

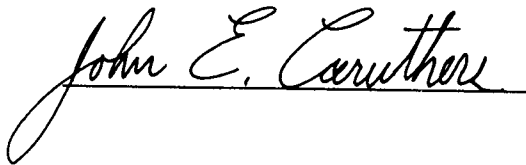
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I am submitting herewith a thesis written by Thomas F. Tibbals entitled "Fatigue Theory Survey and Comparison for Assessing High Cycle Fatigue". I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

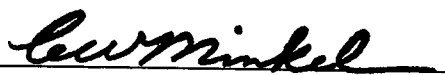

Remi Engels, Major Professor

We have read this thesis
and recommend its acceptance:





Accepted for the Council:


Vice Provost
and Dean of the Graduate School

**FATIGUE THEORY SURVEY and COMPARISON
for
ASSESSING HIGH CYCLE FATIGUE**

**A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville**

**Thomas F. Tibbals
August 1998**

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ABSTRACT

Fatigue is the process by which gradual material damage accumulates (or nucleates) in time to a macrolevel crack and causes yielding or catastrophic failure of a component due to cyclic dynamic stress levels and static stress levels which typically are well below the static strength limit. The goal of theories of fatigue is the explanation of the failure process due to cyclic loads, to establish failure criteria, and to develop material and geometric correlation parameters which characterize the material/geometry/loading scenario sufficiently to predict the potential for failure, the type of failure, and the life expected before failure. Numerous examples of unexpected catastrophic failures have occurred since the industrial revolution which are unexplainable from a static strength design approach alone. The greater number of failures in engineering structures are by fatigue, and relatively few occur by static failure [ref. 1]. Wirsching and Ortiz [ref. 2] report that fatigue from cyclic loading cause 80-90% of observed service failures in mechanical and structural systems, and has a severe economic impact on engineered products.

The approaches to fatigue theory are typically grouped as **Fatigue Analysis Techniques** and **Failure Mechanics Techniques**. Fatigue Analysis Techniques are often referred to as **Safe-Life** techniques, where the material is assumed to be structurally defect-free until failure (crack, yield, etc.). In these approaches, no attempts are made to use a damaged (cracked, or yielded) component regardless of how benign the damage may be. Consequently, these approaches never generated inspection criteria or fault monitoring techniques and nearly all failures were sudden and unexpected. The fatigue analysis safe-life techniques include **Stress-Life** (the oldest, 1800s, where stress is considered the dominant fatigue parameter) and **Strain-Life** (circa 1960, where strain is considered the dominant fatigue parameter). Stress-life is used primarily for long life (high cycle, low load) applications while Strain-life typically is used for predicting initiation of a failure

mechanism (cracking or yielding), and is typically used for short (low cycle, high load) fatigue life assessment. These theories are predominantly experimentally based.

Realizing that failures continue to occur even if strict Fatigue Analysis techniques are applied, competing fatigue approaches have appeared, the most notable that of **Fracture Mechanics** (circa 1950), with the alternative to the safe-life approach being **Damage Tolerance**. This approach is diametrically opposed to the safe-life approaches. Here it is assumed that all materials have defects, which eventually lead to failure if the defect grows to a critical value. Fracture Mechanics attempts to describe life in terms of the propagation of cracks based upon the stress concentration and plastic flow at the edge of the crack in a more theoretically-based manner. However, fracture mechanics is also very dependent upon experimental data and correlation. Typically, fracture mechanics is more conservative than safe-life/fatigue analysis, since Fatigue Analysis considers the life prior to the initiation/nucleation of a crack and subsequent failure, while fracture mechanics predicts life remaining after an assumed initial crack size. The establishment of an inspection interval process is the major accomplishment of Fracture Mechanics. Other competing and more recent fatigue theories include the **Strain Energy Density** (SED, 1980s) failure criterion, **Continuum Damage Mechanics** (circa 1970), **Critical Plane Theory** (1973, rev. 1988), and **Holistic Design** approaches. The objective of this thesis is to summarize and compare fatigue theories as they may apply to turbine engine component life assessment, i.e. on an engineering level as opposed to a theoretical one. The emphasis is in identifying an approach, or methodology, which can be implemented economically by *any* structural analyst, not just one who has an enormous budget, a materials laboratory, and a PhD in material behavior. Such a methodology requires making the most out of the information available, and yet must be based on realistic physics and have at least a limited predictive capability. This thesis will describe in detail each of the broad elements of fatigue alluded to above, highlighting the assumptions, strengths and weaknesses of these approaches. A comparison of these techniques for evaluating high cycle fatigue potential for turbine engine components will

be made. Other factors (some controversial) pertinent to the fatigue problem will be described. Finally, an approach for assessing the potential for fatigue will be described from a test and evaluation perspective.

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DEFINITIONS

The following definitions and nomenclatures were derived from experience and a variety of references, the most notable are *Webster's Dictionary* [ref. 3], the *McGraw-Hill Dictionary of Physics and Mathematics* [ref. 4], and *Primer - Structural Fatigue and Damage Tolerant Design* by Dr. Hoepfner as part of his short course [ref. 5].

Breakage: The physical separation of components into pieces from the whole.

Brittle: A material is said to be brittle if a crack is initiated at the mesoscale without a large amount of plastic deformation. Typically $\epsilon_p/\epsilon_e < 1$, where ϵ_p and ϵ_e are the plastic and elastic strains, respectively. Brittle materials cannot use yielding as a failure criteria. In terms of material slip and separation, a brittle material has more slip resistance than separation resistance.

Continuum: Assumption that the physical properties of a component under a given state of stress/loading are defined at a mathematical point, and each point is materially identical to all other points of the component. Basically, continuum is a matter of scale, where the properties are averaged over a certain volume physically, which is small enough to avoid smoothing of high gradients, but large enough to represent averages of the microprocesses.

Crack: A crack is a discontinuity in the material at the mesoscale level that has two surfaces which have at least one common point or line of points in the region where the surfaces begin.

Creep: Creep is the process of visco-elastic material flow due to sustained loading resulting in permanent deformation. Typically occurs only at elevated temperatures (typically above 1/3 of the melting point).

Damage: The progressive physical process of a material which leads to breakage. At the microscale, it is the progressive creation of microsurfaces of discontinuities through the breaking of atomic bonds and the plastic enlargement of microcavities. This process is sometimes called microcrack/microvoid nucleation.

Damage Tolerance: The design methodology for dynamic components (i.e., susceptible to time-dependent flaw growth) in which fracture mechanics and/or damage mechanics analyses are used to predict remaining life in a component and to quantify inspection intervals for the component which has some form of damage (visible or microscopic cracks/dislocations/voids).

Dislocations: The sudden change in the atomic bond lattice of a material where an atom is missing.

Ductile: A material is said to be ductile if measurable plastic deformations precede crack initiation on the mesoscale. Only ductile materials can use yielding as a failure criteria. In terms of material slip and separation, a ductile material has more separation resistance than slip resistance.

Elasticity: The characteristic of a material to strain with applied stress but exhibit no permanent deformation once the load is removed. Typically, the elastic capability is determined by the linear region of the stress versus strain curve for the material, but this is not a requirement of elasticity, only that it return to the original, undeformed state upon removal of load. Rubber and some polymers are elastic, but not linearly.

Fail-Safe: The design philosophy that recognizes that materials in structures are not homogeneous and continuous, but rather have internal discontinuities or voids which locally can be strained at a higher rate than the overall mean distribution and therefore can cause failure.

Fatigue: The progressive failure process due to cyclic loading causing a permanent structural change of the material state locally at various points within the structure which may result in cracks. The failure of a component material (such as by cracking) as a result of the dynamics of cyclic loading at a stress level usually significantly lower than the static limit.

Fatigue Analysis: For the purposes of this report, based upon indications from literature reviewed, fatigue analysis (as opposed to failure mechanics) will be considered to be the assessment of fatigue life based upon safe-life and strain-life approaches, as differentiated from analyses for critical plane, crack growth and damage potential of the newer theories which address the failure mechanism directly. Fatigue analysis is herein considered to be *purely* experimental data oriented (correlations only), with little supporting theory.

Fracture: The process of crack or voids within a material propagating or nucleating to a critical size which results in catastrophic separation of the whole into pieces or at least a sudden component geometry change.

Fretting: External surface damage/deformation resulting from small amplitude oscillatory motion between two contacting surfaces.

Galling: A seizing, scoring or scuffing action which may occur between two load bearing surfaces. Typically, this is uni-directional wear, as opposed to the fretting oscillatory situation.

Goodman Diagram: In general, the type of diagram used to describe the capability of the material to withstand cyclic loading (alternating stress) as a function of the mean stress (or static load) for a given life (number of cycles). Goodman was only one of several researchers who used this approach, and his was a linear relationship.

High Cycle Fatigue (HCF): Structural damage resulting from generally low cyclic stress/strain levels (relative to the static strength limit). Typically, for HCF to occur, it takes more than 100000 cycles. (See Low Cycle Fatigue definition below.)

Impact Strength: A material property which is a measure of the capacity of a material to absorb energy during sudden fracture.

Isotropic: Material properties are independent of direction.

Low Cycle Fatigue (LCF): Structural damage resulting from high cyclic stress/strain levels (relative to the static strength limit). Typically, LCF results in less than 10000 cycles. Between the LCF upper cycle limit of 10000 and the HCF lower cycle limit of 100000 cycles, there exists LCF-HCF interactions with localized plasticity (typically less than for LCF, but obviously greater than HCF, which typically is purely elastic).

Macroscale: The scale of engineering structures.

Material Failure: The breakdown of the component to perform its intended function due to the material from which it was made. This could be anything from a slight yielding in a tightly fitted system which results in binding, a minute crack which requires the part to be replaced, or catastrophic separation of a component into pieces.

Mesoscale: The scale at which the constitutive relations for mechanics are written. At this scale, the micro-processes are averaged, but high gradients are not smoothed. Per

Lemaitre [ref. 6], the following magnitudes of the Representative Volume Element for the mesoscale are:

- | | |
|---------------------------------|-----------------------|
| • Metals and Ceramics: | 0.001 mm ³ |
| • Polymers and most Composites: | 1 mm ³ |
| • Wood: | 10 mm ³ |
| • Concrete: | 100 mm ³ |

Microscale: The scale of mechanisms used to consider strains and damage within a material.

Nucleate: To form into a nucleus, to bring together, to act as a nucleus.

Plasticity: The characteristic of a stressed material to exhibit a non-linear stress-strain relationship and permanent deformation (after load removal) beyond some threshold of stress/strain called the yield point.

Safe-Life: For a given stress/strain level, the material will not fail during its design life. Typically associated with the assumption of a material continuum where it is assumed that no cracks or internal damage sites exist.

Self-Similar Process: A process in which the changes to the parameters describing the process are such as to preserve the original parametric relationships prior to the initiation of the process.

Slip: The irreversible plastic flow of dislocations within the material which results in no appreciable volume change, typically due to a shearing force.

Small Cracks: Cracks on the order of microstructural periodicity (grain diameter), typically 5 - 10x grain diameter is considered *small*, dimensionally, this is from $1\mu\text{m}$ to 1mm (.0394").

S-N Curve: The curve depicting the allowable stress as a function of fatigue cycles, where S is the bulk stress (applied load divided by the original area of load distribution) and N is the number of cycles.

Spalling: A wear process where microscopic particles separate from a surface into flakes or chips. Typically due to contact stresses such as in gears. In general, spalling is a direct result of a high amplitude compressive wave encountering a free surface (like the rupture surface opposite the point of impact of a projectile striking a thick plate). A compressive wave reaching a free surface generates a reflected tensile wave (due to the radical change in the index of refraction) of the same amplitude. If this stress amplitude exceeds the tensile strength of the material, a layer near the free surface will spall off. The corresponding plane of failure must be below the surface since the peak tensile stress can occur only after the reflected wave starts to travel back towards the original point of impact.

Strain: The per unit change in linear dimensions of a body due to load. Dimensionless quantity.

Strain-Life: Fatigue design concept that is based upon the use of a strain versus number of cycles (S-N or ϵ -N) diagram, typically generated by cyclic uniaxial testing of standard specimens to failure where strain was controlled instead of load.

Stress: The differential ratio of applied load to area (effective) over which the load is distributed that describes the intensity and direction of the resistance to this load. The resistance to deformation as a result of a load.

Stress-Life: Fatigue design concept that is based upon the use of a stress versus number of cycles (S-N or σ -N) diagram, typically generated by cyclic uniaxial testing of standard specimens to failure where load was controlled.

Tensile Ultimate Strength: A material property which is the largest engineering stress at moderate ($\sim$ room) temperatures (well below the region of creep for the material) that can be statically maintained without failure. (Calculated using the original undeformed cross sectional area, i.e., without regard to necking causing area reduction prior to failure, which is a function of the material's ductility.)

Tensile Yield Strength: The stress at which the strain in the non-linear (plastic) portion of the stress-strain curve generally (but not always) at a strain offset of 0.002 is reached.

Visco-elastic: A material which exhibits both elastic and viscous [time (strain-rate) dependent] straining under stress.

Wear: Material removed from contacting surfaces due to relative mechanical motion between the surfaces.

1. INTRODUCTION

Safe and failure free designs require precise understanding of the actual failure mechanism or process and the ability to describe the failure mechanism in terms of its relationship to loads, component geometry, and material response to load. An understanding of the environment in which the component or system must operate, including the effects of the degree of uncertainty in these relations [ref. Boresi & Sidebottom] must also be known. This is a major responsibility for the structural designer and analyst, which must be coupled with the realities of economics which include using minimum factors of safety to produce a structurally and economically successful product. Numerous examples of unexpected catastrophic failures have occurred since the industrial revolution which are unexplainable from a static strength design approach alone. In this paper, the emphasis will be on the fatigue considerations as just one of the mechanisms of failure.

The greater number of failures in engineering structures are by fatigue, and relatively few occur by static failure [ref. 1]. Wirsching and Ortiz [ref. 2] report that fatigue from cyclic loading cause 80-90% of observed service failures in mechanical and structural systems, and has a severe economic impact on engineered products. Fatigue generates cracks which typically appear first at high stress concentration areas (such as fastener holes, corners of extruded or formed components, snapping notches, keyway slots, etc.) that are inherent in any engineering structure or at void/discontinuities found in most engineering materials resulting in a local stress intensity gradient. The magnitude of the stress required to produce failure decreases as the number of cycles of stress increases [ref. 7], suggesting a *life-time* similar to human life. Cyclic loading gradually damages engineering materials near microscopic defects in the material. These defects can be surface irregularities, internal discontinuities in the micro grain structure, and internal voids. **Fatigue is the process by which this gradual damage accumulates (or**

nucleates) over time into a macrolevel crack and causes yielding or catastrophic failure of a component due to the inability to continue to support the load in the presence of the crack at a structurally critical plane at dynamic stress levels which typically are well below the static strength limit. The goal of a theory of fatigue is to explain the failure process due to cyclic loads, to establish failure criteria, and to develop material and geometric correlation parameters which characterize the material/geometry/loading scenario sufficiently to predict the potential for failure, the type of failure, and the life expected before failure.

Fatigue has been studied since the early 1800s [ref. 5]. The first studies were the result of train wheel failures. It was noticed that the material appeared to have crystallized due to vibration and cracks formed near sharp corners. Generally, two zones can be distinguished in a fatigue fracture: first there is a zone of gradual crack development (sometimes identified by concentric rings like beach marks on a shore); secondly there is the region of sudden fracture (which typically exhibits brittle failure, i.e., fracture due to exceeding the material cohesive force) [ref. 7]. Consequently, the brittle fracture that was originally thought to be a crystalization process is really due to the particular loading and the eventual overcoming of the cohesive forces in a tensile manner resulting in a brittle fracture in materials characterized as ductile [ref. 7]. August Woehler (also spelled Wöhler) was the first to systematically characterize materials based upon allowable cyclic stress versus number of cycles to failure (the S-N or Woehler curve). Later, Ewing and Humphrey (1903) used optical microscopy to examine the process of fatigue [ref. 8]:

“The course of the breakdown was as follows: The first examination, made after a few reversals of stress, showed slip-lines on some of the crystals, the slip-lines were quite similar in appearance to those which are seen when a simple tensile stress exceeding the elastic limit is applied. After many reversals, they changed into comparatively wide bands with rather hazily defined

edges. As the number of reversals increased, this process of broadening continued, and some parts of the surface became almost covered with dark markings. When this stage was reached, it was found that some of the crystals had cracked. The cracks occurred along broadened slipbands. In some instances, they were first seen on a single crystal, but soon they joined up from crystal to crystal, until finally a long continuous crack was developed across the surface of the specimen." They also observed, "Once an incipient crack begins to form across a certain set of crystals, the effect of further reversals is mainly confined to the neighborhood of the crack."

The above quote summarizes not only the process of failure, but the path fatigue researchers have taken to cope with the problem. In fatigue analysis, material test results and empirical correlations are used to estimate the incremental fatigue damage (crack growth and localized plastic flow) or cycles to failure (life). The bulk of experimentation and theoretical studies of fatigue have centered around the identification of parameters relative to the material and loading properties by which the life of a component can be predicted. While no definitive fatigue theory currently exists, several techniques have emerged as practical, cost effective, and relatively easy design methods (or guides) that can be reconciled with physical observations of failure mechanisms.

The above quotation clearly describes the fatigue process as a progression of damage based upon the nucleation of damage zones into the form of a crack, which becomes a very localized process within the material. It also highlights the fact that the surface of the part plays an important role in the process. In 1924, A. A. Griffith published his classic paper [ref. 2, 9] on fatigue based upon energy principles in which it was shown that the propagation of a crack will occur if the energy of the system is lowered with crack propagation, i.e., the change in elastic strain energy due to crack extension is larger

than the energy required to create new crack surfaces (or stated another way, the change in strain energy that results from incremental crack growth is sufficient to overcome the surface energy of the material [ref. 10]). In addition, Griffith clearly showed that the last cycle of crack growth was nothing more than brittle fracture caused by cyclic growth of the fatigue crack to an unstable length. This energy approach was initially valid only for brittle materials, but was later extended to ductile materials in the late 1940's by Irwin [ref. 10]. The concept of a critical crack length leads one to concentrate on the properties of the crack itself instead of bulk material strength properties. Coffin and Mason in the 1950's established quantitative relationships between plastic strain and fatigue life [ref. 9]. In the 1960's, Irwin and others developed fracture mechanics where the emphasis shifted to characterizing materials based upon resistance to crack propagation as opposed to defining safe life from bulk strength properties of the material. The most notable result of this effort was the Paris *Law* [ref. 10] for crack growth. Since then, standards for material characterization based upon S-N curves and crack growth with multi-axial loading effects have been the emphasis, while attempts to define the initiation parameters for the crack have resulted in Damage Mechanics as a bridge between the strength approach and the crack propagation approach.

One of the most important physical observations is that the fatigue process can be generally broken down into two distinct phases - initiation life and propagation life [ref. 9]. Prior to the Ewing and Humphrey experiments, investigators didn't make this distinction, and worked from the stress approach to defined static shear and tensile yield and ultimate strengths for a variety of engineering materials to aid in component design. These guidelines were idealized in that the material was assumed homogeneous in a statistical sense and isotropic such that uni-axial loading was all that was necessary and therefore all that was used to characterize material strengths. It was also assumed that the material was continuous such that load distributes continuously from one material point to another within the component. Later, experiments on some materials showed that, while the static strength limits had to be reduced for cyclic use, the allowable cyclic stress

becomes asymptotic to an endurance limit value with increasing number of cycles. This leads one to the prediction of infinite life. Woehler curves of stress versus cycles (S-N diagrams) were the basic correlation developed by these early researchers. Eventually, sensitivities to specimen geometry were evaluated to explain unexpected failures due to geometric discontinuities. This resulted in the plethora of stress concentration factors [ref. 11] for design purposes in dynamic and high stress/strain gradient situations. These approaches are typically grouped as **Fatigue Analysis** (as opposed to failure mechanics) and resulted in what is known as **Safe-Life** techniques, where the material is assumed to be structurally defect free until failure (crack, yield, etc.). In these approaches, no attempts are made to use a damaged (cracked, or yielded) component regardless of how benign the damage may be. Typically, though, the tests which generated the fatigue correlation for the material took the material to complete failure (i.e., two pieces). Consequently, these approaches never generated inspection criteria or fault monitoring techniques, and nearly all failures were sudden and unexpected.

The fatigue analysis safe-life techniques include ***Stress-Life*** (the first fatigue analysis method as described above) and ***Strain-Life*** (circa 1960) approaches to fatigue. These approaches describe the failure mechanism in terms of ultimate strength, yield strength, and endurance limit of the material, assuming the material to be a homogeneous isotropic continuum. Stress-life is used primarily for long life (high cycle, low load) applications where stresses and strains are purely elastic, and deals only with total life [ref. 9], ignoring the distinction between initiation life and propagation life. Strain-life typically is used for predicting initiation of a failure mechanism (crack or yielding) where plastic deformation exists, and is typically used for short (low cycle, high load) fatigue life assessment. A variety of failure theories exist, and are applied for the stress or strain analysis dependent upon the anticipated failure mechanism, whether uni-axial or multi-axial stress/strain situations exist, and whether essentially a static problem or one of considerable load fluctuation. For instance, Maximum Tensile Stress Theory is generally used for static, brittle materials, the Maximum Tensile Yield Theory is typically used for

static, ductile materials, the Maximum Shear Stress Theory can be used for static or cyclic, uni-axial, ductile failure scenarios, and the Maximum Distortion Energy or Octahedral Shear Stress Theory are generally used for multi-axial, static or cyclic, ductile failure scenarios, etc. Basically, all of these theories lead to what has come to be known as the Goodman Diagram, even though Goodman was only one of several researchers who used similar diagrams to correlate a fatigue capability for a material. This diagram describes the capability of a material to withstand cyclic loading (alternating stress or strain) as a function of the mean stress/strain (or static load) for a given life (number of cycles). Basically, safe-life approaches assume that if the component is operated at a stress level below the strength (yield or ultimate depending upon the *safe* criteria) of the material it is composed of for its allowable life, then the component will never fail. Consequently, safe-life approaches quantify the critical combinations of only two variables: applied load and material strength.

The alternative to fatigue analysis is **Fracture Mechanics** and the alternative to the safe-life approach is **Damage Tolerance**. This approach is diametrically opposed to the safe-life approaches. Here it is assumed that all materials have defects which eventually lead to failure if the defect grows, or will function adequately with the damage if it doesn't propagate within the design life. This violates the continuum assumption upon which elasticity and plasticity theories are based. Fracture Mechanics (circa 1950) attempts to describe life in terms of the propagation of cracks based upon the stress concentration and plastic flow at the edge of the crack in a more theoretically-based manner. In this sense, elasticity/plasticity theory is still an integral part of fracture mechanics, but only in the small region ahead of a crack. However, fracture mechanics is also very dependent upon experimental data and correlations. In this approach, one either calculates the critical crack length that can lead to failure, or one calculates the growth rate (change in crack length per cycle) to establish inspection intervals appropriate for monitoring structural integrity. Typically, fracture mechanics is more conservative than safe-life/fatigue analysis, since fatigue analysis considers the life prior to the initiation/nucleation of a

crack and subsequent failure, while fracture mechanics predicts life remaining after an assumed initial crack size. Fracture mechanics can be coupled with the strain-life approach to predict the total life of the component for low cycle fatigue, from pre-initiation to complete failure or the generation of a critical crack size. Basically, fracture mechanics replaces the material strength property of fatigue analysis with a fracture toughness material property and adds flaw size as a parameter of the analysis to determine structural suitability. Therefore fracture mechanics quantifies the critical combinations of three variables: applied load, fracture toughness, and flaw size.

The Strain Energy Density (SED) failure criterion is a newer approach to fatigue which was developed as an attempt to overcome the limitations of Fracture Mechanics. SED is very similar to a safe-life approach based upon strain, however, it does not assume the material is crack free, nor does it require the presence of cracks or the knowledge of the crack size/direction to be of value as a structural analysis tool. The SED criterion was first proposed as a failure criteria and material characterization approach in the early 1980s by G.C. Sih, and developed extensively at the Institute of Fracture and Solid Mechanics of Lehigh University [ref. 12]. The SED criterion is an attempt to overcome the dependencies of specimen size, geometry, and loading on material characterizations, especially with respect to the symmetry between the applied load and the crack plane, which is typically violated for multi-axial stress fields and ductile materials. The fundamental quantity of SED criterion is the strain energy density which includes the thermodynamic irreversible and dilatational strain. This criterion allows for strain energy to be present without a stress field, which cannot be so with the previous stress or strain based criteria (which assume a stress-strain relationship). For plane stress problems (setting all z-components of stress equal to zero), the SED is directly proportional to von Mises' (equivalent octahedral) stress, and therefore is equivalent in this case to the more basic safe life criteria. The SED goes beyond fracture mechanics in that it handles cracked or uncracked ductile materials.

Finally, there is an approach that tries to bridge the gap between the discontinuous fracture mechanics and the continuum mechanics of safe-life approaches. This failure mechanics theory is called **Continuum Damage Mechanics** (circa 1970), and uses thermodynamics coupled to elasticity/plasticity theory to establish the initiation/nucleation of cracks within a continuum based upon the irreversibility of strain energy at the microscale. This approach builds upon original energy concepts of Griffith and establishes a continuous damage variable which represents the deterioration of the material before initiation of the macrocrack [ref. 6]. This establishes the boundary condition for fracture mechanics, namely, the initial crack size (a typical value for an engineering size crack is .1 inch) for life/inspection-interval analysis, though it is not necessary to couple it with fracture mechanics since it is a stand alone predictive theory of material degradation to failure. Damage mechanics couples the strength material property of fatigue analysis with a damage parameter and adds the kinetic damage parameter to establish the initiation of a crack. Damage mechanics compliments and goes beyond fracture mechanics. Therefore, damage mechanics quantifies the critical combinations of four variables: applied load, damage strength material parameter, damage plastic strain threshold, and critical damage toughness material parameter.

There is a vast amount of literature on fatigue. While all of the material listed in the bibliography were examined, they were done so with the intent of picking out the general notions of these theories. Some of the papers were specific formulations or theoretical extensions for narrow regions of application. These will not be addressed here. The objective of this report is to summarize this information as it may apply to turbine engine component life assessment, i.e. on an engineering level as opposed to a theoretical one. This report will describe in detail each of the broad elements of fatigue alluded to above, highlighting the assumptions, strengths and weaknesses of these approaches. A comparison of these techniques for evaluating high cycle fatigue potential for turbine engine components will be made. Other factors (some controversial) pertinent to the fatigue problem will be described. Finally, an approach for assessing the potential for

fatigue will be described from a test and evaluation perspective. The emphasis is in identifying an approach or methodology which can be implemented economically by *any* structural analyst, not just one who has an enormous budget and a materials laboratory. Such a methodology requires making the most out of the information available, and yet is based on realistic physics and has at least a limited predictive capability.

2. STRENGTH of MATERIALS

The strength of materials approach to fatigue resulted in what is known as *safe-life* techniques, where the component is assumed to have infinite life at design load levels below a threshold called the endurance limit, and the material is assumed to be structurally defect free until failure (crack, yield, etc.) due to an overload. Only an overload can cause failure in safe-life, and loading adequate to propagate inherent material discontinuities, cracks, or voids is considered an overload condition. The material is assumed homogeneous in a statistical sense, isotropic such that uni-axial loading fully defines the stress condition and therefore all that is necessary to characterize material strengths, and continuous such that load distributes continuously from one material point to another within the component. In these approaches, the component will neither fail catastrophically nor require repair prior to the design life-time [ref. 5]. Consequently, no attempts are made to quantify damage or use a damaged (cracked, or yielded) component regardless of how benign the damage may be. Typically, the tests that generated the fatigue correlation for the material took the material to complete failure (i.e., two pieces). Finally, these approaches never generated inspection criteria or fault monitoring techniques, and hence nearly all failures were sudden (broke into two pieces or sudden increase in deformation) and unexpected.

The fatigue analysis safe-life techniques include stress-life (the first fatigue analysis method, circa mid-1800's) and strain-life (circa 1960) approaches to fatigue. These approaches emphasize stress and strain, respectively, as the controlling parameter for failure. These techniques describe the failure mechanism in terms of an overloading condition correlated to a given failure criteria based upon ultimate strength, yield strength, and the endurance limit of the material, assuming the material to be a homogeneous isotropic continuum. Consequently, safe-life approaches quantify the

critical combinations of only two variables: applied load and material strength (stress or strain).

Stress-Life

Stress-life is used primarily for long life (high cycle, low load) applications where stresses and strains are purely elastic, and deals only with total life [ref. 9], ignoring the distinction between crack initiation life and crack propagation life, since the concept of *crack* does not exist for safe-life approaches. One of the major drawbacks of the stress-life approach is that it ignores true stress-strain behavior and treats all strains as elastic [ref. 9]. This is probably a good assumption for the HCF regime, but fatigue cracks form due to local plastic straining of the material, which can occur even in low load level cases, and therefore this approach ignores the crack formation mechanism itself.

The basis of the stress-life approach to fatigue is the S-N or σ -N diagram, which is an empirical plot of alternating stress (symbolized as either S or σ) versus number of totally reversed cycles (N) to failure. A generalized S-N curve for wrought steels is shown in Figure 2-1.

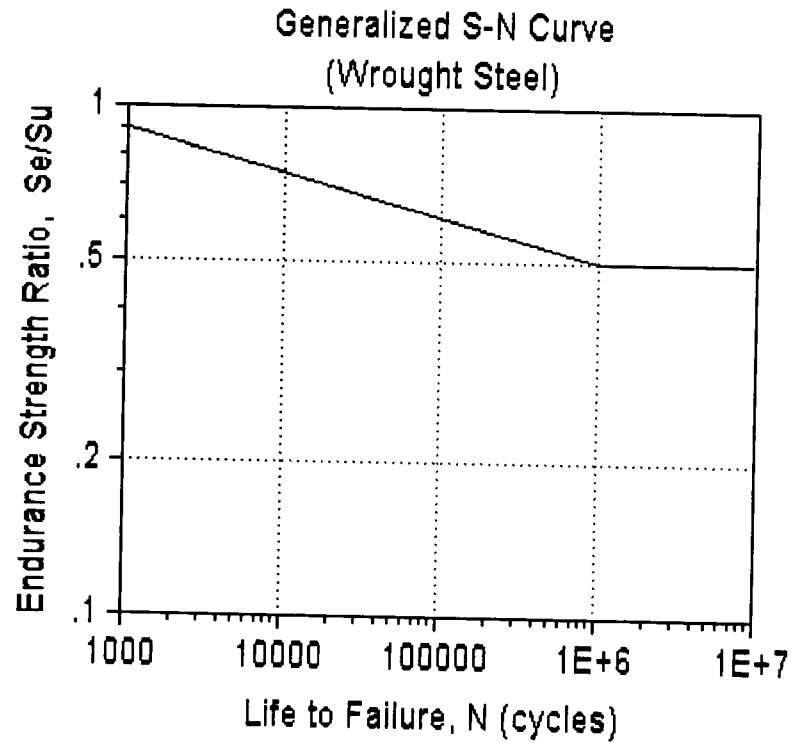


Figure 2-1. Generalized S-N Curve for Wrought Steels [ref. 9].

Typically, these plots are generated by a rotating bending test for the material specimen of interest. This type of test generates a fully reversed uni-axial tensile loading scenario at the surface of the specimen, decreasing linearly in amplitude towards the center or neutral axis (see Figure 2-2).

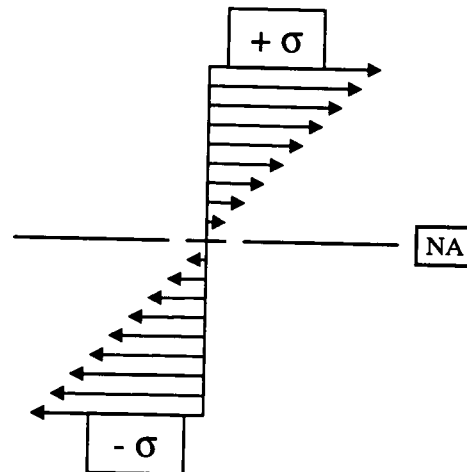


Figure 2-2. Stress Distribution Across Cross-Section for Rotating Beam Testing of Materials.

The stress at the surface of the specimen is used in the plot since this is the region where the failure will initiate. This stress level, σ , is calculated by the simple beam equation:

$$\sigma = \frac{Mc}{I}$$

where M is the bending moment, c is the distance from the neutral axis, and I is the area moment of inertia relative to the bending axis. The bulk of material characterization plots are of this form (Figure 2-2) and are generated in this manner. A problem with this form of material testing is that the presence of internal voids/cracks are masked in the low stress region internal to the specimens, thereby having a diminishing effect on the results as discontinuities approach the center of the specimen. Cyclic tensile tests are better in this respect since the stress profile across the cross-sectional area is flat (see Figure 2-3) such that a void will have a direct effect on the results. Another problem is that if the rotational speed is excessive, significant heating can occur in the specimen which is typically ignored, or at least uncontrolled, but yet can increase the plastic straining leading to an early failure.

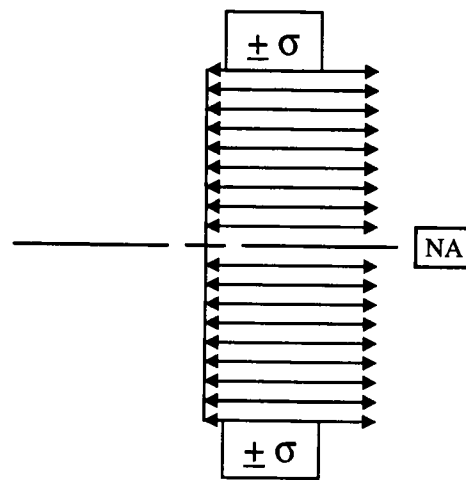


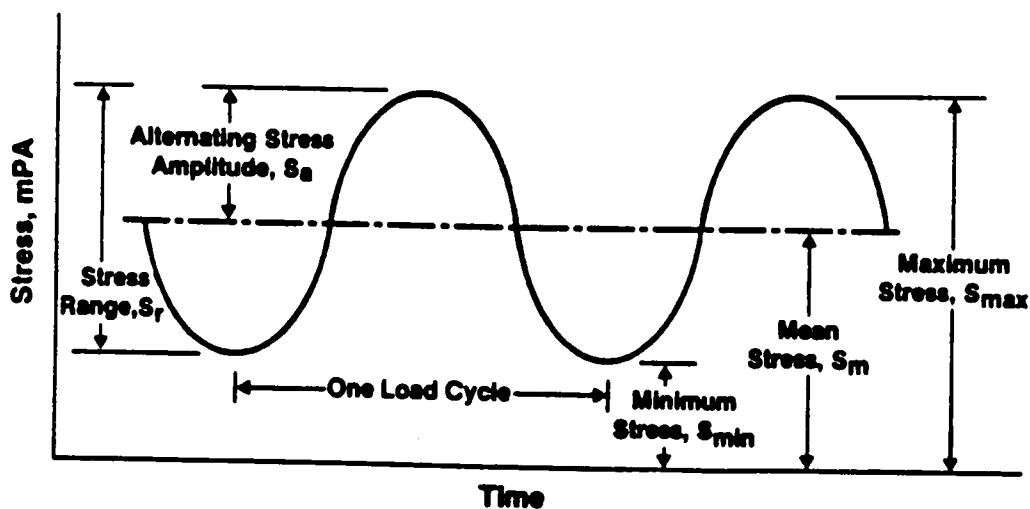
Figure 2-3. Stress Distribution Across Cross-Section for Cyclic Tensile Testing of Materials.

Finally, a problem with the definition of failure is also an issue for these type tests. Older tests took the specimen to complete failure (i.e., resulting in two distinct pieces), whereas

more recent tests defined failure as a drop in load carrying capacity (such as a 10% drop in axial load or torsional load capacity [ref. 13] or the length of visible crack).

Consequently, considerable scatter exists in these S-N plots, depending upon the type of test, the definition of failure, the speed at which the tests were performed, and the presence of dislocations in the proper orientation within the specimen.

Basically, under stress-life approaches, the intent is to correlate a fatigue factor to simple tensile strengths. A problem with such an approach is that the mean stress and not just the alternating (fully reversed) stress plays a role which simple rotating beam tests ignore. The Goodman Diagram describes the capability of a material to withstand cyclic loading (alternating stress or strain) as a function of the mean stress/strain (or static load) for a given life (number of cycles). See Figure 2-4 for a definition of these parameters.



Note that: $S_m = \frac{S_{\max} + S_{\min}}{2}$, $S_r = S_{\max} - S_{\min}$, $S_a = \frac{S_r}{2} = \frac{S_{\max} - S_{\min}}{2}$

R or A = Stress Ratio $R = \frac{S_{\min}}{S_{\max}}$, $A = \frac{S_a}{S_m}$ And $R = \frac{1 - A}{1 + A}$, $A = \frac{1 - R}{1 + R}$

Figure 2-4. Definitions of Cyclic Stress Parameters.

Various approaches to correcting rotating beam and cyclic tensile test data for mean stress have been proposed. The first was that of Gerber (1874), which is a parabolic fit to the data based upon

$$\frac{K\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_u}\right)^2 = \frac{1}{FS}$$

The next was that of Goodman (1899), which is a linear fit described as

$$\frac{K\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = \frac{1}{FS}$$

Soderberg adjusted for mean stress based upon yield, such that

$$\frac{K\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{FS}$$

where $\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$, $\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$, K = Stress Concentration Factor,

$S_e = S_n$ = endurance limit for that number of cycles, S_u = ultimate strength, S_y = yield

strength, and FS is the Factor of Safety, $FS = \frac{P_f}{P_w}$, where P_f is the failure load

(theoretically or experimentally based) and P_w is the working load. It can be shown that in general, the design allowable alternating load for a given mean stress is

$$\sigma_{aa} = \frac{S_e}{K} \left[\frac{1}{FS} - \left(\frac{\sigma_m}{S_u} \right)^N \right]$$

where $N = 1$ gives the Goodman criterion, and $N = 2$ gives the Gerber criterion.

Of these three approaches, relative to Goodman, the Soderberg approach is ultra-conservative, and the Gerber approach is too liberal. Consequently, the Goodman approach is the most widely used. Figure 2-5 illustrates these design curves for a typical steel [ref. 14].

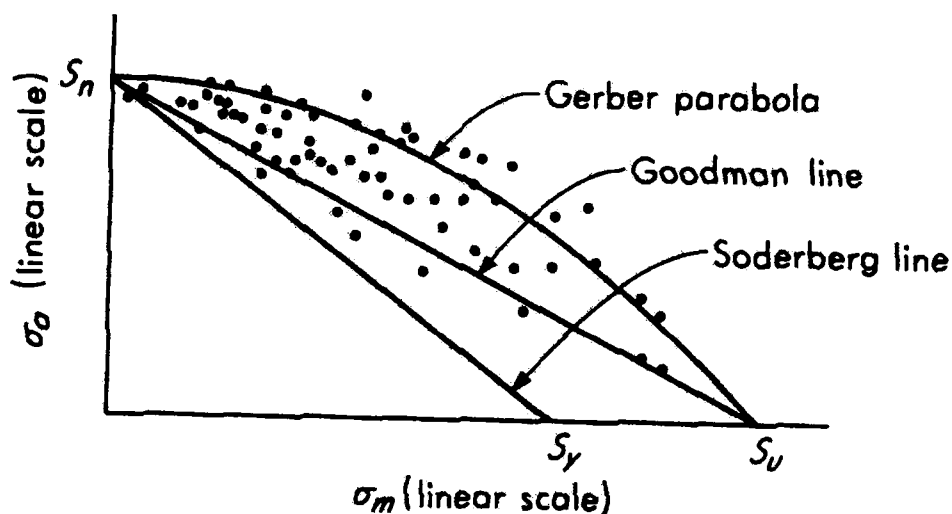


Figure 2-5. Goodman Diagram.

A modification to the Goodman diagram (see Figure 2-6) is typically made to include yielding as a criterion. This combines the Soderberg and Goodman approaches. In this case, a line joining S_y on both the mean stress (x) axis and the alternating stress (y) axis intersects the typical Goodman line, with the intent being that designs should stay below the lower of either the Goodman line or this line subtended by S_y . This intersection point has a mean stress of

$$\sigma_{\text{int}} = \frac{\sigma_y - \sigma_e}{1 - \frac{\sigma_e}{\sigma_u}}$$

In this case, the Goodman interaction equation for the segment from the normal Goodman line intersection defined above to S_y on the mean stress (x) axis is

$$\frac{K \sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{FS}$$

Figure 2-5 illustrates a modified Goodman diagram.

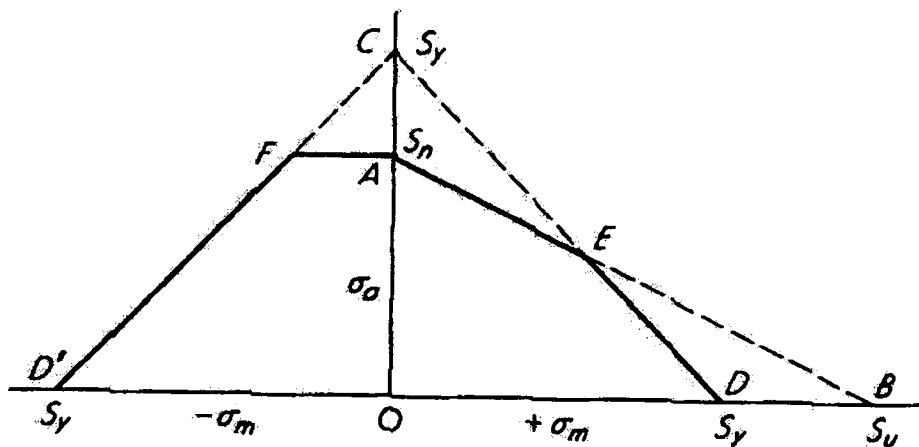


Figure 2-6. Modified Goodman Diagram.

The Goodman-type diagrams are considered *constant life* diagrams in that each point on the curve has the same life (usually 10^7 for engine HCF evaluations). The following plot, Figure 2-7 [ref. 5], illustrates the generation of a constant life diagram from S-N diagrams.

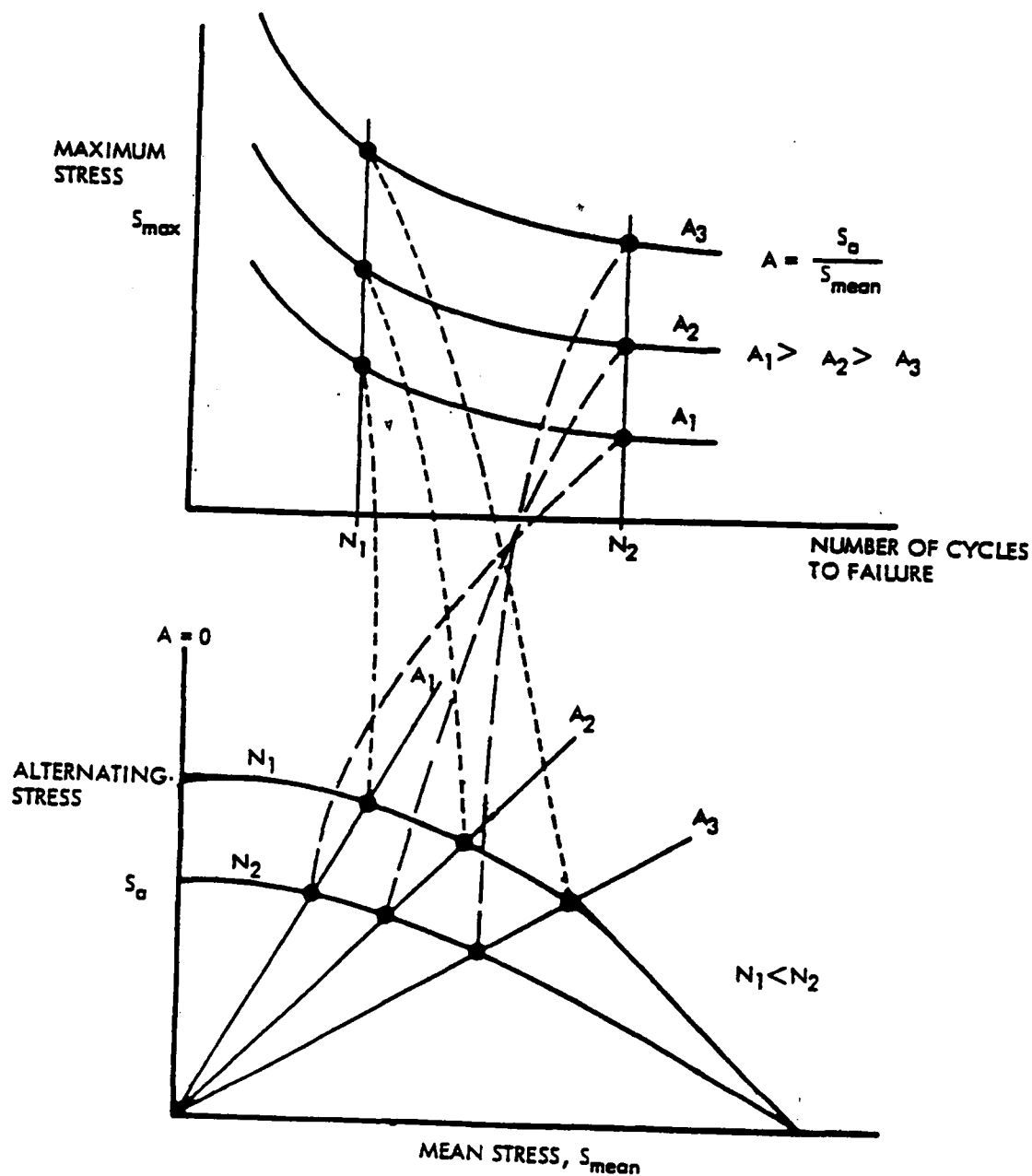


Figure 2-7. Generation of Constant Life Diagram from S-N Diagram.

The above relations correlate the fatigue specimen data, but in a typical application, there are several choices as to which stress is utilized for comparison to the specimen data. Also, a variety of failure theories exist, which are applied for the stress or strain analysis dependent upon the anticipated failure mechanism, whether uni-axial or multi-axial stress/strain situations exist, and whether the problem is essentially a static situation or one of considerable load fluctuation. The following is a discussion of several theories developed to correlate standard tensile (typically uni-axial) test data.

Maximum Normal Tensile Stress Theory is credited to W.J.M. Rankine (1802-1872) and simply states that a material subjected to any combination of loads will [ref. 14]:

- **yield** whenever the greatest positive principal stress exceeds the tensile yield strength of the material or the greatest negative principal stress exceeds the compressive yield strength of the material. Therefore, to prevent yield, this theory requires

$$|\sigma_{\max}| < |S_y|$$

- **fracture** whenever the greatest tensile or compressive principal stress exceeds the tensile ultimate or compressive ultimate strength of the material, respectively. Therefore, to prevent fracture, this theory requires

$$|\sigma_{\max}| < |S_u|$$

This theory is based upon the assumption that a single stress parameter controls failure, the largest normal, and is independent of the other principal stresses. This theory led to the uni-axial specimen test as the sufficient test for characterizing materials. This theory is best used for static unitary load situations and brittle materials (no plastic deformation). This theory cannot predict strength for ductile failure due to biaxial stresses [ref. 14], and is typically non-conservative in these situations. Figure 2-8 defines the design envelope for the Maximum Normal Stress Theory, based upon yields for tension and compression (similar for ultimates in tension and compression).

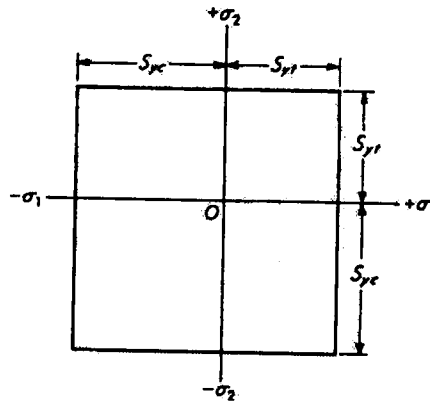


Figure 2-8. Maximum Normal Stress Theory Design Envelope for Yielding.

Maximum Shear Stress Theory is probably the oldest theory of failure and is attributed to C.A. Coulomb (1736-1806). This theory is sometimes called the Tresca theory due to an important paper he wrote related to the maximum shear stress theory in 1864, and sometimes called the Guest Theory due to the confirming tests J.J. Guest performed in 1900. This theory states that a material subjected to any combination of loads will fail (by yielding or fracture) whenever the maximum shear stress exceeds the shear strength (yield or ultimate) of the material [ref. 14]. The shear strength is typically taken as one-half the tensile strength, as indicated by the extremum points on the shear diagonal. Consequently, for design purposes,

$$\tau_{\max} < .5S_y$$

Therefore, maximum shear stress theory states that a material is 50% as strong in shear as it is in tension as shown by the shear diagonal [ref. 14]. This failure theory predicts that failure **cannot** occur due to pure hydrostatic loading. This theory is applied in both static and cyclic loading cases for ductile materials (@ long life), and requires compressive strength data in addition to the tensile strength data. Figure 2-9 defines the design envelope for the Maximum Shear Stress Theory, assuming yield strength is the same for both tension and compression. This envelope is defined by [ref. 15] as

$$S_y = \pm \max(|\sigma_1 - \sigma_2|, |\sigma_1|, |\sigma_2|)$$

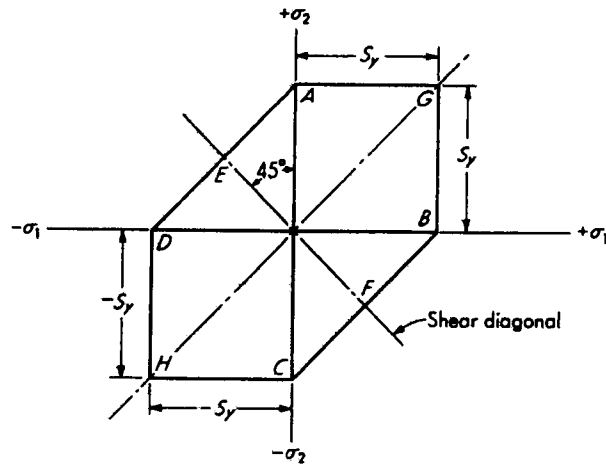


Figure 2-9. Maximum Shear Stress Theory Design Envelope.

Mohr Theory, described by Otto Mohr, states that the envelope generated by drawing Mohr's circles for each test condition be taken as the boundary representing the stress states on the verge of failure. This theory applies to multi-axial and uni-axial load situations since it is based upon the Mohr's circle, and reduces to the Maximum Shear Stress Theory if only uni-axial tension and compression data are available. Reytó in 1897 showed that the Mohr theory is identical to the Internal-Friction Theory, which assumes sliding along slip planes is inhibited by friction associated with compressive stresses acting on the slip planes [ref. 14]. This theory is typically more conservative than the octahedral shear stress theory described below. Figure 2-10 defines the design envelope for the Mohr/Internal Friction Theory (sometimes called the Coulomb-Mohr Theory) for yielding (similar for fracture assuming ultimate stresses). This criteria is defined (in the linear case for fracture at ultimate stresses, see Figure 2-11) by

$$\tau_u = \frac{1}{2} |\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2) \quad \text{where} \quad m = \frac{\mu}{\sqrt{1 + \mu^2}} = \cos(\phi) = \frac{\sigma_{uc} + \sigma_{ut}}{\sigma_{uc} - \sigma_{ut}}, \quad \phi \text{ is the}$$

tangent point angle based upon tension and compression data from uni-axial tests.

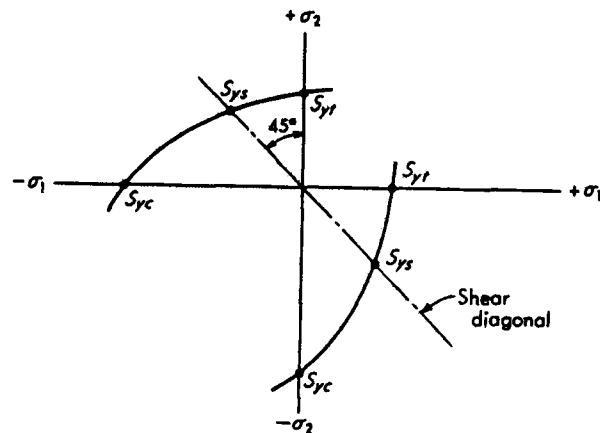


Figure 2-10. Mohr/Internal-Friction Theory Design Envelope.

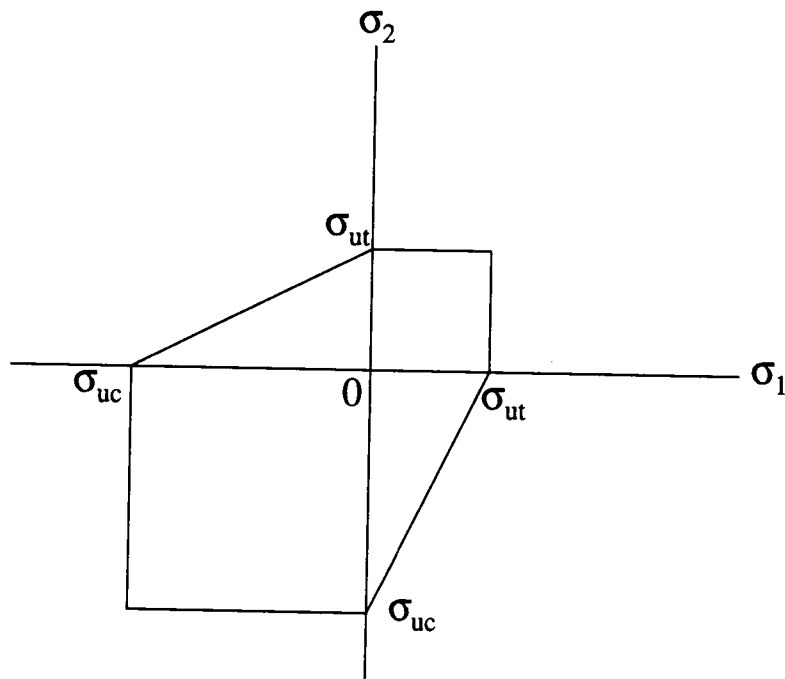


Figure 2-11. Linear Mohr/Internal-Friction Theory Design Envelope.

Maximum Distortion Energy or Octahedral Shear Stress Theory was developed by M.T. Hueber of Poland around 1904. It is sometimes called the von Mises or Mises-Hencky Theory due to further developments and explanations by R. von Mises in 1913

and Hencky in 1925. This theory states that yielding will occur whenever shear stresses on octahedral shear planes exceeds a critical value. An octahedral plane is a plane within a cube generated by collapsing a diagonal line of one face of the cube to a point at the furthest opposing corner of the cube (see Figure 2-12).

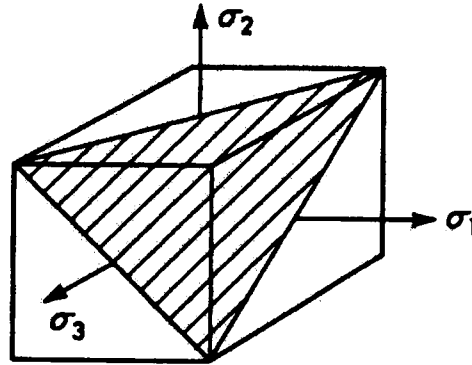


Figure 2-12. Octahedral Plane.

For a uniaxial tensile test used to characterize the material of interest, this critical value is

$$\tau_{oct_{max}} = \frac{\sqrt{2}}{3} S_y = .47140 S_y$$

This theory considers contributions from all principal stresses, and is used for materials which have similar strengths in tension and compression, for multi-axial, static or cyclic, loading and where ductile yielding is the expected failure mechanism. Hydrostatic deformation is assumed to have no effect on failure [ref. 14]. Figure 2-13 defines the design envelope for the Octahedral Shear Stress Theory, and is determined by

$$S_y = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2}.$$

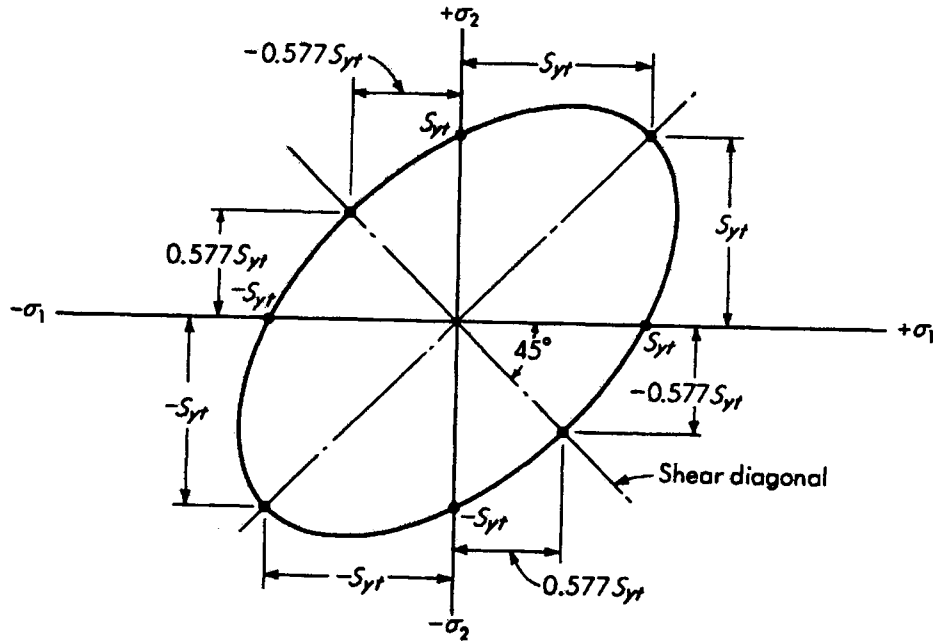


Figure 2-13. Octahedral Shear Stress Theory Design Envelope.

For three-dimensional states of stress, the fatigue **alternating and mean stresses** are typically calculated using the Von Mises distortion energy (or octahedral) criterion equivalent uni-axial stress (i.e., uni-axial tensile stress which produces the same level of octahedral shear stress as the actual combination of principal stresses)

$$\sigma_e = \frac{1}{\sqrt{2}} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \right]^{\frac{1}{2}}$$

$$\tau_{oct_{max}} = \frac{\sqrt{2}}{3} \sigma_e$$

which is then compared to S_y for failure (yielding) potential. For pure biaxial shear, (i.e., $\sigma_1 = +\tau$, $\sigma_2 = 0$, and $\sigma_3 = -\tau$), then

$$\tau_{crit} = \frac{S_y}{\sqrt{3}} = 0.57735 S_y$$

Therefore, Octahedral Shear Stress Theory states that a material is 57.7% as strong in shear as it is in tension as shown by the shear diagonal [ref. 14], which is slightly less conservative than the Maximum Shear Stress Theory. However, Maximum Shear Stress

Theory defines maxima on two planes, whereas the octahedral maxima occur on four planes, and therefore doubles the probability of detection of critical planes for favorable slip [ref. 15] (see Figure 2-14). Figure 2-15 illustrates the design envelopes of the Max Normal, Max Shear, and Max Octahedral Shear stress criteria for several common materials.

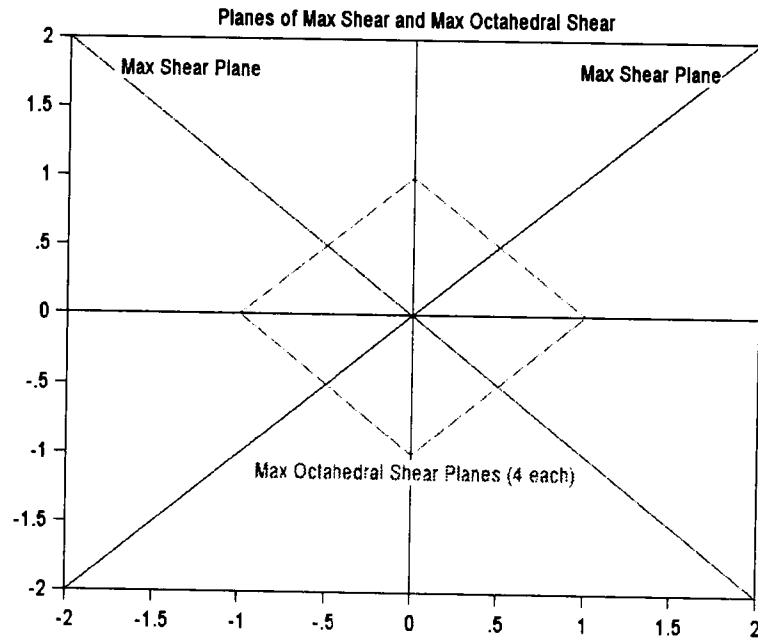


Figure 2-14. Planes of Maximum Shear and Maximum Octahedral Shear.

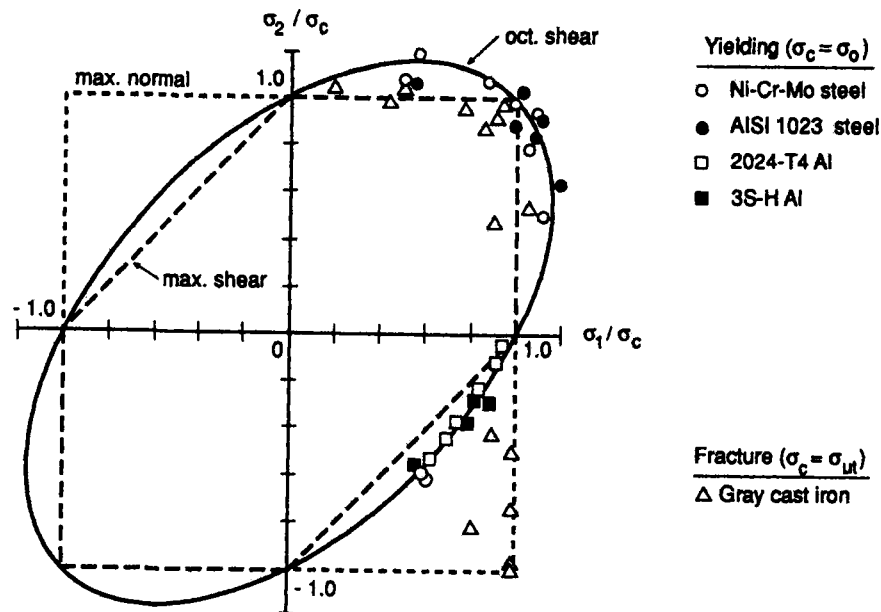


Figure 2-15. Comparison of Max Normal, Max Shear and Octahedral Shear Envelopes.

Alternatively, the **Modified Basquin Equation** (circa 1910) describes the effect of the mean stress as a reduction of the monotonic true fracture strength σ_f as [ref. 16]

$$\frac{\Delta\sigma}{2FS} = (\sigma_f - \sigma_m)(2N_f)^b$$

where $\frac{\Delta\sigma}{2}$ is the true stress amplitude, N_f is the cycles to failure such that $2N_f$ then describes the number of half-cycles (or stress reversals) consistent with various techniques for multi-spectrum cyclic loading (see the *Strain-Life* section for more discussion of the Basquin equation). This approach defines a fatigue strength exponent b as the linear slope of the log-log plot of this function.

All of the above theories were contrived to add some physics to the explanations of observations from material characterization experiments. Therefore, they are only as good as the original data and none can be used if the failure mode is significantly

different than that for which the data applies (i.e., none of these theories will suffice for brittle failure of normally ductile steels [ref. Juvinall, also see Timoshenko&Young for an example of this type of failure]).

Stress life approaches have generated significantly dangerous correlations. The foremost is that the endurance limit can be related to Brinell hardness number (BHN)

($S_e \approx .25 * BHN$ for $BHN \leq 400$) and the other relationships of endurance limit to

ultimate tensile strength ($S_e \approx .47 * S_u$) and endurance limit corrections due to loading

(eg., $S_{e,Torsion} \approx \frac{1}{\sqrt{3}} * S_{e,Bending}$). Consequently, these relationships have had a detrimental

effect on design by leading analysts to eliminating the test aspects of fatigue analysis and discouraging the material science aspects of mechanical engineering curriculums.

Because it has been shown that low cycle fatigue situations exhibit significant plastic strains, generally, the S-N (Stress-Life) approach should not be used to estimate lives below 1000 cycles [ref. 9], and component testing should be used to verify results of the analysis. The stress-life approach remains widely used today for HCF situations, in fact, the NASA-Marshall alternate turbopump development (ATD) program [1992] for the space shuttle main engine (SSME) utilized this approach. In general, the von Mises multi-axial equivalent stress correlates well with experience when applied to fatigue conditions using uni-axial specimen data for un-notched components, at least for the mid to high cycle range of the S-N curve.

The key features of Stress-Life as a fatigue theory are outlined below:

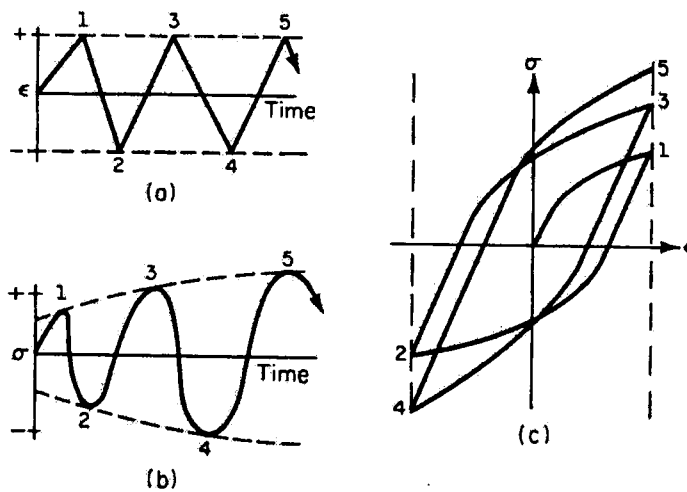
Key Features:

- I. Assumptions:
 - A. Material is isotropic homogeneous continuum
 - B. Fatigue directly related to tensile strength

- C. Infinite life is possible based upon stress level alone
- II. Applications Areas:
 - A. Lower strength ductile materials
 - B. Constant amplitude fatigue
 - C. Low strain, long life, high cycle fatigue (insignificant plastic deformation)
 - D. Materially and geometrically smooth specimens
- III. Advantages:
 - A. Simple to use, provides initial rough estimates easily
 - B. Simple stress (load) controlled material characterization experiments/data
 - C. Wealth of material data
- IV. Disadvantages:
 - A. Completely empirical correlation
 - B. Inconsistency in how material characterization data was gathered
 - C. Stress concentration factors are not well correlated for multi-axial fatigue
 - D. Does not account for plastic strain (i.e., assumes linear stress-strain relationship), therefore short life not accurately predicted
 - E. Simplicity loses sight of underlying failure mechanism; crack initiation and propagation processes not distinguished

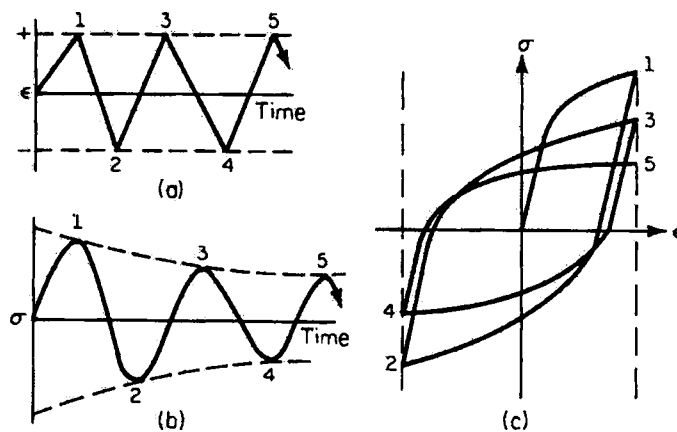
Strain-Life

Strain plays an equally important role in material fatigue endurance as does stress. The characterization of the material response (stress-vs-strain) curve is actually the locus of the stress-strain hysteresis loops observed during testing. If one looks closely at the cyclic stress-strain curves as shown in Figure 2-16, it is apparent that fatigue strength is dependent on how the material responds to early straining. In fact, this is why shot peening and work hardening aid in the fatigue resistance capability of strain-hardening materials.



Cyclic Hardening

- A.) Constant Strain Amplitude
- B.) Stress Response Increasing
- C.) Cyclic Stress – Strain Response Loop Climbing in Stress



Cyclic Softening

- A.) Constant Strain Amplitude
- B.) Stress Response Decreasing
- C.) Cyclic Stress – Strain Response Loop Falling in Stress

Figure 2-16. Early Life Cyclic Strain Response Curves.

However, it was not until about 1960 that strain was identified as a more suitable parameter for low cycle fatigue (LCF, low cycles but high loads) data correlations. This is ironic since typically significant plastic deformation in the regions of stress

concentrations is present in LCF failures, but separating the plastic strain from the total strain is a very difficult process and few characterizations of materials based upon plastic strain have been made. In addition, the designer seldom knows the amount of plastic strain occurring at critical component locations.

Nonetheless, strain-life is used for predicting initiation of a failure mechanism (crack or yielding) where plastic deformation exists, and is generally used for short fatigue life assessment (< 1000 cycles). The strain-life approach is very similar to the stress-life approach in that it assumes the material to be a homogeneous isotropic continuum, and still deals with quantifying the critical combinations of only two variables: applied strain and material strength for a given number of cycles to failure. Strain-life and stress-life are identical for long life, low load situations where plastic strains are negligible.

The same theories and failure criteria apply to strain-life as described above for stress-life. Typically, the generalized Hooke's Law is used to convert the stresses to strains, but it must be remembered that where there is failure, there is plastic strain except for the most brittle of materials, and therefore Hooke's law really doesn't apply since operation is in the non-linear region of the stress-strain curve.

The Maximum Normal Strain Theory (sometimes referred to as St. Venant's Failure Criterion) is the counterpart to the Maximum Normal Stress Theory, and states that failure will result with inelastic action at a point when the maximum strain for uniaxial tension reaches or exceeds

$$\epsilon_{\max} \geq \frac{S_y}{E}$$

Figure 2-17 illustrates the boundaries of the Maximum Normal Strain Theory.

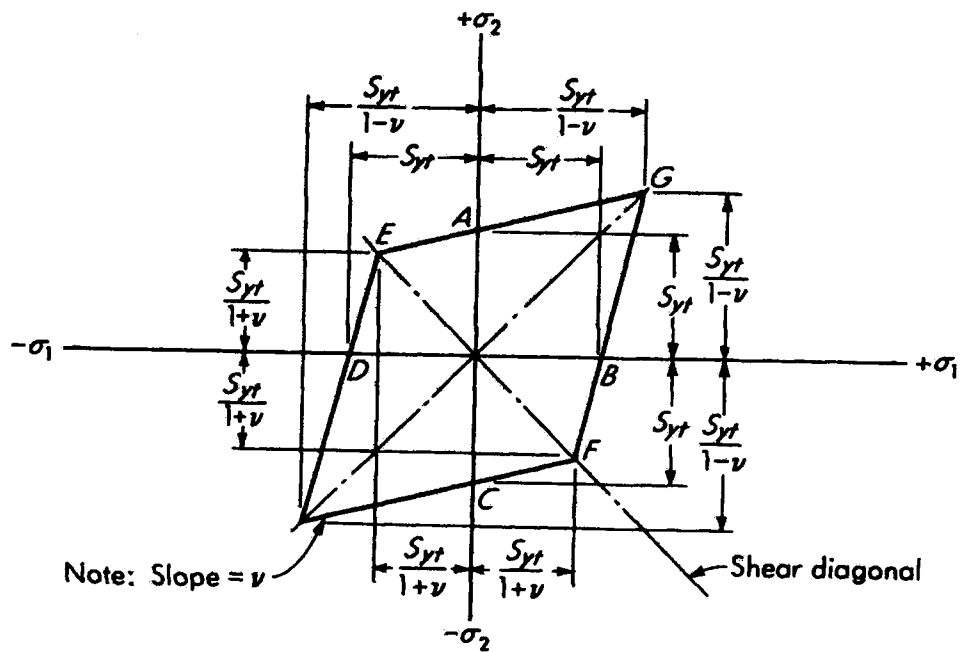


Figure 2-17. Maximum Normal Strain Theory Design Envelope for Yielding.

The Maximum Shear Strain Amplitude Theory is the counterpart to the Maximum Shear Stress Theory, and states that the failure will result due to damage on the plane of maximum shear strain amplitude defined by

$$\gamma_{\max} = (\epsilon_1 - \epsilon_3)_{\max}$$

The Octahedral Shear Strain (Maximum Distortion Strain Energy) Theory is essentially the same with principal strains substituted for principal stresses (for both **alternating and mean strains**), such that

$$\gamma_{oct} = \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_1 - \epsilon_3)^2 + (\epsilon_2 - \epsilon_3)^2}$$

A suitable fix to Hooke's Law for the plastic region is to define an effective Poisson's ratio which lies somewhere between pure elasticity ($\nu_e = .29$) for the material (steel in this case) and perfect plasticity ($\nu_p = .5$, incompressible), such that transverse shears for multi-axial loadings can be determined by

$$\epsilon_{transverse} = \nu_{eff} \epsilon = \nu_e \epsilon_e + \nu_p \epsilon_p$$

This establishes the transverse strain values required. Then the Maximum Shear Strain Theory for combined tension and torsion is represented by

$$\gamma_{max} = \sqrt{(1 + \nu_{eff})^2 \epsilon^2 + \gamma^2}$$

and the Octrahedral Shear Strain (Maximum Distortion Strain Energy) Theory for combined tension and torsion is represented by

$$\gamma_{oct} = \frac{\sqrt{2}}{3} \sqrt{4(1 + \nu_{eff})^2 \epsilon^2 + 3\gamma^2}$$

Zamrik and Frishmuth proposed an empirical total strain parameter defined as [ref. 13]

$$\epsilon_T = \sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2}$$

which showed good agreement with in-phase (all peaks of multi-axial loadings align) and out-of-phase (more critical since out-of-phase loadings produce more cycles of a diverse spectrum) data from tension-torsion experiments.

The Basquin equation describes both the stress and strain fatigue failure relationships (stress or strain versus number of cycles) based upon slopes of a log-log plot. For stress, the Basquin equation is

$$\frac{\Delta \sigma}{2} = \sigma_f (2N_f)^b$$

Coffin and Manson, working independently around 1950 [ref. 9], showed that the plastic strain-life data can be correlated by

$$\frac{\Delta \epsilon_p}{2} = \epsilon_f (2N_f)^c$$

Consequently, an alternative to the Basquin equation for strain-life can be defined by including both the elastic and plastic strain (i.e., $\epsilon_{Total} = \epsilon_e + \epsilon_p$) as

$$\frac{\Delta \epsilon}{2} = \frac{\sigma_f}{E} (2N_f)^b + \epsilon_f (2N_f)^c$$

where $\frac{\Delta\epsilon_e}{2} = \frac{\sigma_f}{E} (2N_f)^b$ is the true elastic strain amplitude associated with the failure,

$\frac{\Delta\epsilon_p}{2} = \epsilon_f (2N_f)^c$ is the true plastic strain amplitude associated with the failure, N_f is the cycles to failure, σ_f is the fatigue strength coefficient, ϵ_f is a fatigue ductility coefficient, b is the fatigue strength exponent, and c is a fatigue ductility exponent. σ_f , ϵ_f , b , and c are material properties. This approach defines the fatigue strain exponent c as the linear slope of the log-log plot of this function (see Figure 2-18). To account for mean stresses, the Modified Basquin equation of the *Stress-Life* section may be substituted for σ_f in the elastic term.

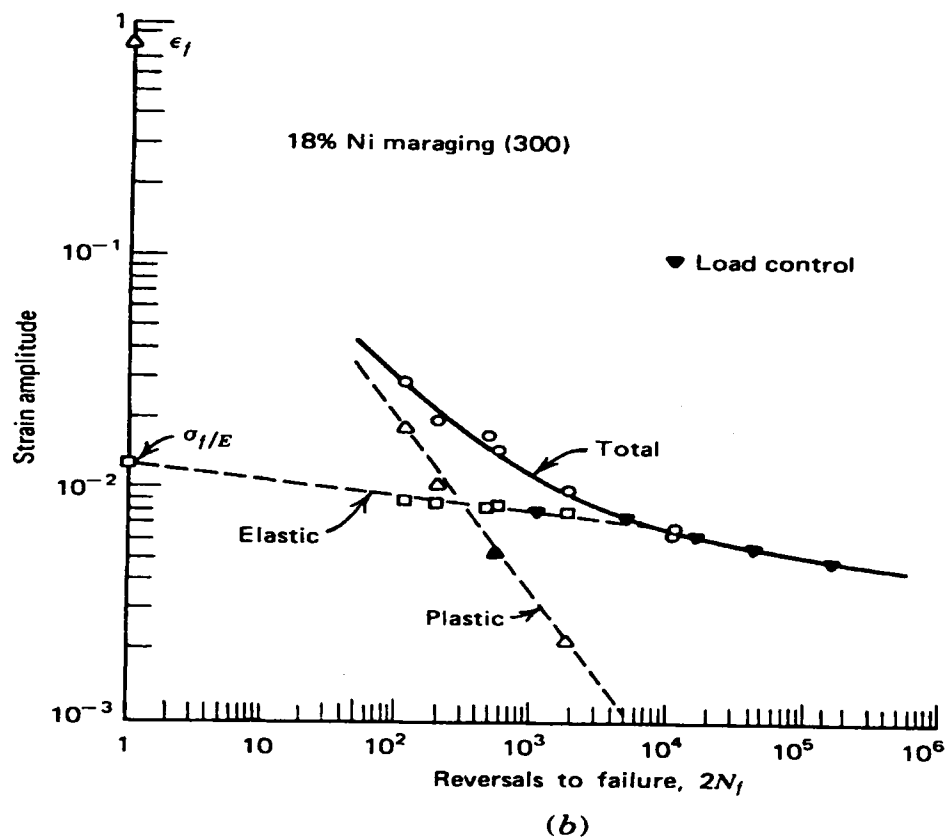
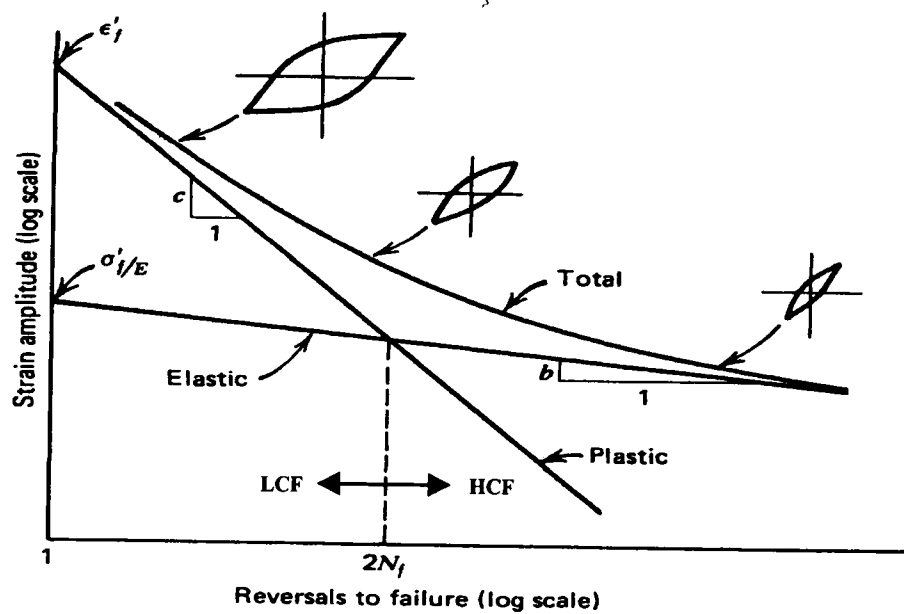


Figure 2-18. Total Strain-Life Curve [ref 15].

Manson also developed a strain-life equation known as the *method of universal slopes* which describes the life in terms of a strain range as

$$\Delta\epsilon = 3.5 \frac{S_u}{E} N^{-0.12} + \epsilon_f N^{-0.6}$$

where $\epsilon_f = \ln \frac{1}{1-RA}$ is the true fracture ductility, $RA = \frac{A_0 - A_f}{A_0}$ is the reduction in area from the original to final state at fracture. This is a potentially dangerous equation in that it defines the strain range in terms of purely uni-axial test data and leads analysts to believe that testing is not required.

The key features of Strain-Life as a fatigue theory are outlined below:

Key Features:

- I. Assumptions:
 - A. Lower strength ductile materials
 - B. Material is isotropic homogeneous continuum
 - C. Fatigue directly related to plastic strain
 - D. Infinite life is possible based upon strain level alone
- II. Applications Areas:
 - A. Plastic strain (which leads to crack initiation) accurately modeled
 - B. High strain, short life, low cycle fatigue (significant plastic deformation)
 - C. Materially and geometrically smooth specimens
- III. Advantages:
 - A. Simple to use
 - B. Can be used to predict the initiation of a crack
 - C. Handles cumulative damage under variable amplitude loading
 - D. May be used for high temperature/creep situations

- E. More applicable to situations with complicated geometries where no stress concentration factors are available
- IV. Disadvantages:
- A. Notch root analysis complicates technique, simple Neuber's Rule not always adequate
 - B. Requires strain controlled material characterization experiments/data
 - C. Simple constants (K_T) approach for surface finish/treatments doesn't work, requiring further testing and therefore more cost

For a given general mode of failure, each of the above theories (or failure criterion as known to some) identifies a significant single quantity that can be correlated to failure data (i.e., assumed to *cause* the failure) based upon this quantity reaching a critical level. It is assumed that in each case a tension test is adequate for determining the critical value of these significant quantities [ref. 17].

Critical Plane Theories

In addition to the strain versions of the stress failure theories, there are strain theories which try to include the plastic strain of critical planes based upon loading condition. These typically give better results than the strain versions of the stress failure theories cited above. These theories postulate that cracks initiate and grow on certain planes, and that the normal stresses or strains to those planes assist in the fatigue crack growth process [ref. 13]. Advantages of these theories are their physical interpretation of the fatigue damage process and the inclusion of hydrostatic or mean stress/strain effects through the normal stress/strain terms. Brown and Miller argued that since crack initiation is a slip process governed by plastic deformation, maximum shear strain is the controlling parameter and that the normal strain across the maximum shear strain amplitude plane assists in propagation [ref. 13]. This is also re-iterated by Fatemi and Socie: "It is widely accepted that fatigue crack initiation involves localized plastic deformation in persistent slip bands even in high cycle fatigue region. It has been

experimentally observed that the direction of these persistent slip bands are closely aligned with that of the maximum shear strain direction and fatigue cracks have always been found to initiate on the maximum shear strain planes under different loading situations" [ref. 18]. Brown and Miller postulated that

$$\frac{1}{2}(\epsilon_1 - \epsilon_3) + \frac{S}{2}(\epsilon_1 + \epsilon_3) = \text{Constant}$$

where S is a material constant found by fitting the pure torsion data to the uni-axial data.

Fatemi and Socie [ref. 18] have come up with what is referred to as the *Socie-Parameter*, which is due to similar critical plane arguments, but allows the inclusion of in-phase and out-of-phase loading conditions. This is a critical capability since "non-proportional or out-of-phase loading has most often been found to be more damaging than equivalent in-phase loading in the low cycle region..." [ref. 18]. This parameter (plotted in Figure 2-18) is defined as

$$\gamma_{\max} \left(1 + n \frac{\sigma_n^{\max}}{\sigma_y} \right) = \text{Constant}$$

where n is a constant found by fitting the uniaxial data against pure torsion data, σ_y is the yield strength of the material, and $\sigma_n^{\max}$ is the maximum normal stress on the plane of $\gamma_{\max}$. Combining this with the low cycle fatigue strain-life relation yields

$$\gamma_{\max} \left(1 + n \frac{\sigma_n^{\max}}{\sigma_y} \right) = (1 + \nu_e) \frac{\sigma_f}{E} (2N_f)^b + \frac{n}{2} (1 + \nu_e) \frac{\sigma_f^2}{E\sigma_y} + (2N_f)^{2b} + (1 + \nu_p) \epsilon \sigma_f (2N_f)^c + \frac{n}{2} (1 + \nu_p) \frac{\epsilon_f \sigma_f}{\sigma_y} (2N_f)^{(b+c)}$$

where ν_e is the elastic Poisson ratio and ν_p is the plastic Poisson ratio ($\nu_p = 0.5$ assuming incompressibility of plastic strains), with the other parameters defined previously. The correlation parameters can be found from uni-axial testing, however, surface finish, geometry, and size effects must be handled separately, similar to simple S-N based evaluations.

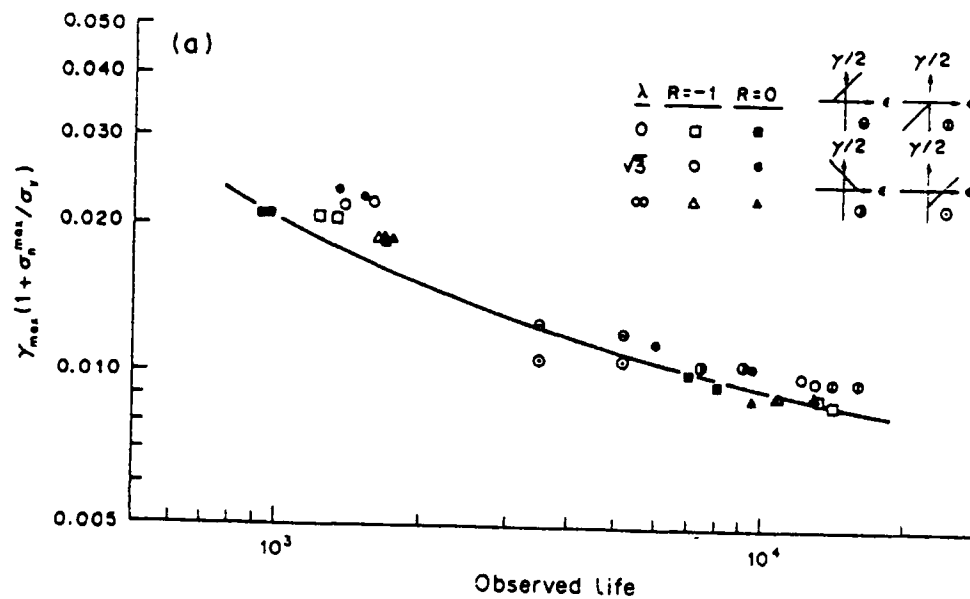


Figure 2-19. Socie-Parameter (critical plane) Fatigue-Life Curve for Inconel 718.

The above Socie equation assumes shear is dominating the cracking process. For tension dominating the crack process, the Smith-Watson-Topper (SWT) criteria is similar, but tends to correlate the fatigue data much better. The SWT parameter is defined as

$$\sigma_{\max} \epsilon_a = \frac{\sigma_f^2}{E} (2N_f)^{2b} + \frac{n}{2} (1 + \nu_e) \sigma_f \epsilon_f (2N_f)^{2b+c}.$$

The key features of Critical Plane fatigue theory are outlined below:

Key Features:

- I. Assumptions:
 - A. Brittle and ductile materials
 - B. Material is isotropic homogeneous continuum
 - C. Fatigue directly related to plastic strain on critical planes

- II. Applications Areas:
 - A. Plastic strain (which leads to crack initiation) accurately modeled
 - B. HCF and LCF adequately predicted
 - C. Materially and geometrically smooth specimens
- III. Advantages:
 - A. Simple to use
 - B. Can be used to predict the initiation of a crack
 - C. Handles cumulative damage under multi-axial non-proportional variable amplitude loading
 - D. More applicable to situations with complicated geometries where no stress concentration factors are available if testing is performed
- IV. Disadvantages:
 - A. Requires strain controlled material characterization experiments/data
 - B. Simple constants (K_T) approach for surface finish/treatments doesn't work, requiring further testing and therefore more cost

In summary, safe-life approaches assume that if the component is operated at a stress/strain level below the *Goodman* stress/strain/Socie-parameter strength of the material it is composed of for its allowable life, then the component will never fail. Safe-life approaches quantify the critical combinations of only two variables: applied load and material strength (stress, strain, or Socie-parameter). Again, safe life approaches do not allow for use of anything but defect free material, an idealization that is becoming less applicable as safety margins are reduced in favor of performance.

3. FRACTURE MECHANICS

From a comprehensive investigation and review of structural failures, E.E. Gdoutos made the following general remarks, quoted verbatim [ref. 12]:

- Most fractures were mainly brittle in the sense that they were accompanied by very little plastic deformation, although they were made of materials with ductile behavior at ambient temperatures.
- Most brittle failures occurred in low temperatures.
- Usually, the nominal stress in the structure was well below the yield stress of the material at the moment of failure.
- Most failures originated from structural discontinuities including holes, notches, re-entrant corners, etc.
- The origin of most failures, excluding those due to poor design, was pre-existing defects and flaws, such as cracks, accidentally introduced into the structure. In many cases, the flaws that triggered fracture were clearly identified.
- The structures that were susceptible to brittle fracture were mostly made of high-strength materials which have low notch or crack toughness (ability of the material to resist loads in the presence of notches or cracks).
- Fractures usually propagated at high speeds which, for steel structures, were in the order of 1000 m/s. The observed crack speeds were a fraction of the longitudinal sound waves in the medium.

Consequently, many instances of structural failures have occurred which could not be explained by standard failure criteria based upon a strength of materials approach, either statically or under fatigue conditions. Such failures are now attributed to the existence of microscopic crack-like voids and discontinuities in the material. These microscopic entities are not necessarily considered flaws, but merely part of the structure of the material itself. These microscopic crack-like voids or discontinuities can be either

inherent in the material or generated via overstraining (Dr. Hoepfner at University of Utah has a video which shows the initial formation of striations in the microstructure due to initial straining of the material). The voids/discontinuities magnify their presence by resulting in what is known as the *size-effect* of material strength, where as early as Leonardo da Vinci (1452-1519, ref. 12) it was shown that longer wires had reduced strength for the same tensile load conditions than shorter ones. More recently, Griffith [ref. 12] showed that not only did the strength vary with the amount of material (see Figure 3-0), but this strength was from 1% - 5% of the theoretical based upon internal molecular forces and bond strengths. Consequently, emphasis on the crack-like discontinuities is appropriate, and a theory which includes the scale of the discontinuities as a governing parameter is desired. In addition, this data tends to indicate that strength of a material is probabilistic in nature since the probability of defects is directly tied to the amount of material as well as the processing of the material.

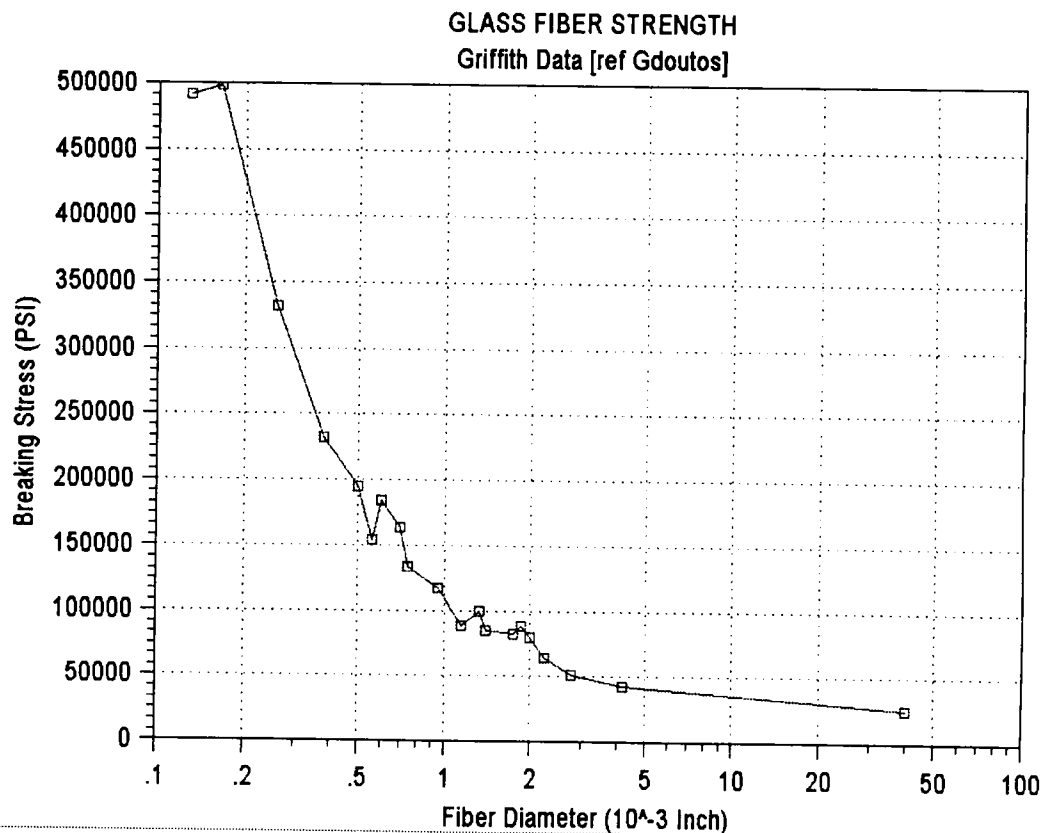


Figure 3-0. Glass Fiber Strength as a Function of Diameter (Material Mass).

The alternative to fatigue analysis is **Fracture Mechanics** (circa 1950) and the alternative to the safe-life approach is **Damage Tolerance**. This approach is diametrically opposed to the safe-life approaches. Here it is assumed that **all** materials have crack-like voids or discontinuities, sometimes referred to as defects or damage, which eventually lead to failure if the defect grows, or will function adequately with the damage if it doesn't propagate to a critical length within the design life. Cracks or any heterogeneity violates the continuum assumption upon which elasticity and plasticity theories are based. Fracture Mechanics attempts to describe life in terms of the propagation of cracks based upon the stress concentration and plastic flow at the edge of the crack in a more theoretically-based manner. In this sense, elasticity/plasticity theory is still an integral part of fracture mechanics, but only in the small region ahead of a crack. No attempt is made in Fracture Mechanics to model the crack itself, merely its propagation, for which a continuum in the immediate vicinity of the crack is assumed. However, fracture mechanics is also very dependent upon experimental data and correlations. In this approach, one either calculates the critical crack length that can lead to failure, or one calculates the growth rate (change in crack length per cycle) to establish inspection intervals appropriate for monitoring structural integrity. Typically, fracture mechanics is more conservative than fatigue analysis safe-life, since fatigue analysis considers the total life, including that prior to the initiation/nucleation of a crack and subsequent failure, while fracture mechanics predicts life remaining to propagate a crack to failure from an assumed initial crack size. Fracture mechanics can be coupled with the strain-life approach described previously to predict the total life of the component for low cycle fatigue, from pre-initiation to complete failure or the generation of a critical crack size.

There are two fracture mechanics formulations. The simplest is Linear Elastic Fracture Mechanics (LEFM), which relates stress and crack size to obtain the stress intensity at the crack tip and assumes stress is proportional to strain. This works very well for brittle materials or brittle failure of ductile materials, and can be applied if plastic strain in the

region of the crack tip is small to insignificant. For cases of significant plastic strain, Elastic/Plastic Fracture Mechanics (EPFM) must be used to obtain the same relations in a non-linear form.

Linear Elastic Fracture Mechanics

Following a development by Professor Engels of UTSI, during a Fracture Mechanics course, consider the following simple example to contrast LEFM against a Strength of Materials approach. Suppose a cantilever beam of length L , with a rectangular cross-section of $B \times H$, is statically loaded by load P at its free end. The stress distribution at the support end through the cross-section will be linearly varying as shown in the free-body diagram of Figure 3-1.

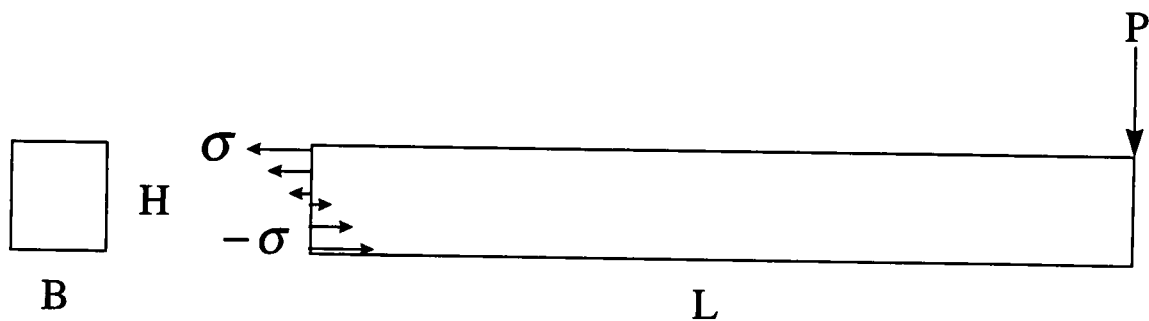


Figure 3-1. Free-Body Diagram of Cantilever.

From Strength of Materials, the beam will safely support the load P if σ does not exceed the yield strength adjusted for some factor of safety (i.e., $\frac{\sigma_y}{FS}$)

or $P < \frac{B * H^2}{6 * FS * L} \sigma_y$. Suppose there exists a crack of depth a at the support end of the beam across the whole width, as shown in Figure 3-2.

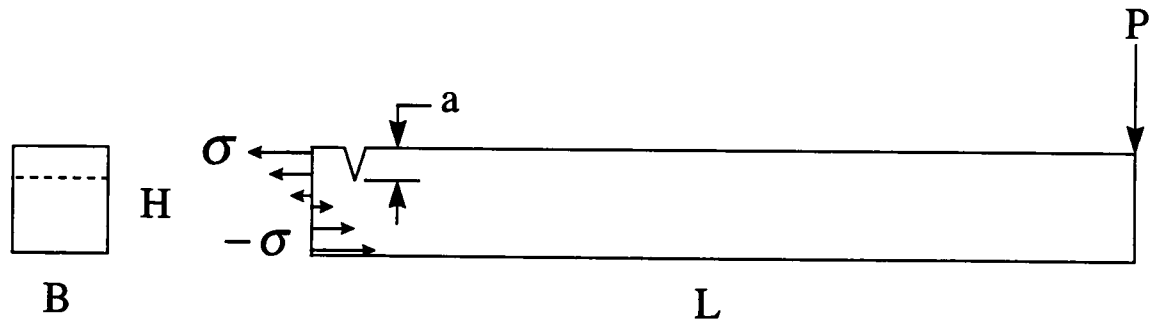


Figure 3-2. Free-Body Diagram of *Cracked* Cantilever.

From Fracture Mechanics, ignoring edge effects and finite width effects, the beam will safely support the load P if σ does not exceed the maximum stress defined as

$\sigma_{\max} = \frac{K}{1.12\sqrt{a\pi}}$, where K is a material parameter. Again adjusting for some factor of

safety (i.e., $\frac{\sigma_{\max}}{FS}$), $P < \frac{B * H^2}{6 * FS * L} \frac{K}{1.12\sqrt{a\pi}}$. It can easily be seen that the functional

relationships between these two equations is similar, with the geometry entering the equations in exactly the same way. However, the material properties are characterized slightly different, i.e., the Strength of Materials approach characterizes the material in terms of an allowable yield stress σ_y/FS , while the Fracture Mechanics approach characterizes the material in terms of a stress intensity factor K , and a new parameter, the crack length a . Basically, fracture mechanics replaces the strength material property of fatigue analysis with a fracture toughness material property and adds flaw size as a parameter of the analysis to determine structural suitability. Therefore fracture mechanics quantifies the critical combinations of three variables: applied load, fracture toughness, and flaw size.

Fracture Mechanics is “the applied mechanics of crack growth” [ref. 19], and as such, it can only deal with a crack situation. However, if one considers that a fracture cannot occur unless the bonds on the atomic level are broken (i.e., the stress at the atomic level exceeds the cohesive strength of the material), then the difference between the theoretical

strength and the actual strength of materials is an indication of the presence of cracks or flaws instead of true material homogeneity as in the elasticity theory sense. Typically, fracture strengths for brittle materials are three to four orders of magnitude less than the theoretical strength based upon chemical/atomic bonds alone [ref. 10]. Thus the effect of the presence of flaws is to lower the global strength of the material by magnifying the stress locally [ref. 10]. This gives rise to the concept of stress concentration factors found throughout structural analysis. Inglis (1913) was the first to give quantitative evidence for the stress concentration effect of flaws, by analyzing elliptical holes in flat plates, as shown in Figure 3-3.

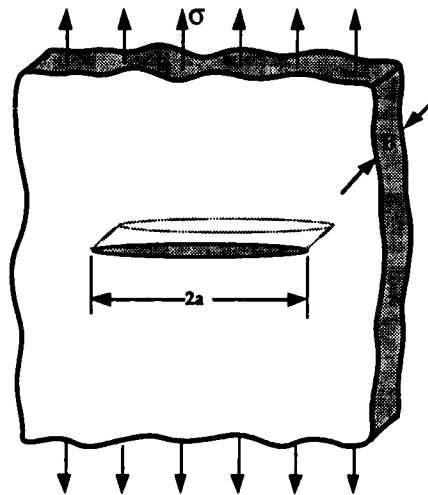


Figure 3-3. Elliptical Hole (Crack) in a Flat Plate. (Anderson Fig 2.3)

The stress at the tip of the elliptical hole of $2a \times 2b$ dimensions in an infinite plate of thickness B is given by

$$\sigma_A = \sigma_\infty \left(1 + \frac{2a}{b} \right),$$

where $k_t = \left(1 + \frac{2a}{b} \right)$ is the stress concentration factor ($k_t = 3.0$ for a circular, $a = b$, hole). Notice that as the hole flattens to a crack (i.e., $a \gg b$) then it is more appropriate to use the radius of curvature form

$$\sigma_A = \sigma_\infty \left(1 + 2\sqrt{\frac{a}{\rho}} \right)$$

where $\rho = \frac{b^2}{a}$ is the radius of curvature. Note that as the radius of the hole decreases, the stress tends toward infinity, such that at the edge of a sharp crack, infinite stress is predicted. Since no material can withstand infinite stress, it implies that the cracked part has no load carrying capability at all! This paradox caused A. A. Griffith (1921) to formalize a fracture theory based upon energy instead of local stress, where plastic flow coupled with crack surface separation within the material locally rounds the crack tip, thus reducing the stress concentration to a real value, albeit, still very high!

Griffith was the first to propose the energy balance approach to the study of the fracture phenomenon in cracked bodies. His basic premise was that unstable propagation of a crack takes place if an increment of crack growth results in more stored energy being released than is absorbed by the creation of new crack surface [ref. 20]. This is a very fundamental statement of the First Law of Thermodynamics which states that there is a net loss of energy as a system goes from a non-equilibrium state to a state of equilibrium [ref. 10], in agreement with the fact that energy seeks a minimum condition. Consider the total energy variation of a thermodynamic particle as it goes from state S to state $S + dS$ along some path C (see Figure 3-4):

$$\frac{dE}{dt} = \frac{dW}{dt} + \frac{dQ}{dt}$$

where $\frac{dW}{dt}$ is the work input to the particle along path C ($\frac{d}{dt}$ denotes a value, not a derivative, i.e., path dependent variation) and $\frac{dQ}{dt}$ is the heat input or release as the particle moves along C .

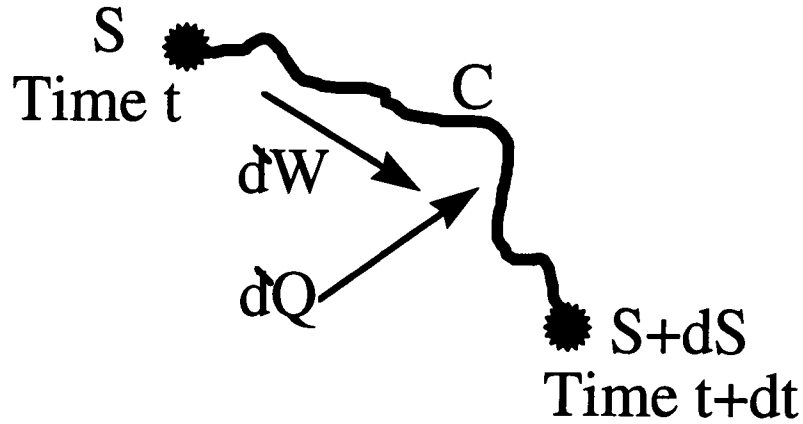


Figure 3-4. Particle Moving from One State to Another.

Further, consider the internal energy of the particle $U \equiv E - K$ where K is the kinetic energy. Then $E = U + K$ and

$$\frac{dE}{dt} = \frac{dU}{dt} + \frac{dK}{dt} = \frac{dW}{dt} + \frac{dQ}{dt}$$

Assuming loads that are *not* time dependent, and that the crack moves slowly enough that the kinetic energy and heating are negligible, then

$$\frac{dU}{dt} = \frac{dW}{dt}$$

Now the internal energy is made up of three components: U^e is the elastic strain energy (reversible); U^p is the plastic strain energy (non-reversible since it leads to permanent deformation); and Γ is the energy loss due to creation of new surfaces as the crack grows (bond breakage, also non-reversible). Then

$$\frac{dU^e}{dt} + \frac{dU^p}{dt} + \frac{d\Gamma}{dt} = \frac{dW}{dt}$$

For conservative loads (constant loads), the total potential energy Π is defined as the difference between the elastic strain energy and the work done, or $\Pi \equiv U^e - W$.

Therefore,

$$\frac{d\Pi}{dt} = \frac{dU^e}{dt} - \frac{dW}{dt} \quad \text{and} \quad \frac{dW}{dt} = \frac{dW}{dt}$$

Substituting results in

$$\frac{d\Pi}{dt} + \frac{dU^p}{dt} + \frac{d\Gamma}{dt} = 0$$

Assuming continuous steady crack growth in time, then via the chain rule $\frac{d(\cdot)}{dt} = \frac{d(\cdot)}{dA} \frac{dA}{dt}$

$$\frac{d\Pi}{dA} \frac{dA}{dt} + \frac{dU^p}{dA} \frac{dA}{dt} + \frac{d\Gamma}{dA} \frac{dA}{dt} = 0$$

and finally

$$-\frac{d\Pi}{dA} = \frac{dU^p}{dA} + \frac{d\Gamma}{dA}$$

This states that the potential energy decreases due to plastic deformation and new crack surfaces form as a result of crack propagation. For an ideally brittle material, no plastic deformation takes place such that, mathematically, Griffith stated the following [ref. 10]:

$$\frac{dE}{d\mathcal{A}} = \frac{d\Pi}{d\mathcal{A}} + \frac{d\Gamma_s}{d\mathcal{A}} = 0$$

where, again, $d\mathcal{A}$ is the incremental growth of the crack area (crack area is the projected area $\mathcal{A} = aB$ of the crack perpendicular to the opening direction, not the surface area which is essentially twice that since the crack has two surfaces), dE is the total energy increment, $d\Pi$ is the potential energy increment due to internal strains and external forces, and $d\Gamma_s$ is the work required to create new surfaces. Consequently, the surface generation energy balances the minimization of the total energy of a brittle material per the First Law of Thermodynamics. Griffith used the work of Inglis [ref. 10] to show that

$$\Pi = \Pi_0 - \frac{\pi\sigma^2 a^2 B}{E}$$

where Π_0 is the potential energy of the un-cracked plate, a is the crack half-length, and B is the plate thickness. The work to create two new crack surfaces is

$$\Gamma_s = 4aB\gamma_s$$

where γ_s is the surface energy of the material. Therefore,

$$\frac{dE}{d\mathcal{A}} = 0 = \frac{-\pi\sigma^2 a}{E} + 2\gamma_s$$

Now $G = -\frac{d\Pi}{d\mathcal{A}} = \frac{\pi \sigma^2 a}{E}$ is the rate of strain energy released as the crack is formed, which is absorbed to create the new surfaces. The strain energy release rate, G , is the driving force for fracture. Solving for the fracture stress σ_f yields

$$\sigma_f = \sqrt{\frac{2\gamma_s E}{a\pi}}$$

for plane stress. Irwin and Orowan [ref. 10] later modified this equation to account for the plastic flow at the crack tip experienced by ductile materials such that

$$\sigma_f = \left(\frac{2E(\gamma_s + \gamma_p)}{a\pi} \right)^{\frac{1}{2}}$$

for plane stress, where γ_p is the plastic work per unit area of surface created, and is typically much larger than the elastic (atomic bond breaking) work γ_s , due to dislocation motion in the vicinity of the crack tip.

Therefore, the theory of fracture mechanics logically rests on two premises. The first is that sufficient stress must be available at the crack tip to operate some mechanism of crack growth, and the second is that sufficient energy must flow to the crack tip to supply the work done in the creation of new surfaces [ref. 19]. Irwin [ref. 20 a & b] showed that the concept of a stress intensity factor, where crack growth occurs at a critical value, simultaneously satisfies both of these conditions. Irwin used plasticity theory and formulations due to Westergaard [ref. 21] to show that the stress distribution normal to the plane of the crack in a plate under tensile load can be stated as

$$\sigma_{yy} = \frac{K_I}{\sqrt{2r\pi}}, \text{ for } r > 0$$

where K_I is the stress intensity factor for crack mode I (crack mode I (one) is pure tensile, crack opening, typically this is the worst case for most engineering materials, i.e., mode II (pure shear) and mode III (tearing) have greater stress intensity allowables for the same material) and r is the linear distance from the crack tip. In addition, the stress

intensity factor for a mode I crack is $K_I = \sigma_\infty \sqrt{a\pi}$, and it is related to the strain energy release rate by $G = \frac{K_I^2}{E}$. Consequently, the Irwin approach results in

$$\sigma_{yy} = \sigma_\infty \sqrt{a\pi} * \frac{1}{\sqrt{2r\pi}} = \sigma_\infty \sqrt{\frac{a}{2r}}.$$

Again, infinite stress at the crack tip ($r = 0$) is predicted, and plastic flow in this region keeps the situation real. However, it is interesting to note that Inglis' relationship showed a crack tip stress dependency of $\sqrt{\frac{a}{\rho}}$, which is a dependence on crack radius and crack length, and the Irwin approach shows a dependency of $\sqrt{\frac{a}{r}}$, which is a dependence on the crack length and distance away from the crack tip! The formulations, while similar in approaches, result in different dependencies for their respective singularities at the crack tip.

K_{Ic} is the critical stress intensity factor where the combination of stress and crack size results in unstable crack growth. When a state of small-scale yielding exists in the neighborhood of the crack tip, then the stress distribution around the crack tip is characterized by the single K_{Ic} parameter *regardless of the geometry or loading* of the specimen [ref. 21]. Consequently, K_{Ic} is a material property, and is always of the form

$$K_{Ic} = \text{Applied Load} * \sqrt{\pi * \text{Crack Half Length}}.$$

K_{Ic} is sometimes called the plane strain fracture toughness, since it is usually determined using thick specimens where the thickness constrains the plastic flow at the crack tip to a small region in the vicinity of the crack and in the plane of the crack.

As the thickness of the material decreases, the zone of plastic deformation increases. This results in the phenomenon of *crack-tip blunting*, indicating that a relatively large

volume of material has deformed to relieve the local stress. Under these conditions, the crack propagation velocities radically reduce and it is possible for the crack to arrest occasionally until further crack extension is forced by increasing the stress beyond the yield point of the material. For these conditions, the fracture toughness defined above does not apply due to the presence plane stress conditions instead of plane strain conditions upon which K_{Ic} is based. An extension to the K_{Ic} concept for such cases has been introduced [ref. 22] where

$$K_C = K_{Ic} \left[1 + \frac{1.4}{B^2} \left(\frac{K_{Ic}}{\sigma_y} \right)^4 \right]^{\frac{1}{2}}$$

where B is the material thickness. Notice that $K_C > K_{Ic}$ to overcome the crack arresting phenomenon. Consequently, the critical crack length ratio becomes

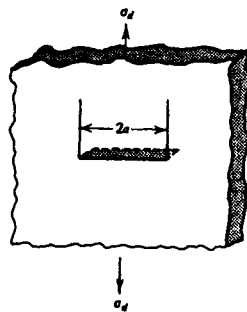
$$\frac{a_C}{a_{Ic}} = \left[1 + \frac{1.4}{B^2} \left(\frac{K_{Ic}}{\sigma_y} \right)^4 \right]$$

indicating that the critical crack length is larger in the plane stress case for the same loading conditions. Determining consistent K_C values is not without serious limitations due to the mixed mode stress field and the large plastic deformation which results. However, some values of K_C have been established for wide panels of thin materials, primarily for the aircraft industry. Another approach to handle large plastic zones is the J-integral approach to be discussed later.

The most valuable contribution of fracture mechanics to the structural analysis community is its quantification of *toughness* as a material characteristic by defining it as the *resistance to crack growth*. The strength of materials approach never truly defined what was meant by material toughness. The stress intensity factor K , is a single parameter description of the elastic stress field in the vicinity of the crack tip [ref. 19] that quantifies the ability of the material to redistribute stresses due to the presence of a crack.

Table 3-1 [ref. 23] contains the critical fracture parameters based upon the assumption that the operating design stress σ_d is one-half the material yield stress σ_{ys} .

Table 3-1. Plane Strain Fracture Toughness Parameters [ref. 23]



$$\sigma_d = \frac{\sigma_{ys}}{2}, K_{IC} = \sigma_d \sqrt{\pi a}$$

$$a_c = \frac{1}{\pi} \left(\frac{K_{IC}}{\sigma_{ys}/2} \right)^2$$

Material	K_{IC}		σ_{ys}		a_c	
	MPa $\sqrt{m}$	ksi $\sqrt{in.}$	MPa	ksi	mm	in.
2014-T651	24.2	22	455	66	3.6	0.14
2024-T3	~44.	~40	345	50	~21.	~0.82
2024-T851	26.4	24	455	66	4.3	0.17
7075-T651	24.2	22	495	72	3.0	0.12
7178-T651	23.1	21	570	83	2.1	0.08
7178-T7651	33.	30	490	71	5.8	0.23
Ti-6Al-4V	115.4	105	910	132	20.5	0.81
Ti-6Al-4V	55.	50	1035	150	3.6	0.14
4340	98.9	90	860	125	16.8	0.66
4340	60.4	55	1515	220	2.	0.08
4335 + V	72.5	66	1340	194	3.7	0.15
17-7PH	76.9	70	1435	208	3.6	0.14
15-7Mo	49.5	45	1415	205	1.5	0.06
H-11	38.5	35	1790	260	<0.6	<0.02
H-11	27.5	25	2070	300	0.23	0.009
350 Maraging	55.	50	1550	225	1.6	0.06
350 Maraging	38.5	35	2240	325	<0.4	<0.02
52100	~14.3	~13	2070	300	~0.06	<0.002

The above description of Fracture Mechanics is analogous to Strength of Materials for static loading. For fatigue, Fracture Mechanics introduces crack growth rate

$\left(\frac{da}{dN}, \text{ where } N \text{ is \#-cycles} \right)$ as a parameter analogous to the endurance limit of Strength

of Materials' $S-N$ diagram, and the variation in the stress intensity ΔK as the

correlating parameter analogous to number of cycles, N . However, while the Goodman-

type diagram plots the alternating stress versus mean stress for a given life, the life plot of Fracture Mechanics is the same $\frac{da}{dN}$ vs ΔK plot, as shown in Figure 3-5.

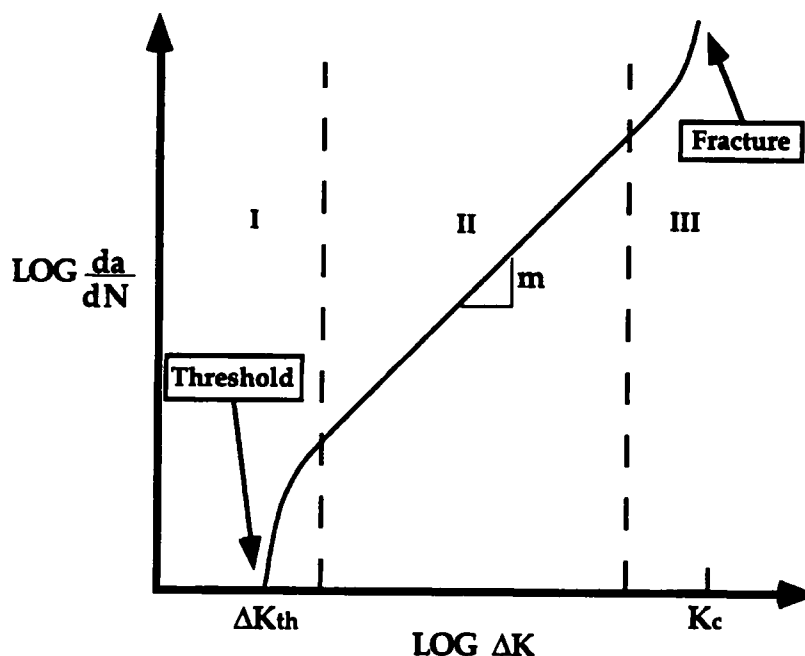


Figure 3-5. Typical Crack Growth Material Characterization Plot

In general, there are three regions of crack growth as shown in Figure 3-5. The first region is the abrupt growth following crack initiation or discontinuity nucleation. Or alternatively, the initial crack growth rate decreases rapidly with decreasing ΔK . This region establishes the threshold stress intensity, ΔK_{th} , as the asymptotic lower extreme. ΔK_{th} is the maximum stress excursion which does not produce fatigue crack propagation under any cyclic loading condition and therefore no increase in material damage [ref. 24]. The initiation/nucleation region can be the longest portion of the component life, which is not accounted for by fracture mechanics (which is why fracture mechanics hasn't had much success for HCF life predictions). The second region is the stable crack growth region identified by a relaxed slope. It is this region where the material is used to its fullest extent, and where one wants to operate for a cyclic strength optimized design

assuming adequate monitoring techniques. This region is where the bulk of the real component life exists assuming a given initial crack/discontinuity size. This region is typically approximated by a straight line characterization of the material based upon Paris' relationship:

$$\frac{da}{dN} = C(\Delta K)^m$$

where C and m are essentially material properties (in reality, other factors, such as loading, mean stress, etc., effect this slope). Typically, $m = 4$ resulting in the *4th Power Law*, and then C alone is a material constant [ref. 12]. The third and final region is the unstable crack growth region, where the crack growth rate increases rapidly with increasing ΔK . This region is most important for 10^3 cycles or less. This region defines the critical stress intensity, ΔK_{Ic} , as the asymptotic upper extreme, and rapid failure typically results when operating in this region.

The Paris relationship does for Fracture Mechanics what the Basquin and Coffin-Manson equations did for the stress and strain relationships of Strength of Materials, i.e., establish a lifing curve based on a controllable material parameter. However, in this case, the material coefficients are descriptors of the potential for cracks to propagate and therefore assumes a crack already exists. The Paris *Law* only holds for the region of stable crack growth, and ignores each end of the $\frac{da}{dN}$ vs ΔK curve (low and high ΔK), where more rapid crack growth rates occur. A variety of modifications to the Paris relationship have been made, with the most notable being that of Forman [ref. 9] who proposed the following to cover the bulk of the curve (regions II and III, Figure 3-5), instead of just the linear portion, and to include the stress intensity ratio $R = \frac{K_{min}}{K_{max}}$:

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1 - R)K_{Ic} - \Delta K}$$

Here R is also a material constant. The quantity $(1 - R)K_{Ic} - \Delta K$ decreases with increasing R and decreasing K_{Ic} , both of which give rise to increasing crack growth rates at a given ΔK level. This compensates for some of the rapid growth rates at each end of the $\frac{da}{dN}$ vs ΔK curve. However, notice that when $K_{max} = K_c$, $\frac{da}{dN}$ is now unbounded. To cover the low ΔK end, Donahue [ref. 12] recommends

$$\frac{da}{dN} = K(\Delta K - \Delta K_{Th})^m$$

where ΔK_{Th} is the threshold value of ΔK below which no crack propagation occurs, which Klesnil and Lucas [ref. 12] define as

$$\Delta K_{Th} = (1 - R)^\gamma \Delta K_{Th(0)}$$

where $\Delta K_{Th(0)}$ is the threshold at $R = 0$ and γ is a material parameter. Finally, a generalized fatigue curve propagation law has been proposed by Erdogan and Ratwani [ref. 12] to cover the whole sigmoidal shape of the fatigue curve, which has the form

$$\frac{da}{dN} = \frac{C(1 + \beta)^n (\Delta K - \Delta K_{Th})^m}{K_{Ic} - (1 + \beta)\Delta K}$$

where C , m , and n are empirical constants, $\beta = \frac{K_{max} + K_{min}}{K_{max} - K_{min}}$ and the $(1 + \beta)^n$ term has been introduced to handle mean stress effects. For variable amplitude loading, ΔK of the above relationships is ambiguous. However, these relationships produce reasonable

results if ΔK is replaced by $\Delta K_{RMS} = \sqrt{\frac{\sum_i (\Delta K_i)^2 n_i}{\sum_i n_i}}$, where n_i is the number of loading amplitudes with a stress intensity range of ΔK_i [ref. 12].

In practical situations, it is difficult to determine the energy state if the crack growth follows considerable plastic deformation in the vicinity of the crack. For purely elastic situations, these problems do not occur, and consequently Griffith's approach is best

suited for brittle materials (glass, hard tool steels, etc.), where all absorbed energy is associated with the classical interpretation of surface energy and not plastic flow. The Griffith model is valid only for bulk elastic material behavior, i.e., plasticity is allowed only in the immediate region of the crack tip and its extent is much less than the length of the crack. In general, LEFM is valid only for situations where the radius of the plastic zone r_p is much less than the crack length ($r_p \ll a$) and the stress σ_∞ is much less than the yield strength of the material ($\sigma_\infty \ll \sigma_y$). Consequently, as the material toughness increases, the applicability of LEFM becomes more suspect since the governing fracture mechanism transitions from brittle fracture to plastic material flow. For very high material toughness, fracture mechanics is not applicable at all, and only a simple load limit analysis is all that is required to predict failure since the failure stress is independent of fracture toughness (see Figure 3-6) [ref. 10].

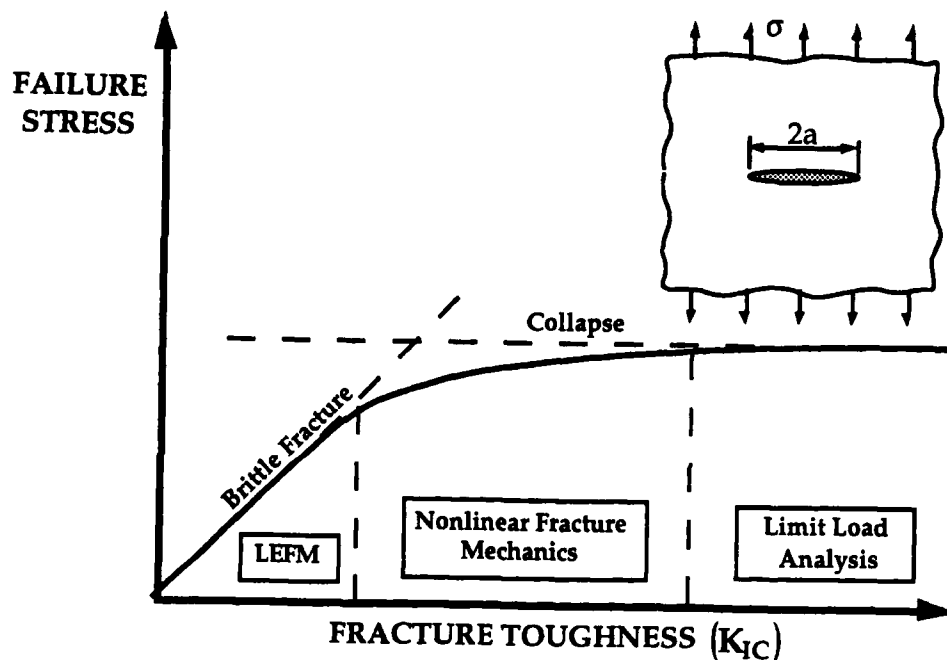


Figure 3-6. Effect of Fracture Toughness on the Governing Failure Mechanism [ref. 10]

To assess the applicability of LEFM to a given problem, the following procedure is recommended:

1. Model the component in question with a finite element method (FEM) tool.
2. For critical von Mises stress locations, impose a crack of engineering dimensions.
3. Re-evaluate the von Mises stress distribution by re-running the FEM.
4. For the stress distribution ahead of the crack, find the contour which is equal to the yield stress. This defines the zone of plasticity. Across this zone towards the crack tip, the perfect elastic/plastic assumption says the stress is a constant equal to the yield stress (no work/strain hardening).
5. If the size of this zone is small relative to the assumed crack length, then LEFM applies.

Figures 3-7 and 3-8 show such a zone for plane strain and plane stress situations, respectively, for the fracture mechanics material characterization specimen shown in Figure 3-9.

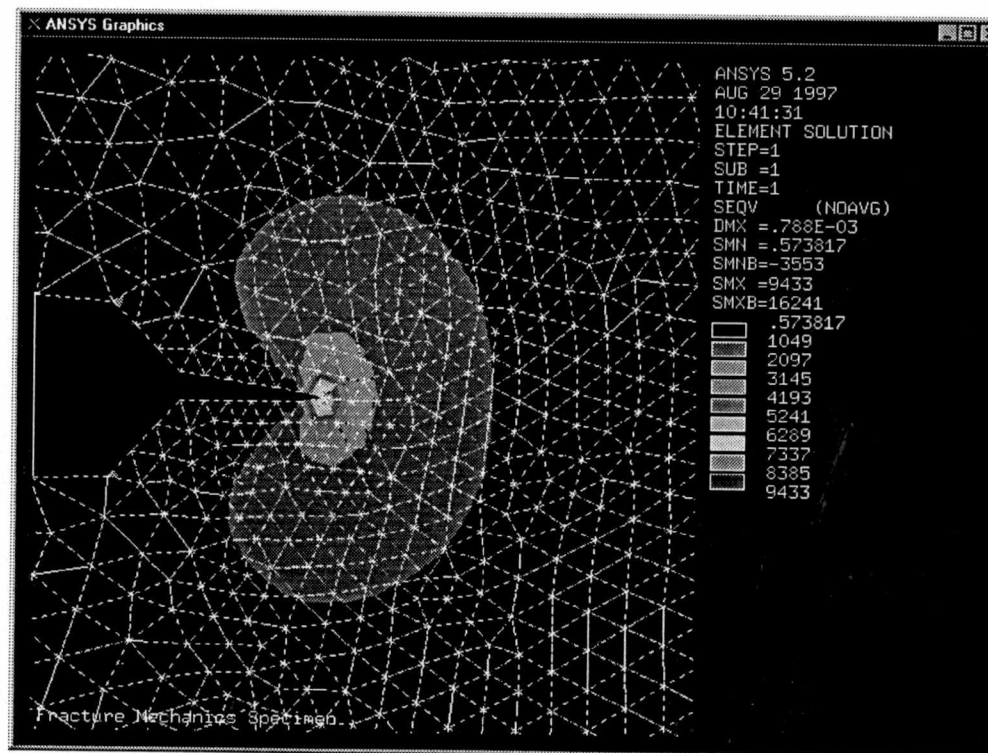


Figure 3-7. Plane Stress Plastic Zone at Crack Tip.

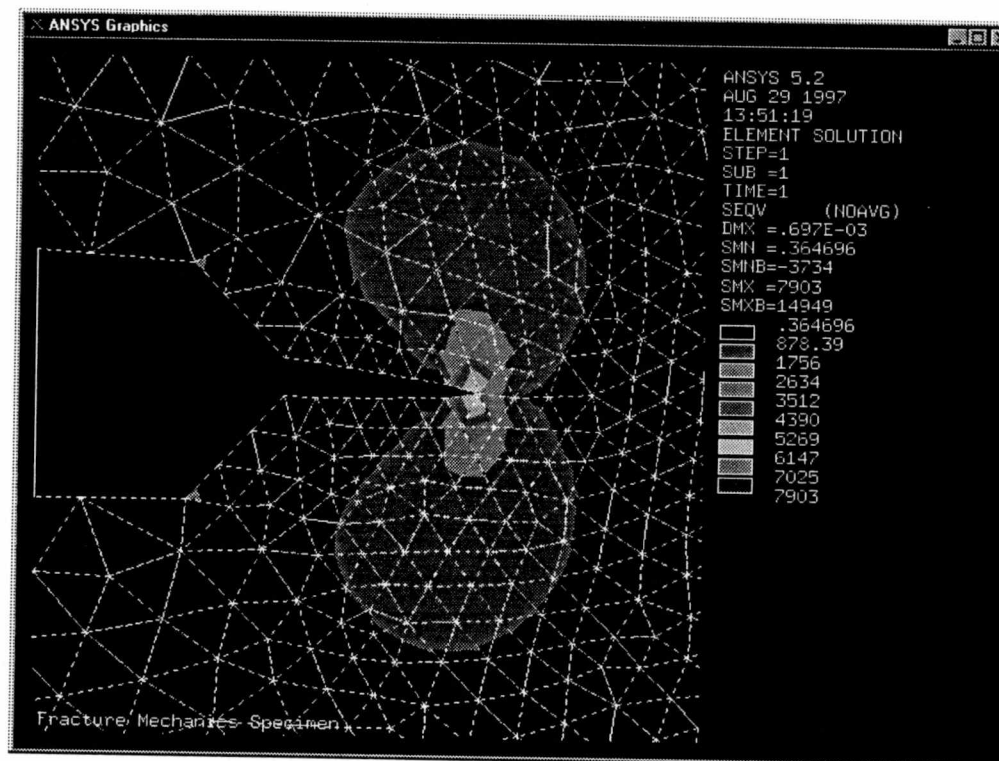


Figure 3-8. Plane Strain Plastic Zone at Crack Tip.

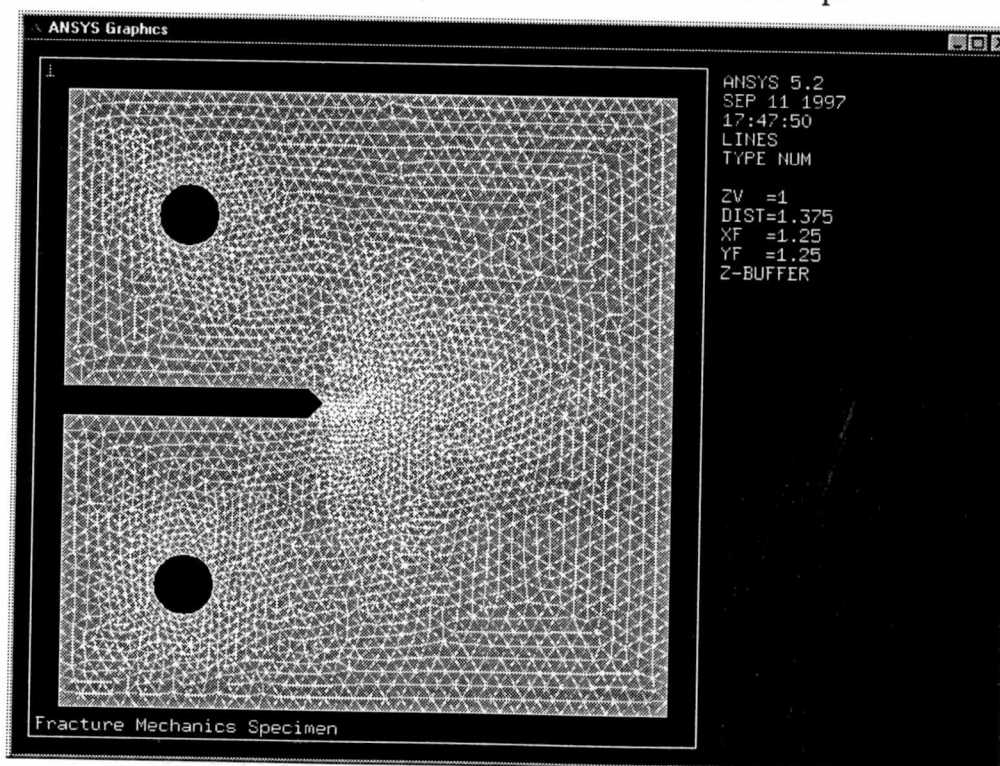


Figure 3-9. Fracture Mechanics Material Characterization Specimen.

The key features of Linear Elastic Fracture Mechanics are outlined below:

Key Features:

I. Assumptions:

- A. Material is isotropic homogeneous continuum in the near-field ahead of the crack
- B. Fatigue is directly related to intensity factor, crack size, and global stress
- C. Infinite life is possible based upon stress intensity and crack growth rate only if below threshold intensity
- D. Assumes linear stress-strain relationship
- E. Insignificant plastic deformation outside of a localized region around the crack tip
- F. Path independent failure process
- G. Self-similar crack growth and symmetric load/crack planes

II. Applications Areas:

- A. Low cycle fatigue (although significant plastic deformation may exist)
- B. Low fracture toughness materials
- C. Large structures where propagation life dominates total component life
- D. Sharply notched components where crack initiation (or nucleation to an *engineering* crack) is only a small portion of component life
- E. Generation of inspection intervals

III. Advantages:

- A. Simple to use
- B. Failure mechanism integral part of theory
- C. Allows damage tolerance concepts
- D. Can be coupled with strain-life to capture initiation life
- E. Provides a method for dealing with non-propagating cracks

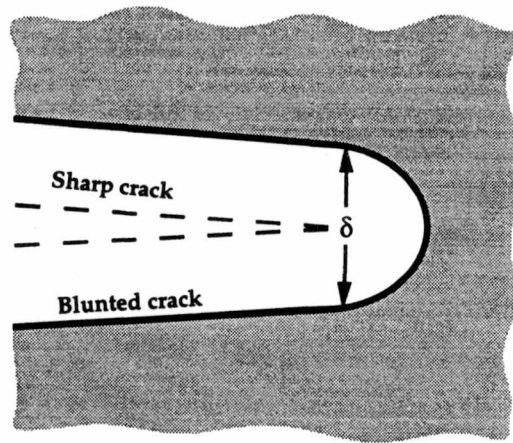
IV. Disadvantages:

- A. Stress intensity factors are not well correlated for multi-axial fatigue

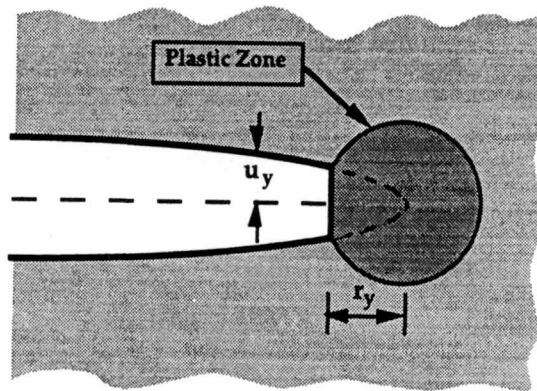
- B. Does not account for large plastic strain (i.e., ductile materials like stainless steel)
- C. Limited material data
- D. Ignores crack initiation life, assumed initial crack size significantly effects predicted life
- E. Inability to model complex loading/crack plane configurations, stress intensity factors must be approximated for complex geometries.

Elastic/Plastic Fracture Mechanics

Elastic-Plastic Fracture Mechanics (EPFM) bridges the gap between LEFM and simple structural collapse for high toughness materials. Per Anderson [ref. 10], EPFM owes its origins to A.A. Wells, G.R. Irwin, and J.R. Rice. Wells (1961) attempted to measure K_{Ic} values for several common structural steels, but realized that these materials were too tough to be characterized by LEFM. Wells explained the situation by showing that the plasticity zone at the end of the crack tip blunted an initially sharp crack, and that the degree of blunting was in direct proportion to the toughness of the material. Irwin (also 1961) showed that this blunting effect was equivalent in behavior as if longer cracks were present (see Figure 3-10). Rice (1968) showed that by idealizing elastic-plastic deformation as non-linear elastic displacement, one can extend the validity of LEFM [ref. 10].



Crack tip opening displacement (CTOD). An initially sharp crack blunts with plastic deformation, resulting in a finite displacement (δ) at the crack tip.



Estimation of CTOD from the displacement of the effective crack in the Irwin plastic zone correction.

Figure 3-10. Equivalence Between Crack Tip Blunting and Increased Effective Crack Length [ref. 10].

Consider the crack tip displacement field in the singularity dominated zone (i.e., close to the tip) for a mode I crack condition described by

$$u_y = K_I \frac{\kappa + 1}{2\mu} \sqrt{\frac{r_y}{2\pi}}$$

where r_y is the dimension from the crack tip in the direction of the load, μ is the shear modulus (to differentiate from G for strain energy used later), and κ is a constant

dependent upon Poisson's ratio ν such that $\kappa = 3 - 4\nu$ for plane strain and $\kappa = \frac{3 - \nu}{1 + \nu}$ for plane stress.

for plane stress [ref. 10]. Now to approximate the size of the plasticity zone, consider K_{Iy} for the material yielding such that

$$r_y = \frac{1}{2\pi} \frac{K_{Iy}^2}{\sigma_y^2}$$

then

$$u_y = K_{Iy} \frac{\kappa+1}{2\mu} \sqrt{\frac{1}{4\pi^2} \frac{K_{Iy}^2}{\sigma_y^2}} = \frac{K_{Iy}^2}{\sigma_y} \frac{\kappa+1}{4\pi\mu} = \frac{K_{Iy}^2}{\sigma_y} \frac{2}{E\pi}$$

where $\mu = \frac{E}{2(1+\nu)}$ has been used. Since the total crack tip opening displacement (CTOD) is $2u_y = \delta$, then

$$\delta_c = CTOD_{critical} = \frac{4}{E\pi} \frac{K_{Iy}^2}{\sigma_y} \text{ for plane stress.}$$

This shows the relationship between $CTOD$ and K_I for small scale yielding is valid for LEFM. Similarly, the $CTOD$ can be shown to be related to the energy release rate G by

$\delta = CTOD = \frac{4}{\pi} \frac{G}{\sigma_y}$, for plane stress, and therefore $G = \frac{K_I^2}{E}$ by equating these two δ equations.

These relationships between the $CTOD$, K_I , and the energy release rate G implies that failure criteria based upon stress intensity factor, strain energy release rate, and crack tip opening displacement are equivalent for small-scale yielding plane stress problems.

Recall from LEFM that the energy release rate was $G = -\frac{d\Pi}{d\mathcal{A}} = \frac{\sigma^2 a \pi}{E} = \frac{K_I^2}{E}$ where again $\mathcal{A}$ is the crack area, Π is the potential energy due to the strains and tractions

$$\Pi = \int_A \frac{dw}{da} dA - \int_{\Gamma} T_k u_k ds$$

where A is the area of the region R and $d\mathcal{A} = Bda$ was used for constant thickness B of the plate, which is ignored. Again, it is concluded from an energy point of view that $K_I = \sigma\sqrt{a\pi}$. However, for plastic (non-linear elastic) deformation, the simple energy release rate G is replaced by an integral J around the deformation zone, such that

$$J = -\frac{d\Pi}{d\mathcal{A}} = -\frac{d\Pi}{da} = \sigma_y \delta = \int_{\Gamma} \left(w dy - T_i \frac{\partial u_i}{\partial x} ds \right) \text{ where } w \text{ is the strain energy density (on}$$

a per volume basis), T_i are components of the traction stress vector (normal to the contour) whose components are $T_i = \sigma_{ij}n_j$ where n_j is a unit normal vector to the contour Γ , u_i are the displacement vector components, and ds is a length increment along contour Γ (see Figure 3-11, see Anderson [ref. 10] A3 and Gdoutos [ref. 12]

sections 5.0 and 5.1 for derivation). The strain energy density is defined as $w = \int_0^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij}$

where σ_{ij} and ϵ_{ij} are the stress and strain tensors, respectively, such that $\frac{\partial w}{\partial \epsilon_{ij}} = \sigma_{ij}$ [ref.

10]. Consider the two integration paths of Figure 3-11, Γ and Γ' around the crack tip. These two line

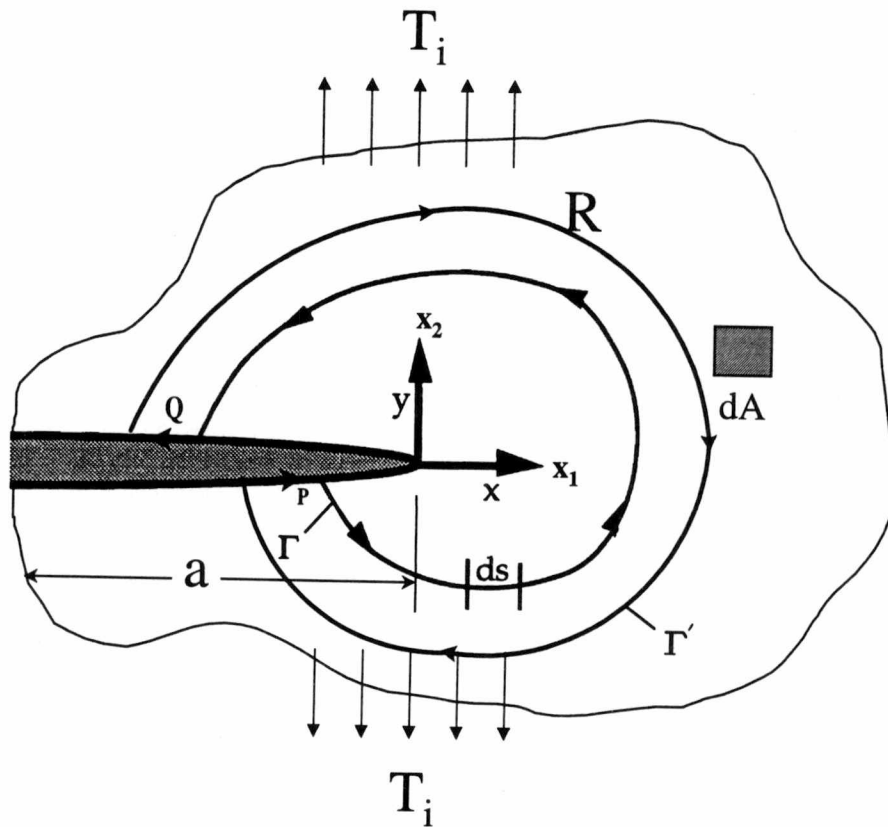


Figure 3-11. J - Integration Path for Plasticity Zone of 2D Cracked Body [ref.10].

contours can be made into a single closed curve by segments P and Q along the crack surfaces. Since this is a closed contour, the J -integral is zero. Also, since no tractions exist on the crack surface ($\sigma_{22} = \sigma_{12} = 0$ along P and Q), and since this is mode I loading ($\sigma_{11} = 0$ on P and Q), then $J = J_C + J_{C'} + J_P + J_Q = 0$, $J_P = J_Q = 0$, which implies $J_C = J_{C'}$. Therefore, J is a path-independent integral provided the stress and strain are continuous (such that calculus techniques are valid) within the region and they both vary as $1/r$ near the crack tip. Since a contour integral is a 2D description, this approach is valid only for planar stress/strain situations in the neighborhood of the crack tip

Attempts have been made to formulate a Paris-type *Law* or relationship involving the J -integral for fatigue, i.e.,

$$\frac{da}{dN} = C(\Delta J)^m$$

However, because deformation plasticity (non-linear elasticity) is an essential element of the J-integral theory, which does not describe the material response during unloading [ref Anderson] (unless no hysteresis exists), the J-integral concept does not apply to elastic-plastic problems where unloading occurs (i.e., fatigue conditions). Consequently, there is no theoretical basis for the above equation [ref. 12]. Further work is needed in this area to satisfy fatigue requirements strictly, but for small-scale yielding in fatigue conditions of constant amplitude loading, since a relationship exists between ΔJ and ΔK [ref. 10] for linear elasticity

$$\Delta J = \frac{(\Delta K_I)^2}{E'}$$

where $E' = E$ for plane stress and $E' = \frac{E}{1-\nu^2}$ for plane strain. Note, however, that while

$\Delta K = (K_{\max} - K_{\min})$ associated directly to the cyclic stress, $\Delta J \neq (J_{\max} - J_{\min})$ since

$$(\Delta K)^2 = K_{\max}^2 - 2K_{\max}K_{\min} + K_{\min}^2.$$

The key features of Elastic-Plastic Fracture Mechanics are outlined below:

Key Features

(essentially same as LEFM, with the addition of the following):

- I. Assumptions:
 - A. Material is isotropic homogeneous continuum in the plastic region ahead of the crack
 - B. Fatigue directly related to J-integral factor, crack tip opening displacement, and global stress
 - C. Small strain theory (strains < 10%)
 - D. Allows non-linear stress-strain relationship
 - E. Insignificant plastic deformation outside crack tip region

- F. Path independent failure process
- G. Self-similar crack growth.
- H. Symmetrical load/crack planes
- II. Applications Areas:
 - A. Low and high cycle fatigue (with significant plastic deformation)
 - B. Moderate to high fracture toughness materials
 - C. High stress concentration regions
- III. Advantages:
 - A. Simple concept
 - B. Failure mechanism integral part of theory
- IV. Disadvantages:
 - A. Hard to use
 - B. J-integral factors are not well correlated for multi-axial fatigue
 - C. Limited material data
 - D. Inability to model complex load/crack configurations

In summary, fracture mechanics approaches assume that if the component is operated at a stress intensity level within the stable crack growth region of the material it is composed of, then the cracked component can be inspected on a period which is predictable based upon the rate of crack growth. It is further assumed that this predictive capability can adequately provide a margin of safety for use until withdrawn from service when the crack has reached a critical length. Fracture Mechanics approaches quantify the critical combinations of three variables: applied load, material toughness, and crack length. However, fracture mechanics does not describe the crack initiation/nucleation process, requires simple load/crack relationships where there exists symmetry between the crack and load planes, and is valid for only self-similar crack growth. Except in the simplest of cases, all of these conditions are typically violated in practical engineering problems.

Once a crack begins to propagate on the macro-level, the component life is typically short. Consider total life (cycles) as

$$N_T = N_I + N_p$$

where N_I is the initiation/nucleation life and N_p is the propagation life. Being a crack **propagation** dependent theory/criteria, fracture mechanics works well for low cycle fatigue (LCF) which is characterized by high stress levels and few cycles (N_p dominant). However, low stress levels and high number of cycles characterize high cycle fatigue (HCF). Consequently, the majority of the time for a high cycle fatigue failure is spent in the initiation/nucleation phase. Therefore, the successful HCF designs are those for which cracks do not propagate or take a long time to initiate (N_I dominant). Given that cracks and other discontinuities exist in real materials, this implies the existence of a threshold below which no cracks propagate, leading back to the concept of an endurance limit of Stress-Life. The efforts of much recent research is to characterize the parameters and processes which define this threshold.

The inability to predict HCF life rests in the data scatter for typical correlations, either based upon an S-N or the crack growth curves. Per McDowell [ref. 25], the scatter corresponds to increasing heterogeneity of the persistent slip bands (PSB) and micro-cracks distribution with decreasing stress amplitude. Due to the low distributions of PSBs and micro-cracks, the HCF failure is typically characterized by a single dominant flaw/discontinuity/micro-crack propagating to failure. Consequently, the propagation of such a crack on a micro-scale can be radically influenced by the micro-structure of the material where the bulk of the crack life is spent. This is observed in the cyclic behavior of the crack propagation rate as a function of crack length (see Figure 3-12). This is the phenomenon known as short crack growth.

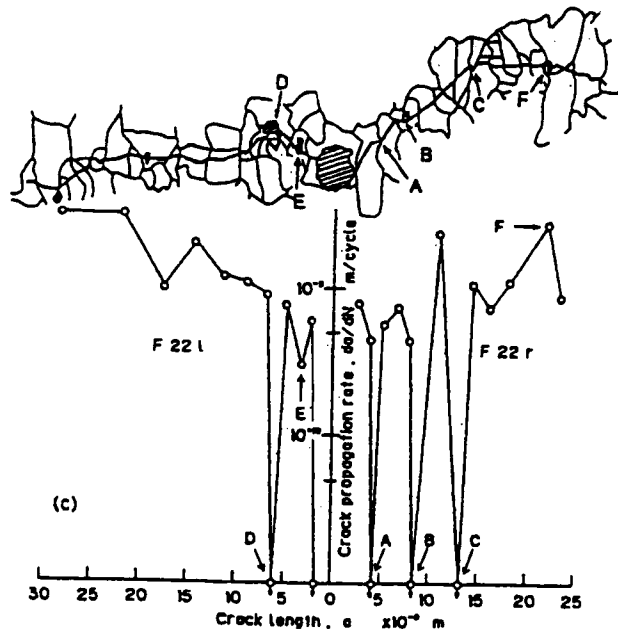


Figure 3-12. Micro-structure interaction effect on the variation of crack Propagation rate versus crack length [ref. 25].

Short Crack Theory

Fracture Mechanics has problems in the short crack region, where microcracks nucleate and rapidly grow. The threshold level for ΔK can be significantly lower for small cracks than for longer cracks, leading to non-conservative life predictions if based upon LEFM. In fact, the concept of a threshold for the small crack regime is questionable (see Figure 3-12) [ref. 25].

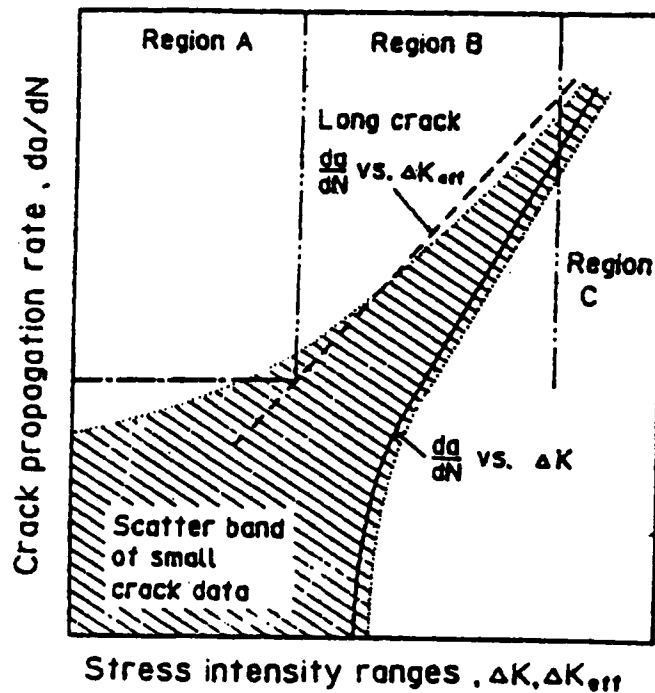


Figure 3-13. Small Crack Growth Data Scatter.

For small cracks ($a \ll 1\text{mm}$), the growth rates appear to be vary random and can far exceed the rate for long cracks ($a > 1\text{mm}$). This is believed to be due to micro-structural effects. Since cracks remain small in HCF for most of the fatigue life of smooth or notched specimens, for HCF the long crack-based LEFM has less relevance. Further evidence of the non-applicability of LEFM can be illustrated by examining the non-linear crack growth as a function of stress level, where typically the higher stress level of LCF conditions exhibit more linear crack growth than the low amplitude stress levels of HCF, which exhibit highly non-linear crack growth (see Figure 3-14).

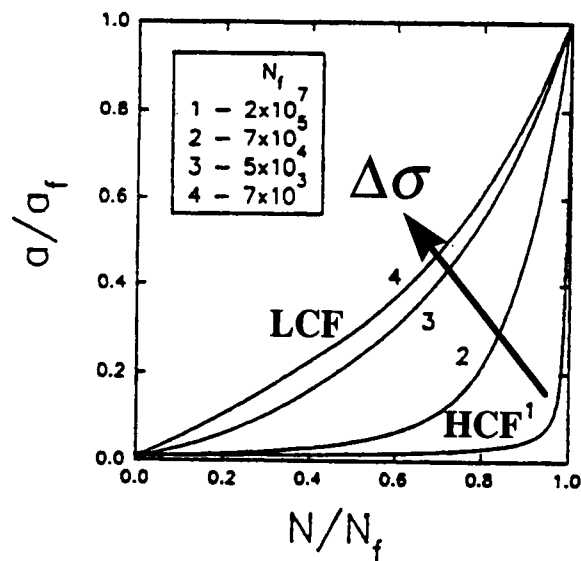


Figure 3-14. Crack Growth Relationships as Function of Stress Level.

McDowell has developed a technique or model to bridge the gap between the initiation avoidance approaches of safe-life and LEFM/EPFM by dividing the crack growth line into three regimes: micro-structural fracture mechanics (MFM), EPFM, and LEFM (see Figure 3-15).

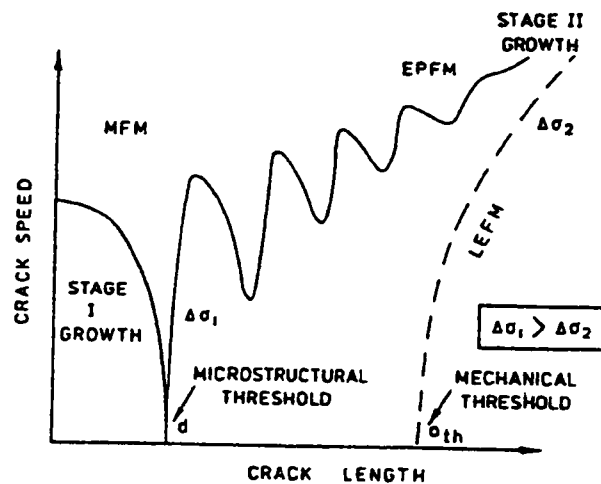


Figure 3-15. Small Crack Growth Characteristics and Theories.

Using this approach, the initiation phase through the stable growth phase of the crack can be adequately modeled, covering both fatigue regimes of LCF and HCF. McDowell essentially uses the appropriate theory for the regime. In other words, he uses MFM where the crack is on the order of the microstructure (swallowed by the plastic zone), EPFM for regions of larger plasticity (crack tip blunting), but which is still focused at the crack tip and the growth is self similar, and LEFM where the plastic zone is small relative to the crack size leading to a brittle-type failure (even if material is ductile). Further study of this method is required before reporting fully herein, however, the approach is to substitute for the typical crack growth law

$$\frac{da}{dN} = C(\Delta K)^m$$

a more detailed microcrack propagation law which includes multi-axial load condition terms of the form

$$\frac{da}{dN} = D_{aN} \left[R_e C_e (\Psi_e)^m + (1 + R_e) C_p (\Psi_p)^m \right] \left(\frac{a}{kd} \right)^m,$$

where Ψ_e and Ψ_p are crack driving force parameters based upon elasticity and plasticity, respectively, both of which depend upon the shear stress, shear strain, and the ratio

$R_n = \frac{\Delta\sigma_n}{\Delta\tau_n}$ on the critical slip plane of crack growth, $R_e = \frac{\Delta\gamma_{\max}^e}{\Delta\gamma_{\max}}$ is the elastic strain

ratio representing the degree of macroscopic elastic straining, a is the crack half-length, kd is the transition crack length as a scale of the grain size d , and the remaining parameters are material constants derived from experiments coupled with classical strain-life relationships (see McDowell & Bennett [ref. 26] for further details). The multiple parameters allow for better fits to the experimental data while maintaining physical significance as the crack grows from micro to macro lengths and the crack growth rate dependency on crack length changes over this same range. The beauty of this approach is the way it details the data to be taken in material characterization experiments to arrive at the material constants. Further details must be studied before more can be said, but this approach is quite analogous to the Socie, et.al., approach described earlier for the strength

of materials characterization. In fact, per discussions with Dr. Socie [ref.verbal discussion with Dr. Mark Sensmeier], the two can be shown to be equivalent.

4. STRAIN ENERGY DENSITY FAILURE CRITERION

The Strain Energy Density (SED) failure criterion is a newer approach to fatigue which was developed as an attempt to overcome the limitations of Fracture Mechanics. SED is very similar to a safe-life approach based upon strain, however, it does not assume the material is crack free, nor does it require the presence of cracks or the knowledge of the crack size/direction to be of value as a structural analysis tool. In some sense, the SED may be considered just another failure criterion applied to fracture mechanics. This is tenable since fracture mechanics includes strain energy in its formulation. However, SED goes well beyond typical fracture mechanics into the domain prior to the initiation of a engineering size crack. The SED criterion was first proposed as a failure criteria and material characterization approach in the early 1980s by G.C. Sih, and developed extensively at the Institute of Fracture and Solid Mechanics of Lehigh University [ref. 12]. The SED criterion is an attempt to overcome the dependencies of specimen size, geometry, and loading on material characterizations, especially with respect to the symmetry between the applied load and the crack plane, which is typically violated for multi-axial stress fields and fracture of ductile materials (due to coning of the material at fracture).

SED theory assumes that a continuum can be viewed as an assembly of small differential elements, each of which has the potential to store a finite amount of energy at a given instance in time. Consequently, the fundamental quantity of SED criterion is the strain energy density

$$\frac{dW}{dV} = \int_0^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij} + f(\Delta T, \Delta C)$$

where W is the strain energy, V is the volume of material at a given instance of time, σ_{ij} are the stress components describing the stress field, ϵ_{ij} are the strain components describing the strain field, and f is a function of material temperature and moisture changes. Note that this criterion allows for strain energy to be present without a stress field (due to $f(\Delta T, \Delta C)$), which cannot be so with the previous stress or strain based criteria (which assume a stress-strain relationship, eg. $\sigma = E\epsilon$). For a linear elastic material, the SED function can be written as [ref. 12]:

$$\frac{dW}{dV} = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E}(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z) + \frac{1}{2G}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)$$

where E is Young's modulus, G is the shear modulus, ν is Poisson's ratio, and the rest are the normal and shear stress components. The strain energy per volume can be further decomposed into

$$\frac{dW}{dV} = \left(\frac{dW}{dV} \right)_d + \left(\frac{dW}{dV} \right)_v$$

where the d subscripted quantity represents the distortional strain energy per unit volume corresponding to the deviatoric stress tensor associated with the distortion of an element with no volume change, while the v subscripted term represents the part of strain energy per unit volume associated with volume change and no distortion (shape) change (dilatational). This separation invokes linearity/superposition principles, however,

$$\left(\frac{dW}{dV} \right)_d = \frac{1+\nu}{6E} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)] = \frac{1+\nu}{3E} \sigma_e^2$$

where σ_e is the equivalent octahedral stress (von Mises stress), and

$$\left(\frac{dW}{dV} \right)_v = \frac{1-2\nu}{6E} (\sigma_x + \sigma_y + \sigma_z)^2.$$

Now for plane stress problems (setting all z-components of stress equal to zero),

$$\left(\frac{dW}{dV} \right)_d = \frac{1+\nu}{6E} [(\sigma_x - \sigma_y)^2 + \sigma_y^2 + \sigma_x^2 + 6\tau_{xy}^2] = \frac{1+\nu}{3E} [\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2] = \frac{1+\nu}{3E} \sigma_{e-PlaneStress}^2$$

Consequently, the octahedral shear stress criterion considers only the distortional strain energy component and ignores the portion of strain energy associated with volumetric changes. In terms of the principal stresses (mutually perpendicular normal stresses in directions of no shear, where $\sigma_1 \geq \sigma_2 \geq \sigma_3$) [ref. 12], for the plane strain condition

$\sigma_3 = \nu(\sigma_1 + \sigma_2)$, the ratio of distortional to dilatational strain energy density is

$$\frac{(dW/dV)_v}{(dW/dV)_d} = \frac{(1-2\nu)(1+\nu)\left[\left(\frac{\sigma_1}{\sigma_2}\right) + 1\right]^2}{\left[\left(\frac{\sigma_1}{\sigma_2}\right) - 1\right]^2 + \left[(1-\nu) - \nu\left(\frac{\sigma_1}{\sigma_2}\right)\right]^2 + \left[(1-\nu)\left(\frac{\sigma_1}{\sigma_2}\right) - \nu\right]^2}$$

Figure 4-1 shows that the most critical SED ratio is for $\sigma_1 = \sigma_2$, corresponding to a two-dimensional hydrostatic stress state [ref. Gdoutos].

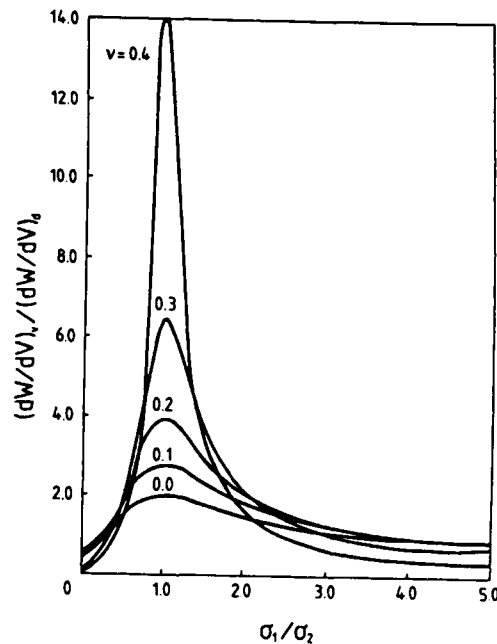


Figure 4-1. Variation of SED Ratio Versus Principal Stress Ratio σ_1/σ_2 .

Per Gdoutos [ref. 12], "Failure of material elements in a solid is caused by permanent deformation or fracture which can be related to shape change (distortion) and volume change (dilatation)". These deformations, due to dilatation and distortion, store energy per the above equations, and are responsible for crack fracture propagation and slip

plane/microcrack generation, respectively. In addition, when the distortional component of SED becomes a maximum while the dilatational component is negligible, the creation of cleavage planes results. Consequently, material *damage* is characterized by **both** components of SED, and “the two processes of yielding (distortion) and fracture (dilatation) are inseparable” [ref. 12]. Therefore, the von Mises yield criterion, which considers only the distortional component of the SED, is incomplete and can never explain the creation of microcracks in the plastic zone without the consideration of dilatation.

A major advantage of the SED criterion is the ability to predict directions of failure. Typically, macrofracture will coincide with the direction where

$$\left. \frac{dW}{dV} \right|_{Dilatation} > \left. \frac{dW}{dV} \right|_{Distortion}$$

and macroyielding will coincide with the direction where

$$\left. \frac{dW}{dV} \right|_{Distortion} > \left. \frac{dW}{dV} \right|_{Dilatation} .$$

In other words, considering the stationary values of the total SED with respect to polar coordinates at the site of failure initiation establishes the failure mechanism and direction.

The relative local minimum of $\frac{dW}{dV}$ corresponds to large volume change (large dV) and is identified with the region dominated by macrodilatation leading to fracture [ref. 12].

Conversely, the local maximum of $\frac{dW}{dV}$ corresponds to large shape change (small dV) and is identified with the region dominated by macrodistortion leading to yielding [ref. 12]. Recall that yielding is influenced by shear (distortional) stresses, which is in agreement with the distortion dominated condition cited above.

SED has three basic hypotheses which are independent of material, loading condition, and structural geometry, with or without initial defects:

1. The location of fracture coincides with the location of relative minimum SED,

$$\left. \frac{dW}{dV} \right|_{\min}, \text{ and yielding with relative maximum of SED, } \left. \frac{dW}{dV} \right|_{\max}.$$

2. Failure by fracture or yielding occurs when $\left. \frac{dW}{dV} \right|_{\min}$ or $\left. \frac{dW}{dV} \right|_{\max}$ reach their respective critical values.
3. The amount of incremental growth $r_1, r_2, \dots, r_j, \dots, r_c$ is governed by

$$\left. \frac{dW}{dV} \right|_{\text{Critical}} = \frac{S_1}{r_1} = \frac{S_2}{r_2} = \dots = \frac{S_j}{r_j} = \dots = \frac{S_c}{r_c}, \text{ where } r \text{ is the radial distance measured from}$$

the site of possible failure initiation and $S = r \frac{dW}{dV}$ is the SED factor [strain field

amplitude and direction (a vector) containing the $\frac{1}{r}$ singularity]. (Note the

dependence of r^{-1} as opposed to the $r^{-\frac{1}{2}}$ dependency of Fracture Mechanics.) r_c is the critical ligament size beyond which unstable fracture or yielding exists. S_c is a material constant and represents the fracture toughness of the material.

In Section 7, *Comparison of Fatigue Approaches*, Table 7-2 shows a comparison of typical fracture toughness values based upon Strength of Materials, Fracture Mechanics, and SED criteria. Note that the most extreme values in each case do not coincide, as shown in Table 7-3.

From a fatigue point of view, SED is very similar to Fracture Mechanics in that a Paris-type law or correlation may be identified:

$$\frac{da}{dN} = C(\Delta S)^m$$

where $\Delta S = S_{\max} - S_{\min}$ along the direction of crack growth. However, in the SED case, this relationship holds for mixed-mode loading conditions. This is not the case for fracture mechanics which is restricted to symmetrical loading/crack plane conditions. (In

fracture mechanics, the crack is assumed to propagate self-similarly in a plane perpendicular to the loading, which is not a restriction in SED criterion.) Figure 4-2 [ref. 12] shows a crack growth curve for a titanium material. Notice that at each end a sigmoidal relationship exists similar to fracture mechanics characterizations. However, notice that the SED curve is independent of $R = \frac{\sigma_{\min}}{\sigma_{\max}}$.

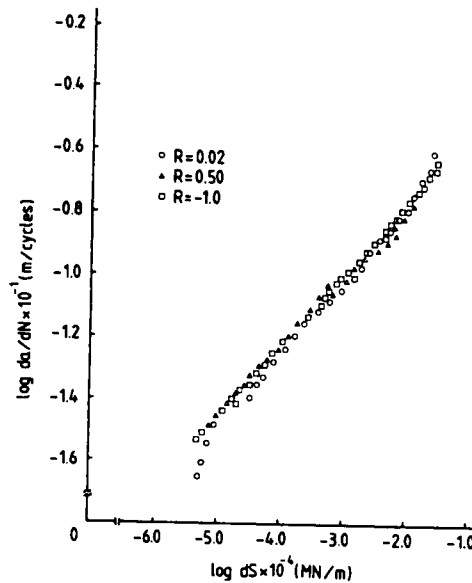


Figure 4-2. SED Crack Growth Rate Curve for Ti-6Al-4V Titanium Material.

Finally, Figure 4-3 [ref 12] shows the prediction of crack growth trajectories for an inclined crack in titanium. Notice the excellent agreement between the actual and SED predictions for both the number of cycles and trajectories.

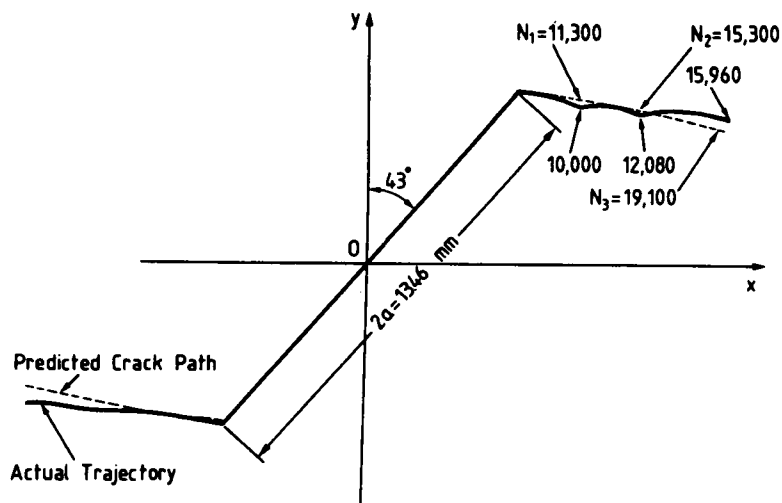


Figure 4-3. SED Predicted Crack Growth Trajectory for Inclined Crack in Ti-6Al-4V Titanium Material ($\sigma_{\max} = 25000$ PSI, $\sigma_{\min} = 2500$ PSI).

SED criterion as presented here is still based upon linear elasticity. A non-linear version has been formulated which takes into account the accumulation of damage during the fatigue process. This approach is still considered preliminary, and further references are being sought.

The key features of Strain Energy Density as a fatigue theory are outlined below:

Key Features:

- I. Assumptions:
 - A. Material is a continuum in the plastic region ahead of the crack
 - B. Fatigue directly related to strain energy density level
 - C. Small strain theory (strains $< 10\%$)
 - D. Allows non-linear stress-strain relationship
 - E. Allows significant plastic deformation outside crack tip region
- II. Applications Areas:

- A. Low and high cycle fatigue
- B. Moderate to high fracture toughness materials
- III. Advantages:
 - A. Simple to use
 - B. Failure mechanism integral part of theory
 - C. Less dependency on load conditions (mixed mode allowed) and component sizes
- IV. Disadvantages:
 - A. Limited material data, though derivable from uni-axial tests
 - B. Limited applications illustrating technique

5. CONTINUUM DAMAGE MECHANICS

Continuum Damage Mechanics (CDM) was originally proposed in the 70's to bridge the gap between the discontinuous fracture mechanics and the continuum mechanics of safe-life approaches. Continuum Damage Mechanics uses thermodynamics coupled to elasticity/plasticity theory to establish the initiation/nucleation of cracks within a continuum based upon the irreversibility of strain energy at the microscale. This approach builds upon original energy concepts of Griffith and establishes a continuous damage variable which represents the deterioration of the material before initiation of the macrocrack [ref. 6]. This establishes the boundary condition for fracture mechanics, namely, the initial crack size (a typical value for an engineering size crack is .1 inch) for life/inspection-interval analysis. Damage mechanics couples the material strength property of fatigue analysis with a damage parameter to characterize the material static state and adds a kinetic damage parameter to characterize the state change as the material deteriorates in order to establish the initiation of a crack. Consequently, damage mechanics compliments fracture mechanics (see Figure 5-1, [ref. 26]). Therefore, damage mechanics quantifies the critical combinations of four variables: applied load, damage strength material parameter, damage plastic strain threshold, and critical damage toughness material parameter. The following discussion should shed some light on these aspects.

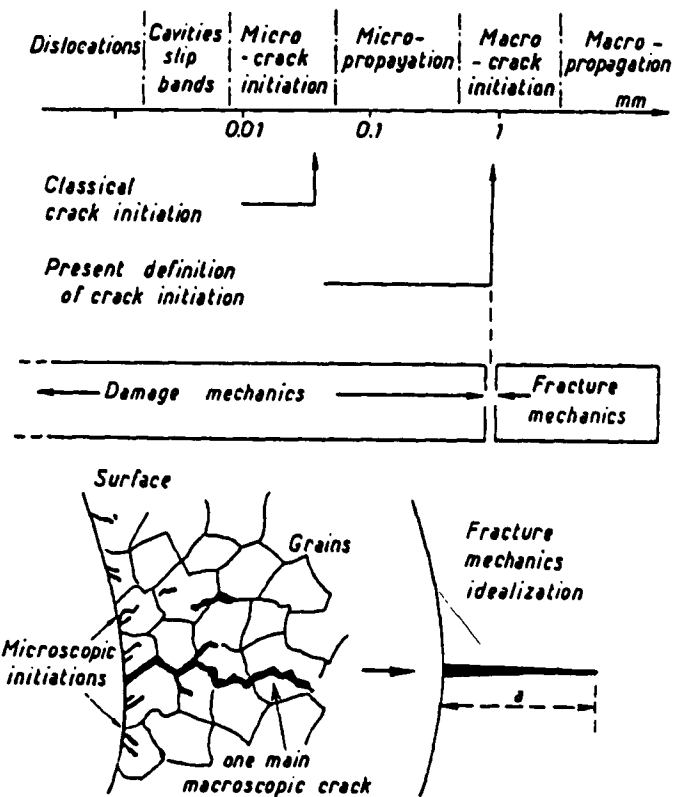


Figure 5-1. Damage Process and Associated Failure Descriptions [REF. 26].

Lemaitre [ref. 6] begins his book on damage mechanics with the following:

“The damage of materials is the progressive process by which they break. The mechanics of damage is the study, through mechanical variables, of the mechanisms involved in this deterioration when the materials are subjected to loading.” The key terms of this quote are *progressive process* and *deterioration*. Damage mechanics attempts to describe the deterioration of the strength of materials as a process which transcends the various length scales inherent in a typical engineering material. These scales are the following:

- Microscale is the scale of individual atoms, where stresses accumulate to break individual bonds. Elasticity/Plasticity takes place on the microscale.

- The mesoscale is the scale where the constitutive equations are valid, which can vary depending upon the material. At the mesoscale, a continuum is assumed.
- Macroscale is the scale of typical engineering structures, and varies with the degree of the gross feature details of the component. Detectable cracks reside at this level, and this is the region of fracture mechanics.

Damage is a parameter, which describes the state of transition from the microscale level of atoms to the mesoscale level for crack initiation/nucleation. *Damage* is the *local* breakdown of continuity, however, in this case the damage is assumed to be homogeneously distributed throughout the material. Damage mechanics is based upon a continuum description, and therefore it is necessary to consider a *representative volume element* (RVE) which is small enough to avoid smoothing of high gradients in a particular material property, yet large enough to represent an average of the microprocesses which produce the gradients such that calculus may be used to describe the mechanics. Table 5-1 [ref 6] is offered as a guide to the definition of RVE.

Table 5-1. Typical Representative Volume Elements	
Material	RVE
Metals & Ceramics	(.1 mm / 0.00394") ³
Polymers & Most Composites	(1.0 mm / 0.0394") ³
Wood	(10 mm / .394") ³
Concrete	(100 mm / 3.94") ³

In addition, one must realize that damage is always more localized than strain, and therefore appears as a stress/strain concentrator. Damage is restricted to a surface (not necessarily external, debonding of internal atoms is a surface generator), while strains occur throughout a volume due to atom movements [ref. 6].

Experimental observations reveal that degradation of material properties on the macroscale “occurs as a result of a complex interaction of two dissipative mechanisms: Propagation of dislocations or line defects versus the nucleation and evolution of microcavities” [ref. 27]. Chow and Lu go on to state “The microcaviting nucleates in planes of local maximum shear stresses where plastic distortion is significant.” In addition, the propagation of dislocations also requires plastic yielding. Consequently, these dissipative mechanisms are coupled by local plastic yielding and surface generation, for which the energy balance can only be thermodynamically described since it is an irreversible non-linear process.

Lemaitre stated that [ref. 6], “...damage may be interpreted at the microscale as the creation of microsurfaces of discontinuities: breaking of atoms and plastic enlargement of microcavities. At the mesoscale, the number of broken bonds or the pattern of microcavities may be approximated in any plane by the area of the intersections of all the *flaws* with that plane. In order to manipulate a dimensionless quantity, this area is scaled by the size of the representative volume element”.

In 1958, L.M. Kachanov published the first paper on a thermodynamic variable called *continuity*, represented by ψ . Based upon the above descriptions of damage, one may conclude

$$D = (1 - \psi)$$

where $0 \leq D \leq 1$, zero being the undamaged state ($\psi = 1$ implying pure continuity) and one being complete failure, i.e., broken into two pieces ($\psi = 0$ implying total discontinuity). Based on the above discussion,

$$D = \frac{\delta A_{SD}}{\delta A_s}$$

where δA_s is the increment of surface area for a plane through the material at the location of the most damage, and δA_{SD} is the increment of surface area occupied by the damage sites that intersect surface S , then

$$\frac{\delta A_{SD}}{\delta A_s} = 1 - \psi \Rightarrow \psi = 1 - \frac{\delta A_{SD}}{\delta A_s} = 1 - D$$

Figure 5-2 illustrates a simple tensile element with damage sites.

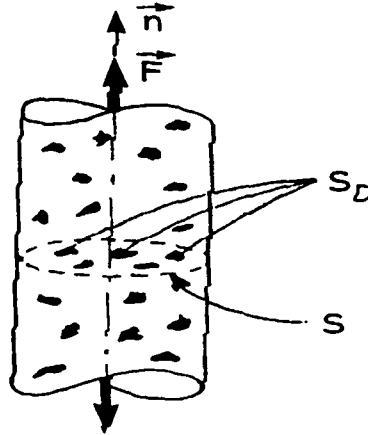


Figure 5-2. Tensile Element with Damage Sites
(S = plane area, S_D = damage area intersecting plane).

For this situation, generally $\sigma = \frac{F}{A_s}$. However, since the defect sites are unstressed, then

$$\sigma_{eff_D} = \frac{F}{A_s - A_{SD}} = \frac{F}{A_s \left(1 - \frac{A_{SD}}{A_s}\right)} = \frac{\sigma}{1 - D} = \frac{\sigma}{\psi}$$

where σ_{eff_D} is the effective stress due to damage $D = \frac{A_{SD}}{A_s}$.

Here it is necessary to point out that this definition of damage assumes homogeneous isotropic materials. However, when combined with a tensor describing the distribution and orientation of the damage field relative to the load, damage then becomes a tensorial quantity and the effective stress is also tensorial in this case. The tensorial damage field allows for anisotropic material characterization, which is a major advantage of CDM.

Notice that this definition of effective stress is at the microscale, since it includes the microstructure (damage sites), and the microstress concentrations resulting from these discontinuities. Also notice that from a material strength viewpoint, the stress capacity is directly related to material continuity. Therefore, only micromechanics can give a precise meaning to this damage concept, which is not of interest to macro-engineering structural problems. In addition, the description basically holds only for brittle conditions since the nucleation and propagation of microcavities is assumed to be the only cause of failure. This does not have to be the case under the tensor formulation.

However, Jean Lemaitre, in 1971, developed the strain equivalence principle to avoid the micromechanical analysis described above by extrapolating the thermodynamic concept of a local state. "In thermodynamics, the method of local state assumes that the thermomechanical state at a point is completely defined by the time values of a set of *continuous* state variables depending upon the point considered" [ref. 6]. This implies that the constitutive equations valid for a microvolume are not influenced by a surrounding microvolume containing a microcrack. At the mesoscale, this implies that the constitutive relationships are not modified by the presence of damage sites, and therefore "Any strain constitutive equation for a damaged material may be derived in the same way as for a virgin material except that the usual stress is replaced by the effective stress" [ref. 6]. Consequently, the effective modulus of elasticity becomes

$$E_{eff_D} = E(1 - D)$$

In addition, for uni-axial elasticity laws, the effective strain becomes

$$\epsilon_{eff_D} = \frac{\sigma}{E_{eff_D}} = \frac{\sigma}{E(1-D)}$$

Figure 5-3 shows the variation of the elastic modulus as a function of damage for copper
(In Figure 5-3, $\tilde{E} = E_{eff_D}$.)

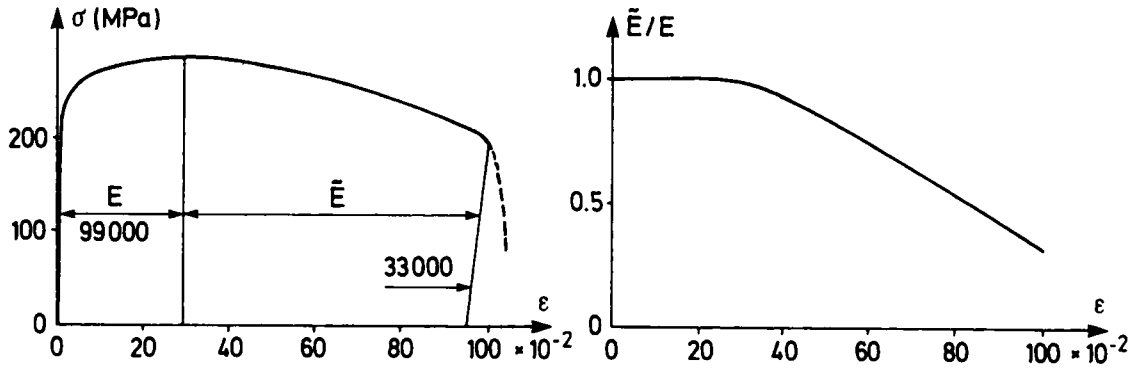


Figure 5-3. 99% Copper Elastic Modulus Variation as a f(Damage) [ref. 6].

Again, σ and D are tensors in the general case, and therefore, a damage effect tensor must be defined which operates to distribute and orient this damage as a field. The simplest way to do this is to assume that one can describe the damage along the principal axes of stress as D_1 , D_2 , and D_3 by appropriate plane intersections.

$$D = \begin{bmatrix} D_1 & & \\ & D_2 & \\ & & D_3 \end{bmatrix} \text{ where } D \text{ includes fatigue plus plastic damage.}$$

Therefore, the effect of this damage elsewhere is based upon a damage operator on the damage tensor $\mathbf{D}$ [ref. 27] defined as

$$M(D) = \begin{bmatrix} \frac{1}{1-D_1} & 0 & 0 & 0 & 0 & 0 \\ \frac{0}{1-D_2} & \frac{1}{1-D_2} & 0 & 0 & 0 & 0 \\ \frac{0}{1-D_3} & \frac{1}{1-D_3} & 1 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2(1-D_2)(1-D_3)}} & \frac{1}{\sqrt{2(1-D_3)(1-D_1)}} & \frac{1}{\sqrt{2(1-D_1)(1-D_2)}} & 0 & 0 & 0 \\ S & & & & & \end{bmatrix}$$

for planar problems, (plane stress or plane strain), the damage effect tensor is simply 3x3. To follow the tensorial development requires a deeper level of detail than intended here for presentation of the CDM concepts. The Chow and Wang [ref. 28, 29] and Chow and Lu [ref. 27] papers present this detail with great clarity, and to attempt to condense it here would not do justice to them. Here the material will be considered purely elastic and isotropic for simplicity such that the salient points and comparisons can be made between all of the fatigue theories discussed.

The coupling between strains and damage is called a state coupling, since the state of the material can be characterized by the elastic modulus, as shown in Table 5-2.

Table 5-2. State Coupling Through Elastic Modulus	
Un-Damaged	Damaged
$D = 0$	$0 < D \leq 1$
$\epsilon_{eff_D} = \frac{\sigma}{E} = \epsilon$	$\epsilon_{eff_D} = \frac{\sigma}{E(1-D)}$

Another coupling takes place, called kinetic coupling, where the kinetic constitutive laws of plasticity are used to couple plastic strain and damage. Consider the yield function f such that

$$f = |\sigma - X| - R - \sigma_y$$

where σ is the actual or effective stress, R is the stress due to isotropic hardening, X is the back stress due to kinematic hardening, and σ_y is the material yield stress. Isotropic hardening is related to the density of the dislocations or flow arrests, while kinematic

hardening is related to the state of the internal microstress concentrations [ref. Lemaitre].

Then the one-dimensional plasticity criterion becomes (recall $\epsilon_{Total} = \epsilon_{Elastic} + \epsilon_{Plastic}$)

$$\exists \quad \dot{\epsilon}_p \neq 0 \text{ if } \begin{cases} f = 0 \\ \text{and} \\ \dot{f} = 0 \end{cases} \quad \text{and} \quad \exists \quad \dot{\epsilon}_p = 0 \text{ if } \begin{cases} f < 0 \\ \text{or} \\ \dot{f} < 0 \end{cases}$$

Therefore, when damage occurs

$$f = \left| \frac{\sigma}{1-D} - X \right| - R - \sigma_y = 0$$

and experiments have shown that damage equally decreases the yield stress, the isotropic strain hardening stress, and the back stress [ref. 6].

One can derive a rupture criterion, where rupture is defined at the mesoscale as the point at which a crack has grown to initiation (or nucleated to a size) as determined by occupying the whole surface of the RVE, i.e., $D = 1$. This process can also be characterized as the sudden decohesion of atoms in the resisting area, corresponding to a critical damage value D_C such that

$$\sigma_{eff_{D_C}} = \frac{\sigma}{1-D_C}$$

where $D_C \equiv 1 - \frac{\sigma}{\sigma_u}$ and σ_u is the ultimate material strength, σ being the uni-axial

stress. $D_C \approx 0$ implies brittle fracture, while $D_C \approx 1$ implies pure ductile fracture.

Typically, $0.2 \leq D_C \leq 0.5$. For a pure monotonic tension test (mode I)

$$D_{IC} = 1 - \frac{\sigma_R}{\sigma_u}$$

where σ_R is the stress at rupture.

In summary, the damage modeled by D is the result of void or dislocation nucleation due to the accumulation of microstress into an initiated microcrack. Therefore, damage is an evolutionary process. Consequently, there exists a plastic strain threshold below which no nucleation onset takes place, and therefore no initiation of microcracks and hence the

arrest of damage evolution. These concepts comprise the basis of damage mechanics [ref. 6] expressed as the following:

$$\epsilon_{eff_D} = \frac{\sigma}{E(1-D)} \text{ for elasticity}$$

$$\left| \frac{\sigma}{1-D} - X \right| - R - \sigma_y = 0 \text{ as the plastic criterion}$$

$$\epsilon_P < \epsilon_{PD} \Rightarrow D = 0 \text{ as the damage threshold}$$

$$D = D_C \Rightarrow \text{crack initiation (or nucleation onset)}$$

It can be shown [ref. 6], that a relationship exists between the strain energy density release rate Y and damage D

$$Y = \frac{1}{2} \frac{dw_e}{dD} \Big|_{\sigma=\text{constant}} = \frac{w_e}{1-D}$$

where w_e is the elastic strain energy density. "This is the energy released by loss of stiffness of the RVE in which damage occurred. The strain energy density release rate, by definition of the associated variable in thermodynamics, is the principal variable which governs the phenomenon of damage" [ref. 6].

Just as von Mises equivalent stress yields the uni-axial equivalent for a three-dimensional stress state, one can also define an equivalent stress which yields the same uni-axial strain energy density as a three-dimensional state:

$$\sigma^* = \sigma_{eq} R_v^{\frac{1}{2}}$$

where σ_{eq} is the von Mises equivalent stress and R_v is the tri-axiality function

$$R_v = \frac{2}{3}(1+\nu) + 3(1-2\nu) \left(\frac{\sigma_H}{\sigma_{eq}} \right)^2$$

where σ_H is the hydrostatic stress, $\sigma_H = \frac{1}{3}\sigma_{kk}$. Note that as the tri-axiality ratio $\frac{\sigma_H}{\sigma_{eq}}$ increases, the tri-axiality function increases. Since $Y = \frac{\sigma_{eq}^2 R_v}{2E(1-D)^2} = \frac{\tilde{\sigma}_{eq}^2 R_v}{2E} = \frac{(\sigma^*)^2}{2E}$, then, as the tri-axiality increases, the material becomes more brittle, as observed in practice. σ^* allows CDM to be applied to multi-axial load/stress situations.

Recall that fracture mechanics also defined a strain energy release rate. This, however, was defined on the mesoscale since this is the domain of fracture mechanics. Comparing the critical strain energy release rate for continuum damage mechanics at the microscale and fracture mechanics at the mesoscale results in the establishment of a characteristic length of the RVE

$$l \equiv \frac{G_c}{Y_c}$$

where G_c is the critical strain energy release rate from fracture mechanics and Y_c is the critical strain energy release rate from continuum damage mechanics. Table 5-3 illustrates representative values for these parameters.

Table 5-3. Representative Material Parameters		
Light Alloys	Parameter	Steels
28.5	$\leq G_c \leq$	285.5 LB _f /In
290	$\leq Y_c \leq$	1450 PSI
.098	$\leq l \leq$	.197 Inch

For fatigue, it is necessary to realize that the damage rate is an increasing function of the strain energy release rate. Just as the crack growth rate of fracture mechanics was dependent upon the material characterizing parameter K_{IC} , the stress intensity, the damage rate leading to the generation of a crack (via either initiation or nucleation) is dependent upon material parameters. These material parameters are the energy strength

of damage S , the damage threshold function, P_D , which combines loading effects and is the irreversible energy difference between the total plastic strain energy and the energy dissipated in the form of heat, and the critical damage D_C .

As an example, for low cycle fatigue, the number of cycles to failure, $N_R < 10K$, is made up of two parts: N_0 to reach the damage threshold plastic strain P_D for microcrack nucleation, and N_D during which damage progresses to initiate a crack at the mesoscale (typically 1mm long), such that $N_R = N_0 + N_D$. Note that N_0 must be calculated by classical elastoplastic constitutive equations where $D = 0$ since for this region no damage has yet occurred, but the process is elastoplastic. Consequently, from

$$\frac{\partial P}{\partial N} = \int_{1 \text{ Cycle}} |\dot{\epsilon}_p| dt = 2\Delta\epsilon_p$$

the accumulated plastic strain for one cycle is obtained, then for N cycles

$$P(N) = 2N\Delta\epsilon_p. \text{ Therefore, at damage point } N_0 = \frac{P_D}{2\Delta\epsilon_{pD}}, \text{ where } P_D = \epsilon_{pD} \frac{\sigma_u - \sigma_f}{\sigma_M - \sigma_f},$$

ϵ_{pD} is the damage plastic strain threshold in pure tension, σ_f is the fatigue limit, σ_u is the ultimate material strength, and σ_M is the maximum stress. Similarly, the calculation of N_D requires the integration of the kinetic damage law over one cycle and then over N cycles:

$$\dot{D} = \frac{\sigma^2}{2ES(1-D)^2} |\dot{\epsilon}_p|$$

where S is the damage strength material parameter. This yields

$$\frac{\partial D}{\partial N} = \int_{1 \text{ Cycle}} \dot{D} dt = \frac{K_p^2}{4ES} \Delta\epsilon_p^{\frac{m+2}{m}}$$

and integrating over N cycles to obtain the critical damage results in

$$N_D = \frac{4ESD_C}{K_p^2} \Delta\epsilon_p^{\frac{-(m+2)}{m}}$$

where D_c is the critical damage level at failure, K_p and m are material constants [ref. 6]. This is the power law relationship including damage, which is similar to the original Manson-Coffin power law typically described as [ref. 30]

$$N_f = \frac{1}{2} \left(\frac{\Delta \epsilon_p}{2 \epsilon'_f} \right)^{\frac{1}{c}}$$

Similar relationships can also be obtained for high cycle fatigue [see Lemaitre].

Continuum Damage Mechanics provides a theoretical basis for crack generation (either via sudden initiation or by damage site nucleation mechanisms) where the failure criterion is based upon the postulation that a microcrack is formed when the overall equivalent damage D , which includes the equivalent fatigue damage D_f and equivalent plastic damage D_p , reaches a critical value D_c [ref. 28]. This allows *total* life of a component to be determined when coupled with Fracture Mechanics for crack propagation. This lifing technique compares favorably with the strength of materials S-N curves for simple specimens. Chaboche and Lesne [ref. 31] have developed a non-linear form of CDM with similar simple formalism which predicts life with startling accuracy. However, the loading, temperature, and other dependencies which generated all of the stress concentration-type factors of the Strength of Materials approach still remain, and, in fact, are common to the Continuum Damage Mechanics approach. CDM appropriately supplements experimental data with theoretical formulation of *Damage Laws* capable of describing the non-linear physical phenomena of material degradation. While better understanding of the failure process is obtained with this formalism, in its current state it offers little benefit to improving material characterization (simple uni-axial tensile testing is still utilized to characterize damage) or analysis methodology to reduce fatigue failures. However, once implemented into a finite element method structures code, this formalism may provide the most accurate modeling of multi-axial fatigue for orthotropic (composite) materials (and also for anisotropic ones) and will be no more complex than current engineering approaches.

The key features of Continuum Damage Mechanics as a fatigue theory are outlined below:

Key Features:

- I. Assumptions:
 - A. Material can be anisotropic non-homogeneous
 - B. Small strain theory (strains $< 10\%$)
 - C. Allows non-linear stress-strain relationship
- II. Applications Areas:
 - A. Low and high cycle fatigue
 - B. Low to high fracture toughness materials
 - C. Crack initiation
 - D. Multi-axial stress situations
- III. Advantages:
 - A. Simple to use (or at least could be if coded)
 - B. Failure mechanism integral part of theory
 - C. Existing material data may be used
 - D. Handles ductile materials
- IV. Disadvantages:
 - A. Limited material data characterizing actual damage
 - B. Complex formalism is frightening
 - C. Lacks implementation into typical engineering structural analysis tools

6. HOLISTIC DESIGN APPROACH

Many instances of structural failures have probably occurred which could not be explained by standard failure criteria based upon a strength of materials approach, fracture mechanics approach, or any other failure theory, either statically or under fatigue conditions. Such failures are generally attributed to the existence of microscopic crack-like voids and discontinuities in the material which may not have existed in the coupon tested during material characterization, or due to loading spectra effects not accounted for during component testing and/or material characterization. The microscopic material entities are not necessarily considered flaws, but merely part of the structure of the material itself, or result from the processing of the material. The voids and/or discontinuities magnify their presence by resulting in what is known as the *size-effect* of material strength, as described in the fracture mechanics section of this report. As stated there, Griffith [ref. 6] showed that not only did the strength vary with the amount of material (again see Figure 3-0), but this strength was from 1% - 5% of the theoretical based upon internal molecular forces and bond strengths. Consequently, this indicates that the *strength* of a material is dependent upon characterization test aspects as well as the true as-used state of the material (all manufacturing processes included). In addition, even discounting procedural induced data scatter typical of material characterizations, the size-effect indicates that the material strength is also probabilistic in nature since the probability of defects is directly tied to the amount of material as well as the processing of the material. These non-deterministic traits make it difficult for any theory to adequately predict the life of a component based upon standard material characterization which may not be the exact material with the exact manufacturing process induced peculiarities as the component in question. Typically, these peculiarities are handled by a suitable factor of safety or other engineering *fudge factor*, however, this approach does not guarantee conservatism, and therefore the unexplained catastrophic failures.

Consider a holistic approach to component design (not related to Holy, but we might do better to invoke this approach earlier in the process too!). For a process to be *holistic*, it should be “emphasizing the organic or fundamental relation between parts and wholes (from *Holism*, a philosophic theory first formulated by J.C. Smuts, that the determining factors in nature are wholes which are irreducible to the sum of their parts. A theory or doctrine according to which a whole cannot be analyzed without residue into the sums of its parts or reduced to discrete elements)” [ref. 3]. With respect to fatigue resistance of a component, this implies that the design methodology must be concerned with

- a failure criterion based upon a sound theory of material fracture and engineering practices;
- material characterization consistent with the failure criterion;
- test methodology for coupons, components, and systems consistent with the material characterization;
- and a clear understanding of the effects of unavoidable deviations in service with respect to theory, material characterization, and development testing on the design, both structurally and performance-wise.

Due to a lack of an absolute failure criterion, a lack of an absolute material characterization which includes effects of load sequence, crack size, and stress state, and the cost/practicality of testing, one is forced to make the design analysis, material selection and processing, and test sequence an iterative process throughout all phases of the component/system life. This ensures assumptions and boundary conditions used to develop the component/system remain valid and therefore the life predicted and inspection intervals established should prevent unexpected catastrophic failures. Nichols [ref. Nichols] illustrates this approach from a test and evaluation (T&E) viewpoint, showing how the T&E phase should feed back the load spectra and various stress ratios in a standardized instrumentation/data protocol to

1. the designer for adjustments in component geometry,
2. the metallurgist for adjustments in material processing and characterization testing,
3. the analyst for model adjustment to actual boundary conditions.

These iterations combine to accomplish the development of a safe component/system with *predictable* life.

In general, the holistic approach to fatigue design is to characterize processed (includes manufacturing aspects) material specimens for a component according to a load spectrum which duplicates the damage effects expected in service (this is not necessarily the exact load spectrum, but rather one that replicates the material damage accumulation). In this way, the whole life of the component can be determined without regard to a complex predictive theory, and the correlations obtained for the fatigue theory assumed will provide a basis for extrapolation within that fatigue theory for other similar components with similar load spectra. This, however, requires that the stress *field* of the component be known and is adequately represented in the material characterization. In addition, the correlations could span various fatigue failure theories such that the data set can be used for low or high cycle fatigue assessments, provided the load spectrums are consistent with these failure regimes. This approach basically assumes that if one tests the material in the state it will be in when the component is manufactured and under service loading conditions, then a typical engineering factor of safety applied to the coupon life will ensure that the component will not fail in service.

To maximize the information upon which design decisions will be based, the holistic approach combines all of the available data from testing coupons, components, and systems into a probabilistic failure analysis utilizing the best theory for the appropriate portion of the total life of the component. The critical stress levels so identified are then compared to a stress probability curve based upon the load sequence. The overlap between these probabilities of failure and probabilities of load excursion (see Figure 6-1) provides an indication of the margin of safety. Miner's Rule (assuming a linear damage accumulation relationship exists) combined with suitable cycle counting techniques allows limited extrapolations to other load and spectra conditions.

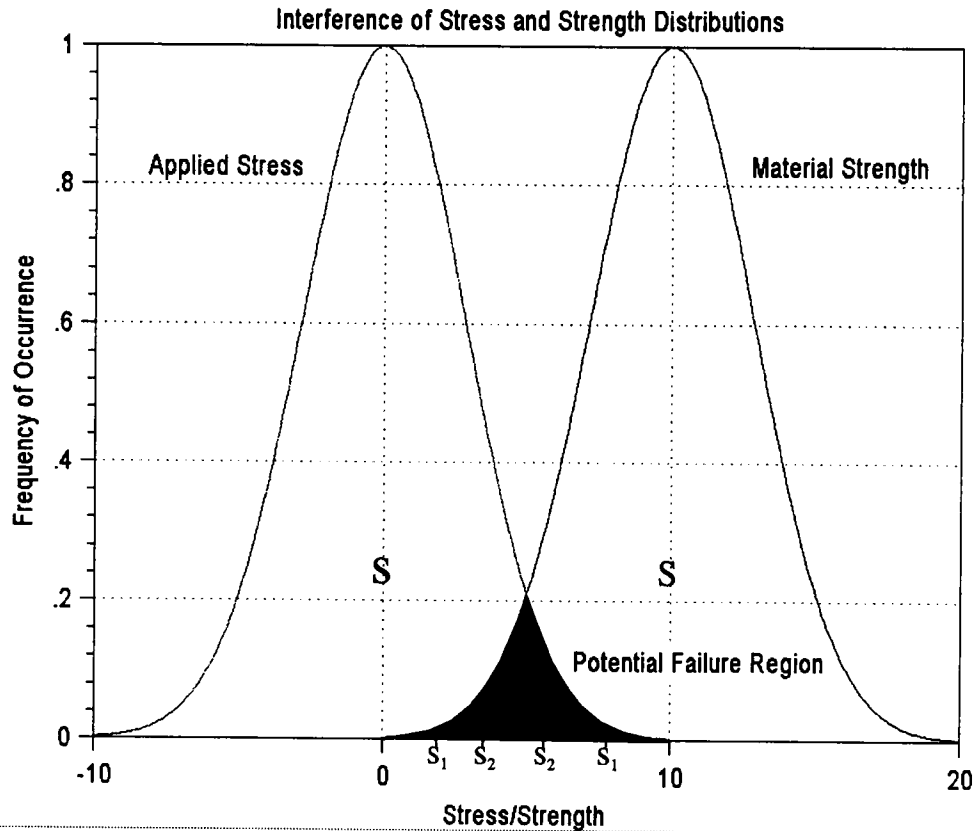


Figure 6-1. Interference of Stress and Strength Probability Distributions (Normalized Frequency of Occurrence) Resulting in Region of Potential Failure

$$\sigma_1 > S_1 \Rightarrow \text{Failure}$$

$$\sigma_2 < S_2 \Rightarrow \text{No Failure}$$

$$\text{Let } \Sigma = S - \sigma, \text{ then } \begin{cases} \Sigma > 0 \Rightarrow \text{OK} \\ \Sigma < 0 \Rightarrow \text{Failure} \end{cases}$$

With respect to Figure 6-1, no overlap of these probabilities is probably too conservative and costly, while complete overlap is just plain stupid since it predicts failure for all occurrences of load excursion. However, the amount of overlap is probably based or optimized more on economics and societal concerns rather than on any real guidance from engineering. The holistic approach is generally more costly since it requires more testing (materials, components, and system just to characterize the material in addition to

normal proof of performance/capability testing), but the life cycle cost can be reduced since inspection cycles and removal-for-cause programs can minimize catastrophic failures (based upon the above region of overlap).

The key features of the Holistic Design Approach to fatigue are outlined below:

Key Features:

- I. Assumptions:
 - A. Material is isotropic homogeneous continuum in the plastic region ahead of the crack to allow use of LEFM tools
 - B. Material characterization is load spectra dependent
 - C. Fatigue data of specimens directly correlate to component failures for same load spectra
 - D. Non-determinism of fatigue can be overcome with appropriate statistical material characterization data
- II. Applications Areas:
 - A. Low and high cycle fatigue
 - B. Any range of fracture toughness
- III. Advantages:
 - A. Simple to use
 - B. Failure mechanism hidden in data analysis
- IV. Disadvantages:
 - A. Material data must be gathered for specific load case
 - B. Predictive modeling is restricted to the load case and material characterized
 - C. Ignores physics of failure in exchange for pure correlations with experiments

7. COMPARISON of FATIGUE APPROACHES

Each of the fatigue theories discussed in this report have advantages and disadvantages as shown in the *Key Features* of each appropriate section. However, since each is based upon different premises, a comparison requires that a criteria for comparison be established. The goal of any fatigue theory is first to explain experience and second to be able to predict the potential for failure of a component. Consequently, the failure criterion of each theory must be part of the comparison criteria. In addition, since the intent is to use the theory for design guidance, the applicability of the theory to real engineering structural problems and its ease of use must also be part of the comparison criteria. Finally, realizing that all is not known about the fatigue phenomenon, the capability of a theory to adapt to new situations and knowledge must be included as a comparison criteria. While these are the primary criteria for comparison, the successful choice of a fatigue theory involves many others as shown in the following table.

Table 7-1 Fatigue Theory Comparison

Criterion	Strength of Materials	Fracture Mechanics	Strain Energy Density	Continuum Damage Mechanics	Socie-Parameter Approaches	Holistic Design Approach
Failure	Yield $\sigma > \sigma_Y$ Fracture $\sigma > \sigma_U$ Fatigue $\sigma > \sigma_e$	$K \geq K_{Ic}$	$\frac{dW}{dV} > \frac{S_c}{r_c}$	$D \geq D_c$	$\gamma = \left(1 + n \frac{\sigma_e^m}{\sigma_s^m}\right) = \text{Constant}$	Coupon Test
Failure Diagram	Goodman	$\frac{da}{dN}$ vs ΔK	$\frac{da}{dN}$ vs ΔS	% D vs ϵ	Socie-N	ALL
Size Effects	C_D	N/A	N/R	N/R	N/R	N/A
Multi-Axial Eq.	Von Mises σ_e					
Ductile/Brittle	N/A	Brittle	Both	Both	Both	N/R
Material Characterization Data	Uni-axial	Uni-axial	Uni-axial	Uni-axial	Bi-Axial	Actual Stress Field
Stress Concentrations	K_T	K_T	N/R	K_T	K_T	N/R
Loading Effects (how handled)	Uni-Axial σ_{eq}	Symmetric* (NOT)	N/R	σ_{eq}	N/R	Load Sequence
Environment Effects	K_T	N/A	??	Included	K_T	??
Strain Rate Effects	N/A	N/A	Included	N/A	N/A	N/R
Ease of Use	1 (Easiest)	2	4	5 (Hardest)	3	3
Predictive Capability	5 (Least)	4	1 (Most)	2	1	3
Application	HCF	LCF	LCF&HCF	LCF&HCF	LCF&HCF	LCF&HCF
Assumptions:	■■■■■	■■■■■	■■■■■	■■■■■	■■■■■	■■■■■
Continuum	✓	✓	Discrete	✓	✓	Don't Care
Homogeneous	✓	✓	NO	✓ Damage	✓	Don't Care
Isotropic	✓	✓	NO	NO	NO	Don't Care
Linear Elastic*	✓	✓ (LEFM)	NO	NO	NO	Don't Care
Initial Crack Size*	NONE	✓ (Long)	NO	NO	✓ (Any)	Don't Care
Defect Free	✓	NO	NO	NO	NO	Don't Care

N/A $\Rightarrow$ Not Addressed

N/R $\Rightarrow$ Not Required

* Also self-similar crack growth

The “Don’t Care” entries in Table 7-1 for the Holistic Design Approach are intended to convey the fact that correlations derived from *as-built* coupon tests for the components of interest exposed to the *same* stress field make no such assumptions, and the tests are applicable regardless of the theory used to correlate the data.

Another way to compare these theories is to compare their failure parameters based upon standard specimen tests. While data isn’t available for the CDM and Holistic approaches, Tables 7-2 and 7-3 [ref. 6] compare fracture toughness results for some common materials.

Table 7-2 Fracture Toughness Comparison			
Material	Strength of Materials σ_u (KSI)	Fracture Mechanics K_{Ic} (KSI/$\sqrt{\text{In}}$)	Strain Energy Density S (Lb/In)
A517F Steel	120	170	95.8
AISI 4340 Steel	260	45	6.7
300M Steel	260	45	6.7
H-11 Steel	300	40	5.3
12Ni-5Cr-3Mo Steel	190	220	160.5
18Ni Maraging Steel *	240	105	41.3
9Ni-4Co-0.3C Steel	260	60	11.9
AL2014-T651	70	23	3.6
AL2024-T851	65	23	3.6
AL2219-T851	66	33	7.4
AL2618-T651	64	32	7.0
AL7001-T75	90	25	4.2
AL7075-T651	83	26	4.6
AL7079-T651	78	29	5.7
AL7178-T651	83	24	3.9

* $\Rightarrow$ Average

Table 7-3 Fracture Toughness Extremes Comparison			
Extreme	Strength of Materials σ_u (KSI)	Fracture Mechanics K_{Ic} (KSI/$\sqrt{\text{In}}$)	Strain Energy Density S (Lb/In)
Minimum Value	64 (AL 2618-T651)	23 (AL 2014-T651) (AL 2024-T851)	3.0 (H-11 Steel)
Maximum Value	320 (H-11 Steel)	220 (12Ni-5Cr-3Mo Steel)	160.5 (12Ni-5Cr-3Mo Steel)
Span	63 - 320	23 - 220	3 - 160.5

Consider Figures 7-1 and 7-2, which plot the fracture toughness data of Table 7-1. For aluminum, the SED and Fracture Mechanics curves at least have the same trends (SED more pronounced due to a change in scale), but the Strength of Materials' correlation shows no similar trend, and in fact appears to be essentially 180 out of phase! For AISI 4340 steel, the trends appear to be the same. Here, however, it is very clear that the fracture toughness increases with decreasing ultimate strength, emphasizing the fact that fatigue problems are most severe for high strength materials. The close trend correlation between SED and FM may lead one to believe that they are essentially equivalent descriptions, but these data are for *uni-axial fracture specimens*. While it is true that the strain energy density can be related to the stress intensity factor, FM runs into trouble for ductile materials when the strain rate is low and the stress is low (slow crack growth of HCF), when the loading is non-symmetric with the plane of the crack, and when the crack is not self-similar. SED handles these conditions adequately. In fact, other than the Holistic Approach (which is more a methodology than a theory), the SED is the obvious choice for fatigue assessments.

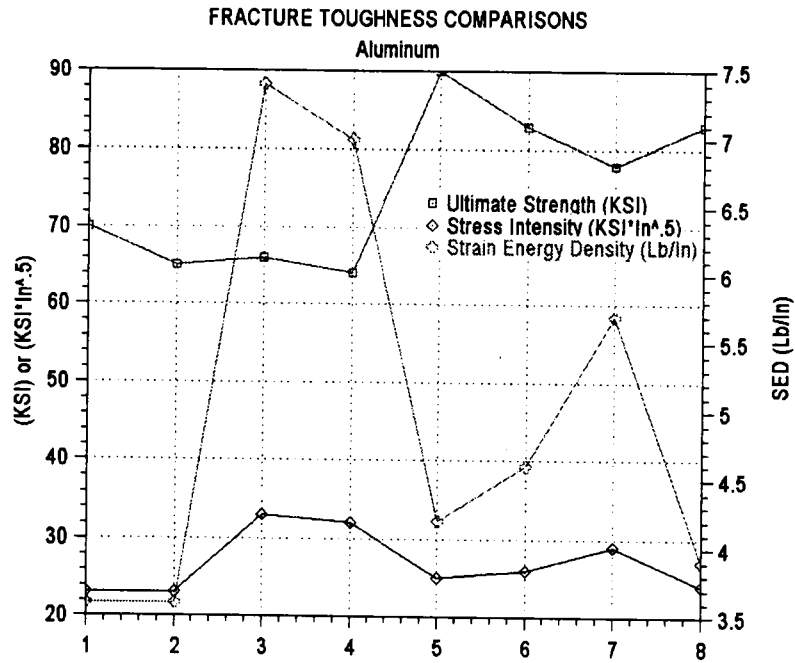


Figure 7-1. Fracture Toughness Comparison for Various Aluminums
(Note: X-axis is notional only, Y-axis is the comparison.).

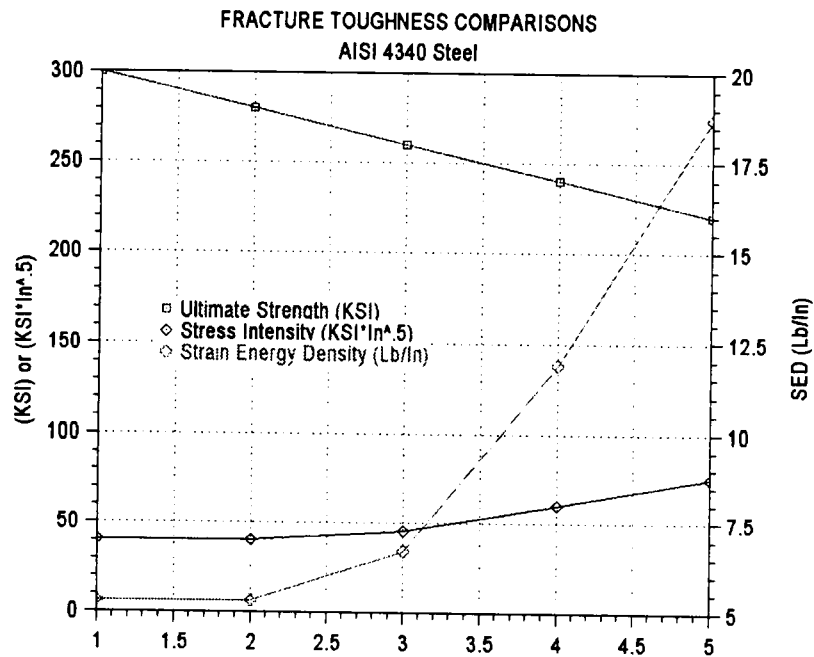


Figure 7-2. Fracture Toughness Comparison for AISI 4340 Steels
(Note: X-axis is notional only, Y-axis is the comparison.).

8. OTHER FACTORS AFFECTING FATIGUE

Stress Concentration Factors

Stress concentrations arise at any discontinuity in the normal distribution of stress, such as [ref. Hoepfner short course]:

- Reduction in cross-section,
- Flow of load (displacement) around discontinuity,
- Change in stress state at the discontinuity.

Stress concentration factors attempt to adjust conditions for areas which are not accounted for by theory. These factors were originally experimentally derived, but some can be derived from advanced elasticity theory while others can be determined from Finite Element Method (FEM) analysis. A major compendium of stress concentration factors is that of Peterson [ref. 11]. Basically, a *theoretical* or geometric stress concentration factor is defined as

$$K_T = \frac{\sigma_{Max_Observed}}{\sigma_{Nominal}}$$

where the maximum observed stress is in the immediate vicinity of the stress concentration while the nominal stress is the distributed stress outside the region of the stress concentration. Consequently, K_T varies with bulk geometry, geometric discontinuity size/shape, and load application or stress state. Figure 8-1 illustrates the concept of stress concentration due to geometric discontinuities and also shows how the stress state has been altered (biaxial in the region of the notch, see vectors in inset) due to the stress concentration field.

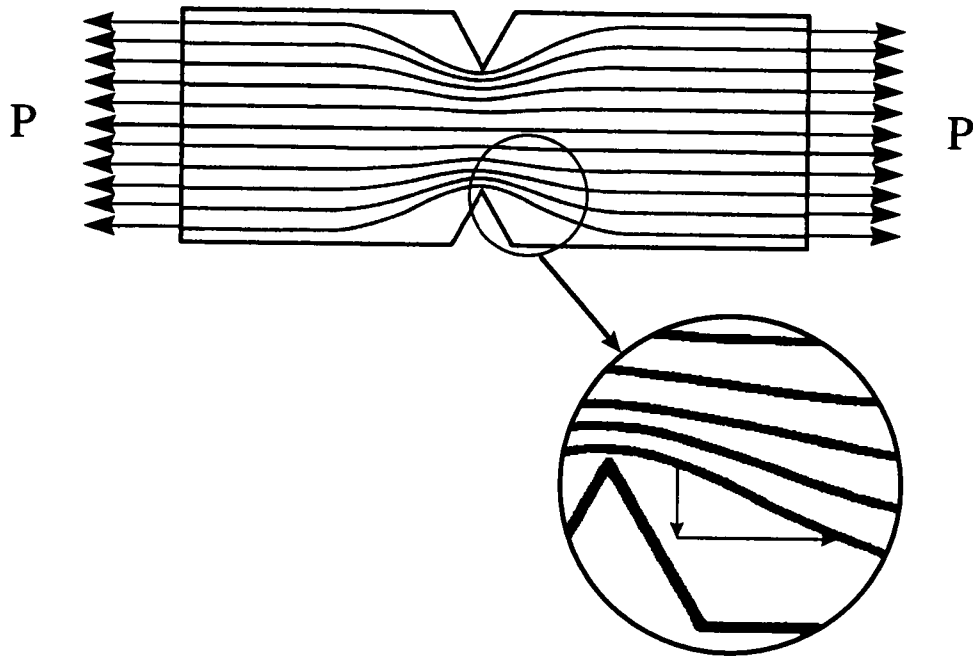


Figure 8-1. Geometric Stress Concentration.

The stress concentration factors are applied to the effective bulk stress for the net cross-section of the component to account for stress riser regions, i.e.,

$$\sigma_{\max} = \sigma_{\text{bulk}} K_T$$

or K_T can be applied to the strength of the material such that

$$S_N = \frac{\hat{S}_N}{K_T}$$

where $\hat{S}_N$ is the tensile strength of the material determined by standard specimen tensile tests. Neuber's Rule specifies a theoretical stress concentration based upon a geometric mean of the actual stress and strain concentration factors K_σ and K_ϵ , respectively, as

$$K_T = (K_\sigma K_\epsilon)^{\frac{1}{2}}$$

where $K_\sigma = K_T$ as defined above, and K_ϵ is similarly defined for the strains. Explicitly, Neuber's Rule is

$$\sigma \epsilon = \frac{(K_T S)^2}{E}$$

where σ and ϵ are the notch root stress and strain, respectively, and S is the nominal stress away from the notch.

There is also a fatigue stress concentration factor K_f for adjustment of the fatigue strength for notch sensitivity, which is typically less than K_T for a notch, especially for coarse-grained materials. Topper, et. Al., modified Neuber's Rule for cyclic conditions (fatigue) by substituting $K_\sigma = \frac{\Delta \sigma}{\Delta S}$ and $K_\epsilon = \frac{\Delta \epsilon}{\Delta e}$ where $\Delta \sigma$ and $\Delta \epsilon$ are the stress and strain ranges, respectively, at the root of the notch and ΔS and Δe are the nominal stress and strain ranges, respectively. In this case, K_f is used instead of K_T , however, Neuber, Peterson, and others [ref. 14] have shown that K_T and K_f are related.

Therefore

$$K_f = \frac{S_{Unnotched}}{S_{Notched}}$$

$$\sigma_{\max \text{ Alternating}} = \sigma_{\max \text{ Range}} K_f$$

$$S_e = \frac{\hat{S}_e}{K_f}$$

where $K_f = 1 + \frac{K_T - 1}{1 + \sqrt{\frac{a}{r}}}$ (Peterson's Equation [ref. 14]), S_e is the unnotched endurance

strength of the material at the desired number of cycles (life), a is the Neuber constant for the material, r is the radius at the root of the notch, and $\sigma_{\text{Range}} = \sigma_{\max} - \sigma_{\min}$.

Peterson also defines a *notch sensitivity factor* for the material as $q = \frac{K_f - 1}{K_T - 1}$ which ranges from zero (no notch effect of the stress riser) to one (fully sensitive to the notch

effect of the stress riser). Then $K_f = 1 + (K_T - 1)q$. Typically, $q = .5$ for aluminum and titanium alloys and $q = .35$ for steel and nickel-based alloys [ref. 26].

In addition, for fatigue in the absence of data for an endurance strength, the tensile strength of the material itself must be adjusted for the loading condition, size, and surface finish relative to the standard specimen and test method used to obtain the tensile strength of the material. For a typical 10^N cycle strength,

$$S_N = \hat{S}_N C_L C_D C_S$$

where C_L , C_D , and C_S are the load, dimension, and surface coefficients, respectively.

Typical values for these coefficients are shown in Table 8-1.

Table 8-1. Strength Coefficients			
	Load Type		
Factor	Bending	Torsion	Axial
C_L	1.0 (Mc/I)	.58 (Tc/J)	.9
C_D	1.0 if $D \leq .4''$.9 if $.4'' < D \leq 2.0''$	1.0 if $D \leq .4''$.9 if $.4'' < D \leq 2.0''$	1.0
C_S	*	*	*

* Function of material, but generally range from .1 for corroded surfaces to .9 for commercial polishing, with machined surfaces ranging from .5 to .8.

Finally, the S-N curve can be generated by $S_N = \frac{\hat{S}_N C_L C_D C_S}{K_f}$. However, adjusting the

static strength of a material S to arrive at $\hat{S}_N$ via some generalized method and then adjusting this to account for loading, dimensions, and surface to arrive at the fatigue strength is very dangerous and inaccurate. This approach should only be used for *ballparking* the design. There is NO substitute for actual component testing or, as a minimum, coupon testing of the material in the *as-processed* state.

Cyclic Stress Ratio

The ratio of minimum to maximum cyclic stress is called the stress ratio defined as

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

This ratio, along with the stress concentration factor, can radically effect the fatigue life, as shown in Figure 8-2 [ref. 2].

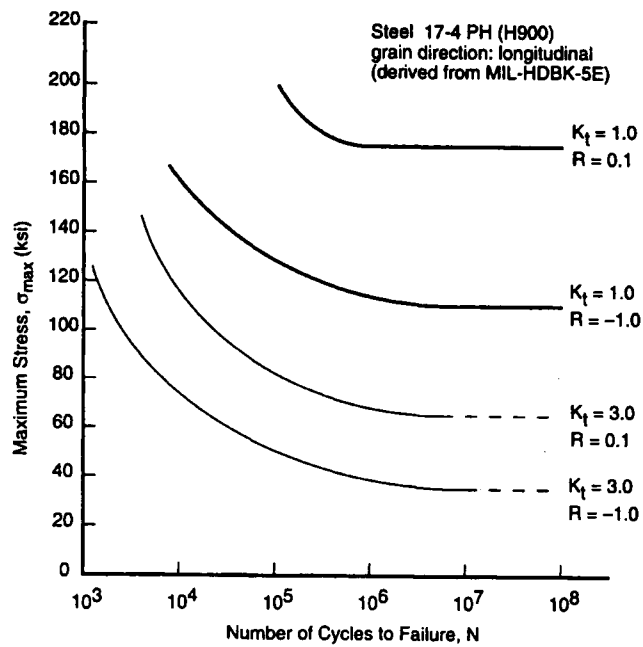


Figure 8-2. Stress Ratio and Concentration Effect on Fatigue Life.

Cumulative Damage - Miner's Rule and Cycle Counting

For variable amplitude alternating stress conditions, the life of the component can be thought of as the summation of the life fractions at each stress level. Consequently, these fractions sum to the total damage D of the component, for which failure occurs when $D = 1$. This formulation is due to Palmgren-Miner [ref. 14], and is typically called Miner's Rule:

$$D = \sum_i \frac{n_i}{N_i} = 1$$

where n_i is the number of cycles at stress range i , and N_i is the life in cycles for stress range i from an S-N curve. For random loading, Miner's Rule will *usually* give conservative results, but failures have been known to occur for

$$0.61 \leq D \leq 2.2$$

and, typically, Miner's Rule works better for aluminum and titanium alloys than for steel. For non-random testing, depending upon whether the stress range is progressively increasing or decreasing, failure has been known to occur for the range

$$0.18 \leq D \leq 23.0$$

In addition, the sequence of the stress state (torsional versus uni-axial) can significantly effect the results from Miner's rule. For increasing fatigue life, the departure from Miner's rule is severe for such sequences, and is potentially highly nonconservative [ref. 26]. Figure 8-3 [ref. 26] shows the results of sequences of torsional loading followed by uni-axial loading and vice versa. Consequently, there is no substitute for actual testing where the real load spectrum for the component is used in either coupon or actual component fatigue tests.

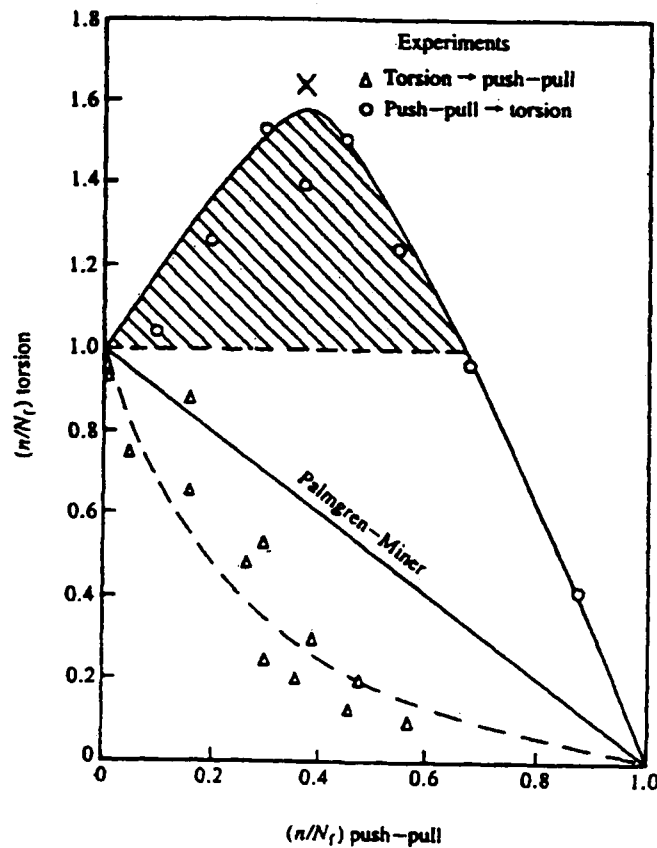


Figure 8-3. Miner's Rule Comparison for Torsion/Uni-Axial Sequence Loading.

Various techniques exist for cycle counting for variable loading conditions. The most common methods are [ref. 32]

- Load level crossings
- Peak or trough counting
- Range counting or range pair counting
- Level distribution counting
- Counting extreme peaks (highest and deepest) between two mean level crossings

These techniques are graphically described in Figure 8-4.

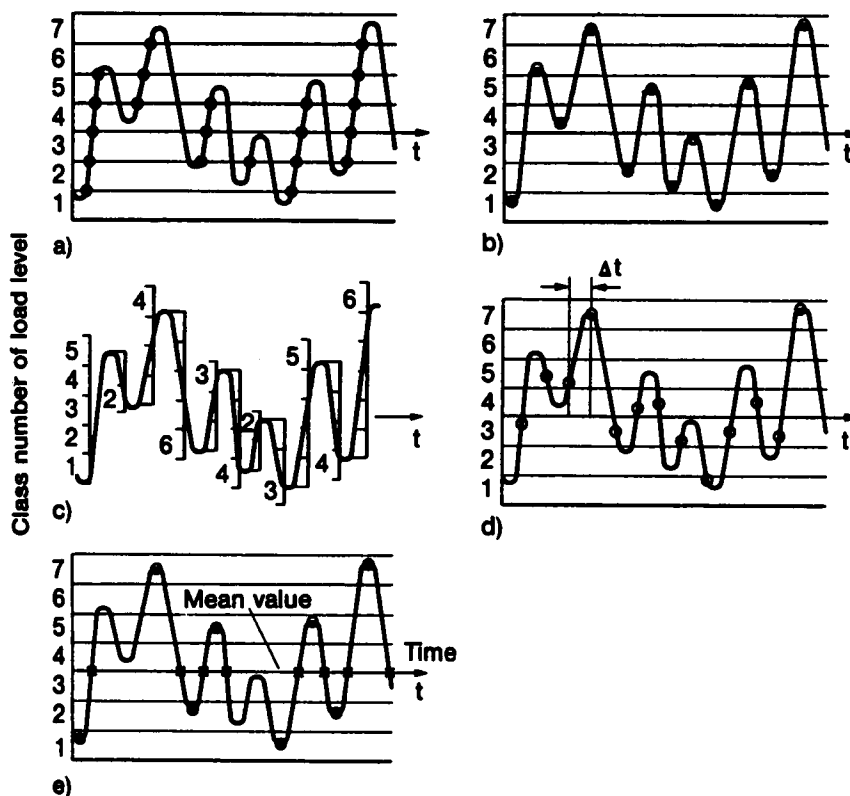


Figure 8-4. Counting methods: (a) Level Crossing Counting, (b) Peak Counting, (c) Range Pair Counting, (d) Level Distribution Counting, (e) Extremes between Mean Crossings Counting.

These methods are considered single parameter methods, and are not generally well suited for fatigue strength evaluations [ref. 32]. The most notable method is a two parameter method called the rainflow method, which will be described here.

The purpose of any cycle counting method is to extract the significant damaging effects of a irregular load/stress/strain history which can be compared to the constant amplitude material fatigue characterization data. The concept of a cycle in an irregular history is difficult to define. However, a reversal can be determined as a swap in the sign of the local slope of the history trace. A constant amplitude sinusoid has two reversals per cycle. The apparent reason for the superior performance of the rainflow technique is that it combines load reversals in a manner that defines a cycle as a closed hysteresis loop

[ref. 16]. Each closed hysteresis loop captures the range and mean of the load/stress/strain. These parameters can be directly compared to the material characterization data and the cumulative damage assessed by the Miner's Rule. The rainflow method takes its name from imagining rain falling from a pagoda roof. This method counts the hysteresis loops, as shown in Figure 8-4 [ref. 32], by localizing a *period* as the time for the history to return to a reference value (usually the mean) with the same slope sign (see Figure 8-5 and compare slope signs for segments 0-1 and 8-9).

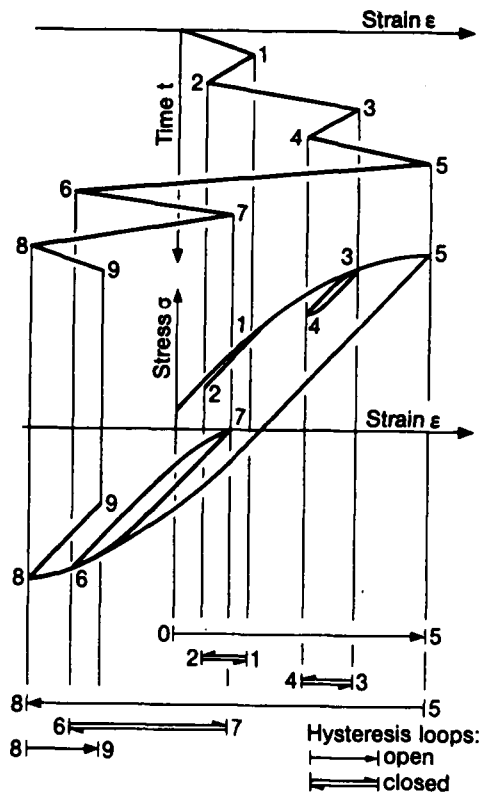


Figure 8-5. The Rainflow Hysteresis Loop Counting Method.

This method can be programmed for automatic spectrum cycle counting in near real time, and a simple C code for the rainflow technique may be found in Appendix A.

The rainflow approach does well at determining the cycles at a given stress level, however, the sequence of occurrence is lost. To preserve the sequence of over/under-loads while smoothing the spectra to reduce the peculiarities without reducing the significant reversals, the racetrack approach is commonly used. This technique basically copies the actual spectrum and displaces it equally plus and minus about the original. It is then imagined that a race car is driving between the displaced spectra. The smoothing then is dependent upon the width of the displacement (shown below as a threshold for smoothing, R_{th}) such that the *race driver* can negotiate the turns with minimal braking (see Figure 8-6).

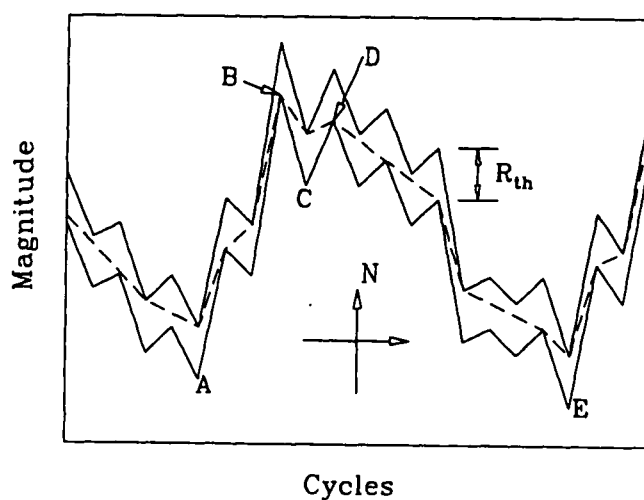


Figure 8-6. Racetrack Approach to Spectrum Smoothing.

The peaks and valley excursions for the smoothed spectrum are then captured to a file in the exact order of occurrence for use as the load spectrum.

Fretting/Corrosion

When two components are in contact and a tangential oscillatory motion ensues, then pitting and scaring of the contact surfaces results. This surface deformation is caused by abrasion based upon the surface roughness, debris present in the contact zone [ref. 33], and microscopic welding due to the heat generated. Waterhouse [ref. 34] provides the most complete description and definitions of the terms associated with this phenomena:

Fretting is the small amplitude oscillatory movement which may occur between contacting surfaces that are usually nominally at rest. One of the immediate consequences of the process in normal atmospheric conditions is the production of oxide debris; hence the term 'fretting wear' or 'fretting corrosion' is applied to the phenomenon. Usually, the movement is the result of external vibration, but in many cases it is the consequence of one of the members of the contact being subjected to a cyclic stress, i.e. fatigue, which gives rise to another and usually more damaging aspect of fretting, namely the early initiation of fatigue cracks. This is termed 'fretting fatigue' or 'contact fatigue'. Fatigue cracks may also be initiated in the absence of an externally applied cyclic stress where the contacting surfaces are under a very heavy normal load or where there is a static tensile stress in one of the surfaces. There are cases where the movement is not simply tangential but is complicated by the normal force also oscillating, to the extent that the surfaces lose contact in each cycle. This leads to a hammering effect which is termed 'impact fatigue'. In this case, the phase relationship between the two motions may be an important factor.

It is unclear whether fretting as a damage mechanism results in a unique type of damage, or that the fretting process merely generates damage sites [ref. 35] which would fail similarly even if the fretting was removed immediately after initiating the damage, but the stress field remained.

Fretting is usually accounted for through a Goodman diagram approach where the fatigue strength follows a reduced curve for fretting as opposed to non-fretting. Typically, fretting reduces the strength of the material by about one third of the non-fretting case. Figure 8-7 illustrates the reduction in fatigue strength due to fretting for Ti-6Al-4V titanium. The upper plot of Figure 8-7 shows the strength of Ti-6Al-4V for fretting and non-fretting cases, while the lower plot shows that the pressure effect was minimal for the values tested.

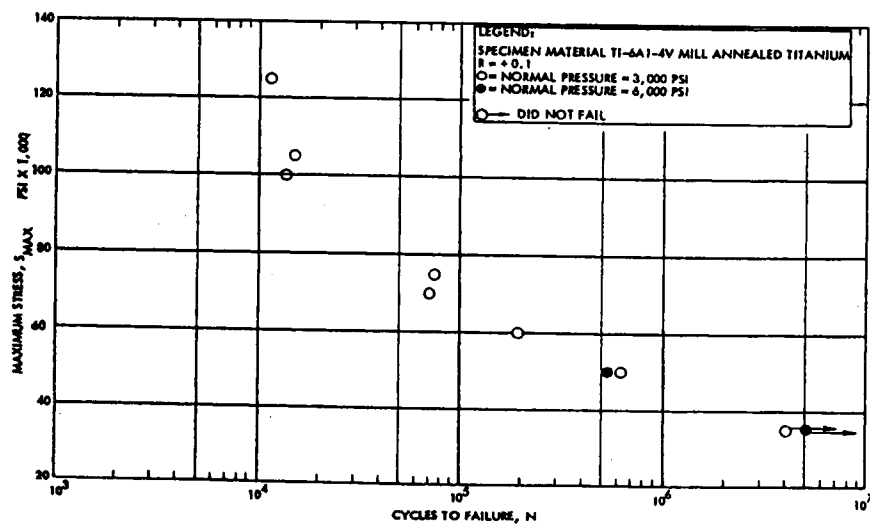
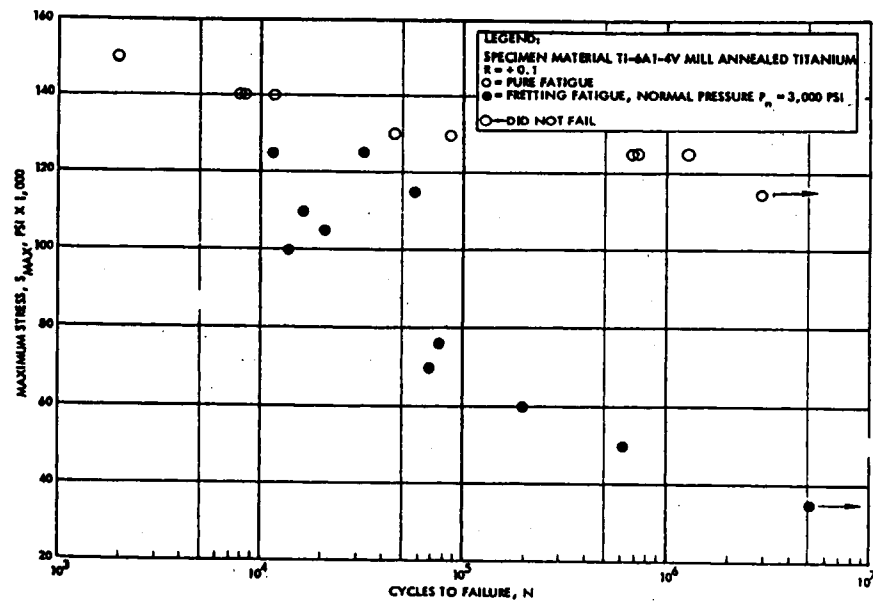


Figure 8-7. S-N Curve Reduction due to Fretting for Ti-6Al-4V [ref. 35].

However, recent research has shed further light on the fretting process. Faanes and Fernando [ref. 36] have shown that the higher fatigue crack growth rate near the fretting zone can be satisfactorily predicted by taking the bulk and contact loading into account (see Figure 8-8). Notice how the rapid short crack growth in this figure appears to be the mean of the short crack phenomenon depicted in Figure 3-14.

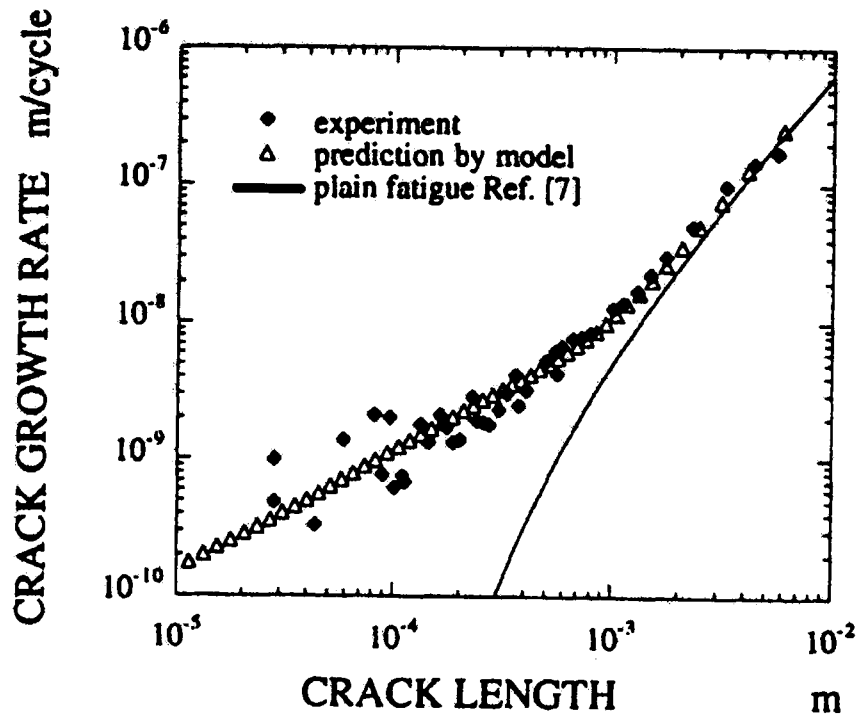


Figure 8-8. Typical Fretting Fatigue Crack Growth Rate

Adibnazari and Hoeppner [ref. 37] showed that there exists a fretting fatigue threshold such that while fretting causes damage within the first cycles, this damage is not irreparable. This means that fretting leads to an early failure only if a generated damage site produces a nucleating crack (see Figure 8-9). While most engineering metals exhibit this threshold for some conditions, it is clearly a function of material and normal pressure. However, more importantly, there exists a normal pressure threshold also [ref.

37], where further increases in normal pressure have no effect on the total life of the component (see Figure 8-10).

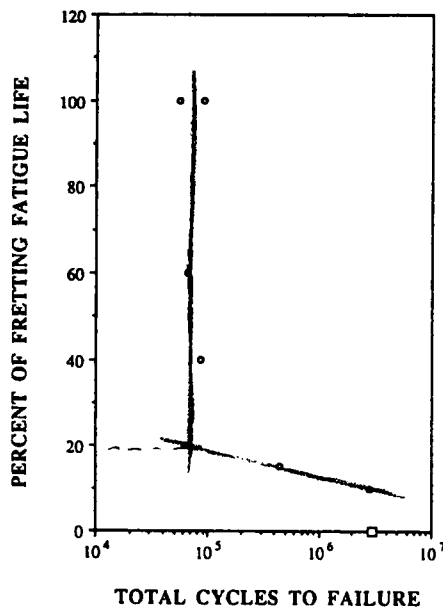


Figure 8-9. Fretting Fatigue Threshold

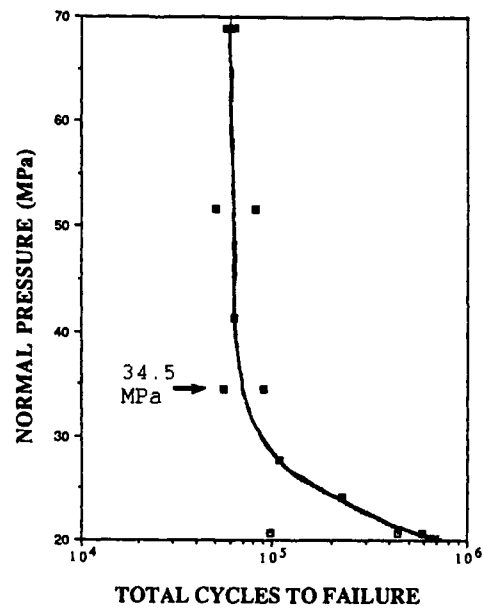


Figure 8-10. Normal Pressure Threshold

The above results are significant for turbo-machinery in that it provides a mechanism to reduce the effects of fretting. First, if parts (disk and blade) can be 'locked-up' quickly after engine start, then this could approach an interrupted fretting situation and the fretting damage may not lead to a propagating crack. Second, if the normal pressure can be maintained below a threshold (lightened blade to reduce centrifugal load), the early failure may be controlled.

Fretting and corrosion appear to be closely related. Corrosion and the fretting processes are similar in the fact that the surface is pitted and micro-particles are present which can affect the surface from a wear viewpoint. The failure mechanism is also similar in that the small cracks initiate at the pits and other such stress concentration areas. Finally, both are surface processes which generate and promote damage sites. In the case of fretting, this damage tends to grow into the metal along shear sensitive planes.

Corrosion is also more likely to occur at the contact points between two components, with cleavage cracking resulting from the tri-axiality of the stress state below the contact zone.

Standardized Load Spectra

To properly assess the life of a component experimentally, it is necessary to subject the component or coupon representative of the component to the same load and environmental conditions expected to be experienced by the component in service. This statement is probably one of the most obvious, but it is also a difficult thing to accomplish in practice. Many times only the most gross conditions are known, and other times the conditions change as the mission or purpose of the system for which the component is a part of changes. Yet it is known that load sequence, frequency, maximum amplitude and mean, and phasing have a significant effect on fatigue life of most engineering materials. Once one abandons (or at least wants to develop a more realistic correlation to) constant-amplitude fatigue tests (which is completely defined by merely two parameters, maximum stress amplitude and mean stress amplitude), then an infinite load spectra possibilities emerge. Consequently, to address the mission effects in a consistent manner such that material/design choices may be made, it is necessary to develop standardized load sequences applicable to the type of system, component, and mission the component is to perform in. Basically, this is a fatigue similarity requirement for testing, since it is not practical to test every component to exact mission conditions prior to fielding the system.

The basis of a meaningful standardized load-time history are stress, strain, and load measurements for the component in question from actual service, suitably averaged to minimize the effects of a single peculiarity of an individual component or test. From multiple time-histories, the average constitutes the common features or similitude of the loading and its effect on the stress and strain fields time histories from which a lifing assessment can be made. For systems which have multiple mission profiles, several such

averages may have to be merged to arrive at a representative whole-life spectrum. In addition, the averages must include multiple vendor or firms and laboratory test data sets to ensure adequate comparisons are possible. Several standardized spectra are available as listed in Table 8-2, all of which are European!

Table 8-2 Standard Load Sequences [ref. Potter & Watanabe]	
Standardized Load History	Application
Twist	Transport Aircraft Wing Standard
Gauss	Gaussian Sequence
FALSTAFF	Fatigue loading standard for fighter aircraft
Felix	Helicopter fixed rotor loading standard
Helix	Helicopter hinged rotor loading standard
Cold TURBISTAN	Cold compressor disk loading standard for fighter engines
Hot TURBISTAN	Hot compressor and turbine disk loading standard for fighter engines
WASH	Wave Action Standard History, off-shore structures loading standard
Walz	Steel-Mill drive loading standard
Wisper	Wind Turbine Spectrum Reference, wind turbine loading standard
ENSTAFF	Environmental conditions added to FALSTAFF
Carlos	Car component loading standard

It is noteworthy that no standardized load spectra has been developed for the internal components of a turbine engine, which may be affected by other loading conditions not captured in a standardized spectrum like TURBISTAN. For instance, the distortion in the inlet or exhaust may have an affect on the fatigue of fan/compressor or exhaust nozzle components, which is not accounted for in TURBISTAN. While it is desirable to not have these distortions, the practicality is that they do exist and therefore must be accounted for in the load spectrum.

9. TURBINE ENGINE TEST and EVALUATION APPROACHES to FATIGUE

To have confidence that a component will not fail in service, testing must be performed. The testing of complicated components like a turbine engine requires some novel approaches to acquisition and analysis of the data. Ideally, one would want to measure the complete strain field in critical high stress gradient regions. However, this is generally impractical if not impossible. Even if one could get the large amount of data channels through a slipring, it is doubtful that these measurements could be made without affecting the field of interest and the modes of oscillation with the over-abundance of strain gages. It is hard enough to accomplish this for a flat plate on a shaker table, let alone the radically shaped rotating blades within a turbine engine. However, the parameters that are typically monitored to make an attempt at verifying the life of the components are:

- Test cell pressures, temperatures, flow rates, and distortion index (i.e., the operating environment)
- Engine power lever angle (throttle position)
- Revolutions Per Minute (RPM)
- Inlet and Interstage Guide Vane (IGV) position and scheduling
- Strain gage outputs as strains (preferably rosettes)
- Non-interference Stress Measurement System (NSMS^{*}) outputs as component displacement and stage nodal pattern
- Material characteristics (if changing due to load or environment)

These parameters are then combined to provide a frequency based stress analysis with engine order tracking assessments to determine where and under what operating

^{*} A time-of-arrival based laser sensor system, which provides blade deflection data derived from a model of where the blade monitoring point should be versus where it is.

conditions a potential may exist for structural/fatigue failure and to assess possible causes for the forcing function.

Typically, the potential for fatigue failure occurs when the natural frequency of the component (blade) is excited by some forcing function. This can be observed on a Campbell diagram as a blossoming of the signal as the engine order line crosses the natural frequency of the component for integral vibrations (see Figure 9-1). In this case the engine order line gives an indication of what may be causing the resonance (e.g., the n th EO-line excitation resulting from wakes of the n -inlet guide vanes). Another example is that of non-integral vibration (such as buffeting at constant frequency) which blossoms at engine order line crossings due to becoming pumped by the forcing function synchronously with the component's natural frequency (see Figure 9-2).

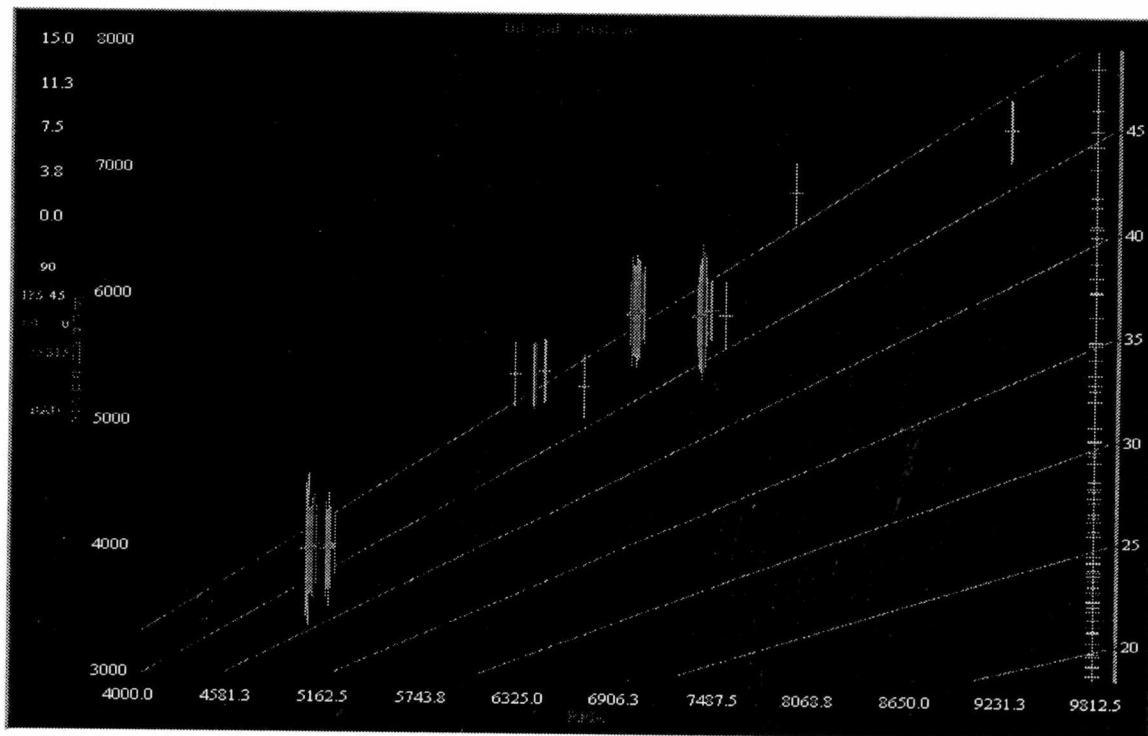


Figure 9-1. Campbell Diagram for Integral Vibration.

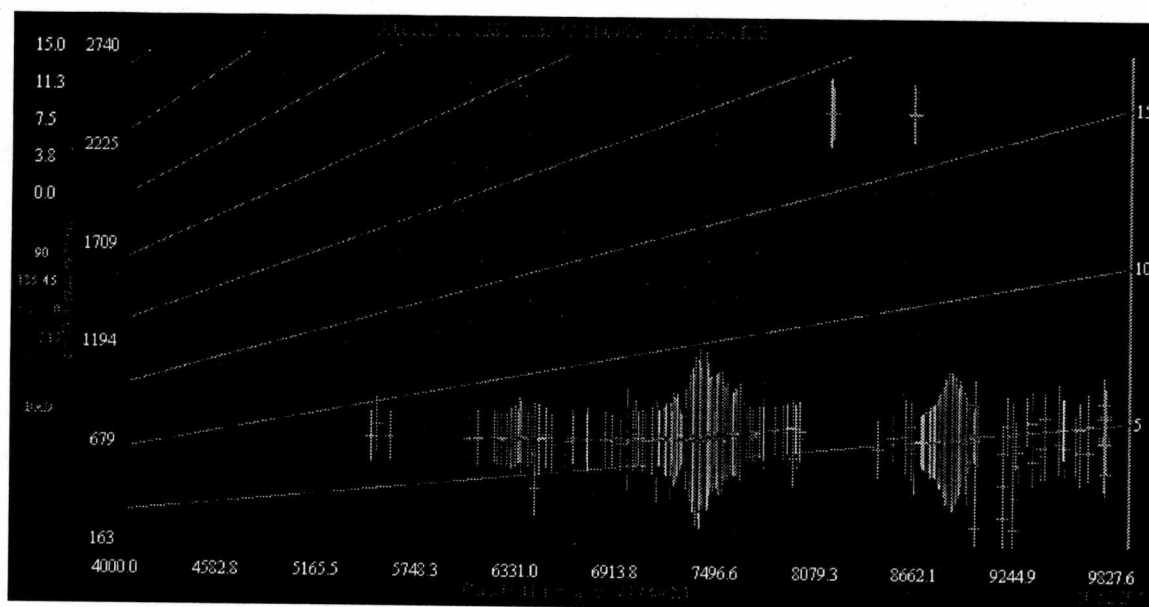


Figure 9-2. Campbell Diagram for Non-Integral Vibration.

Strain gage placement is critical if true stress maximums are to be accurately determined during testing. A strain gage placement optimization routine (BladeOPS) has been developed [ref. 38] to maximize the sensitivity of the gages for critical locations for the maximum number of modes. This routine should be used for all phases of prototype and system testing so comparisons as a function of test approach (static, dynamic, installed, etc.) can be made accurately.

Even with BladeOPS, seldom will it be that the strain gages are precisely located at the points of maximum stress for all modes of vibration. Consequently, a component model must be available which can correlate the measured stresses at some locations to the stresses in critical but non-instrumented areas. Techniques have been developed to tune a finite element method component model based upon known (test data) displacements and/or strains [ref. 39]. Coupling this capability with high-speed data acquisition and display systems and with the fatigue criteria described herein will allow on-line near-real-time fatigue potential assessment and trending during engine operation.

With these tools to properly place and monitor stress/strain fields during testing, excluding material and manufacturing variability, the final source of component life assessment variability can be tackled. This is the test itself, and it is at the heart of the holistic approach. If the engine was operated according to a standardized loading sequence comparable to that used to characterize the material, then the capability to predict life of similar components as the instrumented ones from previous testing becomes possible. Hence the database developed during testing would allow trade-off studies for design improvements or fixes when problems occur in the field.

Therefore, the above Test and Evaluation requirements, and the conduct of the test itself, demand a test protocol which allows the integration of material characterization data, test rig prototype evaluation data, and system test and evaluation data for the purpose of determining the component life in the most accurate manner. Figure 9-3 illustrates such a design scenario, which capitalizes on information provided by testing through the use of a test protocol. Notice that the test protocol becomes a consideration at the component and system levels, and is dependent upon design and testing philosophies which may be proprietary to the manufacturer (i.e., the component's reliability becomes a marketing advantage, consequently the techniques used to arrive at this reliability throughout the component's development become trade secrets). In addition, the test protocol defines the instrumentation requirements, and how the analyses will be performed to assess the structural integrity of the initial component or for a later modification to the component for structural or performance reasons. For an independent assessment of the structural integrity by some outside agency, the test protocol must include the data necessary for manufacturer's and independent agency's assessment criteria.

Generic Design Process

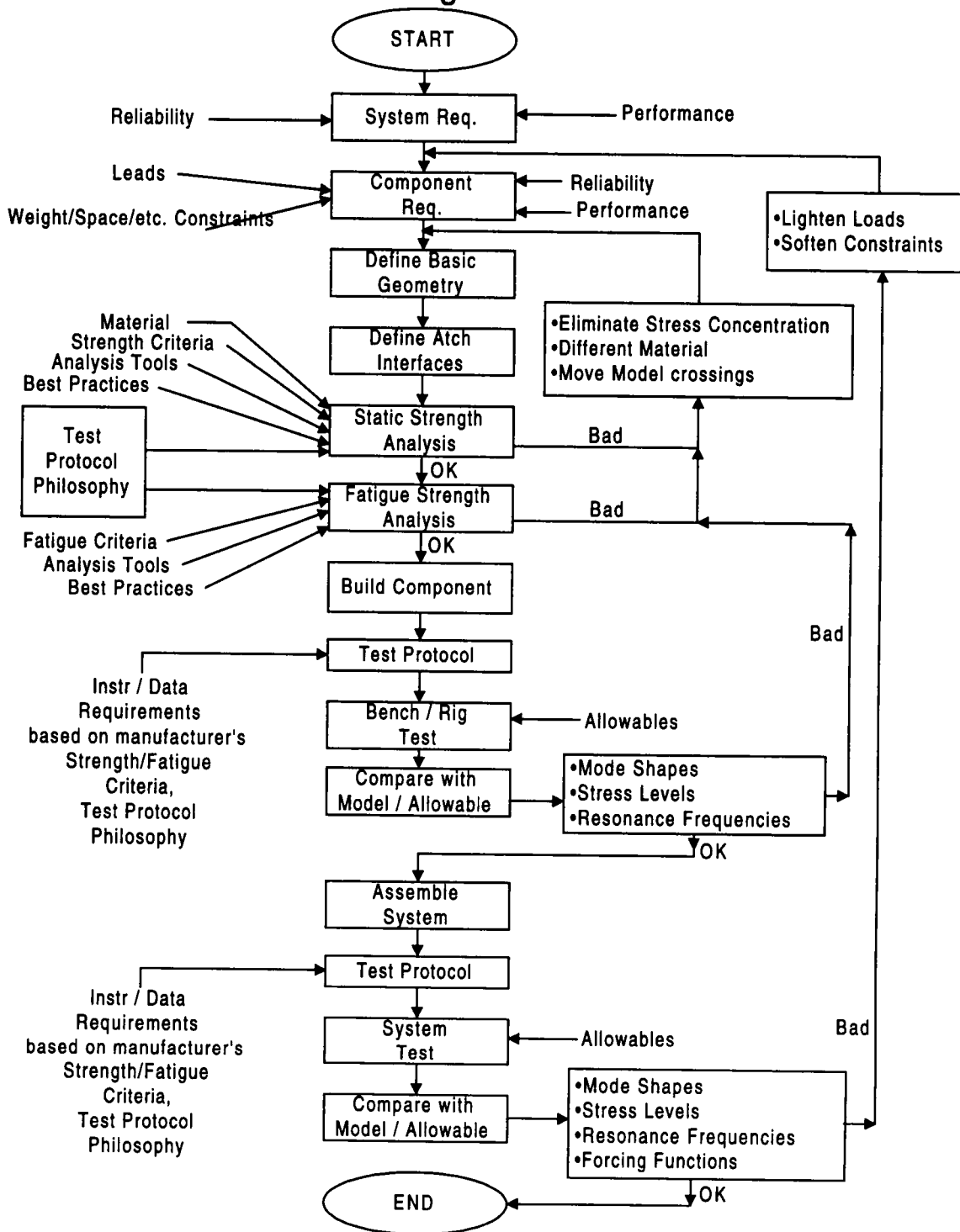


Figure 9-3. Generic Design Process

The major components of a test protocol are:

- Standardized Terminology
- Amalgamation of structural integrity criteria/best practices (a standard here is probably not realizable since technology isn't mature enough)
- Standardized data acquisition schemes
- Standardized operational envelope evaluation methodology (operational sensitivity assessment criteria)
- Standardized load/mission profiles to capture operational sensitivities and variability
- Standardized inlet distortion sensitivity assessment
- Standardized data processing methodology and processed data output formats
- Standardized statistical approaches to system, component, operation, and test variability
- Standardized simulation model input/output formats
- Systematic roadmap for improvement of structural integrity assessment through T&E integrated with design.

10. SUMMARY

Fatigue of materials has been an area of study since the mid 1800s. The initial work generated material characterizations where fatigue or endurance limits were defined in terms of cycles of symmetrical alternating stresses. This characterization, however was not solely material dependent, but also included influences of material condition, presence of voids/discontinuities in microstructure, loading rate, mean load, specimen geometry and size, temperature, environment, etc. Since these early material characterizations, refinements have been attempted, with limited success, to remove those dependencies such that pure material properties and analysis methodologies can be established to aid the designer without the need for expensive test/design iterations. The Goodman diagram is a result of accounting for mean stress, while Miner's rule is an attempt to account for non-steady, non-symmetric, quasi-periodic loading spectra. The plethora of stress concentration factors is an attempt to correct for geometry, surface condition, and notch sensitivity of the materials. Up to this point, the microstructure was ignored since the materials were assumed to be continuous, homogeneous, isotropic, and defect free, with stresses/strains distributed continuously. These approaches worked best for moderate loadings and high number of cycles, but both ends of the load-cycle spectrum (i.e., LCF and HCF) were not adequately modeled, nor was conservatism assured.

As materials and crack propagation could be inspected more closely and critically due to advances in instrumentation, it became obvious that the fatigue/fracture theories were lacking since the failure mechanism of crack propagation was not included in the theories. This gave birth to the field of fracture mechanics, where it was assumed that all materials contained voids/discontinuities, whether considered defects or not, which propagated into cracks prior to failure. Linear Elastic Fracture Mechanics (LEFM), however, is energy and elasticity based, and therefore continues to make the same

restrictive material assumptions, but now only in the region ahead of the crack. Microcracks are still ignored, as well as the process of crack initiation/nucleation. The Fracture Mechanics stress intensity factor, coupled with a given crack size, is used to characterize materials that fail in a brittle fashion, based on a Paris-type correlation of crack growth. However, the stress intensity factor is still loading, load rate, and crack orientation dependent, and therefore the material characterization includes influences which make it difficult for the designer to apply globally in situations not approximate to the way the material was characterized. In addition, the crack size must be known or assumed and must grow in a self-similar manner for fracture mechanics to be valid. LEFM is most applicable to LCF where crack growth is more pronounced due to the high stresses.

For ductile materials, a large amount of plastic deformation takes place in the neighborhood of the crack tip, and therefore LEFM is augmented with plasticity theory (or at least non-linear elasticity theory) to account for this plastic deformation. Elastic-Plastic Fracture Mechanics (EPFM) characterizes the material by assuming that the strain field is uniquely described by the J contour integral and that the crack tip opening displacement (CTOD) parameters adequately define the potential for the crack to propagate. However, here too problems arise since the J -integral is only path independent for a perfectly elastic material, and is valid only for planar stress/strain field conditions since the J -integral is a planar concept which ignores the dilatation aspects that are pronounced in ductile materials. In addition, the crack direction and size must be known apriori for the approach to be useful. However, even with all its problems, Fracture Mechanics made significant contributions by establishing a theoretically based methodology for defining inspection intervals, allowing the *fearless* use of *damaged* components to reduce operational and manufacturing costs, and is much more successful in defining *fracture toughness* as a material characterization. Fracture Mechanics has essentially solved the LCF problem, but short cracks have not been properly addressed.

Fracture Mechanics with short crack enhancements, such as those of McDowell, and critical plane approaches based upon a more classical Strength of Materials approach, such as the Socie-Parameter, basically, constitute the state-of-the-art (SOA) today for structural design material characterization and analysis. To overcome the limitations of Fracture Mechanics with the hope of more pure material characterizations independent of the host of influences discussed above, a variety of new fatigue/fracture theories have been proposed. These new theories are still too young to know if their final formulation will be successful, but their different approaches can only enhance the rich foundations of Strength of Materials and Fracture Mechanics upon which they build. The primary theories in this category are the strain energy density (SED) criterion of failure and the continuum damage mechanics (CDM) approaches. Both of these approaches recognize a fundamental damage assessment approach is necessary to remove the dependencies of the material characterization. The SED approach appears to be readily applicable since the material characterization required is derivable from the classical Strength of Materials data and the finite element programs provide strain energy assessment capabilities. The CDM approach is fundamentally a new look at the fatigue problem. Before this approach can become viable, the structural analysis programs will have to accommodate it and material characterizations must be performed to assess the basic damage of engineering materials. While both of these approaches still need work, they provide some valuable insight into the mechanisms of fatigue damage assessment.

Much work remains before a truly suitable fatigue theory will be available which is capable of predicting component life under all the realistic conditions of multi-axial loading, fretting, small cracks, natural and processed induced anisotropy, non-homogeneity, etc. The need to model the micro-structural effects on a macro-physical basis such that design engineers won't have to be material scientists is a loft goal. Figure 3-12 illustrates the complexity of trying to define the toughness/strength of a material which includes the micro-structural effects on crack propagation.

11. CONCLUSIONS and RECOMMENDATIONS

Conclusions

1. The complexity of the fatigue process makes it very difficult to develop a theory of fatigue and crack growth based upon experimental correlations alone, and attention to the microstructure is necessary. However, when microstructure is included explicitly, the requirement of knowledge of the microstructure in the area of high stress intensity of a processed component makes such formulation useless to the general structural design engineer. There may be hope if the microstructure can be quantified more generally on a macroscale based upon statistical approaches, but such characterization will undoubtedly be expensive to cover all loading and geometrical conditions superimposed on the crystallographic plane orientations. The critical plane approaches have identified a parameter which more closely accounts for such conditions, but complete material characterization based upon this parameter is lacking.
2. Based upon immediate applicability on a global scale, the holistic approach is the obvious choice. The combination of S-N/Goodman for crack initiation coupled with fracture mechanics for damage tolerance provides a complete assessment. However, since the predictive capability is limited, it is felt that this is an interim solution only. The attraction of the holistic approach is that it correlates design parameters *directly*; geometry, load, crack length, etc. The disadvantage is that the dependency on crack length in the short crack regime makes the holistic approach expensive since the total life assessment requires twice the material characterization testing to correlate the short and long crack regions.
3. The most practical and theoretically consistent fatigue theory appears to be the strain energy density (SED) theory. Since it can use existing material data and existing finite element method (FEM) codes can provide strain energy plots, this theory could

be readily applied now. However, the theory has problems in the short crack region for multi-axial load conditions.

4. The most all encompassing theoretical formulation identified is that of continuum damage mechanics (CDM). This theory can also use existing material data, however CDM has not been incorporated into existing structural FEM codes, and therefore general application is further in the future. In addition, short crack growth is a concern. It would be interesting to merge the continuum damage mechanics approach with the critical plane approaches. Such a theory would recognize the obvious that all materials are damaged, yet may allow better damage characterization on a global level for use by the structural engineer.
5. The critical plane approach of McDowell, et.al., is the most promising approach for combining fracture mechanics and stress/strain-life into a predictive tool for the whole life of the component. This is also most promising for fretting situations, where short crack growth dominates. An advantage of this technique is its addressing of the micro-structure with simple correlation parameters. A disadvantage is that the required material characterization data is lacking.
6. There appears to be a need for a standardized load spectra for engine components which may see more operational load conditions than accounted for in TURBISTAN and other load sequence standards.

Recommendations

1. To test the various theories discussed herein under common test conditions with common material lots, it is recommended that various specimens covering the whole range from ductile to brittle engineering materials be evaluated for fatigue response over both the small and long crack regimes for uni-axial, multi-axial (tension - torsion), and fretting load conditions. It is especially important to include titanium as

one of the materials, and induced cracks should span a variety of situations relative to their alignment to the principal axes.

2. The holistic approach and any approach to be used by a responsible structural engineer requires that the load sequence be known. A methodology has not been identified which explicitly guides one to determination of the load sequence to be used in material characterization from an anticipated mission profile, which will allow comparison with existing material characterization data that includes overload cycles which can induce hardening. Along similar lines, non-linear damage accumulation laws need to be identified for use where the Miner's Rule doesn't hold (e.g., HCF).
3. The controlling parameters for fatigue based on the discussed theories should be used in an on-line monitoring system to assess the potential for fatigue and the consistency in identifying potential fatigue problem areas during upcoming engine tests. The load sequences for a complete test program should be compared to a typical mission profile to evaluate the effectiveness of the load sequence for HCF and LCF potential.
4. A standardized load sequence for internal engine components which includes inlet and exhaust flow effects should be developed such that a standard data base can be established to capture lessons learned through an engine development program for use on the next engine program.

APPENDIX A

Rainflow Computer Program

```

/*      This program's name is RAINFLOW.c
This program calculates the effective number of cycles and their stress
levels and periods for real (non-sinusoidal) data files.    03/19/97
*/

#include <graph.h>
#include <stdlib.h>
#include <stdio.h>
#include <math.h>
#include <conio.h>

#define NO_OUT
#undef TEST

char alpha[] = {'9', 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I'};
char c[50];
int n_points;
double data[2][50];
char fn_in[_MAX_PATH], fn_out[_MAX_PATH]; /*      Allow room for      */
char drive[_MAX_DRIVE], dir[_MAX_DIR];    /* drive:\dir\subdir\filename.ext */
char fname[_MAX_FNAME], ext[_MAX_EXT];
FILE *fp_in, *fp_out;

void file_handler( int state );
int get_data( int n );
void find_period( void );
void rainflow( void );

/* ----- */
void file_handler( int state )
{
    if( state == 0 )
        _clearscreen( _GCLEARSCREEN );
    if( state == 0 )
    {
        printf( "\n\nRetrieving Files...\n\n" );
        printf( "Input Data File_Name: " );
        scanf( "%s", fn_in );
        fp_in = fopen( fn_in, "rt" );

/* Check for proper INPUT file OPENING or Check for non-existent INPUT file;
EXIT if 'fopen' failed. */

        if ( fp_in == NULL )
        {
            printf ( "No INPUT file %s found\n", fn_in );
            exit ( 9 );
        }
    }

    #if !defined( NO_OUT )
        if( state == 1 )
        {

```

```

        printf( "\n\nInput File_Name for STORAGE: " );
        scanf( "%s", fn_out );
        fp_out = fopen( fn_out, "wt" );

/* Check for proper OUTPUT file OPENING; EXIT if 'fopen' failed. */

        if ( fp_out == NULL )
        {
                printf ( "CANNOT OPEN OUTPUT file %s\n", fn_out );
                exit ( 9 );
        }
}
#endif

        if( state == 9 )
        {
                printf( "\n\nClosing Files...\n\n" );
                /* fflush( fp_out ); */
                fclose( fp_in );
                /* fclose( fp_out ); */
        }

        return;
}

/* ----- */
int get_data( int n )
{
        static int i = 1, j;
        int k;

#if !defined( TEST )
        fscanf( fp_in, "%lf %lf", &data[0][n], &data[1][n] );
#else
        fscanf( fp_in, "%lf %lf", &data[0][n], &data[1][n] );
        c[n] = alpha[i];
        i++;
#endif
        if( feof( fp_in ) != 0 )
                k = 1;
        else
                k = 0;

        return( k );
}

/* ----- */
void find_period( void )
{
        return;
}

```

```

/* ----- */
void rainflow( void ) /* Ref: Bannantine, Comer, Handrock;
                        "Fundamentals of Metal Fatigue Analysis, Printice Hall, 1990 */
{
    int i=1, j, k, n;
    double x, xmean, y, range;

    n = 0;
one:   n = n + 1;
    k = get_data( n );
    if( k == 1 )
    {
        printf( "\n\n REACHED EOF(%s)\n\n", fn_in );
        fclose( fp_in );
    }
#ifdef TEST
    for( j = 0; j < 10; j++ )
        printf( "%d ", j );
    printf( "\n" );
    for( j = 0; j < 10; j++ )
        printf( "%c ", alpha[j] );
#endif
    exit( 99 );
}
two:   if( n < 3 )
        goto one;
    x = fabs( data[0][n] - data[0][n-1] );
    y = fabs( data[0][n-1] - data[0][n-2] );
#ifdef TEST
    printf( "n = %d  X = %c - %c  Y = %c - %c\n", n, c[n], c[n-1], c[n-1], c[n-2] );
#endif
three: if( x < y )
        goto one;
four:  range = y;
    xmean = ( data[0][n-1] - data[0][n-2] ) / 2.0;
#ifdef TEST
    printf( "i = %4d  Xmean = %10.3lf  Range = %10.3lf  [%6.2lf - %6.2lf]\n",
            i++, xmean, range, data[1][n-2], data[1][n-1] );
#else
    printf( "i = %4d  Xmean = %10.3lf  Range = %10.3lf  [%6.2lf - %6.2lf]  [%c - %c]\n",
            i++, xmean, range, data[1][n-2], data[1][n-1], c[n-2], c[n-1] );
#endif
#ifdef TEST
    getch();
    n = n - 2;
    data[0][n] = data[0][n+2]; /* AMPLITUDE */
    data[1][n] = data[1][n+2]; /* TIME */
    c[n] = c[n+2];
    goto two;
#endif

    return;
}

/* ----- */

```

```
void main( void )
{
    file_handler( 0 ); /* OPEN Input File */
    find_period();
    rainflow();
    file_handler( 9 ); /* CLOSE All Files */
    exit( 0 );
}
```

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REFERENCES

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VITA

Thomas Floyd Tibbals was born August 10, 1950 in the small northern Michigan town of Alpena on the beautiful shores of Lake Huron. He attended parochial elementary and high schools, and was more interested in all things mechanical rather than school. He graduated from Catholic Central High School in June 1968, and was not interested in a four year college, nor was he college material at that point. His interest in jet engine mechanics was squashed when the program was not offered at the local community college. That November he entered the United States Air Force where he received training in aircraft hydraulic and pneumatic systems and later was placed in charge of all in-shop repair. Following a tour of duty in Thailand in support of the Viet Nam War, he was released from active duty as an E-5 in time to attend Alpena Community College in August 1972, with a strong ambition to become an engineer due to the exposure to complex aircraft systems.

Due to economic conditions, he moved to Cleveland, Ohio where he obtained a job as a hydraulic and pneumatic component assembler in order to establish residency for entrance into Cuyahoga Community College in September 1973. He received an Associate of Arts degree and entered Cleveland State University in September 1975 while continuing to work for the same small firm as a hydraulic and pneumatic component designer. In June 1977 he graduated Cum Laude with an Engineering Science/Physics degree, and in August 1977 was also awarded the Bachelor of Science degree in Mathematics.

In May 1978 he entered the United States Air Force Officer Training School, and was commissioned in August 1978. He was stationed at Edwards AFB, California as a flight test engineer involved in evaluating hydraulic systems, icing characteristics, and arresting hook dynamics. In August 1982, he was released from active duty and accepted the position of Chief Engineer and General Manager of a small hydraulic and pneumatic controls design firm in Cleveland, Ohio. In December 1986, he accepted his current position as a Senior Engineer with Sverdrup Technology, Inc. (AEDC Group) in the Propulsion Testing Technology section at Arnold Engineering Development Center near Tullahoma, Tennessee. He is also Major in the USAF Reserves.

The above details are presented to illustrate that one can still become an engineer even if his high school grades are not appropriate, and perhaps the better engineers are lost in our school system because we cater to those who would make it anyway, the academically gifted, even if their engineering insight is lacking, while leaving the shop students unchallenged. We must stop regarding shop courses as for those who have failed academically, and challenge these individuals to excell in their interests. Einstein stated that imagination was more important than knowledge, and in many cases, a skilled laborer

will have the imagination to make something work when the engineer's knowledge has failed to solve the problem. My father (a welder by trade) and my grandfather (a tool and die maker by trade) demonstrated this to me several times. Our schools do not recognize the imagination needed to create an item from raw materials or fix an item to work when the engineer's design is not adequate. Our politicians need not wonder why America is losing its manufacturing capability, it is due to the lack of respect for the skilled laborer and the enormous emphasis on professional careers. I wonder what good an engineer is if there is no machinist or welder or other craft or technician to build and keep running what he designed! For all those skilled laborers who encouraged me and assisted me with my bicycle, go-kart, and car modifications as a young tinkerer, you will always have my respect, THANKS!