aluminum tabs. These tabs would provide the grips with something to clamp to, while hopefully not damaging the specimens or creating a large stress intensity at the transition region.

Building the Specimens

The tabs were made from one sixteenth inch thick aluminum plate. The plate was sheared into squares of 1.5 by 1.5 inches for the Kevlar and the S2-Glass and 1.25 by 1.25 inches squares for the screen wire. To help the transition from the tabs to the materials a taper was ground into the tabs over approximately 0.25 inches using a belt sander (See Figure 2.7). This initial process left the tapered edge of the tabs rounded toward the inside with a sharp edge which would have tended to damage the material in this region. This problem was resolved by touching the inside of the tab to the belt sander forcing this edge toward the outside where it would not damage the specimens.

Once the tabs had been cut, they had to be prepared for use with the epoxy. This was accomplished by sanding the specimens with a slight acidic solution and some fine

sand paper. Then a neutralizer was applied to the tabs and they were thoroughly cleaned. An epoxy resin 882 was used to form the bond between the specimens and the tabs.

The screen wire specimens proved to be the least difficult to construct. The screen wire was cut into strips that were approximately five inches long and one inch wide. The one inch wide specimens corresponded to sixteen wires in the load line direction. Since the wires in the transverse direction would not help to carry the load but merely transfer load between adjacent wires, the edges were left in the rough cut state. To construct the final screen wire specimens, a thin layer of the epoxy was applied to the tabs. The screen wire was then placed in the center of the tab and additional epoxy was applied to fill the gaps in the screen wire. Enough additional epoxy was applied at the tapered edge to force some epoxy to fill the gaps in the screen wire just beyond the tabs when the tabs were clamped together. This was done to further reduce the stress concentrations at the tabs by extending the transition region. Both sides were constructed this way and then clamped together and the epoxy was allowed to cure at room temperature.

To construct the Kevlar specimens, the material was rough cut a little larger than the five inches long and one inch wide size of the final specimen. This was done to allow for some excess fibers to be removed because of the difficulty in hand cutting this material to exact dimensions. The excess fibers were unraveled leaving the material in the desired size. Twenty-four fibers remained in the load line direction corresponding to a specimen one inch wide. Again, the transverse fibers would only transfer load between axial fibers but would directly carry any load, so the edges were left in the rough cut state. The final specimens were constructed by applying a thin layer of epoxy to the tabs and positioning the Kevlar on one of the tabs. Extra epoxy was then applied directly to the Kevlar so the material would absorb the epoxy and make a complete bond across the thickness of the weave from tab to tab. Again, extra epoxy was added at the taper edge so the excess would soak into the fibers just beyond the tabs and help reduce the stress concentrations in the

transition region. Once this had been done to both sides the specimen was clamped and the epoxy was allowed to cure at room temperature.

The S2-Glass specimens proved to be difficult to construct. When the material was cut, the edges would unravel. This made it difficult to obtain the desired dimensions. The material was cut to a much larger size than the desired final size. The unwanted fibers were then removed one at a time. The same process used to construct the Kevlar specimens was used to construct the S2-Glass specimens. This process was not quite as effective on the S2-Glass. First, the amount of unwanted material on the edges was much greater and secondly, the material continued to unravel. This unraveling made it extremely difficult to determine the number of fibers that were actually carrying load and the number that were just being straightened. Several specimens were constructed this way to be tested. The edges of the specimens were trimmed down by coating the edges with a nail polish hardener which allowed the excess to be trimmed off. Since this was only done to the transverse fibers, this should not have effected the results of the tests.

Initial Testing of Tensile Specimens

To determine the viability of the described procedures for testing the tension specimens, several initial tests were performed. All of these tests were run at a displacement rate of 0.001 inches per second. The initial specimens were held in the grips with approximately 6700 psi of pressure. Due to the flexibility of the specimen, the specimens were not taut when first placed in the machine. This slack was taken up in the early part of tests and removed from the final data by subtracting out the displacement that occurred with no load.

The screen wire specimen fractured well away from the tabs. The results from this particular test were promising. The Kevlar specimen fractured partially at the tab and

partially at the edge of the epoxy. Several specimens were tested this way and all fractured at or near the tabs due to the stress concentrations. The S2-Glass specimen never fractured in the usual sense but nonetheless failed. The material loaded up and then the load necessary to further strain the material was more than an order of magnitude below the maximum load. The S2-Glass did not show any negative effects from the tabs.

From these initial tests, it was determined the method for the screen wire specimens was adequate. The S2-Glass method needed to be refined to keep the edges from raveling because this prevented a good specimen from being constructed. Except for the edge problems, the method used to build the S2-Glass specimens was effective. Since the Kevlar specimen failed at the tabs, probably caused by a stress concentration, it was necessary to devise a new method to test these specimens.

Final Tensile Specimens

Another screen wire specimen was constructed using the previous method. Careful attention was given to ensure that the epoxy would fill all the gaps between the wires and enough would be pressed out of the taper to coat the first layer of the specimen.

The ideal way to cut the S2-Glass would have been to use a laser. This process would have given a clean smooth cut, as well as fusing the fibers at the edge to prevent raveling. In the absence of this capability, an alternative was devised. An acetylene micro-torch was found to perform the cutting. This torch had a small tip that produced an extremely small flame. This flame would fuse the fibers and cut the material at the same time. The cut obtained from this process would not be as clean as from the laser but would suffice in its absence.

Several techniques were tried to cut the S2-Glass with the micro-torch. First, the material was placed over two blocks with a small area open underneath the area to be cut.

Two blocks were then placed on top of the material so they lined up with the bottom blocks. The micro-torch was then used to cut the material. The open area had to be fairly large to prevent choking out the micro-torch. This produced a fairly good cut but on both sides of the cut a fair amount of material had been discolored and fused together due to the heat transfer and the large area left open to prevent extinguishing the torch. Different opening sizes were tried without a great deal of success. To get the heat to transfer away from the specimen, only one side of the material was put between blocks. The other side was left free-hanging. This was done so that the majority of the heat would be transferred away from the blocks to the free hanging material, thereby minimizing the damage to the specimen area. This technique proved to be extremely effective and was used to prepare all subsequent S2-Glass specimens.

The basic idea for building the Kevlar specimens was adequate except for the stress concentration caused at the tabs. When metals are tested in uniaxial tension, the center of the specimen is slightly smaller than the rest of the specimen ensuring failure in the center of the specimen. Because a fabric is made of a constant number of load carrying fibers which can not be tapered, this technique is not applicable. In spite of this fact, the fundamental idea was adapted. To incorporate this idea, two smaller pieces of Kevlar were epoxied to the tabs and to the test specimen. These extra pieces extend beyond the tabs (See Figure 2.8). The epoxy should help transfer enough load between the layers to reduce the stress concentration at the tabs. To ensure the three layers bonded together, the layers were placed between Teflon sheets with a weight placed on top. This weight provided a compressive load which forced the layers to be in complete contact, thereby allowing a strong bond to be formed

Final Testing of Tensile Specimens

The test procedure used on the new specimens was identical to the initial tests. The screen wire specimen showed nearly identical behavior to the initial specimen. The maximum load was extremely close to that of the first test, and the pattern of fracture was nearly identical. The S2-Glass specimens had slightly different lengths and widths but their behavior was very similar. The Kevlar specimen loaded up to a higher load than the previous tests and failed at the interface between the three layers of Kevlar and the single layer. The epoxy was relatively weak compared to the strength of the Kevlar so this was an effective test procedure.

Results for In-Plane Tension

Load versus strain plots were generated from the data collected from these tests. A logarithmic strain measure was used. Plotting load versus strain rather than load versus displacement will eliminate the problem of different length specimens. Figure 2.9 shows the load versus strain curve for the screen wire. For the S2-Glass, the specimens were slightly different widths. To compensate for this, load per width is plotted versus strain in Figure 2.10.

Due to the construction of the final Kevlar specimen, the gauge length to be used was difficult to determine. Therefore, the load versus strain relationship was determined from the initial tests since the data was reliable up to the point of failure. This curve will be extrapolated to the maximum load obtained from the final test. These results are shown in Figure 2.11.

To determine how the multi-layer specimen of screen wire and Kevlar would behave, the result from the single layer specimens was combined in the appropriate ratio;

thirty layers of screen wire to two layers of Kevlar. Figure 2.12 shows the behavior for thirty layers of screen wire and two layers of Kevlar. Figure 2.13 shows the behavior of the composite by adding the stress of the Kevlar to the stress in the screen wire at the same strain since they are in parallel. As can be seen, the screen wire will dominate the early part of the test and fracture then the Kevlar will load up and fracture. Since there is such a wide separation between the region where the individual materials were loaded, it was determined that the multi-layer specimen would not be constructed or tested.

CHAPTER 3

Perforated Plate Study

Introduction

Perforated plate was considered as an alternative material for the impact limiter. In order to model perforated plate with a continuum constitutive model, the general behavior of the plates was needed. A continuum constitutive model will be used so that the holes will not need to be represented in the mesh but will be accounted for in the constitutive model.

The particular plate that was analyzed as a possible substitute, was 0.04 inches thick, had a hexagonal symmetry with the holes 0.2 inches apart, center to center, and had hole diameters of one eighth of an inch (See Figure 3.1). The plate was made from aluminum and the assumed properties for the aluminum are shown in Table 3.1.

Table 3.1 Assumed Material Properties for Aluminum.

Young's Modulus	10,000,000 psi
Poisson's Ratio	0.3
Yield Stress	30,000 psi
Hardening Modulus	100,000 psi
Mass Density	0.245E-03 lb*s ² /in ⁴

Since a large number of perforated plate samples were not available for testing, it was decided to obtain the necessary data using the finite element method. Because the plate will be used as an impact limiter, it will undergo large plastic strains in its application and the finite element code selected must have that capability.

SPECTROM331 (Key, 1987), a large strain non-linear dynamic finite element code was used to perform all the analyses. These analyses modeled pseudo-static tests meant to represent actual test conditions. A dynamic code was used because no large strain non-linear static code was available at the time. The loads were applied sufficiently slow so that the inertial effects did not influence the results. Inertial effects were kept small by monitoring oscillations of state parameters and reducing loading rates as necessary. A Goel and Malvern constitutive model (Goel and Malvern, 1970) with only isotropic hardening was used to perform all analyses.

All H8 elements were used to construct the meshes. SPECTROM331 uses one gauss point to perform the integration over the elements. One gauss point under integrates the elements leaving zero eigenvalue modes of deformation called keystoneing (Key, 1987). SPECTROM331 uses artificial stiffness and artificial viscosity, which is sensitive only to the keystone modes, to control these modes of deformation. This was effective for most of the analyses, but some keystoneing remained as will be seen in subsequent sections.

To characterize the behavior of the plate, both in-plane tensile and compressive properties and normal-to-the-plane compressive properties were needed. For the in-plane behavior, the material must be characterized for any direction of tension and compression. For the normal-to-the-plane compression, the material needs to be characterized for a random orientation of the holes in adjacent stacked layers. For all tests, the plates were assumed to be of dimensions such that end effects were not important to the results.

In-Plane Behavior

Introduction

The first step to characterizing this material was to determine the in-plane behavior for large strains. Large strains are necessary because of the use of this material as an impact limiter. Both tensile and compressive properties were needed. The tests chosen to determine these properties were two compressive tests performed in orthogonal directions and two tensile tests in these same two orthogonal directions.

The in-plane behavior was determined by characterizing a single plate. From elasticity, when a plate with a central hole is pulled in the plane, a stress field is created around the hole which is tensile at certain locations and compressive at other locations. The plates near the edges of the holes would warp to relieve the stress in the out-of-plane direction. But many plates will be stacked together to create the impact limiter, and the faces of the plates will be forced to remain flat by the surrounding plates. In addition, the problem was further simplified by recognizing the in-plane symmetries for an infinite plate. A unit cell can be isolated which is representative of all regions of the plate. Figure 3.2 shows the coordinate system and the section of the plate that was used for all the in-plane analyses. Figure 3.3 shows the undeformed mesh and the coordinate system. Since the stress is constant over the plate thickness and has no out-of-plane shear, only one element was used in the thickness direction. A total of 530 nodes and 224 elements were used to mesh this section of the plate.

For all in-plane problems, the x- and y-displacements of the outside edges were prescribed while the edges of the holes were free to displace. To determine only the in-plane behavior, all in-plane analyses were run in plane stress. Because the plate must remain flat, plane stress was imposed in an integrated sense. This was achieved by

driving the face of the plate in a such a way so that the integrated stress in an area normal to the plane was approximately zero. Since achieving zero at every load was impossible, it was decided that the stress in the out-of-plane direction should be less than 5% of the axial stress. This 5% margin was used for all subsequent comparisons.

Four independent problems were run to determine the in-plane behavior. First, tension in the x-direction with no y-displacement of the adjacent edge was analyzed. Then, tension in the y-direction with no x-displacement of the adjacent edge was analyzed. Finally, the same two problems were run in compression instead of tension.

General Analysis

ANTIPASTO, a two dimensional mesh generator (Nielan, et. al., 1990), was used to generate the mesh. The two dimensional mesh was then extruded using a small FORTRAN code to give a three dimensional mesh with one element through the thickness. Because these problems needed to be analyzed as quasi-static tests where inertia does not influence the results, each case had to be run at a sufficiently slow strain rate.

From some initial test runs, it was determined that the elastic oscillations caused by the inertial effects could easily mask the final stress values. To eliminate these oscillations, artificial dynamic relaxation was added to the code to reduce the inertial effects. This was accomplished by multiplying the velocities at the end of each time step by a factor of 0.999. Other values for the damping factor were also attempted but produced very similar results so the value of 0.999 was used for all subsequent analyses. This damping factor did not appear to affect the results significantly, it appeared to only damp out the residual oscillations as required.

For the compression in the x-direction test, shear distortion between the top and bottom planes in the plane of the plate caused severe problems. The apparent keystoneing caused the nodes on the free circular edge to undergo unrealistic displacements. Because the displacements should be constant through the thickness of the plate, nodes opposite each other across the plate thickness were constrained to have the same in-plane displacements. This eliminated the keystone mode that was causing problems.

The desired results from these analyses was to determine the parameters necessary for a constitutive model with volumetric and deviatoric yielding that will model the perforated plate. This was done so that the holes would not need to be represented in the mesh but would be accounted for in the constitutive model. The analyses were then to give the stress-strain behavior of a continuum model with the effects of the holes “smeared” over the entire area of the plate. Because of these facts, the stress is averaged over the entire region, including the hole area, not just the actual material area. The strain is a uniaxial measure based on the change in length in the direction of the loading. A logarithmic strain and a Cauchy stress measure were used.

Both the initial yield stress as well as the large strain behavior were desired from these analyses. The yield point was obtained by extrapolating the stress versus plastic strain relation back to a plastic strain of zero. The plastic strain was calculated using

$$\varepsilon^p = \varepsilon - \frac{\sigma}{E} \quad (3.1)$$

Figure 3.4 shows a hypothetical stress-strain curve and the yield stress that would be predicted using this method. An effective Young’s modulus based on the entire area was calculated as

$$E_{eff} = \frac{\int (\Delta\sigma) dA}{\Delta\epsilon} \quad (3.2)$$

The data for this calculation was obtained at very small strains before any element underwent yielding. Because elastic oscillations are dominant at these small strains, these analyses were performed at strain rates much slower than were used for the remainder of the analysis when plasticity effectively damps the oscillations.

An effective Poisson's ratio was also determined. To get this value, consider Hooke's Law:

$$\epsilon_{ij} = -\frac{\nu}{E} \sigma_{kk} \delta_{ij} + \frac{1+\nu}{E} \sigma_{ij} \quad (3.3)$$

Because these problems were run with no displacement in the transverse direction, the strain in the lateral direction is zero. Equation (3.3) is expanded for the strain in the lateral direction to give:

$$\epsilon_{22} = -\frac{\nu}{E} (\sigma_{11} + \sigma_{22} + \sigma_{33}) + \frac{1+\nu}{E} \sigma_{22} = 0 \quad (3.4)$$

The stress in the direction normal to the plane of the plate is forced to be approximately zero. Using this in Eq. (3.4) and solving for Poisson's ratio we obtain the desired relationship

$$\nu = \frac{\sigma_{22}}{\sigma_{11}} = \frac{\sigma_t}{\sigma_a} \quad (3.5)$$

To obtain the desired stresses and the strains from these analyses, a FORTRAN program was written to perform the necessary calculations. For the strain values, the appropriate nodal displacements were found in the program and then the logarithmic strain was found from the displacements. To get the stress, the area of each element face on the appropriate edge was calculated using standard finite element algorithms. The area was multiplied by the appropriate stress component to get the force in the desired direction. Then all the forces were summed to get the total load. Finally, the stress was calculated by dividing the total force by the entire cell area, including the open hole area. The program was tested with initial meshes of known area and constant stress in each element. In every case, the margin of error in the program was within the bounds of approximating circles with straight line segments or round-off errors.

Results

Figure 3.5 shows the deformed shape with stress contours of the axial stress from the tension in the y-direction test. Actual displacements pictured here vividly illustrate the large strains involved. Figure 3.6 shows the same plot for the tension in the x-direction test. Figures 3.7 and 3.8 show the same plots for compression in the y-direction and x-direction tests respectively. All plots are at an axial strain of approximately 0.4 except for the compression in the y-direction which is at a strain of approximately 0.25 because keystoneing corrupted the results for this particular test.

All data was reduced as described above. The stress-strain data for the tension in the y-direction test is shown in Figure 3.9. The stress strain data for the tension in the x-direction test is shown in Figure 3.10. Similar plots are shown for the compressive tests in the x- and y-directions in Figures 3.11 and 3.12, respectively.

An elastic modulus was calculated for each test by fitting the early part of the stress-strain curve with a straight line, where Young's modulus is the slope of this line. An elastic Poisson's ratio was calculated using Eq. (3.5). In order to determine the yield stress for each test, the axial stress was plotted versus plastic strain as calculated by Eq. (3.1). This data was fit with polynomials and extrapolated back to a plastic strain of zero. The yield stress was then defined as the stress at zero plastic strain. Table 3.2 shows these results.

From these results it is apparent that the properties are very nearly the same in both tension and compression. This is to be expected for small strain behavior where the geometry is nearly the same in both cases.

Table 3.2 Elastic constants and yield stress from the in-plane tests.

Direction	Young's Modulus (psi)	Poisson's Ratio	Yield Stress (psi)
Tension Y	4,218,000	0.3306	15,660
Tension X	3,996,000	0.3478	14,610
Compression Y	4,218,000	0.3306	15,490
Compression X	3,996,000	0.3478	14,160

Compression Normal-to-the-Plane

Introduction

To fully characterize the behavior of perforated plate, the normal-to-the-plane compressive behavior was needed for stacked sheets. The behavior needed to be characterized for a random orientation of adjacent plates. Because it was impossible to analyze every possibility, the two extreme cases were analyzed. These cases were for all holes exactly lined up for a pair of stacked plates and the case where the holes in the adjacent plates had the least overlap possible.

Holes Lined Up

Dynamic relaxation was found to be unnecessary for this highly constrained problem. Because many plates will be stacked together, the plates will be forced to remain flat away from the edges. Because of the symmetries, only one plate with one element through the thickness was analyzed. Again, symmetries were considered and a unit cell was isolated. Figure 3.13 shows the section of the plate that was analyzed. Figure 3.14 show the undeformed mesh. This mesh consisted of 330 nodes and 136 elements.

The boundary conditions for this simulated uniaxial strain test were rollers around the edges except at the holes and no out-of-plane displacement of the bottom face. To simulate the compression of the test, the top face was forced down with a prescribed linear displacement as a function of time.

It was not possible to compress the plate to solid aluminum because keystoneing corrupted the results. Figure 3.15 shows the deformed mesh as keystoneing began. The

large strains present at the onset of keystoneing are evident from this figure. The mesh is viewed from above so that the keystoneing at the two free edges at the holes is apparent. Figure 3.16 shows the stress-strain relation for this particular problem. As a comparison, the same mesh was run with no strain hardening (i.e., an elastic-perfectly plastic material); Figure 3.17 shows the results from this analysis. As can be seen, even a perfectly plastic underlying material shows strain hardening when considered as a continuum because the volumetric yielding increases the cross sectional area of the material. Strain hardening of the underlying material greatly increased the stiffness of the plate as seen by comparing Figures 3.16 and 3.17.

The effective elastic moduli were also needed. To determine these quantities, the problem was run up to a certain strain level and then the plate was partially unloaded. During these unloads, the plate was not allowed to yield in tension. By ensuring that the unloading is totally elastic, then on reloading the plate will begin yielding from the point it started unloading. This was verified by comparing the stress-strain curves from the analyses where the plate was unloaded and from the analyses where the plate was not unloaded. In each case the results were identical. From these unloads, it was possible to determine an elastic unloading Young's modulus and an elastic unloading Poisson's ratio. The process for determining these quantities is described below.

The exact point at the start of the unload is known. All stress components were plotted versus the total volume strain which is also the axial strain since there is no strain in the other two directions. Due to elastic vibrations during the unloading process, it was necessary to perform a straight line curve fit to the unload data. This fit was required to pass exactly through the point where the unload started. The straight line is defined as

$$\sigma = a \epsilon + b \quad (3.6)$$

where a and b are the constants that need to be determined. This is forced to pass through the initial point so

$$\sigma_0 = a \varepsilon_0 + b \quad (3.7)$$

where σ_0 and ε_0 are the stress and strain at the start of the unload. Eq.(3.7) is then solved for b as

$$b = \sigma_0 - a \varepsilon_0 \quad (3.8)$$

This is then substituted into Eq. (3.6) to give

$$\sigma - \sigma_0 = a (\varepsilon - \varepsilon_0) \quad (3.9)$$

A least squares technique is now employed to find the unknown constant a in Eq. (3.9). Start by defining the error as the sum of the squares of the difference in the actual value and the value predicted by the curve fit. This is written mathematically as

$$S_r = \sum_{i=1}^n [(\sigma_i - \sigma_0) - a(\varepsilon_i - \varepsilon_0)]^2 \quad (3.10)$$

where n is the number of points. To minimize the error, take the derivative of S_r with respect to a and set the derivative equal to zero to obtain

$$0 = 2 \sum_{i=1}^n [(\sigma_i - \sigma_0) - a(\varepsilon_i - \varepsilon_0)] (\varepsilon_i - \varepsilon_0) \quad (3.11)$$

This equation is now solved for a to obtain

$$a = \frac{\sum_{i=1}^n (\epsilon_i - \epsilon_0)(\sigma_i - \sigma_0)}{\sum_{i=1}^n (\epsilon_i - \epsilon_0)^2} \quad (3.12)$$

This procedure was performed on all three stress components to give a slope of each curve. It should be noted that this slope is equal to the change in stress divided by the change in strain. To see this, write Eq (3.9) for two different stress-strain points as

$$\sigma_i - \sigma_0 = a(\epsilon_i - \epsilon_0) \quad (3.13a)$$

$$\text{and } \sigma_j - \sigma_0 = a(\epsilon_j - \epsilon_0) \quad (3.13b)$$

where i and j represent two different points. Now subtract Eq. (3.13b) from Eq. (3.13a) to obtain

$$\sigma_i - \sigma_j = a(\epsilon_i - \epsilon_j) \quad (3.14)$$

which can also be written as

$$\Delta\sigma = a \Delta\epsilon \quad (3.15)$$

In order to determine the elastic constants, first consider the following elastic relation between stress and strain:

$$\epsilon_{ij} = -\frac{\nu}{E} \sigma_{kk} \delta_{ij} + \frac{1+\nu}{E} \sigma_{ij} \quad (3.16)$$

Eq (3.16) is now expressed in an incremental form as

$$\Delta \epsilon_{ij} = -\frac{\nu}{E} \Delta \sigma_{kk} \delta_{ij} + \frac{1+\nu}{E} \Delta \sigma_{ij} \quad (3.17)$$

Because no displacements are allowed in the x- or y-directions, the strains in the x- and y-directions are zero. Therefore Eq (3.17) is written for these two components as

$$0 = -\frac{\nu}{E} (\Delta \sigma_{22} + \Delta \sigma_{33}) + \frac{1}{E} \Delta \sigma_{11} \quad (3.18a)$$

$$\text{and} \quad 0 = -\frac{\nu}{E} (\Delta \sigma_{11} + \Delta \sigma_{33}) + \frac{1}{E} \Delta \sigma_{22} \quad (3.18b)$$

Eqs. (3.18a) and (3.18b) are now solved for ν as

$$\nu = \frac{\Delta \sigma_{11}}{\Delta \sigma_{22} + \Delta \sigma_{33}} \quad (3.19a)$$

$$\text{and} \quad \nu = \frac{\Delta \sigma_{22}}{\Delta \sigma_{11} + \Delta \sigma_{33}} \quad (3.19b)$$

Now, Eq (3.17) is written for the change in strain in the z-direction to obtain

$$\Delta \epsilon_{33} = -\frac{\nu}{E} (\Delta \sigma_{11} + \Delta \sigma_{22}) + \frac{1}{E} \Delta \sigma_{33} \quad (3.20)$$

Eq (3.20) is now solve for E as

$$E = \frac{\Delta \sigma_{33} - \nu (\Delta \sigma_{11} + \Delta \sigma_{22})}{\Delta \epsilon_{33}} \quad (3.21)$$

These equations can now be used to determine the effective unloading elastic constants. Because there are two independent definitions for Poisson's ratio (i. e.

Eqs. (19a) and (19b)), two values for Poisson's ratio were calculated for each unload. These two values never differed by more than ten percent. Because this data is to be fit to an isotropic constitutive model, the average of the two values was used. In addition, on the unload at a strain of -0.4, the effective unloading Young's modulus was greater than the Young's modulus of the underlying material. Because this is physically unrealistic, the modulus was taken to be that of the underlying material used to make the plate. It should be noted that one of the two Poisson's ratio gave a reasonable effective Young's modulus. Table 3.2 shows the effective unloading moduli.

Table 3.3 Effective Elastic Constants for Compression Normal-to-the-Plane;
Hole Lined Up

Volume Strain, ϵ_v	Poisson's Ratio, ν	Young's Modulus, E (psi)
0.0	0.208	6,820,000
-0.0861	0.222	8,109,000
-0.1803	0.235	8,797,000
-0.2843	0.257	9,529,000
-0.4004	0.291	10,000,000

Holes Off Center

Because many plates will be stacked together, the region of the plate away from the edges will be forced to remain flat. The edges of the plate were not considered in this analysis so the results only apply to an infinite array. Due to this fact, only two half

plates needed to be analyzed. Figure 3.18 shows the two sections of the plate that are stacked on top of each other. Figures 3.19 and 3.20 show the mesh viewed from above and below, respectively. The mesh of the two plates consists of 692 nodes and 287 elements. For each plate, one element through the thickness was used. Roller boundary conditions were employed at the outside edges, excluding the holes, and the boundary condition for the bottom face was rollers with no displacement normal to the plane. The edges of the holes were free to displace. The compression was accomplished by forcing the top face of the top plate with a prescribed linear displacement in the z-direction as a function of time.

To allow the two plates to transmit load but not penetrate one another, SPECTROM331 provides sliding interfaces. Both symmetric as well as master-slave algorithms are provided. Both algorithms were tried and both displayed problems at large strains. In each case, the mesh at edge of the holes was badly distorted thereby destroying the results. This limited the axial strain for this analysis to about 11%. This is roughly 100 times the strain at yield.

Due to oscillations in the results for this problem, some dynamic relaxation was necessary. Several values were used to determine which would be the most effective. Values of 0.9985, 0.999 and 0.99925 were tried. The results from all three cases were very similar as shown in Figure 3.21. The jagged region starting at a strain of 0.06 is due to problem encountered in the sliding interfaces. At this strain level, nodes from one plate start to push off the edge of the other plate. This apparently causes problems in the code. Once all the nodes in this region are pushed off the edge at a strain of about 0.08 the results settle down. This same problem occurs at a strain of about 0.12 but the solution does not recover. Therefore, results can only be obtained to a strain level of 0.11. Since the dynamic relaxation factor did not greatly affect the results, the value of 0.999 was used, similar to the previous runs.

A contour plot of stress in the out-of-plane direction is shown in Figure 3.22. The effective stress-strain relationship is shown in Figure 3.23. This stress-strain relationship was obtained by integrating the stress in the out-of-plane direction over the entire face to obtain the net force on the section. This force was then divided by the total area of the section including the holes to get an effective stress. The strain measure is a logarithmic strain calculated by looking at the change in volume of the entire section including the holes.

CHAPTER 4

Constitutive Model

Introduction

The constitutive model described here models a volumetrically yielding material with variable elastic constants. This model is developed for layered screen wire and Kevlar although it can be used for other materials that fit in the general class with layered screen wire and Kevlar. Because of the in-plane tensile behavior of the screen wire and Kevlar, this material is anisotropic. But, the in-plane tensile behavior caused by the fibers of Kevlar and the screen wire are independent of the volume crush and will be treated independently in this model. Therefore, the model will consist of an isotropic crush model and a model to describe the in-plane tensile behavior. The model to describe the tensile behavior is only a function of the strain along the axes of the fibers. In addition, the tensile model will not support any shear.

Isotropic Crush Model

General Description

Much of the work for the isotropic crush model also appears in the final report to Sandia National Laboratories (Krieg and Brown, 1992). The constitutive model for the screen wire is an elastic-plastic model with both volumetric and deviatoric yielding. The

yield surface is assumed to be made of a cone, capped by a ellipsoidal end. This is shown in Figure 4.1. The intersection of the two surfaces is chosen to be the pressure at the center of the ellipsoid. This means that there is a slight discontinuity in slope at the intersection. Non-associated flow rules are used on both yield surfaces, although both flow rules are associative in the limited deviatoric stress space. The plasticity theory will first be described in detail and then the numerical solution method is developed for layered screen wire and Kevlar in Chapter 5.

Yield Function

The cone part of the yield surface is described as:

$$f_c = \sqrt{J_2} - (a_0 + a_1 p) = 0 \quad (4.1)$$

$$J_2 = \frac{1}{2} S_{ij} S_{ij} \quad (4.2)$$

$$\frac{dJ_2}{dS_{ij}} = S_{ij} \quad (4.3)$$

The yield ellipse is described as:

$$f_{eps} = C^2 J_2 + (p - p_c)^2 - R^2 = 0 \quad (4.4)$$

where C is a constant.

The surfaces are to be such that they join at $p = p_c$ with no gaps, i.e., it is a continuous surface. This constraint allows one of the constants in (4.1) and (4.4) to be eliminated. In Eq. (A8) of Appendix A it is shown that this condition produces:

$$R = C(a_0 + a_1 p_c) \quad (4.5)$$

so that Eq. (4.4) becomes:

$$f_{eps} = C^2 J_2 + (p - p_c)^2 - C^2 (a_0 + a_1 p_c)^2 \quad (4.6)$$

which is zero on the yield surface. One additional constraint concerns yield surface growth. The minor and major axis diameters are to remain in the same ratio as the ellipsoidal yield surface grows. The equation of an ellipsoid in $(p - p_c) - \sqrt{J_2}$ space is :

$$A_1^2 J_2 + A_2^2 (p - p_c)^2 = R^2 \quad (4.7)$$

where R/A_1 and R/A_2 are the major and minor radii (or *vice versa*). By examination of Eq. (4.5) we note that R changes as p_c changes. This is as it should be since p_c describes both the cone angle and the radius of the cap which must cover the end of the cone.

The final form of the ellipsoidal part of the yield surface is then:

$$f_{eps} = C^2 J_2 + (p - p_c)^2 - C^2 (a_0 + a_1 p_c)^2 \quad (4.8)$$

which is zero on the yield surface. The parameter C is the ratio of the pressure to $\sqrt{J_2}$ radii of the yield ellipse. The parameter C should be determined from experimental data. If no experimental data is available to determine the value of C for a given material, a value of C is chosen such that a uniaxial stress path is normal to the ellipsoidal cap at their intersection. The parameter C is derived in Appendix B after the constants a_0 and a_1 have been chosen.

In summary, Eqs. (4.1) and (4.8) describe the yield or loading surface for the material in $(p - p_c) - \sqrt{J_2}$ space. The variable p_c changes as the material crushes, and is the pressure where the yield surface transitions from a cone to an ellipsoid.

Decomposition of the Stretching Tensor

The total rate of deformation or stretching tensor is additively decomposed into elastic and plastic parts as:

$$d_{ij} = d_{ij}^e + d_{ij}^p \quad (4.9)$$

$$\text{where } d_{ij}^e = E_{ijkl}^{-1} \dot{\sigma}_{kl} \quad (4.10)$$

$$\text{and } d_{ij}^p = \lambda V_{ij} \quad (4.11)$$

where the flow direction tensor, V_{ij} is to be specified in the sections that follow. Equations (4.9) - (4.11) are combined to give:

$$\dot{\sigma}_{ij} = E_{ijkl} (d_{ij} - \lambda V_{ij}) \quad (4.12a)$$

This is also written for an isotropic material as:

$$\dot{\sigma}_{ij} = 2G (d_{ij_{dev}} - \lambda V_{ij_{dev}}) + K (d_{kk} - \lambda V_{kk}) \delta_{ij} \quad (4.12b)$$

where deviatoric quantities are used in the first parentheses.

Variable Elastic Constants

The material will be taken to be elastically isotropic, both before and during crush, so the elasticity matrix is taken to be conventional in form and the yield surface is isotropic as described above. An important attribute of the elasticity matrix, however, is that the elastic moduli are functions of the crush which has taken place. Uncrushed screen wire can be deformed easily by hand, but after it has been compressed to almost solid aluminum, it is almost as hard as solid aluminum as well. It should be realized that the elastic moduli at a given crush state is the change in stress per unit strain which is applied, but this is a stress which does not cause plasticity. To emphasize this point, it can be regarded as an "unloading modulus" and is observed as such in experimental tests.

The elasticity matrix will then be taken to be exactly the same form as a conventional linear elastic matrix except the moduli will be taken to be a function of the volumetric plastic strain. Data is available on the elastic unloading Young's modulus as a function of the axial strain in a uniaxial stress test. An elastic Poisson's ratio is also taken to be a function of the plastic volume strain. Although Young's modulus and Poisson's ratio are fitted to experimental data, the model is expressed in terms of the bulk and shear moduli. These are related to Young's modulus and Poisson's ratio as

$$G = \frac{E}{2(1 + \nu)} \quad (4.13)$$

$$\text{and } K = \frac{E}{3(1 - 2\nu)} \quad (4.14)$$

The rate of change of the unrotated Cauchy stress if no plasticity is taking place is given by the expression:

$$\dot{\sigma}_{ij} = \frac{E}{1 + \nu} \left(d_{ij} + \frac{\nu}{1 - 2\nu} d_{kk} \delta_{ij} \right) \quad (4.15)$$

and if plasticity is present, Eq. (4.12) is applied to give:

$$\dot{\sigma}_{ij} = \frac{E}{1 + \nu} \left[(d_{ij} - \lambda V_{ij}) + \frac{\nu}{1 - 2\nu} (d_{kk} - \lambda V_{kk}) \delta_{ij} \right] \quad (4.16)$$

Young's modulus will be taken to be of the form:

$$E_U = b_0 \exp \{ - [((-b_4 - b_3 e_{v_D}) e_{v_D} + b_2 e_{v_D}) e_{v_D} + b_1] e_{v_D} \} \quad (4.17)$$

and Poisson's ratio will be taken to be of the form:

$$\nu = [b_6 + (b_7 + b_8 e_{v_P}) e_{v_P}] e_{v_P} \quad (4.18)$$

Again, these elastic moduli are unloading elastic moduli rather than loading moduli.

Flow Rules

The direction of the plastic stretching tensor is specified by the flow rule, or in this case, flow rules, since there are two parts to the yield surface. An associative flow rule is used for both surfaces in the limited deviatoric stress space. For the case of yielding on the side of the cone, the plastic stretching tensor is assumed to have no volumetric component. The normal in the π -plane, however, is used on the cone surface to construct deviatoric plastic strain increment directions. For the case of yielding on the ellipsoidal cap, the direction of the plastic stretching tensor is much more complicated and will be discussed below.

Cone

The method used here to develop the first flow rule is to find the normal to the cone. This normal is

$$\frac{df_c}{d\sigma_{ij}} = \frac{\frac{dJ_2}{d\sigma_{ij}}}{2\sqrt{J_2}} - a_1 \frac{dp}{d\sigma_{ij}} \quad (4.21)$$

The two derivatives in the expression are found using Eqs. (4.2) and (4.3) and $p = -\sigma_{kk}/3$ and are given as:

$$\frac{dJ_2}{d\sigma_{ij}} = \frac{dJ_2}{dS_{ij}} = S_{ij} \quad (4.22)$$

$$\frac{dp}{d\sigma_{ij}} = -\frac{1}{3} \frac{d\sigma_{kk}}{d\sigma_{ij}} = -\frac{1}{3} \delta_{ij} \quad (4.23)$$

Equations (4.22) and (4.23) are substituted into Eq. (4.21) to give:

$$\frac{df_c}{d\sigma_{ij}} = \frac{S_{ij}}{2\sqrt{J_2}} + \left(\frac{a_1}{3}\right) \delta_{ij} \quad (4.24)$$

The resulting direction of the normal on the side of the cone is given by the expression:

$$(V_{ij})_c = \frac{S_{ij}}{2\sqrt{J_2}} + \left(\frac{a_1}{3}\right) \delta_{ij} \quad (4.25)$$

The deviatoric part of Eq. (4.25) is needed since the plastic stretching tensor is taken to be normal to the conical part in the restricted deviatoric space. By applying the definition of a deviatoric quantity as

$$(q_{ij})_{dev} = q_{ij} - \frac{1}{3} q_{kk} \delta_{ij} \quad (4.26)$$

the deviatoric part of Eq. (4.25) is found to be

$$[(V_{ij})_c]_{dev} = \frac{S_{ij}}{2\sqrt{J_2}} \quad (4.27)$$

Ellipsoidal Cap

The conical yield surface is capped by a ellipsoidal end. Instead of using Eq. (4.16) to determine the final stress state, the final stress state will be calculated as

$$\sigma_{ijF} = \tilde{\lambda} \sigma_{ijT} \quad (4.28)$$

where $\tilde{\lambda}$ is used to differentiate it from the λ used in Eq. (4.16). This flow rule can also be written in the form of Eq. (4.16) as

$$\sigma_{ijF} = \sigma_{ijT} - 2G \lambda V_{ijdev} - K \lambda V_{kk} \delta_{ij} \quad (4.29)$$

Equating Eq (4.28) and (4.29) the following expression is obtained

$$(1 - \tilde{\lambda}) \sigma_{ijT} = 2G \lambda V_{ijdev} + K \lambda V_{kk} \delta_{ij} \quad (4.30)$$

Equating the deviatoric and volumetric components of Eq. (4.30) the following expressions are obtained

$$(1 - \tilde{\lambda}) S_{ijT} = 2G \lambda V_{ijdev} \quad (4.31a)$$

$$(1 - \tilde{\lambda}) \sigma_{kkT} = 3 K \lambda V_{kk} \quad (4.31b)$$

The plastic strain increment is given as:

$$\Delta \epsilon_{p_{ij}} = \lambda V_{ij} \quad (4.32)$$

Therefore, the plastic volume strain increment is given as

$$\Delta \epsilon_{v_p} = \lambda V_{kk} \quad (4.33)$$

Solving Eq. (4.31b) for λV_{kk} and substituting into Eq. (4.33) the following expression is obtained

$$\Delta \epsilon_{v_p} = \frac{(1 - \tilde{\lambda}) \sigma_{kkT}}{3 K} = \frac{(\tilde{\lambda} - 1) p_T}{K} \quad (4.34)$$

Eqs. (4.28) and (4.34) will be used when the model is implemented because they yield simpler expressions.

This flow rule is associative in the limited deviatoric space. To show this, an associative flow rule is found by differentiating the potential function with respect to the total stress state. The potential function for the ellipsoid is given in Eq. (4.8) as:

$$f_{eps} = C^2 J_2 + (p - p_c)^2 - C^2 (a_0 + a_1 p_c)^2 \quad . \quad (4.35a)$$

This expression is differentiated with respect to the stress. The result is given as:

$$(V_{ij})_{eps} = C^2 S_{ij} - \frac{2}{3} (p - p_c) \delta_{ij} \quad (4.35b)$$

The deviatoric and volumetric parts are:

$$[(V_{ij})_{eps}]_{dev} = C^2 S_{ij} \quad (4.36a)$$

$$\text{and} \quad (V_{kk})_{eps} = -2 (p - p_c) . \quad (4.36b)$$

Comparing Eq. (4.31a) and (4.36a), the expressions for V_{ijdev} only differ by a constant factor. This factor is unimportant, only the ratio of the various components is important. Because they both have the same ratio of components, the flow rule is associative in the limited deviatoric space.

Hardening Rules

The expressions for the yield surface parts included a single parameter which describes the growth of the surface, viz., p_c . This parameter must grow in such a manner as to correspond correctly to the results of the uniaxial compression tests, i. e., the stress must increase or "harden" in a realistic manner. Uniaxial stress experiments were run on the screen wire to determine the stress as a function of strain. For such a test, the various scalar quantities are:

$$\sigma_{ij} = \bar{\sigma} \delta_{i1} \delta_{j1} \quad (4.37)$$

$$p = - \frac{\bar{\sigma}}{3} \quad (4.38)$$

$$J_2 = \frac{1}{3} \bar{\sigma}^2 \quad (4.39)$$

where $\bar{\sigma}$ is the axial stress. Equation (4.8) is now written for this case as:

$$f_{eps} = C^2 \frac{\bar{\sigma}^2}{3} + \left(-\frac{\bar{\sigma}}{3} - p_c \right)^2 - C^2 (a_0 + a_1 p_c)^2 = 0 \quad (4.40)$$

Through a data fitting process, we later find the hardening rules to be:

$$p_c = p_c(e_{vp}) \quad (4.41)$$

$$K = K(e_{vp}) \quad (4.42)$$

and $\nu = \nu(e_{vp})$

The shear modulus is then related to the bulk unloading modulus and the unloading Poisson's ratio as

$$G = \frac{3 K (1 - 2\nu)}{2 (1 + \nu)} \quad (4.43)$$

The details of the data fitting process are given in Chapter 6.

In-Plane Tension Model

The in-plane tension behavior for the screen wire and Kevlar is decomposed into a component for the screen wire and a component for the Kevlar as

$$\sigma_T = \sigma_{sw} + \sigma_K \quad (4.44)$$

where σ_T is the total tensile stress in the direction of the fibers, σ_{sw} is the tensile stress in the screen wire, and σ_K is the tensile stress in the Kevlar fibers. This is done for each of the two orthogonal directions, and each is treated independently. All equations developed for this model will be employed twice, once for each of the orthogonal directions. It should be noted that orthogonal here does not mean the fiber angles are necessarily ninety degrees. Orthogonality here just means that the two directions are independent of one another.

The Kevlar is treated as a nonlinear elastic material up to fracture. Therefore, only the total strain is necessary to completely describe the state of stress in the fibers. The fibers are not allowed to carry any compression and they are allowed to fracture and cease to carry tensile loads at a given fracture strain. A curve fit is performed on experimental data to obtain an expression of the form

$$\sigma_K = \sigma_K(\epsilon) \quad (4.45)$$

The screen wire is treated as an elastic-plastic material up to fracture. For a monotonically loading stress path, the stress is fit as a function of the total strain to obtain an expression of the form

$$\sigma_{sw} = \sigma_{sw}(\varepsilon) \quad (4.46)$$

If the material is unloading, it will unload with a modulus equal to the initial tangent modulus. Upon reloading, it will follow the elastic unloading curve until it undergoes more plasticity, at which time it will again follow Eq. (4.46). The model will not allow the unloading to cause a compressive stress. If the fibers are unloaded such that an elastic stress would be compressive, the model returns a stress of zero but follows the unloading curve along the zero stress path upon reloading.

CHAPTER 5

Numerical Implementation of the Model for Screen Wire and Kevlar

Introduction

The equations presented are to be solved numerically. The material behavior is isolated to a single subroutine which, when given the present state of the material and the strain increment as input, produce the new state of the material as output. With the present material model, the state variables are: e_v^* , e_{vp} , ϵ_1 , ϵ_2 , ϵ_3 , $flagr$, $flagt$, $\sigma_{T_e}^r$, $\sigma_{T_e}^\theta$, σ_T^r , and σ_T^θ where e_v^* is the volumetric tensile strain since tensile fracture occurred (if fracture has taken place) and e_{vp} is the logarithmic volume strain. All other state variables are only used in the tensile fiber model. ϵ_1 , ϵ_2 , and ϵ_3 are the three principal strains in the material axes. $flagr$ and $flagt$ are flags in the R and Theta directions, respectively, to indicate if fracture has occurred in the fibers. $\sigma_{T_e}^r$ and $\sigma_{T_e}^\theta$ are the previous elastic trial tensile stress in the screen wire in the R and Theta directions, respectively, and σ_T^r and σ_T^θ are the total tensile stress added to the isotropic model from the tensile model in the R and Theta directions, respectively.

Upon entering the subroutine, the tensile stresses in the in-plane tensile model due to axial stresses in the fibers must be removed from the current stresses, since the isotropic model is expressed in an incremental form. Once this is accomplished, new stresses are calculated for the isotropic crush model and then the in-plane tensile model and added together to give the final stress state. Finding the new stresses for each model is discussed below. Because the model is anisotropic rather than isotropic, the initial

orientation of the material axes relative to the global axes is needed. This can be accomplished by initializing the rotation tensor $[R]$ to the appropriate values. This is also discussed below.

Isotropic Crush Model - Solution of Equations

General Derivations and Logic

There are four cases which must be covered for the isotropic model – the elastic case, the fracture case, yielding on the side of the conical yield surface, and yielding on the ellipsoidal yield surface. The method of separating the various cases is as follows. The first case concerns fracture and separation.

$$\text{If } (e_v^*)_0 = 0 \quad (5.1)$$

then the material is initially intact. The trial stress must be determined in order to find whether the material has fractured in this time step. That is discussed below.

$$\text{If } (e_v^*)_0 > 0 \quad (5.2)$$

then a trial value of volume fracture strain is found as:

$$(e_v^*)_T = (e_v^*)_0 + d_{kk} \Delta t \quad (5.3)$$

$$\text{If } (e_v^*)_T \geq 0 \quad (5.4)$$

then the material has fractured and the fracture surface is still open so that a tensile volume strain is indicated.

$$\text{If } (e_v^*)_T < 0 \quad (5.5)$$

then the fracture surface has closed and the stretching tensor has caused compression of the sample only over part of the time step, such that, we set:

$$d_{ij} \Delta t = \frac{d_{ij} (e_v^*)_T}{d_{kk}} \quad (5.6)$$

$$\text{and } e_v^* = 0. \quad (5.7)$$

If Eq. (5.1) or inequality (5.5) is true then the values of the elastic moduli are determined and an elastic trial stress is found by integrating Eq. (4.15) to obtain:

$$(\sigma_{ij})_T = (\sigma_{ij})_0 + \frac{E}{1+\nu} \left(d_{ij} \Delta t + \frac{\nu}{1-2\nu} d_{kk} \Delta t \delta_{ij} \right) \quad (5.8a)$$

$$\text{and } p_T = - \frac{(\sigma_{kk})_T}{3} \quad (5.8b)$$

These quantities are used to separate the remaining cases. In particular, this stress is used to evaluate J_2 using Eq. (4.2).

$$\text{If } p_T \leq p_c \quad (5.9)$$

then the state is on the conical end of the yield surface and f_c is calculated using Eq. (4.1).

$$\text{If } f_c \leq 0 \quad (5.10)$$

then the material is elastic. On the other hand,

$$\text{if } f_c > 0 \quad (5.11)$$

then the material is yielding on the conical yield surface and fracture may have occurred.

The case of fracture is identified as follows.

$$\text{If } p_T \leq p_{frac} \quad (5.12)$$

then the material has fractured. In that case, the value of e_v^* must be updated as described in Appendix C to the new value:

$$e_v^* = d_{kk}\Delta t - \left(\frac{p - p_{frac}}{K} \right) \quad (5.13)$$

On the other hand,

$$\text{if } p_r > p_{frac} \quad (5.14)$$

then fracture has not occurred and then the state is simply on the side of the conical yield surface.

$$\text{If } p_r > p_c \quad (5.15)$$

then the state is on the ellipsoidal end of the yield surface and f_{eps} is calculated using Eq. (4.8).

$$\text{If } f_{eps} \leq 0 \quad (5.16)$$

then the material is elastic.

$$\text{If } f_{eps} > 0 \quad (5.17)$$

then the material is yielding on the ellipsoidal part of the yield surface.

The calculations which must be performed in each of these four cases are covered next.

Elastic Case

The state can only be elastic if Eq. (5.1) is satisfied or if both Eqs. (5.2) and (5.5) are satisfied. In addition, either Eqs. (5.9) and (5.14) and Eq. (5.10) or Eqs. (5.15) and (5.16) must also be satisfied. In those cases, the final stress state is simply the trial state as calculated in Eq. (5.8) as

$$\sigma_{ij} = (\sigma_{ij})_T \quad (5.18)$$

No other state variables need to be updated in this case.

Fracture Case

Fracture has taken place if Eqs. (5.2) and (5.4) hold or if Eqs. (5.9) and (5.12) as well as either Eq. (5.1) or Eqs. (5.2) and (5.5) hold. In the case of fracture, the stress is simply given as:

$$\sigma_{ij} = -p_{frac} \delta_{ij} \quad (5.19)$$

The value of e_v^* must also be updated and accumulated at each time step if fracture has taken place and the crack is open. This monitoring is necessary since the crack may close later and the material again begins to take compressive strains. It is necessary to know exactly when this closure takes place. For the case where Eqs. (5.2) and (5.4) hold, the crack is initially open at the beginning of the time step and is still open at the end of the step. In that case the volume opening strain is accumulated as in Eq. (C6) as:

$$e_v^* = (e_v^*)_0 + d_{kk}\Delta t \quad (5.20)$$

In the other case, where fracture takes place during the time step and e_v^* is zero at the start of the time step, then the volume opening strain is given by Eq. (C5) as:

$$e_v^* = d_{kk}\Delta t - \left(\frac{p - p_{frac}}{K} \right) \quad (5.21)$$

Case of Yielding on the Side of the Cone

Yielding on the side of the conical yield surface only occurs when no fracture is taking place so Eq. (5.1) or Eqs. (5.2) and (5.5) are satisfied. In addition, Eqs. (5.9) and (5.11)

and (5.14) must all be satisfied. The model described has a fixed cone angle so the shape of the yield surface does not change. Eq. (4.12a) is applied over the time step to give:

$$(\sigma_{ij})_F = (\sigma_{ij})_0 + \int_0^{\Delta t} E_{ijkl} (d_{kl} - d_{kl}^P) dt \quad (5.22)$$

A non-associated flow rule is used so that no plastic volume strain takes place during yielding on the side of the cone. In other words, all volume strain is elastic volume strain so the final pressure is given by

$$p_r = p_0 - K d_{kk} \Delta t \quad (5.23)$$

The elastic moduli do not change over the time step in this case so the Young's modulus can be calculated at the start of the time step.

As in any displacement finite element program, the strain rate, or in this case the stretching tensor, is taken to be constant over the time step, so:

$$\int_0^{\Delta t} d_{ij} dt = d_{ij} \Delta t \quad (5.24)$$

From Eq. (4.11), the plastic stretching tensor is given by

$$\int_0^{\Delta t} d_{ij}^P dt = \int_0^{\Delta t} \lambda V_{ij} dt \quad (5.25)$$

The direction of the plastic stretching tensor is approximated as the direction of the value of V_{ij} at the trial state so that

$$\int_0^{\Delta t} d_{ij}^p dt = \lambda (V_{ij})_T \quad (5.26)$$

Equations (5.23) and (5.26) are substituted into Eq. (5.22) to give:

$$(\sigma_{ij})_F = (\sigma_{ij})_0 + E_{ijkl} (d_{kl} \Delta t - \lambda (V_{kl})_T) \quad (5.27)$$

Equation (5.8) is now substituted to give:

$$(\sigma_{ij})_F = (\sigma_{ij})_T - \lambda E_{ijkl} (V_{kl})_T \quad (5.28)$$

This is an expression for the final stress state in terms of known quantities and the unknown λ . This unknown is calculated using the compatibility condition. In the finite form used here, the compatibility condition is simply the requirement that the yield condition hold at the final state. Equation (5.8) is used to calculate the trial state and (4.27) to find the direction of the integrated plastic stretching tensor in (5.28). Equation (4.1) for the conical yield condition is now stated in terms of the final state as:

$$\sqrt{J_{2F}} - (a_0 + a_1 p_F) = 0 \quad (5.29)$$

This is now expanded in terms of λ using Eq. (5.28). It should be noted that since there is no plastic volume strain then p_c will not change. Eq. (4.2) and Eq. (5.27) are substituted into Eq. (5.29) and the result is written as:

$$\sqrt{\frac{1}{2}[(S_{ij})_T - 2G\lambda(V_{ijdev})_T][(S_{ij})_T - 2G\lambda(V_{ijdev})_T]} = (a_0 + a_1 p_F) \quad (5.32)$$

This is expanded and solved for the unknown using only the smallest positive root (since the other root is on the opposite side of the conical yield surface). Upon substituting Eq. (4.27) into Eq. (5.32) and solving, the following equation is obtained:

$$\lambda = \frac{\sqrt{J_{2T}} - (a_0 + a_1 p_F)}{G} = \frac{f_c}{G} \quad (5.33)$$

Eq. (5.23), Eq. (5.33) and Eq. (5.28) are combined to give the final total stress as:

$$(\sigma_{ij})_F = (\sigma_{ij})_T - \lambda E_{ijkl} (V_{kl})_T \quad (5.34)$$

The value of λ found in Eq. (5.33) is used to give the final stress state.

The approximate value of V_{ij} used in Eq. (5.28) causes no difficulty in locating on the final yield surface because it simply tells the relative position for the final stress on the yield surface. A single iteration could be used to reduce this error, i.e., the "final" stress value determined in Eq. (5.33) could be used on the right side of Eq. (5.30) and a better value of final stress state could be found. This is felt to be unnecessary and will not be done here.

Case of Yielding on the Ellipsoidal Cap

The case of yielding on the ellipsoidal end of the yield surface will only occur when no fracture is taking place so Eq. (5.1) or Eqs. (5.2) and (5.5) are satisfied. In addition, Eqs. (5.15) and (5.17) must also be satisfied. The plasticity relations given in Eqs. (5.22), (5.24), and (5.25) apply in this case also. At this point the present case becomes more complicated because during a plastic process, hardening is represented by the simultaneous change in the elastic moduli, plastic change of the deviatoric stress,

plastic change of the pressure, movement of the ellipsoidal cap, and an increase in the size of the cap. Both the growth in size and the location of the surface on the pressure axis are represented by the single parameter, p_c . Equation (4.8) describes the loading surface and Eqs. (4.31a) and (4.31b) give the direction of the plastic stretching tensor, V_{ij} . As described in Chapter 4, the final stress state will be found using Eq (4.28).

The big problem with the model presented here is that the elastic moduli are a function of the volumetric plastic strain. Then "trial" stresses cannot be calculated in the spirit of the radial return method, or more correctly, the projection operator method. Thus in the plasticity process on the conical part of the yield surface, all quantities were calculated using the elastic moduli at the initial state. Since the plastic volume strain was assumed to be zero, the initial and final elastic moduli are the same. In the present case, the final elastic moduli may be much different than the moduli at the beginning of the step.

The reason that the moduli at the end of the time step must be used is illustrated in Figure 5.1. A loading curve is schematically shown for a hardening material such as screen wire. Three unloading curves are also shown where the unloading is from a different stress point on the loading curve. The lowest curve is the case where unloading began at 5000 psi, the next curve is where unloading began at 20000 psi, and the third where unloading began at 45000 psi. The unloading modulus is taken to be tangent to the loading curve at the point where unloading begins. This behavior is roughly that seen in the screen wire.

Now consider the case where the material is loaded to a strain of -3. Note that the material passed through many states with a different elastic modulus associated with each state during the loading process. The material is now unloaded along the dashed straight line shown. At that point we note that the plastic strain remaining in the material is the current strain of -1.5. If the material is now reloaded it will follow the dashed line until a

strain of -3. This loading process is completely elastic until it reaches the strain -3, so that the plastic strain in the sample remains at -1.5 during the process. When the material loads beyond a strain of -3, the plastic strain will again increase. The point to note is that if -3 is the maximum strain to which the material has been subjected, then the plastic strain is -1.5 and is calculated as

$$\epsilon_p = \epsilon_{max} - \frac{p_{max}}{(Slope)_{max}} \quad (5.35)$$

Equation (4.12b) then should only involve the elastic modulus associated with the maximum compression to which the material has been subjected.

The final stress state is written as

$$\sigma_{ijF} = \tilde{\lambda} \sigma_{ijT} \quad (5.36)$$

The plastic parameter $\tilde{\lambda}$ is found by setting the loading function of Eq. (4.4) at the final state equal to zero as:

$$C^2 J_{2F} + (p_F - p_{CF})^2 - C^2 (a_0 + a_1 p_{CF})^2 = 0 \quad (5.37)$$

This now presents a computational dilemma. The elastic moduli needed to calculate the proper trial stress state cannot be found until the plastic volume strain at the end state is known. This dilemma is avoided with an estimate of the plastic volume strain increment over the time step. In Appendix D is shown how the plastic strain increment can be approximated with the aid of a hypothetical uniaxial stress case with the same value of f_{eps} .

The approximate plastic strain increment is found from Appendix D and the plastic volume strain is estimated as:

$$\epsilon_{vp} \equiv \epsilon_{vp0} + \Delta\epsilon_{vp} \quad (5.38)$$

This estimated plastic volume strain is used to find new values of the elastic moduli and p_c . A new trial stress state is found using the new moduli so that new values of J_{2T} and V_{ij} can be found. Because equation (5.37) must hold at the end of the time step, the final values are expanded in terms of these trial values and the volumetric plastic increment.

The final pressure and the second invariant of the deviatoric stress are calculated using Eq (5.36) as:

$$p_F = \tilde{\lambda} p_T \quad (5.39)$$

$$\text{and } J_{2F} = \tilde{\lambda}^2 J_{2T} \quad (5.40)$$

Now p_{cF} is expressed in terms of the estimated state and $\tilde{\lambda}$ in the form:

$$p_{cF} = \left(p_{ce} - \left(\frac{dp_c}{de_{vp}} \right)_e \Delta e_{vp_{est}} \right) + \left(\frac{dp_c}{de_{vp}} \right)_e \Delta e_{vp} \quad (5.41)$$

Eq. (4.34) is used for the plastic volume strain increment and substituted into Eq. (5.41) to give:

$$p_{cF} = \left(p_{ce} - \left(\frac{dp_c}{de_{vp}} \right)_e \Delta e_{vp_{est}} \right) + \left(\frac{dp_c}{de_{vp}} \right)_e \frac{(\tilde{\lambda} - 1) p_T}{K} \quad (5.42a)$$

which is simplified to:

$$p_{cF} = p_{c0} - Q p_T + \tilde{\lambda} Q p_T \quad (5.42b)$$

where Q and p_c are defined as:

$$Q = \frac{1}{K} \left(\frac{d p_c}{d e_{vp}} \right)_e \quad (5.43)$$

$$p_{c0} = p_{ce} - Q K \Delta e_{vp_e} \quad (5.44)$$

The derivative of p_c with respect to the plastic volume strain is evaluated at the estimated plastic volume strain at the end of the time step.

Now Eqs. (5.39), (5.40), and (5.42) are substituted into Eq. (5.37) to obtain:

$$C_0 \tilde{\lambda}^2 + C_1 \tilde{\lambda} + C_2 = 0 \quad (5.45)$$

where:

$$C_0 = C^2 J_{2T} + [p_T(1-Q)]^2 - (C a_1 p_T Q)^2 \quad (5.46)$$

$$C_1 = -p_T(1-Q)(p_{co}-Qp_T) - C^2 p_T Q a_1 (a_0 + a_1 [p_{co}-Qp_T]) \quad (5.47)$$

$$\text{and } C_2 = (p_{co}-Qp_T)^2 - C^2 (a_0 + a_1 [p_{co}-Qp_T])^2 \quad (5.48)$$

Eq. (5.45) is a quadratic equation for $\tilde{\lambda}$ and is solved as:

$$\tilde{\lambda} = \frac{-C_1 - \sqrt{C_1^2 - C_0 C_2}}{C_0} \quad (5.49)$$

The final stress state is now calculated using $\tilde{\lambda}$ and Eq. (5.36). The value for the plastic volume strain increment can now be calculated using Eq (4.34):

$$\Delta\epsilon_{v_p} = \frac{(\tilde{\lambda} - 1) p_T}{K} \quad (5.50)$$

As for the conical case, this process will not produce a stress which is exactly on the yield surface but it should be very near. If more precision is needed, the new plastic volume strain produced by the above process can be used as a better approximation and the above process repeated. This was not deemed necessary and therefore was not done.

Tensile Fiber Model

In Chapter 2, the tensile properties for screen wire and Kevlar were determined. These effects are superimposed on top of the isotropic crush model to give the total behavior of the composite of layered screen wire and Kevlar. The tensile behavior in the screen wire and the Kevlar are treated independently, although they both depend on the strain in axial direction of the fibers or wires in the case of the screen wire. The screen wire will be treated as an elastic-plastic material, while the Kevlar is treated as a non-linear elastic material. There is no in-plane shear in the model and the two fiber directions are treated independently so the two fiber directions are orthogonal because there is no coupling between them.

For both the screen wire and the Kevlar, a fracture strain is defined and the fibers cease to carry any tensile load after fracture has occurred. This requires that a gauss point storage be kept for both directions to keep track if the fibers have fractured. The state variables *flagr* and *flagt* are used for this purpose.

Since the Kevlar is treated as a non-linear elastic material, the strain completely describes its stress state. This is not the case for the screen wire because it is considered an elastic-plastic material. To obtain the stress state for the screen wire, assuming fracture has not taken place in these fibers, an elastic trial stress is calculated. This trial stress is calculated as the previous final stress in the fibers plus an elastic increment calculated as the strain step times an unloading modulus. The modulus used for this calculation is the initial slope of the stress strain curve. A plastic stress is then calculated using Eq. (6.15). The final state is the minimum of the elastic trial stress and the plastic stress. If the elastic trial stress is less than zero then zero is returned for the stress from the screen wire. This is necessary to ensure the fibers never carry any compression. This algorithm requires gauss point storage of the elastic trial stress in both directions and the total tensile stress predicted from both the screen wire and the Kevlar so that it can be removed from the isotropic model upon entering the model for the next time step.

Anisotropic Material Description

The woven Kevlar reinforcing material is very anisotropic in a special sense. The properties along the orthogonal weave directions, say the x- and y-directions, are the same but the shear stiffness σ_{xy} is so small as to be negligible. Since these materials are fairly brittle, a reasonable constitutive description would be

$$\sigma_{xx} = f_x(\epsilon_{xx}) \quad (5.51)$$

$$\sigma_{yy} = f_y(\epsilon_{yy}) \quad (5.52)$$

$$\text{and } \sigma_{xy} = 0 \quad (5.53)$$

A description of this type would produce no shear stress for a given shear strain. It should be noted, however, that shear stresses in other coordinate systems can be developed. Mohr's circle for stresses has nothing to do with constitutive models, it is merely a relation between stresses in different coordinate systems. The anisotropy arises because of the relation between stresses and strains. This particular type of anisotropy can cause severe numerical problems for structural finite element codes. In particular, it causes the global stiffness matrix to be singular. It will also produce element eigenvectors with zero eigenvalues. For a code such as PRONTO2D or PRONTO3D (Flanagan and Taylor, 1987), which can operate with singular stiffness matrices, it will evidence itself in much the same way as when hourglass modes (Flanagan and Belytschko, 1981) are not restrained by anti-hour glassing stiffness or viscosity. With the fabric description, the troublesome eigenmodes for an initially square element with the weave aligned with the element would be diamond shaped eigenmodes.

Anisotropic materials can be easily treated in a code such as PRONTO (Taylor and Flanagan, 1987) or SANTOS (Stone, 1992) or JAC (Biffle, 1981) which use the unrotated stretching tensor, the unrotated Cauchy stress, and the Green-Naghdi-Dienes stress flux (Dienes, 1979). With that set of stress and strain measures, the Cauchy stress and the stretching tensor in the fixed global rectangular coordinate system are rotated to a coordinate system which is peculiar to each particle and is such as to be aligned with the original axes. This orthogonal rotation matrix is taken to be the matrix $[R]$ associated with the polar decomposition (Malvern, 1969) of the deformation gradient which is written as:

$$[F] = [V][R] \quad (5.54)$$

The stretching tensor and Cauchy stress and other state variables as well as the stress flux are then transformed as:

$$[D]_{unrot} = [R]^T [D] [R] \quad (5.55)$$

In order that the rotation matrix can be updated in an accurate manner, both matrices $[V]$ and $[R]$ are required. The velocity gradient is also used in updating these quantities.

For the anisotropic material, it is helpful to state constitutive equations in terms of axes lined up with the material directions. All material state variables would be stored in the material coordinates. At a new load step, the stress and stretching tensor would be unrotated from the global coordinates to the local unrotated directions with the rotation tensor which we call $[R_{g \rightarrow ur}]$. They would then be rotated by an additional amount to the local material directions with the rotation tensor $[R_{ur \rightarrow m}]$. Both of these rotation tensors are unique for each particle (or Gauss point). They both must be stored, although only $[R_{g \rightarrow ur}]$ is updated. The total rotation applied to the stress and stretching tensor is then given by the product:

$$[R_{g \rightarrow m}] = [R_{g \rightarrow ur}] [R_{ur \rightarrow m}] \quad (5.56)$$

It appears, however, that this process can be shortened considerably. The matrices $[R]$ and $[V]$ used for the global to local unrotated coordinates are ordinarily both initialized as identity matrices. In the present case, the matrix $[V]$ would still be initialized as the identity but the rotation matrix would be initialized as $[R_{ur \rightarrow m}]$ and could be initially regarded as $[R_{g \rightarrow m}]$. This matrix would then be used in the updating process. In the finite element code then it would be rotated as the material rotates.

CHAPTER 6

Data Fitting for the Constitutive Model

Screen Wire and Kevlar

Isotropic Crush Model

The tests on screen wire were fitted with various functions to use in the constitutive subroutine. These procedures for determining the functions are outlined here. The data fitting process is tailored to the assumptions for the yield surfaces and the flow rules chosen.

Because no experimental data is available to determine the shape of the cone, it is chosen such that a uniaxial stress path intersects the ellipsoidal cap rather than the cone. Appendix E shows that there is a restraint on what these parameters can be. In addition, the cone should not be too steep. Therefore, the constants a_0 and a_1 are chosen to be zero and two respectively. This choice of a_0 and a_1 implies that the tensile fracture stress is zero. With this choice of a_0 and a_1 we now find the constant C to be 0.342374064 in Appendix B.

The functions that are needed are K , ν , p_c , and the derivative of the total volume strain with respect to the plastic volume strain all fitted as a function of the plastic volume strain. K and ν are the traditional bulk modulus and Poisson's ratio. As noted in Chapter 5 these are elastic unloading quantities. p_c describes the yield surface as a function of the plastic volume strain. The derivative of the total volume strain with

respect to the plastic volume strain is needed for the estimate of the plastic volume strain at the end of the step as described in Appendix D.

The process of determining the necessary functions is started by defining a regular progression of axial compressive strains. The lateral strain is then calculated using the function

$$\varepsilon_2 = 0.07085 (\varepsilon_1)^2 \quad (6.1)$$

which was fitted from the uniaxial stress tests. Because the material is assumed to be isotropic, the total volume strain is now calculated as the axial strain plus twice the lateral strain. Now the axial stress is calculated using the fitted expression from the uniaxial stress tests as:

$$\sigma_1 = -300.8 (e^{-2.86339 \varepsilon_1} - 1) \quad (6.2)$$

The unloading elastic Young's modulus is calculated using the expression:

$$E_U = 3000. + \exp \left(\frac{-(13.18 - 1.385 \times 10^{-5} \sigma_1) \sigma_1}{86.81 - \sigma_1} \right) \quad (6.3)$$

The elastic unloading modulus was measured by unloading from various points on a compressive uniaxial loading test and measuring the slope of the unloading curves and plotting them versus the stress at which loading took place. The compressive loading was then resumed and the process repeated at other load levels. The data for this fit was provided by Pierce (1992). Figure 6.1 shows the difference between the elastic unloading modulus and the tangent or loading modulus.

Because of the choice of flow rule for the ellipsoidal cap, Poisson's ratio must be defined such that the proper lateral expansion is obtained for a uniaxial stress path. The choice of the flow rule implies that all lateral expansion is elastic for the uniaxial stress path. Therefore, Poisson's ratio is calculated as:

$$\nu = - \frac{\Delta \epsilon_2}{\Delta \epsilon_1} \quad (6.4)$$

The unloading bulk modulus is then calculated using the elastic relation

$$K = \frac{E}{3(1-2\nu)} \quad (6.5)$$

For the uniaxial compressive tests, the total volume strain increment can be written as the sum of plastic and elastic volume strain increments where the elastic strain increment is computed using the elastic unloading modulus, or more specifically:

$$\Delta \epsilon_{vp} = \Delta \epsilon_v - \frac{\Delta \sigma_{mean}}{K} \quad (6.6)$$

The mean stress is simply one third the axial stress since no stress is developed in the lateral directions. The plastic volume strain is now calculated for each axial strain value using Eq. (6.2). The unloading elastic bulk modulus and Poisson's ratio can now be fit as a function of the plastic volume strain to obtain

$$\nu = [-0.19169 + (-0.08115 - .073289 \epsilon_{vp}) \epsilon_{vp}] \epsilon_{vp} \quad (6.7)$$

$$\text{and } K_u = 577.44 e^{((-21.557 + (-42.550 + (-59.141 + (-46.650 - 14.732 \epsilon_{vp}) \epsilon_{vp}) \epsilon_{vp}) \epsilon_{vp})} \quad (6.8)$$

Figure 6.2 shows a plot of Poisson's ratio as a function of plastic volume strain and the functional fit of this data. Figure 6.3 shows a plot of the elastic unloading bulk modulus as a function of plastic volume strain and the functional fit of this data.

In the estimate for the plastic volume increment the derivative of the total volume strain with respect to the plastic volume strain is needed. This derivative is now calculated by dividing the change in total volume strain by the change in plastic volume strain. This is now fit in terms of the plastic volume strain to obtain

$$\frac{d \epsilon_v}{d \epsilon_{vp}} = 1.6206 + (5.769 + (20.9695 + (37.035 + (33.525 + (14.647 + 2.4000 \epsilon_{vp}) \epsilon_{vp}) \epsilon_{vp}) \epsilon_{vp}) \epsilon_{vp}) \epsilon_{vp} \quad (6.9)$$

Figure 6.4 shows a plot of this derivative as a function of plastic volume strain and the functional fit.

One other variable which is needed is p_c , the center of the ellipsoidal cap. This variable is found by looking at a uniaxial stress path. It should be noted that the yield ellipse is initially at the origin, therefore p_c moves as soon as the material is strained. The uniaxial compressive yield condition on the ellipsoidal cap is

$$C^2 \frac{\sigma_1^2}{3} + \left(\frac{-\sigma_1}{3} - p_c \right)^2 - C^2 (a_0 + a_1 p_c)^2 = 0 \quad (6.10)$$

This equation is solved for p_c and the positive root used to give

$$p_c = \frac{-\left(\frac{\sigma_1}{3} - C^2 a_0 a_1\right) + \sqrt{\left(\frac{\sigma_1}{3} - C^2 a_0 a_1\right)^2 - (1 - a_1^2 C^2) \left[\frac{\sigma_1^2}{9} (3C^2 + 1) - a_0^2 C^2 \right]}}{1 - a_1^2 C^2} \quad (6.12)$$

The constants a_0 , a_1 , and C have been determined so Eq (6.12) is reduced to the following relationship between p_c and σ which is

$$p_c = -0.294261 \sigma_1 \quad (6.13)$$

Now p_c is fit as a function of the plastic volume strain to give

$$p_c = 87.5 \left[e^{(-4.0328 + (-1.8323 - 1.5507 \epsilon_{vp}) \epsilon_{vp})} - 1 \right] \quad (6.14)$$

Figure 6.5 shows a plot of p_c as a function of plastic volume strain and the functional fit.

Some intermediate steps were omitted from the discussion above. A more detailed explanation of the steps involved is included in Appendix F. In this appendix, the spread sheet used for all calculations is included.

In-Plane Tension Model

The experimental data for the in-plane tensile behavior for screen wire and Kevlar, from Chapter 2, was used to determine the necessary parameters for the model. Because the tensile behavior in one direction is assumed to be independent of the other direction, the stress is fit only as a function of the axial strain. The fit for the axial behavior of the screen wire is

$$\sigma_{sw} = \frac{(7092.7 + 58055.4 \epsilon) \epsilon}{.0069978 + \epsilon} \quad (6.15)$$

and the fit for the axial behavior of the Kevlar is

$$\sigma_K = 99480.0 \epsilon [1 - e^{(-29.2 \epsilon)}] \quad (6.16)$$

Perforated Plate

The data for perforated plate must be fit with various functions to be used in the constitutive model. The procedures for performing this data reduction are outlined in this section. Again, the data fitting procedures are tailored to the choice of yield surfaces and flow rules.

The first step to fitting perforated plate to the constitutive subroutine is to determine how stacked perforated plate behaves for a random orientation of adjacent plates. Begin by considering a control volume that represents two adjacent stacked plates. The appropriate control volume for the top plate is shown in Figure 6.6. The plate beneath is assumed to take every possible position under the constraint of no plate rotation. This control volume represents every section of two stacked plates when the bottom plate is translated with respect to the top plate. Rotations are not considered here because when an impact limiter is constructed, the plates will be stacked one on top of another but will not be rotated with respect to one another.

In Chapter 3, data was obtained for the maximum and the minimum hole overlap between adjacent plates. For other hole overlaps, assume that the functions can be written as

$$\varphi = w \varphi_1 + (1 - w) \varphi_2 \quad (6.17)$$

where φ is the function for a particular hole overlap, w is a weighting factor, φ_1 is the function for the minimum hole overlap, and φ_2 is the function for the maximum hole overlap. It should be noted that φ could represent stresses, unloading moduli, or any other quantity. The distribution of the assumed weighting factor is shown in Figure 6.7.

An average ϕ for a random orientation of adjacent plates is needed to perform the data reduction. The top plate is considered fixed while the bottom plate is translated with respect to the top plate. The orientation of the bottom plate with respect to the top plate is defined by considering a single point in the bottom plate. The location of that single point, in the control volume of the top plate, completely defines the orientation of the bottom plate with respect to the top plate.. In a discrete sense, the average can then be calculated by defining a set of points equally spaced throughout the control volume. At each point, ϕ is calculated using Eq. (6.17). An average value for all possible orientations between the two adjacent plates can then be calculated as the sum of the ϕ 's at each of the points in the control volume divided by the number of points. This is expressed mathematically as:

$$\phi_{ave} = \frac{\sum_{i=1}^n \sum_{j=1}^n \phi_{ij}}{\sum_{i=1}^n \sum_{j=1}^n 1} \quad (6.18a)$$

where ϕ_{ij} is the value of ϕ at the point ij , n is the number of points. By taking the limit as n goes to infinity, the summation signs are replaced by integrals and the following expression is obtained:

$$\phi_{ave} = \frac{\int \int \phi \, dx \, dy}{\int \int dx \, dy} \quad (6.18b)$$

where ϕ_{ave} is the average function for a random orientation of hole overlaps and the integral is performed over the entire control volume. Even if this integration can not be performed analytically, it can certainly be performed numerically using Eq. (6.18a). This procedure yields an average stress-strain relationship for a random orientation of hole overlap from the data obtained in Chapter 3.

Because no data is available to describe the initial shapes of the yield surfaces, the general values chosen for screen wire will be used here. These values are

$$a_0 = 0 \quad (6.19a)$$

$$a_1 = 2 \quad (6.19b)$$

$$\text{and } C = 0.342374 \quad (6.19c)$$

Because the data shows that this material behaves elastically for small strains rather than yielding, the data reduction process must represent this fact. From the “experimental” data in Chapter 3, it was determined that yield occurs at a total volume strain of -0.007562. Because data reduction concerns plastic processes only, the process only needs to be performed for strains larger in magnitude than -0.007562. For strains below this value, the process will be elastic and is determined entirely by the initial matrix of elastic constants.

Because of the difficulties in obtaining data for the minimum hole overlap, the above procedure of finding an average value will not be used here. After eliminating the elastic part of the stress strain curves, the curves for the maximum hole overlap, from Chapter 3, can be fit to obtain:

$$\sigma_{x_{max}} = -12054 + (60593 + (-618480 + (-2332000 - 6333600 \epsilon_v) \epsilon_v) \epsilon_v) \epsilon_v \quad (6.19a)$$

$$\sigma_{y_{max}} = -12501 + (63180 + (-617300 + (-2311900 - 6294200 \epsilon_v)\epsilon_v)\epsilon_v) \quad (6.19b)$$

$$\text{and} \quad \sigma_{z_{max}} = -24590 + (-222720 + (-393810 - 2213200 \epsilon_v)\epsilon_v)\epsilon_v \quad (6.19c)$$

where $\sigma_{x_{min}}$, $\sigma_{y_{min}}$, and $\sigma_{z_{min}}$ are the x, y, and z stresses for the minimum hole overlap and ϵ_v is the total volume strain. By comparing the yield stress in the z-direction between the minimum and maximum hole overlap in Chapter 3, it was determined that an average stress-strain curve for a random hole orientation would be approximately 0.712 times the maximum hole overlap stress-strain curve. This value will be used to perform the data reduction.

The data fitting procedure is begun by defining a regular progression of strains. It should be noted that this strain is both axial strain as well as volumetric strains since these problems were analyzed in uniaxial strain. This progression of strains starts at the yield strain since all stress states below this strain are elastic and do not influence the data reduction process. The three stress components are now calculated for a random hole overlap using Eq. (6.19) and the scaling factor described above. The mean stress is now calculated as one third the sum of the x, y, and z stress components.

The data for the elastic unloading Young's modulus and Poisson's ratio for the maximum hole overlap will be used for the random orientation of holes since no other data is available. These moduli are fit as a function of the volumetric strain as

$$E = 6839800 + (-16879000 + (-37414000 - 37556000 \epsilon_v)\epsilon_v)\epsilon_v \quad (6.20a)$$

$$\text{and} \quad \nu = 0.20819 + (-0.16307 + (-0.18611 - 0.73932 \epsilon_v)\epsilon_v)\epsilon_v \quad (6.20b)$$

These elastic moduli are now calculated for the progression of volumetric strains. Now G and K are calculated using Eq. (6.20) and the elastic relations

$$K = \frac{E}{3(1-2\nu)} \quad (6.21a)$$

and $G = \frac{E}{3(1-2\nu)} \quad (6.21b)$

The plastic volume strain, ϵ_{vp} , can now be calculated using Eq. (6.6). G and K are fit as functions of the plastic volume strain to obtain

$$K = 3984600 + (-12490000 + (-26362000 - 54536000 \epsilon_{vp})\epsilon_{vp})\epsilon_{vp} \quad (6.22)$$

and $G = 4402700 + (-11627000 + (-24995000 - 33697000 \epsilon_{vp})\epsilon_{vp})\epsilon_{vp} \quad (6.23)$

Figure 6.8 shows the data and the curve fit for the elastic unloading bulk modulus as a function of the plastic volume strain and Figure 6.9 shows the data and the curve fit for the elastic unloading shear modulus as a function of the plastic volume strain. The derivative of the total volume strain with respect to the plastic volume strain is calculated as

$$\frac{d\epsilon_v}{d\epsilon_{vp}} = \frac{\Delta\epsilon_v}{\Delta\epsilon_{vp}} \quad (6.21)$$

and fit as a function of the plastic volume strain to obtain

$$\frac{d\epsilon_v}{d\epsilon_{vp}} = 1.0220 + (-0.0095229 + (0.0078002 - 0.0014245 \epsilon_{vp})\epsilon_{vp})\epsilon_{vp} \quad (6.22)$$

Figure 6.10 shows the data and the curve fit for the derivative of the total volume strain with respect to the plastic volume strain as a function of the plastic volume strain.

The remaining quantity to be calculated is the center of the ellipsoidal cap. Begin by considering the yield function for the ellipsoidal cap, which is

$$f_{eps} = C^2 J_2 + (p - p_c)^2 - C^2 (a_0 + a_1 p_c)^2 \quad (6.22)$$

Since this is zero on the yield surface, set Eq. (6.22) equal to zero and solve for p_c , to obtain

$$p_c = \frac{(p + a_0 a_1 C^2) \pm \sqrt{(p + a_0 a_1 C^2)^2 - (1 - a_1^2 C^2)(C^2 J_2 + p^2 - a_0^2 C^2)}}{C^2 J_2 + p^2 - a_0^2 C^2} \quad (6.23)$$

There are two ellipses that will satisfy Eq. (6.23). One is centered at a lower pressure than the current stress state and one is centered at a higher pressure than the current stress state. The one needed for this data fitting procedure is the one centered at a lower pressure than the current stress state, so the negative root of Eq. (6.23) is used. The pressure, p , is calculated as minus one-third the sum of the x, y, and z stresses. J_2 is calculated using Eq. (4.2). Since the other constants in Eq. (6.23) have been determined, p_c can be calculated and fit as a function of the plastic volume strain to obtain

$$p_c = 7187.7 + (-76966 + (-209960 - 1131800 \epsilon_{v_p}) \epsilon_{v_p}) \epsilon_{v_p} \quad (6.24)$$

Figure 6.11 shows the data and the curve fit for the center of the ellipsoidal cap as a function of the plastic volume strain.

The in-plane behavior of perforated plate is much different from that of the layered screen wire and Kevlar. First of all, perforated plate can carry in-plane shear. Furthermore, there are not two independent in-plane directions. The in-plane behavior

can be modeled using a conventional plane stress metal plasticity constitutive model where the direction of plane stress is in the out-of-plane direction. This will not be done here.

Some intermediate steps were omitted from the discussion above. A more detailed explanation of the steps involved is included in Appendix G. In this appendix, a discussion for using a spread sheet to perform the calculations is included.

CHAPTER 7

Verification of the Constitutive Model

Introduction

To verify the constitutive model, it was necessary to demonstrate that the model would predict the experimental results used to construct the model. Because of the very complicated data reduction process and model, this is not a trivial exercise. Because very little experimental data is available, there are only a few test problems which can be run. The first of these problems is a uniaxial stress problem. The other problem to which experimental data is available is a pure axial tension test in the plane of the fibers. Both of these problems were run with the model using a driver to give the model the appropriate strain rates, stress states, and state variables. Because very few meaningful problems can be run this way and because this model will be used to predict the behavior of high speed impact limiters during plane crashes, this constitutive model was also implemented in the finite element code PRONTO2D (Taylor and Flanagan, 1987). Only the uniaxial compressive test was modeled in PRONTO2D.

Uniaxial Compression Test Problem

As discussed in Chapter 6, the tensile part of the model must be deactivated because the edges are free and can not support the tension that would be predicted. The

model is implemented as outlined in Chapter 5. A FORTRAN listing of the model including the in-plane tensile model is given in Appendix H.

The finite element method adds several complications to the process of testing the constitutive model. The first and most obvious problem is to ensure the model is properly implemented in the finite element code. In addition, the finite element method adds keystone stiffness and artificial bulk viscosity (Taylor and Flanagan, 1987) which only complicates testing the model. The finite element method must also predict a critical time step for the code to be stable (Taylor and Flanagan, 1987). If a larger time step is taken, the results can become unstable even though the constitutive model is behaving properly. To avoid these complications for the verification of the model, it was decided to exercise the model with a driver rather than implement it in a finite element code.

In order to exercise the constitutive model without a finite element code, a FORTRAN driver was written. The driver's sole purpose is to give the constitutive model the appropriate state variables, the initial state of stress, and the stretching tensor. To obtain a uniaxial stress path, the appropriate stretching tensor components must be sent to the model. The axial component of the stretching tensor is arbitrary and may be chosen to give the desired strain step. The lateral components of the stretching tensor must be chosen such that the model predicts no stress in the lateral directions. The difficulty here is that the lateral components are not known such that the model will predict no stress in the lateral direction. To avoid this difficulty, an iterative process is used. For the first iteration, the lateral components of the stretching tensor are set to zero and the axial component is set to the desired value. The model then predicts the stresses from the stretching tensor and the state variables. This will not be uniaxial behavior. The lateral components of the stretching tensor are changed so that the model will predict results closer to uniaxial compression using the equation

$$d_{2_{n+1}} = d_{2_n} - \frac{\sigma_{2_n}}{E \Delta t} \quad (7.1)$$

where $d_{2_{n+1}}$ is the guess for the lateral component of the stretching at the next iteration, d_{2_n} is the previous guess for the lateral component of the stretching tensor, σ_{2_n} is the predicted stress in the lateral direction, E is an elastic unloading modulus, and Δt is the time step. The elastic unloading modulus E is taken to be a very large number so that the guess for the new lateral component of the stretching tensor converges in a monotonic manner. If E is chosen too large it will just increase the number of iterations required to converge. The axial component of the stretching tensor is never changed. This iterative process is repeated until the results converge to a uniaxial behavior.

This problem was run with a variety of strain step sizes ranging from -0.001 up to -0.5. All results for small strain steps provided almost identical results. In the next three figures, compressive strains and compressive stresses are taken as positive. Figure 7.1 shows a plot of axial stress versus axial strain for several step sizes and the data fit from the experimental data. The figure shows that for small strain steps, the model almost exactly reproduces the experimental data used to construct the model. As this strain step gets larger, the error in the model increases. For a strain step of -0.5, the stress in the axial direction is under predicted by 15.7% at a total strain of -1.5. To achieve more accurate results, strain steps should be limited to about -0.1. Figure 7.2 shows a plot of lateral strain versus axial strain for several step sizes and the data fit from the experimental data. Again, as the strain step size increases, the error in the lateral strain increases. For a strain step of -0.5, the lateral strain is under predicted by 8.2% at a total strain of -1.5. Figure 7.3 shows a plot of axial stress versus strain with unloadings which shows the unloading modulus increases as a function of plastic volume strain.

For small strain steps, the results obtained from the model were extremely close to the experimental data used to construct it. All variables were plotted and shown to be in agreement with the experimental data. In addition, it is clear that the model is almost step size independent for yielding on the ellipse for strain steps smaller than 0.1. One reason for the discrepancy in results for very large strain steps comes from the fact that an estimate is made for the final plastic volume strain and this is used for all calculations. As the steps become very large, this estimate does not give a good enough approximation. This could be partially circumvented by iterating so that the second time through, the guess for the plastic volume strain would be very close. Some care must be taken for other types of loading which yield on the cone and then yield on the ellipse rather than just yielding on the ellipsoidal end. The model was constructed such that it should always give results. But, if the strain step is too large, the results will be inaccurate.

Because the end use for this constitutive model is in a large strain non-linear finite element code, it was necessary to test the model under such an environment. PRONTO2D was used to perform this analysis. The same model used by the driver, which has the tensile model disabled, was implemented in PRONTO2D. A one element, axisymmetric problem was run with the outer boundary free so that the code should predict uniaxial compression. Because PRONTO2D is a dynamic code the results will oscillate slightly rather than being exactly uniaxial. PRONTO2D was forced to have a very small time step thereby reducing the oscillations. The results from this analysis are shown in Figure 7.4. These results are very similar to those predicted with the model by itself and to the experimental data. This demonstrates the model's ability to be used in conjunction with a finite element code.

In-Plane Tension Tests

The other problem to which experimental data was available was in-plane tension for the screen wire and Kevlar with no crushing normal to the plane. The driver described above was used to exercise the model. The driver was modified so that it just strained the material in the directions of the fibers causing only the in-plane tensile model to produce stresses, rather than iterating to uniaxial compression. For this case the isotropic crush model predicted a volume fracture as described in Chapters 4 and 5.

Figure 7.5 shows a plot of the predicted axial stress versus axial strain and the curve that was fit to the experimental data. This figure shows that the fibers do fracture at the appropriate strains and cease to carry load after that point. These results verify that both the theory and implementation of the in-plane tensile model is accurate.

CHAPTER 8

Summary and Conclusions

In medical and defense applications it is necessary to transport nuclear materials by airplane. The containers which carry this material must be able to survive the most severe airplane crash without exposing the public to any radiation. The tests to demonstrate the survivability of a package of this type are extremely expensive and difficult to perform. To supplement the physical tests to demonstrate survivability, the finite element method is used to numerically analyze a given design. In order to accurately model a given design, the proper material description or constitutive model must be used. This thesis documents the development of a volumetrically yielding constitutive model with variable elastic constant to model the materials used in this type of package.

Material test were performed on screen wire, Kevlar, and S2-Glass to determine the necessary material properties needed to define a constitutive model. Both normal-to-the-plane crush tests as well as in-plane tension tests were performed. The data from these tests was reduced to fit the necessary parameters for the constitutive model. Certain material properties were also determined for perforated plate using the finite element method. Normal-to-the-plane compression tests were modeled as well as in-plane tension and compression tests. The data from these analyses was also fit to the constitutive model but the model was not exercised for this particular material.

A volumetrically and deviatorically yielding constitutive model was developed with elastic constants that are functions of the plastic volume crush. The model consists of an isotropic crush model and an anisotropic in-plane tensile model. The isotropic

crush model consists of a cone and an ellipsoidal cap. Non-associative flow rules are used on both surfaces, although the flow rules are associative in the limited deviatoric space.

The model was implemented for wrapped screen wire and Kevlar. The model was then used to predict how the material would behave in a uniaxial crush test. The predicted results from the model were extremely close to the experimental data used to construct the model for small strain steps. The model was then implemented in the large strain dynamic code PRONTO2D and shown to behave properly in this environment.

The procedures for testing layered screen wire and Kevlar cloth can be adapted to test other similar materials. Although each material will have its own peculiarities, the general approaches should be applicable to similar types of materials. Many flat materials may be held in the same way the tensile and screen wire samples were held.

The model developed in this thesis should apply to a large class of volumetrically yielding materials including those that do not have variable elastic constants. The model can be tailored to fit materials of this class by the proper choice of the model parameters. The shape of the cone and the ellipsoidal cap must be determined. A method is presented for choosing parameters if no data is available. Experimental data must be reduced to find how the center of the yield surface grows as a function of the plastic volume crush, how the elastic constants change as the material is crushed, and an approximation is necessary for the derivative of the plastic volume strain with respect to the total volume strain. These processes were demonstrated for both wrapped screen wire and Kevlar cloth and for perforated plate.

In order to completely describe the behavior of wrapped screen wire and Kevlar cloth, more testing is required. The data collected on this material was from uniaxial tests. Multi-axial tests should provide better data to fit to the constitutive model. In addition, triaxial tests and other tests should be performed to determine the true shapes of

the yield surfaces rather than using the assumptions of Chapter 6. No data was available to determine if layered screen wire and Kevlar is strain rate dependent. At modest strain rates, metals are not very strain rate sensitive, but due to the high impact velocities it is possible, if not probable, that layered screen wire and Kevlar will have some strain rate dependence. Further testing is needed to verify this. This model can be adapted to include a third invariant to change the shape of the yield surface (Krieg and Brown, 1992). Further testing is required to determine if this type of model is needed for layered screen wire and Kevlar.

The isotropic crush model presented in this work implicitly assumes that the material can never carry a tensile pressure greater than some initial fracture pressure. This is not a realistic material description. Once the material has undergone some plastic volume strain, the material can carry some tensile load. In fact, once screen wire has been crushed to the density of solid aluminum it should be able to carry almost as much tensile load as solid aluminum. This effect needs to be incorporated by changing the cone angle or the intersection of the cone with the pressure axis by having a_0 and a_1 be functions of the plastic volume strain.

Although not presented in this work, it was determined that there is an optimal hole density for perforated plate (Krieg and Brown, 1992). This was not the plate analyzed in this work. In order to properly compare perforated plate to layered screen wire and Kevlar, it would be necessary to use the optimal perforated plate rather than the one described here.

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APPENDICES

APPENDIX A

Joining of Conical and Ellipsoidal Yield Surfaces

Transformation of Variables

The two yield functions are given by Eqs. (4.1) and (4.4) as:

$$f_c = \sqrt{J_2} - (a_0 + a_1 p) = 0 \quad (\text{A1})$$

$$f_{eps} = C^2 J_2 + (p - p_c)^2 - R^2 = 0 \quad (\text{A2})$$

A new variable can be defined as:

$$y = \sqrt{J_2} \quad (\text{A3})$$

so the two yield functions are rewritten in terms of y as:

$$y = (a_0 + a_1 p) \quad (\text{A4})$$

$$\text{and} \quad C^2 y^2 + (p - p_c)^2 = R^2 \quad (\text{A5})$$

Matching at the Center of the Ellipsoid– Case 1

The two surfaces can be made to match at a pressure p_c which is the pressure coordinate at the center of the ellipsoid. Eqs. (A4) and (A5) are evaluated at $p = p_c$ to obtain the values:

$$y_c = (a_0 + a_1 p_c) \quad (\text{A6})$$

and $C y_c = R \quad (\text{A7})$

The value of y_c is eliminated between the two expressions to give the required relation between the radius of the ellipsoid and the cone parameters:

$$R = C (a_0 + a_1 p_c) \quad (\text{A8})$$

This relation will be used in our model. This choice gives continuity, i. e., no gaps, at the intersection of the cone and the ellipsoid. There is a discontinuity in the slopes in the surfaces at the intersection and in a direction along the pressure axis. This discontinuity in slope could be avoided if, instead, the procedure of the next section were followed. Note that for Case 1 the pressure was selected at which the intersection occurs. In the next case that pressure is determined in the process.

Matching the Surfaces and the Slopes – Case 2

In this case the cone and ellipsoid are required to be continuous and the intersection is to be smooth, i.e., the slope is continuous across the intersection. The change of variables in Eq (A3) is again carried out to result in Eqs. (A4) and (A5) which are also used here. These equations are differentiated with respect to p to obtain the slopes on the cone and ellipsoid as:

$$\frac{dy}{dp} = a_1 \quad (\text{A9})$$

and
$$C^2 y \frac{dy}{dp} + (p - p_c) = 0 \quad (\text{A10})$$

The slope is eliminated by substituting Eq (A9) into (A10) to obtain:

$$a_1 C^2 y + p - p_c = 0 \quad (\text{A11})$$

Equation (A4) is substituted into (A5) and (A11) to eliminate y and give:

$$C^2 (a_0 + a_1 p)^2 + (p - p_c)^2 = R^2 \quad (\text{A12})$$

and
$$a_1 C^2 (a_1 p + a_0) + p - p_c = 0 \quad (\text{A13})$$

Equation (A13) is solved for p to give:

$$p = \frac{p_c - a_0 a_1 C^2}{a_1^2 C^2 + 1} \quad (\text{A14})$$

This is then substituted into Eq. (A12) to give a constraint for R analogous to Eq. (A8) found for Case 1. The substitution gives:

$$(-a_0 a_1 C^2 + p_c - p_c t)^2 + a_2^2 [(-a_0 a_1 C^2 + p_c) a_1 + a_0 t]^2 = R^2 t^2 \quad (\text{A15})$$

where $t = a_1^2 C^2 + 1$ (A16)

Equation (A15) is then solved to give an expression analogous to Eq. (A8) as:

$$R^2 = \left(\frac{-a_0 a_1 C^2 + p_c}{t} - p_c \right)^2 + C^2 \left[\frac{(-a_0 a_1 C^2 + p_c) a_1}{t} + a_0 \right]^2 \quad (\text{A17})$$

The method developed in Case 2 will not be used. First, this method makes it very difficult to determine if yielding is occurring on the cone or the ellipse. This must be determined by calculating a normal at the intersection of the two surfaces. Determining the intersection of these surfaces is not a trivial procedure. Secondly, associative flow rules are not used for either surface so matching the slopes is not necessary. Therefore, the method developed in case 1 will be used.

APPENDIX B

Derivation of the Parameter - C

The constant C is chosen such that a uniaxial stress path is normal to the ellipsoidal cap at their intersection. Begin by considering a uniaxial test such that

$$\begin{aligned}\sigma_1 &= \bar{\sigma} \\ \sigma_2 &= \sigma_3 = \sigma_{12} = \sigma_{23} = \sigma_{13} = 0\end{aligned}\tag{B1}$$

where $\bar{\sigma}$ is taken as positive for compression, since only compression is of interest. We now write the volumetric and deviatoric components of this stress state and obtain

$$p = \frac{\bar{\sigma}}{3} \qquad J_2 = \frac{\bar{\sigma}^2}{3}\tag{B2a,b}$$

The line of uniaxial stress can be written in a vector form in $p\text{-}\sqrt{J_2}$ space as

$$\tilde{V}_{uni} = \bar{\sigma} \left(\frac{1}{3} \hat{e}_1 + \frac{\sqrt{3}}{3} \hat{e}_2 \right)\tag{B3}$$

where $\hat{e}_1$ and $\hat{e}_2$ are unit vectors along the p and $\sqrt{J_2}$ axis respectively.

Consider the equation for the ellipsoidal cap at an arbitrary center p_c

$$\frac{V_2^2}{(a_0 + a_1 p_c)^2} + \frac{(V_1 - p_c)^2}{C^2 (a_0 + a_1 p_c)^2} = 1\tag{B4}$$

where V_1 and V_2 replace p and $\sqrt{J_2}$ respectively. and a_0 and a_1 are the constants defined in Chapter 6. Now the yield function for the ellipsoidal cap can be written as

$$f_{eps} = C^2 V_2^2 + (V_1 - p_c)^2 - C^2 (a_0 + a_1 p_c)^2 \quad (B5)$$

To find the normal to the ellipsoidal cap take the partial derivative of the yield function with respect to the stress or in this case the vector $\tilde{V}$. Performing this differentiation, the desired expression for the normal to the yield surface is obtained as

$$\frac{\partial f_{eps}}{\partial \tilde{V}} = 2 C^2 V_2 \hat{e}_2 + 2 (V_1 - p_c) \hat{e}_1 \quad (B6)$$

In order to force the normal to the yield surface to be in the direction of the uniaxial stress path, the slopes of the two vectors (B6) and (B3) must be the same. This yields the following equation upon changing back to the original variables

$$\frac{C^2 \sqrt{J_2}}{(p - p_c)} = \sqrt{3} \quad (B7)$$

Now using the uniaxial stress case, write the $\sqrt{J_2}$ in terms of p as

$$\sqrt{J_2} = \sqrt{3} p \quad (B8)$$

This is now substituted into Eq. (B7) and solved for C^2 , to give

$$C^2 = 1 - \frac{p}{p_c} \quad (B9)$$

Now go back and express p in terms of p_c , C^2 , a_0 , and a_1 . Find the intersection of the uniaxial stress path and the ellipse by substituting Eq. (B8) into Eq. (B4) to obtain

$$\frac{3 p^2}{(a_0 + a_1 p_c)^2} + \frac{(p - p_c)^2}{C^2 (a_0 + a_1 p_c)^2} = 1 \quad (\text{B10})$$

Eq. (B10) is now solved for p to obtain

$$p = \frac{2 p_c \pm \sqrt{4 p_c^2 - 4 (3 C^2 + 1) [p_c^2 - C^2 (a_0 + a_1 p_c)^2]}}{2 (3 C^2 + 1)} \quad (\text{B11})$$

The positive root to Eq. (B11) is the one needed since the line intersects the ellipse both at a pressure lower than p_c and one greater than p_c and the one needed is the one greater than p_c . Now substitute Eq. (B11) back into Eq. (B9) to obtain

$$C^2 = 1 - \frac{2 (3 C^2 + 1) p_c}{2 p_c + \sqrt{4 p_c^2 - 4 (3 C^2 + 1) [p_c^2 - C^2 (a_0 + a_1 p_c)^2]}} \quad (\text{B12})$$

Using the values of a_0 and a_1 chosen in Chapter 6 of 0.0 and 2.0, respectively, and reducing Eq. (B12) we obtain the following

$$C^2 = 1 - \frac{2 (3 C^2 + 1)}{2 + \sqrt{4 - 4 (3 C^2 + 1) [1 - C^2 (2)^2]}} \quad (\text{B13})$$

It should be noted at this point that if a_0 is not chosen as zero then Eq. (B13) would still be a function of p_c . By expanding Eq. (B13), the following polynomial in C^2 is obtained:

$$C^2(12C^6 - 23C^4 - 6C^2 + 1) = 0 \quad (\text{B13})$$

The four roots for C^2 in Eq. (B13) are -0.3333, 0.0, 0.11722, and 2.1328. When these values for C^2 are substituted back into Eq. (B12) the only real value which satisfies the equation is 0.11722. The extraneous roots were obtained due to the necessity to square the equation several times to reduce it to the form of Eq. (B13). Therefore, the desired value of the constant C is 0.0342374064. This value insures that the uniaxial stress path is always normal to the ellipsoidal cap. As previously discussed, if a_0 were some other value than zero, the parameter C would not be constant and in fact would be a function of the pressure position of the center of the ellipsoidal cap.

APPENDIX C

Expression for Updated Crack Opening Strain

When the material is not initially fractured and fractures during a strain step, part of the step is unloading elastically and part of the step is volume fracture. To determine when the material has closed back up after fracture, the total volume since fracture must be calculated. When the total volume strain since fracture is zero or negative then the crack has closed and the material will begin to load elastically until it reaches one of the yield surfaces. This appendix describes how the volume strain since fracture is calculated.

For the case of no initial volume fracture the trial elastic pressure is given as

$$p_T = p_o - K d_{kk} \Delta t \quad (C1)$$

But the pressure for fracture occurs during this strain step. This pressure is calculated by applying Eq. (C1) for some fraction of the time as given by:

$$p_{frac} = p_o - f K d_{kk} \Delta t \quad (C2)$$

This is solved for the fraction as:

$$f = \frac{p_o - p_{frac}}{K d_{kk} \Delta t} \quad (C3)$$

The remaining strain during this time step is used to open the crack. This strain must be accumulated every time step from the time fracture begins, just in case the crack closes and the material again develops compressive stresses. This opening volume strain is found for this initial time step as:

$$e_v^* = (1 - f) d_{kk} \Delta t \quad (C4)$$

The expression for f from Eq. (C3) is substituted to give the expression to calculate the volume strain since fracture for an initially closed crack as

$$e_v^* = d_{kk} \Delta t - \left(\frac{p - p_{frac}}{K} \right) \quad (C5)$$

If the material is initially fractured then the value of the opening volume strain is found as:

$$e_v^* = (e_v^*)_o + d_{kk} \Delta t \quad (C6)$$

APPENDIX D

Estimate of the Plastic Volume Strain Increment

In the solution logic for the constitutive model, a trial state is found using the moduli at the beginning of the time step. If yielding on the ellipsoidal end is indicated, then the value of f_{eps} will have been determined and found to be greater than zero. In order to estimate the change of the plastic volume strain in this time step, we will construct an equivalent compressive uniaxial stress (ECUS) case. This case is constructed in such a way that the values of the yield function, f_{eps} , the plastic volume strain, ϵ_{vp} , the pressure at the center of the ellipsoidal part of the yield surface, p_c , and the elastic modulus, E are the same as for the actual case. The initial stress for the ECUS case can be easily found if the stress is assumed to be on the yield surface. The trial elastic stress can be found corresponding to the same f_{eps} . The value of $\Delta\epsilon_v$ for the ECUS case can then be found since the trial stress for both the ECUS case and the actual case are elastic. A function has been fitted to the experimental compressive uniaxial stress data for the derivative of ϵ_{vp} with respect to ϵ_v as a function of ϵ_{vp} . In Chapter 6, this derivative was found to be fitted by the expression:

$$\frac{d\epsilon_v}{d\epsilon_{vp}} = 1.6206 + (5.769 + (20.9695 + (37.035 + (33.525 + (14.647 + 2.4000\epsilon_{vp})\epsilon_{vp})\epsilon_{vp})\epsilon_{vp})\epsilon_{vp})\epsilon_{vp} \quad (D1)$$

The change of ϵ_{vp} can then be found for this ECUS case as:

$$\Delta\epsilon_{vp} = \Delta\epsilon_v \left(\frac{d\epsilon_{vp}}{d\epsilon_v} \right) \quad (D2)$$

This increment of plastic volume strain for the ECUS case is assumed to be the same as for the actual case. The value of ϵ_{vp} at the end of the time step for the actual case is then found by adding the increment from Eq (D2) to the actual value of ϵ_{vp} at the start of the time step. The details of this process are given below.

For the ECUS case we would write Eq. (5.2.9) in terms of the axial stress σ_1 as:

$$f_{eps} = C^2 \frac{\sigma_1^2}{3} + \left(\frac{\sigma_1}{3} + p_c \right)^2 - C^2 (a_0 + a_1 p_c)^2 \quad (D3)$$

We solve for the ECUS trial stress as:

$$\sigma_{1T} = \frac{-p_c - \sqrt{p_c^2 - 3 \left(C^2 + \frac{1}{3} \right) (p_c^2 - C^2 (a_0 + a_1 p_c)^2 - f_{eps})}}{C^2 + \frac{1}{3}} \quad (D4)$$

We will assume that the stress state at the beginning of the ECUS case is exactly on the yield surface at the beginning of the time step and evaluate Eq. (D3) as:

$$\sigma_{10} = \frac{-p_c - \sqrt{p_c^2 - 3 \left(C^2 + \frac{1}{3} \right) (p_c^2 - C^2 (a_0 + a_1 p_c)^2)}}{C^2 + \frac{1}{3}} \quad (D5)$$

The difference in these stress states can be used to find the 11-component of the strain increment as:

$$\Delta \epsilon_1 = \frac{\sigma_{1T} - \sigma_{10}}{E} \quad (D6)$$

From experimental data it was found that the total lateral strains were small. Since high accuracy is not need here, the lateral strain will be assumed to be zero. The total volume strain increment for the ECUS case is then:

$$\Delta \varepsilon_v = \Delta \varepsilon_1 \quad (D7)$$

Equations (D3) and (D4) are now substituted into Eq. (D5) and (D6) to give:

$$\Delta \varepsilon_v = \frac{\sqrt{A + M f_{eps}} - \sqrt{A}}{M E} \quad (D8a)$$

$$\text{where} \quad A = p_c^2 - M (p_c^2 - C^2 (a_0 + a_1 p_c)^2) \quad (D8b)$$

$$\text{and} \quad M = 3 C^2 + 1 \quad (D8c)$$

Note that the total volume strain increment is negative since f_{eps} is positive. Equations (D8) are used then to find the change in total volume strain for this equivalent case if the initial uniaxial state is on the yield surface. But the incremental plastic volume strain is needed in order to find the elastic unloading modulus and p_c since they are functions of ε_{vp} . Equation (D2) above is used to make this transition from Eq (D8) to the plastic volume strain increment. The procedure for finding this derivative and all other data reduction is explained in Chapter 6.

APPENDIX E

Restraints on Parameters

There are two restraints on the values of parameters which are chosen to describe the yield surfaces. Of course, if experimental data is available, the parameters are made to fit the data. But if no experimental data is available, there are some restraints to assist in the choices which are made.

The first condition is to make sure that the uniaxial compressive stress path falls inside the conical yield surface rather than outside. If that condition is not satisfied, then the material would be conceptually awkward, to say the least. The yield cone is given in Eq (5.2.1) as

$$f_c = \sqrt{J_2} - (a_0 + a_1 p) \quad (E1)$$

The uniaxial compressive stress test is characterized as:

$$J_2 = \frac{\sigma^2}{3} = 3 p^2 \quad (E2)$$

so that Eq (E1) can be rewritten as:

$$f_c = p (\sqrt{3} - a_1) - a_0 \quad (E3)$$

The condition that f_c is negative, i.e., there is no yielding, gives the constraint for uniaxial stress:

$$p(\sqrt{3} - a_1) < a_0 \quad (\text{E4})$$

Ordinarily, a_0 is small compared to the maximum pressure which can be imposed, and because pressure is positive then the condition is that the terms in parentheses must be negative. This condition is:

$$a_1 > \sqrt{3} \quad (\text{E5})$$

A second condition which can be used involves the parameter p_c . Equation (6.12) is an equation which states that the uniaxial stress state is on the ellipsoidal yield surface. It gives the value of p_c which satisfies that condition. The terms inside the square root must be non-negative in order to solve the equation. That is a restraint on the parameters which can be used. That restraint condition is:

$$\left(\frac{\sigma_1}{3} - a_0 a_1 C^2\right)^2 > (1 - a_1^2 C^2) \left[\frac{\sigma_1^2}{9} (3C^2 + 1) - a_0^2 C^2 \right] \quad (\text{E6})$$

If this condition is to hold regardless of the (compressive) stress imposed, then the terms added or subtracted from terms with stresses are dropped to give the modified restraint:

$$a_1^2 > \frac{3}{3C^2 + 1} \quad (\text{E7})$$

We now expand this result to give:

$$a_1 > \sqrt{\frac{3}{3C^2 + 1}} \quad (E8)$$

Note the similarity between Eqs (E5) and (E8). Because the denominator of Eq. E8 is greater than one, the right side of Eq (E8) is always less than the right side of Eq (E5). Equation (E5) is always more restrictive than Eq (E8), so Eq (E8) can be ignored.

APPENDIX F

Detailed Explanation of Data Fitting Process for Screen Wire

Many steps were used to fit the experimental uniaxial compressive data to the parameters needed in the constitutive model. These procedures are outlined here step by step. A spreadsheet was used to perform all calculations.

The process is begun by defining a regular progression of axial strains; this is entered in column 1 of the spreadsheet. Column 2 is the lateral strain calculated from fitted experimental data as a function of the axial strain using Eq (6.1). The total volume strain is now calculated as the axial strain plus twice the lateral strain; this is entered in column 3 of the spreadsheet. The axial stress is now calculated as a function of the axial strain using Eq. (6.2) and entered into column 4.

The elastic unloading modulus is now calculated using Eq. (6.3) and entered into column 5. Now an elastic Poisson's ratio is needed and is defined by Eq. (6.4). Two intermediate quantities must first be calculated before applying Eq (6.4), namely the change in axial strain and the change in lateral strain. The change in axial strain is calculated by differencing adjacent axial strains in column 1 and entering the result in column 6. The change in lateral strain is calculated by differencing adjacent lateral strains in column 2 and entering the result in column 7. An elastic Poisson's ratio is now calculated using Eq (6.4) by dividing the negative of the change in lateral strain in column 7 by the change in axial strain in column 6 and entering the result in column 8. The unloading elastic bulk modulus is now calculated using Eq (6.5) and columns 5 and 8 and entered into column 9.

Because all parameters are needed as functions of the plastic volume strain, this strain must be calculated for every axial strain in the spread sheet. The plastic volume strain is then calculated using Eq (6.6). Before this calculation can be performed, several intermediate quantities are calculated. The mean stress is calculated as simply one third the axial stress in column 4 and entered in column 10. The change in total volume strain is calculated by differencing adjacent terms in column 3 and entered into column 11. The change in mean stress is calculated by differencing adjacent terms in column 10 and entering the result in column 12. The change in plastic volume strain is then calculated using Eq. (6.6) and columns 11, 12 and 9 and the result entered into column 13. The total plastic volume strain is then calculated by adding up adjacent terms in column 13 and entered into column 14.

Two other quantities still remain to be calculated. These quantities are p_c , the center of the ellipsoidal cap, and the derivative of the total volume strain with respect to the plastic volume strain. The derivative is calculated by dividing the change in total volume strain in column 11 by the change in plastic volume strain in column 13 and entering the result in column 15. The position of the center of the ellipsoidal cap is calculated using equation (6.13) and the quantities in column 4 and the result entered into column 16.

At this point all quantities needed are known so that all that remains is to perform the curve fits using traditional regression methods. The results of these fits are given in Chapter 6. Figure F1 shows the actual spread sheet used. It should be noted that more strain values were used than are shown in Figure F1.

APPENDIX G

Detailed Explanation of Data Fitting Process for Perforated Plate

Many steps are necessary to fit the data for perforated plate to the parameters needed in the constitutive model. The procedures are outlined step by step assuming a spread sheet will be used to perform the calculations. In this appendix, it is assumed that curve fits have been found for all “experimental” data. In addition, it is assumed that the properties for a random orientation of holes from adjacent plates has been determined using the steps outlined in Chapter 6.

The process is begun by defining a regular progression of strains which are entered in column 1. It should be noted that these strain are both the axial strain as well as the volume strain since these results were obtained from uniaxial strain tests. As discussed in Chapter 6, this progression of strains is started at -0.007562. The average x, y, and z stress component for a random orientation of holes from adjacent plates are calculated from the strain in column 1 and entered into columns 2, 3, and 4, respectively. These stresses are calculated using the minimum hole overlap stresses in Eq. (6.19) and the appropriate scaling factor. The mean stress is now calculated as one third the sum of the three stress components in columns 2, 3, and 4 and entered into column 5. The unloading elastic Young's modulus and Poisson's ratio are calculated using Eq. (6.20) and entered in columns 6 and 7, respectively. The unloading shear and bulk moduli are calculated from the elastic unloading Young's modulus in column 6 and the unloading elastic Poisson's ratio in column 7 and Eq. (6.21) and entered in columns 8 and 9, respectively.

In order to calculate the plastic volume strain, several intermediate quantities are needed. First the change in volume strain is calculated by differencing adjacent terms in column 1 and entered into column 10. The change in mean stress is then calculated by differencing adjacent terms in column 5 and entered in column 11. Finally, the change in plastic volume strain is calculated using Eq. (6.6) and columns 8, 10, and 11 and entered in column 12. The total plastic volume strain is now calculated by adding adjacent terms in column 12 and entered in column 13.

The two remaining quantities needed are the derivative of the total volume strain with respect to the plastic volume strain and the center of the ellipsoidal cap. The derivative is calculated by dividing the change in total volume strain in column 10 by the change in plastic volume strain in column 12 and entered in column 14. In order to calculate the center of ellipsoidal cap, several intermediate quantities must be calculated. First the pressure is calculated as minus one-third the sum of the x, y, and z stresses in columns 2, 3, and 4 and entered into column 15. The second invariant of the deviatoric stress tensor is calculated as

$$J_{2D} = \frac{1}{6} [(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2)] \quad (G1)$$

where σ_x , σ_y , and σ_z are the quantities in columns 2, 3, and 4 while σ_{xy} , σ_{xz} , and σ_{yz} are zero. The quantity J_2 is entered in column 15. The center of the ellipsoidal cap is not calculated using Eq. (6.23) and entered in column 16.

At this point all quantities needed are known so that all that remains is to perform the curve fits using traditional regression methods. Figure G1 shows the spread sheet that would be used to perform the necessary calculations. It should be noted that more strain values were used than are shown in Figure G1.

APPENDIX H

FORTTRAN Listing of the Constitutive Model

SUBROUTINE MAT18 (NE,NINSV,Dt,SIG,D,SV)

```

*****
*
*           Professor Raymond D. Krieg
*           Kevin H. Brown, Graduate Student
*           University of Tennessee
*           Engineering Science and Mechanics Department
*           July 1992
*
*   SUBROUTINE MAT18 is a model for layered aluminum screen
*   wire and Kevlar sheets. The model consists of an isotropic
*   model for the screen wire superimposed on an anisotropic
*   model for the in-plane tensile behavior of the screen wire
*   and the Kevlar.
*   For the screen wire model, the bulk modulus K and Poisson's
*   ratio are taken as a function of the plastic volume strain.
*   The yield surface consists of a cone joined to an ellipsoidal
*   cap. An associative flow rule is used in deviatoric space
*   while it is non-associative in total stress space.
*   The in-plane tensile model assumes that the fibers can only
*   carry tension in the directions of the fibers. There is no
*   shear strength in this model and the directions are assumed
*   to be independent. The two directions of the fibers are hard
*   wired into the constitutive model. The stress is calculated
*   using a curve that was fit to experimental data. If the
*   fibers are strained beyond their fracture point, the fibers
*   cease to carry any load.
*
*   Reference:   Final Report to Sandia National Lab
*               Contract # 98-816
*               "Computer Modeling of Wrapped Impact Limiters"
*
*   by:         Raymond D. Krieg
*               Kevin H. Brown
*
*   INPUT--- All material constants have been hard-wired in.
*****

```

INCLUDE 'params.blk'

```

REAL Evp,EvpNew,C,Csqrd,Evt,Pfrac,Dkk,SIG,SIGt
REAL DelEij,DelEkk,b0,b1,b2,b3,b4,b5,b6,b7,b8,a0,a1,a2,a3
REAL a4,a5,a6,a7,a8,a9,a10,a11,a12,c3,c4,c5,c6,c7,SWstrain,c0,c1
REAL c2,Rt3,DelEvp,Q,StressSWe,EEsw
REAL G,Pc,Pco,Pce,Third,Rt27d2,Four9th,Four3rd,Two3rd,Thirty13th
REAL SIGmean,Pt,Pc,Feps,RtJ2t,S,A,DelEv,dEvpDev,dPcdEvp
REAL Ek,Esw,StressK,StressSW
REAL Lamda,K,J2t,NU,M,Kstrain
DIMENSION SIGt(4),S(4),DelEij(4)
DIMENSION SIG(NSYMM,NEBLK),D(NSYMM,NEBLK),SV(NINSV,NEBLK)
PARAMETER(Rt27d2=2.59807621136)      ! 0.5*SQRT(27)
PARAMETER(Third=0.333333333333)      ! 1/3
PARAMETER(Four9th=0.444444444444)    ! 4/9
PARAMETER(Four3rd=1.333333333333)    ! 4/3
PARAMETER(Two3rd=0.666666666666)     ! 2/3
PARAMETER(Thirty13th=2.307692308)    ! 30/13
PARAMETER(Rt3=1.732050808)           ! SQRT(3.0)
PARAMETER(a0=0.0,a1=2.0)
PARAMETER(a2=87.5,a3=-4.0328,a4=-1.8323,a5=-1.5507)
PARAMETER(a6=1.6206,a7=5.769,a8=20.9695,a9=37.035,a10=33.525)

```

```

PARAMETER(a11=14.647,a12=2.4000)
PARAMETER(b0=577.4373779,b1=-21.557,b2=-42.550,b3=-59.141)
PARAMETER(b4=-46.650,b5=-14.732,b6=-0.19169,b7=-.08115)
PARAMETER(b8=-.073289,c3=99480.0,c4=29.2)
PARAMETER(c5=7092.69804,c6=58055.3838,c7=0.00699782)
PARAMETER(Pfrac=0.0,C=0.342374064)
PARAMETER(Kstrain=0.065,SWstrain=0.0159851593)
PARAMETER(EEsw=720000.0)

```

```

Model for a 2-D axisymmetric code

```

```

a0-a1      Constants for conical yield surface
a2-a5      Constants for Pc
a6-a12     Constants for dEvp/dEv
b0-b5      Material constants for the bulk modulus
b6-b8      Constants for Poisson's Ratio
C          Material constant for ellipsoidal yield surface
c0-c2      Coefficients in equation for LAMBDA
c3-c4      Constants used in tensile load for KEVLAR
c5-c7      Constants used in tensile load for SCREEN WIRE
Csqrd      C*C
DelEij     Change in volumetric strain
DelEv      Change in total volumetric strain
DelEvp     Change in plastic volumetric strain
Delt       Time step increment
Dij        Deviatoric stretching tensor
Dkk        Trace of the stretching tensor
dPcdEvp    Derivative of Pc with respect to Evp
E          Elastic Youngs Modulus
Ek         Elastic Youngs Modulus in the Kevlar
Esw        Elastic Youngs Modulus in the Screen Wire
Ev         Volumetric tensile strain since tensile fracture
Evp        Plastic volumetric strain
EvpNew     Estimate for final plastic volume strain on ellipsoidal end
Evt        Trial volumetric strain
Feps       Yield function for the ellipse
Fc         Yield function for the cone
G          Shear modulus
J2t        Trial 2nd invariant
K          Bulk modulus
Kstrain    Strain cut off for tensile load in KEVLAR
Lamda      Constant to scale stress back to the yield surface
NU         Poisson's Ratio
Pc         Pressure intersection of the cone and the ellipse
Pfrac      Fracture Pressure = -a0/a1
Pt         Trial pressure
Q          dPcdEvp/K
S          Deviatoric stress tensor
SIG        Initial and final stress state
SIGmean    Mean value of the trial stress
SIGt       Trial stress
SV(1,j)    Ev
SV(2,j)    Evp
SV(3,j)    Eps1
SV(4,j)    Eps2
SV(5,j)    Eps3
SV(6,j)    Flag for fracture in the R direction
SV(7,j)    Flag for fracture in the Theta direction
SV(8,j)    Elastic tensile stress in the screen wire in the R direction
SV(9,j)    Elastic tensile stress in the screen wire in the Theta direction
SV(10,j)   Tensile stress for the fibers in the R direction
SV(11,j)   Tensile stress for the fibers in the Theta direction
SWstrain   Strain cut off for tensile load in SCREEN WIRE

```

```

Csqrd=C*C
DO j=1,NE

```

```

Remove the tensile stress from the fibers for the isotropic crush model

```

```

SIG(1,j)=SIG(1,j)-SV(10,j)
SIG(3,j)=SIG(3,j)-SV(11,j)

```

```

Begin isotropic crush model

```

```

Evp=SV(2,j)

```

```

      IF (SV(1,j) .GT. 0.0) THEN
c   Case for initially open fracture                                     ! Eq (5.2)
      Dkk=D(1,j)+D(2,j)-D(3,j)
      Evt=SV(1,j)+Dkk*Dt                                             ! Eq (5.4)
      IF (Evt .GT. 0.0) THEN
c   Case when fracture crack is still open.
      K=b0*Exp((b1+(b2-(b3+(b4+b5*Evp)*Evp)*Evp)*Evp)*Evp)
      NU=(b6+(b7+b8*Evp)*Evp)*Evp
      G=K*3.0*(1.0-2.0*NU)/(2.0*(1.0+NU))
      DO i=1,4
        DelEij(i)=D(i,j)*Dt
      END DO
      DO i=1,3
        SIG(i,j)=-Pfrac
      END DO
      SIG(4,j)=0.0
      SV(1,j)=Evt
      GOTO 100
    ELSE
c   Case when the fracture has closed and compression takes place over
c   only part of time step                                           ! Eq (5.5)
      DO i=1,3
        DelEij(i) = D(i,j)*Evt/Dkk                                   ! Eq (5.6)
      END DO
      DelEij(4) = D(4,j) * Dt
      DelEkk = DelEij(1) + DelEij(2) + DelEij(3)
      SV(1,j) = 0.0                                                  ! Eq (5.7)
      DO i=1,4
        SIG(i,j)=0.0
      END DO
    END IF
  ELSE
c   Case when no initial fracture opening is present                 ! Eq (5.1)
      DO i=1,4
        DelEij(i) = D(i,j)*Dt
      END DO
      DelEkk = DelEij(1) + DelEij(2) + DelEij(3)
    END IF
c
c
c   Main case - No open Fracture
c
      K=b0*Exp((b1+(b2+(b3+(b4+b5*Evp)*Evp)*Evp)*Evp)*Evp)         ! Eq (6.8)
      NU=(b6+(b7+b8*Evp)*Evp)*Evp                                   ! Eq (6.7)
      G=K*3.0*(1.0-2.0*NU)/(2.0*(1.0+NU))                          ! Eq (4.43)
      Pc=a2*(-1.0+EXP((a3+(a4-a5*Evp)*Evp)*Evp))                  ! Eq (6.14)
      DO i=1,3
        SIGt(i)=SIG(i,j)+2*G*(DelEij(i)-Third*DelEkk)+K*DelEkk    ! Eq (5.8a)
      END DO
      SIGt(4)=SIG(4,j)+2*G*DelEij(4)
c   Calculate the deviatoric part of the trial stress
      SIGmean=(SIGt(1)+SIGt(2)-SIGt(3))/3.0
      DO i=1,3
        S(i)=SIGt(i)-SIGmean
      END DO
      S(4)=SIGt(4)
      J2t=0.5*(S(1)*S(1)+S(2)*S(2)+S(3)*S(3)+2.0*(S(4)*S(4)))      ! Eq (4.2)
      Pt=-SIGmean                                                    ! Eq (5.8b)
c
      IF (Pt .LE. Pc) THEN                                           ! Eq (5.9)
c   On the conical yield surface or beyond tensile end
        IF (Pt .LE. Pfrac) THEN                                     ! Eq (5.12)
c   Fracture has occurred this time step
          SV(1,j)=DelEkk-(Pt-Pfrac)/K                               ! Eq (5.13)
          DO i=1,3
            SIG(i,j) = - Pfrac
          END DO
          SIG(4,j)=0.0
          GOTO 100
        ELSE
c   Evaluate the conical yield function
          RtJ2t=SQRT(J2t)
          Fc = RtJ2t-(a0+a1*Pt)                                     ! Eq (4.1)
          IF (Fc .LE. 0.0) THEN                                     ! Eq (5.10)
c   Elastic case on conical end
            DO i=1,4
              SIG(i,j)=SIGt(i)
            END DO
          END IF
        END IF
      END IF
    END IF
  END IF

```

```

        END DO
        GOTO 100
    ELSE
c Plastic case on conical end
        Lamda=Fc/G
        DO i=1,4
            SIG(i,j)=SIGt(i) - 2.*G*Lamda*S(i)/(2.*RtJ2t)
        END DO
        GOTO 100
    END IF
END IF
ELSE
c On the ellipsoidal yield surface
        Feps=Csqrd*J2t+(Pt-Pc)**2-(C*(a0+a1*Pc))**2
        IF (Feps.LE. 0.0) THEN
c Elastic case on ellipsoidal end
            DO i=1,4
                SIG(i,j)=SIGt(i)
            END DO
            GOTO 100
        ELSE
c Plastic case on ellipsoidal end
            M=3.0*Csqrd+1.0
            A=Pc*Pc-M*(Pc*Pc-Csqrd*(a0+a1*Pc)**2)
            DelEv=- (SQRT(A+M*Feps)-SQRT(A)) / (M*K*3.0*(1.0-2.0*NU))
            dEvpdEv=1.0/(a6+(a7+(a8+(a9+(a10+(a11+a12*Evp)*Evp)*
            * Evp)*Evp)*Evp)*Evp)
            DelEvp=DelEv*dEvpdEv
            EvpNew=Evp+DelEvp
            K=b0*Exp((b1+(b2+(b3+(b4+b5*EvpNew)*EvpNew)*EvpNew)*
            * EvpNew)*EvpNew)
            NU=(b6+(b7+b8*EvpNew)*EvpNew)*EvpNew
            G=K*3.0*(1.0-2.0*NU)/(2.0*(1.0+NU))
            Pce=a2*(-1.0+EXP((a3+(a4+a5*EvpNew)*EvpNew)*EvpNew))
            dPcdEvp=(Pce+a2)*(a3+(2.*a4+3.0*a5*EvpNew)*EvpNew)
            Pco=Pce-dPcdEvp*DelEvp
            DO i=1,3
                SIGt(i)=SIG(i,j)+2.*G*(DelEij(i)-THIRD*DelEkk)+K*DelEkk
            END DO
            SIGt(4)=SIG(4,j)+2.*G*DelEij(4)
            SIGmean=(SIGt(1)+SIGt(2)+SIGt(3))/3.0
            Pt = -SIGmean
            DO i=1,3
                S(i)=SIGt(i)-SIGmean
            END DO
            S(4)=SIGt(4)
            J2t=0.5*(S(1)*S(1)+S(2)*S(2)+S(3)*S(3)+2.0*(S(4)*S(4)))
            Feps=J2t*Csqrd+((Pt-Pco)**2)-(C*(a0+a1*Pco))**2
        END IF
c Scale the stress back to the yield surface
c
        Q=dPcdEvp/K
        c0=Csqrd*J2t+Pt**2*((1.-Q)**2-Csqrd*(a1*Q)**2)
        c1=-Pt*((1.-Q)*(Pco-Q*Pt)+Csqrd*(a0+a1*(Pco-Q*Pt))*a1*Q)
        c2=(Pco-Q*Pt)**2-Csqrd*(a0+a1*(Pco-Q*Pt))**2
        Lamda=(-c1+SQRT(c1**2-c0*c2))/(c0)
        DelEvp=((Lamda-1.)*Pt)/K
        DO i=1,4
            SIG(i,j)=Lamda*SIGt(i)
        END DO
        SV(2,j)=Evp+DelEvp
        GOTO 100
    END IF
100 CONTINUE
c
c This is for the in plane tension in the 1-3 (R,theta) direction.
c The calculations below take care of the in-plane tension
c for the screen wire and Kevlar.
c
c The Kevlar is treated as a non-linear elastic material.
c The screen wire is treated as an elastic-plastic material.
c Both materials are allowed to fracture and never carry any stress
c after that point.
c
c SV(6,j) is the flag to determine which material if any has fractured
c in the R direction.

```

```

c   SV(7,j) is the flag to determine which material if any has fractured
c       in the Theta direction.
c
c   Flag=0      No fracture in any material
c   Flag=1      Fracture for screen wire.
c   Flag=2      Fracture for both materials
c
c   **NOTE** The screen wire must break before the Kevlar so there is no
c       need to check for the case of Kevlar broken and the screen
c       wire still intact.
c
c   SV(8) contains the previous elastic stress in the screen wire for R
c   SV(9) contains the previous elastic stress in the screen wire for Theta
c   SV(10) contains the previous tensile stress for all fibers for R
c   SV(11) contains the previous tensile stress for all fibers for Theta
c
c
c   SV(3,j)=SV(3,j)+D(1,j)*Dt
c   SV(4,j)=SV(4,j)+D(2,j)*Dt
c   SV(5,j)=SV(5,j)+D(3,j)*Dt
c   SV(10,j)=0.0
c   SV(11,j)=0.0
c   IF (SV(3,j).GT.Kstrain) THEN
c       SV(6,j)=2.0
c   ELSE IF (SV(3,j).GT.SWstrain) THEN
c       SV(6,j)=MAX(SV(6,j),1.0)
c   END IF
c   IF (SV(5,j).GT.Kstrain) THEN
c       SV(7,j)=2.0
c   ELSE IF (SV(5,j).GT.SWstrain) THEN
c       SV(7,j)=MAX(SV(7,j),1.0)
c   END IF
c   IF (SV(3,j).GT.0.0) THEN
c       IF (SV(6,j).LT.1.5) THEN
c           Ek=0.0
c           Esw=0.0
c           StressK=0.0
c           StressSW=0.0
c           IF (SV(6,j).LT.0.5) THEN
c c       Calculate an elastic trial stress in case the material has unloaded.
c           StressSWe=SV(8,j)+EEsw*D(1,j)*Dt
c           Esw=(c5-2.0*c6*SV(3,j))/(c7+SV(3,j))
c           *
c           StressSW=((c5-c6*SV(3,j))*SV(3,j))/(c7+SV(3,j))
c           ! Eq (6.15)
c c       The MIN takes either the elastic solution or the correct plastic
c c       solution if the material is plastic.
c           SV(8,j)=MIN(StressSW,StressSWe)
c c       The MAX insures the fibers never carry a compressive load.
c           StressSW=MAX(0.0,SV(8,j))
c           END IF
c           Ek=c3*(1.0+(c4*SV(3,j)-1.0)*EXP(-c4*SV(3,j)))
c           StressK=c3*SV(3,j)*(1.0-EXP(-c4*SV(3,j)))
c           SV(10,j)=(StressK+StressSW)*EXP(-SV(4,j)-SV(5,j))
c           SIG(1,j)=SIG(1,j)+SV(10,j)
c           ! Eq (6.16)
c           END IF
c       END IF
c       IF (SV(5,j).GT.0.0) THEN
c           IF (SV(7,j).LT.1.5) THEN
c               Ek=0.0
c               Esw=0.0
c               StressK=0.0
c               StressSW=0.0
c               IF (SV(7,j).LT.0.5) THEN
c c       Calculate an elastic trial stress in case the material has unloaded.
c               StressSWe=SV(9,j)+EEsw*D(3,j)*Dt
c               Esw=(c5-2.0*c6*SV(5,j))/(c7+SV(5,j))
c               *
c               StressSW=((c5-c6*SV(5,j))*SV(5,j))/(c7+SV(5,j))
c               ! Eq (6.15)
c c       The MIN takes either the elastic solution or the correct plastic
c c       solution if the material is plastic.
c               SV(9,j)=MIN(StressSW,StressSWe)
c c       The MAX insures the fibers never carry a compressive load.
c               StressSW=MAX(0.0,SV(9,j))
c               END IF
c               Ek=c3*(1.0+(c4*SV(5,j)-1.0)*EXP(-c4*SV(5,j)))
c               StressK=c3*SV(5,j)*(1.0-EXP(-c4*SV(5,j)))
c               SV(11,j)=(StressK+StressSW)*EXP(-SV(3,j)-SV(4,j))
c               SIG(3,j)=SIG(3,j)+SV(11,j)
c               ! Eq (6.16)

```

```
      END IF  
END IF  
END DO  
RETURN  
END
```

Appendix I

Figures

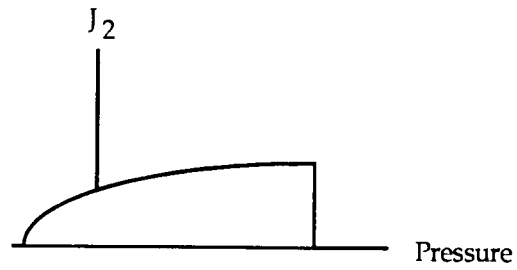


Figure 1.1: Yield surface in the pressure- J_2 space for Krieg model

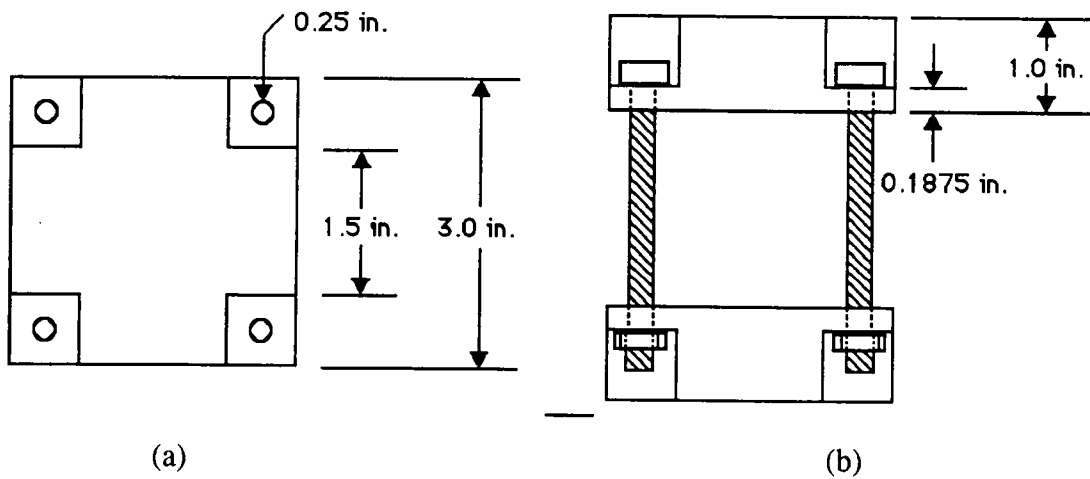


Figure 2.1: Compression test fixture: (a) Top view, (b) Side view

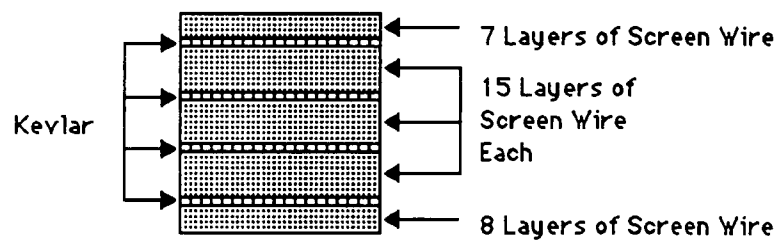


Figure 2.2: Layered screen wire and Kevlar for compression tests

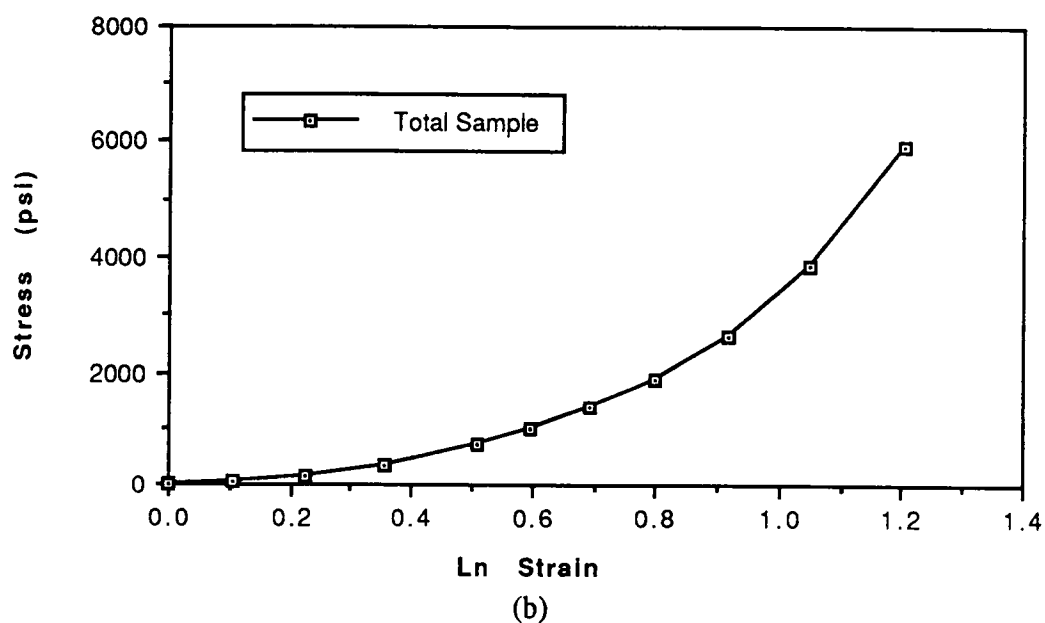
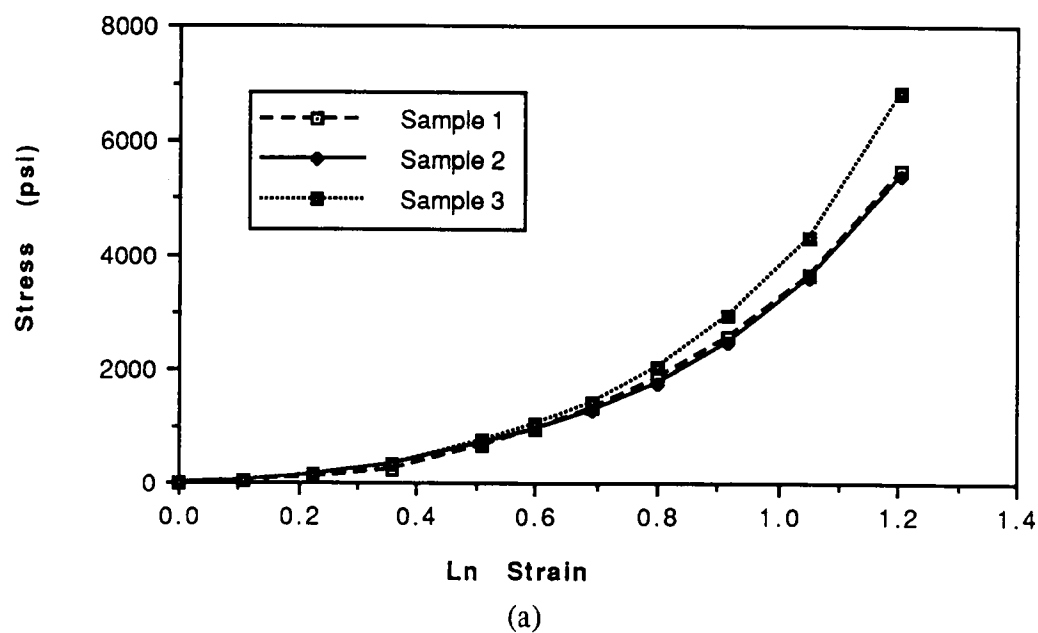


Figure 2.3: Stress-strain curves for initial testing normal to the plane: (a) Individual samples, (b) Total sample by superposition

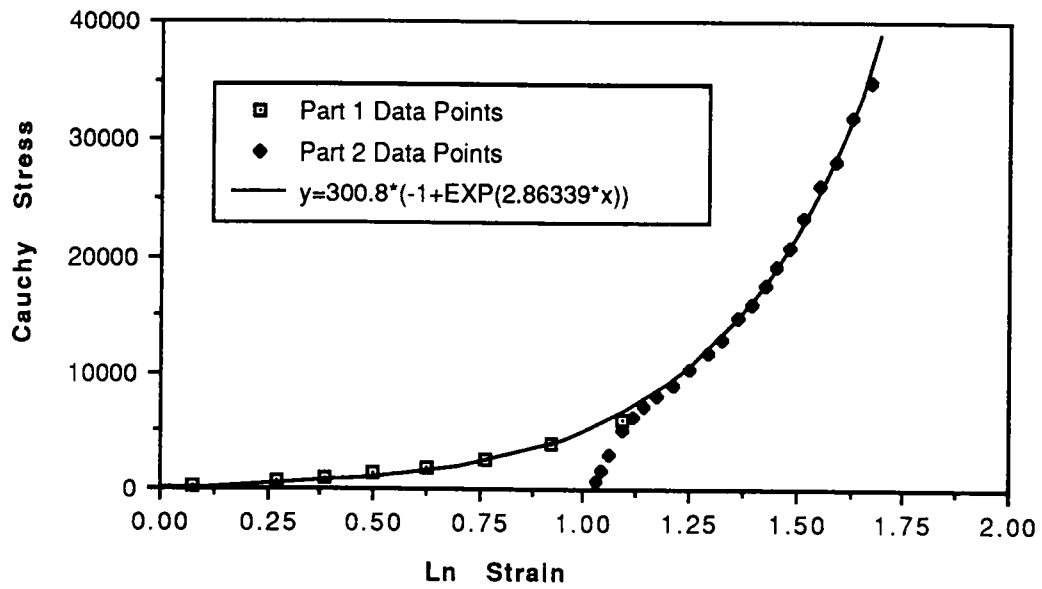


Figure 2.4: Normal to the plane, compression tests for screen wire and Kevlar

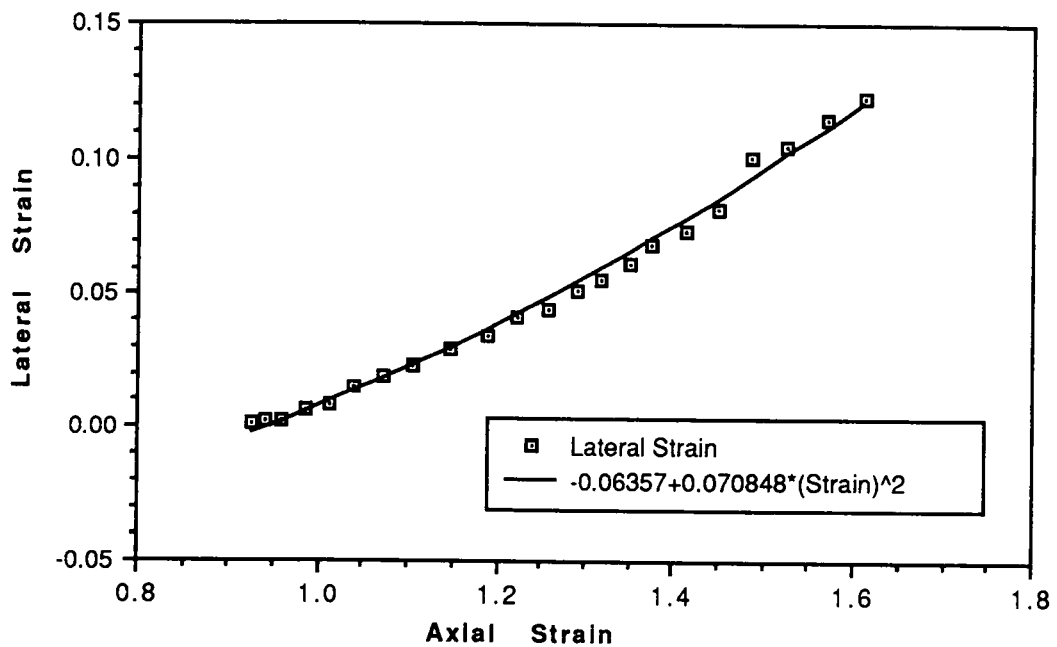


Figure 2.5: Normal to the plane, compression tests; lateral expansion and curve fit

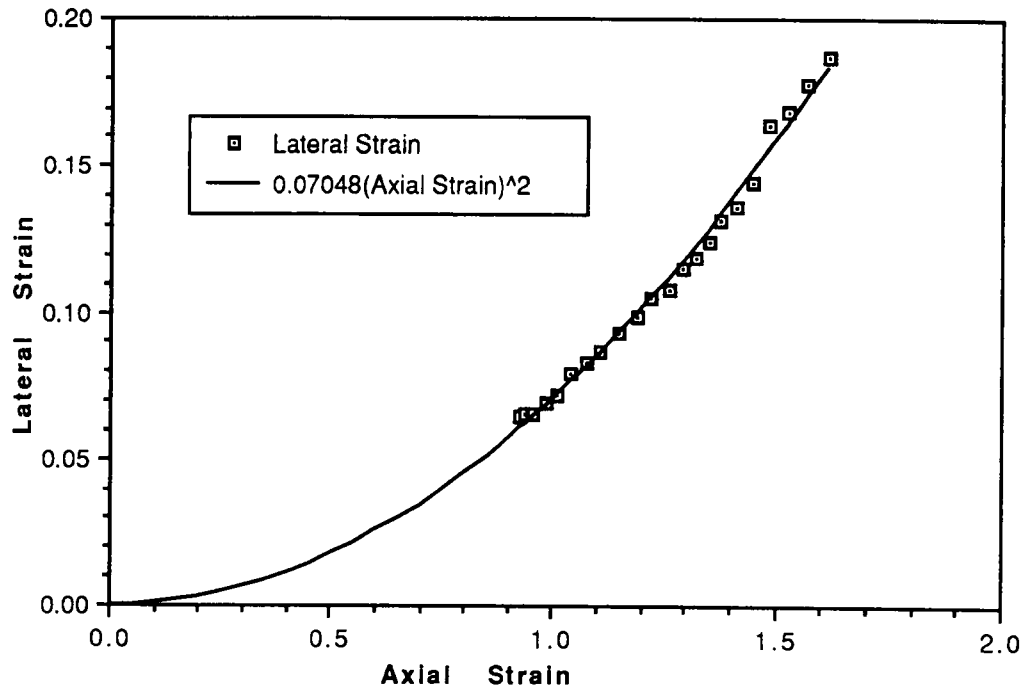


Figure 2.6: Shifted lateral expansion and curve fit

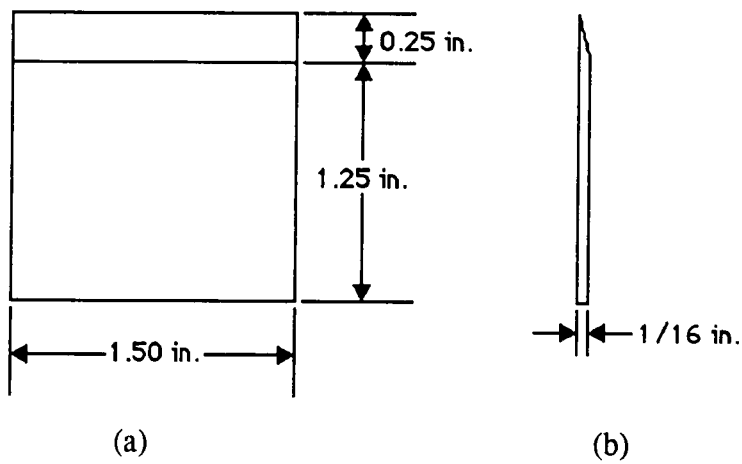


Figure 2.7: Tabs for tensile tests: (a) Top view, (b) Side View



Figure 2.8: Modified single layer Kevlar sample

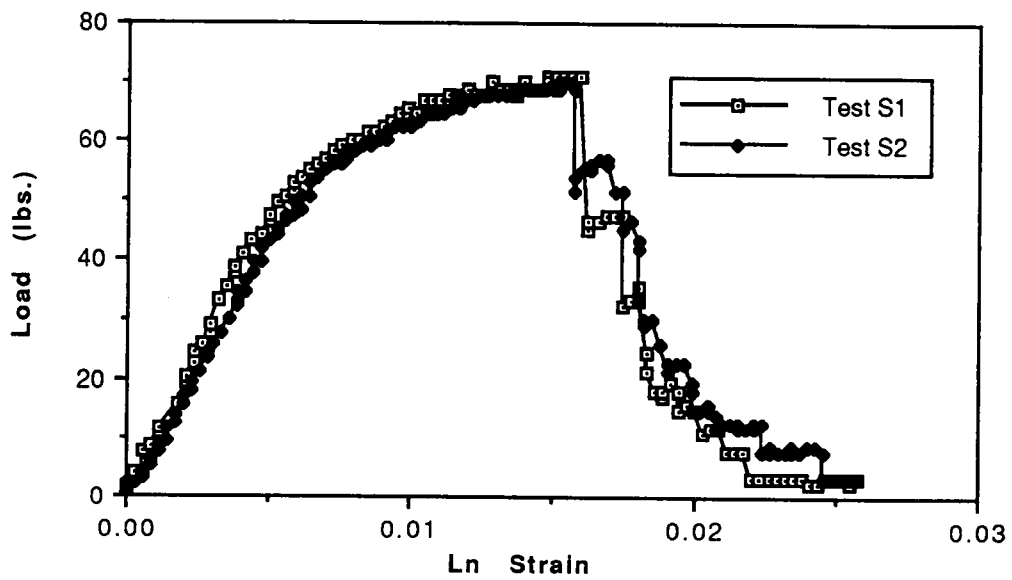


Figure 2.9: Single layer screen wire tension tests (16 wires)

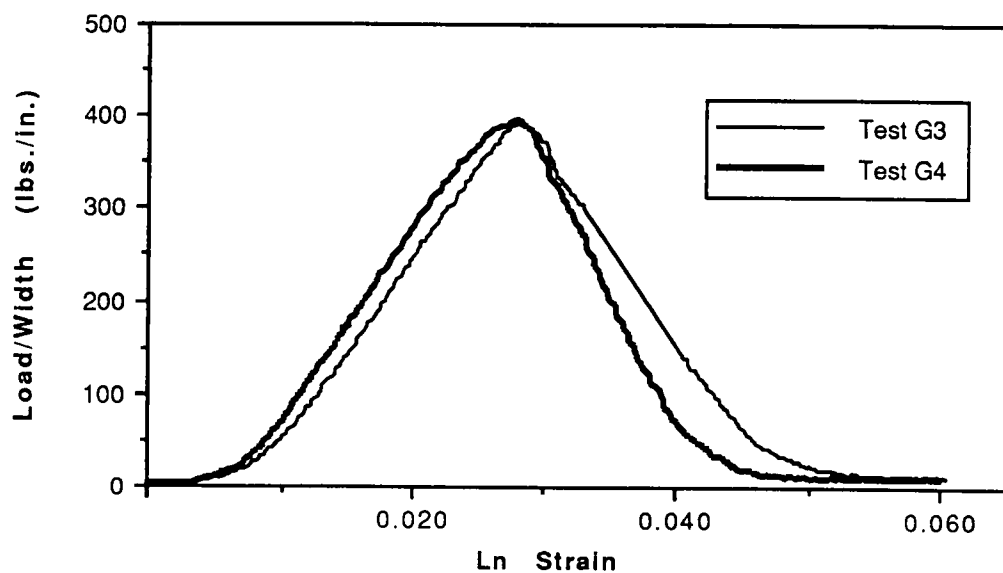


Figure 2.10: Single layer S2-Glass tension tests

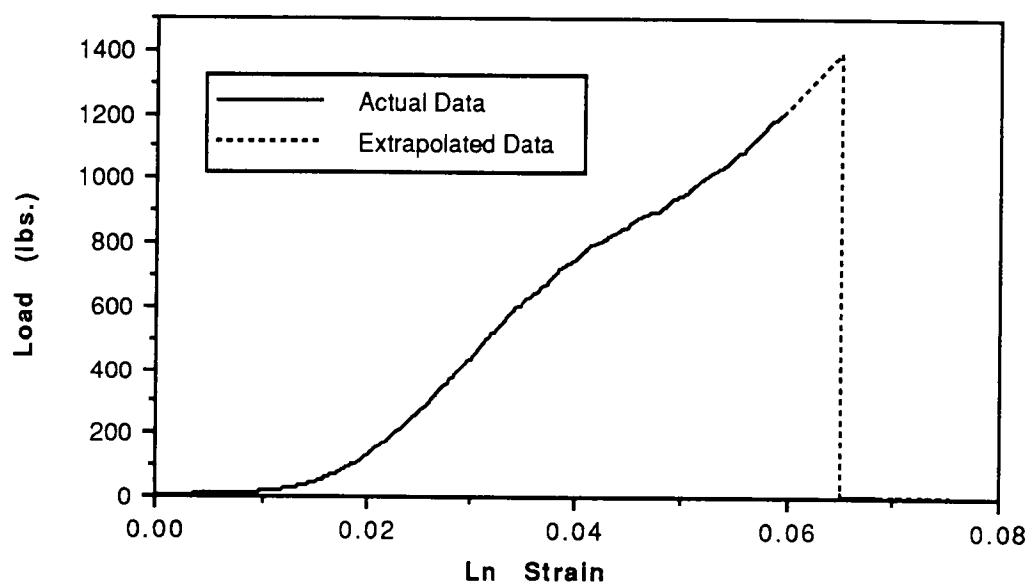


Figure 2.11: Single layer Kevlar tension test

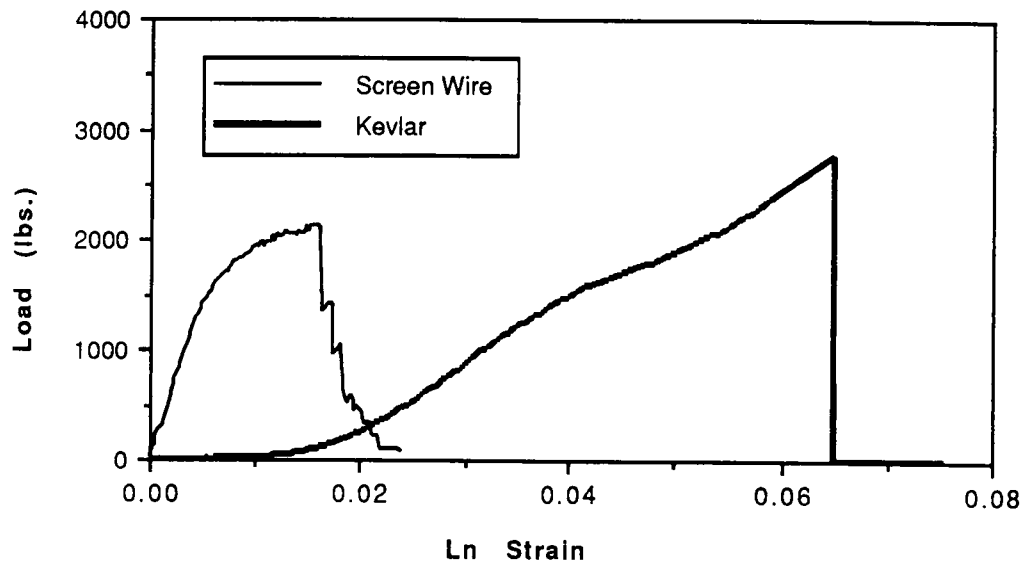


Figure 2.12: Screen wire and Kevlar tension data; 30 layers of screen wire & 2 layers of Kevlar

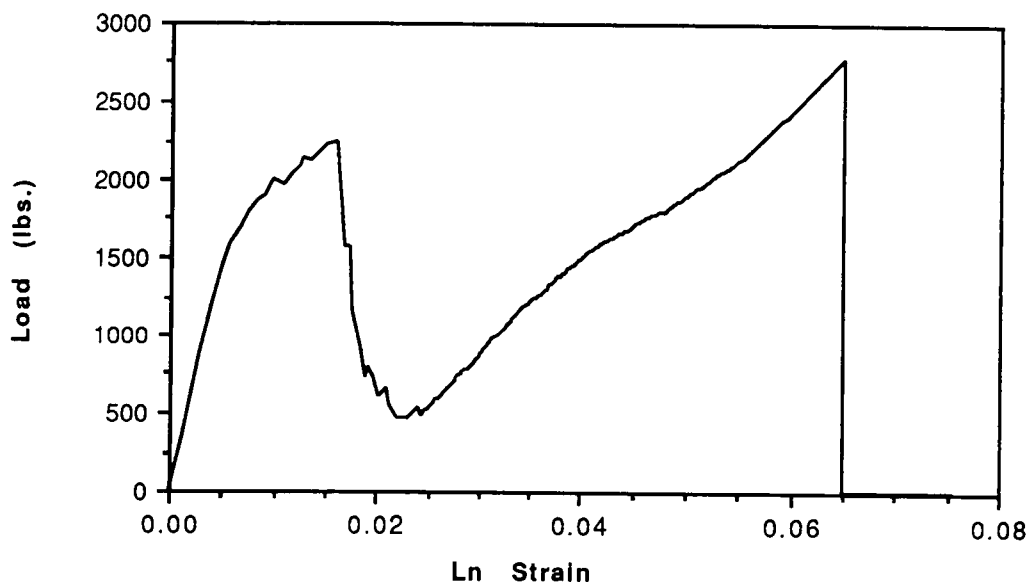


Figure 2.13: Multi-layer tension data

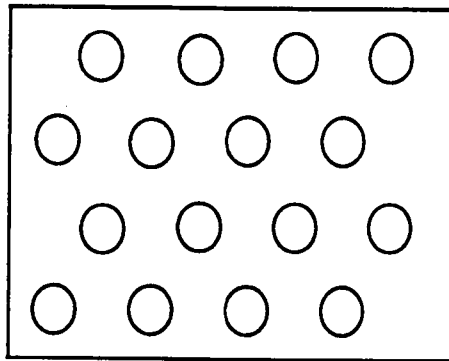


Figure 3.1: Perforated plate; 0.04 inches thick, holes 0.2 inches center to center, one eighth inch hole diameters

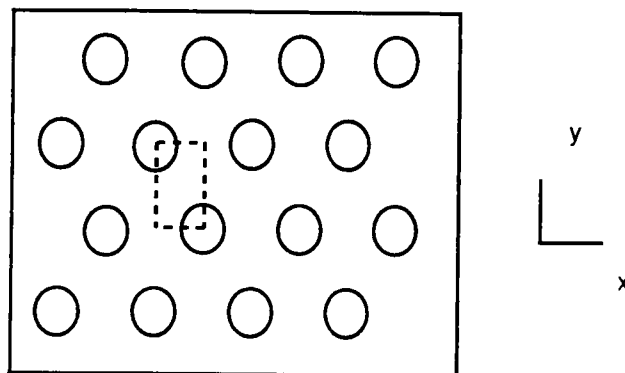


Figure 3.2: Analyzed section of perforated plate for in-plane tests; outlined

Perforated_Plate.in plane tension and compression model

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1 OF 1

TIME 0.3760E-3

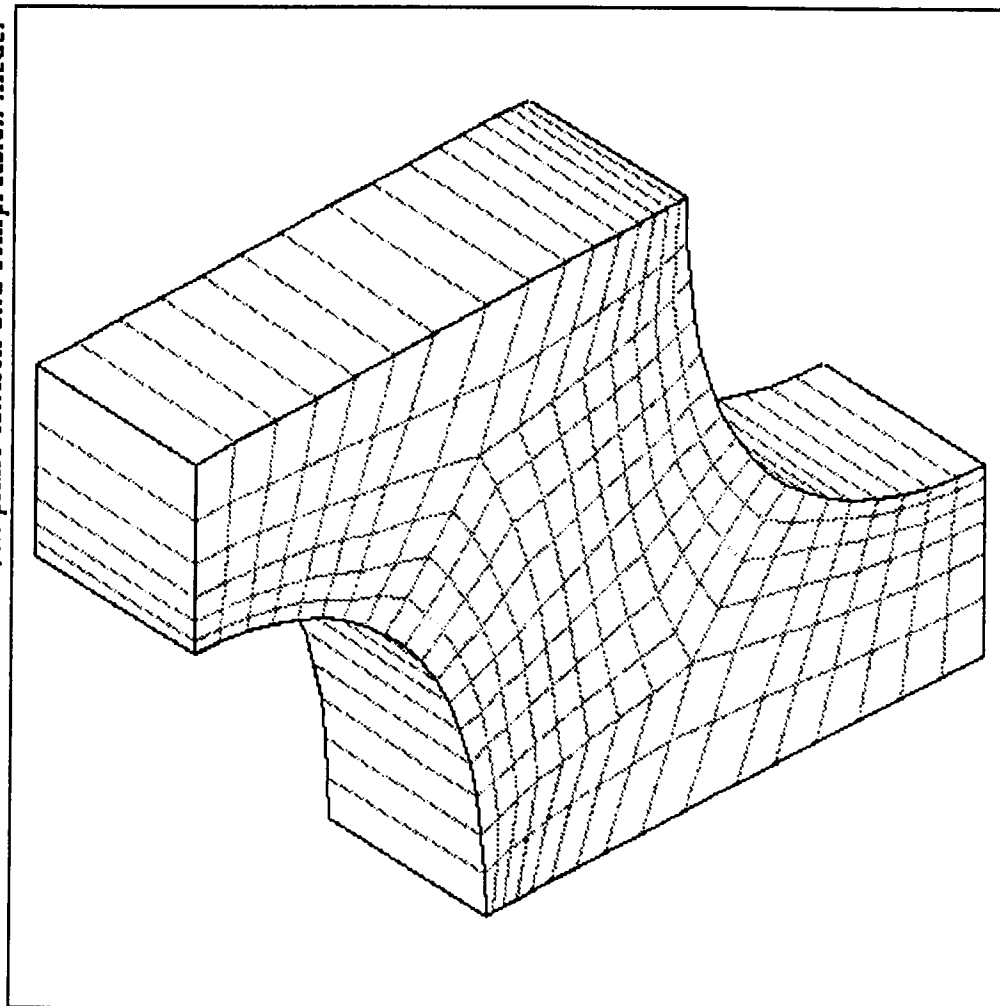
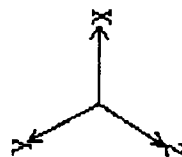


Figure 3.3: Undeformed mesh with coordinate system for in-plane tension and compression tests

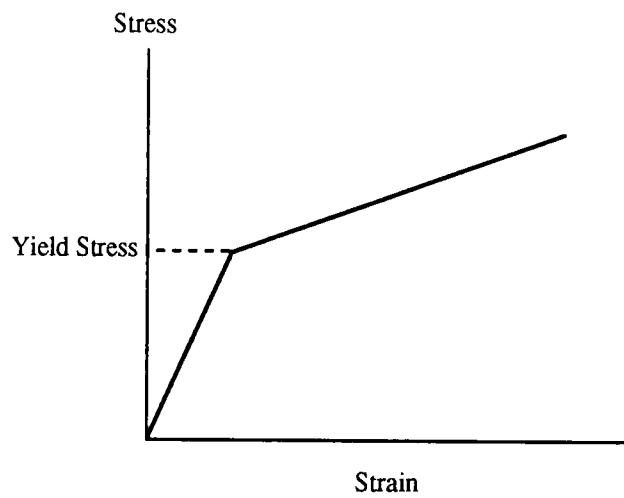


Figure 3.4: A hypothetical stress-strain showing the yield stress predicted by extrapolating the stress versus the plastic strain to a plastic strain of zero

Perforated_Plate, in plane tension and compression model

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 1 OF 1

SIG_Y
 -7.5E+3
 15.0E+3
 37.5E+3
 60.0E+3
 82.5E+3
 105.0E+3
 127.5E+3
 CS = -0.4243E+0
 RS = 124.8E+3

TIME 0.6900E-3

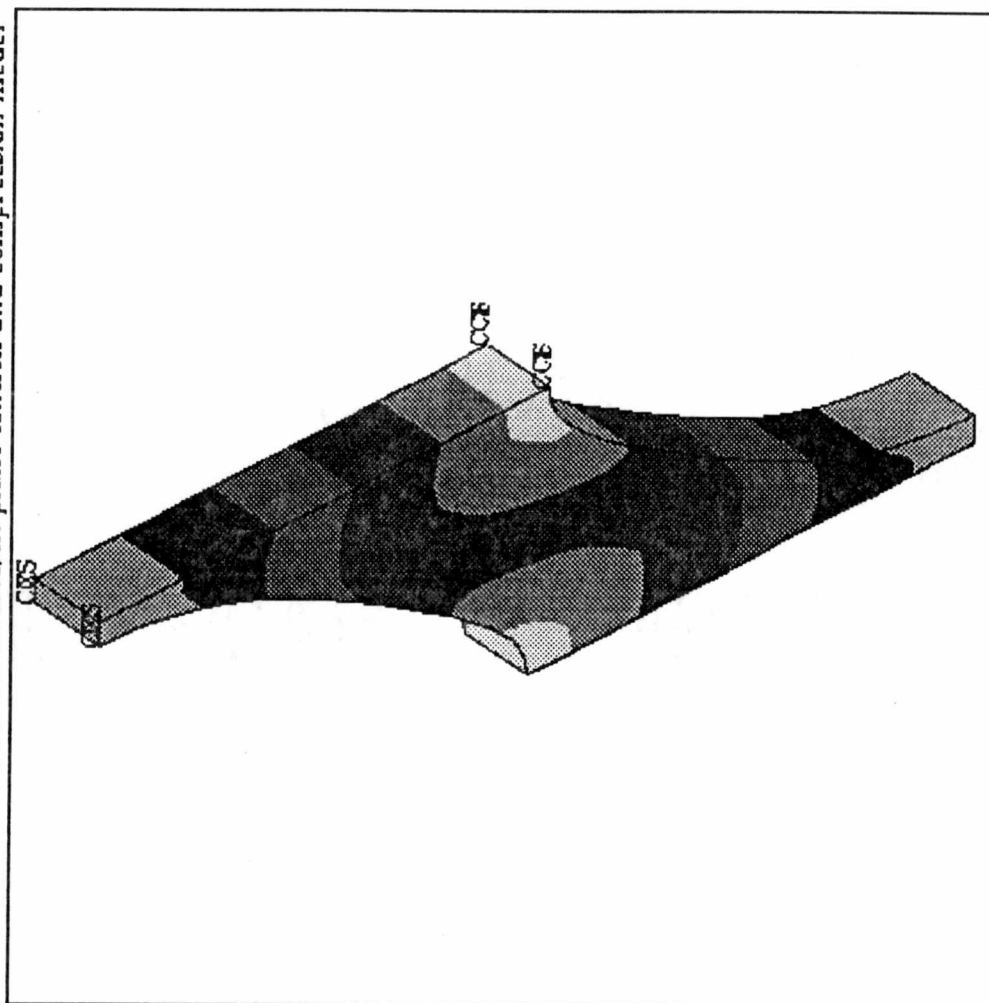
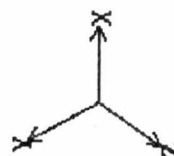


Figure 3.5: Stress contours of the axial stress for the tension in the y-direction test

Perforated_Plate, in plane tension and compression mode]

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 1 OF 1

SIG_X
 0.0E+3
 20.0E+3
 40.0E+3
 60.0E+3
 80.0E+3
 100.0E+3
 120.0E+3
 CF = 0.5E+3
 CS = 106.2E+3

TIME 0.8280E-3

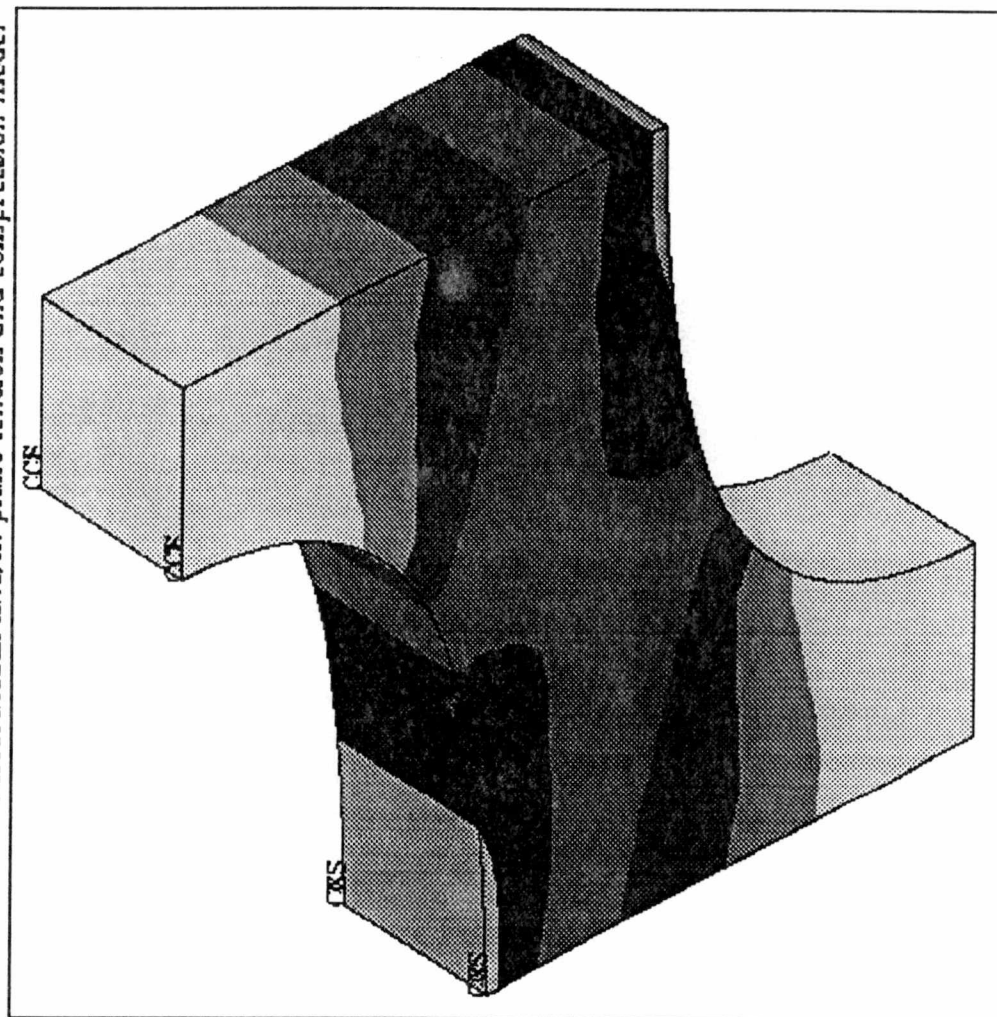
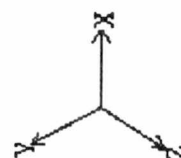


Figure 3.6: Stress contours of the axial stress for the tension in the x-direction test

Perforated_Plate, in plane tension and compression model

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 1 OF 1

SIG_Y
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 -61.67E+3
 -48.33E+3
 -35.00E+3
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 -8.33E+3
 5.00E+3
 CS = -72.73E+3
 KS = 0.22E+3

TIME 0.3904E-3

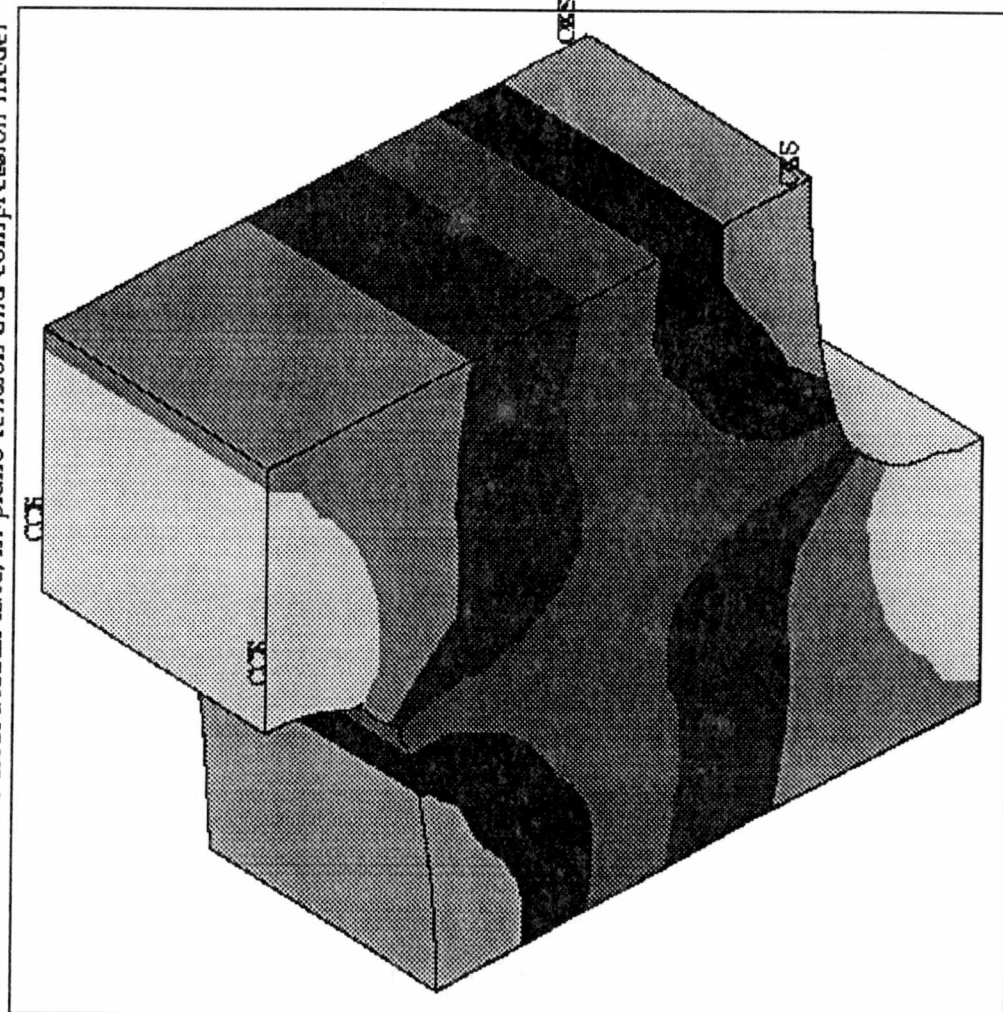
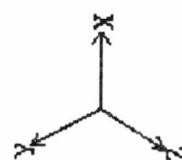


Figure 3.7: Stress contours of the axial stress for the compression in the y-direction test

Perforated_Plate, in plane tension and compression model

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ELEMENT BLOCKS ACTIVE:
1 OF 1

SIG_X

-120.0E+3
-100.0E+3
-80.0E+3
-60.0E+3
-40.0E+3
-20.0E+3
0.0E+3

CS = -110.2E+3
KS = -0.6E+3

TIME 0.3100E-3

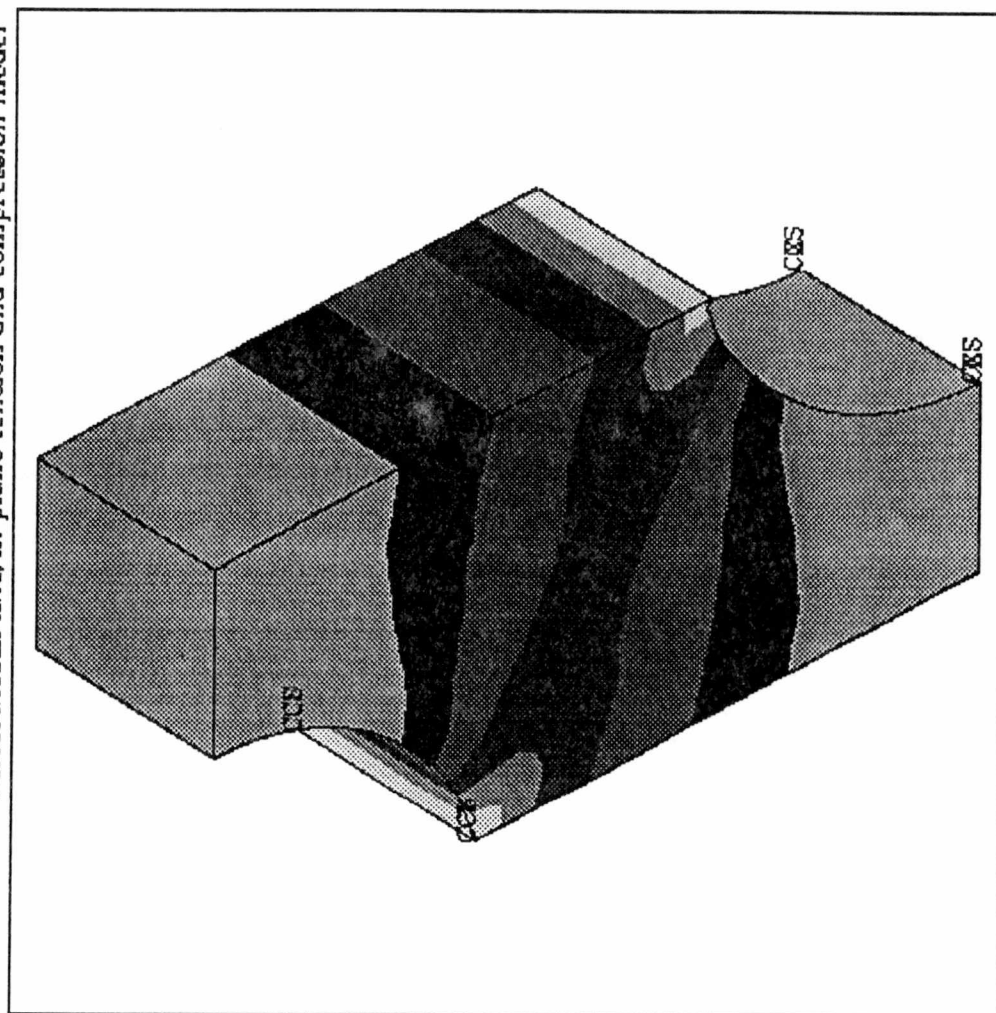
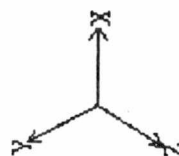


Figure 3.8: Stress contours of the axial stress for the compression in the x-direction test

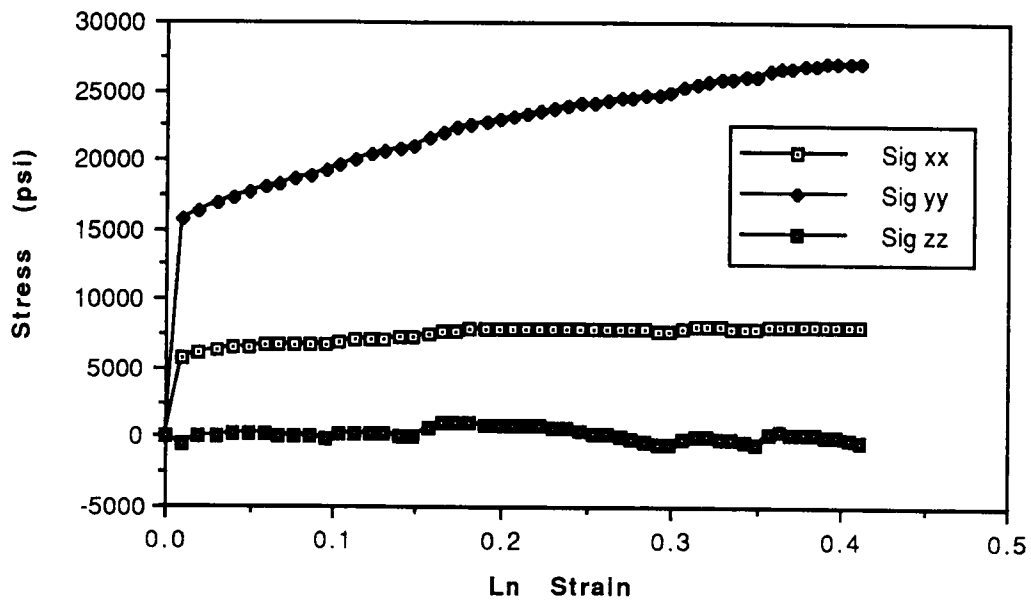


Figure 3.9: Stress strain data from the tension in the y direction test

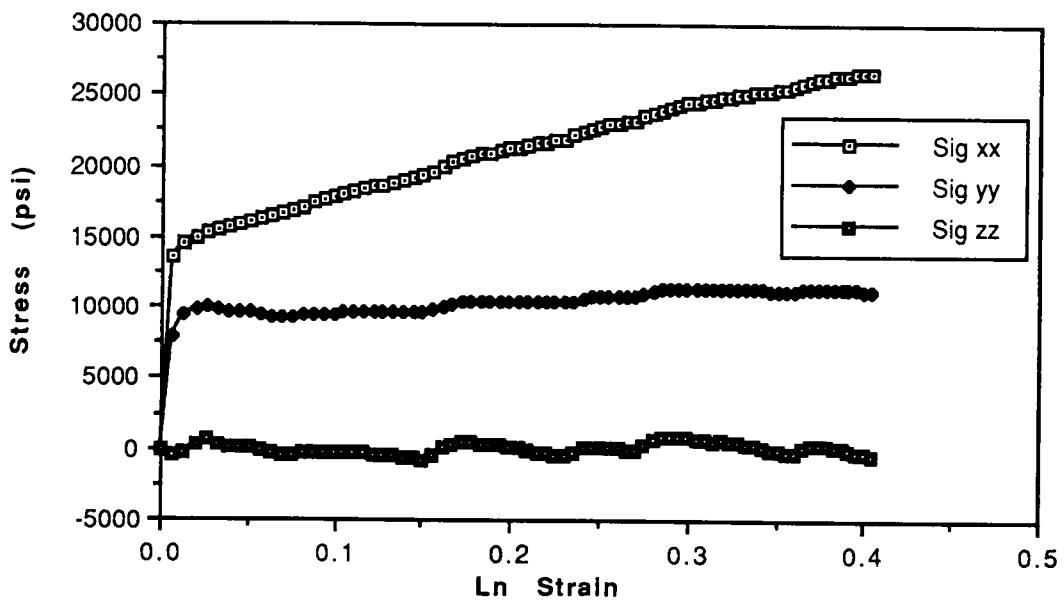


Figure 3.10: Stress strain data from the tension in the x direction test

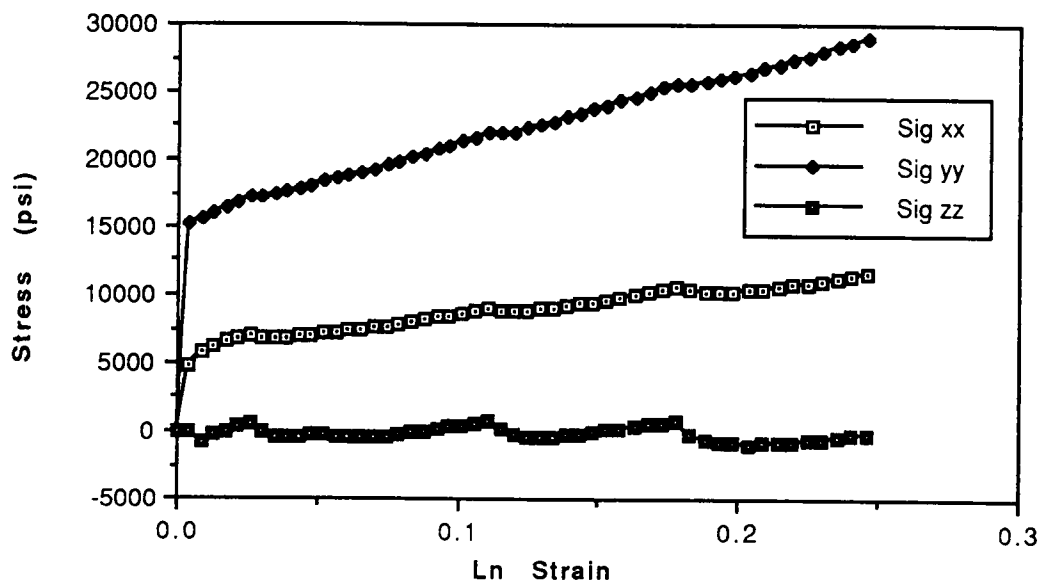


Figure 3.11: Stress strain data from the compression in the y direction test

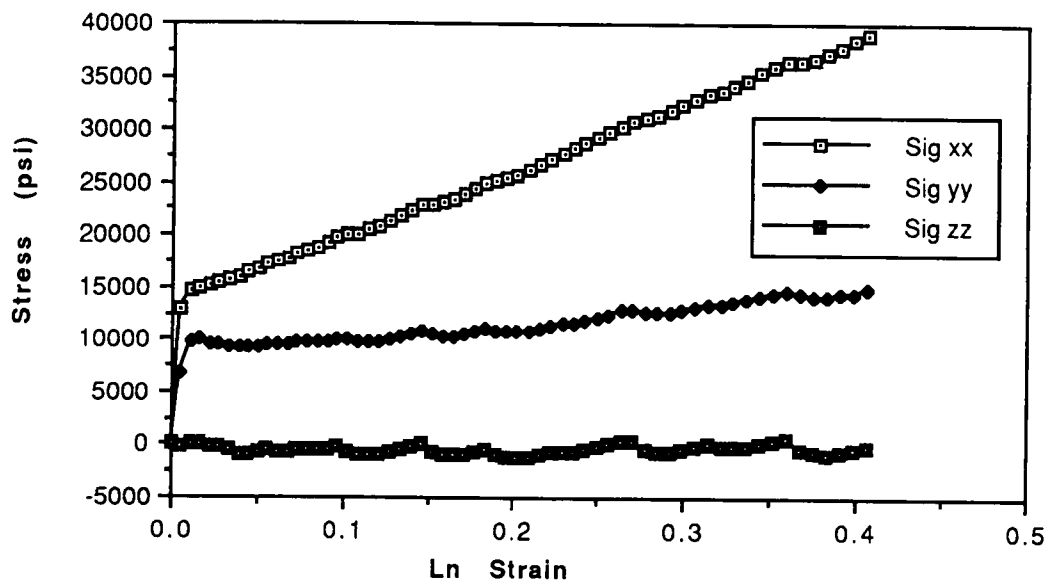


Figure 3.12: Stress strain data from the compression in the x direction test

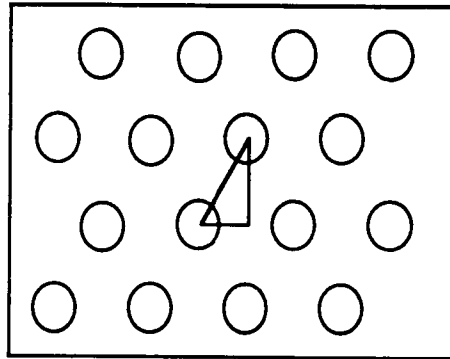


Figure 3.13: Analyzed section of perforated plate for holes lined up

Perforated Plate: holes lined up-pressure normal to plane.

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TIME 1801E-8

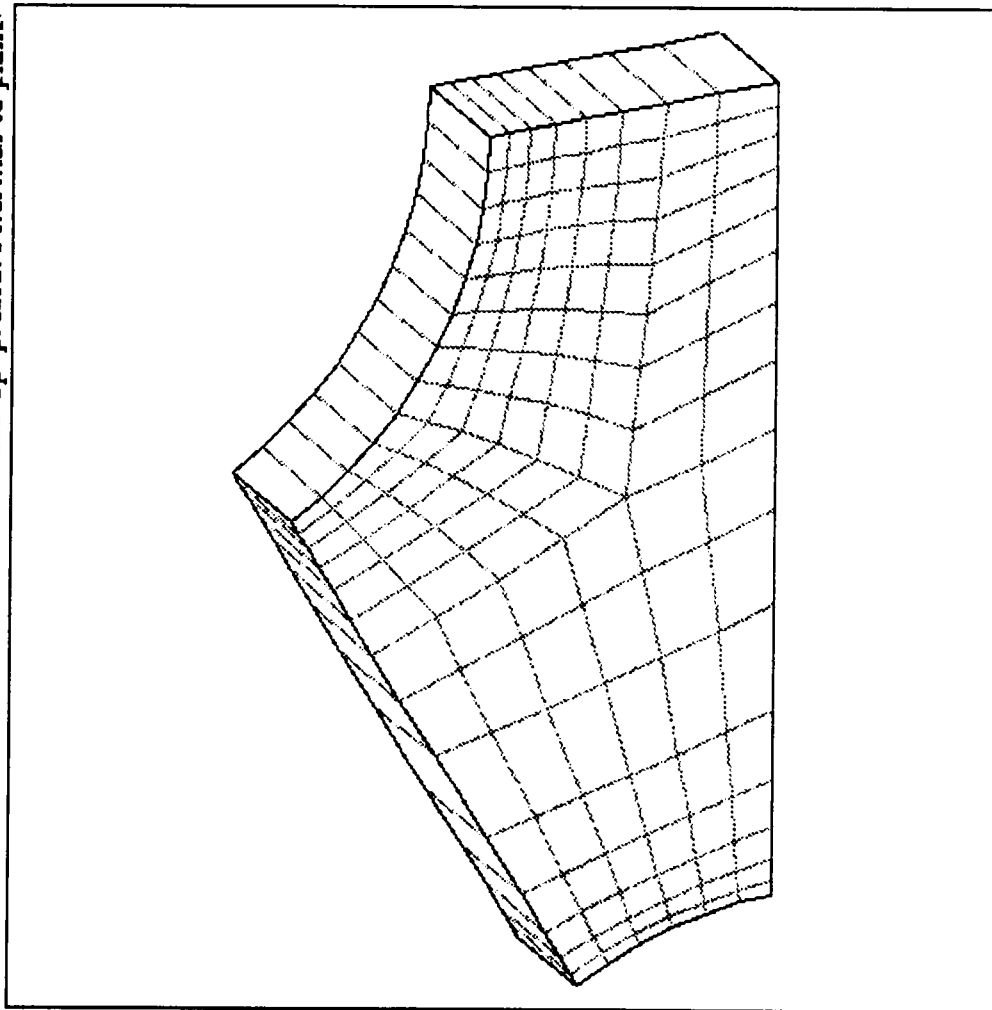
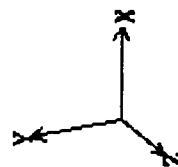


Figure 3.14: Undeformed mesh with coordinate system for compression normal-to-the-plane with the holes lined up

Perforated Plate: holes lined up—pressure normal to plane.

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SIG_Z
 -200.0E+3
 -183.3E+3
 -165.7E+3
 -150.0E+3
 -133.3E+3
 -116.7E+3
 -100.0E+3
 CS = -193.8E+3
 SS = -108.0E+3

TIME 90.40E-6

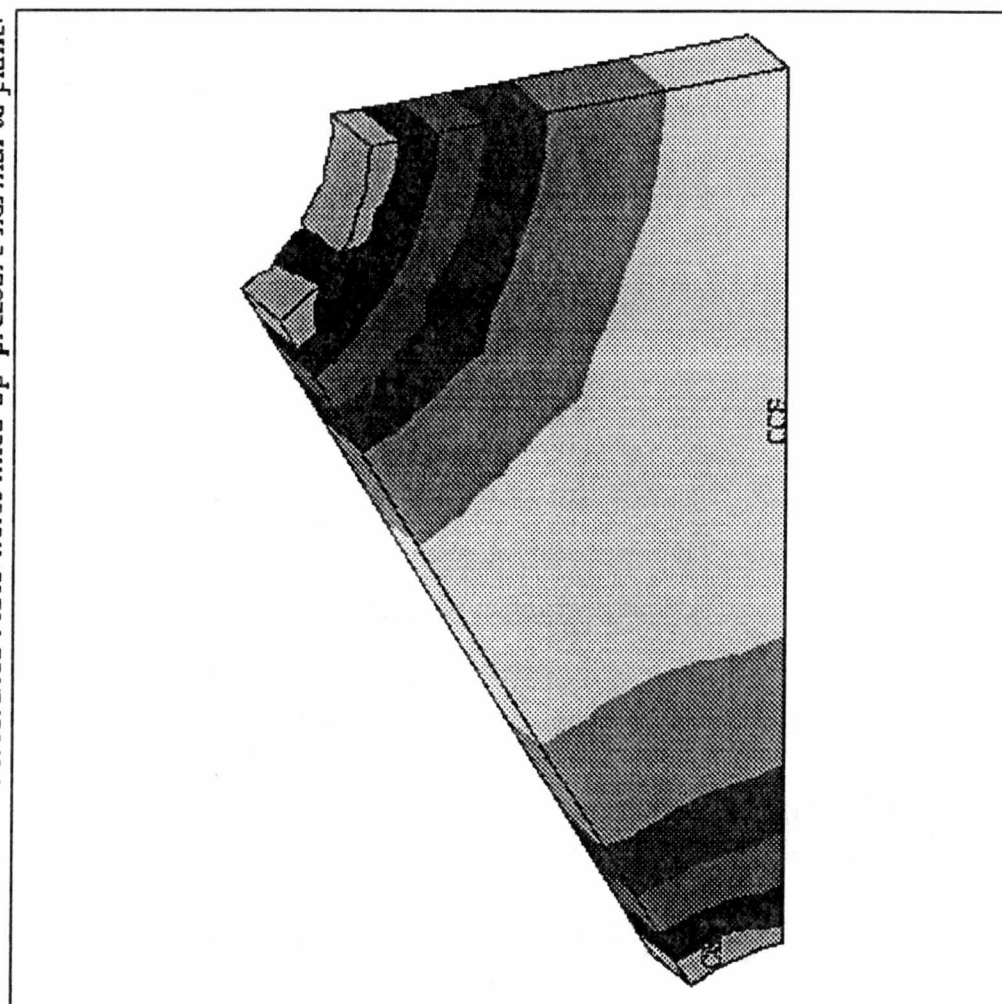
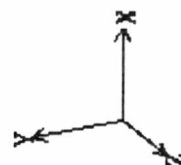


Figure 3.15: Stress contours of the normal-to-the-plane stress as keystoneing begins

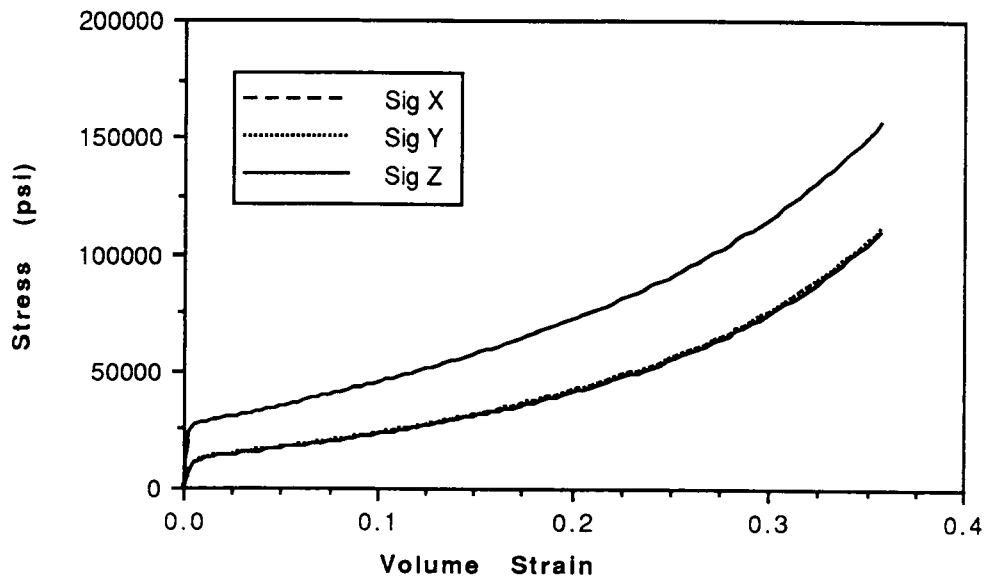


Figure 3.16: Stress-strain relation for compression normal-to-the-plane with holes lined up; hardening material

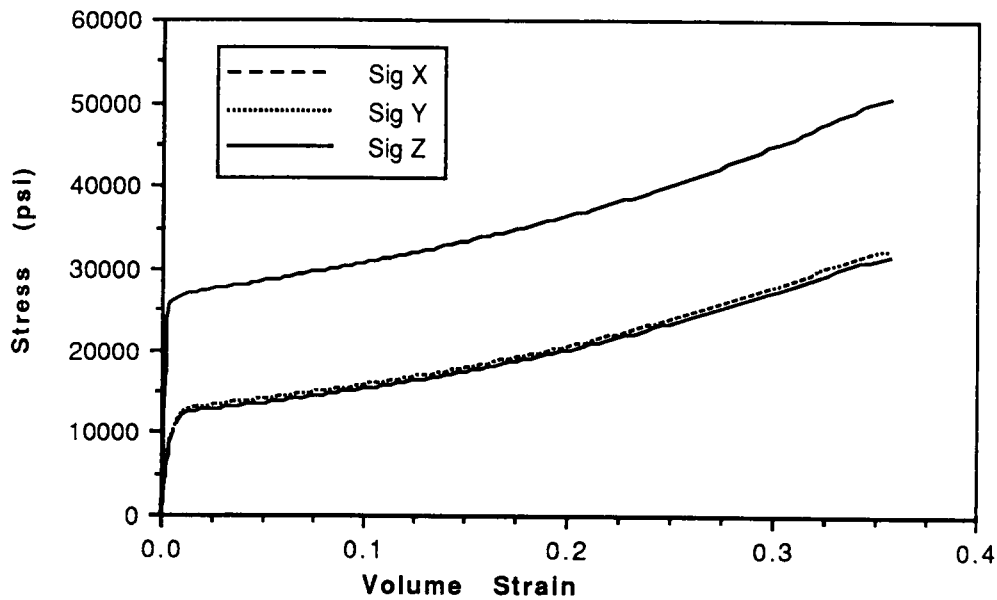
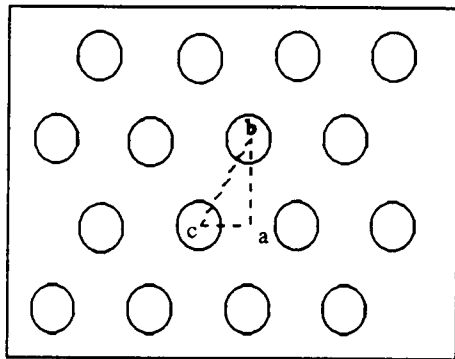
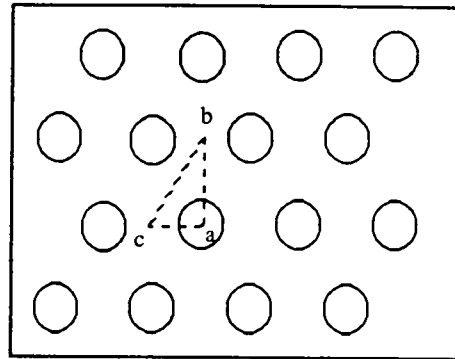


Figure 3.17: Stress-strain relation for compression normal-to-the-plane with holes lined up; elastic-perfectly plastic material.



(a)



(b)

Figure 3.18: Analyzed section of plate for holes off center analysis: (a) Plate with first section used for holes off center analysis; (b) Plate with second section used for holes off center analysis.

Perforated_Plate, holes off center - comp out of plane

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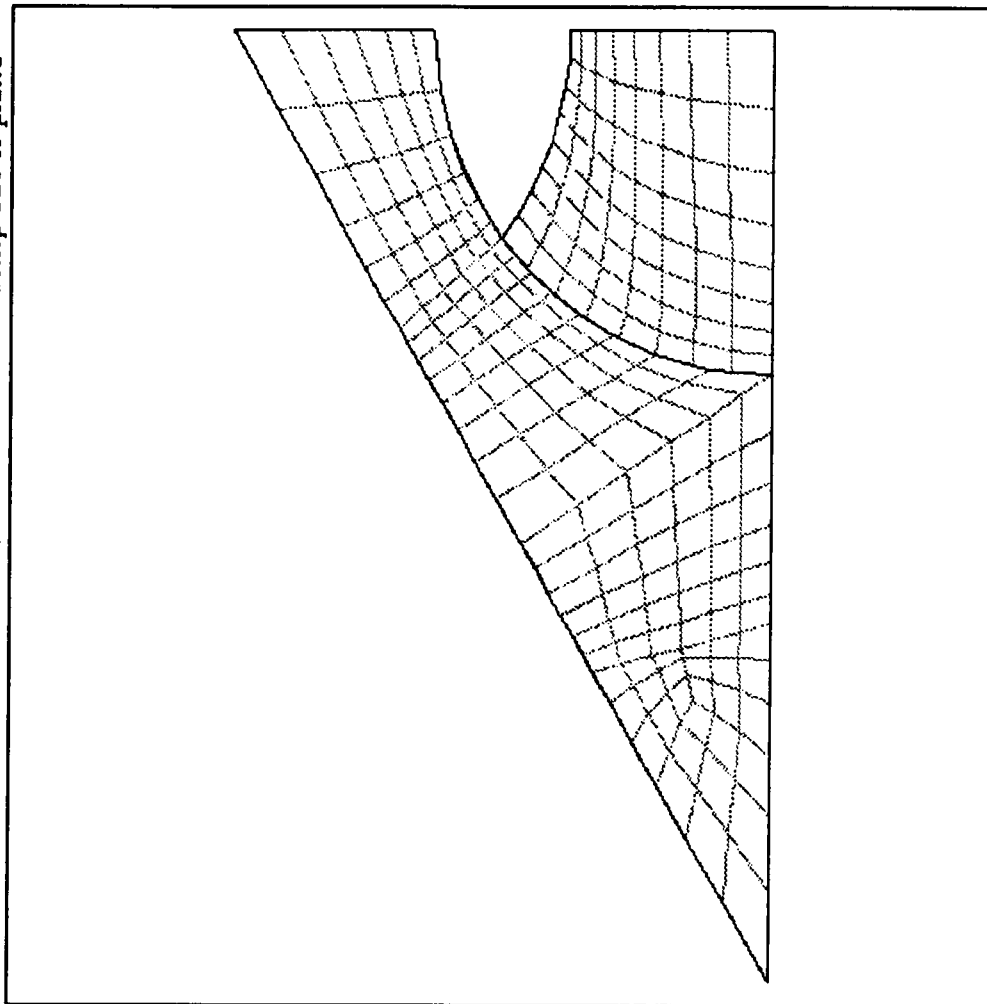


Figure 3.19: Undeformed mesh for the compression normal-to-the-plane with the holes off center; viewed from above

Perforated_Plate, holes off center - comp out of plane

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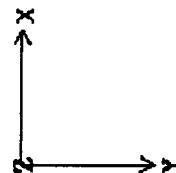
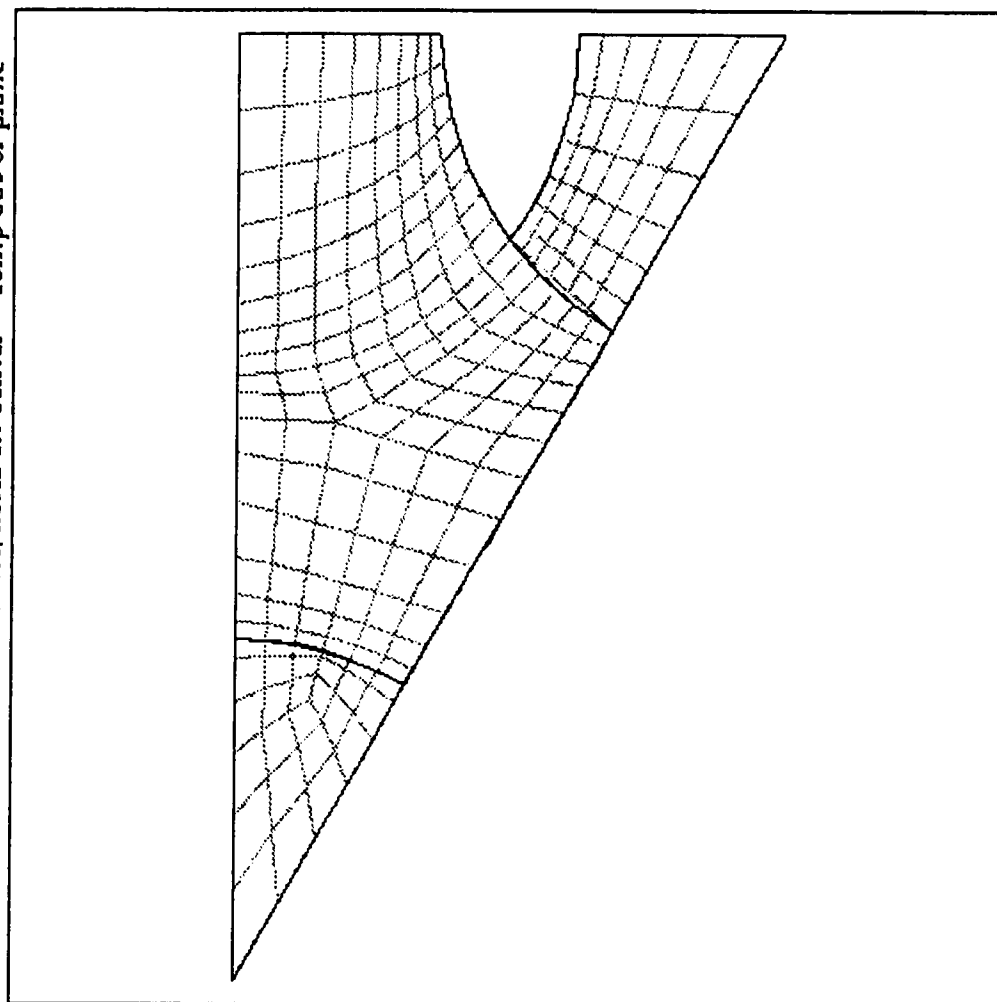


Figure 3.20: Undeformed mesh for the compression normal-to-the-plane with the holes off center, viewed from below

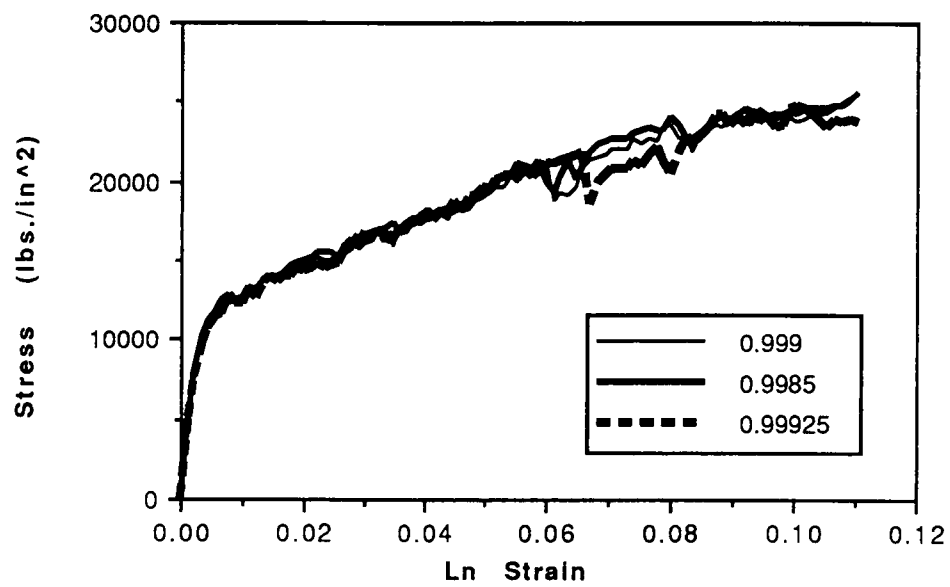


Figure 3.21: Comparison of results for different dynamic relaxation factors

Perforated_Plate; holes off center - comp out of plane

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ELEMENT BLOCKS ACTIVE
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SIG_Z
-120.0E+3
-98.3E+3
-76.7E+3
-55.0E+3
-33.3E+3
-11.7E+3
10.0E+3
CS = -117.2E+3
SS = 6.6E+3

TIME 0.3400E-3

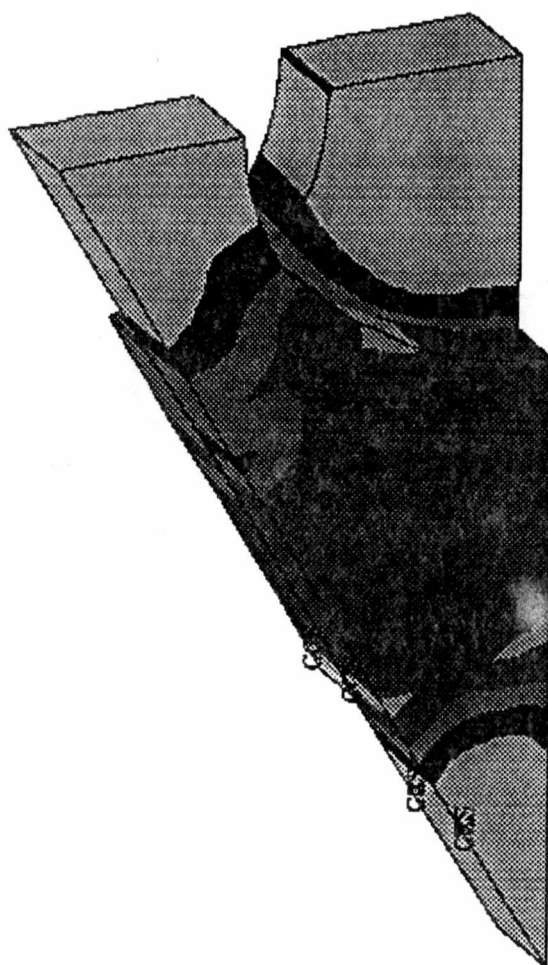


Figure 3.22: Contours of the stress in the normal-to-the-plane direction

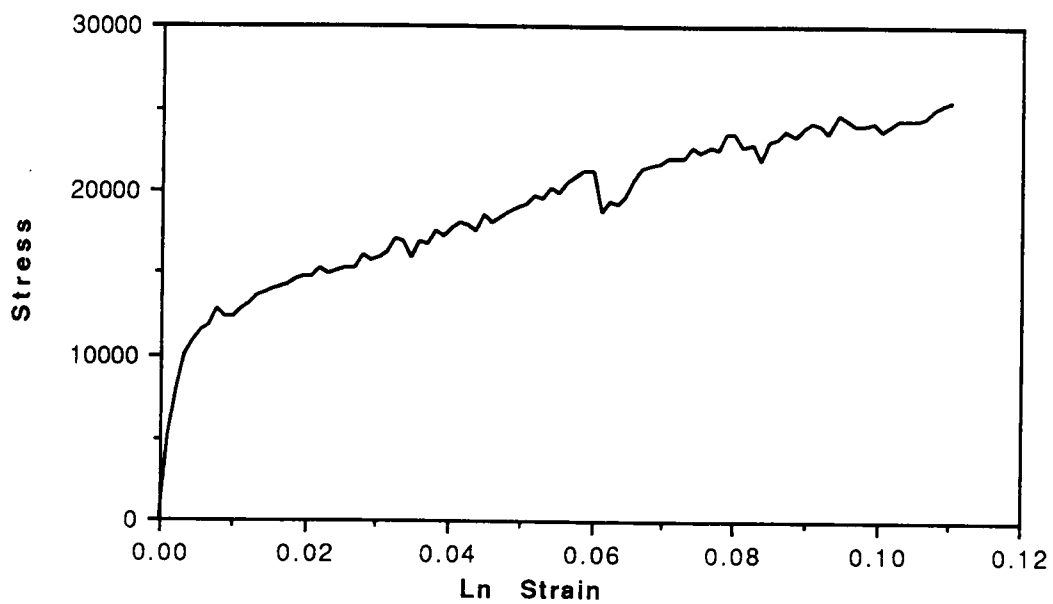


Figure 3.23: Stress-strain relation for compression normal-to-the-plane; minimum hole overlap.

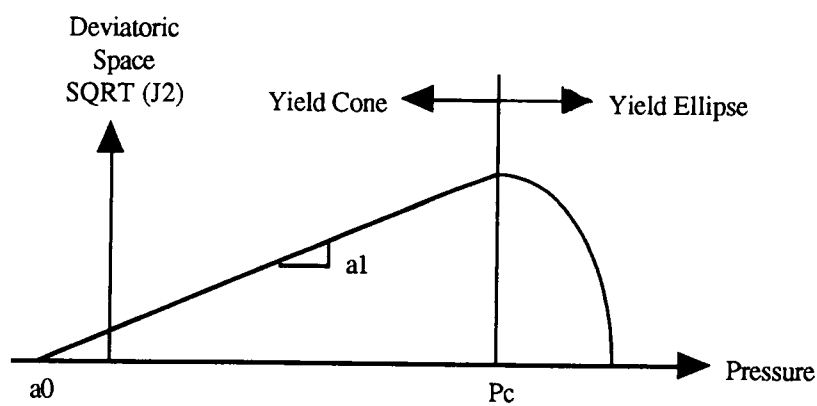


Figure 4.1: Yield surfaces for the isotropic crush model.

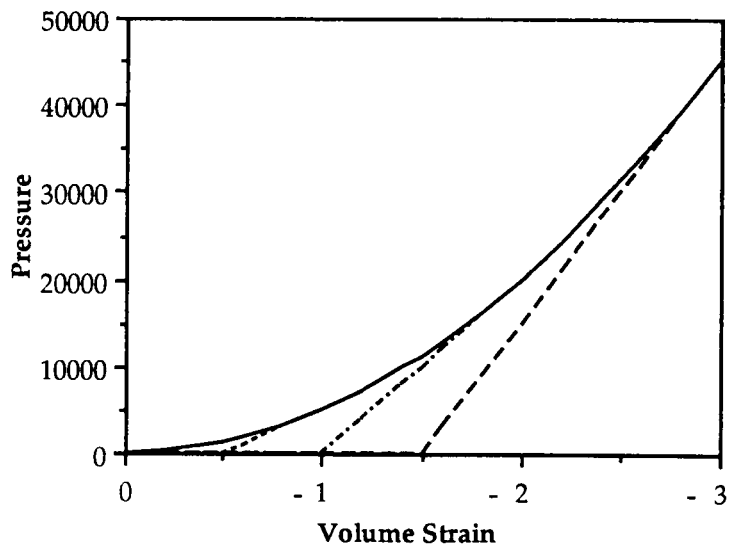


Figure 5.1: Elastic unloadings for variable elastic constants

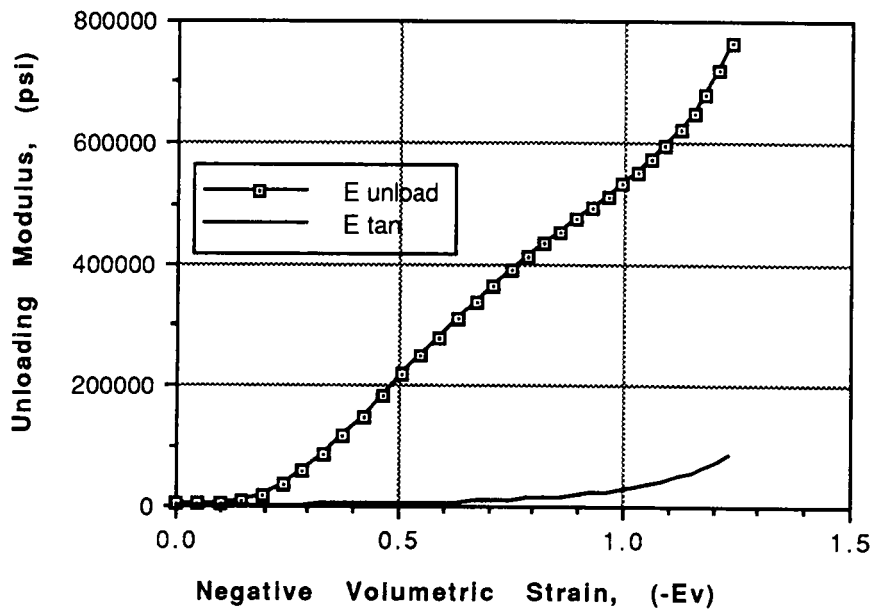


Figure 6.1: Tangent elastic loading modulus versus elastic unloading modulus

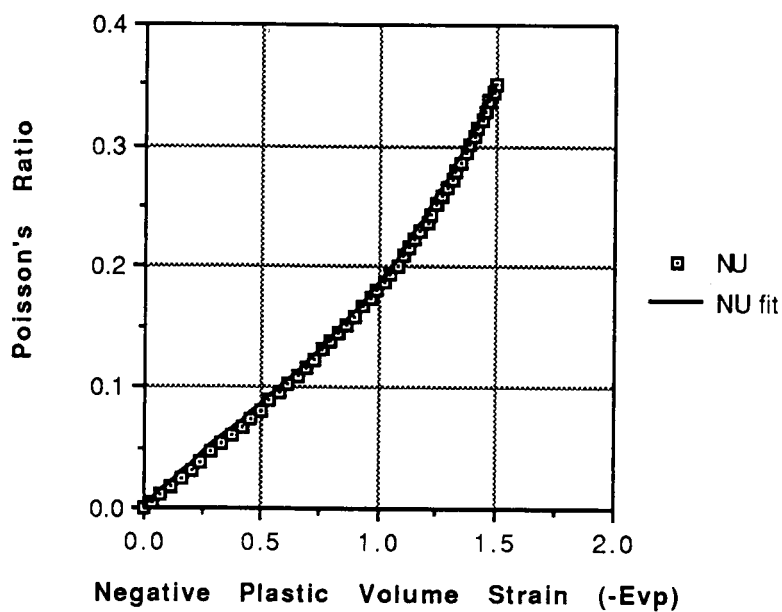


Figure 6.2: Poisson's ratio as a function of plastic volume strain and curve fit for screen wire.

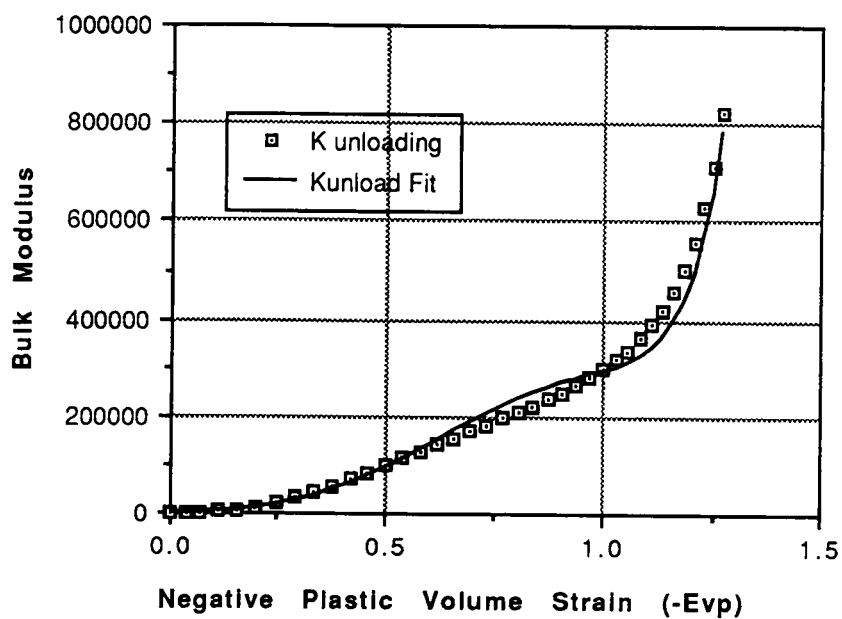


Figure 6.3: Unloading bulk modulus as a function of plastic volume strain and curve fit for screen wire.

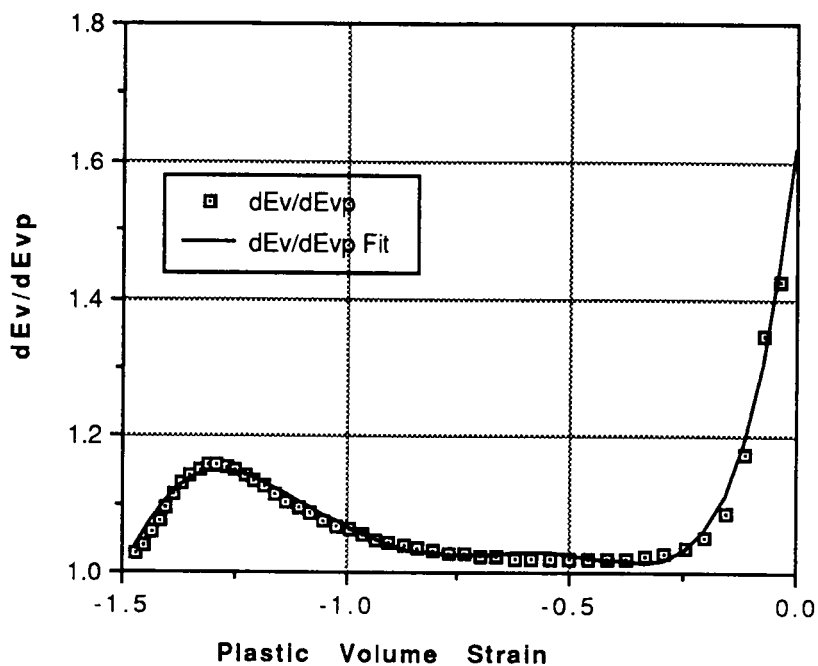


Figure 6.4: Derivative of the total volume strain with respect to the plastic volume strain and curve fit for screen wire.

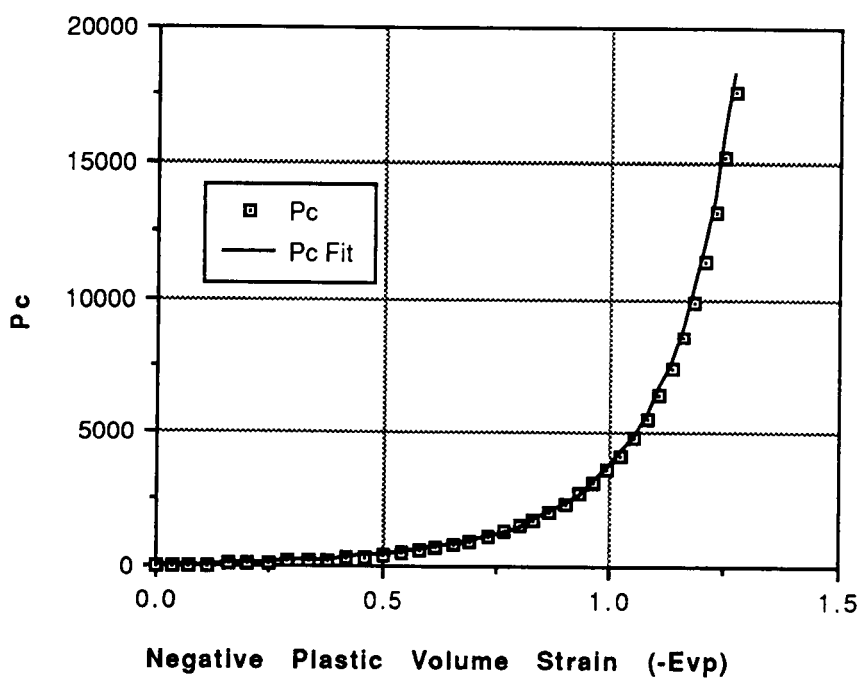


Figure 6.5: P_c as a function of plastic volume strain and curve fit for screen wire.

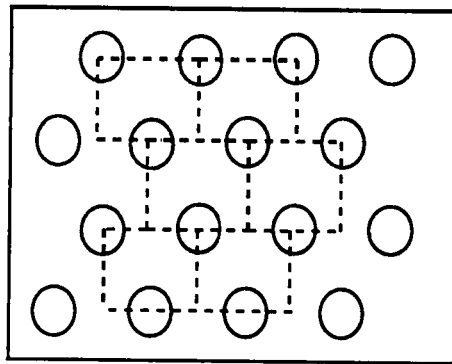


Figure 6.6: Perforated plate control volume; outlined with dashed lines.

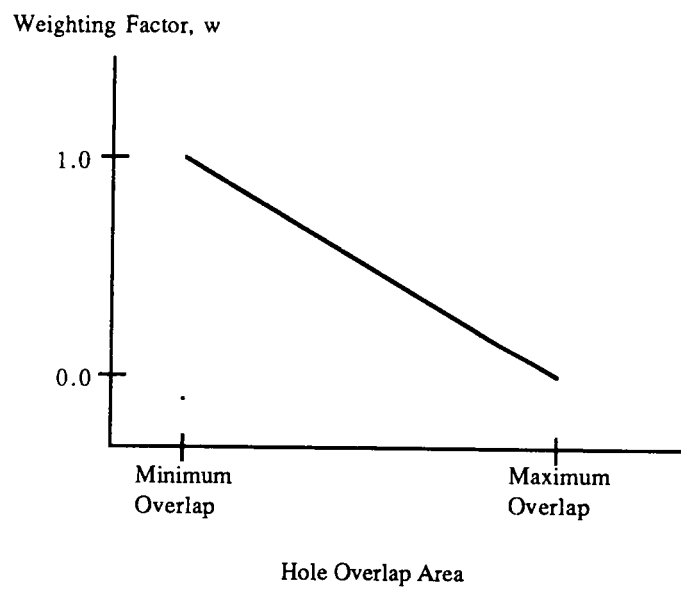


Figure 6.7: Weighting factor versus hole overlap area.

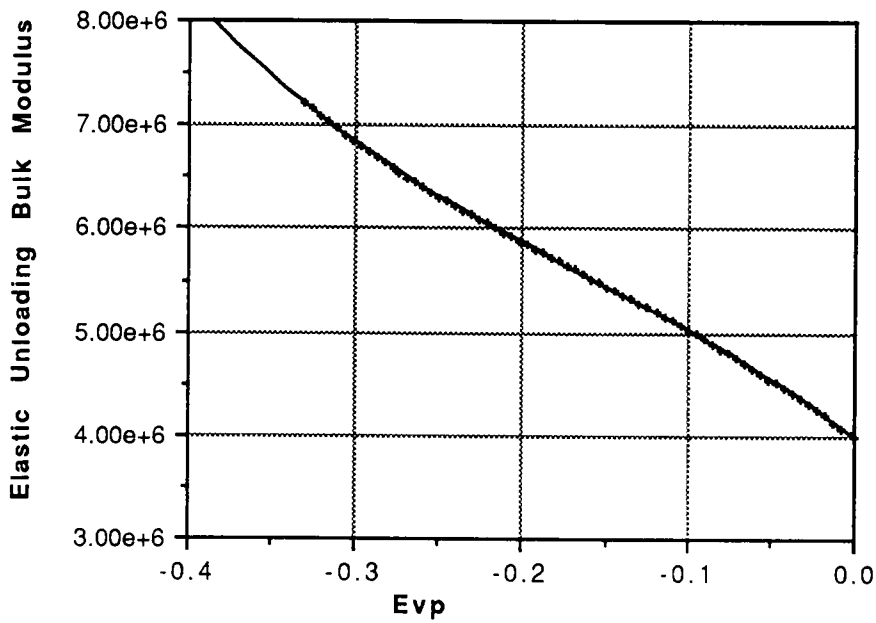


Figure 6.8: Elastic unloading bulk modulus as a function of plastic volume strain and curve fit for perforated plate.

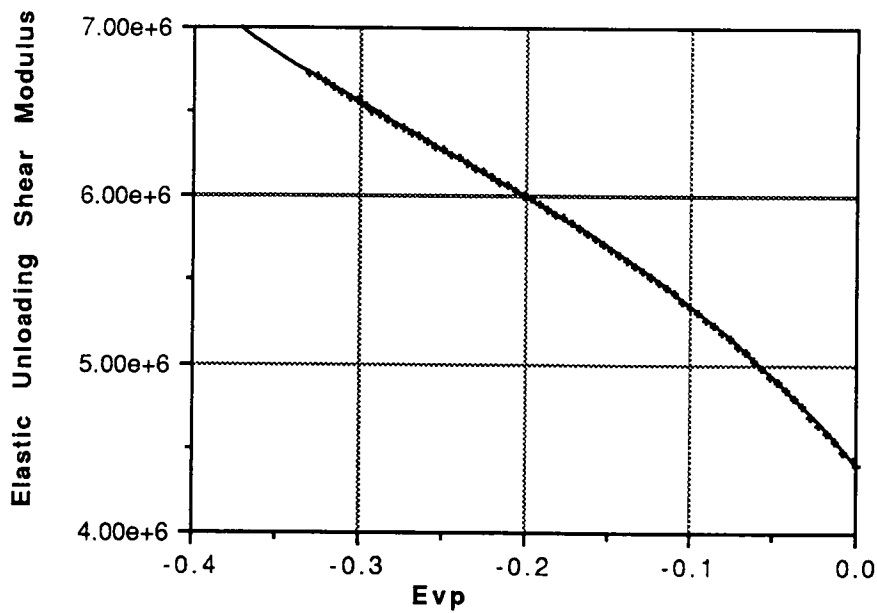


Figure 6.9: Elastic unloading shear modulus as a function of plastic volume strain and curve fit for perforated plate.

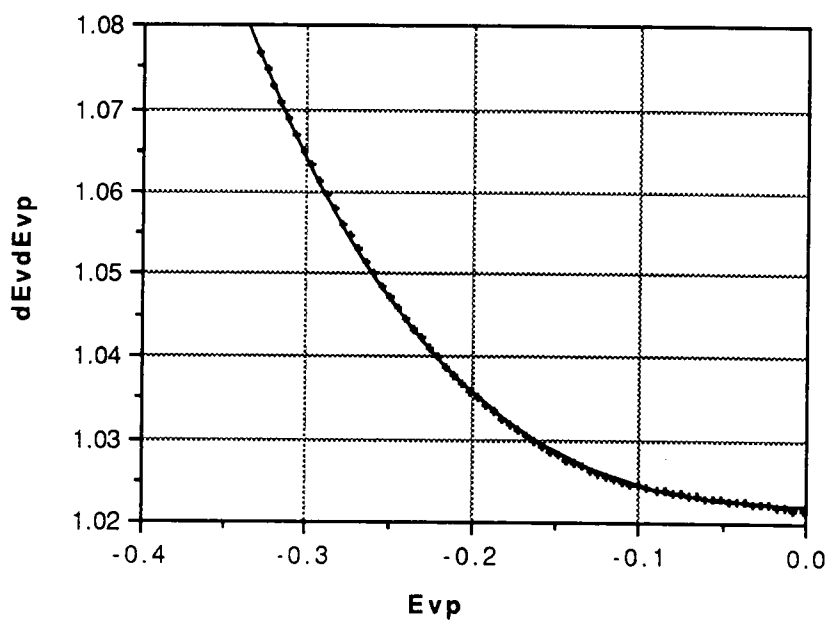


Figure 6.10: The derivative of total volume strain with respect to plastic volume strain as a function of plastic volume strain and curve fit for perforated plate.

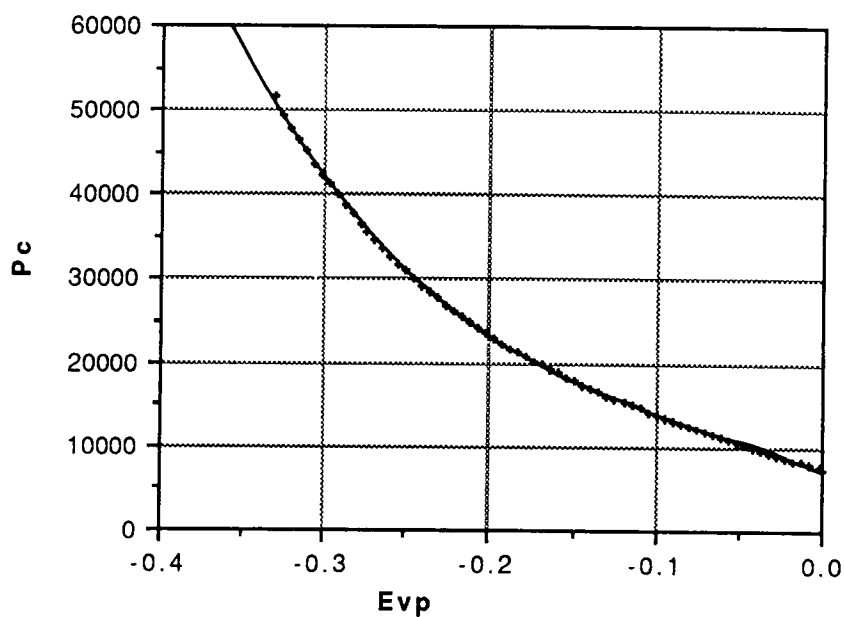


Figure 6.11: P_c as a function of plastic volume strain and curve fit for perforated plate.

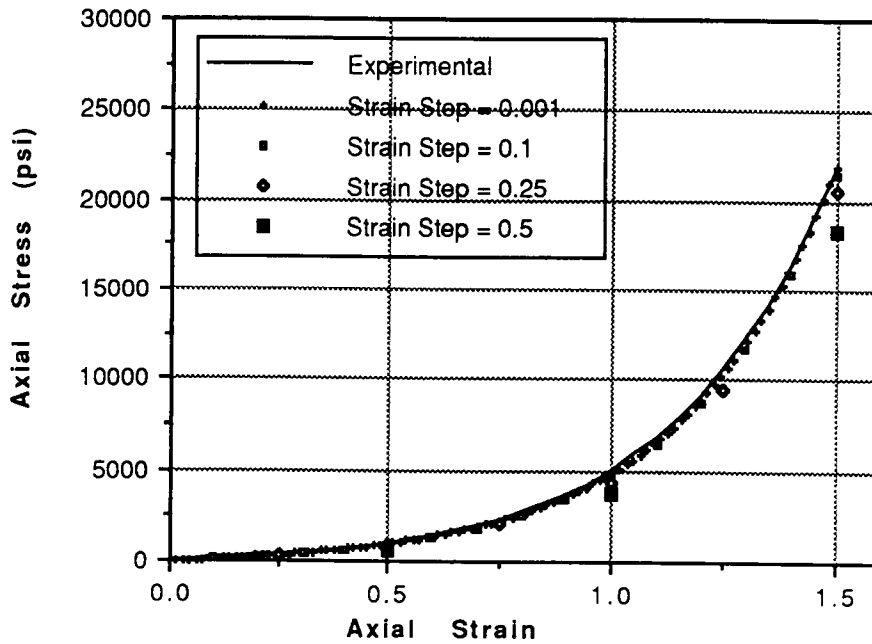


Figure 7.1: Axial stress versus axial strain for various strain steps.

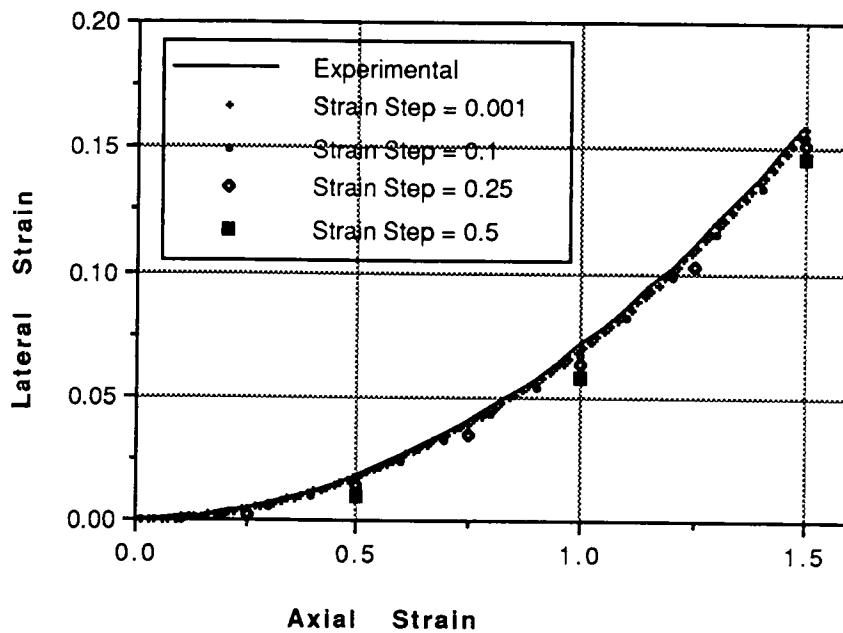


Figure 7.2: Lateral strain vs. axial strain for various strain steps.

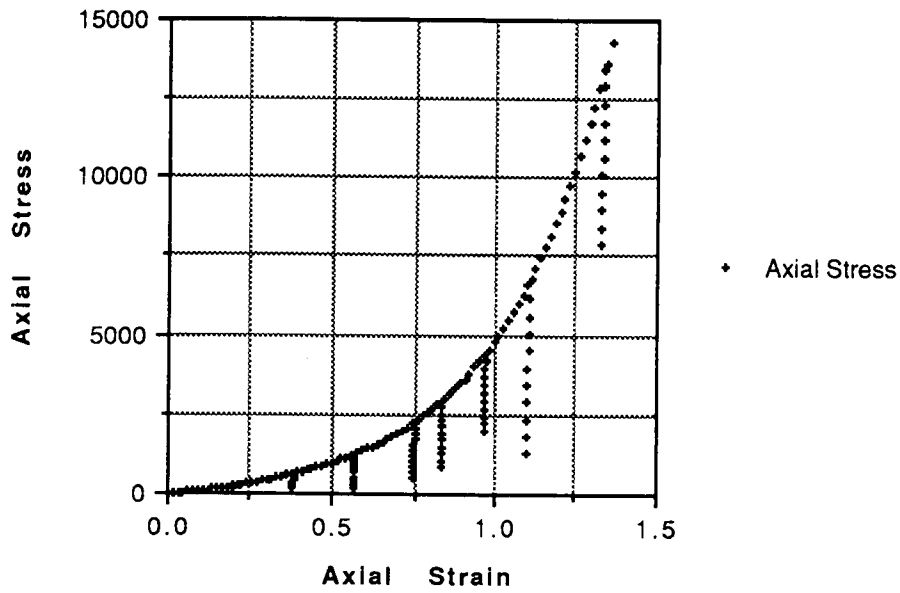


Figure 7.3: Unloading during compression for the isotropic crush model

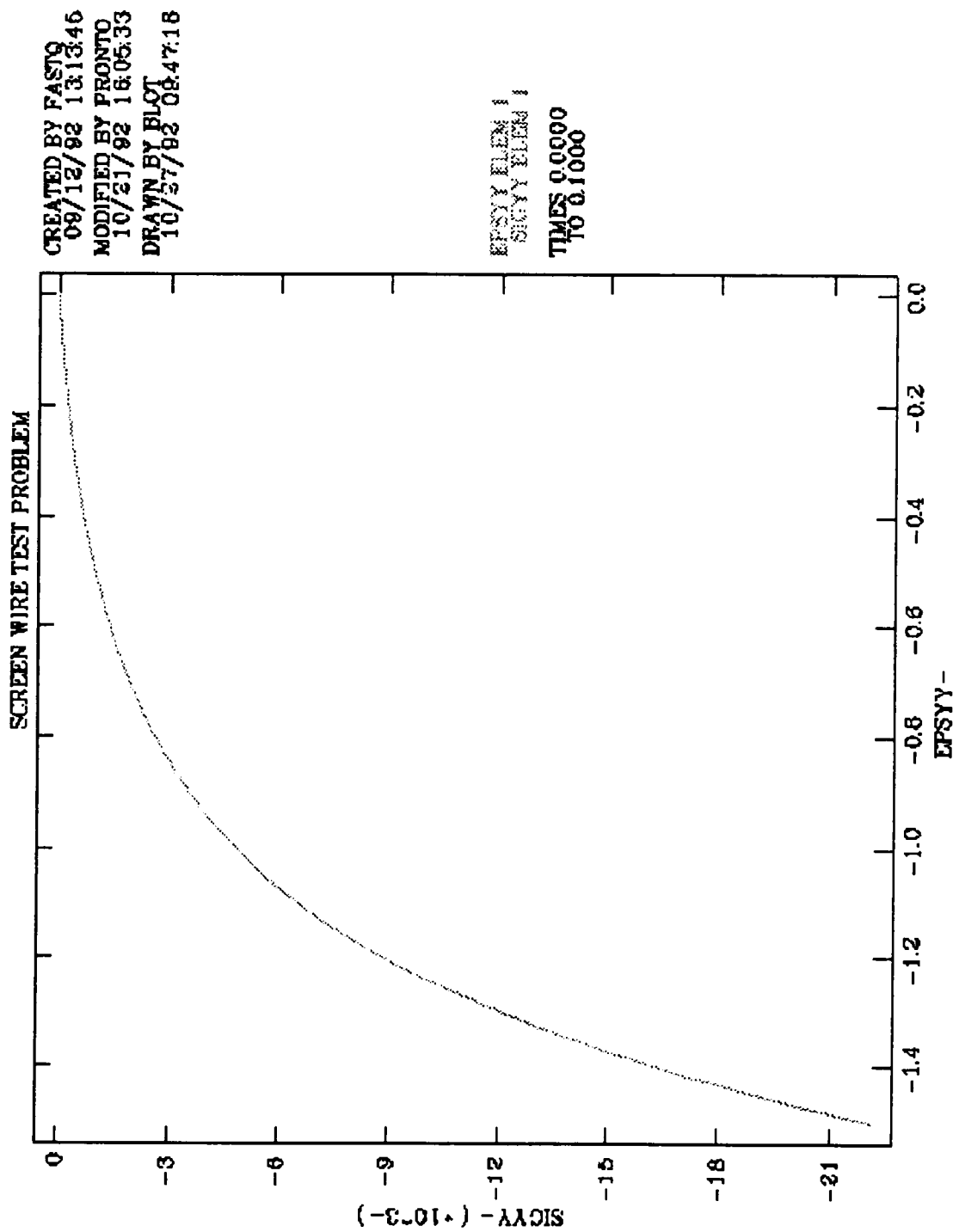


Figure 7.4: Axial stress versus axial strain predicted in PRONTO2D

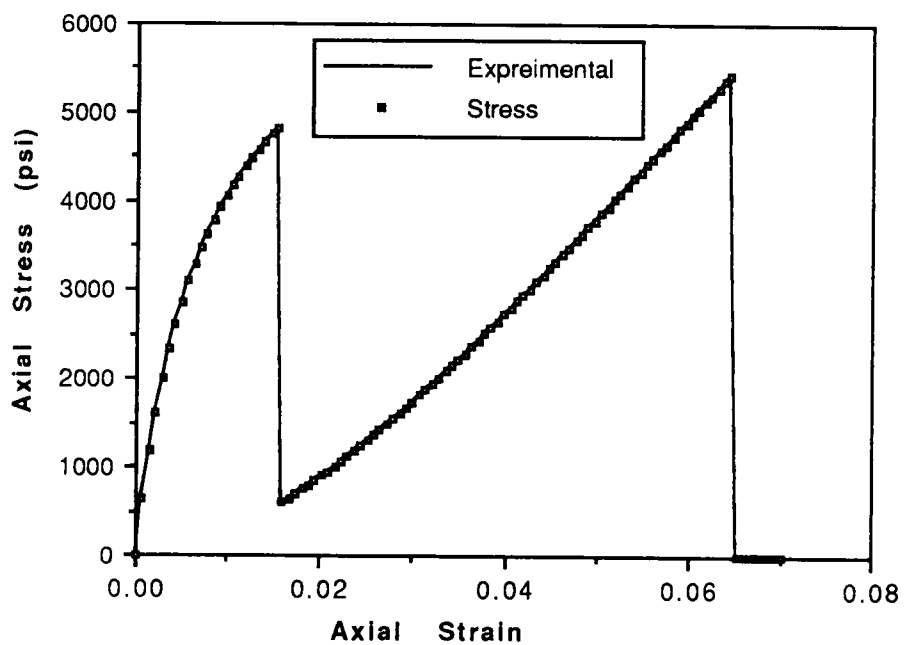


Figure 7.5: In-plane tensile behavior of the model.

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8
Eps1	Eps2	Ev	Sig 1	Eunload	Delta Eps1	Delta Eps2	NU
0.00	0.0000	0.0000	0.0	3001			0.0000
-0.05	0.0002	-0.0496	-46.3	3098	-0.05	0.0002	0.0035
-0.10	0.0007	-0.0986	-99.7	4150	-0.05	0.0005	0.0106
-0.15	0.0016	-0.1468	-161.4	8279	-0.05	0.0009	0.0177
-0.20	0.0028	-0.1943	-232.5	17755	-0.05	0.0012	0.0248
-0.25	0.0044	-0.2411	-314.6	33735	-0.05	0.0016	0.0319
-0.30	0.0064	-0.2872	-409.3	56030	-0.05	0.0019	0.0390
-0.35	0.0087	-0.3326	-518.6	83533	-0.05	0.0023	0.0461
-0.40	0.0113	-0.3773	-644.8	114742	-0.05	0.0027	0.0531
-0.45	0.0143	-0.4213	-790.3	148139	-0.05	0.0030	0.0602
-0.50	0.0177	-0.4646	-958.3	182402	-0.05	0.0034	0.0673
-0.55	0.0214	-0.5071	-1152.1	216483	-0.05	0.0037	0.0744
-0.60	0.0255	-0.5490	-1375.7	249622	-0.05	0.0041	0.0815
-0.65	0.0299	-0.5901	-1633.8	281315	-0.05	0.0044	0.0886
-0.70	0.0347	-0.6306	-1931.6	311271	-0.05	0.0048	0.0956
-0.75	0.0399	-0.6703	-2275.2	339365	-0.05	0.0051	0.1027

(a)

Column 9	Column 10	Column 11	Column 12	Column 13	Column 14	Column 15	Column 16
Kunloading	Sig mean	Delta Ev	Delta Sigmean	Delta Evp	Evp	dEvdEvp	Pc
1000	0.0				0.0000		0.0
1040	-15.4	-0.0496	-15.4	-0.0348	-0.0348	1.4264	13.6
1413	-33.2	-0.0489	-17.8	-0.0363	-0.0711	1.3468	29.3
2861	-53.8	-0.0482	-20.6	-0.0410	-0.1122	1.1750	47.5
6227	-77.5	-0.0475	-23.7	-0.0437	-0.1559	1.0871	68.4
12011	-104.9	-0.0468	-27.4	-0.0445	-0.2004	1.0512	92.6
20255	-136.4	-0.0461	-31.6	-0.0445	-0.2450	1.0350	120.5
30669	-172.9	-0.0454	-36.4	-0.0442	-0.2892	1.0269	152.6
42795	-214.9	-0.0447	-42.0	-0.0437	-0.3329	1.0225	189.7
56142	-263.4	-0.0440	-48.5	-0.0431	-0.3760	1.0200	232.6
70258	-319.4	-0.0433	-56.0	-0.0425	-0.4185	1.0188	282.0
84774	-384.0	-0.0426	-64.6	-0.0418	-0.4603	1.0182	339.0
99406	-458.6	-0.0419	-74.5	-0.0411	-0.5014	1.0182	404.8
113956	-544.6	-0.0411	-86.0	-0.0404	-0.5418	1.0187	480.8
128300	-643.9	-0.0404	-99.3	-0.0397	-0.5814	1.0195	568.4
142375	-758.4	-0.0397	-114.5	-0.0389	-0.6203	1.0207	669.5

(b)

Figure F1: Spread sheet for data reduction of screen wire data

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8
Strain	Sig X	Sig Y	Sig Z	Mean Stress	E unload	Nu	K unload
-0.00756	-8927.4	-9259.3	-18679.8	-12288.8	6965315.9	0.2094	3994966.2
-0.01000	-9050.4	-9386.8	-19055.1	-12497.4	7004886.2	0.2098	4023061.4
-0.01500	-9317.3	-9662.8	-19816.1	-12932.1	7084693.6	0.2106	4080057.6
-0.02000	-9602.7	-9957.3	-20566.8	-13375.6	7162714.8	0.2114	4136226.5
-0.02500	-9905.6	-10269.4	-21308.1	-13827.7	7238978.1	0.2122	4191580.9
-0.03000	-10225.2	-10598.1	-22041.3	-14288.2	7313511.4	0.2129	4246134.9
-0.03500	-10560.6	-10942.6	-22767.6	-14757.0	7386343.1	0.2137	4299902.8
-0.04000	-10910.9	-11302.1	-23488.2	-15233.7	7457501.2	0.2145	4352900.0
-0.04500	-11275.5	-11675.9	-24204.2	-15718.5	7527013.9	0.2152	4405142.1
-0.05000	-11653.6	-12063.2	-24916.8	-16211.2	7594909.5	0.2160	4456645.8
-0.05500	-12044.6	-12463.5	-25627.2	-16711.8	7661216.0	0.2167	4507428.2
-0.06000	-12448.0	-12876.2	-26336.6	-17220.3	7725961.7	0.2175	4557507.1
-0.06500	-12863.4	-13301.0	-27046.2	-17736.9	7789174.7	0.2182	4606900.9
-0.07000	-13290.3	-13737.4	-27757.1	-18261.6	7850883.1	0.2189	4655628.7
-0.07500	-13728.5	-14185.0	-28470.5	-18794.7	7911115.2	0.2197	4703710.2
-0.08000	-14177.5	-14643.7	-29187.6	-19336.3	7969899.1	0.2204	4751165.7
-0.08500	-14637.4	-15113.2	-29909.7	-19886.7	8027262.9	0.2212	4798016.0

(a)

Column 9	Column 10	Column 11	Column 12	Column 13	Column 14	Column 15	Column 16	Column 17
G unload	Delta Ev	Delta Sig Mean	Delta Evp	Evp	dEvdEvp	Pressure	SQRT J2	Pc
4405153.6				0		12288.8	5648.4	7518.0
4432366.0	-0.0024	-208.6	-0.002386	-0.002386	1.0217	12497.4	5908.9	7658.9
4487372.5	-0.0050	-434.6	-0.004893	-0.007280	1.0218	12932.1	6188.2	7931.4
4541313.3	-0.0050	-443.5	-0.004893	-0.012172	1.0219	13375.6	6451.1	8207.6
4594204.6	-0.0050	-452.1	-0.004892	-0.017065	1.0220	13827.7	6699.0	8487.6
4646063.1	-0.0050	-460.5	-0.004892	-0.021956	1.0222	14288.2	6933.0	8771.2
4696905.8	-0.0050	-468.7	-0.004891	-0.026847	1.0223	14757.0	7154.2	9058.4
4746749.6	-0.0050	-476.8	-0.004890	-0.031738	1.0224	15233.7	7363.9	9349.2
4795612.1	-0.0050	-484.8	-0.004890	-0.036627	1.0225	15718.5	7563.1	9643.8
4843510.9	-0.0050	-492.7	-0.004889	-0.041517	1.0226	16211.2	7752.9	9942.1
4890463.8	-0.0050	-500.6	-0.004889	-0.046406	1.0227	16711.8	7934.3	10244.3
4936489.0	-0.0050	-508.5	-0.004888	-0.051294	1.0228	17220.3	8108.3	10550.5
4981604.7	-0.0050	-516.6	-0.004888	-0.056182	1.0229	17736.9	8275.9	10860.8
5025829.6	-0.0050	-524.7	-0.004887	-0.061069	1.0231	18261.6	8437.9	11175.5
5069182.3	-0.0050	-533.1	-0.004887	-0.065956	1.0232	18794.7	8595.3	11494.6
5111681.9	-0.0050	-541.6	-0.004886	-0.070842	1.0233	19336.3	8748.8	11818.4
5153347.6	-0.0050	-550.4	-0.004885	-0.075727	1.0235	19886.7	8899.2	12147.1

(b)

Figure G1: Spread sheet for data reduction of perforated plate data

VITA

Kevin Herbert Brown was born on February 16, 1969 in Chattanooga, Tennessee. He moved to Knoxville, Tennessee in 1974. He attended elementary schools in the Knox County school district. After his junior year of high school at Bearden Senior High School, he entered the University of Tennessee at Knoxville under an early admissions program in January of 1987. He subsequently graduated from Bearden High School in 1988. He received his Bachelor's of Science from the University of Tennessee in May 1991. He then entered the Master's of Science program at the University of Tennessee where he received his Master's of Science degree in December 1992.