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I am submitting herewith a dissertation written by John Dirk Richards entitled "Study of Shape Coexistence in Nuclei by Alpha Decay Fine Structure Studies." I have examined the final copy of this dissertation for form and content and recommended that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Physics.

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Study of Shape Coexistence in Nuclei by Alpha Decay Fine Structure Studies

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Presented for the

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John Dirk Richards

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**This dissertation
is
dedicated
to my wife
Susan**

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ABSTRACT

Alpha-decay fine structure, occurring in the light-Pb region, has possibilities to serve as a probe to study the phenomenon of shape-coexistence. The α -decay of mass separated ^{190}Pb was studied at the UNISOR on-line facility and fine structure was observed. Hindrance factors (HF values), which determine the degree at which nuclear structure inhibits feeding to excited states, are calculated for the corresponding fine structure relative intensities to the 2_1^+ and 0_2^+ states. When compared to HF values obtained from α -decay studies of neighboring light-Pb region nuclei, an erratic systematics is observed. Although the energies of the α fed excited levels also change in a random fashion, no apparent systematics of level energies and HF seems evident, which is in contrast to associated systematics observed in the actinide region.

In this work it is sought to demonstrate the importance of shape-coexistence for understanding the trends of HF values in the light-Pb region. It has long been accepted that the nuclear process of α -decay can be treated as two separate processes, that of formation and that of barrier penetration. The model of α -decay used in this work has been proposed in the literature as a means to incorporate the effects of collectivity in the daughter nucleus, in the part of the α decay process commonly associated with the process of barrier penetration. In this prescription one is freed from having to study the formation process in order to model α -decay relative intensities. Such a prescription might serve as a probe for extracting physical information concerning mixing deformed configurations (shape coexistence) in the daughter nucleus in future studies.

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CHAPTER 1

Introduction

1.1 Shape coexistence and its relationship to α decay

In this work we provide results of experimental α decay spectroscopic data belonging to neutron-deficient Pb region nuclei. All Pb nuclei have a closed proton shell, which causes them to be spherical in their ground states. However when the neutron number N is near 110, which is midway between two magic numbers, low-spin excited states occur in which a proton core is broken. Two protons are excited into the next shell and result in a shape deformation. Furthermore, the excited and the ground configurations with the same angular momentum, or spin, interact with each other, which results in states that are admixtures of the unperturbed configurations. This phenomenon is a manifestation of shape-coexistence [1]. Shape coexistence occurs in a number of nuclear regions near closed shells, and a complete discussion will not be made in this dissertation. A thorough review of the subject, as applied to even mass nuclei can be found by J.L. Wood, *et al.* [1]. This phenomenon as applied to odd-mass nuclei is discussed by K. Heyde, *et al.* [2]. Other regions that exhibit “coexistence” phenomena are light doubly closed-shell nuclei, mid-shell tin, and lanthanide and medium to heavy actinides [1].

Nuclear α -decay connects a parent nucleus to a daughter nucleus with two less protons and two less neutrons. The emitted α particle having a spin of 0 is constituted of a proton pair, coupled to 0 spin, and similarly a neutron pair. The daughter nucleus generally differs from the parent nucleus by a proton pair and

a neutron pair. Transitions between parent and daughter nuclei differing in this way are generally the fastest and are called unhindered transitions. This frequently occurs for ground state to ground state transitions between even-even nuclei. If an excited state cannot be populated by simple removal of a proton pair and neutron pair, but requires in addition some recoupling of the remaining nucleons, the decay to that state will be slower. If they occur at all, the resulting peaks in the spectrum are called fine structure. If the excited state has an admixture of the ground state (g.s.) configuration, this will naturally affect the probability of the α decay to that state. Further discussion of these concepts is contained in [3, 4]. In this work it is our primary objective to investigate this relationship further.

The α decay process has proven throughout the history of the subject to be very difficult to interpret precisely, as well as understand fully. Therefore, an overview of the theory of α decay is discussed in order to clearly establish those parts of the theory we will attempt to relate to the phenomenon of shape-coexistence.

In 1928 the qualitative features of α decay could be reproduced using a semi-classical treatment, whereby a preformed α particle could be described as "tunneling" through a barrier [5]. With the discovery of the neutron it was realized that the α particle was composed of constituent nucleons, and this resulted in more aspects of the problem that needed to be considered. In particular the question of how the α particle formed began to be addressed in conjunction with the problem of how it was able to penetrate a nuclear barrier. The solution to the formation problem is much more difficult and has for the most part remained an unsolved problem. The phenomenon of α decay is a many-body problem, and is still not exactly solvable, even by modern computing techniques. In addition, it was not originally thought that the process of formation and the process of release were problems that could be justifiably solved as separate problems. This resulted in many early theoretical attempts, where the α particle was seen to be simultaneously formed and ejected,

such that these two processes were inextricably linked in the decay process [6].

In 1954 Winslow introduced the concept of the *preformation factor* in connection with his "surface well potential" [7]. In the surface well the α -particle exists for a short time, as compared to the time it takes for its formation, during which it performs one-body motion, immediately prior to and during barrier penetration. In 1959 Rasmussen [8] solved the one-body decay problem using the Woods-Saxon (WS) potential [9]. The WS potential had been successful with describing α scattering of nuclei. Here the WKB integrand is given by the expression:

$$P = \frac{1}{T_o} e^{-\int_{R_i}^{R_o} \frac{(2\mu)^{\frac{1}{2}}}{\hbar} \left[V(r) + \frac{2Ze^2}{r} + \frac{\hbar^2}{2\mu r^2} l(l+1) - E \right]^{\frac{1}{2}} dr} \quad (1.1)$$

where R_i and R_o are the inner and outer turning radii, μ is the reduced mass of the α particle, E is the energy of the α -particle, $V(r)$ is the nuclear potential of the α particle, and T_o is the quasiclassical period of motion for the preformed α -particle inside the nucleus. This quantity was used to calculate the penetrability, which was intended to serve as a precise description of the one-body escape process. The escape of the α -particle and the formation process were expressed as two separate processes in the following way:

$$\lambda = P\delta^2/\hbar. \quad (1.2)$$

This could allow the experimentalist to extract the nuclear spectroscopic effects empirically as these were absorbed into the formation factor, δ^2 . To do this the experimental value for the decay constant, λ , is equated to the right side of Eqn. 1.2, while the penetrability P is calculated from WKB theory.

From this point forth theoretical attempts have either been concerned with *i)* furthering our understanding of the effects of the nuclear potential on the preformed α -particle and *ii)* continued efforts at describing the form factor theoretically. Following Rasmussen's result were efforts to include many-body effects. Here the

formation of the α particle was seen to be an overlap of an excited state decay with the time varying parent wavefunction. The parent wavefunction was constructed from single particle shell-model wavefunctions. Some trends in fine structure data were convincingly reproduced [10, 11, 12, 13]. The same conceptual background was applied using a number of different nuclear models to describe the parent state, depending on the nuclear region [14, 15]. Attempts were made to incorporate effects of a deformed potential [16]. A number of these models used the WKB approximation.

In efforts made subsequently with developments using the WKB approximation, α decay was viewed from a reaction formalism. The methodology which was adopted by several authors from the procedure can be found as outlined in Goldberger and Watson [17]. The primary difference between this theory and the WKB theory is in the expression for the penetrability, which in the reaction theory is formulated from purely quantum mechanical principles, whereas in the WKB theory the formulation is semiclassical. Jackson and Rhoades-Brown provided an overview of all efforts involving the reaction theory in their 1977 paper [18]. They carefully outlined how a cluster model is viewed and developed a quantal expression for the decay width. The significance of this paper is that a consistent way of constructing a microscopic potential for the preformed α particle was proposed and carried out.

Further efforts involved more exacting treatment of the microscopic aspects of the decaying parent nucleus. Penetrabilities are either calculated using the WKB or reaction expressions. Recent papers tend to be aimed at calculating formation probabilities. In early work emphasis was placed on methods that extract nuclear configurations likely to contribute to the formation of the α particle [11, 12]. This was continued by incorporation of important nuclear configurations [19], whose resulting interactions are described using first order field theory perturbations [20, 21, 22, 23]. These and further efforts were aimed at viewing the

likelihood for occurrences of α configurations arising from particular single particle nuclear configurations. Sometimes this is referred to as an “ α cluster model” by which the decaying parent state is expanded in terms of wave functions, which easily project out α configurations that feed known α decay daughter configurations, matching those found from spectroscopic studies. Further such efforts emphasized the cluster picture [24, 25, 26].

One important aspect of excited α decay parent and daughter configurations is that when moving away from closed shells their single particle configurations become very difficult to describe. If the nucleus prefers to deform, this further complicates the shell-model picture of the nucleus. Therefore, as the problem of formation is being considered, it is also important to investigate models which represent the nuclear states as deformed, or mixed deformed states where applicable. The earliest such attempt was by Fröman [27] where the influence of an anisotropic barrier on the evaluated expression for the penetration factor was calculated for nuclei exhibiting quadrupole deformations. More recently Berggren formulated the problem of decays involving deformed states in the daughter [28] while tailoring expressions derived using reaction theory [18] in the particle-plus-rotor model. In these approaches the effect of deformation on the penetration of the α particle through the nuclear barrier is investigated. This is opposite to the picture where deformed configurations, while described in the single particle picture, are viewed as influencing the formation probability. More recent papers are intended to describe mixed-deformed or shape-coexistent configurations in a single particle basis with a proton-neutron interaction [29, 4]. Because the formation is calculated, the experimental width is more nearly reproduced. The prescription introduced by Berggren is not as of yet tailored to include formation explicitly. Other work attempting to link the deformation to effects of α decay relative feeding intensities can also be found in the literature [30, 4, 31, 32, 33, 34].

1.2 Outline of the dissertation

The topics to cover are as follows. In Chapter 2 we will present spectroscopic results from a study of the α decay of ^{190}Pb , which revealed fine structure feeding to excited states of ^{186}Hg . An analysis in terms of the degree of nuclear spectroscopic hindrances, or hindrance factors (HF), of decay to these states, as observed through their relative intensities, is performed. In addition a HF analysis of all neighboring decays, which exhibit fine structure relative intensities, is provided to help facilitate further discussions.

In Chapter 3 an in-depth study is carried out to investigate the relation between the phenomenon of shape-coexistence and α decay fine structure, using an α -rotor model to describe the decaying parent state in terms of admixtures of diabatic states in the daughter, which interact with the preformed α particle immediately prior to decay.

Chapter 4 contains results of an experiment that was performed at the European Center for Nuclear Research (CERN). At CERN a study was carried out to investigate the feasibility of performing a spectroscopic study of ^{194}Pb , to search for its superdeformed (SD) bandhead energy. This chapter mostly concentrates on experimental procedure and data analysis. However, the relevance for performing such an experiment to investigate α decays in the region of light Pb nuclei is also discussed.

Chapter 5 contains concluding remarks, as well as suggestions for further studies based on this work.

CHAPTER 2

Observation of fine structure in ^{190}Pb

2.1 Motivation for study of α decay of light Pb region nuclei

Because of considerable attention that has been given to the study of heavy actinides, a large amount of information on their nuclear decay has been made available in the nuclear data sheets. There are many α -decay transitions to excited states of the daughters. In addition, heavy actinides, well beyond the double-magic shell, have level structure that changes relatively smoothly with atomic mass. In particular the ground states of these even-even nuclides are well deformed so that the even parity excited states built upon them appear, very nearly, as classical rotors. On the neutron-deficient side of the $N=126$ magic shell, however, the behavior of the level spacings between the even-parity states of the ground band exhibit a variety of behavior and change more rapidly between neighboring nuclei. Furthermore, with regards to even-even nuclei, coexisting states of different shape and structure are seen to be present, each with low band-head energies [1]. It has been suggested that the probability of α -decay to excited states is affected by the degree of mixing between ground state bands and excited state configurations in the daughter nuclei. In this chapter we present the first observation of fine structure in ^{190}Pb α -decay and then discuss briefly the differing systematics of such transitions on either side of the $N=126$ closed shell.

2.2 Experimental setup

The Pb isotopes were produced by bombarding natural W with beams of 170 MeV ^{16}O from the Holifield Heavy Ion Research Facility 25-MV tandem accelerator. The $A=190$ products were mass-separated utilizing the UNISOR on-line facility and collected onto an automated tape transport system. Since the half-life of ^{190}Pb is 1.2 min [35], the collection spot was moved every 4 min in successive fashion to two counting stations to optimize statistics. At the first station, there were two Ge(Li) detectors to observe γ -rays and two cooled Si(Li) detectors were used to detect both electrons and α particles. A Si surface-barrier α -particle detector and two Ge(Li) γ -ray detectors were positioned at the second station. Multiscaled α and γ singles data and α - γ - t and α - e - t coincidence data were taken at both stations. The detectors were calibrated using an NBS standardized source for γ rays and a ^{207}Bi source for electrons. The α detector was calibrated using a ^{241}Am and an absolutely calibrated ^{244}Pu source.

2.3 Results

Portions of the α -decay spectra obtained in this experiment are shown in Figure 2.1 from the first (a) and second (b) collection station. In our data the 5577-keV ground-state-to-ground-state α transition of ^{190}Pb decayed with a half-life of 71(1) s in agreement with the adopted value in Ref. [35].

2.3.1 Results of measurement of fine structure relative intensity to the first excited 2_1^+ state

The α line at 5181(5) keV has a half-life of 62(14) s. Based on its half-life and the recoil corrected energy difference with the 5577-keV line of ^{190}Pb , namely 405(5) keV, we assign it as being the transition feeding the 2_1^+ state at 405.2-keV

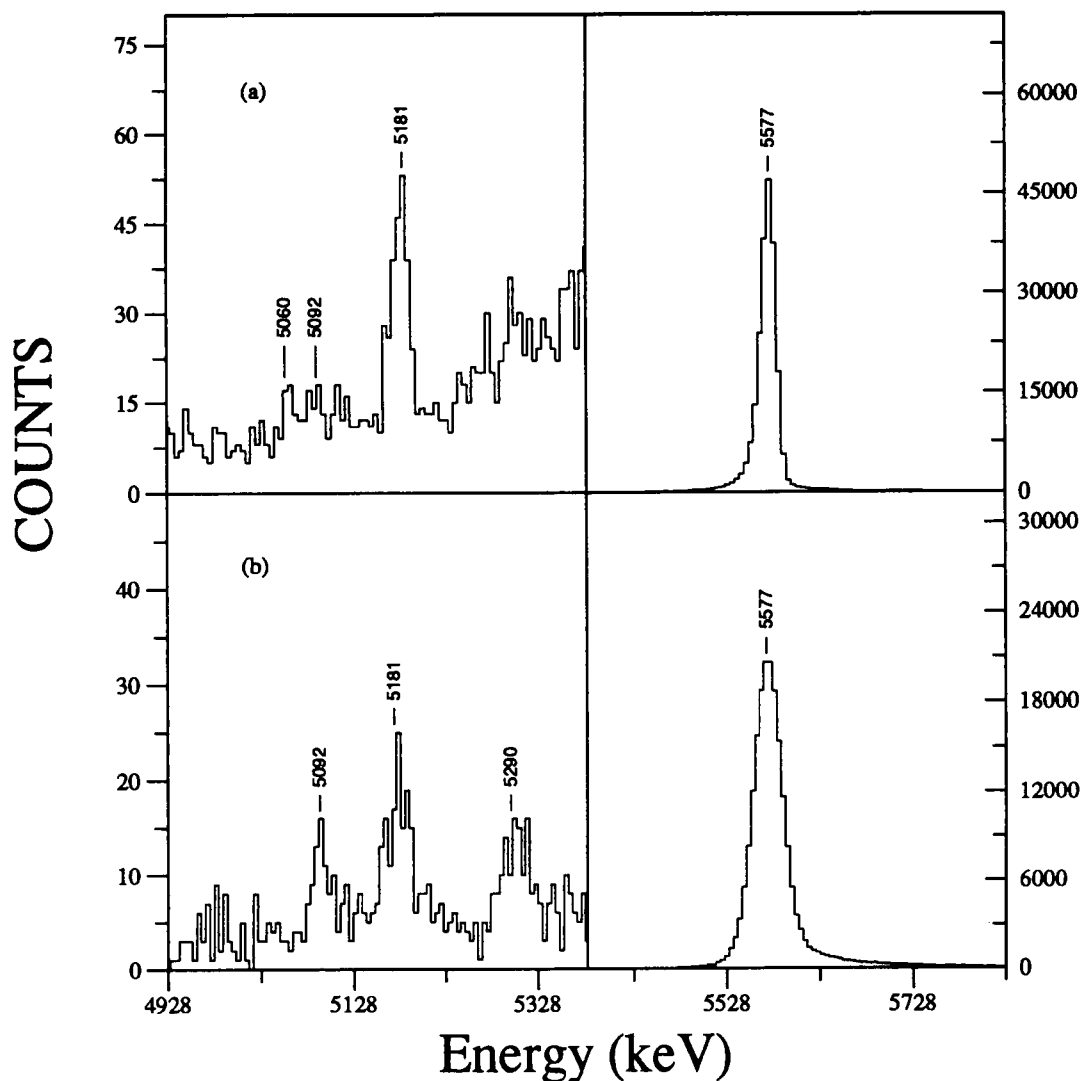


Figure 2.1: Alpha-particle spectrum taken at the first [(a)] and second [(b)] collection station.

[36]. The 405.2 keV γ ray or Hg X rays could not be seen in the spectrum gated on the fine structure peak, due to insufficient efficiency for α - γ coincidences. The expected 405.2 keV peak in the singles γ -ray spectra is obscured by the 404.0 keV γ ray in the $ec+\beta^+$ decay of ^{190}Tl .

Because a second station was used to count α -particles, the collection time to count alpha particles in the fine-structure peak was doubled. Also, the collection

efficiency was better, thus considerably improving the statistics. By summing counts from the two stations we find the relative intensity of ^{190}Pb α -decay to the excited 2^+ state, as compared to transitions to ground state α intensity, to be $8.4(15)\times 10^{-4}$.

2.3.2 Results of measurement of fine structure relative intensity to the second excited 0_2^+ state

The data between 5050 and 5150 keV (see Figure 2.1) were carefully fit and the broad peak revealed two components: a right one at 5092(15) keV, which is attributed to the α decay of ^{186}Hg (5094(15) keV[37]), and a left component at 5060(12) keV. The 5092-keV α line is somewhat broadened due to the fraction of daughter nuclei that recoiled into the detector. The energy of the 5060-keV line yields 528(12) keV for the energy of the level it feeds, probably the 0_2^+ state at 522.7 keV in ^{186}Hg [38]. Furthermore, its half-life behavior is similar to that of the 5577-keV line, as opposed to the 5092-keV α line which, from comparison of Figures 2.1(a) and 2.1(b), grows in. Therefore, we assign the 5060-keV α line as the α branch feeding the 0_2^+ state in ^{186}Hg , with a relative intensity of $1.4(6)\times 10^{-4}$.

2.4 Overview of all nuclear fine structure relative intensities, feeding the first excited 2_1^+ states

In Figure 2.2 the hindrance factors for α -decay transitions and corresponding energies of the first 2^+ excited state in even-even nuclei for all such known transitions, plotted as a function of atomic mass. Here we use the prescription of Preston [39] where no angular momentum induced reduction of the nuclear barrier is taken into account, to be in keeping with the convention adopted by the nuclear data sheets. Beginning in the transitional ^{208}Pb region and extending into the actinides, the HF values follow a smooth path. The excited 2_1^+ energies also follow

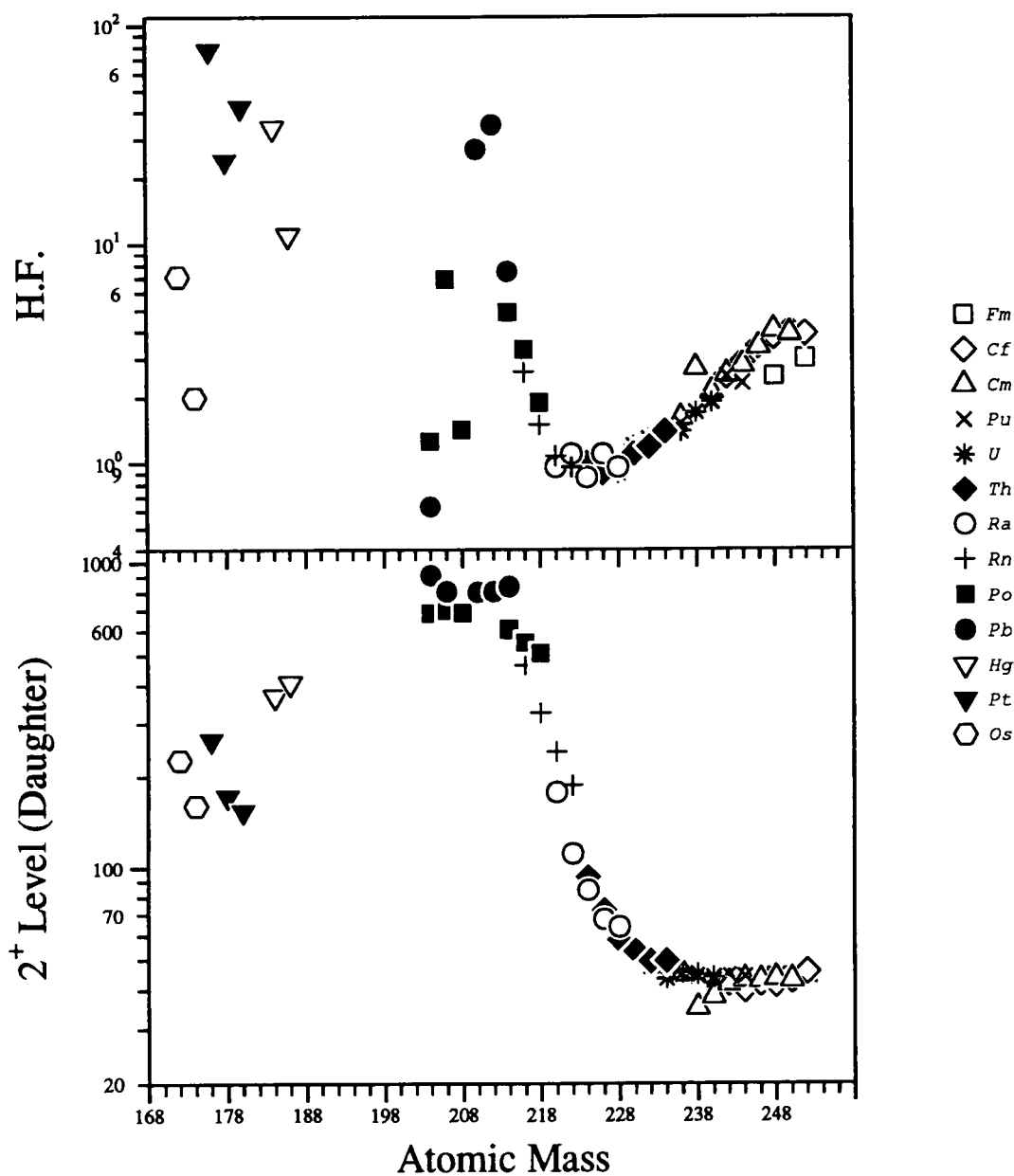


Figure 2.2: Hindrance factors and daughter 2_1^+ energy levels for all known even-even α decays, feeding the first excited 2_1^+ states. Larger uncertainties are present in the light-Pb HF values. However these uncertainties are not great enough to account for the difference in their values.

a decreasing trend indicative of the onset of deformation as the neutron number is increased above the closed shell at $N=126$. It has been suggested that α -clustering may be prevalent in lanthanide, transuranic, and light-Pb isotopes [40]. A test of an α -clustering model was made by reproducing trends of HF to populate excited 2_1^+ states in the actinide region when approaching the doubly closed magic shell of ^{208}Pb [41]. The HF values on the neutron-deficient side of the neutron-closed shell, however, are no longer situated in a neat pattern. Differences between the nature of level structures in this region and the actinides may help explain the behavior of these HF values. One difference, as has been pointed out already, is that the structure of individual rotational bands changes dramatically, perhaps due to mixing.

2.5 HF analysis of the new α transitions

The hindrance factors, or HF values, are computed following the prescription of Rasmussen [8] where the angular momentum component of the nuclear barrier is taken into account. The α -decay constant λ in Eqn. (1.1) is obtained from the experimental values of the half-life $t_{1/2}$ and the branching ratio for α -decay, i.e., $\lambda = \frac{0.693}{t_{1/2}} B_\alpha$. The barrier penetration is calculated via a WKB calculation and the reduced width δ^2 is obtained for each fine structure line. It is assumed that the $0^+ \rightarrow 0^+_{\text{g.s.}}$ α transition is unhindered and the HF for an excited state is taken as $\delta^2_{\text{g.s.}}/\delta^2_{\text{e.s.}}$. The resulting values for α -decay from ^{190}Pb to the 2_1^+ and 0_2^+ states in ^{186}Hg are to our knowledge the first in the literature.

We find the HF to be 6(1) for α decay to the 2_1^+ state in ^{186}Hg . This is a striking difference from the corresponding case found for the first 2_1^+ state of ^{184}Hg , where the HF is found to be 24, and the lower limit of the case for ^{182}Hg of 29 [3]. These values represent hindered transitions, but perhaps for the case of ^{190}Pb α -decay the considerably lower HF value indicates that the 2_1^+ state in ^{186}Hg

is a more pure state, being in line with the ground state rotational band. If the 2_1^+ states in ^{184}Hg and ^{182}Hg contain greater admixtures of configurations belonging to the deformed rotational band, the HF value could be higher, because the more deformed component the 2_1^+ state would be less populated by α -decay. This occurs since there is a smaller overlap of the 2_1^+ state for these two cases, with the ground state of Pb, which is generally thought to be spherical in all three cases. The excited bands of ^{184}Hg and ^{182}Hg are considerably lower in energy than the excited band of ^{186}Hg . In particular, the 2^+ levels of each band may be the result of mixing 2^+ configurations, as has already been indicated in the literature [1]. An in-depth band-mixing calculation will be carried out in Chapter 3, when a calculation of relative intensities using a reaction-theory prescription [28] is performed.

The HF for the α decay of ^{190}Pb to the 0_2^+ state in ^{186}Hg [18(7)] fits, within experimental uncertainty, into the HF systematics of the corresponding decays of $^{186,184}\text{Pb}$, which have HF values of 21 [3, 42].

2.6 Overview of HF for α -decay to excited states in the light lead region of nuclei

Table 2.1 contains hindrance factors for fine structure α -decay transitions in neighboring nuclei taken from the literature [3, 43]. In a neighboring set of nuclear decay sequences of ^{180}Hg , ^{182}Hg and ^{184}Hg , the situation may be more complicated. In ^{178}Pt and ^{180}Pt the two lowest states are thought to belong to the strongly deformed band whereas the first excited level, 0_2^+ , is thought to be a weakly deformed level. In ^{176}Pt , the two lowest levels are thought to belong to a weakly deformed band. The next excited state is possibly the 0_2^+ level of the strongly deformed band [1]. However, it is very close in energy to the 4^+ state of the same band. The 2_2^+ state is not seen, perhaps due to the mixing [1]. The HF values to the first excited

Table 2.1: Alpha-decay energies and HF values for population of excited states in light-Pb region nuclei

Parent Nuclide	Transition	α Decay Energy	Hindrance Factor
^{190}Pb	0_2^+	5066(5) ^a	18(7) ^a
	2_1^+	5179(5) ^a	6.3(1.0) ^a
^{188}Pb	0_2^+	5614(10) ^b	21(3) ^b
	2_1^+	5633(10) ^b	24 ^c
^{186}Pb	0_2^+	6014(10) ^b	21(4) ^b
	2_1^+		> 29 ^b
^{184}Hg	0_2^+	5067 ^b	3.5(6) ^b
	2_1^+	5385 ^b	23 ^b
^{182}Hg	0_2^+	5446(7) ^b	3.5(6) ^b
	2_1^+	5689(7) ^b	11 ^b
^{180}Hg	0_2^+	5689(5) ^b	17(5) ^b
	2_1^+	5862(5) ^b	46 ^b
^{194}Po	0_2^+	6194(5) ^b	1.1(1) ^b
^{196}Po	0_2^+	5769(5) ^b	2.5(1) ^b
^{198}Po	0_2^+	5273(5) ^b	2.8(1) ^b
^{200}Rn	0_2^+	6355(6) ^c	85(7) ^c
^{200}Rn	2_1^+	6454(14) ^c	180(60) ^c
^{202}Rn	0_2^+	5836(5) ^b	19(6) ^b

^aValues from this work.

^bValues from J. Wauters et al. [3]

^cValues from N. Bijmens et al. [43]

2^+ states might be following the overall trend of the absolute experimental reduced widths [42] for α decay of these Hg isotopes to the corresponding Pt ground states. However, the fact that the hindrance is much higher for ^{176}Pt could suggest that the 2^+ state mixes considerably with the deformed configuration as its energy is lowered in relation to the 2^+ levels of the weakly deformed band of Pt nuclei [1]. In addition, Total Routhian Surfaces (TRS) have been calculated for the ^{176}Pt and its neighboring isomers, which revealed β_2 - and γ -softness in ^{176}Pt [44]. Therefore α -decay relative intensities may be more difficult to interpret for the decay of ^{180}Hg , if only mixing quadrupole-deformed shapes are considered as being responsible for

the low-spin members of the yrast band of ^{176}Pt . The HF values of 3.5(6) and 2.4(2) for decay of ^{182}Hg and ^{184}Hg to the weakly deformed 0_2^+ states in ^{178}Pt and ^{180}Pt indicate unhindered transitions, which one might expect because the ground states of the Hg parents are weakly deformed. With the roles reversed for the strongly and weakly deformed bands of ^{176}Pt , it is not difficult to interpret the high hindrance to the excited 0_2^+ state, since, aside from the trend mentioned above, the change in the absolute reduced widths for ground state decays is not as sudden.

2.7 Plans for investigation of the influence of daughter physics on relative intensity fine structure

We will continue the discussion in Chapter 3, in which a calculation of relative feeding intensities of alpha decay to excited states in ^{190}Pb and neighboring nuclei will be carried out using reaction theory in a particle-plus-rotor description of the decaying parent state. Here the particle is the alpha particle and the rotor is the alpha decay daughter. Furthermore, since the excited band of the daughter is thought to have a different shape, but is also thought to mix with the ground band, the effects of the experimentally attained mixing coefficients are input into the calculation. From this it is sought to ascertain whether differing shape and mixing in the daughter states can explain the dramatic variation in relative intensities of α feedings in this region of nuclei.

CHAPTER 3

Modeling of even-even nuclear α decay fine structure

3.1 Use of reaction theory

In this chapter we discuss a calculation to describe our experimental α decay fine-structure data and other α decay data that have been reported in the literature. Because ^{190}Pb and neighboring nuclei are thought to exhibit shape coexistence, an α -decay theory was sought which could take nuclear shape into account. However, many of the nuclei we wish to model occur in regions where the single particle structure has become cumbersome, especially away from the closed magic proton shell $Z=82$. In addition the nuclei of interest are situated at mid-shell in the neutrons, and because of shape coexisting levels in this region, it may be appropriate to treat the nuclei in terms of collective states which interact with each other.

Because of its conceptual simplicity the WKB method of calculating transmission coefficients has been widely used to calculate α decay probabilities [8, 11, 15, 16]. In this approach, α decay is treated in terms of a quantum-mechanical barrier penetration problem. However the prescription uses semiclassical wavefunctions to calculate the transmission probability. For this reason, we decided to perform our modeling in a reaction formalism, which uses purely quantum mechanical wavefunctions. This is intended to minimize uncertainties in the calculation of the transmission probability when effects of a deformed α -core potential are incorporated into the “one-body” escape process.

In this chapter a basic overview of the reaction theory is explained. A theory of α decay which admits decay to or from collective states has been formulated [28], and it is utilized in this chapter. Therefore, following an overview of this theory it will be explained how mixed deformed configurations are input into the theory, which will be used to calculate α -decay relative intensities from light-Pb decays. Finally the results of the calculations are presented along with a discussion of these results.

3.2 Overview of reaction theory concepts

In the historical overview in Chapter 1 it was explained that the problem of α decay has been customarily separated into two problems. One has to do with how an α particle is formed in the nucleus. The remaining problem is how the α particle escapes from the nucleus. Furthermore the two processes have been treated as acting back to back in nature, which has, to a high degree of accuracy given good agreement with empirical relative intensities.

The model we are using treats the second of these processes, namely the release of an already formed α particle. However, a new aspect is consideration of the interaction of the preformed α particle with the daughter core before the release of the α particle. In the nuclear region considered in this work it is now widely accepted that shape coexistence is ever present. The purpose of this model is to show the connection between α -decay fine structure relative intensities and the structure of the daughter nucleus which is being probed by the α before emission. A major goal of this chapter will be to investigate which collective phenomena are at work in driving the so called one-body decay process.

We now present a simplified overview of the concepts pertinent to our model. A more detailed and rigorous overview is already provided in the founding paper by Jackson and Rhoades-Brown [18]. The summary that follows here is in-

tended to be sufficient to orient the reader from the premise of intuitive clarity while providing sufficient reference to the more rigorous treatment. This will facilitate a practical starting point for someone being introduced to the problem of α decay for the first time.

3.2.1 The cluster expansion of the parent wavefunction

The parent wavefunction is constructed in a convenient manner, in order to eventually facilitate its overlap with the relative motion of the α particle, as it escapes the parent system, i.e., an α particle orbiting in a nuclear core (the daughter). The Hamiltonian is written as

$$H = H_A + H_\alpha + V_{\alpha A} + T_R + V_C \quad (3.1)$$

where H_A and H_α are the intrinsic Hamiltonians of the daughter and the α -particle. T_R is the kinetic energy operator for the motion of the α -particle relative to the residual nucleus. $V_{\alpha A}$ is the many-body operator representing the total nuclear interaction between the nucleons and the α -particle potential, and V_C is the Coulomb potential. If we let A be the atomic mass of the daughter, then we can identify H as the Hamiltonian of the nucleus with mass $A + 4$ and

$$H\Psi^{n_{A+4}} = E^{n_{A+4}}\Psi^{n_{A+4}}. \quad (3.2)$$

where $E^{n_{A+4}}$ is the eigenvalue of the wave function $\Psi^{n_{A+4}}$. To establish the cluster interpretation the parent wave function is expanded in terms of wave functions of the daughter, Ψ^{n_A} , and the α -particle, ϕ^{n_α} , which are solutions to

$$H_A\Psi^{n_A} = E^{n_A}\Psi^{n_A}, H_\alpha\phi^{n_\alpha} = E^{n_\alpha}\phi^{n_\alpha}. \quad (3.3)$$

with E^{n_A} being the allowed energies of the daughter system and E^{n_α} being the allowed energy states of the α particle. The α particle and the residual nucleus A

interact through $V_{\alpha A}$. The traditional way of describing the parent system is to stipulate the single particle coordinates, $r_i = 1, 2, \dots, A + 4$. For the cluster description, these coordinates are redefined as

$$\begin{aligned} R_\alpha &= \frac{1}{4} \sum_{k=1}^4 r_{A+k}, \quad R_A = \frac{1}{A} \sum_{i=1}^A r_A, \quad R = R_\alpha - R_A, \\ \eta_i &= r_i - R_A, \quad \zeta_k = r_{A+k} - R_\alpha, \quad \sum_{i=1}^A \eta_i = 0, \quad \sum_{k=1}^4 \zeta_k = 0 \end{aligned} \quad (3.4)$$

Part of the energy of the parent system is in terms of kinetic energy T_R . Therefore, as a result of a convenient coordinate transformation, it naturally follows that we can ascribe an additional wave function to our product basis, which signifies the energy states of the orbiting $A - \alpha$ system. Thus, we arrive at the following form for the wavefunction in the new coordinate system

$$\Psi_o = \sum_n C_n(n_A n_\alpha) \Psi'^{n_A}(\eta) \phi'^{n_\alpha}(\zeta) \chi_\alpha^n(R) \quad (3.5)$$

where $\sum_n C_n(n_A n_\alpha)$ implies summation over all appropriate quantum numbers, and $\chi_\alpha^n(R)$ describes the relative motion of the α -particle and the residual nucleus.

3.2.2 The physical picture of the interaction for decay

In the picture so far described the parent state is comprised of a linear combination of microscopic states of the residual nucleus and the α -particle. The solution to our Hamiltonian can be considered, up to this point, as being exact to the degree that the particulate coordinates of the nucleons are clearly indicated in our wave functions. However, because a many-body problem is typically intractable if treated exactly, one customarily attempts to choose a coordinate that best represents the global dynamical features of the system to replace several coordinates. In this paper we are concerned with modeling the decay of the system after the α particle is formed. Therefore, it is natural to think in terms of the coordinate that describes

the global separation of the α particle and the residual nucleus. For this we have already chosen $R = R_A - R_\alpha$. Furthermore, we make the approximation that the force between the constituent nucleons of the α particle and the nucleons of the residual nucleus can be adequately described as an aggregate force of the group of α nucleons on the group of residual nucleons in terms of the separation distance R . Therefore we make the first approximation by supposing that a potential $V(R)$ can be constructed from the many body potential function $V_{\alpha A}$. Sometimes such potentials are referred to as optical potentials. The details of this procedure were carried out by Jackson and Rhoades-Brown [18] utilizing procedures and results from the work of DeVries *et al.* [45], among others [46, 47]. The central potential that will be used in this paper is of the Woods-Saxon form

$$V(R) = \frac{V_{ws}}{1 + \exp[(R - r_{ws})/a_{ws}]}. \quad (3.6)$$

where V_{ws} gives the depth, R_{ws} is the Woods-Saxon radius and a_{ws} the diffuseness of the potential.

Since we are viewing the part of the decay process after preformation, we note that the coordinate R , by virtue of the massiveness of the α and the residual nucleus, is a slowly changing coordinate compared to the individual η_i and ζ_i . When quantum mechanical coordinates can be distinguished as slow and fast, one is justified in treating the fast coordinates around small time instances of the slow coordinates. It thus follows that the new basis wavefunctions of coordinate R , $\xi_\alpha^n(R)$, are decoupled at small instances from the corresponding microscopic wavefunctions $\Psi'^{n_A}(\eta)$ and $\phi'^{n_\alpha}(\zeta)$. Therefore, the averaging procedure proposed by Jackson and Rhoades-Brown [18] is perhaps justifiable from this simple picture.

A means to view the parent nucleus following the formation of an α particle is presented above. From this point forth we discuss the process of escape of the preformed α particle from the parent nucleus. We adopt the picture presented in a paper by Gurvitz and Kalbermann [48], as well in Jackson and Rhoades-Brown

[18]. If the metastable state of a particle is deeply enough bound then one is justified in expanding the time-dependent wavefunction of the decaying parent in terms of bound states of a Hamiltonian which has the same shape as the original metastable Hamiltonian for the radial values $0 \leq r_\alpha \leq r_b$, beyond which K has the value of the nuclear potential at the barrier radius r_b . We call the original Hamiltonian H , while the bound state Hamiltonian is called K . The associated bound state potential that will be used in this work is of the same form as adopted in Eqn. (3.6)

$$H = K + (H - K) = K + V \quad (3.7)$$

The bound state eigenvalue equation is written as

$$K\Psi^{n_A+4} = E^{n_A+4}\Psi^{n_A+4}, \quad (3.8)$$

where

$$K = H_A + H_\alpha + T(R) + U(R)\theta(r_b - R) + U_b\theta(R - r_b). \quad (3.9)$$

$U(R)$ is the optical potential of the α -particle in the vicinity of the residual nucleus. The perturbing potential is

$$V = H - K = (U(R) - U_b)\theta(R - r_b). \quad (3.10)$$

With the alpha particle already formed, the problem is set up in a time dependent perturbation theory. The result is that the decay probability is proportional to the absolute square of the matrix element between the initial bound state wave function and the outgoing scattered wave function of the alpha particle with the perturbing potential. This is expressed as

$$\omega_{J_e} = (4\mu/\hbar^3 k) \left| \int F_L^{uc} V u_{nL} dR' \right|^2 \quad (3.11)$$

where F_L^{uc} is the regular solution for the Coulomb wave equation that has been given a scattering phase shift to account for the difference between the Coulomb potentials from the point charge and the uniform distribution and u_{nL} is the radial component of the solution of the α -particle wavefunction in the bound system.

3.3 Tailoring reaction theory to deformed nuclei

A fully quantal formulation of the theory of α -decay of deformed and/or oriented nuclei based on a reaction-theoretical foundation [18] was suggested some years ago [28]. The original version, implementing the idea of an alpha-particle-plus-rotor model with quadrupole-quadrupole coupling between the deformed core and the alpha particle, used a harmonic-oscillator basis for the single-particle alpha-orbitals. This program was later employed to analyze the anisotropy measurement data [49] for odd-mass At and Po isotopes [50]. A more recent version based on Woods-Saxon wave functions was first used to re-analyze the same data so as to confirm its applicability to a number of light actinides [51], especially those showing indications of octupole deformation. It is important to stress that the model is basically a phenomenological one. The formulation in terms of the particle-rotor model is dictated by the need to take angular momentum conservation (*rotational invariance*) into account. Also the *translational invariance* is strictly observed here. When using the model, empirical data must be used to fix the parameters of the model. Thus, for instance, potential depths must be adjusted so as to reproduce the decay energy (Q -value) as given by empirical nuclide masses, because this energy has a very strong influence on the decay rate (or equivalently the half-life) of the nucleus considered. The theoretical validity of the model is to be sought in its ability to use general quantal principles and structural ideas to reveal relationships between different empirical data. Hopefully, this will facilitate the physical interpretation of the data. On the other hand it implies that the model cannot be used for predicting,

e.g., the half-life for a parent nucleus if its mass and that of the daughter are unknown and is unreliable for calculating intensity relations if the low-lying states of the daughter are unknown. It must also be pointed out that the Pauli principle can only be imposed in an approximate way, because the alpha particle, a composite particle is treated as a pointlike object. The corresponding structural model for nucleons in a nucleus solves this problem by introducing single hole states and coupling them to the particles via a pairing potential with pair gap and Fermi level as governing parameters. To the author's knowledge no similar model has been formulated and adapted for the alpha-core problem.

Because we deal with nuclei with oblate or prolate deformation, a suitable approximation is the quadrupole term of a multipole expansion of this potential. The deformed potential is of the form

$$V(r) = \frac{V_{ws}}{1 + \exp[(R - R(\theta))/a_{ws}]} \quad (3.12)$$

where

$$R(\theta) = R_{ws}\{1 + \beta_2 Y_{20}(\cos\theta)\}. \quad (3.13)$$

The expansion results in a potential of the form

$$V(R) = U(R) + \kappa_2 \sum_{m=-2}^2 (-1)^m Q_m^2(\mathbf{r}) Q_{-m}^2(\Omega). \quad (3.14)$$

To find the bound state solutions, which we need to evaluate Eqn. (3.11) for the decay width we define a basis of rotor-plus- α states as

$$\phi_{(nK)M}^{(LJ)I} = \sum_m \langle L, m, J, M - m | I, M \rangle \chi_m^{nL}(R) X_{KM-m}^J(\Omega) \quad (3.15)$$

where $\chi_m^{nL}(R)$ are orbital wavefunctions of an α cluster. The quantum numbers n, L are composed from the single particle orbital quantum numbers used to construct the α particle by invoking the oscillator rule, namely

$$2(N-1) + L = \sum_{i=1}^4 [2(n_i - 1) + l_i]. \quad (3.16)$$

The $X_{KM-m}^J(\Omega)$ are the wavefunctions representing core eigenstates.

Because the nuclei we observe decaying are not prepared with a preferred orientation, in forming the matrix equation we integrate over all angles, while taking the inner product of the bound state Hamiltonian to obtain the following equation of motion.

$$E a_{nLJ}^{KI} = (E_{JK} + \epsilon_{nL} + U_b) a_{nLJ}^{KI} + \sum_{n'L'J'} \langle nKLJ : I || \lambda Q \cdot Q || n'KL'J' : I \rangle a_{n'L'J'}^{KI}. \quad (3.17)$$

Consider a system A (the nucleus ${}^AZ$) formed by a spheroidal core C and an alpha particle. The interaction between the mass-centre of the core and the α particle is expressed as a sum of a spherical core- α potential, $U(r)$, and the quadrupole-quadrupole term describing the coupling between the quadrupole moment $Q_m^{(2)}(\Omega)$ of the core and that of the alpha orbital, $Q_m^{(2)}(\mathbf{r})$. We redefine A to be the mass of the parent, as opposed to $A + 4$ used previously. The Hamiltonian H_A of the parent of mass A may be written as

$$\begin{aligned} H_A = & H_C + H_\alpha + T_\alpha + U(r) \\ & + \kappa_2 \sum_{m=-2}^2 (-1)^m Q_m^{(2)}(\mathbf{r}) Q_{-m}^{(2)}(\Omega). \end{aligned} \quad (3.18)$$

Here H_C is the intrinsic Hamiltonian of the core and H_α that of the alpha particle. The principal dynamical variables in this model are (i) the alpha-core relative coordinate vector $\mathbf{r} \equiv \mathbf{R}_\alpha - \mathbf{R}_C$ and (ii) the orientation Ω (given e.g. by the Euler angles α, β, γ) of the intrinsic core reference system with respect to the laboratory frame system. These variables have the corresponding conjugate momenta $\mathbf{p} = -i\hbar \nabla$ and $\mathbf{J}_C$, the latter being the core spin. The individual intrinsic coordinates of the core and the alpha particle are ignored, but their intrinsic *states* can be included

in the formulation, if necessary. If we choose T_α to be the *relative* kinetic energy of the (centre-of-mass of the) alpha particle and the core, then H_A is the intrinsic Hamiltonian of the nucleus A .

The quadrupole moment of the α -particle orbital is

$$Q_m^{(2)}(\mathbf{r}) = f(r) \sqrt{\frac{4\pi}{5}} Y_{2m}(\hat{r}), \quad (3.19)$$

where $f(r)$ is a radial form factor. The parameter κ_2 is determined from the particle-particle interaction underlying Eqn.(3.19). As a consequence of the rotational invariance the form factor $f(r)$ must be entirely independent of angles. In calculations we approximate the quadrupole moment of the core by that of an axial ellipsoid with half-axes a and c : $Q_0^{(2)} = 2(c^2 - a^2)/5 = 4\langle r^2 \rangle_{\text{core}} \delta / (3 - 2\delta)$, where δ is the deformation parameter originally introduced by Nilsson [52].

The formulation in terms of a quadrupole-quadrupole force as in Eqn. (3.18) can be shown to be equivalent to using a deformed α -core potential whose symmetry axis coincides in every instant with the symmetry axis of the core (i.e., the daughter nucleus). This potential then generates α orbitals analogous to the Nilsson nucleon orbitals [52] corrected for the Coriolis force, which is an essential force in the intrinsic (core-fixed) system without counterpart in the laboratory system. Since we need the wave functions in the laboratory system, whose asymptotic parts well beyond the nuclear barrier radius carry information on the orbitals from which the alpha particle is constructed, we perform the entire calculation in the laboratory system. As the interaction is expressed from the start in terms of multipole-multipole operators, the orientation dependence of the interaction presents no further problems.

The form factor $f(r)$ in Eqn. (3.19) can be obtained from the first-order term of a Taylor expansion in β_2 of an axially-symmetric, quadrupole-deformed Woods-Saxon(Ws) potential [53], whereby it is assumed that the half-value radius

is given by Eqn. (3.13). The coupling constant κ_2 is thus given by

$$\kappa_2 = \frac{V_{\text{WS}} R_{\text{WS}}}{2a_{\text{WS}} \langle r^2 \rangle_{\text{core}}}.$$

The angle between the symmetry axis of the core and the position vector $\mathbf{r}$ of the α particle relative to the centre of mass of the core is θ , while a_{WS} and V_{WS} are the diffuseness and the strength of the WS potential, respectively. It is of course the *product* of the coupling constant and the form factor that is determined by the WS strength and shape. Numerical factors can thus be moved between these factors according to which convention one might prefer.

More general types of deformed alpha-core interactions

In a more general mean-field approach we would base the alpha dynamics on an alpha-core interaction $V(\mathbf{r}, \Omega)$ given in Eqn (3.12) with the angle-dependent half-value radius $R(\theta)$ given by

$$R(\theta) = R_{\text{WS}} \left\{ 1 + \sum_{\lambda_{\min}}^{\lambda_{\max}} \beta_{\lambda} Y_{\lambda 0}(\theta) \right\}.$$

Here θ is the angle between the position vector $\mathbf{r}$ for the alpha particle and the symmetry axis of the core. In the laboratory system this angle is orientation-dependent. We define (following, e.g., Möller and Nix [54]) the (electric) 2^λ -pole moment Q_0^λ for a uniform (charge) distribution with charge radius R_C :

$$Q_0^\lambda = 2 \left(\frac{3Z_C}{4\pi R_C} \right) \int r^\lambda P_\lambda(\cos \theta) d^3r \quad (3.20)$$

where the volume of integration is bounded by the surface $(R_C/R_{\text{WS}})R(\theta)$. Then we get, to first order in the β 's, the relationship between the deformation parameter β_λ and the multipole moment $Q_0^\lambda(\text{intr.})$

$$\beta_\lambda \approx \frac{1}{Z_C \langle r^\lambda \rangle_C R_C^2} \frac{\sqrt{\pi(2\lambda+1)}}{\lambda+3} Q_0^\lambda(\text{intr.}). \quad (3.21)$$

Here the average of r^λ to first order in the β 's is

$$\langle r^\lambda \rangle_C = \frac{3R_C^\lambda}{\lambda + 3}.$$

We expand Eqn. (3.12) to first order in β_λ :

$$\begin{aligned} V(\mathbf{r}, \theta) = & \frac{V_{WS}}{1 + \exp[(r - R_{WS})/a_{WS}]} \\ & + \sum_\lambda \frac{V_{WS} R_{WS}}{a_{WS}} \frac{\exp[(r - R_{WS})/a_{WS}]}{\{1 + \exp[(r - R_{WS})/a_{WS}]\}^2} Y_{\lambda 0}(\theta) \beta_\lambda. \end{aligned} \quad (3.22)$$

If we wish to go to higher orders in β then we also have to derive more accurate formulas than Eqn. (3.21) for the multipole moments (see, e.g., ref. [55]). Since the deformed WS potential is itself an approximation to the mean field, the gain in numerical accuracy with such an effort would not entail a corresponding gain in physical accuracy. We therefore drop the higher order terms and consider the resulting expansion "exact" (in a phenomenological sense) as it stands. If improvements are needed, they may be implemented for each multipole individually.

Accordingly, we assume that the non-spherical part of the alpha-core interaction is in general *exactly* described by a *multipole-multipole expansion* rather than a deformed local potential moving with the core. Thus we generalize the Hamiltonian of the parent state to

$$\begin{aligned} H_A = & H_C + H_\alpha + T_\alpha + U(r) \\ & + \sum_\lambda \kappa_\lambda \sum_{m=-\lambda}^{\lambda} (-1)^m Q_m^{(\lambda)}(\mathbf{r}) Q_{-m}^{(\lambda)}(\Omega) \end{aligned} \quad (3.23)$$

with orbital 2^λ -pole moment

$$Q_m^{(\lambda)}(\mathbf{r}) = f(r) \sqrt{\frac{4\pi}{2\lambda + 1}} Y_{\lambda m}(\hat{r}),$$

orientation-dependent core 2^λ -pole moment

$$Q_m^{(\lambda)}(\Omega) = Q_0^{(\lambda)}(\text{intr.}) \mathcal{D}_{m0}^{\lambda*}(\Omega),$$

and 2^λ -pole coupling constant

$$\kappa_\lambda = \frac{1}{Z_C} \frac{2\lambda + 1}{\lambda + 3} \frac{V_{ws} R_{ws}}{2a_{ws} \langle r^\lambda \rangle_C},$$

where Z_C is the charge of the core or residual nucleus.

3.3.1 The effect of shape coexistence

The problem presented by the data on light Hg, Pb and Po isotopes [43, 30] requires a different modification of the Hamiltonian used. These nuclei are examples of shape coexistence showing rotational (and/or vibrational) bands which, although they have the *same parity*, have *quite different intrinsic structures, hence different deformation* parameters. Transitions within the band show “normal” decay rates and decay patterns, which is why this group of states are characterized as a “band”, while transitions between members of different bands are strongly hindered.

The calculations performed for these nuclei are based on the hypothesis that the excited (coexisting) band is coupled to the ground band by means of a monopole-monopole interaction which, by analogy with Eqn. (3.23), can be written

$$\kappa_0 Q_0^0(\mathbf{r}) Q_0^0(\xi). \quad (3.24)$$

Here the variable ξ would in the most general case stand for the full set of intrinsic variables of the core. We shall use ξ to denote a property which sufficiently distinguishes the bands from each other; to specify it much further is outside the scope of the present work (See, however, subsect. 3.3.3). According to Eqn. (3.20) the monopole moment of the core is

$$Q_0^0(\xi) = 2 \left(\frac{3Z_C}{4\pi R_C} \right) \int P_0(\cos \theta) d^3r = 2Z_C R_C^2 \quad (3.25)$$

To good approximation, for the WS (or Fermi) distribution

$$R_C^2 = R_{ws}^2 \left(1 + \frac{7\pi^2 a_{ws}^2}{R_{ws}^2} \right) = R_{ws}^2 + 7\pi^2 a_{ws}^2 \quad (3.26)$$

(see e.g. Appendix C of ref. [56]). We see that the monopole moment of a WS distribution depends on both R_{WS} and a_{WS} .

Naturally we identify the lowest-order monopole-monopole terms within the ground band ($\xi = 0$) and within the coexistent band ($\xi = 1$) with the respective WS potentials, the strengths of which are determined from the observed bandhead energies in a systematic search procedure. The monopole strength of interest for the band coupling is the one associated with a transition between members of different bands. Due to conservation of angular momentum only members of the two bands having the same value of J_C can be coupled via such an operator. This means that only the *radial* components of the mass and charge distributions can be affected by the monopole coupling. Consequently, in the present model only the WS parameters R_{WS} and a_{WS} can change as a direct result of the monopole transition.

A suitable dimensionless “radial deformation parameter” for the monopole case could be constructed as the fractional change in the core monopole moment when the radial WS parameters are changed, i.e.

$$\Delta\xi = \frac{R_{\text{WS}}\Delta R_{\text{WS}} + 7\pi^2 a_{\text{WS}}\Delta a_{\text{WS}}}{R_C^2}$$

in view of the result (3.26). In most of the calculations we have only considered $\Delta R_{\text{WS}} \neq 0$ and $\Delta a_{\text{WS}} = 0$, resulting in $\Delta\xi = \Delta R_{\text{WS}}/R_{\text{WS}}$.

The orbital monopole moment operator is, by analogy,

$$Q_0^0(\mathbf{r}) = f(r)\sqrt{4\pi}Y_0^0 \equiv f(r),$$

The formal analog of the core multipole *operator* would be the rotationally invariant operator.

$$Q_0^0(\xi) = 2Z_C R_C^2 \sum_{J_C, M_C} \{ |J_C, M_C, \text{g.s.}\rangle \langle J_C, M_C, \text{c.x.}| + |J_C, M_C, \text{c.x.}\rangle \langle J_C, M_C, \text{g.s.}| \}$$

where ‘g.s.’ and ‘c.x.’ refer to states in the ground band and the coexisting band, respectively. Alternatively, we will use a numerical label $k_x = 0$ or $k_x = 1$ for the

same purpose. Finally, the coupling constant κ_0 and the form factor $f(r)$ to be used in conjunction with the monopole moment operator are given by

$$\begin{aligned}\kappa_0 f(r) &\equiv \frac{\partial V(r, R_{WS}, a_{WS})}{\partial R_{WS}} \Delta R_{WS} \\ &= \frac{V_{WS}}{a_{WS}} \frac{\exp[(r - R_{WS})/a_{WS}]}{\{1 + \exp[(r - R_{WS})/a_{WS}]\}^2} \Delta R_{WS}\end{aligned}\quad (3.27)$$

Eqn. (3.24) is of course a hypothesis to be tested. The dynamical cause of the assumed change ΔR_{WS} or Δa_{WS} of the radial shape parameters of the potential that generates the alpha orbitals is not specified. As discussed in the introduction, the predominant view at present of the underlying microscopic structure is that the coexisting configuration differs from that of the ground-state band by a two-nucleon-two-hole excitation[1].

It is a reasonable expectation that an alpha particle in an $L = 0$ orbit surrounding the core in its 0^+ ground state may interact with the core and excite it to its next 0^+ state, and that such a change of the core state should also have consequences for the alpha-core potential and therefore on the alpha-core orbital. The models proposed here are primarily constructed to satisfy these expectations qualitatively and to serve as a starting point for more quantitative approaches. The validity of the quantitative implementation of these ideas should then of course be subjected to severe tests and critically examined. The calculations presented here should be viewed as the first step in this procedure.

3.3.2 Calculation of the decay rate

We now define a model Hamiltonian H_0 as

$$\begin{aligned}H_0 &= H_C + H_\alpha + T_\alpha + \theta(R_b - r)U(r) \\ &\quad + \theta(r - R_b)U_b + \sum_{\lambda=0}^{\lambda_{\max}} \kappa_\lambda \mathbf{Q}^\lambda \cdot \mathbf{Q}^\lambda,\end{aligned}\quad (3.28)$$

where the last term is the multipole-multipole interaction. Note that the monopole term ($\lambda = 0$) is also included. The potential term here is constant $U = U_b$, for

$r \geq R_b$, R_b being the barrier radius. Therefore, H_0 may have bound states ϕ_E with energies in the interval $0 < E \leq U_b$. These states may be used as “initial states” in a process leading to eigenstates Ψ_E of the true Hamiltonian, Eqn. 3.28, in accordance with the Jackson–Rhoades–Brown formalism [18, 28]. Since the true α -particle states Ψ_E are unbound and represented asymptotically by free outgoing Coulomb waves, they describe the decay of the initial state ϕ_E .

The solutions to $H_0\phi_M^I = E\phi_M^I$ with well-defined I and M are obtained by diagonalizing H_0 in the weak-coupling basis

$$\phi_{(nK_C\xi)M}^{(\ell J_C)I}(\mathbf{r}, \Omega) = \sum_m \langle \ell, m, J_C, M - m | I, M \rangle [\chi_{n\ell m}(\mathbf{r}) X_{K_C\xi}^{J_C M_C}(\Omega)]_M^I, \quad (3.29)$$

resulting in coefficients $a_{n\ell J_C\xi}^{K_C\zeta I}$. Here $\chi_{n\ell m}(\mathbf{r})$ is the wave function of the α orbital and $X_{K_C\xi}^{J_C M_C}$ is a wave function of the core Hamiltonian corresponding to the diabatic configuration ξ [the quantum number ξ introduced in Sec. 3.3.1 distinguishes between the ground band ($\xi=0$) and the coexisting excited band ($\xi=1$)]. The quantum number K_C is the projection of the core spin J_C on the symmetry axis of the core, and ζ stands for other quantum numbers labeling the core states.

Using the formalism of Refs. [18, 28] with the model eigenstates ϕ_M^I as initial states, one obtains states Ψ_M^I which asymptotically behave as outgoing waves, and the radial current (α -particle flux) at large distances can easily be deduced [28]. The (partial) decay rate, $\omega_{J_C K_C \zeta} = \Gamma_{J_C K_C \zeta} / \hbar$, for populating the daughter state (labeled by its spin J_C , bandhead spin K_C , and band label ζ) is obtained by integrating the current over all directions of emission. This results in the following expression for the partial decay rate

$$\omega_{J_C K_C \zeta} = \frac{4\mu}{\pi \hbar^2 k_{J_C K_C}} \sum_{n\ell\xi} |a_{n\ell J_C\xi}^{K_C\zeta I}|^2 |T_{n\ell\xi}|^2, \quad (3.30)$$

where

$$T_{n\ell\xi} = \exp[i(\sigma_\ell - \ell\pi/2)] \int F_\ell V u_{n\ell\xi} dr \quad (3.31)$$

is the α -transition amplitude (“transmission coefficient”) in the $(n\ell)$ channel. The function $u_{n\ell\xi}(r)$ is the radial wave function for the α particle. It is generated in a spherical WS potential which is different in the two bands because R_{WS} depends on ξ . [As discussed in the following section, the label ξ distinguishes between the *diabatic* (non-interacting) core states while ζ stands for the physical *adiabatic* (mixed) core states.] The influence of deformations of various multipolarities is incorporated through the coefficients $a_{n\ell J_C \xi}^{K_C \zeta I}$.

The model described here, or its nearest predecessors, has previously been applied to a variety of nuclei with masses in the range $200 < A < 230$. To interpret and judge the results one must understand how the analysis is done. Initially the Hamiltonian (3.28) is diagonalized assuming that one of the eigenvectors $\{a_{n\ell J_C \xi}^{K_C \zeta I}\}$ is the one that corresponds to the “ground state” of the parent nucleus. It must be elaborated that although only one solution is thought to be physically relevant, it will be seen that this choice is not always obvious when performing calculations. This is an important point that will be taken up more thoroughly in Sec. 3.5.3. The solutions are labeled according to their relative energy after diagonalization but before their individual tuning to adjust the potential depth. Diagonalization and readjusting are performed until the correct decay energy Q_α is obtained for a particular solution. When this solution has the correct decay energy, the relative feedings to excited states of the daughter nucleus are calculated.

For computational reasons we usually restrict the α orbitals to one or two major shells, *viz.* those compatible with the assumed single-nucleon constituents of the α particle according to the so-called oscillator rule quoted in Ref. [18],

$$N_\alpha = N_{p_1} + N_{p_2} + N_{n_1} + N_{n_2}. \quad (3.32)$$

In deformed nuclei this rule loses precision because the single-nucleon orbitals there have components from several spherical shells, but it remains useful as a rule-of-thumb for setting the major quantum number N_α . The oscillator rule is also an

approximate way of satisfying the Pauli principle for the nucleons in the α particle with respect to those in the core. The assumption made here is that the formation has already taken place when the decay occurs; it is an α particle that penetrates the Coulomb barrier, not four separate nucleons which come together during or after the penetration.

3.3.3 Mixing of core states.

A model of how bands of different kind can interact in a nucleus and give rise to modified bands with mixed properties was suggested by Dickmann and Dietrich [57] and later applied by several groups [58, 59] for structure considerations. It has also been used by Wauters, *et al.* [3] in α -decay analysis.

The two-band model assumes that the states of the even-even daughter nuclide $A-4(Z-2)$ can be approximately described as belonging to either of two bands, the ‘ground band’ or the ‘intruder band’. We need not go further into the physical nature of these states; we simply assume that they are eigenstates of a model core Hamiltonian H_{C_0} and have different and distinguishable properties. The ‘true’ core Hamiltonian H_C differs from the model Hamiltonian H_{C_0} by an interaction $H_C - H_{C_0} \equiv V$, coupling the bands such that

$$\langle X_\xi^J | V | X_{\xi'}^{J'} \rangle = V_J \delta_{JJ'} (1 - \delta_{\xi\xi'}). \quad (3.33)$$

The ‘true’ daughter states $X_{\zeta,J}$ are eigenstates of $H_C = H_{C_0} + V$; they are linear combinations of the model core states X_0^J and X_1^J :

$$X_{\zeta,J} = \alpha_{\zeta,J} X_{\xi=0}^J + \beta_{\zeta,J} X_{\xi=1}^J, \quad \zeta = 0, 1. \quad (3.34)$$

The interaction V is usually associated with the $E0$ transitions between spherical and deformed states in the core system. As discussed in, e.g., Ref. [59], the observed $E0$ transition strengths result from the mixing of states with different shapes.

The decay theory presented in the preceding sections describes how the “true” core states $\Psi_{n,J}$ are populated when an α particle in the parent nucleus makes a transition from a metastable state ϕ_M^I , a quasi-bound eigenstate of the model Hamiltonian H_0 , Eqn. (3.28), to the final state $\Psi_{\zeta,J} W_L^+(k_f r) Y_L(\hat{r})$, where $W_L^+(k_f r)$ is the outgoing Coulomb wave of the α particle with appropriate wave number k_f . However, in developing the decay formalism, we have tacitly assumed that the core states are such that the matrix elements of the multipole operators can be calculated as if the core states are pure rotor (or pure vibrator) eigenfunctions. The actual values of the matrix elements are specified in terms of deformations which are input parameters for the calculation.

The amount of mixing, i.e., the parameters $\alpha_{\zeta,J}$ and $\beta_{\zeta,J}$, is calculated based on experimental data. We should clearly distinguish between the ‘true’ core state labels $\zeta = 0, 1$ and the labels $\xi = 0, 1$ which define the diabatic (non-interacting) core states. The coefficients $a_{n\ell J \zeta \xi}^{K_C \zeta I}$ which specify the solutions to the eigenvalue problem $H_0 \phi_M^I = E \phi_M^I$ can then be easily expressed in terms of the mixing parameters $\alpha_{\zeta,J}$ and $\beta_{\zeta,J}$.

3.4 Band mixing

Band interactions V_J and mixing coefficients $\alpha_{\zeta,J}$ and $\beta_{\zeta,J}$ have been extracted for the daughter nuclei $^{176,178,180}\text{Pt}$ and $^{182,184,186}\text{Hg}$ using the two-level model of Sec. 3.3.3. Above spin $I=6$, the well-deformed prolate band becomes yrast in these nuclei [1]. The diabatic bands ($\xi=0$ and 1) can be parameterized using the standard expansion

$$E_{J,\xi} = A_\xi + B_\xi J(J+1) + C_\xi J^2(J+1)^2. \quad (3.35)$$

Due to the lack of experimental data, an assumption has been made that the interaction between the 0^+ states and the 2^+ states in $^{182,184}\text{Hg}$ are equal. The

results of band mixing calculations for ^{182}Hg are illustrated in Fig. 3.1, where the $I=6-12$ band members of the deformed intruder band were considered in the fit, Eqn. (3.35). The resulting band-mixing parameters for all Pt and Hg nuclei

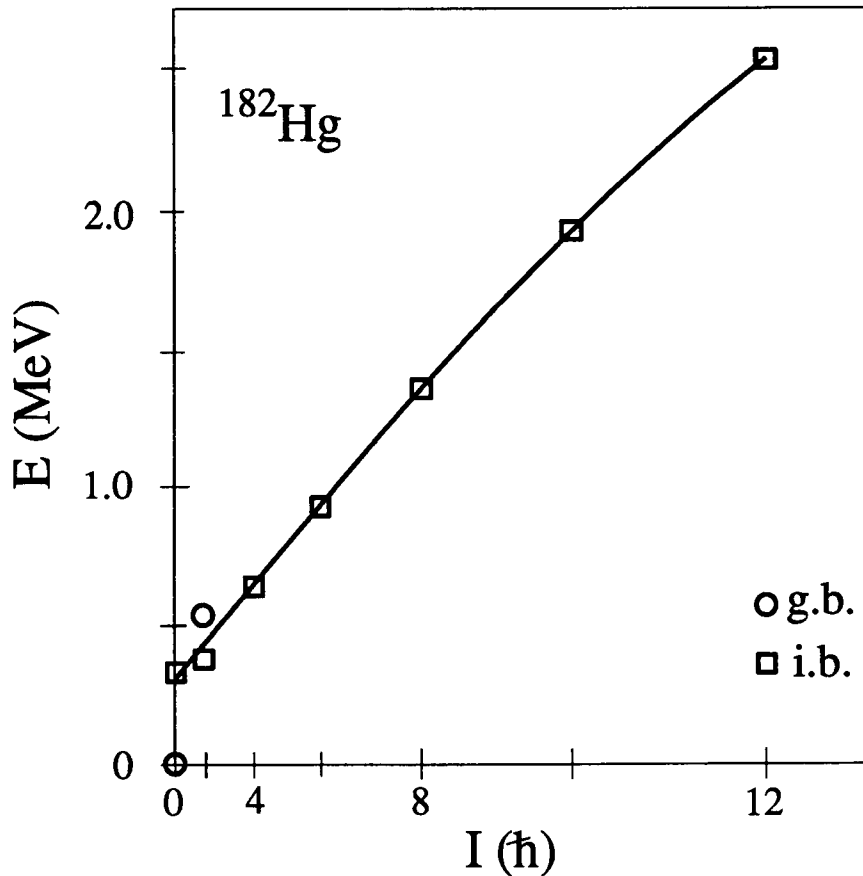


Figure 3.1: Energies of the ground-state and coexisting low-lying rotational band of ^{182}Hg as a function of angular momentum. The solid line is the fit of the deformed band by Eqn. (3.35).

considered in our study are listed in Table 3.1.

For $^{190,192,194}\text{Pb}$, the band mixing parameters were found in Ref. [58]; for completeness they are shown in Table 3.2. As the ground states of the Pb isotopes are spherical, a deformation of $\beta_2=0.01$ was assigned to the ground band, necessary to perform the calculation.

Table 3.1: Results of band mixing calculations. Experimental excitation energies have been taken from Ref. [60] except for the values indicated by the superscript "a" taken from Ref. [30]. Numbers in brackets were obtained by assuming $V_{J_C}=V_0$ (see text). The mixing amplitude α_{J_C} [Eq. (3.34)] always corresponds to a weakly deformed (or spherical) structure. For instance, for the Hg isotopes α_{J_C} decreases from the value close to one at $J_C=0$ (oblate ground state) to small values at $J_C=4$ (prolate band becomes yrast).

Nucleus	J_C	unperturbed energy (keV)	experimental energy (keV)	$ V $ (keV)	$ \alpha_{J_C} $
^{186}Hg	0	16	0	90.0	.985
	2	506.7	522.7	58.8	.959
		422.9	405.4		
		603.7	621.1		
	4	1060.5	808.2	71.3	.272
^{184}Hg	0	19.8	0	83.9	.973
	2	355.3	375.1	83.9	.701
		451.8	366.5		
		449.1	534.4		
	4	1078.1	653.3	66.8	.155
^{182}Hg	0	23.4	0	84.5	.964
	2	304.6	328 ^a	84.5	.494
		500.7	351.8		
		399.8	548.6		
	4	616.9	613.2	[84.5]	.0435
^{180}Pt	0	292	0	233.5	.624
	2	186.7	478.7	265.5	.411
		741.6	153.3		
		273.1	861.4		
	4	1186.6	410.8	219.8	.273
^{178}Pt	0	217.8	421.9	210.8	.696
	2	204.1	0	[210.8]	[.490]
		[288.5]	170.1		
		[483.1]	427.1		
	4			[210.8]	[.257]
^{176}Pt	0	75.2	0	166.3	.90
	2	367.8	443 ^a	[166.3]	[.703]
		[449.8]	263.9		
			564.1		
	4	[638.4]		[166.3]	[.325]

Table 3.2: Band mixing parameters for $^{190,192,194}\text{Pb}$ taken from Ref. [58].

Nucleus	J_C	unperturbed energies (keV)	experimental energies (keV)	$ V $ (keV)	$ \alpha_{J_C} $
^{194}Pb	0	2.7	0	50.9	.997
		927.9	930.6		
	2	1066.1	965.0	157	.834
		1207	1308.2		
^{192}Pb	0	3.5	0	52.0	.995
		764.9	768.5		
	2	1066.9	853.8	191	.668
		1025	1237.8		
^{190}Pb	0	>9	0	>80	<0.99
		<649	658		
	2	871	773	[191]	.457
		1078.1	[1176]		

3.5 Choice of phenomenological parameters

3.5.1 Results

As was discussed earlier in Sec. 3.3.2, only one eigenstate of the Hamiltonian (3.28) is considered as the physical solution representing the ground state of the α -parent nucleus. Initially, the calculations were performed using WS parameters found by DeVries in the α -scattering experiments on ^{208}Pb [45] (set A: $r_{\text{WS}}=1.32$ fm, $a_{\text{WS}}=0.65$ fm). Subsequently, for each nucleus, the two parameters, i.e., the quadrupole coupling strength κ_2 and the monopole “radial shift”, ΔR_{WS} , were adjusted until the theoretical relative intensities best matched experimental values (local fit). We found that these parameters fall into separate ranges for each group of isotopes. Therefore, the next step was to calculate a global least squares fit with one set of parameters for all the isotopes of each element. The resulting relative intensities are listed in Table 3.3. The associated κ_2 and ΔR_{WS} parameter sets are listed in Table 3.4.

For the Hg decays, the quadrupole coupling strength κ_2 obtained in the

Table 3.3: Calculated α -decay relative intensities (in %) obtained in local and global fits (see text). Experimental relative intensities (I_{exp}) [30,3] are included for comparison.

Daughter	J_C^π	I_{exp}	$I_{\text{th}}^{\text{local}}$	$I_{\text{th}}^{\text{global}}$	$t_{1/2}^{\text{Exp}}$	$t_{1/2}^{\text{Theory}}$
^{186}Hg	2_1^+	8.4E-2	8.4E-2	1.1E-1	1.2 min	3.2E-1 s
	0_2^+	1.5E-2	1.5E-2	1.0E-2		
	2_2^+		1.4E-3	1.2E-3		
	4_1^+		1.3E-7	1.5E-5		
^{184}Hg	2_1^+	6.0E-2	6.0E-2	1.3E-1	24.5 s	4.7E-3 s
	0_2^+	9.5E-2	9.5E-2	1.7E-1		
	2_2^+		1.1E-2	2.2E-2		
	4_1^+		3.4E-4	7.0E-4		
^{182}Hg	2_1^+	<7E-2	7.0E-2	4.3E-3	4.79 s	1.9E-4 s
	0_2^+	2E-1	2.0E-1	2.5E-1		
	2_2^+		2.8E-1	2.3E-1		
	4_1^+		1.4E-3	8.0E-4		
^{180}Pt	2_1^+	4.2E-1	4.2E-1	6.4E-1	30.6 s	6.0E-2 s
	4_1^+		1.5E-2	2.9E-2		
	0_2^+	1.6E-1	1.6E-1	1.5E-2		
^{178}Pt	2_1^+	8.4E-1	6.7E-1	2.1E-1	10.83 s	2.1E-3 s
	4_1^+		9.2E-4	8.8E-3		
	0_2^+	3.4E-1	1.8E-1	5.8E-2		
^{176}Pt	2_1^+	1.0E-1	2.7E-1	7.0E-1	2.8 s	2.6E-4 s
	4_1^+		8.1E-5	4.3E-3		
	0_2^+	7.8E-2	2.5E-1	9.3E-1		
^{194}Pb	2_1^+	<3.6E-4	3.7E-4	2.8E-4	1.76 min	6.1E-3 s
	0_2^+	1.3E-3	1.7E-3	1.3E-3		
	2_2^+		3.5E-5	2.7E-5		
	4_1^+		8.5E-11	1.4E-10		
^{192}Pb	2_1^+	8.4E-3	8.4E-3	1.3E-3	5.8 s	4.7E-4 s
	0_2^+	2.2E-2	2.4E-2	3.2E-2		
	2_2^+		4.6E-4	1.6E-4		
	4_1^+		1.2E-8	5.5E-9		
^{190}Pb	2_1^+		3.0E-2	6.1E-3	0.44 s	4.3E-6 s
	0_2^+	2.4E-1	3.0E-1	2.6E-1		
	2_2^+		2.3E-3	9.7E-4		
	4_1^+		1.3E-6	6.6E-7		

Table 3.4: The coupling constants κ_2 and ΔR_{ws} obtained in local and global fit performed in this work.

Parent nucleus	κ_2 (MeV/fm ²)	ΔR_{ws} (fm)
¹⁹⁰ Pb	0.029	0.030
¹⁸⁸ Pb	0.024	0.015
¹⁸⁶ Pb	0.045	0.047
Global Pb	0.037	0.035
¹⁸⁴ Hg	0.028	0.161
¹⁸² Hg	0.016	0.077
¹⁸⁰ Hg	0.010	0.001
Global Hg	0.052	0.041
¹⁹⁸ Po	0.025	0.110
¹⁹⁶ Po	0.033	0.142
¹⁹⁴ Po	0.020	0.126
Global Po	0.035	0.118

local fit has a fairly similar value for all three decays. However, ΔR_{ws} becomes very small in order to best fit the decay data of ¹⁸⁰Hg. This might be due to the fact that the ground state of the daughter nucleus, ¹⁷⁶Pt, has different deformation as compared to the 2⁺ and 4⁺ yrast states. Indeed, according to Ref. [44], the ground state of ¹⁷⁶Pt is triaxial and not oblate as has been assumed in our calculations. Consequently, the global fit for the Hg decays is in worse agreement with experiment than for the other isotopic chains considered. As mentioned above, the α -rotor model does not predict half-lives (i.e., absolute α -decay widths) correctly due to the neglect of the α -formation probability in the assumed parent state ϕ_M^I , but it is able to reproduce the relative intensities to a rather good accuracy. With the exception of the exaggerated feeding to the excited 2₂⁺ state for decay of ¹⁸⁶Pb all the other cases are easily made to fit well with experiment, both locally and globally.

Table 3.5: Relative intensities of α decay fine structure lines predicted for various mixing ratios for $^{182,184}\text{Hg}$ decay daughters $^{178,180}\text{Pt}$.

^{178}Pt α_2^2	0.1	0.15	0.2	0.25	0.30
2_1^+	3.78E-1	3.33E-1	2.72E-1	1.92E-1	9.53E-2
4_1^+	3.03E-2	2.09E-2	1.36E-2	7.75E-3	3.12E-3
0_2^+	5.49E-2	5.62E-2	5.73E-2	5.83E-2	5.92E-2
^{180}Pt α_2^2	0.1	0.15	0.2	0.25	0.30
2_1^+	6.19E-1	6.35E-1	6.46E-1	6.49E-1	6.37E-1
4_1^+	3.88E-2	3.16E-2	2.57E-2	2.06E-2	1.61E-2
0_2^+	1.43E-2	1.45E-2	1.47E-2	1.50E-2	1.52E-2

3.5.2 Effects of mixing on relative intensities of fine structure α decays

In this section the effect of mixing on the predicted relative intensities is discussed. While holding all other variables constant, the variation in mixing of the Hg decay daughters $^{178,180}\text{Pt}$ was investigated and is shown in Table 3.5. It is noted that the variation in α_2^2 produces significant variations of $I_{2_1^+}$ and $I_{4_1^+}$, but little variation in $I_{0_2^+}$ in the decay of ^{182}Hg . In the case of ^{184}Hg only $I_{4_1^+}$ shows significant variation.

The effects of mixing variation for Pb decays is shown in Table 3.6. Ranges of mixing values change for each nucleus to better represent variation around their most likely admixtures. Mixing has the most effect on the decay of ^{186}Pb . The dependence of the relative intensities on the mixing in the 2^+ states of ^{182}Hg is shown in Figure 3.2. Here, however, the local fit parameters, κ_2 and ΔR_{WS} , are used to make a better visual comparison with the experimental data. It is clear from this figure that the relative intensities are quite sensitive to the mixing.

This procedure of varying mixing strengths was carried out for polonium decays. The effect of changing the mixing does not have a large effect on the solution

Table 3.6: Calculated relative intensities as a function of 2^+ mixing of $^{186,188,190}\text{Pb}$ decay daughters $^{182,184,186}\text{Hg}$.

^{186}Hg					
α_2^2	0.50	0.60	0.70	0.80	0.90
2_1^+	1.16E-1	1.14E-1	1.08E-1	1.03E-1	1.04E-1
0_2^+	4.48E-2	2.87E-2	1.93E-2	1.38E-2	1.05E-2
2_2^+	1.37E-3	1.49E-3	1.48E-3	1.40E-3	1.26E-3
4_1^+	5.59E-6	2.62E-6	1.21E-7	1.76E-6	1.17E-5
^{184}Hg					
α_2^2	0.50	0.60	0.70	0.80	0.90
2_1^+	1.31E-1	1.58E-1	1.90E-1	2.26E-1	2.71E-1
0_2^+	1.62E-1	1.37E-1	1.19E-1	1.04E-1	9.22E-2
2_2^+	2.16E-2	1.67E-2	1.33E-2	1.08E-2	8.65E-3
4_1^+	7.13E-4	8.05E-4	8.20E-4	7.23E-4	4.43E-4
^{182}Hg					
α_2^2	0.25	0.35	0.45	0.55	0.65
2_1^+	4.43E-2	6.85E-2	1.03E-1	1.47E-1	1.98E-1
0_2^+	2.49E-1	2.83E-1	2.92E-1	2.90E-1	2.81E-1
2_2^+	2.23E-1	1.35E-1	9.33E-2	6.87E-2	5.29E-2
4_1^+	8.09E-5	1.15E-3	1.47E-3	1.78E-3	1.96E-3

that is taken to model the data, which is displayed in Figure 3.3. This is probably a result of how the ground band has been modeled, as we are treating it as a very slightly prolate deformed rotor [61]. Therefore, at this stage the model is much more sensitive to changes in the κ_2 and ΔR_{WS} parameters. For Po decays it should be noted that mixing coefficients are available for only two spin members, and the mixing coefficients between 2^+ states are more from isotope systematics and less from spectroscopic information and could be less reliable, but this does not present a problem, because the calculated relative intensities are not strongly sensitive to them.

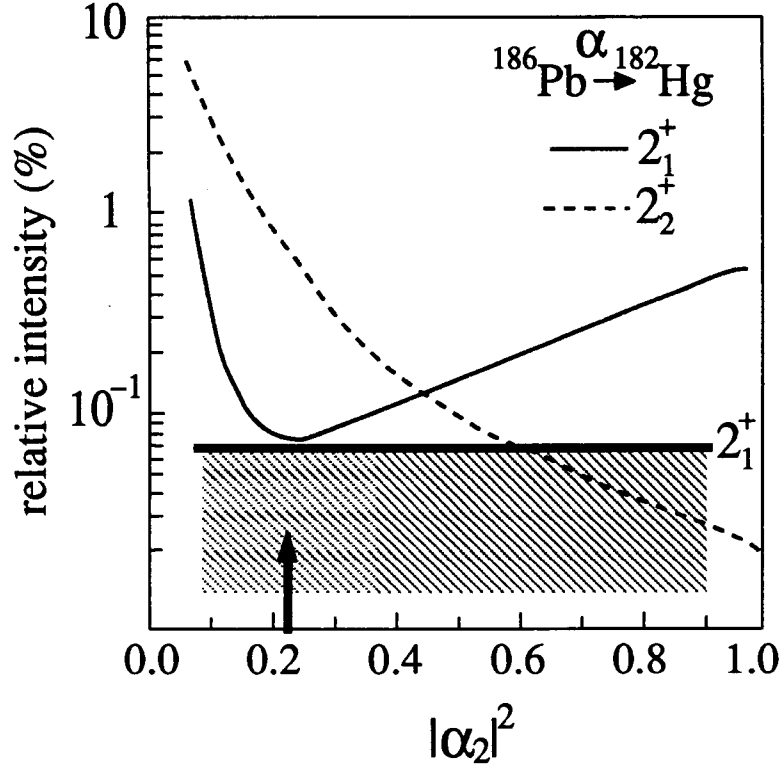


Figure 3.2: Relative intensities of ^{186}Pb decay versus the mixing amplitude α_2^2 of the 2^+ states in ^{182}Hg . The dashed and light solid lines represent calculation. The arrow on the abscissa shows the calculated mixing amplitude from Table 3.1. The upper limit on the experimental relative intensity for the 2_1^+ feeding is also shown. In the calculations, the parameters κ_2 and ΔR_{WS} obtained from the local fit (Table 3.4) were used.

3.5.3 Choice of Woods-Saxon parameters

To investigate the influence of the optical model parameters on relative intensities, a second set of WS parameters proposed in Ref [62] (set *B*: $r_{\text{WS}}=1.05$ fm and $a_{\text{WS}}=0.70$ fm) was employed. The main difference between the sets *A* and *B* is a much smaller WS radius in set *B*.

A least squares minimization was performed, for a set of isotopes, for both parameter sets to establish optimal values of κ_2 and ΔR_{WS} . For the intermedi-

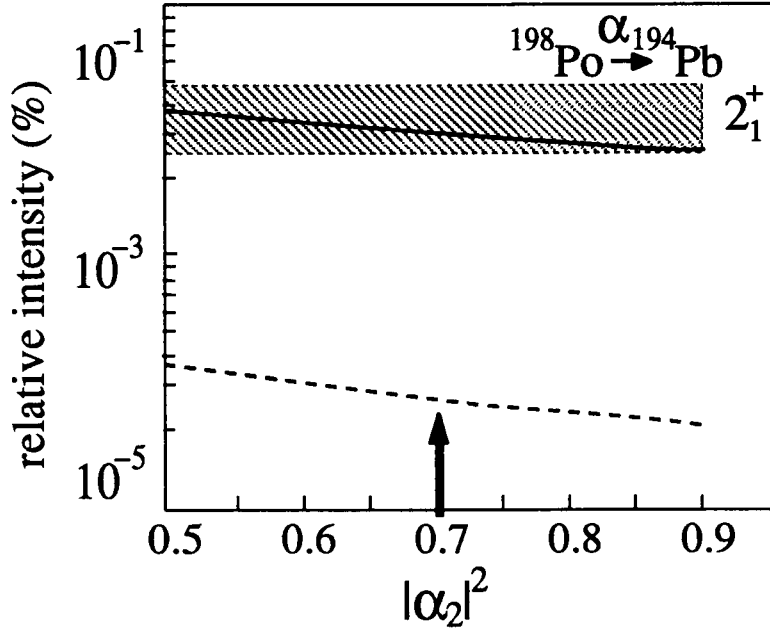


Figure 3.3: Relative intensities of ^{198}Po decay versus α_2^2 the mixing amplitude of the 2^+ states in ^{194}Pb . See previous figure for symbols. The gray bar stands for the experimental value; its width corresponds to the experimental uncertainty.

ate values of r_{WS} , the calculations were carried out by linearly interpolating other parameters between the two limiting “best-fit” values. For the Pb nuclei, the parameters κ_2 and ΔR_{WS} were found to change very little. The variation of the relative intensities for the Pb decays as a function of the WS parameters, is shown in Fig. 3.4. The relative intensities show relatively little variation as a function of WS parameters. However, the calculations show that when using WS parameters close to those of set B , better agreement with experiment is obtained for the eigenstate of Hamiltonian (3.28) corresponding to the solution “2” rather than solution “3”,

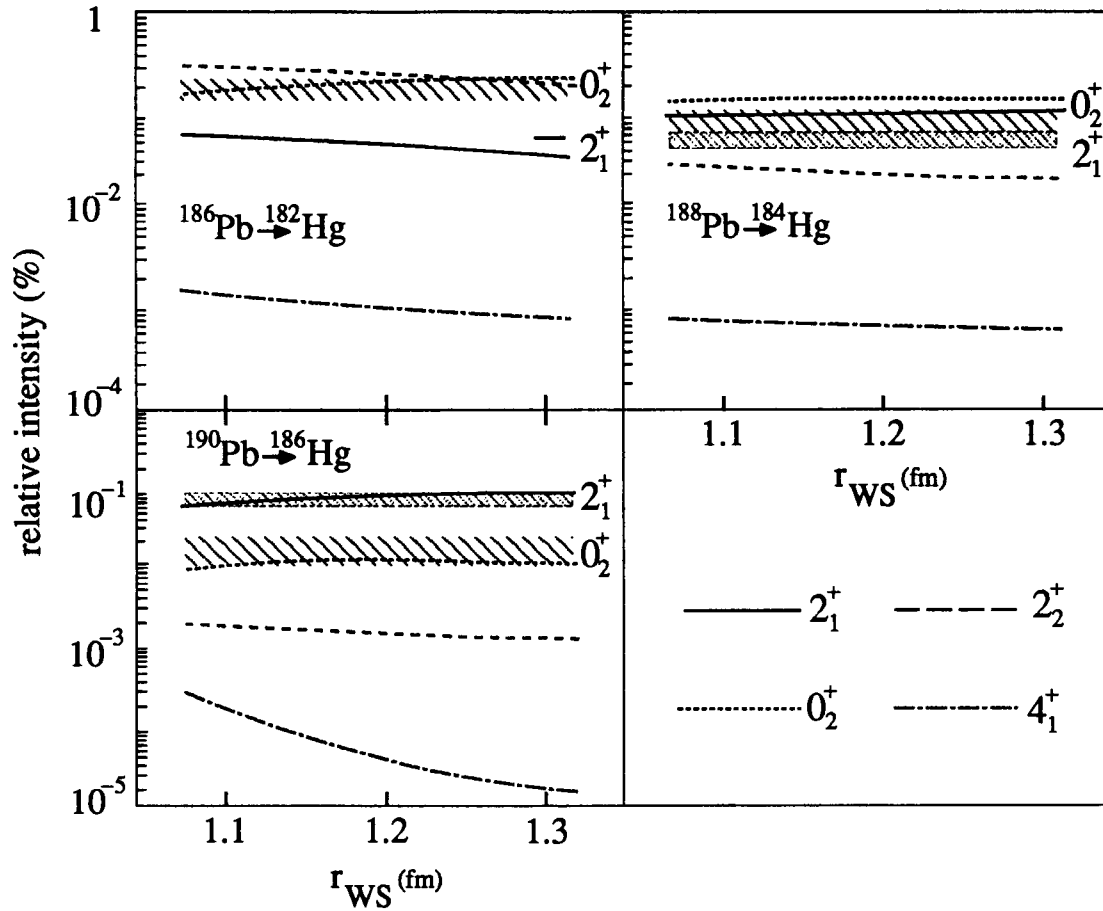


Figure 3.4: Dependence of the relative intensities of $^{186,188,190}\text{Pb}$ on the WS radius R_{WS}

the latter being optimal for the Pb decays using A parametrization. This behavior is illustrated in Fig. 3.5. The major difference between the solutions is that the ΔR_{WS} parameter becomes two times larger for solution “3”, tuned to match experimental data with a local fit, than for solution “2” for the WS parameter set B . Table 3.7 provides this information.

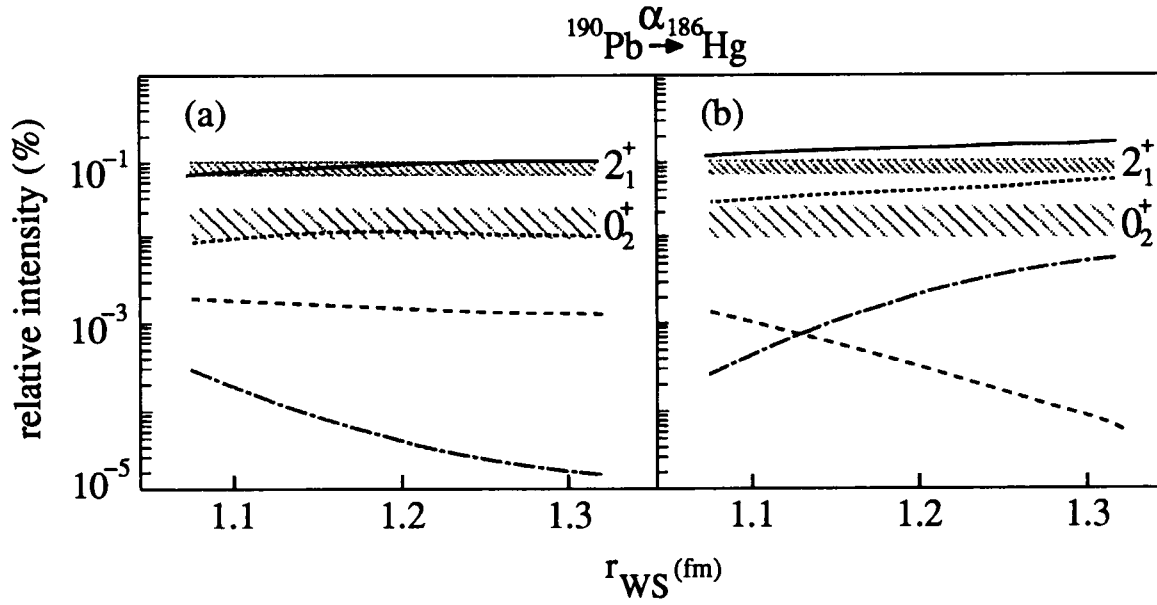


Figure 3.5: Same as in Figure 3.4 but for two solutions of ^{190}Pb , namely solution “3” (a) and solution “2” (b).

Table 3.7: Set of ΔR_{WS} and κ_2 parameters for Soln. “2” vs Soln. “3” for local fit of ^{190}Pb using WS parameter set *B*.

Solution	ΔR_{WS}	κ_2	Relt. Int.
2	0.01768	0.024675	8.40E-2
			1.50E-2
			7.37E-4
			1.03E-4
			3.99E-6
3	0.02876	0.0350	8.40E-2
			1.50E-2
			2.74E-3
			2.37E-4
			5.78E-6

3.6 Discussion concerning choice of solution

Part of the analysis in this work has been concerned with the choice of a particular solution. Fortunately, in most cases only one solution provides a reasonable fit of the experimental data. For instance, for the ^{186}Pb decay, only solution number "3" can match experimental relative intensities. Moreover, this choice is supported by the fact that this solution is rather sensitive to the mixing between 2^+ states, in agreement with what is observed experimentally [30, 63]. Similarly, solution "3" agrees best with experimental data for the α decays of ^{188}Pb and ^{190}Pb .

Figure 3.6 gives the results for Po decays. Figure 3.6 (a) are the solutions which agree best with experiment. Although the solutions were stable, tuning to the same solution required a radical change in ΔR_{WS} . In addition and, in contrast to the modeling of the Pb and Hg decays, the neighboring solutions with Po decays are not as clearly distinguished as the solution found to easily fit with the experimental data. For this reason it was decided to search for a complimentary set of κ_2 and ΔR_{WS} parameters that caused a complimentary set of solutions that agreed with experiment globally. The motivation for this study is from the question as to what governs the choice of solution if they both fit the experimental relative intensities. For a much higher ΔR_{WS} parameter the next higher solutions in Po occur for all three decays, as is represented by Figure 3.6 (b). Furthermore, these same solutions can be maximized for the WS parameter set B and interpolated as above.

The pattern that emerges is that in the WS variation, κ_2 is stable. As with the tuning of the previous solutions ΔR_{WS} changes when going from parameter set A to parameter set B . Overall the ΔR_{WS} parameter is larger by a factor of about 3 to tune the next higher solution number into the closest agreement with experiment globally. Furthermore this appears to be analogous to the situation of two emerging solutions in ^{190}Pb . This we interpret as resulting from the change in the nuclear surface condition for the alpha particle. As ΔR_{WS} increases the model

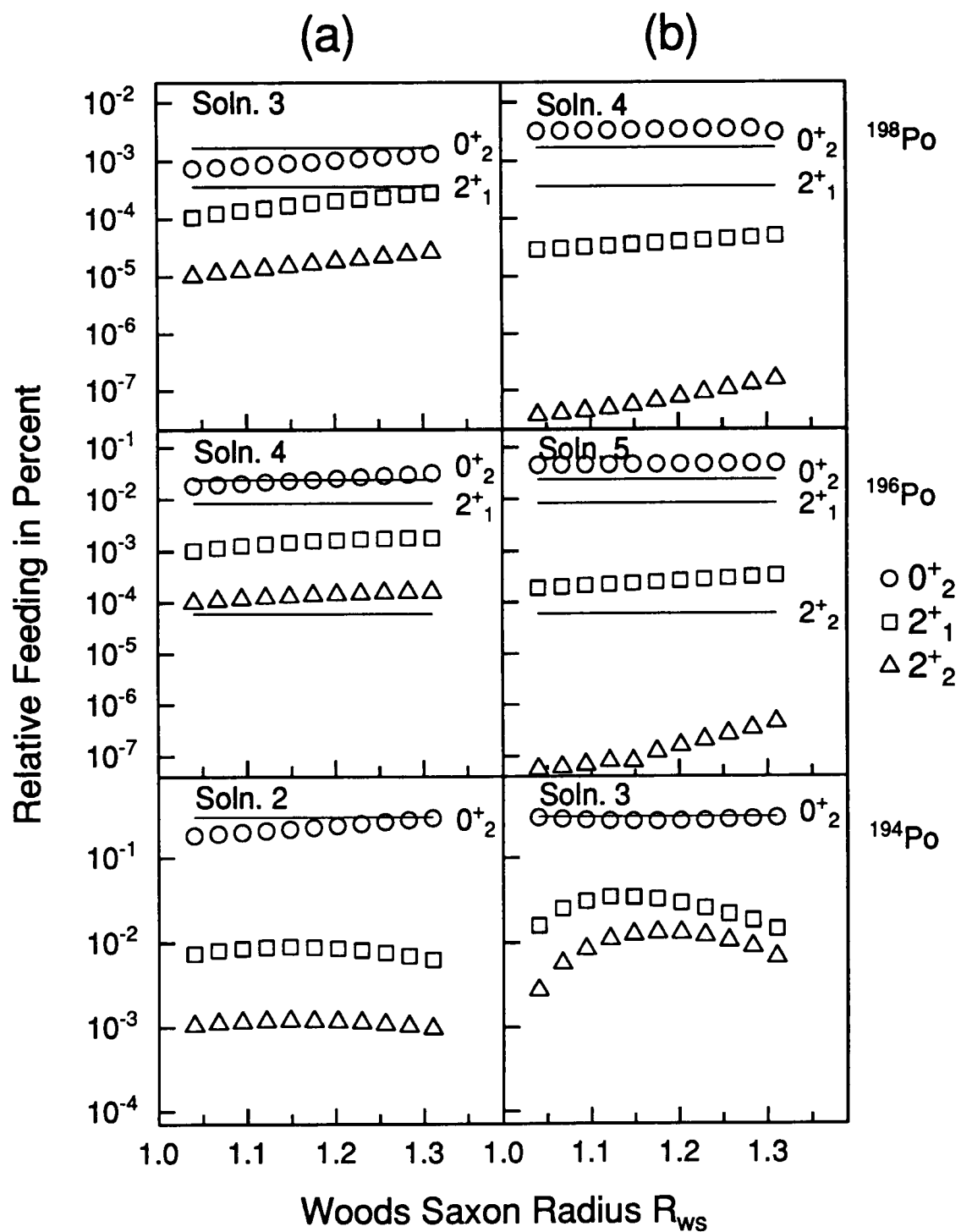


Figure 3.6: The relative intensities for the α -decay of Po isotopes to the excited Pb states as a function of WS radius r_{ws}

prefers a higher energy solution in order to keep the alpha particle at a certain level about the nuclear surface. With increasing nuclear radius in the coexisting core the single-nucleon orbitals may begin to have components from several different shells. The lower energy α orbital becomes composed of single-nucleon orbitals which are the same as some of the occupied nuclear orbitals. Therefore, the Pauli-exclusion principle has more chance of being disobeyed, and the higher energy solution is needed to model the α orbital correctly.

Part of the analysis has been concerned with what reasons there are for choosing a particular solution. Sometimes only one solution can be made to fit to experimental data. In modeling the decay of ^{186}Pb , for instance, only solution number “3” presents itself as being sufficiently tractable for matching experimental intensities. However, we see evidence that this solution has validity, because its only fit of the 2^+ mixing is near that found from our band mixing calculation. In proceeding to model ^{188}Pb and ^{190}Pb we also find that solution number “3” in energy is the only solution that can model the experimental data. Coupled to this is the fact that the resulting κ_2 and ΔR_{WS} parameters are close to those for modeling the decay of ^{186}Pb . In ^{190}Pb another solution presents itself when using WS parameter set B. However the ΔR_{WS} parameter needed to fit this solution to experiment is 1/3 the size of the local ΔR_{WS} parameter. The recipe that presents itself in modeling this isotopic chain is that one chooses solutions that fit experimental data and which respond in an expected fashion to changing physical input.

3.7 Discussion

First the effect of mixing variation was studied. It is seen that mixing has a profound effect on Hg and Pb relative feeding. It has more of an effect for the lower neutron number members of these groups of isotopic decays. For ^{190}Pb the effect of changing mixing is less pronounced. For polonium decays the mixing has less

effect, which enabled a study of other physical effects more intensively. Therefore, it could be shown that shape coexistence in the daughter nuclei does in fact drive the feeding to excited states.

Finally the WS parameters, which change the conditions for the α particle were varied. To this end Hg decay and Pb decay solutions were found to be relatively constant, since with global tuning the phenomenological parameters did not change to a large degree. The Po nuclei, however, do require changes of k_2 and ΔR_{WS} in order to maintain global solutions with good relative intensities. This quality of the solutions of ^{190}Po decay is also true, and in both cases the relative intensities change less with mixing coefficients. There is an additional property of this restricted case within this group of nuclear decays, which is that there are more than one solution available that can be tuned to be in agreement with experimentally measured relative intensities. The critical parameter for attaining these fits is ΔR_{WS} , which controls the monopole mixing. In the Pb nuclei the monopole interaction is thought to cause the 0^+ mixing, which in turn is thought to affect the feeding to the excited 0_2^+ states [58].

In Chapter 2 an analysis of α -decay relative intensities in terms of so-called "HF values" was presented. Furthermore Figure 2.2 seemed to indicate an anomaly in HF values for light-Pb region nuclear decays to the first excited 2_1^+ states. The blame for the apparent variation in the HF values was placed on the change in formation probabilities. However in this chapter it has been demonstrated that much of the variation in HF values can be removed, if experimentally determined non-spherical α -core potentials are incorporated in the calculation of one-body escape probabilities for the preformed α -particle.

CHAPTER 4

Results of feasibility study in search of a superdeformed bandhead

4.1 Introduction: experimental objective at ISOLDE

A superdeformed (SD) nucleus is an interesting example of a structure whose single particle configuration balances delicately with an accompanied deformation. Hence, the nuclear mode is simultaneously collective with a corresponding single particle structure. SD shapes have been observed at low angular momentum only for actinide nuclei, which are also referred to as fission isomers [64, 65, 66, 67]. A number of other SD states have been identified in recent years at higher angular momenta, $I \geq 16$ in lanthanide nuclei and for $I \geq 10$ in the neutron-deficient Hg-Pb region [68, 69]. The nuclear potential which produces a SD configuration can be viewed as having a localized secondary minimum, of elevated energy compared to the ground state configuration. When low-spin SD states inside a second well of the nuclear potential are populated, these states have to deexcite by tunneling through one of the barriers of the nuclear potential. For actinides, one of the modes of decay is to “fission”, in which case the nucleus severs into two different nuclei. In the neutron-deficient Pb-Hg region, SD bands are observed primarily to feed normal deformed and ground bands. In addition there is some speculation that the neutron deficient Pb region nuclei can also have isomers produced, which fission [70].

Our experiment performed at ISOLDE was aimed at making a determination of isomeric ratios of nuclear activity, which would be likely candidates for

feeding a SD bandhead at low spin. SD configurations have been observed at low angular momentum only for actinide nuclei, in the form of identified transitions, of specified energy to ground mode configurations. Although minima are predicted to occur in the nuclear potential for superdeformed shapes at low spins in light mercury, lead and polonium nuclei [61], these have not yet been established experimentally. Previous experiments have been aimed at populating SD bands for the purpose of searching for connecting transitions out of the SD band to lower lying configurations. Perhaps due to lack of sensitivity these attempts have thus far been inconclusive [71]. Recently however, due to increased sensitivity made possible by increased multiplicity of large detector arrays, such as GAMMASPHERE, γ -ray transitions connecting SD bands to normal deformed states have been observed in several nuclei. [72, 73, 74, 75].

The present experiment at ISOLDE was an attempt to measure the efficiency by which activity could be produced, which would feed candidate nuclei for an observable SD bandhead. Candidate nuclei for low-lying SD states were ^{198}Po , ^{194}Pb and ^{190}Hg . The last species involved one of the performed experiments. Its objective was to populate SD states in ^{190}Hg via production and $ec+\beta^+$ decay of ^{190}Tl . Another experiment was devoted to the production of ^{198}At and ^{194}Bi which would feed ^{198}Po and ^{194}Pb via $ec+\beta^+$ decay. These activities were produced indirectly through a decay chain from the single nuclear activity of ^{202}Fr . A third experiment measured ^{204}Fr activity. While the nuclei fed by this decay chain were not thought to contain SD structures, this measurement was carried out in order to observe the ratio of low spin to high spin abundances under the conditions of a high production rate of ^{204}Fr . A short calibration run was carried out, while making ^{212}Fr .

The primary emphasis of this thesis is the study of nuclear α decay. It was put forth in our proposal to ISOLDE that one way to detect SD bandheads is by

observing a hypothetical α decay transition, which would connect the SD bandhead to either the ground state of the daughter or the perchance existing SD band of the daughter. This proposed mechanism involves connecting α decay transitions between highly deformed nuclei, and therefore is interesting from the theoretical standpoint of how shapes of the daughter and parent might influence the relative feedings between the normal deformed bands and the SD bands.

Although alpha decays were observed in our test run at ISOLDE, they are in odd-odd nuclei that seem to exhibit single particle behavior. However, these nuclei eventually feed even-even nuclei exhibiting collective properties via $ec+\beta^+$ transition out of the alpha decay chain. Finally there is another aspect of the ISOLDE results, which was a study of the background products that occurred during the experiment, which has relevance for future attempts at performing experiments in this nuclear region at ISOLDE.

4.2 Experimental setup

The primary activity was produced directly by proton spallation of thorium. The Proton Synchrotron Booster at CERN provided 1-GeV proton pulses of 2×10^{13} protons with a variable repetition time in multiples of 1.2 s between pulses. In three separate experiments, mass 190, 202 and 204 nuclei were directed through the low-mass beam line of the General Purpose Separator. These activities were collected on a movable tape transport provided by the ISOLDE facility. A measuring station was fabricated at the University of Tennessee and installed on site at the ISOLDE facility. The placement of the detectors is indicated in Figure 4.1.

Short-lived activity was measured for α and γ radiation at the collection station, using an ORTEC model Tb-19-200-500 Si(Au) α detector, and a CFG GMX-23195 Ge γ detector. Longer-lived activity was moved via the tape transport

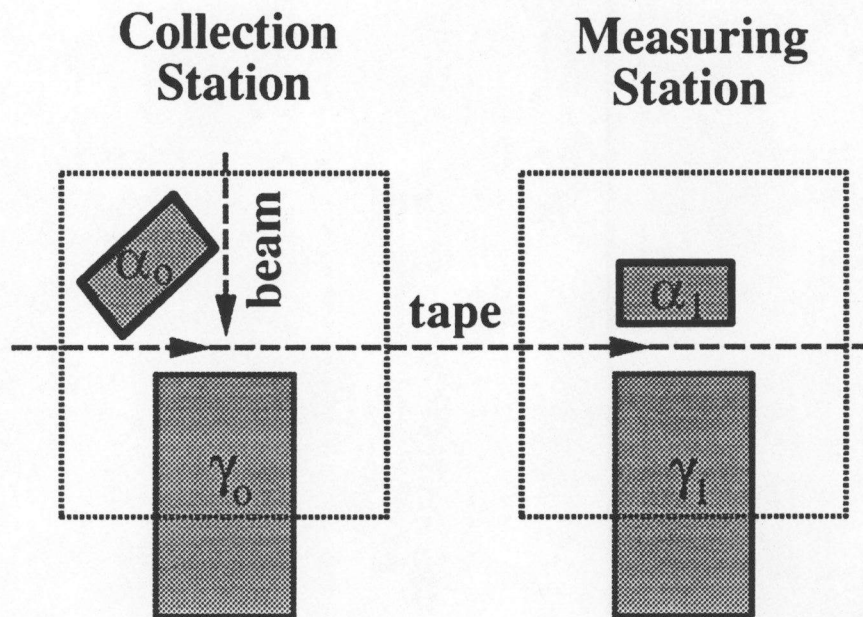


Figure 4.1: Schematic diagram of the detector setup

system to the measuring station, where two additional detectors were in place. A TB-20-300-700 Si(Au) detector was used to measure α radiation and a Silena model GEM 25195 P6 Ge detector was used to measure γ radiation of energies beyond X-rays.

Pulses of 1-GeV protons were produced at 1.2 s intervals. There were 12 pulses that were sent to different parts of the facility in a 12 cycle period of 14.4 seconds. The ISOLDE facility target typically received 5 of these pulses. For each pulse, activity was produced and directed to the collection station. A beam gate opened when a proton pulse arrived at the target and remained open for 0.6 s, as activity emerged from the source, and then closed until the next pulse came. For ^{204}Fr the beam gate stayed open approximately 0.075 seconds longer.

The ^{202}Fr activity has a half-life which is shorter than the interval between pulses. Therefore, when looking at its activity over time, it was possible to follow the buildup and decay of ^{202}Fr during each beam pulse cycle. Figure 4.2 shows the

time profile of the pulsed activity of ^{202}Fr , measured during the experiment. The periodic behavior of the pulses over 14.4 s is depicted in Figure 4.3.

The electronics were set up to admit signals from the detectors at the first station for 1.2 seconds following the arrival of a pulse of protons. Since the pulses from the facility were separated by 1.2 s sometimes the acquisition system would admit signals in consecutive intervals, and at other times, there were periods of no counting when the proton beam had gone to other experiments. However, all data from the 1.2 s collection intervals were multiscaled into 16 spectra. The α spectra, displayed in Figure 4.2 clearly exhibit the buildup of ^{202}Fr activity during the first 4 or 5 bins and the decay for the remaining bins.

For the ^{202}Fr experiment activity was moved from the collection station every 200 s to the measuring station, depicted in Figure 4.1. The collection time of 200 seconds was chosen to maximize the buildup of the low and high spin isomers of ^{194}Bi , whose half-lives are around 2 minutes. The α and γ spectra were multiscaled in 8 separate time bins, in order to help in identifying activity decaying with the ^{194}Bi half-life. The discriminators for signals coming into the ADC from the measuring station detector were set to exclude low-energy γ -rays and X-rays. This was to facilitate greater sensitivity to higher energy transitions, indicative of activity from decay of bismuth.

4.3 External calibration

The energy and efficiency calibration for γ rays was done with a calibrated source from Damri Scientific. The α calibration utilized the facility's production of ^{212}Fr , where the α efficiency was determined using the branching ratio of ^{212}Fr found from the literature [76]. The results for γ efficiencies found for the γ detectors at the collecting and measuring stations (also referred to as the first and second stations) are given in Table 4.1.

Multiscaled activity of mass 202

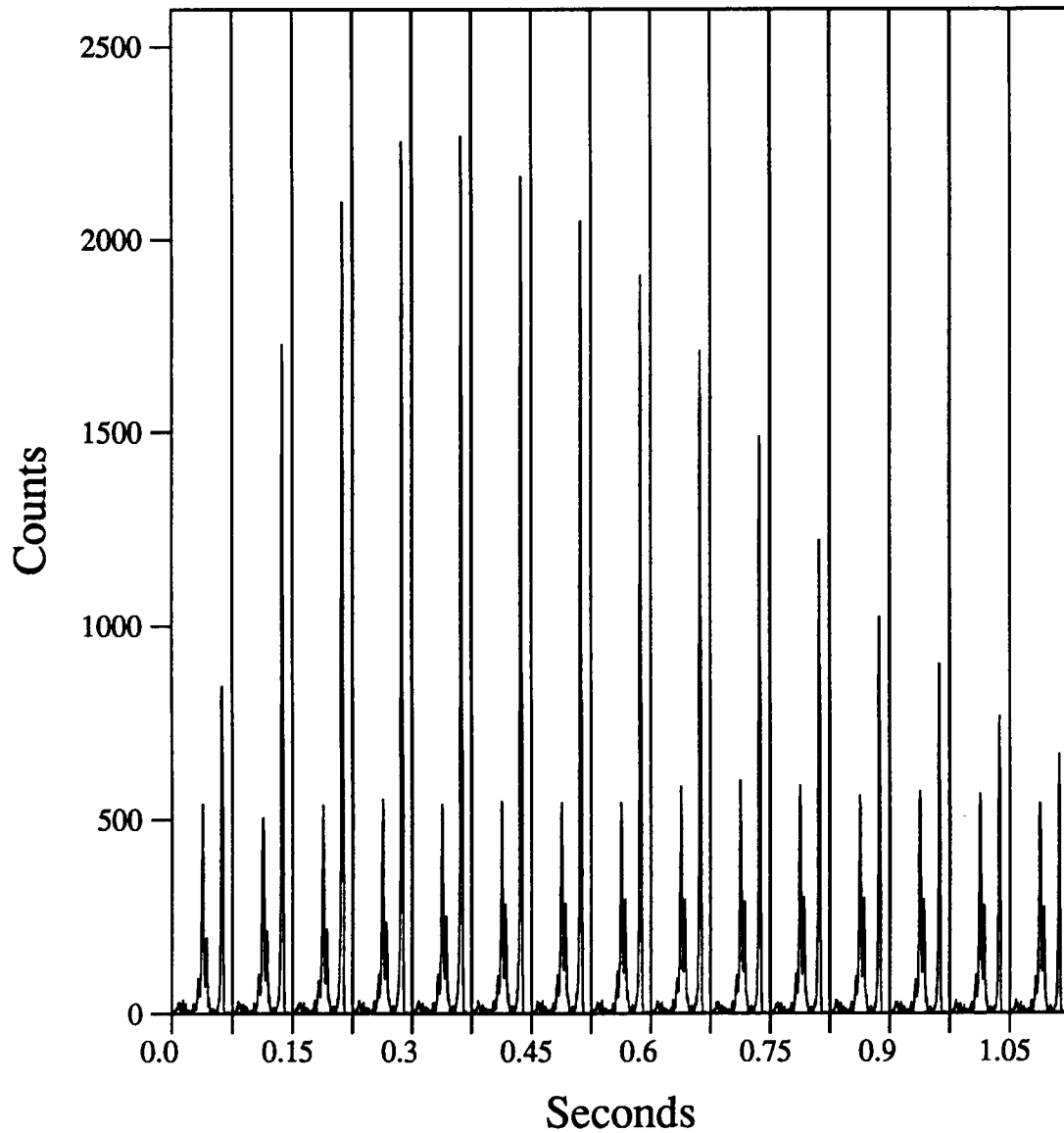


Figure 4.2: Illustration of multiscaled pulsed activity for the mass 202 run. Each slice, or plane represents successive 0.075 second collection periods. The spectra are individual α decay spectra of activity primarily from ^{202}Fr , which shows the largest peak growing as the beam gate is open, and decaying as the beam gate is closed. Also visible are the activities from ^{198}At , whose isomers have longer $t_{1/2}$ than ^{202}Fr .

Periodic behavior within supercycle

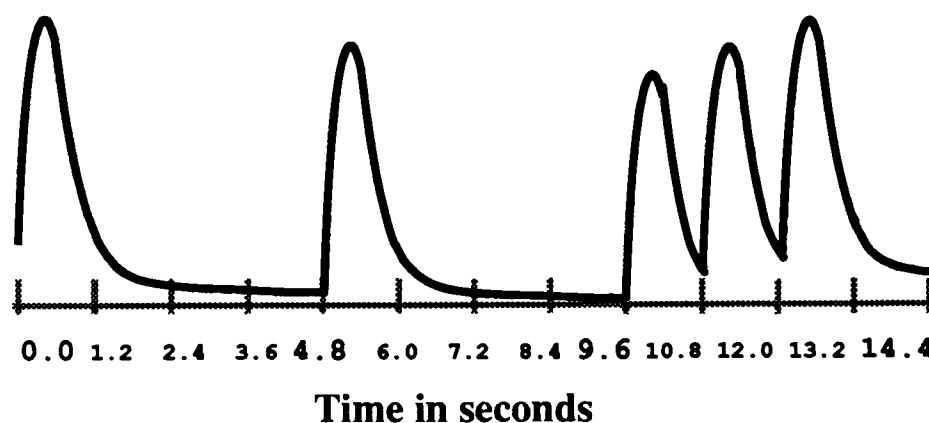


Figure 4.3: This is an illustration of the approximate periodic behavior of activity arriving in synchronization from the proton booster. The individual peaks result from the build-up and decay after each arriving pulse.

Table 4.1: Data needed for efficiency calibration of the γ detectors.

Transition	1st Stat. Cnts/s	2nd Stat. Cnts/s	Source Cnts/s	1st Stat. Eff. %	2nd Stat. Eff. %
122.1	19.21	8.99	345	5.57	2.61
166.3	13.75	7.45	311	4.42	2.40
391.6	24.73	15.01	1204	2.05	1.25
661.2	30.70	18.41	2058	1.49	0.89
897.6	18.48	11.72	1974	0.94	0.59
1172.8	19.61	13.94	2880	0.68	0.48
1332.3	17.88	12.53	2883	0.62	0.43
1835.9	8.82	6.75	2085	0.42	0.32

Table 4.2: Empirical counts and relative intensities for the decay of ^{212}Fr .

Branch	Transition in KeV	Intensity per 100 Decays	1st Stat. Counts	2nd Stat. Counts
^{212}Fr ϵ decay	227.72	100×0.426	3.90E+5	1.35E+5
^{212}Fr α decay	6383.3 6406.2	24.0×0.43 22.0×0.43	1.02E+5 1.03E+5	3.11E+4 2.68E+4

Table 4.3: Alpha detector absolute efficiencies

	1st Stat.	2nd Stat.
Absolute α eff.	3.9%	1.9%

The decay branches for ^{212}Fr from the nuclear data sheets [76] are listed in Table 4.2 for convenience. The branching ratio for α -decay is 0.43, and that for $\epsilon\epsilon+\beta^+$ is 0.57. Note that the factor per 100 decays for α 's plus γ 's does not add up to 100, because the 227.72 γ conversion coefficient of 0.339. Hence $(1.339 \times 0.426) + 0.43 = 1.00$. The 6383.3 keV and 6406.2 keV lines were used because they are well resolved. Counts in the 6261.9 keV line are not used, as this transition occurs in a doublet with the 6264 keV α decay transition from the decay of ^{212}Rn . The efficiencies of the α detectors are now calculated according to Eqn. (4.1) using the information in the above tables.

$$\epsilon_{\alpha} = \frac{(C_{nts.\alpha})\epsilon_{\gamma}0.426(100.0)}{(C_{nts.\gamma})0.43(24.0+22.0)} \quad (4.1)$$

In Table 4.3 are listed the α detector absolute efficiencies found in this manner.

4.4 Decay chain of ^{202}Fr

The α spectrum for the mass=202 run is shown in Figure 4.4. The

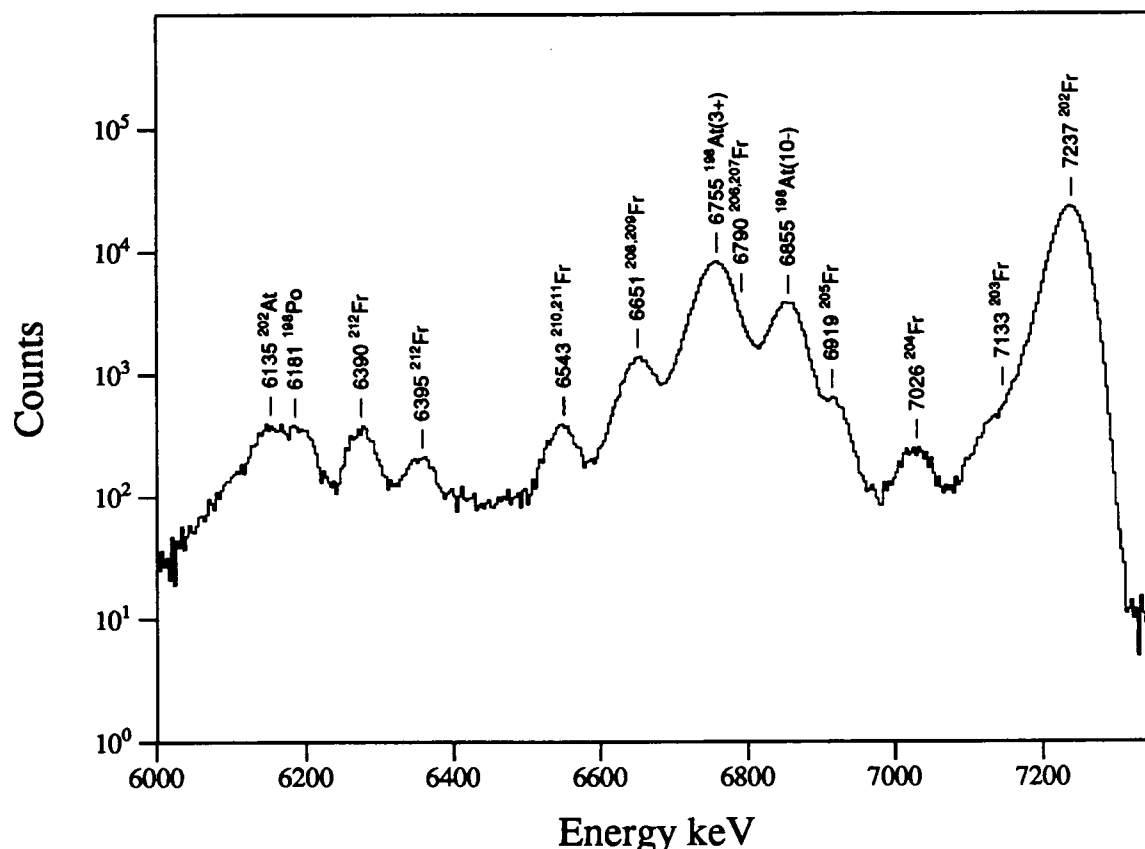


Figure 4.4: Alpha spectrum for mass 202 activity. Depicted are the suggested nuclide assignments indicating the primary activity being mostly from that of ^{202}Fr and ^{198}At .

transitions labeled ^{202}Fr , $^{198}\text{At}(3^+)$ and $^{198}\text{At}(10^-)$ are known transitions found in a previous work [77]. In addition there are provided other assignments for transitions that are thought to be impurities from the source. These assignments are based on energetics and $t_{1/2}$ information.

The objective was to establish *i)* the $ec+\beta^+$ decay branches for ^{198}Po and *ii)* the ratio of production of low-spin ^{194}Bi to high-spin ^{194}Bi . The decay scheme for

^{202}Fr is provided in Figure 4.5. The activity of ^{202}Fr was found to have a half-life

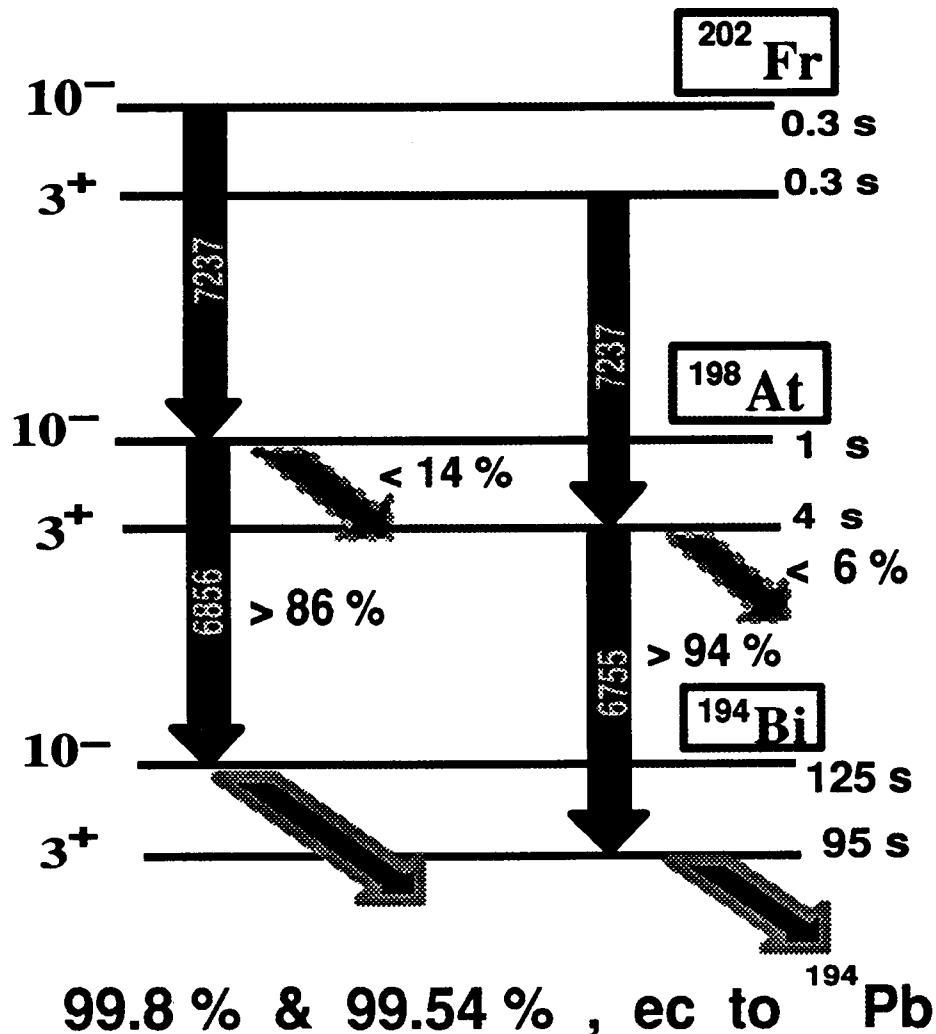


Figure 4.5: Partial decay chain for ^{202}Fr . The electromagnetic transitions and α -decay transitions are shown as vertical lines with energies in keV and branching ratios given for each. Slanted arrows represent $ec+\beta^+$ decay and the branch for this mode is indicated. The spin assignments and the half-lives of the two isomers in ^{202}Fr , ^{198}At and ^{194}Bi are given.

of 0.30(4) s, which is in agreement with literature values [77, 78]. In Table 4.4 the half-lives are given along with values for half-lives found in the literature [77, 78]. In earlier work [77] the peak at 7237 keV was assigned as a doublet, from decay of

Table 4.4: Half-life information from this work, and from available literature.

Nuclide	Exp. $t_{1/2}$	$t_{1/2}$ [77]	$t_{1/2}$ [78]
$^{202}\text{Fr}(10^-)$	0.30(4) s	0.34(4) s	$0.23 \pm^{+0.14}_{-0.05}$
$^{202}\text{Fr}(3^+)$	0.30(4) s	0.34(4) s	$0.23 \pm^{+0.08}_{-0.04}$
$^{198}\text{At}(10^-)$	1.5(6) s	1.0(2) s	$1.3 \pm^{+0.8}_{-0.3}$
$^{198}\text{At}(3^+)$	5.0(10) s	4.2(3) s	$4.6 \pm^{+1.8}_{-1.0}$

two isomers of ^{202}Fr . This is because the daughter nuclei also subsequently α decay, and have assigned to them spins 3^+ and 10^- . The isomers of ^{202}Fr appear, to within experimental uncertainty, to be decaying with the same half-life of 0.3 s. Because these states decay with the same half-life, the change of spin is not expected to be different for either decay. Furthermore, the difference in the α -decay reduced width between ^{202}Fr and ^{204}Fr is similar to the differences between ^{198}Rn , ^{200}Rn and ^{202}Rn . This suggests that no spin change is occurring in the decay from the isomers in ^{202}Fr , as mentioned in Huyse [77]. Therefore, these states were also assigned to 3^+ and 10^- levels.

In order to determine a branching ratio for ^{198}At , known γ transitions in ^{198}Po were sought. However none were found conclusively. Therefore $ec+\beta^+$ branches must be evaluated entirely from α decay data.

4.4.1 Modeling the α decay data

First the parent activity is modeled with decay equations. These equations are derived from the well-known decay equation

$$\frac{dN(t)}{dt} = R(t) - \lambda N(t) \quad (4.2)$$

where $N(t)$ is the abundance of a substance as a function of time and $R(t)$ is the rate at which it is being produced. λ is the decay constant, being equal to the ratio

$\ln 2/t_{1/2}$, where $t_{1/2}$ is the half-life of the substance.

During the experiment, the period of production was governed by when the beam gate was open. The abundance of a substance present during this period is referred to as $N_c(t)$. When the beam gate was closed, the abundance can be referred to as $N_d(t)$ where the subscript d stands for the period of pure decay. In this way we can write the production-decay equations for ^{202}Fr in the following form:

$$N_{c_{\text{Fr}}}(t) = N_{c_{\text{Fr}}}(0)e^{-\lambda_{\text{Fr}}t} + e^{-\lambda_{\text{Fr}}t} \int_0^t R(t')e^{\lambda_{\text{Fr}}t'} dt' \quad (4.3)$$

$$N_{d_{\text{Fr}}}(t) = N_{d_{\text{Fr}}}(0)e^{-\lambda_{\text{Fr}}t} \quad (4.4)$$

with the boundary condition required of $N(t)$ such that

$$N_{d_{\text{Fr}}}(0) = N_{c_{\text{Fr}}}(t_g) \quad (4.5)$$

where t_g is the time at which the beam gate is closed, to stop the arrival of additional activity.

In Figure 4.6 the approximate situation that we are modeling is illustrated, where the rate of production $R(t)$ preceeds the observed decays, referred to in the diagram as simply $\alpha(t)$. Note that for a 100% α decay branching ratio the area under the two curves should be equal. The reproduction of the rate of production function was not as important for ^{202}Fr isomers as it was for modeling the subsequent decay of ^{198}At daughters. Modeling was important for At because the data acquisition system only counted in 1.2 second intervals straddling the incoming beam pulse. Furthermore, the ISOLDE facility only received about 5 out of a 12 pulse supercycle, as indicated in Figure 4.3. Since the $t_{1/2}$ of ^{198}At is comparable or longer than each isolated collection period, this resulted in much of the activity occurring when the data acquisition system was turned off. Therefore our modeling effort was necessary to estimate the total counts for ^{202}Fr and ^{198}At as these sometimes occurred outside the established collection window in different ratios of observed to total abundances.

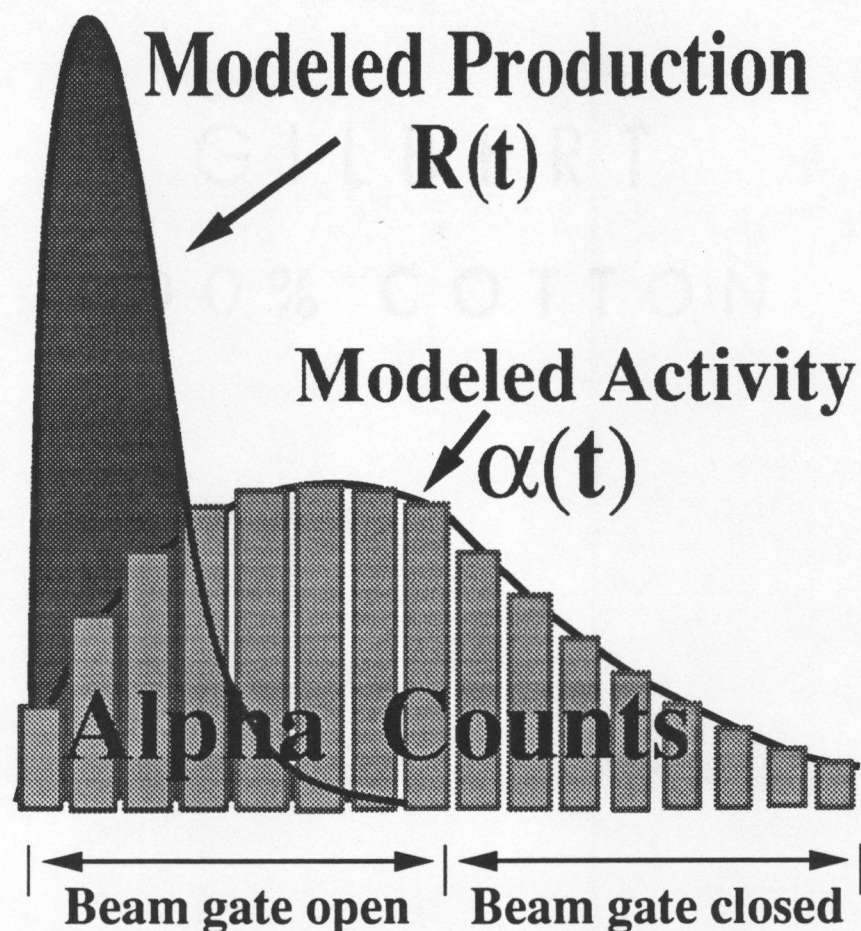


Figure 4.6: Illustration of the relation between buildup and decay of a short lived nucleus and the observed activity. Note that with a correct modeling procedure the areas under the two curves should be equal, if the corresponding branching ratio factor is taken into account.

An analytic function for the number of ^{202}Fr nuclei produced was constructed from the α counts, resulting from the decay of ^{202}Fr . Since an $\text{ec}+\beta^+$ branch was not observed, the fraction of ^{202}Fr produced which would decay by this mechanism is not considered here. One technique that appeared suitable for doing this was to first fit an analytic function, a simple polynomial, to the total number of incoming counts from the decay of ^{202}Fr . The choice of the polynomial as a fitting function is a matter of convenience. The form of the analytic function is

Table 4.5: ^{202}Fr α counts taken at the collection station during the 0.6 second build-up period.

Plane 1 0.075s	Plane 2 0.150s	Plane 3 0.225s	Plane 4 0.300s	Plane 5 0.370s	Plane 6 0.450s	Plane 7 0.525s	Plane 8 0.600s
13114	25834	31953	33962	34410	33037	30963	29160

not considered to be important, as long as the function varies smoothly. Another procedure is to determine the rate $R(t)$, for a particular nuclide being deposited on the tape, from the corresponding α decay data. This procedure is used in the upcoming section, which studies slightly longer-lived activity from ^{204}Fr .

The data for the first eight 0.075 second channels, which comprised the production period with the beam gate open, are listed in Table 4.5. To arrive at an analytic function for $N_c(t)$, we make note of the fact that the total counts, $\alpha(t)$, equal the integral of the quantity $\lambda N_c(t)$, or

$$\alpha(t) = \int_0^t \lambda N_c(t') dt' \quad (4.6)$$

where λ is the decay constant for the ^{202}Fr activity and $N_c(t)$ is the number of ^{202}Fr nuclei present on the collection tape which subsequently α decay. By fitting the total number of α counts to a smooth function representing $\alpha(t)$, we can differentiate the result to obtain a relation in terms of $N_c(t)$.

$$\frac{d\alpha(t)}{dt} = \lambda N_c(t) \quad (4.7)$$

It can be noted that at this stage of our treatment it is not necessary to utilize the expression for the rate of production of ^{202}Fr , $R(t)$, in proceeding further in modeling the subsequent decay of ^{198}At . This can be seen by the expressions for the abundances of ^{198}At as follows:

$$N_{c_{At}}(t) = N_{c_{At}}(0)e^{-\lambda_{At}t} + e^{-\lambda_{At}t} \int_0^t \lambda_{Fr} N_{c_{Fr}}(t') e^{\lambda_{At}t'} dt' \quad (4.8)$$

$$N_{d_{At}}(t) = N_{d_{At}}(0)e^{-\lambda_{At}t} + e^{-\lambda_{At}t} \int_0^t \lambda_{Fr} N_{d_{Fr}}(t') e^{\lambda_{At}t'} dt' \quad (4.9)$$

where, as with Eqn. (4.5), continuity in the activity's population is required with

$$N_{c_{At}}(0) = N_{d_{At}}(t_g) \quad (4.10)$$

A detailed treatment of the activity profile of the Fr and At isomers is needed in order to determine the ratio of the observed activity to the activity that was not observed. We define a quantity T_f as the following:

$$T_f = \frac{\text{Observed decays}}{\text{Total decays; observed and unobserved}} \quad (4.11)$$

With this quantity and the detector efficiency one can obtain the amount of total activity of a decay mode of a particular isomer.

^{202}Fr is observed as an α decay doublet, decaying to the 3^+ and 10^- isomers of ^{198}At . By treating the subsequent decays of ^{198}At properly it is possible to obtain an estimate of the proportion of high-spin to low-spin ^{202}Fr produced, as the spectroscopic information of ^{198}At is more clearly discernable. While taking these matters into consideration, we obtain the following expressions.

$$\frac{\alpha^{202Fr} f_{10^-}}{T_{f_{10^-}}^{202Fr}} = \frac{\alpha_{10^-}^{198At} (1 + Br_{ec+\beta^+}^{198At^{10^-}} / Br_{\alpha}^{198At^{10^-}})}{T_{f_{10^-}}^{198At}} \quad (4.12)$$

$$\frac{\alpha^{202Fr} f_{3^+}}{T_{f_{3^+}}^{202Fr}} = \frac{\alpha_{3^+}^{198At} (1 + Br_{ec+\beta^+}^{198At^{3^+}} / Br_{\alpha}^{198At^{3^+}})}{T_{f_{3^+}}^{198At}} \quad (4.13)$$

The factors f_{10^-} and f_{3^+} are the corresponding percentages of high-spin and low-spin α -decay activity of ^{202}Fr produced and must sum to 1. α^{202Fr} is the total number of observed ^{202}Fr α decay events. The T_f factors are defined in Eqn. (4.11) and are labeled for each corresponding nuclear activity. The Br terms are the $ec+\beta^+$ and α branching ratios for ^{198}At isomers. Therefore, each side of the equation represents the total number of components of α decaying activities of ^{202}Fr . The right side expression assumes that the period of detection is much longer than the half-lives

Table 4.6: Observed and modeled α counts for activity from the 3^+ and 10^- isomers of ^{198}At . Data from the 0.075 second time bins are compressed to 0.15 second time bins for easier comparison of the observed and modeled activity.

Source	isomer	0.15s	0.30s	0.45s	0.60s	0.75s	0.90s	1.05s
Exp.	3^+	15056	15734	16034	16695	16818	17204	17026
Theory	3^+	14337	14491	15289	16749	17725	17735	17648
Exp.	10^-	6118	7117	7946	8710	8954	9264	8875
Theory	10^-	4795	5128	6603	9163	10648	10316	9823

of the ^{202}Fr and ^{198}At isomers and that the $\text{ec}+\beta^+$ branching ratios for the ^{198}At isomers are small.

The equations for the ^{198}At abundances (4.8,4.9,4.10) are propagated, using the software *Mathematica* through the 14.4 second supercycle indicated in Figure 4.3. The profile of the resulting modeled ^{198}At α counts are scaled for comparison to decays that appear as α counts in the detection window are compared with those seen during the experiment in Table 4.6. The fitting assures us that the modeled counts have a time varying profile and are a realistic reproduction of time profile of the experimental counts. This is a crucial step in determining whether a realistic estimate of T_f can be found for the At isomers. The apparent discrepancy in the calculation and experimental counts for the 10^- isomer is perhaps due to an uncertainty in the measured half-life. This discrepancy provides us with the uncertainty, which is reflected in the percentage yield uncertainties for the two ^{198}At isomers. The T_f factors that result for the ^{202}Fr run are provided in Table 4.7. The factor for ^{202}Fr was simply estimated as the amount of decays remaining, after the data acquisition system was turned off after each pulse. These factors will be used in Eqns. (4.12) and (4.13) to determine f_{10^-} and f_{3^+} .

In the spectrum of ^{202}Fr displayed in Figure 4.4, there is a peak which is located at the same energy as the α decay of ^{198}Po . A global least-squares fit of this peak and all the other peaks in the region yields approximately 6300 counts for

Table 4.7: T_f factors for the mass 202 run.

Nuclide	Isomer	T_f
^{202}Fr	$3^+, 10^-$	0.90(03)
^{198}At	10^-	0.48(04)
^{198}At	3^+	0.43(04)

Table 4.8: Percentages of isomeric yields for ^{202}Fr

Isomer	Percent of Total
$^{202}\text{Fr } 10^-$	33(3)%
$^{202}\text{Fr } 3^+$	67(3)%

this peak. The $t_{1/2}$ of ^{198}Po well exceeds the 1.2 second detection window of our detectors and therefore appears as having a flat count rate with respect to time. While taking this into consideration, we arrived at the upper limit for $ec+\beta^+$ decay of ^{198}At to be less than 6 percent for the 3^+ isomer and less than 14 percent for the 10^- isomer. This is indicated in Figure 4.5, as well as in the rates table mentioned above. Equations (4.12) and (4.13) can now be used to determine the amount of high-spin and low spin ^{202}Fr produced. The results for f_{10^-} and f_{3^+} are provided in Table 4.8.

4.4.2 ^{194}Bi activity yields

The final objective of the ^{202}Fr run was to determine the production of low-spin ^{194}Bi compared to high-spin ^{194}Bi . The result should be, in principle, in close agreement with the percentages of low-spin and high-spin ^{202}Fr produced.

The collection time between tape moves was 200 seconds, chosen to maximize the production and measurement of ^{194}Bi whose $t_{1/2}$ is around 100 seconds for each of the isomers to be studied.

However, even though the yields of ^{194}Bi at each station were similar, there appeared to be much more background activity in the γ spectrum taken at the collection station. In Figure 4.7 is provided the γ spectrum taken at the collection station. Gamma-ray transitions measured at the second station are shown in Figure

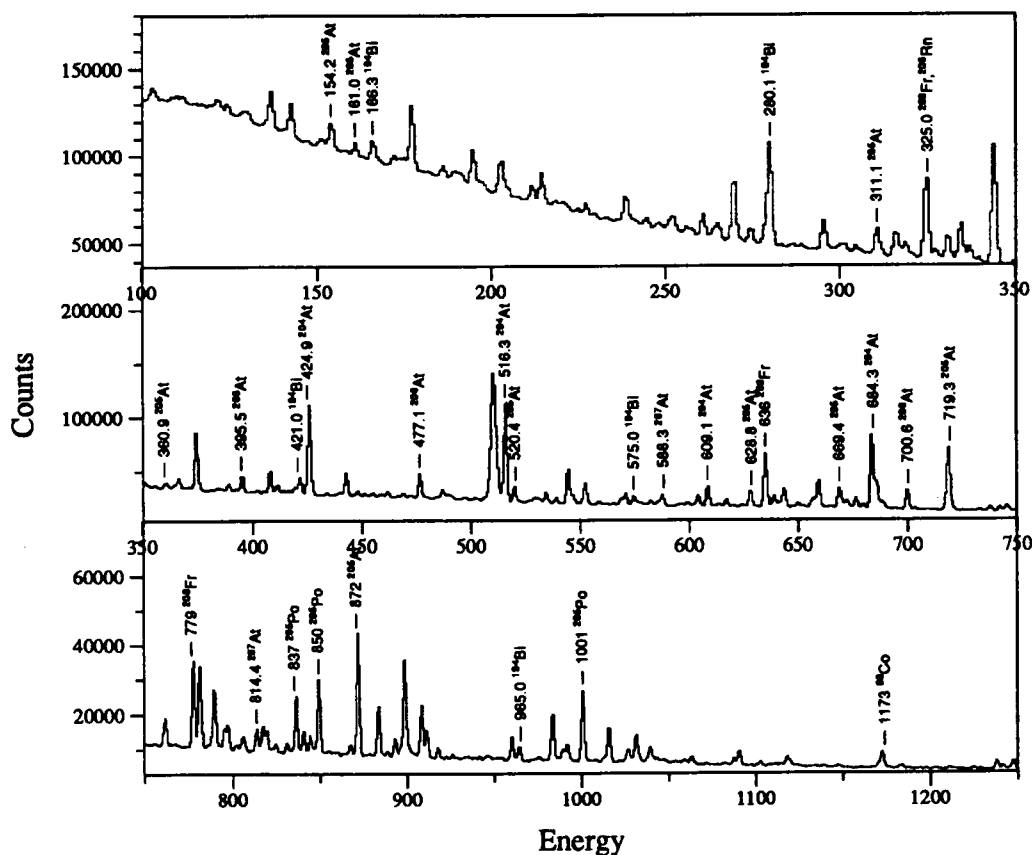


Figure 4.7: Gamma-ray spectrum taken at the collection station for the mass 202 run with suggested assignments for background activity indicated.

4.8. By comparing the spectra in the two figures it is seen that the measuring station data are less encumbered by background than are the collection station

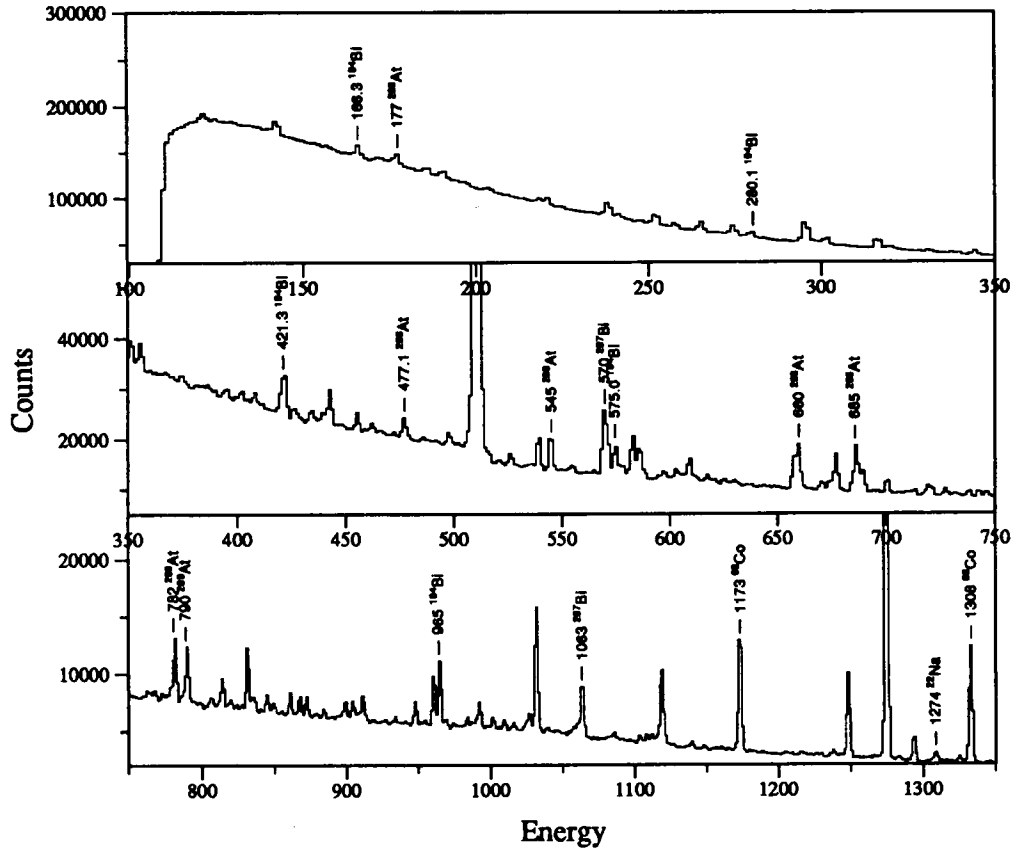


Figure 4.8: Gamma-ray spectrum taken at the measuring station for the mass 202 run with transitions in ^{194}Pb from the decay of ^{194}Bi labeled.

data. Therefore, we use the counts taken at the measuring station to extract the production rates of ^{194}Bi . The data reveal relative intensities shown in Table 4.9. It is seen that this experiment produced a greater percentage of low-spin ^{194}Bi as compared to a study in which ^{194}Bi was produced directly [79]. We can further note that this result is also substantiated by the calculated relative yields of α decay from low-spin vs high-spin isomers of ^{198}At . In the last section of this chapter the yields of the activity produced in this phase of the experiment are provided as well.

Table 4.9: Relative intensities for transitions in ^{194}Pb

Energy	Spin Change	Present I_γ	Published ^a I_γ
166.3	$(9)^- \rightarrow (7)^-$	11.8	36.6
280.3	$(5)^- \rightarrow (4)^+$	20.5	75.7
421.0	$(7)^- \rightarrow (5)^-$	13.1	61.6
575.0	$(4)^+ \rightarrow 2^+$	62.5	99.4
965.0	$2^+ \rightarrow 0^+$	100	100
1308.3	$2^+ \rightarrow 0^+$	21.0	6.4

^aValues from P. Van Duppen *et al.* [79]

4.5 Decay chain of ^{204}Fr

Study of the activity of ^{202}Fr , particularly the determination of the $ec+\beta^+$ branch of ^{198}Po and of low-spin ^{194}Bi comprised the original intended purpose of the experiment at ISOLDE. In the analysis of ^{202}Fr , it was determined that a greater percentage of low-spin ^{194}Bi is produced as compared to high-spin ^{194}Bi . However, because a non-conventional and experimental source utilizing a thin Th target was being used, the yields of ^{202}Fr were far lower than anticipated by the staff at ISOLDE. Furthermore, a considerable amount of background lines from additional unwanted activity accompanied the ^{202}Fr activity. Therefore, it was difficult to determine if an experiment to perform sensitive spectroscopic measurements on the decay of the low-spin isomer of ^{194}Bi would have been feasible with the relatively high amount of background. For this reason it was decided to perform an additional test experiment to observe activity from the production of ^{204}Fr , as the same reaction that produced predominantly low-spin ^{202}Fr should do the same for ^{204}Fr . In addition, from the facility's experience it was clear that much more ^{204}Fr could be produced than ^{202}Fr , even with the thinner target. Therefore byproducts of the production of Fr nuclei could be studied while obtaining additional spectroscopic information on

^{204}Fr . This would help to justify a future experiment at ISOLDE to perform the intended experiment on the decay of the low-spin state of ^{194}Bi .

The total length of the test run was about 4 hrs. However, the facility demonstrated a high production rate for mass 204, for which detuning of the beam was necessary in order to bring the activity to acceptable limits for our detectors. This in turn permitted the first time observation of non- α branches from the isomers of ^{204}Fr . Therefore, the analysis of ^{204}Fr has two primary objectives: i) to study the ratio of production of low-spin isomers to that of high-spin isomers, in order to have future reference for the overall performance of the detection system and ii) to extract new spectroscopic information from the activity of ^{204}Fr .

The α spectrum taken from the collection station is shown in Figure 4.9. Because of an excessive count rate due to a very effective production of ^{204}Fr activity, the detectors were somewhat overloaded, which resulted in the tailing of the peaks and poorer resolution. As in the mass 202 run impurities result from neighboring Francium activity. The experiment reproduced the previously found α decay transitions from isomers in ^{204}Fr [77].

The α -decay spectrum from mass 204 activity taken at the measuring station is provided in Figure 4.10. Because of the better resolution of the α detector at this station the peaks are more easily identifiable. Several proposed assignments are provided in this figure, most of which are based on energetics and proximity in mass to the corresponding activity observed in the γ spectra. It is perhaps noteworthy that a sizable peak results from the decay of ^{200}Po , while there seems to be little activity at lower energies. Therefore, there exists a possibility to include a search for α decay fine structure from decay of this nucleus in further experiments where ^{204}Fr is produced.

The γ spectrum taken at the collection station is provided in Figure 4.11. Many of these transitions have been measured previously; we utilize them to make

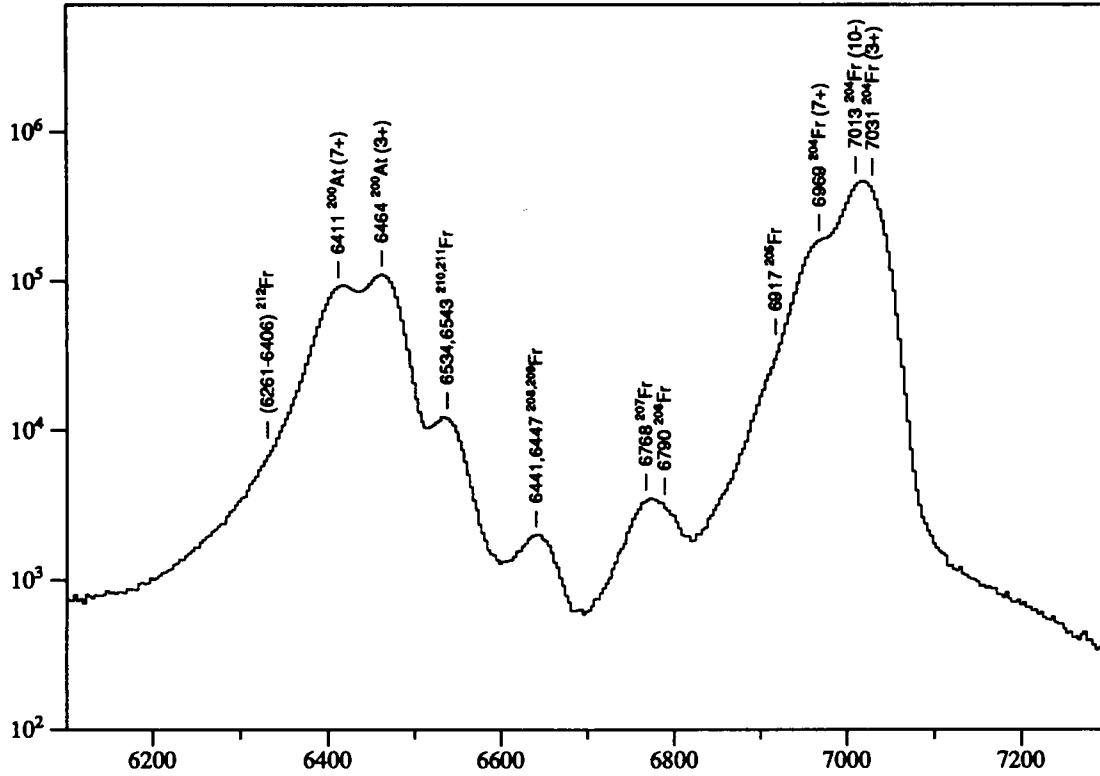


Figure 4.9: Alpha-spectrum taken at the collection station for the mass 204 run. Some suggested background assignments, as well as the main activity from ^{204}Fr and ^{200}At , are labeled.

an internal calibration of the γ detector. This enables more precise energy determinations for the the new transitions. Some of the previously observed α and $ec+\beta^+$ decay γ transitions [77] are labeled. In addition, some γ transitions in ^{204}Rn , previously observed in an in-beam experiment [80] are labeled. The γ transitions are listed in Table 4.10. With this information the energy of the previously unreported isomeric transition between the 10^- and 7^+ isomers of ^{204}Fr , which has been predicted could be determined [77]. Therefore in Table 4.10 the γ energies are listed for the three nuclear decays as they are found in the literature, in addition to the energies found from the calibration referred to in the beginning of this chapter. The discrepancy between the two calibrations is enough to justify an additional internal

Table 4.10: Gamma counts taken at the collection station for the mass 204 run. The transition energies found using the experimental calibration are provided. However, the energy of the IT transition from the ^{204}Fr isomer is determined by using an internal calibration, using the corresponding transition energies found in the literature. The high count rate observed during the mass 204 run may explain the discrepancy in the two sets of calibrations.

Activity	Published E_γ Citation [80]	channel	counts	Exp. Calibration
^{204}Fr	260.2(3)	367.7	41586	260.0
	(275) citation[77]	386.9	239412	275.6
	298.9(3)	419.6	33261	298.6
	542.9(3)	765.8	158748	542.5
	588.5(3)	830.2	111025	587.8
	641.3(3)	905.0	33616	640.5
	674.8(3)	952.4	32293	673.9
	Citation [77]			
^{200}At	230.9(2)	322.8	336091	230.5
	323.7(2)	456.5	50902	324.6
	374.0(2)	526.6	98590	374.0
	484.5(2)	682.3	234104	483.7
	534.5(2)	753.8	93504	534.0
	565.0(2)	797.1	62083	565.3
	611.1(2)	862.7	417087	610.7
	666.0(3)	942.3	537571	665.4
	Citation [79]			
^{196}Bi	138.4(2)	190.6	22294	137.3
	306.9(3)	430.1	5144	306.0
	337.5(2)	474.0	10302	337.0
	372.2(2)	523.4	25857	371.7
	421.7(2)	594.0	12662	421.5
	689.3(2)	973.2	95711	688.6
	846.7(2)	1195.9	8677	845.5
	942.4(2)	1332.1	15328	941.3
	1049.4(2)	1484.1	158268	1048.4
	1449.7(3)	1627.1	2560	1149.1

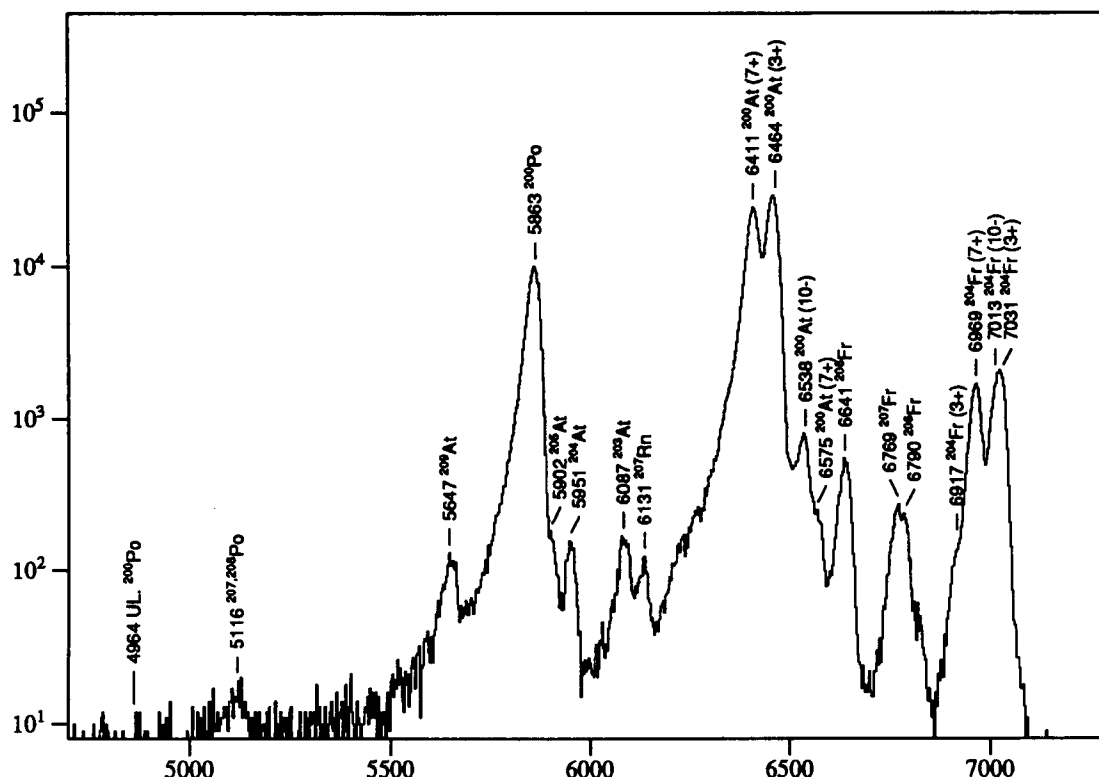


Figure 4.10: Alpha-spectrum taken at the measuring station for the mass 204 run. The indicated suggested background assignments are more easily distinguished, as a result of the better detector resolution of the measuring station.

calibration. Using this calibration we determine an energy of 276.1 keV for the newly discovered IT transition between the 10^- and 7^+ isomers of ^{204}Fr .

Because the half-lives of the ^{204}Fr isomers are comparable to the period between pulses from the booster the multiscaling of the activity reveals a humped time profile as is seen in the ^{202}Fr run. The known transitions in ^{204}Rn and the transition with the predicted IT energy when viewed in the multiscaled mode of collection exhibit the same behavior as the multiscaled α decay, and hence, are assigned to the decay of ^{204}Fr .

In addition, because of the high count rate it was believed that the detec-

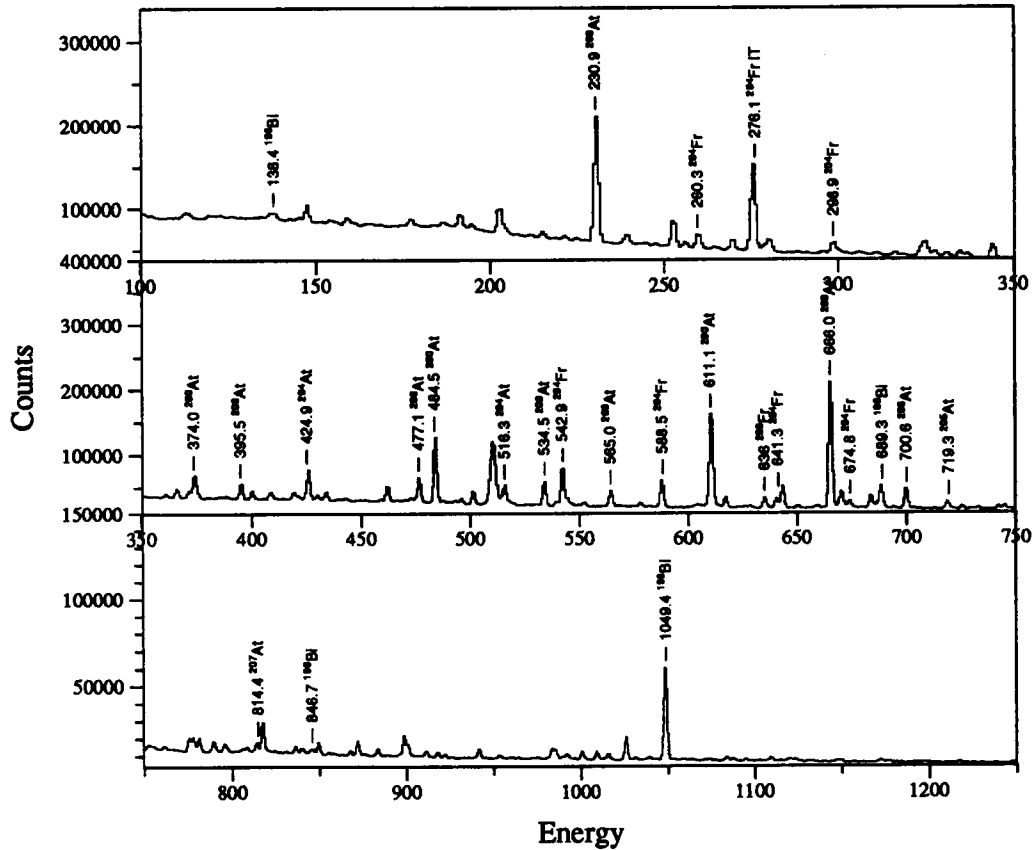
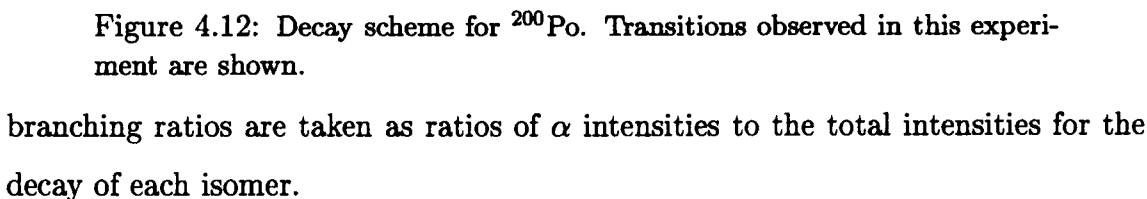


Figure 4.11: Gamma-spectrum taken at the collection station for the mass 204 run. The ^{204}At and ^{200}At activity are indicated along with other suggested background assignments.

tion efficiencies found from the calibration runs would not allow us to determine the branching ratios accurately. Therefore, it was decided to calibrate the detectors at the first station internally using the α decay branching ratios for ^{200}At previously found by M. Huyse *et al.* [77].

4.5.1 Extraction of previously unreported ^{204}Fr branching ratios

In Figure 4.12 is provided the level scheme from the work of M. Huyse *et al.* [77]. From the level scheme we deduce the means to determine the number of decays fed from the 7^+ and 3^+ isomers of ^{200}At via $ec+\beta^+$ transitions. The



$$Br_{\alpha 7^+} = \frac{I_{\alpha 7^+}}{I_{\alpha 3^+} + (I_{\gamma 534.5} + I_{\gamma 484.5})} \quad (4.15)$$

In Table 4.11 are provided the information needed to calculate the α decay and γ transition intensities from the decay of the 7^+ and 3^+ isomers. The branching ratios found using Eqns. (4.14) and (4.15) with this data are given in Table (4.12), along with results from a previous study [77].

It is found while using data from both stations that the branching ratios are somewhat larger than the results of the previous work by Huyse *et al.* [77]. There

Table 4.11: Total γ and α counts from decay of ^{200}At for both detector stations, which were used in our calculation of the $\text{ec}+\beta^+$ branch.

Transition	Efficiency 2nd Stat.	Counts 2nd Stat.	Efficiency 1st Stat.	Counts 1st Stat.
666.0	0.85%	156680	1.30%	537571
534.5	1.00%	27789	1.63%	93504
484.0	1.22%	75176	1.82%	234104
6411.8	1.9 %	257689	3.9 %	1183969
6464.9	1.9 %	320795	3.9 %	1753414
6537.8	1.9 %	not present	3.8 %	183170

Table 4.12: Resulting $\text{ec}+\beta^+$ branches for ^{200}At , compared with corresponding values found in the literature.

Branching Ratio	3^+ Isomer	7^+ Isomer
1st Stat.	66%	62%
2nd Stat.	64%	60%
M. Huyse [77]	57%	43%

are several sources for this discrepancy. One possible difficulty is the method used to determine our branching estimates, where it needs to be stressed that the level scheme for ^{200}Po may not be complete, thereby resulting in possible unaccounted feedings from the 10^- isomer of ^{200}At . No estimate was made for feeding from the 10^- isomer, as no transitions between high-spin states in ^{200}Po are observed, either to or from levels known to have spins at or near 10 units of angular momentum.

A complimentary calculation based entirely on α counts, to and from ^{200}At isomers, was attempted. However, this procedure gave inconsistent results based on the difficulties in analyzing the profile of the beam pulses, because of the longer

half-life and poor detector resolution of the first station α detector. Nevertheless, the T_f factors that were calculated will still be utilized in order to estimate the rate of production of the individual ^{204}Fr isomers, in the last section of this chapter.

Another possible source for mismeasurement of our ^{200}At branching values is the excessive count rate at the collection station during the mass 204 run. This is evidenced by the appearance of the visibly broadened α peaks shown in Figure 4.9. The count rate in the gamma detector and data acquisition system was high enough to affect the dead time of the ADCs that were processing signals from the γ detectors; a deadtime of 30 percent was observed during periods of high count rates. Therefore, it was decided to determine a new relative calibration factor for the γ counts in the mass 204 run. The suggested change in efficiency can be offered as a complimentary efficiency to be taken into consideration, for further decay branching calculations to be made in this chapter.

To calibrate the γ detection efficiency internally, we note the formula to obtain a scale factor, S_{cf} .

$$3^+ \quad S_{cf} = \frac{(I_{\gamma 666.0} - I_{\gamma 534.5} - I_{\gamma 484.5})}{I_{\alpha 3+} \left(\frac{1}{Br[\alpha]^{3+}} - 1 \right)} \quad (4.16)$$

$$7^+ \quad S_{cf} = \frac{(I_{\gamma 534.5} + I_{\gamma 484.5})}{I_{\alpha 7+} \left(\frac{1}{Br[\alpha]^{7+}} - 1 \right)} \quad (4.17)$$

Using the counts in Table 4.11 we obtain an average reduction of relative efficiency to be $S_{cf} = 0.6(1)$. We note that by using the counts in Table 4.10 we can nearly reproduce the previously determined E3 IT branch from the 10^- to the 7^+ isomers of ^{200}At . A value of 88.5(25)% is obtained which compares favorably with the result of 89.5(2)% given in the previous work by M. Huyse [77].

This scale factor is now used to determine a complimentary estimate for newly observed branches from ^{204}Fr . As in the previous section we take note of the level scheme of ^{204}Rn provided in Fig. 4.13 [80]. Using this level scheme we arrive

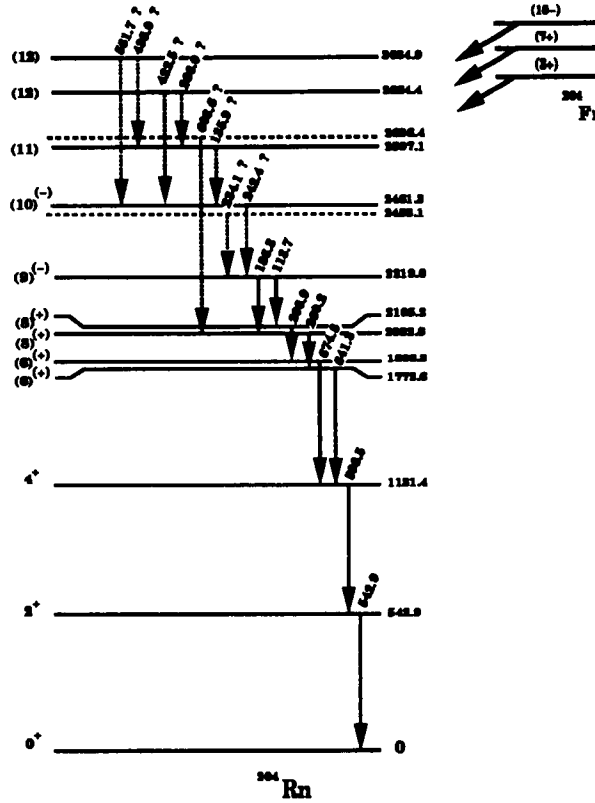


Figure 4.13: Level scheme for ^{204}Rn . Solid vertical lines indicate transitions observed during the mass 204 run. The dashed vertical lines indicate transitions, which have been observed in in-beam data

at the following formula for the $ec+\beta^+$ branches.

$$Br_{ec3^+} = \frac{(I_{\gamma 542.9} - I_{\gamma 674.8} - I_{\gamma 641.3})}{I_{\alpha 3^+} + I_{\gamma 542.9} - (I_{\gamma 674.8} + I_{\gamma 641.3})} \quad (4.18)$$

$$Br_{ec7^+} = \frac{(I_{\gamma 674.8} + I_{\gamma 641.3})}{I_{\alpha 3^+} + (I_{\gamma 674.8} + I_{\gamma 641.3})} \quad (4.19)$$

In Table 4.13 are provided the information needed to calculate the α decay and γ transition intensities from the decay of the 10^- , 7^+ and 3^+ isomers. The branching ratios found with these data are given in Table 4.14. The larger uncertainties for the 10^- and 3^+ α decay doublet indicated in Figure 4.9.

The decay chain for ^{204}Fr is given in Figure 4.14. Two interesting possi-

Table 4.13: Total γ and α counts from decay of ^{204}Fr taken at the collection station.

Transition keV	Efficiency 1st Stat.	Counts 1st Stat.
276.1	2.5(06)%	239412
542.9	1.3(03)%	158748
641.3	1.1(03)%	33616
484.0	1.05(03)%	32293
6969	3.9%	1995653
7013	3.9%	2041477
7031	3.9%	5352693

Table 4.14: The $\text{ec}+\beta^+$ and IT $E3$ branches of ^{204}Fr resulting from data taken at the collection station.

Isomer	Branch Type	Branch Value
10^-	276.1 keV IT	30(7)%
7^+	$\text{ec}+\beta^+$	11(3)%
3^+	$\text{ec}+\beta^+$	5(2)%

bilites arise from the observation of new branching mechanisms. Since ^{204}Rn predominantly α decays, the considerable $\text{ec}+\beta^+$ branch of ^{204}Fr introduces the possibility of performing α spectroscopy on ^{204}Rn . The same experimental possibility exists for ^{200}Po . It is both fed by a considerable $\text{ec}+\beta^+$ branch from ^{200}At and by the α decay transition from ^{204}Rn . Therefore, with the knowledge of these branches, production of ^{204}Fr may be a feasible way to search for α decay fine structure from the decay of these two nuclides. In Table 4.15 the observed α counts from ^{200}Po , as well as the upper limit for a fine structure transition to the 1049 keV level of ^{196}Pb taken at the measuring station are given. However, the hindrance factor for

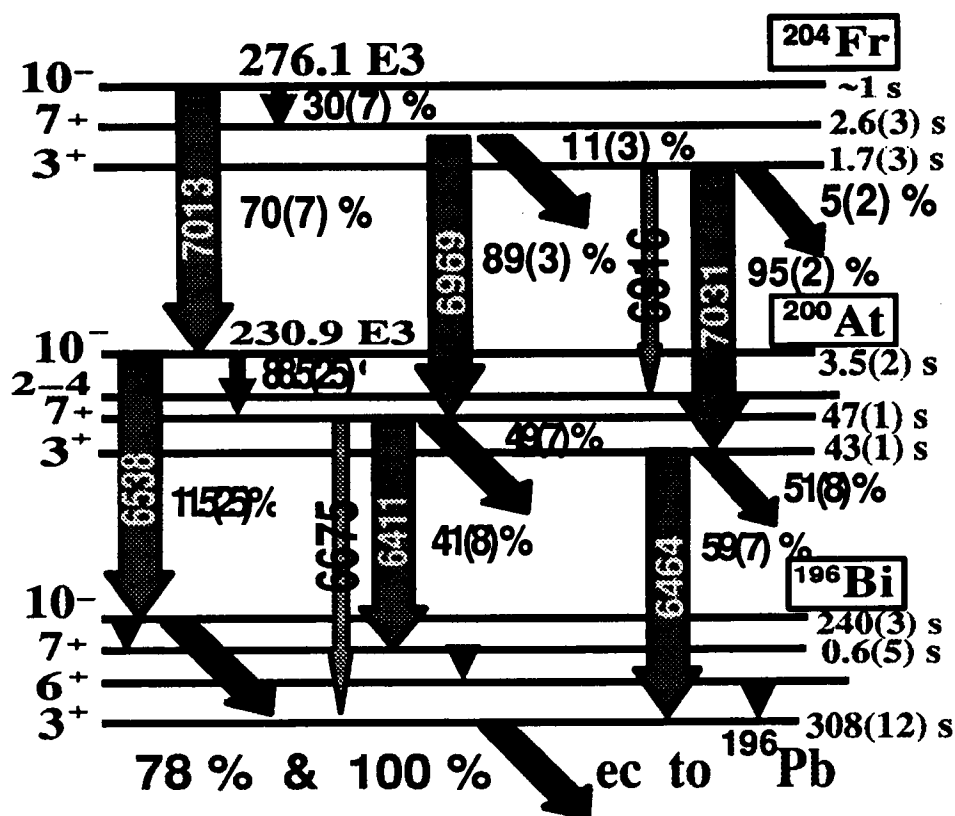


Figure 4.14: Decay Chain for ^{204}Fr . The electromagnetic transitions and α -decay transitions are shown as vertical lines with energies in keV and branching ratios given for each. Slanted arrows represent $ec+\beta^+$ decay, and the branch for this mode is indicated. The spin assignments and the half-lives of the three isomers in ^{204}Fr , ^{200}At and ^{196}Bi are given.

Table 4.15: Counts from the α decay of ^{200}Po . Indicated is our best estimate of an upper limit for an α decay fine structure relative intensity.

Daughter level in ^{196}Pb	α energy keV	Counts	Intensity relative to ground transition
0 keV	5863	112909	100%
1049 keV	4864	≤ 16	$\leq 1.4\text{e-}4\%$

a transition of this relative intensity upper limit is .03, which is an unrealistic transition strength considering that transitions are rarely this unhindered. Therefore, this upper limit is not of spectroscopic value. However, considering that the run time for this mass=204 experiment was just under 4 hours, it still demonstrates the possibility to perform a more in-depth spectroscopy with this nucleus. No feasible upper limit could be obtained for the α decay of ^{204}Rn to the first excited state of ^{200}Po due to the presence of activity from the α decay of ^{200}At .

The second item of interest is the first time measurement of the E3 transition in ^{204}Fr . Using the branch and the previously estimated half-life [77] we deduce a $B(\text{E3})$ of 5.4×10^{-4} Weisskopf units. This is somewhat larger than the upper limit found by Huyse *et al.* [77] for the same transition in ^{206}Fr ($\leq 5.2 \times 10^{-5}$ W.u.). The transition rates for the Fr isotopes discussed here and also for the α -decay daughters shown in Figure 4.14 [77] are about 5 orders of magnitude smaller than those in the trans-lead region with higher neutron numbers [81]. Since the energy difference of the 10^- and 7^+ isomers are very similar to the energy difference of the $13/2^+$ and $5/2^-$ neutron states ($1\nu i_{13/2}$ and $2\nu f_{5/2}$ single particle orbitals) in the odd-A Pb isotopes in this region, it is likely that the 10^- and 7^+ isomers are formed through coupling of these two neutron orbitals to the $\pi(h_{9/2})_{9/2}^n$ orbitals. The E3 transition between the two configurations would be j forbidden ($\Delta j=4 > L=3$). The extreme slowness of the E3 transitions indicates that the configurations are rather pure with little admixture of, e.g., $\nu f_{7/2}$ coupled with the $h_{9/2}$ protons.

4.5.2 Extracting yields for the production of ^{196}Bi

One of the main objectives in producing neutron-deficient Fr isotopes was to determine the effectiveness of producing low-spin Bi isomers. The mass=204 granddaughter activity is seen in the spectrum provided in Figure 4.15.

The information needed to determine the yield of ^{196}Bi , taken at the 2nd

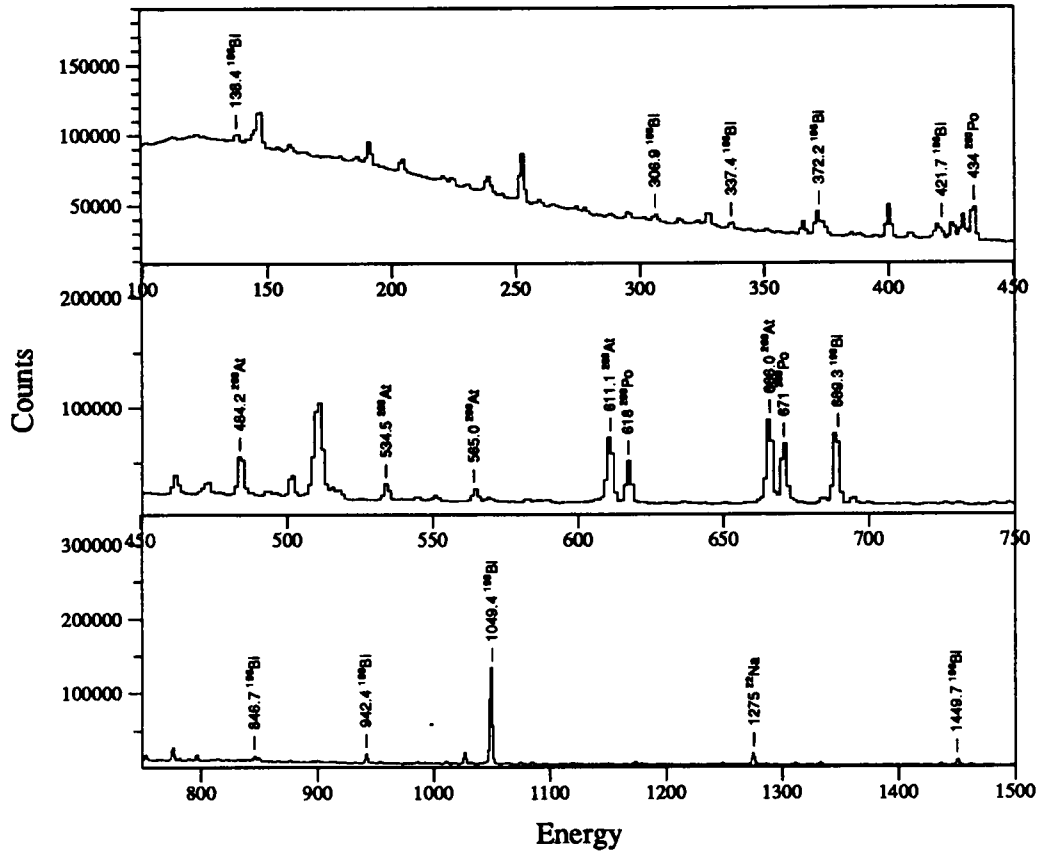


Figure 4.15: Gamma-spectrum taken at the measuring station for the mass 204 run. The observed transitions in ^{196}Pb from the decay of ^{196}Bi are marked. Also some suggested background assignments are labeled.

station, and is provided in Table 4.16. Using this information, we were able to determine that 95% of the Bi produced was the low-spin isomer. This result is used to determine the rate of production of both the Bi isomers and is provided in Table 4.19 in the last section of this chapter.

4.6 Impurities

In this experiment background activity from products of the (p+Th) spallation reaction, other than those selected by the low-mass separator, were able to reach the collection station. Although these activities were weak, the desired signals

Table 4.16: Relative intensities for transitions in ^{196}Pb

Energy	Spin Change	Present I_γ	Published ^a I_γ
138.4	$(9)^- \rightarrow (7)^-$	0.8	10.1
306.9	$2^+ \rightarrow 0^+$	1.1	1.0
337.5	$8^+ \rightarrow 9^-$	1.0	15.2
372.2	$7^- \rightarrow 5^-$	4.2	45.0
421.7	$8^+ \rightarrow 7^-$	1.3	7.7
689.3	$4^+ \rightarrow 2^+$	32.8	63.7
748.8	$5^- \rightarrow 2^+$	1.1	3.3
753.4	$(1,2)^+ \rightarrow 0^+$	3.9	2.2
776.6	$(3,4)^+ \rightarrow 2^+$	9.6	5.7
846.7	$(1,2)^+ \rightarrow 2^+$	4.7	2.4
942.4	$3^- \rightarrow 2^+$	8.2	4.0
1011.1	$(1,2)^+ \rightarrow 2^+$	2.1	1.2
1049.4	$2^+ \rightarrow 0^+$	100	100
1075.0	$? \rightarrow 0^+$	1.5	0.7
1449.7	$2^+ \rightarrow 0^+$	8.2	4.6
1896.3	$(1,2)^+ \rightarrow 0^+$	1.7	0.9

^aValues from P. Van Duppen *et al.* [79]

were often weak as well. Therefore, background activity showed the potential to be problematic for future efforts. It is therefore worthwhile to make an assessment of the dependence of the background on the rates of desired activity.

How to go about this was not clear initially and it was difficult to make a positive identification of all the background activity. Much of the same impurities occur in both the mass 202 and mass 204 runs, which had much different overall count rates. In the mass 202 run, which had a low production rate, the background lines were more obtrusive than in the mass 204 run. This demonstrates the impurity count rates at ISOLDE did not increase linearly with the rates of the desired activity.

We have restricted our background analysis to the impurities found in both the mass 204 run and the mass 202 run. The main impurities are from the region of

the chart of nuclides shown in Figure 4.16. The spectra showing these background

Hypothesized Background Candidates

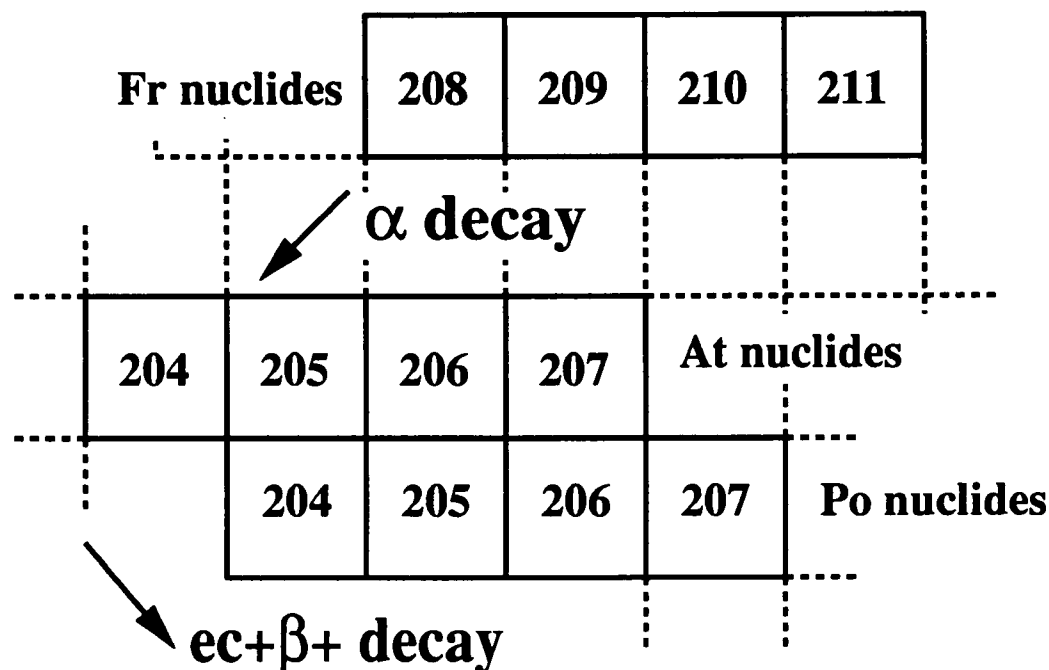


Figure 4.16: Nuclear region considered as primary background nuclides in the mass 202 and 204 runs.

lines are shown in Figures 4.7, 4.8, 4.11 and 4.15. The lines appearing in both are given in Table 4.17. In Table 4.17 are listed the overlapping activity. In addition the discrepancy between counts in the first and second stations can be explained from the fact that these counts are from background activity caught in various parts of the entrance aperture. Since the aperture was close to the γ detector at the collection station, the γ -ray background is much higher than that at station 2 which was far away and was somewhat shielded from the aperture. To help substantiate this hypothesis, it can be noted that the number of α decays seen from the background parent activity indicated in Figure 4.16 is consistent with the number of daughter

Table 4.17: Background counts used for comparative background analysis, between the mass 202 and 204 runs. The stars indicate that counts occur in a 477keV doublet. Therefore, the same numbers of counts appear for transitions in $^{206,207}\text{At}$.

Nuclide	E_γ (keV)	202 Cnts,1	202 Cnts,2	204 Cnts,1	204 Cnts,2
^{204}At	684	178,850	1421	55,697	12,205
	516	216,484	3000	97,480	18,000
	425	195,335	2803	112,813	26,480
^{205}At	719	157,376	3266	29,861	3658
	670	47,539	3892	59,464	unresolved
	629	37,992	2036	8303	2429
^{206}At	701	49,115	5045	80,850	5703
	477	53,134*	7405*	89,308*	3425*
	396	34,610	2713	62,887	2364
^{207}At	814	4896	5931	13,849	6554
	588	24,859	2067	111,597	unresolved
	301	10,000	12,519	8925	4768
	477	53,134*	7405*	89,308*	3425*

decays, observed in the form of γ transitions seen in the second station. For example, the total number of α transitions occurring at the energy, which is in agreement with the energies for the decay of $^{208,209}\text{Fr}$ is approximately 20,000 counts. If a decay rate calculation is carried out, while taking into consideration the detection efficiencies for α and γ detectors and the wait period of 200 seconds between movement of the activity from one station to the other, it is determined that only about 5000 $\text{ec}+\beta^+$ decays can occur from the α daughters. This is consistent with the number of γ transitions occurring from the $\text{ec}+\beta^+$ decays of $^{204,205}\text{At}$, as seen in Table 4.17.

The quantity needed to represent the rate of desired activity are the rates of production of ^{202}Fr and ^{204}Fr activity respectively, which are found in Table 4.19 in the last section of this chapter. A comparison of the rate of background activity common to both experiments, by looking at Table 4.19 is similar. Although the counts in mass 202 are up to three times the amount of counts in the mass 204 run,

one must consider that the mass 202 experiment ran three times as long as the mass 204 experiment. Therefore the background rate did not rise appreciably as compared to the production rate of the desired mass activity. This clearly suggests that if the experiment could be performed again with a better working source, background would not hinder the experiment excessively. Therefore, it is possible that a mass 202 run with better count rates could be feasibly performed at the ISOLDE facility.

4.7 Production rates

For the last section of this chapter we list the resulting production rates for nuclei produced from the source and their decay products. These results taken together with the impurities can be used to make an assessment concerning the feasibility of performing an elaborate decay study at the facility in the future.

In Table 4.18 are provided the information needed to compute the rates of production of the activity measured at ISOLDE. In Table 4.19 are provided all the resulting measured rates to facilitate an easy comparison.

Table 4.18: Summary of absolute numbers of observed decays. Also provided are the T_f factors and branching ratios, which are used to estimate the total number of disintegrations.

Isotope	Observed	Station	Absolute Decays	Calculated T_f factor	Branching Ratio
^{202}Fr	α	Coll.	9.1E+6	0.90	100%
$^{198}\text{At}(3^+)$	α	Coll.	3.1E+6	0.43(4)	>94%
$^{198}\text{At}(10^-)$	α	Coll.	1.5E+6	0.48(4)	>86%
$^{194}\text{Bi}(3^+)$	γ	Meas.	1.5E+6	1.00	99.8 % ^a
$^{194}\text{Bi}(10^-)$	γ	Meas.	6.6E+5	1.00	99.54% ^a
$^{204}\text{Fr}(3^+)$	α	Coll.	1.4(2)E+8	0.55(5)	95(2)%
$^{204}\text{Fr}(7^+)$	α	Coll.	5.1(5)E+7	0.53(5)	88.5(25)%
$^{204}\text{Fr}(10^-)$	α	Coll.	5.1(20)E+7	0.60(5)	70(7)%
$^{204}\text{Fr}(10^-)$	IT	Coll.	2.3(7)E+7	0.60(5)	30(7)%
$^{204}\text{Fr}(3^+)$	γ	Coll.	1.1(3)E+7	0.55(5)	5(2)%
$^{204}\text{Fr}(7^+)$	γ	Coll.	1.1(3)E+7	0.53(5)	11.5(25)%
$^{200}\text{At}(3^+)$	α	Coll.	4.5E+8	0.42(1)	59(7)%
$^{200}\text{At}(7^+)$	α	Coll.	3.0E+8	0.42(1)	51(8)%
$^{200}\text{At}(10^-)$	α	Coll.	4.7E+6	0.39(2)	10.5%
$^{200}\text{At}(10^-)$	IT	Coll.	3.6(9)E+7	0.39(2)	89.5%
$^{200}\text{At}(3^+)$	γ	Coll.	5.0(12)E+7	0.42(1)	41(8)%
$^{200}\text{At}(7^+)$	γ	Coll.	2.5(6)E+7	0.42(1)	49(7)%
$^{196}\text{Bi}(3^+)$	γ	Coll.	3.3(8)E+7	0.42(1)	100% ^a
$^{196}\text{Bi}(10^-)$	γ	Coll.	2.5(6)E+6	0.42(1)	78% ^a

^aValues from P. Van Duppen *et al.* [79]

Table 4.19: Summary of observed rates of production for observed activities.

Isotope	Observed	Station	Dis/sec	Branch	Net Dis/sec
^{202}Fr	α	Coll.	234(20)	100	234(20)
$^{198}\text{At}(3^+)$	α	Coll.	174(20)	>94	174(20)
$^{198}\text{At}(10^-)$	α	Coll.	81(20)	>86	81(20)
$^{194}\text{Bi}(3^+)$	γ	Meas.	140(20)	99.54(25) ^a	140(20)
$^{194}\text{Bi}(10^-)$	γ	Meas.	25(10)	99.8(7) ^a	25(10)
$^{204}\text{Fr}(3^+)$	α	Coll.	17,100(4000)	95(2)%	18,000(4000)
$^{204}\text{Fr}(7^+)$	α	Coll.	6,700(800)	88.5(25)%	7,500(1000)
$^{204}\text{Fr}(10^-)$	α	Coll.	5,900(1000)	70(7)%	8,500(1000)
$^{204}\text{Fr}(10^-)$	IT	Coll.	3,100(1000)	30(7)%	10,400(1000)
$^{204}\text{Fr}(3^+)$	γ	Coll.	1,300(300)	5(2)%	25,500(6000)
$^{204}\text{Fr}(7^+)$	γ	Coll.	1,400(300)	11.5(25)%	12,500(3000)
$^{200}\text{At}(3^+)$	α	Coll.	7,500(1000)	59	12,700(1800)
$^{200}\text{At}(7^+)$	α	Coll.	5,000(800)	51	9,900(1300)
$^{200}\text{At}(10^-)$	α	Coll.	840(200)	11.5	7,300(1000)
$^{200}\text{At}(10^-)$	IT	Coll.	6,400(500)	88.5	7,200(1000)
$^{200}\text{At}(3^+)$	γ	Coll.	8,300(3000)	41	20,300(5000)
$^{200}\text{At}(7^+)$	γ	Coll.	4,200(1000)	49	8,500(2000)
$^{196}\text{Bi}(3^+)$	γ	Meas.	11,000(2000)	100 ^a	11,000(2000)
$^{196}\text{Bi}(10^-)$	γ	Meas.	700(300)	78 ^a	700(300)

^aValues from P. Van Duppen *et al.* [79]

CHAPTER 5

Conclusion

In this dissertation, data revealing fine structure relative intensities in ^{190}Pb were presented. HF values were calculated and compared to other HF values for α decay in the light-Pb region. It was seen that the HF values vary widely. A discussion followed presenting reasons for attributing these variations to the changing degree of mixing between coexisting 0^+ and 2^+ states in the α -decay daughters. This discussion, though qualitative, provided us motivation to find a theory where quantitative calculations could be carried out, in order to reveal the sensitivity of α -decay relative intensities to mixing of excited daughter states.

In Chapter 3 we complete our study by probing shape-coexistence using α -decay fine structure. The reaction formalism of Berggren and Olanders was used in conjunction with the particle-plus-rotor model to study fine structure in the α decay of even-even neutron-deficient Pb region nuclei. The effect of configuration mixing, which was mentioned in Chapter 2, was incorporated in the particle-core formalism. The experimentally extracted two-band-mixing interactions were incorporated into the theory to describe the parent nucleus immediately before decay. Here the daughter configurations which were constructed from experimental data were input into the α -decay code directly.

Our analysis demonstrated that configuration mixing has a significant effect on the relative intensities, except for the Po decays. All experimental relative intensities were reproduced within one order of magnitude, except for ^{180}Hg , whose

α -decay daughter ^{176}Pt configuration is thought to have a triaxial ground state and a low-spin shape change.

In addition, some other properties of the model were discussed. The influence of the WS parameters was studied. The Hg and Pb decays were found to be relatively independent with respect to the variation of the optical model parameters. For the Po isotopes, however, considerable changes in multipole coupling strengths were needed in order to fit the experimental relative intensities. Interestingly, a change in the radius of the optical model potential, while having a significant effect on the calculated α -decay half-life, had considerably less effect on the relative intensities.

In Chapter 4 an experiment was discussed, whose original purpose was to serve as a pilot study for a future experiment to search for connecting transitions from superdeformed states to the ground band in ^{190}Hg , ^{194}Pb or ^{198}Po . It was found that a higher ratio of low-spin isomers to high-spin isomers of ^{194}Bi and ^{196}Bi , could be produced at the ISOLDE facility at CERN than with heavy-ion fusion reactions, which have yielded mostly high-spin light-Bi nuclei.

The significance of our findings at CERN for our study is that nuclear decay spectroscopy can possibly be performed on the light-Bi $ec+\beta^+$ daughters, which are known to exhibit shape-coexistence through interacting bands at low-spin. This would provide a useful alternative to studying shape-coexistence, as opposed to probing decays to the same nuclei by observing variations in the associated α decay fine structure.

In summation it was demonstrated that the α particle-plus rotor model has applicability for calculating fine structure in α decay to nuclei that exhibit shape coexistence. Based on this it is expected that the model can be used to calculate the relative intensities in α decay of nuclei further away from the $Z=82$ shell closure where fine structure has been observed, e.g., Pt and Rn decays. Therefore, progress

has been made in probing shape-coexistence by the study of α decay fine structure.

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On August 18, 1996, he married Susan Lynn Daye, who was born in Miami, Florida on October 13, 1962. She attended Farragut High School and graduated from St. Mary's School of Nursing in Knoxville. She has worked at Miami Children's Hospital since 1983 and will also receive her B.S.N. from the University of Miami in December 1996.