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APPLICATION OF THE DISCRETE-ELEMENT METHOD TO FAST-TRANSIENT THREE-DIMENSIONAL INCOMPRESSIBLE FLOW WITH FREE-SURFACE CONDITIONS IN REGIONS OF ARBITRARY GEOMETRY

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ABSTRACT

A fast-transient, three-dimensional, discrete-element computational model is presented for simulating free-surface flow conditions of water (constant density fluid) in regions with geometrically complex shoreline boundaries.

The geometrical discretization technique of the general three-dimensional discrete-element method (DEM) considers the interior elements as rectangular cells and the boundary elements as truncated rectangular cells which are defined by the impermeable shoreline boundaries of the flow region. Therefore, the rates of mass and momentum transport along any orthogonal axis of a variable-size grid in the Cartesian coordinate system is always accross an enclosure surface area that is normal to the axis. Hence, the computational efficiency of the discrete-element method (DEM) is equivalent to the finite-difference method (FDM) which is significantly superior to the finite-element method (FEM); and it's spatial resolution accuracy is equivalent to the finite-element method (FEM), since the actual geometrical characteristics of the boundary elements are completely retained in the formulation.

The development of the discrete-element computational system utilizes directly the integral forms of the mass conservation and momentum principles, without resorting to the continuum partial differential equations that are used in the development of the conventional numerical solution methods.

The discrete-element formulation of the momentum principle considers two half-elements, within each element, for the calculation of one

component of velocity in the horizontal (x,y) plane. This new formulation requires the computation of the velocity components at approximately twice as many solution locations in comparison to the solution locations required for the computation of pressure. The new formulation exhibits excellent numerical stability characteristics comparable to the conventional staggered mesh systems. Furthermore, it enables the geometrical discretization of the flow region according to a reasonably course spatial resolution grid system for pressure (water-surface elevation) but with twice as high spatial resolution for computationally more critical velocity conditions.

The computational algorithm of the transient, three-dimensional discrete-element method uses an explicit timewise-integration technique, which does not require the implicit solution of any elliptic system of equations. The numerical integration method is based on an explicit, composite time-spliting algorithm which requires only the satisfaction of CFL (Courant, Friedrichs and Lewy) stability criterion for free-surface flow conditions.

The general development of the three-dimensional discrete-element method clearly identifies the individual terms associated with the convection-defect transport and with eddy-turbulent transport which collectively constitute the general nonconvective momentum transport. The general discrete-element computational system is independent of the particular phenomenological nonconvective transport model for the Reynolds stress components which is generally required for the closure of the conventional mathematical system. The present formulation of the nonconvective momentum transport (Reynolds stress) considers a local

eddy-turbulence model based on element Reynolds number and a Blasius power-law form for the evaluation of the momentum eddy diffusivity tensor components on the enclosure surfaces of the elements and half-elements. The particular formulation enables the establishment of a pseudo unified-eddy-turbulence model applicable to different turbulence scales, including geophysical-meso-scale and laboratory-model-scale flow conditions in different regions.

The results of application of the associated computer code HYDROL of the fast-transient, three-dimensional, discrete-element hydrodynamic model to a test case for the simulation of the flow conditions in a symmetric laboratory-model-scale flow region are presented to illustrate the accuracy of the computational algorithm. The computer simulation results indicate that the required symmetry and asymmetry conditions associated with five different test cases were all satisfied in the application of the model. In all test cases the numerical solutions also exhibit all the expected characteristics based on considerations of the physical conditions. The results of application of the associated computer code HYDROL to the simulation of the flow conditions in the actual laboratory-model-scale flow region of the Browns Ferry physical model are also presented. The computer simulation results for the Browns Ferry physical model also illustrate the special capabilities of the fast-transient, three-dimensional, discrete-element model in predicting the occurrence of oscillatory surface-gravity-wave type flow conditions in flow regions with closed enclosure boundaries.

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gradation between -8 mm minimum and 8 mm maximum relative to MWL. 819

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VOLUME 1

1. INTRODUCTION

One of the critical issues related to the licensing of a large electric power generating station (or any large industrial plant) on a lake, river, reservoir, estuary, or coastal zone is the public concern about the potential impact of its operation on the site-specific ecology of the water body.

The federal water quality standards recommended by the Environmental Protection Agency (EPA) and the state regulations applicable to water quality standards generally specify the limiting criteria for the tolerable levels of power plant impact on the ecologically important physical characteristics of the water body. To determine whether the potential impact of the operation of a proposed power plant will comply with applicable federal and state water quality standards, it becomes necessary to complete detailed studies that must consider the hydrodynamic, thermal, and water quality conditions of the water body in the geophysical flow region both without the effects of the power plant (generally referred to as ambient conditions) and with the effects of different operational conditions of the power plant. The results of these studies constitute an essential part of the environmental statement required for the licensing of the proposed power plant.

A large electric power generating station (or any large industrial plant) utilizes a lake, river, reservoir, estuary, or a coastal zone as the common water body for supplying cooling water to the intakes and for receiving the heated water from the discharges of its condenser cooling system. The circulation of large rates of cooling water through the

plant equipment, which includes intake screens, pumps and high temperature condenser pipes, can result in the destruction of large numbers of aquatic organisms due to impingement at the intake structures and due to passage through the plant condenser cooling system. Therefore, in an ecologically important water body, the entrainment and impingement impact of the operation of a large power plant, with a once-through condenser cooling system, can result in reductions in the populations of the early life stages of fish (e.g., egg, yolk-sac larva, post-yolk-sac larva, juveniles, etc.) and other aquatic biota (e.g., commercial crustacean species). To determine whether the potential entrainment and impingement impact of the operation of a proposed power plant will result in tolerable levels of reduction in the populations of the early life stages of the ecologically and commercially important fish and other aquatic biota, it also becomes necessary to complete studies that must consider the population density distributions in the water body in the geophysical flow region both without the effects of the power plant (generally referred to as the ambient population conditions) and with the effects of entrainment and impingement caused by the different operational conditions of the power plant. The results of these studies also constitue an essential part of the environmental statement required for the licencing of the proposed power plant.

The level of overall impact (i.e., consisting of the hydrodynamic, thermal, water quality, entrainment and impingement impacts) of the operation of a large electric power generating station (or any large industrial plant) on the aquatic ecology of a lake, river, reservoir, estuary or coastal zone depends critically on the flow conditions of the

receiving water body. Hence, the studies concerned with the assessment of the potential impact of the operation of a proposed power plant must be based on detailed and accurate considerations of the flow conditions of the water body in the geophysical flow region.

1.1. Discussion of the Flow Conditions in Meso-Scale Geophysical Flow Regions

A lake, river, reservoir, estuary, or coastal zone, that can be used as the common water body for supplying cooling water to the intakes and for receiving the heated water from the discharges of the condenser cooling system of a large electric power generating station (or any large industrial power plant), can be classified as a meso-scale geophysical flow region.

The wind-driven flow conditions in a large lake are controlled by approximately uniform, intermittent, or approximately periodic variations of the wind conditions above the water surface which depend on the temporal and spatial variations of the atmospheric conditions in the climatological region. Furthermore, the flow conditions in a large lake can be controlled by internally generated currents which are driven by water density gradients that result from the temporal and spatial variations of the water temperature conditions in the geophysical flow region.

The flow conditions in the main flow channel of a river are controlled by the intermittent runoff and drainage which depend on the temporally and spatially varying rainfall conditions on the drainage basin of the river in the climatological region. Furthermore, the flow

conditions in a section of a river, between controlled-flow dams, or in a reservoir, can be controlled by the intermittent or approximately periodic variations of the controlled-flow conditions at the dams which generally depend on the operational characteristics of the hydroelectric power generating units.

The flow conditions in the main flow channel of an estuary are usually controlled by multiharmonic tidal phenomena which usually overcome all other additional effects that influence the flow conditions during most stages of the tidal period and in most parts of the main flow channel in a nontidal river. Furthermore, the water density gradients which result from temporal and spatial variations of salinity (predominantly along the vertical direction) in the estuary could control the flow conditions in different layers in the main flow channel.

The flow conditions in a coastal zone are controlled by the multiharmonic tidal phenomena and by the geostrophic currents in the geophysical flow region. Futhermore, intermittent or approximately periodic variations of the wind conditions can also control the flow conditions in the surface layer and in subregions near the shorelines of the coastal zone.

Hence, a typical meso-scale geophysical flow region (e.g., a lake, river, reservoir, estuary, or coastal zone), that can serve as the common water body for supplying cooling water to the intakes and for receiving the heated water from the discharges of the condenser cooling systems of large electric power generating stations (or large industrial power plants), always exhibit transient natural flow conditions which are controlled by transient geophysical phenomena.

The operational conditions of the condenser cooling system of a large electric power generating station depends on the electric power output rate which changes intermittently or with approximately periodic variations, depending on the power demand conditions in the electrical grid network. Therefore, the flow conditions at the intakes and discharges of the different units of a large electric power generating station also vary intermittently or with approximately periodic variations, depending on the operational conditions of the pumps and the condensers in the plant cooling water system.

Hence, the flow conditions in a typical meso-scale geophysical flow region (e.g., a lake, river, reservoir, estuary, or a coastal zone) can also exhibit transient plant-induced flow conditions which result from the transient flow conditions at the intakes and the discharges of the operating electric power generating stations (or large industrial power plants).

The natural flow conditions always exhibit strong spatial variations, both in the horizontal plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) and in the vertical direction (in the general sense normal to the geophysical surface plane and the free-surface plane of the flow region), in a geophysical flow region which can serve as the common water body for supplying the water to the intakes and for reveiving the heated water from the discharges of the condenser cooling system of a large electric power generating station.

The wind-driven flow conditions in a large lake could vary significantly over the horizontal plane area of the lake, since the wind condi-

tions on the water surface could exhibit strong spatial variations in the climatological region. Furthermore, the effects of the Coriolis acceleration (force) could also result in multi-layer Ekman currents which produce large scale circulations which represent significant spatial variations in the flow conditions. The effect of the Coriolis acceleration (force), interacting with the wind stress on the water surface, also produces multi-layer Ekman currents which automatically represent significant vertical variations in the flow conditions. Furthermore, the flow conditions in a large lake can also exhibit significant spatial variations both in the horizontal plane and also in the vertical direction as the result of the internally generated currents caused by the temporal and spatial variations of the water temperature conditions in the geophysical flow region.

The flow conditions in a river could also vary significantly over the horizontal plane area of the main flow channel, depending on the spatial variations of the normal cross-sectional areas along the longitudinal direction. Furthermore, depending on the bottom shear stress conditions and the shear stress conditions along the banks of the main flow channel, the flow conditions could exhibit significant spatial variations, along the vertical and lateral directions in a large river or a reservoir.

The flow conditions in an estuary also vary significantly over the horizontal plane area of the main flow channel, depending on the spatial variations of the normal cross-sectional areas along the longitudinal direction, and more importantly, depending on the variations of the tidal conditions along the longitudinal direction. Furthermore, in

addition to effects of the bottom shear stress conditions and the shear stress conditions along the banks of the main flow channel, the effects of water density gradients, which result from the temporal and spatial variations of salinity.(predominantly along the vertical direction), cause multi-layer currents which result in significant spatial variations of the flow conditions in the vertical and lateral directions in a large estuary.

The flow conditions vary significantly over the horizontal plane area of a coastal zone due to the spatial variations of the tidal flow conditions and the geostrophic currents in the geophysical flow region. Furthermore, the wind-driven flow conditions could vary significantly over the horizontal plane area of the coastal zone, since the wind conditions on the water surface could exhibit strong spatial variations in the climatological region. In a large coastal zone, the effect of the Coriolis acceleration (force) could result in multi-layer Ekman currents which produce large scale circulations which automatically represent significant spatial variations in the flow conditions. The effect of the Coriolis acceleration (force), interacting with the wind stress on the water surface, also produces multi-layer Ekman flows which represent significant vertical variations in the coastal zone.

The plant-induced flow conditions automatically represent spatial variations both in the horizontal plane (in the general, sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) and also in the vertical direction (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region). The flow conditions at the intake of a large electric power

generating station (or a large industrical plant) can significantly influence the flow conditions particularly in the vicinity (near-field zone) of the intake structure. However, the influence of the intake flow conditions decrease rapidly outside of the near-field zone; and the natural flow conditions generally dominate the flow conditions in the far-field zone away from the immediate vicinity of the intake structure in a meso-scale geophysical flow region. Similarly, the flow conditions at the discharge of a large electric generating station (or a large industrial plant) dominate the flow conditions in the vicinity of the discharge (near-field zone). However, the influence of the discharge flow conditions decrease outside the near-field zone; and the natural flow conditions generally dominate the flow conditions in the far-field zone away from the vicinity of the discharge structure in a meso-scale geophysical flow region. The changes that occur in the flow fields, during the transition from the flow conditions dominated by the intake and discharge conditions in the near-field zone to the flow conditions dominated by the natural flow conditions in the far-field zone, introduce significant spatial variations in the plant-induced flow conditions in a meso-scale geophysical flow region.

Hence, considering the fact that the studies concerned with the assessment of the potential impact (i.e., consisting of hydrodynamic, thermal, water quality, entrainment and impingement impacts) of the operation of a large electric power generating station (or any large industrial plant) must be based on detailed and accurate information about the flow conditions of the water body which generally exhibits significant temporal and spatial variations, both in the horizontal

plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) and in the vertical direction (in the general sense, normal to the geophysical surface plane and the free-surface plane), it usually becomes necessary to utilize accurate and reliable fast-transient, three-dimensional hydrodynamic models for the simulation of the natural and plant-induced flow conditions in a meso-scale geophysical flow region.

1.2. Review of the Existing Multi-Dimensional Hydrodynamic Models for Meso-Scale Flow Regions

The applications of the numerical methods to the analysis of the flow conditions in geophysical flow regions were initiated for the simulation of the transient, one-dimensional hydrodynamic conditions in controlled rivers and reservoir systems. An excellent survey of these earlier numerical methods for transient open-channel flow conditions was completed by Miller (1971). The applications of the earlier one dimensional models to the simulation of the hydrodynamic, thermal, salinity, water quality and biological conditions in estuaries were also discussed in a survey report prepared for the assessment of the state-of-the-art in modeling of estuarine systems (Ward and Espey, 1971).

Advanced tidal-transient, one-dimensional numerical models for the simulation of hydrodynamic, thermal and salinity conditions in rivers and estuaries were developed by Thatcher and Harleman (1972), Harleman, Brocard and Najarian (1973), and Eraslan (1974). The ultimate fast-transient, one-dimensional discrete-element model for simultaneously predicting the hydrodynamic, thermal and salinity conditions in

controlled rivers and tidal estuaries was completed by Eraslan (1981). The model was applied to the simulation of the conditions in the Hudson River estuary for two six-month periods (1 April - 30 September) in 1973 and 1974. The computer simulation results of the mathematical model was in excellent agreement with the available field-measured data for flow, water temperature and salinity conditions along the Hudson River during the two six-month periods of simulations (Barnthouse, et al., 1977, and Eraslan, Erhan and Lin, 1980). The computer simulation results of the application of the model to the prediction of the hydrodynamic and water temperature conditions in the Conowing reservoir on the Susquehanna River were also in excellent agreement with the available field-measured data (Eraslan and Kim, 1978).

The application of the finite-difference method (FDM) to the simulation of tidal-transient two-dimensional hydrodynamic conditions in the horizontal plane (in the general sense parallel to the geophysical surface plane and the free-surface plane of the flow region) of shallow estuaries and coastal zones was first introduced by Dronkers (1964, 1969). Advanced tidal-transient, two-dimensional models based on the finite-difference method (FDM) for the simulation of hydrodynamic and water quality conditions in the horizontal plane of shallow estuaries and coastal zones were developed and successfully applied by Leendertse (1967, 1970) and Leendertse and Gritton (1971). Transient, two-dimensional models based on the finite-difference method (FDM) were also applied to the simulation of the flow and water temperature conditions in the vicinities of power plant discharges in shallow flow regions by Wada (1967a, 1967b, 1972) and by Codell (1973). A detailed survey of

the two-dimensional mathematical models for the simulation of the hydrodynamic and water temperature conditions in the vicinities of power plant discharge was completed in studies by Paddock, et al., (1973), Dunn, Policastro and Poddock (1975) and by Jirka, Abraham and Harleman (1975). A computer simulation model for predicting fast-transient, two-dimensional storm-surge flow conditions in coastal bay regions was developed by Reid and Bodine (1968). Recent applications of advanced, tidal-transient, two-dimensional models, based on the finite-difference method (FDM) to the prediction of the tidal flow conditions in the Lagoon of Venice were also presented by Sguazzero, et al., (1978).

Fast-transient, two-dimensional discrete-element hydrodynamic models were developed for simulating the tidal flow conditions in horizontal planes of estuaries and coastal zones in studies concerned with the assesment of thermal impact of power plant operations (Eraslan, 1976). These models were applied successfully for generating the necessary flow conditions as input data to the transient, two-dimensional discrete-element thermal transport models which were utilized for the assessment of the thermal impact of power plant operations in reservoirs, estuaries and coastal zones (Eraslan, 1973, 1975a, 1975b, Eraslan and Siman-Tov, 1974, Eraslan, et al., 1975, Eraslan, 1976, Eraslan, et al., 1976, Eraslan, Kim and Harris, 1977, Barnthouse, et al., 1977, Eraslan and Kim, 1978).

Transient, two-dimensional numerical models for simulating the flow conditions in the vertical plane (in the general sense normal to the geophysical surface plane and free-surface plane of the flow region) were developed for the studies concerned with the prediction of strati-

fied flow conditions in lakes, reservoirs and estuaries. The earlier multi-layer, finite-difference computational model developed by Norton, King and Orlob (1973) was successfully applied to the simulation of multi-layer flow conditions which were influenced by the density gradients in large, relatively deep stratified reservoirs.

A tidal-transient, two-dimensional model, based on finite-difference method (FDM), was developed by Hamilton (1975) for simulating the hydrodynamic and salinity conditions in the vertical plane of an estuary with approximately rectangular cross-sectional areas with varying width and depth only along the longitudinal direction. A more general, tidal-transient, two-dimensional model, based on finite-difference method (FDM), was developed by Blumberg (1975) for simulating the hydrodynamic and salinity conditions in the vertical plane of an estuary, by laterally averaging the flow conditions in the main flow channel. Hence, this model could approximately incorporate the effects of the actual nonrectangular contours of the cross-sectional areas of the main flow channel by considering varying width both in the lateral and vertical directions.

Both models developed by Hamilton (1975) and Blumberg (1975) utilized the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964) which effectively eliminated the effects of density variations on the transport of mass and momentum and considered only the effect of the density variation for modifying the gravity force along the vertical direction in the vertical component of the momentum equation. Furthermore, both models considered the assumption that pressure was strictly hydrostatic, and hence, effectively eliminated the

dynamic solution of the vertical component of the momentum equation. The eddy viscosity coefficients were defined as local quantities based on the local flow conditions; and the vertical eddy viscosity coefficient was modified to include the stratification effects based on the Richardson number (e.g., Schlicting, 1960).

Both models utilized a geometrical discretization technique based on staggered-grid systems, which considered the numerical solutions of the different flow variables at different locations. Both models evaluated the convective terms based on second-upwind-differencing interpolation technique (e.g., Roache, 1972). The transient, two-dimensional model developed by Hamilton (1975) utilized a pseudo explicit timewise-integration numerical scheme with modified semi-implicit formulation for the diffusion terms. The transient, two-dimensional model developed by Blumberg (1975) utilized an explicit timewise-integration numerical scheme based on three-level, centered-differencing algorithm.

Both transient, two-dimensional models developed by Hamilton (1975) and Blumberg (1975) could be classified as fast-transient models, since their timewise explicit numerical integration algorithms could directly incorporate the effect of the surface gravity wave in the solution of the flow conditions in the vertical plane. Consequently, the stability of the numerical solutions was dependent on the maximum allowable time step of the timewise-integration algorithm which was limited by Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972).

A transient, two-dimensional model, based on the finite-difference method (FDM), was developed by Roberts and Street, similar to the

Marker-and Cell (MAC) method, for simulating the hydrodynamic and water temperature conditions in the vertical plane of a reservoir. The model also considered the Boussinesq approximation (e.g., Schlichting, 1960, Birt, Stewart and Lightfoot, 1964) and the hydrostatic pressure assumption in the development of the governing mathematical equations. Hence, it also effectively eliminated the dynamic solution of the computational equation associated with the vertical component of momentum.

The geometrical discretization scheme of the model was according to a staggered grid system; and the solutions were obtained based on an explicit timewise-integration numerical method. Hence, the transient, two-dimensional model developed by Roberts and Street (1975), based on the finite-difference method (FDM), for simulating the hydrodynamic and water temperature conditions in the vertical plane of a reservoir was mathematically and computationally similar to the tidal-transient, two-dimensional models developed by Hamilton (1975) and Blumberg (1975), based on the finite-difference method, for simulating hydrodynamic and salinity conditions in the vertical plane of an estuary.

A transient, two-dimensional model, based on the finite-difference method, was also developed by Waldrop and Farmer (1976) for simulating the hydrodynamic and water temperature conditions in the vertical plane for the assessment of the thermal impact of submerged discharges in lakes, rivers and reservoirs. This model was developed as a simplified version of the transient, three-dimensional model (Waldrop and Farmer, 1976, and Waldrop and Tatom, 1976). The model did not consider the hydrostatic pressure assumption in the formulation of the vertical component of the momentum equation; and it included all the convective

transport terms and the viscous stress terms in the calculation of the local pressure.

The timewise-integration numerical scheme utilized an algorithm which considered two previous time levels to obtain the solution at the new time level, which was considerably different than the well established multi-time-level formulations used in explicit methods, (e.g., time-spliting methods, leapfrog method, etc., Roache, 1972). The computational algorithm obtained the pressure conditions at the grid points by numerically integrating the vertical momentum equation for pressure, downward from the free-surface of the water with known pressure conditions (zero relative pressure). The time rate of change of the vertical component of velocity was either neglected or assumed as known from the previous time step of calculations. Hence, the method effectively eliminated the dynamic solution of the vertical component of the momentum equation.

This transient, two-dimensional model developed by Waldrop and Tatom (1976), based on the finite-difference method (FDM), could also be classified as a fast-transient model, since its timewise explicit numerical integration algorithm could directly incorporate the effect of the surface gravity wave in the solution of the flow conditions in the vertical plane. Consequently, the stability of the numerical solutions had to be dependent on the maximum allowable time step of the timewise-integration algorithm which was limited by Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1976, Roache, 1972).

A transient, two-dimensional model was developed by Edinger and Buchak (1979), based on the finite-difference method (FDM), for simulating the hydrodynamic and water temperature conditions in the vertical plane of a relatively deep reservoir. This model also considered the Boussineq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964) and the hydrostatic pressure assumption in the development of the governing mathematical equations. Hence, it also effectively eliminated the dynamic solution of the vertical component of the momentum equation.

Although the model developed by Edinger and Buchak (1979) was mathematically similar to the previously discussed transient, two-dimensional models for simulating the hydrodynamic and water temperature conditions in the vertical plane (e.g., Roberts and Street, 1975, Waldrop and Farmer, 1976, and Waldrop and Tatom, 1976), it's timewise-integration numerical scheme was completely different than the ones utilized in the other models. The computational algorithm of the model utilized an implicit timewise-integration scheme for the solution of the water surface elevation at the top layer from an artificially constructed, discretized, hyperbolic form of equation, based on the substitution of the vertically integrated, finite-difference form of the horizontal component of the momentum equation in the incompressible, finite-difference form of the continuity equation for the top layer. Hence, the implicit timewise-integration scheme of the model eliminated the effect of the surface gravity wave in the solution of the flow conditions in the vertical plane. Consequently, the stability of the numerical solution was dependent on the maximum allowable time step of

the timewise-integration algorithm which was limited only by the Courant criteria (e.g., Roache, 1972) which was based on the flow velocity conditions in the vertical plane.

For flow conditions in a deep reservoir, with relatively slow flow conditions, this implicit timewise-integration scheme allowed for the use of significantly larger time steps, based on Courant stability criterion (e.g., Roache, 1972), than the maximum allowable time step of the explicit timewise-integration schemes, based on Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972). However, the implicit timewise-integration scheme also required the solution of a tri-diagonal matrix at every time step which required a significatly larger number of computational steps in the numerical solution algorithm in comparison to the number of computational steps required in the numerical solution algorithm based on an explicit timewise-integration scheme.

Furthermore, since the implicit timewise-integration scheme completely eliminated the effect of the surface gravity wave in the solution of the flow condition, the model could be classified as a fast-transient model that could simulate the important effect of the surface gravity wave in shallow flow regions with closed shoreline enclosure boundaries.

All previously discussed transient, two-dimensional models, based on the finite-difference method, for simulating the flow conditions in the vertical plane (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region) avoid the dynamic solution of the vertical component of momentum equation (e.g.,

Hamilton, 1975, Blumberg, 1975, Roberts and Street, 1975, Waldrop and Farmer, 1976, Waldrop and Tatom, 1976, Edinger and Buchak, 1979).

A transient, two-dimensional model (NUMAC) for simulating flow conditions in the vertical plane (in the general sense, normal to the geophysical surface plane and free-surface plane of the flow region) for the interaction of two fluids with different densities was developed by Slotta, et al., (1969) based on the Marker-and-Cell (MAC) method (Welch, et al., 1966).

A mathematically and computationally similar, transient, two-dimensional model (SUMMAC) for simulating the behaviour of shallow waves on sloping beaches was also developed by Chan and Street (1970) by extensively modifying and improving the mathematical formulation of the computational algorithm of the Marker-and-Cell (MAC) method (Welch, et al., 1966).

A transient, two-dimensional, computer simulation model Oak-Ridge-Simlified-Marker-and-Cell (ORSMAC) was developed by Parks and Cross (1979) as an extension of the Simplified-Marker-and-cell (SMAC model (Hirt and Shannon, 1968) for the simulation of the flow and water temperature conditions in the vicinities of submerged jet discharges.

The common numerical solution algorithm of all these models, developed by Slotta, et al., (1969), Chan and Street (1970) and Parks and Cross (1979), included the complete dynamic solution of the vertical component of the momentum equation, simultaneously with the dynamic solution of the horizontal component of the momentum equation. Hence, the numerical solution algorithms obtained simultaneously both the horizontal and vertical components of velocity from the two components

of the momentum equation. Consequently, the solution of the pressure conditions at every time step were obtained from a discretized form of an elliptic equation which was developed by the combination of the differentiated forms of the components of the momentum equation and the continuity equation for incompressible flow conditions, (e.g., Welch, et al., 1966). THe iterative solution of the finite-difference form of this elliptic equation for pressure conditions, which in a sense represented an artificial formulation from physical consideration with a compensatory (artificial) term for the mass conservation error, usually required more (in certain cases ten times more) computational time than required for the solutions of the vertical and horizontal components of velocity according to the explicit timewise-integration method of the computational algorithm.

These transient, two-dimensional, modified Marker-and-Cell (MAC) computer simulation models developed by Slotta, et al., (1969), Chan and Street (1970), and Parks and Cross (1979), could all be classified as fast-transient models, since their timewise-explicit numerical integration algorithms could directly incorporate the surface gravity wave effect in the solution of the flow conditions in the vertical plane. Therefore, the stability of the numerical solutions depended on the maximum allowable time step of the timewise-integration method which was limited by Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1976, Roache, 1972). Hence, from consideration of excessive computer times required for the iterative numerical solution of pressure at each time step of the computational algorithm, these models were impractical for relatively long-term simulations of

the transient hydrodynamic conditions in the vertical plane (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region) in an actual lake, river, reservoir, estuary or coastal zone.

Transient, three-dimensional hydrodynamic computer simulation models were developed by Liggett (1970), Hirt and Cook (1972) and Nichols and Hirt (1973) based on modified and improved extensions of the original Marker-and-Cell (MAC) method (Welch, et al., 1966).

The transient, three-dimensional hydrodynamic computer simulation model developed by Liggett (1970) utilized an explicit timewise integration numerical scheme for the solution of the two horizontal components of velocity from the two horizontal components of the momentum principle in the horizontal plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region). The vertical velocity components were obtained by applying the three-dimensional continuity equations to each computational cell, starting with the bottom cell and proceeding with each cell along the vertical direction. The method included the conventional hydrostatic pressure assumption and the additional assumption of imposed rigid lid conditions on the fluid surface, instead of the more realistic free-surface conditions. The pressure conditions were obtained from a discretized form of an elliptic equation which was developed by the combination of the differential forms of the components of the momentum equation and continuity equation for the incompressible flow conditions. The transient, three-dimensional hydrodynamic numerical model was applied only to a limited number of test cases to compare the results of the computer

simulations with the results of the approximate solutions for simple three-dimensional problems with geometrically simple boundary conditions.

The transient, three-dimensional computer simulation model developed by Hirt and Cook (1972) used a modified extension of the Simplified-Marker-and-Cell (SMAC) method (Hirt and Shannon, 1968) for the simulation of the flow conditions with density gradients over irregularly shaped objects. The model included the restrictive assumption of the imposed rigid-lid condition on the top layer instead of the more realistic free-surface condition for simulating the flow conditions in the flow regions.

The transient, three-dimensional, computer simulation model developed by Nichols and Hirt (1973) also used a modified extension of the Simplified-Marker-and-Cell (SMAC) method (Hirt and Shannon, 1968) for the simulations of flow conditions, with density gradients, over irregularly shaped objects. The general formulation of this transient, three-dimensional model was similar to the one developed by Hirt and Cook (1972); however, it considered the realistic assumption of free-surface condition for simulating the flow conditions in the flow regions.

An advanced, fast-transient, three-dimensional mathematical model, which utilized the staggered-mesh space-differencing scheme similar to Marker-and-Cell (MAC) method, was developed by Spraggs and Street (1975) for the simulation of thermally influenced hydrodynamic conditions in large scale impoundments. The model included the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964) which effectively reduced the governing mathematical system into an incom-

pressible flow formulation with the effect of the variation of water density included only as the body force term in the vertical component of the momentum equation.

The discretized formulations of the convective transport terms were based on second-order upwind-differencing technique (Fromm, 1970) which reduced the effect of the numerical dispersion in the computer simulations.

The development of subgrid-scale turbulent transport submodel was based on a formulation which utilized local grid-size characteristic length and velocity gradients for determining the eddy viscosity coefficients. The values of the eddy viscosity coefficients were determined based on the simulations of flow conditions in a large rectangular basin. The formulation considered the thermal eddy diffusivity coefficients as directly related to the eddy viscosity coefficient based on turbulent Prandtl number (Schlichting, 1960). The development of the subgrid-scale turbulent transport submodel did not include any modification for local stratification conditions which could substantially reduce the vertical component of turbulent transport in thermally stratified reservoirs.

The mathematical model developed by Spraggs and Street (1975) must also be classified as a fast-transient model, since its timewise-explicit numerical integration algorithm can incorporate the effect of the surface gravity wave in the solution of the flow conditions. Consequently, the stability of the numerical solutions must be dependent on the maximum allowable time step of the timewise-integration algorithm

which was limited by Courant, Fridrichs and Lewy (CFL) criterion (Courant, Friedrichs and Lewy, 1928,1967, Roache, 1972).

The aforementioned fast-transient three-dimensional models developed by Hirt and Cook (1972), Nichols and Hirt (1973) and Spraggs and Street (1975) considered all three components of the momentum equation for simultaneous solutions of all three components of velocity in the three-dimensional flow regions. Consequently, the solution of the pressure conditions at every time step were obtained from a discretized form of a three-dimensional elliptic equation which was developed by the combination of the differentiated forms of the three components of the momentum equation and the continuity equation for incompressible flow conditions, similar to the formulations developed earlier for two-dimensional methods (e.g., Welch, et al., 1966). The iterative solution of the discretized form of this elliptic equation, based on the finite-element method, by utilizing successive-over-relaxation (SOR) methods, at every time step usually required more (in certain cases ten times more) computational time than required for the solutions of the three components of velocity and water temperature according to the explicit time-wise integration methods of the general computational algorithms of the models. Hence, these three-dimensional models were generally considered to be impractical based on computer cost requirements in applications to actual site-specific problems for the simulation of flow conditions in actual lakes, rivers, reservoirs, estuaries and coastal zones for realistically long time periods.

An alternate formulation for transient, three-dimensional mathematical models for geophysical flow regions was first introduced by Simons

(1973). The development of the model was based on considerations of the two horizontal components of the momentum equation for the solution of the two horizontal components of velocity in each specified layer of the three-dimensional flow region. The vertical component of velocity was solved by spatially integrating the incompressible form of the continuity equation along the upward vertical direction from the location of the bottom of the flow region toward the free-surface. The model included the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Steward and Lightfoot, 1964) which effectively reduced the governing mathematical system into an incompressible flow formulation with the effect of the variation of water density included only as the body force term in the vertical component of the momentum equation. The pressure conditions were solved by spatially integrating the vertical component of the momentum equation with the hydrostatic pressure assumption, which discarded all the momentum transport terms, along the downward vertical direction, from the location of the free-surface toward the bottom of the flow region.

The particular formulation suggested by Simons (1973) represented in a sense, a transient two-dimensional multi-layer model, since the development of the mathematical model only considered two horizontal components of the momentum equation in each layer with moving boundaries. The hydrostatic pressure assumption effectively eliminated the effect of the vertical component of momentum in the solution of the three-dimensional velocity conditions in the flow region. Hence, the mathematical model was in a sense limited to the simulation of pseudo three-dimensional flow conditions where the vertical component of velocity and the

gradients of the vertical component of velocity would be generally negligible for predominantly two dimensional flow conditions in multiple horizontal layers.

The formulation of the turbulent transport submodel was based on constant eddy viscosity and thermal eddy diffusivity coefficients. The effect of thermal stratification conditions were included in the model by considering temperature-dependent water density and by considering reasonable modifications for reducing or increasing the vertical eddy viscosity coefficients and thermal eddy diffusivity coefficient according to the local stability conditions based on the vertical gradient of water density.

The alternate formulation for transient, three-dimensional mathematical model introduced by Simons (1973) was significantly superior to other fast-transient, three-dimensional mathematical models developed by Hirt and Cook (1972), Nichols and Hirt (1973) and Spraggs and Street (1975), based on considerations of computational efficiency of the solutions. The particular formulation proposed by Simons (1973) completely eliminated the requirement for the solution of the pressure conditions at every time step based on the solution of a discretized form of the three-dimensional elliptic equation by successive-over-relaxation (SOR) methods. Therefore, the mathematical model considered only (1) the sotion of two horizontal components of velocity from the two horizontal components of the momentum equation by a timewise-explicit, multi-time level numerical integration technique, similar to the leapfrog method (e.g., Roache, 1972), (2) the solution of the vertical component of velocity from an algebraic equation representing the discretized form of

the incompressible continuity equation, and (3) the solution of the pressure from an algebraic equation representing the discretized form of the vertical component of the momentum equation which was simplified based on the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964) and the hydrostatic pressure assumption.

Another important aspect of the transient, three-dimensional mathematical model developed by Simons (1973) was the consideration of two different time steps in the numerical integration method of its general computations algorithm. The solution of water-surface-elevation conditions was separated from the solution of the multi-layer values of the flow variables by considering in a sense horizontally two-dimensional mathematical models based on the integration of the two-components of the momentum equation along the vertical direction. The solution of the water-surface elevation conditions was obtained by utilizing a small time step of numerical integration which was limited by Courant, Friedrichs and Lewy (CFL) criterion (Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972) according to the speed of the surface gravity wave based on depth of the flow region. The flow conditions in the internal layers were obtained by numerically integrating the three-dimensional equations according to a relatively large time step which was limited only by the Courant criterion (e.g., Roache, 1972) based on the maximum speed of the two horizontal components of velocity, since the water-surface-elevation conditions were assumed to remain specified during the time period of the integration which corresponded to the large time step.

The transient, three-dimensional mathematical model developed by Simons (1973) can also be classified as a fast-transient model, since the two-dimensional formulation of the water-surface-elevation conditions can incorporate the surface gravity wave according to the time-wise-explicit numerical integration method used in its general computational algorithm.

The fundamental advantage of the mathematical model developed by Simons (1973) was its efficient computational algorithm. Since the flow conditions in the internal layers were solved only at intervals depending on the large time step which was limited only by the Courant criterion (e.g., Roache, 1972) based on the maximum speed of the two horizontal velocity components, the mathematical model could be utilized very efficiently in simulating thermally stratified flow conditions in deep meso-scale lakes and reservoirs which would require the consideration of numerous layers along the vertical direction. However, it must be also emphasized that the special computational algorithm of the model cannot be considered as a complete three-dimensional model, since the solutionsdoes not include entirely the three-dimensional flow conditions of the internal layers.

The transient, three-dimensional model developed by Simons (1973) was successfully applied to the simulations of the stratified flow conditions in Lake Ontario.

A fast-transient, three-dimensional mathematical model, similar to the one proposed by Simons (1973), was developed by Leendertse (1973) for simulating hydrodynamic and salinity conditions in estuaries and coastal zones.

The formulation of the model, based on the finite-difference method (FDM), considered vertically integrated forms of the two components of the momentum equation over specified fixed depths of specified layers. The water-surface layer was considered with the free surface. The development of the model included the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964) which effectively reduced the governing mathematical system into an incompressible flow formulation with the effect of the variation of the water density with salinity included only as the body force term in the vertical component of the momentum equation. The vertical component of the momentum equation was further simplified by considering the hydrostatic pressure assumption which discarded all the terms representing the contribution of momentum transport along the vertical direction.

The computational equations of the model were developed based on a special discretization technique, based on the finite-differenced method (FDM), which considered a staggered-grid system with different flow variables computed at different locations in the horizontal layers, similar to the finite-difference discretization schemes which were utilized earlier in the formulations of transient, two-dimensional models (Leendertse, 1967, 1971).

The discretized formulation of the convective transport terms were based on direct second-upwind-differencing technique (e.g., Roache, 1972) which could introduce numerical dispersion in the solutions of the flow variables.

The development of the subgrid-scale turbulent transport submodel was based on simplified formulations of the turbulent transport fluxes

along the two horizontal directions in terms of constant eddy viscosity coefficients and eddy mass diffusivity coefficients with possibly different, specified values for the two different horizontal directions. The eddy viscosity coefficients and eddy mass diffusivity coefficients for vertical turbulent transport fluxes included the effect of stratification based on the vertical density gradient according to Richardson number (e.g., Schlichting, 1960).

The solution algorithm of the fast-transient, three-dimensional model developed by Leendertse (1973) was fundamentally similar to the computational algorithm of the transient, three-dimensional model developed by Simons (1973). However, the computational algorithm of the model developed by Leendertse (1973) did not separate the numerical solution into two different numerical integration procedures with two different time steps which was utilized in the computational algorithm of the model developed by Simons (1973). Two horizontal components of velocity in each layer were solved by timewise-explicit numerical integration of the two horizontal components of the momentum equation, according to a composite, multi-time-level method. The vertical component of velocity was solved by spatially integrating the incompressible form of the continuity equation along upward vertical direction, from the bottom of the flow region toward the surface. The pressure conditions were solved by spatially integrating the simplified form of the vertical component of the momentum equation, with the hydrostatic assumption which discarded all the momentum transport terms, along downward vertical direction, from the location of the free surface toward the bottom of the flow region.

The mathematical model developed by Leendertse (1973) for estuaries and coastal zones must be classified as a fast-transient model, since its timewise-explicit numerical integration algorithm can incorporate the effect of the surface gravity wave in the solution of the flow conditions. Consequently, the stability of the numerical solutions must be dependent on the maximum allowable time step of the timewise-integration algorithm which is limitted by the Courant, Friedrichs and Lewy (CFL) criterion (Courant, Friedrichs and Lewy,1928, 1967, Roache, 1972).

The mathematical development of the fast-transient, three-dimensional model and the results of its application to tidal flow conditions in an actual estuary was also presented by Leendertse and Liu (1975).

A transient, three-diemnsional mathematical model, which is different in the formulation of the presure conditions than the models introduced by Simons (1973) and Leendertse (1973), was developed by Lick (1976) for simulating the hydrodynamic and thermal condition in the vicinities of thermal discharges.

The transient, three-dimensional mathematical model developed by Lick (1976) imposed the rigid-lid conditions on the water surface, which effectively eliminited the terms associated with the gradients of water-surface elevation in the horizontal components of the momentum equation. Hence, the general formulation of the model discarded the effect of the surface gravity wave in the solution of the flow conditions. According to the rigid-lid assumption, the water surface was treated as an impermeable plane with an unknown pressure distribution which depended on the flow conditions at the surface plane.

The development of the model included the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964), which effectively reduced the governing mathematical system into an incompressible flow formulation with the effect of the variation of water density included only as the body force term in the vertical component of the momentum equation . THe vertical component of the momentum equation was further simplified by considering the hydrostatic pressure approximation which discarded all the terms representing the contributions of the momentum transport.

The mathematical system for the solution of the pressure distribution on the rigid-lid plane of the water surface was formulated by considering the combination of vertically integrated components of the momentum equation and the incompressible form of the continuity equation which resulted in a two-dimensional, spatially-discretized form of the elliptic partial differential equation. The formulation of the elliptic equation for the pressure distribution on the rigid-lid plane of the water surface was similar to the formulations of the earlier two-dimensional models (e.g., Welch, et al., 1966).

The subgrid-scale turbulent transport submodel was developed based on constant eddy viscosity coefficients and thermal eddy diffusivity coefficients for the components of turbulent transport along the two horizontal directions with possibly different values for the constant values of the eddy viscosity coefficients and thermal eddy diffusivity coefficients along the two different horizontal directions. The eddy viscosity coefficients and thermal eddy diffusivity coefficients for the components of turbulent transport along the vertical direction were

modified by including the vertical gradient of water temperature which approximately represented the effect of the thermal stratification conditions in the vertical direction.

The geometrical discretization scheme of the three-dimensional model developed by Lick (1976) first utilized a linear transformation of the vertical z-coordinate which mapped a flow region with a sloping bottom condition into a rectangular computational region. The transformed equations were then spatially discretized according to a staggered-grid finite-difference method (FDM) after eliminating certain cross-derivative terms which included the spatial variations of depth in the formulation of the turbulent transport fluxes. The spatially-discretized forms of the computational equations were timewise integrated according to a single-step, two-time-level, forward-time algorithm for the solution of the two horizontal components of velocity from the two horizontal components of the momentum equation.

The pressure distribution on the rigid-lid plane of the water surface was determined at each time step by the solution of the spatially-discretized elliptic partial differential equation according to an alternating direction implicit (ADI) method. The pressure conditions below the surface layer were solved by spatially integrating the simplified form of the vertical component of the momentum equation, with the hydrostatic pressure assumption which discarded all momentum terms, along the transformed, downward vertical direction from the location of the rigid-lid plane of the water surface toward the location of bottom of the flow region.

The vertical component of velocity was solved by spatially integrating the incompressible form of the continuity equation along the transformed, upward vertical direction form the location of the bottom of the flow region toward the location of the water surface.

The transient, three-dimensional mathematical model developed by Lick (1976) is not a fast-transient model, since the particular formulation of its computational algorithm based on the rigid-lid assumption on the water surface eliminates the effect of the surface gravity wave in the solution of the flow conditions. Consequently, the stability of the numerical solution, in general, depends on the maximum allowable time step of the timewise-explicit integration algorithm which is only limited by the Courant criterion (e.g., Roache, 1972), based only on the maximum speed associated with the two horizontal components of velocity. For certain flow condition which are generated by strong densimetric effects in deep flow regions, the stability of the numerical solutions could again depend on a special type of Courant, Friedrichs, Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy 1928, 1967, Roache, 1972) based on the speed of internally generated densimetric gravity wave conditions.

Hence, the transient, three-dimensional model developed by Lick (1976) requires a much larger time step than the fast-transient, three-dimensional model developed by Leendertse (1973), for the timewise-explicit numerical integration of the two horizontal components of velocity in the flow region. However, the solution of the discretized form of the elliptic partial differential equation for the pressure distribution on the rigid-lid plane of the water surface introduced additional

computation time for solution of the flow conditions, even though the alternating-direction-implicit method (ADI) was generally stable for reasonably well behaved and directionally similar flow conditions.

Different versions of a fast-transient, three-dimensional mathematical model for simulating hydrodynamic and water temperature conditions in the vicinities of thermal discharges were developed by Waldrop and Tatom (1976) and Tatom and Smith (1979). The development of the model included the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964) which effectively reduced the governing mathematical system into an incompressible flow formulation with the effect of the variation of water density with temperature included only as the body force term in the vertical component of the momentum equation. The first version of the model (Waldrop and Tatom, 1976) further simplified the vertical component of the momentum equation by considering the hydrostatic pressure approximation which discarded all the terms representing the contributions of the momentum transport along the vertical direction. The second version of the model (Tatom and Smith, 1979) did not include the hydrostatic pressure approximation and included all the momentum transport terms in the formulation of the vertical component of the momentum equation.

The general mathematical formulation of the fast-transient, three-dimensional mathematical model (Waldrop and Tatom, 1976, and Tatom and Smith, 1979) was developed in generalized orthogonal curvilinear coordinates with the intention to allow for more accurate representations of the flow conditions in rivers with curved shoreline boundaries for the banks. However, the use of generalized orthogonal curvilinear

coordinates also introduced numerous cross-derivative terms in the formulation of the governing mathematical system. The repeated calculations of these terms in the computational algorithm resulted in excessive computer times for the solution of the flow conditions.

The development of the subgrid-scale turbulent transport submodel was based on the formulations of the turbulent transport fluxes along the two horizontal directions in terms of variable eddy viscosity coefficients and thermal eddy diffusivity coefficients. The eddy viscosity coefficients and thermal eddy diffusivity coefficients for the turbulent fluxes along the two horizontal directions were developed according to the same formulation which was based on the simplified mixing length hypothesis for open-channel flow conditions. The eddy viscosity coefficients and thermal eddy diffusity coefficients for the turbulent transport fluxes along two different directions were calculated as identical values according to a simplified formula based on the local vertical location and the local gradient of the magnitude of the horizontal components of velocity. Hence, the turbulent submodel included the isotropic turbulence assumption in the horizontal plane. The formulations of the eddy viscosity coefficients and thermal eddy diffusivity coefficients for the vertical components of turbulent transport were modified according to the local Richardson number (e.g., Schlichting, 1960) which included the effect of the local thermal stratification.

The discretized computational system of the fast-transient, three-dimensional model (Waldrop and Tatom, 1976, Tatom and Smith, 1979) was developed according to a non-staggered finite-difference method (FDM)

which solved for all the flow variables at the same computational points of the unequally spaced grid system. Considering that non-staggered spatial discretization schemes generally tend to cause instablities in timewise-explicit numerical integration methods, the computational algorithm of the model attemped to control the inherent unstable characteristics of the solutions by incorporating a three-time-level timewise-integration method which was based on evaluating the new values at the advanced time level based on two previous time-level values of the time-derivative of the solutions. The general stability criterion for this particular computational algorithm was not established in the development of the model.

The different versions of the transient, three-dimensional model developed by Waldrop and Tatom (1976) and Tatom and Smith (1979) must be classified as fast-transient mathematical models, since their timewise-explicit numerical integration algorithm can incorporate the effect of the surface gravity wave in the solution of the flow conditions. Consequently, the stability of the numerical solutions must be dependent on the maximum allowable time step of the timewise-integration algorithm which is limited by the Courant, Friedrichs and Lewy (CFL) criterion (Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972). However, it must be emphasized that the particular non-staggered spatial-discretization technique used in the different versions of the model could require more stringent criteria for the establishment of stable numerical solutions.

A fast-transient, three-dimensional model was recently developed by Blumberg and Mellor (1978), based on the finite-difference method (FDM),

for simulating hydrodynamic, thermal and salinity conditions in coastal zones.

The development of the model included the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964) which effectively reduced the governing mathematical system into an incompressible flow formulation with the effect of the variations of water density with temperature and salinity included only as a body force term in the vertical component of the momentum equation. The vertical component of the momentum equation was further simplified by considering the hydrostatic pressure approximation which discarded all the terms representing the contribution of momentum transport along the vertical direction.

The most important aspect of the fast-transient, three-dimensional mathematical model developed by Blumberg and Mellor (1978) was that the formulation of the turbulent transport submodel was based on a second-moment turbulence closure model which included the turbulent kinetic energy and turbulent macroscale (mixing length). The formulations of the horizontal and vertical components of the turbulent transport terms were based on the second-moment closure model proposed by Mellor and Yamada (1977). Hence, the mathematical model included two additional partial differntial equations for the solutions of the additional variables as turbulent kinetic energy and turbulent macroscale (mixing length).

Considering the simplifying assumptions already incorporated in the mathematical development of the model proposed by Blumberg and Mellor (1978), including (1) the Bousinessq approximation (Schlichting, 1960,

Bird, Stewart and Lightfoot, 1964) which effectively eliminated the effects of the variations of water density with temperature and salinity in the formulations of mass, momentum and energy transport terms except as a body force in the vertical direction, and (2) the hydrostatic pressure approximation which eliminated all the terms representing the contribution of the momentum transport in the vertical direction, it was not evident that the inclusion of a second-moment closure formulation for the turbulent transport submodel was indeed warranted for improving the predictive capabilities of the model in simulating the flow conditions in coastal zones.

The classical phenomenological turbulent transport submodels which are based on the assumption that the turbulence phenomena can be formulated based on local, mean-time-averaged flow conditions could be considered as deficient for coastal zones. The formulation of the turbulent transport submodel based on second-moment closure hypothesis definitively avoids the restrictive assumption that the turbulence phenomenon can be formulated based on local, mean-time-averaged flow conditions, since it incorporates the transport of the characteristic properties of turbulence, turbulent kinetic energy and turbulent macroscale (mixing length), based on a governing mathematical system which includes the turbulent shear production, convection and dissipation effects in the entire flow region. However, the additional partial differential equations for the solutions of the turbulent kinetic energy and turbulent macroscale (mixing length) requires additional parameters which cannot be specified or formulated accurately for general, transient, three-dimensional flow conditions. Hence, at pre-

sent, it can not be guaranteed that the predictive capabilities of a transient, three-dimensional model for coastal zones can be improved by the inclusion of a second-moment closure formulation for the turbulent transport model.

The discretized formulation of the fast-transient, three-dimensional model presented by Blumberg and Mellor (1978), in a sense utilized a finite-difference formulation in the horizontal plane and a transformed multi-layer formulation along the vertical direction which allowed for a high resolution vertical accuracy in shallow subregions, since the scaled vertical coordinate always considered the same specified number of layers independent of the local depth in the flow region.

The computational algorithm of the fast-transient, three-dimensional mathematical model developed by Blumberg and Mellor (1978) also considered two different time steps in the numerical integration, similar to the one proposed by Simons (1973). The solution of the water-surface elevation conditions were separated from the solution of the multi-layer values of the flow variables by considering a horizontally two dimensional mathematical model which considered only vertically averaged values of the two-horizontal components of velocity. The turbulent transport submodel for the two-dimensional mathematical model did not utilize the second-moment closure formulation; and it only included simple phenomenological formulations of the bottom shear stress based on the depth-averaged values of the two components of velocity in the horizontal plane. The solution of the two-dimensional model utilized a small time step for the timewise-explicit numerical integration method which was limited by Courant, Friedrichs and Lewy

(CFL) criterion (Courant, Friedrichs and Lewy, 1928,1967, Roache, 1972) according to the speed of the surface gravity wave based on the depth of the flow region. The flow conditions in the internal layers were solved from a three-dimensional mathematical model which incorporated the second-moment closure formulation for the turbulent transport submodel. The flow conditions in the internal layers were obtained by numerically integrating the discretized forms of the three-dimensional equations for (1) two horizontal components of velocity, (2) water temperature, (3) salinity, (4) turbulent kinetic energy, and (5) turbulent macroscale (mixing length), according to a relatively large time step which was limited only by the Courant criterion (e.g., Roache, 1972) based on the maximum speed of the two horizontal components of velocity, since the water-surface elevation conditions were assumed to remain specified during the time period of integration corresponding to the large time step.

The transient, three-dimensional mathematical model developed by Blumberg and Mellor (1978) can also be classified as a fast-transient model, since its two-dimensional formulation of the water-surface-elevation conditions can incorporate the surface gravity wave according to the timewise-explicit, leap-frog numerical integration method (e.g., Roache, 1972) used in its general computational algorithm.

The preliminary version of the associated computer code of the mathematical model developed by Blumberg and Mellor (1978) was applied to a limited number of problems for simulating the flow conditions in coastal and offshore zones, including the Middle Atlantic Bight under hypothetical wind, water temperature and salinity boundary conditions.

An excellent review of the computational models for predicting the flow conditions in reservoirs was recently completed by Johnson (1981). This report also included the computer simulation results of various laterally-averaged, two-dimensional and three-dimensional mathematical models to the prediction of the bottom-density-current problem in the Generalized Reservoir Hydrodynamics (GRH) flume at Waterways Experiment Station (WES). The results of this study clearly indicated that some of the well known three-dimensional models developed by Spraggs and Street (1975) and Waldrop and Tatom (1976) were unable to predict the actual flow conditions in the GRH flume for the bottom density current problem. Furthermore, the simulations with these three-dimensional models required excessive computer times which were well beyond the acceptable limits for practical computer applications to the prediction of hydrodynamic conditions in reservoirs.

All presently existing, transient, three-dimensional mathematical models (Liggett, 1970, Hirt and Cook, 1972, Nichols and Hirt, 1973, Simons, 1973, Leendertse, 1973, Spraggs and Street, 1975, Lick, 1976, Waldrop and Tatom, 1976, Tatom and Smith, 1979, and Blumberg and Mellor, 1978) utilize the finite-difference method (FDM) either directly or in modified forms for improving the vertical resolution (e.g., Lick, 1976 and Blumberg and Mellor, 1978). Hence, complex geometrical characteristics of the shoreline enclosure boundaries of the flow region cannot be considered accurately in these mathematical models since their practical applications require reasonably course finite-difference grid systems for improving the computer times required for realistic, real-time simulations of the flow conditions in meso-scale flow regions.

At present, application of the finite-element method (FEM) transient, three-dimensional problems in meso-scale flow regions is not practical from computational considerations, since the introduction of the additional cross-derivative terms associated with formulations in generalized curvilinear coordinates substantially increases the required computer times of the simulations of actual site-specific flow conditions.

Considering the mathematical and computational characteristics of the presently available, transient, three-dimensional models, it can be concluded that there does not exist a mathematical model, with its associated general-usage computer code, which can be applied universally for the prediction of the flow conditions in meso-scale flow regions, including lakes, rivers, reservoirs, estuaries and coastal zones, with geometrically complex shoreline boundaries.

1.3. Summary of the Special Features of the Transient, Three-dimensional Discrete-Element Mathematical Model

The fast-transient, three-dimensional discrete-element computational model represents an advanced state-of-the-art mathematical model which is universally applicable for accurate and reliable predictions of natural and plant-induced flow conditions in meso-scale flow regions, including lakes, rivers, reservoirs, estuaries and coastal zones, with geometrically complex shoreline boundaries.

The development of the model incorporates certain special features associated with the general mathematical formulation and the computational solution which are more advanced than the ones used in most other presently available transient, three-dimensional models (e.g., Liggett,

1970, Hirt and Cook, 1972, Nichols and Hirt, 1973, Simons, 1973, Leendertse, 1973, Spraggs and Street, 1975, Lick, 1976, Waldrop and Tatom, 1976, Tatom and Smith, 1979 and Blumberg and Mellor, 1978).

1. The mathematical model is based on the Discrete-Element Method (DEM) (e.g., Eraslan, 1973, 1975a, 1975b, Eraslan and Siman-Tov, 1974, Eraslan, et al., 1975, Eraslan, 1976, Eraslan, 1977, Eraslan and Kim, 1978) which represents a direct computer simulation technique specifically developed for numerical solutions of multi-dimensional laminar-turbulent free-surface flow problems in meso-scale geophysical flow regions with geometrically complex shoreline boundaries. The discrete-element method (DEM) has the capability to consider accurately the detailed geometrical characteristics of the shoreline boundaries in a variable-resolution rectangular grid system without requiring any significant increase in the complexity of the computational algorithm in comparison to the finite-difference method (FDM).

2. The mathematical formulation of the fast-transient, discrete-element model does not include the hydrostatic pressure assumption (e.g., Liggett, 1970, Simons, 1973, Leendertse, 1973, Lick, 1976, Blumberg and Mellor, 1978). Therefore, the mathematical formulation considers all the terms associated with the vertical momentum transport in the discrete-element equation for the vertical component of the momentum principle. Hence, the fast-transient discrete-element model has significantly superior capabilities than most of the presently available transient, three-dimensional models (e.g., Liggett, 1970, Simons, 1973, Leendertse, 1973, Lick, 1976, Blumberg and Mellor,

1978) for simulating flow conditions with realistically high vertical components of velocity which occur in the vicinities of vertical intakes and discharges.

3. The mathematical formulation of the fast-transient, discrete-element model includes all the terms associated with the Coriolis acceleration (force) in a generalized Cartesian coordinate system which allows for any specified orientation of the selected discrete-element grid system over the flow region. The general formulation includes all the terms associated with the Coriolis acceleration (force) in meso-scale flow regions in all three discrete-element equations for the components of the momentum principle. Hence, the fast-transient, discrete-element mathematical model can realistically simulate all the phenomena associated with the Coriolis acceleration (force) effects in geophysical flow regions, including vertical upwelling conditions in large lakes and coastal zones.
4. The mathematical development of the turbulent transport submodel is based on a phenomenological formulation which does not require the specification of any eddy viscosity coefficients or eddy diffusivity coefficients. Hence, the fast-transient, three-dimensional discrete-element model has the capability to simulate flow conditions in all geophysical flow regions without requiring the specification of any site-specific values for eddy viscosity coefficients or eddy diffusivity coefficients. Futhermore, the model has the capability to simulate flow conditions in laboratory-model-scale flow regions again without requiring the specification of any site-specific values for eddy viscosity coefficients or eddy diffusivity coefficients for the

subgrid-scale (SGS) turbulence phenomena. The special mathematical formulation of the turbulent transport submodel is also applicable to computer simulations for reduced two- or one-dimensional flow conditions without requiring any special modification. Hence, the fast-transient discrete-element mathematical model is superior to most other transient three-dimensional models (e.g., Liggett, 1970, Hirt and Cook, 1972, Nichols and Hirt, 1973, Simons, 1973, Leendertse, 1973, Spraggs and Street, 1975, Lick, 1976, Waldrop and Tatom, 1976 and Tatom and Smith, 1979) in formulating the three-dimensional turbulent transport phenomenon in meso-scale flow regions.

5. The computational algorithm of the fast-transient, three-dimensional discrete-element mathematical model utilizes a composite, multi-time-step, timewise-explicit, numerical integration method. The numerical integration of the convective transport terms are formulated according to a single-time-step, two-time-level algorithm, with second-order, transportive-upwind-differencing which minimizes the numerical dispersion effects (including artificial viscosity). The numerical integration of the nonconvective transport terms are formulated according to a two-time-step, three-time-level algorithm which garantees the stability of the contribution of the subgrid-scale (SGS) eddy-turbulent transport terms to the solutions of the flow conditions.

Furthermore, the special geometrical discretization algorithm of the mathematical model, which utilizes twice as many locations for the calculations of the horizontal components of velocity as for the calculations of pressure in the flow region, guarantees the stability

of the numerical solutions provided that the maximum allowable time step of numerical integration satisfies the Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972). Therefore, the transient, three-dimensional discrete-element model is a fast-transient model, since its timewise-explicit numerical integration algorithm can incorporate the effect of the surface gravity wave in the solution of the flow conditions.

6. The computational algorithm of the fast-transient discrete-element mathematical model utilizes the two horizontal components of the momentum principle for the solution of the two horizontal components of velocity according to a special timewise-explicit, composite, multi-time-step numerical integration method. The vertical component of velocity is obtained by algebraically integrating the discrete-element form of the conservation of mass principle for water along the vertical direction. The pressure conditions are obtained by algebraically integrating the discrete-element form of the vertical component of the momentum principle along the downward vertical direction. Therefore, the computational algorithm of the fast-transient, three-dimensional discrete-element mathematical model does not require the solution of the discretized form of an elliptic equation for pressure at every time level. Hence, the computational algorithm of the fast-transient, three-dimensional discrete-element mathematical model requires substantially less computational effort for the solution of the flow conditions in comparison to the computational algorithms of various other transient, three-dimensional mathematical

models (e.g., Hirt and Cook, 1972, Nichols and Hirt, 1973, Spraggs and Street, 1976).

7. The special mathematical development of the fast-transient, three-dimensional discrete-element model and its computational algorithm has a unique capability, which is different than all other existing three-dimensional models (e.g., Liggett, 1970, Hirt and Cook, 1972, Nichols and Hirt, 1973, Simons, 1973, Leendertse, 1973, Spraggs and Street, 1975, Lick, 1976, Waldrop and Tatom, 1976, Tatom and Smith, 1979 and Blumberg and Mellor, 1978). The general computational algorithm of the fast-transient, three-dimensional discrete-element model automatically allows for applications in reduced, two- or one-dimensional forms to flow regions without requiring any modification whatsoever in the general structure of its computer code. This special feature of the model is also important from computational cost considerations, since it can be used readily for incorporating the special, computationally efficient numerical integration scheme which utilizes two different time steps, one for the solution of the water-surface-elevation conditions and another one for the solution of the flow conditions in the internal layers (e.g., Simons, 1973 and Blumberg and Mellor, 1978).

Hence, the fast-transient, three-dimensional discrete-element mathematical model and its associated computer code can be considered as the state-of-the-art model for predicting hydrodynamic conditions in meso-scale flow regions.

2. GENERAL DEVELPOMENT OF THE DISCRETE-ELEMENT METHOD FOR TRANSIENT THREE-DIMENSIONAL FLOWS

The discrete-element method represents a direct computer simulation technique which was specifically developed for numerical solutions of transient, multi-dimensional, laminar-turbulent, free-surface flow problems in meso-scale geophysical regions with geometrically complex boundaries, including lakes, rivers, reservoirs, estuaries and costal zones. The method is called the "discrete-element method" (DEM) because the development of the mathematical system of computational equations for the solution of the flow variables is based on the application of integral forms of the physical principles directly to "discrete elements" with finite volumes and finite enclosure surfaces. The developement of the mathematical system in no way utilizes the continuum limit, partial differential forms of conventional mathematical physics which represent the infinitesimal limit representations of the physical principles.

2.1. Discussion of the Discrete-Element Method

It is well established that any idealized class of partial differential equations (PDE)s associated with transport phenomena, can be formulated into mathematically identical discretized systems of computational equations by applying either the finite-difference method (FDM) or the finite-element method (FEM), provided the temporal and spatial discretization procedures of the two different methods are based on identical (or equivalent) orders of interpolation approximations. However, in applications to actual, non-idealized, transient, multi-dimen-

sional flow problems, associated with transport phenomena, the computational characteristics of a computer program based on the finite-difference method (FDM) can be vastly different than the computational characteristics of a computer program based on the finite-element method (FEM). Similarly, it is expected that the application of the discrete-element method (DEM) to any idealized class of integral forms of the physical principles, associated with transport phenomena, could reproduce a discretized system of computational equations which could also be obtained by the application of either the finite-difference method (FDM) or the finite-element method (FEM) to the corresponding idealized class of partial differential equations (PDE)s that represent the same physical principles. However, again, in applications to actual non-idealized, transient, multi-dimensional flow problems, associated with transport phenomena, the computational characteristics of a computer program based on the discrete-element method (DEM) can be vastly different than (generally superior to) the computational characteristics of a computer program based on the finite-difference method (FDM) or the finite-element method (FEM). Hence, possible establishement of the mathematical equivalence of the forms of the final discretized systems of computational equations, obtained by the applications of the finite-difference method (FDM), the finite-element method (FEM), and the discrete-element method (DEM) to the same idealized class of flow problem does not conclude the establishment of the equivalence of the computational characteristics of the three numerical methods in application to actual, general classes of flow problems associated with all transport phenomena.

The general discrete-element method (DEM) is conceptually, mathematically and computationally different than the finite-difference method (FDM) and the finite-element method (FEM) in applications to transient, multi-dimensional problems associated with incompressible or compressible, laminar-turbulent flow and convective heat transfer problems in regions with geometrically complex boundries. However, the discrete-element method (DEM) includes certain particular mathematical properties and computational features which are similar to the corresponding ones associated with the finite-difference method (FDM) and the finite-element method (FEM). These important differences and similarities are discussed separately in order to illustrate the unique conceptual, mathematical and computational characteristics of the discrete-element method (DEM).

1. The development of the discrete-element method (DEM) is based on the generalized control volume approach (Eraslan, 1975a, 1975b) which highlights the direct computational discretization process that does not require the consideration of continuum limit partial differential equations forms of the physical principles.

Hence, the discrete-element method is conceptually similar to earlier direct numerical simulation methods developed at Los alamos Scientific Laboratory including Particle-in-Cell (PIC) method by Evans and Harlow (1957), Marker-and-Cell (MAC) method by Harlow and Welch (1965,1966) and Welch, et al., (1966), and particularly, Fluid-in-Cell (FLIC) method by Gentry, Martin and Daly (1966). The details of the development of these earlier "direct numerical simulation methods" and their numerous successful applications were presented in reports by

Harlow (1969), Amsden and Harlow (1971) and in a text on computational fluid mechanics by Roache (1972).

The "segment" or "box" type of formulations of the water-quality models suggested by Thomann (1963) and O'Connor (1966) are also conceptually similar to the discrete-element method (DEM) and the Fluid-in-Cell (FLIC) method (Gentry, Martin and Daly, 1966); and they are extremely efficient from computational considerations for the numerical solutions of the constituent mass conservation equations in large flow regions of estuaries and coastal bays with specified input data for the flow conditions.

The development of the discrete-element method (DEM) utilizes directly the integral forms of the physical principles in formulating the discretized system of computational equations, by considering discrete elements and half-elements. The infinitesimal-limit, continuum partial-differential-equation forms of the physical principles are in no way utilized in the formulation. Conversely, the finite-difference method (FDM) and the finite-element method (FEM) start the development of the discretized system of computational equations from the infinitesimal-limit, continuum partial-differential-equation forms of the physical principles. Hence, the discrete-element method (DEM) is conceptually different than the finite-difference method (FDM) and the finite-element method (FEM) based on the considerations of the development of the discretized systems of computational equations that represent the physical principles.

2. The finite-difference method (FDM) and the finite-element method (FEM) consider the flow variables as continuous functions in certain,

specified domains of existence of the solutions. The numerical solutions for the flow variables are obtained at certain specified solutions locations in space (e,g, grid points, nodal points etc.) where the discrete numerical solution values represent the values of the continuous functions for the flow variables at the specified solution locations in space. Hence, the numerical solutions of the finite-difference method (FDM) and the finite-element method (FEM) represent the values of the flow variables at distinct, specified locations in space (e.g., grid points, nodal points, etc.), corresponding to the values of the continuous functions which represent, based on assumption, the flow variables in the specified domains of existence of the solutions.

Since the discrete-element method (DEM) utilizes the contral volume approach, it considers the spatially averaged values of the flow variables over the volumes and the enclosure surface areas of the elements and half-elements. Therefore, the numerical solution obtained from the discretized system of computational equations of the discrete-element method (DEM) correspond to the values of the flow variables that represent the spatially averaged values of the flow variables over the parts of the flow region contained in the elements and half-elements. The numerical solutions of the discrete-element method (DEM) do not represent the values of the flow variables at certian specified locations in space (e.g., grid points, nodal points, etc.); but they represent the spatially averaged values of the flow variables in the volumes of the elements and half-elements with specified locations in space. This spatially averaged property of the numerical solutions of the flow variables, according to the formulation of the discrete-element method (DEM)

is also common to the formulations of the Fluid-in-Cell (FLIC) method (Gentry, Martin and Daly, 1966) and the segment or box type of formulations of the water-quality models (e.g., Thomann, 1963, and O'Connor, 1966).

Hence, the discrete-element method (DEM), which considers the solutions of the flow variables as spatially-averaged values over the volumes of the elements and half-elements, is conceptually and computationally different than the finite-difference method (FDM) and the finite-element method (FEM) which consider the solutions of the flow variables at specified locations in space (e.g., grid points, nodal points, etc.).

3. The geometrical discretization technique of the discrete-element method (DEM) considers each interior element in the flow region as a complete rectangular cell with six rectangular enclosure surface areas which are always normal to one of the three orthogonal axes x, y, or z of a specified Cartesian coordinate system. Therefore, from considerations of the geometrical discretization techniques, the discrete-element method (DEM) is similar to the finite-difference method (FDM) in the formulation of the discretized system of computational equations for the interior elements.

The geometrical discretization technique of the discrete-element method (DEM) retains the actual geometrical characteristics of the sections of the enclosure boundaries of the flow region in the boundary elements. Therefore, the discrete-element method (DEM) automatically has a spatial resolution accuracy comparable (generally superior) to the finite-element method (FEM) and significantly superior to the finite-

difference method (FDM) in the formulation of the discretized system of computational equations for the boundary elements.

Hence, from considerations of the geometrical discretization techniques, the discrete-element method (DEM) represents a geometrically hybrid method which can selectively incorporate the computationally efficient part of the geometrical discretization technique of the finite-difference method (FDM) and also the spatially high-resolution part of the geometrical discretization technique of the finite-element method (FEM).

4. The integral forms of the physical principles, associated with the general transport phenomena are formulated in terms of the set of "fundamental flow variables" which include (1) the "principal flow variables" (e.g., velocity components and pressure in the fluid), and (2) "the auxiliary flow variables" (e.g., components of the stress tensor in the fluid). The mathematical system associated with the integral forms of the physical principles does not constitute a mathematically consistent system, since it does not include the necessary closure submodels (laminar or turbulent), consisting of constitutive laws and subsidiary equations, which eliminate the "auxiliary flow variables" (e.g., components of the stress tensor in the fluid) in terms of the "principal flow variables" (e.g., velocity components and pressure in the fluid).

The conventional development of the discretized systems of computational equations of the finite-difference method (FDM) and the finite-element method (FEM) are based on the infinitesimal-limits, continuum partial-differential-equation forms of the physical principles which are formulated (in the most general form) in terms of a set of "principal

flow variables" (e.g., velocity components and pressure in the fluid), which includes the "primitive variables" for the hydrodynamic conditions (e.g., Roache, 1972). The "auxiliary flow variables" (e.g., components of the stress tensor in the fluid) are always eliminated in terms of the "principal flow variables" (e.g., velocity components and pressure in the fluid), based on a selected closure submodel (laminar or turbulent), before the development of the discretized system of computational equations of the finite-difference method (FDM) or the finite-element method (FEM) from the infinitesimal-limits, continuum partial-differential-equation forms of the physical principles. Therefore, the formulation of the discretized system of computational equations of the finite-difference method (FDM) and the finite-element method (FEM) depend on the specific closure submodel (laminar or turbulent) which is used in the elimination of the "auxiliary flow variables" (e.g., components of the stress tensor in the fluid), in terms of the "principal flow variables" (e.g., velocity components and pressure in the fluid).

Conversely, the development of the discretized system of computational equations of the discrete-element method (DEM) is based on the integral form of the physical principles which are formulated in terms of the set of "fundamental flow variables", without considering any closure submodel (laminar or turbulent) for eliminating the "auxiliary flow variables" (e.g., components of the stress tensor in the fluid) in terms of the "principal flow variables" (e.g., velocity components and pressure in the fluid). Therefore, the formulation of the discretized system of computational equations of the discrete-element method (DEM),

in terms of the complete set of the "fundamental flow variables", is independent of any closure submodel (laminar or turbulent) which can be utilized subsequently in discretized form for the elimination of the auxilary flow variables" (e.g., components of the stress tensor in the fluid) in terms of the "principal flow variables" (e.g., velocity components and pressure in the fluid). Consequently, the discretized system of the computational equations of the discrete-element method (DEM) can be utilized with any realistic closure submodel (laminar or turbulent) in contrast to the discretized systems of computational equations of the finite-difference method (FDM) and the finite-element method (FEM) which are limited by the particular closure submodel (laminar or turbulent) which is included in the conventional, infinitesimal-limits, continuum partial-differential-equation forms of the physical principles.

Hence, the mathematical formulation of the discrete-element method (DEM) is distinctly different than the mathematical formulations of the finite-difference method (FDM) and finite-element method (FEM) in the development of the discretized system of computational equations associated with general transport phenomena.

5. The discrete-element method rigorously incorporates the subgrid-scale (SGS) variations of the flow variables, based on careful considerations of the temporal and spatial scales of the flow conditions, in the development of the spatially discretized governing system of equations.

The standard formulations of the finite-difference-method (FDM) and the finite-element-method (FEM) automatically assume the particular form

of the nonconvective transport submodel (laminar or turbulent) associated with the subgrid-scale transport phenomena either as a local phenomenological submodel or a higher-order-moment closure submodel before the discretization of the governing system of partial differential equations (PDE)s.

The development of the discrete-element method (DEM) assumes completely arbitrary temporal and spatial variations of the flow variables in terms of (1) temporally primary-scale and spatially primary-scale variations, which in a sense represent the grid-scale variations, and (2) temporally subscale and spatially subscale variations, which in a sense represent the subgrid-scale (SGS) variations. Mathematically rigorous integration of the control surface integrals of the integral forms of the physical principles over the enclosure-surface areas of the elements and half-elements, by considering the temporally subscale and spatially subscale variations of the flow variables, results in the development of explicit terms for the convection-defect transport rates and for the eddy-turbulent transport rates which constitute the nonconvective transport rates. Hence, the formulation of the discrete-element method explicitly identifies the convection-defect transport terms, which result from the spatially subscale variations of the flow variables over the grid-scale enclosure-surface areas of the specified grid system, separatly from the eddy-turbulent transport terms which results from the temporally subscale and spatially subscale eddy-turbulent fluctuations of the flow variables. Consequently, the discrete-element method (DEM) can incorporate higher-order spatial discretization approximations (e.g., Arakawa, 1966, also Roache, 1972) by subsequently

formulating the explicitly identified convection-defect transport terms in the general development of three-dimensional mathematical models.

The mathematically rigorous formulation of the subgrid-scale (SGS) transport phenomena is particularly important in the applications of the discrete-element method (DEM) to the development of models in reduced dimensions, including one or two-dimensional depth-averaged or laterally averaged transport models for meso-scale flow regions. The explicit formulations of the convection-defect transport terms can readily incorporate the effects of the variations of the flow variables along the neglected secondary dimension(s) on the transport along the retained primary direction(s). This unique capability of the mathematical formulation of the discrete-element method (DEM) (e.g., Eraslan, 1981a) completely eliminates the need for specifying site-specific values and/or spatial variations of the questionable but commonly employed "dispersion coefficients".

Hence, the discrete-element method (DEM) is both conceptually and mathematically different and significantly more advanced than the finite-difference method (FDM) and the finite-element method (FEM) in formulating the nonconvective transport rates associated with the subgrid-scale (SGS) transport phenomena.

6. The applications of the finite-difference method (FDM) and the finite-element method (FEM) consider the infinitesimal-limits, continuum partial differential equations (PDE)s which can be formulated either in more commonly used conservative forms or in seldom used nonconservative forms. Certain special discretization techniques used by the finite-difference method (FDM) and the finite-element method (FEM) can lead to

discretized computational systems of equations which do not conserve the mass of the water in the flow region (e.g., discussions in Leendertse, 1971, Roache, 1972), depending on the particular forms used in the discretization of the convective transport terms for the mass of water.

According to the general formulation of the discrete-element method (DEM), the rigorous integration of the control surface integral of the integral form of the conservation principle for water mass, over the enclosure surface areas of the elements, guarantees that the final computational forms of the discrete-element equations conserve the mass of water in every element and also in the entire flow region.

7. In the applications of the finite-difference method (FDM) and the finite-element method (FEM), the commonly employed conservative forms of partial differential equations (PDE)s, associated with the transport of momentum are spatially discretized into "conservative forms" of computational equations which in a sense calculate the extensive forms of the flow variables by including the mass of water in the computational values. Hence, in the numerical solutions of the intensive flow variables, including the components of velocity, water temperature and water quality conditions, the computational errors (e.g., truncation errors, numerical dispersion errors) that occur in mass conservation of water directly introduce computational errors in the solutions of all flow variables.

The general mathematical formulation of the discrete-element method (DEM) is based on a unique "intensive form" (e.g., Eraslan, 1975a, 1976, 1977, 1981a, 1981b) of computational discrete-element equations for all the intensive flow variables, including the components of velocity,

water temperature and water quality conditions. The "intensive form" of development of the computational system of discrete-element equations minimizes the effect of the computational error in mass conservation of water on the solutions of the intensive flow variables.

Hence, the discrete-element method (DEM) is distinctly different than the finite-difference method (FDM) and the finite-element method (FEM) in formulating the governing system of computational equations for the solution of the flow variables.

8. In applications of the finite-difference method (FDM) to the development of timewise-explicit discretized computational system of equations for the numerical solution of transient, three-dimensional flow conditions, it is usually required to consider staggered grid systems where different flow variables are computed at different locations in the flow region (e.g., Liggett, 1970 , Hirt and Cook, 1972, Nichols and Hirt, 1973, Leendertse, 1973, Spraggs and Street, 1975, Blumberg and Mellor, 1978, Welch, et al., 1966, Roache, 1972). The formulation of the computational equations for the three-dimensional flow conditions in a staggered grid system always requires the specification of the velocity conditions on the shoreline boundary enclosure surfaces based on the impermeable wall boundary hypothesis for the normal velocity components and the hypothesis of nonslip velocity conditions for the tangential velocity components. The formulation of the shear stress conditions on the shoreline boundary enclosure surfaces requires the extrapolation of the slope of the velocity conditions which becomes extremely questionable for turbulent and/or laminar subgrid-scale transport phenomena near the wall. Same problem also exists for the formula-

tion of the shear stress conditions on the shoreline boundary enclosure surfaces in the applications of the finite-element method (FEM) to the development of timewise-explicit, spatially discretized computational equations. The specification of the slopes of the velocity components at the shoreline boundary enclosure surfaces, based on extrapolation of the velocity components at the interior computational points in the flow region for evaluating the shear stress conditions on the shoreline boundary enclosure surfaces, usually introduces physically unrealistic instability conditions in the solutions of the flow variables due to the improper modeling of the subgrid-scale (SGS), turbulent and/or laminar phenomena near the solid boundaries in the spatial resolution of the grid scale of the discretized computational system. Hence, in applications of the finite-difference method (FDM) and the finite-element method (FEM) to the solution of flow conditions, it becomes necessary to consider different formulations for the shear stress conditions at the shoreline boundary enclosure surfaces than the ones used for the shear stress conditions in the interior of the flow region.

The general formulation of the discrete-element method (FDM) specifically excludes the consideration of the flow variables at the shoreline boundary enclosure surfaces, since all the computational locations for the principal flow variables are considered at locations interior to the flow region. The shear stress conditions on the shoreline boundary enclosure surfaces are formulated automatically as one of the auxilary flow variables of the discrete-element equations based on appropriate assumptions about the subgrid-scale (SGS) turbulent phenomena in the flow region.

Hence, the discrete-element method (DEM) is different than the finite-difference method (FDM) and the finite-element method (FEM) in formulating the shear stress conditions on the shoreline boundary enclosure surfaces of the flow region.

9. The applications of the spatial discretization techniques used in the finite-difference method (FDM) and the finite-element method (FEM) on the partial differential equations (PDE)s which represent the physical principles result in computational systems of equations which numerically solve for the components of the velocity at approximately the same number of computational locations as for the other principal flow variables, including the pressure.

The spatial discretization technique of the discrete-element method (DEM), considers elements and half-elements in the formulations of the discrete-element forms of the physical principles. Therefore, the computational system of discrete-element equations solves for the components of velocity at approximately twice the number of computational locations as for other principal flow variables, including pressure. This unique feature of the discrete-element method (DEM) allows for accurate and high resolution solutions of the components of velocity in the flow region, with extremely stable characteristics, equal or better than the staggered grid formulations associated with the finite-difference method (FDM).

Hence, the discrete-element method (FDM) is significantly different than the finite-difference method (FDM) and the finite-element method (FEM) in formulating the spatially discretized, governing system of computational discrete-element equations for the solutions of the prin-

cipal flow variables, particularly the components of the velocity in the flow region.

10. The formulations of the governing system of discretized computational equations with timewise-explicit computational algorithms, based on the finite-difference method (FDM), generally utilize higher-order upwind-differencing interpolation techniques which tend to reduce the numerical dispersion in the solutions of the flow variables (e.g., Fromm, 1971, Roache, 1972, Spraggs and Street, 1975).

The general computational algorithm of the discrete-element method (DEM) includes the capability to incorporate all first-order or higher order upwind-differencing interpolation techniques for minimizing the numerical dispersion introduced by the timewise-explicit numerical integration of the discrete-element equations. The general formulation of the discrete-element method (DEM) is not restricted to any one particular upwind-differencing interpolation technique which is reguired for the stability of the timewise-explicit numerical integration method.

Hence, the discrete-element method (DEM) has all the necessary capabilities for incorporating the higher-order upwind-differencing interpolation techniques which are commonly used in the timewise-explicit formulations of the finite-difference method (FDM) and the finite-element method (FEM).

11. The most important feature of the discrete-element method (DEM) is that the governing computational system of discrete-element equations are in a sense computationally equivalent to the finite-difference method (FDM) in all the interior computational locations. Since, the discrete-element equations, which are formulated in the rectangular

coordinate system, similar to the discretized computational equations of the finite-difference method (FDM), do not contain any additional term which could result from discretized formulations in curvilinear coordinates, the computational solutions of the principal flow variables require substantially less storage and computer time in comparison to the storage and computer time required for the solutions of the flow variables according to a discretized system of computational equations with approximately the same spatial resolution based on the finite-element method (FEM).

Furthermore, since the development of the discrete-element equations retains the actual geometrical characteristics of the shoreline boundary enclosure surfaces, the numerical solutions of the principal flow variables in the boundary elements have the same computational accuracy or better than the numerical solutions that can be obtained from a discretized system of computational equations with approximately same spatial resolution based on the finite-delement method (FEM).

Considering the special geometrical discretization technique associated with its mathematically rigorous development based on the integral forms of the physical principles and the special features of its general computational algorithm, it can be concluded that the discrete-element method (DEM) is superior to both the finite-difference method (FDM) and the finite-element method (FEM) for the solution of fast-transient, three-dimensional (also two- and one-dimensional) flow conditions in meso-scale flow regions with geometrically complex shoreline boundaries.

2.2. Integral Forms of Physical Principles

The development of the fast-transient, three-dimensional hydrodynamic model is based on the application of the discrete-element method (DEM) which directly employs the integral forms of statements of the physical principles (e.g., Shames, 1962, Eraslan, 1975) for the fluid contained in a control volume. The fluid is considered as water which represents approximately a binary mixture consisting of mainly water and dilute concentrations of salt (including various chemical components found in sea water). The integral forms of the two physical principles for the water contained in a control volume are:

1. mass conservation principle for water,

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} \rho dv + \iint_{A_{cv}} \rho \vec{V} \cdot \hat{n} dA = 0 , \tag{2.1}$$

2. momentum principle (second law of motion) for water,

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} (\rho \vec{V}) dv + \iint_{A_{cv}} (\rho \vec{V}) \vec{V} \cdot \hat{n} dA = \iiint_{V_{cv}} \vec{f}_b dv + \iint_{A_{cv}} \vec{f}_s dA . \tag{2.2}$$

The variables used in the statements of the integral forms of the physical principles [Eqs. (2.1) and (2.2)] represent,

t = time

V_{cv} = volume of the control volume,

A_{cv} = enclosure surface area of the control volume,

ρ = mass density of water,

$\vec{V}$ = velocity (vector) of water,

$\hat{n}$ = unit normal vector (outward positive) on the enclosure surface area of the control volume,

$\vec{f}_b$ = body force (vector) acting on per unit volume of water,

$\vec{f}_s$ = surface force (vector) acting on per unit area in water.

The general statements of the integral forms of the two physical principles Eqs. (2.1) and (2.2) can be simplified by considering the physical conditions of the water in estuaries and coastal regions which contain only dilute concentrations of salt in water.

Considering that water is incompressible, it.s mass density does not depend on pressure, but depends only on water temperature and dissolved chemical constituents (e.g., salinity). Furthermore, in estuaries and coastal regions, the variations of the mass density of water, due to the variations in water temperature and dissolved chemical constituents, are not appreciable in comparison to the constant representative mass density of fresh water. Hence, the mass density of water can be represented as

$$\rho = \rho(T, C_{sl}) = \rho_w + \Delta\rho(T, C_{sl}) = \rho_w\left[1 + \frac{\Delta\rho(T, C_{sl})}{\rho_w}\right], \tag{2.3a}$$

or,

$$\rho = \rho_w(1+\alpha) = \rho_w\left[1 + \alpha(T, C_{sl})\right], \tag{2.3b}$$

where,

$$\alpha \simeq \alpha(T, C_{sl}) = \frac{\Delta\rho(T, C_{sl})}{\rho_w}. \tag{2.4}$$

The variables used in the formulation of the mass density of water Eqs. (2.3a), (2.3b) and (2.4) represent,

ρ_ω = constant representative mass density of fresh water (1 gm/cm^3 or 1000 kg/m^3 at 4 °c)

T = water temperature,

C_{sl} = dissolve chemical constituents (or salinity) as the ratio of the mass of salt to the mass of fresh water with constant representative mass density ρ_ω,

α = densimetric factor as the percent difference of the mass density of water relative to the constant representative mass density of fresh water ρ_ω.

The definition of salinity C_{sl} in the formulation of the mass density of water [Eqs. (2.3) and (2.4)] is slightly different from the conventional definition of salinity. The present definition is based on the ratio of the mass of dissolved chemical constituents (e.g., salt) to the mass of fresh water with constant representative mass density ρ_ω, which can be represented in terms of mass densities as

$$C_{sl}=\frac{\rho_{sl}}{\rho_w}=\frac{\text{(mass density of salt)}}{\text{(constant representative mass density of water)}}. \tag{2.5}$$

The conventional definition of salinity is based on the ratio of the mass of dissolve substances (salt) to the mass of water, which can be represented in terms of mass densities as,

$$(C_{sl})_c=\frac{\rho_{sl}}{\rho}=\frac{\text{(mass density of salt)}}{\text{(mass density of water)}}. \tag{2.6}$$

The conventional definition of salinity [Eq. (2.6)] can be expressed in terms of the present definition of salinity [Eq. (2.5)] by considering the formulation of the mass density of mixture [Eq. 2.3b)] as,

$$(C_{sl})_c=\frac{\rho_{sl}}{\rho_w(1+\alpha)}=\frac{C_{sl}}{(1+\alpha)}, \tag{2.7a}$$

or for $\alpha \ll 1$, as,

$$(C_{sl})_c=C_{sl}[1-\alpha+\alpha^2-\cdots+(-1)^n\alpha^n+\cdots]=C_{sl}(1-\alpha). \tag{2.7b}$$

In general, considering that the variations of the mass density of water in estuaries and coastal regions are very small relative to the constant representative mass density of fresh water (i.e., $\alpha \leq 0.02$), the differences between the values of the present definition of salinity [Eq. (2.5)] and the conventional definition of salinity [Eq. (2.6)] can be neglected; hence,

$$(C_{sl})_c = C_{sl} \quad \text{for } |\alpha| \leq 0.02 \, . \tag{2.8}$$

Substituting the formulation of the mass density of water [Eq. (2.3b)] in the original integral form of the mass conservation principle for water [Eq. (2.1)], the modified integral form of the mass conservation principle for water becomes,

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} (1+\alpha) dv + \iint_{A_{cv}} (1+\alpha)\vec{V}\cdot\hat{n} dA = 0 \, . \tag{2.9}$$

For the present study, the fast-transient three-dimensional discrete-element method for predicting flows with free surface conditions is applied by assuming that the variations of water density with temperature and salinity variations are negligible. Hence, for constant water density, $\rho = \rho_\omega$, and hence $\alpha = 0$, the integral form of the mass conservation principle for water [Eq. (2.9)] becomes

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} dv + \iint_{A_{cv}} \vec{V}\cdot\hat{n} dA = 0 \, . \tag{2.10}$$

The integral form of the momentum principle [Eq. (2.2)] represents a vector equation which has three components along the directions of a specified coordinate system. The discrete-element formulation of the momentum principle considers a general Cartersian coordinate system

(x,y,z) located on the geophysical surface plane of the earth. Since the analysis of flow conditions in a meso-scale region usually does not require the consideration of the curvature effects of the geophysical surface coordinates, a general Cartesian coordinate system can be conveniently used in the development of the discrete-element equations for the momentum principle.

In the analysis of flow conditions in a region with geometrically complex enclosure boundaries, it is usually convenient to select a Cartesian coordinate system such that the orientation of one of the axes lies along the general direction of the maximum dimension of the region. Furthermore, in the analysis of the flow conditions in fast flowing rivers or streams on inclined slopes, it is usually more convenient to select a Cartesian coordinate system such that the orientation of the vertical axis is generally along the normal to the water surface (and therefore generally along the normal to the inclined plane) rather than being along the direction of the true vertical direction which is normal to the geophysical surface plane. Hence, the general Cartesian coordinate system used in the development of the discrete-element equations for the momentum principle does not necessarily correspond to the conventional coordinate systems used in the analysis of the geophysical flow conditions in meso-scale regions. The conventional Cartesian coordinate systems for meso-scale regions usually specify that the three axes are along the general directions of longitude, latitude and the true vertical on the geophysical surface plane.

Considering that the orientation of the general Cartesian coordinate system used in discrete-element formulation of the momentum princi-

ple [Eq. (2.2)] does not necessarily correspond to the geographically consistent orientation of a conventional geophysical coordinate system, the development of the body force acting on per unit volume of water $\vec{f}_b$ [in Eq. (2.2)] requires generalized formulations of the gravity force and the Coriolis acceleration (force) (e.g., Dronkers, 1964). Hence, the three components of the body force acting on per unit volume of water $\vec{f}_b$ [in Eq. (2.2)], in the general Cartesian coordinate system, become

$$f_{bx} = f_{grv,x} + f_{cor,x} = -g_a \rho \cos\theta_{n,x} + \rho(\Omega_{cor,N,y} V_z - \Omega_{cor,N,z} V_y), \quad (2.11a)$$

$$f_{by} = f_{grv,y} + f_{cor,y} = -g_a \rho \cos\theta_{n,y} + \rho(\Omega_{cor,N,z} V_x - \Omega_{cor,N,x} V_z), \quad (2.11b)$$

$$f_{bz} = f_{grv,z} + f_{cor,z} = -g_a \rho \cos\theta_{n,z} + \rho(\Omega_{cor,N,x} V_y - \Omega_{cor,N,y} V_x). \quad (2.11c)$$

The variables used in the definitions of the components of the body force acting on per unit volume of water [in Eqs. (2.11a) through (2.11c)] represent,

x,y = coordinates in the horizontal (in the general sense) geophysical surface plane,

z = coordinate along (in the general sense) the true vertical to the geophysical surface plane,

f_{bx} = x-component of body force per unit volume of water,

f_{by} = y-component of body force per unit volume of water,

f_{bz} = z-component of body force per unit volume of water,

$f_{grv,x}$ = x-component of gravity force per unit volume of water,

$f_{grv,y}$ = y-component of gravity force per unit volume of water,

$f_{grv,z}$ = z-component of gravity force per unit volume of water,

$f_{cor,x}$ = x-component of Coriolis acceleration (force) per unit volume of water,

$f_{cor,y}$ = y-component of Coriolis acceleration (force) per unit volume of water,

$f_{cor,z}$ = z-component of Coriolis acceleration (force) per unit volume of water,

g_0 = gravitational acceleration constant,

$\theta_{n,x}$ = angle between the x-axis and the true vertical direction normal to the geophysical surface plane,

$\theta_{n,y}$ = angle between the y-axis and the true vertical direction normal to the geophysical surface plane,

$\theta_{n,z}$ = angle between the z-axis and the true vertical direction normal to the geophysical surface plane,

$\Omega_{cor,N,x}$ = factor associated with the x-component of angular velocity (rotation) of the earth,

$\Omega_{cor,N,y}$ = factor associated with the y-component of angular velocity (rotation) of the earth,

$\Omega_{cor,N,z}$ = factor associated with the z-component of angular velocity (rotation) of the earth,

V_x = x-component of velocity in the horizontal plane (in the general sense),

V_y = y-component of velocity in the horizontal plane (in the general sense),

V_z = z-component of velocity along the vertical direction to the horizontal plane (in the general sense),

The generalized forms for the components of the gravity force per unit volume of water [in Eqs. (2.11a) through (2.11c)] reduce to the conventional forms for the special case of the general Cartesian coordinate system which corresponds to a geographical coordinate system that has the z-axis along the true vertical direction normal to the geophysical surface plane, as

$$f_{grv,x}=0, \quad \text{for } \theta_{n,x}=90^{\circ}, \tag{2.12a}$$

$$f_{grv,y}=0, \quad \text{for } \theta_{n,y}=90^{\circ}, \tag{2.12b}$$

$$f_{grv,z}=-g_0\rho, \quad \text{for } \theta_{n,z}=0^{\circ}. \tag{2.12c}$$

Similarly, for the conventional Cartesian coordinate system for meso-scale flow regions, which considers the special orientations of the three axis x,y,z along the directions of longitude, latitude and the true vertical, respectively, on the geophysical surface plane, the three factors associated with the three components of angular velocity (rotation) of the earth reduce to the conventional forms as

$$\Omega_{cor,N,x}=0, \tag{2.13a}$$

$$\Omega_{cor,N,y}=2\omega\cos\beta, \tag{2.13b}$$

$$\Omega_{cor,N,z}=2\omega\sin\beta. \tag{2.13c}$$

where

ω = angular velocity (rotation) of the earth,

β = geographical latitude angle.

Hence, for the conventional Cartesian coordinate system for meso-scale flow regions, which considers the special orientations of the three axis x,y,z along the directions of longitude, latitude and the true vertical, respectively, on the geophysical surface plane, the three components of the body force per unit volume of water $\vec{f}_b$ [in Eq. (2.2)], reduce to the conventional forms as

$$f_{bx} = (2\omega \cos\beta)V_z - (2\omega \sin\beta)V_y \ , \tag{2.14a}$$

$$f_{by} = (2\omega \sin\beta)V_z \ , \tag{2.14b}$$

$$f_{bz} = -g_o \rho - (2\omega \cos\beta)V_x \ . \tag{2.14c}$$

The effect of the small variations of the geographical latitude angle β [in Eqs. (2.14a) through (2.14c)] over a meso-scale flow region is usually negligible in calculating the components of the Coriolis acceleration (force) on the water. Hence, the generalized factors associated with the components of angular velocity (rotation) of earth, $\Omega_{cor,N,x}$, $\Omega_{cor,N,y}$ and $\Omega_{cor,N,z}$ [in Eqs. (2.11a) through (2.11c)], in the general Cartesian coordinate system can also be assumed to remain as constants over a meso-scale flow region.

The surface force acting on per unit area in water $\vec{f}_s$ [in Eq.(2.2)] can be separated into two parts as

$$\vec{f}_s = p(-\hat{n}) + \vec{\sigma}_s \ , \tag{2.15}$$

where

p = pressure on a surface in the flow region of water,

$\vec{\sigma}_s$ = the force resulting from the viscous (molecular level) stress components acting on a surface in the flow region of water.

Hence, the components of the surface force acting on per unit area in water $\vec{f}_s$ [in Eq. (2.2)] can be represented as,

$$f_{sx} = [p(-\hat{n})]_x + \sigma_{sx} , \tag{2.16a}$$

$$f_{sy} = [p(-\hat{n})]_y + \sigma_{sy} , \tag{2.16b}$$

$$f_{sz} = [p(-\hat{n})]_z + \sigma_{sz} , \tag{2.16c}$$

where

f_{sx} = x-component of surface force acting on per unit area in the flow region of water,

f_{sy} = y-component of surface force acting on per unit area in the flow region of water,

f_{sz} = z-component of surface force acting on per unit area in the flow region of water,

σ_{sx} = x-component of viscous (molecular level) stress acting on a surface in the flow region of water,

σ_{sy} = y-component of viscous (molecular level) stress acting on a surface in the flow region of water,

σ_{sz} = z-component of viscous (molecular level) stress acting on a surface in the flow region of water,

Hence, substituting the formulations of mass density of water ρ [Eq. (2.5)], the three components of the body force action on per unit volume of water, f_{bx}, f_{by} and f_{bz} [Eqs. (2.11a) through (2.10c)], and the three components of the surface force acting on per unit area in the flow region of water, f_{sx}, f_{sy} and f_{sz} [Eqs. (2.16a) through (2.16c)], into Eq. (2.2), the three components of the integral form of the momentum principle (second law of motion for water) [Eq. (2.2)] become

$$\frac{\partial}{\partial t}\iiint_{V_{cv}}(1+\alpha)V_x dv+\iint_{A_{cv}}[(1+\alpha)V_x]\vec{V}\cdot\hat{n}dA=-g_a\cos\Theta_{n,x}\iiint_{V_{cv}}(1+\alpha)dv$$
$$+\Omega_{cor,N,y}\iiint_{V_{cv}}(1+\alpha)V_z dv-\Omega_{cor,N,z}\iiint_{V_{cv}}(1+\alpha)V_y dv$$
$$+\iint_{A_{cv}}[\frac{p}{\rho_w}(-\hat{n})]_x dA+\iint_{A_{cv}}[\frac{\sigma_{sx}}{\rho_w}]dA\ , \qquad (2.17)$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}}(1+\alpha)V_y dv+\iint_{A_{cv}}[(1+\alpha)V_y]\vec{V}\cdot\hat{n}dA=-g_a\cos\Theta_{n,y}\iiint_{V_{cv}}(1+\alpha)dv$$
$$+\Omega_{cor,N,z}\iiint_{V_{cv}}(1+\alpha)V_x dv-\Omega_{cor,N,x}\iiint_{V_{cv}}(1+\alpha)V_z dv$$
$$+\iint_{A_{cv}}[\frac{p}{\rho_w}(-\hat{n})]_y dA+\iint_{A_{cv}}[\frac{\sigma_{sy}}{\rho_w}]dA\ , \qquad (2.18)$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}}(1+\alpha)V_z dv+\iint_{A_{cv}}[(1+\alpha)V_z]\vec{V}\cdot\hat{n}dA=-g_a\cos\Theta_{n,z}\iiint_{V_{cv}}(1+\alpha)dv$$
$$+\Omega_{cor,N,x}\iiint_{V_{cv}}(1+\alpha)V_y dv-\Omega_{cor,N,y}\iiint_{V_{cv}}(1+\alpha)V_x dv$$
$$+\iint_{A_{cv}}[\frac{p}{\rho_w}(-\hat{n})]_z dA+\iint_{A_{cv}}[\frac{\sigma_{sz}}{\rho_w}]dA\ . \qquad (2.19)$$

For the present study, the fast-transient three-dimensional discrete-element method for predicting flows with free surface conditions is applied by assuming that the variations of water density with temperature and salinity variations are negligible. Hence, for constant water density, $\rho = \rho_\omega$, and hence $\alpha = 0$, the three components of the integral form of the momentum principle (second law of motion) for water [Eqs. (2.17) through (2.19)] becomes

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_x dv + \iint_{A_{cv}} [V_x]\vec{V}\cdot\hat{n}dA = -g_a \cos\theta_{n,x}\iiint_{V_{cv}} dv + \Omega_{cor,N,y}\iiint_{V_{cv}} V_z dv$$
$$-\Omega_{cor,N,z}\iiint_{V_{cv}} V_y dv + \iint_{A_{cv}} [\frac{p}{\rho_w}(-\hat{n})]_x dA + \iint_{A_{cv}} [\frac{\sigma_{sx}}{\rho_w}]dA \quad , \quad (2.20)$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_y dv + \iint_{A_{cv}} [V_y]\vec{V}\cdot\hat{n}dA = -g_a \cos\theta_{n,y}\iiint_{V_{cv}} dv + \Omega_{cor,N,z}\iiint_{V_{cv}} V_x dv$$
$$-\Omega_{cor,N,x}\iiint_{V_{cv}} V_z dv + \iint_{A_{cv}} [\frac{p}{\rho_w}(-\hat{n})]_y dA + \iint_{A_{cv}} [\frac{\sigma_{sy}}{\rho_w}]dA \quad , \quad (2.21)$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_z dv + \iint_{A_{cv}} [V_z]\vec{V}\cdot\hat{n}dA = -g_a \cos\theta_{n,z}\iiint_{V_{cv}} dv + \Omega_{cor,N,x}\iiint_{V_{cv}} V_y dv$$
$$-\Omega_{cor,N,y}\iiint_{V_{cv}} V_x dv + \iint_{A_{cv}} [\frac{p}{\rho_w}(-\hat{n})]_z dA + \iint_{A_{cv}} [\frac{\sigma_{sz}}{\rho_w}]dA \quad . \quad (2.22)$$

The three-component representation [Eqs. (2.20) through (2.22)] of the integral form of the momentum principle is applicable to most general, transient, three-dimensional flow conditions in all meso-scale and micro-scale geophysical flow regions.

Simplified, restricted integral forms of the the momentum principle can be obtained directly based on the comparisons of the orders of

magnitudes of the individual terms of the three component equations [Eqs. (2.17) through (2.19)] by considering a characteristic length Lc and a characteristic magnitude of velocity V_c defined as,

L_c = characteristic length (generally associated with the characteristic depth of the water in the flow region),

V_c = characteristic magnitude of velocity [generally associated by the horizontal components of velocity in the (x,y) plane].

Hence, similar to the order of magnitude analysis employed in the comparisons of the individual terms of the differential form of the momentum principle (Navier-Stoke's equations) (e.g., Schlichting, 1960), the comparisons of the order of magnitudes of the various terms of the integral form of the momentum principle [Eqs. (2.17) through (2.19)] can be formulated based on the characteristic length L_c and the characteristic magnitude of velocity V_c.

Hence, the order of magnitude of the inertia (force) terms in Eqs. (2.17) through (2.19) becomes

$$O\{\text{inertia}\} = O\left\{\iint_{A_{cv}} [V_{(x,y,z)}]\vec{V}\cdot\hat{n}dA\right\} = O\{V_c^2 L_c^2\}\ , \qquad (2.23)$$

where the subscript notation in parenthesis (x,y,z) for the velocity components represents,

$V_{(x,y,z)}$ = any one of the three (x-, y-, or z-) components of velocity.

Similarly, the order of magnitude of the gravity force terms in Eqs. (2.17) through (2.19) becomes

$$O\{\text{gravity}\} = O\left\{g_a \cos\theta_{n,(x,y,z)} \iiint_{V_{cv}} dv\right\} = O\{g_a L_c^3\}\ , \qquad (2.24)$$

where the subscript notation in parenthesis (x,y,z) for the angles represents,

$\theta_{n,(x,y,z)}$ = any one of the three angles between any one of the (x-, y-, or z-) axes and the true vertical direction normal to the geophysical surface plane.

Similarly, the order of magnitude of the Coriolis acceleration (force) terms in Eqs. (2.17) through (2.19) becomes

$$O\{\text{Coriolis}\} = O\{\Omega_{cor,N,(y,z,x)} \iiint_{V_{cv}} V_{(z,x,y)} dv - \Omega_{cor,N,(z,x,y)} \iiint_{V_{cv}} V_{(y,z,x)} dv\}$$

$$= O\{\Omega_{cor,N} V_c L_c^3\} \cdot \qquad (2.25)$$

where the subscript notations in parenthesis (y,z,x) and (z,x,y) for the factors associated with components of the angular velocity (rotation of the earth) represent the particular two of the general permutational subscript form (x,y,z) as

$\Omega_{cor,N,(z,x,y)}$ = factor associated with any one of the three (x-, y-, or z-) components of angular velocity (rotation of the earth),

and

$\Omega_{cor,N}$ = magnitude of the factors associated with any one of the three (x-, y-, or z-) components of angular velocity (rotation of the earth).

Considering that the characteristic pressure for free surface flow conditions can be represented as the hydrostatic pressure in terms of the characteristic depth length L_c, the order of magnitude of the pressure force terms in Eqs. (2.17) through (2.19) becomes

$$O\{\text{pressure}\} = O\left\{\iint_{A_{cv}} \left[\frac{p}{\rho_w}(-\hat{n})\right]_{(x,y,z)} dA\right\} = O\{g_o L_c^3\}\ , \qquad (2.26)$$

where the subscript notation in the parenthesis (x,y,z) represents any one of the three components of the pressure force along x-, y-, or z-direction.

The components of the viscous stress acting on a surface in the flow region of water can be formulated in terms of the standard, molecular viscosity, with a characteristic constant value as

= constant representative viscosity of water (0.01 $g.sec^{-1}.cm^{-1}$ or 0.001 $kg.sec^{-1}.m^{-1}$ at 20°C).

Hence, the order of magnitude of the viscous force terms in Eqs. (2.17) through (2.19) becomes

$$O\{\text{viscous}\} = O\left\{\iint_{A_{cv}} \left[\frac{\sigma_{s,(x,y,z)}}{\rho_w}\right] dA\right\} = O\left\{\frac{1}{\rho_w}\left[\frac{\mu_w V_c}{L_c}\right] L_c^2\right\} = O\left\{\frac{\mu_w V_c L_c}{\rho_w}\right\}\ , \qquad (2.27)$$

where the subscript notation in the parenthesis (x,y,z) for the components of the viscous stress represents

$\sigma_{s,(x,y,z)}$ = any one of the three (x-, y-, or z-) components of viscous stress acting on a surface in the flow region of water.

The comparison of the inertia (force) effects and the gravity force effects can be considered based on the ratio of the order of magnitude of the inertia (force) terms $O\{\text{inertia}\}$ [Eq. (2.23)] to the order of magnitude of the gravity force terms $O\{\text{gravity}\}$ [Eq. (2.24)] as

$$\frac{O\{\text{inertia}\}}{O\{\text{gravity}\}} = \frac{O\{V_c^2 L_c^2\}}{O\{g_o L_c^3\}} = O\left\{\frac{V_c^2}{g_o L_c}\right\} = O\{Fr_c\}\ , \qquad (2.28)$$

where,

$$Fr_c = \frac{V_c^2}{g_o L_c} = \text{characteristic Froude number} \quad \text{e.g., Daily and Harleman (1966)} . \tag{2.29}$$

Similarly, the comparison of the inertia (force) effects and the Coriolis acceleration (force) effects can be considered based on the ratio of the order of magnitude of the inertia (force) terms $O\{\text{inertia}\}$ [Eq.(2.23)] to the order of magnitude of the Coriolis acceleration force terms $O\{\text{Coriolis}\}$ [Eq. (2.25)] as

$$\frac{O\{\text{inertia}\}}{O\{\text{Coriolis}\}} = \frac{O\{V_c^2 L_c^2\}}{O\{\Omega_{cor,N} V_c L_c^3\}} = O\left\{\frac{V_c}{\Omega_{cor,N} L_c}\right\} = O\{Rs_c\} , \tag{2.30}$$

where,

$$Rs_c = \frac{V_c}{\Omega_{cor,N} L_c} = \text{characteristic modified Rosby number} \quad \text{e.g., Robinson (1970)} . \tag{2.31}$$

Similarly, the comparison of the inertia (force) effects and the hydrostatic pressure force effects can be considered based on the ratio of the order of magnitude of the inertia (force) terms $O\{\text{inertia}\}$ [Eq. (2.23)] to the order of magnitude of the pressure force terms $O\{\text{pressure}\}$ [Eq. (2.26)] as

$$\frac{O\{\text{inertia}\}}{O\{\text{pressure}\}} = \frac{O\{V_c^2 L_c^2\}}{O\{g_o L_c^3\}} = O\left\{\frac{V_c^2}{g_o L_c}\right\} = O\{Fr_c\} . \tag{2.32}$$

Similarly, the comparison of the inertia (force) effects and the viscous force effects can be considered based on the ratio of the order of magnitude of the inertia (force) terms $O\{\text{inertia}\}$ [Eq. (2.23)] to the order of magnitude of the viscous force terms $O\{\text{viscous}\}$ [Eq. (2.27)] as

$$\frac{O\{\text{inertia}\}}{O\{\text{viscous}\}} = \frac{O\{V_c^2 L_c^2\}}{O\left\{\frac{\mu_w V_c L_c}{\rho_w}\right\}} = O\left\{\frac{\rho_w V_c L_c}{\mu_w}\right\} = O\{Re_c\} , \tag{2.33}$$

where

$$Re_c = \frac{\rho_w V_c L_c}{\mu_w} = \text{characteristic Reynolds number (e.g., Schlichting, 1955).} \tag{2.34}$$

The variations of the density of water with the variations of the water temperature conditions in a typical geophysical flow region (e.g., an estuary or a coastal zone) is generally small; hence, the magnitude of the densimetric factor α for water temperature variations is generally,

$$|\alpha| \leq 0.02 \quad \text{for } 0^\circ \leq T \leq 60^\circ c \,. \tag{2.35}$$

Similarly, the variations of the density of water with the variations of the dissolved chemical constituent (salinity) conditions in a typical geophysical flow region is also small; hence the magnitude of the densimetric factor a for salinity variations is gererally,

$$|\alpha| \leq 0.02 \quad \text{for } 0 \leq C_{s|} \leq 0.03 \text{ ppp (parts per part)}. \tag{2.36}$$

Therefore, the effect of density variations can be neglected in the three-component representation of the integral form of the momentum principle [Eqs. (2.17) through (2.19)] based on the comparison of the order of magnitude of the gravity force terms $O\{\text{gravity}\}$ [Eq. (2.24)] and the order of magnitude of the inertia (force) terms $O\{\text{inertia}\}$ [Eq. (2.23)]. The ratio of the order of magnitude of the gravity force term $O\{\text{gravity}\}$ [Eq. (2.24)] to the order of magnitude of the inertia (force) term $O\{\text{inertia}\}$ Eq. (2.23) becomes

$$\frac{O\{\text{gravity}\}}{O\{\text{inertia}\}} = \frac{O\{g_a L_c^3\}}{O\{V_c^2 L_c^2\}} = O\left\{\frac{g_a L_c}{V_c^2}\right\} = O\left\{\frac{1}{Fr_c}\right\} . \tag{2.37}$$

For relatively slow flow conditions, $V_c \leq 1$ m/sec, in a relatively deep, $L_c \geq 10$m, geophysical flow region, the characteristic Froude number is

$$Fr_c \leq 0.01 \quad \text{for } V_c \leq 1 \text{ m/sec and } L_c > 10 \text{ m} . \tag{2.38}$$

Therefore, the effects of the density variations can be neglected in all the terms except in the gravity force terms in the general, three-component representation of the integral form of the momentum principle, as

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_x dv + \iint_{A_{cv}} (V_x)\vec{V}\cdot\hat{n}dA = -g_a \cos\Theta_{n,x}\iiint_{V_{cv}} (1+\alpha)dv + \Omega_{cor,N,y}\iiint_{V_{cv}} V_z dv$$
$$-\Omega_{cor,N,z}\iiint_{V_{cv}} V_y dv + \iint_{A_{cv}} [\frac{p}{\rho_w}(-\hat{n})]_x dA + \iint_{A_{cv}} [\frac{\sigma_{sx}}{\rho_w}]dA, \tag{2.39a}$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_y dv + \iint_{A_{cv}} (V_y)\vec{V}\cdot\hat{n}dA = -g_a \cos\Theta_{n,y}\iiint_{V_{cv}} (1+\alpha)dv + \Omega_{cor,N,z}\iiint_{V_{cv}} V_x dv$$
$$-\Omega_{cor,N,x}\iiint_{V_{cv}} V_z dv + \iint_{A_{cv}} [\frac{p}{\rho_w}(-\hat{n})]_y dA + \iint_{A_{cv}} [\frac{\sigma_{sy}}{\rho_w}]dA, \tag{2.39b}$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_z dv + \iint_{A_{cv}} (V_z)\vec{V}\cdot\hat{n}dA = -g_a \cos\Theta_{n,z}\iiint_{V_{cv}} (1+\alpha)dv + \Omega_{cor,N,x}\iiint_{V_{cv}} V_y dv$$
$$-\Omega_{cor,N,y}\iiint_{V_{cv}} V_x dv + \iint_{A_{cv}} [\frac{p}{\rho_w}(-\hat{n})]_z dA + \iint_{A_{cv}} [\frac{\sigma_{sz}}{\rho_w}]dA . \tag{2.39c}$$

For the conventional Cartesian coordinate system for meso-scale flow regions, which considers special orientations for the three axes (x,y,z) along the directions of longitude, latitude and the true

vertical, respectively, on the geophysical plane, with the simplified forms of the gravity force terms [Eqs. (2.12a) through (2.12c)] and the Coriolis acceleration (force) terms [Eqs. (2.13a), (2.13b) and (2.13c)], the simplified three-component representation of the integral form of the momentum principle [Eqs. (2.39a), through (2.39c)] reduce to

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_x dv + \iint_{A_{cv}} (V_x)\vec{V}\cdot\hat{n}dA = 2\omega\cos\beta \iiint_{V_{cv}} V_z dv - 2\omega\sin\beta \iiint_{V_{cv}} V_y dv + \iint_{A_{cv}} \left[\frac{p}{\rho_w}(-\hat{n})\right]_x dA + \iint_{A_{cv}} \left[\frac{\sigma_{sx}}{\rho_w}\right] dA , \qquad (2.40a)$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_y dv + \iint_{A_{cv}} (V_y)\vec{V}\cdot\hat{n}dA = 2\omega\sin\beta \iiint_{V_{cv}} V_x dv + \iint_{A_{cv}} \left[\frac{p}{\rho_w}(-\hat{n})\right]_y dA + \iint_{A_{cv}} \left[\frac{\sigma_{sy}}{\rho_w}\right] dA , \qquad (2.40b)$$

$$\frac{\partial}{\partial t}\iiint_{V_{cv}} V_z dv + \iint_{A_{cv}} (V_z)\vec{V}\cdot\hat{n}dA = -g_a\cos\Theta_{n,z} \iiint_{V_{cv}} (1+\alpha)dv - 2\omega\cos\beta \iiint_{V_{cv}} V_x dv + \iint_{A_{cv}} \left[\frac{p}{\rho_w}(-\hat{n})\right]_z dA + \iint_{A_{cv}} \left[\frac{\sigma_{sz}}{\rho_w}\right] dA . \qquad (2.40c)$$

The simplified forms of equations, which consider the effects of density variations based on the densimetric factor α only in the gravity force terms [Eqs. (2.39a) through (2.39c) and (2.40a) through (2.40c)], represent the three-component representation of the integral form of the momentum principle with the Boussinesq approximation (e.g., Schlichting, 1960, Bird, Stewart and Lightfoot, 1964, Spraggs and Street, 1975).

It is important to note that the actual, local flow conditions depend on the actual, local Froude number conditions in a flow region. In a meso-scale geophysical flow region (e.g., an estuary or a coastal zone), the local depth and the local velocity conditions could vary significantly (temporally and/or spatically) in the flow region. Therefore, the actual, local Froude number conditions could be significantly different from the representative value of the characteristic Froude number Frc based on the characteristic length (depth) L_c and the characteristic magnitude of velocity V_c [Eq. (2.29)] for the complete flow region. In deep subregions with relatively low local velocity conditions, the values of the actual local Froude number could be substantially lower than one (1); whereas, in shallow subregions near the shorelines with relatively high local velocity conditions, the values of the local Froude number could exceed one (1). Hence, in the development of a fast-transient, hydrodynamic model for simulating flow conditions in a meso-scale geophysical flow region, where the local Froude number conditions could vary substantially (temporally and/or spatially), the simplification of the general integral form of the momentum principle [Eqs. (2.20) through (2.22)] into the forms which include the Boussineq approximation [Eqs. (2.39a)] through (2.39c), (2.40a) through (2.40c) could not be achived based on the value of the characteristic Froude number [Eq. (2.29)].

The importance of the Coriolis acceleration (force) effect on the flow conditions in a geophysical flow region depends on the comparison of the inertia (force) effects and the Coriolis acceleration (force) effects according to the definition of the characteristic modified Rosby

number R_{sc} [Eq. (2.31)]. In micro-scale flow regions (e.g., near-field zones with characteristic lengths $L_c < 100$ m) with relatively high velocity conditions (e.g., intake or discharge flows with characteristic magnitudes of velocity $V_c \geq 1$ m/sec), the values of the characteristic modified Rosby number are generally $R_{sc} > 100$; and hence, the Coriolis acceleration (force) terms can be neglected in the formulation of the three-component representation [Eqs. (2.16) through (2.18)] of the integral form of the momentum principle. However, in meso-scale or macro-scale geophysical flow regions (e.g., large lakes, estuaries and coastal zones with characteristic lengths $L_c > 10{,}000$ m), with relatively low velocity conditions (e.g., tidal flows, geostrophic currents with characteristic magnitudes of velocity $V_c < 1$ m/sec), the values of the characteristic modified Rosby number can be $R_{sc} \leq 1$; and hence, the Coriolis acceleration (force) terms must be retained in the formulation of the three-component representation [Eqs. (2.17) through (2.19)] of the integral form of the momentum principle. In the present development of the fast-transient, three-dimensional, discrete-element hydrodynamic model, the general forms of the Coriolis acceleration (force) terms are retained in the three-component representation [Eqs. (2.20) through (2.22)] of the integral form of the momentum principle; hence, the hydrodynamic model is applicable to most general, transient, three-dimensional, constant-density (incompressible) free-surface flow conditions in all meso-scale and micro-scale geophysical flow regions (e.g., lakes, reserviors, estuaries, coastal zones and near-field zones of intakes and discharges in the vicinities of power plants).

The three-component representation [Eqs. (2.20) through (2.22)] of the integral form of the momentum principle explicitly includes only the viscous (molecular level) stress terms acting on the surfaces in the flow region of water. The turbulent stress effects are not represented explicitly in the formulation; but they are included implicitly in the surface integral representations of the inertia terms. Hence, the importance of the viscous effects on the flow conditions in a geophysical flow region can be considered based on the comparison of the inertia (force) effects and the viscous force effects according to the definition of the characteristic Reynolds number [Eq. (2.34)]. In typical meso-scale geophysical flow regions (e.g., lakes, etuaries and coastal zones, with characteristic lengths $L_c > 10$ m), with typical velocity conditions (e.g., with characteristic magnitudes of velocity $V_c \cong 1$ m/sec), the values of the characteristic Reynolds number are generally $R_{ec} > 10^6$ which indicate that the flow conditions are predominantly turbulent. However, it is important to note that the actual, local flow conditions depend on the actual, local Reynolds number conditions in the flow region. In a meso-scale (or micro-scale) geophysical flow region (e.g., a lake, a reservioir, and estuary or a coastal zone), the local depth and the local velocity conditions could vary significantly (temporally and spatially) in the flow region. Therefore, the actual local Reynolds number conditions could be significantly different than the representative value of the characteristic Reynolds number R_{ec} based on the characteristic length (depth) L_c and the characteristic magnitude of velocity V_c [Eq. (2.34)] for the complete flow region. In shallow subregions (e.g., with actual, local

depths less than 0.1 m) with relatively low velocity conditions (e.g., with actual, local magnitudes of velocity less than 0.01 m/sec), the values of the actual, local Reynolds number could be less than 1000 which indicate that the actual, local flow conditions could be laminar. In the present development of the fast-transient, three-dimensional, discrete-element hydrodynamic model, all the components of the viscous stress terms are retained in the three-component representation [Eqs. (2.20) through (2.22)] of the integral form of the momentum principle; hence, the mathematical model is applicable to most general, transient, three-dimensional, constant-density (incompressible), locally-laminar or locally-turbulent free-surface flow conditions in all meso-scale and micro-scale geophysical regions (e.g., lakes, estuaries, coastal zones and near-field zones of intakes and discharges in the vicinties of power plants).

2.3. Geometrical Definitions of Elements and Half-Elements

The application of the discrete-element method (DEM) to the mathematical modeling of hydrothermal problems in geophysical flow regions includes a special geometrical discretization technique which was specifically developed for areas with simply or multiply connected, geometrically complex shoreline enclosure boundaries in the horizontal planes (in the general sense, parallel to the geophysical surface plane). The three-dimensional elements are generated according to a procedure which consists of two parts.

First, the area in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane) of the free-surface of

the flow region is divided into two-dimensional planar elements, according to a specified, nonuniform (or uniform) rectangular grid system which automatically allows for different spatial resolution scales in different subregions (Fig. 2.1; p.89). The two-dimensional planar elements can be obtained by utilizing the automated discrete-element-generator computer code PREPR2 (Holdeman, 1977) or its modified version PREPAR (Lin, Erhan, and Eraslan, 1978).

The general development of the discrete-element method (DEM) can also utilize different nonuniform (or uniform) rectangular grid systems in different subregions of the flow region with discontinuous grid lines in adjacent subregions (Eraslan, 1974). The discretization of the area in the horizontal (x,y) plane of the free surface of the flow region according to different rectangular grid systems in different subregions is more efficient from geometrical considerations since it can readily generate imbedded rectangular grid systems with different resolution scales. However, the matching of the boundary conditions between adjacent subregions with different resolution scales becomes mathematically cumbersome in the development of the transport rates across the enclosure surface areas of the elements (Eraslan, 1974). Therefore, in the present development of the discrete-element hydrodynamic model for geophysical flow regions, only a single, specified, nonuniform (or uniform) rectangular grid system will be used in the generation of the two-dimensional planar elements.

Each two-dimensional planar element in the horizontal (x,y)plane of the free-surface of the flow region represents the top enclosure surface

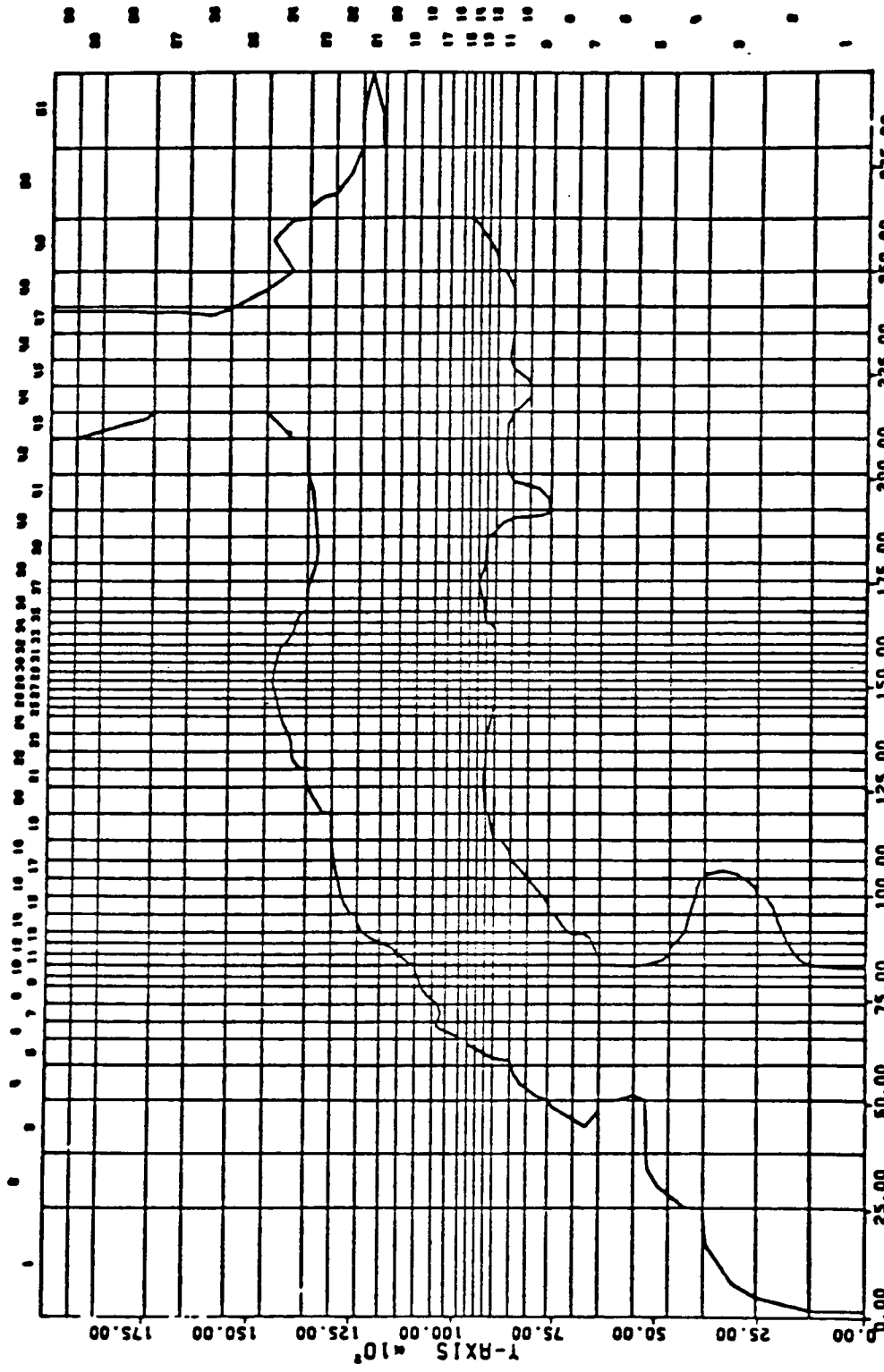

Figure 2.1. A typical computer generated discrete-element grid system for the section of the Hudson River in the vicinity of Indian Point.

of a three dimensional column extending from the bottom to the free-surface of the flow region.

Second, the flow region is divided in the vertical z-direction (in the general sense normal to the geophysical surface plane) into multiple layers, between horizontal (x,y) planes (in the general sense), parallel to the geophysical surface plane and the free-surface plane), according to a specified set of nonuniform (or uniform) depths for the layers. Since the division in the vertical direction z considers multiple horizontal (x,y) planes parallel to the horizontal (x,y) plane of the free-surface of the entire flow region, the number of vertically stacked elements, in each three-dimensional column that extends from the bottom to the free surface, depends on the specified depth for each two-dimensional planar element.

In applications of the general discretization technique of the discrete-element method to the simulation of free-surface flow conditions in a geophysical flow region, the specified depth of the top layer, which contains the free-surface, must be deeper than the maximum temporal and spatial variations of the water surface elevation conditions in the flow region, during the period of the simulations. This restriction on the specification of the depth of the top layer guarantees that all the elements in the top layer contain water during the simulations; and hence, it eliminates the need for "marking" the elements either as (1) water containing elements or as (2) empty elements. The present formulation of the discrete-element method is intended for the simulation of flow conditions when the free-surface layer remains generally in the horizontal (x,y) plane of the flow

region, and it is not intended for complex free-surface flow conditions which can be simulated by marker and cell (MAC), simplified marker and cell (SMAC) and particle in cell (PIC) methods (e.g., Harlow and Welch, 1965, Amsden and Harlow, 1970, Evans and Harlow, 1957, Gentry, Martin and Daley, 1967). However, it is important to note that the simplifying assumption, which restricts the specification of the depth of the top layer to include all temporal and spatial variations of the water-surface elevation in the horizontal (x,y) plane (in general sense, parallel to the geophysical surface plane), allows the discrete-element method to utilize an extremely efficient computational algorithm for the solution of the pressure conditions in the flow region (Sec. 3.3).

According to the general discretization technique of the discrete-element method, each interior element in the flow region represents a complete rectangular cell with six rectangular enclosure surface areas which are always normal to one of the three principal, orthogonal axes x, y or z (Figs. 2.2 through 2.15, p.92 through p.105; and Figs. 2.25 through 2.33, p.115 through p.123). Each boundary element represents a truncated rectangular cell with the only nonrectangular enclosure surface areas, that are possibly not normal to any one of the three principal, orthogonal axes (x,y or z), consisting of the sections of the generally impermeable shoreline boundaries of the flow region (Figs. 2.16 through 2.24, p.106 through p.114; and Figs. 2.34 through 2.42, p.124 through p.132). Therefore, any common enclosure surface area between any two elements is always normal to only a single component of velocity along only one of the principle, orthogonal axes (x, y or z). Hence, the discretized terms associated with the transport of mass of

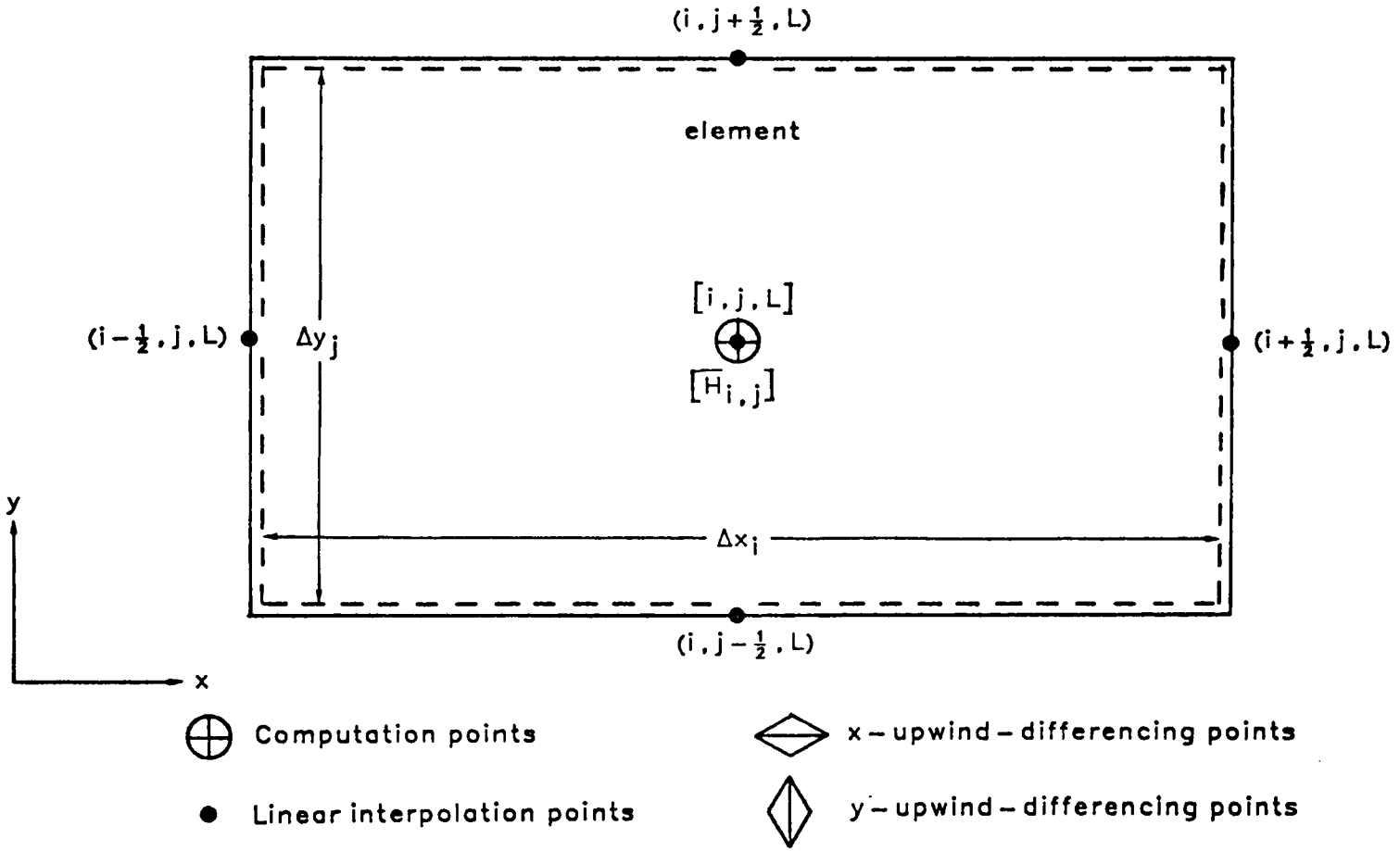

Figure 2.2. Locations used in the computation of water-surface elevation in an interior or boundary element in the horizontal (x,y) plane (i,j,L) above the bottom layer, in the water-surface layer L [top view in (x,y) plane].

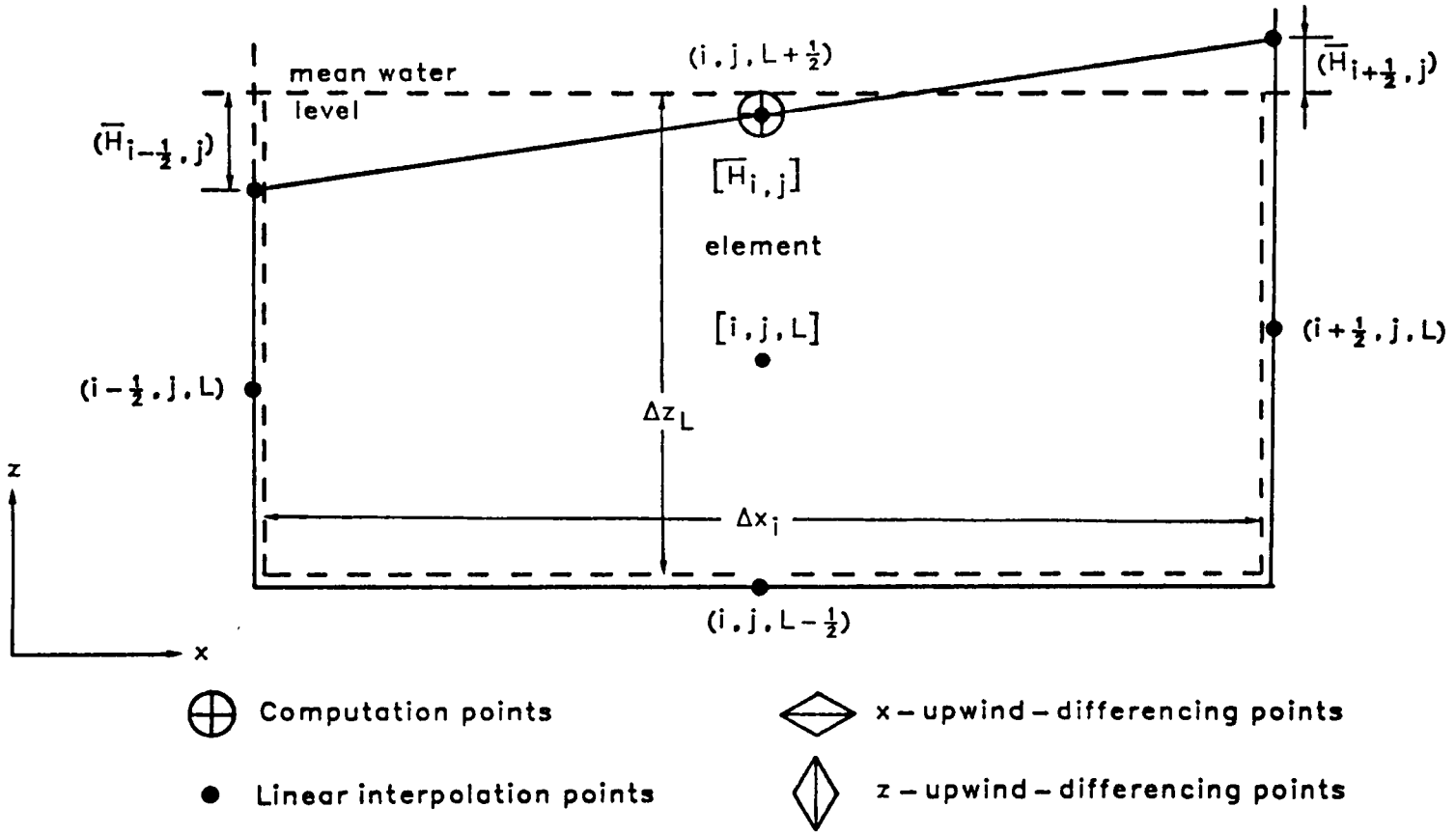

Figure 2.3. Locations used in the computation of water-surface elevation in an interior or boundary element in the horizontal (x,y) plane (i,j,L) above the bottom layer, in the water-surface layer L [side view in (x,z) plane].

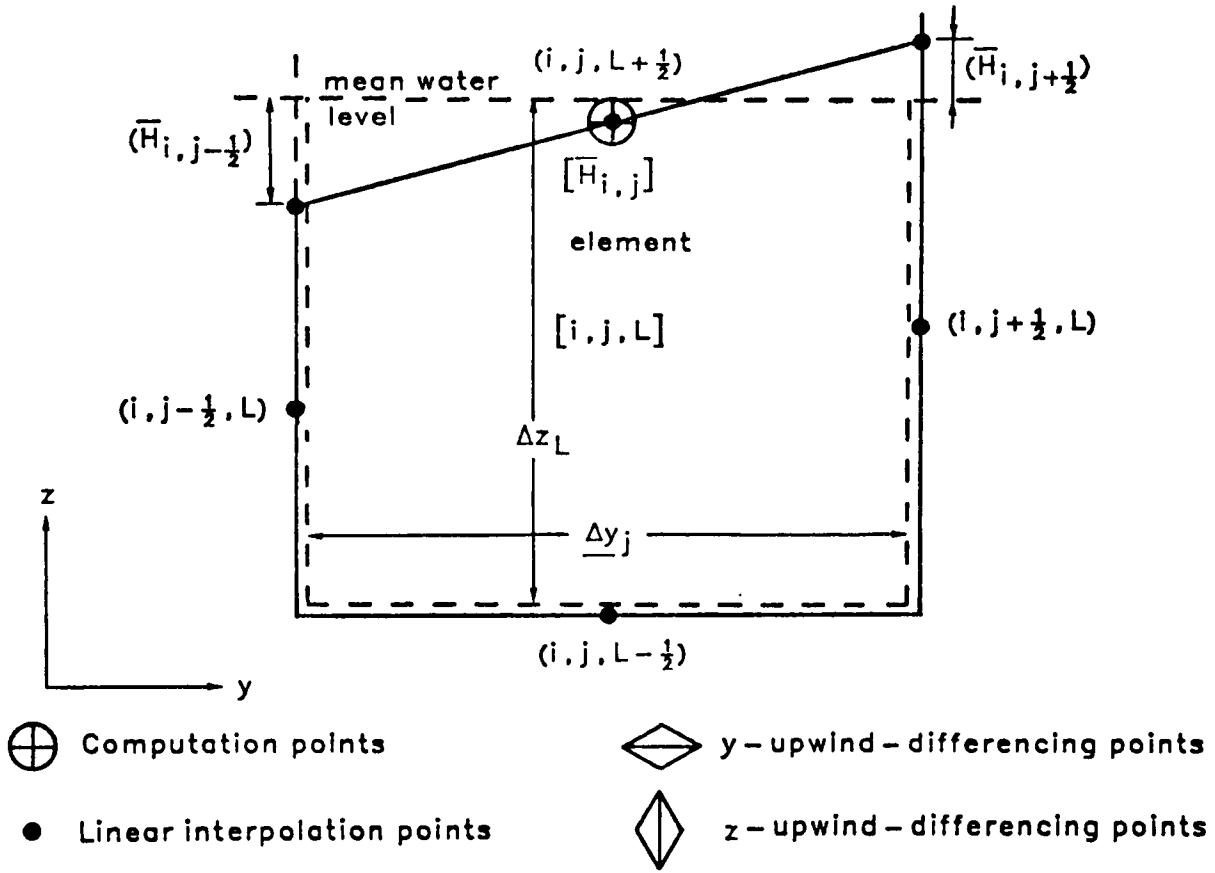

Figure 2.4. Locations used in the computation of water-surface elevation in an interior or boundary element in the horizontal (x,y) plane (i,j,L) above the bottom layer, in the water-surface layer L [side view in (y,z) plane].

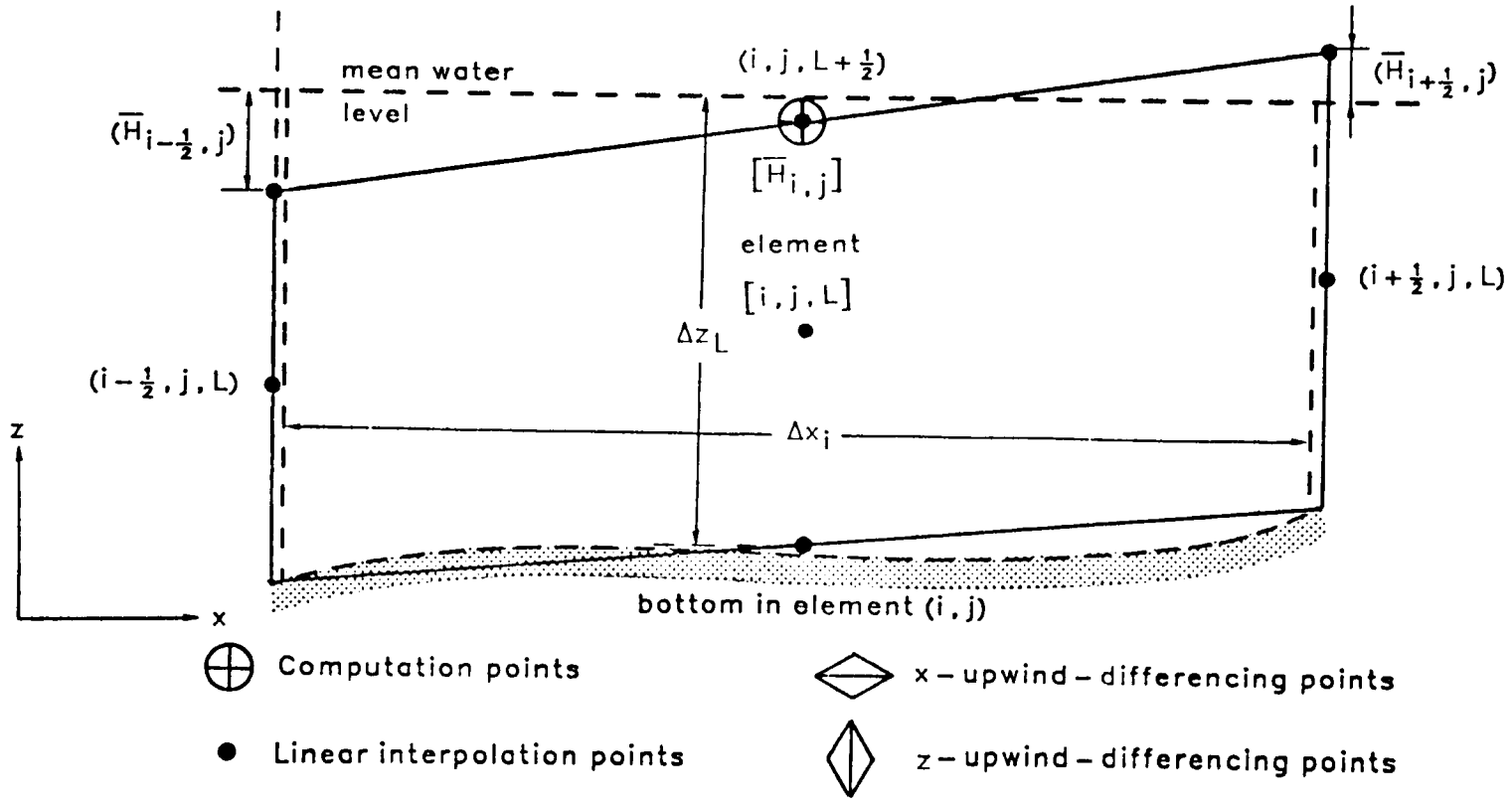

Figure 2.5. Locations used in the computation of water-surface elevation in an interior or boundary element in the horizontal (x,y) plane [i,j,l(bot;i,j)] = (i,j,L), in the bottom layer l(bot;i,j) and also in the water-surface layer L .side view in (x,z) plane..

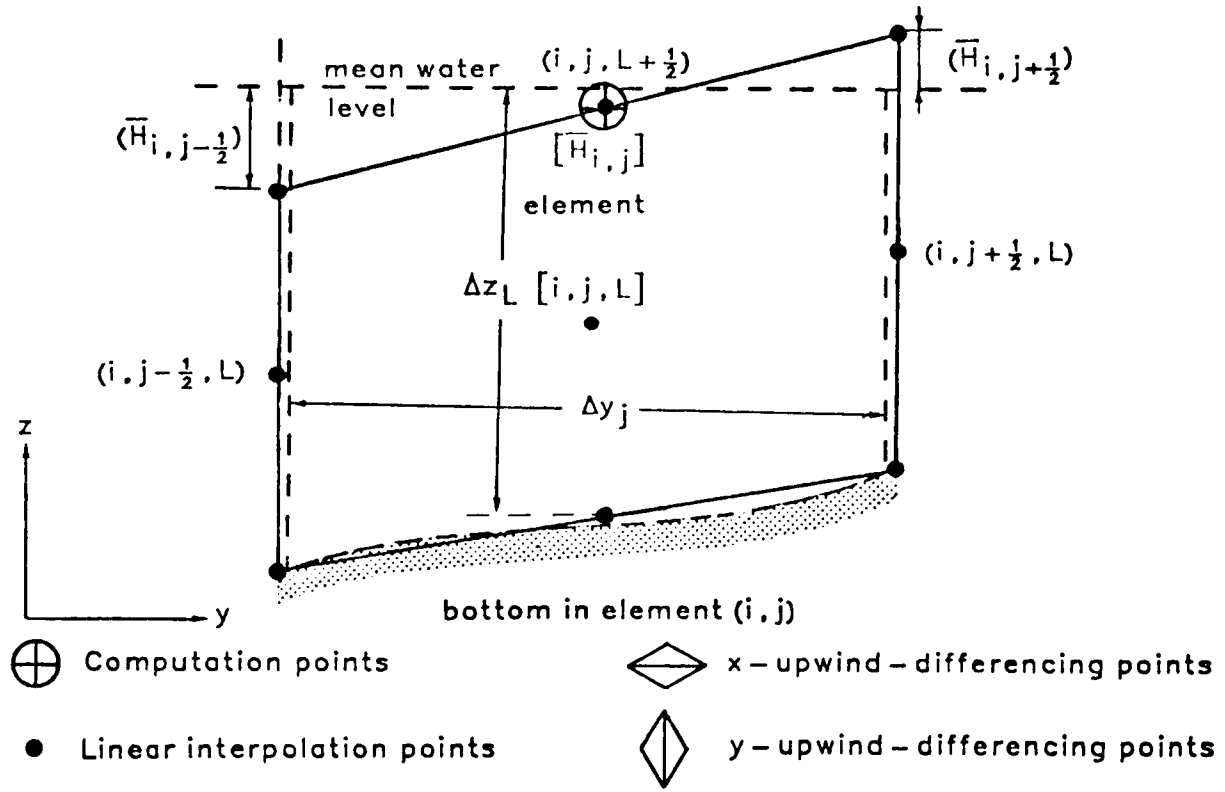

Figure 2.6. Locations used in the computation of water-surface elevation in an interior or boundary element in the horizontal (x,y) plane [i,j,l(bot;i,j)] = (i,j,L), in the bottom layer l(bot;i,j) and also in the water-surface layer L [side view in (y,z) plane].

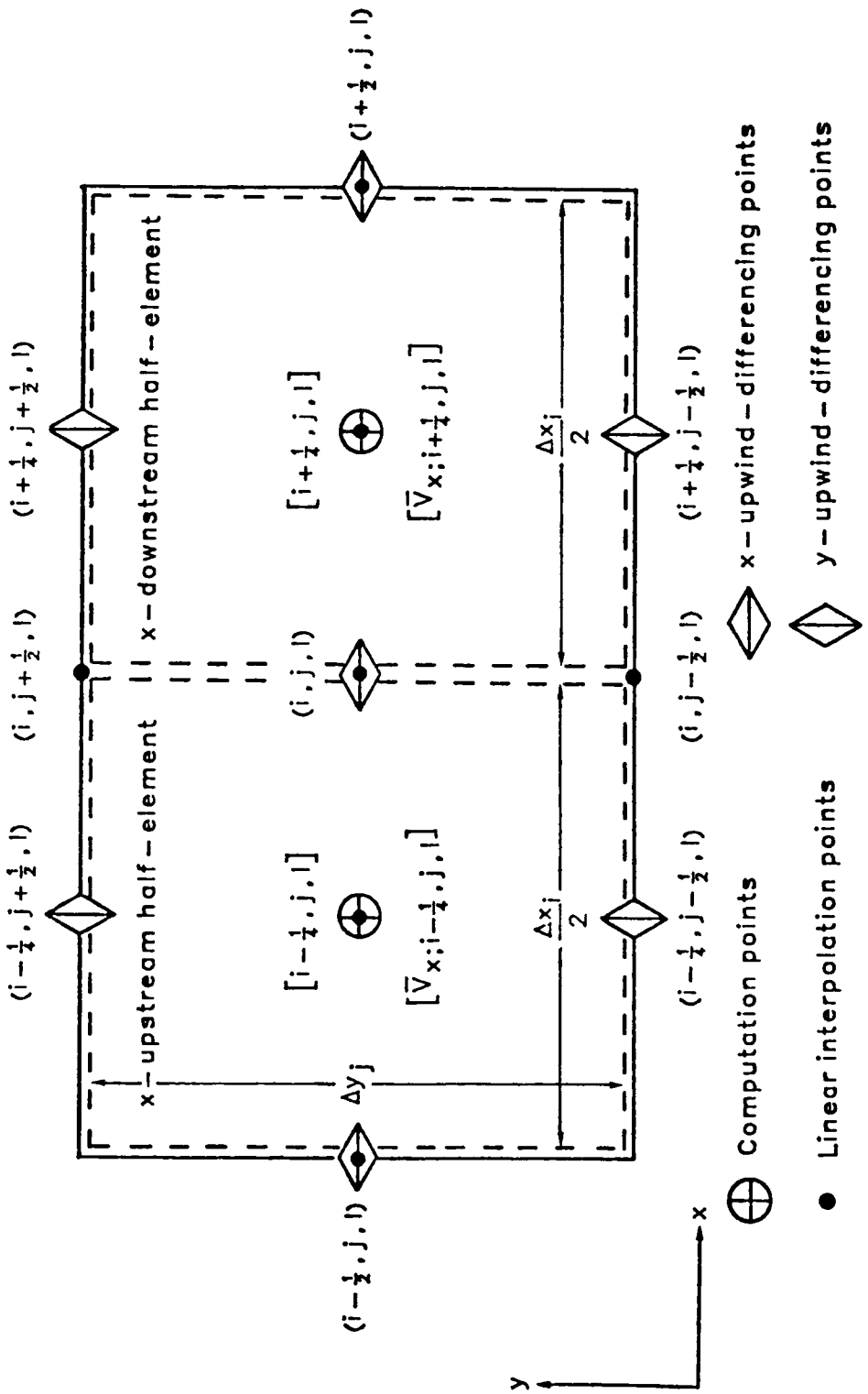

Figure 2.7. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,1) which is also a vertically interior element [top view in (x,y) plane].

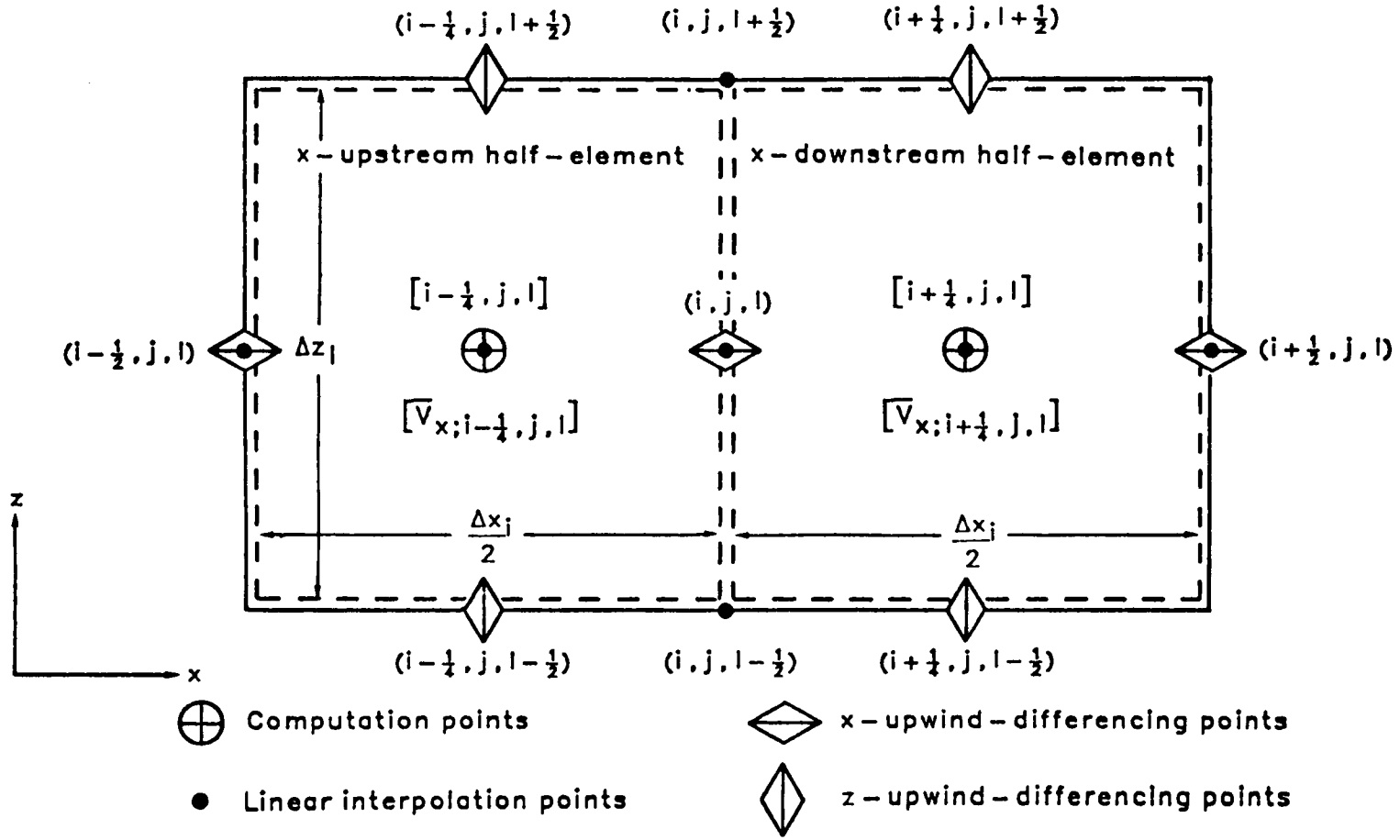

Figure 2.8. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,1) which is also a vertically interior element [side view in (x,z) plane].

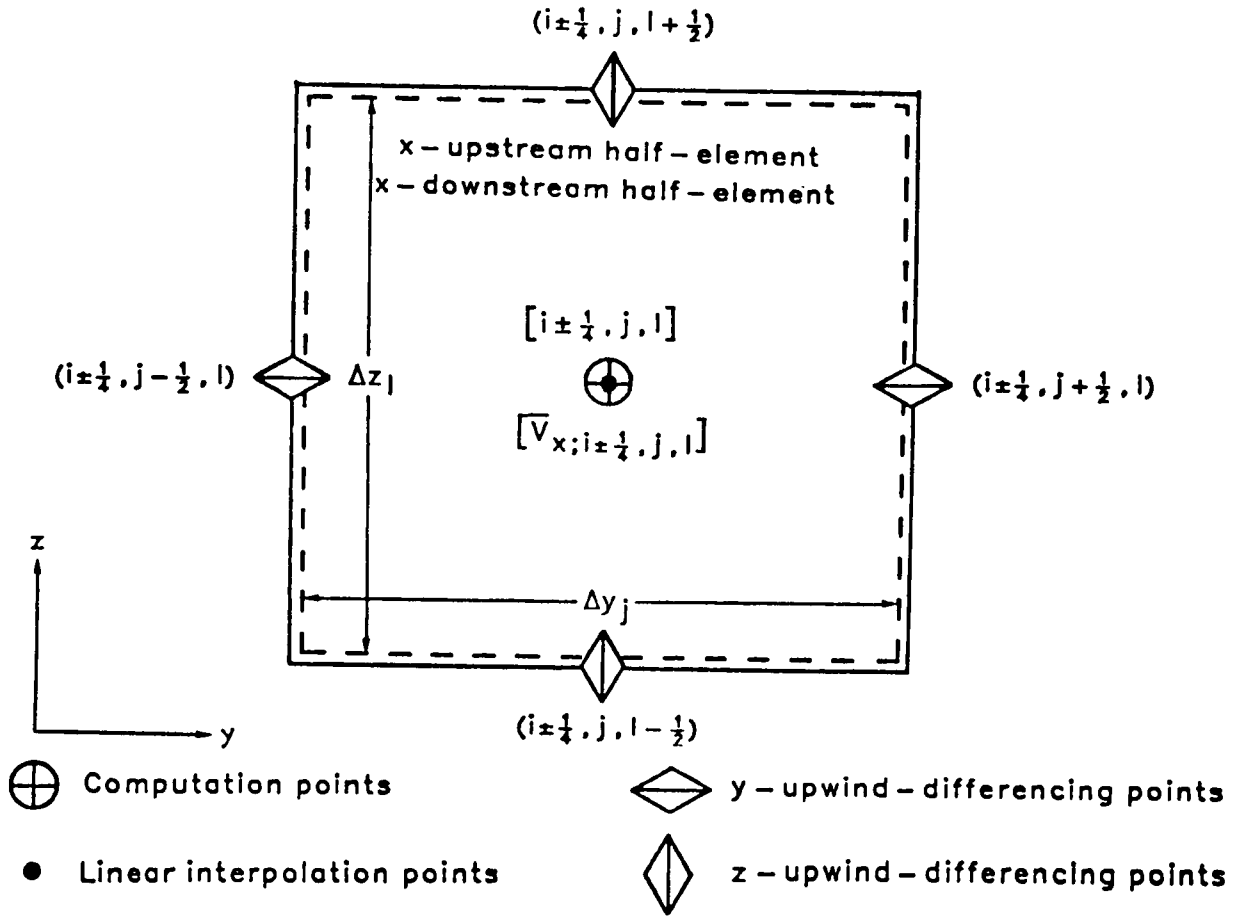

Figure 2.9. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,1) which is also a vertically interior element [side view in (y,z) plane].

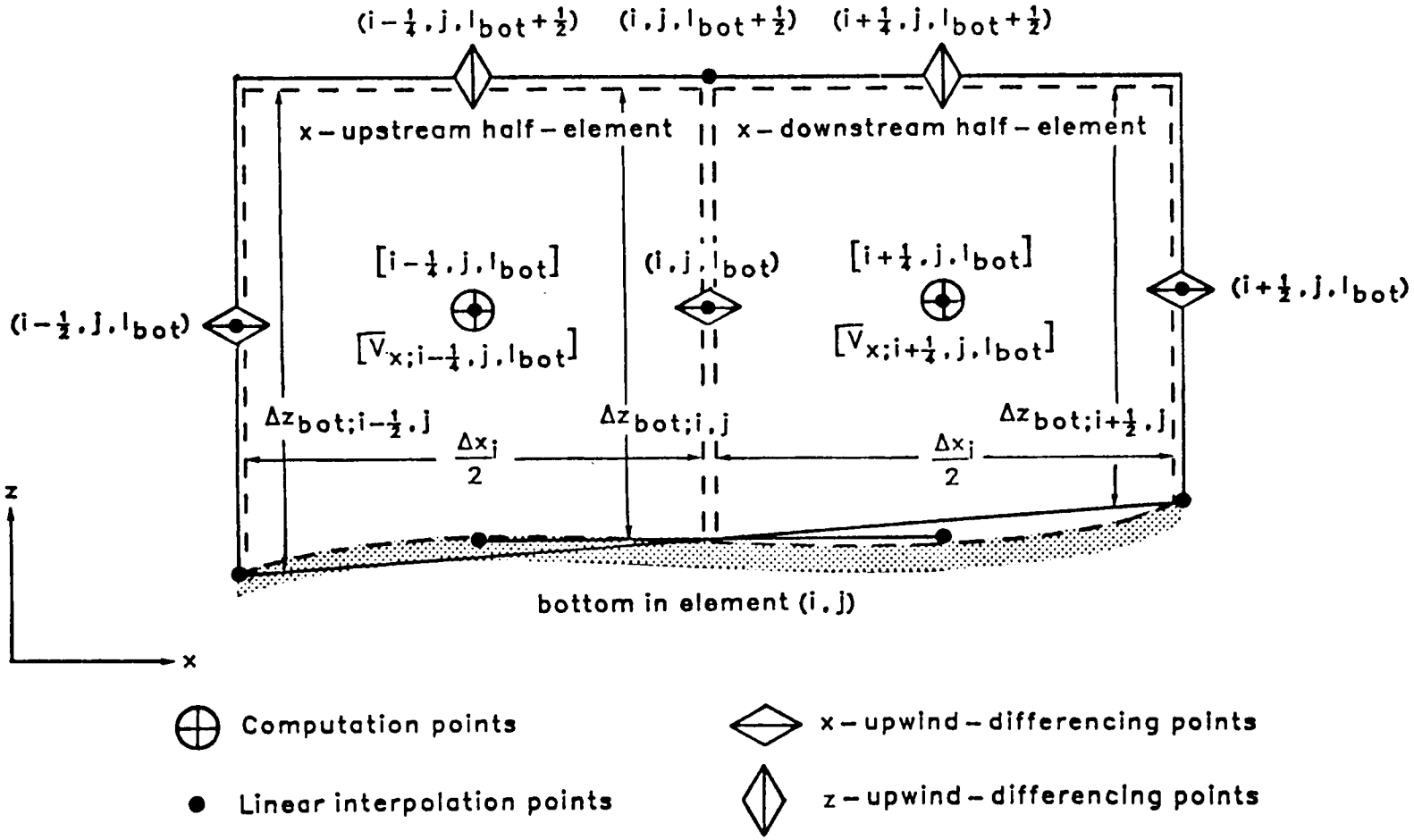

Figure 2.10. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,l(bot;i,j)], in the bottom layer l(bot;i,j), below the water-surface layer [side view in (x,z) plane].

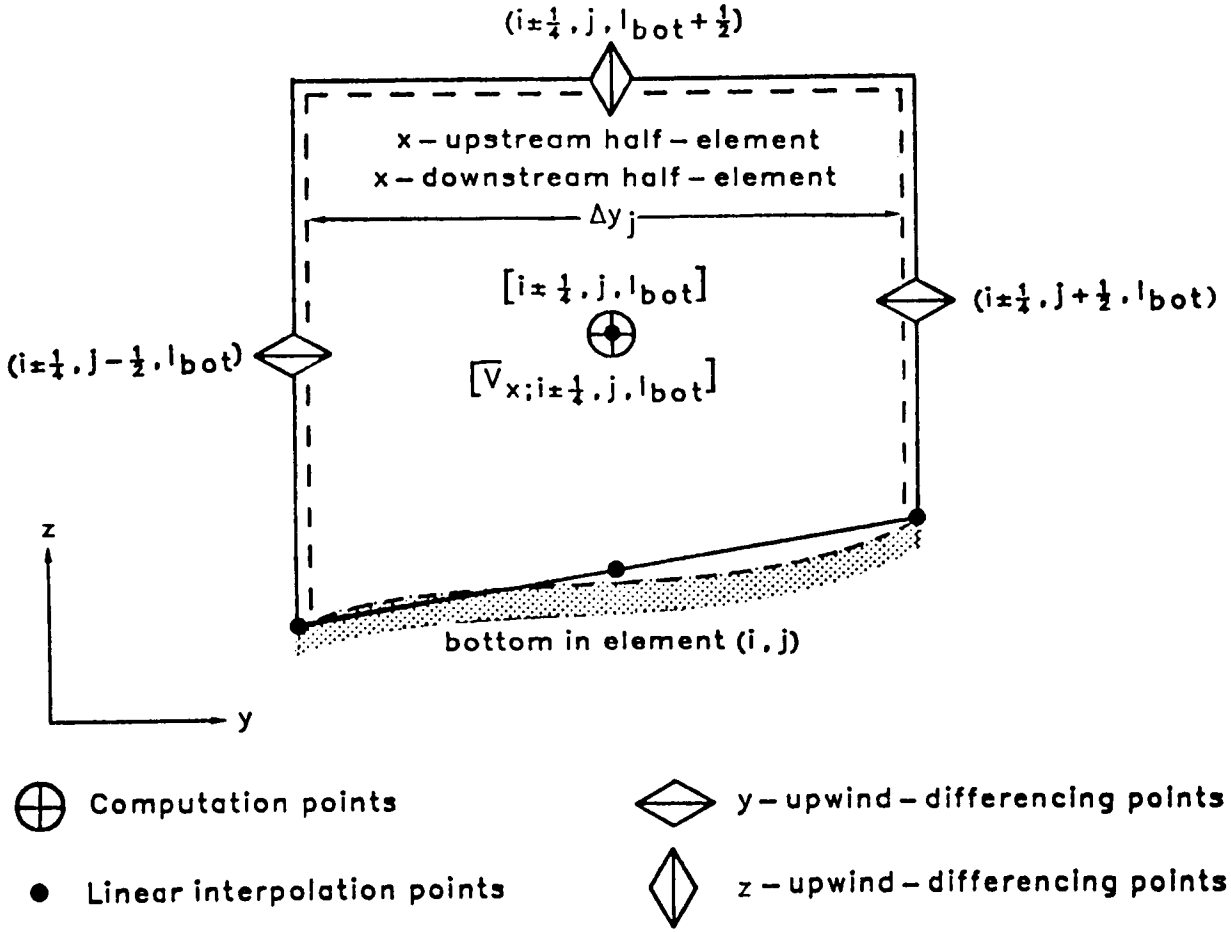

Figure 2.11. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,l(bot;i,j)], in the bottom layer l(bot;i,j), below the water-surface layer [side view in (y,z) plane].

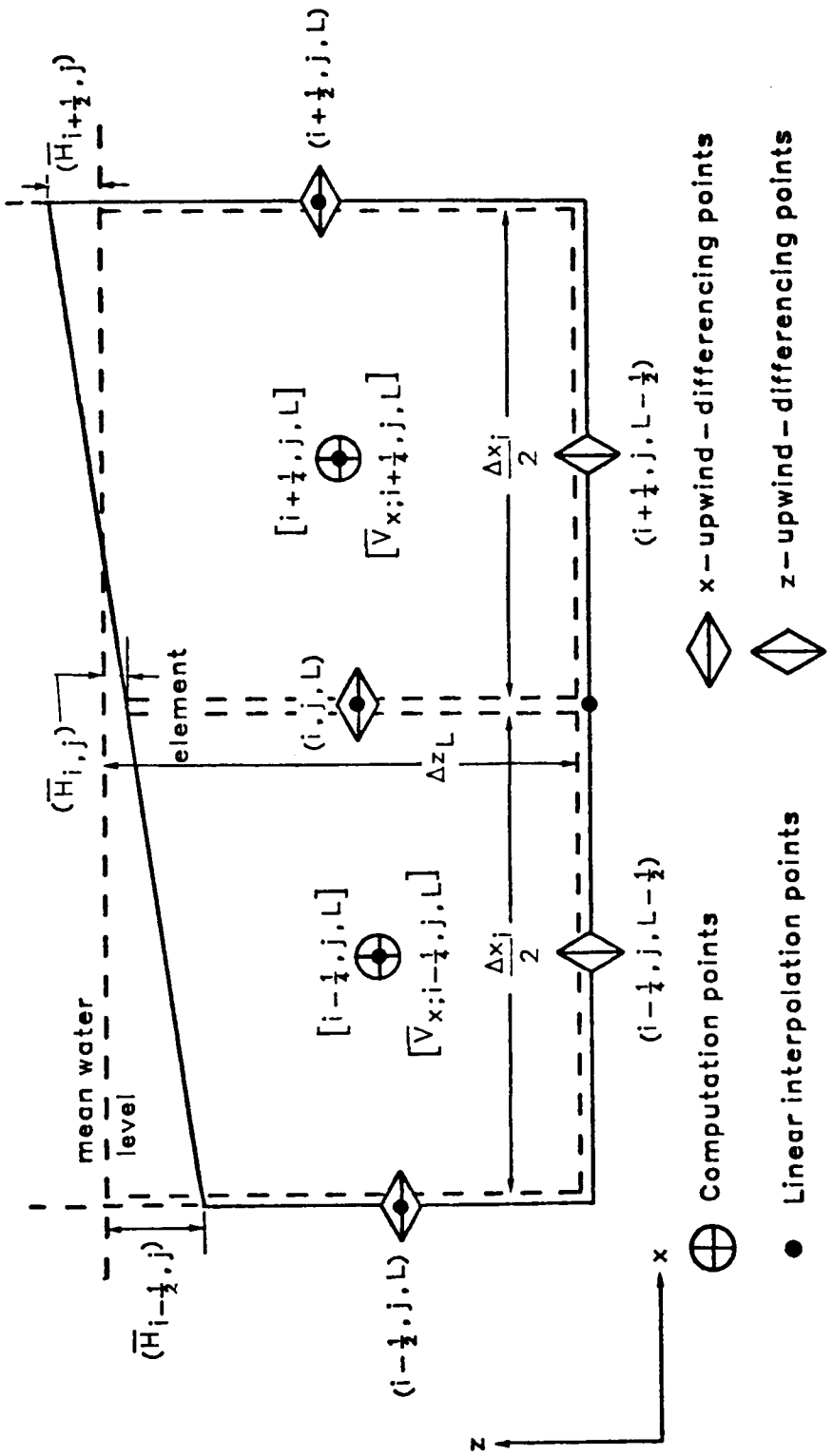

Figure 2.12. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,L), above the bottom layer, in the water-surface layer L [side view in (x,z) plane].

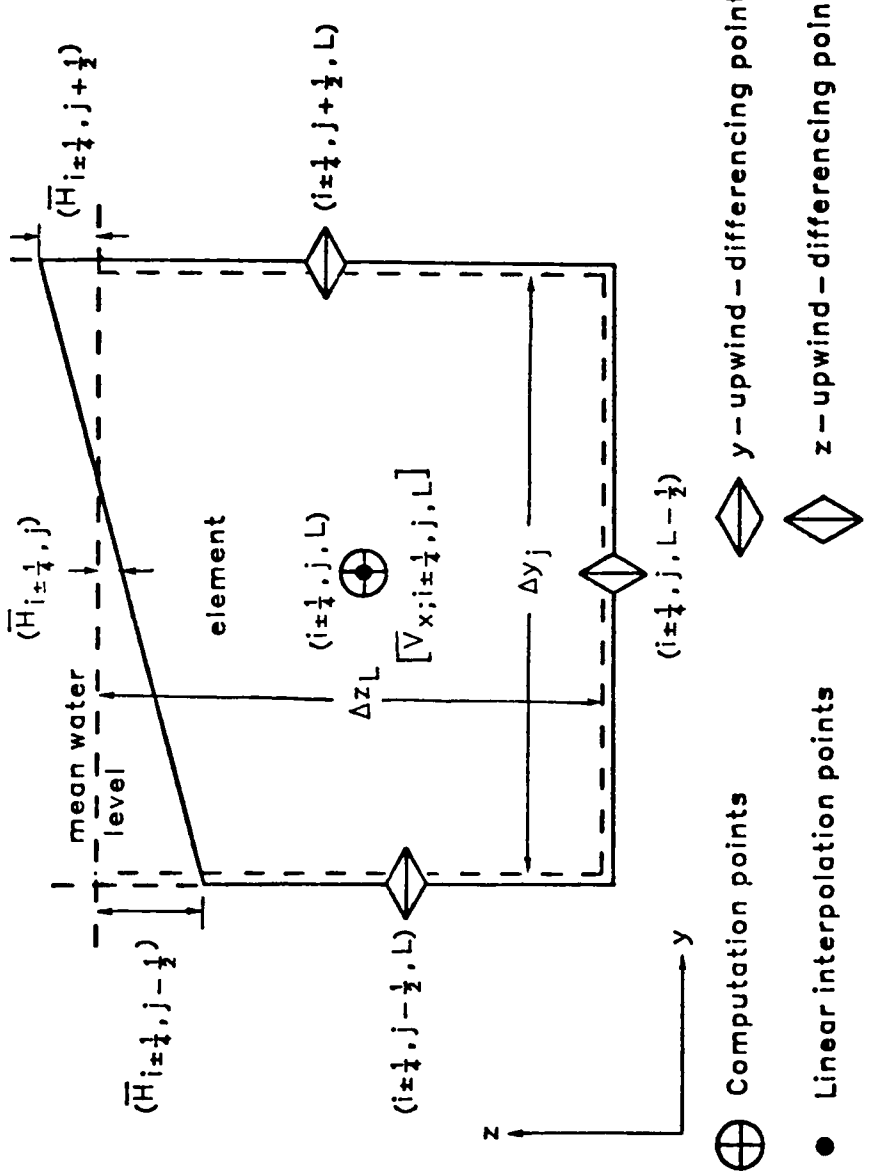

Figure 2.13. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,L), above the bottom layer, in the water-surface layer L [side view in (y,z) plane].

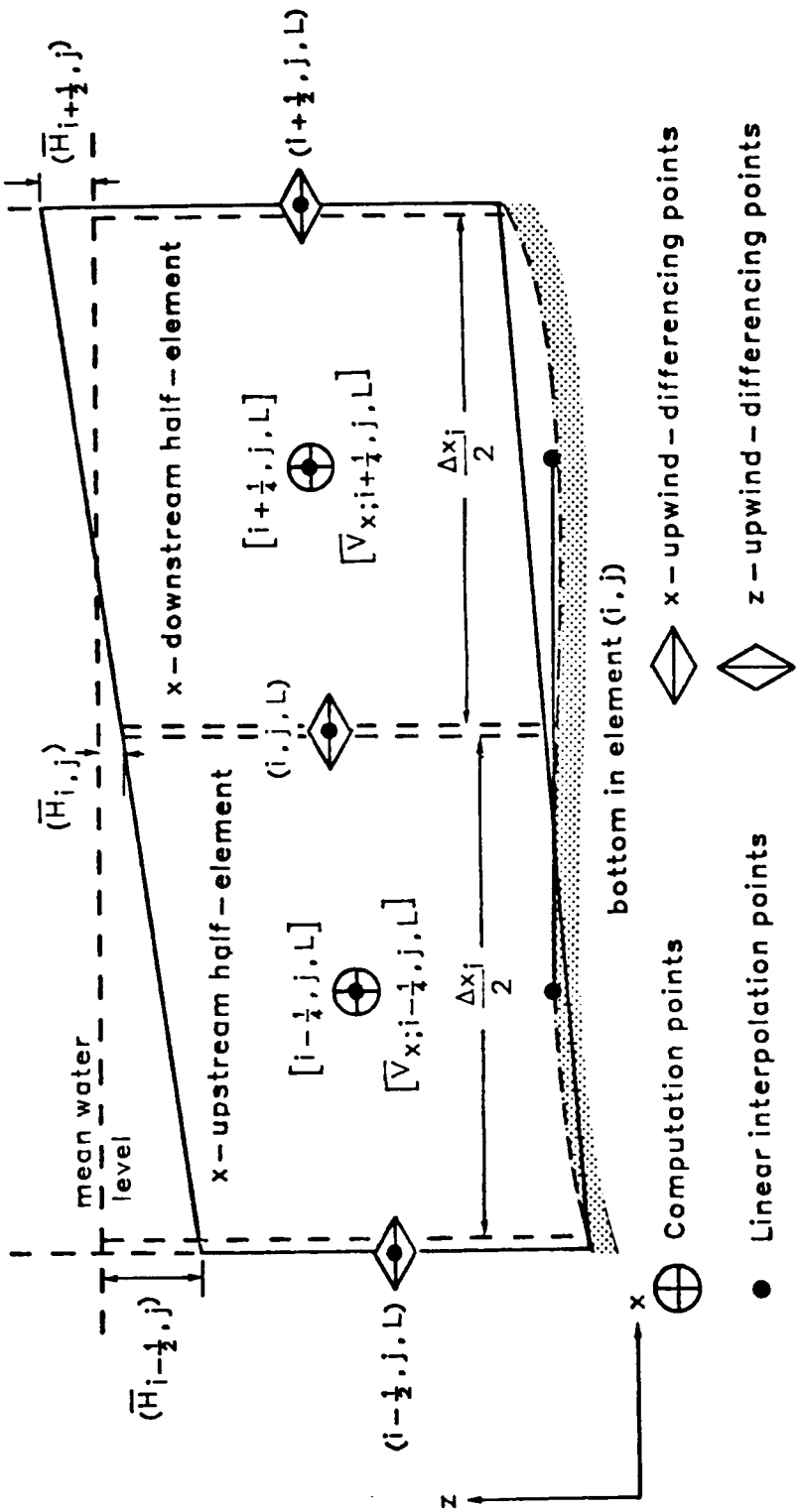

Figure 2.14. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,1(bot;i,j)] = (i,j,L), in the bottom layer 1(bot;i,j) and also in the water-surface layer L [side view in (x,z) plane].

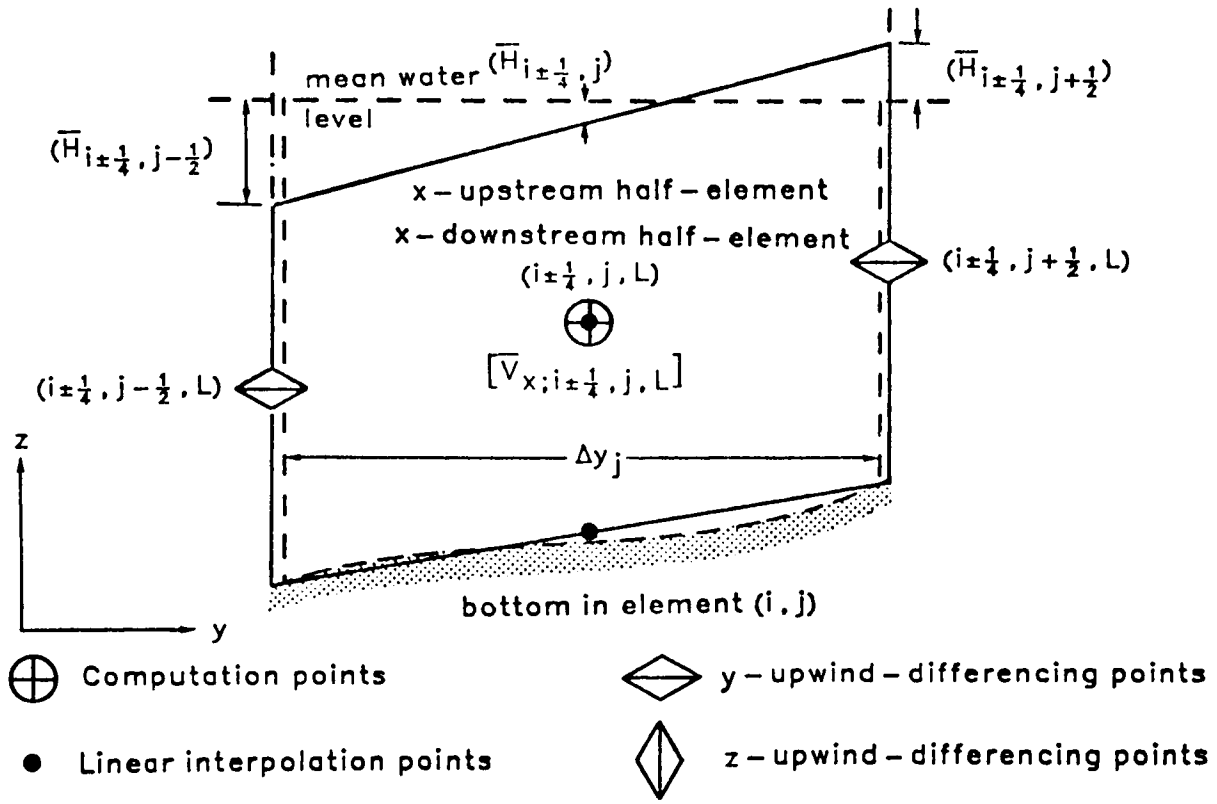

Figure 2.15. Locations used in the computations of x-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,l(bot;i,j)] = (i,j,L), in the bottom layer l(bot;i,j) and also in the water-surface layer L [side view in (y,z) plane].

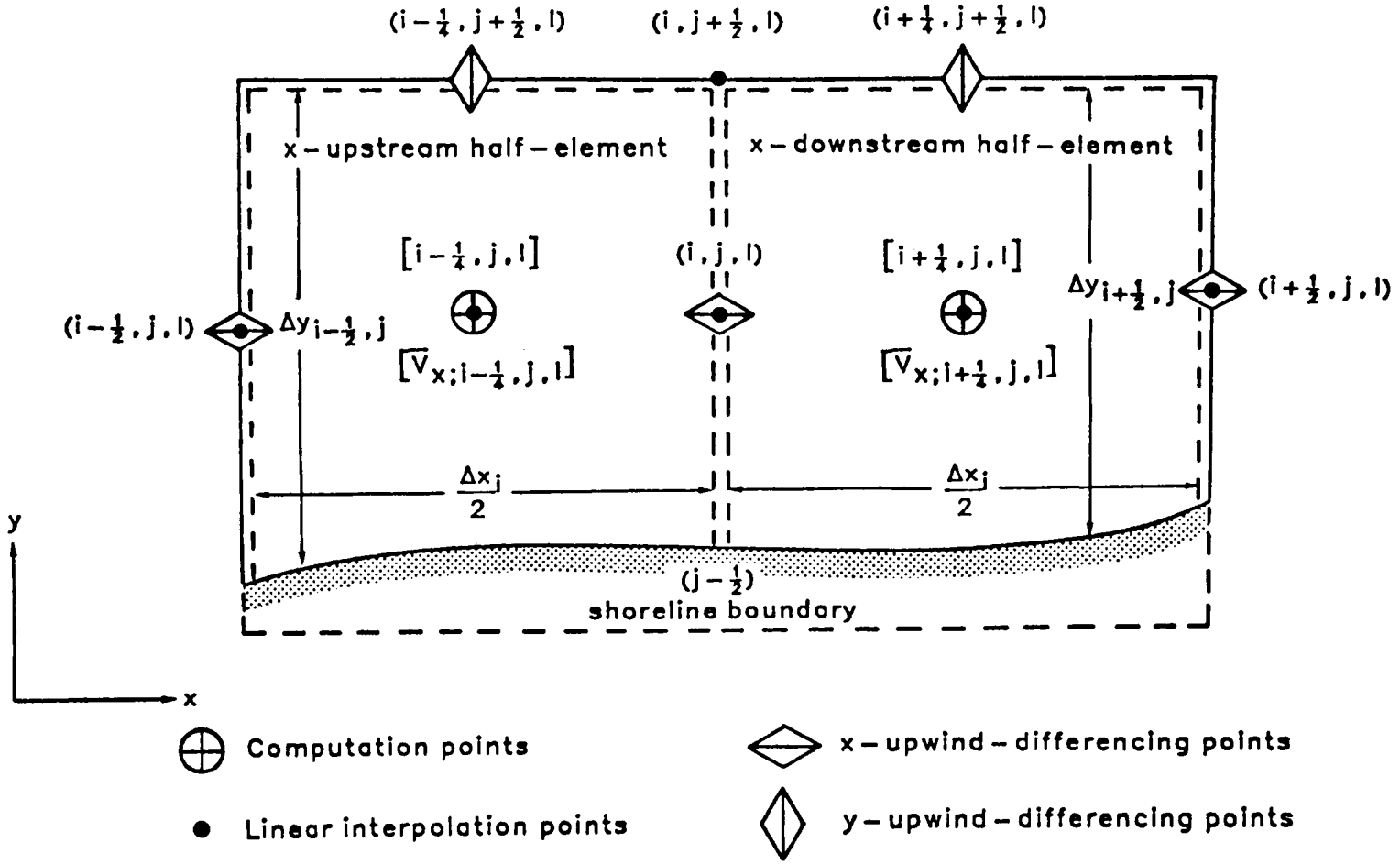

Figure 2.16. Locations used in the computations of x-component of velocity in two half-elements of a boundary element in the horizontal (x,y) plane [I(i,j),l] with a shoreline boundary at y-upstream enclosure surface at (i,j-½,l) [top view in (x,y) plane].

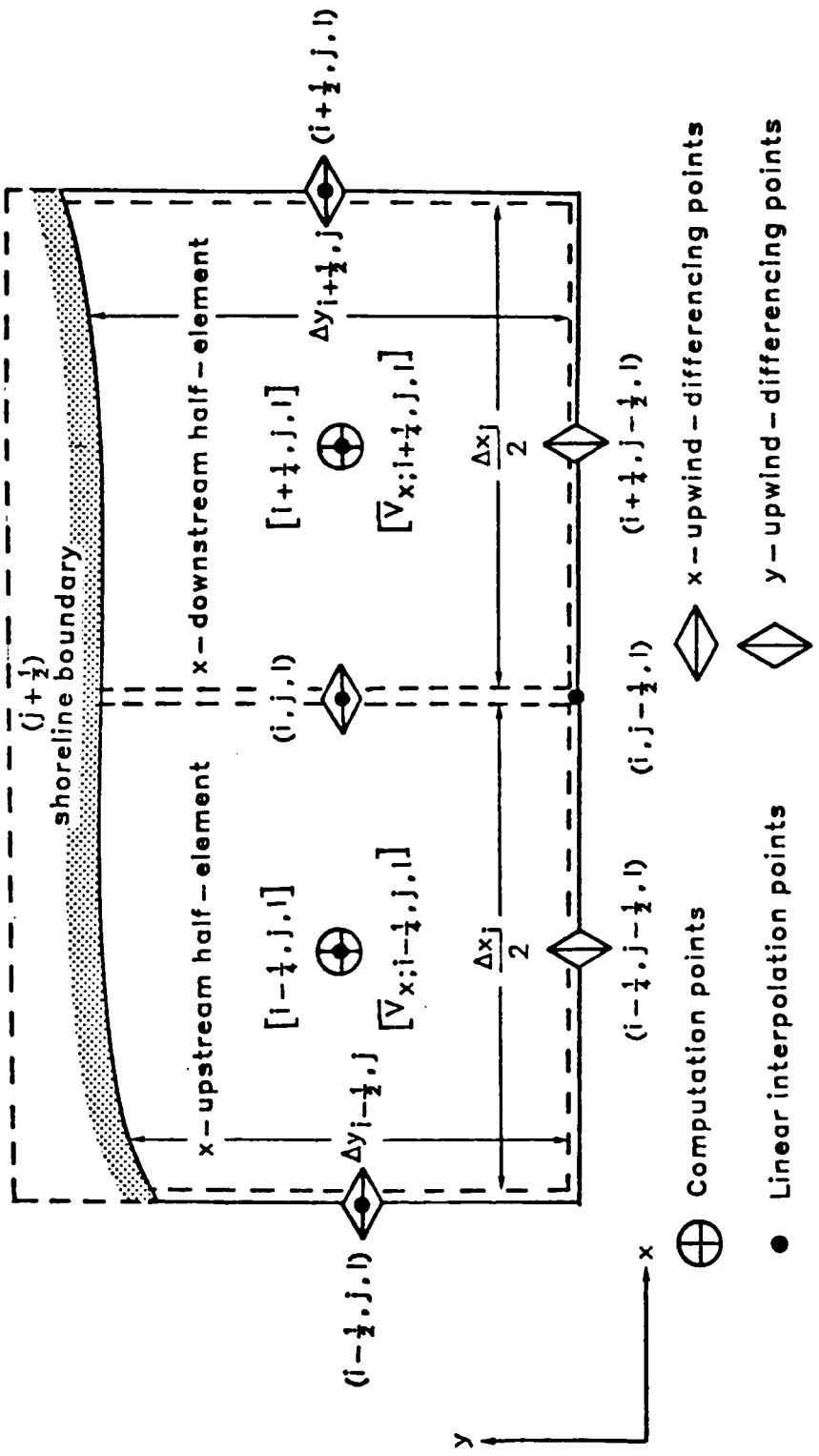

Figure 2.17. Locations used in the computations of x-component of velocity in two half-elements of a boundary element in the horizontal (x,y) plane [I((i,j),1] with a shoreline boundary at y-downstream enclosure surface at $(i,j+\frac{1}{2},1)$ [top view in (x,y) plane].

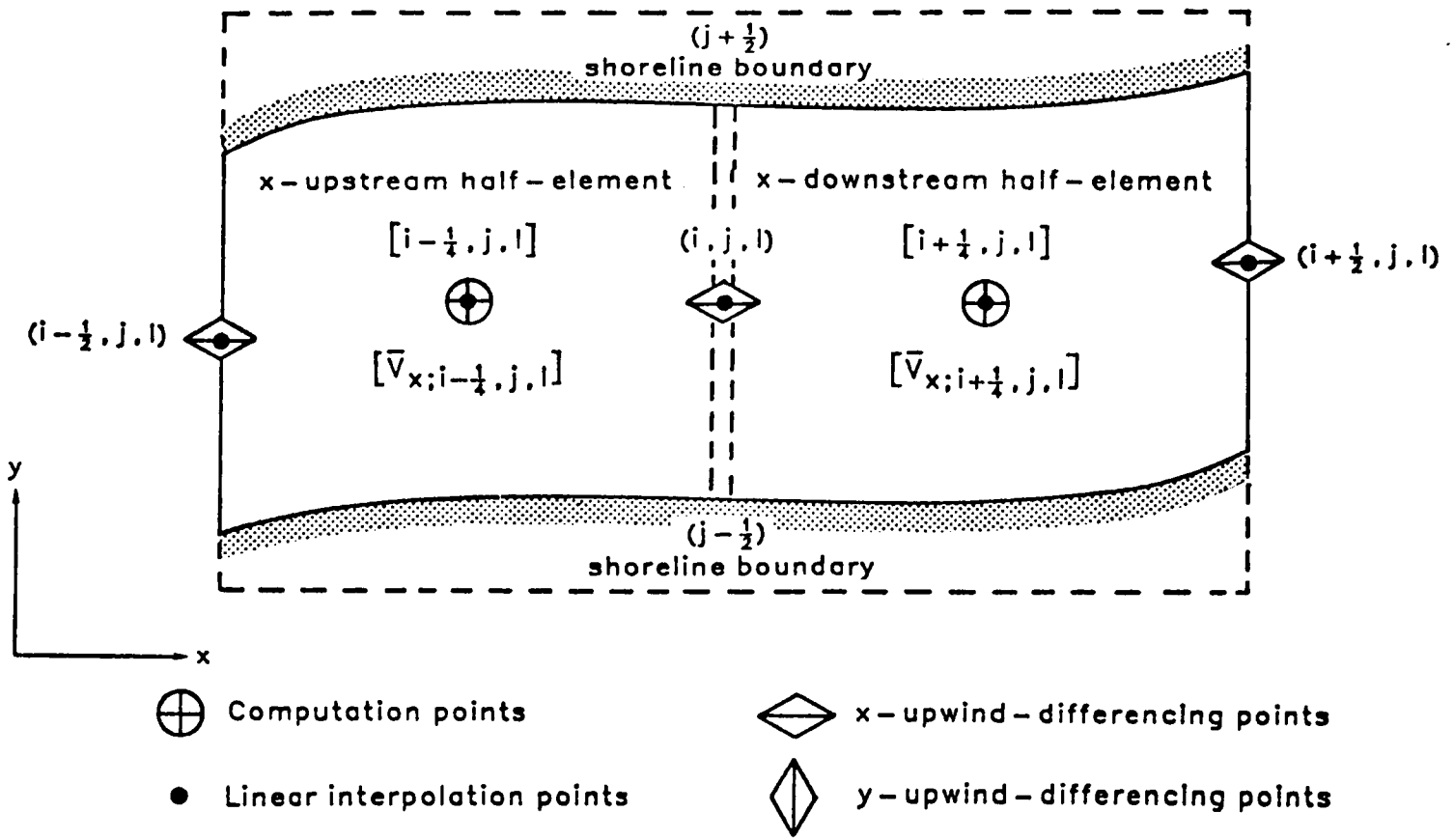

Figure 2.18. Locations used in the computations of x-component of velocity in two half-elements of a boundary element in the horizontal (x,y) plane [I(i,j),1] with shoreline boundaries at the y-upstream enclosure surface at (i,j-½,1) and also at the y-downstream enclosure surface at (i,j+½,1) [top view in (x,y) plane].

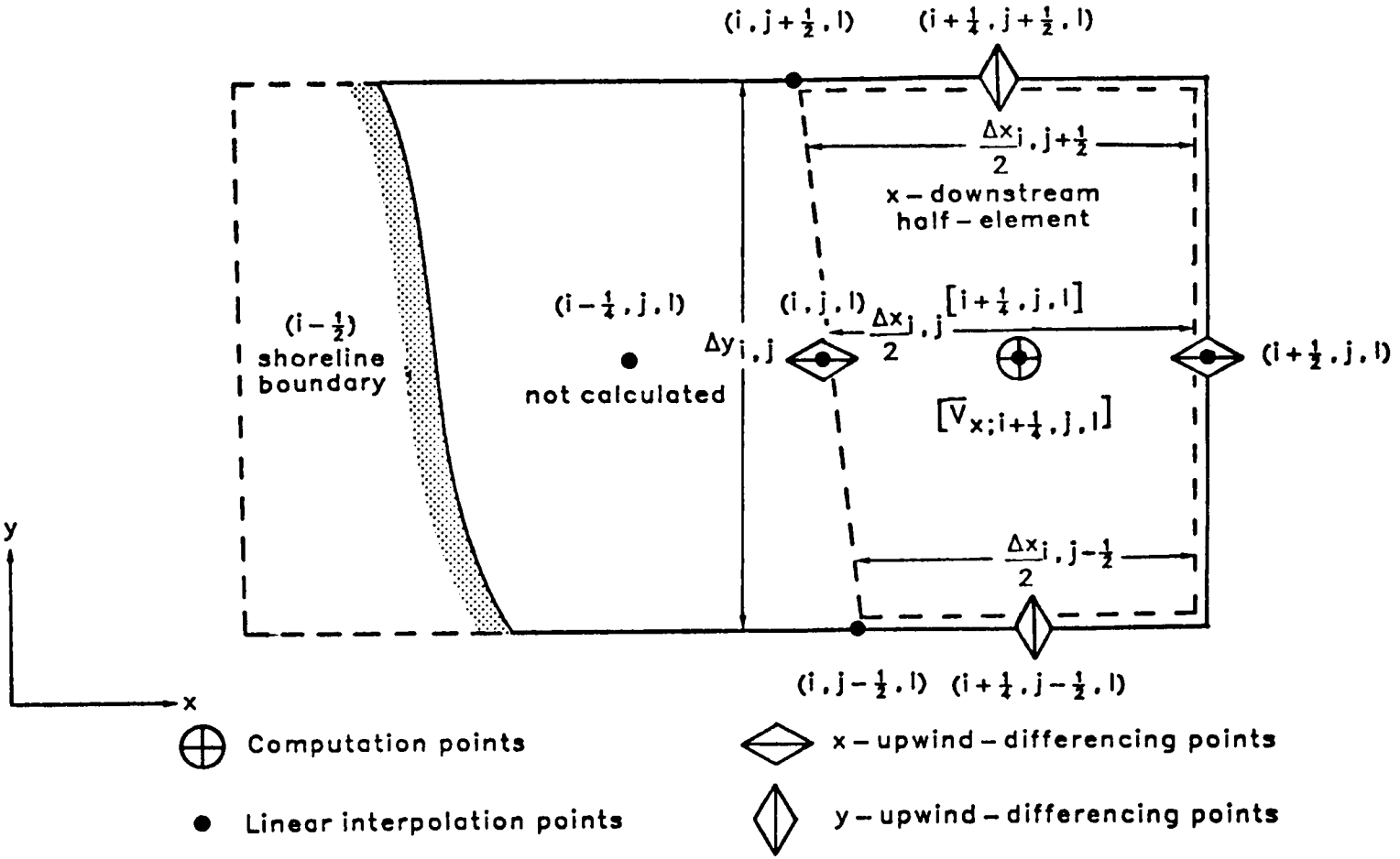

Figure 2.19. Locations used in the computations of x-component of velocity in a half-element of a boundary element in the horizontal plane [I(i,j),l] with a shoreline boundary at the x-upstream enclosure surface at (i-½,j,l) [top view in (x,y) plane].

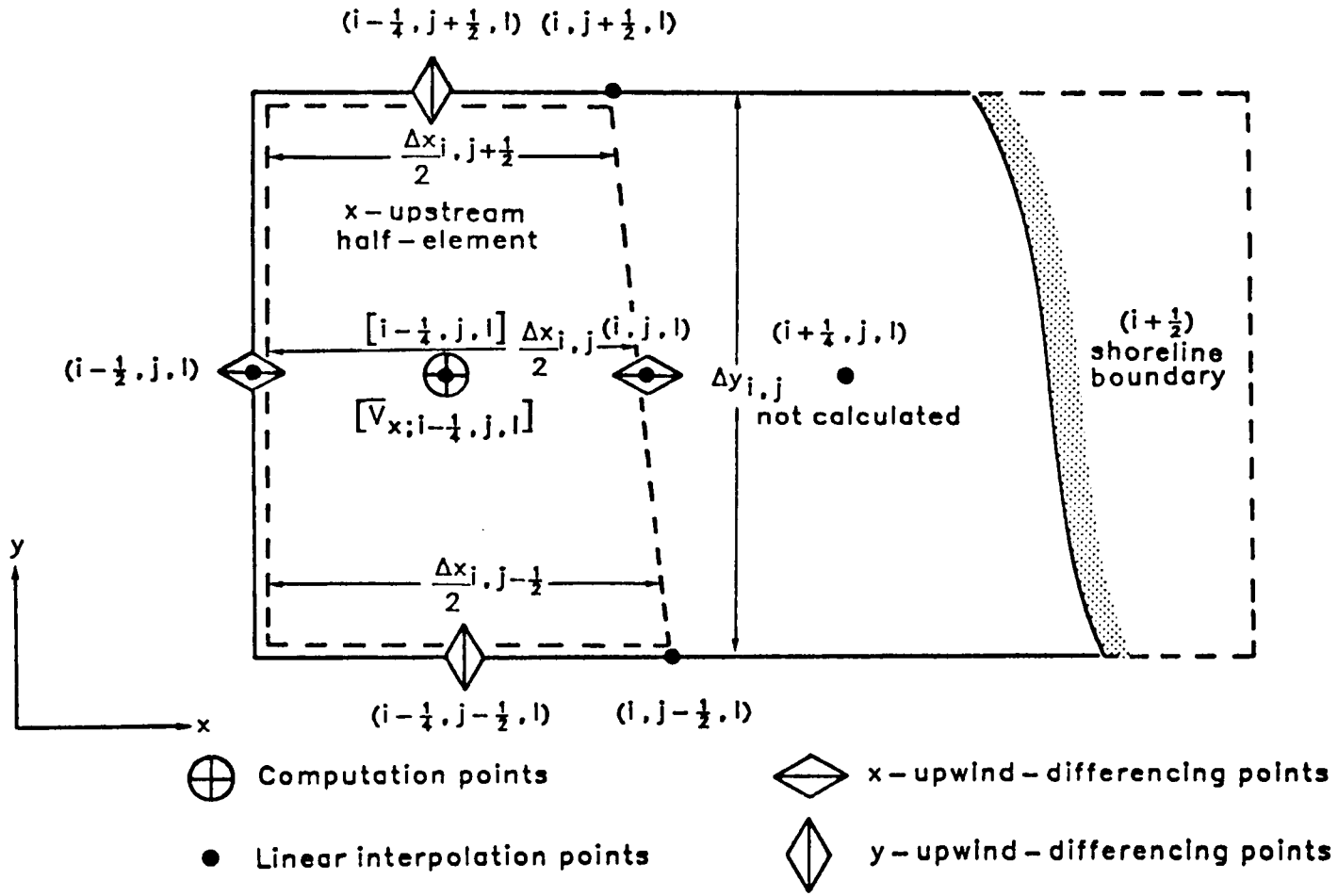

Figure 2.20. Locations used in the computations of x-component of velocity in a half-element of a boundary element in the horizontal plane [I(i,j),l] with a shoreline boundary at the x-downstream enclosure surface at (i+½,j,l) [top view in (x,y) plane].

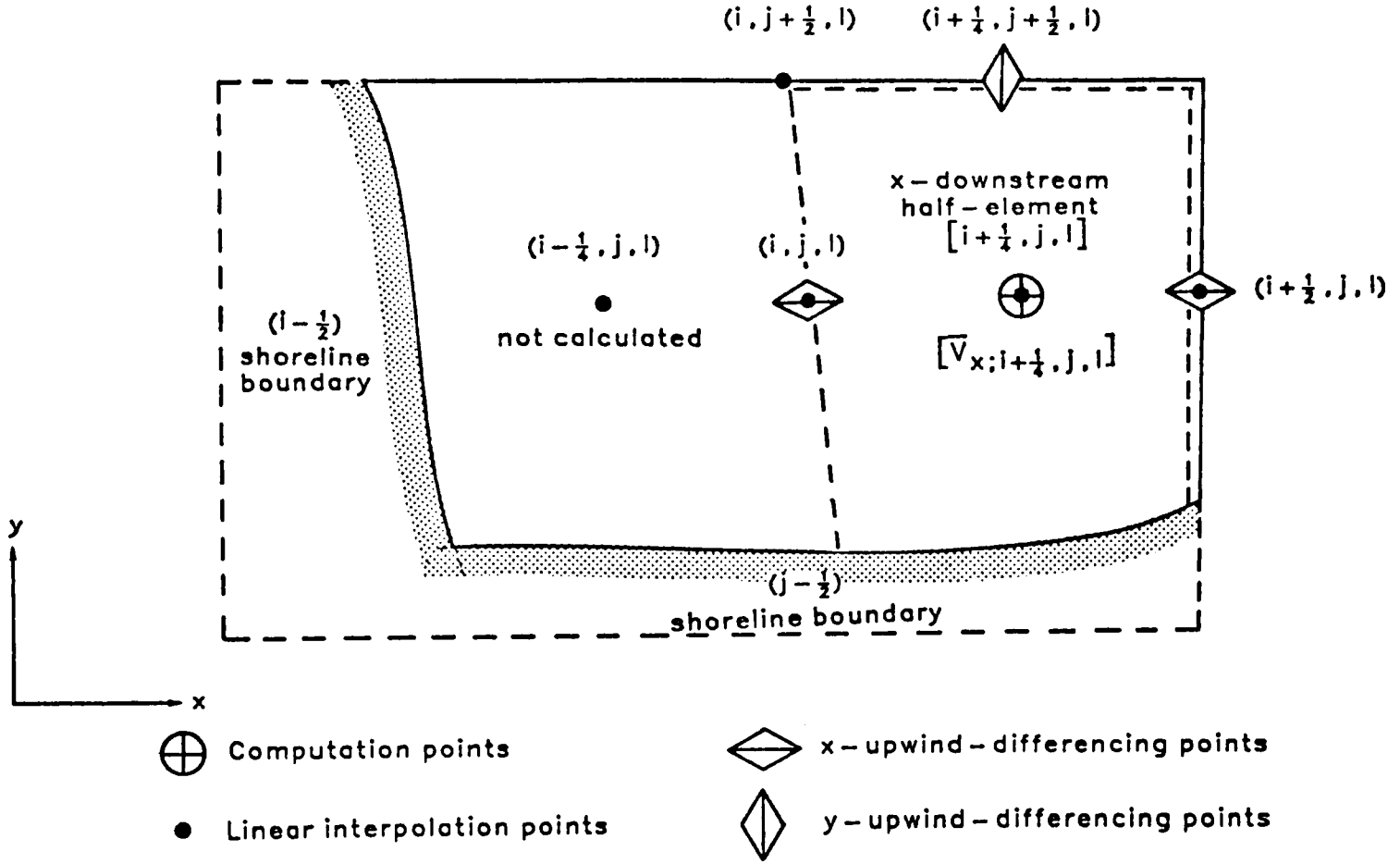

Figure 2.21. Locations used in the computation of x-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I(i,j),l] with shoreline boundaries at the x-upstream enclosure surface at (i-½,j,l) and at the y-upstream enclosure surface at (i,j-½,l) [top view in (x,y) plane].

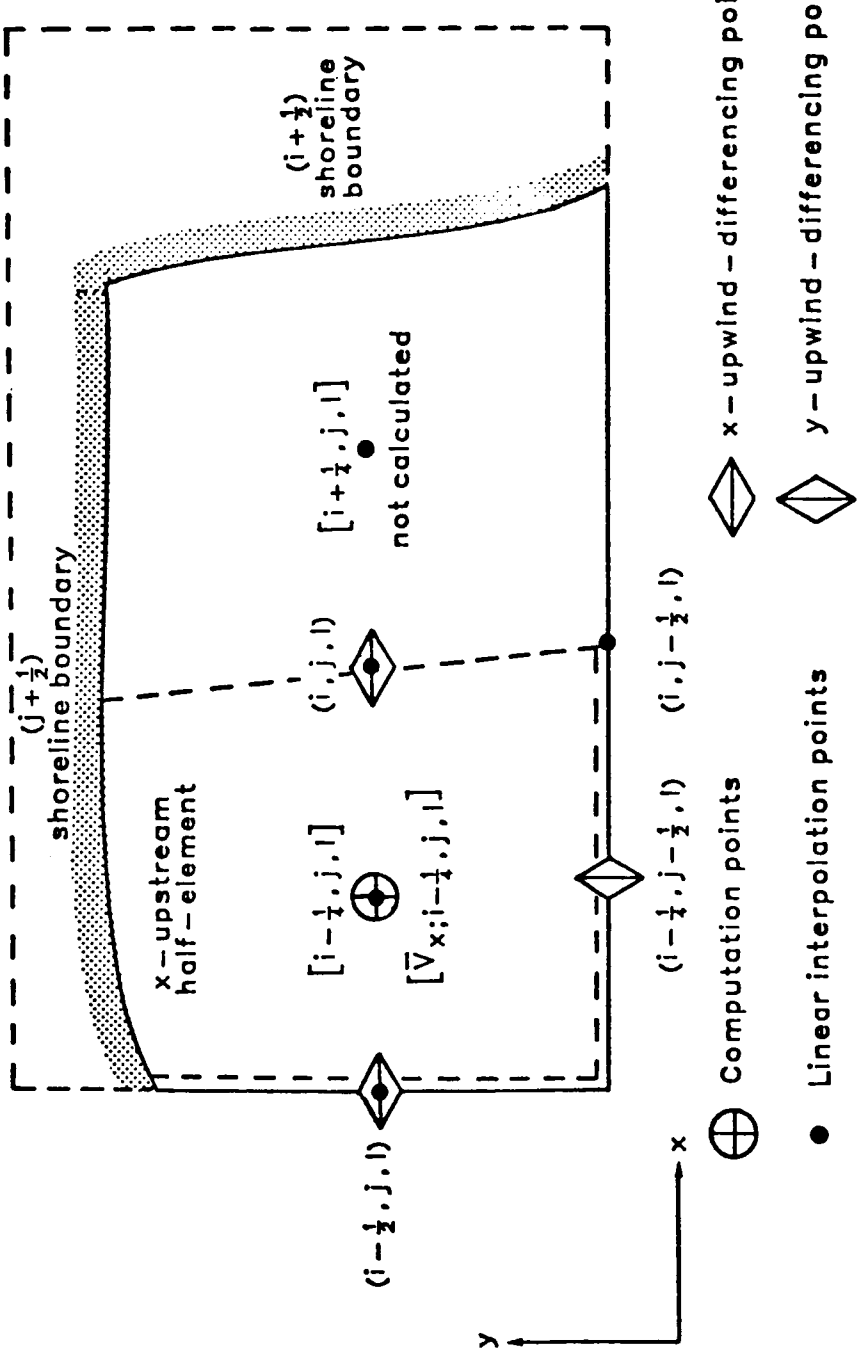

Figure 2.22. Locations used in the computation of x-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I((i,j),1] with shoreline boundaries at the x-downstream enclosure surface at $(i+\frac{1}{2},j,1)$ and at the y-downstream enclosure surface at $(i,j+\frac{1}{2},1)$ [top view in (x,y) plane].

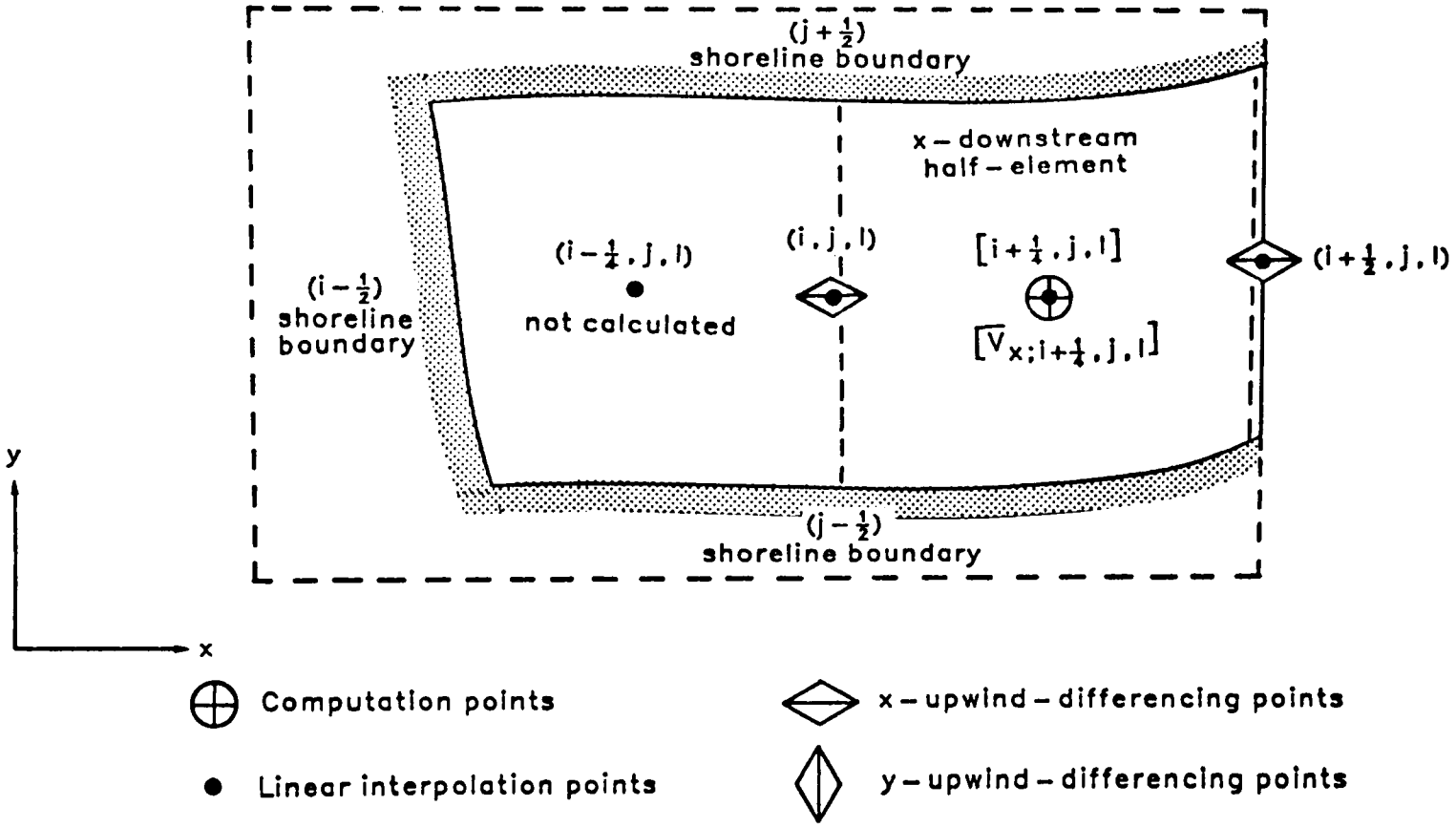

Figure 2.23. Locations used in the computation of x-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I(i,j),l] with shoreline boundaries at the x-upstream enclosure surface at (i-½,j,l), at the y-upstream enclosure surface at (i,j-½,l) and at the y-downstream enclosure surface at (i,j+½,l) [top view in (x,y) plane].

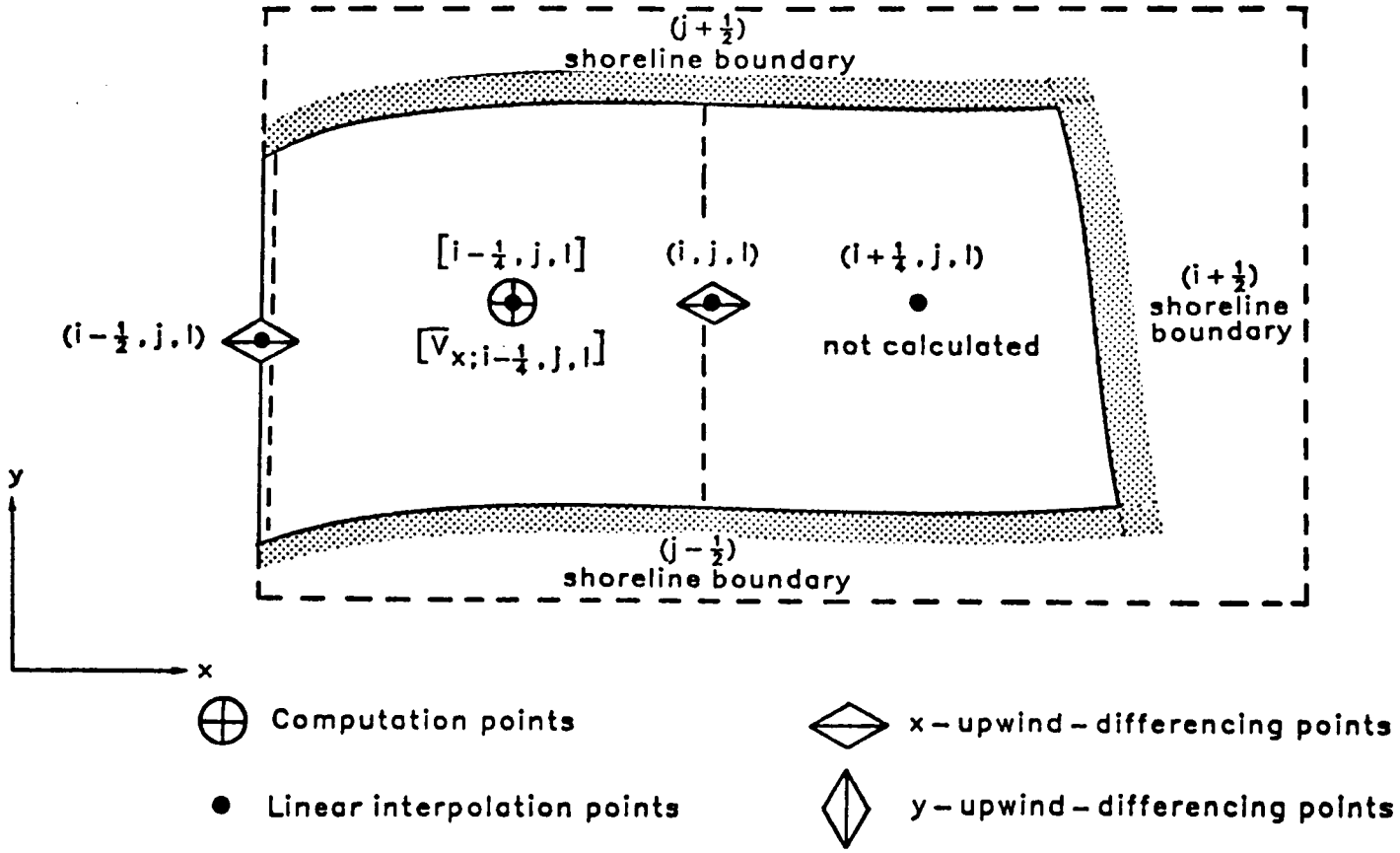

Figure 2.24. Locations used in the computation of x-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I(i,j),l] with shoreline boundaries at the x-downstream enclosure surface at (i+½,j,l), at the y-upstream enclosure surface at (i,j-½,l) and at the y-downstream enclosure surface at (i,j+½,l) [top view in (x,y) plane].

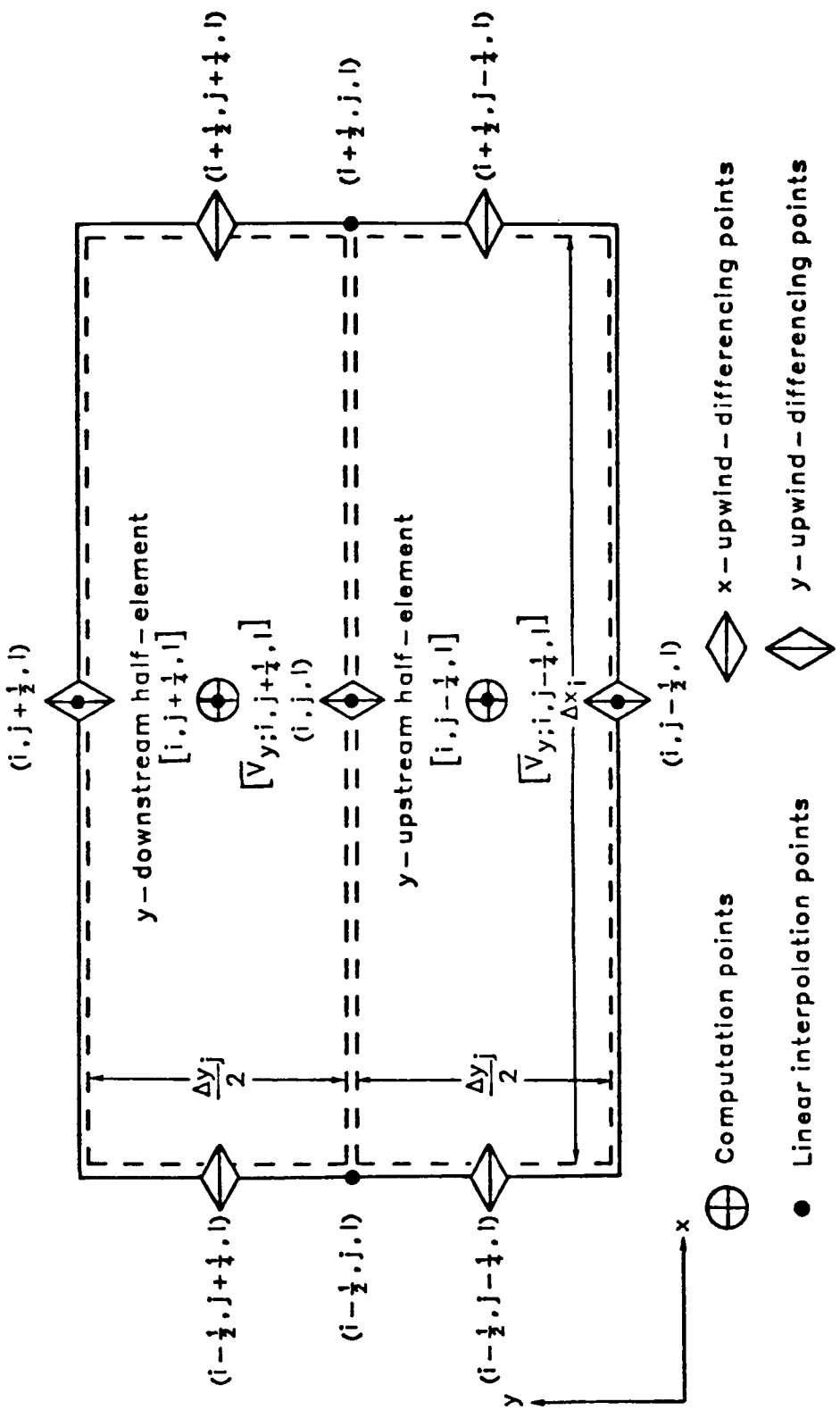

Figure 2.25. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,l) which is also a vertically interior element [top view in (x,y) plane].

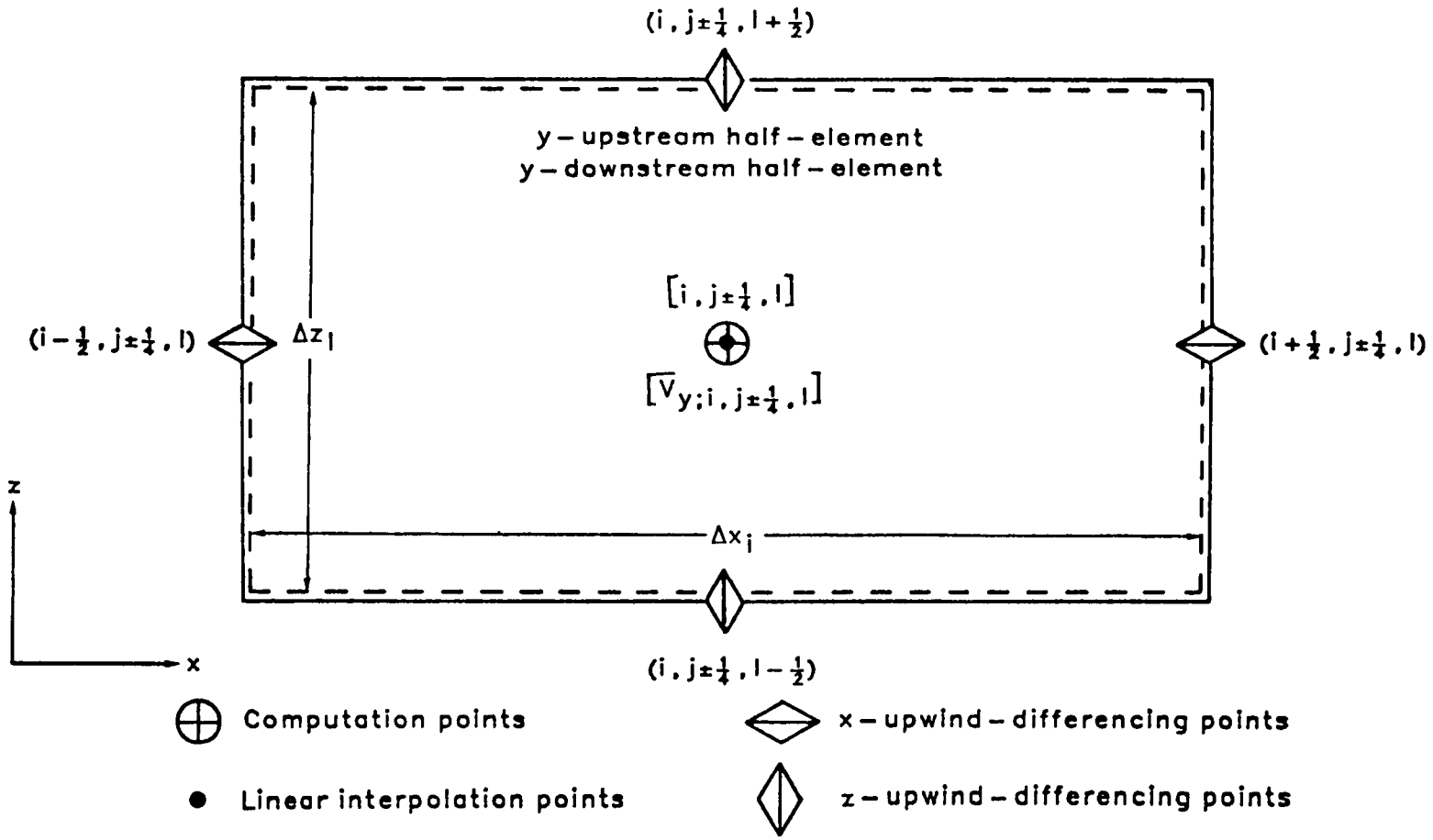

Figure 2.26. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,l) which is also a vertically interior element [side view in (x,z) plane].

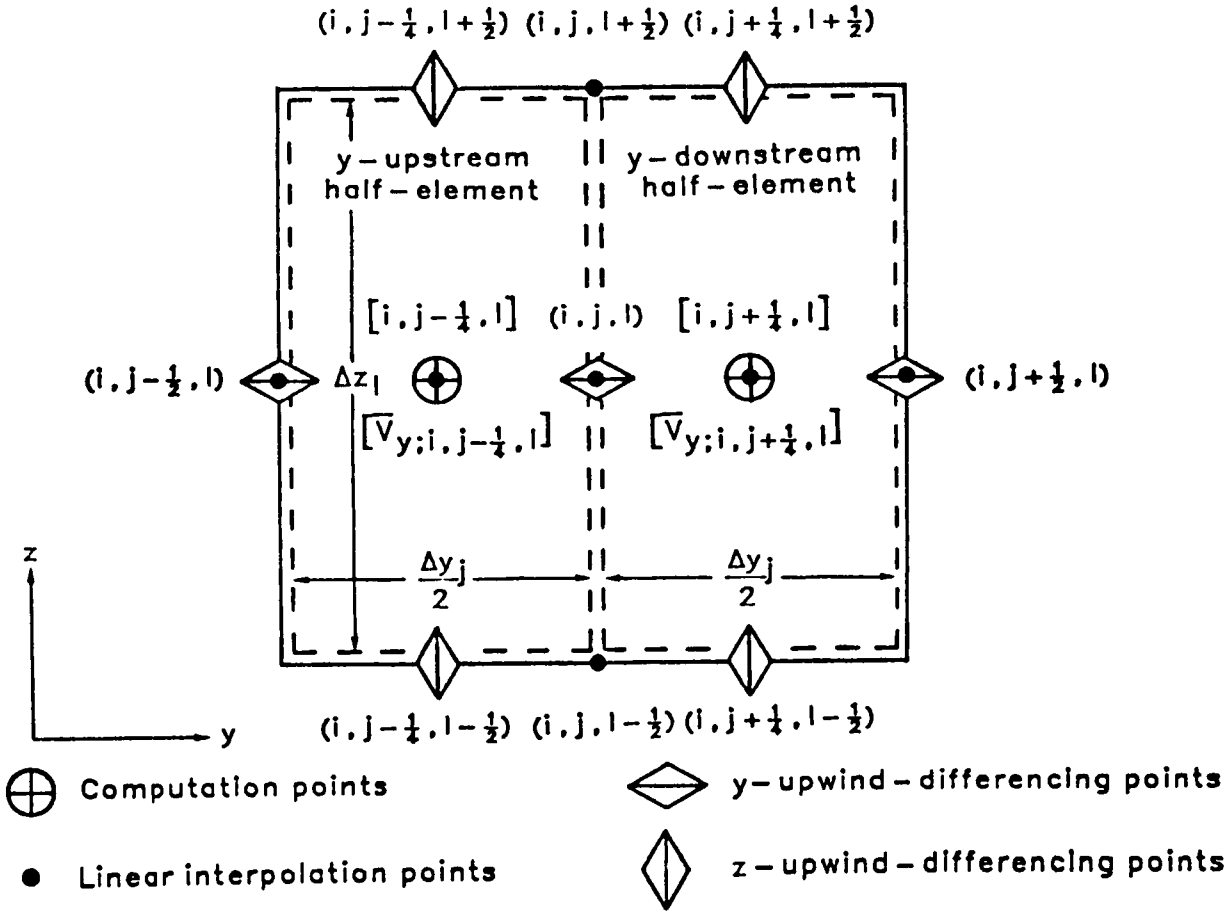

Figure 2.27. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,l) which is also a vertically interior element [side view in (y,z) plane].

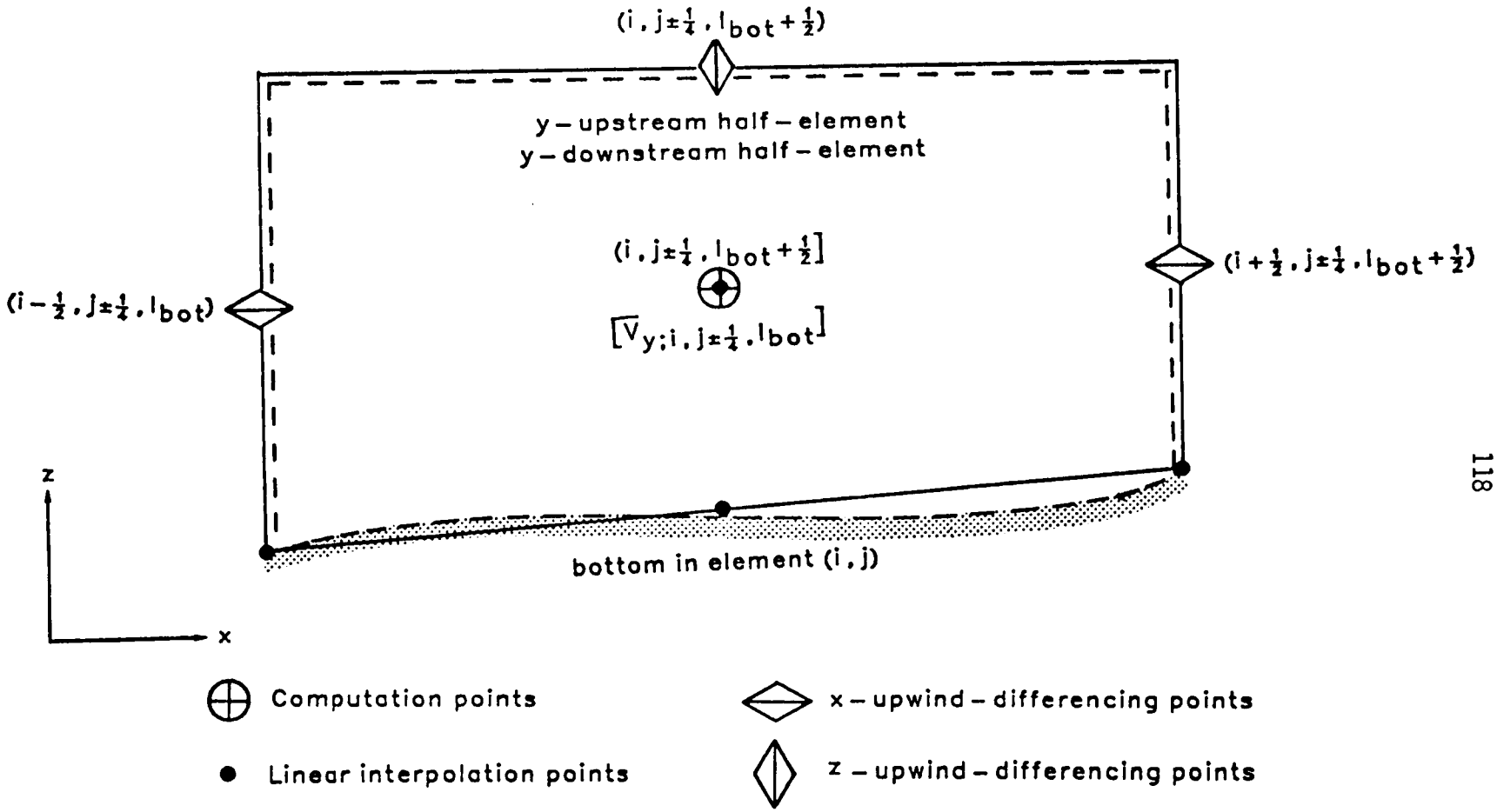

Figure 2.28. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,l(bot;i,j)], in the bottom layer l(bot;i,j), below the water-surface layer [side view in (x,z) plane].

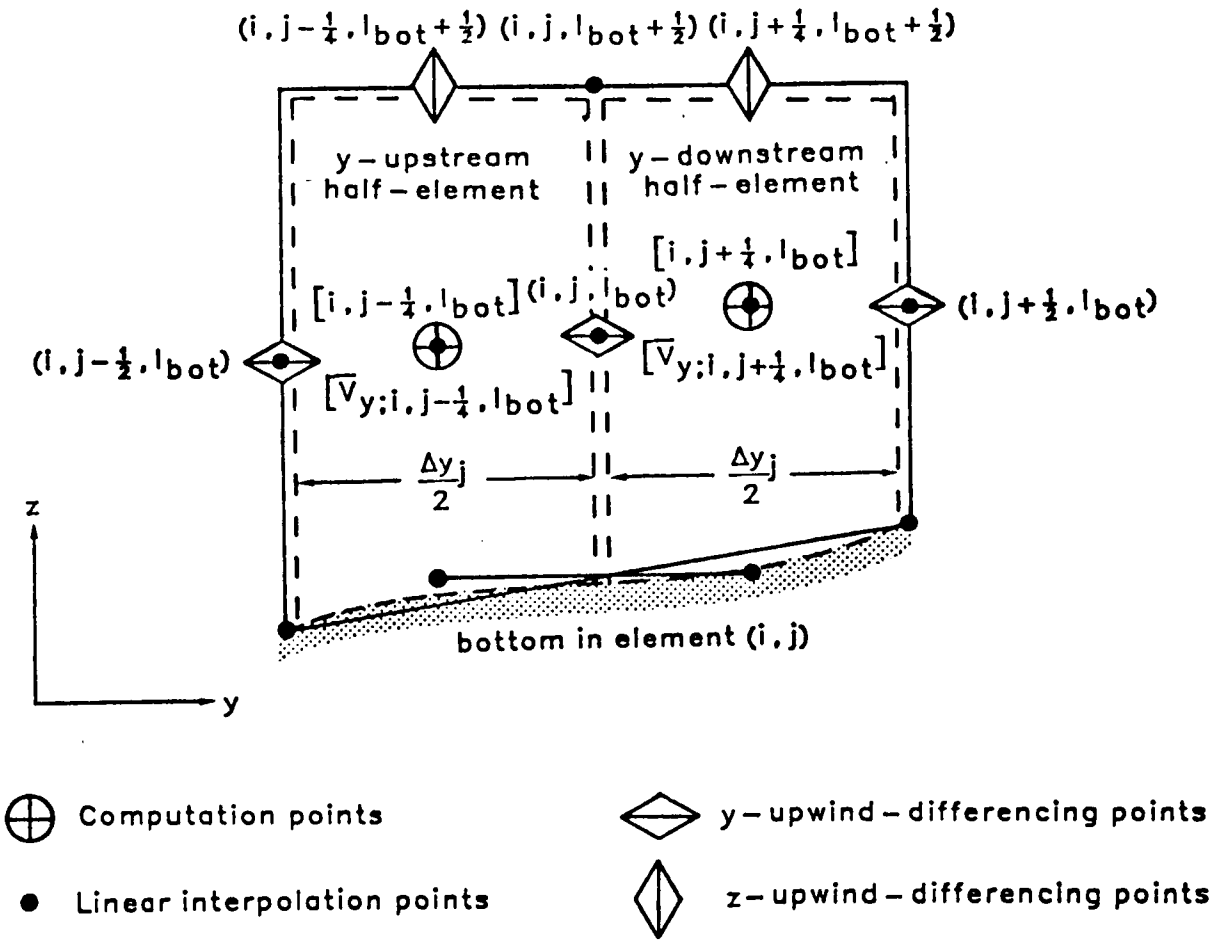

Figure 2.29. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,l(bot;i,j)], in the bottom layer l(bot;i,j), below the water-surface layer [side view in (y,z) plane].

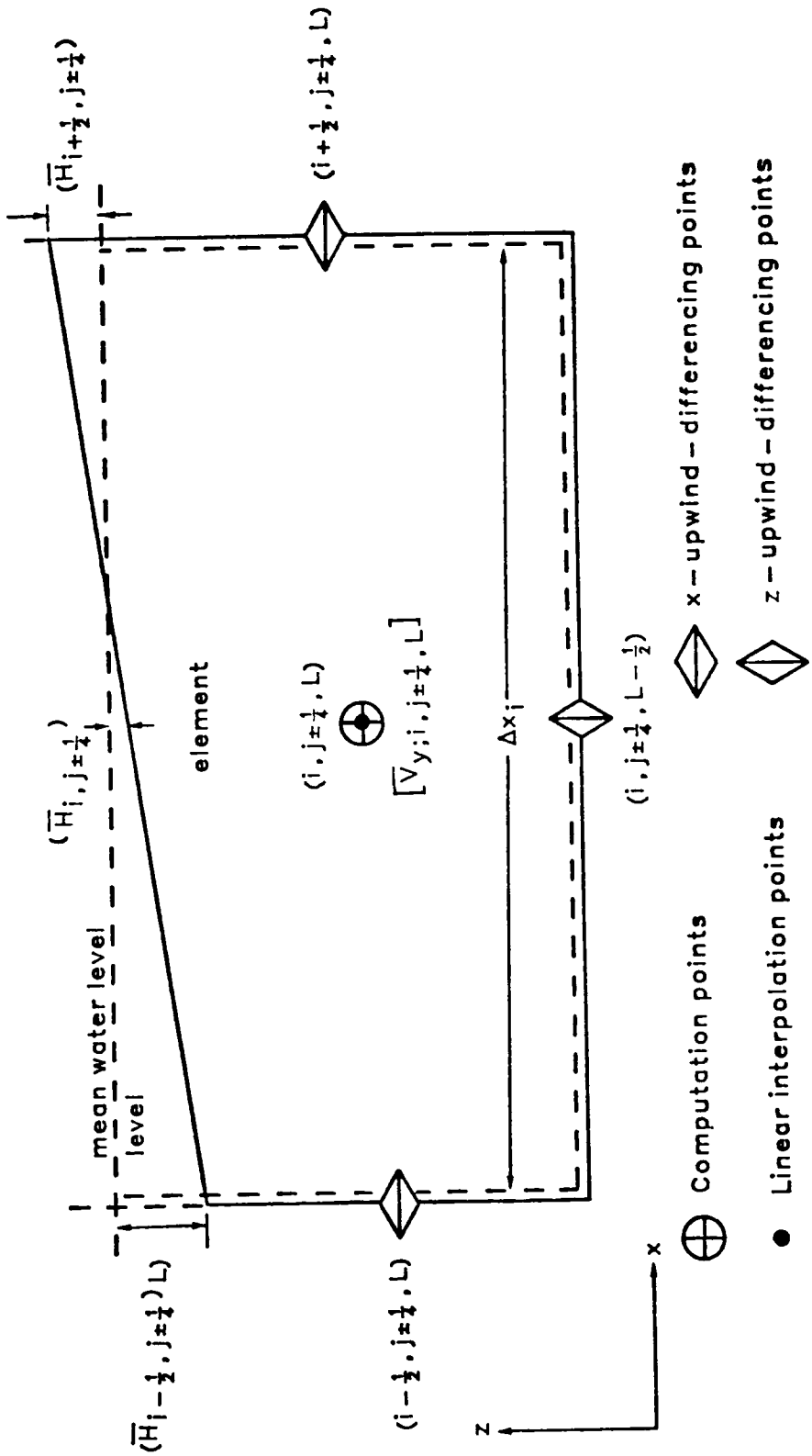

Figure 2.30. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,L), above the bottom layer, in the water-surface layer L [side view in (x,z) plane].

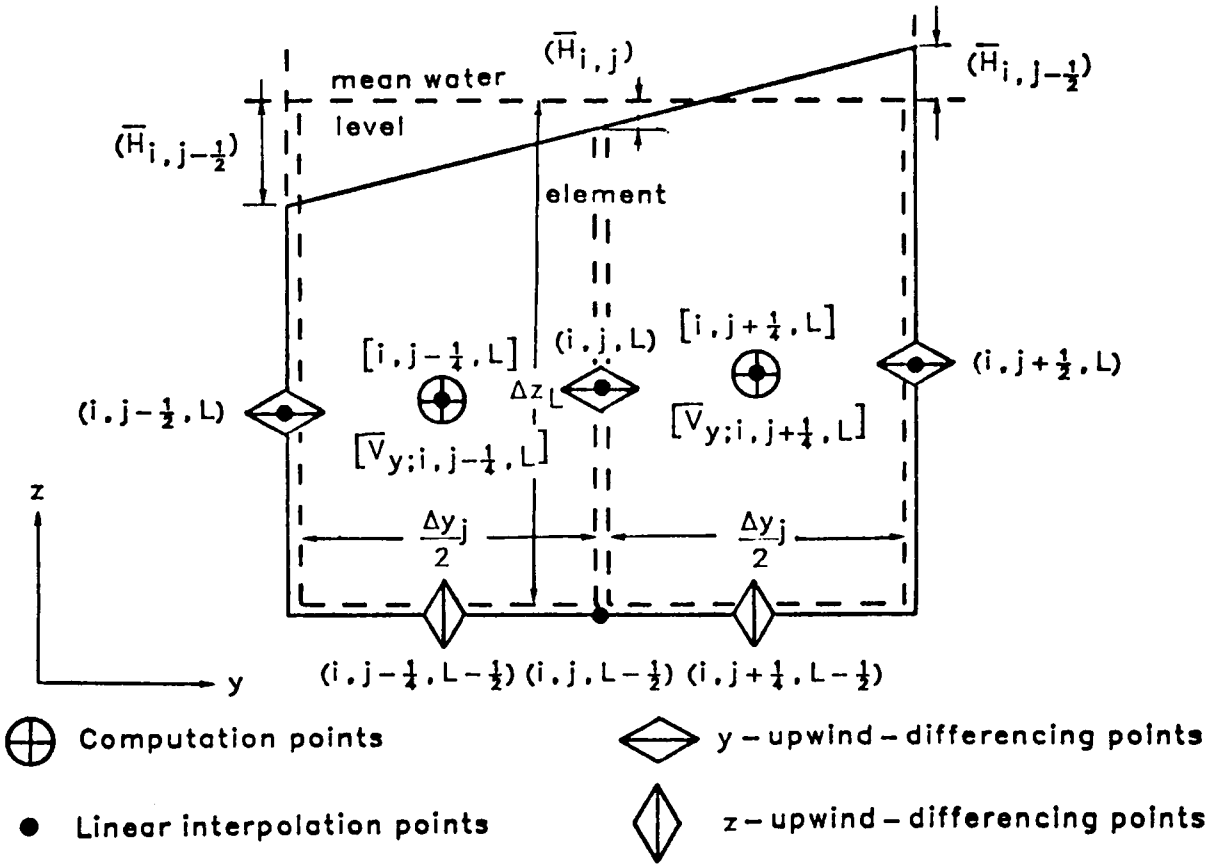

Figure 2.31. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane (i,j,L), above the bottom layer, in the water-surface layer L [side view in (y,z) plane].

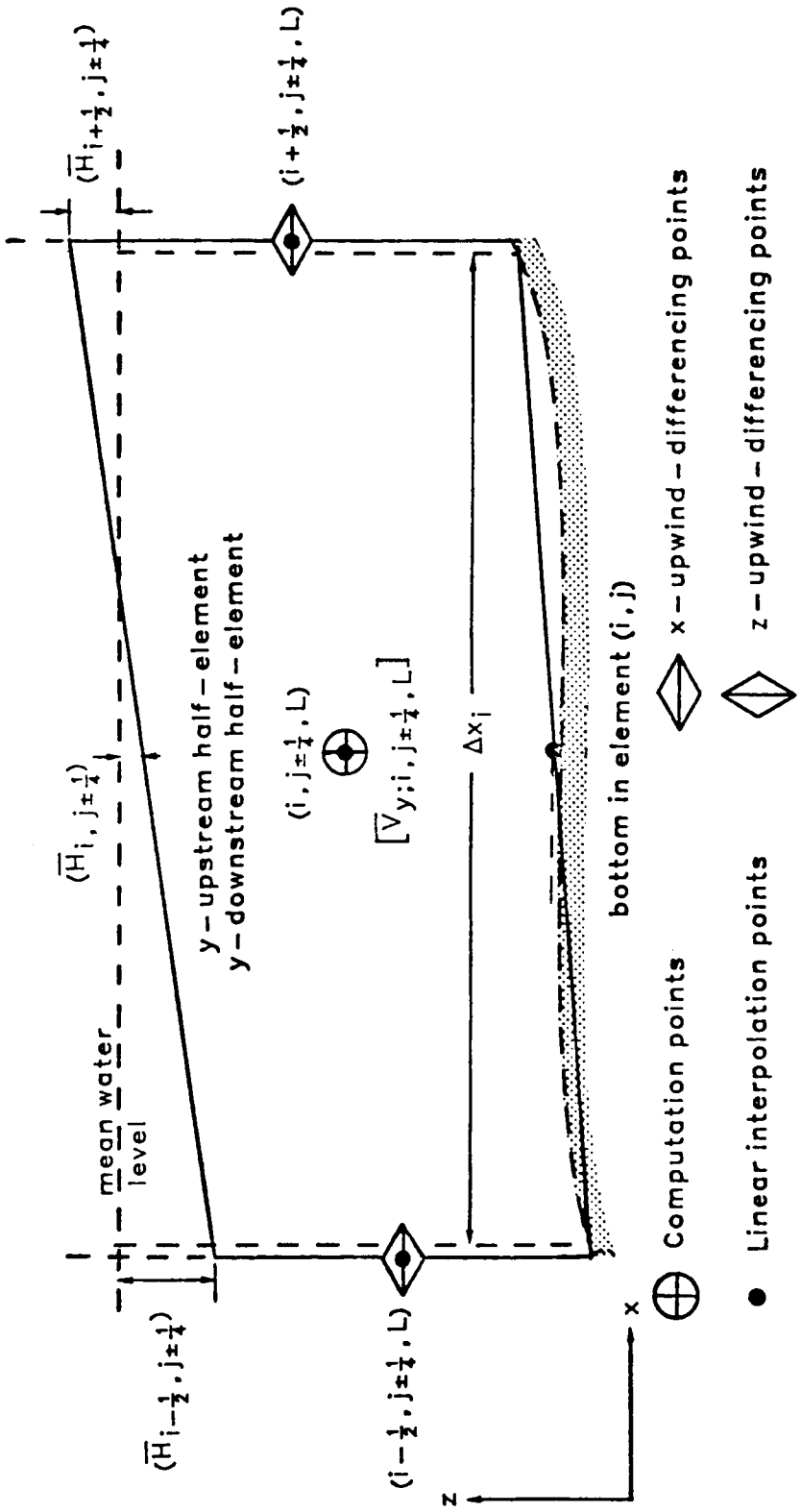

Figure 2.32. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,1(bot;i,j)] = (i,j,L), in the bottom layer 1(bot;i,j) and also in the water-surface layer L [side view in (x,z) plane].

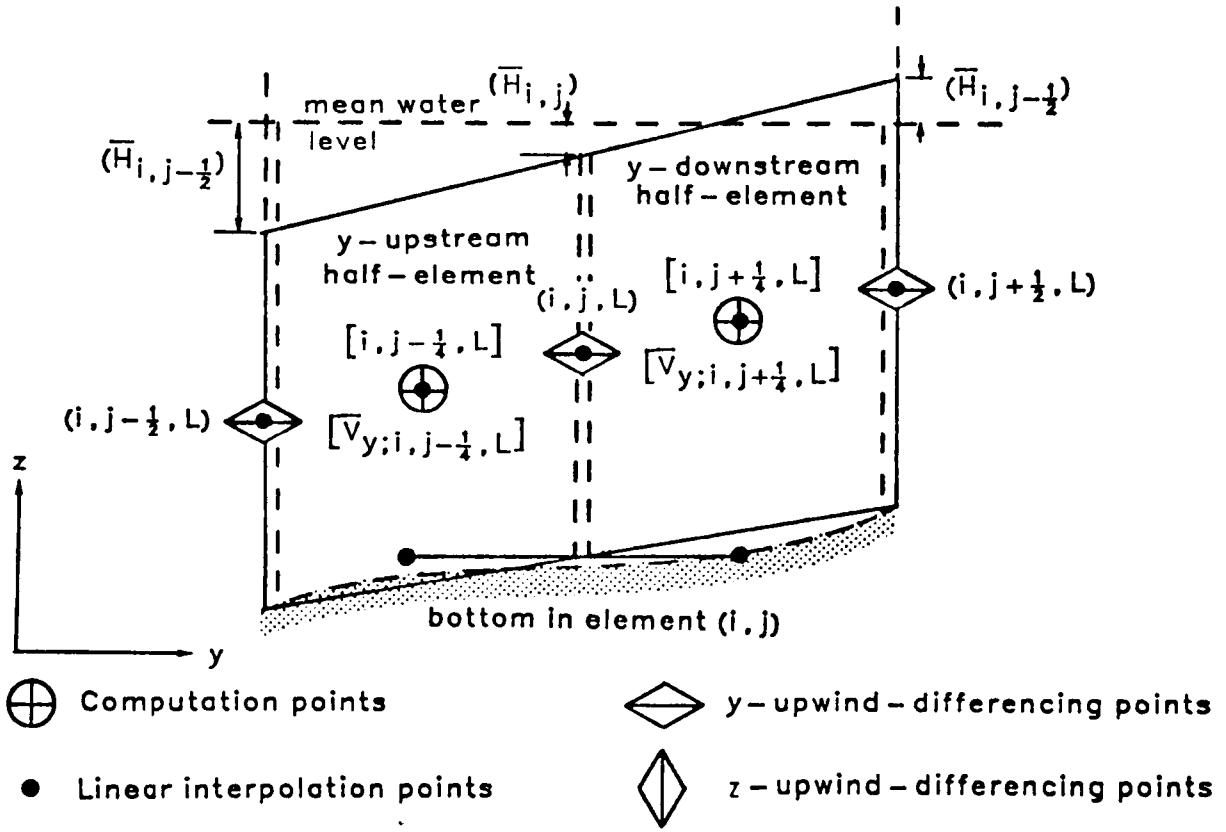

Figure 2.33. Locations used in the computations of y-component of velocity in two half-elements of an interior element in the horizontal (x,y) plane [i,j,l(bot;i,j)] = (i,j,L), in the bottom layer l(bot;i,j) and also in the water-surface layer L [side view in (y,z) plane].

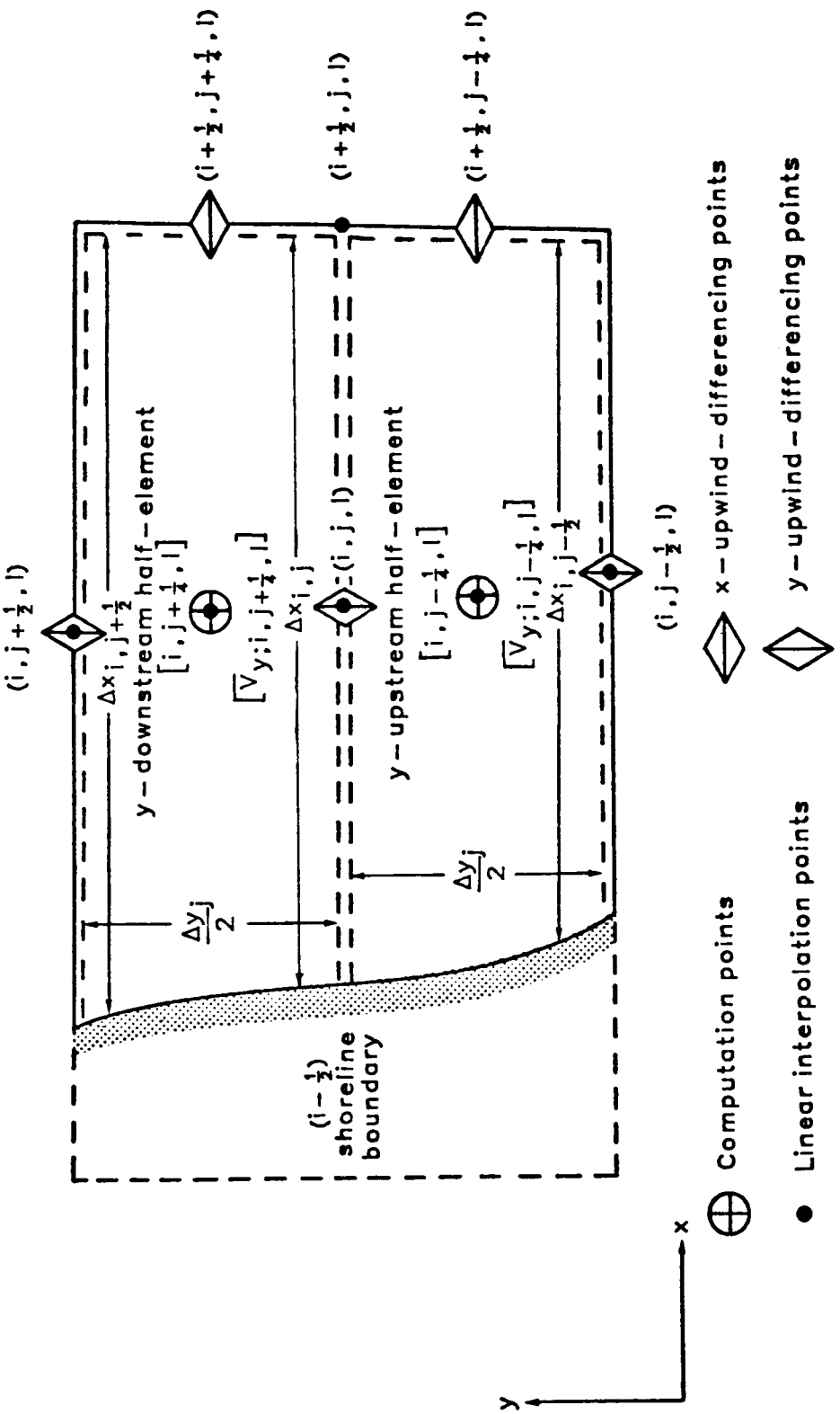

Figure 2.34. Locations used in the computations of y-component of velocity in two half-elements of a boundary element in the horizontal (x,y) plane [I(i,j),1] with a shoreline boundary at x-upstream enclosure surface at (i-½,j,1) [top view in (x,y) plane].

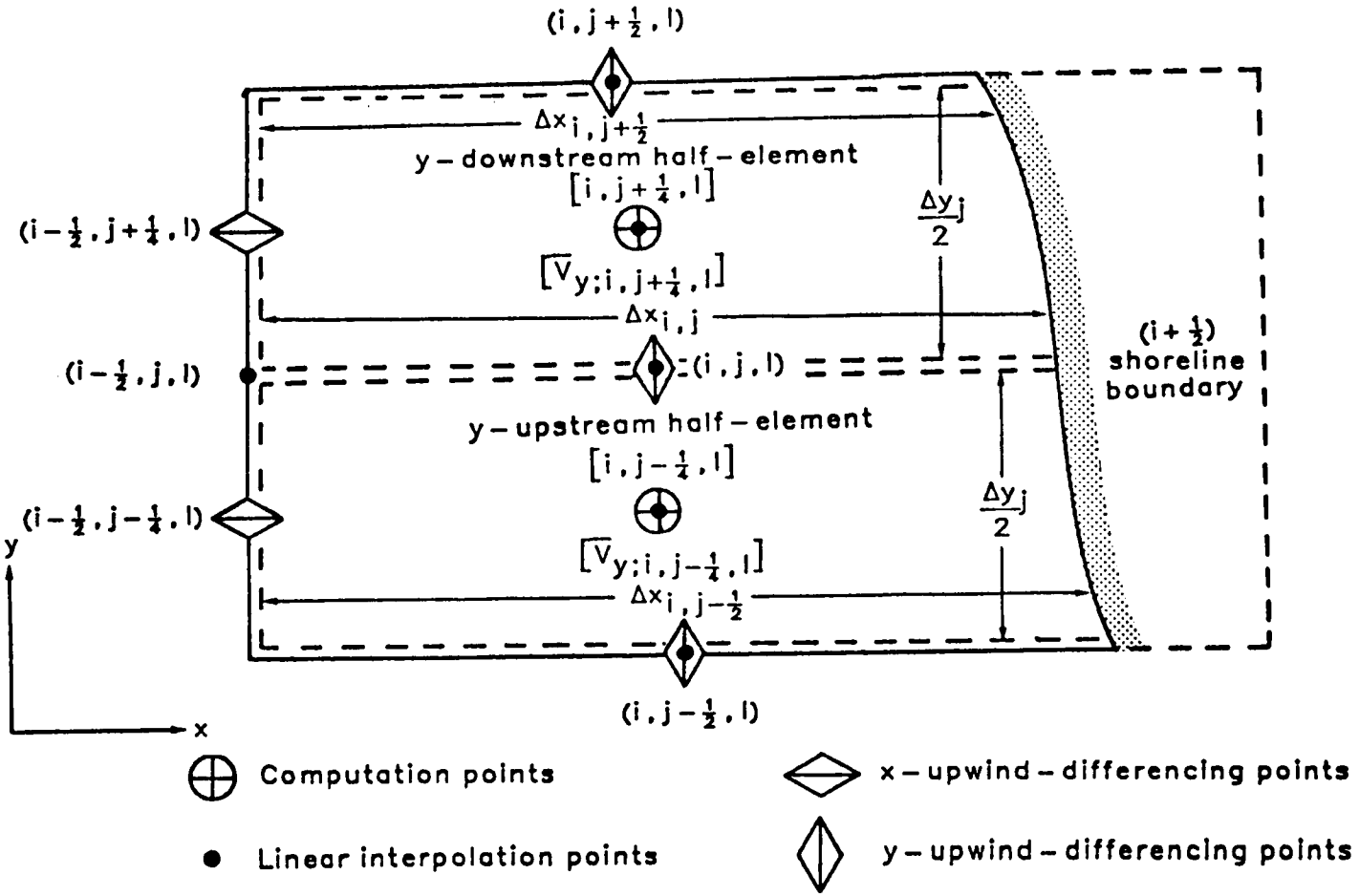

Figure 2.35. Locations used in the computations of y-component of velocity in two half-elements of a boundary element in the horizontal (x,y) plane [I(i,j),l] with a shoreline boundary at x-downstream enclosure surface at (i+½,j,l) [top view in (x,y) plane].

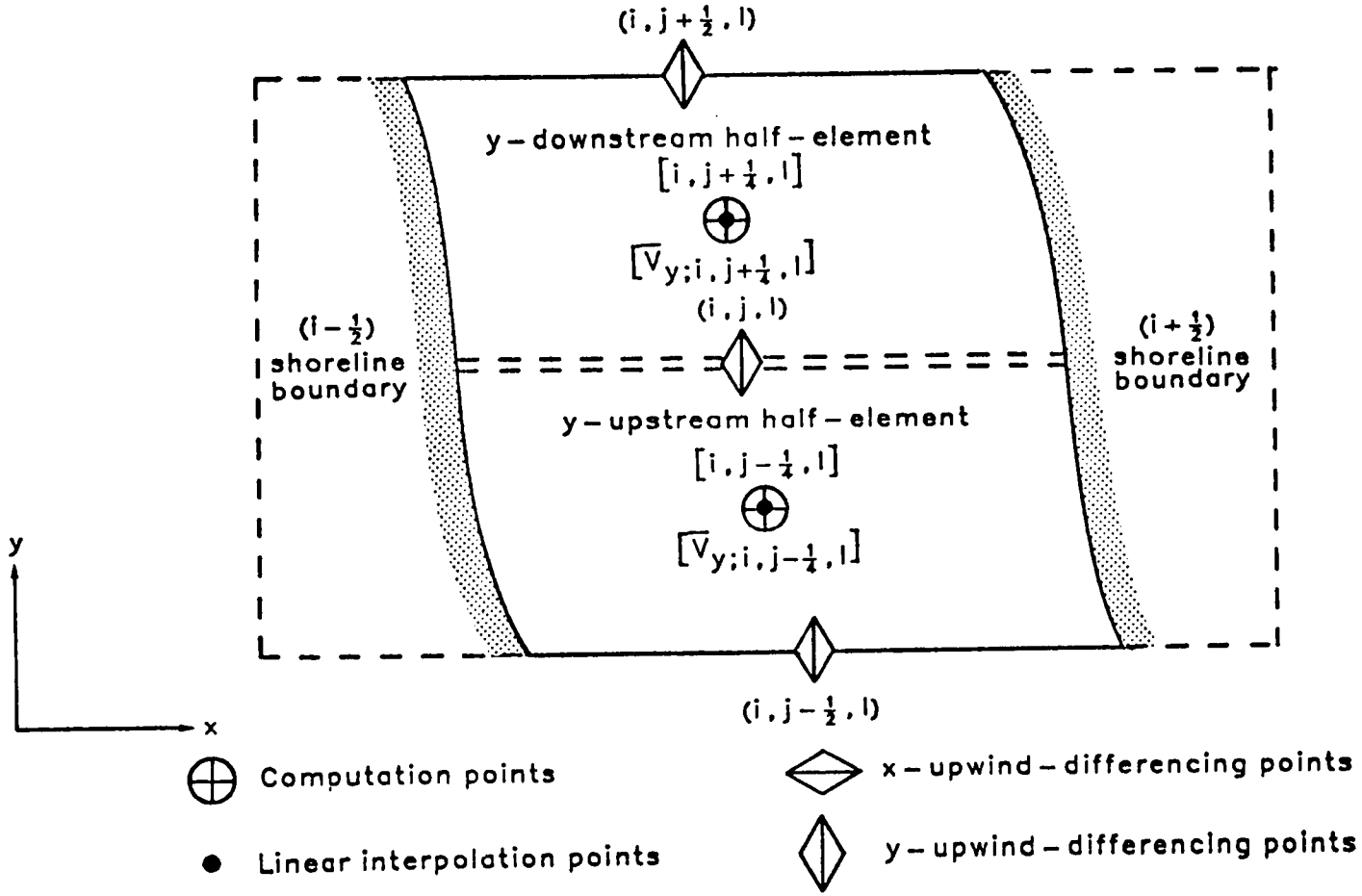

Figure 2.36. Locations used in the computations of y-component of velocity in two half-elements of a boundary element in the horizontal (x,y) plane .I(i,j),l. with shoreline boundaries at the x-upstream enclosure surface at (i-½,j,l) and also at the x-downstream enclosure surface at (i+½,j,l) .top view in (x,y) plane..

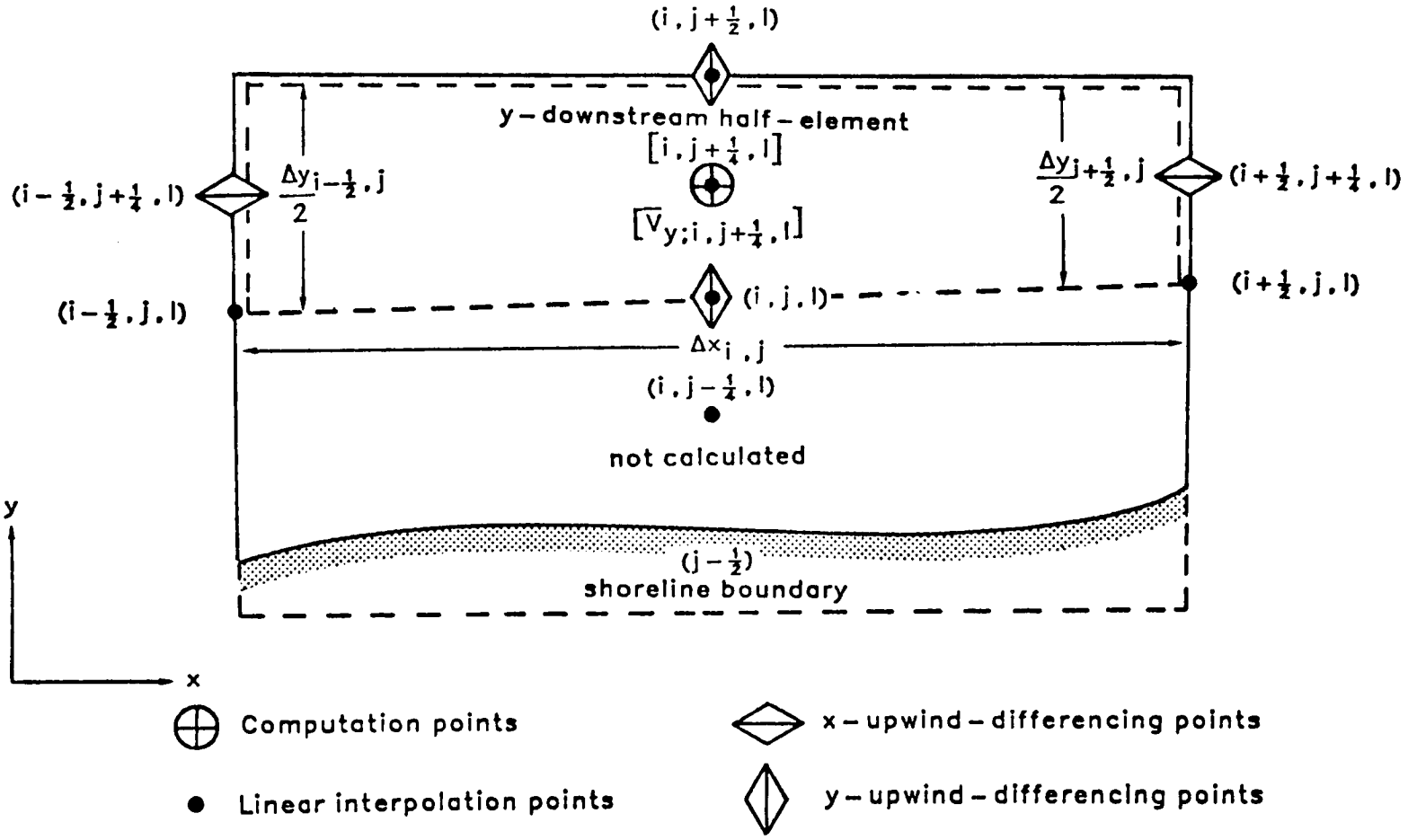

Figure 2.37. Locations used in the computations of y-component of velocity in a half-element of a boundary element in the horizontal plane [I(i,j),1] with a shoreline boundary at the y-upstream enclosure surface at (i,j-½,1) [top view in (x,y) plane].

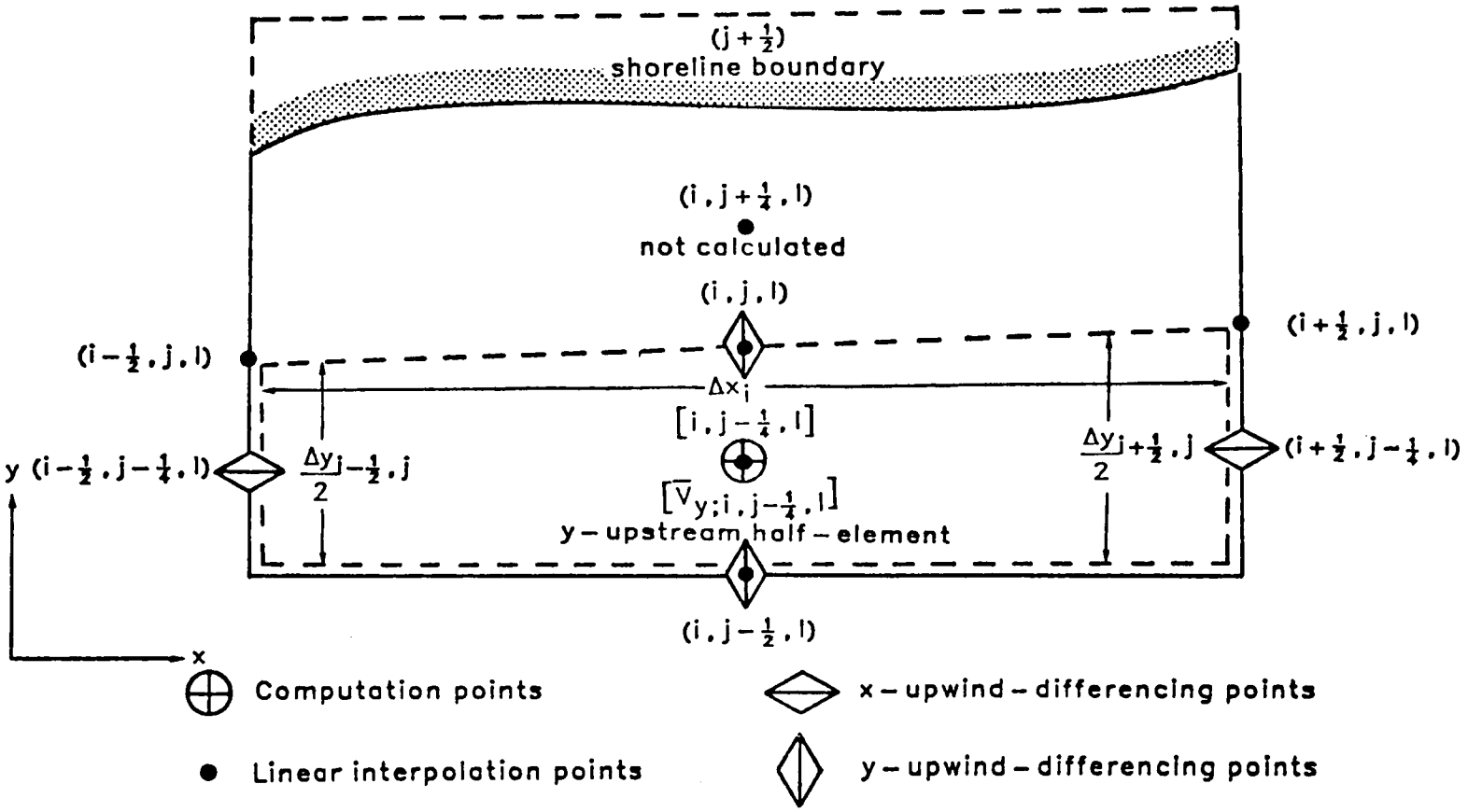

Figure 2.38. Locations used in the computations of y-component of velocity in a half-element of a boundary element in the horizontal plane [I(i,j),l] with a shoreline boundary at the y-downstream enclosure surface at (i,j+½,l) [top view in (x,y) plane].

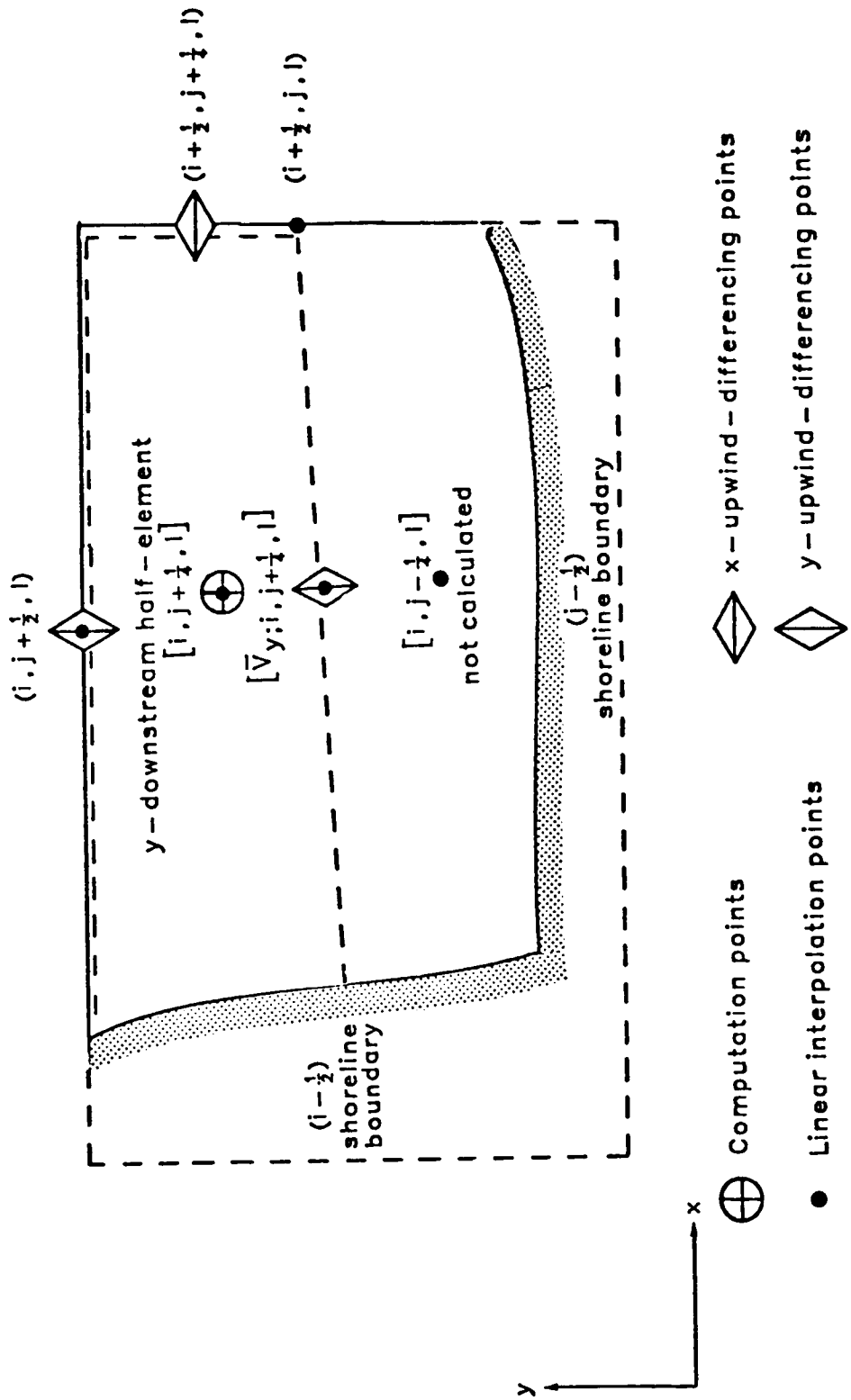

Figure 2.39. Locations used in the computation of y-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I(i,j),1] with shoreline boundaries at the x-upstream enclosure surface at (i-½,j,1) and at the y-upstream enclosure surface at (i,j-½,1) [top view in (x,y) plane].

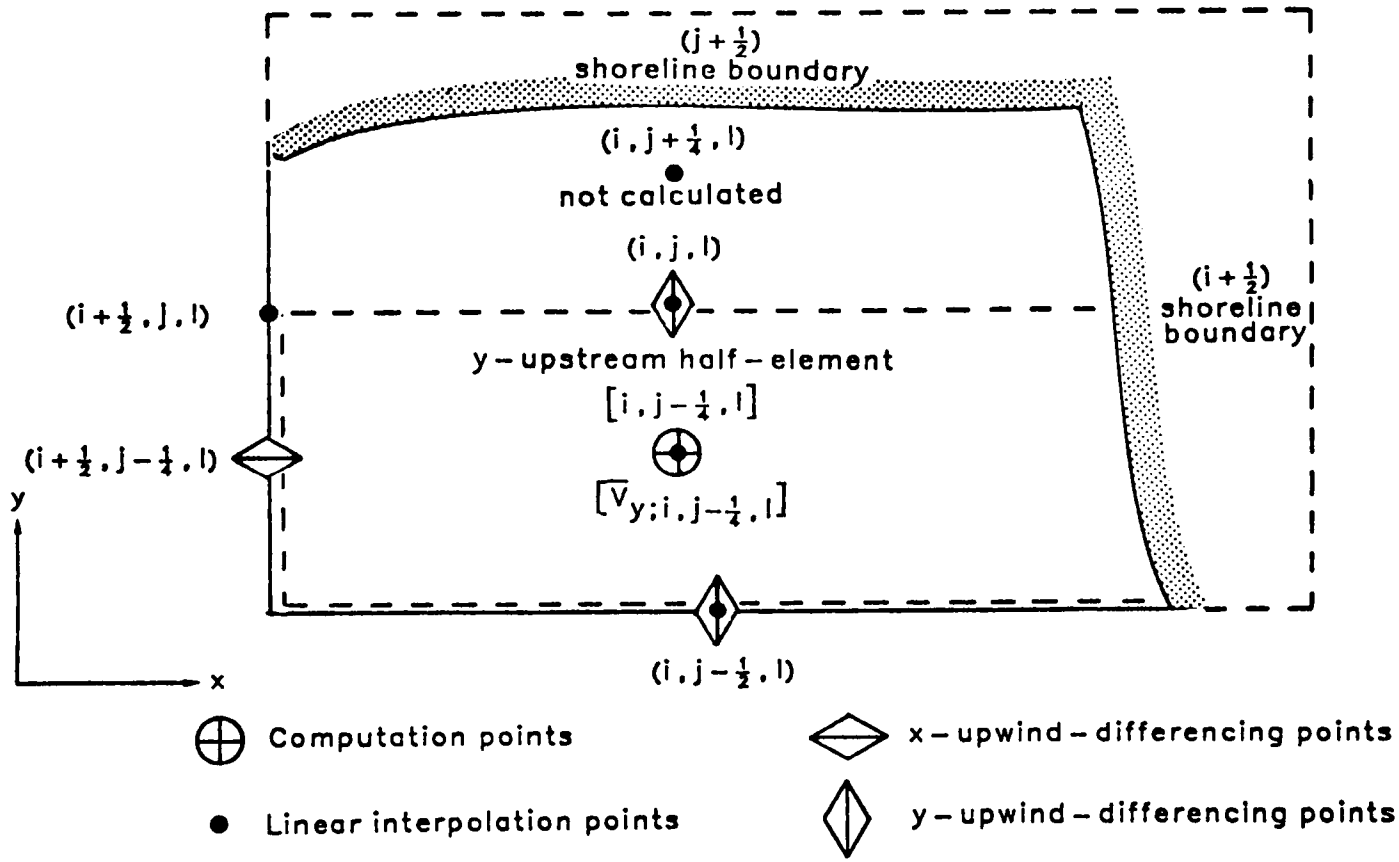

Figure 2.40. Locations used in the computation of y-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I(i,j),l] with shoreline boundaries at the x-downstream enclosure surface at (i+½,j,l) and at the y-downstream enclosure surface at (i,j+½,l) [top view in (x,y) plane].

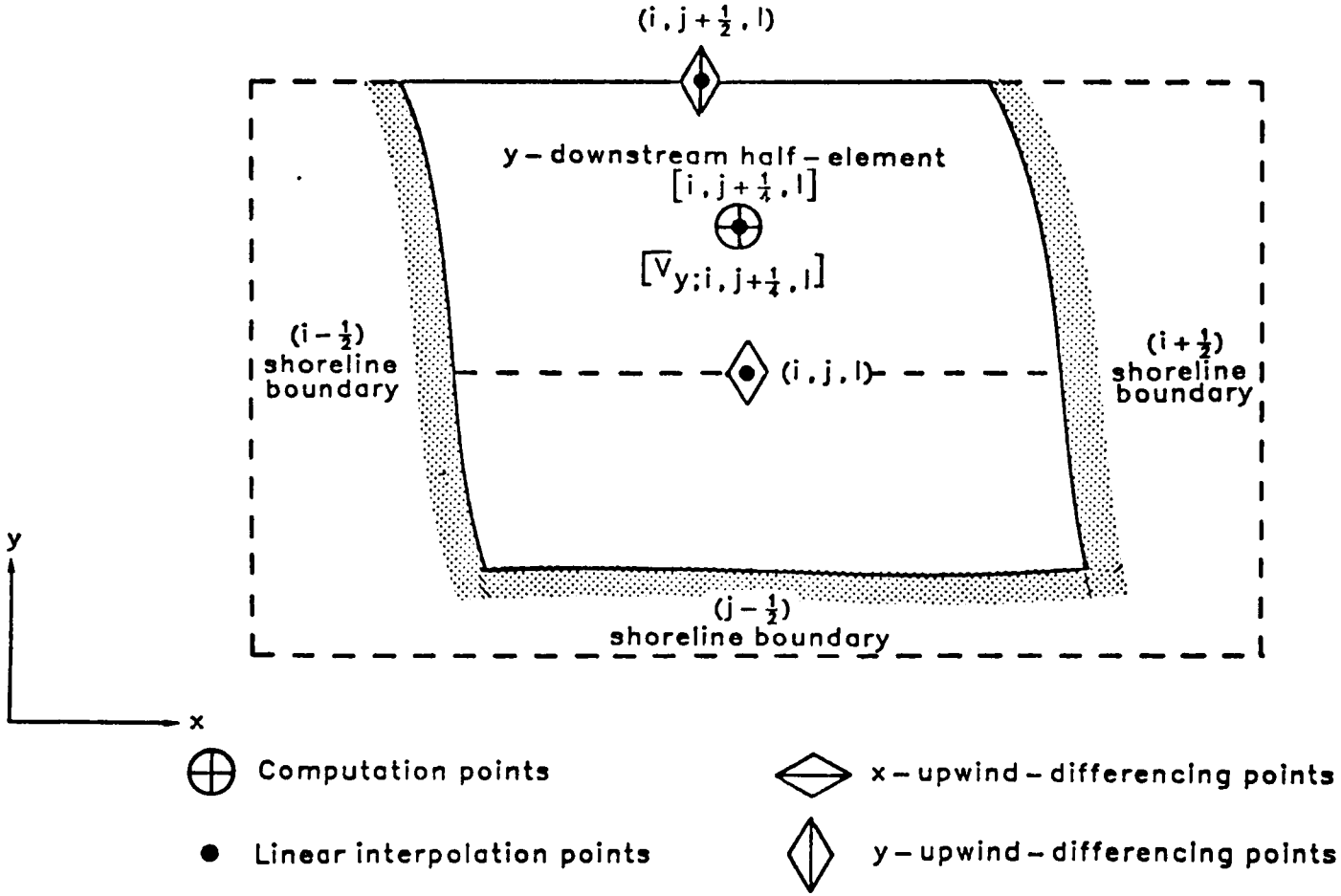

Figure 2.41. Locations used in the computation of y-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I(i,j),l] with shoreline boundaries at the y-upstream enclosure surface at (i,j-½,l), the x-upstream enclosure surface at (i-½,j,l) and at the x-downstream enclosure surface at (i+½,j,l) [top view in (x,y) plane].

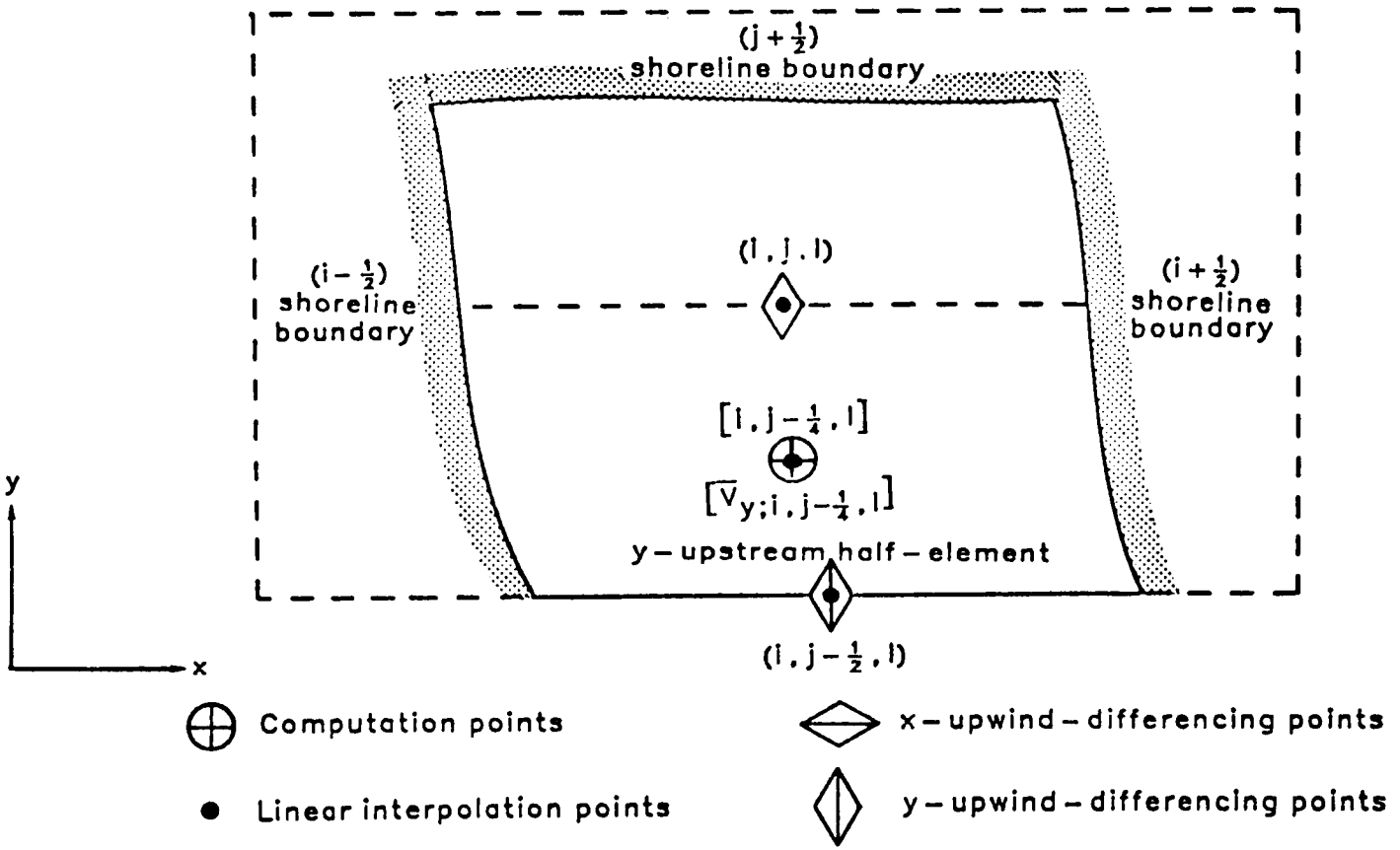

Figure 2.42. Locations used in the computation of y-component of velocity in a half-element of a boundary element in the horizontal (x,y) plane [I(i,j),1] with shoreline boundaries at the y-downstream enclosure surface at (i,j+½,1), the x-upstream enclosure surface at (i-½,j,1) and at the x-downstream enclosure surface at (i+½,j,1) [top view in (x,y) plane].

water and momentum can be evaluated according to extremely efficient computational algorithms, similar to the ones used in finite-difference method (FDM), which are significantly superior to the ones used in finite-element method (FEM) that include the evaluation of the discretized forms of the cross derivative terms associated with the differential forms of the transport equations. Furthermore, since each boundary element retains the actual geometrical characteristics of the sections of the shoreline enclosure boundaries of flow region, the discrete-element method (DEM) automatically has a spatial resolution accuracy comparable (or superior) to the finite-element method (FEM) which is significantly superior to the finite-difference method (FDM). All the detailed three-dimensional geometrical data required for the boundary elements can be readily obtained from the automated discrete-element-generator computer code PREPR2 (Holdeman, 1977), or its modifier version PREPAR (Lin, Erhan and Eraslan, 1978), and the modular subroutine REGION of the associated computer codes of the transient, three-dimensional discrete-element mathematical transport models (e.g., Appendix).

The mathematical formulation of the discrete-element forms of the momentum principle [Eqs. (2.20) through (2.22)] considers a new discretization method which utilizes four overlapping half-elements in each element (Figs. 2.7 through 2.42, p.97 through p.132). THe horizontal planar area (in the general sense, parallel to the geophysical surface plane) of each element is divided into two equal parts by a vertical plane normal to the x axis, through the center of the element, to form the two half-elements for the computation of the x-component of velocity (Figs. 2.7 through 2.24, p.97 through p.114). Similarly, the horizontal

planar area of each element is divided into two equal parts by a vertical plane normal to the y axis, through the center of the element, to form the two half-elements for the computation of the y-component of velocity (Figs. 2.25 through 2.42, p.115 through p.132). The three-dimensional discrete-element method utilizes the element for the formulation of the discrete-element equations for (1) vertical z-component (in the general sense, normal to the geophysical surface plane) of the velocity or flow rate (Figs. 2.43 through 2.46, p.135 through p.138), (2) water-surface elevation (Figs. 2.2 through 2.6, p.92 through p.96), (3) pressure (Figs. 2.47 through 2.54, p.139 through p.146). However, it utilizes (1) two half-elements along x-direction (Figs. 2.7 through 2.24, p.97 through p.114) for the formulation of discrete-element equations for the x-component of velocity and (2) two-half elements along y-direction (Figs. 2.25 through 2.42, p.115 through p.132) for the formulation of the discrete-element equations for the y-component of velocity. Hence, in each layer with N number of elements, which represent the discretized form of a horizontal plane (in the general sense, parallel to the geophysical surface plane and parallel to the free-surface plane) of the flow region, the three-dimensional discrete-element method calculates (1) the x-component of velocity V_x in 2N half-elements and (2) the y-component of velocity V_y in 2N half-elements; whereas, it calculates the pressure p only in N elements. This new discretization technique is distinctly different from the ones used in the conventional staggered-grid computational methods (e.g., Harlow and Welch, 1965, Welch, et al., 1966, Chan and Street, 1970, Leendertse and Gritton, 1971, Leendertse, 1971, Spraggs and Street, 1975, and Roache,

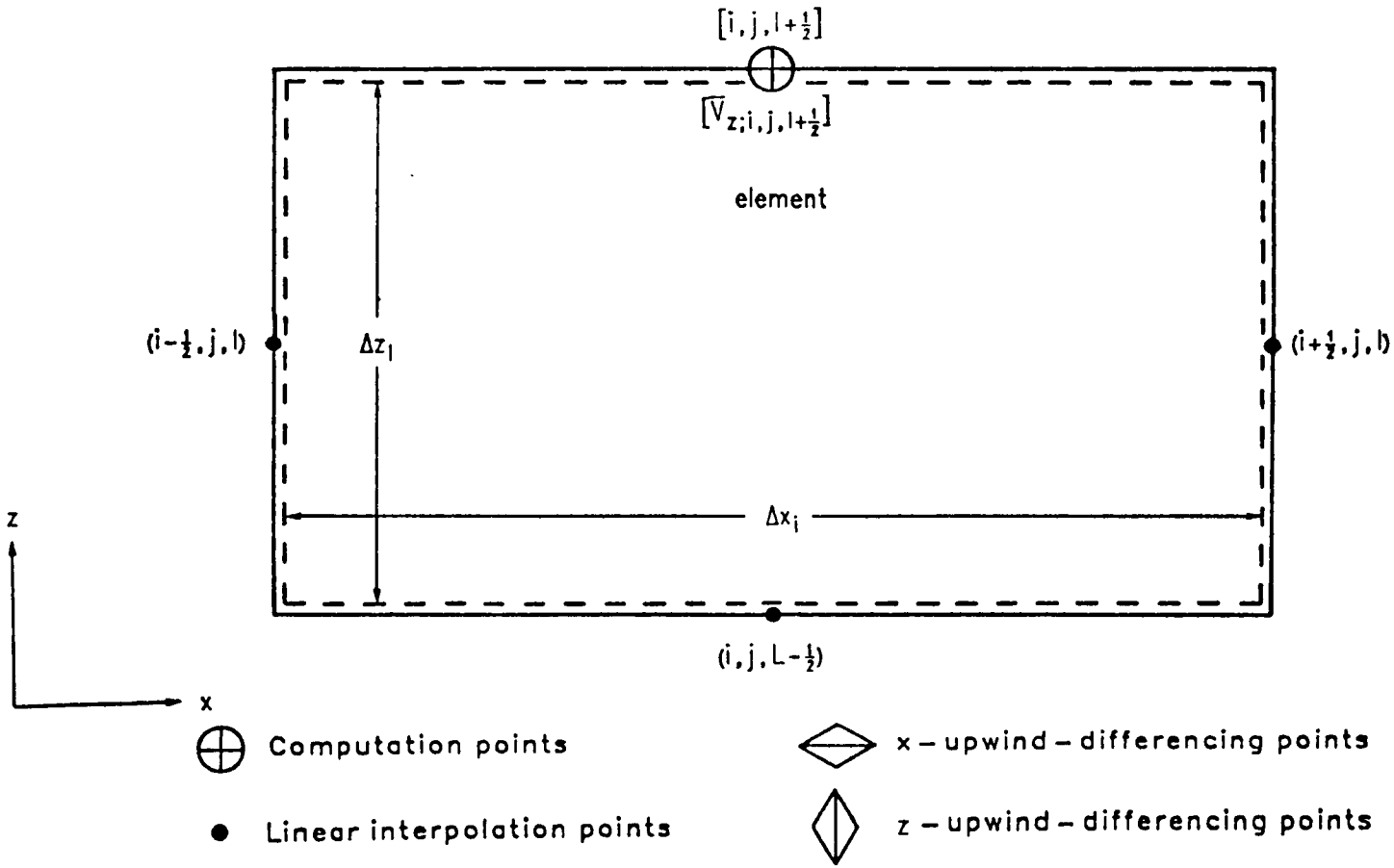

Figure 2.43. Locations used in the computation of z-component of velocity for an interior or boundary element in the horizontal (x,y) plane (i,j,l), which is also a vertically interior element [side view in (x,z) plane].

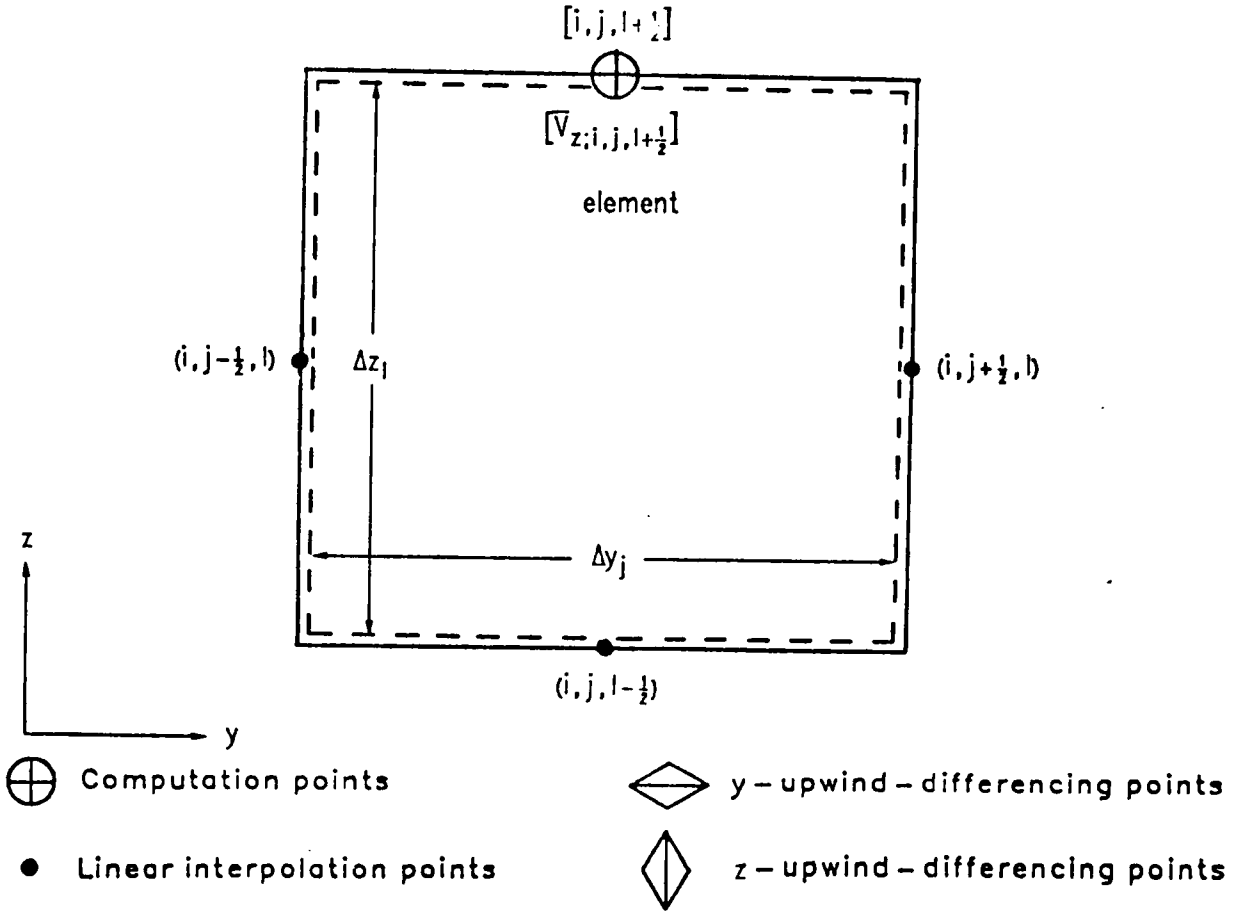

Figure 2.44. Locations used in the computation of z-component of velocity for an interior or boundary element in the horizontal (x,y) plane (i,j,l), which is also a vertically interior element [side view in (y,z) plane].

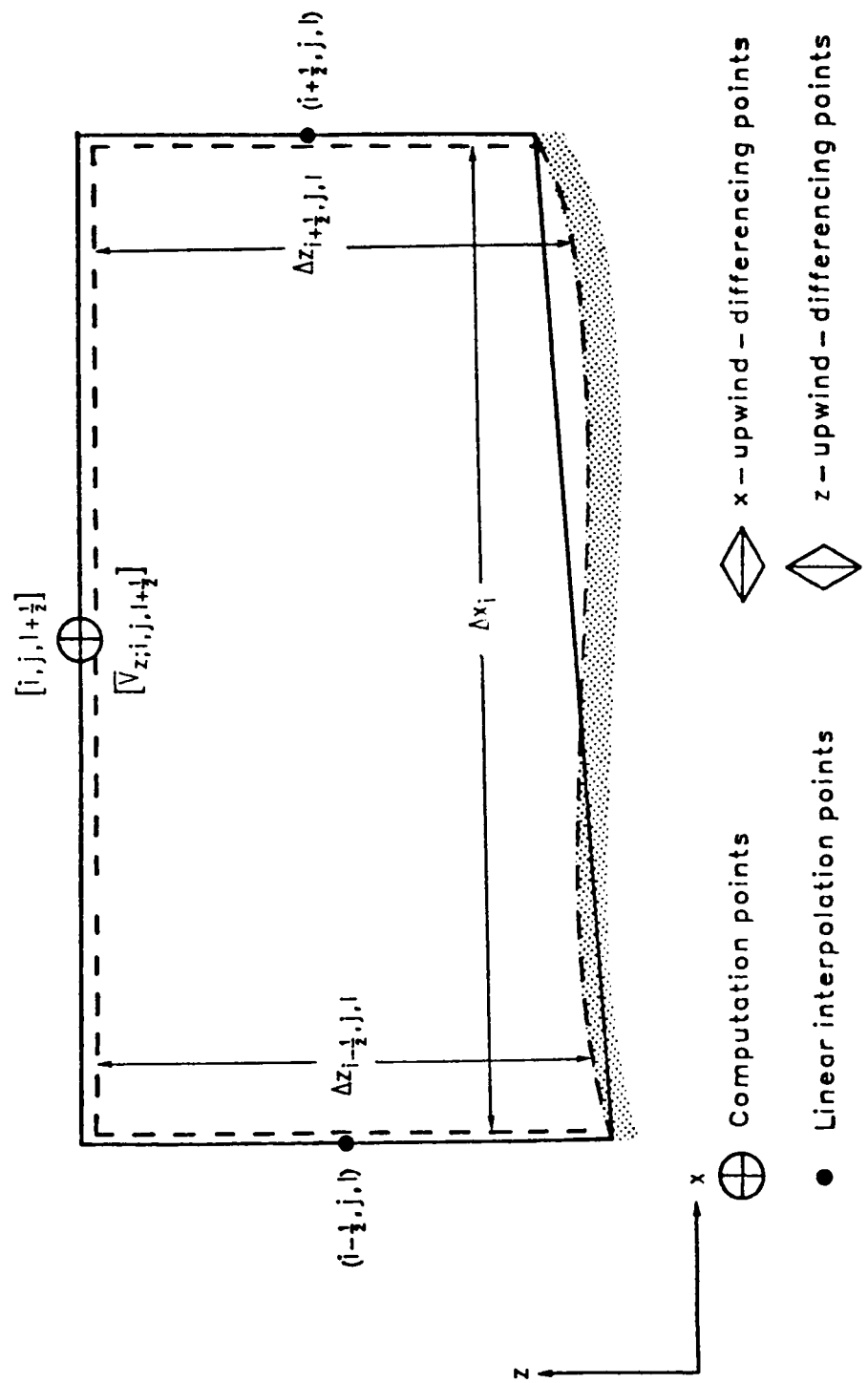

Figure 2.45. Locations used in the computation of z-component of velocity for an interior or boundary element in the horizontal (x,y) plane [i,j,l(bot;i,j)] in the bottom layer l(bot;i,j), below the water surface layer [side view in (x,z) plane].

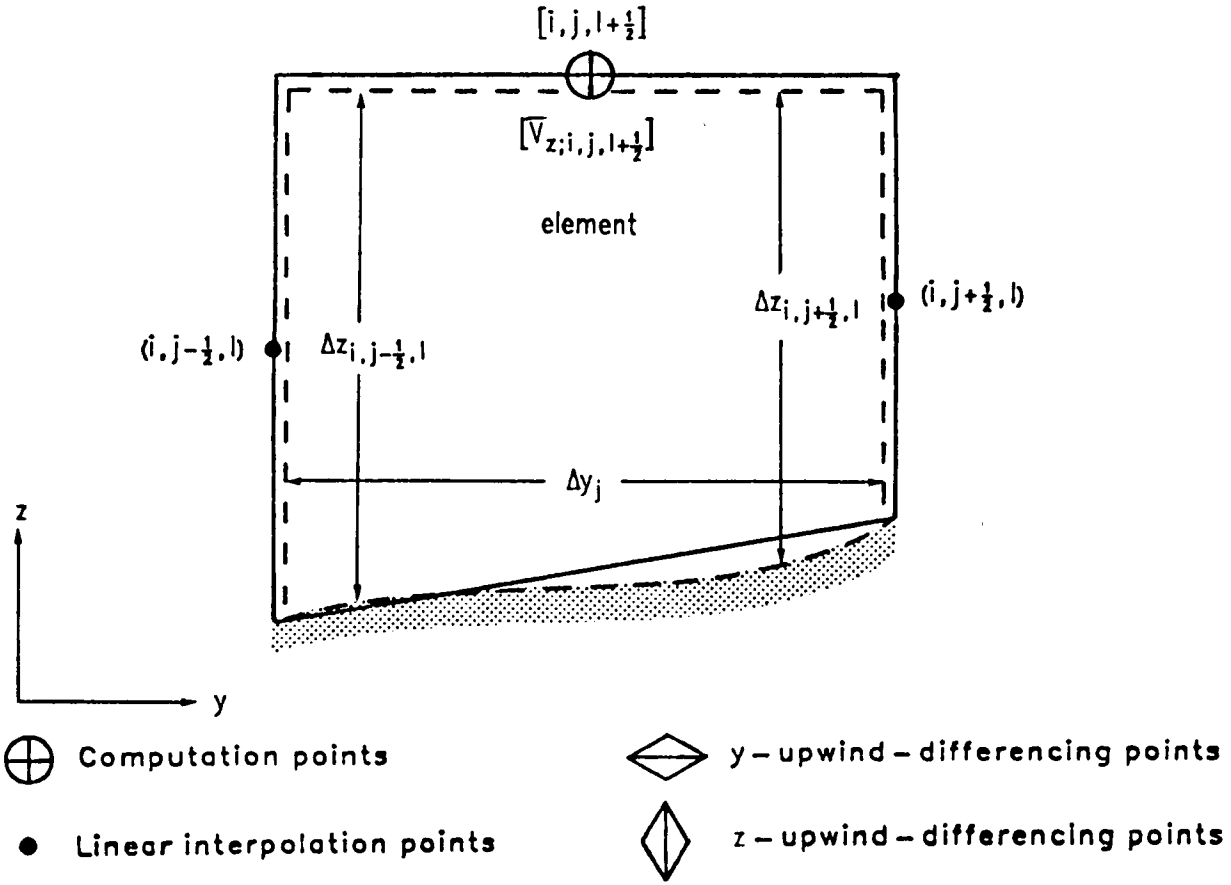

Figure 2.46. Locations used in the computation of z-component of velocity for an interior or boundary element in the horizontal (x,y) plane [i,j,l(bot;i,j)] in the bottom layer l(bot;i,j), below the water surface layer [side view in (y,z) plane].

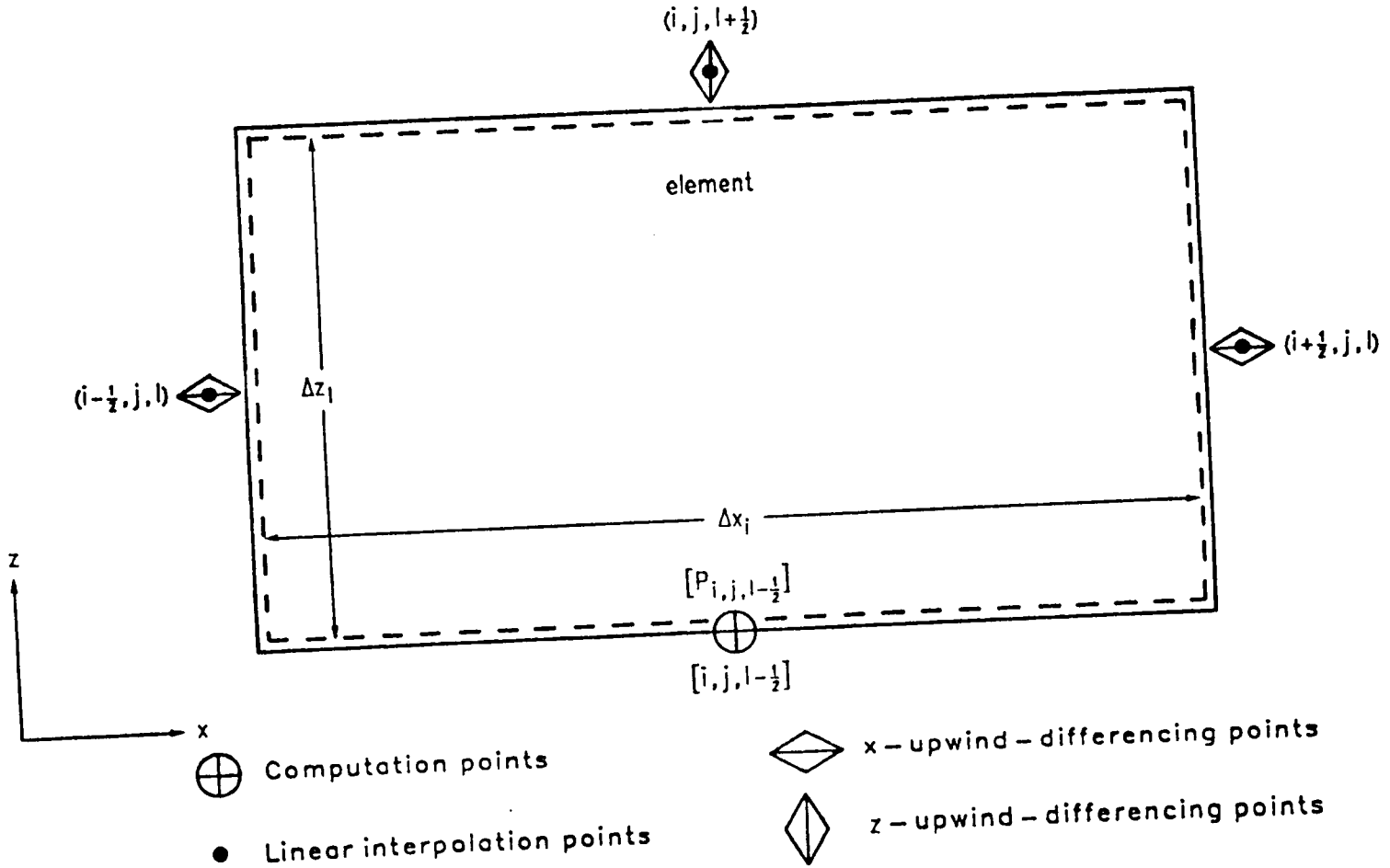

Figure 2.47. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane (i,j,L), above the bottom layer, in the water-surface layer L [side view in (x,z) plane].

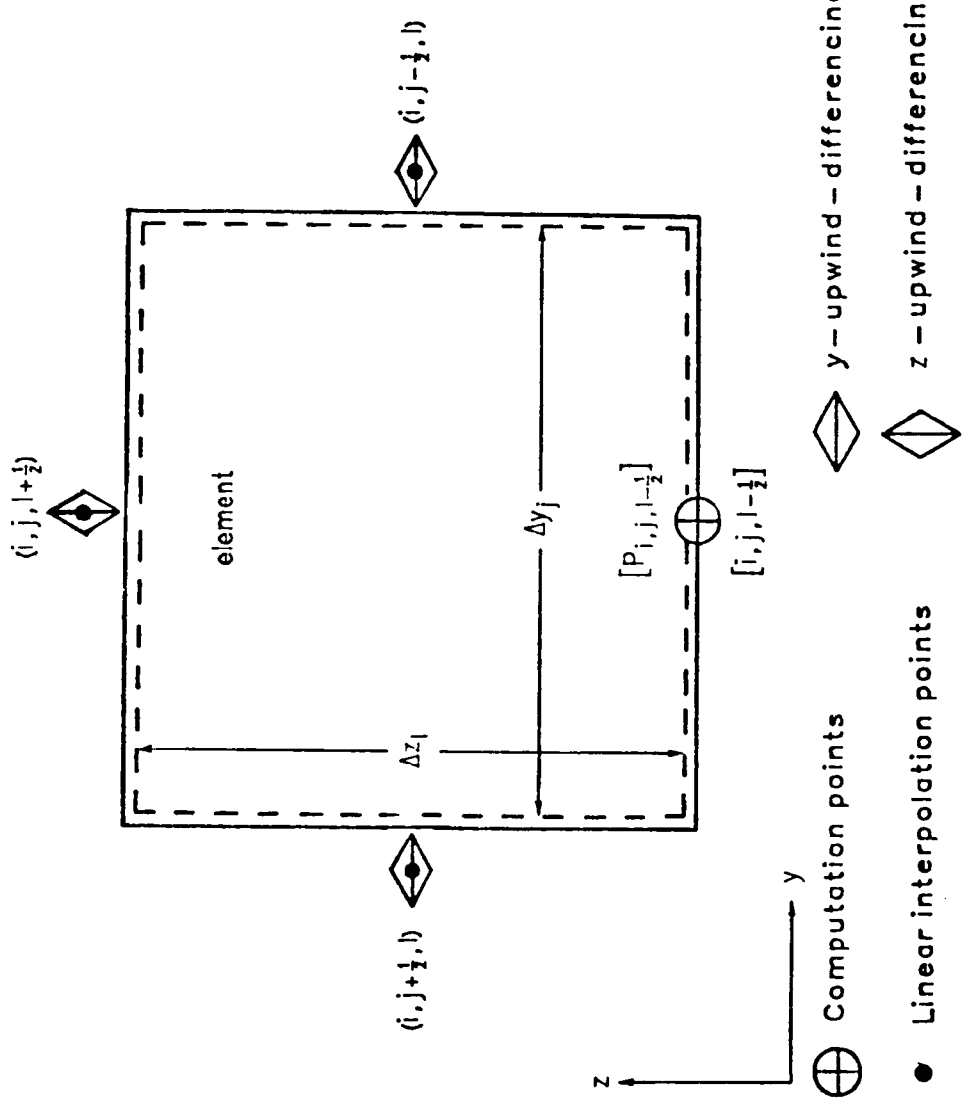

Figure 2.48. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane (i,j,L), above the bottom layer, in the water-surface layer L [side view in (y,z) plane].

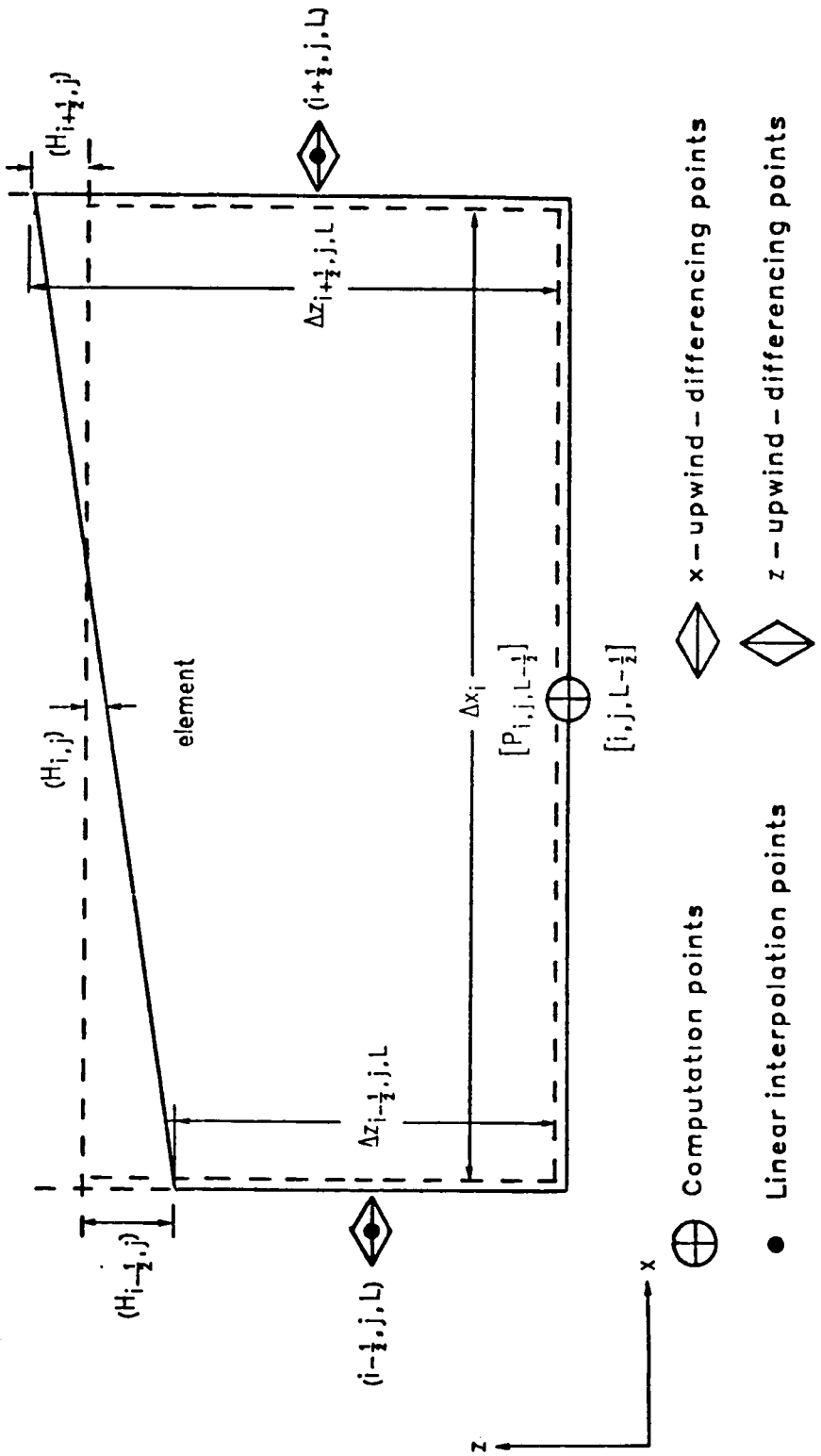

Figure 2.49. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane (i,j,1) which is also a vertically interior elemnt [side view in (x,z) plane].

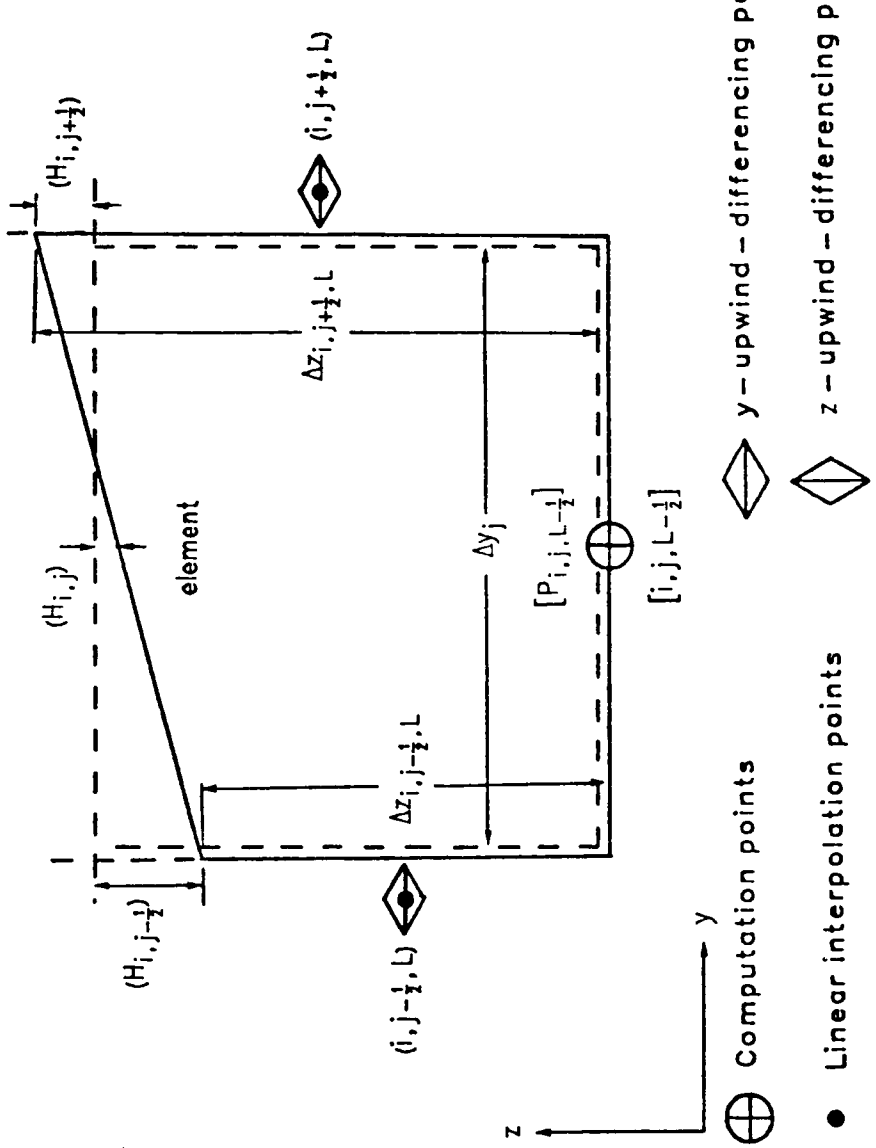

Figure 2.50. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane (i,j,1) which is also a vertically interior elemnt [side view in (y,z) plane].

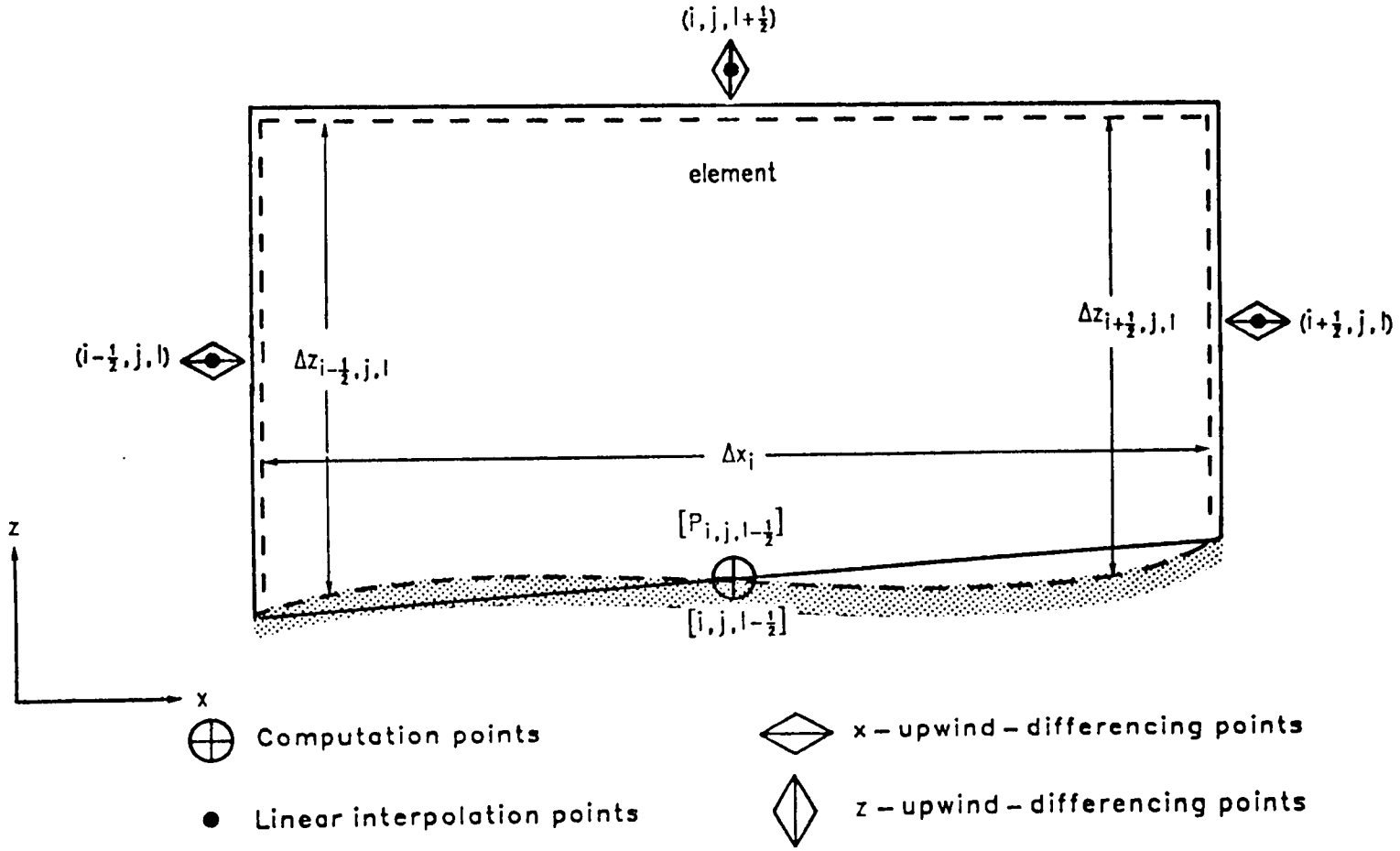

Figure 2.51. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane [i,j,l(bot;i,j)], in the bottom layer l(bot;i,j) and in the water-surface layer L [side view (x,z) plane].

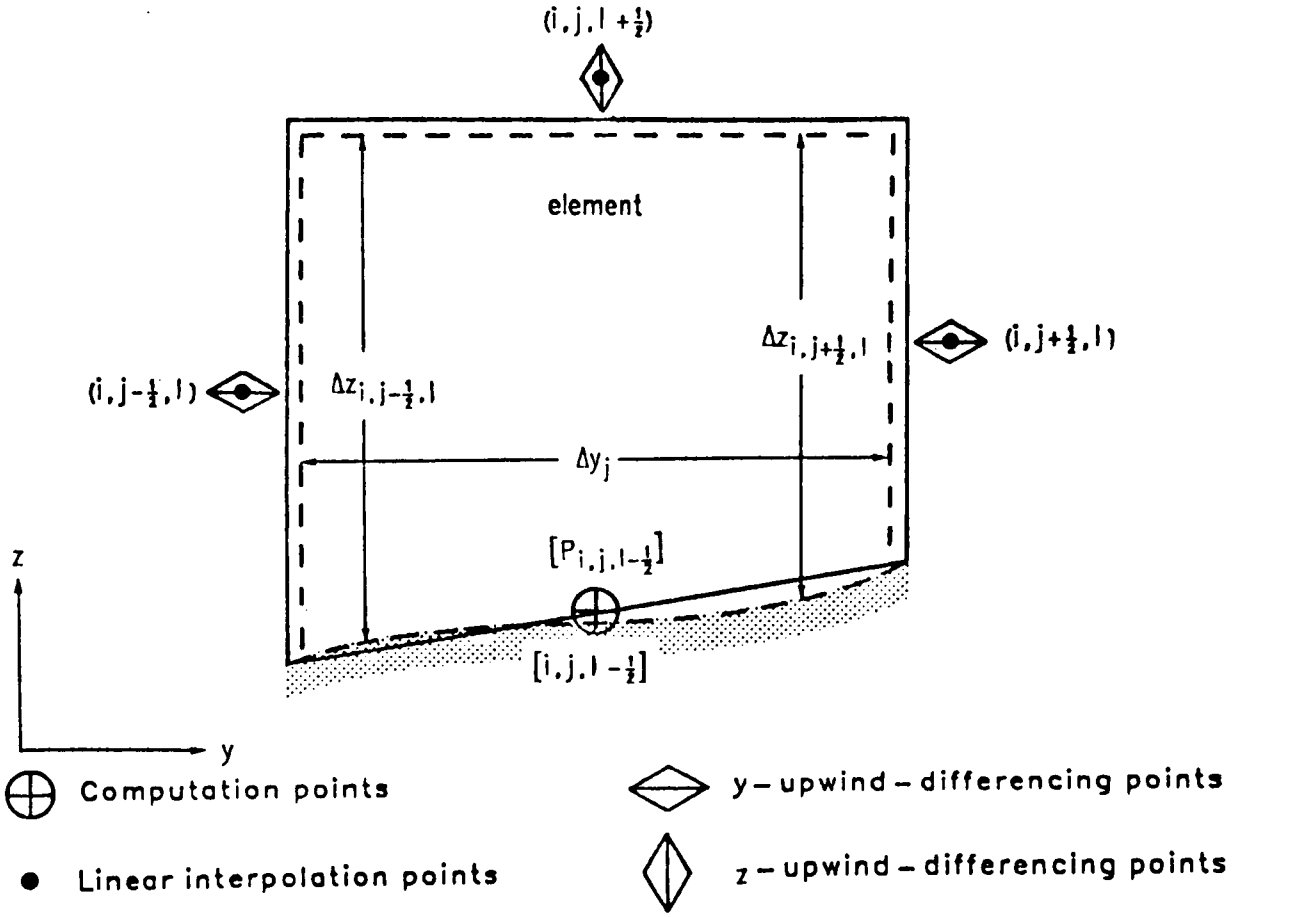

Figure 2.52. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane [i,j,l(bot;i,j)], in the bottom layer l(bot;i,j) and in the water-surface layer L [side view (y,z) plane].

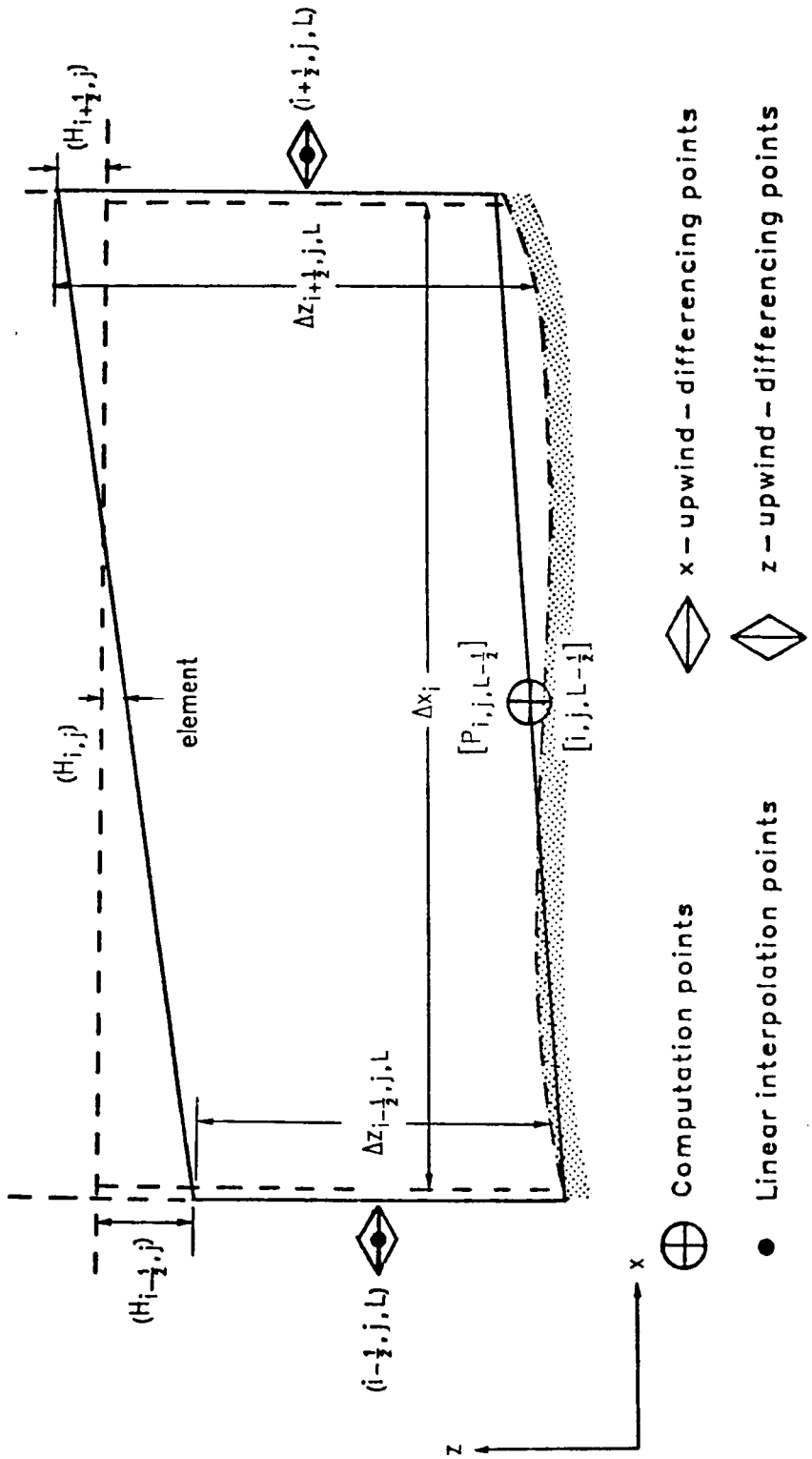

Figure 2.53. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane, in the bottom layer [i,j,l(bot;i,j)], below the water-surface layer [side view in (x,z) plane].

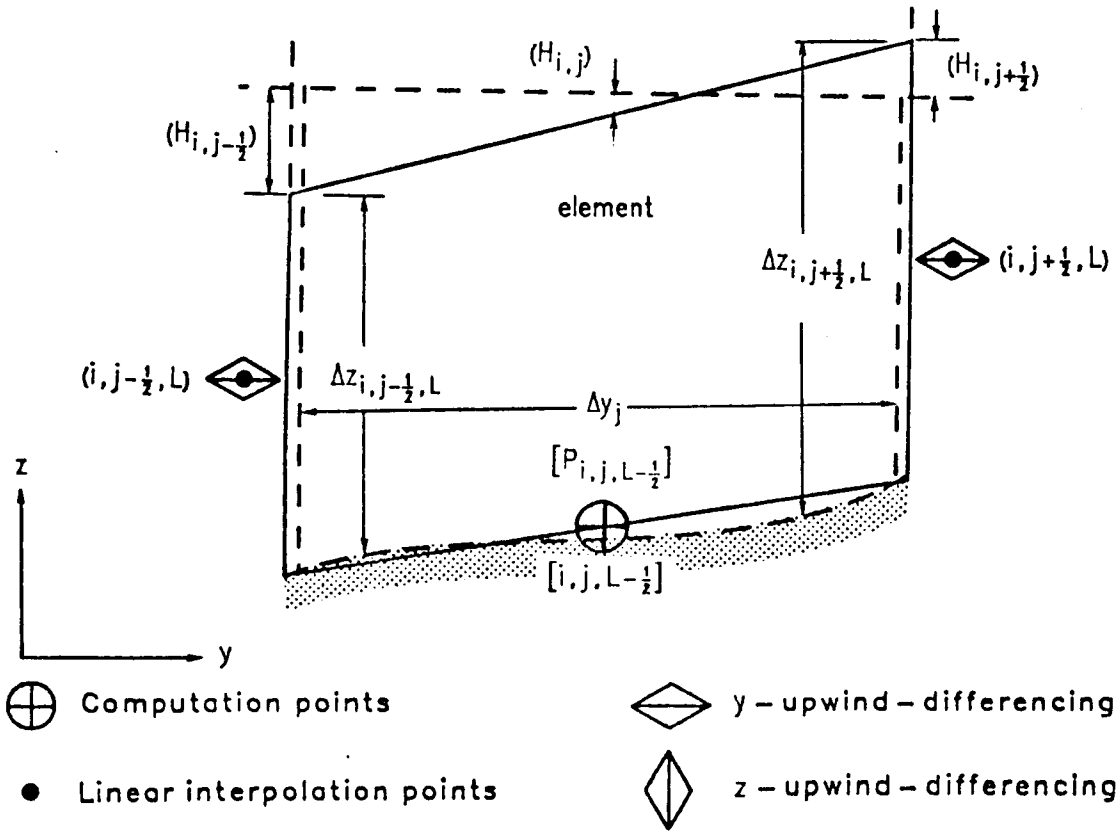

Figure 2.54. Locations used in the computation of pressure for an interior or boundary element in the horizontal (x,y) plane, in the bottom layer [i,j,l(bot;i,j)], below the water-surface layer [side view in (y,z) plane].

1972). The new discretization technique (e.g. Eraslan, 1974, 1976, 1977, 1981a, 1981b, 1981c) allows the discrete-element mehtod to include arbitrarily sized elements, according to a completely generalized nonuniform grid systems; and it establishes extremely stable behaviour characteristics for the numerical solutions of the velocity components and pressure, according to the special, explicit timewise integration algorithm (Sec. 5).

The general formulation of the three-dimensional discrete-element method (DEM) considers the elements and half-elements as discrete control volumes. According to the geometrical definitions of the elements and half-element, the volume integrals and the closed enclosure surface integrals associated with the integral forms of the physical principles, (1) mass conservation principle for water [Eq. (2.9)], (2) three components of the momentum principle (second law of motion) for water [Eqs. (2.16) through (2.18)], can be formulated for each element and half-element in the flow region (e.g., Eraslan, 1974, 1975a).

Each element in the flow region is identified based on the location of its center according to the indices along the three orthogonal axes (x,y,z) (Figs. 2.1 through 2.54, p.89, p.92 through p.132, p.135 through p.146) as

(i,j,l) = index location of the element with its center located at (i,j) in the horizontal (x,y) plane (in the general sense, parallel to the geophyscial surface plane) and in layer l along the vertical z-axis (in the general sense, normal to the geophysical surface plane).

Since the discrete-element method (DEM) considers nonrectangular boundary elements of arbitrary size and shape, according to the actual contour of the shoreline enclosure boundaries, the location of an element (i,j,1), in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), in any layer 1, is defined by the distances of the center of the element from the origin of a specified Cartesian coordinate system along x and y axes (Fig. 2.1, p.89) as

$x_{i,j}$ = location (as distance) along x axis of the center of the element (i,j,1) in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane) is layer 1,

$y_{i,j}$ = location (as distance) along y axis of the center of the element (i,j,1) in the horizontal (x,y) plane in layer 1.

It is important to note that the specification of the general position of an element (i,j,1) in the horizontal (x,y) plane in layer 1 requires double indices i,j for specifying the distances $x_{i,j}$ and $y_{i,j}$, since the center (corresponding to the centroid of the two-dimensional planar area) of a general nonrectangular boundary element is determined according to the specified grid lines of the Cartesian coordinate system.

For an interior element in the horizontal (x,y) plane which is not adjacent to any section of the shoreline enclosure boundaries of the flow region, each position coordinate of the center of the element, in the horizontal (x,y) plane, can be specified by the single index of the rectangular grid system (Figs. 2.1 through 2.15, p.89, p.92 through p.105, Figs. 2.25 through 2.33, p.115 through p.123, and Figs. 2.43 through 2.54, p.135 through p.146) as

$$(x_{i,j})_{interior} = x_i \quad \text{(horizontally interior element)}, \tag{2.41a}$$

$$(y_{i,j})_{interior} = y_j \quad \text{(horizontally interior element)}. \tag{2.41b}$$

For a boundary element in the horizontal (x,y) plane, which is adjacent to any section of the shoreline enclosure boundaries of the flow region, each position coordinate of the center of the element in the horizontal (x,y) plane is specified in terms a single index I(i,j) which identifies the boundary element (Figs. 2.16 through 2.24, p.106 through p.114; and Figs. 2.34 through 2.42, p.124 through p.132) as

$$(x_{i,j})_{boundary} = x_{I(i,j)} \quad \text{(horizontally boundary element)}, \tag{2.42a}$$

$$(y_{i,j})_{boundary} = y_{I(i,j)} \quad \text{(horizontally boundary element)}, \tag{2.42b}$$

where

I(i,j) = index associated with the identification of the boundary element in the horizontal (x,y) plane of the flow region.

Hence, according to the discretization technique of the three-dimensional discrete-element method (DEM), the computational algorithm considers the position of the center of an element (x_i, y_j) for an internal element, or $[x_{I(i,j)}, y_{I(i,j)}]$ for a boundary element, which require specifications of only a single index for each coordinate x or y.

Since the discrete-element method (DEM) considers multiple layers between horizontal (x,y) planes (in the general sense, parallel to the geophysical surface plane and the free surface), according to a specified set of nonuniform (or uniform) depths for the layers, the vertical

location along the vertical z-axis (in the general sense, normal to the geophysical surface plane) of the center of an element depends on the location of the bottom of the flow region in the element (Fig. 2.5, p.95; Fig. 2.6, p.96; Fig. 2.10, p.100; Fig. 2.11, p.101; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.28, p.118; Fig. 2.29, p.119; Fig. 2.32, p.122; Fig.2.33, p.123; Fig. 2.45, p.137; Fig. 2.46, p.138; Fig. 2.51, p.143; Fig. 2.52, p.144; Fig. 2.53, p.145; Fig. 2.54, p.146), i.e.,

$z_{i,j,l}$ = location (as distance) along z-axis of the center of the element (i,j,l) along the vertical direction (in the general sense, normal to the geophysical surface plane) in layer l, [as measured from a horizontal datum plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane) below the bottom of the flow region].

For a vertically interior element, which is not adjacent to the bottom enclosure surface of the flow region, or for a surface element, which includes a part of the free-surface, the vertical position coordinate $z_{i,j,l}$ of the center of the element can be specified based on the single index of the specified layer which contains the element (Fig. 2.3, p.93; Fig. 2.4 ,p.94; Fig. 2.8, p.98; Fig. 2.9, p.99; Fig. 2.12, p.102; Fig. 2.13, p.103; Fig. 2.26, p.116; Fig. 2.27, p.117; Fig. 2.30, p.120; Fig. 2.31, p.121; Fig. 2.43, p.135; Fig. 2.44, p.136; Fig. 2.47, p.139; Fig. 2.48, p.041; Fig. 2.49, p.141; Fig. 2.50, p.142),

$$(z_{i,j,l})_{interior} = z_l \quad \text{(vertically interior element)}. \tag{2.43}$$

For a bottom element, which is adjacent to the bottom enclosure surface of the flow region, the vertical position coordinate z of the

center of the element is specified in term of the double index (i,j) of the element in the horizontal (x,y) plane (Fig. 2.5, p.95; Fig. 2.6, p.96; Fig. 2.10, p.100; Fig. 2.11, p.101; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.28, p.118; Fig. 2.29, p.119; Fig. 2.32, p.122; Fig.2.33, p.123; Fig. 2.45, p.137; Fig. 2.46, p.138; Fig. 2.51, p.143; Fig. 2.52, p.144; Fig. 2.53, p.145; Fig. 2.54, p.146),

$$(Z_{i,j,l})_{bottom} = Z_{bot;i,j} \quad \text{(bottom element)}, \tag{2.44}$$

which depends on the specified depth in the two-dimensional planar element in the horizontal (x,y) plane of the free-surface of the flow region,i.e.,

$D_{i,j}$ = depth of the two-dimensional planar element in the horizontal (x,y) plane, measured from the mean water level (MWL) of the free-sureface plane (in the general sense, parallel to the geophysical surface plane) of the flow region.

All the necessary computations required for determining the locations of the elements in the flow region are completed in the modular subroutine REGION of the associated computer codes of the transient, three-dimensional, discrete-element mathematical transport models (e.g., Appendix A), by utilizing the results of the automated discrete-element generator code PREPR2 (Holdeman, 1977) or its modified version PREPAR (Lin, Erhan, and Eraslan, 1978).

Since the discrete-element method (DEM) considers nonrectangular boundary elements of arbitrary size and shape, according to the actual contours of the impermeable shoreline enclosure boundaries, the dimensions of an element (i,j,l) in the horizontal (x,y) plane (in the

general sense, parallel to the geophysical surface plane) in any layer l, are defined by specifying double indeces (i,j), for the lengths through the center of the element (Figs. 2.1 through 2.54, p.89, p.92 through p.132, p.135 through p.146), as

$\Delta x_{i,j}$ = horizontal length along x-axis of the element (i,j,l), through the center of the element in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), in layer l,

$\Delta y_{i,j}$ = horizontal length along y-axis of the element (i,j,l), through the center of the element in the horizontal (x,y) plane, in layer l.

For an interior element in the horizontal (x,y) plane, which is not adjacent to any section of the shoreline enclosure boundaries of the flow region, each horizontal length of the element, through the center of the element, in the horizontal (x,y) plane, can be specified by the single index of the rectangular grid system (Figs. 2.1 through 2.15, p.89, p.92 through p.105; Figs. 2.25 through 2.33, p.115 through p.123; and Figs. 2.43 through 2.54, p.135 through p.146), as

$$(\Delta x_{i,j})_{interior} = \Delta x_i \quad [\text{interior element in the horizontal (x,y) plane}], \tag{2.45a}$$

$$(\Delta y_{i,j})_{interior} = \Delta y_j \quad [\text{interior element in the horizontal (x,y) plane}]. \tag{2.45b}$$

For a boundary element in the horizontal (x,y) plane, which is adjacent to any section of the shoreline enclosure boundaries of the flow region, each horizontal length of the element, through the center of the element, in the horizontal (x,y) plane, is specified in temrs of

a single index I(i,j), which identifies the boundary element (Figs. 2.16 through 2.24, p.106 through p.114; and Figs. 2.34 through 2.42, p.124 through p.132), as

$$(\Delta x_{i,j})_{boundary} = \Delta x_{I(i,j)} \quad [\text{boundary element in the horizontal } (x,y) \text{ plane}], \tag{2.46a}$$

$$(\Delta y_{i,j})_{boundary} = \Delta y_{I(i,j)} \quad [\text{boundary element in the horizontal } (x,y) \text{ plane}]. \tag{2.46b}$$

Hence, according to the discretization technique of the three-dimensional discrete-element method (DEM), the computational algorithm considers the horizontal lengths of an element, through the center of the element ($\Delta x_{i,j}$, $\Delta y_{i,j}$), in the horizontal (x,y) plane, based on either (Δx_i, Δy_j), for and internal element, or $[\Delta x_{I(i,j)}$, $\Delta y_{I(i,j)}]$, for a boundary element, which require the specification of only a single index for each coordinate x or y.

Since the discrete-element method (DEM) considers multiple layers between horizontal (x,y) planes (in the general sense, parallel to the geophysical surface plane and the free surface), according to the specified set of nonuniform (or uniform) depths for the layers, the vertical dimension, along the vertical z-axis (in the general sense, normal to the geophysical surface plane) depends on the location of the bottom of the flow region (Fig. 2.2, p.92; Fig. 2.6, p.96; Fig. 2.10, p.100; Fig. 2.11, p.101; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.28, p.118; Fig. 2.29, p.119; Fig. 2.32, p.122; Fig.2.33, p.123; Fig. 2.45, p.137; Fig. 2.46, p.138; Fig. 2.51, p.143; Fig. 2.52, p.144; Fig. 2.53, p.145; Fig. 2.54, p.146) and the location of the free-surface layer (Fig. 2.3, p.93; Fig. 2.4, p.94; Fig. 2.5, p.95; Fig. 2.6, p.96; Fig.

2.12, p.102; Fig. 2.13, p.103; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.30, p.120; Fig. 2.31, p.121; Fig. 2.32, p.122; Fig. 2.33, p.123; Fig. 2.49, p.141; Fig. 2.50, p.142; Fig. 2.51, p.143; Fig. 2.52, p.144) in the element, i.e.,

$\Delta z_{i,j,l}$ = vertical length of the element (i,j,l) along z-axis (in the general sense, normal to the geophysical surface plane), in layer l.

For a vertically interior element, which is not adjacent to the bottom enclosure surface of the flow region, and which does not include any part of the free-surface, the vertical length of the element can be specified by the single index of the specified layer l, which contains the element (Fig. 2.8, p.98; Fig. 2.9, p99; Fig. 2.26, p.116; Fig. 2.27, p.117; Fig. 2.43, p.135; Fig. 2.44, p.136; Fig. 2.47, p.139; and Fig. 2.48, p.140), as

$$(\Delta z_{i,j,l})_{interior} = \Delta z_l \quad \text{(vertically interior element)}. \tag{2.47}$$

For a bottom element, which is adjacent to the bottom enclosure surface of the flow region, the vertical length of the element is specified in terms of the double index (i,j) of the element in the horizontal (x,y) plane (Fig. 2.5, p.95; Fig. 2.6, p.96; Fig. 2.10, p.100; Fig. 2.11, p.101; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.28, p.118; Fig. 2.29, p.119; Fig. 2.32, p.122; Fig.2.33, p.123; Fig. 2.45, p.137; Fig. 2.46, p.138; Fig. 2.51, p.143; Fig. 2.52, p.144; Fig. 2.53, p.145; Fig. 2.54, p.146) based on the depth $D_{i,j}$ in the two-dimensional planar element (i,j) in the horizontal (x,y) plane of the free-surface of the flow region, i.e.,

$$(\Delta z_{i,j,l})_{bottom} = \Delta z_{bot;i,j} \quad \text{(bottom element)}. \tag{2.48}$$

The vertical length of the bottom element $\Delta z_{bot;i,j}$ depends on the specified depths of the layers Δz_l and the specified depth $D_{i,j}$ as

$$\Delta z_{bot;i,j} = \left[\sum_{l=l(bot;i,j)}^{L} \Delta z_l \right] - D_{i,j} , \tag{2.49}$$

where,

$l(bot;i,j)$ = index indicating the specified layer which contains the bottom of the flow region in the two-dimensional planar element (i,j) in the horizontal (x,y) plane of the free surface of the flow region,

L = index indicating the specified top layer which contains the free surface of the flow region.

For a surface element (i,j,L) in the specified top layer L, which contains the free surface of the flow region, the vertical length of the element depends on the water-surface elevation in the element (Figs. 2.3 through 2.6, p.93 through p.96; Figs. 2.12 through 2.15, p.102 through p.105; Figs. 2.30 through 2.33, p.120 through p.123; Figs. 2.49 through 2.52, p.141 through p.144), as

$$(\Delta z_{i,j,L})_{surface} = \Delta z_L + H_{i,j}, \quad \text{(surface element)}, \tag{2.50}$$

where

$H_{i,j}$ = water-surface elevation in the surface element (i,j,L) relative to the mean-water-level (MWL) which corresponds to the top horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane) of the specified top layer L in the flow region.

All the necessary computations required for determining the lengths (dimensions) of the elements, through the centers of the elements, in the flow region, are completed by utilizing the automated discrete-element-generator computer code PREPR2 (Holdeman, 1977), or its modified version PREPAR (Lin and Erhan, and Eraslan, 1978), and the computational algorithm of the modular subroutine REGION of the associated computer codes of the transient, three-dimensional, discrete-element mathematical transport models (e.g., Appendix).

The locations of the centers of the six enclosure surfaces for each element (i,j,l) are defined relative to the index location of the center of the element at (i,j,l), according to the index notation (Figs. 2.1, through 2.54, p.89, p.92 through p.132, p.135 through p.146), as

$(i-\frac{1}{2},j,l)$ = index location of the x-upstream enclosure surface of the element (i,j,l),

$(i+\frac{1}{2},j,l)$ = index location of the x-downstream enclosure surface of the element (i,j,l),

$(i,j-\frac{1}{2},l)$ = index location of the y-upstream enclosure surface of the element (i,j,l),

$(i,j+\frac{1}{2},l)$ = index location of the y-downstream enclosure surface of the element (i,j,l),

$(i,j,l-\frac{1}{2})$ = index location of the z-upstream enclosure surface of the element (i,j,l),

$(i,j,l+\frac{1}{2})$ = index location of the z-downstream enclosure surface of the element (i,j,l).

Since the discrete-element method (DEM) considers nonrectangular boundary elements of arbitrary size and shape, according to the actual contours of the shoreline enclosure boundaries, the dimensions of the four enclosure boundaries of an element (i,j,l) in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), in any layer l, could be different than the dimensions of the element, through the center of the element. Hence, the dimension of the four enclosure surfaces of an element (Figs. 2.1, through 2.54, p.89, p.92 through p.132, p.135 through p.146) are defined as

$\Delta y_{i-\frac{1}{2},j,l}$ = horizontal length along y-axis of the x-upstream enclosure surface of the element (i,j,l) in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), in layer l,

$\Delta z_{i-\frac{1}{2},j,l}$ = vertical length of the x-upstream enclosure surface of the element (i,j,l) along z-axis (in the general sense, normal to the geophysical surface plane), in layer l,

$\Delta y_{i+\frac{1}{2},j,l}$ = horizontal length along y-axis of the x-downstream enclosure surface of the element (i,j,l) in the horizontal (x,y) plane, in layer l,

$\Delta z_{i+\frac{1}{2},j,l}$ = vertical length of the x-downstream enclosure surface of the element (i,j,l) along z-axis, in layer l,

$\Delta x_{i,j-\frac{1}{2},l}$ = horizontal length along x-axis of the y-upstream enclosure surface of the element (i,j,l) in the horizontal (x,y) plane, in layer l,

$\Delta z_{i,j-\frac{1}{2},l}$ = vertical length of the y-upstream enclosure surface of the element (i,j,l) along z-axis, in layer l,

$\Delta x_{i,j+\frac{1}{2},l}$ = horizontal length along x-axis of the y-downstream enclosure surface of the element (i,j,l) in the horizontal (x,y) plane, in layer l,

$\Delta z_{i,j+\frac{1}{2},l}$ = vertical length of the y-downstream enclosure surface of the element (i,j,l) along z-axis, in layer l,

For an interior element in the horizontal (x,y) plane which is not adjacent to any section of the shoreline enclosure boundaries of the flow region, the horizontal length along a specified axis of an enclosure surface of the element is the same as the horizontal length, along the same specified axis, through the center of the element, since the element is a complete rectangular cell (Figs. 2.1 through 2.15, p.89, p.92 through p.105; Figs. 2.25 through 2.33, p.115 through p.123; and Figs. 2.43 through 2.54, p.135 through p.146).

Hence, for x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ or for x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$,

$$(\Delta y_{i\pm\frac{1}{2},j,l})_{interior}=\Delta y_j \quad \text{[interior element in the horizontal (x,y) plane]}, \tag{2.51a}$$

and, for y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ or for y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$,

$$(\Delta x_{i,j\pm\frac{1}{2},l})_{interior}=\Delta x_i \quad \text{[interior element in the horizontal (x,y) plane]}. \tag{2.51b}$$

For a boundary element in the horizontal (x,y) plane which is adjacent to any section of the shoreline enclosure boundaries of the flow region, each horizontal length of an enclosure surface of an element in the horizontal (x,y) plane is specified in terms of a single index I(i,j) which identifies the boundary element (Figs. 2.16 through

2.24, p.106 through p.114; and Figs. 2.34 through 2.42, p.124 through p.132).

Hence, for x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ or for x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$,

$$(\Delta y_{i\pm\frac{1}{2},j,l})_{boundary}=\Delta y_{l(i,j);i\pm\frac{1}{2}} \quad \text{[boundary element in the horizontal (x,y) plane]}, \tag{2.52a}$$

and, for y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ or for y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$,

$$(\Delta x_{i,j\pm\frac{1}{2},l})_{boundary}=\Delta x_{l(i,j);j\pm\frac{1}{2}} \quad \text{[boundary element in the horizontal (x,y) plane]}. \tag{2.52b}$$

For a vertically interior element, which is not adjacent to a bottom enclosure surface of the flow region and which does not include any part of the free surface, the vertical length of an enclosure surface of the element is the same as the vertical length of the element, through the center of the element, since the element is a complete rectangular cell (Fig. 2.8, p.98; Fig. 2.9, p.99; Fig. 2.26, p.116; Fig. 2.27, p.117; Fig. 2.43, p.135; Fig. 2.44, p.136; Fig. 2.47, p.139; and Fig. 2.48, p.140).

Hence, for x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ or for x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$,

$$(\Delta z_{i\pm\frac{1}{2},j,l})_{interior}=\Delta z_l \quad \text{(vertically interior element)}, \tag{2.53a}$$

and for y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ or for y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$,

$$(\Delta z_{i,j\pm\frac{1}{2},l})_{interior}=\Delta z_l \quad \text{(vertically interior element)}. \tag{2.53b}$$

For a bottom element, which is adjacent to the bottom enclosure surface of the flow region, the vertical length at any enclosure surface of the element is evaluated by linear interpolation, based on the vertical length of the bottom element and the vertical length of the adjacent bottom element (Fig. 2.5, p.95; Fig. 2.6, p.96; Fig. 2.10, p.100; Fig. 2.11, p.101; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.28, p.118; Fig. 2.29, p.119; Fig. 2.32, p.122; Fig.2.33, p.123; Fig. 2.45, p.137; Fig. 2.46, p.138; Fig. 2.51, p.143; Fig. 2.52, p.144; Fig. 2.53, p.145; Fig. 2.54, p.146).

Hence, for x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ or for x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$

$$(\Delta z_{i\pm\frac{1}{2},j,l})_{bottom}=\Delta z_{bot;i\pm\frac{1}{2},j} \quad \text{(bottom element)}, \tag{2.54a}$$

where

$$\Delta z_{bot;i\pm\frac{1}{2},j}=\frac{(\Delta x_{i,j}\Delta z_{bot;i\pm1,j}+\Delta x_{i\pm1,j}\Delta z_{bot;i,j})}{(\Delta x_{i,j}+\Delta x_{i\pm1,j})}, \tag{2.54a}$$

and for y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ or for y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$,

$$(\Delta z_{i,j\pm\frac{1}{2},l})_{bottom}=\Delta z_{bot;i,j\pm\frac{1}{2}} \quad \text{(bottom element)}, \tag{2.54b}$$

where

$$\Delta z_{bot;i,j\pm\frac{1}{2}}=\frac{(\Delta y_{i,j}\Delta z_{bot;i,j\pm1}+\Delta y_{i,j\pm1}\Delta z_{bot;i,j})}{(\Delta y_{i,j}+\Delta y_{i,j\pm1})}. \tag{2.54b}$$

For a surface element (i,j,L) in the specified top layer L, which contains the free surface of the flow region, the vertical length of any

enclosure surface of the element depends on the water-surface elevation at the location of the enclosure surface (Figs. 2.3 through 2.6, p.93 through p.96; Figs. 2.12 through 2.15, p.102 through p.105; Figs. 2.30 through 2.33, p.120 through p.123; Figs. 2.49 through 2.52, p.141 through p.144).

Hence, for x-upstream enclosure surface at $(i-\frac{1}{2},j,1)$ or for x-downstream enclosure surface at $(i+\frac{1}{2},j,1)$

$$(\Delta z_{i\pm\frac{1}{2},j,L})_{surface}=\Delta z_L+H_{i\pm\frac{1}{2},j} \quad \text{(surface element)}, \tag{2.55a}$$

and for y-upstream enclosure surface at $(i,j-\frac{1}{2},1)$ or for y-downstream enclosure surface at $(i,j+\frac{1}{2},1)$

$$(\Delta z_{i,j\pm\frac{1}{2},L})_{surface}=\Delta z_L+H_{i,j\pm\frac{1}{2}} \quad \text{(surface element)}. \tag{2.55b}$$

The water-surface-elevation $H_{i\pm\frac{1}{2},j}$ or $H_{i,j\pm\frac{1}{2}}$ at the location of any enclosure surface of the element [Eqs. (2.55a) and (2.55b)] is evaluated by linear interpolation based on the water-surface elevation in the element $H_{i,j}$ and the water-surface elevation in the adjacent element $H_{i\pm1,j}$ or $H_{i,j\pm1}$ (Figs. 2.3 through 2.6, p.93 through p.96; Figs. 2.12 through 2.15, p.102 through p.105; Figs. 2.30 through 2.33, p.120 through p.123; Figs. 2.49 through 2.52, p.141 through p.144).

Hence, for x-upstream enclosure surface at $(i-\frac{1}{2},j,1)$ or for x-downstream enclosure surface at $(i+\frac{1}{2},j,1)$

$$H_{i\pm\frac{1}{2},j}=\frac{(\Delta x_{i,j}H_{i\pm1,j}+\Delta x_{i\pm1,j}H_{i,j})}{(\Delta x_{i,j}+\Delta x_{i\pm1,j})}, \tag{2.56a}$$

and for y-upstream enclosure surface at $(i,j-\frac{1}{2},1)$ or for y-downstream enclosure surface at $(i,j+\frac{1}{2},1)$

$$H_{i,j\pm\frac{1}{2}}=\frac{(\Delta y_{i,j}H_{i,j\pm1}+\Delta y_{i\pm1,j}H_{i,j})}{(\Delta y_{i,j}+\Delta y_{i,j\pm1})} \quad . \tag{2.56b}$$

In certain subregions, the specified depth of the flow region in an element $D_{i,j}$ can be less than or equal to the specified depth of the top layer Δz_L. Hence, in these single-layer subregions, an element (i,j,L) represents both a bottom element and also a surface element, with a vertical length in the element (Fig. 2.5, p.95; Fig. 2.6, p.96; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.32, p.122; Fig. 2.33, p.123; Fig. 2.51, p.143; and Fig. 2.52, p.144) as

$$(\Delta z_{i,j,L})_{bottom/surface}=\Delta z_{bot;i,j}+H_{i,j} \quad , \tag{2.57}$$

where $\Delta z_{bot;i,j}$ represents the vertical length of the bottom element, without including the effect of the free-surface [Eq. (2.49)] and $H_{i,j}$ represents the water-surface elevation in the surface element (i,j,L) [Eq. (2.50)].

Similarly, in these single-layer subregions, the vertical length on any enclosure surface of an element is the sum of (1) the vertical length of the bottom element, without including the effect of the free surface, and (2) the water-surface elevation at the location of the enclosure surface (Fig. 2.5, p.95; Fig. 2.6, p.96; Fig. 2.14, p.104; Fig. 2.15, p.105; Fig. 2.32, p.122; Fig. 2.33, p.123; Fig. 2.51, p.143; and Fig. 2.52, p.144). Hence, for x-upstream enclosure surface at $(i-\frac{1}{2},j,L)$ or for x-downstream enclosure surface at $(i+\frac{1}{2},j,L)$

$$(\Delta z_{i\pm\frac{1}{2},j,L})_{bottom/surface}=\Delta z_{bot;i\pm\frac{1}{2},j}+H_{i\pm\frac{1}{2},j} \quad , \tag{2.58}$$

where $\Delta z_{bot;i\pm\frac{1}{2},j}$ represents the vertical length of the bottom element, which is determined by linear interpolation [Eq. (2.54a)] for the enclosure surface at $(i\pm\frac{1}{2},j,L)$, and $H_{i\pm\frac{1}{2},j}$ represents the water-surface elevation, which is also determined by linear interpolation [Eq. (2.56a)] at the location of the enclosure surface at $(i\pm\frac{1}{2},j,L)$.

Similarly, for y-upstream enclosure surface at $(i,j-\frac{1}{2},L)$ or for y-downstream enclosure surface at $(i,j+\frac{1}{2},L)$

$$(\Delta z_{i,j\pm\frac{1}{2},L})_{bottom/surface} = \Delta z_{bot;i,j\pm\frac{1}{2}} + H_{i,j\pm\frac{1}{2}} \quad , \tag{2.59}$$

where, $\Delta z_{bot;i,j\pm\frac{1}{2}}$, represents the vertical length of the bottom, which is determined by linear interpolation[Eq. (2.55b)]for the enclosure surface at $(i,j\pm\frac{1}{2},L)$, and $H_{i,j\pm\frac{1}{2}}$ represents the water-surface elevation, which is also determined by linear interpolation [Eq. (2.56b)] at the location of the enclosure surface at $(i,j\pm\frac{1}{2},L)$.

The dimensions of the remaining two enclosure surfaces, normal to the vertical z-axis, of the element (i,j,l) (Fig. 2.2, p.92; Fig. 2.7, p.97; Fig. 2.16 through 2.25, p.106 through p.115; and 2.34 through 2.42, p.124 through p.132) are defined as

$\Delta x_{i,j,l-\frac{1}{2}}$ = horizontal length along x-axis of the z-upstream enclosure surface of the element (i,j,l) in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), at $l-\frac{1}{2}$,

$\Delta y_{i,j,l-\frac{1}{2}}$ = horizontal length along y-axis of the z-upstream enclosure surface of the element (i,j,l) in the horizontal (x,y) plane, at $l-\frac{1}{2}$,

$\Delta x_{i,j,l+\frac{1}{2}}$ = horizontal length along x-axis of the z-upstream enclosure

surface of the element (i,j,l) in the horizontal (x,y) plane, at $l+\frac{1}{2}$,

$\Delta y_{i,j,l+\frac{1}{2}}$ = horizontal length along y-axis of the z-upstream enclosure surface of the element (i,j,l) in the horizontal (x,y) plane, at $l+\frac{1}{2}$,

Since the geometrical discretization technique of the discrete-element method (DEM) divides the flow region into multiple layers in the vertical z-direction, between horizontal (x,y) planes (in the general sense, parallel to the geophysical surface plane), any element which is an interior element in the horizontal (x,y) plane, and which is also a vertically interior element, has the identical rectangular size and shape for the z-upstream enclosure surface at $l-\frac{1}{2}$ and for the z-downstream enclosure surface at $l+\frac{1}{2}$.

Furthermore, in the application of the geometrical discretization technique to a geophysical flow region, the dimensions of an element, in the horizontal (x,y) plane (in the general sense, parrallel to the geophysical surface plane), consisting of $x_{i,j}$ along x-axis and $y_{i,j}$ along y-axis, are generally much larger than the vertical (in the general sense, normal to the geophysical surface plane) dimension $z_{i,j,l}$ along the z-axis. Hence, for a boundary element in the horizontal (x,y) plane, which is also a bottom element, the enclosure surface of the element, which is adjacent to the shoreline boundary enclosure surface, and which is also adjacent to the bottom of the flow region, is only modified according to the contour of the shoreline boundary enclosure surface in the horizontal (x,y) plane. The contour of the bottom surface of the flow region is used only to specify the depth conditions

at the center of the two-dimensional planar elements in the horizontal (x,y) plane of the free-surface of the flow region. Therefore, for any element, the z-upstream enclosure surface can either be adjacent to another element or adjacent to the bottom surface of the flow region. If the z-upstream enclosure surface is adjacent to another element; then the geometrical discretization technique automatically assumes that the dimensions of the z-upstream enclosure surface at $(i,j,1-\frac{1}{2})$ are approximately the same as the dimensions of the element, $\Delta x_{i,j}$ and $\Delta y_{i,j}$, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane). If the z-upstream enclosure surface is adjacent to the bottom surface of the flow region, the geometrical discretization technique automatically identifies the enclosure surface at $(i,j,1-\frac{1}{2})$ as a bottom enclosure surface which can either be impermeable or can have specified transport according to the specified bottom seepage flow conditions.

The application of the discrete-element method to the simulation of hydrothermal conditions in a geophysical flow region assumes that in any element in the flow region the vertical depth between the bottom surface and the free-surface is completely occupied with water, without any part of the shoreline enclosure boundary intersecting the line between the bottom surface and the free-surface. Hence, the presentformulation of the discrete-element method (DEM) for geophysical flow regions cannot simulate flow conditions in submerged cavities which are closed by the shoreline boundary enclosure surface at the z-downstream enclosure surface at $(i,j,1+\frac{1}{2})$. Therefore, for any element in the flow region, the z-downstream enclosure surface $(i,j,1+\frac{1}{2})$ is always either adjacent

to another element or it corresponds to the free-surface in the element. Again, considering that the dimensions of an element in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), consisting of $\Delta x_{i,j}$ along x-axis and $\Delta y_{i,j}$ along y-axis, are generally much larger than the vertical (in the general sense, normal to the geophysical plane) dimension $\Delta z_{i,j,l}$ along z-axis, the geometrical discretization technique automatically assumes that the dimensions of the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ are approximately the same as the dimensions of the element, $\Delta x_{i,j}$ and $\Delta y_{i,j}$, in the horizontal (x,y) plane.

Hence, for z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ or for z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$

$$\Delta x_{i,j,l-\frac{1}{2}} = \Delta x_{i,j} \quad \text{(for all element)}, \tag{2.60a}$$

$$\Delta y_{i,j,l-\frac{1}{2}} = \Delta y_{i,j} \quad \text{(for all element)}, \tag{2.60b}$$

$$\Delta x_{i,j,l+\frac{1}{2}} = \Delta x_{i,j} \quad \text{(for all element)}, \tag{2.60c}$$

$$\Delta y_{i,j,l+\frac{1}{2}} = \Delta y_{i,j} \quad \text{(for all element)}, \tag{2.60d}$$

where the dimensions of the element (i,j,l) in the horizontal (x,y) plane, $\Delta x_{i,j}$ along x-axis and $\Delta y_{i,j}$ along y-axis, are determined according to the location of the element either as an interior element in the horizontal (x,y) plane [Eqs. (2.45a) and (2.45b)] or as a boundary element in the horizontal (x,y) plane [Eqs. (2.46a) and (2.46b)].

Each element in the flow region is divided into either two half-elements along x-direction (Fig. 2.7, p.97; Fig. 2.8, p.98; Fig. 2.10, p.100; Fig. 2.12, p.102; Fig. 2.14, p.104; and Figs. 2.16 through 2.24, p.106 through p.114) or into two half-elements along y-direction (Fig. 2.25, p.115; Fig. 2.27, p.117; Fig. 2.29, p.119; Fig. 2.31, p.121; and Figs. 2.33 through 2.42, p.123 through p.132). This new discretization technique is novel to the applications of the transient, multi-dimensional discrete-element method (DEM) for the solutions of hydrodynamic conditions in flow regions with geometrically complex enclosure boundaries (e.g., Eraslan, 1974, 1976, 1977, 1981a, 1981b, 1981c). Each half-element in the flow region is identified based on the location of its center according to the indices along the three ortho gonal axes (x,y,z) (Figs. 2.7 through 2.42, p.97 through p.132), as

$(i-\frac{1}{4},j,l)$ = index location of the x-upstream half-element with its center located at $(i-\frac{1}{4},j)$ in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane) and in layer l along the vertical z-axis (in the general sense, normal to the geophysical surface plane),

$(i+\frac{1}{4},j,l)$ = index location of the x-downstream half-element with its center located at $(i+\frac{1}{4},j)$ in the horizontal (x,y) plane and in layer l along the vertical z-axis,

$(i,j-\frac{1}{4},l)$ = index location of the y-upstream half-element with its center located at $(i,j-\frac{1}{4})$ in the horizontal (x,y) plane and in layer l along the vertical z-axis,

$(i,j+\frac{1}{4},l)$ = index location of the y-downstream half-element with its center located at $(i,j+\frac{1}{4})$ in the horizontal (x,y) plane and in layer l along the vertical z-axis,

The locations of the half-elements in the horizontal (x,y) plane, as distances of their centers from the origin of the specified Cartesian coordinate system along x and y axes, are defined relative to the location of the center of the element in the horizontal (x,y) plane that contains the half-elements (Figs. 2.7 through 2.42, p.97 through p.132) as

$x_{i-\frac{1}{4},j}$ = location (as distance) along x-axis of the center of the x-upstream half-element $(i-\frac{1}{4},j,l)$ in the horizontal (x,y) plane (in the general sense, parallel to the geographical surface plane) in layer l,

where

$$x_{i-\frac{1}{4},j}=x_{i,j}-\frac{\Delta x_{j,j}}{4} \quad \text{(for all elements)}, \tag{2.61a}$$

$y_{i-\frac{1}{4},j}$ = location (as distance) along y-axis of the center of the x-upstream half-element $(i-\frac{1}{4},j,l)$ in the horizontal (x,y) plane in layer l,

where

$$y_{i-\frac{1}{4},j}=y_{i,j} \quad \text{(for all elements)}, \tag{2.61b}$$

$x_{i+\frac{1}{4},j}$ = location (as distance) along x-axis of the center of the x-downstream half-element $(i+\frac{1}{4},j,l)$ in the horizontal (x,y) plane in layer l,

where

$$x_{i+\frac{1}{4},j}=x_{i,j}+\frac{\Delta x_{j,j}}{4} \quad \text{(for all elements)}, \tag{2.61c}$$

$y_{i+\frac{1}{4},j}$ = location (as distance) along y-axis of the center of the x-downstream half-element $(i+\frac{1}{4},j,1)$ in the horizontal (x,y) plane in layer 1,

where

$$y_{i+\frac{1}{4},j}=y_{i,j} \quad \text{(for all elements)}, \tag{2.61d}$$

$x_{i,j-\frac{1}{4}}$ = location (as distance) along x-axis of the center of the y-upstream half-element $(i,j-\frac{1}{4},1)$ in the horizontal (x,y) plane in layer 1,

where

$$x_{i,j-\frac{1}{4}}=x_{i,j} \quad \text{(for all elements)}, \tag{2.61e}$$

$y_{i,j-\frac{1}{4}}$ = location (as distance) along y-axis of the center of the y-upstream half-element $(i,j-\frac{1}{4},1)$ in the horizontal (x,y) plane in layer 1,

where

$$y_{i,j-\frac{1}{4}}=y_{i,j}-\frac{\Delta y_j}{4} \quad \text{(for all elements)}, \tag{2.61f}$$

$x_{i,j+\frac{1}{4}}$ = location (as distance) along x-axis of the center of the y-downstream half-element $(i,j+\frac{1}{4},1)$ in the horizontal (x,y) plane in layer 1,

where

$$x_{i,j+\frac{1}{4}}=x_{i,j} \quad \text{(for all elements)}, \tag{2.61g}$$

$y_{i,j+\frac{1}{4}}$ = location (as distance) along y-axis of the center of the y-downstream half-element $(i,j+\frac{1}{4},1)$ in the horizontal (x,y) plane in layer 1,

where

$$y_{i,j+\frac{1}{4}} = y_{i,j} + \frac{\Delta y_{j,j}}{4} \quad \text{(for all elements).} \qquad (2.61h)$$

Hence, the locations (as distances) of the center of the half-elements in the horizontal (x,y) plane in layer 1 [Eqs. (2.61a) and (2.61b)] in an element are defined in terms of (1) the locations (as distances) of the center of the element (i,j,1) in the horizontal (x,y) plane [Eq. (2.42a) and (2.42b) for an interior element in the horizontal (x,y) plane, or Eqs. (2.43a) and (2.43b) for a boundary element in the horizontal (x,y) plane] and (2) the horizontal dimensions (lengths) through the center of the element (i,j,1) in the horizontal (x,y) plane [Eqs. (2.45a) and (2.45b) for an interior element in the horizontal (x,y) plane, or Eqs. (2.46a) and (2.46b) for boundary element in the horizontal (x,y) plane].

The locations (as distances) along z-axis of the centers of the half-elements, along the vertical direction (in the general sense, normal to the geophysical surface plane) are assumed to be identically the same as the location (as distance) along z-axis of the center of the element (i,j,1), along the vertical direction (Figs. 2.7 through 2.15, p.97 through p.105; Figs. 2.25 through 2.33, p.115 through p.123), as

$z_{i-\frac{1}{4},j,1}$ = location (as distance) along z-axis of the center of the x-upstream half-element (i-¼,j,1) along the vertical direction (in the general sense, normal to the geophysical surface plane), in layer 1 [as measured from a horizontal datum plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane) below the bottom of the flow region],

where

$$z_{i-\frac{1}{4},j,l} = z_{i,j,l} \qquad \text{(for all elements)}, \tag{2.62a}$$

$z_{i+\frac{1}{4},j,l}$ = location (as distance) along z-axis of the center of the x-downstream half-element $(i+\frac{1}{4},j,l)$ along the vertical direction, in layer l,

where

$$z_{i+\frac{1}{4},j,l} = z_{i,j,l} \qquad \text{(for all elements)}, \tag{2.62b}$$

$z_{i,j-\frac{1}{4},l}$ = location (as distance) along z-axis of the center of the y-downstream half-element $(i,j-\frac{1}{4},l)$ along the vertical direction, in layer l,

where

$$z_{i,j-\frac{1}{4},l} = z_{i,j,l} \qquad \text{(for all elements)}, \tag{2.62c}$$

$z_{i,j+\frac{1}{4},l}$ = location (as distance) along z-axis of the center of the y-downstream half-element $(i,j+\frac{1}{4},l)$ along the vertical direction, in layer l,

where

$$z_{i,j+\frac{1}{4},l} = z_{i,j,l} \qquad \text{(for all elements)}, \tag{2.62d}$$

Hence, the locations (as distances) along z-axis of the centers of the half-elements along the vertical direction, in layer l [Eqs. (2.62a) through (2.62d)] in an element are assumed to be the same as the location as distance along z-axis of the center of the element (i,j,l) along the vertical direction, in layer l [Eq. (2.43) for a vertically interior element or a surface element, or Eq. (2.44) for a bottom element].

The horizontal dimensions (lengths) in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane) of any two pairs of half-elements, through the centers of the elements, in an element, are determined as halves of the dimension (length) of the element, through the center of the element along the axis which is normal to the two half-elements (Figs. 2.7 through 2.42, p.97 through p.132), as

$\Delta x_{i-\frac{1}{4},j,l}$ = horizontal length along x-axis of the x-upstream half-element ($i-\frac{1}{4},j,1$), through the center of the half-element in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), in layer l,

where

$$\Delta x_{i-\frac{1}{4},j,l}=\frac{\Delta x_{i,j}}{2} \quad \text{(for all elements)}, \tag{2.63a}$$

$\Delta y_{i-\frac{1}{4},j,l}$ = horizontal length along y-axis of the x-upstream half-element ($i-\frac{1}{4},j,1$), through the center of the half-element in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i-\frac{1}{4},j,l}=\Delta y_{j,j} \quad \text{[interior element in the horizontal (x,y) plane]}, \tag{2.63b}$$

or

$$\Delta y_{i-\frac{1}{4},j,l} = \tfrac{1}{2}[\Delta y_{l(i,j)} + \Delta y_{l(i,j);i-\frac{1}{2}}]$$

[boundary element in the horizontal (x,y) plane], (2.63c)

$\Delta x_{i+\frac{1}{4},j,l}$ = horizontal length along x-axis of the x-downstream half-element ($i+\frac{1}{4},j,1$), through the center of the half-element in the horizontal (x,y) plane, in layer l,

where

$$\Delta x_{i+\frac{1}{4},j,l}=\frac{\Delta x_{j,j}}{2}\ , \quad \text{(for all elements)}, \tag{2.63d}$$

$\Delta y_{i+\frac{1}{4},j,l}$ = horizontal length along y-axis of the x-upstream half-element $(i+\frac{1}{4},j,l)$, through the center of the half-element in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i+\frac{1}{4},j,l}=\Delta y_{i,j} \quad \text{[interior element in the horizontal (x,y) plane]}, \tag{2.63e}$$

or

$$\Delta y_{i+\frac{1}{4},j,l} = \tfrac{1}{2}[\Delta y_{I(i,j)} + \Delta y_{I(i,j);i+\frac{1}{2}}]$$

[boundary element in the horizontal (x,y) plane], (2.63f)

$\Delta x_{i,j-\frac{1}{4},l}$ = horizontal length along x-axis of the y-upstream half-element $(i,j-\frac{1}{4},l)$, through the center of the half-element in the horizontal (x,y) plane, in layer l,

where

$$\Delta x_{i,j-\frac{1}{4},l}=\Delta x_{i,j} \quad \text{[(interior element in the horizontal (x,y) plane]}, \tag{2.63g}$$

or

$$\Delta x_{i,j-\frac{1}{4},l} = \tfrac{1}{2}[\Delta x_{I(i,j)} + \Delta x_{I(i,j);j-\frac{1}{2}}]$$

[boundary element in the horizontal (x,y) plane], (2.63h)

$\Delta y_{i,j-\frac{1}{4},l}$ = horizontal length along y-axis of the y-upstream half-element $(i,j-\frac{1}{4},l)$, through the center of the half-element in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i,j-\frac{1}{4},l}=\frac{\Delta y_{i,j}}{2} \quad \text{(for all elements)}, \tag{2.63i}$$

$\Delta x_{i,j+\frac{1}{4},l}$ = horizontal length along x-axis of the y-downstream half-element $(i,j+\frac{1}{4},l)$, through the center of the half-element in the horizontal (x,y) plane, in layer l,

where

$$\Delta x_{i,j+\frac{1}{4},l}=\Delta x_{i,j} \quad \text{[interior element in the horizontal (x,y) plane]}, \tag{2.63j}$$

or

$$\Delta x_{i,j+\frac{1}{4},l} = \tfrac{1}{2}[\Delta x_{l(i,j)} + \Delta x_{l(i,j);j+\frac{1}{2}}]$$

[boundary element in the horizontal (x,y) plane], (2.63k)

$\Delta y_{i,j+\frac{1}{4},l}$ = horizontal length along y-axis of the y-upstream half-element $(i,j+\frac{1}{4},l)$, through the center of the half-element in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i,j+\frac{1}{4},l}=\frac{\Delta y_{i,j}}{2} \quad \text{(for all elements)}. \tag{2.63l}$$

Hence, the horizontal dimensions (lengths) of the half-elements, through the centers of the half-elements, in the horizontal (x,y) plane, in layer l [Eqs. (2.63a) through (2.63l)], in an element, are defined in terms of the horizontal dimensions (lengths) of the element (i,j,l), through the center of the element in the horizontal (x,y) plane, in layer l [Eqs. (2.45a) and (2.45b) for an interior element in the horizontal (x,y) plane, or Eqs. (2.46a) and (2,46b) for a boundary element in the horizontal (x,y) plane].

The vertical dimensions (lengths) of the half-elements along the vertical z-axis (in the general sense, normal to the geophysical surface plane), through the centers of the elements, in an element, are assumed to be the same as the vertical dimension (length) of the element, along the z-axis, through the center of the element (i,j,l) (Figs. 2.7 through 2.15, p.97 through p.105; Fig. 2.25 through 2.33, p.115 through p.123), as

$\Delta z_{i-\frac{1}{4},j,l}$ = vertical length of the x-upstream half-element $(i-\frac{1}{4},j,l)$ along z-axis (in the general sense, normal to the geophysical surface plane), in layer l,

where

$$\Delta z_{i-\frac{1}{4},j,l}=\Delta z_{i,j,l} \qquad \text{(all elements)}, \tag{2.64a}$$

$\Delta z_{i+\frac{1}{4},j,l}$ = vertical length of the x-downstream half-element $(i+\frac{1}{4},j,l)$ along z-axis, in layer l,

where

$$\Delta z_{i+\frac{1}{4},j,l}=\Delta z_{i,j,l} \qquad \text{(all elements)}, \tag{2.64b}$$

$\Delta z_{i,j-\frac{1}{4},l}$ = vertical length of the y-upstream half-element $(i,j-\frac{1}{4},l)$ along z-axis, in layer l,

where

$$\Delta z_{i,j-\frac{1}{4},l}=\Delta z_{i,j,l} \qquad \text{(all elements)}, \tag{2.64c}$$

$\Delta z_{i,j+\frac{1}{4},l}$ = vertical length of the y-downstream half-element $(i,j+\frac{1}{4},l)$ along z-axis, in layer l,

where

$$\Delta z_{i,j+\frac{1}{4},l}=\Delta z_{i,j,l} \qquad \text{(all elements)}. \tag{2.64d}$$

Hence, the vertical dimensions (lengths) of the half-elements, along z-axis, through the centers of the elements, in layer 1 [Eqs. (2.64a) through (2.64d)] are assumed to be the same as the vertical dimension (length) of the element (i,j,1), along z-axis, through the center of the element, in layer 1 [Eq. (2.47) for a vertically interior element, [Eq. (2.48)] for a bottom element, and Eq. (2.50) for a surface element].

Since, according to the geometrical discretization technique of the discrete-element method (DEM), each element consists of two half-elements, one downstream or upstream enclosure surface of a half-element always corresponds to one downstream or upstream enclosure surface of the element (Figs. 2.7 through 2.42, p.97 through p.132). Furthermore, since two half-elements equally divide an element into two parts by a vertical plane through the center of the element, one upstream or downstream enclosure surface of a half-element always corresponds to the area of the vertical plane through the center of the element (Figs. 2.7 through 2.42, p.97 through p.132). Therefore, the locations of the centers of the remaining four enclosure surfaces of each one of the four half-elements in each element are defined by the index locations of the centers of the half-elements $(i\pm \frac{1}{4},j,1)$ or $(i,j\pm \frac{1}{4},1)$, according to the general index notation, as

$(i-\frac{1}{4},j-\frac{1}{2},l)$ = index location of the y-upstream enclosure surface of the x-upstream half-element $(i-\frac{1}{4},j,1)$,

$(i-\frac{1}{4},j+\frac{1}{2},l)$ = index location of the y-downstream enclosure surface of the x-upstream half-element $(i-\frac{1}{4},j,1)$,

$(i-\frac{1}{4},j,l-\frac{1}{2})$ = index location of the z-upstream enclosure surface of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$(i-\frac{1}{4},j,l+\frac{1}{2})$ = index location of the z-downstream enclosure surface of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$(i+\frac{1}{4},j-\frac{1}{2},l)$ = index location of the y-upstream enclosure surface of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$(i+\frac{1}{4},j+\frac{1}{2},l)$ = index location of the y-downstream enclosure surface of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$(i+\frac{1}{4},j,l-\frac{1}{2})$ = index location of the z-upstream enclosure surface of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$(i+\frac{1}{4},j,l+\frac{1}{2})$ = index location of the z-downstream enclosure surface of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$(i-\frac{1}{2},j-\frac{1}{4},l)$ = index location of the x-upstream enclosure surface of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$(i+\frac{1}{2},j-\frac{1}{4},l)$ = index location of the x-downstream enclosure surface of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$(i,j-\frac{1}{4},l-\frac{1}{2})$ = index location of the z-upstream enclosure surface of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$(i,j-\frac{1}{4},l+\frac{1}{2})$ = index location of the z-downstream enclosure surface of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$(i-\frac{1}{2},j+\frac{1}{4},l)$ = index location of the x-upstream enclosure surface of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$(i+\frac{1}{2},j+\frac{1}{4},l)$ = index location of the x-downstream enclosure surface of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$(i,j+\frac{1}{4},l-\frac{1}{2})$ = index location of the z-upstream enclosure surface of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$(i,j+\frac{1}{4},l+\frac{1}{2})$ = index location of the z-downstream enclosure surface of the y-downstream half-element $(i,j+\frac{1}{4},l)$.

According to the geometrical discretization technique of the discrete-element method (DEM), which divides each element into four overlapping elements (into two-equal size half-elements along x-direction and also into two equal-size half-elements along y-direction), only two dimensions (lengths), in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), of the six enclosure surfaces of a half-element are different from the four dimensions (lengths), in the horizontal plane, of the six enclosure surfaces of the element, which contains the half-element (Figs. 2.7 through 2.42, p.97 throughn p.132). Hence, the dimension (Lengths), in horizontal (x,y) plane, of the six enclosure surfaces, of each of the four half-elements which are different than the dimensions (lengths), in the horizontal (x,y) plane, of the six enclosure surfaces, of the element which contains the four elements, are defined as

$\Delta x_{i-\frac{1}{4},j-\frac{1}{2},l}$ = horizontal length along x-axis of the y-upstream enclosure surface of the x-upstream half-element $(i-\frac{1}{4},j,l)$ in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane), in layer l,

where

$$\Delta x_{i-\frac{1}{4},j-\frac{1}{2},l}=\frac{\Delta x_{j,j-\frac{1}{2}}}{2} \qquad \text{(all elements)}, \tag{2.65a}$$

$\Delta x_{i-\frac{1}{4},j+\frac{1}{2},l}$ = horizontal length along x-axis of the y-downstream enclosure surface of the x-upstream half-element $(i-\frac{1}{4},j,l)$ in the horizontal (x,y) plane, in layer l,

where

$$\Delta x_{i-\frac{1}{4},j+\frac{1}{2},l} = \frac{\Delta x_{j,j+\frac{1}{2}}}{2} \qquad \text{(all elements)}, \tag{2.65b}$$

$\Delta x_{i+\frac{1}{4},j-\frac{1}{2},l}$ = horizontal length along x-axis of the y-upstream enclosure surface of the x-downstream half-element $(i+\frac{1}{4},j,l)$ in the horizontal (x,y) plane, in layer l,

where

$$\Delta x_{i+\frac{1}{4},j-\frac{1}{2},l} = \frac{\Delta x_{j,j-\frac{1}{2}}}{2} \qquad \text{(all elements)}, \tag{2.65c}$$

$\Delta x_{i+\frac{1}{4},j+\frac{1}{2},l}$ = horizontal length along x-axis of the y-downstream enclosure surface of the x-downstream half-element $(i+\frac{1}{4},j,l)$ in the horizontal (x,y) plane, in layer l,

where

$$\Delta x_{i+\frac{1}{4},j+\frac{1}{2},l} = \frac{\Delta x_{i,j+\frac{1}{2}}}{2} \qquad \text{(all elements)}, \tag{2.65d}$$

$\Delta y_{i-\frac{1}{2},j-\frac{1}{4},l}$ = horizontal length along y-axis of the x-upstream enclosure surface of the y-upstream half-element $(i,j-\frac{1}{4},l)$ in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i-\frac{1}{2},j-\frac{1}{4},l} = \frac{\Delta y_{j-\frac{1}{2},j}}{2} \qquad \text{(all elements)}, \tag{2.65e}$$

$\Delta y_{i+\frac{1}{2},j-\frac{1}{4},l}$ = horizontal length along y-axis of the x-downstream enclosure surface of the y-upstream half-element $(i,j-\frac{1}{4},l)$ in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i+\frac{1}{2},j-\frac{1}{4},l} = \frac{\Delta y_{j+\frac{1}{2},j}}{2} \qquad \text{(all elements)}, \tag{2.65f}$$

$\Delta y_{i+\frac{1}{2},j-\frac{1}{4},l}$ = horizontal length along y-axis of the x-upstream enclosure surface of the y-downstream half-element $(i,j+\frac{1}{4},l)$ in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i+\frac{1}{2},j-\frac{1}{4},l} = \frac{\Delta y_{j-\frac{1}{2},j}}{2} \qquad \text{(all elements)}, \tag{2.65g}$$

$\Delta y_{i-\frac{1}{2},j+\frac{1}{4},l}$ = horizontal length along y-axis of the x-downstream enclosure surface of the y-downstream half-element $(i+\frac{1}{2},j,l)$ in the horizontal (x,y) plane, in layer l,

where

$$\Delta y_{i-\frac{1}{2},j+\frac{1}{4},l} = \frac{\Delta y_{j+\frac{1}{2},j}}{2} \qquad \text{(all elements)}. \tag{2.65h}$$

Hence, for the four half-elements in an element, the dimensions (lengths) in the horizontal (x,y) plane, in layer l, of the enclosure surfaces of the half-elements, which are different from the dimensions (lengths) in the horizontal (x,y) plane, in layer l, of the enclosure surfaces of the element [Eqs. (2.65a) through (2.65h)], are determined in terms of the dimensions (lengths) in the horizontal (x,y) plane, in layer l, of the enlcosure surfaces of the element (i,j,l) [Eqs. (2.51a) and (2.51b) for an interior element in the horizontal (x,y) plane, or Eqs. (2.52a) and (2.52b) for a boundary element in the horizontal (x,y) plane].

The general formulation of the transient, three-dimensional discrete-element method considers five sets of discrete control volumes, consisting of (1) element (i,j,l), (2) x-upstream half-elements $(i-\frac{1}{4},j,l)$, (3) x-downstream half-elements $(i+\frac{1}{4},j,l)$, (4) y-upstream half-element $(i,j-\frac{1}{4},l)$, and (5) y-downstream half-element $(i,j+\frac{1}{4},l)$, in the applications of the integral forms of the two physical principles.

The control volume of each element (i,j,l) is determined , in the general form (Figs. 2.2 through 2.6, p.92 through p.96), as

$$v_{i,j,l}=A_{z;i,j}\Delta z_{i,j,l}\ , \tag{2.66}$$

where

$A_{z;i,j}$ = two dimensional planar area, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region), of the element (i,j).

For an interior element in the horizontal (x,y) plane, which represents an undistorted rectangular area, according to the specified non-uniform (or uniform) rectangular grid system for the flow region (Figs. 2.1 through 2.6, p.89, p.92 through 2.96), the area of an element, in the horizontal (x,y) plane, becomes

$$(A_{z;i,j})_{interior}=\Delta x_i \Delta y_j \quad [\text{interior element in the horizontal (x,y) plane}]. \tag{2.67}$$

For a boundary element in the horizontal (x,y) plane, which represents a trancated rectangular area with sections of the actual contours of the shoreline enclosure boundaries of the flow region (Figs. 2.16 through 2.24, p.106 through p.114; and Figs. 2.34 through 2.42, p.124 through p.132), the two-dimensional planar area, in the horizontal (x,y) plane, of an element is specified as input data as

$$(A_{z;i,j})_{boundary}=A_{I(i,j)} \quad [\text{boundary element in the horizontal (x,y) plane}]. \tag{2.68}$$

The two-dimensional planar area, in the horizontal (x,y) plane, for the boundary elements are obtained by utilizing the automated discrete-element-generator computer code PREPR2 (Holdeman, 1977) or its modified version PREPAR (Lin, Erhan and Eraslan, 1978).

Hence, the control volume of each element (i,j,l), $v_{i,j,l}$ [Eq. (2.66)], is determined based on the area of the element in the horizontal (x,y) plane, $A_{z;i,j}$ [Eqs. (2.67) and (2.68)] and the depth of the element in layer l, $\Delta z_{i,j,l}$ [Eqs. (2.47), (2.48), (2.49) and (2.50)].

The control volumes of the x-upstream half-element (i-¼,j,l) or the control volume of the x-downstream half-element (i+¼,j,l) is determined in the general form (Figs. 2.7 through 2.24, p.97 through p.114), as

$$v_{i\pm\frac{1}{4},j,l}=A_{z;i\pm\frac{1}{4},j}\Delta z_{i,j,l} \ , \tag{2.69}$$

where

$A_{z;i\pm\frac{1}{4},j}$ = two-dimensional planar area in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region), of the x-upstream half-element (i-¼,j) or x-downstream half-element (i+¼,j).

For an interior element in the horizontal (x,y) plane, which represents an undistorted rectangular area, according to the specified nonuniform (or uniform) rectangular grid system for the flow region (Figs. 2.1 through 2.6, p.89, p.92 through p.96), the two-dimensional planar areas in the horizontal (x,y) plane of the x-upstream half-element (i-¼,j,l) and x-downstream half-element (i+¼,j,l) are determined as two equal halves of the two-dimensional area in the horizontal (x,y) plane of the element (i,j) as

$$(A_{z;i\pm\frac{1}{4},j})_{interior}=\frac{\Delta x_j \Delta y_j}{2} \quad [\text{interior element in the horizontal (x,y) plane}]. \tag{2.70}$$

For a boundary element in the horizontal (x,y) plane, which represents a truncated rectangular area with sections of the actual contours of the shoreline enclosure boundaries of the flow region (Figs. 2.16 through 2.24, p.106 through p.114; and Figs. 2.34 through 2.42, p.124 through p.132), the two-dimensional planar areas in the horizontal (x,y) plane of the x-upstream half-element (i-¼,j,1) and x-downstream half-element (i+¼,j,1) are determined by considering trapezoidal approximations as

$$(A_{z;i\pm\frac{1}{4},j})_{boundary} = \frac{1}{2}\left[\Delta y_{I(i,j);i\pm\frac{1}{2}} + \Delta y_{I(i,j)}\right]\left[\frac{\Delta x_{I(i,j)}}{2}\right]$$

[boundary element in the horizontal (x,y) plane]. (2.71)

Hence, the control volume of each x-upstream half-element (i-¼,j,1) and the control volume of each x-downstream half-element (i+¼,j,1), $v_{i\pm\frac{1}{4},j,1}$ [Eq. (2.69)], are determined based on (1) the two-dimensional planar areas in the horizontal (x,y) plane of the half-elements, $A_{z;i\pm\frac{1}{4},j}$ [Eqs. (2.70) and (2.71)], which depend on the horizontal dimensions (lengths) of the element (i,j,1), in the horizontal (x,y) plane [Eqs. (2.45a), (2.45b) and (2.46a)], and/or the horizontal dimensions (lengths) in the horizontal (x,y) plane of the enclosure surfaces of the element (i,j,1) [Eq. (2.52a)], and (2) the depth of the element (i,j,1), in layer 1, $\Delta z_{i,j,1}$ [Eqs. (2.47), (2.48), (2.49) and (2.50)].

Similarly, the control volume of the y-upstream half-element (i,j-¼,1) or the control volume of the y-downstream half-element (i,j+¼,1) is determined in the general form (Figs. 2.25 through 2.42, p.115 through p.132), as

$$v_{i,j\pm\frac{1}{4},1} = A_{z;i,j\pm\frac{1}{4}}\Delta z_{i,j,1}, \qquad (2.72)$$

where

$A_{z;i,j\pm\frac{1}{4}}$ = two-dimensional planar area, and the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) of the y-upstream half-element $(i,j-\frac{1}{4},1)$ or y-downstream half-element $(i,j+\frac{1}{4},1)$.

For an interior element, in the horizontal (x,y) plane, which represents an undistorted rectangular area, according to the specified nonuniform (or uniform) rectangular grid system for the flow region (Figs. 2.1 through 2.6, p.89, p.92 through p.96), the two-dimensional planar areas in the horizontal (x,y) plane of the y-upstream half-element $(i,j-\frac{1}{4},1)$ and y-downstream half-element $(i,j+\frac{1}{4},1)$ are determined as two equal halves of the two-dimensional planar area, in the horizontal (x,y) plane of the element (i,j) as

$$(A_{z;i,j\pm\frac{1}{4}})_{interior}=\Delta x_i\frac{\Delta y_j}{2} \quad \text{interior element in the horizontal (x,y) plane .} \tag{2.73}$$

For a boundary element in the horizontal (x,y) plane, which represents a truncated rectangular area with sections of the actual contours of the shoreline enclosure boundaries of the flow region (Figs. 2.16 through 2.24, p.106 through p.114; and 2.34 through 2.42, p.124 through p.132), the two-dimensional planar areas in the horizontal (x,y) plane of the y-upstream half-element $(i,j-\frac{1}{4},1)$ and y-downstream half-element $(i,j+\frac{1}{4},1)$ are determined by considering trapezoidal approximations as

$$(A_{z;i,j\pm\frac{1}{4}})_{boundary}=\frac{1}{2}\left[\Delta x_{l(i,j);j\pm\frac{1}{2}}+\Delta x_{l(i,j)}\right]\left[\frac{\Delta y_{l(i,j)}}{2}\right]$$

[boundary element in the horizontal (x,y) plane]. (2.74)

Hence, the control volume of each y-upstream half-element $(i,j-\frac{1}{4},1)$ and the control volume of each y-downstream half-element $(i,j+\frac{1}{4},1)$,

$V_{i,j\pm\frac{1}{4},l}$ [Eq. (2.74)] are determined based on (1) the two-dimensional planar areas in the horizontal (x,y) plane, of the half-elements, $A_{z;i,j\pm\frac{1}{4}}$ [Eqs. (2.73) and (2.74)], which depend on the horizontal dimensions (lengths) of the element (i,j,l) in the horizontal (x,y) plane [Eqs. (2.45a), (2.45b) and (2.46b)] and/or the horizontal dimensions (lengths) in the horizontal (x,y) plane of the enclosure surfaces of the element (i,j,l) [Eq. (2.52b)] and (2) the depth of the element (i,j,l) in layer l, $\Delta z_{i,j,l}$ [Eq. (2.47) for a vertically interior element or Eq. (2.48) for a bottom element or Eq. (2.50) for a surface element].

The six enclosure surface areas of an element (i,j,l) are defined in terms of the dimensions (lengths) of the enclosure surfaces of the element (Figs. 2.2 through 2.42, p.92 through p.132) [Eqs. (2.51a) through (2.60b), (2.67) and (2.68)] as

$A_{x;i-\frac{1}{2},j,l}$ = x-upstream enclosure surface area of the element (i,j,l),

$$A_{x;i-\frac{1}{2},j,l}=\Delta y_{i-\frac{1}{2},j}\Delta z_{i-\frac{1}{2},j,l}, \tag{2.75a}$$

$A_{x;i+\frac{1}{2},j,l}$ = x-downstream enclosure surface area of the element (i,j,l),

$$A_{x;i+\frac{1}{2},j,l}=\Delta y_{i+\frac{1}{2},j}\Delta z_{i+\frac{1}{2},j,l}, \tag{2.75b}$$

$A_{y;i,j-\frac{1}{2},l}$ = y-upstream enclosure surface area of the element (i,j,l),

$$A_{y;i,j-\frac{1}{2},l}=\Delta x_{i,j-\frac{1}{2}}\Delta z_{i,j-\frac{1}{2},l}, \tag{2.75c}$$

$A_{y;i,j+\frac{1}{2},l}$ = y-downstream enclosure surface area of the element (i,j,l),

$$A_{y;i,j+\frac{1}{2},l}=\Delta x_{i,j+\frac{1}{2}}\Delta z_{i,j+\frac{1}{2},l}, \tag{2.75d}$$

$A_{z;i,j,l-\frac{1}{2}}$ = z-upstream enclosure surface area of the element (i,j,l),

$$A_{z;i,j,l-\frac{1}{2}} = A_{z;i,j} \ , \tag{2.75e}$$

$A_{z;i,j,l+\frac{1}{2}}$ = z-downstream enclosure surface area of the element (i,j,l),

$$A_{z;i,j,l+\frac{1}{2}} = A_{z;i,j} \ . \tag{2.75f}$$

The six enclosure-surface areas of a x-upstream half-element $(i-\frac{1}{4},j,l)$ includes the common x-upstream enclosure surface area $A_{x;i-\frac{1}{2},j,l}$ of the element (i,j,l). The remaining five enclosure surface areas are defined in terms of the dimensions (lengths) of the enclosure surfaces of the x-upstream half-element (Figs. 2.7 through 2.24, p.97 through p.114) [Eqs. (2.51a) through (2.60b), (2.70) and (2.71)] as

$A_{x;i,j,l}$ = x-downstream enclosure surface area of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$$A_{x;i,j,l} = \Delta y_{i,j} \Delta z_{i,j,l} \ , \tag{2.76a}$$

$A_{y;i-\frac{1}{4},j-\frac{1}{2},l}$ = y-upstream enclosure surface area of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$$A_{y;i-\frac{1}{4},j-\frac{1}{2},l} = \frac{\Delta x_{j,j-\frac{1}{2}} \Delta z_{i,j-\frac{1}{2},l}}{2} \ , \tag{2.76b}$$

$A_{y;i-\frac{1}{4},j+\frac{1}{2},l}$ = y-downstream enclosure surface area of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$$A_{y;i-\frac{1}{4},j+\frac{1}{2},l} = \frac{\Delta x_{j,j+\frac{1}{2}} \Delta z_{i,j+\frac{1}{2},l}}{2} \ , \tag{2.76c}$$

$A_{z;i-\frac{1}{4},j,l-\frac{1}{2}}$ = z-upstream enclosure surface area of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$$A_{z;i-\frac{1}{4},j,l-\frac{1}{2}} = A_{z;i-\frac{1}{4},j} \ , \tag{2.76d}$$

$A_{z;i-\frac{1}{4},j,l+\frac{1}{2}}$ = z-downstream enclosure surface area of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$$A_{z;i-\frac{1}{4},j,l+\frac{1}{2}} = A_{z;i-\frac{1}{4},j} \ . \tag{2.76e}$$

The six enclosure surface areas of a x-downstream half-element $(i+\frac{1}{4},j,l)$ includes the common x-downstream enclosure surface area $A_{x;i+\frac{1}{2},j,l}$ of the element (i,j,l) and the common x-downstream enclosure surface area $A_{x;i,j,l}$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$. The remaining four enclosure surface areas are defined in terms of the dimensions (lengths) of the enclosure surfaces of the x-downstream half-element (Figs. 2.7 through 2.24, p.97 through p.114) [Eqs. (2.51a) through (2.60b), (2.70) and (2.71)] as

$A_{y;i+\frac{1}{4},j-\frac{1}{2},l}$ = y-upstream enclosure surface area of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$$A_{y;i+\frac{1}{4},j-\frac{1}{2},l} = \frac{\Delta x_{j,j-\frac{1}{2}} \Delta z_{i,j-\frac{1}{2},l}}{2} \ , \tag{2.77a}$$

$A_{y;i+\frac{1}{4},j+\frac{1}{2},l}$ = y-downstream enclosure surface area of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$$A_{y;i+\frac{1}{4},j+\frac{1}{2},l} = \frac{\Delta x_{j,j+\frac{1}{2}} \Delta z_{i,j+\frac{1}{2},l}}{2} \ , \tag{2.77b}$$

$A_{z;i+\frac{1}{4},j,l-\frac{1}{2}}$ = z-upstream enclosure surface area of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$$A_{z;i+\frac{1}{4},j,l-\frac{1}{2}} = A_{z;i+\frac{1}{4},j} \ , \tag{2.77c}$$

$A_{z;i+\frac{1}{4},j,l+\frac{1}{2}}$ = z-downstream enclosure surface area of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$$A_{z;i+\frac{1}{4},j,l+\frac{1}{2}} = A_{z;i+\frac{1}{4},j} \ . \tag{2.77d}$$

The six enclosure surface areas of a y-upstream half-element $(i,j-\frac{1}{4},l)$ includes the common y-upstream enclosure surface area $A_{y;i,j-\frac{1}{2},l}$ of the element (i,j,l). The remaining five enclosure surface areas are defined in terms of the dimensions (lengths) of the enclosure surfaces of the y-upstream half-element (Figs. 2.25 through 2.42, p.115 through p.143) [Eqs. (2.51a) through (2.60b), (2.73) and (2.74)] as

$A_{y;i,j,l}$ = y-downstream enclosure surface area of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$$A_{y;i,j,l} = \Delta x_{i,j} \Delta z_{i,j,l} \ , \tag{2.78a}$$

$A_{x;i-\frac{1}{2},j-\frac{1}{4},l}$ = x-upstream enclosure surface area of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$$A_{x;i-\frac{1}{2},j-\frac{1}{4},l} = \frac{\Delta y_{j-\frac{1}{2},j}}{2} \Delta z_{i-\frac{1}{2},j,l} \ , \tag{2.78b}$$

$A_{x;i+\frac{1}{2},j-\frac{1}{4},l}$ = x-downstream enclosure surface area of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$$A_{x;i+\frac{1}{2},j-\frac{1}{4},l} = \frac{\Delta y_{j+\frac{1}{2},j}}{2} \Delta z_{i+\frac{1}{2},j,l} \ , \tag{2.78c}$$

$A_{z;i,j-\frac{1}{4},l-\frac{1}{2}}$ = z-upstream enclosure surface area of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$$A_{z;i,j-\frac{1}{4},l-\frac{1}{2}} = A_{z;i,j-\frac{1}{4}} \,, \tag{2.78d}$$

$A_{z;i,j-\frac{1}{4},l+\frac{1}{2}}$ = z-downstream enclosure surface area of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$$A_{z;i,j-\frac{1}{4},l+\frac{1}{2}} = A_{z;i,j-\frac{1}{4}} \,. \tag{2.78e}$$

The six enclosure surface areas of a y-downstream half-element $(i,j+\frac{1}{4},l)$ includes the common y-downstream enclosure surface area $A_{y;i,j+\frac{1}{2},l}$ of the element (i,j,l) and the common y-downstream enclosure surface area $A_{y;i,j,l}$ of the y-upstream half-element $(i-\frac{1}{4},j,l)$. The remaining four enclosure surface areas are defined in terms of the dimensions (lengths) of the enclosure surfaces of the y-downstream half-element (Figs. 2.25 through 2.42, p.115 through p.132) [Eqs. (2.51a) through (2.60b), (2.73) and (2.74)] as

$A_{x;i-\frac{1}{2},j+\frac{1}{4},l}$ = x-upstream enclosure surface area of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$$A_{x;i-\frac{1}{2},j+\frac{1}{4},l} = \frac{\Delta y_{j-\frac{1}{2}}, j \Delta z_{i-\frac{1}{2},j,l}}{2} \,, \tag{2.79a}$$

$A_{x;i+\frac{1}{2},j+\frac{1}{4},l}$ = x-downstream enclosure surface area of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$$A_{x;i+\frac{1}{2},j+\frac{1}{4},l} = \frac{\Delta y_{j+\frac{1}{2}}, j \Delta z_{i+\frac{1}{2},j,l}}{2} \,, \tag{2.79b}$$

$A_{z;i,j+\frac{1}{4},l-\frac{1}{2}}$ = z-upstream enclosure surface area of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$$A_{z;i,j+\frac{1}{4},l-\frac{1}{2}} = A_{z;i,j+\frac{1}{4}} \,, \tag{2.79c}$$

$A_{z;i,j+\frac{1}{4},l+\frac{1}{2}}$ = z-downstream enclosure surface area of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$$A_{z;i,j+\frac{1}{4},l+\frac{1}{2}} = A_{z;i,j+\frac{1}{4}} \ . \tag{2.79d}$$

Hence, the closed enclosure surface area of each element (i,j,l) and the closed enclosure surface areas of the four half-elements, consisting of (1) x-upstream half-element $(i-\frac{1}{4},j,l)$, (2) x-downstream half-element $(i+\frac{1}{4},j,l)$, (3) y-upstream half-element $(i,j-\frac{1}{4},l)$, and (4) y-downstream half-element $(i,j+\frac{1}{4},l)$ can be fomulated in terms of the dimensions of the enclosure surfaces of the elements and half-element [Eqs. (2.75a) through (2.79d)].

According to the general discretization technique of the three-dimensional discrete-element method (DEM), the indices associated with the elements range,

$$i=1,2,\ldots,N_x \quad (\text{along x-direction}), \tag{2.80a}$$

$$j=1,2,\ldots,N_y \quad (\text{along y-direction}), \tag{2.80b}$$

$$l=1,2,\ldots,N_z \quad (\text{along z-direction}), \tag{2.80c}$$

based on a specified conventional nonuniform (or uniform) rectangular grid system in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane) (Fig. 2.1, p.89) and a specified set of depths for the layers in the vertical z-direction (in the general sense, normal to the geophysical surface plane). However,

the computational algorithm of the general discrete-element method (DEM) utilizes only a single index array in the horizontal (x,y) plane as

$$I(i,j) = 1, 2, \ldots, N(N_x, N_y), \qquad (2.81)$$

which includes only the internal computational elements in the flow region consisting of only the interior elements and the boundary elements. Hence, the computational algorithm automatically eliminates the storage for the unused external elements, which are outside of the region; and therefore, from computational storage considerations it is at least as efficient as any used in the finite-element method (FEM) in the horizontal (x,y) plane. The range of the vertical index l is determined for in each two-dimensional planar element (i,j) in the horizontal (x,y) plane of the free surface of the flow region, based on the location of the bottom element, by another single index array, as

$N_{z,bot}(i,j)$ = the minimum vertical index l corresponding to the location of the bottom element in the two-dimensional planar element (i,j) in the horizontal (x,y) plane of the free-surface of the flow region [also l(bot;i,j), eq.(2.49)].

Therefore, the computational algorithm of the general three-dimensional discrete-element method (DEM) considers the range of the vertical index l as

$$l = N_{z,bot}(i,j), \ldots, N_z \quad \text{(along z-direction)}. \qquad (2.82)$$

Hence, again, the range of the index l in the vertical z-direction includes only the interior elements.

Therefore, from three-dimensional considerations of the geometrical discretization technique, the general discrete-element method (DEM) utilizes a computational storage algorithm which is at least as efficient as any used in the three-dimensional finite-element method (FEM).

The two single index arrays I(i,j) and $N_{z,bot}(i,j)$ of the general, three-dimensional discrete-element method (DEM) automatically provide all the necessary information about the locations of all the elements and half-elements that are adjacent to an element or a half-element in the flow region. Hence, the two-single-index arrays [I(i,j) and $N_{z,bot}(i,j)$] represent the location indeces of the general three-dimensional discrete-element method (DEM); and, in a sense, they conceptually correspond to the connectivity matrix of a finite-element method (FEM).

It is important to note that in the calculations of the volumes and the enclosure surface areas of the elements and half-elements, the general three-dimensional discrete-element method (DEM) requires arrays with double indeces only for the boundary elements which represent truncated rectangular cells. All the dimensions (lengths) of the interior elements are determined without storage, based on the specified dimensions (lengths) of the nonuniform (or uniform) rectangular grid system, since the interior elements represent strictly rectangular cells. Hence, the storage algorithm of the geometrical discretization technique of the general three-dimensional discrete-element method (DEM) is superior to any used in the finite-element method (FEM).

2.4. Discrete-Element Formulations of the Volume Integrals and the Surface Integrals of the Physical Principles

The development of the transient, three-dimensional discrete element method (DEM) is based on mathematically rigorous evaluation of the control volume integrals and the closed control surface integrals associated with the integral forms of the two physical principles, consisting of (1) the integral form of mass conservation principle for water and (2) the three components of the integral form of the momentum principle for water [Eqs. (2.1), (2.2), Eqs. (2.10), (2.20) through (2.22)]. This novel approach (Eraslan, 1974, 1976, 1977, 1981a, 1981b, 1981c) leads to the appropriate formulations of the transport rates, associated with (1) mass of water, (2) momentum of water, which include explicitely (a) the convection-defect transport rates and (b) the eddy-turbulent transport rates across the enclosure surface areas of the elements and half-elements.

The general volume integrals of the control volumes associated with the integral forms of the statements of the physical principles [Eqs. (2.1) through (2.2), or Eqs. (2.10), (2.20) through (2.22)] can be formulated for each element or half-element in terms of its dimensions (lengths) (Sec. 2.3) which were determined based on the geometrical discretization technique (Sec. 2.3) of the general three-dimensional discrete-element method (DEM).

The general volume integral for the control volume of the element (i,j,l) can be formulated as

$$\iiint_{V_{i,j,l}} dv = \iint_{A_{z;i,j}} \int_{-\frac{\Delta z_{i,j,l}}{2}}^{\frac{\Delta z_{i,j,l}}{2}} dx\, dy\, dz, \tag{2.83}$$

which is in terms of (1) the two-dimensional planar area $A_{z;i,j}$, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) of the element (i,j,1) [Eq. (2.67) for an interior element in the horizontal (x,y) plane, or Eq. (2.68) for a boundary element in the horizontal (x,y) plane], and (2) the depth of the element (i,j,1), in layer 1 $\Delta z_{i,j,l}$, as the vertical dimension (length), along the z-axis (in the general sense, normal to the geophysical surface plane) [Eq. (2.47) for a vertically interior element, or Eq. (2.48) for a bottom element, or Eq. (2.50) for a surface element].

Similarly, the general volume integral for the control volume of the x-upstream half-element (i-¼,j,1) or x-downstream half-element (i+¼,j,1) can be formulated as

$$\iiint_{V_{i\pm\frac{1}{4},j,l}} dv = \iint_{A_{z;i\pm\frac{1}{4},j}} \int_{-\frac{\Delta z_{j\pm\frac{1}{4},j,l}}{2}}^{\frac{\Delta z_{i\pm\frac{1}{4},j,l}}{2}} dx\, dy\, dz, \tag{2.84a}$$

or

$$\iiint_{V_{i\pm\frac{1}{4},j,l}} dv = \iint_{A_{z;i\pm\frac{1}{4},j}} \int_{-\frac{\Delta z_{j,j,l}}{2}}^{\frac{\Delta z_{i,j,l}}{2}} dx\, dy\, dz\,. \tag{2.84b}$$

which is in terms of (1) the two-dimensional planar area $A_{z;i\pm\frac{1}{4},j}$, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) of the x-upstream half-element (i-¼,j,1) or x-downstream half-element (i+¼,j,1) [Eq. (2.70) for an interior element in the horizontal (x,y)

plane, or Eq. (2.71) for a boundary element in the horizontal (x,y) plane], and (2) the depth of the x-upstream half-element $(i-\frac{1}{4},j,l)$ or the x-downstream half-element $(i+\frac{1}{4},j,l)$, in layer l as the vertical dimension (length), along the z-axis (in the general sense, normal to the geophysical surface plane), $\Delta z_{i\pm\frac{1}{4},j,l}$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.64a)] or x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.64b)], which is assumed to be equal to the depth of the element (i,j,l), in layer l, as the vertical dimension (length), along the z-axis, $\Delta z_{i,j,l}$ [Eq. (2.47) for a vertically interior element, or Eq. (2.48) for a bottom element, or Eq. (2.50) for a surface element].

Similarly, the general volume integral for the control volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$ can be formulated as

$$\iiint_{V_{i,j\pm\frac{1}{4},l}} dv = \iint_{A_{z;i,j\pm\frac{1}{4}}} \int_{-\frac{\Delta z_{i,j\pm\frac{1}{4},l}}{2}}^{\frac{\Delta z_{i,j\pm\frac{1}{4},l}}{2}} dx\,dy\,dz\,. \tag{2.85a}$$

or

$$\iiint_{V_{i\pm\frac{1}{4},j,l}} dv = \iint_{A_{z;i\pm\frac{1}{4},j}} \int_{-\frac{\Delta z_{i,j,l}}{2}}^{\frac{\Delta z_{i,j,l}}{2}} dx\,dy\,dz\,. \tag{2.85b}$$

which is in terms of (1) the two-dimensional planar area , $A_{z;i,j\pm\frac{1}{4}}$, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) of the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq.(2.73) for an interior element in the horizontal (x,y) plane, or Eq. (2.74) for a boundary element in the horizontal (x,y) plane], and (2) the depth of the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$, in layer l, as the vertical

dimension (length), along the z-axis, (in the general sense, normal to the geophysical surface plane), $\Delta z_{i,j\pm\frac{1}{4},l}$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.64c)] or y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.64d)], which is assumed to be equal to the depth of the element (i,j,l), in layer l, as the vertical dimension (length) along the z-axis, $\Delta z_{i,j,l}$ [Eq. (2.47) for a vertically interior element, or Eq. (2.48) for a bottom element, or Eq. (2.50) for a surface element].

For completely interior elements, which are both interior in the horizontal (x,y) plane and vertically interior, the general volume integrals for (1) the control volume of the element (i,j,l) [Eq. (2.83)], (2) the control volume of the x-upstream half-element $(i-\frac{1}{4},j,l)$ or x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eqs. (2.12a) and (2.12b)], and (3) the control volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eqs. (2.13a) and (2.13b)] reduce to the simple forms as

$$\left[\iiint_{V_{i,j,l}} dv\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{2}}^{\frac{\Delta x_j}{2}} \int_{-\frac{\Delta y_j}{2}}^{\frac{\Delta y_j}{2}} \int_{-\frac{\Delta z_j}{2}}^{\frac{\Delta z_j}{2}} dx\, dy\, dz, \tag{2.86a}$$

$$\left[\iiint_{V_{i\pm\frac{1}{4},j,l}} dv\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{4}}^{\frac{\Delta x_j}{4}} \int_{-\frac{\Delta y_j}{2}}^{\frac{\Delta y_j}{2}} \int_{-\frac{\Delta z_j}{2}}^{\frac{\Delta z_j}{2}} dx\, dy\, dz, \tag{2.86b}$$

$$\left[\iiint_{V_{i,j\pm\frac{1}{4},l}} dv\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{2}}^{\frac{\Delta x_j}{2}} \int_{-\frac{\Delta y_j}{4}}^{\frac{\Delta y_j}{4}} \int_{-\frac{\Delta z_j}{2}}^{\frac{\Delta z_j}{2}} dx\, dy\, dz\,. \tag{2.86c}$$

Hence, the volume integrals of the control volumes of all the elements and half-elements in the flow region can be formulated in terms

of the dimensions (lengths) of the elements and half-elements which are determined based on the geometrical discretization technique (Sec. 2.3) of the general three-dimensional discrete-element method (DEM).

The general forms for the closed control-surface integrals of the control volumes associated with the integral forms of the physical principles [Eqs. (2.1) through (2.2), or Eqs. (2.10), (2.20) through (2.22)] can be formulated for each element and half-element in terms of it dimensions (lengths) (Sec. 2.3) which are determined based on the geometrical discretization technique of the general three-dimensional discrete-element method (DEM).

The enclosure-surface areas which correspond to the intake cross sections or to the discharge cross sections can be included in two forms in the formulation of the general closed control-surface integrals of the control volumes. Large intake or discharge cross-section areas, relative to the enclosure surface areas of the elements and half-elements, can be considered as open shoreline enclosure surfaces or bottom enclosure surfaces, with specified flow conditions. Small intake or discharge cross-section areas, relative to the enclosure surface areas of the elements and half-elements, can be considered as separate areas, located in the elements and/or half-elements, which constitute small parts of the closed control-surface integrals of the control volumes.

Hence, the general closed control-surface integral for the control volume of the element (i,j,l) can be formulated as

$$\iint_{A_{i,j,l}} dA = \iint_{A_{x;i-\frac{1}{2},j,l}} dA + \iint_{A_{x;i+\frac{1}{2},j,l}} dA + \iint_{A_{y;i,j-\frac{1}{2},l}} dA + \iint_{A_{y;i,j+\frac{1}{2},l}} dA$$

$$+\iint_{A_{z;i,j,l-\frac{1}{2}}} dA + \iint_{A_{z;i,j,l+\frac{1}{2}}} dA + \delta_{int,m;i,j,l} \iint_{A_{int,m;i,j,l}} dA$$

$$+\delta_{dis,m;i,j,l} \iint_{A_{dis,m;i,j,l}} dA \,. \qquad (2.87)$$

where,

$\delta_{int,m;i,j,l}$ = Kronecker delta for indicating the location of the intake of power plant m in the element (i,j,l), i.e.,

= 0 : location of the intake of power plant m is not in element (i,j,l),

= 1 : location of the intake of power plant m is in element (i,j,l),

$\delta_{dis,m;i,j,l}$ = Kronecker delta for indicating the location of the discharge of power plant m in the element (i,j,l), i.e.,

= 0 : location of the discharge of power plant m is not in element (i,j,l),

= 1 : location of the discharge of power plant m is in element (i,j,l),

$A_{int,m;i,j,l}$ = cross-section area of the intake of power plant m in the element (i,j,l),

$A_{dis,m;i,j,l}$ = cross-section area of the discharge of power plant m in the element (i,j,l).

Similarly, the general closed control-surface integral for the control volume of x-upstream half-element (i-¼,j,l) or x-downstream half-element (i+¼,j,l) can be formulated as,

$$\iint_{A_{i\pm\frac{1}{4},j,l}} dA = \iint_{A_{x;i\pm\frac{1}{2},j,l}} dA + \iint_{A_{x;i,j,l}} dA + \iint_{A_{y;i\pm\frac{1}{4},j-\frac{1}{2},l}} dA + \iint_{A_{y;i\pm\frac{1}{4},j+\frac{1}{2},l}} dA$$

(continue)

$$+\iint_{A_{z;i\pm\frac{1}{4},j,l-\frac{1}{2}}} dA+\iint_{A_{z;i\pm\frac{1}{4},j,l+\frac{1}{2}}} dA+\delta_{int,m;i\pm\frac{1}{4},j,l}\iint_{A_{int,m;i\pm\frac{1}{4},j,l}} dA$$

$$+\delta_{dis,m;i\pm\frac{1}{4},j,l}\iint_{A_{dis,m;i\pm\frac{1}{4},j,l}} dA\ , \qquad (2.88)$$

where

$\delta_{int,m;i\pm\frac{1}{4},j,l}$ = Kronecker delta for indicating the location of the intake of power plant m in the x-upstream half-element ($i-\frac{1}{4},j,l$) or x-downstream half-element ($i+\frac{1}{4},j,l$), i.e.,

= 0 : location of the intake of power plant m is not in the x-upstream half-element ($i-\frac{1}{4},j,l$) or x-downstream half-element ($i+\frac{1}{4},j,l$),

= 1 : location of the intake of power plant m is in the x-upstream half-element ($i-\frac{1}{4},j,l$) or x-downstream half-element ($i+\frac{1}{4},j,l$),

$\delta_{dis,m;i\pm\frac{1}{4},j,l}$ = Kronecker delta for indicating the location of the discharge of power plant m in the x-upstream half-element ($i-\frac{1}{4},j,l$) or x-downstream half-element ($i+\frac{1}{4},j,l$), i.e.,

= 0 : location of the discharge of power plant m is not in the x-upstream half-element ($i-\frac{1}{4},j,l$) or x-downstream half-element ($i+\frac{1}{4},j,l$),

= 1 : location of the discharge of power plant m is in the x-upstream half-element ($i-\frac{1}{4},j,l$) or x-downstream half-element ($i+\frac{1}{4},j,l$),

$A_{int,m;i\pm\frac{1}{4},j,l}$ = cross-section area of the intake of power plant m in the x-upstream half-element $(i-\frac{1}{4},j,l)$ or x-downstream half-element $(i+\frac{1}{4},j,l)$,

$A_{dis,m;i\pm\frac{1}{4},j,l}$ = cross-section area of the discharge of power plant m in the x-upstream half-element $(i-\frac{1}{4},j,l)$ or x-downstream half-element $(i+\frac{1}{4},j,l)$,

Similarly, the general closed control-surface integral for the control volume of y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$ can be formulated as

$$\iint_{A_{i,j\pm\frac{1}{4},l}} dA = \iint_{A_{x;i-\frac{1}{2},j\pm\frac{1}{4},l}} dA + \iint_{A_{x;i-\frac{1}{2},j\pm\frac{1}{4},l}} dA + \iint_{A_{y;i,j\pm\frac{1}{4},l}} dA + \iint_{A_{y;i,j,l}} dA$$
$$+ \iint_{A_{z;i,j\pm\frac{1}{4},l-\frac{1}{2}}} dA + \iint_{A_{z;i,j\pm\frac{1}{4},l+\frac{1}{2}}} dA + \delta_{int,m;i,j\pm\frac{1}{4},l} \iint_{A_{int,m;i,j\pm\frac{1}{4},l}} dA$$
$$+ \delta_{dis,m;i,j\pm\frac{1}{4},l} \iint_{A_{dis,m;i,j\pm\frac{1}{4},l}} dA. \qquad (2.89)$$

where

$\delta_{int,m;i,j\pm\frac{1}{4},l}$ = Kronecker delta for indicating the location of the intake of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$, i.e.,

= 0 : location of the intake of power plant m is not in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$,

= 1 : location of the intake of power plant m is in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$,

$\delta_{dis,m;i,j\pm\frac{1}{4},l}$ = Kronecker delta for indicating the location of the discharge of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$, i.e.,

= 0 : location of the discharge of power plant m is not in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$,

= 1 : location of the discharge of power plant m is in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$,

$A_{int,m;i,j\pm\frac{1}{4},l}$ = cross-section area of the intake of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$,

$A_{dis,m;i,j\pm\frac{1}{4},l}$ = cross-section area of the discharge of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$ or y-downstream half-element $(i,j+\frac{1}{4},l)$

The enclosure-surface integrals associated with the upstream and downstream enclosure-surface areas of the elements and half-elements [Eqs. (2.87) through (2.89)] are formulated according to dimensions (length) of the elements and half-elements (Sec. 2.3).

The enclosure-surface integral associated with the x-upstream enclosure-surface area at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream enclosure-surface area at $(i-\frac{1}{2},j,l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.75a)] becomes

$$\iint_{A_{x;i-\frac{1}{2},j,l}} dA = \int_{-\frac{\Delta y_{j-\frac{1}{2},j}}{2}}^{\frac{\Delta y_{j-\frac{1}{2},j}}{2}} \int_{-\frac{\Delta z_{j-\frac{1}{2},j,l}}{2}}^{\frac{\Delta z_{j-\frac{1}{2},j,l}}{2}} dy\, dz . \qquad (2.90a)$$

Similarly, the enclosure-surface integral associated with the x-downstream enclosure-surface area at (i,j,l) of the x-upstream half-element (i-¼,j,l) and the x-upstream enclosure-surface area at (i,j,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.75a)] becomes

$$\iint_{A_{x;i,j,l}} dA = \int_{-\frac{\Delta y_{j,j}}{2}}^{\frac{\Delta y_{j,j}}{2}} \int_{-\frac{\Delta z_{j,j,l}}{2}}^{\frac{\Delta z_{j,j,l}}{2}} dy\, dz \,. \tag{2.90b}$$

Similarly, the enclosure-surface integral associated with the x-downstream enclosure-surface area at (i+½,j,l) of the element (i,j,l) and the x-downstream enclosure-surface area at (i+½,j,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.75b)] becomes

$$\iint_{A_{x;i+\frac{1}{2},j,l}} dA = \int_{-\frac{\Delta y_{j+\frac{1}{2},j}}{2}}^{\frac{\Delta y_{j+\frac{1}{2},j}}{2}} \int_{-\frac{\Delta z_{j+\frac{1}{2},j,l}}{2}}^{\frac{\Delta z_{j+\frac{1}{2},j,l}}{2}} dy\, dz \,. \tag{2.90c}$$

Similarly, the enclosure-surface integral associated with the y-upstream enclosure-surface area at (i-¼,j-½,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.76b)] becomes

$$\iint_{A_{y;i-\frac{1}{4},j-\frac{1}{2},l}} dA = \int_{-\frac{\Delta x_{j,j-\frac{1}{2}}}{4}}^{\frac{\Delta x_{j,j-\frac{1}{2}}}{4}} \int_{-\frac{\Delta z_{j,j-\frac{1}{2},l}}{2}}^{\frac{\Delta z_{j,j-\frac{1}{2},l}}{2}} dx\, dz \,. \tag{2.90d}$$

Similarly, the enclosure-surface integral associated with the y-downstream enclosure-surface area at (i-¼,j+½,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.76c)] becomes

$$\iint_{A_{y;i-\frac{1}{4},j+\frac{1}{2},l}} dA = \int_{-\frac{\Delta x_{j,j+\frac{1}{2}}}{4}}^{\frac{\Delta x_{j,j+\frac{1}{2}}}{4}} \int_{-\frac{\Delta z_{j,j+\frac{1}{2},l}}{2}}^{\frac{\Delta z_{j,j+\frac{1}{2},l}}{2}} dx\, dz \,. \tag{2.90e}$$

Similarly, the enclosure-surface integral associated with the y-upstream enclosure-surface area at $(i+\frac{1}{4},j-\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.77a)] becomes

$$\iint_{A_{y;i+\frac{1}{4},j-\frac{1}{2},l}} dA = \int_{-\frac{\Delta x_{j,j-\frac{1}{2}}}{4}}^{\frac{\Delta x_{j,j-\frac{1}{2}}}{4}} \int_{-\frac{\Delta z_{j,j-\frac{1}{2},l}}{2}}^{\frac{\Delta z_{j,j-\frac{1}{2},l}}{2}} dx\,dz\,. \tag{2.90f}$$

Similarly, the enclosure-surface integral associated with the y-downstream enclosure-surface area at $(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.77b)] becomes

$$\iint_{A_{y;i+\frac{1}{4},j+\frac{1}{2},l}} dA = \int_{-\frac{\Delta x_{j,j+\frac{1}{2}}}{4}}^{\frac{\Delta x_{j,j+\frac{1}{2}}}{4}} \int_{-\frac{\Delta z_{j,j+\frac{1}{2},l}}{2}}^{\frac{\Delta z_{j,j+\frac{1}{2},l}}{2}} dx\,dz\,. \tag{2.90g}$$

Similarly, the enclosure-surface integral associated with the y-upstream enclosure-surface area at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream enclosure-surface area at $(i,j-\frac{1}{2},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.75c)] becomes

$$\iint_{A_{y;i,j-\frac{1}{2},l}} dA = \int_{-\frac{\Delta x_{j,j-\frac{1}{2}}}{2}}^{\frac{\Delta x_{j,j-\frac{1}{2}}}{2}} \int_{-\frac{\Delta z_{j,j-\frac{1}{2},l}}{2}}^{\frac{\Delta z_{j,j-\frac{1}{2},l}}{2}} dx\,dz\,. \tag{2.90h}$$

Similarly, the enclosure-surface integral associated with the y-downstream enclosure-surface area at (i,j,l) of the y-upstream half-element $(i,j-\frac{1}{4},l)$ and the y-upstream enclosure-surface area at (i,j,l) of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.78a)] becomes

$$\iint_{A_{y;i,j,l}} dA = \int_{-\frac{\Delta x_{j,j}}{2}}^{\frac{\Delta x_{j,j}}{2}} \int_{-\frac{\Delta z_{j,j,l}}{2}}^{\frac{\Delta z_{j,j,l}}{2}} dx\,dz\,. \tag{2.90i}$$

Similarly, the enclosure-surface integral associated with the y-downstream enclosure-surface area at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream enclosure-surface area at $(i,j+\frac{1}{2},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.75d)] becomes

$$\iint_{A_{y;i,j+\frac{1}{2},l}} dA = \int_{-\frac{\Delta x_{j,j+\frac{1}{2}}}{2}}^{\frac{\Delta x_{j,j+\frac{1}{2}}}{2}} \int_{-\frac{\Delta z_{j,j+\frac{1}{2},l}}{2}}^{\frac{\Delta z_{j,j+\frac{1}{2},l}}{2}} dx\,dz . \tag{2.90j}$$

Similarly, the enclosure-surface integral associated with the x-upstream enclosure-surface area at $(i-\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.78b)] becomes

$$\iint_{A_{x;i-\frac{1}{2},j-\frac{1}{4},l}} dA = \int_{-\frac{\Delta y_{j-\frac{1}{2},j}}{4}}^{\frac{\Delta y_{j-\frac{1}{2},j}}{4}} \int_{-\frac{\Delta z_{j-\frac{1}{2},j,l}}{2}}^{\frac{\Delta z_{j-\frac{1}{2},j,l}}{2}} dy\,dz . \tag{2.90k}$$

Similarly, the enclosure-surface integral associated with the x-downstream enclosure-surface area at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.78c)] becomes

$$\iint_{A_{x;i+\frac{1}{2},j-\frac{1}{4},l}} dA = \int_{-\frac{\Delta y_{j+\frac{1}{2},j}}{4}}^{\frac{\Delta y_{j+\frac{1}{2},j}}{4}} \int_{-\frac{\Delta z_{j+\frac{1}{2},j,l}}{2}}^{\frac{\Delta z_{j+\frac{1}{2},j,l}}{2}} dy\,dz . \tag{2.90l}$$

Similarly, the enclosure-surface integral associated with the x-upstream enclosure-surface area at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.79a)] becomes

$$\iint_{A_{x;i-\frac{1}{2},j+\frac{1}{4},l}} dA = \int_{-\frac{\Delta y_{j-\frac{1}{2},j}}{4}}^{\frac{\Delta y_{j-\frac{1}{2},j}}{4}} \int_{-\frac{\Delta z_{j-\frac{1}{2},j,l}}{2}}^{\frac{\Delta z_{j-\frac{1}{2},j,l}}{2}} dy\,dz . \tag{2.90m}$$

Similarly, the enclosure-surface integral associated with the x-downstream enclosure-surface area at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.79b)] becomes

$$\iint_{A_{x;i+\frac{1}{2},j+\frac{1}{4},l}} dA = \int_{-\frac{\Delta y_{j+\frac{1}{2},j}}{4}}^{\frac{\Delta y_{j+\frac{1}{2},j}}{4}} \int_{-\frac{\Delta z_{j+\frac{1}{2},j,l}}{2}}^{\frac{\Delta z_{j+\frac{1}{2},j,l}}{2}} dy\, dz \,. \tag{2.90n}$$

Similarly, the enclosure-surface integrals associated with the z-upstream enclosure-surface area at $(i,j,l-\frac{1}{2})$ and the z-downstream enclosure-surface area at $(i,j,l+\frac{1}{2})$ of the element (i,j,l) [Eqs. (2.75e) and (2.75f)] become

$$\iint_{A_{z;i,j,l\pm\frac{1}{2}}} dA = \iint_{A_{z;i,j}} dA \,, \tag{2.91a}$$

which is in terms of the two-dimensional planar area $A_{z;i,j}$, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region), in the element (i,j,l) [Eq. (2.67) for an interior element in the horizontal (x,y) plane or Eq. (2.68) for a boundary element in the horizontal (x,y) plane] (see Sec. 2.3).

Similarly, the enclosure-surface integrals associated with the z-upstream enclosure-surface area at $(i-\frac{1}{4},j,l-\frac{1}{2})$ and z-downstream enclosure-surface area at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eqs. (2.76d) and (2.76e)] become

$$\iint_{A_{z;i-\frac{1}{4},j,l\pm\frac{1}{2}}} dA = \iint_{A_{z;i-\frac{1}{4},j}} dA \,, \tag{2.91b}$$

and the enclosure-surface integrals associated with the z-upstream enclosure-surface area at $(i+\frac{1}{4},j,l-\frac{1}{2})$ and z-downstream enclosure-surface

area at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eqs. (2.77c) and (2.77d)] become

$$\iint_{A_{z;i+\frac{1}{4},j,l\pm\frac{1}{2}}} dA = \iint_{A_{z;i+\frac{1}{4},j}} dA \,, \tag{2.91c}$$

which are in terms of the two-dimensional planar areas $A_{z;i-\frac{1}{4},j}$, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) in the x-upstream half-element $(i-\frac{1}{4},j,l)$ or the two-dimensional planar area $A_{z;i+\frac{1}{4},j}$, in the horizontal (x,y) plane of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.70) for an interior element in the horizontal (x,y) plane or Eq. (2.71) for a boundary element in the horizontal (x,y) plane], respectively (see Sec. 2.3).

Similarly, the enclosure-surface integrals associated with the z-upstream enclosure-surface area at $(i,j-\frac{1}{4},l-\frac{1}{2})$ and z-downstream enclosure-surface area at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eqs. (2.78d) and (2.78e)] become

$$\iint_{A_{z;i,j-\frac{1}{4},l\pm\frac{1}{2}}} dA = \iint_{A_{z;i,j-\frac{1}{4}}} dA \,, \tag{2.91d}$$

and the enclosure-surface integrals associated with the z-upstream enclosure-surface area at $(i,j+\frac{1}{4},l-\frac{1}{2})$ and z-downstream enclosure-surface area at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eqs. (2.79c) and (2.79d)] become

$$\iint_{A_{z;i,j+\frac{1}{4},l\pm\frac{1}{2}}} dA = \iint_{A_{z;i,j+\frac{1}{4}}} dA \,, \tag{2.91e}$$

which are in terms of the two-dimensional planar area $A_{z;i,j-\frac{1}{4}}$ in the horizontal (x,y) plane (in the general sense, parallel to the geophy-

sical surface plane and the free-surface plane of the flow region) of the y-upstream half-element $(i,j-\frac{1}{4},1)$ or the two-dimensional planar area $A_{z;i,j+\frac{1}{4}}$, in the horizontal (x,y) plane, of the y-downstream half-element $(i,j+\frac{1}{4},1)$ [Eq. (2.73) for an interior element in the horizontal (x,y) plane or Eq. (2.74) for a boundary element in the horizontal (x,y) plane], respectively (see Sec. 2.3).

The areas of the intakes and the discharges [in Eqs. (2.87), (2.88) and (2.89)] depend on the specified geometrical characteristics of the intake and/or discharge structures in the element (i,j,1), in the x-upstream half-element $(i-\frac{1}{4},j,1)$, in the x-downstream half-element $(i+\frac{1}{4},j,1)$, in the y-upstream half-element $(i,j-\frac{1}{4},1)$, and in the y-downstream half-element $(i,j+\frac{1}{4},1)$. A large intake or discharge area, in comparison to the enclosure surface areas of an element or a half-element, on a part of the shoreline enclosure boundaries of the flow region, can be considered as a part of the enclosure surface area of the element or half-element corresponding to the part of the shoreline enclosure boundaries of the flow region. However, a small intake or discharge area, in comparison to the enclosure surface areas of an element or half-element, in an element or a half-element of the flow region can also be considered based on only the geometrical characteristics of the intake structure or the discharge structure by assuming that the structure does not sufficiently distort the control volume or the enclosure-surface areas of the element or half-element.

For completely interior elements, which are both interior in the horizontal (x,y) plane and vertically interior, the general forms of all the enclosure surface integrals associated with the enclosure surface

areas of the elements and half-elements [Eqs. (2.90a) through (2.91e)] reduce to,

$$\left[\iint_{A_{x;i,j,l}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta y_j}{2}}^{\frac{\Delta y_j}{2}} \int_{-\frac{\Delta z_l}{2}}^{\frac{\Delta z_l}{2}} dy\, dz\,, \tag{2.92a}$$

$$\left[\iint_{A_{x;i\pm\frac{1}{2},j,l}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta y_j}{2}}^{\frac{\Delta y_j}{2}} \int_{-\frac{\Delta z_l}{2}}^{\frac{\Delta z_l}{2}} dy\, dz\,, \tag{2.92b}$$

$$\left[\iint_{A_{y;i\pm\frac{1}{4},j\pm\frac{1}{2},l}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{4}}^{\frac{\Delta x_j}{4}} \int_{-\frac{\Delta z_l}{2}}^{\frac{\Delta z_l}{2}} dx\, dz\,, \tag{2.92c}$$

$$\left[\iint_{A_{y;i,j,l}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{2}}^{\frac{\Delta x_j}{2}} \int_{-\frac{\Delta z_l}{2}}^{\frac{\Delta z_l}{2}} dx\, dz\,, \tag{2.92d}$$

$$\left[\iint_{A_{y;i,j\pm\frac{1}{2},l}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{2}}^{\frac{\Delta x_j}{2}} \int_{-\frac{\Delta z_l}{2}}^{\frac{\Delta z_l}{2}} dx\, dz\,, \tag{2.92e}$$

$$\left[\iint_{A_{x;i\pm\frac{1}{2},j\pm\frac{1}{4},l}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta y_j}{4}}^{\frac{\Delta y_j}{4}} \int_{-\frac{\Delta z_l}{2}}^{\frac{\Delta z_l}{2}} dy\, dz\,, \tag{2.92f}$$

$$\left[\iint_{A_{z;i,j,l\pm\frac{1}{2}}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{2}}^{\frac{\Delta x_j}{2}} \int_{-\frac{\Delta y_j}{2}}^{\frac{\Delta y_j}{2}} dx\, dy\,, \tag{2.92g}$$

$$\left[\iint_{A_{z;i\pm\frac{1}{4},j,l\pm\frac{1}{2}}} dA\right]_{\text{interior}} = \int_{-\frac{\Delta x_j}{4}}^{\frac{\Delta x_j}{4}} \int_{-\frac{\Delta y_j}{2}}^{\frac{\Delta y_j}{2}} dx\, dy\,, \tag{2.92h}$$

$$\left[\iint_{A_{z;i,j\pm\frac{1}{4},l\pm\frac{1}{2}}} dA\right]_{interior} = \int_{-\frac{\Delta x_j}{2}}^{\frac{\Delta x_j}{2}} \int_{-\frac{\Delta y_j}{4}}^{\frac{\Delta y_j}{4}} dx\, dy. \qquad (2.92i)$$

Hence, the enclosure surface integrals of the control volumes of all the elements and half-elements in the flow region can be formulated in terms of the dimensions of the elements and half-elements which are determined based on the geometrical discretization technique (Sec. 2.3) of the general three-dimensional discrete-element method (DEM).

It is important to note that for completely interior elements, the simplified forms of the enclosure surface integrals [Eqs.(2.92a) through (2.92i)]are evaluated over rectangular areas which are always normal to the three orthogonal coordinate directions (x,y,z). Hence, in completely interior elements, the geometrical discretization technique of the general three-dimensional discrete-element method (DEM) is geometrically equivalent to the geometrical discretization technique of the finite-difference method (FDM), which is significantly simpler and computationally more efficient than the finite-element method (FEM).

2.5. Temporal and Spatial Variations of the Principal Flow Variables

The final integral forms of the two physical principles, consisting of (1) the integral form of the mass conservation principle for water [Eq. (2.10)] and (2) the three components of the integral form of the momentum principle for water [Eqs. (2.20) through (2.22)], are utilized directly in the formulation of the transient, three-dimensional discrete-element equations based on the geometrical discretization technique which determines the elements and half-elements in the flow region.

Hence, similar to the formulations of all other existing numerical solution methods, including the finite-difference method (FDM) and finite-element method (FEM), the discrete-element method (DEM) also has certain limitations on the spatial and temporal accuracies of its solutions of the principal flow variables, according to the specified spatial scales and temporal scales of its geometrical discretization technique.

In applications of the discrete-element method to geophysical flow conditions, the dimensions of the elements can be of the order of kilometers in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane) and of the order of meters in the vertical z-axis (in the general sense, normal to the geophysical surface plane). Similarly, in applications of the discrete-element method to the laboratory-model scale flow conditions, the dimensions of the elements can be of the order of meters in the horizontal (x,y) plane and of the order of centimeters in the vertical z-axis. Hence, in the formulation of a discretized numerical solution method, the flow phenomena (in the more general sense, the transport phenomena) which is spatially smaller scale than the spatial scale of the elements must be averaged over the surfaces of the elements (e.g., Deardorff, 1970). Furthermore, the flow phenomena (in the more general sense, the transport phenomena) which is temporally smaller scale than the specified temporal resolution scale of the numerical solution of the discrete-element method must also be averaged over the characteristic time (in the general sense) of the specified temporal resolution scale of the discretized mathematical system (e.g., Reynolds, 1970).

The temporal-averaging process associated with the formulation of three-dimensional partial differential equations for hydrodynamic conditions in geophysical flow regions are usually formulated according to a two-term representation of the instantanuous velocity which consists of (1) " the ensemble-average velocity" (temporally-averaged over the high-frequency fluctuations) and (2) the "turbulent departure (high-frequency fluctuations) of the instantaneous velocity from its ensemble average" (e.g., Pritchard, 1971). Alternately, the spatial-averaging process associated with the formulation of one-dimensional partial differential equations for longitudinal transport in estuaries is also usually formulated according to a two-term representation of the "time-averaged (over high-frequency fluctuations) longitudinal velocity component, associated with turbulent flow", which consists of (1) the "real-time longitudinal tidal velocity averaged over cross section and (2) the "spatial derivation of the longitudinal velocity component from the cross-section mean value due to the vertical and lateral velocity distribution" (e.g., Harleman, 1971).

It is important to note that the spatial-averaging process is distinctly different than the temporal-averaging process in the formulation of a discretized numerical model. The spatial-averaging process is inherently dependent on the characteristics of the actual subgrid-scale continuous variations of the flow variables which can be accounted for in a spatially-averaged form according to the spatial discretization accuracy (order) of the numerical transport method (e.g., Arakawa, 1966). Hence, the spatial-averaging process should not be combined (or used interchangeably) with the temporal-averaging process associated

with high-frequency, turbulent fluctuations of the flow variables (e.g., Spraggs and Street, 1975).

THe proper formulation of the spatially-averaged and temporally-averaged forms of the discrete-element mathematical models can be obtained by considering three-term representations of the instantaneous flow variables (Eraslan, 1974, 1977, 1981a, 1981b) in one- or two-dimensions. This approach clearly identifies each term associated with the spatial-averaging process of the subelement-scale spatial variations of the flow variables separately from each term associated with the temporal-averaging process associated with the high-frequency turbulent fluctuations of the flow variables. Hence, the present development of the transient three-dimensional discrete-element model is based on the three-term representations of the flow variables which are included in the integral forms of the two physical principles, consisting of (1) the integral form of the mass conservation principle for water [Eqs. (2.10)] and (2) the three components of the integral form of the momentum principle for water [Eqs. (2.20) through (2.22)].

Considering the hierarchy of spatial scales, the assumption of considering numerical solutions for the flow conditions in the elements and half-elements requires that spatially primary-scale flow phenomena be associated with the variations of the flow conditions with a spatial resolution not higher than the dimensions of the elements and half-elements. Consequently, in developing the three-dimensional discrete-element model, to be consistent with the assumption about the spatially primary-scale phenomena, all flow variables must be described as spatially-averaged values over the enclosure-surface areas of the element

or half-element, normal to the three coordinate axes. Furthermore, the spatial variations of the flow variables over the enclosure-surface areas of the elements and half-elements must necessarily be considered as spatially subscale flow phenomena in the form of deviations (departures) of the local flow conditions from the spatially primary-scale, enclosure-surface-averaged values of the flow variables on the enclosure surfaces of the elements and half-elements (Fig. 2.55, p.214). Consideration of the variations of the flow variables over the enclosure-surface areas as spatially subscale flow phenomena is conceptually similar to the considerations of higher order spatial approximations in other higher-order numerical methods (e.g. Arakawa, 1966) for three-dimensional formulations. It is also conceptually similar to the consideration of the vertical and lateral variations of the flow conditions as spatial deviations (departures) from the cross-section-averaged values in the formulations of the convection-defect longitudinal transport rates (Eraslan, 1981a, 1981b) or "longitudinal dispersion transport rates" (e.g., Harleman, 1971) of the conventional one-dimensional models.

Considering the transient, three-dimensional flow conditions in meso-scale geophysical flow regions (e.g., lakes, rivers, estuaries and coastal zones), it is usually assumed that temporally primary-scale flow phenomena consists of (1) uniform or intermittently varying flow conditions controlled by operating dams, power plants and pump-storage facilities (with characteristic times of variations greater than minutes or hours), (2) multiharmonic tidal flow conditions (with characteristic times of variations greater than hours), (3) geostrophic or wind induced

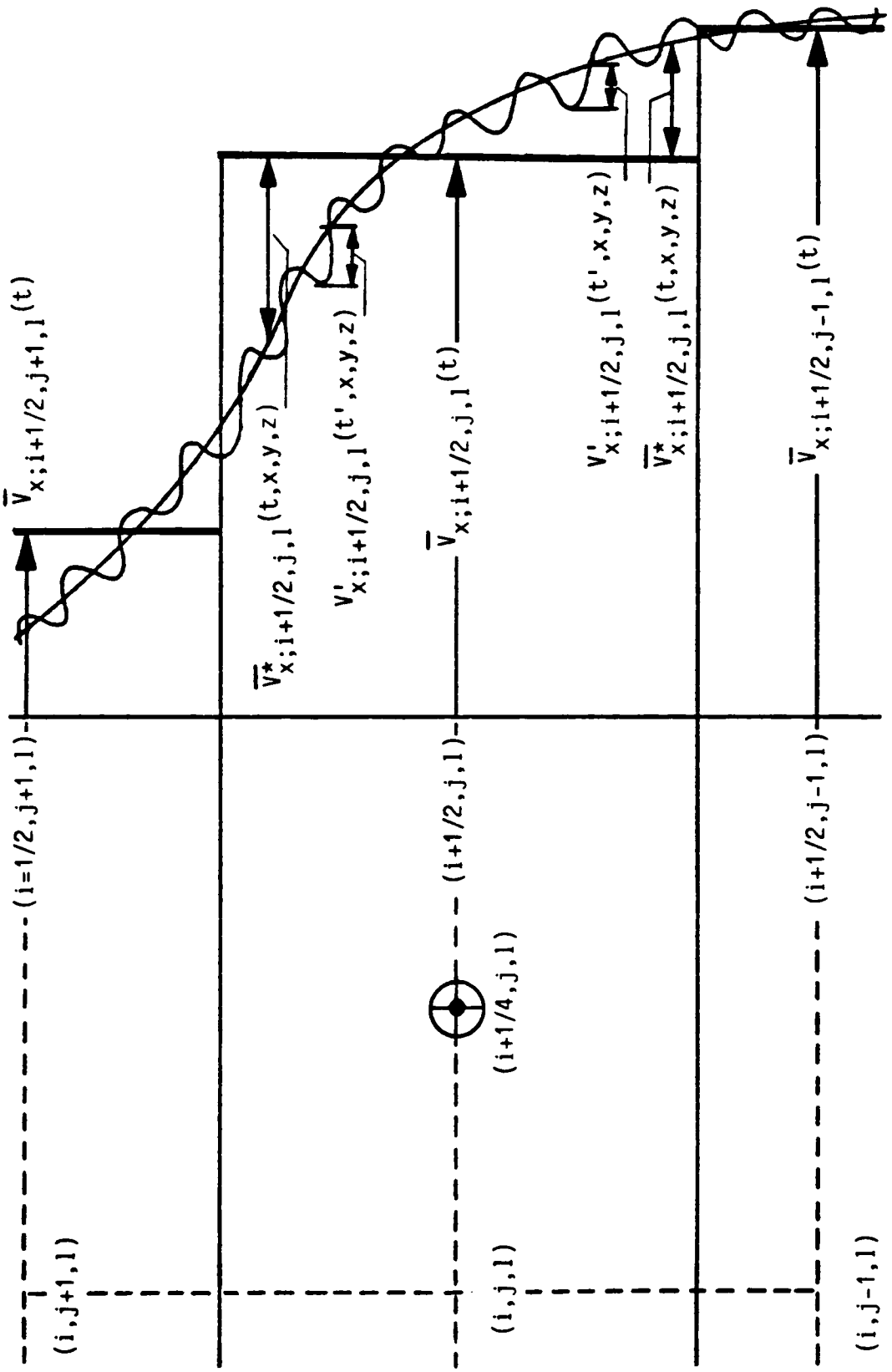

Figure 2.55. A schematic representation of the typical spatial variations of the flow variables over the enclosure-surface areas of the elements and half-elements.

currents (with characteristic times of variation greater than minutes, hours or days). Consequently, to be consistent with these assumptions about the temporally primary-scale flow phenomena, all other high-frequency turbulence phenomena, consisting of eddies with smaller characteristic times of temporal variations, in the elements and half-elements or over the enclosure-surface areas of the elements and half-elements, must be considered as the temporally subscale flow phenomena which represent the deviations (departures) of the instantaneous, local flow conditions from the temporally primary-scale flow conditions in the elements or half-elements or over the enclosure-surface areas of the elements or half-elements. Consideration of the high-frequency flow conditions, consisting of the eddies in the elements and half-elements or over the enclosure-surface areas of the elements and half-elements, as temporally subscale flow phenomena is again conceptually similar to the consideration of high-frequency eddies as turbulent fluctuations in formulating the "turbulent transport rates" (e.g., Pritchard, 1971, Harleman, 1971, Eraslan, 1974,1981b) of the conventional one-dimensional or multi-dimensional models.

The mathematical formulation of the transient, three-dimensional discrete-element transport model considers three-term representations (Eraslan, 1974, 1981a, 1981b) of the instantaneous, local values of all the transportive intensive flow variables (Fig. 2.55, p.214) which are included in the integral forms of the two physical principles, consisting of (1) the integral form of the mass conservation principle for water [Eq. (2.10)] and (2) the three components of the integral form of monemtum principle for water [Eqs. (2.20) through (2.22)]. According to

the hierarchy of the spatial and temporal scales, the spatially primary-scale and temporally primary-scale principle flow variables are represented as functions of primary-scale time t only, since they are assumed to be spatially subscale-averaged (over the enclosure-surface areas of the elements and half-elements) and also temporally subscale-averaged over the characteristic time periods of the high frequency turbulent fluctuations. Similarly, according to the hierarchy of temporal scales, the temporally primary-scale, but spatially subscale parts of the flow variables are represented as functions of primary-scale time t and spatial coordinated (x,y,z), since they are assumed to be temporally subscale-averaged over the characteristic time periods of the high frequency turbulent fluctuations, but with spatially subscale variations over the enclosure surface areas of the elements and half-elements.

Hence, the mathematical formulation of the instantaneous, local values of the transportive intensive flow variables, consisting of the three velocity components along the three orthogonal Cartesian coordinate axes (x,y,z), are represented in three parts (see Fig. 2.55, p.214), according to the hierarchy of spatial and temporal scales, as

$$V_x(t,t',x,y,z)=\overline{V}_x(t)+\overline{V_x^*}(t,x,y,z)+V_x'(t',x,y,z), \tag{2.93}$$

$$V_y(t,t',x,y,z)=\overline{V}_y(t)+\overline{V_y^*}(t,x,y,z)+V_y'(t',x,y,z), \tag{2.94}$$

$$V_z(t,t',x,y,z)=\overline{V}_z(t)+\overline{V_z^*}(t,x,y,z)+V_z'(t',x,y,z), \tag{2.95}$$

where

$\overline{V}_x(t)$ = temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) x-component of velocity associated with an element or half-element (principal flow variable of solution),

$\overline{V_x^*}(t,x,y,z)$ = difference between (1) temporally primary-scale (i.e., temporally subscale-averaged), but spatially subscale, three-dimensional distribution of x-component of velocity and (2) temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) x-component of velocity associated with an element or half-element (auxiliary flow variable assocoated with the formulations of all convection-defect transport fluxes),

$V_x'(t',x,y,z)$ = temporally subscale (i.e., high-frequency) and spatially subscale (eddy formations smaller than the dimensions of the elements and half-elements) turbulent variations of the x-component of velocity (auxiliary flow variable associated with the formulations of all turbulent transport fluxes),

$\overline{V}_y(t)$ = temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) y-component of velocity associated with an element or half-element (principal flow variable of solution),

$\overline{V_y^*}(t,x,y,z)$ = difference between (1) temporally primary-scale (i.e., temporally subscale-averaged), but spatially subscale, three-dimensional distribution of y-component of velocity and (2) temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) y-component of velocity associated with an element or half-element (auxiliary flow variable associated with the formulations of all convection-defect transport fluxes),

$V_y'(t',x,y,z)$ = temporally subscale (i.e., high-frequency) and spatially subscale (eddy formations smaller than the dimensions of the elements and half-elements) turbulent variations of the y-component of velocity (auxiliary flow variable associated with the formulations of all turbulent transport fluxes),

$\overline{V}_z(t)$ = temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) z-component of velocity associated with and element or half-element (principal flow variable of solution),

$\overline{V_z^*}(t,x,y,z)$ = difference between (1) temporally primary-scale (i.e., temporally subscale-averaged), but spatially subscale, three-dimensional distribution of z-component of velocity and (2) temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) z-component of velocity

associated with an element or half-element (auxilary flow variable assocoated with the formulations of all convection-defect transport fluxes),

$V'_z(t',x,y,z)$ = temporally subscale (i.e., high-frequency) and spatially subscale (eddy formations smaller than the dimensions of the elements and half-elements) turbulent variations of the z-component of velocity (auxiliary flow variable associated with the formulations of all turbulent transport fluxes),

According to the mathematical formulation of the discrete-element method, each element or half-element represents a control volume v_{cv} enclosed by a closed control surface A_{cv}. Hence, excluding the elements and half-elements which contain the free-surface water layer, all other elements and half-elements represent control volumes which remain fixed in space (position and shape) and in time. The elements and half-elements which contain the free-surface water layer represent modified control volumes which are partially filled with water; or alternately they represent modified control volumes with moving boundaries. Hence, the instantaneous local values of water surface elevation $H(t,t',x,y,z)$ in the elements and half-elements can also control indirectly the instantaneous local values, with temporal and spatial variations, of the other flow variables associated with the elements and half-elements.

The present formulation of the transient, three-dimensional discrete-element model assumes that the instantaneous, local values of water surface elevation $H(t,t',x,y,z)$ can be approximated by the spatially primary-scale and temporally primary-scale water surface

elevation H(t) associated with elements and half-elements. Hence, the formulation eliminates both spatially subscale variations and temporally subscale variations of water surface elevations associated with the control volumes and enclosure surfaces, which constitute the closed control surfaces, of the elements which contain the free-surface water layer,i.e.,

$$H(t,t',x,y,z)=\overline{H}(t), \tag{2.96}$$

where

$\overline{H}(t)$ = temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) water surface elevation associated with an element or half-element which contains the free-surface water layer (Principal flow variable of solution).

It is important to note that the assumption about the water-surface-elevation conditions is needed to simplify the evaluation of the volume integrals and the closed control-surface integrals, associated with the control volumes of the elements and half-elements. As the consequence of the assumption, all the volumes and the closed control-surface areas of the elements and half-elements vary only according to the spatially primary-scales and temporally primary-scales of the discrete-element formulation.

According to the three-term definitions of the instantaneous, local values of the transportive intensive flow variables, consisting of the three velocity components along the three orthogonal Cartesian coordinate axes (x,y,z), [Eqs. (2.93) through (2.95)], the different parts associated with different temporal scales of formulations obey the

following relations for temporal averaging over the characteristic periods of the high-frequency turbulent fluctuations:

$$\overline{V_x(t,t',x,y,z)}=\overline{V}_x(t)+\overline{V}_x^*(t,x,y,z), \qquad \overline{V_x'(t',x,y,z)}=0, \tag{2.97a}$$

$$\overline{V_y(t,t',x,y,z)}=\overline{V}_y(t)+\overline{V}_y^*(t,x,y,z), \qquad \overline{V_y'(t',x,y,z)}=0, \tag{2.97b}$$

$$\overline{V_z(t,t',x,y,z)}=\overline{V}_z(t)+\overline{V}_z^*(t,x,y,z), \qquad \overline{V_z'(t',x,y,z)}=0. \tag{2.97c}$$

Similarly, according to the three-term definitions of the instantaneous local values of the transportive intensive flow variables, consisting of the three velocity components along the three orthogonal Cartesian coordinate axes (x,y,z), [Eqs. (2.93) through (2.95)], the different terms associated with different spatial scales of formulations obey the following relations for spatial averaging over any enclosure-surface area of an element of half-element:

$$\iint_A \overline{V_x^*}(t,x,y,z)dA=0, \tag{2.98a}$$

$$\iint_A \overline{V_y^*}(t,x,y,z)dA=0, \tag{2.98b}$$

$$\iint_A \overline{V_z^*}(t,x,y,z)dA=0. \tag{2.98c}$$

As a consequence of the constraint relations associated with (1) the temporal averaging over the characteristic periods of the high-frequency turbulent fluctuations [Eqs. (2.97a) through (2.97c)] and (2) the spatial averaging over any enclosure-surface area of an element or half-element [Eqs. (2.98a) through (2.98c)], the temporally subscale-

averaged (over the characteristic periods of the high-frequency turbulent fluctuations) values of the integrals of the instantaneous, local values of the transportive intensive flow variables, consisting of the three velocity components along the three orthogonal Cartesian coordinate axes (x,y,z), over any enclosure surface area of an element or half-element, become

$$\overline{\iint_A V_x(t,t',x,y,z)dA} = \iint_{\bar{A}} \overline{V_x(t,t',x,y,z)}dA$$

$$= \iint_{\bar{A}} \overline{[\bar{V}_x(t) + \bar{V}_x^*(t,x,y,z) + V_x'(t',x,y,z)]}dA$$

$$= \iint_{\bar{A}} \bar{V}_x(t)dA + \iint_{\bar{A}} \bar{V}_x^*(t,x,y,z)dA + \iint_{\bar{A}} \overline{V_x'(t',x,y,z)}]dA$$

$$= \bar{A}\bar{V}_x(t) = \bar{A}\bar{V}_x \,, \qquad (2.99a)$$

$$\overline{\iint_A V_y(t,t',x,y,z)dA} = \iint_{\bar{A}} \overline{V_y(t,t',x,y,z)}dA$$

$$= \iint_{\bar{A}} \overline{[\bar{V}_y(t) + \bar{V}_y^*(t,x,y,z) + V_y'(t',x,y,z)]}dA$$

$$= \iint_{\bar{A}} \bar{V}_y(t)dA + \iint_{\bar{A}} \bar{V}_y^*(t,x,y,z)dA + \iint_{\bar{A}} \overline{V_y'(t',x,y,z)}]dA$$

$$= \bar{A}\bar{V}_y(t) = \bar{A}\bar{V}_y \,, \qquad (2.99b)$$

$$\overline{\iint_A V_z(t,t',x,y,z)dA} = \iint_{\bar{A}} \overline{V_z(t,t',x,y,z)}dA$$

$$= \iint_{\bar{A}} \overline{[\bar{V}_z(t) + \bar{V}_z^*(t,x,y,z) + V_z'(t',x,y,z)]}dA$$

(continue)

$$= \iint_{\bar{A}} \bar{V}_z(t)dA + \iint_{\bar{A}} \bar{V}_z^*(t,x,y,z)dA + \iint_{\bar{A}} \overline{V_z'(t',x,y,z)}]dA$$

$$= \bar{A}\bar{V}_z(t) = \bar{A}\bar{V}_z \,. \qquad (2.99c)$$

According to the definitions of the temporal and spatial scales of the transportive intensive flow variables, consisting of the three velocity components along the three orthogonal Cartesian coordinate axes (x,y,z), [Eqs. (2.93) through (2.95)], which also represent the principal flow variables of the numerical solution, the governing mathematical system of the discrete-element method must be based on the temporally primary-scale (i.e., temporally subscale-averaged) and spatially primary-scale (i.e., spatially subscale-averaged) formulations of the integral forms of the two physical principles, consisting of (1) the integral form of the mass conservation principle for water [Eq. (2.10)] and (2) the three-components of the integral form of the momentum principle for water [Eqs. (2.20) through (2.22)]. Furthermore, considering the assumption that only the temporally primary-scale and spatially primary-scale part of the water surface elevation determines the control volumes and the enclosure-surface areas, which constitue the closed control-surface areas, of the elements and half-elements that contain the free-surface water layer, the integral forms of the two physical principles [Eqs. (2.10), (2.20) through (2.22)] must be formulated for elements and half-elements with control volumes and closed control-surface areas which are independent of the spatially subscale variations and also independent of temporally subscale variations of the water surface elevation.

Hence, the temporally primary-scale and spatially primary-scale modified integral form of the mass conservation principle for water [Eq. (2.10)] becomes

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{cv}} d\overline{v} + \iint_{\overline{A}_{cv}} \overline{\vec{V}\cdot\hat{n}}\, d\overline{A} = 0 \,. \tag{2.100}$$

Similarly, the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eqs. (2.20) through (2.22)] for the three velocity components $\overline{V}_x$, $\overline{V}_y$ and $\overline{V}_z$, along the three orthogonal Cartesian Coordinate axes (x,y,z),

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{cv}} \overline{V}_x\, d\overline{v} + \iint_{\overline{A}_{cv}} \overline{[V_x]\vec{V}\cdot\hat{n}}\, d\overline{A} = -g_a \cos\theta_{n,x} \iiint_{\overline{V}_{cv}} d\overline{v} + \Omega_{cor,N,y} \iiint_{\overline{V}_{cv}} \overline{V}_z\, d\overline{v}$$

$$- \Omega_{cor,N,z} \iiint_{\overline{V}_{cv}} \overline{V}_y\, d\overline{v} + \iint_{\overline{A}_{cv}} \overline{\left[\frac{p}{\rho_w}(-\hat{n})\right]_x} d\overline{A} + \iint_{\overline{A}_{cv}} \overline{\left[\frac{\sigma_{sx}}{\rho_w}\right]} d\overline{A}, \tag{2.101}$$

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{cv}} \overline{V}_y\, d\overline{v} + \iint_{\overline{A}_{cv}} \overline{[V_y]\vec{V}\cdot\hat{n}}\, d\overline{A} = -g_a \cos\theta_{n,y} \iiint_{\overline{V}_{cv}} d\overline{v} + \Omega_{cor,N,z} \iiint_{\overline{V}_{cv}} \overline{V}_x\, d\overline{v}$$

$$- \Omega_{cor,N,x} \iiint_{\overline{V}_{cv}} \overline{V}_z\, d\overline{v} + \iint_{\overline{A}_{cv}} \overline{\left[\frac{p}{\rho_w}(-\hat{n})\right]_y} d\overline{A} + \iint_{\overline{A}_{cv}} \overline{\left[\frac{\sigma_{sy}}{\rho_w}\right]} d\overline{A}, \tag{2.102}$$

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{cv}} \overline{V}_z\, d\overline{v} + \iint_{\overline{A}_{cv}} \overline{[V_z]\vec{V}\cdot\hat{n}}\, d\overline{A} = -g_a \cos\theta_{n,z} \iiint_{\overline{V}_{cv}} d\overline{v} + \Omega_{cor,N,x} \iiint_{\overline{V}_{cv}} \overline{V}_y\, d\overline{v}$$

$$- \Omega_{cor,N,y} \iiint_{\overline{V}_{cv}} \overline{V}_x\, d\overline{v} + \iint_{\overline{A}_{cv}} \overline{\left[\frac{p}{\rho_w}(-\hat{n})\right]_z} d\overline{A} + \iint_{\overline{A}_{cv}} \overline{\left[\frac{\sigma_{sz}}{\rho_w}\right]} d\overline{A}. \tag{2.103}$$

The temporally primary-scale and spatially primary scale, integral forms of the two physical principles for water [Eqs. (2.100) through (2.103)] represent the final integral forms which are employed in the formulation of the transient, three-dimensional discrete-element hydrodynamic model.

2.6. General Formulations of the Temporally Primary-Scale and Spatially Primary-Scale Volume Integrals and Surface Integral Associated with the Control Volumes

According to the assumptions about the temporal scales and the spatial scales of the transportive intensive flow variables, consisting of the three velocity components along the three orthogonal Cartesian coordinate axes (x,y,z) [Eqs. (2.93) through (2.95)], which also represent the principle flow variables of the numerical solution, the discrete-element formulations of the volume integrals and the surface integrals (Sec. 2.4) must also be evaluated according to the temporally primary scale and spatially primary scale integral forms of the two physical principles, consisting of (1) the integral form of the mass conservation principle for water [Eq. (2.100)] and (2) the three components of the integral form of the momentum principle for water [Eqs. (2.101) through (2.103)].

Considering the discrete-element formulations of the volume integrals [Sec. 2.4, Eqs. (2.83) through (2.86)] and the constraint conditions associated with the three parts of the definitions of the different temporal scales and spatial scales of the transportive intensive flow variables [Sec. 2.5, Eqs. (2.97a) through (2.99c)], the temporally primary-scale and spatially primary-scale (i.e., temporally subscale-averaged and spatially subscale-averaged) formulations of the volume integrals of the temporally primary scale and spatially primary scale integral forms of the two physical principles [Eqs. (2.100) through (2.103)] can be obtained in general forms for any control volume associated with an element or half-element.

Hence, the volume integral associated with the control volume of the temporally primary-scale and spatially primary-scale, integral form of the mass conservation principle for water in the elements and half-elements [Eq. (2.101)] becomes

$$\iiint_{\overline{V}_{cv}} d\overline{V} = \int_{-\frac{\overline{\Delta z}}{2}}^{\frac{\overline{\Delta z}}{2}} \iint_{A_z} dA\, dz = A_z \overline{\Delta z}. \tag{2.104}$$

where z represents the depth as the dimension (length) along z-direction of an element or half-element which can depend on the water surface elevation H in the free-surface water layer L, i.e.,

$$\overline{\Delta z} = \Delta z + \overline{H}(\text{free-surface water layer L}). \tag{2.105}$$

Similarly, the volume integrals associated with the control volumes of the temporally primary-scale and spatially primary-scale three components of the integral form of the momentum principle for water in the elements and half-elements [Eqs. (2.101) through (2.103)] become

$$\begin{aligned}
\iiint_{\overline{V}_{cv}} \overline{V_{(x,y,z)}}\, d\overline{V} &= \int_{-\frac{\overline{\Delta z}}{2}}^{\frac{\overline{\Delta z}}{2}} \iint_{A_z} \left[\overline{\overline{V}_{(x,y,z)} + \overline{V}^{*}_{(x,y,z)} + V'_{(x,y,z)}}\right] dA\, dz \\
&= \overline{\Delta z}\left[\iint_{A_z} \overline{\overline{V}_{(x,y,z)}}\, dA + \iint_{A_z} \overline{\overline{V}^{*}_{(x,y,z)}}\, dA + \iint_{A_z} \overline{\overline{V}'_{(x,y,z)}}\, dA\right] \\
&= \left[A_z \overline{\Delta z}\right] \overline{V}_{(x,y,z)},
\end{aligned} \tag{2.106}$$

where the subscript (x,y,z) designates any one of the three x-, y- or z-component of velocity (Sec. 2.2).

The formulation of the closed control-surface integrals associated with the control volumes of the temporally primary-scale and spatially primary scale integral forms of the two physical principles, consisting of (1) the integral form of mass conservation principle for water [Eqs. (2.100)] and (2) the three components of the integral form of the momentum principle for water [Eqs. (2.101) through (2.103)], according to the general discrete-element formulations of the enclosure surface integrals [Sec. 2.4, Eqs. (2.87) through (2.89)], associated with the elements and half-elements, requires careful considerations. Each closed control-surface integral of an element or a half-element consists of (1) two enclosure-surface integrals over the two enclosure-surface areas normal to x-direction [Eqs. (2.90a) through (2.90c), (2.90k) through (2.90n), (2.92a), (2.92b) and (2.92f)], (2) two enclosure-surface integrals over the two enclosure-surface areas normal to y-direction [Eqs. (2.90d) through (2.90j) and (2.92c) through (2.92e)], and (3) two enclosure-surface integrals over two enclosure-surface areas normal to z-direction [Eqs. (2.91a) through (2.91e) and (2.92g) through (2.92i)], (4) surface integral(s) over the cross-section area(s) of the intake of the power plant(s) m, and (5) surface integral(s) over the cross-se ction area(s) of the discharge of the power plant(s) m [Eqs. (2.87) through (2.89)].

The six enclosure-surface integrals that constitue the closed control-surface integral, associated with the convective transport across the closed control surface of the control volume of an element or half-element of the temporally primary-scale and spatially primary-scale, integral forms of the two physical principles [Eqs. (2.100)

through (2.103)] can be formulated according to the discrete-element formulations of the enclosure-surface integrals [Sec. 2.4, Eqs. (2.87) through (2.89)], and the constraint conditions associated with the three parts of the definitions of the different temporal scales and spatial scales of the transportive intensive flow variables [Sec. 2.5, Eqs. (2.97a) through (2.99c)].

Hence, the general representation of the six enclosure-surface integrals which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, integral form of the mass conservation principle for water in the elements and half-elements [Eq. (2.100)] becomes

$$\iint_{\bar{A}_{(x,y,z)}} \overline{V_{(x,y,z)}}\,dA - \iint_{\bar{A}_{(x,y,z)}} \overline{[\bar{V}_{(x,y,z)} + \bar{V}^{*}_{(x,y,z)} + V'_{(x,y,z)}]}\,dA$$

$$= \iint_{\bar{A}_{(x,y,z)}} \overline{\bar{V}_{(x,y,z)}}\,dA + \iint_{\bar{A}_{(x,y,z)}} \overline{\bar{V}^{*}_{(x,y,z)}}\,dA + \iint_{\bar{A}_{(x,y,z)}} \overline{\bar{V}'_{(x,y,z)}}\,dA]$$

$$-\bar{A}_{(x,y,z)}\bar{V}_{(x,y,z)} = \bar{G}_{(x,y,z)}, \qquad (2.107)$$

where

$\bar{A}_{(x,y,z)}$ = enclosure-surface area normal to x-, y- or z- direction,

and

$\bar{G}_{(x,y,z)}$ = flow rate along x-, y- or z- direction, across the enclosure-surface area $\bar{A}_{(x,y,z)}$ which is normal to x-, y- or z-direction, respectively,

and which is defined as

$$\overline{G}_{(x,y,z)}=\overline{A}_{(x,y,z)}\overline{V}_{(x,y,z)}. \tag{2.108}$$

Similarly, the general representation of the six enclosure-surface integrals which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle for water in the elements and half-elements [Eq. (2.101)] becomes

$$\iint_{\overline{A}_{(x,y,z)}}\overline{[V_x]V_{(x,y,z)}}\,dA=\iint_{\overline{A}_{(x,y,z)}}\overline{(\overline{V}_x+\overline{V}^*_x+V'_x)[\overline{V}_{(x,y,z)}+\overline{V}^*_{(x,y,z)}+V'_{(x,y,z)}]}\,dA$$

$$=\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}_x\overline{V}_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}_x\overline{V}^*_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}_xV'_{(x,y,z)}}\,dA$$

$$+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}^*_x\overline{V}_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}^*_x\overline{V}^*_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}^*_xV'_{(x,y,z)}}\,dA$$

$$+\iint_{\overline{A}_{(x,y,z)}}\overline{V'_x\overline{V}_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{V'_x\overline{V}^*_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{V'_xV'_{(x,y,z)}}\,dA$$

$$=\overline{V}_x\overline{V}_{(x,y,z)}\iint_{\overline{A}_{(x,y,z)}}dA+\overline{V}_x\iint_{\overline{A}_{(x,y,z)}}\overline{V}^*_{(x,y,z)}\,dA+\overline{V}_x\iint_{\overline{A}_{(x,y,z)}}\overline{V}'_{(x,y,z)}\,dA$$

$$+\overline{V}_{(x,y,z)}\iint_{\overline{A}_{(x,y,z)}}\overline{V}^*_x\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}^*_x\overline{V}^*_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}^*_xV'_{(x,y,z)}}\,dA$$

$$+\overline{V}_{(x,y,z)}\iint_{\overline{A}_{(x,y,z)}}\overline{V}'_x\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{V'_x\overline{V}^*_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{V'_xV'_{(x,y,z)}}\,dA$$

$$=\overline{V}_x\overline{V}_{(x,y,z)}\iint_{\overline{A}_{(x,y,z)}}dA+\iint_{\overline{A}_{(x,y,z)}}\overline{\overline{V}^*_x\overline{V}^*_{(x,y,z)}}\,dA+\iint_{\overline{A}_{(x,y,z)}}\overline{V'_xV'_{(x,y,z)}}\,dA$$

(continue)

$$-[\overline{A}_{(x,y,z)}\overline{V}_{(x,y,z)}]\overline{V}_x + \iint_{\overline{A}_{(x,y,z)}} \overline{V}_x^* \overline{V}^*_{(x,y,z)} dA + \iint_{\overline{A}_{(x,y,z)}} \overline{V_x' V'_{(x,y,z)}} dA$$

$$= \overline{G}_{(x,y,z)}]\overline{V}_x + \overline{A}_{(x,y,z)}\overline{\sigma}^{**}_{(x,y,z),x} + \overline{A}_{(x,y,z)}\overline{\sigma}''_{(x,y,z),x}$$

$$= \overline{G}_{(x,y,z)}\overline{V}_x + \overline{A}_{(x,y,z)}[\overline{\sigma}^{**}_{(x,y,z),x} + \overline{\sigma}''_{(x,y,z),x}]$$

$$= \overline{G}_{(x,y,z)}\overline{V}_x + \overline{A}_{(x,y,z)}\overline{\sigma}^{*'}_{(x,y,z),x} , \qquad (2.109)$$

where,

$\overline{\sigma}^{**}_{(x,y,z),x}$ = convection-defect stress component (component of convection-defect momentum transport flux) acting on the enclosure-surface area $\overline{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along x-direction,

which is defined as

$$\overline{\sigma}^{**}_{(x,y,z),x} = \frac{1}{\overline{A}_{(x,y,z)}} \iint_{\overline{A}_{(x,y,z)}} \overline{V}_x^* \overline{V}^*_{(x,y,z)} dA , \qquad (2.110)$$

and

$\overline{\sigma}''_{(x,y,z),x}$ = eddy-turbulent stress component (equivalent to the component of Reynolds stress) acting on the enclosure-surface area $\overline{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along x-direction,

which is defined as

$$\overline{\sigma}''_{(x,y,z),x} = \frac{1}{\overline{A}_{(x,y,z)}} \iint_{\overline{A}_{(x,y,z)}} \overline{V}'_x \overline{V}'_{(x,y,z)} dA , \qquad (2.111)$$

and

$\overline{\sigma}^{*'}_{(x,y,z),x}$ = combined convection-defect and eddy-turbulent stress component (excluding only the component of viscous stress) acting on the enclosure-surface area $\overline{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along x-direction,

which is defined as

$$\overline{\sigma}^{*'}_{(x,y,z),x} = \overline{\sigma}^{**}_{(x,y,z),x} + \overline{\sigma}^{''}_{(x,y,z),x} . \tag{2.112}$$

Similarly, the general representation of the six enclosure-surface integrals which constitute the closed control-surface integrals of the temporally primary-scale and spatially primary-scale y-component of the integral form of the momentum principle for water in the elements and half-elements [Eq. (2.102)] becomes

$$\begin{aligned}
\iint_{\overline{A}_{(x,y,z)}} \overline{[V_y]V_{(x,y,z)}}\,dA &= \overline{V}_y \overline{V}_{(x,y,z)} \iint_{\overline{A}_{(x,y,z)}} dA + \iint_{\overline{A}_{(x,y,z)}} \overline{V}^*_y \overline{V}^*_{(x,y,z)}\,dA + \iint_{\overline{A}_{(x,y,z)}} \overline{V'_y V'_{(x,y,z)}}\,dA \\
&= \overline{[A}_{(x,y,z)} \overline{V}_{(x,y,z)}]\overline{V}_y + \iint_{\overline{A}_{(x,y,z)}} \overline{V}^*_y \overline{V}^*_{(x,y,z)}\,dA + \iint_{\overline{A}_{(x,y,z)}} \overline{V'_y V'_{(x,y,z)}}\,dA \\
&= \overline{G}_{(x,y,z)}\overline{]V}_y + \overline{A}_{(x,y,z)}\overline{\sigma}^{**}_{(x,y,z),y} + \overline{A}_{(x,y,z)}\overline{\sigma}^{''}_{(x,y,z),y} \\
&= \overline{G}_{(x,y,z)}\overline{V}_y + \overline{A}_{(x,y,z)}[\overline{\sigma}^{**}_{(x,y,z),y} + \overline{\sigma}^{''}_{(x,y,z),y}] \\
&= \overline{G}_{(x,y,z)}\overline{V}_y + \overline{A}_{(x,y,z)}\overline{\sigma}^{*'}_{(x,y,z),y} .
\end{aligned} \tag{2.113}$$

where,

$\overline{\sigma}^{**}_{(x,y,z),y}$ = convection-defect stress component (component of convection-defect momentum transport flux) acting on the enclo-

sure-surface area $\bar{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along y-direction,

which is defined as

$$\bar{\sigma}^{**}_{(x,y,z),y} = \frac{1}{\bar{A}_{(x,y,z)}} \iint_{\bar{A}_{(x,y,z)}} \bar{V}^{*}_{y} \bar{V}^{*}_{(x,y,z)} dA , \tag{2.114}$$

and

$\bar{\sigma}''_{(x,y,z),y}$ = eddy-turbulent stress component (equivalent to the component of Reynolds stress) acting on the enclosure-surface area $\bar{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along y-direction,

which is defined as

$$\bar{\sigma}''_{(x,y,z),y} = \frac{1}{\bar{A}_{(x,y,z)}} \iint_{\bar{A}_{(x,y,z)}} \bar{V}'_{y} \bar{V}'_{(x,y,z)} dA , \tag{2.115}$$

and

$\bar{\sigma}^{*'}_{(x,y,z),y}$ = combined convection-defect and eddy-turbulent stress component (excluding only the component of viscous stress) acting on the enclosure-surface area $\bar{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along y-direction,

which is defined as

$$\bar{\sigma}^{*'}_{(x,y,z),y} = \bar{\sigma}^{**}_{(x,y,z),y} + \bar{\sigma}''_{(x,y,z),y} . \tag{2.116}$$

Similarly, the general representation of the six enclosure-surface integrals which constitute the closed control-surface integrals of the temporally primary-scale and spatially primary-scale z-component of the integral form of the momentum principle for water in the elements and half-elements [Eq. (2.103)] becomes

$$\iint_{\overline{A}_{(x,y,z)}} \overline{[V_z]V}_{(x,y,z)}\,dA = \overline{V}_z\overline{V}_{(x,y,z)}\iint_{\overline{A}_{(x,y,z)}} dA + \iint_{\overline{A}_{(x,y,z)}} \overline{V}^*_z\overline{V}^*_{(x,y,z)}\,dA + \iint_{\overline{A}_{(x,y,z)}} \overline{V'_z V'}_{(x,y,z)}\,dA$$

$$= [\overline{A}_{(x,y,z)}\overline{V}_{(x,y,z)}]\overline{V}_z + \iint_{\overline{A}_{(x,y,z)}} \overline{V}^*_z\overline{V}^*_{(x,y,z)}\,dA + \iint_{\overline{A}_{(x,y,z)}} \overline{V'_z V'}_{(x,y,z)}\,dA$$

$$= \overline{G}_{(x,y,z)}\overline{V}_z + \overline{A}_{(x,y,z)}\overline{\sigma}^{**}_{(x,y,z),z} + \overline{A}_{(x,y,z)}\overline{\sigma}''_{(x,y,z),z}$$

$$= \overline{G}_{(x,y,z)}\overline{V}_z + \overline{A}_{(x,y,z)}[\overline{\sigma}^{**}_{(x,y,z),z} + \overline{\sigma}''_{(x,y,z),z}]$$

$$= \overline{G}_{(x,y,z)}\overline{V}_z + \overline{A}_{(x,y,z)}\overline{\sigma}^{*'}_{(x,y,z),z}\,, \qquad (2.117)$$

where,

$\overline{\sigma}^{**}_{(x,y,z),z}$ = convection-defect stress component (component of convection-defect momentum transport flux) acting on the enclosure-surface area $\overline{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along z-direction,

which is defined as

$$\overline{\sigma}^{**}_{(x,y,z),z} = \frac{1}{\overline{A}_{(x,y,z)}}\iint_{\overline{A}_{(x,y,z)}} \overline{V}^*_z\overline{V}^*_{(x,y,z)}\,dA\,, \qquad (2.118)$$

and

$\overline{\sigma}''_{(x,y,z),z}$ = eddy-turbulent stress component (equivalent to the component of Reynolds stress) acting on the enclosure-surface area $\overline{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along z-direction,

which is defined as

$$\overline{\sigma}''_{(x,y,z),z} = \frac{1}{\overline{A}_{(x,y,z)}}\iint_{\overline{A}_{(x,y,z)}} \overline{V}'_z\overline{V}'_{(x,y,z)}\,dA\,, \qquad (2.119)$$

and

$\overline{\sigma}^{*'}_{(x,y,z),z}$ = combined convection-defect and eddy-turbulent stress component (excluding only the component of viscous stress) acting on the enclosure-surface area $\overline{A}_{(x,y,z)}$, normal to x-, y- or z-direction, and acting along z-direction,

which is defined as

$$\overline{\sigma}^{*'}_{(x,y,z),z} = \overline{\sigma}^{**}_{(x,y,z),z} + \overline{\sigma}''_{(x,y,z),z} \cdot \tag{2.120}$$

The formulation of the surface integrals associated with the cross-sectional areas of the intakes and discharges of a power plant m does not require the consideration of the temporally subscale and spatially subscale variations of the principal flow variables. The cross-sectional areas of the intakes and discharges are usually assumed to be sufficiently small, such that their contribution to the closed control-surface area of the control volume can be assumed to be negligible in the formulation of the intake and discharge flow conditions within the control volume of an element or half-element (Sec. 2.4). Hence, the contribution of the temporally subscale and spatially subscale variations of the principal flow variables over the cross-sectional areas of the intakes and discharges can be neglected in the formulation of the momentum transport across the closed control-surface area of the control volume of an element or half-element. However, it is important to note that large intake and discharge cross-sectional areas can readily be considered as boundaries of the flow region with specified flow conditions; and therefore, the consideration of intake and discharge flow conditions with large cross-sectional areas does not require any

simplification in the general formulation of the discrete-element method.

Hence, the general representation of the surface integral, associated with the cross-sectional area of the intake of a power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water in the element or half-element [Eq. (2.100)] becomes

$$\iint_{\overline{A}_{int,m}} \overline{[\vec{V}\cdot\hat{n}]}_{int,m}\, dA = -\overline{V}_{int,m}\overline{A}_{int,m} = -\overline{G}_{int,m}\,, \tag{2.121}$$

where

$\overline{V}_{int,m}$ = cross-section-averaged normal velocity of the intake (normal to the cross-sectional area of the intake) of power plant m (assumed as inward positive to the flow region),

$\overline{G}_{int,m}$ = intake flow rate of power plant m (assumed positive for flow conditions into the flow region),

which is defined as

$$\overline{G}_{int,m} = \overline{A}_{int,m}\overline{V}_{int,m}\,. \tag{2.122}$$

Similarly, the general representation of the surface integral, associated with the cross-sectional area of the discharge of a power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale, integral form of the mass conservation principle for water in the element or half-element [Eq. (2.100)] becomes

$$\iint_{\bar{A}_{dis,m}} \overline{[\vec{V}\cdot\hat{n}]}_{dis,m}\,dA = -\bar{V}_{dis,m}\bar{A}_{dis,m} = -\bar{G}_{dis,m}\,, \tag{2.123}$$

where

$\bar{V}_{dis,m}$ = cross-section-averaged normal velocity of the discharge (normal to the cross-sectional area of the discharge) of power plant m (assumed as inward positive to the flow region),

$\bar{G}_{dis,m}$ = discharge flow rate of power plant m (assumed positive for flow conditions into the flow region),

which is defined as

$$\bar{G}_{dis,m} = \bar{A}_{dis,m}\bar{V}_{dis,m}\,. \tag{2.124}$$

Similarly, the general representation of the surface integral, associated with the cross-sectional area of the intake of a power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale three components of the integral form of the momentum principle for water in the element or half-element [Eqs. (2.101) through (2.103)] becomes

$$\iint_{\bar{A}_{int,m}} \overline{\{[V_x]\vec{V}\cdot\hat{n}\}_{int,m}}\,dA = -\bar{V}_{(x,y,z)int,m}\bar{A}_{int,m}\bar{V}_{int,m} = -\bar{G}_{int,m}\bar{V}_{(x,y,z),int,m}\,, \tag{2.125}$$

where

$\bar{V}_{(x,y,z),int,m}$ = cross-section-averaged velocity along x-, y- or z-direction of the intake flow of power plant m.

Similarly, the general representation of the surface integral, associated with the cross-sectional area of the discharge of a power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale

three components of the integral form of the momentum principle for water in the element or half-element [Eqs. (2.101) through (2.103)] becomes

$$\iint_{\overline{A}_{dis,m}} \overline{\{[V_x]\vec{V}\cdot\hat{n}\}_{dis,m}}\,dA = -\overline{V}_{(x,y,z)dis,m}\overline{A}_{dis,m}\overline{V}_{dis,m} = -\overline{G}_{dis,m}\overline{V}_{(x,y,z),dis,m}\,. \tag{2.126}$$

where

$\overline{V}_{(x,y,z),dis,m}$ = cross-section-averaged velocity along x-, y- or z-direction of the discharge of power plant m.

The general representations of (1) the control volume integrals [Eqs. (2.104) and (2.106)], (2) the sets of six enclosure-surface integrals that constitue the closed control-surface integrals [Eqs. (2.107), (2.109), (2.113), and (2.117)], (3) the surface integrals associated with the cross-sectional area of the intake of a power plant m [Eqs. (2.121) and (2.125)], and sectional area of the discharge of a power plant m [Eqs. (2.123) and (2.126)] clearly identify all the convective transport terms and the modified nonconvective (modified, combined convection-defect and eddy-turbulent transport) terms which result from the discrete-element formulation of the temporally primary-scale and spatially primary-scale integral forms of the physical principles [Eqs. (2.100) through (2.104)].

3. FORMULATION OF THE DISCRETE-ELEMENT EQUATIONS

The discrete-element equations which represent the computational system associated with the discrete-element forms of the two physical principles are formulated by considering the temporally primary-scale and spatially primary-scale integral forms of the two physical principles, consisting of (1) the integral form of the mass conservation principle for water [Eq. (2.100)] and (2) the three components of the integral form of the momentum principle for water [Eqs. (2.101) through (2.103)] for the elements and half-elements in the flow region (Sec. 2.3).

The temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.100)] is formulated for the element (i,j,1). The temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle [Eq. (2.101)] is formulated for the x-upstream half-element ($i-\frac{1}{4}$,j,1) and for the x-downstream half-element ($i+\frac{1}{4}$,j,1). The temporally primary-scale and spatially primary-scale y-component of the integral form of the momentum principle [Eq. (2.102)] is formulated for the y-upstream half-element (i,$j-\frac{1}{4}$,1) and for the y-downstream half-element (i,$j+\frac{1}{4}$,1). The temporally primary-scale and spatially primary-scale z-component of the integral form of the momentum principle [Eq. (2.103)] is formulated for the element (i,j,1). The formulations of the temporally primary-scale and spatially primary-scale integral forms of the two physical principles [Eqs. (2.100) through (2.103)] directly lead to the computational forms of the discrete-element equations which are used

in the numerical solutions of the principal flow variables in the elements and half-elements of the flow region.

3.1. General Discussion of Upwind-Differencing Interpolation

The formulations of the parts of the closed control-surface integrals (Sec. 2.4) of the temporally primary-scale and spatially primary-scale integral forms of the two physical principles [Eqs. (2.100) through (2.103)] as surface integrals over the enclosure-surface areas of the elements and half-elements (Sec. 2.6) result in the temporally primary-scale and spatially primary-scale values of the flow variables as convective transport terms across the enclosure-surfaces between the elements and half-elements. It is well known that for any timewise-explicit numerical integration technique (e.g., Roache, 1972), used in the solution of convective transport equations, the values of the transportive, intrinsic flow variables of the solution at the locations between two solution locations, cannot be obtained directly by linear or spatially higher-order interpolation methods. The stability of the solutions of the timewise-explicit numerical integration technique can only be obtained by considering upwind-differencing, either second-upwind-differencing or temporally and spatially higher-order interpolation methods (e.g., Harlow and Welch, 1965, Amsder and Harlow, 1970, Evans and Harlow, 1957, Gentry, Martin and Daley, 1966, Fromm, 1964, Roache, 1972, Spraggs and Street, 1975). Therefore, in the formulations of the closed control-surface integrals, associated with the temporally primary-scale and spatially primary-scale integral forms of the two physical principles [Eqs. (2.100) through (2.103)], over the

enclosure-surface areas of the elements and half-elements, the definitions of the temporally primary-scale and spatially primary-scale transportive intensive flow variables (Sec. 2.5) must be appropriately modified to include the required upwind-differencing interpolation. Hence, the modified forms which indicate the upwind-differencing interpolation of the transportive, intensive flow variables can be defined, as

$\tilde{V}_{(x,y,z)}(t)$ = upwind-differenced value of the temporally primary-scale and spatially primary-scale x-, y- or z-component of velocity at the location of an enclosure-surface of an element or half-element,

In the general sense, the surface of the cross-sectional area of an intake also represents an enclosure-surface area of an element or half-element. Hence, the definitions of the temporally primary-scale and spatially primary-scale transportive intensive flow variables, associated with the intake flow conditions of a power plant m (Sec. 2.6) must also be modified to indicate the upwind-differencing interpolation, as

$\tilde{V}_{(x,y,z),int,m}$ = upwind-differenced value of the cross-section-averaged velocity along x-, y- or z-direction of the intake flow of power plant m,

The transportive, intensive flow variables associated with the discharge flow of a power plant must not be modified according to the upwind-differencing interpolation, since the discharge conditions are specified based on the operational conditions of the power plant which are generally independent of the conditions of the transportive intensive flow variables at the location of the discharge in the flow region.

The detailed development of the upwind-differencing interpolation formulas for the transportive intensive flow variables will be presented in Sec. 4.

3.2. Discrete-Element Equations Associated with the Integral Form of the Mass Conservation Principle for Water

The temporally primary-scale and spatially primary scale integral form of the mass conservation principle for water [Eq. (2.104)] can be formulated for the control volume of the element (i,j,l), as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i,j,l}} dv + \iint_{\overline{A}_{i,j,l}} \overline{\vec{V}\cdot\hat{n}}\,dA = 0 . \tag{3.1}$$

The control-volume integral in Eq.(3.1) can be formulated according to (1) the definition of the general volume integral for the control volume of the element (i,j,l) [Eqs. (2.83) and (2.86a)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.104)], as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i,j,l}} dv = A_{z;i,j}\ \overline{\Delta z}_{i,j,l} . \tag{3.2}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$A_{z;i,j}$ = two-dimensional planar area, in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) of the element (i,j,l) [Eqs. (2.83), (2.67) and (2.68)],

$\Delta\bar{z}_{i,j,l}$ = vertical length of the element (i,j,l) along z-axis (in the general sense, normal to the geophysical surface plane) in layer l [Eqs. (2.83), (2.47), (2.48), (2.49) and (2.50)].

Similarly, the closed control-surface integral in Eq. (3.1) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the element (i,j,l) [Eq. (2.87)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the element (i,j,l) [Eqs. (2.90a), (2.90c), (2.90h), (2.90j), (2.91a), (2.92b), (2.92e) and (2.92g)], (3) the formulation of the general representation of the six enclosure-surface integrals which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.107)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.121)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed contral-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.123)], as

$$\iint_{\bar{A}_{i,j,l}} \overline{\vec{V}\cdot\hat{n}}\,dA = -\tilde{G}_{x;i-\frac{1}{2},j,l} + \tilde{G}_{x;i+\frac{1}{2},j,l} - \tilde{G}_{y;i,j-\frac{1}{2},l} + \tilde{G}_{y;i,j+\frac{1}{2},l} - \tilde{G}_{z;i,j,l-\frac{1}{2}} + \tilde{G}_{z;i,j,l+\frac{1}{2}}$$

(continue)

$$-\delta_{int,m;i,j,l}\widetilde{G}_{int,m;i,j,l} - \delta_{dis,m;i,j,l}\overline{G}_{dis,m;i,j,l} . \qquad (3.3)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\widetilde{G}_{x;i-\frac{1}{2},j,l}$ = flow rate along x-direction, at x-upstream enclosure-surface at (i-½,j,l) of the element (i,j,l), [Eq (2.108)],

$\widetilde{G}_{x;i+\frac{1}{2},j,l}$ = flow rate along x-direction, at x-downstream enclosure-surface at (i+½,j,l) of the element (i,j,l), [Eq. (2.108)],

$\widetilde{G}_{y;i,j-\frac{1}{2},l}$ = flow rate along y-direction, at y-upstream enclosure-surface at (i,j-½,l) of the element (i,j,l), [Eq. (2.108)],

$\widetilde{G}_{y;i,j+\frac{1}{2},l}$ = flow rate along y-direction, at y-downstream enclosure-surface at (i,j+½,l) of the element (i,j,l), [Eq. (2.108)],

$\widetilde{G}_{z;i,j,l-\frac{1}{2}}$ = flow rate along z-direction, at z-upstream enclosure-surface at (i,j,l-½) of the element (i,j,l), [Eq. (2.108)],

$\widetilde{G}_{z;i,j,l+\frac{1}{2}}$ = flow rate along z-direction, at z-downstream enclosure-surface at (i,j,l+½) of the element (i,j,l), [Eq. (2.108)],

$\widetilde{G}_{int,m;i,j,l}$ = intake flow rate of power plant m in the element (i,j,l) (assumed positive for flow conditions into the flow region) [Eq. (2.122)],

$\overline{G}_{dis,m;i,j,l}$ = discharge flow rate of power plant m in the element (i,j,l) (assumed positive for flow conditions into the flow region) [Eq. (2.124)],

Substituting the control-volume integral [Eq. (3.2)] and the closed control-surface integral [Eq. (3.3)] and rearranging, the temporally

primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the element (i,j,l) [Eq. (3.1)], becomes

$$\frac{d}{dt}[A_{z;i,j}\ \overline{\Delta z}_{i,j,l}] - \widetilde{G}_{x;i-\frac{1}{2},j,l} - \widetilde{G}_{x;i+\frac{1}{2},j,l} + \widetilde{G}_{y;i,j-\frac{1}{2},l} - \widetilde{G}_{y;i,j+\frac{1}{2},l} + \widetilde{G}_{z;i,j,l-\frac{1}{2}}$$

$$- \widetilde{G}_{z;i,j,l+\frac{1}{2}} + \delta_{int,m;i,j,l}\widetilde{G}_{int,m;i,j,l} + \delta_{dis,m;i,j,l}\overline{G}_{dis,m;i,j,l}. \quad (3.4)$$

This form of temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the element (i,j,l) [Eq. (3.4)] represents the "discrete-element form of the mass conservation principle for water".

The area $A_{z;i,j,}$ in the horizontal (x,y) plane (in the general sense parallel to the free-surface plane of the flow region and the geophysical surface plane of the two-dimensional planar element (i,j) does not vary with time, according to the geometrical discretization technique of the discrete-element method (Sec. 2.3).

For an element below the free-surface layer (l<L), which does not contain the free-surface layer of the water in the flow region, the temporally primary-scale and spatially primary-scale vertical length $\overline{\Delta z}_{i,j,l}$ of the element (i,j,l) is a specified constant $\Delta z_{i,j,l}$ and hence, the time-derivative term in the discrete-element form of the mass conservation principle for water [Eq. (3.4)] can be simplified, as for $l(bot;i,j) \leq l<L$:

$$\frac{d}{dt}[A_{z;i,j}\ \overline{\Delta z}_{i,j,l}] = 0, \quad (3.5)$$

Substituting Eq. (3.5) in Eq.(3.4) and rearranging, the discrete-element form of the mass conservation principle for water can be formulated as

for $l(bot;i,j) \leq l < L$:

$$\widetilde{G}_{z;i,j,l+\frac{1}{2}} = \widetilde{G}_{z;i,j,l-\frac{1}{2}} + \widetilde{G}_{x;i-\frac{1}{2},j,l} - \widetilde{G}_{x;i+\frac{1}{2},j,l} + \widetilde{G}_{y;i,j-\frac{1}{2},l} - \widetilde{G}_{y;i,j+\frac{1}{2},l}$$

$$+ \delta_{int,m;i,j,l} \widetilde{G}_{int,m;i,j,l} + \delta_{dis,m;i,j,l} \overline{G}_{dis,m;i,j,l} \,. \tag{3.6}$$

For a bottom element $[i,j,l(bot;i,j,l)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure-surface at $[i,j,l(bot;i,j)-\frac{1}{2}]$, the discrete-element form of the mass conservation principle for water [Eq. (3.6)] becomes

for $l=l(bot;i,j)$:

$$\widetilde{G}_{z;i,j,l+\frac{1}{2}} = \widetilde{G}_{z,bot;i,j} + \widetilde{G}_{x;i-\frac{1}{2},j,l} - \widetilde{G}_{x;i+\frac{1}{2},j,l} + \widetilde{G}_{y;i,j-\frac{1}{2},l} - \widetilde{G}_{y;i,j+\frac{1}{2},l}$$

$$+ \delta_{int,m;i,j,l} \widetilde{G}_{int,m;i,j,l} + \delta_{dis,m;i,j,l} \overline{G}_{dis,m;i,j,l} \,. \tag{3.7}$$

where the temporally primary-scale and spatially primary-scale flow variable is defined as

$\widetilde{G}_{z,bot;i,j}$ = bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i,j,l(bot;i,j)-\frac{1}{2}]$ of the element $[i,j,l(bot;i,j)]$ (assumed positive for flow conditions into the flow region).

The discrete-element form of the mass conservation principle for water for the elements below the water-surface layer ($l<L$) [Eqs. (3.6) and (3.7)] represents the "discrete-element equation for flow rate along

z-direction". The discrete-element equation for flow rate along z-direction represents an algebraic equation for the systematic solution of the flow rates $\tilde{G}_{z;i,j,l-\frac{1}{2}}$ at z-upstream enclosure-surfaces at $(i,j,l+\frac{1}{2})$ of the elements (i,j,l), according to the general computational algorithm of the transient, three-dimensional discrete-element method (Sec. 6). The schematic representation of the solution location and the locations used in the computation of the flow rate $\tilde{G}_{z;i,j,l+\frac{1}{2}}$ are indicated in Figs. 2.43 through 2.46, p.135 through p.148, in Sec. 2.3.

For an element (i,j,L) in the water-surface layer L, which contains the free-surface layer of the water in the flow region, the vertical length $\overline{\Delta z}_{i,j,L}$ of the element (i,j,L) depends on the water-surface elevation $\overline{H}_{i,j}$ [Eq. (2.50)] which can vary with time (Figs. 2.2 through 2.6, p.92 through p.96). Hence the time-derivative-term in the discrete-element form of the mass conservation principle for water [Eq. (3.4)] can, be expanded, as

for l=L:

$$\frac{d}{dt}[A_{z;i,j}\,\overline{\Delta z}_{i,j,L}]=\frac{d}{dt}[A_{z;i,j}\,(\overline{\Delta z}_{i,j,L}+\overline{H}_{i,j})]=A_{z;i,j}\frac{d}{dt}\overline{H}_{i,j}\,. \qquad (3.8)$$

Substituting Eq.(3.8) in Eq. (3.4) and rearranging, the discrete-element form of the mass conservation principle for water for an element in the water-surface layer L can be formulated as

for l=L:

$$\frac{d\overline{H}_{i,j}}{dt}=\frac{1}{A_{z;i,j}}[\tilde{G}_{z;i,j,L-\frac{1}{2}}+\tilde{G}_{x;i-\frac{1}{2},j,L}-\tilde{G}_{x;i+\frac{1}{2},j,L}+\tilde{G}_{y;i,j-\frac{1}{2},L}-\tilde{G}_{y;i,j+\frac{1}{2},L}]$$

$$+\frac{\delta_{int,m;i,j,L}}{A_{z;i,j}}\tilde{G}_{int,m;i,j,L}+\frac{\delta_{dis,m;i,j,L}}{A_{z;i,j}}\overline{G}_{dis,m;i,j,L}+\overline{g}_{rnf}\,. \qquad (3.9)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$\bar{g}_{rnf}$ = rainfall rate per unit area on the water-surface of the flow region (assumed positive for rainfall conditions into the flow region).

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element in the horizontal (x,y) plane can be less than or equal to the specified depth of the top layer $\Delta\bar{z}_L$ (Sec. 2.3). In these single-layer subregions, an element (i,j,1) can represent a bottom element [i,j,1(bot;i,j)] and also an element in the water-surface layer (i,j,L). For these special elements [i,j,1(bot;i,j)] and also (i,j,L), the discrete-element form of the mass conservation principle for water becomes

for 1(bot;i,j)=1=L:

$$\frac{d\bar{H}_{i,j}}{dt}=\frac{1}{A_{z;i,j}}\tilde{G}_{z,bot;i,j}+\frac{1}{A_{z;i,j}}[\tilde{G}_{x;i-\frac{1}{2},j,L}-\tilde{G}_{x;i+\frac{1}{2},j,L}+\tilde{G}_{y;i-\frac{1}{2},j,L}-\tilde{G}_{y;i+\frac{1}{2},j,L}]$$

$$+\frac{\delta_{int,m;i,j,L}}{A_{z;i,j}}\tilde{G}_{int,m;i,j,L}+\frac{\delta_{dis,m;i,j,L}}{A_{z;i,j}}\bar{G}_{dis,m;i,j,L}+\bar{g}_{rnf}. \qquad (3.10)$$

The discrete-element form of the mass conservation principle for water for the elements in the water-surface layer (i,j,L) [Eqs. (3.9) and (3.10)] represent the "discrete-element equation for water-surface elevation". The discrete-element equation for water-surface elevation represents an ordinary differential equation for the systematic solution of the water-surface elevation $\bar{H}_{i,j}$ in the element (i,j,L) in the water-surface layer L, according to the general computational algorithm of the transient, three-dimensional discrete-element method (Sec. 5).

The schematic representation of the solution location and the locations used in the computation of the water-surface elevation $H_{i,j}$ are indicated in Figs. 2.2 through 2.6, p.92 through p.96.

The discrete-element equation for flow rate along z-direction [Eqs. (3.6) and (3.7)] and the discrete-element equation for water-surface elevation [Eqs. (3.9) and (3.10)] represent two principal computational equations of the general computational algorithm of the transient three-dimensional discrete-element method (Sec. 5) which can be obtained by formulating the temporally primary-scale and spatially primary-scale, integral form of the mass conservation principle for water [Eq. (2.100)] for the elements in the flow region.

According to the general mathematical development of the discrete-element method, four additional formulations of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.100)] are also needed for the half-elements as auxilary discrete-element forms of the mass conservation principle for water for (1) x-upstream half-element $(i-\frac{1}{4},j,1)$, (2) x-downstream half-element $(i+\frac{1}{4},j,1)$, (3) y-upstream half-element $(i,j-\frac{1}{4},1)$ and (4) y-downstream half-element $(i,j+\frac{1}{4},1)$.

The temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.104)] can be formulated for the control volume of the x-upstream half-element $(i-\frac{1}{4},j,1)$, as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i-\frac{1}{4},j,1}} dv + \iint_{\bar{A}_{i-\frac{1}{4},j,1}} \overline{\vec{V}\cdot\hat{n}}\,dA = 0 \qquad (3.11)$$

The control-volume integral in Eq. (3.11) can be formulated according to (1) the definition of the general volume integral for the control volume of the x-upstream half-element $(i-\frac{1}{4},j,1)$ [Eqs. (2.84a), (2.84b) and (2.86b)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.104)], as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i-\frac{1}{4},j,l}} dv = A_{z;i-\frac{1}{4},j}\ \bar{\Delta z}_{i-\frac{1}{4},j,l} \tag{3.12}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$A_{z;i-\frac{1}{4},j}$ = two-dimensional planar area in the horizontal (x,y) plane (in the general sense, paraller to the geophysical surface plane and the free-surface plane of the flow region) of the x-upstream half-element $(i-\frac{1}{4},j,1)$ [Eqs. (2.84a), (2.84b), (2.70) and (2.71)],

$\bar{\Delta z}_{i-\frac{1}{4},j,l}$ = vertical length of the x-upstream half-element $(i-\frac{1}{4},j,1)$ along z-axis (in the general sense, normal to the geophysical surface plane) in layer l [Eqs. (2.84a), (2.84b) and (2.64a)].

Similarly, the closed control-surface integral in Eq. (3.11) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the x-upstream half-element $(i-\frac{1}{4},j,1)$ [Eq. (2.88)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the x-upstream half-element $(i-\frac{1}{4},j,1)$ [Eqs. (2.90a), (2.90b), (2.90d), (2.90e) and (2.91b)],

(3) the formulation of the general representation of the six enclosure-surface integrals which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.107)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitues a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.121)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed contral-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.123)], as

$$\iint_{\overline{A}_{i-\frac{1}{4},j,l}} \overline{\vec{V}\cdot\hat{n}}\,dA = -\widetilde{G}_{x;i-\frac{1}{2},j,l} + \widetilde{G}_{x;i,j,l}$$

$$-\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l} + \widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l} - \widetilde{G}_{z;i-\frac{1}{4},j,l-\frac{1}{2}} + \widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}$$

$$-\delta_{int,m;i-\frac{1}{4},j,l}\overline{G}_{int,m;i-\frac{1}{4},j,l} - \delta_{dis,m;i-\frac{1}{4},j,l}\overline{G}_{dis,m;i-\frac{1}{4},j,l} \qquad (3.13)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\widetilde{G}_{x;i-\frac{1}{2},j,l}$ = flow rate along x-direction, at x-upstream enclosure-surface at $(i-\frac{1}{2},j,l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.108)],

$\widetilde{G}_{x;i,j,l}$ = flow rate along x-direction, at x-downstream enclosure-surface at (i,j,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.108)],

$\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l}$ = flow rate along y-direction, at y-upstream enclosure-surface at (i-¼,j-½,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.108)],

$\widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l}$ = flow rate along y-direction, at y-downstream enclosure-surface at (i-¼,j+½,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.108)],

$\widetilde{G}_{z;i-\frac{1}{4},j,l-\frac{1}{2}}$ = flow rate along z-direction, at z-upstream enclosure-surface at (i-¼,j,l-½) of the x-upstream half-element (i-¼,j,l) [Eq. (2.108)],

$\widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}$ = flow rate along z-direction, at z-downstream enclosure-surface at (i-¼,j,l+½) of the x-upstream half-element (i-¼,j,l) [Eq. (2.108)],

$\widetilde{G}_{int,m;i-\frac{1}{4},j,l}$ = intake flow rate of power plant m in the x-upstream half-element (i-¼,j,l) (assumed as positive for flow conditions into the flow region) [Eq. (2.122)],

$\overline{G}_{dis,m;i-\frac{1}{4},j,l}$ = discharge flow rate of power plant m in the x-upstream half-element (i-¼,j,l) (assumed as positive for flow conditions into the flow region) [Eq. (2.124)].

Substituting the control-volume integral [Eq. (3.12). and the closed control-surface integral [Eq. (3.13)] and rearranging, the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the x-upstream half-element (i-¼,j,l) [Eq. (3.11)] becomes

$$\frac{d}{dt}[A_{z;i-\frac{1}{4},j}\ \overline{\Delta z}_{i-\frac{1}{4},j,l}] = \widetilde{G}_{x;i-\frac{1}{2},j,l} - \widetilde{G}_{x;i,j,l}$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l} - \widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l} + \widetilde{G}_{z;i-\frac{1}{4},j,l-\frac{1}{2}} - \widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}$$

$$+\delta_{int,m;i-\frac{1}{4},j,l}\widetilde{G}_{int,m;i-\frac{1}{4},j,l} + \delta_{dis,m;i-\frac{1}{4},j,l}\overline{G}_{dis,m;i-\frac{1}{4},j,l} \quad (3.14)$$

This form of temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (3.14)] represents the "auxiliary discrete-element form of the mass conservation principle for water in the x-upstream half-element".

For a bottom x-upstream half-element $[i-\frac{1}{4},j,l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure surface at $[i-\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$, the auxiliary discrete-element form of the mass conservation principle for water in the x-upstream half-element [Eq. (3.14)] becomes

for $l=l(bot;i,j)$:

$$\frac{d}{dt}[A_{z;i-\frac{1}{4},j}\ \overline{\Delta z}_{bot;i-\frac{1}{4},j}] = \widetilde{G}_{x;i-\frac{1}{2},j,l} - \widetilde{G}_{x;i,j,l}$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l} - \widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l} + \widetilde{G}_{z,bot;i-\frac{1}{4},j} - \widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}$$

$$+\delta_{int,m;i-\frac{1}{4},j,l}\widetilde{G}_{int,m;i-\frac{1}{4},j,l} + \delta_{dis,m;i-\frac{1}{4},j,l}\overline{G}_{dis,m;i-\frac{1}{4},j,l} \quad (3.15)$$

where the temporally primary-scale and spatially primary-scale flow variable is defined as

$\widetilde{G}_{z,bot;i-\frac{1}{4},j}$ = bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i-\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$ of the

x-upstream half-element $[i-\frac{1}{4},j,1(bot;i,j)]$ (assumed positive for flow conditions into the flow region).

For a x-upstream half-element in the water-surface layer $(i-\frac{1}{4},j,L)$, which contains the free-surface layer of the water in the flow region, the auxiliary discrete-element form of the mass conservation principle for water in the x-upstream half-element [Eq. (3.14)] becomes

for $l=L$:

$$\frac{d}{dt}[A_{z;i-\frac{1}{4},j}\ \overline{\Delta z}_{i-\frac{1}{4},j,L}] = \widetilde{G}_{x;i-\frac{1}{2},j,L} - \widetilde{G}_{x;i,j,L}$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},L} - \widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},L} + \widetilde{G}_{z;i-\frac{1}{4},j,L-\frac{1}{2}} - A_{z;i-\frac{1}{4},j}\overline{g}_{rnf}$$

$$+\delta_{int,m;i-\frac{1}{4},j,L}\widetilde{G}_{int,m;i-\frac{1}{4},j,L} + \delta_{dis,m;i-\frac{1}{4},j,L}\overline{G}_{dis,m;i-\frac{1}{4},j,L} \quad (3.16)$$

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer Δz_L (Sec. 2.3). In these single-layer subregion, a x-upstream half-element $(i-\frac{1}{4},j,1)$ can represent a bottom x-upstream half-element $[i-\frac{1}{4},j,1(bot;i,j)]$ and also a x-upstream half-element in the water-surface layer $(i-\frac{1}{4},j,1)$. For these special x-upstream half-elements $[i-\frac{1}{4},j,1(bot;i,j)]$ and also $(i-\frac{1}{4},j,L)$, the auxiliary discrete-element form of the mass conservation principle for water [Eq. (3.14)], becomes

for $l(bot;i,j)=l=L$:

$$\frac{d}{dt}[A_{z;i-\frac{1}{4},j}\ \overline{\Delta z}_{i-\frac{1}{4},j,L}] = \widetilde{G}_{x;i-\frac{1}{2},j,L} - \widetilde{G}_{x;i,j,L}$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},L} - \widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},L} + \widetilde{G}_{z,bot;i-\frac{1}{4},j} - A_{z;i-\frac{1}{4},j}\overline{g}_{rnf}$$

(continue)

$$+\delta_{int,m;i-\frac{1}{4},j,L}\widetilde{G}_{int,m;i-\frac{1}{4},j,L}+\delta_{dis,m;i-\frac{1}{4},j,L}\overline{G}_{dis,m;i-\frac{1}{4},j,L} \qquad (3.17)$$

The temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.100)] can be formulated for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$, as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i+\frac{1}{4},j,l}} dv + \iint_{\overline{A}_{i+\frac{1}{4},j,l}} \overline{\vec{V}\cdot\hat{n}}\,dA = 0 \qquad (3.18)$$

The control-volume integral in Eq. (3.18) can be formulated according to (1) the definitions of the general volume integral for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eqs. (2.84a), (2.84b) and (2.86b)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.104)], as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i+\frac{1}{4},j,l}} dv = A_{z;i+\frac{1}{4},j}\ \overline{\Delta z}_{i+\frac{1}{4},j,l} \qquad (3.19)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$A_{z;i+\frac{1}{4},j}$ = two-dimensional planar area in the horizontal (x,y) plane (in the general sense, parallel to the geophysical sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) of the x-downstream half-element $(i+\frac{1}{4},j)$ [Eqs. (2.84a), (2.84b), (2.70) and (2.71)],

$\overline{\Delta z}_{i+\frac{1}{4},j,l}$ = vertical length of the x-downstream half-element $(i+\frac{1}{4},j,l)$ along z-axis (in the general sense, normal to the geophysical surface plane) in layer l [Eqs. (2.84a), (2.84b) and (2.64a)].

Similarly, the closed control-surface integral in Eq. (3.18) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.88)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eqs. (2.90a), (2.90b), (2.90d), (2.90e) and (2.91b)], (3) the formulation of the general representation of the six enclosure-surface integrals which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, integral form of the mass conservation principle for water [Eq. (2.107)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitues a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.121)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.123)], as

$$\iint_{\overline{A}_{i+\frac{1}{4},j,l}} \overline{\vec{V}\cdot\hat{n}}\,dA = -\widetilde{G}_{x;i,j,l} + \widetilde{G}_{x;i+\frac{1}{2},j,l}$$

$$-\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l} + \widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l} - \widetilde{G}_{z;i+\frac{1}{4},j,l-\frac{1}{2}} + \widetilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}$$

$$-\delta_{int,m;i+\frac{1}{4},j,l}\widetilde{G}_{int,m;i+\frac{1}{4},j,l} - \delta_{dis,m;i+\frac{1}{4},j,l}\overline{G}_{dis,m;i+\frac{1}{4},j,l} \qquad (3.20)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\tilde{G}_{x;i,j,l}$ = flow rate along x-direction, at x-upstream enclosure-surface at (i,j,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.113)],

$\tilde{G}_{x;i+\frac{1}{2},j,l}$ = flow rate along x-direction, at x-downstream enclosure-surface at (i+½,j,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.108)],

$\tilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l}$ = flow rate along y-direction, at y-upstream enclosure-surface at (i+¼,j-½,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.108)],

$\tilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l}$ = flow rate along y-direction, at y-downstream enclosure-surface at (i+¼,j+½,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.108)],

$\tilde{G}_{z;i+\frac{1}{4},j,l-\frac{1}{2}}$ = flow rate along z-direction, at z-upstream enclosure-surface at (i+¼,j,l-½) of the x-downstream half-element (i+¼,j,l) [Eq. (2.108)],

$\tilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}$ = flow rate along z-direction, at z-downstream enclosure-surface at (i+¼,j,l+½) of the x-downstream half-element (i+¼,j,l) [Eq. (2.108)],

$\tilde{G}_{int,m;i+\frac{1}{4},j,l}$ = intake flow rate of power plant m in the x-downstream half-element (i+¼,j,l) (assumed as positive for flow conditions into the flow region) [Eq. (2.122)],

$\overline{G}_{dis,m;i+\frac{1}{4},j,l}$ = discharge flow rate of power plant m in the x-downstream half-element (i+¼,j,l) (assumed as positive for flow conditions into the flow region) [Eq. (2.124)].

Substituting the control-volume integral [Eq. (3.19)] and the closed control-surface integral [Eq. (3.20)] and rearranging, the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (3.18)] becomes

$$\frac{d}{dt}[A_{z;i+\frac{1}{4},j}\ \overline{\Delta z}_{i+\frac{1}{4},j,l}] - \widetilde{G}_{x;i,j,l} - \widetilde{G}_{x;i+\frac{1}{2},j,l}$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l} - \widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l} + \widetilde{G}_{z;i+\frac{1}{4},j,l-\frac{1}{2}} - \widetilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}$$

$$+\delta_{int,m;i+\frac{1}{4},j,l}\widetilde{G}_{int,m;i+\frac{1}{4},j,l} + \delta_{dis,m;i+\frac{1}{4},j,l}\overline{G}_{dis,m;i+\frac{1}{4},j,l} \qquad (3.21)$$

This form of temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (3.21)] represents the "auxiliary discrete-element form of the mass conservation principle for water the x-downstream half-element".

For a bottom x-downstream half-element $[i+\frac{1}{4},j,l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom closure-surface at $[i+\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$, the auxiliary discrete-element form of the mass conservation principle for water in the x-downstream half-element [Eq. (3.21)] becomes

for $l=l(bot;i,j)$:

$$\frac{d}{dt}[A_{z;i+\frac{1}{4},j}\ \overline{\Delta z}_{bot;i+\frac{1}{4},j}] - \widetilde{G}_{x;i,j,l} - \widetilde{G}_{x;i+\frac{1}{2},j,l}$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l} - \widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l} + \widetilde{G}_{z,bot;i+\frac{1}{4},j} - \widetilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}$$

$$+\delta_{int,m;i+\frac{1}{4},j,l}\widetilde{G}_{int,m;i+\frac{1}{4},j,l} + \delta_{dis,m;i+\frac{1}{4},j,l}\overline{G}_{dis,m;i+\frac{1}{4},j,l} \qquad (3.22)$$

where the temporally primary-scale and spatially primary-scale flow variable, associated with the bottom flow conditions, is defined as

$\widetilde{G}_{z,bot;i+\frac{1}{4},j}$ = bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i+\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$ of the x-downstream half-element $[i+\frac{1}{4},j,l(bot;i,j)]$ (assumed positive for flow conditions into the flow region).

For a x-downstream half-element in the water-surface layer $(i+\frac{1}{4},j,L)$, which contains the free-surface layer of the water in the flow region, the auxiliary discrete-element form of the mass conservation principle for water in the x-downstream half-element [Eq. (3.21)] becomes

for l=L:

$$\frac{d}{dt}[A_{z;i+\frac{1}{4},j}\ \overline{\Delta z}_{i+\frac{1}{4},j,L}] = \widetilde{G}_{x;i,j,L} - \widetilde{G}_{x;i+\frac{1}{2},j,L}$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},L} - \widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},L} + \widetilde{G}_{z;i+\frac{1}{4},j,L-\frac{1}{2}} - A_{z;i+\frac{1}{4},j}\overline{g}_{rnf}$$

$$+\delta_{int,m;i+\frac{1}{4},j,L}\widetilde{G}_{int,m;i+\frac{1}{4},j,L} + \delta_{dis,m;i+\frac{1}{4},j,L}\overline{G}_{dis,m;i+\frac{1}{4},j,L} \quad (3.23)$$

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer Δz_L (Sec. 2.3). In these single-layer subregions, a x-downstream half-element $(i+\frac{1}{4},j,1)$ can represent a bottom x-downstream half-element $[i+\frac{1}{4},j,l(bot;i,j)]$ and also a x-downstream half-element in the water-surface layer $(i+\frac{1}{4},j,1)]$. For these special x-downstream half-elements $[i+\frac{1}{4},j,l(bot;i,j)]$ and also $(i+\frac{1}{4},j,L)$, the auxiliary discrete-element form of the mass conservation principle for water [Eq. (3.21)] becomes

for l(bot;i,j)=l=L:

$$\frac{d}{dt}[A_{z;i+\frac{1}{4},j}\ \overline{\Delta z}_{i+\frac{1}{4},j,L}] = \widetilde{G}_{x;i,j,L} - \widetilde{G}_{x;i+\frac{1}{2},j,L}$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},L} - \widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},L} + \widetilde{G}_{z,bot;i+\frac{1}{4},j} - A_{z;i+\frac{1}{4},j}\overline{g}_{rnf}$$

$$+\delta_{int,m;i+\frac{1}{4},j,L}\widetilde{G}_{int,m;i+\frac{1}{4},j,L} + \delta_{dis,m;i+\frac{1}{4},j,L}\overline{G}_{dis,m;i+\frac{1}{4},j,L} \quad (3.24)$$

The temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.104)] can be formulated for the control volume of the y-upstream half-element (i,j-¼,l), as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i,j-\frac{1}{4},l}} dv + \iint_{\overline{A}_{i,j-\frac{1}{4},l}} \overline{\vec{V}\cdot\hat{n}}dA = 0 \quad (3.25)$$

The control-volume integral in Eq. (3.25) can be formulated according to (1) the definitions of the general volume integral for the control volume of the y-upstream half-element (i,j-¼,l) [Eqs. (2.85a), (2.85b) and (2.86c)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.108)], as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i,j-\frac{1}{4},l}} dv = A_{z;i,j-\frac{1}{4}}\ \overline{\Delta z}_{i,j-\frac{1}{4},l} \quad (3.26)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$A_{z;i,j-\frac{1}{4}}$ = two-dimensional planar area in the area in the horizontal (x,y) plane (in the general sense, parallel to the geophy-

sical surface plane and the free-surface plane of the flow region) of the y-upstream half-element $(i,j-\frac{1}{4})$ [Eqs. (2.85a), (2.85b), (2.73) and (2.74)],

$\overline{\Delta z}_{i,j-\frac{1}{4},l}$ = vertical length of the y-upstream half-element $(i,j-\frac{1}{4},l)$ along the vertical z-axis (in the general sense, normal to the geophysical surface plane) in layer l [Eqs. (2.85a), (2.85b) and (2.64a)].

Similarly, the control-surface integral in Eq. (3.25) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.89)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eqs. (2.90h), (2.90i), (2.90k), (2.90l) and (2.91d)], (3) the formulation of the general representation of the six enclosure-surface integrals which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, integral form of the mass conservation principle for water [Eq. (2.107)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitues a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.121)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed control-surface integral of

the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.123)], as

$$\iint_{\overline{A}_{i,j-\frac{1}{4},l}} \overline{\vec{V}\cdot\hat{n}}\,dA = -\widetilde{G}_{y;i,j-\frac{1}{2},l}+\widetilde{G}_{y;i,j,l}$$

$$-\widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}+\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},l} \; -\widetilde{G}_{z;i,j-\frac{1}{4},l-\frac{1}{2}}+\widetilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}$$

$$-\delta_{int,m;i,j-\frac{1}{4},l}\widetilde{G}_{int,m;i,j-\frac{1}{4},l} \; -\delta_{dis,m;i,j-\frac{1}{4},l}\overline{G}_{dis,m;i,j-\frac{1}{4},l} \qquad (3.27)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$\widetilde{G}_{y;i,j-\frac{1}{2},l}$ = flow rate along y-direction, at y-upstream enclosure-surface at $(i,j-\frac{1}{2},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{y;i,j,l}$ = flow rate along y-direction, at y-downstream enclosure-surface at (i,j,l) of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}$ = flow rate along x-direction, at x-upstream enclosure-surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},l}$ = flow rate along x-direction, at x-downstream enclosure-surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{z;i,j-\frac{1}{4},l-\frac{1}{2}}$ = flow rate along z-direction, at z-upstream enclosure-surface at $(i,j-\frac{1}{4},l-\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}$ = flow rate along z-direction, at z-downstream enclosure-surface at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half-

element $(i,j-\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{int,m;i,j-\frac{1}{4},l}$ = intake flow rate of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$ (assumed as positive for flow conditions into the flow region) [Eq. (2.122)],

$\overline{G}_{dis,m;i,j-\frac{1}{4},l}$ = discharge flow rate of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$ (assumed as positive for flow conditions into the flow region) [Eq. (2.124)].

Substituting the control-volume integral [Eq. (3.26)] and the closed control-surface integral [Eq. (3.27)] and rearranging, the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (3.25)] becomes

$$\frac{d}{dt}[A_{z;i-\frac{1}{4},j}\ \overline{\Delta z}_{i-\frac{1}{4},j,l}]=\widetilde{G}_{y;i,j-\frac{1}{2},l}-\widetilde{G}_{y;i,j,l}$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}-\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},l}+\widetilde{G}_{z;i,j-\frac{1}{4},l-\frac{1}{2}}-\widetilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}$$

$$+\delta_{int,m;i,j-\frac{1}{4},l}\widetilde{G}_{int,m;i,j-\frac{1}{4},l}+\delta_{dis,m;i,j-\frac{1}{4},l}\overline{G}_{dis,m;i,j-\frac{1}{4},l} \qquad (3.28)$$

This form of temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (3.28)] represents the "auxiliary discrete-element form of the mass conservation principle for water in the y-upstream half-element".

For a bottom y-upstream half-element $[i,j-\frac{1}{4},l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure surface at $[i,j-\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$, the auxiliary discrete-element

form of the mass conservation principle for water in the y-upstream half-element [Eq. (3.28)] becomes

for l=l(bot;i,j):

$$\frac{d}{dt}[A_{z;i,j-\frac{1}{4}}\ \overline{\Delta z}_{bot;i,j-\frac{1}{4}}]=\tilde{G}_{y;i,j-\frac{1}{2},l}-\tilde{G}_{y;i,j,l}$$

$$+\tilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}-\tilde{G}_{x;i+\frac{1}{2},j,l}\ +\tilde{G}_{z,bot;i,j-\frac{1}{4}}-\tilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}$$

$$+\delta_{int,m;i,j-\frac{1}{4},l}\tilde{G}_{int,m;i,j-\frac{1}{4},l}+\delta_{dis,m;i,j-\frac{1}{4},l}\overline{G}_{dis,m;i,j-\frac{1}{4},l} \qquad (3.29)$$

where the temporally primary-scale and spatially primary-scale flow variable is defined as

$\tilde{G}_{z,bot;i,j-\frac{1}{4}}$ = bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i,j-\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$ of the y-upstream half-element $[i,j-\frac{1}{4},l(bot;i,j)]$ (assumed positive for flow conditions into the flow region).

For a y-upstream half-element in the water-surface layer $(i,j-\frac{1}{4},L)$, which contains the free-surface layer of the water in the flow region, the auxiliary discrete-element form of the mass conservation principle for water in the y-upstream half-element [Eq. (3.28)] becomes

for l=L:

$$\frac{d}{dt}[A_{z;i,j-\frac{1}{4}}\ \overline{\Delta z}_{i,j-\frac{1}{4},L}]=\tilde{G}_{y;i,j-\frac{1}{2},L}-\tilde{G}_{y;i,j,L}$$

$$+\ \tilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},L}-\tilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},L}+\tilde{G}_{z;i,j-\frac{1}{4},L-\frac{1}{2}}-A_{z;i,j-\frac{1}{4}}\overline{g}_{rnf}$$

$$+\delta_{int,m;i,j-\frac{1}{4},L}\tilde{G}_{int,m;i,j-\frac{1}{4},L}+\delta_{dis,m;i,j-\frac{1}{4},L}\overline{G}_{dis,m;i,j-\frac{1}{4},L} \qquad (3.30)$$

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer Δz_L (Sec. 2.3). In these single-layer subregions, a y-upstream half-element $(i,j-\frac{1}{4},L)$ can represent a bottom y-upstream half-element $[i,j-\frac{1}{4},L(bot;i,j)]$ and also a y-upstream half-element in the water-surface layer $(i,j-\frac{1}{4},1)$. For these special y-upstream half-elements $[i,j-\frac{1}{4},1(bot;i,j)]$ and also (i,j,L), the auxiliary discrete-element form of the mass conservation principle for water [Eq. (3.28)] becomes

for $1(bot;i,j)=1=L$:

$$\frac{d}{dt}[A_{z;i,j-\frac{1}{4}}\ \overline{\Delta z}_{i,j-\frac{1}{4},L}] = \widetilde{G}_{y;i,j-\frac{1}{2},L} - \widetilde{G}_{y;i,j,L}$$

$$+ \widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},L} - \widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},L} + \widetilde{G}_{z,bot;i,j-\frac{1}{4}} - A_{z;i,j-\frac{1}{4}}\overline{g}_{rnf}$$

$$+ \delta_{int,m;i,j-\frac{1}{4},L}\widetilde{G}_{int,m;i,j-\frac{1}{4},L} + \delta_{dis,m;i,j-\frac{1}{4},L}\overline{G}_{dis,m;i,j-\frac{1}{4},L} \qquad (3.31)$$

The temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.100)] can be formulated for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},1)$, as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i,j+\frac{1}{4},1}} dv + \iint_{\overline{A}_{i,j+\frac{1}{4},1}} \overline{\vec{V}\cdot\hat{n}}\,dA = 0 \qquad (3.32)$$

The control-volume integral in Eq. (3.32) can be formulated according to (1) the definition of the general volume integral for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},1)$ [Eqs. (2.85a), (2.85b) and (2.86c)] and (2) the formulation of the control-volume inte-

gral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.104)], as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} dv = A_{z;i,j+\frac{1}{4}}\ \overline{\Delta z}_{i,j+\frac{1}{4},l} \tag{3.33}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$A_{z;i,j+\frac{1}{4}}$ = two-dimensional planar area in the horizontal (x,y) plane (in general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) of the y-downstream half-element $(i,j+\frac{1}{4},L)$ [Eqs. (2.85a), (2.85b), (2.73) and (2.74)],

$\overline{\Delta z}_{i,j+\frac{1}{4},l}$ = vertical length of the y-downstream half-element $(i,j+\frac{1}{4},l)$ along the vertical z-axis (in the general sense, normal to the geophysical surface plane) in layer l [Eqs. (2.85a), (2.85b) and (2.64b)].

Similarly, the control-surface integral in Eq. (3.32) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.89)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eqs. (2.90i), (2.90j), (2.90m), (2.90m) and (2.91e)], (3) the formulation of the general representation of the six enclosure-surface integrals which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, integral form of

the mass conservation principle for water [Eq. (2.107)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitues a small part of the control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.121)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the control-surface integral of the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water [Eq. (2.123)], as

$$\iint_{\bar{A}_{i,j+\frac{1}{4},l}} \overline{\vec{V}\cdot\hat{n}}\,dA = -\widetilde{G}_{y;i,j,l} + \widetilde{G}_{y;i,j+\frac{1}{2},l}$$

$$-\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l} + \widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l} - \widetilde{G}_{z;i,j+\frac{1}{4},l-\frac{1}{2}} + \widetilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}$$

$$-\delta_{int,m;i,j+\frac{1}{4},l}\widetilde{G}_{int,m;i,j+\frac{1}{4},l} - \delta_{dis,m;i,j+\frac{1}{4},l}\widetilde{G}_{dis,m;i,j+\frac{1}{4},l} \qquad (3.34)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$\widetilde{G}_{y;i,j,l}$ = flow rate along y-direction, at y-upstream enclosure-surface at (i,j,l) of the y-downstream half-element (i,j+¼,l) [Eq. (2.108)],

$\widetilde{G}_{y;i,j+\frac{1}{2},l}$ = flow rate along y-direction, at y-downstream enclosure-surface at (i,j+½,l) of the y-downstream half-element (i,j+¼,l) [Eq. (2.108)],

$\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l}$ = flow rate along x-direction, at x-upstream enclosure-surface at (i-½,j+¼,l) of the y-downstream half-

element $(i,j+\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l}$ = flow rate along x-direction, at x-downstream enclosure-surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{z;i,j+\frac{1}{4},l-\frac{1}{2}}$ = flow rate along z-direction, at z-upstream enclosure-surface at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.108)],

$\widetilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}$ = flow rate along z-direction, at z-downstream enclosure-surface at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.108)],

$\overline{G}_{int,m;i,j+\frac{1}{4},l}$ = intake flow rate of power plant m in the y-downstream half-element $(i,j+\frac{1}{4},l)$ (assumed as positive for flow conditions into the flow region) [Eq. (2.122)],

$\overline{G}_{dis,m;i,j+\frac{1}{4},l}$ = discharge flow rate of power plant m in the y-downstream half-element $(i,j+\frac{1}{4},l)$ (assumed as positive for flow conditions into the flow region) [Eq. (2.124)].

Substituting the control-volume integral [Eq. (3.33)] and the closed control-surface integral [Eq. (3.34)] and rearranging, the temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (3.32)] becomes

$$\frac{d}{dt}[A_{z;i,j+\frac{1}{4}}\ \overline{\Delta z}_{i,j+\frac{1}{4},l}] = \widetilde{G}_{y;i,j,l} - \widetilde{G}_{y;i,j+\frac{1}{2},l}$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l} - \widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l} + \widetilde{G}_{z;i,j+\frac{1}{4},l-\frac{1}{2}} - \widetilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}$$

$$+\delta_{int,m;i,j+\frac{1}{4},l}\overline{G}_{int,m;i,j+\frac{1}{4},l} + \delta_{dis,m;i,j+\frac{1}{4},l}\overline{G}_{dis,m;i,j+\frac{1}{4},l} \qquad (3.35)$$

This form of temporally primary-scale and spatially primary-scale integral form of the mass conservation principle for water for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (3.35)] represents the "auxiliary discrete-element form of the mass conservation principle for water in the y-downstream half-element".

For a bottom y-downstream half-element $[i,j+\frac{1}{4},l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure surface at $[i,j+\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$, the auxiliary discrete-element form of the mass conservation principle for water in the y-downstream half-element [Eq. (3.28)] becomes

for $l=l(bot;i,j)$:

$$\frac{d}{dt}[A_{z;i,j+\frac{1}{4}}\ \overline{\Delta z}_{bot;i,j+\frac{1}{4}}] = \widetilde{G}_{y;i,j,l} - \widetilde{G}_{y;i,j+\frac{1}{2},l}$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l} - \widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l} + \widetilde{G}_{z,bot;i,j+\frac{1}{4}} - \widetilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}$$

$$+\delta_{int,m;i,j+\frac{1}{4},l}\widetilde{G}_{int,m;i,j+\frac{1}{4},l} + \delta_{dis,m;i,j+\frac{1}{4},l}\overline{G}_{dis,m;i,j+\frac{1}{4},l} \qquad (3.36)$$

where the temporally primary-scale and spatially primary-scale flow variable is defined as

$\widetilde{G}_{z,bot;i,j+\frac{1}{4}}$ = bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i,j+\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$ of the y-downstream half-element $[i,j+\frac{1}{4},l(bot;i,j)]$ (assumed positive for flow conditions into the flow region).

For a y-downstream half-element in the water-surface layer $(i,j+\frac{1}{4},L)$, which contains the free-surface layer of the water in the flow region, the auxiliary discrete-element form of the mass conserva-

tion principle for water in the y-downstream half-element [Eq. (3.35)] becomes

for l=L:

$$\frac{d}{dt}[A_{z;i,j+\frac{1}{4}}\ \overline{\Delta z}_{i,j+\frac{1}{4},L}] - \widetilde{G}_{y;i,j,L} - \widetilde{G}_{y;i,j+\frac{1}{2},L}$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},L} - \widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},L} + \widetilde{G}_{z;i,j+\frac{1}{4},L-\frac{1}{2}} - A_{z;i,j+\frac{1}{4}}\overline{g}_{rnf}$$

$$+\delta_{int,m;i,j+\frac{1}{4},L}\widetilde{G}_{int,m;i,j+\frac{1}{4},L} + \delta_{dis,m;i,j+\frac{1}{4},L}\overline{G}_{dis,m;i,j+\frac{1}{4},L} \quad (3.37)$$

In certain subregions, the specified depth $D_{l,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer Δz_L (Sec. 2.3). In these single-layer subregions, a y-downstream half-element $(i,j+\frac{1}{4},l)$ can represent a bottom y-downstream half-element $[i,j+\frac{1}{4},l(bot;i,j)]$ and also a y-downstream half-element in the water-surface layer $(i,j+\frac{1}{4},l)$. For these special y-downstream half-elements $[i,j+\frac{1}{4},l(bot;i,j)]$ and also $(i,j+\frac{1}{4},L)$, the auxiliary discrete-element form of the mass conservation principle for water [Eq. (3.35)] becomes

for l(bot;i,j)=l=L:

$$\frac{d}{dt}[A_{z;i,j+\frac{1}{4}}\ \overline{\Delta z}_{i,j+\frac{1}{4},L}] - G_{y;i,j,L} - G_{y;i,j+\frac{1}{2},L}$$

$$+G_{x;i-\frac{1}{2},j+\frac{1}{4},L} - G_{x;i+\frac{1}{2},j+\frac{1}{4},L} + G_{z,bot;i,j+\frac{1}{4}} - A_{z;i,j+\frac{1}{4}}\overline{g}_{rnf}$$

$$+\delta_{int,m;i,j+\frac{1}{4},L}G_{int,m;i,j+\frac{1}{4},L} + \delta_{dis,m;i,j+\frac{1}{4},L}\overline{G}_{dis,m;i,j+\frac{1}{4},L} \quad (3.38)$$

The discrete-element formulation of the temporally primary-scale and spatially primary-scale, integral form of the mass conservation

principle for water [Eq. (2.100)] leads to (1) two principal computational equations, consisting of (a) the discrete-element equation for flow rate along z-direction [Eqs. (3.6) and (3.7)], which is used for the solution of the flow rates along z-direction $\tilde{G}_{z;i,j,l+\frac{1}{2}}$ at the z-downstream enclosure-surfaces at $(i,j,l+\frac{1}{2})$ of all the elements that are below the water-surface layer $(i,j,l<L)$, and (b) the discrete-element equation for water-surface elevation [Eqs. (3.9), (3.10)], which is used for the solution of the water-surface elevation Hi,j in all the elements which are in the water-surface layer (i,j,L), and (2) four auxiliary discrete-element forms of the mass conservation principle for water [Eqs. (3.14) through (3.17), (3.21) through (3.24), (3.28) through (3.31), and (3.35) through (3.38)] which are used in the discrete-element formulations of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eqs. (2.101) through (2.103)] for the x-upstream half-element $(i-\frac{1}{4},j,l)$, x-downstream half-element $(i+\frac{1}{4},j,l)$, y-upstream half-element $(i,j-\frac{1}{4},l)$ and y-downstream half-element $(i,j+\frac{1}{4},l)$.

3.3 Discrete-Element Equations Associated with the Three Components of the Integral Form Statement of the Momentum Principle

According to the discrete-element formulation of the temporally primary-scale and spatially primary-scale three components of the integral form of the momentum principle [Eqs. (2.101) through (2.103)], it is computationally convinient to consider modified pressure and modified viscous stress components defined, as

$$p_w = \frac{1}{\rho_\omega}(p - p_a) \tag{3.39}$$

$$\sigma_{\nu,s(x,y,z)} = \frac{1}{\rho_\omega}\sigma_{s(x,y,z)} \tag{3.40}$$

where

p_a = spatially uniform atmospheric pressure over the flow region,

p_w = modified pressure (dimensions consistent with the formulation in terms of time rate of change of momentum),

$\sigma_{s(x,y,z)}$ = viscous (molecular) stress component acting on generalized unit area of the closed control surface, along x-, y-, or z-direction,

$\sigma_{\nu,s(x,y,z)}$ = modified viscous (molecular) stress component acting on generalized unit area of the closed control surface along x-, y-, or z-direction (dimensions consistent with the formulation in terms of time rate of change of momentum.

Hence, the temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle [Eq. (2.101)] can be formulated for the control-volume of the x-upstream half-element $(i-\frac{1}{4},j,l)$, as

$$\begin{aligned}\frac{\partial}{\partial t}\iiint_{\bar{V}_{i-\frac{1}{4},j,l}} \overline{V_x}\,dv + \iint_{\bar{A}_{i-\frac{1}{4},j,l}} \overline{[V_x]\vec{V}\cdot\hat{n}}\,dA &= -g_a\cos\Theta_{n,x}\iiint_{\bar{V}_{i-\frac{1}{4},j,l}} dv \\ &+ \Omega_{cor,N,y}\iiint_{\bar{V}_{i-\frac{1}{4},j,l}} \overline{V_z}\,dv - \Omega_{cor,N,z}\iiint_{\bar{V}_{i-\frac{1}{4},j,l}} \overline{V_y}\,dv \\ &+ \iint_{\bar{A}_{i-\frac{1}{4},j,l}} \overline{[p_w(-\hat{n})]_x}\,dA + \iint_{\bar{A}_{i-\frac{1}{4},j,l}} \overline{\sigma_{\nu,sx}}\,dA\end{aligned} \tag{3.41}$$

The control-volume integral on the left-hand side of Eq. (3.41) can be formulated according to (1) the definition of the general volume integral for the control volume of the x-upstream half-element (i-¼,j,l) [Eqs. (2.84a), (2.84b) and (2.86b)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)], as

$$\iiint_{\bar{V}_{i-\frac{1}{4},j,l}} \bar{V}_x dv = [A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]\bar{V}_{x;i-\frac{1}{4},j,l}, \tag{3.42}$$

where the temporally primary-scale and spatially primary-scale flow variable is defined, as

$\bar{V}_{x;i-\frac{1}{4},j,l}$ = x-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] in the x-upstream half-element (i-¼,j,l).

The time derivative of the control-volume integral [Eq. (3.42)], on the left-hand side of Eq. (3.41), can be explanded, as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i-\frac{1}{4},j,l}} \bar{V}_x dv = \frac{\partial}{\partial t}\{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]\bar{V}_{x;i-\frac{1}{4},j,l}\}$$

$$= [A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]\frac{\partial \bar{V}_{x;i-\frac{1}{4},j,l}}{\partial t} + \bar{V}_{x;i-\frac{1}{4},j,l}\frac{\partial}{\partial t}[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}] . \tag{3.43}$$

Hence, multiplying the auxiliary discrete-element form of the mass conservation principle for water in the x-upstream half-element (i-¼,j,l) [Eq. (3.14)] with the x-component of velocity $\bar{V}_{x;i+\frac{1}{4},j,l}$ in the x-upstream half-element (i-¼,j,l) and substituting in Eq. (3.43),

the time derivative of the control-volume integral [Eq. (3.43)] on the left-hand side of Eq. (3.41) becomes

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i-\frac{1}{4},j,l}} \overline{V_x}\,dV = [A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]\frac{\partial}{\partial t}\overline{V}_{x;i-\frac{1}{4},j,l}$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j,l}\overline{V}_{x;i-\frac{1}{4},j,l}-\widetilde{G}_{x;i,j,l}\overline{V}_{x;i-\frac{1}{4},j,l}$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l}\overline{V}_{x;i-\frac{1}{4},j,l}-\widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l}\overline{V}_{x;i-\frac{1}{4},j,l}$$

$$+\widetilde{G}_{z;i-\frac{1}{4},j,l-\frac{1}{2}}\overline{V}_{x;i-\frac{1}{4},j,l}-\widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}\overline{V}_{x;i-\frac{1}{4},j,l}$$

$$+\delta_{int,m;i-\frac{1}{4},j,l}\widetilde{G}_{int,m;i-\frac{1}{4},j,l}\overline{V}_{x;i-\frac{1}{4},j,l}$$

$$+\delta_{dis,m;i-\frac{1}{4},j,l}\widetilde{G}_{dis,m;i-\frac{1}{4},j,l}\overline{V}_{x;i-\frac{1}{4},j,l}\,. \qquad (3.44)$$

The closed control-surface integral on the left-hand side of Eq. (3.41) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the x-upstream half-element (i-¼,j,l) [Eq. (2.88)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the x-upstream half element (i-¼,j,l) [Eqs. (2.90a), (2.90b), (2.90d), (2.90e) and (2.91b)], (3) the formulation of the general representation of the six enclosure-surface integral which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, x-component of the integral form of the momentum principle [Eq. (2.109)], (4) the formulation of the general representation of the surface integral, asso-

ciated with the cross-sectional area of the intake of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle [Eq. (2.125)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle [Eq. (2.126)], and by considering the general definitions of the upwind-differencing interpolation forms for the transportive, intensive flow variables, as

$$\begin{aligned}
\iint_{\overline{A}_{i-\frac{1}{4},j,l}} \overline{[V_x]\vec{V}\cdot\hat{n}}\,dA = &-\widetilde{G}_{x;i-\frac{1}{2},j,l}\widetilde{V}_{x;i-\frac{1}{2},j,l}+\overline{A}_{x;i-\frac{1}{2},j,l}\overline{\sigma}^{*'}_{x,x;i-\frac{1}{2},j,l} \\
&+\widetilde{G}_{x;i,j,l}\widetilde{V}_{x;i,j,l}-\overline{A}_{x;i,j,l}\overline{\sigma}^{*'}_{x,x;i,j,l} \\
&-\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l}\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l}+\overline{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}\overline{\sigma}^{*'}_{y,x;i-\frac{1}{4},j-\frac{1}{2},l} \\
&+\widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l}\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}-\overline{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}\overline{\sigma}^{*'}_{y,x;i-\frac{1}{4},j+\frac{1}{2},l} \\
&-\widetilde{G}_{z;i-\frac{1}{4},j,l-\frac{1}{2}}\widetilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}}+\overline{A}_{z;i-\frac{1}{4},j}\overline{\sigma}^{*'}_{z,x;i-\frac{1}{4},j,l-\frac{1}{2}} \\
&+\widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}\widetilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}-\overline{A}_{z;i-\frac{1}{4},j}\overline{\sigma}^{*'}_{z,x;i-\frac{1}{4},j,l+\frac{1}{2}} \\
&-\delta_{int,m;i-\frac{1}{4},j,l}\widetilde{G}_{int,m;i-\frac{1}{4},j,l}\widetilde{V}_{x,int,m;i-\frac{1}{4},j,l}
\end{aligned}$$

(continue)

$$-\delta_{dis,m;i-\frac{1}{4},j,l}\overline{G}_{dis,m;i-\frac{1}{4},j,l}\,\overline{V}_{x,dis,m;i-\frac{1}{4},j,l}\ , \qquad (3.45)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\widetilde{V}_{x;i-\frac{1}{2},j,l}$ = upwind-differenced value of x-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] at x-upstream enclosure-surface at (i-½,j,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.109)],

$\overline{\sigma}^{*'}_{x,x;i-\frac{1}{2},j,l}$ = combined convection-defect and eddy-turbulent stress component (excluding only the component of viscous stress) acting on the x-upstream enclosure-surface area $\overline{A}_{x;i-\frac{1}{2},j,l}$ at (i-½,j,l) of the x-upstream half-element (i-¼,j,l), along x-direction [Eq. (2.112)],

$\widetilde{V}_{x;i,j,l}$ = upwind-differenced value of x-component of velocity at x-downstream enclosure surface at (i,j,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.109)],

$\overline{\sigma}^{*'}_{x,x;i,j,l}$ = combined convection-defect and eddy-turbulent stress component acting on the x-downstream enclosure-surface area $\overline{A}_{x;i,j,l}$ at (i,j,l) of the x-upstream half-element (i-¼,j,l), along x-direction [Eq. (2.112)],

$\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l}$ = upwind-differenced value of x-component of velocity at y-upstream enclosure-surface at (i-¼,j-½,l) of the x-upstream half-element (i-¼,j,l) [Eq. (2.109)].

$\overline{\sigma}^{*'}_{y,x;i-\frac{1}{4},j+\frac{1}{2},l}$ = combined convection-defect and eddy-turbulent stress component acting on the y-upstream enclosure-surface

area $\bar{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}$ at $(i-\frac{1}{4},j-\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction [Eq. (2.112)],

$\tilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}$ = upwind-differenced value of x-component of velocity at y-downstream enclosure-surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.109)],

$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$ = combined convection-defect and eddy-turbulent stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}$ at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction [Eq. (2.112)],

$\tilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}}$ = upwind-differenced value of x-component of velocity at z-upstream enclosure-surface at $(i-\frac{1}{4},j,l-\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.109)],

$\bar{\sigma}^{*'}_{z,x;i-\frac{1}{4},j,l-\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-upstream enclosure-surface area $A_{z;i-\frac{1}{4},j}$ at $(i-\frac{1}{4},j,l-\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction [Eq. (2.112)],

$\tilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}$ = upwind-differenced value of x-component of velocity at z-downstream enclosure-surface at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.109)],

$\bar{\sigma}^{*'}_{z,x;i-\frac{1}{4},j,l+\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-downstream enclosure-surface area $A_{z;i-\frac{1}{4},j}$ at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction [Eq. (2.112)],

$\tilde{V}_{x,int,m;i-\frac{1}{4},j,l}$ = upwind-differenced value of x-component of velocity associated with the intake flow of power plant m in the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$\overline{V}_{x,dis,m;i-\frac{1}{4},j,l}$ = x-component of velocity associated with the discharge flow of power plant m in the x-upstream half-element $(i-\frac{1}{4},j,l)$.

The first control-volume integral on the right-hand side of Eq. (3.41), associated with the gravity force term along x-direction, is already evaluated for the discrete-element formulation of the temporally primary-scale and spatially primary-scale integral form of the conservation of mass principle for water [Eq. (3.12)].

The control-volume integrals on the right-hand side of Eq. (3.41), associated with the Coriolis acceleration (force) terms are also formulated according to (1) the definition of the general volume integral for the control-volume of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eqs. (2.84a), (2.84b) and (2.86b)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)], as

$$\iiint_{\overline{V}_{i-\frac{1}{4},j,l}} \overline{V}_z dv = [A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]\overline{V}_{z;i-\frac{1}{4},j,l}, \tag{3.46}$$

$$\iiint_{\overline{V}_{i-\frac{1}{4},j,l}} \overline{V}_y dv = [A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]\overline{V}_{y;i-\frac{1}{4},j,l}. \tag{3.47}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\overline{V}_{z;i-\frac{1}{4},j,l}$ = z-component of velocity along the vertical direction (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region) in the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.106)],

$\bar{V}_{y;i-\frac{1}{4},j,l}$ = y-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] in the x-upstream half-element ($i-\frac{1}{4},j,l$) [Eq. (2.106)].

The discrete-element formulation of the closed control-surface integral associated with the pressure force term on the right-hand side of Eq. (3.41) requires careful considerations of the geometrical properties of the elements, particularly the boundary elements which represent nonrectangular (truncated rectangular) cells with enclosure-surface areas not necessarily normal to any of the three principal orthogonal axes x, y, or z (Figs. 2.16 through 2.24, p.106 through p.114; Fig. 2.51, p.143; Fig. 2.53, p.145). It is important to note that on these boundary enclosure-surface areas, the pressure forces result in components along x-direction depending on the angle (in the general sense) between the unit normal vector to the boundary enclosure-surface and the x-axis. The geometrical effects of the boundary enclosure-surface areas in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) can be incorporated by considering the area $\bar{A}_{x;i-\frac{1}{4},j,l}$, normal to the x-direction, at the center of the x-upstream half-element ($i-\frac{1}{4},j,l$) (e.g., Shames, 1962, Eraslan, 1975, 1977,1981a), for calculating the net pressure force acting on the x-upstream half-element. However, the variation of the pressure, particularly the hydrostatic pressure part, along the z-direction must be carefully considered in the formulation of the pressure force term acting along x-direction on the x-upstream half-element ($i-\frac{1}{4},j,l$). The important consequences of the variation of pres-

sure along z-direction will be considered in detail in the discussions of the interpolation formulas for the pressure conditions on the enclosure-surface areas of the half-elements (Sec. 4.3).

Hence, the control-surface integral associated with the pressure force term on the right-hand side of Eq. (3.41) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.88)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and downstream enclosure-surface areas of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eqs. (2.90a), (2.90b), (2.90d), (2.90e) and (2.91b)], and by considering the components of the presure forces, along the x-direction, on all enclosure-surface areas of the x-upstream half-element $(i-\frac{1}{4},j,l)$, including the ones which are the parts of the nonrectangular boundary elements that represent truncated rectangular cells (Sec. 2, Figs. 2.16 through 2.24, p.106 through p.114; Fig. 2.51, p.143; Fig. 2.53, p.145), as

$$\iint_{\bar{A}_{i-\frac{1}{4},j,l}} \overline{[p_w(-\hat{n})]_x} dA = \bar{A}_{x;i-\frac{1}{4},j,l}(\bar{p}_{w;i-\frac{1}{2},j,l} - \bar{p}_{w;i,j,l}), \tag{3.48}$$

where the area $\bar{A}_{x;i-\frac{1}{4},j,l}$, normal to the x-direction, at the center of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [in Eq. (3.48)] is defined, as

$$\bar{A}_{x;i-\frac{1}{4},j,l} = \Delta y_{i-\frac{1}{4},j} \overline{\Delta z}_{i-\frac{1}{4},j,l}, \tag{3.49}$$

and the temporally primary-scale and spatially primary-scale values of the modified pressure are defined, as

$\bar{p}_{w;i-\frac{1}{2},j,l}$ = modified pressure at the location of the x-upstream enclosure-surface at $(i-\frac{1}{2},j,l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$,

$\bar{p}_{w;i,j,l}$ = modified pressure at the location of the x-downstream enclosure-surface at (i,j,l) of the x-upstream half-element $(i-\frac{1}{4},j,l)$.

The discrete-element formulation of the control-surface integral, associated with the pressure force term for the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (3.48)] is applicable to all x-upstream half-elements, including (1) bottom x-upstream half-element $[i-\frac{1}{4},j,l(bot;i,j)]$, (2) vertically interior x-upstream half-elements $[i-\frac{1}{4},j,l(bot;i,j)\frac{1}{4}l\frac{1}{4}L]$, and (3) x-upstream half-elements in the water-surface layer $(i-\frac{1}{4},j,L)$. Furthermore, according to the discrete-element formulation of the control-surface integral [Eq. (3.48)], the pressure force term acting on the x-upstream half-element $(i-\frac{1}{4},j,l)$ is completely independent of the value of the spatially uniform atmospheric pressure on the free-surface of the water in the flow region, since it includes only the difference of the two pressure values at the upstream enclosure-surface at $(i-\frac{1}{2},j,l)$ and at the downstream enclosure-surface at (i,j,l).

The control-surface integral associated with the modified viscous (molecular) force term, on the right-hand side of Eq. (3.41) can be evaluated according to (1) the definitions of the general closed control-surface integral for the control-volume of x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (2.88)] and (2) the definitions of the enclosure-surface

integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eqs. (2.90a), (2.90b), (2.90d), (2.90e) and (2.91b)] , as

$$\iint_{\bar{A}_{i-\frac{1}{4},j,l}} \overline{\sigma_{\nu,sx}} dA \doteq -\bar{A}_{x;i-\frac{1}{2},j,l}\bar{\sigma}_{\nu,xx;i-\frac{1}{2},j,l} + \bar{A}_{x;i,j,l}\bar{\sigma}_{\nu,xx;i,j,l}$$

$$-\bar{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}\bar{\sigma}_{\nu,yx;i-\frac{1}{4},j-\frac{1}{2},l} + \bar{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}\bar{\sigma}_{\nu,yx;i-\frac{1}{4},j+\frac{1}{2},l}$$

$$-A_{z;i-\frac{1}{4},j}\bar{\sigma}_{\nu,zx;i-\frac{1}{4},j,l-\frac{1}{2}} + A_{z;i-\frac{1}{4},j}\bar{\sigma}_{\nu,zx;i-\frac{1}{4},j,l+\frac{1}{2}} , \tag{3.50}$$

where the temporally primary-scale and spatially primary-scale viscous stress components are defined, as

$\bar{\sigma}_{\nu,xx;i-\frac{1}{2},j,l}$ = modified viscous stress component acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j,l}$ at $(i-\frac{1}{2},j,l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

$\bar{\sigma}_{\nu,xx;i,j,l}$ = modified viscous stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i,j,l}$ at (i,j,l) of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

$\bar{\sigma}_{\nu,yx;i-\frac{1}{4},j-\frac{1}{2},l}$ = modified viscous stress component acting on the y-upstream enclosure-surface area $\bar{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}$ at $(i-\frac{1}{4},j-\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

$\bar{\sigma}_{\nu,yx;i-\frac{1}{4},j+\frac{1}{2},l}$ = modified viscous stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}$ at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

$\bar{\sigma}_{\nu,zx;i-\frac{1}{4},j,l-\frac{1}{2}}$ = modified viscous stress component acting on the z-upstream enclosure-surface area $A_{z;i-\frac{1}{4},j}$ at $(i-\frac{1}{4},j,l-\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

$\bar{\sigma}_{\nu,zx;i-\frac{1}{4},j,l+\frac{1}{2}}$ = modified viscous stress component acting on the z-downstream enclosure-surface area $A_{z;i-\frac{1}{4},j}$ at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction.

Hence, substituting the discrete-element formulations of (1) the time derivative of the control-volume integral [Eq. (3.44)] of the left-hand side of Eq. (3.41), (2) the control-surface inteqral [Eq. (3.45)] of the left-hand side of Eq. (3.41), (3) the control-volume integral associated with the gravity force term along x-direction [Eq. (3.12)] of the right-hand side of Eq. (3.41), (4) the two control-volume integrals associated with the Coriolis acceleration (force) term [Eqs. (3.46) and (3.47)] of the right-hand side of Eq. (3.41), (5) the control-surface integral associated with the modified pressure force term [Eq. (3.48)] of the right-hand side of Eq. (3.41), and (6) the control-surface integral associated with the modified viscous force term [Eq. (3.49)] of the right-hand side of Eq. (3.41), in the temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle for the control-volume of the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (3.41)], combining the terms and rearranging, the discrete-element form of the x-component of the momentum principle for the x-upstream half-element $(i-\frac{1}{4},j,l)$ becomes,

$$\frac{\partial}{\partial t}\overline{V}_{x;i-\frac{1}{4},j,l} = \frac{1}{\left[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}\right]}\left[\widetilde{G}_{x;i-\frac{1}{2},j,l}(\widetilde{V}_{x;i-\frac{1}{2},j,l}-\overline{V}_{x;i-\frac{1}{4},j,l})\right.$$

$$-\widetilde{G}_{x;i,j,l}(\widetilde{V}_{x;i,j,l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l}(\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$-\widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l}(\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$+\widetilde{G}_{z;i-\frac{1}{4},j,l-\frac{1}{2}}(\widetilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$\left.-\widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}(\widetilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}-\overline{V}_{x;i-\frac{1}{4},j,l})\right]$$

$$+\frac{\overline{A}_{x;i-\frac{1}{4},j,l}}{\left[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}\right]}(\overline{p}_{w;i-\frac{1}{2},j,l}-\overline{p}_{w;i,j,l})$$

$$-g_a\cos\theta_{n,x}+\Omega_{cor,N,y}\overline{V}_{z;i-\frac{1}{4},j,l}-\Omega_{cor,N,z}\overline{V}_{y;i-\frac{1}{4},j,l}$$

$$+\frac{1}{\left[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}\right]}\left[-\overline{A}_{x;i-\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,l}+\overline{A}_{x;i,j,l}\overline{\sigma}^{*'}_{\nu,x,x;i,j,l}\right.$$

$$-\overline{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$$

$$+\overline{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$$

$$-A_{z;i-\frac{1}{4},j}\overline{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l-\frac{1}{2}}$$

$$\left.+A_{z;i-\frac{1}{4},j}\overline{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l+\frac{1}{2}}\right]$$

(continue)

$$+\frac{\delta_{int,m;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}\widetilde{G}_{int,m;i-\frac{1}{4},j,l}(\widetilde{V}_{x,int,m;i-\frac{1}{4},j,l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$+\frac{\delta_{dis,m;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}\overline{G}_{dis,m;i-\frac{1}{4},j,l}(\overline{V}_{x,dis,m;i-\frac{1}{4},j,l}-\overline{V}_{x;i-\frac{1}{4},j,l}). \tag{3.51}$$

where the temporally primary-scale and spatially primary-scale stress components are defined, as

$\overline{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,l}$ = modified stress component (consisting of viscous stress component, and combined convection-defect and eddy-turbulent stress component) acting on the x-upstream enclosure-surface area $\overline{A}_{x;i-\frac{1}{2},j,l}$ at (i-½,j,l) of the x-upstream half-element (i-¼,j,l), along the x-direction,

which represents

$$\overline{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,l}=\overline{\sigma}_{\nu,xx;i-\frac{1}{2},j,l}+\overline{\sigma}^{*'}_{x,x;i-\frac{1}{2},j,l}, \tag{3.52a}$$

$\overline{\sigma}^{*'}_{\nu,x,x;i,j,l}$ = modified stress component acting on the x-downstream enclosure-surface area $\overline{A}_{x;i,j,l}$ at (i,j,l) of the x-upstream half-element (i-¼,j,l), along x-direction,

which represents

$$\overline{\sigma}^{*'}_{\nu,x,x;i,j,l}=\overline{\sigma}_{\nu,xx;i,j,l}+\overline{\sigma}^{*'}_{x,x;i,j,l}, \tag{3.52b}$$

$\overline{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$ = modified stress component acting on the y-upstream enclosure-surface area $\overline{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}$ at (i-¼,j-½,l) of the x-upstream half-element (i-¼,j,l), along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l} = \bar{\sigma}_{\nu,yx;i-\frac{1}{4},j-\frac{1}{2},l} + \bar{\sigma}^{*'}_{y,x;i-\frac{1}{4},j-\frac{1}{2},l} , \qquad (3.52c)$$

$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j+\frac{1}{2},l}$ = modified stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}$ at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j+\frac{1}{2},l} = \bar{\sigma}_{\nu,yx;i-\frac{1}{4},j+\frac{1}{2},l} + \bar{\sigma}^{*'}_{y,x;i-\frac{1}{4},j+\frac{1}{2},l} , \qquad (3.52d)$$

$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l-\frac{1}{2}}$ = modified stress component acting on the z-upstream enclosure-surface area $A_{z;i-\frac{1}{4},j}$ at $(i-\frac{1}{4},j,l-\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l-\frac{1}{2}} = \bar{\sigma}_{\nu,zx;i-\frac{1}{4},j,l-\frac{1}{2}} + \bar{\sigma}^{*'}_{z,x;i-\frac{1}{4},j,l-\frac{1}{2}} , \qquad (3.52e)$$

$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l+\frac{1}{2}}$ = modified stress component acting on the z-downstream enclosure-surface area $A_{z;i-\frac{1}{4},j}$ at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l+\frac{1}{2}} = \bar{\sigma}_{\nu,zx;i-\frac{1}{4},j,l+\frac{1}{2}} + \bar{\sigma}^{*'}_{z,x;i-\frac{1}{4},j,l+\frac{1}{2}} . \qquad (3.52f)$$

This discrete-element form of the temporally primary-scale and spatially primary-scale integral form of the x-component of the momentum principle for the x-upstream half-element $(i-\frac{1}{4},j,l)$ [Eq. (3.51)] represents the first "discrete-element equation for x-component of velocity".

For a bottom x-upstream half-element [i-¼,j,l(bot;i,j)], which includes the bottom of the flow region as the z-upstream the enclosure-surface at [i-¼,j,l(bot;i,j)-½] , the discrete-element equation for the x-component of velocity [Eq. (3.51)] becomes

for l = l(bot;i,j):

$$\frac{\partial}{\partial t}\overline{V}_{x;i-\frac{1}{4},j,l} = \frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}[\widetilde{G}_{x;i-\frac{1}{2},j,l}(\widetilde{V}_{x;i-\frac{1}{2},j,l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$-\widetilde{G}_{x;i,j,l}(\widetilde{V}_{x;i,j,l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l}(\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$-\widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l}(\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$+\widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}(\widetilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}-\overline{V}_{x;i-\frac{1}{4},j,l})]$$

$$+\frac{\overline{A}_{x;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}(\overline{p}_{w;i-\frac{1}{2},j,l}-\overline{p}_{w;i,j,l})$$

$$-g_o\cos\theta_{n,x}+\Omega_{cor,N,y}\overline{V}_{z;i-\frac{1}{4},j,l}-\Omega_{cor,N,z}\overline{V}_{y;i-\frac{1}{4},j,l}$$

$$+\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}[-\overline{A}_{x;i-\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,l}+\overline{A}_{x;i,j,l}\overline{\sigma}^{*'}_{\nu,x,x;i,j,l}$$

$$-\overline{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$$

$$+\overline{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$$

(continue)

$$+A_{z;i-\frac{1}{4},j}\overline{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l+\frac{1}{2}}]$$

$$+\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}[\widetilde{G}_{z,bot;i-\frac{1}{4},j}(\widetilde{V}_{x,bot;i-\frac{1}{4},j}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$-A_{z;i-\frac{1}{4},j}\overline{\sigma}^{*'}_{\nu,x,bot;i-\frac{1}{4},j}]$$

$$+\frac{\delta_{int,m;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}\widetilde{G}_{int,m;i-\frac{1}{4},j,l}(\widetilde{V}_{x,int,m;i-\frac{1}{4},j,l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$+\frac{\delta_{dis,m;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}\overline{G}_{dis,m;i-\frac{1}{4},j,l}(\overline{V}_{x,dis,m;i-\frac{1}{4},j,l}-\overline{V}_{x;i-\frac{1}{4},j,l}). \quad (3.53)$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the bottom flow conditions, are defined as

$\widetilde{V}_{x,bot;i-\frac{1}{4},j}$ = upwind-differenced value of x-component of velocity associated with the bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i-\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$ of the x-upstream half-element $[i-\frac{1}{4},j,l(bot;i,j)]$,

$\widetilde{G}_{z,bot;i-\frac{1}{4},j}$ = modified bottom shear stress component acting on the z-upstream bottom enclosure-surface area $A_{z;i-\frac{1}{4},j}$ at $[i-\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$ of the x-upstream half-element $[i-\frac{1}{4},j,l(bot;i,j)]$, along x-direction.

For a x-upstream half-element in the water-surface layer $(i-\frac{1}{4},j,L)$, which contains the free-surface layer of the water in the flow region, the discrete-element equation for x-component of velocity [Eq. (3.51)] becomes

for $l = L$:

$$\frac{\partial}{\partial t}\bar{V}_{x;i-\frac{1}{4},j,L}=\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}[\tilde{G}_{x;i-\frac{1}{2},j,L}(\tilde{V}_{x;i-\frac{1}{2},j,L}-\bar{V}_{x;i-\frac{1}{4},j,L})$$

$$-\tilde{G}_{x;i,j,L}(\tilde{V}_{x;i,j,L}-\bar{V}_{x;i-\frac{1}{4},j,L})$$

$$+\tilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},L}(\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},L}-\bar{V}_{x;i-\frac{1}{4},j,L})$$

$$+\tilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},L}(\tilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},L}-\bar{V}_{x;i-\frac{1}{4},j,L})$$

$$+\tilde{G}_{z;i-\frac{1}{4},j,L-\frac{1}{2}}(\tilde{V}_{x;i-\frac{1}{4},j,L-\frac{1}{2}}-\bar{V}_{x;i-\frac{1}{4},j,L})]$$

$$+\frac{\bar{A}_{x;i-\frac{1}{4},j,L}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}(\bar{p}_{w;i-\frac{1}{2},j,L}-\bar{p}_{w;i,j,L})$$

$$-g_o\cos\theta_{n,x}+\Omega_{cor,N,y}\bar{V}_{z;i-\frac{1}{4},j,L}-\Omega_{cor,N,z}\bar{V}_{y;i-\frac{1}{4},j,L}$$

$$+\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}[-\bar{A}_{x;i-\frac{1}{2},j,L}\bar{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,L}+\bar{A}_{x;i,j,L}\bar{\sigma}^{*'}_{\nu,x,x;i,j,L}$$

$$-\bar{A}_{y;i-\frac{1}{4},j-\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},L}$$

$$+\bar{A}_{y;i-\frac{1}{4},j+\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},L}$$

$$-A_{z;i-\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,L-\frac{1}{2}}]$$

$$+\frac{1}{\overline{\Delta z}_{i-\frac{1}{4},j,L}}[\bar{g}_{rnf}(\bar{V}_{x,rnf}-\bar{V}_{x;i-\frac{1}{4},j,L})+\bar{\sigma}^{*'}_{\nu,x,wnd;i-\frac{1}{4},j}]$$

$$+\frac{\delta_{int,m;i-\frac{1}{4},j,L}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}\tilde{G}_{int,m;i-\frac{1}{4},j,L}(\tilde{V}_{x,int,m;i-\frac{1}{4},j,L}-\bar{V}_{x;i-\frac{1}{4},j,L})$$

(continue)

$$+\frac{\delta_{dis,m;i-\frac{1}{4},j,L}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}\overline{G}_{dis,m;i-\frac{1}{4},j,L}(\overline{V}_{x,dis,m;i-\frac{1}{4},j,L}-\overline{V}_{x;i-\frac{1}{4},j,L}). \tag{3.54}$$

where the temporally primary-scale and spatially primary-scale flow variables, assocaited with the surface flow conditions, are defined as

$\overline{V}_{x,rnf}$ = x-component of velocity associated with the rainfall on the water surface of the flow region.

$\overline{\sigma}^{*'}_{\nu,x,wnd;i-\frac{1}{4},j}$ = modified wind-shear stress component acting on the z-downstream water enclosure-surface at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, along x-direction.

It is important to note that the wind-shear stress component, along x-direction, depends on the free-surface flow conditions on the water enclosure-surface at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$ as well as the wind conditions over the water surface of the flow region.

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer $\overline{\Delta z}_L$ (Sec. 2.3). In these single layer subregions, a x-upstream half-element $(i-\frac{1}{4},j,L)$ can represent a bottom x-upstream half-element $i-\frac{1}{4},j,l(bot;i,j)$ and also a x-upstream half-element in the water-surface layer $(i-\frac{1}{4},j,L)$. For these special x-upstream half-elements $[i-\frac{1}{4},j,l(bot;i,j)]$ and also $(i-\frac{1}{4},j,L)$ the discrete-element equation for x-component of velocity [Eq. (3.51)] becomes

for $l(bot;i,j)=l=L$:

$$\frac{\partial}{\partial t}\overline{V}_{x;i-\frac{1}{4},j,L}=\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}[\widetilde{G}_{x;i-\frac{1}{2},j,L}(\widetilde{V}_{x;i-\frac{1}{2},j,L}-\overline{V}_{x;i-\frac{1}{4},j,L})$$

$$-\widetilde{G}_{x;i,j,L}(\widetilde{V}_{x;i,j,L}-\overline{V}_{x;i-\frac{1}{4},j,L})$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},L}(\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},L}-\overline{V}_{x;i-\frac{1}{4},j,L})$$

$$+\widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},L}(\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},L}-\overline{V}_{x;i-\frac{1}{4},j,L})]$$

$$+\frac{\overline{A}_{x;i-\frac{1}{4},j,L}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}(\overline{p}_{w;i-\frac{1}{2},j,L}-\overline{p}_{w;i,j,L})$$

$$-g_o\cos\theta_{n,x}+\Omega_{cor,N,y}\overline{V}_{z;i-\frac{1}{4},j,L}-\Omega_{cor,N,z}\overline{V}_{y;i-\frac{1}{4},j,L}$$

$$+\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}[-\overline{A}_{x;i-\frac{1}{2},j,L}\overline{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,L}+\overline{A}_{x;i,j,L}\overline{\sigma}^{*'}_{\nu,x,x;i,j,L}$$

$$-\overline{A}_{y;i-\frac{1}{4},j-\frac{1}{2},L}\overline{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},L}$$

$$+\overline{A}_{y;i-\frac{1}{4},j+\frac{1}{2},L}\overline{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},L}$$

$$+\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,1}]}[\widetilde{G}_{z,bot;i-\frac{1}{4},j}(\widetilde{V}_{x,bot;i-\frac{1}{4},j}-\overline{V}_{x;i-\frac{1}{4},j,1})$$

$$-A_{z;i-\frac{1}{4},j}\overline{\sigma}^{*'}_{\nu,x,bot;i-\frac{1}{4},j}]$$

$$+\frac{1}{\overline{\Delta z}_{i-\frac{1}{4},j,L}}[\overline{g}_{rnf}(\overline{V}_{x,rnf}-\overline{V}_{x;i-\frac{1}{4},j,L})+\overline{\sigma}^{*'}_{\nu,x,wnd;i-\frac{1}{4},j}]$$

(continue)

$$+\frac{\delta_{int,m;i-\frac{1}{4},j,L}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}\widetilde{G}_{int,m;i-\frac{1}{4},j,L}(\widetilde{V}_{x,int,m;i-\frac{1}{4},j,L}-\overline{V}_{x;i-\frac{1}{4},j,L})$$

$$+\frac{\delta_{dis,m;i-\frac{1}{4},j,L}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,L}]}\widetilde{G}_{dis,m;i-\frac{1}{4},j,L}(\widetilde{V}_{x,dis,m;i-\frac{1}{4},j,L}-\overline{V}_{x;i-\frac{1}{4},j,L}). \tag{3.55}$$

Similarly, the temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle [Eq. (102)] can be formulated for the control-volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$, as

$$\frac{\partial}{\partial t}\iiint_{\overline{V}_{i+\frac{1}{4},j,l}}\overline{V_x}dv+\iint_{\overline{A}_{i+\frac{1}{4},j,l}}\overline{[V_x]\vec{V}\cdot\hat{n}}dA=-g_a\cos\Theta_{n,x}\iiint_{\overline{V}_{i+\frac{1}{4},j,l}}dv$$
$$+\Omega_{cor,N,y}\iiint_{\overline{V}_{i+\frac{1}{4},j,l}}\overline{V_z}dv-\Omega_{cor,N,z}\iiint_{\overline{V}_{i+\frac{1}{4},j,l}}\overline{V_y}dv$$
$$+\iint_{\overline{A}_{i+\frac{1}{4},j,l}}\overline{[p_w(-\hat{n})]_x}dA+\iint_{\overline{A}_{i+\frac{1}{4},j,l}}\overline{\sigma_{\nu,sx}}dA\,. \tag{3.56}$$

The control-volume integral on the left-hand side of Eq. (3.56) can be formulated according to (1) the definition of the general volume integral for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eqs. (2.84a), (2.84b) and (2.86b)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)], as

$$\iiint_{\overline{V}_{i+\frac{1}{4},j,l}}\overline{V_x}dv=[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]\overline{V}_{x;i+\frac{1}{4},j,l}\,, \tag{3.57}$$

where the temporally primary-scale and spatially primary-scale flow variable is defined, as

$\bar{V}_{x;i+\frac{1}{4},j,l}$ = x-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geopysical surface plane and the free-surface plane of the flow region)] in the x-downstream half-element $(i+\frac{1}{4},j,l)$.

The time derivative of the control-volume integral [Eq. (3.57)] on the left-hand side of Eq. (3.56), can be expanded, as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i+\frac{1}{4},j,l}} \bar{V}_x dv = \frac{\partial}{\partial t}\left\{ [A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]\bar{V}_{x;i+\frac{1}{4},j,l} \right\}$$

$$= [A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]\frac{\partial}{\partial t}\bar{V}_{x;i+\frac{1}{4},j,l} + \bar{V}_{x,i+\frac{1}{4},j,l}\frac{\partial}{\partial t}[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]. \quad (3.58)$$

Hence, multiplying the auxilary discrete-element form of the mass conservation principle for water in the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (3.21)] with the x-component of velocity $\bar{V}_{x;i+\frac{1}{4},j,l}$ in the x-downstream half-element $(i+\frac{1}{4},j,l)$ and substituting in Eq. (3.58), the time derivative of the control-volume integral [Eq. (3.58)] on the left-hand side of Eq. (3.56) becomes

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i+\frac{1}{4},j,l}} \bar{V}_x dv = [A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]\frac{\partial}{\partial t}\bar{V}_{x;i+\frac{1}{4},j,l}$$

$$+\tilde{G}_{x;i,j,l}\bar{V}_{x;i+\frac{1}{4},j,l} - \tilde{G}_{x;i+\frac{1}{2},j,l}\bar{V}_{x;i+\frac{1}{4},j,l}$$

$$+\tilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l}\bar{V}_{x;i+\frac{1}{4},j,l} - \tilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l}\bar{V}_{x;i+\frac{1}{4},j,l}$$

$$+\tilde{G}_{z;i+\frac{1}{4},j,l-\frac{1}{2}}\bar{V}_{x;i+\frac{1}{4},j,l} - \tilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}\bar{V}_{x;i+\frac{1}{4},j,l}$$

$$+\delta_{int,m;i+\frac{1}{4},j,l}\tilde{G}_{int,m;i+\frac{1}{4},j,l}\bar{V}_{x;i+\frac{1}{4},j,l} \quad \text{(continue)}$$

$$+\delta_{dis,m;i+\frac{1}{4},j,l}\overline{C}_{dis,m;i+\frac{1}{4},j,l}\overline{V}_{x;i+\frac{1}{4},j,l}. \qquad (3.59)$$

The closed control-surface integral on the left-hand side of Eq. (3.56) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.88)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the x-downstream half element $(i+\frac{1}{4},j,l)$ [Eqs. (2.90b), (2.90c), (2.90f), (2.90g) and (2.91c)], (3) the formulation of the general representation of the six enclosure-surface integral which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, x-component of the integral form of the momentum principle [Eq. (2.109)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale, x-component of the integral form of the momentum principle [Eq. (2.125)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale, x-component of the integral form of the momentum principle [Eq. (2.126)], and by considering the general definitions of the upwind-differencing interpolation forms for the transportive, intensive flow variables, as

$$\iint_{\bar{A}_{i+\frac{1}{4},j,l}} \overline{[V_x]\vec{V}\cdot\hat{n}}\,dA = -\tilde{G}_{x;i,j,l}\tilde{V}_{x;i,j,l}\oplus\bar{A}_{x;i,j,l}\bar{\sigma}^{*'}_{x,x;i,j,l}$$

$$+\tilde{G}_{x;i+\frac{1}{2},j,l}\tilde{V}_{x;i+\frac{1}{2},j,l}-\bar{A}_{x;i+\frac{1}{2},j,l}\bar{\sigma}^{*'}_{x,x;i+\frac{1}{2},j,l}$$

$$-\tilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l}\tilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l}+\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},l}\bar{\sigma}^{*'}_{y,x;i+\frac{1}{4},j-\frac{1}{2},l}$$

$$+\tilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l}\tilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l}-\bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},l}\bar{\sigma}^{*'}_{y,x;i+\frac{1}{4},j+\frac{1}{2},l}$$

$$-\tilde{G}_{z;i+\frac{1}{4},j,l-\frac{1}{2}}\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}}+A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{z,x;i+\frac{1}{4},j,l-\frac{1}{2}}$$

$$+\tilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}-A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{z,x;i+\frac{1}{4},j,l+\frac{1}{2}}$$

$$-\delta_{int,m;i+\frac{1}{4},j,l}\tilde{G}_{int,m;i+\frac{1}{4},j,l}\,\tilde{V}_{x,int,m;i+\frac{1}{4},j,l}$$

$$-\delta_{dis,m;i+\frac{1}{4},j,l}\bar{G}_{dis,m;i+\frac{1}{4},j,l}\,\bar{V}_{x,dis,m;i+\frac{1}{4},j,l}\,. \qquad (3.60)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\tilde{V}_{x;i,j,l}$ = upwind-differenced value of x-component of velocity in the horizontal (x,y) plane [in the general sense, parallel to the geopysical surface plane and the free-surface plane of the flow region)] at x-upstream enclosure-surface at (i,j,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.109)],

$\bar{\sigma}^{*'}_{x,x;i,j,l}$ = combined convection-defect and eddy-turbulent stress component (excluding only the component of viscous

stress) acting on the x-upstream enclosure-surface area $\bar{A}_{x;i,j,l}$ at (i,j,l) of the x-downstream half-element (i+¼,j,l), along x-direction [Eq. (2.112)],

$\tilde{V}_{x;i+\frac{1}{2},j,l}$ = upwind-differenced value of x-component of velocity at x-downstream enclosure surface at (i+½,j,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.109)],

$\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,l}$ = combined convection-defect and eddy-turbulent stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j,l}$ at (i+½,j,l) of the x-downstream half-element (i+¼,j,l), along x-direction [Eq. (2.112)],

$\bar{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l}$ = upwind-differenced value of x-component of velocity at y-upstream enclosure surface at (i+¼,j-½,l) of the x-downstream half-elements (i+¼,j,l) [Eq. (2.109)]

$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$ = combined convection-defect and eddy-turbulent stress component acting on the y-upstream enclosure-surface area $\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},l}$ at (i+¼,j-½,l) of the x-downstream half-element (i+¼,j,l), along x-direction [Eq. (2.112)],

$\bar{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l}$ = upwind-differenced value of x-component of velocity at y-downstream enclosure-surface at (i+¼,j+½,l) of the x-downstream half-element (i+¼,j,l) [Eq. (2.109)],

$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$ = combined convection-defect and eddy-turbulent stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},l}$ at (i+¼,j+½,l) of the x-downstream half-element (i+¼,j,l), along x-direction [Eq. (2.112)],

$\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}}$ = upwind-differenced value of x-component of velocity at z-upstream enclosure-surface at $(i+\frac{1}{4},j,l-\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.109)],

$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l-\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-upstream enclosure-surface area $A_{z;i+\frac{1}{4},j}$ at $(i+\frac{1}{4},j,l-\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, along x-direction [Eq. (2.112)],

$\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}$ = upwind-differenced value of x-component of velocity at z-downstream enclosure-surface at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.109)],

$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l+\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-downstream enclosure-surface area $A_{z;i+\frac{1}{4},j}$ at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstreAM half-element $(i+\frac{1}{4},j,l)$ along x-direction [Eq. (2.112)],

$\tilde{V}_{x,int,m;i+\frac{1}{4},j,l}$ = upwind-differenced value of x-component of velocity associated with the intake flow of power plant m in the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$\bar{V}_{x,dis,m;i+\frac{1}{4},j,l}$ = x-component of velocity associated with the discharge flow of power plant m in the x-downstream half-element $(i+\frac{1}{4},j,l)$.

The first control-volume integral on the right-hand side of Eq. (3.56), associated with the gravity force term along x-direction, is already evaluated for the discrete-element formulation of the temporally primary-scale and spatially primary-scale, integral form of the conservation of mass principle for water [Eq. (3.19)].

The control-volume integrals on the right-hand side of Eq. (3.56), associated with the Coriolis acceleration (force) terms are also formulated according to (1) the definition of the general control-volume integral for the control-volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eqs. (2.84a),(2.84b) and (2.86b)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)], as

$$\iiint_{\bar{V}_{i+\frac{1}{4},j,l}} \bar{V}_z dv = [A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]\bar{V}_{z;i+\frac{1}{4},j,l}, \tag{3.61}$$

$$\iiint_{\bar{V}_{i+\frac{1}{4},j,l}} \bar{V}_y dv = [A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]\bar{V}_{y;i+\frac{1}{4},j,l}. \tag{3.62}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\bar{V}_{z;i-\frac{1}{4},j,l}$ = z-component of velocity along the vertical direction (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region) in the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.106)],

$\bar{V}_{y;i-\frac{1}{4},j,l}$ = y-component of velocity [in the horizontal (x,y,) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) in the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (2.106)].

The discrete-element formulation of the closed control-surface integral associated with the pressure force term on the right-hand side of Eq. (3.56) also requires careful considerations of the geometrical

properties of the elements, particularly the boundary elements which represent nonrectangular (truncated rectangular) cells with enclosure-surface areas not necessarily normal to any of the three principle orthogonal axes x, y, or z (Figs. 2.16 through 2.24, p.106 through 2.114; Fig. 2.51, p.143; Fig. 2.53, p.145). It is important to note that on these boundary enclosure-surface areas, the pressure forces result in components along x-direction depending on the angle (in the general sense) between the unit normal vector to the boundary-enclosure-surface and the x-axis. The geometrical effects of the boundary enclosure-surface areas in the horizontal (x,y) plane (in the general sense, parallel to the geopysical surface plane and the free-surface plane of the flow region) can be incorporated by considering the area $\overline{A}_{x;i+\frac{1}{4},j,l}$, normal to the x-direction, at the center of the x-downstream half-element $(i+\frac{1}{4},j,l)$ (e.g., Shames, 1962, Eraslan, 1975, 1977,1981a), for calculating the net pressure force acting on the x-downstream half-element. However, the variation of the pressure, particularly the hydrostatic pressure part, along the z-direction must be carefully considered in the formulation of the pressure force term acting along x-direction on the x-downstream half-element $(i+\frac{1}{4},j,l)$. The important consequences of the variation of pressure along z-direction will be considered in detail in the discussions of the interpolation formulas for the pressure conditions on the enclosure-surface areas of the half-elements (Sec. 4.3).

Hence, the control-surface integral associated with the pressure force term, on the right-hand side of Eq. (3.56) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the x-downstream half-element $(i+\frac{1}{4},j,l)$

[Eq. (2.88)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface and downstream enclosure-surface areas of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eqs. (2.90b), (2.90c), (2.90f), (2.90g) and (2.91c)], and by considering the components of the presure forces along the x-direction, on all enclosure-surface areas of the x-downstream half-element $(i+\frac{1}{4},j,l)$, including the ones which are parts of the nonrectangular boundary elements that represent truncated rectangular cells (Sec. 2, Figs. 2.16 through 2.24, p.106 through 2.114; Fig. 2.51, p.143; Fig. 2.53, p.145), as

$$\iint_{\bar{A}_{i+\frac{1}{4},j,l}} \overline{[p_w(-\hat{n})]_x} dA = \bar{A}_{x;i+\frac{1}{4},j,l}(\bar{p}_{w;i,j,l} - \bar{p}_{w;i+\frac{1}{2},j,l}), \tag{3.63}$$

where the area $Ax;i+\frac{1}{4},j,l$, normal to the x-direction, at the center of the x-downstream half-element $(i+\frac{1}{4},j,l)$ [in Eq. (3.63)] is defined, as

$$\bar{A}_{x;i+\frac{1}{4},j,l} = \Delta y_{i+\frac{1}{4},j} \overline{\Delta z}_{i+\frac{1}{4},j,l}. \tag{3.64}$$

and the temporally primary-scale and spatially primary-scale values of the modified pressure are defined, as

$\bar{p}_{w;i-\frac{1}{2},j,l}$ = modified pressure at the location of the x-upstream enclosure-surface at (i,j,l) of the x-downstream half-element $(i+\frac{1}{4},j,l)$,

$\bar{p}_{w;i,j,l}$ = modified pressure at the location of the x-downstream enclosure-surface at $(i+\frac{1}{2},j,l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$.

The discrete-element formulation of the control-surface integral, associated with the pressure force term for the x-downstream

half-element $(i+\frac{1}{4},j,1)$ [Eq. (3.63)] is applicable to all x-downstream half-elements, including (1) bottom x-downstream half-element $[i+\frac{1}{4},j,1(bot;i,j)]$, (2) vertically interior x-downstream half-elements $[i+\frac{1}{4},j,1(bot;i,j) \leq 1 < L]$, and (3) x-downstream half-elements in the water-surface layer $(i+\frac{1}{4},j,L)$. Furthermore, according to the discrete-element formulation of the control-surface integral [Eq. (3.63)], the pressure force term acting on the x-downstream half-element $(i+\frac{1}{4},j,1)$ is completely independent of the value of the spatially uniform atmospheric pressure on the free-surface of the water in the flow region, since it includes only the difference of the two pressure values at the upstream enclosure-surface at $(i,j,1)$ and at the downstream enclosure-surface at $(i+\frac{1}{2},j,1)$.

The control-surface integral associated with the viscous (molecular) force term, on the right-hand side of Eq. (3.56) can be evaluated according to (1) the definitions of the general closed control-surface integral for the control-volume of x-downstream half-element $(i+\frac{1}{4},j,1)$ [Eq. (2.88)] and (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure surface areas of the x-downstream half-element $(i+\frac{1}{4},j,1)$ [Eqs. (2.90b), (2.90c), (2.90f), (2.90g) and (2.91c)], as

$$\iint_{\bar{A}_{i+\frac{1}{4},j,1}} \overline{\sigma_{\nu,sx}} dA = -\bar{A}_{x;i,j,1}\bar{\sigma}_{\nu,xx;i,j,1} + \bar{A}_{x;i+\frac{1}{2},j,1}\bar{\sigma}_{\nu,xx;i+\frac{1}{2},j,1}$$

$$-\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},1}\bar{\sigma}_{\nu,yx;i+\frac{1}{4},j-\frac{1}{2},1} + \bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},1}\bar{\sigma}_{\nu,yx;i+\frac{1}{4},j+\frac{1}{2},1}$$

$$-A_{z;i+\frac{1}{4},j}\bar{\sigma}_{\nu,zx;i+\frac{1}{4},j,1-\frac{1}{2}} + A_{z;i+\frac{1}{4},j}\bar{\sigma}_{\nu,zx;i+\frac{1}{4},j,1+\frac{1}{2}} , \qquad (3.65)$$

where the temporally primary-scale and spatially primary-scale viscous stress components are defined, as

$\overline{\sigma}_{\nu,xx;i,j,l}$ = modified viscous stress component acting on the x-upstream enclosure-surface area $\overline{A}_{x;i,j,l}$ at (i,j,l) of the x-downstream half-element (i+¼,j,l), along x-direction,

$\overline{\sigma}_{\nu,xx;i+\frac{1}{2},j,l}$ = modified viscous stress component acting on the x-downstream enclosure-surface area $\overline{A}_{x;i+\frac{1}{2},j,l}$ at (i+½,j,l) of the x-downstream half-element (i+¼,j,l), along x-direction,

$\overline{\sigma}_{\nu,yx;i+\frac{1}{4},j-\frac{1}{2},l}$ = modified viscous stress component acting on the y-upstream enclosure-surface area $\overline{A}_{y;i+\frac{1}{4},j-\frac{1}{2},l}$ at (i+¼,j-½,l) of the x-downstream half-element (i+¼,j,l), along x-direction,

$\overline{\sigma}_{\nu,yx;i+\frac{1}{4},j+\frac{1}{2},l}$ = modified viscous stress component acting on the y-downstream enclosure-surface area $\overline{A}_{y;i+\frac{1}{4},j+\frac{1}{2},l}$ at (i+¼,j+½,l) of the x-downstream half-element (i+¼,j,l) along x-direction,

$\overline{\sigma}_{\nu,zx;i+\frac{1}{4},j,l-\frac{1}{2}}$ = modified viscous stress component acting on the z-upstream enclosure-surface area $A_{z;i+\frac{1}{4},j}$ at (i+¼,j,l-½) of the x-downstream half-element (i+¼,j,l), along x-direction,

$\overline{\sigma}_{\nu,zx;i+\frac{1}{4},j,l+\frac{1}{2}}$ = modified viscous stress component acting on the z-downstream enclosure-surface area $A_{z;i+\frac{1}{4},j}$ at (i+¼,j,l+½) of the x-downstream half-element (i+¼,j,l), along x-direction.

Hence, substituting the discrete-element formulations of (1) the time derivative of the control-volume integral [Eq. (3.59)] of the left-hand side of Eq. (3.56), (2) the control-surface inteqral [Eq. (3.60)] of the left-hand side of Eq. (3.56), (3) the control-volume integral associated with the gravity force term along x-direction [Eq. (3.19)] of the right-hand side of Eq. (3.56), (4) the two control-volume integrals associated with the Coriolis acceleration (force) term [Eqs. (3.61)] and (3.62) of the right-hand side of [Eq. (3.56)], (5) the control-surface integral associated with the modified pressure force term [Eq. (3.63)] of the right-hand side of Eq. (3.56), and (6) the control-surface integral associated with the modified viscous force term [Eq. (3.64)] of the right-hand side of Eq. (3.56), in the temporally primary-scale and spatially primary-scale x-component of the integral form of the momentum principle for the control-volume of the x-downstream half-element $(i+\frac{1}{4},j,1)$ [Eq. (3.56)], combining terms and rearranging, the discrete-element form of the momentum principle for the x-downstream half-element $(i+\frac{1}{4},j,1)$ becomes,

$$\frac{\partial}{\partial t}\overline{V}_{x;i+\frac{1}{4},j,1} = \frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,1}]}\Big[\widetilde{G}_{x;i,j,1}(\widetilde{V}_{x;i,j,1}-\overline{V}_{x;i+\frac{1}{4},j,1})$$

$$-\widetilde{G}_{x;i+\frac{1}{2},j,1}(\widetilde{V}_{x;i+\frac{1}{2},j,1}-\overline{V}_{x;i+\frac{1}{4},j,1})$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},1}(\widetilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},1}-\overline{V}_{x;i+\frac{1}{4},j,1})$$

$$-\widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},1}(\widetilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},1}-\overline{V}_{x;i+\frac{1}{4},j,1})$$

(continue)

$$+\tilde{G}_{z;i+\frac{1}{4},j,l-\frac{1}{2}}(\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}}-\bar{V}_{x;i+\frac{1}{4},j,l})$$

$$-\tilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}(\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}-\bar{V}_{x;i+\frac{1}{4},j,l})]$$

$$+\frac{\bar{A}_{x;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}(\bar{p}_{w;i,j,l}-\bar{p}_{w;i+\frac{1}{2},j,l})$$

$$-g_o\cos\theta_{n,x}+\Omega_{cor,N,y}\bar{V}_{z;i+\frac{1}{4},j,l}-\Omega_{cor,N,z}\bar{V}_{y;i+\frac{1}{4},j,l}$$

$$+\frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}[-\bar{A}_{x;i,j,l}\bar{\sigma}^{*'}_{\nu,x,x;i,j,l}+\bar{A}_{x;i+\frac{1}{2},j,l}\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,l}$$

$$-\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$$

$$+\bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$$

$$-A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l-\frac{1}{2}}$$

$$+A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l+\frac{1}{2}}]$$

$$+\frac{\delta_{int,m;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}\tilde{G}_{int,m;i+\frac{1}{4},j,l}(\tilde{V}_{x,int,m;i+\frac{1}{4},j,l}-\bar{V}_{x;i+\frac{1}{4},j,l})$$

$$+\frac{\delta_{dis,m;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}\bar{G}_{dis,m;i+\frac{1}{4},j,l}(\bar{V}_{x,dis,m;i+\frac{1}{4},j,l}-\bar{V}_{x;i+\frac{1}{4},j,l}). \tag{3.66}$$

where the temporally primary-scale and spatially primary-scale stress components are defined, as

$\bar{\sigma}^{*'}_{\nu,x,x;i,j,l}$ = modified stress component (consisting of viscous stress component, and combined convection-defect and eddy-

turbulent stress component) acting on the x-upstream enclosure-surface area $\bar{A}_{x;i,j,l}$ at (i,j,l) of the x-downstream half-element (i+¼,j,l), along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,l} = \bar{\sigma}_{\nu,xx;i+\frac{1}{2},j,l} + \bar{\sigma}^{*'}_{x,x;i+\frac{1}{2},j,l}, \tag{3.67a}$$

$\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,l}$ = modified stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j,l}$ at (i+½,j,l) of the x-downstream half-element (i+¼,j,l), along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,x;i,j,l} = \bar{\sigma}_{\nu,xx;i,j,l} + \bar{\sigma}^{*'}_{x,x;i,j,l}, \tag{3.67b}$$

$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$ = modified stress component acting on the y-upstream enclosure-surface area $\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},l}$ at (i+¼,j-½,l) of the x-downstream half-element (i+¼,j,l), along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l} = \bar{\sigma}_{\nu,yx;i+\frac{1}{4},j-\frac{1}{2},l} + \bar{\sigma}^{*'}_{y,x;i+\frac{1}{4},j-\frac{1}{2},l}, \tag{3.67c}$$

$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j+\frac{1}{2},l}$ = modified stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},l}$ at (i+¼,j+½,l) of the x-downstream half-element (i+¼,j,l), along x-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j+\frac{1}{2},l} = \bar{\sigma}_{\nu,yx;i+\frac{1}{4},j+\frac{1}{2},l} + \bar{\sigma}^{*'}_{y,x;i+\frac{1}{4},j+\frac{1}{2},l}, \tag{3.67d}$$

$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l-\frac{1}{2}}$ = modified stress component acting on the z-upstream enclosure-surface area $A_{z;i+\frac{1}{4},j}$ at $(i+\frac{1}{4},j,l-\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, along x-direction, which represents

$$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l-\frac{1}{2}} = \bar{\sigma}_{\nu,zx;i+\frac{1}{4},j,l-\frac{1}{2}} + \bar{\sigma}^{*'}_{z,x;i+\frac{1}{4},j,l-\frac{1}{2}}, \qquad (3.67e)$$

$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l+\frac{1}{2}}$ = modified stress component acting on the z-downstream enclosure-surface area $A_{z;i+\frac{1}{4},j}$ at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, along x-direction, which represents

$$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l+\frac{1}{2}} = \bar{\sigma}_{\nu,zx;i+\frac{1}{4},j,l+\frac{1}{2}} + \bar{\sigma}^{*'}_{z,x;i+\frac{1}{4},j,l+\frac{1}{2}}. \qquad (3.67f)$$

This discrete-element form of the temporally primary-scale and spatially primary-scale integral form of the momentum principle for the x-downstream half-element $(i+\frac{1}{4},j,l)$ [Eq. (3.66)] represents the second "discrete-element equation for x-component of velocity".

For a bottom x-downstream half-element $[i+\frac{1}{4},j,l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure-surface at $[i+\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$, the discrete-element equation for x-component of velocity [Eq. (3.66)] becomes

for $l = l(bot;i,j)$:

$$\frac{\partial}{\partial t}\bar{V}_{x;i+\frac{1}{4},j,l} = \frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}[\tilde{G}_{x;i,j,l}(\tilde{V}_{x;i,j,l} - \bar{V}_{x;i+\frac{1}{4},j,l})$$

$$-\tilde{G}_{x;i+\frac{1}{2},j,l}(\tilde{V}_{x;i+\frac{1}{2},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l})$$

(continue)

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l}(\widetilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l}-\overline{V}_{x;i+\frac{1}{4},j,l})$$

$$-\widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l}(\widetilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l}-\overline{V}_{x;i+\frac{1}{4},j,l})$$

$$+\widetilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}(\widetilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}-\overline{V}_{x;i+\frac{1}{4},j,l})]$$

$$+\frac{\overline{A}_{x;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}(\overline{p}_{w;i,j,l}-\overline{p}_{w;i+\frac{1}{2},j,l})$$

$$-g_a\cos\theta_{n,x}+\Omega_{cor,N,y}\overline{V}_{z;i+\frac{1}{4},j,l}-\Omega_{cor,N,z}\overline{V}_{y;i+\frac{1}{4},j,l}$$

$$+\frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}[-\overline{A}_{x;i,j,l}\overline{\sigma}^{*'}_{\nu,x,x;i,j,l}+\overline{A}_{x;i+\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,l}$$

$$-\overline{A}_{y;i+\frac{1}{4},j-\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$$

$$+\overline{A}_{y;i+\frac{1}{4},j+\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$$

$$+A_{z;i+\frac{1}{4},j}\overline{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l+\frac{1}{2}}]$$

$$+\frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}[\widetilde{G}_{z,bot;i+\frac{1}{4},j}(\widetilde{V}_{x,bot;i+\frac{1}{4},j}-\overline{V}_{x;i+\frac{1}{4},j,l})$$

$$-A_{z;i+\frac{1}{4},j}\overline{\sigma}^{*'}_{\nu,x,bot;i+\frac{1}{4},j}]$$

$$+\frac{\delta_{int,m;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}\widetilde{G}_{int,m;i+\frac{1}{4},j,l}(\widetilde{V}_{x,int,m;i+\frac{1}{4},j,l}-\overline{V}_{x;i+\frac{1}{4},j,l})$$

$$+\frac{\delta_{dis,m;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}\widetilde{G}_{dis,m;i+\frac{1}{4},j,l}(\widetilde{V}_{x,dis,m;i+\frac{1}{4},j,l}-\overline{V}_{x;i+\frac{1}{4},j,l})\,. \tag{3.68}$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the bottom flow conditions, are defined, as

$\widetilde{V}_{x,bot;i+\frac{1}{4},j}$ = upwind-differenced value of x-component of velocity associated with the bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i+\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$ of the x-downstream half-element $[i+\frac{1}{4},j,l(bot;i,j)]$,

$\overline{\sigma}^{*'}_{\nu,x,bot;i+\frac{1}{4},j}$ = modified bottom shear stress component acting on the z-upstream bottom enclosure-surface area $A_{z;i+\frac{1}{4},j}$ at $[i+\frac{1}{4},j,l(bot;i,j)-\frac{1}{2}]$ of the x-downstream half-element $[i+\frac{1}{4},j,l(bot;i,j)]$, along x-direction.

For a x-downstream half-element in the water-surface layer $(i+\frac{1}{4},j,L)$, which contains the free-surface layer of the water in the flow region, the discrete-element equation for x-component of velocity [Eq. (3.66)] becomes

for l = L:

$$\frac{\partial}{\partial t}\overline{V}_{x;i+\frac{1}{4},j,L} = \frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}[\widetilde{G}_{x;i,j,L}(\widetilde{V}_{x;i,j,L}-\overline{V}_{x;i+\frac{1}{4},j,L})$$

$$-\widetilde{G}_{x;i+\frac{1}{2},j,L}(\widetilde{V}_{x;i+\frac{1}{2},j,L}-\overline{V}_{x;i+\frac{1}{4},j,L})$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},L}(\widetilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},L}-\overline{V}_{x;i+\frac{1}{4},j,L})$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},L}(\widetilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},L}-\overline{V}_{x;i+\frac{1}{4},j,L})$$

(continue)

$$+\tilde{G}_{z;i+\frac{1}{4},j,L-\frac{1}{2}}(\tilde{V}_{x;i+\frac{1}{4},j,L-\frac{1}{2}}-\bar{V}_{x;i+\frac{1}{4},j,L})]$$

$$+\frac{\bar{A}_{x;i+\frac{1}{2},j,L}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}(\bar{p}_{w;i+\frac{1}{2},j,L}-\bar{p}_{w;i,j,L})$$

$$-g_o\cos\theta_{n,x}+\Omega_{cor,N,y}\bar{V}_{z;i+\frac{1}{4},j,1}-\Omega_{cor,N,z}\bar{V}_{y;i+\frac{1}{4},j,1}$$

$$+\frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}[-\bar{A}_{x;i,j,L}\bar{\sigma}^{*'}_{\nu,x,x;i,j,L}+\bar{A}_{x;i+\frac{1}{2},j,L}\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,L}$$

$$-\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},L}$$

$$+\bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},L}$$

$$-A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,L-\frac{1}{2}}]$$

$$+\frac{1}{\overline{\Delta z}_{i+\frac{1}{4},j,L}}[\bar{g}_{rnf}(\bar{V}_{x,rnf}-\bar{V}_{x;i+\frac{1}{4},j,L})+\bar{\sigma}^{*'}_{\nu,x,wnd;i+\frac{1}{4},j}]$$

$$+\frac{\delta_{int,m;i+\frac{1}{4},j,L}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}\tilde{G}_{int,m;i+\frac{1}{4},j,L}(\tilde{V}_{x,int,m;i+\frac{1}{4},j,L}-\bar{V}_{x;i+\frac{1}{4},j,L})$$

$$+\frac{\delta_{dis,m;i+\frac{1}{4},j,L}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}\tilde{G}_{dis,m;i+\frac{1}{4},j,L}(\tilde{V}_{x,dis,m;i+\frac{1}{4},j,L}-\bar{V}_{x;i+\frac{1}{4},j,L}). \tag{3.69}$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the surface flow conditions, are defined, as

$\bar{V}_{x,rnf}$ = x-component of velocity associated with the rainfall on the water surface of the flow region.

$\bar{\sigma}^{*'}_{\nu,x,wnd;i+\frac{1}{4},j}$ = modified wind-shear stress component acting on the

z-downstream water enclosure-surface at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, along x-direction.

It is important to note that the wind-shear stress component, along x-direction, depends on the free-surface flow conditions on the water enclosure-surface at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$ as well as the wind conditions over the water surface of the flow region.

In certain subregions, the specified depth $D_{I,J}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer $\overline{\Delta z}_L$ (Sec. 2.3). In these single layer subregions, a x-downstream half-element $(i+\frac{1}{4},j,l)$ can represent a bottom x-downstream half-element $[i+\frac{1}{4},j,l(bot;i,j)]$ and also a x-downstream half-element in the water-surface layer $(i+\frac{1}{4},j,L)$. For these special x-downstream half-elements $[i+\frac{1}{4},j,l(bot;i,j)]$ and also $(i+\frac{1}{4},j,l)$, the discrete-element equation for x-componen velocity [Eq. (3.66)] becomes for $l(bot;i,j)=l=L$:

$$\frac{\partial}{\partial t}\overline{V}_{x;i+\frac{1}{4},j,L} = \frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}\Big[\widetilde{G}_{x;i,j,L}(\widetilde{V}_{x;i,j,L}-\overline{V}_{x;i+\frac{1}{4},j,L})$$

$$-\widetilde{G}_{x;i+\frac{1}{2},j,L}(\widetilde{V}_{x;i+\frac{1}{2},j,L}-\overline{V}_{x;i+\frac{1}{4},j,L})$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},L}(\widetilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},L}-\overline{V}_{x;i+\frac{1}{4},j,L})$$

$$+\widetilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},L}(\widetilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},L}-\overline{V}_{x;i+\frac{1}{4},j,L})\Big]$$

(continue)

$$+\frac{\bar{A}_{x;i+\frac{1}{4},j,L}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}(\bar{p}_{w;i,j,L}-\bar{p}_{w;i+\frac{1}{2},j,L})$$

$$-g_o\cos\Theta_{n,x}+\Omega_{cor,N,y}\bar{V}_{z;i+\frac{1}{4},j,L}-\Omega_{cor,N,z}\bar{V}_{y;i+\frac{1}{4},j,L}$$

$$+\frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}[-\bar{A}_{x;i,j,L}\bar{\sigma}^{*'}_{\nu,x,x;i,j,L}+\bar{A}_{x;i+\frac{1}{2},j,L}\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,L}$$

$$-\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},L}$$

$$+\bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},L}]$$

$$+\frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}[\tilde{G}_{z,bot;i+\frac{1}{4},j}(\tilde{V}_{x,bot;i+\frac{1}{4},j}-\bar{V}_{x;i+\frac{1}{4},j,l})$$

$$-A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,x,bot;i+\frac{1}{4},j}]$$

$$+\frac{1}{\overline{\Delta z}_{i+\frac{1}{4},j,L}}[\bar{g}_{rnf}(\bar{V}_{rnf}-\bar{V}_{x;i+\frac{1}{4},j,L})+\bar{\sigma}^{*'}_{\nu,x,wnd;i+\frac{1}{4},j}]$$

$$+\frac{\delta_{int,m;i+\frac{1}{4},j,L}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}\tilde{G}_{int,m;i+\frac{1}{4},j,L}(\tilde{V}_{x,int,m;i+\frac{1}{4},j,L}-\bar{V}_{x;i+\frac{1}{4},j,L})$$

$$+\frac{\delta_{dis,m;i+\frac{1}{4},j,L}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,L}]}\bar{G}_{dis,m;i+\frac{1}{4},j,L}(\bar{V}_{x,dis,m;i+\frac{1}{4},j,L}-\bar{V}_{x;i+\frac{1}{4},j,L}).$$

(3.70)

Similarily, the temporally primary-scale and spatially primary-scale y-component of the integral form of the momentum principle [Eq. (102)]can be formulated for the control-volume of the y-upstream half-element(i,j-¼,l) as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j-\frac{1}{4},l}}\overline{V_y}dv + \iint_{\bar{A}_{i,j-\frac{1}{4},l}}\overline{[V_y]\vec{V}\cdot\hat{n}}dA = -g_a\cos\Theta_{n,y}\iiint_{\bar{V}_{i,j-\frac{1}{4},l}}dv$$

$$+\Omega_{cor,N,z}\iiint_{\bar{V}_{i,j-\frac{1}{4},l}}\overline{V_x}dv - \Omega_{cor,N,x}\iiint_{\bar{V}_{i,j-\frac{1}{4},l}}\overline{V_z}dv$$

$$+\iint_{\bar{A}_{i,j-\frac{1}{4},l}}\overline{[p_w(-\hat{n})]_y}dA + \iint_{\bar{A}_{i,j-\frac{1}{4},l}}\overline{\sigma_{\nu,sy}}dA\,. \qquad (3.71)$$

The control-volume integral on the left-hand side of Eq. (3.71) can be formulated according to (1) the definition of the general control-volume integral for the control volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eqs. (2.85a), (2.85b) and (2.86c)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)] as

$$\iiint_{\bar{V}_{i,j-\frac{1}{4},l}}\overline{V_y}dv = [A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]\overline{V}_{y;i,j-\frac{1}{4},l}\,, \qquad (3.72)$$

where the temporally primary-scale and spatially primary-scale flow variable is defined, as

$\overline{V}_{y;i,j-\frac{1}{4},l}$ = y-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] in the y-upstream half-element $(i,j-\frac{1}{4},l)$.

The time derivative of the control-volume integral [Eq. (3.72)] on the left-hand side of Eq. (3.71), can be expanded, as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j-\frac{1}{4},l}} \bar{V}_y dv = \frac{\partial}{\partial t}\left\{ [A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]\bar{V}_{y;i,j-\frac{1}{4},l} \right\}$$

$$= [A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]\frac{\partial}{\partial t}\bar{V}_{y;i,j-\frac{1}{4},l} + \bar{V}_{y;i,j-\frac{1}{4},l}\frac{\partial}{\partial t}[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l} . \quad (3.73)$$

Hence, multiplying the auxiliary discrete-element form of the mass conservation principle for water in the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (3.28)] with the y-component of velocity $\bar{V}_{y;i,j-\frac{1}{4},l}$ in the y-upstream half-element $(i,j-\frac{1}{4},l)$ and substituting in Eq. (3.73), the time derivative of the control-volume integral [Eq. (3.73)] on the left-hand side of Eq. (3.71) becomes

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j-\frac{1}{4},l}} \bar{V}_y dv = [A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]\frac{\partial}{\partial t}\bar{V}_{y;i,j-\frac{1}{4},l}$$

$$+ \tilde{G}_{y;i,j-\frac{1}{2},l}\bar{V}_{y;i,j-\frac{1}{4},l} - \tilde{G}_{y;i,j,l}\bar{V}_{y;i,j-\frac{1}{4},l}$$

$$+ \tilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}\bar{V}_{y;i,j-\frac{1}{4},l} - \tilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},l}\bar{V}_{y;i,j-\frac{1}{4},l}$$

$$+ \tilde{G}_{z;i,j-\frac{1}{4},l-\frac{1}{2}}\bar{V}_{y;i,j-\frac{1}{4},l} - \tilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}\bar{V}_{y;i,j-\frac{1}{4},l}$$

$$+ \delta_{int,m;i,j-\frac{1}{4},l}\tilde{G}_{int,m;i,j-\frac{1}{4},l}\bar{V}_{y;i,j-\frac{1}{4},l}$$

$$+ \delta_{dis,m;i,j-\frac{1}{4},l}\bar{G}_{dis,m;i,j-\frac{1}{4},l}\bar{V}_{y;i,j-\frac{1}{4},l} . \quad (3.74)$$

The closed control-surface integral on the left-hand side of Eq. (3.71) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.89)], (2) the definitions of the enclo-

sure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the y-upstream half element $(i,j-\frac{1}{4},l)$ [Eqs. (2.90h), (2.90i), (2.90k), (2.90l) and (2.91d)], (3) the formulation of the general representation of the six enclosure-surface integral which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, y-component of the integral form of the momentum principle [Eq. (2.113)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale, y-component of the integral form of the momentum principle [Eq. (2.125)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale, y-component of the integral form of the momentum principle [Eq. (2.126)], and by considering the general definitions of the upwind-differencing interpolation forms for the transportive, intensive flow variables, as

$$\iint_{\overline{A}_{i,j-\frac{1}{4},l}} \overline{[V_y]\vec{V}\cdot\hat{n}}\,dA = -\widetilde{G}_{y;i,j-\frac{1}{2},l}\widetilde{V}_{y;i,j-\frac{1}{2},l} + \overline{A}_{y;i,j-\frac{1}{2},l}\overline{\sigma}^{*'}_{y,y;i,j-\frac{1}{2},l}$$

$$+\widetilde{G}_{y;i,j,l}\widetilde{V}_{y;i,j,l} - \overline{A}_{y;i,j,l}\overline{\sigma}^{*'}_{y,y;i,j,l}$$

$$-\widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}\widetilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} + \overline{A}_{x;i-\frac{1}{2},j-\frac{1}{4},l}\overline{\sigma}^{*'}_{x,y;i-\frac{1}{2},j-\frac{1}{4},l}$$

(continue)

$$+\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},l}\widetilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l}-\overline{A}_{x;i+\frac{1}{2},j-\frac{1}{4},l}\overline{\sigma}^{*'}_{x,y;i+\frac{1}{2},j-\frac{1}{4},l}$$

$$-\widetilde{G}_{z;i,j-\frac{1}{4},l-\frac{1}{2}}\widetilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}}+A_{z;i,j-\frac{1}{4}}\overline{\sigma}^{*'}_{z,y;i,j-\frac{1}{4},l-\frac{1}{2}}$$

$$+\widetilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}\widetilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}-A_{z;i,j-\frac{1}{4}}\overline{\sigma}^{*'}_{z,y;i,j-\frac{1}{4},l+\frac{1}{2}}$$

$$-\delta_{int,m;i,j-\frac{1}{4},l}\widetilde{G}_{int,m;i,j-\frac{1}{4},l}\widetilde{V}_{y,int,m;i,j-\frac{1}{4},l}$$

$$-\delta_{dis,m;l,j-\frac{1}{4},l}\widetilde{G}_{dis,m;l,j-\frac{1}{4},l}\overline{V}_{y,dis,m;i,j-\frac{1}{4},l}, \qquad (3.75)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\widetilde{V}_{y;i,j-\frac{1}{2},l}$ = upwind-differenced value of y-component of velocity in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) at y-upstream enclosure-surface at $(i,j-\frac{1}{2},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.113)],

$\overline{\sigma}^{*'}_{y,y;i,j-\frac{1}{2},l}$ = combined convention-defect and eddy-turbulent stress component (excluding only the component of modified viscous stress) acting on the y-upstream enclosure-surface area $\overline{A}_{y;i,j-\frac{1}{2},l}$ at $(i,j-\frac{1}{2},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\widetilde{V}_{y;i,j,l}$ = upwind-differenced value of y-component of velocity at y-downstream enclosure surface at (i,j,l) of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.113)],

$\bar{\sigma}^{*'}_{y,y;i,j,l}$ = combined convection-defect and eddy-turbulent stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i,j,l}$ at (i,j,l) of the y-upstream half-element (i,j-¼,l), along y-direction [Eq. (2.116)] ,

$\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l}$ = upwind-differenced value of y-component of velocity at x-upstream enclosure surface at (i-½,j-¼,l) of the y-upstream half-element (i,j-¼,l) [Eq. (2.113)]

$\bar{\sigma}^{*'}_{x,y;i-\frac{1}{2},j-\frac{1}{4},l}$ = combined convection-defect and eddy-turbulent stress component acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j-\frac{1}{4},l}$ at (i-½,j-¼,l) of the y-upstream half-element (i,j-¼,l), along y-direction [Eq. (2.116)],

$\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l}$ = upwind-differenced value of y-component of velocity at x-downstream enclosure-surface at (i+½,j-¼,l) of the y-upstream half-element (i,j-¼,l) [Eq. (2.113)],

$\bar{\sigma}^{*'}_{x,y;i+\frac{1}{2},j-\frac{1}{4},l}$ = combined convection-defect and eddy-turbulent stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j-\frac{1}{4},l}$ at (i+½,j-¼,l) of the y-upstream half-element (i,j-¼,l), along y-direction [Eq. (2.116)] ,

$\tilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}}$ = upwind-differenced value of y-component of velocity at z-upstream enclosure-surface at (i,j-¼,l-½) of the y-upstream half-element (i,j-¼,l) [Eq. (2.113)],

$\bar{\sigma}^{*'}_{z,y;i,j-\frac{1}{4},l-\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-upstream enclosure-surface

area $A_{z;i,j-\frac{1}{4}}$ at $(i,j-\frac{1}{4},l-\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}$ = upwind-differenced value of y-component of velocity at z-downstream enclosure-surface at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.113)],

$\bar{\sigma}^{*'}_{z,y;i,j-\frac{1}{4},l+\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-downstream enclosure-surface area $A_{z;i,j-\frac{1}{4}}$ at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\tilde{V}_{y,int,m;i,j-\frac{1}{4},l}$ = upwind-differenced value of y-component of velocity associated with the intake flow of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$\bar{V}_{y,dis,m;i,j-\frac{1}{4},l}$ = y-component of velocity associated with the discharge flow of power plant m in the y-upstream half-element $(i,j-\frac{1}{4},l)$.

The first control-volume integral on the left side of Eq. (3.71), associated with the gravity force term along y-direction, is already evaluated for the discrete-element formulation of the temporally primary-scale and spatially primary-scale integral form of the conservation of mass principle for water [Eq. (3.26)].

The control-volume integrals on the left-hand side of Eq. (3.71), associated with the Coriolis acceleration (force) terms are also formulated according to (1) the definition of the general control-volume integral for the control-volume of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eqs. (2.85a), (2.85b) and (2.86c)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially

primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)] as

$$\iiint_{\bar{V}_{i,j-\frac{1}{4},l}} \bar{V}_z dv = [A_{z;i,j-\frac{1}{4}} \overline{\Delta z}_{i,j-\frac{1}{4},l}] \bar{V}_{z;i,j-\frac{1}{4},l}, \tag{3.76}$$

$$\iiint_{\bar{V}_{i,j-\frac{1}{4},l}} \bar{V}_x dv = [A_{z;i-\frac{1}{4},j} \overline{\Delta z}_{i,j-\frac{1}{4},l}] \bar{V}_{x;i,j-\frac{1}{4},l}. \tag{3.77}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\bar{V}_{z;i,j-\frac{1}{4},l}$ = z-component of velocity along the vertical direction (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region) in the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.106)],

$\bar{V}_{x;i,j-\frac{1}{4},l}$ = x-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] in the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (2.106)].

The discrete-element formulation of the closed control-surface integral associated with the pressure force term on the right-hand side of Eq. (3.71) requires caretul considerations of the geometrical properties of the elements, particularly the boundary elements which represent nonrectangular (truncated rectangular) cells with enclosure-surface areas not necessarily normal to any of the three principal orthogonal axes x, y, or z (Figs. 2.34 through 2.42, p.124 through p.132; Fig. 2.52, p.144; Fig. 2.54, p.146). It is important to note that on these boundary enclosure-surface areas, the modified pressure forces result in components along y-direction depending on the angle (in the general

sense) between the unit normal vector to the boundary enclosure-surface and the y-axis. The geometrical effects of the boundary enclosure-surface areas in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) can be incorporated by considering the area $\bar{A}_{y;i,j-\frac{1}{4},l}$, normal to the y-direction, at the center of the y-upstream half-element (i,j-¼,l) (e.g., Shames, 1962, Eraslan, 1975, 1977,1981a), for calculating the net modified pressure force acting on the y-upstream half-element. However, the variation of the modified pressure, particularly the hydrostatic modified pressure part, along the z-direction must be carefully considered in the formulation of the modified pressure force term acting along y-direction on the y-upstream half-element (i,j-¼,l). The important consequences of the variation of modified pressure along z-direction will be considered in detail in the discussions of the interpolation formulas for the modified pressure conditions on the enclosure-surface areas of the half-elements (Sec. 4.3).

Hence, the control-surface integral associated with the pressure force term on the right-hand side of Eq. (3.71) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the y-upstream half-element (i,j-¼,l) [Eq. (2.89)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and downstream enclosure-surface areas of the y-upstream half-element (i,j-¼,l) [Eqs. (2.90h), (2.90i), (2.90k), (2.90l) and (2.91d)], and by considering the components of the presure forces along the y-direction, on all enclosure-surface areas of the y-upstream half-element (i,j-¼,l),

including the ones which are parts of the nonrectangular boundary elements that represent truncated rectangular cells (Sec. 2, Fig. 2.34 through 2.42, p.124 through p.132; Fig. 2.52, p.144; Fig. 2.54, p.146), as

$$\iint_{\bar{A}_{i,j-\frac{1}{4},l}} \overline{[p_w(-\hat{n})]_y}\,dA = \bar{A}_{y;i,j-\frac{1}{4},l}(\bar{p}_{w;i,j-\frac{1}{2},l} - \bar{p}_{w;i,j,l}). \qquad (3.78)$$

where the area $A_{y;i,j-\frac{1}{4},l}$, normal to the y-direction, at the center of the y-upstream half-element $(i,j-\frac{1}{4},l)$ [in Eq. (3.78)] is defined, as

$$\bar{A}_{y;i,j-\frac{1}{4},l} = \Delta x_{i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l} . \qquad (3.79)$$

and the temporally primary-scale and spatially primary-scale values of the modified pressure are defined, as

$\bar{p}_{w;i,j-\frac{1}{2},l}$ = modified pressure at the location of the y-upstream enclosure-surface at $(i,j-\frac{1}{2},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$,

$\bar{p}_{w;i,j,l}$ = modified pressure at the location of the y-downstream enclosure-surface at (i,j,l) of the y-upstream half-element $(i,j-\frac{1}{4},l)$.

The discrete-element formulation of the control-surface integral, associated with the pressure force term for the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (3.78)] is applicable to all y-upstream half-element, including (1) bottom y-upstream half-element $[i,j-\frac{1}{4},l(bot;i,j)]$, (2) vertically interior y-upstream half-elements $[i,j-\frac{1}{4},l(bot;i,j)\frac{1}{4}l\frac{1}{4}l]$, and (3) y-upstream half-elements in the water-surface layer $(i,j-\frac{1}{4},L)$. Furthermore, according to the discrete-element formulation of the control-

surface integral Eq. (3.78) , the pressure force term acting on the y-upstream half-element $(i,j-\frac{1}{4},1)$ is completely independent of the value of the spatially uniform atmospheric pressure on the free-surface of the water in the flow region, since it includes only the difference of the two pressure values at the upstream enclosure-surface at $(i,j-\frac{1}{2},1)$ and at the downstream enclosure-surface at $(i,j,1)$.

The surface integral associated with the modified viscous (molecular) force term, on the right-hand side of Eq. (3.71) can be evaluated according to (1) the definitions of the general closed control-surface integral for the control-volume of y-upstream half-element $(i,j-\frac{1}{4},1)$ [Eq. (2.89)] and (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the y-upstream half-element $(i,j-\frac{1}{4},1)$ [Eqs. (2.90h), (2.90i), (2.90k), (2.90l) and (2.91d)] , as

$$\iint_{\overline{A}_{i,j-\frac{1}{4},1}} \overline{\sigma_{\nu,sy}}\,dA = -\overline{A}_{y;i,j-\frac{1}{2},1}\overline{\sigma}_{\nu,yy;i,j-\frac{1}{2},1} + \overline{A}_{y;i,j,1}\overline{\sigma}_{\nu,yy;i,j,1}$$

$$-\overline{A}_{x;i-\frac{1}{2},j-\frac{1}{4},1}\overline{\sigma}_{\nu,xy;i-\frac{1}{2},j-\frac{1}{4},1} + \overline{A}_{x;i+\frac{1}{2},j-\frac{1}{4},1}\overline{\sigma}_{\nu,xy;i+\frac{1}{2},j-\frac{1}{4},1}$$

$$-A_{z;i,j-\frac{1}{4}}\overline{\sigma}_{\nu,zy;i,j-\frac{1}{4},1-\frac{1}{2}} + A_{z;i,j-\frac{1}{4}}\overline{\sigma}_{\nu,zy;i,j-\frac{1}{4},1+\frac{1}{2}} \,. \tag{3.80}$$

where the temporally primary-scale and spatially primary-scale modified viscous stress components are defined as

$\overline{\sigma}_{\nu,yy;i,j-\frac{1}{2},1}$ = modified viscous stress component acting on the y-upstream enclosure-surface area $\overline{A}_{y;i,j-\frac{1}{2},1}$ at $(i,j-\frac{1}{2},1)$ of the y-upstream half-element $(i,j-\frac{1}{4},1)$, along y-direction,

$\bar{\sigma}_{\nu,yy;i,j,l}$ = modified viscous stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i,j,l}$ at (i,j,l) of the y-upstream half-element (i,j-¼,l), along y-direction,

$\bar{\sigma}_{\nu,xy;i-\frac{1}{2},j-\frac{1}{4},l}$ = modified viscous stress component acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j-\frac{1}{4},l}$ at (i-½,j-¼,l) of the y-upstream half-element (i,j-¼,l), along y-direction,

$\bar{\sigma}_{\nu,xy;i+\frac{1}{2},j-\frac{1}{4},l}$ = modified viscous stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j-\frac{1}{4},l}$ at (i+½,j-¼,l) of the y-upstream half-element (i,j-¼,l), along y-direction,

$\bar{\sigma}_{\nu,zy;i,j-\frac{1}{4},l-\frac{1}{2}}$ = modified viscous stress component acting on the z-upstream enclosure-surface area $A_{z;i,j-\frac{1}{4}}$ at (i,j-¼,l-½) of the y-upstream half-element (i,j-¼,l), along y-direction,

$\bar{\sigma}_{\nu,zy;i,j-\frac{1}{4},l+\frac{1}{2}}$ = modified viscous stress component acting on the z-downstream enclosure-surface area $A_{z;i,j-\frac{1}{4}}$ at (i,j-¼,l+½) of the y-upstream half-element (i,j-¼,l), along y-direction.

Hence, substituting the discrete-element formulations of (1) the time derivative of the control-volume integral [Eq. (3.74)] of the left-hand side of Eq. (3.71), (2) the control-surface inteqral [Eq. (3.75)] of the left-hand side of Eq. (3.71), (3) the control-volume integral associated with the gravity force term along y-direction Eq. (3.26) of the right-hand side of Eq. (3.71), (4) the two control-volume integrals associated with the Coriolis acceleration (force) term

[Eqs. (3.76) and (3.77)] of the right-hand side of Eq. (3.71), (5) the control-surface integral associated with the modified pressure force term [Eq. (3.78)] of the right-hand side of Eq. (3.71), and (6) the control-surface integral associated with the modified viscous force term [Eq. (3.79)] of the right-hand side of Eq. (3.71), in the temporally primary-scale and spatially primary-scale y-component of the integral form of the momentum principle for the control-volume of the y-upstream half-element $(i,j-\frac{1}{4},1)$ [Eq. (3.71)], combining the terms and rearranging, the discrete-element form of the momentum principle for the y-upstream half-element $(i,j-\frac{1}{4},1)$ becomes,

$$\frac{\partial}{\partial t}\overline{V}_{y;i,j-\frac{1}{4},1} = \frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},1}]}\Big[\widetilde{G}_{y;i,j-\frac{1}{2},1}(\widetilde{V}_{y;i,j-\frac{1}{2},1}-\overline{V}_{y;i,j-\frac{1}{4},1})$$

$$-\widetilde{G}_{y;i,j,1}(\widetilde{V}_{y;i,j,1}-\overline{V}_{y;i,j-\frac{1}{4},1})$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},1}(\widetilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},1}-\overline{V}_{y;i,j-\frac{1}{4},1})$$

$$-\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},1}(\widetilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},1}-\overline{V}_{y;i,j-\frac{1}{4},1})$$

$$+\widetilde{G}_{z;i,j-\frac{1}{4},1-\frac{1}{2}}(\widetilde{V}_{y;i,j-\frac{1}{4},1-\frac{1}{2}}-\overline{V}_{y;i,j-\frac{1}{4},1})$$

$$-\widetilde{G}_{z;i,j-\frac{1}{4},1+\frac{1}{2}}(\widetilde{V}_{y;i,j-\frac{1}{4},1+\frac{1}{2}}-\overline{V}_{y;i,j-\frac{1}{4},1})$$

$$+\frac{\overline{A}_{y;i,j-\frac{1}{4},1}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},1}]}(\overline{p}_{w;i,j-\frac{1}{2},1}-\overline{p}_{w;i,j,1})$$

(continue)

$$-g_o\cos\theta_{n,y}+\Omega_{cor,N,z}\bar{V}_{x;i,j-\frac{1}{4},l}-\Omega_{cor,N,x}\bar{V}_{z;i,j-\frac{1}{4},l}$$

$$+\frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}[-\bar{A}_{y;i,j-\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,y;i,j-\frac{1}{2},l}+\bar{A}_{y;i,j,l}\bar{\sigma}^{*'}_{\nu,y,y;i,j,l}$$

$$-\bar{A}_{x;i-\frac{1}{2},j-\frac{1}{4},l}\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},l}$$

$$+\bar{A}_{x;i+\frac{1}{2},j-\frac{1}{4},l}\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},l}$$

$$-A_{z;i,j-\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l-\frac{1}{2}}$$

$$+A_{z;i,j-\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l+\frac{1}{2}}]$$

$$+\frac{\delta_{int,m;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\tilde{G}_{int,m;i,j-\frac{1}{4},l}(\tilde{V}_{y,int,m;i,j-\frac{1}{4},l}-\bar{V}_{y;i,j-\frac{1}{4},l})$$

$$+\frac{\delta_{dis,m;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\bar{G}_{dis,m;i,j-\frac{1}{4},l}(\bar{V}_{y,dis,m;i,j-\frac{1}{4},l}-\bar{V}_{y;i,j-\frac{1}{4},l}). \tag{3.81}$$

where the temporally primary-scale and spatially primary-scale stress components are defined as

$\bar{\sigma}^{*'}_{\nu,y,y;i,j-\frac{1}{2},l}$ = modified stress component (consisting of modified viscous stress component, and modified convection-defect and eddy-turbulent stress component) acting on the y-upstream enclosure-surface area $\bar{A}_{y;i,j-\frac{1}{2},l}$ at $(i,j-\frac{1}{2},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,y;i,j-\frac{1}{2},l}=\bar{\sigma}_{\nu,yy;i,j-\frac{1}{2},l}+\bar{\sigma}^{*'}_{y,y;i,j-\frac{1}{2},l}, \tag{3.82a}$$

$\bar{\sigma}^{*'}_{\nu,y,y;i,j,l}$ = modified stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i,j,l}$ at (i,j,l) of the y-upstream half-element (i,j-¼,l), along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,y;i,j,l} = \bar{\sigma}_{\nu,yy;i,j,l} + \bar{\sigma}^{*'}_{y,y;i,j,l}, \tag{3.82b}$$

$\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},l}$ = modified stress component acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j-\frac{1}{4},l}$ at (i-½,j-¼,l) of the y-upstream half-element (i,j-¼,l), along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},l} = \bar{\sigma}_{\nu,xy;i-\frac{1}{2},j-\frac{1}{4},l} + \bar{\sigma}^{*'}_{x,y;i-\frac{1}{2},j-\frac{1}{4},l}, \tag{3.82c}$$

$\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j-\frac{1}{4},l}$ = modified stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j-\frac{1}{4},l}$ at (i+½,j-¼,l) of the y-upstream half-element (i,j-¼,l), along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j-\frac{1}{4},l} = \bar{\sigma}_{\nu,xy;i+\frac{1}{2},j-\frac{1}{4},l} + \bar{\sigma}^{*'}_{x,y;i+\frac{1}{2},j-\frac{1}{4},l}, \tag{3.82d}$$

$\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l-\frac{1}{2}}$ = modified stress component acting on the z-upstream enclosure-surface area $A_{z;i,j-\frac{1}{4}}$ at (i,j-¼,l-½) of the y-upstream half-element (i,j-¼,l), along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l-\frac{1}{2}} = \bar{\sigma}_{\nu,zy;i,j-\frac{1}{4},l-\frac{1}{2}} + \bar{\sigma}^{*'}_{z,y;i,j-\frac{1}{4},l-\frac{1}{2}} \ , \qquad (3.82e)$$

= modified stress component acting on the z-downstream enclosure-surface area $A_{z;i,j-\frac{1}{4}}$ at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l+\frac{1}{2}} = \bar{\sigma}_{\nu,zy;i,j-\frac{1}{4},l+\frac{1}{2}} + \bar{\sigma}^{*'}_{z,y;i,j-\frac{1}{4},l+\frac{1}{2}} \ . \qquad (3.82f)$$

This discrete-element form of the temporally primary-scale and spatially primary-scale integral form of the momentum principle for the y-upstream half-element $(i,j-\frac{1}{4},l)$ [Eq. (3.81)] represents the first "discrete-element equation for y-component of velocity".

For a bottom y-upstream half-element $[i,j-\frac{1}{4},l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure-surface at $[i,j-\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$, the discrete-element equation for y-component of velocity [Eq. (3.81)] becomes

for $l = l(bot;i,j)$:

$$\frac{\partial}{\partial t}\bar{V}_{y;i,j-\frac{1}{4},l} = \frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\Big[\widetilde{G}_{y;i,j-\frac{1}{2},l}(\widetilde{V}_{y;i,j-\frac{1}{2},l} - \bar{V}_{y;i,j-\frac{1}{4},l})$$

$$- \widetilde{G}_{y;i,j,l}(\widetilde{V}_{y;i,j,l} - \bar{V}_{y;i,j-\frac{1}{4},l})$$

$$+ \widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}(\widetilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l})$$

(continue)

$$+\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},l}(\overline{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l}-\overline{V}_{y;i,j-\frac{1}{4},l})$$

$$+\widetilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}(\overline{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}-\overline{V}_{y;i,j-\frac{1}{4},l})]$$

$$+\frac{\overline{A}_{y;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}(\overline{p}_{w;i,j-\frac{1}{2},l}-\overline{p}_{w;i,j,l})$$

$$-g_o\cos\Theta_{n,y}+\Omega_{cor,N,z}\overline{V}_{x;i,j-\frac{1}{4},l}-\Omega_{cor,N,x}\overline{V}_{z;i,j-\frac{1}{4},l}$$

$$+\frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}[-\overline{A}_{y;i,j-\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,y;i,j-\frac{1}{2},l}+\overline{A}_{y;i,j,l}\overline{\sigma}^{*'}_{\nu,y,y;i,j,l}$$

$$-\overline{A}_{x;i-\frac{1}{2},j-\frac{1}{4},l}\overline{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},l}$$

$$+A_{x;i+\frac{1}{2},j-\frac{1}{4}}\ \overline{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j-\frac{1}{4},l}$$

$$+A_{z;i,j-\frac{1}{4}}\overline{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l+\frac{1}{2}}]$$

$$+\frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}[\widetilde{G}_{z,bot;i,j-\frac{1}{4}}(\overline{V}_{y,bot;i,j-\frac{1}{4}}-\overline{V}_{y;i,j-\frac{1}{4},l})$$

$$-A_{z;i,j-\frac{1}{4}}\overline{\sigma}^{*'}_{\nu,y,bot;i,j-\frac{1}{4}}]$$

$$+\frac{\delta_{int,m;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\widetilde{G}_{int,m;i,j-\frac{1}{4},l}(\overline{V}_{y,int,m;i,j-\frac{1}{4},l}-\overline{V}_{y;i,j-\frac{1}{4},l})$$

$$+\frac{\delta_{dis,m;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\overline{G}_{dis,m;i,j-\frac{1}{4},l}(\overline{V}_{y,dis,m;i,j-\frac{1}{4},l}-\overline{V}_{y;i,j-\frac{1}{4},l}), \tag{3.83}$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the bottom conditions, are defined, as

$V_{y,bot;i,j-\frac{1}{4}}$ = upwind-differenced value of y-component of velocity associated with the bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i,j-\frac{1}{4},1(bot;i,j)-\frac{1}{2}]$ of the y-upstream half-element $[i,j-\frac{1}{4},1(bot;i,j)]$,

$\bar{\sigma}^{*'}_{\nu,y,bot;i,j-\frac{1}{4}}$ = modified bottom shear stress component acting on the z-upstream bottom enclosure-surface area $A_{z;i,j-\frac{1}{4}}$ at $[i,j-\frac{1}{4},1(bot;i,j)-\frac{1}{2}]$ of the y-upstream half-element $[i,j-\frac{1}{4},1(bot;i,j)]$.

For a y-upstream half-element in the water-surface layer $(i,j-\frac{1}{4},L)$ which contains the free-surface layer of the water in the flow region, the discrete-element equation for y-component of velocity [Eq. (3.81)] becomes

for $l = L$:

$$\frac{\partial}{\partial t}\bar{V}_{y;i,j-\frac{1}{4},L} = \frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},L}]}[\tilde{G}_{y;i,j-\frac{1}{2},L}(\tilde{V}_{y;i,j-\frac{1}{2},L}-\bar{V}_{y;i,j-\frac{1}{4},L})$$

$$-\tilde{G}_{y;i,j,L}(\tilde{V}_{y;i,j,L}-\bar{V}_{y;i,j-\frac{1}{4},L})$$

$$+\tilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},L}(\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},L}-\bar{V}_{y;i,j-\frac{1}{4},L})$$

$$+\tilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},L}(\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},L}-\bar{V}_{y;i,j-\frac{1}{4},L})$$

$$+\tilde{G}_{z;i,j-\frac{1}{4},L-\frac{1}{2}}(\tilde{V}_{y;i,j-\frac{1}{4},L-\frac{1}{2}}-\bar{V}_{y;i,j-\frac{1}{4},L})]$$

$$+\frac{\bar{A}_{y;i,j-\frac{1}{4},L}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},L}]}(\bar{p}_{w;i,j-\frac{1}{2},L}-\bar{p}_{w;i,j,L})$$

(continue)

$$-g_o\cos\Theta_{n,y}+\Omega_{cor,N,z}\bar{V}_{x;i,j-\frac{1}{4},L}-\Omega_{cor,N,x}\bar{V}_{z;i,j-\frac{1}{4},L}$$

$$+\frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},L}]}[-\bar{A}_{y;i,j-\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,y;i,j-\frac{1}{2},L}+\bar{A}_{y;i,j,L}\bar{\sigma}^{*'}_{\nu,y,y;i,j,L}$$

$$-\bar{A}_{x;i-\frac{1}{2},j-\frac{1}{4},L}\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},L}$$

$$+\bar{A}_{x;i+\frac{1}{2},j-\frac{1}{4},L}\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j-\frac{1}{4},L}$$

$$-A_{z;i,j-\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},L-\frac{1}{2}}]$$

$$+\frac{1}{\overline{\Delta z}_{i,j-\frac{1}{4},L}}[\bar{g}_{rnf}(\bar{V}_{y,rnf,}-\bar{V}_{y;i,j-\frac{1}{4},L})+\bar{\sigma}^{*'}_{\nu,y,wnd;i,j-\frac{1}{4}}]$$

$$+\frac{\delta_{int,m;i,j-\frac{1}{4},L}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},L}]}\tilde{G}_{int,m;i,j-\frac{1}{4},L}(\tilde{V}_{y,int,m;i,j-\frac{1}{4},L}-\bar{V}_{y;i,j-\frac{1}{4},L})$$

$$+\frac{\delta_{dis,m;i,j-\frac{1}{4},L}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},L}]}\bar{G}_{dis,m;i,j-\frac{1}{4},L}(\bar{V}_{y,dis,m;i,j-\frac{1}{4},L}-\bar{V}_{y;i,j-\frac{1}{4},L}), \tag{3.84}$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the surface conditions, are defined, as

$\bar{V}_{y,rnf}$ = y-component of velocity associated with the rainfall on the water surface of the flow region.

$\bar{\sigma}^{*'}_{\nu,y,wnd;i,j-\frac{1}{4}}$ = modified wind-shear stress component acting on the z-downstream water enclosure-surface at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, along y-direction.

It is important to note that the wind-shear stress component, along y-direction, depends on the free-surface flow conditions on the water enclosure-surface at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$ as well as the wind conditions over the water surface of the flow region.

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer $\overline{\Delta z}_L$ (Sec. 2.3). In these single layer subregions, a y-upstream half-element $(i,j-\frac{1}{4},L)$ can represent a bottom y-upstream half-element $[i,j-\frac{1}{4},l(bot;i,j)]$ and also a y-upstream half-element in the water-surface layer $(i,j-\frac{1}{4},L)$. For these special y-upstream half-elements $[i,j-\frac{1}{4},l(bot;i,j)]$ and also $(i,j-\frac{1}{4},L)$ the discrete-element equation for y-component of the discrete-element equation for y-component velocity [Eq. (3.81)] becomes

for $l(bot;i,j)=l=L$:

$$\frac{\partial}{\partial t}\overline{V}_{y;i,j-\frac{1}{4},L} = \frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},L}]}[\widetilde{G}_{y;i,j-\frac{1}{2},L}(\widetilde{V}_{y;i,j-\frac{1}{2},L}-\overline{V}_{y;i,j-\frac{1}{4},L})$$

$$-\widetilde{G}_{y;i,j,L}(\widetilde{V}_{y;i,j,L}-\overline{V}_{y;i,j-\frac{1}{4},L})$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},L}(\widetilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},L}-\overline{V}_{y;i,j-\frac{1}{4},L})$$

$$+\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},L}(\widetilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},L}-\overline{V}_{y;i,j-\frac{1}{4},L})]$$

$$+\frac{\overline{A}_{y;i,j-\frac{1}{4},L}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},L}]}(\overline{p}_{w;i,j-\frac{1}{2},L}-\overline{p}_{w;i,j,L})$$

(continue)

$$-g_0\cos\theta_{n,y}+\Omega_{cor,N,z}\bar{V}_{x;i,j-\frac{1}{4},L}-\Omega_{cor,N,x}\bar{V}_{z;i,j-\frac{1}{4},L}$$

$$+\frac{1}{[A_{z;i,j-\frac{1}{4}}\bar{\Delta z}_{i,j-\frac{1}{4},L}]}[-\bar{A}_{y;i,j-\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,y;i,j-\frac{1}{2},L}+\bar{A}_{y;i,j,L}\bar{\sigma}^{*'}_{\nu,y,y;i,j,L}$$

$$-\bar{A}_{x;i-\frac{1}{2},j-\frac{1}{4},L}\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},L}$$

$$+\bar{A}_{x;i+\frac{1}{2},j-\frac{1}{4},L}\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j-\frac{1}{4},L}]$$

$$+\frac{1}{[A_{z;i,j-\frac{1}{4}}\bar{\Delta z}_{i,j-\frac{1}{4},1}]}[\tilde{G}_{z,bot;i,j-\frac{1}{4}}(\tilde{V}_{y,bot;i,j-\frac{1}{4}}-\bar{V}_{y;i,j-\frac{1}{4},1})$$

$$-A_{z;i,j-\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,y,bot;i,j-\frac{1}{4}}]$$

$$+\frac{1}{\bar{\Delta z}_{i,j-\frac{1}{4},L}}[\bar{g}_{rnf}(\bar{V}_{rnf}-\bar{V}_{y;i,j-\frac{1}{4},L})+\bar{\sigma}^{*'}_{\nu,y,wnd;i,j-\frac{1}{4}}]$$

$$+\frac{\delta_{int,m;i,j-\frac{1}{4},L}}{[A_{z;i,j-\frac{1}{4}}\bar{\Delta z}_{i,j-\frac{1}{4},L}]}\tilde{G}_{int,m;i,j-\frac{1}{4},L}(\tilde{V}_{y,int,m;i,j-\frac{1}{4},L}-\bar{V}_{y;i,j-\frac{1}{4},L})$$

$$+\frac{\delta_{dis,m;i,j-\frac{1}{4},L}}{[A_{z;i,j-\frac{1}{4}}\bar{\Delta z}_{i,j-\frac{1}{4},L}]}\bar{G}_{dis,m;i,j-\frac{1}{4},L}(\bar{V}_{y,dis,m;i,j-\frac{1}{4},L}-\bar{V}_{y;i,j-\frac{1}{4},L}). \tag{3.85}$$

Similarly, the temporally primary-scale and spatially primary-scale y-component of the integral form of the momentum principle [Eq. (102)] can be formulated for the control-volume of the y-downstream half-element $(i,j+\frac{1}{4},1)$, as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \bar{V}_y dv + \iint_{\bar{A}_{i,j+\frac{1}{4},l}} \overline{[V_y]\vec{V}\cdot\hat{n}}\,dA = -g_0 \cos\theta_{n,y} \iiint_{\bar{V}_{i,j+\frac{1}{4},l}} dv$$

$$+\Omega_{cor,N,z}\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \bar{V}_x dv - \Omega_{cor,N,x}\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \bar{V}_z dv$$

$$+\iint_{\bar{A}_{i,j+\frac{1}{4},l}} \overline{[p_w(-\hat{n})]_y}\,dA + \iint_{\bar{A}_{i,j+\frac{1}{4},l}} \overline{\sigma_{\nu,sy}}\,dA \,. \qquad (3.86)$$

The control-volume integral on the left-hand side of Eq. (3.86) can be formulated according to (1) the definition of the general control-volume integral for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eqs. (2.85a), (2.85b) and (2.86c)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)], as

$$\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \bar{V}_y dv = [A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]\overline{V}_{y;i,j+\frac{1}{4},l} \,, \qquad (3.87)$$

where the temporally primary-scale and spatially primary-scale flow variable is defined, as

$\overline{V}_{y;i,j+\frac{1}{4},l}$ = y-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] in the y-downstream half-element $(i,j+\frac{1}{4},l)$.

The time derivative of the control-volume integral [Eq. (3.87)] on the left-hand side of Eq. (3.86), can be expanded, as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \bar{V}_y dv = \frac{\partial}{\partial t}\left\{ [A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]\bar{V}_{y;i,j+\frac{1}{4},l} \right\}$$

$$-[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]\frac{\partial}{\partial t}\bar{V}_{y;i,j+\frac{1}{4},l} + \bar{V}_{y;i,j+\frac{1}{4},l}\frac{\partial}{\partial t}[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]. \quad (3.88)$$

Hence, multiplying the auxiliary discrete-element form of the mass conservation principle for water in the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (3.35)] with the y-component of velocity $\bar{V}_{y;i,j+\frac{1}{4},l}$ in the y-downstream half-element $(i,j+\frac{1}{4},l)$ and substituting in Eq. (3.88), the time derivative of the control-volume integral [Eq. (3.88)] on the left-hand side of Eq. (3.86) becomes

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \bar{V}_y dv = [A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]\frac{\partial}{\partial t}\bar{V}_{y;i,j+\frac{1}{4},l}$$

$$+\tilde{G}_{y;i,j,l}\bar{V}_{y;i,j+\frac{1}{4},l} - \tilde{G}_{y;i,j+\frac{1}{2},l}\bar{V}_{y;i,j+\frac{1}{4},l}$$

$$+\tilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l}\bar{V}_{y;i,j+\frac{1}{4},l} - \tilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l}\bar{V}_{y;i,j+\frac{1}{4},l}$$

$$+\tilde{G}_{z;i,j+\frac{1}{4},l-\frac{1}{2}}\bar{V}_{y;i,j+\frac{1}{4},l} - \tilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}\bar{V}_{y;i,j+\frac{1}{4},l}$$

$$+\delta_{int,m;i,j+\frac{1}{4},l}\tilde{G}_{int,m;i,j+\frac{1}{4},l}\bar{V}_{y;i,j+\frac{1}{4},l}$$

$$+\delta_{dis,m;i,j+\frac{1}{4},l}\tilde{G}_{dis,m;i,j+\frac{1}{4},l}\bar{V}_{y;i,j+\frac{1}{4},l} . \quad (3.89)$$

The closed control-surface integral on the left-hand side of Eq. (3.86) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.89)], (2) the definitions of the

enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the y-downstream half element $(i,j+\frac{1}{4},l)$ [Eqs. (2.90i), (2.90j), (2.90m), (2.90n) and (2.91e)], (3) the formulation of the general representation of the six enclosure-surface integral which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, y-component of the integral form of the momentum principle [Eq. (2.113)], (4) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the intake of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale, y-component of the integral form of the momentum principle [Eq. (2.125)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale, y-component of the integral form of the momentum principle [Eq. (2.126)], and by considering the general definitions of the upwind-differencing interpolation forms for the transportive, intensive flow variables, as

$$\iint_{\underline{A}_{i,j+\frac{1}{4},l}} \overline{[V_y]\vec{V}\cdot\hat{n}}\,dA = -\widetilde{G}_{y;i,j,l}\widetilde{V}_{y;i,j,l} + \overline{A}_{y;i,j,l}\overline{\sigma}^{*'}_{y,y;i,j,l}$$

$$+ \widetilde{G}_{y;i,j+\frac{1}{2},l}\widetilde{V}_{y;i,j+\frac{1}{2},l} - \overline{A}_{y;i,j+\frac{1}{2},l}\overline{\sigma}^{*'}_{y,y;i,j+\frac{1}{2},l}$$

$$- \widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l}\widetilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l} + \overline{A}_{x;i-\frac{1}{2},j+\frac{1}{4},l}\overline{\sigma}^{*'}_{x,y;i-\frac{1}{2},j+\frac{1}{4},l}$$

(continue)

$$+\widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l}\widetilde{V}_{y;l+\frac{1}{2},j+\frac{1}{4},l}-\overline{A}_{x;i+\frac{1}{2},j+\frac{1}{4},l}\overline{\sigma}^{*'}_{x,y;i+\frac{1}{2},j+\frac{1}{4},l}$$

$$-\widetilde{G}_{z;i,j+\frac{1}{4},l-\frac{1}{2}}\widetilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}}+\overline{A}_{z;i,j+\frac{1}{4}}\overline{\sigma}^{*'}_{z,y;i,j+\frac{1}{4},l-\frac{1}{2}}$$

$$+\widetilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}\widetilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}}-\overline{A}_{z;i,j+\frac{1}{4}}\overline{\sigma}^{*'}_{z,y;i,j+\frac{1}{4},l+\frac{1}{2}}$$

$$-\delta_{int,m;i,j+\frac{1}{4},l}\widetilde{G}_{int,m;i,j+\frac{1}{4},l}\widetilde{V}_{y,int,m;i,j+\frac{1}{4},l}$$

$$-\delta_{dis,m;l,j+\frac{1}{4},l}\overline{G}_{dis,m;l,j+\frac{1}{4},l}\overline{V}_{y,dis,m;i,j+\frac{1}{4},l}\ , \qquad (3.90)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\widetilde{V}_{y;i,j,l}$ = upwind-differenced value of y-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] at y-upstream enclosure-surface at (i,j,l) of the y-downstream half-element (i,j+¼,l) [Eq. (2.113)],

$\overline{\sigma}^{*'}_{y,y;i,j,l}$ = combined convention-defect and eddy-turbulent stress component (excluding only the component of modified viscous stress) acting on the y-upstream enclosure-surface area $\overline{A}_{y;i,j,l}$ at (i,j,l) of the y-downstream half-element (i,j+¼,l), along y-direction [Eq. (2.116)],

$\overline{V}_{y;i,j+\frac{1}{2},l}$ = upwind-differenced value of y-component of velocity at y-downstream enclosure surface at (i,j+½,l) of the y-downstream half-element (i,j+¼,l) [Eq. (2.113)],

$\overline{\sigma}^{*'}_{y,y;i,j+\frac{1}{2},l}$ combined convection-defect and eddy-turbulent stress component acting on the y-downstream enclosure-surface area $\overline{A}_{y;i,j+\frac{1}{2},l}$ at $(i,j+\frac{1}{2},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\widetilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l}$ = upwind-differenced value of y-component of velocity at x-upstream enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-elements $(i,j+\frac{1}{4},l)$ [Eq. (2.113)]

$\overline{\sigma}^{*'}_{x,y;i-\frac{1}{2},j+\frac{1}{4},l}$ = combined convection-defect and eddy-turbulent stress component acting on the x-upstream enclosure-surface area $\overline{A}_{x;i-\frac{1}{2},j+\frac{1}{4},l}$ at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\widetilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l}$ = upwind-differencied value of y-component of velocity at x-downstream enclosure-surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.113)],

$\overline{\sigma}^{*'}_{x,y;i+\frac{1}{2},j+\frac{1}{4},l}$ = combined convection-defect and eddy-turbulent stress component acting on the x-downstream enclosure-surface area $\overline{A}_{x;i+\frac{1}{2},j+\frac{1}{4},l}$ at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\widetilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}}$ = upwind-differenced value of y-component of velocity at z-upstream enclosure-surface at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, [Eq. (2.113)],

$\overline{\sigma}^{*'}_{z,y;i,j+\frac{1}{4},l-\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-upstream enclosure-surface

area $A_{z;i,j+\frac{1}{4}}$ at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\overline{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}}$ = upwind-differenced value of y-component of velocity at z-downstream enclosure-surface at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.113)],

$\overline{\sigma}^{*'}_{z,y;i,j+\frac{1}{4},l+\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-downstream enclosure-surface area $A_{z;i,j+\frac{1}{4}}$ at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction [Eq. (2.116)],

$\widetilde{\overline{V}}_{y,int,m;i,j+\frac{1}{4},l}$ = upwind-differenced value of y-component of velocity associated with the intake flow of power plant m in the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$\overline{V}_{y,dis,m;i,j+\frac{1}{4},l}$ = y-component of velocity associated with the discharge flow of power plant m in the y-downstream half-element $(i,j+\frac{1}{4},l)$.

The first control-volume integral on the left side of Eq. (3.86), associated with the gravity force term along y-direction, is already evaluated for the discrete-element formulation of the temporally primary-scale and spatially primary-scale integral form of the conservation of mass principle for water [Eq. (3.33)].

The control-volume integrals on the right-hand side of Eq. (3.86), associated with the Coriolis acceleration (force) terms are also formulated according to (1) the definition of the general control-volume integral for the control-volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eqs. (2.85a), (2.85b) and (2.86c)] and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially pri-

mary-scale, three components of the integral form of the momentum principle [Eq. (2.106)], as

$$\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \overline{V_z} dv = [A_{z;i,j+\frac{1}{4}} \overline{\Delta z}_{i,j+\frac{1}{4},l}] [\overline{V}_{z;i,j+\frac{1}{4},l}] , \tag{3.91}$$

$$\iiint_{\bar{V}_{i,j+\frac{1}{4},l}} \overline{V_x} dv = [A_{z;i,j+\frac{1}{4}} \overline{\Delta z}_{i,j+\frac{1}{4},l}] [\overline{V}_{x;i,j+\frac{1}{4},l}] , \tag{3.92}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\overline{V}_{z;i,j+\frac{1}{4},l}$ = z-component of velocity along the vertical direction (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region) in the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.106)],

$\overline{V}_{x;i,j+\frac{1}{4},l}$ = x-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] in the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.106)].

The discrete-element formulation of the closed control-surface integral associated with the pressure force term on the right-hand side of Eq. (3.86) also requires careful considerations of the geometrical properties of the elements, particularly the boundary elements which represent nonrectangular (truncated rectangular) cells with enclosure-surface areas not necessarily normal to any of the three principal orthogonal axes x, y, or z (Figs. 2.34 through 2.42, p.124 through p.132; Fig. 2.52, p.144; Fig. 2.54, p.146). It is important to note that on these boundary enclosure-surface areas, the pressure forces result in

components along y-direction depending on the angle (in the general sense) between the unit nirmal vector to the boundary enclosure-surface and the y-axis. The geometrical effects of the boundary enclosure-surface areas in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region) can be incorporated by considering the area $\bar{A}_{y;i,j+\frac{1}{4},l}$ normal to the y-direction, at the center of the y-downstream half-element $(i,j+\frac{1}{4},l)$ (e.g., Shames, 1962, Eraslan, 1975, 1977,1981a), for calculating the net pressure force acting on the y-downstream half-element. However, the variation of the pressure, particularly the hydrostatic pressure part, along the z-direction must be carefully considered in the formulation of the pressure force term acting along y-direction on the y-downstream half-element $(i,j+\frac{1}{4},l)$. The important consequences of the variation of pressure along z-direction will be considered in detail in the discussions of the interpolation formulas for the pressure conditions on the enclosure-surface areas of the half-elements (Sec. 4.4).

Hence, the control-surface integral associated with the pressure force term on the right-hand side of Eq. (3.86) can be formulated according to (1) the definitions of the general closed control-surface integral for the control volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (2.89)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and down-stream enclosure-surface areas of the y-downstream half-element $(i,j+\frac{1}{4},l)$]Eqs. (2.90i), (2.90j), (2.90m), (2.90n) and (2.91e)], and by considering the components of the presure forces along the y-direction, on all enclo-

sure-surface areas of the y-downstream half-element $(i,j+\frac{1}{4},l)$, including the ones which are parts of the nonrectangular boundary elements that represent truncated rectangular cells (Sec. 2, Figs. 2.34 through 2.42, p.124 through 146; Fig. 2.52, p.144; Fig. 2. 54; p.146), as

$$\iint_{\bar{A}_{i,j+\frac{1}{4},l}} \overline{[p_w(-\hat{n})]}_y dA = \bar{A}_{y;i,j+\frac{1}{4},l}(\bar{p}_{w;i,j,l} - \bar{p}_{w;i,j+\frac{1}{2},l}), \qquad (3.93)$$

where the area $\bar{A}_{y;i,j+\frac{1}{4},l}$, normal to the y-direction, at the center of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [in Eq. (3.93)] is defined, as

$$\bar{A}_{y;i,j+\frac{1}{4},l} = \Delta x_{i,j+\frac{1}{4}} \overline{\Delta z}_{i,j+\frac{1}{4},l}, \qquad (3.94)$$

and the temporally primary-scale and spatially primary-scale values of the modified pressure are defined, as

$\bar{p}_{w;i,j,l}$ = modified pressure at the location of the y-upstream enclosure-surface at (i,j,l) of the y-downstream half-element $(i,j+\frac{1}{4},l)$,

$\bar{p}_{w;i,j+\frac{1}{2},l}$ = modified pressure at the location of the y-downstream enclosure-surface at $(i,j+\frac{1}{2},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$.

The discrete-element formulation of the control-surface integral, associated with the pressure force term for the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (3.93)] is applicable to all y-downstream half-elements, including (1) bottom y-downstream half-element $[i,j+\frac{1}{4},l(bot;i,j)]$, (2) vertically interior y-downstream half-elements $[i,j+\frac{1}{4},l(bot;i,j)\frac{1}{4}l\frac{1}{4}L]$, and (3) y-downstream half-elements in the water-

surface layer $(i,j+\frac{1}{4},1)$. Furthermore, according to the discrete-element formulation of the control-surface integral [Eq. (3.93)], the pressure force term acting on the y-downstream half-element $(i,j+\frac{1}{4},L)$ is completely independent of the value of the spatially uniform atmospheric pressure on the free-surface of the water in the flow region, since it includes only the difference of the two pressure values at the upstream enclosure-surface at $(i,j,1)$ and at the downstream enclosure-surface at $(i,j+\frac{1}{2},1)$.

The control-surface integral associated with the modified viscous (molecular) force term, on the right-hand side of Eq. (3.86) can be evaluated according to (1) the definitions of the general closed control-surface integral for the control-volume of y-downstream half-element $(i,j+\frac{1}{4},1)$ [Eq. (2.89)] and (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and the downstream enclosure surface areas of the y-downstream half-element $(i,j+\frac{1}{4},1)$ [Eqs. (2.90i), (2.90j), (2.90m), (2.90n) and (2.91e)], as

$$\iint_{\bar{A}_{i,j+\frac{1}{4},1}} \overline{\sigma_{\nu,sy}}\,dA = -\bar{A}_{y;i,j,1}\bar{\sigma}_{\nu,yy;i,j,1} + \bar{A}_{y;i,j+\frac{1}{2},1}\bar{\sigma}_{\nu,yy;i,j+\frac{1}{2},1}$$

$$-\bar{A}_{x;i-\frac{1}{2},j+\frac{1}{4},1}\bar{\sigma}_{\nu,xy;i-\frac{1}{2},j+\frac{1}{4},1}+\bar{A}_{x;i+\frac{1}{2},j+\frac{1}{4},1}\bar{\sigma}_{\nu,xy;i+\frac{1}{2},j+\frac{1}{4},1}$$

$$-\bar{A}_{z;i,j+\frac{1}{4},1-\frac{1}{2}}\bar{\sigma}_{\nu,zy;i,j+\frac{1}{4},1-\frac{1}{2}}+\bar{A}_{z;i,j+\frac{1}{4},1}\bar{\sigma}_{\nu,zy;i,j+\frac{1}{4},1+\frac{1}{2}}\ , \tag{3.95}$$

where the temporally primary-scale and spatially primary-scale modified viscous stress components are defined, as

$\bar{\sigma}_{\nu,yy;i,j,1}$ = modified viscous stress component acting on the y-upstream enclosure-surface area $\bar{A}_{y;i,j,1}$ at $(i,j,1)$ of the

y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction,

$\bar{\sigma}_{\nu,yy;i,j+\frac{1}{2},l}$ = modified viscous stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i,j+\frac{1}{2},l}$ at $(i,j+\frac{1}{2},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction,

$\bar{\sigma}_{\nu,xy;i-\frac{1}{2},j+\frac{1}{4},l}$ = modified viscous stress component acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j+\frac{1}{4},l}$ at $(i-\frac{1}{2},j+1)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction,

$\bar{\sigma}_{\nu,xy;i+\frac{1}{2},j+\frac{1}{4},l}$ = modified viscous stress component acting on the y-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j+\frac{1}{4},l}$ at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$ along y-direction,

$\bar{\sigma}_{\nu,zy;i,j+\frac{1}{4},l-\frac{1}{2}}$ = modified viscous stress component acting on the z-upstream enclosure-surface area $A_{z;i,j+\frac{1}{4}}$ at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y- direction,

$\bar{\sigma}_{\nu,zy;i,j+\frac{1}{4},l+\frac{1}{2}}$ = modified viscous stress component acting on the z-downstream enclosure-surface area $A_{z;i,j+\frac{1}{4}}$ at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction.

Hence, substituting the discrete-element formulations of (1) the time derivative of the control-volume integral [Eq. (3.89)] of the left-hand side of Eq. (3.86), (2) the control-surface inteqral [Eq. (3.90)] of the left-hand side of Eq. (3.86), (3) the control-volume integral associated with the gravity force term along y-direction [Eq. (3.33)] of

the right-hand side of Eq. (3.86), (4) the two control-volume integrals associated with the Coriolis acceleration (force) term [Eqs. (3.91) and (3.92)] of the right-hand side of Eq. (3.86), (5) the control-surface integral associated with the modified pressure force term [Eq. (3.93)] of the right-hand side of Eq. (3.86), and (6) the control-surface integral [Eq. (3.94)] associated with the modified viscous force term [Eq. (3.94)] of the right-hand side of Eq. (3.86), in the temporally primary-scale and spatially primary-scale y-component of the integral form of the momentum principle for the control-volume of the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (3.86)], combining terms and reorranging, the discrete-element form of the momentum principle for the y-downstream half-element $(i,j+\frac{1}{4},l)$ becomes,

$$\frac{\partial}{\partial t}\bar{V}_{y;i,j+\frac{1}{4},l} = \frac{1}{[A_{z;i,j+\frac{1}{4}}\Delta z_{i,j+\frac{1}{4},l}]}\Big[\tilde{G}_{y;i,j,l}(\tilde{V}_{y;i,j,l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$-\tilde{G}_{y;i,j+\frac{1}{2},l}(\tilde{V}_{y;i,j+\frac{1}{2},l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$+\tilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l}(\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$-\tilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l}(\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$+\tilde{G}_{z;i,j+\frac{1}{4},l-\frac{1}{2}}(\tilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$-\tilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}(\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$+\frac{\bar{A}_{y;i,j+\frac{1}{2},l}}{[A_{z;i,j+\frac{1}{4}}\Delta z_{i,j+\frac{1}{4},l}]}(\bar{P}_{w;i,j,l}-\bar{P}_{w;i,j+\frac{1}{2},l})$$

(continue)

$$-g_0\cos\theta_{n,y}+\Omega_{cor,N,z}\overline{V}_{x;i,j+\frac{1}{4},l}-\Omega_{cor,N,x}\overline{V}_{z;i,j+\frac{1}{4},l}$$

$$+\frac{1}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}[-\overline{A}_{y;i,j,l}\overline{\sigma}^{*'}_{\nu,y,y;i,j,l}+\overline{A}_{y;i,j+\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},l}$$

$$-\overline{A}_{x;i-\frac{1}{2},j+\frac{1}{4},l}\overline{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l}$$

$$+\overline{A}_{x;i+\frac{1}{2},j+\frac{1}{4},l}\overline{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l}$$

$$-A_{z;i,j+\frac{1}{4}}\overline{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l-\frac{1}{2}}$$

$$+A_{z;i,j+\frac{1}{4}}\overline{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l+\frac{1}{2}}]$$

$$+\frac{\delta_{int,m;i,j+\frac{1}{4},l}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}\widetilde{G}_{int,m;i,j+\frac{1}{4},l}(V_{y,int,m;i,j+\frac{1}{4},l}-\overline{V}_{y;i,j+\frac{1}{4},l})$$

$$+\frac{\delta_{dis,m;i,j+\frac{1}{4},l}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}\overline{G}_{dis,m;i,j+\frac{1}{4},l}(\overline{V}_{y,dis,m;i,j+\frac{1}{4},l}-\overline{V}_{y;i,j+\frac{1}{4},l}), \quad (3.96)$$

where the temporally primary-scale and spatially primary-scale stress components are defined, as

$\overline{\sigma}^{*'}_{\nu,y,y;i,j,l}$ = modified stress component (consisting of modified viscous stress component, and combined convection- defect and eddy-turbulent stress component) acting on the y-upstream enclosure-surface area $\overline{A}_{y;i,j,l}$ at (i,j,l) of the y-downstream half-element (i,j+¼,l), along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,y;i,j,l} = \bar{\sigma}_{\nu,yy;i,j,l} + \bar{\sigma}^{*'}_{y,y;i,j,l} \,, \tag{3.97a}$$

$\bar{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},l}$ = modified stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i,j+\frac{1}{2},l}$ at $(i,j+\frac{1}{2},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},l} = \bar{\sigma}_{\nu,yy;i,j+\frac{1}{2},l} + \bar{\sigma}^{*'}_{y,y;i,j+\frac{1}{2},l} \,, \tag{3.97b}$$

$\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l}$ = modified stress component acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j+\frac{1}{4},l}$ at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l} = \bar{\sigma}_{\nu,xy;i-\frac{1}{2},j+\frac{1}{4},l} + \bar{\sigma}^{*'}_{x,y;i-\frac{1}{2},j+\frac{1}{4},l} \,, \tag{3.97c}$$

$\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j+\frac{1}{4},l}$ = modified stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j+\frac{1}{4},l}$ at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j+\frac{1}{4},l} = \bar{\sigma}_{\nu,xy;i+\frac{1}{2},j+\frac{1}{4},l} + \bar{\sigma}^{*'}_{x,y;i+\frac{1}{2},j+\frac{1}{4},l} \,, \tag{3.97d}$$

$\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l-\frac{1}{2}}$ = modified stress component acting on the z-upstream enclosure-surface area $A_{z;i,j+\frac{1}{4}}$ at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the

y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction, which represents

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l-\frac{1}{2}} = \bar{\sigma}_{\nu,zy;i,j+\frac{1}{4},l-\frac{1}{2}} + \bar{\sigma}^{*'}_{z,y;i,j+\frac{1}{4},l-\frac{1}{2}} , \qquad (3.97e)$$

$\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l+\frac{1}{2}}$ = modified stress component acting on the z-downstream enclosure-surface area $A_{z;i,j+\frac{1}{4}}$ at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, along y-direction, which represents

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l+\frac{1}{2}} = \bar{\sigma}_{\nu,zy;i,j+\frac{1}{4},l+\frac{1}{2}} + \bar{\sigma}^{*'}_{z,y;i,j+\frac{1}{4},l+\frac{1}{2}} . \qquad (3.97f)$$

This discrete-element form of the temporally primary-scale and spatially primary-scale integral form of the momentum principle for the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eq. (3.96)] represents the second "discrete-element equation for y-component of velocity".

For a bottom y-downstream half-element $[i,j+\frac{1}{4},l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure-surface at $[i,j+\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$, the discrete-element equation for y-component of velocity [Eq. (3.96)] becomes

for $l = l(bot;i,j)$:

$$\frac{\partial}{\partial t}\bar{V}_{y;i,j+\frac{1}{4},l} = \frac{1}{\left[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}\right]}\left[\tilde{G}_{y;i,j,l}(\tilde{V}_{y;i,j,l} - \bar{V}_{y;i,j+\frac{1}{4},l})\right.$$

$$\left. - \tilde{G}_{y;i,j+\frac{1}{2},l}(\tilde{V}_{y;i,j+\frac{1}{2},l} - \bar{V}_{y;i,j+\frac{1}{4},l})\right.$$

(continue)

$$+\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l}(\widetilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l}-\overline{V}_{y;i,j+\frac{1}{4},l})$$

$$-\widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l}(\widetilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l}-\overline{V}_{y;i,j+\frac{1}{4},l})$$

$$+\widetilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}(\widetilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}}-\overline{V}_{y;i,j+\frac{1}{4},l})]$$

$$+\frac{\overline{A}_{y;i,j+\frac{1}{4},l}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}(\overline{p}_{w;i,j,l}-\overline{p}_{w;i,j+\frac{1}{2},l})$$

$$-g_o\cos\Theta_{n,y}+\Omega_{cor,N,z}\overline{V}_{x;i,j+\frac{1}{4},l}-\Omega_{cor,N,x}\overline{V}_{z;i,j+\frac{1}{4},l}$$

$$+\frac{1}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}[-\overline{A}_{y;i,j,l}\overline{\sigma}^{*'}_{\nu,y,y;i,j,l}+\overline{A}_{y;i,j+\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},l}$$

$$-\overline{A}_{x;i-\frac{1}{2},j+\frac{1}{4},l}\overline{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l}$$

$$+\overline{A}_{x;i+\frac{1}{2},j+\frac{1}{4},l}\overline{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j+\frac{1}{4},l}$$

$$+A_{z;i,j+\frac{1}{4}}\overline{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l+\frac{1}{2}}]$$

$$+\frac{1}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}[\widetilde{G}_{z,bot;i,j+\frac{1}{4}}(\widetilde{V}_{y,bot;i,j+\frac{1}{4}}-\overline{V}_{y;i,j+\frac{1}{4},l})$$

$$-A_{z;i,j+\frac{1}{4}}\overline{\sigma}^{*'}_{\nu,y,bot;i,j+\frac{1}{4}}]$$

$$+\frac{\delta_{int,m;i,j+\frac{1}{4},l}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}\widetilde{G}_{int,m;i,j+\frac{1}{4},l}(\widetilde{V}_{y,int,m;i,j+\frac{1}{4},l}-\overline{V}_{y;i,j+\frac{1}{4},l})$$

$$+\frac{\delta_{dis,m;i,j+\frac{1}{4},l}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}\widetilde{G}_{dis,m;i,j+\frac{1}{4},l}(\overline{V}_{y,dis,m;i,j+\frac{1}{4},l}-\overline{V}_{y;i,j+\frac{1}{4},l}), \tag{3.98}$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the bottom flow conditions, are defined, as

$\widetilde{V}_{y,bot;i,j+\frac{1}{4}}$ = upwind-differenced value of y-component of velocity associated with the bottom seepage flow rate at the z-upstream bottom enclosure-surface at $[i,j+\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$ of the y-downstream half-element $[i,j+\frac{1}{4},l(bot;i,j)]$,

$\overline{\sigma}^{*'}_{\nu,y,bot;i,j+\frac{1}{4}}$ = modified bottom shear stress component acting on the z-upstream bottom enclosure-surface area $Az;i+\frac{1}{4},j$ at $[i,j+\frac{1}{4},l(bot;i,j)-\frac{1}{2}]$ of the y-downstream half-element $[i,j+\frac{1}{4},l(bot;i,j)]$, along y-direction.

For a y-downstream half-element in the water-surface layer $(i,j+\frac{1}{4},L)$ which contains the free-surface layer of the water in the flow region, the discrete-element equation for y-component of velocity [Eq. (3.96)] becomes

for $l = L$:

$$\frac{\partial}{\partial t}\overline{V}_{y;i,j+\frac{1}{4},L} = \frac{1}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}]}\Big[\widetilde{G}_{y;i,j,L}(\widetilde{V}_{y;i,j,L}-\overline{V}_{y;i,j+\frac{1}{4},L})$$

$$-\widetilde{G}_{y;i,j+\frac{1}{2},L}(\widetilde{V}_{y;i,j+\frac{1}{2},L}-\overline{V}_{y;i,j+\frac{1}{4},L})$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},L}(\widetilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},L}-\overline{V}_{y;i,j+\frac{1}{4},L})$$

$$-\widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},L}(\widetilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},L}-\overline{V}_{y;i,j+\frac{1}{4},L})\Big]$$

$$+\widetilde{G}_{z;i,j+\frac{1}{4},L-\frac{1}{2}}(\widetilde{V}_{y;i,j+\frac{1}{4},L-\frac{1}{2}}-\overline{V}_{y;i,j+\frac{1}{4},L})\Big]$$

(continue)

$$+\frac{\overline{A}_{y;i,j+\frac{1}{4},L}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}]}(\overline{p}_{w;i,j,L}-\overline{p}_{w;i,j+\frac{1}{2},L})$$

$$-g_a\cos\theta_{n,y}+\Omega_{cor,N,z}\overline{V}_{x;i,j+\frac{1}{4},L}-\Omega_{cor,N,x}\overline{V}_{z;i,j+\frac{1}{4},L}$$

$$+\frac{1}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}]}[-\overline{A}_{y;i,j,L}\overline{\sigma}^{*'}_{\nu,y,y;i,j,L}+\overline{A}_{y;i,j+\frac{1}{2},L}\overline{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},L}$$

$$-\overline{A}_{x;i-\frac{1}{2},j+\frac{1}{4},L}\overline{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},L}$$

$$+\overline{A}_{x;i+\frac{1}{2},j+\frac{1}{4},L}\overline{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j+\frac{1}{4},L}$$

$$-\overline{A}_{z;i,j+\frac{1}{4}}\overline{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},L-\frac{1}{2}}]$$

$$+\frac{1}{\overline{\Delta z}_{i,j+\frac{1}{4},L}}[\overline{g}_{rnf}(\overline{V}_{rnf}-\overline{V}_{y;i,j+\frac{1}{4},L})+\overline{\sigma}^{*'}_{\nu,y,wnd;i,j+\frac{1}{4}}]$$

$$+\frac{\delta_{int,m;i,j+\frac{1}{4},L}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}]}\widetilde{G}_{int,m;i,j+\frac{1}{4},L}(\widetilde{V}_{y,int,m;i,j+\frac{1}{4},L}-\overline{V}_{y;i,j+\frac{1}{4},L})$$

$$+\frac{\delta_{dis,m;i,j+\frac{1}{4},L}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}]}\overline{G}_{dis,m;i,j+\frac{1}{4},L}(\overline{V}_{y,dis,m;i,j+\frac{1}{4},L}-\overline{V}_{y;i,j+\frac{1}{4},L}), \tag{3.99}$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the surface conditions, are defined, as

$\overline{V}_{y,rnf;i,j+\frac{1}{4}}$ = y-component of velocity associated with the rainfall on the water surface of the flow region.

$\overline{\sigma}^{*'}_{\nu,y,wnd;i,j-\frac{1}{4}}$ = modified wind-shear stress component acting on the z-downstream water enclosure-surface at $(i,j+\frac{1}{4},1+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},1)$, along y-direction.

It is important to note that the wind-shear stress component, along y-direction, depends on the free-surface flow conditions on the water enclosure-surface at $(i,j+\frac{1}{4},1+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},1)$ as well as the wind conditions over the water surface of the flow region.

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer $\overline{\Delta z}_L$ (Sec. 2.3). In these single layer subregion, a y-downstream half-element $(i,j+\frac{1}{4},1)$ can represent a bottom y-downstream half-element $[i,j+\frac{1}{4},1(bot;i,j)]$ and also a y-downstream half-element in the water-surface layer $(i,j+\frac{1}{4},L)$. For these special y-downstream half-elements $[i,j+\frac{1}{4},1(bot;i,j)]$ and also $(i,j+\frac{1}{4},L)$, the discrete- element equation for y-component of the discrete-element equation for y-component velocity [Eq. (3.96)] becomes

for $1(bot;i,j)=1=L$:

$$\frac{\partial}{\partial t}\overline{V}_{y;i,j+\frac{1}{4},L} = \frac{1}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}]}[\widetilde{G}_{y;i,j,L}(\widetilde{V}_{y;i,j,L}-\overline{V}_{y;i,j+\frac{1}{4},L})$$

$$-\widetilde{G}_{y;i,j+\frac{1}{2},L}(\widetilde{V}_{y;i,j+\frac{1}{2},L}-\overline{V}_{y;i,j+\frac{1}{4},L})$$

$$+\widetilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},L}(\widetilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},L}-\overline{V}_{y;i,j+\frac{1}{4},L})$$

$$-\widetilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},L}(\widetilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},L}-\overline{V}_{y;i,j+\frac{1}{4},L})]$$

$$+\frac{\overline{A}_{y;i,j+\frac{1}{4},L}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}]}(\overline{p}_{w;i,j,L}-\overline{p}_{w;i,j+\frac{1}{2},L})$$

(continue)

$$-g_o\cos\Theta_{n,y}+\Omega_{cor,N,z}\bar{V}_{x;i,j+\frac{1}{4},L}-\Omega_{cor,N,x}\bar{V}_{z;i,j+\frac{1}{4},L}$$

$$+\frac{1}{\left[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}\right]}\left[-\bar{A}_{y;i,j,L}\bar{\sigma}^{*'}_{\nu,y,y;i,j,L}+\bar{A}_{y;i,j+\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},L}\right.$$

$$-\bar{A}_{x;i-\frac{1}{2},j+\frac{1}{4},L}\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},L}$$

$$\left.+\bar{A}_{x;i+\frac{1}{2},j+\frac{1}{4},L}\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j+\frac{1}{4},L}\right]$$

$$+\frac{1}{\left[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},1}\right]}\left[\tilde{G}_{z,bot;i,j+\frac{1}{4}}(\tilde{V}_{y,bot;i,j+\frac{1}{4}}-\bar{V}_{y;i,j+\frac{1}{4},1})\right.$$

$$\left.-\bar{A}_{z;i,j+\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,y,bot;i,j+\frac{1}{4}}\right]$$

$$+\frac{1}{\overline{\Delta z}_{i,j+\frac{1}{4},L}}\left[\bar{g}_{rnf}(\bar{V}_{rnf}-\bar{V}_{y;i,j+\frac{1}{4},L})+\bar{\sigma}^{*'}_{\nu,y,wnd;i,j+\frac{1}{4}}\right]$$

$$+\frac{\delta_{int,m;i,j+\frac{1}{4},L}}{\left[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}\right]}\tilde{G}_{int,m;i,j+\frac{1}{4},L}(\tilde{V}_{y,int,m;i,j+\frac{1}{4},L}-\bar{V}_{y;i,j+\frac{1}{4},L})$$

$$+\frac{\delta_{dis,m;i,j+\frac{1}{4},L}}{\left[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},L}\right]}\bar{G}_{dis,m;i,j+\frac{1}{4},L}(\bar{V}_{y,dis,m;i,j+\frac{1}{4},L}-\bar{V}_{y;i,j+\frac{1}{4},L}). \tag{3.100}$$

The general computational algorithm of the transient, three-dimensional discrete-element model simultaneously solves for the two velocity components at four locations in (1) the half-element equation for x-component of velocity in the x-upstream half-element (i-¼,j,l) [Eqs. (3.51), (3.53), (3.54) and (3.55)], (2) the discrete-element equation for x-component of velocity in the x-downstream half-element (i+¼,j,l) [Eqs. (3.66), (3.68), (3.69) and (3.70)] , (3) the discrete-element equation for y-component of velocity in the y-upstream half-element

$(i,j-\frac{1}{4},1)$ [Eqs. (3.81), (3.83), (3.84) and (3.85)], and (4) the discrete-element equation for y-component of velocity in the y-downstream half-element $(i,j+\frac{1}{4},1)$ [Eqs. (3.96), (3.98), (3.99) and (3.100)].

It is important to note that this special formulation of the discrete-element method (Eraslan, 1976, 1977, 1981a, 1981b, 1981c) is completely different than all other existing, transient, multi-dimensional hydrodynamic models (e.g., Roache, 1972, Johnson, 1981). The special formulation of the discrete-element equations for the velocity components $\bar{V}_x$ and $\bar{V}_y$ in the horizontal (x,y) plane (in the general sense parallel to the geophysical surface plane and the free-surface plane of the flow region) has two distinct computational advantages over all existing numerical methods [i.e., finite-difference method (FDM) and finite-element method (FEM)]. First, since the half-elements are within an element and they divide the element into two approximately equal-size control-volumes, the geometrical computations are minimized for any variable grid system; and furthermore the interpolation formulas are vastly simpler in comparison to other numerical methods which utilize variable grid system. Second, and more importantly, the special formulation of the discrete-element method computes the velocity components strictly in the interior parts of the flow region. The computational algorithm does not include any computational points on the shoreline enclosure boundaries. Hence, the special formulation of the discrete-element completely eliminate the possiblility of error propagation from the non-slip boundaries of the flow region (e.g., Roache, 1972).

The most important feature of the discrete-element equations for the velocity component $\bar{V}_x$ [Eqs. (3.51), (3.53), (3.54), (3.55), (3.66),

(3.68), (3.69) and (3.70) and for the velocity component $\bar{V}_y$ [Eqs. (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99) and (3.100)] is that its mathematical formulation is distinctly different than the conventional "conservative" or "nonconservative" momentum principle. This special discrete-element form of the momentum equations is referred to as the "intensive" form of the discretized momentum equation (Eraslan, 1974, 1976, 1981a, 1981b, 1981c). Since, each convective transport term represents an intensive form of the momentum transport rate as the product of the flow rate and the difference between velocity of the water of either the inflow rate or the outflow rate and the velocity in the half-element, each convective momentum transport rate is strictly based on relative, intensive quantity as being the velocity difference. The numerical solution of the special intensive form of discrete-element equations guarantees that the solutions of the discretized forms of the momentum equations cannot be directly affected by the computational errors (e.g., truncation errors, numerical dispersion errors, etc.) that can be introduced in the solution of the velocity conditions (e.g., Eraslan, 1981a, 1981b).

The computational algorithm of the three-dimensional discrete-element method simultaneously solves for the velocity components $\bar{V}_x$ and $\bar{V}_y$ in the horizontal (x,y) plane in all computational half-elements by numerically integrating the discrete-element intensive form of the momentum principle [Eqs. (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100)].

Considering the definitions of modified pressure [Eq. (3.39)] and modified viscous stress components [Eq. (3.40)], the temporally primary-scale and spatially primary-scale z-component of the integral form of the momentum principle [Eq. (2.104)] is formulated for the control volume of the element (i,j,1) as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j,1}} \overline{V_z}\,dv + \iint_{\bar{A}_{i,j,1}} \overline{[V_z]\vec{V}\cdot\hat{n}}\,dA = -g_a \cos\theta_{n,z} \iiint_{\bar{V}_{i,j,1}} dv$$
$$+\Omega_{cor,N,x}\iiint_{\bar{V}_{i,j,1}} \overline{V_y}\,dv - \Omega_{cor,N,y}\iiint_{\bar{V}_{i,j,1}} \overline{V_x}\,dv$$
$$+\iint_{\bar{A}_{i,j,1}} \overline{[p_w(-\hat{n})]_z}\,dA + \iint_{\bar{A}_{i,j,1}} \overline{\sigma_{\nu,sz}}\,dA \,. \qquad (3.101)$$

The control-volume integral on the left-hand side of Eq. (3.101) can be formulated according to (1) the definition of the general control-volume integral for the control volume of the element (i,j,1) [Eqs. (2.83)] and (2.86a) and (2) the formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)] as

$$\iiint_{\bar{V}_{i,j,1}} \overline{V_z}\,dv = [A_{z;i,j}\overline{\Delta z}_{i,j,1}]\overline{V}_{z;i,j,1} \,, \qquad (3.102)$$

where the temporally primary-scale and spatially primary-scale flow variable is defined, as

$\overline{V}_{z;i,j,1}$ = z-component of velocity (in the general sense, normal to geophysical surface plane and the free-surface plane of the flow region) in the element (i,j,1).

The time derivative of the control-volume integral [Eq. (3.102)], on the left-hand side of Eq. (3.101), can be expanded, as

$$\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j,l}} \bar{V}_z dv = \frac{\partial}{\partial t}\{[A_{z;i,j}\overline{\Delta z}_{i,j,l}]\bar{V}_{z;i,j,l}\}$$

$$= [A_{z;i,j}\overline{\Delta z}_{i,j,l}]\frac{\partial}{\partial t}\bar{V}_{z;i,j,l} + \bar{V}_{z;i,j,l}\frac{\partial}{\partial t}[A_{z;i,j}\overline{\Delta z}_{i,j,l}]. \qquad (3.103)$$

Hence, multiplying the discrete-element form of the mass conservation principle in water for the element (i,j,l) [Eq. (3.4)] with the z-component of velocity $\overline{\Delta z}_{i,j,l}$ in the element (i,j,l) and substituting in Eq. (3.103), the time derivative of the control-volume integral [Eq. (3.103)] on the left-hand side of Eq. (3.101) becomes

$$\begin{aligned}\frac{\partial}{\partial t}\iiint_{\bar{V}_{i,j,l}} \bar{V}_z dv = {} & [A_{z;i,j}\overline{\Delta z}_{i,j,l}]\frac{\partial}{\partial t}\bar{V}_{z;i,j,l} \\ & + \tilde{G}_{x;i-\frac{1}{2},j,l}\bar{V}_{z;i,j,l} - \tilde{G}_{x;i+\frac{1}{2},j,l}\bar{V}_{z;i,j,l} \\ & + \tilde{G}_{y;i,j-\frac{1}{2},l}\bar{V}_{z;i,j,l} - \tilde{G}_{y;i,j+\frac{1}{2},l}\bar{V}_{z;i,j,l} \\ & + \tilde{G}_{z;i,j,l-\frac{1}{2}}\bar{V}_{z;i,j,l} - \tilde{G}_{z;i,j,l+\frac{1}{2}}\bar{V}_{z;i,j,l} \\ & + \delta_{int,m;i,j,l}\tilde{G}_{int,m;i,j,l}\bar{V}_{z;i,j,l} \\ & + \delta_{dis,m;i,j,l}\bar{G}_{dis,m;i,j,l}\bar{V}_{z;i,j,l}. \end{aligned} \qquad (3.104)$$

The closed control-surface integral on the left-hand side of Eq. (3.101) can be formulated according to (1) the definitions of the general closed control-surface integral for the control-volume of the

element (i,j,l) [Eq. (2.87)], and the free-surface plane of the flow regiongrals associated with the upstream enclosure-surface areas and the downstream enclosure-surface areas of the element (i,j,l), [Eqs. (2.90a), (2.90c), (2.90h), (2.90j), (2.91a), (2.92b) and (2,92e)], (3) the formulation of the general representation of the six enclosure-surface integral which constitute the closed control-surface integral of the temporally primary-scale and spatially primary-scale, z-component of the integral form of the momentum principle [Eq. (2.117)], (4) the formulation of the general representation of the surface integral associated with the cross-sectional area of the intake of power plant m, which con- stitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale z-component of the integral form of the momentum principle [Eq. (2.125)], and (5) the formulation of the general representation of the surface integral, associated with the cross-sectional area of the discharge of power plant m, which constitutes a small part of the closed control-surface integral of the temporally primary-scale and spatially primary-scale z-component of the integral form of the momentum principle [Eq. (2.126)], and by considering the general definitions of the upwind-differencing interpolation forms for the transportive, intensive flow variables, as

$$\iint_{\bar{A}_{i,j,l}} \overline{[V_z]\vec{V}\cdot\hat{n}}\,dA = -\widetilde{G}_{x;i-\frac{1}{2},j,l}\widetilde{V}_{z;i-\frac{1}{2},j,l} + \bar{A}_{x;i-\frac{1}{2},j,l}\bar{\sigma}^{*'}_{x,z;i-\frac{1}{2},j,l}$$

$$+\widetilde{G}_{x;i+\frac{1}{2},j,l}\widetilde{V}_{z;i+\frac{1}{2},j,l} - \bar{A}_{x;i+\frac{1}{2},j,l}\bar{\sigma}^{*'}_{x,z;i+\frac{1}{2},j,l}$$

(continue)

$$-\widetilde{G}_{y;i,j-\frac{1}{2},l}\widetilde{V}_{z;i,j-\frac{1}{2},l}+\overline{A}_{y;i,j-\frac{1}{2},l}\overline{\sigma}^{*'}_{y,z;i,j-\frac{1}{2},l}$$

$$+\widetilde{G}_{y;i,j+\frac{1}{2},l}\widetilde{V}_{z;i,j+\frac{1}{2},l}-\overline{A}_{y;i,j+\frac{1}{2},l}\overline{\sigma}^{*'}_{y,z;i,j+\frac{1}{2},l}$$

$$-\widetilde{G}_{z;i,j,l-\frac{1}{2}}\widetilde{V}_{z;i,j,l-\frac{1}{2}}+A_{z;i,j}\overline{\sigma}^{*'}_{z,z;i,j,l-\frac{1}{2}}$$

$$+\widetilde{G}_{z;i,j,l+\frac{1}{2}}\widetilde{V}_{z;i,j,l+\frac{1}{2}}-A_{z;i,j}\overline{\sigma}^{*'}_{z,z;i,j,l+\frac{1}{2}}$$

$$-\delta_{int,m;i,j,l}\widetilde{G}_{int,m;i,j,l}\widetilde{V}_{z,int,m;i,j,l}$$

$$-\delta_{dis,m;i,j,l}\overline{G}_{dis,m;i,j,l}\overline{V}_{z,dis,m;i,j,l}\,, \qquad (3.105)$$

where the temporally primary-scale and spatially primary-scale flow variables are defined, as

$\widetilde{V}_{z;i-\frac{1}{2},j,l}$ = upwind-differenced value of z-component of velocity (in the general sense, normal to the geophysical surface plane and the free-surface plane of the flow region) at x-upstream enclosure-surface at (i-½,j,l) of the element (i,j,l) [Eq. (2.117)],

$\overline{\sigma}^{*'}_{x,z;i-\frac{1}{2},j,l}$ = combined convention-defect and eddy-turbulent stress component (excluding only the component of modified viscous stress) acting on the x-upstream enclosure-surface area $\overline{A}_{x;i-\frac{1}{2},j,l}$ at (i-½,j,l) of the element (i,j,l), along z-direction [Eq. (2.118)],

$\widetilde{V}_{z;i+\frac{1}{2},j,l}$ = upwind-differenced value of z-component of velocity at x-downstream enclosure surface at (i+½,j,l) of the element (i,j,l) [Eq. (2.117)],

$\overline{\sigma}^{*'}_{x,z;i+\frac{1}{2},j,l}$ = combined convection-defect and eddy-turbulent stress component acting on the x-downstream enclosure-surface area $\overline{A}_{x;i+\frac{1}{2},j,l}$ at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), along z-direction [Eq. (2.118)] ,

$\overline{V}_{z;i,j-\frac{1}{2},l}$ = upwind-differenced value of z-component of velocity at y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the elements (i,j,l) [Eq. (2.117)]

$\overline{\sigma}^{*'}_{y,z;i,j-\frac{1}{2},l}$ = combined convection-defect and eddy-turbulent stress component acting on the y-upstream enclosure-surface area $\overline{A}_{y;i,j-\frac{1}{2},l}$ at $(i,j-\frac{1}{2},l)$ of the element (i,j,l), along z-direction [Eq. (2.118)],

$\widetilde{V}_{z;i,j+\frac{1}{2},l}$ = upwind-differenced value of z-component of velocity at y-downstream enclosure-surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) [Eq. (2.117)],

$\overline{\sigma}^{*'}_{y,z;i,j+\frac{1}{2},l}$ = combined convection-defect and eddy-turbulent stress component acting on the y-downstream enclosure-surface area $\overline{A}_{y;i,j+\frac{1}{2},l}$ at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), along z-direction [Eq. (2.118)] ,

$\widetilde{V}_{z;i,j,l-\frac{1}{2}}$ = upwind-differenced value of z-component of velocity at z-upstream enclosure-surface at $(i,j,l-\frac{1}{2})$ of the x-upstream element (i,j,l) [Eq. (2.117)],

$\overline{\sigma}^{*'}_{z,z;i,j,l-\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-upstream enclosure-surface area $A_{z;i,j}$ at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), along z-direction [Eq. (2.118)],

$\tilde{\bar{V}}_{z;i,j,l+\frac{1}{2}}$ = upwind-differenced value of z-component of velocity at z-downstream enclosure-surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l) [Eq. (2.117)],

$\bar{\sigma}^{*'}_{z,z;i,j,l+\frac{1}{2}}$ = combined convection-defect and eddy-turbulent stress component acting on the z-downstream enclosure-surface area $A_{z;i,j}$ at $(i,j,l+\frac{1}{2})$ of the element (i,j,l), along z-direction [Eq. (2.118)],

$\tilde{\bar{V}}_{z,int,m;i,j,l}$ = upwind-differenced value of z-component of velocity associated with the intake flow of power plant m in the element (i,j,l)

$\bar{V}_{z,dis,m;i,j,l}$ = z-component of velocity associated with the discharge flow of power plant m in the element (i,j,l).

The first control-volume integral on the right-hand side of Eq. (3.101), associated with the gravity force term along z-direction, is already evaluated for the discrete-element formulation of the temporally primary-scale and spatially primary-scale, integral form of the conservation of mass principle for water [Eq. (3.2)].

The control-volume integrals on the right-hand side of Eq. (3.101), associated with the Coriolis acceleration (force) terms are also formulated according to (1) the definition of the general control-volume integral for the control volume of the element (i,j,l) [Eqs. (2.83) and (2.86)] and (2) formulation of the control-volume integral of the temporally primary-scale and spatially primary-scale, three components of the integral form of the momentum principle [Eq. (2.106)] as

$$\iiint_{\bar{V}_{i,j,l}} \bar{V}_y dv = [A_{z;i,j}\overline{\Delta z}_{i,j,l}]\bar{V}_{y;i,j,l}, \qquad (3.106)$$

$$\iiint_{\bar{V}_{i,j,l}} \bar{V}_x dv = [A_{z;i,j} \overline{\Delta z}_{i,j,l}] \bar{V}_{x;i,j,l}, \tag{3.107}$$

where the temporally primary-scale and spatially primary-scale flow variables are defined as

$\bar{V}_{y;i,j,l}$ = y-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the geophysical surface plane and the free-surface plane of the flow region)] in the element (i,j,l) [Eq. (2.106)],

$\bar{V}_{x;i,j,l}$ = x-component of velocity [in the horizontal (x,y) plane (in the general sense, parallel to the flow region and the geophysical surface plane)] in the element (i,j,l) [Eq. (2.106)].

According to the geometrical discretization technique of the three-dimensional discrete-element method (Sec. 2.3) for geophysical flow regions, the z-upstream enclosure-surface area at $(i,j,l-\frac{1}{2})$ is approximately equal to the z-downstream enclosure-surface area at $(i,j,l+\frac{1}{2})$ as $A_{z;i,j}$ of the element (i,j,l). Hence, in boundary elements, which represent nonrectangular enclosure-surface areas are assumed to be parallel to the z-direction. Therefore, the only z-components of the modified pressure force that can act on the on the enclosure surface areas of the element (i,j,l) act on the z-upstream enclosure-surface at $(i,j,l-\frac{1}{2})$ and on the z-downstream enclosure-surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l).

Hence, the control-surface integral associated with the modified pressure-force term on the right-hand side of Eq. (3.101) can be formulated according to (1) the definitions of the general closed control-

surface integral for the control control volume of the element (i,j,l) [Eq. (2.87)], (2) the definitions of the enclosure-surface integrals associated with the upstream enclosure-surface areas and downstream enclosure-surface areas of the element (i,j,l) [Eqs. (2.90a), (2.90c), (2.90h), (2.90j), (2.91a), (2.92b), (2.92e) and (2.92g)], and by considering the z-components of the modified pressure forces act only on the enclosure-surface areas normal to the z-direction, as

$$\iint_{\bar{A}_{i,j,l}} \overline{[p_w(-\hat{n})]_z} dA = A_{z;i,j}(\bar{p}_{w;i,j,l-\frac{1}{2}} - \bar{p}_{w;i,j,l+\frac{1}{2}}), \tag{3.108}$$

the temporally primary-scale and spatially primary-scale values of the modified pressure are defined, as

$\bar{p}_{w;i,j,l-\frac{1}{2}}$ = modified pressure at the location of the z-upstream enclosure-surface at (i,j,l-½) of the element (i,j,l),

$\bar{p}_{w;i,j,l+\frac{1}{2}}$ = modified pressure at the location of the z-downstream enclosure-surface at (i,j,l+½) of the element (i,j,l).

The discrete-element formulation of the control-surface integral, associated with the modified pressure force term for the element (i,j,l) [Eq. (3.108)] is applicable to all element, including (1) bottom element [i,j,l(bot;i,j)], (2) vertically interior elements [i,j,l(bot;i,j)¼l¼L], and (3) elements in the water-surface layer (i,j,L). Furthermore, according to the discrete-element formulation of the control-surface integral [Eq. (3.108)], the modified pressure force term acting on the element (i,j,l) is completely independent of the value of the spatially uniform atmospheric pressure on the free-surface of the water in the flow region, since it includes only the difference of the two pressure

values at the upstream enclosure-surface at $(i-\frac{1}{2},j,l)$ and at the downstream enclosure-surface at (i,j,l).

The control-surface integral associated with the modified viscous (molecular) force term, on the right-hand side of Eq. (3.101) can be evaluated according to (1) the definitions of the general closed control-surface integral for the control-volume of element (i,j,l) [Eq. (2.87)] and (2) the definitions of the enclosure-surface integrals, associated with the upstream enclosure-surface areas and the downstream enclosure surface areas of the element (i,j,l) [Eqs. (2.90a), (2.90c), (2.90h), (2.90j), (2.91a), (3.92b), (2.92e) and (2.92g)], as

$$\iint_{\bar{A}_{i,j,l}} \overline{\sigma_{\nu,\ast x}} dA = -\bar{A}_{x;i-\frac{1}{2},j,l}\bar{\sigma}_{\nu,xz;i-\frac{1}{2},j,l} + \bar{A}_{x;i+\frac{1}{2},j,l}\bar{\sigma}_{\nu,xz;i+\frac{1}{2},j,l}$$

$$-\bar{A}_{y;i,j-\frac{1}{2},l}\bar{\sigma}_{\nu,yz;i,j-\frac{1}{2},l} + \bar{A}_{y;i,j+\frac{1}{2},l}\bar{\sigma}_{\nu,yz;i,j-\frac{1}{2},l}$$

$$-A_{z;i,j}\bar{\sigma}_{\nu,zz;i,j,l-\frac{1}{2}} + A_{z;i,j}\bar{\sigma}_{\nu,zz;i,j,l+\frac{1}{2}}, \qquad (3.109)$$

where the temporally primary-scale and spatially primary-scale modified viscous stress components are defined, as

$\bar{\sigma}_{\nu,xz;i-\frac{1}{2},j,l}$ = modified viscous stress component acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j,l}$ at $(i-\frac{1}{2},j,l)$ of the element (i,j,l), along z-direction,

$\bar{\sigma}_{\nu,xz;i+\frac{1}{2},j,l}$ = modified viscous stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j,l}$ at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), along z-direction,

$\bar{\sigma}_{\nu,yz;i,j-\frac{1}{2},l}$ = modified viscous stress component acting on the y-upstream enclosure-surface area $\bar{A}_{y;i,j-\frac{1}{2},l}$ at $(i,j-\frac{1}{2},l)$ of

the element (i,j,l), along z-direction,

$\bar{\sigma}_{\nu,yz;i,j-\frac{1}{2},l}$ = modified viscous stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i,j+\frac{1}{2},l}$ at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), along z-direction,

$\bar{\sigma}_{\nu,zz;i,j,l-\frac{1}{2}}$ = modified viscous stress component acting on the z-upstream enclosure-surface area $A_{z;i,j}$ at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), along z-direction,

$\bar{\sigma}_{\nu,zz;i,j,l+\frac{1}{2}}$ = modified viscous stress component acting on the z-downstream enclosure-surface area $A_{z;i,j}$ at $(i,j,l+\frac{1}{2})$ of the element (i,j,l), along z-direction.

Hence, substituting the discrete-element formulations of (1) the time derivative of the control-volume integral [Eq. (3.104)] of the left-hand side of Eq. (3.101), (2) the control-surface inteqral [Eq. (3.105)] of the left-hand side of Eq. (3.101), (3) the control-volume integral associated with the gravity force term along z-direction [Eq. (3.2)] of the right-hand side of Eq. (3.101), (4) the two control-volume integrals associated with the Coriolis acceleration (force) term [Eqs. (3.106) and (3.107)] of the right-hand side of Eq. (3.101), (5) the control-surface integral associated with the modified pressure force term [Eq. (3.108)] of the right-hand side of Eq.(3.101), and (6) the control-surface integral [Eq. (3.109)] associated with the modified viscous force term [Eq. (3.109)] of the right-hand side of Eq. (3.101), in the temporally primary-scale and spatially primary-scale z-component of the integral form of the momentum principle for the control-volume of the element (i,j,l) [Eq. (3.101)], combining terms and rearranging, the

discrete-element form of the z-component of the momentum principle for the element (i,j,l) becomes,

$$\overline{P}_{w;i,j,l-\frac{1}{2}} = \overline{P}_{w;i,j,l+\frac{1}{2}} + \overline{\Delta z}_{i,j,l}\frac{\partial}{\partial t}\overline{V}_{z;i,j,l}$$

$$-\frac{1}{A_{z;i,j}}\left[\widetilde{G}_{x;i-\frac{1}{2},j,l}(\widetilde{V}_{z;i-\frac{1}{2},j,l}-\overline{V}_{z;i,j,l}) - \widetilde{G}_{x;i,j,l}(\widetilde{V}_{z;i+\frac{1}{2},j,l}-\overline{V}_{z;i,j,l})\right.$$

$$+\widetilde{G}_{y;i,j-\frac{1}{2},l}(\widetilde{V}_{z;i,j-\frac{1}{2},l}-\overline{V}_{z;i,j,l}) - \widetilde{G}_{y;i,j+\frac{1}{2},l}(\widetilde{V}_{z;i,j+\frac{1}{2},l}-\overline{V}_{z;i,j,l})$$

$$\left.+\widetilde{G}_{z;i,j,l-\frac{1}{2}}(\widetilde{V}_{z;i,j,l-\frac{1}{2}}-\overline{V}_{z;i,j,l}) - \widetilde{G}_{z;i,j,l+\frac{1}{2}}(\widetilde{V}_{z;i,j,l+\frac{1}{2}}-\overline{V}_{z;i,j,l})\right]$$

$$+g_0\cos\theta_{n,z}\overline{\Delta z}_{i,j,l} + \overline{\Delta z}_{i,j,l}\left[\Omega_{cor,N,y}\overline{V}_{x;i,j,l} - \Omega_{cor,N,x}\overline{V}_{y;i,j,l}\right]$$

$$+\frac{1}{A_{z;i,j}}\left[+\overline{A}_{x;i-\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,l} - \overline{A}_{x;i+\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,l}\right.$$

$$+\overline{A}_{y;i,j-\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},l} - \overline{A}_{y;i,j+\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,z;i,j+\frac{1}{2},l}$$

$$\left.+A_{z;i,j}\overline{\sigma}^{*'}_{\nu,z,z;i,j,l-\frac{1}{2}} - A_{z;i,j}\overline{\sigma}^{*'}_{\nu,z,z;i,j,l+\frac{1}{2}}\right]$$

$$-\frac{\delta_{int,m;i,j,l}}{A_{z;i,j}}\widetilde{G}_{int,m;i,j,l}(\widetilde{V}_{z,int,m;i,j,l}-\overline{V}_{z;i,j,l})$$

$$-\frac{\delta_{dis,m;i,j,l}}{A_{z;i,j}}\overline{G}_{dis,m;i,j,l}(\overline{V}_{z,dis,m;i,j,l}-\overline{V}_{z;i,j,l}), \qquad (3.110)$$

where the temporally primary-scale and spatially primary-scale stress components are defined, as

$\bar{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,l}$ = modified stress component (consisting of modified viscous stress component, and convection-defect and eddy-turbulent stress component) acting on the x-upstream enclosure-surface area $\bar{A}_{x;i-\frac{1}{2},j,l}$ at $(i-\frac{1}{2},j,l)$ of the element (i,j,l), along z-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,l} = \bar{\sigma}_{\nu,xz;i-\frac{1}{2},j,l} + \bar{\sigma}^{*'}_{x,z;i-\frac{1}{2},j,l}, \qquad (3.111a)$$

$\bar{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,l}$ = modified stress component acting on the x-downstream enclosure-surface area $\bar{A}_{x;i+\frac{1}{2},j,l}$ at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), along z-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,l} = \bar{\sigma}_{\nu,xz;i+\frac{1}{2},j,l} + \bar{\sigma}^{*'}_{x,z;i+\frac{1}{2},j,l}, \qquad (3.111b)$$

$\bar{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},l}$ = modified stress component acting on the y-upstream enclosure-surface area $\bar{A}_{y;i,j-\frac{1}{2},l}$ at $(i,j-\frac{1}{2},l)$ of the x-upstream element (i,j,l), along z-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},l} = \bar{\sigma}_{\nu,yz;i,j-\frac{1}{2},l} + \bar{\sigma}^{*'}_{y,z;i,j-\frac{1}{2},l}, \qquad (3.111c)$$

$\bar{\sigma}^{*'}_{\nu,y,z;i,j+\frac{1}{2},l}$ = modified stress component acting on the y-downstream enclosure-surface area $\bar{A}_{y;i,j+\frac{1}{2},l}$ at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), along z-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,y,z;i,j+\frac{1}{2},l} = \bar{\sigma}_{\nu,yz;i,j+\frac{1}{2},l} + \bar{\sigma}^{*'}_{y,z;i,j+\frac{1}{2},l} \ , \tag{3.111d}$$

$\bar{\sigma}^{*'}_{\nu,z,z;i,j,l-\frac{1}{2}}$ = modified stress component acting on the z-upstream enclosure-surface area $A_{z;i,j}$ at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), along z-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,z,z;i,j,l-\frac{1}{2}} = \bar{\sigma}_{\nu,zz;i,j,l-\frac{1}{2}} + \bar{\sigma}^{*'}_{z,z;i,j,l-\frac{1}{2}} \ , \tag{3.111e}$$

$\bar{\sigma}^{*'}_{\nu,z,z;i,j,l+\frac{1}{2}}$ = modified stress component acting on the z-downstream enclosure-surface area $A_{z;i,j}$ at $(i,j,l+\frac{1}{2})$ of the x-upstream element (i,j,l), along z-direction,

which represents

$$\bar{\sigma}^{*'}_{\nu,z,z;i,j,l+\frac{1}{2}} = \bar{\sigma}_{\nu,zz;i,j,l+\frac{1}{2}} + \bar{\sigma}^{*'}_{z,z;i,j,l+\frac{1}{2}} \ . \tag{3.111f}$$

This discrete-element form of the temporally primary-scale and spatially primary-scale integral form of the z-component of the momentum principle for the element (i,j,l) [Eq. (3.110)] represents the "discrete-element equation for modified pressure".

For a bottom element $[i,j,l(bot;i,j)]$, which includes the bottom of the flow region as the z-upstream bottom enclosure-surface at $[i,j,l(bot;i,j)-\frac{1}{2}]$, the discrete-element equation for modified pressure [Eq. (3.110)] becomes

for $l = l(bot;i,j)$:

$$\overline{p}_{w;i,j,l-\frac{1}{2}} = \overline{p}_{w;i,j,l+\frac{1}{2}} + \overline{\Delta z}_{i,j,l}\frac{\partial}{\partial t}\overline{V}_{z;i,j,l}$$

$$-\frac{1}{A_{z;i,j}}\left[\widetilde{G}_{x;i-\frac{1}{2},j,l}(\widetilde{V}_{z;i-\frac{1}{2},j,l}-\overline{V}_{z;i,j,l}) - \widetilde{G}_{x;i,j,l}(\widetilde{V}_{z;i+\frac{1}{2},j,l}-\overline{V}_{z;i,j,l})\right.$$

$$+\widetilde{G}_{y;i,j-\frac{1}{2},l}(\widetilde{V}_{z;i,j-\frac{1}{2},l}-\overline{V}_{z;i,j,l}) - \widetilde{G}_{y;i,j+\frac{1}{2},l}(\widetilde{V}_{z;i,j+\frac{1}{2},l}-\overline{V}_{z;i,j,l})$$

$$\left.-\widetilde{G}_{z;i,j,l+\frac{1}{2}}(\widetilde{V}_{z;i,j,l+\frac{1}{2}}-\overline{V}_{z;i,j,l})\right]$$

$$+g_0\cos\theta_{n,z}\overline{\Delta z}_{i,j,l} + \overline{\Delta z}_{i,j,l}\left[\Omega_{cor,N,y}\overline{V}_{x;i,j,l} - \Omega_{cor,N,x}\overline{V}_{y;i,j,l}\right]$$

$$+\frac{1}{A_{z;i,j}}\left[+\overline{A}_{x;i-\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,l} - \overline{A}_{x;i+\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,l}\right.$$

$$+\overline{A}_{y;i,j-\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},l} - \overline{A}_{y;i,j+\frac{1}{2},l}\overline{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},l}$$

$$\left.-A_{z;i,j}\overline{\sigma}^{*'}_{\nu,z,z;i,j,l+\frac{1}{2}}\right]$$

$$-\frac{1}{A_{z;i,j}}\left[\widetilde{G}_{z,bot;i,j}(\widetilde{V}_{z,bot;i,j}-\overline{V}_{z;i,j,l})\right]$$

$$-\frac{\delta_{int,m;i,j,l}}{A_{z;i,j}}\widetilde{G}_{int,m;i,j,l}(\widetilde{V}_{z,int,m;i,j,l}-\overline{V}_{z;i,j,l})$$

$$-\frac{\delta_{dis,m;i,j,l}}{A_{z;i,j}}\overline{G}_{dis,m;i,j,l}(\overline{V}_{z,dis,m;i,j,l}-\overline{V}_{z;i,j,l}), \qquad (3.112)$$

where the temporally primary-scale and spatially primary-scale flow variables, associated with the bottom flow conditions, are defined as

$\widetilde{V}_{z,bot;i,j}$ = upwind-differenced value of z-component of velocity associated with the bottom seepage flow rate at the z-up-

stream bottom enclosure-surface at $[i,j,1(bot;i,j)-\frac{1}{2}]$ of the element $(i,j,1)$

It is important to note that the normal component of stress at the bottom enclosure-surface area consists only of the modified pressure $\overline{P}_{w;i,j,1-\frac{1}{2}}$. Hence, the discrete-element equation for modified pressure for the bottom element $[i,j,1(bot;i,j)]$ [Eq. (3.112)] does not include the modified stress component $\overline{\sigma}^{*'}_{\nu,z,z;i,j,1-\frac{1}{2}}$ acting on the z-upstream bottom enclosure-surface at $[i,j,1(bot;i,j)-\frac{1}{2}]$; since the contribution of the z-component of velocity associated with the bottom seepage flow rate is assumed to be negligible.

For a surface element in the water-surface layer (i,j,L), which contains the free-surface layer of the water in the flow region, the discrete-element equation for modified pressure [Eq. (3.110)] becomes, for $1 = L$:

$$\overline{P}_{w;i,j,L-\frac{1}{2}} = \overline{\Delta z}_{i,j,L}\frac{\partial}{\partial t}\overline{V}_{z;i,j,L}$$

$$-\frac{1}{A_{z;i,j}}\left[\widetilde{G}_{x;i-\frac{1}{2},j,L}(\widetilde{V}_{z;i-\frac{1}{2},j,L}-\overline{V}_{z;i,j,L})-\widetilde{G}_{x;i,j,L}(\widetilde{V}_{z;i+\frac{1}{2},j,L}-\overline{V}_{z;i,j,L})\right.$$

$$+\widetilde{G}_{y;i,j-\frac{1}{2},L}(\widetilde{V}_{z;i,j-\frac{1}{2},L}-\overline{V}_{z;i,j,L})-\widetilde{G}_{y;i,j+\frac{1}{2},L}(\widetilde{V}_{z;i,j+\frac{1}{2},L}-\overline{V}_{z;i,j,L})$$

$$\left.+\widetilde{G}_{z;i,j,L-\frac{1}{2}}(\widetilde{V}_{z;i,j,L-\frac{1}{2}}-\overline{V}_{z;i,j,L})\right]$$

$$+g_a\cos\theta_{n,z}\overline{\Delta z}_{i,j,L}+\overline{\Delta z}_{i,j,L}[\Omega_{cor,N,y}\overline{V}_{x;i,j,L}-\Omega_{cor,N,x}\overline{V}_{y;i,j,L}]$$

(continue)

$$+\frac{1}{A_{z;i,j}}\left[+\bar{A}_{x;i-\frac{1}{2},j,L}\bar{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,L}-\bar{A}_{x;i+\frac{1}{2},j,L}\bar{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,L}\right.$$

$$+\bar{A}_{y;i,j-\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},L}-\bar{A}_{y;i,j+\frac{1}{2},L}\bar{\sigma}^{*'}_{\nu,y,z;i,j+\frac{1}{2},L}$$

$$\left.+\bar{A}_{z;i,j}\bar{\sigma}^{*'}_{\nu,z,z;i,j,L-\frac{1}{2}}\right]$$

$$-\bar{g}_{rnf}(\bar{V}_{z;rnf}-\bar{V}_{z;i,j,L})$$

$$-\frac{\delta_{int,m;i,j,L}}{A_{z;i,j}}\tilde{G}_{int,m;i,j,L}(\tilde{V}_{z,int,m;i,j,L}-\bar{V}_{z;i,j,L})$$

$$-\frac{\delta_{dis,m;i,j,L}}{A_{z;i,j}}\bar{G}_{dis,m;i,j,L}(\bar{V}_{z,dis,m;i,j,L}-\bar{V}_{z;i,j,L})\,, \tag{3.113}$$

where the temporally primary-scale and spatially primary-scale rainfall condition is defined as

$\bar{V}_{z;rnf}$ = z-component of velocity associated with the rainfall on the water surface of the flow region.

It is again important to note that the normal component of stress at the water surface area is not included. The modified pressure [Eq. (3.39)] identically vanishes on the water surface for specified spatially uniform atmospheric conditions over the flow region. Furthermore, the normal modified viscous stress component along z-direction on the water surface due to the z-component of rainfall velocity is negligible.

The general formulation of the discrete-element equation for modified pressure for a surface element in the water surface layer (i,j,L) includes the effect of the vertical component of the momentum transport associated with the rainfall which corresponds, in a sense, to the stag-

nation pressure effect of the rainfall on the water surface. Considering the "intensive form" of the discrete-element formulation of the z-component of the momentum principle, the local effect of spatially uniform rainfall over the flow region can be important on the local flow conditions in shallow subregions.

In certain subregions, the specified depth $D_{i,j}$ of the flow region in a planar element can be less than or equal to the specified depth of the top layer $\overline{\Delta z}_L$ (Sec. 2.3). In these single layer subregions, an element (i,j,1) can represent a bottom element [i,j,1(bot;i,j)] and also an element in the water-surface layer (i,j,L). For these special elements [i.j,1(bot;i,j)] and also (i,j,L), the discrete-element equation for modified pressure [Eq. (3.110)] becomes

for 1(bot:i,j) = 1 = L:

$$\overline{p}_{w;i,j,L-\frac{1}{2}} = \overline{\Delta z}_{i,j,L}\frac{\partial}{\partial t}\overline{V}_{z;i,j,L}$$

$$-\frac{1}{A_{z;i,j}}\left[\widetilde{G}_{x;i-\frac{1}{2},j,L}(\widetilde{V}_{z;i-\frac{1}{2},j,L}-\overline{V}_{z;i,j,L})-\widetilde{G}_{x;i,j,L}(\widetilde{V}_{z;i+\frac{1}{2},j,L}-\overline{V}_{z;i,j,L})\right.$$

$$\left.+\widetilde{G}_{y;i,j-\frac{1}{2},L}(\widetilde{V}_{z;i,j-\frac{1}{2},L}-\overline{V}_{z;i,j,L})-\widetilde{G}_{y;i,j+\frac{1}{2},L}(\widetilde{V}_{z;i,j+\frac{1}{2},L}-\overline{V}_{z;i,j,L})\right]$$

$$+g_o\cos\Theta_{n,z}\overline{\Delta z}_{i,j,L}+\overline{\Delta z}_{i,j,L}\left[\Omega_{cor,N,y}\overline{V}_{x;i,j,L}-\Omega_{cor,N,x}\overline{V}_{y;i,j,L}\right]$$

$$+\frac{1}{A_{z;i,j}}\left[+\overline{A}_{x;i-\frac{1}{2},j,L}\overline{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,L}-\overline{A}_{x;i+\frac{1}{2},j,L}\overline{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,L}\right.$$

$$\left.+\overline{A}_{y;i,j-\frac{1}{2},L}\overline{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},L}-\overline{A}_{y;i,j+\frac{1}{2},L}\overline{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},L}\right]$$

(continue)

$$-\frac{1}{A_{z;i,j}}\left[\tilde{G}_{z,bot;i,j}(\tilde{V}_{z,bot;i,j}-\bar{V}_{z;i,j,L})\right]$$

$$-\bar{g}_{rnf}(\bar{V}_{z;rnf}-\bar{V}_{z;i,j,L})$$

$$-\frac{\delta_{int,m;i,j,L}}{A_{z;i,j}}\tilde{G}_{int,m;i,j,L}(\tilde{V}_{z,int,m;i,j,L}-\bar{V}_{z;i,j,L})$$

$$-\frac{\delta_{dis,m;i,j,L}}{A_{z;i,j}}\bar{G}_{dis,m;i,j,L}(\bar{V}_{z,dis,m;i,j,L}-\bar{V}_{z;i,j,L}). \qquad (3.114)$$

It is important to note that the discrete-element equation for modified pressure [Eqs. (3.110), (3.112), (3.113) and (3.114)] includes all the terms of the discrete-element formulation of the vertical component of the momentum principle without resorting to the conventional hydrostatic pressure approximation.

3.4. Discussion of the Discrete-Element Equations

The general discrete-element formulation of the mathematical model considers the solutions of seven spatially primary-scale and temporally primary-scale flow variables at various locations in the elements and half-elements and on the enclosure surfaces of the elements and half-elements which represent the principal flow variables of solution of the general discrete-element system of computational equations.

The principal flow variable of solution $G_{z;i,j,l+\frac{1}{2}}$ flow rate along z-direction, is solved for on the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l), according to the algebric equation which represents the discrete-element form of the mass conservation principle for water in the element (i,j,l) [Eqs (3.5), (3.6) and (3.7)].

The principal flow variable of solution $\overline{H}_{i,j}$, water surface elevation, is solved for in the element (i,j,L) in the water surface layer (l = L), according to the first-order, ordinary differential equation which represents the discrete-element form of the mass conservation principle for water in the element (i,j,L) in the water surface layer (l = L) [Eqs. (3.9) and (3.10)].

The principal flow variable of solution $\overline{V}_{x;i-\frac{1}{4},j,l}$, x-component of velocity, is solved for in the x-upstream half-element (i-¼,j,l), according to the first-order, ordinary differential equation which represents the discrete-element form of the x-component of the momentum principle for the x-upstream half-element (i-¼,j,l) [Eqs. (3.51), (3.53), (3.54) and (3.55)].

The principal flow variable of solution $\overline{V}_{x;i+\frac{1}{4},j,l}$, x-component of velocity, is also solved for in the x-downstream half-element (i+¼,j,l), according to the first-order, ordinary differential equation which represents the discrete-element form of the x-component of the momentum principle for the x-downstream half-element (i+¼,j,l) Eqs. (3.66), (3.68), (3.69) and (3.100) .

The principal flow variable of solution $\overline{V}_{y;i,j-\frac{1}{4},l}$, y-component of velocity, is solved for in the y-upstream half-element (i,j-¼,l), according to the first-order, ordinary differential equation which represents the discrete-element form of the y-component of the momentum principle for the y-upstream half-element (i,j-¼,l) [Eqs. (3.81), (3.83), (3.84) and (3.85)].

The principal flow variable of solution $\overline{V}_{y;i,j+\frac{1}{4},l}$, y-component of velocity, is also solved for in the y-downstream half-element (i,j+¼,l),

according to the first-order, ordinary differential equation which represents the discrete-element form of the y-component of the momentum principle for the y-downstream half-element $(i,j+\frac{1}{4},l)$ [Eqs. (3.96), (3.98), (3.99) and (3.100)].

The principal flow variable of solution $\bar{p}_{w;i,j,l-\frac{1}{2}}$, pressure, is solved for on the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), according to the algebric equation which represents the discrete-element form of the z-component of the momentum principle for the element (i,j,l) Eqs. (3.110), (3.112), (3.113) and (3.114) .

The governing system of discrete-element computational equations are not consistent for the solution of the seven principal flow variables of the solution, since the discrete-element equations also contain the values of the flow variables at auxiliary locations, not corresponding to the solution locations, and other auxiliary flow variables acting on the enclosure elements. The closure of the governing system of discrete-element computational equations is presented in the subsequent section.

4. CLOSURE OF THE COMPUTATIONAL SYSTEM OF DISCRETE-ELEMENT

The values of the flow variables at auxiliary locations which do not correspond to the solution locations of the principal flow variables and the auxiliary flow variables which appear in the governing computational system of discrete-element equations [Eqs. (3.5), (3.6), (3.7), (3.9), (3.10), (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113) and (3.114)] are eliminated by considering various formulations associated with the general closure procedure for the mathematical model.

4.1. Standard Linear-Interpolation formulation for the Auxiliary Values of the Geometrical Variables

The geometrical variables of the discrete-element formulation of the mathematical model includes the auxiliary values of the flow variable water-surface elevation $\bar{H}$ at auxiliary locations which do not correspond to the solution locations for the principal flow variable $\bar{H}_{i,j}$, water-surface elevation in the element (i,j,L) in the water-surface layer (l=L). All auxiliary values of the water surface elevation at the auxiliary locations are formulated according to the standard linear-interpolation formulas based on the sizes of the adjacent elements (see Sec. 2.3).

The auxiliary value of water-surface elevation $\bar{H}_{i-\frac{1}{2},j}$ at the x-upstream enclosure surface at $(i-\frac{1}{2},j,L)$ of the element (i,j,L) and the

x-upstream half-element ($i-\frac{1}{4},j,L$) in the water-surface layer ($l=L$) is formulated, as:

$$\overline{H}_{i-\frac{1}{2},j} = \overline{H}_{i,j} + \frac{\Delta x_{i,j}}{(\Delta x_{i,j}+\Delta x_{i-1,j})} (\overline{H}_{i,j}-\overline{H}_{i-1,j}). \tag{4.1a}$$

The auxiliary value of water-surface elevation $\overline{H}_{i-\frac{1}{4},j}$ in the x-upstream half-element ($i-\frac{1}{4},j,L$) in the water-surface layer ($l=L$) is formulated, as:

$$\overline{H}_{i-\frac{1}{4},j} = \frac{1}{2}(\overline{H}_{i,j}+\overline{H}_{i-\frac{1}{2},j}). \tag{4.1b}$$

The auxiliary value of water-surface elevation $\overline{H}_{i+\frac{1}{2},j}$ at the x-downstream enclosure surface at ($i+\frac{1}{2},j,L$) of the element (i,j,L) and the x-downstream half-element ($i+\frac{1}{4},j,L$) in the water-surface layer ($l=L$) is formulated, as:

$$\overline{H}_{i+\frac{1}{2},j} = \overline{H}_{i,j} + \frac{\Delta x_{i,j}}{(\Delta x_{i,j}+\Delta x_{i+1,j})} (\overline{H}_{i+1,j}-\overline{H}_{i,j}). \tag{4.1c}$$

The auxiliary value of water-surface elevation $\overline{H}_{i+\frac{1}{4},j}$ in the x-downstream half-element ($i+\frac{1}{4},j,L$) in the water-surface layer ($l=L$) is formulated, as:

$$\overline{H}_{i+\frac{1}{4},j} = \frac{1}{2}(\overline{H}_{i,j}+\overline{H}_{i+\frac{1}{2},j}). \tag{4.1d}$$

The auxiliary values of water-surface elevation $\overline{H}_{i,j-\frac{1}{2}}$ at the y-upstream enclosure surface at ($i,j-\frac{1}{2},L$) of the element (i,j,L) and the y-upstream half-element ($i,j-\frac{1}{4},L$) in the water-surface layer ($l=L$) is formulated, as:

$$\overline{H}_{i,j-\frac{1}{2}} = \overline{H}_{i,j} - \frac{\Delta y_{i,j}}{(\Delta y_{i,j}+\Delta y_{i,j-1})} (\overline{H}_{i,j}-\overline{H}_{i,j-1}) \ . \tag{4.1e}$$

The auxiliary value of water-surface elevation $\overline{H}_{i,j-\frac{1}{4}}$ in the y-upstream half-element $(i,j-\frac{1}{4},L)$ in the water-surface layer $(l=L)$ is formulated, as:

$$\overline{H}_{i,j-\frac{1}{4}} = \frac{1}{2}(\overline{H}_{i,j}+\overline{H}_{i,j-\frac{1}{2}}) \ . \tag{4.1f}$$

The auxiliary values of water-surface elevation $\overline{H}_{i,j+\frac{1}{2}}$ at the y-downstream enclosure surface at $(i,j+\frac{1}{2},L)$ of the element (i,j,L) and the y-downstream half-element $(i,j+\frac{1}{4},L)$ in the water-surface layer $(l=L)$ is formulated, as:

$$\overline{H}_{i,j+\frac{1}{2}} = \overline{H}_{i,j} + \frac{\Delta y_{i,j}}{(\Delta y_{i,j}+\Delta y_{i,j+1})} (\overline{H}_{i,j+1}-\overline{H}_{i,j}) \ . \tag{4.1g}$$

The auxiliary value of water-surface elevation $\overline{H}_{i,j+\frac{1}{4}}$ in the y-downstream half-element $(i,j+\frac{1}{4},L)$ in the water-surface layer $(l=L)$ is formulated, as:

$$\overline{H}_{i,j+\frac{1}{4}} = \frac{1}{2}(\overline{H}_{i,j}+\overline{H}_{i,j+\frac{1}{2}}) \ . \tag{4.1h}$$

The auxiliary values of depth in the elements and half-elements at auxiliary locations, which do not correspond to the locations where the actual depths of the elements and half-elements are specified or calculated, are also formulated according to the standard linear-interpolation formulas based on the sizes of the adjacent elements (see Sec. 2).

The auxiliary value of depth $\overline{\Delta z}_{i-\frac{1}{2},j,l}$ at the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and x-upstream half-element $(i-\frac{1}{4},j,l)$ is formulated, as:

$$\overline{\Delta z}_{i-\frac{1}{2},j,l} = \overline{\Delta z}_{i,j,l} - \frac{\Delta x_{i,j}}{(\Delta x_{i,j}+\Delta x_{i-1,j})} (\overline{\Delta z}_{i,j,l} - \overline{\Delta z}_{i-1,j,l}) . \qquad (4.2a)$$

The auxiliary value of depth $\overline{\Delta z}_{i-\frac{1}{4},j,l}$ in the x-upstream half-element (i-¼,j,l) is formulated, as:

$$\overline{\Delta z}_{i-\frac{1}{4},j,l} = \overline{\Delta z}_{i,j,l} . \qquad (4.2b)$$

The auxiliary value of depth $\overline{\Delta z}_{i+\frac{1}{2},j,l}$ at the x-downstream enclosure surface at (i+½,j,l) of the element (i,j,l) and the x-downstream half-element (i+¼,j,l) is formulated, as:

$$\overline{\Delta z}_{i+\frac{1}{2},j,l} = \overline{\Delta z}_{i,j,l} + \frac{\Delta x_{i,j}}{(\Delta x_{i,j}+\Delta x_{i+1,j})} (\overline{\Delta z}_{i+1,j,l} - \overline{\Delta z}_{i,j,l}) . \qquad (4.2c)$$

The auxiliary value of depth $\overline{\Delta z}_{i+\frac{1}{4},j,l}$ in the x-downstream half-element (i+¼,j,l) is formulated, as:

$$\overline{\Delta z}_{i+\frac{1}{4},j,l} = \overline{\Delta z}_{i,j,l} . \qquad (4.2d)$$

The auxiliary value of depth $\overline{\Delta z}_{i,j-\frac{1}{2},l}$ at the y-upstream enclosure surface at (i,j-½,l) of the element (i,j,l) and the y-upstream half-element (i,j-¼,l) is formulated, as:

$$\overline{\Delta z}_{i,j-\frac{1}{2},l} = \overline{\Delta z}_{i,j,l} - \frac{\Delta y_{i,j}}{(\Delta y_{i,j}+\Delta y_{i,j-1})} (\overline{\Delta z}_{i,j,l} - \overline{\Delta z}_{i,j-1,l}) . \qquad (4.2e)$$

The auxiliary value of depth $\overline{\Delta z}_{i,j-\frac{1}{4},l}$ in the y-upstream half-element (i,j-¼,l) is formulated, as:

$$\overline{\Delta z}_{i,j-\frac{1}{4},l} = \overline{\Delta z}_{i,j,l} . \qquad (4.2f)$$

The auxiliary value of depth $\overline{\Delta z}_{i,j+\frac{1}{2},l}$ at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half-element $(i,j+\frac{1}{4},l)$ is formulated, as:

$$\overline{\Delta z}_{i,j+\frac{1}{2},l} = \overline{\Delta z}_{i,j,l} + \frac{\Delta y_{i,j}}{(\Delta y_{i,j}+\Delta y_{i,j+1})} (\overline{\Delta z}_{i,j+1,l} - \overline{\Delta z}_{i,j,l}) . \tag{4.2g}$$

The auxiliary value of depth $\overline{\Delta z}_{i,j+\frac{1}{4},l}$ in the y-downstream half-element $(i,j+\frac{1}{4},l)$ is formulated, as:

$$\overline{\Delta z}_{i,j+\frac{1}{4},l} = \overline{\Delta z}_{i,j,l} . \tag{4.2h}$$

The formulations of the auxiliary values of the water-surface elevation [Eqs. (4.1a) through (4.1h)] and the auxiliary values of depth in the elements [Eqs. (4.2a) through (4.2h)], according to standard linear-interpolation formulas, eliminate all the auxiliary values of the geometrical variables (see Sec. 2.3) associated with the governing computational system of discrete-element equations [Eqs. (3.5), (3.6), (3.7), (3.9), (3.10), (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113) and (3.114)].

4.2. Standard Linear-Interpolation Formulation for the Auxiliary Values of the Normal Components of Velocity Associated with Flow Rates on the Enclosure surfaces of the Elements and Half Elements

The general discrete-element formulation of the mathematical model includes the auxiliary values of the velocity components which do not correspond to the solution locations of (1) the principal flow variable $\overline{V}_{x;i\pm\frac{1}{4},j,l}$, x-component of velocity, in the x-upstream half element

$(i-\frac{1}{4},j,l)$ or x-downstream half element $(i+\frac{1}{4},j,l)$, and (2) the principal flow variable $\bar{V}_{y;i,j-\frac{1}{4},l}$, y-component of velocity, in the y-upstream half element $(i,j-\frac{1}{4},l)$ or y-downstream half element $(i,j+\frac{1}{4},l)$. The auxiliary values of the velocity components which represent the normal components of the velocity vector on the enclosure surfaces of the elements and half elements specify the flow conditions across the enclosure surfaces of the elements and half elements. All the auxiliary values of the velocity components which are normal to the enclosure surfaces of the elements and half elements are formulated according to the standard linear-interpolation formulas based on the sizes of the adjacent elements (see Sec. 2.3).

The auxiliary value of the x-component of velocity $\bar{V}_{x;i-\frac{1}{2},j,l}$, normal to the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, as:

$$\bar{V}_{x;i-\frac{1}{2},j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} - \frac{\Delta x_{i,j}}{(\Delta x_{i,j}+\Delta x_{i-1,j})}[\bar{V}_{x;i-\frac{1}{4},j,l}-\bar{V}_{x;(i-1)+\frac{1}{4},j,l}] \ . \qquad (4.3a)$$

The auxiliary value of the x-component of velocity $\bar{V}_{x;i,j,l}$, normal to the x-downstream enclosure surface at (i,j,l) of the x-upstream half element $(i-\frac{1}{4},j,l)$ and normal to the x-upstream enclosure surface at (i,j,l) of the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, as:

$$\bar{V}_{x;i,j,l} = \frac{1}{2}(\bar{V}_{x;i-\frac{1}{4},j,l}+\bar{V}_{x;i+\frac{1}{4},j,l}) \ . \qquad (4.3b)$$

The auxiliary value of the x-component of velocity $\bar{V}_{x;i+\frac{1}{2},j,l}$, normal to the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, as:

$$\bar{V}_{x;i+\frac{1}{2},j,l} = \bar{V}_{x;i+\frac{1}{4},j,l} + \frac{\Delta x_{i,j}}{(\Delta x_{i,j}+\Delta x_{i+1,j})}\left[\bar{V}_{x;(i+1)-\frac{1}{4},j,l}-\bar{V}_{x;i+\frac{1}{4},j,l}\right]. \quad (4.3c)$$

The auxiliary value of the y-component of velocity $\bar{V}_{y;i,j-\frac{1}{2},l}$, normal to the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, as:

$$\bar{V}_{y;i,j-\frac{1}{2},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{\Delta y_{i,j}}{(\Delta y_{i,j}+\Delta y_{i,j-1})}\left[\bar{V}_{y;i,j-\frac{1}{4},l}-\bar{V}_{y;i,(j-1)+\frac{1}{4},l}\right]. \quad (4.3d)$$

The auxiliary value of the y-component of velocity $\bar{V}_{y;i,j,l}$, normal to the y-downstream enclosure surface at (i,j,l) of the y-upstream half element $(i,j-\frac{1}{4},l)$ and normal to the y-upstream enclosure surface at (i,j,l) of the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, as:

$$\bar{V}_{y;i,j,l} = \frac{1}{2}(\bar{V}_{y;i,j-\frac{1}{4},l}+\bar{V}_{y;i,j+\frac{1}{4},l}). \quad (4.3e)$$

The auxiliary value of the y-component of velocity $\bar{V}_{y;i,j+\frac{1}{2},l}$, normal to the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, as:

$$\bar{V}_{y;i,j+\frac{1}{2},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{\Delta y_{i,j}}{(\Delta y_{i,j}+\Delta y_{i,j+1})}\left[\bar{V}_{y;i,(j+1)-\frac{1}{4},l}-\bar{V}_{y;i,j+\frac{1}{4},l}\right]. \quad (4.3f)$$

The formulations of the auxiliary values of (1) the x-copmonent of velocity normal to the enclosure surfaces of the elements and half elements [Eqs. (4.3a) through (4.3c)] and (2) the y-component of velocity normal to the enclosure surfaces of the elements and half elements [Eqs. (4.3d) through (4.3f)] determine the flow rates normal to the enclosure surfaces of the elements and half elements [Eq. (2.108)], based on the definitions of the enclosure surface areas

of the elements and half elements of the general discretization technique of the discrete element method (Sec. 2.3).

The auxiliary value of the flow rate along x-direction $\overline{G}_{x;i-\frac{1}{2},j,l}$, normal to the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, as:

$$\overline{G}_{x;i-\frac{1}{2},j,l} = \overline{A}_{x;i-\frac{1}{2},j,l}\,\overline{V}_{x;i-\frac{1}{2},j,l}\,. \tag{4.4a}$$

The auxiliary value of the flow rate along x-direction $\overline{G}_{x;i,j,l}$, normal to the x-downstream enclosure surface at (i,j,l) of the x-upstream half element $(i-\frac{1}{4},j,l)$ and normal to the x-upstream enclosure surface at (i,j,l) of the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, as:

$$\overline{G}_{x;i,j,l} = \overline{A}_{x;i,j,l}\,\overline{V}_{x;i,j,l}\,. \tag{4.4b}$$

The auxiliary value of the flow rate along x-direction $\overline{G}_{x;i+\frac{1}{2},j,l}$, normal to the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, as:

$$\overline{G}_{x;i+\frac{1}{2},j,l} = \overline{A}_{x;i+\frac{1}{2},j,l}\,\overline{V}_{x;i+\frac{1}{2},j,l}\,. \tag{4.4c}$$

The auxiliary value of the flow rate along y-direction $\overline{G}_{y;i,j-\frac{1}{2},l}$, normal to the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, as:

$$\overline{G}_{y;i,j-\frac{1}{2},l} = \overline{A}_{y;i,j-\frac{1}{2},l}\,\overline{V}_{y;i,j-\frac{1}{2},l}\,. \tag{4.4d}$$

The auxiliary value of the flow rate along y-direction $\overline{G}_{y;i,j,l}$, normal to the y-downstream enclosure surface at (i,j,l) of the y-upstream half element $(i,j-\frac{1}{4},l)$ and normal to the y-upstream enclosure

surface at (i,j,l) of the y-downstream half element (i,j+¼,l), is formulated, as:

$$\overline{G}_{y;i,j,l} \quad \overline{A}_{y;i,j,l}\,\overline{V}_{y;i,j,l} \cdot \tag{4.4e}$$

The auxiliary value of the flow rate along y-direction $\overline{G}_{y;i,j+\frac{1}{2},l}$, normal to the y-downstream enclosure surface at (i,j+½,l) of the element (i,j,l) and the y-downstream half element (i,j+¼,l), is formulated, as:

$$\overline{G}_{y;i,j+\frac{1}{2},l} = \overline{A}_{y;i,j+\frac{1}{2},l}\,\overline{V}_{y;i,j+\frac{1}{2},l} \cdot \tag{4.4f}$$

The formulations of the auxiliary flow variables, the flow rate along x-direction [Eqs. (4.4a) through (4.4c)] and the flow rate along y-direction [Eqs. (4.4d) through (4.4f)] at the auxiliary locations corresponding to the locations of the enclosure surfaces of the elements and half elements, based on standard linear-interpolation formulations of the x-component of velocity [Eqs. (4.3a) through (4.3c)] and the y-component of velocity [Eqs. (4.3d) through (4.3f)], respectively, eliminate all the terms associated with the auxiliary flow variables, the flow rates, associated with the governing computational system of discrete-element equations [Eqs. (3.5), (3.6), (3.7), (3.9), (3.10), (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113) and (3.114)].

4.3. Upwind-Differencing Formulation for the Auxiliary Values of the Velocity Components Associated with the Momentum Transport across the Enclosure Surfaces of the Elements and Half Elements

The convergent (consistent and stable) solution of the governing computational system of discrete-element equations according to a

timewise-explicit numerical integration method requires upwind-differencing interpolation for all the convective transport terms (e.g., Roache, 1972). The fast-transient, three-dimensional, discrete-element hydrodynamic model can incorporate three types of upwind-differencing interpolation for the convective transport terms in the general formulations of the governing computational system of discrete-element equations.

The general discrete-element formulation of the mathematical model includes the auxiliary values of all three velocity components associated with the momentum transport across the enclosure surfaces of the elements and half elements. All the auxiliary values of all three velocity components associated with momentum transport are formulated according to any one of the three types of the upwind-differencing interpolation formulas which consider the direction of flow across the enclosure surfaces of the elements and half elements.

4.3.1. Standard Second-Upwind-Differencing Interpolation for the Auxiliary Values of the Velocity Components Associated with the Momentum Transport across the Enclosure Surfaces of the Elements and Half Elements

The formulation of the governing computational system of discrete-element equations can incorporate the modified, discrete-element forms of the standard second-upwind-differencing interpolation (e.g., Eraslan, 1974) which is similar to the form used in the classical finite-difference formulations of the cell methods (e.g., Roache, 1972).

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{2},j,l}$, associated with the x-component of momentum transport, at the x-upstream

enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{2},j,l} = \bar{V}_{x;(i-1)+\frac{1}{4},j,l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 \text{ ,} \tag{4.5a}$$

$$\tilde{V}_{x;i-\frac{1}{2},j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 \text{ .} \tag{4.5b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i-1,j-\frac{1}{4},l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 \text{ ,} \tag{4.6a}$$

$$\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 \text{ .} \tag{4.6b}$$

The auxiliary values of the y-component of velocity $\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i-1,j+\frac{1}{4},l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 \text{ ,} \tag{4.7a}$$

$$\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 \text{ .} \tag{4.7b}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i-\frac{1}{2},j,l}$, associated with the z-component of momentum transport, at the x-upstream

enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l), is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i-\frac{1}{2},j,l} = \bar{V}_{z;i-1,j,l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 \text{ ,} \tag{4.8a}$$

$$\tilde{V}_{z;i-\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} \text{ , for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 \text{ .} \tag{4.8b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i,j,l}$, associated with the x-component of momentum transport, at the x-downstream enclosure surface at (i,j,l) of the x-upstream half element $(i-\frac{1}{4},j,l)$ and at the x-upstream enclosure surface at (i,j,l) of the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i,j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} \text{ , for } \bar{V}_{x;i,j,l} > 0 \text{ ,} \tag{4.9a}$$

$$\tilde{V}_{x;i,j,l} = \bar{V}_{x;i+\frac{1}{4},j,l} \text{ , for } \bar{V}_{x;i,j,l} \leq 0 \text{ .} \tag{4.9b}$$

Similarly, the auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{2},j,l}$, associated with the x-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{2},j,l} = \bar{V}_{x;i+\frac{1}{4},j,l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 \text{ ,} \tag{4.10a}$$

$$\tilde{V}_{x;i+\frac{1}{2},j,l} = \bar{V}_{x;(i+1)-\frac{1}{4},j,l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 \text{ .} \tag{4.10b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 \text{ ,} \tag{4.11a}$$

$$\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i+1,j-\frac{1}{4},l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 \text{ .} \tag{4.11b}$$

The auxiliary values of the y-component of velocity $\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 \text{ ,} \tag{4.12a}$$

$$\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i+1,j+\frac{1}{4},l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 \text{ .} \tag{4.12b}$$

The auxiliary value of z-component of velocity $\tilde{V}_{z;i+\frac{1}{2},j,l}$, associated with the z-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i+\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 \text{ ,} \tag{4.13a}$$

$$\tilde{V}_{z;i+\frac{1}{2},j,l} = \bar{V}_{z;i+1,j,l} \text{ , for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 \text{ .} \tag{4.13b}$$

Similarly, the auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j-\frac{1}{2},l}$, associated with the y-component of momentum transport, at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j-\frac{1}{2},l} = \bar{V}_{y;i,(j-1)+\frac{1}{4},l}, \text{ for } \bar{V}_{y;i,j-\frac{1}{2},l} > 0. \tag{4.14a}$$

$$\tilde{V}_{y;i,j-\frac{1}{2},l} = \bar{V}_{y;i,j-\frac{1}{4},l}, \text{ for } \bar{V}_{y;i,j-\frac{1}{2},l} \leq 0. \tag{4.14b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-upstream enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i-\frac{1}{4},j-1,l}, \text{ for } \bar{V}_{y;i,j-\frac{1}{2},l} > 0. \tag{4.15a}$$

$$\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i-\frac{1}{4},j,l}, \text{ for } \bar{V}_{y;i,j-\frac{1}{2},l} \leq 0. \tag{4.15b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-upstream enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j-1,l}, \text{ for } \bar{V}_{y;i,j-\frac{1}{2},l} > 0, \tag{4.16a}$$

$$\tilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j,l}, \text{ for } \bar{V}_{y;i,j-\frac{1}{2},l} \leq 0. \tag{4.16b}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j-\frac{1}{2},l}$, associated with the z-component of momentum transport, at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l), is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j-\frac{1}{2},l} = \bar{V}_{z;i,j,l} \text{ , for } \bar{V}_{y;i,j-\frac{1}{2},l} \leq 0 \text{ .} \tag{4.17a}$$

$$\tilde{V}_{z;i,j-\frac{1}{2},l} = \bar{V}_{z;i,j-1,l} \text{ , for } \bar{V}_{y;i,j-\frac{1}{2},l} > 0 \text{ .} \tag{4.17b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j,l}$, associated with the y-component of momentum transport, at the y-downstream enclosure surface at (i,j,l) of the y-upstream half element $(i,j-\frac{1}{4},l)$ and at the y-upstream enclosure surface at (i,j,l) of the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j,l} = \bar{V}_{y;i,j-\frac{1}{4},l} \text{ , for } \bar{V}_{y;i,j,l} > 0 \text{ .} \tag{4.18a}$$

$$\tilde{V}_{y;i,j,l} = \bar{V}_{y;i,j+\frac{1}{4},l} \text{ , for } \bar{V}_{y;i,j,l} \leq 0 \text{ .} \tag{4.18b}$$

Similarly, the auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j+\frac{1}{2},l}$, associated with the y-component of momentum transport, at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j+\frac{1}{2},l} = \bar{V}_{y;i,j+\frac{1}{4},l} \text{ , for } \bar{V}_{y;i,j+\frac{1}{2},l} \geq 0 \text{ ,} \tag{4.19a}$$

$$\widetilde{V}_{y;i,j+\frac{1}{2},l} = \overline{V}_{y;i,(j+1)-\frac{1}{4},l} \text{ , for } \overline{V}_{y;i,j+\frac{1}{2},l} < 0 . \qquad (4.19b)$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-downstream enclosure surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l} = \overline{V}_{x;i-\frac{1}{4},j,l} \text{ , for } \overline{V}_{y;i,j+\frac{1}{2},l} \geq 0 , \qquad (4.20a)$$

$$\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l} = \overline{V}_{x;i-\frac{1}{4},j+1,l} \text{ , for } \overline{V}_{y;i,j+\frac{1}{2},l} < 0 . \qquad (4.20b)$$

The auxiliary values of the x-component of velocity $\widetilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-downstream enclosure surface at $(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\widetilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l} = \overline{V}_{x;i+\frac{1}{4},j,l} \text{ , for } \overline{V}_{y;i,j+\frac{1}{2},l} \geq 0 , \qquad (4.21a)$$

$$\widetilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l} = \overline{V}_{x;i+\frac{1}{4},j+1,l} \text{ , for } \overline{V}_{y;i,j+\frac{1}{2},l} < 0 . \qquad (4.21b)$$

The auxiliary value of the z-component of velocity $\widetilde{V}_{z;i,j+\frac{1}{2},l}$, associated with the z-component of momentum transport, at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j+\frac{1}{2},l} = \bar{V}_{z;i,j,l} \text{ , for } \bar{V}_{y;i,j+\frac{1}{2},l} \geq 0 \text{ ,} \tag{4.22a}$$

$$\tilde{V}_{z;i,j+\frac{1}{2},l} = \bar{V}_{z;i,j+1,l} \text{ , for } \bar{V}_{y;i,j+\frac{1}{2},l} < 0 \text{ .} \tag{4.22b}$$

Similarly, the auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j,l-\frac{1}{2}}$, associated with the z-component of momentum transport, at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j,l-\frac{1}{2}} = \bar{V}_{z;i,j,l-1} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 \text{ ,} \tag{4.23a}$$

$$\tilde{V}_{z;i,j,l-\frac{1}{2}} = \bar{V}_{z;i,j,l} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 \text{ .} \tag{4.23b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-upstream enclosure surface at $(i-\frac{1}{4},j,l-\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l-1} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 \text{ ,} \tag{4.24a}$$

$$\tilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 \text{ .} \tag{4.24b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-upstream enclosure surface at $(i+\frac{1}{4},j,l-\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i+\frac{1}{4},j,l-1} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 \text{ ,} \tag{4.25a}$$

$$\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i+\frac{1}{4},j,l} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 \text{ .} \tag{4.25b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-upstream enclosure surface at $(i,j-\frac{1}{4},l-\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}} = \bar{V}_{y;i,j-\frac{1}{4},l-1} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 \text{ ,} \tag{2.26a}$$

$$\tilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}} = \bar{V}_{y;i,j-\frac{1}{4},l} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 \text{ .} \tag{4.26b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-upstream enclosure surface at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l-1} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 \text{ ,} \tag{4.27a}$$

$$\tilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} \text{ , for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 \text{ .} \tag{4.27b}$$

Similarly, the auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j,l+\frac{1}{2}}$, associated with the z-component of momentum transport, at the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l), is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j,l+\frac{1}{2}} = \bar{V}_{z;i,j,l} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 \text{ ,} \tag{4.28a}$$

$$\tilde{V}_{z;i,j,l+\frac{1}{2}} = \bar{V}_{z;i,j,l+1} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 \text{ .} \tag{4.28b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-downstream enclosure surface at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 \text{ ,} \tag{4.29a}$$

$$\tilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l+1} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 \text{ .} \tag{4.29b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-downstream enclosure surface at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i+\frac{1}{4},j,l} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 \text{ ,} \tag{4.30a}$$

$$\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i+\frac{1}{4},j,l+1} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 \text{ .} \tag{4.30b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-down-

stream enclosure surface at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j-\frac{1}{4},l} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 \text{ ,} \tag{4.31a}$$

$$\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j-\frac{1}{4},l+1} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 \text{ .} \tag{4.31b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-downstream enclosure surface at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the standard second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 \text{ ,} \tag{4.32a}$$

$$\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l+1} \text{ , for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 \text{ .} \tag{4.32b}$$

The formulations of the auxiliary values of the components of velocity, associated with the components of momentum transport, at the enclosure surfaces of the elements and half elements, according to the standard second-upwind-differencing interpolation Eqs. (4.5a) through (4.32b) , can eliminate all the auxiliary values of the components of velocity which appear in the terms, associated with momentum transport in the governing computational system of discrete-element equations [Eqs. (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113), (3.114)].

The standard second-upwind-differencing interpolation formulation of the components of velocity, associated with the components of momentum transport, guarantees the computational stability of the numerical solutions, according to a timewise-explicit computational algorithm, provided that the local values of the Courant number remain less than or equal to one (1) everywhere in the flow region (e.g., Roache, 1972). Furthermore,in elements and/or half elements, where the local value of the Courant number become less than one (1), the standard second-upwind-differencing interpolation of the components of momentum transport automatically introduces the effect of artificial viscosity (as a part of the general numerical dispersion effect) which can be substantial for small values of the Courant number (e.g., Roache, 1972). The effect of local artificial viscosity can be tolerated in the numerical solutions of the governing computational system of discrete-element equations, provided the local values of Courant number remain relatively close to one (1) during most of the time of the numerical solutions in the flow region.

The numerical solution of the fast-transient, three-dimensional, discrete-element hydrodynamic model, according to a timewise explicit computational algorithm, is only stable provided that the Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972) is satisfied in every element and half element in the flow region. Since the Courant, Friedrichs and Lewy (CFL) criterion for free-surface flow conditions depends on the speed of the surface gravity wave, for relatively deep flow regions, the time step of the numerical integration of the timewise-explicit computational algo-

rithm is generally limited to values much smaller than the allowable time step of the numerical integration based on the Courant criterion (e.g., Roache, 1972). Hence, the numerical solution of the governing computational system of discrete-element equations, with the allowable time step of numerical integration based on the Courant, Friedrichs and Lewy (CFL) criterion, can include substantial amounts of error due to the effect of the artificial viscosity (as a part of the general numerical dispersion effect) introduced by the standard second-upwind-differencing interpolation of the components of velocity, associated with the components of momentum transport.

In order to minimize the effect of the artificial viscosity, two additional upwind-differencing interpolation formulations of the components of viscosity were considered in the development of the fast-transient, three-dimensional, discrete-element hydrodynamic model.

4.3.2. Weighted-Second-Upwind-Differencing Interpolation for the Auxiliary Values of the Velocity Components Associated with the Momentum Transport across the Enclosure Surfaces of Elements and Half Elements

The formulation of the governing computational system of discrete-element equations can also incorporate a weighted-second-upwind-differencing interpolation which represents a modified form of the standard second-upwind-differencing interpolation, based on three specified weighting parameters, as:

$\alpha_{x,ups}$ = specified parameter for weighted-second-upwind-differencing interpolation along x-direction,

$\alpha_{y,ups}$ = specified parameter for weighted-second-upwind-differencing interpolation along y-direction,

$\alpha_{z,ups}$ = specified parameter for weighted-second-upwind-differencing interpolation along z-direction,

The weighted-second-upwind-differencing interpolation of the auxiliary values of the components of velocity, associated with the components of the momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, are formulated, by considering the average size $\Delta x_{i-\frac{1}{2},j}$ of the two adjacent elements, as:

$$\Delta x_{i-\frac{1}{2},j} = \frac{1}{2}(\Delta x_{i,j} + \Delta x_{i-1,j}) . \tag{4.33}$$

The auxiliary value of the x-component of velocity $\tilde{\bar{V}}_{x;i-\frac{1}{2},j,l}$, associated with the x-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{x;i-\frac{1}{2},j,l} = \bar{V}_{x;i-\frac{1}{2},j,l} - \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} [\, \bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;(i-1)+\frac{1}{4},j,l} \,],$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 , \tag{4.34a}$$

$$\tilde{\bar{V}}_{x;i-\frac{1}{2},j,l} = \bar{V}_{x;i-\frac{1}{2},j,l} - \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i,j}}{\Delta x_{i-\frac{1}{2},j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i-\frac{1}{2},j}})} [\, \bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j,l} \,],$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} < 0 , \tag{4.34b}$$

$$\tilde{V}_{x;i-\frac{1}{2},j,l} = \bar{V}_{x;i-\frac{1}{2},j,l} - \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} [\, \bar{V}_{x;i-\frac{1}{2},j,l} - \bar{V}_{x;(i-1)+\frac{1}{2},j,l} \,],$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 \,. \tag{4.34b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i-1,j-\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 \,, \tag{4.35a}$$

$$\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i-1,j-\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 \,. \tag{4.35b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{y;i-\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i-1,j-\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 , \tag{4.36a}$$

$$\tilde{\bar{V}}_{y;i-\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} - \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i-1,j+\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 . \tag{4.36b}$$

The auxiliary value of the z-component of velocity $\tilde{\bar{V}}_{z;i-\frac{1}{2},j,l}$, associated with the z-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{z;i-\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} - \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i-1,j,l}),$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} > 0 , \tag{4.37a}$$

$$\tilde{\bar{V}}_{z;i-\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} - \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i-\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i-1,j,l}),$$

$$\text{for } \bar{V}_{x;i-\frac{1}{2},j,l} \leq 0 . \tag{4.37b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i,j,l}$, associated with the x-component of momentum transport, at the x-downstream enclosure surface at (i,j,l) of the x-upstream half element ($i-\frac{1}{4},j,l$) and at the x-upstream enclosure surface at (i,j,l) of the x-downstream half element ($i+\frac{1}{4},j,l$), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i,j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{1}{2}(1 - \alpha_{x,ups})\,(\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;(i-\frac{1}{4},j,l)},$$

$$\text{for } \bar{V}_{x;i,j,l} > 0\,. \tag{4.38a}$$

$$\tilde{V}_{x;i,j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{1}{2}(1 + \alpha_{x,ups})\,(\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;(i-\frac{1}{4},j,l)},$$

$$\text{for } \bar{V}_{x;i,j,l} \leq 0\,. \tag{4.38b}$$

Similarly, the weighted-second-upwind-differencing interpolation of the auxiliary values of the components of velocity, associated with the components of the momentum transport, at the x-downstream enclosure surface at ($i+\frac{1}{2},j,l$) of the element (i,j,l) and the x-downstream half element ($i+\frac{1}{4},j,l$), are formulated, by considering the average size $\Delta x_{i+\frac{1}{2},j}$ of the two adjacent elements, as:

$$\Delta x_{i+\frac{1}{2},j} = \frac{1}{2}(\Delta x_{i,j} + \Delta x_{i+1,j})\,. \tag{4.39}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{2},j,l}$, associated with the x-component of momentum transport, at the x-downstream enclosure surface at ($i+\frac{1}{2},j,l$) of the element (i,j,l) and the x-downstream half element ($i+\frac{1}{4},j,l$), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{x;i+\frac{1}{2},j,l} = \bar{V}_{x;i+\frac{1}{2},j,l} + \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})} [\, \bar{V}_{x;(i+1)-\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l} \,],$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 . \tag{4.40a}$$

$$\tilde{\bar{V}}_{x;i+\frac{1}{2},j,l} = \bar{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})} [\, \bar{V}_{x;(i+1)-\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l} \,],$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 . \tag{4.40b}$$

The auxiliary value of the y-component of velocity $\tilde{\bar{V}}_{y;i+\frac{1}{2},j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{y;i+\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i+1,j-\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 . \tag{4.41a}$$

$$\tilde{\bar{V}}_{y;i+\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})} [\, \bar{V}_{y;i+1,j-\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l} \,],$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 . \tag{4.41b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})}(\bar{V}_{y;i+1,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 , \tag{4.42a}$$

$$\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})}(\bar{V}_{y;i+1,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 . \tag{4.42b}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i+\frac{1}{2},j,l}$, associated with the z-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i+\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} + \frac{(1 - \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})}(\bar{V}_{z;i+1,j,l} - \bar{V}_{z;i,j,l}),$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} \geq 0 , \tag{4.43a}$$

$$\tilde{V}_{z;i+\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} + \frac{(1 + \alpha_{x,ups}\frac{\Delta x_{i+\frac{1}{2},j}}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})} (\bar{V}_{z;i+1,j,l} - \bar{V}_{z;i,j,l}),$$

$$\text{for } \bar{V}_{x;i+\frac{1}{2},j,l} < 0 . \tag{4.43b}$$

Similary, the weighted-second-upwind-differencing interpolation of the auxiliary values of the components of velocity, associated with the components of the momentum transport, at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the x-upstream half element $(i,j-\frac{1}{4},l)$, are formulated, by considering the average size $\Delta y_{i,j-\frac{1}{2}}$ of the two adjacent elements, as:

$$\Delta y_{i,j-\frac{1}{2}} = \frac{1}{2}(\Delta y_{i,j} + \Delta y_{i,j-1}) . \tag{4.44}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j-\frac{1}{2},l}$, associated with the y-component of momentum transport, at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j-\frac{1}{2},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} [\, \bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i,(j-1)+\frac{1}{4},l} \,],$$

$$\text{for } \bar{V}_{y;i,j-\frac{1}{2},l} > 0 , \tag{4.45a}$$

$$\widetilde{V}_{y;i,j-\frac{1}{2},l} = \overline{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} [\, \overline{V}_{y;i,j-\frac{1}{4},l} - \overline{V}_{y;i,(j-1)+\frac{1}{4},l} \,],$$

$$\text{for } \overline{V}_{y;i,j-\frac{1}{2},l} \leq 0 . \tag{4.45b}$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-upstream enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l} = \overline{V}_{x;i-\frac{1}{4},j,l} - \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} (\overline{V}_{x;i-\frac{1}{4},j,l} - \overline{V}_{x;i-\frac{1}{4},j-1,l}),$$

$$\text{for } \overline{V}_{y;i,j-\frac{1}{2},l} > 0 , \tag{4.46a}$$

$$\widetilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l} = \overline{V}_{x;i-\frac{1}{4},j,l} - \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} (\overline{V}_{x;i-\frac{1}{4},j,l} - \overline{V}_{x;i-\frac{1}{4},j-1,l}),$$

$$\text{for } \overline{V}_{y;i,j-\frac{1}{2},l} \leq 0 . \tag{4.46b}$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-upstream enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{x;i+\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j,l} - \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} (\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j-1,l}),$$

$$\text{for } \bar{V}_{y;i,j-\frac{1}{2},l} > 0 , \tag{4.47a}$$

$$\tilde{\bar{V}}_{x;i+\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j,l} - \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} (\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j-1,l}),$$

$$\text{for } \bar{V}_{y;i,j-\frac{1}{2},l} \leq 0 . \tag{4.47b}$$

The auxiliary value of the z-component of velocity $\tilde{\bar{V}}_{z;i,j-\frac{1}{2},l}$, associated with the z-component of momentum transport at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{z;i,j-\frac{1}{2},l} = \bar{V}_{z;i,j,l} - \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j-1,l}),$$

$$\text{for } \bar{V}_{y;i,j-\frac{1}{2},l} > 0 , \tag{4.48a}$$

$$\tilde{\bar{V}}_{z;i,j-\frac{1}{2},l} = \bar{V}_{z;i,j,l} - \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j-\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j-1,l}),$$

$$\text{for } \bar{V}_{y;i,j-\frac{1}{2},l} \leq 0 . \tag{4.48b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j,l}$, associated with the y-component of momentum transport, at the y-downstream enclosure surface at (i,j,l) of the y-upstream half element $(i,j-\frac{1}{4},l)$ and at the y-upstream enclosure surface at (i,j,l) of the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j,l} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{1}{2}(1 - \alpha_{y,ups}) \quad (\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{y;i,j,l} > 0 . \tag{4.49a}$$

$$\tilde{V}_{y;i,j,l} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{1}{2}(1 + \alpha_{y,ups}) \quad (\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{y;i,j,l} \leq 0 . \tag{4.49b}$$

Similarly, the weighted-second-upwind-differencing interpolation of the auxiliary values of the components of velocity, associated with the components of the momentum transport, at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half element $(i+\frac{1}{4},j,l)$, are formulated, by considering the average size $\Delta y_{i,j+\frac{1}{2}}$ of the two adjacent elements, as:

$$\Delta y_{i,j+\frac{1}{2}} = \frac{1}{2}(\Delta y_{i,j} + \Delta y_{i,j+1}) . \tag{4.50}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j+\frac{1}{2},l}$, associated with the y-component of momentum transport, at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j+\frac{1}{2},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} [\, \bar{V}_{y;i,(j+1)-\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l} \,],$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} \geq 0 , \tag{4.51a}$$

$$\tilde{V}_{y;i,j+\frac{1}{2},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} [\, \bar{V}_{y;i,(j+1)-\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l} \,],$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} < 0 . \tag{4.51b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-downstream enclosure surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{x;i-\frac{1}{4},j+1,l} - \bar{V}_{x;i-\frac{1}{4},j,l}),$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} \geq 0 , \tag{4.52a}$$

$$\tilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{x;i-\frac{1}{4},j+1,l} - \bar{V}_{x;i-\frac{1}{4},j,l}),$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} < 0 . \tag{4.52b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-downstream enclosure surface at $(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{x;i+\frac{1}{4},j+1,l} - \bar{V}_{x;i+\frac{1}{4},j,l}),$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} \geq 0 , \tag{4.53a}$$

$$\tilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{x;i+\frac{1}{4},j+1,l} - \bar{V}_{x;i+\frac{1}{4},j,l}),$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} < 0 . \tag{4.53b}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j+\frac{1}{2},l}$, associated with the z-component of momentum transport, at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j+\frac{1}{2},l} = \bar{V}_{z;i,j,l} + \frac{(1 - \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{z;i,j+1,l} - \bar{V}_{z;i,j,l}),$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} \geq 0 , \tag{4.54a}$$

$$\tilde{V}_{z;i,j+\frac{1}{2},l} = \bar{V}_{z;i,j,l} + \frac{(1 + \alpha_{y,ups}\frac{\Delta y_{i,j+\frac{1}{2}}}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{z;i,j+1,l} - \bar{V}_{z;i,j,l}),$$

$$\text{for } \bar{V}_{y;i,j+\frac{1}{2},l} < 0 . \tag{4.54b}$$

Similarly, the weighted-second-upwind-differencing interpolation of the auxiliary values of the components of velocity, associated with the components of the momentum transport, at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), are formulated, by considering the average size $\Delta z_{i,j,l-\frac{1}{2}}$ of the two adjacent elements, as:

$$\Delta z_{i,j,l-\frac{1}{2}} = \frac{1}{2}(\Delta z_{i,j,l} + \Delta z_{i,j,l-1}) . \tag{4.55}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j,l-\frac{1}{2}}$, associated with the z-component of momentum transport, at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j-\frac{1}{2},l} = \bar{V}_{z;i,j,l} - \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j,l-1}),$$

$$\text{for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 , \tag{4.56a}$$

$$\tilde{V}_{z;i,j,l-\frac{1}{2}} = \bar{V}_{z;i,j,l} - \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j,l-1}),$$

$$\text{for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 . \tag{4.56b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-upstream enclosure surface at $(i-\frac{1}{4},j,l-\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} - \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})}(\bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j,l-1}),$$

$$\text{for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 , \tag{4.57a}$$

$$\tilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} - \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})}(\bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j,l-1}),$$

$$\text{for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 . \tag{4.57b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-upstream enclosure surface at $(i+\frac{1}{4},j,l-\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i+\frac{1}{4},j,l} - \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})}(\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l-1}),$$

$$\text{for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 , \tag{4.58a}$$

$$\widetilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}} = \overline{V}_{x;i+\frac{1}{4},j,l} - \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\overline{V}_{x;i+\frac{1}{4},j,l} - \overline{V}_{x;i+\frac{1}{4},j,l-1}),$$

$$\text{for } \overline{V}_{z;i,j,l-\frac{1}{2}} \leq 0 . \tag{4.58b}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-upstream enclosure surface at $(i,j-\frac{1}{4},l-\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\widetilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}} = \overline{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\overline{V}_{y;i,j-\frac{1}{4},l} - \overline{V}_{y;i,j-\frac{1}{4},l-1}),$$

$$\text{for } \overline{V}_{z;i,j,l-\frac{1}{2}} > 0 , \tag{4.59a}$$

$$\widetilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}} = \overline{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\overline{V}_{y;i,j-\frac{1}{4},l} - \overline{V}_{y;i,j-\frac{1}{4},l-1}),$$

$$\text{for } \overline{V}_{z;i,j,l-\frac{1}{2}} \leq 0 . \tag{4.59b}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-upstream enclosure surface at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} - \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l-1}),$$

$$\text{for } \bar{V}_{z;i,j,l-\frac{1}{2}} > 0 , \tag{4.60a}$$

$$\tilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} - \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l-\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l-1}),$$

$$\text{for } \bar{V}_{z;i,j,l-\frac{1}{2}} \leq 0 . \tag{4.60b}$$

Similarly, the weighted-second-upwind-differencing interpolation of the auxiliary values of the components of velocity, associated with the components of the momentum transport, at the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l), are formulated, by considering the average size $\Delta z_{i,j,l+\frac{1}{2}}$ of the two adjacent elements, as:

$$\Delta z_{i,j,l+\frac{1}{2}} = \frac{1}{2}(\Delta z_{i,j,l+1} + \Delta z_{i,j,l}) . \tag{4.61}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j,l+\frac{1}{2}}$, associated with the z-component of momentum transport, at the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l), is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j,l+\frac{1}{2}} = \bar{V}_{z;i,j,l} + \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{z;i,j,l+1} - \bar{V}_{z;i,j,l}),$$

$$\text{for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 , \tag{4.62a}$$

$$\tilde{V}_{z;i,j,l+\frac{1}{2}} = \bar{V}_{z;i,j,l} + \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{z;i,j,l+1} - \bar{V}_{z;i,j,l}),$$

$$\text{for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 . \tag{4.62b}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-downstream enclosure surface at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{x;i-\frac{1}{4},j,l+1} - \bar{V}_{x;i-\frac{1}{4},j,l}),$$

$$\text{for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 , \tag{4.63a}$$

$$\tilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{x;i-\frac{1}{4},j,l+1} - \bar{V}_{x;i-\frac{1}{4},j,l}),$$

$$\text{for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 . \tag{4.63b}$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-downstream enclosure surface at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\widetilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}} = \overline{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\overline{V}_{x;i+\frac{1}{4},j,l+1} - \overline{V}_{x;i+\frac{1}{4},j,l}),$$

$$\text{for } \overline{V}_{z;i,j,l+\frac{1}{2}} \geq 0 , \tag{4.64a}$$

$$\widetilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}} = \overline{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\overline{V}_{x;i+\frac{1}{4},j,l+1} - \overline{V}_{x;i+\frac{1}{4},j,l}),$$

$$\text{for } \overline{V}_{z;i,j,l+\frac{1}{2}} < 0 . \tag{4.64b}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-downstream enclosure surface at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\widetilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}} = \overline{V}_{y;i,j-\frac{1}{4},l} + \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\overline{V}_{y;i,j-\frac{1}{4},l+1} - \overline{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for } \overline{V}_{z;i,j,l+\frac{1}{2}} \geq 0 , \tag{4.65a}$$

$$\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{y;i,j-\frac{1}{4},l+1} - \bar{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 . \tag{4.65b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-downstream enclosure surface at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the weighted-second-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 - \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{y;i,j+\frac{1}{4},l+1} - \bar{V}_{y;i,j+\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{z;i,j,l+\frac{1}{2}} \geq 0 , \tag{4.66a}$$

$$\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 + \alpha_{z,ups}\frac{\Delta z_{i,j,l+\frac{1}{2}}}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{y;i,j+\frac{1}{4},l+1} - \bar{V}_{y;i,j+\frac{1}{4},l}),$$

$$\text{for } \bar{V}_{z;i,j,l+\frac{1}{2}} < 0 . \tag{4.66b}$$

The formulations of the auxiliary values of the components of velocity, associated with the components of of momentum transport at the enclosure surfaces of the elements and half elements, according to the weighted-second-upwind-differencing interpolation Eqs. (4.33) through (4.66b) , can eliminate all the auxiliary values of the components of

velocity which appear in the terms, associated with momentum transport, in the governing computational system of discrete-element equations [Eqs. (3.53), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113), and (3.114)].

The weighted-second-upwind-differencing interpolation of the components of velocity, associated with the components of momentum transport, establishes the required stability of the numerical solutions, according to the timewise-explicit computational algorithm, for certain specified values of the parameters $\alpha_{x,ups}$, $\alpha_{y,ups}$ and $\alpha_{z,ups}$ of the weighted-second-upwind-differencing interpolation along x-, y-, and z-direction, respectively. The specified values of the parameters depend on the specified time step of integration and the requirement for obtaining the local values of the Courant number (e.g., Roache, 1972) approximately equal to one (1) in order to minimize the effect of the artificial viscosity (as a part of the general numerical dispersion effect). Hence, the values of the parameter for the weighted-second-upwind-differencing interpolation can be specified as:

$$0 < \alpha_{x,ups} \leq 1, \tag{4.67a}$$

$$0 < \alpha_{y,ups} \leq 1, \tag{4.67b}$$

$$0 < \alpha_{z,ups} \leq 1. \tag{4.67c}$$

The specified values of the parameters for the weighted-second-upwind-differencing interpolation can only approximately reduce the effect of the artificial viscosity, particularly for the cases when the

specified time step of numerical integration, according to the Courant, Friedrichs and Lewy (CFL) criterion (Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972), results in local values of Courant number (e.g., Roache, 1972) much less (order of magnitude smaller) than one (1). However, the weighted-second-upwind-differencing interpolation cannot minimize the local values of artificial viscosity (as a part of the general numerical dispersion effect) in all the elements and half elements of the flow region.

4.3.3. Transportive-Upwind Differencing Interpolation for the Auxiliary Values of the Velocity Components Associated with the Momentum Transport across the Enclosure Surfaces of the Elements and Half Elements

The formulation of the governing computational system of discrete-element equations can also incorporate a transportive-upwind-differencing interpolation, which represents a modified form of the one-dimensional, second-order approximation for the convective terms (e.g., Welch, et al., 1966, Fromm, 1968), and which was specifically developed for the discrete-element method (e.g., Eraslan, 1976, 1981b). The transportive-upwind-differencing interpolation considers three specified parameters as:

$\beta_{x,ups}$ = specified parameter for transportive-upwind differencing interpolation along x-direction,

$\beta_{y,ups}$ = specified parameter for transportive-upwind differencing interpolation along y-direction,

$\beta_{z,ups}$ = specified parameter for transportive-upwind differencing interpolation along z-direction,

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{2},j,l}$, associated with the x-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{2},j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} - \frac{(1 + \beta_{x,ups}\frac{\bar{V}_{x;i-\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} [\, \bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;(i-1)+\frac{1}{4},j,l} \,],$$

$$\text{for all } \bar{V}_{x;i-\frac{1}{2},j,l}. \tag{4.68}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 + \beta_{x,ups}\frac{\bar{V}_{x;i-\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i-1,j-\frac{1}{4},l}),$$

$$\text{for all } \bar{V}_{x;i-\frac{1}{2},j,l}. \tag{4.69}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} - \frac{(1 + \beta_{x,ups}\frac{\bar{V}_{x;i-\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i-1,j+\frac{1}{4},l}),$$

$$\text{for all } \bar{V}_{x;i-\frac{1}{2},j,l}. \tag{4.70}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i-\frac{1}{2},j,l}$, associated with the z-component of momentum transport, at the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l), is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i-\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} - \frac{(1 + \beta_{x,ups}\frac{\bar{V}_{x;i-\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i-1,j}}{\Delta x_{i,j}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i-1,j,l}),$$

$$\text{for all } \bar{V}_{x;i-\frac{1}{2},j,l}. \tag{4.71}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i,j,l}$, associated with the x-component of momentum transport, at the x-downstream enclosure surface at (i,j,l) of the x-upstream half element $(i-\frac{1}{4},j,l)$ and at the x-upstream enclosure surface at (i,j,l) of the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i,j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} + (1 - \beta_{x,ups}\frac{\bar{V}_{x;i,j,l}\Delta t}{\Delta x_{i,j}}) (\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j,l}),$$

$$\text{for all } \bar{V}_{x;i,j,l}. \tag{4.72}$$

Similarly, the auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{2},j,l}$, associated with the x-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{2},j,l} = \bar{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 - \beta_{x,ups}\frac{\bar{V}_{x;i+\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})}[\bar{V}_{x;(i+1)+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l}],$$

$$\text{for all } \bar{V}_{x;i+\frac{1}{2},j,l}. \tag{4.73}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{(1 - \beta_{x,ups}\frac{\bar{V}_{x;i+\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})}(\bar{V}_{y;i+1,j-\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for all } \bar{V}_{x;i+\frac{1}{2},j,l}. \tag{4.74}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 - \beta_{x,ups}\frac{\bar{V}_{x;i+\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})} (\bar{V}_{y;i+1,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}),$$

$$\text{for all } \bar{V}_{x;i+\frac{1}{2},j,l} . \tag{4.75}$$

The auxiliary value of z-component of velocity $\tilde{V}_{z;i+\frac{1}{2},j,l}$, associated with the z-component of momentum transport, at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i+\frac{1}{2},j,l} = \bar{V}_{z;i,j,l} + \frac{(1 - \beta_{x,ups}\frac{\bar{V}_{x;i+\frac{1}{2},j,l}\Delta t}{\Delta x_{i,j}})}{(1 + \frac{\Delta x_{i+1,j}}{\Delta x_{i,j}})} (\bar{V}_{z;i+1,j,l} - \bar{V}_{z;i,j,l}),$$

$$\text{for all } \bar{V}_{x;i+\frac{1}{2},j,l} . \tag{4.76}$$

Similarly, the auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j-\frac{1}{2},l}$, associated with the y-component of momentum transport, at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j-\frac{1}{2},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 + \beta_{y,ups}\frac{\bar{V}_{y;i,j-\frac{1}{2},l}\Delta t}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})} [\, \bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i,(j-1)-\frac{1}{4},l} \,],$$

$$\text{for all } \bar{V}_{x;i-\frac{1}{2},j,l} . \tag{4.77}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-upstream enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i-\frac{1}{4},j,l} - \frac{\left(1 + \beta_{y,ups}\frac{\bar{V}_{y;i,j-\frac{1}{2},l}\Delta t}{\Delta y_{i,j}}\right)}{\left(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}}\right)} \left(\bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j-1,l}\right),$$

$$\text{for all } \bar{V}_{y;i,,j-\frac{1}{2},l}. \tag{4.78}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-upstream enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j,l} - \frac{\left(1 + \beta_{y,ups}\frac{\bar{V}_{y;i,j-\frac{1}{2},l}\Delta t}{\Delta y_{i,j}}\right)}{\left(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}}\right)} \left(\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j-1,l}\right),$$

$$\text{for all } \bar{V}_{y;i,,j-\frac{1}{2},l}. \tag{4.79}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j-\frac{1}{2},l}$, associated with the z-component of momentum transport, at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l), is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{z;i,j-\frac{1}{2},l} = \bar{V}_{z;i,j,l} - \frac{(1 + \beta_{y,ups}\frac{\bar{V}_{y;i,j-\frac{1}{2},l}\Delta t}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j}})}(\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j-1,l}),$$

$$\text{for all } \bar{V}_{y;i,,j-\frac{1}{2},l}\,. \tag{4.80}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j,l}$, associated with the y-component of momentum transport, at the y-downstream enclosure surface at (i,j,l) of the y-upstream half element (i,j-¼,l) and at the y-upstream enclosure surface at (i,j,l) of the y-downstream half element (i,j+¼,l), is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j,l} = \bar{V}_{y;i,j-\frac{1}{4},l} + (1 - \beta_{y,ups}\frac{\bar{V}_{y;i,j,l}\Delta t}{\Delta y_{i,j}})\,(\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for all } \bar{V}_{y;i,j,l}\,. \tag{4.81}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j+\frac{1}{2},l}$, associated with the y-component of momentum transport, at the y-downstream enclosure surface at (i,j+½,l) of the element (i,j,l) and the y-downstream half element (i,j+¼,l), is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j+\frac{1}{2},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 - \beta_{y,ups}\frac{\bar{V}_{y;i,j+\frac{1}{2},l}\Delta t}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})}[\,\bar{V}_{y;i,(j+1)-\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}\,],$$

$$\text{for all } \bar{V}_{y;i,,j+\frac{1}{2},l}\,. \tag{4.82}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-downstream enclosure surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{(1 - \beta_{y,ups}\frac{\bar{V}_{y;i,j+\frac{1}{2},l}\Delta t}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{x;i-\frac{1}{4},j+1,l} - \bar{V}_{x;i-\frac{1}{4},j,l}),$$

$$\text{for all } \bar{V}_{y;i,,j+\frac{1}{2},l} \,. \tag{4.83}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l}$, associated with the x-component of momentum transport, at the y-downstream enclosure surface at $(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l} = \bar{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 - \beta_{y,ups}\frac{\bar{V}_{y;i,j+\frac{1}{2},l}\Delta t}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{x;i+\frac{1}{4},j+1,l} - \bar{V}_{x;i+\frac{1}{4},j,l}),$$

$$\text{for all } \bar{V}_{y;i,,j+\frac{1}{2},l} \,. \tag{4.84}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,j+\frac{1}{2},l}$, associated with the z-component of momentum transport, at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{z;i,j+\frac{1}{2},l} = \bar{V}_{z;i,j,l} + \frac{(1 - \beta_{y,ups}\frac{\bar{V}_{y;i,j+\frac{1}{2},l}\Delta t}{\Delta y_{i,j}})}{(1 + \frac{\Delta y_{i,j+1}}{\Delta y_{i,j}})} (\bar{V}_{z;i,j+1,l} - \bar{V}_{z;i,j,l}),$$

$$\text{for all } \bar{V}_{y;i,,j+\frac{1}{2},l} . \tag{4.85}$$

Similarly, the auxiliary value of the z-component of velocity $\tilde{\bar{V}}_{z;i,j,l-\frac{1}{2}}$, associated with the z-component of momentum transport, at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ of the element (i,j,l) is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{z;i,j,l-\frac{1}{2}} = \bar{V}_{z;i,j,l} - \frac{(1 + \beta_{z,ups}\frac{\bar{V}_{z;i,j,l-\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j,l-1}),$$

$$\text{for all } \bar{V}_{z;i,j,l-\frac{1}{2}} . \tag{4.86}$$

The auxiliary value of the x-component of velocity $\tilde{\bar{V}}_{x;i-\frac{1}{4},j,l-\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-upstream enclosure surface at $(i-\frac{1}{4},j,l-\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{x;i-\frac{1}{4},j,l-\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} - \frac{(1 + \beta_{z,ups}\frac{\bar{V}_{z;i,j,l-\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j,l-1}),$$

$$\text{for all } \bar{V}_{z;i,j,l-\frac{1}{2}} . \tag{4.87}$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-upstream enclosure surface at $(i+\frac{1}{4},j,l-\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\widetilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}} = \overline{V}_{x;i+\frac{1}{4},j,l} - \frac{(1 + \beta_{z,ups}\frac{\overline{V}_{z;i,j,l-\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\overline{V}_{x;i+\frac{1}{4},j,l} - \overline{V}_{x;i+\frac{1}{4},j,l-1}),$$

$$\text{for all } \overline{V}_{z;i,j,l-\frac{1}{2}} . \tag{4.88}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-upstream enclosure surface at $(i,j-\frac{1}{4},l-\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\widetilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}} = \overline{V}_{y;i,j-\frac{1}{4},l} - \frac{(1 + \beta_{z,ups}\frac{\overline{V}_{z;i,j,l-\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})} (\overline{V}_{y;i,j-\frac{1}{4},l} - \overline{V}_{y;i,j-\frac{1}{4},l-1}),$$

$$\text{for all } \overline{V}_{z;i,j,l-\frac{1}{2}} . \tag{4.89}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-upstream enclosure surface at $(i,j+\frac{1}{4},l-\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{y;i,j+\frac{1}{4},l-\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} - \frac{(1 + \beta_{z,ups}\frac{\bar{V}_{z;i,j,l-\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l}})}(\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l-1}),$$

$$\text{for all } \bar{V}_{z;i,j,l-\frac{1}{2}} \tag{4.90}$$

Similarly, the auxiliary value of the z-component of velocity $\tilde{\bar{V}}_{z;i,j,l+\frac{1}{2}}$, associated with the z-component of momentum transport, at the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l) is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{z;i,j,l+\frac{1}{2}} = \bar{V}_{z;i,j,l} + \frac{(1 - \beta_{z,ups}\frac{\bar{V}_{z;i,j,l+\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})}(\bar{V}_{z;i,j,l+1} - \bar{V}_{z;i,j,l}),$$

$$\text{for all } \bar{V}_{z;i,j,l+\frac{1}{2}} \tag{4.91}$$

The auxiliary value of the x-component of velocity $\tilde{\bar{V}}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-downstream enclosure surface at $(i-\frac{1}{4},j,l+\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{x;i-\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i-\frac{1}{4},j,l} + \frac{(1 - \beta_{z,ups}\frac{\bar{V}_{z;i,j,l+\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})}(\bar{V}_{x;i-\frac{1}{4},j,l+1} - \bar{V}_{x;i-\frac{1}{4},j,l}),$$

$$\text{for all } \bar{V}_{z;i,j,l+\frac{1}{2}} \tag{4.92}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}$, associated with the x-component of momentum transport, at the z-downstream enclosure surface at $(i+\frac{1}{4},j,l+\frac{1}{2})$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}} = \bar{V}_{x;i+\frac{1}{4},j,l} + \frac{(1 - \beta_{z,ups}\frac{\bar{V}_{z;i,j,l+\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})}(\bar{V}_{x;i+\frac{1}{4},j,l+1} - \bar{V}_{x;i+\frac{1}{4},j,l}),$$

$$\text{for all } \bar{V}_{z;i,j,l+\frac{1}{2}}. \tag{4.93}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-downstream enclosure surface at $(i,j-\frac{1}{4},l+\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{(1 - \beta_{z,ups}\frac{\bar{V}_{z;i,j,l+\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})}(\bar{V}_{y;i,j-\frac{1}{4},l+1} - \bar{V}_{y;i,j-\frac{1}{4},l}),$$

$$\text{for all } \bar{V}_{z;i,j,l+\frac{1}{2}}. \tag{4.94}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}}$, associated with the y-component of momentum transport, at the z-downstream enclosure surface at $(i,j+\frac{1}{4},l+\frac{1}{2})$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, according to the transportive-upwind-differencing interpolation, as:

$$\tilde{\bar{V}}_{y;i,j+\frac{1}{4},l+\frac{1}{2}} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{(1 - \beta_{z,ups}\frac{\bar{V}_{z;i,j,l+\frac{1}{2}}\Delta t}{\Delta z_{i,j,l}})}{(1 + \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l}})} (\bar{V}_{y;i,j+\frac{1}{4},l+1} - \bar{V}_{y;i,j+\frac{1}{4},l}),$$

$$\text{for all } \bar{V}_{z;i,j,l+\frac{1}{2}} . \tag{4.95}$$

The formulation of the auxiliary values of the components of velocity, associated with the components of momentum transport, at the enclosure surfaces of the elements and half elments, according to the transportive-upwind-differencing interpolation [Eqs. (4.68) through (4.95)], can eliminate all the auxiliary values of the components of the velocity which appear in the terms, associated with the momentum transport, in the governing computational system of discrete-element equations [Eqs. (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113), (3.114)].

The transportive-upwind-differencing interpolation of the components of velocity, associated with the components of momentum transport, establishes the required stability of the numerical solutions, according to the timewise-explicit computational algorithm, for the specified values of the parameters $\beta_{x,ups}$, $\beta_{y,ups}$ and $\beta_{z,ups}$ of the transportive-upwind-differencing interpolation, along x-, y- and z-direction, respectively, all equal to one (1). Furthermore, for the specified values of the parameters $\beta_{x,ups}$, $\beta_{y,ups}$ and $\beta_{z,ups}$ of the transportive-upwind-differencing interpolation, along x-, y- and z-direction, respectively, all equal to one (1), the effect of the artificial

viscosity (as a part of the general numerical dispersion effect) identically vanishes, from first-order mathematical considerations of the discretization technique of the general discrete-element method. However, the higher order effects, associated with the discretization of the convective transport terms, could result in small, oscillatory errors which could not be avoided for the values of the parameters $\beta_{x,ups}$, $\beta_{y,ups}$ and $\beta_{z,ups}$ of the transportive-upwind-differencing interpolation, along x-, y- and z-direction, respectively, all equal to one (1). Therefore, it is usually more convient to consider certain ranges for the specified values of the parameters $\beta_{x,ups}$, $\beta_{y,ups}$ and $\beta_{z,ups}$ of the transportive-upwind-differencing interpolations along x-, y- and z-direction, respectively, as:

$$0.8 \leq \beta_{x,ups} \leq 1.2, \tag{4.96a}$$

$$0.8 \leq \beta_{y,ups} \leq 1.2, \tag{4.96b}$$

$$0.8 \leq \beta_{z,ups} \leq 1.2. \tag{4.96c}$$

The appropriate specified values of the parameters $\beta_{x,ups}$, $\beta_{y,ups}$ and $\beta_{z,ups}$ of the transportive-upwind-differencing interpolation along x-, y- and z-direction, respectively, and within the suggested ranges [Eqs. (4.96a) through (4.96c)], can be determined based on the computer simulation results of the fast-transient, three-dimensional discrete-element hydrodynamic model in application to site-specific problems in actual flow regions.

4.4. Interpolation of the Auxiliary Values of Pressure Associated with the Forces Acting on the Enclosure Surfaces of the Half Elements

The general formulation of the fast-transient, three-dimensional, discrete-element hydrodynamic model includes the auxiliary values of pressure which are necessary to determine the pressure forces acting on the half-elements [Eqs. (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100)]. The formulation of these auxiliary values of pressure requires careful considerations of the general discretization technique used in the development of the governing computational system of discrete-element equations.

The required formulaton for determining the auxiliary values of pressure, used in the formulation of the pressure force terms acting on the half elements must guarantee that the effects of the varying depth conditions could not generate any artificial force, due to the interpolation algorithm, which could result in unrealistic velocity conditions in bottom layers in the flow region.

Furthermore, the required formulation for determining the auxiliary values of pressure, used in the formulation of the pressure force terms acting on the half elements, must guarantee that the effects of the variable sizes of the adjacent elements and half elements could not generate any artificial force, due to the spatial interpolation algorithm, which could result in unrealistic velocity conditions in the different layers of the flow region.

Hence, a special, three-step, linear-interpolation formulation was developed for determining the auxiliary values of the modified pressure

pw associated with the pressure forces acting on the half elements. The special formulation is applicable to all layers in the flow region, including the bottom layer $l = l(bot,i,j)$ and the surface layer $l = L$ for all adjacent elements.

The auxiliary values of modified pressure $\bar{p}_{w;i-\frac{1}{2},j,l}$ associated with the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is determined, according to the special, three-step, linear-interpolation formulation.

First, the auxiliary value of modified pressure $\bar{p}_{w;i-\frac{1}{2},j,l+\frac{1}{2}}$ at the x-upstream enclosure surface, at the top of the layer, at $(i-\frac{1}{2},j,l+\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to standard linear-interpolation, as:

$$\bar{p}_{w;i-\frac{1}{2},j,l+\frac{1}{2}} = \bar{p}_{w;i,j,l+\frac{1}{2}} - \frac{\Delta x_{i,j}}{(\Delta x_{i,j} + \Delta x_{i-1,j})} (\bar{p}_{w;i,j,l+\frac{1}{2}} - \bar{p}_{w;i-1,j,l+\frac{1}{2}}) . \quad (4.97)$$

Second, the auxiliary value of modified pressure $\bar{p}_{w;i-\frac{1}{2},j,l-\frac{1}{2}}$ at the x-upstream enclosure surface at the bottom of the layer at $(i-\frac{1}{2},j,l-\frac{1}{2})$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated, according to standard linear-interpolation, as:

$$\bar{p}_{w;i-\frac{1}{2},j,l-\frac{1}{2}} = \bar{p}_{w;i,j,l-\frac{1}{2}} - \frac{\Delta x_{i,j}}{(\Delta x_{i,j} + \Delta x_{i-1,j})} (\bar{p}_{w;i,j,l-\frac{1}{2}} - \bar{p}_{w;i-1,j,l-\frac{1}{2}}) . \quad (4.98)$$

Third, the auxiliary value of modified pressure $\bar{p}_{w;i-\frac{1}{2},j,l}$, associated with the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the x-upstream half element

$(i-\frac{1}{4},j,l)$, is formulated, according to standard linear-interpolation, along the z-direction, as:

$$\bar{p}_{w;i-\frac{1}{2},j,l} = \bar{p}_{w;i-\frac{1}{2},j,l+\frac{1}{2}} + \frac{\Delta z_{i,j,l}}{2\Delta z_{i-\frac{1}{2},j,l}} \left(\bar{p}_{w;i-\frac{1}{2},j,l-\frac{1}{2}} - \bar{p}_{w;i-\frac{1}{2},j,l+\frac{1}{2}}\right). \quad (4.99)$$

Similarly, the auxiliary value of modified pressure $\bar{p}_{w;i+\frac{1}{2},j,l}$, associated with the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is determined, according to the special, three-step, linear-interpolation formulation.

First, the auxiliary value of modified pressure $\bar{p}_{w;i+\frac{1}{2},j,l+\frac{1}{2}}$ at the x-downstream enclosure surface, at the top of the layer at $(i+\frac{1}{2},j,l+\frac{1}{2})$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated, according to standard linear-interpolation, as:

$$\bar{p}_{w;i+\frac{1}{2},j,l+\frac{1}{2}} = \bar{p}_{w;i,j,l+\frac{1}{2}} + \frac{\Delta x_{i,j}}{(\Delta x_{i,j} + \Delta x_{i+1,j})} \left(\bar{p}_{w;i+1,j,l+\frac{1}{2}} - \bar{p}_{w;i,j,l+\frac{1}{2}}\right). \quad (4.100)$$

Second, the auxiliary value of modified pressure $\bar{p}_{w;i+\frac{1}{2},j,l-\frac{1}{2}}$ at the x-downstream enclosure surface at the bottom of the layer at $(i+\frac{1}{2},j,l-\frac{1}{2})$ of the x-downstream half element $(i+\frac{1}{4},j,l)$ is formulated, according to the standard linear-interpolation, as:

$$\bar{p}_{w;i+\frac{1}{2},j,l-\frac{1}{2}} = \bar{p}_{w;i,j,l-\frac{1}{2}} + \frac{\Delta x_{i,j}}{(\Delta x_{i,j} + \Delta x_{i+1,j})} \left(\bar{p}_{w;i+1,j,l-\frac{1}{2}} - \bar{p}_{w;i,j,l-\frac{1}{2}}\right). \quad (4.101)$$

THird, the auxiliary value of modified pressure $\bar{p}_{w;i+\frac{1}{2},j,l}$, associated with the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the x-downstream half

element $(i+\frac{1}{4},j,l)$, is formulated, according to standard linear-interpolation, along the z-direction, as:

$$\bar{p}_{w;i+\frac{1}{2},j,l} = \bar{p}_{w;i+\frac{1}{2},j,l+\frac{1}{2}} + \frac{\Delta z_{i,j,l}}{2\Delta z_{i+\frac{1}{2},j,l}} \left(\bar{p}_{w;i+\frac{1}{2},j,l-\frac{1}{2}} - \bar{p}_{w;i+\frac{1}{2},j,l+\frac{1}{2}}\right). \quad (4.102)$$

Similarly, the auxiliary value of modified pressure $\bar{p}_{w;i,j-\frac{1}{2},l}$, associated with the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is determined, according to the special, three-step, linear-interpolation formulation.

First, the auxiliary value of modified pressure $\bar{p}_{w;i,j+\frac{1}{2},l-\frac{1}{2}}$ at the y-upstream enclosure surface, at the top of the layer at $(i,j-\frac{1}{2},l+\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to standard linear-interpolation, as:

$$\bar{p}_{w;i,j-\frac{1}{2},l+\frac{1}{2}} = \bar{p}_{w;i,j,l+\frac{1}{2}} - \frac{\Delta y_{i,j}}{(\Delta y_{i,j} + \Delta y_{i,j-1})} \left(\bar{p}_{w;i,j,l+\frac{1}{2}} - \bar{p}_{w;i,j-1,l+\frac{1}{2}}\right). \quad (4.103)$$

Second, the auxiliary value of modified pressure $\bar{p}_{w;i,j-\frac{1}{2},l-\frac{1}{2}}$ at the y-upstream enclosure surface at the bottom of the layer at $(i,j-\frac{1}{2},l-\frac{1}{2})$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is formulated, according to standard linear-interpolation, as:

$$\bar{p}_{w;i,j-\frac{1}{2},l-\frac{1}{2}} = \bar{p}_{w;i,j,l-\frac{1}{2}} - \frac{\Delta y_{i,j}}{(\Delta y_{i,j} + \Delta y_{i,j-1})} \left(\bar{p}_{w;i,j,l-\frac{1}{2}} - \bar{p}_{w;i,j-1,l-\frac{1}{2}}\right). \quad (4.104)$$

Third, the auxiliary value of modified pressure $\bar{p}_{w;i,j-\frac{1}{2},l}$, associated with the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the y-upstream half

element $(i,j-\frac{1}{4},l)$, is formulated, according to standard linear-interpolation, along the z-direction, as:

$$\bar{p}_{w;i,j-\frac{1}{4},l} = \bar{p}_{w;i,j-\frac{1}{4},l+\frac{1}{2}} + \frac{\Delta z_{i,j,l}}{2\Delta z_{i,j-\frac{1}{4},l}} \left(\bar{p}_{w;i,j-\frac{1}{4},l-\frac{1}{2}} - \bar{p}_{w;i,j-\frac{1}{4},l+\frac{1}{2}}\right). \quad (4.105)$$

Similarly, the auxiliary value of modified pressure $\bar{p}_{w;i,j+\frac{1}{2},l}$, associated with the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is determined, according to the special, three-step, linear-interpolation formulation.

First, the auxiliary value of modified pressure $\bar{p}_{w;i,j+\frac{1}{2},l+\frac{1}{2}}$ at the y-downstream enclosure surface, at the top of the layer at $(i,j+\frac{1}{2},l+\frac{1}{2})$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, according to standard linear-interpolation, as:

$$\bar{p}_{w;i,j+\frac{1}{2},l+\frac{1}{2}} = \bar{p}_{w;i,j,l+\frac{1}{2}} + \frac{\Delta y_{i,j}}{(\Delta y_{i,j} + \Delta y_{i,j+1})} \left(\bar{p}_{w;i,j+1,l+\frac{1}{2}} - \bar{p}_{w;i,j,l+\frac{1}{2}}\right). \quad (4.106)$$

Second, the auxiliary value of modified pressure $\bar{p}_{w;i,j+\frac{1}{2},l-\frac{1}{2}}$ at the y-downstream enclosure surface at the bottom of the layer at $(i,j+\frac{1}{2},l-\frac{1}{2})$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated, according to standard linear-interpolation, as:

$$\bar{p}_{w;i,j+\frac{1}{2},l-\frac{1}{2}} = \bar{p}_{w;i,j,l-\frac{1}{2}} + \frac{\Delta y_{i,j}}{(\Delta y_{i,j} + \Delta y_{i,j+1})} \left(\bar{p}_{w;i,j+1,l-\frac{1}{2}} - \bar{p}_{w;i,j,l-\frac{1}{2}}\right) \quad (4.107)$$

Third, the auxiliary value of modified pressure $\bar{p}_{w;i,j+\frac{1}{2},l}$, associated with the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ and the bottom enclosure surface for $l = l(bot;i,j)$ of the y-downstream half

element $(i,j+\frac{1}{4},l)$, is formulated, according to standard linear-interpolation, along the z-direction, as:

$$\bar{p}_{w;i,j+\frac{1}{2},l} = \bar{p}_{w;i,j+\frac{1}{2},l+\frac{1}{2}} + \frac{\Delta z_{i,j,l}}{2\Delta z_{i,j+\frac{1}{2},l}} \left(\bar{p}_{w;i,j+\frac{1}{2},l-\frac{1}{2}} - \bar{p}_{w;i,j+\frac{1}{2},l+\frac{1}{2}}\right) . \tag{4.108}$$

The auxiliary value of the modified pressure $\bar{p}_{w;i,j,l}$, at the center of the element (i,j,l), which corresponds also to the auxiliary value of the modified pressure (1) at the x-downstream enclosure surface at (i,j,l) of the x-upstream half element $(i-\frac{1}{4},j,l)$, (2) at the x-upstream enclosure surface at (i,j,l) of the x-downstream half element $(i+\frac{1}{4},j,l)$, (3) at the y-downstream enclosure surface at (i,j,l) of the y-upstream half element $(i,j-\frac{1}{4},l)$, (4) at the y-upstream enclosure surface at (i,j,l) of the y-downstream half element $(i,j+\frac{1}{4},l)$, is formulated according to standard linear-interpolation, along the z-direction, as:

$$\bar{p}_{w;i,j,l} = \frac{1}{2}\left(\bar{p}_{w;i,j,l+\frac{1}{2}} + \bar{p}_{w;i,j,l-\frac{1}{2}}\right) . \tag{4.109}$$

The formulations of the auxiliary value of modified pressure, associated with pressure forces acting on the enclosure surfaces of the half elements, according to the special, three-step, linear-interpolation formulation [Eqs. (4.97) through (4.109)], eliminate all the auxiliary value of modified pressure, associated with the terms of the discrete-element equations for the components of velocity, of the governing computational system of discrete-element equations [Eqs.

(3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100)].

4.5. Formulation of Boundary Conditions on the Enclosure Surfaces of the Elements and Half Elements Corresponding to the Enclosure Boundaries of the Flow Region

The auxiliary values of the principal flow variables at the enclosure surfaces of the elements and half elements, which correspond to the parts of the enclosure boundary of the flow region, require special formulation, since these auxiliary values cannot be determined based on interpolation.

In the general formulation of the fast-transient, three-dimensional, discrete-element hydrodynamic model, three special types of enclosure surfaces are considered for representing the parts of the enclosure boundary of the flow region, as:

1. shoreline-boundary enclosure surface,
2. specified-boundary enclosure surface,
3. open-region enclosure surface.

The general formulations of the conditions for the auxiliary values of the flow variables on the boundary-enclosure surfaces can incorporlate all possible flow conditions that need to be specified on the enclosure boundary of a geophysical flow region.

4.5.1. Boundary Conditions for Shoreline-Boundary Enclosure Surfaces of the Elements and Half Elements

The automated discrete-element generater computer code PREPR2 (Holdeman, 1977) or its modified version PREPAR (Lin, Erhan and Eraslan, 1978) can directly identify all the enclosure surfaces of the elements which correspond to the parts of the existing shoreline boundary of the flow region.

The conditions for the auxiliary values of the flow variables at the shoreline-boundary enclosure surfaces of the elements and half elements are determined directly from the impermeable conditions associated with the shoreline boundary of the flow region. In general, all the auxiliary values at a shoreline-boundary enclosure surface, associated with the flow along the normal to the enclosure surface, and associated with the components of momentum transport across the enclosure surface are considered as zero based on the impermeable surface conditions of the shoreline.

One of the important features of the general discrete-element method is that the computational algorithm does not calculate the velocity component normal to a shoreline-boundary enclosure surface in the half element adjacent to the shoreline boundary. Hence, the general formulation of the discrete-element method completely eliminates the possibility of numerical instability problems which could be generated due to the discontinuous (in the discretized sense) reduction of a velocity component from a finite value to zero within a single half element (e.g., Roache, 1972).

The auxiliary values of the flow variables at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l)

and the x-upstream half-element $(i-\frac{1}{4},j,l)$ are formulated, based on the impermeability conditions associated with the shoreline boundary.

The auxiliary value of the flow rate $\overline{G}_{x;(shr,i-\frac{1}{2}),j,l}$ at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half-element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\overline{G}_{x;i-\frac{1}{2},j,l} = \overline{G}_{x;(shr,i-\frac{1}{2}),j,l} = 0, \text{ for all time } t. \tag{4.110a}$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;(shr,i-\frac{1}{2}),j,l}$, associated with the x-component of momentum transport, at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\widetilde{V}_{x;i-\frac{1}{2},j,l} = \widetilde{V}_{x;(shr,i-\frac{1}{2}),j,l} = 0, \text{ for all time } t. \tag{4.110b}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;(shr,i-\frac{1}{2}),j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l} = \widetilde{V}_{y;(shr,i-\frac{1}{2}),j-\frac{1}{4},l} = 0, \text{ for all time } t. \tag{4.110c}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;(shr,i-\frac{1}{2}),j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l} = \widetilde{V}_{y;(shr,i-\frac{1}{2}),j+\frac{1}{4},l} = 0, \text{ for all time } t. \tag{4.110d}$$

The auxiliary value of the z-component of velocity $\widetilde{V}_{z;(shr,i-\frac{1}{2}),j,l}$, associated with the z-component of momentum transport, at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,l)$ of element (i,j,l), is specified, as:

$$\widetilde{V}_{z;i-\frac{1}{2},j,l} = \widetilde{V}_{z;(shr,i-\frac{1}{2}),j,l} = 0, \text{ for all time } t. \tag{4.110e}$$

According to the general formulation of the discrete-element method, the solution of the x-component of velocity $\overline{V}_{x;(shr,i-\frac{1}{4}),j,l}$ in the x-upstream half element $(i-\frac{1}{4},j,l)$ adjacent to the shoreline boundary, which corresponds to the location of the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,l)$, is not included in the computations, i.e.,

$$\overline{V}_{x;i-\frac{1}{4},j,l} = \overline{V}_{x;(shr,i-\frac{1}{4}),j,l} \quad \text{not calculated, for all time } t. \tag{4.111}$$

Similarly, the auxiliary values of the flow variables at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half-element $(i+\frac{1}{4},j,l)$ are formulated, based on the impermeability conditions associated with the shoreline boundary.

The auxiliary value of the flow rate $\overline{G}_{x;(shr,i+\frac{1}{2}),j,l}$ at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half-element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\overline{G}_{x;i+\frac{1}{2},j,l} = \overline{G}_{x;(shr,i+\frac{1}{2}),j,l} = 0, \text{ for all time } t. \tag{4.112a}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;(shr,i+\frac{1}{2}),j,l}$, associated with the x-component of momentum transport, at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of the x-downstream half-element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x;i+\frac{1}{2},j,l} = \tilde{V}_{x;(shr,i+\frac{1}{2}),j,l} = 0, \text{ for all time } t. \tag{4.112b}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;(shr,i+\frac{1}{2}),j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, is specified, as:

$$\tilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l} = \tilde{V}_{y;(shr,i+\frac{1}{2}),j-\frac{1}{4},l} = 0, \text{ for all time } t. \tag{4.112c}$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;(shr,i+\frac{1}{2}),j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l} = \tilde{V}_{y;(shr,i+\frac{1}{2}),j+\frac{1}{4},l} = 0, \text{ for all time } t. \tag{4.112d}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;(shr,i+\frac{1}{2}),j,l}$, associated with the z-component of momentum transport, at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of element (i,j,l), is specified, as:

$$\tilde{V}_{z;i+\frac{1}{2},j,l} = \tilde{V}_{z;(shr,i+\frac{1}{2}),j,l} = 0, \text{ for all time } t. \tag{4.112e}$$

Similarly, according to the general formulation of the discrete-element method, the solution of the x-component of velocity $\bar{V}_{x;(shr,i+\frac{1}{4}),j,l}$ in the x-downstream half element $(i+\frac{1}{4},j,l)$ adjacent to the shoreline boundary, which corresponds to the location of the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$, is not included in the computations, i.e.,

$$\bar{V}_{x;i+\frac{1}{4},j,l} = \bar{V}_{x;(shr,i+\frac{1}{4}),j,l} \text{ not calculated, for all time } t . \qquad (4.113)$$

The auxiliary values of the flow variables at the y-upstream shoreline-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half-element $(i,j-\frac{1}{4},l)$ are formulated, based on the impermeability conditions associated with the shoreline boundary.

The auxiliary value of the flow rate $\bar{G}_{y;i,(shr,j-\frac{1}{2}),l}$ at the y-upstream shoreline-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half-element $(i,j-\frac{1}{4},l)$, is specified, as:

$$\bar{G}_{y;i,j-\frac{1}{2},l} = \bar{G}_{y;i,(shr,j-\frac{1}{2}),l} = 0, \text{ for all time } t . \qquad (4.114a)$$

The auxiliary value of the y-component of velocity $\tilde{V}_{y;i,(shr,j-\frac{1}{2}),l}$, associated with the y-component of momentum transport, at the y-upstream shoreline-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the y-upstream half-element $(i,j-\frac{1}{4},l)$, is specified, as:

$$\tilde{V}_{y;i,j-\frac{1}{2},l} = \tilde{V}_{y;i,(shr,j-\frac{1}{2}),l} = 0, \text{ for all time } t . \qquad (4.114b)$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i-\frac{1}{4},(shr,j-\frac{1}{2}),l}$, associated with the x-component of momentum

transport, at the y-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l} = \tilde{V}_{x;i-\frac{1}{4},(shr,j-\frac{1}{2}),l} = 0, \quad \text{for all time } t. \tag{4.114c}$$

The auxiliary value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},(shr,j-\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-upstream shoreline-boundary enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l} = \tilde{V}_{x;i+\frac{1}{4},(shr,j-\frac{1}{2}),l} = 0, \quad \text{for all time } t. \tag{4.114d}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,(shr,j-\frac{1}{2}),l}$, associated with the z-component of momentum transport, at the y-upstream shoreline-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of element (i,j,l), is specified, as:

$$\tilde{V}_{z;i,j-\frac{1}{2},l} = \tilde{V}_{z;i,(shr,j-\frac{1}{2}),l} = 0, \text{ for all time } t. \tag{4.114e}$$

Similarly, according to the general formulation of the discrete-element method, the solution of the y-component of velocity $\bar{V}_{y;i,(shr,j-\frac{1}{4}),l}$ in the y-upstream half element $(i,j-\frac{1}{4},l)$ adjacent to the shoreline boundary, which corresponds to the location of the y-upstream shoreline-boundary enclosure surface at $(i,j-\frac{1}{2},l)$, is not included in the computations, i.e.,

$$\bar{V}_{y;i,j-\frac{1}{4},l} = \bar{V}_{y;i,(shr,j-\frac{1}{4}),l} \text{ not calculated, for all time } t. \tag{4.115}$$

The auxiliary values of the flow variables at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half-element $(i,j+\frac{1}{4},l)$ are formulated, based on the impermeability conditions associated with the shoreline boundary.

The auxiliary value of the flow rate $\overline{G}_{y;i,(shr,j+\frac{1}{2}),l}$ at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half-element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\overline{G}_{y;i,j+\frac{1}{2},l} = \overline{G}_{y;i,(shr,j+\frac{1}{2}),l} = 0, \text{ for all time } t. \tag{4.116a}$$

The auxiliary value of the y-component of velocity $\widetilde{V}_{y;i,(shr,j+\frac{1}{2}),l}$, associated with the y-component of momentum transport, at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the y-downstream half-element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y;i,j+\frac{1}{2},l} = \widetilde{V}_{y;i,(shr,j+\frac{1}{2}),l} = 0, \text{ for all time } t. \tag{4.116b}$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;i-\frac{1}{4},(shr,j+\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-downstream shoreline-boundary enclosure surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half-element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l} = \widetilde{V}_{x;i-\frac{1}{4},(shr,j+\frac{1}{2}),l} = 0, \text{ for all time } t. \tag{4.116c}$$

The auxiliary value of the x-component of velocity $\widetilde{V}_{x;i+\frac{1}{4},(shr,j+\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-downstream shoreline-boundary enclosure surface at

$(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half-element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l} = \tilde{V}_{x;i+\frac{1}{4},(shr,j+\frac{1}{2}),l} = 0, \text{ for all time } t. \tag{4.116d}$$

The auxiliary value of the z-component of velocity $\tilde{V}_{z;i,(shr,j+\frac{1}{2}),l}$, associated with the z-component of momentum transport, at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of element (i,j,l), is specified, as:

$$\tilde{V}_{z;i,j+\frac{1}{2},l} = \tilde{V}_{z;i,(shr,j+\frac{1}{2}),l} = 0, \text{ for all time } t. \tag{4.116e}$$

Similarly, according to the general formulation of the discrete-element method, the solution of the y-component of velocity $\bar{V}_{y;i,(shr,j+\frac{1}{4}),l}$ in the y-downstream half element $(i,j+\frac{1}{4},l)$ adjacent to the shoreline boundary, which corresponds to the location of the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$, is not included in the computations, i.e.,

$$\bar{V}_{y;i,j+\frac{1}{4},l} = \bar{V}_{y;i,(shr,j+\frac{1}{4}),l} \text{ not calculated, for all time } t. \tag{4.117}$$

4.5.2. Boundary Conditions for Specified-Boundary Enclosure Surface of the Elements and Half Elements

The general mathematical formulation of the fast-transient, three-dimensional, discrete-element hydrodynamic model can incorporate any form of temporally varying specified flow conditions on the enclosure surfaces of the elements and half elements. The specified boundary conditions can incude (1) temporally varying flow conditions from rivers and (2) tributaries into the main flow region and temporally

varying flow conditions from large intakes and discharges with opening areas to the flow region at specified locations along the shoreline enclosure boundaries.

The discrete elements which contain the enclosure surfaces with specified boundary conditions can be identified by the index Ispb which represents the numbers assigned to the specified-boundary elements. The locations of the specified-boundary enclosure surfaces are formulated in terms of the indices of the discrete-element system and the lowest and top layers of the openings of the area of the boundary where the flow conditions are specified. In each specified-boundary element, the specified flow conditions can be assigned to only one of the two enclosure surfaces normal to the x-direction and/or only to one of the two enclosure surfaces normal to the y-direction.

The locations of the specified-boundary elements are specified, as:

$$i_{spb}(I_{spb}) \text{ specified, for all time t,} \tag{4.118a}$$

$$j_{spb}(I_{spb}) \text{ specified, for all time t,} \tag{4.118b}$$

$$l_{spb,x,low}(I_{spb}) \text{ specified, for all time t,} \tag{4.119a}$$

$$l_{spb,x,top}(I_{spb}) \text{ specified, for all time t,} \tag{4.119b}$$

$$l_{spb,y,low}(I_{spb}) \text{ specified, for all time t,} \tag{4.120a}$$

$$l_{spb,y,top}(I_{spb}) \text{ specified, for all time t.} \tag{4.120b}$$

The length along y-direction $\Delta y_{(spb,i\pm\frac{1}{2}),j}(I_{spb})$ of the specified-boundary enclosure surface, for all layers, is specified, as:

$$\Delta y_{(spb,i\pm\frac{1}{2}),j}(I_{spb}) \text{ specified, for all time t.} \tag{4.121a}$$

Similarly, the length along x-direction $\Delta x_{i,(spb,j\pm\frac{1}{2})}(I_{spb})$ of the specified-boundary enclosure surface, for all layers, is defined, as:

$$\Delta x_{i,(spb,j\pm\frac{1}{2})}(I_{spb}) \text{ specified, for all time t.} \tag{4.121b}$$

The value of the x-component of velocity $\bar{V}_{x;(spb,i\pm\frac{1}{2}),j,l}$, associated with the flow rate, at the x-upstream specified-boundary enclosure surface at (i-½,j,l) of the element (i,j,l) and the x-upstream half element (i-¼,j,l), or at the x-downstream specified-boundary enclosure surface at (i+½,j,l) of the element (i,j,l) and the x-downstream half element (i+¼,j,l), is specified, as:

$$\bar{V}_{x;(spb,i\pm\frac{1}{2}),j,l} = \left[\bar{V}_x(t;I_{spb})\right]_{(spb,i\pm\frac{1}{2}),j,l},$$

$$\text{for } l_{spb,x,low}(I_{spb}) \leq l \leq l_{spb,x,top}(I_{spb}). \tag{4.122a}$$

The value of the x-component of velocity $\tilde{V}_{x;(spb,i\pm\frac{1}{2}),j,l}$, associated with the x-component of momentum transport, at the x-upstream specified-boundary enclosure surface at (i-½,j,l) of the x-upstream half element (i-¼,j,l), or at the x-downstream specified-boundary enclosure surface at (i+½,j,l) of the x-downstream half element (i+¼,j,l), is specified, as:

$$\widetilde{V}_{x;(spb,i\pm\frac{1}{2}),j,l} = \left[\widetilde{V}_x(t;I_{spb})\right]_{(spb,i\pm\frac{1}{2}),j,l} ,$$

$$\text{for } I_{spb,x,low}(I_{spb}) \leq I \leq I_{spb,x,top}(I_{spb}) . \qquad (4.122b)$$

The value of the y-component of velocity $\widetilde{V}_{y;(spb,i\pm\frac{1}{2}),j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$ or at the x-downstream specified-boundary enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y;(spb,i\pm\frac{1}{2}),j-\frac{1}{4},l} = \left[\widetilde{V}_{y;j-\frac{1}{4}}(t;I_{spb})\right]_{(spb,i\pm\frac{1}{2}),j-\frac{1}{4},l} ,$$

$$\text{for } I_{spb,x,low}(I_{spb}) \leq I \leq I_{spb,x,top}(I_{spb}) . \qquad (4.122c)$$

The value of the y-component of velocity $\widetilde{V}_{y;(spb,i\pm\frac{1}{2}),j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$ or at the x-downstream specified-boundary enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y;(spb,i\pm\frac{1}{2}),j+\frac{1}{4},l} = \left[\widetilde{V}_{y;j+\frac{1}{4}}(t;I_{spb})\right]_{(spb,i\pm\frac{1}{2}),j+\frac{1}{4},l} ,$$

$$\text{for } I_{spb,x,low}(I_{spb}) \leq I \leq I_{spb,x,top}(I_{spb}) . \qquad (4.122d)$$

The value of the z-component of velocity $\widetilde{V}_{z;(spb,i\pm\frac{1}{2}),j,l}$, associated with the z-component of momentum transport, at the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j,l)$ or at the x-downstream

specified-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), is specified, as:

$$\tilde{V}_{z;(spb,i\pm\frac{1}{2}),j,l} = \left[\tilde{V}_z(t;I_{spb})\right]_{(spb,i\pm\frac{1}{2}),j,l},$$

$$\text{for } l_{spb,x,low}(I_{spb}) \leq l \leq l_{spb,x,top}(I_{spb}). \tag{4.122e}$$

In the case that the specified-boundary element is in the water surface layer $l = L$, the water-surface elevation $\bar{H}_{(spb,i\pm\frac{1}{2}),j}$ at the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j,L)$ or at the x-downstream specified-boundary enclosure surface at $(i+\frac{1}{2},j,L)$ of the element (i,j,L), is specified, as:

$$\bar{H}_{(spb,i\pm\frac{1}{2}),j} = \left[\bar{H}(t;I_{spb})\right]_{(spb,i\pm\frac{1}{2}),j}. \tag{4.123}$$

Similarly, the value of the y-component of velocity $\tilde{V}_{y;i,(spb,j\pm\frac{1}{2}),l}$, associated with the flow rate, at the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, or at the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\bar{V}_{y;i,(spb,j\pm\frac{1}{2}),l} = \left[\bar{V}_y(t;I_{spb})\right]_{(spb,i,j\pm\frac{1}{2}),l},$$

$$\text{for } l_{spb,y,low}(I_{spb}) \leq l \leq l_{spb,y,top}(I_{spb}). \tag{4.124a}$$

The value of the y-component of velocity $\bar{V}_{y;i,(spb,j\pm\frac{1}{2}),l}$, associated with the y-component of momentum transport, at the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the y-upstream half

element $(i,j-\frac{1}{4},l)$, or at the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y;i,(spb,j\pm\frac{1}{2}),l} = \left[\widetilde{V}_y(t;I_{spb})\right]_{i,(spb,j\pm\frac{1}{2}),l} ,$$

$$\text{for } I_{spb,y,low}(I_{spb}) \leq l \leq I_{spb,y,top}(I_{spb}) . \qquad (4.124b)$$

The value of the x-component of velocity $\widetilde{V}_{x;i-\frac{1}{4},(spb,j\pm\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-upstream specified-boundary enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$ or at the y-downstream specified-boundary enclosure surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\widetilde{V}_{x;i-\frac{1}{4},(spb,j\pm\frac{1}{2}),l} = \left[\widetilde{V}_{x;i-\frac{1}{4}}(t;I_{spb})\right]_{i-\frac{1}{4},(spb,j\pm\frac{1}{2}),l} ,$$

$$\text{for } I_{spb,y,low}(I_{spb}) \leq l \leq I_{spb,y,top}(I_{spb}) . \qquad (4.124c)$$

The value of the x-component of velocity $\widetilde{V}_{x;i+\frac{1}{4},(spb,j\pm\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-upstream specified-boundary enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$ or at the y-downstream specified-boundary enclosure surface at $(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\widetilde{V}_{x;i+\frac{1}{4},(spb,j\pm\frac{1}{2}),l} = \left[\widetilde{V}_{x;i+\frac{1}{4}}(t;I_{spb})\right]_{i+\frac{1}{4},(spb,j\pm\frac{1}{2}),l} ,$$

$$\text{for } I_{spb,y,low}(I_{spb}) \leq l \leq I_{spb,y,top}(I_{spb}) . \qquad (4.124d)$$

The value of the z-component of velocity $\tilde{V}_{z;i,(spb,j\pm\frac{1}{2}),l}$, associated with the z-component of momentum transport, at the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ or at the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), is specified, as:

$$\tilde{V}_{z;i,(spb,j\pm\frac{1}{2}),l} = \left[\tilde{V}_z(t;I_{spb})\right]_{i,(spb,j\pm\frac{1}{2}),l},$$

$$\text{for } l_{spb,y,low}(I_{spb}) \leq l \leq l_{spb,y,top}(I_{spb}). \qquad (4.124e)$$

Similarly, in the case that the specified-boundary element is in the water-surface layer $l = L$, the water-surface elevation $\bar{H}_{i,(spb,j\pm\frac{1}{2})}$, at the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ or at the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},L)$ of the element (i,j,L), is specified, as:

$$\bar{H}_{i,(spb,j\pm\frac{1}{2}),l} = \left[\bar{H}(t;I_{spb})\right]_{i,(spb,j\pm\frac{1}{2}),l}. \qquad (4.125)$$

The x-component of velocity $\bar{V}_{x;(spb,i-\frac{1}{4}),j,l}$ in the x-upstream half element, which has the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j,l)$, is not calculated from the general, governing computational system of discrete-element equations, since the momentum conditions in the half element adjacent to the specified-boundary enclosure surface is controlled directly by the specified flow conditions. Therefore, the x-component of velocity $\tilde{V}_{x;(spb,i-\frac{1}{4}),j,l}$ in the x-upstream half element $(i-\frac{1}{4},j,l)$, which has the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j,l)$, is determined, based on linear-interpolation, as:

$$\bar{V}_{x;(spb,i-\frac{1}{4}),j,l} = \tilde{V}_{x;(spb,i-\frac{1}{2}),j,l} + \frac{1}{3}\left[\bar{V}_{x;i+\frac{1}{4},j,l} - \tilde{V}_{x;(spb,i-\frac{1}{2}),j,l}\right] . \quad (4.126)$$

The value of the flow rate along x-direction $\bar{G}_{x;(spb,i-\frac{1}{2}),j,l}$, normal to the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half-element $(i-\frac{1}{4},j,l)$, is formulated, as:

$$\bar{G}_{x;(spb,i-\frac{1}{2}),j,l} = \bar{A}_{x;i-\frac{1}{2},j,l}\,\bar{V}_{x;(spb,i-\frac{1}{2}),j,l} . \quad (4.127)$$

Consistent with the general upwind-differencing formulation, the specified flow conditions at the x-upstream specified-boundary enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$ are corrected, based on the standard second-upwind-differencing interpolation for outflow conditions, i.e.,

for $\bar{V}_{x;(spb,i-\frac{1}{2}),j,l} \leq 0$:

$$\tilde{V}_{y;(spb,i-\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} , \quad (4.128a)$$

$$\tilde{V}_{y;(spb,i-\frac{1}{2}),j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} , \quad (4.128b)$$

$$\tilde{V}_{z;(spb,i-\frac{1}{2}),j,l} = \bar{V}_{z;i,j,l} . \quad (4.128c)$$

Similarly, the x-component of Velocity $\bar{V}_{x;(spb,i+\frac{1}{4}),j,l}$ in the x-downstream half element, which has the x-downstream specified-boundary enclosure surface at $(i+\frac{1}{2},j,l)$, is not calculated from the general, governing computational system of discrete-element equations, since the momentum conditions in the half element adjacent to the specified-

boundary enclosure surface is controlled directly by the specified flow conditions. Therefore, the x-component of velocity $\bar{V}_{x;(spb,i+\frac{1}{4}),j,l}$ in the x-downstream half element $(i+\frac{1}{4},j,l)$, which has the x-downstream specified-boundary enclosure surface at $(i+\frac{1}{2},j,l)$, is determined, based on linear-interpolation, as:

$$\bar{V}_{x;(spb,i+\frac{1}{4}),j,l} = \tilde{V}_{x;(spb,i+\frac{1}{2}),j,l} - \frac{1}{3}\left[\bar{V}_{x;(spb,i+\frac{1}{2}),j,l} - \tilde{V}_{x;i-\frac{1}{4},j,l}\right]. \quad (4.129)$$

The value of the flow rate along x-direction $\bar{G}_{x;(spb,i+\frac{1}{2}),j,l}$, normal to the x-downstream specified-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half-element $(i+\frac{1}{4},j,l)$, is formulated, as:

$$\bar{G}_{x;(spb,i+\frac{1}{2}),j,l} = \bar{A}_{x;i+\frac{1}{2},j,l}\,\bar{V}_{x;(spb,i+\frac{1}{2}),j,l}. \quad (4.130)$$

Similarly, consistent with the general upwind-differencing formulation, the specified flow conditions at the x-downstream specified-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$ are determined, based on the standard second-upwind-differencing interpolation for outflow conditions, i.e.,

for $\bar{V}_{x;(spb,i+\frac{1}{2}),j,l} \geq 0$:

$$\tilde{V}_{y;(spb,i+\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l}. \quad (4.131a)$$

$$\tilde{V}_{y;(spb,i+\frac{1}{2}),j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l}. \quad (4.131b)$$

$$\widetilde{V}_{z;(spb,i+\frac{1}{2}),j,l} = \overline{V}_{z;i,j,l} \cdot \qquad (4.131c)$$

Similarly, the y-component of Velocity $\overline{V}_{y;i,(spb,j-\frac{1}{4}),l}$ in the y-upstream half element, which has the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$, is not calculated from the general, governing computational system of discrete-element equations, since the momentum conditions in the half element adjacent to the specified-boundary enclosure surface is controlled directly by the specified flow conditions. Therefore, the y-component of velocity $\widetilde{V}_{y;i,(spb,j-\frac{1}{2}),l}$ in the y-upstream half element $(i,j-\frac{1}{4},l)$, which has the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$, is determined, based on linear-interpolation, as:

$$\overline{V}_{y;i,(spb,j-\frac{1}{4}),l} = \widetilde{V}_{y;i,(spb,j-\frac{1}{2}),l} + \frac{1}{3}\left[\overline{V}_{y;i,j+\frac{1}{4},l} - \widetilde{V}_{y;i,(spb,j-\frac{1}{2}),l}\right] \cdot \qquad (4.132)$$

The value of the flow rate along y-direction $\overline{G}_{y;i,(spb,j-\frac{1}{2}),l}$, normal to the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half-element $(i-\frac{1}{4},j,l)$, is formulated, as:

$$\overline{G}_{y;i,(spb,j-\frac{1}{2}),l} = \overline{A}_{y;i,j-\frac{1}{2},l}\,\overline{V}_{y;i,(spb,j-\frac{1}{2}),l} \cdot \qquad (4.133)$$

Similarly, consistent with the general upwind-differencing formulation, the specified flow conditions at the y-upstream specified-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$ are corrected, based on the standard second-upwind-differencing interpolation for outflow conditions, i.e.,

for $\bar{V}_{y;i,(spb,j-\frac{1}{2}),l} \leq 0$:

$$\tilde{V}_{x;i-\frac{1}{4},(spb,j-\frac{1}{2}),l} = \bar{V}_{x;i-\frac{1}{4},j,l} \,, \tag{4.134a}$$

$$\tilde{V}_{x;i+\frac{1}{4},(spb,j-\frac{1}{2}),,l} = \bar{V}_{x;i+\frac{1}{4},j,l} \,, \tag{4.134b}$$

$$\tilde{V}_{z;i,(spb,j-\frac{1}{2}),l} = \bar{V}_{z;i,j,l} \,. \tag{4.134c}$$

Similarly, the y-component of Velocity $\bar{V}_{y;i,(spb,j+\frac{1}{4}),l}$ in the y-downstream half element, which has the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},l)$, is not calculated from the general, governing computational system of discrete-element equations, since the momentum conditions in the half element adjacent to the specified-boundary enclosure surface is controlled directly by the specified flow conditions. Therefore, the y-component of velocity $\bar{V}_{y;i,(spb,j+\frac{1}{2}),l}$ in the y-downstream half element $(i,j+\frac{1}{4},l)$, which has the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},l)$, is determined, based on linear-interpolation, as:

$$\bar{V}_{y;i,(spb,j+\frac{1}{4}),l} = \tilde{V}_{y;i,(spb,j+\frac{1}{2}),l} - \frac{1}{3}\left[\bar{V}_{y;i,(spb,j+\frac{1}{2}),l} - \tilde{V}_{y;i,j-\frac{1}{4},l}\right]. \tag{4.135}$$

The value of the flow rate along y-direction $\bar{G}_{y;i,(spb,j+\frac{1}{2}),l}$, normal to the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half-element $(i,j+\frac{1}{4},l)$, is formulated, as:

$$\bar{G}_{y;i,(spb,j+\frac{1}{2}),l} = \bar{A}_{y;i,j+\frac{1}{2},l}\,\bar{V}_{y;i,(spb,j+\frac{1}{2}),l} \,. \tag{4.136}$$

Similarly, consistent with the general upwind-differencing formulation, the specified flow conditions at the y-downstream specified-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half element $(i,j+\frac{1}{4},l)$ are corrected, based on the standard second-upwind-differencing interpolation for outflow conditions, i.e.,

for $\bar{V}_{y;i,(spb,j+\frac{1}{2}),j,l} \leq 0$:

$$\tilde{V}_{x;i-\frac{1}{4},(spb,j+\frac{1}{2}),l} = \bar{V}_{x;i-\frac{1}{4},j,l} , \tag{4.137a}$$

$$\tilde{V}_{x;i+\frac{1}{4},(spb,j+\frac{1}{2}),l} = \bar{V}_{x;i+\frac{1}{4},j,l} , \tag{4.137b}$$

$$\tilde{V}_{z;i,(spb,j+\frac{1}{2}),l} = \bar{V}_{z;i,j,l} . \tag{4.137c}$$

Hence, the auxiliary values of the components of the flow rate and the components of velocity, associated with the components of momentum transport, at the specified-boundary enclosure surfaces can be represented as temporal variations of the flow conditions [Eqs. (4.122a) through (4.137c)] which effectively eliminate all the auxiliary values of the components of flow rate and the components of velocity which appear in the terms associated with the momentum transport in the governing computational system of discrete-element equations [Eqs. (3.5), (3.6), (3.7), (3.9), (3.10), (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3,96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113) and (3.114)].

4.5.3. Boundary Conditions for Open-Region Enclosure Surfaces of the Elements and Half Elements

The general mathematical formulation of the fast-transient, three-dimensional, discrete-element hydrodynamic model can also incorporate approximately specified flow conditions at the open-region enclosure surfaces of the elements and half-elements.

In general, the boundary conditions for the components of velocity, associated with the components of momentum transport, cannot be specified precisely in open regions immediately external to the flow region, in the vicinities of the open-region enclosure surfaces, since the flow conditions in the flow region interact with the flow conditions in the external flow region on the two sides of the hypothetical open-region enclosure surface. The only flow variable that can be specified precisely in an open region is the water-surface elevation which can be obtained readily as temporal variations in most large water bodies, including rivers, reservoirs, lakes, estuaries and coastal regions. In general, the information about the components of velocity, associated with the components of momentum transport, can only be estimated as values between upper and lower limits.

The boundary conditions for the components of velocity at the open-region enclosure surfaces are formulated based on the extrapolation method which was developed for estuaries and coastal regions (Eraslan, 1974, 1976, 1981a, 1981b). The method for establishing the boundary conditions is completely general and it can be applied to any type of open-region enclosure surface based on specified, approximate minimum

and maximum conditions which exist in the open region external to the flow region.

The discrete elements which contain the open-region enclosure surfaces can be identified by the index I_{opr}, which is a special subset of the general index used for the specified-boundary elements I_{spb}, for representing the numbers assigned to the open-region elements. The locations of the open-region enclosure surfaces are formulated in terms of the indices of the discrete-element system. In each open-region boundary element the open-region boundary conditions can only be assigned to only one of the two enclosure-surfaces normal to the x-direction and/or only to one of the two enclosure-surfaces normal to the y-direction.

Hence, the locations of the open-region elements are specified, as:

$$i_{opr}(I_{opr}) \text{ specified, for all time,} \tag{4.138a}$$

$$j_{opr}(I_{opr}) \text{ specified, for all time.} \tag{4.138b}$$

The length along y-direction $\Delta y_{(opr,i\pm\frac{1}{2}),j}(I_{opr})$ of the open-region enclosure surface, for all layers, is specified, as:

$$\Delta y_{(opr,i\pm\frac{1}{2}),j}(I_{opr}) \text{ specified, for all time.} \tag{4.139a}$$

Similarly, the length along x-direction $\Delta x_{i,(opr,j\pm\frac{1}{2})}(I_{opr})$ of the open-region enclosure surface, for all layers, is specified, as:

$$\Delta x_{i,(opr,j\pm\frac{1}{2})}(I_{opr}) \text{ specified, for all time.} \tag{4.139b}$$

The open area of the open-region enclosure surface to the external region is assumed to extend from the bottom layer l(bot;i,j) to the water-surface layer L.

The value of the water-surface elevation $\overline{H}_{(opr,i\pm1),j}$, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\overline{H}_{(opr,i\pm1),j} = \left[\overline{H}(t;I_{opr}) \right]_{(opr,i\pm1),j} . \tag{4.140}$$

Approximate minimum value of the x-component of velocity $\tilde{V}_{x,min;(opr,i\pm1),j,l}$, associated with the x-component of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x,min;(opr,i\pm1),j,l} = \left[\tilde{V}_{x,min}(t;I_{opr}) \right]_{(opr,i\pm1),j,l} . \tag{4.141a}$$

Approximate maximum value of the x-component of velocity $\tilde{V}_{x,max;(opr,i\pm1),j,l}$, associated with the x-component of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x,max;(opr,i\pm1),j,l} = \left[\tilde{V}_{x,max}(t;I_{opr}) \right]_{(opr,i\pm1),j,l} . \tag{4.141b}$$

Approximate minimum value of the y-component of velocity $\tilde{V}_{y,min;(opr,i\pm1),j-\frac{1}{4},l}$, associated with the y-component of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is specified, as:

$$\tilde{V}_{y,min;(opr,i\pm1),j-\frac{1}{4},l} = \left[\tilde{V}_{y,min;j-\frac{1}{4}}(t;I_{opr}) \right]_{(opr,i\pm1),j-\frac{1}{4},l} . \quad (4.141c)$$

Approximate maximum value of the y-component of velocity $\tilde{V}_{y,max;(opr,i\pm1),j-\frac{1}{4},l}$, associated with the y-component of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is specified, as:

$$\tilde{V}_{y,max;(opr,i\pm1),j-\frac{1}{4},l} = \left[\tilde{V}_{y,max;j-\frac{1}{4}}(t;I_{opr}) \right]_{(opr,i\pm1),j-\frac{1}{4},l} . \quad (4.141d)$$

Approximate minimum value of the y-component of velocity $\tilde{V}_{y,min;(opr,i\pm1),j+\frac{1}{4},l}$, associated with the y-component of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\tilde{V}_{y,min;(opr,i\pm1),j+\frac{1}{4},l} = \left[\tilde{V}_{y,min;j+\frac{1}{4}}(t;I_{opr}) \right]_{(opr,i\pm1),j+\frac{1}{4},l} . \quad (4.141e)$$

Approximate maximum value of the y-component of velocity $\tilde{V}_{y,max;(opr,i\pm1),j+\frac{1}{4},l}$, associated with the y-component of momentum

transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y,max;(opr,i\pm1),j+\frac{1}{4},l} = \left[\widetilde{V}_{y,max;j+\frac{1}{4}}(t;I_{opr}) \right]_{(opr,i\pm1),j+\frac{1}{4},l} . \quad (4.141f)$$

Approximate minimum value of the z-component of velocity $\widetilde{V}_{z,min;(opr,i\pm1),j,l}$, associated with the z-component of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), is specified, as:

$$\widetilde{V}_{z,min;(opr,i\pm1),j,l} = \left[\widetilde{V}_{z,min}(t;I_{opr}) \right]_{(opr,i\pm1),j,l} . \quad (4.141g)$$

Approximate maximum value of the z-component of velocity $\widetilde{V}_{z,max;(opr,i\pm1),j,l}$, associated with the z-component of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$, or external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), is specified, as:

$$\widetilde{V}_{z,max;(opr,i\pm1),j,l} = \left[\widetilde{V}_{z,max}(t;I_{opr}) \right]_{(opr,i\pm1),j,l} . \quad (4.141h)$$

Similarly, the value of the water-surface elevation $\overline{H}_{i,(opr,j\pm1)}$, external to the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, or external to the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\overline{H}_{i,(opr,j\pm1)} = \left[\overline{H}(t;I_{opr}) \right]_{i,(opr,j\pm1)} . \qquad (4.142)$$

Approximate minimum value of the y-component of velocity $\widetilde{V}_{y,min;i,(opr,j\pm1),l}$, associated with the y-component of momentum transport, external to the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, or external to the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y,min;i,(opr,j\pm1),l} = \left[\widetilde{V}_{y,min}(t;I_{opr}) \right]_{i,(opr,j\pm1),l} . \qquad (4.143a)$$

Approximate maximum value of the y-component of velocity $\widetilde{V}_{y,max;i,(opr,j\pm1),l}$, associated with the y-component of momentum transport, external to the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, or external to the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is specified, as:

$$\widetilde{V}_{y,max;i,(opr,j\pm1),l} = \left[\widetilde{V}_{y,max}(t;I_{opr}) \right]_{i,(opr,j\pm1),l} . \qquad (4.143b)$$

Approximate minimum value of the x-component of velocity $\widetilde{V}_{x,min;i-\frac{1}{4},(opr,j\pm1),l}$, associated with the x-component of momentum transport, external to the y-upstream open-region enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$, or external to the y-downstream open-region enclosure surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\widetilde{V}_{x,min;i-\frac{1}{4},(opr,j\pm1),l} = \left[\widetilde{V}_{x,min,i-\frac{1}{4}}(t;I_{opr}) \right]_{i-\frac{1}{4},(opr,j\pm1),l} . \qquad (4.143c)$$

Approximate maximum value of the x-component of velocity $\tilde{V}_{x,max;i-\frac{1}{4},(opr,j\pm1),l}$, associated with the x-component of momentum transport, external to the y-upstream open-region enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$, or external to the y-downstream open-region enclosure surface at $(i-\frac{1}{4},j+\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x,max;i-\frac{1}{4},(opr,j\pm1),l} = \left[\tilde{V}_{x,max,i-\frac{1}{4}}(t;I_{opr}) \right]_{i-\frac{1}{4},(opr,j\pm1),l} . \quad (4.143d)$$

Approximate minimum value of the x-component of velocity $\tilde{V}_{x,min;i+\frac{1}{4},(opr,j\pm1),l}$, associated with the x-component of momentum transport, external to the y-upstream open-region enclosure surface at $(i+\frac{1}{4},j,l)$, or external to the y-downstream open-region enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x,min;i+\frac{1}{4},(opr,j\pm1),l} = \left[\tilde{V}_{x,min,i+\frac{1}{4}}(t;I_{opr}) \right]_{i+\frac{1}{4},(opr,j\pm1),l} . \quad (4.143e)$$

Approximate maximum value of the x-component of velocity $\tilde{V}_{x,max;i+\frac{1}{4},(opr,j\pm1),l}$, associated with the x-component of momentum transport, external to the y-upstream open-region enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$, or external to the y-downstream open-region enclosure surface at $(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is specified, as:

$$\tilde{V}_{x,max;i+\frac{1}{4},(opr,j\pm1),l} = \left[\tilde{V}_{x,max,i+\frac{1}{4}}(t;I_{opr}) \right]_{i+\frac{1}{4},(opr,j\pm1),l} . \quad (4.143f)$$

Approximate minimum value of the z-component of velocity $\tilde{V}_{z,min;i,(opr,j\pm1),l}$, associated with the z-component of momentum

transport, external to the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},l)$, or external to the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), is specified, as:

$$\tilde{V}_{z,min;i,(opr,j\pm1),l} = \left[\tilde{V}_{z,min}(t;I_{opr}) \right]_{i,(opr,j\pm1),l} . \qquad (4.143g)$$

Approximate maximum value of the z-component of velocity $\tilde{V}_{z,max;i,(opr,j\pm1),l}$, associated with the z-component of momentum transport, external to the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},l)$, or external to the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), is specified, as:

$$\tilde{V}_{z,max;i,(opr,j\pm1),l} = \left[\tilde{V}_{z,max}(t;I_{opr}) \right]_{i,(opr,j\pm1),l} . \qquad (4.143h)$$

The value of the water-surface elevation $\bar{H}_{(opr,i-\frac{1}{2}),j}$, at the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,L)$ of the element (i,j,L) and the x-upstream half element $(i-\frac{1}{4},j,L)$ is determined, based on linear-interpolation, as:

$$\bar{H}_{(opr,i-\frac{1}{2}),j} = \frac{1}{2}\left[\bar{H}_{(opr,i-1),j} + \bar{H}_{i,j} \right] . \qquad (4.144)$$

The value of the x-component of velocity $\bar{V}_{x;[opr,(i-1)+\frac{1}{4}],j,l}$ external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$ is determined, as the value of the x-component of velocity in the x-upstream half element $(i-\frac{1}{4},j,l)$, i.e.,

$$\bar{V}_{x;[opr,(i-1)+\frac{1}{4}],j,l} = \bar{V}_{x;i-\frac{1}{4},j,l} . \qquad (4.145)$$

For a x-upstream open-region element, which is adjacent to an external x-downstream element $(i+1,j,l)$, the values of the components of

velocity, associated with the components of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$, are first approximated as equal to the values of the components of velocity in the element (i,j,l), the y-upstream half element $(i,j-\frac{1}{4},l)$ and the y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$\bar{V}_{y;(opr,i-1),j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} \,. \tag{4.146a}$$

$$\bar{V}_{y;(opr,i-1),j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} \,. \tag{4.146b}$$

$$\bar{V}_{z;(opr,i-1),j,l} = \bar{V}_{z;i,j,l} \,. \tag{4.146c}$$

For a x-upstream open-region element, which is adjacent to an internal x-downstream element $(i+1,j,l)$, the values of the components of velocity, associated with the components of momentum transport, external to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$, are first approximated, based on linear-interpolation between the values of the components of velocity in the elements (i,j,l) and $(i+1,j,l)$, in the y-upstream half elements $(i,j-\frac{1}{4},l)$ and $(i+1,j-\frac{1}{4},l)$, and in the y-downstream half elements $(i,j+\frac{1}{4},l)$ and $(i+1,j+\frac{1}{4},l)$, as:

$$\bar{V}_{y;(opr,i-1),j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} - \frac{\Delta x_{i,j}}{\Delta x_{i+\frac{1}{2},j}}\left(\bar{V}_{y;i+1,j-\frac{1}{4},l} - \bar{V}_{y;i,j-\frac{1}{4},l}\right) . \tag{4.147a}$$

$$\bar{V}_{y;(opr,i-1),j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} - \frac{\Delta x_{i,j}}{\Delta x_{i+\frac{1}{2},j}}\left(\bar{V}_{y;i+1,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}\right) , \tag{4.147b}$$

$$\bar{V}_{z;(opr,i-1),j,l} = \bar{V}_{z;i,j,l} - \frac{\Delta x_{i,j}}{\Delta x_{i+\frac{1}{2},j}}\left(\bar{V}_{z;i+1,j,l} - \bar{V}_{z;i,j,l}\right) . \tag{4.147c}$$

The value of the flow rate along x-direction $\overline{G}_{x;(opr,i-\frac{1}{2}),j,l}$, normal to the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, is formulated as:

$$\overline{G}_{x;(opr,i-\frac{1}{2}),j,l} = \overline{A}_{x;i-\frac{1}{2},j,l}\,\overline{V}_{x;(opr,i-\frac{1}{2}),j,l}$$

$$= \overline{A}_{x;i-\frac{1}{2},j,l}\,\overline{V}_{x;i-\frac{1}{4},j,l}\,. \qquad (4.148)$$

The value of the x-component of velocity $\widetilde{V}_{x;(opr,i-\frac{1}{2}),j,l}$, associated with the x-component of momentum transport, at the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l) and the x-upstream half element $(i-\frac{1}{4},j,l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \overline{V}_{x;[opr,(i-1)+\frac{1}{4}],j,l} < \widetilde{\overline{V}}_{x,min;(opr,i-1),j,l}:$$

$$\widetilde{V}_{x;(opr,i-\frac{1}{2}),j,l} = \widetilde{\overline{V}}_{x,min;(opr,i-1),j,l}\,. \qquad (4.149a)$$

$$\text{if } \overline{V}_{x;[opr,(i-1)+\frac{1}{4}],j,l} > \widetilde{\overline{V}}_{x,max;(opr,i-1),j,l}:$$

$$\widetilde{V}_{x;(opr,i-\frac{1}{2}),j,l} = \widetilde{\overline{V}}_{x,max;(opr,i-1),j,l}\,, \qquad (4.149b)$$

or

$$\widetilde{V}_{x;(opr,i-\frac{1}{2}),j,l} = \widetilde{\overline{V}}_{x;[opr,(i-1)+\frac{1}{4}],j,l}\,. \qquad (4.149c)$$

The value of the y-component of velocity $\widetilde{V}_{y;(opr,i-\frac{1}{2}),j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half

element $(i,j-\frac{1}{4},l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{y;(opr,i-1),j-\frac{1}{4},l} < \bar{V}_{y,min;(opr,i-1),j-\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i-\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y,min;(opr,i-1),j-\frac{1}{4},l}, \tag{4.150a}$$

$$\text{if } \bar{V}_{y;(opr,i-1),j-\frac{1}{4},l} > \bar{V}_{y,max;(opr,i-1),j-\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i-\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y,max;(opr,i-1),j-\frac{1}{4},l}, \tag{4.150b}$$

or

$$\tilde{V}_{y;(opr,i-\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y;(opr,i-1),j-\frac{1}{4},l}. \tag{4.150c}$$

The value of the y-component of velocity $\tilde{V}_{y;(opr,i-\frac{1}{2}),j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{y;(opr,i-1),j+\frac{1}{4},l} < \bar{V}_{y,min;(opr,i-1),j+\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i-\frac{1}{2}),j+\frac{1}{4},l} = \bar{V}_{y,min;(opr,i-1),j+\frac{1}{4},l}. \tag{4.151a}$$

$$\text{if } \bar{V}_{y;(opr,i-1),j+\frac{1}{4},l} > V_{y,max;(opr,i-1),j+\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i-\frac{1}{2}),j+\frac{1}{4},l} = \bar{V}_{y,max;(opr,i-1),j+\frac{1}{4},l}, \tag{4.151b}$$

or

$$\widetilde{V}_{y;(opr,i-\frac{1}{2}),j+\frac{1}{4},l} = \overline{V}_{y;(opr,i-1),j+\frac{1}{4},l} \cdot \tag{4.151c}$$

The value of the z-component of velocity $\widetilde{V}_{z;(opr,i-\frac{1}{2}),j,l}$, associated with the z-component of momentum transport, at the x-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$ of the element (i,j,l), is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \overline{V}_{z;(opr,i-1),j,l} < \overline{V}_{z,min;(opr,i-1),j,l}:$$

$$\widetilde{V}_{z;(opr,i-\frac{1}{2}),j,l} = \overline{V}_{z,min;(opr,i-1),j,l} \cdot \tag{4.152a}$$

$$\text{if } \overline{V}_{z;(opr,i-1),j,l} > \overline{V}_{z,max;(opr,i-1),j,l}:$$

$$\widetilde{V}_{z;(opr,i-\frac{1}{2}),j,l} = \overline{V}_{z,max;(opr,i-1),j,l} \cdot \tag{4.152b}$$

or

$$\widetilde{V}_{z;(opr,i-\frac{1}{2}),j,l} = \overline{V}_{z;(opr,i-1),j,l} \cdot \tag{4.152c}$$

Similarly, the value of the water-surface elevation $\overline{H}_{(opr,i+\frac{1}{2}),j}$, at the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,L)$ of the element (i,j,L) and the x-downstream half element $(i+\frac{1}{4},j,L)$, is determined, based on linear-interpolation, as:

$$\overline{H}_{(opr,i+\frac{1}{2}),j} = \frac{1}{2}\left[\ \overline{H}_{(opr,i+1),j} + \overline{H}_{i,j}\ \right] \cdot \tag{4.153}$$

The value of the x-component of velocity $\overline{V}_{x;[opr,(i+1)-\frac{1}{4}],j,l}$ external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$,

is determined, as the value of the x-component of velocity in the x-downstream half element $(i+\frac{1}{4},j,l)$, i.e.,

$$\bar{V}_{x;[opr,(i+1)-\frac{1}{4}],j,l} = \bar{V}_{x;i+\frac{1}{4},j,l} \cdot \tag{4.154}$$

For a x-downstream open-region element, which is adjacent to an external x-upstream element $(i-1,j,l)$, the values of the components of velocity, associated with the components of momentum transport, external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$, are first approximated as equal to the values of the components of velocity in the element (i,j,l), the y-upstream half element $(i,j-\frac{1}{4},l)$ and the y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$\bar{V}_{y;(opr,i+1),j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} \cdot \tag{4.155a}$$

$$\bar{V}_{y;(opr,i+1),j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} \cdot \tag{4.155b}$$

$$\bar{V}_{z;(opr,i+1),j,l} = \bar{V}_{z;i,j,l} \cdot \tag{4.155c}$$

For a x-downstream open-region element, which is adjacent to an internal x-upstream element $(i-1,j,l)$, the values of the components of velocity, associated with the components of momentum transport, external to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$, are first approximatedi, based on linear-interpolation between the values of the components of velocity in the elements (i,j,l) and $(i-1,j,l)$, in the y-upstream half elements $(i,j-\frac{1}{4},l)$ and $(i-1,j-\frac{1}{4},l)$, and in the y-downstream half elements $(i,j+\frac{1}{4},l)$ and $(i-1,j+\frac{1}{4},l)$, as:

$$\bar{V}_{y;(opr,i+1),j-\frac{1}{4},l} = \bar{V}_{y;i,j-\frac{1}{4},l} + \frac{\Delta x_{i,j}}{\Delta x_{i-\frac{1}{2},j}}(\bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i-1,j-\frac{1}{4},l}) \,. \tag{4.156a}$$

$$\bar{V}_{y;(opr,i+1),j+\frac{1}{4},l} = \bar{V}_{y;i,j+\frac{1}{4},l} + \frac{\Delta x_{i,j}}{\Delta x_{i-\frac{1}{2},j}}(\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i-1,j+\frac{1}{4},l}) \,, \tag{4.156b}$$

$$\bar{V}_{z;(opr,i+1),j,l} = \bar{V}_{z;i,j,l} + \frac{\Delta x_{i,j}}{\Delta x_{i-\frac{1}{2},j}}(\bar{V}_{z;i,j,l} - \bar{V}_{z;i-1,j,l}) \,. \tag{4.156c}$$

The value of the flow rate along x-direction $\bar{G}_{x;(opr,i+\frac{1}{2}),j,l}$, normal to the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, is formulated as:

$$\begin{aligned}\bar{G}_{x;(opr,i+\frac{1}{2}),j,l} &= \bar{A}_{x;i+\frac{1}{2},j,l}\,\bar{V}_{x;(opr,i+\frac{1}{2}),j,l} \\ &= \bar{A}_{x;i+\frac{1}{2},j,l}\,\bar{V}_{x;i+\frac{1}{4},j,l} \,.\end{aligned} \tag{4.157}$$

The value of the x-component of velocity $\tilde{V}_{x;(opr,i+\frac{1}{2}),j,l}$, associated with the x-component of momentum transport, at the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{x;[opr,(i+1)-\frac{1}{4}],j,l} < \bar{V}_{x,min;(opr,i+1),j,l}:$$

$$\tilde{V}_{x;(opr,i+\frac{1}{2}),j,l} = \bar{V}_{x,min;(opr,i+1),j,l}, \tag{4.158a}$$

$$\text{if } \bar{V}_{x;[opr,(i+1)-\frac{1}{4}],j,l} > \bar{V}_{x,max;(opr,i+1),j,l}:$$

$$\tilde{V}_{x;(opr,i+\frac{1}{2}),j,l} = \bar{V}_{x,max;(opr,i+1),j,l}, \tag{4.158b}$$

or

$$\tilde{V}_{x;(opr,i+\frac{1}{2}),j,l} = \bar{V}_{x;[opr,(i+1)-\frac{1}{4}],j,l}. \tag{4.158c}$$

The value of the y-component of velocity $\tilde{V}_{y;(opr,i+\frac{1}{2}),j-\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j-\frac{1}{4},l)$ of the y-upstream half element $(i,j-\frac{1}{4},l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{y;(opr,i+1),j-\frac{1}{4},l} < \bar{V}_{y,min;(opr,i+1),j-\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i+\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y,min;(opr,i+1),j-\frac{1}{4},l}, \tag{4.159a}$$

$$\text{if } \bar{V}_{y;(opr,i+1),j-\frac{1}{4},l} > V_{y,max;(opr,i+1),j-\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i+\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y,max;(opr,i+1),j-\frac{1}{4},l}, \tag{4.159b}$$

or

$$\tilde{V}_{y;(opr,i+\frac{1}{2}),j-\frac{1}{4},l} = \bar{V}_{y;(opr,i+1),j-\frac{1}{4},l}. \tag{4.159c}$$

The value of the y-component of velocity $\tilde{V}_{y;(opr,i+\frac{1}{2}),j+\frac{1}{4},l}$, associated with the y-component of momentum transport, at the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j+\frac{1}{4},l)$ of the y-downstream half element $(i,j+\frac{1}{4},l)$, is determined by considering the

specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{y;(opr,i+1),j+\frac{1}{4},l} < \bar{V}_{y,min;(opr,i+1),j+\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i+\frac{1}{2}),j+\frac{1}{4},l} = \bar{V}_{y,min;(opr,i+1),j+\frac{1}{4},l}, \tag{4.160a}$$

$$\text{if } \bar{V}_{y;(opr,i+1),j+\frac{1}{4},l} > \bar{V}_{y,max;(opr,i+1),j+\frac{1}{4},l}:$$

$$\tilde{V}_{y;(opr,i+\frac{1}{2}),j+\frac{1}{4},l} = \bar{V}_{y,max;(opr,i+1),j+\frac{1}{4},l}, \tag{4.160b}$$

or

$$\tilde{V}_{y;(opr,i+\frac{1}{2}),j+\frac{1}{4},l} = \bar{V}_{y;(opr,i+1),j+\frac{1}{4},l}. \tag{4.160c}$$

The value of the z-component of velocity $\tilde{V}_{z;(opr,i+\frac{1}{2}),j,l}$, associated with the z-component of momentum transport, at the x-downstream open-region enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l), is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{z;(opr,i+1),j,l} < \bar{V}_{z,min;(opr,i+1),j,l}:$$

$$\tilde{V}_{z;(opr,i+\frac{1}{2}),j,l} = \bar{V}_{z,min;(opr,i+1),j,l}, \tag{4.161a}$$

$$\text{if } \bar{V}_{z;(opr,i+1),j,l} > \bar{V}_{z,max;(opr,i+1),j,l}:$$

$$\tilde{V}_{z;(opr,i+\frac{1}{2}),j,l} = \bar{V}_{z,max;(opr,i+1),j,l}, \tag{4.161b}$$

or

$$\tilde{V}_{z;(opr,i+\frac{1}{2}),j,l} = \bar{V}_{z;(opr,i+1),j,l}. \tag{4.161c}$$

Similarly, the value of the water-surface elevation $\overline{H}_{i,(opr,j-\frac{1}{2})}$, at the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},L)$ of the element (i,j,L) and the y-upstream half element $(i,j-\frac{1}{4},L)$, is determined, based on linear-interpolation, as:

$$\overline{H}_{i,(opr,j-\frac{1}{2})} = \frac{1}{2}\left[\ \overline{H}_{i,(opr,j-1)} + \overline{H}_{i,j}\ \right] . \qquad (4.162)$$

The value of the y-component of velocity $\overline{V}_{y;i,[opr,(j-1)+\frac{1}{4}],l}$, external to the y-upstream open-region enclosure surface at $(i-\frac{1}{2},j,l)$, is determined, as the value of the y-component of velocity in the y-upstream half element $(i,j-\frac{1}{4},l)$, i.e.,

$$\overline{V}_{y;i,[opr,(j-1)+\frac{1}{4}],l} = \overline{V}_{y;i,j-\frac{1}{4},l} . \qquad (4.163)$$

For a y-upstream open-region element, which is adjacent to an external y-downstream element $(i,j+1,l)$, the values of the components of velocity, associated with the components of momentum transport, external to the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},l)$, are first approximated as equal to the values of the components of velocity in the element (i,j,l), the x-upstream half element $(i-\frac{1}{4},j,l)$ and the x-downstream half element $(i+\frac{1}{4},j,l)$, as:

$$\overline{V}_{x;i-\frac{1}{4},(opr,j-1),l} = \overline{V}_{x;i-\frac{1}{4},j,l} . \qquad (4.164a)$$

$$\overline{V}_{x;i+\frac{1}{4},(opr,j-1),l} = \overline{V}_{x;i+\frac{1}{4},j,l} . \qquad (4.164b)$$

$$\overline{V}_{z;i,(opr,j-1),l} = \overline{V}_{z;i,j,l} . \qquad (4.164c)$$

For a y-upstream open-region element, which is adjacent to an internal y-downstream element (i,j+1,l), the values of the components of velocity, associated with the components of momentum transport, external to the y-upstream open-region enclosure surface at (i,j-½,l), are first approximated, based on linear-interpolation between the values of the components of velocity in the elements (i,j,l) and (i,j+1,l), in the x-upstream half elements (i-¼,j,l) and (i-¼,j+1,l), and in the x-downstream half elements (i+¼,j,l) and (i+¼,j+1,l), as:

$$\overline{V}_{x;i-\frac{1}{4},(opr,j-1),l} = \overline{V}_{x;i-\frac{1}{4},j,l} - \frac{\Delta y_{i,j}}{\Delta y_{i,j+\frac{1}{2}}}(\overline{V}_{x;i-\frac{1}{4},j+1,l} - \overline{V}_{x;i-\frac{1}{4},j,l}) , \tag{4.165a}$$

$$\overline{V}_{x;i+\frac{1}{4},(opr,j-1),l} = \overline{V}_{x;i+\frac{1}{4},j,l} - \frac{\Delta y_{i,j}}{\Delta y_{i,j+\frac{1}{2}}}(\overline{V}_{x;i+\frac{1}{4},j+1,l} - \overline{V}_{x;i+\frac{1}{4},j,l}) , \tag{4.165b}$$

$$\overline{V}_{z;i,(opr,j-1),l} = \overline{V}_{z;i,j,l} - \frac{\Delta y_{i,j}}{\Delta y_{i,j+\frac{1}{2}}}(\overline{V}_{z;i,j+1,l} - \overline{V}_{z;i,j,l}) . \tag{4.165c}$$

The value of the flow rate along y-direction $\overline{G}_{y;i,(opr,j-\frac{1}{2}),l}$, normal to the y-upstream open-region enclosure surface at (i,j-½,l) of the element (i,j,l) and the y-upstream half element (i,j-¼,l), is formulated as:

$$\begin{aligned}\overline{G}_{y;i,(opr,j-\frac{1}{2}),l} &= \overline{A}_{y;i,j-\frac{1}{2},l}\,\overline{V}_{y;i,(opr,j-\frac{1}{2}),l} \\ &= \overline{A}_{y;i,j-\frac{1}{2},l}\,\overline{V}_{y;i,j-\frac{1}{4},l} .\end{aligned} \tag{4.166}$$

The value of the y-component of velocity $\widetilde{V}_{y;i,(opr,j-\frac{1}{2}),l}$, associated with the y-component of momentum transport, at the y-upstream

open-region enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \overline{V}_{y;i,[opr,(j-1)+\frac{1}{4}],l} < \overline{V}_{y,min;i,(opr,j-1),l}:$$

$$\widetilde{V}_{y;i,(opr,j-\frac{1}{2}),l} = \overline{V}_{y,min;i,(opr,j-1),l}, \tag{4.167a}$$

$$\text{if } \overline{V}_{y;i,[opr,(j-1)+\frac{1}{4}],l} > \overline{V}_{y,max;i,(opr,j-1),l}:$$

$$\widetilde{V}_{y;i,(opr,j-\frac{1}{2}),l} = \overline{V}_{y,max;i,(opr,j-1),l}, \tag{4.167b}$$

or

$$\widetilde{V}_{y;i,(opr,j-\frac{1}{2}),l} = \overline{V}_{y;i,[opr,(j-1)+\frac{1}{4}],l}. \tag{4.167c}$$

The value of the x-component of velocity $\widetilde{V}_{x;i-\frac{1}{4},(opr,j-\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-upstream open-region enclosure surface at $(i-\frac{1}{4},j-\frac{1}{2},l)$ of the x-upstream half element $(i-\frac{1}{4},j,l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \overline{V}_{x;i-\frac{1}{4},(opr,j-1),l} < \overline{V}_{x,min;i-\frac{1}{4},(opr,j-1),l}:$$

$$\widetilde{V}_{x;i-\frac{1}{4},(opr,j-\frac{1}{2}),l} = \overline{V}_{x,min;i-\frac{1}{4},(opr,j-1),l}, \tag{4.168a}$$

$$\text{if } \overline{V}_{x;i-\frac{1}{4},(opr,j-1),l} > \overline{V}_{x,max;i-\frac{1}{4},(opr,j-1),l}:$$

$$\widetilde{V}_{x;i-\frac{1}{4},(opr,j-\frac{1}{2}),l} = \overline{V}_{x,max;i-\frac{1}{4},(opr,j-1),l}, \tag{4.168b}$$

or

$$\tilde{V}_{x;i-\frac{1}{4},(opr,j-\frac{1}{2}),l} = \bar{V}_{x;i-\frac{1}{4},(opr,j-1),l} \cdot \tag{4.168c}$$

The value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},(opr,j-\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-upstream open-region enclosure surface at $(i+\frac{1}{4},j-\frac{1}{2},l)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{x;i+\frac{1}{4},(opr,j-1),l} < \bar{V}_{x,min;i+\frac{1}{4},(opr,j-1),l}:$$

$$\tilde{V}_{x;i+\frac{1}{4},(opr,j-\frac{1}{2}),l} = \bar{V}_{x,min;i+\frac{1}{4},(opr,j-1),l} \cdot \tag{4.169a}$$

$$\text{if } \bar{V}_{x;i+\frac{1}{4},(opr,j-1),l} > \bar{V}_{x,max;i+\frac{1}{4},(opr,j-1),l}:$$

$$\tilde{V}_{x;i+\frac{1}{4},(opr,j-\frac{1}{2}),l} = \bar{V}_{x,max;i+\frac{1}{4},(opr,j-1),l}, \tag{4.169b}$$

or

$$\tilde{V}_{x;i+\frac{1}{4},(opr,j-\frac{1}{2}),l} = \bar{V}_{x;i+\frac{1}{4},(opr,j-1),l} \cdot \tag{4.169c}$$

The value of the z-component of velocity $V_{z;i,(opr,j-\frac{1}{2}),l}$, associated with the z-component of momentum transport, at the y-upstream open-region enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l), is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{z;i,(opr,j-1),l} < \bar{V}_{z,min;i,(opr,j-1),l}:$$

$$\tilde{V}_{z;i,(opr,j-\frac{1}{2}),l} = \bar{V}_{z,min;i,(opr,j-1),l}, \tag{4.170a}$$

$$\text{if } \bar{V}_{z;i,(opr,j-1),l} > \bar{V}_{z,max;i,(opr,j-1),l}:$$

$$\tilde{V}_{z;i,(opr,j-\frac{1}{2}),l} = \bar{V}_{z,max;i,(opr,j-1),l}, \tag{4.170b}$$

or

$$\tilde{V}_{z;i,(opr,j-\frac{1}{2}),l} = \bar{V}_{z;i,(opr,j-1),l}. \tag{4.170c}$$

Similarly, the value of the water-surface elevation $\bar{H}_{i,(opr,j+\frac{1}{2})}$, at the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},L)$ of the element (i,j,L) and the y-downstream half element $(i,j+\frac{1}{4},L)$, is determined, based on linear-interpolation, as:

$$\bar{H}_{i,(opr,j+\frac{1}{2})} = \frac{1}{2}\left[\ \bar{H}_{i,(opr,j+1)} + \bar{H}_{i,j}\ \right]. \tag{4.171}$$

The value of the y-component of velocity $\bar{V}_{y;i,[opr,(j+1)-\frac{1}{4}],l}$, external to the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$, is determined, as the value of the y-component of velocity in the y-downstream half element $(i+\frac{1}{4},j,l)$, i.e.,

$$\bar{V}_{y;i,[opr,(j+1)-\frac{1}{4}],l} = \bar{V}_{y;i,j+\frac{1}{4},l}. \tag{4.172}$$

For a y-downstream open-region element, which is adjacent to an external y-upstream element $(i,j-1,l)$, the values of the components of velocity, associated with the components of momentum transport, external to the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$ are first approximated as equal to the values of the components of velocity in the

element (i,j,l), the x-upstream half element (i-¼,j,l) and the x-downstream half element (i+¼,j,l), as:

$$\overline{V}_{x;i-\frac{1}{4},(opr,j+1),l} = \overline{V}_{x;i-\frac{1}{4},j,l} , \tag{4.173a}$$

$$\overline{V}_{x;i+\frac{1}{4},(opr,j+1),l} = \overline{V}_{x;i+\frac{1}{4},j,l} , \tag{4.173b}$$

$$\overline{V}_{z;i,(opr,j+1),l} = \overline{V}_{z;i,j,l} . \tag{4.173c}$$

For a y-downstream open-region element, which is adjacent to an internal y-upstream element (i,j-1,l), the values of the components of velocity, associated with the components of momentum transport, external to the y-downstream open-region enclosure surface at (i,j+1,l), are first approximated, based on linear-interpolation between the values of the components of velocity in the elements (i,j,l) and (i,j-1,l), in the x-upstream half elements (i-¼,j,l) and (i-¼,j-1,l), and in the x-downstream half elements (i+¼,j,l) and (i+¼,j-1,l), as:

$$\overline{V}_{x;i-\frac{1}{4},(opr,j+1),l} = \overline{V}_{x;i-\frac{1}{4},j,l} + \frac{\Delta y_{i,j}}{\Delta y_{i,j-\frac{1}{2}}} \left(\overline{V}_{x;i-\frac{1}{4},j,l} - \overline{V}_{x;i-\frac{1}{4},j-1,l} \right) , \tag{4.174a}$$

$$\overline{V}_{x;i+\frac{1}{4},(opr,j+1),l} = \overline{V}_{x;i+\frac{1}{4},j,l} + \frac{\Delta y_{i,j}}{\Delta y_{i,j-\frac{1}{2}}} \left(\overline{V}_{x;i+\frac{1}{4},j,l} - \overline{V}_{x;i+\frac{1}{4},j-1,l} \right) , \tag{4.174b}$$

$$\overline{V}_{z;i,(opr,j+1),l} = \overline{V}_{z;i,j,l} + \frac{\Delta y_{i,j}}{\Delta y_{i,j-\frac{1}{2}}} \left(\overline{V}_{z;i,j,l} - \overline{V}_{z;i,j-1,l} \right) . \tag{4.174c}$$

The value of the flow rate along y-direction $\overline{G}_{y;i,(opr,j+\frac{1}{2}),l}$, normal to the y-downstream open-region enclosure surface at (i,j+½,l) of

the element (i,j,1) and the y-downstream half element (i,j+¼,1), is formulated as:

$$\overline{G}_{y;i,(opr,j+\frac{1}{2}),l} = \overline{A}_{y;i,j+\frac{1}{2},l}\,\overline{V}_{y;i,(opr,j+\frac{1}{2}),l}$$

$$= \overline{A}_{y;i,j+\frac{1}{2},l}\,\overline{V}_{y;i,j+\frac{1}{4},l}\,. \qquad (4.175)$$

The value of the y-component of velocity $\widetilde{V}_{y;i,(opr,j+\frac{1}{2}),l}$, associated with the y-component of momentum transport, at the y-downstream open-region enclosure surface at (i,j+½,1) of the element (i,j,1) and the y-downstream half element (i,j+¼,1), is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \overline{V}_{y;i,[opr,(j+1)-\frac{1}{4}],l} < \overline{V}_{y,min;i,(opr,j+1),l}:$$

$$\widetilde{V}_{y;i,(opr,j+\frac{1}{2}),l} = \overline{V}_{y,min;i,(opr,j+1),l}\,, \qquad (4.176a)$$

$$\text{if } \overline{V}_{y;i,[opr,(j+1)-\frac{1}{4}],l} > \overline{V}_{y,max;i,(opr,j+1),l}:$$

$$\widetilde{V}_{y;i,(opr,j+\frac{1}{2}),l} = \overline{V}_{y,max;i,(opr,j+1),l}\,, \qquad (4.176b)$$

or

$$\widetilde{V}_{y;i,(opr,j+\frac{1}{2}),l} = \overline{V}_{y;i,[opr,(j+1)-\frac{1}{4}],l}\,. \qquad (4.176c)$$

The value of the y-component of velocity $\widetilde{V}_{x;i-\frac{1}{4},(opr,j+\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the y-downstream open-region enclosure surface at (i-¼,j+½,1) of the x-upstream half element (i-¼,j,1), is determined by considering the

specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{x;i-\frac{1}{4},(opr,j+1),l} < \bar{V}_{x,min;i-\frac{1}{4},(opr,j+1),l}:$$

$$\tilde{V}_{x;i-\frac{1}{4},(opr,j+\frac{1}{2}),l} = \bar{V}_{x,min;i-\frac{1}{4},(opr,j+1),l}, \tag{4.177a}$$

$$\text{if } \bar{V}_{x;i-\frac{1}{4},(opr,j+1),l} > \bar{V}_{x,max;i-\frac{1}{4},(opr,j+1),l}:$$

$$\tilde{V}_{x;i-\frac{1}{4},(opr,j+\frac{1}{2}),l} = \bar{V}_{x,max;i-\frac{1}{4},(opr,j+1),l}, \tag{4.177b}$$

or

$$\tilde{V}_{x;i-\frac{1}{4},(opr,j+\frac{1}{2}),l} = \bar{V}_{x;i-\frac{1}{4},(opr,j+1),l}. \tag{4.177c}$$

The value of the x-component of velocity $\tilde{V}_{x;i+\frac{1}{4},(opr,j+\frac{1}{2}),l}$, associated with the x-component of momentum transport, at the x-downstream open-region enclosure surface at $(i+\frac{1}{4},j+\frac{1}{2},l)$ of the x-downstream half element $(i+\frac{1}{4},j,l)$, is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{x;i+\frac{1}{4},(opr,j+1),l} < \bar{V}_{x,min;i+\frac{1}{4},(opr,j+1),l}:$$

$$\tilde{V}_{x;i+\frac{1}{4},(opr,j+\frac{1}{2}),l} = \bar{V}_{x,min;i+\frac{1}{4},(opr,j+1),l}, \tag{4.178a}$$

$$\text{if } \bar{V}_{x;i+\frac{1}{4},(opr,j+1),l} > V_{x,max;i+\frac{1}{4},(opr,j+1),l}:$$

$$\tilde{V}_{x;i+\frac{1}{4},(opr,j+\frac{1}{2}),l} = \bar{V}_{x,max;i+\frac{1}{4},(opr,j+1),l}, \tag{4.178b}$$

or

$$\tilde{V}_{x;i+\frac{1}{4},(opr,j+\frac{1}{2}),l} = \bar{V}_{x;i+\frac{1}{4},(opr,j+1),l}. \tag{4.178c}$$

The value of the z-component of velocity $\tilde{V}_{z;i,(opr,j+\frac{1}{2}),l}$, associated with the z-component of momentum transport, at the y-downstream open-region enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l), is determined by considering the specified minimum and maximum limiting values of the particular component of velocity in the external open region, i.e.,

$$\text{if } \bar{V}_{z;i,(opr,j+1),l} < \bar{V}_{z,min;i,(opr,j+1),l}:$$

$$\tilde{V}_{z;i,(opr,j+\frac{1}{2}),l} = \bar{V}_{z,min;i,(opr,j+1),l}, \tag{4.179a}$$

$$\text{if } \bar{V}_{z;i,(opr,j+1),l} > \bar{V}_{z,max;i,(opr,j+1),l}:$$

$$\tilde{V}_{z;i,(opr,j+\frac{1}{2}),l} = \bar{V}_{z,max;i,(opr,j+1),l}, \tag{4.179b}$$

or

$$\tilde{V}_{z;i,(opr,j+\frac{1}{2}),l} = \bar{V}_{z;i,(opr,j+1),l}. \tag{4.179c}$$

Hence, the auxiliary values of the water-surface elevation, the components of flow rate and the components of velocity, associated with the components of momentum transport, at the open-region enclosure surfaces can be formulated as temporal variations of the flow conditions [Eqs. (4.138a) through (4.179c)] which effectively eliminate all the auxiliary values of the water-surface elevation, the components of flow rate and the components of velocity which appear in the terms associated with the mass transport and momentum transport in the governing computational system of discrete-element equations [Eqs. (3.5), (3.6), (3.7) (3.9), (3.10), (3.51), (3.53), (3.53), (3.55), (3.66), (3.68)

(3.69), (3.70), (3.81), (3.83), (3.83), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113) and (3.113)].

4.6. Formulation of the Turbulent Transport Submodel for the Combined Stress Components Acting on the Enclosure Surfaces of the Elements and Half Elements

The formulation of the combined stress components acting on the enclosure surfaces of the elements and half elements (Sec. 3) is based on a new phenomenological turbulent transport model specifically developed for the discrete-element method (DEM) by Eraslan (1973, 1974, 1976,1981a). The subgrid-scale (SGS) phenomena which contributes to the momentum transort is formulated according to a submodel which is significantly different from the "Mean Velocity Field (MVF)" closure models proposed earlier for the representation of the turbulent stress components for discretized computational algorithms (e.g., Reynolds, 1970, Deardorff, 1970, Schumann, 1973, Spraggs and Street, 1975).

The present development of the model is based on a modified form of the Blasius power law type formulation (e.g., Goldstein, 1938) which was successfully used in the formulations of fast-transient, one- and two-dimensional hydrodynamic, thermal and chemical constituent transport mathematical models for river, reservoirs, lakes, estuaries and coastal zones (Eraslan, 1973, 1976, 1977, 1981a, 1981b). The three-dimensional application of the Eraslan's phenomenological turbulent transport model enables the general fast-transient, three-dimensional hydrodynamic model to simulate flow conditions under reduced resolution conditions in one- and two-dimensonal applications without requiring any modification of

the general computational algorithm. Furthermore, since the formulation of the phenomenological turbulet transport model is independent of the spatial scale of turbulence and spatial scale of discretization technique used in the representation of the flow region, it enables the fast-transient, three-dimensional hydrodynamic model to simulate the flow conditions both in the prototype-scale and also in laboratory-scale flow regions without requiring any modifications of the general governing computational system of discrete-element equations.

The combined stress components acting on an internal x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$ is formulated by considering the relative shear effect on both sides of the enclosure surface normal to the x-direction. Since two adjacent elements along the x-direction can be of different sizes, the development of the relative shear effect requires the formulation of the velocity conditions at equal distances from the location of the x-upstream enclosure surface at $(i-\frac{1}{2},j,l)$, along the x-direction.

The upstream value $\bar{V}_{y;(-,i-\frac{1}{2}),j,l}$ and the downstream value $\bar{V}_{y;(+,i-\frac{1}{2}),j,l}$ of the y-component of velocity and the upstream value $\bar{V}_{z;(-,i-\frac{1}{2}),j,l}$ and the downstream value $\bar{V}_{z;(+,i-\frac{1}{2}),j,l}$ of the z-component of velocity on both sides of the x-uptream enclosure at $(i-\frac{1}{2},j,l)$ are formulated, based on linear interpolation which considers the sizes of the adjacent elements (i,j,l) and $(i-1,j,l)$, along the x-direction.

$$\text{for } \Delta x_{i,j} \leq \Delta x_{i-1,j}:$$

$$\bar{V}_{y;(-,i-\frac{1}{2}),j,l} = \bar{V}_{y;i,j,l} - \frac{\Delta x_{i,j}}{\Delta x_{i-\frac{1}{2},j}} \left(\bar{V}_{y;i,j,l} - \bar{V}_{y;i-1,j,l}\right), \tag{4.180a}$$

$$\overline{V}_{z;(-,i-\frac{1}{2}),j,l} = \overline{V}_{z;i,j,l} - \frac{\Delta x^{*}_{i,j}}{\Delta x_{i-\frac{1}{2},j}} \left(\overline{V}_{z;i,j,l} - \overline{V}_{z;i-1,j,l}\right) , \tag{4.180b}$$

$$\overline{V}_{y;(+,i-\frac{1}{2}),j,l} = \overline{V}_{y;i,j,l} , \tag{4.180c}$$

$$\overline{V}_{z;(+,i-\frac{1}{2}),j,l} = \overline{V}_{z;i,j,l} , \tag{4.180d}$$

for $\Delta x_{i,j} > \Delta x_{i-1,j}$:

$$\overline{V}_{y;(-,i-\frac{1}{2}),j,l} = \overline{V}_{y;i-1,j,l} , \tag{4.180e}$$

$$\overline{V}_{z;(-,i-\frac{1}{2}),j,l} = \overline{V}_{z;i-1,j,l} , \tag{4.180f}$$

$$\overline{V}_{y;(+,i-\frac{1}{2}),j,l} = \overline{V}_{y;i-1,j,l} + \frac{\Delta x_{i-1,j}}{\Delta x_{i-\frac{1}{2},j}} \left(\overline{V}_{y;i,j,l} - \overline{V}_{y;i-1,j,l}\right) , \tag{4.180g}$$

$$\overline{V}_{z;(+,i-\frac{1}{2}),j,l} = \overline{V}_{z;i-1,j,l} + \frac{\Delta x_{i-1,j}}{\Delta x_{i-\frac{1}{2},j}} \left(\overline{V}_{z;i,j,l} - \overline{V}_{z;i-1,j,l}\right) . \tag{4.180h}$$

The magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;i-\frac{1}{2},j,l}$ at the x-upstream enclosure surface at (i-½,j,l) is defined as:

$$\left| \overline{\Delta V}_{y,z;i-\frac{1}{2},j,l} \right| = \left\{ \left[\overline{V}_{y;(+,i-\frac{1}{2}),j,l} - \overline{V}_{y;(-,i-\frac{1}{2}),j,l}\right]^2 + \left[\overline{V}_{z;(+,i-\frac{1}{2}),j,l} - \overline{V}_{z;(-,i-\frac{1}{2}),j,l}\right]^2 \right\}^{\frac{1}{2}} . \tag{4.181}$$

The local value of Reynolds number $\overline{Re}_{i-\frac{1}{2},j,l}$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;i-\frac{1}{2},j,l}$, at the x-upstream enclosure surface at (i-½,j,l), is formulated by considering the sizes of the adjacent elements (i,j,l) and (i-1,j,l), along the x-direction.

$$\text{for } \Delta x_{i,j} \leq \Delta x_{i-1,j}:$$

$$\overline{Re}_{i-\frac{1}{2},j,l} = \Delta x_{i,j} \frac{|\overline{\Delta V}_{y,z;i-\frac{1}{2},j,l}|}{\nu}, \tag{4.182a}$$

$$\text{for } \Delta x_{i,j} > \Delta x_{i-1,j}:$$

$$\overline{Re}_{i-\frac{1}{2},j,l} = \Delta x_{i-1,j} \frac{|\overline{\Delta V}_{y,z;i-\frac{1}{2},j,l}|}{\nu}. \tag{4.182b}$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with a critical value of Reynolds number assumed as 4000. For values of the local Reynolds number $\overline{Re}_{i-\frac{1}{2},j,l}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i-\frac{1}{2},j,l}$ at the x-upstream enclosure surface at (i-½,j,l) is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i-\frac{1}{2},j,l} \leq Re_{crt} = 4000:$$

$$\bar{\varepsilon}_{i-\frac{1}{2},j,l} = \nu \quad \text{(laminar flow conditions)}. \tag{4.183}$$

For the values of local Reynolds number $\overline{Re}_{i-\frac{1}{2},j,l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i-\frac{1}{2},j,l}$ at the x-upstream enclosure surface at (i-½,j,l) is determined, based on a

modified form of the Blasius power-law type formulation, n=8, β=0.2 (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i-\frac{1}{2},j,l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i-\frac{1}{2},j,l} = \nu + \bar{\varepsilon}_{trb;i-\frac{1}{2},j,l} ,$$

$$\bar{\varepsilon}_{i-\frac{1}{2},j,l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}_{i-\frac{1}{2},j,l}^{\frac{n-1}{n+1}} . \qquad (4.184)$$

The combined stress components (Sec. 3), acting on the internal x-upstream enclosure surface at (i-½,j,1), are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the internal x-upstream enclosure surface at (i-½,j,1) of the element (i-½,j,1) and the x-upstream half element (i-¼,j,1), as:

$$\bar{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,l} = -2\bar{\varepsilon}_{i-\frac{1}{2},j,l} \left[\frac{\bar{V}_{x;i-\frac{1}{2},j,l} - \bar{V}_{x;(i-1)+\frac{1}{2},j,l}}{\Delta x_{i-\frac{1}{2},j}} \right] , \qquad (4.185a)$$

$$\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j-\frac{1}{4},l} = -\bar{\varepsilon}_{i-\frac{1}{2},j,l} \left(\frac{\bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i-1,j-\frac{1}{4},l}}{\Delta x_{i-\frac{1}{2},j}} \right) , \qquad (4.185b)$$

$$\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l} = -\bar{\varepsilon}_{i-\frac{1}{2},j,l} \left(\frac{\bar{V}_{y;i,j+\frac{1}{4},l} - \bar{V}_{y;i-1,j+\frac{1}{4},l}}{\Delta x_{i-\frac{1}{2},j}} \right) , \qquad (4.185c)$$

$$\bar{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,l} = -\bar{\varepsilon}_{i-\frac{1}{2},j,l} \left(\frac{\bar{V}_{z;i,j,l} - \bar{V}_{z;i-1,j,l}}{\Delta x_{i-\frac{1}{2},j}} \right) . \qquad (4.185d)$$

The formulation of the combined stress components acting on a x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,i)$ requires special modifications in order to compensate for the nonexisting x-downstream element $(i+1,j,1)$. Hence, the magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;(shr,i-\frac{1}{2}),j,l}$ at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,1)$ of the element $(i,j,1)$ and the x-upstream half-element $(i-\frac{1}{4},j,1)$ is formulated, as:

$$\left| \overline{\Delta V}_{y,z;(shr,i-\frac{1}{2}),j,l} \right| = \left(\bar{V}^2_{y;i,j,l} + \bar{V}^2_{z;i,j,l} \right)^{\frac{1}{2}} . \tag{4.186}$$

The local value of Reynolds number $\overline{Re}_{(shr,i-\frac{1}{2}),j,l}$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;(shr,i-\frac{1}{2}),j,l}$ at the x-upstream shoreline-boundary enclosure surface, is formulated, as:

$$\overline{Re}_{(shr,i-\frac{1}{2}),j,l} = \left(\frac{\Delta x_{i,j}}{2}\right) \frac{\left| \overline{\Delta V}_{y,z;(shr,i-\frac{1}{2}),j,l} \right|}{\nu} . \tag{4.187}$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with the assumed critical value of Reynolds number. For the values of the local Reynolds number $\overline{Re}_{(shr,i-\frac{1}{2}),j,l}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j,l}$ at the x-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,1)$ is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{(shr,i-\frac{1}{2}),j,l} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j,l} = \nu \quad \text{(laminar flow conditions)} . \tag{4.188a}$$

For the values of the local Reynolds number $\overline{Re}_{(shr,i-\frac{1}{2}),j,l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j;l}$ at the x-upstream enclosure surface at (i-½,j,l) is determined, based on the modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{(shr,i-\frac{1}{2}),j,l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j;l} = \nu + \bar{\varepsilon}_{trb;(shr,i-\frac{1}{2}),j,l} ,$$

$$\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j,l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}^{\frac{n-1}{n+1}}_{(shr,i-\frac{1}{2}),j,l} . \qquad (4.188b)$$

The combined stress components (Sec. 3), acting on the x-upstream shoreline-boundary enclosure surface at (i-½,j,l), are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the x-upstream shoreline-boundary enclosure surface at (i-½,j,l) of the element (i,j,l) and the x-upstream half element (i-¼,j,l), as:

$$\bar{\sigma}^{*'}_{\nu,x,x;(shr,i-\frac{1}{2}),j,l} = 0 , \qquad (4.189a)$$

$$\bar{\sigma}^{*'}_{\nu,x,y;(shr,i-\frac{1}{2}),j-\frac{1}{4},l} = -2\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j,l} \frac{\bar{V}_{y;(shr,i),j-\frac{1}{4},l}}{\Delta x_{i,j-\frac{1}{4}}} , \qquad (4.189b)$$

$$\bar{\sigma}^{*'}_{\nu,x,y;(shr,i-\frac{1}{2}),j+\frac{1}{4},l} = -2\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j,l} \frac{\bar{V}_{y;(shr,i),j+\frac{1}{4},l}}{\Delta x_{i,j+\frac{1}{4}}} , \qquad (4.189c)$$

$$\bar{\sigma}^{*'}_{\nu,x,z;(shr,i-\frac{1}{2}),j,l} = -2\bar{\varepsilon}_{(shr,i-\frac{1}{2}),j,l} \frac{\bar{V}_{z;(shr,i),j,l}}{\Delta x_{i,j}} . \qquad (4.189d)$$

Similarly, the upstream value $\bar{V}_{y;(-,i+\frac{1}{2}),j,l}$ and the downstream value $\bar{V}_{y;(+,i+\frac{1}{2}),j,l}$ of the y-component of velocity and the upstream value $\bar{V}_{z;(-,i+\frac{1}{2}),j,l}$ and the downstream value $\bar{V}_{z;(+,i+\frac{1}{2}),j,l}$ of the z-component of velocity on both sides of the x-downstream enclosure at $(i+\frac{1}{2},j,l)$ are formulated, based on linear interpolation which considers the sizes of the adjacent elements (i,j,l) and (i+1,j,l), along the x-direction.

$$\text{for } \Delta x_{i,j} \leq \Delta x_{i+1,j}:$$

$$\bar{V}_{y;(-,i+\frac{1}{2}),j,l} = \bar{V}_{y;i,j,l}\,, \tag{4.190a}$$

$$\bar{V}_{z;(-,i+\frac{1}{2}),j,l} = \bar{V}_{z;i,j,l}\,, \tag{4.190b}$$

$$\bar{V}_{y;(+,i+\frac{1}{2}),j,l} = \bar{V}_{y;i,j,l} + \frac{\Delta x_{i,j}}{\Delta x_{i+\frac{1}{2},j}} \left(\bar{V}_{y;i+1,j,l} - \bar{V}_{y;i,j,l}\right), \tag{4.190c}$$

$$\bar{V}_{z;(+,i+\frac{1}{2}),j,l} = \bar{V}_{z;i,j,l} + \frac{\Delta x_{i,j}}{\Delta x_{i+\frac{1}{2},j}} \left(\bar{V}_{z;i+1,j,l} - \bar{V}_{z;i,j,l}\right), \tag{4.190d}$$

$$\text{for } \Delta x_{i,j} > \Delta x_{i+1,j}:$$

$$\bar{V}_{y;(-,i+\frac{1}{2}),j,l} = \bar{V}_{y;i+1,j,l} + \frac{\Delta x_{i+1,j}}{\Delta x_{i+\frac{1}{2},j}} \left(\bar{V}_{y;i+1,j,l} - \bar{V}_{y;i,j,l}\right), \tag{4.190e}$$

$$\bar{V}_{z;(-,i+\frac{1}{2}),j,l} = \bar{V}_{z;i+1,j,l} + \frac{\Delta x_{i+1,j}}{\Delta x_{i+\frac{1}{2},j}} \left(\bar{V}_{z;i+1,j,l} - \bar{V}_{z;i,j,l}\right), \tag{4.190f}$$

$$\overline{V}_{y;(+,i+\frac{1}{2}),j,l} = \overline{V}_{y;i+1,j,l} \,. \tag{4.190g}$$

$$\overline{V}_{z;(+,i+\frac{1}{2}),j,l} = \overline{V}_{z;i+1,j,l} \,. \tag{4.190h}$$

The magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;i+\frac{1}{2},j,l}$ at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ is defined as:

$$\left| \overline{\Delta V}_{y,z;i+\frac{1}{2},j,l} \right| = \left\{ \left[\overline{V}_{y;(+,i+\frac{1}{2}),j,l} - \overline{V}_{y;(-,i+\frac{1}{2}),j,l} \right]^2 + \left[\overline{V}_{z;(+,i+\frac{1}{2}),j,l} - \overline{V}_{z;(-,i+\frac{1}{2}),j,l} \right]^2 \right\}^{\frac{1}{2}} . \tag{4.191}$$

The local value of Reynolds number $\overline{Re}_{i+\frac{1}{2},j,l}$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;i+\frac{1}{2},j,l}$, at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$, is formulated by considering the sizes of the adjacent elements (i,j,l) and (i+1,j,l), along the x-direction.

$$\text{for } \Delta x_{i,j} \leq \Delta x_{i+1,j} :$$

$$\overline{Re}_{i+\frac{1}{2},j,l} = \Delta x_{i,j} \frac{\left| \overline{\Delta V}_{y,z;i+\frac{1}{2},j,l} \right|}{\nu} \,. \tag{4.192a}$$

$$\text{for } \Delta x_{i,j} > \Delta x_{i+1,j} :$$

$$\overline{Re}_{i+\frac{1}{2},j,l} = \Delta x_{i+1,j} \frac{\left| \overline{\Delta V}_{y,z;i+\frac{1}{2},j,l} \right|}{\nu} \,. \tag{4.192b}$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with a critical value of Reynolds number assumed as 4000. For values of the local Reynolds

number $\overline{Re}_{i+\frac{1}{2},j,l}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i+\frac{1}{2},j,l}$ at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i+\frac{1}{2},j,l} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i+\frac{1}{2},j,l} = \nu \quad \text{(laminar flow conditions)} . \tag{4.193}$$

For the values of local Reynolds number $\overline{Re}_{i+\frac{1}{2},j,l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i+\frac{1}{2},j,l}$ at the x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ is determined, based on a modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i+\frac{1}{2},j,l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i+\frac{1}{2},j,l} = \nu + \bar{\varepsilon}_{trb;i+\frac{1}{2},j,l} ,$$

$$\bar{\varepsilon}_{i+\frac{1}{2},j,l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}^{\frac{n-1}{n+1}}_{i+\frac{1}{2},j,l} . \tag{4.194}$$

The combined, stress components (Sec. 3), acting on the internal x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$, are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the internal x-downstream enclosure surface at $(i+\frac{1}{2},j,l)$ of the element $(i+\frac{1}{2},j,l)$, and the x-downstream half element $(i+\frac{1}{4},j,l)$, as:

$$\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,l} = -2\bar{\varepsilon}_{i+\frac{1}{2},j,l} \left[\frac{\bar{V}_{x;(i+1)+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l}}{\Delta x_{i+\frac{1}{2},j}} \right] . \tag{4.195a}$$

$$\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j-\frac{1}{4},l} = -\bar{\varepsilon}_{i+\frac{1}{2},j,l} \left(\frac{\bar{V}_{y;i,j-\frac{1}{4},l} - \bar{V}_{y;i+1,j-\frac{1}{4},l}}{\Delta x_{i+\frac{1}{2},j}} \right) , \qquad (4.195b)$$

$$\bar{\sigma}^{*'}_{\nu,x,y;i+\frac{1}{2},j+\frac{1}{4},l} = -\bar{\varepsilon}_{i+\frac{1}{2},j,l} \left(\frac{\bar{V}_{y;i+1,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}}{\Delta x_{i+\frac{1}{2},j}} \right) , \qquad (4.195c)$$

$$\bar{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,l} = -\bar{\varepsilon}_{i+\frac{1}{2},j,l} \left(\frac{\bar{V}_{z;i+1,j,l} - \bar{V}_{z;i,j,l}}{\Delta x_{i+\frac{1}{2},j}} \right) . \qquad (4.195d)$$

The formulation of the combined stress components acting on a x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,i)$ requires special modifications in order to compensate for the nonexisting x-upstream element $(i-1,j,l)$. Hence, the magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;(shr,i+\frac{1}{2}),j,l}$ at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half-element $(i+\frac{1}{4},j,l)$ is formulated, as:

$$\left| \overline{\Delta V}_{y,z;(shr,i+\frac{1}{2}),j,l} \right| = \left(\bar{V}^2_{y;i,j,l} + \bar{V}^2_{z;i,j,l} \right)^{\frac{1}{2}} . \qquad (4.196)$$

The local value of Reynolds number $\overline{Re}_{(shr,i+\frac{1}{2}),j,l}$ associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{y,z;(shr,i+\frac{1}{2}),j,l}$ at the x-downstream shoreline-boundary enclosure surface, is formulated, as:

$$\overline{Re}_{(shr,i+\frac{1}{2}),j,l} = \left(\frac{\Delta x_{i,j}}{2} \right) \frac{\left| \overline{\Delta V}_{y,z;(shr,i+\frac{1}{2}),j,l} \right|}{\nu} . \qquad (4.197)$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with the assumed critical value

of Reynolds number. For values of the the local Reynolds number $\overline{Re}_{(shr,i+\frac{1}{2}),j,l}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l}$ at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{(shr,i+\frac{1}{2}),j,l} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l} = \nu \quad \text{(laminar flow conditions)} . \tag{4.198a}$$

For the values of the local Reynolds number $\overline{Re}_{(shr,i+\frac{1}{2}),j,l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l}$ at the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$ is determined, based on the modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{(shr,i+\frac{1}{2}),j,l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l} = \nu + \bar{\varepsilon}_{trb;(shr,i+\frac{1}{2}),j,l} ,$$

$$\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}^{\frac{n-1}{n+1}}_{(shr,i+\frac{1}{2}),j,l} . \tag{4.198b}$$

The combined stress components (Sec. 3), acting on the x-downstream shoreline-boundary enclosure surface at $(i+\frac{1}{2},j,l)$, are formulated based on appropriate, discretized forms of the gradients of the components of velocity at the x-downstream shoreline-boundary enclosure surface at

$(i+\frac{1}{2},j,l)$ of the element (i,j,l) and the x-downstream half element $(i+\frac{1}{4},j,l)$, as:

$$\bar{\sigma}^{*'}_{\nu,x,x;(shr,i+\frac{1}{2}),j,l} = 0, \tag{4.199a}$$

$$\bar{\sigma}^{*'}_{\nu,x,y;(shr,i+\frac{1}{2}),j-\frac{1}{4},l} = -2\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l} \frac{\bar{V}_{y;(shr,i),j-\frac{1}{4},l}}{\Delta x_{i,j-\frac{1}{4}}}, \tag{4.199b}$$

$$\bar{\sigma}^{*'}_{\nu,x,y;(shr,i+\frac{1}{2}),j+\frac{1}{4},l} = -2\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l} \frac{\bar{V}_{y;(shr,i),j+\frac{1}{4},l}}{\Delta x_{i,j+\frac{1}{4}}}, \tag{4.199c}$$

$$\bar{\sigma}^{*'}_{\nu,x,z;(shr,i+\frac{1}{2}),j,l} = -2\bar{\varepsilon}_{(shr,i+\frac{1}{2}),j,l} \frac{\bar{V}_{z;(shr,i),j,l}}{\Delta x_{i,j}}. \tag{4.199d}$$

Similarly, the upstream value $\bar{V}_{x;i\,(-,j-\frac{1}{2}),l}$ and the downstream value $\bar{V}_{x;i,(+,j-\frac{1}{2}),l}$ of the x-component of velocity and the upstream value $\bar{V}_{z;i,(-,j-\frac{1}{2}),l}$ and the downstream value $\bar{V}_{z;i,(+,j-\frac{1}{2}),l}$ of the z-component of velocity on both sides of the y-upstream enclosure at $(i,j-\frac{1}{2},l)$ are formulated, based on linear interpolation which considers the sizes of the adjacent elements (i,j,l) and $(i,j-1,l)$, along the y-direction.

$$\text{for } \Delta y_{i,j} \leq \Delta y_{i,j-1}:$$

$$\bar{V}_{x;i\,(-,j-\frac{1}{2}),l} = \bar{V}_{x;i,j,l} - \frac{\Delta y_{i,j}}{\Delta y_{i,j-\frac{1}{2}}} (\bar{V}_{x;i,j,l} - \bar{V}_{x;i,j-1,l}), \tag{4.200a}$$

$$\bar{V}_{z;i,(-,j-\frac{1}{2}),l} = \bar{V}_{z;i,j,l} - \frac{\Delta y_{i,j}}{\Delta y_{i,j-\frac{1}{2}}} (\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j-1,l}), \tag{4.200b}$$

$$\overline{V}_{x;i,(+,j-\frac{1}{2}),l} = \overline{V}_{x;i,j,l} \,, \tag{4.200c}$$

$$\overline{V}_{z;i,(+,j-\frac{1}{2}),l} = \overline{V}_{z;i,j,l} \,, \tag{4.200d}$$

$$\text{for } \Delta y_{i,j} > \Delta y_{i,j-1} :$$

$$\overline{V}_{x;i,(-,j-\frac{1}{2}),l} = \overline{V}_{x;i,j-1,l} \,, \tag{4.200e}$$

$$\overline{V}_{z;i,(-,j-\frac{1}{2}),l} = \overline{V}_{z;i,j-1,l} \,, \tag{4.200f}$$

$$\overline{V}_{x;i,(+,j-\frac{1}{2}),l} = \overline{V}_{x;i,j-1,l} + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j-\frac{1}{2}}} \left(\overline{V}_{x;i,j,l} - \overline{V}_{x;i,j-1,l}\right) , \tag{4.200g}$$

$$\overline{V}_{z;i,(+,j-\frac{1}{2}),l} = \overline{V}_{z;i,j-1,l} + \frac{\Delta y_{i,j-1}}{\Delta y_{i,j-\frac{1}{2}}} \left(\overline{V}_{z;i,j,l} - \overline{V}_{z;i,j-1,l}\right) . \tag{4.200h}$$

The magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,j-\frac{1}{2},l}$ at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ is defined as:

$$\left| \overline{\Delta V}_{x,z;i,j-\frac{1}{2},l} \right| = \left\{ \left[\overline{V}_{x;i,(+,j-\frac{1}{2}),l} - \overline{V}_{x;i,(-,j-\frac{1}{2}),l}\right]^2 + \left[\overline{V}_{z;i,(+,j-\frac{1}{2}),l} - \overline{V}_{z;i,(-,j-\frac{1}{2}),l}\right]^2 \right\}^{\frac{1}{2}} . \tag{4.201}$$

The local value of Reynolds number $\overline{Re}_{i,j-\frac{1}{2},l}$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,j-\frac{1}{2},l}$, at the y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$, is formulated by considering

the sizes of the adjacent elements (i,j,l) and (i,j-1,l), along the y-direction.

$$\text{for } \Delta y_{i,j} \leq \Delta y_{i,j-1}:$$

$$\overline{Re}_{i,j-\frac{1}{2},l} = \Delta y_{i,j} \frac{|\overline{\Delta V}_{x,z;i,j-\frac{1}{2},l}|}{\nu}, \tag{4.202a}$$

$$\text{for } \Delta y_{i,j} > \Delta y_{i,j-1}:$$

$$\overline{Re}_{i,j-\frac{1}{2},l} = \Delta y_{i,j-1} \frac{|\overline{\Delta V}_{x,z;i,j-\frac{1}{2},l}|}{\nu}. \tag{4.202b}$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with a critical value of Reynolds number assumed as 4000. For values of the local Reynolds number $\overline{Re}_{i,j-\frac{1}{2},l}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j-\frac{1}{2},l}$ at the y-upstream enclosure surface at (i,j-½,l) is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i,j-\frac{1}{2},l} \leq Re_{crt} = 4000:$$

$$\bar{\varepsilon}_{i,j-\frac{1}{2},l} = \nu \quad \text{(laminar flow conditions)}. \tag{4.203}$$

For the values of local Reynolds number $\overline{Re}_{i,j-\frac{1}{2},l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j-\frac{1}{2},l}$ at the y-upstream enclosure surface at (i,j-½,l) is determined, based on a modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i,j-\frac{1}{2},l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,j-\frac{1}{2},l} = \nu + \bar{\varepsilon}_{trb;i,j-\frac{1}{2},l} ,$$

$$\bar{\varepsilon}_{i,j-\frac{1}{2},l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}^{\frac{n-1}{n+1}}_{i,j-\frac{1}{2},l} . \qquad (4.204)$$

The combined stress components (Sec. 3), acting on the internal y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$, are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the internal y-upstream enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, as:

$$\bar{\sigma}^{*'}_{\nu,y,y;i,j-\frac{1}{2},l} = -2\bar{\varepsilon}_{i,j-\frac{1}{2},l} \left[\frac{\bar{V}_{y;i,j-\frac{1}{2},l} - \bar{V}_{y;i,(j-1)+\frac{1}{4},l}}{\Delta y_{i,j-\frac{1}{2},j}} \right] , \qquad (4.205a)$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l} = -\bar{\varepsilon}_{i,j-\frac{1}{2},l} \left(\frac{\bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j-1,l}}{\Delta y_{i,j-\frac{1}{2}}} \right) , \qquad (4.205b)$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l} = -\bar{\varepsilon}_{i,j-\frac{1}{2},l} \left(\frac{\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j-1,l}}{\Delta y_{i,j-\frac{1}{2}}} \right) , \qquad (4.205c)$$

$$\bar{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},l} = -\bar{\varepsilon}_{i,j-\frac{1}{2},l} \left(\frac{\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j-1,l}}{\Delta y_{i,j-\frac{1}{2}}} \right) . \qquad (4.205d)$$

The formulation of the combined stress components acting on a y-upstream shoreline-boundary enclosure surface at $(i-\frac{1}{2},j,i)$ requires special modifications in order to compensate for the nonexisting

x-downstream element (i+1,j,l). Hence, the magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,(shr,j-\frac{1}{2}),l}$ at the y-upstream shoreline-boundary enclosure surface at (i,j-½,l) of the element (i,j,l) and the y-upstream half-element (i-¼,j,l) is formulated, as:

$$\left| \overline{\Delta V}_{x,z;i,(shr,j-\frac{1}{2}),l} \right| = \left(\overline{V}^2_{x;i,j,l} + \overline{V}^2_{z;i,j,l} \right)^{\frac{1}{2}} . \qquad (4.206)$$

The local value of Reynolds number $\overline{Re}_{i,(shr,j-\frac{1}{2}),l}$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,(shr,j-\frac{1}{2}),l}$ at the y-upstream shoreline-boundary enclosure surface, is formulated, as:

$$\overline{Re}_{i,(shr,j-\frac{1}{2}),l} = \left(\frac{\Delta y_{i,j}}{2}\right) \frac{\left| \overline{\Delta V}_{x,z;i,(shr,j-\frac{1}{2}),l} \right|}{\nu} . \qquad (4.207)$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with the assumed critical value of Reynolds number. For values of the local Reynolds number $\overline{Re}_{i,(shr,j-\frac{1}{2}),l}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l}$ at the y-upstream shoreline-boundary enclosure surface at (i,j-½,l) is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i,(shr,j-\frac{1}{2}),l} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l} = \nu \quad (\text{laminar flow conditions}) . \qquad (4.208a)$$

For the values of the local Reynolds number $\overline{Re}_{i,(shr,j-\frac{1}{2}),l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l}$ at the y-upstream shoreline-boundary enclosure surface at

$(i,j-\frac{1}{2},l)$ is determined, based on the modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i,(shr,j-\frac{1}{2}),l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l} = \nu + \bar{\varepsilon}_{trb;i,(shr,j-\frac{1}{2}),l} ,$$

$$\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}_{i,(shr,j-\frac{1}{2}),l}^{\frac{n-1}{n+1}} . \qquad (4.208b)$$

The combined stress components (Sec. 3), acting on the y-upstream shoreline-boundary enclosure surface at $(i,j-\frac{1}{2},l)$, are formulated based on appropriate, discretized forms of the gradients of the components of velocity at the y-upstream shoreline-boundary enclosure surface at $(i,j-\frac{1}{2},l)$ of the element (i,j,l) and the y-upstream half element $(i,j-\frac{1}{4},l)$, as:

$$\bar{\sigma}^{*'}_{\nu,y,y;i,(shr,j-\frac{1}{2}),l} = 0, \qquad (4.209a)$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},(shr,j-\frac{1}{2}),,l} = -2\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l} \frac{\bar{V}_{x;i-\frac{1}{4},(shr,j),l}}{\Delta y_{i-\frac{1}{4},j}} , \qquad (4.209b)$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},(shr,j-\frac{1}{2}),,l} = -2\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l} \frac{\bar{V}_{x;i+\frac{1}{4},(shr,j),l}}{\Delta y_{i+\frac{1}{4},j}} , \qquad (4.209c)$$

$$\bar{\sigma}^{*'}_{\nu,y,z;i,(shr,j-\frac{1}{2}),l} = -2\bar{\varepsilon}_{i,(shr,j-\frac{1}{2}),l} \frac{\bar{V}_{z;i,(shr,j),l}}{\Delta y_{i,j}} . \qquad (4.209d)$$

Similarly, the upstream value $\overline{V}_{\lambda;i,(-,j+\frac{1}{2}),l}$ and the downstream value $\overline{V}_{x;i,(+,j+\frac{1}{2}),l}$ of the x-component of velocity and the upstream value $\overline{V}_{z;i,(-,j+\frac{1}{2}),l}$ and the downstream value $\overline{V}_{z;i,(+,j+\frac{1}{2}),l}$ of the z-component of velocity on both sides of the y-downstream enclosure at $(i,j+\frac{1}{2},l)$ are formulated, based on linear interpolation which considers the sizes of the adjacent elements (i,j,l) and (i,j+1,l), along the y-direction.

$$\text{for } \Delta y_{i,j} \leq \Delta y_{i,j+1}:$$

$$\overline{V}_{x;i,(-,j+\frac{1}{2}),l} = \overline{V}_{x;i,j,l}\,, \tag{4.210a}$$

$$\overline{V}_{z;i,(-,j+\frac{1}{2}),l} = \overline{V}_{z;i,j,l}\,, \tag{4.210b}$$

$$\overline{V}_{x;i(+,j+\frac{1}{2}),l} = \overline{V}_{x;i,j,l} + \frac{\Delta y_{i,j}}{\Delta y_{i,j+\frac{1}{2}}}\,(\overline{V}_{x;i,j+1,l} - \overline{V}_{x;i,j,l})\,, \tag{4.210c}$$

$$\overline{V}_{z;i,(+,j+\frac{1}{2}),l} = \overline{V}_{z;i,j,l} + \frac{\Delta y_{i,j}}{\Delta y_{i,j+\frac{1}{2}}}\,(\overline{V}_{z;i,j+1,l} - \overline{V}_{z;i,j,l})\,, \tag{4.210d}$$

$$\text{for } \Delta y_{i,j} > \Delta y_{i,j+1}:$$

$$\overline{V}_{x;i,(+,j+\frac{1}{2}),l} = \overline{V}_{x;i,j+1,l}\,, \tag{4.210e}$$

$$\overline{V}_{z;i,(+,j+\frac{1}{2}),l} = \overline{V}_{z;i,j+1,l}\,, \tag{4.210f}$$

$$\overline{V}_{x;i,(-,j+\frac{1}{2}),l} = \overline{V}_{x;i,j+1,l} - \frac{\Delta y_{i,j+1}}{\Delta y_{i,j+\frac{1}{2}}}\,(\overline{V}_{x;i,j+1,l} - \overline{V}_{x;i,j,l})\,, \tag{4.210g}$$

$$\overline{V}_{z;i,(-,j+\frac{1}{2}),l} = \overline{V}_{z;i,j+1,l} - \frac{\Delta y_{i,j+1}}{\Delta y_{i,j+\frac{1}{2}}}\,(\overline{V}_{z;i,j+1,l} - \overline{V}_{z;i,j,l})\,. \tag{4.210h}$$

The magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,j+\frac{1}{2},l}$ at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ is defined as:

$$\left| \overline{\Delta V}_{x,z;i,j+\frac{1}{2},l} \right| = \left\{ \left[\overline{V}_{x;i,(+,j+\frac{1}{2}),l} - \overline{V}_{x;i,(-,j+\frac{1}{2}),l} \right]^2 + \left[\overline{V}_{z;i,(+,j+\frac{1}{2}),l} - \overline{V}_{z;i,(-,j+\frac{1}{2}),l} \right]^2 \right\}^{\frac{1}{2}} . \qquad (4.211)$$

The local value of Reynolds number $Rei,j+\frac{1}{2},l$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,j+\frac{1}{2},l}$, at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$, is formulated by considering the sizes of the adjacent elements (i,j,l) and (i,j+1,l), along the y-direction.

$$\text{for } \Delta y_{i,j} \leq \Delta y_{i,j+1} :$$

$$\overline{Re}_{i,j+\frac{1}{2},l} = \Delta y_{i,j} \frac{\left| \overline{\Delta V}_{x,z;i,j+\frac{1}{2},l} \right|}{\nu} , \qquad (4.212a)$$

$$\text{for } \Delta y_{i,j} > \Delta y_{i,j+1} :$$

$$\overline{Re}_{i,j+\frac{1}{2},l} = \Delta y_{i,j+1} \frac{\left| \overline{\Delta V}_{x,z;i,j+\frac{1}{2},l} \right|}{\nu} . \qquad (4.212b)$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with a critical value of Reynolds number assumed as 4000. For values of the local Reynolds number $\overline{Re}_{i,j+\frac{1}{2},l}$ less than the critical value, the combined coefficient of viscosity $\overline{\varepsilon}_{i,j+\frac{1}{2},l}$ at the y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i,j+\frac{1}{2},l} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,j+\frac{1}{2},l} = \nu \quad \text{(laminar flow conditions)} . \tag{4.213}$$

For the values of local Reynolds number $\overline{Re}_{i,j+\frac{1}{2},l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j+\frac{1}{2},l}$ at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ is determined, based on a modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i,j+\frac{1}{2},l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,j+\frac{1}{2},l} = \nu + \bar{\varepsilon}_{trb;i,j+\frac{1}{2},l} \cdot$$

$$\bar{\varepsilon}_{i,j+\frac{1}{2},l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}_{i,j+\frac{1}{2},l}^{\frac{n-1}{n+1}} . \tag{4.214}$$

The combined stress components (Sec. 3), acting on the internal y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$, are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the internal y-downstream enclosure surface at $(i,j+\frac{1}{2},l)$ of the element $(i,j+\frac{1}{2},l)$, as:

$$\bar{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},l} = -2\bar{\varepsilon}_{i,j+\frac{1}{2},l} \left[\frac{\bar{V}_{y;i,(j+1)-\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}}{\Delta y_{i,j+\frac{1}{2},j}} \right] , \tag{4.215a}$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j+\frac{1}{2},l} = -\bar{\varepsilon}_{i,j+\frac{1}{2},l} \left(\frac{\bar{V}_{x;i-\frac{1}{4},j+1,l} - \bar{V}_{x;i-\frac{1}{4},j,l}}{\Delta y_{i,j+\frac{1}{2}}} \right) , \tag{4.215b}$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j+\frac{1}{2},l} = -\bar{\varepsilon}_{i,j+\frac{1}{2},l} \left(\frac{\bar{V}_{x;i+\frac{1}{4},j+1,l} - \bar{V}_{x;i+\frac{1}{4},j,l}}{\Delta y_{i,j+\frac{1}{2}}} \right) , \tag{4.215c}$$

$$\bar{\sigma}^{*'}_{\nu,y,z;i,j+\frac{1}{2},l} = -\bar{\varepsilon}_{i,j+\frac{1}{2},l} \left(\frac{\bar{V}_{z;i,j+1,l} - \bar{V}_{z;i,j,l}}{\Delta y_{i,j+\frac{1}{2}}}\right) . \qquad (4.215d)$$

The formulation of the combined stress components acting on a y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},i)$ requires special modifications in order to compensate for the nonexisting y-downstream element $(i,j+1,l)$. Hence, the magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,(shr,j+\frac{1}{2}),l}$ at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ of the element (i,j,l) and the y-downstream half-element $(i,j+\frac{1}{4},l)$ is formulated, as:

$$\left| \overline{\Delta V}_{x,z;i,(shr,j+\frac{1}{2}),l} \right| = \left(\bar{V}^2_{x;i,j,l} + \bar{V}^2_{z;i,j,l} \right)^{\frac{1}{2}} . \qquad (4.216)$$

The local value of Reynolds number $\overline{Re}_{i,(shr,j+\frac{1}{2}),l}$ associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{x,z;i,(shr,j+\frac{1}{2}),l}$ at the y-downstream shoreline-boundary enclosure surface, is formulated, as:

$$\overline{Re}_{i,(shr,j+\frac{1}{2}),l} = \left(\frac{\Delta y_{i,j}}{2}\right) \frac{\left| \overline{\Delta V}_{x,z;i,(shr,j+\frac{1}{2}),l} \right|}{\nu} . \qquad (4.217)$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with the assumed critical value of Reynolds number. For the values of the local Reynolds number $\overline{Re}_{i,(shr,j+\frac{1}{2}),l}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l}$ at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},l)$ is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i,(shr,j+\frac{1}{2}),l} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l} = \nu \quad \text{(laminar flow conditions)} . \tag{4.218a}$$

For the values of the local Reynolds number $\overline{Re}_{i,(shr,j+\frac{1}{2}),l}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l}$ at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},1)$ is determined, based on the modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i,(shr,j+\frac{1}{2}),l} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l} = \nu + \bar{\varepsilon}_{trb;i,(shr,j+\frac{1}{2}),l} ,$$

$$\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}^{\frac{n-1}{n+1}}_{i,(shr,j+\frac{1}{2}),l} . \tag{4.218b}$$

The combined stress components (Sec. 3), acting on the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},1)$, are formulated based on appropriate, discretized forms of the gradients of the components of velocity at the y-downstream shoreline-boundary enclosure surface at $(i,j+\frac{1}{2},1)$ of the element $(i,j,1)$ and the y-downstream half element $(i,j+\frac{1}{4},1)$, as:

$$\bar{\sigma}^{*'}_{\nu,y,y;i,(shr,j+\frac{1}{2}),l} = 0, \tag{4.219a}$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},(shr,j+\frac{1}{2}),,l} = 2\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l} \frac{\bar{V}_{x;i-\frac{1}{4},(shr,j),l}}{\Delta y_{i-\frac{1}{4},j}} , \tag{4.219b}$$

$$\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},(shr,j+\frac{1}{2}),,l} = 2\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l} \frac{\bar{V}_{x;i+\frac{1}{4},(shr,j),l}}{\Delta y_{i+\frac{1}{4},j}} , \tag{4.219c}$$

$$\bar{\sigma}^{*'}_{\nu,y,z;i,(shr,j+\frac{1}{2}),l} = 2\bar{\varepsilon}_{i,(shr,j+\frac{1}{2}),l} \frac{\bar{V}_{z;i,(shr,j),l}}{\Delta y_{i,j}} . \tag{4.219d}$$

The upstream value $\bar{V}_{x;i,j,(+,l-\frac{1}{2})}$ and the downstream value $\bar{V}_{x;i,j,(-,l-\frac{1}{2})}$ of the x-component of velocity and the upstream value $\bar{V}_{y;i,j,(-,l-\frac{1}{2})}$ and the downstream value $\bar{V}_{y;i,j,(+,l-\frac{1}{2})}$ of the y-component of velocity on both sides of the z-upstream enclosure at (i,j,l-½) are formulated, based on linear interpolation which considers the sizes of the adjacent elements (i,j,l) and (i,j,l-1), along the z-direction.

$$\text{for } \Delta z_{i,j,l} \leq \Delta z_{i,j,l} :$$

$$\bar{V}_{x;i,j,(-,l-\frac{1}{2})} = \bar{V}_{x;i,j,l} - \frac{\Delta z_{i,j,l}}{\Delta z_{i,j,l-\frac{1}{2}}} (\bar{V}_{x;i,j,l} - \bar{V}_{x;i,j,l-1}) , \tag{4.220a}$$

$$\bar{V}_{y;i,j,(-,l-\frac{1}{2})} = \bar{V}_{y;i,j,l} - \frac{\Delta z_{i,j,l}}{\Delta z_{i,j,l-\frac{1}{2}}} (\bar{V}_{y;i,j,l} - \bar{V}_{y;i,j,l-1}) , \tag{4.220b}$$

$$\bar{V}_{x;i,j,(+,l-\frac{1}{2})} = \bar{V}_{x;i,j,l} , \tag{4.220c}$$

$$\overline{V}_{y;i,j,(+,l-\frac{1}{2})} = \overline{V}_{z;i,j,l} \,, \tag{4.220d}$$

for $\Delta z_{i,j,l} > \Delta z_{i,j,l-1}$:

$$\overline{V}_{y;i,j,(-,l-\frac{1}{2})} = \overline{V}_{y;i,j,l-1} \,, \tag{4.220e}$$

$$\overline{V}_{x;i,j,(-,l-\frac{1}{2})} = \overline{V}_{x;i,j,l-1} \,, \tag{4.220f}$$

$$\overline{V}_{x;i,j,(+,l-\frac{1}{2})} = \overline{V}_{x;i,j,l-1} + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l-\frac{1}{2}}} \left(\overline{V}_{x;i,j,l} - \overline{V}_{x;i,j,l-1}\right) . \tag{4.220g}$$

$$\overline{V}_{y;i,j,(+,l-\frac{1}{2})} = \overline{V}_{y;i,j,l-1} + \frac{\Delta z_{i,j,l-1}}{\Delta z_{i,j,l-\frac{1}{2}}} \left(\overline{V}_{y;i,j,l} - \overline{V}_{y;i,j,l-1}\right) . \tag{4.220h}$$

The magnitude of the relative shear velocity $V_{x,y;i,j,l-\frac{1}{2}}$ at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ is defined as:

$$\left| \overline{\Delta V}_{x,y;i,j,l-\frac{1}{2}} \right| = \left\{ \left[\overline{V}_{x;i,j,(+,l-\frac{1}{2})} - \overline{V}_{x;i,j,(-,l-\frac{1}{2})}\right]^2 + \left[\overline{V}_{y;i,j,(+,l-\frac{1}{2})} - \overline{V}_{y;i,j,(-,l-\frac{1}{2})}\right]^2 \right\}^{\frac{1}{2}} . \tag{4.221}$$

The local value of Reynolds number $Re_{i,j,l-\frac{1}{2}}$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{x,y;i,j,l-\frac{1}{2}}$, at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$, is formulated by considering the sizes of the adjacent elements (i,j,l) and (i,j,l-1), along the z-direction.

for $\Delta z_{i,j,l} \leq \Delta z_{i,j,l-1}$:

$$\overline{Re}_{i,j,l-\frac{1}{2}} = \Delta z_{i,j,l} \frac{| \overline{\Delta V}_{x,y;i,j,l-\frac{1}{2}} |}{\nu} , \tag{4.222a}$$

for $\Delta z_{i,j,l} > \Delta z_{i,j,l-1}$:

$$\overline{Re}_{i,j,l-\frac{1}{2}} = \Delta z_{i,j,l-1} \frac{| \overline{\Delta V}_{x,y;i,j,l-\frac{1}{2}} |}{\nu} . \tag{4.222b}$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with a critical value of Reynolds number assumed as 4000. For values of the local Reynolds number $\overline{Re}_{i,j,l-\frac{1}{2}}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j,l-\frac{1}{2}}$ at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ is assumed as the molecular coefficient of viscosity ν, i.e.,

for $\overline{Re}_{i,j,l-\frac{1}{2}} \leq Re_{crt} = 4000$:

$$\bar{\varepsilon}_{i,j,l-\frac{1}{2}} = \nu \quad \text{(laminar flow conditions)} . \tag{4.223}$$

For the values of local Reynolds number $\overline{Re}_{i,j,l-\frac{1}{2}}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j,l-\frac{1}{2}}$ at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ is determined, based on a modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

for $\overline{Re}_{i,j,l-\frac{1}{2}} > Re_{crt} = 4000$:

$$\bar{\varepsilon}_{i,j,l-\frac{1}{2}} = \nu + \bar{\varepsilon}_{trb;i,j,l-\frac{1}{2}} ,$$

(continue)

$$\bar{\varepsilon}_{i,j,l-\frac{1}{2}} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{n+1} \beta^{n+1} \right] \overline{Re}_{i,j,l-\frac{1}{2}}^{\frac{n-1}{n+1}} \cdot \tag{4.224}$$

The combined stress components (Sec. 3), acting on the internal z-upstream enclosure surface at (i,j,l-½), are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the internal z-upstream enclosure surface at (i,j,l-½) of the element (i,j,l), x-upstream half element (i-¼,j,l), x-downstream half element (i+¼,j,l), y-upstream half element (i,j-¼,l), y-downstream half element (i,j+¼,l), as:

$$\bar{\sigma}^{*'}_{\nu,z,z;i,j,l-\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l-\frac{1}{2}} \left[\frac{\bar{V}_{z;i,j,l} - \bar{V}_{z;i,j,l-1}}{\Delta z_{i,j,l-\frac{1}{2}}} \right] , \tag{4.225a}$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l-\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l-\frac{1}{2}} \left(\frac{\bar{V}_{x;i-\frac{1}{4},j,l} - \bar{V}_{x;i-\frac{1}{4},j,l-1}}{\Delta z_{i,j,l-\frac{1}{2}}} \right) , \tag{4.225b}$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l-\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l-\frac{1}{2}} \left(\frac{\bar{V}_{x;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l-1}}{\Delta z_{i,j,l-\frac{1}{2}}} \right) , \tag{4.225c}$$

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l-\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l-\frac{1}{2}} \left(\frac{\bar{V}_{x;i,j-\frac{1}{4},l} - \bar{V}_{x;i,j-\frac{1}{4},l-1}}{\Delta z_{i,j,l-\frac{1}{2}}} \right) , \tag{4.225d}$$

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l-\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l-\frac{1}{2}} \left(\frac{\bar{V}_{x;i,j+\frac{1}{4},l} - \bar{V}_{x;i,j+\frac{1}{4},l-1}}{\Delta z_{i,j,l-\frac{1}{2}}} \right) . \tag{4.225e}$$

The formulation of the combined stress components acting on a z-upstream bottom-boundary enclosure surface at $(i,j,l-\frac{1}{2})$ requires special modifications in order to compensate for the nonexisting z-downstream element $(i,j,l+1)$. Hence, the magnitude of the relative shear velocity $\overline{\Delta V}_{x,y;i,j,(bot,l-\frac{1}{2})}$ at the z-upstream bottom-boundary enclosure surface at $(i,j,l-\frac{1}{2})$ of the element (i,j,l) is formulated, as:

$$\left| \overline{\Delta V}_{x,y;i,j,(bot,l-\frac{1}{2})} \right| = \left(\bar{V}^2_{x;i,j,l} + \bar{V}^2_{y;i,j,l} \right)^{\frac{1}{2}} . \qquad (4.226)$$

The local value of Reynolds number $\overline{Re}_{i,j,(bot,l-\frac{1}{2})}$ associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{x,y;i,j,(bot,l-\frac{1}{2})}$ at the z-upstream bottom-boundary enclosure surface, is formulated, as:

$$\overline{Re}_{i,j,(bot,l-\frac{1}{2})} = \left(\frac{\Delta z_{i,j,l}}{2}\right) \frac{\left| \overline{\Delta V}_{x,y;i,j,(bot,l-\frac{1}{2})} \right|}{\nu} . \qquad (4.227)$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with the assumed critical value of Reynolds number. For values of the local Reynolds number $\overline{Re}_{i,j,(bot,l-\frac{1}{2})}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})}$, at the z-upstream bottom-boundary enclosure surface at $(i,j,l-\frac{1}{2})$ is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i,j,(bot,l-\frac{1}{2})} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})} = \nu \quad \text{(laminar flow conditions)} , \qquad (4.228a)$$

For the values of the local Reynolds number $\overline{Re}_{i,j,(bot,l-\frac{1}{2})}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})}$ at the z-upstream enclosure surface at $(i,j,l-\frac{1}{2})$ is determined, based on the modified form of the Blasius power-law type formulation (e.g., Goldstein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i,j,(bot,l-\frac{1}{2})} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})} = \nu + \bar{\varepsilon}_{trb;i,j,(bot,l-\frac{1}{2})} ,$$

$$\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}^{\frac{n-1}{n+1}}_{i,j,(bot,l-\frac{1}{2})} . \quad (4.228b)$$

The combined stress components (Sec. 3), acting on the z-upstream bottom-boundary enclosure surface at $(i,j,l-\frac{1}{2})$, are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the z-upstream bottom-boundary enclosure surface at $(i,j,l-\frac{1}{2})$ of the element (i,j,l), the x-upstream half element $(i-\frac{1}{4},j,l)$, the x-downstream half element $(i+\frac{1}{4},j,l)$, the y-upstream half element $(i,j-\frac{1}{4},l)$, the y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$\bar{\sigma}^{*'}_{\nu,z,z;i,j,(bot,l-\frac{1}{2})} = 0, \quad (4.229a)$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,(bot,l-\frac{1}{2})} = -2\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})} \frac{\bar{V}_{x;i-\frac{1}{4},j,(bot,l)}}{\Delta z_{i,j,l}}, \quad (4.229b)$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,(bot,l-\frac{1}{2})} = -2\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})} \frac{\bar{V}_{x;i+\frac{1}{4},j,(bot,l)}}{\Delta z_{i,j,l}}, \quad (4.229c)$$

$$\bar{\sigma}^{*}_{\nu,z,y;i,j-\frac{1}{4},(bot,l-\frac{1}{2})} = -2\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})} \frac{\bar{V}_{y;i,j-\frac{1}{4},(bot,l)}}{\Delta z_{i,j,l}} . \qquad (4.229d)$$

$$\bar{\sigma}^{*}_{\nu,z,y;i,j+\frac{1}{4},(bot,l-\frac{1}{2})} = -2\bar{\varepsilon}_{i,j,(bot,l-\frac{1}{2})} \frac{\bar{V}_{y;i,j+\frac{1}{4},(bot,l)}}{\Delta z_{i,j,l}} . \qquad (4.229e)$$

Similarly, the upstream value $\bar{V}_{x;i,j,(-,l+\frac{1}{2})}$ and the downstream value $\bar{V}_{x;i,j,(+,l+\frac{1}{2})}$ of the x-component of velocity and the upstream value $\bar{V}_{y;i,j,(-,l+\frac{1}{2})}$ and the downstream value $\bar{V}_{y;i,j,(+,l+\frac{1}{2})}$ of the y-component of velocity on both sidess of the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ are formulated, based on linear interpolation which considers the sizes of the adjacent elements (i,j,l) and (i,j,l+1), along the z-direction.

$$\text{for } \Delta z_{i,j,l} \leq \Delta z_{i,j,l} :$$

$$\bar{V}_{x;i,j,(-,l+\frac{1}{2})} = \bar{V}_{x;i,j,l} , \qquad (4.230a)$$

$$\bar{V}_{y;i,j,(-,l+\frac{1}{2})} = \bar{V}_{z;i,j,l} , \qquad (4.230b)$$

$$\bar{V}_{x;i,j,(+,l+\frac{1}{2})} = \bar{V}_{x;i,j,l} + \frac{\Delta z_{i,j,l}}{\Delta z_{i,j,l+\frac{1}{2}}} (\bar{V}_{x;i,j,l+1} - \bar{V}_{x;i,j,l}) , \qquad (4.230c)$$

$$\bar{V}_{y;i,j,(+,l+\frac{1}{2})} = \bar{V}_{y;i,j,l} + \frac{\Delta z_{i,j,l}}{\Delta z_{i,j,l+\frac{1}{2}}} (\bar{V}_{y;i,j,l+1} - \bar{V}_{y;i,j,l}) , \qquad (4.230d)$$

for $\Delta z_{i,j,l} > \Delta z_{i,j,l+1}$:

$$\overline{V}_{x;i,j(-,l+\frac{1}{2})} = \overline{V}_{x;i,j,l+1} - \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l+\frac{1}{2}}} (\overline{V}_{x;i,j,l+1} - \overline{V}_{x;i,j,l}) . \tag{4.230e}$$

$$\overline{V}_{y;i,j(-,l+\frac{1}{2})} = \overline{V}_{y;i,j,l+1} - \frac{\Delta z_{i,j,l+1}}{\Delta z_{i,j,l+\frac{1}{2}}} (\overline{V}_{y;i,j,l+1} - \overline{V}_{y;i,j,l}) , \tag{4.230f}$$

$$\overline{V}_{y;i,j,(+,l+\frac{1}{2})} = \overline{V}_{y;i,j,l+1} . \tag{4.230g}$$

$$\overline{V}_{x;i,j,(+,l+\frac{1}{2})} = \overline{V}_{x;i,j,l+1} . \tag{4.230h}$$

The magnitude of the relative shear velocity $\overline{\Delta V}_{x,y;i,j,l+\frac{1}{2}}$ at the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ is defined as:

$$\left| \overline{\Delta V}_{x,y;i,j,l+\frac{1}{2}} \right| = \left\{ \left[\overline{V}_{x;i,j,(+,l+\frac{1}{2})} - \overline{V}_{x;i,j,(-,l+\frac{1}{2})} \right]^2 + \left[\overline{V}_{y;i,j,(+,l+\frac{1}{2})} - \overline{V}_{y;i,j,(-,l+\frac{1}{2})} \right]^2 \right\}^{\frac{1}{2}} . \tag{4.231}$$

The local value of Reynolds number $\overline{Re}_{i,j,l+\frac{1}{2}}$, associated with the magnitude of the relative shear velocity $\overline{\Delta V}_{x,y;i,j,l+\frac{1}{2}}$, at the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$, is formulated by considering the sizes of the adjacent elements (i,j,l) and $(i,j,l+1)$, along the z-direction.

for $\Delta z_{i,j,l} \leq \Delta z_{i,j,l+1}$:

$$\overline{Re}_{i,j,l+\frac{1}{2}} = \Delta z_{i,j,l} \frac{\left| \overline{\Delta V}_{x,y;i,j,l+\frac{1}{2}} \right|}{\nu} , \tag{4.232a}$$

$$\text{for } \Delta z_{i,j,l} > \Delta z_{i,j,l+1} :$$

$$\overline{Re}_{i,j,l+\frac{1}{2}} = \Delta z_{i,j,l+1} \frac{| \overline{\Delta V}_{x,y;i,j,l+\frac{1}{2}} |}{\nu} . \qquad (4.232b)$$

The local turbulence conditions are checked, based on the comparison of the local Reynolds number with the critical value of Reynolds number assumed as 4000. For values of the local Reynolds number $\overline{Re}_{i,j,l+\frac{1}{2}}$ less than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j,l+\frac{1}{2}}$ at the z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ is assumed as the molecular coefficient of viscosity ν, i.e.,

$$\text{for } \overline{Re}_{i,j,l+\frac{1}{2}} \leq Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,j,l+\frac{1}{2}} = \nu \quad \text{(laminar flow conditions)} . \qquad (4.233)$$

For the values of the local Reynolds number $\overline{Re}_{i,j,l+\frac{1}{2}}$ greater than the critical value, the combined coefficient of viscosity $\bar{\varepsilon}_{i,j,l+\frac{1}{2}}$ at the z-downstream enclosure at $(i,j,l+\frac{1}{2})$ is determined, based on the modified form of the Blasius power-law type formulation (e.g., Goldestein, 1938, Eraslan, 1981b), i.e.,

$$\text{for } \overline{Re}_{i,j,l+\frac{1}{2}} > Re_{crt} = 4000 :$$

$$\bar{\varepsilon}_{i,j,l+\frac{1}{2}} = \nu + \bar{\varepsilon}_{trb;i,j,l+\frac{1}{2}} ,$$

$$\bar{\varepsilon}_{i,j,l+\frac{1}{2}} = \nu + \nu \left[\left(\frac{n+1}{n}\right)^{\frac{2}{n+1}} \beta^{\frac{2n}{n+1}} \right] \overline{Re}_{i,j,l+\frac{1}{2}}^{\frac{n-1}{n+1}} . \qquad (4.234)$$

The combined stress components (Sec. 3), acting on the internal z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ are formulated, based on appropriate, discretized forms of the gradients of the components of velocity at the internal z-downstream enclosure surface at $(i,j,l+\frac{1}{2})$ of the element (i,j,l), x-upstream half element $(i-\frac{1}{4},j,l)$, x-downstream half element $(i+\frac{1}{4},j,l)$, y-upstream half element $(i,j-\frac{1}{4},l)$ and y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$\bar{\sigma}^{*'}_{\nu,z,z;i,j,l+\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l+\frac{1}{2}} \left[\frac{\bar{V}_{z;i,j,l+1} - \bar{V}_{z;i,j,l}}{\Delta z_{i,j,l+\frac{1}{2}}} \right] , \qquad (4.235a)$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l+\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l+\frac{1}{2}} \left(\frac{\bar{V}_{x;i-\frac{1}{4},j,l+1} - \bar{V}_{x;i-\frac{1}{4},j,l}}{\Delta z_{i,j,l+\frac{1}{2}}} \right) , \qquad (4.235b)$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l+\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l+\frac{1}{2}} \left(\frac{\bar{V}_{x;i+\frac{1}{4},j,l+1} - \bar{V}_{x;i+\frac{1}{4},j,l}}{\Delta z_{i,j,l+\frac{1}{2}}} \right) , \qquad (4.235c)$$

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},l+\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l+\frac{1}{2}} \left(\frac{\bar{V}_{x;i,j-\frac{1}{4},l+1} - \bar{V}_{x;i,j-\frac{1}{4},l}}{\Delta z_{i,j,l+\frac{1}{2}}} \right) , \qquad (4.235d)$$

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l+\frac{1}{2}} = -\bar{\varepsilon}_{i,j,l+\frac{1}{2}} \left(\frac{\bar{V}_{x;i,j+\frac{1}{4},l+1} - \bar{V}_{x;i,j+\frac{1}{4},l}}{\Delta z_{i,j,l+\frac{1}{2}}} \right) . \qquad (4.235e)$$

The formulation of the combined stress components (Sec. 3), acting on an air-boundary enclosure of the water, requires detailed information about the atmospheric conditions above the water surface of the flow region. Hence, according to the general formulation of the discrete-

element turbulent transport model, the combined stress components on the z-upstream air-boundary enclosure surface at $(i,j,l+\frac{1}{2})$ can be specified directly from an auxiliary submodel for the surface-wind stress conditions, as:

$$\bar{\sigma}^{*'}_{\nu,z,z;i,j,(air,l+\frac{1}{2})} = 0,$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,(air,l+\frac{1}{2})} = \bar{\sigma}_{wnd,x;i-\frac{1}{4},j} \text{ specified}, \tag{4.236a}$$

$$\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,(air,l+\frac{1}{2})} = \bar{\sigma}_{wnd,x;i+\frac{1}{4},j} \text{ specified}, \tag{4.236b}$$

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j-\frac{1}{4},(air,l+\frac{1}{2})} = \bar{\sigma}_{wnd,y;i,j-\frac{1}{4}} \text{ specified}, \tag{4.236c}$$

$$\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},(air,l+\frac{1}{2})} = \bar{\sigma}_{wnd,y;i,j+\frac{1}{4}} \text{ specified}. \tag{4.236d}$$

Hence, the formulation of the combined stress components, according to the discrete-element turbulent transport closure submodel [Eqs. (4.180a) through (4.236d)], completes the elimination of all the auxiliary flow variables which appear in the governing computational system of discrete-element equations [Eqs. (3.5), (3.6), (3.7), (3.9), (3.10), (3.51), (3.53), (3.54), (3.55), (3.66), (3.68), (3.69), (3.70), (3.81), (3.83), (3.84), (3.85), (3.96), (3.98), (3.99), (3.100), (3.110), (3.112), (3.113) and (3.114)].

5. MATHEMATICAL PROPERTIES, COMPUTATIONAL ALGORITHM, AND STABILITY CRITERIA OF THE NUMERICAL SOLUTION METHOD

The development of the numerical solution method and the computational algorithm of the fast-transient, three-dimensional discrete-element hydrodynamic model is based on a special, state-of-the-art, consistent and stable numerical integration technique for spatially discretized systems of simultaneous first-order, ordinary differential equatious which must be considered in conjuction with a set of algebraic constraint equation.

5.1. Mathematical Properties of the Governing Computational System of Discrete-Element Equations

The governing computational system of discrete-element equations (Sec. 3) can be represented as a mathematically simplified system, consisting of a set of first-order, ordinary differential equations and a set of algebraic constraint equations.

The set of discrete-element equations associated with the mass conservation principle for water [Sec. 3.2, Eqs. (3.6), (3.7),(3.9) and (3.10)] can be represented as a subset of algebraic equations and a subset of first-order, ordinary differential equations, as:

for $l(bot;i,j) \leq l \leq L-1$:

$$\tilde{G}_{z;i,j,l+\frac{1}{2}} = \tilde{G}_{z;i,j,l-\frac{1}{2}} + [\bar{F}_{flo,x,y}(\tilde{G})]_{i,j,l} + [\bar{F}_{plt,x,y}(\tilde{G})]_{i,j,l} \quad , \qquad (5.1)$$

for $l = L$:

$$\frac{d\bar{H}_{i,j}}{dt} = [\ \bar{F}_{(\bar{H})}\ ]_{i,j,L}$$

$$= [\ \bar{F}_{flo,x,y(\bar{H})}\ ]_{i,j,L} + [\ \bar{F}_{plt(\bar{H})}\ ]_{i,j,L} + [\ \bar{F}_{srf(\bar{H})}\ ]_{i,j,L}\ , \qquad (5.2)$$

where the right-hand-side solution function for water-surface elevation $[\ \bar{F}(\bar{H})\ ]_{i,j,l}$ represents all the terms that appear on the right-hand side of the original discrete-element equation for water-surface elevation [Eqs. (3.9) and (3.10)]. The right-hand-side solution function for water-surface elevation $[\ \bar{F}_{(\bar{H})}\ ]_{i,j,L}$ represents the special form, for the water-surface layer $l = L$, of the more general right-hand-side solution function, for the flow rate along the z-direction, which can be seperated into three parts.

The part of the right-hand-side solution function for the flow rate along the z-direction, associated with the flow rates in the horizontal x-y plane, $[\bar{F}_{flo,x,y(\tilde{G})}]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for flow rate, along the z-direction [Eqs. (3.6) and (3.7)], in the element (i,j,l), as:

$$[\bar{F}_{flo,x,y(\tilde{G})}]_{i,j,l} = \tilde{G}_{x;i-\frac{1}{2},j,l} - \tilde{G}_{x;i+\frac{1}{2},j,l} + \tilde{G}_{y;i,j-\frac{1}{2},l} - \tilde{G}_{y;i,j+\frac{1}{2},l}\ . \qquad (5.3)$$

The part of the right-hand-side solution function for the flow rate along the z-direction, associated with the flow conditions at the power plants, $[\bar{F}_{plt,x,y(\tilde{G})}]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for flow rate, along the z-direction [Eqs. (3.6) and (3.7)], in the element(i,j,l), as:

$$[\bar{F}_{plt(\tilde{G})}]_{i,j,l} = \delta_{int,m;i,j,l}\tilde{G}_{int,m;i,j,l} + \delta_{dis,m;i,j,l}\bar{G}_{dis,m;i,j,l} \cdot \qquad (5.4)$$

The part of the right-hand-side solution function for water surface elevation, associated with the flow rates in the horizontal x-y plane, $[\bar{F}_{flo,x,y(\bar{H})}]_{i,j,L}$, represents a part of the right-hand side of the discrete-element equation for water-surface elevation [Eqs. (3.9) and (3.10)] in the element (i,j,L), in the water-surface layer l=L, as:

$$[\bar{F}_{flo,x,y(\bar{H})}]_{i,j,L} = \frac{1}{A_{z;i,j}}[\tilde{G}_{z;i,j,L-\frac{1}{2}} + \tilde{G}_{x;i-\frac{1}{2},j,L} - \tilde{G}_{x;i+\frac{1}{2},j,L} + \tilde{G}_{y;i,j-\frac{1}{2},L} - \tilde{G}_{y;i,j+\frac{1}{2},L}] \cdot \qquad (5.5)$$

The part of the right-hand-side solution function for water surface elevation, associated with the flow conditions at the power plants, $[\bar{F}_{plt(\bar{H})}]_{i,j,L}$, represents a part of the right-hand side of the discrete-element equation for water-surface elevation [Eqs. (3.9) and (3.10)] in the element (i,j,l), in the water-surface layer l = L, as:

$$[\bar{F}_{plt(\bar{H})}]_{i,j,l} = \frac{\delta_{int,m;i,j,L}}{A_{z;i,j}}\tilde{G}_{int,m;i,j,L} + \frac{\delta_{dis,m;i,j,L}}{A_{z;i,j}}\bar{G}_{dis,m;i,j,L} \cdot \qquad (5.6)$$

The part of the right-hand-side solution function for water surface elevation, associated with the rainfall conditions on the water-surface, $[\bar{F}_{srf(\bar{H})}]_{i,j,L}$, represents a part of the right-hand side of the discrete-element equation for water surface elevation [Eqs. (3.9) and (3.10)] in the element (i,j,l), in the water-surface layer l=L, as:

$$[\bar{F}_{srf(\bar{H})}]_{i,j,l} = \bar{g}_{rnf} \cdot \qquad (5.7)$$

Similarly, the set of discrete-element equations associated with the x-component of the momentum principle [Sec. 3.3., Eqs. (3.51), (3.53), (3.54) and (3.55)] can be represented as a subset of first-order, ordinary differential equations, as:

$$\frac{\partial}{\partial t}\bar{V}_{x;i-\frac{1}{4},j} = [\ \bar{F}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l}$$

$$= [\ \bar{F}_{con}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l} + [\ \bar{F}_{for}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l}$$

$$+ [\ \bar{F}_{trb}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l} + [\ \bar{F}_{bot}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l}$$

$$+ [\ \bar{F}_{srf}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l} + [\ \bar{F}_{plt}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l}\ , \qquad (5.8)$$

where the right-hand-side solution function for x-component of velocity $[\ \bar{F}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l}$ consists of six parts.

The part of the right-hand-side solution function for x-component of velocity, associated with the convective momentum transport, along the x-direction, $[\ \bar{F}_{con}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for the x-component of velocity [Eqs. (3.51),(3.53),(3.54) and (3.55)], in the x-upstream half element $(i-\frac{1}{4},j,l)$, as:

$$[\ \bar{F}_{con}(\bar{V}_x)\ ]_{i-\frac{1}{4},j,l} = \frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}\ [\tilde{G}_{x;i-\frac{1}{2},j,l}(\tilde{V}_{x;i-\frac{1}{2},j,l} - \bar{V}_{x;i-\frac{1}{4},j,l})$$

$$- \tilde{G}_{x;i,j,l}(\tilde{V}_{x;i,j,l} - \bar{V}_{x;i-\frac{1}{4},j,l})$$

$$+ \tilde{G}_{y;i-\frac{1}{4},j-\frac{1}{2},l}(\tilde{V}_{x;i-\frac{1}{4},j-\frac{1}{2},l} - \bar{V}_{x;i-\frac{1}{4},j,l})$$

(continue)

$$-\widetilde{G}_{y;i-\frac{1}{4},j+\frac{1}{2},l}(\widetilde{V}_{x;i-\frac{1}{4},j+\frac{1}{2},l}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$+\widetilde{G}_{z;i-\frac{1}{4},j,l-\frac{1}{2}}(\widetilde{V}_{x;i-\frac{1}{4},j,l-\frac{1}{2}}-\overline{V}_{x;i-\frac{1}{4},j,l})$$

$$-\widetilde{G}_{z;i-\frac{1}{4},j,l+\frac{1}{2}}(\widetilde{V}_{x;i-\frac{1}{4},j,l+\frac{1}{2}}-\overline{V}_{x;i-\frac{1}{4},j,l})]\,. \quad (5.9)$$

The part of the right-hand-side solution function for x-component of velocity, associated with the net effect of the forces acting along the x-direction, $[\,\overline{F}_{for}(\overline{V}_x)\,]_{i-\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for the x-component of velocity [Eqs.(3.51), (3.53),(3.54) and (3.55)], in the x-upstream half element (i-¼,j,l), as:

$$[\,\overline{F}_{for}(\overline{V}_x)\,]_{i-\frac{1}{4},j,l}=\frac{\overline{A}_{x;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}(\overline{p}_{w;i-\frac{1}{2},j,l}-\overline{p}_{w;i,j,l})$$

$$-g_o\cos\theta_{n,x}+\Omega_{cor,N,y}\overline{V}_{z;i-\frac{1}{4},j,l}-\Omega_{cor,N,z}\overline{V}_{y;i-\frac{1}{4},j,l}\,. \quad (5.10)$$

The part of the right-hand-side solution function for x-component of velocity, associated with the net effect of the combined stress components, acting along the x-direction, $[\,\overline{F}_{trb}(\overline{V}_x)\,]_{i-\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for the x-component of velocity [Eqs. (3.51), (3.53), (3.54) and (3.55)], in the x-upstream half element (i-¼,j,l), as:

$$[\,\overline{F}_{trb}(\overline{V}_x)\,]_{i-\frac{1}{4},j,l}=\frac{1}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}$$

$$[-\overline{A}_{x;i-\frac{1}{2},j,l}\overline{\sigma}^{*'}_{\nu,x,x;i-\frac{1}{2},j,l}+\overline{A}_{x;i,j,l}\overline{\sigma}^{*'}_{\nu,x,x;i,j,l}\,.$$

(continue)

$$-\bar{A}_{y;i-\frac{1}{4},j-\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$$

$$+\bar{A}_{y;i-\frac{1}{4},j+\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,x;i-\frac{1}{4},j-\frac{1}{2},l}$$

$$-A_{z;i-\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l-\frac{1}{2}}$$

$$+A_{z;i-\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i-\frac{1}{4},j,l+\frac{1}{2}}\Big] . \qquad (5.11)$$

The part of the right-hand-side solution function for x-component of velocity, associated with the momentum transport along the x-direction, at the bottom z-upstream enclosure surface, $[\bar{F}_{bot(\bar{V}_x)}]_{i-\frac{1}{4},j,l}$, represents a part of right-hand side of the discrete-element equa- tion for the x-component of velocity [Eqs (3.53) and (3.55)], in the x-upstream half element [i-¼,j,l(bot;i,j)], in the bottom layer l = l(bot;i,j), as:

$$[\bar{F}_{bot(\bar{V}_x)}]_{i-\frac{1}{4},j,l} = \frac{\delta_{l,l(bot;i,j)}}{[A_{z;i-\frac{1}{4},j}\overline{\Delta z}_{i-\frac{1}{4},j,l}]}\Big[\tilde{G}_{z,bot;i-\frac{1}{4},j}(\tilde{V}_{x,bot;i-\frac{1}{4},j}-\bar{V}_{x;i-\frac{1}{4},j,l})$$

$$-A_{z;i-\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,x,bot;i-\frac{1}{4},j}\Big] , \qquad (5.12)$$

where $\delta_{l,l(bot;i,j)}$ represents the Kronecker delta for indicating that the particular term is considered only in the bottom layers l = l(bot;i,j) of the flow region.

The part of the right-hand-side solution function for x-component of velocity, associated with the momentum transport by rainfall and the surface wind force, along the x-direction, at the water-surface, z-downstream enclosure surface, $[\bar{F}_{srf(\bar{V}_x)}]_{i-\frac{1}{4},j,l}$, represents a part of

the right-hand side of the discrete-element equation for x-component of velocity Eqs. (3.54) and (3.55), in the x-upstream half element $(i-\frac{1}{4},j,L)$, in the water-surface layer $l = L$, as:

$$[\,\bar{F}_{srf}(\bar{V}_x)\,]_{i-\frac{1}{4},j,l} = \frac{\delta_{l,L}}{\Delta z_{i-\frac{1}{4},j,L}}[\bar{g}_{rnf}(\bar{V}_{x,rnf}-\bar{V}_{x;i-\frac{1}{4},j,L})+\bar{\sigma}^{*'}_{\nu,x,wnd;i-\frac{1}{4},j}], \quad (5.13)$$

where $\delta_{l,L}$ represents the Kronecker delta for indicating that the particular term is considered only in the water-surface layer $l = L$ of the flow region.

The part of the right-hand-side solution function for x-component of velocity, associated with the momentum transport controlled by the power plant operational conditions, along the x-direction, $[\,\bar{F}_{plt}(\bar{V}_x)\,]_{i-\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for x-component of velocity [Eqs. (3.51), (3.53), (3.54) and (3.55)] in the x-upstream half element $(i-\frac{1}{4},j,l)$, as:

$$[\,\bar{F}_{plt}(\bar{V}_x)\,]_{i-\frac{1}{4},j,l} = \frac{\delta_{int,m;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\Delta z_{i-\frac{1}{4},j,l}]}\tilde{G}_{int,m;i-\frac{1}{4},j,l}(\tilde{V}_{x,int,m;i-\frac{1}{4},j,l}-\bar{V}_{x;i-\frac{1}{4},j,l})$$

$$+\frac{\delta_{dis,m;i-\frac{1}{4},j,l}}{[A_{z;i-\frac{1}{4},j}\Delta z_{i-\frac{1}{4},j,l}]}\bar{G}_{dis,m;i-\frac{1}{4},j,l}(\bar{V}_{x,dis,m;i-\frac{1}{4},j,l}-\bar{V}_{x;i-\frac{1}{4},j,l}). \quad (5.14)$$

Similarly, the set of discrete-element equations associated with the x-component of the momentum principle [Sec. 3.3. Eqs. (3.66), (3.68), (3.69) and (3.70)] can be represented as a subset of first-order, ordinary differential equations, as:

$$\frac{\partial}{\partial t}\bar{V}_{x;i+\frac{1}{4},j,l} = [\,\bar{F}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l}$$

$$[\,\bar{F}_{con}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l} + [\,\bar{F}_{for}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l}$$

$$+ [\,\bar{F}_{trb}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l} + [\,\bar{F}_{bot}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l}$$

$$+ [\,\bar{F}_{srf}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l} + [\,\bar{F}_{plt}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l}, \tag{5.15}$$

where the right-hand-side solution function for x-component of velocity $[\,\bar{F}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l}$ consists of six parts.

The part of the right-hand-side solution function for x-component of velocity, associated with the convective momentum transport, along the x-direction, $[\,\bar{F}_{con}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l}$, represents a part of right-hand side of the discrete-element equation for x-component of velocity [Eqs. (3.66),(3.68),(3.69) and (3.70)], in the x-downstream half element $(i+\frac{1}{4},j,l)$, as:

$$[\,\bar{F}_{con}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l} = \frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}$$

$$\cdot[\tilde{G}_{x;i,j,l}(\bar{V}_{x;i,j,l} - \bar{V}_{x;i+\frac{1}{4},j,l})$$

$$-\tilde{G}_{x;i+\frac{1}{2},j,l}(\bar{V}_{x;i+\frac{1}{2},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l})$$

$$+\tilde{G}_{y;i+\frac{1}{4},j-\frac{1}{2},l}(\bar{V}_{x;i+\frac{1}{4},j-\frac{1}{2},l} - \bar{V}_{x;i+\frac{1}{4},j,l})$$

$$-\tilde{G}_{y;i+\frac{1}{4},j+\frac{1}{2},l}(\bar{V}_{x;i+\frac{1}{4},j+\frac{1}{2},l} - \bar{V}_{x;i+\frac{1}{4},j,l})$$

(continue)

$$+\tilde{G}_{z;i+\frac{1}{4},j,l-\frac{1}{2}}(\tilde{V}_{x;i+\frac{1}{4},j,l-\frac{1}{2}}-\bar{V}_{x;i+\frac{1}{4},j,l})$$

$$-\tilde{G}_{z;i+\frac{1}{4},j,l+\frac{1}{2}}(\tilde{V}_{x;i+\frac{1}{4},j,l+\frac{1}{2}}-\bar{V}_{x;i+\frac{1}{4},j,l})]. \tag{5.16}$$

The part of the right-hand-side solution function for x-component of velocity, associated with the net effect of the forces acting along the x-direction, $[\bar{F}_{for}(\bar{V}_x)]_{i+\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for the x-component of velocity [Eqs.(3.66), (3.68),(3.69) and (3.70)], in the x-downstream half element $(i+\frac{1}{4},j,l)$, as:

$$[\bar{F}_{for}(\bar{V}_x)]_{i+\frac{1}{4},j,l}=\frac{\bar{A}_{x;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}(\bar{p}_{w;i,j,l}-\bar{p}_{w;i-\frac{1}{2},j,l})$$

$$-g_o\cos\theta_{n,x}+\Omega_{cor,N,y}\bar{V}_{z;i+\frac{1}{4},j,l}-\Omega_{cor,N,z}\bar{V}_{y;i+\frac{1}{4},j,l}. \tag{5.17}$$

The part of the right-hand-side solution function for x-component of velocity, associated with the net effect of the combined stress components, acting along the x-direction, $[\bar{F}_{trb}(\bar{V}_x)]_{i+\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for the x-component of velocity [Eqs. (3.66), (3.68), (3.69) and (3.70)], in the x-downstream half element $(i+\frac{1}{4},j,l)$, as:

$$[\bar{F}_{trb}(\bar{V}_x)]_{i+\frac{1}{4},j,l}=\frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}$$

$$\cdot[-\bar{A}_{x;i,j,l}\bar{\sigma}^{*'}_{\nu,x,x;i,j,l}+\bar{A}_{x;i+\frac{1}{2},j,l}\bar{\sigma}^{*'}_{\nu,x,x;i+\frac{1}{2},j,l}$$

(continue)

$$-\bar{A}_{y;i+\frac{1}{4},j-\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$$

$$+\bar{A}_{y;i+\frac{1}{4},j+\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,x;i+\frac{1}{4},j-\frac{1}{2},l}$$

$$-A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l-\frac{1}{2}}$$

$$+A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,z,x;i+\frac{1}{4},j,l+\frac{1}{2}}\Big]. \qquad (5.18)$$

The part of the right-hand-side solution function for x-component of velocity, associated with the momentum transport along the x-direction, at the bottom, z-upstream enclosure surface, $[\bar{F}_{bot}(\bar{V}_x)]_{i+\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for x-component of velocity [Eqs (3.68) and (3.70)], in the x-downstream half element $[i+\frac{1}{4},j,l(bot;i,j)]$, in the bottom layer $l = l(bot;i,j)$, as:

$$[\bar{F}_{bot}(\bar{V}_x)]_{i+\frac{1}{4},j,l} = \frac{1}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}\Big[\bar{G}_{z,bot;i+\frac{1}{4},j}(\bar{V}_{x,bot;i+\frac{1}{4},j}-\bar{V}_{x;i+\frac{1}{4},j,l})$$

$$-A_{z;i+\frac{1}{4},j}\bar{\sigma}^{*'}_{\nu,x,bot;i+\frac{1}{4},j}\Big]. \qquad (5.19)$$

The part of the right-hand-side solution function for x-component of velocity, associated with the momentum transport by rainfall and the surface wind force, along the x-direction, at the water-surface, z-downstream enclosure surface, $[\bar{F}_{srf}(\bar{V}_x)]_{i+\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for x-component of velocity [Eqs. (3.69) and (3.70)], in the x-downstream half element $(i+\frac{1}{4},j,L)$, in the water-surface layer $l = L$, as:

$$[\,\bar{F}_{srf}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l} = \frac{1}{\overline{\Delta z}_{i+\frac{1}{4},j,L}}[\bar{g}_{rnf}(\bar{V}_{x,rnf} - \bar{V}_{x;i+\frac{1}{4},j,L}) + \bar{\sigma}^{*'}_{\nu,x,wnd;i+\frac{1}{4},j}]. \quad (5.20)$$

The part of the right-hand-size solution function for x-component of velocity, associated with the momentum transport controlled by the power plant operational conditions, along the x-direction, $[\,\bar{F}_{plt}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l}$, represents a part of the right-hand side of the discrete-element equation for x-component of velocity [Eqs. (3.66), (3.68), (3.69) and (3.70)], in the x-downstream half element $(i+\frac{1}{4},j,1)$, as:

$$[\,\bar{F}_{plt}(\bar{V}_x)\,]_{i+\frac{1}{4},j,l} = \frac{\delta_{int,m;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}\tilde{G}_{int,m;i+\frac{1}{4},j,l}(\tilde{V}_{x,int,m;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l})$$

$$+\frac{\delta_{dis,m;i+\frac{1}{4},j,l}}{[A_{z;i+\frac{1}{4},j}\overline{\Delta z}_{i+\frac{1}{4},j,l}]}\bar{G}_{dis,m;i+\frac{1}{4},j,l}(\bar{V}_{x,dis,m;i+\frac{1}{4},j,l} - \bar{V}_{x;i+\frac{1}{4},j,l}). \quad (5.21)$$

Similarly, the set of discrete-element equations associated with the y-component of the momentum principle [Sec. 3.3. Eqs. (3.81), (3.83), (3.84) and (3.85)] can be represented as a subset of first-order, ordinary differential equations, as:

$$\frac{\partial}{\partial t}\bar{V}_{y;i,j-\frac{1}{4},l} = [\,\bar{F}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l}$$

$$= [\,\bar{F}_{con}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l} + [\,\bar{F}_{for}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l}$$

$$+ [\,\bar{F}_{trb}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l} + [\,\bar{F}_{bot}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l}$$

$$+ [\,\bar{F}_{srf}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l} + [\,\bar{F}_{plt}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l}, \quad (5.22)$$

where the right-hand-side solution function for y-component of velocity $[\,\overline{F}(\overline{V}_y)\,]_{i,j-\frac{1}{4},l}$ consists of six parts.

The part of the right-hand-side solution function for y-component of velocity, associated with the convective momentum transport, along the y-direction, $[\,\overline{F}_{con}(\overline{V}_y)\,]_{i,j-\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.81),(3.83),(3.84) and (3.85)], in the y-upstream half element $(i,j-\frac{1}{4},l)$, as:

$$
\begin{aligned}
[\,\overline{F}_{con}(\overline{V}_y)\,]_{i,j-\frac{1}{4},l} = & \frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]} \\
& [\widetilde{G}_{y;i,j-\frac{1}{2},l}(\widetilde{V}_{y;i,j-\frac{1}{2},l}-\overline{V}_{y;i,j-\frac{1}{4},l}) \\
& -\widetilde{G}_{y;i,j,l}(\widetilde{V}_{y;i,j,l}-\overline{V}_{y;i,j-\frac{1}{4},l}) \\
& +\widetilde{G}_{x;i-\frac{1}{2},j-\frac{1}{4},l}(\widetilde{V}_{y;i-\frac{1}{2},j-\frac{1}{4},l}-\overline{V}_{y;i,j-\frac{1}{4},l}) \\
& -\widetilde{G}_{x;i+\frac{1}{2},j-\frac{1}{4},l}(\widetilde{V}_{y;i+\frac{1}{2},j-\frac{1}{4},l}-\overline{V}_{y;i,j-\frac{1}{4},l}) \\
& +\widetilde{G}_{z;i,j-\frac{1}{4},l-\frac{1}{2}}(\widetilde{V}_{y;i,j-\frac{1}{4},l-\frac{1}{2}}-\overline{V}_{y;i,j-\frac{1}{4},l}) \\
& -\widetilde{G}_{z;i,j-\frac{1}{4},l+\frac{1}{2}}(\widetilde{V}_{y;i,j-\frac{1}{4},l+\frac{1}{2}}-\overline{V}_{y;i,j-\frac{1}{4},l})] \,. \qquad (5.23)
\end{aligned}
$$

The part of the right-hand-side solution function for y-component of velocity, associated with the net effect of the forces acting along the y-direction, $[\,\overline{F}_{for}(\overline{V}_y)\,]_{i,j-\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity

[Eqs.(3.81), (3.83),(3.84) and (3.85)], in the y-upstream half element $(i,j-\frac{1}{4},l)$, as:

$$[\,\bar{F}_{for}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l} = \frac{\bar{A}_{y;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\,(\bar{p}_{w;i,j-\frac{1}{2},l}-\bar{p}_{w;i,j,l})$$

$$-g_a\cos\theta_{n,y}+\Omega_{cor,N,z}\bar{V}_{x;i,j-\frac{1}{4},l}-\Omega_{cor,N,x}\bar{V}_{z;i,j-\frac{1}{4},l}\,. \tag{5.24}$$

The part of the right-hand-side solution function for y-component of velocity, associated with the net effect of the combined stress components, acting along the y-direction, $[\,\bar{F}_{trb}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.81), (3.83), (3.84) and (3.85)], in the y-upstream half element $(i,j-\frac{1}{4},l)$, as:

$$[\,\bar{F}_{trb}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l} = \frac{1}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}$$

$$[-\bar{A}_{y;i,j-\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu},_{y,y;i,j-\frac{1}{2},l}+\bar{A}_{y;i,j,l}\bar{\sigma}^{*'}_{\nu},_{y,y;i,j,l}$$

$$-\bar{A}_{x;i-\frac{1}{2},j-\frac{1}{4},l}\bar{\sigma}^{*'}_{\nu},_{x,y;i-\frac{1}{2},j-\frac{1}{4},l}$$

$$+\bar{A}_{x;i+\frac{1}{2},j-\frac{1}{4},l}\bar{\sigma}^{*'}_{\nu},_{x,y;i-\frac{1}{2},j-\frac{1}{4},l}$$

$$-A_{z;i,j-\frac{1}{4}}\bar{\sigma}^{*'}_{\nu},_{z,y;i,j-\frac{1}{4},l-\frac{1}{2}}$$

$$+A_{z;i,j-\frac{1}{4}}\bar{\sigma}^{*'}_{\nu},_{z,y;i,j-\frac{1}{4},l+\frac{1}{2}}]\,. \tag{5.25}$$

The part of the right-hand-side solution function for y-component of velocity, associated with the momentum transport along the y-direction, at the bottom, z-upstream enclosure surface, $[\bar{F}_{bot}(\bar{V}_y)]_{i,j-\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs (3.83) and (3.85)], in the y-upstream half element $[i,j-\frac{1}{4},l(bot;i,j)]$, in the bottom layer $l = l(bot;i,j)$, as:

$$[\bar{F}_{bot}(\bar{V}_y)]_{i,j-\frac{1}{4},l} = \frac{\delta_{l,l(bot;i,j)}}{[A_{z;i,j-\frac{1}{4}}\Delta z_{i,j-\frac{1}{4},l}]}[\bar{G}_{z,bot;i,j-\frac{1}{4}}(\bar{V}_{y,bot;i,j-\frac{1}{4}} - \bar{V}_{y;i,j-\frac{1}{4},l}) - A_{z;i,j-\frac{1}{4}}\bar{\sigma}'^{*}_{\nu,y,bot;i,j-\frac{1}{4}}]. \quad (5.26)$$

The part of the right-hand-side solution function for y-component of velocity, associated with the momentum transport by rainfall and the surface wind force, along the y-direction, at the water-surface, z-downstream enclosure surface, $[\bar{F}_{srf}(\bar{V}_y)]_{i,j-\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.84) and (3.85)], in the y-upstream half element $(i,j-\frac{1}{4},l)$, in the water-surface layer $l = L$, as:

$$[\bar{F}_{srf}(\bar{V}_y)]_{i,j-\frac{1}{4},l} = \frac{\delta_{l,L}}{\Delta z_{i,j-\frac{1}{4},L}}[\bar{g}_{rnf}(\bar{V}_{y,rnf} - \bar{V}_{y;i,j-\frac{1}{4},L}) + \bar{\sigma}'^{*}_{\nu,y,wnd;i,j-\frac{1}{4}}]. \quad (5.27)$$

The part of the right-hand-size solution function for y-component of velocity, associated with the momentum transport controlled by the power plant operational conditions, along the y-direction, $[\bar{F}_{plt}(\bar{V}_y)]_{i,j-\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.81), (3.83), (3.84) and (3.85)], in the y-upstream half element $(i,j-\frac{1}{4},l)$, as:

$$[\,\bar{F}_{plt}(\bar{V}_y)\,]_{i,j-\frac{1}{4},l} = \frac{\delta_{int,m;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\,\bar{G}_{int,m;i,j-\frac{1}{4},l}(\bar{V}_{y,int,m;i,j-\frac{1}{4},l}-\bar{V}_{y;i,j-\frac{1}{4},l})$$

$$+\frac{\delta_{dis,m;i,j-\frac{1}{4},l}}{[A_{z;i,j-\frac{1}{4}}\overline{\Delta z}_{i,j-\frac{1}{4},l}]}\,\bar{G}_{dis,m;i,j-\frac{1}{4},l}(\bar{V}_{y,dis,m;i,j-\frac{1}{4},l}-\bar{V}_{y;i,j-\frac{1}{4},l}). \tag{5.28}$$

Similarly, the set of discrete-element equations associated with the y-component of the momentum principle [Sec. 3.3. Eqs. (3.96), (3.98), (3.99) and (3.100)] can be represented as a subset of first-order, ordinary differential equations, as:

$$\frac{\partial}{\partial t}\bar{V}_{y;i,j+\frac{1}{4},l} = [\,\bar{F}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l}$$

$$= [\,\bar{F}_{con}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l} + [\,\bar{F}_{for}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l}$$

$$+ [\,\bar{F}_{trb}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l} + [\,\bar{F}_{bot}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l}$$

$$+ [\,\bar{F}_{srf}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l} + [\,\bar{F}_{plt}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l}\ , \tag{5.29}$$

where the right-hand-side solution function for y-component of velocity $[\,\bar{F}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l}$ consists of six parts.

The part of the right-hand-side solution function for y-component of velocity, associated with the convective momentum transport, along the y-direction, $[\,\bar{F}_{con}(\bar{V}_y)\,]_{i,j+\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.96),(3.98), (3.99) and (3.100)], in the y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$[\bar{F}_{con}(\bar{V}_y)]_{i,j+\frac{1}{4},l} = \frac{1}{[A_{z;i,j+\frac{1}{4}}\Delta z_{i,j+\frac{1}{4},l}]}$$

$$[\tilde{G}_{y;i,j,l}(\tilde{V}_{y;i,j,l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$-\tilde{G}_{y;i,j+\frac{1}{2},l}(\tilde{V}_{y;i,j+\frac{1}{2},l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$+\tilde{G}_{x;i-\frac{1}{2},j+\frac{1}{4},l}(\tilde{V}_{y;i-\frac{1}{2},j+\frac{1}{4},l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$-\tilde{G}_{x;i+\frac{1}{2},j+\frac{1}{4},l}(\tilde{V}_{y;i+\frac{1}{2},j+\frac{1}{4},l}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$+\tilde{G}_{z;i,j+\frac{1}{4},l-\frac{1}{2}}(\tilde{V}_{y;i,j+\frac{1}{4},l-\frac{1}{2}}-\bar{V}_{y;i,j+\frac{1}{4},l})$$

$$-\tilde{G}_{z;i,j+\frac{1}{4},l+\frac{1}{2}}(\tilde{V}_{y;i,j+\frac{1}{4},l+\frac{1}{2}}-\bar{V}_{y;i,j+\frac{1}{4},l})]. \qquad (5.30)$$

The part of the right-hand-side solution function for y-component of velocity, associated with the net effect of the forces acting along the y-direction, $[\bar{F}_{for}(\bar{V}_y)]_{i,j+\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.96), (3.98),(3.99) and (3.100)], in the y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$[\bar{F}_{for}(\bar{V}_y)]_{i,j+\frac{1}{4},l} = \frac{\bar{A}_{y;i,j+\frac{1}{2},l}}{[A_{z;i,j+\frac{1}{4}}\Delta z_{i,j+\frac{1}{4},l}]}(\bar{P}_{w;i,j,l}-\bar{P}_{w;i,j+\frac{1}{2},l})$$

$$-g_o\cos\theta_{n,y}+\Omega_{cor,N,z}\bar{V}_{x;i,j+\frac{1}{4},l}-\Omega_{cor,N,x}\bar{V}_{z;i,j+\frac{1}{4},l}. \qquad (5.31)$$

The part of the right-hand-side solution function for y-component of velocity, associated with the net effect of the combined stress

components, acting along the y-direction, $[\bar{F}_{trb}(\bar{V}_y)]_{i,j+\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.96), (3.98), (3.99) and (3.100)], in the y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$[\bar{F}_{trb}(\bar{V}_y)]_{i,j+\frac{1}{4},l} = \frac{1}{[A_{z;i,j+\frac{1}{4}}\Delta z_{i,j+\frac{1}{4},l}]}$$

$$[-\bar{A}_{y;i,j,l}\bar{\sigma}^{*'}_{\nu,y,y;i,j,l} + \bar{A}_{y;i,j+\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,y;i,j+\frac{1}{2},l}$$

$$-\bar{A}_{x;i-\frac{1}{2},j+\frac{1}{4},l}\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l}$$

$$+\bar{A}_{x;i+\frac{1}{2},j+\frac{1}{4},l}\bar{\sigma}^{*'}_{\nu,x,y;i-\frac{1}{2},j+\frac{1}{4},l}$$

$$-A_{z;i,j+\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l-\frac{1}{2}}$$

$$+A_{z;i,j+\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,z,y;i,j+\frac{1}{4},l+\frac{1}{2}}] \; . \tag{5.32}$$

The part of the right-hand-side solution function for y-component of velocity, associated with the momentum transport along the y-direction, at the bottom, z-upstream enclosure surface, $[\bar{F}_{bot}(\bar{V}_y)]_{i,j+\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs (3.98) and (3.100)], in the y-downstream half element $[i,j+\frac{1}{4},l(bot;i,j)]$, in the bottom layer $l = l(bot;i,j)$, as:

$$[\bar{F}_{bot}(\bar{V}_y)]_{i,j+\frac{1}{4},l} = \frac{\delta_{l,l(bot;i,j)}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}[\bar{G}_{z,bot;i,j+\frac{1}{4}}(\bar{V}_{y,bot;i,j+\frac{1}{4}} - \bar{V}_{y;i,j+\frac{1}{4},l}) - A_{z;i,j+\frac{1}{4}}\bar{\sigma}^{*'}_{\nu,y,bot;i,j+\frac{1}{4}}] . \quad (5.33)$$

The part of the right-hand-side solution function for y-component of velocity, associated with the momentum transport by rainfall and the surface wind force, along the y-direction, at the water-surface, z-downstream enclosure surface, $[\bar{F}_{srf}(\bar{V}_y)]_{i,j+\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.99) and (3.100)], in the y-downstream half element $(i,j+\frac{1}{4},l)$, in the water-surface layer $l = L$, as:

$$[\bar{F}_{srf}(\bar{V}_y)]_{i,j+\frac{1}{4},l} = \frac{\delta_{l,L}}{\overline{\Delta z}_{i,j+\frac{1}{4},L}}[\bar{g}_{rnf}(\bar{V}_{rnf} - \bar{V}_{y;i,j+\frac{1}{4},L}) + \bar{\sigma}^{*'}_{\nu,y,wnd;i,j+\frac{1}{4}}]. \quad (5.34)$$

The part of the right-hand-size solution function for y-component of velocity, associated with the momentum transport controlled by the power plant operational conditions, along the y-direction, $[\bar{F}_{plt}(\bar{V}_y)]_{i,j+\frac{1}{4},l}$, represents a part of the right-hand side of the discrete-element equation for y-component of velocity [Eqs. (3.96), (3.98), (3.99) and (3.100)], in the y-downstream half element $(i,j+\frac{1}{4},l)$, as:

$$[\bar{F}_{plt}(\bar{V}_y)]_{i,j-\frac{1}{4},l} = \frac{\delta_{int,m;i,j+\frac{1}{4},l}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}\bar{G}_{int,m;i,j+\frac{1}{4},l}(\bar{V}_{y,int,m;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}) + \frac{\delta_{dis,m;i,j+\frac{1}{4},l}}{[A_{z;i,j+\frac{1}{4}}\overline{\Delta z}_{i,j+\frac{1}{4},l}]}\bar{G}_{dis,m;i,j+\frac{1}{4},l}(\bar{V}_{y,dis,m;i,j+\frac{1}{4},l} - \bar{V}_{y;i,j+\frac{1}{4},l}). \quad (5.35)$$

The discrete-element equation for pressure [Eqs. (3.110), (3.112), (3.113) and (3.114)] can be rearranged into an algebraic form of equation, as:

$$\bar{P}_{w;i,j,l-\frac{1}{2}} = \bar{P}_{w;i,j,l+\frac{1}{2}} + \overline{\Delta z}_{i,j,l} \frac{\partial}{\partial t} \bar{V}_{z;i,j,l} + [\, \bar{F}(\bar{V}_z) \,]_{i,j,l}$$

$$= \bar{P}_{w;i,j,l+\frac{1}{2}} + \overline{\Delta z}_{i,j,l} \frac{\partial}{\partial t} \bar{V}_{z;i,j,l}$$

$$+ [\, \bar{F}_{con}(\bar{V}_z) \,]_{i,j,l} + [\, \bar{F}_{for}(\bar{V}_z) \,]_{i,j,l}$$

$$+ [\, \bar{F}_{trb}(\bar{V}_z) \,]_{i,j,l} + [\, \bar{F}_{bot}(\bar{V}_z) \,]_{i,j,l}$$

$$+ [\, \bar{F}_{srf}(\bar{V}_z) \,]_{i,j,l} + [\, \bar{F}_{plt}(\bar{V}_z) \,]_{i,j,l} \,. \qquad (5.36)$$

The modified form of the right-hand-side solution function for the z-component of velocity $[\, \bar{F}(\bar{V}_z) \,]_{i,j,l}$, associated with the algebric form of the discrete-element equation for pressure [Eq. (5.36)], consists of six parts.

The part of the right-hand-side solution function for z-component of velocity in the algebraic form of the discrete-element equation for pressure [Eq. (5.36)], associated with the convective momentum transport, along the z-direction, $[\, \bar{F}_{con}(\bar{V}_z) \,]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for pressure [Eqs. (3.110), (3.112), (3.113) and (3.114)], as:

$$[\, \bar{F}_{con}(\bar{V}_z) \,]_{i,j,l} = -\frac{1}{A_{z;i,j}} [\tilde{G}_{x;i-\frac{1}{2},j,l} (\tilde{V}_{z;i-\frac{1}{2},j,l} - \bar{V}_{z;i,j,l})$$

$$- \tilde{G}_{x;i,j,l} (\tilde{V}_{z;i+\frac{1}{2},j,l} - \bar{V}_{z;i,j,l})$$

(continue)

$$+\widetilde{G}_{y;i,j-\frac{1}{2},l}(\widetilde{V}_{z;i,j-\frac{1}{2},l}-\overline{V}_{z;i,j,l})$$

$$-\widetilde{G}_{y;i,j+\frac{1}{2},l}(\widetilde{V}_{z;i,j+\frac{1}{2},l}-\overline{V}_{z;i,j,l})$$

$$+\widetilde{G}_{z;i,j,l-\frac{1}{2}}(\widetilde{V}_{z;i,j,l-\frac{1}{2}}-\overline{V}_{z;i,j,l})$$

$$-\widetilde{G}_{z;i,j,l+\frac{1}{2}}(\widetilde{V}_{z;i,j,l+\frac{1}{2}}-\overline{V}_{z;i,j,l})]. \tag{5.37}$$

The part of the right-hand-side solution function for z-component of velocity in the algebraic form of the discrete-element equation for pressure [Eq. (5.36)], associated with the net effect of the forces acting along the z-direction, $[\overline{F}_{for}(\overline{V}_z)]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for pressure [Eqs. (3.110), (3.112), (3.113) and (3.114)], as:

$$[\overline{F}_{for}(\overline{V}_z)]_{i,j,l} = g_0\cos\theta_{n,z}\overline{\Delta z}_{i,j,l} + \overline{\Delta z}_{i,j,l}[\Omega_{cor,N,y}\overline{V}_{x;i,j,l} - \Omega_{cor,N,x}\overline{V}_{y;i,j,l}]. \tag{5.38}$$

The part of the right-hand-side solution function for the z-component of velocity in the algebraic form of the discrete-element equation for pressure [Eq. (5.36)], associated with the net effect of the combined stress components, acting along the z-direction, $[\overline{F}_{trb}(\overline{V}_z)]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for pressure [Eqs. (3.110), (3.112), (3.113) and (3.114)], as:

$$[\bar{F}_{trb}(\bar{V}_z)]_{i,j,l} = \frac{1}{A_{z;i,j}}[+\bar{A}_{x;i-\frac{1}{2},j,l}\bar{\sigma}^{*'}_{\nu,x,z;i-\frac{1}{2},j,l} - \bar{A}_{x;i+\frac{1}{2},j,l}\bar{\sigma}^{*'}_{\nu,x,z;i+\frac{1}{2},j,l}$$

$$+\bar{A}_{y;i,j-\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,z;i,j-\frac{1}{2},l} - \bar{A}_{y;i,j+\frac{1}{2},l}\bar{\sigma}^{*'}_{\nu,y,z;i,j+\frac{1}{2},l}$$

$$+A_{z;i,j}\bar{\sigma}^{*'}_{\nu,z,z;i,j,l-\frac{1}{2}} - A_{z;i,j}\bar{\sigma}^{*'}_{\nu,z,z;i,j,l+\frac{1}{2}}]. \quad (5.39)$$

The part of the right-hand-side solution function for z-component of velocity in the algebraic form of the discrete-element equation for pressure [Eq. (5.36)], associated with the momentum transport along the z-direction, at the bottom, z-upstream enclosure surface, $[\bar{F}_{bot}(\bar{V}_z)]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for pressure [Eqs. (3.112) and (3.114), for the bottom layer $l = l(bot;i,j)$, as:

$$[\bar{F}_{bot}(\bar{V}_z)]_{i,j,l} = -\frac{\delta_{l,l(bot;i,j)}}{A_{z;i,j}}[\tilde{G}_{z,bot;i,j}(\bar{V}_{z,bot;i,j} - \bar{V}_{z;i,j,l})]. \quad (5.40)$$

The part of the right-hand-side solution function for z-component of velocity in the algebraic form of the discrete-element equation for pressure [Eq. (5.36), associated with the momentum transport by rainfall, along the z-direction, at the water-surface z-downstream enclosure surface, $[\bar{F}_{srf}(\bar{V}_z)]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for pressure [Eqs. (3.113) and (3.114)], for the water-surface layer $l = L$, as:

$$[\bar{F}_{srf}(\bar{V}_z)]_{i,j,l} = -\bar{g}_{rnf}(\bar{V}_{z;rnf} - \bar{V}_{z;i,j,L}). \quad (5.41)$$

The part of the right-hand-side solution function for z-component of velocity in the algebraic form of the discrete-element equation for

pressure [Eq. (5.36)], associated with the momentum transport controlled by the power plant operational conditions, along the z-direction, $[\bar{F}_{plt}(\bar{V}_z)]_{i,j,l}$, represents a part of the right-hand side of the discrete-element equation for pressure [Eqs. (3.110), (3.112), (3.113) and (3.114)], as:

$$[\bar{F}_{plt}(\bar{V}_z)]_{i,j,l} = -\frac{\delta_{int,m;i,j,l}}{A_{z;i,j}} \tilde{C}_{int,m;i,j,l}(\tilde{V}_{z,int,m;i,j,l} - \bar{V}_{z;i,j,l}) - \frac{\delta_{dis,m;i,j,l}}{A_{z;i,j}} \bar{C}_{dis,m;i,j,l}(\bar{V}_{z,dis,m;i,j,l} - \bar{V}_{z;i,j,l}). \quad (5.42)$$

According to the general discretization technique of the discrete-element method (DEM), a flow region which is divided by a rectangular grid system with Nx length increments along the x-direction, Ny length increments along the y-direction, and L length increments along the z-direction, consists of a maximum possible number of elements (NxNyL).

The principal flow variables of solution in the discretized flow region consists of

(1) Maximum possible number of (NxNy) water surface elevations $\bar{H}_{i,j}$ in the elements (i,j,L),

(2) maximum possible number of (2NxNyL) x-components of velocity $\bar{V}_{x;i\pm\frac{1}{4},j,l}$ in the x-upstream half elements (i-¼,j,1) and in the x-downstream half elements (i+¼,j,1),

(3) maximum possible number of (2NxNyL) y-components of velocity $\bar{V}_{y;i,l\pm\frac{1}{4},l}$ in the y-upstream half elements (i,j-¼,1) and in the y-downstream half elements (i,j+¼,1),

which represent collectively a total maximum possible number of [(1+4L)NxNy] unknowns.

The governing computational system of discrete-element equations include the subset of ordinary first-order differential equations consisting of:

(1) maximum possible number of (NxNy) discrete-element equations for water-surface elevation $\overline{H}_{i,j}$ in the elements (i,j,L) [Eq. (5.2)],

(2) maximum possible number of (2NxNy) discrete-element equations for x-component of velocity $\overline{V}_{x;i\pm\frac{1}{4},j,l}$ in the x-upstream half elements ($i-\frac{1}{4}$,j,l) and in the x-downstream half elements ($i+\frac{1}{4}$,j,l) [Eqs. (5.8) and (5.15)],

(3) maximum possible number of (2NxNy) discrete-element equations for y-component of velocity $\overline{V}_{y;i,j\pm\frac{1}{4},l}$ in the y-upstream half elements ($i,j-\frac{1}{4}$,l) and in the y-downstream half elements ($i,j+\frac{1}{4}$,l) [Eqs. (5.22) and (5.29)],

which also represent collectively a total maximum possible number of [(1+4L)NxNy] equations.

Hence, the subset of a total maximum possible number of [(1+4L)NxNy] first-order, ordinary differential equations [Eqs. (5.2), (5.8), (5.15), (5.22) and (5.29)] for the total maximum possible number of [(1+4L)NxNy] unknown principal flow variables constitutes a mathematically consistent system.

The governing computational system of discrete-element equations also includes:

(1) a subset of a maximum possible number of (NxNyL) algebraic equations [Eq. (5.1)] for the maximum possible number of (NxNyL) additional, nonprincipal flow variables, flow rates along z-direction $\overline{G}_{z;i,j,l+\frac{1}{2}}$,

(2) a subset of a maximum possible number of (NxNyL) algebraic equations [Eqs. (5.36)] for the maximum possible number of (NxNyL) additional, nonprincipal flow variables, modified pressures $\bar{P}_{w;i,j,l-\frac{1}{2}}$.

The additional, nonprincipal flow variables, the flow rates along z-direction $\bar{G}_{z;i,j,l+\frac{1}{2}}$ and the modified pressures $\bar{P}_{w;i,j,l+\frac{1}{2}}$, in the mathematical sense, represent the two sets of parameters of the subset of a maximum possible number of [(1+4L)NxNy] first-order, ordinary differential equations [Eqs. (5.2), (5.8), (5.15), (5.22) and (5.29)] for the solution of the maximum possible number of [(1+4L)NxNy] principal flow variables, (1) water-surface elevations $\bar{H}_{i,j}$, (2) x-components of velocity $\bar{V}_{x;i\pm\frac{1}{4},j,l}$, and (3) y-components of velocity $\bar{V}_{y;i,j\pm\frac{1}{4},l}$.

Therefore the governing computational system of discrete element equations requires a set of initial conditions for the maximum possible number of [(1+4L)NxNy] principal flow variables at any arbitrarily specified time to.

The subset of maximum possible number of (NxNy) initial conditions for the water-surface elevations in the elements (i,j,L) must be specified, as:

$$\bar{H}_{i,j}(t_o) = (H_{i,j})_o \ . \tag{5.43a}$$

The subset of maximum possible number of (NxNyL) initial conditions for the x-components of velocity in the x-upstream half elements ($i-\frac{1}{4}$,j,l) must be specified, as:

$$\bar{V}_{x;i-\frac{1}{4},j,l}(t_o) = (V_{x;i-\frac{1}{4},j,l})_o \ . \tag{5.43b}$$

The subset of maximum possible number of (NxNyL) initial conditions for the x-components of velocity in the x-downstream half elements $(i+\frac{1}{4},j,l)$ must be specified, as:

$$\overline{V}_{x;i+\frac{1}{4},j,l}(t_o) = (V_{x;i+\frac{1}{4},j,l})_o \ . \qquad (5.43c)$$

The subset of maximum possible number of (NxNyL) initial conditions for the y-components of velocity in the y-upstream half elements $(i,j-\frac{1}{4},l)$ must be specified, as:

$$\overline{V}_{y;i,j-\frac{1}{4},l}(t_o) = (V_{y;i,j-\frac{1}{4},l})_o \ . \qquad (5.43d)$$

The subset of maximum possible number of (NxNyL) initial conditions for the y-components of velocity in the y-downstream half elements $(i,j+\frac{1}{4},l)$ must be specified, as:

$$\overline{V}_{y;i,j+\frac{1}{4},l}(t_o) = (V_{y;i,j+\frac{1}{4},l})_o \ . \qquad (5.43e)$$

Hence, the governing computational system of discrete-element equations [Eqs. (5.1), (5.2), (5.8), (5.15), (5.22), (5.29) and (5.36)] can be used to obtain the solutions for the maximum possiblenumber of [(1+4L)NxNy] principal flow variables by numerically integrating the subset of the maximum possible number of [(1+4L)NxNy] first-order, ordinary differential equations [Eqs. (5.2), (5.8), (5.15), (5.22) and (5.29)], as extension in time of the set of the maximun possible number of [(1+4L)NxNy] initial conditions [Eqs. (5.43a) through 5.43e)].

5.2. Computational Algorithm of the Numerical solution of the Governing Computational System of Discrete-Element Equations

The fast-transient, three-dimensional, discrete-element hydrodynamic model includes a special composite time-splitting algorithm for the numerical integration of the subset of the maximum possible number of [(1+4L)NxNy] first-order, ordinary differential equations. This new computational algorithm was specifically developed for obtaining stable and convergent numerical solutions with minimum numerical dispersion for fast-transient simulations of hydrodynamic, thermal and salinity conditions in geophysical flow regions (Eraslan, 1981a)

The numerical integration of the governing computational system of discrete-element equations obtains the solutions of the principal flow variables at the new time level t_{n+1}, based on the already known solutions of the principal flow variables at the old time level t_n, by dividing the time step of integration t into two equal parts at the intermediate time level $t_{n+\frac{1}{2}}$, defined as:

$$t_{n+\frac{1}{2}} = t_n + \frac{\Delta t}{2} \quad . \tag{5.44}$$

The intermediate values of the flow variables at the intermediate time level $t_{n+\frac{1}{2}}$ are determined based on seven operational steps as:

$$\tilde{G}^{n}_{z;i,j,l+\frac{1}{2}} = \tilde{G}^{n}_{z;i,j,l-\frac{1}{2}} + \left[\bar{F}_{flo,x,y}(\tilde{G}) \right]^{n}_{i,j,l} + \left[\bar{F}_{plt}(\tilde{G}) \right]^{n}_{i,j,l} , \tag{5.45a}$$

$$\bar{H}^{n+\frac{1}{2}}_{i,j} = \bar{H}^{n}_{i,j} + \frac{\Delta t}{2} \left[\bar{F}(\bar{H}) \right]^{n}_{i,j,L} , \tag{5.45b}$$

$$\bar{P}^{-n,n+\frac{1}{2}}_{w;i,j,l-\frac{1}{2}} = \bar{P}^{-n,n+\frac{1}{2}}_{w;i,j,l+\frac{1}{2}} + \Delta z^{n+\frac{1}{2}}_{i,j,l} \left(\frac{\partial}{\partial t} \bar{V}_z \right)^{n-1,n}_{i,j,l} + \left[\bar{F}(\bar{V}_z) \right]^{n}_{i,j,l} . \tag{5.45c}$$

$$\bar{V}^{n+\frac{1}{2}}_{x;i-\frac{1}{4},j,l} = \bar{V}^{n}_{x;i-\frac{1}{4},j,l} + \frac{\Delta t}{2}\left[\,\bar{F}(\bar{V}_x)\,\right]^{n,n+\frac{1}{2}}_{i-\frac{1}{4},j,l}, \tag{5.45d}$$

$$\bar{V}^{n+\frac{1}{2}}_{x;i+\frac{1}{4},j,l} = \bar{V}^{n}_{x;i+\frac{1}{4},j,l} + \frac{\Delta t}{2}\left[\,\bar{F}(\bar{V}_x)\,\right]^{n,n+\frac{1}{2}}_{i+\frac{1}{4},j,l}, \tag{5.45e}$$

$$\bar{V}^{n+\frac{1}{2}}_{y;i,j-\frac{1}{4},l} = \bar{V}^{n}_{y;i,j-\frac{1}{4},l} + \frac{\Delta t}{2}\left[\,\bar{F}(\bar{V}_y)\,\right]^{n,n+\frac{1}{2}}_{i,j-\frac{1}{4},l}, \tag{5.45f}$$

$$\bar{V}^{n+\frac{1}{2}}_{y;i,j+\frac{1}{4},l} = \bar{V}^{n}_{y;i,j+\frac{1}{4},l} + \frac{\Delta t}{2}\left[\,\bar{F}(\bar{V}_y)\,\right]^{n,n+\frac{1}{2}}_{i,j+\frac{1}{4},l}. \tag{5.45g}$$

The superscripts $n-1,n$,$n+\frac{1}{2}$ indicate the time levels t_{n+1}, tn and $tn+\frac{1}{2}$, respectively, used in obtaining the intermediate values of the flow variables at the intermediate time level $t_{n+\frac{1}{2}}$. The superscripts $n-1$, n indicate that the value of the particular term was obtained by using the values of the flow variables at two time levels t_{n-1} and t_n. In general, all the terms used in the calculations of the intermediate values of the modified pressure $\bar{P}^{n,n+\frac{1}{2}}_{w;i,j,l-\frac{1}{2}}$ Eq. (5.45c), the x-component of velocity $\bar{V}^{n+\frac{1}{2}}_{x;i\pm\frac{1}{4},j,l}$ [Eqs. (5.45d) and (5.45e)] and the y-component of velocity $\bar{V}^{n+\frac{1}{2}}_{y;i,j\pm\frac{1}{4},l}$ [Eqs. (5.45f) and (5.45g)] use the already computed intermediate values of the water surface elevation $\bar{H}^{n+\frac{1}{2}}_{i,j}$. Therefore, the double superscript notation $n,n+\frac{1}{2}$ indicates the two different time levels tn and tn+1 associated with the solutions of the flow variables.

The solutions of the flow variables at the final time level t_n are obtained by considering two separate parts of the computational algorithm.

The first part of the computational algorithm directly computes the flow rate along z-direction $\tilde{G}^{n+\frac{1}{2}}_{z;i,j,l+\frac{1}{2}}$, the water-surface elevation $\bar{H}^{n+1}_{i,j}$ and the modified pressure $\bar{P}^{n+\frac{1}{2},n+1}_{w;i,j,l-\frac{1}{2}}$ as:

$$\tilde{G}^{n+\frac{1}{2}}_{z;i,j,l+\frac{1}{2}} = \tilde{G}^{n+\frac{1}{2}}_{z;i,j,l-\frac{1}{2}} + [\ \bar{F}_{flo,x,y}(\tilde{G})\ ]^{n+\frac{1}{2}}_{i,j,l} + [\ \bar{F}_{plt}(\tilde{G})\ ]^{n+\frac{1}{2}}_{i,j,l}\ , \qquad (5.46a)$$

$$\bar{H}^{n+1}_{i,j} = \bar{H}^{n}_{i,j} + \Delta t\ [\ \bar{F}(\bar{H})\ ]^{n+\frac{1}{2}}_{i,j,L}\ , \qquad (5.46b)$$

$$\bar{P}^{n+\frac{1}{2},n+1}_{w;i,j,l-\frac{1}{2}} = \bar{P}^{n+\frac{1}{2},n+1}_{w;i,j,l+\frac{1}{2}} + \Delta z^{n+1}_{i,j,l}\left(\frac{\partial \bar{V}_z}{\partial t}\right)^{n,n+\frac{1}{2}}_{i,j,l} + [\ \bar{F}(\bar{V}_z)\ ]^{n+\frac{1}{2}}_{i,j,l}\ . \qquad (5.46c)$$

The second part of the computational algorithm can consider two different options in evaluating the values of the parts of the right-hand-side solution functions for advancing the solutions of the components of the velocity to the final time level t_n.

The first option is based on evaluating the values of all the parts of the right-hand-side solution functions by utilizing the already computed final values of the water surface elevations $\bar{H}^{n+1}_{i,j}$ at the final time level t_n and the intermediate values of all other flow variables at the intermediate time level $t_{n+\frac{1}{2}}$, as:

$$\begin{aligned}[\ \bar{F}(\bar{V}_x)\ ]^{n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l} &= [\ \bar{F}_{con}(\bar{V}_x)\ ]^{n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l} + [\ \bar{F}_{for}(\bar{V}_x)\ ]^{n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l}\ , \\ &+ [\ \bar{F}_{trb}(\bar{V}_x)\ ]^{n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l} + [\ \bar{F}_{bot}(\bar{V}_x)\ ]^{n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l}\ , \\ &+ [\ \bar{F}_{srf}(\bar{V}_x)\ ]^{n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l} + [\ \bar{F}_{plt}(\bar{V}_x)\ ]^{n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l}\ , \qquad (5.47a)\end{aligned}$$

$$\begin{aligned}[\ \bar{F}(\bar{V}_y)\ ]^{n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l} &= [\ \bar{F}_{con}(\bar{V}_y)\ ]^{n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l} + [\ \bar{F}_{for}(\bar{V}_y)\ ]^{n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l}\ , \\ &+ [\ \bar{F}_{trb}(\bar{V}_y)\ ]^{n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l} + [\ \bar{F}_{bot}(\bar{V}_y)\ ]^{n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l}\ , \\ &+ [\ \bar{F}_{srf}(\bar{V}_y)\ ]^{n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l} + [\ \bar{F}_{plt}(\bar{V}_y)\ ]^{n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l}\ . \qquad (5.47b)\end{aligned}$$

The second option is based on the evaluation of the values of the different parts of the right-hand-side solution functions by utilizing different values of the flow variables at different time levels according to the composite time-splitting technique. This option evaluates the values of the parts of the right-hand-side solution functions, associated with the convective momentum transport, $[\,\bar{F}_{con}(\bar{V}_x)\,]^{n,n+1}_{i\pm\frac{1}{4},j,l}$ and $[\,\bar{F}_{con}(\bar{V}_y)\,]^{n,n+1}_{i,j\pm\frac{1}{4},l}$ by utilizing the final values of the water-surface elevation $\bar{H}^{n}_{i,j}$ at the final time level tn and the initial values of the components of velocity $\bar{V}^{n}_{x;i-\frac{1}{4},j,l}$ and $\bar{V}^{n}_{y;i,j-\frac{1}{4},l}$ at the initial time level t_n, as:

$$[\,\bar{F}(\bar{V}_x)\,]^{n,n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l} = [\,\bar{F}_{con}(\bar{V}_x)\,]^{n,n+1}_{i\pm\frac{1}{4},j,l} + [\,\bar{F}_{for}(\bar{V}_x)\,]^{n,n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l}$$

$$+ [\,\bar{F}_{trb}(\bar{V}_x)\,]^{n,n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l} + [\,\bar{F}_{bot}(\bar{V}_x)\,]^{n,n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l}$$

$$+ [\,\bar{F}_{srf}(\bar{V}_x)\,]^{n,n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l} + [\,\bar{F}_{plt}(\bar{V}_x)\,]^{n,n+\frac{1}{2},n+1}_{i\pm\frac{1}{4},j,l}\,, \qquad (5.48a)$$

$$[\,\bar{F}(\bar{V}_y)\,]^{n,n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l} = [\,\bar{F}_{con}(\bar{V}_y)\,]^{n,n+1}_{i,j\pm\frac{1}{4},l} + [\,\bar{F}_{for}(\bar{V}_y)\,]^{n,n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l}$$

$$+ [\,\bar{F}_{trb}(\bar{V}_y)\,]^{n,n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l} + [\,\bar{F}_{bot}(\bar{V}_y)\,]^{n,n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l}$$

$$+ [\,\bar{F}_{srf}(\bar{V}_y)\,]^{n,n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l} + [\,\bar{F}_{plt}(\bar{V}_y)\,]^{n,n+\frac{1}{2},n+1}_{i,j\pm\frac{1}{4},l}\,. \qquad (5.48b)$$

The particular advantage of the composite time-splitting algorithm [Eqs. (5.48a) and (5.48b)] is that it minimizes the effects of the artificial viscosity (a part of the general numerical dispersion

effect), associated with the convective transport terms and in conjuction with the transportive-upwind-differencing interpolation (Sec. 4.3.3), without introducing any instability associated with the turbulent transport terms which represent of the net effects of the combined stress components.

Hence, according to the first option, the final values of the components of velocity at the final time level tn are evaluated, as:

$$\bar{V}^{n+1}_{x;i-\frac{1}{4},j,l} = \bar{V}^{n}_{x;i-\frac{1}{4},j,l} + \Delta t \left[\bar{F}(\bar{V}_x) \right]^{n+\frac{1}{2},n+1}_{i-\frac{1}{4},j,l}, \tag{5.49a}$$

$$\bar{V}^{n+1}_{x;i+\frac{1}{4},j,l} = \bar{V}^{n}_{x;i+\frac{1}{4},j,l} + \Delta t \left[\bar{F}(\bar{V}_x) \right]^{n+\frac{1}{2},n+1}_{i+\frac{1}{4},j,l}, \tag{5.49b}$$

$$\bar{V}^{n+1}_{y;i,j-\frac{1}{4},l} = \bar{V}^{n}_{y;i,j-\frac{1}{4},l} + \Delta t \left[\bar{F}(\bar{V}_y) \right]^{n+\frac{1}{2},n+1}_{i,j-\frac{1}{4},l}, \tag{5.49c}$$

$$\bar{V}^{n+1}_{y;i,j+\frac{1}{4},l} = \bar{V}^{n}_{y;i,j+\frac{1}{4},l} + \Delta t \left[\bar{F}(\bar{V}_y) \right]^{n+\frac{1}{2},n+1}_{i,j+\frac{1}{4},l}. \tag{5.49d}$$

Similarly, according to the second option, the final values of the components of velocity at the final time level t_n are evaluated, as:

$$\bar{V}^{n+1}_{x;i-\frac{1}{4},j,l} = \bar{V}^{n}_{x;i-\frac{1}{4},j,l} + \Delta t \left[\bar{F}(\bar{V}_x) \right]^{n,n+\frac{1}{2},n+1}_{i-\frac{1}{4},j,l}, \tag{5.50a}$$

$$\bar{V}^{n+1}_{x;i+\frac{1}{4},j,l} = \bar{V}^{n}_{x;i+\frac{1}{4},j,l} + \Delta t \left[\bar{F}(\bar{V}_x) \right]^{n,n+\frac{1}{2},n+1}_{i+\frac{1}{4},j,l}, \tag{5.50b}$$

$$\bar{V}^{n+1}_{y;i,j-\frac{1}{4},l} = \bar{V}^{n}_{y;i,j-\frac{1}{4},l} + \Delta t \left[\bar{F}(\bar{V}_y) \right]^{n,n+\frac{1}{2},n+1}_{i,j-\frac{1}{4},l}, \tag{5.50c}$$

$$\bar{V}^{n+1}_{y;i,j+\frac{1}{4},l} = \bar{V}^{n}_{y;i,j+\frac{1}{4},l} + \Delta t \left[\bar{F}(\bar{V}_y) \right]^{n,n+\frac{1}{2},n+1}_{i,j+\frac{1}{4},l}. \tag{5.50d}$$

The composite time-splitting computational algorithm represents a new numerical integration method which can minimize the computational errors caused by the temporal and spatial discretization techniques used in the development of fast-transient, three-dimensional, discrete-element transport models for simulating flow and water quality conditions in geophysical flow regions.

5.3. Stability Criteria of the Numerical Solution Method for the Governing Computational system of Discrete-Element Equations

The fast-transient, three-dimensional, discrete-element hydrodynamic model utilizes a timewise-explicit numerical integration algorithm for the solution of the governing computational system of discrete-element equations [Eqs. (5.1), (5.2), (5.8), (5.15), (5.22), (5.29) and (5.36)]. Hence, the stability of the numerical solutions requires that the selected time step of numerical integration must satisfy the Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972, Eraslan, 1981b) for free-surface flow conditions.

The local depth in the flow region can be obtained as the sum of the layers along the z-direction, i.e.,

$$\overline{D}_{i,j} = \sum_{l=l(bot;i,j)}^{L} \overline{\Delta z}_{i,j,l} \quad . \tag{5.51}$$

The local value of the surface gravity wave is defined as:

$$\overline{c}_{sgw;i,j} = \sqrt{g_a \overline{D}_{i,j}} \quad . \tag{5.52}$$

which, in a sense, represents the principal speed of propogation of disturbance in the flow region.

Hence, an effective critical value for the maximum speed of propagation of disturbance in the entire flow region can be defined as:

$$c_{max;i,j,l} = max(c_{sgw;i,j}) + max\left[|\bar{V}_{x;i\pm\frac{1}{4},j,l}| + |\bar{V}_{y;i,j\pm\frac{1}{4},l}|\right] . \tag{5.53}$$

The stability of the numerical solutions, according to the composite time-splitting computational algorithm, can be guaranteed by selecting a time step such that two stability criteria could be satisfied simultaneously during the numerical integration of the solutions.

First, the selected time step Δt must satisfy the generalized Courant, Friedrichs and Lewy (CFL) criterion (e.g., Courant, Friedrichs and Lewy, 1928, 1967, Roache, 1972) for free-surface flow conditions as a sufficient condition, i.e.,

$$\Delta t < \frac{\min(\Delta x_{i,j} \text{ or } \Delta y_{i,j})}{\bar{c}_{max;i,j,l}} . \tag{5.54}$$

Furthermore, the selected time step Δt must also satisfy the Courant criterion (e.g., Roache, 1972, Eraslan, 1974), based on the minimum vertical size of the elements along the z-direction and the maximum z-component of velocity in the flow region, again as a sufficient condition, i.e.,

$$\Delta t < \frac{\min(\Delta z_{i,j,l})}{\max(|\bar{V}_{z;i,j,l}|)} . \tag{5.55}$$

According to the composite time-splitting computational algorithm, if the selected time step of the numerical integration Δt satisfies both of stability criteria Eqs. (5.54) and (5.55) as sufficient conditions, the results of the numerical integration of the governing computational system of discrete-element equations represent the necessary convergent solutions which can be utilized as the results of the fast-transient, three-dimensional, discrete-element hydrodynamic model for simulating the flow conditions in geophysical flow regions, including lakes, rivers, reservoirs, estuaries and coastal zones.

APPLICATION OF THE DISCRETE-ELEMENT METHOD TO FAST-TRANSIENT THREE-DIMENSIONAL INCOMPRESSIBLE FLOW WITH FREE-SURFACE CONDITIONS IN REGIONS OF ARBITRARY GEOMETRY

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6. APPLICATIONS OF THE DISCRETE-ELEMENT MODEL

The discrete-element hydrodynamic model is specifically designed for application to both prototype-scale and laboratory-model-scale flow problems. One-dimensional and two-dimensional discrete-element transport models were applied successfully to estuaries and coastal regions (Eraslan, et al., 1976, Eraslan, Erhan, Lin, 1980). The results of these studies indicated very good agreement between the computer simulations and the available field-measured data for the prototype-scale flow conditions. The main objective of the application part of this study is to illustrate the applicability of the three-dimensional discrete-element hydrodynamic model to the simulation of flow conditions in the laboratory-model-scale, without necessitating any modification in the formulation of the transport model that was used in the simulations of the flow conditions in prototype-scale problems.

Two different types of laboratory-model-scale applications were considered. The first type of applications were used to check the reliability of the mathematical formulation of the discrete-element hydrodynamic model and the numerical accuracy of the associated HYDROL computer code in simulating the flow condition in a longitudinally and laterally symmetric flow region with specified open-region boundary conditions. The second type of application was used for simulating the flow conditions with specified-flow boundary conditions in an actual laboratory-model-scale experiment (Browns Ferry physical model) which was constructed for studying the flow conditions in the Wheeler Reservoir on the Tennessee River.

Detailed discussions of these applications are presented in the subsequent Sections based on the computer simulation results for transient water-surface-elevation and three-dimensional velocity conditions in the laboratory-model-scale flow region.

6.1 Applications to Test Cases

A special laboratory-model-scale flow region with geometrically symmetric enclosure boundaries was selected for all the test cases (Fig. 6.1, p.550) used in checking the mathematical formulations of the discrete-element hydrodynamic model and the accuracy of the associated HYDROL computer code. The computer code was applied to simulate the flow conditions in the laboratory-model-scale flow region for the following five test case studies:

1. Symmetric Unidirectional Flow (inflow and outflow along longitudinal direction),
2. Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction),
3. Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction),
4. Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method),
5. Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind differencing interpolation method).

6.1.1 Discussion of the Geometry of the Laboratory-Model-Scale Flow Region for the Test Cases

The dimensions of the laboratory-model-scale flow region were specified as 44 m along the longitudinal direction x and 24 m along the lateral direction y (Fig. 6.1, p.550). The geometry of the flow region was assumed to be symmetric: (a) about an axis along longitudinal direction x at y = 12 m, (b) about an axis along lateral direction y at x = 22 m.

The discrete-element system was constructed based on a rectangular grid with 14 elements along x direction and 6 elements along y direction. Hence, the laboratory-model-scale flow region was divided into 34 internal elements and 34 boundary elements (Fig. 6.1, p.550). The general mathematical formulation of the discrete-element and the computational algorithm of HYDROL computer code automatically discarded the external elements without requiring any storage, according to the preprocessing discrete-element-generation program PREPR2 (Holdeman, 1977). Hence, the flow region consisted of a total number of ~~68~~ 64 computational elements. The deep main flow channel of the flow region consisted of two parts perpendicular to each other. The first part was specified as a subregion with a width of 4 m, along the longitudinal direction, between y = 10 m and y = 14 m, and with a uniform depth of 0.4 m. The second part was specified as a subregion with a width of 8 m, along the lateral direction between x = 18 m and x = 26 m, and with a uniform depth of 0.4 m. The elements between x =14 m and x = 18 m and between x = 26 m and x = 30 m, outside of the deep main flow channel, were speci-

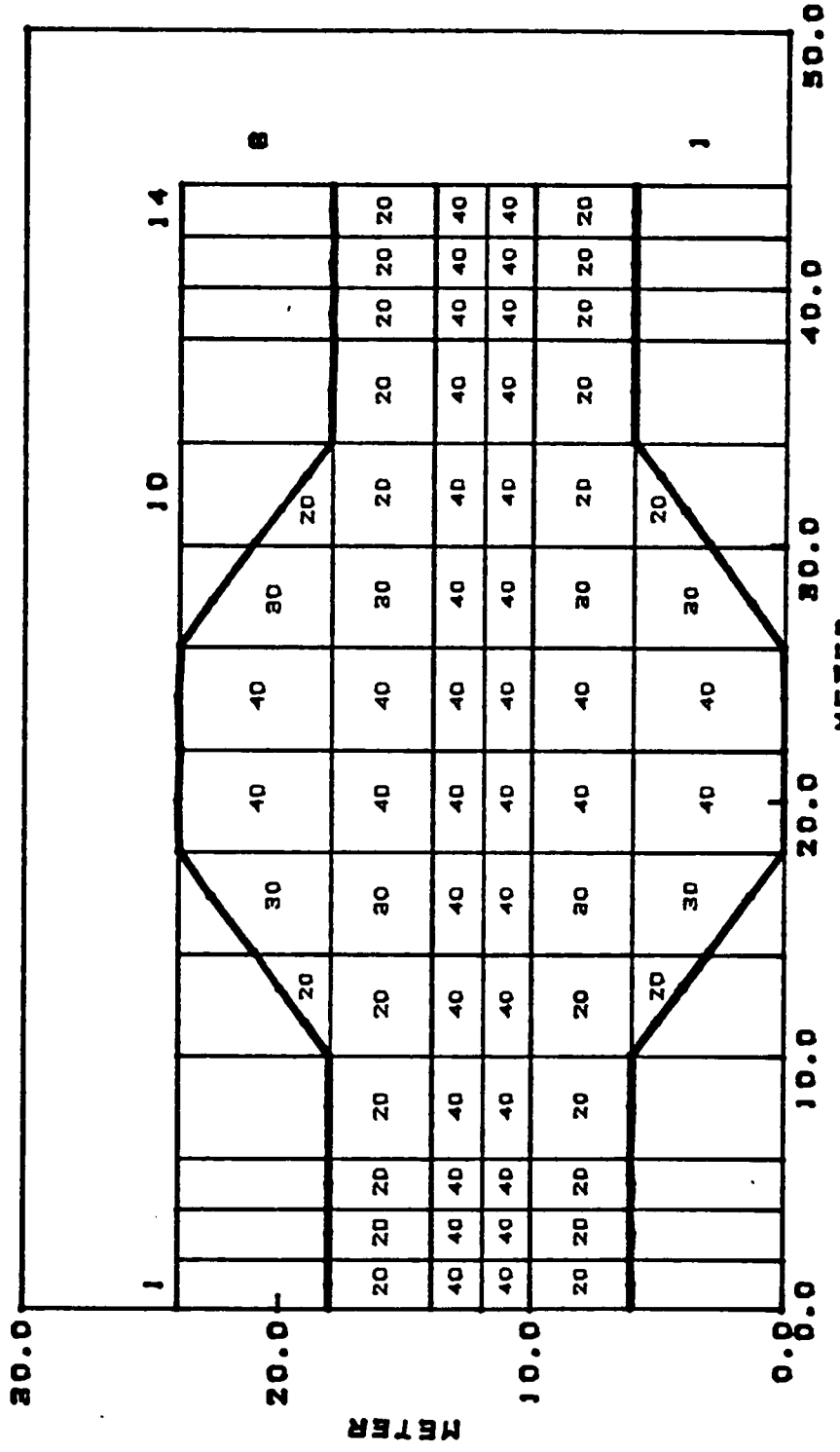

Figure 6.1. Discrete-element geometry of the flow region used for test cases 1 through 5. Outside boundary dimensions are 44 m by 24 m. Numbers represent the depth (in cm) in the elements.

fied to have a depth of 0.3 m. The rest of the elements were specified to have a depth of 0.2 m.

6.1.2 Computer Simulation Results for Five Test Cases

The discrete-element hydrodynamic model and its associated HYDROL computer code was applied to simulate the flow conditions in the symmetric, laboratory-model-scale flow region for five test cases. The results of the computer simulations were prepared in computer graphics format, at different time levels of computer simulations as (1) constant elevation difference (relative to mean water level) contours (with 2mm gradation between -10 mm minimum and 10 mm maximum), and (2) velocity conditions, represented as two-dimensional vectors in the horizontal (x,y) planes for four specified layers. The four layers were specified according to the convention that the first layer is at the bottom of the deep main flow channel (l = 1) with a depth of 0.1 m. The two internal layers (l = 2 and l = 3) were assumed to have the same depth of 0.1 m, and the surface layer (l = 4) was assumed to have a depth of 0.1 m at undisturbed mean water level (MWL) conditions in the flow region.

The computer simulation results for the five test cases were determined based on specified upstream (inflow) and/or downstream (outflow) conditions at four enclosure surfaces of the laboratory-model-scale flow region (Fig. 6.1, p.550) as:

1. west-end enclosure surface at x = 0 and between y = 6 m and y = 18 m (in elements i=1; j=2,3,4,5),
2. east-end enclosure surface at x = 44 m and between y = 6 m and y = 18 m (in elements i=14; j=2,3,4,5),

3. south-end enclosure surface at y = 0 and between x = 18 m and x = 24 m (in elements i=7,8; j=1),

4. north-end enclosure surface at y = 24 m and between x = 18 m and x = 24 m (in elements i=7,8; j= 6).

The flow conditions for the five test cases were based on the specification of different combinations of open-region boundary conditions at there four enclosure surfaces. It is important to note that the two enclosure surfaces normal to the longitudinal x axis, west-end enclosure surface at x = 0 and east-end enclosure surface at x = 44 m, have equal widths as 12 m along the lateral y axis, whereas the two enclosure surfaces normal to the lateral y axis, south-end enclosure surface at y = 0 and north-end enclosure surface at y = 24 m, have equal widths as 8 m along the longitudinal x axis (Fig. 6.1, p.550). Hence, the same specified water-surface-elevation conditions, associated with the same specified open-region boundary conditions, at different enclosure surfaces could result in different values for increased or decreased flow areas at the four different enclosure surfaces (Fig. 6.1, p.550).

6.1.2.1 Test case 1: Symmetric Unidirectional Flow (inflow and outflow along longitudinal direction). The first test case was considered for the simulation of the symmetric unidirectional flow conditions between the west-end upstream enclosure surface and the east-end downstream enclosure surface, along the x axis of the laboratory-model-scale flow region. The necessary input data to the HYDROL computer code was prepared according to the following conditions associated with the mathematical formulation of the discrete element hydrodynamic model:

a. Boundary Conditions

The test case was run by considering open-region boundary conditions (1) at the west-end upstream enclosure surface (at x = 0) and (2) at the east-end downstream enclosure surface (at x = 44 m) of the flow region. All the enclosure surfaces along y-axis were assumed to be closed, impermeable shoreline boundary surfaces.

West-end upstream condition for water-surface-elevation at x = 0 (relative to MWL):

$\overline{H}_{(opr,i-1),j}$ = 0.01 m (above MWL), along lateral open-region enclosure surface, upstream of the four boundary elements i=1; j=2,3,4,5; l=4.

West-end upstream control conditions on velocity components at x = 0:

longitudinal velocity component

$\widetilde{V}_{x,min;(opr,i-1),j,l}$ = -0.5 m/sec, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\widetilde{V}_{x,max;(opr,i-1),j,l}$ = 0.5 m/sec, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

lateral velocity component

$\widetilde{V}_{y,min;(opr,i-1),j-\frac{1}{4},l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\widetilde{V}_{y,max;(opr,i-1),j-\frac{1}{4},l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

vertical velocity component

$\widetilde{V}_{z,min;(opr,i-1),j,l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\tilde{V}_{z,max;(opr,i+1),j,l} = 0$, for all elements i=1; j=2,5; l=3,4;
and i=1; j=3,4; l=1,2,3,4.

East-end downstream condition for the surface elevation at x = 44 m (relative to MWL):

$\bar{H}_{(opr,i+1),j} = -0.01$ m (below MWL), along lateral open-region enclosure surface, downstream of the four boundary elements i=14; j=2,3,4,5; l=4.

East-end downstream control conditions on velocity components at x = 44 m:

longitudinal velocity component

$\tilde{V}_{x,min;(opr,i+1),j,l} = -0.5$ m/sec, for all elements i=14; j=2,5; l=3,4, and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{x,max;(opr,i+1),j,l} = 0.5$ m/sec, for all elements i=14; j=2,5; l=3,4, and i=14; j=3,4; l=1,2,3,4.

lateral velocity component

$\tilde{V}_{y,min;(opr,i+1),j-\frac{1}{4},l} = 0$, for all elements i=14; j=2,5; l=3,4, and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{y,max;(opr,i+1),j-\frac{1}{4},l} = 0$, for all elements i=14; j=2,5; l=3,4, and i=14; j=3,4; l=1,2,3,4.

vertical velocity component

$\tilde{V}_{z,min;(opr,i+1),j,l} = 0$, for all elements i=14; j=2,5; l=3,4, and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{z,max;(opr,i+1),j,l} = 0$, for all elements i=14; j=2,5; l=3,4, and i=14; j=3,4; l=1,2,3,4.

The computer simulation results for test case 1 were obtained by considering the open-region boundary conditions which required the spe-

cification of water-surface-elevation conditions on both boundaries at the west-end upstream enclosure surface of the longitudinal axis and at the east-end downstream enclosure surface of the longitudinal axis of the flow region. Therefore, according to the particular option for specifying open-region boundary conditions (Sec. 4.5), the velocity components outside of the west-end upstream open-region enclosure surface of the flow region were determined by linear extrapolation, based on the values of the velocity components in the two boundary half-elements along the x axis, in the element immediately adjacent to the west-end upstream open-region enclosure surface. The control conditions were used for eliminating possible convergence errors that may be caused by overestimating (or underestimating) the velocity conditions outside of the upstream open-region enclosure surface. This option for open-region boundary conditions were included in the discrete-element hydrodynamic model as a general capability for incorporating approximate control conditions at open-region enclosure surfaces where exact values for the velocity components could not be specified. In the case when the velocity components could be determined exactly at the west-end open-region enclosure surface, the specification of Vx,ups,min and Vx,ups,max as equal to the actual exact values of the velocity components could simulate automatically the required boundary conditions for the velocity components. The velocity components that were not normal to the west-end upstream open-region enclosure surface at x = 0 were specified as $\tilde{V}_{y,min} = \tilde{V}_{y,max} = 0$ and as $\tilde{V}_{z,min} = \tilde{V}_{z,max} = 0$.

The velocity components for the outflow at the east-end downstream open-region enclosure surface at x = 44 m were not used for extrapolat-

ing to the outside of the flow region, since general second-upwind-differencing formulation automatically determines the values of the velocity components based on the velocity components in the boundary half-elements along the x-axis, immediately adjacent to the east-end downstream open-region enclosure surface at x = 44 m.

b. Interpolation Option

The computer simulation results for test case 1 were obtained by applying the HYDROL computer code with simple second-upwind-differencing interpolation option. It was noted that this interpolation option could introduce numerical dispersion effects in the elements and half-elements where the local Courant number was much less than one (1) (see Sec. 4.3).

c. Numerical Solution Option

The computer simulation results for the test case 1 were obtained by applying the HYDROL computer code with the standard default option which ultilized the special, composite time-splitting method in the numerical integration of the governing system of spatially discretized ordinary differential equations (see Sec. 5.2).

d. Discussion of the Computer Simulation Results

The results of the computer simulations for the water-surface-elevation and velocity conditions are presented in Figs. 6.2 throuhg 6.21, p.557 through p.576. The surface elevation conditions are presented as (1) numbers in the elements which represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL) and (2) constant water-surface-elevation contours which correspond to

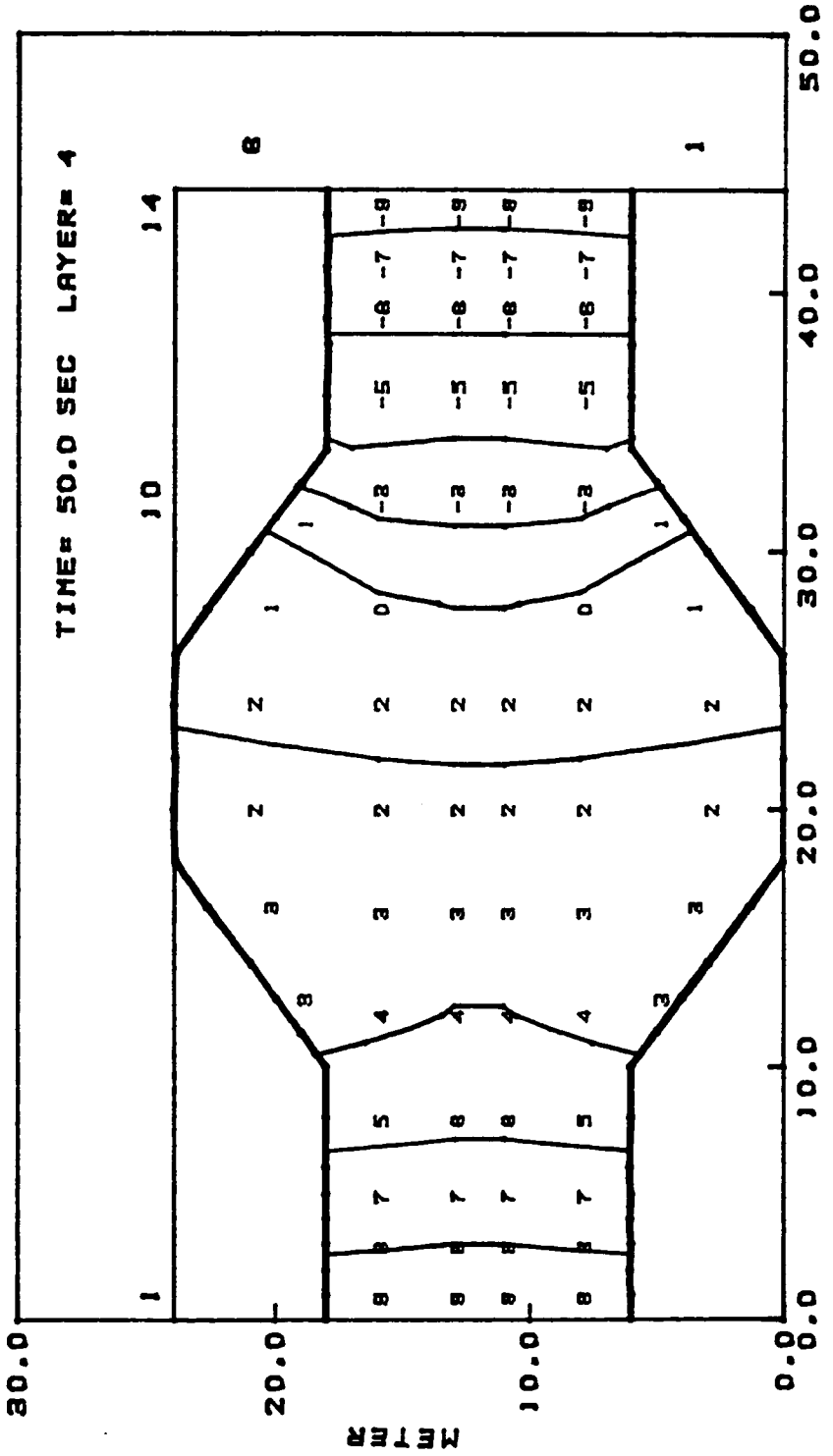

Figure 6.2. Water-surface-elevation conditions at time t = 50 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

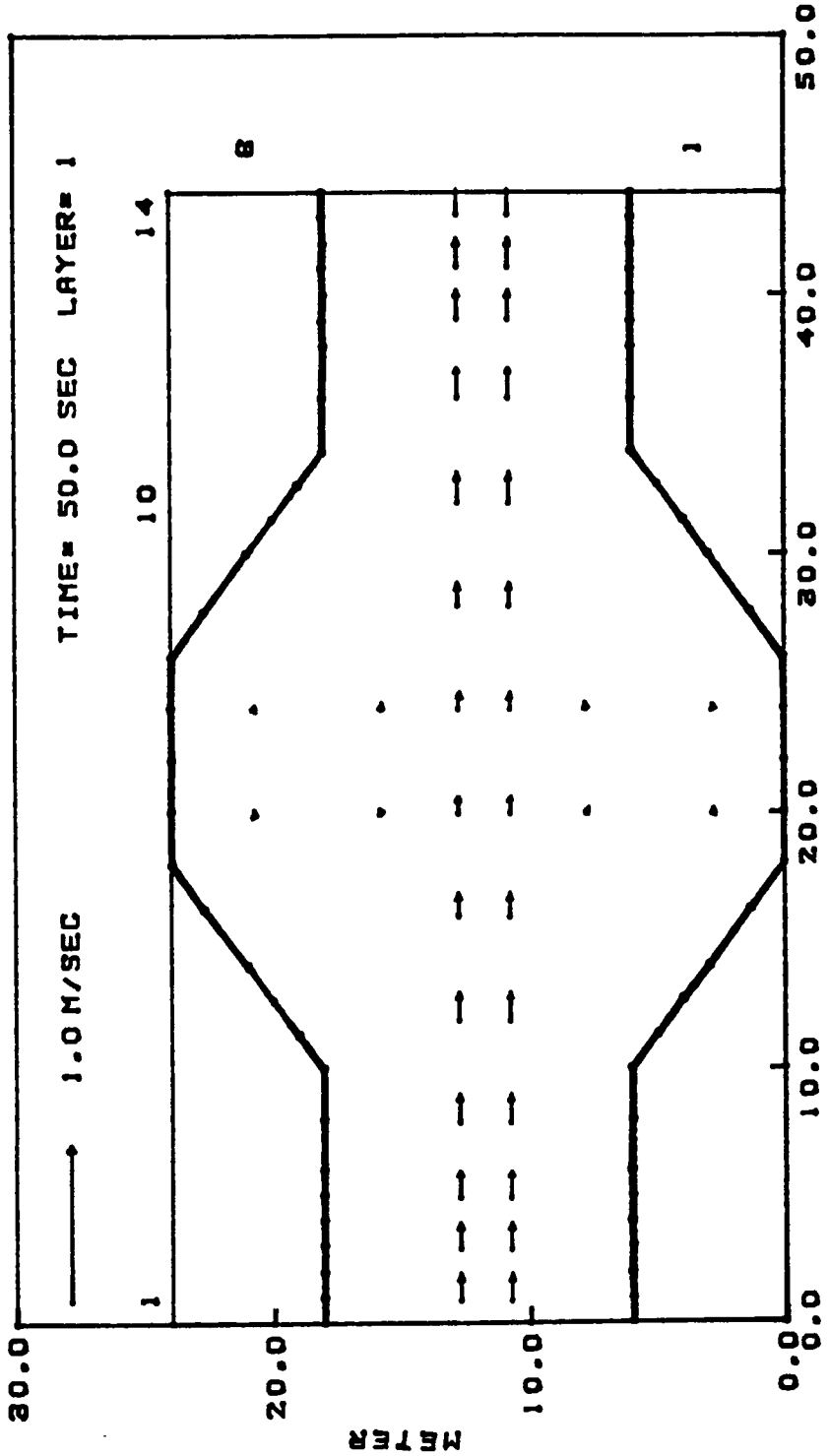

Figure 6.3. Velocity conditions in the bottom layer l = 1, at time t = 50 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

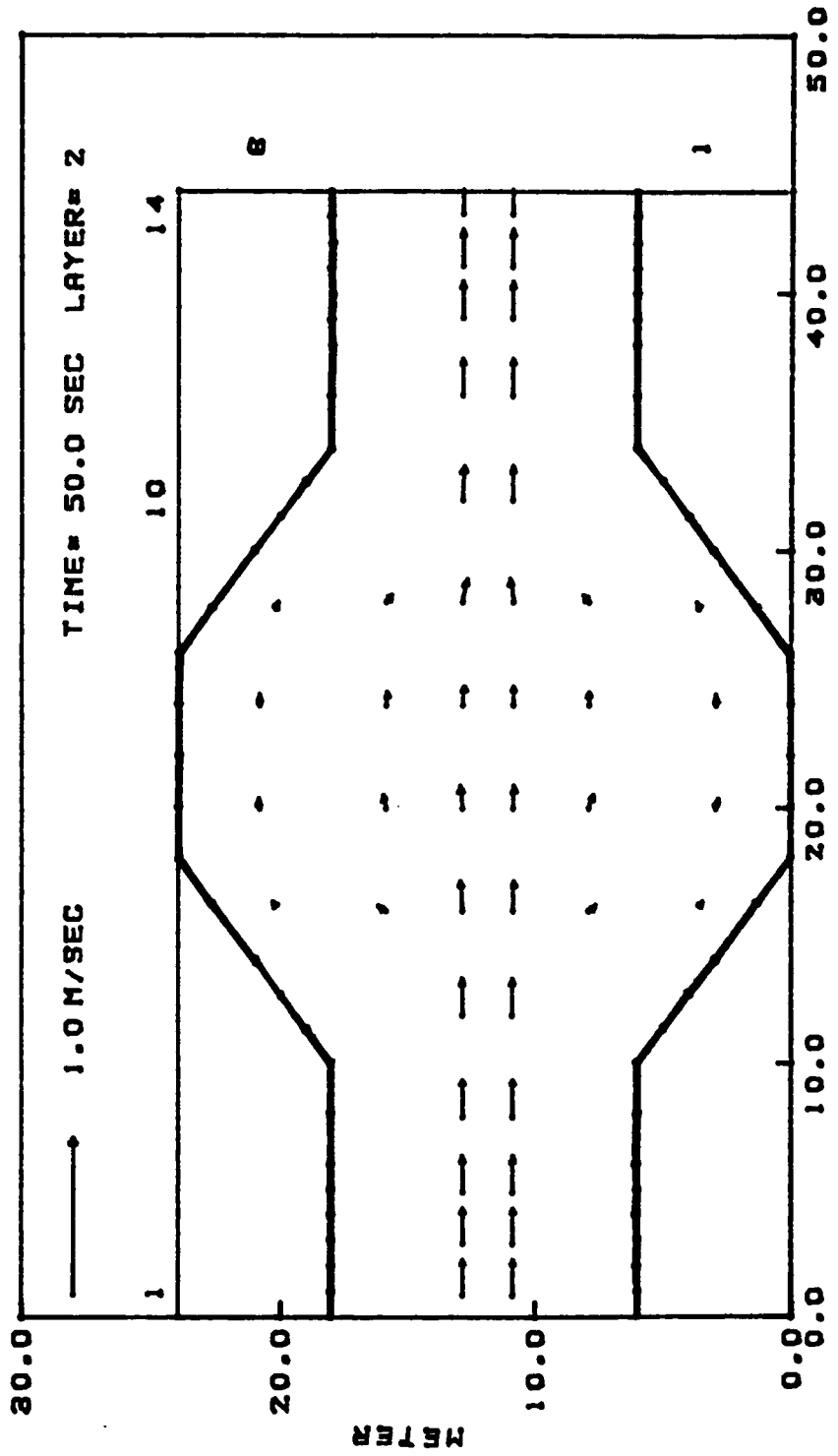

Figure 6.4. Velocity conditions in the internal layer l = 2, at time t = 50 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

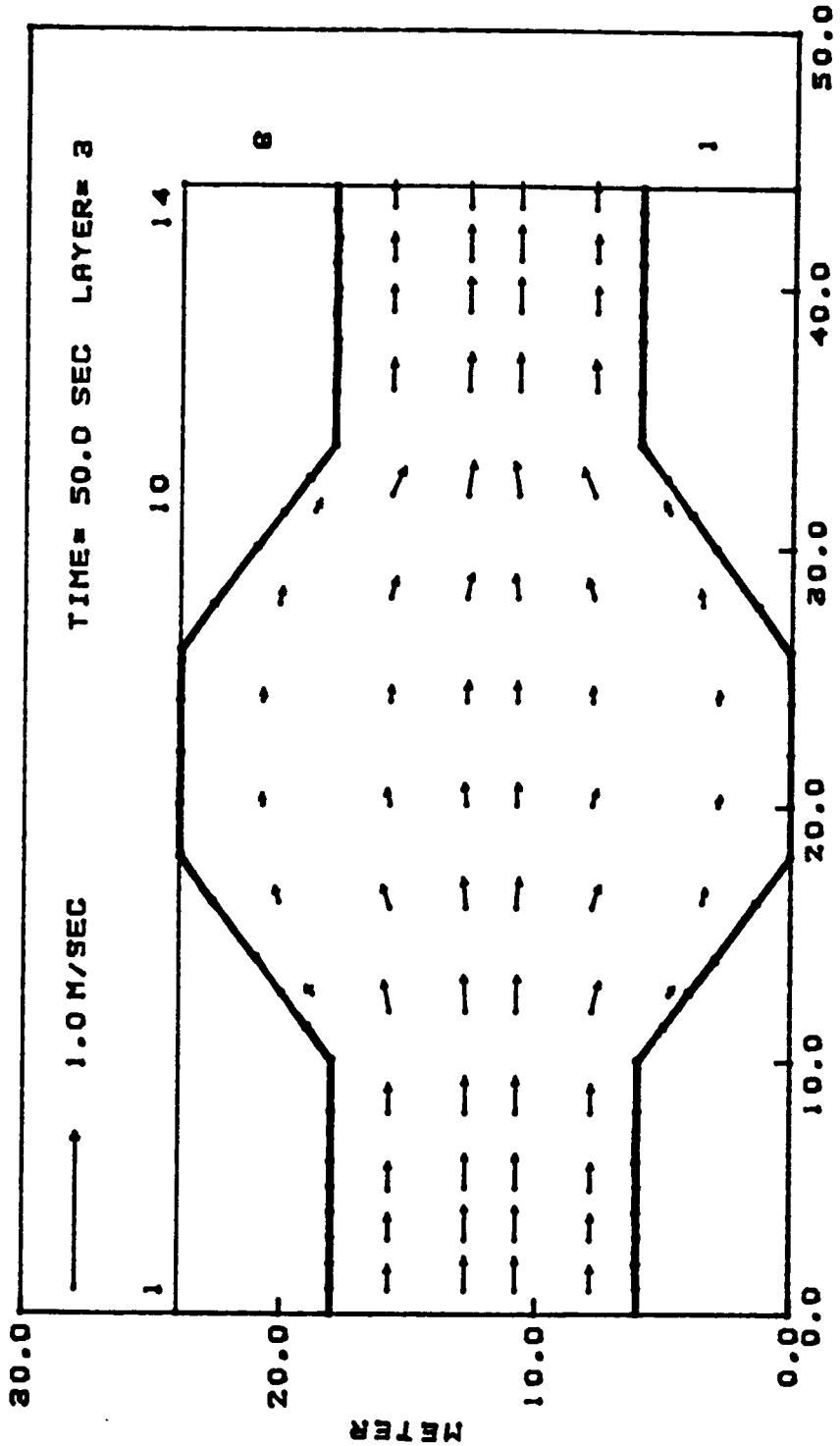

Figure 6.5. Velocity conditions in the internal layer l = 3, at time t = 50 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

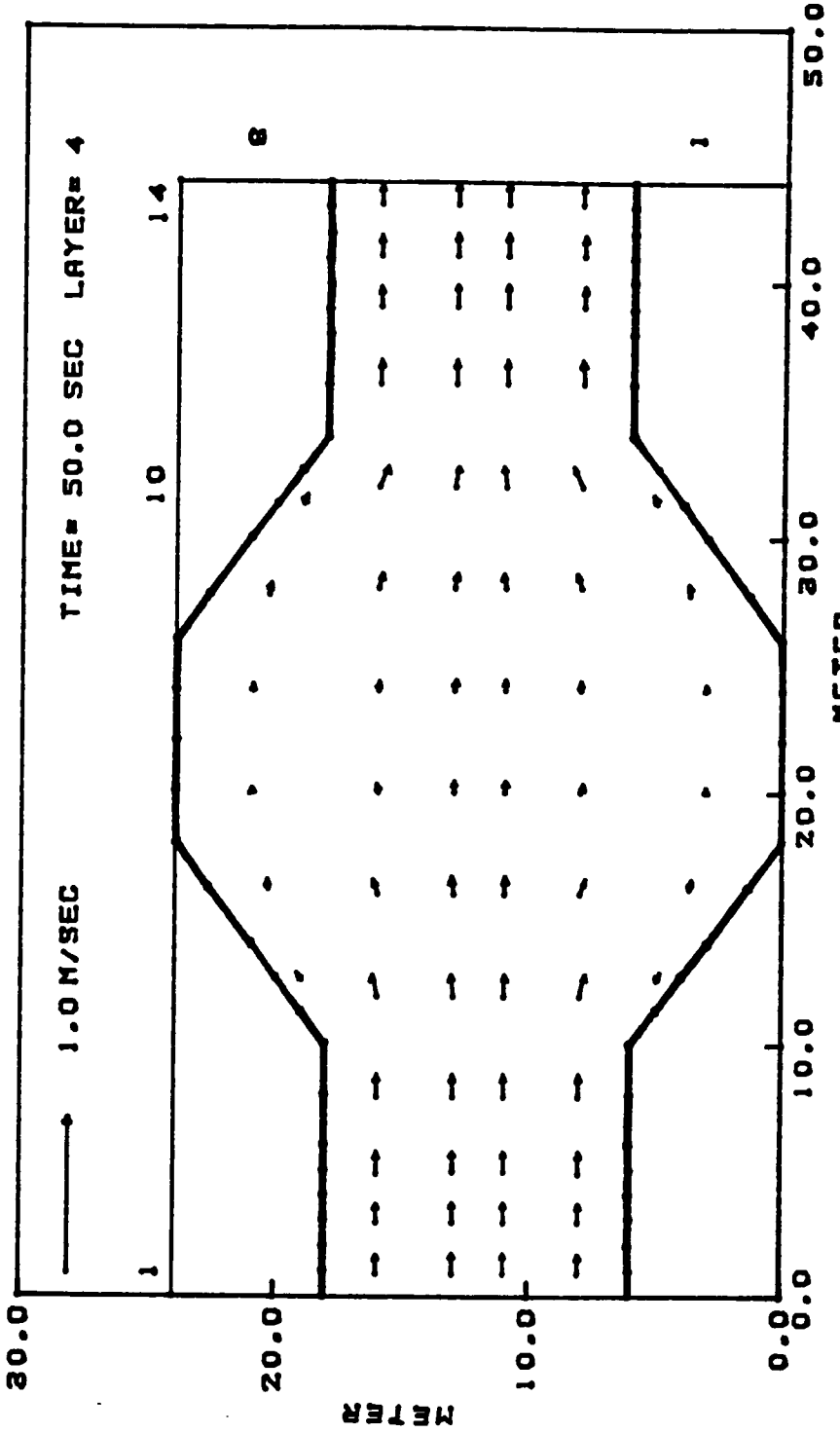

Figure 6.6. Velocity conditions in the surface layer l = 4, at time t = 50 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

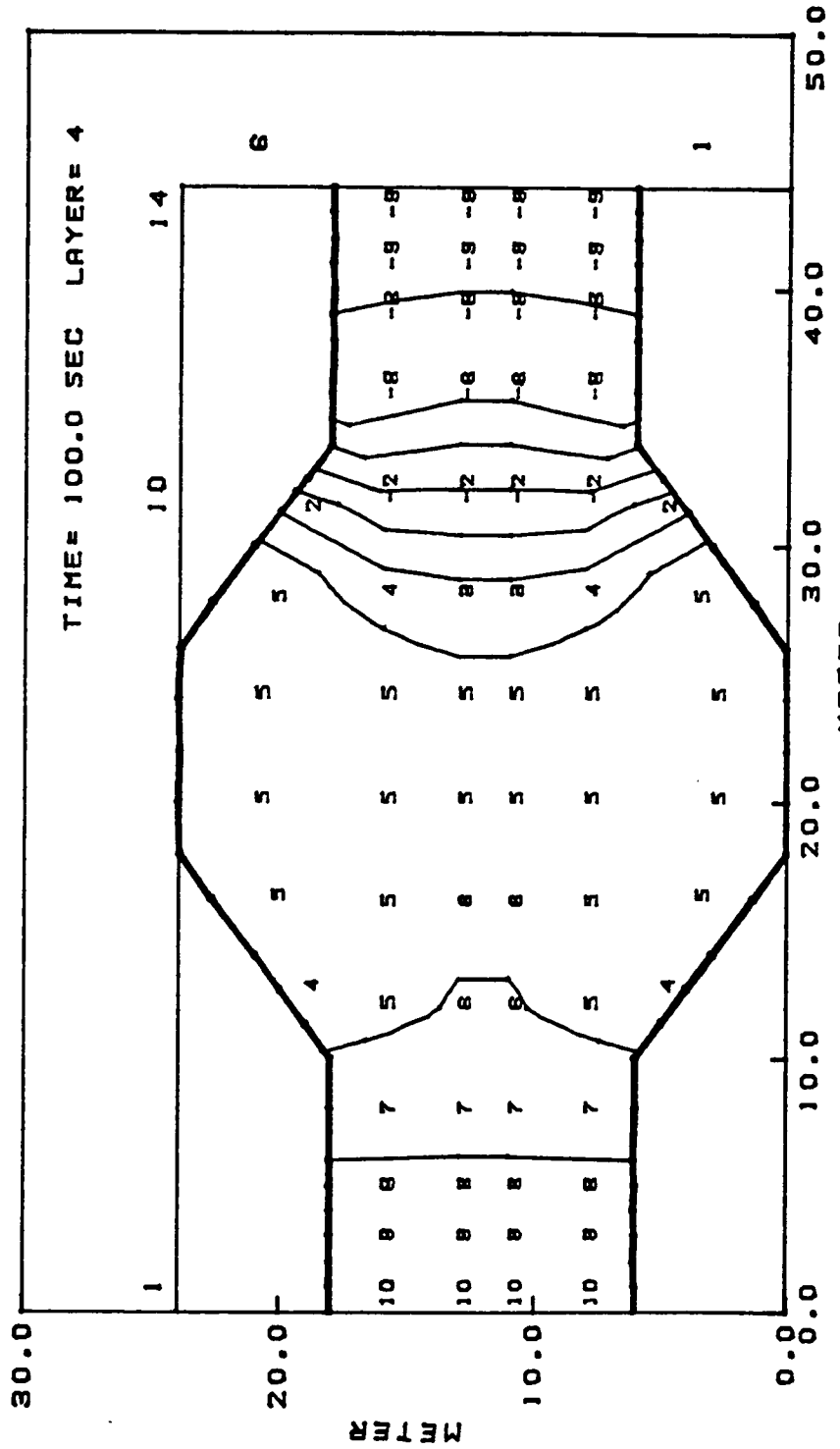

Figure 6.7. Water-surface-elevation conditions at time t = 100 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

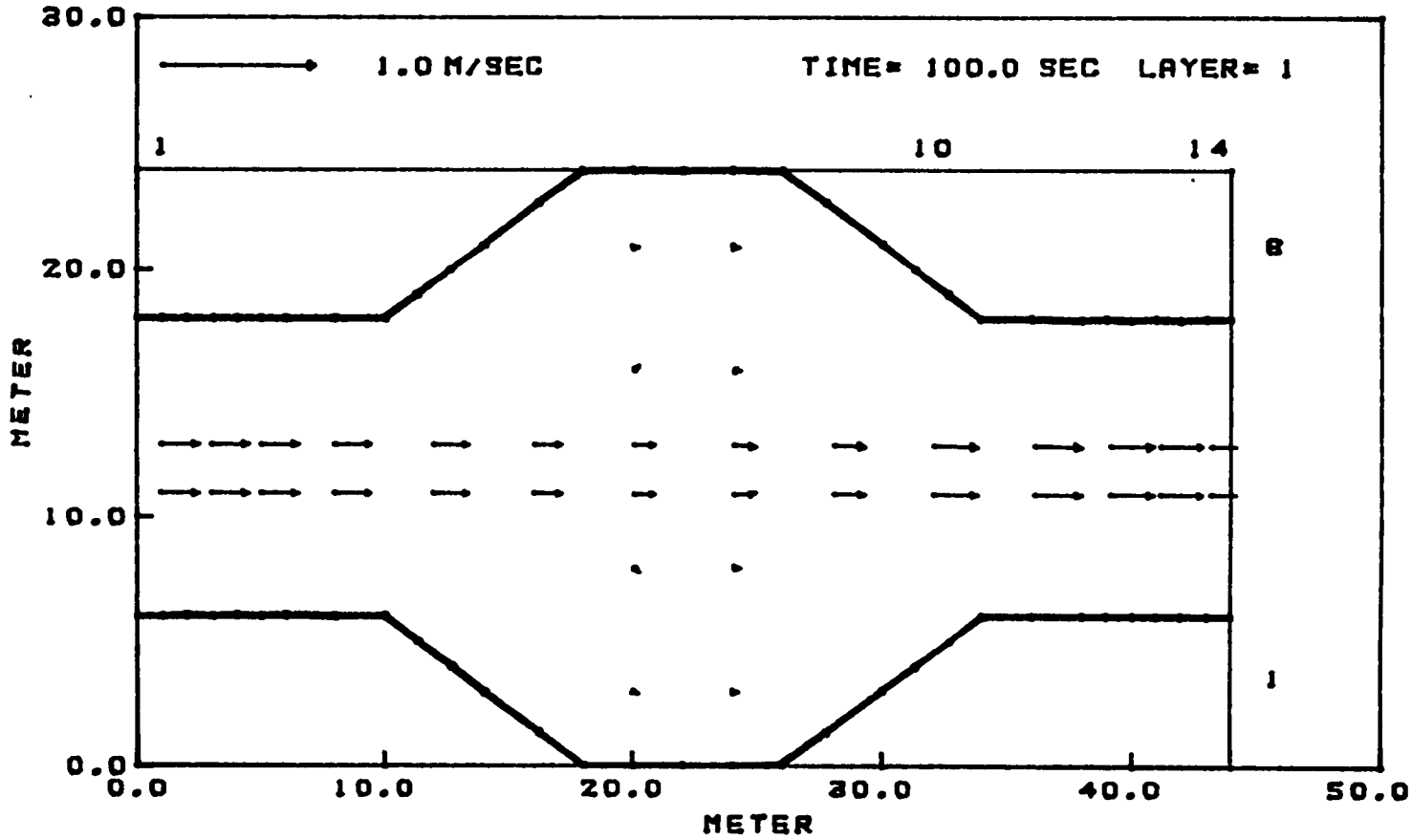

Figure 6.8. Velocity conditions in the bottom layer l = 1, at time t = 100 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

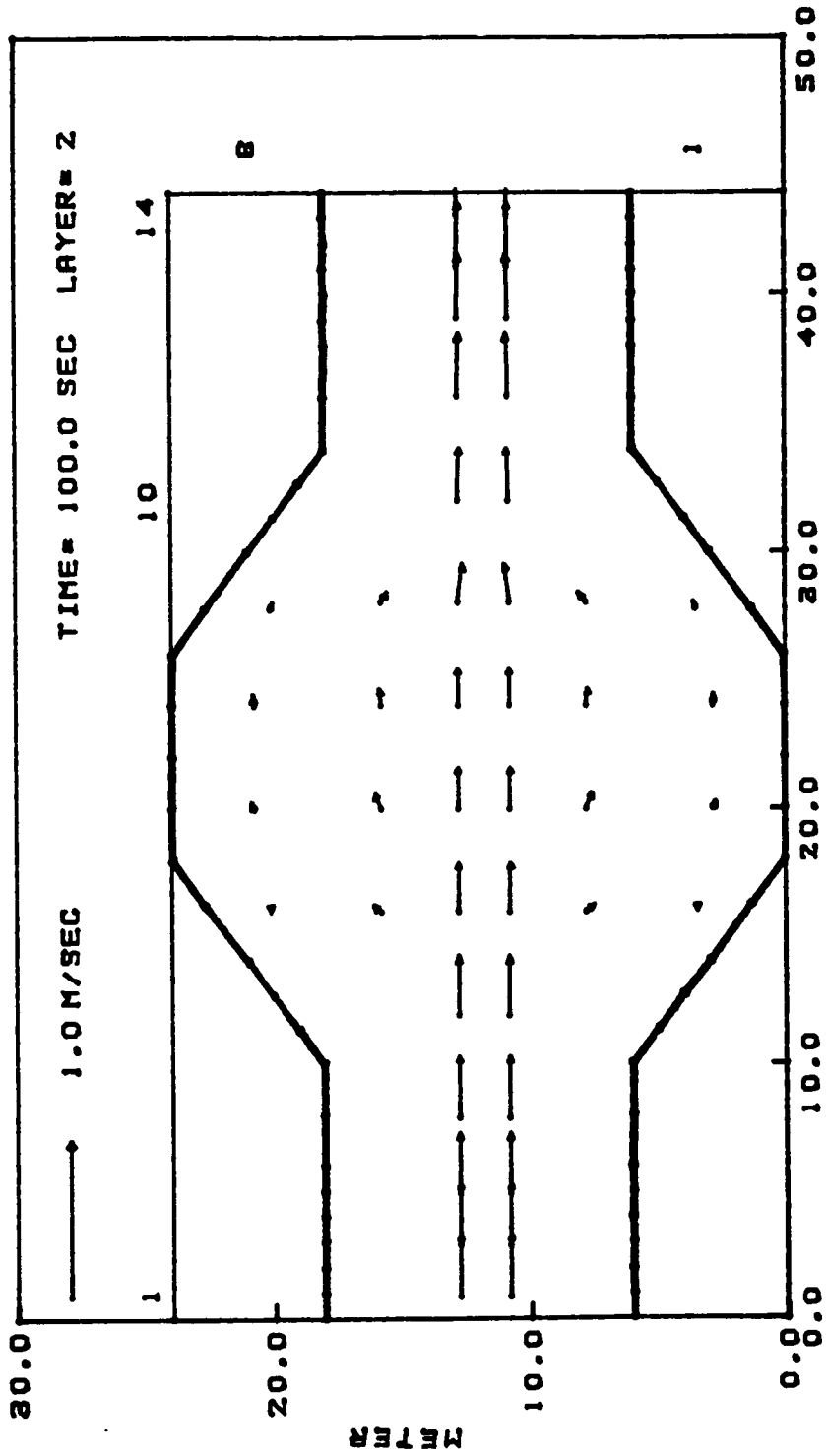

Figure 6.9. Velocity conditions in the internal layer l = 2, at time t = 100 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

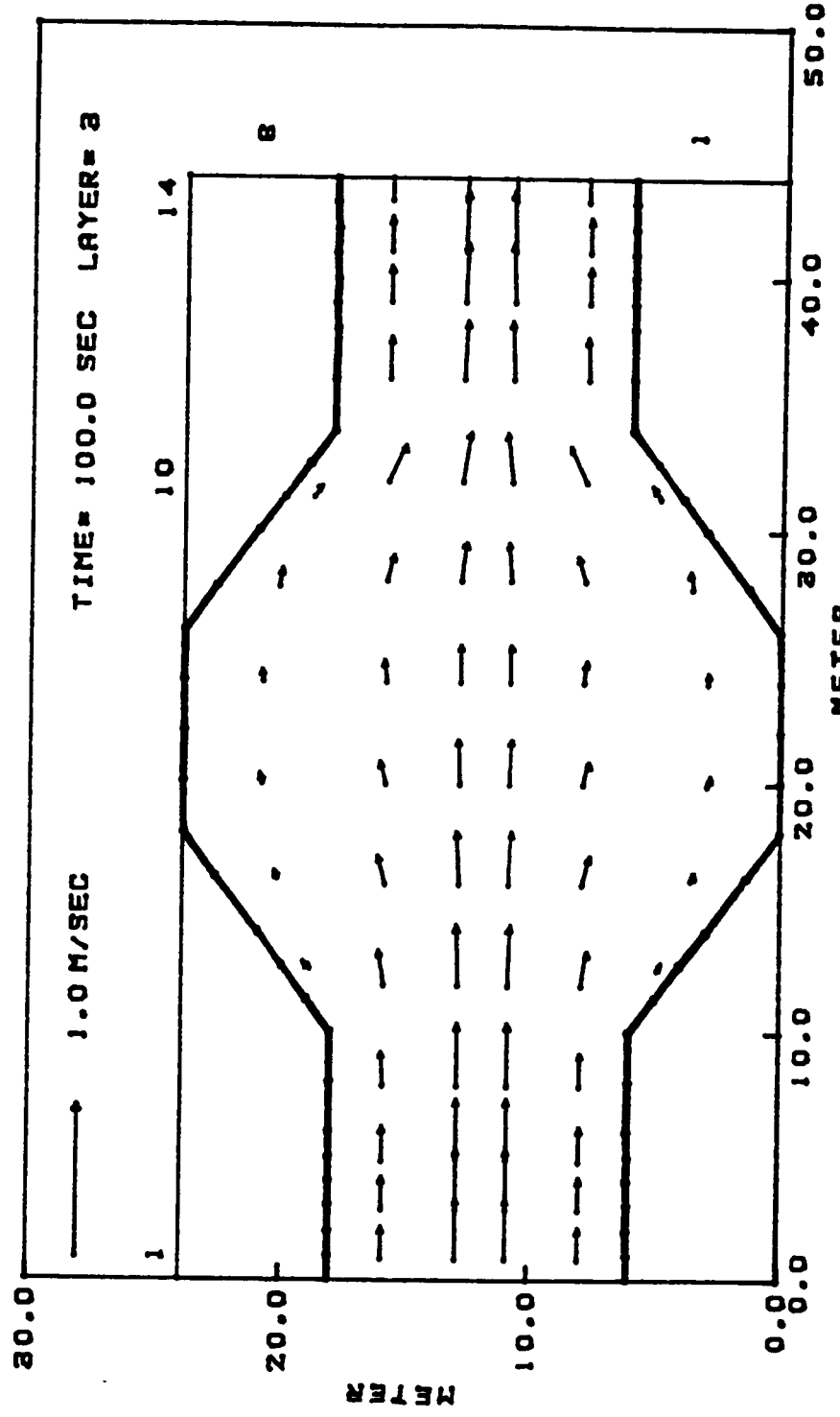

Figure 6.10. Velocity conditions in the internal layer l = 3, at time t = 100 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

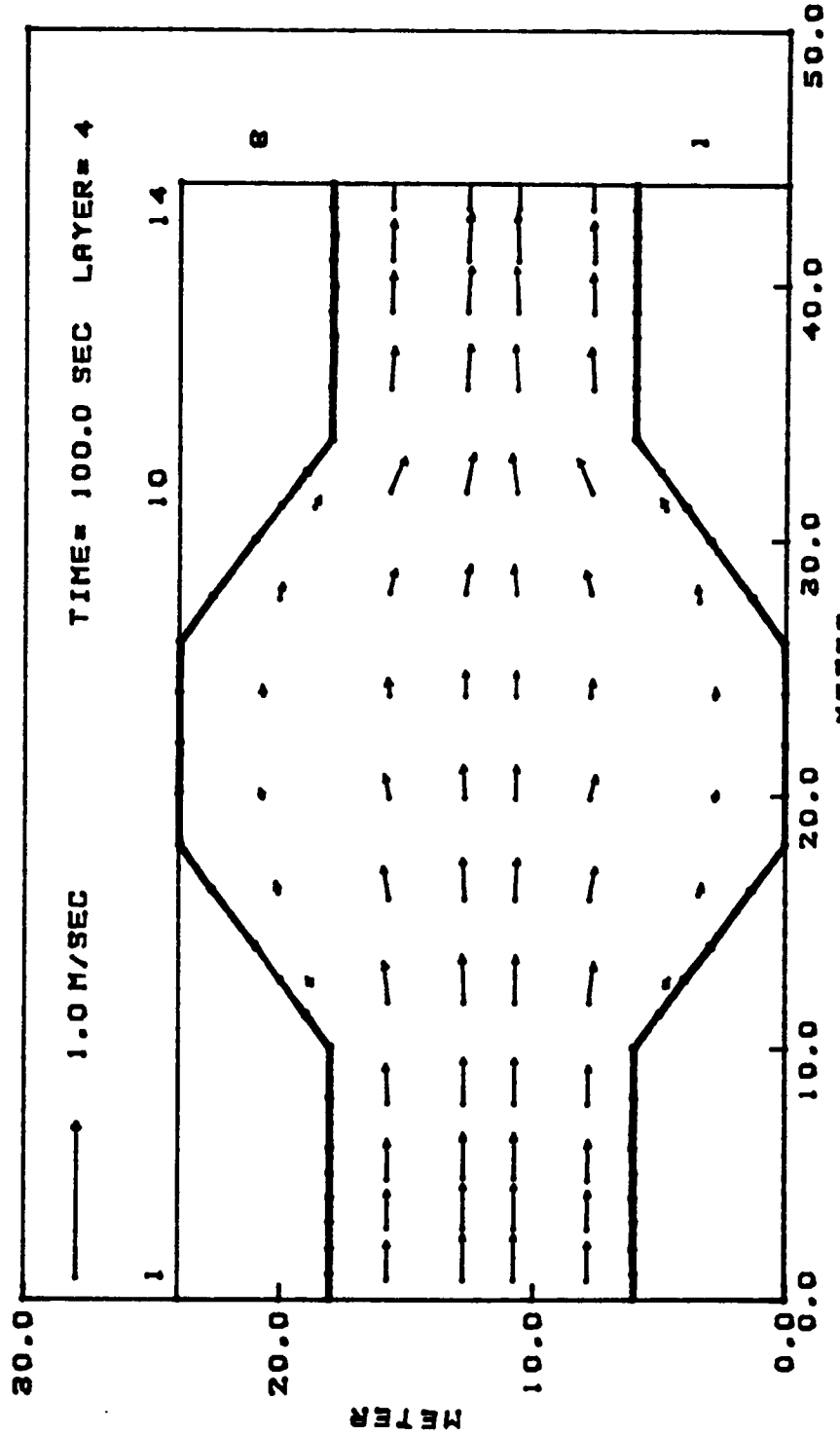

Figure 6.11. Velocity conditions in the surface layer l = 4, at time t = 100 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

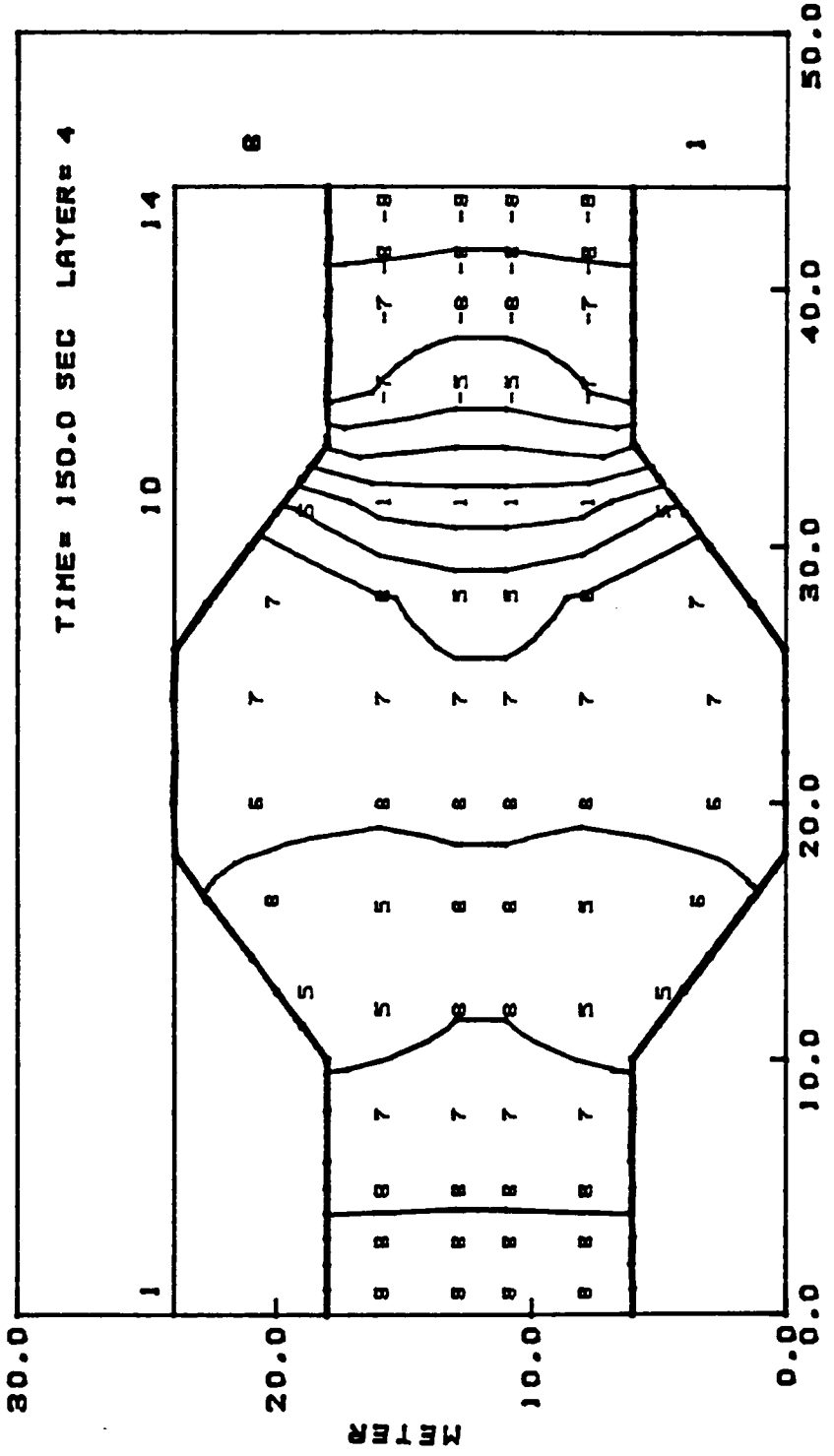

Figure 6.12. Water-surface-elevation conditions at time t = 150 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

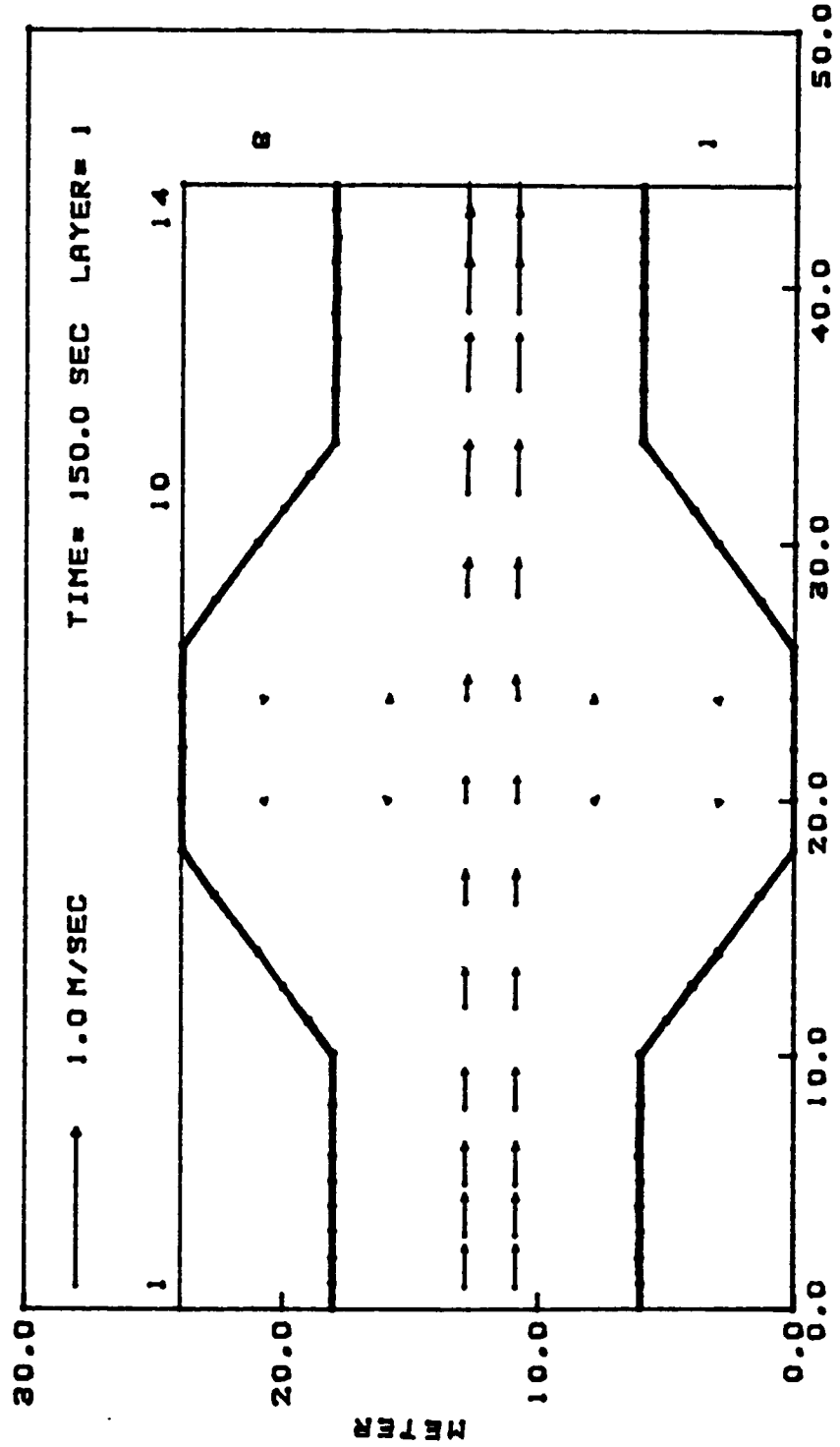

Figure 6.13. Velocity conditions in the bottom layer l = 1, at time t = 150 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

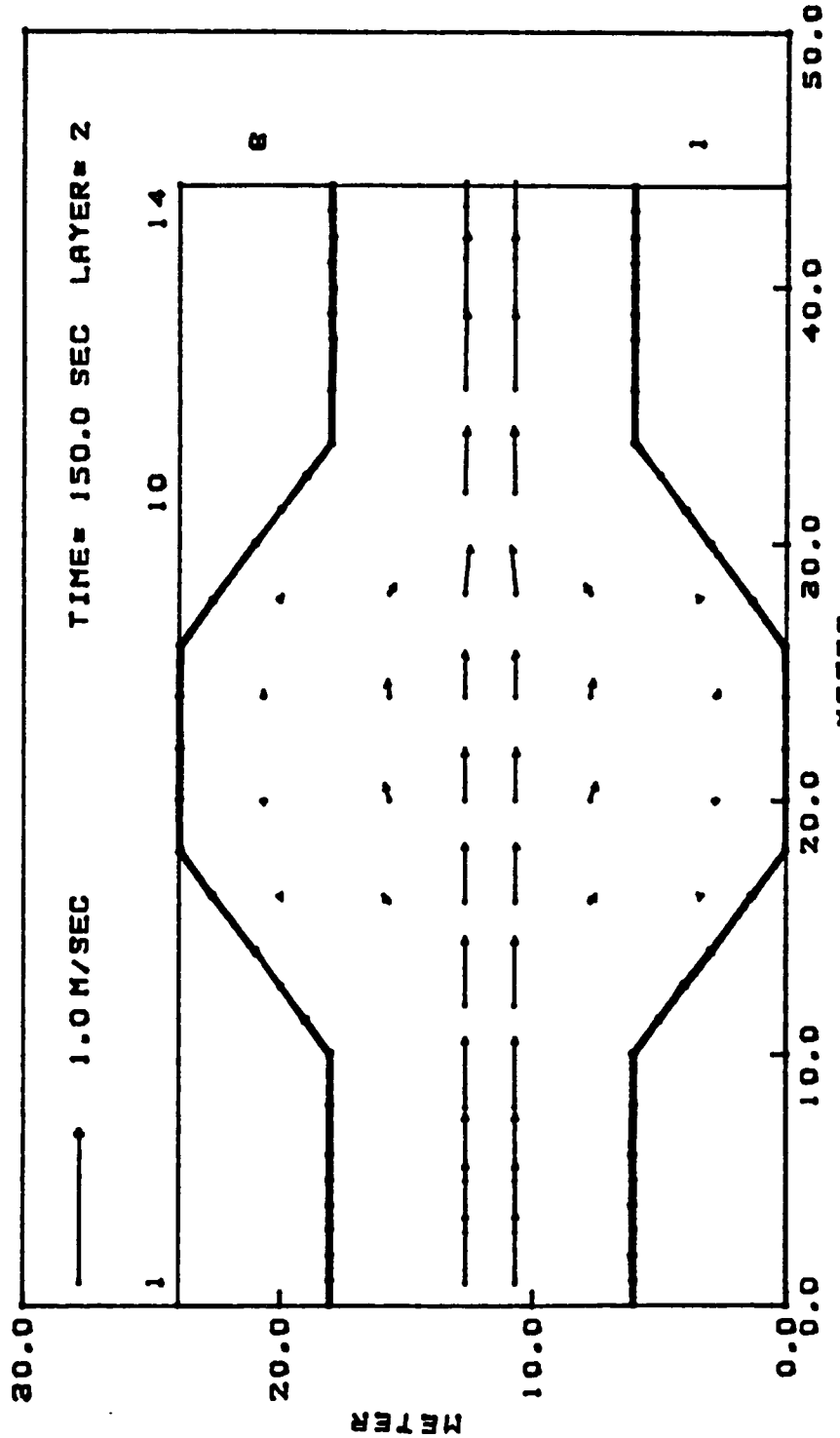

Figure 6.14. Velocity conditions in the internal layer l = 2, at time t = 150 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

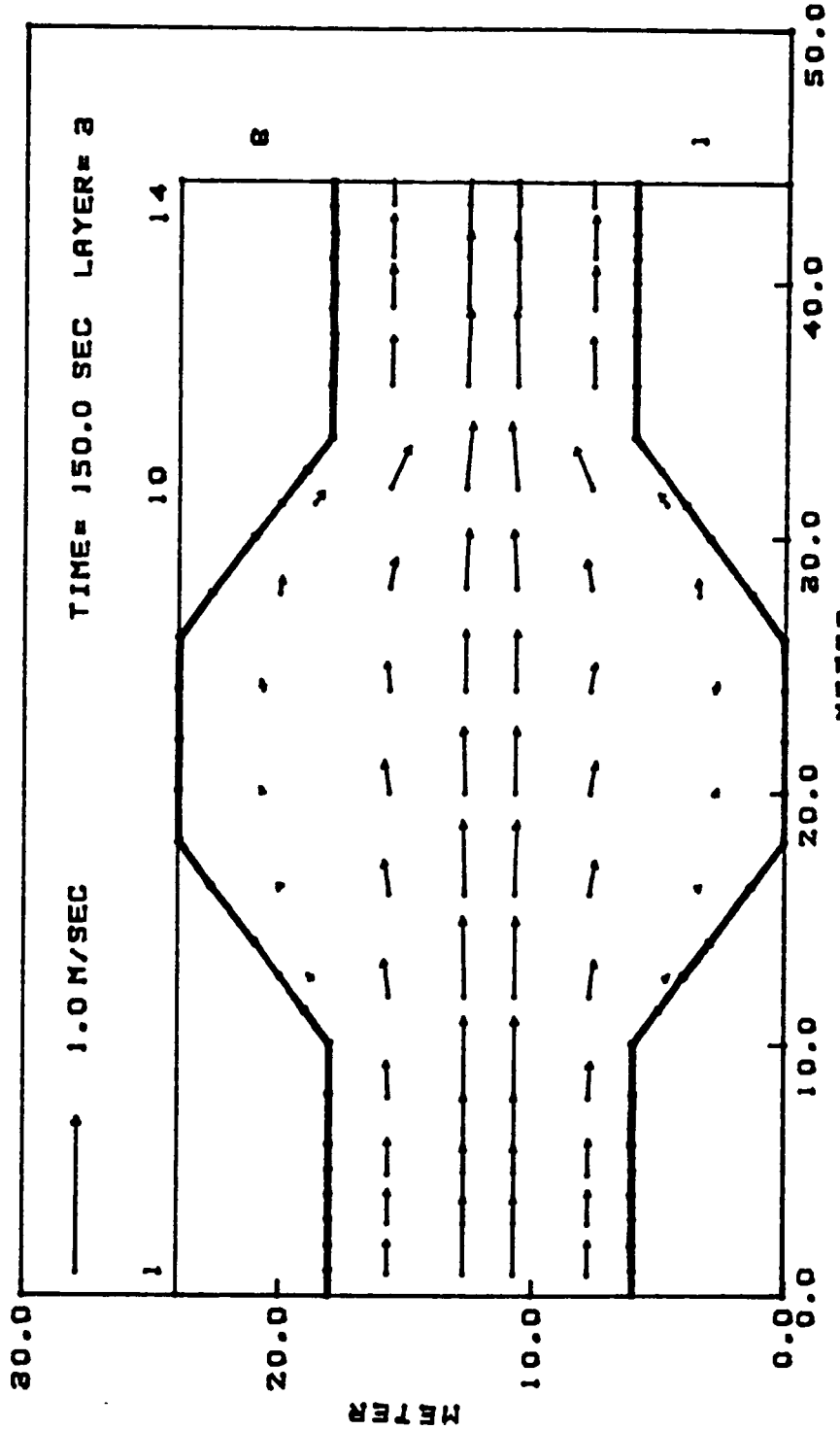

Figure 6.15. Velocity conditions in the internal layer l = 3, at time t = 150 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

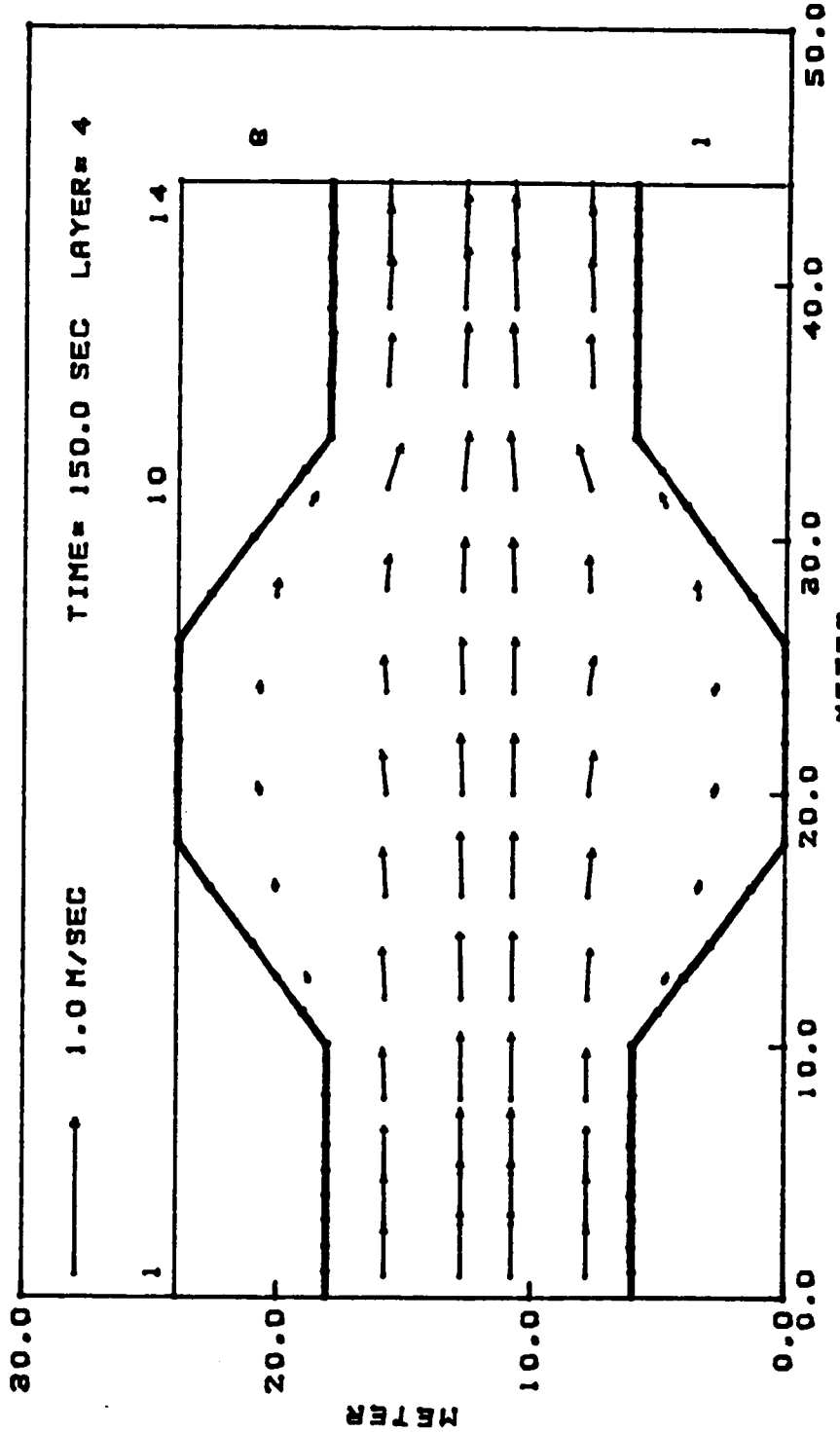

Figure 6.16. Velocity conditions in the surface layer l = 4, at time t = 150 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

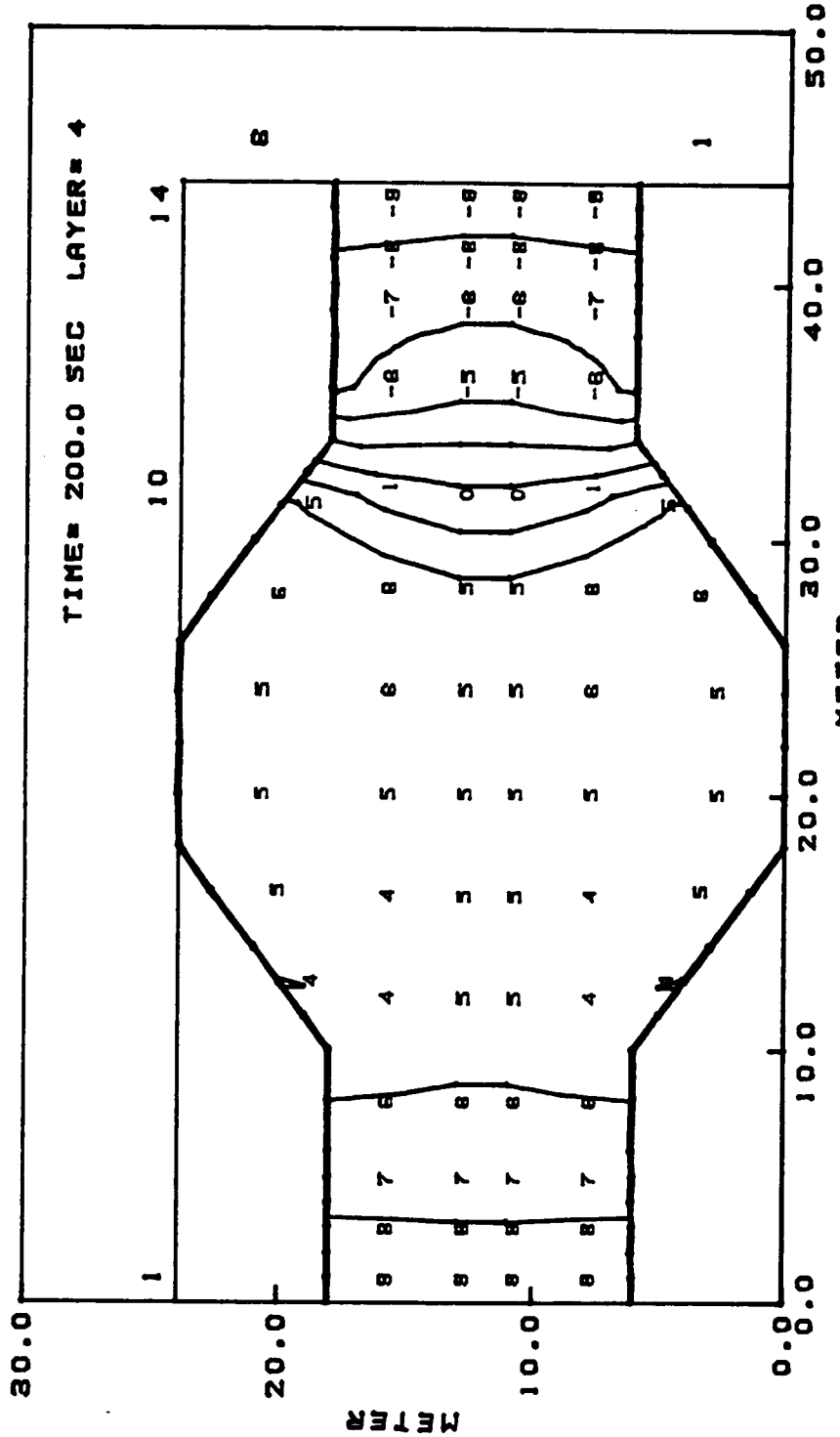

Figure 6.17. Water-surface-elevation conditions at time t = 200 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

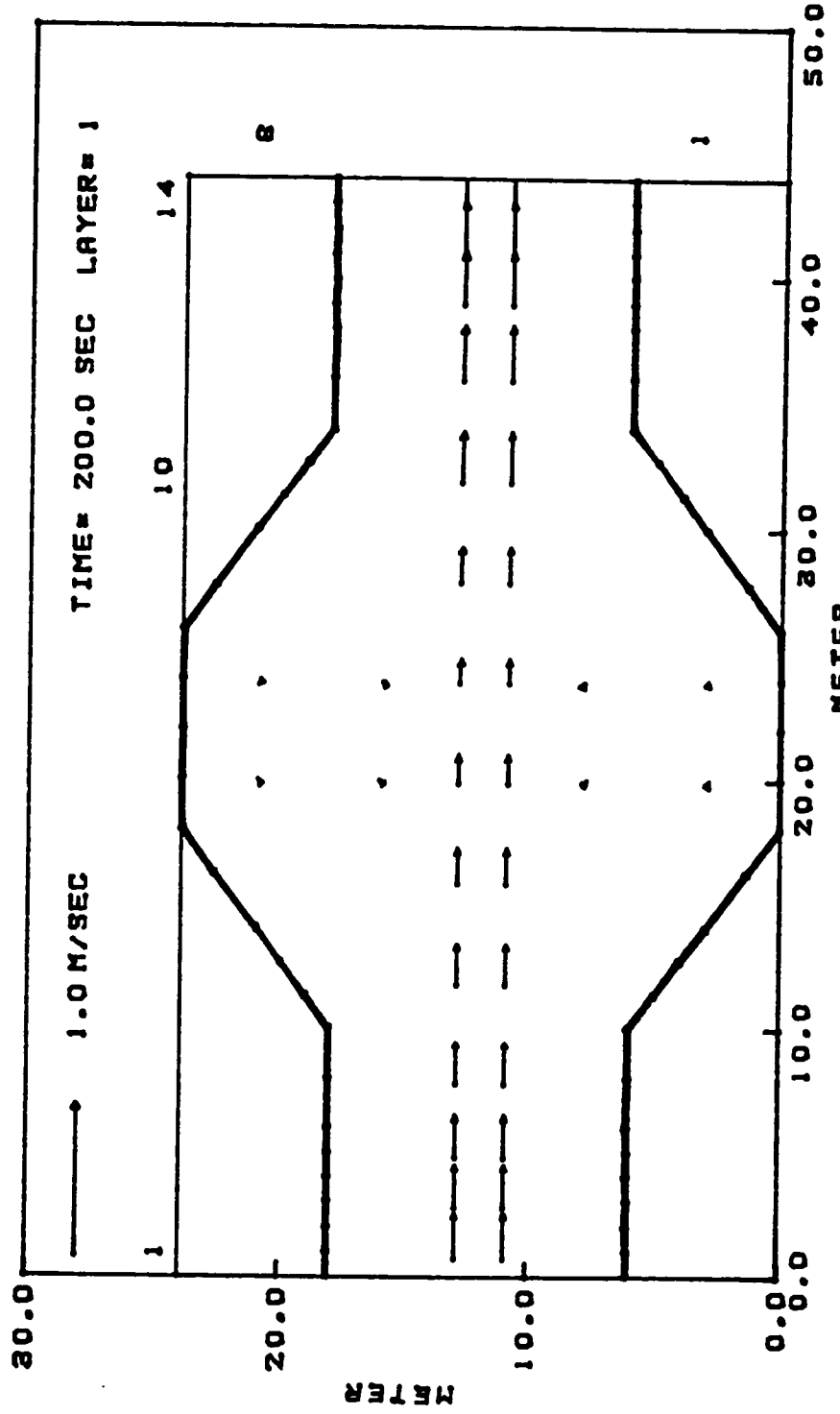

Figure 6.18. Velocity conditions in the bottom layer l = 1, at time t = 200 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

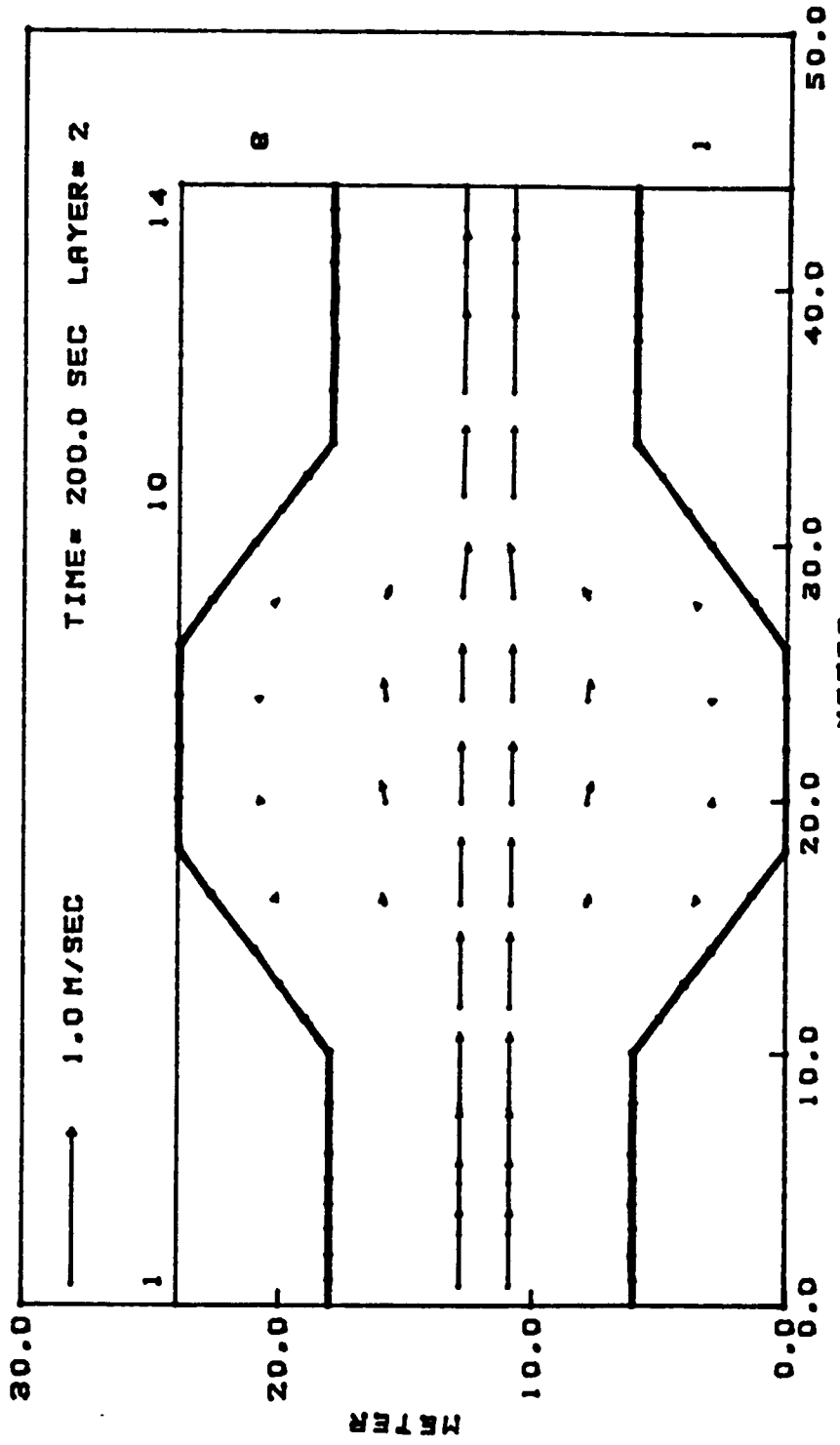

Figure 6.19. Velocity conditions in the internal layer l = 2, at time t = 200 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

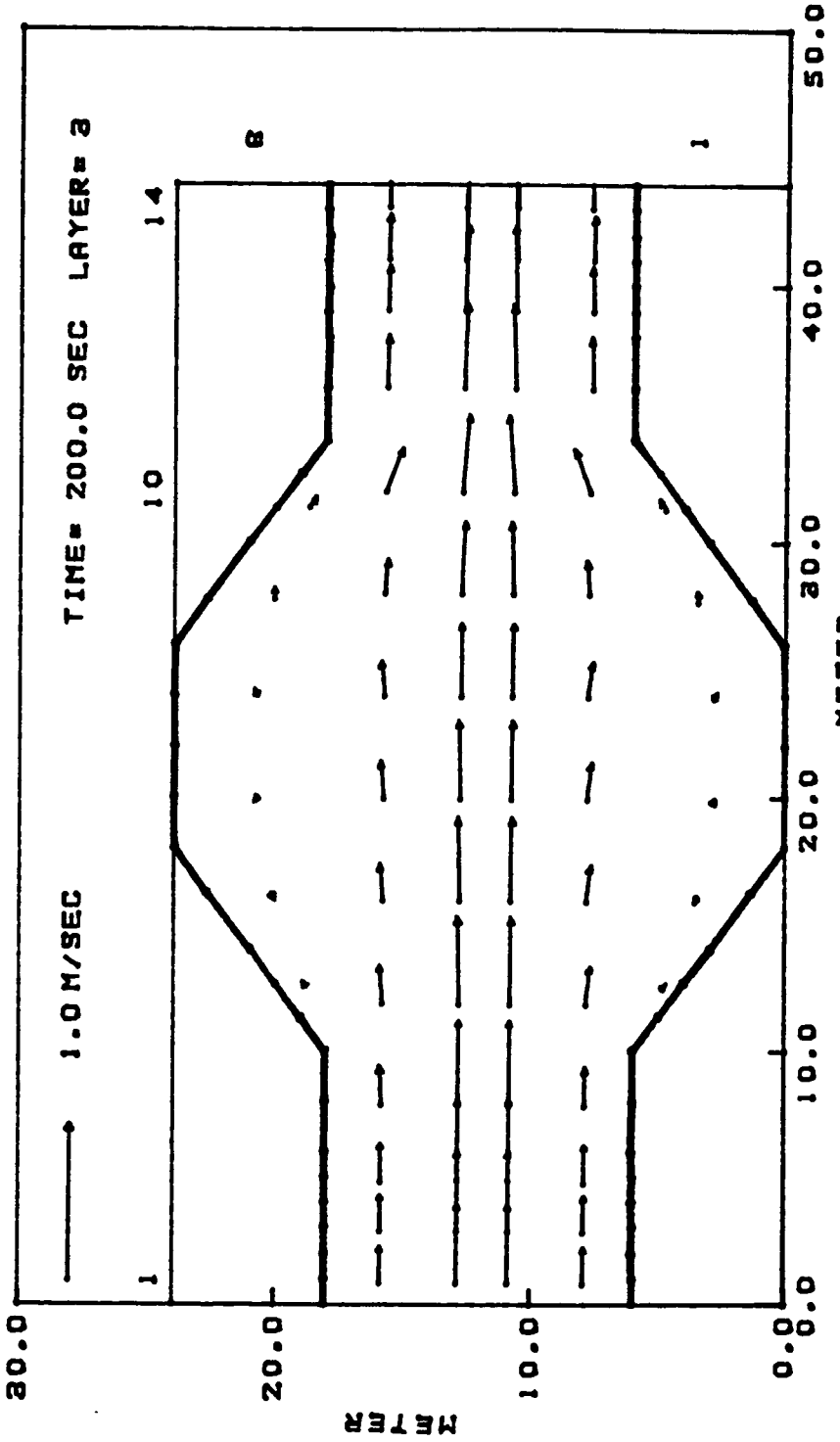

Figure 6.20. Velocity conditions in the internal layer l = 3, at time t = 200 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

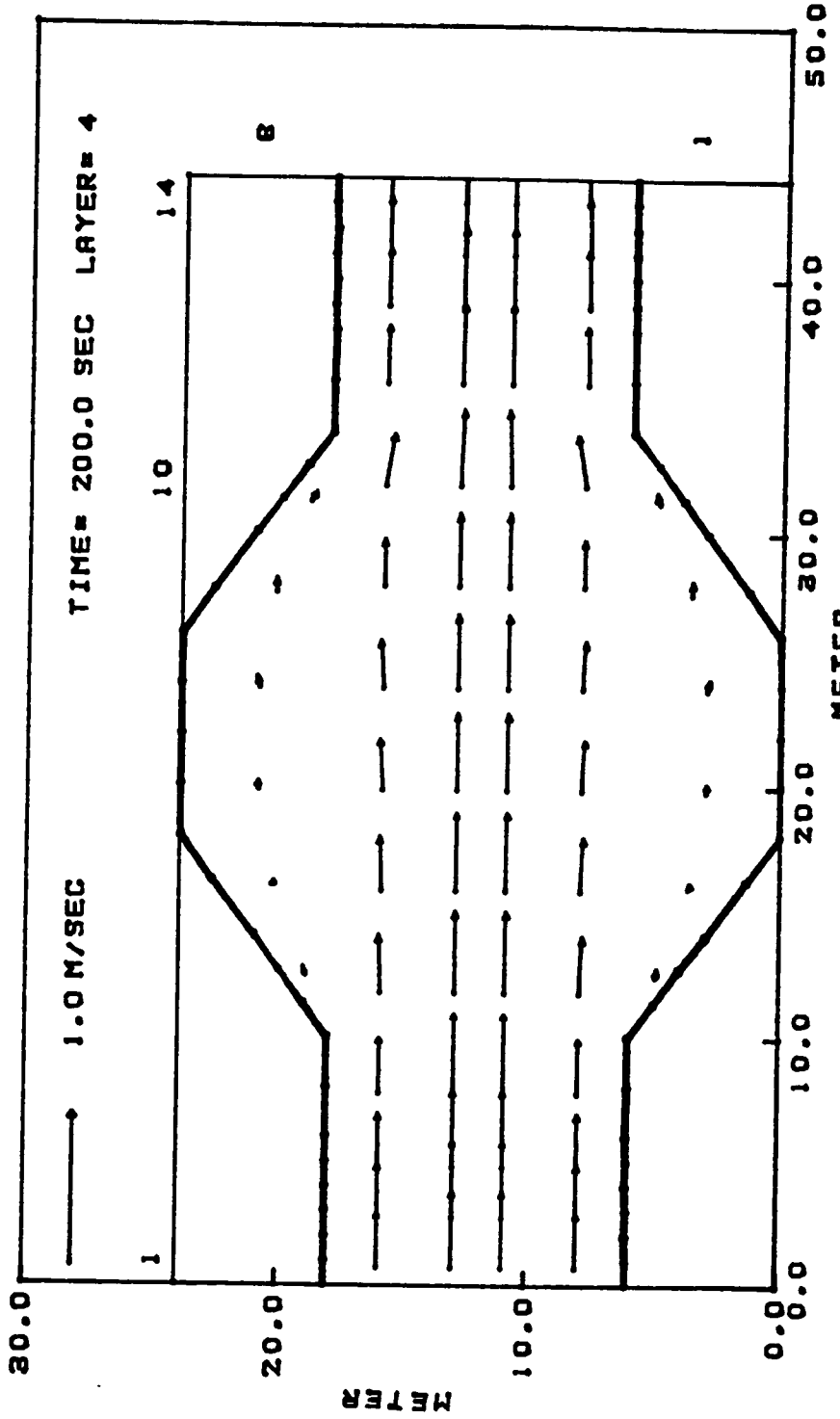

Figure 6.21. Velocity conditions in the surface layer l = 4, at time t = 200 sec for test case 1: Unidirectional Flow (inflow and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL. The velocity conditions are presented as two-dimensional vectors which correspond to two-component representations of velocity in the horizontal (x,y) planes of the four specified layers.

The results of the computer simulations for the flow conditions in the symmetric laboratory-model-scale flow region indicated that the water-surface-elevation and velocity conditions were generally consistent with the conditions that would exist in a similar prototype-scale flow region with a similar configuration for the deep main flow channel.

The special, transient flow characteristics of the water-surface-elevation and velocity conditions for test case 1 are discussed in detail below in order to illustrate the important correspondance between the results of the computer simulations and certain special characteristics of the physical phenomena that control the flow conditions in the laboratory-model-scale flow region.

The results for constant-water-surface-elevation contours (Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567; and Fig. 6.17, p.572) indicate that there is a gradual accumulation of water in most parts of the flow region except in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. The results indicate that, at time level t = 50 sec, the constant-water-surface-elevation contour with zero value, which corresponds to the specified initial mean water level (MWL), is already relocated at approximately x = 30 m, which is 8 m farther downstream from the central y-axis at x = 22 m (Fig. 6.2, p.557). As the flow development progresses, during time

levels t = 100 sec and t = 150 sec, the constant-water-surface-elevation contour with zero value gradually moves farther downstream (Fig. 6.7, p.562; and Fig. 6.12, p.567); and at time level t = 200 sec, it becomes located at approximately x = 34 m (Fig. 6.17, p.572) which corresponds approximately to the location of the upstream enclosure surface of the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. The results for constant-water-surface-elevation contours also indicate that the gradual accumulation of water in the middle of the flow region stops after the time level t = 150 sec, (Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567; and Fig. 6.17, p.572). At time level t = 200 sec, the water-surface-elevation contours in the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m), in the middle of the flow region, reach the pseudo steady-state conditions with the water-surface-elevation values varying between 4 mm and 6 mm above the mean water level (MWL) of the specified initial conditions (Fig. 6.17, p.572).

The results of the computer simulations for water-surface-elevation conditions (Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567; and Fig. 6.17, p.572) can be explained readily by considering various physical conditions which control the flow in the laboratory-model-scale flow region. Rapid accumulation of water in the middle of the region, during the time period before time level t = 150 sec (Fig. 6.2, p.557; Fig. 6.7, p.562; and Fig. 6.15, p.570), can be explained by considering two physical characteristics of the simulated flow conditions.

First, since the inflow conditions at the inlet plane at the west-end upstream enclosure surface at $x = 0$, can attain relatively high x-component of momentum (control condition specified as $\tilde{V}_{x,max,(opr,i-1),j,l}$ 0.5 m/sec), the inflowing water can rapidly push the water, that was initially in the narrow subregion (between $x = 0$ and $x = 10$ m and between $y = 6$ m and $y = 18$ m), downstream of the inlet plane at the west-end upstream enclosure surface at $x = 0$, into the large subregion (between $x = 10$ m and $x = 34$ m and between $y = 6$ m and $y = 18$ m), in the middle of the flow region. The water that flows out of this narrow subregion undergoes rapid spreading due to the divergent geometrical shape of the boundary enclosure surfaces of the upstream part of the large subregion (between $x = 10$ m and $x = 18$m and between $y = 0$ and $y = 24$ m). However, the water that flows out of the downstream part of the large subregion (between $x = 26$ m and $x = 34$ m and between $y = 0$ and $y = 24$ m) first has to undergo rapid contraction due to the convergent geometrical shape of the boundary enclosure surfaces of this part of the large subregion. Considering the effect of retardation of flow in the relatively shallow subregions (between $x = 26$ m and $x = 34$ m, between $y = 0$ and $y = 10$ m and between $y = 14$ m and $y = 24$ m), outside of the deep main flow channel along the longitudinal x axis, the accumulated water cannot freely flow into the narrow subregion (between $x = 34$ m and $x = 44$ m and between $y = 6$ m and $y = 18$ m), upstream of the discharge plane at the east-end downstream enclosure surface at $x = 44$ m. Hence, the water accumulates in the large subregion (between $x = 10$ m and $x = 34$ m and between $y = 0$ and $y = 24$ m) in the middle of the flow region (Fig. 6.2, p.557; Fig. 6.7, p.562; and Fig. 6.12, p.567), since the

water can flow easier into the upstream part of the large subregion (between x = 10 m and x = 18 m and between y = 0 and y = 24 m), from the narrow subregion (between x = 0 and x = 10 m and between y = 6 m and y = 18 m), downstream of the inlet plane at the west-end upstream enclosure surface at x = 0, than it can flow out of the downstream part of the large subregion (between x = 26 m and x = 34 m and between y = 0 and y = 24 m), into the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. Furthermore, since the x-component of momentum of the outflow conditions at the discharge plane at the east-end downstream enclosure surface at x = 44 m depends (from physical considerations) on the x-component of the momentum of the water upstream of the discharge plane, and since the flow in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane, at the east-end downstream enclosure surface at x = 44 m, is unidirectional along the downstream-x direction, the x-component of momentum of the water in the narrow subregion can not increase progressively along the downstream-x direction. Hence, the flow conditions in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m, in a sense also retard the flow of water out of the downstream part of the large subregion (between x = 26 m and x = 44 m and between y = 0 and y = 24 m) in the middle of the flow region.

Second, the imposed water-surface-elevation condition, at the west-end upstream open-region enclosure surface at x = 0, as 0.01 m (10 mm)

above the specified initial mean water level (MWL), represents an imposed positive hydrostatic pressure differential which initiates a compression type of surface gravity wave that travels along the downstream-x direction with the predominantly unidirectional flow which is also along the downstream-x direction in the flow region (see Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567 and Fig. 6.17, p.572). On the other hand, the imposed water-surface-elevation condition, at the east-end downstream open-region enclosure surface at x = 44 m, as -0.01 m (10 mm) below the specified initial mean water level (MWL), represents an imposed negative hydrostatic pressure differential which initiates a rarefaction type of surface gravity wave that travels along the upstream-x direction against the predominantly unidirectional flow which is along the downstream-x direction in the flow region (see Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567; and Fig. 6.17, p.572). Therefore, the propagation of higher water-surface-elevation condition is more rapid along the downstream-x direction (since the surface gravity wave velocity and the flow velocity are both along the same downstream-x direction) than the propagation of lower water-surface-elevation condition along the upstream-x direction (since the surface gravity wave velocity along the upstream-x direction opposes the flow velocity along the downstream-x direction). Hence, during the time period before the establishment of pseudo steady-state flow conditions, the water-surface-elevation conditions have to increase in most parts of the flow region (between x = 0 and x = 30 m, approximately), except in the narrow sub-region (between x = 34 m and x = 44 m), upstream of the discharge plane

at the east-end downstream enclosure surface at x = 44 m (Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567 and Fig. 6.17, p.572).

It is important to note that the gradual accumulation of water in the flow region (Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567 Fig. 6.17, p.572) is also a consequence of the open-region boundary conditions that were used in the computer simulations. The specified increased water-surface-elevation condition, at the west-end upstream open-region enclosure surface at x = 0, as 0.01 m (10 mm) above the specified initial mean water level (MWL) results in an increase of 0.12 m^2 of inflow area, since the width of the upstream enclosure surface is 12 m. Similarly, the specified decreased water-surface-elevation condition, at the east-end downstream open-region enclosure surface at x = 44 m, as -0.01 m (-10 mm) below the specified initial MWL results in a decrease of -0.12 m^2 of outflow area, since the width of the downstream enclosure surface is also 12 m. Therefore, effectively, the inflow conditions at the upstream open-region enclosure surface at x = 0 have an available flow area which is 0.24 m^2 larger than the available flow area for the outflow conditions at the downstream open-region enclosure surface at x = 44 m.

The inflow conditions at the inlet plane at the west-end upstream enclosure surface at x = 0, can attain relatively high values for the x-component of velocity (control condition specified as $\bar{V}_{x,max}$ = 0.5 m/sec), according to the specified open-region boundary conditions, immediately after the start of the flow conditions in the flow region. On the other hand, since the viscous effects in the flow region always tend to retard the general flow conditions along the x direc-

tion, the outflow conditions at the discharge plane at the east-end down-stream enclosure surface at x = 44 m cannot attain values for the x-component of velocity higher than the values for the x-component of velocity at the west-end upstream enclosure surface at x = 0, until the increased water-surface-elevation conditions in the flow region can cause sufficiently high hydrostatic pressure gradient conditions that can accelerate the flow along the x direction. Therefore, during the period of flow development, the inflow rate at the inlet plane at the west-end upstream enclosure surface at x = 0, with effectively increased value of 0.12 m^2 available flow area, is higher than the outflow rate at the discharge plane at the east-end downstream enclosure surface at x = 44 m, with effectively decreased value of -0.12 m^2 available flow area. Hence, during the period of flow development, the inflow rate is effectively higher than the outflow rate, and hence, the water-surface-elevation conditions gradually increase in most parts of the flow region.

The considerations of the physical characteristics of the flow conditions clearly indicate that the computer simulation results of the discrete-element hydrodynamic model (HYDROL) for the transient water-surface-elevation conditions are realistic and representative of conditions that would exist for test case 1 : Symmetric Unidirectional Flow (inflow and outflow along longitudinal direction) in an actual laboratory-model-scale flow region.

The computer simulation results for velocity conditions (Figs. 6.3 through 6.6, p.558 through p.561; Figs. 6.8 through 6.11, p.563 through p.566; Figs. 6.13 through 6.16, p.568 through p.571; and Figs. 6.18 through 6.21, p.573 through p.576) indicate that the three-dimensional

flow is predominantly in the subregion of the deep main flow channel (between x = 0 and x = 44 m and between y = 10 m and y = 14 m) along the downstream-x direction in the flow region. The flow conditions in shallow regions in layers l=3,4 contribute little to the overall flow conditions in the flow region.

These results based on the computer simulations of the velocity conditions in the laboratory-model-scale flow region are consistent with the generally observed velocity conditions that occur in prototype-scale flow regions with deep main flow channels. Since the overall frictional effect on the flow in the deep main flow channel is substantially less than the frictional effects on the flow in shallow regions, the velocity conditions are generally higher in the deep main flow channel. Hence, based on the considerations of the physical characteristics that control the flow conditions in a flow region with a deep main flow channel, the computer simulation results of the discrete-element hydrodynamic model (HYDROL) are realistic and representative of the transient, three-dimensional velocity conditions that would exist in an actual laboratory-model-scale flow region.

The results of the computer simulations agree perfectly with the required conditions of symmetry and asymmetry in the flow region at all time levels, t = 50 sec through t = 200 sec (Figs. 6.2 through 6.21, p.557 through p.576). The water-surface-elevation conditions exhibit perfectly symmetric spatial distributions about the central longitudinal axis along x at y = 12 m (Fig. 6.2, p.557; Fig. 6.7, p.562; Fig. 6.12, p.567; and Fig. 6.17, p.572). The x-component of velocity also exhibits perfectly symmetric spatial distributions about the central longitudinal

axis along x at y = 12m (Figs. 6.3 through 6.6, p.558 through p.561; Figs. 6.8 through 6.11, p.563 through p.566; Figs. 6.13 through 6.16, p.568 through p.571; and Figs. 6.18 through 6.21, p.573 through p.576). Furthermore, the y-component of the velocity exhibits perfectly symmetric spatial distributions about the central longitudinal axis along x at y = 12 m (Figs. 6.3 through 6.6, p.558 through p.561; Figs. 6.8 through 6.11, p.563 through p.566; Figs. 6.13 through 6.16, p.568 through p.571; and Figs. 6.18 through 6.21, p.573 through p.576).

Hence, the results of the computer simulations of the water-surface-elevation and three-dimensional velocity conditions in the laboratory-model-scale flow region for the test case 1: Symmetric Unidirectional Flow (inflow and outflow along the longitudinal direction), clearly illustrates the reliability of the mathematical formulation of the transient, three-dimensional, discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of its associated HYDROL computer code.

6.1.2.2 Test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). The second test case was considered for the simulation of the nonsymmetric flow conditions in the laboratory-model-scale flow region, with inflow along y axis (lateral direction) and outflow along x axis (longitudinal direction), between the north-end upstream enclosure surface and the east-end downstram enclosure surface. The necessary input data to the HYDROL computer code was prepared according to the following conditions associated with the mathematical formulation of the discrete element hydrodynamic model.

a. Boundary Conditions

The test case was run by considering open-region boundary conditions at the north-end upstream enclosure surface (at y = 24 m) and at the east-end downstream enclosure surface (at x = 44 m) of the flow region. The enclosure surface at x = 0, the enclosure surface at y = 0 and all other enclosure boundaries were assumed to be closed, impermeable shoreline boundary surfaces.

North-end upstream condition for water surface elevation at y = 24 m (relative to MWL):

$\overline{H}_{i,(opr,j\pm1)}$ = 0.01 m (above MWL), along longitudinal open-region enclosure surface, upstream of the two boundary elements i=7,8; j=6; l=4.

North-end upstream control conditions on velocity components at y = 24 m:

lateral velocity component

$\widetilde{V}_{y,min;i,(opr,j\pm1),l}$ = -0.5 m/sec, for all elements i=7,8; j=6; l=1,2,3,4.

$\widetilde{V}_{y,max;i,(opr,j\pm1),l}$ = 0.5 m/sec, for all elements i=7,8; j=6; l=1,2,3,4.

longitudinal velocity component

$\widetilde{V}_{x,min;i+\frac{1}{4},(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

$\widetilde{V}_{x,max;i+\frac{1}{4},(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

vertical velocity component

$\widetilde{V}_{z,min;i,(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

$\widetilde{V}_{z,max;i,(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

East-end downstream condition for the surface elevation at x = 44 m (relative to MWL):

$\bar{H}_{(opr,i+1),j}$ = -0.01 m (below MWL), along lateral open-region enclosure surface, downstream of the four bounary elements i=14; j=2,3,4,5; l=4.

East-end downstream control conditions on velocity components at x = 44 m:

longitudinal velocity component

$\tilde{V}_{x,min;(opr,i+1),j,l}$ = -0.5 m/sec, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{x,max;(opr,i+1),j,l}$ = 0.5 m/sec, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

lateral velocity component

$\tilde{V}_{y,min;(opr,i+1),j-\frac{1}{2},l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{y,max;(opr,i+1),j-\frac{1}{2},l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

vertical velocity component

$\tilde{V}_{z,min;(opr,i+1),j,l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{z,max;(opr,i+1),j,l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

The computer simulation results for test case 2 were obtained by considering the open-region enclosure boundary conditions which required the specification of water-surface-elevation conditions on both boundaries at the north-end upstream enclosure surface of the lateral axis

and at the east-end downstream enclosure surface of the longitudinal axis of the flow region. Therefore, according to the particular option for specifying open-region boundary conditions (Sec. 4.5), the velocity components outside of the north-end upstream open-region enclosure surface of the flow region were determined by linear extrapolation, based on the values of the velocity components in the two boundary half-elements along the y axis, in the elements immediately adjacent to the north-end upstream open-region enclosure surface. The control conditions were used for eliminating possible convergence errors that may be caused by overestimating (or underestimating) the velocity conditions outside of the upstream open-region enclosure surface. This option for open-region boundary conditions were included in the discrete-element hydrodynamic model as a general capability for incorporating approximate control conditions at open-region enclosure surfaces where exact values for the velocity components could not be specified. In the case when the velocity components could be determined exactly at the north-end open-region enclosure surface, the specification of $\tilde{V}_{y,min}$ and $\tilde{V}_{y,max}$ as equal to the actual exact values of the velocity components could simulate automatically the required boundary conditions for the velocity components. The velocity components that were not normal to the north-end upstream open-region enclosure surface at y = 24 m were specified as $\tilde{V}_{x,min} = \tilde{V}_{x,max} = 0$ and as $\tilde{V}_{z,min} = \tilde{V}_{z,max} = 0$.

The velocity components for the outflow at the east-end downstream open-region enclosure surface at x = 44 m were not used for extrapolating to the outside of the flow region, since general second-upwind-

differencing formulation automatically determines the values of the velocity components based on the velocity components in the boundary half-elements along the x-axis, immediately adjacent to the east-end downstream open-region enclosure surface at x = 44 m.

b. Interpolation Option

The computer simulation results for test case 2 were obtained by applying the HYDROL computer code with simple second-upwind-differencing interpolation option. It was noted that this interpolation option could introduce numerical dispersion effects in the elements and half-elements where the local Courant number was much less than 1 (see Sec. 4.3).

c. Numerical Solution Option

The computer simulation results for the test case 2 were obtained by applying the HYDROL computer code with the standart default option which ultilized the special, composite time-splitting method in the numerical integration of the governing system of spatially discretized ordinary differential equations (see Sec. 5.2).

d. Discussion of the Computer Simulation Results

The results of the computer simulations for the water surface elevation and velocity conditions are presented in Figs. 6.22 through 6.41, p.590 through p.609. The surface elevation conditions are presented as (1) numbers in the elements which represent water surface elevation (in mm) relative to specified initial mean water level (MWL) and (2) constant water surface elevation contours which correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL. The velocity conditions were presented as two-dimensional vectors which corres-

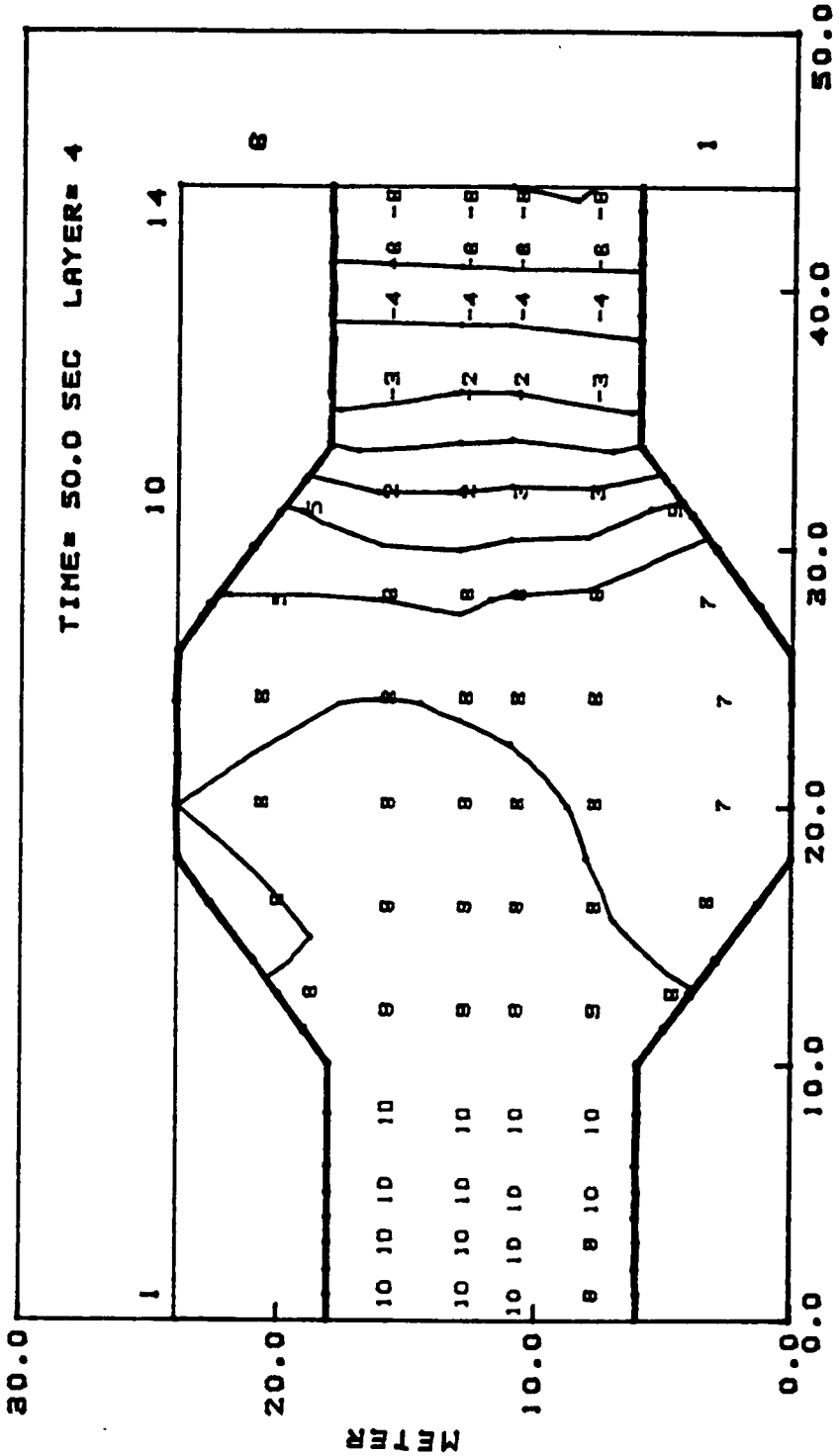

Figure 6.22. Water-surface-elevation conditions at time t = 50 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

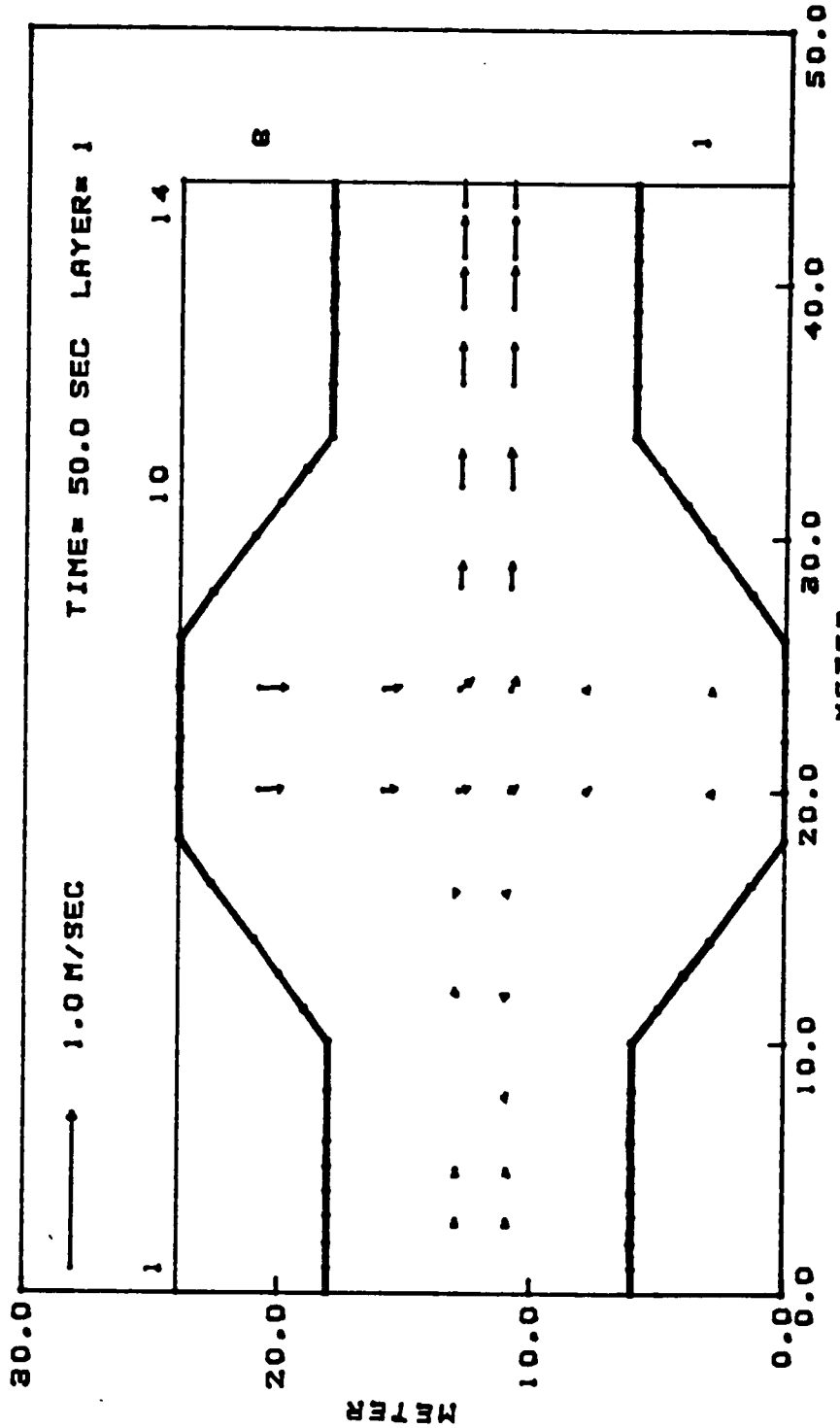

Figure 6.23. Velocity conditions in the bottom layer l = 1, at time t = 50 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

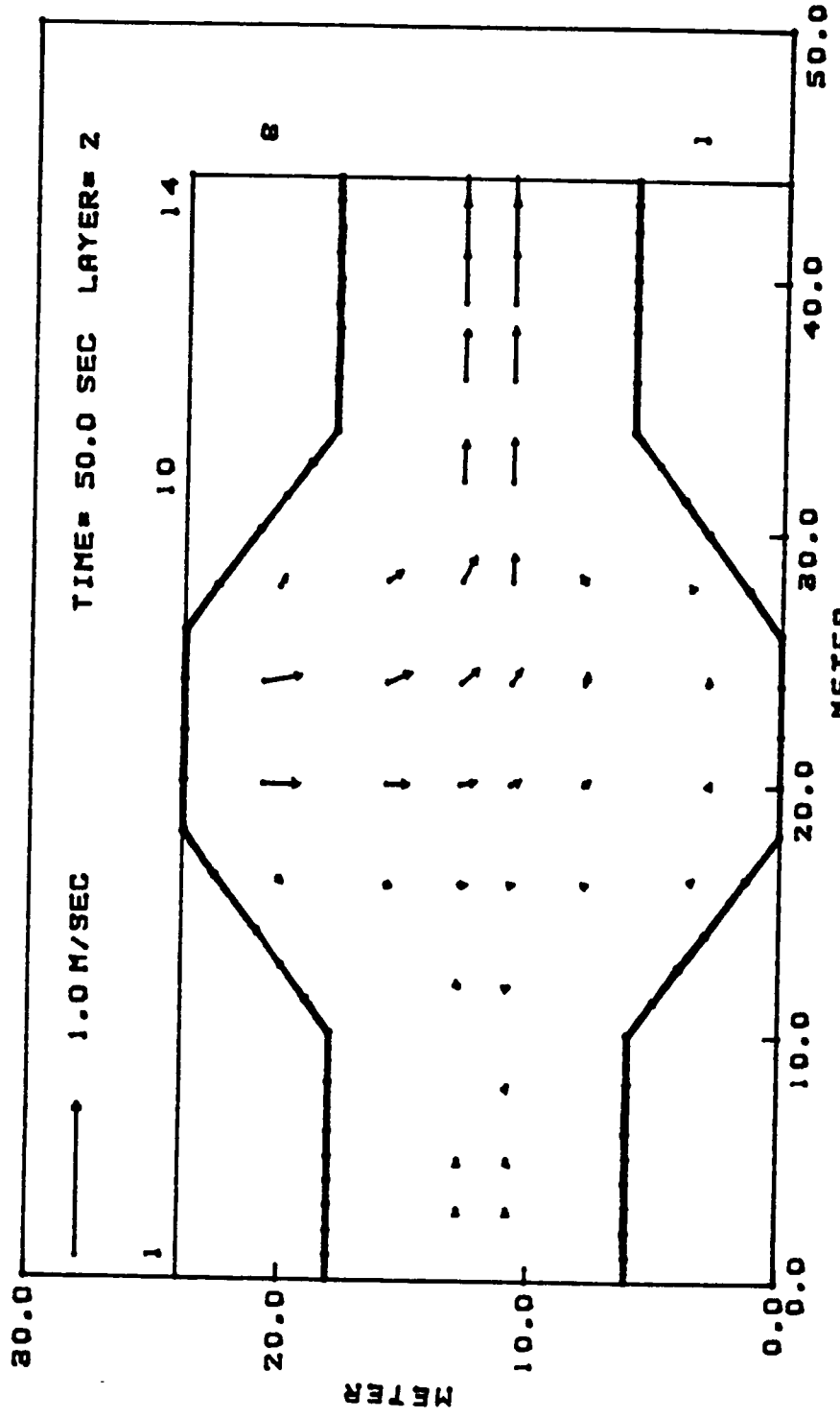

Figure 6.24. Velocity conditions in the internal layer l = 2, at time t = 50 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

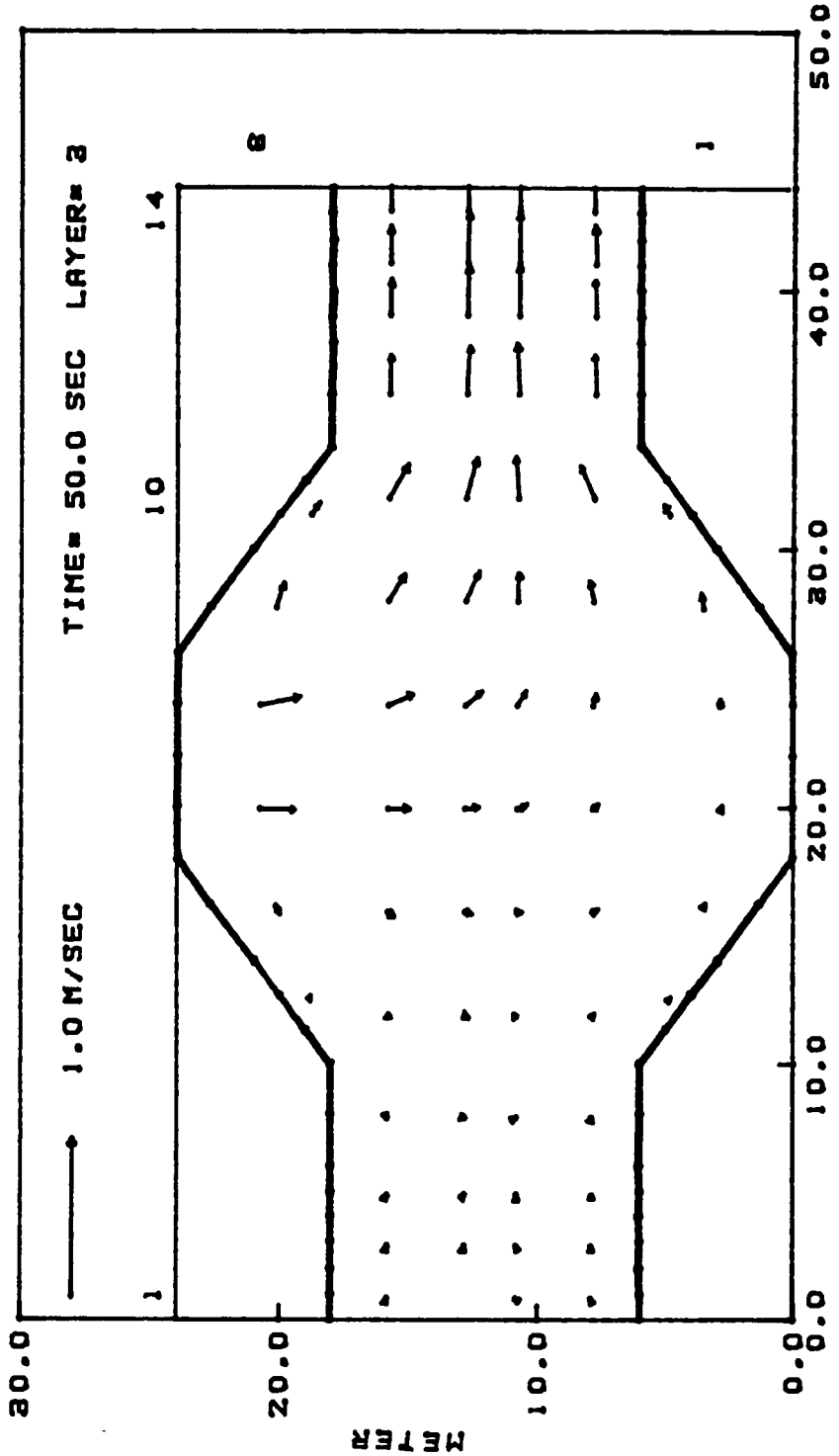

Figure 6.25. Velocity conditions in the internal layer l = 3, at time t = 50 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

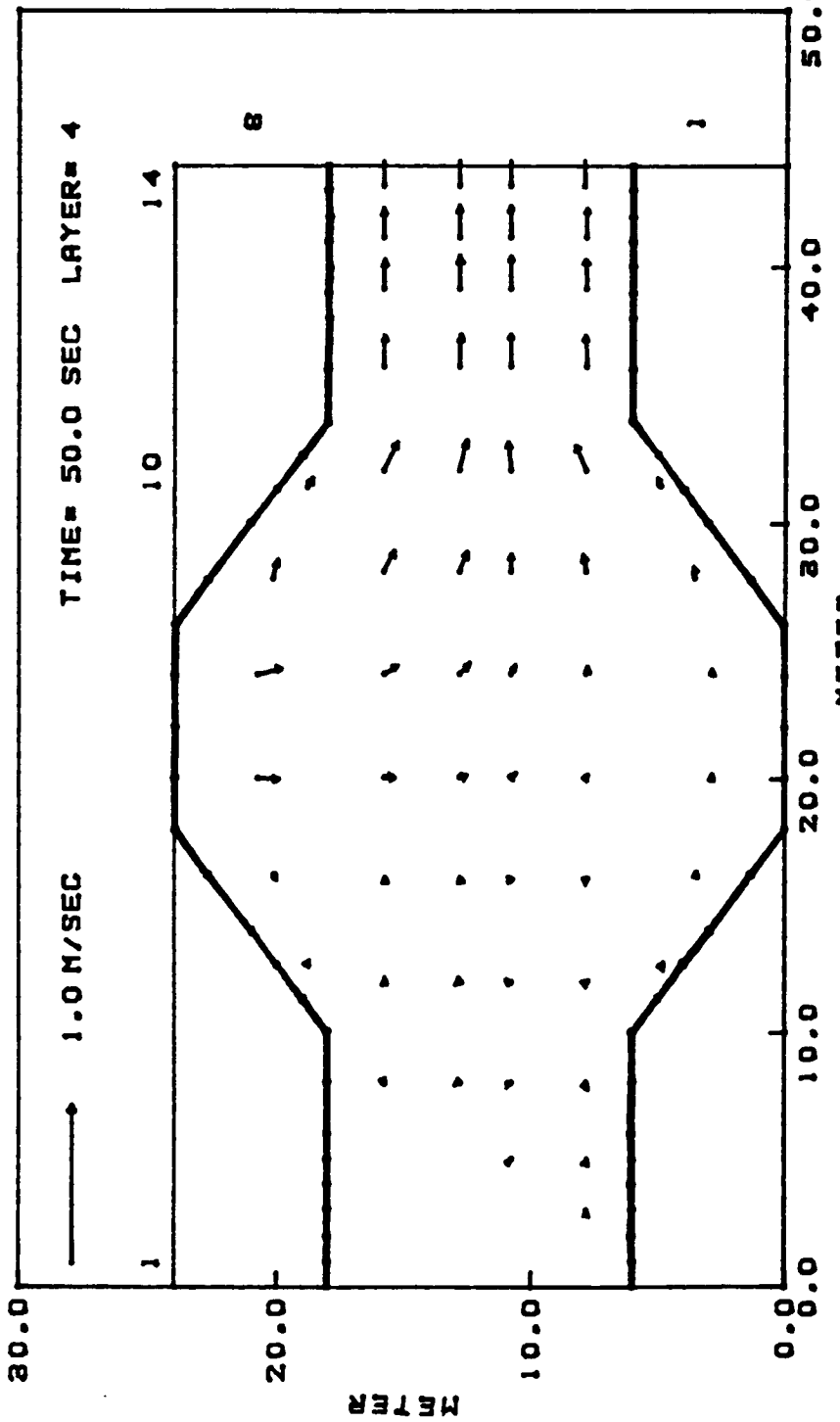

Figure 6.26. Velocity conditions in the surface layer l = 4, at time t = 50 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

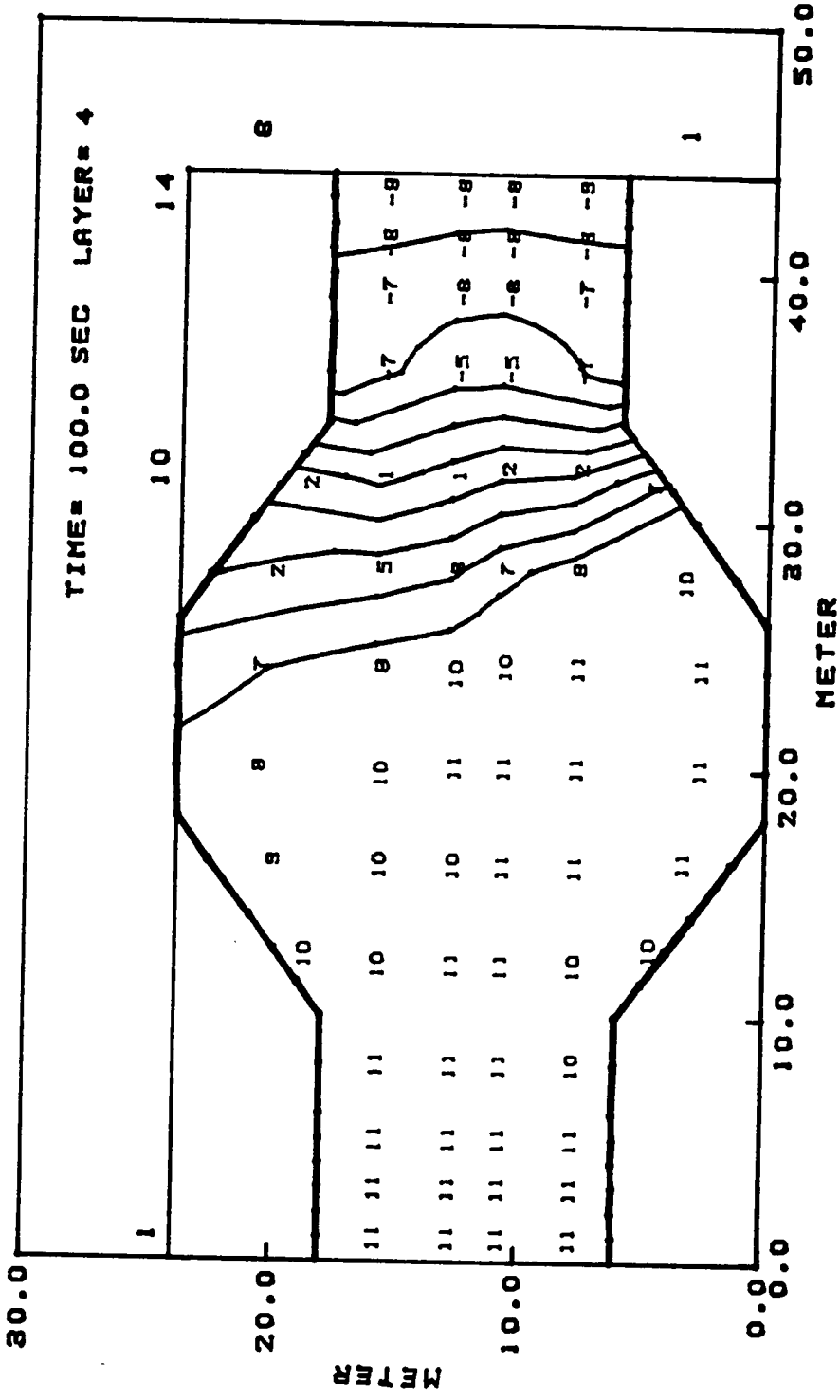

Figure 6.27. Water-surface-elevation conditions at time t = 100 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

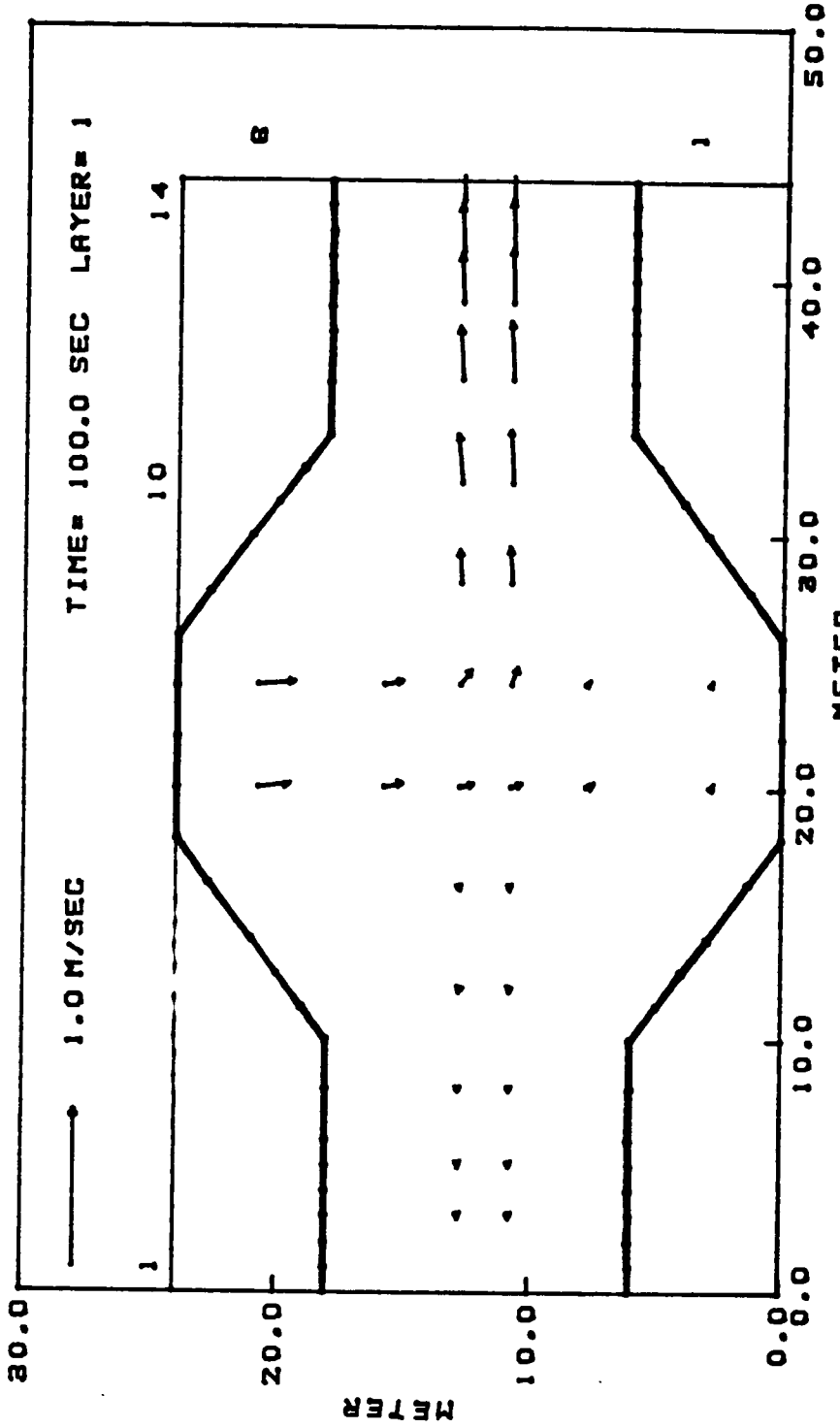

Figure 6.28. Velocity conditions in the bottom layer l = 1, at time t = 100 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

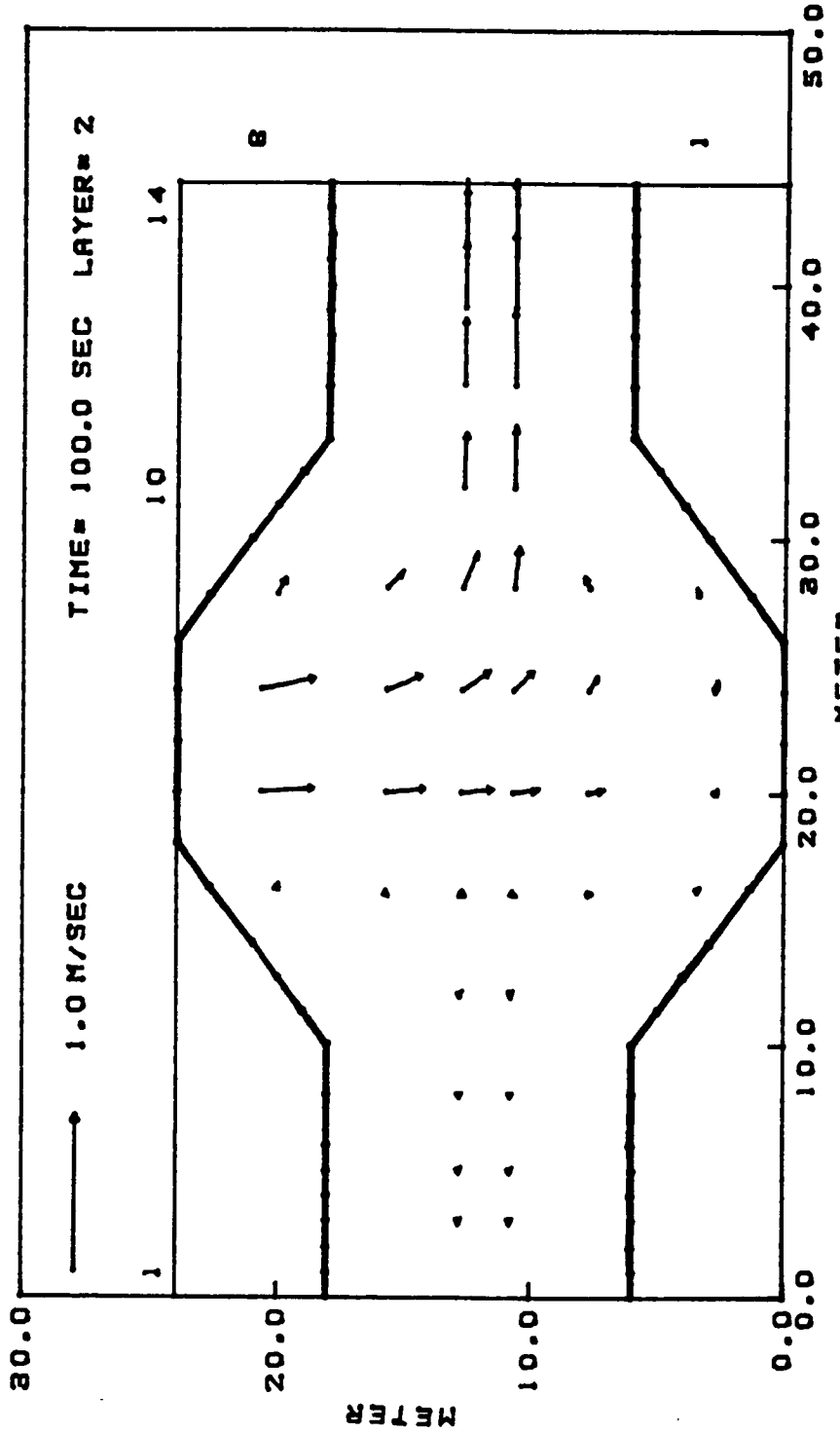

Figure 6.29. Velocity conditions in the internal layer l = 2, at time t = 100 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

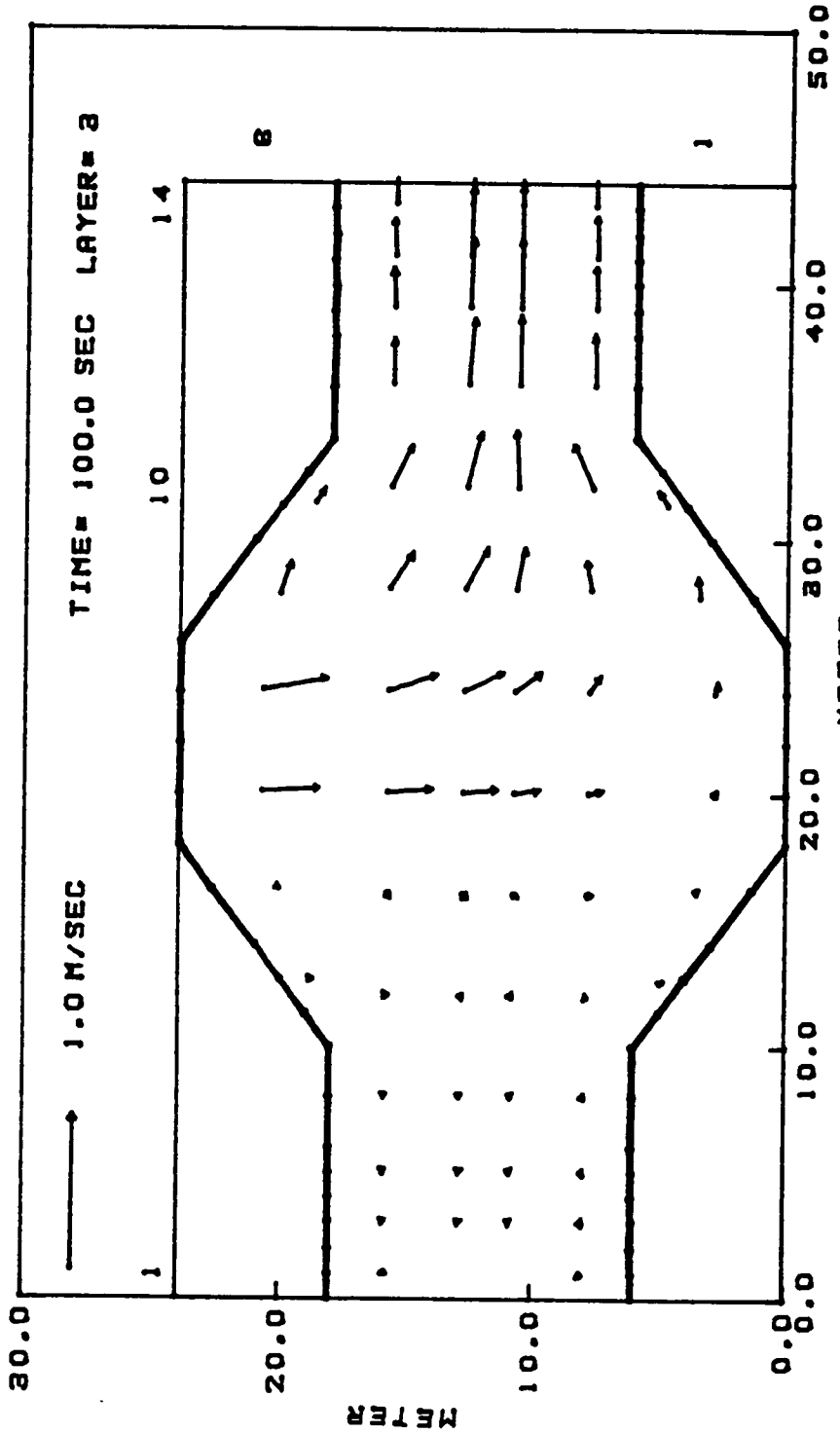

Figure 6.30. Velocity conditions in the internal layer l = 3, at time t = 100 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

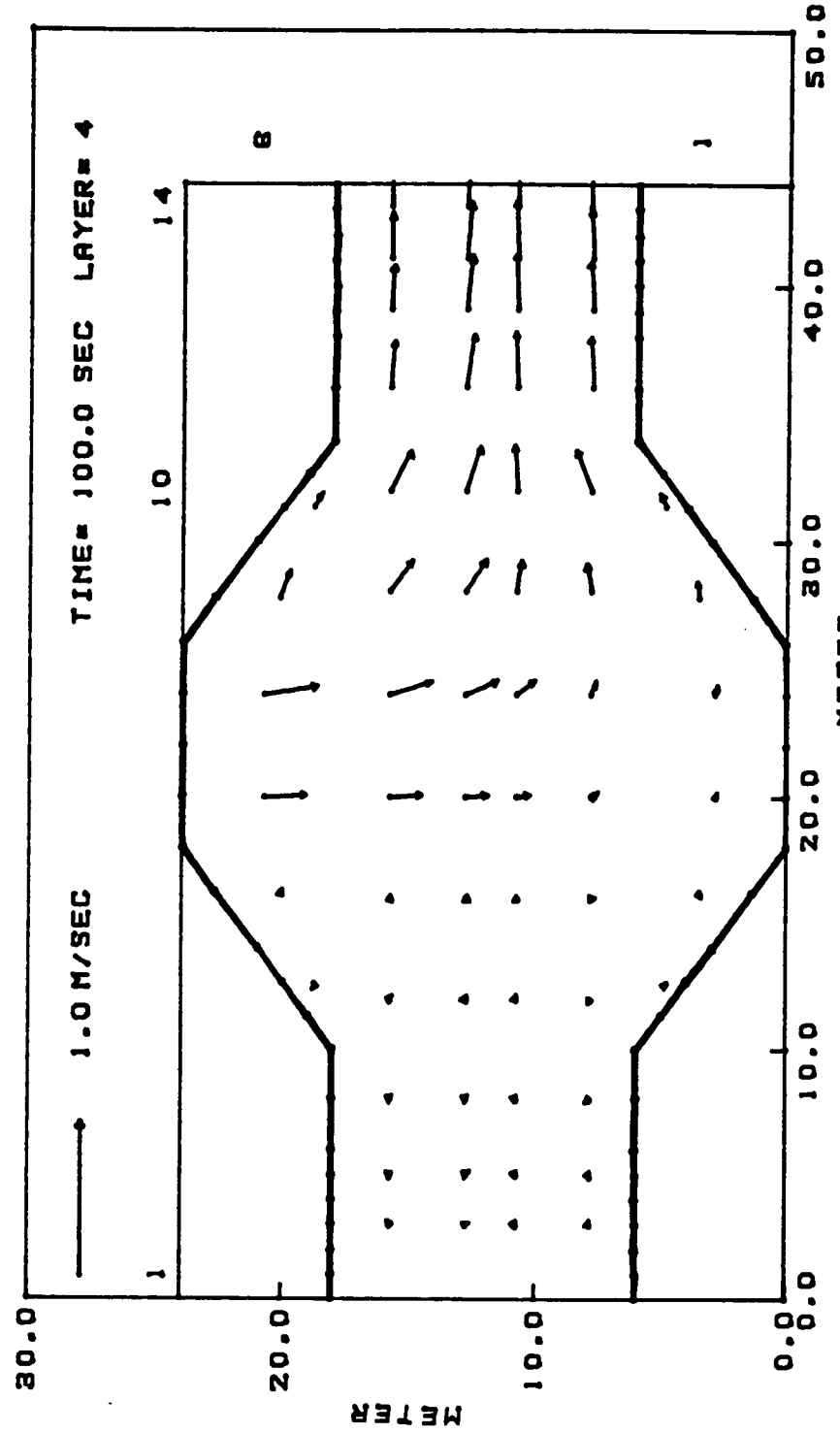

Figure 6.31. Velocity conditions in the surface layer l = 4, at time t = 100 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

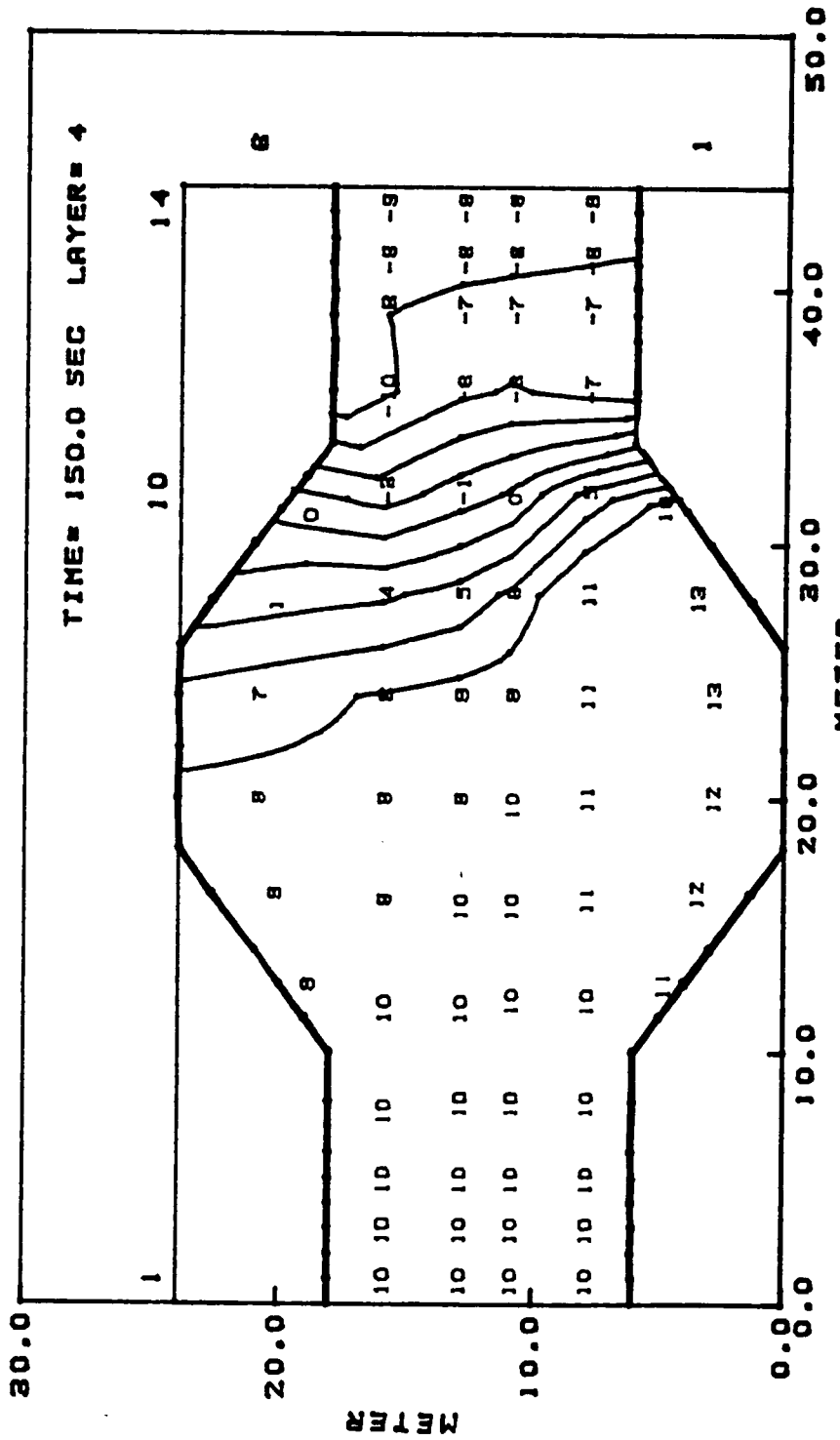

Figure 6.32. Water-surface-elevation conditions at time t = 150 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

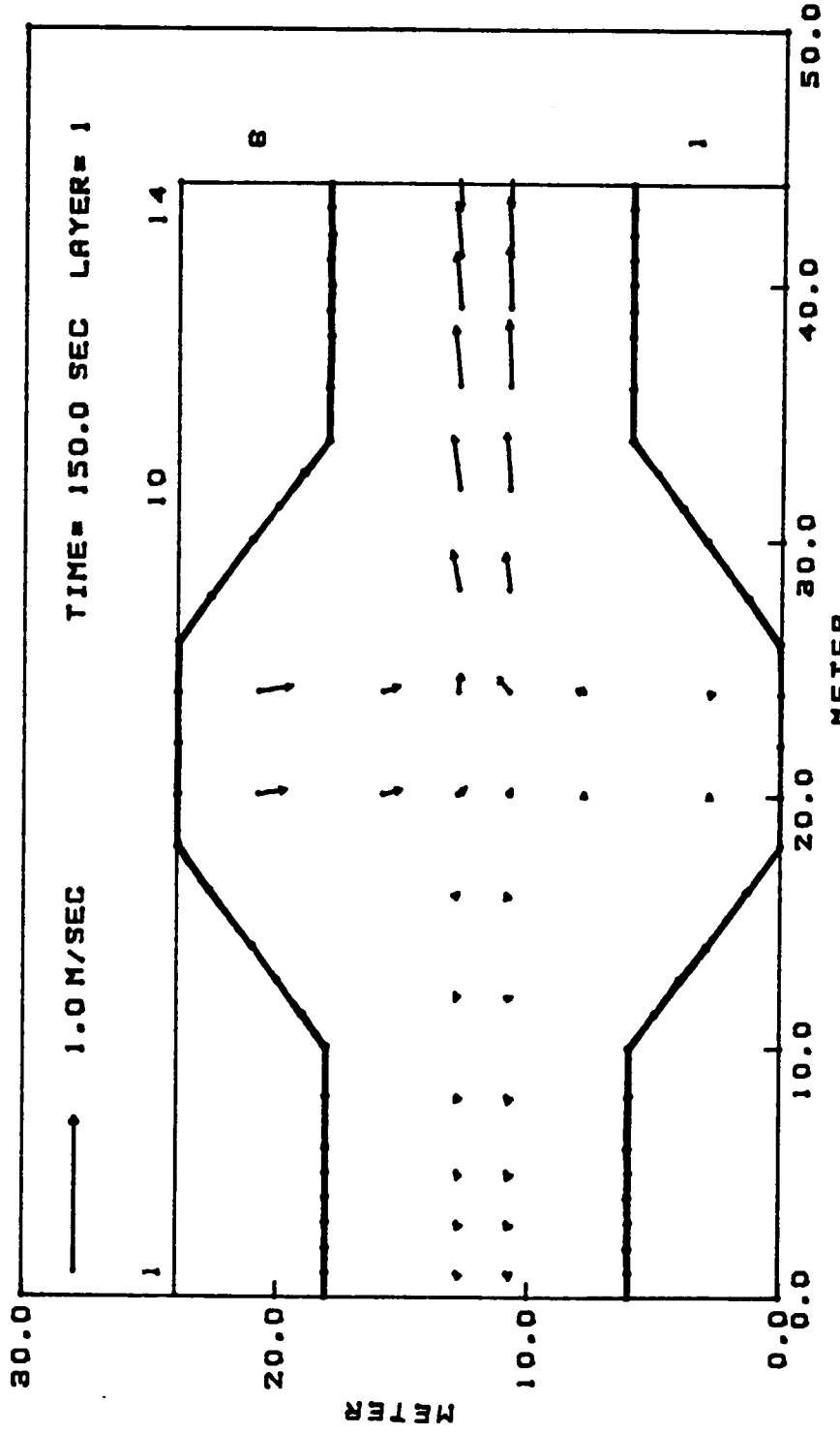

Figure 6.33. Velocity conditions in the bottom layer l = 1, at time t = 150 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

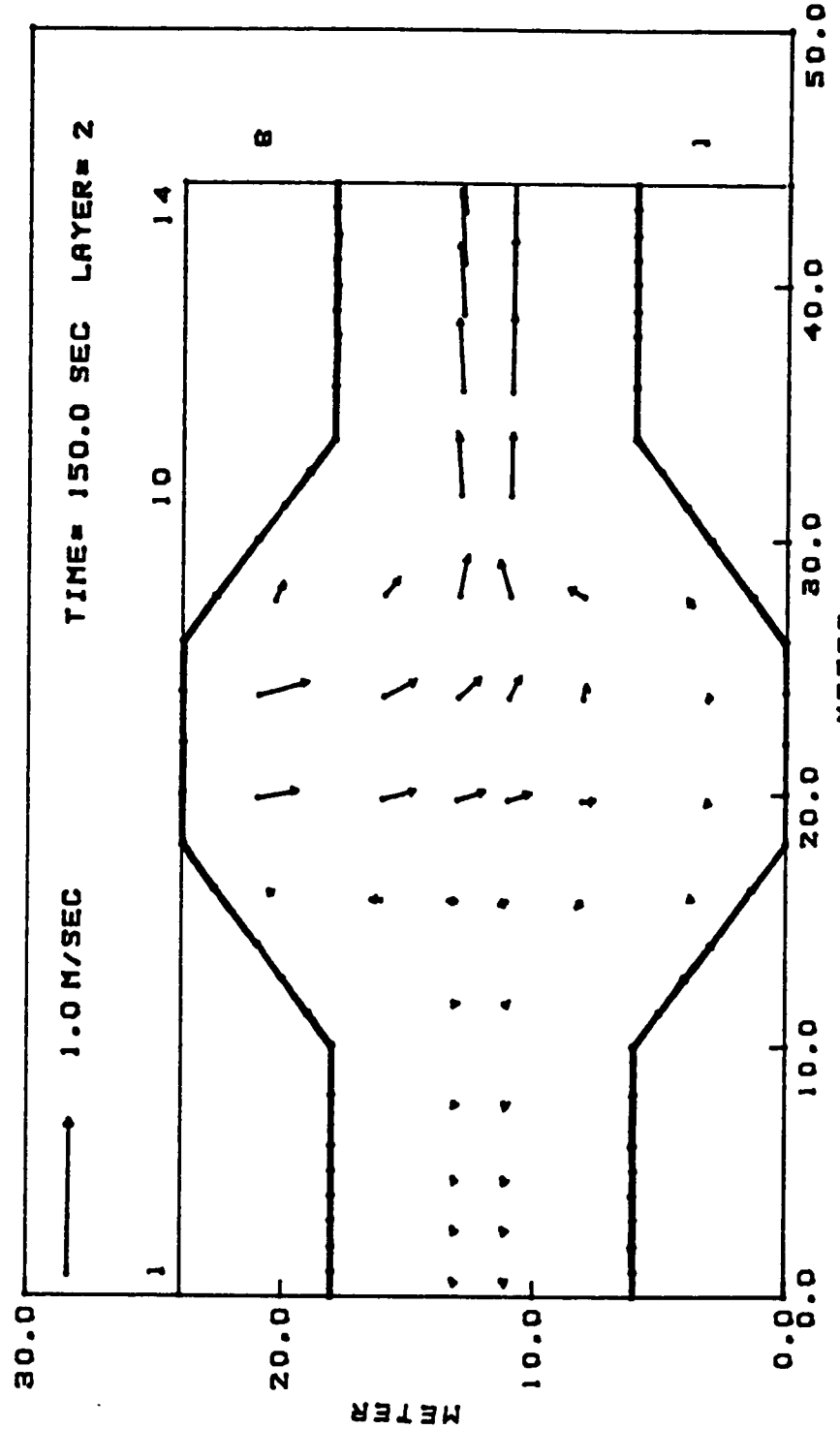

Figure 6.34. Velocity conditions in the internal layer 1 = 2, at time t = 150 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

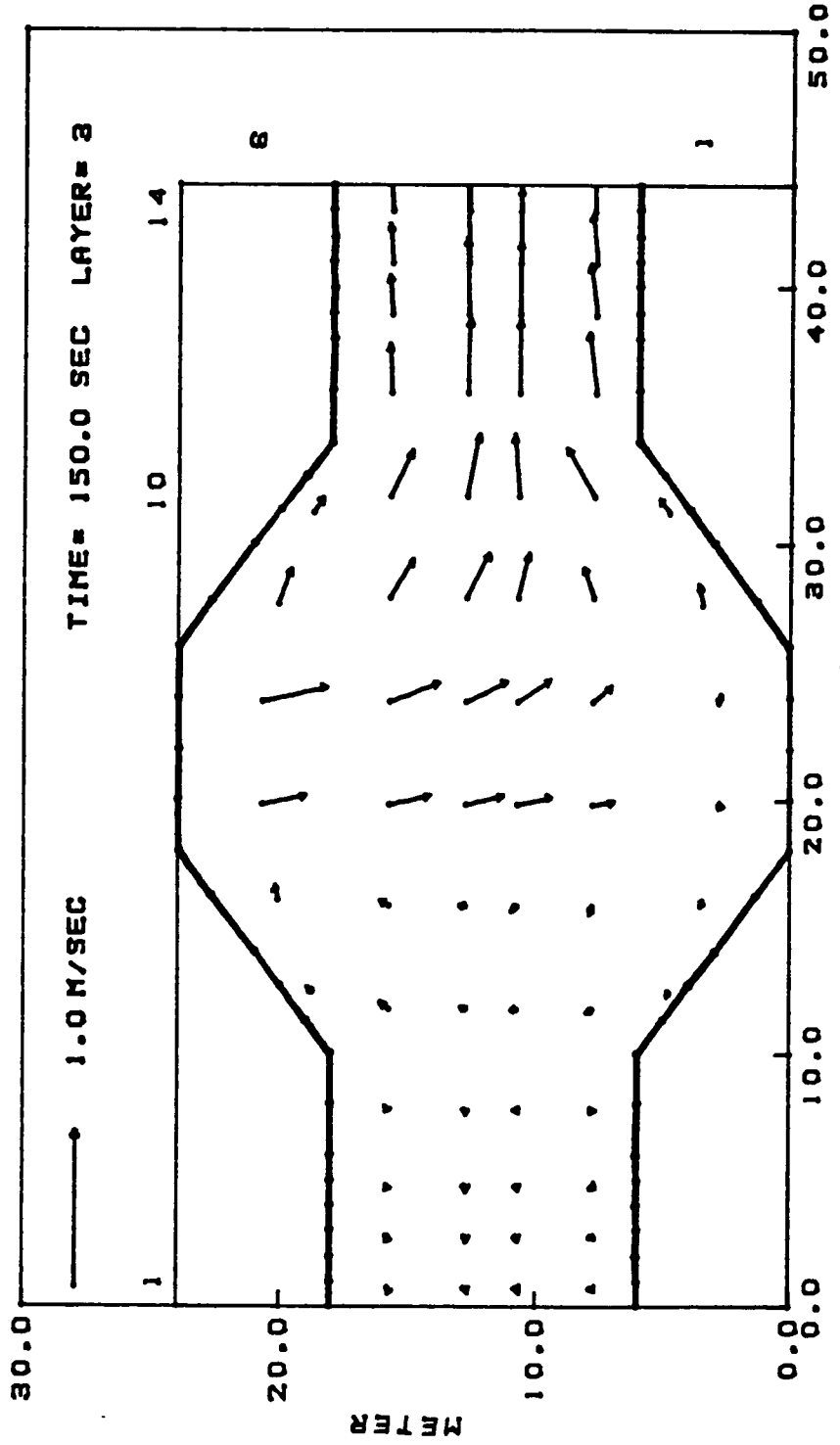

Figure 6.35. Velocity conditions in the internal layer l = 3, at time t = 150 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

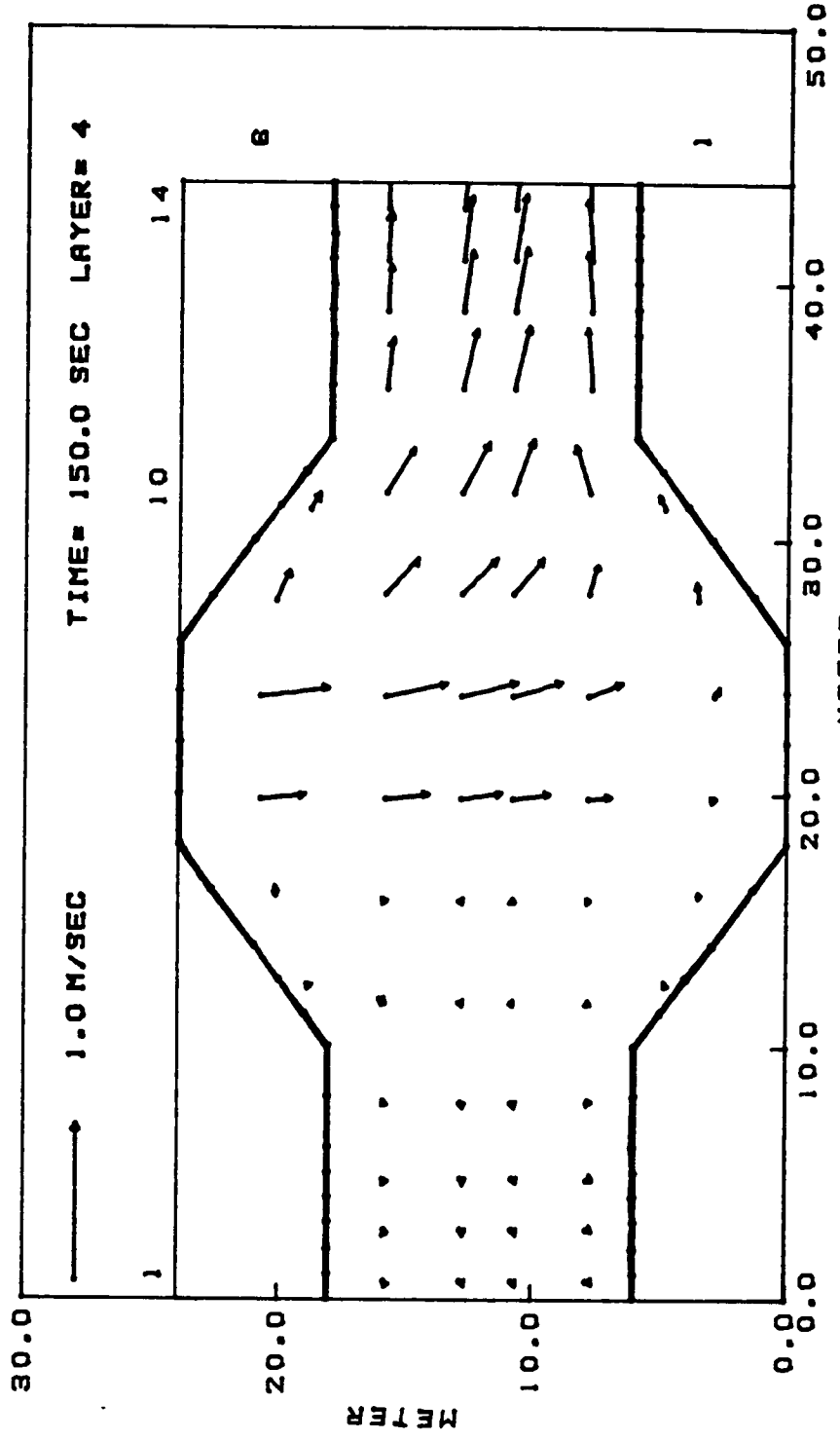

Figure 6.36. Velocity conditions in the surface layer l = 4, at time t = 150 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

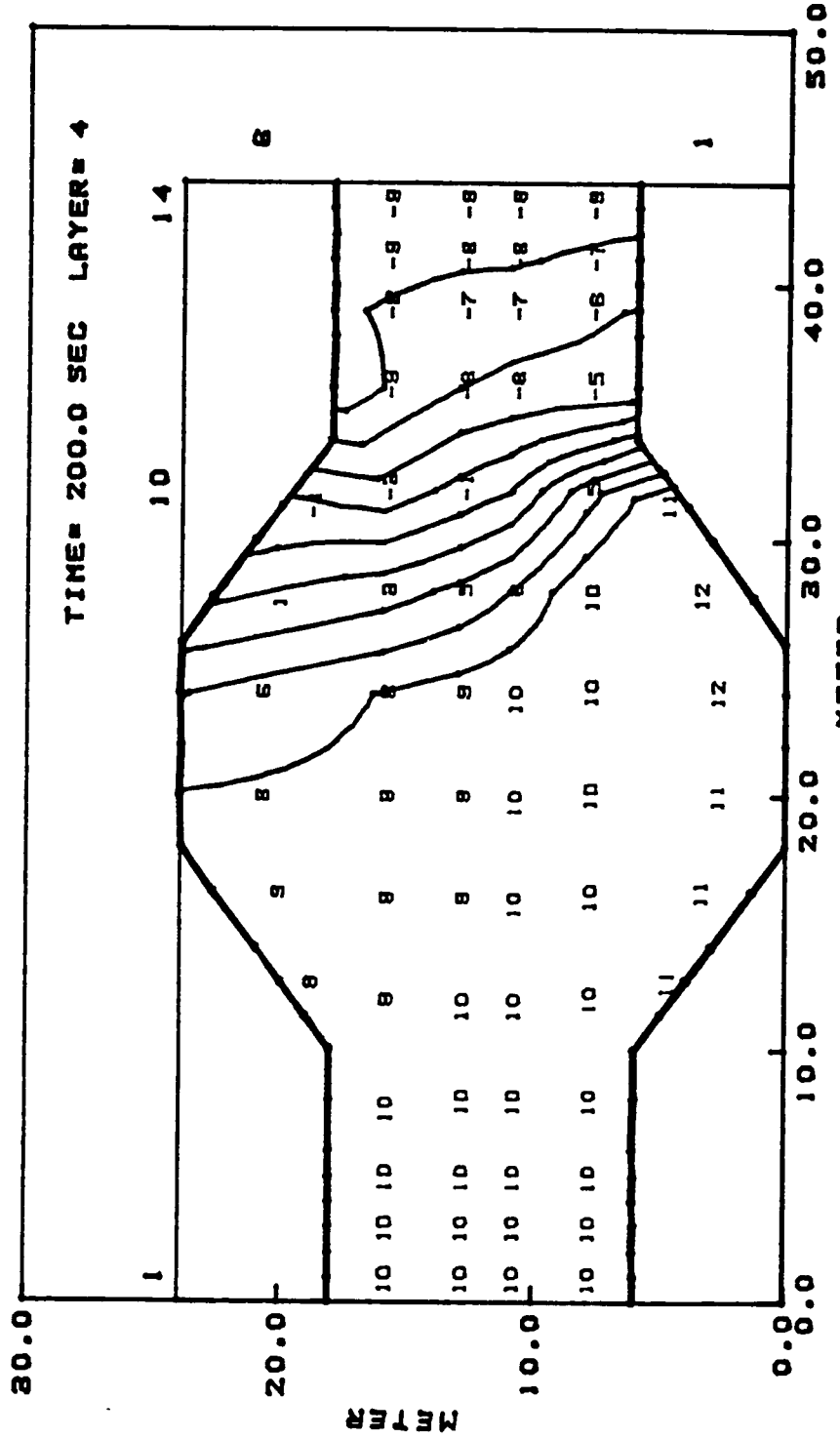

Figure 6.37. Water-surface-elevation conditions at time t = 200 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

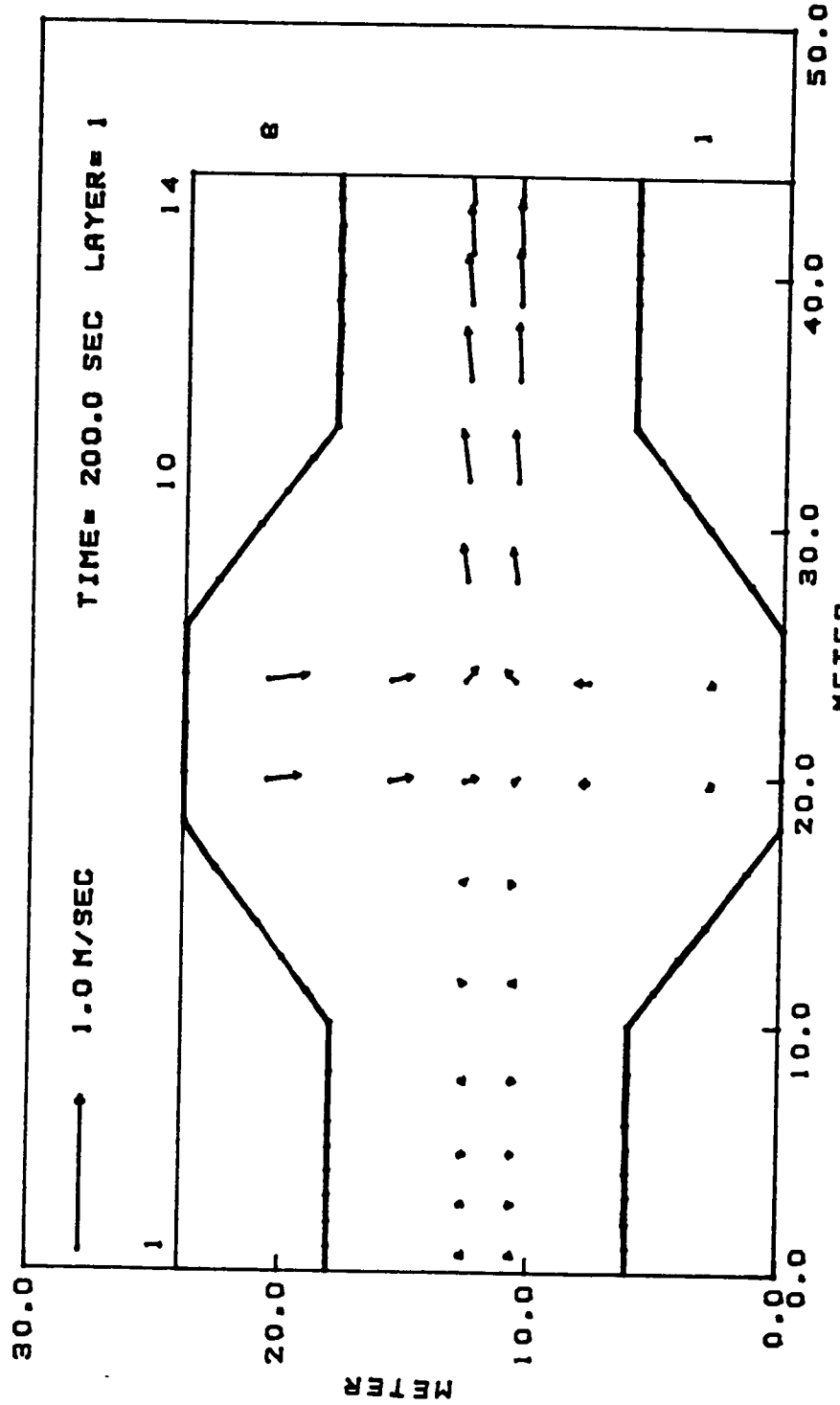

Figure 6.38. Velocity conditions in the bottom layer l = 1, at time t = 200 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

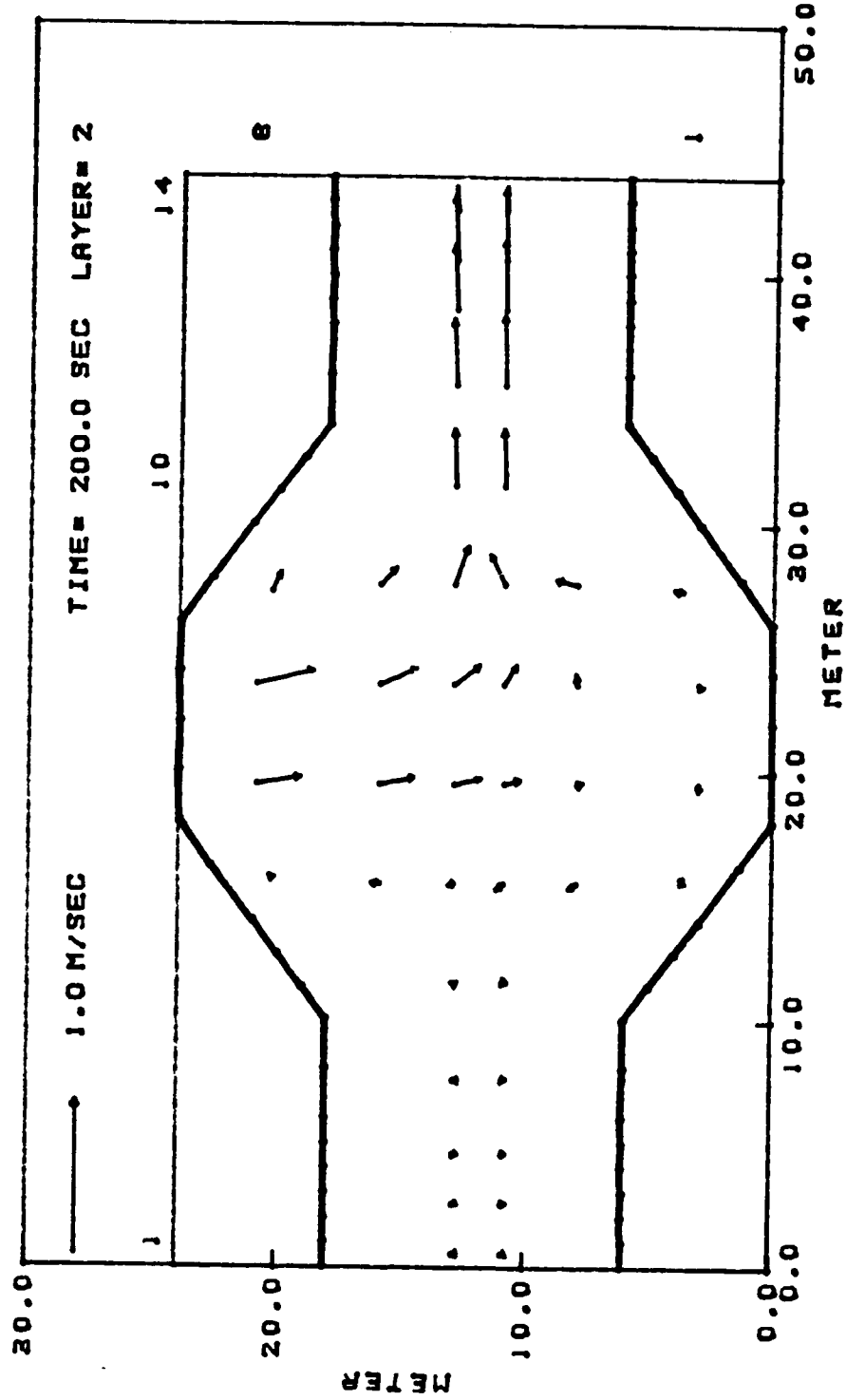

Figure 6.39. Velocity conditions in the internal layer l = 2, at time t = 200 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

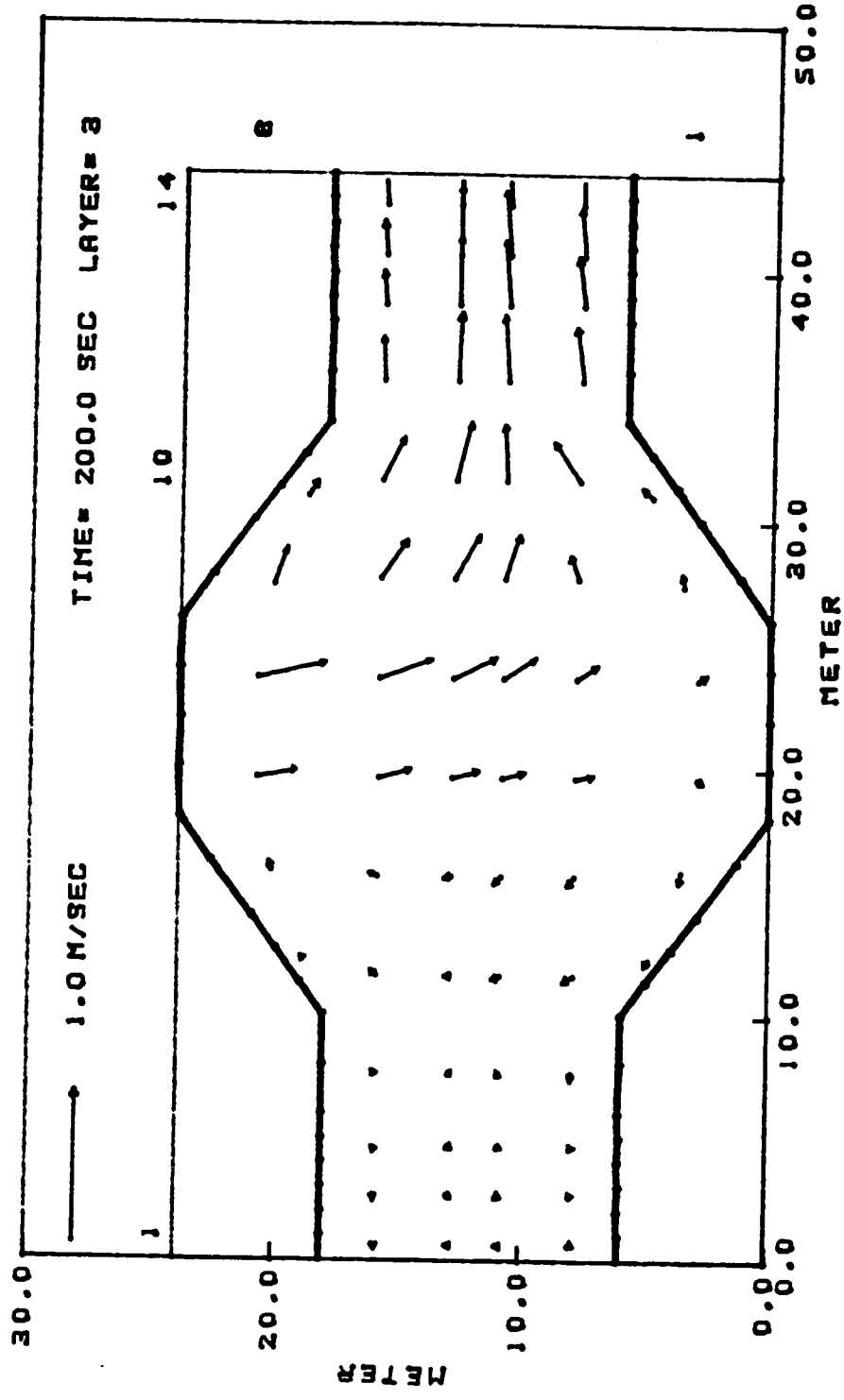

Figure 6.40. Velocity conditions in the internal layer l = 3, at time t = 200 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

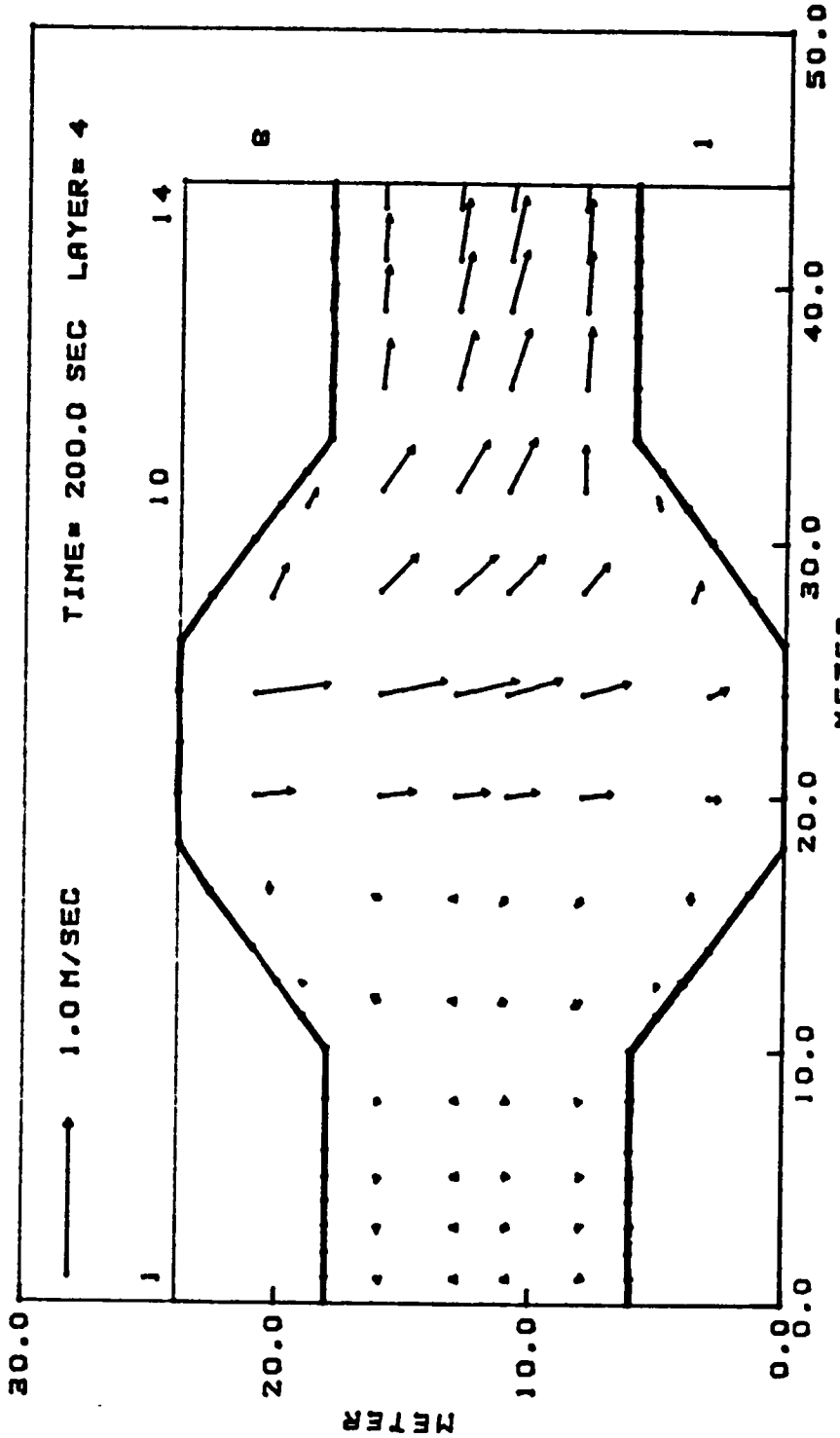

Figure 6.41. Velocity conditions in the surface layer l = 4, at time t = 200 sec for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

pond to two-component representations of velocity in the horizontal (x,y) planes of the four specified layers.

The results of the computer simulations for the flow conditions in the asymmetric laboratory-model-scale flow region again indicated that the water surface elevation and velocity conditions were generally consistent with the conditions that would exist in a similar prototype-scale flow region with a similar configuration for the deep main flow channel.

The special, transient flow characteristics of the water surface elevation and velocity conditions for test case 2 are discussed in detail below in order to illustrate the important correspondance between the results of the computer simulations and certain special characteristics of the physical phenomena that control the flow conditions in the laboratory-model-scale flow region.

The results for constant-water-surface-elevation contours (Fig. 6.22, p.590; Fig. 6.27, p.595; Fig. 6.32, p.600; and Fig. 6.37, p.605) indicate that there is a rapid accumulation of water in most parts of the flow region except in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. The results indicate that at time level t = 50 sec, the constant-water-surface-elevation contours with zero value, which corresponds to the specified initial mean water level (MWL), is already relocated to approximately x = 34 m at the upstream enclosure surface of the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x

= 44 m (Fig. 6.22, p.590). The water-surface-elevation conditions in the narrow subregion (between x = 0 and x = 10 m and between y = 6 m and y = 18 m), near the west-end closed shoreline boundary (Fig. 6.22, p.590), reach approximately 10 mm above the specified initial mean water level (MWL) at time level t = 50 sec. Furthermore, the water-surface-elevation conditions in the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m), in the middle of the flow region, reach values varying between 9 mm and 5 mm above the MWL at time level t = 50 sec (Fig. 6.22, p.590). The computer simulation results indicate that 50 seconds after the start of the flow conditions, the inflowing water through the inlet plane at the north-end upstream enclosure surface at y = 24 m cannot outflow, at the same rate of the inflow, through the discharge plane at the east-end downstream enclosure surface at x = 44 m. Hence, the water rapidly accumulates in most parts of the flow region during the period until time level t = 50 sec and particularly in the narrow subregion (between x = 0 and x = 10 m and between y = 6 m and y = 18 m) near the west-end closed shoreline boundary (Fig. 6.22, p.590).

The results of the computer simulations for water-surface elevation also indicate that the rapid accumulation of water in most parts of the flow region reaches the maximum conditions at time level t = 100 sec (Fig. 6.27, p.595; Fig. 6.32, p.600; and Fig. 6.37, p.605). The water-surface-elevation conditions in the narrow subregion (between x = 0 and x = 10 m and between y = 6 m and y = 18 m), near the west-end closed shoreline boundary, reach 11 mm above the specified initial mean water level (MWL), in the flow region (Fig. 6.27, p.595). As the flow deve-

lopment progresses during time levels t = 100 sec and t = 150 sec (Fig. 6.27, p.595; and Fig. 6.32, p.600), the water-surface-elevation conditions gradually decrease. At time level t = 200 sec, the water-surface-elevation conditions in the flow region attain the pseudo steady-state conditions (Fig. 6.37, p.605), with the values varying between 10 mm, above the specified initial mean water level (MWL), in the narrow subregion (between x = 0 and x = 10 m and between y = 6 m and y = 18 m) near the west-end closed shoreline boundary, and -9 mm below MWL in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. It is important to note that at approximately pseudo steady-state conditions at time level t = 200 sec (Fig. 6.37, p.605), the water-surface-elevation conditions remain well above the specified initial mean water level (MWL), in most parts of the flow region execpt in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m.

The results of the computer simulations for water-surface-elevation conditions (Fig. 6.22, p.590; Fig. 6.27, p.595; Fig. 6.32, p.600; and Fig. 6.37, p.605) can be explained readily by considering various physical conditions which control the flow conditions in the laboratory-model-scale flow region. Accumulation of the water in the narrow subregion (between x = 0 and x = 10 m and between y = 6 m and y = 18 m), near the west-end closed shoreline boundary, and in most parts of the large subregion (between x = 10 m and x = 30 m and between y = 0 and y = 24 m) in the middle of the flow region, during the development of the

pseudo steady-state flow conditions, between time levels t = 100 sec and t = 200 sec (Fig. 6.27, p.595; Fig. 6.32, p.600; and Fig. 6.37, p.605), can be explained by considering two physical characteristics of the simulated flow conditions.

First, since the inflow conditions, at the inlet plane at the north-end upstream enclosure surface at y = 24 m, can attain relatively high y-component of momentum (control condition specified as $\bar{V}_{y,max;i,(opr,j\pm1),l}$ = 0.5 m/sec), the inflowing water can rapidly flood the total area of the divergent geometrical shape of the subregion (between x = 10 m and x = 34 m and y = 18 m and y = 24 m), downstream of the inlet plane of the north-end upstream enclosure surface at y = 24 m. However, the water that flows out of the downstream part of the large subregion (between x = 26 m and x = 34 m and between y = 0 and y = 24 m) first has to undergo rapid contraction due to the convergent geometrical shape of the boundary enclosure surfaces of this part of the subregion. Considering the effects of retardation of flow in the relatively shallow subregions (between x = 26 m and x = 34 m; between y = 0 and y = 10 m and between y = 14 m and y = 24 m), outside of the deep main flow channel along the longitudinal x axis, the accumulated water cannot freely flow into the narrow subregion (between x = 34 m and x = 44 m and y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. Hence, the water has the tendency to accumulate in the flow region (Fig. 6.22, p.590; Fig. 6.27, p.595; Fig. 6.32, p.600; and Fig. 6.37, p.605), since it can flow easier into the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m), downstream of the inlet plane at the north-end upstream

enclosure surface at y = 24 m, than it can flow out of the downstream part of the large subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. Furthermore, since the x-component of momentum of the outflow conditions at the discharge plane at the east-end downstream enclosure surface at x = 44 m depends (from physical considerations) on the x-component of the momentum of the water upstream of the discharge plane, and since the flow in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane, at the east-end downstream enclosure surface at x = 44 m, is approximately unidirectional along the downstream-x direction, the x-component of momentum of the water in the narrow subregion cannot increase progressively along the downstream-x direction. Hence the flow conditions in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at east-end downstream enclosure surface at x = 44 m, in a sense, also retardes the flow of water out of the downstream part of the large subregion (between x = 26 m and x = 34 m and between y = 0 and y = 24 m) in the flow region.

Second, the imposed water-surface-elevation condition, at the north-end upstream open-region enclosure surface at y = 24 m, as 0.01 m (10 mm) above the specified initial mean water level (MWL), represents an imposed positive hydrostatic pressure differential which initiates a compression type of surface gravity wave that travels along the downstream y-direction (from north toward south). The surface gravity wave rapidly spreads over the entire area of the flow region due to the

divergent geometrical characteristics of the enclosure boundaries of the subregion (between x = 10 m and x = 34 m and between y = 18 m and y = 24 m), downstream of the inlet plane at north-end upstream enclosure surface at y = 24 m. Similarily, the imposed water-surface-elevation condition, at the east-end downstream open-region enclosure surface at x = 44 m, as -0.01 m (-10 mm) below the specified initial mean water level (MWL), represents an imposed negative hydrostatic pressure differential which initiates a rarefaction type of surface gravity wave that travels along the upstream-x direction against the predominantly unidirectional flow which is along the downstream-x direction in the narrow subregion (between x = 34 m and x = 44 m and y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. Since the two surface gravity waves travel generally along orthogonal direction, the compression type of surface gravity wave, propagating along the downstream-y direction, and initiating from the inlet plane at the north-end upstream enclosure surface at y = 24 m, intersects the rarefaction type of surface gravity wave, propagating along the upstream-x direction, and initiating from the discharge plane at the east-end downstream enclosure surface at x = 44 m, in the subregion (between x = 20 m and x = 30 m and y = 6 m and y = 24 m). Consequently, the remaining part of the flow region, consisting of the large subregion (between x = 0 and x = 25 m and between y = 0 and y = 16 m) does not get influenced directly by the hydrostatic pressure differential caused by the compression type and the rarefaction type of surface gravity waves, which intersect downstream along x and upstream along y, relative to the middle of the flow region, as the pseudo steady-state conditions become

established between time levels t = 100 sec and t = 200 sec (Fig. 6.27, p.595; Fig. 6.32, p.600; and Fig. 6.37, p.605). Hence, the water already accumulated in the large subregion (between x = 0 and x = 25 m and between y = 0 and y = 16 m), during the short time period at the beginning of the flow conditions, between the time levels t = 0 and t = 100 sec (Fig. 6.22, p.590; and Fig. 6.27, p.595), becomes trapped in the large subregion and results in high water-surface-elevation conditions, varying between 12 mm and 8 mm above the initial specified mean water level (MWL) in the flow region.

The computer simulation results also indicate that , as the pseudo steady-state flow conditions start being established in the flow region, the maximum water-surface-elevation conditions occur in a subregion (between x = 14 m and x = 30 m and between y = 0 and y = 8 m) near the south-end closed shoreline boundary (Fig. 6.32, p.600; and Fig. 6.37, p.605). This result is to be expected since the subregion (between x = 14 m and x = 30 m and between y = 0 and y = 8 m) is directly oppsite to the inlet plane at the north-end upstream enclosure surface at y = 24 m, along the lateral direction of the y axis. Since the inflow conditions, at the inlet plane at the north-end upstream enclosure surface at y = 24 m, can attain relatively high y-component of momentum (control condition specified as $\bar{V}_{y,max;i,(opr,j\pm1),l}$ = 0.5 m/sec), the inflowing water along the y-direction can partially overcome the hydrostatic pressure effect, which turns the flow toward the x-direction and into the narrow subregion (between x = 34 m and x = 44 m and y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m (Fig. 6.35, p.603; Fig. 6.36, p.604; Fig. 6.40,

p.608; and Fig. 6.41, p.609). Hence, the inflowing water, from the north-end upstream enclosure surface at y = 24 m, can reach the subregion (between x = 14 m and x = 30 m and between y = 0 and y = 8 m) near the south-end closed shoreline boundary, which is directly opposite to the inlet plane along the y-direction; and hence, it can result in higher water-surface-elevation conditions in the subregion (Fig. 6.32, p.600; and Fig. 6.37, p.605).

It is again important to note that the gradual accumulation of water in the flow region (Fig. 6.22, p.590; Fig. 6.27, p.595; Fig. 6.32, p.600; and Fig. 6.37, p.605) is also a consequence of the open-region boundary conditions that were used in the computer simulations. The specified increased water-surface-elevation condition, at the north-end upstream open-region enclosure surface at y = 24 m, as 0.01 m (10 mm) above the specified initial mean water level (MWL), results in an increase of 0.08 m^2 of inflow area, since the width of the upstream enclosure surface is 8 m. Similarly, the specified decreased water-surface-elevation condition, at the east-end downstream open-region enclosure surface at x = 44 m, as -0.01 m (-10 mm) below the specified initial MWL, results in a decrease of -0.12 m^2 of outflow area, since the width of the downstream enclosure surface is 12 m. Therefore, effectively, the inflow conditions at the upstream open-region enclosure surface at y = 24 m have an available flow area which is 0.20 m^2 larger than the available flow area for the outflow conditions at the downstream open-region enclosure surface at x = 44 m.

The inflow conditions at the inlet plane at the north-end upstream enclosure surface at y = 24 m, can attain relatively high values for the

y-component of velocity (control condition specified as $\tilde{V}_{y,min;i,(opr,j\pm1),l} = -0.5$ m/sec), according to the specified open-region boundary conditions, immediately after the start of the flow conditions in the flow region. On the other hand, the outflow conditions at the discharge plane at the east-end downstream enclosure surface at x = 44 m cannot attain a value for the x-component of velocity with a magnitude higher than the magnitude of the y-component of velocity at the north-end upstream enclosure surface at y = 24 m, until the increased water-surface-elevation conditions in the flow region can cause sufficiently high hydrostatic pressure gradient conditions that can accelerate the flow along the x direction. Therefore, during the period of flow development, the inflow rate at the inlet plane at the north-end upstream enclosure surface at y = 24 m, with effectively increased value of 0.12 m^2 available flow area, is higher than the outflow rate at the discharge plane at the east-end downstream enclosure surface at x = 44 m, with effectively decreased value of -0.12 m^2 available flow area. Hence, during the period of flow development, the inflow rate again is effectively higher than the outflow rate, and hence, the water-surface-elevation conditions again gradually increase in most parts of the flow region.

The consideration of the physical characteristics of the flow conditions clearly indicate that the computer simulation results of the discrete-element hydrodynamic model (HYDROL) for the transient water-surface-elevation conditions are realistic and representative of the conditions that would exist for test case 2: Nonsymmetric Flow (inflow

along lateral direction and outflow along longitudinal direction) in an actual laboratory-model-scale flow region.

The computer simulation results for velocity conditions (Figs. 6.23 through 6.26, p.591 through p.594; Figs. 6.28 through 6.31, p.596 through p.599; Figs. 6.33 through 6.36, p.601 through p.604; and Figs. 6.38 through 6.41, p.606 through p.609) indicate that the three-dimensional flow generally turns a quadrant angle from the inflow conditions along the y-direction, at the inlet plane at the north-end upstream enclosure surface at y = 24 m, to the outflow conditions along the x-direction at the discharge plane at the east-end downstream enclosure surface at x = 44 m. The primary flow conditions generally occur in approximately half of the flow region (between x = 18 m and x = 44 m and between y = 0 and y = 24 m), where the velocity conditions change direction in the downstream-x part of the large subregion in the middle of the flow region and in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end enclosure surface at x = 44 m (Figs. 6.23 through 6.26, p.591 through p.594; Figs. 6.28 through 6.31, p.596 through p.599; Figs. 6.33 through 6.36, p.601 through p.604; and Figs. 6.38 through 6.41, p.606 through p.609). Three-dimensional primary flow is predominantly in the subregion of the deep main flow channel (between x = 20 m and x = 28 m and between y = 6 m and y = 24 m along y-direction and between x = 20 m and x = 34 m and between y = 10 m and y = 14 m along x -direction) and in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m.

The computer simulation results indicate that the velocity conditions in the upper layers l = 3,4, associated with the primary flow conditions, develop rapidly into pseudo steady-state conditions in the flow region approximately at time level t = 100 sec (Fig. 6.30, p.598; and Fig. 6.31, p.599). The two-dimensional components of velocity of the primary flow in the upper layer l = 3,4 do not vary appreciably during the remaining time period of the computer simulations between time levels t = 100 sec and t = 200 sec (Fig. 6.30, p.598; Fig. 6.31, p.599; Fig. 6.35, p.603; Fig. 6.36, p.604; Fig. 6.40, p.608; and Fig. 6.41, p.609)

The computer simulation results of the three-dimensional velocity conditions for test case 2: Nonsymmetric flow (inflow along lateral direction and outflow along longitudinal direction) also indicate the existence of two types of secondary flows. The two tpyes of secondary flows indicate the formation of secondary vortices in the horinontal and vertical planes of the flow region in certain subregions where the hydrostatic effects and the three-dimensional viscous effects collectively enhance the possibility of vortex formation.

First, at time levels t = 150 sec and t = 200 sec, the computer simulation results indicate a reversal of the velocity conditions in the lower layers l = 1,2 of the subregion (between x = 20 m and x = 30 m and between y = 6 m and y = 10 m), between the south-end closed shoreline boundary and the middle of the flow region (Figs. 6.33, p.601; Fig. 6.34, p.602; Fig. 6.38, p.606; and Fig. 6.39, p.607). The two-component representations of the velocity vectors in the horixontal (x,y) planes of the lower layers l = 1,2 indicate that the flow in the lower layers

of the subregion is generally against the direction of the inflow which is along the lateral y-direction from the north-end toward the south-end of the flow region during the time period between the time levels t = 150 sec and t = 200 sec (Figs. 6.33, p.601; Fig. 6.34, p.602; Fig. 6.38, p.606; and Fig. 6.39, p.607). However, the two-component representations of the velotiy vectors in the horizontal (x,y) planes of the upper layers l = 3,4 indicate that the flow in the upper layers of the subregion (between x = 20 m and x = 30 m and between y = 6 m and y = 10 m) is generally along the inflow direction from the north-end toward the south-end of the flow region during the same time period between time levels t = 150 sec and t = 200 sec (Fig. 6.35, p.603; Fig. 6.36, p.604; Fig. 6.40, p.608; and Fig. 6.41, p.609). Hence, the computer simulation results indicate that there exist a two-layer reversed flow in the subregion (between x = 20 m and x = 30 m and between y = 6 m and y = 10 m), during the time period between time levels t = 150 sec and t = 200 sec. The existence of the two-layer reversed flow conditions in the subregion (between x = 20 m and x = 30 m and between y = 6 m and y = 10 m) at time level t = 200 sec, indicates the possible formulation of a secondary-flow vortex in the vertical (y,z) plane in the subregion, under pseudo steady-state conditions in the flow region. The formation of a secondary-flow vortex in the vertical (y,z) plane, according to the results of the computer simulations, can be readily explained by considering the physical conditions that control the flow in the subregion (between x = 20 m and x = 30 m and between y = 6 m and y = 10 m), between the south-end closed shoreline boundary and the middle of the flow region.

The inflow conditions at the inlet plane at the north-end upstream enclosure surface at y = 24 m can attain relatively high y-component of momentum (control condition specified as $\tilde{V}_{y,max;i,(opr,j\pm1),l}$ =0.5 m/sec). Hence, the inflowing water rapidly reaches the south-end closed shoreline boundary of the flow region which is directly opposite the inlet plane along y-direction; and it results in higher water-surface-elevation conditions in the subregion (between x = 14 m and x = 30 m and between y = 0 and y = 8 m) between the south-end closed shoreline boundary and the middle of the flow region (Fig. 6.32, p.600; and Fig. 6.37, p.605). The hydrostatic pressure effect, which results from increased water-surface-elevation conditions in the subregion (between x = 14 m and x = 30 m and between y = 0 and y = 8 m), cannot readily overcome the effect of the y-component of momentum in the upper layers l = 3,4, since the viscous effects have relatively small influence in retarding the flow in the upper layers l = 3,4, of the flow region, particularly in the subregion of the deep main flow channel (between x = 18 m and x = 26 m). However, the visicous effects in the lower layers l = 1,2 have relatively high influence in retrading the flow; hence the y-component of momentum of flow in the lower layers l = 1,2, reaching the subregion (between x = 14m and x = 30 m and between y = 0 and y = 8 m), is generally much lower than the y-component of momentum of flow in the upper layers l = 3,4. Hence, the hydrostatic pressure effect (gradient along lateral y-direction), resulting from increased water-surface-elevation conditions in the subregion (between x = 14 m and x = 30 m and between y = 0 and y = 8 m) causes the flow in the lower layers l = 1,2, particularly in the subregion of the deep main flow channel, to

reverse and flow from the south end toward the north end of the flow region, against the north-end upstream enclosure surface at y = 24 m (Fig. 6.38, p.606; and Fig. 6.39, p.607).

Second, at time levels t = 150 sec and t = 200 sec the simulations for two-component representations of velocity conditions indicate the formations of a secondary-flow vortex in the subregion (between x = 10 m and x = 20 m and between y = 0 and y = 24 m) in the horizontal (x,y) planes of the upper three layers l = 2,3,4 of the flow region (Figs. 6.34 through 6.36, p.602 through p.604; and Figs. 6.39 through 6.41, p.607 through p.609). The formation of this secondary-flow vortex in the subregion (Fig. 6.42, p.624; and Fig. 6.43, p.625) in the horizontal (x,y) planes of the upper layers l = 2,3,4 can also be explained by considering the physical conditions associated with the viscous effects which collectively control the flow conditions in the flow region. Since the inflow conditions at the inlet plane at the north-end upstream enclosure surface at y = 24 m can attain relatively high y-component of momentum (control condition specified as $\tilde{V}_{y,min;i,(opr,j\pm1),l}$ = -0.5 m/sec), the three-dimensional inlet flow is, in a sense, similar to the conditions that would be introduced by a jet discharge from the north-end upstream enclosure surface into the flow region. Hence, the viscous effects in the vicinity of the inlet plane of the discharge jet (at y = 24 m) generate the entrainment flow which is along the transverse lateral axis of the jet (i.e., along x-direction of the flow region). Furthermore, the flow of the discharge jet is restricted along its longitudinal axis (i.e., along y-direction of the flow region) by the impermeable, south-end shoreline boundary at y = 0 which forces the flow

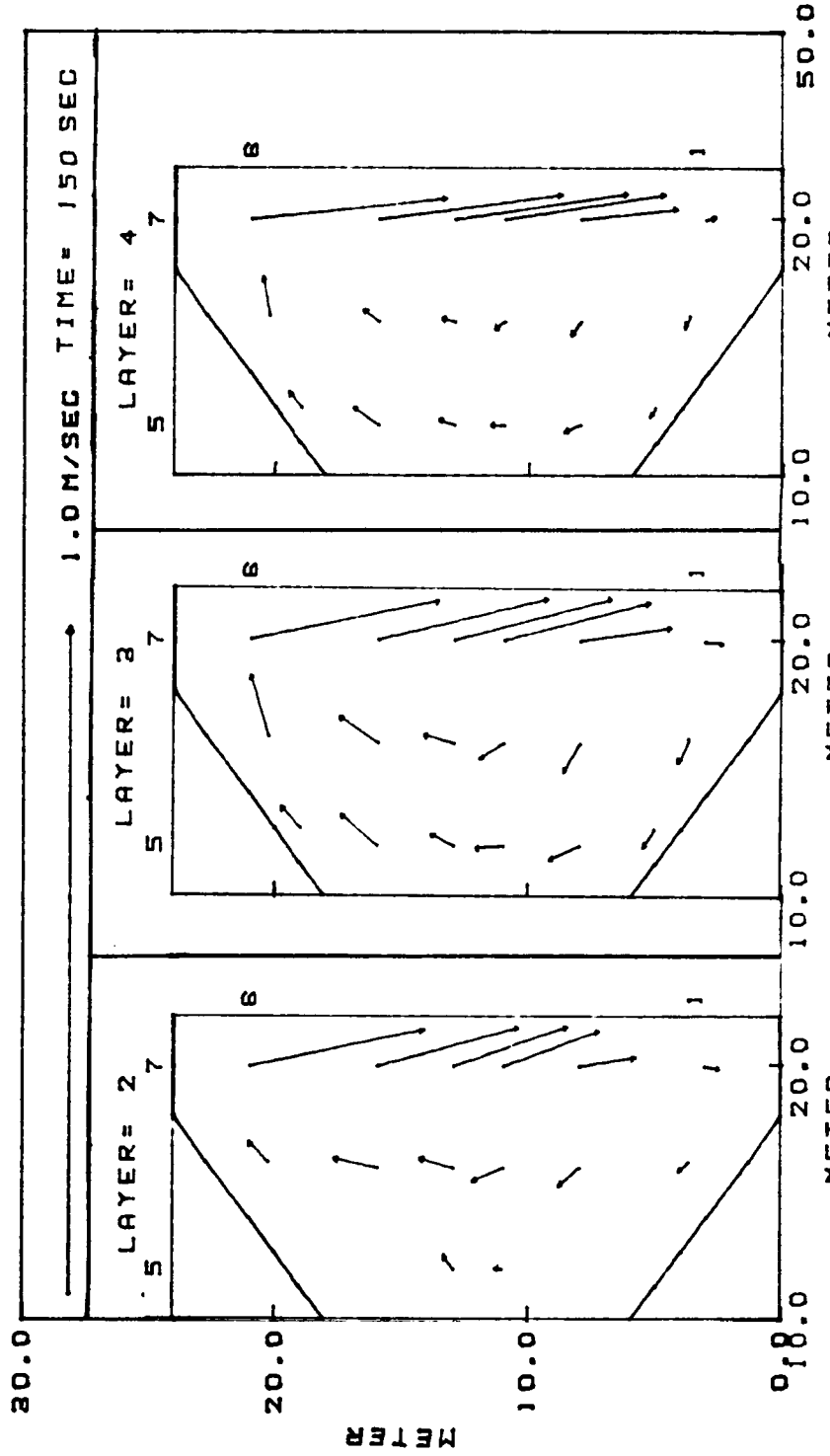

Figure 6.42. Exageraled representation of velocity conditions indicating the formation of secondary-flow vortex in the three upper layers I = 2,3,4 at time t = 150 sec for test 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vector correspond to two-component representations of velocity in the horizontal (x,y) planes of the upper layers I = 2,3,4 in the subregion.

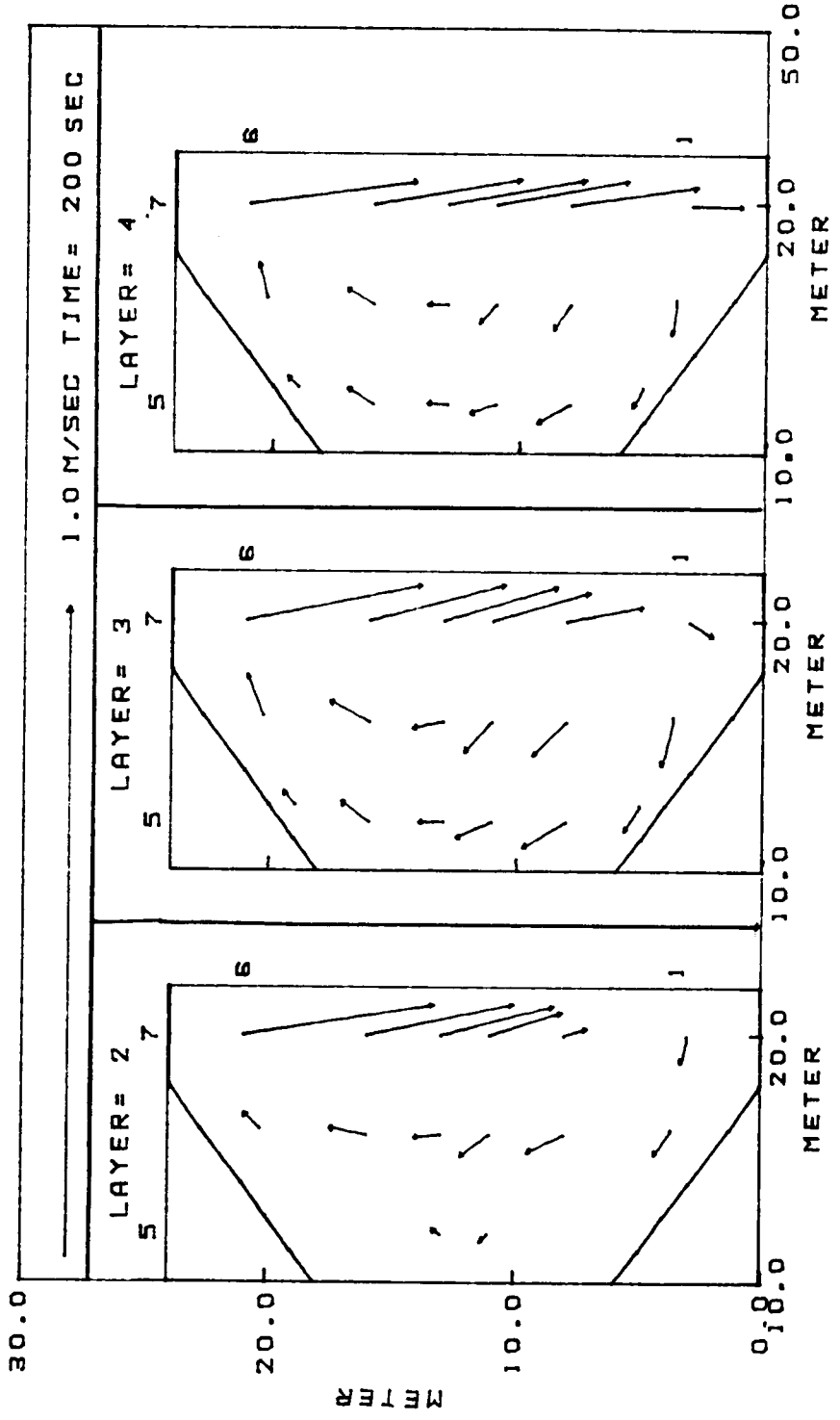

Figure 6.43. Exageraled representation of velocity conditions indicating the formation of secondary-flow vortex in the three upper layers l = 2,3,4 at time t = 200 sec for test 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction). Vector correspond to two-component representations of velocity in the horizontal (x,y) planes of the upper layers l = 2,3,4 in the subregion.

to turn toward a direction normal to the orginal direction of the discharge flow (i.e., toward the direction of the x-axis of the flow region). Hence, consistent with the behavior of restricted jet discharges (e.g., Jirka, Abrahams and Harleman, 1975, Eraslan and Lin, 1981), the combined effect of the entrainment flow in the vicinity of the north-end upstream enclosure surface at y = 24 m and the turning flow in the vicinity of the south-end closed shoreline boundary at y = 0, contributes to the formation of the secondary-flow vortex in the subregion (between x = 10 m and x = 20 m and between y = 0 and y = 24 m) in the horizontal planes of the upper three layers l = 2,3,4 of the flow region (Figs. 6.34 through 6.36, p.602 through p.604; and Figs. 6.39 through 6.41, p.607 through p.609).

Also, at time levels t = 150 sec and t = 200 sec, the computer simulation results for water-surface elevation (Fig. 6.32, p.600; and Fig. 6.37, p.605) indicate that the accumulated water levels in the vicinity of the south-end closed shoreline boundary at y = 0 are relatively higher than the north-end upstream enclosure surface at y = 24 m in the subregion (between x = 10 m and x = 20 m and between y = 0 and y = 24 m). The difference in the accumulated water levels in the subregion result in a negative hydrostatic pressure gradient along the y-direction from the south-end closed shoreline boundary at y = 0 toward the north-end upstream enclosure surface at y = 24 m. Therefore, the effect of the hydrostatic pressure gradient, caused by the differences in water-surface-elevation conditions (Fig. 6.32, p.600; and Fig. 6.37, p.605), also contributes to the formation of the secondary-flow vortex (Figs. 6.34 through 6.36, p.602 through p.604; and Figs. 6.39 through

6.41, p.607 through p.609) in the subregion (between x = 10 m and x = 20 m and between y = 0 and y = 24 m).

Hence, based on considerations of the physical effects, the results of the computer simulations of the water-surface-elevation and three-dimensional velocity conditions in the laboratory-model-scale flow region for test case 2: Nonsymmetric Flow (inflow along lateral direction and outflow along longitudinal direction), again illustrates the reliability of the mathematical formulation of the transient, three-dimensional, discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of its associated HYDROL computer code.

6.1.2.3 Test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). The third test case and the subsequent fourth and fifth test cases were considered for the simulation of the symmetric flow conditions in the laboratory-model-scale flow region. The symmetric opposing flow cases were considered specifically to test the reliability of the mathematical formulation of the transient, three-dimensional, discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of the HYDROL computer code under extremely demanding flow simulation conditions. It is important to note that since the specified inflow conditions are in exactly opposite directions, and since the simulations are transient, any possible unstable characteristic of the mathematical formulation and/or the computational algorithm of the discrete-element hydrodynamic model would tend to destroy (1) the exact symmetry of the water surface elevation conditions about the central x-axis and the

central y-axis and (2) the exact symmetry or asymmetry of the velocity components about the xentral x-axis or the central y-axis associated with the simulated flow conditions in the symmetric flow region.

The necessary input data to the HYDROL computer code for the third test case was prepared according to the following conditions associated with the mathematical formulation of the discrete-element hydrodynamic model.

a. Boundary Conditions

The test case was run by considering open-region boundary conditions (1) at two upstream enclosure surfaces, (a) the south-end upstream enclosure surface at y = 0 and (b) the north-end upstream enclosure surface at y = 24 m, and (2) at two downstream enclosure surfaces, (c) the west-end downstream enclosure surface at at x = 0 and (d) the east-end downstream enclosure surface at x = 44 m, of the flow region. All other enclosure boundaries were assumed to be closed, impermeable shoreline boundary surfaces.

South-end upstream condition for water surface elevation at y = 0 (relative to MWL):

$\bar{H}_{i,(opr,j\pm1)}$ = 0.01 m (above MWL), along longitudinal open-region enclosure surface, upstream of the two boundary elements i=7,8; j=1; l=4.

South-end upstream control conditions on velocity components at y = 0:

lateral velocity component

$\bar{V}_{y,min;i,(opr,j\pm1),l}$ = -0.5 m/sec, for all elements i=7,8; j=1; l=1,2,3,4.

$\tilde{V}_{y,max;i,(opr,j\pm1),l}$ = 0.5 m/sec, for all elements i=7,8; j=1; l=1,2,3,4.

longitudinal velocity component

$\tilde{V}_{x,min;i-\frac{1}{2},(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

$\tilde{V}_{x,max;i-\frac{1}{2},(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

vertical velocity component

$\tilde{V}_{z,min;i,(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

$V_{z,ups,max;i,j=1,l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

North-end upstream condition for water surface elevation at y = 24 m (relative to MWL):

$\bar{H}_{i,(opr,j\pm1)}$ = 0.01 m (above MWL), along longitudinal open-region enclosure surface, upstream of the two boundary elements i=7,8; j=6; l=4.

North-end upstream control conditions on velocity components at y = 24 m:

lateral velocity component

$\tilde{V}_{y,min;i,(opr,j\pm1),l}$ = -0.5 m/sec, for all elements i=7,8; j=6; l=1,2,3,4.

$\tilde{V}_{y,max;i,(opr,j\pm1),l}$ = 0.5 m/sec, for all elements i=7,8; j=6; l=1,2,3,4.

longitudinal velocity component

$\tilde{V}_{x,min;i-\frac{1}{2},(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

$\tilde{V}_{x,max;i-\frac{1}{2},(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

vertical velocity component

$\tilde{V}_{z,min;i,(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

$\tilde{V}_{z,max;i,(opr,j\pm1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

West-end downstream condition for the surface elevation at x = 0 (relative to MWL):

$\overline{H}_{(opr,i-1),j}$ = -0.01 m (below MWL), along lateral open-region enclosure surface, downstream of the four boundary elements i=1; j=2,3,4,5; l=4.

West-end downstream control conditions on velocity components at x = 0:

longitudinal velocity component

$\widetilde{V}_{x,min;(opr,i-1),j,l}$ = -0.5 m/sec, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\widetilde{V}_{x,max;(opr,i-1),j,l}$ = 0.5 m/sec, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

lateral velocity component

$\widetilde{V}_{y,min;(opr,i-1),j-\frac{1}{4},l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\widetilde{V}_{y,max;(opr,i-1),j-\frac{1}{4},l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

vertical velocity component

$\widetilde{V}_{z,min;(opr,i-1),j,l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\widetilde{V}_{z,max;(opr,i-1),j,l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

East-end downstream condition for the surface elevation at x = 44 m (relative to MWL):

$\overline{H}_{(opr,i+1),j}$ = -0.01 m (below MWL), along lateral open-region enclosure surface, downstream of the four boundary elements i=14; j=2,3,4,5; l=4.

East-end downstream control conditions on velocity components at x = 44 m:

longitudinal velocity component

$\tilde{V}_{x,min;(opr,i+1),j,l} = -0.5$ m/sec, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{x,max;(opr,i+1),j,l} = 0.5$ m/sec, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

lateral velocity component

$\tilde{V}_{y,min;(opr,i\pm1),j-\frac{1}{4},l} = 0$, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{y,max;(opr,i\pm1),j-\frac{1}{4},l} = 0$, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

vertical velocity component

$\tilde{V}_{z,min;(opr,i\pm1),j,l} = 0$, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\tilde{V}_{z,max;(opr,i\pm1),j,l} = 0$, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

The computer simulation results for test case 3 were obtained by considering the open-region boundary conditions which required the specification of water-surface-elevation conditions on all four boundary enclosure surfaces of the flow region, (1) at two upstream enclosure surfaces, (a) the south-end upstream enclosure surface at y = 0 and (b) the norht-end upstream enclosure surface at y = 24 m, and (2) at two downstream enclosure surfaces, (c) the west-end downstream enclosure surface at x = 0 and (d) the east-end downstream enclosure surface at x = 44 m. Therefore, according to the particular option for specifying

open-region boundary conditions (Sec. 4.5), the velocity components outside of the south-end upstream open-region enclosure surface of the flow region were determined by linear extrapolation, based on the values of the velocity components in the two boundary half-elements along the y axis, in the element immediately adjacent to the south-end upstream open-region enclosure surface. Similarly, the velocity components outside of the north-end upstream open-region enclosure surface of the flow region were determined by linear extrapolation, based on the values of the velocity components in the two boundary half-elements along the y axis, in the element immediately adjacent to the north-end upstream open-region enclosure surface. The control conditions were used for eliminating possible convergence errors that may be caused by overestimating (or underestimating) the velocity conditions outside of the two upstream open-region enclosure surfaces. This option for open-region boundary conditions were included in the discrete-element hydrodynamic model as a general capability for incorporating approximate control conditions at open-region enclosure surfaces when exact values for the velocity components could not be specified. In the case when the velocity components could be determined exactly at the south-end open-region enclosure surfaces and at the north-end open-region enclosure surface, the specification of $\tilde{V}_{y,min}$ and $\tilde{V}_{y,max}$ as equal to the actual exact values of the velocity components could simulate automatically the required boundary conditions for the velocity components. The velocity components that were not normal to the south-end upstream open-region enclosure surface at $y = 0$ and to the north-end upstream open-region

enclosure surface at y = 24 m, were all specified as Vx,ups,min = Vx,ups,max = 0 and as Vz,ups,min = Vz,ups,max = 0.

The velocity components (1) for the outflow at the west-end downstream open-region enclosure surface at x = 0 and (2) for the outflow at the east-end downstream open-region enclosure surface at x = 44 m, were not used for extrapolating to outside of the flow region, since general second-upwind-differencing formulation automatically determines the values of the velocity components based on the velocity components in the boundary half-elements along the x-axis, (1) immediately adjacent to the west-end downstream open-region enclosure surface at x = 0 and (2) immediately adjacent to the east-end downstream open-region enclosure surfaces at x = 44m.

b. Interpolation Option

The computer simulation results for test case 3 were obtained by applying the HYDROL computer code with simple second-upwind-differencing interpolation option. It was noted that this interpolation option could introduce numerical dispersion effects in the elements and half-elements where the local Courant number was much less than one (1) (Sec. 4.3).

c. Numerical Solution Option

The computer simulation results for the test case 3 were obtained by applying the HYDROL computer code with the standart default option which ultilized the special, composite time-splitting method in the numerical integration of the governing system of spatially discretized ordinary differential equations (Sec. 5.2).

d. Discussion of the Computer Simulation Results

The results of the computer simulations for the water surface elevation and velocity conditions are presented in Figs. 6.44 through 6.63, p.635 through p.654. The surface elevation conditions were presented as (1) numbers in the elements which represent water surface elevation (mm) relative to specified initial mean water level (MWL) and (2) constant water surface elevation contours which correspond to 2mm gradation between -8 mm minimum and 8mm maximum relative to MWL. The velocity conditions were presented as two-dimensional vectors which correspond to two-component representations of velocity in the horizontal (x,y) planes of the four specified layers.

The results of the computer simulations for the flow conditions in the symmetric laboratory-model-scale flow region indicated that the water surface elevation and velocity conditions were generally consistent with the conditions that would exist in a similar prototype-scale flow region with a similar configuration for the deep main flow channel.

The special transient flow characteristics of the water surface elevation and velocity conditions for test case 3 are discussed in detailed below in order to illustrate the important correspondance between the results of the computer simulations and certain special characteristics of the physical phenomena that control the flow conditions in the laboratory-model-scale flow region.

The results for constant-water-surface-elevation contours (Fig. 6.44, p.635; Fig. 6.49, p.640; Fig. 6.54, p.645; and Fig. 6.59, p.650) indicate that there is a gradual accumulation of water in most parts of the flow region except (1) in the narrow subregion (between x = 0 and x

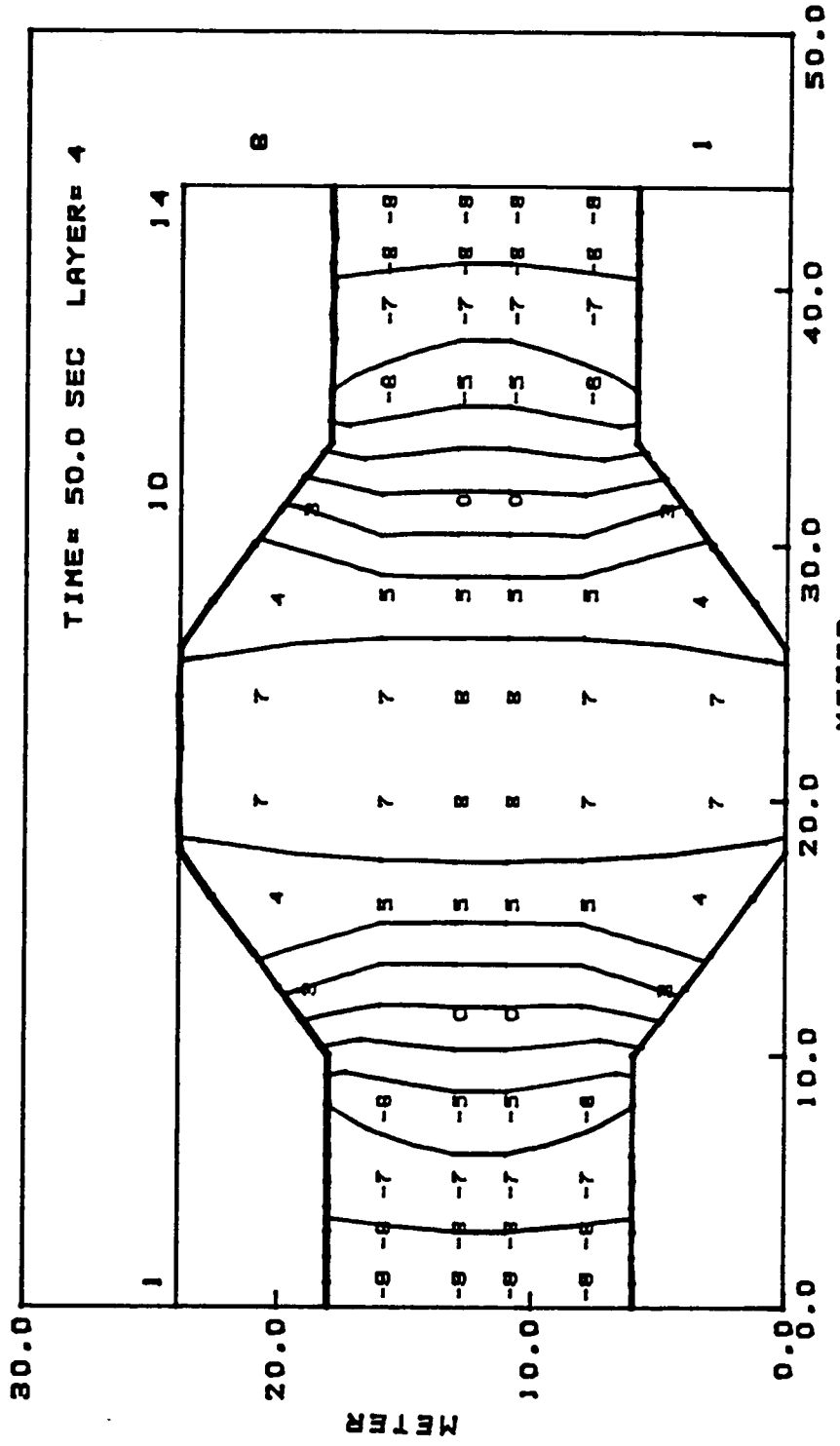

Figure 6.44. Water-surface-elevation conditions at time t = 50 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

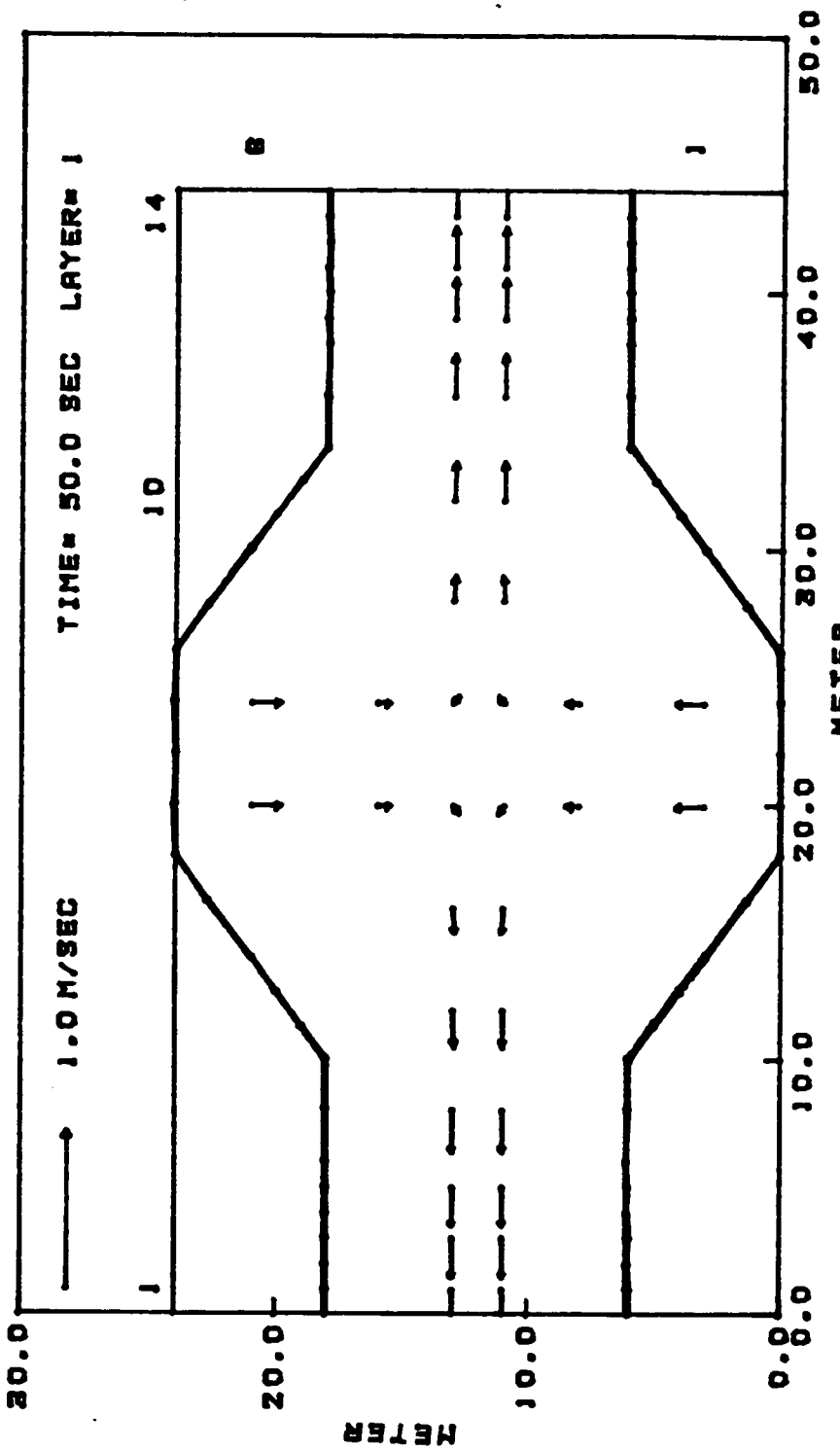

Figure 6.45. Velocity conditions in the bottom layer l = 1, at time t = 50 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

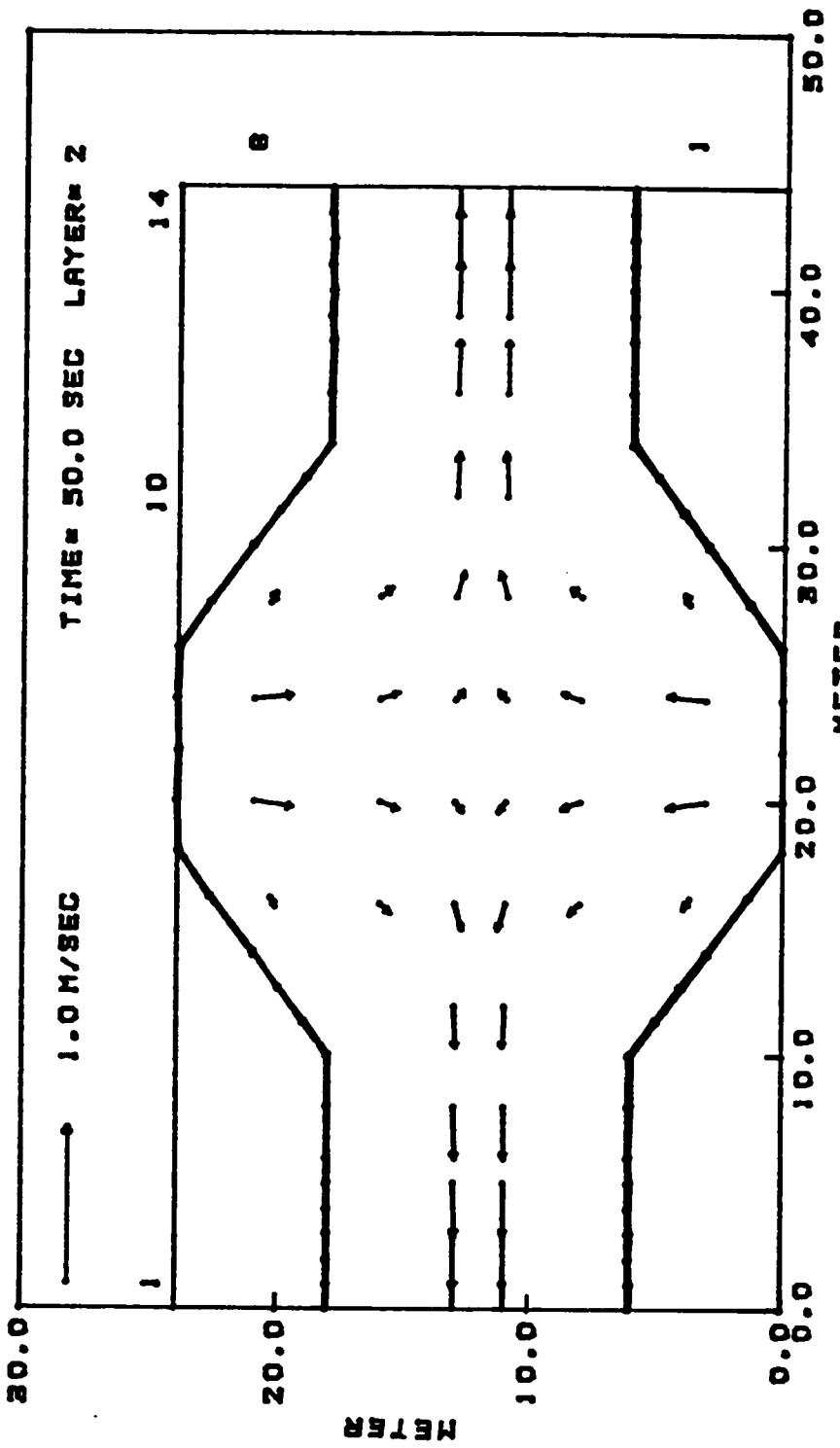

Figure 6.46. Velocity conditions in the internal layer l = 2, at time t = 50 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

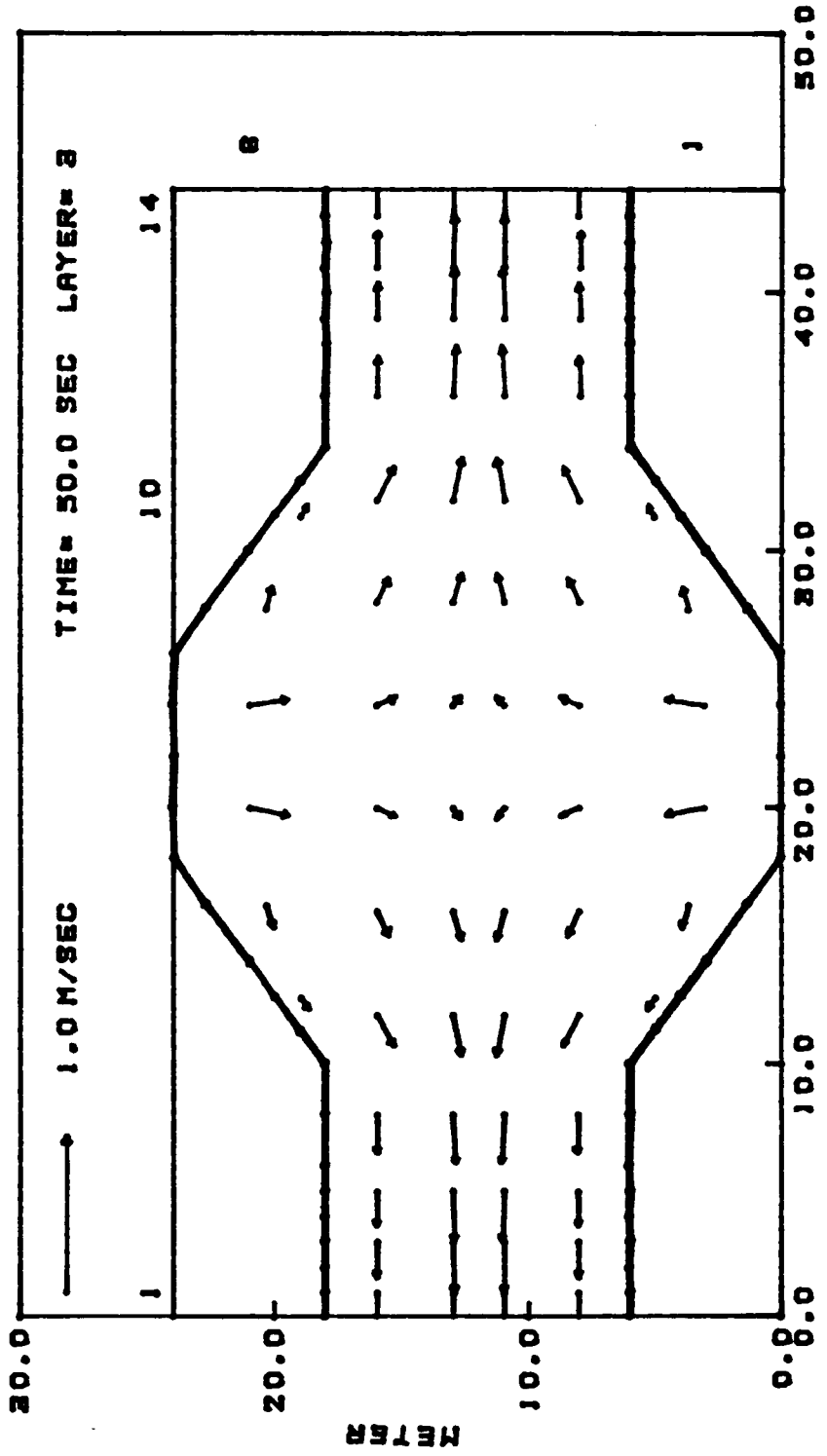

Figure 6.47. Velocity conditions in the internal layer l = 3, at time t = 50 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

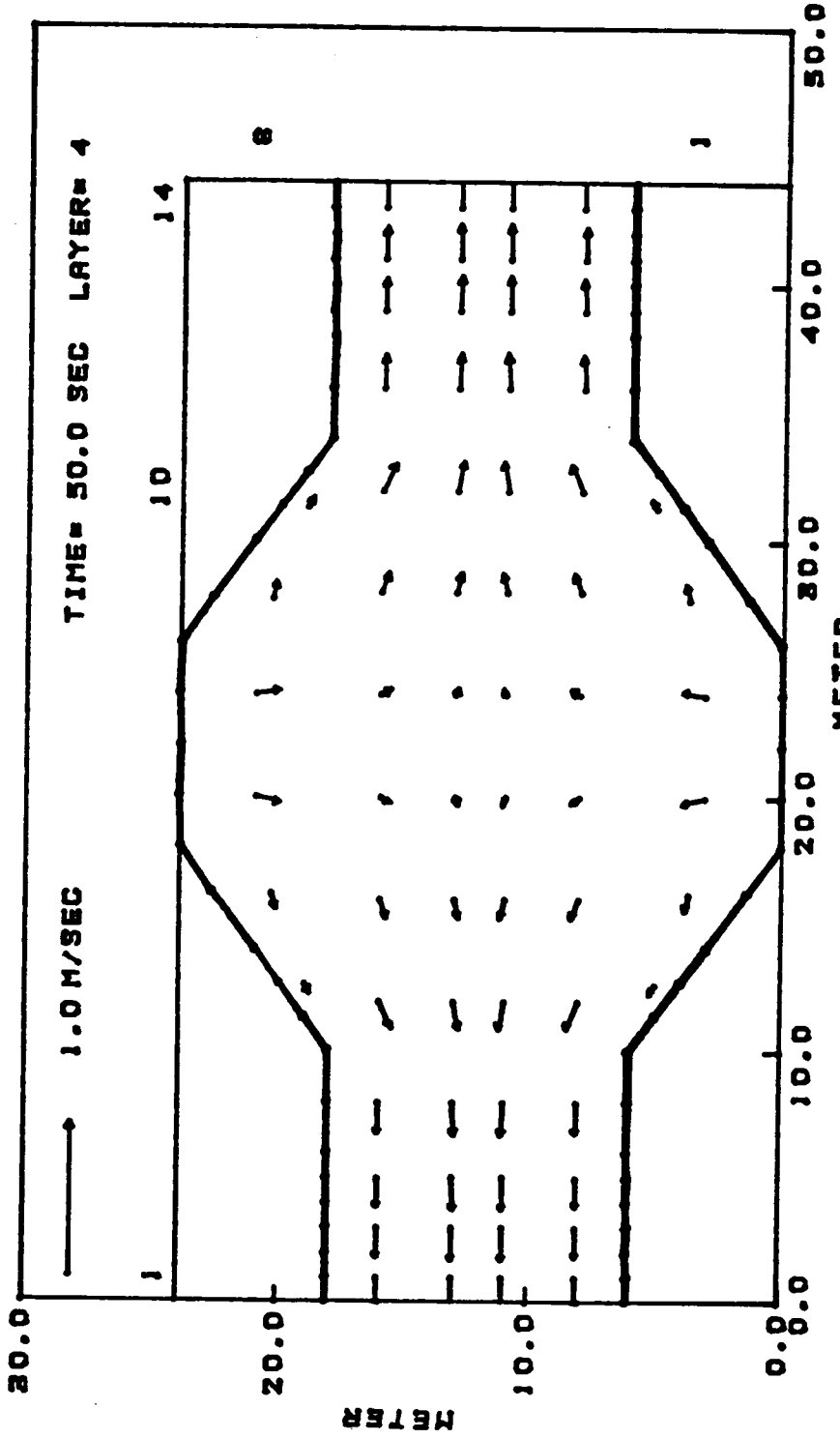

Figure 6.48. Velocity conditions in the surface layer l = 4, at time t = 50 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

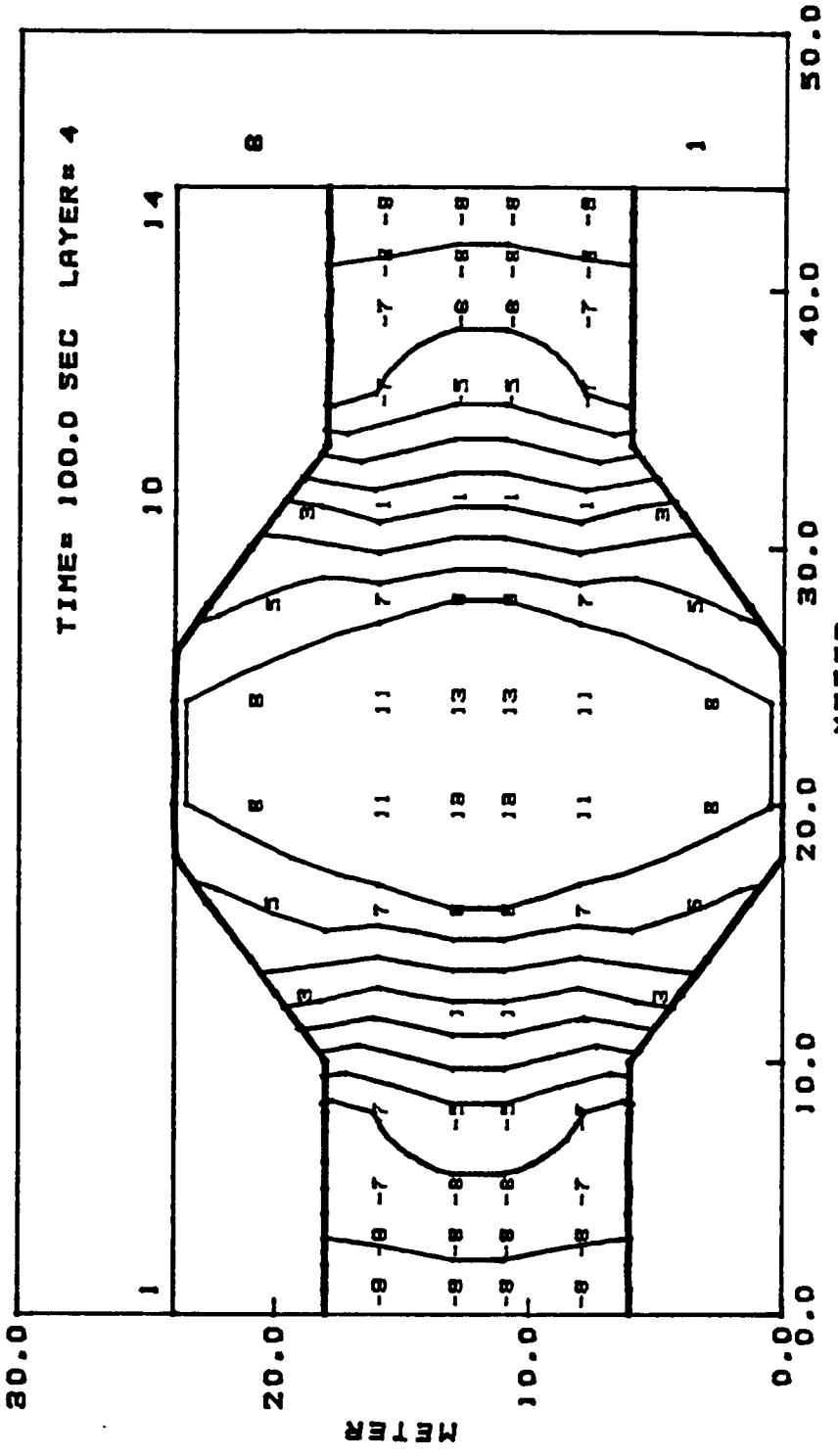

Figure 6.49. Water-surface-elevation conditions at time t = 100 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

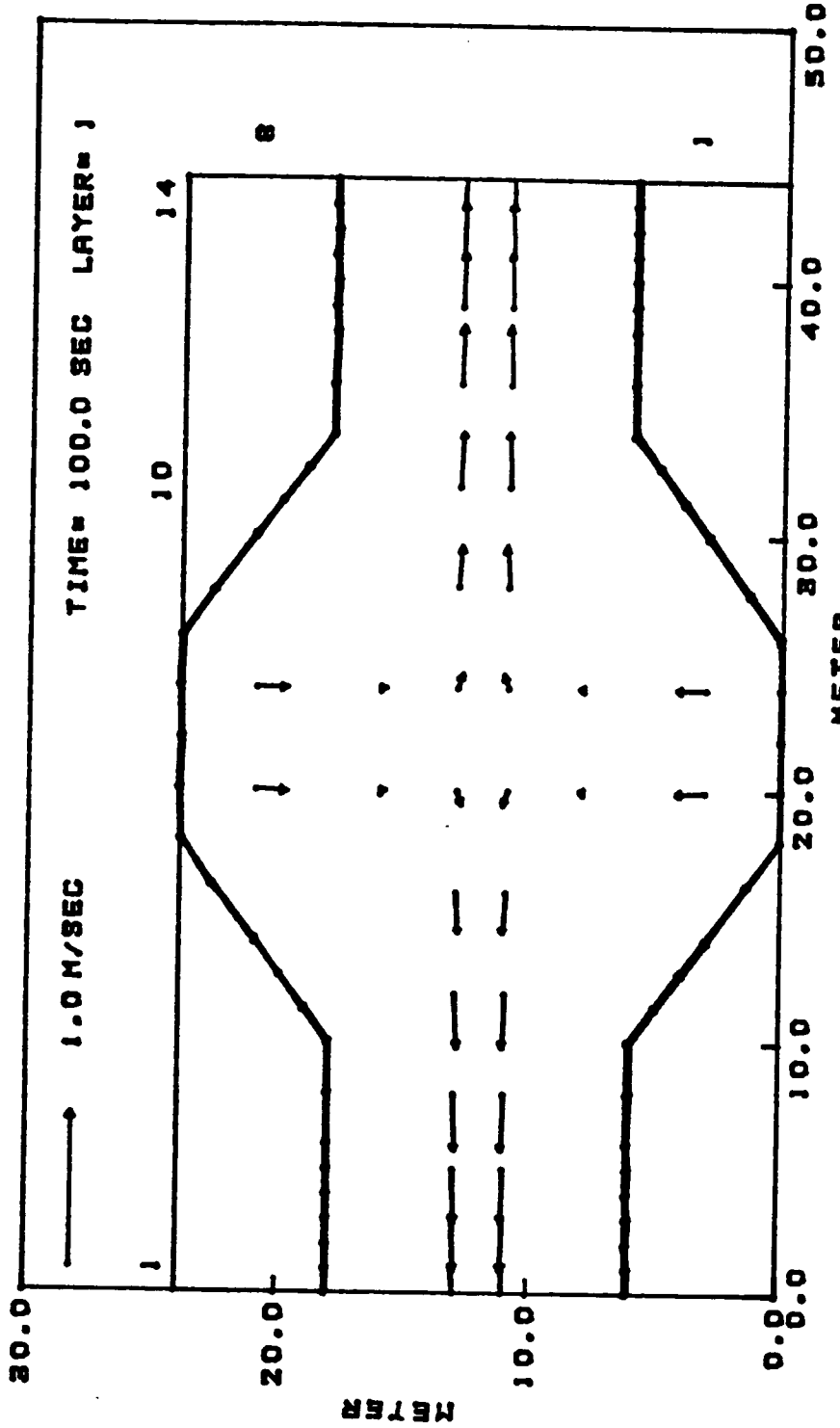

Figure 6.50. Velocity conditions in the bottom layer l = 1, at time t = 100 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

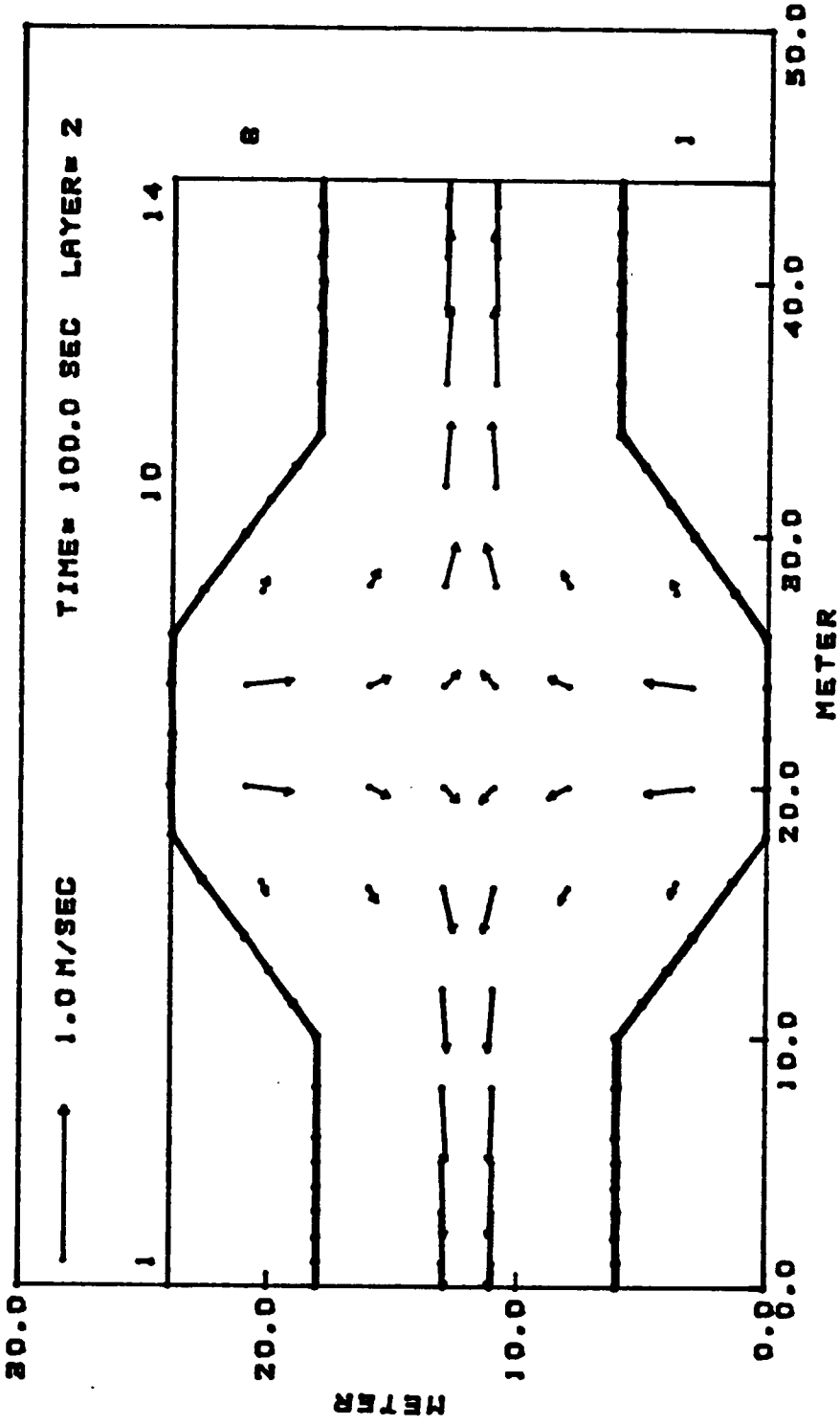

Figure 6.51. Velocity conditions in the internal layer l = 2, at time t = 100 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

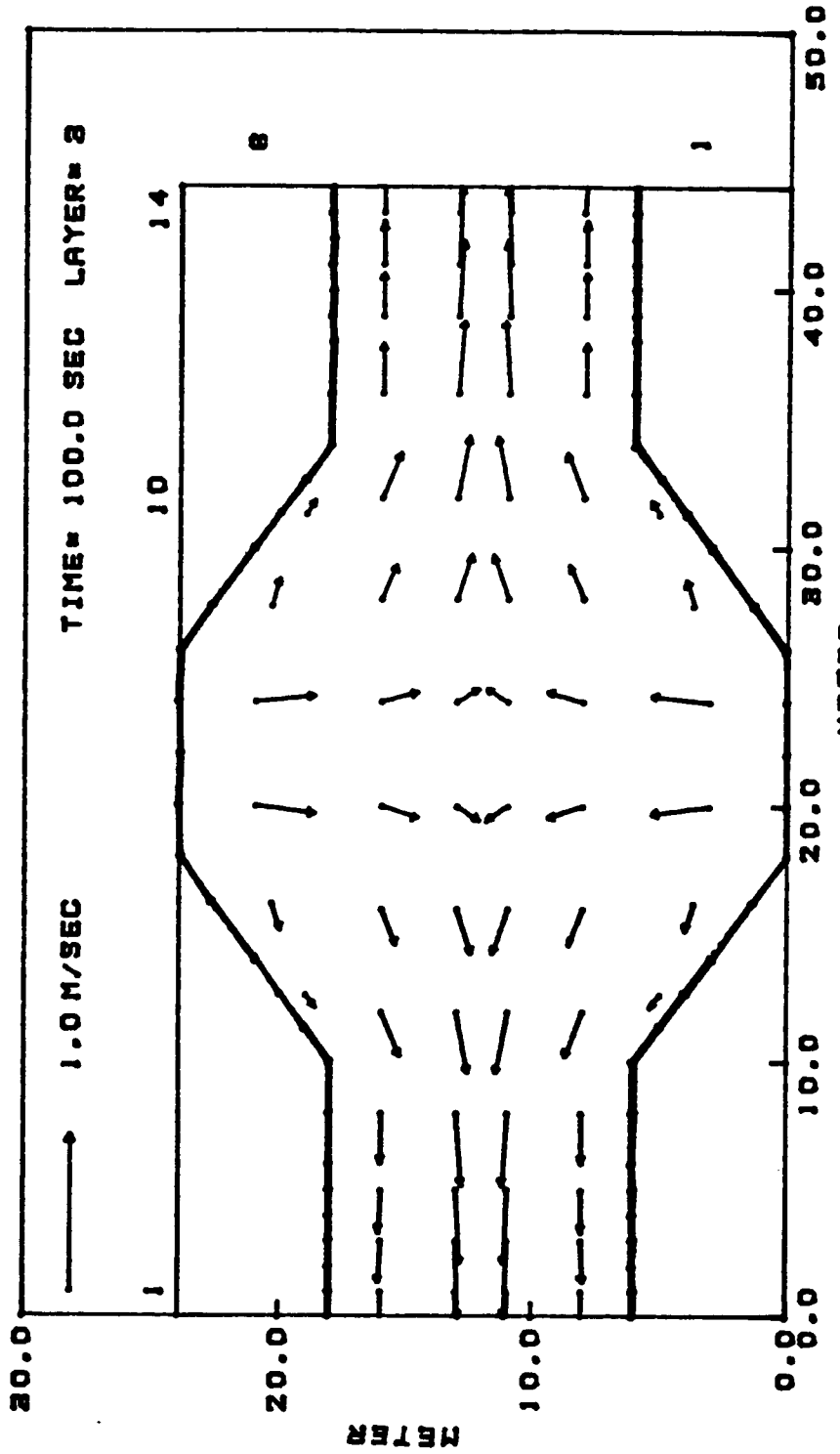

Figure 6.52. Velocity conditions in the internal layer l = 3, at time t = 100 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

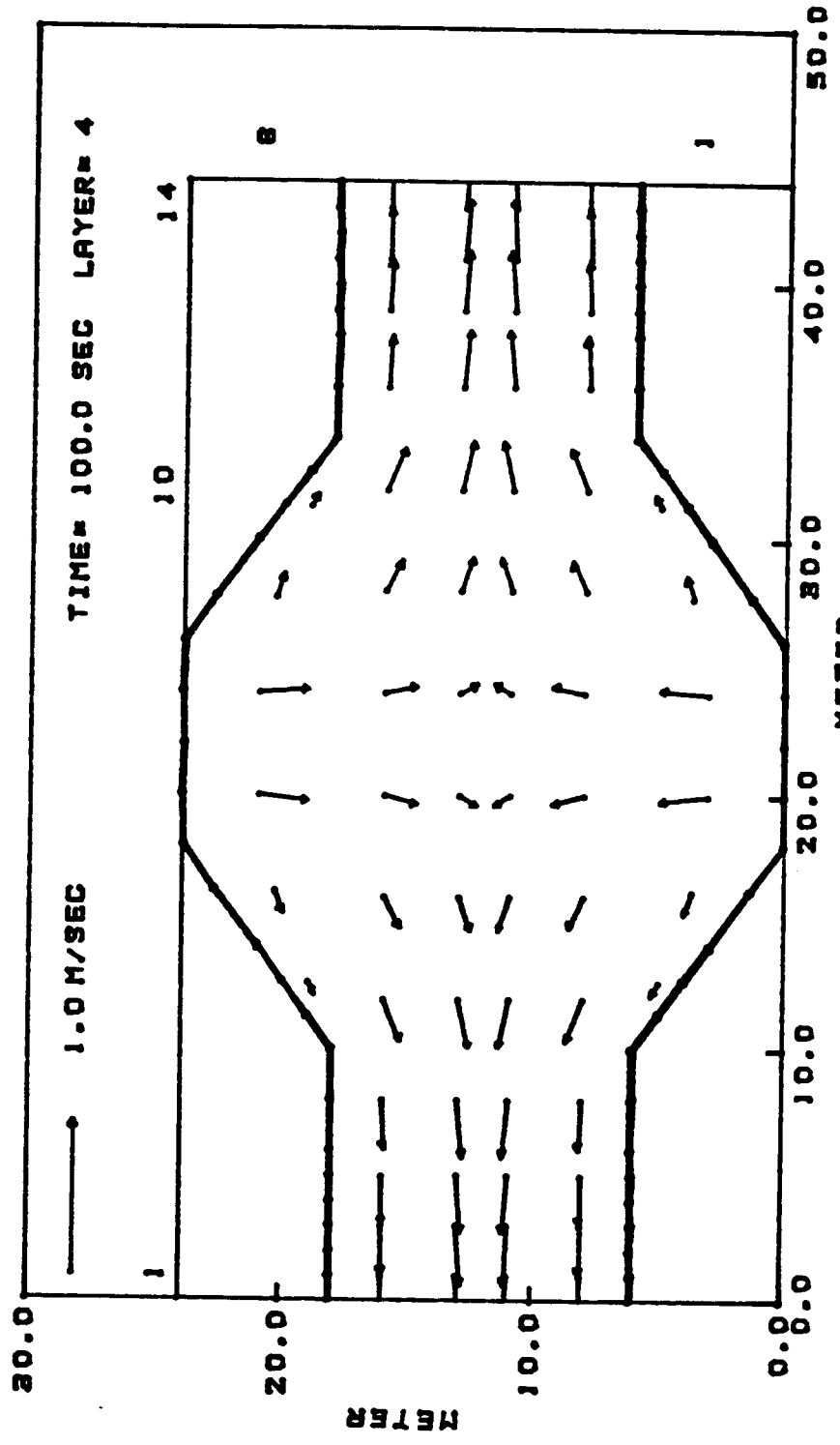

Figure 6.53. Velocity conditions in the surface layer l = 4, at time t = 100 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

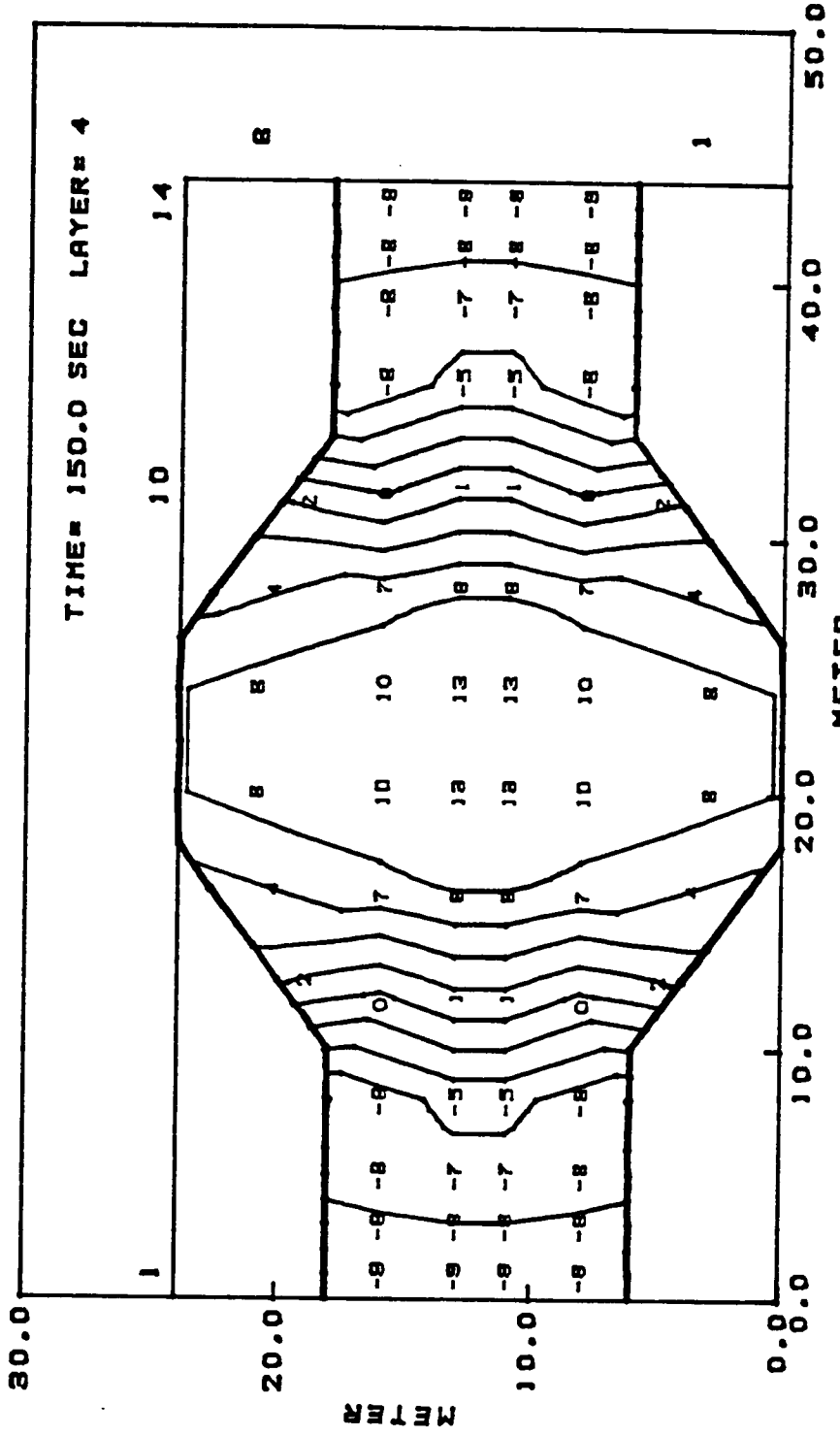

Figure 6.54. Water-surface-elevation conditions at time t = 150 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

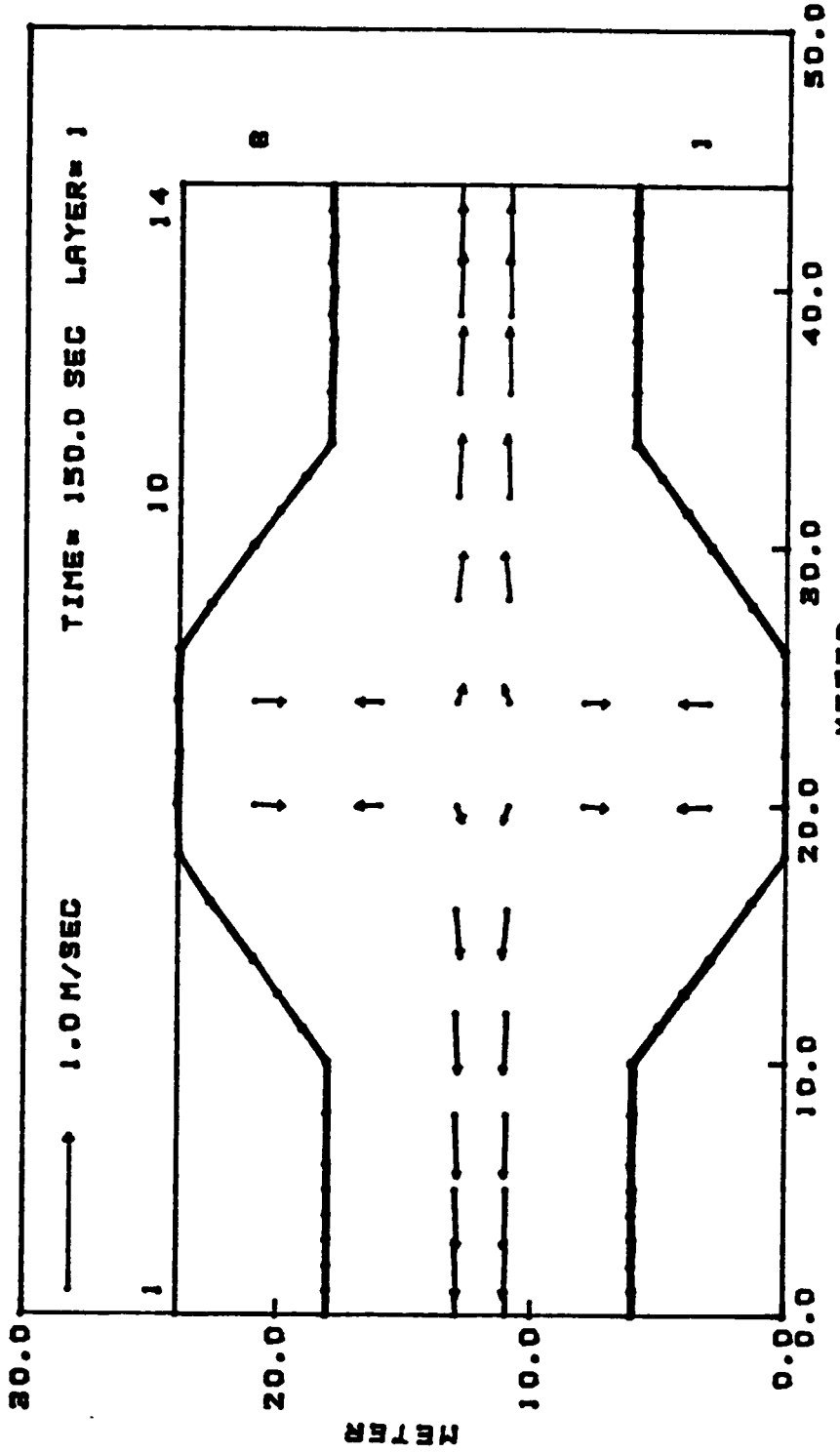

Figure 6.55. Velocity conditions in the bottom layer l = 1, at time t = 150 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

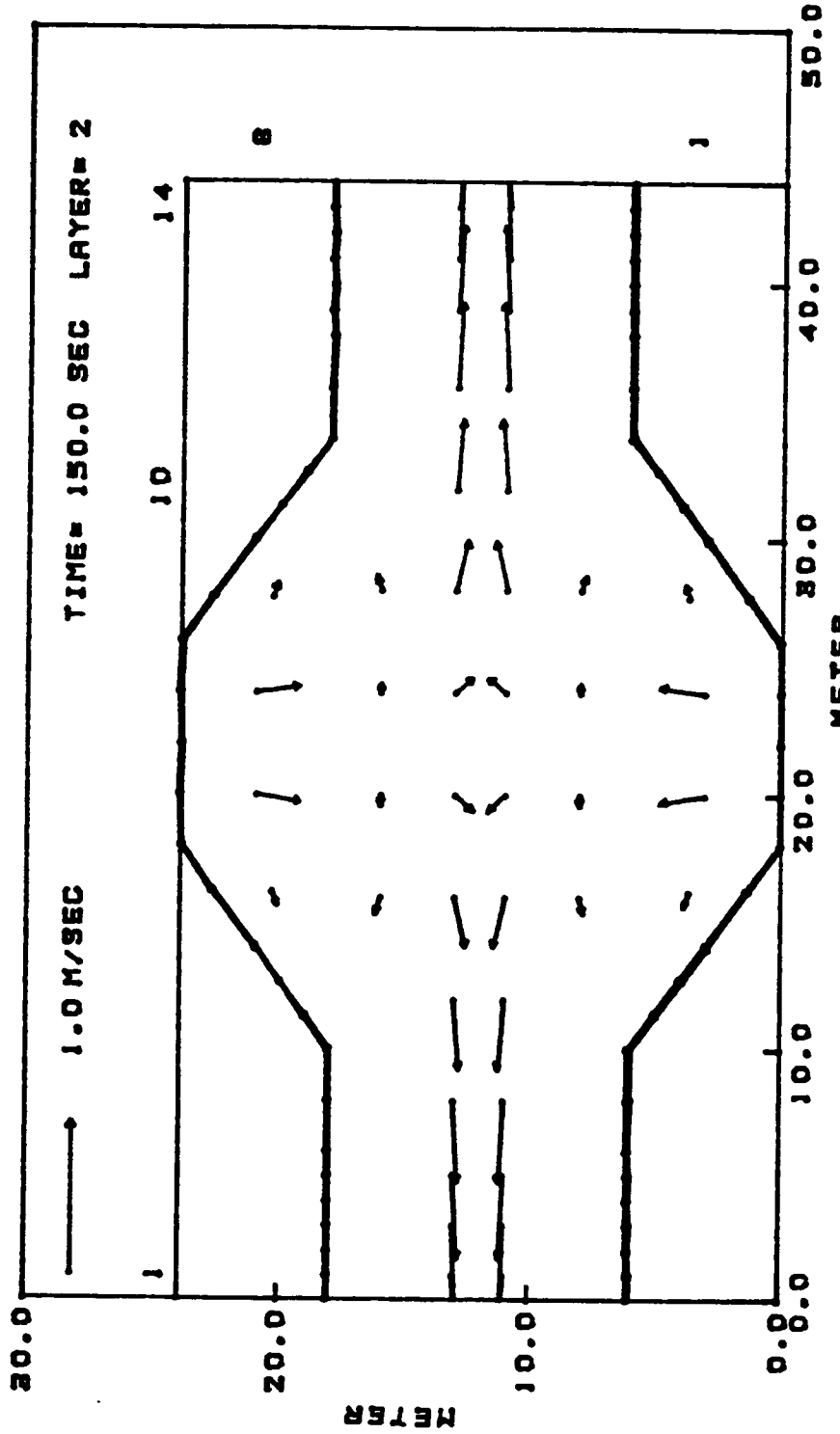

Figure 6.56. Velocity conditions in the internal layer l = 2, at time t = 150 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

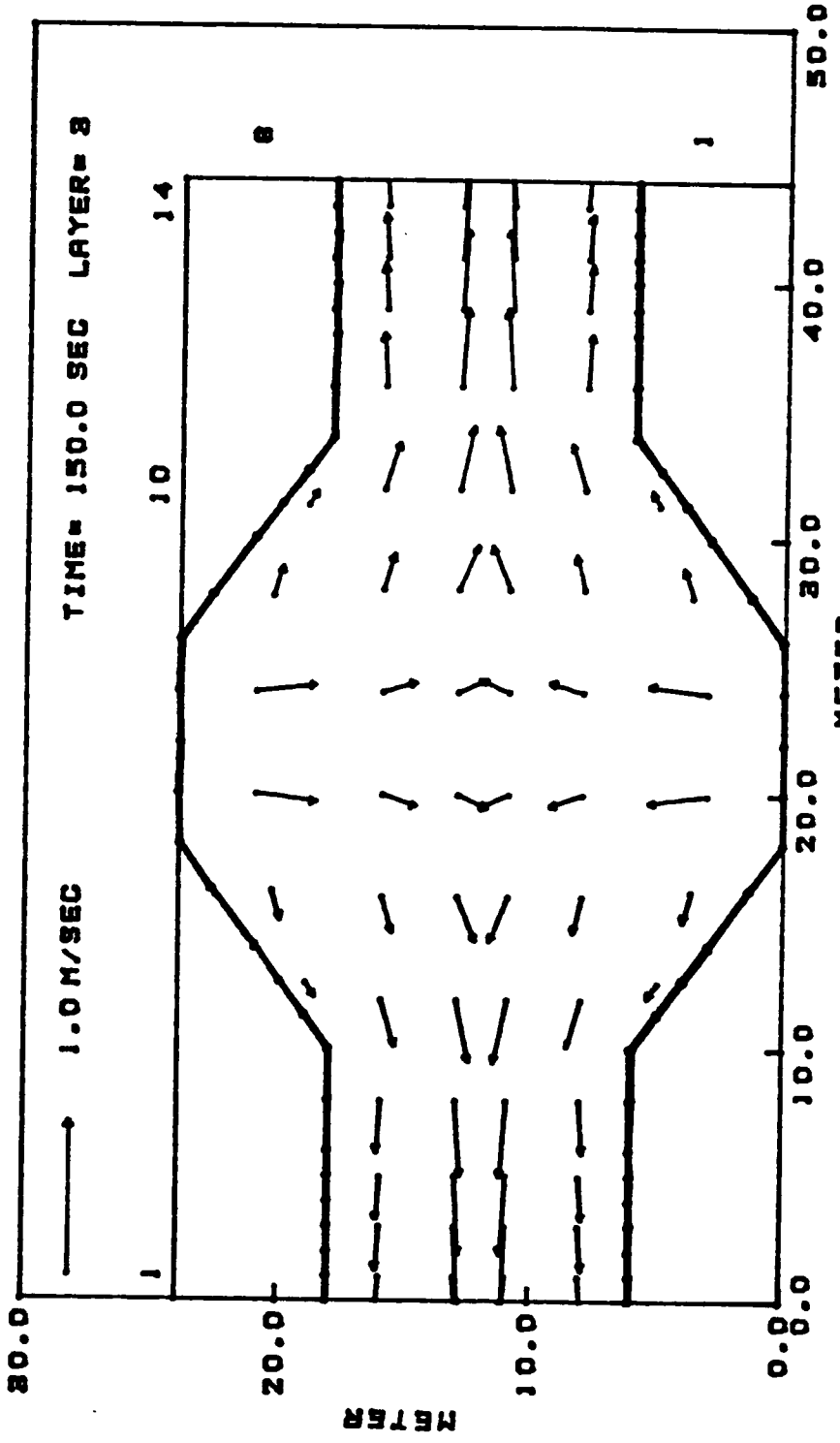

Figure 6.57. Velocity conditions in the internal layer l = 3, at time t = 150 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

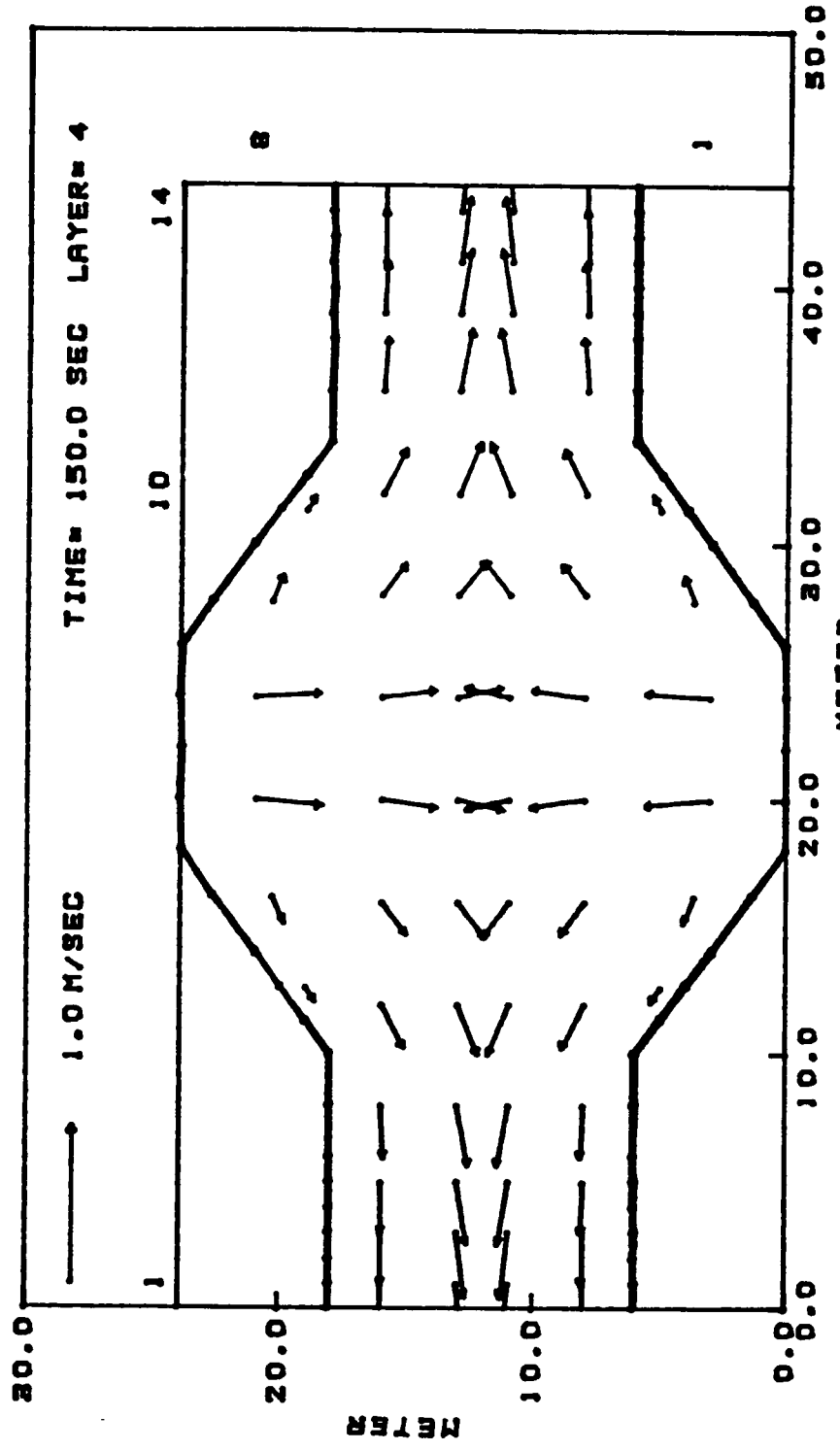

Figure 6.58. Velocity conditions in the surface layer l = 4, at time t = 150 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

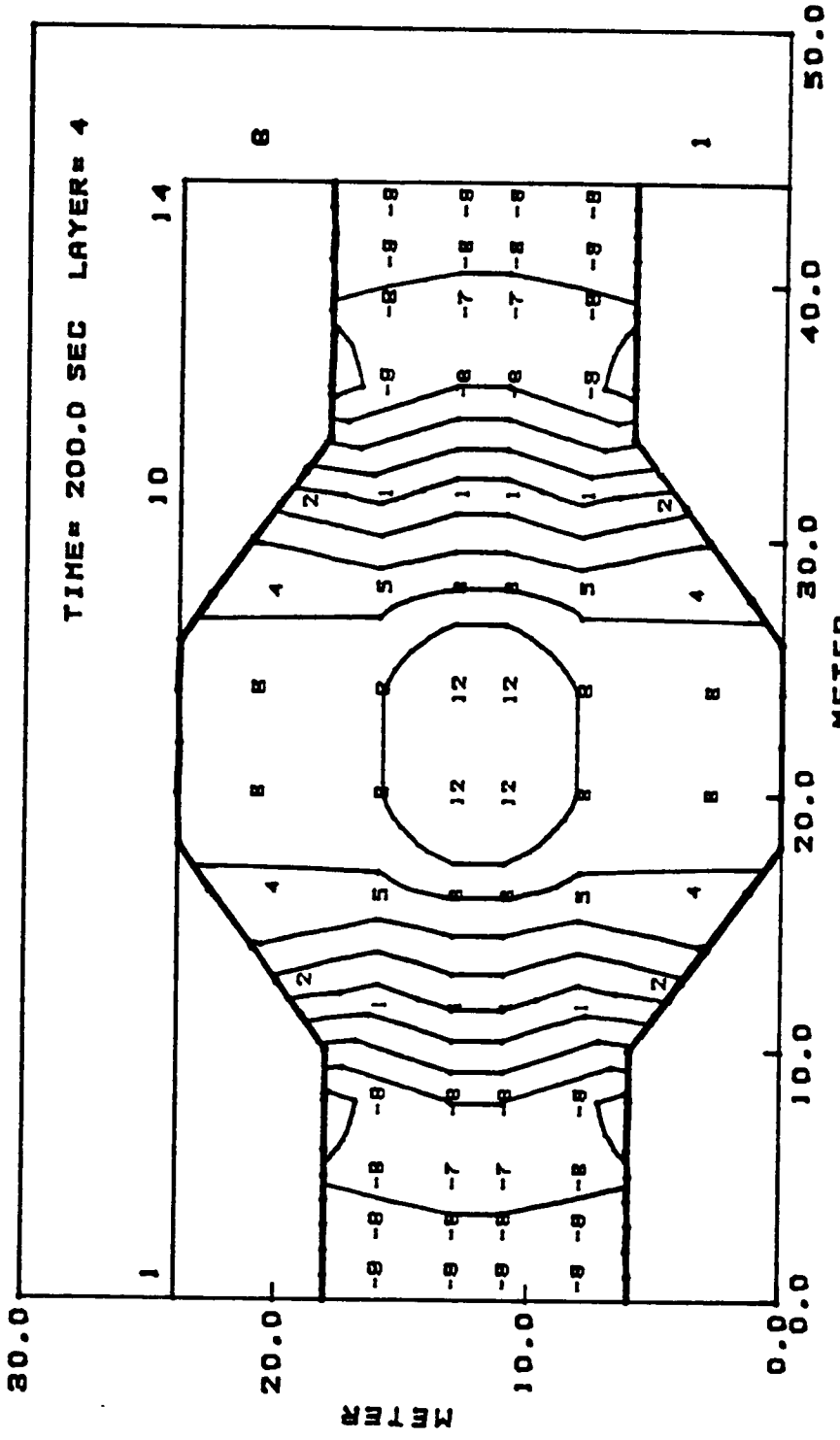

Figure 6.59. Water-surface-elevation conditions at time t = 200 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

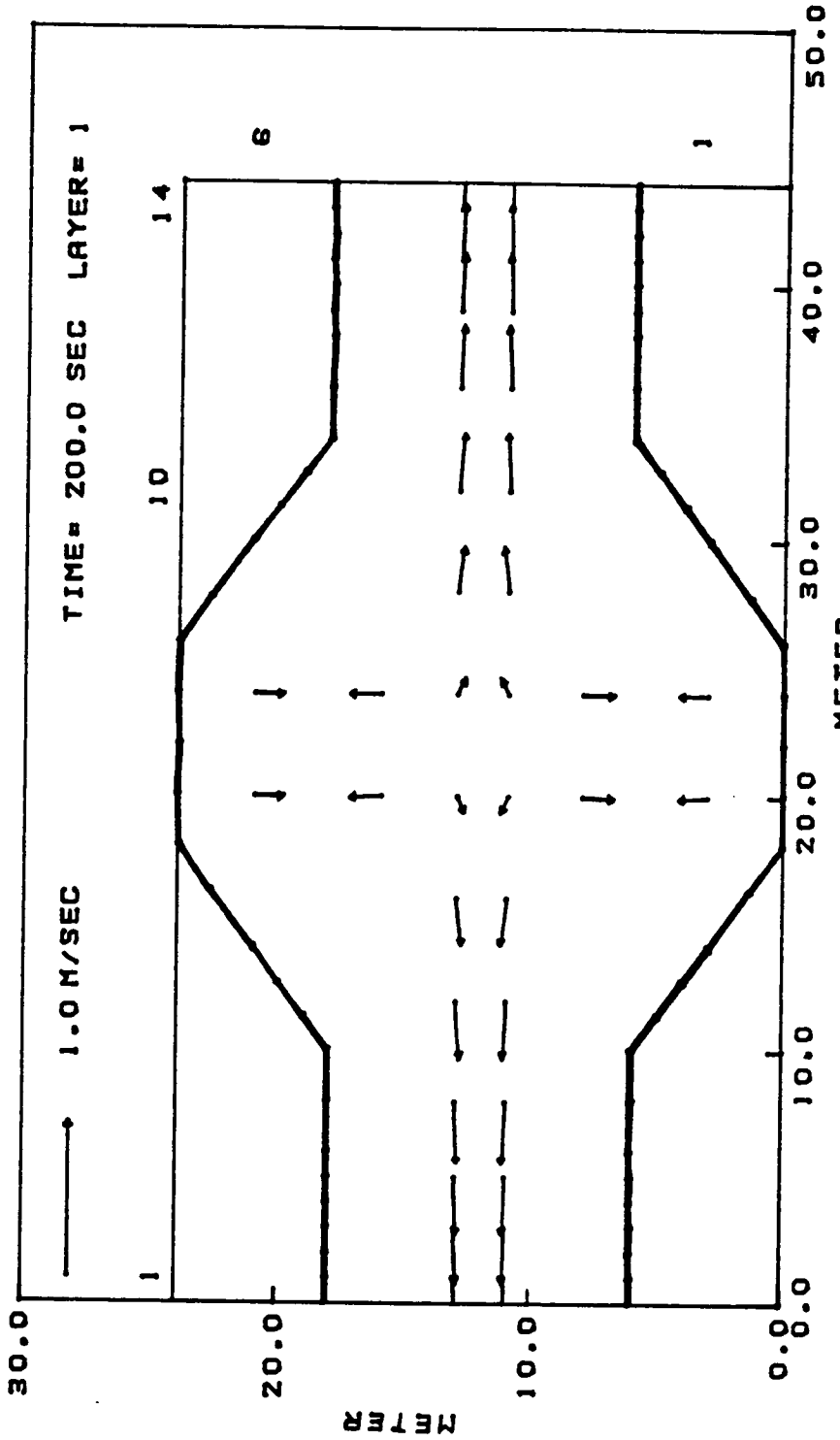

Figure 6.60. Velocity conditions in the bottom layer l = 1, at time t = 200 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

= 200 sec.

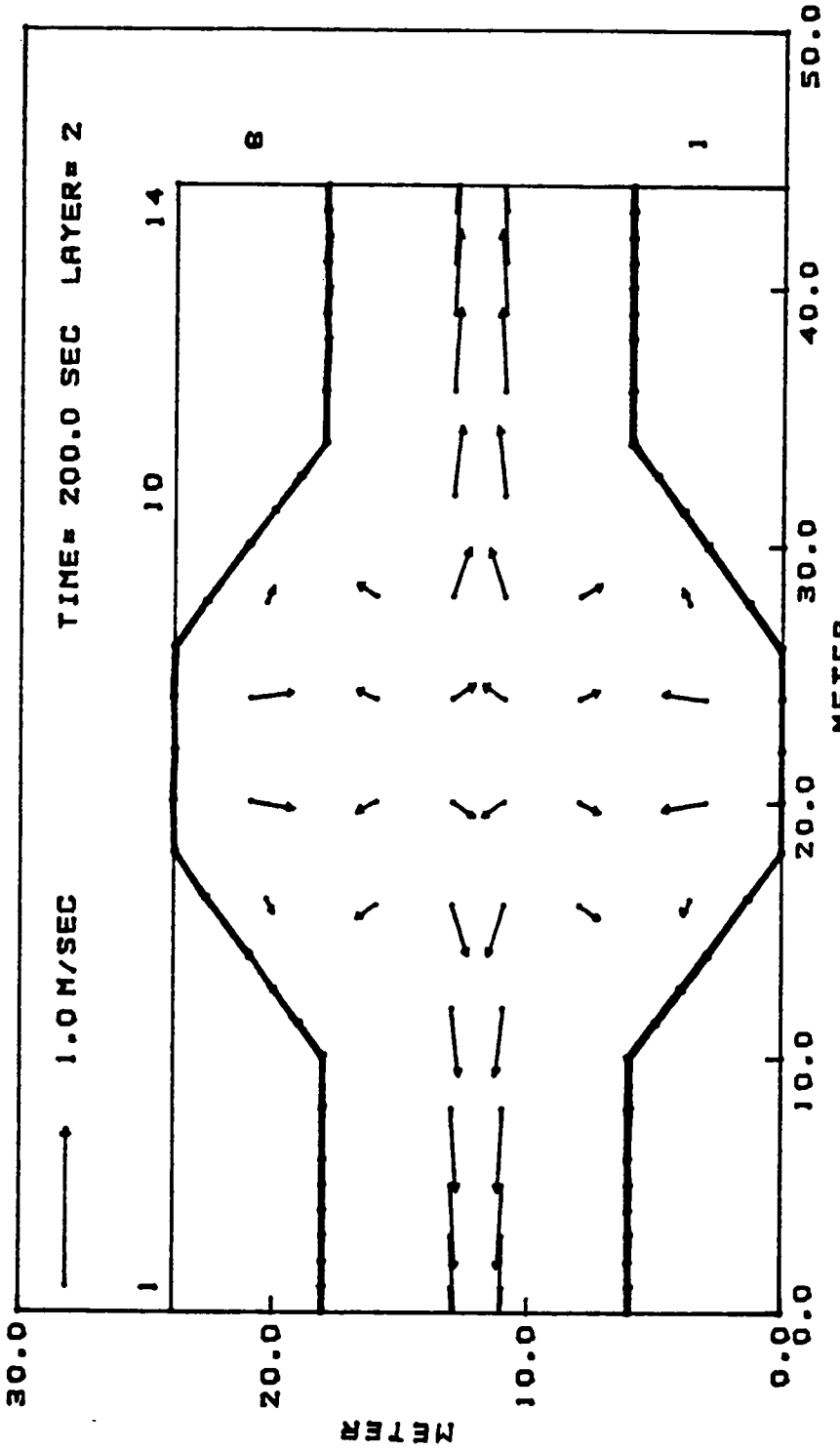

Figure 6.61. Velocity conditions in the internal layer l = 2, at time t = 200 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

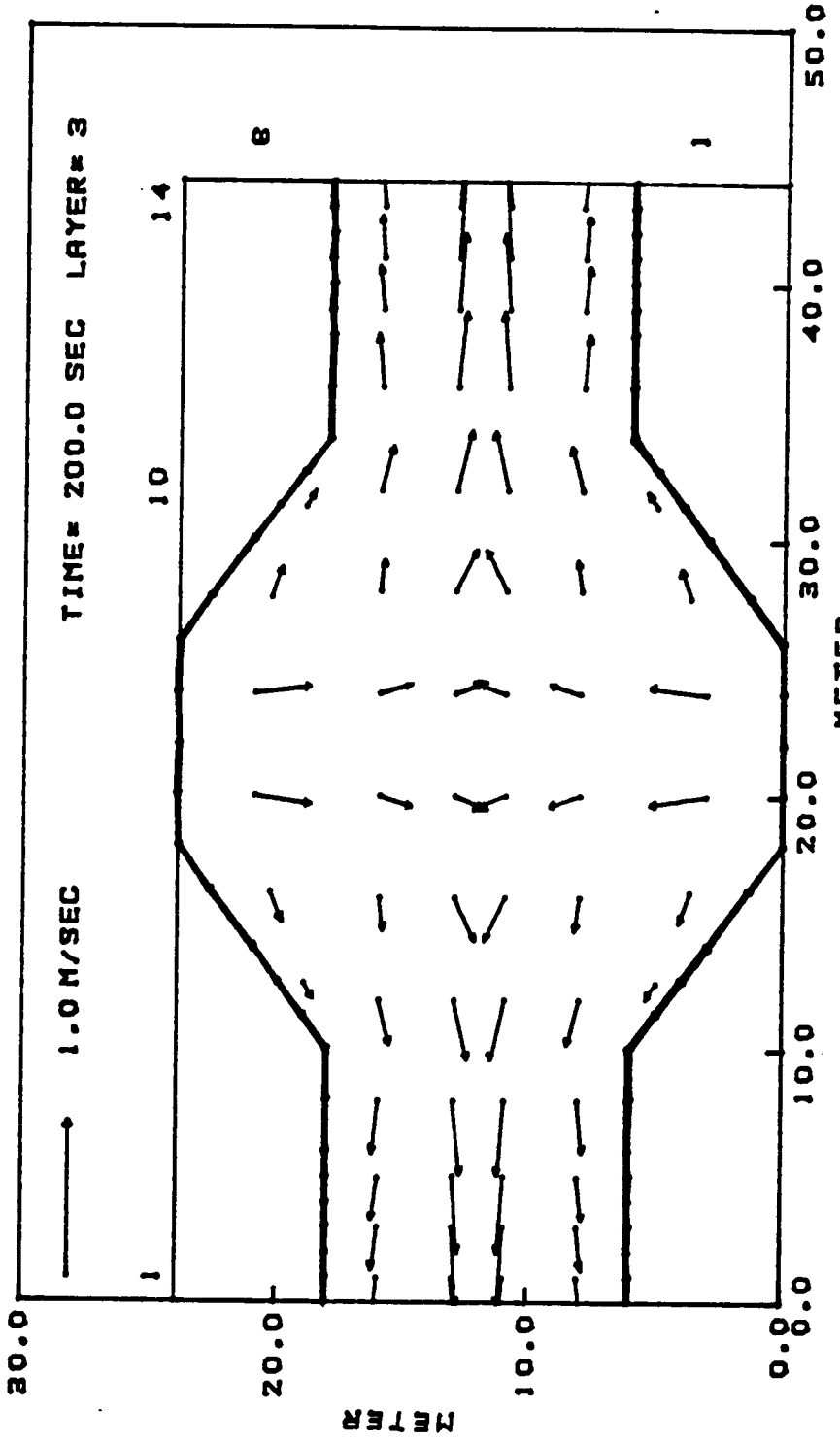

Figure 6.62. Velocity conditions in the internal layer l = 3, at time t = 200 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

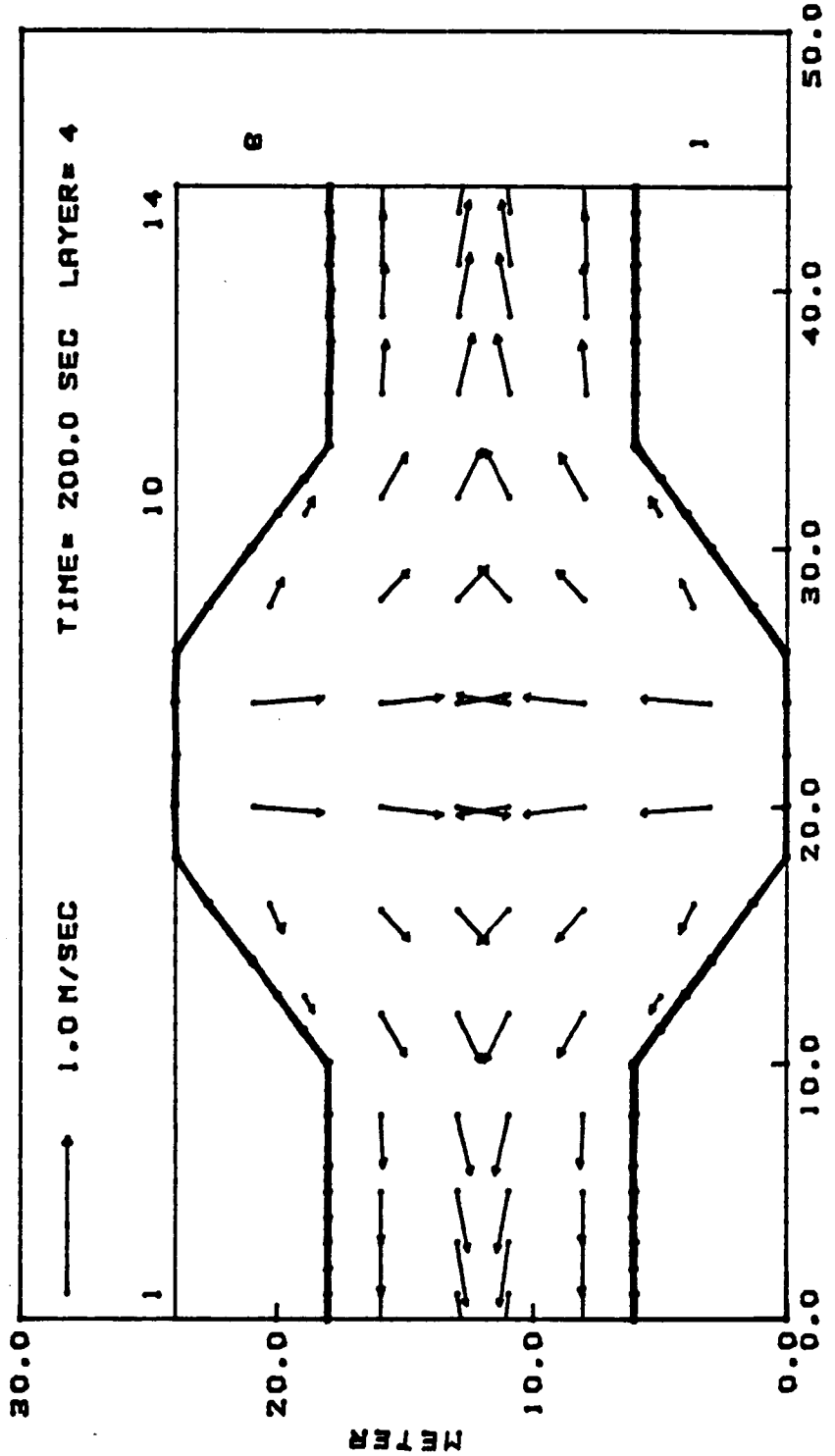

Figure 6.63. Velocity conditions in the surface layer l = 4, at time t = 200 sec for test case 3: Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

= 14 m and between y = 6 m and y = 18 m), downstream of the inlet plane at the west-end downstream enclosure surface at x = 0, and (2) in the narrow subregion (between x = 34 m and x = 44 m and between y = 6 m and y = 18 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. The results indicate that, at time level t = 50 sec, two constant-water-surface-elevation contours with zero value, which corresponds to the specified initial mean water level (MWL), are already relocated at two positions, (1) at approximately x = 12 m which is 10 m farther upstream from the central y-axis at x = 22 m and (2) at approximately x = 32 m which is 10 m farther downstream from the central y-axis at x = 22 m (Fig. 6.44, p.635). As the flow development progresses, during time levels t = 100 sec, t = 150 sec and t = 200 sec, the location of the two constant-water-surface-elevation contours with zero value remain approximately at the same two positions as during time level t = 50 sec (Fig. 6.49, p.640; Fig. 6.54, p.645; and Fig. 6.59, p.650). The results for constant-water-surface-elevation contours also indicate that the gradual accumulation of water in the middle of the flow region stops after the time level t = 150 sec (Fig. 6.44, p.635; Fig. 6.49, p.640; Fig. 6.54, p.645; and Fig. 6.59, p.650). At time level t = 200 sec, the water-surface-elevation contours in the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) reach the pseudo-steady-state conditions with the water-surface-elevation values remaining at approximately 12 mm, above the mean water level (MWL) of the specified initial conditions, in the middle of the flow region (Fig. 6.59, p.650).

The results of the computer simulations for water-surface-elevation conditions (Fig. 6.44, p.635; Fig. 6.49, p.640; Fig. 6.54, p.645; and Fig. 6.59, p.650) can be explained readily by considering various physical conditions which control the flow in the laboratory-model-scale flow region. Rapid accumulation of water in the middle of the region, during the time period before time level t = 150 sec (Fig. 6.44, p.635; Fig. 6.49, p.640; and Fig. 6.54, p.645), can be explained by considering two physical characteristics of the simulated flow conditions.

First, the inflow conditions at the inlet plane at the south-end enclosure surface at y = 0 can attain relatively high positive y-component of momentum (control condition specified as $\tilde{V}_{y,max;i,(opr,j-1),l}$ = 0.5 m/sec) and similarly the inflow conditions at the inlet plane at the north-end enclosure surface at y = 24 m can attain relatively high negative y-component of momentum (control condition specified as $\tilde{V}_{y,min;i,(opr,j-1),l}$ = -0.5 m/sec). Hence, the inflowing water from the south-end enclosure surface and from the north-end enclosure surface rapidly flood the central subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) which consists of two opposing divergent conditions are along opposing y-directions, the y-component of velocity in the middle of the flow region must necessarily vanish; and therefore the opposing y-components of momentum of the inflow conditions must result in hydrostatic pressure gradients along opposing y-directions with the maximum hydrostatic pressure conditon occuring in the middle of the flow channel. Hence, the water-surface-elevation conditions in the central subregion (between x = 18 m and x = 26 m and between y = 10 m and y = 14 m) must attain the maximum values (Fig. 6.44, p.635; Fig.

6.49, p.640; Fig. 6.54, p.645; and Fig. 6.59, p.650), since the hydrostatic pressure conditions must be maximum in the subregion, due to the occurrance of stagnation flow conditions in the middle of the flow region. Furthermore, the water that flows out of the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m), in the middle of the flow region, first has to undergo rapid contraction due to the convergent geometrical shapes of the boundary enclosure surfaces which lead to the two narrow outflow subregions, (1) narrow subregion (between x = 0 and x = 10 m), upstream of the discharge plane at the west-end downstream enclosure surface at x = 0, and (2) narrow subregion between x = 34 m and x = 44 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m). Considering the effect of the retardation of flow in the relatively shallow subregions (between x = 10 m and x = 18m, between y = 0 and y = 10m and between y = 14 m and y = 24 m; and between x = 26 m and x = 34 m, between y = 0 and y = 10 m and between y = 14 m and y = 24 m), outside of the deep main flow channel along the longitudinal x axis, the accumulated water cannot freely flow into the two narrow subregions, (1) narrow subregion (between x = 0 and x = 10 m), upstream of the discharge plane at the west-end downstream enclosure surface at x = 0, and (2) narrow subregion (between x = 34 m and x = 44 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m. Hence, the water accumulates in the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) in the middle of the flow region (Fig. 6.44, p.635; Fig. 6.49, p.640; Fig. 6.54, p.645; and Fig. 6.59, p.650), since the water can flow easier into the subregion, from (1) the south-end

upstream enclosure surface at y = 0 and (2) the north-end upstream enclosure surface at y = 24 m, than it can flow out of the subregion through the two narrow subregions, (a) narrow subregion (between x = 0 and x = 10 m), upstream of the discharge plane at the west-end downstream enclosure surface at x = 0, and (b) narrow subregion between x = 34 m and x = 44 m), upstream of the discharge plane at the east-end downstream enclosure surface at x = 44 m.

The computer simulation results indicate that, as the pseudo-steady-state flow conditions start being established in the flow region, the maximum water-surface-elevation conditions occur in a small subregion (between x = 18 m and x = 26 m and between y = 8 m and y = 16 m) in the middle of the flow region (Fig. 6.59, p.650). The water-surface-elevation conditions in the subregion vary between 8 mm and 12 mm above the specified initial mean water level (MWL), with the maximum water-surface-elevation occuring in the middle of the flow region (Fig. 6.59, p.650). This result is to be expected based on the consideration of the stagnation flow conditions that must exist in the middle of the flow region. Since the specified inflow conditions, at the south-end upstream enclosure surface at y = 0 and at the north-end upstream enclosure surface at y = 24 m, are in opposite directions along the lateral y axis and the specified outflow conditions, at the west-end downstream enclosure surface at x = 0 and at the east-end downstream enclosure surface at x = 44 m, are also in opposite directions along the longitudinal x axis, the stagnation point must be exactly in the middle of the symmetric flow region. Hence, under pseudo-steady-state flow conditions the maximum hydrostatic pressure conditions must be in the

middle of the symmetric flow region; and therefore, the maximum water-surface-elevation conditions must also occur in the middle of the symmetric flow region (Fig. 6.59, p.650).

It is again important to note that the gradual accumulation of water in the flow region (Fig. 6.44, p.635; Fig. 6.49, p.649; Fig. 6.54, p.645; and Fig. 6.59, p.650) is also a consequence of the open-region boundary conditions that were used in the computer simulations. The specified increased water-surface-elevation conditions, at the south-end upstream enclosure surface at $y = 0$ and at the north-end upstream enclosure surface at $y = 24$ m, as 0.01 m (10 mm) above the specified initial mean water level (MWL), result in increases of 0.08 m^2 of inflow areas, since the widths of the upstream enclosure surface are 8 m. Similarly, the specified decreased water-surface-elevation conditions, at the west-end downstream enclosure surface at $x = 0$ and at the east-end downstream enclosure surface at $x = 44$ m, as 0.01 m (10 mm) below the specified initial MWL, result in decreases of -0.12 m^2 of outflow area, since the widths of the downstream enclosure surfaces are 12 m. Therefore, effectively, the inflow conditions at the upstream enclosure surfaces have an available cumulative flow area which is 0.08 m2 larger than the available cumulative flow area for the outflow conditions at the downstream enclosure surfaces.

The inflow conditions at the inlet plane at the south-end upstream enclosure surface at $y = 0$ can attain relatively high values for the y-component of velocity (control condition specified as $\tilde{V}_{y,max;i,(opr,j-1),l} = 0.5$ m/sec) according to the specified open-region boundary conditions, immediately after the start of the flow conditions in the flow region.

Similarly, the inflow conditions at the inlet plane at the north-end upstream enclosure surface at y = 24 m can attain relatively high values for the y-component of velocity (control condition specified as $\tilde{V}_{y,min;i,(opr,j+1),l}$ = -0.5 m/sec), according to the specified open-region boundary conditions, again immediately after the start of the flow conditions in the flow region. Hence, the y-components of the velocity conditions at the inlet planes of the upstream open-region enclodure surfaces can attain relatively high values, immediately after the start of the flow conditions, which can lead to relatively high inflow rates into the flow region. On the other hand, the ouflow conditions at the discharge plane at the west-end downstream enclosure surface at x = 0 cannot attain a value for the x-component of velocity with a magnitude higher than the magnitudes of the y-components of velocities at the south-end upstream enclosure surface at y = 0 and at the north-end upstream enclosure surface at y = 24 m, until the increased water-surface-elevation conditons in the flow region can cause sufficiently high hydrostatic pressure gradient conditions that can accelerate the flow along the x direction. Similarly, the outflow conditions at the discharge plane at the east-end downstream enclosure surface at x = 44 m cannot attain a value for the x-component of velocity with a magnitude higher than the magnitudes of the y-components of velocities at the south-end upstream enclosure surface at y = 0 and at the north-end upstream enclosure surface at y= 24 m, until the increased water-surface-elevation conditions in the flow region can cause sufficiently high hydrostatic pressure gradient conditions that can accelerate the flow along the x direction. THerefore, during the

period of flow development, the cumulative in flow rate, resulting from the inflow rate at the inlet plane at the south-end upstream enclosure surface at $y = 0$ and from the inflow rate at the inlet plane at the north-end upstream enclosure surface at $y = 24$ m, with effectively increased value of 0.24 m^2 total available flow area, is higher than the cumulative outflow rate, resulting from the outflow rate at the discharge plane at the west-end downstream enclosure surface at $x = 0$ and from the outflow rate at the discharge plane at the east-end downstream enclosure surface at $x = 44$ m, with effectively decreased value of -0.16 m^2 total available flow area. Hence, during the period of flow development, the cumulative inflow rate is effectively higher than the cumulative outflow rate, and hence, the water-surface-elevation conditions gradually increase in most parts of the flow region.

The most important result of the computer simulations of water-surface-elevation conditions is that during the complete period of flow development, between time levels $t = 50$ sec and $t = 200$ sec, the temporal variations of the water-surface-elevation conditions retain exactly the required spatially-symmetric distributions about both central x-axis and central y-axis of the flow region (Fig. 6.44, p.635; Fig. 6.49, p.640; Fig. 6.54, p.645; and Fig. 6.59, p.650).

Hence, the considerations of the physical characteristics of the flow conditions clearly indicate that the computer simulation results of the discrete-element hydrodynamic model (HYDROL) for the transient water-surface-elevation conditions are realistic and representative of the conditions that showed exist for test case 3 : Symmetric Opposing

Flow 1 (inflow along lateral direction and outflow along longitudinal direction) in the laboratory-model-scale flow region.

The computer simulation results for velocity conditions (Figs. 6.45 through 6.48, p.636 through p.639; Figs. 6.50 through 6.53, p.641 through p.644 Figs. 6.55 through 6.58, p.646 through p.649; and Figs. 6.60 through 6.63, p.651 through p.654) indicate that the three-dimensional opposing inflow conditions along the y direction and the opposing outflow conditions along the x direction in the flow region develop as expected for the specified open-region water-surface-elevation boundary conditions (1) at the inlet planes (a) at the south-end upstream enclosure surface at $y = 0$ and (b) at the north-end upstream enclosure surface at $y = 24$ m, and (2) at the discharge planes (c) at the west-end downstream enclosure surface at $x = 0$ and (d) at the east-end downstream enclosure surface at $x = 44$ m.

During the early period of flow development, between time levels $t = 50$ sec and $t = 100$ sec, the two-dimensional vector representations of the velocity conditions in the lower layers $l = 1,2$ indicate that the inflow conditions along the y direction suddenly change into outflow conditions along the x direction in the small subregion (between $x = 18$ m and $x = 26$ m and between $y = 10$ m and $y = 14$ m) corresponding to the subregion of stagnation flow conditions in the middle of the flow region (Fig. 6.45, p.636; Fig. 6.46, p.637; Fig. 6.50, p.641; and Fig. 6.51, p.642). During the same period, between time levels $t = 50$ sec and $t = 100$ sec, the two-dimensional vector representations of the velocity conditions in the upper layers $l = 3,4$ indicate that inflow conditions along the y direction, originating from the inlet planes (1) at

the south-end upstream enclosure surface at y = 0 and (2) at the north-end upstream enclousre surface at y = 24 m, change gradually into outflow conditions along the x direction by spreading over the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) which is enclosed by divergent shoreline boundaries in the middle of the flow region (Fig. 6.47, p.638; Fig. 6.48, Fig. 6.52, p.643; and Fig. 6.53, p.644).

During the later period of flow development, between time levels t = 150 sec and t = 200 sec, the two-dimensional vector representations of the velocity conditions of the velocity conditions in the lower layers l = 1,2 indicate reversed flow conditions along the y-direction, in (1) the subregion (between x = 18 m and x = 26 m and between x = 0 and y = 10 m) and (2) the subregion (between x = 18 m and x = 26 m and y = 14 m and y = 24 m), which contain the parts of the deep main flow channel along the lateral central y axis of the flow region. However, during the same period, between the time levels t = 150 sec and t = 200 sec, the two-dimensional vector representations of the velocity conditions in the upper layers l = 3,4 indicate that the inflow conditions along the y-direction, originating from the inlet planes (1) at the south-end upstream enclosure surface at y = 0 and (2) at the north-end upstream enclosure surface at y = 24 m, still change gradually into outflow conditions along the x direction, without any reversed-flow conditions along the y direction, by spreading over the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) which is enclosed by the divergent shoreline boundaries in the middle of the flow region (Fig. 6.57, p.648; Fig. 6.58, p.649; Fig.6.62, p.653; and Fig. 6.63,

p.654). Hence, the computer simulation indicate the formation of two small secondary-flow vortices in the vertical (y,z) plane in the parts of the deep main flow channel slong the lateral central y axis in (1) the subregion (between x = 18 m and x = 26 m and betweeen y = 4 m and y = 10 m) and (2) the subregion (between x = 18 m and x = 26 m and between y = 14m and y = 20 m).

The formation of the two small secondary flow vortices in the vertical (y,z) plane can be explained by considering the water-surface-elevation conditions in the middle of the flow region during the later period of flow development, between the time levels t = 150 sec and t = 200 sec (Fig. 6.57, p.648; and Fig. 6.58, p.649). As the flow develops into pseudo-steady-state sonditions, between time levels t = 150 sec and t = 200 sec, the maximum water-surface-elevation conditions occur in a small subregion (between x = 18 m and x = 26 m and y = 8 m and y = 16 m) in the middle of the flow region (Fig. 6.54, p.645; and Fig. 6.59, p.650). Hence, the hydrostatic pressure gradients along the y direction, which indicate decreasing hydrostatic pressure conditions from the middle of the flow region toward (1) the south-end upstream enclosure surface at y = 0 and (2) the north-end upstream enclosure surface at y = 0 and (2) the north-end upstream enclosure surface at y = 24 m, can induce reversed flow against the inflow conditions along the y axis. In the upper layers l = 3,4, the existing hydrostatic pressure gradient along the direction cannot overcome the momentum of the inflow along the y direction, since the viscous effects in the upper layers of the deep main flow channel is not sufficiently strong to retard the inflow conditions toward the center of the flow region. However, in the

lower layers l = 1,2, the viscous effects in the deep main flow channel are stronger; and consequently they can retard substantially the inflow conditions toward the center of the flow region. Hence, the existing hydrostatic pressure gradient along the y-direction can overcome the momentum of the inflow along the y-direction; and it can cause reversed-flow conditions in the lower layers l = 1,2 in the deep main flow channel in (1) the subregion (between x = 18 m and x = 26 m and between y = 4 m and y =10 m) and (2) in the subregion (between x = 18 m and x = 26 m and between y = 14 m and y = 20 m).

The most important result of the computer simulations of the velocity conditions is that during the complete period of flow development, between time levels t = 50 sec and t = 200 sec, the temporal variations of all the components of the three-dimensional velocity vectors retained perfectly the required spatially-symmetric and spatially-asymmetric distributions about both central x-axis and centrl y-axis of the flow region (Figs. 6.45 through 6.48, p.636 through p.639; Figs. 6.50 through 6.53, p.641 through p.644, Figs. 6.55 through 6.58, p.646 through p.649; and Figs. 6.60 through 6.63, p.651 through p.654).

Hence, the results of the computer simulations of the water-surface-elevation and three-dimensional velocity conditions in the laboratory-model-scale flow region for test case 3 : Symmetric Opposing Flow 1 (inflow along lateral direction and outflow along longitudinal direction), again illustrates the reliability of the mathematical formulation of the transient, three-dimensional discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of its associated HYDROL computer code.

6.1.2.4 Test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). The fourth test case was considered specifically to test the reliability of the mathematical, formulation of the transient, three-dimensional, discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of the HYDROL computer code under extremely demanding flow simulation conditions. It is again important to note that since the specified inflow conditions are again in exactly opposite directions, and since the simulations are transient, any possible unstable characteristic of the mathematical formulation and/or computational algorithm of the discrete-element hydrodynamic model would tend to destroy (1) the exact symmetry of the water surface elevation conditions about the central x axis and the central y axis and (2) the exact symmetry or asymmetry of the velocity components about the central x axis or the central y axis associated with the simulated flow conditions in the symmetric flow region.

The necessary input data to the HYDROL computer code for the fourth test case was prepared according to the following conditions associated with the mathematical formulation of the discrete-element hydrodynamic model.

a. Boundary Conditions

The test case was run by considering open-region boundary conditions (1) at two upstream enclosure surfaces, (a) the west-end upstream enclosure surface at $x = 0$ and (b) the east-end upstream enclosure surface at $x = 44$ m, and (2) at two downstream enclosure surfaces, (c) the south-end downstream enclosure surface at $y = 0$ and (d) the

north-end downstream enclosure surface at y = 24 m, of the flow region. All other enclosure boundaries were assumed to be closed, impermeable shoreline boundary surfaces.

West-end upstream condition for the surface elevation at x = 0 (relative to MWL):

$\overline{H}_{(opr,i-1),j}$ = 0.01 m (above MWL), along lateral open-region enclosure surface, upstream of the four boundary elements i=1; j=2,3,4,5; l=4.

West-end upstream control conditions on velocity components at x = 0:

longitudinal velocity component

$\overline{V}_{x,min;(opr,i-1),j,l}$ = -0.5 m/sec, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\overline{V}_{x,max;(opr,i-1),j,l}$ = 0.5 m/sec, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

lateral velocity component

$\overline{V}_{y,min;(opr,i-1),j-\frac{1}{2},l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\overline{V}_{y,max;(opr,i-1),j-\frac{1}{2},l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

vertical velocity component

$\overline{V}_{z,min;(opr,i-1),j,l}$ = 0, for all elements i=1; j=2,5 l=3,4; and i=1; j=3,4; l=1,2,3,4.

$\overline{V}_{z,max;(opr,i-1),j,l}$ = 0, for all elements i=1; j=2,5; l=3,4; and i=1; j=3,4; l=1,2,3,4.

East-end upstream condition for the surface elevation at x = 44 m (relative to MWL):

$\overline{H}_{(opr,i+1),j}$ = 0.01 m (above MWL), along lateral open-region enclosure surface, upstream of the four boundary elements i=14; j=2,3,4,5; l=4.

East-end upstream control conditions on velocity components at x = 44 m:

longitudinal velocity component

$\overline{V}_{x,min;(opr,i+1),j,l}$ = -0.5 m/sec, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\overline{V}_{x,max;(opr,i+1),j,l}$ = 0.5 m/sec, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

lateral velocity component

$\overline{V}_{y,min;(opr,i+1),j-\frac{1}{2},l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\overline{V}_{y,max;(opr,i+1),j-\frac{1}{2},l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

vertical velocity component

$\overline{V}_{z,min;(opr,i+1),j,l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

$\overline{V}_{z,max;(opr,i+1),j,l}$ = 0, for all elements i=14; j=2,5; l=3,4; and i=14; j=3,4; l=1,2,3,4.

South-end downstream condition for water surface elevation at y = 0 (relative to MWL):

$\overline{H}_{i,(opr,j-1)}$ = -0.01 m (below MWL), along longitudinal open-region enclosure surface, downstream of the two boundary elements i=7,8; j=1; l=4.

South-end downstream control conditions on velocity components at y = 0:

lateral velocity component

$\bar{V}_{y,min;i,(opr,j-1),l}$ = -0.5 m/sec, for all elements i=7,8; j=1; l=1,2,3,4.

$\bar{V}_{y,max;i,(opr,j-1),l}$ = 0.5 m/sec, for all elements i=7,8; j=1; l=1,2,3,4.

longitudinal velocity component

$\bar{V}_{x,min;i-\frac{1}{4},(opr,j-1),l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

$\bar{V}_{x,max;i-\frac{1}{4},(opr,j-1),l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

vertical velocity component

$\bar{V}_{z,min;i,(opr,j-1),l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

$\bar{V}_{z,max;i,(opr,j-1),l}$ = 0, for all elements i=7,8; j=1; l=1,2,3,4.

North-end downstream condition for water surface elevation at y = 24 m (relative to MWL):

$\bar{H}_{i,(opr,j+1)}$ = -0.01 m (below MWL), along longitudinal open-region enclosure surface, downstream of the two boundary elements i=7,8; j=6; l=4.

North-end downstream control conditions on velocity components at y = 24 m:

lateral velocity component

$\bar{V}_{y,min;i,(opr,j+1),l}$ = -0.5 m/sec, for all elements i=7,8; j=6; l=1,2,3,4.

$\bar{V}_{y,max;i,(opr,j+1),l}$ = 0.5 m/sec, for all elements i=7,8; j=6; l=1,2,3,4.

longitudinal velocity component

$\bar{V}_{x,min;i-\frac{1}{4},(opr,j+1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

$\bar{V}_{x,max;i-\frac{1}{4},(opr,j+1),l}$ = 0, for all elements i=7,8; j=6; l=1,2,3,4.

vertical velocity component

$\bar{V}_{z,min;i,(opr,j+1),l} = 0$, for all elements i=7,8; j=6; l=1,2,3,4.

$\bar{V}_{z,max;i,(opr,j+1),l} = 0$, for all elements i=7,8; j=6; l=1,2,3,4.

The computer simulation results for test case 4 were obtained by considering the open-region boundary conditions which required the specification of water-surface-elevation on all four boundary enclosure surfaces of the flow region, (1) at two upstream enclosure surfaces, (a) the west-end upstream enclosure surface at x = 0 and (b) the east-end upstream enclosure surface at x = 44 m, and (2) at two downstream enclosure surfaces, (c) the south-end downstream enclosure surface at y = 0 and (d) the norht-end upstream enclosure surface at y = 24 m. Therefore, according to the particular option for specifying open-region boundary conditions (Sec. 4.5), the velocity components outside of the west-end upstream open-region enclosure surface of the flow region were determined by linear extrapolation, based on the values of the velocity components in the two boundary half-elements along the x axis, in the element immediately adjacent to the west-end upstream open-region enclosure surface. Similarly, the velocity components outside of the east-end upstream open-region enclosure surface of the flow region were determined by linear extrapolation, based on the values of the velocity components in the two boundary half-elements along the x axis, in the element immediately adjacent to the east-end upstream open-region enclosure surface. The control conditions were used for eliminating possible convergence errors that may be caused by overestimating (or underestimating) the velocity conditions outside of the upstream open-region enclosure surface. This option for open-region boundary conditions were included in the discrete-element hydrodynamic model as a general capabi-

lity for incorporating approximate control conditions at open-region enclosure surfaces when exact values for the velocity components could not be specified. In the case when the velocity components could be determined exactly at the west-end open-region enclosure surface and at the east-end open-region enclosure surface, the specification of $\bar{V}_{x,min}$ and $\bar{V}_{x,max}$ as equal to the actual exact values of the velocity components could simulate automatically the required boundary conditions for the velocity components. The velocity components that were not normal to the west-end upstream open-region enclosure surface at $x = 0$ and to the east-end upstream open-region enclosure surfaces at $x = 44$ m, were all specified as $\bar{V}_{y,min} = \bar{V}_{y,max} = 0$ and $\bar{V}_{z,min} = \bar{V}_{z,max} = 0$.

The velocity components (1) for the outflow at the south-end downstream open-region enclosure surface at $y = 0$ and (2) for the outflow at the north-end downstream open-region enclosure surfaces at $y = 24$ m, were not used for extrapolating to outside of the flow region, since general second-upwind-differencing formulation automatically determines the values of the velocity components based on the velocity components in the boundary half-elements along the y axis, (1) immediately adjacent to the south-end downstream open-region enclosure surfaces at $y = 0$ and (2) immediately adjacent to the north-end downstream open-region enclosure surface at $y = 24$ m.

b. Interpolation Option

The computer simulation results for test case 4 were obtained by applying the HYDROL computer code with simple second-upwind-differencing interpolation option. It was noted that this interpolation option could

introduce numerical dispersion effects in the elements and half-elements where the local Courant number was much less than 1 (see Sec. 4.3).

c. Numerical Solution Option

The computer simulation results for the test case 4 were obtained by applying the HYDROL computer code with the standart default option which ultilized the special, composite time-splitting method in the numerical integration of the governing system of spatially discretized ordinary differential equations (see Sec. 5.2).

d. Discussion of the Computer Simulation Results

The results of the computer simulations for the water surface elevation and velocity conditions are presented in Figs. 6.64 through 6.83, p.673 through p.692. The surface elevation conditions were presented as (1) numbers in the elements which represent water surface elevation (mm) relative to specified initial mean water level (MWL) and (2) constant water surface elevation contours which correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL. The velocity conditions were presented as two-dimensional vectors which correspond to two-component representations of velocity in the horizontal (x,y) planes of the four specified layers.

The results of the computer simulations for the flow conditions in the symmetric laboratory-model-scale flow region indicated that the water surface elevation and velocity conditions were generally consistent with the conditions that would exist in a similar prototype-scale flow region with a similar configuration for the deep main flow channel.

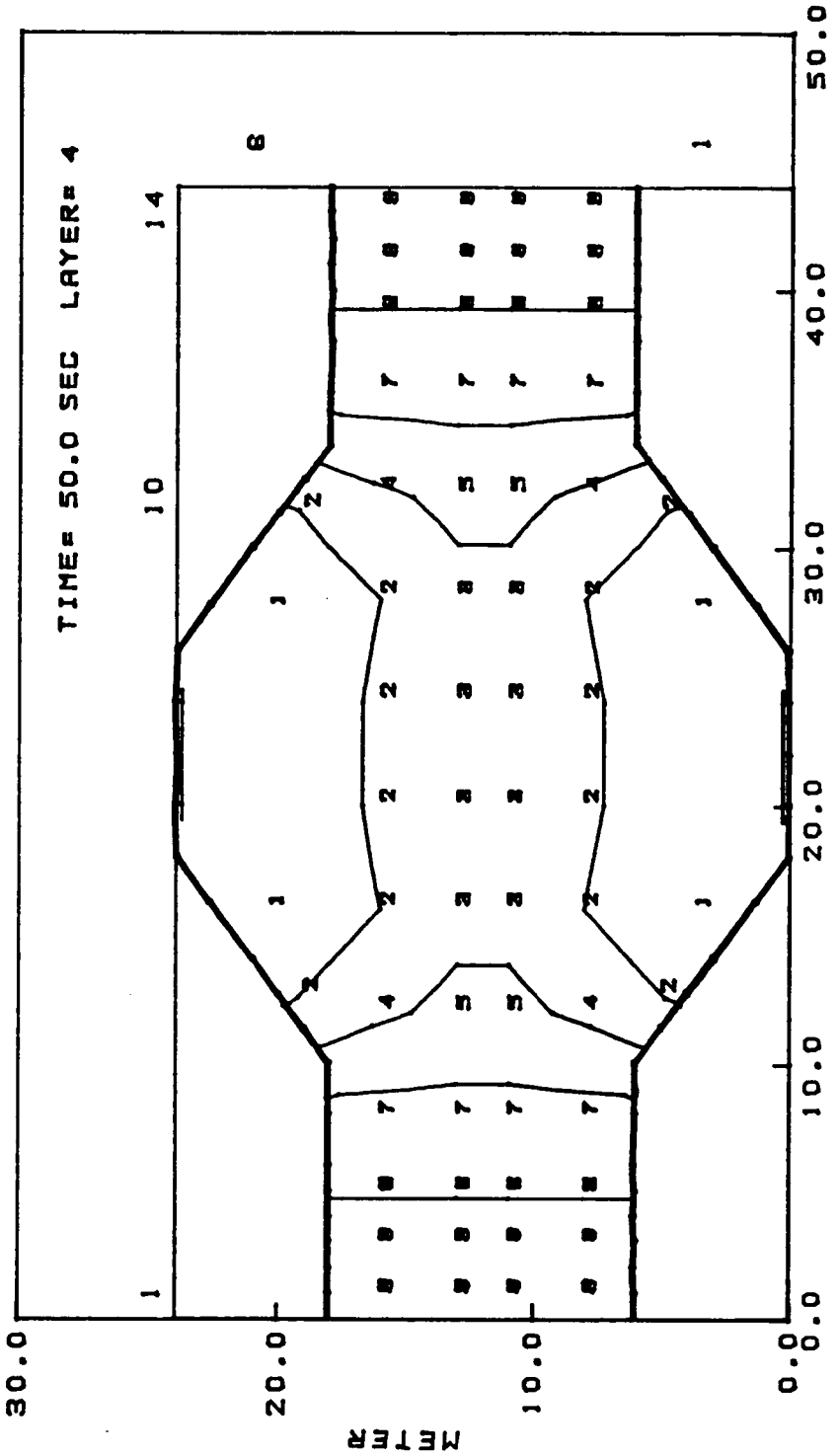

Figure 6.64. Water-surface-elevation conditions at time t = 50 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

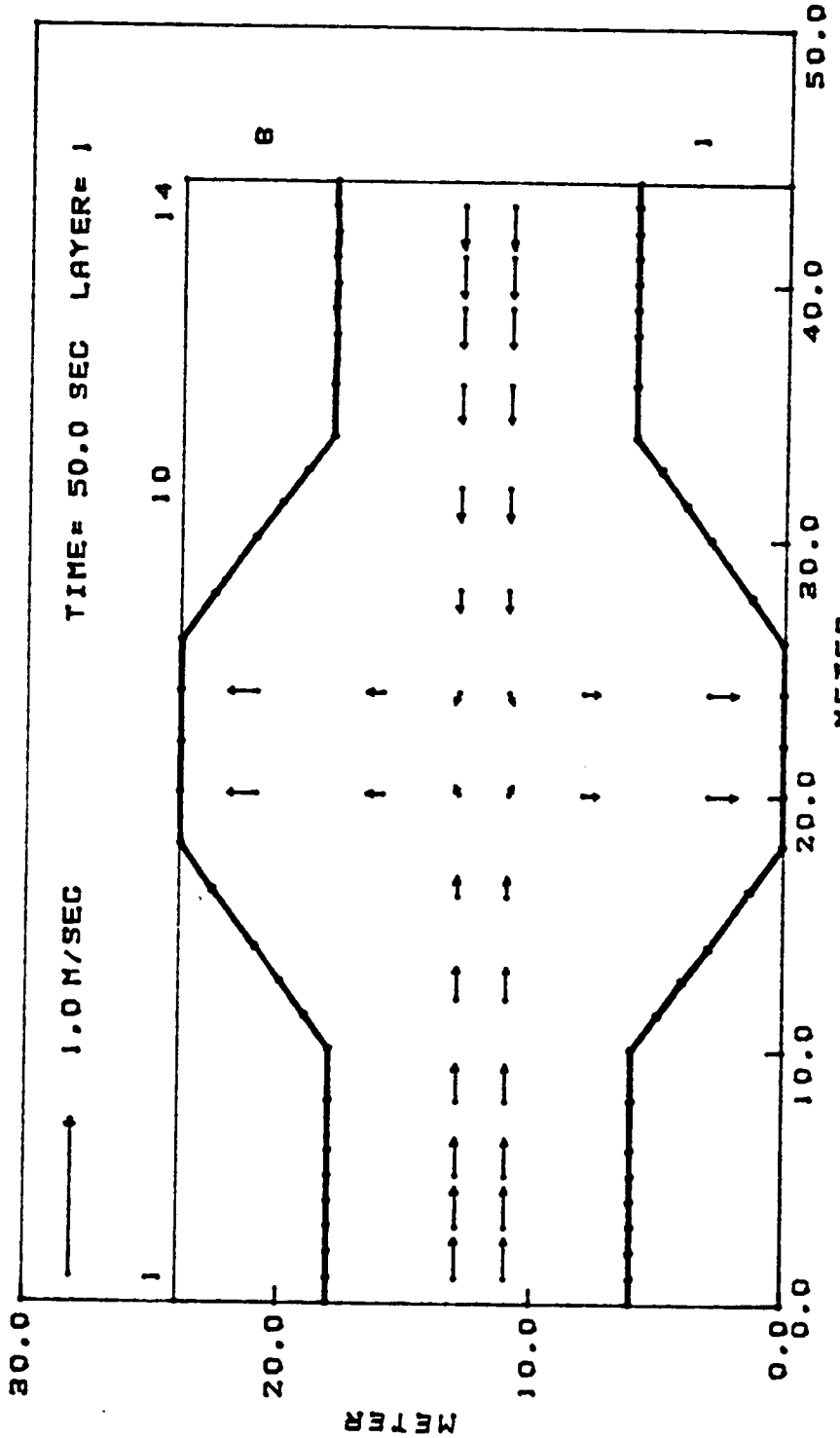

Figure 6.65. Velocity conditions in the bottom layer l = 1, at time t = 50 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

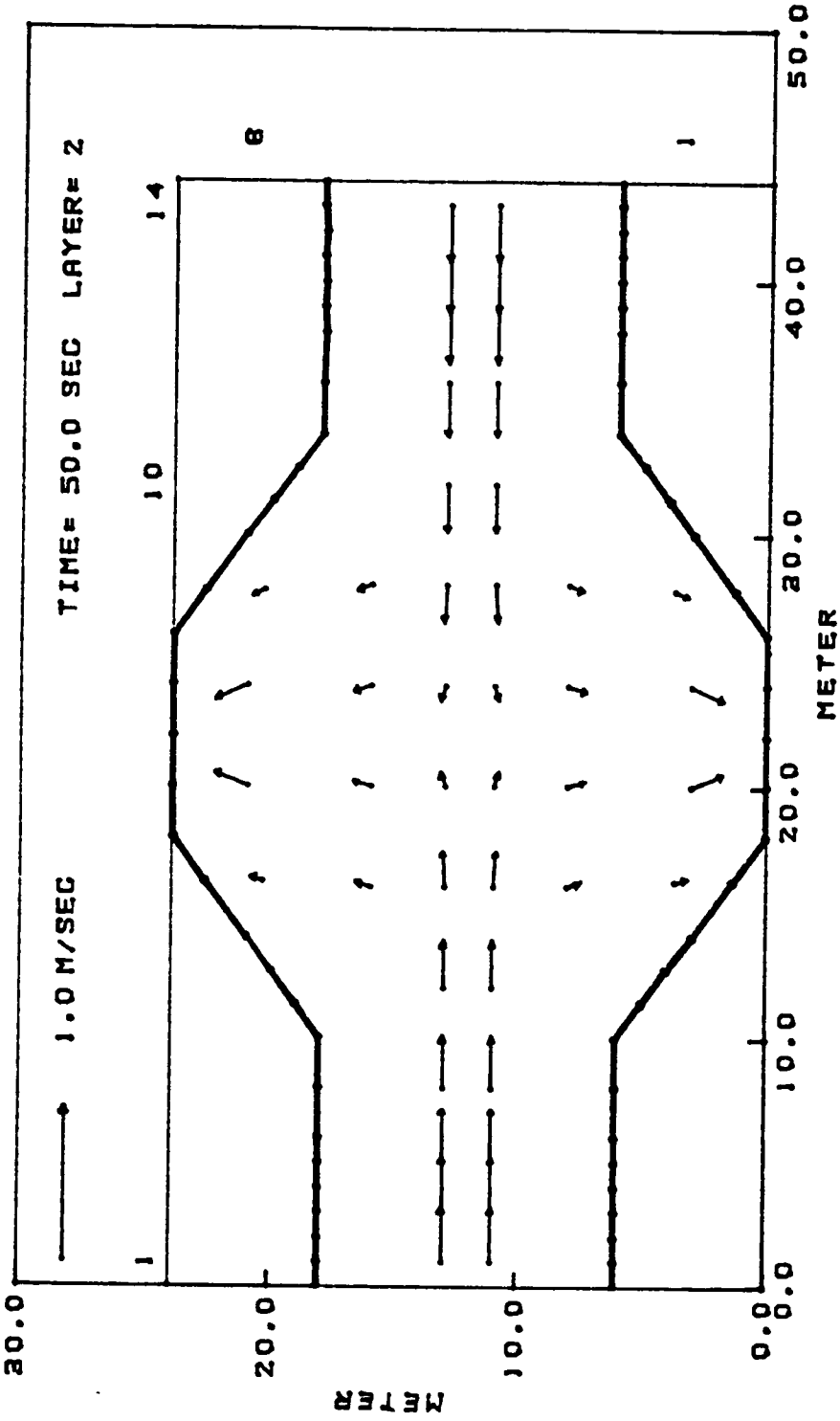

Figure 6.66. Velocity conditions in the internal layer l = 2, at time t = 50 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

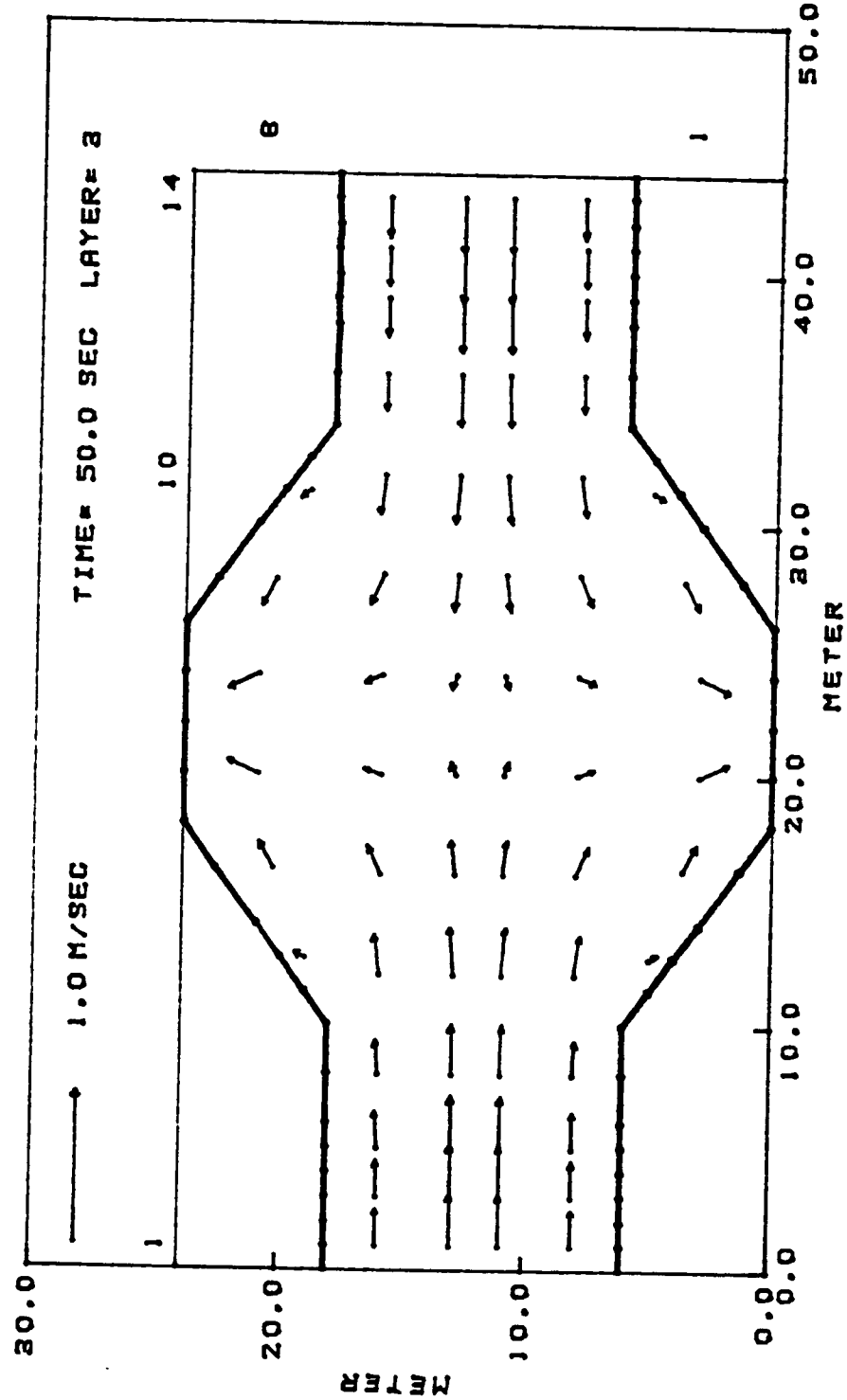

Figure 6.67. Velocity conditions in the internal layer l = 3, at time t = 50 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the interanl layer.

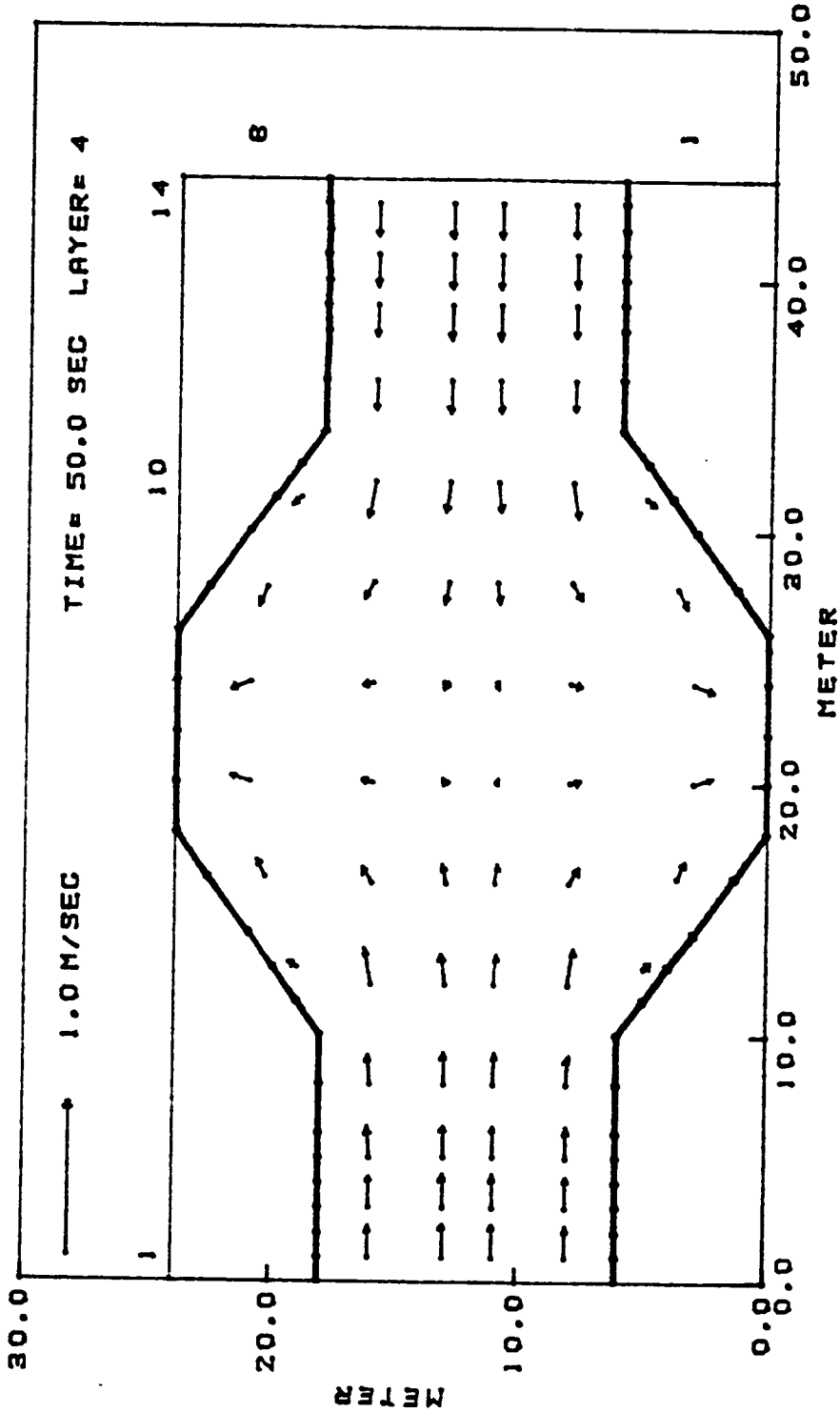

Figure 6.68. Velocity conditions in the surface layer l = 4, at time t = 50 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

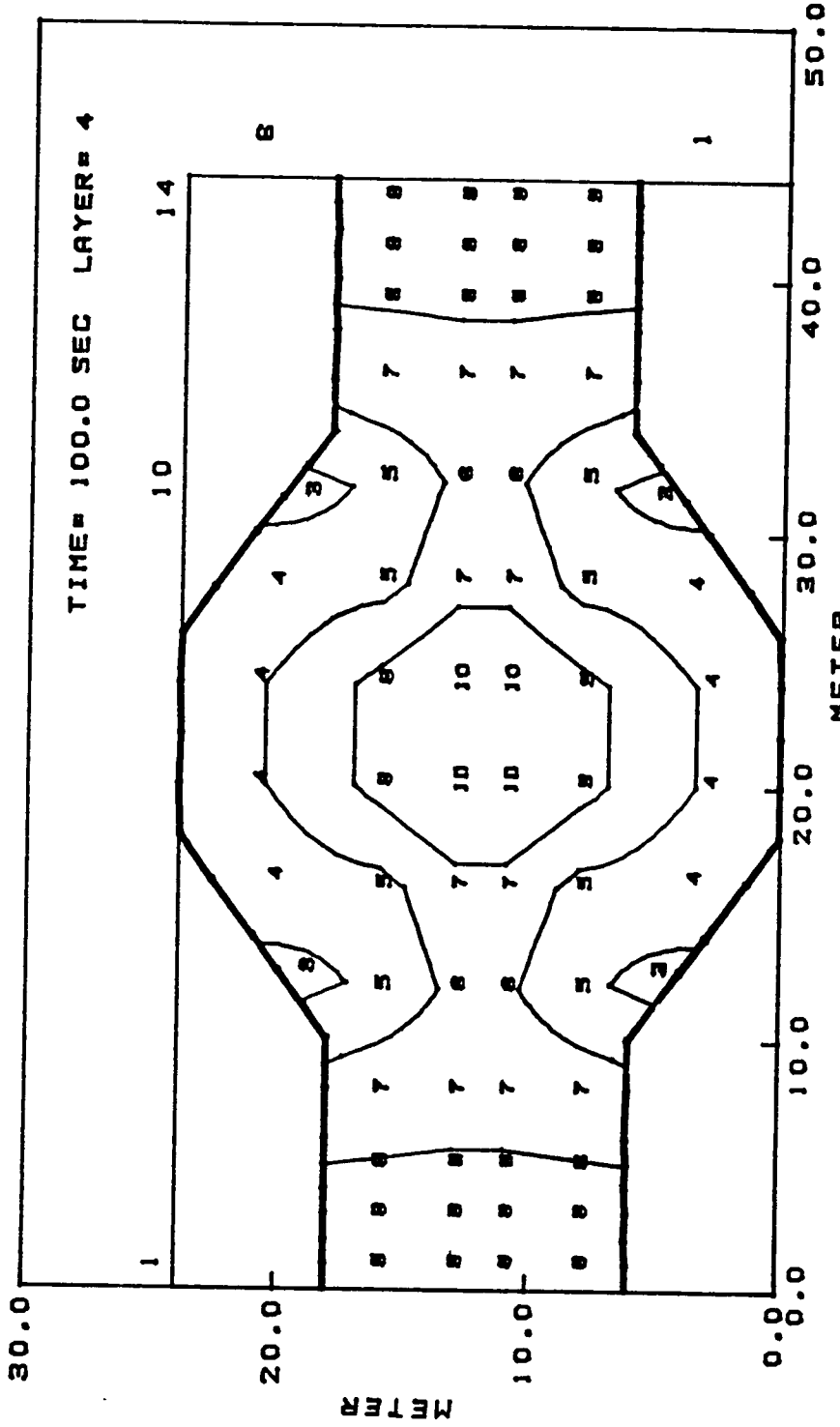

Figure 6.69. Water-surface-elevation conditions at time t = 100 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

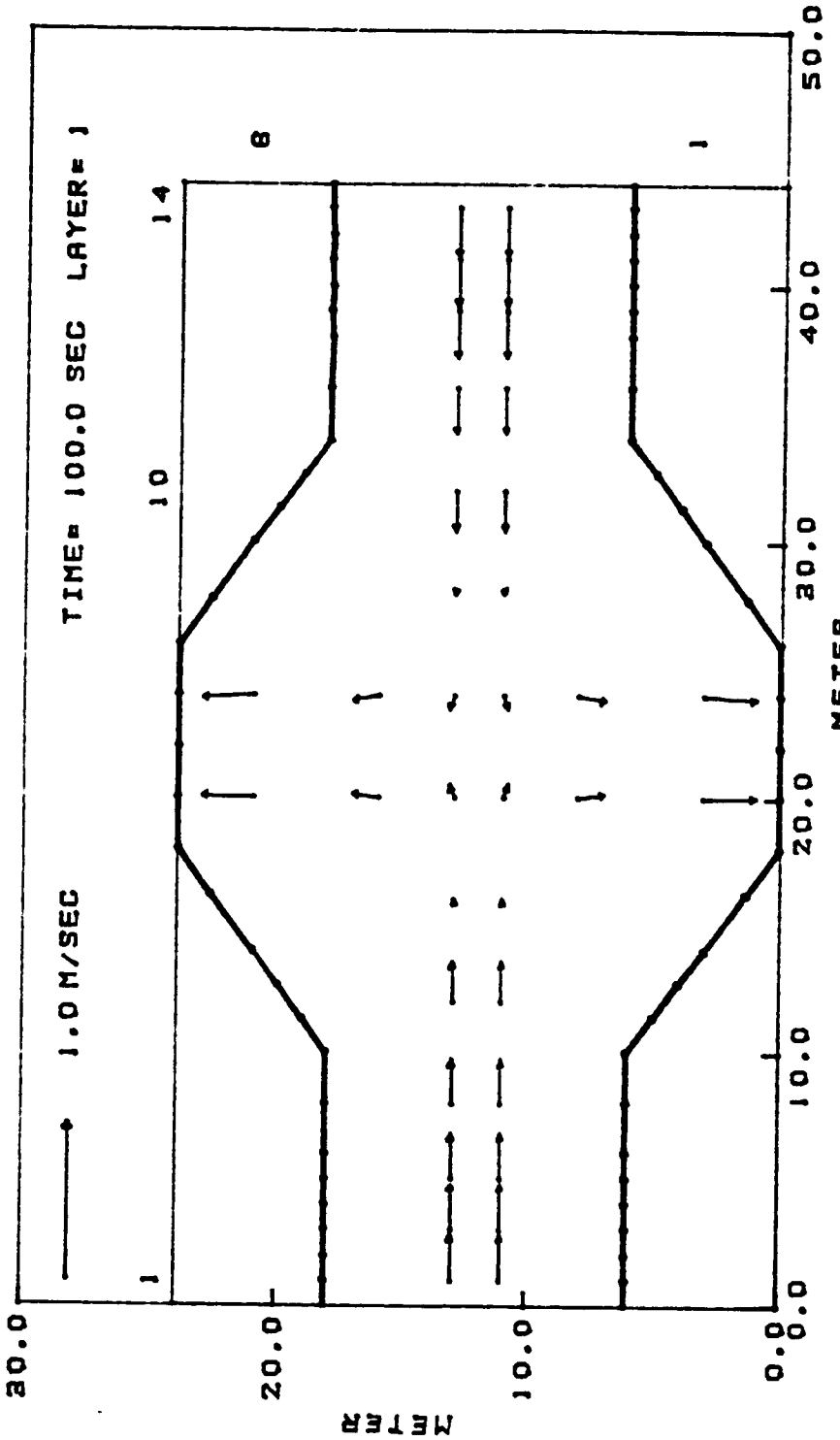

Figure 6.70. Velocity conditions in the bottom layer l = 1, at time t = 100 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

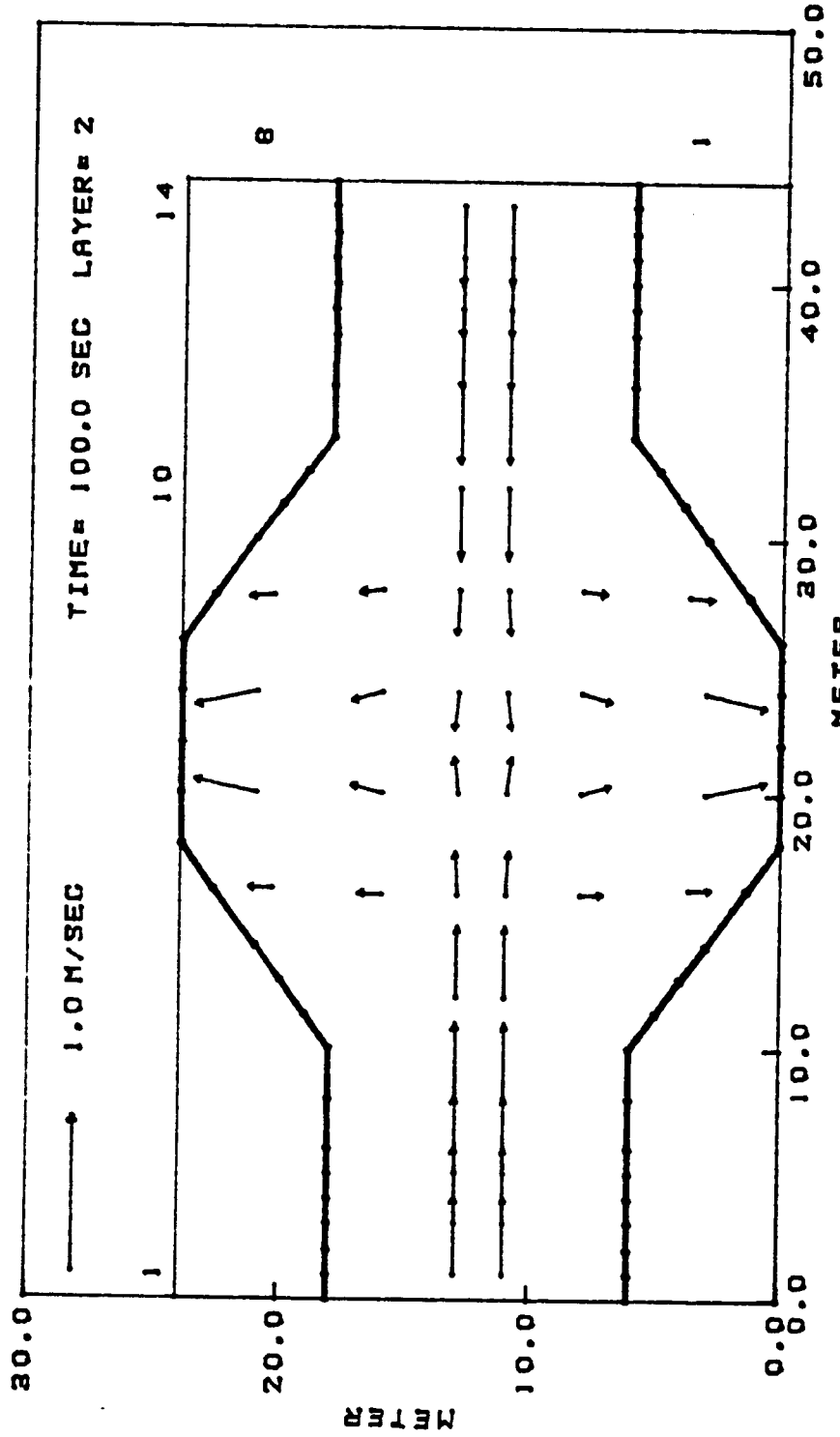

Figure 6.71. Velocity conditions in the internal layer l = 2, at time t = 100 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

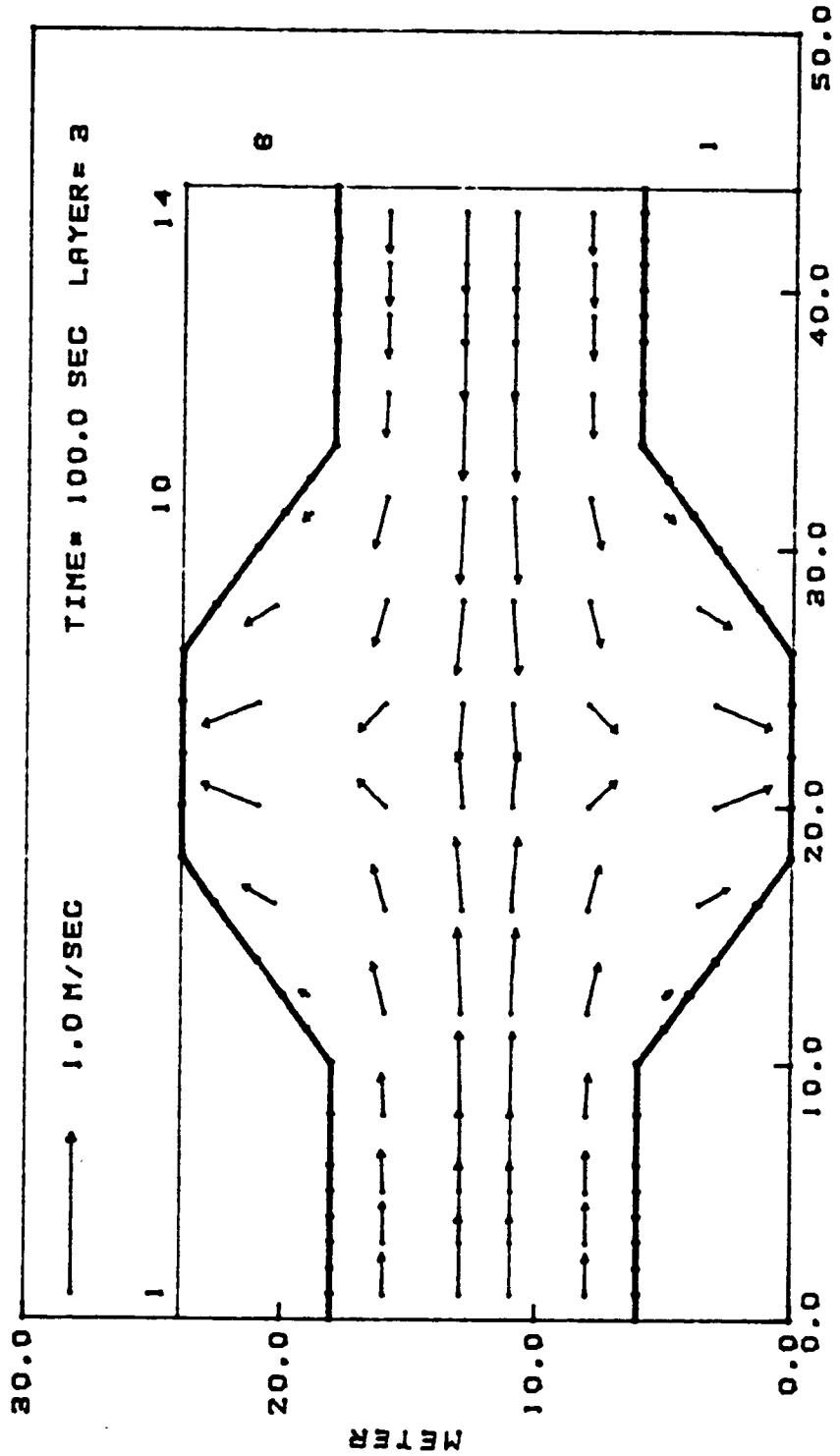

Figure 6.72. Velocity conditions in the internal layer l = 3, at time t = 100 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

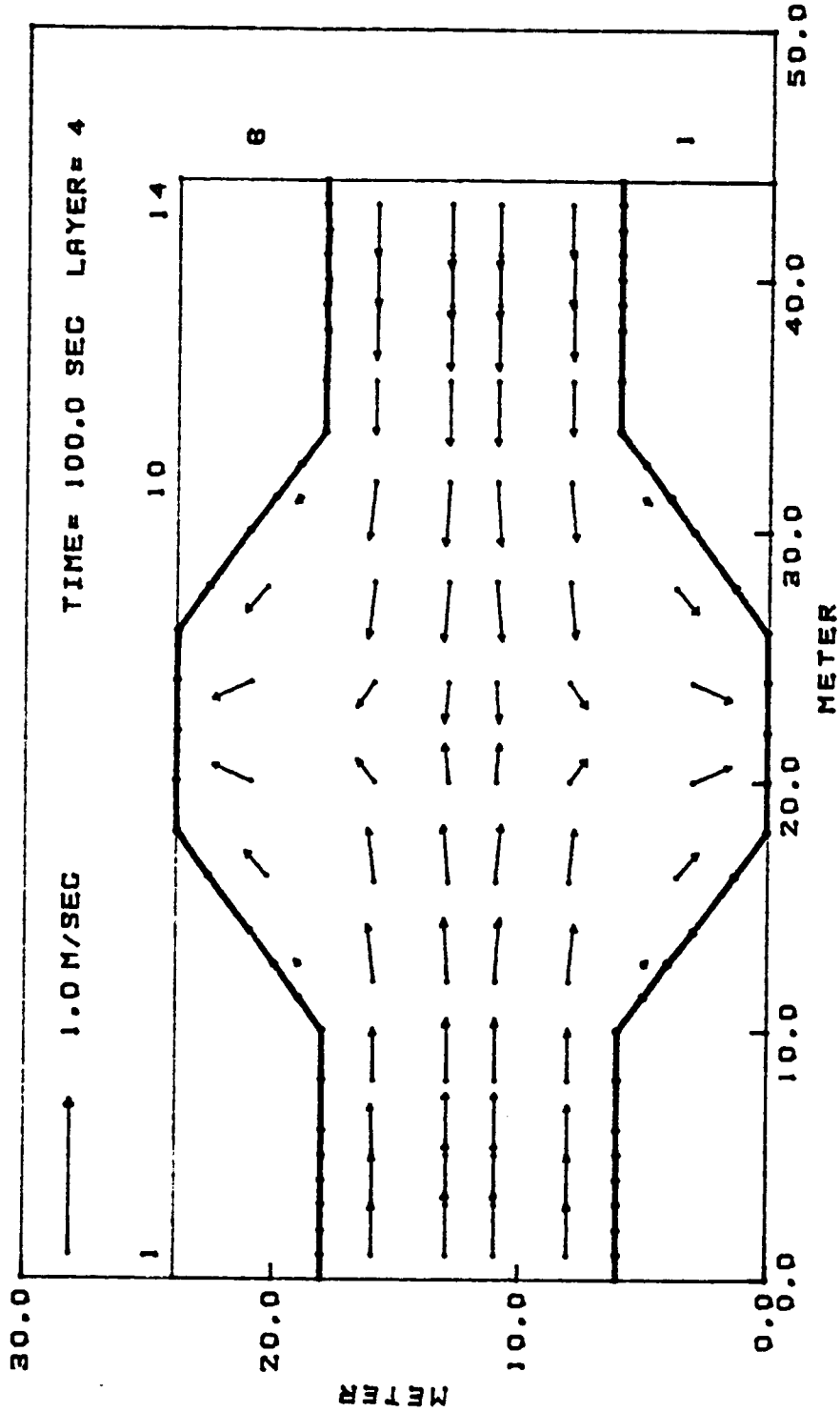

Figure 6.73. Velocity conditions in the surface layer l = 4, at time t = 100 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

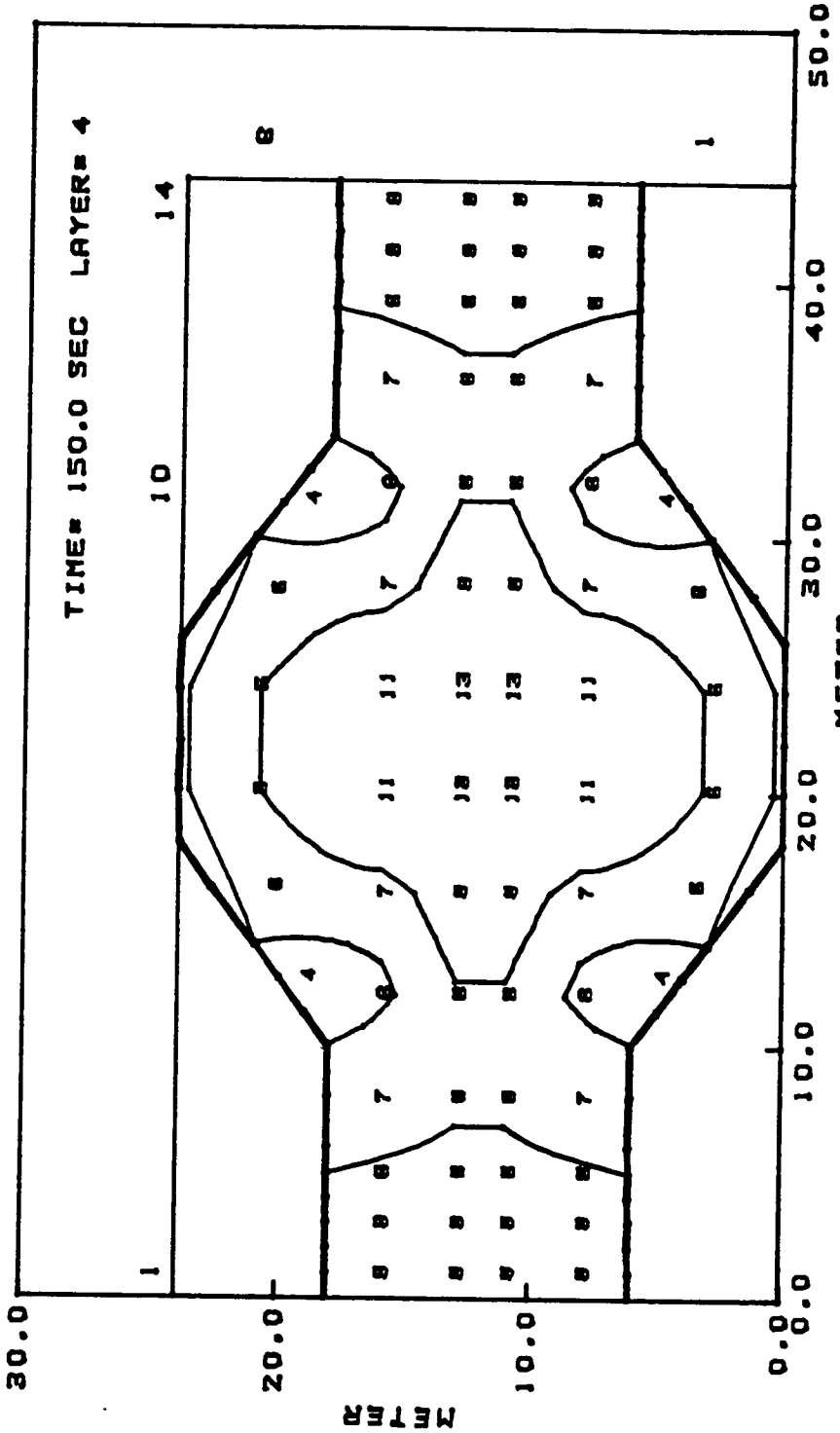

Figure 6.74. Water-surface-elevation conditions at time t = 150 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

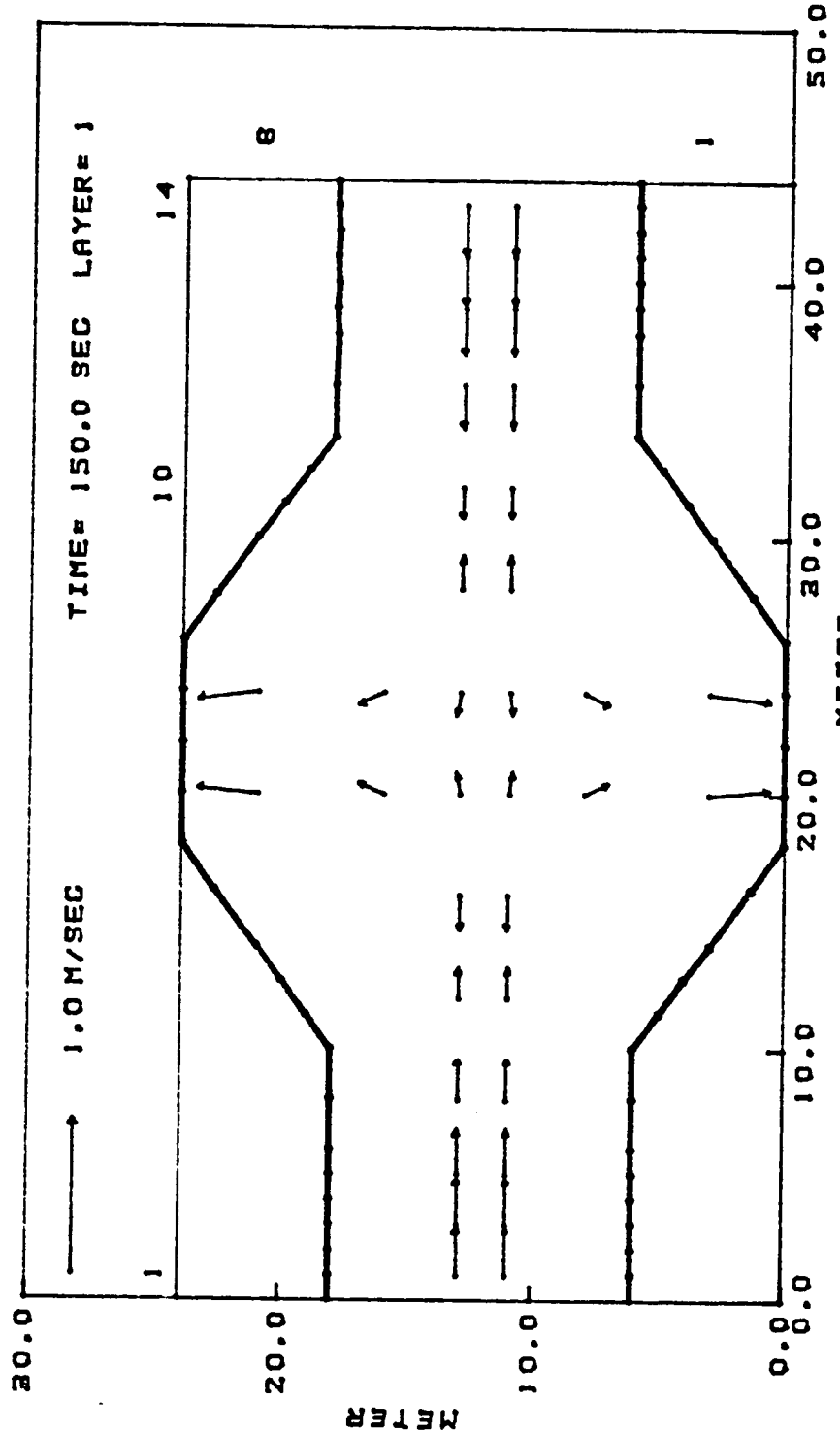

Figure 6.75. Velocity conditions in the bottom layer l = 1, at time t = 150 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

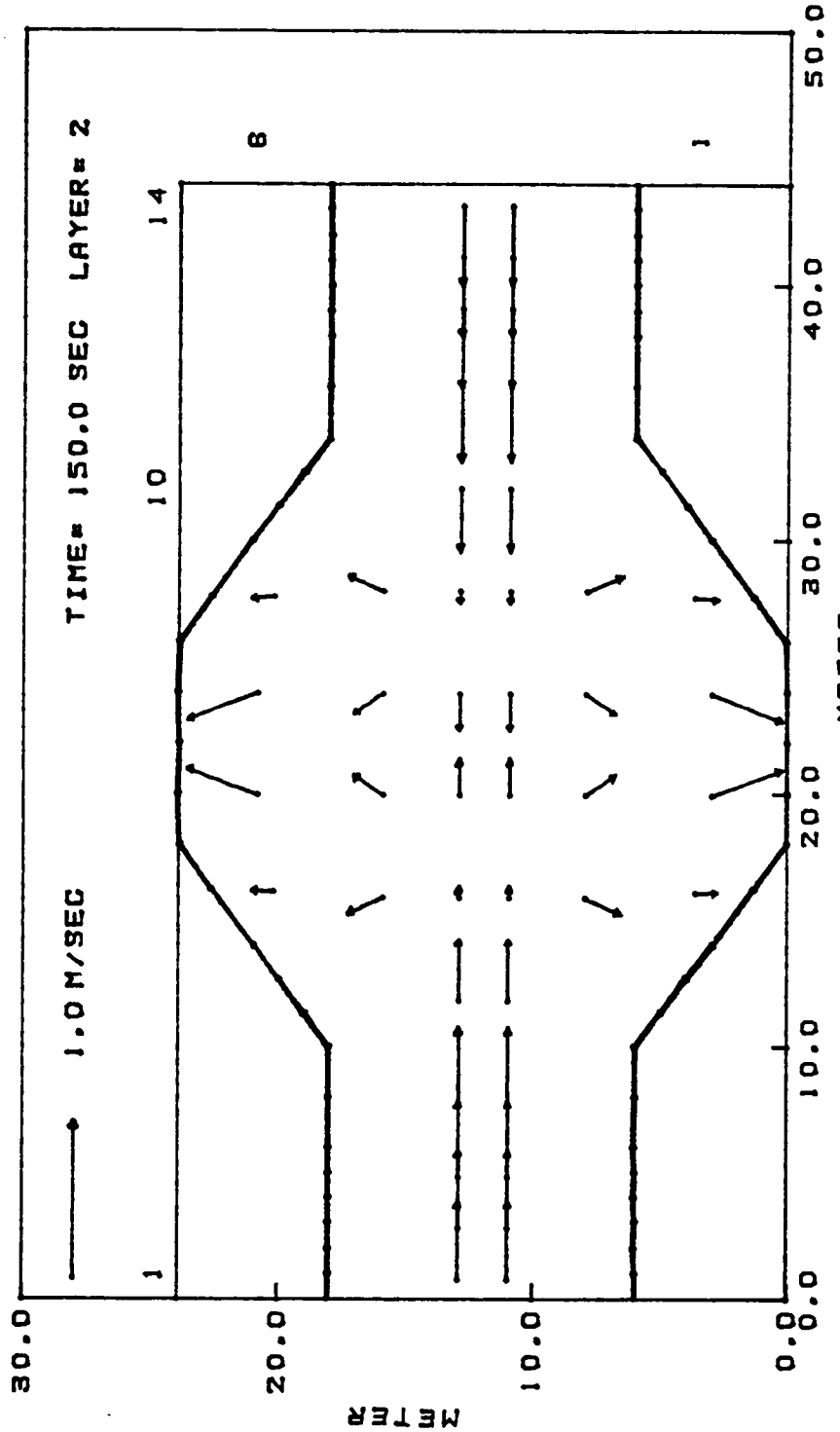

Figure 6.76. Velocity conditions in the internal layer l = 2, at time t = 150 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

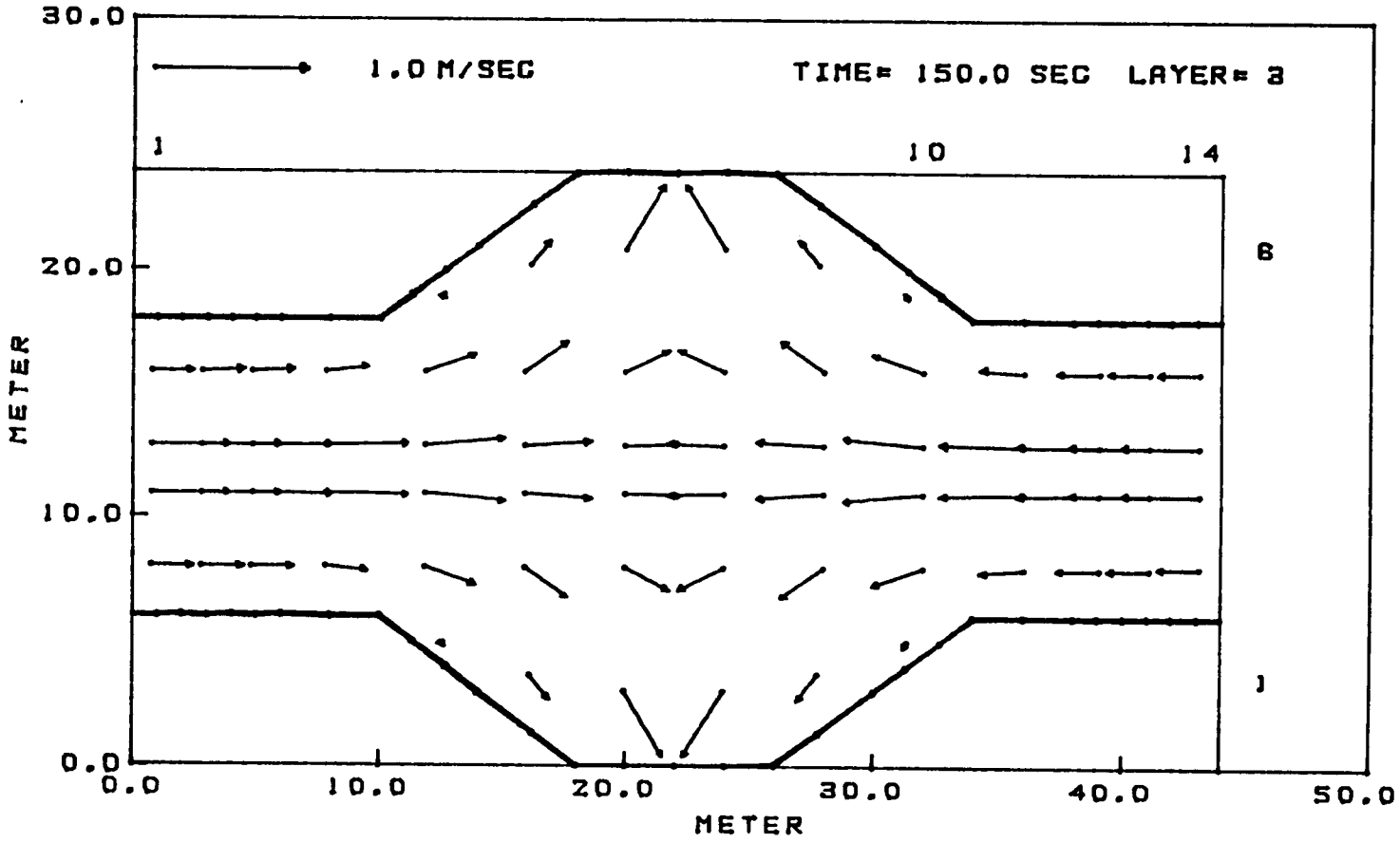

Figure 6.77. Velocity conditions in the internal layer l = 3, at time t = 150 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

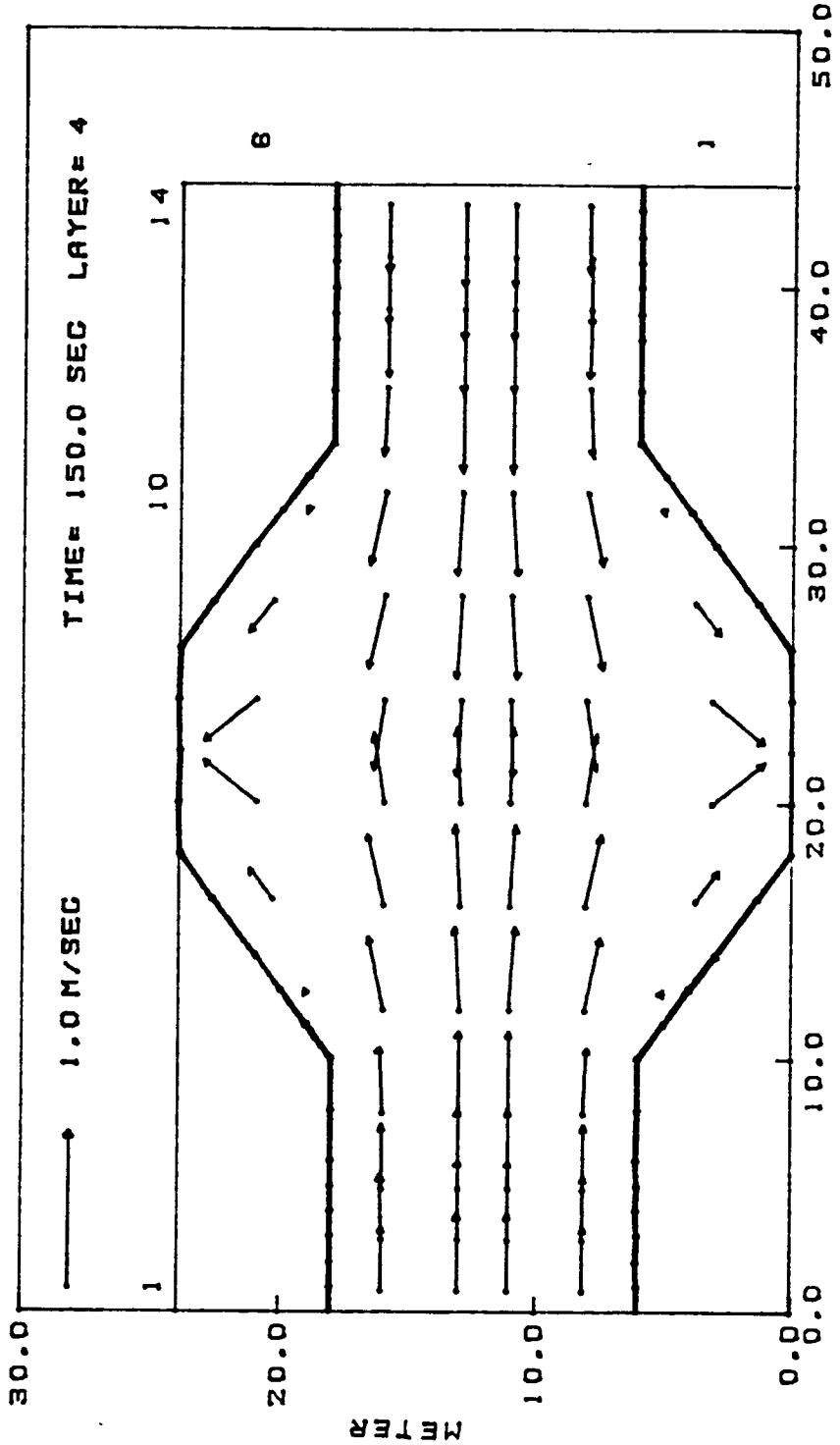

Figure 6.78. Velocity conditions in the surface layer l = 4, at time t = 150 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

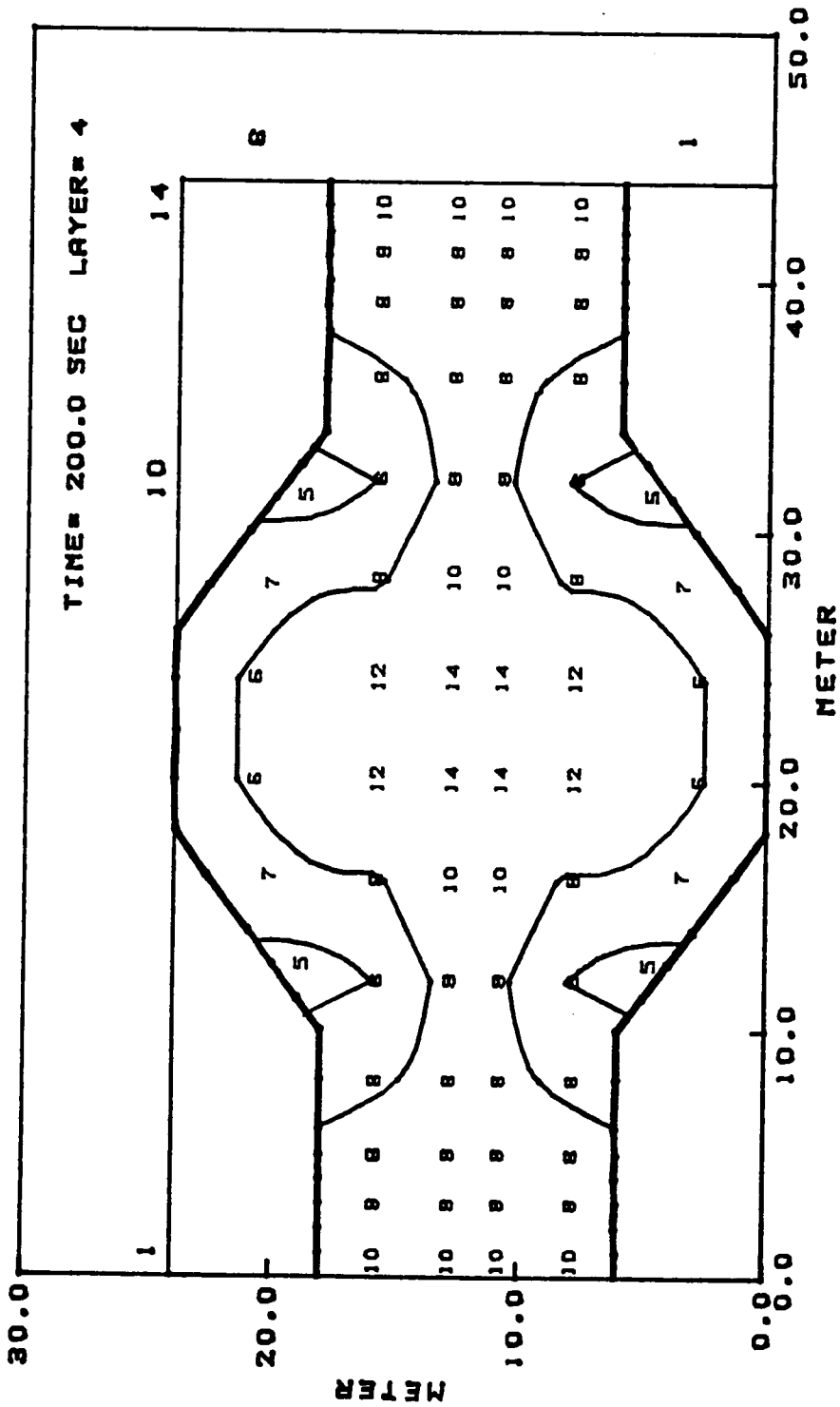

Figure 6.79. Water-surface-elevation conditions at time t = 200 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

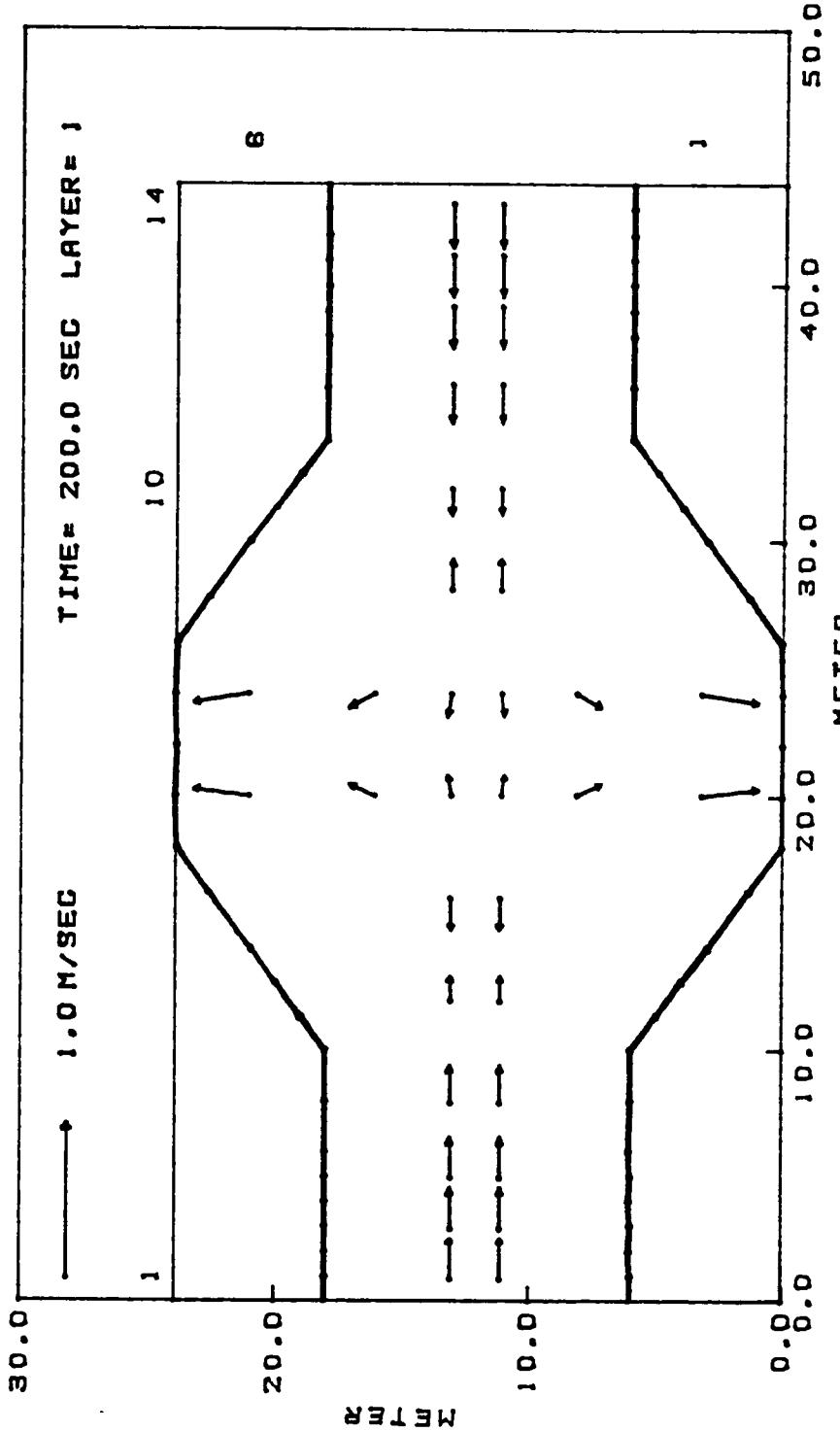

Figure 6.80. Velocity conditions in the bottom layer l = 1, at time t = 200 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

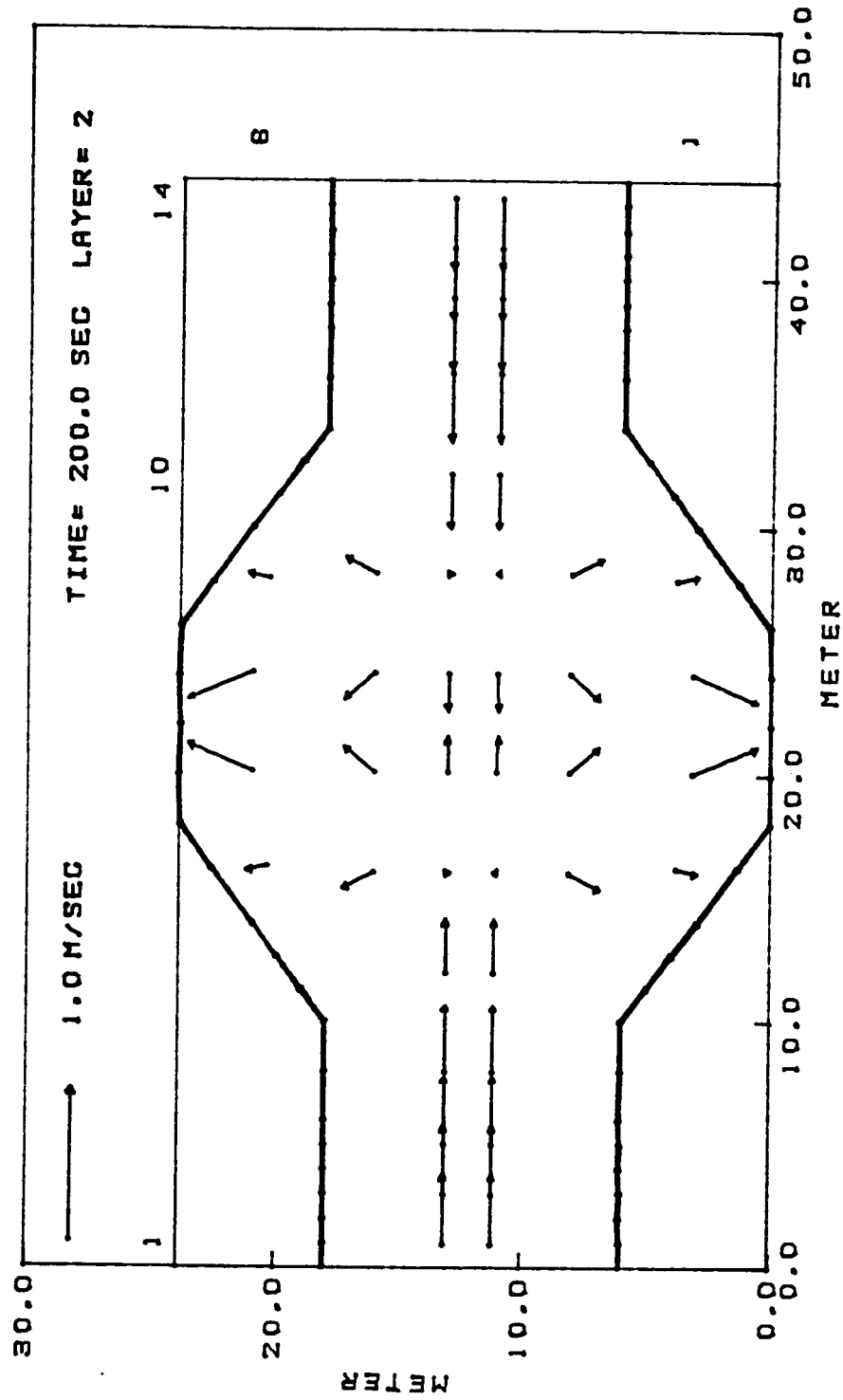

Figure 6.81. Velocity conditions in the internal layer l = 2, at time t = 200 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

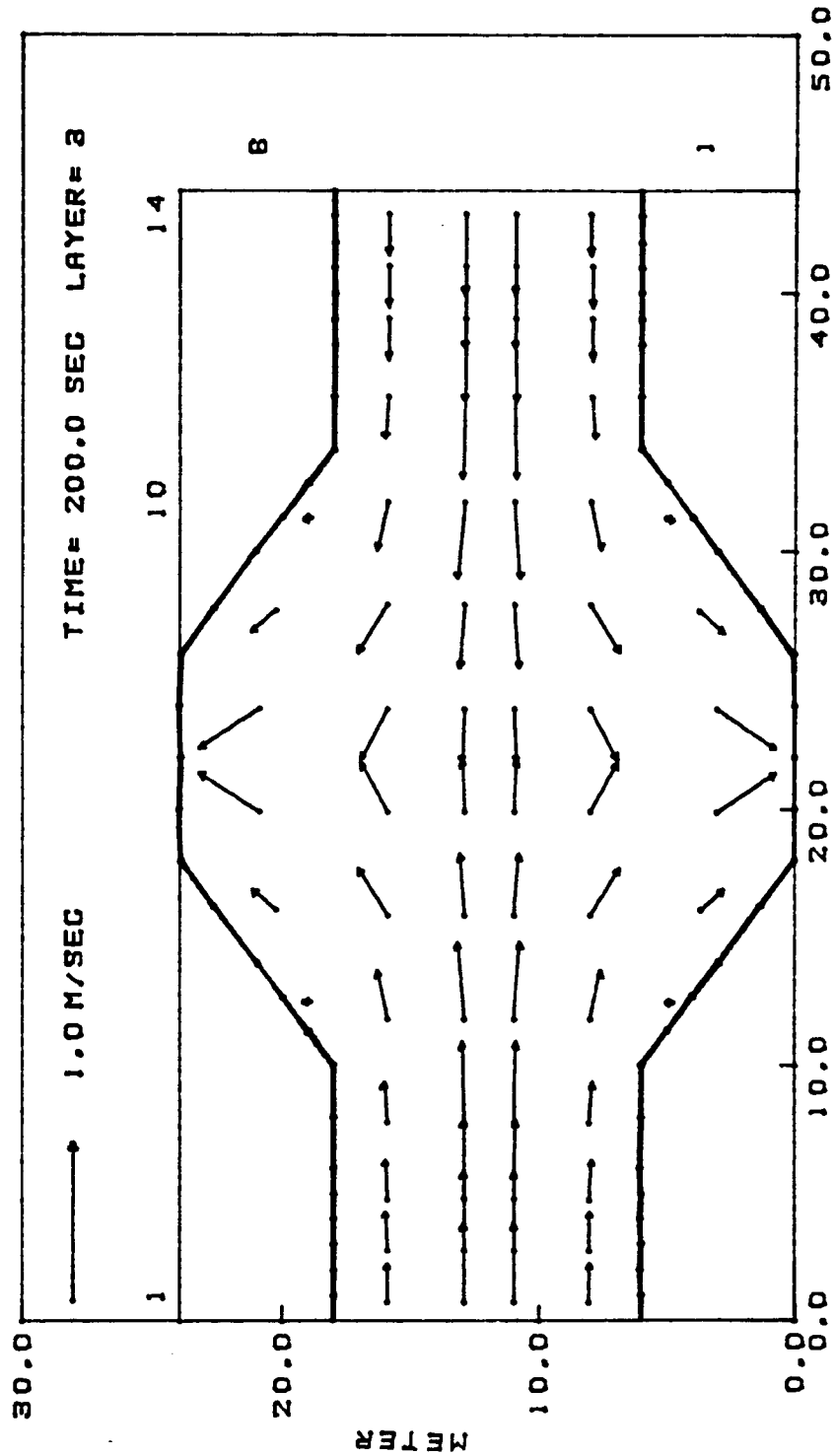

Figure 6.82. Velocity conditions in the internal layer l = 3, at time t = 200 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

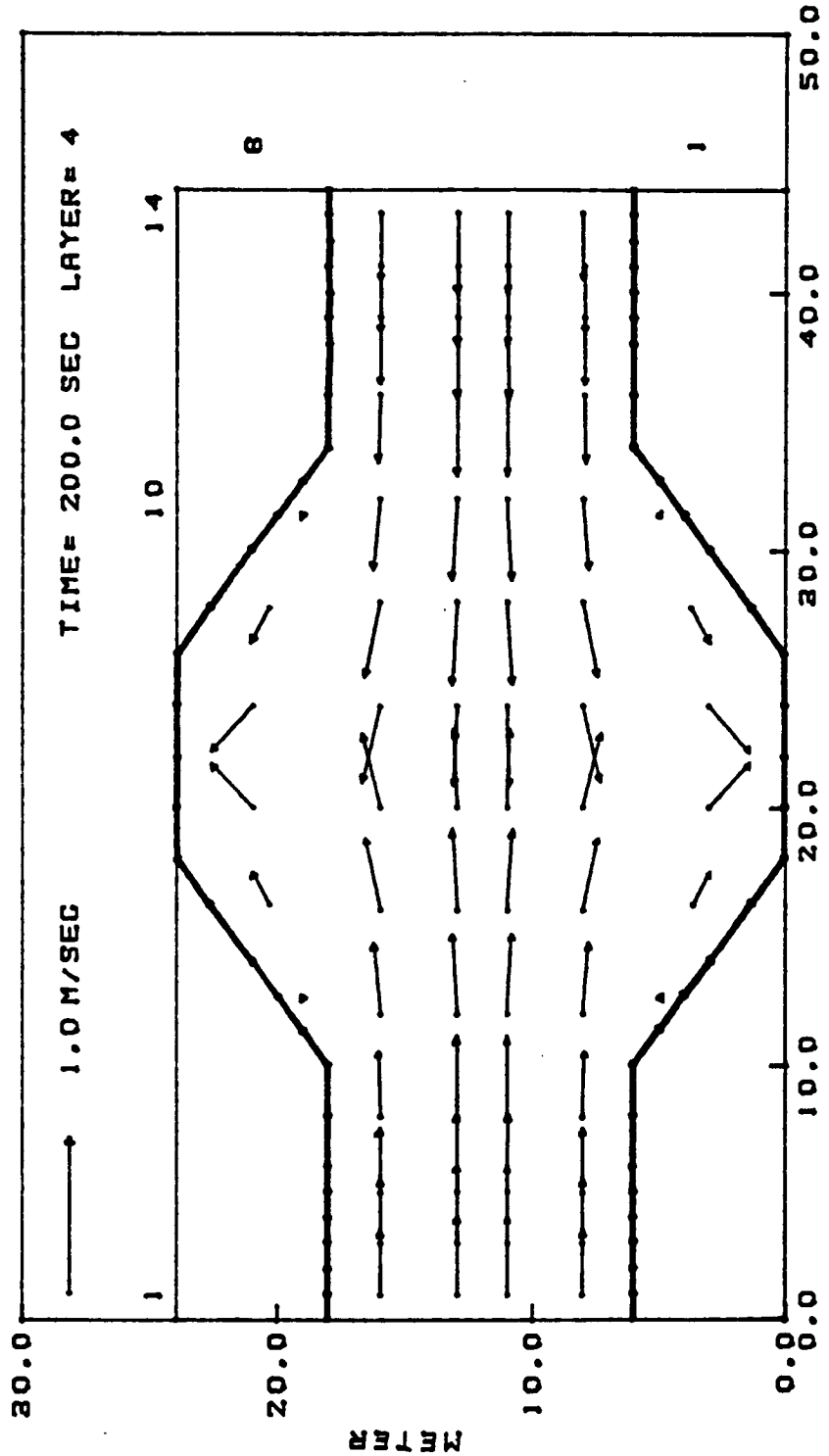

Figure 6.83. Velocity conditions in the surface layer l = 4, at time t = 200 sec for test case 4: Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

The special transient flow characteristics of the water surface elevation and velocity conditions for test case 4 are discussed in detailed below in order to illustrate the important correspondance between the results of the computer simulations and certain special characteristics of the physical phenomena that control the flow conditions in the laboratory-model-scale flow region.

The results for constant-water-surface-elevation contours (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688) indicate that there is a gradual accumulation of water in all parts of the flow region. The results indicate that, at time level t = 50 sec, the two constant-water-surface-elevation contours with zero value, which correspond to the specified initial mean water level (MWL), are already located in the immediate vicinities of (1) the discharge plane at the south-end downstream enclosure surface at y = 0 and (2) the discharge plane at the north-end downstream enclosure surface at y = 24 m (Fig. 6.64, p.673). As the flow development progresses, during time levels t = 100 sec, t = 150 sec and t = 200 sec, the water-surface-elevation conditions exceed the specified initial MWL everywhere in the flow region (Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688). At approximately pseudo-steady-state conditions, at time level t = 200 sec, the minimum value of constant-water-surface-elevation contour is 6 mm above the specified initial MWL and the maximum water-surface-elevation conditions reach 14 mm, above the specified initial MWL, in the small subregion (between x = 18 m and x = 26 m and between y = 10 m and y = 14 m, in the middle of the flow region (Fig. 6.79, p.688).

The results of computer simulations for water-surface-elevation conditions (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688) indicate two important special characteristics associated with the free-surface conditions. First, although the open-region boundary conditions at the inlet planes, (1) at the west-end upstream enclosure surface at $x = 0$ and (2) at the east-end upstream enclosure surface at $x = 44$ m, are limited to 0.01 m (10 mm) as the specified water-surface-elevation conditions, above the specified initial mean water level (MWL), the water-surface-elevation conditions in the flow region exceed the specified boundary conditions, with maximum water-surface-elevation reaching 0.014 m (14 mm), above the specified initial MWL, which is 0.004 m (4mm) above the value of the specified open-region boundary conditions at the inlet planes. Second, although the open-region boundary conditions at the discharge planes, (1) at the south-end downstream enclosure surface at $y = 0$ and (2) at the north-end downstream enclosure surface at $y = 24$ m, are specified as water-surface-elevation conditions -0.01 m (-10mm), below the specified initial MWL, the water surface elevation conditions in the flow region, under pseudo-steady-state conditions, remain higher than 0.006 m (6 mm), above the specified initial MWL, which is 0.016 m (16 mm) above the value of the specified open-region boundary conditions at the discharge planes. These special characteristics of the water-surface-elevation conditions, which indicate rapid accumulation of water in the flow region, during the period of flow development, between time levels $t = 50$ sec and $t = 200$ sec (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688) can be explained by considering the physical charac-

teristics of the simulated flow conditions and the special geometrical configuration of the closed shoreline boundaries of the laboratory-model-scale flow region.

First, the inflow conditions at the inlet plane at the west-end upstream enclosure surface at x = 0 can attain relatively high positive x-component of momentum (control condition specified as $\bar{V}_{x,max;(opr,i-1),j,l}$ = 0.5 m/sec); and similarly the inflow conditions at the inlet plane at the east-end upstream enclosure surface at x = 44 m can attain relatively high negative x-component of momentum (control condition specified as $\bar{V}_{x,min;(opr,i-1),j,l}$ = -0.5 m/sec). Hence, the inflowing water from the west-end enclosure surface and from the east-end enclosure surface rapidly flood the central subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) which consists two opposing divergent channel sections relative to the inflow conditions. Since the two inflow conditions are along opposing x-directions, the x-component of velocity in the middle of the flow region must necessarily vanish; and the inflow conditions must result in hydrostatic pressure gradients along the opposing x-directions, with the maximum hydrostatic pressure conditions occurring in the middle of the flow channel. Hence, the water-surface-elevation conditions in the central subregion (between x = 18 m and x = 26 m and between y = 10 m and y = 14 m) must attain maximum values (Fig. 6.64, p.673, Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688), since the hydrostatic pressure conditions must be maximum in the subregion due to the occurrance of stagnation flow conditions in the flow region.

Second, the water that flows out of the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m), in the middle of the flow region, must first undergo rapid contraction due to the convergent geometrical shapes of the boundary enclosure surfaces which lead to the discharge planes, (1) at the south-end downstream enclosure surface at y = 0 and (2) at the north-end downstream enclosure surface at y = 24 m. Considering the free surface flow conditions in the flow region (which are similar to compressible flow conditions in a convergent channel), the two discharge planes, (1) at the south-end downstream enclosure surface at y = 0 and (2) at the north-end downstream enclosure surface at y = 24 m, behave as two throat planes where the outflowing water could reach velocity conditions with magnitudes approximately equal to the speed of the surface gravity wave (which is similar to the speed of sound for compressible flow conditions). The computer simulation results for water-surface-elevation conditions in the laboratory-model-scale flow region indicate that the two discharge planes, located at the end of the two convergent sections, behave as two throat planes where the free surface flow conditions become "choked" to the propagation of disturbances along the upstream directions relative to the outflow conditions. Consequently, from qualitative considerations, the specified lower hydrostatic pressure conditions at the two discharge planes, (1) at the south-end downsteam enclosure surface at y = 0 and (2) at the north-end downstream enclosure surface at y = 24 m, as -0.01 m (-10 mm) below the specified initial mean water level (MWL), do not control the outflow conditions along the lateral y direction of the flow region. Therefore, the effects of the lower hydrostatic pressure

conditions at the two discharge planes cannot propagate against the outflow directions; and hence, the water-surface-elevation conditions in the flow region do not gradually decrease to the specified open-region boundary conditions as -0.01 m (-10 mm) below the specified initial MWL in the vicinities of the two discharge planes, (1) at the south-end downstream enclosure surface at y = 0, and (2) at the north-end downstream enclosure surface at y = 24 m (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and 6.79, p.688). It is important to note that the conclusion based on "choked" free-surface flow conditions can only be considered as qualitative, since the quantitative formulations of the "choked" conditions are based on steady-state free-surface-flow conditions in a convergent conduit, whereas the computer simulation results represent strictly transient conditions during the period of flow development in the geometrically complex, divergent and convergent subregions of the symmetrici flow region.

It is again important to note that the gradual accumulation of water in the flow region (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688) is also a consequence of the open-region boundary conditions that were used in the computer simulations. The specified increased water-surface-elevation conditions, at the west-end upstream enclosure surface at x = 0 and at the east-end upstream enclosure surface at x = 44 m, as 0.01 m (10 mm) above the specified initial mean water level (MWL), result in increases of 0.12 m^2 of inflow areas, since the widths of the upstream enclosure surface are 12 m. Similarly, the specified decreased water-surface-elevation conditions, at the south-end downstream enclosure surface at y = 0 and at the

north-end downstream enclosure surface at y = 24 m, as -0.01 m (-10 mm) below the specified initial MWL, result in decreases of -0.08 m^2 of outflow area, since the widths of the downstream enclosure surfaces are 8 m. Therefore, effectively, the inflow conditions at the upstream enclosure surfaces have an available cumulative flow area which is 0.08 m^2 larger than the available cumulative flow area for the outflow conditions at the downstream enclosure surfaces.

The inflow conditions at the inlet plane at the west-end upstream enclosure surface at x = 0 can attain relatively high values for the x-component of velocity (control condition specified as $\bar{V}_{x,max;(opr,i-1),j,l}$ = 0.5 m/sec) according to the specified open-region boundary conditions, immediately after the start of the flow conditions in the flow region. Similarly, the inflow conditions at the inlet plane at the east-end upstream enclosure surface at x = 44 m can attain relatively high values for the x-component of velocity (control condition specified as $\bar{V}_{x,min;(opr,i-1),j,l}$ = -0.5 m/sec), according to the specified open-region boundary conditions, again immediately after the start of the flow conditions in the flow region. Hence, the x-components of the velocity conditions at the inlet planes of the upstream open-region enclosure surfaces can attain relatively high values, immediately after the start of the flow conditions, which can lead to relatively high inflow rates into the flow region. On the other hand, the ouflow conditions at the discharge plane at the south-end downstream enclosure surface at y = 0 cannot attain a value for the y-component of velocity with a magnitude higher than the magnitudes of the x-components of velocities at the west-end upstream enclosure surface at x = 0 and at

the east-end upstream enclosure surface at x = 44 m, until the increased water-surface-elevation conditions in the flow region can cause sufficiently high hydrostatic pressure gradient conditions that can accelerate the flow along the y direction. Similarly, the outflow conditions at the discharge plane at the north-end downstream enclosure surface at y = 24 m cannot attain a value for the y-component of velocity with a magnitude higher than the magnitudes of the x-components of velocities at the west-end upstream enclosure surface at x = 0 and at the east-end upstream enclosure surface at x = 44 m, until the increased water-surface-elevation conditions in the flow region can cause sufficiently high hydrostatic pressure gradient conditions that can accelerate the flow along the y direction. THerefore, during the period of flow development, the cumulative inflow rate, resulting from the inflow rate at the inlet plane at the west-end upstream enclosure surface at x = 0 and from the inflow rate at the inlet plane at the east-end upstream enclosure surface at x = 44 m, with effectively increased value of 0.24 m^2 total available flow area, is higher than the cumulative outflow rate, resulting from the outflow rate at the discharge plane at the south-end downstream enclosure surface at y = 0 and from the outflow rate at the discharge plane at the north-end downstream enclosure surface at y = 24 m, with effectively decreased value of -0.16 m^2 total available flow area. Hence, during the period of flow development, the cumulative inflow rate is effectively higher than the cumulative outflow rate, and hence, the water-surface-elevation conditions gradually increase in most parts of the flow region.

Again, the most important result of the computer simulations of water-surface-elevation conditions is that during the complete period of flow development, between time levels t = 50 sec and t = 200 sec, the temporal variations of the water-surface-elevation conditions retain exactly the required spatially-symmetric distributions about both central x-axis and central y-axis of the flow region (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688).

Hence, the considerations of the physical characteristics of the flow conditions clearly indicate that the computer simulation results of the discrete-element hydrodynamic model (HYDROL) for the transient water-surface-elevation conditions are realistic and representative of the conditions that should exist for test case 4 : Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method) in the laboratory-model-scale flow region.

The computer simulation results for velocity conditions (Figs. 6.65 through 6.68, p.674 through p.677; Figs. 6.70 through 6.73, p.679 through p.682; Figs. 6.75 through 6.78, p.684 through p.687; and Figs. 6.80 through 6.83, p.689 through p.692) indicate that the three-dimensional opposing inflow conditions along the x direction and the opposing outflow conditions along the y direction in the flow region develop as expected for the specified open-region water-surface-elevation boundary conditions (1) at the inlet planes (a) at the west-end upstream enclosure surface at x = 0 and (b) at the east-end upstream enclosure surface at y = 44 m, and (2) at the discharge planes (c) at the south-end down-

stream enclosure surface at y = 0 and (d) at the north-end downstream enclosure surface at y = 24 m.

During the complete period of flow development, between time levels t = 50 sec and t = 200 sec, the two-dimensional vector representations of the velocity conditions in all four layers l = 1,2,3,4 indicate that the inflow conditions along the x direction gradually change into outflow conditions along the y direction in the subregions with convergent shoreline enclosure boundaries (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) in the middle of the flow region.

In the lower layers l = 1,2, which consists mainly of the lower parts of the sections of the deep main flow channel, the inflow conditions along the x direction change into outflow conditions along y direction in the small subregion (between x = 18 m and x = 26 m and between y = 10 m and y = 14 m) corresponding to the subregion of stagnation flow conditions in the middle of the region, during the complete period of the flow development, between time levels t = 50 sec and t = 200 sec (Fig. 6.65, p.674; Fig. 6.66, p.675; Fig. 6.70, p.679; Fig. 6.71, p.680; Fig. 6.75, p.684; Fig. 6.76, p.685; Fig. 6.80, p.689; and Fig. 6.81, p.690).

The inflow conditions in the upper layers l = 3,4 of the subregion (between x = 0 and x = 44 m and between y = 10 m and y = 14 m), which contains the section of the deep main flow channel along x direction, indicate that the velocity conditions along opposing x directions remain relatively high throughout the subregion, between the middle of the flow region and the two inlet planes, (1) at the west-end upstream enclosure surface at x = 0 and (2) at the east-end upstream enclosure surface at x

= 44 , during the complete period of flow development, between time levels t = 50 and t = 200 sec (Fig. 6.67, p.676; Fig. 6.68, p.677; Fig. 6.72, p.681; Fig. 6.73, p.682; Fig. 6.77, p.686; Fig. 6.78, p.687; Fig. 6.82, p.691;and Fig. 6.83, p.692). However, the outflow conditions in the upper layers l = 3,4 of the subregion (between x = 18 m and x = 26 m and between y = 0 and y = 24 m), which contains the section of the deep main flow channel along y direction, indicate that the velocity conditions along opposing y directions remain relatively low throughout the subregion, between the middle of the flow region and the two discharge planes, (1) at the south-end downstream enclosure surface at y = 0 and (2) at the north-end downstream enclosure surface at y = 24 m, between time levels t = 50 sec and t = 200 sec (Fig. 6.67, p.676; Fig. 6.68, p.677; Fig. 6.72, p.681; Fig. 6.73, p.682; Fig. 6.77, p.686; Fig. 6.78, p.687; Fig. 6.82, p.691; and Fig. 6.83, p.692). Hence, the computer simulation results indicate that the inflow conditions along the x direction, originating from the inlet planes, (1) at the west-end upstream enclosure surface at x = 0 and (2) at the east-end upstream enclosure at x = 44 m, change gradually into outflow conditions along the y direction by spreading over the large subregion (between x = 10 m and x = 34 m and between y = 0 and y = 24 m) which is enclosed by the convergent shoreline boundaries (relative to the direction of flow) in the middle of the flow region (Fig. 6.67, p.676; Fig. 6.68, p.677; Fig. 6.72, p.681; Fig. 6.73, p.682; Fig. 6.77, p.686; Fig. 6.78, p.687; Fig. 6.82, p.691; and Fig. 6.83, p.692).

The computer simulation results for two-dimensional representations of the velocity conditions, in the horizontal (x,y) planes of the four

layers l = 1,2,3,4, indicate that the inflow conditions change uniformly into outflow conditions, without forming any secondary-flow vortex in the flow region, at any time during the simulated period of flow development, between time levels t = 50 sec and t = 200 sec (Figs. 6.65 through 6.68, p.674 through p.677; Figs. 6.70 through 6.73, p.679 through p.682; Figs. 6.75 through 6.78, p.684 through p.687; and Figs. 6.80 through 6.83, p.689; through p.692).

The most important result of the computer simulations of the velocity conditions is that during the complete period of flow development, between time levels t = 50 sec and t = 200 sec, the temporal variations of all the components of the three-dimensional velocity vectors retained perfectly the required spatially-symmetric and spatially-asymmetric distributions about both central x-axis and centrl y-axis of the flow region (Figs. 6.65 through 6.68, p.674 through p.677; Figs. 6.70 through 6.73, p.679 through p.682; Figs. 6.75 through 6.78, p.684 through p.687; and Figs. 6.80 through 6.83, p.689; through p.692).

Hence, the results of the computer simulations of the water-surface-elevation and three-dimensional velocity conditions in the laboratory-model-scale flow region for test case 4 : Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on simple second-upwind-differencing interpolation method), again illustrates the reliability of the mathematical formulation of the transient, three-dimensional discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of its associated HYDROL computer code.

6.1.2.5 Test Case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). The fifth test case was also considered specifically to test the realiability of the mathematical formulation of the fast-transient, three-dimensional, discrete-element hydrodynamic model and the accuracy of the computational algorithm of the HYDROL computer code under most demanding flow simulation conditions. Since test case 5 considers transportive-upwind-differencing interpolation, the effect of the artificial viscosity (as a part of the general numerical dispersion effect) becomes minimized in the computer simulations. Hence, any possible instability that could result from the mathematical development of the governing computational system of discrete-element equations, or from the structure of the computational algorithm of the HYDROL computer code, would automatically be accentuated in the simulations of the flow conditions in the symmetric flow region.

The necessary input data to the HYDROL computer code for the fifth test case was prepared according to the following conditions associated with the mathematical formulation of the discrete-element hydrodynamic model.

a. Boundary Conditions

Boundary conditions for Test case 5 was assumed as the same boundary conditions for Test Case 4.

b. Interpolation Option

The computer simulation results for test case 5 were obtained by applying the HYDROL computer code with transportive-upwind-differencing interpolation option. The values of the parameters for the transportive-upwind-differencing interpolation were all selected as one (1) such that local values of Courant number (E.q., Roache, 1972, Eraslan, 1974) remained as one (1) at the enclosure surface of all the elements and half elements (see Sec. 4.3.3). Hence, the computer simulation results were obtained with effectively zero one-dimensional artificial viscosity along the individual directions of flow in the symmetric flow region.

c. Numerical Solution Option

The computer simulation results for test case 5 were obtained by applying the HYDROL computer code with the standard default option which utilized the special, composite time-splitting method in the numerical integration of the governing system of spatially discretized ordinary differential equations (see Sec. 5).

d. Discussion of the Computer Simulation Results

The results of the computer simulations for the water surface elevation and velocity conditions are presented in Figs. 6.84 through 6.103, p.706 through p.725. The surface elevation conditions were presented as (1) numbers in the elements which represent water surface elevation (mm) relative to specified initial mean water level (MWL) and (2) constant water surface elevation contours which correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL. The velocity conditions were presented as two-dimensional vectors which

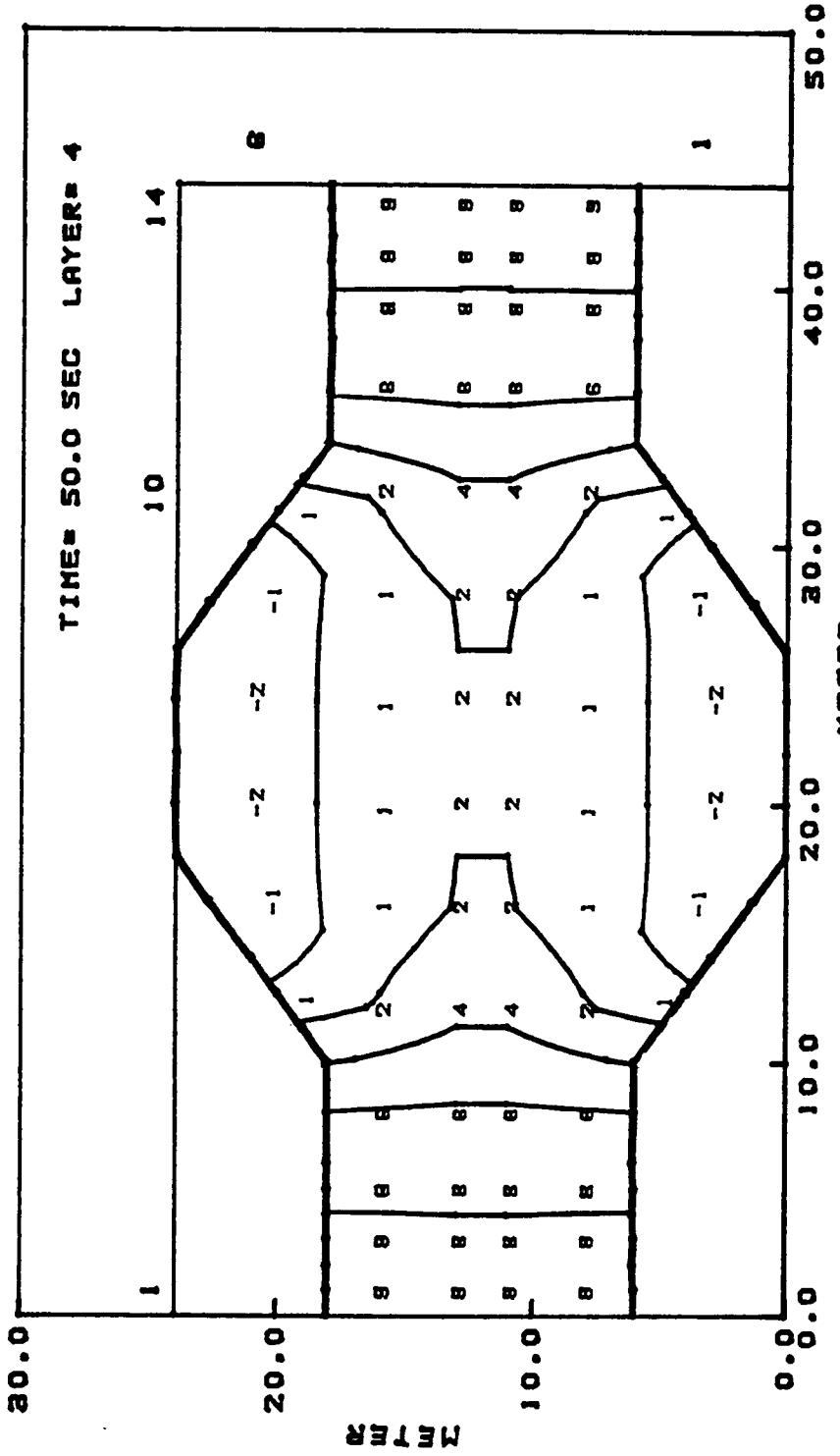

Figure 6.84. Water-surface-elevation conditions at time t = 50 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

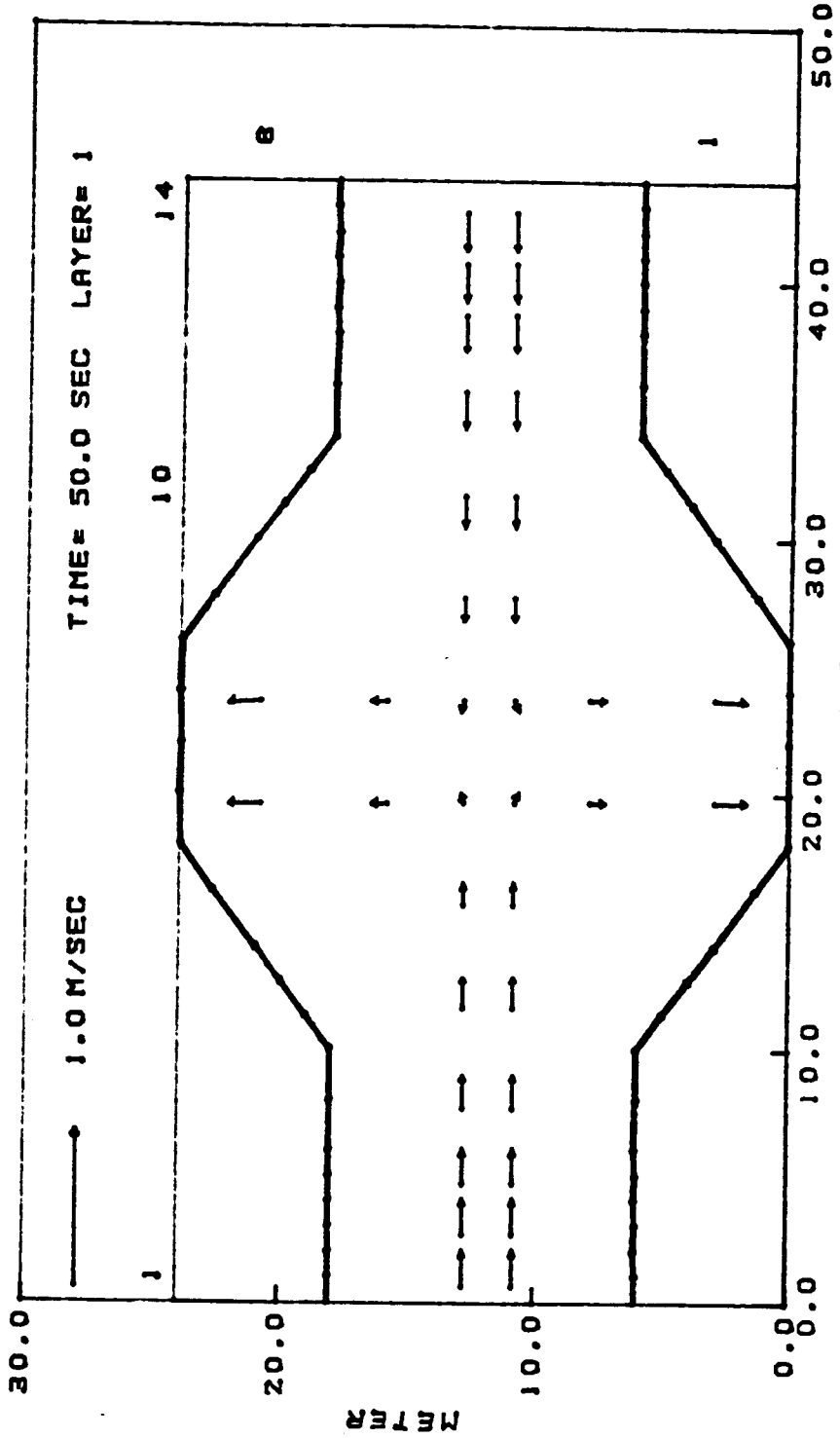

Figure 6.85. Velocity conditions in the bottom layer l = 1, at time t = 50 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

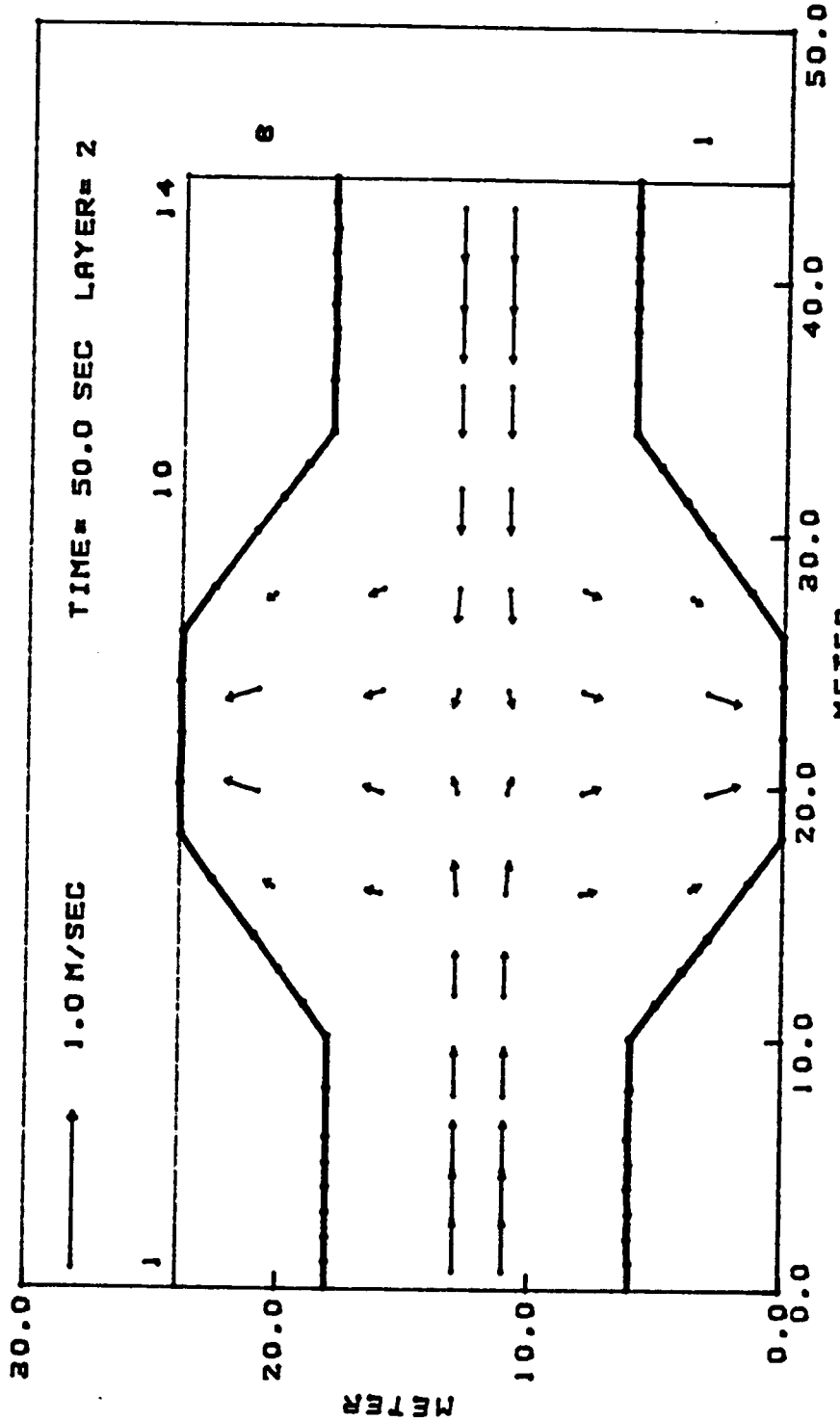

Figure 6.86. Velocity conditions in the internal layer l = 2, at time t = 50 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

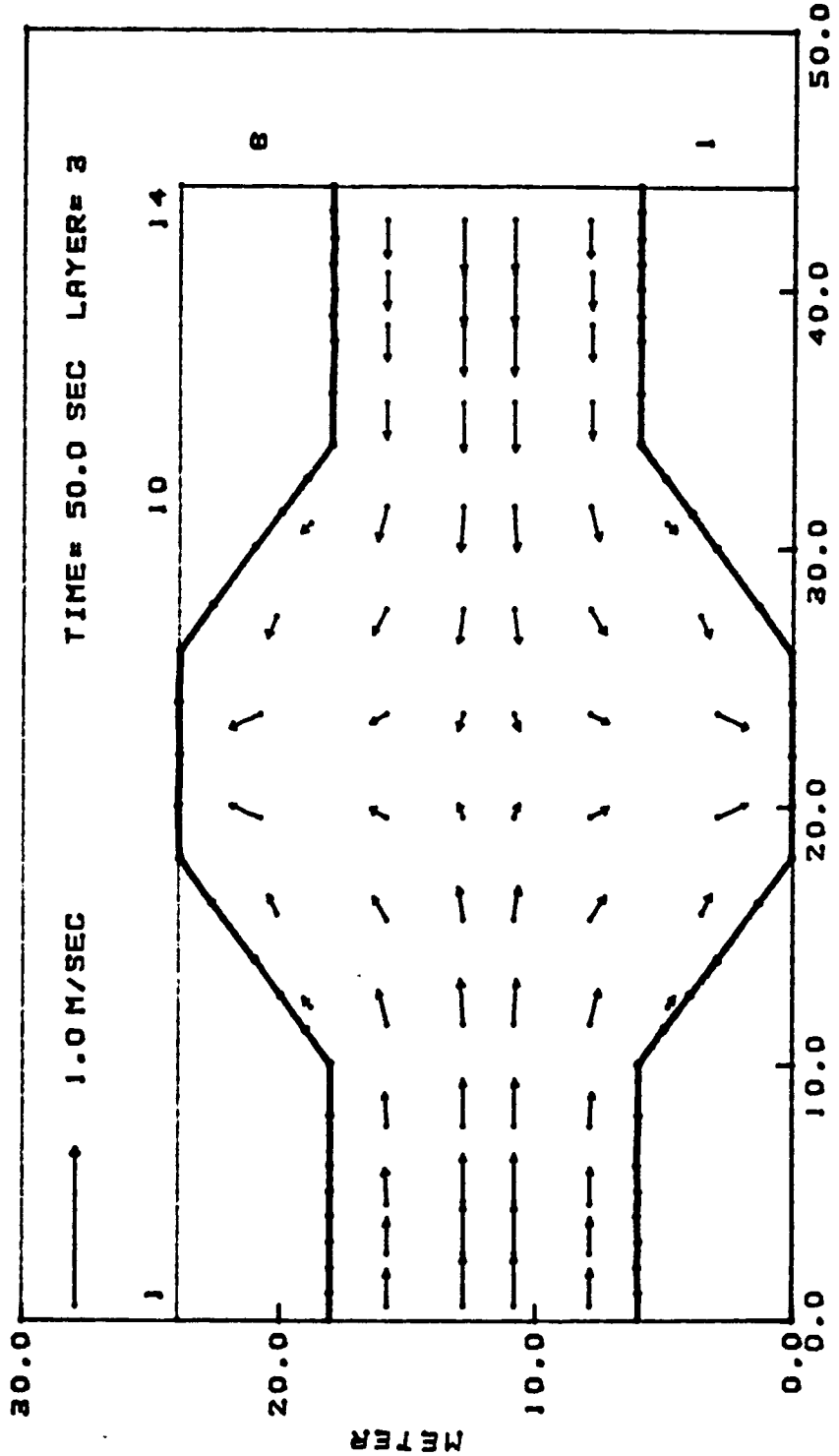

Figure 6.87. Velocity conditions in the internal layer l = 3, at time t = 50 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

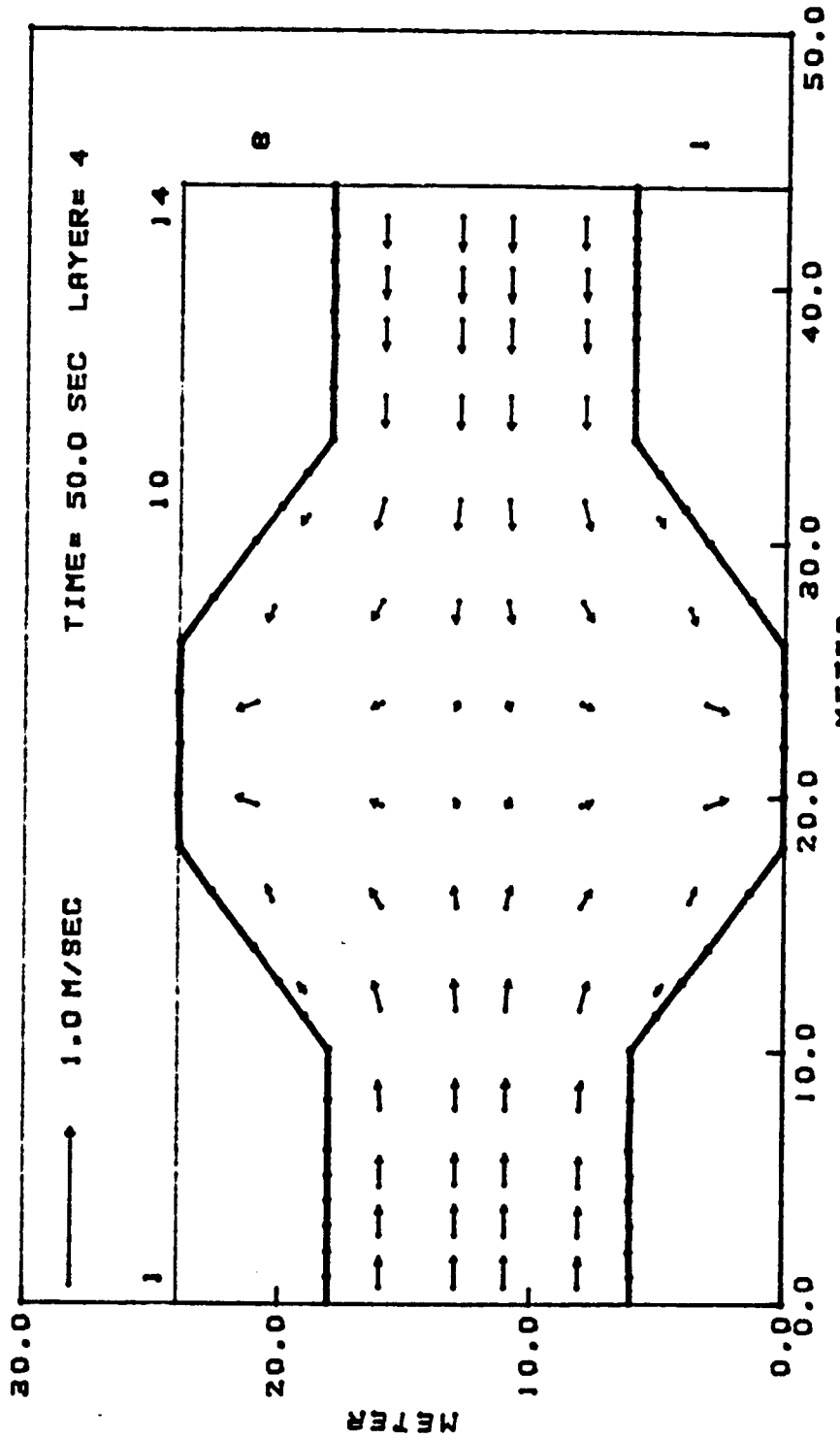

Figure 6.88. Velocity conditions in the surface layer l = 4, at time t = 50 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

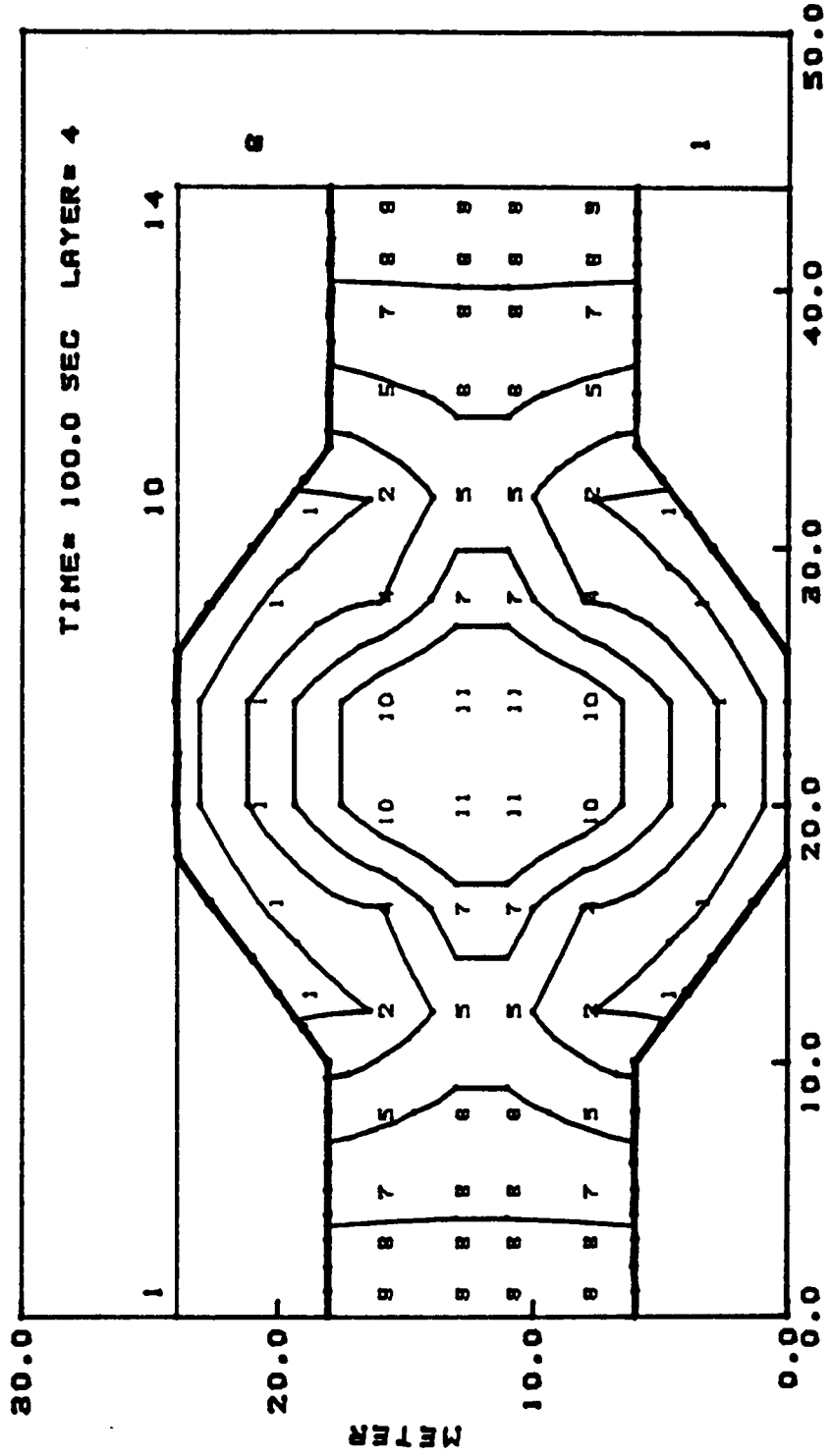

Figure 6.89. Water-surface-elevation conditions at time t = 100 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

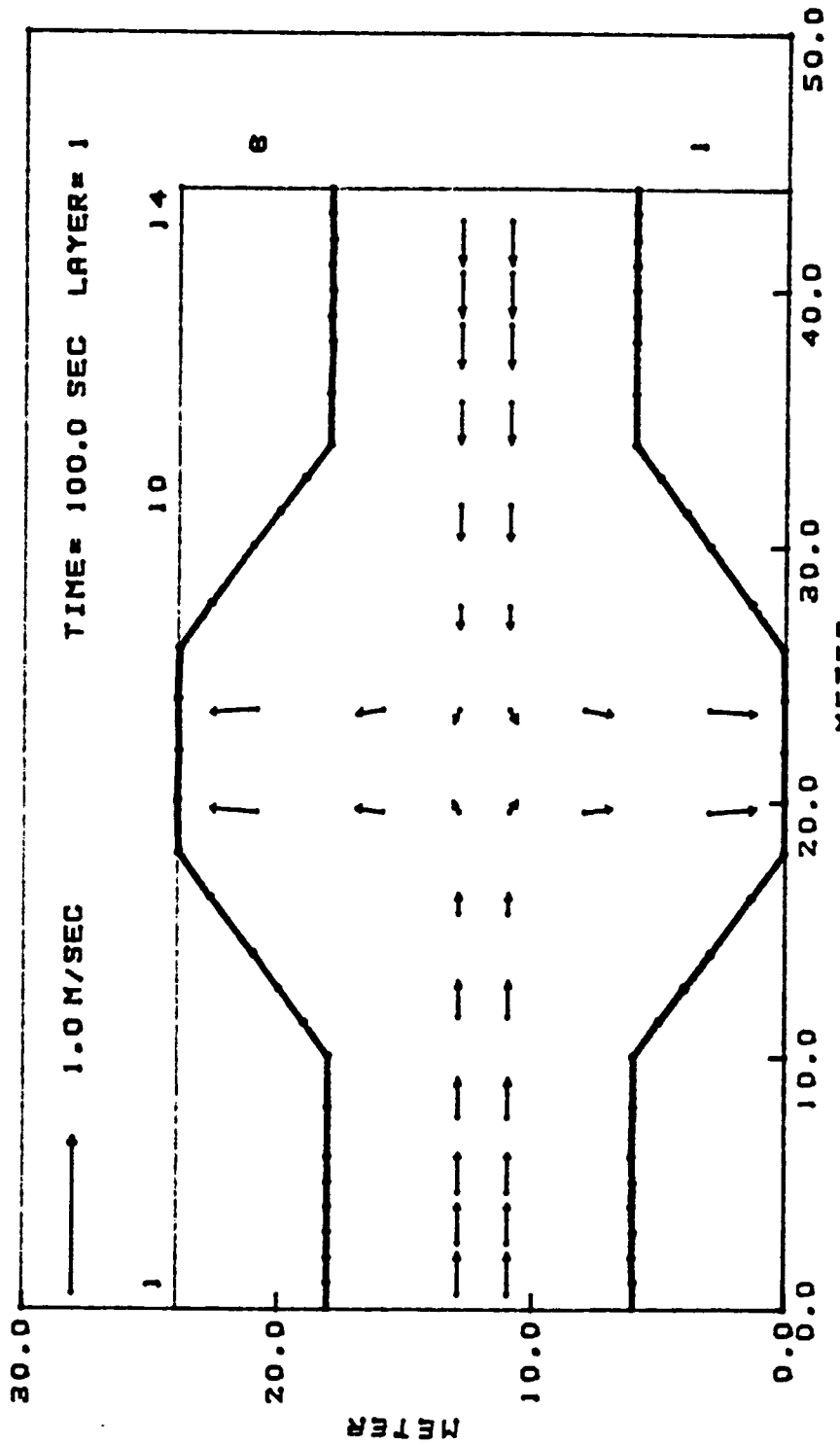

Figure 6.90. Velocity conditions in the bottom layer l = 1, at time t = 100 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

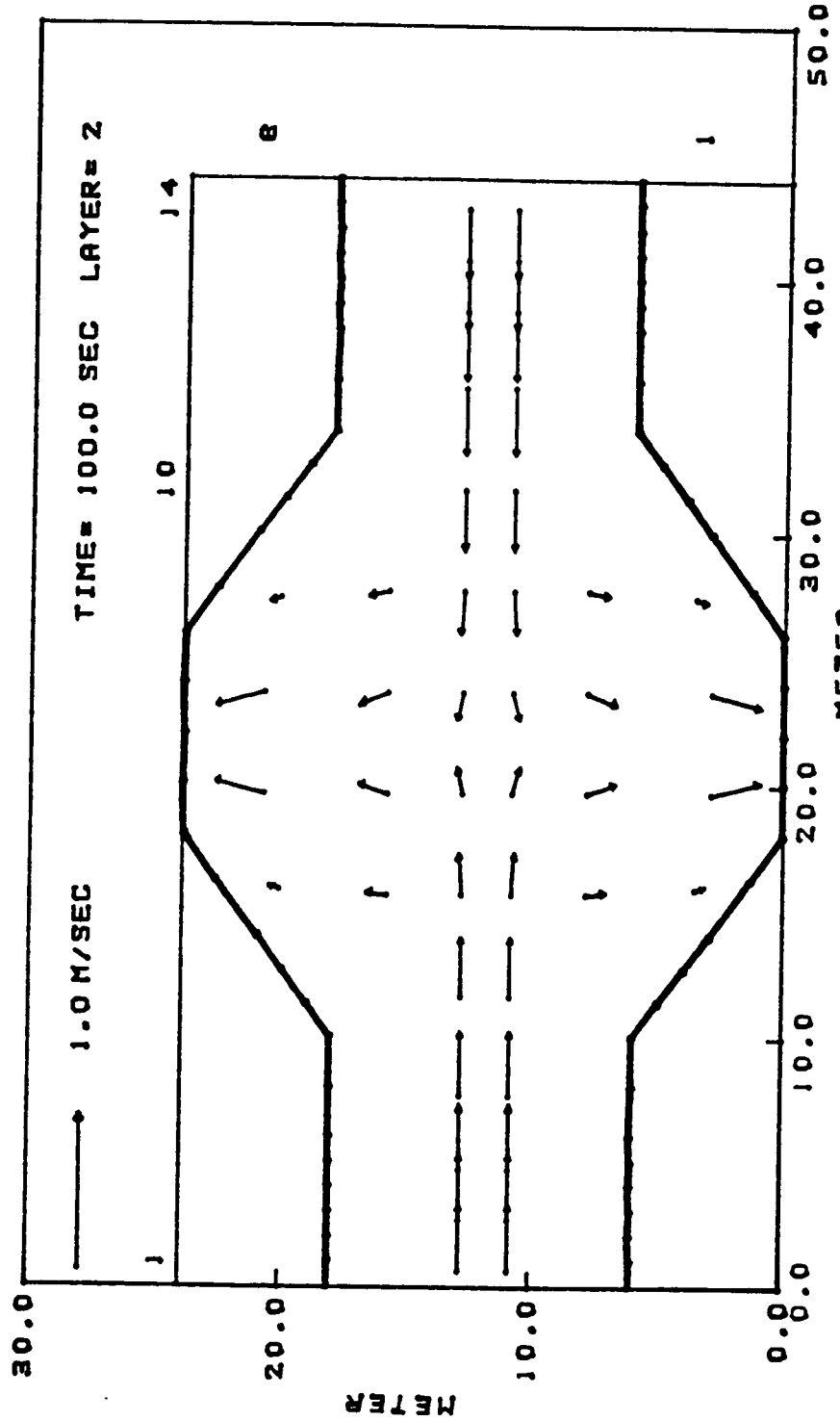

Figure 6.91. Velocity conditions in the internal layer l = 2, at time t = 100 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

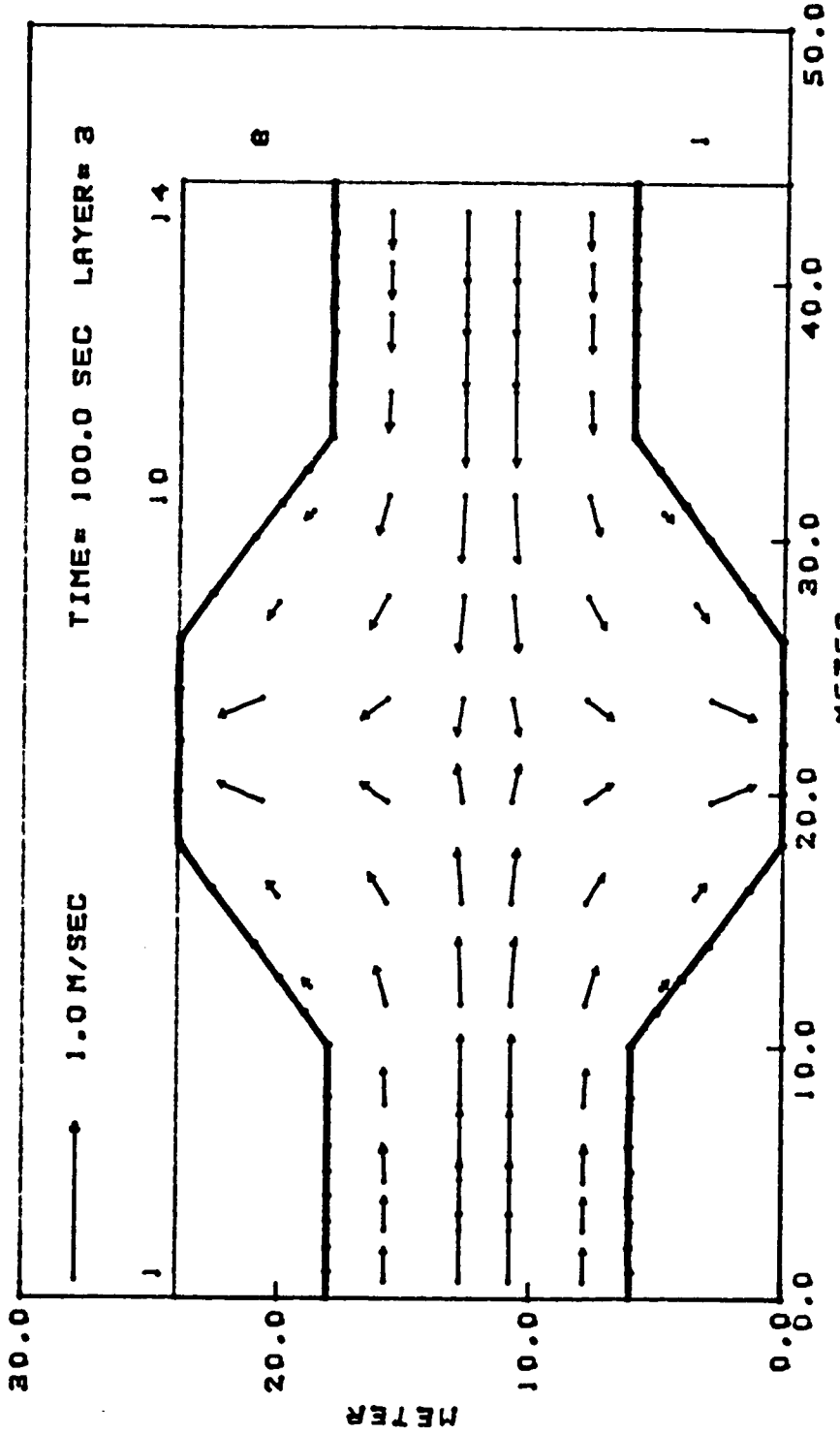

Figure 6.92. Velocity conditions in the internal layer l = 3, at time t = 100 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

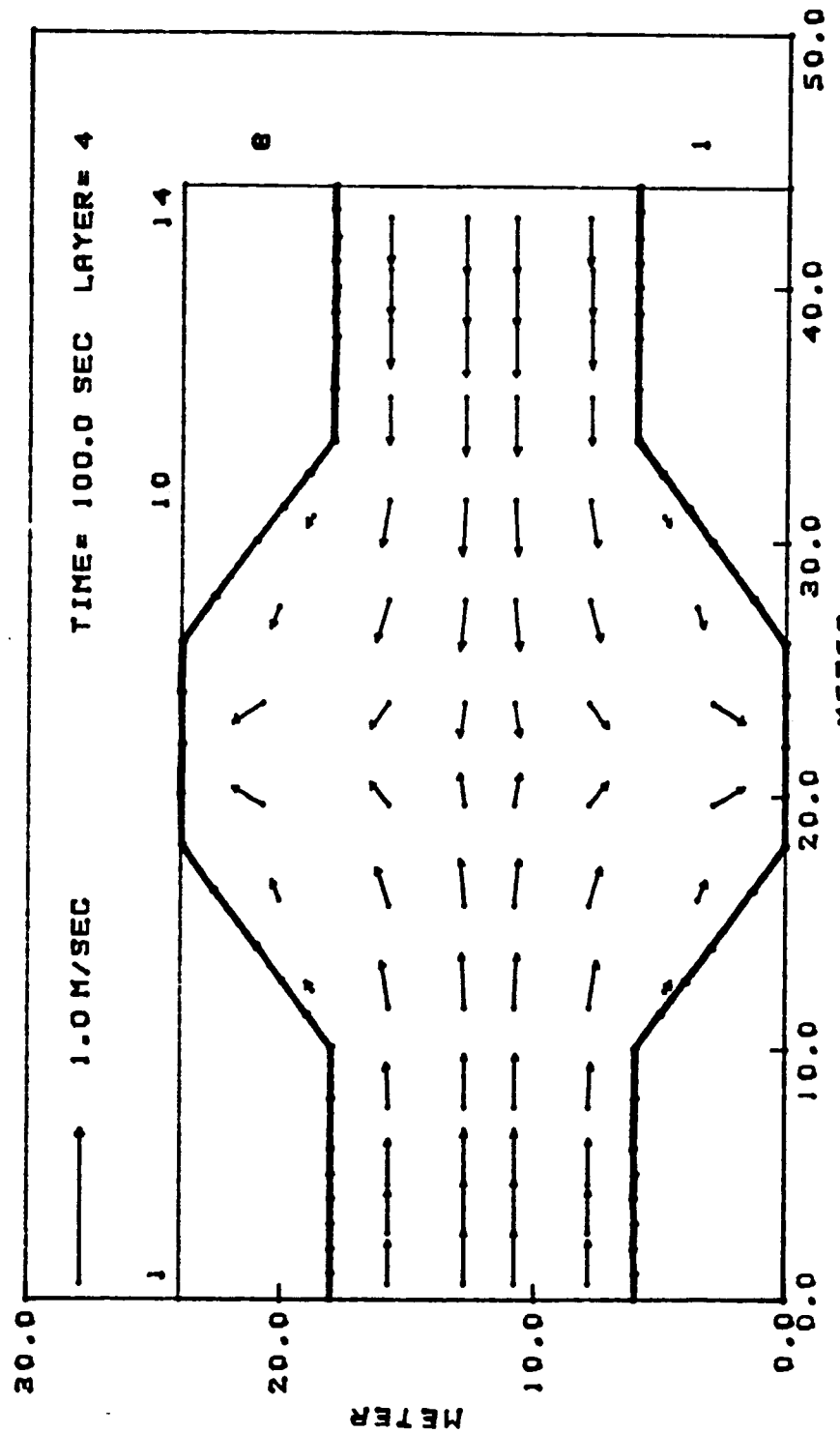

Figure 6.93. Velocity conditions in the surface layer l = 4, at time t = 100 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

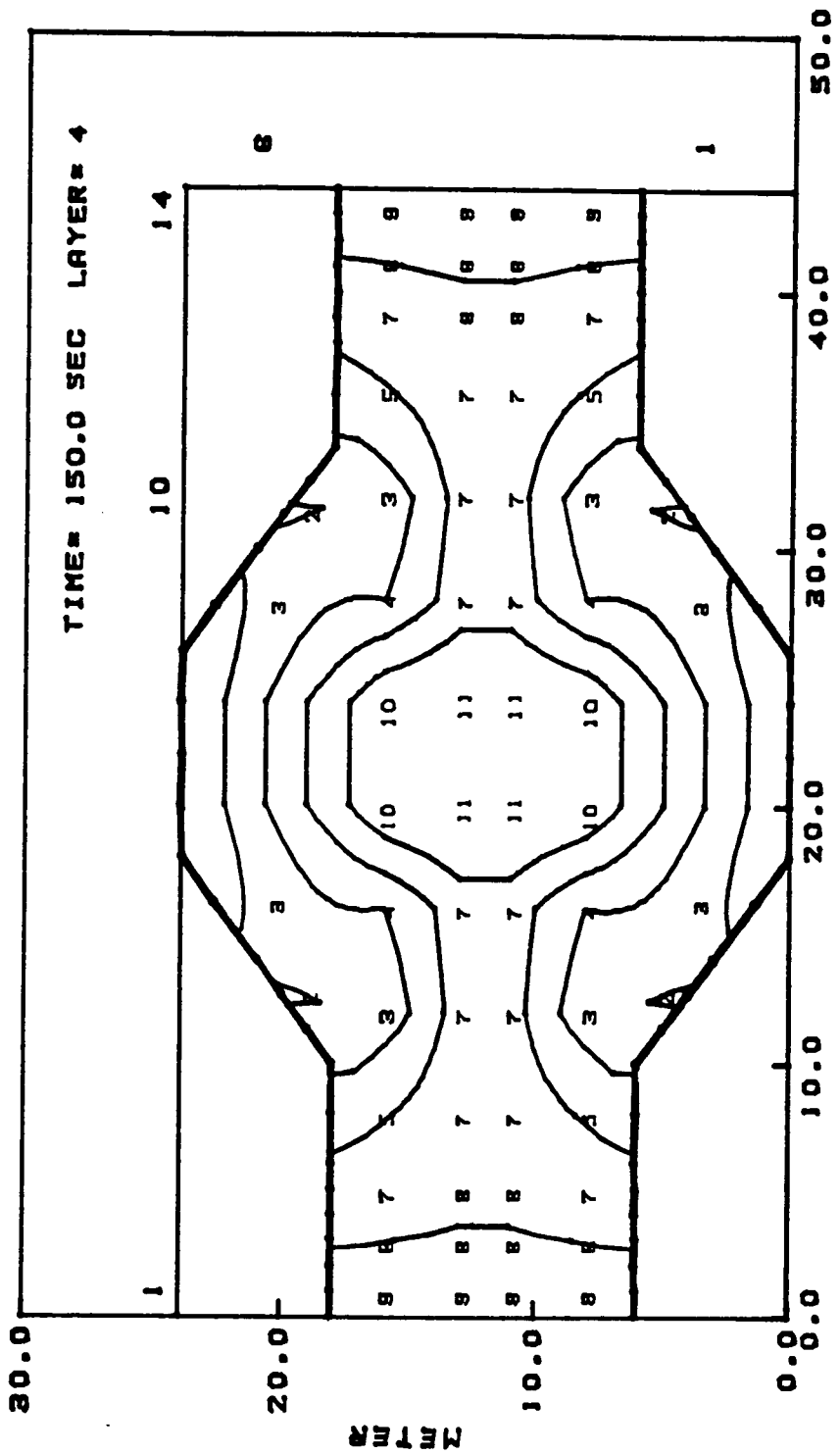

Figure 6.94. Water-surface-elevation conditions at time t = 150 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

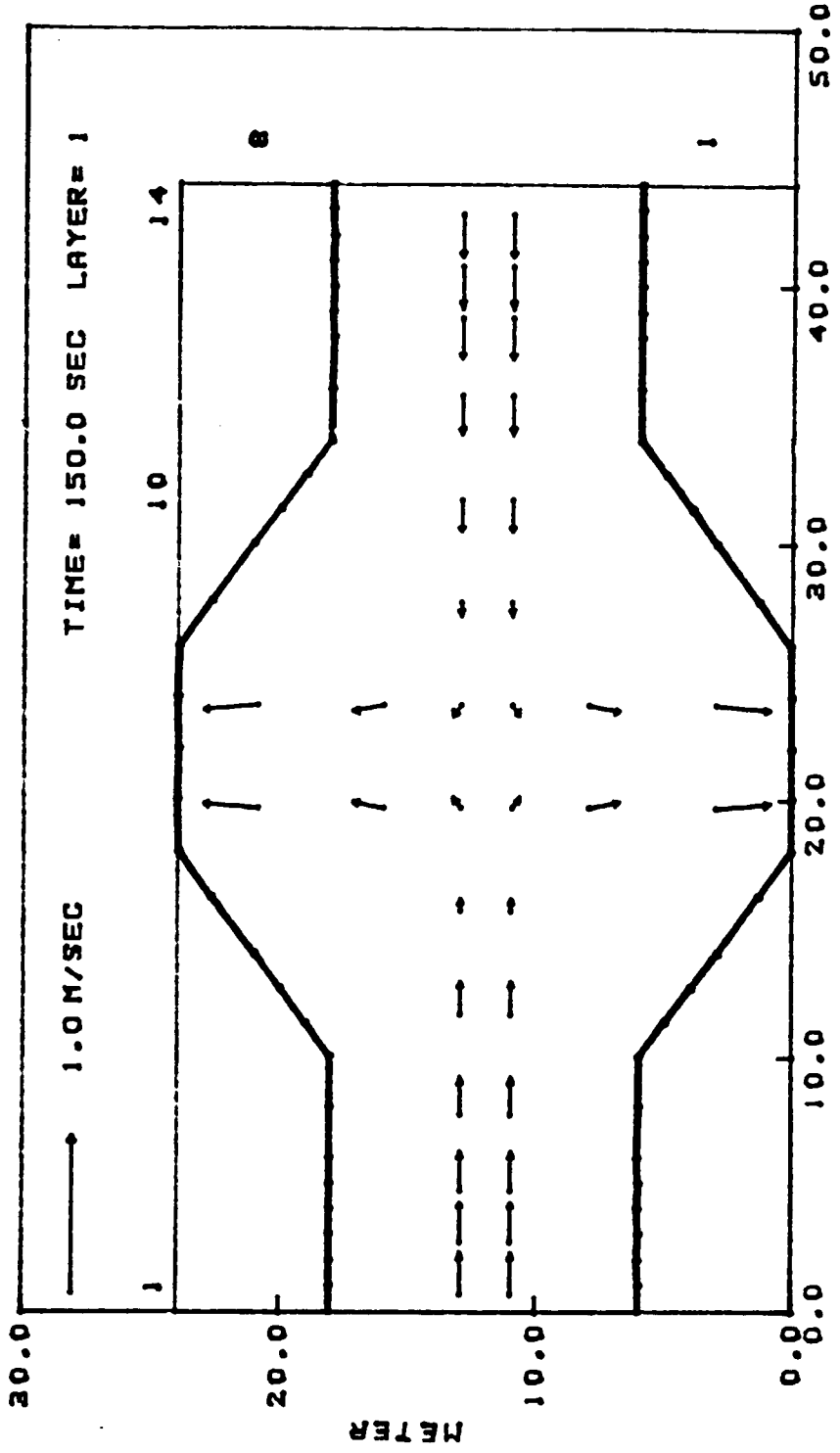

Figure 6.95. Velocity conditions in the bottom layer l = 1, at time t = 150 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

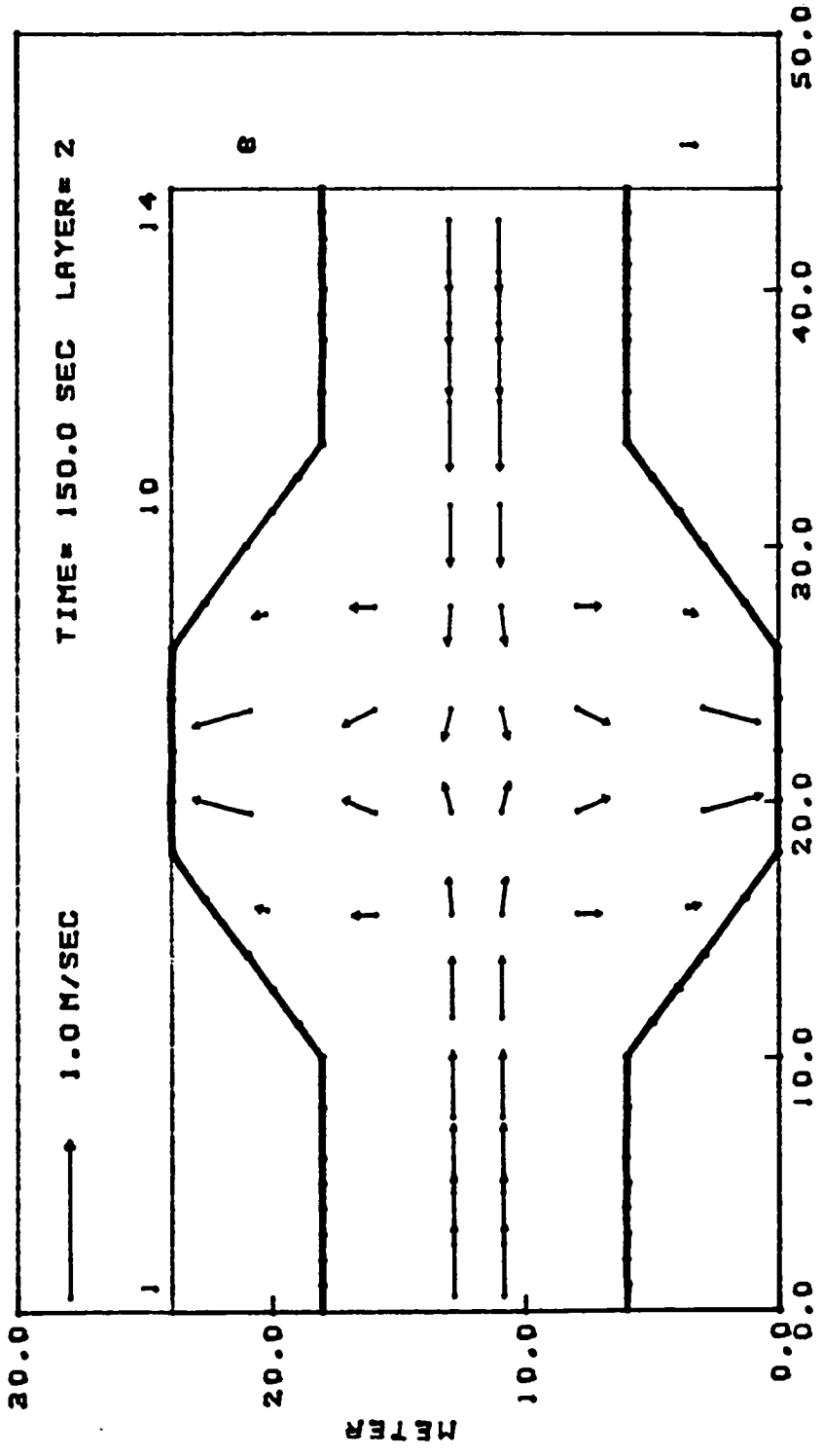

Figure 6.96. Velocity conditions in the internal layer l = 2, at time t = 150 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

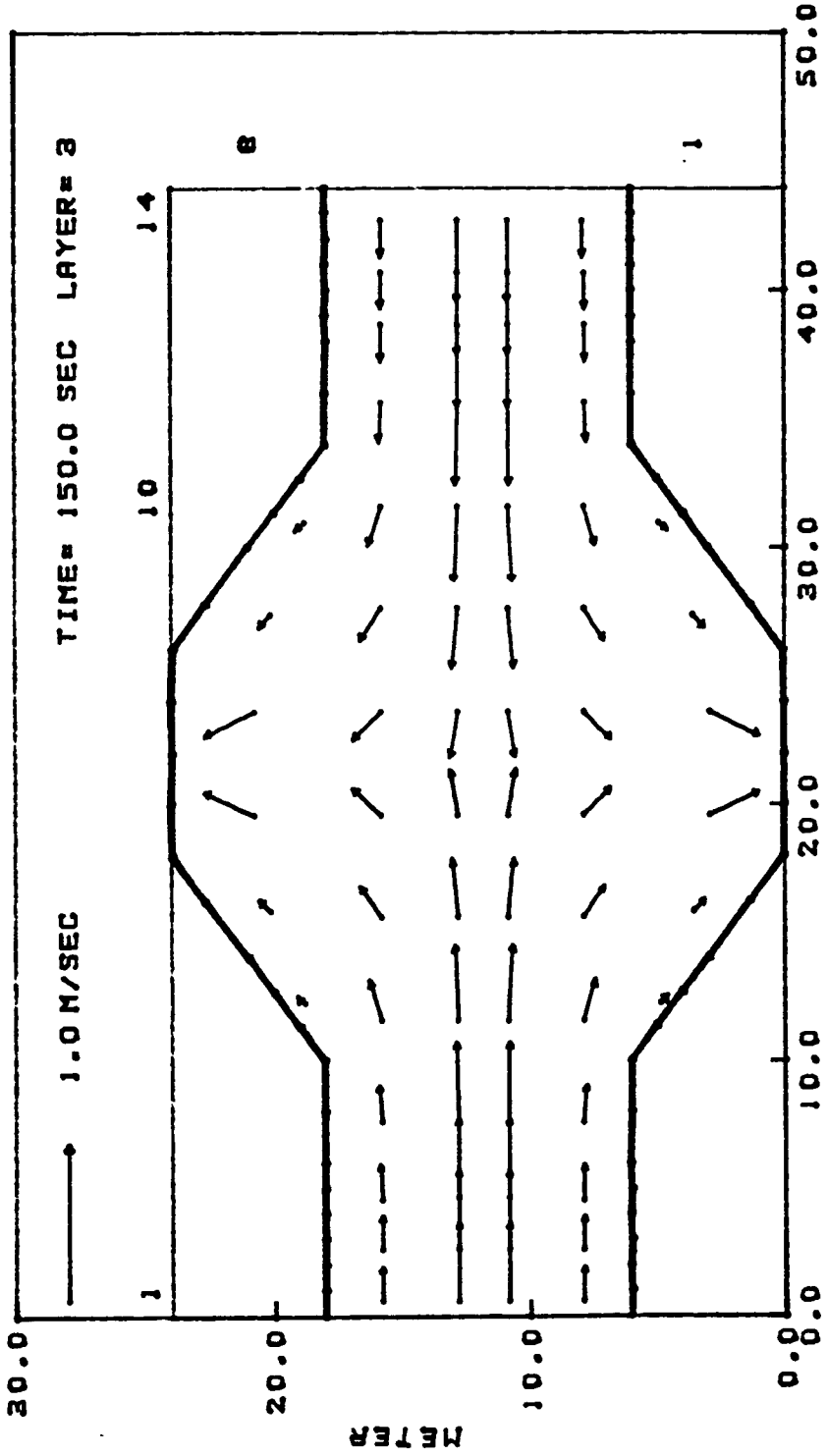

Figure 6.97. Velocity conditions in the internal layer l = 3, at time t = 150 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

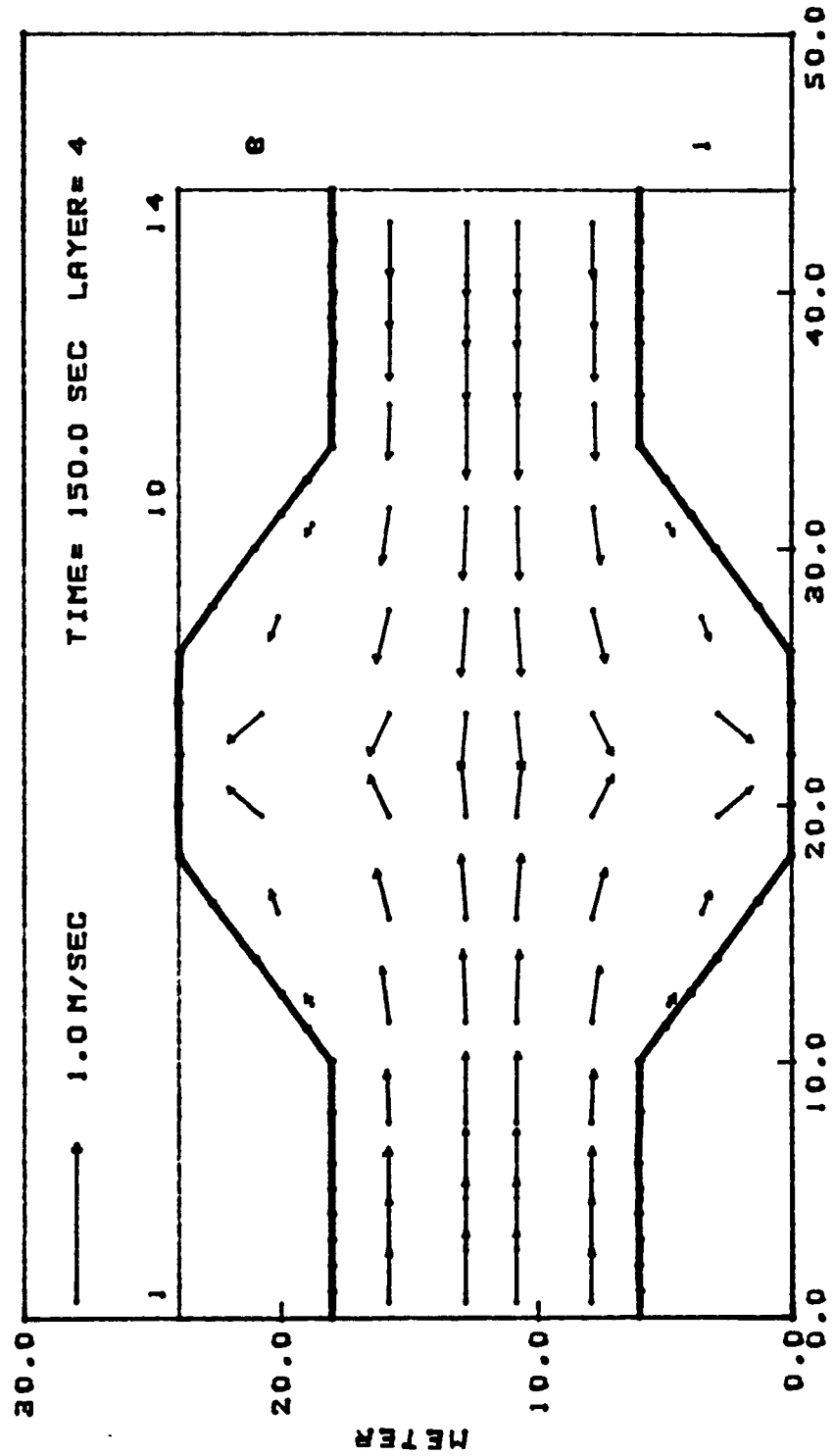

Figure 6.98. Velocity conditions in the surface layer l = 4, at time t = 150 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

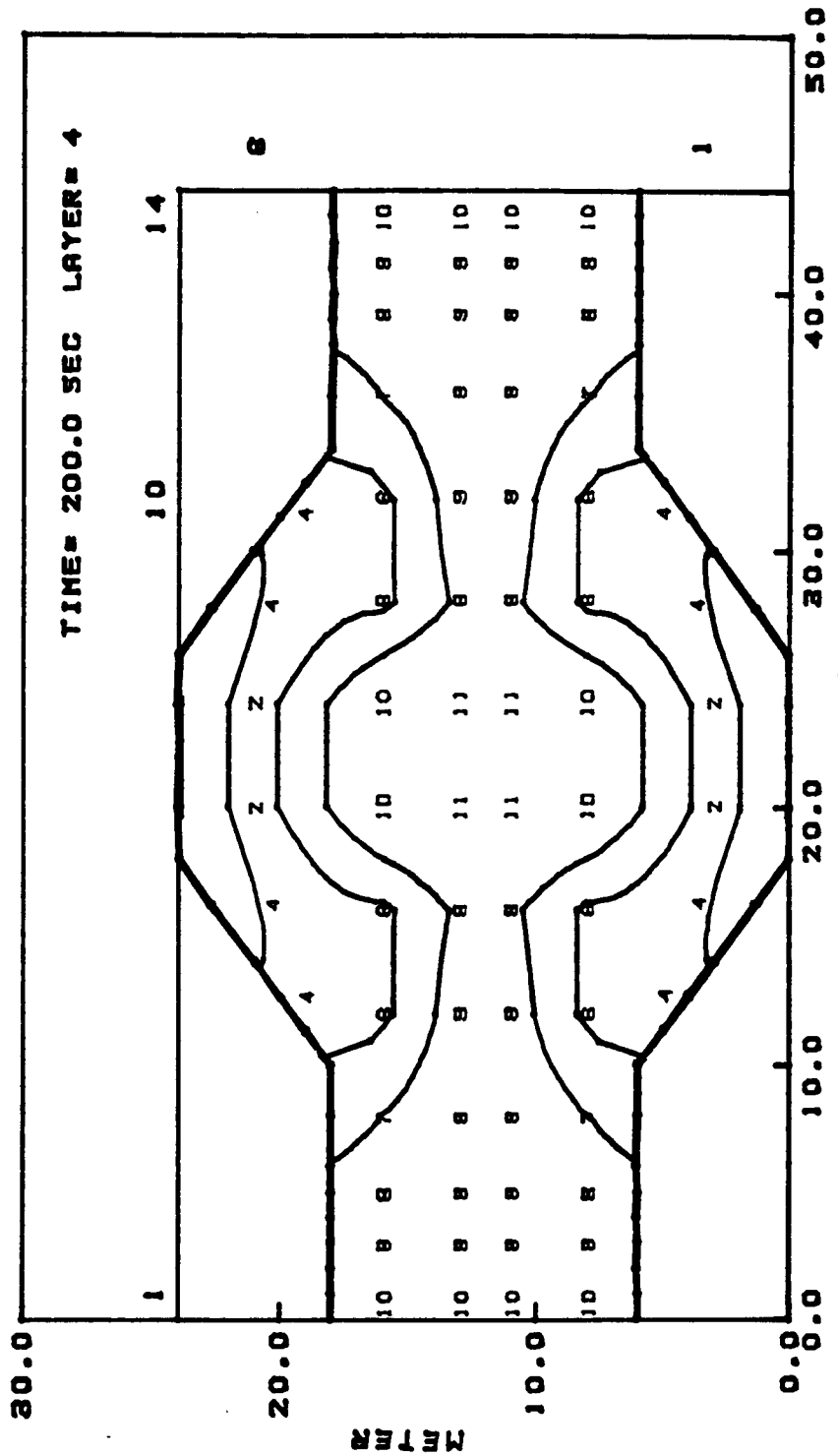

Figure 6.99. Water-surface-elevation conditions at time t = 200 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Numbers in elements represent water-surface-elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

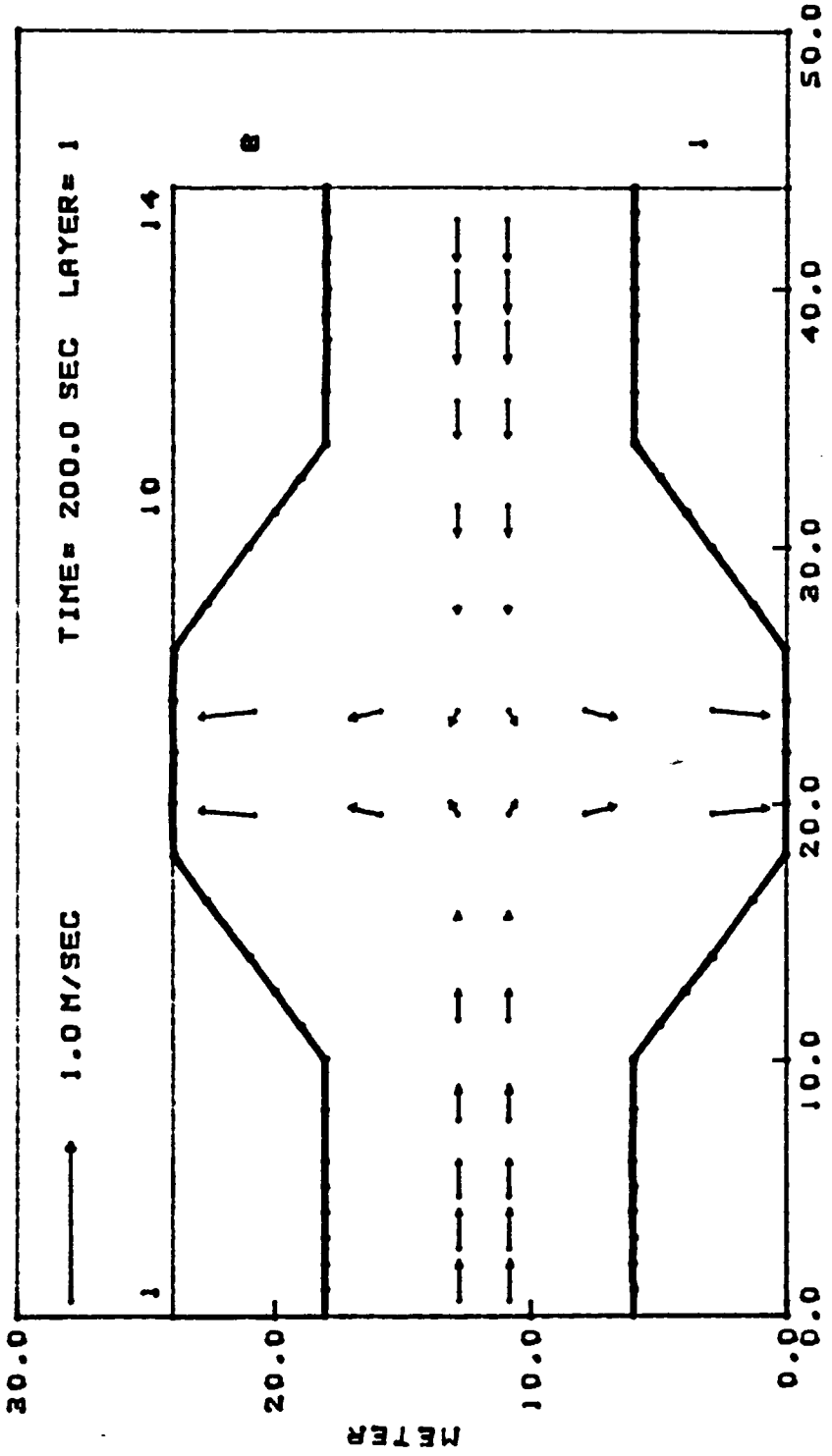

Figure 6.100. Velocity conditions in the bottom layer l = 1, at time t = 200 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

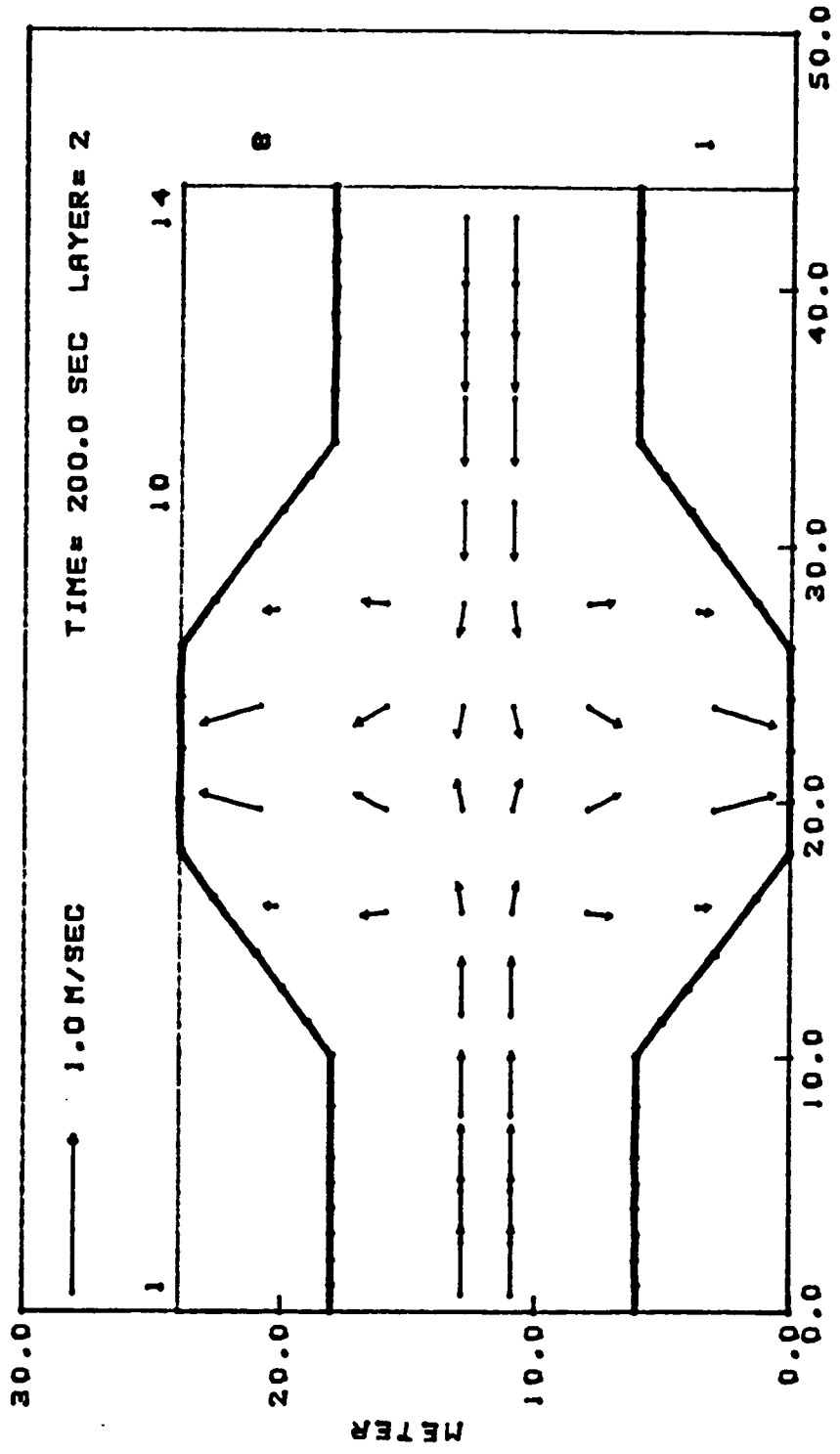

Figure 6.101. Velocity conditions in the internal layer l = 2, at time t = 200 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

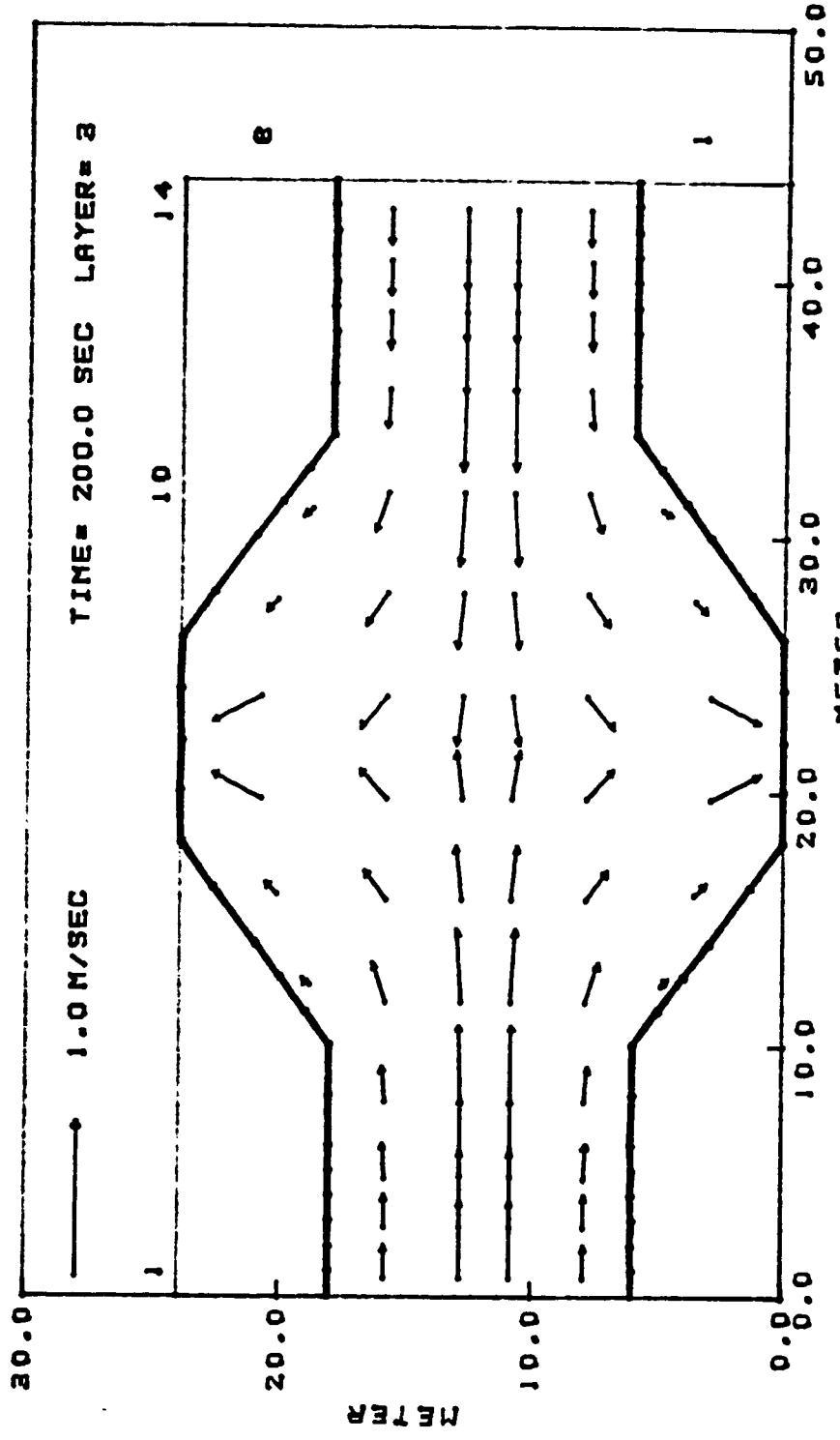

Figure 6.102. Velocity conditions in the internal layer l = 3, at time t = 200 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

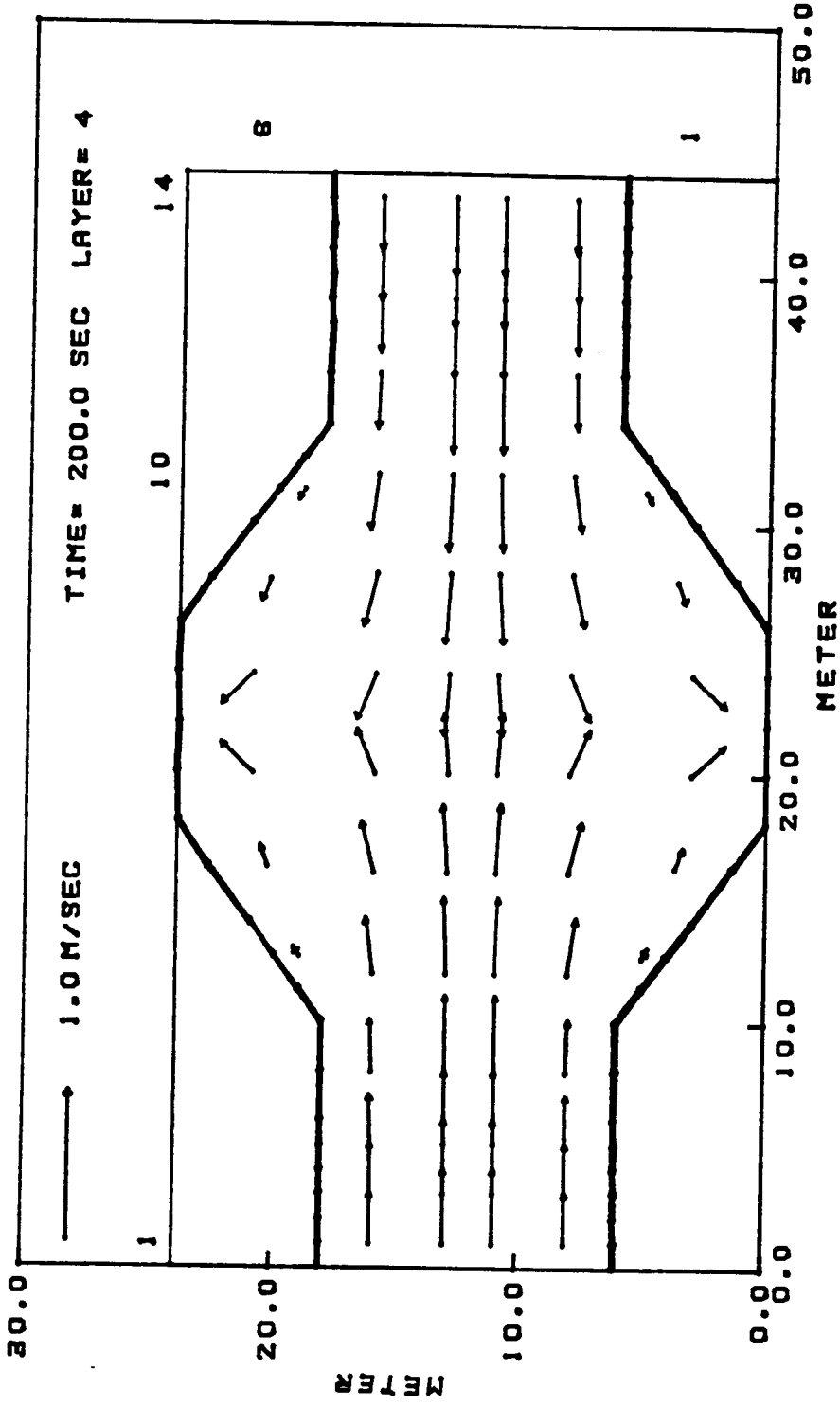

Figure 6.103. Velocity conditions in the surface layer l = 4, at time t = 200 sec for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method). Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

correspond to two-component representations of velocity in the horizontal (x,y) planes of the four specified layers.

The results of the computer simulations for the flow conditions in the symmetric laboratory-model-scale flow region indicated that the water-surface elevation conditions for test case 5 (Fig. 6.84, p.706; Fig. 6.89, p.711; Fig. 6.94, p.716; and Fig. 6.99, p.721) were significantly different than the results for water-surface elevation conditions for test case 4 (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688). However, the results for the velocity conditions for test case 5 were generally similar to the results for the velocity conditions for test case 4 (Figs. 6.65 through 6.68, p.674 through p.677; Figs. 6.70 through 6.73, p.679 through p.682; Figs. 6.75 through 6.78, p.684 through p.687; and Figs. 6.80 through 6.83, p.689 through p.692). This was expected, since the effect of the artificial viscosity, which influenced the results of the simulations for test case 4, could only retard the velocity conditions in the flow region without causing significant alterations in the transient behavior or in the spatial distributions of the velocity field. However, since the effect of the artificial viscosity directly influences the pressure gradients in the flow region, considering the predominant effect of the hydrostatic pressure conditions, it directly influences the spatial gradients of the water-surface elevation. Therefore, since the effect of the artificial viscosity influences the spatial gradients of the water-surface elevation, it can substantially alter the spatial distribution of the water-surface elevation conditions in the laboratory-model-scale flow region. Hence, the results of the computer

simulations for test case 5 illustrate the effect of the artificial viscosity through the results for the water-surface elevation conditions, rather than the results for the velocity conditions.

The comparison of the results for constant-water-surface-elevation contours for test case 5 at time level t = 50 sec (Fig. 6.84, p.706) with the results for constant-water-surface-elevation contours for test case 4 at time level t = 50 sec (Fig. 6.64, p.673) clearly illustrates the decreased effect of the artificial viscosity (as a part of the general numerical dispersion effect) in the simulation results for test case 5. The results for test case 5 indicate that at time level t = 50 sec, the two constant-water-surface-elevation contours with zero value, which correspond to the specified initial mean water level (MWL), are located more than 5 m away from (1) the discharge plane at the south-end downstream enclosure surface at y = 0, and (2) the discharge plane at the north-end downstream enclosure surface at y = 24 m (Fig. 6.84, p.706), in contrast to the results for test case 4, at time level t = 50 sec, when the two constant-water-surface-elevation contours with zero value are already located at the immediate vicinities of the two discharge planes (Fig. 6.64, p.673). The results for test case 5 also indicate that at time level t = 50 sec, the water-surface-elevation conditions drop below the zero value, which corresponds to the specified initial mean water level (MWL), in the vicinity of (1) the discharge plane at the south-end downstream enclosure surface at y = 0, and (2) the discharge plane at the north-end downstream enclosure surface at y = 24 m (Fig. 6.84, p.706), again in contrast to the results of test case 4 at time level t = 50 sec which indicate zero drop in the

water-surface-elevation conditions in the vicinities of the two discharge planes (Fig. 6.64, p.673). This result is a direct consequence of the effect of the reduced artificial viscosity for test case 5. Since for test case 5, the effect of artificial viscosity is minimized, the water has the tendency to flow rapidly out of (1) the discharge plane at the south-end downstream enclosure surface at $y = 0$, and (2) the discharge plane at the north-end downstream enclosure surface at $y = 24$ m.

The results for test case 5, for all time levels, $t = 50$ sec through $t = 200$ sec, indicate that the water-surface-elevation conditions in the flow region (Fig. 6.84, p.706; Fig. 6.89 p.711; Fig. 6.94, p.716; and Fig. 6.99, p.721) remain significantly lower than the results for water-surface elevation conditions for test case 4 (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688). This result is again a direct consequence of the effect of the reduced artificial viscosity for test case 5. Since for test case 5 the effect of the artificial viscosity is minimized at all time, the water can flow with much less resistance into the flow region through (1) the inlet plane of the west-end upstream enclosure surface at $x = 0$ and (2) the inlet plane of the east-end upstream enclosure surface at $x = 44$ m, and it can flow out of the flow region, also with much less resistance, through (3) the discharge plane of the south-end downstream enclosure surface at $y = 0$ and (4) the discharge plane of the north-end downstream enclosure surface at $y = 24$ m (Fig. 6.84, p.706; Fig. 6.89, p.711; Fig. 6.94, p.716; and Fig. 6.99, p.721). Therefore, the accumulated water surface elevation conditions remain at a level, much less than the

accumulated water-surface elevation conditions for test case 4 (Fig. 6.64, p.673; Fig. 6.69, p.678; Fig. 6.74, p.683; and Fig. 6.79, p.688) at all time levels, t = 50 sec through t = 200 sec, according to the results of the computer simulations of the flow conditions.

Hence, the results of the computer simulations of the water-surface-elevation and three-dimensional velocity conditions in the laboratory-model-scale flow region for test case 5: Alternate Symmetric Opposing Flow 2 (inflow along longitudinal direction and outflow along lateral direction, based on transportive-upwind-differencing interpolation method), again illustrate the reliability of the mathematical formulation of the transient, three-dimensional, discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of the associated HYDROL computer code, in applications with the special interpolation options which can minimize the effect of the artificial viscosity (as a part of the general numerical dispersion effect).

6.2. Application to the Simulation of Flow Conditions in Browns Ferry Physical Model

The fast-transient, three-dimensional discrete-element hydrodynamic model was also applied to the simulation of the flow development in the Browns Ferry physical model which was located in the Engineering Laboratory of the Water System Development Branch, Division of Water Management, the Tennessee Valley Authority, Norris, Tennessee.

Browns Ferry physical model was originally constructed as a laboratory model for studying the effect of the operation of the Browns Ferry Nuclear Power Plant on the Wheeler Resevoir of the Tennessee

River. The physical model was constructed as a distorted scale model of approximately a 5-mile stretch of the Tennessee River. The horizontal scale was specified as 1:250 and the vertical scale was specified as 1:50 in the physical model in comparison to the prototype flow region. The distorted scale model was selected to maintain the same Froude number between the physical laboratory-scale model and the protype flow region. The details of the construction of the model were presented in a TVA (Tennessee Valley Authority) report (1972).

In the applications of the fast-transient, three-dimensional, discrete-element hydrodynamic model, actual dimensions of the flow region of the Browns Ferry physical model were used in the computer simulations of the flow conditions.

6.2.1. Discussion of the Geometry and Depth Conditions of the Browns Ferry Physical Model

In the application of the discrete-element method to the Browns Ferry physical model, the longitudinal x-axis was selected to extend along approximately east to west direction and the lateral y-axis was selected to extend along approximately north to south direction relative to the actural configuration of the Wheeler Reservoir on the Tennessee River. The dimensions of the flow region of the physical model were approximately 38.4 m along the longitudinal x-direction and approximately 18.3 m along the lateral y-direction of the discrete-element grid system.

The flow region was discretized by using a variable rectangular grid system with 18 length increments along the x-axis and 12 length increments along the y-axis (Fig. 6.104, p.731). The variable

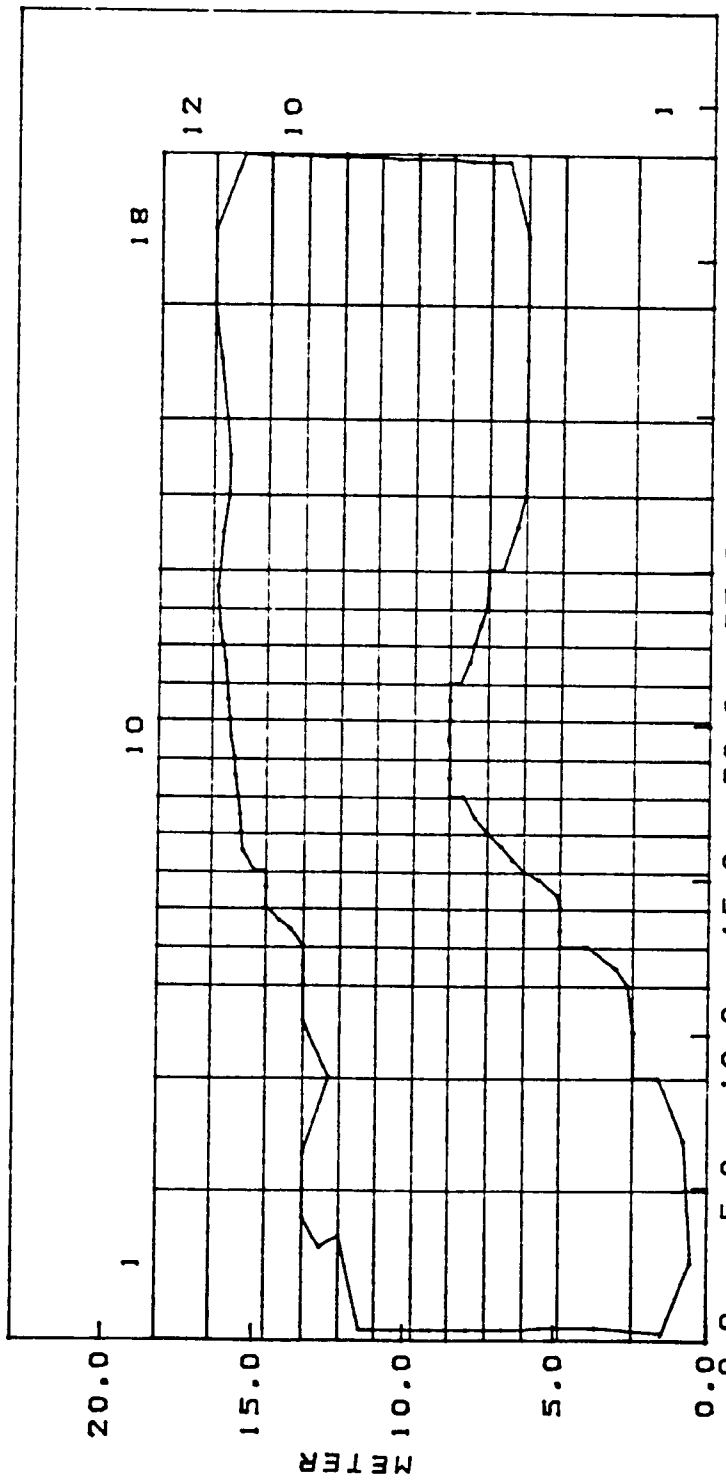

Figure 6.104. Discrete-element, variable-size, rectangular grid system for the Browns Ferry physical model. Length increments along x-direction vary between 1.22 m and 2.44 m and length increments along y-direction vary between 1.22 m and 1.83 m.

rectangular grid system considered length increments varying between 1.22 m and 2.44 m along the x-direction and varying between 1.22 m and 1.83 m along the y-direction. The variable rectangular grid was selected to obtain higher spatial resolution in the vicinity of the power plant for the computer simulations of the flow conditions. The planar elements in the horizontal (x,y) plane, corresponding to the free-surface plane of the flow region, were obtained by utilizing the automated discrete-element generator computer code PREPR2 (Holdeman, 1977). The flow region was divided into a total of 136 planar elements in the (x,y) plane, consisting of 87 internal planar elements and 49 boundary planar elements (Fig. 6.104, p.731).

The depth condition in each planar element was specified approximately to represent the depth in the actual laboratory-model-scale Browns Ferry physical model at the location corresponding to the location of the element. The values of the local depth varied between 1 to 21 cm, relative to the mean water level in the flow region (Fig. 6.105, p.732). The x-upstream part of the flow region, extending approximately between x = 0 and x = 25 m and extending from the y-downstream shoreline to y-upstream shoreline, represented a channel with shallow subregions along the two banks and a deep main-flow channel approximately between x = 0 and x = 25 m and between y = 6 m and y= 10 m, along the x-direction in the center of the flow region (Fig. 6.105, p.733). The deep main-flow channel separated into two main-flow channels at approximately x = 25 m, with the first part of the main-flow channel extending along the y-direction, approximately between y = 6 m and y = 9 m, and the second part of the main-flow channel extending along the y-direction

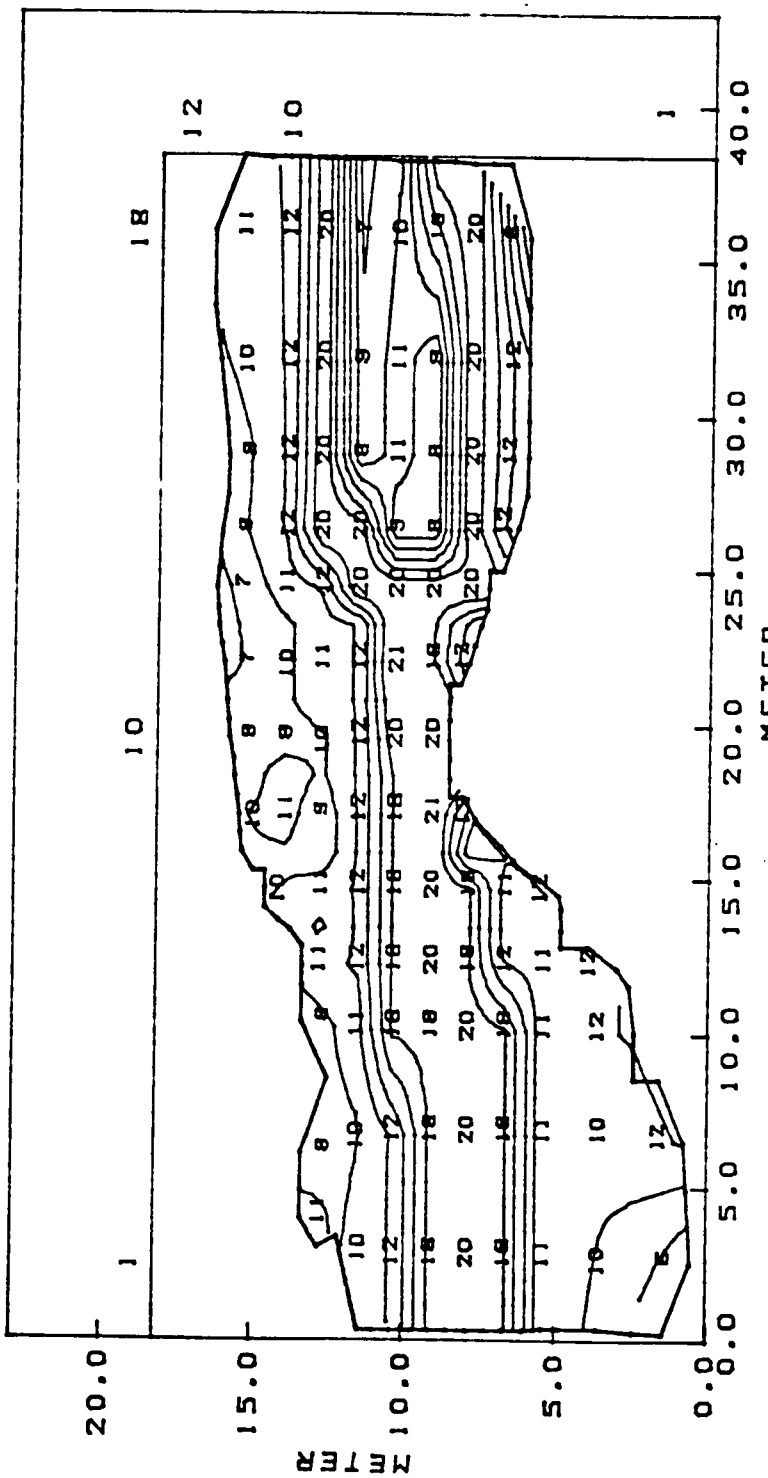

Figure 6.105. Depth conditions at specified initial mean water level (MWL) for the flow region of the Browns Ferry physical model. Gradations of constant-depth contours as 2 cm (0.02 m), between minimum 2 cm (0.02 m) and maximum 20 cm (0.2 m).

approximately between y = 12 m and y = 14 m (Fig. 6.105, p.733). Both parts of the main-flow channels were along the x-direction in the flow region (Fig. 6.105, p.733).

The flow region was divided into five specified layers (L = 5) along the vertical direction, with the depths of layers specified, as:

l = 1: 0.1524 m (bottom layer),

l = 2: 0.0254 m (internal layer),

l = 3: 0.0254 m (internal layer),

l = 4: 0.0254 m (internal layer),

l = 5: 0.0127 m (water-surface layer).

The particular specification of the depths of the layers were based on the set of layers which were considered in a separate experimental study conducted for the measurement of the flow conditions in the Browns Ferry physical model (Remenyik, et al., 1980).

According to the specified depths of the layers, the general discretization technique of the discrete-element method divided the flow region into:

(1) 47 elements in the bottom layer l = 1, corresponding to the bottom of the deep main-flow channel,

(2) 130 elements in the internal layer l = 2, corresponding to the relatively deep parts of the flow region,

(3) 136 elements in the internal layer l = 3,

(4) 136 elements in the internal layer l = 4,

(5) 136 elements in the water-surface layer l = 5.

Hence, the application of the fast-transient, three-dimensional, discrete-element hydrodynamic model considered a total of 945 elements

in representing the laboratory-model-scale flow region of the Browns Ferry phsical model. The variable, rectangular grid system used in the discretization of the flow region allowed for accurate, high-resolution results in the vicinity of the locations of the intake structure and the submerged diffuser of the Browns Ferry Nuclear Power Plant in the distorted-scale physical model.

6.2.2. Discussion of the Specified Boundary Conditions for the Browns Ferry Physical Model

The fast-transient, three-dimensional, discrete-element hydrodynamic model was applied to the simulation of the water-surface elevation and velocity conditons in the laboratory-model-scale flow region of the Browns Ferry physical model. The computer simulation results were obtained to study the development of the flow conditions in the Browns Ferry physical model from the initial conditions of stagnant water in the flow region, after starting the inflow from the x-upstream end and simultaneously starting the outflow from the x-downstream end of the flow region.

The specified inflow rate and outflow rate to the flow region were selected based on the results of the experimental studies for measuring the flow conditions in the Browns Ferry physical model (Remenyik, et al., 1980). The inflow rate at the x-upstream boundary of the flow region was selected as 0.01032 m^3/sec and the outflow rate at the x-downstream boundary of the flow region was selected also as 0.01032 m^3/sec. The necessary input data to the HYDROL computer code was prepared by considering specified-boundary enclosure-surface conditions

(see Sec. 4.5) at the x-upstream boundary at x = 0 and at the x-downstream boundary at x = 38.4 m.

The specified conditions for the flow rate along x-direction and the momentum transport along x-direction at the x-upstream boundary at x = 0 were determined by considering a uniform velocity distribution over all x-upstream specified-boundary enclosure surfaces of the elements and half elements, such that the sum of the flow rates, across the x-upstream specified boundary enclosure surfaces corresponding to the x-upstream boundary of the flow region, was exactly equal to the specified inflow rate of 0.01032 m3/sec.

longitudinal x-component of velocity, associated with the flow rate along x-direction at x = 0:

$\bar{V}_{x;(spb,i-\frac{1}{2}),j,l}$ = 0.006110 m/sec for all elements i=1;j=1 through 9; l=1 through 5.

longitudinal x-component of velocity, associated with the momentum transport along x-direction at x = 0:

$\tilde{V}_{x;(spb,i-\frac{1}{2}),j,l}$ =0.006110 m/sec for all element s i=1; j=1 through 9; l=1 through 5.

lateral y-component of velocity, associated with the momentum transport along y-direction at x = 0:

$\tilde{V}_{y;(spb,i-\frac{1}{2}),j-\frac{1}{4},l}$ = 0 for all elements i=1 through 9; l=1 through 5.

vertical z-component of velocity, associated with the momentum transport along z-direction at x = 0:

$\tilde{V}_{z;(spb,i-\frac{1}{2}),j,l}$ =0 for all elements i=1; j=1 through 9; l=1 through 5.

The specified conditions for the flow rate along x-direction and the momentum transport along x-direction at the x-downstream boundary at x = 38.4 m were determined by considering a uniform velocity distribution over all x-downstream specified-boundary enclosure surfaces of the elements and half elements, such that the sum of the flow rates across the x-downstream boundary of the flow region, was exactly equal to the specified outflow rate of 0.01031 m^3/sec.

longitudinal x-component of velocity, associated with the flow rate along x-direction at x = 38.4 m:

$\bar{V}_{x;(spb,i+\frac{1}{2}),j,l}$ = 0.007096 m/sec for all elements i=18; j=4 through 11 l=1 through 5.

longitudinal x-component of velocity, associated with the momentum transport along x-direction at x = 38.4 m:

$\tilde{V}_{x;(spb,i+\frac{1}{2}),j,l}$ = 0.007096 m/sec for all elements i=18; j=4 through 11; l=1 through 5.

lateral y-component of velocity, associated with the momentum transport along y-direction at x = 38.4 m:

$\tilde{V}_{y;(spb,i+\frac{1}{2}),j+\frac{1}{4},l}$ = 0 for all elements i=18 j=4 through 11, l=1 through 5.

vertical z-component of velocity, associated with the momentum transport along z-direction at x = 38.4 m:

$\tilde{V}_{z;(spb,i+\frac{1}{2}),j,l}$ = 0 for all elements i=18, j=4 through 11; l=1 through 5.

The specified water-surface-elevation conditions at the x-upstream boundary at x = 0 and at the x-downstream boundary at x = 38.4 m were

both specified as zero for all the elements and half elements which included the specified-boundary enclosure surfaces.

It is important to note that the specified-boundary conditions used in the simulation of the flow conditions in the Browns Ferry physical model are significantly different than the open-region conditions used for the simulation of the flow conditions for the test cases. Since the velocity conditions, associated with the flow rates and the momentum transport, are specified at the specified-boundary enclosure surfaces, the water-surface-elevation conditions in the specified boundary elements are determined as the consequences of the solutions in the flow region.

6.2.3. Discussion of the Computer Simulation Options for the Brown Ferry Physical Model

The results of the computer simulations of the flow conditions for the Browns Ferry physical model were also used to check the computational accuracy of the HYDROL computer code by considering two cases with two different interpolation algorithms. However, the parameters in one of the interpolation algorithms were selected in such a manner that the computer simulation results had to reproduce the identical results of the second case with the different interpolation algorithm.

a. Interpolation option

The computer simulation results for the Browns Ferry physical model were obtained by applying the HYDROL computer code with two interpolation options.

The weighted-upwind-differencing interpolation option was first considered with all weighting coefficients along x-, y- and z-directions as equal to 1.0. The simple second-upwind-differencing interpolation option was considered next. Since, all weighting coefficients along x-, y- and z-directions for the weighted-upwind-differencing were assumed as 1.0, the particular option was effectively identical to the simple second-upwind-differencing interpolation option.

The computer simulation results for the two options were identical, except for the truncation errors, which indicated that the general computational algorithms for the weighted-upwind-differencing and for the simple second-upwind differencing were appropriately formulated in the HYDROL computer code. It was noted that both interpolation options could introduce artificial viscosity (numerical dispersion) effects in the elements and half elements where the local Courant numbers were much less than one (see Sec. 4.3).

b. Numerical Solution Option

The computer simulation results for the Browns Ferry physical model, for both interpolation options, were obtained by applying the HYDROL computer code with the standard default option which utilized the special, composite time-splitting method in the numerical integration of the governing system of spatially discretized, ordinary differential equations (see Sec. 5.2)

6.2.4. Discussion of the Computer Simulation Results for the Flow Conditions in the Browns Ferry Physical Model

The results of the computer simulations for the water-surface elevation and velocity conditions are presented in Figs. 6.106 through 6.153, p.741 through p.788. The water-surface elevation conditions are presented as (1) numbers in the elements which represent water-surface -elevation (in mm), relative to the initial specified mean water level, and (2) constant water-surface-elevation contours which correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL. The velocity conditions are presented as two-dimensional vectors which correspond to two-component representations of velocity in the horizontal (x,y) planes of the five specified layers.

The computer simulation results for the water-surface elevation and velocity conditions in the five layers, between the initial time t = 0 and the time level t = 30 sec (Figs. 6.106 through 6.123, p.741 through p.758) indicate the gradual development of the flow conditons in the Browns Ferry physical model after the initiation of the temporally uniform inflow conditions at the x-upstream end at x = 0 and outflow conditions at the x-downstream end at x = 38.4 m.

The computer simulation results for the water-surface-elevation conditions, between the inital time t = 0 and the time level t = 30 sec (Fig. 6.106, p.741; Fig. 6.112, p.747; and Fig. 6.118, p.753) indicate the generally expected conditions for the development of the free-surface flow conditions. The water-surface-elevation conditions gradually rise to approximately 10 mm, above the specified initial mean water level (MWL), in the vicinity of the x-upstream specified-boundary enclosure surface where temporally uniform inflow conditions are specified; and the water-surface-elevation conditions gradually drop to approxi-

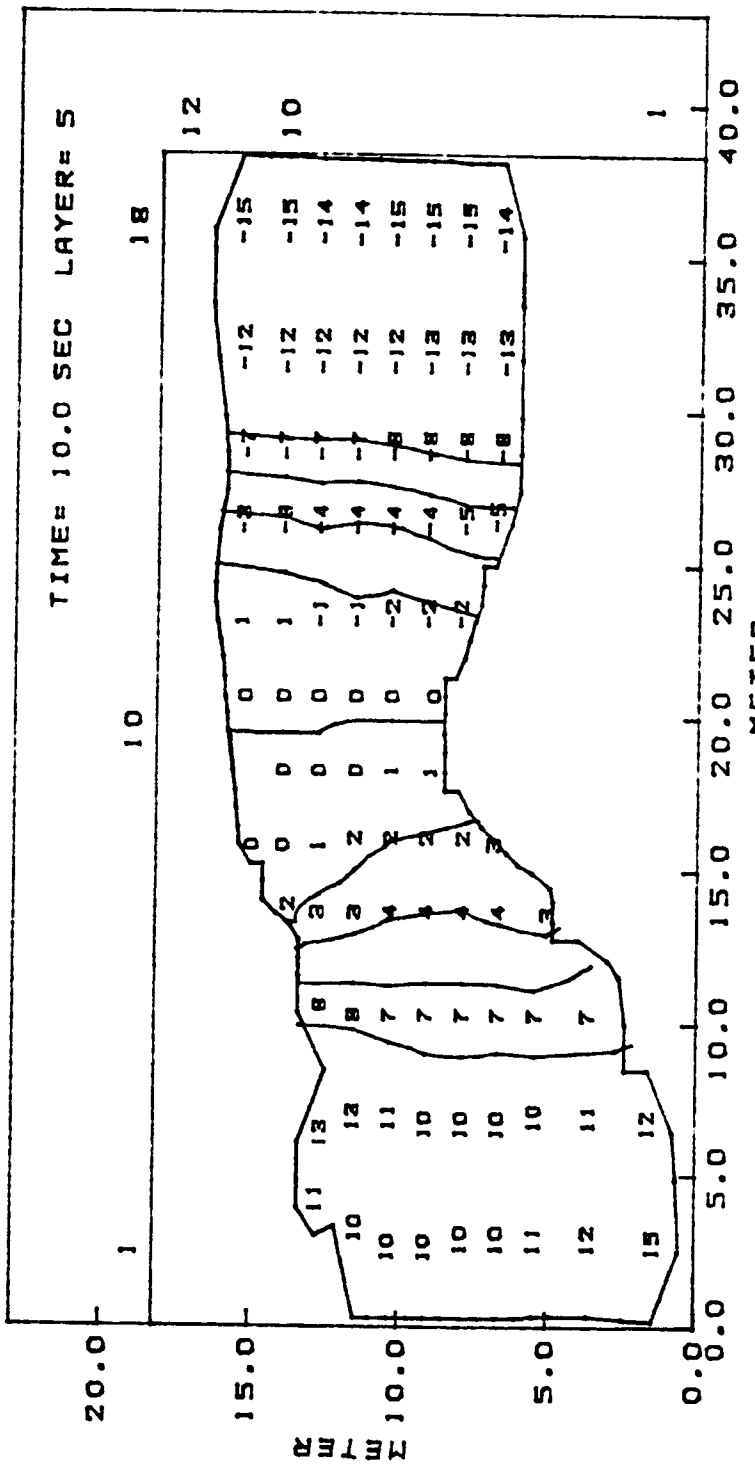

Figure 6.106. Water-surface-elevation conditions at time t = 10 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

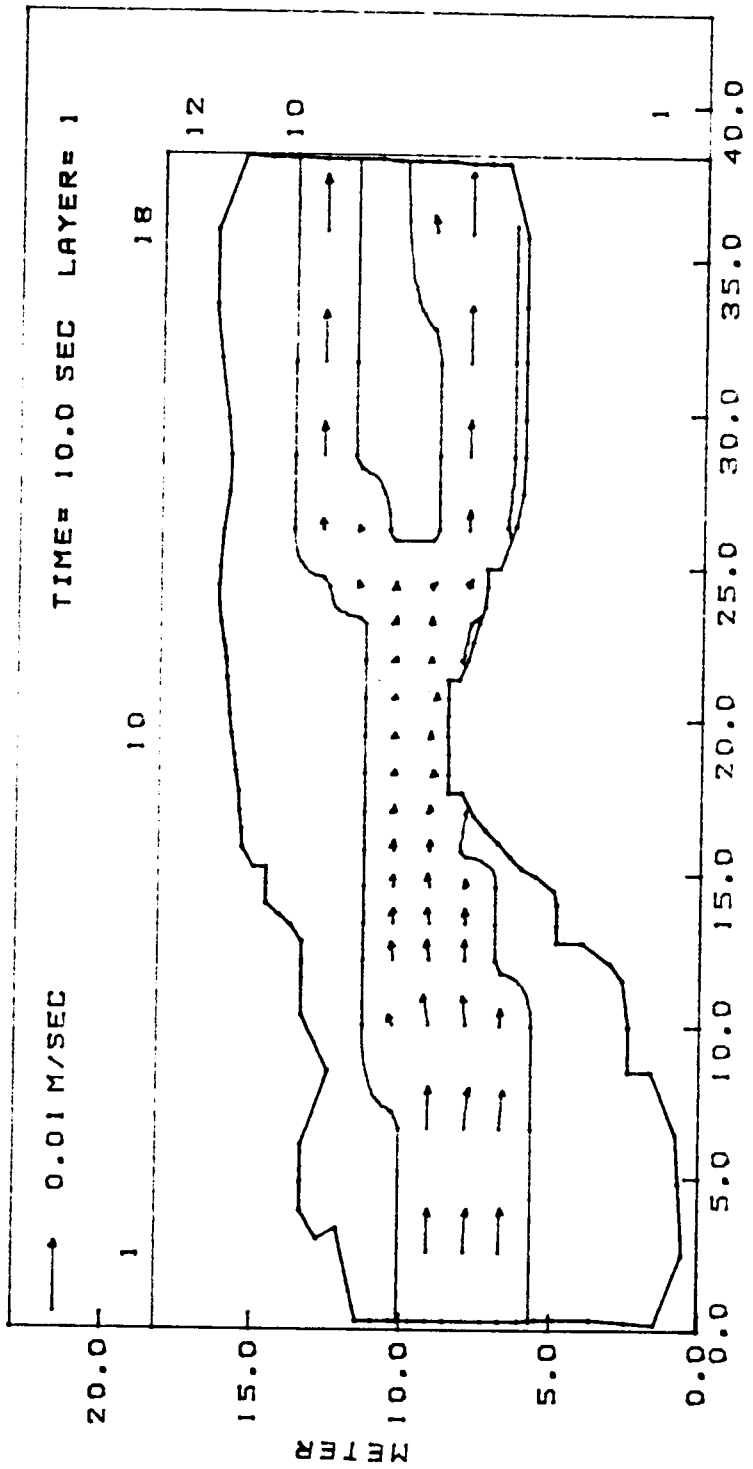

Figure 6.107. Velocity conditions in the bottom layer l = 1, at time t = 10 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

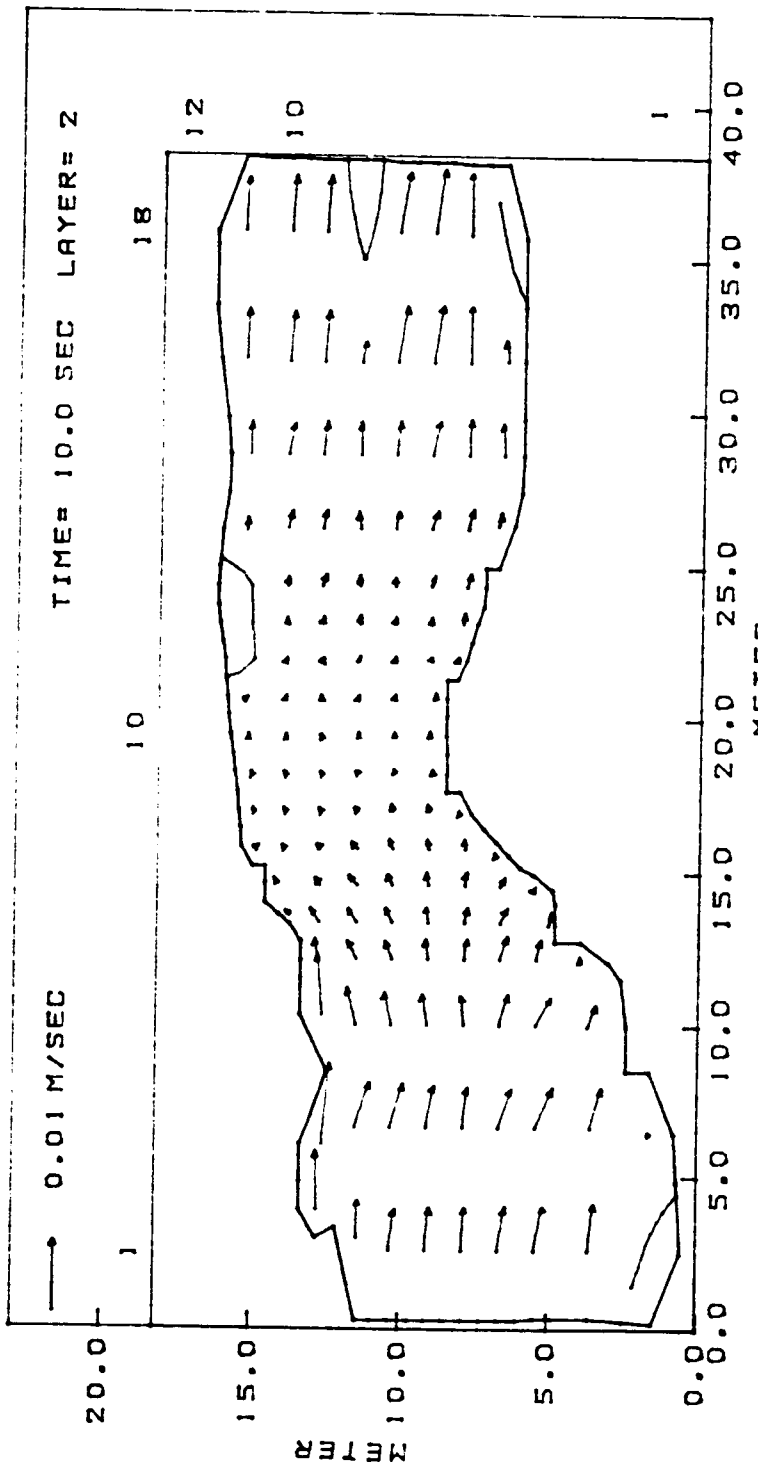

Figure 6.108. Velocity conditions in the internal layer l = 2, at time t = 10 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

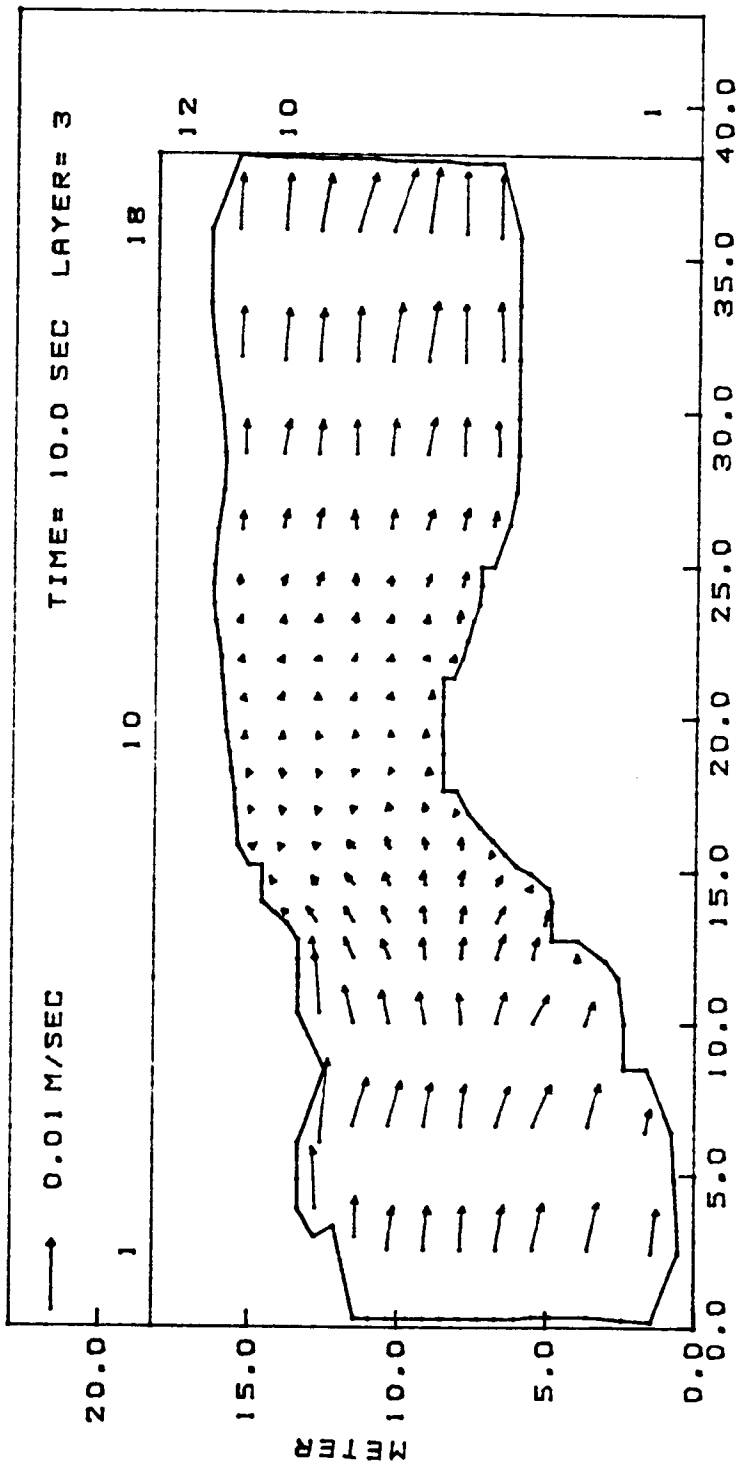

Figure 6.109. Velocity conditions in the internal layer l = 3, at time t = 10 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

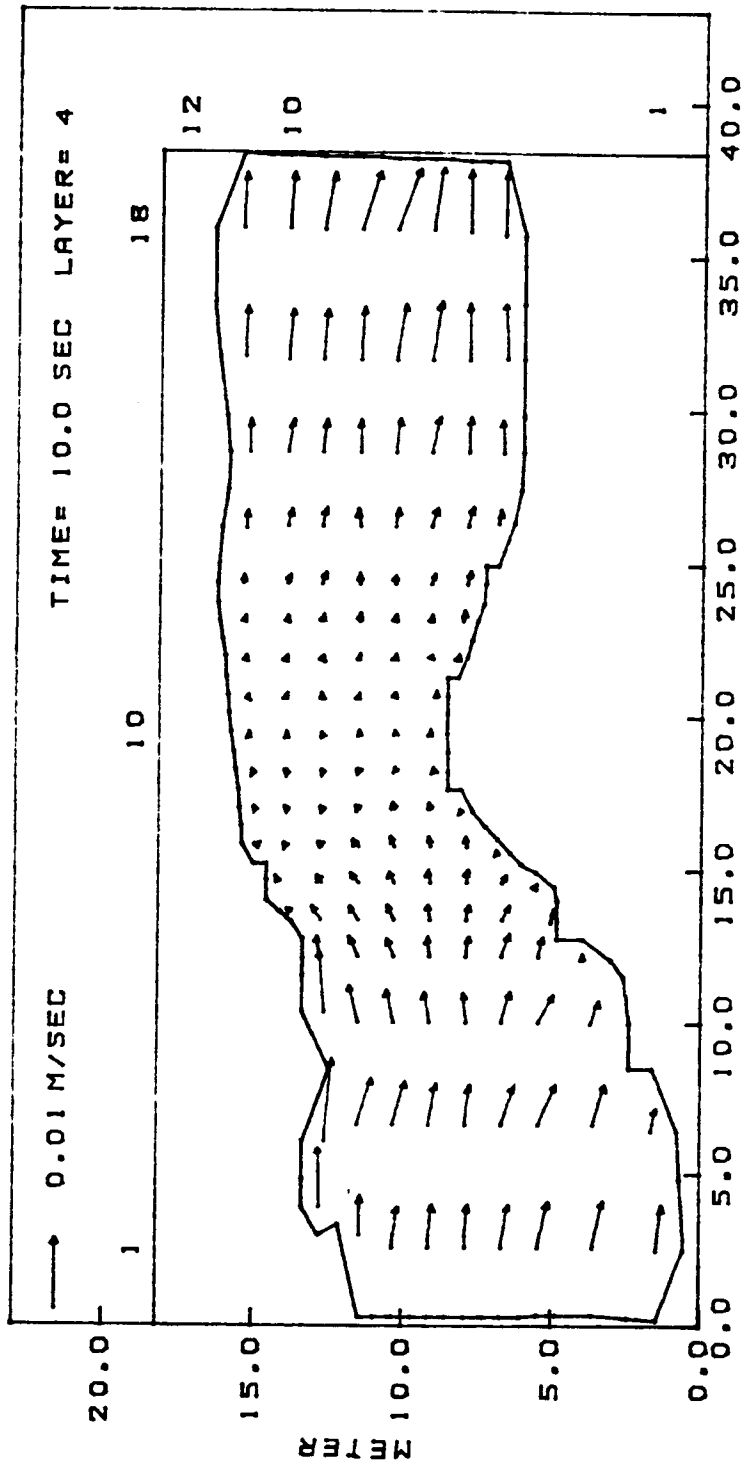

Figure 6.110. Velocity conditions in the internal layer l = 4, at time t = 10 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

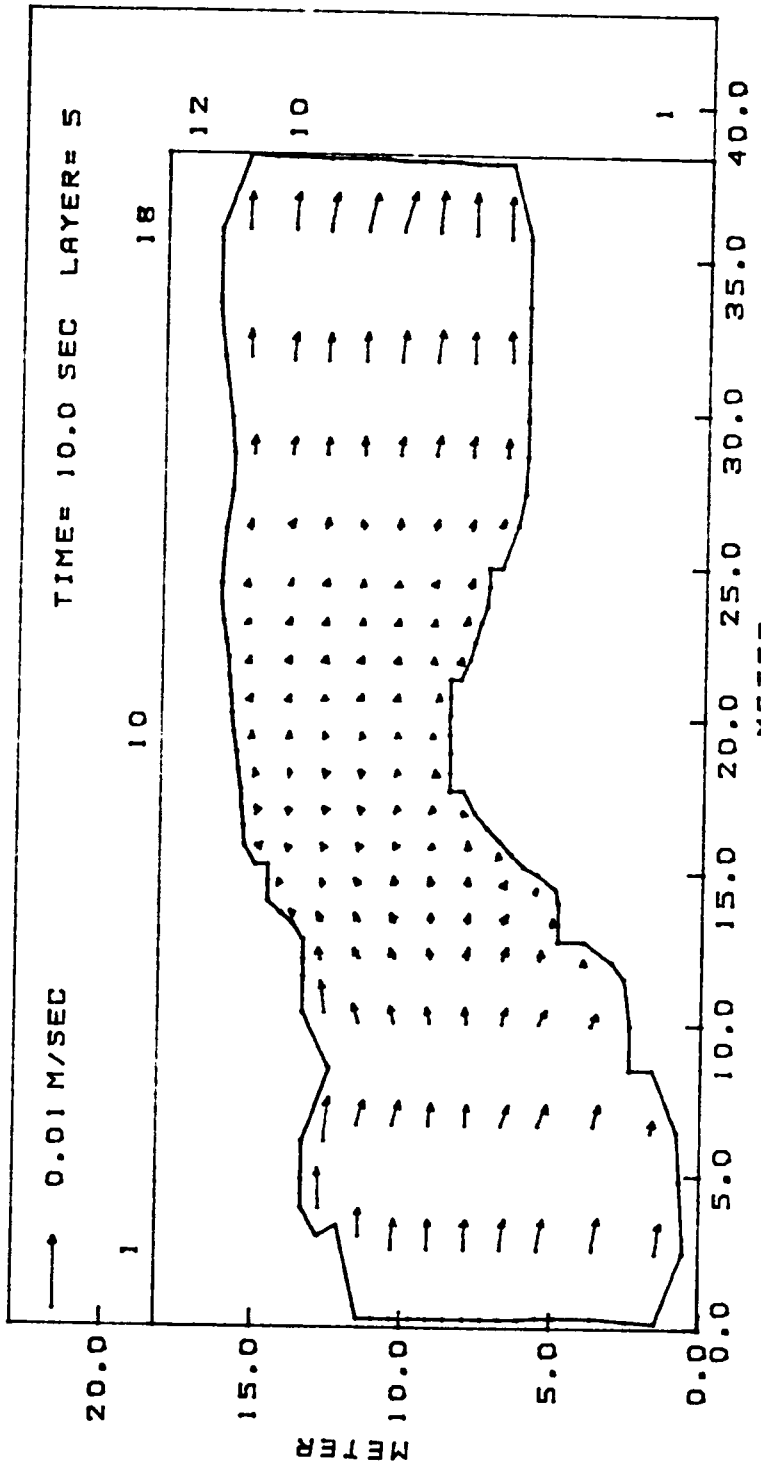

Figure 6.111. Velocity conditions in the surface layer l = 5, at time t = 10 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

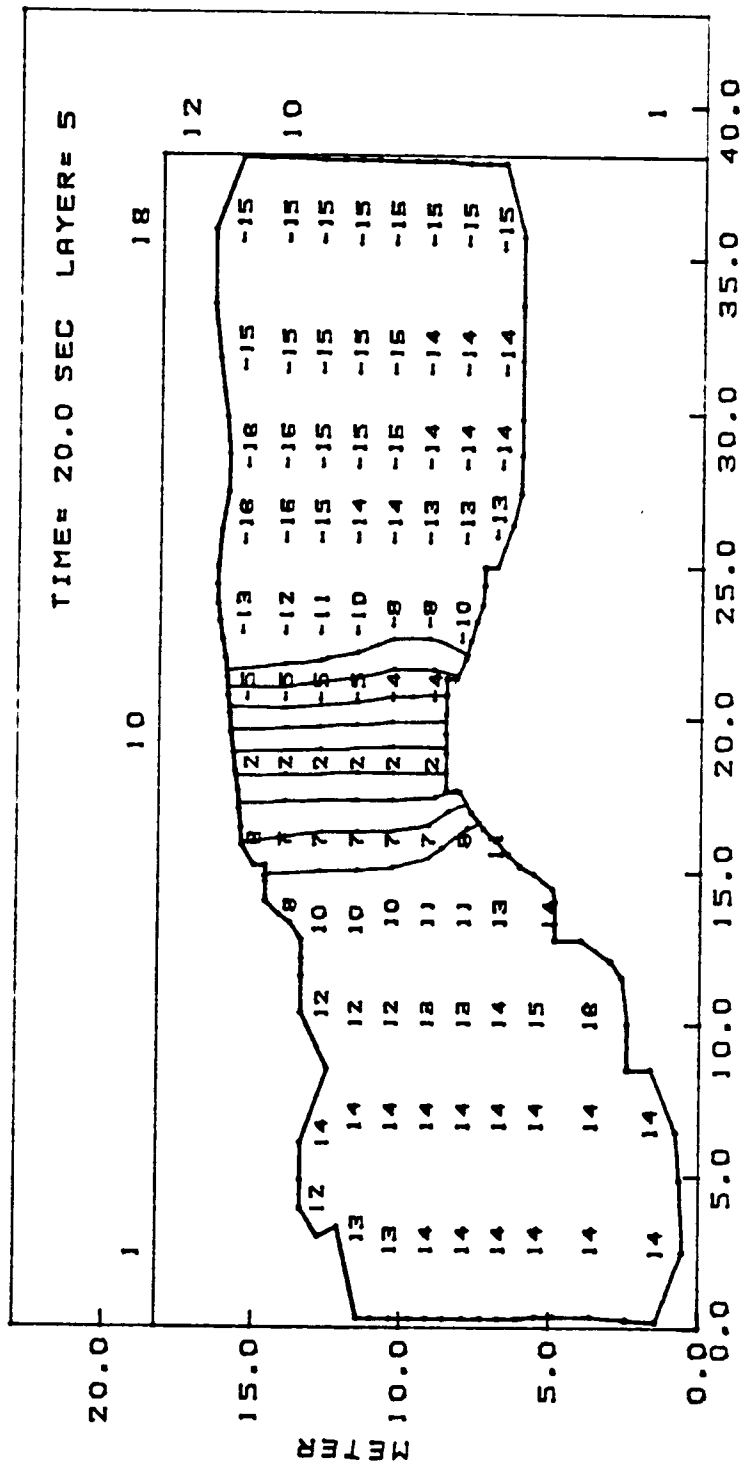

Figure 6.112. Water-surface-elevation conditions at time t = 20 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

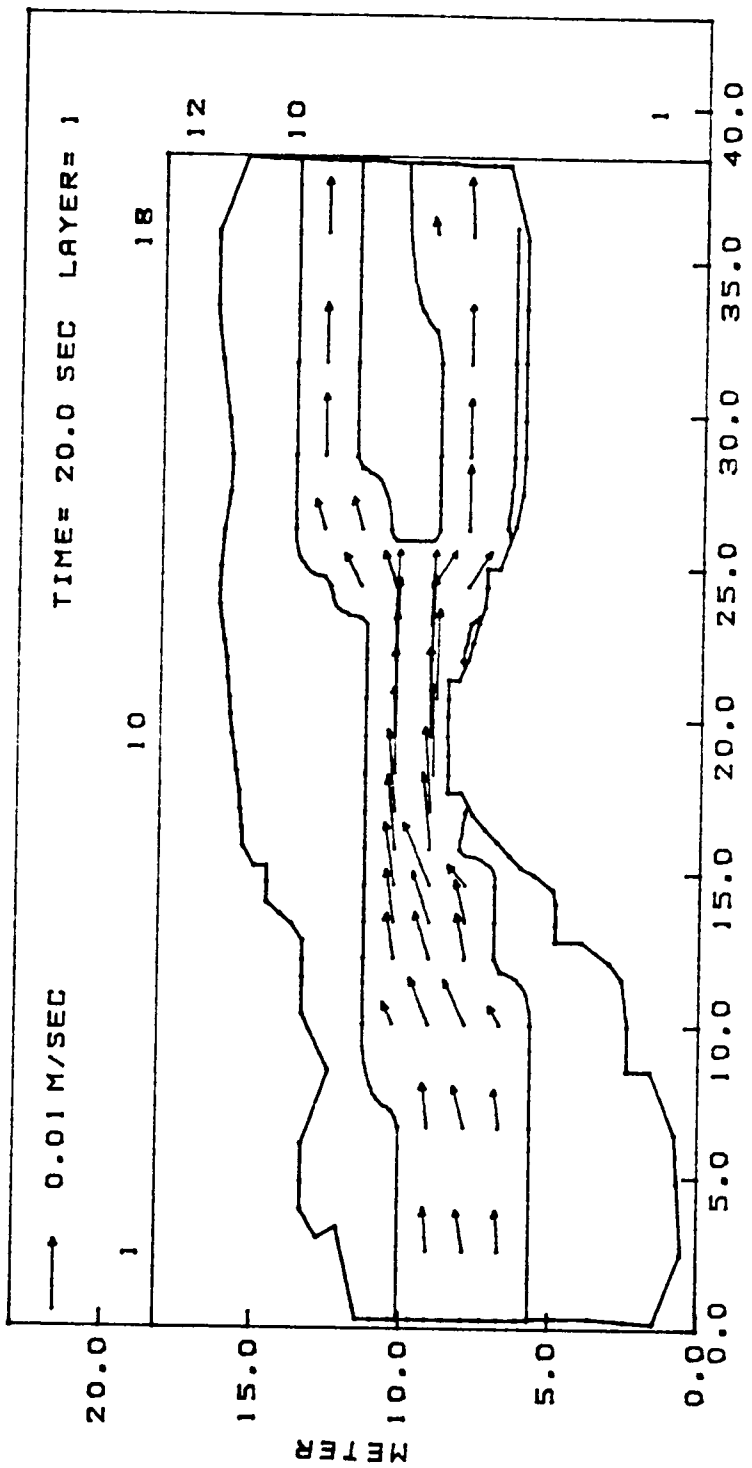

Figure 6.113. Velocity conditions in the bottom layer l = 1, at time t = 20 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

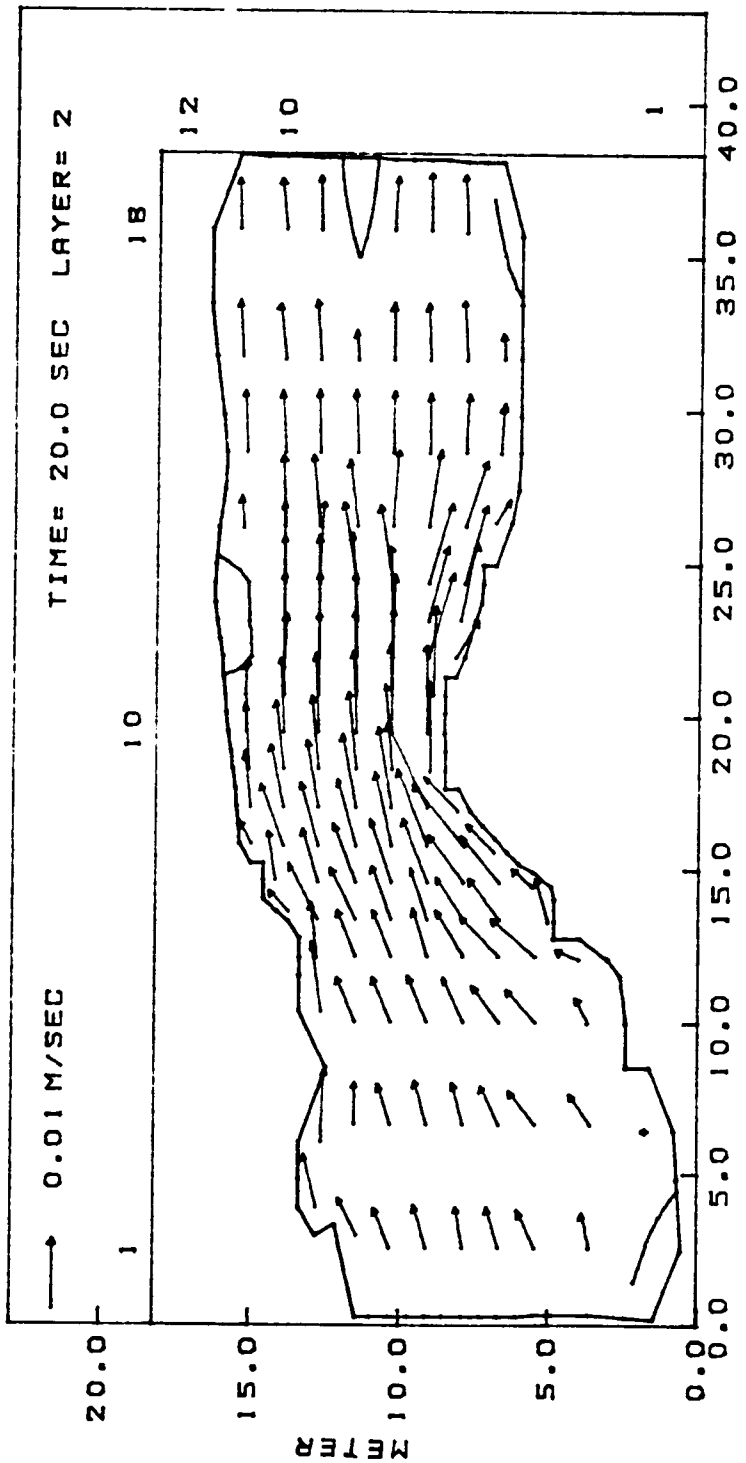

Figure 6.114. Velocity conditions in the internal layer l = 2, at time t = 20 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

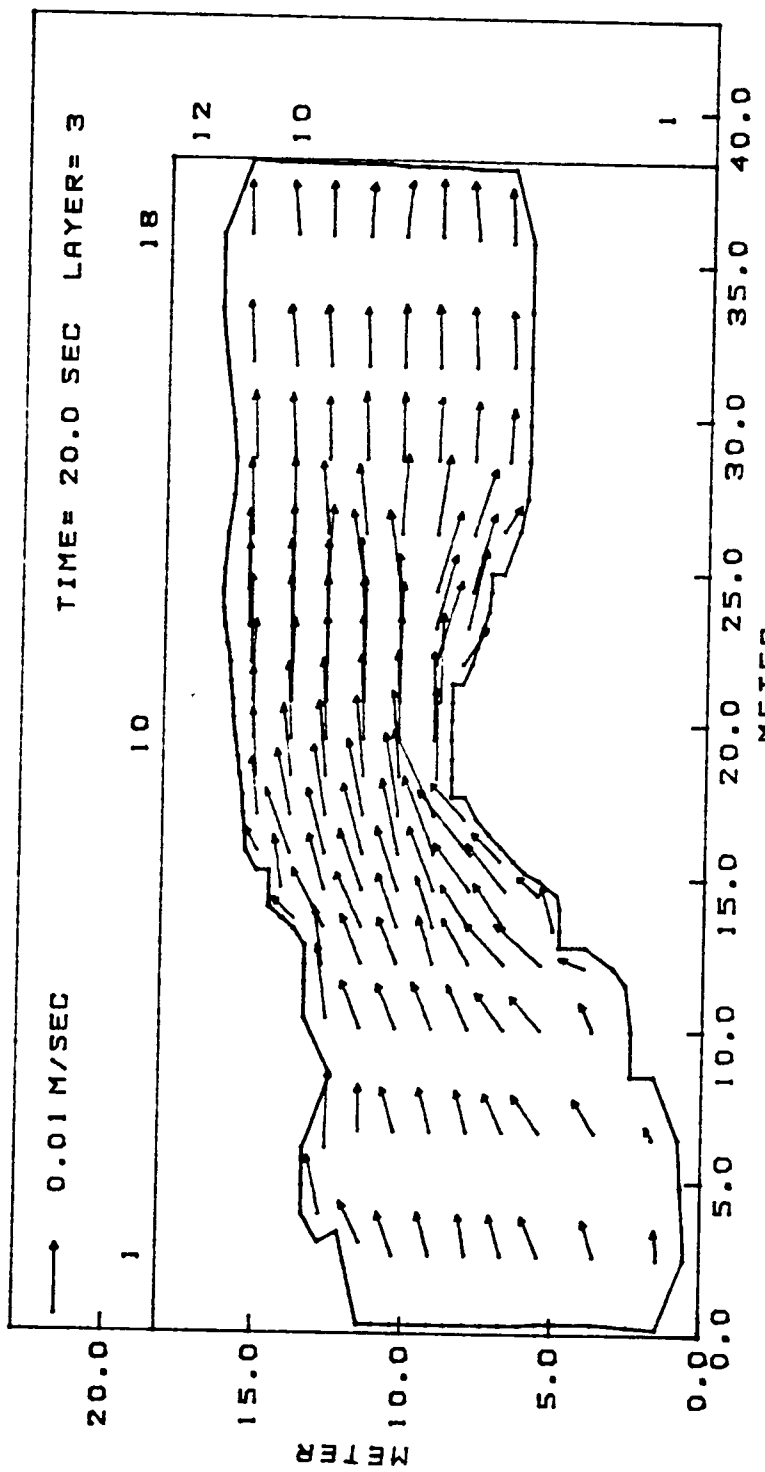

Figure 6.115. Velocity conditions in the internal layer l = 3, at time t = 20 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

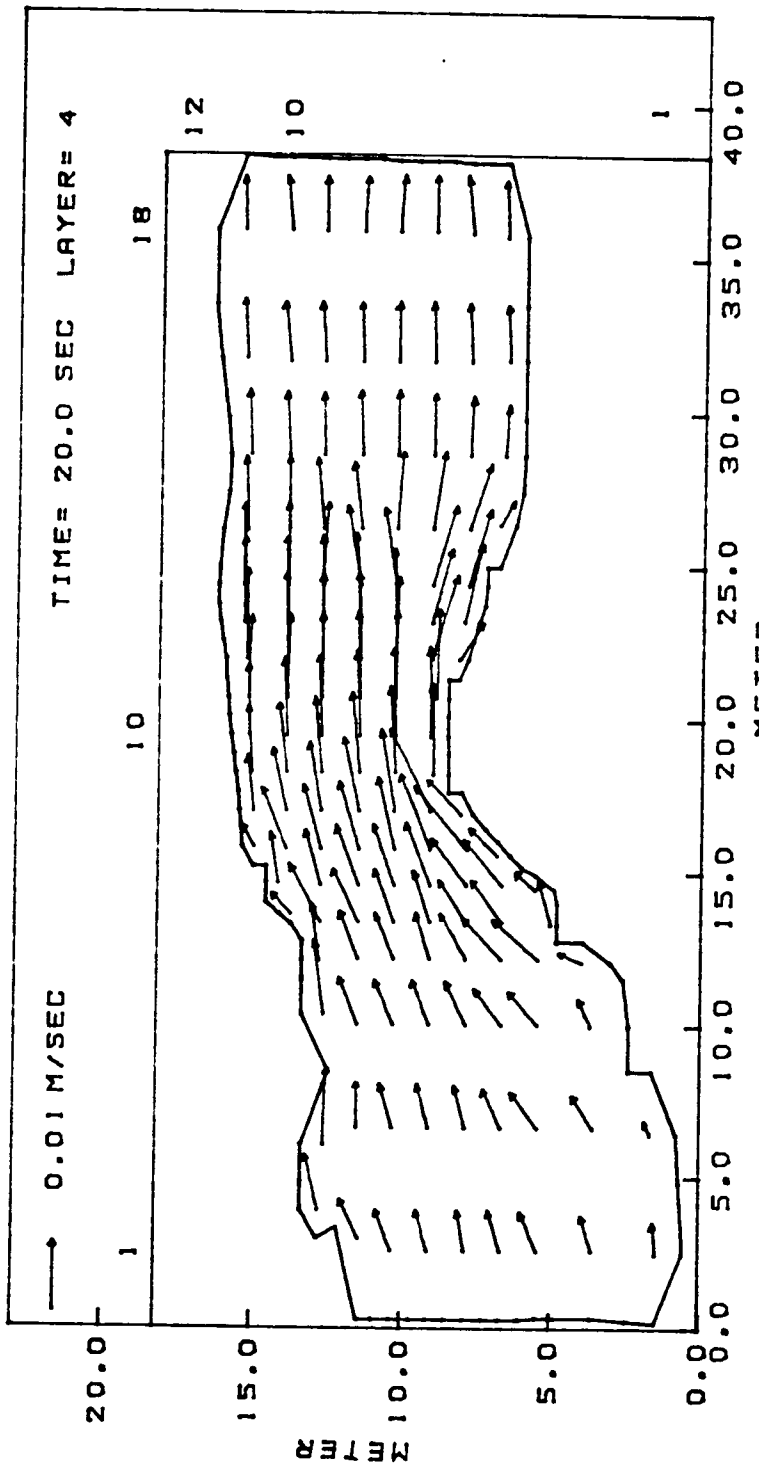

Figure 6.116. Velocity conditions in the internal layer l = 4, at time t = 20 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

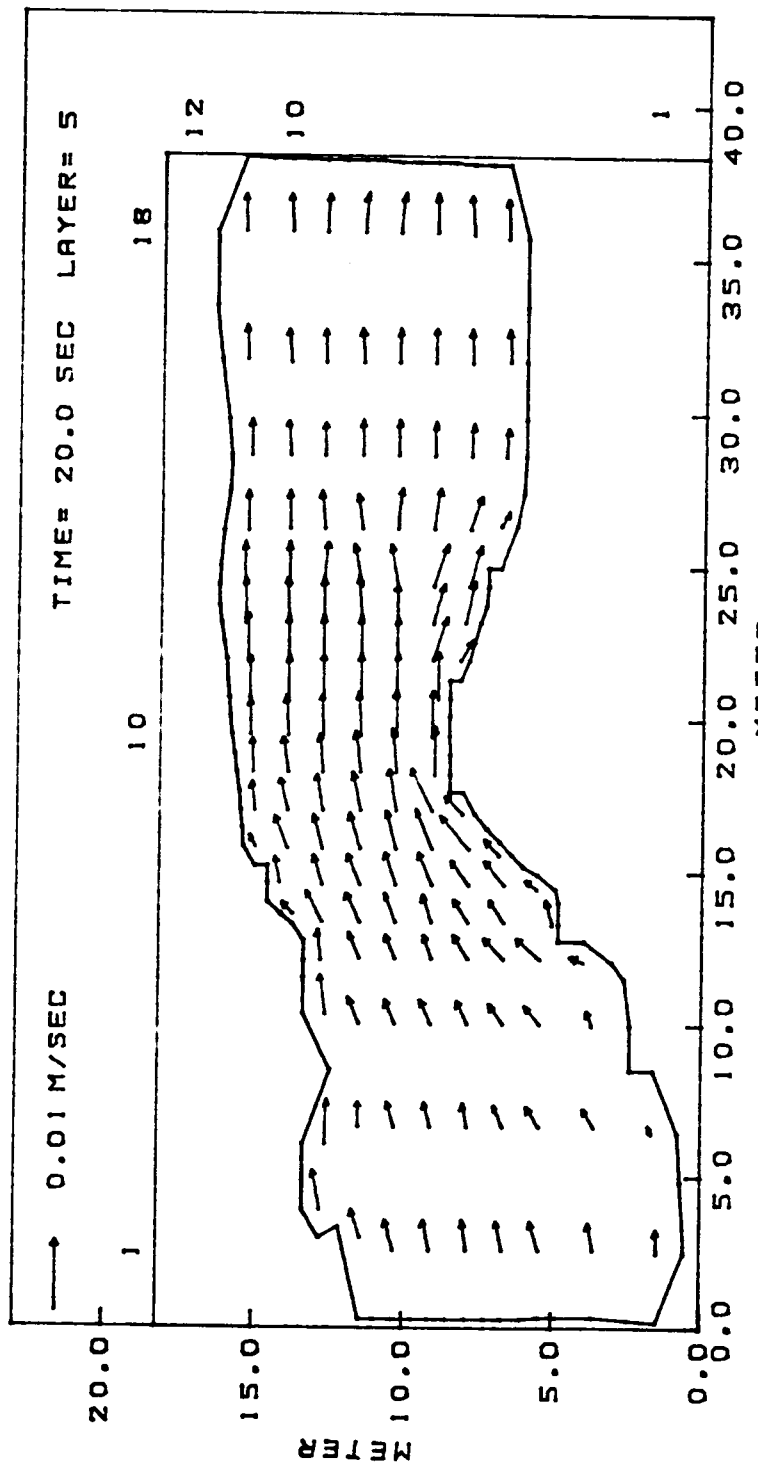

Figure 6.117. Velocity conditions in the surface layer l = 5, at time t = 20 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

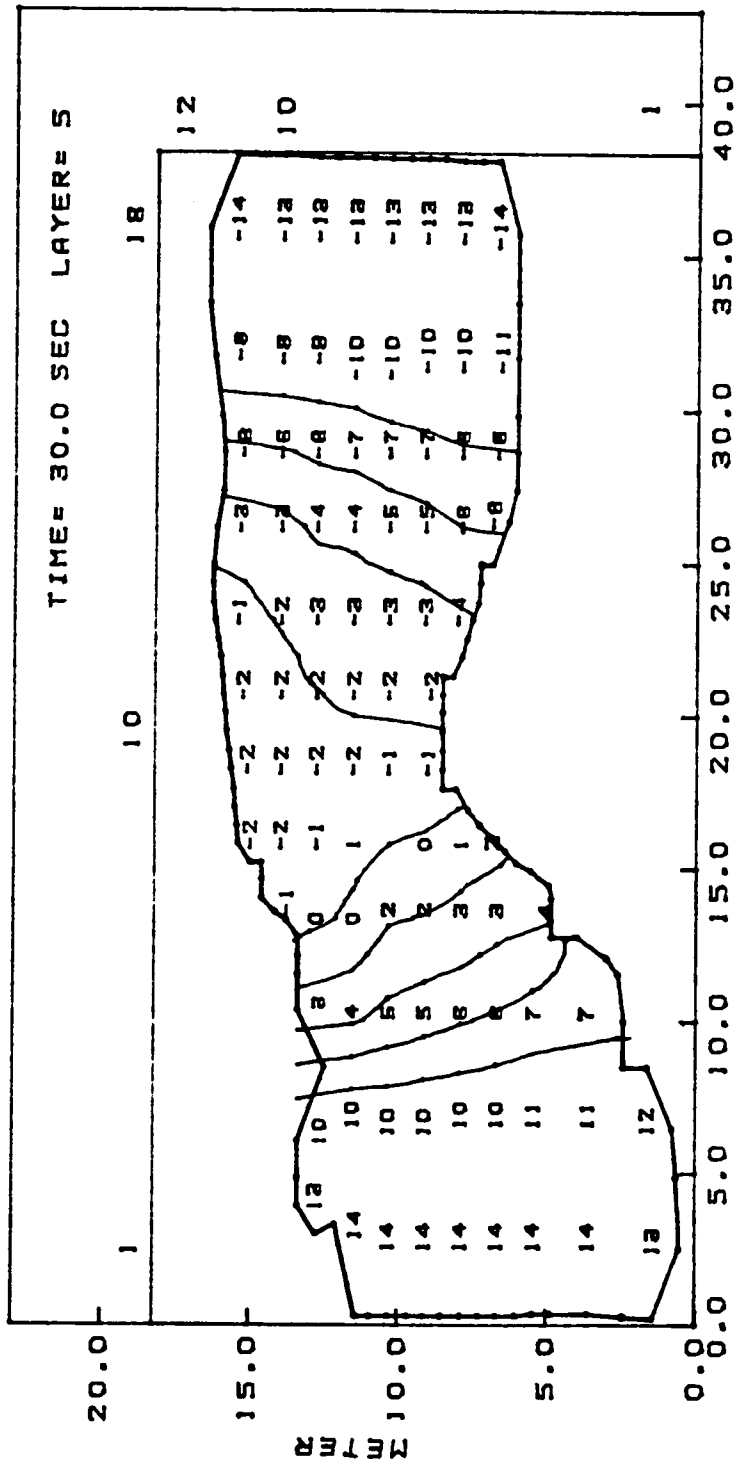

Figure 6.118. Water-surface-elevation conditions at time t = 30 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

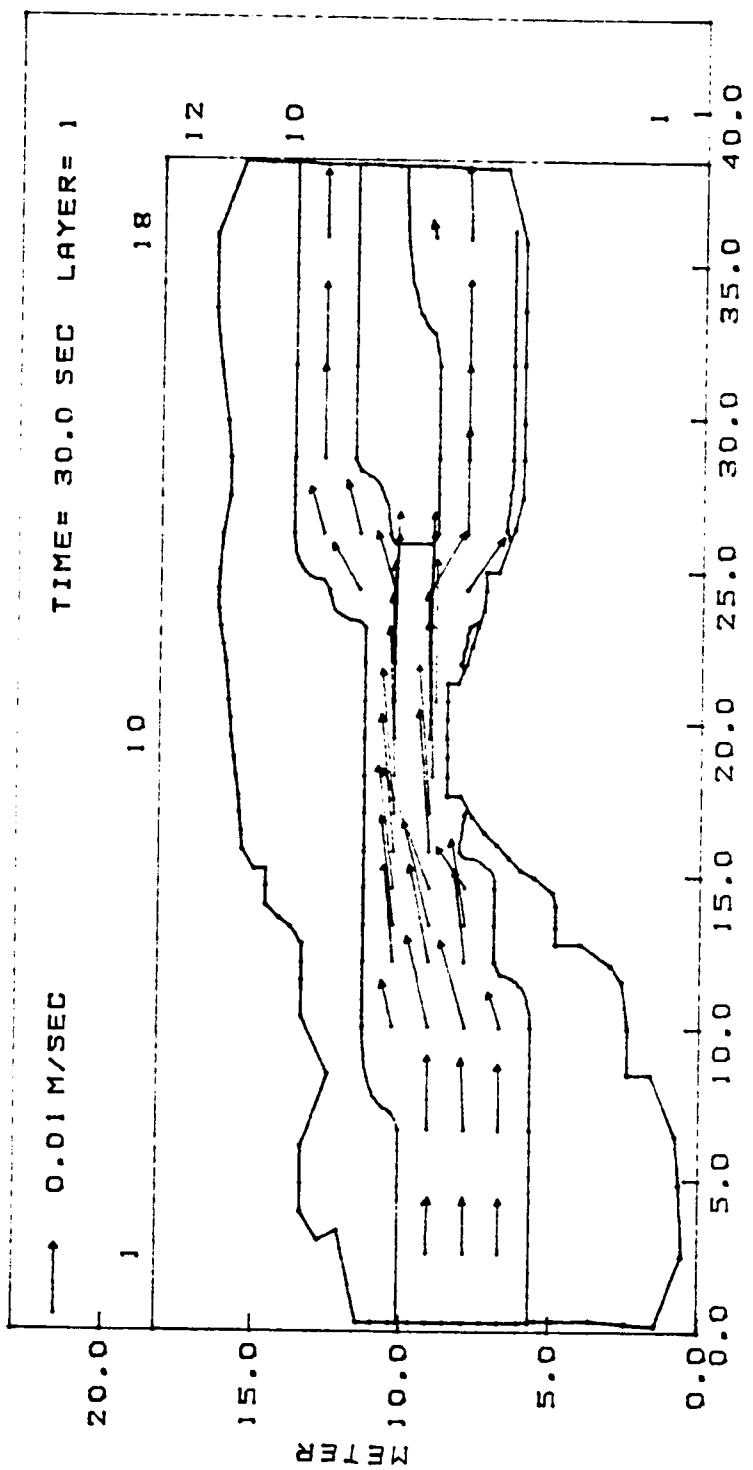

Figure 6.119. Velocity conditions in the bottom layer l = 1, at time t = 30 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

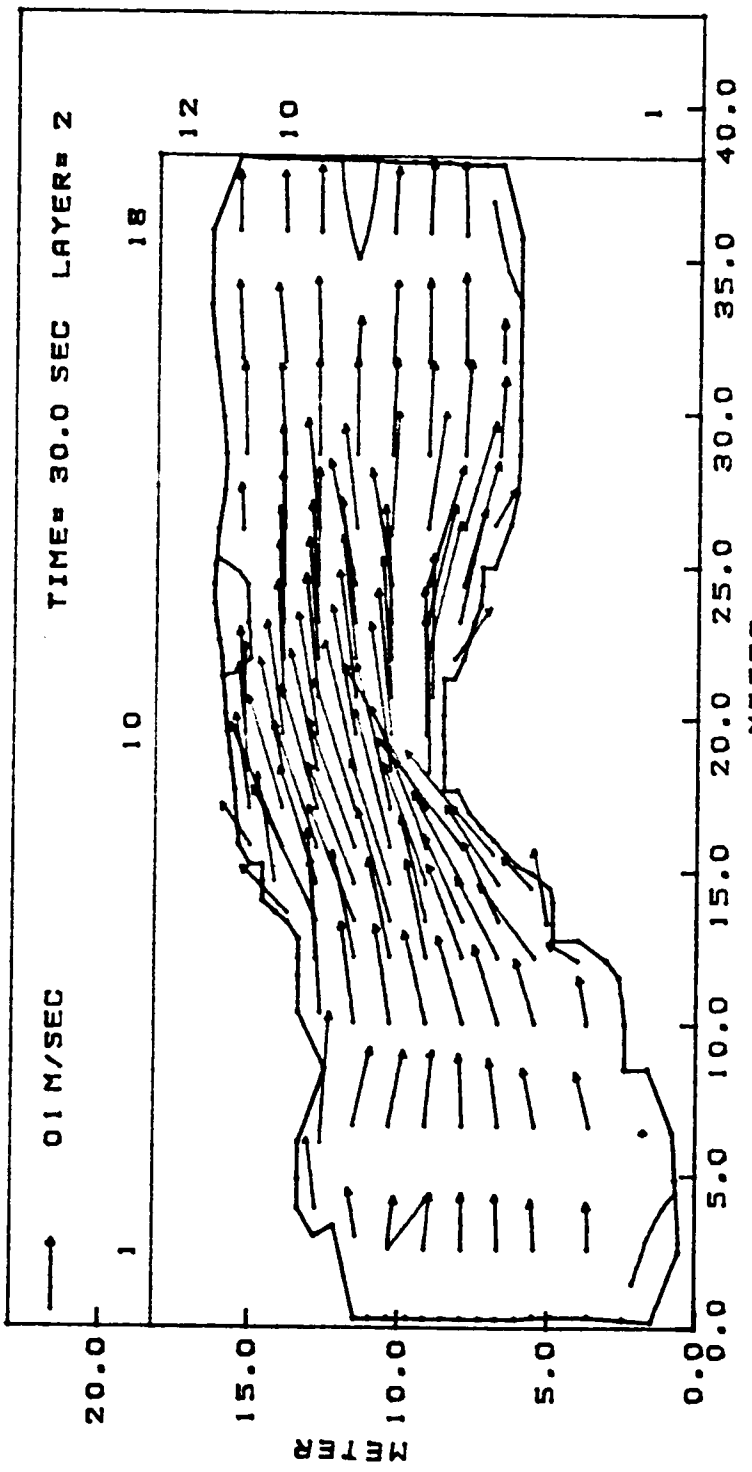

Figure 6.120. Velocity conditions in the internal layer l = 2, at time t = 30 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

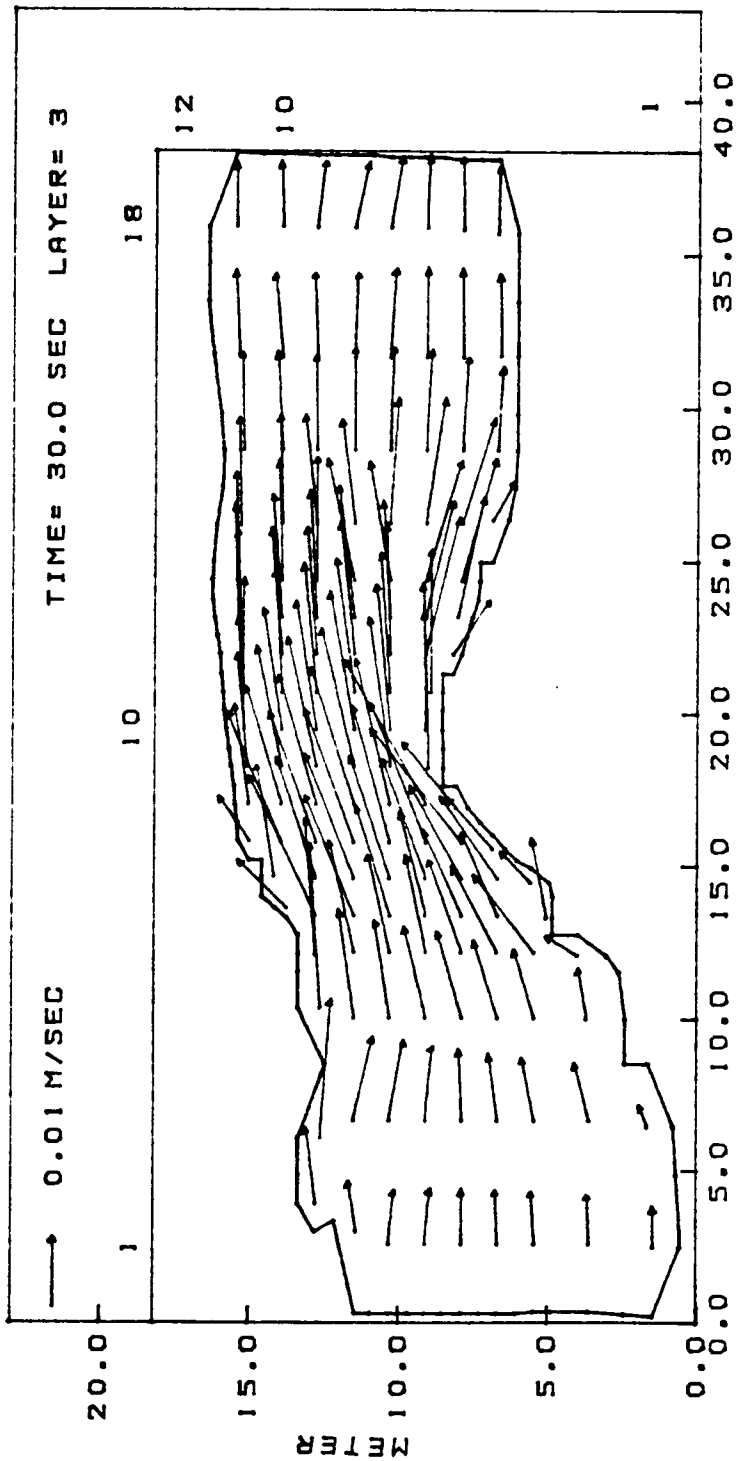

Figure 6.121. Velocity conditions in the internal layer l = 3, at time t = 30 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

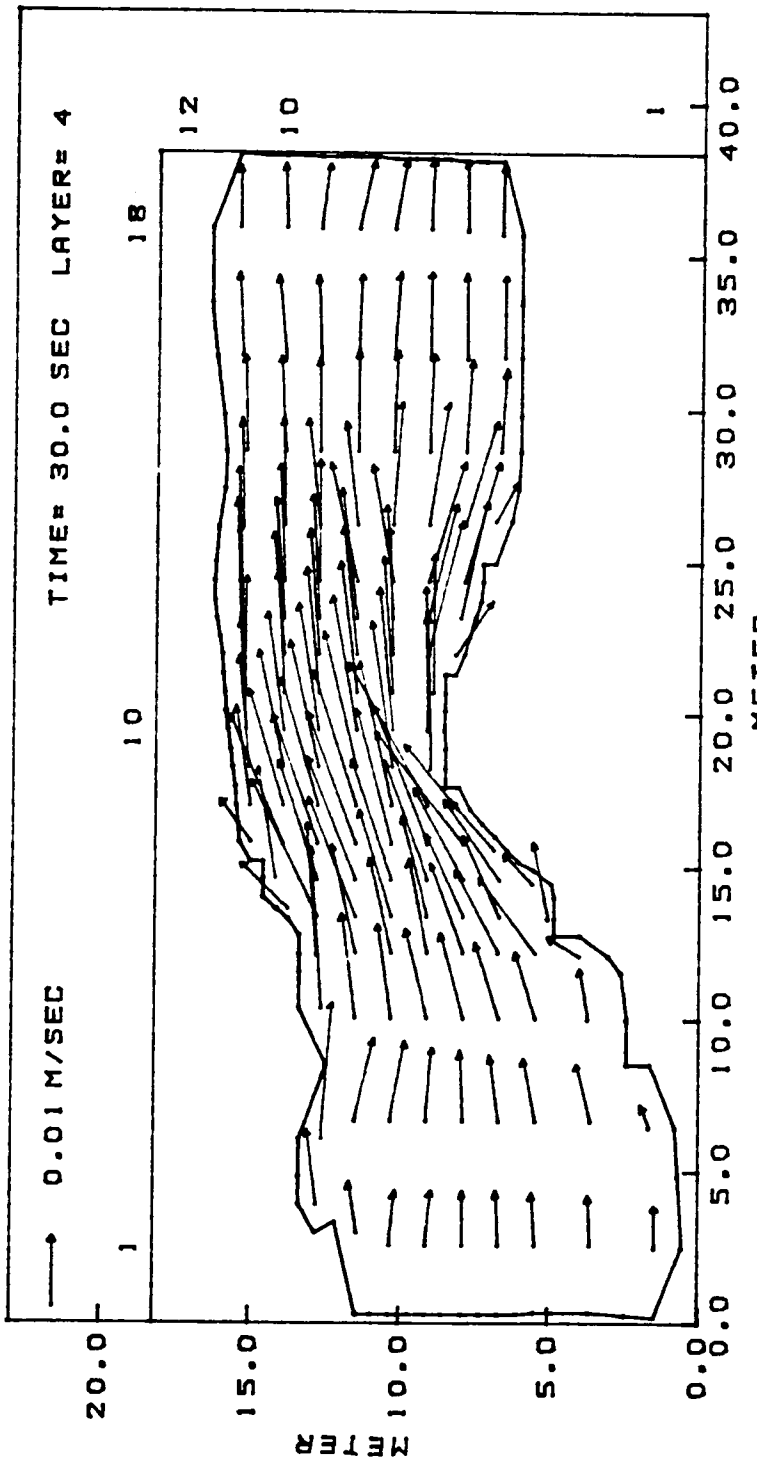

Figure 6.122. Velocity conditions in the internal layer l = 4, at time t = 30 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

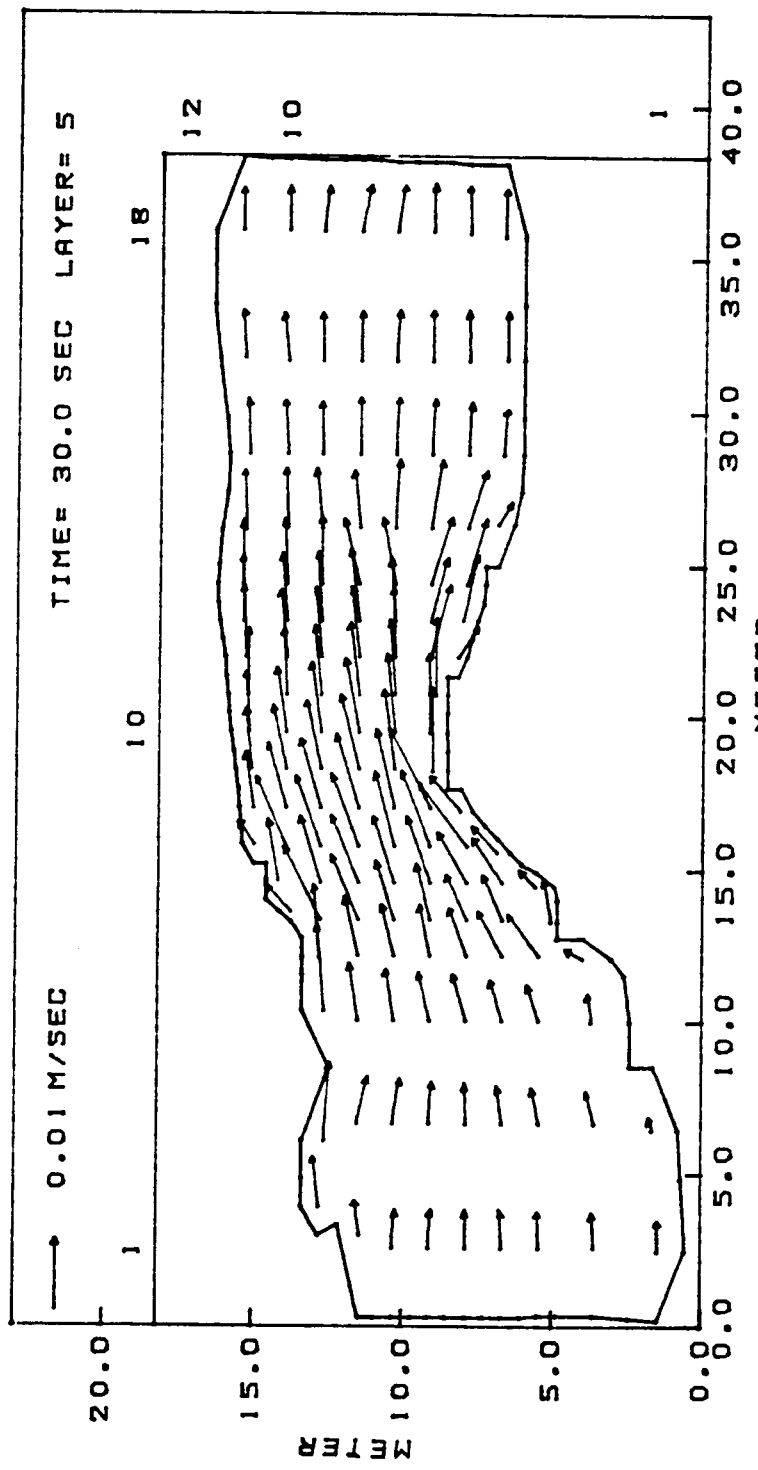

Figure 6.123. Velocity conditions in the surface layer l = 5, at time t = 30 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

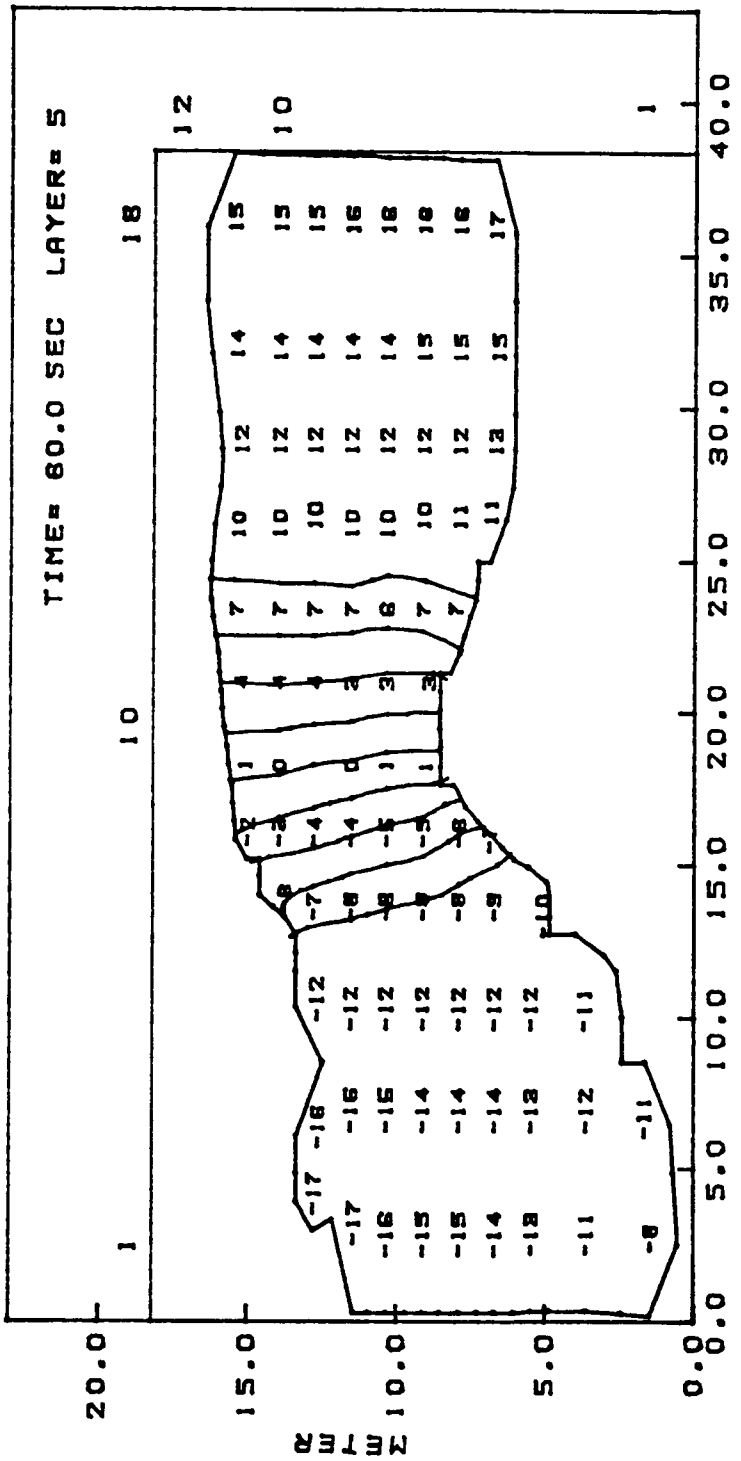

Figure 6.124. Water-surface-elevation conditions at time t = 60 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

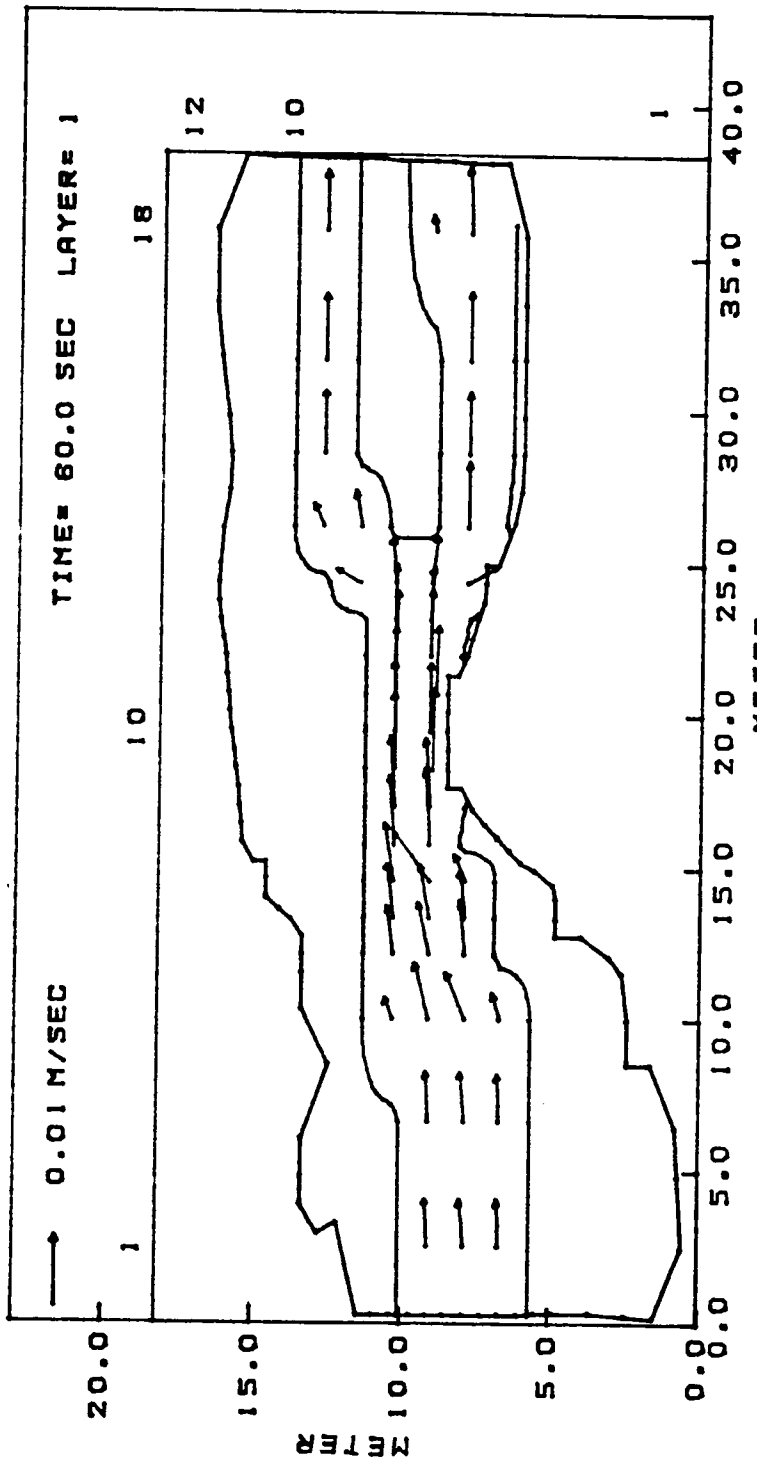

Figure 6.125. Velocity conditions in the bottom layer l = 1, at time t = 60 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

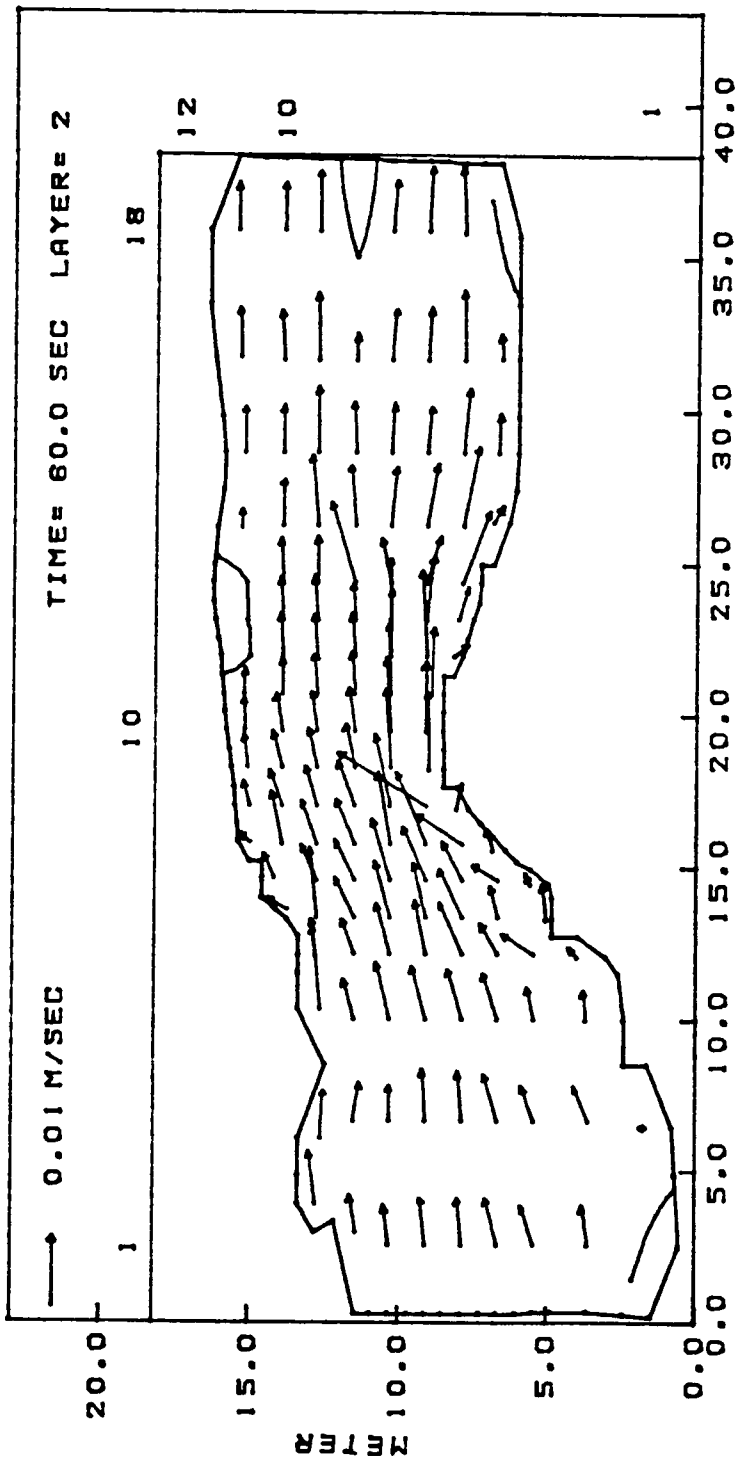

Figure 6.126. Velocity conditions in the internal layer l = 2, at time t = 60 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

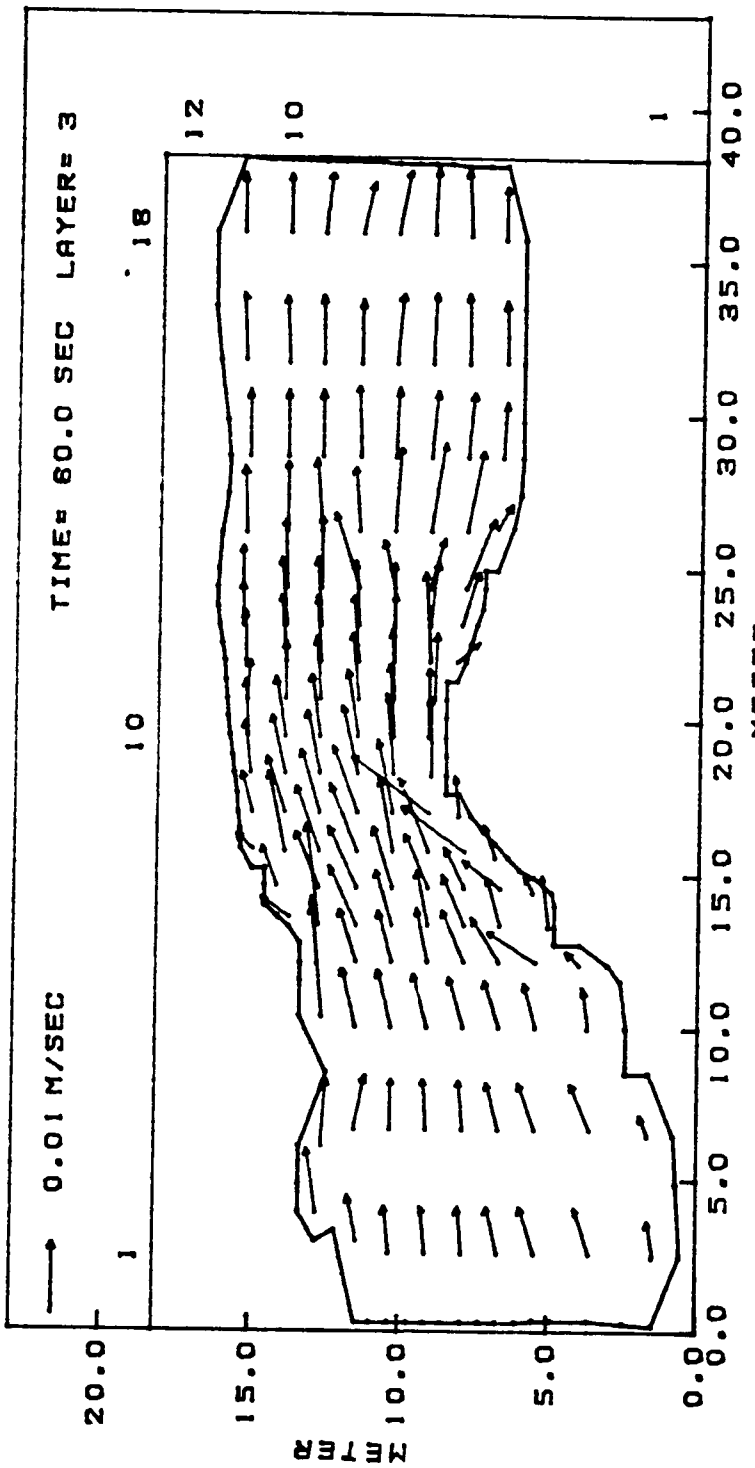

Figure 6.127. Velocity conditions in the internal layer l = 3, at time t = 60 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

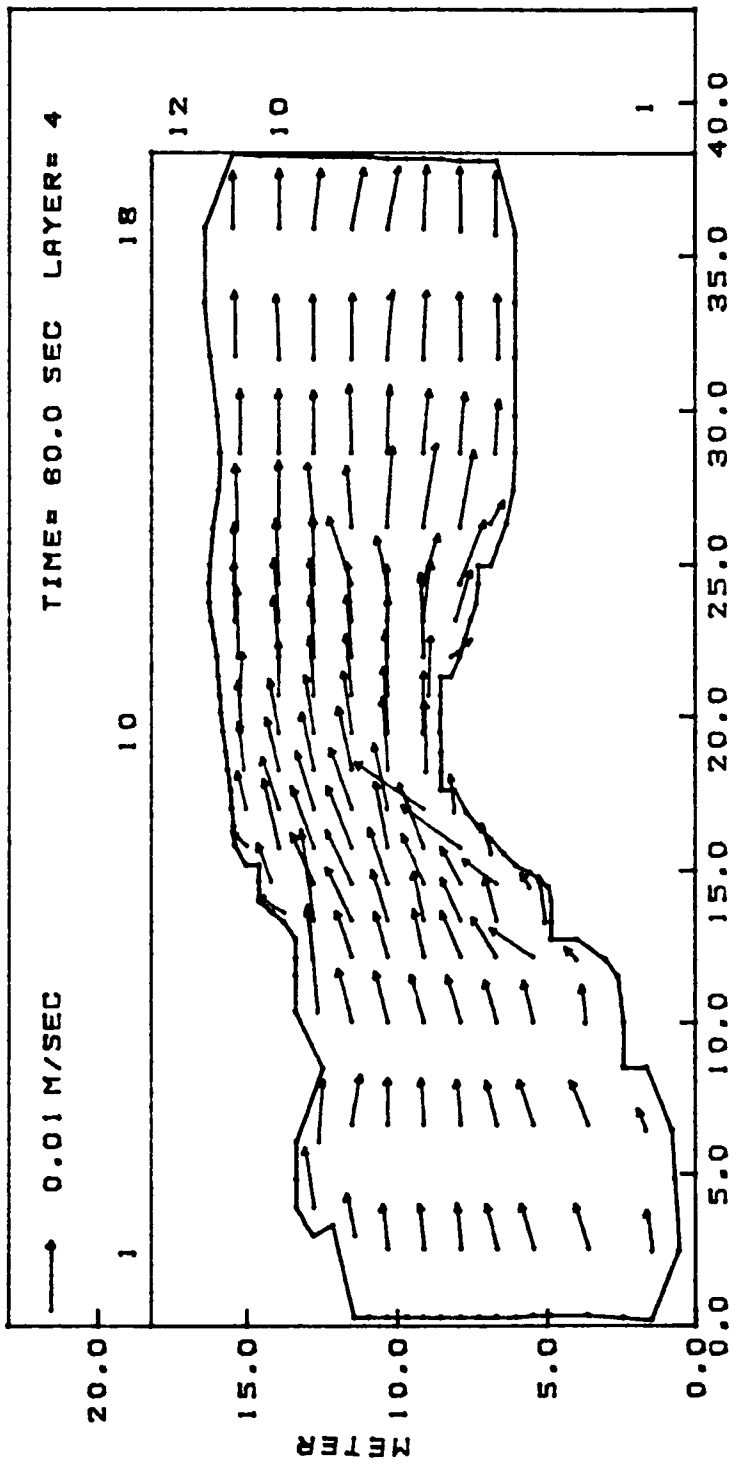

Figure 6.128. Velocity conditions in the internal layer l = 4, at time t = 60 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

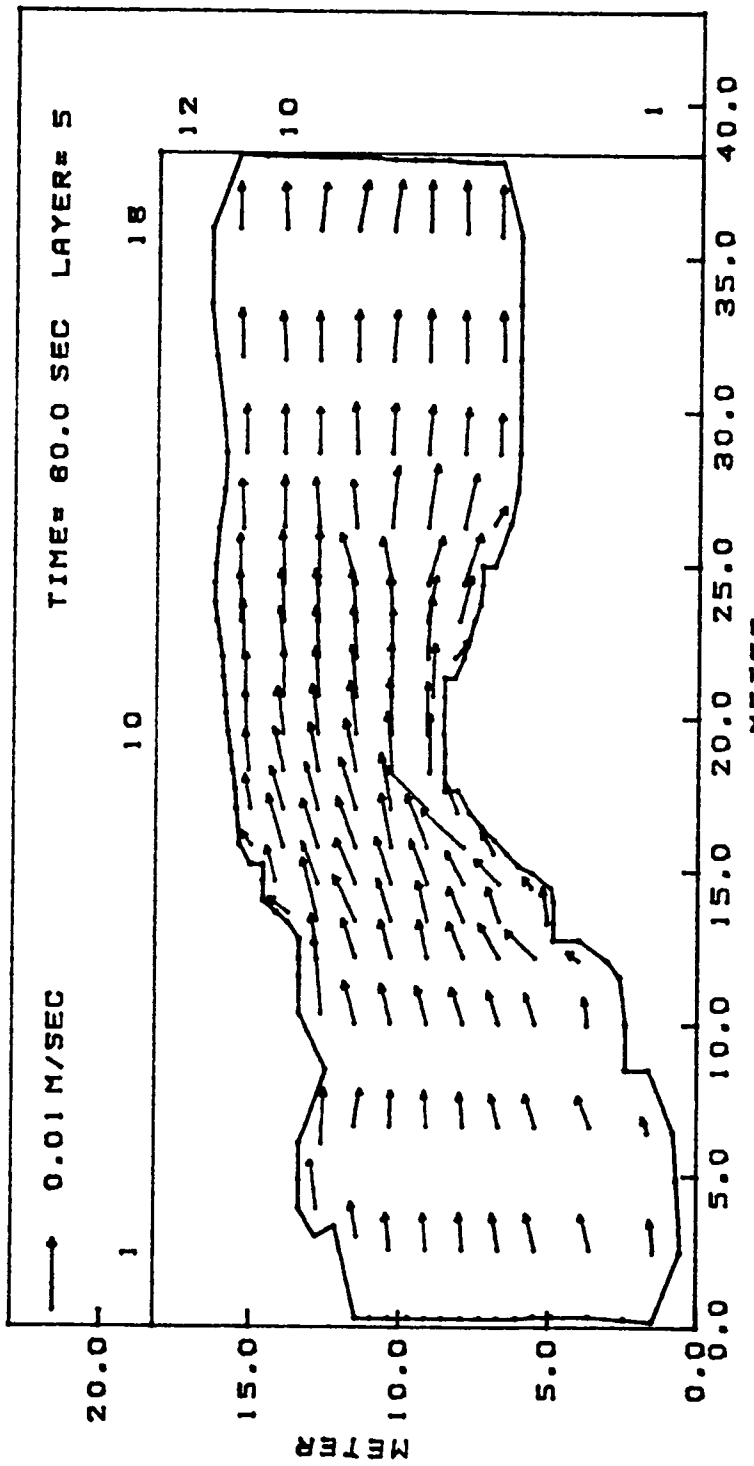

Figure 6.129. Velocity conditions in the surface layer l = 5, at time t = 60 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

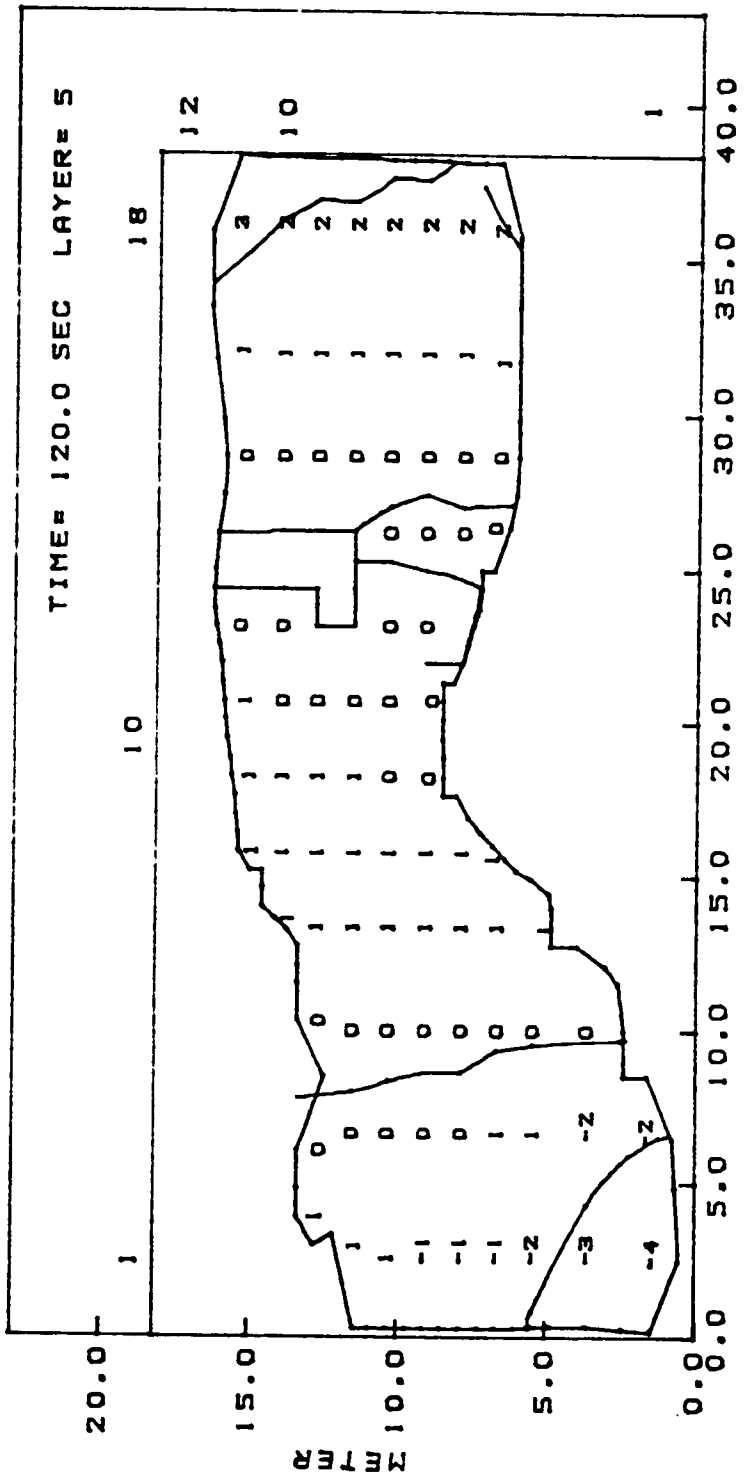

Figure 6.130. Water-surface-elevation conditions at time t = 120 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

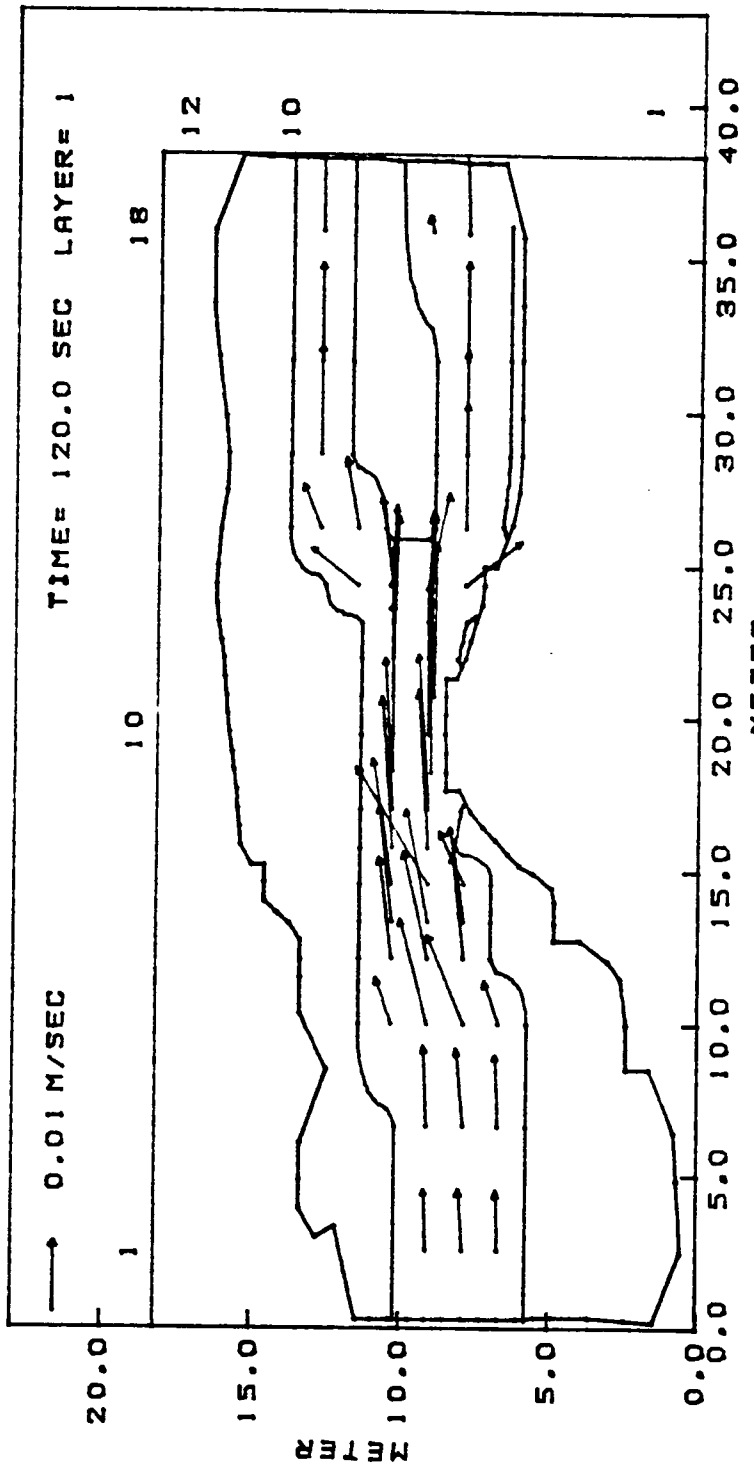

Figure 6.131. Velocity conditions in the bottom layer l = 1, at time t = 120 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

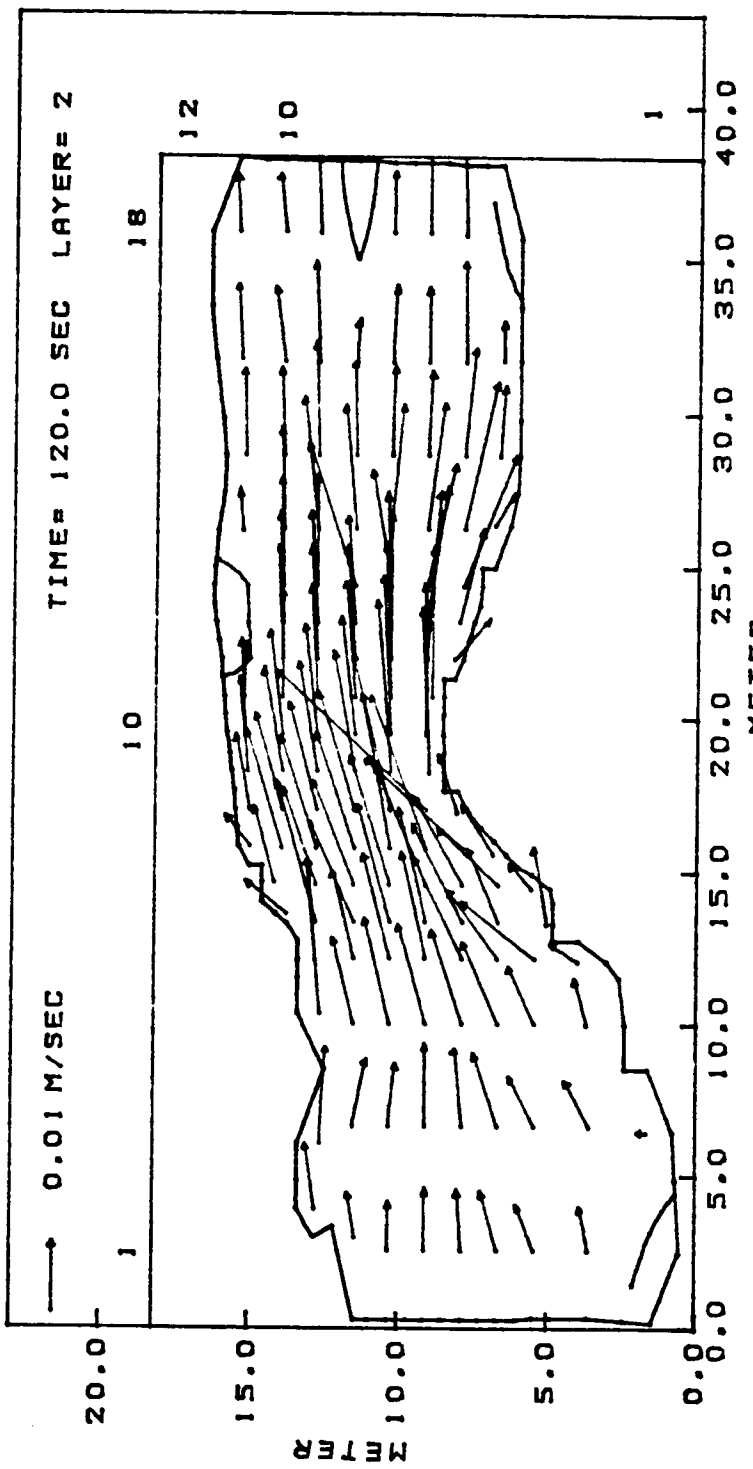

Figure 6.132. Velocity conditions in the internal layer l = 2, at time t = 120 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

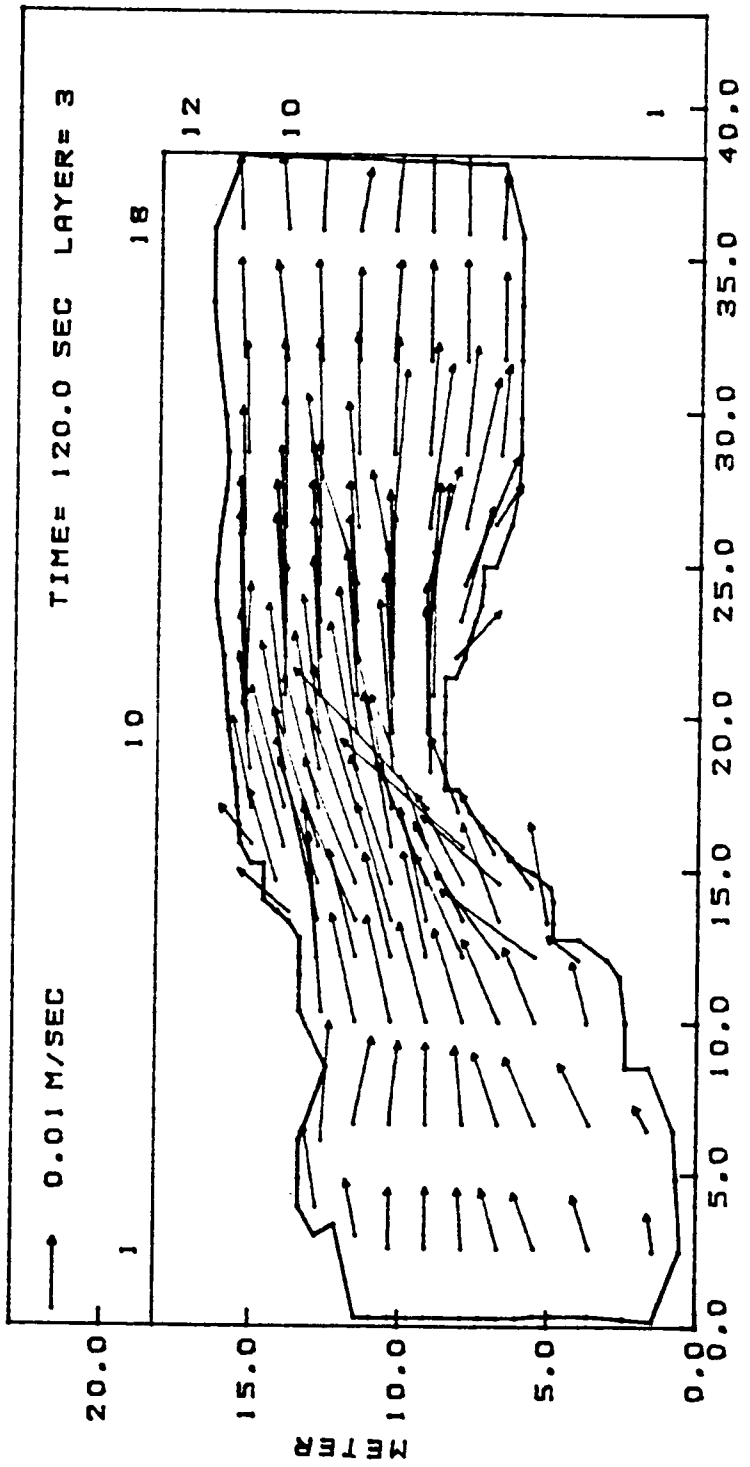

Figure 6.133. Velocity conditions in the internal layer l = 3, at time t = 120 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

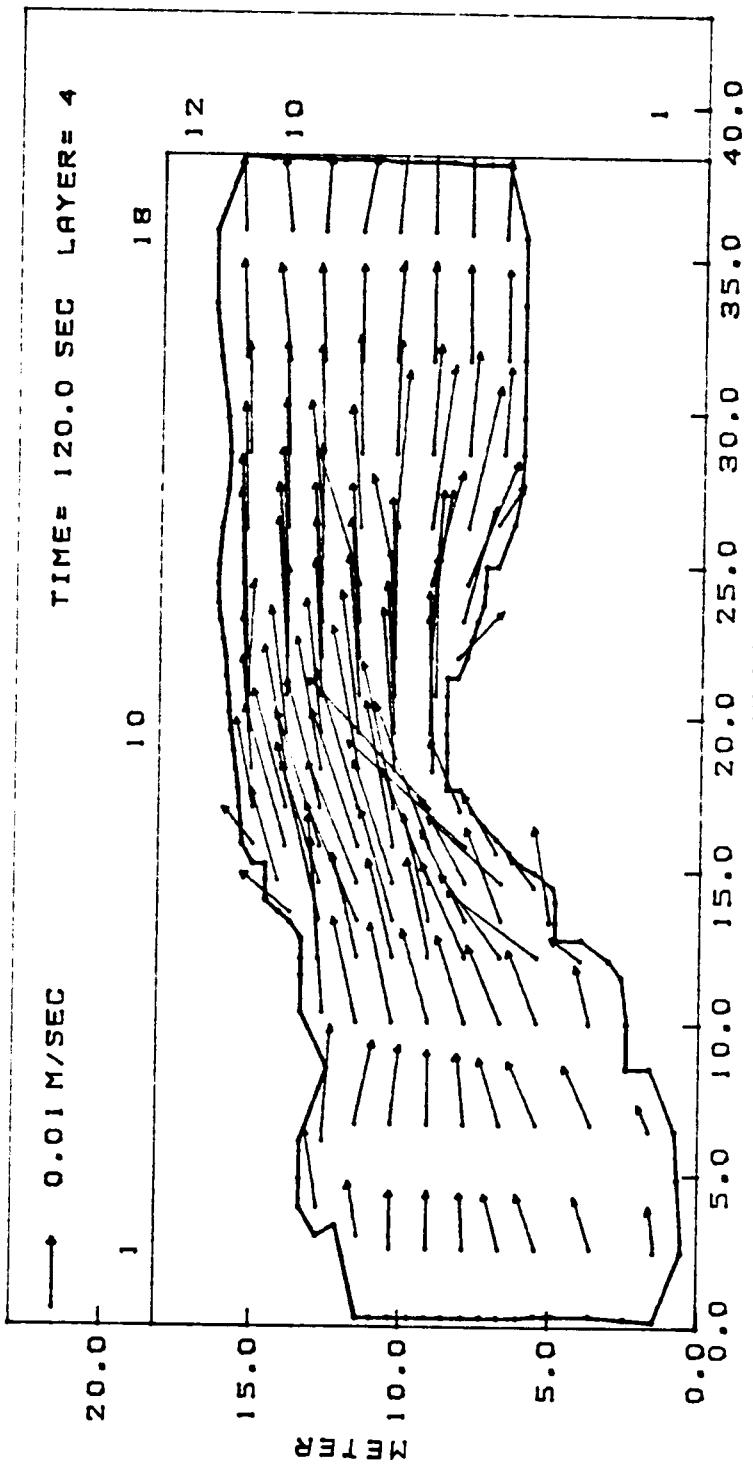

Figure 6.134. Velocity conditions in the internal layer l = 4, at time t = 120 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

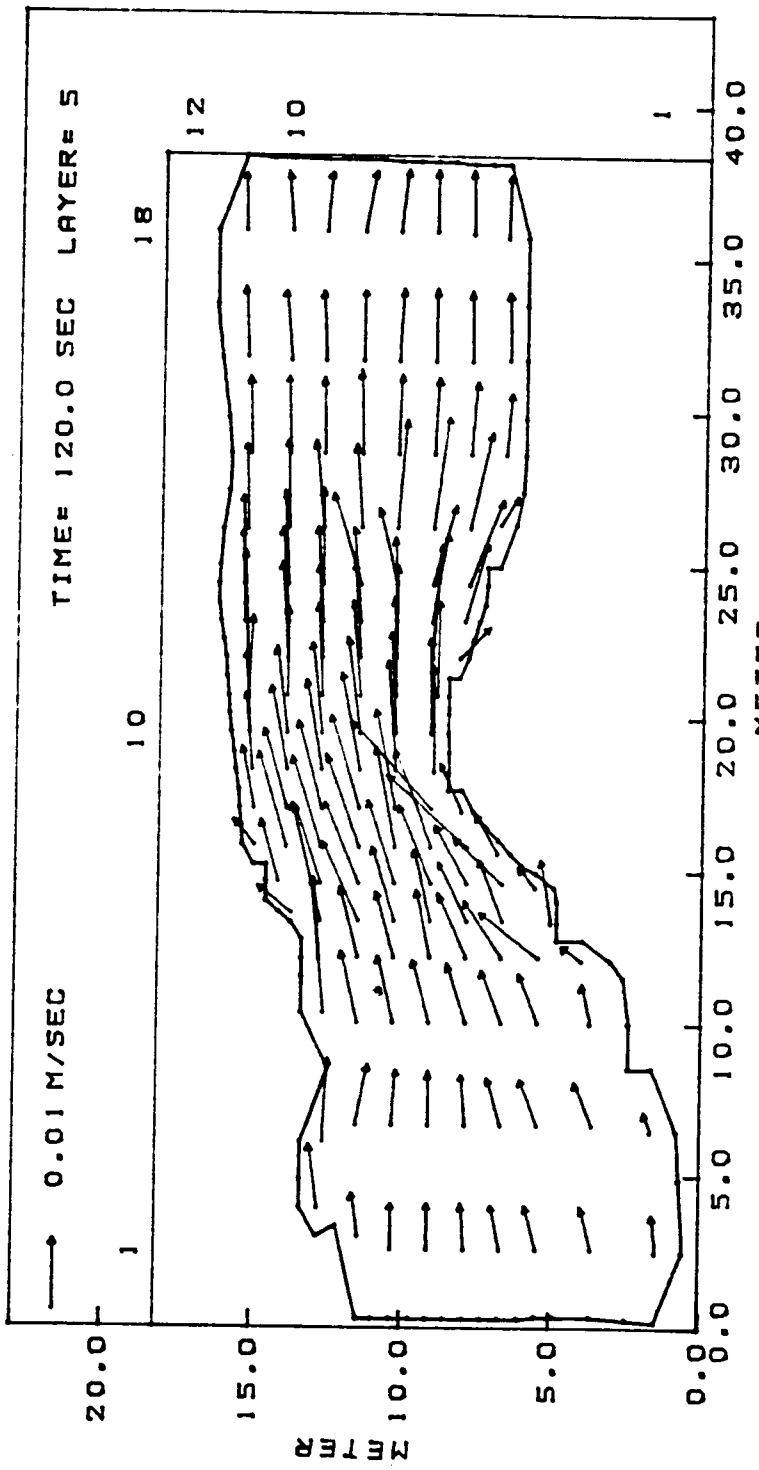

Figure 6.135. Velocity conditions in the surface layer l = 5, at time t = 120 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

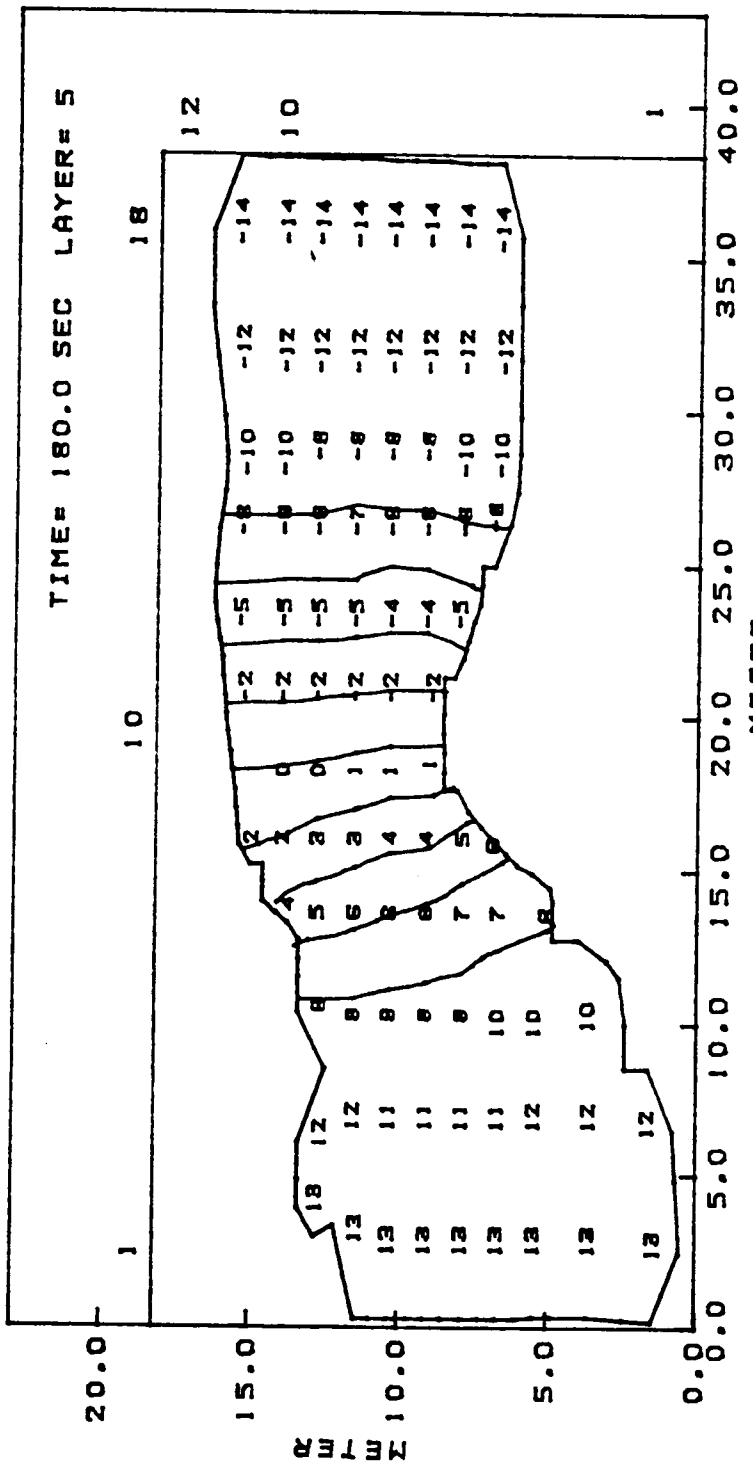

Figure 6.136. Water-surface-elevation conditions at time t = 180 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

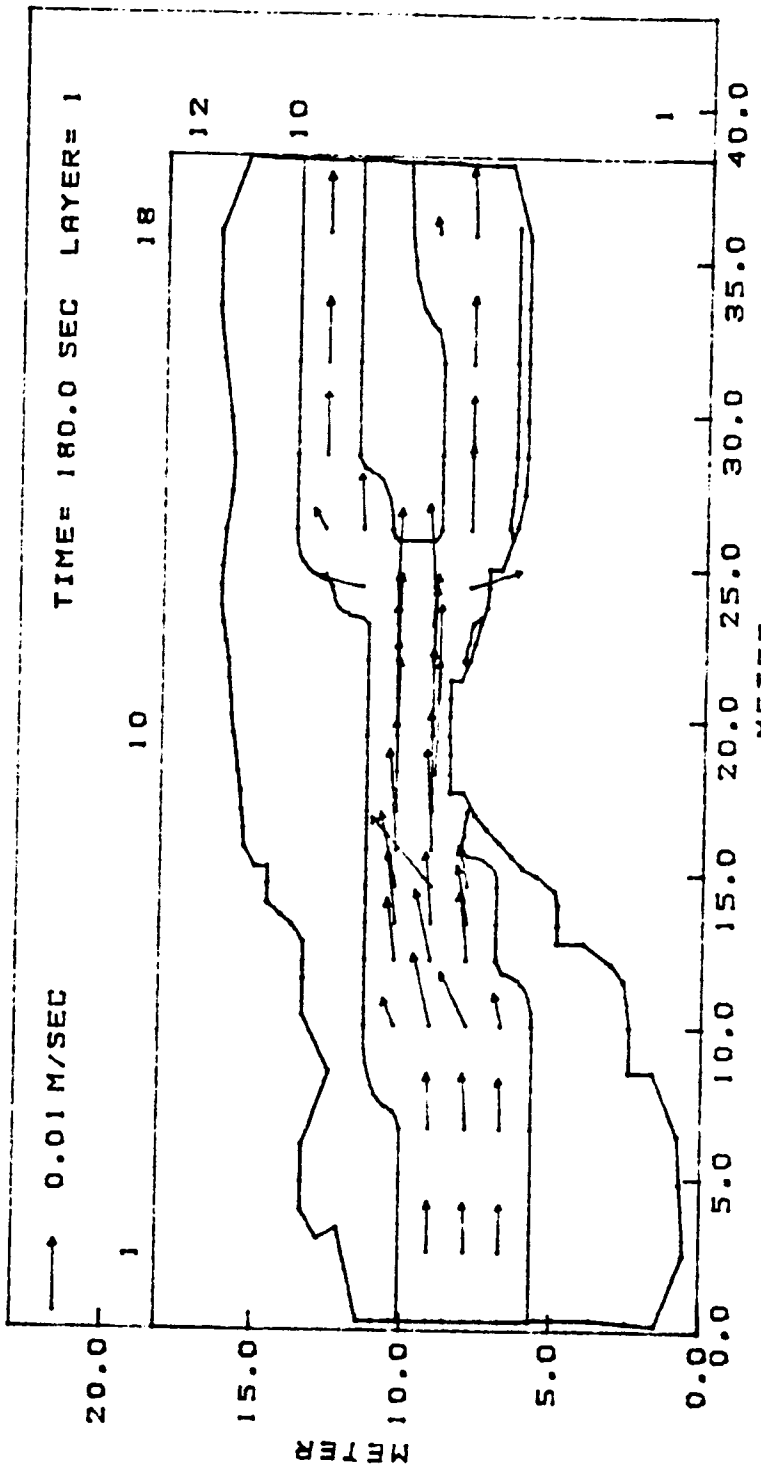

Figure 6.137. Velocity conditions in the bottom layer l = 1, at time t = 180 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

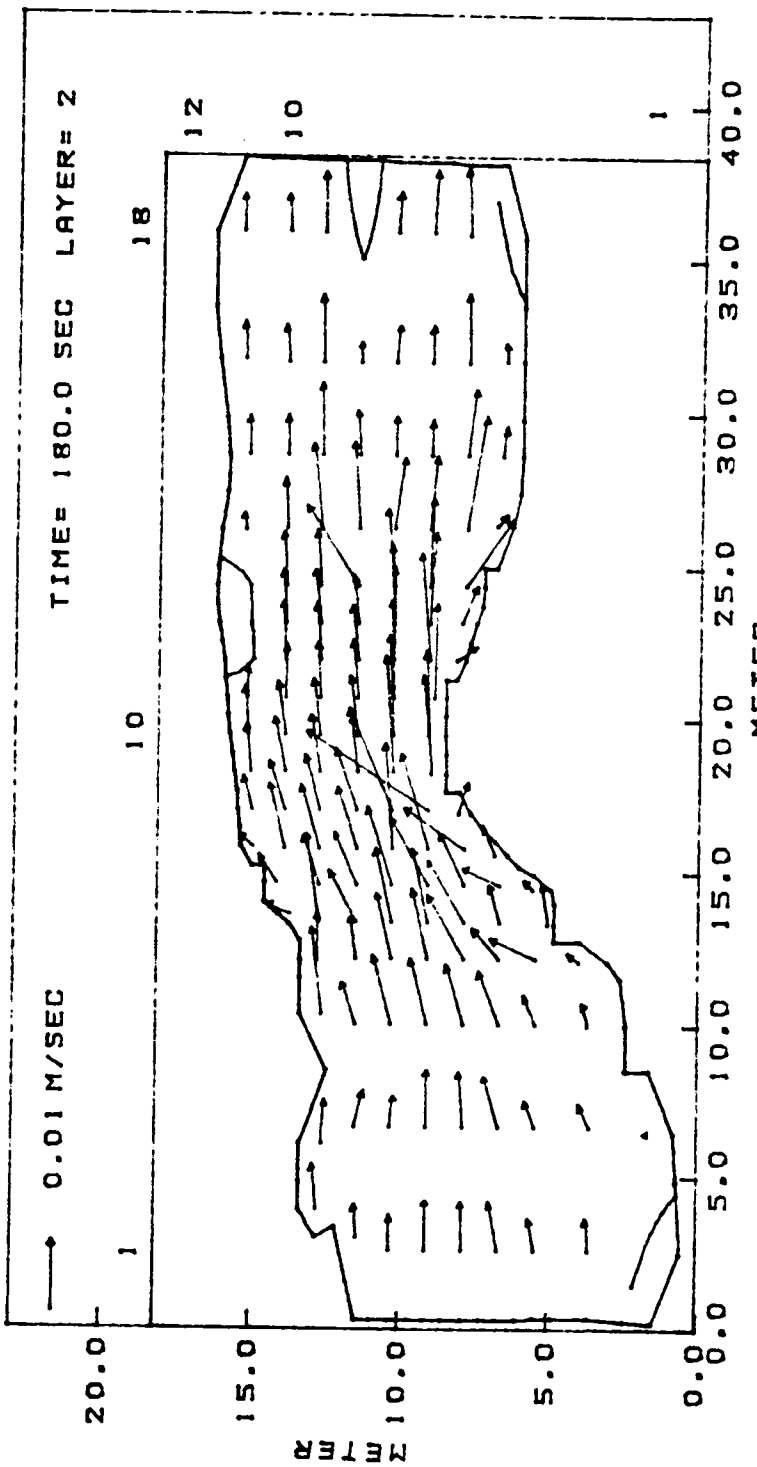

Figure 6.138. Velocity conditions in the internal layer l = 2, at time t = 180 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

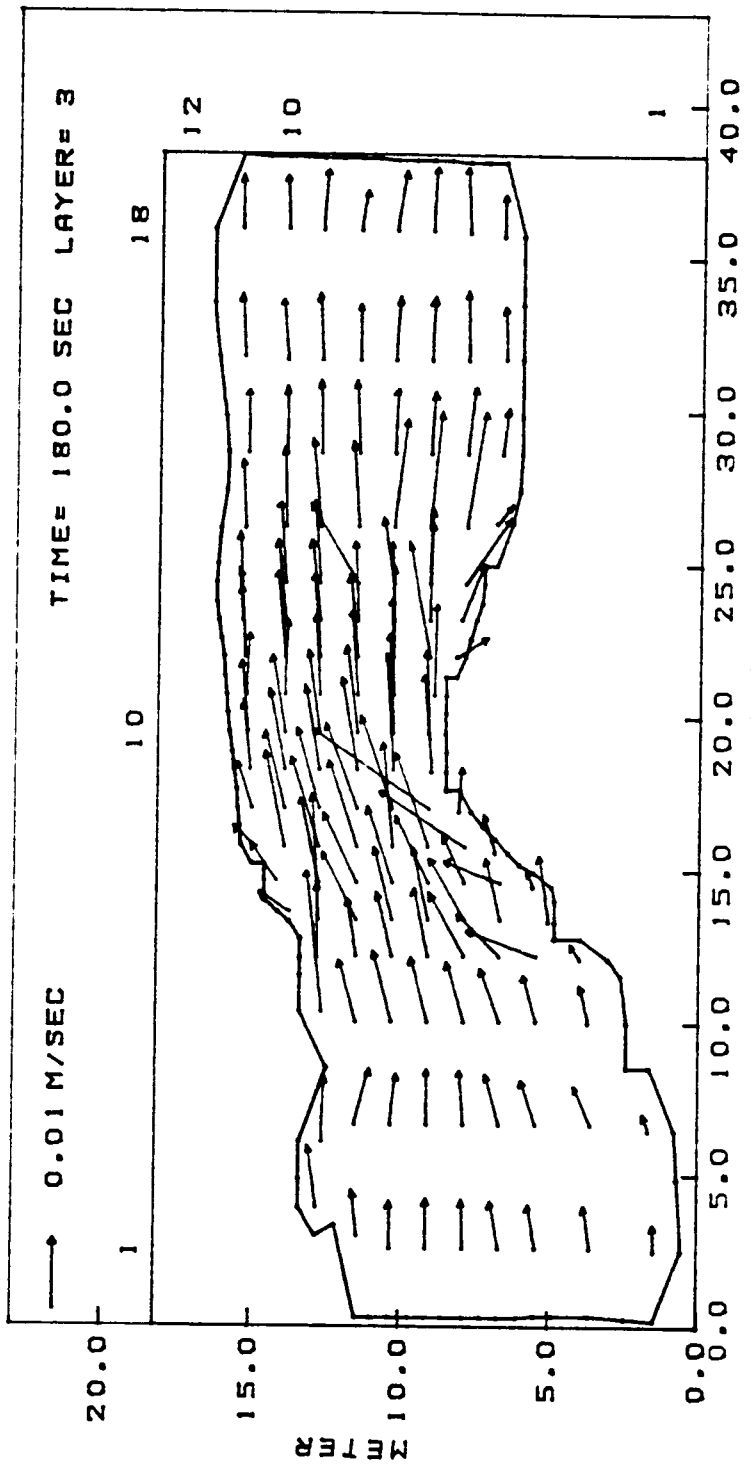

Figure 6.139. Velocity conditions in the internal layer l = 3, at time t = 180 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

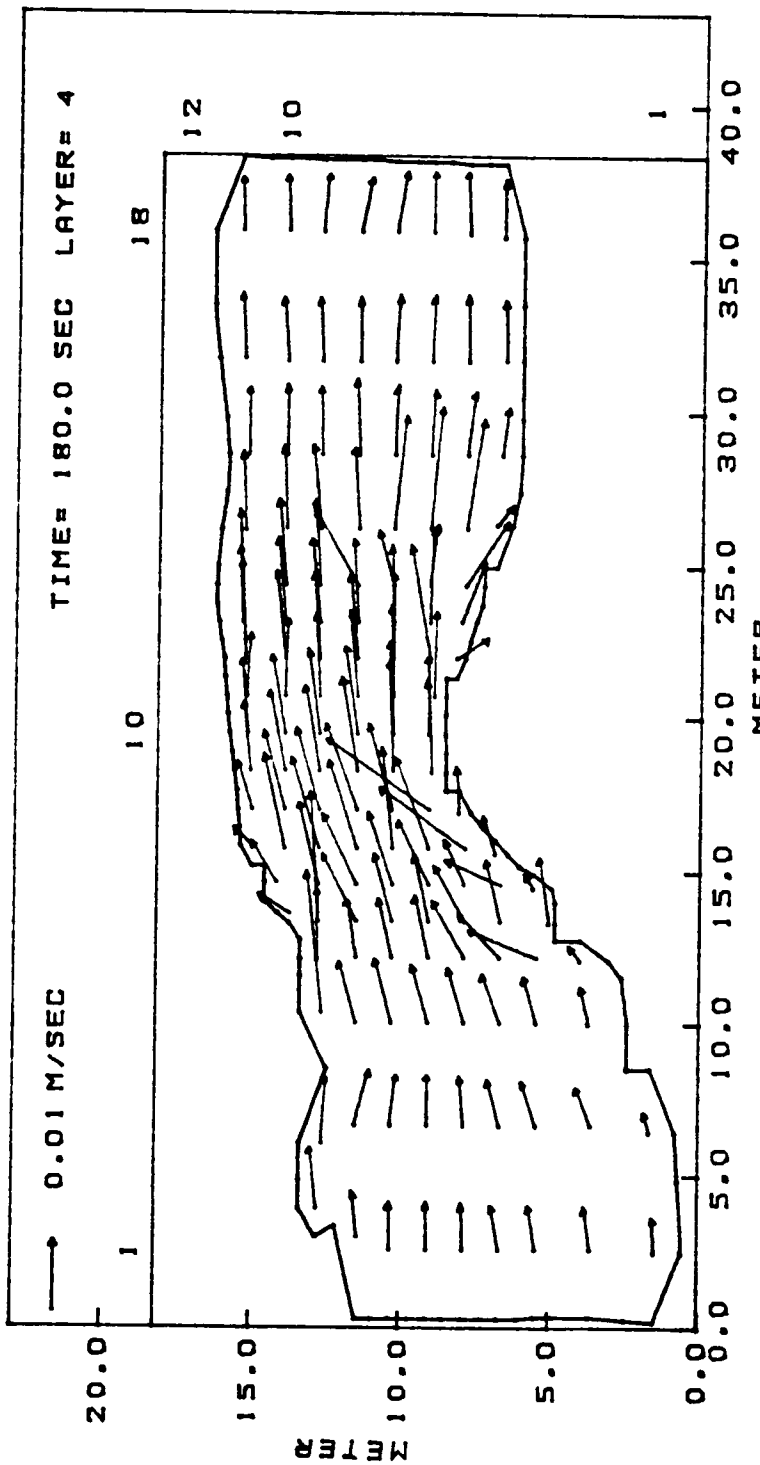

Figure 6.140. Velocity conditions in the internal layer l = 4, at time t = 180 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

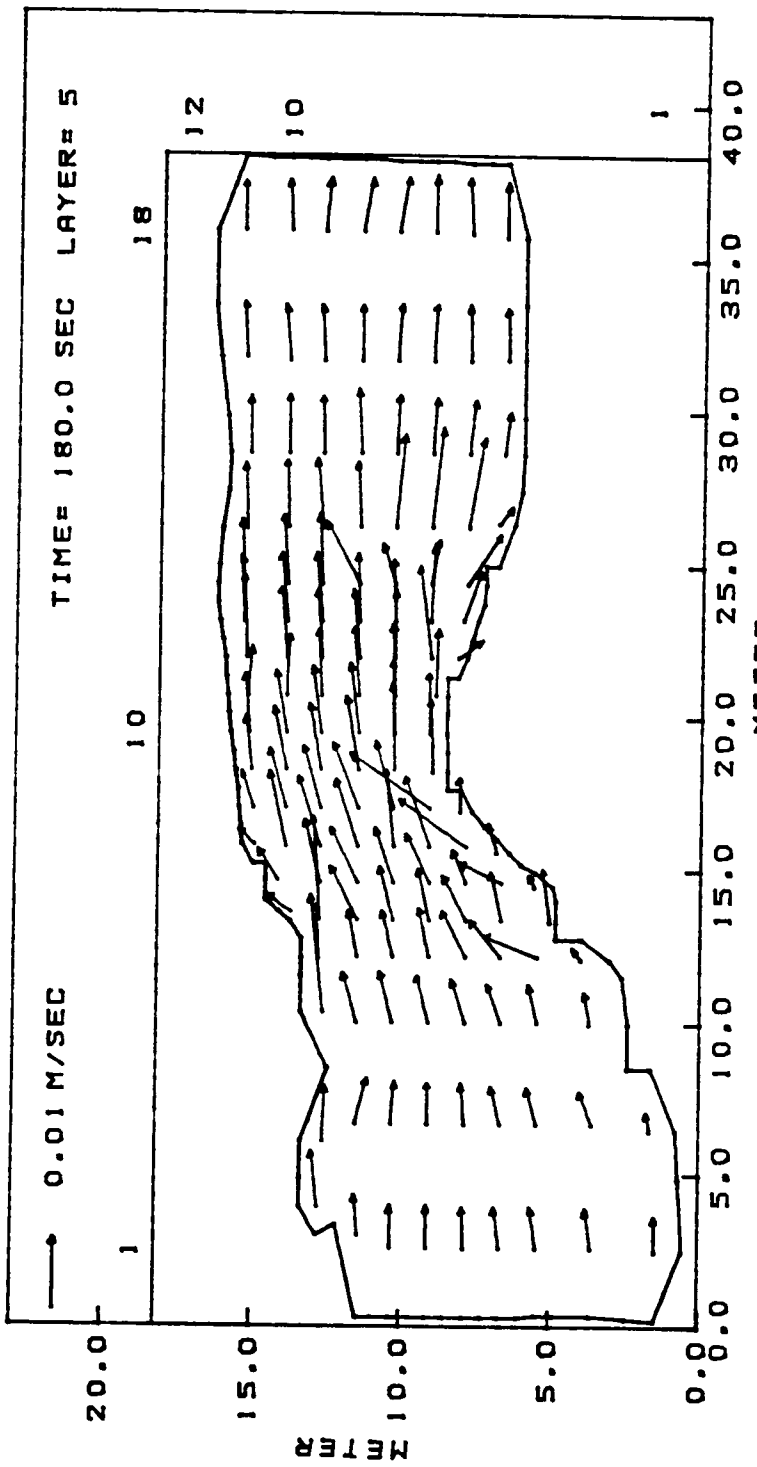

Figure 6.141. Velocity conditions in the surface layer l = 5, at time t = 180 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

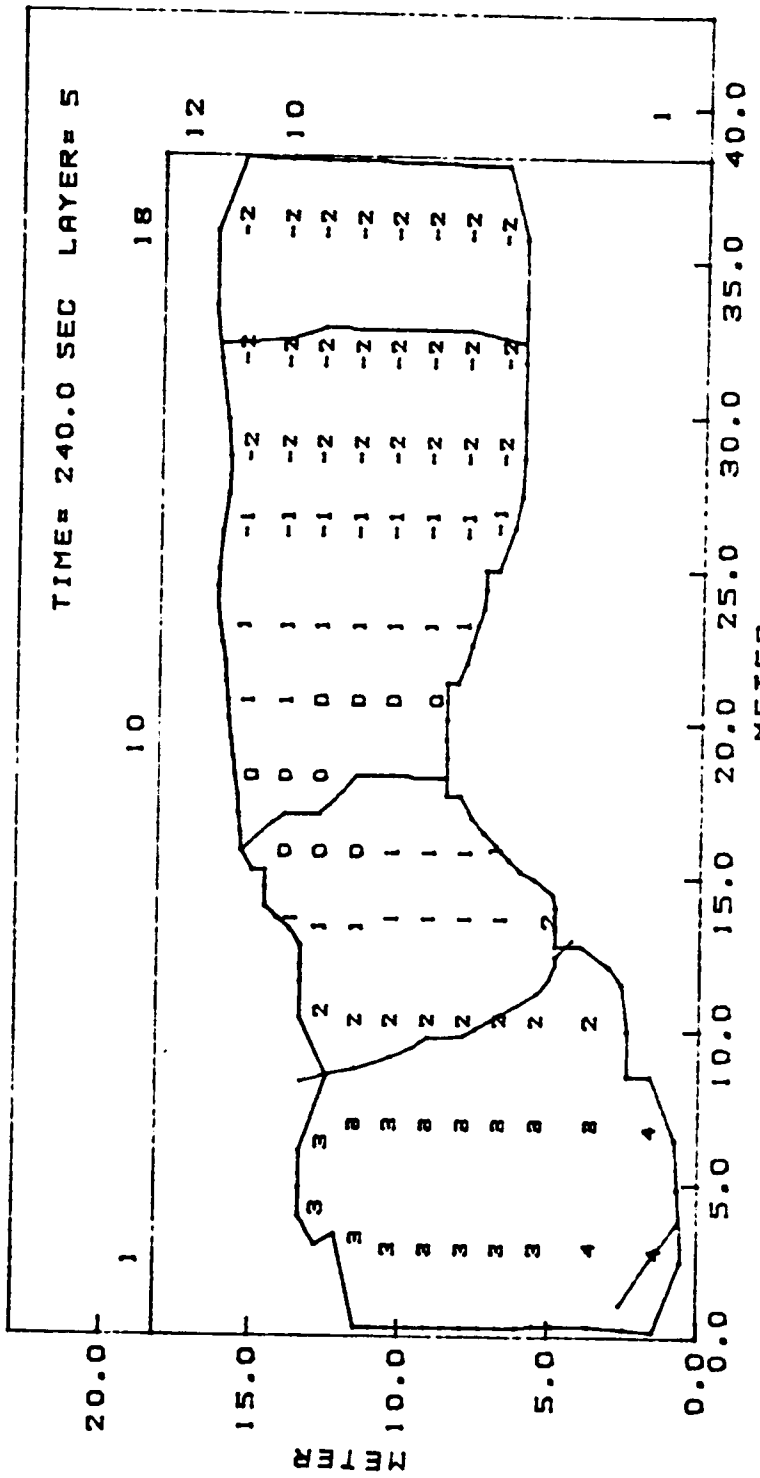

Figure 6.142. Water-surface-elevation conditions at time t = 240 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

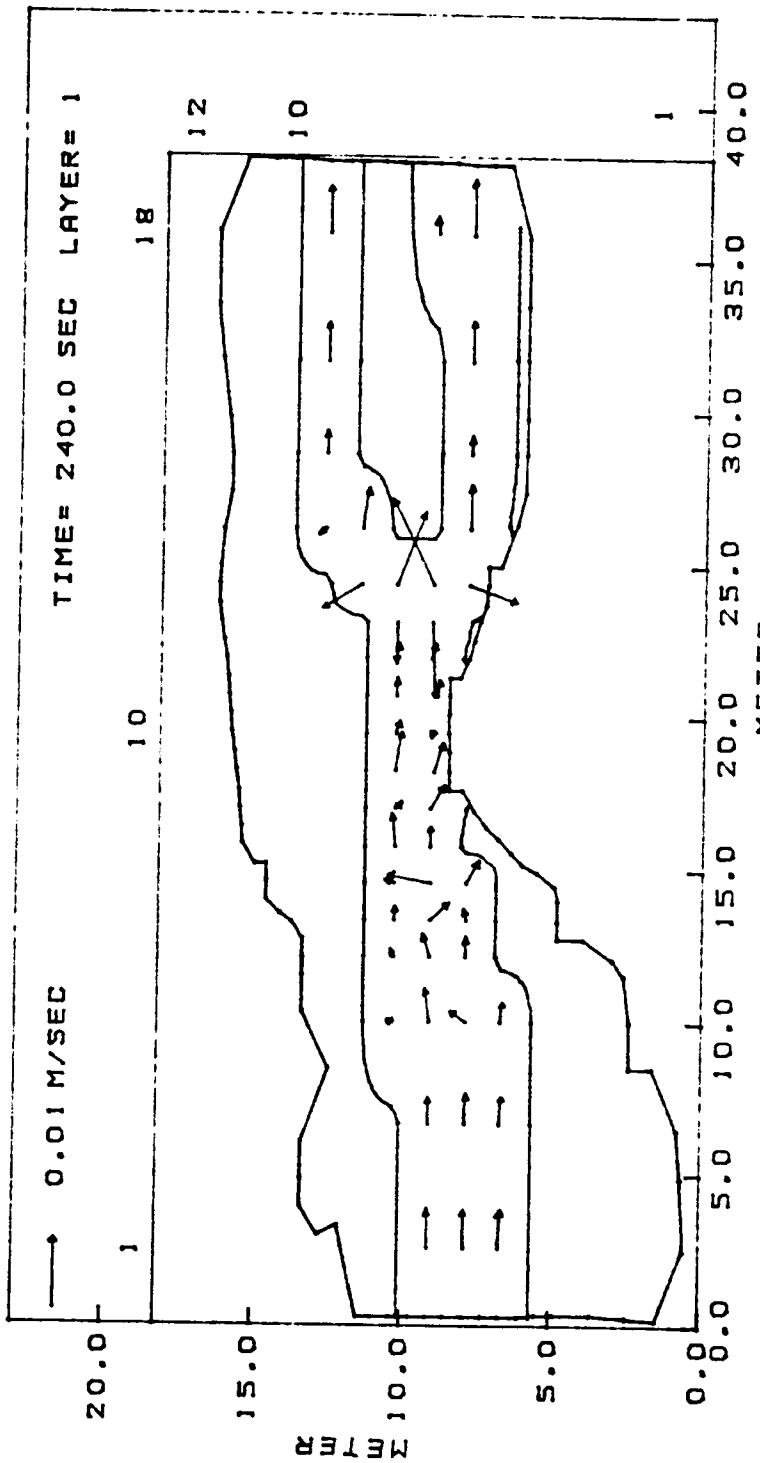

Figure 6.143. Velocity conditions in the bottom layer l = 1, at time t = 240 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

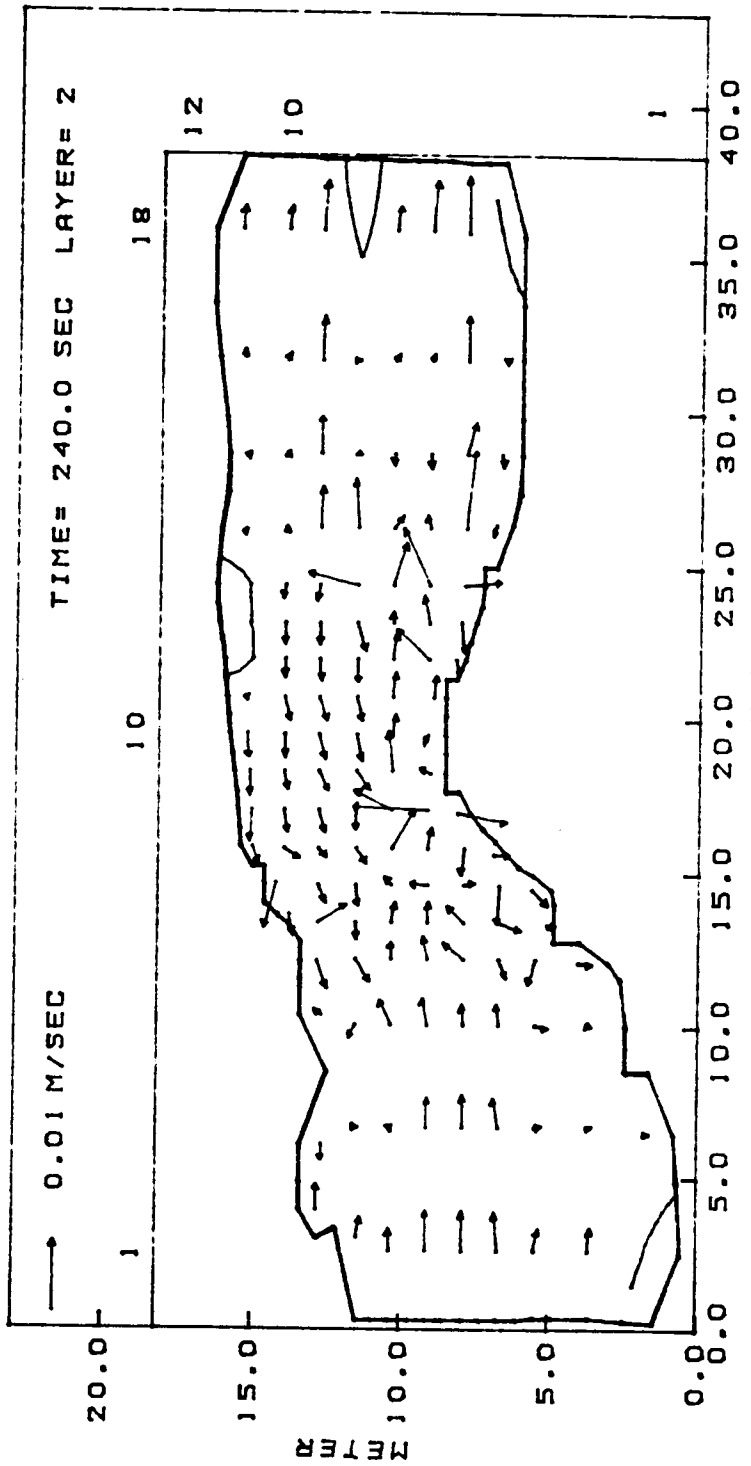

Figure 6.144. Velocity conditions in the internal layer l = 2, at time t = 240 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

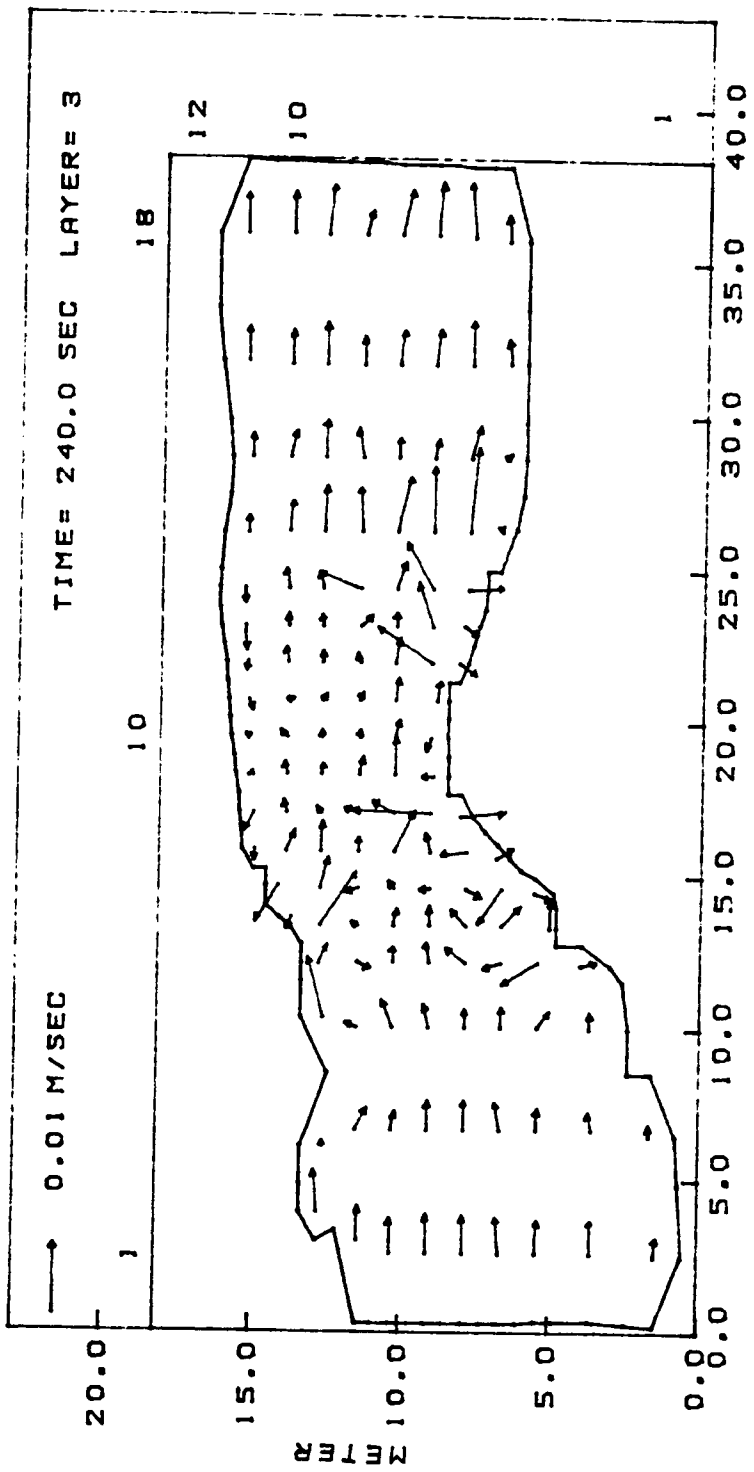

Figure 6.145. Velocity conditions in the internal layer l = 3, at time t = 240 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

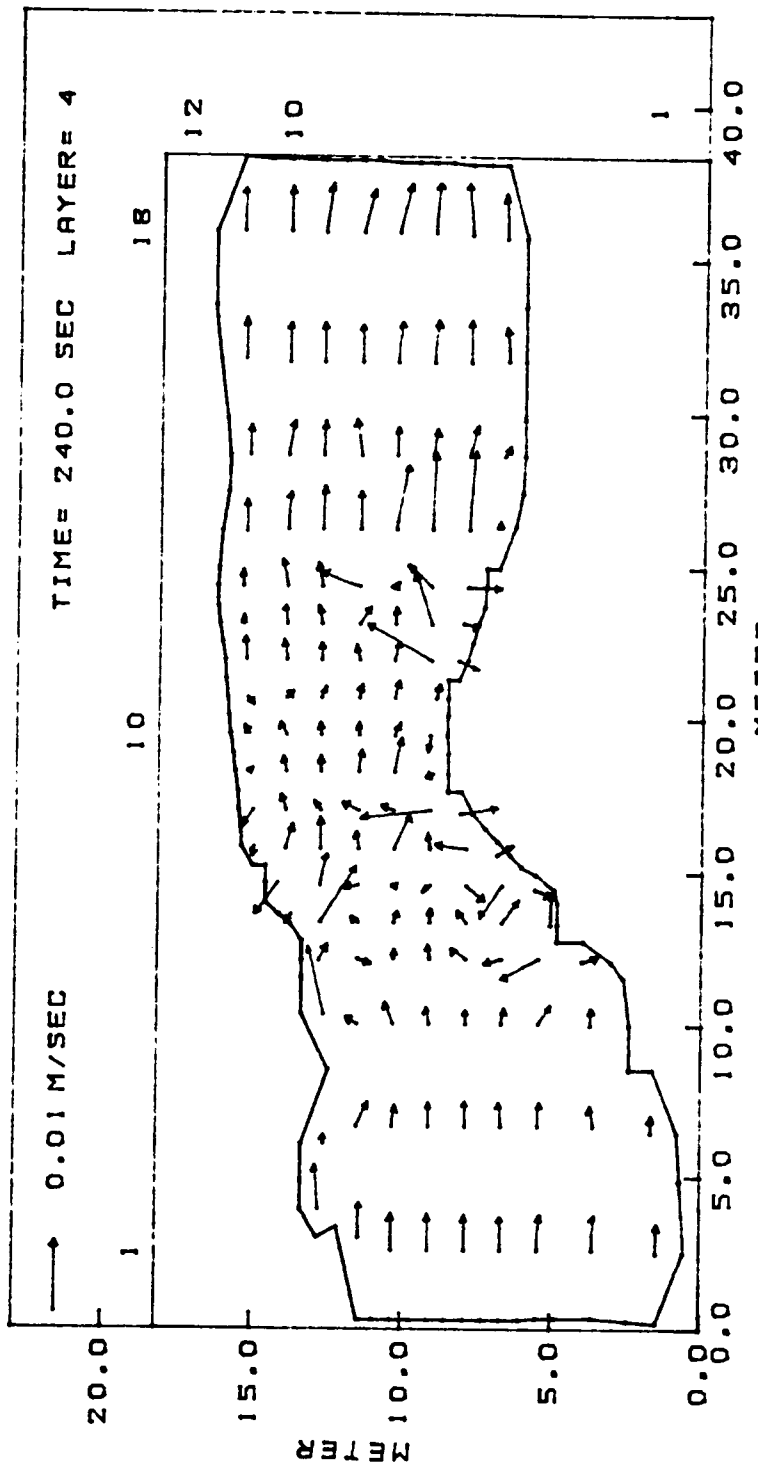

Figure 6.146. Velocity conditions in the internal layer l = 4, at time t = 240 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

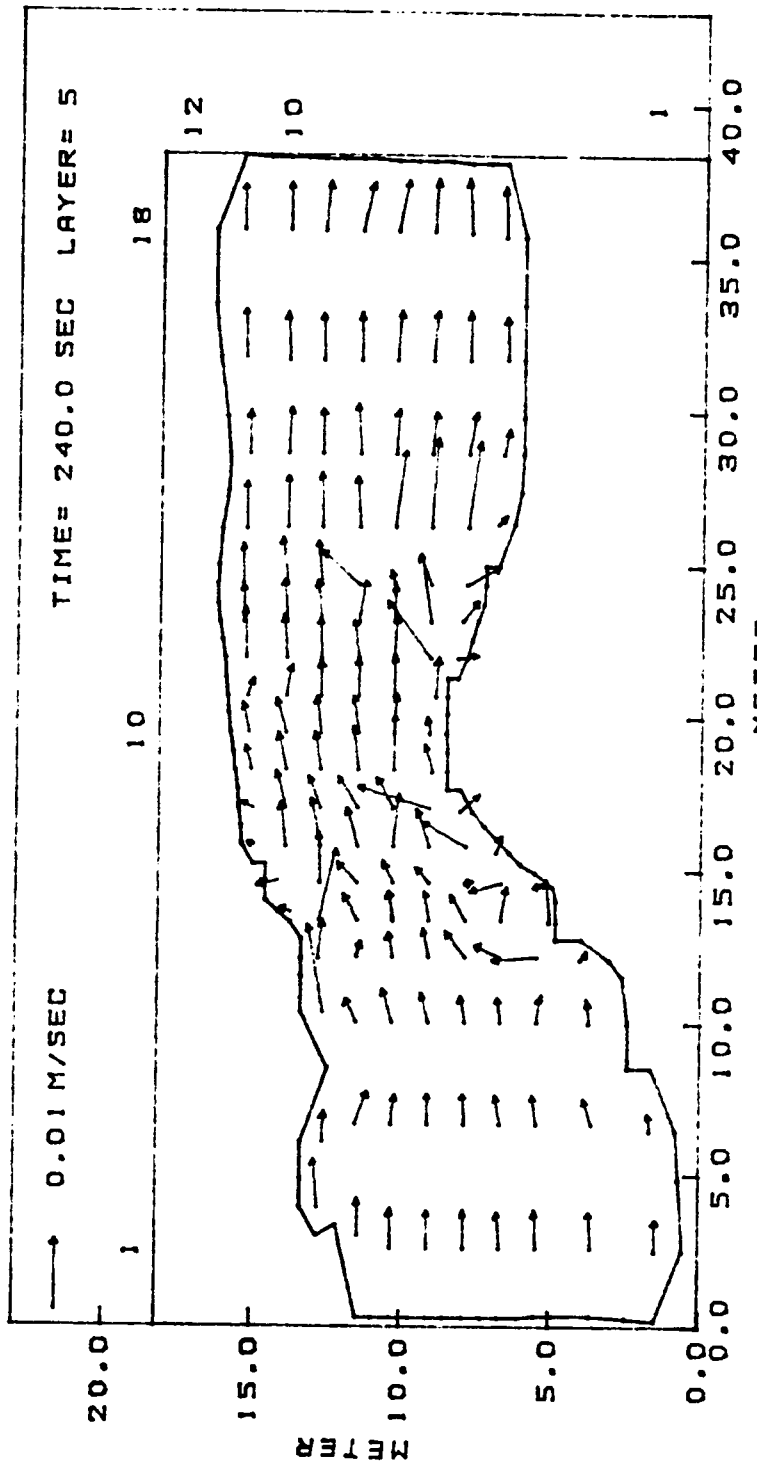

Figure 6.147. Velocity conditions in the surface layer l = 5, at time t = 240 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

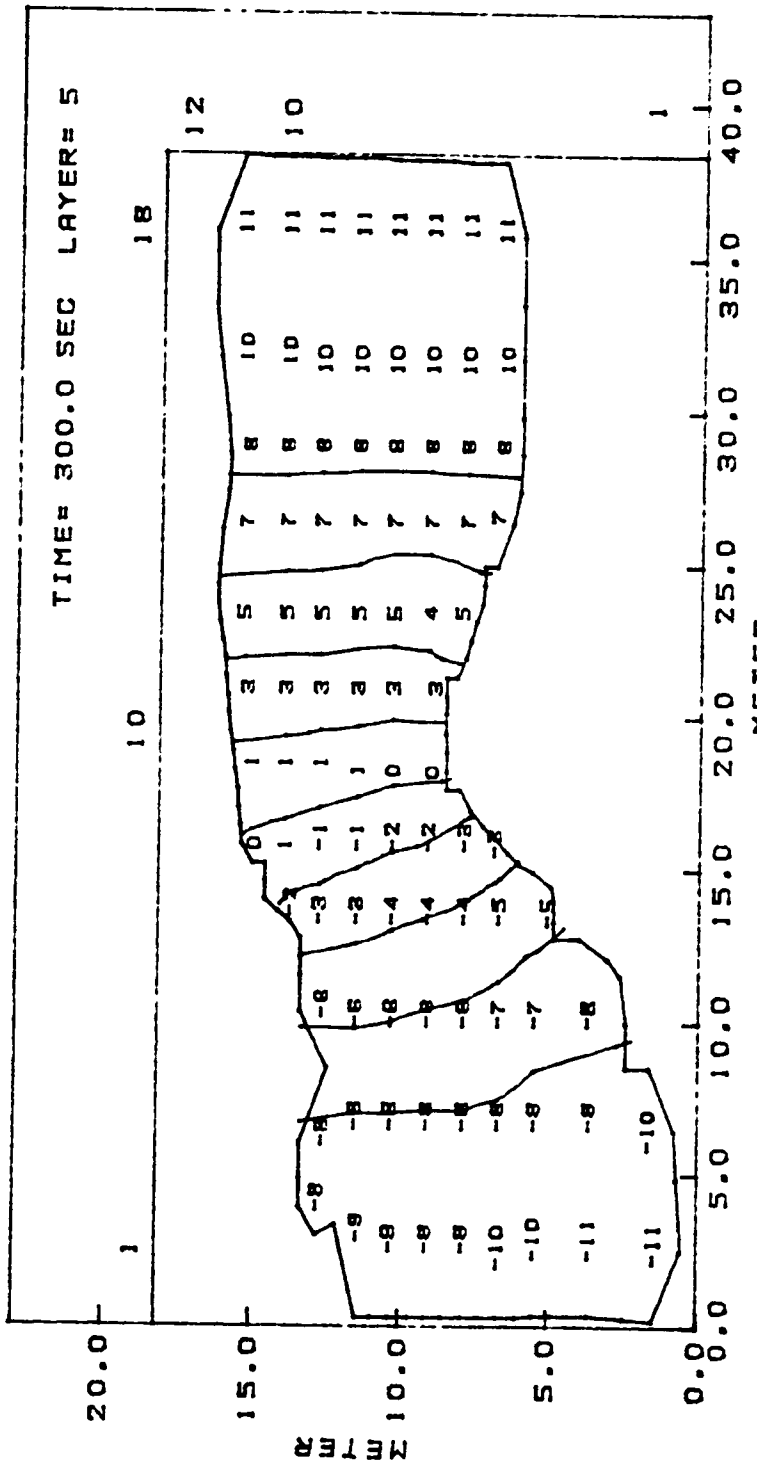

Figure 6.148. Water-surface-elevation conditions at time t = 300 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

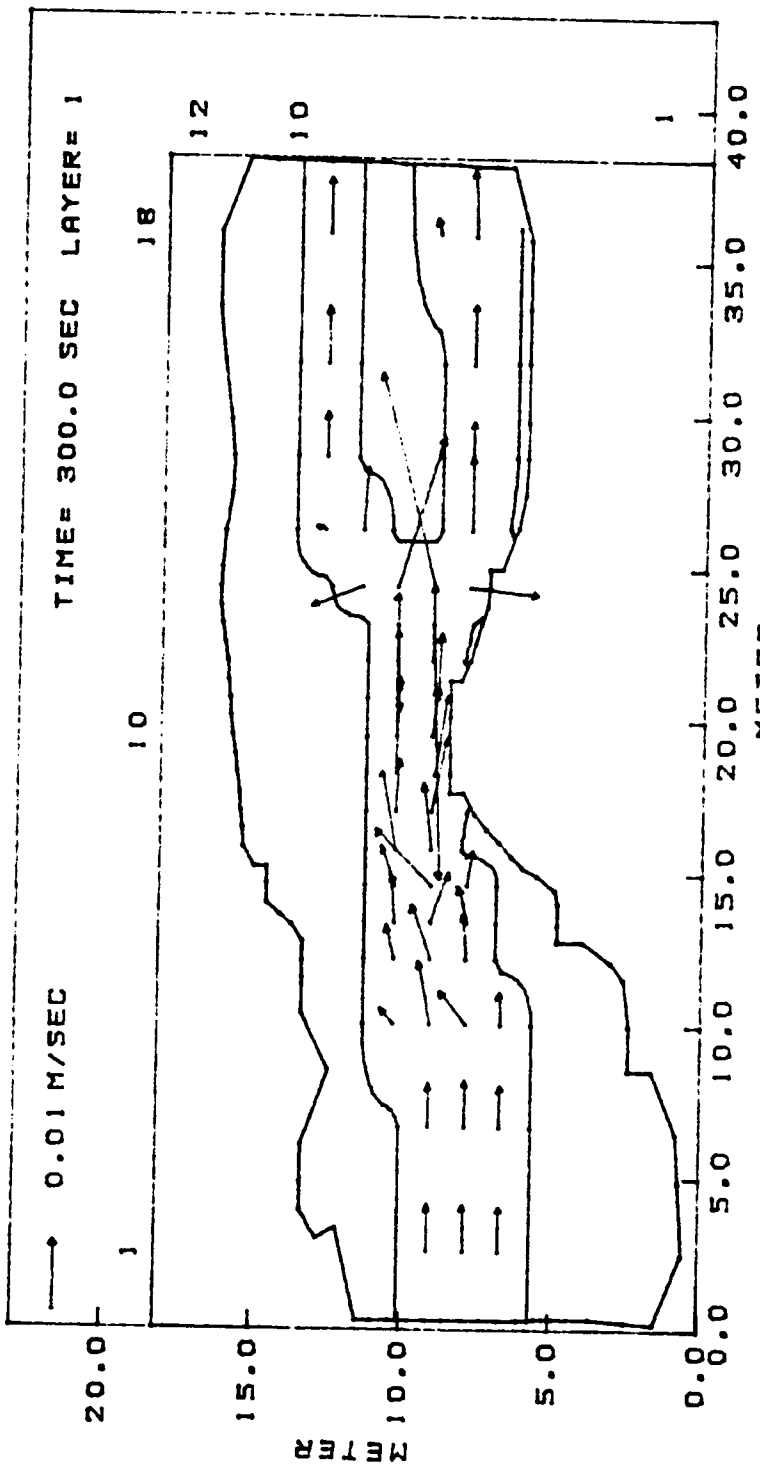

Figure 6.149. Velocity conditions in the bottom layer l = 1, at time t = 300 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the bottom layer.

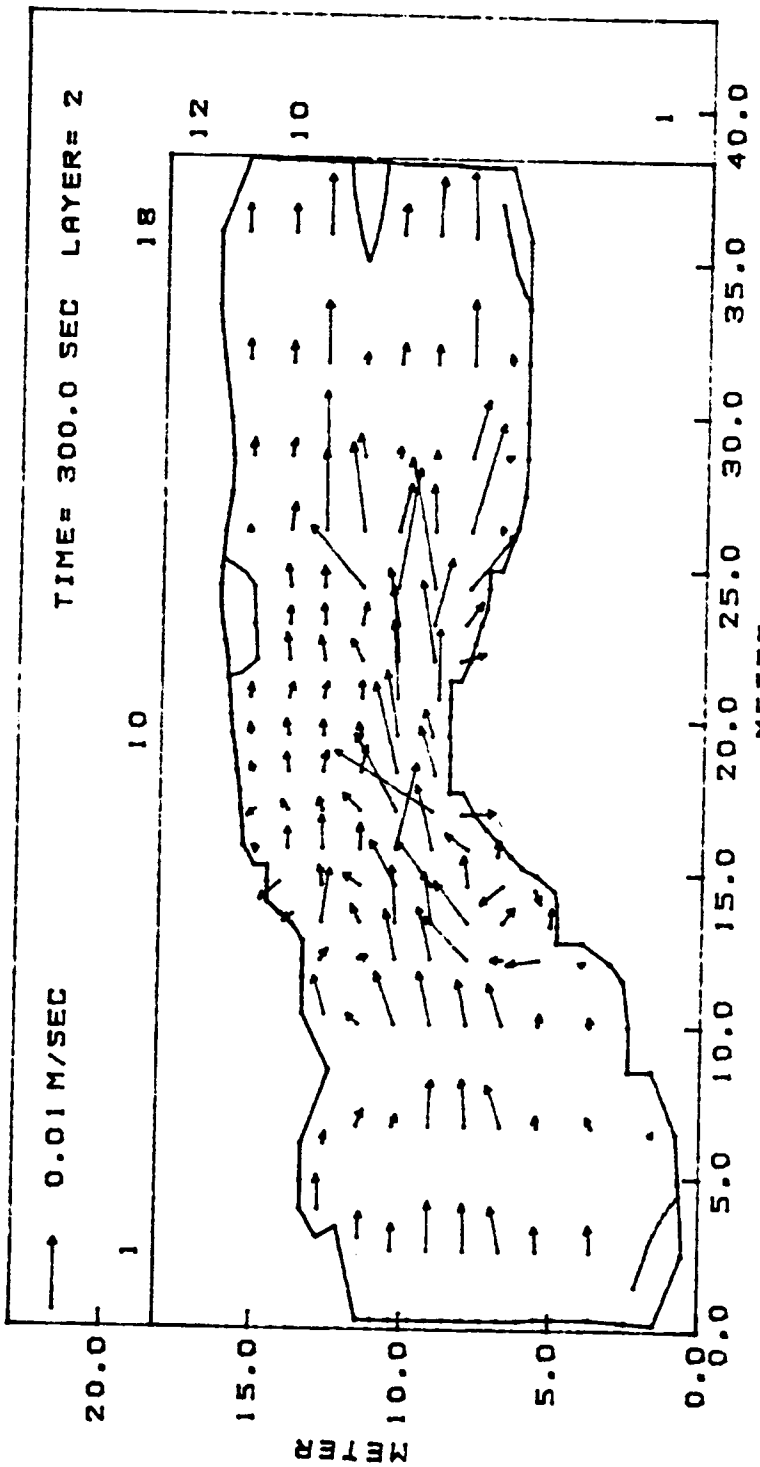

Figure 6.150. Velocity conditions in the internal layer l = 2, at time t = 300 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

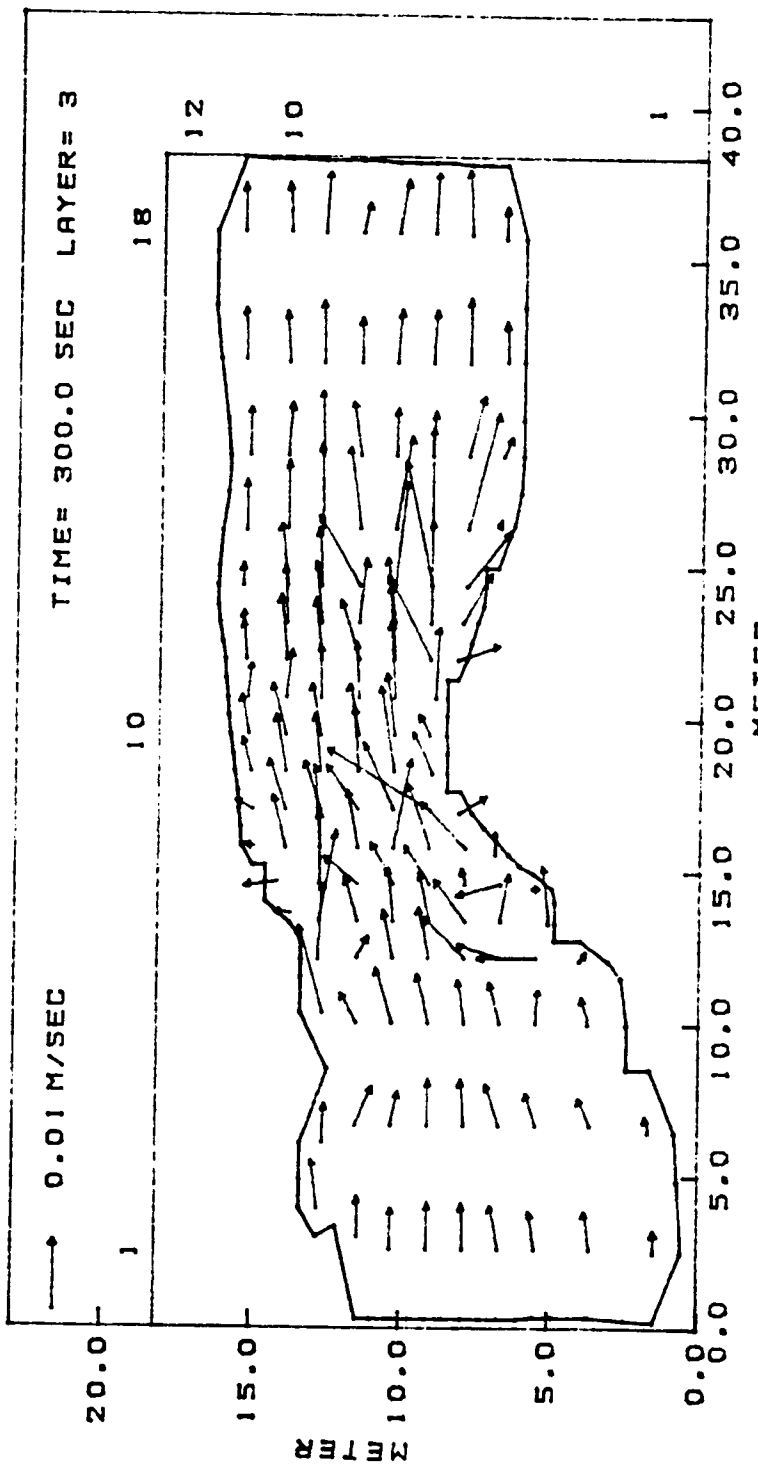

Figure 6.151. Velocity conditions in the internal layer l = 3, at time t = 300 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

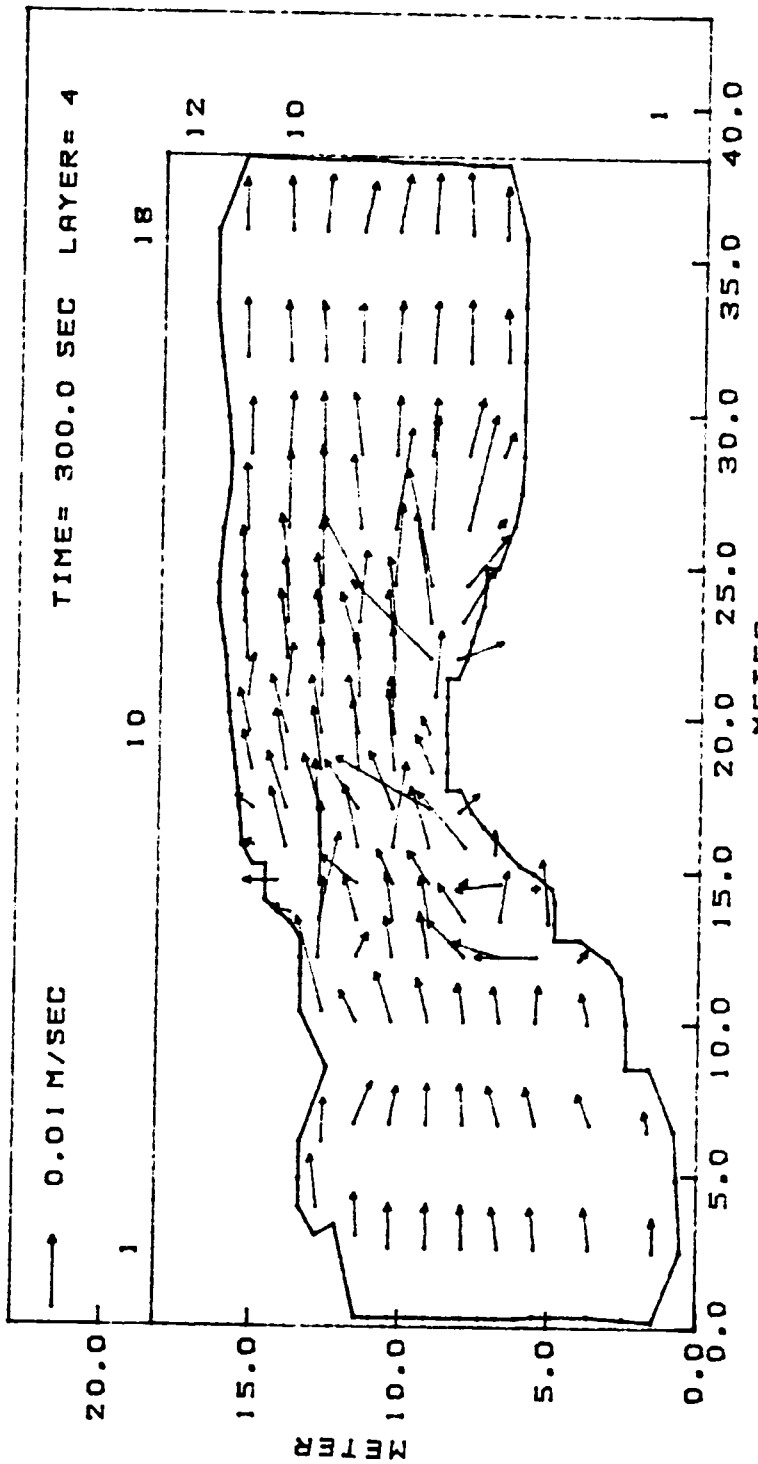

Figure 6.152. Velocity conditions in the internal layer l = 4, at time t = 300 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the internal layer.

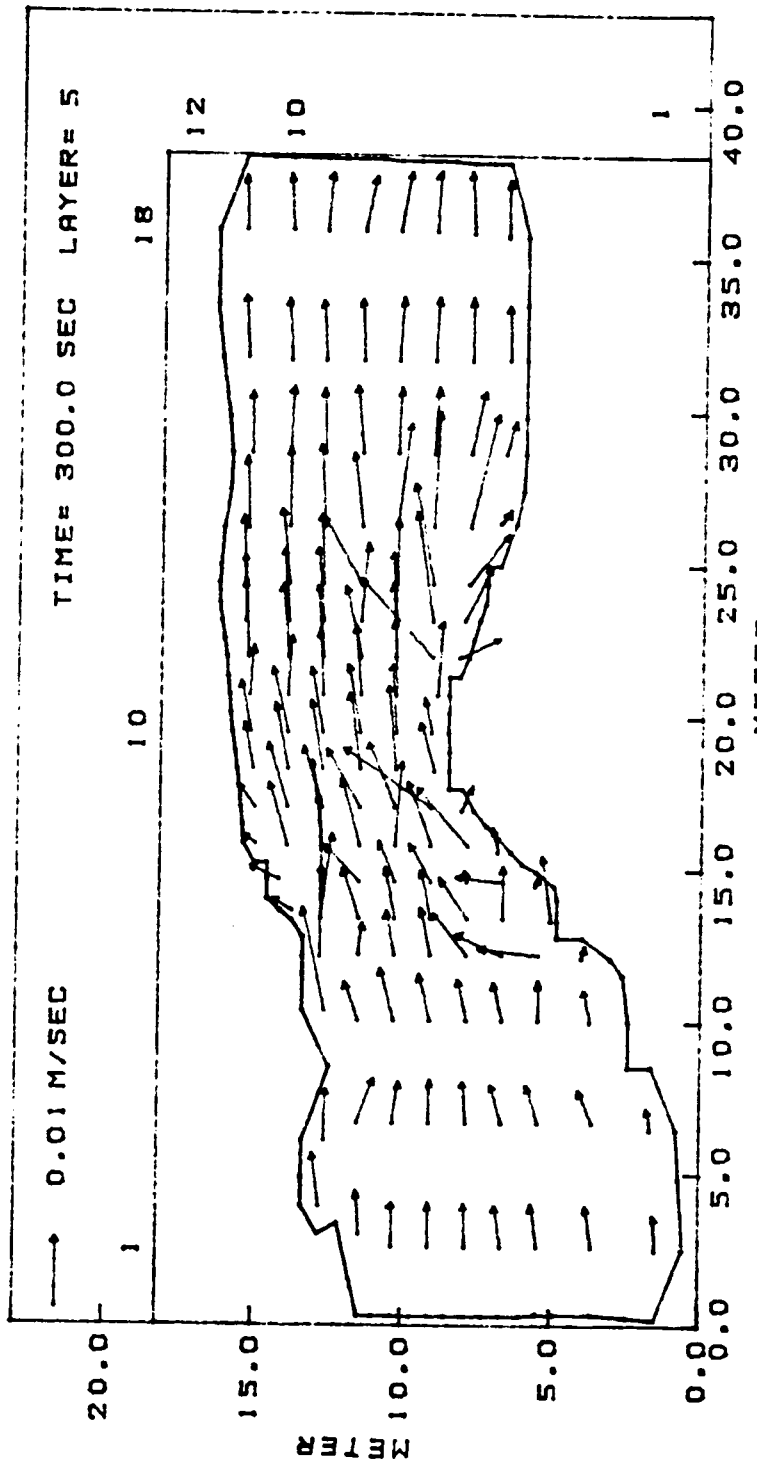

Figure 6.153. Velocity conditions in the surface layer l = 5, at time t = 300 sec in Browns Ferry physical model. Vectors correspond to two-component representations of velocity in the horizontal (x,y) plane of the surface layer.

mately -15 mm, below the specified initial MWL, in the vicinity of the x-downstream specified-boundary enclosure surface where temporally uniform outflow conditions are specified (Fig. 6.106, p.741; Fig. 6.112, p.747; and Fig. 6.118, p.753).

The computer simulation results for the velocity conditions in the five layers, l = 1 through l = 5, of the flow region, between the initial time t = 0 and the time level t = 30 sec (Figs. 6.107 through 6.111, p.742 through p.746; Figs. 6.113 through 6.117, p.748 through p.752; and Figs. 6.119 through 6.123, p.754 through p.758) also indicate the generally expected conditions for the development of the free-surface flow conditions. The velocity conditions in all layers l = 1 through 5, gradually increase throughout the flow region, with the magnitude of the maximum velocity condition exceeding 1 cm/sec (0.01m/sec) in the flow region (Figs. 6.119 through 6.123, p.754 through p.758).

However, the computer simulation results for the water-surface-elevation conditions at the later time level t = 60 sec indicate a certain characteristic which is definitely not consistent with the generally expected developed flow conditions in the flow region of the Browns Ferry physical model (Fig. 6.104, p.731). The distribution of the water-surface elevation conditions indicate a complete reversal in the water-surface-elevation distribution over the flow region, with the maximum value of the water-surface elevation, approximately 16 mm above the specified initial mean water level (MWL), being in the vicinity of the x-downstream specified-boundary enclosure surface at x = 38.4 m, where the temporally uniform outflow conditions are specified; and the

minimum value of the water surface elevation, approximately -15 mm below the specified initial MWL, being in the vicinity of the x-upstream specified-boundary enclosure surface at x = 0, where temporally uniform inflow conditions are specified (Fig. 6.104, p.731).

The computer simulation results for the velocity conditions in the five layers, l = 1 through 5, at the time level t = 60 sec (Figs. 6.125 through 6.129, p.760 through p.764) indicate the flow is still predominantly along the expected direction, from the x-upstream specified-boundary enclosure surface at x = 0, toward the x-downstream specified-boundary enclosure surface at x = 38.4 m. Hence, at the time level t = 60 sec, the velocity conditions in all five layers, l = 1 through 5, in the Browns Ferry physical model (Figs. 6.125 through 6.129, p.760 through p.764) indicate that the flow is against the general direction of the gradient of the water-surface elevation (Fig. 6.124, p.759).

The computer simulation results at the time level t = 60 sec (Figs. 6.124 through 6.129, p.759 through p.764) clearly indicate that the flow conditions have not developed into steady state conditions, since the gradients of the water-surface elevation opposes the direction of flow, along the main-flow channel; From the plane of the inflow at the x-upstream specified-boundary enclosure surface at x = 0, toward the plane of the outflow at the x-downstream specified-boundary enclosure surface at x = 38.4 m. It is also important to note that since the x-component of velocity, associated with momentum transport along x-direction, is specified as 0.006110 m/sec at the x-upstream specified-boundary enclosure surface at x = 0, and the x-component of velocity,

associated with momentum transport along x-direction, is specified as 0.007096 m/sec at the x-downstream specified-boundary enclosure surface at x = 38.4 m, considering the overall effect of convective momentum transport along the main-flow channel, the direction of the velocity conditions can be realistically along the opposite direction to the effective gradient of the water-surface elevation in the flow region, under transient flow conditions.

The computer simulation results for the water surface-elevation conditions at the later time levels, t = 120 sec (Fig. 6.130, p.765), t = 180 sec (Fig. 6.136, p.771), t = 240 sec (Fig. 6.142, p.777) and t = 300 sec (Fig. 6.148, p.783), also indicate that the flow conditions exhibit oscillatory characteristics between the initial time t = 0 and the last time level of the simulations t = 300 sec. The oscillatory characteristics of the water-surface elevation conditions also affect the velocity conditions in the flow region at certain times, particularly at the time level t = 240 sec when the predominantly unidirectional flow in the main-flow channel is altered significantly in the vicinity of the location of the power plant intake and discharge in the Browns Ferry physical model (Figs. 6.143 through 6.147, p.778 through p.782). At the time level t = 240 sec, in the second layer from the bottom l = 2, a reversal in flow direction occurs in the main-flow channel in the vicinity of the shoreline-boundary enclosure surface, immediately across the water from the location of the power plant. The intermittent formation of this vortex is caused by the unsteady oscillatory behavior of the water-surface-elevation conditions in the main-flow channel (Fig. 6.105, p.733); and it usually observed during

the start-up procedures for initiating the flow conditions in the actual Browns Ferry physical model.

In order to understand the transient, oscillatory behavior of the flow conditions, the detailed results of the computer simulations of the water-surface-elevation conditions in the Browns Ferry Physical model can be considered at more frequent time intervals (10 sec) during the start-up of the flow conditions (Figs. 6.154 through 6.183, p.793 through p.822). The detailed computer simulation results for the water-surface-elevation conditions (Figs. 6.154 through 6.183, p.154 through p.822) clearly indicate the formation of a traveling-surface-wave phenomenon in the flow region during the start-up of the flow conditions in the Browns Ferry physical model.

Since the inital conditions for the computer simulations were assumed as zero flow in the entire region of the Browns Ferry physical model, after the start of the temporally constant inflow conditions at the x-upstream specified-boundary enclosure surface at x = 0 and the outflow conditions at the x-downstream specified-boundary enclosure surface at x = 38.4 m, effectively, two sudden-disturbance fronts are introduced in the flow region. During the first 30 seconds of the flow start-up, a compressive type surface-gravity wave, initiated at the inflow plane at the x-upstream specified-boundary enclosure surface at x = 0, progresses along the downstream x-direction and simultaneously a rarefaction type surface-gravity wave, initiated at the outflow plane at the x-downstream specified-boundary enclosure surface at x = 38.4 m, progresses along the upstream x-direction. After 30 seconds the two surface-gravity waves cross each other at approximately midway between

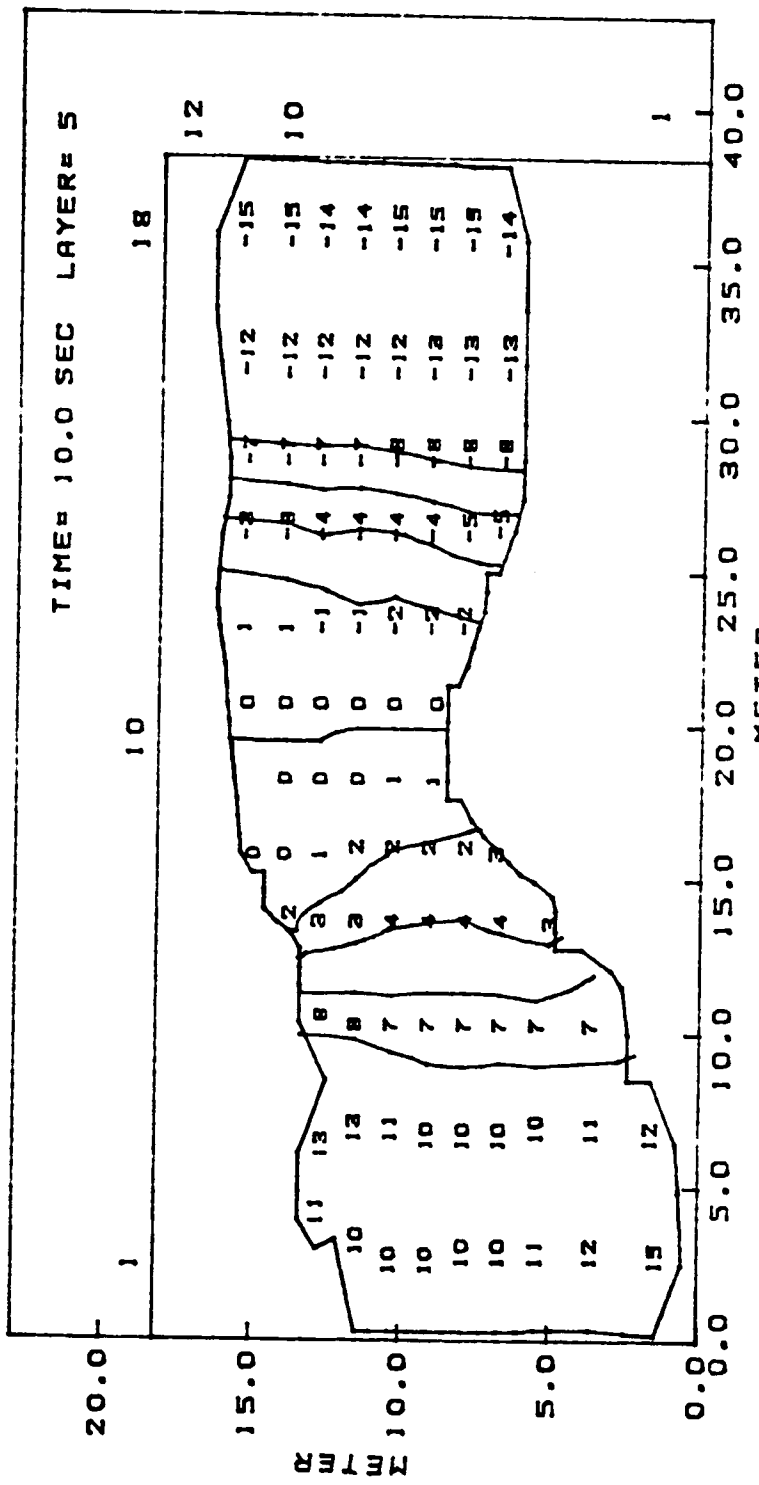

Figure 6.154. Water-surface-elevation conditions at time t = 10 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

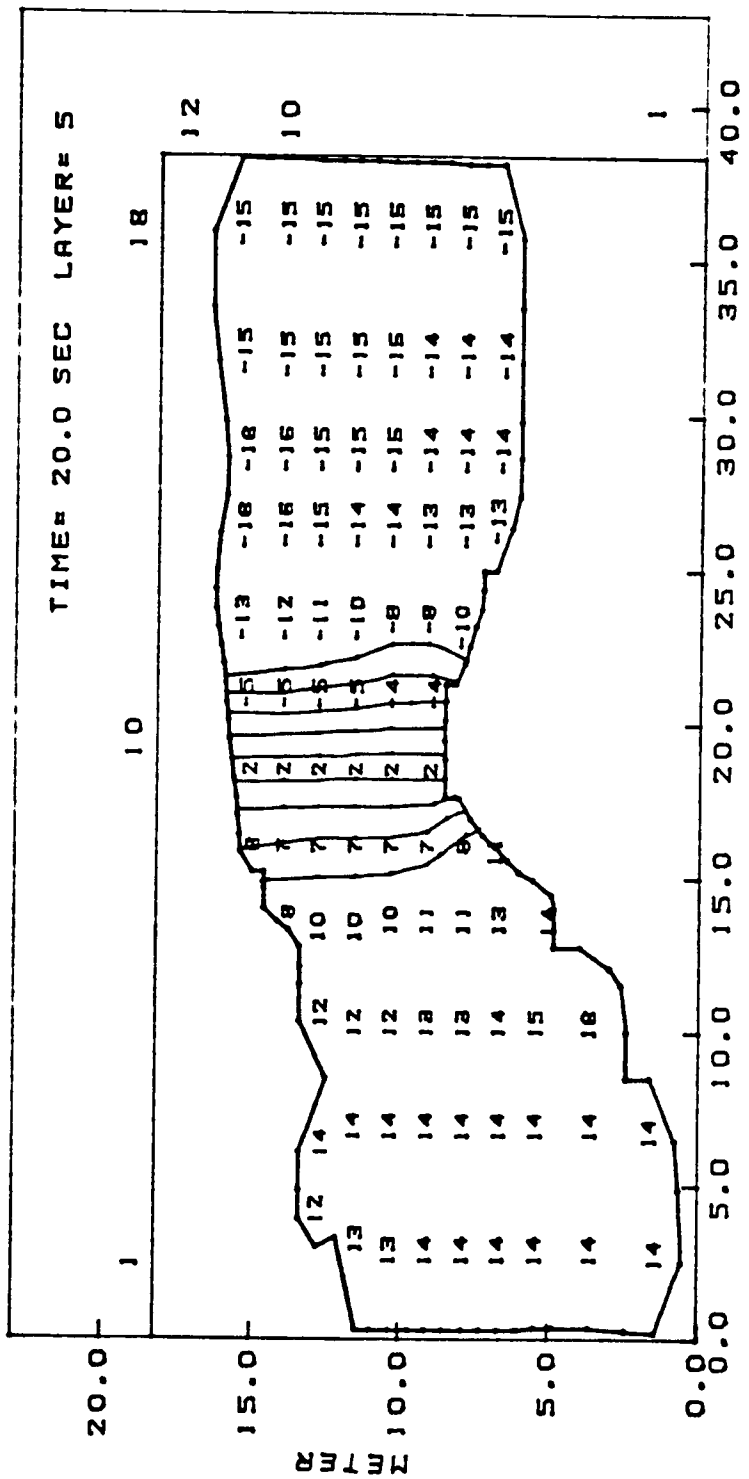

Figure 6.155. Water-surface-elevation conditions at time t = 20 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

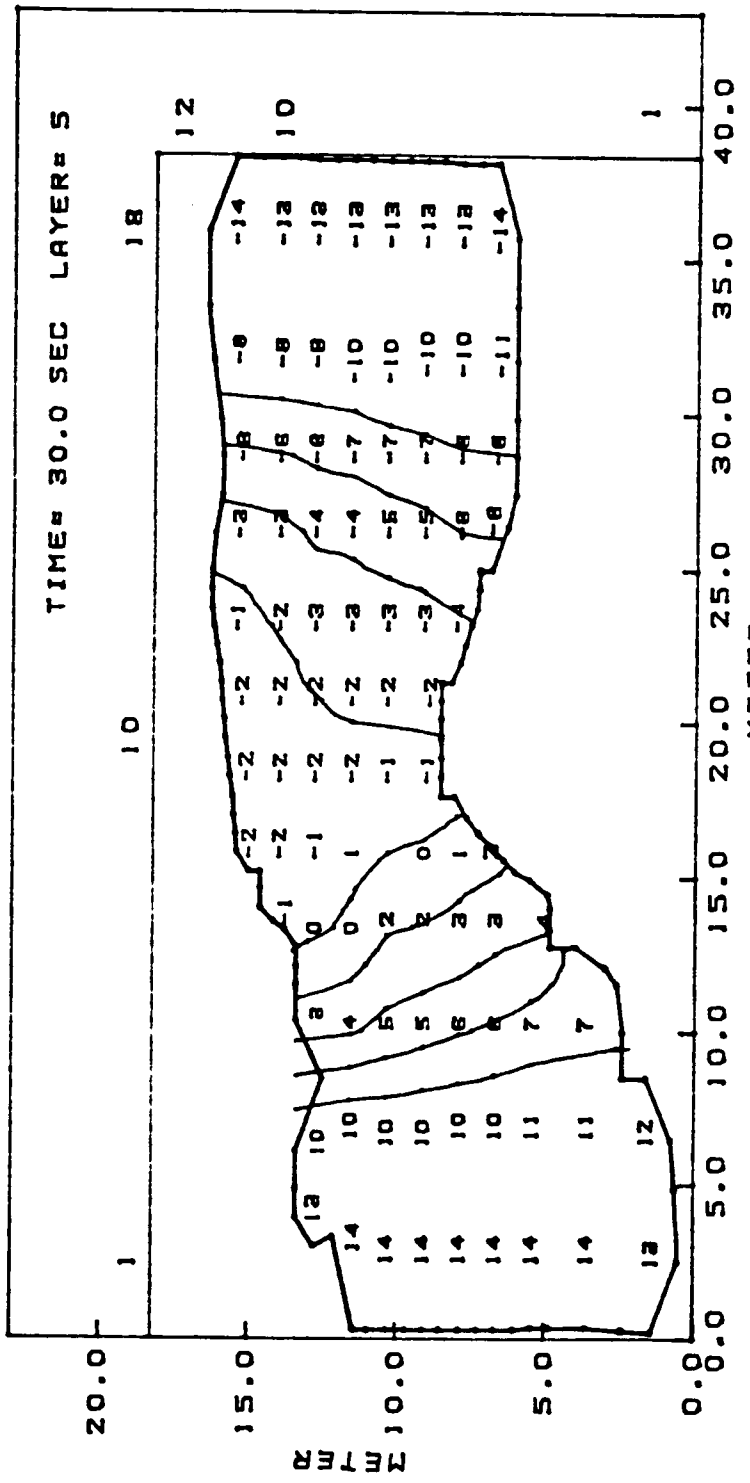

Figure 6.156. Water-surface-elevation conditions at time t = 30 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

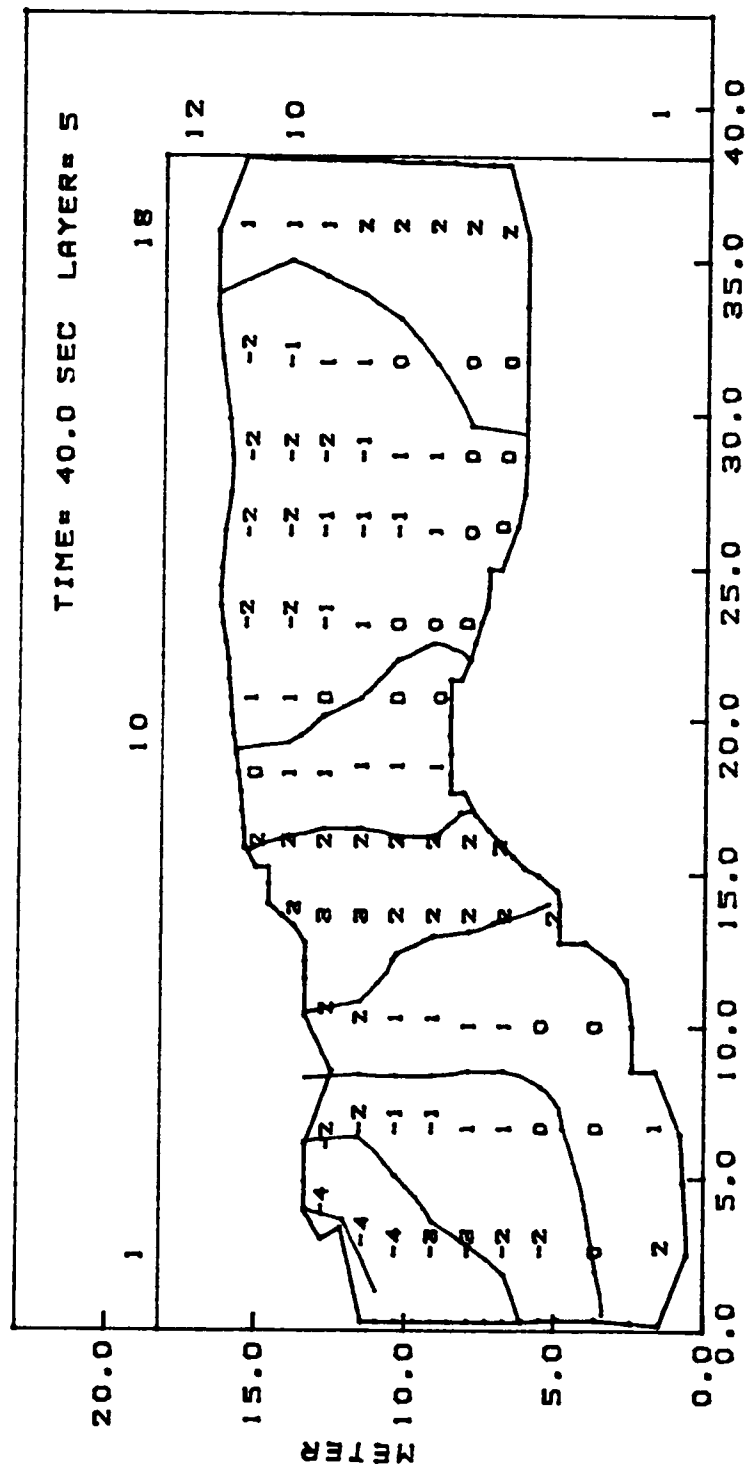

Figure 6.157. Water-surface-elevation conditions at time t = 40 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

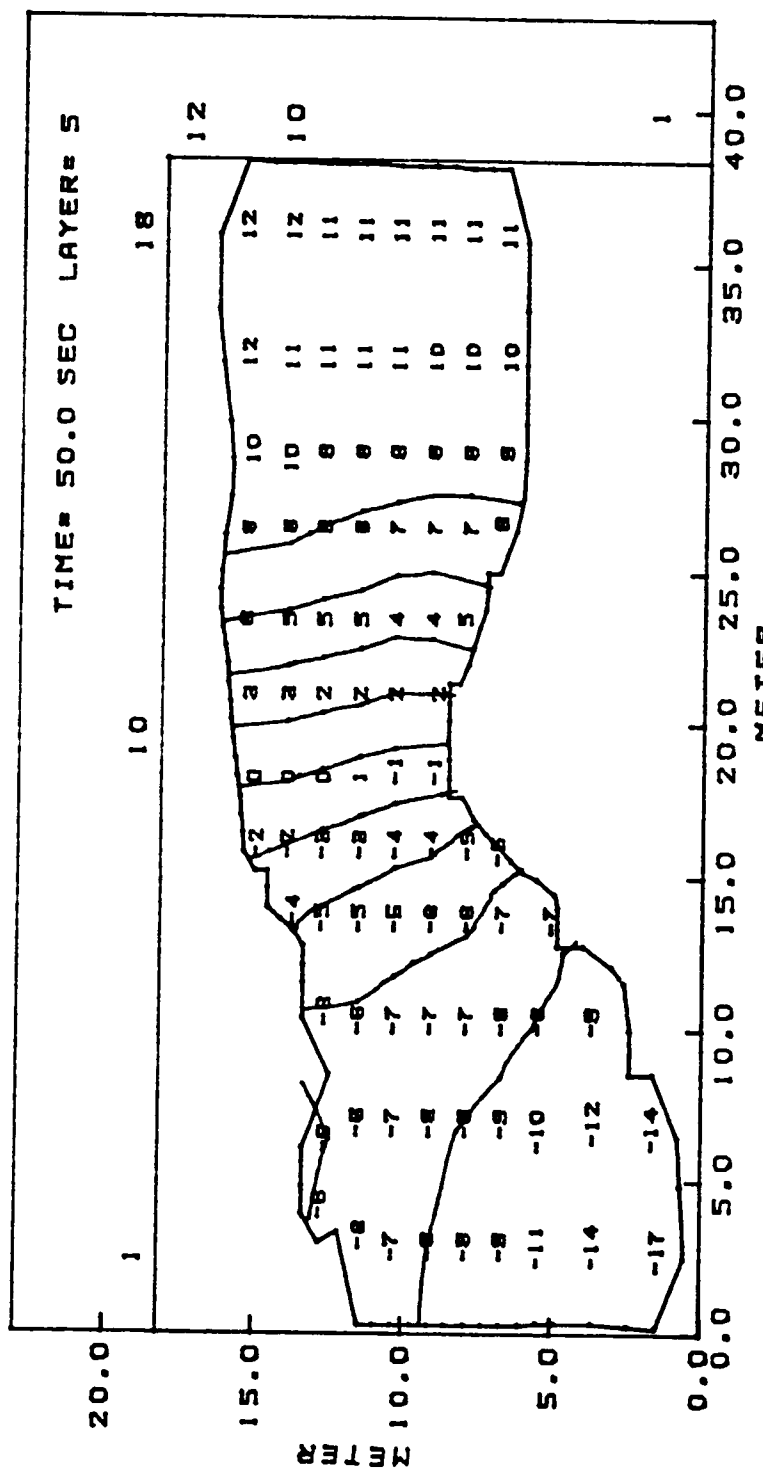

Figure 6.158. Water-surface-elevation conditions at time t = 50 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

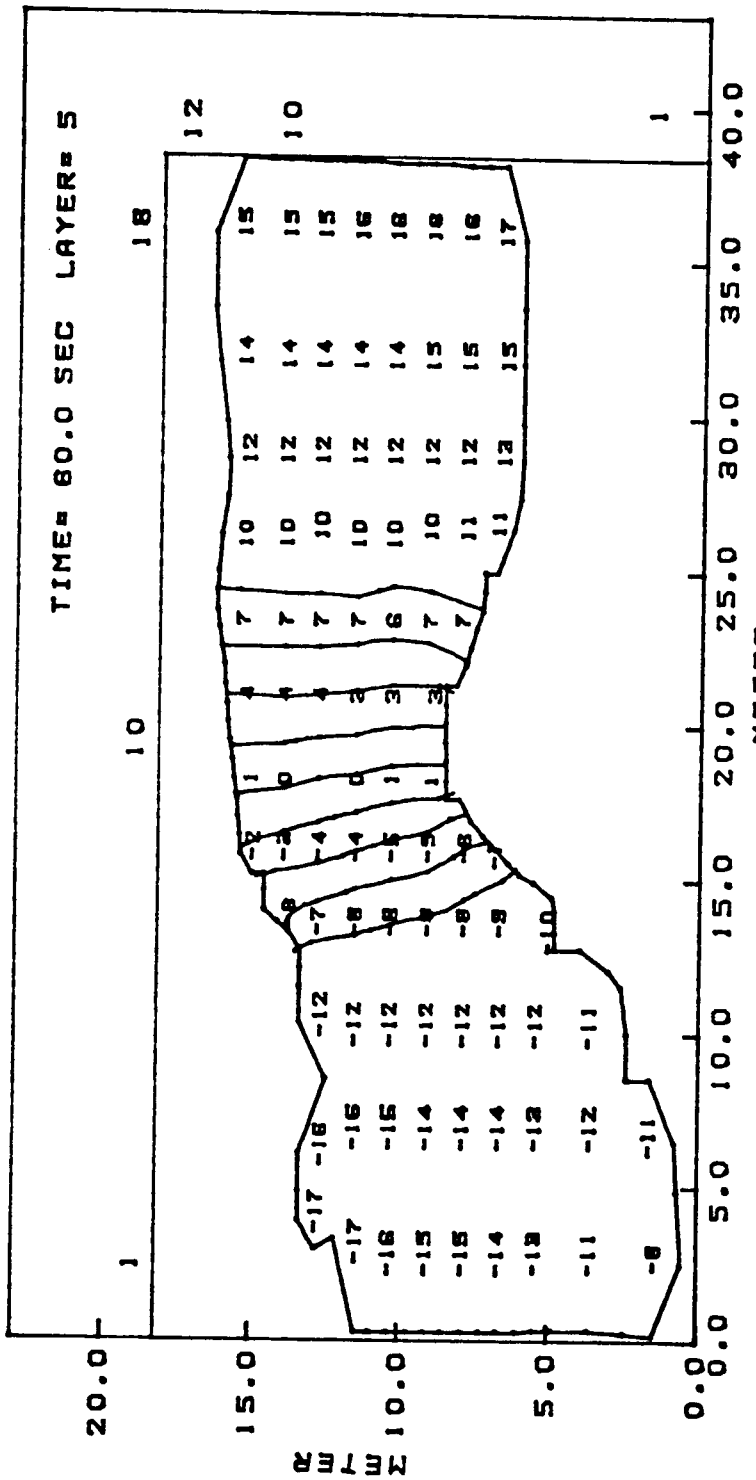

Figure 6.159. Water-surface-elevation conditions at time t = 60 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

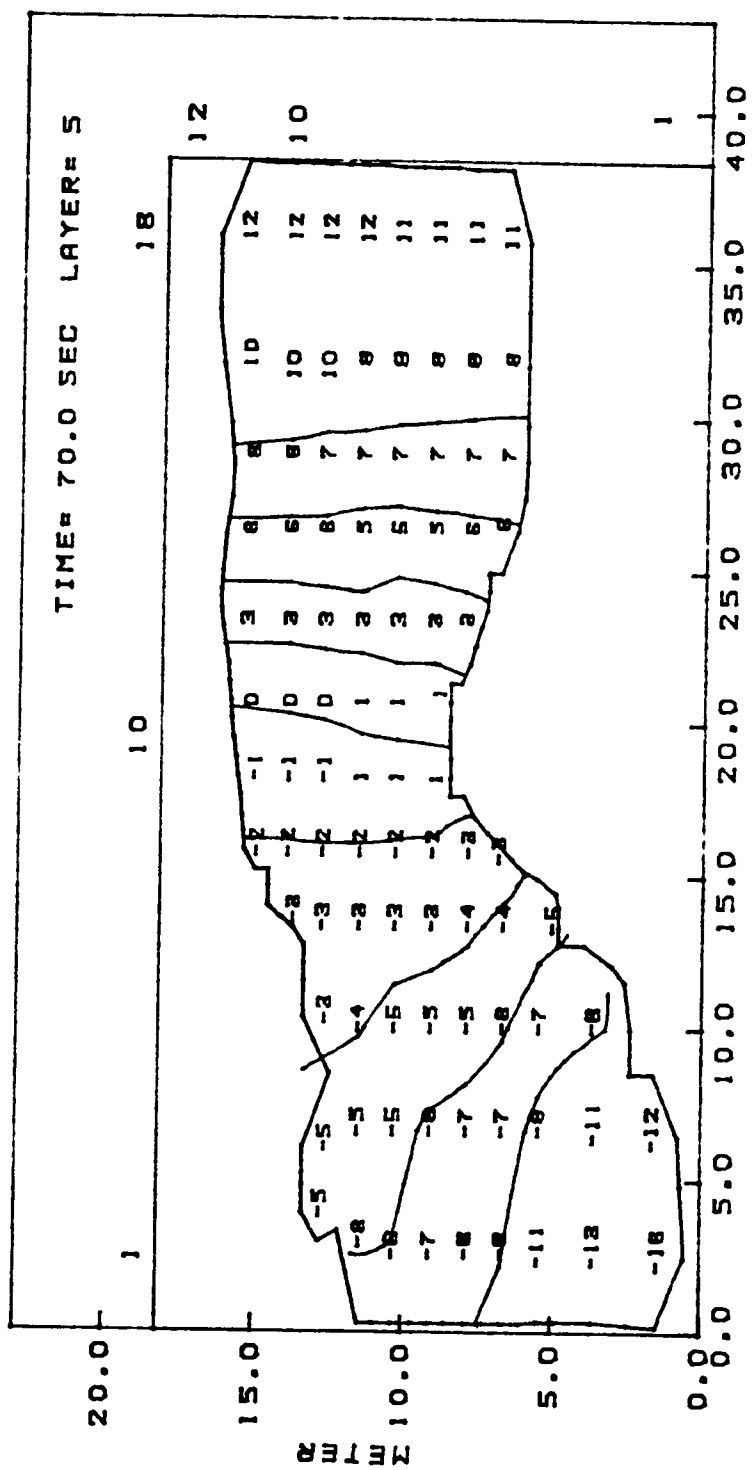

Figure 6.160. Water-surface-elevation conditions at time t = 70 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

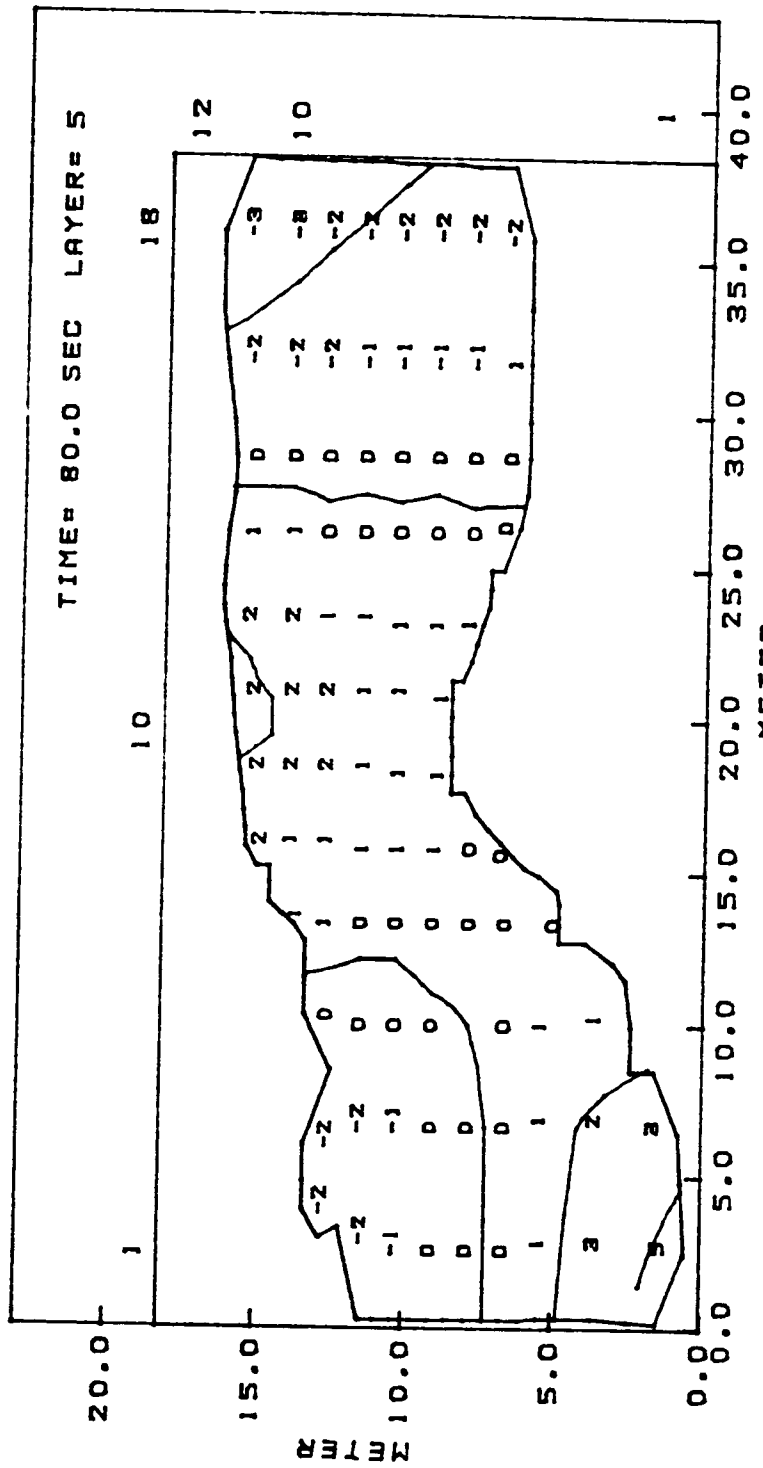

Figure 6.161. Water-surface-elevation conditions at time t = 80 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

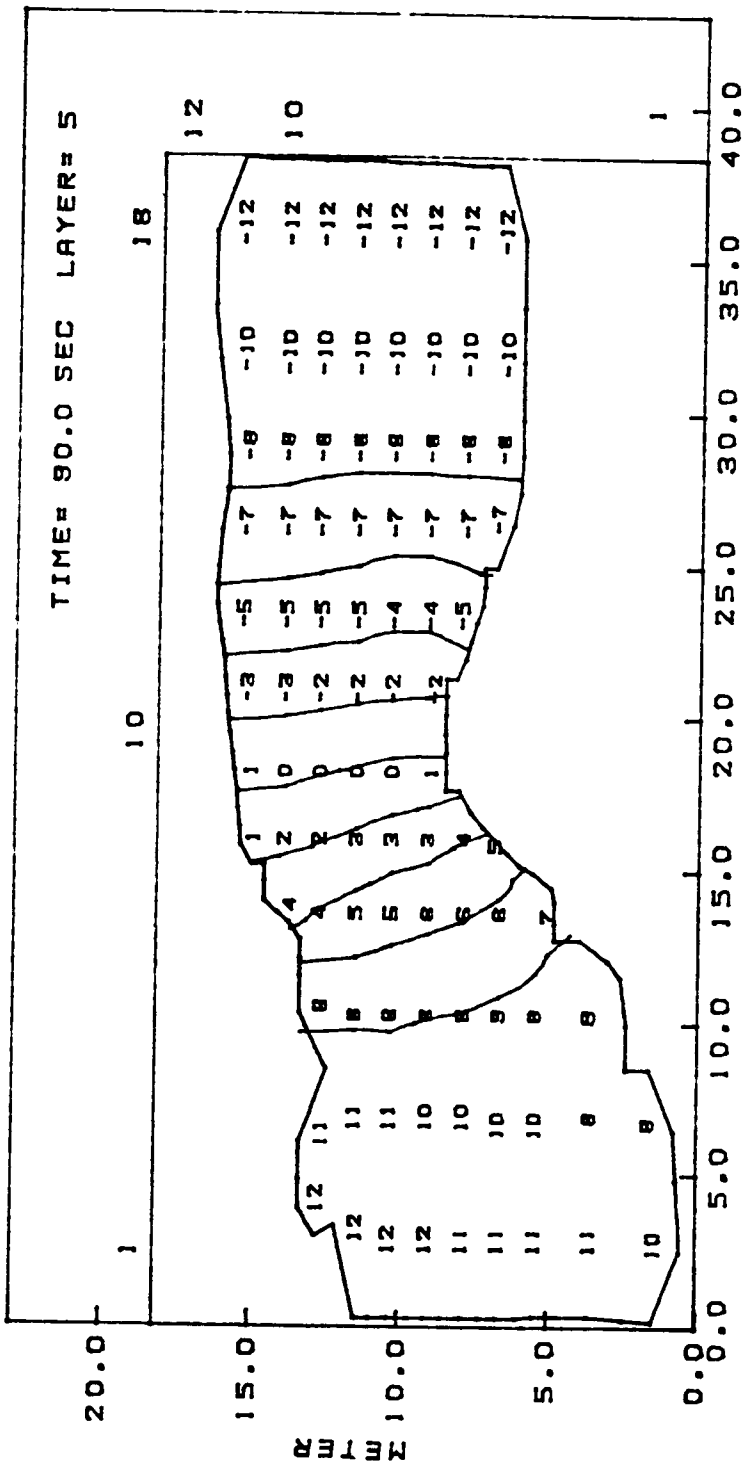

Figure 6.162. Water-surface-elevation conditions at time t = 90 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

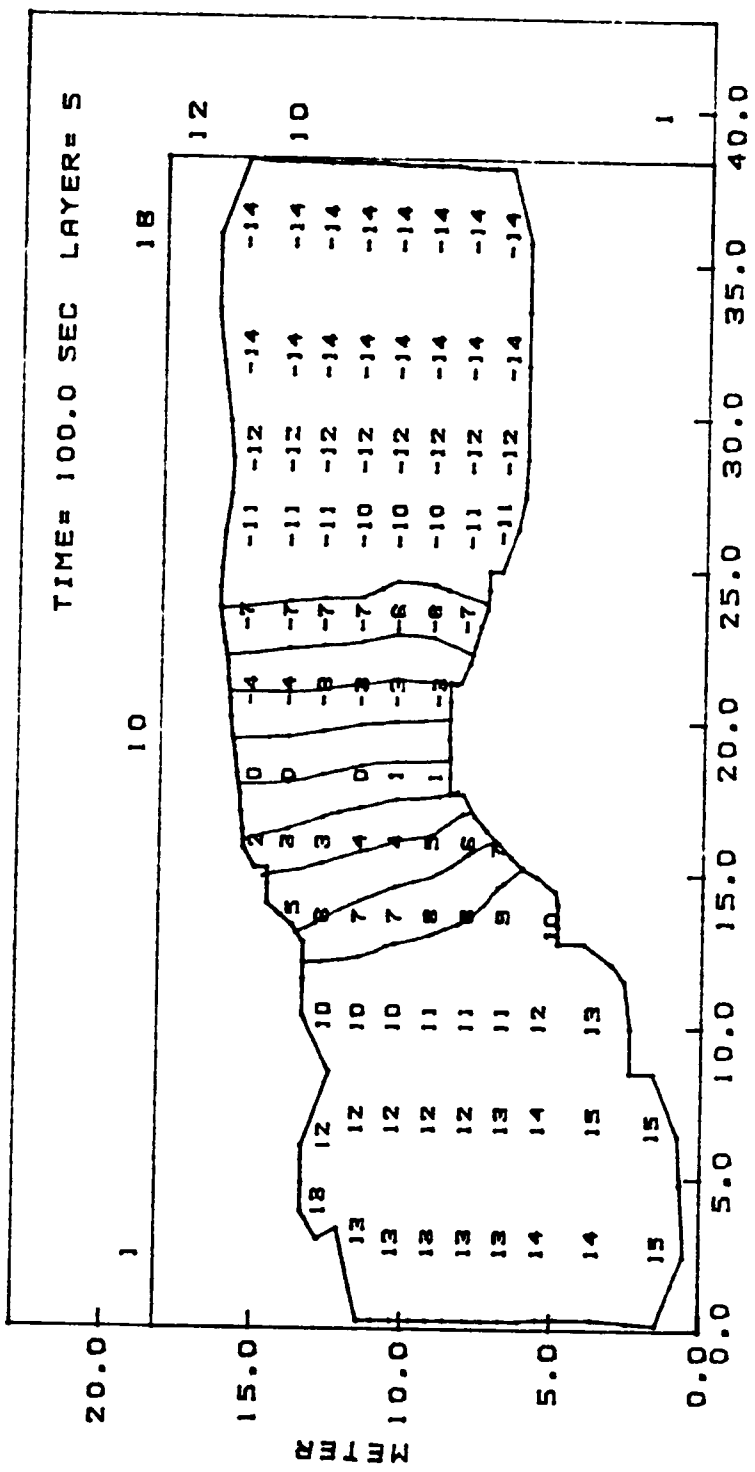

Figure 6.163. Water-surface-elevation conditions at time t = 100 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

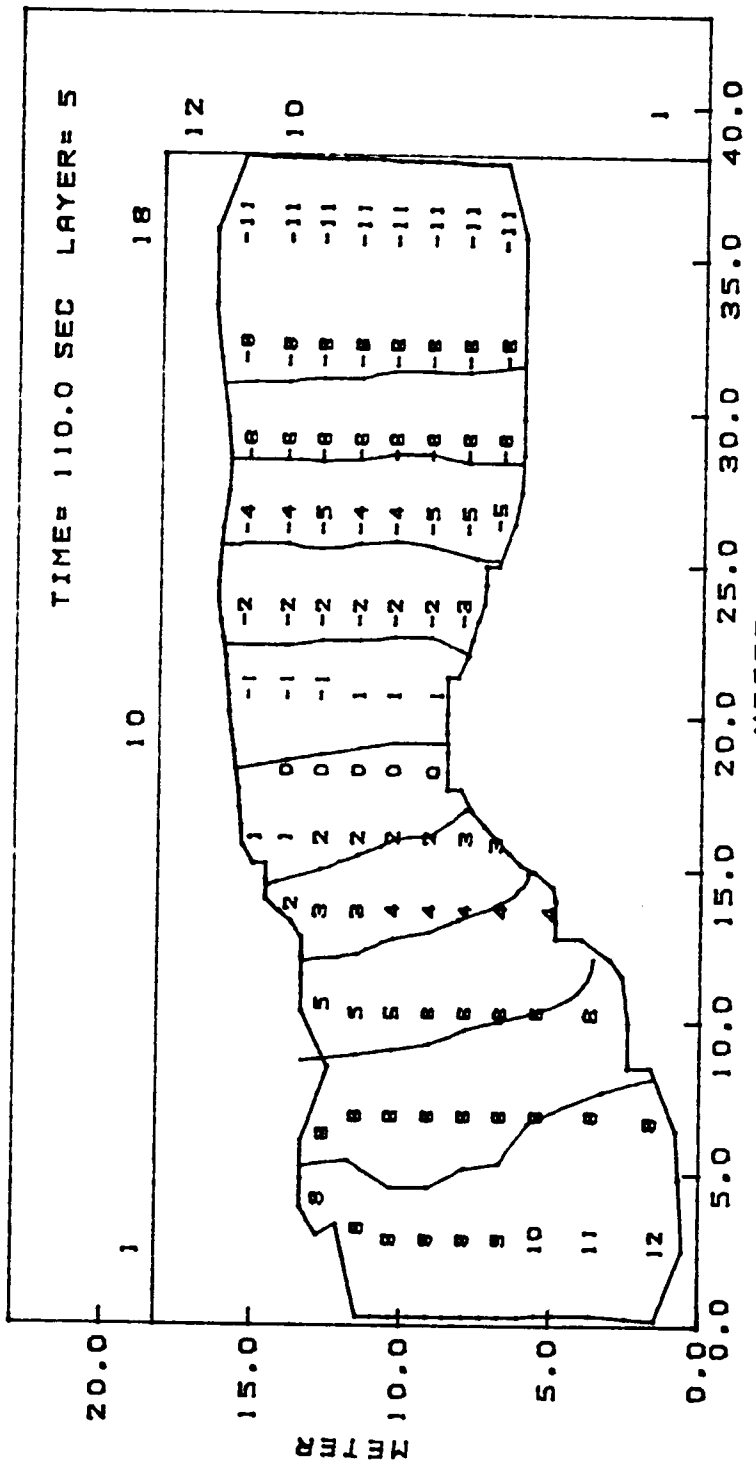

Figure 6.164. Water-surface-elevation conditions at time t = 110 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

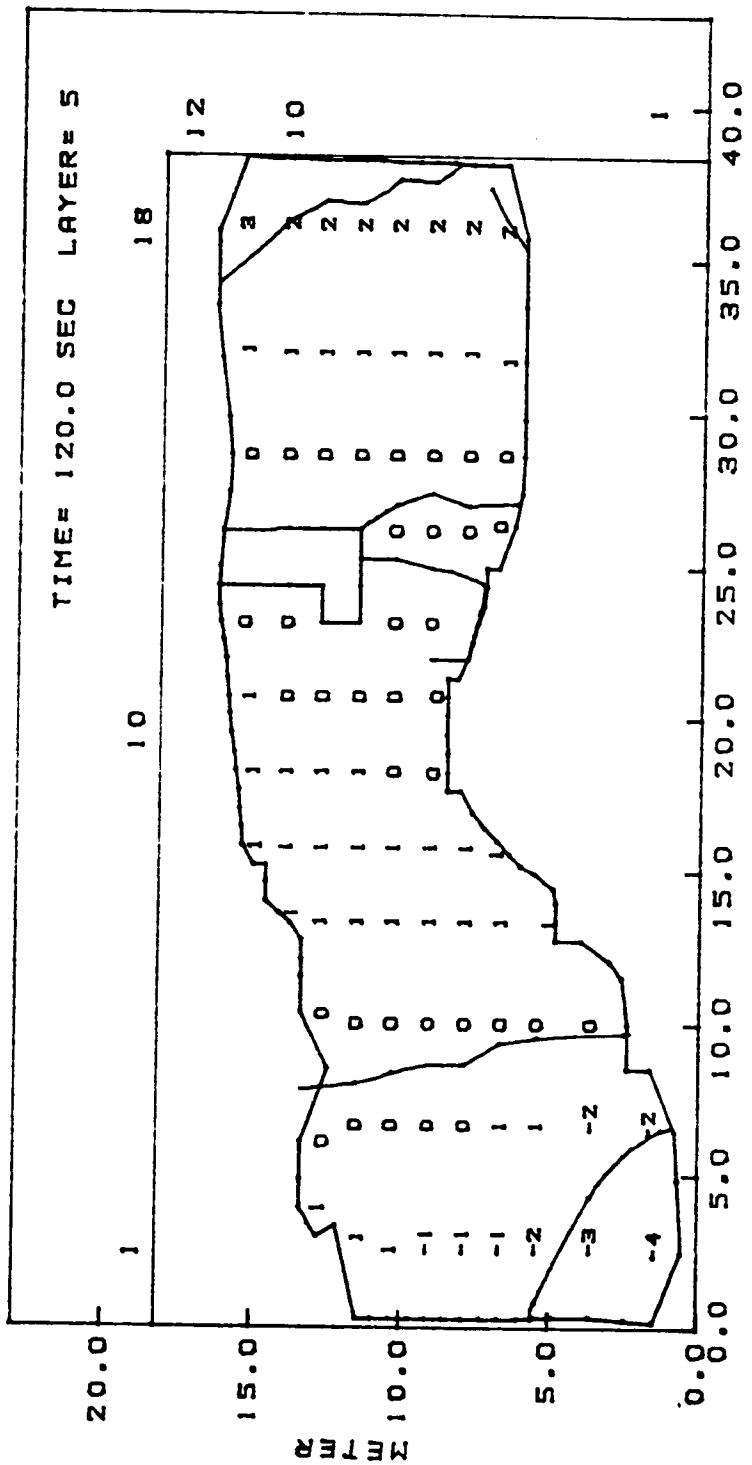

Figure 6.165. Water-surface-elevation conditions at time t = 120 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

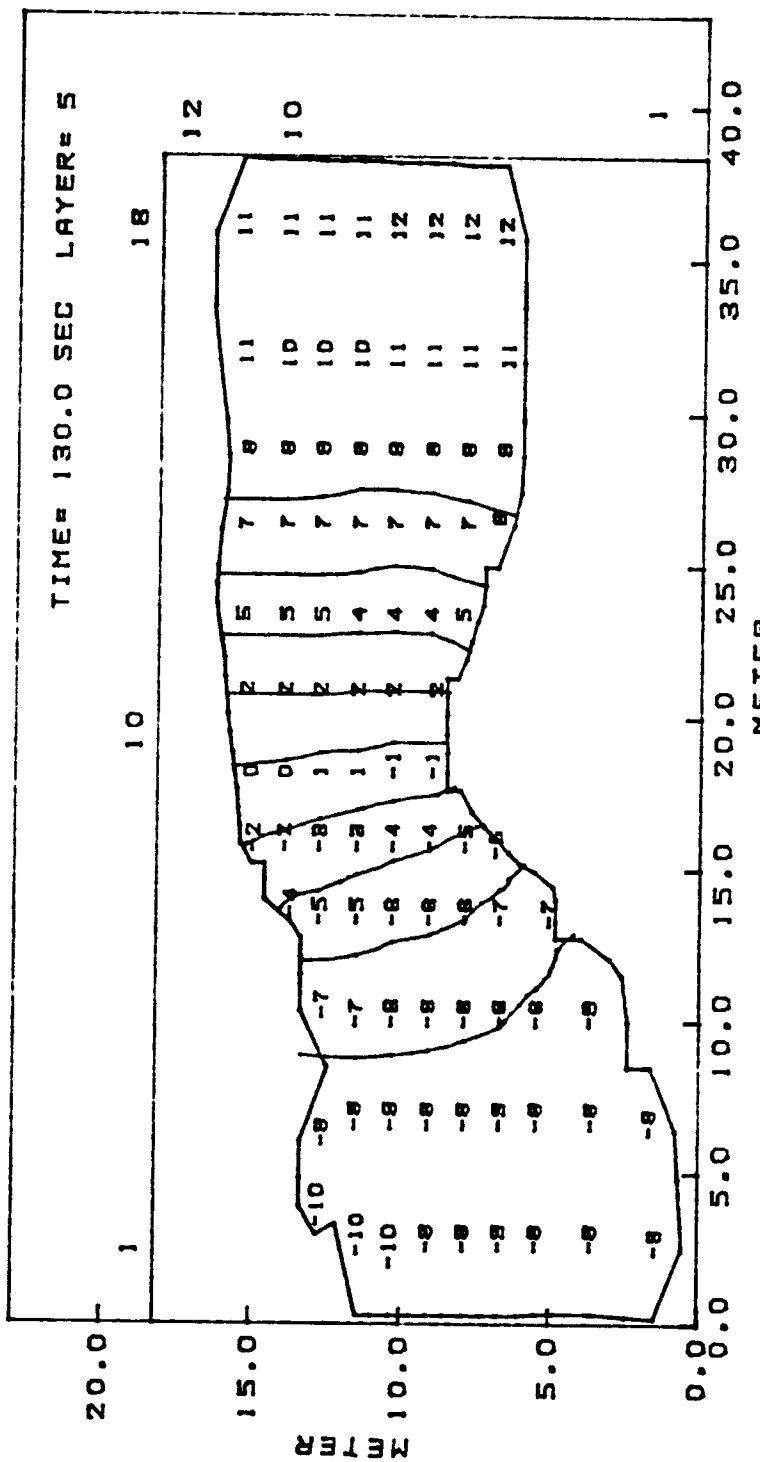

Figure 6.166. Water-surface-elevation conditions at time t = 130 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

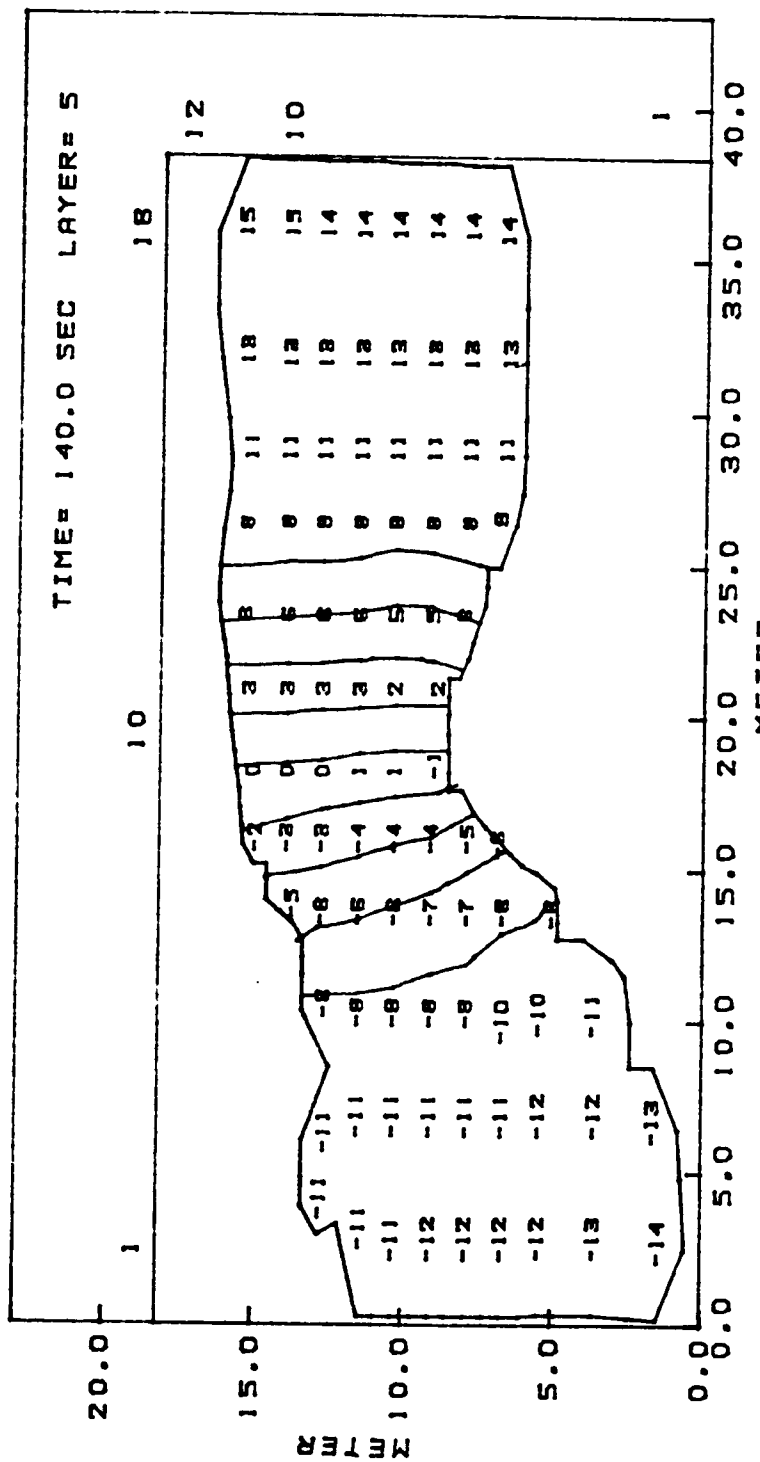

Figure 6.167. Water-surface-elevation conditions at time t = 140 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

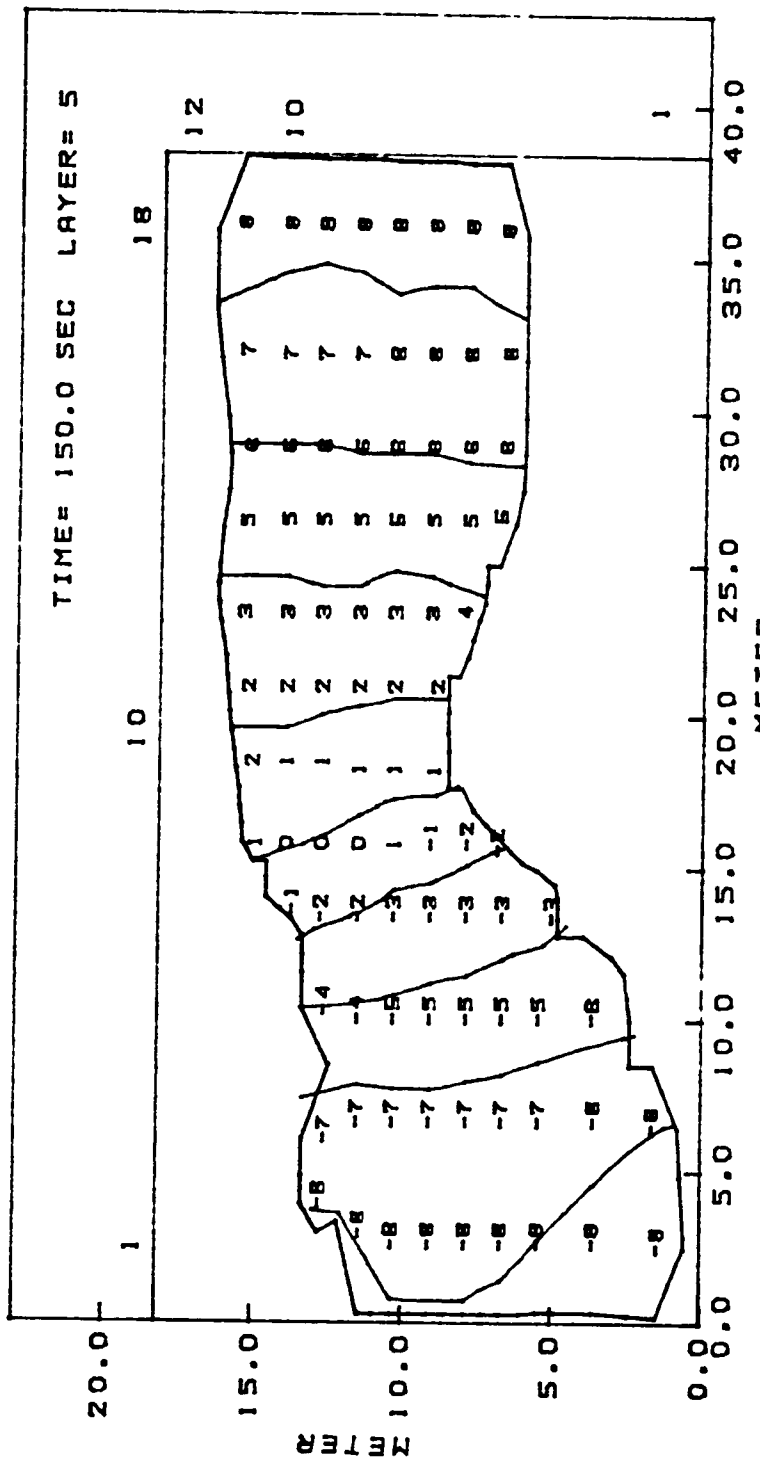

Figure 6.168. Water-surface-elevation conditions at time t = 150 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

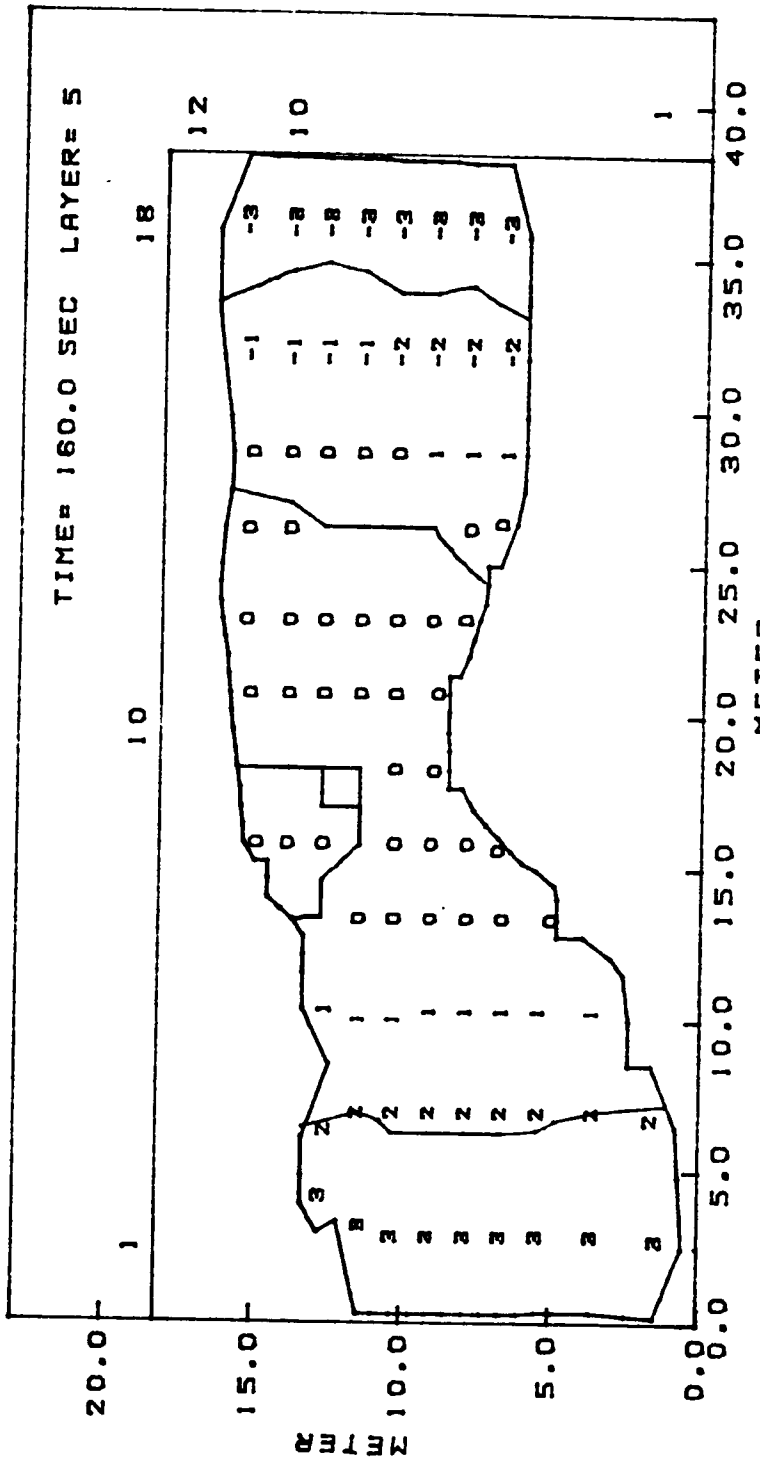

Figure 6.169. Water-surface-elevation conditions at time t = 160 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

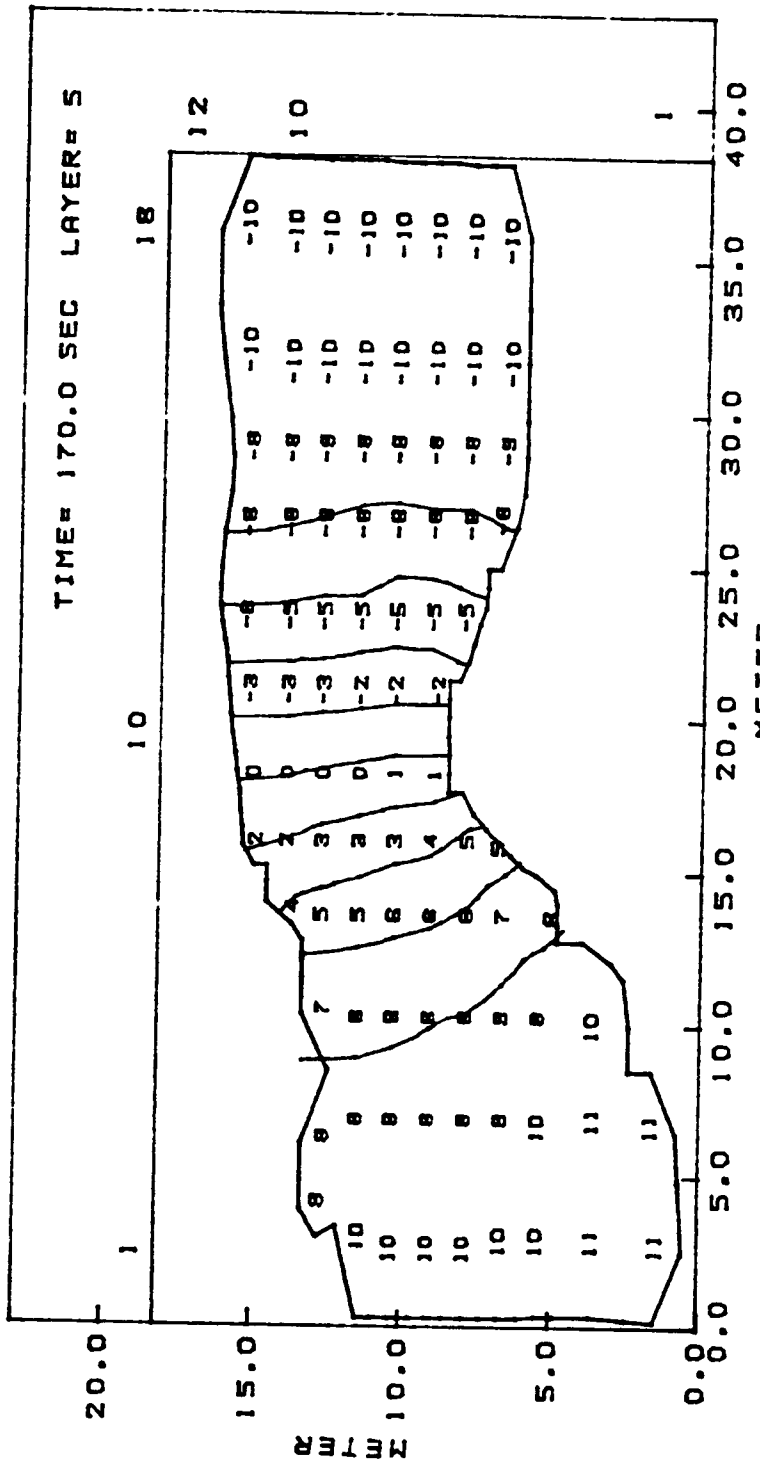

Figure 6.170. Water-surface-elevation conditions at time t = 170 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

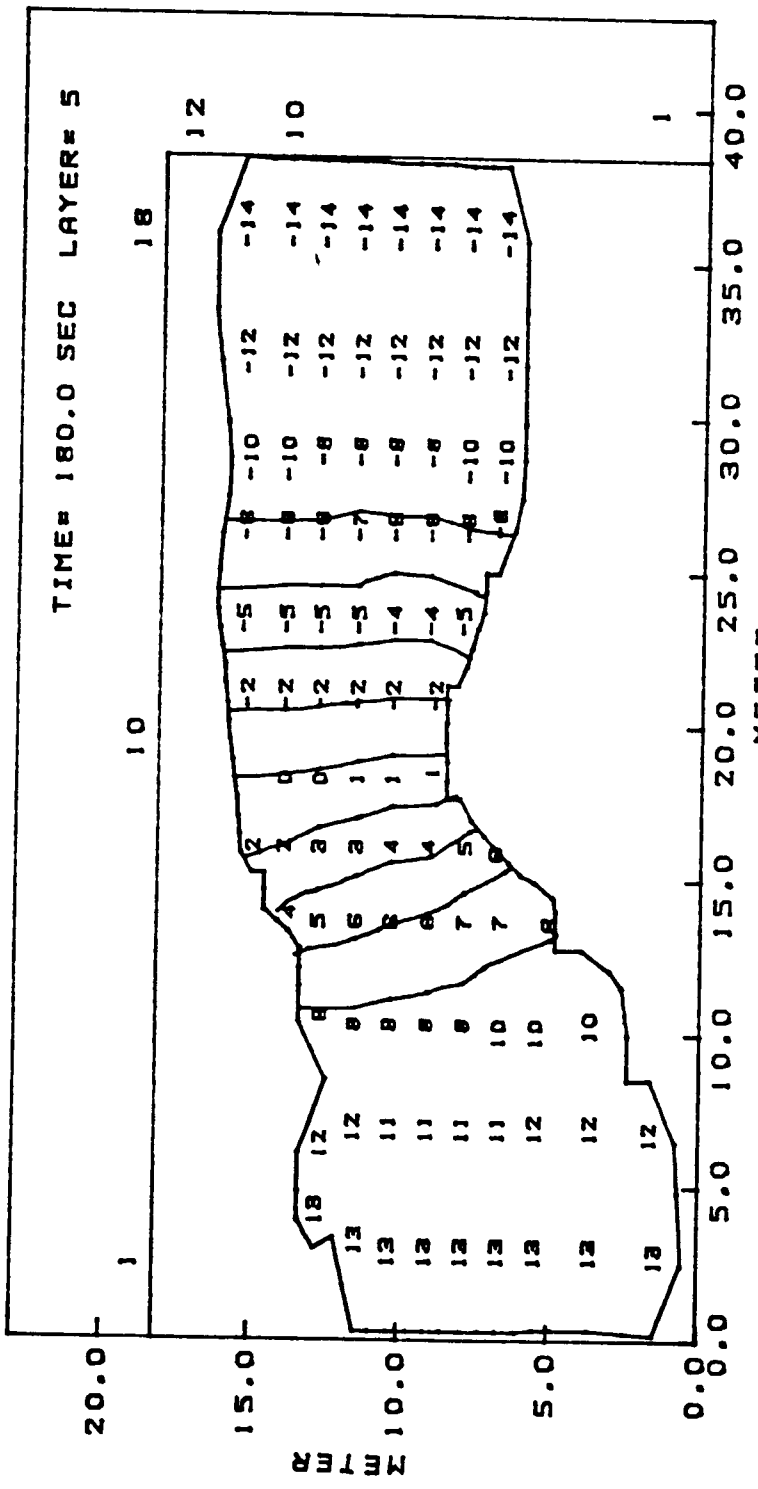

Figure 6.171. Water-surface-elevation conditions at time t = 180 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

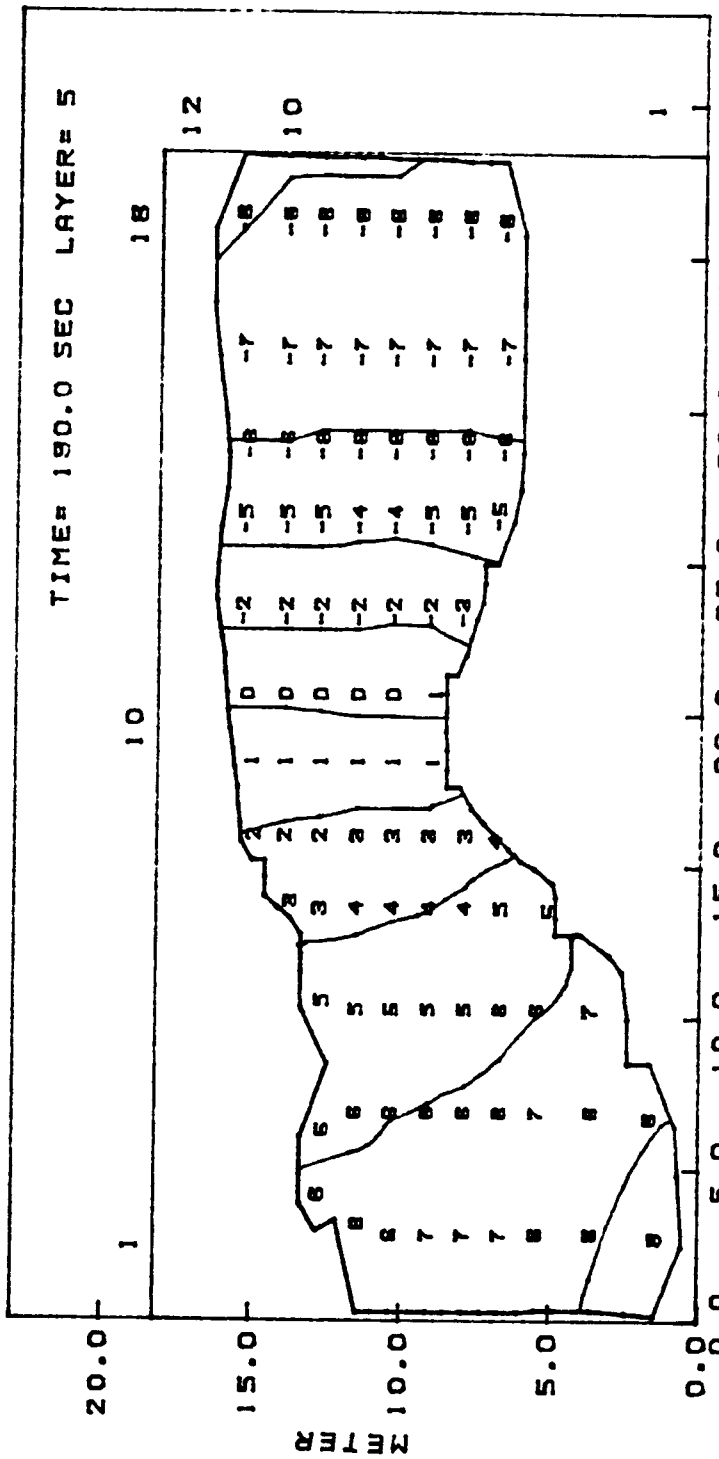

Figure 6.172. Water-surface-elevation conditions at time t = 190 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

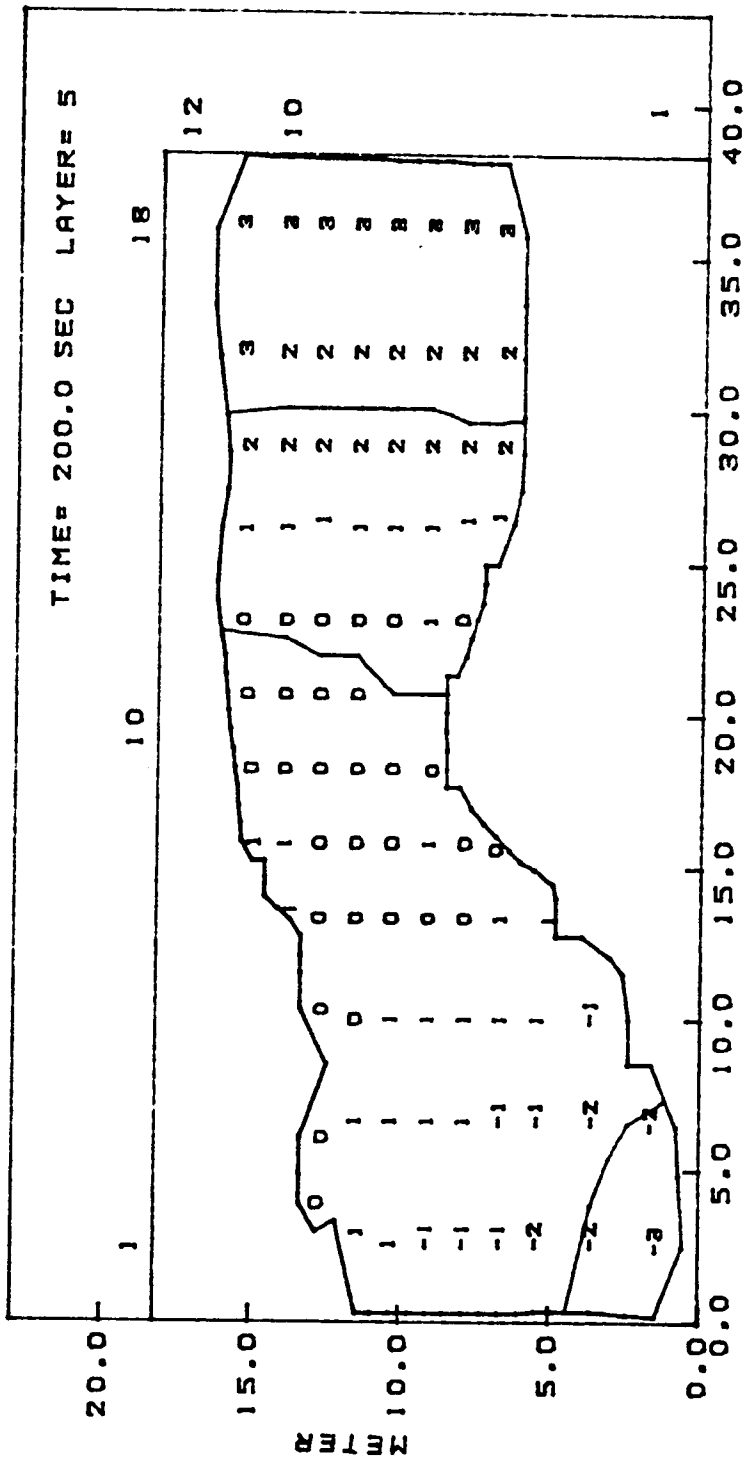

Figure 6.173. Water-surface-elevation conditions at time t = 200 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

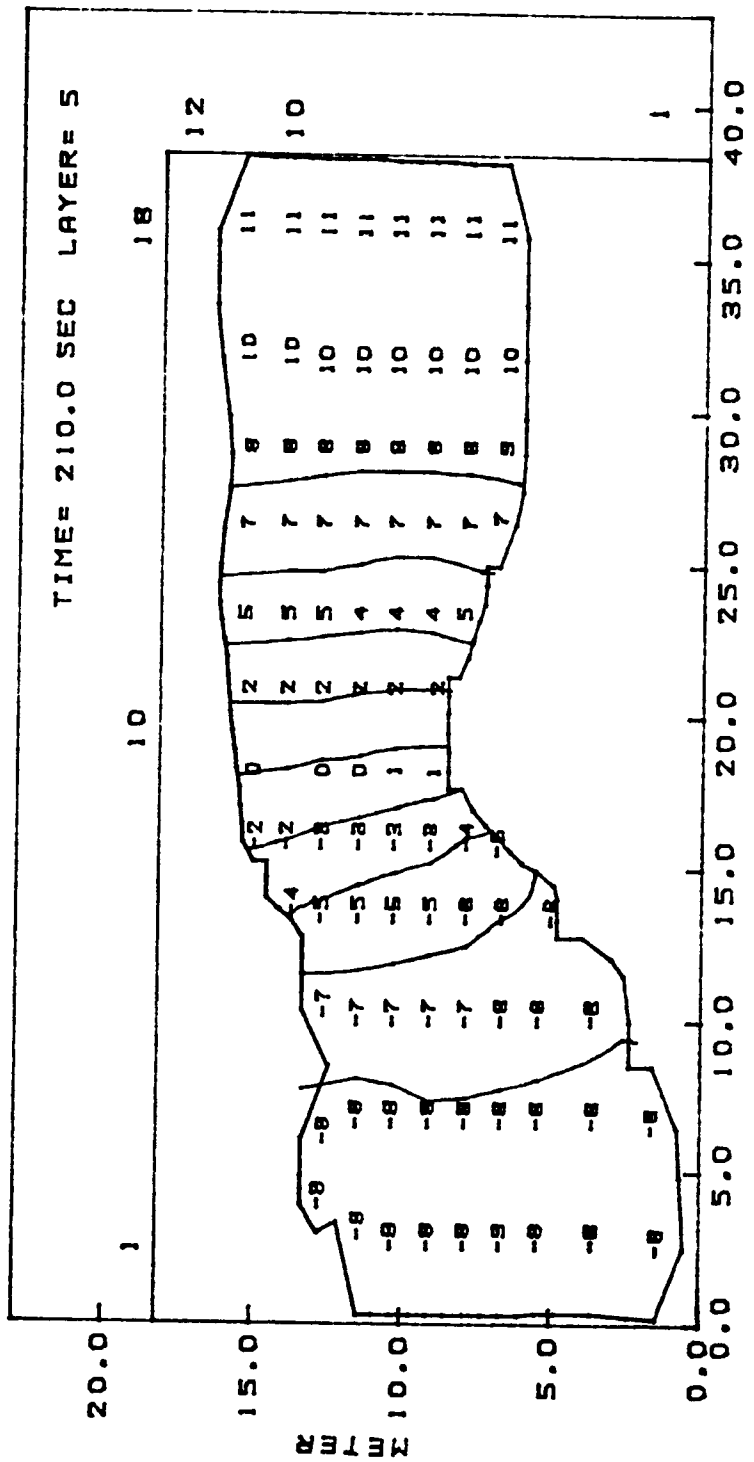

Figure 6.174. Water-surface-elevation conditions at time t = 210 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

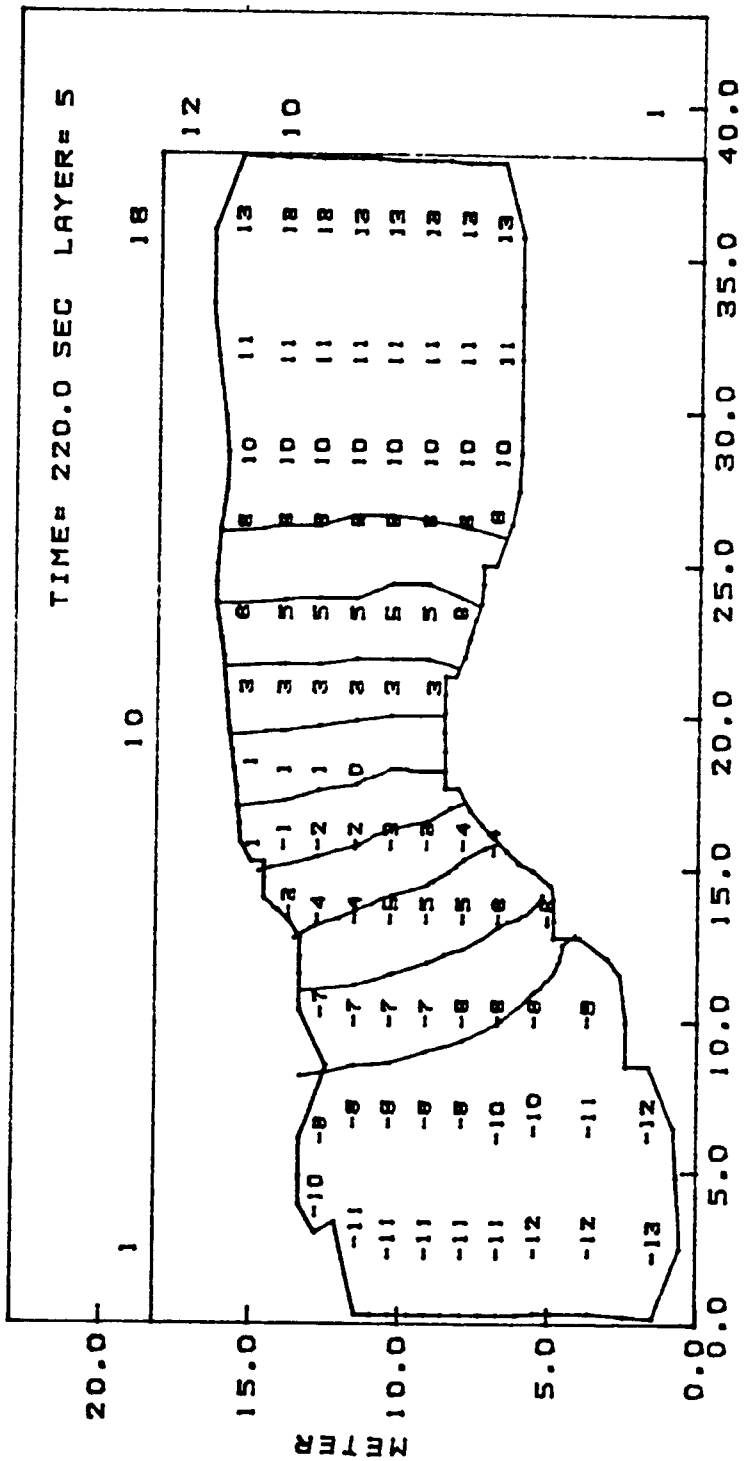

Figure 6.175. Water-surface-elevation conditions at time t = 220 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

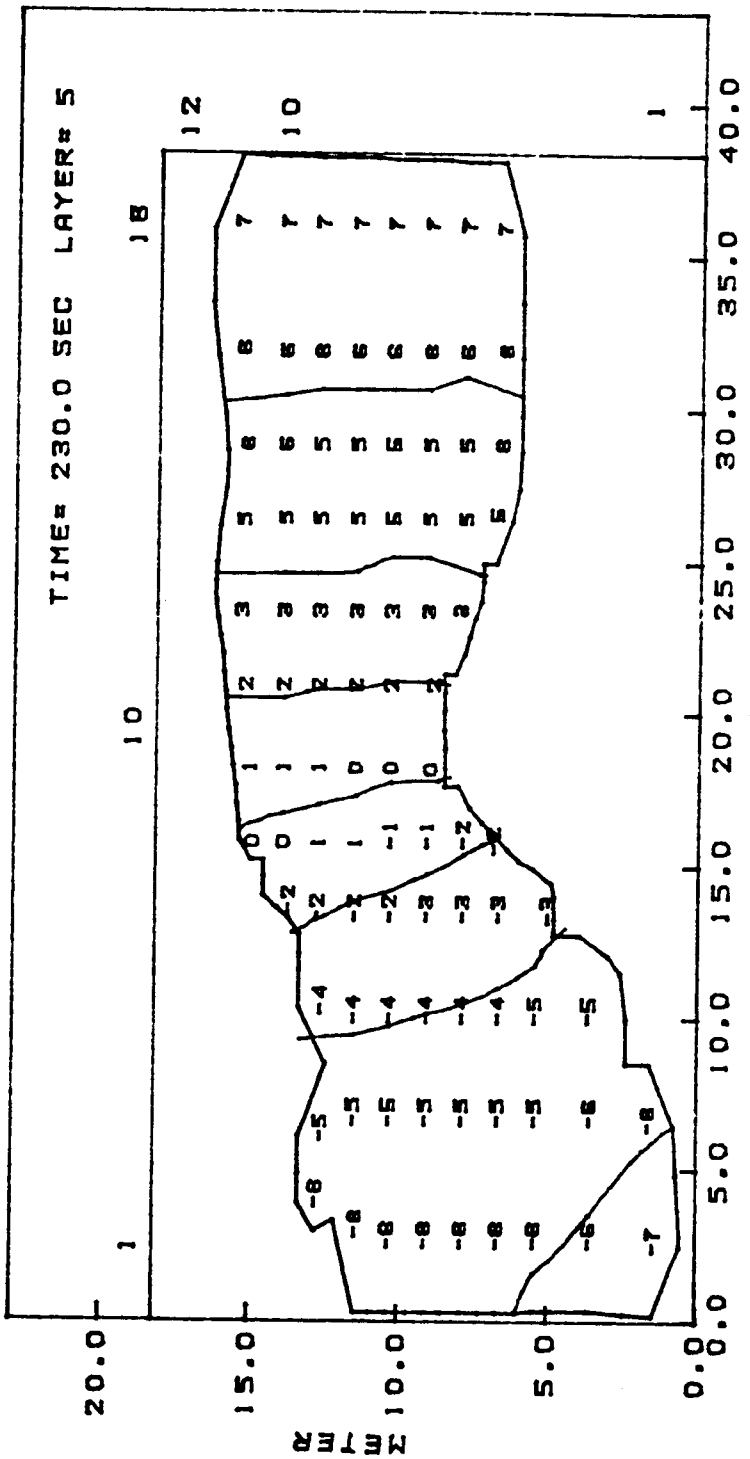

Figure 6.176. Water-surface-elevation conditions at time t = 230 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

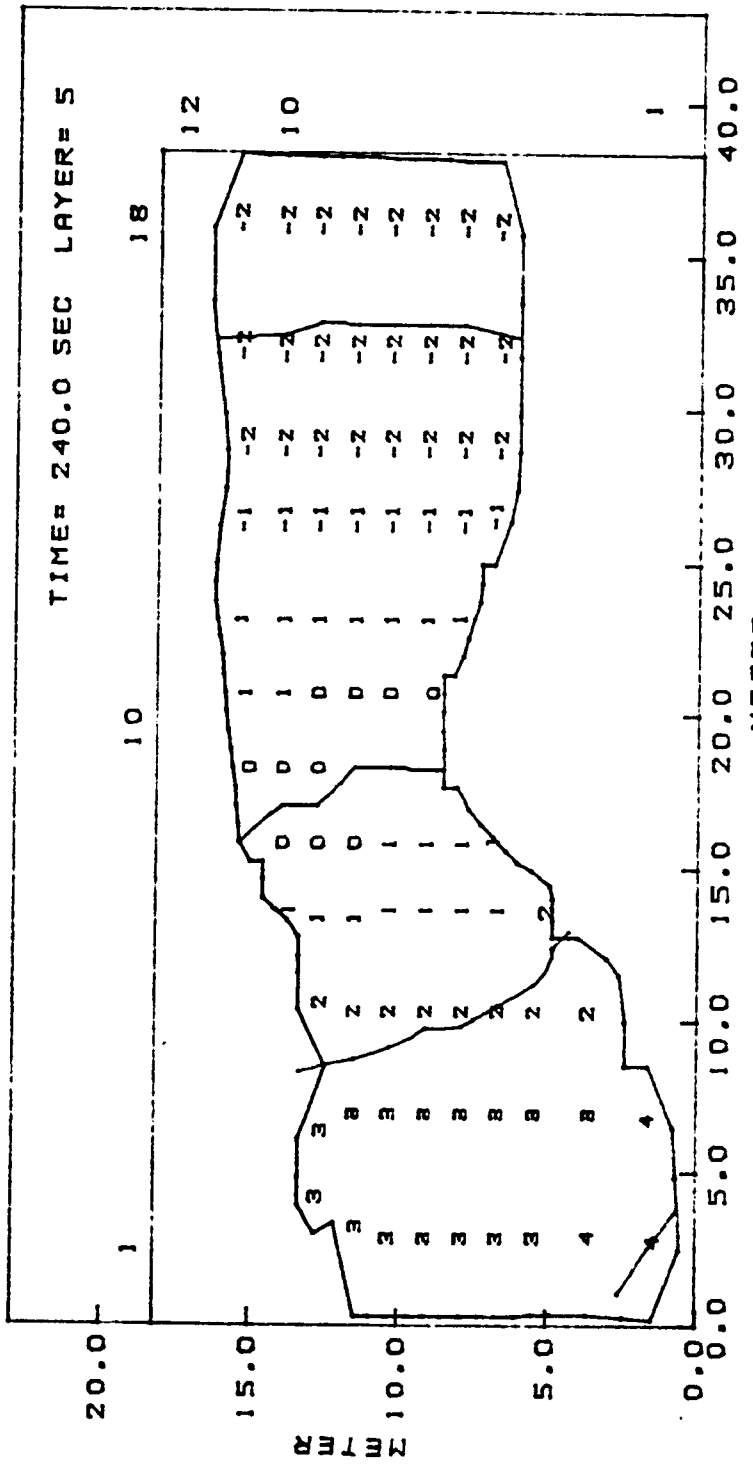

Figure 6.177. Water-surface-elevation conditions at time t = 240 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

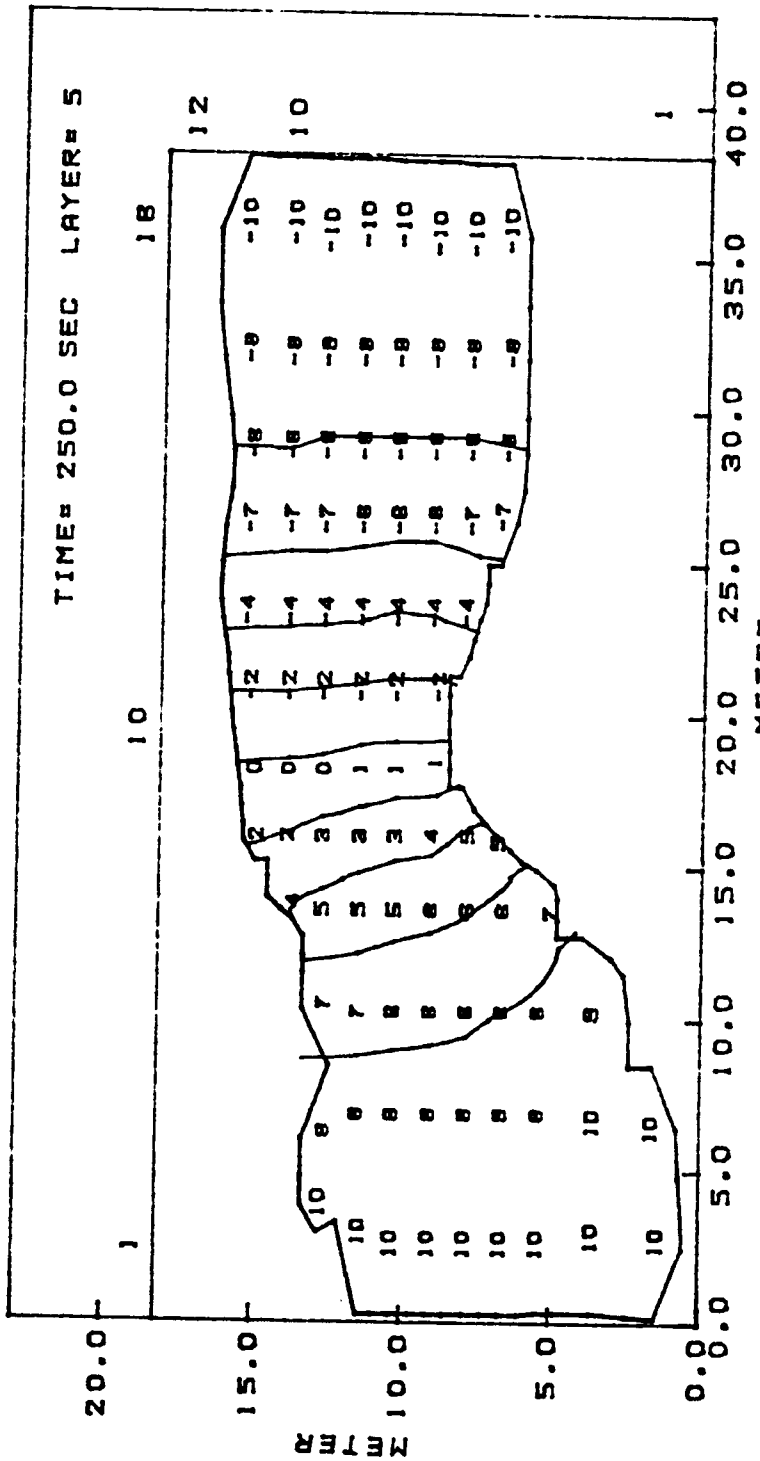

Figure 6.178. Water-surface-elevation conditions at time t = 250 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

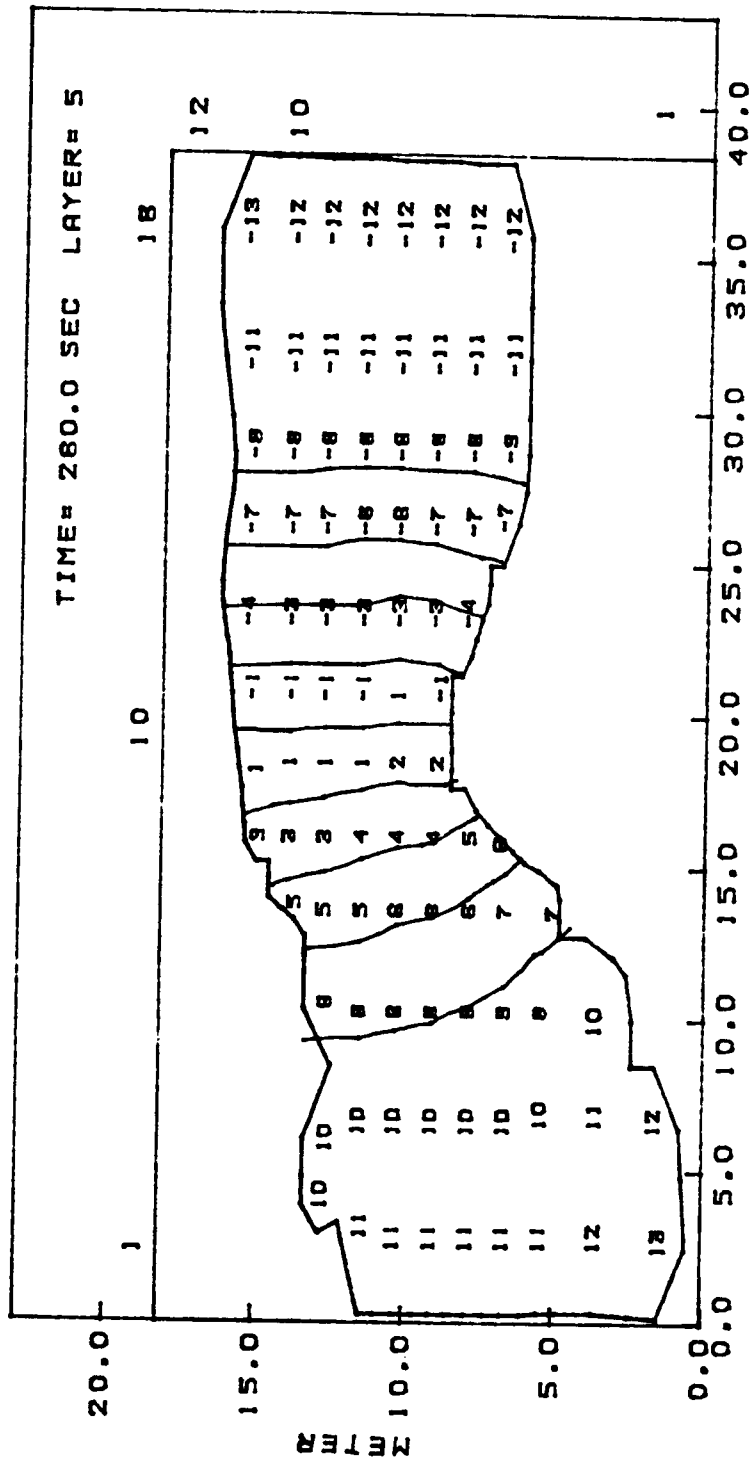

Figure 6.179. Water-surface-elevation conditions at time t = 260 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

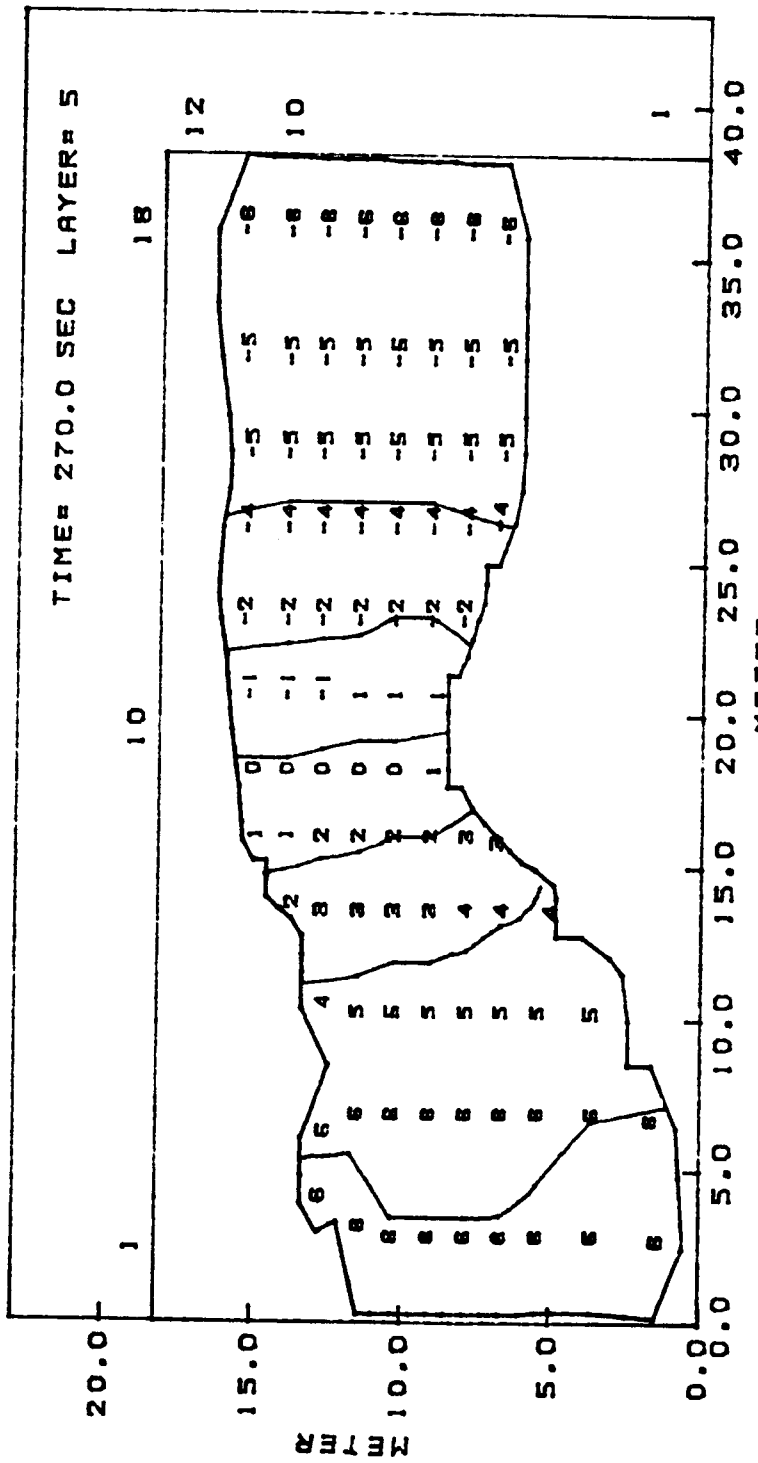

Figure 6.180. Water-surface-elevation conditions at time t = 270 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

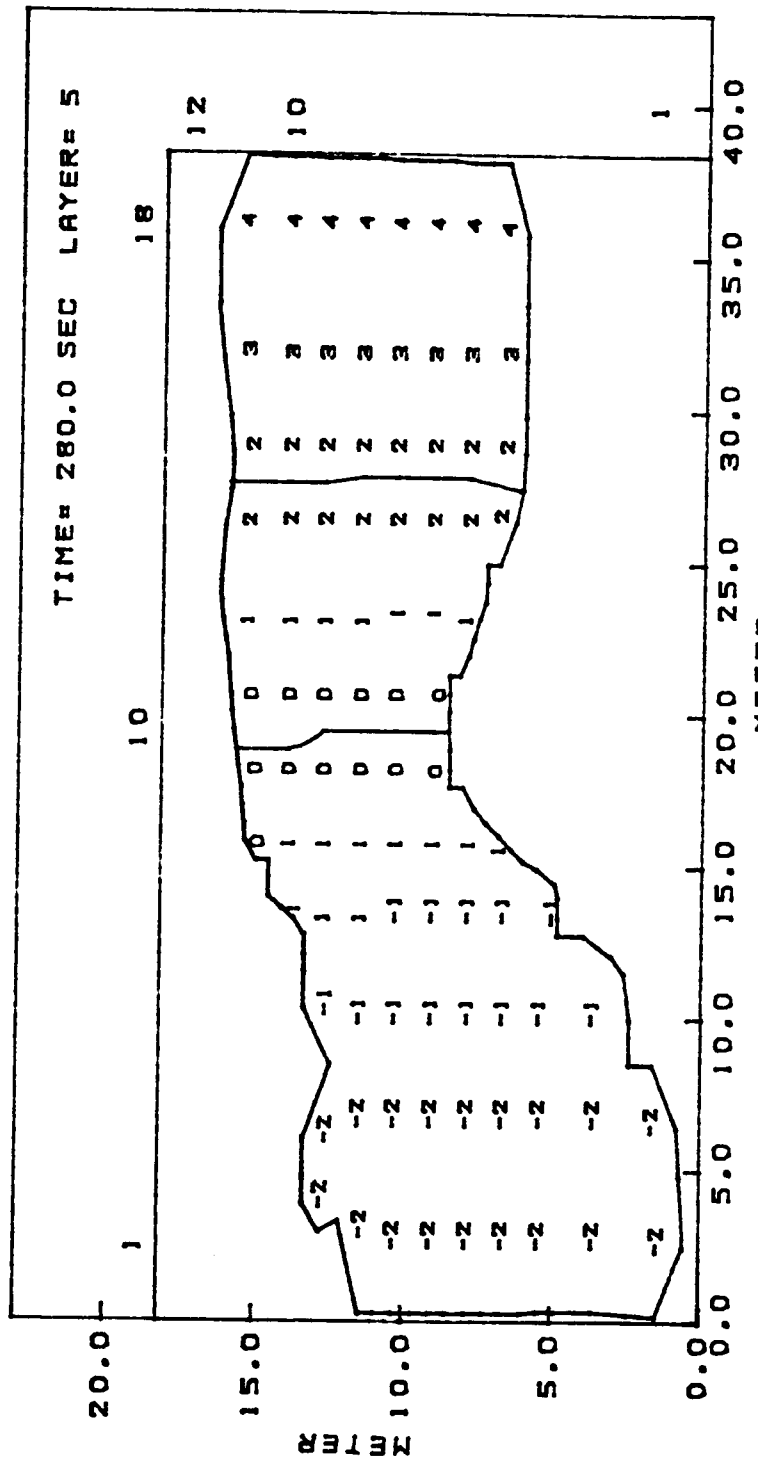

Figure 6.181. Water-surface-elevation conditions at time t = 280 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

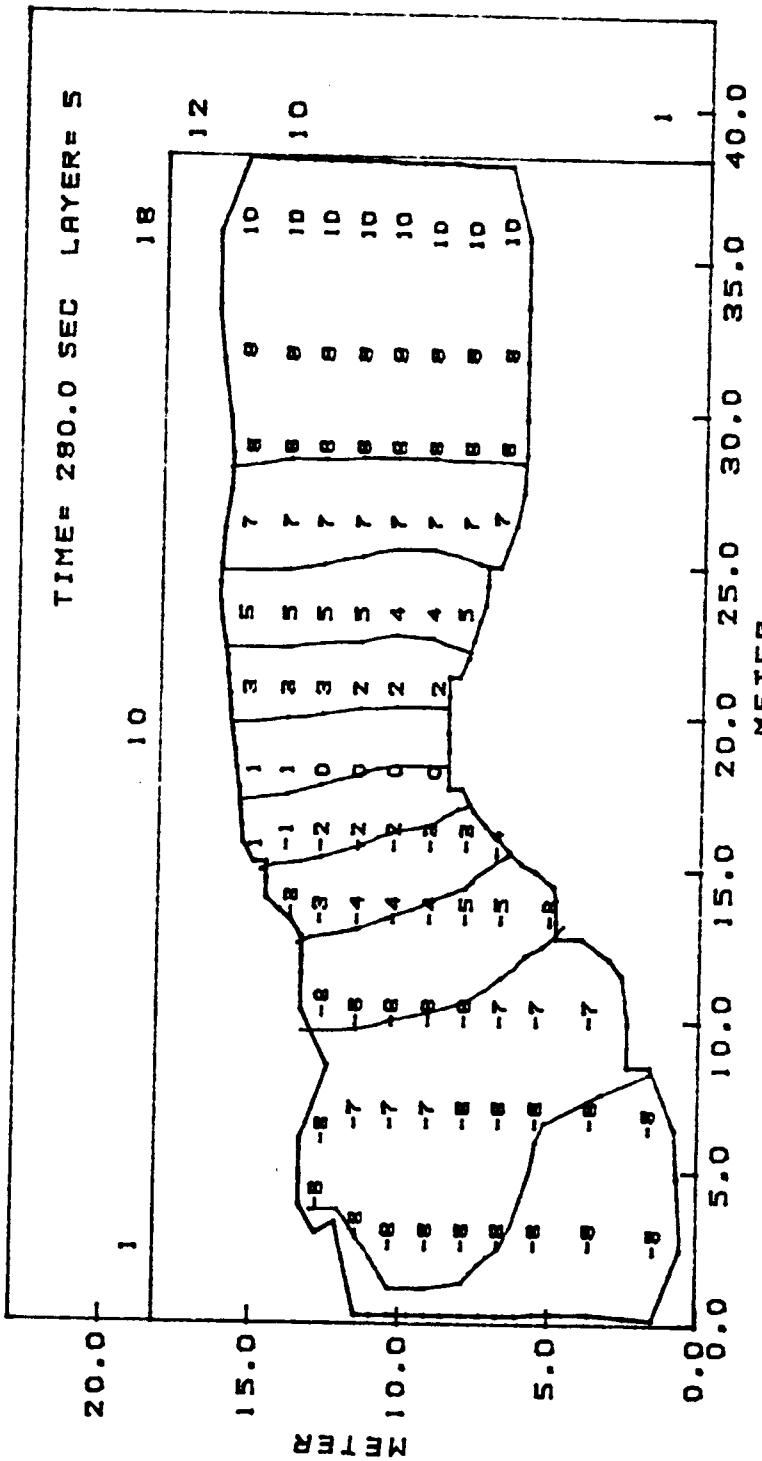

Figure 6.182. Water-surface-elevation conditions at time t = 290 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

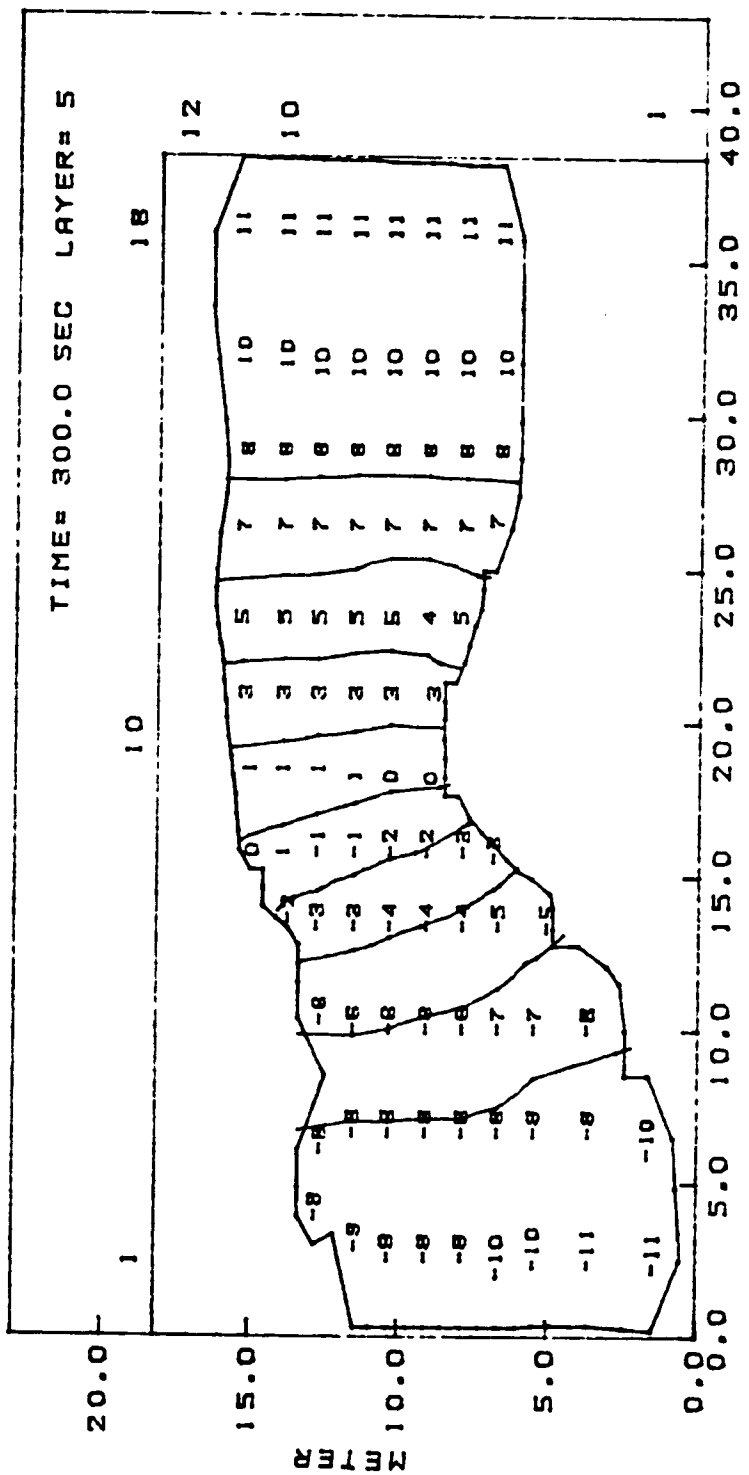

Figure 6.183. Water-surface-elevation conditions at time t = 300 sec in Browns Ferry physical model. Numbers in elements represent water-surface elevation (in mm) relative to specified initial mean water level (MWL). Constant water-surface-elevation contours correspond to 2 mm gradation between -8 mm minimum and 8 mm maximum relative to MWL.

the x-upstream and x-downstream specified-boundary enclosure surfaces of the flow region (Fig. 6.104, p.731) and they proceed along the opposing directions in the main-flow channel. Approximately, at the time level t = 60 sec, the compressive type surface-gravity wave, initiated at the inflow plane at the x-upstream specified-boundary enclosure surface at x = 0, reaches the x-downstream plane at x = 38.4 m, and simultaneously, the rarefaction type surface-gravity wave, initiated at the outflow plane at the x-downstream specified-boundary enclosure surface at x = 38.4 m, reaches the x-upstream plane at x = 0. After the time level t = 70 sec, the compressive type surface-gravity wave, initiated at the inflow plane at the x-upstream specified-boundary enclosure surface at x = 0, reflects from the x-downstream plane at x = 38.4 m and starts traveling along the upstream x-direction, and the rarefaction type surface-gravity wave, initiated at the outflow plane at the x-downstream specified-boundary enclosure surface at x = 38.4 m, reflects from the x-upstream plane at x = 0 and starts traveling along the downstream x-direction (Fig. 6.160, p.799 and Fig.6.161, p.800). Approximately, at the time level t = 100 sec, the compressive type surface-gravity wave, initiated at the inflow plane at the x-upstream specified-boundary enclosure surface at x = 0, returns to the x-upstream plane at x = 0, and approximately simultaneously, the rarefaction type surface-gravity wave, initiated at the outflow plane at the x-downstream specified-boundary enclosure surface at x = 38.4 m, returns to the x-downstream plane at x = 38.4 m (Fig. 6.163,p.802).

The detailed results of the computer simulation for the water-surface-elevation conditions in the Browns Ferry physical model (Figs.

6.184 through 6.189, p.825 through p.830) indicate that traveling-wave phenomena, associated with the compression type surface-gravity wave, initiated at the inflow plane at the x-upstream specified-boundary enclosure surface at x= 0, and the rarefaction type surface gravity wave, initiated at the outflow plane at the x-downstream specified-boundary enclosure surface at x = 38.4 m, traveling in opposite directions, are continuously repeated during the 300 sec duration of the start-up period which corresponds to the period of the computer simulations.

The detailed results of the computer simulations for the water-surface-elevation conditions, which illustrate the existence of two traveling-surface-gravity waves during the start-up period of the flow conditions in the Browns Ferry physical model (Figs. 6.184 through 6.189, p.825 through p.830) can also be used to illustrate the reliability of the fast-transient, three-dimensional, discrete-element hydrodynamic model and its associated HYDROL computer code in predicting the actual physical flow phenomena. In order to emphasize the effect of the simulated surface-gravity wave flow conditions in the laboratory-model-scale flow region, 12 elements, approximately along the main-flow channel of the Browns Ferry physical model, are considered (Figs. 6.184 through 6.189, p.825 through p.830) for presenting the transient variations of the local water-surface-elevation conditions.

The temporal variations of the water-surface-elevation conditions in the selected x-upstream element (i = 1, j = 4) and in the selected x-downstream element (i = 18, j = 8) indicate that the maximum amplitude of the surface-gravity wave is approximately 1.5 mm (Fig.6.184, p.825).

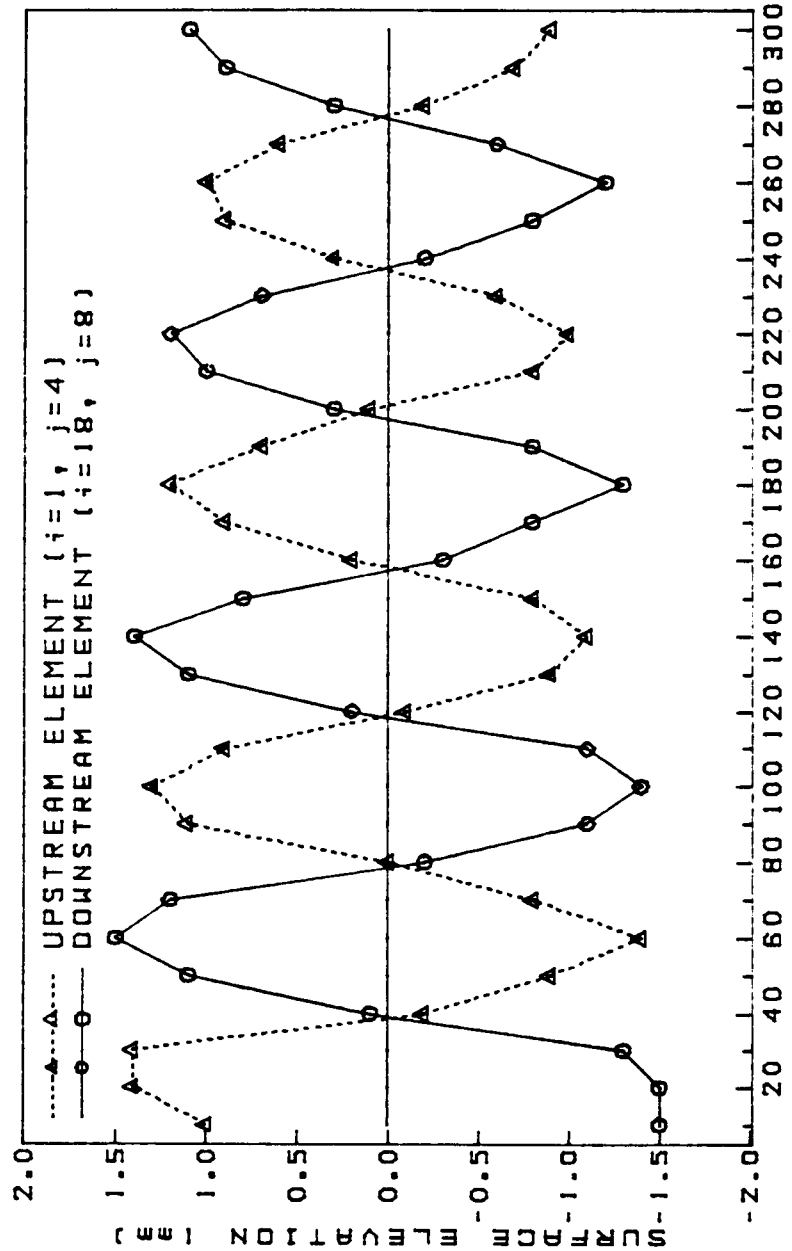

Figure 6.184. Variation of the local water-surface-elevation conditions in Browns Ferry physical model in the upstream element (i = 1, j = 4) and in the downstream element (i = 18, j = 8). Water-surface-elevation values relative to specified initial mean water level (MWL).

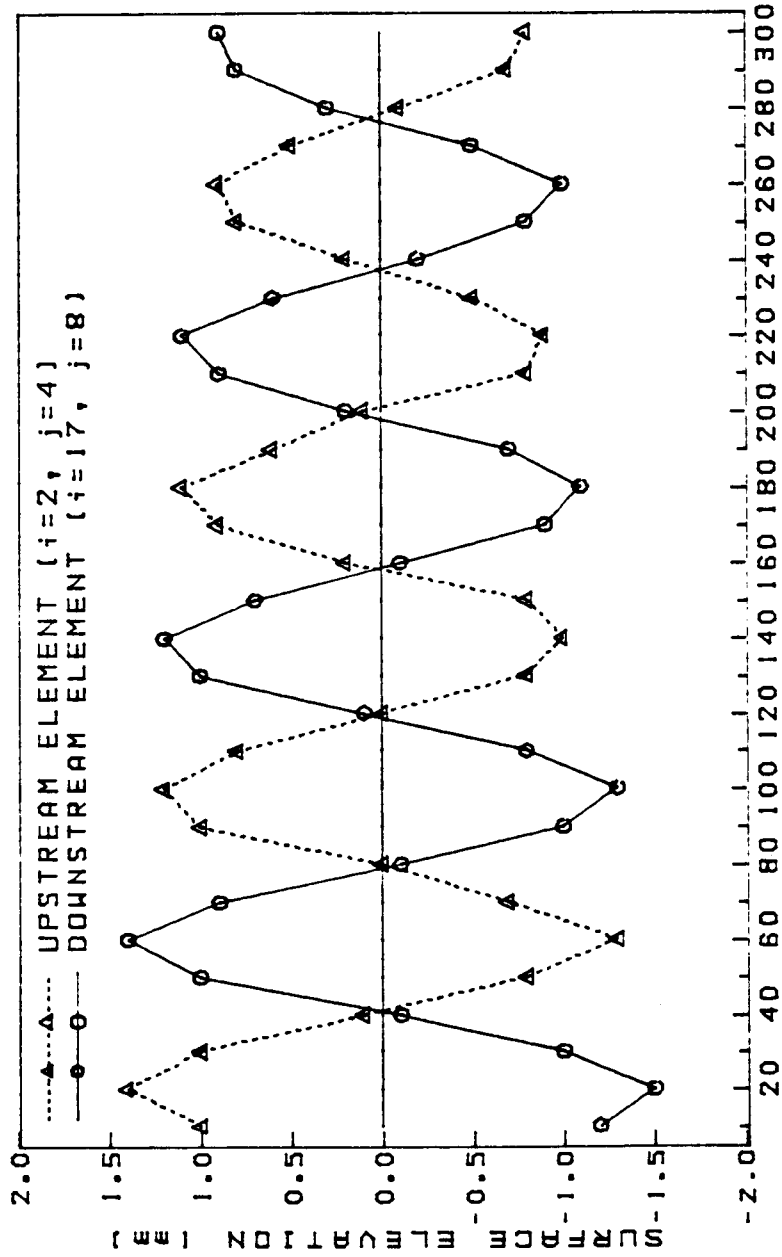

Figure 6.185. Variation of the local water-surface-elevation conditions in Browns Ferry physical model in the upstream element (i = 2, j = 4) and in the downstream element (i = 17, j = 8). Water-surface-elevation values relative to specified initial mean water level (MWL).

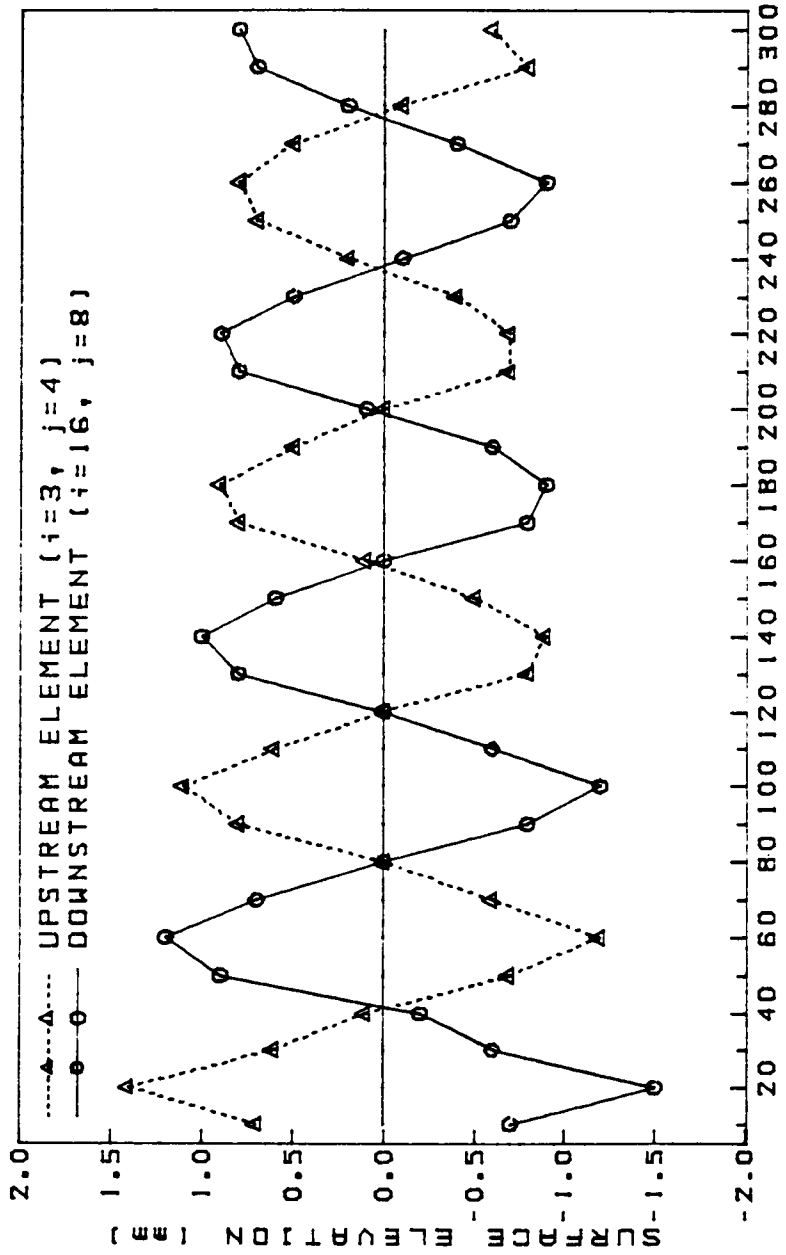

Figure 6.186. Variation of the local water-surface-elevation conditions in Browns Ferry physical model in the upstream element (i = 3, j = 4) and in the downstream element (i = 16, j = 8). Water-surface-elevation values relative to specified initial mean water level (MWL).

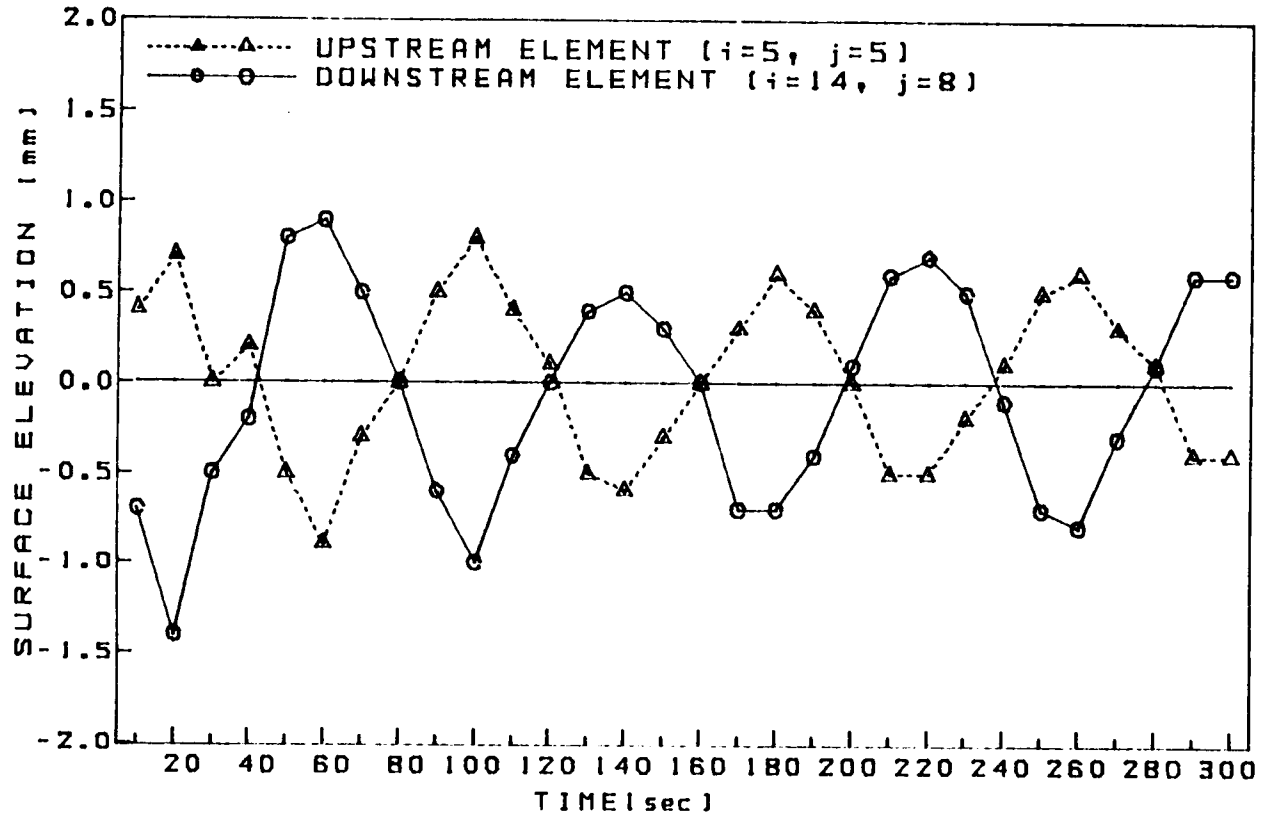

Figure 6.187. Variation of the local water-surface-elevation conditions in Browns Ferry physical model in the upstream element (i = 5, j = 5) and in the downstream element (i = 14, j = 8). Water-surface-elevation values relative to specified initial mean water level (MWL).

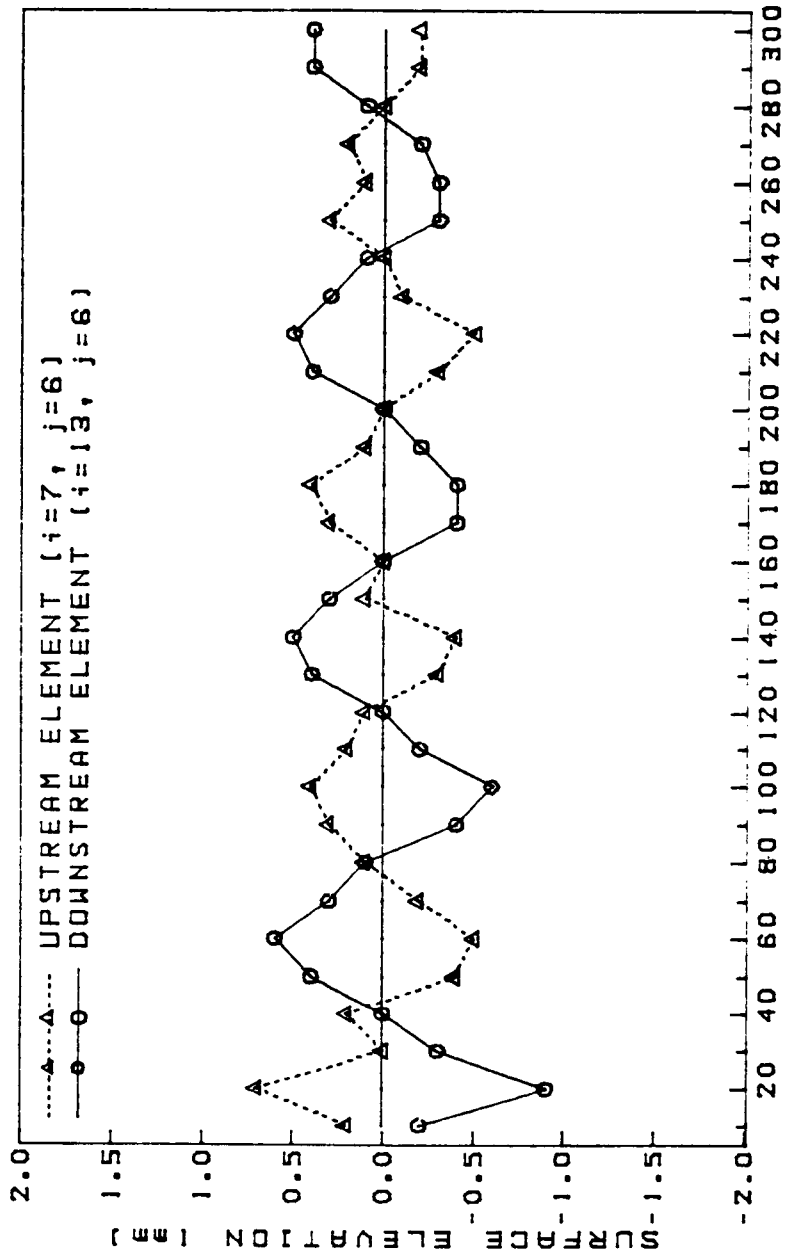

Figure 6.188. Variation of the local water-surface-elevation conditions in Browns Ferry physical model in the upstream element (i = 7, j = 6) and in the downstream element (i = 13, j = 6). Water-surface-elevation values relative to specified initial mean water level (MWL).

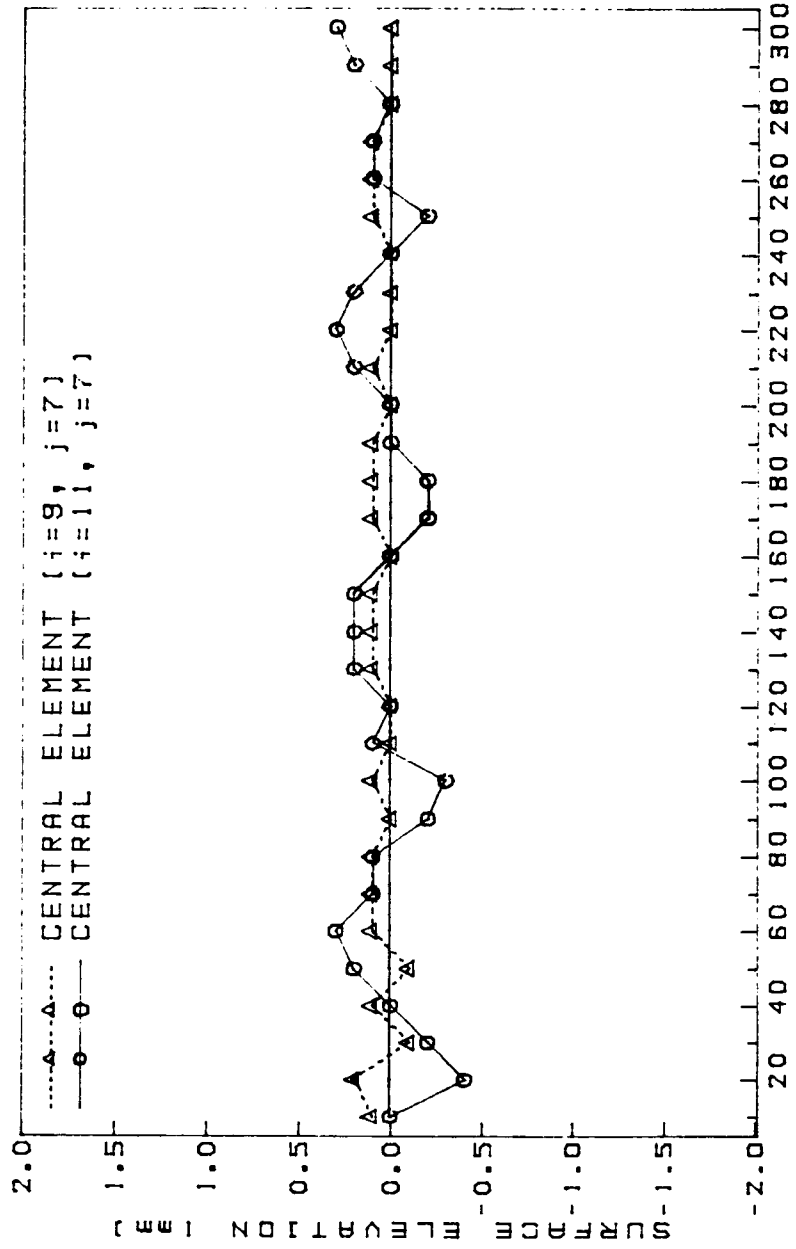

Figure 6.189. Variation of the local water-surface-elevation conditions in Browns Ferry physical model in the upstream element (i = 9, j = 6) and in the downstream element (i = 11, j = 7). Water-surface-elevation values relative to specified initial mean water level (MWL).

The temporal variations of the water-surface-elevation conditions in the selected interior elements, with successively increasing distances from the x-upstream specified-boundary enclosure surface at x = 0 and from the x-downstream specified-boundary enclosure surface at x = 38.4 m, indicate that, as expected, the local amplitudes of the surface-gravity wave gradually decrease toward the middle of the main-flow channel, along the x-direction (Figs. 6.184 through 6.189, p.825 through p.830).

An important quantative verification of the reliability of the fast-transient, three-dimensional, discrete-element hydrodynamic model and its associated HYDROL computer code can be achieved by considering the time periods required for the traveling surface-gravity waves to return to the locations where they were initiated, after traveling between the plane of inflow conditions at the x-upstream specified-boundary enclosure surface at x = 0 and the plane of the outflow conditions at the x-downstream specified-boundary enclosure surface at x = 38.4 m.

The necessary time period for the compressive type surface-gravity wave to complete a cycle, by (1) starting at the plane of the inflow at the x-upstream specified-boundary enclosure surface at x = 0, (2) reflecting from the plane of the outflow at the x-downstream specified-boundary enclosure surface at x = 38.4 m, and (3) returning to the x-upstream enclosure surface, is determined, from the transient variations of the water-surface-elevation conditions in the selected elements (Figs. 6.184 through 6.189, p.825 through p.830), as approximately 77 sec. Similarly, the necessary time period for the rarefaction type surface-gravity wave to complete a cycle, by (1)

starting at the plane of the outflow at the x-downstream specified-boundary enclosure surface at x = 38.4 m, (2) reflecting from the plane of the inflow at the x-upstream specified-boundary enclosure surface at x = 0, and (3) returning to the x-downstream enclosure surface, is also determined, from the transient variations of the water-surface-elevation conditions in the selected elements (Figs. 6.184 through 6.189, p.825 through p.830), as approximately as 77 sec. Hence, during the period of the computer simulations of 300 seconds for the start-up of flow conditions, the results from the HYDROL computer code indicate that a traveling surface-gravity wave would require approximately 77 seconds to traverse twice the length of the main-flow channel between the x-upstream enclosure surface at x = 0 and the x-downstream enclosure surface at x = 38.4 m of the Browns Ferry physical model.

The representative values of the surface-gravity wave can be formulated in four different forms in the flow region of the Browns Ferry physical model. The representative values of the surface-gravity wave can be used to estimate the time required for the wave to traverse twice the length of the main-flow channel independently from the results of the computer simulations obtained by the application of the transient, three-dimensional, discrete-element hydrodynamic model.

The first form of estimating the representative speed of the surface-gravity wave considers the mean depth, based on the areas of the planar elements in the horizontal (x,y) plane of the initial free-surface, as:

$$\overline{D} = \frac{1}{A} \sum_{i,j} \overline{A}_{z;i,j} \overline{D}_{i,j} = 0.1301 \text{ m} \ . \qquad (6.1)$$

Hence, the speed of the surface-gravity wave c1 becomes

$$\bar{C}_1 = \sqrt{g\bar{D}_1} = \sqrt{(9.82\ m/sec^2)(0.1301\ m)} = 1.13\ m/sec\ . \qquad (6.2)$$

Therefore, based on the calculated speed of the surface-gravity wave $\bar{C}_1$ [Eq. (6.2)], the time required for the wave to traverse twice the length of the main-flow channel becomes

$$\bar{t}_1 = \frac{2 \times 38.4\ m}{\bar{C}_1} = \frac{2 \times 38.4\ m}{1.13\ m/sec} \cong 68\ sec\ . \qquad (6.3)$$

The second form of estimating the representative speed of the surface-gravity wave first considers the individual, local values of the speed of the surface-gravity wave $\bar{C}_{i,j}$ in each element, as

$$\bar{C}_{i,j} = \sqrt{g\bar{D}_{i,j}}\ . \qquad (6.4)$$

The representative speed of the surface-gravity wave $\bar{C}_2$ is then estimated as the mean speed of the surface-gravity wave, based on the areas of the planar elements in the horizontal (x,y) plane of the initial free-surface, as:

$$\bar{C}_2 = \frac{1}{A} \sum_{i,j} A_{z;i,j} \bar{C}_{i,j} = \frac{1}{A} \sum_{i,j} A_{z;i,j} \sqrt{g\bar{D}_{i,j}} = 1.1137\ m/sec\ . \qquad (6.5)$$

Therefore, based on the calculated speed of the surface-gravity wave $\bar{C}_2$ [Eq. (6.5)], the time required for the wave to traverse twice the length of the main-flow channel becomes

$$\bar{t}_2 = \frac{2 \times 38.4\ m}{\bar{C}_2} = \frac{2 \times 28.4\ m}{1.1137\ m/sec} = 69\ sec\ . \qquad (6.6)$$

The third form of estimating the representative speed of the surface-gravity wave first considers the mean depth, based on the

dimensions along the x-direction of the planar elements in the horizontal (x,y) plane of the initial free-surface, as:

$$\bar{D}_3 = \frac{1}{L_x} \sum_{i,j} \Delta x_{i,j} \bar{D}_{i,j} = 0.1337 \text{ m.} \qquad (6.7)$$

where L_x represents the length of the main-flow channel along the x-direction as 38.4 m.

Hence, the speed of the surface-gravity wave $\bar{C}_3$ becomes

$$\bar{C}_3 = \sqrt{g\bar{D}_3} = \sqrt{(9.82 \text{ m/s}_2)(0.1337 \text{ m})} = 1.1459 \text{ m/sec.} \qquad (6.8)$$

Therefore, based on the calculated speed of the surface-gravity wave $\bar{C}_3$ [Eq. (6.8)], the time required for the wave to traverse twice the length of the main-flow channel becomes

$$\bar{t}_3 = \frac{2 \times 38.4 \text{ m}}{\bar{C}_3} = \frac{2 \times 38.4 \text{ m}}{1.1459 \text{ m/sec}} = 67 \text{ sec.} \qquad (6.9)$$

Finally, the fourth form of estimating the representative speed of the surface-gravity wave first considers the individual, local values of the speed of the surface-gravity wave $\bar{C}_{i,j}$ in each element [Eq. (6.4)]; and then estimates the mean speed of the surface-gravity wave $\bar{C}_4$, based on the dimensions along the x-direction of the planar elements in the horizontal (x,y) plane of the initial free-surface, as:

$$\bar{C}_4 = \frac{1}{L_x} \sum_{i,j} \Delta x_{i,j} \bar{C}_{i,j} = 1.1297 \text{ m/sec.} \qquad (6.10)$$

Therefore, based on the calculated speed of the surface-gravity wave $\bar{C}_4$ [Eq. (6.10)], the time required for the wave to traverse twice the length of the main-flow channel becomes

$$\bar{t}_4 = \frac{2 \times 38.4\ m}{\bar{C}_4} = \frac{2 \times 38.4\ m}{1.1297\ m/sec} = 68\ sec. \qquad (6.11)$$

The representative values of the speed of the surface-gravity wave, estimated according to four different formulations, are within 2 seconds of each other as 68 sec, 69 sec, 67 sec and 68 sec. The average estimated value of the representative time of the surface-gravity wave to traverse twice the length of the main-flow channel as 68 seconds is 9 seconds less than the approximate effective time of the surface-gravity waveto traverse twice the length of the main-flow channel as 77 seconds, based on the results of the computer simulations of the water-surface-elevation conditions during the start-up period of the flow conditions in the Browns Ferry physical model. The difference of 9 seconds can be related to the frictional effects and the nonlinear interaction effects which tend to slow the traveling speed of the surface-gravity wave in the flow region. Since the results of the computer simulations by the HYDROL computer code includes all the frictional and the nonlinear interaction effects, according to the general formulation of the fast-transient, three-dimensional hydrodynamic model, the effective time of the surface-gravity wave to traverse twice the length of the main flow channel as 77 sec must be higher than the average estimated value of the representative time of the surface-gravity wave to traverse twice the length of the main-flow channel as 68 sec, based only on the depth conditions in the flow region of the Browns Ferry physical model.

The computer simulation results for the transient variations of the water-surface-elevation conditions in the selected elements (Figs. 6.184

through 6.189, p.825 through p.830) also illustrate the existing effect of friction (laminar and turbulent), by indicating the gradually decreasing trend in the amplitudes of the oscillations of the water-surface-elevation in the flow region of the Browns Ferry physical model.

Hence, based on the considerations of the physical effects, the results of the computer simulations of the water-surface-elevation and three-dimensional velocity conditions in the Browns Ferry physical model again illustrate the reliability of the mathematical formulation of the fast-transient, three-dimensioal, discrete-element hydrodynamic model and the numerical accuracy of the computational algorithm of its associated HYDROL computer code in predicting the complex flow conditions which can include the effects of the traveling-waves in laboratory-model-scale flow regions.

7. CONCLUSIONS AND RECOMMENDATIONS

The results of the study concluded that the discrete-element method (DEM) can be successfully applied to the development of a fast-transient, three-dimensional hydrodynamic model for simulating free-surface flow conditions of water (as an incompressible fluid) in regions with geometrically complex enclosure boundaries.

The special geometrical discretization technique of the discrete-element method (DEM) was applied with special modifications for formulating the governing computational system of discrete-element equations for the solutions of the velocity components. The study concluded that the application of the special geometrical discretization technique to the components of the integral form of the momentum principle resulted in a governing computational system of discrete-element equations which was mathematically consistent and extremely stable for numerical integration of the solutions for fast-transient flow conditions.

The formulation of the fast-transient, three-dimensional discrete-element hydrodynamic model included three separate forms of upwind-differencing-interpolation which could be used selectively in its applications to different problems without requiring any modification of its general discrete-element formulation. The special formulation of the transportive-upwind-differencing integration allowed for the minimization of the local, one-dimensional effects of artificial viscosity (as a part of the general numerical dispersion effect) in the numerical integration of the solutions of the velocity components.

The development of the fast-transient, three-dimensional discrete-element hydrodynamic model carefully considered the temporally sub-scale and spatially sub-scale variations of the velocity components, over the enclosure-surface areas of the elements and half-elements, in formulating the nonconvective momentum transport rates. Hence, the general formulation of the discrete-element hydrodynamic model was independent of the particular closure submodel required for the elimination of the turbulent momentum transport rates (turbulent stress components) in terms of the principal flow variables of the solution which include the velocity components. A special closure submodel, based on the local element Reynolds number and the Blasius power-law type distributions, was considered for the formulations of the turbulent stress components acting on the enclosure-surface areas of the elements and half-elements. The special formulation of the turbulent stress components allowed the fast-transient, three-dimensional discrete-element hydrodynamic model to be universally applicable to the simulation of the flow conditions in flow regions with vastly different spatial scales, including micro-scale laboratory models and meso-scale geophysical regions associated with lakes, rivers, reservoirs, estuaries and coastal zones. Furthermore, the special closure submodel for the turbulent stress components allowed the general three-dimensional form to be applicable also to the simulations of reduced one- or two-dimensional flow conditions without requiring any modification of the fast-transient, three-dimensional discrete-element hydrodynamic model.

The development of the numerical integration algorithm of the associated HYDROL computer code of the fast-transient, three-dimensional

discrete-element hydrodynamic model considered a special explicit, composite-time-splitting technique. The special numerical integration algorithm allowed for the minimization of the effects of artificial viscosity, which could be caused by the computations of the convective momentum transport terms, without introducing instability characteristics, which could be caused by the computations of the terms associated with the turbulent momentum transport (turbulent stress components).

The development of the fast-transient, three-dimensional, discrete-element hydrodynamic model also included all the necessary formulations for considering the most general forms of specified conditions at the enclosure boundary surfaces, including (1) temporally varying specified flow conditions on the specified-boundary enclosure surfaces and (2) temporally varying water-surface-elevation conditions, associated with tidal conditions, on the open-region enclosure surfaces.

The results of applications of the fast-transient, three-dimensional discrete-element hydrodynamic model to the simulation of five test cases of flow conditions in a laboratory-model-scale flow region clearly established the reliability of the mathematical formulation and the computational accuracy of its associated HYDROL computer code. The simulations also indicated all the special symmetric and asymmetric characteristics of the flow conditions which were expected to occur in the special symmetric flow region.

The results of application of the fast-transient, three-dimensional discrete-element hydrodynamic model to the actual laboratory-model-scale Browns Ferry physical model clearly illustrated its special capability for simulating the fast-transient flow conditions associated with

traveling surface-gravity waves which occur during the start-up of the flow from initially stagnant conditions in the flow region.

The present form of the fast-transient, three-dimensional discrete-element hydrodynamic model can be readily extended to include the densimetric effects which result from the temporal and spatial variations of water temperature and salinity in stratified lakes, reservoirs, estuaries and coastal zones. The modifications can be incorporated readily by including the necessary product terms associated with the variations of water density with water temperature and salinity in (1) the discrete-element equation for water-surface elevation, (2) the discrete-element equations for the velocity components and (3) the discrete-element equation for pressure. The additional discrete-element equations for water temperature and salinity can be developed readily by applying the general discretization technique of the discrete-element method (DEM) to (1) the principle of conservation of energy and (2) the principle of conservation of salt mass, respectively.

The general formulation of the fast-transient,three-dimensional discrete-element hydrodynamic model can also be applied with minor modifications to the simulation of flow conditions that occur in a large number of problems in engineering, associated with transient flows in elastic tubes (bioengineering) and transient compressible flows in ducts (aerospace engineering). Hence, the discrete-element method (DEM) can be used as a reliable and computationally efficient mathematical tool in the analysis of fast-transient flow problems in engineering and other fields of phycial sciences.

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LIST OF REFERENCES

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APPENDIX

APPENDIX

LISTING OF HYDROL COMPUTER CODE

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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            COMMON/CONTRL/KRUN,T                                          MAIN0001
   ISN 0003            COMMON/CALLER/INCONV,INTURB,IBOUND,INCBOT,NPLANT,INCSUR       MAIN0002
   ISN 0004            COMMON/REGINT/NX,NY,NZ,NIND,NBND,INTEXT(18,12),IJIXJY(18,12),  MAIN0003
                      1                  NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)  MAIN0004
   ISN 0005            COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ( 6),DHZBOT(136)  MAIN0005
   ISN 0006            COMMON/GEOBND/IXBND( 49),JYBND( 49),                          MAIN0006
                      1                  DXBND( 49),DYBND( 49),APHBND( 49)          MAIN0007
   ISN 0007            COMMON/ENCBND/IMHBND( 49),DYBIMH( 49),IPHBND( 49),DYBIPH( 49),  MAIN0008
                      1                  JMHBND( 49),DXBJMH( 49),JPHBND( 49),DXBJPH( 49)  MAIN0009
   ISN 0008            COMMON/HSRSOL/HSR(136),PREHSR(136),ENCHSR(136)                MAIN0010
   ISN 0009            COMMON/VLXSOL/VLX(272, 6),PREVLX(272, 6),ENCVLX(272, 6)       MAIN0011
   ISN 0010            COMMON/VLYSOL/VLY(272, 6),PREVLY(272, 6),ENCVLY(272, 6)       MAIN0012
   ISN 0011            COMMON/FLOVLZ/QLZLZH(136, 6),ENCQVZ(136, 6)                   MAIN0013
   ISN 0012            COMMON/UPWIND/IUWIND,MAWIND,MYWIND,MZWIND                     MAIN0014
   ISN 0013            COMMON/ANGLES/SINBNX,SINBNY,COSBN7                            MAIN0015
   ISN 0014            COMMON/BNDCON/ISPBND( 49),IXBSPB(17),JYBSPB(17),              MAIN0016
                      1                  HSRBXH(17),QFXBXH(17, 6),IVXBXH(17, 6),    MAIN0017
                      2                  VLXBXH(17, 6),VMYBXH(17, 6),VPYBXH(17, 6), MAIN0018
                      3                  VLZBXH(17, 6),FPRBXH(17, 6),               MAIN0019
                      4                  HSRBYH(17),QFYBYH(17, 6),IVYBYH(17, 6),    MAIN0020
                      5                  VLYBYH(17, 6),VMXBYH(17, 6),VPXBYH(17, 6), MAIN0021
                      6                  VLZBYH(17, 6),FPRBYH(17, 6)                MAIN0022
   ISN 0015             KRUN=0                                                      MAIN0023
   ISN 0016             READ(20,2001)NT,NTW,DTHR,INPUTS                             MAIN0024
   ISN 0017        2001 FORMAT(3X,I6,2X,I6,4X,E13.6,4X,I2)                          MAIN0025
   ISN 0018             CALL REGION(INPUTS)                                         MAIN0026
   ISN 0019             CALL SOLVER(0.0,0.0)                                        MAIN0027
   ISN 0020             CALL OUTPUT                                                 MAIN0028
   ISN 0021             CALL INITAL                                                 MAIN0029
   ISN 0022             CALL BOUNDS                                                 MAIN0030
   ISN 0023             CALL BOTTOM                                                 MAIN0031
   ISN 0024             CALL FLOWUP(0.0)                                            MAIN0032
   ISN 0025             CALL CONVEC(0.0)                                            MAIN0033
   ISN 0026             CALL PLANTS(0)                                              MAIN0034
   ISN 0027             CALL SURFAC(0)                                              MAIN0035
   ISN 0028             CALL TURBUL                                                 MAIN0036
   ISN 0029             CALL FORCER                                                 MAIN0037
   ISN 0030             KRUN=1                                                      MAIN0038
   ISN 0031             ITW=0                                                       MAIN0039
   ISN 0032             DT=DTHR                                                     MAIN0040
   ISN 0033             DO 2000 IT=1,NT                                             MAIN0041
   ISN 0034             TS=T                                                        MAIN0042
   ISN 0035             ITW=ITW+1                                                   MAIN0043
   ISN 0036             CALL SOLVER(TS,DT)                                          MAIN0044
   ISN 0037             IF(ITW.LT.NTW)GO TO 2000                                    MAIN0045
   ISN 0039             ITW=0                                                       MAIN0046
   ISN 0040             CALL OUTPUT                                                 MAIN0047
   ISN 0041        2000 CONTINUE                                                    MAIN0048
   ISN 0042             KRUN=2                                                      MAIN0049
   ISN 0043             CALL OUTPUT                                                 MAIN0050
   ISN 0044        9999 CONTINUE                                                    MAIN0051
   ISN 0045             STOP                                                        MAIN0052
   ISN 0046             END                                                         MAIN0053

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
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*STATISTICS*  NO  DIAGNOSTICS GENERATED
****** END OF COMPILATION ******
*STATISTICS*  NO  DIAGNOSTICS THIS STEP
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
 ISN 0002           SUBROUTINE REGION(INPUTS)                                    RGON0001
 ISN 0003           COMMON/CONTRL/KRUN,T                                         RGON0002
 ISN 0004           COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,12),IJIXJY(18,12),RGON0003
                   1                NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)RGON0004
 ISN 0005           COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ( 6),DHZBOT(136) RGON0005
 ISN 0006           COMMON/GEOBND/IXBND( 49),JYBND( 49),                         RGON0006
                   1                DXBND( 49),DYBND( 49),ARHBND( 49)           RGON0007
 ISN 0007           COMMON/ENGBND/IHHBND( 49),DYBIHH( 49),IPHBND( 49),DYBIPH( 49),RGON0008
                   1                JMHBND( 49),DXBJHH( 49),JPHBND( 49),DXBJPH( 49)RGON0009
 ISN 0008           DIMENSION DHZTRX(12)                                         RGON0010
 ISN 0009           READ(20,1001)NX,NY,NZ,NBND,NINJ                              RGON0011
 ISN 0010      1001 FORMAT(6X,3(2X,I3),4X,I4,4X,I5)                              RGON0012
 ISN 0011           READ(20,1002)(DZ(LZ),LZ=1,NZ)                                RGON0013
 ISN 0012      1002 FORMAT(6(1X,E11.4))                                          RGON0014
 ISN 0013           READ(20,1003)ICNDHZ,DHZMIN,DHZRAT                            RGON0015
 ISN 0014      1003 FORMAT(8X,I2,2X,E11.4,2X,F5.2)                               RGON0016
 ISN 0015           NXYMIN=MINO(NX,NY)                                           RGON0017
 ISN 0016           NXYMAX=MAXO(NX,NY)                                           RGON0018
 ISN 0017           DO 1100 IJ=1,NXYMIN                                          RGON0019
 ISN 0018           IX=IJ                                                        RGON0020
 ISN 0019           JY=IJ                                                        RGON0021
 ISN 0020           READ(20,1101)DX(IX),NREACH(IX),              ((NJYMIN(IX,JRCH),RGON0022
                   1NJYMAX(IX,JRCH)),JRCH=1,6),DY(JY)                           RGON0023
 ISN 0021      1101 FORMAT(6X,E13.6,1X,I2,2X,6(1X,2I3),1X,E13.6)                 RGON0024
 ISN 0022      1100 CONTINUE                                                     RGON0025
 ISN 0023           IF(NX.LT.NY)GO TO 1299                                       RGON0026
 ISN 0025           NXMIN=NXYMIN+1                                               RGON0027
 ISN 0026           DO 1200 IX=NXMIN,NX                                          RGON0028
 ISN 0027           READ(20,1201)DX(IX),NREACH(IX),              .(NJYMIN(IX,JRCH),RGON0029
                   1NJYMAX(IX,JRCH)),JRCH=1,6)                                  RGON0030
 ISN 0028      1201 FORMAT(6X,E13.6,1X,I2,2X,6(1X,2I3),1X,E13.6)                 RGON0031
 ISN 0029      1200 CONTINUE                                                     RGON0032
 ISN 0030      1299 CONTINUE                                                     RGON0033
 ISN 0031           IF(NX.GE.NY)GO TO 1399                                       RGON0034
 ISN 0033           NYMIN=NXYMIN+1                                               RGON0035
 ISN 0034           DO 1300 JY=NYMIN,NY                                          RGON0036
 ISN 0035           READ(20,1301)DY(JY)                                          RGON0037
 ISN 0036      1301 FORMAT(67X,E13.6)                                            RGON0038
 ISN 0037      1300 CONTINUE                                                     RGON0039
 ISN 0038      1399 CONTINUE                                                     RGON0040
 ISN 0039           DO 1400 IBND=1,NBND                                          RGON0041
 ISN 0040           READ(20,1401)IXBND(IBND),JYBND(IBND),              DXBND(IBND),RGON0042
                   1DYBND(IBND),ARHBND(IBND)                                    RGON0043
 ISN 0041      1401 FORMAT(6X,2(2X,I3),4X,2(2X,E13.6),6X,E13.6)                  RGON0044
 ISN 0042           READ(20,1402)IHHBND(IBND),DYBIHH(IBND),IPHBND(IBND),DYBIPH(IBND),RGON0045
                   1                JMHBND(IBND),DXBJHH(IBND),JPHBND(IBND),DXBJPH(IBND)RGON0046
 ISN 0043      1402 FORMAT(6X,2(2X,I2,1X,E13.6),2X,2(2X,I2,1X,E13.6))            RGON0047
 ISN 0044      1400 CONTINUE                                                     RGON0048
 ISN 0045           DO 1500 IX=1,NX                                              RGON0049
 ISN 0046           DO 1520 JY=1,NY                                              RGON0050
 ISN 0047           INTEXT(IX,JY)=0                                              RGON0051
 ISN 0048           IJIXJY(IX,JY)=0                                              RGON0052
 ISN 0049      1520 CONTINUE                                                     RGON0053
 ISN 0050      1500 CONTINUE                                                     RGON0054
 ISN 0051           DO 1540 IJ=1,NINJ                                            RGON0055
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ISN 0052          DHZBOT(IJ)=-0.3333E+33                                        RGDN0056
ISN 0053       1540 CONTINUE                                                    RGDN0057
ISN 0054          IJ=0                                                          RGDN0058
ISN 0055          DO 1600 IX=1,NX                                               RGDN0059
ISN 0056          NRCH=NREACH(IX)                                               RGDN0060
ISN 0057          IF(NRCH.LT.1)GO TO 1600                                       RGDN0061
ISN 0059          DO 1620 IRCH=1,NRCH                                           RGDN0062
ISN 0060          NYMIN=NJYMIN(IX,IRCH)                                         RGDN0063
ISN 0061          NYMAX=NJYMAX(IX,IRCH)                                         RGDN0064
ISN 0062          NJ=NYMAX-NYMIN+1                                              RGDN0065
ISN 0063          READ(20,1641)(DHZTRX(JJ),JJ=1,NJ)                             RGDN0066
ISN 0064       1641 FORMAT(8X,6(1X,E11.4))                                      RGDN0067
ISN 0065          DO 1660 JJ=1,NJ                                               RGDN0068
ISN 0066          IJ=IJ+1                                                       RGDN0069
ISN 0067          JY=JJ+NYMIN-1                                                 RGDN0070
ISN 0068          IJIXJY(IX,JY)=IJ                                              RGDN0071
ISN 0069          INTEXT(IX,JY)=-1                                              RGDN0072
                C********** DHZBOT(IJ) USED TO STORE MWL-DEPTH IN THE REGION  **********RGDN0073
ISN 0070          DHZBOT(IJ)=DHZTRX(JJ)                                         RGDN0074
ISN 0071       1660 CONTINUE                                                    RGDN0075
ISN 0072       1620 CONTINUE                                                    RGDN0076
ISN 0073       1600 CONTINUE                                                    RGDN0077
ISN 0074          DO 1700 IBND=1,NBND                                           RGDN0078
ISN 0075          IX=IXBND(IBND)                                                RGDN0079
ISN 0076          JY=JYBND(IBND)                                                RGDN0080
ISN 0077          INTEXT(IX,JY)=IBND                                            RGDN0081
ISN 0078       1700 CONTINUE                                                    RGDN0082
                C********** START CALCULATING THE DEPTH OF THE BOTTOM ELEMENTS DHZBOT(IJ)RGDN0083
ISN 0079          DO 2100 IX=1,NX                                               RGDN0084
ISN 0080          DO 2200 JY=1,NY                                               RGDN0085
ISN 0081          IF(INTEXT(IX,JY).EQ.0)GO TO 2200                              RGDN0086
ISN 0083          IJ=IJIXJY(IX,JY)                                              RGDN0087
ISN 0084          DHZIAJ=DHZBOT(IJ)                                             RGDN0088
ISN 0085          DHZSUM=0.0                                                    RGDN0089
ISN 0086          DO 2220 LZD=1,NZ                                              RGDN0090
ISN 0087          LZ=NZ-LZD+1                                                   RGDN0091
ISN 0088          DHZSUM=DHZSUM+DZ(LZ)                                          RGDN0092
ISN 0089          IF(DHZSUM.GE.DHZIAJ)GO TO 2229                                RGDN0093
ISN 0091       2220 CONTINUE                                                    RGDN0094
ISN 0092       2229 CONTINUE                                                    RGDN0095
ISN 0093          NLZBOT(IJ)=LZ                                                 RGDN0096
ISN 0094          DHZBOT(IJ)=DHZIAJ-DHZSUM+DZ(LZ)                               RGDN0097
ISN 0095          IF(ICNDHZ.LT.1)GO TO 2200                                     RGDN0098
ISN 0097          IF(ICNDHZ.GT.1)GO TO 2240                                     RGDN0099
ISN 0099          IF(DHZBOT(IJ).GE.DHZMIN)GO TO 2200                            RGDN0100
ISN 0101          LZ=NLZBOT(IJ)                                                 RGDN0101
ISN 0102          IF(LZ.EQ.NZ)GO TO 9005                                        RGDN0102
ISN 0104          NLZBOT(IJ)=LZ+1                                               RGDN0103
ISN 0105          DHZBOT(IJ)=DZ(LZ+1)                                           RGDN0104
ISN 0106          GO TO 2200                                                    RGDN0105
ISN 0107       2240 CONTINUE                                                    RGDN0106
ISN 0108          LZ=NLZBOT(IJ)                                                 RGDN0107
ISN 0109          RATDHZ=DHZBOT(IJ)/DZ(LZ)                                      RGDN0108
ISN 0110          IF(RATDHZ.GT.DHZRAT)GO TO 2245                                RGDN0109
ISN 0112          IF(LZ.EQ.NZ)GO TO 9005                                        RGDN0110
ISN 0114          NLZBOT(IJ)=LZ+1                                               RGDN0111
ISN 0115          DHZBOT(IJ)=DZ(LZ+1)                                           RGDN0112
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ISN 0116            GO TO 2200                                                    RGON0113
ISN 0117         2245 CONTINUE                                                    RGON0114
ISN 0118            DHZBOT(IJ)=DZ(LZ)                                             RGON0115
ISN 0119         2200 CONTINUE                                                    RGON0116
ISN 0120         2100 CONTINUE                                                    RGON0117
ISN 0121            IF(INPUTS.LT.1)GO TO 9999                                     RGON0118
                 C********** START WRITING GEOMETRY DATA                **********RGON0119
ISN 0123            DO 3000 LZ=1,NZ                                               RGON0120
ISN 0124            WRITE(6,3001)                                                 RGON0121
ISN 0125            IF(LZ.GT.1)GO TO 3010                                         RGON0122
ISN 0127            WRITE(6,3002)LZ,LZ,DZ(LZ)                                     RGON0123
ISN 0128            WRITE(6,3003)                                                 RGON0124
ISN 0129            GO TO 3050                                                    RGON0125
ISN 0130         3010 CONTINUE                                                    RGON0126
ISN 0131            IF(LZ.EQ.NZ)GO TO 3020                                        RGON0127
ISN 0133            WRITE(6,3012)LZ,LZ,DZ(LZ)                                     RGON0128
ISN 0134            WRITE(6,3003)                                                 RGON0129
ISN 0135            GO TO 3050                                                    RGON0130
ISN 0136         3020 CONTINUE                                                    RGON0131
ISN 0137            WRITE(6,3022)LZ,LZ,DZ(LZ)                                     RGON0132
ISN 0138            WRITE(6,3003)                                                 RGON0133
ISN 0139         3001 FORMAT(1H1,25X,10(1H*),1X,55HDISCRETE ELEMENTS LISTED IN LAYERS ONRGON0134
                     1 HORIZONTAL PLANES,1X,10(1H*))                           RGON0135
ISN 0140         3002 FORMAT(1H1,10X,8(1H*),1X,4HLZ =,I3,1X,4(1H*),1X,34HBOTTOM LAYER WIRGON0136
                     1TH LAYER DEPTH, DZ(,I3,3H) =,E11.4,1X,10(1H*))           RGON0137
ISN 0141         3003 FORMAT(1HO,3X,4H*IX*,4X,4H*JY*,4X,4H*IJ*,3X,15H*INTEXT(IX,JY)*,6X,RGON0138
                     18H*XLGRID*,6X,8H*YLGRID*,6X,7H*DXIAJ*,7X,7H*DYIAJ*,7X,8H*ARHIAJ*,RGON0139
                     2/)                                                       RGON0140
ISN 0142         3012 FORMAT(1H1,10X,8(1H*),1X,4HLZ =,I3,1X,4(1H*),1X,36HINTERIOR LAYER RGON0141
                     1WITH LAYER DEPTH, DZ(,I3,3H) =,E11.4,1X,10(1H*))         RGON0142
ISN 0143         3022 FORMAT(1H1,10X,8(1H*),1X,4HLZ =,I3,1X,4(1H*),1X,35HSURFACE LAYER WRGON0143
                     11TH LAYER DEPTH, DZ(,I3,3H) =,E11.4,1X,10(1H*))          RGON0144
ISN 0144         3050 CONTINUE                                                    RGON0145
ISN 0145            NLTIJ=0                                                       RGON0146
ISN 0146            XLGRID=0.0                                                    RGON0147
ISN 0147            DO 3100 IX=1,NX                                               RGON0148
ISN 0148            XLGRID=XLGRID+0.5*DX(IX)                                      RGON0149
ISN 0149            YLGRID=0.0                                                    RGON0150
ISN 0150            DO 3200 JY=1,NY                                               RGON0151
ISN 0151            YLGRID=YLGRID+0.5*DY(JY)                                      RGON0152
ISN 0152            IJ=IJIXJY(IX,JY)                                              RGON0153
ISN 0153            IF(INTEXT(IX,JY).EQ.0)GO TO 3500                              RGON0154
ISN 0155            NLZMIN=NLZBOT(IJ)                                             RGON0155
ISN 0156            IF(LZ.LT.NLZMIN)GO TO 3900                                    RGON0156
ISN 0158            DHZIAJ=DHZBOT(IJ)                                             RGON0157
ISN 0159            IF(INTEXT(IX,JY).GT.0)GO TO 3300                              RGON0158
                 C********** INTERNAL ELEMENT GEOMETRIC CONDITIONS           **********RGON0159
ISN 0161            NLTIJ=NLTIJ+1                                                 RGON0160
ISN 0162            DXIAJ=DX(IX)                                                  RGON0161
ISN 0163            DYIAJ=DY(JY)                                                  RGON0162
ISN 0164            ARHIAJ=DXIAJ*DYIAJ                                            RGON0163
ISN 0165            WRITE(6,3201)IX,JY,IJ,INTEXT(IX,JY),          XLGRID,YLGRID,  RGON0164
                    1DXIAJ,DYIAJ,ARHIAJ                                        RGON0165
ISN 0166         3201 FORMAT(1HO,4X,I3,3X,I3,4X,I5,6X,I2,11H :INTERIOR,2X,6(3X,E11.4)) RGON0166
ISN 0167            WRITE(6,3202)                                                 RGON0167
ISN 0168         3202 FORMAT(1H ,10X,11HENCLOSURE :,30H* ALL FOUR SURFACES INTERIOR *,RGON0168
                     131H* STANDARD ELEMENT DIMENSIONS *)                      RGON0169
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ISN 0169          IF(IJ.LT.1)GO TO 9000                                                  RGON0170
ISN 0171          GO TO 3900                                                             RGON0171
ISN 0172       3300 CONTINUE                                                             RGON0172
               C********** BOUNDARY ELEMENT GEOMETRIC CONDITIONS          **********RGON0173
ISN 0173          IBND=INTEXT(IX,JY)                                                     RGON0174
ISN 0174          NLTIJ=NLTIJ+1                                                          RGON0175
ISN 0175          DXIAJ=DXBND(IBND)                                                      RGON0176
ISN 0176          DYIAJ=DYBND(IBND)                                                      RGON0177
ISN 0177          XLCBND=XLGRID+0.5*DXIAJ-0.5*DX(IX)                                     RGON0178
ISN 0178          YLCBND=YLGRID+0.5*DYIAJ-0.5*DY(JY)                                     RGON0179
ISN 0179          ARHIAJ=ARHBND(IBND)                                                    RGON0180
ISN 0180          WRITE(6,3301)IX,JY,IJ,INTEXT(IX,JY),          XLCBND,YLCBND,           RGON0181
                 1DXIAJ,DYIAJ,ARHIAJ                                                     RGON0182
ISN 0181       3301 FORMAT(1HO,4X,I3,3X,I3,4X,I5,6X,I2,11H : BOUNDARY,2X,6(3X,E11.4)) RGON0183
ISN 0182          WRITE(6,3302)IBND,IPHBND(IBND),DYEIPH(IBND),                           RGON0184
                 1                IPHBND(IBND),DYEIPH(IBND),                           RGON0185
                 2                JMHBND(IBND),DXEJMH(IBND),                           RGON0186
                 3                JPHBND(IBND),DXEJPH(IBND)                            RGON0187
ISN 0183       3302 FORMAT(1H ,1X,I4,5X,11HENCLOSURE :,                                  RGON0188
                14(3X,2H**,1X,I2,1X,2H**,3X,E11.4,1X))                                   RGON0189
ISN 0184          NLTBND=IBND                                                            RGON0190
ISN 0185          IF(IJ.LT.1)  GO TO 9000                                                RGON0191
ISN 0187          GO TO 3900                                                             RGON0192
ISN 0188       3500 CONTINUE                                                             RGON0193
               C********** EXTERNAL ELEMENT INTEGERS                    **********RGON0194
ISN 0189          WRITE(6,3501)IX,JY,IJ,INTEXT(IX,JY)                                    RGON0195
ISN 0190       3501 FORMAT(1HO,4X,I3,3X,I3,4X,I5,6X,I2,45H : EXTERIOR ELEMENT, NOT USERGON0196
                1D IN CALCULATIONS)                                                      RGON0197
ISN 0191          WRITE(6,3502)                                                          RGON0198
ISN 0192       3502 FORMAT(1H ,10X,11HENCLOSURE :,104(1H*))                              RGON0199
ISN 0193          IF(IJ.GT.0)GO TO 9000                                                  RGON0200
ISN 0195       3900 CONTINUE                                                             RGON0201
ISN 0196          YLGRID=YLGRID+0.5*DY(JY)                                               RGON0202
ISN 0197       3200 CONTINUE                                                             RGON0203
ISN 0198          XLGRID=XLGRID+0.5*DX(IX)                                               RGON0204
ISN 0199       3100 CONTINUE                                                             RGON0205
ISN 0200       3000 CONTINUE                                                             RGON0206
ISN 0201          WRITE(6,4001)                                                          RGON0207
ISN 0202       4001 FORMAT(1HO,4X,10(1H*),1X,17HREGION DATA CHECK,1X,10(1H*))            RGON0208
ISN 0203          IF(NLTIJ-NINJ)4010,4020,4030                                           RGON0209
ISN 0204       4010 CONTINUE                                                             RGON0210
ISN 0205          WRITE(6,4011)NINJ,NLTIJ                                                RGON0211
ISN 0206       4011 FORMAT(1HO,4X,4(1H*),1X,34HNUMBER OF COMPUTATIONAL ELEMENTS :,1X,    RGON0212
                117HSPECIFIED, NINJ =,I5,1X,4(1H*),1X,23HUSED IN REGION, NLTIJ =,        RGON0213
                215)                                                                     RGON0214
ISN 0207          WRITE(6,4012)                                                          RGON0215
ISN 0208       4012 FORMAT(1HO,4X,8(1H*),1X,54HWARNING : CHECK NUMBER OF COMPUTATIONALRGON0216
                1 ELEMENTS, NINJ,1X,4(1H*),1X,18HEXECUTION POSSIBLE,1X,4(1H*))          RGON0217
ISN 0209          GO TO 4039                                                             RGON0218
ISN 0210       4020 CONTINUE                                                             RGON0219
ISN 0211          WRITE(6,4011)NINJ,NLTIJ                                                RGON0220
ISN 0212          WRITE(6,4022)                                                          RGON0221
ISN 0213       4022 FORMAT(1HO,4X,8(1H*),1X,45HNUMBER OF COMPUTATIONAL ELEMENTS, NINJ RGON0222
                1CHECKS,1X,4(1H*),1X,18HEXECUTION POSSIBLE,1X,4(1H*))                    RGON0223
ISN 0214          GO TO 4039                                                             RGON0224
ISN 0215       4030 CONTINUE                                                             RGON0225
ISN 0216          WRITE(6,4011)NINJ,NLTIJ                                                RGON0226
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   ISN 0217            WRITE(6,4032)                                                    RGON0227
   ISN 0218       4032 FORMAT(1HO,4X,8(1H*),1X,93HFATAL ERROR: NUMBER OF COMPUTATIONAL EL RGON0228
                      1EMENTS USED, NLTIJ IS MORE THAN NUMBER SPECIFIED, NINJ,1X,4(1H*), RGON0229
                      21X,20HEXECUTION IMPOSSIBLE,1X,3(1H*))                           RGON0230
   ISN 0219            GO TO 9009                                                       RGON0231
   ISN 0220       4039 CONTINUE                                                         RGON0232
   ISN 0221            IF(NLTBND-NBND)4050,4060,4070                                    RGON0233
   ISN 0222       4050 CONTINUE                                                         RGON0234
   ISN 0223            WRITE(6,4051)NBND,NLTBND                                         RGON0235
   ISN 0224       4051 FORMAT(1HO,4X,4(1H*),1X,29HNUMBER OF BOUNDARY ELEMENTS :,1X,     RGON0236
                      117HSPECIFIED, NBND =,I4,1X,4(1H*),1X,24HUSED IN REGION, NLTBND =, RGON0237
                      2I4)                                                             RGON0238
   ISN 0225            WRITE(6,4052)                                                    RGON0239
   ISN 0226       4052 FORMAT(1HO,4X,8(1H*),1X,49HWARNING : CHECK NUMBER OF BOUNDARY ELEMRGON0240
                      1ENTS, NBND,1X,4(1H*),1X,18HEXECUTION POSSIBLE,1X,4(1H*))        RGON0241
   ISN 0227            GO TO 9999                                                       RGON0242
   ISN 0228       4060 CONTINUE                                                         RGON0243
   ISN 0229            WRITE(6,4051)NBND,NLTBND                                         RGON0244
   ISN 0230            WRITE(6,4062)                                                    RGON0245
   ISN 0231       4062 FORMAT(1HO,4X,8(1H*),1X,40HNUMBER OF BOUNDARY ELEMENTS, NBND CHECKRGON0246
                      1S,1X,4(1H*),1X,18HEXECUTION POSSIBLE,1X,4(1H*))                 RGON0247
   ISN 0232            GO TO 9999                                                       RGON0248
   ISN 0233       4070 CONTINUE                                                         RGON0249
   ISN 0234            WRITE(6,4051)NBND,NLTBND                                         RGON0250
   ISN 0235            WRITE(6,4072)                                                    RGON0251
   ISN 0236       4072 FORMAT(1HO,4X,8(1H*),1X,90HFATAL ERROR : NUMBER OF BOUNDARY ELEMENRGON0252
                      1TS USED, NLTBND IS MORE THAN NUMBER SPECIFIED, NBND,1X,4(1H*),1X, RGON0253
                      22HEXECUTION IMPOSSIBLE,1X,4(1H*))                               RGON0254
   ISN 0237            GO TO 9009                                                       RGON0255
   ISN 0238       9000 CONTINUE                                                         RGON0256
   ISN 0239            WRITE(6,9001)                                                    RGON0257
   ISN 0240       9001 FORMAT(1HO,4X,8(1H*),1X,61HFATAL ERROR DETECTED IN GEOMETRY INPUT RGON0258
                      1DATA AT THE LOCATION :,1X,4(1H*))                               RGON0259
   ISN 0241            WRITE(6,9002)IX,JY,IJ                                            RGON0260
   ISN 0242       9002 FORMAT(1HO,4X,8(1H*),1X,4HIX =,I3,1X,4(1H*),1X,4HJY =,I3,1X,4(1H*)RGON0261
                      1,1X,4HIJ =,I5,1X,4(1H*))                                        RGON0262
   ISN 0243            WRITE(6,9003)                                                    RGON0263
   ISN 0244       9003 FORMAT(1HO,4X,8(1H*),1X,20HEXECUTION TERMINATED,1X,4(1H*),1X,    RGON0264
                      120HEXECUTION IMPOSSIBLE,1X,4(1H*))                              RGON0265
   ISN 0245            GO TO 9009                                                       RGON0266
   ISN 0246       9005 CONTINUE                                                         RGON0267
   ISN 0247            WRITE(6,9006)IJ,NLZBOT(IJ),NZ,DHZBOT(IJ),DZ(NZ)                  RGON0268
   ISN 0248       9006 FORMAT(1HO,4X,8(1H*),1X,4HIJ =,I3,2X,21HBOTTOM LAYER NLZBOT =,I3, RGON0269
                      1 1X,23HEQUAL TO TOP LAYER NZ =,I3,2X,1H*,/,13X,                 RGON0270
                      2 41HDEPTH APPROXIMATION PROBLEM: DHZBOT(IJ) =,E11.4,2X,8HDZ(NZ) =,RGON0271
                      3 E11.4)                                                         RGON0272
   ISN 0249       9009 CALL EXIT                                                        RGON0273
   ISN 0250            STOP                                                             RGON0274
   ISN 0251       9999 RETURN                                                           RGON0275
   ISN 0252            END                                                              RGON0276

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*      SOURCE STATEMENTS =      251 ,PROGRAM SIZE =      6266

*STATISTICS*  NO  DIAGNOSTICS GENERATED
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****** END OF COMPILATICN ******

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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
 ISN 0002            SUBROUTINE SOLVER(TNSTEP,DTSTEP)                                  SLVR0001
 ISN 0003            COMMON/CONTRL/KRUN,T                                              SLVR0002
 ISN 0004            COMMON/CALLER/INCONV,INTURB,IBOUNC,INCBOT,NPLANT,INCSUR           SLVR0003
 ISN 0005            COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,12),IJIXJY(18,12),     SLVR0004
                    1              NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)  SLVR0005
 ISN 0006            COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ(6),DHZBOT(136)       SLVR0006
 ISN 0007            COMMON/GEOBND/IXBND(49),JYBND(49),                                SLVR0007
                    1              DXBND(49),DYBND(49),ARHBND(49)                    SLVR0008
 ISN 0008            COMMON/ENCBND/IMHBND(49),DYBIMH(49),IPHBND(49),DYBIPH(49),        SLVR0009
                    1              JMHBND(49),DXBJMH(49),JPHBND(49),DXBJPH(49)       SLVR0010
 ISN 0009            COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)                    SLVR0011
 ISN 0010            COMMON/VLXSOL/VLX(272,6),PREVLX(272,6),FNCVLX(272,6)              SLVR0012
 ISN 0011            COMMON/VLYSOL/VLY(272,6),PREVLY(272,6),FNCVLY(272,6)              SLVR0013
 ISN 0012            COMMON/FLOVLZ/GWZLZH(136,6),FNCGWZ(136,6)                         SLVR0014
 ISN 0013            COMMON/UPWIND/INWIND,RXWIND,RYWIND,RZWIND                         SLVR0015
 ISN 0014            COMMON/ANGLES/SINRNX,SINRNY,COSRNZ                                SLVR0016
 ISN 0015            COMMON/BNDCON/ISPBND(49),IXBSPB(17),JYBSPB(17),                   SLVR0017
                    1              HSRBXH(17),GFXBXH(17,6),IVXBXH(17,6),             SLVR0018
                    2              VLXBXH(17,6),VMYBXH(17,6),VPYBXH(17,6),           SLVR0019
                    3              VLZBXH(17,6),FPRBXH(17,6),                        SLVR0020
                    4              HSRBYH(17),GFYBYH(17,6),JVYBYH(17,6),             SLVR0021
                    5              VLYBYH(17,6),VMXBYH(17,6),VPXBYH(17,6),           SLVR0022
                    6              VLZBYH(17,6),FPRBYH(17,6)                         SLVR0023
 ISN 0016            IF(KRUN.GT.0)GO TO 2                                              SLVR0024
 ISN 0018            READ(21,1001)INCONV,INTURB                                        SLVR0025
 ISN 0019       1001 FORMAT(8X,I2,2X,I2)                                               SLVR0026
 ISN 0020            READ(21,1002)NONMID,INWIND,RXWIND,RYWIND,RZWIND                   SLVR0027
 ISN 0021       1002 FORMAT(8X,2X,4X,I2,4X,I2,2X,3(2X,F5.2))                           SLVR0028
 ISN 0022            DO 1100 IJ=1,NINJ                                                 SLVR0029
 ISN 0023            IMX=2*IJ-1                                                        SLVR0030
 ISN 0024            IPX=IMX+1                                                         SLVR0031
 ISN 0025            JMY=IMX                                                           SLVR0032
 ISN 0026            JPY=IPX                                                           SLVR0033
 ISN 0027            PREHSR(IJ)=0.0                                                    SLVR0034
 ISN 0028            FNCHSR(IJ)=0.0                                                    SLVR0035
 ISN 0029            DO 1110 LZ=1,NZ                                                   SLVR0036
 ISN 0030            PREVLX(IMX,LZ)=0.0                                                SLVR0037
 ISN 0031            FNCVLX(IMX,LZ)=0.0                                                SLVR0038
 ISN 0032            PREVLX(IPX,LZ)=0.0                                                SLVR0039
 ISN 0033            FNCVLX(IPX,LZ)=0.0                                                SLVR0040
 ISN 0034            PREVLY(JMY,LZ)=0.0                                                SLVR0041
 ISN 0035            FNCVLY(JMY,LZ)=0.0                                                SLVR0042
 ISN 0036            PREVLY(JPY,LZ)=0.0                                                SLVR0043
 ISN 0037            FNCVLY(JPY,LZ)=0.0                                                SLVR0044
 ISN 0038       1110 CONTINUE                                                          SLVR0045
 ISN 0039       1100 CONTINUE                                                          SLVR0046
 ISN 0040            GO TO 9999                                                        SLVR0047
 ISN 0041          2 CONTINUE                                                          SLVR0048
 ISN 0042            DO 2100 IJ=1,NINJ                                                 SLVR0049
 ISN 0043            IMX=2*IJ-1                                                        SLVR0050
 ISN 0044            IPX=IMX+1                                                         SLVR0051
 ISN 0045            JMY=IMX                                                           SLVR0052
 ISN 0046            JPY=IPX                                                           SLVR0053
 ISN 0047            PREHSR(IJ)=HSR(IJ)                                                SLVR0054
 ISN 0048            NLZMIN=NLZBOT(IJ)                                                 SLVR0055
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ISN 0049            DO 2110 LZ=NLZMIN,NZ                                    SLVR0056
ISN 0050            PREVLX(IMX,LZ)=VLX(IMX,LZ)                              SLVR0057
ISN 0051            PREVLX(IPX,LZ)=VLX(IPX,LZ)                              SLVR0058
ISN 0052            PREVLY(JMY,LZ)=VLY(JMY,LZ)                              SLVR0059
ISN 0053            PREVLY(JPY,LZ)=VLY(JPY,LZ)                              SLVR0060
ISN 0054         2110 CONTINUE                                              SLVR0061
ISN 0055         2100 CONTINUE                                              SLVR0062
ISN 0056            T=TNSTEP+0.5*DTSTEP                                     SLVR0063
ISN 0057            IF(IBOUND.GT.0)  CALL BOUNDS                            SLVR0064
ISN 0059            IF(INCBOT.GT.0)  CALL BOTTOM                            SLVR0065
ISN 0061            DT=0.5*DTSTEP                                           SLVR0066
ISN 0062            CALL FLOWUP(DT)                                         SLVR0067
ISN 0063            IF(NPLANT.GT.0)  CALL PLANTS(0)                         SLVR0068
ISN 0065            IF(INCSUR.GT.0)  CALL SURFAC(0)                         SLVR0069
ISN 0067            DO 2200 IJ=1,NINJ                                       SLVR0070
ISN 0068            HSR(IJ)=PREHSR(IJ)+DT*FNCHSR(IJ)                        SLVR0071
ISN 0069         2200 CONTINUE                                              SLVR0072
ISN 0070            CALL FORCER                                             SLVR0073
ISN 0071            IF(INTURB.GT.0)  CALL TURBUL                            SLVR0074
ISN 0073            IF(INCONV.GT.0)  CALL CONVEC(DT)                        SLVR0075
ISN 0075            IF(NPLANT.GT.0)  CALL PLANTS(1)                         SLVR0076
ISN 0077            IF(INCSUR.GT.0)  CALL SURFAC(1)                         SLVR0077
ISN 0079            DO 2400 IJ=1,NINJ                                       SLVR0078
ISN 0080            IMX=2*IJ-1                                              SLVR0079
ISN 0081            IPX=IMX+1                                               SLVR0080
ISN 0082            JMY=IMX                                                 SLVR0081
ISN 0083            JPY=IPX                                                 SLVR0082
ISN 0084            NLZMIN=NLZBOT(IJ)                                       SLVR0083
ISN 0085            DO 2410 LZ=NLZMIN,NZ                                    SLVR0084
ISN 0086            VLX(IMX,LZ)=PREVLX(IMX,LZ)+DT*FNCVLX(IMX,LZ)            SLVR0085
ISN 0087            FNCVLX(IMX,LZ)=0.0                                      SLVR0086
ISN 0088            VLX(IPX,LZ)=PREVLX(IPX,LZ)+DT*FNCVLX(IPX,LZ)            SLVR0087
ISN 0089            FNCVLX(IPX,LZ)=0.0                                      SLVR0088
ISN 0090            VLY(JMY,LZ)=PREVLY(JMY,LZ)+DT*FNCVLY(JMY,LZ)            SLVR0089
ISN 0091            FNCVLY(JMY,LZ)=0.0                                      SLVR0090
ISN 0092            VLY(JPY,LZ)=PREVLY(JPY,LZ)+DT*FNCVLY(JPY,LZ)            SLVR0091
ISN 0093            FNCVLY(JPY,LZ)=0.0                                      SLVR0092
ISN 0094            FNCGWZ(IJ,LZ)=0.0                                       SLVR0093
ISN 0095         2410 CONTINUE                                              SLVR0094
ISN 0096         2400 CONTINUE                                              SLVR0095
ISN 0097            IF(IBOUND.GT.0)  CALL BOUNDS                            SLVR0096
ISN 0099            IF(INCBOT.GT.0)  CALL BOTTOM                            SLVR0097
ISN 0101            DT=DTSTEP                                               SLVR0098
ISN 0102            CALL FLOWUP(DT)                                         SLVR0099
ISN 0103            IF(NPLANT.GT.0)  CALL PLANTS(0)                         SLVR0100
ISN 0105            IF(INCSUR.GT.0)  CALL SURFAC(0)                         SLVR0101
ISN 0107            DO 2500 IJ=1,NINJ                                       SLVR0102
ISN 0108            HSR(IJ)=PREHSR(IJ)+DT*FNCHSR(IJ)                        SLVR0103
ISN 0109         2500 CONTINUE                                              SLVR0104
ISN 0110            CALL FORCER                                             SLVR0105
ISN 0111            IF(INTURB.GT.0)  CALL TURBUL                            SLVR0106
ISN 0113            IF(NONMID.LT.1)GO TO 2699                               SLVR0107
ISN 0115            DO 2600 IJ=1,NINJ                                       SLVR0108
ISN 0116            IMX=2*IJ-1                                              SLVR0109
ISN 0117            IPX=IMX+1                                               SLVR0110
ISN 0118            JMY=IMX                                                 SLVR0111
ISN 0119            JPY=IPX                                                 SLVR0112
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE BOTTOM                                          BTTM0001
   ISN 0003            COMMON/CONTRL/KRUN,T                                       BTTM0002
   ISN 0004            COMMON/CALLER/INCONV,INTURB,IBOUND,INCBOT,NPLANT,INCSUR    BTTM0003
   ISN 0005            COMMON/REGINT/NX,NY,NZ,NINJ,NBND, INTEXT(18,12),IJIXJY(18,12),  BTTM0004
                      1                NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)  BTTM0005
   ISN 0006            COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ( 6),DHZBOT(136)  BTTM0006
   ISN 0007            COMMON/GEOBND/IXBND( 49),JYBND( 49),                       BTTM0007
                      1                DXBND( 49),DYBND( 49),ARHBND( 49)          BTTM0008
   ISN 0008            COMMON/FLOVLZ/GWZLZH(136, 6),FNCGWZ(136, 6)                BTTM0009
   ISN 0009            IF(KRUN.GT.0)GO TO 2                                       BTTM0010
   ISN 0011            DO 1800 IJ=1,NINJ                                          BTTM0011
   ISN 0012            LZ=NLZBOT(IJ)                                              BTTM0012
   ISN 0013            GWZLZH(IJ,LZ)=0.0                                          BTTM0013
   ISN 0014       1800 CONTINUE                                                   BTTM0014
   ISN 0015            INCBOT=0                                                   BTTM0015
   ISN 0016            GO TO 9999                                                 BTTM0016
   ISN 0017          2 CONTINUE                                                   BTTM0017
   ISN 0018       9999 RETURN                                                     BTTM0018
   ISN 0019            END                                                        BTTM0019

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       18 ,PROGRAM SIZE =      302

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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ISN 0120            NLZMIN=NLZBOT(IJ)                                          SLVR0113
ISN 0121            DO 2610 LZ=NLZMIN,NZ                                       SLVR0114
ISN 0122            VLX(IMX,LZ)=PREVLX(IMX,LZ)                                 SLVR0115
ISN 0123            VLX(IPX,LZ)=PREVLX(IPX,LZ)                                 SLVR0116
ISN 0124            VLY(JMY,LZ)=PREVLY(JMY,LZ)                                 SLVR0117
ISN 0125            VLY(JPY,LZ)=PREVLY(JPY,LZ)                                 SLVR0118
ISN 0126       2610 CONTINUE                                                   SLVR0119
ISN 0127       2600 CONTINUE                                                   SLVR0120
ISN 0128       2699 CONTINUE                                                   SLVR0121
ISN 0129            IF(INCONV.GT.0)  CALL CONVEC(DT)                           SLVR0122
ISN 0131            IF(NPLANT.GT.0)  CALL PLANTS(1)                            SLVR0123
ISN 0133            IF(INCSUR.GT.0)  CALL SURFAC(1)                            SLVR0124
ISN 0135            DO 2800 IJ=1,NINJ                                          SLVR0125
ISN 0136            IMX=2*IJ-1                                                 SLVR0126
ISN 0137            IPX=IMX+1                                                  SLVR0127
ISN 0138            JMY=IMX                                                    SLVR0128
ISN 0139            JPY=IPX                                                    SLVR0129
ISN 0140            NLZMIN=NLZBOT(IJ)                                          SLVR0130
ISN 0141            DO 2810 LZ=NLZMIN,NZ                                       SLVR0131
ISN 0142            VLX(IMX,LZ)=PREVLX(IMX,LZ)+DT*FNCVLX(IMX,LZ)               SLVR0132
ISN 0143            FNCVLX(IMX,LZ)=0.0                                         SLVR0133
ISN 0144            VLX(IPX,LZ)=PREVLX(IPX,LZ)+DT*FNCVLX(IPX,LZ)               SLVR0134
ISN 0145            FNCVLX(IPX,LZ)=0.0                                         SLVR0135
ISN 0146            VLY(JMY,LZ)=PREVLY(JMY,LZ)+DT*FNCVLY(JMY,LZ)               SLVR0136
ISN 0147            FNCVLY(JMY,LZ)=0.0                                         SLVR0137
ISN 0148            VLY(JPY,LZ)=PREVLY(JPY,LZ)+DT*FNCVLY(JPY,LZ)               SLVR0138
ISN 0149            FNCVLY(JPY,LZ)=0.0                                         SLVR0139
ISN 0150            FNCGWZ(IJ,LZ)=0.0                                          SLVR0140
ISN 0151       2810 CONTINUE                                                   SLVR0141
ISN 0152       2800 CONTINUE                                                   SLVR0142
ISN 0153            T=TNSTEP*DTSTEP                                            SLVR0143
ISN 0154       9999 RETURN                                                     SLVR0144
ISN 0155            END                                                        SLVR0145

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      154 ,PROGRAM SIZE =     2716

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE SURFAC(ISURFC)                                  SRFC0001
   ISN 0003            COMMON/CONTRL/KRUN,T                                       SRFC0002
   ISN 0004            COMMON/CALLER/INCONV,INTURB,IBOUNC,INCBOT,NPLANT,INCSUR    SRFC0003
   ISN 0005            IF(KRUN.GT.0)GO TO 2                                       SRFC0004
   ISN 0007            INCSUR=0                                                   SRFC0005
   ISN 0008            GO TO 9999                                                 SRFC0006
   ISN 0009          2 CONTINUE                                                   SRFC0007
   ISN 0010       9999 CONTINUE                                                   SRFC0008
   ISN 0011            RETURN                                                     SRFC0009
   ISN 0012            END                                                        SRFC0010

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       11 ,PROGRAM SIZE =       230

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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 LEVEL 21.8 ( JUN 74 )                          OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE PLANTS(INCPLT)                                  PLNT0001
   ISN 0003            COMMON/CONTRL/KRUN,T                                       PLNT0002
   ISN 0004            COMMON/CALLER/INCONV,INTURB,IBOUNC,INCBOT,NPLANT,INCSUR    PLNT0003
   ISN 0005            IF(KRUN.GT.0)GO TO 2                                       PLNT0004
   ISN 0007            NPLANT=0                                                   PLNT0005
   ISN 0008            GO TO 9999                                                 PLNT0006
   ISN 0009          2 CONTINUE                                                   PLNT0007
   ISN 0010       9999 CONTINUE                                                   PLNT0008
   ISN 0011            RETURN                                                     PLNT0009
   ISN 0012            END                                                        PLNT0010

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       11 ,PROGRAM SIZE =       230

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
 ISN 0002            SUBROUTINE BOUNDS                                          BNDS0001
 ISN 0003            COMMON/CONTRL/KRUN,T                                       BNDS0002
 ISN 0004            COMMON/CALLER/INCONV,INTURB,IBOUNC,INCBOT,NPLANT,INCSUR    BNDS0003
 ISN 0005            COMMON/REGINT/NX,NY,NZ,NINJ,NRND,INTEXT(18,12),IJIXJY(18,12),  BNDS0004
                    1              NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)  BNDS0005
 ISN 0006            COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ( 6),DHZBOT(136)  BNDS0006
 ISN 0007            COMMON/GEOBND/IXBND( 49),JYBND( 49),                       BNDS0007
                    1              DXBND( 49),DYBND( 49),ARHBND( 49)           BNDS0008
 ISN 0008            COMMON/ENCBND/IMHBND( 49),DYBIMH( 49),IPHBND( 49),DYBIPH( 49),  BNDS0009
                    1              JMHBND( 49),DXBJMH( 49),JPHBND( 49),DXBJPH( 49)  BNDS0010
 ISN 0009            COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)             BNDS0011
 ISN 0010            COMMON/VLXSOL/VLX(272, 6),PREVLX(272, 6),FNCVLX(272, 6)    BNDS0012
 ISN 0011            COMMON/VLYSOL/VLY(272, 6),PREVLY(272, 6),FNCVLY(272, 6)    BNDS0013
 ISN 0012            COMMON/FLCVLZ/GWZLZH(136, 6),FNCGWZ(136, 6)                BNDS0014
 ISN 0013            COMMON/UPWIND/INWIND,RXWIND,RYWIND,RZWIND                  BNDS0015
 ISN 0014            COMMON/ANGLES/SINRNX,SINRNY,COSRNZ                         BNDS0016
 ISN 0015            COMMON/BNDCON/ISPBND( 49),IXBSPB(17),JYBSPB(17),           BNDS0017
                    1              HSRBXH(17),GFXBXH(17, 6),IVXBXH(17, 6),     BNDS0018
                    2              VLXBXH(17, 6),VMYBXH(17, 6),VPYBXH(17, 6),  BNDS0019
                    3              VLZBXH(17, 6),FPRBXH(17, 6),                BNDS0020
                    4              HSRBYH(17),GFYBYH(17, 6),JVYBYH(17, 6),     BNDS0021
                    5              VLYBYH(17, 6),VMXBYH(17, 6),VPXBYH(17, 6),  BNDS0022
                    6              VLZBYH(17, 6),FPRBYH(17, 6)                 BNDS0023
 ISN 0016            DIMENSION LZXLOW(17),LZXTOP(17),LZYLOW(17),LZYTOP(17)      BNDS0024
 ISN 0017            DIMENSION HBXINI(17),                                      BNDS0025
                    1          VLXBXI(17, 6),VLXMXI(17, 6),VMYBXI(17, 6),VMYMXI(17, 6),  BNDS0026
                    2          VPYBXI(17, 6),VPYMXI(17, 6),VLZBXI(17, 6),VLZMXI(17, 6)  BNDS0027
 ISN 0018            DIMENSION HBYINI(17),                                      BNDS0028
                    1          VLYBYI(17, 6),VLYMYI(17, 6),VMXBYI(17, 6),VMXMYI(17, 6),  BNDS0029
                    2          VPXBYI(17, 6),VPXMYI(17, 6),VLZBYI(17, 6),VLZMYI(17, 6)  BNDS0030
 ISN 0019            DIMENSION NSPPER(17),TSPPER(17, 6)                         BNDS0031
 ISN 0020            DIMENSION VLXTTX(17, 6,1),VLXTMX(17, 6,1),                 BNDS0032
                    1          VMYTTX(17, 6,1),VMYTMX(17, 6,1),                BNDS0033
                    2          VPYTTX(17, 6,1),VPYTMX(17, 6,1),                BNDS0034
                    3          VLZTTX(17, 6,1),VLZTMX(17, 6,1)                 BNDS0035
 ISN 0021            DIMENSION VLYTTY(17, 6,1),VLYTMY(17, 6,1),                 BNDS0036
                    1          VMXTTY(17, 6,1),VMXTMY(17, 6,1),                BNDS0037
                    2          VPXTTY(17, 6,1),VPXTMY(17, 6,1),                BNDS0038
                    3          VLZTTY(17, 6,1),VLZTMY(17, 6,1)                 BNDS0039
 ISN 0022            DIMENSION HBXSPB(17, 6),NRXHBX(17, 6),                     BNDS0040
                    1          AMXHBX(17, 6,1),TLXHBX(17, 6,1),PTXHBX(17, 6,1),  BNDS0041
                    2          HBYSPB(17, 6),NRYHBY(17, 6),                    BNDS0042
                    3          AMYHBY(17, 6,1),TLYHBY(17, 6,1),PTYHBY(17, 6,1)  BNDS0043
 ISN 0023            DIMENSION NRXVLX(17, 6,1),                                 BNDS0044
                    1          AMXVLX(17, 6,1,1),PTXVLX(17, 6,1,1),TLXVLX(17, 6,1,1)  BNDS0045
 ISN 0024            DIMENSION NRXVMY(17, 6,1),                                 BNDS0046
                    1          AMXVMY(17, 6,1,1),PTXVMY(17, 6,1,1),TLXVMY(17, 6,1,1)  BNDS0047
 ISN 0025            DIMENSION NRXVPY(17, 6,1),                                 BNDS0048
                    1          AMXVPY(17, 6,1,1),PTXVPY(17, 6,1,1),TLXVPY(17, 6,1,1)  BNDS0049
 ISN 0026            DIMENSION NRXVLZ(17, 6,1),                                 BNDS0050
                    1          AMXVLZ(17, 6,1,1),PTXVLZ(17, 6,1,1),TLXVLZ(17, 6,1,1)  BNDS0051
 ISN 0027            DIMENSION NRYVLY(17, 6,1),                                 BNDS0052
                    1          AMYVLY(17, 6,1,1),PTYVLY(17, 6,1,1),TLYVLY(17, 6,1,1)  BNDS0053
 ISN 0028            DIMENSION NRYVMX(17, 6,1),                                 BNDS0054
                    1          AMYVMX(17, 6,1,1),PTYVMX(17, 6,1,1),TLYVMX(17, 6,1,1)  BNDS0055
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ISN 0029          DIMENSION NRYVPX(17, 6,1),                                                BNDS0056
                 1          AMYVPX(17, 6,1,1),PTYVPX(17, 6,1,1),TLYVPX(17, 6,1,1)          BNDS0057
                                                                                           BNDS0058
ISN 0030          DIMENSION NRYVLZ(17, 6,1),                                               BNDS0059
                 1          AMYVLZ(17, 6,1,1),PTYVLZ(17, 6,1,1),TLYVLZ(17, 6,1,1)          BNDS0060
ISN 0031          DIMENSION ISPPER(17),TSPTWO(17)                                          BNDS0061
ISN 0032          DIMENSION IVXSPX(17, 6),                                                 BNDS0062
                 1          VLXSPX( 6),VLXMXX( 6),VPYSPX( 6),VMYMXX( 6),                  BNDS0063
                 2          VPYSPX( 6),VPYMXX( 6),VLZSPX( 6),VLZMXX( 6)                    BNDS0064
ISN 0033          DIMENSION JVYSPY(17, 6),                                                 BNDS0065
                 1          VLYSPY( 6),VLYMXY( 6),VPXSPY( 6),VMXMXY( 6),                  BNDS0066
                 2          VPXSPY( 6),VPXMXY( 6),VLZSPY( 6),VLZMXY( 6)                    BNDS0067
ISN 0034          DIMENSION VMXIAJ( 6),VPXIAJ( 6),VLZIAJ( 6),DHZIAJ( 6),                   BNDS0068
                 1          VMYIAJ( 6),VPYIAJ( 6)                                         BNDS0069
ISN 0035          IF(KRUN.GT.0)GO TO 2                                                     BNDS0070
ISN 0037          PI=3.1415927                                                             BNDS0071
ISN 0038          READ(22,1001)NSPBND,INVARY,INHARM,LINEAR,INPUTS                          BNDS0072
ISN 0039       1001 FORMAT(4X,I4,3(4X,I2),8X,I2)                                           BNDS0073
ISN 0040          IBOUND=NSPBND                                                            BNDS0074
ISN 0041          IF(NSPBND.GT.0)GO TO 1040                                                BNDS0075
                 C********** CHECK FOR POSSIBLE NON-SPECIFIED OPEN-REGION      **********BNDS0076
                 C********** BOUNDARY CONDITIONS                              **********BNDS0077
ISN 0043          DO 1010 IX=1,NX                                                          BNDS0078
ISN 0044          DO 1020 JY=1,NY                                                          BNDS0079
ISN 0045          IF(INTEXT(IX,JY).LT.1)GO TO 1020                                         BNDS0080
ISN 0047          IBND=INTEXT(IX,JY)                                                       BNDS0081
ISN 0048          IF(IMHBND(IBND).GT.0)GO TO 9010                                          BNDS0082
ISN 0050          IF(IPHBND(IBND).GT.0)GO TO 9010                                          BNDS0083
ISN 0052          IF(JMHBND(IBND).GT.0)GO TO 9010                                          BNDS0084
ISN 0054          IF(JPHBND(IBND).GT.0)GO TO 9010                                          BNDS0085
ISN 0056       1020 CONTINUE                                                               BNDS0086
ISN 0057       1010 CONTINUE                                                               BNDS0087
ISN 0058          GO TO 9999                                                               BNDS0088
ISN 0059       1040 CONTINUE                                                               BNDS0089
ISN 0060          DO 1050 IBND=1,NBND                                                      BNDS0090
ISN 0061          ISPBND(IBND)=0                                                           BNDS0091
ISN 0062       1050 CONTINUE                                                               BNDS0092
                 C********** START READING DATA FOR SPECIFIED CONDITIONS AT SPECIFIED ***BNDS0093
                 C********** BOUNDARY ELEMENTS ACCORDING TO SPECIFIED ISPB,IBND,IX AND JYBNDS0094
                 C********** READ INPUT GEOMETRY DATA FOR THE ELEMENTS WITH     **********BNDS0095
                 C********** SPECIFIED BOUNDARY CONDITIONS                      **********BNDS0096
ISN 0063          DO 1100 ISPB=1,NSPBND                                                    BNDS0097
ISN 0064          READ(22,1101)IXBSPB(ISPB),JYBSPB(ISPB),          IMHSPB,DYMSPB,BNDS0098
                 1IPHSPB,DYPSPB,              LZXLOW(ISPB),LZXTOP(ISPB),                  BNDS0099
                 2          JMHSPB,DXMSPB,JPHSPB,DXPSPB,          LZYLOW(ISPB)BNDS0100
                 3,LZYTOP(ISPB)                                                           BNDS0101
ISN 0065       1101 FORMAT(4X,2(1X,I4),/,4X,2(1X,2(1X,I2,E11.4),1X,2(1X,I3)))            BNDS0102
ISN 0066          IX=IXBSPB(ISPB)                                                          BNDS0103
ISN 0067          JY=JYBSPB(ISPB)                                                          BNDS0104
ISN 0068          IF(INTEXT(IX,JY).LT.1)GO TO 9110                                         BNDS0105
ISN 0070          IBND=INTEXT(IX,JY)                                                       BNDS0106
ISN 0071          IF(IMHSPB.LT.1)GO TO 1103                                                BNDS0107
ISN 0073          IF(IMHBND(IBND).LT.0)GO TO 9120                                          BNDS0108
ISN 0075          IMHBND(IBND)=IMHSPB                                                      BNDS0109
ISN 0076          DYBIMH(IBND)=DYMSPB                                                      BNDS0110
ISN 0077       1103 CONTINUE                                                               BNDS0111
ISN 0078          IF(IPHSPB.LT.1)GO TO 1104                                                BNDS0112
ISN 0080          IF(IPHBND(IBND).LT.0)GO TO 9120
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ISN 0082            IPHBND(IBND)=IPHSPB                                                  BNDS0113
ISN 0083            DYBJPH(IBND)=DYPSPB                                                  BNDS0114
ISN 0084          1104 CONTINUE                                                          BNDS0115
ISN 0085            IF(JMHSPB.LT.1)GO TO 1105                                            BNDS0116
ISN 0087            IF(JMHBND(IBND).LT.0)GO TO 9120                                      BNDS0117
ISN 0089            JMHBND(IBND)=JMHSPB                                                  BNDS0118
ISN 0090            DXBJMH(IBND)=DXMSPB                                                  BNDS0119
ISN 0091          1105 CONTINUE                                                          BNDS0120
ISN 0092            IF(JPHSPB.LT.1)GO TO 1106                                            BNDS0121
ISN 0094            IF(JPHBND(IBND).LT.0)GO TO 9120                                      BNDS0122
ISN 0096            JPHBND(IBND)=JPHSPB                                                  BNDS0123
ISN 0097            DXBJPH(IBND)=DXPSPB                                                  BNDS0124
ISN 0098          1106 CONTINUE                                                          BNDS0125
ISN 0099            IJ=IJIXJY(IX,JY)                                                     BNDS0126
ISN 0100            NLZMIN=NLZBOT(IJ)                                                    BNDS0127
                  C********** CLOSE BOUNDARY CONDITIONS IN LAYERS BELOW LZXLOW AND LZYLOW BNDS0128
                  C********** AND ABOVE LZXTOP AND LZYTOP BASED ON SPECIFIED CONDITIONS ** BNDS0129
                                                                                       BNDS0130
ISN 0101            IF(IMHBND(IBND).GT.0)GO TO 1108                                      BNDS0131
ISN 0103            IF(IPHBND(IBND).GT.0)GO TO 1108                                      BNDS0132
ISN 0105            GO TO 1139                                                           BNDS0133
ISN 0106          1108 CONTINUE                                                          BNDS0134
ISN 0107            IF(LZXLOW(ISPB).LT.NLZMIN)GO TO 9130                                 BNDS0135
ISN 0109            IF(LZXTOP(ISPB).GT.NZ)GO TO 9130                                     BNDS0136
ISN 0111            IF(LZXLOW(ISPB).EQ.NLZMIN)GO TO 1119                                 BNDS0137
ISN 0113            NLZOUT=LZXLOW(ISPB)-1                                                BNDS0138
ISN 0114            DO 1110 LZ=NLZMIN,NLZOUT                                             BNDS0139
ISN 0115            IVXSPX(ISPB,LZ)=0                                                    BNDS0140
ISN 0116          1110 CONTINUE                                                          BNDS0141
ISN 0117          1119 CONTINUE                                                          BNDS0142
ISN 0118            IF(LZXTOP(ISPB).EQ.NZ)GO TO 1129                                     BNDS0143
                  C********** FOR OPEN-REGION X-ENCLOSURE BOUNDARY SURFACES THE ********** BNDS0144
                  C********** CONDITIONS MUST BE SPECIFIED FROM A LOW LEVEL     ********** BNDS0145
                  C********** LZXLOW TO THE TOP LAYER,INCLUDING LZXTOP=NZ       ********** BNDS0146
ISN 0120            IF(IMHBND(IBND).GT.1)GO TO 9140                                      BNDS0147
ISN 0122            IF(IPHBND(IBND).GT.1)GO TO 9140                                      BNDS0148
ISN 0124            NLZOUT=LZXTOP(ISPB)+1                                                BNDS0149
ISN 0125            DO 1120 LZ=NLZOUT,NZ                                                 BNDS0150
ISN 0126            IVXSPX(ISPB,LZ)=0                                                    BNDS0151
ISN 0127          1120 CONTINUE                                                          BNDS0152
ISN 0128          1129 CONTINUE                                                          BNDS0153
ISN 0129            NLZLOW=LZXLOW(ISPB)                                                  BNDS0154
ISN 0130            NLZTOP=LZXTOP(ISPB)                                                  BNDS0155
ISN 0131            DO 1130 LZ=NLZLOW,NLZTOP                                             BNDS0156
ISN 0132            IVXSPX(ISPB,LZ)=1                                                    BNDS0157
ISN 0133          1130 CONTINUE                                                          BNDS0158
ISN 0134          1139 CONTINUE                                                          BNDS0159
ISN 0135            IF(JMHBND(IBND).GT.0)GO TO 1148                                      BNDS0160
ISN 0137            IF(JPHBND(IBND).GT.0)GO TO 1148                                      BNDS0161
ISN 0139            GO TO 1179                                                           BNDS0162
ISN 0140          1148 CONTINUE                                                          BNDS0163
ISN 0141            IF(LZYLOW(ISPB).LT.NLZMIN)GO TO 9150                                 BNDS0164
ISN 0143            IF(LZYTOP(ISPB).GT.NZ)GO TO 9150                                     BNDS0165
ISN 0145            IF(LZYLOW(ISPB).EQ.NLZMIN)GO TO 1159                                 BNDS0166
ISN 0147            NLZOUT=LZYLOW(ISPB)-1                                                BNDS0167
ISN 0148            DO 1150 LZ=NLZMIN,NLZOUT                                             BNDS0168
ISN 0149            JVYSPY(ISPB,LZ)=0                                                    BNDS0169
ISN 0150          1150 CONTINUE
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ISN 0151      1159 CONTINUE                                                    BNDS0170
ISN 0152           IF(LZYTOP(ISPB).EQ.NZ)GO TO 1169                            BNDS0171
              C********** FOR OPEN-REGION Y-ENCLOSURE BOUNDARY SURFACES THE ********** BNDS0172
              C********** CONDITIONS MUST BE SPECIFIED FROM A LOW LEVEL      ********** BNDS0173
              C********** LZYLOW TO THE TOP LAYER,INCLUDING LZYTOP=NZ        ********** BNDS0174
ISN 0154           IF(JMHBND(IBND).GT.1)GO TO 9160                             BNDS0175
ISN 0156           IF(JPHBND(IBND).GT.1)GO TO 9160                             BNDS0176
ISN 0158           NLZOUT=LZYTOP(ISPB)+1                                       BNDS0177
ISN 0159           DO 1160 LZ=NLZOUT,NZ                                        BNDS0178
ISN 0160           JVYSPY(ISPB,LZ)=0                                           BNDS0179
ISN 0161      1160 CONTINUE                                                    BNDS0180
ISN 0162      1169 CONTINUE                                                    BNDS0181
ISN 0163           NLZLOW=LZYLOW(ISPB)                                         BNDS0182
ISN 0164           NLZTOP=LZYTOP(ISPB)                                         BNDS0183
ISN 0165           DO 1170 LZ=NLZLOW,NLZTOP                                    BNDS0184
ISN 0166           JVYSPY(ISPB,LZ)=1                                           BNDS0185
ISN 0167      1170 CONTINUE                                                    BNDS0186
ISN 0168      1179 CONTINUE                                                    BNDS0187
              C********** SET THE SPECIFIED BOUNDARY ELEMENT INDEX ISPBND(IBND) ******* BNDS0188
              C********** FOR CORRECTLY SPECIFIED ELEMENTS                   ********** BNDS0189
ISN 0169           ISPBND(IBND)=ISPB                                           BNDS0190
ISN 0170      1100 CONTINUE                                                    BNDS0191
              C********** READ INPUT DATA FOR THE INITIAL AND TIME-VARYING   ********** BNDS0192
              C********** SPECIFIED BOUNDARY CONDITIONS                       ********** BNDS0193
ISN 0171           DO 1200 ISPB=1,NSPBND                                       BNDS0194
ISN 0172           IX=IXBSPB(ISPB)                                             BNDS0195
ISN 0173           JY=JYBSPB(ISPB)                                             BNDS0196
ISN 0174           IBND=INTEXT(IX,JY)                                          BNDS0197
              C********** AT LEAST ONE PERIOD IS NECESSARY FOR ALL THE        ********** BNDS0198
              C********** SPECIFIED BOUNDARY CONDITIONS                       ********** BNDS0199
ISN 0175           READ(22,1201)NSPPER(ISPB),HBXINI(ISPB),HBYINI(ISPB)         BNDS0200
ISN 0176      1201 FORMAT(8X,I3,2X,2(2X,E11.4))                                BNDS0201
              C********** READ INPUT DATA FOR THE INITIAL VALUES OF THE       ********** BNDS0202
              C********** NON-HARMONIC SPECIFIED BOUNDARY CONDITIONS          ********** BNDS0203
              C********** WITH MEAN AVERAGE OR MINIMUM VALUES AS (BXI/BYI)    ********** BNDS0204
              C********** AND MAXIMUM VALUES AS (MXI/MYI)                     ********** BNDS0205
ISN 0177           IF(IMHBND(IBND).GT.0)GO TO 1212                             BNDS0206
ISN 0179           IF(IPHBND(IBND).GT.0)GO TO 1212                             BNDS0207
ISN 0181           GO TO 1229                                                  BNDS0208
ISN 0182      1212 CONTINUE                                                    BNDS0209
ISN 0183           NLZLOW=LZXLOW(ISPB)                                         BNDS0210
ISN 0184           NLZTOP=LZXTOP(ISPB)                                         BNDS0211
ISN 0185           DO 1220 LZ=NLZLOW,NLZTOP                                    BNDS0212
ISN 0186           READ(22,1221)VLXBXI(ISPB,LZ),VLXMXI(ISPB,LZ),               BNDS0213
                  1              VMYBXI(ISPB,LZ),VMYMXI(ISPB,LZ),             BNDS0214
                  2              VPYBXI(ISPB,LZ),VPYMXI(ISPB,LZ),             BNDS0215
                  3              VLZBXI(ISPB,LZ),VLZMXI(ISPB,LZ)              BNDS0216
ISN 0187      1221 FORMAT(5X,3(1X,2(1X,E11.4)),/,5X,1X,2(1X,E11.4))            BNDS0217
ISN 0188      1220 CONTINUE                                                    BNDS0218
ISN 0189      1229 CONTINUE                                                    BNDS0219
ISN 0190           IF(JMHBND(IBND).GT.0)GO TO 1232                             BNDS0220
ISN 0192           IF(JPHBND(IBND).GT.0)GO TO 1232                             BNDS0221
ISN 0194           GO TO 1249                                                  BNDS0222
ISN 0195      1232 CONTINUE                                                    BNDS0223
ISN 0196           NLZLOW=LZYLOW(ISPB)                                         BNDS0224
ISN 0197           NLZTOP=LZYTOP(ISPB)                                         BNDS0225
ISN 0198           DO 1240 LZ=NLZLOW,NLZTOP                                    BNDS0226
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ISN 0199          READ(22,1221)VLYBYI(ISPB,LZ),VLYMYI(ISPB,LZ),                  BNDS0227
                 1             VMXBYI(ISPB,LZ),VMXMYI(ISPB,LZ),                  BNDS0228
                 2             VPXBYI(ISPB,LZ),VPXMYI(ISPB,LZ),                  BNDS0229
                 3             VLZBYI(ISPB,LZ),VLZMYI(ISPB,LZ)                   BNDS0230
ISN 0200          1240 CONTINUE                                                  BNDS0231
ISN 0201          1249 CONTINUE                                                  BNDS0232
                 C********** READ INPUT DATA BY PERIODS FOR VARYING VALUES OF THE ********BNDS0233
                 C********** NON-HARMONIC SPECIFIED CONDITIONS WITH MEAN AVERAGE ********BNDS0234
                 C********** OR MINIMUM VALUES AS (TTX/TTY) AND MAXIMUM VALUES **********BNDS0235
                 C********** AS (TMX/TMY)                                    **********BNDS0236
                                                                                BNDS0237
ISN 0202          IF(INVARY.LT.1)GO TO 1200                                     BNDS0238
                 C********** AT LEAST ONE PERIOD IS NECESSARY FOR ALL THE      **********BNDS0239
                 C********** SPECIFIED BOUNDARY CONDITIONS                     **********BNDS0240
ISN 0204          NPER=NSPPER(ISPB)                                             BNDS0241
ISN 0205          DO 1250 IPER=1,NPER                                           BNDS0242
ISN 0206          READ(22,1251)TSPPER(ISPB,IPER),          HBXSPB(ISPB,IPER),   BNDS0243
                 1HEYSPB(ISPB,IPER)                                             BNDS0244
ISN 0207          1251 FORMAT(12X,E13.6,2(2X,E11.4))                            BNDS0245
ISN 0208          IF(IMHBND(IBND).GT.0)GO TO 1252                               BNDS0246
ISN 0210          IF(IPHBND(IBND).GT.0)GO TO 1252                               BNDS0247
ISN 0212          GO TO 1269                                                    BNDS0248
ISN 0213          1252 CONTINUE                                                 BNDS0249
ISN 0214          NLZLOW=LZXLOW(ISPB)                                           BNDS0250
ISN 0215          NLZTOP=LZXTOP(ISPB)                                           BNDS0251
ISN 0216          DO 1260 LZ=NLZLOW,NLZTOP                                      BNDS0252
ISN 0217          READ(22,1221)VLXTTX(ISPB,LZ,IPER),VLXTMX(ISPB,LZ,IPER),       BNDS0253
                 1             VMYTTX(ISPB,LZ,IPER),VMYTMX(ISPB,LZ,IPER),     BNDS0254
                 2             VPYTTX(ISPB,LZ,IPER),VPYTMX(ISPB,LZ,IPER),     BNDS0255
                 3             VLZTTX(ISPB,LZ,IPER),VLZTMX(ISPB,LZ,IPER)      BNDS0256
ISN 0218          1260 CONTINUE                                                 BNDS0257
ISN 0219          1269 CONTINUE                                                 BNDS0258
ISN 0220          IF(JMHBND(IBND).GT.0)GO TO 1272                               BNDS0259
ISN 0222          IF(JPHBND(IBND).GT.0)GO TO 1272                               BNDS0260
ISN 0224          GO TO 1289                                                    BNDS0261
ISN 0225          1272 CONTINUE                                                 BNDS0262
ISN 0226          NLZLOW=LZYLOW(ISPB)                                           BNDS0263
ISN 0227          NLZTOP=LZYTOP(ISPB)                                           BNDS0264
ISN 0228          DO 1280 LZ=NLZLOW,NLZTOP                                      BNDS0265
ISN 0229          READ(22,1221)VLYTTY(ISPB,LZ,IPER),VLYTMY(ISPB,LZ,IPER),       BNDS0266
                 1             VMXTTY(ISPB,LZ,IPER),VMXTMY(ISPB,LZ,IPER),     BNDS0267
                 2             VPXTTY(ISPB,LZ,IPER),VPXTMY(ISPB,LZ,IPER),     BNDS0268
                 3             VLZTTY(ISPB,LZ,IPER),VLZTMY(ISPB,LZ,IPER)      BNDS0269
ISN 0230          1280 CONTINUE                                                 BNDS0270
ISN 0231          1289 CONTINUE                                                 BNDS0271
ISN 0232          1250 CONTINUE                                                 BNDS0272
ISN 0233          IF(INHARM.LT.1)GO TO 1200                                     BNDS0273
                 C********** READ INPUT DATA BY PERIODS FOR VARYING VALUES OF  **********BNDS0274
                 C********** THE HARMONIC SPECIFIED CONDITIONS BASED ON NUMBER **********BNDS0275
                 C********** OF HARMONICS, AMPLITUDES, HARMONIC PERIODS AND LAG TIMES ***BNDS0276
                 C********** FOR SPECIFYING HARMONIC CONDITIONS AT LEAST ONE PERIOD *****BNDS0277
                 C********** IS REQUIRED WITHOUT LINEAR INTERPOLATION          **********BNDS0278
ISN 0235          DO 1300 IPER=1,NPER                                           BNDS0279
ISN 0236          IF(IMHBND(IBND).GT.0)GO TO 1312                               BNDS0280
ISN 0238          IF(IPHBND(IBND).GT.0)GO TO 1312                               BNDS0281
ISN 0240          GO TO 1319                                                    BNDS0282
ISN 0241          1312 CONTINUE                                                 BNDS0283
ISN 0242          READ(22,1313)NRXHBX(ISPB,IPER)
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ISN 0243      1313 FORMAT(12X,I2)                                                    BNDS0284
ISN 0244           IF(NRXHBX(ISPB,IPER).LT.1)GO TO 1319                              BNDS0285
ISN 0246           NHR=NRXHBX(ISPB,IPER)                                             BNDS0286
ISN 0247           DO 3110 IHR=1,NHR                                                 BNDS0287
ISN 0248           READ(22,1315)AMXHBX(ISPB,IPER,IHR),PTXHBX(ISPB,IPER,IHR),         BNDS0288
                  1              TLXHBX(ISPB,IPER,IHR)                              BNDS0289
ISN 0249      1315 FORMAT(12X,3(1X,E11.4))                                           BNDS0290
ISN 0250      3110 CONTINUE                                                          BNDS0291
ISN 0251      1319 CONTINUE                                                          BNDS0292
ISN 0252           IF(JMHBND(IBND).GT.0)GO TO 1322                                   BNDS0293
ISN 0254           IF(JPHBND(IBND).GT.0)GO TO 1322                                   BNDS0294
ISN 0256           GO TO 1329                                                        BNDS0295
ISN 0257      1322 CONTINUE                                                          BNDS0296
ISN 0258           READ(22,1313)NRYHBY(ISPB,IPER)                                    BNDS0297
ISN 0259           IF(NRYHBY(ISPB,IPER).LT.1)GO TO 1329                              BNDS0298
ISN 0261           NHR=NRYHBY(ISPB,IPER)                                             BNDS0299
ISN 0262           DO 3120 IHR=1,NHR                                                 BNDS0300
ISN 0263           READ(22,1315)AMYHBY(ISPB,IPER,IHR),PTYHBY(ISPB,IPER,IHR),         BNDS0301
                  1              TLYHBY(ISPB,IPER,IHR)                              BNDS0302
ISN 0264      3120 CONTINUE                                                          BNDS0303
ISN 0265      1329 CONTINUE                                                          BNDS0304
ISN 0266           IF(IMHBND(IBND).GT.0)GO TO 1332                                   BNDS0305
ISN 0268           IF(IPHBND(IBND).GT.0)GO TO 1332                                   BNDS0306
ISN 0270           GO TO 1349                                                        BNDS0307
ISN 0271      1332 CONTINUE                                                          BNDS0308
ISN 0272           NLZLOW=LZXLOW(ISPB)                                               BNDS0309
ISN 0273           NLZTOP=LZXTOP(ISPB)                                               BNDS0310
ISN 0274           DO 1340 LZ=NLZLOW,NLZTOP                                          BNDS0311
ISN 0275           READ(22,1343)NRXVLX(ISPB,LZ,IPER),          NRXVMY(ISPB,LZ,       BNDS0312
                  1IPER),NRXVPY(ISPB,LZ,IPER),          NRXVLZ(ISPB,LZ,IPER)       BNDS0313
ISN 0276      1343 FORMAT(10X,4(2X,I2))                                              BNDS0314
ISN 0277           IF(NRXVLX(ISPB,LZ,IPER).LT.1)GO TO 1344                           BNDS0315
ISN 0279           NHR=NRXVLX(ISPB,LZ,IPER)                                          BNDS0316
ISN 0280           DO 3130 IHR=1,NHR                                                 BNDS0317
ISN 0281           READ(22,1315)AMXVLX(ISPB,LZ,IPER,IHR),PTXVLX(ISPB,LZ,IPER,IHR),   BNDS0318
                  1              TLXVLX(ISPB,LZ,IPER,IHR)                           BNDS0319
ISN 0282      3130 CONTINUE                                                          BNDS0320
ISN 0283      1344 CONTINUE                                                          BNDS0321
ISN 0284           IF(NRXVMY(ISPB,LZ,IPER).LT.1)GO TO 1345                           BNDS0322
ISN 0286           NHR=NRXVMY(ISPB,LZ,IPER)                                          BNDS0323
ISN 0287           DO 3140 IHR=1,NHR                                                 BNDS0324
ISN 0288           READ(22,1315)AMXVMY(ISPB,LZ,IPER,IHR),PTXVMY(ISPB,LZ,IPER,IHR),   BNDS0325
                  1              TLXVMY(ISPB,LZ,IPER,IHR)                           BNDS0326
ISN 0289      3140 CONTINUE                                                          BNDS0327
ISN 0290      1345 CONTINUE                                                          BNDS0328
ISN 0291           IF(NRXVPY(ISPB,LZ,IPER).LT.1)GO TO 1346                           BNDS0329
ISN 0293           NHR=NRXVPY(ISPB,LZ,IPER)                                          BNDS0330
ISN 0294           DO 3150 IHR=1,NHR                                                 BNDS0331
ISN 0295           READ(22,1315)AMXVPY(ISPB,LZ,IPER,IHR),PTXVPY(ISPB,LZ,IPER,IHR),   BNDS0332
                  1              TLXVPY(ISPB,LZ,IPER,IHR)                           BNDS0333
ISN 0296      3150 CONTINUE                                                          BNDS0334
ISN 0297      1346 CONTINUE                                                          BNDS0335
ISN 0298           IF(NRXVLZ(ISPB,LZ,IPER).LT.1)GO TO 1347                           BNDS0336
ISN 0300           NHR=NRXVLZ(ISPB,LZ,IPER)                                          BNDS0337
ISN 0301           DO 3160 IHR=1,NHR                                                 BNDS0338
ISN 0302           READ(22,1315)AMXVLZ(ISPB,LZ,IPER,IHR),PTXVLZ(ISPB,LZ,IPER,IHR),   BNDS0339
                  1              TLXVLZ(ISPB,LZ,IPER,IHR)                           BNDS0340
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ISN 0303      3160 CONTINUE                                                                  BNDS0341
ISN 0304      1347 CONTINUE                                                                  BNDS0342
ISN 0305      1340 CONTINUE                                                                  BNDS0343
ISN 0306      1349 CONTINUE                                                                  BNDS0344
ISN 0307           IF(JMHBND(IBND).GT.0)GO TO 1352                                         BNDS0345
ISN 0309           IF(JPHBND(IBND).GT.0)GO TO 1352                                         BNDS0346
ISN 0311           GO TO 1369                                                              BNDS0347
ISN 0312      1352 CONTINUE                                                                  BNDS0348
ISN 0313           NLZLOW=LZYLOW(ISPB)                                                     BNDS0349
ISN 0314           NLZTOP=LZYTOP(ISPB)                                                     BNDS0350
ISN 0315           DO 1360 LZ=NLZLOW,NLZTOP                                                BNDS0351
ISN 0316           READ(22,1343)NRYVLY(ISPB,LZ,IPER),          NRYVMX(ISPB,LZ,             BNDS0352
                  1IPER),NRYVPX(ISPB,LZ,IPER),          NRYVLZ(ISPB,LZ,IPER)              BNDS0353
ISN 0317           IF(NRYVLY(ISPB,LZ,IPER).LT.1)GO TO 1364                                 BNDS0354
ISN 0319           NHR=NRYVLY(ISPB,LZ,IPER)                                                BNDS0355
ISN 0320           DO 3330 IHR=1,NHR                                                       BNDS0356
ISN 0321           READ(22,1315)AMYVLY(ISPB,LZ,IPER,IHR),PTYVLY(ISPB,LZ,IPER,IHR),         BNDS0357
                  1          TLYVLY(ISPB,LZ,IPER,IHR)                                   BNDS0358
ISN 0322      3330 CONTINUE                                                                  BNDS0359
ISN 0323      1364 CONTINUE                                                                  BNDS0360
ISN 0324           IF(NRYVMX(ISPB,LZ,IPER).LT.1)GO TO 1365                                 BNDS0361
ISN 0326           NHR=NRYVMX(ISPB,LZ,IPER)                                                BNDS0362
ISN 0327           DO 3340 IHR=1,NHR                                                       BNDS0363
ISN 0328           READ(22,1315)AMYVMX(ISPB,LZ,IPER,IHR),PTYVMX(ISPB,LZ,IPER,IHR),         BNDS0364
                  1          TLYVMX(ISPB,LZ,IPER,IHR)                                   BNDS0365
ISN 0329      3340 CONTINUE                                                                  BNDS0366
ISN 0330      1365 CONTINUE                                                                  BNDS0367
ISN 0331           IF(NRYVPX(ISPB,LZ,IPER).LT.1)GO TO 1366                                 BNDS0368
ISN 0333           NHR=NRYVPX(ISPB,LZ,IPER)                                                BNDS0369
ISN 0334           DO 3350 IHR=1,NHR                                                       BNDS0370
ISN 0335           READ(22,1315)AMYVPX(ISPB,LZ,IPER,IHR),PTYVPX(ISPB,LZ,IPER,IHR),         BNDS0371
                  1          TLYVPX(ISPB,LZ,IPER,IHR)                                   BNDS0372
ISN 0336      3350 CONTINUE                                                                  BNDS0373
ISN 0337      1366 CONTINUE                                                                  BNDS0374
ISN 0338           IF(NRYVLZ(ISPB,LZ,IPER).LT.1)GO TO 1367                                 BNDS0375
ISN 0340           NHR=NRYVLZ(ISPB,LZ,IPER)                                                BNDS0376
ISN 0341           DO 3360 IHR=1,NHR                                                       BNDS0377
ISN 0342           READ(22,1315)AMYVLZ(ISPB,LZ,IPER,IHR),PTYVLZ(ISPB,LZ,IPER,IHR),         BNDS0378
                  1          TLYVLZ(ISPB,LZ,IPER,IHR)                                   BNDS0379
ISN 0343      3360 CONTINUE                                                                  BNDS0380
ISN 0344      1367 CONTINUE                                                                  BNDS0381
ISN 0345      1360 CONTINUE                                                                  BNDS0382
ISN 0346      1369 CONTINUE                                                                  BNDS0383
ISN 0347      1300 CONTINUE                                                                  BNDS0384
ISN 0348      1200 CONTINUE                                                                  BNDS0385
ISN 0349           IF(INPUTS.LT.1)GO TO 1799                                               BNDS0386
ISN 0351           WRITE(3,1401)                                                           BNDS0387
ISN 0352      1401 FORMAT(1H1,28X,10(1H*),1X,55HSPECIFIED BOUNDARY CONDITIONS FOR THE      BNDS0388
                  1 BOUNDARY ELEMENTS,1X,10(1H*))                                         BNDS0389
ISN 0353           WRITE(3,1402)                                                           BNDS0390
ISN 0354      1402 FORMAT(1HO,10X,8(1H*),1X,80HINPUT GEOMETRY DATA FOR THE BOUNDARY E     BNDS0391
                 1LEMENTS WITH SPECIFIED BOUNDARY CONDITIONS)                             BNDS0392
ISN 0355           WRITE(3,1403)NSPBND                                                     BNDS0393
ISN 0356      1403 FORMAT(1HO,10X,8(1H*),1X,71HNUMBER OF BOUNDARY ELEMENTS WITH SPECI     BNDS0394
                 1FIED BOUNDARY CONDITIONS, NSPBND=,I4)                                   BNDS0395
ISN 0357           IF(NSPBND.LT.1)GO TO 1799                                               BNDS0396
ISN 0359           WRITE(3,1404)                                                           BNDS0397
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ISN 0360      1404 FORMAT(1HO,2X,1H*,4HISPB,1H*,4HIBND,1H*,6HIXBSPB,1H*,6HJYBSPB,     BNDS0398
                  1               1H*,6HIMHSPB,1H*,3X,6HDYMSPB,2X,                  BNDS0399
                  2               1H*,6HIPHSPB,1H*,3X,6HDYPSPB,2X,                  BNDS0400
                  3               1H*,6HLZXLOW,1H*,6HLZXTOP,                        BNDS0401
                  4               1H*,6HJMHSPB,1H*,3X,6HDXMSPB,2X,                  BNDS0402
                  5               1H*,6HJPHSPB,1H*,3X,6HDXPSPB,2X,                  BNDS0403
                  6               1H*,6HLZYLOW,1H*,6HLZYTOP)                        BNDS0404
ISN 0361            DO 1410 ISPB=1,NSPBND                                           BNDS0405
ISN 0362            IX=IXBSPB(ISPB)                                                 BNDS0406
ISN 0363            JY=JYBSPB(ISPB)                                                 BNDS0407
ISN 0364            IBND=INTEXT(IX,JY)                                              BNDS0408
ISN 0365            WRITE(3,1411)ISPB,IBND,IXBSPB(ISPB),JYBSPB(ISPB),               BNDS0409
                  1               IMHBND(IBND),DYBIMH(IBND),IPHBND(IBND),DYBIPH(IBND),  BNDS0410
                  2               LZXLOW(ISPB),LZXTOP(ISPB),              JMHBND(IBND),  BNDS0411
                  3DXBJMH(IBND),JPHBND(IBND),DXBJPH(IBND),                LZYLOW(ISPB),  BNDS0412
                  4LZYTOP(ISPB)                                                     BNDS0413
ISN 0366      1411 FORMAT(1HO,2X,I4,1X,I4,1X,I4,3X,I4,                              BNDS0414
                  1       2(4X,I3,4X,E11.4,1X,I3,4X,E11.4,1X,I3,3X,I4))           BNDS0415
ISN 0367      1410 CONTINUE                                                         BNDS0416
ISN 0368            WRITE(3,1416)                                                   BNDS0417
ISN 0369      1416 FORMAT(1HO,8(1H*),96HINPUT DATA FOR THE INITIAL VALUES FOR THE SPE BNDS0418
                 1CIFIED BOUNDARY CONDITIONS IN THE BOUNDARY ELEMENTS)              BNDS0419
ISN 0370            DO 1400 ISPB=1,NSPBND                                           BNDS0420
ISN 0371            IX=IXBSPB(ISPB)                                                 BNDS0421
ISN 0372            JY=JYBSPB(ISPB)                                                 BNDS0422
ISN 0373            IBND=INTEXT(IX,JY)                                              BNDS0423
ISN 0374            WRITE(3,1421)                                                   BNDS0424
ISN 0375      1421 FORMAT(1HO,4X,1H*,2X,4HISPB,2X,1H*,2X,4HIBND,2X,                 BNDS0425
                  1       1H*,1X,6HIXBSPB,1X,1H*,1X,6HJYBSPB,1X,                  BNDS0426
                  2       1H*,1X,6HIMHBND,1X,1H*,1X,6HIPHBND,1X,                  BNDS0427
                  3       1H*,1X,6HJMHBND,1X,1H*,1X,6HJPHBND,1X,                  BNDS0428
                  4       1H*,1X,2HLZ,1X,1H*)                                     BNDS0429
ISN 0376            WRITE(3,1422)ISPB,IBND,IX,JY,IMHBND(IBND),IPHBND(IBND),         BNDS0430
                  1                            JMHBND(IBND),JPHBND(IBND)          BNDS0431
ISN 0377      1422 FORMAT(1HO,4X,3X,I4,2X,3X,I4,2X,2X,1X,I4,2X,2X,1X,I4,            BNDS0432
                  1       2X,2X,I2,3X,2X,2X,I2,3X,2X,2X,I2,3X,2X,2X,I2)           BNDS0433
ISN 0378            IF(IMHBND(IBND).GT.0)GO TO 1423                                 BNDS0434
ISN 0380            IF(IPHBND(IBND).GT.0)GO TO 1423                                 BNDS0435
ISN 0382            GO TO 1425                                                      BNDS0436
ISN 0383      1423 CONTINUE                                                         BNDS0437
ISN 0384            WRITE(3,1424)HBXINI(ISPB)                                       BNDS0438
ISN 0385      1424 FORMAT(1HO,84X,1H*,1X,8HHBXINI =,E11.4)                          BNDS0439
ISN 0386      1425 CONTINUE                                                         BNDS0440
ISN 0387            IF(JMHBND(IBND).GT.0)GO TO 1426                                 BNDS0441
ISN 0389            IF(JPHBND(IBND).GT.0)GO TO 1426                                 BNDS0442
ISN 0391            GO TO 1429                                                      BNDS0443
ISN 0392      1426 CONTINUE                                                         BNDS0444
ISN 0393            WRITE(3,1427)HBYINI(ISPB)                                       BNDS0445
ISN 0394      1427 FORMAT(1HO,84X,1H*,1X,8HHBYINI =,E11.4)                          BNDS0446
ISN 0395      1429 CONTINUE                                                         BNDS0447
ISN 0396            IF(IMHBND(IBND).GT.0)GO TO 1512                                 BNDS0448
ISN 0398            IF(IPHBND(IBND).GT.0)GO TO 1512                                 BNDS0449
ISN 0400            GO TO 1529                                                      BNDS0450
ISN 0401      1512 CONTINUE                                                         BNDS0451
ISN 0402            NLZLOW=LZXLOW(ISPB)                                             BNDS0452
ISN 0403            NLZTOP=LZXTOP(ISPB)                                             BNDS0453
ISN 0404            DO 1520 LZ=NLZLOW,NLZTOP                                        BNDS0454
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ISN 0405          WRITE(3,1521)LZ,VLXBXI(ISPB,LZ),VLXMXI(ISPB,LZ)                    BNDS0455
ISN 0406       1521 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLXEXI =,E11.4,                    BNDS0456
                   1                  2X,1H*,1X,8HVLXMXI =,E11.4)                  BNDS0457
ISN 0407          WRITE(3,1522)LZ,VMYBXI(ISPB,LZ),VMYMXI(ISPB,LZ)                    BNDS0458
ISN 0408       1522 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVMYEXI =,E11.4,                    BNDS0459
                   1                  2X,1H*,1X,8HVMYMXI =,E11.4)                  BNDS0460
ISN 0409          WRITE(3,1523)LZ,VPYBXI(ISPB,LZ),VPYMXI(ISPB,LZ)                    BNDS0461
ISN 0410       1523 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVPYEXI =,E11.4,                    BNDS0462
                   1                  2X,1H*,1X,8HVPYMXI =,E11.4)                  BNDS0463
ISN 0411          WRITE(3,1524)LZ,VLZBXI(ISPB,LZ),VLZMXI(ISPB,LZ)                    BNDS0464
ISN 0412       1524 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLZEXI =,E11.4,                    BNDS0465
                   1                  2X,1H*,1X,8HVLZMXI =,E11.4)                  BNDS0466
ISN 0413       1520 CONTINUE                                                         BNDS0467
ISN 0414       1529 CONTINUE                                                         BNDS0468
ISN 0415          IF(JMHBND(IBND).GT.0)GO TO 1532                                    BNDS0469
ISN 0417          IF(JPHBND(IBND).GT.0)GO TO 1532                                    BNDS0470
ISN 0419          GO TO 1449                                                         BNDS0471
ISN 0420       1532 CONTINUE                                                         BNDS0472
ISN 0421          NLZLOW=LZYLOW(ISPB)                                                BNDS0473
ISN 0422          NLZTOP=LZYTOP(ISPB)                                                BNDS0474
ISN 0423          DO 1540 LZ=NLZLOW,NLZTOP                                           BNDS0475
ISN 0424          WRITE(3,1541)LZ,VLYBYI(ISPB,LZ),VLYMYI(ISPB,LZ)                    BNDS0476
ISN 0425       1541 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLYEYI =,E11.4,                    BNDS0477
                   1                  2X,1H*,1X,8HVLYMYI =,E11.4)                  BNDS0478
ISN 0426          WRITE(3,1542)LZ,VMXBYI(ISPB,LZ),VMXMYI(ISPB,LZ)                    BNDS0479
ISN 0427       1542 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVMXEYI =,E11.4,                    BNDS0480
                   1                  2X,1H*,1X,8HVMXMYI =,E11.4)                  BNDS0481
ISN 0428          WRITE(3,1543)LZ,VPXBYI(ISPB,LZ),VPXMYI(ISPB,LZ)                    BNDS0482
ISN 0429       1543 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVPXEYI =,E11.4,                    BNDS0483
                   1                  2X,1H*,1X,8HVPXMYI =,E11.4)                  BNDS0484
ISN 0430          WRITE(3,1544)LZ,VLZBYI(ISPB,LZ),VLZMYI(ISPB,LZ)                    BNDS0485
ISN 0431       1544 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLZEYI =,E11.4,                    BNDS0486
                   1                  2X,1H*,1X,8HVLZMYI =,E11.4)                  BNDS0487
ISN 0432       1540 CONTINUE                                                         BNDS0488
ISN 0433       1449 CONTINUE                                                         BNDS0489
ISN 0434          IF(INVARY.GT.0)GO TO 1432                                          BNDS0490
ISN 0436          WRITE(3,1431)INVARY                                                BNDS0491
ISN 0437       1431 FORMAT(1HO,4X,8(1H*),1X,93HALL SPECIFIED BOUNDARY CONDITIONS IN AL BNDS0492
                  1L THE BOUNDARY ELEMENTS REMAIN AT THEIR INITIAL VALUES,/,18X,1H*, BNDS0493
                  21X,18HSPECIFIED INVARY =,I2)                                      BNDS0494
ISN 0438          GO TO 1400                                                         BNDS0495
ISN 0439       1432 CONTINUE                                                         BNDS0496
ISN 0440          WRITE(3,1441)NSPPER(ISPB)                                          BNDS0497
ISN 0441       1441 FORMAT(1HO,4X,8(1H*),1X,106HNUMBER OF SPECIFIED TIME PERIODS FOR TBNDS0498
                  1HE VARYING NON-HARMONIC VALUES OF THE BOUNDARY CONDITIONS, NSPPER BNDS0499
                  2 =,I3)                                                            BNDS0500
ISN 0442          NPER=NSPPER(ISPB)                                                  BNDS0501
ISN 0443          WRITE(3,1442)                                                      BNDS0502
ISN 0444       1442 FORMAT(1HO,54X,1H*,1X,4HIPER,1X,1H*,3X,6HTSPPER,                 BNDS0503
                   1       3X,1H*,3X,2HLZ,1X,1H*)                                  BNDS0504
ISN 0445          DO 1450 IPER=1,NPER                                                BNDS0505
ISN 0446          WRITE(3,1451)IPER,TSPPER(ISPB,IPER)                                BNDS0506
ISN 0447       1451 FORMAT(1HO,54X,2X,I4,1X,1X,E11.4)                                BNDS0507
ISN 0448          IF(IMHBND(IBND).GT.0)GO TO 1452                                    BNDS0508
ISN 0450          IF(IPHBND(IBND).GT.0)GO TO 1452                                    BNDS0509
ISN 0452          GO TO 1455                                                         BNDS0510
ISN 0453       1452 CONTINUE                                                         BNDS0511
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ISN 0454            WRITE(3,1453)HBXSPB(ISPB,IPER)                                  BNDS0512
ISN 0455         1453 FORMAT(1HO,84X,1H*,1X,8HHBXSPB =,E11.4)                       BNDS0513
ISN 0456         1455 CONTINUE                                                      BNDS0514
ISN 0457            IF(JMHBND(IBND).GT.0)GO TO 1456                                 BNDS0515
ISN 0459            IF(JPHBND(IBND).GT.0)GO TO 1456                                 BNDS0516
ISN 0461            GO TO 1459                                                      BNDS0517
ISN 0462         1456 CONTINUE                                                      BNDS0518
ISN 0463            WRITE(3,1457)HBYSPB(ISPB,IPER)                                  BNDS0519
ISN 0464         1457 FORMAT(1HO,84X,1H*,1X,8HHBYSPB =,E11.4)                       BNDS0520
ISN 0465         1459 CONTINUE                                                      BNDS0521
ISN 0466            IF(IMHBND(IBND).GT.0)GO TO 1552                                 BNDS0522
ISN 0468            IF(IPHBND(IBND).GT.0)GO TO 1552                                 BNDS0523
ISN 0470            GO TO 1569                                                      BNDS0524
ISN 0471         1552 CONTINUE                                                      BNDS0525
ISN 0472            NLZLOW=LZXLOW(ISPB)                                             BNDS0526
ISN 0473            NLZTOP=LZXTOP(ISPB)                                             BNDS0527
ISN 0474            DO 1560 LZ=NLZLOW,NLZTOP                                        BNDS0528
ISN 0475            WRITE(3,1561)LZ,VLXTTX(ISPB,LZ,IPER),VLXTMX(ISPB,LZ,IPER)       BNDS0529
ISN 0476         1561 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLXTTX =,E11.4,                 BNDS0530
                    1                  2X,1H*,1X,8HVLXTMX =,E11.4)                 BNDS0531
ISN 0477            WRITE(3,1562)LZ,VMYTTX(ISPB,LZ,IPER),VMYTMX(ISPB,LZ,IPER)       BNDS0532
ISN 0478         1562 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVMYTTX =,E11.4,                 BNDS0533
                    1                  2X,1H*,1X,8HVMYTMX =,E11.4)                 BNDS0534
ISN 0479            WRITE(3,1563)LZ,VPYTTX(ISPB,LZ,IPER),VPYTMX(ISPB,LZ,IPER)       BNDS0535
ISN 0480         1563 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVPYTTX =,E11.4,                 BNDS0536
                    1                  2X,1H*,1X,8HVPYTMX =,E11.4)                 BNDS0537
ISN 0481            WRITE(3,1564)LZ,VLZTTX(ISPB,LZ,IPER),VLZTMX(ISPB,LZ,IPER)       BNDS0538
ISN 0482         1564 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLZTTX =,E11.4,                 BNDS0539
                    1                  2X,1H*,1X,8HVLZTMX =,E11.4)                 BNDS0540
ISN 0483         1560 CONTINUE                                                      BNDS0541
ISN 0484         1569 CONTINUE                                                      BNDS0542
ISN 0485            IF(JMHBND(IBND).GT.0)GO TO 1572                                 BNDS0543
ISN 0487            IF(JPHBND(IBND).GT.0)GO TO 1572                                 BNDS0544
ISN 0489            GO TO 1589                                                      BNDS0545
ISN 0490         1572 CONTINUE                                                      BNDS0546
ISN 0491            NLZLOW=LZYLOW(ISPB)                                             BNDS0547
ISN 0492            NLZTOP=LZYTOP(ISPB)                                             BNDS0548
ISN 0493            DO 1580 LZ=NLZLOW,NLZTOP                                        BNDS0549
ISN 0494            WRITE(3,1581)LZ,VLYTTY(ISPB,LZ,IPER),VLYTMY(ISPB,LZ,IPER)       BNDS0550
ISN 0495         1581 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLYTTY =,E11.4,                 BNDS0551
                    1                  2X,1H*,1X,8HVLYTMY =,E11.4)                 BNDS0552
ISN 0496            WRITE(3,1582)LZ,VMXTTY(ISPB,LZ,IPER),VMXTMY(ISPB,LZ,IPER)       BNDS0553
ISN 0497         1582 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVMXTTY =,E11.4,                 BNDS0554
                    1                  2X,1H*,1X,8HVMXTMY =,E11.4)                 BNDS0555
ISN 0498            WRITE(3,1583)LZ,VPXTTY(ISPB,LZ,IPER),VPXTMY(ISPB,LZ,IPER)       BNDS0556
ISN 0499         1583 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVPXTTY =,E11.4,                 BNDS0557
                    1                  2X,1H*,1X,8HVPXTMY =,E11.4)                 BNDS0558
ISN 0500            WRITE(3,1584)LZ,VLZTTY(ISPB,LZ,IPER),VLZTMY(ISPB,LZ,IPER)       BNDS0559
ISN 0501         1584 FORMAT(1HO,76X,I4,4X,1H*,1X,8HVLZTTY =,E11.4,                 BNDS0560
                    1                  2X,1H*,1X,8HVLZTMY =,E11.4)                 BNDS0561
ISN 0502         1580 CONTINUE                                                      BNDS0562
ISN 0503         1589 CONTINUE                                                      BNDS0563
ISN 0504         1450 CONTINUE                                                      BNDS0564
ISN 0505            IF(INHARM.GT.0)GO TO 1462                                       BNDS0565
ISN 0507            WRITE(3,1461)                                                   BNDS0566
ISN 0508         1461 FORMAT(1HO,4X,8(1H*),1X,90HALL SPECIFIED BOUNDARY CONDITIONS IN ALBNDS0567
                    1L THE BOUNDARY ELEMENTS ARE NON-HARMONIC CONDITIONS)           BNDS0568
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ISN 0509          GO TO 1400                                                                BNDS0569
ISN 0510       1462 CONTINUE                                                                BNDS0570
ISN 0511          NPER=NSPPER(ISPB)                                                         BNDS0571
ISN 0512          WRITE(3,1465)NPER                                                         BNDS0572
ISN 0513       1465 FORMAT(1HO,4X,8(1H*),1X,10HNUMBER OF SPECIFIED TIME PERIODS FOR T      BNDS0573
                   1HE VARYING HARMONIC VALUES OF THE BOUNDARY CONDITIONS, NSPPER =,      BNDS0574
                   2I),I3)                                                                BNDS0575
ISN 0514          DO 1600 IPER=1,NPER                                                       BNDS0576
ISN 0515          WRITE(3,1451)IPER,TSPPER(ISPB,IPER)                                       BNDS0577
ISN 0516          IF(IMHBND(IBND).GT.0)GO TO 1612                                           BNDS0578
ISN 0518          IF(IPHBND(IBND).GT.0)GO TO 1612                                           BNDS0579
ISN 0520          GO TO 1619                                                                BNDS0580
ISN 0521       1612 CONTINUE                                                                BNDS0581
ISN 0522          WRITE(3,1613)NRXHBX(ISPB,IPER)                                            BNDS0582
ISN 0523       1613 FORMAT(1HO,40X,8HNRXHBX =,I2)                                           BNDS0583
ISN 0524          IF(NRXHBX(ISPB,IPER).LT.1)GO TO 1619                                      BNDS0584
ISN 0526          NHR=NRXHBX(ISPB,IPER)                                                     BNDS0585
ISN 0527          WRITE(3,1614)                                                             BNDS0586
ISN 0528       1614 FORMAT(1HO,40X,1H*,1X,3HIPR,1X,1H*,4X,6HAMXHBX,4X,                      BNDS0587
                  1       1H*,4X,6HPTXHBX,4X,1H*,4X,6HTLXHBX,4X,1H*)                     BNDS0588
                                                                                           BNDS0589
ISN 0529          DO 3610 IHR=1,NHR                                                         BNDS0590
ISN 0530          WRITE(3,1615)IHR,AMXHBX(ISPB,IPER,IHR),PTXHBX(ISPB,IPER,IHR),          BNDS0591
                  1                TLXHBX(ISPB,IPER,IHR)                                  BNDS0592
ISN 0531       1615 FORMAT(1HO,40X,3X,I2,2X,1X,E11.4,2X,1X,1X,E11.4,2X,1X,1X,E11.4)       BNDS0593
ISN 0532       3610 CONTINUE                                                               BNDS0594
ISN 0533       1619 CONTINUE                                                               BNDS0595
ISN 0534          IF(JMHBND(IBND).GT.0)GO TO 1622                                           BNDS0596
ISN 0536          IF(JPHBND(IBND).GT.0)GO TO 1622                                           BNDS0597
ISN 0538          GO TO 1629                                                                BNDS0598
ISN 0539       1622 CONTINUE                                                               BNDS0599
ISN 0540          WRITE(3,1623)NRYHBY(ISPB,IPER)                                            BNDS0600
ISN 0541       1623 FORMAT(1HO,40X,8HNRYHBY =,I2)                                           BNDS0601
ISN 0542          IF(NRYHBY(ISPB,IPER).LT.1)GO TO 1629                                      BNDS0602
ISN 0544          NHR=NRYHBY(ISPB,IPER)                                                     BNDS0603
ISN 0545          WRITE(3,1624)                                                             BNDS0604
ISN 0546       1624 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,6HAMYHBY,4X,                      BNDS0605
                  1       1H*,4X,6HPTYHBY,4X,1H*,4X,6HTLYHBY,4X,1H*)                     BNDS0606
ISN 0547          DO 3620 IHR=1,NHR                                                         BNDS0607
ISN 0548          WRITE(3,1615)IHR,AMYHBY(ISPB,IPER,IHR),PTYHBY(ISPB,IPER,IHR),          BNDS0608
                  1                TLYHBY(ISPB,IPER,IHR)                                  BNDS0609
ISN 0549       3620 CONTINUE                                                               BNDS0610
ISN 0550       1629 CONTINUE                                                               BNDS0611
ISN 0551          IF(IMHBND(IBND).GT.0)GO TO 1632                                           BNDS0612
ISN 0553          IF(IPHBND(IBND).GT.0)GO TO 1632                                           BNDS0613
ISN 0555          GO TO 1649                                                                BNDS0614
ISN 0556       1632 CONTINUE                                                               BNDS0615
ISN 0557          NLZLOW=LZXLOW(ISPB)                                                       BNDS0616
ISN 0558          NLZTOP=LZXTOP(ISPB)                                                       BNDS0617
ISN 0559          DO 1640 LZ=NLZLOW,NLZTOP                                                  BNDS0618
ISN 0560          WRITE(3,1641)                                                             BNDS0619
ISN 0561       1641 FORMAT(1HO,40X,1H*,1X,6HNRXVLX,1X,                                     BNDS0620
                  1                1H*,1X,6HNRXVMY,1X,1H*,1X,6HNRXVPY,1X,                BNDS0621
                  2                1H*,1X,6HNRXVLZ,1X,1H*)                                BNDS0622
ISN 0562          WRITE(3,1642)NRXVLX(ISPB,LZ,IPER),          NRXVMY(ISPB,LZ,            BNDS0623
                  1IPER),NRXVPY(ISPB,LZ,IPER),          NRXVLZ(ISPB,LZ,IPER)            BNDS0624
ISN 0563       1642 FORMAT(1HO,40X,2X,2X,I2,2X,3X,2X,I2,2X,3X,2X,I2,2X,                    BNDS0625
                  1                3X,2X,I2)
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ISN 0564            IF(NRXVLX(ISPB,LZ,IPER).LT.1)GO TO 1644                                  BNDS0626
ISN 0566            NHR=NRXVLX(ISPB,LZ,IPER)                                                 BNDS0627
ISN 0567            WRITE(3,1643)                                                            BNDS0628
ISN 0568          1643 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                                BNDS0629
                     1       6HAMXVLX,4X,1H*,4X,6HPTXVLX,4X,1H*,4X,6HTLXVLX,4X,1H*)        BNDS0630
ISN 0569            DO 3630 IHR=1,NHR                                                        BNDS0631
ISN 0570            WRITE(3,1615)IHR,AMXVLX(ISPB,LZ,IPER,IHR),PTXVLX(ISPB,LZ,IPER,IHR)BNDS0632
                    1                  ,TLXVLX(ISPB,LZ,IPER,IHR)                           BNDS0633
ISN 0571          3630 CONTINUE                                                              BNDS0634
ISN 0572          1644 CONTINUE                                                              BNDS0635
ISN 0573            IF(NRXVMY(ISPB,LZ,IPER).LT.1)GO TO 1646                                  BNDS0636
ISN 0575            NHR=NRXVMY(ISPB,LZ,IPER)                                                 BNDS0637
ISN 0576            WRITE(3,1645)                                                            BNDS0638
ISN 0577          1645 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                                BNDS0639
                     1       6HAMXVMY,4X,1H*,4X,6HPTXVMY,4X,1H*,4X,6HTLXVMY,4X,1H*)        BNDS0640
ISN 0578            DO 3650 IHR=1,NHR                                                        BNDS0641
ISN 0579            WRITE(3,1615)IHR,AMXVMY(ISPB,LZ,IPER,IHR),PTXVMY(ISPB,LZ,IPER,IHR)BNDS0642
                    1                  ,TLXVMY(ISPB,LZ,IPER,IHR)                           BNDS0643
ISN 0580          3650 CONTINUE                                                              BNDS0644
ISN 0581          1646 CONTINUE                                                              BNDS0645
ISN 0582            IF(NRXVPY(ISPB,LZ,IPER).LT.1)GO TO 1648                                  BNDS0646
ISN 0584            NHR=NRXVPY(ISPB,LZ,IPER)                                                 BNDS0647
ISN 0585            WRITE(3,1647)                                                            BNDS0648
ISN 0586          1647 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                                BNDS0649
                     1       6HAMXVPY,4X,1H*,4X,6HPTXVPY,4X,1H*,4X,6HTLXVPY,4X,1H*)        BNDS0650
ISN 0587            DO 3670 IHR=1,NHR                                                        BNDS0651
ISN 0588            WRITE(3,1615)IHR,AMXVPY(ISPB,LZ,IPER,IHR),PTXVPY(ISPB,LZ,IPER,IHR)BNDS0652
                    1                  ,TLXVPY(ISPB,LZ,IPER,IHR)                           BNDS0653
ISN 0589          3670 CONTINUE                                                              BNDS0654
ISN 0590          1648 CONTINUE                                                              BNDS0655
ISN 0591            IF(NRXVLZ(ISPB,LZ,IPER).LT.1)GO TO 1652                                  BNDS0656
ISN 0593            NHR=NRXVLZ(ISPB,LZ,IPER)                                                 BNDS0657
ISN 0594            WRITE(3,1651)                                                            BNDS0658
ISN 0595          1651 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                                BNDS0659
                     1       6HAMXVLZ,4X,1H*,4X,6HPTXVLZ,4X,1H*,4X,6HTLXVLZ,4X,1H*)        BNDS0660
ISN 0596            DO 3710 IHR=1,NHR                                                        BNDS0661
ISN 0597            WRITE(3,1615)IHR,AMXVLZ(ISPB,LZ,IPER,IHR),PTXVLZ(ISPB,LZ,IPER,IHR)BNDS0662
                    1                  ,TLXVLZ(ISPB,LZ,IPER,IHR)                           BNDS0663
ISN 0598          3710 CONTINUE                                                              BNDS0664
ISN 0599          1652 CONTINUE                                                              BNDS0665
ISN 0600          1640 CONTINUE                                                              BNDS0666
ISN 0601          1649 CONTINUE                                                              BNDS0667
ISN 0602            IF(JMHBND(IBND).GT.0)GO TO 1656                                          BNDS0668
ISN 0604            IF(JPHBND(IBND).GT.0)GO TO 1656                                          BNDS0669
ISN 0606            GO TO 1669                                                               BNDS0670
ISN 0607          1656 CONTINUE                                                              BNDS0671
ISN 0608            NLZLOW=LZYLOW(ISPB)                                                      BNDS0672
ISN 0609            NLZTOP=LZYTOP(ISPB)                                                      BNDS0673
ISN 0610            DO 1660 LZ=NLZLOW,NLZTOP                                                 BNDS0674
ISN 0611            WRITE(3,1661)                                                            BNDS0675
ISN 0612          1661 FORMAT(1HO,40X,1H*,1X,6HNRYVLY,1X,                                    BNDS0676
                     1                1H*,1X,6HNRYVMX,1X,1H*,1X,6HNRYVPX,1X,                BNDS0677
                     2                1H*,1X,6HNRYVLZ,1X,1H*)                                BNDS0678
ISN 0613            WRITE(3,1642)NRYVLY(ISPB,LZ,IPER),          NRYVMX(ISPB,LZ,            BNDS0679
                    1IPER),NRYVPX(ISPB,LZ,IPER),          NRYVLZ(ISPB,LZ,IPER)              BNDS0680
ISN 0614            IF(NRYVLY(ISPB,LZ,IPER).LT.1)GO TO 1664                                  BNDS0681
ISN 0616            NHR=NRYVLY(ISPB,LZ,IPER)                                                 BNDS0682
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ISN 0670            TSPTWO(ISPB)=TIMTWO                                          BNDS0740
ISN 0671         1800 CONTINUE                                                   BNDS0741
ISN 0672            GO TO 9999                                                   BNDS0742
ISN 0673            2 CONTINUE                                                   BNDS0743
ISN 0674            DO 2000 ISPB=1,NSPBND                                        BNDS0744
ISN 0675            IX=IXBSPB(ISPB)                                              BNDS0745
ISN 0676            JY=JYBSPB(ISPB)                                              BNDS0746
ISN 0677            IBND=INTEXT(IX,JY)                                           BNDS0747
ISN 0678            IF(INVARY.GT.0)GO TO 2100                                    BNDS0748
                   C********** ALL SPECIFIED BOUNDARY CONDITIONS REMAIN AT THEIR INITIAL *BNDS0749
                   C********** SPECIFIED VALUES ACCORDING THE SPECIFIED INITIAL CONDITIONS BNDS0750
ISN 0680            IF(IMHBND(IBND).GT.0)GO TO 2012                              BNDS0751
ISN 0682            IF(IPHBND(IBND).GT.0)GO TO 2012                              BNDS0752
ISN 0684            GO TO 2029                                                   BNDS0753
ISN 0685         2012 CONTINUE                                                   BNDS0754
ISN 0686            HSRSPX=HBXINI(ISPB)                                          BNDS0755
ISN 0687            NLZLOW=LZXLOW(ISPB)                                          BNDS0756
ISN 0688            NLZTOP=LZXTOP(ISPB)                                          BNDS0757
ISN 0689            DO 2020 LZ=NLZLOW,NLZTOP                                     BNDS0758
ISN 0690            VLXSPX(LZ)=VLXBXI(ISPB,LZ)                                   BNDS0759
ISN 0691            VLXMXX(LZ)=VLXMXI(ISPB,LZ)                                   BNDS0760
ISN 0692            VPYSPX(LZ)=VMYBXI(ISPB,LZ)                                   BNDS0761
ISN 0693            VMYMXX(LZ)=VMYMXI(ISPB,LZ)                                   BNDS0762
ISN 0694            VPYSPX(LZ)=VPYBXI(ISPB,LZ)                                   BNDS0763
ISN 0695            VPYMXX(LZ)=VPYMXI(ISPB,LZ)                                   BNDS0764
ISN 0696            VLZSPX(LZ)=VLZBXI(ISPB,LZ)                                   BNDS0765
ISN 0697            VLZMXX(LZ)=VLZMXI(ISPB,LZ)                                   BNDS0766
ISN 0698         2020 CONTINUE                                                   BNDS0767
ISN 0699         2029 CONTINUE                                                   BNDS0768
ISN 0700            IF(JMHBND(IBND).GT.0)GO TO 2032                              BNDS0769
ISN 0702            IF(JPHBND(IBND).GT.0)GO TO 2032                              BNDS0770
ISN 0704            GO TO 2049                                                   BNDS0771
ISN 0705         2032 CONTINUE                                                   BNDS0772
ISN 0706            HSRSPY=HBYINI(ISPB)                                          BNDS0773
ISN 0707            NLZLOW=LZYLOW(ISPB)                                          BNDS0774
ISN 0708            NLZTOP=LZYTOP(ISPB)                                          BNDS0775
ISN 0709            DO 2040 LZ=NLZLOW,NLZTOP                                     BNDS0776
ISN 0710            VLYSPY(LZ)=VLYBYI(ISPB,LZ)                                   BNDS0777
ISN 0711            VLYMXY(LZ)=VLYMYI(ISPB,LZ)                                   BNDS0778
ISN 0712            VMXSPY(LZ)=VMXBYI(ISPB,LZ)                                   BNDS0779
ISN 0713            VMXMXY(LZ)=VMXMYI(ISPB,LZ)                                   BNDS0780
ISN 0714            VPXSPY(LZ)=VPXBYI(ISPB,LZ)                                   BNDS0781
ISN 0715            VPXMXY(LZ)=VPXMYI(ISPB,LZ)                                   BNDS0782
ISN 0716            VLZSPY(LZ)=VLZBYI(ISPB,LZ)                                   BNDS0783
ISN 0717            VLZMXY(LZ)=VLZMYI(ISPB,LZ)                                   BNDS0784
ISN 0718         2040 CONTINUE                                                   BNDS0785
ISN 0719         2049 CONTINUE                                                   BNDS0786
ISN 0720            GO TO 2199                                                   BNDS0787
ISN 0721         2100 CONTINUE                                                   BNDS0788
                   C********** SPECIFIED BOUNDARY CONDITIONS VARY ACCORDING TO  **********BNDS0789
                   C********** THE SPECIFIED PERIODS WITH SPECIFIED CONDITION DATA *********BNDS0790
ISN 0722            TIMTWO=TSPTWO(ISPB)                                          BNDS0791
ISN 0723            IF(T.LE.TIMTWO)GO TO 2115                                    BNDS0792
                   C********** NEW SEARCH REQUIRED TO DETERMINE THE TIME PERIOD  **********BNDS0793
ISN 0725            NPXPER=NSPPER(ISPB)                                          BNDS0794
ISN 0726            IPER=ISPPER(ISPB)                                            BNDS0795
ISN 0727         2110 CONTINUE                                                   BNDS0796
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ISN 0617            WRITE(3,1663)                                                    BNDS0683
ISN 0618         1663 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                         BNDS0684
                     1       6HAMYVLY,4X,1H*,4X,6HPTYVLY,4X,1H*,4X,6HTLYVLY,4X,1H*)  BNDS0685
ISN 0619            DO 3830 IHR=1,NHR                                                BNDS0686
ISN 0620            WRITE(3,1615)IHR,AMYVLY(ISPB,LZ,IPER,IHR),PTYVLY(ISPB,LZ,IPER,IHR) BNDS0687
                    1                  ,TLYVLY(ISPB,LZ,IPER,IHR)                     BNDS0688
ISN 0621         3830 CONTINUE                                                       BNDS0689
ISN 0622         1664 CONTINUE                                                       BNDS0690
ISN 0623            IF(NRYVMX(ISPB,LZ,IPER).LT.1)GO TO 1666                          BNDS0691
ISN 0625            NHR=NRYVMX(ISPB,LZ,IPER)                                         BNDS0692
ISN 0626            WRITE(3,1665)                                                    BNDS0693
ISN 0627         1665 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                         BNDS0694
                     1       6HAMYVMX,4X,1H*,4X,6HPTYVMX,4X,1H*,4X,6HTLYVMX,4X,1H*)  BNDS0695
ISN 0628            DO 3850 IHR=1,NHR                                                BNDS0696
ISN 0629            WRITE(3,1615)IHR,AMYVMX(ISPB,LZ,IPER,IHR),PTYVMX(ISPB,LZ,IPER,IHR) BNDS0697
                    1                  ,TLYVMX(ISPB,LZ,IPER,IHR)                     BNDS0698
ISN 0630         3850 CONTINUE                                                       BNDS0699
ISN 0631         1666 CONTINUE                                                       BNDS0700
ISN 0632            IF(NRYVPX(ISPB,LZ,IPER).LT.1)GO TO 1668                          BNDS0701
ISN 0634            NHR=NRYVPX(ISPB,LZ,IPER)                                         BNDS0702
ISN 0635            WRITE(3,1667)                                                    BNDS0703
ISN 0636         1667 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                         BNDS0704
                     1       6HAMYVPX,4X,1H*,4X,6HPTYVPX,4X,1H*,4X,6HTLYVPX,4X,1H*)  BNDS0705
ISN 0637            DO 3870 IHR=1,NHR                                                BNDS0706
ISN 0638            WRITE(3,1615)IHR,AMYVPX(ISPB,LZ,IPER,IHR),PTYVPX(ISPB,LZ,IPER,IHR) BNDS0707
                    1                  ,TLYVPX(ISPB,LZ,IPER,IHR)                     BNDS0708
ISN 0639         3870 CONTINUE                                                       BNDS0709
ISN 0640         1668 CONTINUE                                                       BNDS0710
ISN 0641            IF(NRYVLZ(ISPB,LZ,IPER).LT.1)GO TO 1672                          BNDS0711
ISN 0643            NHR=NRYVLZ(ISPB,LZ,IPER)                                         BNDS0712
ISN 0644            WRITE(3,1671)                                                    BNDS0713
ISN 0645         1671 FORMAT(1HO,40X,1H*,1X,3HIHR,1X,1H*,4X,                         BNDS0714
                     1       6HAMYVLZ,4X,1H*,4X,6HPTYVLZ,4X,1H*,4X,6HTLYVLZ,4X,1H*)  BNDS0715
ISN 0646            DO 3910 IHR=1,NHR                                                BNDS0716
ISN 0647            WRITE(3,1615)IHR,AMYVLZ(ISPB,LZ,IPER,IHR),PTYVLZ(ISPB,LZ,IPER,IHR) BNDS0717
                    1                  ,TLYVLZ(ISPB,LZ,IPER,IHR)                     BNDS0718
ISN 0648         3910 CONTINUE                                                       BNDS0719
ISN 0649         1672 CONTINUE                                                       BNDS0720
ISN 0650         1660 CONTINUE                                                       BNDS0721
ISN 0651         1669 CONTINUE                                                       BNDS0722
ISN 0652         1600 CONTINUE                                                       BNDS0723
ISN 0653         1400 CONTINUE                                                       BNDS0724
ISN 0654         1799 CONTINUE                                                       BNDS0725
ISN 0655            IF(INVARY.LT.1)GO TO 9999                                        BNDS0726
                 C********** DETERMINE THE PERIODS FOR ALL THE BOUNDARY ELEMENTS  ********BNDS0727
                 C********** CORRESPONDING TO THE INITIAL TIME                    **********BNDS0728
ISN 0657            DO 1800 ISPB=1,NSPBND                                            BNDS0729
ISN 0658            NMXPER=NSPPER(ISPB)                                              BNDS0730
ISN 0659            TIMTWO=0.0                                                       BNDS0731
ISN 0660            IPER=0                                                           BNDS0732
ISN 0661         1810 CONTINUE                                                       BNDS0733
ISN 0662            IPER=IPER+1                                                      BNDS0734
ISN 0663            IF(IPER.GT.NMXPER)GO TO 9980                                     BNDS0735
ISN 0665            TIMONE=TIMTWO                                                    BNDS0736
ISN 0666            TIMTWO=TIMONE+TSPPER(ISPB,IPER)                                  BNDS0737
ISN 0667            IF(T.GT.TIMTWO)GO TO 1810                                        BNDS0738
ISN 0669            ISPPER(ISPB)=IPER                                                BNDS0739
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ISN 0728          IPER=IPER+1                                                    BNDS0797
ISN 0729          IF(IPER.GT.NMXPER)GO TO 9980                                   BNDS0798
ISN 0731          TIMONE=TIMTWO                                                  BNDS0799
ISN 0732          TIMTWO=TIMONE+TSPPER(ISPB,IPER)                                BNDS0800
ISN 0733          IF(T.GT.TIMTWO)GO TO 2110                                      BNDS0801
ISN 0735          ISPPER(ISPB)=IPER                                              BNDS0802
ISN 0736          TSPTWO(ISPB)=TIMTWO                                            BNDS0803
ISN 0737       2115 CONTINUE                                                     BNDS0804
ISN 0738          IPER=ISPPER(ISPB)                                              BNDS0805
ISN 0739          IF(LINEAR.GT.0)GO TO 2150                                      BNDS0806
                  C********** NO LINEAR INTERPOLATION BETWEEN THE DATA POINTS      ******BNDS0807
                  C********** DISCONTINUOUS SPECIFICATION OF THE BOUNDARY CONDITIONS *****BNDS0808
ISN 0741          IF(IMHBND(IBND).GT.0)GO TO 2112                                BNDS0809
ISN 0743          IF(IPHBND(IBND).GT.0)GO TO 2112                                BNDS0810
ISN 0745          GO TO 2129                                                     BNDS0811
ISN 0746       2112 CONTINUE                                                     BNDS0812
ISN 0747          HSRSPX=HBXSPB(ISPB,IPER)                                       BNDS0813
ISN 0748          NLZLOW=LZXLOW(ISPB)                                            BNDS0814
ISN 0749          NLZTOP=LZXTOP(ISPB)                                            BNDS0815
ISN 0750          DO 2120 LZ=NLZLOW,NLZTOP                                       BNDS0816
ISN 0751          VLXSPX(LZ)=VLXTTX(ISPB,LZ,IPER)                                BNDS0817
ISN 0752          VLXMXX(LZ)=VLXTMX(ISPB,LZ,IPER)                                BNDS0818
ISN 0753          VMYSPX(LZ)=VMYTTX(ISPB,LZ,IPER)                                BNDS0819
ISN 0754          VMYMXX(LZ)=VMYTMX(ISPB,LZ,IPER)                                BNDS0820
ISN 0755          VPYSPX(LZ)=VPYTTX(ISPB,LZ,IPER)                                BNDS0821
ISN 0756          VPYMXX(LZ)=VPYTMX(ISPB,LZ,IPER)                                BNDS0822
ISN 0757          VLZSPX(LZ)=VLZTTX(ISPB,LZ,IPER)                                BNDS0823
ISN 0758          VLZMXX(LZ)=VLZTMX(ISPB,LZ,IPER)                                BNDS0824
ISN 0759       2120 CONTINUE                                                     BNDS0825
ISN 0760       2129 CONTINUE                                                     BNDS0826
ISN 0761          IF(JMHBND(IBND).GT.0)GO TO 2132                                BNDS0827
ISN 0763          IF(JPHBND(IBND).GT.0)GO TO 2132                                BNDS0828
ISN 0765          GO TO 2149                                                     BNDS0829
ISN 0766       2132 CONTINUE                                                     BNDS0830
ISN 0767          HSRSPY=HBYSPB(ISPB,IPER)                                       BNDS0831
ISN 0768          NLZLOW=LZYLOW(ISPB)                                            BNDS0832
ISN 0769          NLZTOP=LZYTOP(ISPB)                                            BNDS0833
ISN 0770          DO 2140 LZ=NLZLOW,NLZTOP                                       BNDS0834
ISN 0771          VLYSPY(LZ)=VLYTTY(ISPB,LZ,IPER)                                BNDS0835
ISN 0772          VLYMXY(LZ)=VLYTMY(ISPB,LZ,IPER)                                BNDS0836
ISN 0773          VMXSPY(LZ)=VMXTTY(ISPB,LZ,IPER)                                BNDS0837
ISN 0774          VMXMXY(LZ)=VMXTMY(ISPB,LZ,IPER)                                BNDS0838
ISN 0775          VPXSPY(LZ)=VPXTTY(ISPB,LZ,IPER)                                BNDS0839
ISN 0776          VPXMXY(LZ)=VPXTMY(ISPB,LZ,IPER)                                BNDS0840
ISN 0777          VLZSPY(LZ)=VLZTTY(ISPB,LZ,IPER)                                BNDS0841
ISN 0778          VLZMXY(LZ)=VLZTMY(ISPB,LZ,IPER)                                BNDS0842
ISN 0779       2140 CONTINUE                                                     BNDS0843
ISN 0780       2149 CONTINUE                                                     BNDS0844
ISN 0781          GO TO 2199                                                     BNDS0845
ISN 0782       2150 CONTINUE                                                     BNDS0846
                  C********** LINEAR INTERPOLATION BETWEEN PERIOD-END DATA POINTS ********BNDS0847
                  C********** CONTINUOUS SPECIFICATION OF BOUNDARY CONDITIONS    **********BNDS0848
ISN 0783          DURPER=TSPPER(ISPB,IPER)                                       BNDS0849
ISN 0784          TIMTWO=TSPTWO(ISPB)                                            BNDS0850
ISN 0785          TIMONE=TIMTWO-DURPER                                           BNDS0851
ISN 0786          RDTPER=(T-TIMONE)/DURPER                                       BNDS0852
ISN 0787          IF(IMHBND(IBND).GT.0)GO TO 2152                                BNDS0853
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ISN 0789            IF(IPHBND(IBND).GT.0)GO TO 2152                          BNDS0854
ISN 0791            GO TO 2169                                               BNDS0855
ISN 0792       2152 CONTINUE                                                 BNDS0856
ISN 0793            IF(IPER.GT.1)GO TO 2155                                  BNDS0857
ISN 0795            HBXONE=HBXINI(ISPB)                                      BNDS0858
ISN 0796            GO TO 2158                                               BNDS0859
ISN 0797       2155 CONTINUE                                                 BNDS0860
ISN 0798            HBXONE=HBXSPB(ISPB,IPER-1)                               BNDS0861
ISN 0799       2158 CONTINUE                                                 BNDS0862
ISN 0800            HBXTWO=HBXSPB(ISPB,IPER)                                 BNDS0863
ISN 0801            HBRSPX=HBXONE+RDTPER*(HBXTWO-HBXONE)                     BNDS0864
ISN 0802            NLZLOW=LZXLOW(ISPB)                                      BNDS0865
ISN 0803            NLZTOP=LZXTOP(ISPB)                                      BNDS0866
ISN 0804            DO 2160 LZ=NLZLOW,NLZTOP                                 BNDS0867
ISN 0805            IF(IPER.GT.1)GO TO 2165                                  BNDS0868
ISN 0807            VLXBX1=VLXBXI(ISPB,LZ)                                   BNDS0869
ISN 0808            VLXMX1=VLXMXI(ISPB,LZ)                                   BNDS0870
ISN 0809            VPYBX1=VMYBXI(ISPB,LZ)                                   BNDS0871
ISN 0810            VPYMX1=VMYMXI(ISPB,LZ)                                   BNDS0872
ISN 0811            VPYBX1=VPYBXI(ISPB,LZ)                                   BNDS0873
ISN 0812            VPYMX1=VPYMXI(ISPB,LZ)                                   BNDS0874
ISN 0813            VLZBX1=VLZBXI(ISPB,LZ)                                   BNDS0875
ISN 0814            VLZMX1=VLZMXI(ISPB,LZ)                                   BNDS0876
ISN 0815            GO TO 2168                                               BNDS0877
ISN 0816       2165 CONTINUE                                                 BNDS0878
ISN 0817            VLXBX1=VLXTTX(ISPB,LZ,IPER-1)                            BNDS0879
ISN 0818            VLXMX1=VLXTMX(ISPB,LZ,IPER-1)                            BNDS0880
ISN 0819            VPYBX1=VMYTTX(ISPB,LZ,IPER-1)                            BNDS0881
ISN 0820            VPYMX1=VMYTMX(ISPB,LZ,IPER-1)                            BNDS0882
ISN 0821            VPYBX1=VPYTTX(ISPB,LZ,IPER-1)                            BNDS0883
ISN 0822            VPYMX1=VPYTMX(ISPB,LZ,IPER-1)                            BNDS0884
ISN 0823            VLZBX1=VLZTTX(ISPB,LZ,IPER-1)                            BNDS0885
ISN 0824            VLZMX1=VLZTMX(ISPB,LZ,IPER-1)                            BNDS0886
ISN 0825       2168 CONTINUE                                                 BNDS0887
ISN 0826            VLXBX2=VLXTTX(ISPB,LZ,IPER)                              BNDS0888
ISN 0827            VLXMX2=VLXTMX(ISPB,LZ,IPER)                              BNDS0889
ISN 0828            VPYBX2=VMYTTX(ISPB,LZ,IPER)                              BNDS0890
ISN 0829            VPYMX2=VMYTMX(ISPB,LZ,IPER)                              BNDS0891
ISN 0830            VPYBX2=VPYTTX(ISPB,LZ,IPER)                              BNDS0892
ISN 0831            VPYMX2=VPYTMX(ISPB,LZ,IPER)                              BNDS0893
ISN 0832            VLZBX2=VLZTTX(ISPB,LZ,IPER)                              BNDS0894
ISN 0833            VLZMX2=VLZTMX(ISPB,LZ,IPER)                              BNDS0895
ISN 0834            VLXSPX(LZ)=VLXBX1+RDTPER*(VLXBX2-VLXBX1)                 BNDS0896
ISN 0835            VLXMXX(LZ)=VLXMX1+RDTPER*(VLXMX2-VLXMX1)                 BNDS0897
ISN 0836            VMYSPX(LZ)=VMYBX1+RDTPER*(VMYBX2-VMYBX1)                 BNDS0898
ISN 0837            VMYMXX(LZ)=VMYMX1+RDTPER*(VMYMX2-VMYMX1)                 BNDS0899
ISN 0838            VPYSPX(LZ)=VPYBX1+RDTPER*(VPYBX2-VPYBX1)                 BNDS0900
ISN 0839            VPYMXX(LZ)=VPYMX1+RDTPER*(VPYMX2-VPYMX1)                 BNDS0901
ISN 0840            VLZSPX(LZ)=VLZBX1+RDTPER*(VLZBX2-VLZBX1)                 BNDS0902
ISN 0841            VLZMXX(LZ)=VLZMX1+RDTPER*(VLZMX2-VLZMX1)                 BNDS0903
ISN 0842       2160 CONTINUE                                                 BNDS0904
ISN 0843       2169 CONTINUE                                                 BNDS0905
ISN 0844            IF(JMHBND(IBND).GT.0)GO TO 2172                          BNDS0906
ISN 0846            IF(JPHBND(IBND).GT.0)GO TO 2172                          BNDS0907
ISN 0848            GO TO 2189                                               BNDS0908
ISN 0849       2172 CONTINUE                                                 BNDS0909
ISN 0850            IF(IPER.GT.1)GO TO 2175                                  BNDS0910
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ISN 0852            HBYONE=HBYINI(ISPB)                                          BNDS0911
ISN 0853            GO TO 2178                                                   BNDS0912
ISN 0854         2175 CONTINUE                                                   BNDS0913
ISN 0855            HBYONE=HBYSPB(ISPB,IPER-1)                                   BNDS0914
ISN 0856         2178 CONTINUE                                                   BNDS0915
ISN 0857            HBYTWO=HBYSPB(ISPB,IPER)                                     BNDS0916
ISN 0858            HSRSPY=HBYONE+RDTPER*(HBYTWO-HBYONE)                         BNDS0917
ISN 0859            NLZLOW=LZYLOW(ISPB)                                          BNDS0918
ISN 0860            NLZTOP=LZYTOP(ISPB)                                          BNDS0919
ISN 0861            DO 2180 LZ=NLZLOW,NLZTOP                                     BNDS0920
ISN 0862            IF(IPER.GT.1)GO TO 2185                                      BNDS0921
ISN 0864            VLYBY1=VLYBYI(ISPB,LZ)                                       BNDS0922
ISN 0865            VLYMY1=VLYMYI(ISPB,LZ)                                       BNDS0923
ISN 0866            VMXBY1=VMXBYI(ISPB,LZ)                                       BNDS0924
ISN 0867            VMXMY1=VMXMYI(ISPB,LZ)                                       BNDS0925
ISN 0868            VPXBY1=VPXBYI(ISPB,LZ)                                       BNDS0926
ISN 0869            VPXMY1=VPXMYI(ISPB,LZ)                                       BNDS0927
ISN 0870            VLZBY1=VLZBYI(ISPB,LZ)                                       BNDS0928
ISN 0871            VLZMY1=VLZMYI(ISPB,LZ)                                       BNDS0929
ISN 0872            GO TO 2188                                                   BNDS0930
ISN 0873         2185 CONTINUE                                                   BNDS0931
ISN 0874            VLYBY1=VLYTTY(ISPB,LZ,IPER-1)                                BNDS0932
ISN 0875            VLYMY1=VLYTMY(ISPB,LZ,IPER-1)                                BNDS0933
ISN 0876            VMXBY1=VMXTTY(ISPB,LZ,IPER-1)                                BNDS0934
ISN 0877            VMXMY1=VMXTMY(ISPB,LZ,IPER-1)                                BNDS0935
ISN 0878            VPXBY1=VPXTTY(ISPB,LZ,IPER-1)                                BNDS0936
ISN 0879            VPXMY1=VPXTMY(ISPB,LZ,IPER-1)                                BNDS0937
ISN 0880            VLZBY1=VLZTTY(ISPB,LZ,IPER-1)                                BNDS0938
ISN 0881            VLZMY1=VLZTMY(ISPB,LZ,IPER-1)                                BNDS0939
ISN 0882         2188 CONTINUE                                                   BNDS0940
ISN 0883            VLYBY2=VLYTTY(ISPB,LZ,IPER)                                  BNDS0941
ISN 0884            VLYMY2=VLYTMY(ISPB,LZ,IPER)                                  BNDS0942
ISN 0885            VMXBY2=VMXTTY(ISPB,LZ,IPER)                                  BNDS0943
ISN 0886            VMXMY2=VMXTMY(ISPB,LZ,IPER)                                  BNDS0944
ISN 0887            VPXBY2=VPXTTY(ISPB,LZ,IPER)                                  BNDS0945
ISN 0888            VPXMY2=VPXTMY(ISPB,LZ,IPER)                                  BNDS0946
ISN 0889            VLZBY2=VLZTTY(ISPB,LZ,IPER)                                  BNDS0947
ISN 0890            VLZMY2=VLZTMY(ISPB,LZ,IPER)                                  BNDS0948
ISN 0891            VLYSPY(LZ)=VLYBY1+RDTPER*(VLYBY2-VLYBY1)                     BNDS0949
ISN 0892            VLYMXY(LZ)=VLYMY1+RDTPER*(VLYMY2-VLYMY1)                     BNDS0950
ISN 0893            VMXSPY(LZ)=VMXBY1+RDTPER*(VMXBY2-VMXBY1)                     BNDS0951
ISN 0894            VMXMXY(LZ)=VMXMY1+RDTPER*(VMXMY2-VMXMY1)                     BNDS0952
ISN 0895            VPXSPY(LZ)=VPXBY1+RDTPER*(VPXBY2-VPXBY1)                     BNDS0953
ISN 0896            VPXMXY(LZ)=VPXMY1+RDTPER*(VPXMY2-VPXMY1)                     BNDS0954
ISN 0897            VLZSPY(LZ)=VLZBY1+RDTPER*(VLZBY2-VLZBY1)                     BNDS0955
ISN 0898            VLZMXY(LZ)=VLZMY1+RDTPER*(VLZMY2-VLZMY1)                     BNDS0956
ISN 0899         2180 CONTINUE                                                   BNDS0957
ISN 0900         2189 CONTINUE                                                   BNDS0958
ISN 0901         2199 CONTINUE                                                   BNDS0959
ISN 0902            IF(INHARM.LT.1)GO TO 2269                                    BNDS0960
                 C********** SPECIFIED BOUNDARY CONDITIONS VARY ACCORDING TO  **********BNDS0961
                 C********** THE SPECIFIED HARMONIC CONDITIONS     **********BNDS0962
                 C********** FOR EACH SPECIFIED PERIOD OF THE INPUT DATA  **********BNDS0963
                 C********** AT LEAST ONE INPUT DATA IS REQUIRED FOR INCLUDING THE ***** BNDS0964
                 C********** SPECIFIED HARMONIC CONDITIONS IN THE BOUNDARY ELEMENTS  ***BNDS0965
ISN 0904            TIMTWO=TSPTWO(ISPB)                                          BNDS0966
ISN 0905            IF(T.LE.TIMTWO)GO TO 2215                                    BNDS0967
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                C********** NEW SEARCH REQUIRED TO DETERMINE THE TIME PERIOD  **********BNDS0968
ISN 0907              NPXPER=NSPPER(ISPB)                                                BNDS0969
ISN 0908           2210 CONTINUE                                                         BNDS0970
ISN 0909              IPER=IPER+1                                                        BNDS0971
ISN 0910              IF(IPER.GT.NMXPER)GO TO 9980                                       BNDS0972
ISN 0912              TIMONE=TIMTWO                                                      BNDS0973
ISN 0913              TIMTWO=TIMONE+TSPPER(ISPB,IPER)                                    BNDS0974
ISN 0914              IF(T.GT.TIMTWO)GO TO 2210                                          BNDS0975
ISN 0916              ISPPER(ISPB)=IPER                                                  BNDS0976
ISN 0917              TSPTWO(ISPB)=TIMTWO                                                BNDS0977
ISN 0918           2215 CONTINUE                                                         BNDS0978
ISN 0919              IPER=ISPPER(ISPB)                                                  BNDS0979
                C********** NO LINEAR INTERPOLATION BETWEEN DATA POINTS IN  **********BNDS0980
                C********** CALCULATING THE SPECIFIED HARMONIC CONDITIONS    **********BNDS0981
                C********** DISCONTINUOUS SPECIFICATION OF THE HARMONIC CONDITIONS  ****BNDS0982
ISN 0920              IF(IMHBND(IBND).GT.0)GO TO 2232                                    BNDS0983
ISN 0922              IF(IPHBND(IBND).GT.0)GO TO 2232                                    BNDS0984
ISN 0924              GO TO 2249                                                         BNDS0985
ISN 0925           2232 CONTINUE                                                         BNDS0986
ISN 0926              IF(NRXHBX(ISPB,IPER).LT.1)GO TO 4319                               BNDS0987
ISN 0928              NHR=NRXHBX(ISPB,IPER)                                              BNDS0988
ISN 0929              HBXHRM=0.0                                                         BNDS0989
ISN 0930              DO 4310 IHR=1,NHR                                                  BNDS0990
ISN 0931              PERCOS=PTXHBX(ISPB,IPER,IHR)                                       BNDS0991
ISN 0932              IT=T/PERCOS                                                        BNDS0992
ISN 0933              TI=IT                                                              BNDS0993
ISN 0934              TPRCOS=T-TI*PERCOS-TLXHBX(ISPB,IPER,IHR)                           BNDS0994
ISN 0935              COSHBX=COS((2.*PI/PERCOS)*TPRCOS)                                  BNDS0995
ISN 0936              HBXHRM=HBXHRM+AMXHBX(ISPB,IPER,IHR)*COSHBX                         BNDS0996
ISN 0937           4310 CONTINUE                                                         BNDS0997
ISN 0938              HSRSPX=HSRSPX+HBXHRM                                               BNDS0998
ISN 0939           4319 CONTINUE                                                         BNDS0999
ISN 0940              NLZLOW=LZXLOW(ISPB)                                                BNDS1000
ISN 0941              NLZTOP=LZXTOP(ISPB)                                                BNDS1001
ISN 0942              DO 2240 LZ=NLZLOW,NLZTOP                                           BNDS1002
ISN 0943              IF(NRXVLX(ISPB,LZ,IPER).LT.1)GO TO 4419                            BNDS1003
ISN 0945              NHR=NRXVLX(ISPB,LZ,IPER)                                           BNDS1004
ISN 0946              VLXHRM=0.0                                                         BNDS1005
ISN 0947              DO 4410 IHR=1,NHR                                                  BNDS1006
ISN 0948              PERCOS=PTXVLX(ISPB,LZ,IPER,IHR)                                    BNDS1007
ISN 0949              IT=T/PERCOS                                                        BNDS1008
ISN 0950              TI=IT                                                              BNDS1009
ISN 0951              TPRCOS=T-TI*PERCOS-TLXVLX(ISPB,LZ,IPER,IHR)                        BNDS1010
ISN 0952              COSVLX=COS((2.*PI/PERCOS)*TPRCOS)                                  BNDS1011
ISN 0953              VLXHRM=VLXHRM+AMXVLX(ISPB,LZ,IPER,IHR)*COSVLX                      BNDS1012
ISN 0954           4410 CONTINUE                                                         BNDS1013
ISN 0955              VLXSPX(LZ)=VLXSPX(LZ)+VLXHRM                                       BNDS1014
ISN 0956              VLXMXX(LZ)=VLXMXX(LZ)+VLXHRM                                       BNDS1015
ISN 0957           4419 CONTINUE                                                         BNDS1016
ISN 0958              IF(NRXVMY(ISPB,LZ,IPER).LT.1)GO TO 4429                            BNDS1017
ISN 0960              NHR=NRXVMY(ISPB,LZ,IPER)                                           BNDS1018
ISN 0961              VMYHRM=0.0                                                         BNDS1019
ISN 0962              DO 4420 IHR=1,NHR                                                  BNDS1020
ISN 0963              PERCOS=PTXVMY(ISPB,LZ,IPER,IHR)                                    BNDS1021
ISN 0964              IT=T/PERCOS                                                        BNDS1022
ISN 0965              TI=IT                                                              BNDS1023
ISN 0966              TPRCOS=T-TI*PERCOS-TLXVMY(ISPB,LZ,IPER,IHR)                        BNDS1024
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ISN 0967            COSVMY=COS((2.*PI/PERCOS)*TPRCOS)                          BNDS1025
ISN 0968            VPYHRM=VMYHRM+AMXVMY(ISPB,LZ,IPER,IHR)*COSVMY              BNDS1026
ISN 0969      4420  CONTINUE                                                   BNDS1027
ISN 0970            VPYSPX(LZ)=VMYSPX(LZ)+VMYHRM                               BNDS1028
ISN 0971            VPYMXX(LZ)=VMYMXX(LZ)+VMYHRM                               BNDS1029
ISN 0972      4429  CONTINUE                                                   BNDS1030
ISN 0973            IF(NRXVPY(ISPB,LZ,IPER).LT.1)GO TO 4439                    BNDS1031
ISN 0975            NHR=NRXVPY(ISPB,LZ,IPER)                                   BNDS1032
ISN 0976            VPYHRM=0.0                                                 BNDS1033
ISN 0977            DO 4430 IHR=1,NHR                                          BNDS1034
ISN 0978            PERCOS=PTXVPY(ISPB,LZ,IPER,IHR)                            BNDS1035
ISN 0979            IT=T/PERCOS                                                BNDS1036
ISN 0980            TI=IT                                                      BNDS1037
ISN 0981            TPRCOS=T-TI*PERCOS-TLXVPY(ISPB,LZ,IPER,IHR)                BNDS1038
ISN 0982            COSVPY=COS((2.*PI/PERCOS)*TPRCOS)                          BNDS1039
ISN 0983            VPYHRM=VPYHRM+AMXVPY(ISPB,LZ,IPER,IHR)*COSVPY              BNDS1040
ISN 0984      4430  CONTINUE                                                   BNDS1041
ISN 0985            VPYSPX(LZ)=VPYSPX(LZ)+VPYHRM                               BNDS1042
ISN 0986            VPYMXX(LZ)=VPYMXX(LZ)+VPYHRM                               BNDS1043
ISN 0987      4439  CONTINUE                                                   BNDS1044
ISN 0988            IF(NRXVLZ(ISPB,LZ,IPER).LT.1)GO TO 4449                    BNDS1045
ISN 0990            NHR=NRXVLZ(ISPB,LZ,IPER)                                   BNDS1046
ISN 0991            VLZHRM=0.0                                                 BNDS1047
ISN 0992            DO 4440 IHR=1,NHR                                          BNDS1048
ISN 0993            PERCOS=PTXVLZ(ISPB,LZ,IPER,IHR)                            BNDS1049
ISN 0994            IT=T/PERCOS                                                BNDS1050
ISN 0995            TI=IT                                                      BNDS1051
ISN 0996            TPRCOS=T-TI*PERCOS-TLXVLZ(ISPB,LZ,IPER,IHR)                BNDS1052
ISN 0997            COSVLZ=COS((2.*PI/PERCOS)*TPRCOS)                          BNDS1053
ISN 0998            VLZHRM=VLZHRM+AMXVLZ(ISPB,LZ,IPER,IHR)*COSV' Z             BNDS1054
ISN 0999      4440  CONTINUE                                                   BNDS1055
ISN 1000            VLZSPX(LZ)=VLZSPX(LZ)+VLZHRM                               BNDS1056
ISN 1001            VLZMXX(LZ)=VLZMXX(LZ)+VLZHRM                               BNDS1057
ISN 1002      4449  CONTINUE                                                   BNDS1058
ISN 1003      2240  CONTINUE                                                   BNDS1059
ISN 1004      2249  CONTINUE                                                   BNDS1060
ISN 1005            IF(JMHBND(IBND).GT.0)GO TO 2252                            BNDS1061
ISN 1007            IF(JPHBND(IBND).GT.0)GO TO 2252                            BNDS1062
ISN 1009            GO TO 2269                                                 BNDS1063
ISN 1010      2252  CONTINUE                                                   BNDS1064
ISN 1011            IF(NRYHBY(ISPB,IPER).LT.1)GO TO 4519                       BNDS1065
ISN 1013            NHR=NRYHBY(ISPB,IPER)                                      BNDS1066
ISN 1014            HBYHRM=0.0                                                 BNDS1067
ISN 1015            DO 4510 IHR=1,NHR                                          BNDS1068
ISN 1016            PERCOS=PTYHBY(ISPB,IPER,IHR)                               BNDS1069
ISN 1017            IT=T/PERCOS                                                BNDS1070
ISN 1018            TI=IT                                                      BNDS1071
ISN 1019            TPRCOS=T-TI*PERCOS-TLYHBY(ISPB,IPER,IHR)                   BNDS1072
ISN 1020            COSHBY=((2.*PI/PERCOS)*TPRCOS)                             BNDS1073
ISN 1021            HBYHRM=HBYHRM+AMYHBY(ISPB,IPER,IHR)*COSHBY                 BNDS1074
ISN 1022      4510  CONTINUE                                                   BNDS1075
ISN 1023            HSRSPY=HSRSPY+HBYHRM                                       BNDS1076
ISN 1024      4519  CONTINUE                                                   BNDS1077
ISN 1025            NLZLOW=LZYLOW(ISPB)                                        BNDS1078
ISN 1026            NLZTOP=LZYTOP(ISPB)                                        BNDS1079
ISN 1027            DO 2260 LZ=NLZLOW,NLZTOP                                   BNDS1080
ISN 1028            IF(NRYVLY(ISPB,LZ,IPER).LT.1)GO TO 4619                    BNDS1081
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ISN 1030            NHR=NRYVLY(ISPB,LZ,IPER)                                BNDS1082
ISN 1031            VLYHRM=0.0                                              BNDS1083
ISN 1032            DO 4610 IHR=1,NHR                                       BNDS1084
ISN 1033            PERCOS=PTYVLY(ISPB,LZ,IPER,IHR)                         BNDS1085
ISN 1034            IT=T/PERCOS                                             BNDS1086
ISN 1035            TI=IT                                                   BNDS1087
ISN 1036            TPRCOS=T-TI*PERCOS-TLYVLY(ISPB,LZ,IPER,IHR)             BNDS1088
ISN 1037            COSVLY=((2.*PI/PERCOS)*TPRCOS)                          BNDS1089
ISN 1038            VLYHRM=VLYHRM+AMYVLY(ISPB,LZ,IPER,IHR)*COSVLY           BNDS1090
ISN 1039      4610  CONTINUE                                                BNDS1091
ISN 1040            VLYSPY(LZ)=VLYSPY(LZ)+VLYHRM                            BNDS1092
ISN 1041            VLYMXY(LZ)=VLYMXY(LZ)+VLYHRM                            BNDS1093
ISN 1042      4619  CONTINUE                                                BNDS1094
ISN 1043            IF(NRYVMX(ISPB,LZ,IPER).LT.1)GO TO 4629                 BNDS1095
ISN 1045            NHR=NRYVMX(ISPB,LZ,IPER)                                BNDS1096
ISN 1046            VMXHRM=0.0                                              BNDS1097
ISN 1047            DO 4620 IHR=1,NHR                                       BNDS1098
ISN 1048            PERCOS=PTYVMX(ISPB,LZ,IPER,IHR)                         BNDS1099
ISN 1049            IT=T/PERCOS                                             BNDS1100
ISN 1050            TI=IT                                                   BNDS1101
ISN 1051            TPRCOS=T-TI*PERCOS-TLYVMX(ISPB,LZ,IPER,IHR)             BNDS1102
ISN 1052            COSVMX=((2.*PI/PERCOS)*TPRCOS)                          BNDS1103
ISN 1053            VMXHRM=VMXHRM+AMYVMX(ISPB,LZ,IPER,IHR)*COSVMX           BNDS1104
ISN 1054      4620  CONTINUE                                                BNDS1105
ISN 1055            VMXSPY(LZ)=VMXSPY(LZ)+VMXHRM                            BNDS1106
ISN 1056            VMXMXY(LZ)=VMXMXY(LZ)+VMXHRM                            BNDS1107
ISN 1057      4629  CONTINUE                                                BNDS1108
ISN 1058            IF(NRYVPX(ISPB,LZ,IPER).LT.1)GO TO 4639                 BNDS1109
ISN 1060            NHR=NRYVPX(ISPB,LZ,IPER)                                BNDS1110
ISN 1061            VPXHRM=0.0                                              BNDS1111
ISN 1062            DO 4630 IHR=1,NHR                                       BNDS1112
ISN 1063            PERCOS=PTYVPX(ISPB,LZ,IPER,IHR)                         BNDS1113
ISN 1064            IT=T/PERCOS                                             BNDS1114
ISN 1065            TI=IT                                                   BNDS1115
ISN 1066            TPRCOS=T-TI*PERCOS-TLYVPX(ISPB,LZ,IPER,IHR)             BNDS1116
ISN 1067            COSVPX=((2.*PI/PERCOS)*TPRCOS)                          BNDS1117
ISN 1068            VPXHRM=VPXHRM+AMYVPX(ISPB,LZ,IPER,IHR)*COSVPX           BNDS1118
ISN 1069      4630  CONTINUE                                                BNDS1119
ISN 1070            VPXSPY(LZ)=VPXSPY(LZ)+VPXHRM                            BNDS1120
ISN 1071            VPXMXY(LZ)=VPXMXY(LZ)+VPXHRM                            BNDS1121
ISN 1072      4639  CONTINUE                                                BNDS1122
ISN 1073            IF(NRYVLZ(ISPB,LZ,IPER).LT.1)GO TO 4649                 BNDS1123
ISN 1075            NHR=NRYVLZ(ISPB,LZ,IPER)                                BNDS1124
ISN 1076            VLZHRM=0.0                                              BNDS1125
ISN 1077            DO 4640 IHR=1,NHR                                       BNDS1126
ISN 1078            PERCOS=PTYVLZ(ISPB,LZ,IPER,IHR)                         BNDS1127
ISN 1079            IT=T/PERCOS                                             BNDS1128
ISN 1080            TI=IT                                                   BNDS1129
ISN 1081            TPRCOS=T-TI*PERCOS-TLYVLZ(ISPB,LZ,IPER,IHR)             BNDS1130
ISN 1082            COSVLZ=((2.*PI/PERCOS)*TPRCOS)                          BNDS1131
ISN 1083            VLZHRM=VLZHRM+AMYVLZ(ISPB,LZ,IPER,IHR)*COSVLZ           BNDS1132
ISN 1084      4640  CONTINUE                                                BNDS1133
ISN 1085            VLZSPY(LZ)=VLZSPY(LZ)+VLZHRM                            BNDS1134
ISN 1086            VLZMXY(LZ)=VLZMXY(LZ)+VLZHRM                            BNDS1135
ISN 1087      4649  CONTINUE                                                BNDS1136
ISN 1088      2260  CONTINUE                                                BNDS1137
ISN 1089      2269  CONTINUE                                                BNDS1138
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ISN 1090          IJ=IJIXJY(IX,JY)                                                BNDS1139
ISN 1091          IPX=2*IJ-1                                                      BNDS1140
ISN 1092          IPX=IMX+1                                                       BNDS1141
ISN 1093          JMY=IMX                                                         BNDS1142
ISN 1094          JPY=IPX                                                         BNDS1143
ISN 1095          DXIAJ=DXBND(IBND)                                               BNDS1144
ISN 1096          DYIAJ=DYBND(IBND)                                               BNDS1145
ISN 1097          ARHIAJ=DXIAJ*DYIAJ                                              BNDS1146
ISN 1098          HSRIAJ=HSR(IJ)                                                  BNDS1147
ISN 1099          NLZMIN=NLZBOT(IJ)                                               BNDS1148
ISN 1100          DO 2270 LZ=NLZMIN,NZ                                            BNDS1149
ISN 1101          DHZDUM=DZ(LZ)                                                   BNDS1150
ISN 1102          IF(LZ.EQ.NLZMIN)  DHZDUM=DHZBOT(IJ)                             BNDS1151
ISN 1104          IF(LZ.EQ.NZ)  DHZDUM=DHZDUM+HSRIAJ                              BNDS1152
ISN 1106          DHZIAJ(LZ)=DHZDUM                                               BNDS1153
ISN 1107          VMXIAJ(LZ)=VLX(IMX,LZ)                                          BNDS1154
ISN 1108          VPXIAJ(LZ)=VLX(IPX,LZ)                                          BNDS1155
ISN 1109          VMYIAJ(LZ)=VLY(JMY,LZ)                                          BNDS1156
ISN 1110          VPYIAJ(LZ)=VLY(JPY,LZ)                                          BNDS1157
                  C********** CALCULATE VLZIAJ(LZ)                  **********BNDS1158
ISN 1111          GWZLMH=GWZLZH(IJ,LZ)                                            BNDS1159
ISN 1112          IF(LZ.EQ.NZ)GO TO 2272                                          BNDS1160
ISN 1114          GWZLPH=GWZLZH(IJ,LZ+1)                                          BNDS1161
ISN 1115          GO TO 2275                                                      BNDS1162
ISN 1116       2272 CONTINUE                                                      BNDS1163
ISN 1117          GWZLPH=ARHIAJ*FNCHSR(IJ)                                        BNDS1164
ISN 1118       2275 CONTINUE                                                      BNDS1165
ISN 1119          GWZIAJ=0.5*(GWZLMH+GWZLPH)                                      BNDS1166
ISN 1120          VLZIAJ(LZ)=GWZIAJ/ARHIAJ                                        BNDS1167
ISN 1121       2270 CONTINUE                                                      BNDS1168
                  C********** FLOW CONDITIONS ON THE ENCLOSURE SURFACES  **********BNDS1169
                  C********** OF THE BOUNDARY ELEMENTS ARE DETERMINED FOR SPECIFIED ******BNDS1170
                  C********** AND OPEN BOUNDARY CONDITIONS ACCORDING TO INPUT DATA *******BNDS1171
                  C********** ONLY ONE X-ENCLOSURE SURFACE AND/OR              **********BNDS1172
                  C********** ONLY ONE Y-ENCLOSURE SURFACE CAN HAVE SPECIFIED DATA *******BNDS1173
                  C********** AND/OR OPEN REGION BOUNDARY CONDITIONS           **********BNDS1174
                  C********** TWO FACING ENCLOSURE SURFACES CAN NOT BOTH HAVE SPECIFIED **BNDS1175
                  C********** INPUT DATA AND/OR OPEN REGION BOUNDARY CONDITIONS **********BNDS1176
ISN 1122       2300 CONTINUE                                                      BNDS1177
ISN 1123          IF(IMHBND(IBND).LT.1)GO TO 2400                                 BNDS1178
                  C********** START SPECIFYING IMH BOUNDARY CONDITIONS  **********BNDS1179
ISN 1125          DYIMH=DYBIMH(IBND)                                              BNDS1180
ISN 1126          HSRBXH(ISPB)=HSRSPX                                             BNDS1181
ISN 1127          IF(IMHBND(IBND).GT.1)GO TO 2320                                 BNDS1182
                  C********** SPECIFIED BOUNDARY CONDITIONS ON THE IMH ENCLOSURE SURFACE *BNDS1183
ISN 1129          HSRIMH=HSRSPX                                                   BNDS1184
ISN 1130          DO 2310 LZ=NLZMIN,NZ                                            BNDS1185
ISN 1131          IF(IVXSPX(ISPB,LZ).GT.0)GO TO 2315                              BNDS1186
                  C********** CLOSED IMH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1187
ISN 1133          GFXBXH(ISPB,LZ)=0.0                                             BNDS1188
ISN 1134          IVXBXH(ISPB,LZ)=0                                               BNDS1189
ISN 1135          VLX(IMX,LZ)=0.0                                                 BNDS1190
ISN 1136          VLXBXH(ISPB,LZ)=0.0                                             BNDS1191
ISN 1137          VMYBXH(ISPB,LZ)=0.0                                             BNDS1192
ISN 1138          VPYBXH(ISPB,LZ)=0.0                                             BNDS1193
ISN 1139          VLZBXH(ISPB,LZ)=0.0                                             BNDS1194
                  C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1195
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C********** NEEDED FOR CLOSED IMH BOUNDARY CONDITION          **********BNDS1196
C********** DUMMY PRESSURE (HEAD) SET TO ZERO                  **********BNDS1197
ISN 1140        FPRBXH(ISPB,LZ)=0.0                                        BNDS1198
ISN 1141        GO TO 2310                                                 BNDS1199
ISN 1142     2315 CONTINUE                                                 BNDS1200
C********** OPEN IMH ENCLOSURE SURFACE WITH SPECIFIED CONDITION  ********BNDS1201
ISN 1143        DHZIMH=DHZIAJ(LZ)                                          BNDS1202
ISN 1144        IF(LZ.EQ.NZ)  DHZIMH=DHZIMH-HSRIAJ+HSRIMH                  BNDS1203
ISN 1146        ARXIMH=DYIMH*DHZIMH                                        BNDS1204
ISN 1147        VLXIMH=VLXSPX(LZ)                                          BNDS1205
ISN 1148        GFXBXH(ISPB,LZ)=ARXIMH*VLXIMH                              BNDS1206
C********** HALF-ELEMENT VELOCITY, VLX(IMX,LZ) DETERMINED BY  **********BNDS1207
C********** INTERPOLATING THE SPECIFIED VELOCITY VLXMXX        **********BNDS1208
ISN 1149        IVXBXH(ISPB,LZ)=-1                                         BNDS1209
ISN 1150        VLXIMX=VLXMXX(LZ)+(1./3.)*(VLX(IP),LZ)-VLXMXX(LZ))         BNDS1210
ISN 1151        VLX(IMX,LZ)=VLXIMX                                         BNDS1211
ISN 1152        PREVLX(IMX,LZ)=VLXIMX                                      BNDS1212
C********** SIMPLE UPWIND DIFFERENCING FOR THE TRANSPORTIVE CONDITIONS *BNDS1213
ISN 1153        IF(VLXIMH.GT.0)GO TO 2316                                  BNDS1214
ISN 1155        VLXBXH(ISPB,LZ)=VMXIAJ(LZ)                                 BNDS1215
ISN 1156        VMYBXH(ISPB,LZ)=VMYIAJ(LZ)                                 BNDS1216
ISN 1157        VPYBXH(ISPB,LZ)=VPYIAJ(LZ)                                 BNDS1217
ISN 1158        VLZBXH(ISPB,LZ)=VLZIAJ(LZ)                                 BNDS1218
ISN 1159        GO TO 2318                                                 BNDS1219
ISN 1160     2316 CONTINUE                                                 BNDS1220
ISN 1161        VLXBXH(ISPB,LZ)=VLXSPX(LZ)                                 BNDS1221
ISN 1162        VMYBXH(ISPB,LZ)=VMYSPX(LZ)                                 BNDS1222
ISN 1163        VPYBXH(ISPB,LZ)=VPYSPX(LZ)                                 BNDS1223
ISN 1164        VLZBXH(ISPB,LZ)=VLZSPX(LZ)                                 BNDS1224
ISN 1165     2318 CONTINUE                                                 BNDS1225
C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1226
C********** NEEDED FOR SPECIFIED OPEN IMH BOUNDARY CONDITION  **********BNDS1227
C********** DUMMY PRESSURE (HEAD) SET TO ZERO                  **********BNDS1228
ISN 1166        FPRBXH(ISPB,LZ)=0.0                                        BNDS1229
ISN 1167     2310 CONTINUE                                                 BNDS1230
ISN 1168        GO TO 2500                                                 BNDS1231
ISN 1169     2320 CONTINUE                                                 BNDS1232
C********** START CALCULATING THE OPEN REGION BOUNDARY CONDITIONS ******BNDS1233
C********** ON THE IMH ENCLOSURE SURFACE                       **********BNDS1234
C********** DETERMINE THE EXTRAPOLATED EXTERNAL OPEN REGION CONDITIONS *BNDS1235
C********** WITH COMPLETELY OPEN IMH SURFACE IN ALL LAYERS     **********BNDS1236
ISN 1170        HSRIM1=HSRSPX                                              BNDS1237
ISN 1171        IF(INTEXT(IX+1,JY).NE.0)GO TO 2340                         BNDS1238
C********** ELEMENT AT IX+1 IS EXTERNAL TO THE REGION:         **********BNDS1239
C********** LINEAR EXTRAPOLATION FOR IX-1 IS IMPOSSIBLE        **********BNDS1240
C********** EXTERNAL OPEN REGION CONDITIONS SET EQUAL TO       **********BNDS1241
C********** THE CONDITIONS IN THE BOUNDARY ELEMENT             **********BNDS1242
ISN 1173        DO 2330 LZ=NLZMIN,NZ                                       BNDS1243
ISN 1174        VLXBXH(ISPB,LZ)=VMXIAJ(LZ)                                 BNDS1244
ISN 1175        VMYBXH(ISPB,LZ)=VMYIAJ(LZ)                                 BNDS1245
ISN 1176        VPYBXH(ISPB,LZ)=VPYIAJ(LZ)                                 BNDS1246
ISN 1177        VLZBXH(ISPB,LZ)=VLZIAJ(LZ)                                 BNDS1247
ISN 1178     2330 CONTINUE                                                 BNDS1248
ISN 1179        GO TO 2359                                                 BNDS1249
ISN 1180     2340 CONTINUE                                                 BNDS1250
C********** ELEMENT AT IX+1 IS AN INTERNAL OR A BOUNDARY ELEMENT: ******BNDS1251
C********** LINEAR EXTRAPOLATION FOR IX-1 IS POSSIBLE          **********BNDS1252
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ISN 1181          IJIP1=IJIXJY(IX+1,JY)                                          BNDS1253
ISN 1182          JMYIP1=2*IJIP1-1                                               BNDS1254
ISN 1183          JPYIP1=JMYIP1+1                                                BNDS1255
ISN 1184          IF(INTEXT(IX+1,JY).GT.0)GO TO 2342                             BNDS1256
                 C********** ELEMENT AT IX+1 IS AN INTERNAL ELEMENT              **********BNDS1257
ISN 1186          DXIP1=DX(IX+1)                                                 BNDS1258
ISN 1187          DYIP1=DY(JY)                                                   BNDS1259
ISN 1188          ARHIP1=DXIP1*DYIP1                                             BNDS1260
ISN 1189          GO TO 2345                                                     BNDS1261
ISN 1190         2342 CONTINUE                                                   BNDS1262
                 C********** ELEMENT AT IX+1 IS A BOUNDARY ELEMENT              **********BNDS1263
ISN 1191          IBNIP1=INTEXT(IX+1,JY)                                         BNDS1264
ISN 1192          DXIP1=DXBND(IBNIP1)                                            BNDS1265
ISN 1193          ARHIP1=ARHBND(IBNIP1)                                          BNDS1266
ISN 1194         2345 CONTINUE                                                   BNDS1267
                 C********** LINEAR EXTRAPOLATION TO THE OPEN-REGION BOUNDARY    **********BNDS1268
ISN 1195          CDXIMH=DXIAJ/(DXIAJ+DXIP1)                                     BNDS1269
ISN 1196          NLZIP1=NLZBOT(IJIP1)                                           BNDS1270
ISN 1197          DO 2350 LZ=NLZMIN,NZ                                           BNDS1271
ISN 1198          IF(LZ.GE.NLZIP1)GO TO 2352                                     BNDS1272
                 C********** LINEAR EXTRAPOLATION IS IMPOSSIBLE IN THE LAYERS    **********BNDS1273
                 C********** CLOSED AT IX+1 WITH THE SHORELINE BOUNDARY          **********BNDS1274
                 C********** EXTERNAL OPEN REGION CONDITIONS IN THE LAYER ARE    **********BNDS1275
                 C********** SET EQUAL TO THE CONDITIONS IN THE ELEMENT         **********BNDS1276
ISN 1200          VLXBXH(ISPB,LZ)=VMXIAJ(LZ)                                     BNDS1277
ISN 1201          VMYBXH(ISPB,LZ)=VMYIAJ(LZ)                                     BNDS1278
ISN 1202          VPYBXH(ISPB,LZ)=VPYIAJ(LZ)                                     BNDS1279
ISN 1203          VLZBXH(ISPB,LZ)=VLZIAJ(LZ)                                     BNDS1280
ISN 1204          GO TO 2350                                                     BNDS1281
ISN 1205         2352 CONTINUE                                                   BNDS1282
ISN 1206          VMYIP1=VLY(JMYIP1,LZ)                                          BNDS1283
ISN 1207          VPYIP1=VLY(JPYIP1,LZ)                                          BNDS1284
                 C********** CALCULATE VLZIP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJIP1 *BNDS1285
ISN 1208          GWZLMH=GWZLZH(IJIP1,LZ)                                        BNDS1286
ISN 1209          IF(LZ.EQ.NZ)GO TO 2356                                         BNDS1287
ISN 1211          GWZLPH=GWZLZH(IJIP1,LZ+1)                                      BNDS1288
ISN 1212          GO TO 2358                                                     BNDS1289
ISN 1213         2356 CONTINUE                                                   BNDS1290
ISN 1214          GWZLPH=ARHIP1*FNCHSR(IJIP1)                                    BNDS1291
ISN 1215         2358 CONTINUE                                                   BNDS1292
ISN 1216          GWZIP1=0.5*(GWZLMH+GWZLPH)                                     BNDS1293
ISN 1217          VLZIP1=GWZIP1/ARHIP1                                           BNDS1294
ISN 1218          VLXBXH(ISPB,LZ)=VMXIAJ(LZ)                                     BNDS1295
ISN 1219          VMYBXH(ISPB,LZ)=VMYIAJ(LZ)-CDXIMH*(VMYIP1-VMYIAJ(LZ))          BNDS1296
ISN 1220          VPYBXH(ISPB,LZ)=VPYIAJ(LZ)-CDXIMH*(VPYIP1-VPYIAJ(LZ))          BNDS1297
ISN 1221          VLZBXH(ISPB,LZ)=VLZIAJ(LZ)-CDXIMH*(VLZIP1-VLZIAJ(LZ))          BNDS1298
ISN 1222         2350 CONTINUE                                                   BNDS1299
ISN 1223         2359 CONTINUE                                                   BNDS1300
                 C********** CORRECT THE EXTRAPOLATED VALUES ACCORDING TO THE    **********BNDS1301
                 C********** SPECIFIED MINIMUM AND MAXIMUM VALUES OF THE OPEN    **********BNDS1302
                 C********** REGION CONDITIONS                                   **********BNDS1303
ISN 1224          DO 2360 LZ=NLZMIN,NZ                                           BNDS1304
ISN 1225          IF(IVXSPX(ISPB,LZ).GT.0)GO TO 2365                             BNDS1305
                 C********** CLOSED IMH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1306
ISN 1227          GFXBXH(ISPB,LZ)=0.0                                            BNDS1307
ISN 1228          IVXBXH(ISPB,LZ)=0                                              BNDS1308
ISN 1229          VLX(IMX,LZ)=0.0                                                BNDS1309
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ISN 1230          VLXBXH(ISPB,LZ)=0.0                                                  BNDS1310
ISN 1231          VPYBXH(ISPB,LZ)=0.0                                                  BNDS1311
ISN 1232          VPYBXH(ISPB,LZ)=0.0                                                  BNDS1312
ISN 1233          VLZBXH(ISPB,LZ)=0.0                                                  BNDS1313
                 C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1314
                 C********** NEEDED FOR CLOSED IMH BOUNDARY CONDITION         **********BNDS1315
                 C********** DUMMY PRESSURE (HEAD) SET TO ZERO                **********BNDS1316
ISN 1234          FPRBXH(ISPB,LZ)=0.0                                                  BNDS1317
ISN 1235          GO TO 2360                                                           BNDS1318
ISN 1236           2365 CONTINUE                                                       BNDS1319
                 C********** OPEN-REGION IMH ENCLOSURE SURFACE WITH LIMITING CONDITIONS *BNDS1320
ISN 1237          HSRIMH=0.5*(HSRIM1+HSRIAJ)                                           BNDS1321
ISN 1238          DHZIMH=DHZIAJ(LZ)                                                    BNDS1322
ISN 1239          IF(LZ.EQ.NZ)  DHZIMH=DHZIMH-HSRIAJ+HSRIMH                            BNDS1323
ISN 1241          ARXIMH=DYIMH*DHZIMH                                                  BNDS1324
ISN 1242          GFXBXH(ISPB,LZ)=ARXIMH*VMXIAJ(LZ)                                    BNDS1325
ISN 1243          IVXBXH(ISPB,LZ)=1                                                    BNDS1326
                 C********** CHECK THE SPECIFIED MINIMUM AND MAXIMUM LIMITING CONDITIONS BNDS1327
ISN 1244          VLXDUM=VLXBXH(ISPB,LZ)                                               BNDS1328
ISN 1245          IF(VLXDUM.LT.VLXSPX(LZ))  VLXDUM=VLXSPX(LZ)                          BNDS1329
ISN 1247          IF(VLXDUM.GT.VLXMXX(LZ))  VLXDUM=VLXMXX(LZ)                          BNDS1330
ISN 1249          VLXBXH(ISPB,LZ)=VLXDUM                                               BNDS1331
ISN 1250          VMYDUM=VMYBXH(ISPB,LZ)                                               BNDS1332
ISN 1251          IF(VMYDUM.LT.VMYSPX(LZ))  VMYDUM=VMYSPX(LZ)                          BNDS1333
ISN 1253          IF(VMYDUM.GT.VMYMXX(LZ))  VMYDUM=VMYMXX(LZ)                          BNDS1334
ISN 1255          VMYBXH(ISPB,LZ)=VMYDUM                                               BNDS1335
ISN 1256          VPYDUM=VPYBXH(ISPB,LZ)                                               BNDS1336
ISN 1257          IF(VPYDUM.LT.VPYSPX(LZ))  VPYDUM=VPYSPX(LZ)                          BNDS1337
ISN 1259          IF(VPYDUM.GT.VPYMXX(LZ))  VPYDUM=VPYMXX(LZ)                          BNDS1338
ISN 1261          VPYBXH(ISPB,LZ)=VPYDUM                                               BNDS1339
ISN 1262          VLZDUM=VLZBXH(ISPB,LZ)                                               BNDS1340
ISN 1263          IF(VLZDUM.LT.VLZSPX(LZ))  VLZDUM=VLZSPX(LZ)                          BNDS1341
ISN 1265          IF(VLZDUM.GT.VLZMXX(LZ))  VLZDUM=VLZMXX(LZ)                          BNDS1342
ISN 1267          VLZBXH(ISPB,LZ)=VLZDUM                                               BNDS1343
ISN 1268           2360 CONTINUE                                                       BNDS1344
                 C********** START CALCULATING THE PRESSURE (HEAD) CONDITIONS  **********BNDS1345
                 C********** AT THE OPEN REGION BOUNDARY IN ALL LAYERS WITH OPEN ********BNDS1346
                 C********** IMH ENCLOSURE BOUNDARY                           **********BNDS1347
                 C********** ONLY HYDROSTATIC PRESSURE EFFECTS WITH VARIABLE   **********BNDS1348
                 C********** DENSITY ARE INCLUDED IN CALCULATING THE PRESSURE  **********BNDS1349
                 C********** CONDITIONS ON THE OPEN REGION BOUNDARY           **********BNDS1350
                 C********** HYDROSTATIC PRESSURE CONDITIONS DETERMINED BY SUMMING FROM *BNDS1351
                 C********** THE TOP LAYER NZ TO THE LOWER SPECIFIED LAYER LZXLOW *******BNDS1352
ISN 1269          FPRLPH=0.0                                                           BNDS1353
ISN 1270          NLZD=NZ-LZXLOW(ISPB)+1                                               BNDS1354
ISN 1271          DO 2370 LZD=1,NLZD                                                   BNDS1355
ISN 1272          LZ=NZ-LZD+1                                                          BNDS1356
ISN 1273          DHZIMH=DHZIAJ(LZ)                                                    BNDS1357
ISN 1274          IF(LZ.EQ.NZ)  DHZIMH=DHZIMH-HSRIAJ+HSRIMH                            BNDS1358
ISN 1276          ARXIMH=DYIMH*DHZIMH                                                  BNDS1359
ISN 1277          VLXIMH=GFXBXH(ISPB,LZ)/ARXIMH                                        BNDS1360
                 C********** UPWIND DIFFERENCING ON ARXIMH SURFACE           **********BNDS1351
ISN 1278          IF(VLXIMH.GT.0.)GO TO 2376                                           BNDS1362
                 C********** FOR OUTFLOWING OPEN REGION CONDITIONS           **********BNDS1363
                 C********** SIMPLE UPWIND DIFFERENCING FORM                 **********BNDS1364
ISN 1280           2376 CONTINUE                                                       BNDS1365
ISN 1281          FPRLMH=FPRLPH+COSRNZ*DHZIMH                                          BNDS1366
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ISN 1282          FPRBXH(ISPB,LZ)=FPRLMH                                          BNDS1367
ISN 1283          FPRLPH=FPRLMH                                                   BNDS1368
ISN 1284       2370 CONTINUE                                                      BNDS1369
ISN 1285          GO TO 2500                                                      BNDS1370
ISN 1286       2400 CONTINUE                                                      BNDS1371
ISN 1287          IF(IPHBND(IBND).LT.1)GO TO 2500                                 BNDS1372
                C********** START SPECIFING IPH BOUNDARY CONDITIONS          **********BNDS1373
ISN 1289          DYIPH=DYBIPH(IBND)                                              BNDS1374
ISN 1290          HSRBXH(ISPB)=HSRSPX                                             BNDS1375
ISN 1291          IF(IPHBND(IBND).GT.1)GO TO 2420                                 BNDS1376
                C********** SPECIFIED BOUNDARY CONDITIONS ON THE IPH ENCLOSURE SURFACE *BNDS1377
ISN 1293          HSRIPH=HSRSPX                                                   BNDS1378
ISN 1294          DO 2410 LZ=NLZMIN,NZ                                            BNDS1379
ISN 1295          IF(IVXSPX(ISPB,LZ).GT.0)GO TO 2415                              BNDS1380
                C********** CLOSED IPH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1381
ISN 1297          GFXBXH(ISPB,LZ)=0.0                                             BNDS1382
ISN 1298          IVXBXH(ISPB,LZ)=0                                               BNDS1383
ISN 1299          VLX(IPX,LZ)=0.0                                                 BNDS1384
ISN 1300          VLXBXH(ISPB,LZ)=0.0                                             BNDS1385
ISN 1301          VPYBXH(ISPB,LZ)=0.0                                             BNDS1386
ISN 1302          VPYBXH(ISPB,LZ)=0.0                                             BNDS1387
ISN 1303          VLZBXH(ISPB,LZ)=0.0                                             BNDS1388
                C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1389
                C********** NEEDED FOR CLOSED IPH BOUNDARY CONDITION          **********BNDS1390
                C********** DUMMY PRESSURE (HEAD) SET TO ZERO                 **********BNDS1391
ISN 1304          FPRBXH(ISPB,LZ)=0.0                                             BNDS1392
ISN 1305          GO TO 2410                                                      BNDS1393
ISN 1306       2415 CONTINUE                                                      BNDS1394
                C********** OPEN IPH ENCLOSURE SURFACE WITH SPECIFIED CONDITION  *******BNDS1395
ISN 1307          DHZIPH=DHZIAJ(LZ)                                               BNDS1396
ISN 1308          IF(LZ.EQ.NZ)  DHZIPH=DHZIPH-HSRIAJ+HSRIPH                       BNDS1397
ISN 1310          ARXIPH=DYIPH*DHZIPH                                             BNDS1398
ISN 1311          VLXIPH=VLXSPX(LZ)                                               BNDS1399
ISN 1312          GFXBXH(ISPB,LZ)=ARXIPH*VLXIPH                                   BNDS1400
                C********** HALF-ELEMENT VELOCITY, VLX(IPX,LZ) DETERMINED BY  **********BNDS1401
                C********** INTERPOLATING THE SPECIFIED VELOCITY VLXMXX       **********BNDS1402
ISN 1313          IVXBXH(ISPB,LZ)=-1                                              BNDS1403
ISN 1314          VLXIPX=VLXMXX(LZ)-(1./3.)*(VLXMXX(LZ)-VLX(IMX,LZ))              BNDS1404
ISN 1315          VLX(IPX,LZ)=VLXIPX                                              BNDS1405
ISN 1316          PREVLX(IPX,LZ)=VLXIPX                                           BNDS1406
                C********** SIMPLE UPWIND DIFFERENCING FOR THE TRANSPORTIVE CONDITIONS *BNDS1407
ISN 1317          IF(VLXIPH.LT.0)GO TO 2416                                       BNDS1408
ISN 1319          VLXBXH(ISPB,LZ)=VPXIAJ(LZ)                                      BNDS1409
ISN 1320          VPYBXH(ISPB,LZ)=VMYIAJ(LZ)                                      BNDS1410
ISN 1321          VPYBXH(ISPB,LZ)=VPYIAJ(LZ)                                      BNDS1411
ISN 1322          VLZBXH(ISPB,LZ)=VLZIAJ(LZ)                                      BNDS1412
ISN 1323          GO TO 2418                                                      BNDS1413
ISN 1324       2416 CONTINUE                                                      BNDS1414
ISN 1325          VLXBXH(ISPB,LZ)=VLXSPX(LZ)                                      BNDS1415
ISN 1326          VPYBXH(ISPB,LZ)=VMYSPX(LZ)                                      BNDS1416
ISN 1327          VPYBXH(ISPB,LZ)=VPYSPX(LZ)                                      BNDS1417
ISN 1328          VLZBXH(ISPB,LZ)=VLZSPX(LZ)                                      BNDS1418
ISN 1329       2418 CONTINUE                                                      BNDS1419
                C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1420
                C********** NEEDED FOR SPECIFIED OPEN IPH BOUNDARY CONDITION  **********BNDS1421
                C********** DUMMY PRESSURE (HEAD) SET TO ZERO                 **********BNDS1422
ISN 1330          FFRBXH(ISPB,LZ)=0.0                                             BNDS1423
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ISN 1331      2410 CONTINUE                                                    BNDS1424
ISN 1332           GO TO 2500                                                  BNDS1425
ISN 1333      2420 CONTINUE                                                    BNDS1426
              C********** START CALCULATING THE OPEN REGION BOUNDARY CONDITIONS ******BNDS1427
              C********** ON THE IPH ENCLOSURE SURFACE                         **********BNDS1428
              C********** DETERMINE THE EXTRAPOLATED EXTERNAL OPEN REGION CONDITIONS *BNDS1429
              C********** WITH COMPLETELY OPEN IPH SURFACE IN ALL LAYERS       **********BNDS1430
ISN 1334           HSRIP1=HSRSPX                                               BNDS1431
ISN 1335           IF(INTEXT(IX-1,JY).NE.0)GO TO 2440                          BNDS1432
              C********** ELEMENT AT IX-1 IS EXTERNAL TO THE REGION:           **********BNDS1433
              C********** LINEAR EXTRAPOLATION FOR IX+1 IS IMPOSSIBLE          **********BNDS1434
              C********** EXTERNAL OPEN REGION CONDITIONS SET EQUAL TO         **********BNDS1435
              C********** THE CONDITIONS IN THE BOUNDARY ELEMENT               **********BNDS1436
ISN 1337           DO 2430 LZ=NLZMIN,NZ                                        BNDS1437
ISN 1338           VLXBXH(ISPB,LZ)=VPXIAJ(LZ)                                  BNDS1438
ISN 1339           VMYBXH(ISPB,LZ)=VMYIAJ(LZ)                                  BNDS1439
ISN 1340           VPYBXH(ISPB,LZ)=VPYIAJ(LZ)                                  BNDS1440
ISN 1341           VLZBXH(ISPB,LZ)=VLZIAJ(LZ)                                  BNDS1441
ISN 1342      2430 CONTINUE                                                    BNDS1442
ISN 1343           GO TO 2459                                                  BNDS1443
ISN 1344      2440 CONTINUE                                                    BNDS1444
              C********** ELEMENT AT IX-1 IS AN INTERNAL OR A BOUNDARY ELEMENT: ******BNDS1445
              C********** LINEAR EXTRAPOLATION FOR IX+1 IS POSSIBLE           **********BNDS1446
ISN 1345           IJIM1=IJIXJY(IX-1,JY)                                       BNDS1447
ISN 1346           JMYIM1=2*IJIM1-1                                            BNDS1448
ISN 1347           JPYIM1=JMYIM1+1                                             BNDS1449
ISN 1348           IF(INTEXT(IX-1,JY).GT.0)GO TO 2442                          BNDS1450
              C********** ELEMENT AT IX-1 IS AN INTERNAL ELEMENT               **********BNDS1451
ISN 1350           DXIM1=DX(IX-1)                                              BNDS1452
ISN 1351           DYIM1=DY(JY)                                                BNDS1453
ISN 1352           ARHIM1=DXIM1*DYIM1                                          BNDS1454
ISN 1353           GO TO 2445                                                  BNDS1455
ISN 1354      2442 CONTINUE                                                    BNDS1456
              C********** ELEMENT AT IX-1 IS A BOUNDARY ELEMENT                **********BNDS1457
ISN 1355           IBNIM1=INTEXT(IX-1,JY)                                      BNDS1458
ISN 1356           DXIM1=DXBND(IBNIM1)                                         BNDS1459
ISN 1357           ARHIM1=ARHBND(IBNIM1)                                       BNDS1460
ISN 1358      2445 CONTINUE                                                    BNDS1461
              C********** LINEAR EXTRAPOLATION TO THE OPEN-REGION BOUNDARY     **********BNDS1462
ISN 1359           CDXIPH=DXIAJ/(DXIAJ+DXIM1)                                  BNDS1463
ISN 1360           NLZIM1=NLZBOT(IJIM1)                                        BNDS1464
ISN 1361           DO 2450 LZ=NLZMIN,NZ                                        BNDS1465
ISN 1362           IF(LZ.GE.NLZIM1)GO TO 2452                                  BNDS1466
              C********** LINEAR EXTRAPOLATION IS IMPOSSIBLE IN THE LAYERS     **********BNDS1467
              C********** CLOSED AT IX-1 WITH THE SHORELINE BOUNDARY           **********BNDS1468
              C********** EXTERNAL OPEN REGION CONDITIONS IN THE LAYER ARE     **********BNDS1469
              C********** SET EQUAL TO THE CONDITIONS IN THE ELEMENT           **********BNDS1470
ISN 1364           VLXBXH(ISPB,LZ)=VPXIAJ(LZ)                                  BNDS1471
ISN 1365           VMYBXH(ISPB,LZ)=VMYIAJ(LZ)                                  BNDS1472
ISN 1366           VPYBXH(ISPB,LZ)=VPYIAJ(LZ)                                  BNDS1473
ISN 1367           VLZBXH(ISPB,LZ)=VLZIAJ(LZ)                                  BNDS1474
ISN 1368           GO TO 2450                                                  BNDS1475
ISN 1369      2452 CONTINUE                                                    BNDS1476
ISN 1370           VMYIM1=VLY(JMYIM1,LZ)                                       BNDS1477
ISN 1371           VPYIM1=VLY(JPYIM1,LZ)                                       BNDS1478
              C********** CALCULATE VLZIM1 WITH GWZLMH AND GWZLPH EVALUATED AT IJIM1 *BNDS1479
ISN 1372           GWZLMH=GWZLZH(IJIM1,LZ)                                     BNDS1480
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ISN 1373          IF(LZ.EQ.NZ)GO TO 2456                                                BNDS1481
ISN 1375          GWZLPH=GWZLZH(IJIM1,LZ+1)                                             BNDS1482
ISN 1376          GO TO 2458                                                            BNDS1483
ISN 1377       2456 CONTINUE                                                            BNDS1484
ISN 1378          GWZLPH=ARHIM1*FNCHSR(IJIM1)                                           BNDS1485
ISN 1379       2458 CONTINUE                                                            BNDS1486
ISN 1380          GWZIM1=0.5*(GWZLMH+GWZLPH)                                            BNDS1487
ISN 1381          VLZIM1=GWZIM1/ARHIM1                                                  BNDS1488
ISN 1382          VLXBXH(ISPB,LZ)=VPXIAJ(LZ)                                            BNDS1489
ISN 1383          VMYBXH(ISPB,LZ)=VMYIAJ(LZ)+CDXIPH*(VMYIAJ(LZ)-VMYIM1)                BNDS1490
ISN 1384          VPYBXH(ISPB,LZ)=VPYIAJ(LZ)+CDXIPH*(VPYIAJ(LZ)-VPYIM1)                BNDS1491
ISN 1385          VLZBXH(ISPB,LZ)=VLZIAJ(LZ)+CDXIPH*(VLZIAJ(LZ)-VLZIM1)                BNDS1492
ISN 1386       2450 CONTINUE                                                            BNDS1493
ISN 1387       2459 CONTINUE                                                            BNDS1494
               C********** CORRECT THE EXTRAPOLATED VALUES ACCORDING TO THE  **********BNDS1495
               C********** SPECIFIED MINIMUM AND MAXIMUM VALUES OF THE OPEN  **********BNDS1496
               C********** REGION CONDITIONS                                **********BNDS1497
ISN 1388          DO 2460 LZ=NLZMIN,NZ                                                  BNDS1498
ISN 1389          IF(IVXSPX(ISPB,LZ).GT.0)GO TO 2465                                    BNDS1499
               C********** CLOSED IPH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1500
ISN 1391          GFXBXH(ISPB,LZ)=0.0                                                   BNDS1501
ISN 1392          IVXBXH(ISPB,LZ)=0                                                     BNDS1502
ISN 1393          VLX(IPX,LZ)=0.0                                                       BNDS1503
ISN 1394          VLXBXH(ISPB,LZ)=0.0                                                   BNDS1504
ISN 1395          VMYBXH(ISPB,LZ)=0.0                                                   BNDS1505
ISN 1396          VPYBXH(ISPB,LZ)=0.0                                                   BNDS1506
ISN 1397          VLZBXH(ISPB,LZ)=0.0                                                   BNDS1507
               C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1508
               C********** NEEDED FOR CLOSED IPH BOUNDARY CONDITION         **********BNDS1509
               C********** DUMMY PRESSURE (HEAD) SET TO ZERO                **********BNDS1510
ISN 1398          FPRBXH(ISPB,LZ)=0.0                                                   BNDS1511
ISN 1399          GO TO 2460                                                            BNDS1512
ISN 1400       2465 CONTINUE                                                            BNDS1513
               C********** OPEN-REGION IPH ENCLOSURE SURFACE WITH LIMITING CONDITIONS *BNDS1514
ISN 1401          HSRIPH=0.5*(HSRIAJ+HSRIP1)                                            BNDS1515
ISN 1402          DHZIPH=DHZIAJ(LZ)                                                     BNDS1516
ISN 1403          IF(LZ.EQ.NZ)  DHZIPH=DHZIPH-HSRIAJ+HSRIPH                             BNDS1517
ISN 1405          ARXIPH=DYIPH*DHZIPH                                                   BNDS1518
ISN 1406          GFXBXH(ISPB,LZ)=ARXIPH*VPXIAJ(LZ)                                     BNDS1519
ISN 1407          IVXBXH(ISPB,LZ)=1                                                     BNDS1520
               C********** CHECK THE SPECIFIED MINIMUM AND MAXIMUM LIMITING CONDITIONS BNDS1521
ISN 1408          VLXDUM=VLXBXH(ISPB,LZ)                                                BNDS1522
ISN 1409          IF(VLXDUM.LT.VLXSPX(LZ))  VLXDUM=VLXSPX(LZ)                           BNDS1523
ISN 1411          IF(VLXDUM.GT.VLXMXX(LZ))  VLXDUM=VLXMXX(LZ)                           BNDS1524
ISN 1413          VLXBXH(ISPB,LZ)=VLXDUM                                                BNDS1525
ISN 1414          VMYDUM=VMYBXH(ISPB,LZ)                                                BNDS1526
ISN 1415          IF(VMYDUM.LT.VMYSPX(LZ))  VMYDUM=VMYSPX(LZ)                           BNDS1527
ISN 1417          IF(VMYDUM.GT.VMYMXX(LZ))  VMYDUM=VMYMXX(LZ)                           BNDS1528
ISN 1419          VMYBXH(ISPB,LZ)=VMYDUM                                                BNDS1529
ISN 1420          VPYDUM=VPYBXH(ISPB,LZ)                                                BNDS1530
ISN 1421          IF(VPYDUM.LT.VPYSPX(LZ))  VPYDUM=VPYSPX(LZ)                           BNDS1531
ISN 1423          IF(VPYDUM.GT.VPYMXX(LZ))  VPYDUM=VPYMXX(LZ)                           BNDS1532
ISN 1425          VPYBXH(ISPB,LZ)=VPYDUM                                                BNDS1533
ISN 1426          VLZDUM=VLZBXH(ISPB,LZ)                                                BNDS1534
ISN 1427          IF(VLZDUM.LT.VLZSPX(LZ))  VLZDUM=VLZSPX(LZ)                           BNDS1535
ISN 1429          IF(VLZDUM.GT.VLZMXX(LZ))  VLZDUM=VLZMXX(LZ)                           BNDS1536
ISN 1431          VLZBXH(ISPB,LZ)=VLZDUM                                                BNDS1537
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ISN 1432      2460 CONTINUE                                                    BNDS1538
              C********** START CALCULATING THE PRESSURE (HEAD) CONDITIONS  **********BNDS1539
              C********** AT THE OPEN REGION BOUNDARY IN ALL LAYERS WITH OPEN ********BNDS1540
              C********** IPH ENCLOSURE BOUNDARY                            **********BNDS1541
              C********** ONLY HYDROSTATIC PRESSURE EFFECTS WITH VARIABLE   **********BNDS1542
              C********** DENSITY ARE INCLUDED IN CALCULATING THE PRESSURE  **********BNDS1543
              C********** CONDITIONS ON THE OPEN REGION BOUNDARY            **********BNDS1544
              C********** HYDROSTATIC PRESSURE CONDITIONS DETERMINED BY SUMMING FROM *BNDS1545
              C********** THE TOP LAYER NZ TO THE LOWER SPECIFIED LAYER LZXLOW *******BNDS1546
ISN 1433            FPRLPH=0.0                                                 BNDS1547
ISN 1434            NLZD=NZ-LZXLOW(ISPB)+1                                     BNDS1548
ISN 1435            DO 2470 LZD=1,NLZD                                         BNDS1549
ISN 1436            LZ=NZ-LZD+1                                                BNDS1550
ISN 1437            DHZIPH=DHZIAJ(LZ)                                          BNDS1551
ISN 1438            IF(LZ.EQ.NZ)  DHZIPH=DHZIPH-HSRIAJ+HSRIPH                  BNDS1552
ISN 1440            ARXIPH=DYIPH*DHZIPH                                        BNDS1553
ISN 1441            VLXIPH=GFXBXH(ISPB,LZ)/ARXIPH                              BNDS1554
              C********** UPWIND DIFFERENCING FORM ON ARXIPH SURFACE       **********BNDS1555
ISN 1442            IF(VLXIPH.LT.0.)GO TO 2476                                 BNDS1556
              C********** FOR OUTFLOWING OPEN REGION CONDITIONS            **********BNDS1557
              C********** SIMPLE UPWIND DIFFERENCING FORM                  **********BNDS1558
ISN 1444       2476 CONTINUE                                                   BNDS1559
ISN 1445            FPRLMH=FPRLPH+COSRNZ*DHZIPH                                BNDS1560
ISN 1446            FPRBXH(ISPB,LZ)=FPRLMH                                     BNDS1561
ISN 1447            FPRLPH=FPRLMH                                              BNDS1562
ISN 1448       2470 CONTINUE                                                   BNDS1563
ISN 1449       2500 CONTINUE                                                   BNDS1564
ISN 1450            IF(JMHBND(IBND).LT.1)GO TO 2600                            BNDS1565
              C********** START SPECIFING JMH BOUNDARY CONDITIONS         **********BNDS1566
ISN 1452            DXJMH=DXBJMH(IBND)                                         BNDS1567
ISN 1453            HSRBYH(ISPB)=HSRSPY                                        BNDS1568
ISN 1454            IF(JMHBND(IBND).GT.1)GO TO 2520                            BNDS1569
              C********** SPECIFIED BOUNDARY CONDITIONS ON THE JMH ENCLOSURE SURFACE *BNDS1570
ISN 1456            HSRJMH=HSRSPY                                              BNDS1571
ISN 1457            DO 2510 LZ=NLZMIN,NZ                                       BNDS1572
ISN 1458            IF(JVYSPY(ISPB,LZ).GT.0)GO TO 2515                         BNDS1573
              C********** CLOSED JMH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1574
ISN 1460            GFYBYH(ISPB,LZ)=0.0                                        BNDS1575
ISN 1461            JVYBYH(ISPB,LZ)=0                                          BNDS1576
ISN 1462            VLY(JMY,LZ)=0.0                                            BNDS1577
ISN 1463            VLYBYH(ISPB,LZ)=0.0                                        BNDS1578
ISN 1464            VPXBYH(ISPB,LZ)=0.0                                        BNDS1579
ISN 1465            VPXBYH(ISPB,LZ)=0.0                                        BNDS1580
ISN 1466            VLZBYH(ISPB,LZ)=0.0                                        BNDS1581
              C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1582
              C********** NEEDED FOR CLOSED JMH BOUNDARY CONDITION          **********BNDS1583
              C********** DUMMY PRESSURE (HEAD) SET TO ZERO                 **********BNDS1584
ISN 1467            FPRBYH(ISPB,LZ)=0.0                                        BNDS1585
ISN 1468            GO TO 2510                                                 BNDS1586
ISN 1469       2515 CONTINUE                                                   BNDS1587
              C********** OPEN JMH ENCLOSURE SURFACE WITH SPECIFIED CONDITION  *******BNDS1588
ISN 1470            DHZJMH=DHZIAJ(LZ)                                          BNDS1589
ISN 1471            IF(LZ.EQ.NZ)  DHZJMH=DHZJMH-HSRIAJ+HSRJMH                  BNDS1590
ISN 1473            ARYJMH=DXJMH*DHZJMH                                        BNDS1591
ISN 1474            VLYJMH=VLYSPY(LZ)                                          BNDS1592
ISN 1475            GFYBYH(ISPB,LZ)=ARYJMH*VLYJMH                              BNDS1593
              C********** HALF-ELEMENT VELOCITY, VLY(JMY,LZ) DETERMINED BY  **********BNDS1594
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                C********** INTERPOLATING THE SPECIFIED VELOCITY VLYMXY          **********BNDS1595
ISN 1476              JVYBYH(ISPB,LZ)=-1                                                  BNDS1596
ISN 1477              VLYJMY=VLYMXY(LZ)+(1./3.)*(VLY(JPY,LZ)-VLYMXY(LZ))                  BNDS1597
ISN 1478              VLY(JMY,LZ)=VLYJMY                                                  BNDS1598
ISN 1479              PREVLY(JMY,LZ)=VLYJMY                                               BNDS1599
                C********** SIMPLE UPWIND DIFFERENCING FOR THE TRANSPORTIVE CONDITIONS *BNDS1600
ISN 1480              IF(VLYJMH.GT.0)GO TO 2516                                           BNDS1601
ISN 1482              VLYBYH(ISPB,LZ)=VMYIAJ(LZ)                                          BNDS1602
ISN 1483              VMXBYH(ISPB,LZ)=VMXIAJ(LZ)                                          BNDS1603
ISN 1484              VPXBYH(ISPB,LZ)=VPXIAJ(LZ)                                          BNDS1604
ISN 1485              VLZBYH(ISPB,LZ)=VLZIAJ(LZ)                                          BNDS1605
ISN 1486              GO TO 2518                                                          BNDS1606
ISN 1487             2516 CONTINUE                                                        BNDS1607
ISN 1488              VLYBYH(ISPB,LZ)=VLYSPY(LZ)                                          BNDS1608
ISN 1489              VMXBYH(ISPB,LZ)=VMXSPY(LZ)                                          BNDS1609
ISN 1490              VPXBYH(ISPB,LZ)=VPXSPY(LZ)                                          BNDS1610
ISN 1491              VLZBYH(ISPB,LZ)=VLZSPY(LZ)                                          BNDS1611
ISN 1492             2518 CONTINUE                                                        BNDS1612
                C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1613
                C********** NEEDED FOR SPECIFIED OPEN JMH BOUNDARY CONDITION  **********BNDS1614
                C********** DUMMY PRESSURE (HEAD) SET TO ZERO                 **********BNDS1615
ISN 1493              FPRBYH(ISPB,LZ)=0.0                                                 BNDS1616
ISN 1494             2510 CONTINUE                                                        BNDS1617
ISN 1495              GO TO 2700                                                          BNDS1618
ISN 1496             2520 CONTINUE                                                        BNDS1619
                C********** START CALCULATING THE OPEN REGION BOUNDARY CONDITIONS ******BNDS1620
                C********** ON THE JMH ENCLOSURE SURFACE                      **********BNDS1621
                C********** DETERMINE THE EXTRAPOLATED EXTERNAL OPEN REGION CONDITIONS *BNDS1622
                C********** WITH COMPLETELY OPEN JMH SURFACE IN ALL LAYERS    **********BNDS1623
ISN 1497              HSRJM1=HSRSPY                                                       BNDS1624
ISN 1498              IF(INTEXT(IX,JY+1).NE.0)GO TO 2540                                  BNDS1625
                C********** ELEMENT AT JY+1 IS EXTERNAL TO THE REGION:        **********BNDS1626
                C********** LINEAR EXTRAPOLATION FOR JY-1 IS IMPOSSIBLE       **********BNDS1627
                C********** EXTERNAL OPEN REGION CONDITIONS SET EQUAL TO      **********BNDS1628
                C********** THE CONDITIONS IN THE BOUNDARY ELEMENT            **********BNDS1629
ISN 1500              DO 2530 LZ=NLZMIN,NZ                                                BNDS1630
ISN 1501              VLYBYH(ISPB,LZ)=VMYIAJ(LZ)                                          BNDS1631
ISN 1502              VMXBYH(ISPB,LZ)=VMXIAJ(LZ)                                          BNDS1632
ISN 1503              VPXBYH(ISPB,LZ)=VPXIAJ(LZ)                                          BNDS1633
ISN 1504              VLZBYH(ISPB,LZ)=VLZIAJ(LZ)                                          BNDS1634
ISN 1505             2530 CONTINUE                                                        BNDS1635
ISN 1506              GO TO 2559                                                          BNDS1636
ISN 1507             2540 CONTINUE                                                        BNDS1637
                C********** ELEMENT AT JY+1 IS AN INTERNAL OR A BOUNDARY ELEMENT: ******BNDS1638
                C********** LINEAR EXTRAPOLATION FOR JY-1 IS POSSIBLE         **********BNDS1639
ISN 1508              IJJP1=IJIXJY(IX,JY+1)                                               BNDS1640
ISN 1509              IPXJP1=2*IJJP1-1                                                    BNDS1641
ISN 1510              IPXJP1=IMXJP1+1                                                     BNDS1642
ISN 1511              IF(INTEXT(IX,JY+1).GT.0)GO TO 2542                                  BNDS1643
                C********** ELEMENT AT JY+1 IS AN INTERNAL ELEMENT            **********BNDS1644
ISN 1513              DYJP1=DY(JY+1)                                                      BNDS1645
ISN 1514              DXJP1=DX(IX)                                                        BNDS1646
ISN 1515              ARHJP1=DXJP1*DYJP1                                                  BNDS1647
ISN 1516              GO TO 2545                                                          BNDS1648
ISN 1517             2542 CONTINUE                                                        BNDS1649
                C********** ELEMENT AT JY+1 IS A BOUNDARY ELEMENT            **********BNDS1650
ISN 1518              IBNJP1=INTEXT(IX,JY+1)                                              BNDS1651
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ISN 1519          DYJP1=DYBND(IBNJP1)                                                BNDS1652
ISN 1520          ARHJP1=ARHBND(IBNJP1)                                              BNDS1653
ISN 1521       2545 CONTINUE                                                         BNDS1654
               C********** LINEAR EXTRAPOLATION TO THE OPEN-REGION BOUNDARY  **********BNDS1655
ISN 1522          CDYJMH=DYIAJ/(DYIAJ+DYJP1)                                         BNDS1656
ISN 1523          NLZJP1=NLZBOT(IJJP1)                                               BNDS1657
ISN 1524          DO 2550 LZ=NLZMIN,NZ                                               BNDS1658
ISN 1525          IF(LZ.GE.NLZJP1)GO TO 2552                                         BNDS1659
               C********** LINEAR EXTRAPOLATION IS IMPOSSIBLE IN THE LAYERS  **********BNDS1660
               C********** CLOSED AT JY+1 WITH THE SHORELINE BOUNDARY         **********BNDS1661
               C********** EXTERNAL OPEN REGION CONDITIONS IN THE LAYER ARE   **********BNDS1662
               C********** SET EQUAL TO THE CONDITIONS IN THE ELEMENT         **********BNDS1663
ISN 1527          VLYBYH(ISPB,LZ)=VMYIAJ(LZ)                                         BNDS1664
ISN 1528          VMXBYH(ISPB,LZ)=VMXIAJ(LZ)                                         BNDS1665
ISN 1529          VPXBYH(ISPB,LZ)=VPXIAJ(LZ)                                         BNDS1666
ISN 1530          VLZBYH(ISPB,LZ)=VLZIAJ(LZ)                                         BNDS1667
ISN 1531          GO TO 2550                                                         BNDS1668
ISN 1532       2552 CONTINUE                                                         BNDS1669
ISN 1533          VMXJP1=VLX(IMXJP1,LZ)                                              BNDS1670
ISN 1534          VPXJP1=VLX(IPXJP1,LZ)                                              BNDS1671
               C********** CALCULATE VLZJP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJJP1 *BNDS1672
ISN 1535          GWZLMH=GWZLZH(IJJP1,LZ)                                            BNDS1673
ISN 1536          IF(LZ.EQ.NZ)GO TO 2556                                             BNDS1674
ISN 1538          GWZLPH=GWZLZH(IJJP1,LZ+1)                                          BNDS1675
ISN 1539          GO TO 2558                                                         BNDS1676
ISN 1540       2556 CONTINUE                                                         BNDS1677
ISN 1541          GWZLPH=ARHJP1*FNCHSR(IJJP1)                                        BNDS1678
ISN 1542       2558 CONTINUE                                                         BNDS1679
ISN 1543          GWZJP1=0.5*(GWZLMH+GWZLPH)                                         BNDS1680
ISN 1544          VLZJP1=GWZJP1/ARHJP1                                               BNDS1681
ISN 1545          VLYBYH(ISPB,LZ)=VMYIAJ(LZ)                                         BNDS1682
ISN 1546          VMXBYH(ISPB,LZ)=VMXIAJ(LZ)-CDYJMH*(VMXJP1-VMXIAJ(LZ))             BNDS1683
ISN 1547          VPXBYH(ISPB,LZ)=VPXIAJ(LZ)-CDYJMH*(VPXJP1-VPXIAJ(LZ))             BNDS1684
ISN 1548          VLZBYH(ISPB,LZ)=VLZIAJ(LZ)-CDYJMH*(VLZJP1-VLZIAJ(LZ))             BNDS1685
ISN 1549       2550 CONTINUE                                                         BNDS1686
ISN 1550       2559 CONTINUE                                                         BNDS1687
               C********** CORRECT THE EXTRAPOLATED VALUES ACCORDING TO THE  **********BNDS1688
               C********** SPECIFIED MINIMUM AND MAXIMUM VALUES OF THE OPEN  **********BNDS1689
               C********** REGION CONDITIONS                                 **********BNDS1690
ISN 1551          DO 2560 LZ=NLZMIN,NZ                                               BNDS1691
ISN 1552          IF(JVYSPY(ISPB,LZ).GT.0)GO TO 2565                                 BNDS1692
               C********** CLOSED JMH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1693
ISN 1554          QFYBYH(ISPB,LZ)=0.0                                                BNDS1694
ISN 1555          JVYBYH(ISPB,LZ)=0                                                  BNDS1695
ISN 1556          VLY(JMY,LZ)=0.0                                                    BNDS1696
ISN 1557          VLYBYH(ISPB,LZ)=0.0                                                BNDS1697
ISN 1558          VMXBYH(ISPB,LZ)=0.0                                                BNDS1698
ISN 1559          VPXBYH(ISPB,LZ)=0.0                                                BNDS1699
ISN 1560          VLZBYH(ISPB,LZ)=0.0                                                BNDS1700
               C********** PRESSURE (HEAD) FOR THE FORCEP SUBROUTINE IS NOT  **********BNDS1701
               C********** NEEDED FOR CLOSED JMH BOUNDARY CONDITION         **********BNDS1702
               C********** DUMMY PRESSURE (HEAD) SET TO ZERO                **********BNDS1703
ISN 1561          FPRBYH(ISPB,LZ)=0.0                                                BNDS1704
ISN 1562          GO TO 2560                                                         BNDS1705
ISN 1563       2565 CONTINUE                                                         BNDS1706
               C********** OPEN-REGION JMH ENCLOSURE SURFACE WITH LIMITING CONDITIONS *BNDS1707
ISN 1564          HSRJMH=0.5*(HSRJM1+HSRIAJ)                                         BNDS1708
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ISN 1565          DHZJMH=DHZIAJ(LZ)                                                    BNDS1709
ISN 1566          IF(LZ.EQ.NZ)  DHZJMH=DHZJMH-HSRIAJ+HSRJMH                            BNDS1710
ISN 1568          ARYJMH=DXJMH*DHZJMH                                                  BNDS1711
ISN 1569          GFYBYH(ISPB,LZ)=ARYJMH*VMYIAJ(LZ)                                    BNDS1712
ISN 1570          JVYBYH(ISPB,LZ)=1                                                    BNDS1713
                 C********** CHECK THE SPECIFIED MINIMUM AND MAXIMUM LIMITING CONDITIONS BNDS1714
ISN 1571          VLYDUM=VLYBYH(ISPB,LZ)                                               BNDS1715
ISN 1572          IF(VLYDUM.LT.VLYSPY(LZ))  VLYDUM=VLYSPY(LZ)                          BNDS1716
ISN 1574          IF(VLYDUM.GT.VLYMXY(LZ))  VLYDUM=VLYMXY(LZ)                          BNDS1717
ISN 1576          VLYBYH(ISPB,LZ)=VLYDUM                                               BNDS1718
ISN 1577          VMXDUM=VMXBYH(ISPB,LZ)                                               BNDS1719
ISN 1578          IF(VMXDUM.LT.VMXSPY(LZ))  VMXDUM=VMXSPY(LZ)                          BNDS1720
ISN 1580          IF(VMXDUM.GT.VMXMXY(LZ))  VMXDUM=VMXMXY(LZ)                          BNDS1721
ISN 1582          VMXBYH(ISPB,LZ)=VMXDUM                                               BNDS1722
ISN 1583          VPXDUM=VPXBYH(ISPB,LZ)                                               BNDS1723
ISN 1584          IF(VPXDUM.LT.VPXSPY(LZ))  VPXDUM=VPXSPY(LZ)                          BNDS1724
ISN 1586          IF(VPXDUM.GT.VPXMXY(LZ))  VPXDUM=VPXMXY(LZ)                          BNDS1725
ISN 1588          VPXBYH(ISPB,LZ)=VPXDUM                                               BNDS1726
ISN 1589          VLZDUM=VLZBYH(ISPB,LZ)                                               BNDS1727
ISN 1590          IF(VLZDUM.LT.VLZSPY(LZ))  VLZDUM=VLZSPY(LZ)                          BNDS1728
ISN 1592          IF(VLZDUM.GT.VLZMXY(LZ))  VLZDUM=VLZMXY(LZ)                          BNDS1729
ISN 1594          VLZBYH(ISPB,LZ)=VLZDUM                                               BNDS1730
ISN 1595       2560 CONTINUE                                                           BNDS1731
                 C********** START CALCULATING THE PRESSURE (HEAD) CONDITIONS  **********BNDS1732
                 C********** AT THE OPEN REGION BOUNDARY IN ALL LAYERS WITH OPEN **********BNDS1733
                 C********** JMH ENCLOSURE BOUNDARY                           **********BNDS1734
                 C********** ONLY HYDROSTATIC PRESSURE EFFECTS WITH VARIABLE   **********BNDS1735
                 C********** DENSITY ARE INCLUDED IN CALCULATING THE PRESSURE  **********BNDS1736
                 C********** CONDITIONS ON THE OPEN REGION BOUNDARY           **********BNDS1737
                 C********** HYDROSTATIC PRESSURE CONDITIONS DETERMINED BY SUMMING FROM *BNDS1738
                 C********** THE TOP LAYER NZ TO THE LOWER SPECIFIED LAYER LZYLOW *******BNDS1739
ISN 1596          FPRLPH=0.0                                                           BNDS1740
ISN 1597          NLZD=NZ-LZYLOW(ISPB)+1                                               BNDS1741
ISN 1598          DO 2570 LZD=1,NLZD                                                   BNDS1742
ISN 1599          LZ=NZ-LZD+1                                                          BNDS1743
ISN 1600          DHZJMH=DHZIAJ(LZ)                                                    BNDS1744
ISN 1601          IF(LZ.EQ.NZ)  DHZJMH=DHZJMH-HSRIAJ+HSRJMH                            BNDS1745
ISN 1603          ARYJMH=DXJMH*DHZJMH                                                  BNDS1746
ISN 1604          VLYJMH=GFYBYH(ISPB,LZ)/ARYJMH                                        BNDS1747
                 C********** UPWIND DIFFERENCING ON ARYJMH SURFACE           **********BNDS1748
ISN 1605          IF(VLYJMH.GT.0.)GO TO 2576                                           BNDS1749
                 C********** FOR OUTFLOWING OPEN REGION CONDITIONS           **********BNDS1750
                 C********** SIMPLE UPWIND DIFFERENCING FORM                 **********BNDS1751
ISN 1607       2576 CONTINUE                                                          BNDS1752
ISN 1608          FPRLMH=FPRLPH+COSRNZ*DHZJMH                                          BNDS1753
ISN 1609          FPRBYH(ISPB,LZ)=FPRLMH                                               BNDS1754
ISN 1610          FPRLPH=FPRLMH                                                        BNDS1755
ISN 1611       2570 CONTINUE                                                          BNDS1756
ISN 1612          GO TO 2700                                                           BNDS1757
ISN 1613       2600 CONTINUE                                                          BNDS1758
ISN 1614          IF(JPHBND(IBND).LT.1)GO TO 2700                                      BNDS1759
                 C********** START SPECIFING JPH BOUNDARY CONDITIONS       **********BNDS1760
ISN 1616          DXJPH=DXBJPH(IBND)                                                   BNDS1761
ISN 1617          HSRBYH(ISPB)=HSRSPY                                                  BNDS1762
ISN 1618          IF(JPHBND(IBND).GT.1)GO TO 2620                                      BNDS1763
                 C********** SPECIFIED BOUNDARY CONDITIONS ON THE JPH ENCLOSURE SURFACE *BNDS1764
ISN 1620          HSRJPH=HSRSPY                                                        BNDS1765
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ISN 1621          DO 2610 LZ=NLZMIN,NZ                                                BNDS1766
ISN 1622          IF(JVYSPY(ISPB,LZ).GT.0)GO TO 2615                                  BNDS1767
                 C********** CLOSED JPH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1768
ISN 1624          GFYBYH(ISPB,LZ)=0.0                                                 BNDS1769
ISN 1625          JVYBYH(ISPB,LZ)=0                                                   BNDS1770
ISN 1626          VLY(JPY,LZ)=0.0                                                     BNDS1771
ISN 1627          VLYBYH(ISPB,LZ)=0.0                                                 BNDS1772
ISN 1628          VMXBYH(ISPB,LZ)=0.0                                                 BNDS1773
ISN 1629          VPXBYH(ISPB,LZ)=0.0                                                 BNDS1774
ISN 1630          VLZBYH(ISPB,LZ)=0.0                                                 BNDS1775
                 C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1776
                 C********** NEEDED FOR CLOSED JPH BOUNDARY CONDITION          **********BNDS1777
                 C********** DUMMY PRESSURE (HEAD) SET TO ZERO                **********BNDS1778
ISN 1631          FPRBYH(ISPB,LZ)=0.0                                                 BNDS1779
ISN 1632          GO TO 2610                                                          BNDS1780
ISN 1633         2615 CONTINUE                                                        BNDS1781
                 C********** OPEN JPH ENCLOSURE SURFACE WITH SPECIFIED CONDITION  *******BNDS1782
ISN 1634          DHZJPH=DHZIAJ(LZ)                                                   BNDS1783
ISN 1635          IF(LZ.EQ.NZ)  DHZJPH=DHZJPH-HSRIAJ+HSRJPH                           BNDS1784
ISN 1637          ARYJPH=DXJPH*DHZJPH                                                 BNDS1785
ISN 1638          VLYJPH=VLYSPY(LZ)                                                   BNDS1786
ISN 1639          GFYBYH(ISPB,LZ)=ARYJPH*VLYJPH                                       BNDS1787
                 C********** HALF-ELEMENT VELOCITY, VLY(JPY,LZ) DETERMINED BY  **********BNDS1788
                 C********** INTERPOLATING THE SPECIFIED VELOCITY VLYMXY       **********BNDS1789
ISN 1640          JVYBYH(ISPB,LZ)=-1                                                  BNDS1790
ISN 1641          VLYJPY=VLYMXY(LZ)-(1./3.)*(VLYMXY(LZ)-VLY(JMY,LZ))                  BNDS1791
ISN 1642          VLY(JPY,LZ)=VLYJPY                                                  BNDS1792
ISN 1643          PREVLY(JPY,LZ)=VLYJPY                                               BNDS1793
                 C********** SIMPLE UPWIND DIFFERENCING FOR THE TRANSPORTIVE CONDITIONS *BNDS1794
ISN 1644          IF(VLYJPH.LT.0)GO TO 2616                                           BNDS1795
ISN 1646          VLYBYH(ISPB,LZ)=VPYIAJ(LZ)                                          BNDS1796
ISN 1647          VMXBYH(ISPB,LZ)=VMXIAJ(LZ)                                          BNDS1797
ISN 1648          VPXBYH(ISPB,LZ)=VPXIAJ(LZ)                                          BNDS1798
ISN 1649          VLZBYH(ISPB,LZ)=VLZIAJ(LZ)                                          BNDS1799
ISN 1650          GO TO 2618                                                          BNDS1800
ISN 1651         2616 CONTINUE                                                        BNDS1801
ISN 1652          VLYBYH(ISPB,LZ)=VLYSPY(LZ)                                          BNDS1802
ISN 1653          VMXBYH(ISPB,LZ)=VMXSPY(LZ)                                          BNDS1803
ISN 1654          VPXBYH(ISPB,LZ)=VPXSPY(LZ)                                          BNDS1804
ISN 1655          VLZBYH(ISPB,LZ)=VLZSPY(LZ)                                          BNDS1805
ISN 1656         2618 CONTINUE                                                        BNDS1806
                 C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1807
                 C********** NEEDED FOR SPECIFIED OPEN JPH BOUNDARY CONDITION  **********BNDS1808
                 C********** DUMMY PRESSURE (HEAD) SET TO ZERO                **********BNDS1809
ISN 1657          FPRBYH(ISPB,LZ)=0.0                                                 BNDS1810
ISN 1658         2610 CONTINUE                                                        BNDS1811
ISN 1659          GO TO 2700                                                          BNDS1812
ISN 1660         2620 CONTINUE                                                        BNDS1813
                 C********** START CALCULATING THE OPEN REGION BOUNDARY CONDITIONS ******BNDS1814
                 C********** ON THE JPH ENCLOSURE SURFACE                    **********BNDS1815
                 C********** DETERMINE THE EXTRAPOLATED EXTERNAL OPEN REGION CONDITIONS *BNDS1816
                 C********** WITH COMPLETELY OPEN JPH SURFACE IN ALL LAYERS  **********BNDS1817
ISN 1661          HSRJP1=HSRSPY                                                       BNDS1818
ISN 1662          IF(INTEXT(IX,JY-1).NE.0)GO TO 2640                                  BNDS1819
                 C********** ELEMENT AT JY-1 IS EXTERNAL TO THE REGION:      **********BNDS1820
                 C********** LINEAR EXTRAPOLATION FOR JY+1 IS IMPOSSIBLE     **********BNDS1821
                 C********** EXTERNAL OPEN REGION CONDITIONS SET EQUAL TO    **********BNDS1822
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                C********** THE CONDITIONS IN THE BOUNDARY ELEMENT          **********BNDS1823
ISN 1664              DO 2630 LZ=NLZMIN,NZ                                          BNDS1824
ISN 1665              VLYBYH(ISPB,LZ)=VPYIAJ(LZ)                                    BNDS1825
ISN 1666              VPXBYH(ISPB,LZ)=VMXIAJ(LZ)                                    BNDS1826
ISN 1667              VPXBYH(ISPB,LZ)=VPXIAJ(LZ)                                    BNDS1827
ISN 1668              VLZBYH(ISPB,LZ)=VLZIAJ(LZ)                                    BNDS1828
ISN 1669             2630 CONTINUE                                                  BNDS1829
ISN 1670              GO TO 2659                                                    BNDS1830
ISN 1671             2640 CONTINUE                                                  BNDS1831
                C********** ELEMENT AT JY-1 IS AN INTERNAL OR A BOUNDARY ELEMENT: ******BNDS1832
                C********** LINEAR EXTRAPOLATION FOR JY+1 IS POSSIBLE          **********BNDS1833
ISN 1672              IJJM1=IJIXJY(IX,JY-1)                                         BNDS1834
ISN 1673              IPXJM1=2*IJJM1-1                                              BNDS1835
ISN 1674              IPXJM1=IMXJM1+1                                               BNDS1836
ISN 1675              IF(INTEXT(IX,JY-1).GT.0)GO TO 2642                            BNDS1837
                C********** ELEMENT AT JY-1 IS AN INTERNAL ELEMENT              **********BNDS1838
ISN 1677              DYJM1=DY(JY-1)                                                BNDS1839
ISN 1678              DXJM1=DX(IX)                                                  BNDS1840
ISN 1679              ARHJM1=DXJM1*DYJM1                                            BNDS1841
ISN 1680              GO TO 2645                                                    BNDS1842
ISN 1681             2642 CONTINUE                                                  BNDS1843
                C********** ELEMENT AT JY-1 IS A BOUNDARY ELEMENT               **********BNDS1844
ISN 1682              IBNJM1=INTEXT(IX,JY-1)                                        BNDS1845
ISN 1683              DYJM1=DYBND(IBNJM1)                                           BNDS1846
ISN 1684              ARHJM1=ARHBND(IBNJM1)                                         BNDS1847
ISN 1685             2645 CONTINUE                                                  BNDS1848
                C********** LINEAR EXTRAPOLATION TO THE OPEN-REGION BOUNDARY  **********BNDS1849
ISN 1686              CDYJPH=DYIAJ/(DYIAJ+DYJM1)                                    BNDS1850
ISN 1687              NLZJM1=NLZBOT(IJJM1)                                          BNDS1851
ISN 1688              DO 2650 LZ=NLZMIN,NZ                                          BNDS1852
ISN 1689              IF(LZ.GE.NLZJM1)GO TO 2652                                    BNDS1853
                C********** LINEAR EXTRAPOLATION IS IMPOSSIBLE IN THE LAYERS  **********BNDS1854
                C********** CLOSED AT JY-1 WITH THE SHORELINE BOUNDARY         **********BNDS1855
                C********** EXTERNAL OPEN REGION CONDITIONS IN THE LAYER ARE   **********BNDS1856
                C********** SET EQUAL TO THE CONDITIONS IN THE ELEMENT         **********BNDS1857
ISN 1691              VLYBYH(ISPB,LZ)=VPYIAJ(LZ)                                    BNDS1858
ISN 1692              VMXBYH(ISPB,LZ)=VMXIAJ(LZ)                                    BNDS1859
ISN 1693              VPXBYH(ISPB,LZ)=VPXIAJ(LZ)                                    BNDS1860
ISN 1694              VLZBYH(ISPB,LZ)=VLZIAJ(LZ)                                    BNDS1861
ISN 1695              GO TO 2650                                                    BNDS1862
ISN 1696             2652 CONTINUE                                                  BNDS1863
ISN 1697              VMXJM1=VLX(IMXJM1,LZ)                                         BNDS1864
ISN 1698              VPXJM1=VLX(IPXJM1,LZ)                                         BNDS1865
                C********** CALCULATE VLZJM1 WITH GWZLMH AND GWZLPH EVALUATED AT IJJM1 *BNDS1866
ISN 1699              GWZLMH=GWZLZH(IJJM1,LZ)                                       BNDS1867
ISN 1700              IF(LZ.EQ.NZ)GO TO 2656                                        BNDS1868
ISN 1702              GWZLPH=GWZLZH(IJJM1,LZ+1)                                     BNDS1869
ISN 1703              GO TO 2658                                                    BNDS1870
ISN 1704             2656 CONTINUE                                                  BNDS1871
ISN 1705              GWZLPH=ARHJM1*FNCHSR(IJJM1)                                   BNDS1872
ISN 1706             2658 CONTINUE                                                  BNDS1873
ISN 1707              GWZJM1=0.5*(GWZLMH+GWZLPH)                                    BNDS1874
ISN 1708              VLZJM1=GWZJM1/ARHJM1                                          BNDS1875
ISN 1709              VLYBYH(ISPB,LZ)=VPYIAJ(LZ)                                    BNDS1876
ISN 1710              VPXBYH(ISPB,LZ)=VMXIAJ(LZ)+CDYJPH*(VMXIAJ(LZ)-VMXJM1)         BNDS1877
ISN 1711              VPXBYH(ISPB,LZ)=VPXIAJ(LZ)+CDYJPH*(VPXIAJ(LZ)-VPXJM1)         BNDS1878
ISN 1712              VLZBYH(ISPB,LZ)=VLZIAJ(LZ)+CDYJPH*(VLZIAJ(LZ)-VLZJM1)         BNDS1879
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ISN 1713       2650 CONTINUE                                                    BNDS1880
ISN 1714       2659 CONTINUE                                                    BNDS1881
               C********** CORRECT THE EXTRAPOLATED VALUES ACCORDING TO THE  **********BNDS1882
               C********** SPECIFIED MINIMUM AND MAXIMUM VALUES OF THE OPEN  **********BNDS1883
               C********** REGION CONDITIONS                                 **********BNDS1884
ISN 1715             DO 2660 LZ=NLZMIN,NZ                                       BNDS1885
ISN 1716             IF(JVYSPY(ISPB,LZ).GT.0)GO TO 2665                         BNDS1886
               C********** CLOSED JPH ENCLOSURE SURFACE WITH ZERO FLOW CONDITIONS *****BNDS1887
ISN 1718             QFYBYH(ISPB,LZ)=0.0                                        BNDS1888
ISN 1719             JVYBYH(ISPB,LZ)=0                                          BNDS1889
ISN 1720             VLY(JPY,LZ)=0.0                                            BNDS1890
ISN 1721             VLYBYH(ISPB,LZ)=0.0                                        BNDS1891
ISN 1722             VMXBYH(ISPB,LZ)=0.0                                        BNDS1892
ISN 1723             VPXBYH(ISPB,LZ)=0.0                                        BNDS1893
ISN 1724             VLZBYH(ISPB,LZ)=0.0                                        BNDS1894
               C********** PRESSURE (HEAD) FOR THE FORCER SUBROUTINE IS NOT  **********BNDS1895
               C********** NEEDED FOR CLOSED JPH BOUNDARY CONDITION          **********BNDS1896
               C********** DUMMY PRESSURE (HEAD) SET TO ZERO                 **********BNDS1897
ISN 1725             PFRBYH(ISPB,LZ)=0.0                                        BNDS1898
ISN 1726             GO TO 2660                                                 BNDS1899
ISN 1727       2665 CONTINUE                                                    BNDS1900
ISN 1728             HSRJPH=0.5*(HSRIAJ+HSRJP1)                                 BNDS1901
               C********** OPEN-REGION JPH ENCLOSURE SURFACE WITH LIMITING CONDITIONS *BNDS1902
ISN 1729             DHZJPH=DHZIAJ(LZ)                                          BNDS1903
ISN 1730             IF(LZ.EQ.NZ)  DHZJPH=DHZJPH-HSRIAJ+HSRJPH                  BNDS1904
ISN 1732             ARYJPH=DXJPH*DHZJPH                                        BNDS1905
ISN 1733             QFYBYH(ISPB,LZ)=ARYJPH*VPYIAJ(LZ)                          BNDS1906
ISN 1734             JVYBYH(ISPB,LZ)=1                                          BNDS1907
               C********** CHECK THE SPECIFIED MINIMUM AND MAXIMUM LIMITING CONDITIONS BNDS1908
ISN 1735             VLYDUM=VLYBYH(ISPB,LZ)                                     BNDS1909
ISN 1736             IF(VLYDUM.LT.VLYSPY(LZ))  VLYDUM=VLYSPY(LZ)                BNDS1910
ISN 1738             IF(VLYDUM.GT.VLYMXY(LZ))  VLYDUM=VLYMXY(LZ)                BNDS1911
ISN 1740             VLYBYH(ISPB,LZ)=VLYDUM                                     BNDS1912
ISN 1741             VMXDUM=VMXBYH(ISPB,LZ)                                     BNDS1913
ISN 1742             IF(VMXDUM.LT.VMXSPY(LZ))  VMXDUM=VMXSPY(LZ)                BNDS1914
ISN 1744             IF(VMXDUM.GT.VMXMXY(LZ))  VMXDUM=VMXMXY(LZ)                BNDS1915
ISN 1746             VMXBYH(ISPB,LZ)=VMXDUM                                     BNDS1916
ISN 1747             VPXDUM=VPXBYH(ISPB,LZ)                                     BNDS1917
ISN 1748             IF(VPXDUM.LT.VPXSPY(LZ))  VPXDUM=VPXSPY(LZ)                BNDS1918
ISN 1750             IF(VPXDUM.GT.VPXMXY(LZ))  VPXDUM=VPXMXY(LZ)                BNDS1919
ISN 1752             VPXBYH(ISPB,LZ)=VPXDUM                                     BNDS1920
ISN 1753             VLZDUM=VLZBYH(ISPB,LZ)                                     BNDS1921
ISN 1754             IF(VLZDUM.LT.VLZSPY(LZ))  VLZDUM=VLZSPY(LZ)                BNDS1922
ISN 1756             IF(VLZDUM.GT.VLZMXY(LZ))  VLZDUM=VLZMXY(LZ)                BNDS1923
ISN 1758             VLZBYH(ISPB,LZ)=VLZDUM                                     BNDS1924
ISN 1759       2660 CONTINUE                                                    BNDS1925
               C********** START CALCULATING THE PRESSURE (HEAD) CONDITIONS  **********BNDS1926
               C********** AT THE OPEN REGION BOUNDARY IN ALL LAYERS WITH OPEN ********BNDS1927
               C********** JPH ENCLOSURE BOUNDARY                           **********BNDS1928
               C********** ONLY HYDROSTATIC PRESSURE EFFECTS WITH VARIABLE  **********BNDS1929
               C********** DENSITY ARE INCLUDED IN CALCULATING THE PRESSURE **********BNDS1930
               C********** CONDITIONS ON THE OPEN REGION BOUNDARY           **********BNDS1931
               C********** HYDROSTATIC PRESSURE CONDITIONS DETERMINED BY SUMMING FROM *BNDS1932
               C********** THE TOP LAYER NZ TO THE LOWER SPECIFIED LAYER LZYLOW *******BNDS1933
ISN 1760             FPRLPH=0.0                                                 BNDS1934
ISN 1761             NLZD=NZ-LZYLOW(ISPB)+1                                     BNDS1935
ISN 1762             DO 2670 LZD=1,NLZD                                         BNDS1936
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ISN 1763          LZ=NZ-LZD+1                                                    BNDS1937
ISN 1764          DHZJPH=DHZIAJ(LZ)                                              BNDS1938
ISN 1765          IF(LZ.EQ.NZ)  DHZJPH=DHZJPH-HSRIAJ+HSRJPH                      BNDS1939
ISN 1767          ARYJPH=DXJPH*DHZJPH                                            BNDS1940
ISN 1768          VLYJPH=GFYBYH(ISPB,LZ)/ARYJPH                                  BNDS1941
                 C********** UPWIND DIFFERENCING ON ARYJPH SURFACE              **********BNDS1942
ISN 1769          IF(VLYJPH.LT.0.)GO TO 2676                                     BNDS1943
                 C********** FOR OUTFLOWING OPEN REGION CONDITIONS              **********BNDS1944
                 C********** SIMPLE UPWIND DIFFERENCING FORM                    **********BNDS1945
ISN 1771          2676 CONTINUE                                                  BNDS1946
ISN 1772          2678 CONTINUE                                                  BNDS1947
ISN 1773          FPRLMH=FPRLPH+COSRNZ*DHZJPH                                    BNDS1948
ISN 1774          FPRBYH(ISPB,LZ)=FPRLMH                                         BNDS1949
ISN 1775          FPRLPH=FPRLMH                                                  BNDS1950
ISN 1776          2670 CONTINUE                                                  BNDS1951
ISN 1777          2700 CONTINUE                                                  BNDS1952
ISN 1778          2000 CONTINUE                                                  BNDS1953
ISN 1779          GO TO 9999                                                     BNDS1954
ISN 1780          9010 CONTINUE                                                  BNDS1955
ISN 1781          WRITE(3,9011)ISPB,IX,JY,IBND                                   BNDS1956
ISN 1782          9011 FORMAT(1HO,8(1H*),1X,97HBOUNDARY CONDITION ERROR: SPECIFIED CONDITBNDS1957
                  1IONS ARE REQUIRED FOR THE BOUNDARY THE BOUNDARY ELEMENT,/,     BNDS1958
                  2 9X,6HISPB =,I4,2X,4HIX =,I4,2X,4HJY =,I4,2X,6HIBND =,I4)     BNDS1959
ISN 1783          CALL EXIT                                                      BNDS1960
ISN 1784          9110 CONTINUE                                                  BNDS1961
ISN 1785          WRITE(3,9111)ISPB,IX,JY,INTEXT(IX,JY)                          BNDS1962
ISN 1786          9111 FORMAT(1HO,8(1H*),1X,102HBOUNDARY CONDITION ERROR: AN INTERNAL OR BNDS1963
                  1AN EXTERNAL ELEMENT CANNOT BE SPECIFIED AS A BOUNDARY ELEMENT,/,BNDS1964
                  2 9X,6HISPB =,I4,2X,4HIX =,I4,2X,4HJY =,I4,2X,8HINTEXT =,I4)   BNDS1965
ISN 1787          CALL EXIT                                                      BNDS1966
ISN 1788          9120 CONTINUE                                                  BNDS1967
ISN 1789          WRITE(3,9121)ISPB,IX,JY,IBND                                   BNDS1968
ISN 1790          9121 FORMAT(1HO,8(1H*),1X,99HBOUNDARY CONDITION ERROR: BOUNDARY CONDITIBNDS1969
                  1ONS CANNOT BE SPECIFIED FOR AN INTERNAL ENCLOSURE SURFACE,/,   BNDS1970
                  2 9X,6HISPB =,I4,2X,4HIX =,I4,2X,4HJY =,I4,2X,6HIBND =,I4)     BNDS1971
ISN 1791          CALL EXIT                                                      BNDS1972
ISN 1792          9130 CONTINUE                                                  BNDS1973
ISN 1793          WRITE(3,9131)ISPB,IX,JY,IBND,          LZXLOW(ISPB),NLZMIN,    BNDS1974
                  1LZXTOP(ISPB),NZ                                               BNDS1975
ISN 1794          9131 FORMAT(1HO,8(1H*),1X,78HX-BOUNDARY CONDITION ERROR: SPECIFIED LZ LBNDS1976
                  1EVELS CANNOT EXCEED THE LIMITS OF LZ,/,9X,6HISPB =,I4,2X,      BNDS1977
                  2 4HIX =,I4,2X,4HJY =,I4,2X,6HIBND =,I4,2X,8HLZXLOW =,I3,1X,    BNDS1978
                  3 8HLZYBOT =,I3,2X,8HLZBTOP =,I3,1X,4HNZ =,I3)                 BNDS1979
ISN 1795          9140 CONTINUE                                                  BNDS1980
ISN 1796          WRITE(3,9141)ISPB,IX,JY,IBND,IMHBND(IBND),IPHBND(IBND),        BNDS1981
                  1          LZXTOP(ISPB),NZ                                     BNDS1982
ISN 1797          9141 FORMAT(1HO,8(1H*),1X,46HOPEN-REGION X-ENCLOSURE BOUNDARY SURFACE EBNDS1983
                  1RROR,/,9X,67HCONDITIONS MUST BE SPECIFIED TO THE TOP LAYER, INCLUDBNDS1984
                  2ING LZXTOP =NZ,/,9X,6HISPB =,I4,2X,4HIX =,I4,2X,4HJY =,I4,2X,  BNDS1985
                  36HIBND =,I4,2X,8HIMHBND =,I3,2X,8HIPHBND =,I3,2X,              BNDS1986
                  48HLZXTOP =,I3,1X,4HNZ =,I3)                                   BNDS1987
ISN 1798          CALL EXIT                                                      BNDS1988
ISN 1799          9150 CONTINUE                                                  BNDS1989
ISN 1800          WRITE(3,9151)ISPB,IX,JY,IBND,          LZYLOW(ISPB),NLZMIN,    BNDS1990
                  1LZYTOP(ISPB),NZ                                               BNDS1991
ISN 1801          9151 FORMAT(1HO,8(1H*),1X,78HY-BOUNDARY CONDITION ERROR: SPECIFIED LZ LBNDS1992
                  1EVELS CANNOT EXCEED THE LIMITS OF LZ,/,9X,6HISPB =,I4,2X,      BNDS1993
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                     2 4HIX =,I4,2X,4HJY =,I4,2X,6HIBND =,I4,2X,8HLZYLOW =,I3,1X,        BNDS1994
                     3 8HNLZBOT =,I3,2X,8HLZYTOP =,I3,1),4HNZ =,I3)                     BNDS1995
                                                                                        BNDS1996
ISN 1802              CALL EXIT                                                         BNDS1997
ISN 1803           9160 CONTINUE                                                        BNDS1998
ISN 1804              WRITE(3,9161)ISPB,IX,JY,IBND,JMHBND(IBND),JPHBND(IBND),           BNDS1999
                     1              LZYTOP(ISPB),NZ                                    BNDS2000
ISN 1805           9161 FORMAT(1HO,8(1H*),1X,46HOPEN-REGION Y-ENCLOSURE BOUNDARY SURFACE EBNDS2000
                     1ROR,/,9X,67HCONDITIONS MUST BE SPECIFIED TO THE TOP LAYER, INCLUDBNDS2001
                     2ING LZYTOP =NZ,/,9X,6HISPB =,I4,2X,4HIX =,I4,2X,4HJY =,I4,2X,    BNDS2002
                     3 6HIBND =,I4,2X,8HJMHBND =,I3,2X,8HJPHBND =,I3,2X,               BNDS2003
                     4 8HLZYTOP =,I3,1X,4HNZ =,I3)                                     BNDS2004
                                                                                        BNDS2005
ISN 1806              CALL EXIT                                                         BNDS2006
ISN 1807           9980 CONTINUE                                                        BNDS2007
ISN 1808           9999 RETURN                                                          BNDS2008
ISN 1809              END

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*      SOURCE STATEMENTS =     1808 ,PROGRAM SIZE =     69964

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
  ISN 0002            SUBROUTINE FLOWUP(DTWIND)                                    FLWP0001
  ISN 0003            COMMON/CONTRL/KRUN,T                                         FLWP0002
  ISN 0004            COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,12),IJIXJY(18,12),FLWP0003
                     1            NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136) FLWP0004
  ISN 0005            COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ( 6),DHZBOT(136) FLWP0005
  ISN 0006            COMMON/GEOBND/IXBND(49),JYBND(49),                           FLWP0006
                     1            DXBND(49),DYBND(49),APHBND(49)                   FLWP0007
  ISN 0007            COMMON/ENCBND/IMHBND(49),DYBIMH(49),IPHBND(49),DYBIPH(49),   FLWP0008
                     1            JMHBND(49),DXBJMH(49),JPHBND(49),DXBJPH(49)      FLWP0009
  ISN 0008            COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)               FLWP0010
  ISN 0009            COMMON/VLXSOL/VLX(272, 6),PREVLX(272, 6),FNCVLX(272, 6)      FLWP0011
  ISN 0010            COMMON/VLYSOL/VLY(272, 6),PREVLY(272, 6),FNCVLY(272, 6)      FLWP0012
  ISN 0011            COMMON/FLOVLZ/GWZLZH(136, 6),FNCGWZ(136, 6)                  FLWP0013
  ISN 0012            COMMON/UPWIND/INWIND,RXWIND,RYWIND,RZWIND                    FLWP0014
  ISN 0013            COMMON/BNDCON/ISPBND(49),IXBSPB(17),JYBSPB(17),              FLWP0015
                     1            HSRBXH(17),GFXBXH(17, 6),IVXBXH(17, 6),          FLWP0016
                     2            VLXBXH(17, 6),VMYBXH(17, 6),VPYBXH(17, 6),       FLWP0017
                     3            VLZBXH(17, 6),FPRBXH(17, 6),                     FLWP0018
                     4            HSRBYH(17),GFYBYH(17, 6),JVYBYH(17, 6),          FLWP0019
                     5            VLYBYH(17, 6),VMXBYH(17, 6),VPXBYH(17, 6),       FLWP0020
                     6            VLZBYH(17, 6),FPRBYH(17, 6)                      FLWP0021
  ISN 0014            DIMENSION VMXIAJ( 6),VPXIAJ( 6),VMYIAJ( 6),VPYIAJ( 6)        FLWP0022
  ISN 0015            DIMENSION GWXIMH( 6),GWXIPH( 6),GWYJMH( 6),GWYJPH( 6)        FLWP0023
  ISN 0016            DIMENSION DHZIAJ( 6)                                         FLWP0024
  ISN 0017            DIMENSION GWXTRX(12, 6),GWYTRY( 6)                           FLWP0025
  ISN 0018            IF(KRUN.GT.0)GO TO 2                                         FLWP0026
  ISN 0020            DO 1200 JY=1,NY                                              FLWP0027
  ISN 0021            DO 1210 LZ=1,NZ                                              FLWP0028
  ISN 0022            GWXTRX(JY,LZ)=0.0                                            FLWP0029
  ISN 0023            GWYTRY(LZ)=0.0                                               FLWP0030
  ISN 0024       1210 CONTINUE                                                     FLWP0031
  ISN 0025       1200 CONTINUE                                                     FLWP0032
  ISN 0026            GO TO 9999                                                   FLWP0033
  ISN 0027          2 CONTINUE                                                     FLWP0034
  ISN 0028            DT=DTWIND                                                    FLWP0035
  ISN 0029            DO 2100 IX=1,NX                                              FLWP0036
  ISN 0030            DO 2200 JY=1,NY                                              FLWP0037
  ISN 0031            IF(INTEXT(IX,JY).EQ.0)GO TO 2200                             FLWP0038
  ISN 0033            IJ=IJIXJY(IX,JY)                                             FLWP0039
  ISN 0034            IPX=2*IJ-1                                                   FLWP0040
  ISN 0035            IMX=IPX+1                                                    FLWP0041
  ISN 0036            JMY=IMX                                                      FLWP0042
  ISN 0037            JPY=IPX                                                      FLWP0043
  ISN 0038            IF(INTEXT(IX,JY).GT.0)GO TO 2105                             FLWP0044
                     C********** INTERNAL DISCRETE ELEMENT GEOMETRY CONDITIONS ********** FLWP0045
  ISN 0040            DXIAJ=DX(IX)                                                 FLWP0046
  ISN 0041            DYIAJ=DY(JY)                                                 FLWP0047
  ISN 0042            AREIAJ=DXIAJ*DYIAJ                                           FLWP0048
  ISN 0043            DYIPH=DYIAJ                                                  FLWP0049
  ISN 0044            DXJPH=DXIAJ                                                  FLWP0050
  ISN 0045            GO TO 2109                                                   FLWP0051
  ISN 0046       2105 CONTINUE                                                     FLWP0052
                     C********** BOUNDARY DISCRETE ELEMENT GEOMETRY CONDITIONS ********** FLWP0053
  ISN 0047            IBND=INTEXT(IX,JY)                                           FLWP0054
  ISN 0048            DXIAJ=DXBND(IBND)                                            FLWP0055
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ISN 0049          DYIAJ=DYBND(IBND)                                                  FLWP0056
ISN 0050          ARHIAJ=DXIAJ*DYIAJ                                                 FLWP0057
ISN 0051          DYIMH=DYBIMH(IBND)                                                 FLWP0058
ISN 0052          DYIPH=DYBIPH(IBND)                                                 FLWP0059
ISN 0053          DXJMH=DXBJMH(IBND)                                                 FLWP0060
ISN 0054          DXJPH=DXBJPH(IBND)                                                 FLWP0061
ISN 0055       2109 CONTINUE                                                         FLWP0062
ISN 0056          HSRIAJ=HSR(IJ)                                                     FLWP0063
ISN 0057          NLZMIN=NLZBOT(IJ)                                                  FLWP0064
ISN 0058          DO 2110 LZ=NLZMIN,NZ                                               FLWP0065
ISN 0059          DHZDUM=DZ(LZ)                                                      FLWP0066
ISN 0060          IF(LZ.EQ.NLZMIN)  DHZDUM=DHZBOT(IJ)                                FLWP0067
ISN 0062          IF(LZ.EQ.NZ)  DHZDUM=DHZDUM+HSRIAJ                                 FLWP0068
ISN 0064          DHZIAJ(LZ)=DHZDUM                                                  FLWP0069
ISN 0065          VMXIAJ(LZ)=VLX(IMX,LZ)                                             FLWP0070
ISN 0066          VPXIAJ(LZ)=VLX(IPX,LZ)                                             FLWP0071
ISN 0067          VMYIAJ(LZ)=VLY(JMY,LZ)                                             FLWP0072
ISN 0068          VPYIAJ(LZ)=VLY(JPY,LZ)                                             FLWP0073
ISN 0069       2110 CONTINUE                                                         FLWP0074
ISN 0070          IF(INTEXT(IX,JY).GT.0)GO TO 2300                                   FLWP0075
                C********** START INTERNAL ELEMENT CALCULATIONS              **********FLWP0076
                C********** IAJ ENCLOSURE SURFACE CONVECTION TRANSPORT CALCULATED ******FLWP0077
                C********** IMH AND JMH ENCLOSURE SURFACE CONVECTION TRANSPORT *********FLWP0078
                C********** TRANSFERRED FROM PREVIOUS IPH AND JPH CALCULATIONS *********FLWP0079
                C********** IPH AND JPH ENCLOSURE SURFACE CONVECTION          **********FLWP0080
                C********** TRANSPORT CALCULATED                              **********FLWP0081
                C********** IMH AND JMH ENCLOSURE SURFACE CONVECTION          **********FLWP0082
                C********** TRANSPORT TRANSFERRED FROM PREVIOUS               **********FLWP0083
                C********** IPH AND JPH CALCULATIONS                          **********FLWP0084
ISN 0072          IJIM1=IJIXJY(IX-1,JY)                                                FLWP0085
ISN 0073          NLZIM1=NLZBOT(IJIM1)                                                 FLWP0086
ISN 0074          DO 2150 LZ=NLZMIN,NZ                                                 FLWP0087
ISN 0075          IF(LZ.GE.NLZIM1)GO TO 2155                                           FLWP0088
                C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********FLWP0089
                C********** SHORELINE BOUNDARY                                **********FLWP0090
ISN 0077          GWXIMH(LZ)=0.0                                                       FLWP0091
ISN 0078          GO TO 2159                                                           FLWP0092
ISN 0079       2155 CONTINUE                                                           FLWP0093
                C********** IMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT   **********FLWP0094
ISN 0080          GWXIMH(LZ)=GWXTRX(JY,LZ)                                             FLWP0095
ISN 0081       2159 CONTINUE                                                           FLWP0096
ISN 0082       2150 CONTINUE                                                           FLWP0097
ISN 0083          IJJM1=IJIXJY(IX,JY-1)                                                FLWP0098
ISN 0084          NLZJM1=NLZBOT(IJJM1)                                                 FLWP0099
ISN 0085          DO 2160 LZ=NLZMIN,NZ                                                 FLWP0100
ISN 0086          IF(LZ.GE.NLZJM1)GO TO 2165                                           FLWP0101
                C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********FLWP0102
                C********** SHORELINE BOUNDARY                                **********FLWP0103
ISN 0088          GWYJMH(LZ)=0.0                                                       FLWP0104
ISN 0089          GO TO 2169                                                           FLWP0105
ISN 0090       2165 CONTINUE                                                           FLWP0106
                C********** JMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT   **********FLWP0107
ISN 0091          GWYJMH(LZ)=GWYTRY(LZ)                                                FLWP0108
ISN 0092       2169 CONTINUE                                                           FLWP0109
ISN 0093       2160 CONTINUE                                                           FLWP0110
                C********** START CALCULATIONS FOR INTERNAL IPH SURFACE    **********FLWP0111
ISN 0094          IJIP1=IJIXJY(IX+1,JY)                                                FLWP0112
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ISN 0095          IPXIP1=2*IJIP1-1                                                FLWP0113
ISN 0096          JPYIP1=IMXIP1                                                   FLWP0114
ISN 0097          JPYIP1=JMYIP1+1                                                 FLWP0115
ISN 0098          IF(INTEXT(IX+1,JY).GT.0)GO TO 2212                              FLWP0116
                 C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT          **********FLWP0117
ISN 0100          DXIP1=DX(IX+1)                                                  FLWP0118
ISN 0101          GO TO 2215                                                      FLWP0119
ISN 0102         2212 CONTINUE                                                    FLWP0120
                 C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT          **********FLWP0121
ISN 0103          IBNIP1=INTEXT(IX+1,JY)                                          FLWP0122
ISN 0104          DXIP1=DXBND(IBNIP1)                                             FLWP0123
ISN 0105         2215 CONTINUE                                                    FLWP0124
ISN 0106          DXIPH=0.5*(DXIAJ+DXIP1)                                         FLWP0125
ISN 0107          CDXIPH=0.5*DXIAJ/DXIPH                                          FLWP0126
ISN 0108          HSRIP1=HSR(IJIP1)                                               FLWP0127
ISN 0109          NLZIP1=NLZBOT(IJIP1)                                            FLWP0128
ISN 0110          DO 2220 LZ=NLZMIN,NZ                                            FLWP0129
ISN 0111          IF(LZ.GE.NLZIP1)GO TO 2225                                      FLWP0130
                 C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM            **********FLWP0131
                 C********** SHORELINE BOUNDARY                                  **********FLWP0132
ISN 0113          GBXIPH(LZ)=0.0                                                  FLWP0133
ISN 0114          GO TO 2220                                                      FLWP0134
ISN 0115         2225 CONTINUE                                                    FLWP0135
                 C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT      **********FLWP0136
ISN 0116          DHZIP1=DZ(LZ)                                                   FLWP0137
ISN 0117          IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                          FLWP0138
ISN 0119          IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                              FLWP0139
ISN 0121          DHZIPH=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))                    FLWP0140
ISN 0122          ARXIPH=DYIPH*DHZIPH                                             FLWP0141
ISN 0123          VPXIP1=VLX(IMXIP1,LZ)                                           FLWP0142
ISN 0124          VLXIPH=VPXIAJ(LZ)+CDXIPH*(VMXIP1-VPXIAJ(LZ))                    FLWP0143
                 C********** UPWIND DIFFERENCING ON ARXIPH SURFACE                **********FLWP0144
ISN 0125          IF(INWIND.NE.0)GO TO 2233                                       FLWP0145
                 C********** SIMPLE UPWIND DIFFERENCING FORM                      **********FLWP0146
ISN 0127          IF(VLXIPH.LT.0.)GO TO 2231                                      FLWP0147
ISN 0129          GBXIPH(LZ)=ARXIPH*VLXIPH                                        FLWP0148
ISN 0130          GO TO 2239                                                      FLWP0149
ISN 0131         2231 CONTINUE                                                    FLWP0150
ISN 0132          GBXIPH(LZ)=ARXIPH*VLXIPH                                        FLWP0151
ISN 0133          GO TO 2239                                                      FLWP0152
ISN 0134         2233 CONTINUE                                                    FLWP0153
ISN 0135          RDXIP1=1.+(DXIP1/DXIAJ)                                         FLWP0154
ISN 0136          IF(INWIND.GT.0)GO TO 2235                                       FLWP0155
                 C********** WEIGHTED UPWIND DIFFERENCING FORM                   **********FLWP0156
ISN 0138          DXRIPH=DXIPH/DXIAJ                                              FLWP0157
ISN 0139          IF(VLXIPH.LT.0.)GO TO 2234                                      FLWP0158
ISN 0141          CBXIPH=(1.-RXWIND*DXRIPH)/RDXIP1                                FLWP0159
ISN 0142          GO TO 2236                                                      FLWP0160
ISN 0143         2234 CONTINUE                                                    FLWP0161
ISN 0144          CBXIPH=(1.+RXWIND*DXRIPH)/RDXIP1                                FLWP0162
ISN 0145          GO TO 2236                                                      FLWP0163
ISN 0146         2235 CONTINUE                                                    FLWP0164
                 C********** TRANSPORTIVE UPWIND DIFFERENCING FORM               **********FLWP0165
ISN 0147          CRXIPH=VLXIPH*DT/DXIAJ                                          FLWP0166
ISN 0148          CBXIPH=(1.-2.*RXWIND*CRXIPH)/RDXIP1                             FLWP0167
ISN 0149         2236 CONTINUE                                                    FLWP0168
ISN 0150          GBXIPH(LZ)=ARXIPH*VLXIPH                                        FLWP0169
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ISN 0151      2239 CONTINUE                                                    FLWP0170
ISN 0152      2220 CONTINUE                                                    FLWP0171
             C********** START CALCULATIONS FOR INTERNAL JPH SURFACE           **********FLWP0172
ISN 0153            IJJP1=IJIXJY(IX,JY+1)                                     FLWP0173
ISN 0154            JMYJP1=2*IJJP1-1                                          FLWP0174
ISN 0155            IMXJP1=JMYJP1                                             FLWP0175
ISN 0156            IPXJP1=IMXJP1+1                                           FLWP0176
ISN 0157            IF(INTEXT(IX,JY+1).GT.0)GO TO 2252                        FLWP0177
             C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT           **********FLWP0178
ISN 0159            DYJP1=DY(JY+1)                                            FLWP0179
ISN 0160            GO TO 2255                                                FLWP0180
ISN 0161      2252 CONTINUE                                                    FLWP0181
             C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT            **********FLWP0182
ISN 0162            IBNJP1=INTEXT(IX,JY+1)                                    FLWP0183
ISN 0163            DYJP1=DYBND(IBNJP1)                                       FLWP0184
ISN 0164      2255 CONTINUE                                                    FLWP0185
ISN 0165            DYJPH=0.5*(DYIAJ+DYJP1)                                   FLWP0186
ISN 0166            CDYJPH=0.5*DYIAJ/DYJPH                                    FLWP0187
ISN 0167            HSRJP1=HSR(IJJP1)                                         FLWP0188
ISN 0168            NLZJP1=NLZBOT(IJJP1)                                      FLWP0189
ISN 0169            DO 2260 LZ=NLZMIN,NZ                                      FLWP0190
ISN 0170            IF(LZ.GE.NLZJP1)GO TO 2265                                FLWP0191
             C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM              **********FLWP0192
             C********** SHORELINE BOUNDARY                                    **********FLWP0193
ISN 0172            GWYJPH(LZ)=0.0                                            FLWP0194
ISN 0173            GO TO 2260                                                FLWP0195
ISN 0174      2265 CONTINUE                                                    FLWP0196
             C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT        **********FLWP0197
ISN 0175            DHZJP1=DZ(LZ)                                             FLWP0198
ISN 0176            IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                    FLWP0199
ISN 0178            IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                        FLWP0200
ISN 0180            DHZJPH=DHZIAJ(LZ)+CDYJPH*(DHZJP1-DHZIAJ(LZ))              FLWP0201
ISN 0181            ARYJPH=DXJPH*DHZJPH                                       FLWP0202
ISN 0182            VMYJP1=VLY(JMYJP1,LZ)                                     FLWP0203
ISN 0183            VLYJPH=VPYIAJ(LZ)+CDYJPH*(VMYJP1-VPYIAJ(LZ))              FLWP0204
             C********** UPWIND DIFFERENCING ON ARYJPH SURFACE                 **********FLWP0205
ISN 0184            IF(INWIND.NE.0)GO TO 2273                                 FLWP0206
             C********** SIMPLE UPWIND DIFFERENCING FORM                       **********FLWP0207
ISN 0186            IF(VLYJPH.LT.0.)GO TO 2271                                FLWP0208
ISN 0188            GWYJPH(LZ)=ARYJPH*VLYJPH                                  FLWP0209
ISN 0189            GO TO 2279                                                FLWP0210
ISN 0190      2271 CONTINUE                                                    FLWP0211
ISN 0191            GWYJPH(LZ)=ARYJPH*VLYJPH                                  FLWP0212
ISN 0192            GO TO 2279                                                FLWP0213
ISN 0193      2273 CONTINUE                                                    FLWP0214
ISN 0194            RDYJP1=1.+(DYJP1/DYIAJ)                                   FLWP0215
ISN 0195            IF(INWIND.GT.0)GO TO 2275                                 FLWP0216
             C********** WEIGHTED UPWIND DIFFERENCING FORM                     **********FLWP0217
ISN 0197            DYRJPH=DYJPH/DYIAJ                                        FLWP0218
ISN 0198            IF(VLYJPH.LT.0.)GO TO 2274                                FLWP0219
ISN 0200            CWYJPH=(1.-RYWIND*DYRJPH)/RDYJP1                          FLWP0220
ISN 0201            GO TO 2276                                                FLWP0221
ISN 0202      2274 CONTINUE                                                    FLWP0222
ISN 0203            CWYJPH=(1.+RYWIND*DYRJPH)/RDYJP1                          FLWP0223
ISN 0204            GO TO 2276                                                FLWP0224
ISN 0205      2275 CONTINUE                                                    FLWP0225
             C********** TRANSPORTIVE UPWIND DIFFERENCING FORM                 **********FLWP0226
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ISN 0206          CNYJPH=VLYJPH*DT/DYIAJ                                                FLWP0227
ISN 0207          CWYJPH=(1.-2.*RYWIND*CNYJPH)/RDYJP1                                   FLWP0228
ISN 0208       2276 CONTINUE                                                            FLWP0229
ISN 0209          GWYJPH(LZ)=ARYJPH*VLYJPH                                              FLWP0230
ISN 0210       2279 CONTINUE                                                            FLWP0231
ISN 0211       2260 CONTINUE                                                            FLWP0232
ISN 0212          GO TO 2700                                                            FLWP0233
ISN 0213       2300 CONTINUE                                                            FLWP0234
ISN 0214          IBND=INTEXT(IX,JY)                                                    FLWP0235
               C********** START BOUNDARY ELEMENT CALCULATIONS                ********** FLWP0236
               C********** START IMH BOUNDARY CALCULATIONS                   ********** FLWP0237
ISN 0215          IMHCON=IMHBND(IBND)+2                                                 FLWP0238
ISN 0216          GO TO (2310,2340,2360,2360),IMHCON                                    FLWP0239
ISN 0217       2310 CONTINUE                                                            FLWP0240
               C********** INTERNAL IMH BOUNDARY                              ********** FLWP0241
               C********** IMH ENCLOSURE SURFACE CONVECTION TRANSPORT         ********** FLWP0242
               C********** TRANSFERRED FROM PREVIOUS IPH CALCULATIONS         ********** FLWP0243
ISN 0218          IJIM1=IJIXJY(IX-1,JY)                                                 FLWP0244
ISN 0219          NLZIM1=NLZBOT(IJIM1)                                                  FLWP0245
ISN 0220          DO 2320 LZ=NLZMIN,NZ                                                  FLWP0246
ISN 0221          IF(LZ.GE.NLZIM1)GO TO 2325                                            FLWP0247
               C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          ********** FLWP0248
               C********** SHORELINE BOUNDARY                               ********** FLWP0249
ISN 0223          GWXIMH(LZ)=0.0                                                        FLWP0250
ISN 0224          GO TO 2329                                                            FLWP0251
ISN 0225       2325 CONTINUE                                                            FLWP0252
               C********** IMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT   ********** FLWP0253
ISN 0226          GWXIMH(LZ)=GWXTRX(JY,LZ)                                              FLWP0254
ISN 0227       2329 CONTINUE                                                            FLWP0255
ISN 0228       2320 CONTINUE                                                            FLWP0256
ISN 0229          GO TO 2400                                                            FLWP0257
ISN 0230       2340 CONTINUE                                                            FLWP0258
               C********** SHORELINE IMH BOUNDARY                            ********** FLWP0259
ISN 0231          DO 2350 LZ=NLZMIN,NZ                                                  FLWP0260
ISN 0232          GWXIMH(LZ)=0.0                                                        FLWP0261
ISN 0233       2350 CONTINUE                                                            FLWP0262
ISN 0234          GO TO 2400                                                            FLWP0263
ISN 0235       2360 CONTINUE                                                            FLWP0264
               C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS ********** FLWP0265
               C********** FOR IMH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********* FLWP0266
ISN 0236          ISPB=ISPAND(IBND)                                                     FLWP0267
ISN 0237          HSRIMH=HSRBXH(ISPB)                                                   FLWP0268
ISN 0238          DO 2370 LZ=NLZMIN,NZ                                                  FLWP0269
ISN 0239          IFXIMH=IVXBXH(ISPB,LZ)                                                FLWP0270
ISN 0240          GFXIMH=GFXBXH(ISPB,LZ)                                                FLWP0271
ISN 0241          IF(IFXIMH.LT.1)GO TO 2376                                             FLWP0272
ISN 0243          DHZIMH=DHZIAJ(LZ)                                                     FLWP0273
ISN 0244          IF(LZ.EQ.NZ)  DHZIMH=DHZIMH-HSRIAJ+HSRIMH                             FLWP0274
ISN 0246          ARXIMH=DYIMH*DHZIMH                                                   FLWP0275
ISN 0247          VLXIMH=GFXIMH/ARXIMH                                                  FLWP0276
               C********** UPWIND DIFFERENCING ON ARXIMH SURFACE            ********** FLWP0277
ISN 0248          IF(VLXIMH.GT.0.)GO TO 2376                                            FLWP0278
               C********** FOR OUTFLOWING OPEN REGION CONDITIONS            ********** FLWP0279
               C********** SIMPLE UPWIND DIFFERENCING FORM                  ********** FLWP0280
ISN 0250          GWXIMH(LZ)=GFXIMH                                                     FLWP0281
ISN 0251          GO TO 2379                                                            FLWP0282
ISN 0252       2376 CONTINUE                                                            FLWP0283
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ISN 0253          GBXIMH(LZ)=GFXIMH                                          FLWP0284
ISN 0254       2379 CONTINUE                                                 FLWP0285
ISN 0255       2370 CONTINUE                                                 FLWP0286
ISN 0256       2400 CONTINUE                                                 FLWP0287
               C********** START IPH BOUNDARY CALCULATIONS                  **********FLWP0288
ISN 0257          IPHCON=IPHBND(IBND)+2                                      FLWP0289
ISN 0258          GO TO (2410,2440,2460,2460),IPHCON                         FLWP0290
ISN 0259       2410 CONTINUE                                                 FLWP0291
               C********** INTERNAL IPH BOUNDARY                            **********FLWP0292
ISN 0260          IJIP1=IJIXJY(IX+1,JY)                                      FLWP0293
ISN 0261          IMXIP1=2*IJIP1-1                                           FLWP0294
ISN 0262          JMYIP1=IMXIP1                                              FLWP0295
ISN 0263          JPYIP1=JMYIP1+1                                            FLWP0296
ISN 0264          IF(INTEXT(IX+1,JY).GT.0)GO TO 2412                         FLWP0297
               C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT       **********FLWP0298
ISN 0266          DXIP1=DX(IX+1)                                             FLWP0299
ISN 0267          GO TO 2415                                                 FLWP0300
ISN 0268       2412 CONTINUE                                                 FLWP0301
               C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT        **********FLWP0302
ISN 0269          IBNIP1=INTEXT(IX+1,JY)                                     FLWP0303
ISN 0270          DXIP1=DXBND(IBNIP1)                                        FLWP0304
ISN 0271       2415 CONTINUE                                                 FLWP0305
ISN 0272          DXIPH=0.5*(DXIAJ+DXIP1)                                    FLWP0306
ISN 0273          CDXIPH=0.5*DXIAJ/DXIPH                                     FLWP0307
ISN 0274          HSRIP1=HSR(IJIP1)                                          FLWP0308
ISN 0275          NLZIP1=NLZBOT(IJIP1)                                       FLWP0309
ISN 0276          DO 2420 LZ=NLZMIN,NZ                                       FLWP0310
ISN 0277          IF(LZ.GE.NLZIP1)GO TO 2425                                 FLWP0311
               C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********FLWP0312
               C********** SHORELINE BOUNDARY                               **********FLWP0313
ISN 0279          GBXIPH(LZ)=0.0                                             FLWP0314
ISN 0280          GO TO 2420                                                 FLWP0315
ISN 0281       2425 CONTINUE                                                 FLWP0316
               C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********FLWP0317
ISN 0282          DHZIP1=DZ(LZ)                                              FLWP0318
ISN 0283          IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                     FLWP0319
ISN 0285          IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                         FLWP0320
ISN 0287          DHZIPH=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))               FLWP0321
ISN 0288          ARXIPH=DYIPH*DHZIPH                                        FLWP0322
ISN 0289          VMXIP1=VLX(IMXIP1,LZ)                                      FLWP0323
ISN 0290          VLXIPH=VPXIAJ(LZ)+CDXIPH*(VMXIP1-VPXIAJ(LZ))               FLWP0324
               C********** UPWIND DIFFERENCING ON ARXIPH SURFACE            **********FLWP0325
ISN 0291          IF(INWIND.NE.0)GO TO 2433                                  FLWP0326
               C********** SIMPLE UPWIND DIFFERENCING FORM                  **********FLWP0327
ISN 0293          IF(VLXIPH.LT.0.)GO TO 2431                                 FLWP0328
ISN 0295          GBXIPH(LZ)=ARXIPH*VLXIPH                                   FLWP0329
ISN 0296          GO TO 2439                                                 FLWP0330
ISN 0297       2431 CONTINUE                                                 FLWP0331
ISN 0298          GBXIPH(LZ)=ARXIPH*VLXIPH                                   FLWP0332
ISN 0299          GO TO 2439                                                 FLWP0333
ISN 0300       2433 CONTINUE                                                 FLWP0334
ISN 0301          RDXIP1=1.+(DXIP1/DXIAJ)                                    FLWP0335
ISN 0302          IF(INWIND.GT.0)GO TO 2435                                  FLWP0336
               C********** WEIGHTED UPWIND DIFFERENCING FORM                 **********FLWP0337
ISN 0304          DXRIPH=DXIPH/DXIAJ                                         FLWP0338
ISN 0305          IF(VLXIPH.LT.0.)GO TO 2434                                 FLWP0339
ISN 0307          CBXIPH=(1.-RXWIND*DXRIPH)/RDXIP1                           FLWP0340
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ISN 0308          GC TO 2436                                                    FLWP0341
ISN 0309       2434 CONTINUE                                                    FLWP0342
ISN 0310          CWXIPH=(1.+RXWIND*DXRIPH)/RDXIP1                              FLWP0343
ISN 0311          GC TO 2436                                                    FLWP0344
ISN 0312       2435 CONTINUE                                                    FLWP0345
               C********** TRANSPORTIVE UPWIND DIFFERENCING FORM                **********FLWP0346
ISN 0313          CNXIPH=VLXIPH*DT/DXIAJ                                        FLWP0347
ISN 0314          CWXIPH=(1.-2.*RXWIND*CNXIPH)/RDXIP1                           FLWP0348
ISN 0315       2436 CONTINUE                                                    FLWP0349
ISN 0316          GWXIPH(LZ)=ARXIPH*VLXIPH                                      FLWP0350
ISN 0317       2439 CONTINUE                                                    FLWP0351
ISN 0318       2420 CONTINUE                                                    FLWP0352
ISN 0319          GC TO 2500                                                    FLWP0353
ISN 0320       2440 CONTINUE                                                    FLWP0354
               C********** SHORELINE IPH BOUNDARY                              **********FLWP0355
ISN 0321          DO 2450 LZ=NLZMIN,NZ                                          FLWP0356
ISN 0322          GWXIPH(LZ)=0.0                                                FLWP0357
ISN 0323       2450 CONTINUE                                                    FLWP0358
ISN 0324          GC TO 2500                                                    FLWP0359
ISN 0325       2460 CONTINUE                                                    FLWP0360
               C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS  *********FLWP0361
               C********** FOR IPH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE *********FLWP0362
ISN 0326          ISPB=ISPBND(IBND)                                             FLWP0363
ISN 0327          HSRIPH=HSRBXH(ISPB)                                           FLWP0364
ISN 0328          DO 2470 LZ=NLZMIN,NZ                                          FLWP0365
ISN 0329          IFXIPH=IVXBXH(ISPB,LZ)                                        FLWP0366
ISN 0330          GFXIPH=GFXBXH(ISPB,LZ)                                        FLWP0367
ISN 0331          IF(IFXIPH.LT.1)GO TO 2476                                     FLWP0368
ISN 0333          DHZIPH=DHZIAJ(LZ)                                             FLWP0369
ISN 0334          IF(LZ.EQ.NZ)  DHZIPH=DHZIPH-HSRIAJ+HSRIPH                     FLWP0370
ISN 0336          ARXIPH=DYIPH*DHZIPH                                           FLWP0371
ISN 0337          VLXIPH=GFXIPH/ARXIPH                                          FLWP0372
               C********** UPWIND DIFFERENCING FORM ON ARXIPH SURFACE           **********FLWP0373
ISN 0338          IF(VLXIPH.LT.0.)GO TO 2476                                    FLWP0374
               C********** FOR OUTFLOWING OPEN REGION CONDITIONS                **********FLWP0375
               C********** SIMPLE UPWIND DIFFERENCING FORM                      **********FLWP0376
ISN 0340          GWXIPH(LZ)=GFXIPH                                             FLWP0377
ISN 0341          GO TO 2479                                                    FLWP0378
ISN 0342       2476 CONTINUE                                                    FLWP0379
ISN 0343          GWXIPH(LZ)=GFXIPH                                             FLWP0380
ISN 0344       2479 CONTINUE                                                    FLWP0381
ISN 0345       2470 CONTINUE                                                    FLWP0382
ISN 0346       2500 CONTINUE                                                    FLWP0383
               C********** START JMH BOUNDARY CALCULATIONS                      **********FLWP0384
ISN 0347          JMHCON=JMHBND(IBND)+2                                         FLWP0385
ISN 0348          GO TO (2510,2540,2560,2560),JMHCON                            FLWP0386
ISN 0349       2510 CONTINUE                                                    FLWP0387
               C********** INTERNAL JMH BOUNDARY                               **********FLWP0388
               C********** JMH ENCLOSURE SURFACE CONVECTION TRANSPORT           **********FLWP0389
               C********** TRANSFERRED FROM PREVIOUS JPH CALCULATIONS           **********FLWP0390
ISN 0350          IJJM1=IJIXJY(IX,JY-1)                                         FLWP0391
ISN 0351          NLZJM1=NLZBOT(IJJM1)                                          FLWP0392
ISN 0352          DO 2520 LZ=NLZMIN,NZ                                          FLWP0393
ISN 0353          IF(LZ.GE.NLZJM1)GO TO 2525                                    FLWP0394
               C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM             **********FLWP0395
               C********** SHORELINE BOUNDARY                                  **********FLWP0396
ISN 0355          GWYJMH(LZ)=0.0                                                FLWP0397
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ISN 0356          GO TO 2529                                                    FLWP0398
ISN 0357       2525 CONTINUE                                                    FLWP0399
               C********** JMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT     **********FLWP0400
ISN 0358          GWYJMH(LZ)=GWYTRY(LZ)                                        FLWP0401
ISN 0359       2529 CONTINUE                                                    FLWP0402
ISN 0360       2520 CONTINUE                                                    FLWP0403
ISN 0361          GO TO 2600                                                   FLWP0404
ISN 0362       2540 CONTINUE                                                    FLWP0405
               C********** SHORELINE JMH BOUNDARY                            **********FLWP0406
ISN 0363          DO 2550 LZ=NLZMIN,NZ                                         FLWP0407
ISN 0364          GWYJMH(LZ)=0.0                                               FLWP0408
ISN 0365       2550 CONTINUE                                                    FLWP0409
ISN 0366          GO TO 2600                                                   FLWP0410
ISN 0367       2560 CONTINUE                                                    FLWP0411
               C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS  *********FLWP0412
               C********** FOR JMH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********FLWP0413
ISN 0368          ISPB=ISPBND(IBND)                                            FLWP0414
ISN 0369          HSRJMH=HSRBYH(ISPB)                                          FLWP0415
ISN 0370          DO 2570 LZ=NLZMIN,NZ                                         FLWP0416
ISN 0371          JFYJMH=JVYBYH(ISPB,LZ)                                       FLWP0417
ISN 0372          GFYJMH=GFYBYH(ISPB,LZ)                                       FLWP0418
ISN 0373          IF(JFYJMH.LT.1)GO TO 2576                                    FLWP0419
ISN 0375          DHZJMH=DHZIAJ(LZ)                                            FLWP0420
ISN 0376          IF(LZ.EQ.NZ)  DHZJMH=DHZJMH-HSRIAJ+HSRJMH                    FLWP0421
ISN 0378          ARYJMH=DXJMH*DHZJMH                                          FLWP0422
ISN 0379          VLYJMH=GFYJMH/ARYJMH                                         FLWP0423
               C********** UPWIND DIFFERENCING ON ARYJMH SURFACE              **********FLWP0424
ISN 0380          IF(VLYJMH.GT.0.)GO TO 2576                                   FLWP0425
               C********** FOR OUTFLOWING OPEN REGION CONDITIONS               **********FLWP0426
               C********** SIMPLE UPWIND DIFFERENCING FORM                    **********FLWP0427
ISN 0382          GWYJMH(LZ)=GFYJMH                                            FLWP0428
ISN 0383          GO TO 2579                                                   FLWP0429
ISN 0384       2576 CONTINUE                                                    FLWP0430
ISN 0385          GWYJMH(LZ)=GFYJMH                                            FLWP0431
ISN 0386       2579 CONTINUE                                                    FLWP0432
ISN 0387       2570 CONTINUE                                                    FLWP0433
ISN 0388       2600 CONTINUE                                                    FLWP0434
               C********** START JPH BOUNDARY CALCULATIONS                    **********FLWP0435
ISN 0389          JPHCON=JPHBND(IBND)+2                                        FLWP0436
ISN 0390          GO TO (2610,2640,2660,2660),JPHCON                           FLWP0437
ISN 0391       2610 CONTINUE                                                    FLWP0438
               C********** INTERNAL JPH BOUNDARY                             **********FLWP0439
ISN 0392          IJJP1=IJIXJY(IX,JY+1)                                        FLWP0440
ISN 0393          JPYJP1=2*IJJP1-1                                             FLWP0441
ISN 0394          IMXJP1=JMYJP1                                                FLWP0442
ISN 0395          IPXJP1=IMXJP1+1                                              FLWP0443
ISN 0396          IF(INTEXT(IX,JY+1).GT.0)GO TO 2612                           FLWP0444
               C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT        **********FLWP0445
ISN 0398          DYJP1=DY(JY+1)                                               FLWP0446
ISN 0399          GO TO 2615                                                   FLWP0447
ISN 0400       2612 CONTINUE                                                    FLWP0448
               C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT        **********FLWP0449
ISN 0401          IBNJP1=INTEXT(IX,JY+1)                                       FLWP0450
ISN 0402          DYJP1=DYBND(IBNJP1)                                          FLWP0451
ISN 0403       2615 CONTINUE                                                    FLWP0452
ISN 0404          DYJPH=0.5*(DYIAJ+DYJP1)                                      FLWP0453
ISN 0405          CDYJPH=0.5*DYIAJ/DYJPH                                       FLWP0454
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ISN 0406          HSRJP1=HSR(IJJP1)                                                    FLWP0455
ISN 0407          NLZJP1=NLZBOT(IJJP1)                                                 FLWP0456
ISN 0408          DO 2620 LZ=NLZMIN,NZ                                                 FLWP0457
ISN 0409          IF(LZ.GE.NLZJP1)GO TO 2625                                           FLWP0458
                 C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********FLWP0459
                 C********** SHORELINE BOUNDARY                              **********FLWP0460
ISN 0411          GWYJPH(LZ)=0.0                                                       FLWP0461
ISN 0412          GO TO 2620                                                           FLWP0462
ISN 0413         2625 CONTINUE                                                         FLWP0463
                 C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT  **********FLWP0464
ISN 0414          DHZJP1=DZ(LZ)                                                        FLWP0465
ISN 0415          IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                               FLWP0466
ISN 0417          IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                                   FLWP0467
ISN 0419          DHZJPH=DHZIAJ(LZ)+CDYJPH*(DHZJP1-DHZIAJ(LZ))                         FLWP0468
ISN 0420          ARYJPH=DXJPH*DHZJPH                                                  FLWP0469
ISN 0421          VMYJP1=VLY(JMYJP1,LZ)                                                FLWP0470
ISN 0422          VLYJPH=VPYIAJ(LZ)+CDYJPH*(VMYJP1-VPYIAJ(LZ))                         FLWP0471
                 C********** UPWIND DIFFERENCING ON ARYJPH SURFACE          **********FLWP0472
ISN 0423          IF(INWIND.NE.0)GO TO 2633                                            FLWP0473
                 C********** SIMPLE UPWIND DIFFERENCING FORM                 **********FLWP0474
ISN 0425          IF(VLYJPH.LT.0.)GO TO 2631                                           FLWP0475
ISN 0427          GWYJPH(LZ)=ARYJPH*VLYJPH                                             FLWP0476
ISN 0428          GO TO 2639                                                           FLWP0477
ISN 0429         2631 CONTINUE                                                         FLWP0478
ISN 0430          GWYJPH(LZ)=ARYJPH*VLYJPH                                             FLWP0479
ISN 0431          GO TO 2639                                                           FLWP0480
ISN 0432         2633 CONTINUE                                                         FLWP0481
ISN 0433          RDYJP1=1.+(DYJP1/DYIAJ)                                              FLWP0482
ISN 0434          IF(INWIND.GT.0)GO TO 2635                                            FLWP0483
                 C********** WEIGHTED UPWIND DIFFERENCING FORM              **********FLWP0484
ISN 0436          DYRJPH=DYJPH/DYIAJ                                                   FLWP0485
ISN 0437          IF(VLYJPH.LT.0.)GO TO 2634                                           FLWP0486
ISN 0439          CWYJPH=(1.-RYWIND*DYRJPH)/RDYJP1                                     FLWP0487
ISN 0440          GO TO 2636                                                           FLWP0488
ISN 0441         2634 CONTINUE                                                         FLWP0489
ISN 0442          CWYJPH=(1.+RYWIND*DYRJPH)/RDYJP1                                     FLWP0490
ISN 0443          GO TO 2636                                                           FLWP0491
ISN 0444         2635 CONTINUE                                                         FLWP0492
                 C********** TRANSPORTIVE UPWIND DIFFERENCING FORM          **********FLWP0493
ISN 0445          CNYJPH=VLYJPH*DT/DYIAJ                                               FLWP0494
ISN 0446          CWYJPH=(1.-2.*RYWIND*CNYJPH)/RDYJP1                                  FLWP0495
ISN 0447         2636 CONTINUE                                                         FLWP0496
ISN 0448          GWYJPH(LZ)=ARYJPH*VLYJPH                                             FLWP0497
ISN 0449         2639 CONTINUE                                                         FLWP0498
ISN 0450         2620 CONTINUE                                                         FLWP0499
ISN 0451          GO TO 2700                                                           FLWP0500
ISN 0452         2640 CONTINUE                                                         FLWP0501
                 C********** SHORELINE JPH BOUNDARY                          **********FLWP0502
ISN 0453          DO 2650 LZ=NLZMIN,NZ                                                 FLWP0503
ISN 0454          GWYJPH(LZ)=0.0                                                       FLWP0504
ISN 0455         2650 CONTINUE                                                         FLWP0505
ISN 0456          GO TO 2700                                                           FLWP0506
ISN 0457         2660 CONTINUE                                                         FLWP0507
                 C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS  *********FLWP0508
                 C********** FOR JPH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********FLWP0509
ISN 0458          ISPB=ISPBND(IBND)                                                    FLWP0510
ISN 0459          HSRJPH=HSRBYH(ISPB)                                                  FLWP0511
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ISN 0460            DO 2670 LZ=NLZMIN,NZ                                                    FLWP0512
ISN 0461            JFYJPH=JVYBYH(ISPB,LZ)                                                  FLWP0513
ISN 0462            GFYJPH=GFYBYH(ISPB,LZ)                                                  FLWP0514
ISN 0463            IF(JFYJPH.LT.1)GO TO 2676                                               FLWP0515
ISN 0465            DHZJPH=DHZIAJ(LZ)                                                       FLWP0516
ISN 0466            IF(LZ.EQ.NZ)  DHZJPH=DHZJPH-HSRIAJ+HSRJPH                               FLWP0517
ISN 0468            ARYJPH=DXJPH*DHZJPH                                                     FLWP0518
ISN 0469            VLYJPH=GFYJPH/ARYJPH                                                    FLWP0519
                  C********** UPWIND DIFFERENCING ON ARYJPH SURFACE                  **********FLWP0520
ISN 0470            IF(VLYJPH.LT.0.)GO TO 2676                                              FLWP0521
                  C********** FOR OUTFLOWING OPEN REGION CONDITIONS                  **********FLWP0522
                  C********** SIMPLE UPWIND DIFFERENCING FORM                       **********FLWP0523
ISN 0472            GWYJPH(LZ)=GFYJPH                                                       FLWP0524
ISN 0473            GO TO 2679                                                              FLWP0525
ISN 0474         2676 CONTINUE                                                              FLWP0526
ISN 0475            GWYJPH(LZ)=GFYJPH                                                       FLWP0527
ISN 0476         2679 CONTINUE                                                              FLWP0528
ISN 0477         2670 CONTINUE                                                              FLWP0529
ISN 0478         2700 CONTINUE                                                              FLWP0530
ISN 0479            GWZLPH=GWZLZH(IJ,NLZMIN)                                                FLWP0531
ISN 0480            IF(NLZMIN.EQ.NZ)GO TO 2899                                              FLWP0532
ISN 0482            NZM1=NZ-1                                                               FLWP0533
ISN 0483            DO 2800 LZ=NLZMIN,NZM1                                                  FLWP0534
ISN 0484            GWZLMH=GWZLPH                                                           FLWP0535
ISN 0485            GWZLPH=GWZLMH                                                           FLWP0536
                   1       +GWXIMH(LZ)-GWXIPH(LZ)+GWYJPH(LZ)-GWYJPH(LZ)                   FLWP0537
ISN 0486            GWZLZH(IJ,LZ+1)=GWZLPH                                                  FLWP0538
ISN 0487         2800 CONTINUE                                                              FLWP0539
ISN 0488         2899 CONTINUE                                                              FLWP0540
ISN 0489            GWZLMH=GWZLPH                                                           FLWP0541
ISN 0490            GWZLPH=GWZLMH                                                           FLWP0542
                   1       +GWXIMH(NZ)-GWXIPH(NZ)+GWYJPH(NZ)-GWYJPH(NZ)                   FLWP0543
ISN 0491            FACHSR(IJ)=(1./ARHIAJ)*GWZLPH                                           FLWP0544
ISN 0492         2900 CONTINUE                                                              FLWP0545
ISN 0493            DO 2920 LZ=NLZMIN,NZ                                                    FLWP0546
ISN 0494            GWXTRX(JY,LZ)=GWXIPH(LZ)                                                FLWP0547
ISN 0495            GWYTRY(LZ)=GWYJPH(LZ)                                                   FLWP0548
ISN 0496         2920 CONTINUE                                                              FLWP0549
ISN 0497         2200 CONTINUE                                                              FLWP0550
ISN 0498         2100 CONTINUE                                                              FLWP0551
ISN 0499         9999 RETURN                                                                FLWP0552
ISN 0500            END                                                                     FLWP0553

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =     499 ,PROGRAM SIZE =     6452

*STATISTICS*  NO  DIAGNOSTICS GENERATED

******* END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
  ISN 0002            SUBROUTINE CONVEC(DTWIND)                                        CNVC0001
  ISN 0003            COMMON/CONTRL/KRUN,T                                             CNVC0002
  ISN 0004            COMMON/CALLER/INCONV,INTURB,IBOUNC,INCBOT,NPLANT,INCSUR          CNVC0003
  ISN 0005            COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,..),IJIXJY(18,12),    CNVC0004
                     1            NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)   CNVC0005
  ISN 0006            COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ( 6),DHZBOT(136)     CNVC0006
  ISN 0007            COMMON/GEOBND/IXBND( 49),JYBND( 49),                             CNVC0007
                     1            DXBND( 49),DYBND( 49),ARHBND( 49)                  CNVC0008
  ISN 0008            COMMON/ENGBND/IMHBND( 49),DYBIMH( 49),IPHBND( 49),DYBIPH( 49),   CNVC0009
                     1            JMHBND( 49),DXBJMH( 49),JPHBND( 49),DXBJPH( 49)    CNVC0010
  ISN 0009            COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)                   CNVC0011
  ISN 0010            COMMON/VLXSOL/VLX(272, 6),PREVLX(272, 6),FNCVLX(272, 6)          CNVC0012
  ISN 0011            COMMON/VLYSOL/VLY(272, 6),PREVLY(272, 6),FNCVLY(272, 6)          CNVC0013
  ISN 0012            COMMON/FLOVLZ/GWZLZH(136, 6),FNCGWZ(136, 6)                      CNVC0014
  ISN 0013            COMMON/UPWIND/INWIND,PXWIND,RYWIND,PZWIND                        CNVC0015
  ISN 0014            COMMON/BNDCON/ISPBND( 49),IXBSPB(17),JYBSPB(17),                 CNVC0016
                     1            HSRBXH(17),GFXBXH(17, 6),IVXBXH(17, 6),            CNVC0017
                     2            VLXBXH(17, 6),VMYBXH(17, 6),VPYBXH(17, 6),         CNVC0018
                     3            VLZBXH(17, 6),FPRBXH(17, 6),                       CNVC0019
                     4            HSRBYH(17),GFYBYH(17, 6),JVYBYH(17, 6),            CNVC0020
                     5            VLYBYH(17, 6),VMXBYH(17, 6),VPXBYH(17, 6),         CNVC0021
                     6            VLZBYH(17, 6),FPRBYH(17, 6)                        CNVC0022
  ISN 0015            DIMENSION DHZIAJ( 6),VLZIAJ( 6)                                  CNVC0023
  ISN 0016            DIMENSION IMXIMH( 6),IPXIPH( 6),JPYJMH( 6),JPYJPH( 6)            CNVC0024
  ISN 0017            DIMENSION VMXIAJ( 6),VPXIAJ( 6),VPYIAJ( 6),VPYIAJ( 6)            CNVC0025
  ISN 0018            DIMENSION GFXIMH( 6),GFXIAJ( 6),GFXIPH( 6),                      CNVC0026
                     1          GFYJMH( 6),GFYIAJ( 6),GFYJPH( 6)                     CNVC0027
  ISN 0019            DIMENSION VLXMXH( 6),VLXHXH( 6),VLXPXH( 6),                      CNVC0028
                     1          VMYMXH( 6),VPYMXH( 6),VPYPXH( 6),VPYPXH( 6),         CNVC0029
                     2          VLZMXH( 6),VLZPXH( 6)                                CNVC0030
  ISN 0020            DIMENSION VLYMYH( 6),VLYHYH( 6),VLYPYH( 6),                      CNVC0031
                     1          VMXMYH( 6),VPXMYH( 6),VPXPYH( 6),VPXPYH( 6),         CNVC0032
                     2          VLZMYH( 6),VLZPYH( 6)                                CNVC0033
  ISN 0021            DIMENSION GFXTRX(12, 6),VLXTRX(12, 6),VMYTRX(12, 6),VPYTRX(12, 6),CNVC0034
                     1          VLZTRX(12, 6)                                        CNVC0035
  ISN 0022            DIMENSION GFYTRY( 6),VLYTRY( 6),VPXTRY( 6),VPXTRY( 6),           CNVC0036
                     1          VLZTRY( 6)                                           CNVC0037
  ISN 0023            IF(KRUN.GT.0)GO TO 2                                             CNVC0038
  ISN 0025            DO 1100 IJ=1,NINJ                                                CNVC0039
  ISN 0026            DO 1110 LZ=1,NZ                                                  CNVC0040
  ISN 0027            FNCGWZ(IJ,LZ)=0.0                                                CNVC0041
  ISN 0028      1110 CONTINUE                                                          CNVC0042
  ISN 0029      1100 CONTINUE                                                          CNVC0043
  ISN 0030            DO 1200 JY=1,NY                                                  CNVC0044
  ISN 0031            DO 1210 LZ=1,NZ                                                  CNVC0045
  ISN 0032            GFXTRX(JY,LZ)=0.0                                                CNVC0046
  ISN 0033            VLXTRX(JY,LZ)=0.0                                                CNVC0047
  ISN 0034            VPYTRX(JY,LZ)=0.0                                                CNVC0048
  ISN 0035            VPYTRX(JY,LZ)=0.0                                                CNVC0049
  ISN 0036            VLZTRX(JY,LZ)=0.0                                                CNVC0050
  ISN 0037            GFYTRY(LZ)=0.0                                                   CNVC0051
  ISN 0038            VLYTRY(LZ)=0.0                                                   CNVC0052
  ISN 0039            VPXTRY(LZ)=0.0                                                   CNVC0053
  ISN 0040            VPXTRY(LZ)=0.0                                                   CNVC0054
  ISN 0041            VLZTRY(LZ)=0.0                                                   CNVC0055
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ISN 0042      1210 CONTINUE                                                    CNVC0056
ISN 0043      1200 CONTINUE                                                    CNVC0057
ISN 0044           GO TO 9999                                                  CNVC0058
ISN 0045         2 CONTINUE                                                    CNVC0059
ISN 0046           DT=DTWIND                                                   CNVC0060
ISN 0047           DO 2100 IX=1,NX                                             CNVC0061
ISN 0048           DO 2200 JY=1,NY                                             CNVC0062
ISN 0049           IF(INTEXT(IX,JY).EQ.0)GO TO 2200                            CNVC0063
ISN 0051           IJ=IJIXJY(IX,JY)                                            CNVC0064
ISN 0052           IMX=2*IJ-1                                                  CNVC0065
ISN 0053           IPX=IMX+1                                                   CNVC0066
ISN 0054           JMY=IMX                                                     CNVC0067
ISN 0055           JPY=IPX                                                     CNVC0068
ISN 0056           IF(INTEXT(IX,JY).GT.0)GO TO 2105                            CNVC0069
                  C********** INTERNAL DISCRETE ELEMENT GEOMETRY CONDITIONS    **********CNVC0070
ISN 0058           DXIAJ=DX(IX)                                                CNVC0071
ISN 0059           DYIAJ=DY(JY)                                                CNVC0072
ISN 0060           ARHIAJ=DXIAJ*DYIAJ                                          CNVC0073
ISN 0061           DYIPH=DYIAJ                                                 CNVC0074
ISN 0062           DXJPH=DXIAJ                                                 CNVC0075
ISN 0063           DXIMX=0.5*DXIAJ                                             CNVC0076
ISN 0064           DYIMX=DYIAJ                                                 CNVC0077
ISN 0065           ARHIMX=DXIMX*DYIMX                                          CNVC0078
ISN 0066           DXIPX=0.5*DXIAJ                                             CNVC0079
ISN 0067           DYIPX=DYIAJ                                                 CNVC0080
ISN 0068           ARHIPX=DXIPX*DYIPX                                          CNVC0081
ISN 0069           DYJMY=0.5*DYIAJ                                             CNVC0082
ISN 0070           DXJMY=DXIAJ                                                 CNVC0083
ISN 0071           ARHJMY=DXJMY*DYJMY                                          CNVC0084
ISN 0072           DYJPY=0.5*DYIAJ                                             CNVC0085
ISN 0073           DXJPY=DXIAJ                                                 CNVC0086
ISN 0074           ARHJPY=DXJPY*DYJPY                                          CNVC0087
ISN 0075           GO TO 2109                                                  CNVC0088
ISN 0076      2105 CONTINUE                                                    CNVC0089
                  C********** BOUNDARY DISCETE ELEMENT GEOMETRY CONDITIONS    **********CNVC0090
ISN 0077           IBND=INTEXT(IX,JY)                                          CNVC0091
ISN 0078           DXIAJ=DXBND(IBND)                                           CNVC0092
ISN 0079           DYIAJ=DYBND(IBND)                                           CNVC0093
ISN 0080           ARHIAJ=ARHBND(IBND)                                         CNVC0094
ISN 0081           DYIMH=DYBIMH(IBND)                                          CNVC0095
ISN 0082           DYIPH=DYBIPH(IBND)                                          CNVC0096
ISN 0083           DXJMH=DXBJMH(IBND)                                          CNVC0097
ISN 0084           DXJPH=DXBJPH(IBND)                                          CNVC0098
ISN 0085           DXIMX=0.5*DXIAJ                                             CNVC0099
ISN 0086           DYIMX=0.5*(DYIAJ+DYIMH)                                     CNVC0100
ISN 0087           ARHIMX=DXIMX*DYIMX                                          CNVC0101
ISN 0088           DXIPX=0.5*DXIAJ                                             CNVC0102
ISN 0089           DYIPX=0.5*(DYIAJ+DYIPH)                                     CNVC0103
ISN 0090           ARHIPX=DXIPX*DYIPX                                          CNVC0104
ISN 0091           DYJMY=0.5*DYIAJ                                             CNVC0105
ISN 0092           DXJMY=0.5*(DXIAJ+DXJMH)                                     CNVC0106
ISN 0093           ARHJMY=DXJMY*DYJMY                                          CNVC0107
ISN 0094           DYJPY=0.5*DYIAJ                                             CNVC0108
ISN 0095           DXJPY=0.5*(DXIAJ+DXJPH)                                     CNVC0109
ISN 0096           ARHJPY=DXJPY*DYJPY                                          CNVC0110
ISN 0097      2109 CONTINUE                                                    CNVC0111
ISN 0098           HSRIAJ=HSR(IJ)                                              CNVC0112
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ISN 0099          NLZMIN=NLZBOT(IJ)                                                CNVC0113
ISN 0100          DC 2110 LZ=NLZMIN,NZ                                             CNVC0114
ISN 0101          DHZDUM=DZ(LZ)                                                    CNVC0115
ISN 0102          IF(LZ.EQ.NLZMIN)  DHZDUM=DHZBOT(IJ)                              CNVC0116
ISN 0104          IF(LZ.EQ.NZ)  DHZDUM=DHZDUM+HSRIAJ                               CNVC0117
ISN 0106          DHZIAJ(LZ)=DHZDUM                                                CNVC0118
ISN 0107          ARXIAJ=DYIAJ*DHZDUM                                              CNVC0119
ISN 0108          ARYIAJ=DXIAJ*DHZDUM                                              CNVC0120
ISN 0109          VPXIAJ(LZ)=VLX(IMX,LZ)                                           CNVC0121
ISN 0110          VPXIAJ(LZ)=VLX(IPX,LZ)                                           CNVC0122
ISN 0111          VMYIAJ(LZ)=VLY(JMY,LZ)                                           CNVC0123
ISN 0112          VPYIAJ(LZ)=VLY(JPY,LZ)                                           CNVC0124
                  C********** CALCULATE GFXIAJ(LZ)                                **********CNVC0125
ISN 0113          VLXIAJ=0.5*(VMXIAJ(LZ)+VPXIAJ(LZ))                               CNVC0126
ISN 0114          GFXIAJ(LZ)=ARXIAJ*VLXIAJ                                         CNVC0127
                  C********** CALCULATE GFYIAJ(LZ)                                **********CNVC0128
ISN 0115          VLYIAJ=0.5*(VMYIAJ(LZ)+VPYIAJ(LZ))                               CNVC0129
ISN 0116          GFYIAJ(LZ)=ARYIAJ*VLYIAJ                                         CNVC0130
                  C********** CALCULATE VLZIAJ(LZ)                                **********CNVC0131
ISN 0117          GWZLMH=GWZLZH(IJ,LZ)                                             CNVC0132
ISN 0118          IF(LZ.EQ.NZ)GO TO 2112                                           CNVC0133
ISN 0120          GWZLPH=GWZLZH(IJ,LZ+1)                                           CNVC0134
ISN 0121          GC TO 2115                                                       CNVC0135
ISN 0122   2112 CONTINUE                                                          CNVC0136
ISN 0123          GWZLPH=ARHIAJ*FNCHSR(IJ)                                         CNVC0137
ISN 0124   2115 CONTINUE                                                          CNVC0138
ISN 0125          GWZIAJ=0.5*(GWZLMH+GWZLPH)                                       CNVC0139
ISN 0126          VLZIAJ(LZ)=GWZIAJ/APHIAJ                                         CNVC0140
ISN 0127   2110 CONTINUE                                                          CNVC0141
                  C********** CALCULATE UPWIND-DIFFERENCED VALUES FOR VELOCITY     **********CNVC0142
                  C********** COMPONENTS AT THE CENTER OF THE ELEMENTS            **********CNVC0143
ISN 0128          DC 2120 LZ=NLZMIN,NZ                                             CNVC0144
ISN 0129          VLXIAJ=0.5*(VMXIAJ(LZ)+VPXIAJ(LZ))                               CNVC0145
                  C********** UPWIND DIFFERENCING ON ARXIAJ SURFACE                **********CNVC0146
ISN 0130          IF(INWIND.NE.0)GO TO 2122                                        CNVC0147
                  C********** SIMPLE UPWIND DIFFERENCING FORM                      **********CNVC0148
ISN 0132          IF(VLXIAJ.GT.0.)GO TO 2121                                       CNVC0149
ISN 0134          VLXHXH(LZ)=VPXIAJ(LZ)                                            CNVC0150
ISN 0135          GC TO 2129                                                       CNVC0151
ISN 0136   2121 CONTINUE                                                          CNVC0152
ISN 0137          VLXHXH(LZ)=VMXIAJ(LZ)                                            CNVC0153
ISN 0138          GC TO 2129                                                       CNVC0154
ISN 0139   2122 CONTINUE                                                          CNVC0155
ISN 0140          IF(INWIND.GT.0)GO TO 2124                                        CNVC0156
                  C********** WEIGHTED UPWIND DIFFERENCING FORM                    **********CNVC0157
ISN 0142          IF(VLXIAJ.GT.0.)GO TO 2123                                       CNVC0158
ISN 0144          CWXIAJ=0.5*(1.+RXWIND)                                           CNVC0159
ISN 0145          GC TO 2126                                                       CNVC0160
ISN 0146   2123 CONTINUE                                                          CNVC0161
ISN 0147          CWXIAJ=0.5*(1.-RXWIND)                                           CNVC0162
ISN 0148          GC TO 2126                                                       CNVC0163
ISN 0149   2124 CONTINUE                                                          CNVC0164
                  C********** TRANSPORTIVE UPWIND DIFFERENCING FORM                **********CNVC0165
ISN 0150          CWXIAJ=VLXIAJ*DT/DXIAJ                                           CNVC0166
ISN 0151          CWXIAJ=0.5-2.*RXWIND*CNXIAJ                                      CNVC0167
ISN 0152   2126 CONTINUE                                                          CNVC0168
ISN 0153          VLXHXH(LZ)=VMXIAJ(LZ)+CWXIAJ*(VPXIAJ(LZ)-VMXIAJ(LZ))            CNVC0169
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ISN 0154      2129 CONTINUE                                                    CNVC0170
ISN 0155           VLYIAJ=0.5*(VMYIAJ(LZ)+VPYIAJ(LZ))                          CNVC0171
              C********** UPWIND DIFFERENCING ON ARYIAJ SURFACE          **********CNVC0172
ISN 0156           IF(INWIND.NE.0)GO TO 2132                                   CNVC0173
              C********** SIMPLE UPWIND DIFFERENCING FORM                  **********CNVC0174
ISN 0158           IF(VLYIAJ.GT.0.)GO TO 2131                                  CNVC0175
ISN 0160           VLYHYH(LZ)=VPYIAJ(LZ)                                       CNVC0176
ISN 0161           GO TO 2139                                                  CNVC0177
ISN 0162      2131 CONTINUE                                                    CNVC0178
ISN 0163           VLYHYH(LZ)=VMYIAJ(LZ)                                       CNVC0179
ISN 0164           GO TO 2139                                                  CNVC0180
ISN 0165      2132 CONTINUE                                                    CNVC0181
ISN 0166           IF(INWIND.GT.0)GO TO 2134                                   CNVC0182
              C********** WEIGHTED UPWIND DIFFERENCING FORM                 **********CNVC0183
ISN 0168           IF(VLYIAJ.GT.0.)GO TO 2133                                  CNVC0184
ISN 0170           CNYIAJ=0.5*(1.+RYWIND)                                      CNVC0185
ISN 0171           GO TO 2136                                                  CNVC0186
ISN 0172      2133 CONTINUE                                                    CNVC0187
ISN 0173           CNYIAJ=0.5*(1.-RYWIND)                                      CNVC0188
ISN 0174           GO TO 2136                                                  CNVC0189
ISN 0175      2134 CONTINUE                                                    CNVC0190
              C********** TRANSPORTIVE UPWIND DIFFERENCING FORM              **********CNVC0191
ISN 0176           CNYIAJ=VLYIAJ*DT/DYIAJ                                      CNVC0192
ISN 0177           CNYIAJ=0.5-2.*RYWIND*CNYIAJ                                 CNVC0193
ISN 0178      2136 CONTINUE                                                    CNVC0194
ISN 0179           VLYHYH(LZ)=VMYIAJ(LZ)+CNYIAJ*(VPYIAJ(LZ)-VMYIAJ(LZ))       CNVC0195
ISN 0180      2139 CONTINUE                                                    CNVC0196
ISN 0181      2120 CONTINUE                                                    CNVC0197
ISN 0182           IF(INTEXT(IX,JY).GT.0)GO TO 2300                            CNVC0198
              C********** START INTERNAL ELEMENT CALCULATIONS             **********CNVC0199
              C********** IMH AND JMH ENCLOSURE SURFACE CONVECTION TRANSPORT *********CNVC0200
              C********** TRANSFERRED FROM PREVIOUS IPH AND JPH CALCULATIONS *********CNVC0201
              C********** IPH AND JPH ENCLOSURE SURFACE CONVECTION          **********CNVC0202
              C********** TRANSPORT CALCULATED                             **********CNVC0203
ISN 0184           IJIM1=IJIXJY(IX-1,JY)                                       CNVC0204
ISN 0185           NLZIM1=NLZBOT(IJIM1)                                        CNVC0205
ISN 0186           DO 2150 LZ=NLZMIN,NZ                                        CNVC0206
ISN 0187           IF(LZ.GE.NLZIM1)GO TO 2155                                  CNVC0207
              C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM         **********CNVC0208
              C********** SHORELINE BOUNDARY                               **********CNVC0209
ISN 0189           IMXIMH(LZ)=0                                                CNVC0210
ISN 0190           GFXIMH(LZ)=0.0                                              CNVC0211
ISN 0191           VLXMXH(LZ)=0.0                                              CNVC0212
ISN 0192           VMYMXH(LZ)=0.0                                              CNVC0213
ISN 0193           VPYMXH(LZ)=0.0                                              CNVC0214
ISN 0194           VLZMXH(LZ)=0.0                                              CNVC0215
ISN 0195           GO TO 2159                                                  CNVC0216
ISN 0196      2155 CONTINUE                                                    CNVC0217
              C********** IMH SURFACE AT LZ LEVEL ADJASENT TO AN ELEMENT  **********CNVC0218
ISN 0197           IMXIMH(LZ)=1                                                CNVC0219
ISN 0198           GFXIMH(LZ)=GFXTRX(JY,LZ)                                    CNVC0220
ISN 0199           VLXMXH(LZ)=VLXTRX(JY,LZ)                                    CNVC0221
ISN 0200           VMYMXH(LZ)=VMYTRX(JY,LZ)                                    CNVC0222
ISN 0201           VPYMXH(LZ)=VPYTRX(JY,LZ)                                    CNVC0223
ISN 0202           VLZMXH(LZ)=VLZTRX(JY,LZ)                                    CNVC0224
ISN 0203      2159 CONTINUE                                                    CNVC0225
ISN 0204      2150 CONTINUE                                                    CNVC0226
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ISN 0205          IJJM1=IJIXJY(IX,JY-1)                                          CNVC0227
ISN 0206          NLZJM1=NLZBOT(IJJM1)                                           CNVC0228
ISN 0207          DO 2160 LZ=NLZMIN,NZ                                           CNVC0229
ISN 0208          IF(LZ.GE.NLZJM1)GO TO 2165                                     CNVC0230
                  C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********CNVC0231
                  C********** SHORELINE BOUNDARY                               **********CNVC0232
ISN 0210          JMYJMH(LZ)=0                                                   CNVC0233
ISN 0211          GFYJMH(LZ)=0.0                                                 CNVC0234
ISN 0212          VLYMYH(LZ)=0.0                                                 CNVC0235
ISN 0213          VMXMYH(LZ)=0.0                                                 CNVC0236
ISN 0214          VPXMYH(LZ)=0.0                                                 CNVC0237
ISN 0215          VLZMYH(LZ)=0.0                                                 CNVC0238
ISN 0216          GO TO 2169                                                     CNVC0239
ISN 0217         2165 CONTINUE                                                   CNVC0240
                  C********** JMH SURFACE AT LZ LEVEL ADJASENT TO AN ELEMENT    **********CNVC0241
ISN 0218          JMYJMH(LZ)=1                                                   CNVC0242
ISN 0219          GFYJMH(LZ)=GFYTRY(LZ)                                          CNVC0243
ISN 0220          VLYMYH(LZ)=VLYTRY(LZ)                                          CNVC0244
ISN 0221          VMXMYH(LZ)=VMXTRY(LZ)                                          CNVC0245
ISN 0222          VPXMYH(LZ)=VPXTRY(LZ)                                          CNVC0246
ISN 0223          VLZMYH(LZ)=VLZTRY(LZ)                                          CNVC0247
ISN 0224         2169 CONTINUE                                                   CNVC0248
ISN 0225         2160 CONTINUE                                                   CNVC0249
                  C********** START CALCULATIONS FOR INTERNAL IPH SURFACE       **********CNVC0250
ISN 0226          IJIP1=IJIXJY(IX+1,JY)                                          CNVC0251
ISN 0227          IMXIP1=2*IJIP1-1                                               CNVC0252
ISN 0228          JMYIP1=IMXIP1                                                  CNVC0253
ISN 0229          JPYIP1=JMYIP1+1                                                CNVC0254
ISN 0230          IF(INTEXT(IX+1,JY).GT.0)GO TO 2212                             CNVC0255
                  C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT        **********CNVC0256
ISN 0232          DXIP1=DX(IX+1)                                                 CNVC0257
ISN 0233          DYIP1=DYIAJ                                                    CNVC0258
ISN 0234          ARHIP1=DXIP1*DYIP1                                             CNVC0259
ISN 0235          GO TO 2215                                                     CNVC0260
ISN 0236         2212 CONTINUE                                                   CNVC0261
                  C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT        **********CNVC0262
ISN 0237          IBNIP1=INTEXT(IX+1,JY)                                         CNVC0263
ISN 0238          DXIP1=DXBND(IBNIP1)                                            CNVC0264
ISN 0239          ARHIP1=ARHBND(IBNIP1)                                          CNVC0265
ISN 0240         2215 CONTINUE                                                   CNVC0266
ISN 0241          DXIPH=0.5*(DXIAJ+DXIP1)                                        CNVC0267
ISN 0242          CDXIPH=0.5*DXIAJ/DXIPH                                         CNVC0268
ISN 0243          HSRIP1=HSR(IJIP1)                                              CNVC0269
ISN 0244          NLZIP1=NLZBOT(IJIP1)                                           CNVC0270
ISN 0245          DO 2220 LZ=NLZMIN,NZ                                           CNVC0271
ISN 0246          IF(LZ.GE.NLZIP1)GO TO 2222                                     CNVC0272
                  C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********CNVC0273
                  C********** SHORELINE BOUNDARY                               **********CNVC0274
ISN 0248          IPXIPH(LZ)=0                                                   CNVC0275
ISN 0249          GFXIPH(LZ)=0.0                                                 CNVC0276
ISN 0250          VLXPXH(LZ)=0.0                                                 CNVC0277
ISN 0251          VMYPXH(LZ)=0.0                                                 CNVC0278
ISN 0252          VPYPXH(LZ)=0.0                                                 CNVC0279
ISN 0253          VLZPXH(LZ)=0.0                                                 CNVC0280
ISN 0254          GO TO 2220                                                     CNVC0281
ISN 0255         2222 CONTINUE                                                   CNVC0282
                  C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********CNVC0283
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ISN 0256          IPXIPH(LZ)=1                                                    CNVC0284
ISN 0257          DHZIP1=DZ(LZ)                                                   CNVC0285
ISN 0258          IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                          CNVC0286
ISN 0260          IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                              CNVC0287
ISN 0262          DHZIPH=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))                    CNVC0288
ISN 0263          ARXIPH=DYIPH*DHZIPH                                             CNVC0289
ISN 0264          VMXIP1=VLX(IMXIP1,LZ)                                           CNVC0290
ISN 0265          VLXIPH=VPXIAJ(LZ)+CDXIPH*(VMXIP1-VPXIAJ(LZ))                    CNVC0291
ISN 0266          GFXIPH(LZ)=ARXIPH*VLXIPH                                        CNVC0292
ISN 0267          VMYIP1=VLY(JMYIP1,LZ)                                           CNVC0293
ISN 0268          VPYIP1=VLY(JPYIP1,LZ)                                           CNVC0294
                 C********** CALCULATE VLZIP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJIP1 *CNVC0295
ISN 0269          GWZLMH=GWZLZH(IJIP1,LZ)                                         CNVC0296
ISN 0270          IF(LZ.EQ.NZ)GO TO 2226                                          CNVC0297
ISN 0272          GWZLPH=GWZLZH(IJIP1,LZ+1)                                       CNVC0298
ISN 0273          GO TO 2228                                                      CNVC0299
ISN 0274       2226 CONTINUE                                                      CNVC0300
ISN 0275          GWZLPH=ARHIP1*FNCHSR(IJIP1)                                     CNVC0301
ISN 0276       2228 CONTINUE                                                      CNVC0302
ISN 0277          GWZIP1=0.5*(GWZLMH+GWZLPH)                                      CNVC0303
ISN 0278          VLZIP1=GWZIP1/ARHIP1                                            CNVC0304
                 C********** UPWIND DIFFERENCING ON ARXIPH SURFACE                **********CNVC0305
ISN 0279          IF(INWIND.NE.0)GO TO 2233                                       CNVC0306
                 C********** SIMPLE UPWIND DIFFERENCING FORM                      **********CNVC0307
ISN 0281          IF(VLXIPH.LT.0.)GO TO 2231                                      CNVC0308
ISN 0283          VLXPXH(LZ)=VPXIAJ(LZ)                                           CNVC0309
ISN 0284          VMYPXH(LZ)=VMYIAJ(LZ)                                           CNVC0310
ISN 0285          VPYPXH(LZ)=VPYIAJ(LZ)                                           CNVC0311
ISN 0286          VLZPXH(LZ)=VLZIAJ(LZ)                                           CNVC0312
ISN 0287          GO TO 2239                                                      CNVC0313
ISN 0288       2231 CONTINUE                                                      CNVC0314
ISN 0289          VLXPXH(LZ)=VMXIP1                                               CNVC0315
ISN 0290          VMYPXH(LZ)=VMYIP1                                               CNVC0316
ISN 0291          VPYPXH(LZ)=VPYIP1                                               CNVC0317
ISN 0292          VLZPXH(LZ)=VLZIP1                                               CNVC0318
ISN 0293          GO TO 2239                                                      CNVC0319
ISN 0294       2233 CONTINUE                                                      CNVC0320
ISN 0295          RDXIP1=1.+(DXIP1/DXIAJ)                                         CNVC0321
ISN 0296          IF(INWIND.GT.0)GO TO 2235                                       CNVC0322
                 C********** WEIGHTED UPWIND DIFFERENCING FORM                    **********CNVC0323
ISN 0298          DXRIPH=DXIPH/DXIAJ                                              CNVC0324
ISN 0299          IF(VLXIPH.LT.0.)GO TO 2234                                      CNVC0325
ISN 0301          CWXPXH=(1.-RXWIND*DXRIPH)/RDXIP1                                CNVC0326
ISN 0302          CWXIPH=CWXPXH                                                   CNVC0327
ISN 0303          GO TO 2236                                                      CNVC0328
ISN 0304       2234 CONTINUE                                                      CNVC0329
ISN 0305          CWXPXH=(1.+RXWIND*DXRIPH)/RDXIP1                                CNVC0330
ISN 0306          CWXIPH=CWXPXH                                                   CNVC0331
ISN 0307          GO TO 2236                                                      CNVC0332
ISN 0308       2235 CONTINUE                                                      CNVC0333
                 C********** TRANSPORTIVE UPWIND DIFFERENCING FORM                **********CNVC0334
ISN 0309          CNXIPH=VLXIPH*DT/DXIAJ                                          CNVC0335
ISN 0310          CWXPXH=(1.-4.*RXWIND*CNXIPH)/RDXIP1                             CNVC0336
ISN 0311          CWXIPH=(1.-2.*RXWIND*CNXIPH)/RDXIP1                             CNVC0337
ISN 0312       2236 CONTINUE                                                      CNVC0338
ISN 0313          VLXPXH(LZ)=VPXIAJ(LZ)+CWXPXH*(VMXIP1-VPXIAJ(LZ))                CNVC0339
ISN 0314          VMYPXH(LZ)=VMYIAJ(LZ)+CWXIPH*(VMYIP1-VMYIAJ(LZ))                CNVC0340
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ISN 0315          VPYPXH(LZ)=VPYIAJ(LZ)+CWXIPH*(VPYIP1-VPYIAJ(LZ))                    CNVC0341
ISN 0316          VLZPXH(LZ)=VLZIAJ(LZ)+CWXIPH*(VLZIP1-VLZIAJ(LZ))                    CNVC0342
ISN 0317       2239 CONTINUE                                                          CNVC0343
ISN 0318       2220 CONTINUE                                                          CNVC0344
               C********** START CALCULATIONS FOR INTERNAL JPH SURFACE          **********CNVC0345
ISN 0319          IJJP1=IJIXJY(IX,JY+1)                                               CNVC0346
ISN 0320          JMYJP1=2*IJJP1-1                                                    CNVC0347
ISN 0321          IMXJP1=JMYJP1                                                       CNVC0348
ISN 0322          IPXJP1=IMXJP1+1                                                     CNVC0349
ISN 0323          IF(INTEXT(IX,JY+1).GT.0)GO TO 2252                                  CNVC0350
               C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT          **********CNVC0351
ISN 0325          DYJP1=DY(JY+1)                                                      CNVC0352
ISN 0326          DXJP1=DXIAJ                                                         CNVC0353
ISN 0327          ARHJP1=DXJP1*DYJP1                                                  CNVC0354
ISN 0328          GO TO 2255                                                          CNVC0355
ISN 0329       2252 CONTINUE                                                          CNVC0356
               C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT          **********CNVC0357
ISN 0330          IENJP1=INTEXT(IX,JY+1)                                              CNVC0358
ISN 0331          DYJP1=DYBND(IBNJP1)                                                 CNVC0359
ISN 0332          ARHJP1=ARHBND(IBNJP1)                                               CNVC0360
ISN 0333       2255 CONTINUE                                                          CNVC0361
ISN 0334          DYJPH=0.5*(DYIAJ+DYJP1)                                             CNVC0362
ISN 0335          CCYJPH=0.5*DYIAJ/DYJPH                                              CNVC0363
ISN 0336          HSRJP1=HSR(IJJP1)                                                   CNVC0364
ISN 0337          NLZJP1=NLZBOT(IJJP1)                                                CNVC0365
ISN 0338          DO 2260 LZ=NLZMIN,NZ                                                CNVC0366
ISN 0339          IF(LZ.GE.NLZJP1)GO TO 2262                                          CNVC0367
               C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********CNVC0368
               C********** SHORELINE BOUNDARY                                 **********CNVC0369
ISN 0341          JPYJPH(LZ)=0                                                        CNVC0370
ISN 0342          GFYJPH(LZ)=0.0                                                      CNVC0371
ISN 0343          VLYPYH(LZ)=0.0                                                      CNVC0372
ISN 0344          VPXPYH(LZ)=0.0                                                      CNVC0373
ISN 0345          VPXPYH(LZ)=0.0                                                      CNVC0374
ISN 0346          VLZPYH(LZ)=0.0                                                      CNVC0375
ISN 0347          GO TO 2260                                                          CNVC0376
ISN 0348       2262 CONTINUE                                                          CNVC0377
               C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT     **********CNVC0378
ISN 0349          JPYJPH(LZ)=1                                                        CNVC0379
ISN 0350          DHZJP1=DZ(LZ)                                                       CNVC0380
ISN 0351          IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                              CNVC0381
ISN 0353          IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                                  CNVC0382
ISN 0355          DHZJPH=DHZIAJ(LZ)+CCYJPH*(DHZJP1-DHZIAJ(LZ))                        CNVC0383
ISN 0356          ARYJPH=DXJPH*DHZJPH                                                 CNVC0384
ISN 0357          VMYJP1=VLY(JMYJP1,LZ)                                               CNVC0385
ISN 0358          VLYJPH=VPYIAJ(LZ)+CCYJPH*(VMYJP1-VPYIAJ(LZ))                        CNVC0386
ISN 0359          GFYJPH(LZ)=ARYJPH*VLYJPH                                            CNVC0387
ISN 0360          VMXJP1=VLX(IMXJP1,LZ)                                               CNVC0388
ISN 0361          VPXJP1=VLX(IPXJP1,LZ)                                               CNVC0389
               C********** CALCULATE VLZJP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJJP1 *CNVC0390
ISN 0362          GWZLMH=GWZLZH(IJJP1,LZ)                                             CNVC0391
ISN 0363          IF(LZ.EQ.NZ)GO TO 2266                                              CNVC0392
ISN 0365          GWZLPH=GWZLZH(IJJP1,LZ+1)                                           CNVC0393
ISN 0366          GO TO 2268                                                          CNVC0394
ISN 0367       2266 CONTINUE                                                          CNVC0395
ISN 0368          GWZLPH=ARHJP1*FNCHSR(IJJP1)                                         CNVC0396
ISN 0369       2268 CONTINUE                                                          CNVC0397
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ISN 0370          GWZJP1=0.5*(GWZLMH+GWZLPH)                                  CNVC0398
ISN 0371          VLZJP1=GWZJP1/ARHJP1                                        CNVC0399
                 C********** UPWIND DIFFERENCING ON ARYJPH SURFACE            **********CNVC0400
ISN 0372          IF(INWIND.NE.0)GO TO 2273                                   CNVC0401
                 C********** SIMPLE UPWIND DIFFERENCING FORM                  **********CNVC0402
ISN 0374          IF(VLYJPH.LT.0.)GO TO 2271                                  CNVC0403
ISN 0376          VLYPYH(LZ)=VPYIAJ(LZ)                                       CNVC0404
ISN 0377          VMXPYH(LZ)=VMXIAJ(LZ)                                       CNVC0405
ISN 0378          VPXPYH(LZ)=VPXIAJ(LZ)                                       CNVC0406
ISN 0379          VLZPYH(LZ)=VLZIAJ(LZ)                                       CNVC0407
ISN 0380          GO TO 2279                                                  CNVC0408
ISN 0381       2271 CONTINUE                                                  CNVC0409
ISN 0382          VLYPYH(LZ)=VMYJP1                                           CNVC0410
ISN 0383          VMXPYH(LZ)=VMXJP1                                           CNVC0411
ISN 0384          VPXPYH(LZ)=VPXJP1                                           CNVC0412
ISN 0385          VLZPYH(LZ)=VLZJP1                                           CNVC0413
ISN 0386          GO TO 2279                                                  CNVC0414
ISN 0387       2273 CONTINUE                                                  CNVC0415
ISN 0388          RDYJP1=1.+(DYJP1/DYIAJ)                                     CNVC0416
ISN 0389          IF(INWIND.GT.0)GO TO 2275                                   CNVC0417
                 C********** WEIGHTED UPWIND DIFFERENCING FORM                **********CNVC0418
ISN 0391          DYRJPH=DYJPH/DYIAJ                                          CNVC0419
ISN 0392          IF(VLYJPH.LT.0.)GO TO 2274                                  CNVC0420
ISN 0394          CWYPYH=(1.-RYWIND*DYRJPH)/RDYJP1                            CNVC0421
ISN 0395          CWYJPH=CWYPYH                                               CNVC0422
ISN 0396          GO TO 2276                                                  CNVC0423
ISN 0397       2274 CONTINUE                                                  CNVC0424
ISN 0398          CWYPYH=(1.+RYWIND*DYRJPH)/RDYJP1                            CNVC0425
ISN 0399          CWYJPH=CWYPYH                                               CNVC0426
ISN 0400          GO TO 2276                                                  CNVC0427
ISN 0401       2275 CONTINUE                                                  CNVC0428
                 C********** TRANSPORTIVE UPWIND DIFFERENCING FORM            **********CNVC0429
ISN 0402          CWYJPH=VLYJPH*DT/DYIAJ                                      CNVC0430
ISN 0403          CWYPYH=(1.-4.*RYWIND*CWYJPH)/RDYJP1                         CNVC0431
ISN 0404          CWYJPH=(1.-2.*RYWIND*CWYJPH)/RDYJP1                         CNVC0432
ISN 0405       2276 CONTINUE                                                  CNVC0433
ISN 0406          VLYPYH(LZ)=VPYIAJ(LZ)+CWYPYH*(VMYJP1-VPYIAJ(LZ))            CNVC0434
ISN 0407          VMXPYH(LZ)=VMXIAJ(LZ)+CWYJPH*(VMXJP1-VMXIAJ(LZ))            CNVC0435
ISN 0408          VPXPYH(LZ)=VPXIAJ(LZ)+CWYJPH*(VPXJP1-VPXIAJ(LZ))            CNVC0436
ISN 0409          VLZPYH(LZ)=VLZIAJ(LZ)+CWYJPH*(VLZJP1-VLZIAJ(LZ))            CNVC0437
ISN 0410       2278 CONTINUE                                                  CNVC0438
ISN 0411       2279 CONTINUE                                                  CNVC0439
ISN 0412       2260 CONTINUE                                                  CNVC0440
ISN 0413          GO TO 2700                                                  CNVC0441
ISN 0414       2300 CONTINUE                                                  CNVC0442
                 C********** START BOUNDARY ELEMENT CALCULATIONS              **********CNVC0443
ISN 0415          IBND=INTEXT(IX,JY)                                          CNVC0444
                 C********** START IMH BOUNDARY CALCULATIONS                  **********CNVC0445
ISN 0416          IMHCON=IMHBND(IBND)+2                                       CNVC0446
ISN 0417          GO TO (2310,2340,2360,2360),IMHCON                          CNVC0447
ISN 0418       2310 CONTINUE                                                  CNVC0448
                 C********** INTERNAL IMH BOUNDARY                           **********CNVC0449
                 C********** IMH ENCLOSURE SURFACE CONVECTION TRANSPORT       **********CNVC0450
                 C********** TRANSFERRED FROM PREVIOUS IFH CALCULATIONS       ********** CNVC0451
ISN 0419          IJIM1=IJIXJY(IX-1,JY)                                       CNVC0452
ISN 0420          NLZIM1=NLZBOT(IJIM1)                                        CNVC0453
ISN 0421          DO 2320 LZ=NLZMIN,NZ                                        CNVC0454
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ISN 0422          IF(LZ.GE.NLZIM1)GO TO 2325                                    CNVC0455
                 C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********CNVC0456
                 C********** SHORELINE BOUNDARY                               **********CNVC0457
ISN 0424          IMXIMH(LZ)=0                                                  CNVC0458
ISN 0425          GFXIMH(LZ)=0.0                                                CNVC0459
ISN 0426          VLXMXH(LZ)=0.0                                                CNVC0460
ISN 0427          VMYMXH(LZ)=0.0                                                CNVC0461
ISN 0428          VPYMXH(LZ)=0.0                                                CNVC0462
ISN 0429          VLZMXH(LZ)=0.0                                                CNVC0463
ISN 0430          GO TO 2329                                                    CNVC0464
ISN 0431       2325 CONTINUE                                                    CNVC0465
                 C********** IMH SURFACE AT LZ LEVEL ADJASENT TO AN ELEMENT    **********CNVC0466
ISN 0432          IMXIMH(LZ)=1                                                  CNVC0467
ISN 0433          GFXIMH(LZ)=GFXTRX(JY,LZ)                                      CNVC0468
ISN 0434          VLXMXH(LZ)=VLXTRX(JY,LZ)                                      CNVC0469
ISN 0435          VMYMXH(LZ)=VMYTRX(JY,LZ)                                      CNVC0470
ISN 0436          VPYMXH(LZ)=VPYTRX(JY,LZ)                                      CNVC0471
ISN 0437          VLZMXH(LZ)=VLZTRX(JY,LZ)                                      CNVC0472
ISN 0438       2329 CONTINUE                                                    CNVC0473
ISN 0439       2320 CONTINUE                                                    CNVC0474
ISN 0440          GO TO 2400                                                    CNVC0475
ISN 0441       2340 CONTINUE                                                    CNVC0476
                 C********** SHORELINE IMH BOUNDARY                            **********CNVC0477
ISN 0442          DO 2350 LZ=NLZMIN,NZ                                          CNVC0478
ISN 0443          IMXIMH(LZ)=0                                                  CNVC0479
ISN 0444          GFXIMH(LZ)=0.0                                                CNVC0480
ISN 0445          VLXMXH(LZ)=0.0                                                CNVC0481
ISN 0446          VMYMXH(LZ)=0.0                                                CNVC0482
ISN 0447          VPYMXH(LZ)=0.0                                                CNVC0483
ISN 0448          VLZMXH(LZ)=0.0                                                CNVC0484
ISN 0449       2350 CONTINUE                                                    CNVC0485
ISN 0450          GO TO 2400                                                    CNVC0486
ISN 0451       2360 CONTINUE                                                    CNVC0487
                 C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS  *********CNVC0488
                 C********** FOR IMH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE *********CNVC0489
ISN 0452          ISPB=ISPBND(IBND)                                             CNVC0490
ISN 0453          HSRIMH=HSRBXH(ISPB)                                           CNVC0491
ISN 0454          DO 2370 LZ=NLZMIN,NZ                                          CNVC0492
ISN 0455          IMXIMH(LZ)=IVXBXH(ISPB,LZ)                                    CNVC0493
ISN 0456          GFXIMH(LZ)=GFXBXH(ISPB,LZ)                                    CNVC0494
ISN 0457          IF(IMXIMH(LZ).LT.1)GO TO 2376                                 CNVC0495
ISN 0459          DHZIMH=DHZIAJ(LZ)                                             CNVC0496
ISN 0460          IF(LZ.EQ.NZ)  DHZIMH=DHZIMH-HSRIAJ+HSRIMH                     CNVC0497
ISN 0462          ARXIMH=DYIMH*DHZIMH                                           CNVC0498
ISN 0463          VLXIMH=GFXIMH(LZ)/ARXIMH                                      CNVC0499
                 C********** UPWIND DIFFERENCING ON ARXIMH SURFACE              **********CNVC0500
ISN 0464          IF(VLXIMH.GT.0.)GO TO 2376                                    CNVC0501
                 C********** FOR OUTFLOWING OPEN REGION CONDITIONS             **********CNVC0502
                 C********** SIMPLE UPWIND DIFFERENCING FORM                   **********CNVC0503
ISN 0466          VLXMXH(LZ)=VMXIAJ(LZ)                                         CNVC0504
ISN 0467          VMYMXH(LZ)=VMYIAJ(LZ)                                         CNVC0505
ISN 0468          VPYMXH(LZ)=VPYIAJ(LZ)                                         CNVC0506
ISN 0469          VLZMXH(LZ)=VLZIAJ(LZ)                                         CNVC0507
ISN 0470          GO TO 2379                                                    CNVC0508
ISN 0471       2376 CONTINUE                                                    CNVC0509
ISN 0472          VLXMXH(LZ)=VLXBXH(ISPB,LZ)                                    CNVC0510
ISN 0473          VMYMXH(LZ)=VMYBXH(ISPB,LZ)                                    CNVC0511
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ISN 0474          VPYMXH(LZ)=VPYBXH(ISPB,LZ)                                    CNVC0512
ISN 0475          VLZMXH(LZ)=VLZBXH(ISPB,LZ)                                    CNVC0513
ISN 0476       2378 CONTINUE                                                    CNVC0514
ISN 0477       2379 CONTINUE                                                    CNVC0515
ISN 0478       2370 CONTINUE                                                    CNVC0516
ISN 0479       2400 CONTINUE                                                    CNVC0517
               C********** START IPH BOUNDARY CALCULATIONS          **********CNVC0518
ISN 0480          IPHCON=IPHBND(IBND)+2                                         CNVC0519
ISN 0481          GO TO (2410,2440,2460,2460),IPHCON                            CNVC0520
ISN 0482       2410 CONTINUE                                                    CNVC0521
               C********** INTERNAL IPH BOUNDARY                   **********CNVC0522
ISN 0483          IJIP1=IJIXJY(IX+1,JY)                                         CNVC0523
ISN 0484          IMXIP1=2*IJIP1-1                                              CNVC0524
ISN 0485          JMYIP1=IMXIP1                                                 CNVC0525
ISN 0486          JPYIP1=JMYIP1+1                                               CNVC0526
ISN 0487          IF(INTEXT(IX+1,JY).GT.0)GO TO 2412                            CNVC0527
               C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT  **********CNVC0528
ISN 0489          DXIP1=DX(IX+1)                                                CNVC0529
ISN 0490          DYIP1=DYIAJ                                                   CNVC0530
ISN 0491          ARHIP1=DXIP1*DYIP1                                            CNVC0531
ISN 0492          GO TO 2415                                                    CNVC0532
ISN 0493       2412 CONTINUE                                                    CNVC0533
               C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT  **********CNVC0534
ISN 0494          IBNIP1=INTEXT(IX+1,JY)                                        CNVC0535
ISN 0495          DXIP1=DXBND(IBNIP1)                                           CNVC0536
ISN 0496          ARHIP1=ARHBND(IBNIP1)                                         CNVC0537
ISN 0497       2415 CONTINUE                                                    CNVC0538
ISN 0498          DXIPH=0.5*(DXIAJ+DXIP1)                                       CNVC0539
ISN 0499          CDXIPH=0.5*DXIAJ/DXIPH                                        CNVC0540
ISN 0500          HSRIP1=HSR(IJIP1)                                             CNVC0541
ISN 0501          NLZIP1=NLZBOT(IJIP1)                                          CNVC0542
ISN 0502          DO 2420 LZ=NLZMIN,NZ                                          CNVC0543
ISN 0503          IF(LZ.GE.NLZIP1)GO TO 2422                                    CNVC0544
               C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM  **********CNVC0545
               C********** SHORELINE BOUNDARY                       **********CNVC0546
ISN 0505          IPXIPH(LZ)=0                                                  CNVC0547
ISN 0506          GFXIPH(LZ)=0.0                                                CNVC0548
ISN 0507          VLXPXH(LZ)=0.0                                                CNVC0549
ISN 0508          VMYPXH(LZ)=0.0                                                CNVC0550
ISN 0509          VPYPXH(LZ)=0.0                                                CNVC0551
ISN 0510          VLZPXH(LZ)=0.0                                                CNVC0552
ISN 0511          GO TO 2420                                                    CNVC0553
ISN 0512       2422 CONTINUE                                                    CNVC0554
               C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT **********CNVC0555
ISN 0513          IPXIPH(LZ)=1                                                  CNVC0556
ISN 0514          DHZIP1=DZ(LZ)                                                 CNVC0557
ISN 0515          IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                        CNVC0558
ISN 0517          IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                            CNVC0559
ISN 0519          DHZIPH=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))                  CNVC0560
ISN 0520          ARXIPH=DYIPH*DHZIPH                                           CNVC0561
ISN 0521          VMXIP1=VLX(IMXIP1,LZ)                                         CNVC0562
ISN 0522          VLXIPH=VPXIAJ(LZ)+CDXIPH*(VMXIP1-VPXIAJ(LZ))                  CNVC0563
ISN 0523          GFXIPH(LZ)=ARXIPH*VLXIPH                                      CNVC0564
ISN 0524          VMYIP1=VLY(JMYIP1,LZ)                                         CNVC0565
ISN 0525          VPYIP1=VLY(JPYIP1,LZ)                                         CNVC0566
               C********** CALCULATE VLZIP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJIP1 *CNVC0567
ISN 0526          GWZLMH=GWZLZM(IJIP1,LZ)                                       CNVC0568
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ISN 0527          IF(LZ.EQ.NZ)GO TO 2426                                        CNVC0569
ISN 0529          GWZLPH=GWZLZH(IJIP1,LZ+1)                                     CNVC0570
ISN 0530          GO TO 2428                                                    CNVC0571
ISN 0531       2426 CONTINUE                                                    CNVC0572
ISN 0532          GWZLPH=ARHIP1*FNCHSR(IJIP1)                                   CNVC0573
ISN 0533       2428 CONTINUE                                                    CNVC0574
ISN 0534          GWZIP1=0.5*(GWZLMH+GWZLPH)                                    CNVC0575
ISN 0535          VLZIP1=GWZIP1/ARHIP1                                          CNVC0576
                C********** UPWIND DIFFERENCING ON ARXIPH SURFACE               **********CNVC0577
ISN 0536          IF(INWIND.NE.0)GO TO 2433                                     CNVC0578
                C********** SIMPLE UPWIND DIFFERENCING FORM                     **********CNVC0579
ISN 0538          IF(VLXIPH.LT.0.)GO TO 2431                                    CNVC0580
ISN 0540          VLXPXH(LZ)=VPXIAJ(LZ)                                         CNVC0581
ISN 0541          VMYPXH(LZ)=VMYIAJ(LZ)                                         CNVC0582
ISN 0542          VPYPXH(LZ)=VPYIAJ(LZ)                                         CNVC0583
ISN 0543          VLZPXH(LZ)=VLZIAJ(LZ)                                         CNVC0584
ISN 0544          GO TO 2439                                                    CNVC0585
ISN 0545       2431 CONTINUE                                                    CNVC0586
ISN 0546          VLXPXH(LZ)=VMXIP1                                             CNVC0587
ISN 0547          VMYPXH(LZ)=VMYIP1                                             CNVC0588
ISN 0548          VPYPXH(LZ)=VPYIP1                                             CNVC0589
ISN 0549          VLZPXH(LZ)=VLZIP1                                             CNVC0590
ISN 0550       2432 CONTINUE                                                    CNVC0591
ISN 0551          GO TO 2439                                                    CNVC0592
ISN 0552       2433 CONTINUE                                                    CNVC0593
ISN 0553          RDXIP1=1.+(DXIP1/DXIAJ)                                       CNVC0594
ISN 0554          IF(INWIND.GT.0)GO TO 2435                                     CNVC0595
                C********** WEIGHTED UPWIND DIFFERENCING FORM                   **********CNVC0596
ISN 0556          DXRIPH=DXIPH/DXIAJ                                            CNVC0597
ISN 0557          IF(VLXIPH.LT.0.)GO TO 2434                                    CNVC0598
ISN 0559          CWXPXH=(1.-RXWIND*DXRIPH)/RDXIP1                              CNVC0599
ISN 0560          CWXIPH=CWXPXH                                                 CNVC0600
ISN 0561          GO TO 2436                                                    CNVC0601
ISN 0562       2434 CONTINUE                                                    CNVC0602
ISN 0563          CWXPXH=(1.+RXWIND*DXRIPH)/RDXIP1                              CNVC0603
ISN 0564          CWXIPH=CWXPXH                                                 CNVC0604
ISN 0565          GO TO 2436                                                    CNVC0605
ISN 0566       2435 CONTINUE                                                    CNVC0606
                C********** TRANSPORTIVE UPWIND DIFFERENCING FORM               **********CNVC0607
ISN 0567          CNXIPH=VLXIPH*DT/DXIAJ                                        CNVC0608
ISN 0568          CWXPXH=(1.-4.*RXWIND*CNXIPH)/RDXIP1                           CNVC0609
ISN 0569          CWXIPH=(1.-2.*RXWIND*CNXIPH)/RDXIP1                           CNVC0610
ISN 0570       2436 CONTINUE                                                    CNVC0611
ISN 0571          VLXPXH(LZ)=VPXIAJ(LZ)+CWXPXH*(VMXIP1-VPXIAJ(LZ))              CNVC0612
ISN 0572          VMYPXH(LZ)=VMYIAJ(LZ)+CWXIPH*(VMYIP1-VMYIAJ(LZ))              CNVC0613
ISN 0573          VPYPXH(LZ)=VPYIAJ(LZ)+CWXIPH*(VPYIP1-VPYIAJ(LZ))              CNVC0614
ISN 0574          VLZPXH(LZ)=VLZIAJ(LZ)+CWXIPH*(VLZIP1-VLZIAJ(LZ))              CNVC0615
ISN 0575       2438 CONTINUE                                                    CNVC0616
ISN 0576       2439 CONTINUE                                                    CNVC0617
ISN 0577       2420 CONTINUE                                                    CNVC0618
ISN 0578          GO TO 2500                                                    CNVC0619
ISN 0579       2440 CONTINUE                                                    CNVC0620
                C********** SHORELINE IPH BOUNDARY                             **********CNVC0621
ISN 0580          DO 2450 LZ=NLZMIN,NZ                                          CNVC0622
ISN 0581          IPXIPH(LZ)=0                                                  CNVC0623
ISN 0582          GFXIPH(LZ)=0.0                                                CNVC0624
ISN 0583          VLXPXH(LZ)=0.0                                                CNVC0625
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ISN 0584          VPYPXH(LZ)=0.0                                                CNVC0626
ISN 0585          VPYPXH(LZ)=0.0                                                CNVC0627
ISN 0586          VLZPXH(LZ)=0.0                                                CNVC0628
ISN 0587       2450 CONTINUE                                                    CNVC0629
ISN 0588          GO TO 2500                                                    CNVC0630
ISN 0589       2460 CONTINUE                                                    CNVC0631
               C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS  *********CNVC0632
               C********** FOR IPH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********CNVC0633
ISN 0590          ISPB=ISPBND(IBND)                                             CNVC0634
ISN 0591          HSRIPH=HSRBXH(ISPB)                                           CNVC0635
ISN 0592          DO 2470 LZ=NLZMIN,NZ                                          CNVC0636
ISN 0593          IPXIPH(LZ)=IVXBXH(ISPB,LZ)                                    CNVC0637
ISN 0594          GFXIPH(LZ)=GFXBXH(ISPB,LZ)                                    CNVC0638
ISN 0595          IF(IPXIPH(LZ).LT.1)GO TO 2476                                 CNVC0639
ISN 0597          DHZIPH=DHZIAJ(LZ)                                             CNVC0640
ISN 0598          IF(LZ.EQ.NZ)  DHZIPH=DHZIPH-HSRIAJ+HSRIPH                     CNVC0641
ISN 0600          ARXIPH=DYIPH*DHZIPH                                           CNVC0642
ISN 0601          VLXIPH=GFXIPH(LZ)/ARXIPH                                      CNVC0643
               C********** UPWIND DIFFERENCING ON ARXIPH SURFACE           **********CNVC0644
ISN 0602          IF(VLXIPH.LT.0.)GO TO 2476                                    CNVC0645
               C********** FOR OUTFLOWING OPEN REGION CONDITIONS           **********CNVC0646
               C********** SIMPLE UPWIND DIFFERENCING FORM                 **********CNVC0647
ISN 0604          VLXPXH(LZ)=VPXIAJ(LZ)                                         CNVC0648
ISN 0605          VPYPXH(LZ)=VMYIAJ(LZ)                                         CNVC0649
ISN 0606          VPYPXH(LZ)=VPYIAJ(LZ)                                         CNVC0650
ISN 0607          VLZPXH(LZ)=VLZIAJ(LZ)                                         CNVC0651
ISN 0608          GO TO 2479                                                    CNVC0652
ISN 0609       2476 CONTINUE                                                    CNVC0653
ISN 0610          VLXPXH(LZ)=VLXBXH(ISPB,LZ)                                    CNVC0654
ISN 0611          VPYPXH(LZ)=VMYBXH(ISPB,LZ)                                    CNVC0655
ISN 0612          VPYPXH(LZ)=VPYBXH(ISPB,LZ)                                    CNVC0656
ISN 0613          VLZPXH(LZ)=VLZBXH(ISPB,LZ)                                    CNVC0657
ISN 0614       2478 CONTINUE                                                    CNVC0658
ISN 0615       2479 CONTINUE                                                    CNVC0659
ISN 0616       2470 CONTINUE                                                    CNVC0660
ISN 0617       2500 CONTINUE                                                    CNVC0661
               C********** START JMH BOUNDARY CALCULATIONS              **********CNVC0662
ISN 0618          JMHCON=JMHBND(IBND)+2                                         CNVC0663
ISN 0619          GO TO (2510,2540,2560,2560),JMHCON                            CNVC0664
ISN 0620       2510 CONTINUE                                                    CNVC0665
               C********** INTERNAL JMH BOUNDARY                        **********CNVC0666
               C********** JMH ENCLOSURE SURFACE CONVECTION TRANSPORT    **********CNVC0667
               C********** TRANSFERRED FROM PREVIOUS JPH CALCULATIONS    **********CNVC0668
ISN 0621          IJJM1=IJIXJY(IX,JY-1)                                         CNVC0669
ISN 0622          NLZJM1=NLZBOT(IJJM1)                                          CNVC0670
ISN 0623          DO 2520 LZ=NLZMIN,NZ                                          CNVC0671
ISN 0624          IF(LZ.GE.NLZJM1)GO TO 2525                                    CNVC0672
               C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM      **********CNVC0673
               C********** SHORELINE BOUNDARY                           **********CNVC0674
ISN 0626          JMYJMH(LZ)=0                                                  CNVC0675
ISN 0627          GFYJMH(LZ)=0.0                                                CNVC0676
ISN 0628          VLYMYH(LZ)=0.0                                                CNVC0677
ISN 0629          VPXMYH(LZ)=0.0                                                CNVC0678
ISN 0630          VPXMYH(LZ)=0.0                                                CNVC0679
ISN 0631          VLZMYH(LZ)=0.0                                                CNVC0680
ISN 0632          GO TO 2529                                                    CNVC0681
ISN 0633       2525 CONTINUE                                                    CNVC0682
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                C********** JMH SURFACE AT LZ LEVEL ADJASENT TO AN ELEMENT     **********CNVC0683
ISN 0634              JPYJMH(LZ)=1                                                      CNVC0684
ISN 0635              GFYJMH(LZ)=GFYTRY(LZ)                                             CNVC0685
ISN 0636              VLYMYH(LZ)=VLYTRY(LZ)                                             CNVC0686
ISN 0637              VPXMYH(LZ)=VMXTRY(LZ)                                             CNVC0687
ISN 0638              VPXMYH(LZ)=VPXTRY(LZ)                                             CNVC0688
ISN 0639              VLZMYH(LZ)=VLZTRY(LZ)                                             CNVC0689
ISN 0640           2529 CONTINUE                                                        CNVC0690
ISN 0641           2520 CONTINUE                                                        CNVC0691
ISN 0642              GO TO 2600                                                        CNVC0692
ISN 0643           2540 CONTINUE                                                        CNVC0693
                C********** SHORELINE JMH BOUNDARY                                  **********CNVC0694
ISN 0644              DO 2550 LZ=NLZMIN,NZ                                              CNVC0695
ISN 0645              JPYJMH(LZ)=0                                                      CNVC0696
ISN 0646              GFYJMH(LZ)=0.0                                                    CNVC0697
ISN 0647              VLYMYH(LZ)=0.0                                                    CNVC0698
ISN 0648              VPXMYH(LZ)=0.0                                                    CNVC0699
ISN 0649              VPXMYH(LZ)=0.0                                                    CNVC0700
ISN 0650              VLZMYH(LZ)=0.0                                                    CNVC0701
ISN 0651           2550 CONTINUE                                                        CNVC0702
ISN 0652              GO TO 2600                                                        CNVC0703
ISN 0653           2560 CONTINUE                                                        CNVC0704
                C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS  *********CNVC0705
                C********** FOR JMH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE *********CNVC0706
ISN 0654              ISPB=ISPBND(IBND)                                                 CNVC0707
ISN 0655              HSRJMH=HSRBYH(ISPB)                                               CNVC0708
ISN 0656              DO 2570 LZ=NLZMIN,NZ                                              CNVC0709
ISN 0657              JPYJMH(LZ)=JVYBYH(ISPB,LZ)                                        CNVC0710
ISN 0658              GFYJMH(LZ)=GFYBYH(ISPB,LZ)                                        CNVC0711
ISN 0659              IF(JPYJMH(LZ).LT.1)GO TO 2576                                     CNVC0712
ISN 0661              DHZJMH=DHZIAJ(LZ)                                                 CNVC0713
ISN 0662              IF(LZ.EQ.NZ)  DHZJMH=DHZJMH-HSRIAJ+HSRJMH                         CNVC0714
ISN 0664              ARYJMH=DXJMH*DHZJMH                                               CNVC0715
ISN 0665              VLYJMH=GFYJMH(LZ)/ARYJMH                                          CNVC0716
                C********** UPWIND DIFFERENCING ON ARYJMH SURFACE                  **********CNVC0717
ISN 0666              IF(VLYJMH.GT.0.)GO TO 2576                                        CNVC0718
                C********** FOR OUTFLOWING OPEN REGION CONDITIONS                  **********CNVC0719
                C********** SIMPLE UPWIND DIFFERENCING FORM                        **********CNVC0720
ISN 0668              VLYMYH(LZ)=VMYIAJ(LZ)                                             CNVC0721
ISN 0669              VPXMYH(LZ)=VMXIAJ(LZ)                                             CNVC0722
ISN 0670              VPXMYH(LZ)=VPXIAJ(LZ)                                             CNVC0723
ISN 0671              VLZMYH(LZ)=VLZIAJ(LZ)                                             CNVC0724
ISN 0672              GO TO 2579                                                        CNVC0725
ISN 0673           2576 CONTINUE                                                        CNVC0726
ISN 0674              VLYMYH(LZ)=VLYBYH(ISPB,LZ)                                        CNVC0727
ISN 0675              VPXMYH(LZ)=VMXBYH(ISPB,LZ)                                        CNVC0728
ISN 0676              VPXMYH(LZ)=VPXBYH(ISPB,LZ)                                        CNVC0729
ISN 0677              VLZMYH(LZ)=VLZBYH(ISPB,LZ)                                        CNVC0730
ISN 0678           2578 CONTINUE                                                        CNVC0731
ISN 0679           2579 CONTINUE                                                        CNVC0732
ISN 0680           2570 CONTINUE                                                        CNVC0733
ISN 0681           2600 CONTINUE                                                        CNVC0734
                C********** START JPH BOUNDARY CALCULATIONS                       **********CNVC0735
ISN 0682              JPHCON=JPHBND(IBND)+2                                             CNVC0736
ISN 0683              GO TO (2610,2640,2660,2660),JPHCON                                CNVC0737
ISN 0684           2610 CONTINUE                                                        CNVC0738
                C********** INTERNAL JPH BOUNDARY                                  **********CNVC0739
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ISN 0685          IJJP1=IJIXJY(IX,JY+1)                                          CNVC0740
ISN 0686          JMYJP1=2*IJJP1-1                                               CNVC0741
ISN 0687          IMXJP1=JMYJP1                                                  CNVC0742
ISN 0688          IPXJP1=IMXJP1+1                                                CNVC0743
ISN 0689          IF(INTEXT(IX,JY+1).GT.0)GO TO 2612                             CNVC0744
                 C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT          **********CNVC0745
ISN 0691          DYJP1=DY(JY+1)                                                 CNVC0746
ISN 0692          DXJP1=DXIAJ                                                    CNVC0747
ISN 0693          ARHJP1=DXJP1*DYJP1                                             CNVC0748
ISN 0694          GO TO 2615                                                     CNVC0749
ISN 0695       2612 CONTINUE                                                     CNVC0750
                 C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT          **********CNVC0751
ISN 0696          IBNJP1=INTEXT(IX,JY+1)                                         CNVC0752
ISN 0697          DYJP1=DYBND(IBNJP1)                                            CNVC0753
ISN 0698          ARHJP1=ARHBND(IBNJP1)                                          CNVC0754
ISN 0699       2615 CONTINUE                                                     CNVC0755
ISN 0700          DYJPH=0.5*(DYIAJ+DYJP1)                                        CNVC0756
ISN 0701          CCYJPH=0.5*DYIAJ/DYJPH                                         CNVC0757
ISN 0702          HSRJP1=HSR(IJJP1)                                              CNVC0758
ISN 0703          NLZJP1=NLZBOT(IJJP1)                                           CNVC0759
ISN 0704          DO 2620 LZ=NLZMIN,NZ                                           CNVC0760
ISN 0705          IF(LZ.GE.NLZJP1)GO TO 2622                                     CNVC0761
                 C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM            **********CNVC0762
                 C********** SHORELINE BOUNDARY                                 **********CNVC0763
ISN 0707          JPYJPH(LZ)=0                                                   CNVC0764
ISN 0708          GFYJPH(LZ)=0.0                                                 CNVC0765
ISN 0709          VLYPYH(LZ)=0.0                                                 CNVC0766
ISN 0710          VPXPYH(LZ)=0.0                                                 CNVC0767
ISN 0711          VPXPYH(LZ)=0.0                                                 CNVC0768
ISN 0712          VLZPYH(LZ)=0.0                                                 CNVC0769
ISN 0713          GO TO 2620                                                     CNVC0770
ISN 0714       2622 CONTINUE                                                     CNVC0771
                 C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT     **********CNVC0772
ISN 0715          JPYJPH(LZ)=1                                                   CNVC0773
ISN 0716          DHZJP1=DZ(LZ)                                                  CNVC0774
ISN 0717          IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                         CNVC0775
ISN 0719          IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                             CNVC0776
ISN 0721          DHZJPH=DHZIAJ(LZ)+CCYJPH*(DHZJP1-DHZIAJ(LZ))                   CNVC0777
ISN 0722          APYJPH=DXJPH*DHZJPH                                            CNVC0778
ISN 0723          VMYJP1=VLY(JMYJP1,LZ)                                          CNVC0779
ISN 0724          VLYJPH=VPYIAJ(LZ)+CCYJPH*(VMYJP1-VPYIAJ(LZ))                   CNVC0780
ISN 0725          GFYJPH(LZ)=ARYJPH*VLYJPH                                       CNVC0781
ISN 0726          VPXJP1=VLX(IMXJP1,LZ)                                          CNVC0782
ISN 0727          VPXJP1=VLX(IPXJP1,LZ)                                          CNVC0783
ISN 0728       2624 CONTINUE                                                     CNVC0784
                 C********** CALCULATE VLZJP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJJP1 *CNVC0785
ISN 0729          GWZLMH=GWZLZH(IJJP1,LZ)                                        CNVC0786
ISN 0730          IF(LZ.EQ.NZ)GO TO 2626                                         CNVC0787
ISN 0732          GWZLPH=GWZLZH(IJJP1,LZ+1)                                      CNVC0788
ISN 0733          GO TO 2628                                                     CNVC0789
ISN 0734       2626 CONTINUE                                                     CNVC0790
ISN 0735          GWZLPH=ARHJP1*FNCHSR(IJJP1)                                    CNVC0791
ISN 0736       2628 CONTINUE                                                     CNVC0792
ISN 0737          GWZJP1=0.5*(GWZLMH+GWZLPH)                                     CNVC0793
ISN 0738          VLZJP1=GWZJP1/ARHJP1                                           CNVC0794
                 C********** UPWIND DIFFERENCING ON ARYJPH SURFACE               **********CNVC0795
ISN 0739          IF(INWIND.NE.0)GO TO 2633                                      CNVC0796
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                C********** SIMPLE UPWIND DIFFERENCING FORM                **********CNVC0797
ISN 0741              IF(VLYJPH.LT.0.)GO TO 2631                                  CNVC0798
ISN 0743              VLYPYH(LZ)=VPYIAJ(LZ)                                       CNVC0799
ISN 0744              VMXPYH(LZ)=VMXIAJ(LZ)                                       CNVC0800
ISN 0745              VPXPYH(LZ)=VPXIAJ(LZ)                                       CNVC0801
ISN 0746              VLZPYH(LZ)=VLZIAJ(LZ)                                       CNVC0802
ISN 0747             2631 CONTINUE                                                CNVC0803
ISN 0748              VLYPYH(LZ)=VMYJP1                                           CNVC0804
ISN 0749              VMXPYH(LZ)=VMXJP1                                           CNVC0805
ISN 0750              VPXPYH(LZ)=VPXJP1                                           CNVC0806
ISN 0751              VLZPYH(LZ)=VLZJP1                                           CNVC0807
ISN 0752             2632 CONTINUE                                                CNVC0808
ISN 0753              GO TO 2639                                                  CNVC0809
ISN 0754             2633 CONTINUE                                                CNVC0810
ISN 0755              RDYJP1=1.+(DYJP1/DYIAJ)                                     CNVC0811
ISN 0756              IF(INWIND.GT.0)GO TO 2635                                   CNVC0812
                C********** WEIGHTED UPWIND DIFFERENCING FORM                  **********CNVC0813
ISN 0758              DYRJPH=DYJPH/DYIAJ                                          CNVC0814
ISN 0759              IF(VLYJPH.LT.0.)GO TO 2634                                  CNVC0815
ISN 0761              CWYPYH=(1.-RYWIND*DYRJPH)/RDYJP1                            CNVC0816
ISN 0762              CWYJPH=CWYPYH                                               CNVC0817
ISN 0763              GO TO 2636                                                  CNVC0818
ISN 0764             2634 CONTINUE                                                CNVC0819
ISN 0765              CWYPYH=(1.+RYWIND*DYRJPH)/RDYJP1                            CNVC0820
ISN 0766              CWYJPH=CWYPYH                                               CNVC0821
ISN 0767              GO TO 2636                                                  CNVC0822
ISN 0768             2635 CONTINUE                                                CNVC0823
                C********** TRANSPORTIVE UPWIND DIFFERENCING FORM              **********CNVC0824
ISN 0769              CNYJPH=VLYJPH*DT/DYIAJ                                      CNVC0825
ISN 0770              CWYPYH=(1.-4.*RYWIND*CNYJPH)/RDYJP1                         CNVC0826
ISN 0771              CWYJPH=(1.-2.*RYWIND*CNYJPH)/RDYJP1                         CNVC0827
ISN 0772             2636 CONTINUE                                                CNVC0828
ISN 0773              VLYPYH(LZ)=VPYIAJ(LZ)+CWYPYH*(VMYJP1-VPYIAJ(LZ))            CNVC0829
ISN 0774              VMXPYH(LZ)=VMXIAJ(LZ)+CWYJPH*(VMXJP1-VMXIAJ(LZ))            CNVC0830
ISN 0775              VPXPYH(LZ)=VPXIAJ(LZ)+CWYJPH*(VPXJP1-VPXIAJ(LZ))            CNVC0831
ISN 0776              VLZPYH(LZ)=VLZIAJ(LZ)+CWYJPH*(VLZJP1-VLZIAJ(LZ))            CNVC0832
ISN 0777             2638 CONTINUE                                                CNVC0833
ISN 0778             2639 CONTINUE                                                CNVC0834
ISN 0779             2620 CONTINUE                                                CNVC0835
ISN 0780              GO TO 2700                                                  CNVC0836
ISN 0781             2640 CONTINUE                                                CNVC0837
                C********** SHORELINE JPH BOUNDARY                           **********CNVC0838
ISN 0782              DO 2650 LZ=NLZMIN,NZ                                        CNVC0839
ISN 0783              JPYJPH(LZ)=0                                                CNVC0840
ISN 0784              GFYJPH(LZ)=0.0                                              CNVC0841
ISN 0785              VLYPYH(LZ)=0.0                                              CNVC0842
ISN 0786              VMXPYH(LZ)=0.0                                              CNVC0843
ISN 0787              VPXPYH(LZ)=0.0                                              CNVC0844
ISN 0788              VLZPYH(LZ)=0.0                                              CNVC0845
ISN 0789             2650 CONTINUE                                                CNVC0846
ISN 0790              GO TO 2700                                                  CNVC0847
ISN 0791             2660 CONTINUE                                                CNVC0848
                C********** SPECIFIED FLOW CONDITIONS AND OPEN END CONDITIONS  *********CNVC0849
                C********** FOR JPH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE *********CNVC0850
ISN 0792              ISPB=ISPBND(IBND)                                           CNVC0851
ISN 0793              HSRJPH=HSRBYH(ISPB)                                         CNVC0852
ISN 0794              DO 2670 LZ=NLZMIN,NZ                                        CNVC0853
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ISN 0795            JPYJPH(LZ)=JVYBYH(ISPB,LZ)                                    CNVC0854
ISN 0796            GFYJPH(LZ)=GFYBYH(ISPB,LZ)                                    CNVC0855
ISN 0797            IF(JPYJPH(LZ).LT.1)GO TO 2676                                 CNVC0856
ISN 0799            DHZJPH=DHZIAJ(LZ)                                             CNVC0857
ISN 0800            IF(LZ.EQ.NZ)  DHZJPH=DHZJPH-HSRIAJ+HSRJPH                     CNVC0858
ISN 0802            ARYJPH=DXJPH*DHZJPH                                           CNVC0859
ISN 0803            VLYJPH=GFYJPH(LZ)/ARYJPH                                      CNVC0860
                   C********** UPWIND DIFFERENCING ON ARYJPH SURFACE          **********CNVC0861
ISN 0804            IF(VLYJPH.LT.0.)GO TO 2676                                    CNVC0862
                   C********** FOR OUTFLOWING OPEN REGION CONDITIONS           **********CNVC0863
                   C********** SIMPLE UPWIND DIFFERENCING FORM                 **********CNVC0864
ISN 0806            VLYPYH(LZ)=VPYIAJ(LZ)                                         CNVC0865
ISN 0807            VMXPYH(LZ)=VMXIAJ(LZ)                                         CNVC0866
ISN 0808            VPXPYH(LZ)=VPXIAJ(LZ)                                         CNVC0867
ISN 0809            VLZPYH(LZ)=VLZIAJ(LZ)                                         CNVC0868
ISN 0810            GO TO 2679                                                    CNVC0869
ISN 0811           2676 CONTINUE                                                  CNVC0870
ISN 0812            VLYPYH(LZ)=VLYBYH(ISPB,LZ)                                    CNVC0871
ISN 0813            VMXPYH(LZ)=VMXBYH(ISPB,LZ)                                    CNVC0872
ISN 0814            VPXPYH(LZ)=VPXBYH(ISPB,LZ)                                    CNVC0873
ISN 0815            VLZPYH(LZ)=VLZBYH(ISPB,LZ)                                    CNVC0874
ISN 0816           2678 CONTINUE                                                  CNVC0875
ISN 0817           2679 CONTINUE                                                  CNVC0876
ISN 0818           2670 CONTINUE                                                  CNVC0877
ISN 0819           2700 CONTINUE                                                  CNVC0878
ISN 0820            VLZPZH=0.0                                                    CNVC0879
ISN 0821            VMXPZH=0.0                                                    CNVC0880
ISN 0822            VPXPZH=0.0                                                    CNVC0881
ISN 0823            VMYPZH=0.0                                                    CNVC0882
ISN 0824            VPYPZH=0.0                                                    CNVC0883
                   C********** BOTTOM SEEPAGE TEMPERATURE AND SALINITY CONDITIONS ARE *****CNVC0884
                   C********** ASSUMED TO BE THE SAME AS IN THE DISCRETE ELEMENT **********CNVC0885
ISN 0825            GWZLPH=GWZLZH(IJ,NLZMIN)                                      CNVC0886
ISN 0826            DO 2800 LZ=NLZMIN,NZ                                          CNVC0887
ISN 0827            DHZIJL=DHZIAJ(LZ)                                             CNVC0888
ISN 0828            VLZMZH=VLZPZH                                                 CNVC0889
ISN 0829            VMXMZH=VMXPZH                                                 CNVC0890
ISN 0830            VPXMZH=VPXPZH                                                 CNVC0891
ISN 0831            VMYMZH=VMYPZH                                                 CNVC0892
ISN 0832            VPYMZH=VPYPZH                                                 CNVC0893
ISN 0833            GWZLMH=GWZLPH                                                 CNVC0894
ISN 0834            IF(LZ.LT.NZ)GO TO 2820                                        CNVC0895
ISN 0836            VLZPZH=0.0                                                    CNVC0896
ISN 0837            VMXPZH=0.0                                                    CNVC0897
ISN 0838            VPXPZH=0.0                                                    CNVC0898
ISN 0839            VMYPZH=0.0                                                    CNVC0899
ISN 0840            VPYPZH=0.0                                                    CNVC0900
ISN 0841            GWZLPH=0.0                                                    CNVC0901
ISN 0842            GO TO 2880                                                    CNVC0902
ISN 0843           2820 CONTINUE                                                  CNVC0903
ISN 0844            LZP1=LZ+1                                                     CNVC0904
ISN 0845            GWZLPH=GWZLZH(IJ,LZP1)                                        CNVC0905
                   C********** IN CALCULATING VLZLPH IN UPWIND DIFFERENCING  **********CNVC0906
                   C********** SIMPLE UPWIND DIFFERENCING FORM IS USED FOR WATER DENSITY **CNVC0907
                   C********** ONLY AS AN APPROXIMATION WITH SMALL SECOND ORDER ERROR *****CNVC0908
ISN 0846            IF(GWZLPH.LT.0.)GO TO 2822                                    CNVC0909
ISN 0848            GO TO 2825                                                    CNVC0910
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ISN 0849      2822 CONTINUE                                                    CNVC0911
ISN 0850      2825 CONTINUE                                                    CNVC0912
ISN 0851           VLZLPH=GWZLPH/ARHIAJ                                        CNVC0913
                C********** UPWIND DIFFERENCING ON ARZLPH=ARHIAJ SURFACE ********** CNVC0914
ISN 0852           IF(INWIND.NE.0)GO TO 2862                                   CNVC0915
                C********** SIMPLE UPWIND DIFFERENCING FORM ********** CNVC0916
ISN 0854           IF(VLZLPH.LT.0.)GO TO 2861                                  CNVC0917
ISN 0856           VLZPZH=VLZIAJ(LZ)                                           CNVC0918
ISN 0857           VPXPZH=VMXIAJ(LZ)                                           CNVC0919
ISN 0858           VPXPZH=VPXIAJ(LZ)                                           CNVC0920
ISN 0859           VPYPZH=VMYIAJ(LZ)                                           CNVC0921
ISN 0860           VPYPZH=VPYIAJ(LZ)                                           CNVC0922
ISN 0861           GO TO 2880                                                  CNVC0923
ISN 0862      2861 CONTINUE                                                    CNVC0924
ISN 0863           VLZPZH=VLZIAJ(LZP1)                                         CNVC0925
ISN 0864           VPXPZH=VMXIAJ(LZP1)                                         CNVC0926
ISN 0865           VPXPZH=VPXIAJ(LZP1)                                         CNVC0927
ISN 0866           VPYPZH=VMYIAJ(LZP1)                                         CNVC0928
ISN 0867           VPYPZH=VPYIAJ(LZP1)                                         CNVC0929
ISN 0868      2862 CONTINUE                                                    CNVC0930
ISN 0869           DHZLP1=DHZIAJ(LZP1)                                         CNVC0931
ISN 0870           DHZLPH=0.5*(DHZIJL+DHZLP1)                                  CNVC0932
ISN 0871           RDZLP1=1./(DHZLP1/DHZIJL)                                   CNVC0933
ISN 0872           IF(INWIND.GT.0)GO TO 2864                                   CNVC0934
                C********** WEIGHTED UPWIND DIFFERENCING FORM ********** CNVC0935
ISN 0874           DZRLPH=DHZLPH/DHZIJL                                        CNVC0936
ISN 0875           IF(VLZLPH.LT.0.)GO TO 2863                                  CNVC0937
ISN 0877           CWZLPH=(1.-RZWIND*DZRLPH)/RDZLP1                            CNVC0938
ISN 0878           GO TO 2866                                                  CNVC0939
ISN 0879      2863 CONTINUE                                                    CNVC0940
ISN 0880           CWZLPH=(1.+RZWIND*DZRLPH)/RDZLP1                            CNVC0941
ISN 0881           GO TO 2866                                                  CNVC0942
ISN 0882      2864 CONTINUE                                                    CNVC0943
                C********** TRANSPORTIVE UPWIND DIFFERENCING FORM ********** CNVC0944
ISN 0883           CNZLPH=VLZLPH*DT/DHZIJL                                     CNVC0945
ISN 0884           CWZLPH=(1.-2.*RZWIND*CNZLPH)/RDZLP1                         CNVC0946
ISN 0885      2866 CONTINUE                                                    CNVC0947
ISN 0886           VLZPZH=VLZIAJ(LZ)+CWZLPH*(VLZIAJ(LZP1)-VLZIAJ(LZ))          CNVC0948
ISN 0887           VPXPZH=VMXIAJ(LZ)+CWZLPH*(VMXIAJ(LZP1)-VMXIAJ(LZ))          CNVC0949
ISN 0888           VPXPZH=VPXIAJ(LZ)+CWZLPH*(VPXIAJ(LZP1)-VPXIAJ(LZ))          CNVC0950
ISN 0889           VMYPZH=VMYIAJ(LZ)+CWZLPH*(VMYIAJ(LZP1)-VMYIAJ(LZ))          CNVC0951
ISN 0890           VPYPZH=VPYIAJ(LZ)+CWZLPH*(VPYIAJ(LZP1)-VPYIAJ(LZ))          CNVC0952
ISN 0891      2880 CONTINUE                                                    CNVC0953
ISN 0892           GWXIMH=GFXIMH(LZ)                                           CNVC0954
ISN 0893           GWXIAJ=GFXIAJ(LZ)                                           CNVC0955
ISN 0894           GWXIPH=GFXIPH(LZ)                                           CNVC0956
ISN 0895           GWYJMH=GFYJMH(LZ)                                           CNVC0957
ISN 0896           GWYIAJ=GFYIAJ(LZ)                                           CNVC0958
ISN 0897           GWYJPH=GFYJPH(LZ)                                           CNVC0959
ISN 0898           GFZLMH=GWZLMH                                               CNVC0960
ISN 0899           GFZLPH=GWZLPH                                               CNVC0961
ISN 0900           IF(IMXIMH(LZ).LT.1)GO TO 2882                               CNVC0962
ISN 0902           DNMVMX=ARHIMX*DHZIJL                                        CNVC0963
ISN 0903           FNCVLX(IMX,LZ)=FNCVLX(IMX,LZ)                               CNVC0964
                  1              +(1./DNMVMX)*(GWXIMH*(VLXMXH(LZ)-VMXIAJ(LZ))  CNVC0965
                  2                           -GWXIAJ*(VLXHXH(LZ)-VMXIAJ(LZ))  CNVC0966
                  3                        +0.5*(GWYJMH*(VMXMYH(LZ)-VMXIAJ(LZ))  CNVC0967
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                4                              -GWYJPH*(VMXPYH(LZ)-VMXIAJ(LZ)))      CNVC0968
                5                          +0.5*(GWZLMH*(VMXMZH-VMXIAJ(LZ))       CNVC0969
                6                              -GWZLPH*(VMXPZH-VMXIAJ(LZ)))       CNVC0970
ISN 0904      2882 CONTINUE                                                        CNVC0971
ISN 0905           IF(IPXIPH(LZ).LT.1)GO TO 2884                                   CNVC0972
ISN 0907           DNMVPX=ARHIPX*DHZIJL                                            CNVC0973
ISN 0908           FNCVLX(IPX,LZ)=FNCVLX(IPX,LZ)                                   CNVC0974
                1                +(1./DNMVPX)*(GWXIAJ*(VLXHXH(LZ)-VPXIAJ(LZ))      CNVC0975
                2                              -GWXIPH*(VLXPXH(LZ)-VPXIAJ(LZ))     CNVC0976
                3                          +0.5*(GWYJMH*(VPXMYH(LZ)-VPXIAJ(LZ))    CNVC0977
                4                              -GWYJPH*(VPXPYH(LZ)-VPXIAJ(LZ)))    CNVC0978
                5                          +0.5*(GWZLMH*(VPXMZH-VPXIAJ(LZ))        CNVC0979
                6                              -GWZLPH*(VPXPZH-VPXIAJ(LZ)))        CNVC0980
ISN 0909      2884 CONTINUE                                                        CNVC0981
ISN 0910           IF(JMYJMH(LZ).LT.1)GO TO 2886                                   CNVC0982
ISN 0912           DNMVMY=ARHJMY*DHZIJL                                            CNVC0983
ISN 0913           FNCVLY(JMY,LZ)=FNCVLY(JMY,LZ)                                   CNVC0984
                1                +(1./DNMVMY)*(GWYJMH*(VLYMYH(LZ)-VMYIAJ(LZ))      CNVC0985
                2                              -GWYIAJ*(VLYHYH(LZ)-VMYIAJ(LZ))     CNVC0986
                3                          +0.5*(GWXIMH*(VMYMXH(LZ)-VMYIAJ(LZ))    CNVC0987
                4                              -GWXIPH*(VMYPXH(LZ)-VMYIAJ(LZ)))    CNVC0988
                5                          +0.5*(GWZLMH*(VMYMZH-VMYIAJ(LZ))        CNVC0989
                6                              -GWZLPH*(VMYPZH-VMYIAJ(LZ)))        CNVC0990
ISN 0914      2886 CONTINUE                                                        CNVC0991
ISN 0915           IF(JPYJPH(LZ).LT.1)GO TO 2888                                   CNVC0992
ISN 0917           DNMVPY=ARHJPY*DHZIJL                                            CNVC0993
ISN 0918           FNCVLY(JPY,LZ)=FNCVLY(JPY,LZ)                                   CNVC0994
                1                +(1./DNMVPY)*(GWYIAJ*(VLYHYH(LZ)-VPYIAJ(LZ))      CNVC0995
                2                              -GWYJPH*(VLYPYH(LZ)-VPYIAJ(LZ))     CNVC0996
                3                          +0.5*(GWXIMH*(VPYMXH(LZ)-VPYIAJ(LZ))    CNVC0997
                4                              -GWXIPH*(VPYPXH(LZ)-VPYIAJ(LZ)))    CNVC0998
                5                          +0.5*(GWZLMH*(VPYMZH-VPYIAJ(LZ))        CNVC0999
                6                              -GWZLPH*(VPYPZH-VPYIAJ(LZ)))        CNVC1000
ISN 0919      2888 CONTINUE                                                        CNVC1001
ISN 0920           DNMGWZ=ARHIAJ*DHZIJL                                            CNVC1002
ISN 0921           FNCGWZ(IJ,LZ)=FNCGWZ(IJ,LZ)                                     CNVC1003
                1                +(1./DNMGWZ)*(GWXIMH*(VLZMXH(LZ)-VLZIAJ(LZ))      CNVC1004
                2                              -GWXIPH*(VLZPXH(LZ)-VLZIAJ(LZ))     CNVC1005
                3                              +GWYJMH*(VLZMYH(LZ)-VLZIAJ(LZ))     CNVC1006
                4                              -GWYJPH*(VLZPYH(LZ)-VLZIAJ(LZ))     CNVC1007
                5                              +GWZLMH*(VLZMZH-VLZIAJ(LZ))         CNVC1008
                6                              -GWZLPH*(VLZPZH-VLZIAJ(LZ)))        CNVC1009
ISN 0922      2800 CONTINUE                                                        CNVC1010
ISN 0923           DO 2920 LZ=NLZMIN,NZ                                            CNVC1011
ISN 0924           GFXTRX(JY,LZ)=GFXIPH(LZ)                                        CNVC1012
ISN 0925           VLXTRX(JY,LZ)=VLXPXH(LZ)                                        CNVC1013
ISN 0926           VMYTRX(JY,LZ)=VMYPXH(LZ)                                        CNVC1014
ISN 0927           VPYTRX(JY,LZ)=VPYPXH(LZ)                                        CNVC1015
ISN 0928           VLZTRX(JY,LZ)=VLZPXH(LZ)                                        CNVC1016
ISN 0929           GFYTRY(LZ)=GFYJPH(LZ)                                           CNVC1017
ISN 0930           VLYTRY(LZ)=VLYPYH(LZ)                                           CNVC1018
ISN 0931           VMXTRY(LZ)=VMXPYH(LZ)                                           CNVC1019
ISN 0932           VPXTRY(LZ)=VPXPYH(LZ)                                           CNVC1020
ISN 0933           VLZTRY(LZ)=VLZPYH(LZ)                                           CNVC1021
ISN 0934      2920 CONTINUE                                                        CNVC1022
ISN 0935      2200 CONTINUE                                                        CNVC1023
ISN 0936      2100 CONTINUE                                                        CNVC1024
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   ISN 0937       9999 RETURN                                                    CNVC1025
   ISN 0938            END                                                       CNVC1026

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*      SOURCE STATEMENTS =      937 ,PROGRAM SIZE =     13456

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
  ISN 0002            SUBROUTINE TURBUL                                                  TRBL0001
  ISN 0003            COMMON/CONTRL/KRUN,T                                               TRBL0002
  ISN 0004            COMMON/CALLER/INCONV,INTURB,IBOUND,INCBOT,NPLANT,INCSUR            TRBL0003
  ISN 0005            COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,2),IJIXJY(18,12),       TRBL0004
                     1              NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)   TRBL0005
  ISN 0006            COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ(6),DHZBOT(136)        TRBL0006
  ISN 0007            COMMON/GEOBND/IXBND(49),JYBND(49),                                 TRBL0007
                     1              DXBND(49),DYBND(49),ARHBND(49)                     TRBL0008
  ISN 0008            COMMON/ENCBND/IMHBND(49),DYBIMH(49),IPHBND(49),DYBIPH(49),         TRBL0009
                     1              JMHBND(49),DXBJMH(49),JPHBND(49),DXBJPH(49)        TRBL0010
  ISN 0009            COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)                     TRBL0011
  ISN 0010            COMMON/VLXSOL/VLX(272,6),PREVLX(272,6),FNCVLX(272,6)               TRBL0012
  ISN 0011            COMMON/VLYSOL/VLY(272,6),PREVLY(272,6),FNCVLY(272,6)               TRBL0013
  ISN 0012            COMMON/FLOVLZ/GWZLZH(136,6),FNCGWZ(136,6)                          TRBL0014
  ISN 0013            COMMON/BNDCON/ISPBND(49),IXBSPB(17),JYBSPB(17),                    TRBL0015
                     1              HSRBXH(17),GFXBXH(17,6),IVXBXH(17,6),              TRBL0016
                     2              VLXBXH(17,6),VMYBXH(17,6),VPYBXH(17,6),            TRBL0017
                     3              VLZBXH(17,6),FPRBXH(17,6),                         TRBL0018
                     4              HSRBYH(17),GFYBYH(17,6),JVYBYH(17,6),              TRBL0019
                     5              VLYBYH(17,6),VMXBYH(17,6),VPXBYH(17,6),            TRBL0020
                     6              VLZBYH(17,6),FPRBYH(17,6)                          TRBL0021
  ISN 0014            DIMENSION GFXTRX(12,6),VLXTRX(12,6),VMYTRX(12,6),                  TRBL0022
                     1          VPYTRX(12,6),VLZTRX(12,6)                              TRBL0023
  ISN 0015            DIMENSION GFYTRY(6),VLYTRY(6),VPXTRY(6),                           TRBL0024
                     1          VPXTRY(6),VLZTRY(6)                                    TRBL0025
  ISN 0016            DIMENSION VLYTRX(12,6),GVXTRX(12,6),GMYTRX(12,6),                  TRBL0026
                     1          GPYTRX(12,6),GVZTRX(12,6),                             TRBL0027
                     2          VLXTRY(6),GVYTRY(6),GPXTRY(6),GPXTRY(6),               TRBL0028
                     3          GVZTRY(6)                                              TRBL0029
  ISN 0017            DIMENSION VLXIAJ(6),VMXIAJ(6),VPXIAJ(6),                           TRBL0030
                     1          VLXIMH(6),VLYIMH(6),VLZIMH(6),GVXIMH(6),GMYIMH(6),     TRBL0031
                     2          GPYIMH(6),GVZIMH(6),                                   TRBL0032
                     3          VLYIAJ(6),VMYIAJ(6),VPYIAJ(6),                         TRBL0033
                     4          VLYJMH(6),VLXJMH(6),VLZJMH(6),GVYJMH(6),GMXJMH(6),     TRBL0034
                     5          GPXJMH(6),GVZJMH(6)                                    TRBL0035
  ISN 0018            DIMENSION VLXIPH(6),GVXIPH(6),VLYIPH(6),VLZIPH(6),                 TRBL0036
                     1          GMYIPH(6),GPYIPH(6),GVZIPH(6),                         TRBL0037
                     2          VLYJPH(6),GVYJPH(6),VLXJPH(6),VLZJPH(6),               TRBL0038
                     3          GMXJPH(6),GPXJPH(6),GVZJPH(6)                          TRBL0039
  ISN 0019            DIMENSION DHZIAJ(6),VLZIAJ(6),GVXIAJ(6),GVYIAJ(6)                  TRBL0040
  ISN 0020            DIMENSION IMXIMH(6),IPXIPH(6),JMYJMH(6),JPYJPH(6)                  TRBL0041
  ISN 0021            IF(KRUN.GT.0)GO TO 2                                               TRBL0042
  ISN 0023            GRVTY=9.82                                                         TRBL0043
  ISN 0024            READ(23,1001)INPUT                                                 TRBL0044
  ISN 0025       1001 FORMAT(12X,2X,2X,2X,4X,6X,4X,I2)                                   TRBL0045
  ISN 0026            READ(23,1002)RWDREF                                                TRBL0046
  ISN 0027       1002 FORMAT(10X,E11.4)                                                  TRBL0047
  ISN 0028            IF(INTURB.LT.1)GO TO 9999                                          TRBL0048
  ISN 0030            READ(23,1002)VISREF                                                TRBL0049
  ISN 0031            READ(23,1003)EVLLOW,ETHMOL,ETHLOW,ESLREF,ESLLOW                    TRBL0050
  ISN 0032       1003 FORMAT(10X,E11.4,4X,2(1X,E11.4),4X,2(1X,E11.4))                    TRBL0051
  ISN 0033            READ(23,1004)NFR,BETAFR,REYTRB,INCRIC,EXPRIC                       TRBL0052
  ISN 0034       1004 FORMAT(10X,I3,2X,E11.4,2X,E11.4,4X,I2,2X,E11.4)                    TRBL0053
  ISN 0035            IF(INPUT.LT.1)GO TO 1099                                           TRBL0054
  ISN 0037            WRITE(3,1012)RWDREF                                                TRBL0055
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ISN 0038      1012 FORMAT(1X,8HRWDREF =,1X,E11.4)                                    TRBL0056
ISN 0039           IF(INTURB.LT.1)GO TO 9999                                         TRBL0057
ISN 0041           WRITE(3,1013)VISREF                                               TRBL0058
ISN 0042      1013 FORMAT(1X,8HVISREF =,1X,E11.4)                                    TRBL0059
ISN 0043           WRITE(3,1014)EVLLOW,ETHMOL,ETHLOW,ESLREF,ESLLOW                   TRBL0060
ISN 0044      1014 FORMAT(1X,8HEVLLOW =,1X,E11.4,2X,8HETHMOL =,1X,E11.4,             TRBL0061
                  1       2X,8HETHLOW =,1X,E11.4,2X,8HESLREF =,1X,E11.4,          TRBL0062
                  2       2X,8HESLLOW =,1X,E11.4)                                 TRBL0063
ISN 0045           WRITE(3,1016)NFR,BETAFR,REYTRB                                    TRBL0064
ISN 0046      1016 FORMAT(1X,5HNFR =,1X,I3,2X,8HBETAFR =,1X,E11.4,2X,8HREYTRB =,     TRBL0065
                  1       1X,E11.4)                                               TRBL0066
ISN 0047      1099 CONTINUE                                                         TRBL0067
ISN 0048           IF(EVLLOW.LT.VISREF)  EVLLOW=VISREF                             TRBL0068
ISN 0050           IF(ETHLOW.LT.ETHMOL)  ETHLOW=ETHMOL                             TRBL0069
ISN 0052           IF(ESLLOW.LT.ESLREF)  ESLLOW=ESLREF                             TRBL0070
ISN 0054           PN=NFR                                                          TRBL0071
ISN 0055           PNM1=NFR-1                                                      TRBL0072
ISN 0056           PNP1=NFR+1                                                      TRBL0073
ISN 0057           P22N=2./PNP1                                                    TRBL0074
ISN 0058           P2NN=(2.*PN)/PNP1                                               TRBL0075
ISN 0059           PNFR=PNM1/PNP1                                                  TRBL0076
ISN 0060           CFRNFR=((PNP1/PN)**P22N)*(BETAFR**P2NN)                         TRBL0077
ISN 0061           DO 1100 JY=1,NY                                                 TRBL0078
ISN 0062           DO 1110 LZ=1,NZ                                                 TRBL0079
ISN 0063           VLXTRX(JY,LZ)=0.0                                               TRBL0080
ISN 0064           VLYTRX(JY,LZ)=0.0                                               TRBL0081
ISN 0065           VLZTRX(JY,LZ)=0.0                                               TRBL0082
ISN 0066           GVXTRX(JY,LZ)=0.0                                               TRBL0083
ISN 0067           GMYTRX(JY,LZ)=0.0                                               TRBL0084
ISN 0068           GFYTRX(JY,LZ)=0.0                                               TRBL0085
ISN 0069           GVZTRX(JY,LZ)=0.0                                               TRBL0086
ISN 0070           VLYTRY(LZ)=0.0                                                  TRBL0087
ISN 0071           VLXTRY(LZ)=0.0                                                  TRBL0088
ISN 0072           VLZTRY(LZ)=0.0                                                  TRBL0089
ISN 0073           GVYTRY(LZ)=0.0                                                  TRBL0090
ISN 0074           GMXTRY(LZ)=0.0                                                  TRBL0091
ISN 0075           GFXTRY(LZ)=0.0                                                  TRBL0092
ISN 0076           GVZTRY(LZ)=0.0                                                  TRBL0093
ISN 0077      1110 CONTINUE                                                         TRBL0094
ISN 0078      1100 CONTINUE                                                         TRBL0095
ISN 0079           GO TO 9999                                                      TRBL0096
ISN 0080         2 CONTINUE                                                        TRBL0097
ISN 0081           DO 2100 IX=1,NX                                                 TRBL0098
ISN 0082           DO 2200 JY=1,NY                                                 TRBL0099
ISN 0083           IF(INTEXT(IX,JY).EQ.0)GO TO 2200                                TRBL0100
ISN 0085           IJ=IJIXJY(IX,JY)                                                TRBL0101
ISN 0086           IMX=2*IJ-1                                                      TRBL0102
ISN 0087           IPX=IMX+1                                                       TRBL0103
ISN 0088           JMY=IMX                                                         TRBL0104
ISN 0089           JPY=IPX                                                         TRBL0105
ISN 0090           IF(INTEXT(IX,JY).GT.0)GO TO 2105                                TRBL0106
                C********** INTERNAL DISCRETE ELEMENT GEOMETRY CONDITIONS     **********TRBL0107
ISN 0092           DXIAJ=DX(IX)                                                    TRBL0108
ISN 0093           DYIAJ=DY(JY)                                                    TRBL0109
ISN 0094           ARHIAJ=DXIAJ*DYIAJ                                              TRBL0110
ISN 0095           DYIPH=DYIAJ                                                     TRBL0111
ISN 0096           DXJPH=DXIAJ                                                     TRBL0112
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ISN 0097          DXIMX=0.5*DXIAJ                                                  TRBL0113
ISN 0098          DYIMX=DYIAJ                                                      TRBL0114
ISN 0099          ARHIMX=DXIMX*DYIMX                                               TRBL0115
ISN 0100          DXIPX=0.5*DXIAJ                                                  TRBL0116
ISN 0101          DYIPX=DYIAJ                                                      TRBL0117
ISN 0102          ARHIPX=DXIPX*DYIPX                                               TRBL0118
ISN 0103          DYJMY=0.5*DYIAJ                                                  TRBL0119
ISN 0104          DXJMY=DXIAJ                                                      TRBL0120
ISN 0105          ARHJMY=DXJMY*DYJMY                                               TRBL0121
ISN 0106          DYJPY=0.5*DYIAJ                                                  TRBL0122
ISN 0107          DXJPY=DXIAJ                                                      TRBL0123
ISN 0108          ARHJPY=DXJPY*DYJPY                                               TRBL0124
ISN 0109          GO TO 2109                                                       TRBL0125
ISN 0110       2105 CONTINUE                                                       TRBL0126
               C********** BOUNDARY DISCETE ELEMENT GEOMETRY CONDITIONS ********** TRBL0127
ISN 0111          IBND=INTEXT(IX,JY)                                               TRBL0128
ISN 0112          DXIAJ=DXBND(IBND)                                                TRBL0129
ISN 0113          DYIAJ=DYBND(IBND)                                                TRBL0130
ISN 0114          ARHIAJ=ARHBND(IBND)                                              TRBL0131
ISN 0115          DYIMH=DYBIMH(IBND)                                               TRBL0132
ISN 0116          DYIPH=DYBIPH(IBND)                                               TRBL0133
ISN 0117          DXJMH=DXBJMH(IBND)                                               TRBL0134
ISN 0118          DXJPH=DXBJPH(IBND)                                               TRBL0135
ISN 0119          DXIMX=0.5*DXIAJ                                                  TRBL0136
ISN 0120          DYIMX=0.5*(DYIAJ+DYIMH)                                          TRBL0137
ISN 0121          ARHIMX=DXIMX*DYIMX                                               TRBL0138
ISN 0122          DXIPX=0.5*DXIAJ                                                  TRBL0139
ISN 0123          DYIPX=0.5*(DYIAJ+DYIPH)                                          TRBL0140
ISN 0124          ARHIPX=DXIPX*DYIPX                                               TRBL0141
ISN 0125          DYJMY=0.5*DYIAJ                                                  TRBL0142
ISN 0126          DXJMY=0.5*(DXIAJ+DXJMH)                                          TRBL0143
ISN 0127          ARHJMY=DXJMY*DYJMY                                               TRBL0144
ISN 0128          DYJPY=0.5*DYIAJ                                                  TRBL0145
ISN 0129          DXJPY=0.5*(DXIAJ+DXJPH)                                          TRBL0146
ISN 0130          ARHJPY=DXJPY*DYJPY                                               TRBL0147
ISN 0131       2109 CONTINUE                                                       TRBL0148
ISN 0132          HSRIAJ=HSR(IJ)                                                   TRBL0149
ISN 0133          NLZMIN=NLZBOT(IJ)                                                TRBL0150
ISN 0134          DO 2110 LZ=NLZMIN,NZ                                             TRBL0151
ISN 0135          DHZDUM=DZ(LZ)                                                    TRBL0152
ISN 0136          IF(LZ.EQ.NLZMIN)  DHZDUM=DHZBOT(IJ)                              TRBL0153
ISN 0138          IF(LZ.EQ.NZ)  DHZDUM=DHZDUM+HSRIAJ                               TRBL0154
ISN 0140          DHZIAJ(LZ)=DHZDUM                                                TRBL0155
ISN 0141          ARXIAJ=DYIAJ*DHZDUM                                              TRBL0156
ISN 0142          ARYIAJ=DXIAJ*DHZDUM                                              TRBL0157
ISN 0143          VMXIAJ(LZ)=VLX(IMX,LZ)                                           TRBL0158
ISN 0144          VPXIAJ(LZ)=VLX(IPX,LZ)                                           TRBL0159
ISN 0145          VMYIAJ(LZ)=VLY(JMY,LZ)                                           TRBL0160
ISN 0146          VPYIAJ(LZ)=VLY(JPY,LZ)                                           TRBL0161
               C********** CALCULATE VLXIAJ(LZ) AND VLYIAJ(LZ) ********** TRBL0162
ISN 0147          VLXIAJ(LZ)=0.5*(VMXIAJ(LZ)+VPXIAJ(LZ))                          TRBL0163
ISN 0148          VLYIAJ(LZ)=0.5*(VMYIAJ(LZ)+VPYIAJ(LZ))                          TRBL0164
               C********** CALCULATE VLZIAJ(LZ) ********** TRBL0165
ISN 0149          GWZLMH=GWZLZH(IJ,LZ)                                             TRBL0166
ISN 0150          IF(LZ.EQ.NZ)GO TO 2112                                           TRBL0167
ISN 0152          GWZLPH=GWZLZH(IJ,LZ+1)                                           TRBL0168
ISN 0153          GO TO 2115                                                       TRBL0169
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ISN 0154      2112 CONTINUE                                                    TRBL0170
ISN 0155           GWZLPH=ARHIAJ*FNCHSR(IJ)                                    TRBL0171
ISN 0156      2115 CONTINUE                                                    TRBL0172
ISN 0157           GWZIAJ=0.5*(GWZLMH+GWZLPH)                                  TRBL0173
ISN 0158           VLZIAJ(LZ)=GWZIAJ/ARHIAJ                                    TRBL0174
ISN 0159      2110 CONTINUE                                                    TRBL0175
ISN 0160           IF(INTEXT(IX,JY).GT.0)GO TO 2300                            TRBL0176
              C********** START INTERNAL ELEMENT CALCULATIONS                  **********TRBL0177
              C********** IAJ ENCLOSURE SURFACE TURBULENT TRANSPORT CALCULATED ********TRBL0178
              C********** IMH AND JMH ENCLOSURE SURFACE TURBULENT TRANSPORT    **********TRBL0179
              C********** TRANSFERRED FROM PREVIOUS IPH AND JPH CALCULATIONS ***********TRBL0180
              C********** FOR ENCLOSURE SURFACES THAT ARE NOT CLOSED           **********TRBL0181
              C********** BY THE SHORELINE BOUNDARY IN LZ LEVEL               **********TRBL0182
              C********** IPH AND JPH ENCLOSURE SURFACE TURBULENT             **********TRBL0183
              C********** TRANSPORT CALCULATED                               **********TRBL0184
              C********** START TRANSFERRING OR CALCULATING THE IMH           **********TRBL0185
              C********** TRANSPORT FOR AN INTERNAL ELEMENT                   **********TRBL0186
ISN 0162           IJIM1=IJIXJY(IX-1,JY)                                       TRBL0187
ISN 0163           NLZIM1=NLZBOT(IJIM1)                                        TRBL0188
ISN 0164           DO 2120 LZ=NLZMIN,NZ                                        TRBL0189
ISN 0165           IF(LZ.GE.NLZIM1)GO TO 2125                                  TRBL0190
              C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM            **********TRBL0191
              C********** SHORELINE BOUNDARY                                  **********TRBL0192
ISN 0167           IMXIMH(LZ)=0                                                TRBL0193
ISN 0168           VLXIMH(LZ)=0.0                                              TRBL0194
ISN 0169           VLYIMH(LZ)=0.0                                              TRBL0195
ISN 0170           VLZIMH(LZ)=0.0                                              TRBL0196
ISN 0171           GVXIMH(LZ)=0.0                                              TRBL0197
              C********** TURBULENT SHEAR STRESS COMPONENTS ON IMH BOUNDARY ***********TRBL0198
              C********** SURFACE ARE CALCULATED AS GMYIMH AND GPYIMH          **********TRBL0199
              C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM      **********TRBL0200
              C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM     **********TRBL0201
              C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON        **********TRBL0202
              C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS    **********TRBL0203
              C********** IN THE ELEMENT                                      **********TRBL0204
ISN 0172           VISKIN=VISREF                                               TRBL0205
ISN 0173           EVLIMH=EVLLOW                                               TRBL0206
ISN 0174           VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                          TRBL0207
ISN 0175           IF(VFLSQR.LE.0.)GO TO 2124                                  TRBL0208
ISN 0177           VFLIAJ=SQRT(VFLSQR)                                         TRBL0209
ISN 0178           REYNUM=DXIMX*VFLIAJ/VISKIN                                  TRBL0210
ISN 0179           IF(REYNUM.LT.REYTRB)GO TO 2124                              TRBL0211
ISN 0181           ETRIMH=VISKIN*CFRNFR*(REYNUM**PNFR)                         TRBL0212
ISN 0182           EVLIMH=EVLIMH+ETRIMH                                        TRBL0213
ISN 0183      2124 CONTINUE                                                    TRBL0214
ISN 0184           DHZIMH=DHZIAJ(LZ)                                           TRBL0215
ISN 0185           ARXVMY=DYJMY*DHZIMH                                         TRBL0216
ISN 0186           ARXVPY=DYJPY*DHZIMH                                         TRBL0217
ISN 0187           ARXVLZ=DYIAJ*DHZIMH                                         TRBL0218
ISN 0188           GMYIMH(LZ)=ARXVMY*(-EVLIMH*VMYIAJ(LZ)/(0.5*DXJMY))          TRBL0219
ISN 0189           GPYIMH(LZ)=ARXVPY*(-EVLIMH*VPYIAJ(LZ)/(0.5*DXJPY))          TRBL0220
ISN 0190           GVZIMH(LZ)=ARXVLZ*(-EVLIMH*VLZIAJ(LZ)/DXIMX)                TRBL0221
ISN 0191           GO TO 2120                                                  TRBL0222
ISN 0192      2125 CONTINUE                                                    TRBL0223
              C********** IMH SURFACE AT LZ LEVEL ADJASENT TO AN ELEMENT      **********TRBL0224
ISN 0193           IMXIMH(LZ)=1                                                TRBL0225
ISN 0194           VLXIMH(LZ)=VLXTRX(JY,LZ)                                    TRBL0226
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ISN 0195          VLYJMH(LZ)=VLYTRX(JY,LZ)                                        TRBL0227
ISN 0196          VLZJMH(LZ)=VLZTRX(JY,LZ)                                        TRBL0228
ISN 0197          GVXJMH(LZ)=GVXTRX(JY,LZ)                                        TRBL0229
ISN 0198          GMYJMH(LZ)=GMYTRX(JY,LZ)                                        TRBL0230
ISN 0199          GPYJMH(LZ)=GPYTRX(JY,LZ)                                        TRBL0231
ISN 0200          GVZJMH(LZ)=GVZTRX(JY,LZ)                                        TRBL0232
ISN 0201       2120 CONTINUE                                                      TRBL0233
ISN 0202          IJJM1=IJIXJY(IX,JY-1)                                           TRBL0234
ISN 0203          NLZJM1=NLZBOT(IJJM1)                                            TRBL0235
ISN 0204          DO 2130 LZ=NLZMIN,NZ                                            TRBL0236
ISN 0205          IF(LZ.GE.NLZJM1)GO TO 2135                                      TRBL0237
                  C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM ********** TRBL0238
                  C********** SHORELINE BOUNDARY                                 ********** TRBL0239
ISN 0207          JMYJMH(LZ)=0                                                    TRBL0240
ISN 0208          VLYJMH(LZ)=0.0                                                  TRBL0241
ISN 0209          VLXJMH(LZ)=0.0                                                  TRBL0242
ISN 0210          VLZJMH(LZ)=0.0                                                  TRBL0243
ISN 0211          GVYJMH(LZ)=0.0                                                  TRBL0244
                  C********** TURBULENT SHEAR STRESS COMPONENTS ON THE JMH BOUNDARY ****** TRBL0245
                  C********** SURFACE ARE CALCULATED AS GMXJMH AND GPXJMH          ********** TRBL0246
                  C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM      ********** TRBL0247
                  C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM     ********** TRBL0248
                  C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON       ********** TRBL0249
                  C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS   ********** TRBL0250
                  C********** IN THE ELEMENT                                      ********** TRBL0251
ISN 0212          VISKIN=VISREF                                                   TRBL0252
ISN 0213          EVLJMH=EVLLOW                                                   TRBL0253
ISN 0214          VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                              TRBL0254
ISN 0215          IF(VFLSQR.LE.0.)GO TO 2134                                      TRBL0255
ISN 0217          VFLIAJ=SQRT(VFLSQR)                                             TRBL0256
ISN 0218          REYNUM=DYJMY*VFLIAJ/VISKIN                                      TRBL0257
ISN 0219          IF(REYNUM.LT.REYTRB)GO TO 2134                                  TRBL0258
ISN 0221          ETRJMH=VISKIN*CFRNFR*(REYNUM**PNFR)                             TRBL0259
ISN 0222          EVLJMH=EVLJMH+ETRJMH                                            TRBL0260
ISN 0223       2134 CONTINUE                                                      TRBL0261
ISN 0224          DHZJMH=DHZIAJ(LZ)                                               TRBL0262
ISN 0225          ARYVMX=DXIMX*DHZJMH                                             TRBL0263
ISN 0226          ARYVPX=DXIPX*DHZJMH                                             TRBL0264
ISN 0227          ARYVLZ=DXIAJ*DHZJMH                                             TRBL0265
ISN 0228          GMXJMH(LZ)=ARYVMX*(-EVLJMH*VMXIAJ(LZ)/(0.5*DYIMX))              TRBL0266
ISN 0229          GPXJMH(LZ)=ARYVPX*(-EVLJMH*VPXIAJ(LZ)/(0.5*DYIPX))              TRBL0267
ISN 0230          GVZJMH(LZ)=ARYVLZ*(-EVLJMH*VLZIAJ(LZ)/DYJMY)                    TRBL0268
ISN 0231          GO TO 2130                                                      TRBL0269
ISN 0232       2135 CONTINUE                                                      TRBL0270
                  C********** JMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT     ********** TRBL0271
ISN 0233          JMYJMH(LZ)=1                                                    TRBL0272
ISN 0234          VLYJMH(LZ)=VLYTRY(LZ)                                           TRBL0273
ISN 0235          VLXJMH(LZ)=VLXTRY(LZ)                                           TRBL0274
ISN 0236          VLZJMH(LZ)=VLZTRY(LZ)                                           TRBL0275
ISN 0237          GVYJMH(LZ)=GVYTRY(LZ)                                           TRBL0276
ISN 0238          GMXJMH(LZ)=GMXTRY(LZ)                                           TRBL0277
ISN 0239          GPXJMH(LZ)=GPXTRY(LZ)                                           TRBL0278
ISN 0240          GVZJMH(LZ)=GVZTRY(LZ)                                           TRBL0279
ISN 0241       2130 CONTINUE                                                      TRBL0280
                  C********** START CALCULATIONS FOR INTERNAL IPH SURFACE         ********** TRBL0281
ISN 0242          IJIP1=IJIXJY(IX+1,JY)                                           TRBL0282
ISN 0243          IMXIP1=2*IJIP1-1                                                TRBL0283
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ISN 0244          JPYIP1=IMXIP1                                                  TRBL0284
ISN 0245          JPYIP1=JMYIP1+1                                                TRBL0285
ISN 0246          IF(INTEXT(IX+1,JY).GT.0)GO TO 2212                             TRBL0286
                  C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT       **********TRBL0287
ISN 0248          DXIP1=DX(IX+1)                                                 TRBL0288
ISN 0249          DYIP1=DYIAJ                                                    TRBL0289
ISN 0250          ARHIP1=DXIP1*DYIP1                                             TRBL0290
ISN 0251          GO TO 2215                                                     TRBL0291
ISN 0252           2212 CONTINUE                                                 TRBL0292
                  C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT        **********TRBL0293
ISN 0253          IBNIP1=INTEXT(IX+1,JY)                                         TRBL0294
ISN 0254          DXIP1=DXBND(IBNIP1)                                            TRBL0295
ISN 0255          ARHIP1=ARHBND(IBNIP1)                                          TRBL0296
ISN 0256           2215 CONTINUE                                                 TRBL0297
ISN 0257          DXIPH=0.5*(DXIAJ+DXIP1)                                        TRBL0298
ISN 0258          CDXIPH=0.5*DXIAJ/DXIPH                                         TRBL0299
ISN 0259          HSRIP1=HSR(IJIP1)                                              TRBL0300
ISN 0260          NLZIP1=NDZBOT(IJIP1)                                           TRBL0301
ISN 0261          DO 2220 LZ=NLZMIN,NZ                                           TRBL0302
ISN 0262          IF(LZ.GE.NLZIP1)GO TO 2225                                     TRBL0303
                  C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********TRBL0304
                  C********** SHORELINE BOUNDARY                               **********TRBL0305
ISN 0264          IPXIPH(LZ)=0                                                   TRBL0306
ISN 0265          VLXIPH(LZ)=0.0                                                 TRBL0307
ISN 0266          VLYIPH(LZ)=0.0                                                 TRBL0308
ISN 0267          VLZIPH(LZ)=0.0                                                 TRBL0309
ISN 0268          GVXIPH(LZ)=0.0                                                 TRBL0310
                  C********** TURBULENT SHEAR STRESS COMPONENTS ON IPH BOUNDARY **********TRBL0311
                  C********** SURFACE ARE CALCULATED AS GMYIPH AND GPYIPH       **********TRBL0312
                  C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM   **********TRBL0313
                  C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0314
                  C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0315
                  C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL0316
                  C********** IN THE ELEMENT                                   **********TRBL0317
ISN 0269          VISKIN=VISREF                                                  TRBL0318
ISN 0270          EVLIPH=EVLLOW                                                  TRBL0319
ISN 0271          VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                             TRBL0320
ISN 0272          IF(VFLSQR.LE.0.)GO TO 2224                                     TRBL0321
ISN 0274          VFLIAJ=SQRT(VFLSQR)                                            TRBL0322
ISN 0275          REYNUM=DXIPX*VFLIAJ/VISKIN                                     TRBL0323
ISN 0276          IF(REYNUM.LT.REYTRB)GO TO 2224                                 TRBL0324
ISN 0278          ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFR)                            TRBL0325
ISN 0279          EVLIPH=EVLIPH+ETRIPH                                           TRBL0326
ISN 0280           2224 CONTINUE                                                 TRBL0327
ISN 0281          DHZIPH=DHZIAJ(LZ)                                              TRBL0328
ISN 0282          ARXVMY=DYJMY*DHZIPH                                            TRBL0329
ISN 0283          ARXVPY=DYJPY*DHZIPH                                            TRBL0330
ISN 0284          APXVLZ=DYIAJ*DHZIPH                                            TRBL0331
ISN 0285          GMYIPH(LZ)=ARXVMY*(EVLIPH*VMYIAJ(LZ)/(0.5*DXJMY))              TRBL0332
ISN 0286          GPYIPH(LZ)=ARXVPY*(EVLIPH*VPYIAJ(LZ)/(0.5*DXJPY))              TRBL0333
ISN 0287          GVZIPH(LZ)=ARXVLZ*(EVLIPH*VLZIAJ(LZ)/CXIPX)                    TRBL0334
ISN 0288          GO TO 2220                                                     TRBL0335
ISN 0289           2225 CONTINUE                                                 TRBL0336
                  C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********TRBL0337
                  C********** TURBULENT TRANSPORT RATES ON THE IPH SURFACE ARE  **********TRBL0338
                  C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL0339
                  C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0340
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                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0341
                C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL0342
                C********** COMPONENTS IN THE ADJACENT ELEMENTS               **********TRBL0343
ISN 0290              IFXIPH(LZ)=1                                                      TRBL0344
ISN 0291              DHZIP1=DZ(LZ)                                                     TRBL0345
ISN 0292              IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                            TRBL0346
ISN 0294              IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                                TRBL0347
ISN 0296              DHZIPH=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))                      TRBL0348
ISN 0297              ARXIPH=DYIPH*DHZIPH                                               TRBL0349
ISN 0298              VMXIP1=VLX(IMXIP1,LZ)                                             TRBL0350
ISN 0299              VLXIPH(LZ)=VPXIAJ(LZ)+CDXIPH*(VMXIP1-VPXIAJ(LZ))                  TRBL0351
ISN 0300              VMYIP1=VLY(JMYIP1,LZ)                                             TRBL0352
ISN 0301              VPYIP1=VLY(JPYIP1,LZ)                                             TRBL0353
ISN 0302              VLYIP1=0.5*(VMYIP1+VPYIP1)                                        TRBL0354
ISN 0303              VLYIPH(LZ)=VLYIAJ(LZ)+CDXIPH*(VLYIP1-VLYIAJ(LZ))                  TRBL0355
                C********** CALCULATE VLZIP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJIP1 *TRBL0356
ISN 0304              GWZLMH=GWZLZH(IJIP1,LZ)                                           TRBL0357
ISN 0305              IF(LZ.EQ.NZ)GO TO 2228                                            TRBL0358
ISN 0307              GWZLPH=GWZLZH(IJIP1,LZ+1)                                         TRBL0359
ISN 0308              GO TO 2230                                                        TRBL0360
ISN 0309           2228 CONTINUE                                                        TRBL0361
ISN 0310              GWZLPH=ARHIP1*FNCHSR(IJIP1)                                       TRBL0362
ISN 0311           2230 CONTINUE                                                        TRBL0363
ISN 0312              GWZIP1=0.5*(GWZLMH+GWZLPH)                                        TRBL0364
ISN 0313              VLZIP1=GWZIP1/ARHIP1                                              TRBL0365
ISN 0314              VLZIPH(LZ)=VLZIAJ(LZ)+CDXIPH*(VLZIP1-VLZIAJ(LZ))                  TRBL0366
ISN 0315              IF(DXIAJ.GT.DXIP1)GO TO 2232                                      TRBL0367
ISN 0317              DXHXH=DXIAJ                                                       TRBL0368
ISN 0318              CDXHXH=DXHXH/DXIPH                                                TRBL0369
ISN 0319              VLYMXH=VLYIAJ(LZ)                                                 TRBL0370
ISN 0320              VLZMXH=VLZIAJ(LZ)                                                 TRBL0371
ISN 0321              VLYPXH=VLYIAJ(LZ)+CDXHXH*(VLYIP1-VLYIAJ(LZ))                      TRBL0372
ISN 0322              VLZPXH=VLZIAJ(LZ)+CDXHXH*(VLZIP1-VLZIAJ(LZ))                      TRBL0373
ISN 0323              GO TO 2235                                                        TRBL0374
ISN 0324           2232 CONTINUE                                                        TRBL0375
ISN 0325              DXHXH=DXIP1                                                       TRBL0376
ISN 0326              CDXHXH=DXHXH/DXIPH                                                TRBL0377
ISN 0327              VLYMXH=VLYIP1-CDXHXH*(VLYIP1-VLYIAJ(LZ))                          TRBL0378
ISN 0328              VLZMXH=VLZIP1-CDXHXH*(VLZIP1-VLZIAJ(LZ))                          TRBL0379
ISN 0329              VLYPXH=VLYIP1                                                     TRBL0380
ISN 0330              VLZPXH=VLZIP1                                                     TRBL0381
ISN 0331           2235 CONTINUE                                                        TRBL0382
ISN 0332              VISKIN=VISREF                                                     TRBL0383
ISN 0333              EVLIPH=EVLLOW                                                     TRBL0384
ISN 0334              ETHIPH=ETHLOW                                                     TRBL0385
ISN 0335              ESLIPH=ESLLOW                                                     TRBL0386
ISN 0336              VFLSQR=(VLYPXH-VLYMXH)**2+(VLZPXH-VLZMXH)**2                      TRBL0387
ISN 0337              IF(VFLSQR.LE.0.)GO TO 2238                                        TRBL0388
ISN 0339              VFLHXH=SQRT(VFLSQR)                                               TRBL0389
ISN 0340              REYNUM=DXHXH*VFLHXH/VISKIN                                        TRBL0390
ISN 0341              IF(REYNUM.LT.REYTRB)GO TO 2238                                    TRBL0391
ISN 0343              ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFF)                               TRBL0392
ISN 0344              EVLIPH=EVLIPH+ETRIPH                                              TRBL0393
ISN 0345              ETHIPH=ETHIPH+ETRIPH                                              TRBL0394
ISN 0346              ESLIPH=ESLIPH+ETRIPH                                              TRBL0395
ISN 0347           2238 CONTINUE                                                        TRBL0396
ISN 0348              ARXVMY=DYJMY*DHZIPH                                               TRBL0397
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ISN 0349            APXVPY=DYJPY*DHZIPH                                          TRBL0398
ISN 0350            GVXIPH(LZ)=ARXIPH*(-EVLIPH                                   TRBL0399
                   1          *(VMXIP1-VPXIAJ(LZ))/(0.5*DXIPH))                  TRBL0400
ISN 0351            GMYIPH(LZ)=ARXVMY*(-EVLIPH                                   TRBL0401
                   1                  *(VMYIP1-VMYIAJ(LZ))/DXIPH)               TRBL0402
ISN 0352            GPYIPH(LZ)=ARXVPY*(-EVLIPH                                   TRBL0403
                   1                  *(VPYIP1-VPYIAJ(LZ))/DXIPH)               TRBL0404
ISN 0353            GVZIPH(LZ)=ARXIPH*(-EVLIPH                                   TRBL0405
                   1                  *(VLZIP1-VLZIAJ(LZ))/DXIPH)               TRBL0406
ISN 0354         2220 CONTINUE                                                   TRBL0407
                 C********** START CALCULATIONS FOR INTERNAL JPH SURFACE ********** TRBL0408
ISN 0355            IJJP1=IJIXJY(IX,JY+1)                                        TRBL0409
ISN 0356            JMYJP1=2*IJJP1-1                                             TRBL0410
ISN 0357            IPXJP1=JMYJP1                                                TRBL0411
ISN 0358            IPXJP1=IMXJP1+1                                              TRBL0412
ISN 0359            IF(INTEXT(IX,JY+1).GT.0)GO TO 2252                           TRBL0413
                 C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT ********** TRBL0414
ISN 0361            DYJP1=DY(JY+1)                                               TRBL0415
ISN 0362            DXJP1=DXIAJ                                                  TRBL0416
ISN 0363            ARHJP1=DXJP1*DYJP1                                           TRBL0417
ISN 0364            GO TO 2255                                                   TRBL0418
ISN 0365         2252 CONTINUE                                                   TRBL0419
                 C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT ********** TRBL0420
ISN 0366            IBNJP1=INTEXT(IX,JY+1)                                       TRBL0421
ISN 0367            DYJP1=DYBND(IBNJP1)                                          TRBL0422
ISN 0368            ARHJP1=ARHBND(IBNJP1)                                        TRBL0423
ISN 0369         2255 CONTINUE                                                   TRBL0424
ISN 0370            DYJPH=0.5*(DYIAJ+DYJP1)                                      TRBL0425
ISN 0371            CDYJPH=0.5*DYIAJ/DYJPH                                       TRBL0426
ISN 0372            HSRJP1=HSR(IJJP1)                                            TRBL0427
ISN 0373            NLZJP1=NLZBOT(IJJP1)                                         TRBL0428
ISN 0374            DO 2260 LZ=NLZMIN,NZ                                         TRBL0429
ISN 0375            IF(LZ.GE.NLZJP1)GO TO 2265                                   TRBL0430
                 C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM ********** TRBL0431
                 C********** SHORELINE BOUNDARY ********** TRBL0432
ISN 0377            JPYJPH(LZ)=0                                                 TRBL0433
ISN 0378            VLYJPH(LZ)=0.0                                               TRBL0434
ISN 0379            VLXJPH(LZ)=0.0                                               TRBL0435
ISN 0380            VLZJPH(LZ)=0.0                                               TRBL0436
ISN 0381            GVYJPH(LZ)=0.0                                               TRBL0437
                 C********** TURBULENT SHEAR STRESS COMPONENTS ON JPH BOUNDARY ********** TRBL0438
                 C********** SURFACE ARE CALCULATED AS GMXJPH AND GPXJPH ********** TRBL0439
                 C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM ********** TRBL0440
                 C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM ********** TRBL0441
                 C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON ********** TRBL0442
                 C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS ********** TRBL0443
                 C********** IN THE ELEMENT ********** TRBL0444
ISN 0382            VISKIN=VISREF                                                TRBL0445
ISN 0383            EVLJPH=EVLLOW                                                TRBL0446
ISN 0384            VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                           TRBL0447
ISN 0385            IF(VFLSQR.LE.0.)GO TO 2264                                   TRBL0448
ISN 0387            VFLIAJ=SQRT(VFLSQR)                                          TRBL0449
ISN 0388            REYNUM=DYJPY*VFLIAJ/VISKIN                                   TRBL0450
ISN 0389            IF(REYNUM.LT.REYTRB)GO TO 2264                               TRBL0451
ISN 0391            ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFR)                          TRBL0452
ISN 0392            EVLJPH=EVLJPH+ETRJPH                                         TRBL0453
ISN 0393         2264 CONTINUE                                                   TRBL0454
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ISN 0394          DHZJPH=DHZIAJ(LZ)                                                    TRBL0455
ISN 0395          ARYVMX=DXIMX*DHZJPH                                                  TRBL0456
ISN 0396          ARYVPX=DXIPX*DHZJPH                                                  TRBL0457
ISN 0397          ARYVLZ=DXIAJ*DHZJPH                                                  TRBL0458
ISN 0398          GMXJPH(LZ)=ARYVMX*(EVLJPH*VMXIAJ(LZ)/(0.5*DYIMX))                    TRBL0459
ISN 0399          GPXJPH(LZ)=ARYVPX*(EVLJPH*VPXIAJ(LZ)/(0.5*DYIPX))                    TRBL0460
ISN 0400          GVZJPH(LZ)=ARYVLZ*(EVLJPH*VLZIAJ(LZ)/DYJPY)                          TRBL0461
ISN 0401          GO TO 2260                                                           TRBL0462
ISN 0402       2265 CONTINUE                                                           TRBL0463
               C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********TRBL0464
               C********** TURBULENT TRANSPORT RATES ON THE JPH SURFACE ARE  **********TRBL0465
               C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL0466
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0467
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0468
               C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL0469
               C********** COMPONENTS IN THE ADJACENT ELEMENTS               **********TRBL0470
ISN 0403          JPYJPH(LZ)=1                                                         TRBL0471
ISN 0404          DHZJP1=DZ(LZ)                                                        TRBL0472
ISN 0405          IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                               TRBL0473
ISN 0407          IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                                   TRBL0474
ISN 0409          DHZJPH=DHZIAJ(LZ)+CDYJPH*(DHZJP1-DHZIAJ(LZ))                         TRBL0475
ISN 0410          ARYJPH=DXJPH*DHZJPH                                                  TRBL0476
ISN 0411          VMYJP1=VLY(JMYJP1,LZ)                                                TRBL0477
ISN 0412          VLYJPH(LZ)=VPYIAJ(LZ)+CDYJPH*(VMYJP1-VPYIAJ(LZ))                     TRBL0478
ISN 0413          VMXJP1=VLX(IMXJP1,LZ)                                                TRBL0479
ISN 0414          VPXJP1=VLX(IPXJP1,LZ)                                                TRBL0480
ISN 0415          VLXJP1=0.5*(VMXJP1+VPXJP1)                                           TRBL0481
ISN 0416          VLXJPH(LZ)=VLXIAJ(LZ)+CDYJPH*(VLXJP1-VLXIAJ(LZ))                     TRBL0482
               C********** CALCULATE VLZJP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJJP1 *TRBL0483
ISN 0417          GWZLMH=GWZLZH(IJJP1,LZ)                                              TRBL0484
ISN 0418          IF(LZ.EQ.NZ)GO TO 2268                                               TRBL0485
ISN 0420          GWZLPH=GWZLZH(IJJP1,LZ+1)                                            TRBL0486
ISN 0421          GO TO 2270                                                           TRBL0487
ISN 0422       2268 CONTINUE                                                           TRBL0488
ISN 0423          GWZLPH=ARHJP1*FNCHSR(IJJP1)                                          TRBL0489
ISN 0424       2270 CONTINUE                                                           TRBL0490
ISN 0425          GWZJP1=0.5*(GWZLMH+GWZLPH)                                           TRBL0491
ISN 0426          VLZJP1=GWZJP1/ARHJP1                                                 TRBL0492
ISN 0427          VLZJPH(LZ)=VLZIAJ(LZ)+CDYJPH*(VLZJP1-VLZIAJ(LZ))                     TRBL0493
ISN 0428          IF(DYIAJ.GT.DYJP1)GO TO 2272                                         TRBL0494
ISN 0430          DYHYH=DYIAJ                                                          TRBL0495
ISN 0431          CDYHYH=DYHYH/DYJPH                                                   TRBL0496
ISN 0432          VLXMYH=VLXIAJ(LZ)                                                    TRBL0497
ISN 0433          VLZMYH=VLZIAJ(LZ)                                                    TRBL0498
ISN 0434          VLXPYH=VLXIAJ(LZ)+CDYHYH*(VLXJP1-VLXIAJ(LZ))                         TRBL0499
ISN 0435          VLZPYH=VLZIAJ(LZ)+CDYHYH*(VLZJP1-VLZIAJ(LZ))                         TRBL0500
ISN 0436          GO TO 2275                                                           TRBL0501
ISN 0437       2272 CONTINUE                                                           TRBL0502
ISN 0438          DYHYH=DYJP1                                                          TRBL0503
ISN 0439          CDYHYH=DYHYH/DYJPH                                                   TRBL0504
ISN 0440          VLXMYH=VLXJP1-CDYHYH*(VLXJP1-VLXIAJ(LZ))                             TRBL0505
ISN 0441          VLZMYH=VLZJP1-CDYHYH*(VLZJP1-VLZIAJ(LZ))                             TRBL0506
ISN 0442          VLXPYH=VLXJP1                                                        TRBL0507
ISN 0443          VLZPYH=VLZJP1                                                        TRBL0508
ISN 0444       2275 CONTINUE                                                           TRBL0509
ISN 0445          VISKIN=VISREF                                                        TRBL0510
ISN 0446          EVLJPH=EVLLOW                                                        TRBL0511
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ISN 0447          ETHJPH=ETHLOW                                                  TRBL0512
ISN 0448          ESLJPH=ESLLOW                                                  TRBL0513
ISN 0449          VFLSQR=(VLXPYH-VLXMYH)**2+(VLZPYH-VLZMYH)**2                   TRBL0514
ISN 0450          IF(VFLSQR.LE.0.)GO TO 2278                                     TRBL0515
ISN 0452          VFLHYH=SQRT(VFLSQR)                                            TRBL0516
ISN 0453          REYNUM=DYHYH*VFLHYH/VISKIN                                     TRBL0517
ISN 0454          IF(REYNUM.LT.REYTRB)GO TO 2278                                 TRBL0518
ISN 0456          ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFR)                            TRBL0519
ISN 0457          EVLJPH=EVLJPH+ETRJPH                                           TRBL0520
ISN 0458          ETHJPH=ETHJPH+ETRJPH                                           TRBL0521
ISN 0459          ESLJPH=ESLJPH+ETRJPH                                           TRBL0522
ISN 0460       2278 CONTINUE                                                     TRBL0523
ISN 0461          ARYVMX=DXIMX*DHZJPH                                            TRBL0524
ISN 0462          ARYVPX=DXIPX*DHZJPH                                            TRBL0525
ISN 0463          GVYJPH(LZ)=ARYJPH*(-EVLJPH                                     TRBL0526
                 1           *(VMYJP1-VPYIAJ(LZ))/(0.5*DYJPH))                   TRBL0527
ISN 0464          GPXJPH(LZ)=ARYVMX*(-EVLJPH                                     TRBL0528
                 1                  *(VMXJP1-VMXIAJ(LZ))/DYJPH)                 TRBL0529
ISN 0465          GPXJPH(LZ)=ARYVPX*(-EVLJPH                                     TRBL0530
                 1                  *(VPXJP1-VPXIAJ(LZ))/DYJPH)                 TRBL0531
ISN 0466          GVZJPH(LZ)=ARYJPH*(-EVLJPH                                     TRBL0532
                 1                  *(VLZJP1-VLZIAJ(LZ))/DYJPH)                 TRBL0533
ISN 0467       2260 CONTINUE                                                     TRBL0534
ISN 0468          GO TO 2700                                                     TRBL0535
ISN 0469       2300 CONTINUE                                                     TRBL0536
               C********** START BOUNDARY ELEMENT CALCULATIONS          **********TRBL0537
ISN 0470          IBND=INTEXT(IX,JY)                                             TRBL0538
               C********** START IMH BOUNDARY CALCULATIONS             **********TRBL0539
ISN 0471          IMHCON=IMHBND(IBND)+2                                          TRBL0540
ISN 0472          GO TO (2310,2330,2350,2370),IMHCON                             TRBL0541
ISN 0473       2310 CONTINUE                                                     TRBL0542
               C********** INTERNAL IMH BOUNDARY                      **********TRBL0543
               C********** IMH ENCLOSURE SURFACE TURBULENT TRANSPORT  **********TRBL0544
               C********** TRANSFERRED FROM PREVIOUS IMH CALCULATIONS **********TRBL0545
               C********** FOR ENCLOSURE SURFACES THAT ARE NOT CLOSED  **********TRBL0546
               C********** BY THE SHORELINE BOUNDARY IN LZ LEVEL       **********TRBL0547
ISN 0474          IJIM1=IJIXJY(IX-1,JY)                                          TRBL0548
ISN 0475          NLZIM1=NLZBOT(IJIM1)                                           TRBL0549
ISN 0476          DO 2320 LZ=NLZMIN,NZ                                           TRBL0550
ISN 0477          IF(LZ.GE.NLZIM1)GO TO 2325                                     TRBL0551
               C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM    **********TRBL0552
               C********** SHORELINE BOUNDARY                          **********TRBL0553
ISN 0479          IMXIMH(LZ)=0                                                   TRBL0554
ISN 0480          VLXIMH(LZ)=0.0                                                 TRBL0555
ISN 0481          VLYIMH(LZ)=0.0                                                 TRBL0556
ISN 0482          VLZIMH(LZ)=0.0                                                 TRBL0557
ISN 0483          GVXIMH(LZ)=0.0                                                 TRBL0558
               C********** TURBULENT SHEAR STRESS COMPONENTS ON IMH BOUNDARY **********TRBL0559
               C********** SURFACE ARE CALCULATED AS GPYIMH AND GPYIMH  **********TRBL0560
               C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM **********TRBL0561
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM **********TRBL0562
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON  **********TRBL0563
               C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL0564
               C********** IN THE ELEMENT                               **********TRBL0565
ISN 0484          VISKIN=VISREF                                                  TRBL0566
ISN 0485          EVLIMH=EVLLOW                                                  TRBL0567
ISN 0486          VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                             TRBL0568
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ISN 0487          IF(VFLSQR.LE.0.)GO TO 2324                                    TRBL0569
ISN 0489          VFLIAJ=SQRT(VFLSQR)                                           TRBL0570
ISN 0490          REYNUM=DXIMX*VFLIAJ/VISKIN                                    TRBL0571
ISN 0491          IF(REYNUM.LT.REYTRB)GO TO 2324                                TRBL0572
ISN 0493          ETRIMH=VISKIN*CFRNFR*(REYNUM**PNFR)                           TRBL0573
ISN 0494          EVLIMH=EVLIMH+ETRIMH                                          TRBL0574
ISN 0495       2324 CONTINUE                                                    TRBL0575
ISN 0496          DHZIMH=DHZIAJ(LZ)                                             TRBL0576
ISN 0497          ARXVMY=DYJMY*DHZIMH                                           TRBL0577
ISN 0498          ARXVPY=DYJPY*DHZIMH                                           TRBL0578
ISN 0499          ARXVLZ=DYIAJ*DHZIMH                                           TRBL0579
ISN 0500          GMYIMH(LZ)=ARXVMY*(-EVLIMH*VMYIAJ(LZ)/(0.5*DXJMY))            TRBL0580
ISN 0501          GPYIMH(LZ)=ARXVPY*(-EVLIMH*VPYIAJ(LZ)/(0.5*DXJPY))            TRBL0581
ISN 0502          GVZIMH(LZ)=ARXVLZ*(-EVLIMH*VLZIAJ(LZ)/DXIMX)                  TRBL0582
ISN 0503          GO TO 2320                                                    TRBL0583
ISN 0504       2325 CONTINUE                                                    TRBL0584
                  C********** IMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********TRBL0585
ISN 0505          IMXIMH(LZ)=1                                                  TRBL0586
ISN 0506          VLXIMH(LZ)=VLXTRX(JY,LZ)                                      TRBL0587
ISN 0507          VLYIMH(LZ)=VLYTRX(JY,LZ)                                      TRBL0588
ISN 0508          VLZIMH(LZ)=VLZTRX(JY,LZ)                                      TRBL0589
ISN 0509          GVXIMH(LZ)=GVXTRX(JY,LZ)                                      TRBL0590
ISN 0510          GMYIMH(LZ)=GMYTRX(JY,LZ)                                      TRBL0591
ISN 0511          GPYIMH(LZ)=GPYTRX(JY,LZ)                                      TRBL0592
ISN 0512          GVZIMH(LZ)=GVZTRX(JY,LZ)                                      TRBL0593
ISN 0513       2320 CONTINUE                                                    TRBL0594
ISN 0514          GO TO 2400                                                    TRBL0595
ISN 0515       2330 CONTINUE                                                    TRBL0596
                  C********** SHORELINE IMH BOUNDARY                           **********TRBL0597
ISN 0516          DO 2340 LZ=NLZMIN,NZ                                          TRBL0598
ISN 0517          IMXIMH(LZ)=0                                                  TRBL0599
ISN 0518          VLXIMH(LZ)=0.0                                                TRBL0600
ISN 0519          VLYIMH(LZ)=0.0                                                TRBL0601
ISN 0520          VLZIMH(LZ)=0.0                                                TRBL0602
ISN 0521          GVXIMH(LZ)=0.0                                                TRBL0603
                  C********** TURBULENT SHEAR STRESS COMPONENTS ON IMH BOUNDARY **********TRBL0604
                  C********** SURFACE ARE CALCULATED AS GPYIMH AND GPYIMH       **********TRBL0605
                  C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM   **********TRBL0606
                  C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0607
                  C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0608
                  C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL0609
                  C********** IN THE ELEMENT                                   **********TRBL0610
ISN 0522          VISKIN=VISREF                                                 TRBL0611
ISN 0523          EVLIMH=EVLLOW                                                 TRBL0612
ISN 0524          VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                            TRBL0613
ISN 0525          IF(VFLSQR.LE.0.)GO TO 2344                                    TRBL0614
ISN 0527          VFLIAJ=SQRT(VFLSQR)                                           TRBL0615
ISN 0528          REYNUM=DXIMX*VFLIAJ/VISKIN                                    TRBL0616
ISN 0529          IF(REYNUM.LT.REYTRB)GO TO 2344                                TRBL0617
ISN 0531          ETRIMH=VISKIN*CFRNFR*(REYNUM**PNFR)                           TRBL0618
ISN 0532          EVLIMH=EVLIMH+ETRIMH                                          TRBL0619
ISN 0533       2344 CONTINUE                                                    TRBL0620
ISN 0534          DHZIMH=DHZIAJ(LZ)                                             TRBL0621
ISN 0535          ARXVMY=DYJMY*DHZIMH                                           TRBL0622
ISN 0536          ARXVPY=DYJPY*DHZIMH                                           TRBL0623
ISN 0537          ARXVLZ=DYIAJ*DHZIMH                                           TRBL0624
ISN 0538          GMYIMH(LZ)=ARXVMY*(-EVLIMH*VMYIAJ(LZ)/(0.5*DXJMY))            TRBL0625
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ISN 0539          GPYIMH(LZ)=ARXVPY*(-EVLIMH*VPYIAJ(LZ)/(0.5*DXJPY))                    TRBL0626
ISN 0540          GVZIMH(LZ)=ARXVLZ*(-EVLIMH*VLZIAJ(LZ)/DXIMX)                          TRBL0627
ISN 0541       2340 CONTINUE                                                            TRBL0628
ISN 0542          GO TO 2400                                                            TRBL0629
ISN 0543       2350 CONTINUE                                                            TRBL0630
               C********** SPECIFIED FLOW CONDITIONS FOR IMH BOUNDARY ARE TRANSFFERED *TRBL0631
               C********** FROM THE BOUNDS SUBROUTINE                          **********TRBL0632
ISN 0544          ISPB=ISPBND(IBND)                                                     TRBL0633
ISN 0545          HSRIMH=HSRBXH(ISPB)                                                   TRBL0634
ISN 0546          DO 2360 LZ=NLZMIN,NZ                                                  TRBL0635
ISN 0547          IMXIMH(LZ)=IVXBXH(ISPB,LZ)                                            TRBL0636
ISN 0548          IF(IMXIMH(LZ).LT.0)GO TO 2355                                         TRBL0637
               C********** CLOSED IMH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL0638
ISN 0550          VLXIMH(LZ)=0.0                                                        TRBL0639
ISN 0551          VLYIMH(LZ)=0.0                                                        TRBL0640
ISN 0552          VLZIMH(LZ)=0.0                                                        TRBL0641
ISN 0553          GVXIMH(LZ)=0.0                                                        TRBL0642
               C********** TURBULENT SHEAR STRESS COMPONENTS ON IMH BOUNDARY **********TRBL0643
               C********** SURFACE ARE CALCULATED AS GPYIMH AND GPYIMH        **********TRBL0644
               C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM    **********TRBL0645
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM   **********TRBL0646
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON      **********TRBL0647
               C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS  **********TRBL0648
               C********** IN THE ELEMENT                                     **********TRBL0649
ISN 0554          VISKIN=VISREF                                                         TRBL0650
ISN 0555          EVLIMH=EVLLOW                                                         TRBL0651
ISN 0556          VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                                    TRBL0652
ISN 0557          IF(VFLSQR.LE.0.)GO TO 2364                                            TRBL0653
ISN 0559          VFLIAJ=SQRT(VFLSQR)                                                   TRBL0654
ISN 0560          REYNUM=DXIMX*VFLIAJ/VISKIN                                            TRBL0655
ISN 0561          IF(REYNUM.LT.REYTRB)GO TO 2364                                        TRBL0656
ISN 0563          ETRIMH=VISKIN*CFRNFR*(REYNUM**PNFR)                                   TRBL0657
ISN 0564          EVLIMH=EVLIMH+ETRIMH                                                  TRBL0658
ISN 0565       2364 CONTINUE                                                            TRBL0659
ISN 0566          DHZIMH=DHZIAJ(LZ)                                                     TRBL0660
ISN 0567          ARXVMY=DYJMY*DHZIMH                                                   TRBL0661
ISN 0568          ARXVPY=DYJPY*DHZIMH                                                   TRBL0662
ISN 0569          ARXVLZ=DYIAJ*DHZIMH                                                   TRBL0663
ISN 0570          GPYIMH(LZ)=ARXVMY*(-EVLIMH*VMYIAJ(LZ)/(0.5*DXJMY))                    TRBL0664
ISN 0571          GPYIMH(LZ)=ARXVPY*(-EVLIMH*VPYIAJ(LZ)/(0.5*DXJPY))                    TRBL0665
ISN 0572          GVZIMH(LZ)=ARXVLZ*(-EVLIMH*VLZIAJ(LZ)/DXIMX)                          TRBL0666
ISN 0573          GO TO 2360                                                            TRBL0667
ISN 0574       2355 CONTINUE                                                            TRBL0668
               C********** OPEN IMH ENCLOSURE SURFACE AT LZ LEVEL            **********TRBL0669
ISN 0575          VLXIMH(LZ)=VLXBXH(ISPB,LZ)                                            TRBL0670
ISN 0576          VLYIMH(LZ)=0.5*(VMYBXH(ISPB,LZ)+VPYBXH(ISPB,LZ))                      TRBL0671
ISN 0577          VLZIMH(LZ)=VLZBXH(ISPB,LZ)                                            TRBL0672
ISN 0578          IF(GPXBXH(ISPB,LZ).GT.0)GO TO 2356                                    TRBL0673
               C********** TURBULENT TRANSPORT IS NEGLIGIBLE FOR OUTFLOW TYPE **********TRBL0674
               C********** SPECIFIED BOUNDARY CONDITIONS ON THE IMH SURFACE   **********TRBL0675
ISN 0580          GVXIMH(LZ)=0.0                                                        TRBL0676
ISN 0581          GPYIMH(LZ)=0.0                                                        TRBL0677
ISN 0582          GPYIMH(LZ)=0.0                                                        TRBL0678
ISN 0583          GVZIMH(LZ)=0.0                                                        TRBL0679
ISN 0584          GO TO 2360                                                            TRBL0680
ISN 0585       2356 CONTINUE                                                            TRBL0681
               C********** TURBULENT TRANSPORT IS INCLUDED FOR INFLOW TYPE    **********TRBL0682
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                C********** SPECIFIED BOUNDARY CONDITIONS ON THE IMH SURFACE  **********TRBL0683
ISN 0586              VLXMXH=VLXBXH(ISPB,LZ)                                              TRBL0684
ISN 0587              VMYMXH=VMYBXH(ISPB,LZ)                                              TRBL0685
ISN 0588              VPYMXH=VPYBXH(ISPB,LZ)                                              TRBL0686
ISN 0589              VLZMXH=VLZBXH(ISPB,LZ)                                              TRBL0687
                C********** TURBULENT TRANSPORT RATES ON THE IMH SURFACE ARE  **********TRBL0688
                C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL0689
                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0690
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0691
                C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL0692
                C********** COMPONENTS IN THE ELEMENT AND AT THE              **********TRBL0693
                C********** SPECIFIED INFLOW CONDITIONS                       **********TRBL0694
ISN 0590              VISKIN=VISREF                                                       TRBL0695
ISN 0591              EVLIMH=EVLLOW                                                       TRBL0696
ISN 0592              ETHIMH=ETHLOW                                                       TRBL0697
ISN 0593              ESLIMH=ESLLOW                                                       TRBL0698
ISN 0594              VELSQR=(VLYIAJ(LZ)-VLYIMH(LZ))**2+(VLZIAJ(LZ)-VLZIMH(LZ))**2       TRBL0699
ISN 0595              IF(VELSQR.LE.0.)GO TO 2368                                          TRBL0700
ISN 0597              VELIAJ=SQRT(VELSQR)                                                 TRBL0701
ISN 0598              REYNUM=DXIAJ*VELIAJ/VISKIN                                          TRBL0702
ISN 0599              IF(REYNUM.LT.REYTRB)GO TO 2368                                      TRBL0703
ISN 0601              ETRIMH=VISKIN*CFRNFR*(REYNUM**PNFR)                                 TRBL0704
ISN 0602              EVLIMH=EVLIMH+ETRIMH                                                TRBL0705
ISN 0603              ETHIMH=ETHIMH+ETRIMH                                                TRBL0706
ISN 0604              ESLIMH=ESLIMH+ETRIMH                                                TRBL0707
ISN 0605             2368 CONTINUE                                                        TRBL0708
ISN 0606              DHZIMH=DHZIAJ(LZ)                                                   TRBL0709
ISN 0607              ARXIMH=DYIMH*DHZIMH                                                 TRBL0710
ISN 0608              ARXVMY=DYJMY*DHZIMH                                                 TRBL0711
ISN 0609              ARXVPY=DYJPY*DHZIMH                                                 TRBL0712
ISN 0610              GVXIMH(LZ)=ARXIMH*(-EVLIMH*(VMXIAJ(LZ)                              TRBL0713
                     1                          -VLXMXH)/DXIMX)                         TRBL0714
ISN 0611              GVYIMH(LZ)=ARXVMY*(-EVLIMH*(VMYIAJ(LZ)                              TRBL0715
                     1                          -VMYMXH)/DXJMY)                         TRBL0716
ISN 0612              GVYIMH(LZ)=ARXVPY*(-EVLIMH*(VPYIAJ(LZ)                              TRBL0717
                     1                          -VPYMXH)/DXJPY)                         TRBL0718
ISN 0613              GVZIMH(LZ)=ARXIMH*(-EVLIMH*(VLZIAJ(LZ)                              TRBL0719
                     1                          -VLZMXH)/DXIAJ)                         TRBL0720
ISN 0614             2360 CONTINUE                                                        TRBL0721
ISN 0615              GO TO 2400                                                          TRBL0722
ISN 0616             2370 CONTINUE                                                        TRBL0723
                C********** SECIFIED OPEN REGION CONDITIONS FOR IMH BOUNDARY **********TRBL0724
                C********** ARE TRANSFERRED FROM THE BOUNDS SUBROUTINE       **********TRBL0725
ISN 0617              ISPB=ISPBND(IBND)                                                   TRBL0726
ISN 0618              HSRIM1=HSRBXH(ISPB)                                                 TRBL0727
ISN 0619              HSRIMH=0.5*(HSRIM1+HSRIAJ)                                          TRBL0728
ISN 0620              DO 2380 LZ=NLZMIN,NZ                                                TRBL0729
ISN 0621              IMXIMH(LZ)=IVXBXH(ISPB,LZ)                                          TRBL0730
ISN 0622              IF(IMXIMH(LZ).GT.0.)GO TO 2375                                      TRBL0731
                C********** CLOSED IMH ENCLOSURE SURFACE AT LZ LEVEL         **********TRBL0732
ISN 0624              VLXIMH(LZ)=0.0                                                      TRBL0733
ISN 0625              VLYIMH(LZ)=0.0                                                      TRBL0734
ISN 0626              VLZIMH(LZ)=0.0                                                      TRBL0735
ISN 0627              GVXIMH(LZ)=0.0                                                      TRBL0736
                C********** TURBULENT SHEAR STRESS COMPONENTS ON IMH BOUNDARY **********TRBL0737
                C********** SURFACE ARE CALCULATED AS GPYIMH AND GPYIMH       **********TRBL0738
                C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM   **********TRBL0739
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                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0740
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0741
                C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL0742
                C********** IN THE ELEMENT                                   **********TRBL0743
ISN 0628              VISKIN=VISREF                                                    TRBL0744
ISN 0629              EVLIMH=EVLLOW                                                    TRBL0745
ISN 0630              VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                               TRBL0746
ISN 0631              IF(VFLSQR.LE.0.)GO TO 2384                                       TRBL0747
ISN 0633              VFLIAJ=SQRT(VFLSQR)                                              TRBL0748
ISN 0634              REYNUM=DXIMX*VFLIAJ/VISKIN                                       TRBL0749
ISN 0635              IF(REYNUM.LT.REYTRB)GO TO 2384                                   TRBL0750
ISN 0637              ETRIMH=VISKIN*CFRNFR*(REYNUM**PNFR)                              TRBL0751
ISN 0638              EVLIMH=EVLIMH+ETRIMH                                             TRBL0752
ISN 0639       2384 CONTINUE                                                           TRBL0753
ISN 0640              DHZIMH=DHZIAJ(LZ)                                                TRBL0754
ISN 0641              ARXVMY=DYJMY*DHZIMH                                              TRBL0755
ISN 0642              ARXVPY=DYJPY*DHZIMH                                              TRBL0756
ISN 0643              ARXVLZ=DYIAJ*DHZIMH                                              TRBL0757
ISN 0644              GMYIMH(LZ)=ARXVMY*(-EVLIMH*VMYIAJ(LZ)/(0.5*DXJMY))               TRBL0758
ISN 0645              GPYIMH(LZ)=ARXVPY*(-EVLIMH*VPYIAJ(LZ)/(0.5*DXJPY))               TRBL0759
ISN 0646              GVZIMH(LZ)=ARXVLZ*(-EVLIMH*VLZIAJ(LZ)/DXIMX)                     TRBL0760
ISN 0647              GO TO 2380                                                       TRBL0761
ISN 0648       2375 CONTINUE                                                           TRBL0762
                C********** OPEN IMH ENCLOSURE SURFACE AT LZ LEVEL           **********TRBL0763
ISN 0649              VLXMXH=VLXBXH(ISPB,LZ)                                           TRBL0764
ISN 0650              VLXIMH(LZ)=VLXMXH                                                TRBL0765
ISN 0651              VMYMXH=VMYBXH(ISPB,LZ)                                           TRBL0766
ISN 0652              VPYMXH=VPYBXH(ISPB,LZ)                                           TRBL0767
ISN 0653              VLYIMH(LZ)=0.5*(VMYMXH+VPYMXH)                                   TRBL0768
ISN 0654              VLZMXH=VLZBXH(ISPB,LZ)                                           TRBL0769
ISN 0655              VLZIMH(LZ)=VLZMXH                                                TRBL0770
                C********** TURBULENT TRANSPORT RATES ON THE IMH SURFACE ARE  **********TRBL0771
                C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL0772
                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0773
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0774
                C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL0775
                C********** COMPONENTS IN THE ELEMENTS AND IN THE EXTERNAL    **********TRBL0776
                C********** OPEN REGION ELEMENT WITH SPECIFIED VELOCITY       **********TRBL0777
                C********** AND OTHER FLOW CONDITIONS                         **********TRBL0778
ISN 0656              VISKIN=VISREF                                                    TRBL0779
ISN 0657              EVLIMH=EVLLOW                                                    TRBL0780
ISN 0658              ETHIMH=ETHLOW                                                    TRBL0781
ISN 0659              ESLIMH=ESLLOW                                                    TRBL0782
ISN 0660              VFLSQR=(VLYIAJ(LZ)-VLYIMH(LZ))**2+(VLZIAJ(LZ)-VLZIMH(LZ))**2    TRBL0783
ISN 0661              IF(VFLSQR.LE.0.)GO TO 2388                                       TRBL0784
ISN 0663              VFLIAJ=SQRT(VFLSQR)                                              TRBL0785
ISN 0664              REYNUM=DXIAJ*VFLIAJ/VISKIN                                       TRBL0786
ISN 0665              IF(REYNUM.LT.REYTRB)GO TO 2388                                   TRBL0787
ISN 0667              ETRIMH=VISKIN*CFRNFR*(REYNUM**PNFR)                              TRBL0788
ISN 0668              EVLIMH=EVLIMH+ETRIMH                                             TRBL0789
ISN 0669              ETHIMH=ETHIMH+ETRIMH                                             TRBL0790
ISN 0670              ESLIMH=ESLIMH+ETRIMH                                             TRBL0791
ISN 0671       2388 CONTINUE                                                           TRBL0792
ISN 0672              DHZIMH=DHZIAJ(LZ)                                                TRBL0793
ISN 0673              IF(LZ.EQ.NZ)  DHZIMH=DHZIMH-HSRIAJ+HSRIMH                        TRBL0794
ISN 0675              ARXIMH=DYIMH*DHZIMH                                              TRBL0795
ISN 0676              ARXVMY=DYJMY*DHZIMH                                              TRBL0796
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ISN 0677          ARXVPY=DYJPY*DHZIMH                                          TRBL0797
ISN 0678          GVXIMH(LZ)=ARXIMH*(-EVLIMH*(VMXIAJ(LZ)                       TRBL0798
                 1                              -VLXMXH)/DXIMX)              TRBL0799
ISN 0679          GMYIMH(LZ)=ARXVMY*(-EVLIMH*(VMYIAJ(LZ)                       TRBL0800
                 1                              -VMYMXH)/DXJMY)              TRBL0801
ISN 0680          GFYIMH(LZ)=ARXVPY*(-EVLIMH*(VPYIAJ(LZ)                       TRBL0802
                 1                              -VPYMXH)/DXJPY)              TRBL0803
ISN 0681          GVZIMH(LZ)=ARXIMH*(-EVLIMH*(VLZIAJ(LZ)                       TRBL0804
                 1                              -VLZMXH)/DXIAJ)              TRBL0805
ISN 0682       2380 CONTINUE                                                   TRBL0806
ISN 0683       2400 CONTINUE                                                   TRBL0807
               C********** START IPH BOUNDARY CALCULATIONS                    **********TRBL0808
ISN 0684          IPHCON=IPHBND(IBND)+2                                        TRBL0809
ISN 0685          GO TO (2410,2430,2450,2470),IPHCON                           TRBL0810
ISN 0686       2410 CONTINUE                                                   TRBL0811
               C********** INTERNAL IPH BOUNDARY                             **********TRBL0812
               C********** IPH ENCLOSURE SURFACE TURBULENT TRANSPORT          **********TRBL0813
               C********** CALCULATED FOR ALL ENCLOSURE SURFACES THAT ARE     **********TRBL0814
               C********** NOT CLOSED BY THE SHORELINE BOUNDARY IN LZ LEVEL   **********TRBL0815
ISN 0687          IJIP1=IJIXJY(IX+1,JY)                                        TRBL0816
ISN 0688          IMXIP1=2*IJIP1-1                                             TRBL0817
ISN 0689          JMYIP1=IMXIP1                                                TRBL0818
ISN 0690          JPYIP1=JMYIP1+1                                              TRBL0819
ISN 0691          IF(INTEXT(IX+1,JY).GT.0)GO TO 2412                           TRBL0820
               C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT        **********TRBL0821
ISN 0693          DXIP1=DX(IX+1)                                               TRBL0822
ISN 0694          DYIP1=DYIAJ                                                  TRBL0823
ISN 0695          ARHIP1=DXIP1*DYIP1                                           TRBL0824
ISN 0696          GO TO 2415                                                   TRBL0825
ISN 0697       2412 CONTINUE                                                   TRBL0826
               C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT        **********TRBL0827
ISN 0698          IBNIP1=INTEXT(IX+1,JY)                                       TRBL0828
ISN 0699          DXIP1=DXBND(IBNIP1)                                          TRBL0829
ISN 0700          ARHIP1=ARHBND(IBNIP1)                                        TRBL0830
ISN 0701       2415 CONTINUE                                                   TRBL0831
ISN 0702          DXIPH=0.5*(DXIAJ+DXIP1)                                      TRBL0832
ISN 0703          CCXIPH=0.5*DXIAJ/DXIPH                                       TRBL0833
ISN 0704          HSRIP1=HSR(IJIP1)                                            TRBL0834
ISN 0705          NLZIP1=NLZBOT(IJIP1)                                         TRBL0835
ISN 0706          DO 2420 LZ=NLZMIN,NZ                                         TRBL0836
ISN 0707          IF(LZ.GE.NLZIP1)GO TO 2425                                   TRBL0837
               C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********TRBL0838
               C********** SHORELINE BOUNDARY                                **********TRBL0839
ISN 0709          IPXIPH(LZ)=0                                                 TRBL0840
ISN 0710          VLXIPH(LZ)=0.0                                               TRBL0841
ISN 0711          VLYIPH(LZ)=0.0                                               TRBL0842
ISN 0712          VLZIPH(LZ)=0.0                                               TRBL0843
ISN 0713          GVXIPH(LZ)=0.0                                               TRBL0844
               C********** TURBULENT SHEAR STRESS COMPONENTS ON IPH BOUNDARY **********TRBL0845
               C********** SURFACE ARE CALCULATED AS GMYIPH AND GPYIPH        **********TRBL0846
               C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM    **********TRBL0847
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM   **********TRBL0848
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON      **********TRBL0849
               C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL0850
               C********** IN THE ELEMENT                                    **********TRBL0851
ISN 0714          VISKIN=VISREF                                                TRBL0852
ISN 0715          EVLIPH=EVLLOW                                                TRBL0853
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ISN 0716            VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                                  TRBL0854
ISN 0717            IF(VFLSQR.LE.0.)GO TO 2424                                          TRBL0855
ISN 0719            VFLIAJ=SQRT(VFLSQR)                                                 TRBL0856
ISN 0720            REYNUM=DXIPX*VFLIAJ/VISKIN                                          TRBL0857
ISN 0721            IF(REYNUM.LT.REYTRB)GO TO 2424                                      TRBL0858
ISN 0723            ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFF)                                 TRBL0859
ISN 0724            EVLIPH=EVLIPH+ETRIPH                                                TRBL0860
ISN 0725          2424 CONTINUE                                                         TRBL0861
ISN 0726            DHZIPH=DHZIAJ(LZ)                                                   TRBL0862
ISN 0727            ARXVMY=DYJMY*DHZIPH                                                 TRBL0863
ISN 0728            ARXVPY=DYJPY*DHZIPH                                                 TRBL0864
ISN 0729            ARXVLZ=DYIAJ*DHZIPH                                                 TRBL0865
ISN 0730            GMYIPH(LZ)=ARXVMY*(EVLIPH*VMYIAJ(LZ)/(0.5*DXJMY))                   TRBL0866
ISN 0731            GPYIPH(LZ)=ARXVPY*(EVLIPH*VPYIAJ(LZ)/(0.5*DXJPY))                   TRBL0867
ISN 0732            GVZIPH(LZ)=ARXVLZ*(EVLIPH*VLZIAJ(LZ)/CXIPX)                         TRBL0868
ISN 0733            GO TO 2420                                                          TRBL0869
ISN 0734          2425 CONTINUE                                                         TRBL0870
                  C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********TRBL0871
                  C********** TURBULENT TRANSPORT RATES ON THE IPH SURFACE ARE  **********TRBL0872
                  C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL0873
                  C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0874
                  C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0875
                  C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL0876
                  C********** COMPONENTS IN THE ADJACENT ELEMENTS               **********TRBL0877
ISN 0735            IPXIPH(LZ)=1                                                        TRBL0878
ISN 0736            DHZIP1=DZ(LZ)                                                       TRBL0879
ISN 0737            IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                              TRBL0880
ISN 0738            IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                                  TRBL0881
ISN 0741            DHZIPH=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))                        TRBL0882
ISN 0742            ARXIPH=DYIPH*DHZIPH                                                 TRBL0883
ISN 0743            VPXIP1=VLX(IMXIP1,LZ)                                               TRBL0884
ISN 0744            VLXIPH(LZ)=VPXIAJ(LZ)+CDXIPH*(VMXIP1-VPXIAJ(LZ))                    TRBL0885
ISN 0745            VMYIP1=VLY(JMYIP1,LZ)                                               TRBL0886
ISN 0746            VPYIP1=VLY(JPYIP1,LZ)                                               TRBL0887
ISN 0747            VLYIP1=0.5*(VMYIP1+VPYIP1)                                          TRBL0888
ISN 0748            VLYIPH(LZ)=VLYIAJ(LZ)+CDXIPH*(VLYIP1-VLYIAJ(LZ))                    TRBL0889
                  C********** CALCULATE VLZIP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJIP1 *TRBL0890
ISN 0749            GWZLMH=GWZLZH(IJIP1,LZ)                                             TRBL0891
ISN 0750            IF(LZ.EQ.NZ)GO TO 2427                                              TRBL0892
ISN 0752            GWZLPH=GWZLZH(IJIP1,LZ+1)                                           TRBL0893
ISN 0753            GO TO 2428                                                          TRBL0894
ISN 0754          2427 CONTINUE                                                         TRBL0895
ISN 0755            GWZLPH=ARHIP1*FNCHSR(IJIP1)                                         TRBL0896
ISN 0756          2428 CONTINUE                                                         TRBL0897
ISN 0757            GWZIP1=0.5*(GWZLMH+GWZLPH)                                          TRBL0898
ISN 0758            VLZIP1=GWZIP1/ARHIP1                                                TRBL0899
ISN 0759            VLZIPH(LZ)=VLZIAJ(LZ)+CDXIPH*(VLZIP1-VLZIAJ(LZ))                    TRBL0900
ISN 0760            IF(DXIAJ.GT.DXIP1)GO TO 2432                                        TRBL0901
ISN 0762            DXHXH=DXIAJ                                                         TRBL0902
ISN 0763            CDXHXH=DXHXH/DXIPH                                                  TRBL0903
ISN 0764            VLYMXH=VLYIAJ(LZ)                                                   TRBL0904
ISN 0765            VLZMXH=VLZIAJ(LZ)                                                   TRBL0905
ISN 0766            VLYPXH=VLYIAJ(LZ)+CDXHXH*(VLYIP1-VLYIAJ(LZ))                        TRBL0906
ISN 0767            VLZPXH=VLZIAJ(LZ)+CDXHXH*(VLZIP1-VLZIAJ(LZ))                        TRBL0907
ISN 0768            GO TO 2435                                                          TRBL0908
ISN 0769          2432 CONTINUE                                                         TRBL0909
ISN 0770            DXHXH=DXIP1                                                         TRBL0910
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ISN 0771            CDXHXH=DXHXH/DXIPH                                                  TRBL0911
ISN 0772            VLYMXH=VLYIP1-CDXHXH*(VLYIP1-VLYIAJ(LZ))                            TRBL0912
ISN 0773            VLZMXH=VLZIP1-CDXHXH*(VLZIP1-VLZIAJ(LZ))                            TRBL0913
ISN 0774            VLYPXH=VLYIP1                                                       TRBL0914
ISN 0775            VLZPXH=VLZIP1                                                       TRBL0915
ISN 0776          2435 CONTINUE                                                         TRBL0916
ISN 0777            VISKIN=VISREF                                                       TRBL0917
ISN 0778            EVLIPH=EVLLOW                                                       TRBL0918
ISN 0779            ETHIPH=ETHLOW                                                       TRBL0919
ISN 0780            ESLIPH=ESLLOW                                                       TRBL0920
ISN 0781            VFLSQR=(VLYPXH-VLYMXH)**2+(VLZPXH-VLZMXH)**2                        TRBL0921
ISN 0782            IF(VFLSQR.LE.0.)GO TO 2438                                          TRBL0922
ISN 0784            VFLHXH=SQRT(VFLSQR)                                                 TRBL0923
ISN 0785            REYNUM=DXHXH*VFLHXH/VISKIN                                          TRBL0924
ISN 0786            IF(REYNUM.LT.REYTRB)GO TO 2438                                      TRBL0925
ISN 0788            ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFR)                                 TRBL0926
ISN 0789            EVLIPH=EVLIPH+ETRIPH                                                TRBL0927
ISN 0790            ETHIPH=ETHIPH+ETRIPH                                                TRBL0928
ISN 0791            ESLIPH=ESLIPH+ETRIPH                                                TRBL0929
ISN 0792          2438 CONTINUE                                                         TRBL0930
ISN 0793            ARXVMY=DYJMY*DHZIPH                                                 TRBL0931
ISN 0794            ARXVPY=DYJPY*DHZIPH                                                 TRBL0932
ISN 0795            GVXIPH(LZ)=ARXIPH*(-EVLIPH                                          TRBL0933
                   1           *(VMXIP1-VPXIAJ(LZ))/(0.5*DXIPH))                       TRBL0934
ISN 0796            GMYIPH(LZ)=ARXVMY*(-EVLIPH                                          TRBL0935
                   1                  *(VMYIP1-VMYIAJ(LZ))/DXIPH)                      TRBL0936
ISN 0797            GPYIPH(LZ)=ARXVPY*(-EVLIPH                                          TRBL0937
                   1                  *(VPYIP1-VPYIAJ(LZ))/DXIPH)                      TRBL0938
ISN 0798            GVZIPH(LZ)=ARXIPH*(-EVLIPH                                          TRBL0939
                   1                  *(VLZIP1-VLZIAJ(LZ))/DXIPH)                      TRBL0940
ISN 0799          2420 CONTINUE                                                         TRBL0941
ISN 0800            GO TO 2500                                                          TRBL0942
ISN 0801          2430 CONTINUE                                                         TRBL0943
                  C********** SHORELINE IPH BOUNDARY                  **********TRBL0944
ISN 0802            DO 2440 LZ=NLZMIN,NZ                                              TRBL0945
ISN 0803            IFXIPH(LZ)=0                                                      TRBL0946
ISN 0804            VLXIPH(LZ)=0.0                                                    TRBL0947
ISN 0805            VLYIPH(LZ)=0.0                                                    TRBL0948
ISN 0806            VLZIPH(LZ)=0.0                                                    TRBL0949
ISN 0807            GVXIPH(LZ)=0.0                                                    TRBL0950
                  C********** TURBULENT SHEAR STRESS COMPONENTS ON IPH BOUNDARY **********TRBL0951
                  C********** SURFACE ARE CALCULATED AS GPYIPH AND GPYIPH       **********TRBL0952
                  C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM   **********TRBL0953
                  C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL0954
                  C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL0955
                  C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL0956
                  C********** IN THE ELEMENT                                   **********TRBL0957
ISN 0808            VISKIN=VISREF                                                     TRBL0958
ISN 0809            EVLIPH=EVLLOW                                                     TRBL0959
ISN 0810            VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                                TRBL0960
ISN 0811            IF(VFLSQR.LE.0.)GO TO 2444                                        TRBL0961
ISN 0813            VFLIAJ=SQRT(VFLSQR)                                               TRBL0962
ISN 0814            REYNUM=DXIPX*VFLIAJ/VISKIN                                        TRBL0963
ISN 0815            IF(REYNUM.LT.REYTRB)GO TO 2444                                    TRBL0964
ISN 0817            ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFR)                               TRBL0965
ISN 0818            EVLIPH=EVLIPH+ETRIPH                                              TRBL0966
ISN 0819          2444 CONTINUE                                                       TRBL0967
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ISN 0820          DHZIPH=DHZIAJ(LZ)                                                  TRBL0968
ISN 0821          ARXVMY=DYJMY*DHZIPH                                                TRBL0969
ISN 0822          ARXVPY=DYJPY*DHZIPH                                                TRBL0970
ISN 0823          ARXVLZ=DYIAJ*DHZIPH                                                TRBL0971
ISN 0824          GMYIPH(LZ)=ARXVMY*(EVLIPH*VMYIAJ(LZ)/(0.5*DXJMY))                  TRBL0972
ISN 0825          GPYIPH(LZ)=ARXVPY*(EVLIPH*VPYIAJ(LZ)/(0.5*DXJPY))                  TRBL0973
ISN 0826          GVZIPH(LZ)=ARXVLZ*(EVLIPH*VLZIAJ(LZ)/DXIPX)                        TRBL0974
ISN 0827       2440 CONTINUE                                                         TRBL0975
ISN 0828          GO TO 2500                                                         TRBL0976
ISN 0829       2450 CONTINUE                                                         TRBL0977
               C********** SPECIFIED FLOW CONDITIONS FOR IPH BOUNDARY ARE TRANSFFERED *TRBL0978
               C********** FROM THE BOUNDS SUBROUTINE                 **********TRBL0979
ISN 0830          ISPB=ISPBND(IBND)                                                  TRBL0980
ISN 0831          HSRIPH=HSRBXH(ISPB)                                                TRBL0981
ISN 0832          DO 2460 LZ=NLZMIN,NZ                                               TRBL0982
ISN 0833          IFXIPH(LZ)=IVXBXH(ISPB,LZ)                                         TRBL0983
ISN 0834          IF(IPXIPH(LZ).LT.0)GO TO 2455                                      TRBL0984
               C********** CLOSED IPH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL0985
ISN 0836          VLXIPH(LZ)=0.0                                                     TRBL0986
ISN 0837          VLYIPH(LZ)=0.0                                                     TRBL0987
ISN 0838          VLZIPH(LZ)=0.0                                                     TRBL0988
ISN 0839          GVXIPH(LZ)=0.0                                                     TRBL0989
               C********** TURBULENT SHEAR STRESS COMPONENTS ON IPH BOUNDARY **********TRBL0990
               C********** SURFACE ARE CALCULATED AS GMYIPH AND GPYIPH       **********TRBL0991
               C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM    **********TRBL0992
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM   **********TRBL0993
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON      **********TRBL0994
               C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS  **********TRBL0995
               C********** IN THE ELEMENT                                    **********TRBL0996
ISN 0840          VISKIN=VISREF                                                      TRBL0997
ISN 0841          EVLIPH=EVLLOW                                                      TRBL0998
ISN 0842          VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                                 TRBL0999
ISN 0843          IF(VFLSQR.LE.0.)GO TO 2464                                         TRBL1000
ISN 0845          VFLIAJ=SQRT(VFLSQR)                                                TRBL1001
ISN 0846          REYNUM=DXIPX*VFLIAJ/VISKIN                                         TRBL1002
ISN 0847          IF(REYNUM.LT.REYTRB)GO TO 2464                                     TRBL1003
ISN 0849          ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFR)                                TRBL1004
ISN 0850          EVLIPH=EVLIPH+ETRIPH                                               TRBL1005
ISN 0851       2464 CONTINUE                                                         TRBL1006
ISN 0852          DHZIPH=DHZIAJ(LZ)                                                  TRBL1007
ISN 0853          ARXVMY=DYJMY*DHZIPH                                                TRBL1008
ISN 0854          ARXVPY=DYJPY*DHZIPH                                                TRBL1009
ISN 0855          ARXVLZ=DYIAJ*DHZIPH                                                TRBL1010
ISN 0856          GMYIPH(LZ)=ARXVMY*(EVLIPH*VMYIAJ(LZ)/(0.5*DXJMY))                  TRBL1011
ISN 0857          GPYIPH(LZ)=ARXVPY*(EVLIPH*VPYIAJ(LZ)/(0.5*DXJPY))                  TRBL1012
ISN 0858          GVZIPH(LZ)=ARXVLZ*(EVLIPH*VLZIAJ(LZ)/DXIPX)                        TRBL1013
ISN 0859          GO TO 2460                                                         TRBL1014
ISN 0860       2455 CONTINUE                                                         TRBL1015
               C********** OPEN IPH ENCLOSURE SURFACE AT LZ LEVEL           **********TRBL1016
ISN 0861          VLXIPH(LZ)=VLXBXH(ISPB,LZ)                                         TRBL1017
ISN 0862          VLYIPH(LZ)=0.5*(VMYBXH(ISPB,LZ)+VPYBXH(ISPB,LZ))                   TRBL1018
ISN 0863          VLZIPH(LZ)=VLZBXH(ISPB,LZ)                                         TRBL1019
ISN 0864          IF(GFXBXH(ISPB,LZ).LT.0)GO TO 2456                                 TRBL1020
               C********** TURBULENT TRANSPORT IS NEGLIGIBLE FOR OUTFLOW TYPE **********TRBL1021
               C********** SPECIFIED BOUNDARY CONDITIONS ON THE IPH SURFACE   **********TRBL1022
ISN 0866          GVXIPH(LZ)=0.0                                                     TRBL1023
ISN 0867          GMYIPH(LZ)=0.0                                                     TRBL1024
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ISN 0868          GPYIPH(LZ)=0.0                                                  TRBL1025
ISN 0869          GVZIPH(LZ)=0.0                                                  TRBL1026
ISN 0870          GO TO 2460                                                      TRBL1027
ISN 0871       2456 CONTINUE                                                      TRBL1028
               C********** TURBULENT TRANSPORT IS INCLUDED FOR INFLOW TYPE  **********TRBL1029
               C********** SPECIFIED BOUNDARY CONDITIONS ON THE IPH SURFACE  **********TRBL1030
ISN 0872          VLXPXH=VLXBXH(ISPB,LZ)                                          TRBL1031
ISN 0873          VMYPXH=VMYBXH(ISPB,LZ)                                          TRBL1032
ISN 0874          VPYPXH=VPYBXH(ISPB,LZ)                                          TRBL1033
ISN 0875          VLZPXH=VLZBXH(ISPB,LZ)                                          TRBL1034
               C********** TURBULENT TRANSPORT RATES ON THE IPH SURFACE ARE  **********TRBL1035
               C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL1036
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1037
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1038
               C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL1039
               C********** COMPONENTS IN THE ELEMENT AND AT THE              **********TRBL1040
               C********** SPECIFIED INFLOW CONDITIONS                       **********TRBL1041
ISN 0876          VISKIN=VISREF                                                   TRBL1042
ISN 0877          EVLIPH=EVLLOW                                                   TRBL1043
ISN 0878          ETHIPH=ETHLOW                                                   TRBL1044
ISN 0879          ESLIPH=ESLLOW                                                   TRBL1045
ISN 0880          VFLSQR=(VLYIAJ(LZ)-VLYIPH(LZ))**2+(VLZIAJ(LZ)-VLZIPH(LZ))**2    TRBL1046
ISN 0881          IF(VFLSQR.LE.0.)GO TO 2468                                      TRBL1047
ISN 0883          VFLIAJ=SQRT(VFLSQR)                                             TRBL1048
ISN 0884          REYNUM=DXIAJ*VFLIAJ/VISKIN                                      TRBL1049
ISN 0885          IF(REYNUM.LT.REYTRB)GO TO 2468                                  TRBL1050
ISN 0887          ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFR)                             TRBL1051
ISN 0888          EVLIPH=EVLIPH+ETRIPH                                            TRBL1052
ISN 0889          ETHIPH=ETHIPH+ETRIPH                                            TRBL1053
ISN 0890          ESLIPH=ESLIPH+ETRIPH                                            TRBL1054
ISN 0891       2468 CONTINUE                                                      TRBL1055
ISN 0892          DHZIPH=DHZIAJ(LZ)                                               TRBL1056
ISN 0893          ARXIPH=DYIPH*DHZIPH                                             TRBL1057
ISN 0894          ARXVMY=DYJMY*DHZIPH                                             TRBL1058
ISN 0895          ARXVPY=DYJPY*DHZIPH                                             TRBL1059
ISN 0896        .    GVXIPH(LZ)=ARXIPH*(-EVLIPH*(VLXPXH                          TRBL1060
                    1                          -VPXIAJ(LZ))/DXIPX)              TRBL1061
ISN 0897          GPYIPH(LZ)=ARXVMY*(-EVLIPH*(VMYPXH                            TRBL1062
                  1                          -VMYIAJ(LZ))/DXJMY)               TRBL1063
ISN 0898          GPYIPH(LZ)=ARXVPY*(-EVLIPH*(VPYPXH                            TRBL1064
                  1                          -VPYIAJ(LZ))/DXJPY)               TRBL1065
ISN 0899          GVZIPH(LZ)=ARXIPH*(-EVLIPH*(VLZPXH                            TRBL1066
                  1                          -VLZIAJ(LZ))/DXIAJ)               TRBL1067
ISN 0900       2460 CONTINUE                                                      TRBL1068
ISN 0901          GO TO 2500                                                      TRBL1069
ISN 0902       2470 CONTINUE                                                      TRBL1070
               C********** SECIFIED OPEN REGION CONDITIONS FOR IPH BOUNDARY  **********TRBL1071
               C********** ARE TRANSFERRED FROM THE BOUNDS SUBROUTINE        **********TRBL1072
ISN 0903          ISPB=ISPBND(IBND)                                               TRBL1073
ISN 0904          HSRIP1=HSRBXH(ISPB)                                             TRBL1074
ISN 0905          HSRIPH=0.5*(HSRIAJ+HSRIP1)                                      TRBL1075
ISN 0906          DO 2480 LZ=NLZMIN,NZ                                            TRBL1076
ISN 0907          IPXIPH(LZ)=IVXBXH(ISPB,LZ)                                      TRBL1077
ISN 0908          IF(IPXIPH(LZ).GT.0.)GO TO 2475                                  TRBL1078
               C********** CLOSED IPH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1079
ISN 0910          VLXIPH(LZ)=0.0                                                  TRBL1080
ISN 0911          VLYIPH(LZ)=0.0                                                  TRBL1081
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ISN 0912          VLZIPH(LZ)=0.0                                                    TRBL1082
ISN 0913          GVXIPH(LZ)=0.0                                                    TRBL1083
                 C********** TURBULENT SHEAR STRESS COMPONENTS ON IPH BOUNDARY **********TRBL1084
                 C********** SURFACE ARE CALCULATED AS GPYIPH AND GPYIPH        **********TRBL1085
                 C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM    **********TRBL1086
                 C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM   **********TRBL1087
                 C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON      **********TRBL1088
                 C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL1089
                 C********** IN THE ELEMENT                                     **********TRBL1090
ISN 0914          VISKIN=VISREF                                                     TRBL1091
ISN 0915          EVLIPH=EVLLOW                                                     TRBL1092
ISN 0916          VFLSQR=VLYIAJ(LZ)**2+VLZIAJ(LZ)**2                                TRBL1093
ISN 0917          IF(VFLSQR.LE.0.)GO TO 2484                                        TRBL1094
ISN 0919          VFLIAJ=SQRT(VFLSQR)                                               TRBL1095
ISN 0920          REYNUM=DXIPX*VFLIAJ/VISKIN                                        TRBL1096
ISN 0921          IF(REYNUM.LT.REYTRB)GO TO 2484                                    TRBL1097
ISN 0923          ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFF)                               TRBL1098
ISN 0924          EVLIPH=EVLIPH+ETRIPH                                              TRBL1099
ISN 0925       2484 CONTINUE                                                        TRBL1100
ISN 0926          DHZIPH=DHZIAJ(LZ)                                                 TRBL1101
ISN 0927          ARXVMY=DYJMY*DHZIPH                                               TRBL1102
ISN 0928          ARXVPY=DYJPY*DHZIPH                                               TRBL1103
ISN 0929          ARXVLZ=DYIAJ*DHZIPH                                               TRBL1104
ISN 0930          GMYIPH(LZ)=ARXVMY*(EVLIPH*VMYIAJ(LZ)/(0.5*DXJMY))                 TRBL1105
ISN 0931          GPYIPH(LZ)=ARXVPY*(EVLIPH*VPYIAJ(LZ)/(0.5*DXJPY))                 TRBL1106
ISN 0932          GVZIPH(LZ)=ARXVLZ*(EVLIPH*VLZIAJ(LZ)/DXIPX)                       TRBL1107
ISN 0933          GO TO 2480                                                        TRBL1108
ISN 0934       2475 CONTINUE                                                        TRBL1109
                 C********** OPEN IPH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1110
ISN 0935          VLXPXH=VLXBXH(ISPB,LZ)                                            TRBL1111
ISN 0936          VLXIPH(LZ)=VLXPXH                                                 TRBL1112
ISN 0937          VMYPXH=VMYBXH(ISPB,LZ)                                            TRBL1113
ISN 0938          VPYPXH=VPYBXH(ISPB,LZ)                                            TRBL1114
ISN 0939          VLYIPH(LZ)=0.5*(VMYPXH+VPYPXH)                                    TRBL1115
ISN 0940          VLZPXH=VLZBXH(ISPB,LZ)                                            TRBL1116
ISN 0941          VLZIPH(LZ)=VLZPXH                                                 TRBL1117
                 C********** TURBULENT TRANSPORT RATES ON THE IPH SURFACE ARE  **********TRBL1118
                 C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL1119
                 C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1120
                 C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1121
                 C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL1122
                 C********** COMPONENTS IN THE ELEMENTS AND IN THE EXTERNAL    **********TRBL1123
                 C********** OPEN REGION ELEMENT WITH SPECIFIED VELOCITY       **********TRBL1124
                 C********** AND OTHER FLOW CONDITIONS                         **********TRBL1125
ISN 0942          VISKIN=VISREF                                                     TRBL1126
ISN 0943          EVLIPH=EVLLOW                                                     TRBL1127
ISN 0944          ETHIPH=ETHLOW                                                     TRBL1128
ISN 0945          ESLIPH=ESLLOW                                                     TRBL1129
ISN 0946          VFLSQR=(VLYIAJ(LZ)-VLYIPH(LZ))**2+(VLZIAJ(LZ)-VLZIPH(LZ))**2      TRBL1130
ISN 0947          IF(VFLSQR.LE.0.)GO TO 2488                                        TRBL1131
ISN 0949          VFLIAJ=SQRT(VFLSQR)                                               TRBL1132
ISN 0950          REYNUM=DXIAJ*VFLIAJ/VISKIN                                        TRBL1133
ISN 0951          IF(REYNUM.LT.REYTRB)GO TO 2488                                    TRBL1134
ISN 0953          ETRIPH=VISKIN*CFRNFR*(REYNUM**PNFF)                               TRBL1135
ISN 0954          EVLIPH=EVLIPH+ETRIPH                                              TRBL1136
ISN 0955          ETHIPH=ETHIPH+ETRIPH                                              TRBL1137
ISN 0956          ESLIPH=ESLIPH+ETRIPH                                              TRBL1138
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ISN 0957      2488 CONTINUE                                                    TRBL1139
ISN 0958           DHZIPH=DHZIAJ(LZ)                                          TRBL1140
ISN 0959           IF(LZ.EQ.NZ)  DHZIPH=DHZIPH-HSRIAJ+HSRIPH                  TRBL1141
ISN 0961           ARXIPH=DYIPH*DHZIPH                                        TRBL1142
ISN 0962           ARXVMY=DYJMY*DHZIPH                                        TRBL1143
ISN 0963           ARXVPY=DYJPY*DHZIPH                                        TRBL1144
ISN 0964           GVXIPH(LZ)=ARXIPH*(-EVLIPH*(VLXPXH                         TRBL1145
                  1                          -VPXIAJ(LZ))/DXIPX)              TRBL1146
ISN 0965           GMYIPH(LZ)=ARXVMY*(-EVLIPH*(VMYPXH                         TRBL1147
                  1                          -VMYIAJ(LZ))/DXJMY)              TRBL1148
ISN 0966           GPYIPH(LZ)=ARXVPY*(-EVLIPH*(VPYPXH                         TRBL1149
                  1                          -VPYIAJ(LZ))/DXJPY)              TRBL1150
ISN 0967           GVZIPH(LZ)=ARXIPH*(-EVLIPH*(VLZPXH                         TRBL1151
                  1                          -VLZIAJ(LZ))/DXIAJ)              TRBL1152
ISN 0968      2480 CONTINUE                                                    TRBL1153
ISN 0969      2500 CONTINUE                                                    TRBL1154
              C********** START JMH BOUNDARY CALCULATIONS                     **********TRBL1155
ISN 0970           JMHCON=JMHBND(IBND)+2                                      TRBL1156
ISN 0971           GO TO (2510,2530,2550,2570),JMHCON                         TRBL1157
ISN 0972      2510 CONTINUE                                                    TRBL1158
              C********** INTERNAL JMH BOUNDARY                               **********TRBL1159
              C********** JMH ENCLOSURE SURFACE TURBULENT TRANSPORT           **********TRBL1160
              C********** TRANSFERRED FROM PREVIOUS JPH CALCULATIONS          **********TRBL1161
              C********** FOR ENCLOSURE SURFACES THAT ARE NOT CLOSED          **********TRBL1162
              C********** BY THE SHORELINE BOUNDARY IN LZ LEVEL               **********TRBL1163
ISN 0973           IJJM1=IJIXJY(IX,JY-1)                                      TRBL1164
ISN 0974           NLZJM1=NLZBOT(IJJM1)                                       TRBL1165
ISN 0975           DO 2520 LZ=NLZMIN,NZ                                       TRBL1166
ISN 0976           IF(LZ.GE.NLZJM1)GO TO 2525                                 TRBL1167
              C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM            **********TRBL1168
              C********** SHORELINE BOUNDARY                                  **********TRBL1169
ISN 0978           JMYJMH(LZ)=0                                               TRBL1170
ISN 0979           VLYJMH(LZ)=0.0                                             TRBL1171
ISN 0980           VLXJMH(LZ)=0.0                                             TRBL1172
ISN 0981           VLZJMH(LZ)=0.0                                             TRBL1173
ISN 0982           GVYJMH(LZ)=0.0                                             TRBL1174
              C********** TURBULENT SHEAR STRESS COMPONENTS ON JMH BOUNDARY   **********TRBL1175
              C********** SURFACE ARE CALCULATED AS GPXJMH AND GPXJMH          **********TRBL1176
              C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM      **********TRBL1177
              C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM     **********TRBL1178
              C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON        **********TRBL1179
              C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS    **********TRBL1180
              C********** IN THE ELEMENT                                      **********TRBL1181
ISN 0983           VISKIN=VISREF                                              TRBL1182
ISN 0984           EVLJMH=EVLLOW                                              TRBL1183
ISN 0985           VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                         TRBL1184
ISN 0986           IF(VFLSQR.LE.0.)GO TO 2524                                 TRBL1185
ISN 0988           VFLIAJ=SQRT(VFLSQR)                                        TRBL1186
ISN 0989           REYNUM=DYJMY*VFLIAJ/VISKIN                                 TRBL1187
ISN 0990           IF(REYNUM.LT.REYTRB)GO TO 2524                             TRBL1188
ISN 0992           ETRJMH=VISKIN*CFRNFR*(REYNUM**PNFR)                        TRBL1189
ISN 0993           EVLJMH=EVLJMH+ETRJMH                                       TRBL1190
ISN 0994      2524 CONTINUE                                                    TRBL1191
ISN 0995           DHZJMH=DHZIAJ(LZ)                                          TRBL1192
ISN 0996           ARYVMX=DXIMX*DHZJMH                                        TRBL1193
ISN 0997           ARYVPX=DXIPX*DHZJMH                                        TRBL1194
ISN 0998           ARYVLZ=DXIAJ*DHZJMH                                        TRBL1195
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ISN 0999          GMXJMH(LZ)=ARYVMX*(-EVLJMH*VMXIAJ(LZ)/(0.5*DYIMX))                TRBL1196
ISN 1000          GPXJMH(LZ)=ARYVPX*(-EVLJMH*VPXIAJ(LZ)/(0.5*DYIPX))                TRBL1197
ISN 1001          GVZJMH(LZ)=ARYVLZ*(-EVLJMH*VLZIAJ(LZ)/DYJMY)                      TRBL1198
ISN 1002          GO TO 2520                                                        TRBL1199
ISN 1003       2525 CONTINUE                                                        TRBL1200
               C********** JMH SURFACE AT LZ ADJASENT TO AN ELEMENT          **********TRBL1201
ISN 1004          JMYJMH(LZ)=1                                                      TRBL1202
ISN 1005          VLYJMH(LZ)=VLYTRY(LZ)                                             TRBL1203
ISN 1006          VLXJMH(LZ)=VLXTRY(LZ)                                             TRBL1204
ISN 1007          VLZJMH(LZ)=VLZTRY(LZ)                                             TRBL1205
ISN 1008          GVYJMH(LZ)=GVYTRY(LZ)                                             TRBL1206
ISN 1009          GMXJMH(LZ)=GMXTRY(LZ)                                             TRBL1207
ISN 1010          GPXJMH(LZ)=GPXTRY(LZ)                                             TRBL1208
ISN 1011          GVZJMH(LZ)=GVZTRY(LZ)                                             TRBL1209
ISN 1012       2520 CONTINUE                                                        TRBL1210
ISN 1013          GO TO 2600                                                        TRBL1211
ISN 1014       2530 CONTINUE                                                        TRBL1212
               C********** SHORELINE JMH BOUNDARY                          **********TRBL1213
ISN 1015          DO 2540 LZ=NLZMIN,NZ                                              TRBL1214
ISN 1016          JMYJMH(LZ)=0                                                      TRBL1215
ISN 1017          VLYJMH(LZ)=0.0                                                    TRBL1216
ISN 1018          VLXJMH(LZ)=0.0                                                    TRBL1217
ISN 1019          VLZJMH(LZ)=0.0                                                    TRBL1218
ISN 1020          GVYJMH(LZ)=0.0                                                    TRBL1219
               C********** TURBULENT SHEAR STRESS COMPONENTS ON JMH BOUNDARY **********TRBL1220
               C********** SURFACE ARE CALCULATED AS GPXJMH AND GPXJMH       **********TRBL1221
               C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM   ********** TRBL1222
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1223
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1224
               C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL1225
               C********** IN THE ELEMENT                                   **********TRBL1226
ISN 1021          VISKIN=VISREF                                                     TRBL1227
ISN 1022          EVLJMH=EVLLOW                                                     TRBL1228
ISN 1023          VELSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                                TRBL1229
ISN 1024          IF(VELSQR.LE.0.)GO TO 2544                                        TRBL1230
ISN 1026          VELIAJ=SQRT(VELSQR)                                               TRBL1231
ISN 1027          REYNUM=DYJMY*VELIAJ/VISKIN                                        TRBL1232
ISN 1028          IF(REYNUM.LT.REYTRB)GO TO 2544                                    TRBL1233
ISN 1030          ETRJMH=VISKIN*CFRNFR*(REYNUM**PNEF)                               TRBL1234
ISN 1031          EVLJMH=EVLJMH+ETRJMH                                              TRBL1235
ISN 1032       2544 CONTINUE                                                        TRBL1236
ISN 1033          DHZJMH=DHZIAJ(LZ)                                                 TRBL1237
ISN 1034          ARYVMX=DXIMX*DHZJMH                                               TRBL1238
ISN 1035          ARYVPX=DXIPX*DHZJMH                                               TRBL1239
ISN 1036          ARYVLZ=DXIAJ*DHZJMH                                               TRBL1240
ISN 1037          GMXJMH(LZ)=ARYVMX*(-EVLJMH*VMXIAJ(LZ)/(0.5*DYIMX))                TRBL1241
ISN 1038          GPXJMH(LZ)=ARYVPX*(-EVLJMH*VPXIAJ(LZ)/(0.5*DYIPX))                TRBL1242
ISN 1039          GVZJMH(LZ)=ARYVLZ*(-EVLJMH*VLZIAJ(LZ)/DYJMY)                      TRBL1243
ISN 1040       2540 CONTINUE                                                        TRBL1244
ISN 1041          GO TO 2600                                                        TRBL1245
ISN 1042       2550 CONTINUE                                                        TRBL1246
               C********** SPECIFIED FLOW CONDITIONS FOR JMH BOUNDARY ARE TRANSFFERED *TRBL1247
               C**********  FROM THE BOUNDS SUBROUTINE                     **********TRBL1248
ISN 1043          ISPB=ISPBND(IBND)                                                 TRBL1249
ISN 1044          HSRJMH=HSRBYH(ISPB)                                               TRBL1250
ISN 1045          DO 2560 LZ=NLZMIN,NZ                                              TRBL1251
ISN 1046          JMYJMH(LZ)=JVYBYH(ISPB,LZ)                                        TRBL1252
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ISN 1047          IF(JMYJMH(LZ).LT.0)GO TO 2555                                  TRBL1253
                C********** CLOSED JMH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1254
ISN 1049          VLYJMH(LZ)=0.0                                                 TRBL1255
ISN 1050          VLXJMH(LZ)=0.0                                                 TRBL1256
ISN 1051          VLZJMH(LZ)=0.0                                                 TRBL1257
ISN 1052          GVYJMH(LZ)=0.0                                                 TRBL1258
                C********** TURBULENT SHEAR STRESS COMPONENTS ON JMH BOUNDARY **********TRBL1259
                C********** SURFACE ARE CALCULATED AS GMXJMH AND GPXJMH        **********TRBL1260
                C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM    **********TRBL1261
                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM   **********TRBL1262
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON      **********TRBL1263
                C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL1264
                C********** IN THE ELEMENT                                    **********TRBL1265
ISN 1053          VISKIN=VISREF                                                  TRBL1266
ISN 1054          EVLJMH=EVLLOW                                                  TRBL1267
ISN 1055          VELSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                             TRBL1268
ISN 1056          IF(VELSQR.LE.0.)GO TO 2564                                     TRBL1269
ISN 1058          VELIAJ=SQRT(VELSQR)                                            TRBL1270
ISN 1059          REYNUM=DYJMY*VELIAJ/VISKIN                                     TRBL1271
ISN 1060          IF(REYNUM.LT.REYTRB)GO TO 2564                                 TRBL1272
ISN 1062          ETRJMH=VISKIN*CFRNFR*(REYNUM**PNEF)                            TRBL1273
ISN 1063          EVLJMH=EVLJMH+ETRJMH                                           TRBL1274
ISN 1064       2564 CONTINUE                                                     TRBL1275
ISN 1065          DHZJMH=DHZIAJ(LZ)                                              TRBL1276
ISN 1066          ARYVMX=DXIMX*DHZJMH                                            TRBL1277
ISN 1067          ARYVPX=DXIPX*DHZJMH                                            TRBL1278
ISN 1068          ARYVLZ=DXIAJ*DHZJMH                                            TRBL1279
ISN 1069          GMXJMH(LZ)=ARYVMX*(-EVLJMH*VMXIAJ(LZ)/(0.5*DYIMX))             TRBL1280
ISN 1070          GPXJMH(LZ)=ARYVPX*(-EVLJMH*VPXIAJ(LZ)/(0.5*DYIPX))             TRBL1281
ISN 1071          GVZJMH(LZ)=ARYVLZ*(-EVLJMH*VLZIAJ(LZ)/DYJMY)                   TRBL1282
ISN 1072          GO TO 2560                                                     TRBL1283
ISN 1073       2555 CONTINUE                                                     TRBL1284
                C********** OPEN JMH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1285
ISN 1074          VLYJMH(LZ)=VLYBYH(ISPB,LZ)                                     TRBL1286
ISN 1075          VLXJMH(LZ)=0.5*(VMXBYH(ISPB,LZ)+VPXBYH(ISPB,LZ))               TRBL1287
ISN 1076          VLZJMH(LZ)=VLZBYH(ISPB,LZ)                                     TRBL1288
ISN 1077          IF(GPYBYH(ISPB,LZ).GT.0)GO TO 2556                             TRBL1289
                C********** TURBULENT TRANSPORT IS NEGLIGIBLE FOR OUTFLOW TYPE **********TRBL1290
                C********** SPECIFIED BOUNDARY CONDITIONS ON THE JMH SURFACE  **********TRBL1291
ISN 1079          GVYJMH(LZ)=0.0                                                 TRBL1292
ISN 1080          GMXJMH(LZ)=0.0                                                 TRBL1293
ISN 1081          GPXJMH(LZ)=0.0                                                 TRBL1294
ISN 1082          GVZJMH(LZ)=0.0                                                 TRBL1295
ISN 1083          GO TO 2560                                                     TRBL1296
ISN 1084       2556 CONTINUE                                                     TRBL1297
                C********** TURBULENT TRANSPORT IS INCLUDED FOR INFLOW TYPE   **********TRBL1298
                C********** SPECIFIED BOUNDARY CONDITIONS ON THE JMH SURFACE  **********TRBL1299
ISN 1085          VLYMYH=VLYBYH(ISPB,LZ)                                         TRBL1300
ISN 1086          VMXMYH=VMXBYH(ISPB,LZ)                                         TRBL1301
ISN 1087          VPXMYH=VPXBYH(ISPB,LZ)                                         TRBL1302
ISN 1088          VLZMYH=VLZBYH(ISPB,LZ)                                         TRBL1303
                C********** TURBULENT TRANSPORT RATES ON THE JMH SURFACE ARE  **********TRBL1304
                C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL1305
                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1306
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1307
                C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL1308
                C********** COMPONENTS IN THE ELEMENT AND AT THE              **********TRBL1309
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                C********** SPECIFIED INFLOW CONDITIONS                          **********TRBL1310
ISN 1089              VISKIN=VISREF                                                      TRBL1311
ISN 1090              EVLJMH=EVLLOW                                                      TRBL1312
ISN 1091              ETHJMH=ETHLOW                                                      TRBL1313
ISN 1092              ESLJMH=ESLLOW                                                      TRBL1314
ISN 1093              VFLSQR=(VLXIAJ(LZ)-VLXJMH(LZ))**2+(VLZIAJ(LZ)-VLZJMH(LZ))**2       TRBL1315
ISN 1094              IF(VFLSQR.LE.0.)GO TO 2568                                         TRBL1316
ISN 1096              VFLIAJ=SQRT(VFLSQR)                                                TRBL1317
ISN 1097              REYNUM=DYIAJ*VFLIAJ/VISKIN                                         TRBL1318
ISN 1098              IF(REYNUM.LT.REYTRB)GO TO 2568                                     TRBL1319
ISN 1100              ETRJMH=VISKIN*CFRNFR*(REYNUM**PNFF)                                TRBL1320
ISN 1101              EVLJMH=EVLJMH+ETRJMH                                               TRBL1321
ISN 1102              ETHJMH=ETHJMH+ETRJMH                                               TRBL1322
ISN 1103              ESLJMH=ESLJMH+ETRJMH                                               TRBL1323
ISN 1104             2568 CONTINUE                                                       TRBL1324
ISN 1105              DHZJMH=DHZIAJ(LZ)                                                  TRBL1325
ISN 1106              ARYJMH=DXJMH*DHZJMH                                                TRBL1326
ISN 1107              ARYVMX=DXIMX*DHZJMH                                                TRBL1327
ISN 1108              ARYVPX=DXIPX*DHZJMH                                                TRBL1328
ISN 1109              GVYJMH(LZ)=ARYJMH*(-EVLJMH*(VMYIAJ(LZ)                             TRBL1329
                     1                                -VLYMYH)/DYJMY)                    TRBL1330
ISN 1110              GPXJMH(LZ)=ARYVMX*(-EVLJMH*(VMXIAJ(LZ)                             TRBL1331
                     1                                -VMXMYH)/DYIMX)                    TRBL1332
ISN 1111              GPXJMH(LZ)=ARYVPX*(-EVLJMH*(VPXIAJ(LZ)                             TRBL1333
                     1                                -VPXMYH)/DYIPX)                    TRBL1334
ISN 1112              GVZJMH(LZ)=ARYJMH*(-EVLJMH*(VLZIAJ(LZ)                             TRBL1335
                     1                                -VLZMYH)/DYIAJ)                    TRBL1336
ISN 1113             2560 CONTINUE                                                       TRBL1337
ISN 1114              GO TO 2600                                                         TRBL1338
ISN 1115             2570 CONTINUE                                                       TRBL1339
                C********** SECIFIED OPEN REGION CONDITIONS FOR JMH BOUNDARY  **********TRBL1340
                C********** ARE TRANSFERRED FROM THE BOUNDS SUBROUTINE        **********TRBL1341
ISN 1116              ISPB=ISPBND(IBND)                                                  TRBL1342
ISN 1117              HSRJM1=HSRBYH(ISPB)                                                TRBL1343
ISN 1118              HSRJMH=0.5*(HSRJM1+HSRIAJ)                                         TRBL1344
ISN 1119              DO 2580 LZ=NLZMIN,NZ                                               TRBL1345
ISN 1120              JMYJMH(LZ)=JVYBYH(ISPB,LZ)                                         TRBL1346
ISN 1121              IF(JMYJMH(LZ).GT.0.)GO TO 2575                                     TRBL1347
                C********** CLOSED JMH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1348
ISN 1123              VLYJMH(LZ)=0.0                                                     TRBL1349
ISN 1124              VLXJMH(LZ)=0.0                                                     TRBL1350
ISN 1125              VLZJMH(LZ)=0.0                                                     TRBL1351
ISN 1126              GVYJMH(LZ)=0.0                                                     TRBL1352
                C********** TURBULENT SHEAR STRESS COMPONENTS ON JMH BOUNDARY **********TRBL1353
                C********** SURFACE ARE CALCULATED AS GPXJMH AND GPXJMH        **********TRBL1354
                C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM    **********TRBL1355
                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM   **********TRBL1356
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON      **********TRBL1357
                C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS  **********TRBL1358
                C********** IN THE ELEMENT                                     **********TRBL1359
ISN 1127              VISKIN=VISREF                                                      TRBL1360
ISN 1128              EVLJMH=EVLLOW                                                      TRBL1361
ISN 1129              VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                                 TRBL1362
ISN 1130              IF(VFLSQR.LE.0.)GO TO 2584                                         TRBL1363
ISN 1132              VFLIAJ=SQRT(VFLSQR)                                                TRBL1364
ISN 1133              REYNUM=DYJMY*VFLIAJ/VISKIN                                         TRBL1365
ISN 1134              IF(REYNUM.LT.REYTRB)GO TO 2584                                     TRBL1366
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ISN 1136          ETRJMH=VISKIN*CFRNFR*(REYNUM**PNFF)                                    TRBL1367
ISN 1137          EVLJMH=EVLJMH+ETRJMH                                                   TRBL1368
ISN 1138       2584 CONTINUE                                                             TRBL1369
ISN 1139          DHZJMH=DHZIAJ(LZ)                                                      TRBL1370
ISN 1140          ARYVMX=DXIMX*DHZJMH                                                    TRBL1371
ISN 1141          ARYVPX=DXIPX*DHZJMH                                                    TRBL1372
ISN 1142          ARYVLZ=DXIAJ*DHZJMH                                                    TRBL1373
ISN 1143          GMXJMH(LZ)=ARYVMX*(-EVLJMH*VMXIAJ(LZ)/(0.5*DYIMX))                     TRBL1374
ISN 1144          GPXJMH(LZ)=ARYVPX*(-EVLJMH*VPXIAJ(LZ)/(0.5*DYIPX))                     TRBL1375
ISN 1145          GVZJMH(LZ)=ARYVLZ*(-EVLJMH*VLZIAJ(LZ)/DYJMY)                           TRBL1376
ISN 1146          GO TO 2580                                                             TRBL1377
ISN 1147       2575 CONTINUE                                                             TRBL1378
                  C********** OPEN JMH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1379
ISN 1148          VLYMYH=VLYBYH(ISPB,LZ)                                                 TRBL1380
ISN 1149          VLYJMH(LZ)=VLYMYH                                                      TRBL1381
ISN 1150          VMXMYH=VMXBYH(ISPB,LZ)                                                 TRBL1382
ISN 1151          VPXMYH=VPXBYH(ISPB,LZ)                                                 TRBL1383
ISN 1152          VLXJMH(LZ)=0.5*(VMXMYH+VPXMYH)                                         TRBL1384
ISN 1153          VLZMYH=VLZBYH(ISPB,LZ)                                                 TRBL1385
ISN 1154          VLZJMH(LZ)=VLZMYH                                                      TRBL1386
                  C********** TURBULENT TRANSPORT RATES ON THE JMH SURFACE ARE  **********TRBL1387
                  C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL1388
                  C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1389
                  C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1390
                  C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL1391
                  C********** COMPONENTS IN THE ELEMENTS AND IN THE EXTERNAL    **********TRBL1392
                  C********** OPEN REGION ELEMENT WITH SPECIFIED VELOCITY       **********TRBL1393
                  C********** AND OTHER FLOW CONDITIONS                         **********TRBL1394
ISN 1155          VISKIN=VISREF                                                          TRBL1395
ISN 1156          EVLJMH=EVLLOW                                                          TRBL1396
ISN 1157          ETHJMH=ETHLOW                                                          TRBL1397
ISN 1158          ESLJMH=ESLLOW                                                          TRBL1398
ISN 1159          VFLSQR=(VLXIAJ(LZ)-VLXJMH(LZ))**2+(VLZIAJ(LZ)-VLZJMH(LZ))**2          TRBL1399
ISN 1160          IF(VFLSQR.LE.0.)GO TO 2588                                             TRBL1400
ISN 1162          VFLIAJ=SQRT(VFLSQR)                                                    TRBL1401
ISN 1163          REYNUM=DYIAJ*VFLIAJ/VISKIN                                             TRBL1402
ISN 1164          IF(REYNUM.LT.REYTRB)GO TO 2588                                         TRBL1403
ISN 1166          ETRJMH=VISKIN*CFRNFR*(REYNUM**PNFF)                                    TRBL1404
ISN 1167          EVLJMH=EVLJMH+ETRJMH                                                   TRBL1405
ISN 1168          ETHJMH=ETHJMH+ETRJMH                                                   TRBL1406
ISN 1169          ESLJMH=ESLJMH+ETRJMH                                                   TRBL1407
ISN 1170       2588 CONTINUE                                                             TRBL1408
ISN 1171          DHZJMH=DHZIAJ(LZ)                                                      TRBL1409
ISN 1172          IF(LZ.EQ.NZ)  DHZJMH=DHZJMH-HSRIAJ+HSRJMH                              TRBL1410
ISN 1174          ARYJMH=DXJMH*DHZJMH                                                    TRBL1411
ISN 1175          ARYVMX=DXIMX*DHZJMH                                                    TRBL1412
ISN 1176          ARYVPX=DXIPX*DHZJMH                                                    TRBL1413
ISN 1177          GVYJMH(LZ)=ARYJMH*(-EVLJMH*(VMYIAJ(LZ)                                 TRBL1414
                 1                          -VLYMYH)/DYJMY)                            TRBL1415
ISN 1178          GMXJMH(LZ)=ARYVMX*(-EVLJMH*(VMXIAJ(LZ)                                 TRBL1416
                 1                          -VMXMYH)/DYIMX)                            TRBL1417
ISN 1179          GPXJMH(LZ)=ARYVPX*(-EVLJMH*(VPXIAJ(LZ)                                 TRBL1418
                 1                          -VPXMYH)/DYIPX)                            TRBL1419
ISN 1180          GVZJMH(LZ)=ARYJMH*(-EVLJMH*(VLZIAJ(LZ)                                 TRBL1420
                 1                          -VLZMYH)/DYIAJ)                            TRBL1421
ISN 1181       2580 CONTINUE                                                             TRBL1422
ISN 1182       2600 CONTINUE                                                             TRBL1423
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                C********** START JPH BOUNDARY CALCULATIONS                    **********TRBL1424
ISN 1183              JPHCON=JPHBND(IBND)+2                                          TRBL1425
ISN 1184              GO TO (2610,2630,2650,2670),JPHCON                             TRBL1426
ISN 1185       2610 CONTINUE                                                         TRBL1427
                C********** INTERNAL JPH BOUNDARY                                 **********TRBL1428
                C********** JPH ENCLOSURE SURFACE TURBULENT TRANSPORT             **********TRBL1429
                C********** TRANSFERRED FROM PREVIOUS JPH CALCULATIONS            **********TRBL1430
                C********** FOR ENCLOSURE SURFACES THAT ARE NOT CLOSED            **********TRBL1431
                C********** BY THE SHORELINE BOUNDARY IN LZ LEVEL                **********TRBL1432
ISN 1186              IJJP1=IJIXJY(IX,JY+1)                                          TRBL1433
ISN 1187              JMYJP1=2*IJJP1-1                                               TRBL1434
ISN 1188              IPXJP1=JMYJP1                                                  TRBL1435
ISN 1189              IPXJP1=IMXJP1+1                                                TRBL1436
ISN 1190              IF(INTEXT(IX,JY+1).GT.0)GO TO 2612                             TRBL1437
                C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT          **********TRBL1438
ISN 1192              DYJP1=DY(JY+1)                                                 TRBL1439
ISN 1193              DXJP1=DXIAJ                                                    TRBL1440
ISN 1194              ARHJP1=DXJP1*DYJP1                                             TRBL1441
ISN 1195              GO TO 2615                                                     TRBL1442
ISN 1196       2612 CONTINUE                                                         TRBL1443
                C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT           **********TRBL1444
ISN 1197              IBNJP1=INTEXT(IX,JY+1)                                         TRBL1445
ISN 1198              DYJP1=DYBND(IBNJP1)                                            TRBL1446
ISN 1199              ARHJP1=ARHBND(IBNJP1)                                          TRBL1447
ISN 1200       2615 CONTINUE                                                         TRBL1448
ISN 1201              DYJPH=0.5*(DYIAJ+DYJP1)                                        TRBL1449
ISN 1202              CCYJPH=0.5*DYIAJ/DYJPH                                         TRBL1450
ISN 1203              HSRJP1=HSR(IJJP1)                                              TRBL1451
ISN 1204              NLZJP1=NLZBOT(IJJP1)                                           TRBL1452
ISN 1205              DO 2620 LZ=NLZMIN,NZ                                           TRBL1453
ISN 1206              IF(LZ.GE.NLZJP1)GO TO 2625                                     TRBL1454
                C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM             **********TRBL1455
                C********** SHORELINE BOUNDARY                                   **********TRBL1456
ISN 1208              JPYJPH(LZ)=0                                                   TRBL1457
ISN 1209              VLYJPH(LZ)=0.0                                                 TRBL1458
ISN 1210              VLXJPH(LZ)=0.0                                                 TRBL1459
ISN 1211              VLZJPH(LZ)=0.0                                                 TRBL1460
ISN 1212              GVYJPH(LZ)=0.0                                                 TRBL1461
                C********** TURBULENT SHEAR STRESS COMPONENTS ON JPH BOUNDARY   **********TRBL1462
                C********** SURFACE ARE CALCULATED AS GPXJPH AND GPXJPH          **********TRBL1463
                C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM      **********TRBL1464
                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM     **********TRBL1465
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON        **********TRBL1466
                C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS    **********TRBL1467
                C********** IN THE ELEMENT                                       **********TRBL1468
ISN 1213              VISKIN=VISREF                                                  TRBL1469
ISN 1214              EVLJPH=EVLLOW                                                  TRBL1470
ISN 1215              VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                             TRBL1471
ISN 1216              IF(VFLSQR.LE.0.)GO TO 2624                                     TRBL1472
ISN 1218              VFLIAJ=SQRT(VFLSQR)                                            TRBL1473
ISN 1219              REYNUM=DYJPY*VFLIAJ/VISKIN                                     TRBL1474
ISN 1220              IF(REYNUM.LT.REYTRB)GO TO 2624                                 TRBL1475
ISN 1222              ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFF)                            TRBL1476
ISN 1223              EVLJPH=EVLJPH+ETRJPH                                           TRBL1477
ISN 1224       2624 CONTINUE                                                         TRBL1478
ISN 1225              DHZJPH=DHZIAJ(LZ)                                              TRBL1479
ISN 1226              ARYVMX=DXIMX*DHZJPH                                            TRBL1480
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ISN 1227          ARYVPX=DXIPX*DHZJPH                                                    TRBL1481
ISN 1228          ARYVLZ=DXIAJ*DHZJPH                                                    TRBL1482
ISN 1229          GPXJPH(LZ)=ARYVHX*(EVLJPH*VMXIAJ(LZ)/(0.5*DYIMX))                      TRBL1483
ISN 1230          GPXJPH(LZ)=ARYVPX*(EVLJPH*VPXIAJ(LZ)/(0.5*DYIPX))                      TRBL1484
ISN 1231          GVZJPH(LZ)=ARYVLZ*(EVLJPH*VLZIAJ(LZ)/DYJPY)                            TRBL1485
ISN 1232          GO TO 2620                                                             TRBL1486
ISN 1233       2625 CONTINUE                                                             TRBL1487
               C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********TRBL1488
               C********** TURBULENT TRANSPORT RATES ON THE JPH SURFACE ARE  **********TRBL1489
               C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL1490
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1491
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1492
               C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL1493
               C********** COMPONENTS IN THE ADJACENT ELEMENTS               **********TRBL1494
ISN 1234          JPYJPH(LZ)=1                                                           TRBL1495
ISN 1235          DHZJP1=DZ(LZ)                                                          TRBL1496
ISN 1236          IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                                 TRBL1497
ISN 1238          IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                                     TRBL1498
ISN 1240          DHZJPH=DHZIAJ(LZ)+CDYJPH*(DHZJP1-DHZIAJ(LZ))                           TRBL1499
ISN 1241          ARYJPH=DXJPH*DHZJPH                                                    TRBL1500
ISN 1242          VMYJP1=VLY(JMYJP1,LZ)                                                  TRBL1501
ISN 1243          VLYJPH(LZ)=VPYIAJ(LZ)+CDYJPH*(VMYJP1-VPYIAJ(LZ))                       TRBL1502
ISN 1244          VMXJP1=VLX(IMXJP1,LZ)                                                  TRBL1503
ISN 1245          VPXJP1=VLX(IPXJP1,LZ)                                                  TRBL1504
ISN 1246          VLXJP1=0.5*(VMXJP1+VPXJP1)                                             TRBL1505
ISN 1247          VLXJPH(LZ)=VLXIAJ(LZ)+CDYJPH*(VLXJP1-VLXIAJ(LZ))                       TRBL1506
               C********** CALCULATE VLZJP1 WITH GWZLMH AND GWZLPH EVALUATED AT IJJP1 *TRBL1507
ISN 1248          GWZLMH=GWZLZH(IJJP1,LZ)                                                TRBL1508
ISN 1249          IF(LZ.EQ.NZ)GO TO 2627                                                 TRBL1509
ISN 1251          GWZLPH=GWZLZH(IJJP1,LZ+1)                                              TRBL1510
ISN 1252          GO TO 2628                                                             TRBL1511
ISN 1253       2627 CONTINUE                                                             TRBL1512
ISN 1254          GWZLPH=ARHJP1*FNCHSR(IJJP1)                                            TRBL1513
ISN 1255       2628 CONTINUE                                                             TRBL1514
ISN 1256          GWZJP1=0.5*(GWZLMH+GWZLPH)                                             TRBL1515
ISN 1257          VLZJP1=GWZJP1/ARHJP1                                                   TRBL1516
ISN 1258          VLZJPH(LZ)=VLZIAJ(LZ)+CDYJPH*(VLZJP1-VLZIAJ(LZ))                       TRBL1517
ISN 1259          IF(DYIAJ.GT.DYJP1)GO TO 2632                                           TRBL1518
ISN 1261          DYHYH=DYIAJ                                                            TRBL1519
ISN 1262          CDYHYH=DYHYH/DYJPH                                                     TRBL1520
ISN 1263          VLXMYH=VLXIAJ(LZ)                                                      TRBL1521
ISN 1264          VLZMYH=VLZIAJ(LZ)                                                      TRBL1522
ISN 1265          VLXPYH=VLXIAJ(LZ)+CDYHYH*(VLXJP1-VLXIAJ(LZ))                           TRBL1523
ISN 1266          VLZPYH=VLZIAJ(LZ)+CDYHYH*(VLZJP1-VLZIAJ(LZ))                           TRBL1524
ISN 1267          GO TO 2635                                                             TRBL1525
ISN 1268       2632 CONTINUE                                                             TRBL1526
ISN 1269          DYHYH=DYJP1                                                            TRBL1527
ISN 1270          CDYHYH=DYHYH/DYJPH                                                     TRBL1528
ISN 1271          VLXMYH=VLXJP1-CDYHYH*(VLXJP1-VLXIAJ(LZ))                               TRBL1529
ISN 1272          VLZMYH=VLZJP1-CDYHYH*(VLZJP1-VLZIAJ(LZ))                               TRBL1530
ISN 1273          VLXPYH=VLXJP1                                                          TRBL1531
ISN 1274          VLZPYH=VLZJP1                                                          TRBL1532
ISN 1275       2635 CONTINUE                                                             TRBL1533
ISN 1276          VISKIN=VISREF                                                          TRBL1534
ISN 1277          EVLJPH=EVLLOW                                                          TRBL1535
ISN 1278          ETHJPH=ETHLOW                                                          TRBL1536
ISN 1279          ESLJPH=ESLLOW                                                          TRBL1537
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ISN 1280          VFLSQR=(VLXPYH-VLXMYH)**2+(VLZPYH-VLZMYH)**2                    TRBL1538
ISN 1281          IF(VFLSQR.LE.0.)GO TO 2638                                      TRBL1539
ISN 1283          VFLMYH=SQRT(VFLSQR)                                             TRBL1540
ISN 1284          REYNUM=DYMYH*VFLMYH/VISKIN                                      TRBL1541
ISN 1285          IF(REYNUM.LT.REYTRB)GO TO 2638                                  TRBL1542
ISN 1287          ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFR)                             TRBL1543
ISN 1288          EVLJPH=EVLJPH+ETRJPH                                            TRBL1544
ISN 1289          ETHJPH=ETHJPH+ETRJPH                                            TRBL1545
ISN 1290          ESLJPH=ESLJPH+ETRJPH                                            TRBL1546
ISN 1291       2638 CONTINUE                                                      TRBL1547
ISN 1292          ARYVMX=DXIMX*DHZJPH                                             TRBL1548
ISN 1293          ARYVPX=DXIPX*DHZJPH                                             TRBL1549
ISN 1294          GVYJPH(LZ)=ARYJPH*(-EVLJPH                                      TRBL1550
                 1           *(VMYJP1-VPYIAJ(LZ))/(0.5*DYJPH))                    TRBL1551
ISN 1295          GPXJPH(LZ)=ARYVMX*(-EVLJPH                                      TRBL1552
                 1           *(VMXJP1-VMXIAJ(LZ))/DYJPH)                          TRBL1553
ISN 1296          GPXJPH(LZ)=ARYVPX*(-EVLJPH                                      TRBL1554
                 1           *(VPXJP1-VPXIAJ(LZ))/DYJPH)                          TRBL1555
ISN 1297          GVZJPH(LZ)=ARYJPH*(-EVLJPH                                      TRBL1556
                 1           *(VLZJP1-VLZIAJ(LZ))/DYJPH)                          TRBL1557
ISN 1298       2620 CONTINUE                                                      TRBL1558
ISN 1299          GO TO 2700                                                      TRBL1559
ISN 1300       2630 CONTINUE                                                      TRBL1560
               C********** SHORELINE JPH BOUNDARY                  **********TRBL1561
ISN 1301          DO 2640 LZ=NLZMIN,NZ                                            TRBL1562
ISN 1302          JPYJPH(LZ)=0                                                    TRBL1563
ISN 1303          VLYJPH(LZ)=0.0                                                  TRBL1564
ISN 1304          VLXJPH(LZ)=0.0                                                  TRBL1565
ISN 1305          VLZJPH(LZ)=0.0                                                  TRBL1566
ISN 1306          GVYJPH(LZ)=0.0                                                  TRBL1567
               C********** TURBULENT SHEAR STRESS COMPONENTS ON JPH BOUNDARY **********TRBL1568
               C********** SURFACE ARE CALCULATED AS GPXJPH AND GPXJPH         **********TRBL1569
               C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM     **********TRBL1570
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM    **********TRBL1571
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON       **********TRBL1572
               C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS   **********TRBL1573
               C********** IN THE ELEMENT                                     **********TRBL1574
ISN 1307          VISKIN=VISREF                                                   TRBL1575
ISN 1308          EVLJPH=EVLLOW                                                   TRBL1576
ISN 1309          VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                              TRBL1577
ISN 1310          IF(VFLSQR.LE.0.)GO TO 2644                                      TRBL1578
ISN 1312          VFLIAJ=SQRT(VFLSQR)                                             TRBL1579
ISN 1313          REYNUM=DYJPY*VFLIAJ/VISKIN                                      TRBL1580
ISN 1314          IF(REYNUM.LT.REYTRB)GO TO 2644                                  TRBL1581
ISN 1316          ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFR)                             TRBL1582
ISN 1317          EVLJPH=EVLJPH+ETRJPH                                            TRBL1583
ISN 1318       2644 CONTINUE                                                      TRBL1584
ISN 1319          DHZJPH=DHZIAJ(LZ)                                               TRBL1585
ISN 1320          ARYVMX=DXIMX*DHZJPH                                             TRBL1586
ISN 1321          ARYVPX=DXIPX*DHZJPH                                             TRBL1587
ISN 1322          ARYVLZ=DXIAJ*DHZJPH                                             TRBL1588
ISN 1323          GPXJPH(LZ)=ARYVMX*(EVLJPH*VMXIAJ(LZ)/(0.5*DYIMX))               TRBL1589
ISN 1324          GPXJPH(LZ)=ARYVPX*(EVLJPH*VPXIAJ(LZ)/(0.5*DYIPX))               TRBL1590
ISN 1325          GVZJPH(LZ)=ARYVLZ*(EVLJPH*VLZIAJ(LZ)/DYJPY)                     TRBL1591
ISN 1326       2640 CONTINUE                                                      TRBL1592
ISN 1327          GO TO 2700                                                      TRBL1593
ISN 1328       2650 CONTINUE                                                      TRBL1594
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                C********** SPECIFIED FLOW CONDITIONS FOR JPH BOUNDARY ARE TRANSFFERED *TRBL1595
                C********** FROM THE BOUNDS SUBROUTINE                              **********TRBL1596
ISN 1329              ISPB=ISPBND(IBND)                                                    TRBL1597
ISN 1330              HSRJPH=HSRBYH(ISPB)                                                  TRBL1598
ISN 1331              DO 2660 LZ=NLZMIN,NZ                                                 TRBL1599
ISN 1332              JFYJPH(LZ)=JVYBYH(ISPB,LZ)                                           TRBL1600
ISN 1333              IF(JFYJPH(LZ).LT.0)GO TO 2655                                        TRBL1601
                C********** CLOSED JPH ENCLOSURE SURFACE AT LZ LEVEL                    **********TRBL1602
ISN 1335              VLYJPH(LZ)=0.0                                                       TRBL1603
ISN 1336              VLXJPH(LZ)=0.0                                                       TRBL1604
ISN 1337              VLZJPH(LZ)=0.0                                                       TRBL1605
ISN 1338              GVYJPH(LZ)=0.0                                                       TRBL1606
                C********** TURBULENT SHEAR STRESS COMPONENTS ON JPH BOUNDARY **********TRBL1607
                C********** SURFACE ARE CALCULATED AS GPXJPH AND GPXJPH        **********TRBL1608
                C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM    **********TRBL1609
                C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM   **********TRBL1610
                C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON      **********TRBL1611
                C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS  **********TRBL1612
                C********** IN THE ELEMENT                                     **********TRBL1613
ISN 1339              VISKIN=VISREF                                                        TRBL1614
ISN 1340              EVLJPH=EVLLOW                                                        TRBL1615
ISN 1341              VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                                   TRBL1616
ISN 1342              IF(VFLSQR.LE.0.)GO TO 2664                                           TRBL1617
ISN 1344              VFLIAJ=SQRT(VFLSQR)                                                  TRBL1618
ISN 1345              REYNUM=DYJPY*VFLIAJ/VISKIN                                           TRBL1619
ISN 1346              IF(REYNUM.LT.REYTRB)GO TO 2664                                       TRBL1620
ISN 1348              ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFR)                                  TRBL1621
ISN 1349              EVLJPH=EVLJPH+ETRJPH                                                 TRBL1622
ISN 1350       2664 CONTINUE                                                               TRBL1623
ISN 1351              DHZJPH=DHZIAJ(LZ)                                                    TRBL1624
ISN 1352              ARYVMX=DXIMX*DHZJPH                                                  TRBL1625
ISN 1353              ARYVPX=DXIPX*DHZJPH                                                  TRBL1626
ISN 1354              ARYVLZ=DXIAJ*DHZJPH                                                  TRBL1627
ISN 1355              GMXJPH(LZ)=ARYVMX*(EVLJPH*VMXIAJ(LZ)/(0.5*DYIMX))                    TRBL1628
ISN 1356              GPXJPH(LZ)=ARYVPX*(EVLJPH*VPXIAJ(LZ)/(0.5*DYIPX))                    TRBL1629
ISN 1357              GVZJPH(LZ)=ARYVLZ*(EVLJPH*VLZIAJ(LZ)/DYJPY)                          TRBL1630
ISN 1358              GO TO 2660                                                           TRBL1631
ISN 1359       2655 CONTINUE                                                               TRBL1632
                C********** OPEN JPH ENCLOSURE SURFACE AT LZ LEVEL                     **********TRBL1633
ISN 1360              VLYJPH(LZ)=VLYBYH(ISPB,LZ)                                           TRBL1634
ISN 1361              VLXJPH(LZ)=0.5*(VMXBYH(ISPB,LZ)+VPXBYH(ISPB,LZ))                     TRBL1635
ISN 1362              VLZJPH(LZ)=VLZBYH(ISPB,LZ)                                           TRBL1636
ISN 1363              IF(GFYBYH(ISPB,LZ).LT.0)GO TO 2656                                   TRBL1637
                C********** TURBULENT TRANSPORT IS NEGLIGIBLE FOR OUTFLOW TYPE *********TRBL1638
                C********** SPECIFIED BOUNDARY CONDITIONS ON THE JPH SURFACE   **********TRBL1639
ISN 1365              GVYJPH(LZ)=0.0                                                       TRBL1640
ISN 1366              GMXJPH(LZ)=0.0                                                       TRBL1641
ISN 1367              GPXJPH(LZ)=0.0                                                       TRBL1642
ISN 1368              GVZJPH(LZ)=0.0                                                       TRBL1643
ISN 1369              GO TO 2660                                                           TRBL1644
ISN 1370       2656 CONTINUE                                                               TRBL1645
                C********** TURBULENT TRANSPORT IS INCLUDED FOR INFLOW TYPE    **********TRBL1646
                C********** SPECIFIED BOUNDARY CONDITIONS ON THE JPH SURFACE   **********TRBL1647
ISN 1371              VLYPYH=VLYBYH(ISPB,LZ)                                               TRBL1648
ISN 1372              VMXPYH=VMXBYH(ISPB,LZ)                                               TRBL1649
ISN 1373              VPXPYH=VPXBYH(ISPB,LZ)                                               TRBL1650
ISN 1374              VLZPYH=VLZBYH(ISPB,LZ)                                               TRBL1651
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C********** TURBULENT TRANSPORT RATES ON THE JPH SURFACE ARE  **********TRBL1652
C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL1653
C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1654
C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1655
C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL1656
C********** COMPONENTS IN THE ELEMENT AND AT THE              **********TRBL1657
C********** SPECIFIED INFLOW CONDITIONS                       **********TRBL1658
ISN 1375        VISKIN=VISREF                                             TRBL1659
ISN 1376        EVLJPH=EVLLOW                                             TRBL1660
ISN 1377        ETHJPH=ETHLOW                                             TRBL1661
ISN 1378        ESLJPH=ESLLOW                                             TRBL1662
ISN 1379        VFLSQR=(VLXIAJ(LZ)-VLXJPH(LZ))**2+(VLZIAJ(LZ)-VLZJPH(LZ))**2  TRBL1663
ISN 1380        IF(VFLSQR.LE.0.)GO TO 2668                                TRBL1664
ISN 1382        VFLIAJ=SQRT(VFLSQR)                                       TRBL1665
ISN 1383        REYNUM=DYIAJ*VFLIAJ/VISKIN                                TRBL1666
ISN 1384        IF(REYNUM.LT.REYTPB)GO TO 2668                            TRBL1667
ISN 1386        ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFR)                       TRBL1668
ISN 1387        EVLJPH=EVLJPH+ETRJPH                                      TRBL1669
ISN 1388        ETHJPH=ETHJPH+ETRJPH                                      TRBL1670
ISN 1389        ESLJPH=ESLJPH+ETRJPH                                      TRBL1671
ISN 1390     2668 CONTINUE                                                TRBL1672
ISN 1391        DHZJPH=DHZIAJ(LZ)                                         TRBL1673
ISN 1392        ARYJPH=DXJPH*DHZJPH                                       TRBL1674
ISN 1393        ARYVMX=DXIMX*DHZJPH                                       TRBL1675
ISN 1394        ARYVPX=DXIPX*DHZJPH                                       TRBL1676
ISN 1395        GVYJPH(LZ)=ARYJPH*(-EVLJPH*(VLYPYH                       TRBL1677
               1                           -VPYIAJ(LZ))/DYJPY)           TRBL1678
ISN 1396        GPXJPH(LZ)=ARYVMX*(-EVLJPH*(VMXPYH                       TRBL1679
               1                           -VMXIAJ(LZ))/DYIMX)           TRBL1680
ISN 1397        GPXJPH(LZ)=ARYVPX*(-EVLJPH*(VPXPYH                       TRBL1681
               1                           -VPXIAJ(LZ))/DYIPX)           TRBL1682
ISN 1398        GVZJPH(LZ)=ARYJPH*(-EVLJPH*(VLZPYH                       TRBL1683
               1                           -VLZIAJ(LZ))/DYIAJ)           TRBL1684
ISN 1399     2660 CONTINUE                                                TRBL1685
ISN 1400        GO TO 2700                                                TRBL1686
ISN 1401     2670 CONTINUE                                                TRBL1687
C********** SECIFIED OPEN REGION CONDITIONS FOR JPH BOUNDARY  **********TRBL1688
C********** ARE TRANSFERRED FROM THE BOUNDS SUBROUTINE        **********TRBL1689
ISN 1402        ISPB=ISPBND(IBND)                                         TRBL1690
ISN 1403        HSRJP1=HSRBYH(ISPB)                                       TRBL1691
ISN 1404        HSRJPH=0.5*(HSRIAJ+HSRJP1)                                TRBL1692
ISN 1405        DO 2680 LZ=NLZMIN,NZ                                      TRBL1693
ISN 1406        JFYJPH(LZ)=JVYBYH(ISPB,LZ)                                TRBL1694
ISN 1407        IF(JPYJPH(LZ).GT.0.)GO TO 2675                            TRBL1695
C********** CLOSED JPH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1696
ISN 1409        VLYJPH(LZ)=0.0                                            TRBL1697
ISN 1410        VLXJPH(LZ)=0.0                                            TRBL1698
ISN 1411        VLZJPH(LZ)=0.0                                            TRBL1699
ISN 1412        GVYJPH(LZ)=0.0                                            TRBL1700
C********** TURBULENT SHEAR STRESS COMPONENTS ON JPH BOUNDARY **********TRBL1701
C********** SURFACE ARE CALCULATED AS GPXJPH AND GPXJPH       **********TRBL1702
C********** ACCORDING TO THE GENERALIZED TURBULENT MOMENTUM   **********TRBL1703
C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1704
C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1705
C********** THE MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS **********TRBL1706
C********** IN THE ELEMENT                                    **********TRBL1707
ISN 1413        VISKIN=VISREF                                             TRBL1708
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ISN 1414          EVLJPH=EVLLOW                                                  TRBL1709
ISN 1415          VFLSQR=VLXIAJ(LZ)**2+VLZIAJ(LZ)**2                             TRBL1710
ISN 1416          IF(VFLSQR.LE.0.)GO TO 2684                                     TRBL1711
ISN 1418          VFLIAJ=SQRT(VFLSQR)                                            TRBL1712
ISN 1419          REYNUM=DYJPY*VFLIAJ/VISKIN                                     TRBL1713
ISN 1420          IF(REYNUM.LT.REYTRB)GO TO 2684                                 TRBL1714
ISN 1422          ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFP)                            TRBL1715
ISN 1423          EVLJPH=EVLJPH+ETRJPH                                           TRBL1716
ISN 1424       2684 CONTINUE                                                     TRBL1717
ISN 1425          DHZJPH=DHZIAJ(LZ)                                              TRBL1718
ISN 1426          ARYVMX=DXIMX*DHZJPH                                            TRBL1719
ISN 1427          ARYVPX=DXIPX*DHZJPH                                            TRBL1720
ISN 1428          ARYVLZ=DXIAJ*DHZJPH                                            TRBL1721
ISN 1429          GMXJPH(LZ)=ARYVMX*(EVLJPH*VMXIAJ(LZ)/(0.5*DYIMX))              TRBL1722
ISN 1430          CPXJPH(LZ)=ARYVPX*(EVLJPH*VPXIAJ(LZ)/(0.5*DYIPX))              TRBL1723
ISN 1431          GVZJPH(LZ)=ARYVLZ*(EVLJPH*VLZIAJ(LZ)/DYJPY)                    TRBL1724
ISN 1432          GO TO 2680                                                     TRBL1725
ISN 1433       2675 CONTINUE                                                     TRBL1726
               C********** OPEN JPH ENCLOSURE SURFACE AT LZ LEVEL          **********TRBL1727
ISN 1434          VLYPYH=VLYBYH(ISPB,LZ)                                         TRBL1728
ISN 1435          VLYJPH(LZ)=VLYPYH                                              TRBL1729
ISN 1436          VMXPYH=VMXBYH(ISPB,LZ)                                         TRBL1730
ISN 1437          VPXPYH=VPXBYH(ISPB,LZ)                                         TRBL1731
ISN 1438          VLXJPH(LZ)=0.5*(VMXPYH+VPXPYH)                                 TRBL1732
ISN 1439          VLZPYH=VLZBYH(ISPB,LZ)                                         TRBL1733
ISN 1440          VLZJPH(LZ)=VLZPYH                                              TRBL1734
               C********** TURBULENT TRANSPORT RATES ON THE JPH SURFACE ARE  **********TRBL1735
               C********** CALCULATED ACCORDING TO THE GENERALIZED TURBULENT **********TRBL1736
               C********** TRANSPORT MODEL WITH BLASIUS POWER LAW TYPE FORM  **********TRBL1737
               C********** AND DEPENDING ON THE REYNOLDS NUMBER BASED ON     **********TRBL1738
               C********** THE MAGNITUDE OF THE DIFFERENCES OF THE VELOCITY  **********TRBL1739
               C********** COMPONENTS IN THE ELEMENTS AND IN THE EXTERNAL    **********TRBL1740
               C********** OPEN REGION ELEMENT WITH SPECIFIED VELOCITY       **********TRBL1741
               C********** AND OTHER FLOW CONDITIONS                         **********TRBL1742
ISN 1441          VISKIN=VISREF                                                  TRBL1743
ISN 1442          EVLJPH=EVLLOW                                                  TRBL1744
ISN 1443          ETHJPH=ETHLOW                                                  TRBL1745
ISN 1444          ESLJPH=ESLLOW                                                  TRBL1746
ISN 1445          VFLSQR=(VLXIAJ(LZ)-VLXJPH(LZ))**2+(VLZIAJ(LZ)-VLZJPH(LZ))**2   TRBL1747
ISN 1446          IF(VFLSQR.LE.0.)GO TO 2688                                     TRBL1748
ISN 1448          VFLIAJ=SQRT(VFLSQR)                                            TRBL1749
ISN 1449          REYNUM=DYIAJ*VFLIAJ/VISKIN                                     TRBL1750
ISN 1450          IF(REYNUM.LT.REYTRB)GO TO 2688                                 TRBL1751
ISN 1452          ETRJPH=VISKIN*CFRNFR*(REYNUM**PNFP)                            TRBL1752
ISN 1453          EVLJPH=EVLJPH+ETRJPH                                           TRBL1753
ISN 1454          ETHJPH=ETHJPH+ETRJPH                                           TRBL1754
ISN 1455          ESLJPH=ESLJPH+ETRJPH                                           TRBL1755
ISN 1456       2688 CONTINUE                                                     TRBL1756
ISN 1457          DHZJPH=DHZIAJ(LZ)                                              TRBL1757
ISN 1458          IF(LZ.EQ.NZ)  DHZJPH=DHZJPH-HSRIAJ+HSRJPH                      TRBL1758
ISN 1460          ARYJPH=DXJPH*DHZJPH                                            TRBL1759
ISN 1461          ARYVMX=DXIMX*DHZJPH                                            TRBL1760
ISN 1462          ARYVPX=DXIPX*DHZJPH                                            TRBL1761
ISN 1463          GVYJPH(LZ)=ARYJPH*(-EVLJPH*(VLYPYH                             TRBL1762
                 1                          -VPYIAJ(LZ))/DYJPY)                 TRBL1763
ISN 1464          GMXJPH(LZ)=ARYVMX*(-EVLJPH*(VMXPYH                             TRBL1764
                 1                          -VMXIAJ(LZ))/DYIMX)                 TRBL1765
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ISN 1465          GPXJPH(LZ)=ARYVPX*(-EVLJPH*(VPXPYH                                    TRBL1766
                 1                          -VPXIAJ(LZ))/DYIPX)                       TRBL1767
ISN 1466          GVZJPH(LZ)=ARYJPH*(-EVLJPH*(VLZPYH                                    TRBL1768
                 1                          -VLZIAJ(LZ))/DYIAJ)                       TRBL1769
ISN 1467       2680 CONTINUE                                                           TRBL1770
ISN 1468       2700 CONTINUE                                                           TRBL1771
               C********** START MOMENTUM TRANSPORT CALCULATIONS FOR THE HALF ELEMENTS TRBL1772
               C********** IN THE DISCRETE ELEMENT FOR VLX AND VLY SOLUTIONS **********TRBL1773
ISN 1469          DO 2720 LZ=NLZMIN,NZ                                                 TRBL1774
ISN 1470          VISKIN=VISREF                                                        TRBL1775
ISN 1471          EVLMXP=EVLLOW                                                        TRBL1776
ISN 1472          EVLMYP=EVLLOW                                                        TRBL1777
ISN 1473          VFLSQR=(VLYIPH(LZ)-VLYIMH(LZ))**2+(VLZIPH(LZ)-VLZIMH(LZ))**2        TRBL1778
ISN 1474          IF(VFLSQR.LE.0.)GO TO 2724                                           TRBL1779
ISN 1476          VFLXHX=SQRT(VFLSQR)                                                  TRBL1780
ISN 1477          REYNUM=DXIAJ*VFLXHX/VISKIN                                           TRBL1781
ISN 1478          IF(REYNUM.LT.REYTRB)GO TO 2724                                       TRBL1782
ISN 1480          ETRXHX=VISKIN*CFRNFR*(REYNUM**PNFR)                                  TRBL1783
ISN 1481          EVLMXP=EVLMXP+ETRXHX                                                 TRBL1784
ISN 1482       2724 CONTINUE                                                           TRBL1785
ISN 1483          ARXIAJ=DYIAJ*DHZIAJ(LZ)                                              TRBL1786
ISN 1484          GVXIAJ(LZ)=ARXIAJ*(-EVLMXP                                           TRBL1787
                 1                   *(VPXIAJ(LZ)-VMXIAJ(LZ))/(0.5*DXIAJ))           TRBL1788
ISN 1485          VFLSQR=(VLXJPH(LZ)-VLXJMH(LZ))**2+(VLZJPH(LZ)-VLZJMH(LZ))**2        TRBL1789
ISN 1486          IF(VFLSQR.LE.0.)GO TO 2728                                           TRBL1790
ISN 1488          VFLYHY=SQRT(VFLSQR)                                                  TRBL1791
ISN 1489          REYNUM=DYIAJ*VFLYHY/VISKIN                                           TRBL1792
ISN 1490          IF(REYNUM.LT.REYTRB)GO TO 2728                                       TRBL1793
ISN 1492          ETRYHY=VISKIN*CFRNFR*(REYNUM**PNFR)                                  TRBL1794
ISN 1493          EVLMYP=EVLMYP+ETRYHY                                                 TRBL1795
ISN 1494       2728 CONTINUE                                                           TRBL1796
ISN 1495          ARYIAJ=DXIAJ*DHZIAJ(LZ)                                              TRBL1797
ISN 1496          GVYIAJ(LZ)=ARYIAJ*(-EVLMYP                                           TRBL1798
                 1                   *(VPYIAJ(LZ)-VMYIAJ(LZ))/(0.5*DYIAJ))           TRBL1799
ISN 1497       2720 CONTINUE                                                           TRBL1800
               C********** START CALCULATING THE VERTICAL TURBULENT TRANSPORT *********TRBL1801
               C********** ON THE LMH AND LPH SURFACES AND THE SOLUTION FUNCTIONS *****TRBL1802
               C********** REDUCTION OF THE VERTICAL TRANSPORT DUE TO STRATIFICATION **TRBL1803
               C********** EFFECT IS INCLUDED BY CONSIDERING THE LOCAL RICHARSON ******TRBL1804
               C********** NUMBER FOR SCALING THE LOCAL REYNOLDS NUMBER      **********TRBL1805
               C********** START CALCULATING THE BOTTOM CONDITIONS (LPH)     **********TRBL1806
               C********** DENSITY STRATIFICATION EFFECTS ARE NOT INCLUDED IN *********TRBL1807
               C********** THE BOTTOM LAYER LZ=NLZMIN IN CALCULATING THE SHEAR ********TRBL1808
               C********** STRESS COMPONENTS                                 **********TRBL1809
               C********** TURBULENT SHEAR STRESS COMPONENTS ON BOTTOM(LPH)  **********TRBL1810
               C********** BOUNDARY SURFACE ARE CALCULATED ACCORDING         **********TRBL1811
               C********** TO THE GENERALIZED TURBULENT MOMENTUM TRANSPORT   **********TRBL1812
               C********** MODEL WITH BLASIUS POWER LAW TYPE FORM AND        **********TRBL1813
               C********** DEPENDING ON THE REYNOLDS NUMBER BASED ON THE     **********TRBL1814
               C********** MAGNITUDE OF THE RELATIVE VELOCITY COMPONENTS     **********TRBL1815
               C********** IN THE ELEMENT                                   **********TRBL1816
ISN 1498          LZ=NLZMIN                                                            TRBL1817
ISN 1499          DHZLPH=0.5*DHZIAJ(LZ)                                                TRBL1818
ISN 1500          VISKIN=VISREF                                                        TRBL1819
ISN 1501          EVLLPH=EVLLOW                                                        TRBL1820
ISN 1502          VFLSQR=VLXIAJ(LZ)**2+VLYIAJ(LZ)**2                                   TRBL1821
ISN 1503          IF(VFLSQR.LE.0.)GO TO 2764                                           TRBL1822
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ISN 1505            VFLIAJ=SQRT(VFLSQR)                                                  TRBL1823
ISN 1506            REYNUM=DHZLPH*VFLIAJ/VISKIN                                          TRBL1824
ISN 1507            IF(REYNUM.LT.REYTRB)GO TO 2764                                       TRBL1825
ISN 1509            ETRLPH=VISKIN*CFRNFP*(REYNUM**PNFP)                                  TRBL1826
ISN 1510            EVLLPH=EVLLPH+ETRLPH                                                 TRBL1827
ISN 1511         2764 CONTINUE                                                           TRBL1828
ISN 1512            GMXLPH=ARHIMX*(-EVLLPH*VMXIAJ(LZ)/DHZLPH)                            TRBL1829
ISN 1513            GPXLPH=ARHIPX*(-EVLLPH*VPXIAJ(LZ)/DHZLPH)                            TRBL1830
ISN 1514            GMYLPH=ARHJMY*(-EVLLPH*VMYIAJ(LZ)/DHZLPH)                            TRBL1831
ISN 1515            GPYLPH=ARHJPY*(-EVLLPH*VPYIAJ(LZ)/DHZLPH)                            TRBL1832
ISN 1516            GVZLPH=0.0                                                           TRBL1833
ISN 1517            DO 2800 LZ=NLZMIN,NZ                                                 TRBL1834
ISN 1518            DHZIJL=DHZIAJ(LZ)                                                    TRBL1835
ISN 1519            GMXLMH=GMXLPH                                                        TRBL1836
ISN 1520            GPXLMH=GPXLPH                                                        TRBL1837
ISN 1521            GMYLMH=GMYLPH                                                        TRBL1838
ISN 1522            GPYLMH=GPYLPH                                                        TRBL1839
ISN 1523            GVZLMH=GVZLPH                                                        TRBL1840
ISN 1524            IF(LZ.LT.NZ)GO TO 2820                                               TRBL1841
                 C********** TURBULENT TRANSPORT ON THE WATER SURFACE CALCULATED IN *****TRBL1842
                 C********** THE SURFACE SUBROUTINE BASED ON INPUT DATA FOR SURFACE *****TRBL1843
                 C********** AND ATMOSPHERIC CONDITIONS                  **********TRBL1844
ISN 1526            GMXLPH=0.0                                                           TRBL1845
ISN 1527            GPXLPH=0.0                                                           TRBL1846
ISN 1528            GMYLPH=0.0                                                           TRBL1847
ISN 1529            GPYLPH=0.0                                                           TRBL1848
ISN 1530            GVZLPH=0.0                                                           TRBL1849
ISN 1531            GO TO 2680                                                           TRBL1850
ISN 1532         2820 CONTINUE                                                           TRBL1851
ISN 1533            LZP1=LZ+1                                                            TRBL1852
ISN 1534            DHZLP1=DHZIAJ(LZP1)                                                  TRBL1853
ISN 1535            DHZLPH=0.5*(DHZIJL+DHZLP1)                                           TRBL1854
ISN 1536            CDZLPH=0.5*DHZIJL/DHZLPH                                             TRBL1855
ISN 1537            IF(DHZIJL.GT.DHZLP1)GO TO 2822                                       TRBL1856
ISN 1539            DHZHZH=DHZIJL                                                        TRBL1857
ISN 1540            CDZHZH=DHZHZH/DHZLPH                                                 TRBL1858
ISN 1541            VLXMZH=VLXIAJ(LZ)                                                    TRBL1859
ISN 1542            VLYMZH=VLYIAJ(LZ)                                                    TRBL1860
ISN 1543            VLXPZH=VLXIAJ(LZ)+CDZHZH*(VLXIAJ(LZP1)-VLXIAJ(LZ))                  TRBL1861
ISN 1544            VLYPZH=VLYIAJ(LZ)+CDZHZH*(VLYIAJ(LZP1)-VLYIAJ(LZ))                  TRBL1862
ISN 1545            GO TO 2825                                                           TRBL1863
ISN 1546         2822 CONTINUE                                                           TRBL1864
ISN 1547            DHZHZH=DHZLP1                                                        TRBL1865
ISN 1548            CDZHZH=DHZHZH/DHZLPH                                                 TRBL1866
ISN 1549            VLXMZH=VLXIAJ(LZP1)-CDZHZH*(VLXIAJ(LZP1)-VLXIAJ(LZ))                TRBL1867
ISN 1550            VLYMZH=VLYIAJ(LZP1)-CDZHZH*(VLYIAJ(LZP1)-VLYIAJ(LZ))                TRBL1868
ISN 1551            VLXPZH=VLXIAJ(LZP1)                                                  TRBL1869
ISN 1552            VLYPZH=VLYIAJ(LZP1)                                                  TRBL1870
ISN 1553         2825 CONTINUE                                                           TRBL1871
ISN 1554            VISKIN=VISREF                                                        TRBL1872
ISN 1555            EVLLPH=EVLLOW                                                        TRBL1873
ISN 1556            ETHLPH=ETHLOW                                                        TRBL1874
ISN 1557            ESLLPH=ESLLOW                                                        TRBL1875
ISN 1558            VFLSQR=(VLXPZH-VLXMZH)**2+(VLYPZH-VLYMZH)**2                         TRBL1876
ISN 1559            IF(VFLSQR.LE.0.)GO TO 2838                                           TRBL1877
ISN 1561            VFLHZH=SQRT(VFLSQR)                                                  TRBL1878
ISN 1562            REYNUM=DHZHZH*VFLHZH/VISKIN                                          TRBL1879
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ISN 1563          IF(REYNUM.LT.REYTRB)GO TO 2838                                    TRBL1889
ISN 1565          ETRLPH=VISKIN*CFRNFR*(REYNUM**PNFP)                               TRBL1891
ISN 1566          EVLLPH=EVLLPH+ETRLPH                                              TRBL1892
ISN 1567          ETHLPH=ETHLPH+ETRLPH                                              TRBL1893
ISN 1568          ESLLPH=ESLLPH+ETRLPH                                              TRBL1894
ISN 1569       2838 CONTINUE                                                        TRBL1895
ISN 1570          GMXLPH=ARHIMX*(-EVLLPH*(VMXIAJ(LZF1)                              TRBL1896
                 1                       -VMXIAJ(LZ))/DHZLPH)                      TRBL1897
ISN 1571          GPXLPH=ARHIPX*(-EVLLPH*(VPXIAJ(LZF1)                              TRBL1898
                 1                       -VPXIAJ(LZ))/DHZLPH)                      TRBL1899
ISN 1572          GMYLPH=ARHJMY*(-EVLLPH*(VMYIAJ(LZF1)                              TRBL1890
                 1                       -VMYIAJ(LZ))/DHZLPH)                      TRBL1891
ISN 1573          GPYLPH=ARHJPY*(-EVLLPH*(VPYIAJ(LZF1)                              TRBL1892
                 1                       -VPYIAJ(LZ))/DHZLPH)                      TRBL1893
ISN 1574          GVZLPH=ARHIAJ*(-EVLLPH*(VLZIAJ(LZF1)                              TRBL1894
                 1                       -VLZIAJ(LZ))/DHZLPH)                      TRBL1895
ISN 1575       2880 CONTINUE                                                        TRBL1896
                C********** START MODIFYING THE SOLUTION FUNCTIONS FOR          **********TRBL1897
                C********** TURBULENT TRANSPORT                                  **********TRBL1898
ISN 1576          IF(IMXIMH(LZ).LT.1)GO TO 2882                                     TRBL1899
ISN 1578          DNMVMX=ARHIMX*DHZIJL                                              TRBL1900
ISN 1579          FNCVLX(IMX,LZ)=FNCVLX(IMX,LZ)+(1./DNMVMX)*(GVXIMH(LZ)-GVXIAJ(LZ)  TRBL1901
                 1              +GMXJMH(LZ)-GMXJPH(LZ)+GMXLMH-GMXLPH)             TRBL1902
ISN 1580       2882 CONTINUE                                                        TRBL1903
ISN 1581          IF(IPXIPH(LZ).LT.1)GO TO 2884                                     TRBL1904
ISN 1583          DNMVPX=ARHIPX*DHZIJL                                              TRBL1905
ISN 1584          FNCVLX(IPX,LZ)=FNCVLX(IPX,LZ)+(1./DNMVPX)*(GVXIAJ(LZ)-GVXIPH(LZ)  TRBL1906
                 1              +GPXJMH(LZ)-GPXJPH(LZ)+GPXLMH-GPXLPH)             TRBL1907
ISN 1585       2884 CONTINUE                                                        TRBL1908
ISN 1586          IF(JMYJMH(LZ).LT.1)GO TO 2886                                     TRBL1909
ISN 1588          DNMVMY=ARHJMY*DHZIJL                                              TRBL1910
ISN 1589          FNCVLY(JMY,LZ)=FNCVLY(JMY,LZ)+(1./DNMVMY)*(GVYJMH(LZ)-GVYIAJ(LZ)  TRBL1911
                 1              +GMYIMH(LZ)-GMYIPH(LZ)+GMYLMH-GMYLPH)             TRBL1912
ISN 1590       2886 CONTINUE                                                        TRBL1913
ISN 1591          IF(JPYJPH(LZ).LT.1)GO TO 2888                                     TRBL1914
ISN 1593          DNMVPY=ARHJPY*DHZIJL                                              TRBL1915
ISN 1594          FNCVLY(JPY,LZ)=FNCVLY(JPY,LZ)+(1./DNMVPY)*(GVYIAJ(LZ)-GVYJPH(LZ)  TRBL1916
                 1              +GPYIMH(LZ)-GPYIPH(LZ)+GPYLMH-GPYLPH)             TRBL1917
ISN 1595       2888 CONTINUE                                                        TRBL1918
ISN 1596          DNMGWZ=ARHIAJ*DHZIJL                                              TRBL1919
ISN 1597          FNCGWZ(IJ,LZ)=FNCGWZ(IJ,LZ)+(1./DNMGWZ)*(GVZIMH(LZ)-GVZIPH(LZ)  TRBL1920
                 1              +GVZJMH(LZ)-GVZJPH(LZ)+GVZLMH-GVZLPH)             TRBL1921
ISN 1598       2800 CONTINUE                                                        TRBL1922
ISN 1599          DO 2920 LZ=NLZMIN,NZ                                              TRBL1923
ISN 1600          VLXTRX(JY,LZ)=VLXIPH(LZ)                                          TRBL1924
ISN 1601          VLYTRX(JY,LZ)=VLYIPH(LZ)                                          TRBL1925
ISN 1602          VLZTRX(JY,LZ)=VLZIPH(LZ)                                          TRBL1926
ISN 1603          GVXTRX(JY,LZ)=GVXIPH(LZ)                                          TRBL1927
ISN 1604          GMYTRX(JY,LZ)=GMYIPH(LZ)                                          TRBL1928
ISN 1605          GPYTRX(JY,LZ)=GPYIPH(LZ)                                          TRBL1929
ISN 1606          GVZTRX(JY,LZ)=GVZIPH(LZ)                                          TRBL1930
ISN 1607          VLYTRY(LZ)=VLYJPH(LZ)                                             TRBL1931
ISN 1608          VLXTRY(LZ)=VLXJPH(LZ)                                             TRBL1932
ISN 1609          VLZTRY(LZ)=VLZJPH(LZ)                                             TRBL1933
ISN 1610          GVYTRY(LZ)=GVYJPH(LZ)                                             TRBL1934
ISN 1611          GMXTRY(LZ)=GMXJPH(LZ)                                             TRBL1935
ISN 1612          GPXTRY(LZ)=GPXJPH(LZ)                                             TRBL1936
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   ISN 1613            GVZTRY(LZ)=GVZJPH(LZ)                                    TRBL1937
   ISN 1614         2920 CONTINUE                                               TRBL1938
   ISN 1615         2200 CONTINUE                                               TRBL1939
   ISN 1616         2100 CONTINUE                                               TRBL1940
   ISN 1617         9999 RETURN                                                 TRBL1941
   ISN 1618              END                                                    TRBL1942

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*      SOURCE STATEMENTS =     1617 ,PROGRAM SIZE =     25260

*STATISTICS*  NO  DIAGNOSTICS GENERATED

******* END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
 ISN 0002            SUBROUTINE FORCER                                          FRCR0001
 ISN 0003            COMMON/CONTRL/KRUN,T                                       FRCR0002
 ISN 0004            COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,12),IJIXJY(18,12),  FRCR0003
                    1            NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)  FRCR0004
 ISN 0005            COMMON/GEOPEG/XLREGN,YLREGN,DX(18),DY(12),DZ( 6),DHZBOT(136)  FRCR0005
 ISN 0006            COMMON/GEOBND/IXBND( 49),JYBND( 49),                       FRCR0006
                    1            DXBND( 49),DYBND( 49),ARHBND( 49)              FRCR0007
 ISN 0007            COMMON/ENCBND/IMHBND( 49),DYBIMH( 49),IPHBND( 49),DYBIPH( 49),  FRCR0008
                    1            JMHBND( 49),DXBJMH( 49),JPHBND( 49),DXBJPH( 49)  FRCR0009
 ISN 0008            COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)             FRCR0010
 ISN 0009            COMMON/VLXSOL/VLX(272, 6),PREVLX(272, 6),FNCVLX(272, 6)    FRCR0011
 ISN 0010            COMMON/VLYSOL/VLY(272, 6),PREVLY(272, 6),FNCVLY(272, 6)    FRCR0012
 ISN 0011            COMMON/FLOVLZ/GWZLZH(136, 6),FNCGWZ(136, 6)                FRCR0013
 ISN 0012            COMMON/TMPSOL/TMP(136, 6),PRETMP(136, 6),FNCTMP(136, 6)    FRCR0014
 ISN 0013            COMMON/UPWIND/INWIND,RXWIND,RYWIND,RZWIND                  FRCR0015
 ISN 0014            COMMON/ANGLES/SINRNX,SINRNY,COSRNZ                         FRCR0016
 ISN 0015            COMMON/BNDCON/ISPBND( 49),IXBSPB(17),JYBSPB(17),           FRCR0017
                    1            HSRBXH(17),GFXBXH(17, 6),IVXBXH(17, 6),        FRCR0018
                    2            VLXBXH(17, 6),VMYBXH(17, 6),VPYBXH(17, 6),     FRCR0019
                    3            VLZBXH(17, 6),FPRBXH(17, 6),                   FRCR0020
                    4            HSRBYH(17),GFYBYH(17, 6),JVYBYH(17, 6),        FRCR0021
                    5            VLYBYH(17, 6),VMXBYH(17, 6),VPXBYH(17, 6),     FRCR0022
                    6            VLZBYH(17, 6),FPRBYH(17, 6)                    FRCR0023
 ISN 0016            DIMENSION DHZIAJ( 6),VLZIAJ( 6)                            FRCR0024
 ISN 0017            DIMENSION IMXIMH( 6),IPXIPH( 6),JMYJMH( 6),JPYJPH( 6)      FRCR0025
 ISN 0018            DIMENSION DHZIMH( 6),DHZIPH( 6),DHZJMH( 6),DHZJPH( 6)      FRCR0026
 ISN 0019            DIMENSION VMXIAJ( 6),VLXIAJ( 6),VPXIAJ( 6),                FRCR0027
                    1          VMYIAJ( 6),VLYIAJ( 6),VPYIAJ( 6)                FRCR0028
 ISN 0020            DIMENSION FPMIAJ( 6),FPPIAJ( 6),                           FRCR0029
                    1          FPMIMH( 6),FPMIPH( 6),FPPIMH( 6),FPPIPH( 6),    FRCR0030
                    2          FPMJMH( 6),FPMJPH( 6),FPPJMH( 6),FPPJPH( 6)     FRCR0031
 ISN 0021            DIMENSION DHBTRX(12),DHZTRX(12, 6),FPMTRX(12, 6),FPPTRX(12, 6),  FRCR0032
                    1          DHZTRY( 6),FPMTRY( 6),FPPTRY( 6)                FRCR0033
 ISN 0022            DIMENSION FPRLZH( 136, 6)                                  FRCR0034
 ISN 0023            EQUIVALENCE (FNCGWZ(1,1),FPRLZH(1,1))                      FRCR0035
 ISN 0024            IF(KRUN.GT.0)GO TO 2                                       FRCR0036
 ISN 0026            PI=3.1415927                                               FRCR0037
 ISN 0027            CCRVTY=9.82                                                FRCR0038
                C********** EFFECT OF THE TIME RATE OF CHANGE OF VLZ NOT INCLUDED ******FRCR0039
                C********** IN CALCULATING PRESSURE CONDITIONS                  **********FRCR0040
 ISN 0028            DVLZDT=0.0                                                 FRCR0041
 ISN 0029            READ(21,1001)INLINE,ANGRNX,ANGRNY,ANGRNZ,          INRCOR,  FRCR0042
                    1RNXCOR,RNYCOR,RNZCOR, INPUTS                              FRCR0043
 ISN 0030          1001 FORMAT(8X,I2,3(1X,F6.2),2X,I2,3(1X,E11.4),2X,I2)        FRCR0044
 ISN 0031            SINRNX=0.0                                                 FRCR0045
 ISN 0032            SINRNY=0.0                                                 FRCR0046
 ISN 0033            COSRNZ=1.0                                                 FRCR0047
 ISN 0034            IF(INLINE.LT.1)GO TO 1005                                  FRCR0048
 ISN 0036            IF(ANGRNX.LT.0.0001)GO TO 1002                             FRCR0049
 ISN 0038            RADRNX=(PI/180.)*ANGRNX                                    FRCR0050
 ISN 0039            SINRNX=SIN(RADRNX)                                         FRCR0051
 ISN 0040          1002 CONTINUE                                                FRCR0052
 ISN 0041            IF(ANGRNY.LT.0.0001)GO TO 1004                             FRCR0053
 ISN 0043            RADRNY=(PI/180.)*ANGRNY                                    FRCR0054
 ISN 0044            SINRNY=SIN(RADRNY)                                         FRCR0055
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ISN 0045      1004 CONTINUE                                                    FRCR0056
ISN 0046           IF(ANGRNZ.LT.0.0001)GO TO 1005                              FRCR0057
ISN 0047           RADRNZ=(PI/180.)*ANGRNZ                                     FRCR0058
ISN 0048           COSRNZ=COS(RADRNZ)                                          FRCR0059
ISN 0049      1005 CONTINUE                                                    FRCR0060
ISN 0050           IF(INRCOR.GT.0)GO TO 1009                                   FRCR0061
ISN 0051           RNXCOR=0.0                                                  FRCR0062
ISN 0053           RNYCOR=0.0                                                  FRCR0063
ISN 0054           RNZCOR=0.0                                                  FRCR0064
ISN 0055      1009 CONTINUE                                                    FRCR0065
ISN 0056           DHSTRY=0.0                                                  FRCR0066
ISN 0057           DO 1100 JY=1,NY                                             FRCR0067
ISN 0058           DO 1110 LZ=1,NZ                                             FRCR0068
ISN 0059           DHSTRX(JY)=0.0                                              FRCR0069
ISN 0060           DHZTRX(JY,LZ)=0.0                                           FRCR0070
ISN 0061           FPMTRX(JY,LZ)=0.0                                           FRCR0071
ISN 0062           FPPTRX(JY,LZ)=0.0                                           FRCR0072
ISN 0063           DHZTRY(LZ)=0.0                                              FRCR0073
ISN 0064           FPMTRY(LZ)=0.0                                              FRCR0074
ISN 0065           FPPTRY(LZ)=0.0                                              FRCR0075
ISN 0066      1110 CONTINUE                                                    FRCR0076
ISN 0067      1100 CONTINUE                                                    FRCR0077
ISN 0068           IF(INPUTS.LT.1)GO TO 9999                                   FRCR0078
              C********** WRITE THE INPUT DATA FOR COORDINATE INCLINATION ANGLES *****FRCR0079
              C********** AND THE CORIOLIS ACCELERATION (FORCE) PARAMETERS   **********FRCR0080
ISN 0071           IF(INLINE.GT.0)GO TO 1202                                   FRCR0081
ISN 0073           WRITE(3,1201)                                               FRCR0082
ISN 0074      1201 FORMAT(1HO,4X,8(1H*),1X,58HNO INCLINATION EFFECT ON THE FLOW CONDIFRCR0083
                  1TIONS IN THE REGION,1X,8(1H*))                             FRCR0084
ISN 0075           GO TO 1209                                                  FRCR0085
ISN 0076      1202 CONTINUE                                                    FRCR0086
ISN 0077           WRITE(3,1203)                                               FRCR0087
ISN 0078      1203 FORMAT(1HO,4X,8(1H*),1X,60HINCLINATION EFFECT ON THE FLOW CONDITIOFRCR0088
                  1NS IN THE REGION ARE:)                                     FRCR0089
ISN 0079           WRITE(3,1204)ANGRNX,ANGRNY,ANGRNZ                           FRCR0090
ISN 0080      1204 FORMAT(1HO,8X,4(1H*),1X,20H X-X ANGLE, ANGRNX =,F6.2,/,     FRCR0091
                 1           8X,4(1H*),1X,20H Y-Y ANGLE, ANGRNY =,F6.2,/,     FRCR0092
                 2           8X,4(1H*),1X,20H Z-Z ANGLE, ANGRNZ =,F6.2)       FRCR0093
ISN 0081      1209 CONTINUE                                                    FRCR0094
ISN 0082           IF(INRCOR.GT.0)GO TO 1212                                   FRCR0095
ISN 0084           WRITE(3,1211)                                               FRCR0096
ISN 0085      1211 FORMAT(1HO,4X,8(1H*),1X,55HNO CORIOLIS EFFECT ON THE FLOW CONDITIOFRCR0097
                  1NS IN THE REGION,1X,8(1H*))                                FRCR0098
ISN 0086           GO TO 9999                                                  FRCR0099
ISN 0087      1212 CONTINUE                                                    FRCR0100
ISN 0088           WRITE(3,1213)                                               FRCR0101
ISN 0089      1213 FORMAT(1HO,4X,8(1H*),1X,58HCORIOLIS EFFECTS ON THE FLOW CONDITIONSFRCR0102
                  1 IN THE REGION ARE:)                                       FRCR0103
ISN 0090           WRITE(3,1214)RNXCOR,RNYCOR,RNZCOR                           FRCR0104
ISN 0091      1214 FORMAT(1HO,8X,4(1H*),1X,30HX-CORIOLIS PARAMETER, RNXCOR =,E11.4,/,FRCR0105
                 1           8X,4(1H*),1X,30HY-CORIOLIS PARAMETER, RNYCOR =,E11.4,/,FRCR0106
                 2           8X,4(1H*),1X,30HZ-CORIOLIS PARAMETER, RNZCOR =,E11.4)  FRCR0107
ISN 0092           GO TO 9999                                                  FRCR0108
ISN 0093      2 CONTINUE                                                       FRCR0109
              C********** START CALCULATING PRESSURE AT THE LOWER LMH LEVEL **********FRCR0110
              C********** OF EACH DISCRETE ELEMENT, STARTING FROM THE WATER **********FRCR0111
              C********** SURFACE CORRESPONDING TO NZ                        **********FRCR0112
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                C********** MODIFIED PRESSURE (HEAD) UNIT AS METER            **********FRCR0113
ISN 0094              DO 2000 IX=1,NX                                                  FRCR0114
ISN 0095              DO 2000 JY=1,NY                                                  FRCR0115
ISN 0096              IF(INTEXT(IX,JY).EQ.0)GO TO 2000                                 FRCR0116
ISN 0098              IJ=IJIXJY(IX,JY)                                                 FRCR0117
ISN 0099              HSRIAJ=HSR(IJ)                                                   FRCR0118
ISN 0100              NLZMIN=NLZBOT(IJ)                                                FRCR0119
ISN 0101              DO 2010 LZ=NLZMIN,NZ                                             FRCR0120
ISN 0102              DHZDUM=DZ(LZ)                                                    FRCR0121
ISN 0103              IF(LZ.EQ.NLZMIN)  DHZDUM=DHZBOT(IJ)                              FRCR0122
ISN 0105              IF(LZ.EQ.NZ)  DHZDUM=DHZDUM+HSRIAJ                               FRCR0123
ISN 0107              DHZIAJ(LZ)=DHZDUM                                                FRCR0124
ISN 0108             2010 CONTINUE                                                     FRCR0125
ISN 0109              FPRLPH=0.0                                                       FRCR0126
ISN 0110              NLZD=NZ-NLZMIN+1                                                 FRCR0127
ISN 0111              DO 2020 LZD=1,NLZD                                               FRCR0128
ISN 0112              LZ=NZ-LZD+1                                                      FRCR0129
                C********** CALCULATE THE HYDROSTATIC PRESSURE EFFECT          **********FRCR0130
ISN 0113              FPRLMH=FPRLPH+COSRNZ*DHZIAJ(LZ)                                  FRCR0131
                C********** CALCULATE THE EFFECT OF TIME RATE OF CHANGE OF VLZ **********FRCR0132
ISN 0114              FPRLMH=FPRLMH+(1./(CGRVTY*DHZIAJ(LZ)))*DVLZDT                    FRCR0133
                C********** CALCULATE THE EFFECT OF THE TOTAL CONVECTIVE AND   **********FRCR0134
                C********** NONCONVECTIVE MOMENTUM TRANSPORT ALONG Z-DIRECTION  ********FRCR0135
ISN 0115              FPRLMH=FPRLMH+FNCGWZ(IJ,LZ)/CGRVTY                               FRCR0136
ISN 0116              FPRLZH(IJ,LZ)=FPRLMH                                             FRCR0137
ISN 0117              FPRLPH=FPRLMH                                                    FRCR0138
ISN 0118             2020 CONTINUE                                                     FRCR0139
ISN 0119              IF(INPCOR.LT.1)GO TO 2099                                        FRCR0140
                C********** START CALCULATING THE CORIOLIS ACCELERATION (FORCE) ********FRCR0141
                C********** EFFECT IN THE Z-DIRECTION FOR PRESSURE,          **********FRCR0142
ISN 0121              IMX=2*IJ-1                                                       FRCR0143
ISN 0122              IPX=IMX+1                                                        FRCR0144
ISN 0123              JMY=IMX                                                          FRCR0145
ISN 0124              JPY=IPX                                                          FRCR0146
ISN 0125              DO 2040 LZ=NLZMIN,NZ                                             FRCR0147
ISN 0126              VLXIAJ(LZ)=0.5*(VLX(IMX,LZ)+VLX(IPX,LZ))                         FRCR0148
ISN 0127              VLYIAJ(LZ)=0.5*(VLY(JMY,LZ)+VLY(JPY,LZ))                         FRCR0149
ISN 0128              FPRLZH(IJ,LZ)=FPRLZH(IJ,LZ)+(DHZIAJ(LZ)/CGRVTY)                  FRCR0150
                     1                          *(RNYCOR*VLXIAJ(LZ)-RNXCOR*VLYIAJ(LZ)) FRCR0151
ISN 0129             2040 CONTINUE                                                     FRCR0152
ISN 0130             2099 CONTINUE                                                     FRCR0153
ISN 0131             2000 CONTINUE                                                     FRCR0154
                C********** START CALCULATIONS FOR PRESSURE FORCE EFFECTS ON  **********FRCR0155
                C********** THE X- AND Y- MOMENTUM EQUATIONS                 **********FRCR0156
ISN 0132              DO 2100 IX=1,NX                                                  FRCR0157
ISN 0133              DO 2200 JY=1,NY                                                  FRCR0158
ISN 0134              IF(INTEXT(IX,JY).EQ.0)GO TO 2200                                 FRCR0159
ISN 0136              IJ=IJIXJY(IX,JY)                                                 FRCR0160
ISN 0137              IMX=2*IJ-1                                                       FRCR0161
ISN 0138              IPX=IMX+1                                                        FRCR0162
ISN 0139              JMY=IMX                                                          FRCR0163
ISN 0140              JPY=IPX                                                          FRCR0164
ISN 0141              IF(INTEXT(IX,JY).GT.0)GO TO 2105                                 FRCR0165
                C********** INTERNAL DISCRETE ELEMENT GEOMETRY CONDITIONS   **********FRCR0166
ISN 0143              DXIAJ=DX(IX)                                                     FRCR0167
ISN 0144              DYIAJ=DY(JY)                                                     FRCR0168
ISN 0145              ARHIAJ=DXIAJ*DYIAJ                                               FRCR0169
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ISN 0146          DYIPH=DYIAJ                                                  FRCR0170
ISN 0147          DXJPH=DXIAJ                                                  FRCR0171
ISN 0148          DXIMX=0.5*DXIAJ                                              FRCR0172
ISN 0149          DYIMX=DYIAJ                                                  FRCR0173
ISN 0150          ARHIMX=DXIMX*DYIMX                                           FRCR0174
ISN 0151          DXIPX=0.5*DXIAJ                                              FRCR0175
ISN 0152          DYIPX=DYIAJ                                                  FRCR0176
ISN 0153          APHIPX=DXIPX*DYIPX                                           FRCR0177
ISN 0154          DYJMY=0.5*DYIAJ                                              FRCR0178
ISN 0155          DXJMY=DXIAJ                                                  FRCR0179
ISN 0156          ARHJMY=DXJMY*DYJMY                                           FRCR0180
ISN 0157          DYJPY=0.5*DYIAJ                                              FRCR0181
ISN 0158          DXJPY=DXIAJ                                                  FRCR0182
ISN 0159          ARHJPY=DXJPY*DYJPY                                           FRCR0183
ISN 0160          GO TO 2109                                                   FRCR0184
ISN 0161       2105 CONTINUE                                                   FRCR0185
               C********** BOUNDARY DISCRETE ELEMENT GEOMETRY CONDITIONS      **********FRCR0186
ISN 0162          IBND=INTEXT(IX,JY)                                           FRCR0187
ISN 0163          DXIAJ=DXBND(IBND)                                            FRCR0188
ISN 0164          DYIAJ=DYBND(IBND)                                            FRCR0189
ISN 0165          ARHIAJ=ARHBND(IBND)                                          FRCR0190
ISN 0166          DYIMH=DYBIMH(IBND)                                           FRCR0191
ISN 0167          DYIPH=DYBIPH(IBND)                                           FRCR0192
ISN 0168          DXJMH=DXBJMH(IBND)                                           FRCR0193
ISN 0169          DXJPH=DXBJPH(IBND)                                           FRCR0194
ISN 0170          DXIMX=0.5*DXIAJ                                              FRCR0195
ISN 0171          DYIMX=0.5*(DYIAJ+DYIMH)                                      FRCR0196
ISN 0172          ARHIMX=DXIMX*DYIMX                                           FRCR0197
ISN 0173          DXIPX=0.5*DXIAJ                                              FRCR0198
ISN 0174          DYIPX=0.5*(DYIAJ+DYIPH)                                      FRCR0199
ISN 0175          ARHIPX=DXIPX*DYIPX                                           FRCR0200
ISN 0176          DYJMY=0.5*DYIAJ                                              FRCR0201
ISN 0177          DXJMY=0.5*(DXIAJ+DXJMH)                                      FRCR0202
ISN 0178          ARHJMY=DXJMY*DYJMY                                           FRCR0203
ISN 0179          DYJPY=0.5*DYIAJ                                              FRCR0204
ISN 0180          DXJPY=0.5*(DXIAJ+DXJPH)                                      FRCR0205
ISN 0181          APHJPY=DXJPY*DYJPY                                           FRCR0206
ISN 0182       2109 CONTINUE                                                   FRCR0207
ISN 0183          HSRIAJ=HSR(IJ)                                               FRCR0208
ISN 0184          NLZMIN=NLZBOT(IJ)                                            FRCR0209
ISN 0185          DHBIAJ=DHZBOT(IJ)                                            FRCR0210
ISN 0186          DO 2110 LZ=NLZMIN,NZ                                         FRCR0211
ISN 0187          DHZDUM=DZ(LZ)                                                FRCR0212
ISN 0188          IF(LZ.EQ.NLZMIN)  DHZDUM=DHZBOT(IJ)                          FRCR0213
ISN 0190          IF(LZ.EQ.NZ)  DHZDUM=DHZDUM+HSRIAJ                           FRCR0214
ISN 0192          DHZIAJ(LZ)=DHZDUM                                            FRCR0215
ISN 0193          IF(LZ.LT.NZ)GO TO 2112                                       FRCR0216
ISN 0195          FFPIAJ(LZ)=0.0                                               FRCR0217
ISN 0196          GO TO 2115                                                   FRCR0218
ISN 0197       2112 CONTINUE                                                   FRCR0219
ISN 0198          FFPIAJ(LZ)=FPRLZH(IJ,LZ+1)                                   FRCR0220
ISN 0199       2115 CONTINUE                                                   FRCR0221
ISN 0200          FFMIAJ(LZ)=FPRLZH(IJ,LZ)                                     FRCR0222
ISN 0201          IF(INRCOR.LT.1)GO TO 2110                                    FRCR0223
               C********** CALCULATE THE VELOCITY COMPONENTS FOR CALCULATING **********FRCR0224
               C********** THE CORIOLIS ACCELERATION (FORCE) EFFECTS          **********FRCR0225
ISN 0203          VMXIAJ(LZ)=VLX(IMX,LZ)                                       FRCR0226
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ISN 0204          VPXIAJ(LZ)=VLX(IPX,LZ)                                                FRCR0227
ISN 0205          VMYIAJ(LZ)=VLY(JMY,LZ)                                                FRCR0228
ISN 0206          VPYIAJ(LZ)=VLY(JPY,LZ)                                                FRCR0229
ISN 0207          GWZLMH=GWZLZH(IJ,LZ)                                                  FRCR0230
ISN 0208          IF(LZ.EQ.NZ)GO TO 2116                                                FRCR0231
ISN 0210          GWZLPH=GWZLZH(IJ,LZ+1)                                                FRCR0232
ISN 0211          GO TO 2118                                                            FRCR0233
ISN 0212       2116 CONTINUE                                                            FRCR0234
ISN 0213          GWZLPH=ARHIAJ*FNCHSR(IJ)                                              FRCR0235
ISN 0214       2118 CONTINUE                                                            FRCR0236
ISN 0215          GWZIAJ=0.5*(GWZLMH+GWZLPH)                                            FRCR0237
ISN 0216          VLZIAJ(LZ)=GWZIAJ/ARHIAJ                                              FRCR0238
ISN 0217       2110 CONTINUE                                                            FRCR0239
               C********** START INTERNAL ELEMENT CALCULATIONS                          **********FRCR0240
ISN 0218          IF(INTEXT(IX,JY).GT.0)GO TO 2300                                      FRCR0241
               C********** IMH AND JMH PRESSURE FORCE CONDITIONS                        **********FRCR0242
               C********** TRANSFERRED FROM PREVIOUS IPH AND JPH CALCULATIONS           **********FRCR0243
ISN 0220          IJIM1=IJIXJY(IX-1,JY)                                                 FRCR0244
ISN 0221          NLZIM1=NLZBOT(IJIM1)                                                  FRCR0245
ISN 0222          DHBIMH=DHBTRX(JY)                                                     FRCR0246
ISN 0223          DO 2150 LZ=NLZMIN,NZ                                                  FRCR0247
ISN 0224          IF(LZ.GE.NLZIM1)GO TO 2155                                            FRCR0248
               C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM                     **********FRCR0249
               C********** SHORELINE BOUNDARY                                          **********FRCR0250
ISN 0226          IMXIMH(LZ)=0                                                          FRCR0251
ISN 0227          GO TO 2150                                                            FRCR0252
ISN 0228       2155 CONTINUE                                                            FRCR0253
               C********** IMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT               **********FRCR0254
ISN 0229          IMXIMH(LZ)=1                                                          FRCR0255
ISN 0230          DHZIMH(LZ)=DHZTRX(JY,LZ)                                              FRCR0256
ISN 0231          FPPIMH(LZ)=FPPTRX(JY,LZ)                                              FRCR0257
ISN 0232          FPMIMH(LZ)=FPMTRX(JY,LZ)                                              FRCR0258
ISN 0233       2150 CONTINUE                                                            FRCR0259
ISN 0234          IJJM1=IJIXJY(IX,JY-1)                                                 FRCR0260
ISN 0235          NLZJM1=NLZBOT(IJJM1)                                                  FRCR0261
ISN 0236          DHBJMH=DHBTRY                                                         FRCR0262
ISN 0237          DO 2160 LZ=NLZMIN,NZ                                                  FRCR0263
ISN 0238          IF(LZ.GE.NLZJM1)GO TO 2165                                            FRCR0264
               C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM                     **********FRCR0265
               C********** SHORELINE BOUNDARY                                          **********FRCR0266
ISN 0240          JMYJMH(LZ)=0                                                          FRCR0267
ISN 0241          GO TO 2160                                                            FRCR0268
ISN 0242       2165 CONTINUE                                                            FRCR0269
               C********** JMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT               **********FRCR0270
ISN 0243          JMYJMH(LZ)=1                                                          FRCR0271
ISN 0244          DHZJMH(LZ)=DHZTRY(LZ)                                                 FRCR0272
ISN 0245          FPPJMH(LZ)=FPPTRY(LZ)                                                 FRCR0273
ISN 0246          FPMJMH(LZ)=FPMTRY(LZ)                                                 FRCR0274
ISN 0247       2160 CONTINUE                                                            FRCR0275
               C********** START CALCULATIONS FOR INTERNAL IPH SURFACE                  **********FRCR0276
ISN 0248          IJIP1=IJIXJY(IX+1,JY)                                                 FRCR0277
ISN 0249          IF(INTEXT(IX+1,JY).GT.0)GO TO 2212                                    FRCR0278
               C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT                  **********FRCR0279
ISN 0251          DXIP1=DX(IX+1)                                                        FRCR0280
ISN 0252          GO TO 2215                                                            FRCR0281
ISN 0253       2212 CONTINUE                                                            FRCR0282
               C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT                   **********FRCR0283
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ISN 0254          IBNIP1=INTEXT(IX+1,JY)                                    FRCR0294
ISN 0255          DXIP1=DXBND(IBNIP1)                                       FRCR0295
ISN 0256       2215 CONTINUE                                                FRCR0296
ISN 0257          DXIPH=0.5*(DXIAJ+DXIP1)                                   FRCR0297
ISN 0258          CDXIPH=0.5*DXIAJ/DXIPH                                    FRCR0298
ISN 0259          HSRIP1=HSR(IJIP1)                                         FRCR0299
ISN 0260          NLZIP1=NLZBOT(IJIP1)                                      FRCR0300
ISN 0261          IF(NLZMIN.LT.NLZIP1)GO TO 2219                            FRCR0301
ISN 0263          IF(NLZMIN.GT.NLZIP1)GO TO 2216                            FRCR0302
ISN 0265          DHBIP1=DHZBOT(IJIP1)                                      FRCR0303
ISN 0266          GO TO 2218                                                FRCR0304
ISN 0267       2216 CONTINUE                                                FRCR0305
ISN 0268          DHBIP1=DZ(NLZMIN)                                         FRCR0306
ISN 0269       2218 CONTINUE                                                FRCR0307
ISN 0270          DHBIPH=DHBIAJ+CDXIPH*(DHBIP1-DHBIAJ)                      FRCR0308
ISN 0271       2219 CONTINUE                                                FRCR0309
ISN 0272          DO 2220 LZ=NLZMIN,NZ                                      FRCR0310
ISN 0273          IF(LZ.GE.NLZIP1)GO TO 2225                                FRCR0311
                C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM        **********FRCR0312
                C********** SHORELINE BOUNDARY                             **********FRCR0313
ISN 0275          IPXIPH(LZ)=0                                              FRCR0314
ISN 0276          GO TO 2220                                                FRCR0315
ISN 0277       2225 CONTINUE                                                FRCR0316
                C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT  **********FRCR0317
ISN 0278          IPXIPH(LZ)=1                                              FRCR0318
ISN 0279          DHZIP1=DZ(LZ)                                             FRCR0319
ISN 0280          IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                    FRCR0320
ISN 0282          IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                        FRCR0321
ISN 0284          DHZIPH(LZ)=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))          FRCR0322
ISN 0285          IF(LZ.LT.NZ)GO TO 2226                                    FRCR0323
ISN 0287          FPPIP1=0.0                                                FRCR0324
ISN 0288          GO TO 2228                                                FRCR0325
ISN 0289       2226 CONTINUE                                                FRCR0326
ISN 0290          FPPIP1=FPRLZH(IJIP1,LZ+1)                                 FRCR0327
ISN 0291       2228 CONTINUE                                                FRCR0328
ISN 0292          FPMIP1=FPRLZH(IJIP1,LZ)                                   FRCR0329
ISN 0293          FPPIPH(LZ)=FPPIAJ(LZ)+CDXIPH*(FPPIP1-FPPIAJ(LZ))          FRCR0330
ISN 0294          FPMIPH(LZ)=FPMIAJ(LZ)+CDXIPH*(FPMIP1-FPMIAJ(LZ))          FRCR0331
ISN 0295       2220 CONTINUE                                                FRCR0332
                C********** START CALCULATIONS FOR INTERNAL JPH SURFACE     **********FRCR0333
ISN 0296          IJJP1=IJIXJY(IX,JY+1)                                     FRCR0334
ISN 0297          IF(INTEXT(IX,JY+1).GT.0)GO TO 2252                        FRCR0335
                C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT     **********FRCR0336
ISN 0299          DYJP1=DY(JY+1)                                            FRCR0337
ISN 0300          GO TO 2255                                                FRCR0338
ISN 0301       2252 CONTINUE                                                FRCR0339
                C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT      **********FRCR0340
ISN 0302          IBNJP1=INTEXT(IX,JY+1)                                    FRCR0341
ISN 0303          DYJP1=DYBND(IBNJP1)                                       FRCR0342
ISN 0304       2255 CONTINUE                                                FRCR0343
ISN 0305          DYJPH=0.5*(DYIAJ+DYJP1)                                   FRCR0344
ISN 0306          CDYJPH=0.5*DYIAJ/DYJPH                                    FRCR0345
ISN 0307          HSRJP1=HSR(IJJP1)                                         FRCR0346
ISN 0308          NLZJP1=NLZBOT(IJJP1)                                      FRCR0347
ISN 0309          IF(NLZMIN.LT.NLZJP1)GO TO 2259                            FRCR0348
ISN 0311          IF(NLZMIN.GT.NLZJP1)GO TO 2256                            FRCR0349
ISN 0313          DHBJP1=DHZBOT(IJJP1)                                      FRCR0350
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ISN 0314          GO TO 2258                                                    FRCR0341
ISN 0315       2256 CONTINUE                                                    FRCR0342
ISN 0316          DHBJP1=DZ(NLZMIN)                                             FRCR0343
ISN 0317       2258 CONTINUE                                                    FRCR0344
ISN 0318          DHBJPH=DHBIAJ+CDYJPH*(DHBJP1-DHBIAJ)                          FRCR0345
ISN 0319       2259 CONTINUE                                                    FRCR0346
ISN 0320          DO 2260 LZ=NLZMIN,NZ                                          FRCR0347
ISN 0321          IF(LZ.GE.NLZJP1)GO TO 2265                                    FRCR0348
                C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM           **********FRCR0349
                C**********  SHORELINE BOUNDARY                                **********FRCR0350
ISN 0323          JPYJPH(LZ)=0                                                  FRCR0351
ISN 0324          GO TO 2260                                                    FRCR0352
ISN 0325       2265 CONTINUE                                                    FRCR0353
                C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT     **********FRCR0354
ISN 0326          JPYJPH(LZ)=1                                                  FRCR0355
ISN 0327          DHZJP1=DZ(LZ)                                                 FRCR0356
ISN 0328          IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                        FRCR0357
ISN 0330          IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                            FRCR0358
ISN 0332          DHZJPH(LZ)=DHZIAJ(LZ)+CDYJPH*(DHZJP1-DHZIAJ(LZ))              FRCR0359
ISN 0333          IF(LZ.LT.NZ)GO TO 2266                                        FRCR0360
ISN 0335          FPPJP1=0.0                                                    FRCR0361
ISN 0336          GO TO 2268                                                    FRCR0362
ISN 0337       2266 CONTINUE                                                    FRCR0363
ISN 0338          FPPJP1=FPRLZH(IJJP1,LZ+1)                                     FRCR0364
ISN 0339       2268 CONTINUE                                                    FRCR0365
ISN 0340          FPMJP1=FPRLZH(IJJP1,LZ)                                       FRCR0366
ISN 0341          FPPJPH(LZ)=FPPIAJ(LZ)+CDYJPH*(FPPJP1-FPPIAJ(LZ))              FRCR0367
ISN 0342          FPMJPH(LZ)=FPMIAJ(LZ)+CDYJPH*(FPMJP1-FPMIAJ(LZ))              FRCR0368
ISN 0343       2260 CONTINUE                                                    FRCR0369
ISN 0344          GO TO 2700                                                    FRCR0370
ISN 0345       2300 CONTINUE                                                    FRCR0371
                C********** START BOUNDARY ELEMENT CALCULATIONS                **********FRCR0372
ISN 0346          IEND=INTEXT(IX,JY)                                            FRCR0373
                C********** START IMH BOUNDARY CALCULATIONS                    **********FRCR0374
ISN 0347          IMHCON=IMHBND(IBND)+2                                         FRCR0375
ISN 0348          GO TO (2310,2340,2360,2360),IMHCON                            FRCR0376
ISN 0349       2310 CONTINUE                                                    FRCR0377
                C********** INTERNAL IMH BOUNDARY                              **********FRCR0378
                C********** IMH SURFACE PRESSURE FORCE CONDITION               **********FRCR0379
                C********** TRANSFERRED FROM PREVIOUS IPH CALCULATIONS         **********FRCR0380
ISN 0350          IJIM1=IJIXJY(IX-1,JY)                                         FRCR0381
ISN 0351          NLZIM1=NLZBOT(IJIM1)                                          FRCR0382
ISN 0352          DHBIMH=DHBTRX(JY)                                             FRCR0383
ISN 0353          DO 2320 LZ=NLZMIN,NZ                                          FRCR0384
ISN 0354          IF(LZ.GE.NLZIM1)GO TO 2325                                    FRCR0385
                C********** IMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM           **********FRCR0386
                C**********  SHORELINE BOUNDARY                                **********FRCR0387
ISN 0356          IMXIMH(LZ)=0                                                  FRCR0388
ISN 0357          GO TO 2320                                                    FRCR0389
ISN 0358       2325 CONTINUE                                                    FRCR0390
                C********** IMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT     **********FRCR0391
ISN 0359          IMXIMH(LZ)=1                                                  FRCR0392
ISN 0360          DHZIMH(LZ)=DHZTRX(JY,LZ)                                      FRCR0393
ISN 0361          FPPIMH(LZ)=FPPTRX(JY,LZ)                                      FRCR0394
ISN 0362          FPMIMH(LZ)=FPMTRX(JY,LZ)                                      FRCR0395
ISN 0363       2320 CONTINUE                                                    FRCR0396
ISN 0364          GO TO 2400                                                    FRCR0397
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ISN 0365      2340 CONTINUE                                                    FRCR0393
              C********** SHORELINE IMH BOUNDARY                ********** FRCR0395
ISN 0366            DO 2350 LZ=NLZMIN,NZ                                     FRCR0400
ISN 0367            IMXIMH(LZ)=0                                             FRCR0401
ISN 0368      2350 CONTINUE                                                  FRCR0402
ISN 0369            GO TO 2400                                               FRCR0403
ISN 0370      2360 CONTINUE                                                  FRCR0404
              C********** SPECIFIED PRESSURE CONDITIONS AND OPEN END CONDITIONS ******FRCR0405
              C********** FOR IMH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********FRCR0406
ISN 0371            ISPB=ISPBND(IBND)                                        FRCR0407
ISN 0372            HSRIMH=0.5*(HSRBXH(ISPB)+HSRIAJ)                         FRCR0408
ISN 0373            DHBIMH=DHBIAJ                                            FRCR0409
ISN 0374            DO 2370 LZ=NLZMIN,NZ                                     FRCR0410
ISN 0375            IMXIMH(LZ)=IVXBXH(ISPB,LZ)                               FRCR0411
ISN 0376            DHZDUM=DHZIAJ(LZ)                                        FRCR0412
ISN 0377            IF(LZ.EQ.NZ)  DHZDUM=DHZDUM-HSRIAJ+HSRIMH                FRCR0413
ISN 0379            DHZIMH(LZ)=DHZDUM                                        FRCR0414
ISN 0380            IF(LZ.LT.NZ)GO TO 2372                                   FRCR0415
ISN 0382            FPPIMH(LZ)=0.0                                           FRCR0416
ISN 0383            GO TO 2375                                               FRCR0417
ISN 0384      2372 CONTINUE                                                  FRCR0418
ISN 0385            FPPIMH(LZ)=FPRBXH(ISPB,LZ+1)                             FRCR0419
ISN 0386      2375 CONTINUE                                                  FRCR0420
ISN 0387            FPMIMH(LZ)=FPRBXH(ISPB,LZ)                               FRCR0421
ISN 0388      2370 CONTINUE                                                  FRCR0422
ISN 0389      2400 CONTINUE                                                  FRCR0423
              C********** START IPH BOUNDARY CALCULATIONS          ********** FRCR0424
ISN 0390            IPHCON=IPHBND(IBND)+2                                    FRCR0425
ISN 0391            GO TO (2410,2440,2460,2460),IPHCON                       FRCR0426
ISN 0392      2410 CONTINUE                                                  FRCR0427
              C********** INTERNAL IPH BOUNDARY                  ********** FRCR0428
ISN 0393            IJIP1=IJIXJY(IX+1,JY)                                    FRCR0429
ISN 0394            IF(INTEXT(IX+1,JY).GT.0)GO TO 2412                       FRCR0430
              C********** IPH SURFACE ADJACENT TO AN INTERNAL ELEMENT ********** FRCR0431
ISN 0396            DXIP1=DX(IX+1)                                           FRCR0432
ISN 0397            GO TO 2415                                               FRCR0433
ISN 0398      2412 CONTINUE                                                  FRCR0434
              C********** IPH SURFACE ADJACENT TO A BOUNDARY ELEMENT ********** FRCR0435
ISN 0399            IBNIP1=INTEXT(IX+1,JY)                                   FRCR0436
ISN 0400            DXIP1=DXBND(IBNIP1)                                      FRCR0437
ISN 0401      2415 CONTINUE                                                  FRCR0438
ISN 0402            DXIPH=0.5*(DXIAJ+DXIP1)                                  FRCR0439
ISN 0403            CDXIPH=0.5*DXIAJ/DXIPH                                   FRCR0440
ISN 0404            HSRIP1=HSR(IJIP1)                                        FRCR0441
ISN 0405            NLZIP1=NLZBOT(IJIP1)                                     FRCR0442
ISN 0406            IF(NLZMIN.LT.NLZIP1)GO TO 2419                           FRCR0443
ISN 0408            IF(NLZMIN.GT.NLZIP1)GO TO 2416                           FRCR0444
ISN 0410            DHBIP1=DHZBOT(IJIP1)                                     FRCR0445
ISN 0411            GO TO 2418                                               FRCR0446
ISN 0412      2416 CONTINUE                                                  FRCR0447
ISN 0413            DHBIP1=DZ(NLZMIN)                                        FRCR0448
ISN 0414      2418 CONTINUE                                                  FRCR0449
ISN 0415            DHBIPH=DHBIAJ+CDXIPH*(DHBIP1-DHBIAJ)                     FRCR0450
ISN 0416      2419 CONTINUE                                                  FRCR0451
ISN 0417            DO 2420 LZ=NLZMIN,NZ                                     FRCR0452
ISN 0418            IF(LZ.GE.NLZIP1)GO TO 2425                               FRCR0453
              C********** IPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM ********** FRCR0454
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                C********** SHORELINE BOUNDARY                                **********FRCR0455
ISN 0420              IPXIPH(LZ)=0                                                      FRCR0456
ISN 0421              GO TO 2420                                                        FRCR0457
ISN 0422           2425 CONTINUE                                                       FRCR0458
                C********** IPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********FRCR0459
ISN 0423              IPXIPH(LZ)=1                                                      FRCR0460
ISN 0424              DHZIP1=DZ(LZ)                                                     FRCR0461
ISN 0425              IF(LZ.EQ.NLZIP1)  DHZIP1=DHZBOT(IJIP1)                            FRCR0462
ISN 0427              IF(LZ.EQ.NZ)  DHZIP1=DHZIP1+HSRIP1                                FRCR0463
ISN 0429              DHZIPH(LZ)=DHZIAJ(LZ)+CDXIPH*(DHZIP1-DHZIAJ(LZ))                  FRCR0464
ISN 0430              IF(LZ.LT.NZ)GO TO 2426                                            FRCR0465
ISN 0432              FPPIP1=0.0                                                        FRCR0466
ISN 0433              GO TO 2428                                                        FRCR0467
ISN 0434           2426 CONTINUE                                                       FRCR0468
ISN 0435              FPPIP1=FPRLZH(IJIP1,LZ+1)                                         FRCR0469
ISN 0436           2428 CONTINUE                                                       FRCR0470
ISN 0437              FPMIP1=FPRLZH(IJIP1,LZ)                                           FRCR0471
ISN 0438              FPPIPH(LZ)=FPPIAJ(LZ)+CDXIPH*(FPPIP1-FPPIAJ(LZ))                  FRCR0472
ISN 0439              FPMIPH(LZ)=FPMIAJ(LZ)+CDXIPH*(FPMIP1-FPMIAJ(LZ))                  FRCR0473
ISN 0440           2420 CONTINUE                                                       FRCR0474
ISN 0441              GO TO 2500                                                        FRCR0475
ISN 0442           2440 CONTINUE                                                       FRCR0476
                C********** SHORELINE IPH BOUNDARY                          **********FRCR0477
ISN 0443              DO 2450 LZ=NLZMIN,NZ                                              FRCR0478
ISN 0444              IPXIPH(LZ)=0                                                      FRCR0479
ISN 0445           2450 CONTINUE                                                       FRCR0480
ISN 0446              GO TO 2500                                                        FRCR0481
ISN 0447           2460 CONTINUE                                                       FRCR0482
                C********** SPECIFIED PRESSURE CONDITIONS AND OPEN END CONDITIONS ******FRCR0483
                C********** FOR IPH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********FRCR0484
ISN 0448              ISPB=ISPBND(IBND)                                                 FRCR0485
ISN 0449              HSRIPH=0.5*(HSRIAJ+HSRBXH(ISPB))                                  FRCR0486
ISN 0450              DHBIPH=DHBIAJ                                                     FRCR0487
ISN 0451              DO 2470 LZ=NLZMIN,NZ                                              FRCR0488
ISN 0452              IPXIPH(LZ)=IVXBXH(ISPB,LZ)                                        FRCR0489
ISN 0453              DHZDUM=DHZIAJ(LZ)                                                 FRCR0490
ISN 0454              IF(LZ.EQ.NZ)  DHZDUM=DHZDUM-HSRIAJ+HSRIPH                         FRCR0491
ISN 0456              DHZIPH(LZ)=DHZDUM                                                 FRCR0492
ISN 0457              IF(LZ.LT.NZ)GO TO 2472                                            FRCR0493
ISN 0459              FPPIPH(LZ)=0.0                                                    FRCR0494
ISN 0460              GO TO 2475                                                        FRCR0495
ISN 0461           2472 CONTINUE                                                       FRCR0496
ISN 0462              FPPIPH(LZ)=FPRBXH(ISPB,LZ+1)                                      FRCR0497
ISN 0463           2475 CONTINUE                                                       FRCR0498
ISN 0464              FPMIPH(LZ)=FPRBXH(ISPB,LZ)                                        FRCR0499
ISN 0465           2470 CONTINUE                                                       FRCR0500
ISN 0466           2500 CONTINUE                                                       FRCR0501
                C********** START JMH BOUNDARY CALCULATIONS                **********FRCR0502
ISN 0467              JMHCON=JMHBND(IBND)+2                                             FRCR0503
ISN 0468              GO TO (2510,2540,2560,2560),JMHCON                                FRCR0504
ISN 0469           2510 CONTINUE                                                       FRCR0505
                C********** INTERNAL JMH BOUNDARY                           **********FRCR0506
                C********** JMH SURFACE PRESSURE FORCE CONDITION            **********FRCR0507
                C********** TRANSFERRED FROM PREVIOUS JPH CALCULATIONS      **********FRCR0508
ISN 0470              IJJM1=IJIXJY(IX,JY-1)                                             FRCR0509
ISN 0471              NLZJM1=NLZBOT(IJJM1)                                              FRCR0510
ISN 0472              DHBJMH=DHBTRY                                                     FRCR0511
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ISN 0473          DO 2520 LZ=NLZMIN,NZ                                                  FRCR0512
ISN 0474          IF(LZ.GE.NLZJMI)GO TO 2525                                            FRCR0513
                  C********** JMH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********FRCR0514
                  C********** SHORELINE BOUNDARY                              **********FRCR0515
ISN 0476          JMYJMH(LZ)=0                                                          FRCR0516
ISN 0477          GO TO 2520                                                            FRCR0517
ISN 0478        2525 CONTINUE                                                           FRCR0518
                  C********** JMH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT  **********FRCR0519
ISN 0479          JMYJMH(LZ)=1                                                          FRCR0520
ISN 0480          DHZJMH(LZ)=DHZTRY(LZ)                                                 FRCR0521
ISN 0481          FPPJMH(LZ)=FPPTRY(LZ)                                                 FRCR0522
ISN 0482          FPMJMH(LZ)=FPMTRY(LZ)                                                 FRCR0523
ISN 0483        2520 CONTINUE                                                           FRCR0524
ISN 0484          GO TO 2600                                                            FRCR0525
ISN 0485        2540 CONTINUE                                                           FRCR0526
                  C********** SHORELINE JMH BOUNDARY                          **********FRCR0527
ISN 0486          DO 2550 LZ=NLZMIN,NZ                                                  FRCR0528
ISN 0487          JMYJMH(LZ)=0                                                          FRCR0529
ISN 0488        2550 CONTINUE                                                           FRCR0530
ISN 0489          GO TO 2600                                                            FRCR0531
ISN 0490        2560 CONTINUE                                                           FRCR0532
                  C********** SPECIFIED PRESSURE CONDITIONS AND OPEN END CONDITIONS ******FRCR0533
                  C********** FOR JMH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********FRCR0534
ISN 0491          ISPB=ISPBND(IBND)                                                     FRCR0535
ISN 0492          HSRJMH=0.5*(HSRBYH(ISPB)+HSRIAJ)                                      FRCR0536
ISN 0493          DHBJMH=DHBIAJ                                                         FRCR0537
ISN 0494          DO 2570 LZ=NLZMIN,NZ                                                  FRCR0538
ISN 0495          JMYJMH(LZ)=JVYBYH(ISPB,LZ)                                            FRCR0539
ISN 0496          DHZDUM=DHZIAJ(LZ)                                                     FRCR0540
ISN 0497          IF(LZ.EQ.NZ)  DHZDUM=DHZDUM-HSRIAJ+HSRJMH                             FRCR0541
ISN 0499          DHZJMH(LZ)=DHZDUM                                                     FRCR0542
ISN 0500          IF(LZ.NE.NZ)GO TO 2572                                                FRCR0543
ISN 0502          FPPJMH(LZ)=0.0                                                        FRCR0544
ISN 0503          GO TO 2575                                                            FRCR0545
ISN 0504        2572 CONTINUE                                                           FRCR0546
ISN 0505          FPPJMH(LZ)=FPRBYH(ISPB,LZ+1)                                          FRCR0547
ISN 0506        2575 CONTINUE                                                           FRCR0548
ISN 0507          FPMJMH(LZ)=FPRBYH(ISPB,LZ)                                            FRCR0549
ISN 0508        2570 CONTINUE                                                           FRCR0550
ISN 0509        2600 CONTINUE                                                           FRCR0551
                  C********** START JPH BOUNDARY CALCULATIONS          **********FRCR0552
ISN 0510          JPHCON=JPHBND(IBND)+2                                                 FRCR0553
ISN 0511          GO TO (2610,2640,2660,2660),JPHCON                                    FRCR0554
ISN 0512        2610 CONTINUE                                                           FRCR0555
                  C********** INTERNAL JPH BOUNDARY                          **********FRCR0556
ISN 0513          IJJP1=IJIXJY(IX,JY+1)                                                 FRCR0557
ISN 0514          IF(INTEXT(IX,JY+1).GT.0)GO TO 2612                                    FRCR0558
                  C********** JPH SURFACE ADJACENT TO AN INTERNAL ELEMENT  **********FRCR0559
ISN 0516          DYJP1=DY(JY+1)                                                        FRCR0560
ISN 0517          GO TO 2615                                                            FRCR0561
ISN 0518        2612 CONTINUE                                                           FRCR0562
                  C********** JPH SURFACE ADJACENT TO A BOUNDARY ELEMENT  **********FRCR0563
ISN 0519          IBNJP1=INTEXT(IX,JY+1)                                                FRCR0564
ISN 0520          DYJP1=DYBND(IBNJP1)                                                   FRCR0565
ISN 0521        2615 CONTINUE                                                           FRCR0566
ISN 0522          DYJPH=0.5*(DYIAJ+DYJP1)                                               FRCR0567
ISN 0523          CDYJPH=0.5*DYIAJ/DYJPH                                                FRCR0568
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ISN 0524            HSRJP1=HSR(IJJP1)                                                    FRCR0569
ISN 0525            NLZJP1=NLZBOT(IJJP1)                                                 FRCR0570
ISN 0526            IF(NLZMIN.LT.NLZJP1)GO TO 2619                                       FRCR0571
ISN 0528            IF(NLZMIN.GT.NLZJP1)GO TO 2616                                       FRCR0572
ISN 0530            DHBJP1=DHZBOT(IJJP1)                                                 FRCR0573
ISN 0531            GO TO 2618                                                           FRCR0574
ISN 0532         2616 CONTINUE                                                           FRCR0575
ISN 0533            DHBJP1=DZ(NLZMIN)                                                    FRCR0576
ISN 0534         2618 CONTINUE                                                           FRCR0577
ISN 0535            DHBJPH=DHBIAJ+CDYJPH*(DHBJP1-DHBIAJ)                                 FRCR0578
ISN 0536         2619 CONTINUE                                                           FRCR0579
ISN 0537            DO 2620 LZ=NLZMIN,NZ                                                 FRCR0580
ISN 0538            IF(LZ.GE.NLZJP1)GO TO 2625                                           FRCR0581
                 C********** JPH SURFACE AT LZ LEVEL CLOSED BY BOTTOM          **********FRCR0582
                 C********** SHORELINE BOUNDARY                              **********FRCR0583
ISN 0540            JPYJPH(LZ)=0                                                         FRCR0584
ISN 0541            GO TO 2620                                                           FRCR0585
ISN 0542         2625 CONTINUE                                                           FRCR0586
                 C********** JPH SURFACE AT LZ LEVEL ADJACENT TO AN ELEMENT    **********FRCR0587
ISN 0543            JPYJPH(LZ)=1                                                         FRCR0588
ISN 0544            DHZJP1=DZ(LZ)                                                        FRCR0589
ISN 0545            IF(LZ.EQ.NLZJP1)  DHZJP1=DHZBOT(IJJP1)                               FRCR0590
ISN 0547            IF(LZ.EQ.NZ)  DHZJP1=DHZJP1+HSRJP1                                   FRCR0591
ISN 0549            DHZJPH(LZ)=DHZIAJ(LZ)+CDYJPH*(DHZJP1-DHZIAJ(LZ))                     FRCR0592
ISN 0550            IF(LZ.LT.NZ)GO TO 2626                                               FRCR0593
ISN 0552            FPPJP1=0.0                                                           FRCR0594
ISN 0553            GO TO 2628                                                           FRCR0595
ISN 0554         2626 CONTINUE                                                           FRCR0596
ISN 0555            FPPJP1=FPRLZH(IJJP1,LZ+1)                                            FRCR0597
ISN 0556         2628 CONTINUE                                                           FRCR0598
ISN 0557            FPMJP1=FPRLZH(IJJP1,LZ)                                              FRCR0599
ISN 0558            FPPJPH(LZ)=FPPIAJ(LZ)+CDYJPH*(FPPJP1-FPPIAJ(LZ))                     FRCR0600
ISN 0559            FPMJPH(LZ)=FPMIAJ(LZ)+CDYJPH*(FPMJP1-FPMIAJ(LZ))                     FRCR0601
ISN 0560         2620 CONTINUE                                                           FRCR0602
ISN 0561            GO TO 2700                                                           FRCR0603
ISN 0562         2640 CONTINUE                                                           FRCR0604
                 C********** SHORELINE JPH BOUNDARY                          **********FRCR0605
ISN 0563            DO 2650 LZ=NLZMIN,NZ                                                 FRCR0606
ISN 0564            JPYJPH(LZ)=0                                                         FRCR0607
ISN 0565         2650 CONTINUE                                                           FRCR0608
ISN 0566            GO TO 2700                                                           FRCR0609
ISN 0567         2660 CONTINUE                                                           FRCR0610
                 C********** SPECIFIED PRESSURE CONDITIONS AND OPEN END CONDITIONS ******FRCR0611
                 C********** FOR JPH BOUNDARY TRANSFERRED FROM BOUNDS SUBROUTINE ********FRCR0612
ISN 0568            ISPB=ISPBND(IBND)                                                    FRCR0613
ISN 0569            HSRJPH=0.5*(HSRIAJ+HSRBYH(ISPB))                                     FRCR0614
ISN 0570            DHBJPH=DHBIAJ                                                        FRCR0615
ISN 0571            DO 2670 LZ=NLZMIN,NZ                                                 FRCR0616
ISN 0572            JPYJPH(LZ)=JVYBYH(ISPB,LZ)                                           FRCR0617
ISN 0573            DHZDUM=DHZIAJ(LZ)                                                    FRCR0618
ISN 0574            IF(LZ.EQ.NZ)  DHZDUM=DHZDUM-HSRIAJ+HSRJPH                            FRCR0619
ISN 0576            DHZJPH(LZ)=DHZDUM                                                    FRCR0620
ISN 0577            IF(LZ.LT.NZ)GO TO 2672                                               FRCR0621
ISN 0579            FPPJPH(LZ)=0.0                                                       FRCR0622
ISN 0580            GO TO 2675                                                           FRCR0623
ISN 0581         2672 CONTINUE                                                           FRCR0624
ISN 0582            FPPJPH(LZ)=FPRBYH(ISPB,LZ+1)                                         FRCR0625
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ISN 0583       2675 CONTINUE                                                    FRCR0626
ISN 0584            FPMJPH(LZ)=FPRBYH(ISPB,LZ)                                  FRCR0627
ISN 0585       2670 CONTINUE                                                    FRCR0628
ISN 0586       2700 CONTINUE                                                    FRCR0629
ISN 0587            IF(INLINE.LT.1)GO TO 2799                                   FRCR0630
               C********** CALCULATE THE INCLINED COORDINATE ACCELERATION EFFECTS *****FRCR0631
               C********** ON THE X- AND Y- MOMENTUM EQUATIONS                 **********FRCR0632
ISN 0589            DO 2710 LZ=NLZMIN,NZ                                        FRCR0633
ISN 0590            FNCVLX(IMX,LZ)=FNCVLX(IMX,LZ)-SINFNX*CGRVTY                 FRCR0634
ISN 0591            FNCVLX(IPX,LZ)=FNCVLX(IPX,LZ)-SINFNX*CGRVTY                 FRCR0635
ISN 0592            FNCVLY(JMY,LZ)=FNCVLY(JMY,LZ)-SINFNY*CGRVTY                 FRCR0636
ISN 0593            FNCVLY(JPY,LZ)=FNCVLY(JPY,LZ)-SINFNY*CGRVTY                 FRCR0637
ISN 0594       2710 CONTINUE                                                    FRCR0638
ISN 0595       2799 CONTINUE                                                    FRCR0639
               C********** CALCULATE THE NET PRESSURE FORCE EFFECTS ON         *********FRCR0640
               C********** X- AND Y- MOMENTUM EQUATIONS                       **********FRCR0641
ISN 0596            DO 2810 LZ=NLZMIN,NZ                                        FRCR0642
ISN 0597            IF(IMXIMH(LZ).LT.1)GO TO 2820                               FRCR0643
ISN 0599            DHZIMX=0.5*(DHZIMH(LZ)+DHZIAJ(LZ))                          FRCR0644
ISN 0600            IF(LZ.GT.NLZMIN)GO TO 2815                                  FRCR0645
ISN 0602            FPRIMX=DYIMX*DHZIMX*(FPMIMH(LZ)-FPMIAJ(LZ))                 FRCR0646
ISN 0603            GO TO 2818                                                  FRCR0647
ISN 0604       2815 CONTINUE                                                    FRCR0648
ISN 0605            FPRIMX=DYIMX*DHZIMX*(0.5*(FPMIMH(LZ)+FPPIMH(LZ))            FRCR0649
                   1                          -0.5*(FPMIAJ(LZ)+FPPIAJ(LZ)))    FRCR0650
ISN 0606       2818 CONTINUE                                                    FRCR0651
ISN 0607            DNMVMX=ARHIMX*DHZIMX                                        FRCR0652
ISN 0608            FNCVLX(IMX,LZ)=FNCVLX(IMX,LZ)+(CGFVTY/DNMVMX)*FPRIMX        FRCR0653
ISN 0609       2820 CONTINUE                                                    FRCR0654
ISN 0610            IF(IPXIPH(LZ).LT.1)GO TO 2830                               FRCR0655
ISN 0612            DHZIPX=0.5*(DHZIPH(LZ)+DHZIAJ(LZ))                          FRCR0656
ISN 0613            IF(LZ.GT.NLZMIN)GO TO 2825                                  FRCR0657
ISN 0615            FPRIPX=DYIPX*DHZIPX*(FPMIAJ(LZ)-FPMIPH(LZ))                 FRCR0658
ISN 0616            GO TO 2828                                                  FRCR0659
ISN 0617       2825 CONTINUE                                                    FRCR0660
ISN 0618            FPRIPX=DYIPX*DHZIPX*(0.5*(FPMIAJ(LZ)+FPPIAJ(LZ))            FRCR0661
                   1                          -0.5*(FPMIPH(LZ)+FPPIPH(LZ)))    FRCR0662
ISN 0619       2828 CONTINUE                                                    FRCR0663
ISN 0620            DNMVPX=ARHIPX*DHZIPX                                        FRCR0664
ISN 0621            FNCVLX(IPX,LZ)=FNCVLX(IPX,LZ)+(CGFVTY/DNMVPX)*FPRIPX        FRCR0665
ISN 0622       2830 CONTINUE                                                    FRCR0666
ISN 0623            IF(JMYJMH(LZ).LT.1)GO TO 2840                               FRCR0667
ISN 0625            DHZJMY=0.5*(DHZJMH(LZ)+DHZIAJ(LZ))                          FRCR0668
ISN 0626            IF(LZ.GT.NLZMIN)GO TO 2835                                  FRCR0669
ISN 0628            FPRJMY=DXJMY*DHZJMY*(FPMJMH(LZ)-FPMIAJ(LZ))                 FRCR0670
ISN 0629            GO TO 2838                                                  FRCR0671
ISN 0630       2835 CONTINUE                                                    FRCR0672
ISN 0631            FPRJMY=DXJMY*DHZJMY*(0.5*(FPMJMH(LZ)+FPPJMH(LZ))            FRCR0673
                   1                          -0.5*(FPMIAJ(LZ)+FPPIAJ(LZ)))    FRCR0674
ISN 0632       2838 CONTINUE                                                    FRCR0675
ISN 0633            DNMVMY=ARHJMY*DHZJMY                                        FRCR0676
ISN 0634            FNCVLY(JMY,LZ)=FNCVLY(JMY,LZ)+(CGFVTY/DNMVMY)*FPRJMY        FRCR0677
ISN 0635       2840 CONTINUE                                                    FRCR0678
ISN 0636            IF(JPYJPH(LZ).LT.1)GO TO 2810                               FRCR0679
ISN 0638            DHZJPY=0.5*(DHZJPH(LZ)+DHZIAJ(LZ))                          FRCR0680
ISN 0639            IF(LZ.GT.NLZMIN)GO TO 2845                                  FRCR0681
ISN 0641            FPRJPY=DXJPY*DHZJPY*(FPMIAJ(LZ)-FPMJPH(LZ))                 FRCR0682
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ISN 0642            GO TO 2848                                                  FRCR0683
ISN 0643         2845 CONTINUE                                                  FRCR0684
ISN 0644            FPRJPY=DXJPY*DHZJPY*(C.5*(FPMIAJ(LZ)+FPPIAJ(LZ))           FRCR0685
                   1                        -C.5*(FPMJPH(LZ)+FPPJPH(LZ)))     FRCR0686
ISN 0645         2848 CONTINUE                                                  FRCR0687
ISN 0646            DNMVPY=ARHJPY*DHZJPY                                        FRCR0688
ISN 0647            FNCVLY(JPY,LZ)=FNCVLY(JPY,LZ)+(CGRVTY/DNMVPY)*FPRJPY       FRCR0689
ISN 0648         2810 CONTINUE                                                  FRCR0690
ISN 0649            IF(INRCOR.LT.1)GO TO 2900                                   FRCR0691
                  C********** CALCULATE THE NET CORIOLIS ACCELERATION FORCE EFFECTS ******FRCR0692
                  C********** ON THE X- AND Y- MOMENTUM EQUATIONS               **********FRCR0693
ISN 0651            DO 2860 LZ=NLZMIN,NZ                                        FRCR0694
                  C********** VLX,VLY AND VLZ VELOCITY COMPONENTS ARE NOT       **********FRCR0695
                  C********** SEPERATED INTO HALF-ELEMENT VALUES FOR CALCULATING **********FRCR0696
                  C********** THE CORIOLIS ACCELERATION FORCES                 **********FRCR0697
ISN 0652            FMXCOR=RNYCOR*VLZIAJ(LZ)-RNZCOR*VLYIAJ(LZ)                  FRCR0698
ISN 0653            FPXCOR=FMXCOR                                               FRCR0699
ISN 0654            FMYCOR=-RNXCOR*VLZIAJ(LZ)+RNZCOR*VLXIAJ(LZ)                 FRCR0700
ISN 0655            FPYCOR=FMYCOR                                               FRCR0701
ISN 0656            FNCVLX(IMX,LZ)=FNCVLX(IMX,LZ)+FMXCOR                        FRCR0702
ISN 0657            FNCVLX(IPX,LZ)=FNCVLX(IPX,LZ)+FPXCOR                        FRCR0703
ISN 0658            FNCVLY(JMY,LZ)=FNCVLY(JMY,LZ)+FMYCOR                        FRCR0704
ISN 0659            FNCVLY(JPY,LZ)=FNCVLY(JPY,LZ)+FPYCOR                        FRCR0705
ISN 0660         2860 CONTINUE                                                  FRCR0706
ISN 0661         2900 CONTINUE                                                  FRCR0707
ISN 0662            DHBTRX(JY)=DHBIPH                                           FRCR0708
ISN 0663            DHBTRY=DHBJPH                                               FRCR0709
ISN 0664            DO 2920 LZ=NLZMIN,NZ                                        FRCR0710
ISN 0665            DHZTRX(JY,LZ)=DHZIPH(LZ)                                    FRCR0711
ISN 0666            FPPTRX(JY,LZ)=FPPIPH(LZ)                                    FRCR0712
ISN 0667            FPMTRX(JY,LZ)=FPMIPH(LZ)                                    FRCR0713
ISN 0668            DHZTRY(LZ)=DHZJPH(LZ)                                       FRCR0714
ISN 0669            FPPTRY(LZ)=FPPJPH(LZ)                                       FRCR0715
ISN 0670            FPMTRY(LZ)=FPMJPH(LZ)                                       FRCR0716
ISN 0671         2920 CONTINUE                                                  FRCR0717
ISN 0672         2200 CONTINUE                                                  FRCR0718
ISN 0673         2100 CONTINUE                                                  FRCR0719
ISN 0674         9999 RETURN                                                    FRCR0720
ISN 0675            END                                                         FRCR0721

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =     674 ,PROGRAM SIZE =    11014

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
 ISN 0002           SUBROUTINE OUTPUT                                          OTPT0001
 ISN 0003           COMMON/CONTRL/KRUN,T                                       OTPT0002
 ISN 0004           COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,12),IJIXJY(18,12),   OTPT0003
                   1               NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)  OTPT0004
 ISN 0005           COMMON/GEOREG/XLREGN,YLREGN,DX(18),DY(12),DZ(6),DHZBOT(136)  OTPT0005
 ISN 0006           COMMON/GEOBND/IXBND(49),JYBND(49),                          OTPT0006
                   1               DXBND(49),DYBND(49),ARHBND(49)              OTPT0007
 ISN 0007           COMMON/ENGBND/IMHBND(49),DYBIMH(49),IPHBND(49),DYBIPH(49),  OTPT0008
                   1               JMHBND(49),DXBJMH(49),JPHBND(49),DXBJPH(49)  OTPT0009
 ISN 0008           COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)              OTPT0010
 ISN 0009           COMMON/VLXSOL/VLX(272,6),PREVLX(272,6),FNCVLX(272,6)        OTPT0011
 ISN 0010           COMMON/VLYSOL/VLY(272,6),PREVLY(272,6),FNCVLY(272,6)        OTPT0012
 ISN 0011           COMMON/FLOVLZ/GWZLZH(136,6),FNCGWZ(136,6)                   OTPT0013
 ISN 0012           COMMON/UPWIND/INWIND,PXWIND,RYWIND,RZWIND                   OTPT0014
 ISN 0013           COMMON/ANGLES/SINRNX,SINRNY,COSRNZ                          OTPT0015
 ISN 0014           COMMON/BNDCON/ISPBND(49),IXBSPB(17),JYBSPB(17),             OTPT0016
                   1               HSRBXH(17),GFXBXH(17,6),IVXBXH(17,6),       OTPT0017
                   2               VLXBXH(17,6),VMYBXH(17,6),VPYBXH(17,6),     OTPT0018
                   3               VLZBXH(17,6),FPRBXH(17,6),                  OTPT0019
                   4               HSRBYH(17),GFYBYH(17,6),JVYBYH(17,6),       OTPT0020
                   5               VLYBYH(17,6),VMXBYH(17,6),VPXBYH(17,6),     OTPT0021
                   6               VLZBYH(17,6),FPRBYH(17,6)                   OTPT0022
 ISN 0015           IF(KRUN.GT.0)GO TO 2                                        OTPT0023
 ISN 0017           READ(21,1001)IXPLAN,JYPLAN,LZPLAN,IWRITE,INPLOT,INFLOW      OTPT0024
 ISN 0018      1001 FORMAT(8X,3(2X,I2),2X,2(2X,I2),4X,I2)                       OTPT0025
 ISN 0019           GO TO 9999                                                  OTPT0026
 ISN 0020         2 CONTINUE                                                    OTPT0027
 ISN 0021           IF(KRUN.GT.1)GO TO 3                                        OTPT0028
 ISN 0023           TSEC=T                                                      OTPT0029
 ISN 0024           THOUR=TSEC/3600.                                            OTPT0030
 ISN 0025           NDAY=THOUR/24.                                              OTPT0031
 ISN 0026           THRDAY=THOUR-24.*FLOAT(NDAY)                                OTPT0032
 ISN 0027           IF(IXPLAN.LT.1)GO TO 2299                                   OTPT0033
                  C********** OUTPUT IN Y-Z PLANES FOR INCREASING IX            **********OTPT0034
 ISN 0029           IF(IWRITE.LT.1)GO TO 2105                                   OTPT0035
 ISN 0031           WRITE(3,2102)TSEC,THOUR,NDAY,THRDAY                         OTPT0036
 ISN 0032      2102 FORMAT(1H1,8X,10(1H*),1X,6HTIME =,E13.6,1X,6HSECOND ,1X,1H=,E13.6,OTPT0037
                   1       1X,4HHOUR,1X,1H=,I4,1X,7HDAY AND,F6.2,1X,4HHOUR,1X,10(1H*))OTPT0038
 ISN 0033      2105 CONTINUE                                                    OTPT0039
 ISN 0034           IF(INPLOT.LT.1)GO TO 2109                                   OTPT0040
 ISN 0036           WRITE(25,2106)T                                             OTPT0041
 ISN 0037      2106 FORMAT(8X,E13.6)                                            OTPT0042
 ISN 0038      2109 CONTINUE                                                    OTPT0043
 ISN 0039           DO 2200 IX=1,NX                                             OTPT0044
 ISN 0040           IF(IWRITE.LT.1)GO TO 2205                                   OTPT0045
 ISN 0042           WRITE(3,2202)IX                                             OTPT0046
 ISN 0043      2202 FORMAT(1H0,4X,8(1H*),1X,4HIX =,I4)                          OTPT0047
 ISN 0044           WRITE(3,2203)                                               OTPT0048
 ISN 0045      2203 FORMAT(1H0,2X,2X,2HIJ,2X,1X,1X,2HJY,1X,1X,2HLZ,2X,1X,5X,    OTPT0049
                   1       6HDHZIJL,2X,1X,5X,6HVLXIJL,2X,1X,5X,6HVLYIJL,2X,1X,5X,  OTPT0050
                   2       6HVLZIJL)                                           OTPT0051
 ISN 0046      2205 CONTINUE                                                    OTPT0052
 ISN 0047           IF(INPLOT.LT.1)GO TO 2209                                   OTPT0053
 ISN 0049           WRITE(25,2206)IX                                            OTPT0054
 ISN 0050      2206 FORMAT(4X,I4)                                               OTPT0055
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ISN 0051      2209 CONTINUE                                                    OTPT0056
ISN 0052           DO 2220 JY=1,NY                                             OTPT0057
ISN 0053           IF(IJIXJY(IX,JY).LT.1)GO TO 2220                            OTPT0058
ISN 0055           IJ=IJIXJY(IX,JY)                                            OTPT0059
ISN 0056           IMX=2*IJ-1                                                  OTPT0060
ISN 0057           IPX=IMX+1                                                   OTPT0061
ISN 0058           JMY=IMX                                                     OTPT0062
ISN 0059           JPY=IPX                                                     OTPT0063
ISN 0060           IF(INTEXT(IX,JY).GT.0)GO TO 2222                            OTPT0064
ISN 0062           DXIAJ=DX(IX)                                                OTPT0065
ISN 0063           DYIAJ=DY(JY)                                                OTPT0066
ISN 0064           ARHIAJ=DXIAJ*DYIAJ                                          OTPT0067
ISN 0065           GO TO 2228                                                  OTPT0068
ISN 0066      2222 CONTINUE                                                    OTPT0069
ISN 0067           IEND=INTEXT(IX,JY)                                          OTPT0070
ISN 0068           ARHIAJ=ARHBND(IBND)                                         OTPT0071
ISN 0069      2228 CONTINUE                                                    OTPT0072
ISN 0070           HSRIAJ=HSR(IJ)                                              OTPT0073
ISN 0071           NLZMIN=NLZBOT(IJ)                                           OTPT0074
ISN 0072           DO 2230 LZ=1,NZ                                             OTPT0075
ISN 0073           IF(LZ.LT.NLZMIN)GO TO 2230                                  OTPT0076
ISN 0075           DHZIJL=DZ(LZ)                                               OTPT0077
ISN 0076           IF(LZ.EQ.NLZMIN)  DHZIJL=DHZBOT(IJ)                         OTPT0078
ISN 0078           IF(LZ.EQ.NZ)  DHZIJL=DHZIJL+HSRIAJ                          OTPT0079
ISN 0080           VLXIJL=0.5*(VLX(IMX,LZ)+VLX(IPX,LZ))                        OTPT0080
ISN 0081           VLYIJL=0.5*(VLY(JMY,LZ)+VLY(JPY,LZ))                        OTPT0081
                 C********** CALCULATE VLZIJL                  **********     OTPT0082
ISN 0082           GWZLMH=GWZLZH(IJ,LZ)                                        OTPT0083
ISN 0083           IF(LZ.EQ.NZ)GO TO 2232                                      OTPT0084
ISN 0085           GWZLPH=GWZLZH(IJ,LZ+1)                                      OTPT0085
ISN 0086           GO TO 2235                                                  OTPT0086
ISN 0087      2232 CONTINUE                                                    OTPT0087
ISN 0088           GWZLPH=ARHIAJ*FNCHSR(IJ)                                    OTPT0088
ISN 0089      2235 CONTINUE                                                    OTPT0089
ISN 0090           GWZIJL=0.5*(GWZLMH+GWZLPH)                                  OTPT0090
ISN 0091           VLZIJL=GWZIJL/ARHIAJ                                        OTPT0091
ISN 0092           IF(IWRITE.LT.1)GO TO 2238                                   OTPT0092
ISN 0094           WRITE(3,2236)IJ,JY,LZ,DHZIJL,VLXIJL,VLYIJL,VLZIJL           OTPT0093
ISN 0095      2236 FORMAT(1H ,2X,I4,2X,2(1X,I3),2X,6(2X,E11.4,1X))             OTPT0094
ISN 0096      2238 CONTINUE                                                    OTPT0095
ISN 0097           IF(INPLOT.LT.1)GO TO 2230                                   OTPT0096
ISN 0099           WRITE(25,2239)IJ,JY,LZ,DHZIJL,VLYIJL,VLZIJL                 OTPT0097
ISN 0100      2239 FORMAT(2X,I4,2X,2(1X,I3),2X,5(1X,E11.4))                    OTPT0098
ISN 0101      2230 CONTINUE                                                    OTPT0099
ISN 0102      2220 CONTINUE                                                    OTPT0100
ISN 0103      2200 CONTINUE                                                    OTPT0101
ISN 0104      2299 CONTINUE                                                    OTPT0102
ISN 0105           IF(JYPLAN.LT.1)GO TO 2499                                   OTPT0103
                 C********** OUTPUT IN X-Z PLANES FOR INCREASING JY  **********OTPT0104
ISN 0107           IF(IWRITE.LT.1)GO TO 2305                                   OTPT0105
ISN 0109           WRITE(3,2102)TSEC,THOUR,NDAY,THRDAY                         OTPT0106
ISN 0110      2305 CONTINUE                                                    OTPT0107
ISN 0111           IF(INPLOT.LT.1)GO TO 2309                                   OTPT0108
ISN 0113           WRITE(25,2106)T                                             OTPT0109
ISN 0114      2309 CONTINUE                                                    OTPT0110
ISN 0115           DO 2400 JY=1,NY                                             OTPT0111
ISN 0116           IF(IWRITE.LT.1)GO TO 2405                                   OTPT0112
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ISN 0118          WRITE(3,2402)JY                                                    OTPT0113
ISN 0119       2402 FORMAT(1HO,4X,8(1H*),1X,4HJY =,I4)                               OTPT0114
ISN 0120          WRITE(3,2403)                                                      OTPT0115
ISN 0121       2403 FORMAT(1HO,2X,2X,2HIJ,2X,1X,1X,2HIX,1X,1X,2HLZ,2X,1X,5X,         OTPT0116
                   1       6HDHZIJL,2X,1X,5X,6HVLXIJL,2X,1X,5X,6HVLYIJL,2X,1X,5X,  OTPT0117
                   2       6HVLZIJL)                                                OTPT0118
ISN 0122       2405 CONTINUE                                                         OTPT0119
ISN 0123          IF(INPLOT.LT.1)GO TO 2409                                          OTPT0120
ISN 0125          WRITE(25,2206)JY                                                   OTPT0121
ISN 0126       2409 CONTINUE                                                         OTPT0122
ISN 0127          DO 2420 IX=1,NX                                                    OTPT0123
ISN 0128          IF(IJIXJY(IX,JY).LT.1)GO TO 2420                                   OTPT0124
ISN 0130          IJ=IJIXJY(IX,JY)                                                   OTPT0125
ISN 0131          IMX=2*IJ-1                                                         OTPT0126
ISN 0132          IPX=IMX+1                                                          OTPT0127
ISN 0133          JMY=IMX                                                            OTPT0128
ISN 0134          JPY=IPX                                                            OTPT0129
ISN 0135          IF(INTEXT(IX,JY).GT.0)GO TO 2422                                   OTPT0130
ISN 0137          DXIAJ=DX(IX)                                                       OTPT0131
ISN 0138          DYIAJ=DY(JY)                                                       OTPT0132
ISN 0139          ARHIAJ=DXIAJ*DYIAJ                                                 OTPT0133
ISN 0140          GO TO 2428                                                         OTPT0134
ISN 0141       2422 CONTINUE                                                         OTPT0135
ISN 0142          IBND=INTEXT(IX,JY)                                                 OTPT0136
ISN 0143          ARHIAJ=ARHBND(IBND)                                                OTPT0137
ISN 0144       2428 CONTINUE                                                         OTPT0138
ISN 0145          HSRIAJ=HSR(IJ)                                                     OTPT0139
ISN 0146          NLZMIN=NLZBOT(IJ)                                                  OTPT0140
ISN 0147          DO 2430 LZ=1,NZ                                                    OTPT0141
ISN 0148          IF(LZ.LT.NLZMIN)GO TO 2430                                         OTPT0142
ISN 0150          DHZIJL=DZ(LZ)                                                      OTPT0143
ISN 0151          IF(LZ.EQ.NLZMIN)  DHZIJL=DHZBOT(IJ)                                OTPT0144
ISN 0153          IF(LZ.EQ.NZ)  DHZIJL=DHZIJL+HSRIAJ                                 OTPT0145
ISN 0155          VLXIJL=0.5*(VLX(IMX,LZ)+VLX(IPX,LZ))                               OTPT0146
ISN 0156          VLYIJL=0.5*(VLY(JMY,LZ)+VLY(JPY,LZ))                               OTPT0147
                C********** CALCULATE VLZIJL                          **********     OTPT0148
ISN 0157          GWZLMH=GWZLZH(IJ,LZ)                                               OTPT0149
ISN 0158          IF(LZ.EQ.NZ)GO TO 2432                                             OTPT0150
ISN 0160          GWZLPH=GWZLZH(IJ,LZ+1)                                             OTPT0151
ISN 0161          GO TO 2435                                                         OTPT0152
ISN 0162       2432 CONTINUE                                                         OTPT0153
ISN 0163          GWZLPH=ARHIAJ*FNCHSR(IJ)                                           OTPT0154
ISN 0164       2435 CONTINUE                                                         OTPT0155
ISN 0165          GWZIJL=0.5*(GWZLMH+GWZLPH)                                         OTPT0156
ISN 0166          VLZIJL=GWZIJL/ARHIAJ                                               OTPT0157
ISN 0167          IF(IWRITE.LT.1)GO TO 2438                                          OTPT0158
ISN 0169          WRITE(3,2236)IJ,IX,LZ,DHZIJL,VLXIJL,VLYIJL,VLZIJL                  OTPT0159
ISN 0170       2438 CONTINUE                                                         OTPT0160
ISN 0171          IF(INPLOT.LT.1)GO TO 2430                                          OTPT0161
ISN 0173          WRITE(25,2239)IJ,IX,LZ,DHZIJL,VLXIJL,VLZIJL                        OTPT0162
ISN 0174       2430 CONTINUE                                                         OTPT0163
ISN 0175       2420 CONTINUE                                                         OTPT0164
ISN 0176       2400 CONTINUE                                                         OTPT0165
ISN 0177       2499 CONTINUE                                                         OTPT0166
ISN 0178          IF(LZPLAN.LT.1)GO TO 2699                                          OTPT0167
                C********** OUTPUT IN X-Y PLANES FOR INCREASING LZ          **********OTPT0168
ISN 0180          IF(IWRITE.LT.1)GO TO 2505                                          OTPT0169
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ISN 0182          WRITE(3,2102)TSEC,THOUR,NDAY,THRDAY                                  OTPT0170
ISN 0183       2505 CONTINUE                                                           OTPT0171
ISN 0184          IF(INPLOT.LT.1)GO TO 2509                                            OTPT0172
ISN 0186          WRITE(25,2106)T                                                      OTPT0173
ISN 0187       2509 CONTINUE                                                           OTPT0174
ISN 0188          DO 2600 LZ=1,NZ                                                      OTPT0175
ISN 0189          IF(IWRITE.LT.1)GO TO 2605                                            OTPT0176
ISN 0191          WRITE(3,2602)LZ                                                      OTPT0177
ISN 0192       2602 FORMAT(1HO,4X,8(1H*),1X,4HLZ =,I4)                                 OTPT0178
ISN 0193          WRITE(3,2603)                                                        OTPT0179
ISN 0194       2603 FORMAT(1HO,2X,2X,2HIJ,2X,1X,1X,2HIX,1X,1X,2HJY,2X,1X,5X,           OTPT0180
                   1       6HDHZIJL,2X,1X,5X,6HVLXIJL,2X,1X,5X,6HVLYIJL,2X,1X,5X,   OTPT0181
                   2       6HVLZIJL)                                                OTPT0182
ISN 0195       2605 CONTINUE                                                           OTPT0183
ISN 0196          IF(INPLOT.LT.1)GO TO 2609                                            OTPT0184
ISN 0198          WRITE(25,2206)LZ                                                     OTPT0185
ISN 0199       2609 CONTINUE                                                           OTPT0186
ISN 0200          DO 2610 IX=1,NX                                                      OTPT0187
ISN 0201          DO 2620 JY=1,NY                                                      OTPT0188
ISN 0202          IF(IJIXJY(IX,JY).LT.1)GO TO 2620                                     OTPT0189
ISN 0204          IJ=IJIXJY(IX,JY)                                                     OTPT0190
ISN 0205          IMX=2*IJ-1                                                           OTPT0191
ISN 0206          IPX=IMX+1                                                            OTPT0192
ISN 0207          JMY=IMX                                                              OTPT0193
ISN 0208          JPY=IPX                                                              OTPT0194
ISN 0209          IF(INTEXT(IX,JY).GT.0)GO TO 2622                                     OTPT0195
ISN 0211          DXIAJ=DX(IX)                                                         OTPT0196
ISN 0212          DYIAJ=DY(JY)                                                         OTPT0197
ISN 0213          ARHIAJ=DXIAJ*DYIAJ                                                   OTPT0198
ISN 0214          GO TO 2628                                                           OTPT0199
ISN 0215       2622 CONTINUE                                                           OTPT0200
ISN 0216          IBND=INTEXT(IX,JY)                                                   OTPT0201
ISN 0217          ARHIAJ=ARHBND(IBND)                                                  OTPT0202
ISN 0218       2628 CONTINUE                                                           OTPT0203
ISN 0219          HSRIAJ=HSR(IJ)                                                       OTPT0204
ISN 0220          NLZMIN=NLZBOT(IJ)                                                    OTPT0205
ISN 0221          IF(LZ.LT.NLZMIN)GO TO 2630                                           OTPT0206
ISN 0223          DHZIJL=DZ(LZ)                                                        OTPT0207
ISN 0224          IF(LZ.EQ.NLZMIN)  DHZIJL=DHZBOT(IJ)                                  OTPT0208
ISN 0226          IF(LZ.EQ.NZ)  DHZIJL=DHZIJL+HSRIAJ                                   OTPT0209
ISN 0228          VLXIJL=0.5*(VLX(IMX,LZ)+VLX(IPX,LZ))                                OTPT0210
ISN 0229          VLYIJL=0.5*(VLY(JMY,LZ)+VLY(JPY,LZ))                                OTPT0211
                C********** CALCULATE VLZIJL                          *********       OTPT0212
ISN 0230          GWZLMH=GWZLZH(IJ,LZ)                                                 OTPT0213
ISN 0231          IF(LZ.EQ.NZ)GO TO 2632                                               OTPT0214
ISN 0233          GWZLPH=GWZLZH(IJ,LZ+1)                                               OTPT0215
ISN 0234          GO TO 2635                                                           OTPT0216
ISN 0235       2632 CONTINUE                                                           OTPT0217
ISN 0236          GWZLPH=ARHIAJ*FNCHSR(IJ)                                             OTPT0218
ISN 0237       2635 CONTINUE                                                           OTPT0219
ISN 0238          GWZIJL=0.5*(GWZLMH+GWZLPH)                                           OTPT0220
ISN 0239          VLZIJL=GWZIJL/ARHIAJ                                                 OTPT0221
ISN 0240          IF(IWRITE.LT.1)GO TO 2638                                            OTPT0222
ISN 0242          WRITE(3,2236)IJ,IX,JY,DHZIJL,VLXIJL,VLYIJL,VLZIJL                    OTPT0223
ISN 0243       2638 CONTINUE                                                           OTPT0224
ISN 0244          IF(INPLOT.LT.1)GO TO 2630                                            OTPT0225
ISN 0246          WRITE(25,2239)IJ,IX,JY,DHZIJL,VLXIJL,VLYIJL                          OTPT0226
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ISN 0247      2630 CONTINUE                                                    OTPT0227
ISN 0248      2620 CONTINUE                                                    OTPT0228
ISN 0249      2610 CONTINUE                                                    OTPT0229
ISN 0250      2600 CONTINUE                                                    OTPT0230
ISN 0251      2699 CONTINUE                                                    OTPT0231
ISN 0252           IF(INFLOW.LT.1)GO TO 2999                                   OTPT0232
ISN 0254      2999 CONTINUE                                                    OTPT0233
ISN 0255           GO TO 9999                                                  OTPT0234
ISN 0256         3 CONTINUE                                                    OTPT0235
ISN 0257           WRITE(4,3001)T                                              OTPT0236
ISN 0258      3001 FORMAT(8X,E13.6)                                            OTPT0237
ISN 0259           DO 3100 IJ=1,NINJ                                           OTPT0238
ISN 0260           IMX=2*IJ-1                                                  OTPT0239
ISN 0261           IPX=IMX+1                                                   OTPT0240
ISN 0262           JMY=IMX                                                     OTPT0241
ISN 0263           JPY=IPX                                                     OTPT0242
ISN 0264           NLZMIN=NLZBOT(IJ)                                           OTPT0243
ISN 0265           WRITE(4,3101)IJ,HSR(IJ)                                     OTPT0244
ISN 0266      3101 FORMAT(3X,I4,1X,E11.4)                                      OTPT0245
ISN 0267           DO 3110 LZ=NLZMIN,NZ                                        OTPT0246
ISN 0268           WRITE(4,3111)LZ,VLX(IMX,LZ),VLX(IPX,LZ),VLY(JMY,LZ),VLY(JPY,LZ)  OTPT0247
ISN 0269           WRITE(4,3111)LZ,GWZLZH(IJ,LZ)                               OTPT0248
ISN 0270      3111 FORMAT(3X,I4,1X,4(1X,E11.4))                                OTPT0249
ISN 0271      3110 CONTINUE                                                    OTPT0250
ISN 0272      3100 CONTINUE                                                    OTPT0251
ISN 0273      9999 RETURN                                                      OTPT0252
ISN 0274           END                                                         OTPT0253

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,..JEDIT,ID,NOXREF

*STATISTICS*      SOURCE STATEMENTS =      273 ,PROGRAM SIZE =      4772

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******
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LEVEL 21.8 ( JUN 74 )                    OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE INITAL                                          INTL0001
   ISN 0003            COMMON/CONTRL/KRUN,T                                       INTL0002
   ISN 0004            COMMON/REGINT/NX,NY,NZ,NINJ,NBND,INTEXT(18,12),IJIXJY(18,12),  INTL0003
                      1              NREACH(18),NJYMIN(18,6),NJYMAX(18,6),NLZBOT(136)  INTL0004
   ISN 0005            COMMON/HSRSOL/HSR(136),PREHSR(136),FNCHSR(136)             INTL0005
   ISN 0006            COMMON/VLXSOL/VLX(272, 6),PREVLX(272, 6),FNCVLX(272, 6)    INTL0006
   ISN 0007            COMMON/VLYSOL/VLY(272, 6),PREVLY(272, 6),FNCVLY(272, 6)    INTL0007
   ISN 0008            COMMON/FLOVLZ/GWZLZH(136, 6),FNCGWZ(136, 6)                INTL0008
   ISN 0009            READ(21,1001)ISTART,TSTART                                 INTL0009
   ISN 0010         1001 FORMAT(8X,I2,2X,E13.6,2X,2(1X,E11.4))                     INTL0010
   ISN 0011            IF(ISTART.GT.0)GO TO 2                                     INTL0011
   ISN 0012            T=TSTART                                                   INTL0012
   ISN 0013            DO 1100 IJ=1,NINJ                                          INTL0013
   ISN 0014            IMX=2*IJ-1                                                 INTL0014
   ISN 0015            IPX=IMX+1                                                  INTL0015
   ISN 0016            JMY=IMX                                                    INTL0016
   ISN 0017            JPY=IPX                                                    INTL0017
   ISN 0018            HSP(IJ)=0.0                                                INTL0018
   ISN 0019            NLZMIN=NLZBOT(IJ)                                          INTL0019
   ISN 0020            DO 1110 LZ=NLZMIN,NZ                                       INTL0020
   ISN 0021            VLX(IMX,LZ)=0.0                                            INTL0021
   ISN 0022            VLX(IPX,LZ)=0.0                                            INTL0022
   ISN 0023            VLY(JMY,LZ)=0.0                                            INTL0023
   ISN 0024            VLY(JPY,LZ)=0.0                                            INTL0024
   ISN 0025            GWZLZH(IJ,LZ)=0.0                                          INTL0025
   ISN 0026         1110 CONTINUE                                                 INTL0026
   ISN 0027         1100 CONTINUE                                                 INTL0027
   ISN 0028            GO TO 9999                                                 INTL0028
   ISN 0029            2 CONTINUE                                                 INTL0029
   ISN 0030            READ(24,2001)TSTART                                        INTL0030
   ISN 0031         2001 FORMAT(8X,E13.6)                                         INTL0031
   ISN 0032            T=TSTART                                                   INTL0032
   ISN 0033            DO 2100 IJ=1,NINJ                                          INTL0033
   ISN 0034            IMX=2*IJ-1                                                 INTL0034
   ISN 0035            IPX=IMX+1                                                  INTL0035
   ISN 0036            JMY=IMX                                                    INTL0036
   ISN 0037            JPY=IPX                                                    INTL0037
   ISN 0038            READ(24,2101)HSR(IJ)                                       INTL0038
   ISN 0039         2101 FORMAT(1X,6X,1X,E11.4)                                   INTL0039
   ISN 0040            NLZMIN=NLZBOT(IJ)                                          INTL0040
   ISN 0041            DO 2110 LZ=NLZMIN,NZ                                       INTL0041
   ISN 0042            READ(24,2111)VLX(IMX,LZ),VLX(IPX,LZ),VLY(JMY,LZ),VLY(JPY,LZ)  INTL0042
   ISN 0043            READ(24,2111)GWZLZH(IJ,LZ)                                 INTL0043
   ISN 0044         2111 FORMAT(1X,2X,4X,1X,4(1X,E11.4))                          INTL0044
   ISN 0045         2110 CONTINUE                                                 INTL0045
   ISN 0046         2100 CONTINUE                                                 INTL0046
   ISN 0047         9999 CONTINUE                                                 INTL0047
   ISN 0048            RETURN                                                     INTL0048
   ISN 0049            END                                                        INTL0049

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=02,LINECNT=59,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       49 ,PROGRAM SIZE =     1204
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*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
*STATISTICS*     SOURCE STATEMENTS =       45 ,PROGRAM SIZE =      732
*STATISTICS*  NO  DIAGNOSTICS GENERATED
****** END OF COMPILATION ******
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VITA

Wen-Lianq Lin was born in Chia-Yi, Taiwan on Febuary 22, 1949. He attended elementary school there and was graduated from Chia-Yi High School in Chia-Yi, Taiwan in June 1967. In September 1968, he entered Tamkang College, Tamsui, Taiwan and received a Bachelor of Irrigational Engineering Degree in June, 1972.

After two years military service, he became a teaching assistant in the Department of Hydraulic Engineering in Tamkang College in 1974.

He came to the University of Tennessee in 1975 , to continue his graduate studies in Civil Engineering. He obtained the Master of Science degree in December 1976. He chose to continue studying for the Doctor of Philosophy degree in the Department of Engineering Science and Mechanics at the University of Tennessee. The author is a member of Phi Kappa Phi.

He was married to the former Kuei-Chin Wang of Chia-Yi, Taiwan in 1978. They now have one son, Ted Michael Lin, age two.