To the Graduate Council:

I am submitting herewith a dissertation written by Patricia G. Bailey entitled "The Effects of a Constructivist Instructional Model on the Achievement and Attitude of College Students Enrolled in Intermediate Algebra." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Education.

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THE EFFECTS OF A CONSTRUCTIVIST INSTRUCTIONAL MODEL ON THE ACHIEVEMENT AND ATTITUDE OF COLLEGE STUDENTS ENROLLED IN INTERMEDIATE ALGEBRA

A Dissertation

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ABSTRACT

This study was designed to investigate the effects of a constructivist instructional model on the achievement and attitude of college students enrolled in intermediate algebra.

The study was conducted with four class sections and 41 students enrolled in developmental intermediate algebra at Roane State Community College. Two instructors each taught two classes: one class utilizing the traditional lecture format and the other utilizing a constructivist instructional model designed by the researcher. The design allowed for a comparison of the two different treatments at the Roane County branch campus and at the Oak Ridge branch campus. The null hypotheses tested were that there were no significant differences between the following: the mean achievement scores of the constructivist model group and the lecture format group at each site, and the median attitude scores of the constructivist model group and the lecture format group at each site.

The students in all four sections were given a pretreatment achievement test and attitude survey to determine if the initial achievement levels and mathematics attitudes were comparable. Pretreatment measures of both achievement and attitude were comparable for the two Roane County treatment sections and for the two Oak Ridge treatment sections.

Since pretreatment achievement and attitudes were comparable, posttreatment measures of achievement and attitude were analyzed to test the hypotheses. The results indicated that students who were taught from a constructivist perspective experienced achievement not significantly different from those students who were taught with a traditional lecture format. The largest gain in achievement occurred in the Oak Ridge constructivist

model group. In addition, students who were taught from a constructivist perspective had posttreatment attitudes that were not significantly different from those who were taught with the traditional lecture format. The only decrease in mean attitude occurred in the Roane County lecture section.

This experiment established the practicability of this particular constructivist instructional methodology. In light of the present reform movement in mathematics education, which is calling for adoption of a constructivist pedagogy, the constructivist model should be considered by developmental mathematics educators as a worthy alternative to the lecture format.

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CHAPTER I

INTRODUCTION

According to the Curriculum and Evaluation Standards for School Mathematics (Standards) published in 1989 by the National Council of Teachers of Mathematics (NCTM), educational goals should include ones directed toward the development of mathematically literate students. In the Standards, five goals are listed for all K-12 students:

> (a) that students learn to value mathematics, (b) that they become confident in their ability to do mathematics, (c) that they become mathematical problem solvers, (d) that they learn to communicate mathematically, and (e) that they learn to reason mathematically. (NCTM, 1989, p. 5)

This call for reform in mathematics education provides a vision and a starting point for all students and teachers of mathematics.

The need for reform can be evidenced by the large enrollments in college remedial and developmental courses. Students have spent up to twelve years in mathematics courses, sometimes including several years in college preparatory mathematics, and still lack competencies in basic mathematics. In the state of Tennessee, 49.5% of the recent high school graduates enrolling as first time freshmen at Tennessee Board of Regents' (TBR) institutions (14 community colleges and 6 universities) during the 1993 summer and fall semesters were required to take one or more remedial or developmental mathematics courses. Obviously, a large segment of the college bound population has not become mathematically literate. There is a clear need for math educators at every level to encourage mathematical literacy.

As part of this reform effort in mathematics education, the American Mathematical Association of Two-Year Colleges (AMATYC) addressed specific curriculum and pedagogical reform that is needed in two-year colleges and lower division mathematics programs. In the document, Standards for Curriculum and Pedagogical Reform in Two-Year College and Lower Division Mathematics, goals and standards are recommended with the intention of improving mathematics education at this level. An entire chapter of this reform document is devoted to the vision of a dramatically reformed remedial and developmental mathematics curriculum.

The purpose of remedial mathematics and developmental algebra courses is to prepare students for their entry level college mathematics course. Arithmetic skills and precollege algebra skills are taught in what is usually a three course sequence consisting of remedial arithmetic, developmental elementary algebra, and developmental intermediate algebra. The lecture approach and the programmed study approach are the most widely used methods of instruction (McDonald, 1988). The emphasis is usually on acquisition of basic skills by the student. Thus, the primary goal is not mathematical literacy, it is skill acquisition. This approach is not in keeping with the goals and standards presented by NCTM and AMATYC.

The NCTM and AMATYC are asking math educators to adopt a constructivist approach to math education. Constructivism is "based on the premise that knowledge is something that students must construct for themselves. It cannot be 'given' to them" (AMATYC, 1993, p. 29). Instructional strategies must be developed which reflect this constructive, active view of the learning process (NCTM, 1989, p. 10).

The Problem

Instructional methods which are consistent with the goals and standards of NCTM and AMATYC should be investigated in relation to college remedial and developmental mathematics. An instructional method grounded in constructivism, using a constructivist approach, is consistent with the goal of mathematical literacy.

This study compared two treatments used in a developmental intermediate algebra course:

(a) an instructional methodology utilizing a constructivist model (CM) and

(b) an instructional methodology consisting of the traditional lecture format (LF).

The purpose of this research was to determine the effect of the two treatments on mathematical achievement and mathematical attitudes of the students. The problem investigated in this study addressed the following questions:

1. Do students in a developmental intermediate algebra course unit of study which is taught from a constructivist perspective experience equal or increased achievement compared with students enrolled in the same unit of study taught using a traditional lecture format?
2. Do the students who are taught from a constructivist perspective develop more positive attitudes toward mathematics than those students who are taught using a traditional lecture format?

The following null hypotheses were tested at a .05 level of significance:

H1: There is no significant difference in the means of algebraic achievement for students between the following groups:

(a) class taught utilizing a constructivist model (CM) and

(b) class taught utilizing a traditional lecture format (LF).

H2: There is no significant difference in the medians of mathematical attitude scores between the following groups:

(a) class taught utilizing a constructivist model (CM) and

(b) class taught utilizing a traditional lecture format (LF).

Definitions

The following definitions will be used in this study:

1. Achievement - Achievement is the acquisition of skills and the mastery of concepts as measured by a test. The test used in this study was the Tennessee Board of Regents Academic Assessment and Placement Program (AAPP) Intermediate Algebra Skills Subtest.

2. Attitude - Attitude is a learned predisposition to respond in a consistently favorable or unfavorable manner with respect to a given object (Fishbein & Ajzen, 1975, p. 6).

3. Attitude scale - An attitude scale is an instrument which numerically measures the direction and intensity of an individual's attitude toward an object (Anastasi, 1976). The attitude scale used in this study was the Fennema-Sherman Mathematics

Attitudes Scales.

4. Constructivism - A theory of learning and knowing based on the premise that all knowledge is constructed or created by the learner.

5. CM - CM is an instructional methodology utilizing a constructivist model. This model is described in Chapter III of this document.

6. LF - LF is an instructional methodology utilizing a traditional lecture format. This format is described in Chapter III of this document.

7. Developmental Studies Mathematics DSM 082 - DSM 082 Intermediate Algebra is a course designed to provide students with algebraic skills on a pre-college level.

8. Pair problem solving - A constructivist instructional strategy which calls for one student to listen to another student solve a problem aloud. The listener takes an active role in the process which includes checking for accuracy and demanding constant vocalization from the problem solver.

Importance of the Study

A circulating draft of Standards for Curriculum and Pedagogical Reform for Two-Year Colleges and Lower Division Mathematics, devotes an entire chapter to the call for and recommendations relating to a "dramatically reformed developmental mathematics curriculum" (American Mathematical Association of Two-Year Colleges [AMATYC], 1993, p.34). This document states, "The trademark of the new curriculum must be lively classrooms with students enthusiastically engaged in activities designed to enhance their understanding of mathematics and their ability to solve problems mathematically" (AMATYC, 1993, p.34).

This active view of the teaching and learning process is grounded in constructivist learning theory. However, there appears to be a gap between what math educators advocate in theory and what math educators actually practice in the classroom. George Gadanidis (1994) likens this condition to schizophrenia. Typical mathematics instruction involves a routine of teacher-centered discussion and explanation followed by periods of practice by students working individually while the teacher moves around the classroom answering questions (NCTM, 1991). This method differs greatly from the vision of the mathematics classroom provided by the NCTM and AMATYC. This gap between theory and practice, between our present practices and our vision of the future, must be bridged. Classroom research related to constructivist teaching and learning is needed to assist in the integration of theory and practice.

This study will examine the effect of a constructivist teaching model on achievement and attitudes of students enrolled in an intermediate algebra course. The results of this study will contribute to narrowing the gap between mathematics education theory and practices in the mathematics classroom.

Assumptions

The following assumptions were made in this study:

1. Students will respond honestly when completing the attitude measurement scale.
2. Random samples were obtained through registration procedures.

3. The difference in class meeting times did not affect the results.

Delimitations

The subjects involved in this study were students enrolled in DSM 082 Intermediate Algebra. The students who registered for the four class sections at the chosen times were randomly assigned to the treatment groups via the registration process. Only data from students who completed the course (i.e. completed the posttest) were included in the results of the study.

Organization of the Study

This study is organized into five chapters.

Chapter I provides an introduction and context for the study and contains the following: the problem statement and associated questions, the null hypotheses, operational definitions, the purpose and the need for the study, assumptions, delimitations, and an overview of the organization of the study.

Chapter II is a literature review which is organized around the topics of college-level remedial/developmental mathematics programs and students, constructivism as a theory of learning and knowing, pedagogical constructivism, and constructivism and the remedial student.

Chapter III presents the methods and procedures used in the study. The following topics are included: description of the setting, subjects, sample selection, experimental design, instructors, instructional materials, instructional methodologies, instrumentation, hypotheses, and techniques of analysis of data.

Chapter IV includes a presentation and analysis of the data.

Chapter V provides a summary of the study, findings, limitations, discussion, recommendations for future research, and concluding remarks.

CHAPTER II

REVIEW OF LITERATURE

This study will investigate the effects of a constructivist instructional model on the mathematical achievement and attitudes toward mathematics of college students enrolled in developmental intermediate algebra. Therefore, the review of literature for this study will include the following topics: (a) a discussion of college-level remedial/developmental mathematics programs and students, (b) constructivism as a theory of learning and knowing, (c) pedagogical constructivism, and (d) constructivism and the remedial student.

College-level Mathematical Remediation

Status of Remedial/Developmental Programs

Mathematical remediation at the college level refers to the process of developing the mathematical competence of students enrolled in remedial or developmental college mathematics courses. Remedial and developmental mathematics courses are offered at colleges and universities across the nation. The terms remedial and developmental are usually considered interchangeable when referring to courses or programs in general. When the terms are used to describe the level of mathematics instruction, remedial refers to the lowest level of mathematics instruction. In Tennessee Board of Regents' (TBR) institutions, the remedial level of mathematics is designed to develop basic arithmetic skills. The developmental level of mathematics is designed to provide students with basic algebraic skills on a pre-college level. The remedial and developmental courses are considered to be prerequisites for appropriate college-level mathematics courses and do not count for credit towards a degree at TBR institutions.

Instruments for the testing and placement of students, guidelines governing testing and placement, and the structure of remedial programs vary from state to state and program to program. TBR institutions participate in the Board of Regents' Academic Assessment and Placement Program (AAPP) to assess basic skills. Through this assessment, institutions can identify students who need remediation in reading, writing, and mathematics skills. For applicants under 21 years of age, ACT sub-test scores are used to determine the areas (if any) in which the students will be required to undergo placement assessment. Applicants 21 years of age and older are also required to undergo placement assessment. Enrollment in the courses indicated by the results of the AAPP is mandatory.

Remedial and developmental education programs are an important part of the postsecondary educational experience for students across the nation. A recent study conducted by the Southern Regional Education Board (SREB) found that 85% of the responding higher education institutions in the 15 state region offered at least one remedial course in reading, writing, or mathematics (Abraham,1992). The study also revealed that remedial enrollment at most private and public colleges and universities has increased since 1984, with the two-year colleges showing the largest increase (Abraham, 1991). The SREB states are Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, and West Virginia.

In the state of Tennessee, the fall 1993 academic profile of recent high school graduates enrolling as first-time freshman students in TBR institutions (6 universities and 14 community colleges) reported that 57.2% of those students needed one or more remedial or developmental courses. The TBR profile reported the highest developmental or remedial enrollment in the area of mathematics (49.5%). At Roane State Community College, the setting for this study, 60.7% of the 1993 summer and fall first-time freshmen were enrolled in remedial arithmetic or developmental algebra. Remedial and developmental mathematics are an essential part of the college-level mathematics curriculum.

The goal of mathematical remediation is to prepare students for college-level mathematics. The goal was described by Wepner (1985):

> The primary aim of postsecondary mathematics remediation is to sufficiently improve the mathematical skills of remedial students so they can successfully complete college level mathematics or mathematics dependent courses. The expectation is that successful remediation will allow the same opportunity of success to remedial students as is available to students not requiring remediation. (p. 1)

Criteria for evaluation of remedial and developmental programs flow from this primary goal. These criteria usually include the percentage of students who pass each remedial course and the percentage of students who exit the program and go on to pass subsequent college-level math courses.

Conclusions regarding the effectiveness of developmental mathematics programs are mixed. In a 1986 survey of 165 exemplary programs in developmental education, of those institutions having basic or intermediate algebra classes in a course format, one-third of these institutions reported having 50% to 75% of the students not meeting with success (McDonald, 1988). A meta-analysis of 185 studies (1970-1990) performed by Burley (1993) indicated that college developmental programs do have a positive impact on the achievement, attitude, and persistence of the underprepared college student. In contrast, Koch (1992) stated that high failure rates in developmental courses indicate that current methods and materials are not appropriate for most developmental students.

Blais (1990) stated that assessment of remedial mathematics programs usually does not include an evaluation of the philosophy or theory of instruction that supports the program. He called for a philosophical examination of remedial programs looking at questions concerning how each topic is taught and a justification for teaching it that way.

Most remedial and developmental college mathematics courses are taught utilizing the lecture approach (McDonald, 1988). However, major reform documents such as <u>Curriculum and Evaluation Standards for School Mathematics</u> (NCTM, 1989) and <u>Standards for Curriculum and Pedagogical Reform for Two-Year Colleges and Lower Division Mathematics</u> (AMATYC,1993) call for decreased usage of lecturing as a teaching strategy. The AMATYC document recommends that collegiate mathematics faculty adopt a constructivist approach toward mathematics education. This is based on the premise that students construct their own knowledge; knowledge cannot be given to them.

Blais (1988) concluded that conventional instruction, or indiscriminate use of explanations by the teacher, contributes to remedial processing by the remedial student or novice. He stated:

> Giving a maximum of explanation implies the creation of a listener-follower role for students. Such a role contributes to dependence, eliminates the need to think for oneself, and fosters the growth of learned helplessness. The habit of dependence and a belief in one's own helplessness ensures that the novice will remain just that. (p. 5)

It is reasonable to assume that there will be a continued need for remedial and developmental mathematics programs. These programs play a vital role in providing access to higher education for the underprepared student. However, there is a call for reform which includes changes in both content and methodology. The instructional strategies must meet the needs of the developmental student. Characteristics of the developmental student should be examined. Furthermore, a philosophical and theoretical foundation should be developed which results in a pedagogy that better serves the developmental student.

Characteristics, Attitudes, and Beliefs of the Remedial/Developmental Student

Who is the developmental student? The developmental student is an individual who possesses his or her own unique set of characteristics, attitudes, and beliefs. In order to become better equipped to assist these students in their mathematical development, potential affective variables must be examined.

Hardin (1988) identified six categories of students requiring developmental or remedial courses and discussed the characteristics and needs of these groups. One group consists of students who made poor choices concerning their academic development and future. Included in this group, are the students who selected something other than a college-prep curriculum. Also included are those students who dropped out of high school. Students in this group lack an academic background, but usually do not lack ability.

The adult student also makes up a large segment of the developmental student population. Some of these students also made poor choices earlier in life concerning their education. Other adult learners have good academic backgrounds, but have not used their knowledge and skills in many years. For the adult student, help is needed in gaining knowledge, refreshing and updating previous knowledge, and adjusting to the college environment.

Another group characterized by Hardin are those students who have academic or physical weaknesses that were not detected during high school. Due to these students' passive involvement in the classroom, physical problems such as hearing and vision difficulties may have gone unnoticed, which resulted in limited academic progress. These students had limited interaction with teachers, and found it difficult to ask for help. Developmental courses must provide these students with the opportunity to interact with the teacher and other classmates, and gain a new perspective on the learning process, a process which requires active participation.

The developmental student may not have clearly defined academic goals, but plans to use the educational system for his or her own benefit. This may include collecting financial aid, being enrolled to satisfy parents' wishes, playing sports, avoiding employment, or just enjoying the status of being a college student. Hardin stated that these students need assistance to overcome this limited view of college.

Two other categories of developmental students described by Hardin are the foreign students and the students who have a physical or learning disability. These two groups of students have needs which present a special challenge to the developmental educator. Foreign students often have inadequate communication skills and have difficulty becoming acclimated to the American way of doing things. Learning disabled students may feel frustrated, helpless, and lack self-confidence. The physically handicapped student may have academic deficiencies accompanied with frustration associated with using special equipment or learning aids.

It is apparent that there is much diversity among the developmental student population. In addition to the characteristics described above, the developmental mathematics student enters college with poor mathematical skills, often accompanied by negative attitudes about mathematics, a lack of self-confidence in their mathematical ability, and beliefs about mathematics which are detrimental to the learning process.

Buchanan (1992) explored differences between the remedial math student and the college-level math student. She found the following:

> Remedial math students were found to have a significantly poorer attitude toward mathematics regarding confidence in their ability, feelings about being successful, anxiety levels, motivation to do math, and usefulness of math. The remedial students were significantly more concrete in their learning style while the college-level were more abstract....A significant relationship appeared between math attitudes at the end of the semester and final course grades for both groups of students. (abstract)

Several studies have examined the relationship between affective variables and mathematics achievement for college developmental mathematics students. Eldersveld (1983), in a study of 513 developmental mathematics students, found that attitude toward mathematics and self-assessment of knowledge of mathematics were related to success or failure in developmental mathematics courses. Bassarear (1986/1987) determined that attitudes were a stronger predictor of performance for females enrolled in a college remedial mathematics course than for males. In another study involving developmental mathematics students, self-confidence in math ability was a significant predictor of success (Goolsby,1988). For nontraditional students over 25 years of age enrolled in a developmental mathematics course, causal attributions (perception of the causes for success or failure in mathematics) were the best affective predictors for mathematics achievement (Elliot,1990). Based upon these research studies, it can be concluded that there is a relationship between affective variables such as attitude toward self and mathematics and mathematics performance.

A discussion concerning mathematical attitudes would not be complete without an examination of students' belief systems concerning mathematics. Schoenfeld (1985) called this a "mathematical world view" which students formulate based upon their experiences with mathematics in the real world and the classroom. He explained:

> Belief systems are one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and hard one will work on it, and so on. Beliefs establish the context within which resources, heuristics, and control operate. (p. 45)

Schoenfeld (1985) described three typical student beliefs that could have negative consequences on their mathematical learning. Students believe that formal mathematics has little to do with thinking and problem solving, thus, mathematics is not used by these students in problems that call for discovery. Another belief is that mathematics problems should be solved quickly. This belief causes students to give up if they do not find a solution immediately. They also believe that mathematics is for geniuses. Therefore, they do not attempt to derive information on their own and tend to accept procedures without trying to understand them. Mathematical procedures are believed to be derived by geniuses who pass them down to students. If students hold these beliefs, the consequences for learning are negative.

Koch (1992) discussed the effects of a limited mathematical world view. Students view mathematics as rules and procedures developed by someone else. These students rely heavily on memorization with little emphasis placed on intuition or understanding. As a result, mathematical representations are only partially developed which results in misconceptions and procedural errors.

Gourgey (1992) described the negative consequences of a nonconceptual approach to mathematics:

> A mechanistic, nonconceptual approach to mathematics inevitably leads to difficulties with learning because there is a limit to how far students can progress without understanding the concepts behind the procedures. Because they have implicitly ruled out understanding, students often react to their mistakes by clinging to the security of an algorithm but are unable to correct their mistakes when they misapply it. This leads to a vicious cycle of frustration, more misguided efforts, and feelings of helplessness and stupidity in which students' conceptions of math and math ability virtually insure their failure in it. It is no wonder that they all too often conclude that they just don't have a mind for math, so there is no point in trying to succeed in it. (p. 11)

Blais (1988) used the term novice to describe the mathematics student who relies on rote memorization and algorithmic activity that is separate from the perception of essence. Blais stated that too much guidance and explanation from the teacher contributes to the development of the novice's mathematical behavior. He concluded conventional instruction must give way to an approach that does not perpetuate remedial processing.

A description of remedial mathematics students and a prescription for change was offered by Narode (1985):

> For almost all of our students in remedial math finding the "right formula" is a preconception which borders on obsession. They have no idea how such "formulas" are arrived at and consider most problem solutions as "tricks." That their former education has promulgated these notions is unfortunate indeed. The only way for these students to discover mathematics is to engage in math problem solving while monitoring their thoughts so as to understand each step in their solution--the steps back as well as the steps forward. (p. 9)

Constructivism and its view of knowledge and a constructivist instructional methodology are presented by researchers as an approach to learning and teaching that will effectively address the characteristics, attitudes, and beliefs of the developmental student (Koch, 1992; Narode, 1989; Blais, 1988). An analysis of constructivism as a theory of learning and knowing and a discussion of pedagogical constructivism follows.

Constructivism as a Theory of Learning and Knowing

Jean Piaget is considered the great pioneer of the constructivist theory of knowing (von Glasersfeld, 1990). Ernst von Glasersfeld summarized the principles of constructivism which can be traced to the work of Piaget: (a) knowledge is actively constructed, and (b) the function of cognition is adaptive and serves in the subject's organization of the experiential world.

To explain how mathematical knowledge is constructed, Piaget used the concept of reflective abstraction. Reflective abstraction is a process by which we interiorize our physical operations on objects (Noddings, 1990). Piaget's concepts of assimilation and accommodation describe the active nature of intellectual processes. Assimilation enables one to abstract rules and regularities from experience, and accommodation allows for continual revision of constructions based upon the results of our actions (von Glasersfeld, 1990).

In the following list, generally agreed upon constructivist views are summarized by Noddings (1990):

1. All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
2. There exists cognitive structures that are activated in the processes of

> construction. These structures account for the construction; that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.
>
> 3. Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.
>
> 4. Acknowledgment of constructivism as a cognitive position leads to the adoption of methodological constructivism.
>
> a. Methodological constructivism in research develops methods of study consonant with the assumption of cognitive constructivism.
>
> b. Pedagogical constructivism suggests methods of teaching consonant with cognitive constructivism. (p. 10)

Pedagogical Constructivism

Piaget's theories of cognitive processes and structures lead to methodologies which support this cognitive position. In research, the methodologies might include interviews and observation. "For teachers, methodological constructivism becomes pedagogical constructivism" (Noddings, 1990, p. 15). Teachers following a constructivist approach employ strategies that result in accurate constructions of mathematical knowledge by students, and which allow for investigation of the students' thought processes by the students themselves or by the teacher. "The design of the learning space includes three principal currencies of a mathematics teacher: the posing of situations, the encouragement of reflection, and interactive mathematical communication" (Steffe & D'Ambrosio, 1995, p.156).

This study involves the use of an instructional strategy which encourages metacognition and social interaction through paired problem solving. In the following review of literature, metacognition, problem solving, dyadic cooperative learning, and the role of the teacher in the constructivist classroom are discussed.

Metacognition

Teaching strategies which encourage metacognition during problem solving are constructivist. Flavell (1976) defined metacognition as follows:

> Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them .... Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective. (p. 232)

Schoenfeld (1987) indicated that successful problem solvers monitor and assess their cognitive processes throughout the problem-solving process.

Schoenfeld (1987) described a "kitchen sink" approach to developing metacognitive skills in students. He developed the techniques in his college-level problem-solving courses, but suggested that all of them can be adapted to any level of mathematics instruction. The first technique is the use of videotapes. The students watch and discuss the problem-solving behaviors of the students on the video, and as a result awareness of metacognitive issues is increased. Another technique involves teacher as role model for metacognitive behavior. During role modeling, Schoenfeld (1987) suggested that the teacher reveal "the give-and-take of real problem solving" (p. 200). The third technique is whole-class discussion with the teacher acting as moderator and the students acting as decision makers. About a third of the time, he used a fourth technique, problem solving in small groups. Schoenfeld used a coaching metaphor to describe the teacher's role during small group problem solving. The teacher, acting as coach, watches and listens to students to make on-line corrections in their problem-solving techniques. He told his students that he reserves the right to ask the following questions during problem solving: "What (exactly) are you doing?"; "Why are you doing it?"; and "How does it help you?" (Schoenfeld, 1987, p. 206). Schoenfeld, via observation and analysis of students' problem-solving behavior, established these techniques as effective in improving students' problem-solving ability.

Overt thinking by the student, or thinking out loud with guidance from the teacher, allows the teacher to aid in the development of the student's reflective processes and to reveal student's misconceptions concerning the problem. A study by Confrey in 1983 (cited in Confrey, 1990) involving high school females revealed that instruction involving overt thinking and other constructivist strategies was successful in improving the students' math SAT scores.

Whimbey and Lochhead (1985) analyzed the responses of a good problem solver, and concluded that two characteristics were evident: carefulness and a step-by-step approach. In the step-by-step approach, good problem solvers restate ideas. They "talk to themselves" while thinking. This behavior is not limited to problems that they are asked to do out loud. The authors state that "studies using electronic amplifying equipment (to monitor speech muscle activity) reveal good problems solvers talk to themselves while they solve problems.... This talking, which is not done aloud, is called covert or subvocal speech" (Whimbey & Lochhead, 1985, p. 227).

Problem Solving

The instructional methodologies in the 1983 study by Confrey (cited in Confrey, 1990) also included the paired problem-solving method of Whimbey and Lochhead (1980). In this process, one student talks through the problem and the other student asks questions concerning the problem and the methods employed to solve the problem. This strategy encourages the students' metacognition and overt thinking. It also allows the teacher to observe and assess the students' problem-solving skills.

Silver (1987) advocated explicit instruction related to the metalevel processes of planning, monitoring, and evaluation. In particular, he mentioned paired problem solving as a technique which has been successful at the secondary and college levels for emphasizing the importance of monitoring and evaluation processes as components of mathematical problem solving. The following statement by Silver (1987) reflected his position relative to metalevel processes:

> If a major goal of mathematics instruction is the development of students' problem-solving abilities, metalevel processes need to become an important curricular focus. Until these processes receive explicit attention in the curriculum, we will continue to produce mathematics students who know fairly well what to do in routine and simple problem situations, but have little competence in handling unfamiliar or complex problems. (p. 55)

Paired problem solving and problem solving in small groups are constructivist teaching strategies which allow for social interaction among students. As students interact with their peers, opportunities arise for the construction of mathematical knowledge (Cobb, Wood, & Yackel, 1990). Vygotsky's (1978) view of learning is based upon the belief that social interaction plays a primary role in cognitive development. Research by Petitto (1985, cited in Schoenfeld, 1985) showed that the method utilized by a pair of students in an estimation task differed qualitatively from the method used by either of the students individually. Schoenfeld (1985) offered a Vygotskean hypothesis:

> It seems reasonable that involvement in cooperative problem solving, where one is forced to examine one's ideas when challenged by others, and in turn to keep an eye out for possible mistakes that are made by one's collaborators, is an environment in which one could begin to develop the skills that, internalized, are the essence of good control. (p. 143)

Koch (1992) pointed out that a constructivist instructional model allows for the detection and confrontation of misconceptions and procedural bugs. Many students develop consistent, systematic errors in mathematical algorithms. The use of constructivist strategies such as paired problem solving or small cooperative groups, and communication between the individual student and teacher, allow the teacher to monitor and modify student's present mathematical knowledge.

Cooperative Learning

The paired problem-solving method is a form of cooperative learning, which is defined as "students working together in a group small enough that everyone can participate on a collective task that has been clearly assigned" (Cohen, 1994, p. 3). It is not the purpose of this review to discuss the substantial amount of research related to cooperative learning. However, the effect of cooperative learning on the outcome variables of achievement and attitude and the related constructivist views will be examined.

An abundance of research indicates that cooperative learning experiences promote higher achievement than traditional learning situations. The universal nature of this positive impact on achievement was summarized in the following:

> The results hold for all age levels, for all subject areas, and for tasks involving concept attainment, verbal problem solving, categorization, spatial problem solving, retention and memory, motor performance, and guessing-judging-predicting. For rote-decoding and correcting tasks, cooperation seems to be equally as effective as competitive and individualistic learning procedures. (Johnson, Johnson, Holubec, &

Roy, 1984, p. 15)

In addition, the research shows that cooperative learning promotes the use of critical thinking and higher level reasoning strategies. Johnson and Johnson (cited in Johnson, Johnson, Holubec, & Roy, 1984) concluded that the effectiveness of cooperative learning is maximized when the learning tasks require greater conceptual attainment. This conclusion was confirmed in a study conducted in a college remedial mathematics course. Students in the treatment group which utilized cooperative learning strategies performed better than students in the control sections on the measures involving higher cognitive skills (Dees, 1991).

How does cooperative learning enhance the development of higher order thinking skills? Schwartz, Black, and Strange (cited in Cohen, 1994) offered a constructivist explanation in answering this question. In their study, they found that dyads were far more effective in solving a physical problem related to the effects of multiple gears. Cohen (1994) summarized their conclusions:

> Based on a study of interaction of dyads, they concluded that working pairs required subjects to create an agreed-on representation of the problem in order to communicate with each other. This representation allowed the group to abstract more successfully than single individuals. They recommend that cooperative learning should capitalize on the unique strengths of group learning by selecting tasks that involve abstractions and require and enable representational negotiation. (p. 6)

This explanation agrees with the social constructivism of Vygotsky (1978), which is based upon the belief that social interaction plays a primary role in cognitive development. Furthermore, there are metacognitive benefits involved. When students are working cooperatively, they are thinking out loud. Thus, their thoughts are being examined by others. This process allows for detection and correction of errors. As a result, more accurate mathematical constructions are formed and important components of the mathematical problem-solving process are being reinforced.

Research by Johnson and Johnson (cited in Johnson, Johnson, Holubec, & Roy, 1984) also indicated that cooperative learning promotes more positive attitudes toward the subject area and the instructional experience. Koch (1992) found that developmental arithmetic students who were taught by constructivist methods, which included pair and small group interaction, did develop more positive attitudes toward themselves as learners of mathematics. These students also experienced a decrease in math anxiety. From a constructivist perspective, this may be seen as resulting from a more positive mathematical world view. Students must become empowered and begin to think of mathematics as something that works for them, not against them.

Role of Teacher in the Constructivist Classroom

In a constructivist classroom, the teacher takes on a role that is distinctly different from the role of the teacher involved in traditional, direct instruction. "When one applies constructivism to the issue of teaching, one must reject the assumption that one can simply pass on information to a set of learners and expect that understanding will take place" (Davis, 1990, p. 109).

Narode (1989b) described three roles the teacher plays in the constructivist classroom: classroom manager, clinical interviewer, and problem solver. As classroom manager, the instructor plans and organizes the activities, provides instructions concerning desired behaviors during activities, and administers examinations. In the role of clinical interviewer, the teacher uses questioning to promote metacognition. The teacher assumes the role of the listener as students reflect on their thought processes. Narode also suggested that the teacher be knowledgeable about the problem-solving process and model expert problem solving for students.

Narode (1985) described the pair problem-solving method and the teacher's role as clinical interviewer in the following:

> The pair problem-solving method of Whimbey and Lochhead (1979) is developed to accomplish this task [developing students' metacognitive skills] almost naturally provided the instructor is well trained. The task is not for the teacher to develop the skills in our students but rather for the students to develop metacognitive skills for themselves. According to the methods of Whimbey and Lochhead, the best teacher is the one who doesn't teach. Instead the teacher models the role of a clinical interviewer so that the students see what is expected of them when they engage in pair problem solving. This type of instruction, or rather non-instruction, requires the students to learn to become clinical interviewers themselves. They work in pairs--one student solves a problem while the other listens carefully asking for the other's thoughts, reasonings, and elucidations until they both follow clearly what has been said and solved. They then exchange roles and work another problem. (p. 10)

The constructivist teacher works with students individually, in the context of dyads or larger groups, and with the whole class. In each of these contexts, the teacher employs questioning strategies. Confrey (cited in Narode, 1989b) suggested four such strategies:

> (a) ask students to discuss their interpretation of the problem, (b) ask the students to describe precisely their methods of solution, (c) ask students to defend their answer and their solution, and (d) ask students to retrace the steps in their solution so as to review the process they engaged in to solve the problem. (p. 10)

Questioning in this form promotes problem-solving skills and metacognition, and allows the teacher to detect misconceptions and procedural errors.

Noddings (1990) described the value of constructivist instructional strategies:

> Pedagogical constructivism suggests more sophisticated diagnostic tools--tools that will uncover patterns of thinking, systematic errors, persistent misconceptions. Further, the elaboration required in, say, thinking aloud in the presence of a teacher

> encourages students to concentrate on the question or problem at hand. Conducted well, such a session gives the teacher many opportunities to reassure students that they are doing some things right, that their thinking has some power, that their errors are correctable. Above all such a method can be used to create a mathematical environment--one that will press for mathematical adaptation rather than a form suitable for another environment. (p. 15)

Confrey (1990) examined the practices of one teacher who taught in the constructivist tradition and presented an instructional model based upon these practices. This model consisted of the following six components:

> (a) promotion of autonomy and commitment in the students, (b) development of students' reflective processes, (c) construction of case histories, (d) identification and negotiation of tentative solution paths with the student, (e) retracing of those solution paths, and (f) adherence to the intent of the materials. (p. 115)

Blais (1990) suggested the use of two automatic-response patterns by teachers in the constructivist classroom: retreat-advance and comprehension probe. The retreat-advance response pattern is used instead of an explanation when students ask for help with complex problems. The teacher retreats to a simpler example that is essentially the same and checks for student understanding. The teacher then points out that the simpler example and the more complex one are essentially the same. The comprehension probe response pattern allows the teacher to determine if students really understand the mathematical language they are using. Teachers might ask students to prove understanding via graphical representations, supplying simpler similar examples, or asking the student to explain the concept using natural language. Again, an emphasis on student metacognition, student problem-solving skills, and an emphasis on questioning and listening by the teacher are reinforced by these teacher response patterns.

Since students are involved in overt thinking, the social context must facilitate and support this process. The teacher is responsible for creating a risk-free environment which is conducive to reflective problem solving. "Fumbling, groping, and error" (Blais, 1990, p. 77) must be seen as a positive and normal part of the thinking process. "These students must learn to question and be questioned without the fear of being admonished for not 'knowing' mathematics" (Koch, 1992, p. 14).

Narode (1989a) summarized the goal and associated methods of a constructivist pedagogy in mathematics:

> The main goal of a constructivist pedagogy in mathematics is to help the student to construct powerful constructions which the student will believe and commit to, and which are in agreement with expert opinion. To facilitate this, constructivist theory suggests that 1) students must actively engage their current knowledge in the solution of a problem or task; 2) students must communicate their current knowledge structures in actions, representations, and verbal reports, simultaneously relating one knowledge structure to another and to their immediate solution of the problem; 3) students must commit themselves to their solution and to their justification of the solution after testing their constructions with disequilibrating problems which arise from their own work, from questions posed by the teacher, or from other students, but always upon reflection on their own conflicting schemata. (p. 110)

Constructivism and the Remedial Student

Narode (1985) asserted that pair problem solving assists students in developing their metacognitive skills, and that metacognition is a necessary condition for effective problem solving. He based this assertion (at least partially) on his experience with a remedial math course at the University of Massachusetts, Amherst, in which pair problem solving was the mode of instruction. The following statements by Narode (1985) underscore his position regarding remedial math programming and mathematics instruction in general:

> The effectiveness of a remedial math program rests on the ability of the student to formulate a better understanding of the type of thinking that mathematics entails. They need to develop a mathematical epistemology that goes beyond the rote memorization of formulas matched to sample problem-types. Conceptual understanding must be seen as the goal of math instruction. (p. 13)

Narode (1989a) developed a constructivist program for college remedial mathematics and a basis for evaluation of the program. The following is a description of Narode's constructivist classroom:

> The constructivist classroom is conducted almost entirely within the context of group problem solving, with students in dyads and in larger groups. Lecture is kept to a minimum. The teacher serves as a coach, moving from one student group to another, listening to their discussions, asking selected group members to summarize the group's solution path, and probing student solutions and conceptions with questions rather than answering questions. (pp. 164-165)

Whimbey and Lochhead's (1980) pair problem-solving method served as the model for the cooperative problem solving. The evaluation of the program included the use of a diagnostic test which revealed that the constructivist program did teach students to use algorithms and to apply their mathematical understanding toward effective problem solving.

Blais (1988) analyzed college student performance on a mathematics college placement exam to explore how the remedial student or novice processes mathematical material. Blais (1988) stated the following:

> The natural or acquired tendency to perceive the essence of scientific or mathematical material is the single most important processing difference between novices and experts. Experts perceive essence before they "algorithmate" while novices "algorithmate" without ever perceiving essence. (p. 3)

Blais (1988) contended that conventional instruction, with its emphasis on explanation and transmission of information, often serves to perpetuate remedial processing tendencies. "Conventional instruction permits, allows, and sometimes blatantly encourages algorithmic activity that is separate and isolated from essence" (Blais, 1988, p. 4). Blais concluded that constructivism provides the theoretical and philosophical view of knowledge and learning which may provide us with the methodologies to transform the novice, via education, into an expert.

Utilizing a case study research methodology, Blais (1990) examined three instructors' pedagogical approaches to teaching remedial algebra. The goal of the case studies was to see if the instructors' approaches encouraged expert or remedial processing, and to see if the approaches allowed for opportunities for the instructor to gain insight into the students' thought processes. All three case study teachers used a lecture format and relied on explanation and modeling to transmit the material. He concluded that "those who enter [conventionally taught remedial algebra classes] as remedial processors will exit as remedial processors" (p. 181).

Blais cited his 1990 study as one of the first to link the teaching of algebra and constructivism. He stated:

> This study is motivated by the belief that most remedial mathematics programs at the college level suffer from an inadequate theoretical foundation. The main purpose of this thesis is to present a more adequate theory upon which to base attempts at mathematical remediation. (p. 3)

As a result of this belief, he offered an educational theory grounded in constructivism as an alternative that will address the needs of the remedial mathematics student and help them along their way to becoming an expert processor.

David (1988/1989) examined the effect of metacognitive training upon mathematics performance and attitudes toward mathematics. One of the groups studied was a remedial algebra group. David found that self-communication could be taught to remedial college mathematics students and that this metacognitive strategy was used by students in problem solving at least over the short term. Mathematics performance increased at a .05 level of significance; however, the change in attitude was not statistically significant.

Koch (1992) conducted a teaching experiment utilizing a constructivist teaching model in a developmental arithmetic course at the University of Minnesota. This model consisted of five types of classroom interaction. These interactions involved: (a) the teacher providing directions to the students, (b) the teacher interacting with individuals or small groups, (c) the students interacting in pairs or small groups, (d) the students interacting with mathematical materials, and (e) the teacher and teaching assistant modeling problem solving and questioning techniques. Active problem solving, questioning, listening, and social interaction were encouraged and reinforced through these interactions. As a result, students taught from this constructivist perspective decreased their level of math anxiety, developed more positive attitudes about themselves as mathematics students, and acquired greater proficiency in mathematics. These students also completed the course at higher rates than those enrolled in the control section which was taught by lecture. Koch concluded:

> College developmental mathematics programs cannot change the students' past experiences but can, and should, use those experiences to foster and encourage new mathematical growth. For students who have spent up to 12 years sitting in mathematics classes, and still lack basic mathematics concepts and processes, relearning mathematics through a constructivist approach may provide the impetus that is needed as evidenced from the results of the teaching experiment. (p. 18)

Conclusion

The purpose of this study is to propose an alternative instructional methodology based upon the tenets of constructivism and to examine its effects on the mathematical attitudes and achievement of the developmental student. This review of literature served several purposes. The first purpose was to describe the status of college-level remedial/developmental programs. In addition, the characteristics, attitudes, and beliefs of the remedial/developmental student were examined. This portion of the review of literature established the lecture approach as the most commonly utilized instructional methodology. However, current reform documents are clearly calling for change--a change which incorporates a constructivist approach to mathematics education.

The second purpose of this review was to present constructivism as a theory of learning and knowing and to discuss a resulting pedagogy based upon constructivism. Metacognition, social interaction through dyadic cooperative learning, and problem solving were discussed as constructivist teaching strategies. The role of the teacher in the constructivist classroom was also described.

A third purpose of this review was to examine research involving the remedial/developmental mathematics student in the constructivist classroom. Research indicates that constructivism and the resulting pedagogy can effectively address the needs of the developmental student. Thus, through reviewing the related literature and responding to the calls for reform, a context for this study has been set.

CHAPTER III

METHODS AND PROCEDURES

The purpose of this study was to determine the effect of the two treatments, instructional methodologies consisting of a traditional lecture format and a constructivist instructional model, on mathematical achievement and mathematical attitudes of students enrolled in a developmental algebra course. This chapter describes the experimental design of the study.

Setting

The study was conducted during the 1995 fall semester at Roane State Community College (RSCC). Two RSCC locations, the Roane County campus in Harriman, Tennessee and the Oak Ridge campus in Oak Ridge, Tennessee, were utilized for this study. The Roane County campus which has a headcount of 2,159, and the Oak Ridge campus with a headcount of 2,098, are the two largest branches of RSCC. RSCC has an open-door admissions policy which allows any student with a high school diploma or equivalent to be admitted. In an effort to provide advisement and accurate placement of students, RSCC participates in the Board of Regents' Academic Assessment Placement Program (AAPP) to assess basic skills. Applicants under 21 years of age must submit ACT scores. If the student's mathematics sub-test score is less than 19 on the ACT, the student is required to take the math portion of the placement assessment. All applicants 21 years of age and older must undergo placement assessment in reading, writing, and mathematics skills. Based upon the results of the AAPP, students are placed in remedial, developmental, or college-level classes.

Developmental mathematics consists of two courses, Elementary Algebra (DSM 081) and Intermediate Algebra (DSM 082). The purpose of these courses is to prepare students for their entry level college mathematics courses. This study involved students enrolled in Intermediate Algebra (DSM 082).

Subjects

Four class sections of DSM 082 Intermediate Algebra were chosen for the study. Two of the class sections met on Monday, Wednesday, and Friday at 8:30 and 10:00 a.m. at the Oak Ridge Branch and were taught by the same instructor. The other two class sections met on the same days and at the same times at the Roane County Branch. A second instructor taught the two Roane County sections. The 10:00 a.m. class sections at both locations were taught utilizing the constuctivist instructional model (CM). The 8:30 a.m. class sections at both locations were taught utilizing the traditional lecture format (LF). The students who registered for the four class sections at the chosen times were randomly assigned to the four treatment groups via the registration process. At the beginning of the study, the enrollment was 19 in the Oak Ridge CM group and 24 in the Oak Ridge LF group. The initial enrollment was 19 in the Roane County CM group and 8 in the Roane County LF group. All students were asked whether or not they wished to participate in the study, and all students chose to participate. Subjects were asked to sign a consent form (see Appendix A). Only data from students who completed the course (i.e. completed the posttest) were included in the results of the study. Table 3.1 is a summary of class description information, and reflects the number of students who completed the course.

Design

This study examined the effects of two instructional methodologies on the achievement and attitude of students enrolled in intermediate algebra . In two class sections, the instructional methodology was the traditional lecture format. This is the mode of instruction currently utilized at RSCC at the intermediate algebra level. In the other two treatment class sections, the instructional methodology was a model based upon the tenets of constructivism which emphasized metacognition during problem-solving activities. Students in all four class sections were given the AAPP Intermediate Algebra Skills Subtest and the Fennema-Sherman Mathematics Attitudes Scales as pre and post measures of algebra knowledge and mathematics attitudes.

As stated previously, two class sections were taught by the same instructor at the Oak Ridge Branch. The 8:30 section was taught utilizing the traditional lecture format, and the 10:00 section was taught utilizing the constructivist instructional model. This arrangement was replicated with a second instructor at the Roane County Branch. The replication allowed two separate, parallel studies. Replication was deemed necessary for the following reasons: (a) small class sizes and unknown attrition rates, (b) allowance for the completion of the

Table 3.1

Class Description by Location, Instructor, Time, Mode of Instruction, and Number of Students

Oak Ridge (OR) Branch OR Instructor			Roane County (RC) Branch RC Instructor		
Time	Treatment	(n)	Time	Treatment	(n)
8:30	LF*	10	8:30	LF*	6
10:00	CM**	10	10:00	CM**	15

* LF-Lecture Format
** CM-Constructivist Model

study in the event of termination of a treatment by one of the instructors, and (c) an opportunity to gather more data to test the research hypotheses.

Instructors

Since a potential limitation of this study is the introduction of the teacher as a variable, an effort was made to select teachers with similar attributes who could teach effectively with either treatment modality. Both instructors were females with five years of college teaching experience, and both had previously taught mathematics at the high-school level. The Oak Ridge instructor had a preference for the lecture format, but readily adopted the constructivist instructional methodology in the designated section. The Roane County instructor had an existing preference for a combined lecture and cooperative-learning model. She modified her approach to be purely lecture for the lecture section, and utilized the researcher designed constructivist model for the other section. Both instructors reported that they were able to modify their behaviors according to the experimental design. Regular contact between the researcher and the instructors via telephone and e-mail conversations was maintained to verify the on-going utilization of the treatments.

Instructional Materials

The text used in all sections was Intermediate Algebra, Seventh Edition, by Marvin Bittinger and Mervin Keedy. The syllabus, content, and assignments were the same for all sections (see Appendix B). The same schedule of topics and testing was followed in each section (see Appendix C). The chapter exams used in each section were developed by a curriculum committee, thus covering the same concepts and skills.

Instructional Strategies

Two instructional strategies were utilized in this study: the traditional lecture format, and the constructivist model.

Lecture Format

The usual method of instruction utilized at RSCC at the intermediate algebra level is lecture. In the lecture format, the instructor is seen as lecturer, and the instructor directly provides the students with concepts and basic skills. Mathematics topics are presented by lectures, and basic skills are modeled by the instructor via the working of example problems during the lecture process. The activity in the classroom is teacher-centered, with the instructor as the leader. Student-teacher interaction in the lecture process is limited to the answering of individual student questions and teacher controlled teacher-class discussion.

Constructivist Model

In this treatment, approximately one half of the instructional time was spent in pair problem-solving activities. During group activities, the instructor passed from pair to pair monitoring the students' activities. Individualized assistance was given to students on an as needed basis during the problem-solving time. Thus, any student without a partner joined an existing dyad. Instructions were given to students on how to conduct the pair problem-solving activities which included sample dialogue with the roles of problem solver and listener clearly defined (see Appendix D). During the remainder of the instructional time, the instructor presented key concepts and skills via lecture, moderated classroom discussion, and employed questioning strategies. A description of the teacher's role in the constructivist classroom was provided to the instructor prior to the beginning of the experiment (see Chapter 2 of this study and guidelines in Appendix E).

Instruments of the Study

Instruments were used to measure the students' algebraic achievement and attitudes toward mathematics. These included the Tennessee Board of Regents Academic Assessment and Placement Program (AAPP) Intermediate Algebra Skills Subtest and the Fennema-Sherman Mathematics Attitudes Scales.

Tennessee Board of Regents AAPP

The AAPP Intermediate Algebra Skills Subtest Forms E and F were used to measure the students' algebra knowledge prior to the study and at the end of the study. The test is a 30 item multiple choice exam and consists of the following clusters: algebraic operations (7 items), solutions of equations and inequalities (8 items), geometry (8 items), and applications (7 items). The test is administered in a 30 minute time frame. The reliability coefficient for Form E is .809 and for Form F is .785. The test was developed by The College Board, Multiple Assessment Programs and Services.

Fennema-Sherman Mathematics Attitudes Scales

The Fennema-Sherman Mathematics Attitudes Scales (see Appendix F) were used to measure student attitudes at the beginning and end of the 15 week study. The scales were originally published in 1976 and are currently distributed by the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison. The instrument consists of nine Likert-type scales which measure attitudes related to mathematics learning. The following is a description of the scales provided in the document manual:

> Each scale consists of six positively stated and six negatively stated items with five response alternatives: strongly agree, agree, undecided, disagree and strongly disagree. Each response is given a score from 1-5 and on each scale, except the Male Domain Scale, the weight of 5 is given to the response that is hypothesized to have a positive effect on the learning of mathematics. The person's total score on each of these scales is their cumulative total and the higher the score, the more positive their attitude. (Fennema & Sherman, 1986, p. 7)

The nine domain specific scales include the following:

> (a) confidence in learning mathematics; (b) father, mother, and teacher scales measuring perceptions of attitudes toward one as a learner of mathematics; (c) effective motivation in mathematics; (d) attitude toward success in mathematics; (e)

> mathematics as a male domain; (f) usefulness of mathematics; and (g) mathematics anxiety scale. (Fennema & Sherman, 1986, p. 0)

The scales can be used individually, in any combination, or as a total package. In this study, five of the nine scales were used. Therefore, students responded to a total of 60 statements. The maximum, most positive score, for this study was 300. Each total score was divided by 60, and rounded to the hundredths place, in order for each score to be on a 5-point scale. All 60 statements were randomly distributed within the instrument. A computer program written by the researcher was used to randomize the numbers 1 through 60. Thus, the five scales, as well as the positive and negative statements, were not clustered. The randomization was suggested and termed appropriate by Linda Levi, an assistant researcher at the Wisconsin Center for Education Research (as per telephone conversation 6/21/95). Descriptions of these five scales are as follows:

<u>Attitude Toward Success in Mathematics Scale</u>

> The Attitude Toward Success in Mathematics Scale (AS) is designed to measure the degree to which students anticipate positive or negative consequences as a result of success in mathematics. They evidence this fear by anticipating negative consequences of success as well as by lack of acceptance or responsibility for the success, e.g., "It was just luck." (Fennema & Sherman, 1986, p. 2)

<u>Confidence in Learning Mathematics Scale</u>

> The Confidence in Learning Mathematics Scale (C) is intended to measure confidence in one's ability to learn and to perform well on mathematical tasks. The dimension ranges from distinct lack of confidence to definite confidence. The scale is not intended to measure anxiety and/or mental confusion, interest, enjoyment or zest in problem solving. (Fennema & Sherman, 1986, p. 4)

<u>Mathematics Anxiety Scale</u>

> The Mathematics Anxiety Scale (A) is intended to measure feelings of anxiety, dread, nervousness and associated bodily symptoms related to doing mathematics. The

> dimension ranges from feeling at ease to those of distinct anxiety. The scale is not intended to measure confidence in or enjoyment of mathematics. (Fennema & Sherman, 1986, p. 4)

Effectance Motivation Scale in Mathematics

> The Effectance Motivation Scale in Mathematics (E) is intended to measure effectance as applied to mathematics. The dimension ranges from lack of involvement in mathematics to active enjoyment and seeking of challenge. The scale is not intended to measure interest or enjoyment of mathematics. (Fennema & Sherman, 1986, p. 5)

Mathematics Usefulness Scale

> The Mathematics Usefulness Scale (U) is designed to measure students' beliefs about the usefulness of mathematics currently and in relationship to their future education, vocation, or other activities. (Fennema & Sherman, 1986, p. 5)

Fennema and Sherman obtained individual item and scale statistics by testing two high-school age populations (N=1600). The criteria used for the final versions of the scales are listed in order of importance: "(a) Items which correlated highest with the total score for each sex; (b) Items with higher standard deviations for each sex; (c) Items which yielded results consistent with the theoretical construct of a scale; (d) Items which differentiated mathematics and non-mathematics students" (Fennema & Sherman, 1986, p. 6). Split-half reliabilities were calculated after the final selection of the six positive and six negative items for each scale. The split-half reliabilities for the scales being utilized in this study are as follows: Attitudes Towards Success in Mathematics, .87; Confidence in Learning Mathematics, .93; Effectance Motivation in Mathematics, .87; Usefulness of Mathematics, .88; and Mathematics Anxiety, .89.

Hypotheses

The following hypotheses were tested at the .05 level of significance.

Pretreatment Hypotheses

H1: There is no significant difference between the mean achievement scores of :

(a) the CM group and

(b) the LF group

as measured by the AAPP Intermediate Algebra Skills Subtest Form E.

H2: There is no significant difference between the median scores of :

(a) the CM group and

(b) the LF group

as measured by the Fennema-Sherman Mathematics Attitudes Scales.

Experimental or Posttreatment Hypotheses

H1: There is no significant difference between the mean achievement scores of:

(a) the CM group and

(b) the LF group

as measured by the AAPP Intermediate Algebra Skills Subtest Form F.

H2: There is no significant difference between the median scores of :

(a) the CM group and

(b) the LF group

as measured by the Fennema-Sherman Mathematics Attitudes Scales.

Techniques of Analysis of Data

Using the results from the AAPP Intermediate Algebra Skills Subtest (Form E), pretreatment means were calculated for each group. A *t* test for the difference between two means for independent samples was used to test for equality of means with respect to algebraic achievement for the two Roane County treatment groups and for the two Oak Ridge treatment groups. Calculated *t* values were not significant at the .05 level. Thus, the pretreatment achievement means were not significantly different. Since the pretreatment achievement means were not significantly different, posttreatment achievement means were analyzed utilizing a *t* test. Differences in post achievment means, if found, could be attributed to the treatment.

Using the results from the pretreatment administration of the Fennema-Sherman Mathematics Attitudes Scales, the median test, a sign test for two independent samples, was used to test for equality of medians. The median test showed no significant difference between the medians of the two Oak Ridge treatment groups and no significant difference between the medians of the two Roane County treatment gruops. Thus, the pretreatment attitude medians were not significantly different. Similarly, posttreatment attitude medians were tested. Differences in posttreatment attitude medians, if found, could be attributed to the treatment.

Chapter IV includes a presentation and analysis of the data collected from the pretreatment and posttreatment measurement instruments. Chapter V includes a summary of these findings as well as a discussion of the treatments based upon instructor feedback.

The instructors of this study were asked to report regularly to the researcher on the implementation of the methodologies, and were asked at the end of the study to reflect upon their experiences.

CHAPTER IV

PRESENTATION AND ANALYSIS OF DATA

In this chapter the numerical data obtained in this study are presented, and the statistical analysis for the testing of each hypothesis is given. The data resulted from the pretreatment and posttreatment measures of algebraic achievement and mathematics attitudes. The scores were obtained during the fall semester of 1995 from treatment groups at the Roane County and Oak Ridge campuses of Roane State Community College. Only data from students who completed the course (i.e. completed the posttest) were included in the analysis. The results of the pretreatment achievement test and attitude scale and the posttreatment achievement test and attitude scale are found in Appendix G.

Pretreatment Analysis

Hypothesis 1

H1: There is no significant difference between the mean achievement scores of the CM group and the LF group as measured by the AAPP Intermediate Algebra Skills Subtest Form E.

Scores from the AAPP Intermediate Algebra Subtest Form E were analyzed to test this hypothesis. A *t* test for the difference between two means for independent samples was used to test for equality of means with respect to algebraic achievement for the two Roane County treatment groups and for the two Oak Ridge treatment groups. Assumptions for the *t* test were met based upon the following:

1. The data were interval.

2. It was assumed that random samples were achieved via the registration process.

3. An analysis done at the institutional level determined that the scores from the Intermediate Algebra Subtest are normally distributed.

4. A two sample *F* test for variances showed that the variances were equal for the two Roane County groups and for the two Oak Ridge groups.

Since niether of the calculated *t* values was significant at the .05 level, there was no basis to reject the pretreatment hypothesis and the pretreatment hypothesis 1 was assumed to hold. The results of the *t* test and the related descriptive statistics for pretreatment hypothesis 1 are presented in Table 4.1.

Table 4.1

Means, Standard Deviations, and *t* Test Results for the AAPP Pretest

Treatment (n)	Mean	Standard Deviation	Calculated *t*	*p* value
Oak Ridge				
LF* (10)	11.60	3.878	-0.180	$p>.05$
CM** (10)	11.30	3.164		
Roane County				
LF* (6)	13.0	3.512	0.608	$p>.05$
CM** (15)	14.0	3.120		

* LF-Lecture Format
**CM-Constructivist Model

Hypothesis 2

H2: There is no significant difference between the median scores of the CM group and the LF group as measured by the Fennema-Sherman Mathematics Attitudes Scales.

Scores from the Fennema-Sherman Mathematics Attitudes Scales were analyzed to test this hypothesis. The median test, a sign test for two independent samples, was used. This nonparametric procedure was chosen for the following reasons:

1. The normality and homogeneity of variance assumptions could not be made.

2. The ordinal nature of the data resulting from the Fennema-Sherman Mathematics Attitudes Scales.

The median test showed no significant difference between the medians of the two Oak Ridge treatment groups. Due to the small total sample size, the contingency table for the Oak Ridge data was evaluated utilizing Fisher's exact probability test instead of Chi square (see Appendix H). The calculated p value was 0.414 which was not significant at the .05 level of significance. The results are presented in Table 4.2. (Note: In the Oak Ridge lecture format group, only nine of the ten students completed a posttreatment attitude scale, therefore, n=9 for all attitude measures relating to this group.)

Similarly, the median test showed no significant difference between the medians of the two Roane County treatment groups. The obtained contingency table was evaluated utilizing Fisher's exact probability test, again, due to the small total sample size. The calculated p value was 0.367 which was not significant at the .05 level of significance. The results are presented in Table 4.3. The related descriptive statistics for the pretreatment hypothesis 2 are presented in Table 4.4.

Table 4.2

Median Test for Oak Ridge Pretreatment Attitude Scores

	LF*	CM**	Row Total
Above Median			
Observed Frequency	5	4	9
Expected Frequency	4.263	4.737	
At or Below Median			
Observed Frequency	4	6	10
Expected Frequency	4.737	5.263	
Column Total	9	10	19

p value = 0.414

* LF-Lecture Format
**CM-Constructivist Model

Table 4.3

Median Test for Roane County Pretreatment Attitude Scores

	LF*	CM**	Row Total
Above Median			
Observed Frequency	2	8	10
Expected Frequency	2.857	7.143	
At or Below Median			
Observed Frequency	4	7	11
Expected Frequency	3.143	7.857	
Column Total	6	15	21

p value = 0.367

* LF-Lecture Format
**CM-Constructivist Model

Table 4.4

Means, Standard Deviations, Medians, and Ranges for Pretreatment Attitude Scores

Treatment (n)	Mean	Standard Deviation	Median	Range
<u>Oak Ridge</u>				
LF* (9)	3.496	0.384	3.530	1.120
CM** (10)	3.397	0.456	3.270	1.300
<u>Roane County</u>				
LF* (6)	3.348	0.526	3.255	1.510
CM** (15)	3.557	0.456	3.700	1.650

* LF-Lecture Format
**CM-Constructivist Model

Posttreatment Analysis

The pretreatment mean achievement scores were not significantly different at the .05 level for the two Oak Ridge treatment groups and for the two Roane County treatment groups. Since the pretreatment hypothesis 1 was not rejected, posttreatment mean achievement scores were analyzed to test posttreatment hypothesis 1. The AAPP Intermediate Algebra Skills Subtest Form F served as the posttest measure of algebraic achievement and was administered during the last week of the semester.

<u>Hypothesis 1</u>

H1: There is no significant difference between the mean achievement scores of the CM group and the LF group as measured by the AAPP Intermediate Algebra Skills Subtest Form F.

Scores from the AAPP Intermediate Algebra Subtest Form F were analyzed to test this hypothesis. A *t* test for the difference between two means for independent samples was used to test for equality of means with respect to posttreatment algebraic achievement for the two Roane County treatment groups and for the two Oak Ridge treatment groups. Assumptions for the *t* test were also satisfied for the posttreatment analysis (see pretreatment analysis).

A two-tailed *t* test at the .05 level of significance revealed no significant difference in the posttreatment mean achievement scores for the two Roane County treatment groups and for the two Oak Ridge treatment groups. Therefore, there was no basis for rejection of posttreatment hypothesis 1. Thus, students taught from a constructivist perspective demonstrated equal achievement when compared to students who were taught using a traditional lecture format. The results of the *t* test and related descriptive statistics for posttreatment hypothesis 1 are presented in Table 4.5.

<u>Hypothesis 2</u>

H2: There is no significant difference between the median scores of the CM group and the LF group as measured by the Fennema-Sherman Mathematics Attitudes Scales.

Scores from the Fennema-Sherman Mathematics Attitudes Scales were analyzed to test this hypothesis. The pretreatment attitude measurement instrument and the posttreatment attitude measurement instrument were the same. The posttreatment instrument was administered during the last week of the semester.

The median test, utilizing Fisher's exact probability test, yielded a *p* value of 0.414 which showed no significant difference at the .05 level of significance between the medians

Table 4.5

Means, Standard Deviations, and *t* Test Results for the AAPP Achievement Posttest

Treatment (n)	Mean	Standard Deviation	Calculated *t*	*p* value
Oak Ridge				
LF* (10)	22.70	3.164	0.996	$p>.05$
CM** (10)	24.30	3.635		
Roane County				
LF* (6)	18.17	3.131	-0.280	$p>.05$
CM** (15)	17.67	3.664		

* LF-Lecture Format
**CM-Constructivist Model

of the two Oak Ridge treatment groups. Both Oak Ridge groups showed increases in the mean attitude score when pre and post mean measures were compared. The results are presented in Table 4.6.

Similarly, the median test showed no significant difference between the medians of the two Roane County treatment groups. The calculated *p* value was 0.094 which was not significant at the .05 level of significance. However, the Roane County lecture section did experience a decrease in the mean attitude score. The other three groups involved in the study experienced an increase in mean attitude. The results of the median test are presented in Table 4.7. The related descriptive statistics for the posttreatment hypothesis 2 are presented in Table 4.8. A summary of pre and post mean achievement and attitude scores for all four treatment groups is presented in Table 4.9.

Table 4.6

Median Test for Oak Ridge Posttreatment Attitude Scores

	LF*	CM**	Row Total
Above Median			
Observed Frequency	5	4	9
Expected Frequency	4.263	4.737	
At or Below Median			
Observed Frequency	4	6	10
Expected Frequency	4.737	5.263	
Column Total	9	10	19

p value = 0.414

* LF-Lecture Format
**CM-Constructivist Model

Table 4.7

Median Test for Roane County Posttreatment Attitude Scores

	LF*	CM**	Row Total
Above Median			
Observed Frequency	1	9	10
Expected Frequency	2.857	7.143	
At or Below Median			
Observed Frequency	5	6	11
Expected Frequency	3.143	7.857	
Column Total	6	15	21

p value = 0.094

* LF-Lecture Format
**CM-Constructivist Model

Table 4.8

Means, Standard Deviations, Medians, and Ranges for Posttreatment Attitude Scores

Treatment (n)	Mean	Standard Deviation	Median	Range
Oak Ridge				
LF* (9)	3.720	0.237	3.820	0.670
CM** (10)	3.577	0.398	3.650	1.130
Roane County				
LF* (6)	3.133	0.695	3.215	2.130
CM** (15)	3.580	0.395	3.620	1.620

* LF-Lecture Format
**CM-Constructivist Model

Table 4.9

Summary of Pre and Post Mean Achievement and Attitude Scores

	Pre-Mean Achievement	Post-Mean Achievement	Pre-Mean Attitude	Post-Mean Attitude
Oak Ridge				
LF*	11.60	22.70	3.496	3.720
CM**	11.30	24.30	3.397	3.577
Roane County				
LF*	13.0	18.17	3.348	3.133
CM**	14.0	17.67	3.557	3.580

* LF-Lecture Format
**CM-Constructivist Model

Summary of the Results

The posttreatment testing of hypotheses may be summarized as follows:

H1 (not rejected) : No significant differences exist in the posttreatment mean achievement scores. This hypothesis held for the Roane County treatment sections and the Oak Ridge treatment sections.

H2 (not rejected) : No significant differences exist in the posttreatment median attitude scores. This hypothesis held for the Roane County treatment sections and the Oak Ridge treatment sections.

CHAPTER V

SUMMARY, FINDINGS, LIMITATIONS, DISCUSSION, AND RECOMMENDATIONS

In this chapter, an overview of the study is given and a summary of the statistical findings are discussed in relation to the research questions posed in Chapter I. Several limitations of the study are examined. A discussion which is based upon instructor feedback during and at the end of the study provides further information relating to the effects of the constructivist instructional model. The chapter concludes with recommendations for additional research and remarks concerning the instructional implications of this study.

Summary

This study was designed to investigate the effects of a constructivist instructional model on the achievement and attitude of college students enrolled in intermediate algebra. The present reform movement in school mathematics is calling for a constructivist approach to the teaching and learning of mathematics, and it is this call for reform that sets the context for this research. The study was conducted with four class sections of developmental intermediate algebra during the 1995 fall semester at Roane State Community College (RSCC). Two instructors each taught two classes: one class utilizing the traditional lecture format and the other utilizing a constructivist instructional model designed by the researcher.

The design allowed for a comparison of the two different treatments at the RSCC Roane County Branch and at the RSCC Oak Ridge Branch. The lecture format is the current mode of instruction for the intermediate algebra course at RSCC.

The students in all four sections were given a pretreatment achievement test and attitude survey to determine if the initial achievement levels and mathematics attitudes were comparable. Pretreatment measures of both achievement and attitude were comparable for the two Roane County treatment sections and for the two Oak Ridge treatment sections. The following posttreatment null hypotheses were tested at the .05 level of significance:

H1: There is no significant difference between the mean achievement scores of the constructivist model group (CM) and the lecture format (LF) group as measured by the AAPP Intermediate Algebra Skills Subtest Form F

H2: There is no significant difference between the median scores of the CM group and the LF group as measured by the Fennema-Sherman Mathematics Attitudes Scales.

Findings

Findings in this section relate to posttreatment algebraic achievement and posttreatment mathematical attitude. In this section, posttreatment measures of each will be discussed in relation to each of the null hypotheses tested. Further discussion of nonstatistical findings appears in the discussion section of this chapter.

Algebraic Achievement

Hypothesis 1 relates to the effect of the traditional lecture format and a constructivist instructional model on algebraic achievement. Hypothesis 1 was not rejected. This answered the first question presented under the problem section of this study (p. 3):

Question: Do students in a developmental intermediate algebra course which is taught from a constructivist perspective experience equal or increased achievement compared with students enrolled in the same unit of study taught using a traditional lecture ?

Answer: Students in this study who were taught from a constructivist perspective experienced equal achievement (not significantly different) when compared to students who were taught with a traditional lecture format. This held in the comparison between the Roane County treatment groups and the Oak Ridge treatment groups. The largest gain in achievement took place in the Oak Ridge constructivist model group. However, this post mean achievement score was not significantly different from the mean achievement score of the Oak Ridge lecture group.

Mathematical Attitude

Hypothesis 2 relates to the effect of the traditional lecture format and a constructivist instructional model on mathematical attitude. Hypothesis 2 was not rejected. This answered the second question under the problem section of this study (p. 3):

Question: Do the students who are taught from a constructivist perspective develop more positive attitudes toward mathematics than those students who are taught using a traditional lecture format?

Answer: Students in this study who were taught from a constructivist perspective experienced equal attitudes (not significantly different) when compared to those students who were taught with a lecture format. This held in the comparison between the Roane County treatment groups and the Oak Ridge treatment groups. The only decrease in mean attitude occurred in the Roane County lecture section. However, the medians of the two Roane County posttreatment attitude scores were not significantly different at the .05 level.

Limitations

Instructor Faithfulness To and Attitude Toward Treatments

In this study, each instructor was asked to teach utilizing two different methodologies. There are two potential areas of concern: the teacher's ability to switch from one teaching pattern to another, and the possibility that teacher preference for one method over the other could affect the results. Both instructors reported that they were able to modify their behavior to fulfill the requirements of the experimental design. Both instructors communicated regularly with the researcher, and reported faithfulness to both treatments throughout the semester. (However, the Roane County instructor was absent during week 13 and part of week 14 due to a family emergency. During this time, a substitute teacher lectured in both sections.) In this study, the two methodologies were different and very well defined. The lecture section was teacher-centered, where most of the communication was one-way, from teacher to students. The constructivist methodology revolved around regular usage of pair problem solving which encouraged interaction among students, student metacognition, and reinforcement of step-wise problem solving. Based upon the design and the instructor feedback, it is believed that a faithfulness to the methodologies was achieved.

Did the instructors favor one method over the other, and if so, did the preference affect their delivery of each methodology? The issue of existing preferences and practices of each instructor was discussed by the researcher and instructors prior to the study. Both instructors showed a willingness to "buy into" the experimental design. Feedback from the instructors during and at the end of the study did not indicate the presence of preferences that could significantly impact the results of this study. However, it is recognized that this potential exists in all studies of this nature. The discussion section of this chapter contains a narrative which describes the initial preferences of each instructor and the experiences each had with the instructional methodologies.

Student Readiness

The issue of student readiness in relation to the constructivist model was a concern of both instructors during the study. At the beginning of the study, all students enrolled in the constructivist sections consented to participate in the study. These students were given a brief description of the study, and were given an orientation to the pair problem-solving activities. Thus, students were apparently ready and willing for constructivist activities.

However, the issue of readiness as it relates to algebra skill level and maturity did surface during the semester. Early in the semester, one of the instructors was concerned that the basic algebra skill level of the students was extremely weak, and therefore the weakness would prevent productive pair problem-solving activities. This concern quickly diminished as these students began to effectively implement the pair problem-solving process. The other instructor was concerned about the maturity level of the constructivist group. The constructivist group was less mature in age and behavior than the lecture group. She felt a more mature group would have benefitted more from the small group activities. Several of the students knew each other from high school and may have been more interested in the social nature of the activities rather than the potential intellectual benefits. Although these concerns were voiced, both instructors were able to utilize the constructivist methodology. The effect of these group characteristics can not be assessed for relevance. The random assignment of the treatments and the assumption of randomization via registration were attempts to control for these variables.

Attrition

The DSM 082 Intermediate Algebra course was a competency-based course with special mastery requirements for completion of the course. Students were allowed one retest (if needed) on each of the chapter tests in order to achieve the required 75 percent average on all chapter tests. Only students who completed all chapter tests with the required average were allowed to take the final exam. Students were required to pass the final exam with a minimum score of 70 percent. One retest was also allowed on the final. Only data from students who successfully completed the course are included in this study.

Attrition occurred in each group involved in this study, resulting in small group sizes. The rates of attrition ranged from 21 percent for the Roane County constructivist group to 58 percent for the Oak Ridge lecture group. There is no evidence that any of the attrition was due to the treatments. These attrition rates are within the same range as other developmental algebra courses at RSCC. Achievement measures may have been influenced, since all students had attained certain levels of competency prior to the posttest measure of achievement. In addition, posttreatment measures of attitude were gathered only from students who completed the course, therefore, not including those who dropped out during the semester and those who failed.

Discussion

Students taught from a constructivist perspective experienced equal achievement and had comparable posttreatment mathematical attitudes when compared to students who were taught with a lecture format. An obvious conclusion is that both treatments were equally effective in relation to attitude and achievement. Thus, there is no evidence that can be drawn from this study against the utilization of either treatment.

However, feedback from instructors and students involved in this study provided some valuable insight which is not reflected in the statistical analysis of data. The following discussion is based upon communication during and at the end of the semester with the instructors. The discussion is separated by instructor because each instructor reported different experiences with the constructivist model.

Oak Ridge Instructor

Prior to this study, this instructor utilized the traditional lecture format. Although she had an existing preference for the lecture method, she was ready and willing to experiment with the researcher designed constructivist methodology. She admitted that she had initial reservations, and was nervous about the actual implementation. After pretesting both sections, she was even more concerned about the students' weak basic algebra skills. At this point, she was not convinced that the students in the constructivist section could progress with limited instructor transmitted information. Furthermore, she anticipated less than productive pair problem-solving activities due to limited algebra skills.

The instructor's concerns quickly diminished as the students began to engage in pair problem-solving activities. The students, in her words, "melded together as a family." Friendly competition, as well as pulling for one another on exams, became the norm. As she moved around the during the pair problem solving, she was no longer a teacher dispensing information, she said she was "just a facilitator." During pair problem solving, the instructor listened to the dialogue and utilized questioning strategies to probe student solutions and conceptions. She described student behavior as exhibiting a "total loss of fear." On a few occasions, the dialogue was so loud and the students so absorbed, she could not gain the students' attention. It was during these times that she realized "they didn't need her."

One of the students enrolled in the Oak Ridge constructivist group wrote the following in a paper prepared for another class and shared it with her algebra instructor:

> Students feel there is a greater comfort zone among peers. They are much more open to discussion and aren't afraid to make mistakes. While working with peers you have more of a tranquility just knowing they are on the same wavelength as yourself. Some people think a teacher is a perfect person who doesn't make errors, so there [*sic*] afraid to ask questions. Most students agree that they would much rather discuss a subject among themselves, than have a superior being teach it to them....Most students are able to get much more involved with their subject by a group session. They are not so easily bored with the whole class. By actually helping another student understand, you are teaching yourself without even knowing it. It gives the students more of an independence about themselves and their learning habits. (Oak Ridge student, 1995)

Toward the end of the semester the instructor wrote in an e-mail message to the researcher: "The class as a whole loves the pair working--I was shocked to hear how they feel about it--as they were convincing in their sincerity. I had feared their progress would be less than acceptable--but who knows. Surely their attitudes will have improved."

The Oak Ridge instructor described her lecture section as mostly transmission of information, giving more conceptual explanations and working more examples than in the constructivist section. In the constructivist class, she worked from a concise outline using limited examples, and then utilized the pair problem-solving activities. She went from more than an hour of talking to the students as a group in the lecture section to 20 minutes in the constructivist class. She described the experience of limiting the lecture time and letting the students take over as follows: "I thought that when I let go, they might sink, but they did not." She did a routine one class period review session in both treatment sections for the final exam and was pleased with the post achievement levels for both groups.

Roane County Instructor

The Roane County instructor had an existing preference for a combined lecture and cooperative learning model. For this study, she modified her approach to be purely lecture in the lecture section, and utilized the researcher designed constructivist model for the other section. Her experiences with the constructivist model were quite different from those of the Oak Ridge instructor. The following discussion describes her experiences and perceptions for both the lecture format and the constructivist model.

The Roane County lecture section consisted of 8 females. The instructor described the group as having wonderful comradery, however, the lecture format did not allow her to "use the comradery to its fullest value." The instructor stated that the lecture class was a more mature class than the constructivist class, and expressed these thoughts at the end of the semester, "I feel that maturity is a definite factor in the success of the constructivist model. If it had been up to me to choose the structure for this study, I would have chosen this group for my constructivist group and not the lecture group." Further written comments sent to the researcher at the end of the semester reflect her frustrations with the teacher-dominated lecture format:

> I am a comfortable lecturer and an able communicator, but I found I repeated myself frequently just trying to make sure they were catching it, since I did not reinforce with constructivist activities. Students are much less likely to "talk math" out loud in front of the entire class than they are in a small group setting. This group did very little communicating with each other outside of class. If they were absent, all of their attention was turned to me, since it was my lecture they missed. They all tried very hard to be significantly related to me--not in any unhealthy way--but there was very little value placed on relating to anyone else in the class since I was the expert and I gave the grades and I did most of the talking and I...I...I...I....

The instructor described the constructivist class as follows:

> This class had 19 students who participated in the study. It was made up entirely (one exception) of 19 year olds just out of high school. For most, this was their first semester at RSCC. As a whole, this was an extremely immature group. The constructivist activities worked, but there were attendance problems, flirting problems, a couple of independent types who never wanted to work with anyone else--it was a real mixed bag overall. This group would have benefited from more lecture. I believe it takes a certain amount of maturity to trust in peer tutoring, and I had to spend quite some time selling this system all semester. No, I do not believe constructivist is the end all and be all of education, but I believe it to be a very valuable piece of it that is probably not used nearly enough.

In summation, the instructor had these general comments about both of the methodologies:

> I got tired of the constructivist activities--and really wore thin trying to keep it upbeat and interesting and stimulating. Students are not motivated from within to "talk math", and they don't trust themselves to be good at it, so they would just really rather not talk about it. They would rather hear it from me....I did not tire of lecturing, but longed to get them actively involved with each other and not just with me. I felt like I was personally carrying each one of them--that's a load. I don't mean

> to sound like I'm complaining, but one style or the other is not sufficient by itself. I was frustrated by the weakness of both styles. Since my normal style combines both, it is natural that I would feel this frustration.

It should also be noted that this instructor had a family emergency which required that she be away week 13 and part of week 14 during the semester. A substitute teacher taught both sections utilizing the lecture format during her absence. Due to her absence, the instructor did not have time to have review sessions for the final exam. The effect, if any, on posttreatment measures of attitude and achievement can not be determined.

Conclusions

The results of this study revealed that students do achieve gains in both achievement and attitude when taught from a constructivist perspective. The achievement and attitude levels were comparable to those achieved by students taught with a traditional lecture format. Thus, the results suggest that the constructivist model is as effective as the lecture format (in relation to attitude and achievement) and could be a desirable alternative.

Based upon the above discussion, it is apparent that the instructors and the students had different perceptions and experiences related to the implementation of the constructivist instructional model. The experimental design and the associated statistical analysis provide valuable, however limited, information concerning the overall effect of this constructivist methodology. Mathematical attitude and achievement are only two variables out of many that can be investigated. The comments from the instructors and student are anecdotal, nonetheless provide a foundation for future research.

Recommendations

In light of the results and experiences of this study, the following section includes recommendations for further research and concluding remarks regarding instructional implications.

Further Research

1. This study examined the effect of a constructivist instructional model on algebraic achievement as measured by a standardized test which tested procedural skills and mastery of concepts. A similar study designed to examine the effect on student problem-solving skills would be appropriate. The depth of student understanding of mathematical concepts could also be explored.

2. Further investigations of the hypotheses of this study could be made using larger populations divided into subgroups by ability levels, mathematical backgrounds, and other selected group characteristics (e.g. gender, age, etc.). This study could reveal if there is an optimum match between different student groups and the instructional methodology.

3. A similar study utilizing additional qualitative methods would provide enhanced insight into the effects of the constructivist instructional model on attitudes and achievement. This qualitative approach could include data collection from several sources such as researcher observations, videotape, instructor case studies, and student interviews.

4. This study could be replicated in a non-mastery, non-competency based course. Students failing this course could be included in the study. This would provide more complete data in relation to achievement and attitude measures.

5. The intermediate algebra course could be divided into units of study to determine if the constructivist instructional model had different effects on student achievement related to the particular unit of study.

6. Further research focusing on the role of the instructor and implementation of a constructivist instructional model is necessary. More information is needed in relation to teacher characteristics, preferences, and behaviors and their effect on the implementation of instructional methodologies.

7. A study which included a follow-up component monitoring the students' performance in their college-level mathematics courses could provide additional information concerning the effect of the constructivist model on future achievement.

Concluding Remarks

This study was undertaken in response to the present reform movement in mathematics education. The different voices of reform are asking math educators to adopt a constructivist approach to the teaching and learning of mathematics.

In this study, an instructional model based upon the tenets of constructivism was developed and then implemented in a college developmental algebra course. The constructivist model was developed as an alternative to the traditional lecture format which is the most widely used instructional methodology in developmental and remedial mathematics classes. The results of this study indicate that the constructivist model was equally effective in relation to mathematics attitudes and achievement when compared to the lecture format. Thus, this model should be considered by developmental mathematics educators as a worthy alternative to the lecture format.

It is the opinion of the researcher that this experiment established the practicability of this particular constructivist instructional model. The model was utilized for a semester by two instructors and it worked. Further research is needed to develop even more effective constructivist models for use at the college developmental level.

Within the current reform movement, standards have been adopted to lead math educators toward new goals. In order to reach these goals, teaching strategies must be directed toward meeting these goals. Teaching strategies should be goal directed and should emanate from a philosophical and theoretical perspective. Math educators at all levels must examine and clarify their personal philosophy of teaching and learning and the theoretical foundation upon which it is based. This study presents constructivism as a cognitive position, discusses the resulting pedagogy, and presents an instructional model which is consonant with this cognitive position. Therefore, this research exemplifies the relationship between theory and the resulting pedagogy. Developmental math educators could use this research to assist in a self-analysis to examine the theory and philosophy which supports their beliefs and practices.

The teaching strategies in this study are not new. Furthermore, strategies and exercises alone do not create a constructivist classroom. Grennon Brooks and Brooks write in <u>In Search of Understanding: The Case for Constructivist Classrooms</u> (1993):

> For many educators, becoming a constructivist teacher requires a paradigm shift. Becoming such a teacher means much more than appending new practices to already full repertoires. For many, it requires the willing abandonment of familiar perspectives and practices and the adoption of new ones. (p. 25)
>
> Becoming a teacher who helps students search rather than follow *is* challenging and, in many ways, frightening. Teachers who resist constructivist pedagogy do so for

> understandable reasons: most were not themselves educated in these settings nor trained to teach in these ways. The shift, therefore, seems enormous. And, if current instructional practices are perceived to be working, there is little incentive to experiment with new methodologies--even if the pedagogy undergirding the new methodologies is appealing. (p. 102)

It is the hope of the researcher that this study will assist developmental math educators in achieving a paradigm shift, a shift which is necessary if the vision of the current reform movement is to be realized.

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APPENDICES

APPENDIX A

STUDENT CONSENT FORM

Information about Teaching Study
Intermediate Algebra DSM 082
Fall 1995

During Fall Semester 1995, a teaching study will be conducted in four sections of DSM 082 Intermediate Algebra. The study will examine the effects of an instructional methodology on mathematics achievement and attitudes. You are enrolled in one of the following sections:

1) Lecture Format Group-Lecture is the usual method of instruction utilized at RSCC at the intermediate algebra level. Algebra topics will be presented by lectures and basic skills will be modeled by the instructor via the working of example problems during the lecture process.

2) Constructivist Model Group-In addition to lectures presented by your instructor, you will engage in pair problem-solving activities. You will work in pairs with other students utilizing a step-by-step problem-solving approach with a variety of intermediate algebra concepts and skills.

At the beginning and at the end of the semester you will be asked to take the AAPP Intermediate Algebra Skills Subtest and a mathematics attitudes scale. Statistics will be compiled at the end of the semester. Information gathered will be reported as group data and you will not be identified in the study.

If you decide not to participate, you may transfer to another class section not involved in this study. Counselor assistance is available for this purpose. This decision will not affect your grade in this course.

If you have any questions regarding this study, please ask your instructor or contact Pat Bailey (354-3000, ext. 4480, Academic Development Division).

You are making a decision whether or not to participate in the study described above. Participation in this study is completely voluntary. Your signature indicates that you have read the information provided and decided to participate. You may withdraw at any time without prejudice should you choose to discontinue participation in this study.

Name: (Please print)

____
Last First Middle

Social Security Number: ________________________

Signature: ____________________________________ Date: ________

APPENDIX B

INSTRUCTIONAL MATERIALS

SYLLABUS: ACADEMIC DEVELOPMENT (DSM) 082, FIVE (5) CREDIT HOURS
__________SEMESTER, 19___, **INTERMEDIATE ALGEBRA,** SECTION_____

I. INSTRUCTOR____________________ OFFICE LOCATION__________

II. OFFICE HOURS____________________ TELEPHONE______________

III. COURSE DESCRIPTION:

This course is designed to provide students advanced algebraic skills on a pre-college level. Satisfactory completion of this course or other evidence of competencies in these areas is a prerequisite for appropriate college-level mathematics courses.

IV. COMPETENCIES: Upon completion of the course the student should be able to:

A. Reinforce skills taught in Basic Math and Elementary Algebra

1. Evaluate algebraic expressions by substitution.
2. Perform arithmetic operations on real numbers.
3. Simplify algebraic expressions by using rules for the order of operations.
4. Simplify expressions containing exponential notation.
5. Translate numbers in decimal notation to scientific notation and vice versa.
6. Solve linear equations and inequalities.
7. Solve application problems involving linear equations and inequalities.
8. Solve formulas for a given variable.
9. Graph linear equations and inequalities.
10. Find the slope of the line containing two given points.
11. Find the equation of a line given its slope and a point on the line or given two points on the line.
12. Perform addition, subtraction, multiplication and division of polynomials.
13. Factor polynomials by removing the greatest common factor and by grouping.
14. Factor trinomials of type , trinomial squares and the difference of two squares.
15. Simplify, add, subtract, multiply and divide rational expressions.
16. Solve quadratic equations by factoring.
17. Find the mean, median and mode for a given set of numbers.
18. Read and interpret data from bar graphs, line graphs, and circle graphs.

B. Introduce new algebraic skills and topics in geometry, probability and statistics.

1. Use scientific notation for calculation purposes.
2. Solve equations containing absolute value expressions.
3. Determine whether two given lines are parallel, perpendicular or neither.
4. Given an equation of a line and a point not on the line, find the equation of a line parallel or perpendicular to the line and containing the point.
5. Solve a system of two equations in two variables by graphing, the substitution method, and the elimination method.
6. Solve application problems involving two equations in two variables.
7. Graph systems of two linear inequalities in two variables.

8. Factor the sum or difference of two cubes.
9. Solve equations involving rational expressions.
10. Simplify complex rational expressions.
11. Simplify, add, subtract, multiply, and divide radical expressions.
12. Rationalize the denominator of radical expressions.
13. Write radical expressions as expressions with rational exponents and vice versa.
14. Use laws of exponents to simplify expressions with rational exponents.
15. Given the lengths of two sides of a right triangle, use the Pythagorean theorem to find the length of the unknown side.
16. Write radical expression with negative radicand as an imaginary number using I.
17. Solve quadratic equations by the square root method and the quadratic formula.
18. Graph quadratic equations of type and type
19. Solve quadratic inequalities.
20. Use the Fundamental Counting Principle and the definition of probability to determine the probability that an event will occur.
21. Find measures of angles and lengths of side in similar triangles.
22. Find complements and supplements of given angles.

V. MATERIALS:

TEXTS: Intermediate Algebra, by Bittinger/Keedy, 7th Edition
Addison-Wesley, 1994.

CALCULATOR: A scientific calculator is required. A TI-30X, TI-35X, Casio 300, Casio 115S, or a similar scientific calculator with a fraction key is suggested.

VI. GRADING:

The final grade will be based on:
---70 points - Mastery Quizzes: A student who scores less than 75% will be allowed to take one retest at a time specified by the instructor. Retesting must be done outside of class or on designated test dates. **A student must master each test at a 75% level or average 75% on all tests before taking the final exam.**

--30 points - Exit Exam: Part I is a departmental exam which must be completed with at least 70% mastery before taking Part II. Part II is the Tennessee Board of Regents AAPP post test.

In general, the grading scale for this course is
90-100 = A, 80-89 = B, 70-79 = C, 0-69 = F

Students with a grade of F must re-enroll in the course and start over repeating all units and tests.

VII. ATTENDANCE:

Mandatory attendance in Academic Development classes is required by the Tennessee Board of Regents and Roane State Community College.

VIII. LEARNING AIDS AND COUNSELING:

Counseling is an important element of this course and is available and strongly encouraged for all students. At least one mid-term conference with the instructor will be required of **ALL** students whose progress is **unsatisfactory.** Other conferences may be held informally. Professional counselors are also ready to help. Please inform your instructor early in the course if there is anything he/she can do to aid in your efforts to learn. Preferential seating, cassette-taped materials, computer software, open math labs, etc. are available as extra help to aid the student in learning.

03/95/klm

INTERMEDIATE ALGEBRA ASSIGNED PROBLEMS
BITTINGER/KEEDY, 7TH EDITION

CHAPTER 1: ALGEBRA AND REAL NUMBERS

REVIEW AS NECESSARY -- YOU WILL BE TESTED ON THIS MATERIAL

EXERCISE SET	1.1	PAGES 7-8	/ 1 - 41 ODD
	1.2	PAGES 15-16	/ 25-65 ODD
	1.3	PAGES 23-26	/ 1-23 ODD, 33-91 ODD, 101-127 ODD
	1.4	PAGES 35-38	/ 25-89 ODD, 107-119 ODD

	1.5	PAGES 43-46	/ 1-61 ODD, 101-117 ODD
	1.6	PAGES 53-56	/ 1-57 ODD, 77-99 ODD, 109 - 114
CHAPTER 1 TEST		PAGES 61-62	/ 1-30, 35-55

CHAPTER 2: SOLVING EQUATIONS AND INEQUALITIES

EXERCISE SET	2.1	PAGES 73-76	/ 1-73 ODD, 81-86
	2.2	PAGES 83-85	/ 1-37 ODD
	2.3	PAGES 89-90	/ 1-19 ODD, 29-34
	2.4	PAGES 99-104	/ 1-43 ODD, 69-72, 85-90
	2.6	PAGES 121-122	/ 23-51 ODD, 105 - 110
CHAPTER 2 TEST		PAGES 129-130	/ 1-17, 27, 32, 36-44

CHAPTER 3: GRAPHS OF EQUATIONS AND INEQUALITIES

EXERCISE SET	3.1	PAGES 141-142	/ 1-21 ODD, 25, 27, 30
	3.2	PAGES 145-148	/ 1-27 ODD, 31-33
	3.3	PAGES 155-157	/ 1-4, 9-16, 27-51 ODD, 62, 63
	3.4	PAGES 163-164	/ 1-47 ODD, 49-52
	3.6	PAGES 173-174	/ 1-23 ODD, 25, 26
CHAPTER 3 TEST		PAGES 179-180	/ 1-11, 13-16, 19

CHAPTER 4: SYSTEMS OF EQUATIONS AND INEQUALITIES

EXERCISE SET	4.1	PAGES 189-190	/ 1-21 ODD, 25
	4.2	PAGES 197-198	/ 1-15 ODD, 18, 19, 23, 25, 35-40
	4.3	PAGES 207-209	/ 1-29 ODD
	4.8	PAGES 241-242	/ 1-13 ODD, 21-24
CHAPTER 4 TEST		PAGES 247-248	/ 1-10, 19, 21-24

CHAPTER 5: POLYNOMIALS

EXERCISE SET	5.1	PAGES 259-262 / 1-55 ODD, 63-73 ODD, 81, 82, 84
	5.2	PAGES 267-269 / 1-27 ODD, 35-45 ODD, 61-65 ODD
	5.3	PAGES 273-274 / 1-39 ODD, 45, 49, 50, 52
	5.4	PAGES 283-285 / 1-75 ODD
	5.5	PAGES 291-292 / 1-7, 21-26, 33-39, 67-69
	5.6	PAGES 295-296 / 1-19 ODD, 39-42
	5.7	PAGES 299-300 / 1-31 ODD (OMIT 13, 15, 17, 27, 29), 47, 48
	5.8	PAGES 307-310 / 1-39 ODD, 40, 43, 44, 55, 56, 61-64
	5.9	PAGES 317-318 / 1-13, 35-38
CHAPTER 5 TEST		PAGES 323-324 / 1-25, 29, 30, 32-36

CHAPTER 6: RATIONAL EXPRESSIONS AND EQUATIONS

EXERCISE SET:	6.1	PAGES 335-338 / 1-49 ODD, 53-58
	6.2	PAGES 345-348 / 1-59 ODD, 73-78
	6.3	PAGES 353-354 / 1-25 ODD, 43, 44, 46-48
	6.6	PAGES 373-374 / 1-11 ODD, 23-28
CHAPTER 6 TEST		PAGES 389-390 / 1-6, 8-11, 21-27

CHAPTER 7: RADICAL EXPRESSIONS AND EQUATIONS

EXERCISE SET:	7.1	PAGES 399-400 / 1-35 ODD, 57-62
	7.2	PAGES 405-406 / 1-12, 23-31, 54-59
	7.3	PAGES 409-410 / 1-9, 15-23, 46, 47
	7.4	PAGES 413-416 / 1-23 ODD, 31-61 ODD, 69-72
	7.5	PAGES 421-424 / 1-6, 37-39, 61, 63, 64
	7.6	PAGES 429-430 / 1-31 ODD
	7.8	PAGES 439-442 / 1-11, 23, 28-33
	7.9	PAGES 449-452 / 1-13 ODD, 100
CHAPTER 7 TEST		PAGES 457-458 / 1, 2, 3, 6, 7, 9-12, 15, 22, 23, 25, 26

CHAPTER 8: QUADRATIC EQUATIONS

EXERCISE SET:	8.1	PAGES 471-472 / 1-7, 9, 10, 63-66, 68
	8.2	PAGES 477-478 / 1-29 ODD (OMIT 13, 19, 21)
	8.5	PAGES 491-494 / 1-8, 25-28, 39, 40
	8.6	PAGES 499-502 / 1-13 ODD, 39, 40
	8.7	PAGES 509-512 / 1-29 ODD
	8.8	PAGE 519 / 1-9
	8.9	PAGE 529 / 1-8
CHAPTER 8 SUMMARY		PAGES 533-534 / 1-5, 12, 13, 22, 27, 28, 32, 36, 37

SUPPLEMENTARY BOOKLET

Probability:	PAGES 5-6	/1, 2, 3, 4, 7, 8, 9
Statistics:	PAGES 11-12	/1-17 ODD, 18, 19
Bar and Line Graphs:	PAGES 19-21	/1, 5, 9, 17, 23, 25
Circle Graphs:	PAGES 28-29	/1-11 ODD
Graph Interpretation:	PAGE 27	/1, 2, 3
Similar Triangles:	PAGES 32-33	/1, 3, 4, 9, 11, 13, 14
Complementary and Supplementary Angles:	PAGES 34	/1-8

Review the Pythagorean Theorem

APPENDIX C

SEMESTER SCHEDULE

DSM 082 SEMESTER SCHEDULE

SESSION	UNIT	SESSION	UNIT
1	Introduction, Consent form, AAPP	24	5.9**, Review **
		25	Test 5, 6.1
2	Attitude Scale, 1.1, 1.2	26	6.2 **, 6.3**
3	1.3, 1.4	27	6.6, Review**
4	PPST *, 1.5**	28	Test 6
5	1.6 **, Review **	29	7.1, 7.2
6	Test 1	30	7.3, 7.4 **
7	2.1 **, 2.2 **	31	7.5 **, 7.6
8	2.2 ** Contd.	32	7.8 **, 7.9
9	2.3 **, 2.4 **	33	Review, Test 7
10	2.6, Review **	34	8.1, 8.2 **
11	Test 2, 3.1	35	8.5 **, 8.6 **
12	3.2 **, 3.3**	36	8.7 **, 8.8 **
13	3.4**	37	8.9 **, Review **
14	3.6 **, Review **	38	Test 8
15	Test 3	39	Suppl. Booklet
16	4.1, 4.2 **	40	Suppl. Booklet**
17	4.3 **	41	Test 9
18	4.8 **, Review **	42	Review
19	Test 4	43	Review, Attitude Scale
20	5.1, 5.2	44	Review, AAPP
21	5.3, 5.4 **	45	Final
22	5.5, 5.6 **	46	Exam Period, Final (2nd attempt if needed)
23	5.7 **, 5.8 **		

* Pair Problem-Solving Training (CM sections only)

** Use Pair Problem-Solving Activity (CM sections only)

APPENDIX D

STUDENT PROBLEM-SOLVING

GUIDELINES

STUDENT PROBLEM-SOLVING GUIDE

During this semester, you will be asked to participate in problem-solving activities. The following is information related to problem solving and the pair problem-solving process.

METHODS AND CHARACTERISTICS OF GOOD PROBLEM SOLVERS

1. Positive attitude-Good problem solvers believe that through careful analysis and persistence problems can be solved. Experience with problem solving breeds confidence.

2. Concern for accuracy-Good problem solvers pay attention to detail, sift through the given information for relevant facts, and may reread the problem several times in order to gain the necessary understanding. All calculations are checked and sometimes rechecked.

3. Breaking the problem into parts-Good problem solvers take a step-by-step approach to problem solving. Complex problems are dissected into pieces and a starting point is chosen.

4. Avoid guessing-Good problem solvers do not guess or jump to quick conclusions. Patience and reliance on step-by-step analysis are ingredients of effective problem solving. Problem solving takes time.

5. Activeness in problem solving-Good problem solvers may create a mental or physical picture of the given problem. Good problem solvers will talk to him or herself, asking and answering questions concerning the given information.

PAIR PROBLEM SOLVING

In this course, the pair problem-solving process will be utilized to assist in the development of your problem-solving skills. The pair consists of a problem solver and a listener. The problem solver will think aloud as he or she proceeds along the solution path. The listener will also take an active role which is described below. You will be given the opportunity to experience both roles as you take turns with your partner being the problem solver and the listener. A description of the role of the listener follows:

1. Continually check for accuracy-The listener must work through the problem along with, not independent of, the problem solver. The listener must not allow the problem solver to get ahead. The problem solver and listener must finish the problem at the same time, after actively working together on the problem. The listener may point out an error, but not correct it. If the problem solver "gets stuck", the listener may make a first step or next step suggestion. However, all work on the problem should be performed by the problem solver.

2. Demand constant vocalization-The listener must remind the problem solver to vocalize each step in solving the problem. The listener can not monitor the process unless the problem solver explains each step out loud. If the problem solver skips steps, the listener should interrupt the process and ask for explanation.

Source: Whimbey, A., & Lochhead, J. (1991). *Problem solving and comprehension* (5th ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.

EXAMPLE OF PAIR PROBLEM-SOLVING DIALOGUE DURING THE SOLVING OF AN ALGEBRAIC WORD PROBLEM

Problem Solver: I'll read the problem. A carpenter needs to cut a board so that the length of one piece is seven inches less than the length of the other. The board is 25 inches long. Find the length of each piece. Let me read it again. A carpenter cuts a board so that the length of one piece is seven inches less than the length of the other. I think I'll draw a picture.

Listener: Did the problem say the pieces were equal? Your diagram looks that way.

Problem Solver: OK, I'll change it to make one piece smaller than the other. I need to label each piece. I'll choose the variable X to represent one of the unknown lengths. The other piece is seven inches less, so I'll label it 7 - X.

Listener: Why 7 - X? If something is 7 inches less than something else, wouldn't you take 7 away from it?

Problem Solver: You mean I should label it X - 7? I think that's correct. I remember you need to be careful with subtraction! I'll change it. Now we need an equation. The two pieces should add up to equal 25, so that X + X - 7 = 25. Now, I need to solve. I'll collect like terms, now I have 2X - 7 = 25. I need to get X by itself, so I'll take the - 7 over to the other side as + 7.

Listener: You mean add 7 to both sides?

Problem Solver: Yeah, it's the same thing. Now I have 2X = 32, so X = 16, and we're finished!

Listener: You divided by 2, so show it.

Problem Solver: OK, now we're finished!

Listener: Don't we need to find both lengths?

Problem Solver: Oh yeah. If X = 16, then X - 7 = 9. So the two pieces are 16 inches and 9 inches.

Listener: Does that check out?

Problem Solver: Yes. 9 inches is 7 inches shorter than 16 inches, and 9 and 16 add up to 25.

APPENDIX E

INSTRUCTOR GUIDELINES

INSTRUCTIONAL GUIDELINES FOR FALL 1995 TEACHING EXPERIMENT

LECTURE FORMAT:

- Present concepts and model skills via lecture.
- Employ traditional questioning strategies.
- Do not organize cooperative learning groups during time given to students to complete homework, etc.
- If graphing calculator is used in lecture format (LF), duplicate usage in constructivist model (CM).
- Read description on p. 37 of study.

CONSTRUCTIVIST MODEL:

- Approximately 50% of instructional time should be spent with students in dyads. Present student problem-solving guide and sample dialogue to class in preparation for pair problem-solving activities. Use process to form random pairs and have students exchange roles after each problem.
- Monitor students' problem-solving behaviors. Watch and listen in order to make on-line corrections. Utilize coaching metaphor.
- If a student needs individualized assistance, the remaining partner can join an existing dyad and take on a role of recorder.
- Model metacognitive behavior during problem solving at the board.
- Employ constructivist questioning strategies during class discussions and when monitoring pair problem-solving activities:
  - Ask students to discuss their interpretation of the problem.
  - Ask students to describe precisely their methods of solution.
  - Ask students to defend their answer and their solution.
  - Ask students to retrace the steps in their solution.
  - Allow sufficient wait time for the students' responses.
- Listen to the students' responses.
  Use this as an opportunity to positively reinforce those things that they are

doing correctly.
Create a mathematical environment, one which fosters mathematical growth and adaptation.
Uncover errors in thinking and misconceptions. Detect deficient or nonexistent skills.

- Use the following automatic-response patterns when responding to student questions:

  Retreat-advance response -the teacher retreats to a simpler example that is essentially the same and checks for understanding. Point out that the simpler example is essentially the same as the more complex one.
  Comprehension probe -Ask student to prove understanding by using natural language, providing similar examples, via graphical representations, etc.

- Create risk-free environment. "Wrong" answers and "stupid" questions provide invaluable insight and should be encouraged. Students' questions and answers are providing you with information concerning their present mathematical knowledge and their mathematical world view.

- Read Chapter 2 of study.

- Read Chapter 9 in *In Search of Understanding: The Case for Constructivist Classrooms.*

Notes:
As an incentive to complete the supplemental booklet and the 9th exam, allow students to replace lowest test score with test 9 grade.
Allow 20 minutes for administration of attitude scale.
Utilize the AAPP post test as a review for the final.
E-mail short report of pair problem-solving activities and progress weekly to researcher.
Offer same assistance outside class, retesting options, supplemental material, etc. to both sections.
It is of extreme importance that treatments are adhered to (keep lecture method pure, utilize constructivist strategies only in constructivist treatment section). Report problems or anticipated problems to researcher immediately.
Keep daily attendance records.
Keep records of chapter test and final exam scores.

APPENDIX F

FENNEMA-SHERMAN MATHEMATICS

ATTITUDES SCALES

FENNEMA-SHERMAN MATHEMATICS ATTITUDE SCALES
DIRECTIONS

- The following statements have been designed to permit you to indicate the extent to which you agree or disagree with the ideas expressed. If you neither agree nor disagree, that is you are not certain, then select response C for undecided. Also, if you cannot answer a question, select C. Do not spend much time with any statement. There are no "right or "wrong" answers. The only correct responses are those that are true for you.
- Be sure to respond to every statement.
- Do not mark on this survey. Mark your response on the answer sheet provided.
- Use a #2 pencil.
- In the space used for name on the answer sheet, write your code number and the course section number. Do not put your name there.

THIS INVENTORY IS BEING USED FOR RESEARCH PURPOSES ONLY AND NO ONE WILL KNOW WHAT YOUR RESPONSES ARE.

***

1. I see mathematics as a subject I will rarely use in my daily life as an adult.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

2. A math test would scare me.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

3. I'd be happy to get top grades in mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

4. My mind goes blank and I am unable to think clearly when working mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

5. I'll need mathematics for my future work.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

6. I am sure that I can learn mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

7. If I got the highest grade in math I'd prefer no one knew.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

8. Winning a prize in mathematics would make me feel unpleasantly conspicuous.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

9. Mathematics usually makes me feel uncomfortable and nervous.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

10. Mathematics will not be important to me in my life's work.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

11. Math doesn't scare me at all.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

12. When a math problem arises that I can't immediately solve, I stick with it until I have the solution.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

13. It would be really great to win a prize in mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

14. I don't understand how some people can spend so much time on math and seem to enjoy it.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

15. It would make me happy to be recognized as an excellent student in mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

16. I study mathematics because I know how useful it is.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

17. Generally I have felt secure about attempting mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

18. I like math puzzles.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

19. When a question is left unanswered in math class, I continue to think about it afterward.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

20. I get a sinking feeling when I think of trying hard math problems.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

21. Mathematics makes me feel uneasy and confused.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

22. Math has been my worst subject.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

23. If I had good grades in math, I would try to hide it.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

24. I'm no good in math.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

25. Most subjects I can handle O.K., but I have a knack for flubbing up math.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

26. I would rather have someone give me the solution to a difficult math problem than to have to work it out for myself.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

27. I am challenged by math problems I can't understand immediately.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

28. I'd be proud to be the outstanding student in math.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

29. Being first in a mathematics competition would make me pleased.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

30. Mathematics is of no relevance to my life.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

31. People would think I was some kind of a grind (i.e. someone who studies constantly) if I got A's in math.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

32. I will use mathematics in many ways as an adult.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

33. Taking mathematics is a waste of time.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

34. Knowing mathematics will help me earn a living.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

35. I expect to have little use for mathematics when I get out of school.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

36. I haven't usually worried about being able to solve math problems.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

37. Figuring out mathematical problems does not appeal to me.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

38. It wouldn't bother me at all to take more math courses.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

39. I can get good grades in mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

40. I usually have been at ease during math tests.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

41. I think I could handle more difficult mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

42. I am sure I could do advanced work in mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

43. Being regarded as smart in mathematics would be a great thing.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

44. Mathematics is enjoyable and stimulating to me.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

45. I almost never have gotten shook up during a math test.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

46. I usually have been at ease in math classes.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

47. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

48. I don't think I could do advanced mathematics.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

49. The challenge of math problems does not appeal to me.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

50. For some reason even though I study, math seems unusually hard for me.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

51. Math puzzles are boring.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

52. I'm not the type to do well in math.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

53. Once I start trying to work on a math puzzle, I find it hard to stop.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

54. I'll need a firm mastery of mathematics for my future work.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

55. I have a lot of self-confidence when it comes to math.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

56. I don't like people to think I'm smart in math.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

57. It would make people like me less if I were a really good math student.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

58. I do as little work in math as possible.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

59. In terms of my adult life it is not important for me to do well in mathematics in college.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

60. Mathematics is a worthwhile and necessary subject.

A	B	C	D	E
Strongly agree	Agree	Undecided	Disagree	Strongly disagree

APPENDIX G

STUDENT SCORES

Table A.10

Oak Ridge LF Group Scores

Student	Pretreatment Scores		Posttreatment Scores	
	Achievement	Attitude	Achievement	Attitude
1	12	3.3	25	3.33
2	5	2.85	20	3.38
3	12	3.4	23	3.9
4	10	3.78	15	3.88
5	6	2.95	21	3.6
6	17	3.53	24	3.97
7	10	3.95	24	3.6
8	15	3.97	26	4
9	12	3.73	26	3.82
10	17	3.45	23	*

* student did not complete

Table A.11

Oak Ridge CM Group Scores

Student	Pretreatment Scores		Posttreatment Scores	
	Achievement	Attitude	Achievement	Attitude
1	16	4.03	27	3.93
2	13	2.92	26	2.97
3	6	3.42	19	3.58
4	12	3.12	21	3.28
5	15	3.88	23	3.92
6	9	3.6	29	4
7	8	4.1	29	4.1
8	8	3.1	19	3.04
9	13	2.8	23	3.72
10	13	3	27	3.23

Table A.12

Roane County LF Group Scores

Student	Pretreatment Scores		Posttreatment Scores	
	Achievement	Attitude	Achievement	Attitude
1	17	4.08	23	4.25
2	9	3	17	2.48
3	14	3.93	16	3.52
4	8	3.12	22	3.08
5	13	2.57	15	2.12
6	17	3.4	16	3.35

Table A.13

Roane County CM Group Scores

Student	Pretreatment Scores		Posttreatment Scores	
	Achievement	Attitude	Achievement	Attitude
1	16	3	19	3.57
2	18	3.3	26	3.1
3	16	4	15	3.65
4	12	3.35	17	3.4
5	15	3.77	18	3.93
6	9	3.1	17	3.33
7	10	3.7	19	3.75
8	11	4.43	17	4.6
9	16	3.85	22	3.85
10	14	4.03	15	3.62
11	17	2.78	15	2.98
12	12	3.9	14	3.83
13	18	3.83	23	3.67
14	9	3.32	11	3.35
15	17	3	17	3.07

APPENDIX H

MEDIAN TEST

MEDIAN TEST

MacDougall and Stevens (1994) described the median test as follows:

> The Median Test is used to test the [null hypothesis] that two independent samples were drawn from populations having the same median....It involves finding the overall median of the scores combined across the two samples, and then counting how many scores in each sample fall above and below the overall median. If [the null hypothesis] is false and the two populations have different medians, then the sample drawn from the population with the larger median will tend to have more scores above the overall median for the two samples, while the sample from the population with the smaller median will have the most scores below the overall median. (p. 52)

Siegel (1956) described the considerations which should guide the researcher when choosing between the Fisher test and the chi-square test to evaluate the contingency table which results when analyzing data split at the median:

1) Use chi-square corrected for continuity when the total number across the two samples is larger than 40.
2) When the total number across the two samples is between 20 and 40 and when no cell has an expected frequency of less than 5, use chi-square corrected for continuity. If the smallest expected frequency is less than 5, use the Fisher test.
3) When the total number across the two samples is less than 20, use the Fisher test.

The formula for Fisher's exact probability test as described by MacDougall and Stevens (1994) is:

$$p = \frac{(A+B)!\,(C+D)!\,(A+C)!\,(B+D)!}{N!A!B!C!D!}$$

> where:
> N = total number of scores across the two samples
> A = observed frequency for Sample-1, less than or equal to the overall median for the combined samples,
> B = observed frequency for Sample-1, greater than the overall median for the combined samples,
> C = observed frequency for Sample-2, less than or equal to the overall median for the combined samples, and
> D = observed frequency for Sample-2, greater than the overall median for the combined samples.
> The overall formula is used to compute *p* not only for the obtained contingency table, but also for each of the more extreme (statistically unlikely) "splits" that could occur. The separate *p*'s are then summed to give the overall *p* value for the table. (p. 52)

VITA

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The author is a member of the National Council of Teachers of Mathematics, the Tennessee Mathematics Teachers Association, the National Association for Developmental Education, and the Tennessee Association for Developmental Education. She is currently an associate professor of developmental mathematics at Roane State Community College, where she has been employed since 1979. She currently resides in Knoxville, with her husband, Leonard Bailey, and her cat, Muttley.