To the Graduate Council:

I am submitting herewith a thesis written by Gim Cheng Khor entitled "A Comparision Of Multivariable Controllers: Singular Value Analysis Controller And Multivariable Tuning Regulator." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Chemical Engineering.

Duane D Bruns

Duane D. Bruns, Major Professor

We have read this thesis
and recommend its acceptance:

Charles E. Mann

Joe Wm. Mays

Accepted for the Council:

Vice Provost
and Dean of The Graduate School

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a Master's degree at The University of Tennessee, Knoxville, I agree that the library shall make it available to borrowers under rules of the library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of the source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Head of Interlibrary Services when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my written permission.

Signature Cheng Khor

Date July 31, 1987

A COMPARISON OF MULTIVARIABLE CONTROLLERS : SINGULAR VALUE ANALYSIS CONTROLLER AND MULTIVARIABLE TUNING REGULATOR

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Gim Cheng Khor

August 1987

ACKNOWLEDGMENTS

The author acknowledges with deep gratitude the valuable suggestions and guidance given by her mojor professor, Dr. Duane D. Bruns. The author is also very grateful to Drs. C. F. Moore and T. S. Wang for serving on her committee. Thanks also goes to The Department of Chemical Engineering for financial support to make this research possible.

ABSTRACT

The performance of three alternative multivariable control strategies are investigated. The recently developed SVAC, singular value analysis controller, and MVPIR, multivariable tuning regulator, are compared to the conventional MVSISO, multivariable single - input single - output controller. These controllers can all be characterized as requiring a minimum amount of process information and being relatively straightforward to design. The MVSISO control technique is based on the steady state process gain matrix with the manipulated and controlled variables paired according to the relative gain array analysis. Similary, the SVAC development is based only on the process gain matrix, but accounts for steady state interaction by using matrices from the singular value decomposition of the gain matrix. In contrast, the MVPIR requires the process transfer function matrix or a dynamic state space model as the design takes into account the initial transient interaction, as well as, steady state interaction. The MVSISO controller is implemented using a diagonal PI controller. The SVAC also uses a diagonal PI controller in its structure but it is pre- and post-multiplied by singular value decomposition matrices. The MVPIR uses full P-mode and I-mode matrices.

In order to evaluate the potential of the controllers, a digital simulation study is carried out using four examples based on literature transfer function models excluding their deadtime. The examples were selected to exhibit various degrees of steady state interaction. The

evaluation is made in terms of their response to set point changes with their tuning parameters optimized for the IAE or the IAE with a penalty for manipulated variable movement.

The simulation studies indicate that the three control schemes under study could successfully control non-interacting systems. However, if considerable interactions exist between loops, the MVPIR has the best performance, followed by SVAC. The effectiveness of MVSISO control system is reduced as the interactions between loops increases. Contrarily, the stronger the interactions, the more useful the MVPIR.

TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION	1
II.	LITERATURE REVIEW	5
	Description And Modeling Of Multivariable Processes And Control Systems	6
	Interaction Analysis And MVSISO Design	13
	Design Methodology	18
	Summary	19
III.	MATHEMATICAL PRELIMINARIES	22
	Vector And Matrix Norms	22
	Orthogonal Matrices	25
	Pseudoinverse	26
	Singular Value Decomposition	26
	Interpretation Of Process Control Properties Provided By SVA	28
	Scaling	30
	Computation Of SVD	33
	Summary	34
IV.	MULTIVARIABLE CONTROLLERS AND SIMULATION PROCEDURES	35
	Singular Value Analysis Controller Structure	35
	Multivariable Proportional Integral Tuning Regulator	39
	Multivariable Single Input - Single Output Controller	45
	Comparison Of The Theoretical Basis Of The Studied Controllers	45
	Minimum Realization	47
	Simulation Method	52
	Optimal Tuning Procedure	54

CHAPTER PAGE

V. APPLICATION OF MULTIVARIABLE CONTROLLERS. . . . 56

Selected Models Used In Studies. 56
Multivariable Control Of Model A. 61
Multivariable Control Of Model B. 70
Multivariable Control Of Model C. 82
Multivariable Control Of Model D. 95
Summary. 107

VI. CONCLUSIONS AND RECOMMENDATIONS. 112

Recommendations. 113

BIBLIOGRAPHY. 116

APPENDICES. 125

APPENDIX A - An Example Of Minimum Realization. 126

APPENDIX B - State Space Models. 133

APPENDIX C - Computer Programs. 139

VITA. 153

LIST OF FIGURES

FIGURE		PAGE
2.1	Block diagram of a multivariable process control system composed of a transfer function for each element.	7
2.2	Block diagram of a multivariable process control system composed of matrices of transfer functions.	8
2.3	State space representation of multivariable process control system.	11
3.1	Geometric interpretation of the SVD. Input matrix is partitioned into three parts : input rotational matrix V, scaling matrix Σ, and output rotational matrix U.	29
4.1	Development of singular value analysis controller.	37
4.2	Multivariable proportional plus integral tuning regulator structure.	44
4.3	Multivariable single input - single output controller structure	46
5.1	Evaluation procedure scheme.	57
5.2	Non-unimodal search surface of Model B for MVPIR control system with P_2 fixed.	60
5.3	Schematic diagram of Wood and Berry column.	62
5.4	Model A response to a unit step in set point two using MVSISO.	67
5.5	Model A response to a unit step in set point two using MVPIR.	68

FIGURE PAGE

5.6 Model A response to a unit step in set point two using SVAC. 69

5.7 Response of Model B for step in set point one with MVSISO. 75

5.8 Response of Model B for step in set point one with MVPIR. 76

5.9 Response of Model B for step in set point one with SVAC. 77

5.10 MVSISO control of Model B for step in set point one followed by step in set point two. 79

5.11 MVPIR control of Model B for step in set point one followed by step in set point two. 80

5.12 SVAC control of Model B for step in set point one followed by step in set point two. 81

5.13 Schematic diagram of Ogunnaike and Ray experimental distillation column. 83

5.14 Process inputs for MVSISO to simultaneous steps in set points one and two for Model C. 89

5.15 System response for MVSISO to simultaneous steps in set points one and two for Model C. 90

5.16 Process inputs for MVPIR to simultaneous steps in set points one and two for Model C. 91

FIGURE PAGE

5.17 System response for MVPIR to simultaneous steps in set points one and two for Model C. 92

5.18 Process inputs for SVAC to simultaneous steps in set points one and two for Model C. 93

5.19 System response for SVAC to simultaneous steps in set points one and two for Model C. 94

5.20 Process input for MVSISO to simultaneous steps in set points one and two for Model C with input suppression. 96

5.21 System response for MVSISO to simultaneous steps in set points one and two for Model C with input suppression. 97

5.22 Schematic diagram of Alatiqi and Luyben "LOF" system. . 98

5.23 Response of Model D to a step in set point one with MVSISO. 103

5.24 Response of Model D to a step in set point one with MVPIR. 104

5.25 Response of Model D to a step in set point one with SVAC. 105

5.26 System response for SVAC to a step in set point one for Model D with input suppression. 109

5.27 System response for SVAC to a step in set point one for Model D with new tuning criteria. 110

LIST OF TABLES

TABLE		PAGE
2.1	An overview of multivariable control literature.	20
5.1	Open loop transfer functions of selected models.	59
5.2	Steady state operating conditions and input constraints of Wood and Berry model.	63
5.3	Interaction analysis and design equations for Model A. . . .	64
5.4	IAE tuning of Model A.	66
5.5	Steady state operating conditions and input constraints of Column C model.	71
5.6	Interaction analysis and design equations for Model B. . . .	72
5.7	Tuning parameters of Model B.	74
5.8	Steady state operating conditions and input constraints Ogunnaike and Ray column model.	84
5.9	Interaction analysis and design equations for Model C. . . .	85
5.10	IAE tuning of Model C.	88
5.11	Steady state operating conditions and input constraints of Alatiqi and Luyben model.	99

TABLE PAGE

5.12 Interaction analysis and design equations for Model D. 101

5.13 Tuning parameters for step change in set point one for Model D. 102

5.14 Comparison of SVAC tuning parameters for step change in set point one for Model D. 108

CHAPTER I

INTRODUCTION

The application and importance of multivariable control has increased rapidly in recent years due largely to increasing fuel and feed stock costs and the growing competitiveness of the processing industries. This trend has also been enchanced due to the dropping cost of needed computational resources, availability of supporting software,and development of real time sensors. The terminology "multivariable" refers to processes with multiple inputs and/or multiple outputs. There has been much work in the academic community on the design of multivariable control systems to address the control difficulties presented by multivariable processes. Recently, researchers have used mathematical models of such processes to back calculate a multivariable controller structure. Two examples are the multivariable proportional plus integral tuning regulator and singular value analysis controller. Despite all the developmental academic work,very few multivariable controller applications have been reported to be in use in industry. Some of the reasons for this include excessive engineering manpower requirements, the design procedures model requirements, operating personnel technical requirements and lack of multivariable controller ability to operate under process uncertainties or changing process conditions. Therefore, most multivariable processes are still controlled by establishing several control loops with each control loop individually established by pairing

one output with one input and using a single input - single output controller. This technique is called multivariable single input - single output control (MVSISO). The multivariable single input - single output configuration still remains the bench mark against which all the more complex designs are compared.

The goal of this thesis is to investigate the effectiveness of applying two recent multivariable controller design techniques,the multivariable proportional plus integral tuning regulator (MVPIR) and singular value analysis controller (SVAC) with respect to set point tracking. The MVSISO controller, MVPIR, and SVAC are relatively easy to design because they only require little information. Although both MVSISO and SVAC control techniques are based on steady state process gain matrix, SVAC accounts for steady state interaction. In contrast to both MVSISO and SVAC, the MVPIR is developed from a dynamic state space model. The proportional and integral modes are designed to eliminate initial transient and steady state error respectively. Hence, among the controllers the MVPIR may be ranked as the best with respect to error tracking followed by SVAC and MVSISO.

These controllers will be studied by applying the design techniques to four models and evaluating the design through computer simulation of the models with the multivariable control structures. The MVSISO design is also developed and simulated to provide additional comparisions for these truly multivariable control designs.

The models selected all have the following characteristics;

1. They are linear transfer function models which are taken from literature.
2. Deadtime is neglected in this study as none of the design methods treats deadtime. Thus retaining the deadtime would only mask the comparisions. Another point of view would be that the controllers are applied to the process with good deadtime compensation in place.
3. The transfer functions are all first order lag or second order under or critically damped.

First of all, the thesis is organized by giving a brief description of multivariable control systems in Chapter II. The MVSISO control method is treated in this chapter. The critical step in designing a MVSISO control system is selecting the best pairing between manipulated and controlled variables so that interaction is avoided and the control system gives satisfactory behavior. Several interaction measures which are used to guide in the input - output pairing for the loops such as relative gain array, dynamic gain array and Niederlinski index are discussed. Finally, the history of various well developed multivariable design methodologies is also outlined.

Singular value decomposition is a matrix decomposition technique which expresses a general matrix in terms of a diagonal matrix and two orthonormal matrices. Singular value decomposition of a steady state process gain matrix provides useful information regarding the process structure by analyzing the three decomposition matrices. Robust

numerical algorithms are available to carry out the decomposition. Chapter III reviews some mathematical concepts required in order to understand and interpret singular value decomposition, then gives a summary of singular value analysis and the interpretation of the decomposition.

Chapter IV discusses the development, structure and design details of the multivariable control design techniques under study. To study the effectiveness of the controllers, digital simulation is used. A review of current simulation techniques and the method used in this work for the simulation of the closed loop performance is also presented.

Finally, the results of using multivariable proportional integral tuning regulator and singular value analysis controller are presented in Chapter V. The performance of these controllers is compared to the conventional multivariable single input - single output controller. Chapter VI proposes future work based on the results and questions which arose from this work.

CHAPTER II

LITERATURE REVIEW

Multivariable control systems are considered to be advanced and complicated by control application engineers. As the processing industry continues to develop, process flowsheet complexity has increased greatly. This complexity has arisen from the need to optimize the overall plant to reduce energy and feed stock consumption. In many cases the production scale is also becoming larger. This overall plant integration along with increasingly sophisticated designs requires the multivariable nature of more and more processes to be taken into account. Consequently, the control systems being needed are, in many cases, no longer single variable in nature. Multivariable control techniques hold the promise that they can successfully control some processes containing many coupled variables. In other words, multivariable control system theory is directed at satisfying higher control requirements, such as higher production specifications and tighter enviromental regulations, than those of the conventional analysis theory of single variable control system.

There has been considerable interest in developing process control strategies for multivariable control problems since about 1950 (1). Since then, a variety of advanced control techniques have been developed. This chapter is devoted to the description of multivariable process control systems, a review of the classical MVSISO design approach, and a listing of other multivariable control approaches.

Description And Modeling Of Multivariable Processes And Control Systems

As in the single variable or single input-single output (SISO) control systems, block diagrams are widely applied to analyze multivariable process control systems. The popularity of block diagram exists because of two very important properties. The first is that the transfer function for the overall system can be obtained by means of block diagram manipulation if all the system elements are linear. The second is that the block diagram can be used as a reference for the analytical or the computer solution of the system if the mathematical and functional relationships of all the system elements are known. There are two common formulations of a block diagram: a block diagram composed of a transfer function for each input - output pair and a block diagram composed of matrices of transfer functions, as shown for a two input - two output example in Figures 2.1 and 2.2 respectively. Both of these block diagram representations have the same characteristics and both can be analyzed by using matrices and matrix operations. Therefore, application of matrix algebra is a main mathematical tool in the analysis of multivariable process control systems.

There are two usual ways of modeling a linear dynamic system, the transfer function and state space representation. For a transfer function representation, the output vector is related to the input vector via the transfer function matrix such as:

$$\underline{Y}(s) = \underline{G}(s)\,\underline{U}(s) \qquad (2.1a)$$

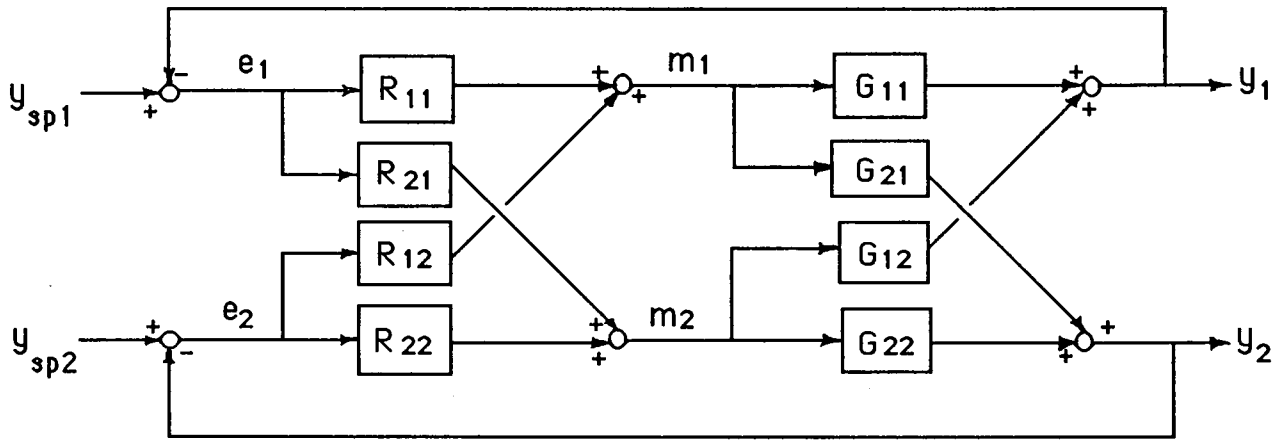


Figure 2.1. Block diagram of a multivariable process control system composed of a transfer function for each element.

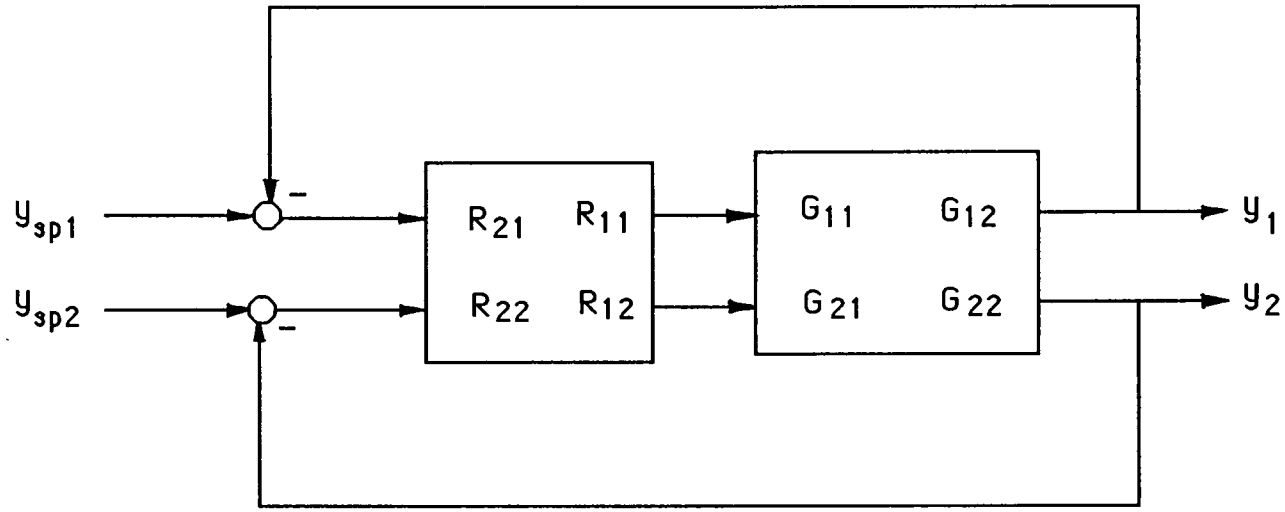


Figure 2.2. Block diagram of a multivariable process control system composed of matrices of transfer functions.

or for a two input - two output example

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \qquad (2.1b)$$

where $\underline{Y}(s)$ = output vector

$\underline{U}(s)$ = input vector

$\underline{G}(s)$ = transfer function matrix

s = Laplace variable

The transfer function matrix may be obtained by appling a known input $\underline{U}$ to the system while measuring the output $\underline{Y}$. In general, $G_{ij}(s)$ is the transfer function relating input j to output i. A diagonal transfer function matrix indicates a system without interactions.

A multivariable control system can also be expressed by using a state space model. Suppose the system is linear with constant coefficients and can be described by a set of first order differential equations:

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \cdots + b_{1p}u_p \\ \dot{x}_2 &= a_{21}x_1 + a_{21}x_2 + \cdots + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + \cdots + b_{2p}u_p \\ &\vdots \\ \dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_{n1}u_1 + b_{n2}x_2 + \cdots + b_{np}u_p \end{aligned} \qquad (2.2)$$

and

$$y_1 = c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n + d_{11}u_1 + d_{12}u_2 + \cdots + d_{1p}u_p$$

$$y_2 = c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_n + d_{21}u_1 + d_{22}u_2 + \cdots + d_{2p}u_p$$

$$\vdots \qquad (2.3)$$

$$y_m = c_{m1}x_1 + c_{m2}x_2 + \cdots + c_{mn}x_n + d_{m1}u_1 + d_{m2}u_2 + \cdots + d_{mp}u_p$$

If written in matrix notation, Equations 2.2 and 2.3 become

$$\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}\underline{U}$$

$$\underline{Y} = \underline{C}\underline{X} + \underline{D}\underline{U} \qquad (2.4)$$

where $\underline{X}$ = state vector of length n

$\underline{U}$ = input vector of length p

$\underline{Y}$ = output vector of length m

$\underline{A}$ = n x n state matrix

$\underline{B}$ = n x p input state matrix

$\underline{C}$ = m x n output state matrix

$\underline{D}$ = m x p direct action matrix

In terms of a state space model or state variables, a general two input -two output multivariable process and control system is expressed in Figure 2.3. The state of a system or set of state variables is defined as the smallest set of variables which determine the state of the system. The set of variables chosen as the state variables are not unique.

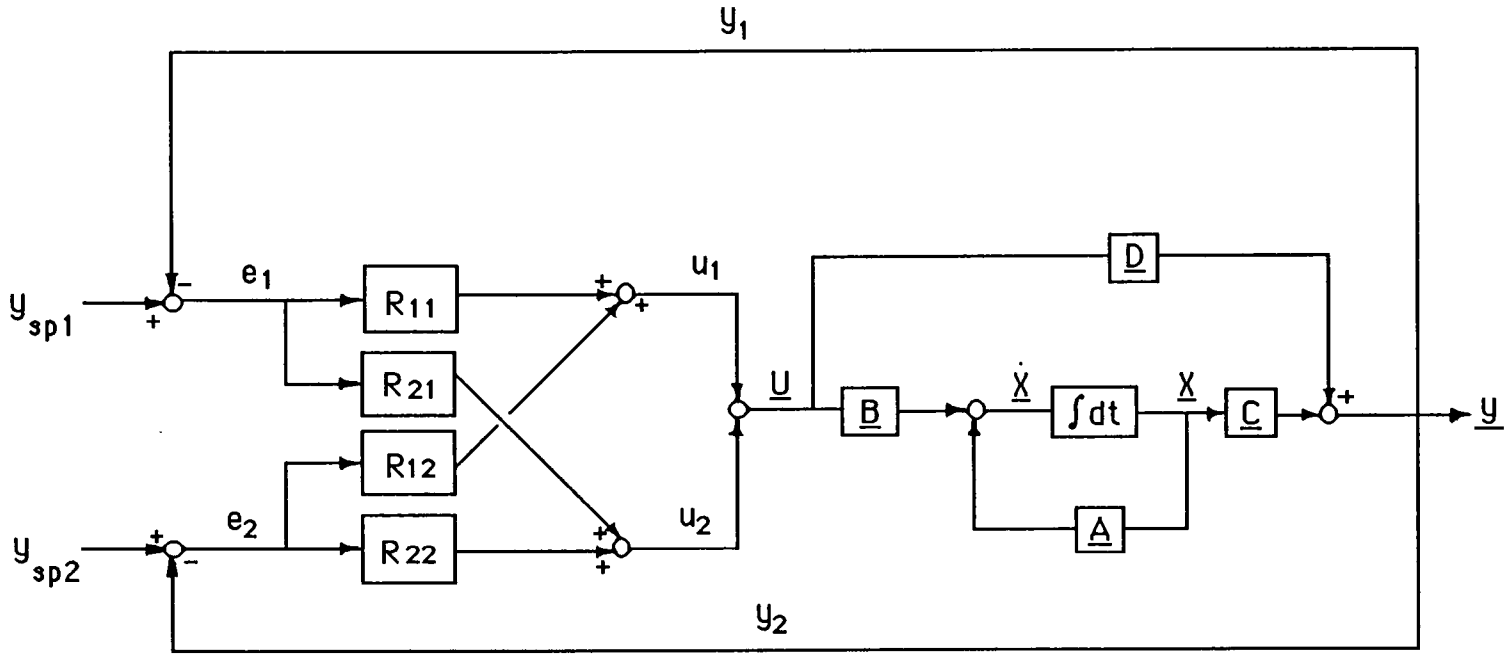


Figure 2.3. State space representation of multivariable process control system.

Although a state variable may be physically measurable or unmeasurable, it is better to choose the measurable variables as state variables. The state variable analysis method has seen rapid development recently and become a strong method for multivariable process control system design and analysis.

By taking the Laplace transform of Equation 2.4 and with $\underline{X}(0)=0$, it becomes

$$\begin{aligned} s\underline{X}(s) &= \underline{A}\underline{X}(s) + \underline{B}\underline{U}(s) \\ \underline{Y}(s) &= \underline{C}\underline{X}(s) + \underline{D}\underline{U}(s) \end{aligned} \tag{2.5a}$$

Rearrangement of Equation 2.5a yields

$$\underline{Y}(s) = \underline{C}(Is-\underline{A})^{-1}\,\underline{B}\underline{U}(s) \tag{2.5b}$$

Comparing this equation with Equation 2.1a, it is obvious that

$$\underline{G}(s) = \underline{C}(Is-\underline{A})^{-1}\,\underline{B} \tag{2.6}$$

Thus, given the state space model, the corresponding transfer function model can be generated which is unique. It should be mentioned that given a transfer function model a state space model can also be obtained but it is not unique. One procedure for developing a state space model from a transfer function model is presented in Chapter 4.

Interaction Analysis And MVSISO Design

Multivariable processes are typically much more difficult to operate optimally than single variable processes because of the interactions that occur between the controlled and manipulated variables. Multivariable control designs must take into account this interaction. As briefly discussed above, there are two approaches to the problem of interaction. The most common and simplest approach is to pair process inputs and outputs using SISO controllers such that the interaction is minimized; then treating each loop as a SISO control system. This is the MVSISO approach. In the case of significant interaction, a decoupling scheme might be used to compensate for the interaction and then proceed with a MVSISO design but then the robust is not guaranteed (2). Another approach is to design truly multivariable controllers which do not use a given manipulated variable to control a specific output but based on the errors in all of the outputs to make adjustments in all of the manipulated variables, a multi-input multi-output controller.

In the case of the MVSISO design approach, the key decision of designing such a multiloop control system is to determine the proper pairing of controlled and manipulated variables to minimize interaction between the SISO control loops. If an incorrect pairing is used or if there is not a good pairing scheme, the control system may perform very poorly or even be inoperable. Thus, the correct choice of pairing with suitable regulator gains may be vital to reflect minimum loop interaction. Therefore, a significant amount of research has been done on interaction

analysis and variable pairing for multivariable process control system design (3-11).

Since Bristol (4) proposed, in1966, the relative gain array (RGA) as a means of determining pairing between controlled and manipulated variables and predicting interactions between loops of a multivariable single input - single output system, the technique has gained considerable practical acceptance (6,7,8,11,12). The RGA is defined as the ratio of two steady-state gains : the steady state gain of the process when no control is applied to the system and the steady gain of the process with all loops under perfect control except the one under study (5). The definition can be mathematically represented as

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{u_k,\, k\neq j}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{y_k,\, k\neq i}} \qquad (2.7a)$$

$$\underline{\Lambda} = \begin{bmatrix} \lambda_{11} & \cdots & \cdots & \lambda_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{1m} & \cdots & \cdots & \lambda_{mn} \end{bmatrix} \qquad m = n \qquad (2.7b)$$

where $\underline{\Lambda}$ = RGA matrix

u_j = j^{th} process input

y_i = i^{th} process output

λ_{ij} = relative gain element between u_j and y_i

Loop pairing is suggested by the RGA depending on the numerical values of the elements in the RGA matrix. Note that the RGA analysis requires an equal number of inputs and outputs. In general, the RGA is calculated from the process steady state gain matrix by

$$\underline{\Lambda} = \underline{K} \; [*] \; (\underline{K}^{-1})^T \qquad (2.8)$$

where [*] corresponds to Hadamard multiplication (13). When $\underline{K}$ is near singular, SVD is a good technique to calculate the inverse. The Hadamard multiplication means that the corresponding elements of $\underline{K}$ and $(\underline{K}^{-1})^T$ are multiplied.

Bristol (4) discussed several useful properties of the RGA which are listed below:

1. The sum of the elements of each row and each column is one.
2. Any permutation of rows and columns in the process gain matrix results in the same permutation in the RGA.
3. The RGA is invariant to scaling.
4. Large elements are an indication of high sensitivity to modeling error.
5. If the process gain matrix is diagonal, then the RGA matrix will be diagonal and the system is said to be decoupled.

McAvoy (6) has also performed research extensively on the RGA and suggested improved pairing rules. The improved pairing rules are:

1. Positive RGA elements closet to one should be considered for pairing.
2. The resulting pairings for the diagonal controller should be checked for stability using Niederlinski's theorem. The process inputs and outputs and thus the steady state gain matrix need to be reordered so the RGA pairing is between u_1 - y_1, . . . , u_n - y_n. Niederlinski's theorem then states a necessary condition for the closed loop system resulting from the pairings u_1 - y_1, ·· ··· , u_n - y_n to be stable is

$$\frac{|K|}{\prod_{i=1}^{n} K_{ii}} > 0 \qquad (2.9)$$

where $|K|$ is the determinant of steady state gain matrix.

If Niederlinsk's theorem indicates instability, choose another set of positive pairings with values next closest to one.

3. Negative pairings should be avoided.
4. The pairing elements in the range of $0.67 < \lambda < 1.5$, a series of single loop controllers should result in good performance.
5. For $\lambda > 25$, alternate control strategies should be considered over the RGA pairing design.

The RGA would appear to be ideal because it can be calculated using only steady state process gains. Unfortunately, especially for higher order systems, RGA appears to recommed unstable pairing and fails to predict the interaction of the system (7).

The drawbacks of RGA has led to the development of alternative interaction analysis. In 1977, Witcher and McAvoy (8) proposed the dynamic gain array (DGA) to overcome the problems of RGA. The DGA is defined as

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{s,\, u_k,\, k\neq j}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{s,\, y_k,\, k\neq i}} \qquad s=jw \qquad (2.10)$$

Just like the RGA, the elements are dimensionless and the sum of elements of each row and column is one. However, λ_{ij} of DGA is a quantity with both amplitude and phase: the amplitude of λ_{ij} is a measurement of interaction degree between u_j and y_i at some finite frequency while the phase is an indirect measurement of interaction effects. Witcher and McAvoy stated that interaction was small if the magnitude of the main diagonal elements of DGA were close to one over a range of frequencies. In other words, the DGA should be a diagonally dominant matrix.

Tung and Edgar (9) used both state space and frequency domain approaches to develop a relative dynamic gain array (RDGA). They concluded that the pairing by RDGA was capable of moving the closed loop poles farther to the left in the complex plane than was pairing by RGA. However, this interaction analysis requires a state space model,and the model is assumed to be a decoupled model.

In addition, Gagnepain and Seborg (10) extended the idea of Witcher and McAvoy to develop the average dynamic gain array (ADGA). Jensen, Fisher, and Shah (11) reviewed the above interaction measures and suggested that direct Nyquist array (DNA) was an excellent basis for interaction analysis.

Design Methodology

As reviewed in the previous section a very common design method for multivariable control systems is to decompose the systems into a set of single input - single output (SISO) systems. In this case, the interaction measures described above are utilized to determine the best pairing for single loop controllers between the controlled and manipulated variables. However, the multivariable single input - single output (MVSISO) will give acceptable control only if the interaction between control loops is weak. When the interaction between control loops is too strong to give acceptable control, some other control techniques will be required.

The existing literature on multivariable control can be grouped into several areas:

1. The dominant interaction or MVSISO method proposed and developed by Bristol (4) in 1966.
2. The non-interacting (decoupled) control suggested by Boksenbom and Hood (14) in 1949.
3. The inverse and direct Nyquist array method introduced by Rosenbrock (15) in 1969.
4. The characteristic loci method developed by MacFarlane and Belletrutti (16,17) in 1971.
5. The optimal control method proposed by Athans (18) in 1966.
6. The pole placement method introduced by Rosenbrock (19) in 1962.
7. The adaptive control method developed by Kalman (20) in 1958.
8. The predictive control strategies (MAC, DMC, IMC, etc) developed in the late 1970's.

The first four methods are frequency domain approaches while the last three are state space or time domain approaches. Rijnsdorp and Seborg (21) and Ray (22) reported on the industrial applications of these techniques. Moreover, MacFarlane (1) also summarized the advantages and disadvantages of these design techniques. Tables 2.1 provides an overview of the literature on multivariable design methods.

Summary

For the control of multivariable systems, several methodologies for controller systhesis and design have been developed. Each of the modern control techniques has its advantages and drawbacks. It is

Table 2.1. An overview of multivariable control literature.

Design Technique	General Literature
Dominant Interaction	4 , 23
Non-interacting (decoupled) Control	24 - 27
Inverse and Direct Nyquist Array	28 - 30
Characteristic Loci	15-16, 31 - 33
Optimal Control	18, 34 - 37
Pole Placement	38 - 40
Adaptive Control	41 - 44
Predictive Control	45 - 53

difficult to give an exact answer as to which is the best method because each application or problem has different considerations and demands. This work addresses two relatively new design techniques, multivariable PI tuning regulator and the singular value analysis controller based on the singular value decomposition (SVD) of the steady state gain matrix. So the next chapter will discuss the mathematical development of singular value analysis (SVA).

CHAPTER III

MATHEMATICAL PRELIMINARIES

Singular value analysis (SVA) has rapidly become a widespread tool for process analysis and control design. It provides convenient quantitative measures for assessing steady state interaction and singularity of a system. Computational efficiency of SVA is ensured since effective numerical algorithms exist for calculating the singular value decomposition (SVD). Recently, many papers have appeared in the literature investigating application of this analysis to different problems in process control and design (12, 54-60). This chapter will focus on concepts of linear algebra which are related to SVD and interpretation of SVD.

Vector And Matrix Norms

Vector norms are used to give the notion of the "length" of a vector, and a vector norm may be defined in many ways. The general case of p-norm of a finite-dimensional vector is defined as:

$$\| \underline{X} \|_p = (\sum_i | X_i |^p)^{1/p} \qquad (3.1)$$

where p a positive integer denotes the power to which the elements of a vector are raised in forming the norm. A commonly used vector norm from this family of p-norms is the 2- or Euclidean norm. The Euclidean

norm is defined according to Equation 3.1 as:

$$\| \underline{X} \|_2 = \left(\sum_i | X_i |^2 \right)^{1/2} \qquad (3.2)$$

In addition to 2-norm, two other commonly used norms are the 1- and ∞ -norms. These norms are defined as:

$$\| \underline{X} \|_1 = \sum_i | X_i | \qquad \text{1-norm} \qquad (3.3)$$

$$\| \underline{X} \|_\infty = \max_i | X_i | \qquad \infty\text{-norm} \qquad (3.4)$$

Properties of the norm are:

$$\| \underline{X} \| > 0 \qquad \text{for } \underline{X} \neq 0 \qquad (3.5a)$$

$$\| \underline{X} \| = 0 \qquad \text{for } \underline{X} = 0 \qquad (3.5b)$$

$$\| a \underline{X} \| = | a | \, \| \underline{X} \| \qquad a = \text{a scalar} \qquad (3.5c)$$

$$\| \underline{X} + \underline{Y} \| \leq \| \underline{X} \| + \| \underline{Y} \| \qquad \text{triangle inequality} \qquad (3.5d)$$

The norms for vectors can be extended to matrices. So, there are properties that a matrix norm must have such as:

$$\| \underline{A} \| > 0 \qquad \text{for all } \underline{A} \neq 0 \qquad (3.6a)$$

$$\| \underline{A} \| = 0 \qquad \text{for } \underline{A} = 0 \qquad (3.6b)$$

$$\| b \underline{A} \| = | b | \, \| \underline{A} \| \qquad b = \text{a scalar} \qquad (3.6c)$$

$$\| \underline{A} + \underline{B} \| \leq \| \underline{A} \| + \| \underline{B} \| \tag{3.6d}$$

$$\| \underline{A}\,\underline{B} \| \leq \| \underline{A} \| \; \| \underline{B} \| \tag{3.6e}$$

Matrix norms which satisfy the above properties and are based on the vector 1-p norm are called induced, natural, or operator norms. They are defined as:

$$\| \underline{A} \|_p = \sup_{\|\underline{X}\| \neq 0} \frac{\| \underline{A}\underline{X} \|_p}{\| \underline{X} \|_p} \quad \text{for all } \underline{X} \tag{3.7}$$

where sup is the least upper bound on $\| \underline{X} \|$. The four most common norms are the one, infinity, Euclidean and two norms. These norms are defined as:

$$\| \underline{A} \|_1 \triangleq \max_j \left(\sum_{i=1} | a_{ij} | \right) \tag{3.8}$$

$$\| \underline{A} \|_\infty \triangleq \max_i \left(\sum_{j=1} | a_{ij} | \right) \tag{3.9}$$

$$\| \underline{A} \|_E \triangleq \left(\sum_{i,j} | a_{ij} |^2 \right) \tag{3.10}$$

$$\| \underline{A} \|_2 = \left(\lambda_{max} (\underline{A}^T \underline{A}) \right)^{1/2} \tag{3.11}$$

where λ is the eigenvalue of $(\underline{A}^T \underline{A})$.

The Euclidean and 2-vector norms are equivalent but the Euclidean and 2-matrix norms are different. Vector and matrix norms are used with SVD to provide bounds relating manipulated variables to controlled variables (60).

Orthogonal Matrices

An orthogonal matrix $\underline{Q}$ is a square matrix with orthogonal columns. Thus, the inverse of an orthogonal matrix is its transpose or

$$\underline{Q}^{-1} = \underline{Q}^{T} \tag{3.12}$$

and

$$\underline{Q}^{T}\underline{Q} = \underline{I} \tag{3.13a}$$

$$\underline{Q}\,\underline{Q}^{T} = \underline{I} \tag{3.13b}$$

The above expressions indicate that both the columns and the rows of an orthogonal matrix form an orthogonal set of vectors. Moreover, the 2-norm of an orthogonal matrix is one. As a result, the norm of a matrix is unchanged when multiplied by an orthogonal matrix, thus

$$\| \underline{A}\,\underline{Q} \| = \| \underline{Q}\,\underline{A} \| = \| \underline{A} \| \, \| \underline{Q} \| = \| \underline{A} \| \tag{3.14a}$$

$$\| \underline{A}\,\underline{Q}^{T} \| = \| \underline{Q}^{T}\underline{A} \| = \| \underline{A} \| \, \| \underline{Q}^{T} \| = \| \underline{A} \| \tag{3.14b}$$

Therefore, orthogonal matrices may be viewed as rotation and reflection matrices or permutation matrices when the entries are either 1 or 0 (62). These properties illustrate that orthogonal matrices, when used in conjunction with the 2-norm, provide analytical and computational advantages.

Pseudoinverse

The pseudoinverse of a mxn matrix A is computed as:

$$\underline{A}^{+} = \underline{V}\,\Sigma^{+}\,\underline{U}^{T} \qquad (3.15)$$

where $$\Sigma^{+} = \begin{cases} \frac{1}{\sigma_i} & \sigma_i > 0 \\ 0 & \sigma_i = 0 \end{cases}$$

$\underline{A}^{+}$ = the pseudoinverse of $\underline{A}$

$\underline{U}$ = an mxm orthogonal matrix

$\underline{V}$ = an nxn orthogonal matrix

σ = singular value of $\underline{A}$

It is useful for solving problems that are under-determined, singular, and over-determined. For a least squares problem $\underline{A}\underline{x} = \underline{b}$ where $\underline{A}$ and $\underline{b}$ are given, the solution found with the pseudoinverse is the mininum norm of all possible solutions that minimizes the norm $|| \underline{A}\underline{x} - \underline{b} ||$. Moreover, for square matrices that are full rank, the pseudoinverse is identical to the standard inverse.

Singular Value Decomposition

Singular value analysis (SVA) is a term used to classify process analysis based upon interpretation of the singular value decomposition (SVD) of a matrix of interest. SVD expresses a general matrix in terms of the products of three matrices as (61):

$$\underline{A} = \underline{U}\,\Sigma\,\underline{V}^T \tag{3.16}$$

where $\underline{A}$ = an n x m matrix

$\underline{U}$ = an n x n orthonormal matrix

Σ = an n x m matrix with positive singular values along the main diagonal

$\underline{V}$ = an m x m orthonormal matrix

An alternative expression for Equation 3.16 is to write $\underline{A}$ as a sum of dyads:

$$\underline{A} = \sum_{i=1}^{r} \sigma_i \, U_i \, V_i^T \tag{3.17}$$

where r is the rank of $\underline{A}$ and $\underline{U}_i$ and $\underline{V}_i$ are the ith column of $\underline{U}$ and $\underline{V}^T$ matrices respectively. Thus, the three matrices in the SVD when operating on a vector may be viewed as three transformations for the input vector to obtain the output vector:

1. The first orthonormal matrix, $\underline{V}^T$, provides a rotation for the input vector.
2. The second matrix, Σ, scales the rotated vector along the orthogonal basis vectors of V.

3. Finally, the third matrix, $\underline{U}$, provides a second rotation to the scaled vector into the output space giving the output vector.

This geometric interpretation of SVD is illustrated in Figure 3.1 (54,60).

Interpretation Of Process Control Properties Provided By SVA

The condition number of a matrix plays an important role in SVA. For any nonsingular mxn matrix $\underline{A}$, condition number is defined as (62):

$$K_\beta(\underline{A}) = \| \underline{A} \|_\beta \| \underline{A}^{-1} \|_\beta \qquad (3.18)$$

where $\| \quad \|_\beta$ is any matrix norm. If the 1-2 norm is used, the condition number is defined as the ratio of the maximum and minimum nonzero singular values of the matrix. Mathematically, it is expressed as:

$$K_2(\underline{A}) = \frac{\sigma_{max}(\underline{A})}{\sigma_{min}(\underline{A})} \quad \text{where} \quad \sigma_{max}, \sigma_{min} \neq 0 \qquad (3.19)$$

The singular values, $\sigma_1, \sigma_2, \ldots, \sigma_n$, are defined as the positive square roots of the eigenvalues of $\underline{A}^*\underline{A}$ where $\underline{A}^*$ is the complex conjugate transpose of $\underline{A}$. The relative ratios between the various singular values provide a relative sensitivity measure of the system outputs to perturbation in the system inputs. A large condition number, given as $K_2 > 50$ by McAvoy (6) and $K_2 > 20$ by Moore (56), of a system implies that the relative sensitivity in one direction is very weak compared to that in

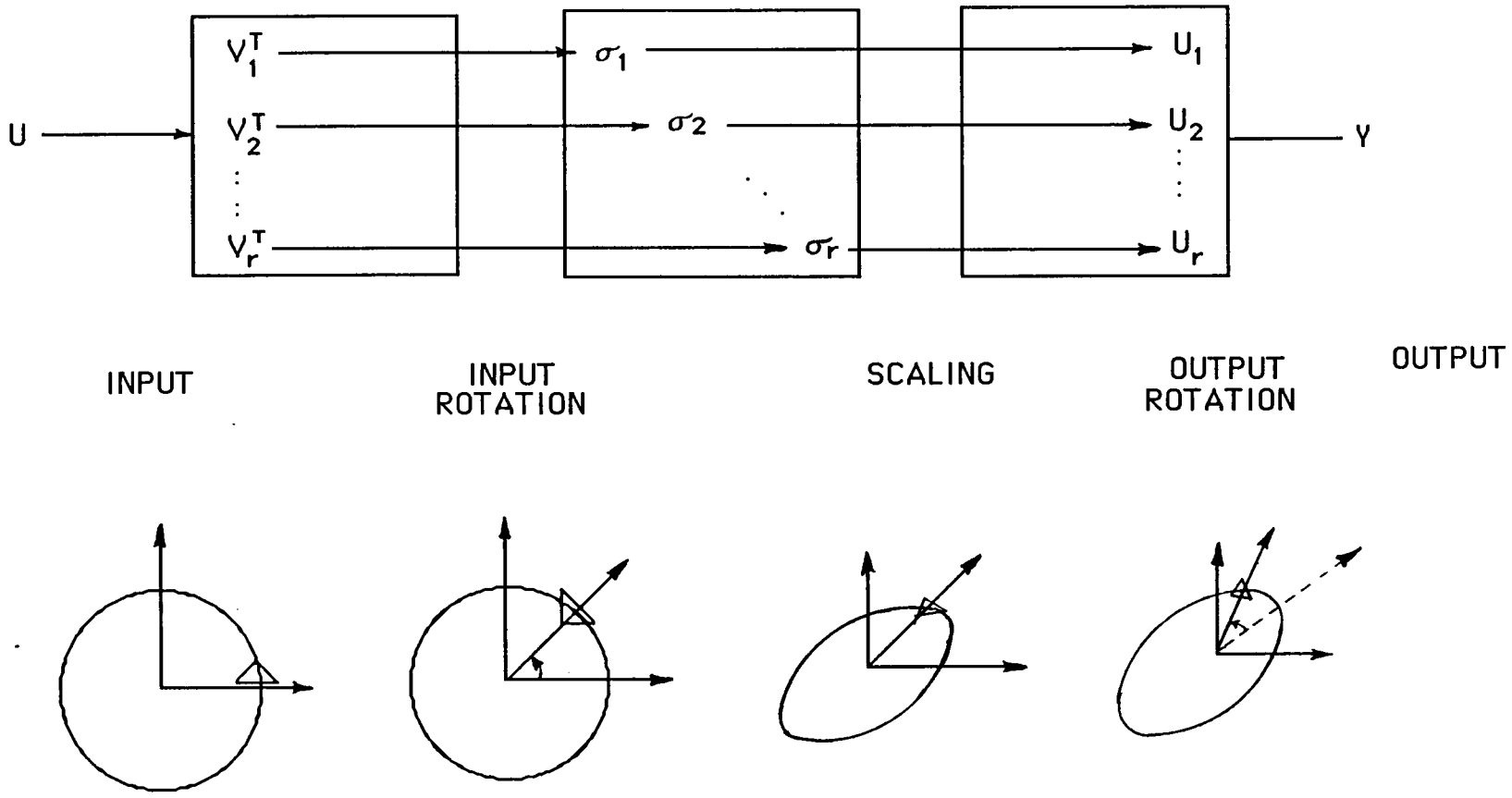


Figure 3.1. Geometric interpretation of the SVD. Input matrix is partitioned into three parts : input rotational matrix V, scaling matrix Σ, and output rotational matrix U.

the other direction. In addition, the mininum singular value, σ_{min}, is a measure of the mininum distance to the nearest singular matrix and thus also a measure of invertibility of the system (58,59). In addition, $1/\sigma_{min}$ may be interpreted as an indicator of control effort needed in the hardest to control direction.

The U column vectors or the so called left singular vectors are related to the principle directions of the outputs of the system. The column vector associated with the largest singular value is the easiest output direction in which the system can be moved in terms of process input direction changes of a given magnitude or vector norm. Correspondingly, the column vector associated with the smallest singular value is the most difficult output direction in which to move the system.

The V column vectors or the right singular vectors provide a useful orthogonal coordinate system for viewing the process inputs or manipulated variables. The description of ease of moving the system is similar to the interpretation of the U column vectors. The principle input direction or V column vector associated with the largest singular value is the input direction that will have the most effect on the process output. Likewise, the column vector associated with the smallest singular value is the direction that will have the least effect on the process output.

Scaling

Singular value decomposition depends on the scaling or the units used for the process inputs and outputs. Thus, the singular value analysis interpretations will depend on scaling. Therefore, the SVA results may be

meaningful only after the process model is scaled "appropriately." Several studies on the effect of scaling and proposed methods to appropriately scale have been made (58-59, 63-67). However, no definitive or satisfying results have been found, if they exist. This short discussion on scaling given in this thesis as the effect of first scaling the process gain matrix on the development of the singular value analysis controller could be an interesting study.

In mathematics, scaling is concerned with accuracy in numerical algorithms that are due to machine computation. However, the purposes of scaling in process control are (63,66,67):

1. To eliminate the arbitrariness of units associated with input, state, and output variables.
2. To keep the relative strength of input-output couplings.

Scaling is equivalent to pre- and post- multiplying the system matrix by diagonal matrices such as:

$$\underline{S}_R \underline{A} \underline{S}_C = \underline{A}_{SC} \tag{3.20}$$

where $\underline{S}_R$ = diag ($s_{r1}, s_{r2}, \ldots, s_{rm}$), row scaling matrix

$\underline{S}_C$ = diag ($s_{c1}, s_{c2}, \ldots, s_{cn}$), column scaling matrix

s_{rj} = scaling factor for row j

s_{ci} = scaling factor for column i

Although optimal scaling methods based on the Euclidean norm do not exist, optimal methods do exist for other norms (64).

Tomlin (65) has applied various scaling methods such as arithmetic mean, equilibration, and geometric mean to form "well scaled matrices", that is, the elements of the matrices are all about the same magnitude. Definition of those techniques discussed by Tomlin are:

1. Arithmetric mean

   The elements of each row are divided by the arithmetic mean of that row, followed by similar treatment for the elements of each column.

2. Equilibration

   All the elements in each row are divided by the largest element in each row, followed by the same procedure for each column. The largest element of each row or column for the scaled matrix is of order unity.

3. Geometric mean

   The elements of each row is divided by the square root of the absolute value of the product of the maximum element and mininum element of each row, followed by a similar scaling of the columns.

These techniques can be applied in any order (i.e. row followed by column or column followed by row) and also can be combined. The scaling done varies depending on the technique used and on the row column order. In one study, geometric mean scaling yield the smallest condition number for all 2x2 process gain matrices studied (66). Recently, Dyer's (67) expanded

research on scaling proved that geometric mean scaling of any row - column order gives the minimum condition number for full (contain no zero elements) 2x2 matrices. She also found that both geometric mean and nonlinear least squares search technique (NLST) were efficient in lowering the condition number, but the NLST was more effective for higher order process gain matrices.

Lau and Jensen (58,59) studied the effects that scaling the steady state process gain matrix has on the results obtained with SVD. They also proposed a scaling method, variable normalization and equation equilibration, based on physical considerations by limiting the variability of the system variables. From their analysis, they concluded that the absolute magnitude of condition number or minimum singular value is not necessarily an appropriate indicator of the controllability of a process control system, but its relative magnitude evaluated over the feasible region.

Computation Of SVD

Computer programs for SVD computation in Fortran are available in several subroutine packages such as IMSL (68), Eispack (69), and Linpack (70). The QR algorithm is used in the above packages. The computation of SVD involves two major steps. First, the given matrix is transformed into an upper Hessenberg, bidiagonal, or tridiagonal form using Householder transformations. Second, the singular values of the transformed matrix are found by a series of orthogonal rotations shifts to zero the superdiagonal row.

Although SVD programs are typically written for the case of a m>n (more rows than columns), SVD of a m<n matrix can be computed by taking the transpose of the matrix first. In this case, the roles of U and V are interchanged from that of the SVD of the original matrix.

Summary

This chapter has discussed some useful properties of matrices which are related to SVA. Concepts of SVA will be employed to formulate the singular value analysis controller (SVAC). This controller, as well as, the multivariable single input - single output controller (MVSISO) and multivariable proportional integral regulator (MVPIR) will be discussed in next chapter.

CHAPTER IV

MULTIVARIABLE CONTROLLERS AND SIMULATION PROCEDURES

The singular value decomposition provides a framework for a multivariable controller struture. Smith (54,55) proposed a controller structure called the singular value analysis controller (SVAC). SVAC is based on the orthogonal matrices that result from the singular value decomposition of the process gain matrix. The development of the SVAC will be discussed first. Then, the structure of multivariable single input - single output controller (MVSISO), as well as, the multivariable proportional integral tuning regulator (MVPIR) will be treated. Finally, the scheme of simulation used in this research will be presented.

Singular Value Analysis Controller Structure

The ultimate goal of any control system is to find the inputs to the process that give the desired character to the process outputs.

$$\underline{Y} = \underline{K}\,\underline{M} \tag{4.1}$$

where $\underline{K}$ = steady state gain matrix

$\underline{Y}$ = desired process output vector

$\underline{M}$ = needed process input vector

To solve this set of equations numerically, the inverse of $\underline{K}$ may be post

multiplied by the desired output, $\underline{Y}$. However, not all systems have steady state gain matrices which are square and nonsingular. For these systems, there still exists an input vector of minimum norm which provides the least error (minimum norm) from the set point. The input vector may be calculated using the pseudo-inverse which has been discussed in Chapter III.

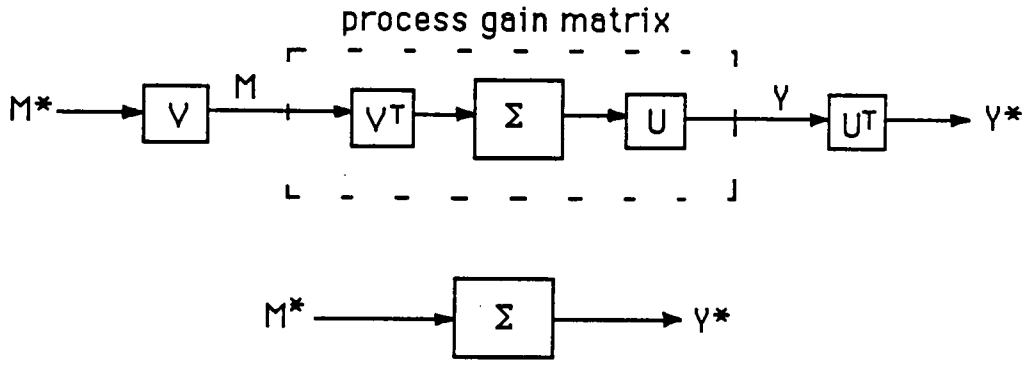
$$\underline{M} = \underline{K}^{+} \underline{Y} \tag{4.2}$$

where $\underline{K}^{+}$ is pseudo-inverse of $\underline{K}$. A robust technique for solving Equation 4.1 is using SVD as the decomposition exists for any general matrix :

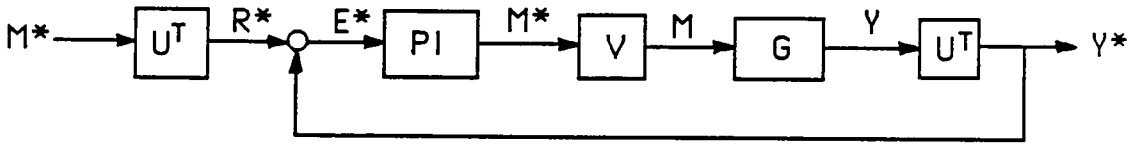
$$\begin{aligned} \underline{M} &= \underline{K}^{+} \underline{Y} \\ &= [\underline{U} \underline{\Sigma} \underline{V}^{T}]^{+} \underline{Y} \\ &= \underline{V} \underline{\Sigma}^{+} \underline{U}^{T} \underline{Y} \end{aligned} \tag{4.3}$$

A controller based on the SVD structure of the process gain matrix can be developed through the use of block diagram manipulation. The first step is to determine the SVD of the steady state gain matrix. Figure 4.1 is a block diagram representation of the development of SVAC structure. In Figure 4.1A the block diagram shows the process signals being transformed. The signals with superscript * have been defined by Smith (54,55) as structured variables. The structured variables are defined in terms of the process variables and SVD orthogonal matrices as

A) Process Diagonalization



B) Closed Loop Structure



C) Closed Loop Form In Terms Of Physical Variables

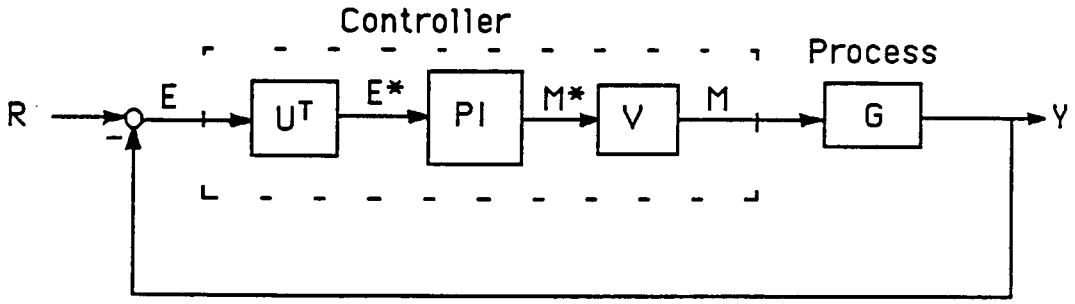


Figure 4.1. Development of singular value analysis controller.

$$\underline{M}^* = \underline{V}^T \underline{M} \qquad \text{or} \qquad \underline{M} = \underline{V}\,\underline{M}^* \tag{4.4}$$

$$\underline{Y}^* = \underline{U}^T \underline{Y} \qquad \text{or} \qquad \underline{Y} = \underline{U}\,\underline{Y}^* \tag{4.5}$$

Pre-multipling the process equation, Equation 4.1, by $\underline{U}^T$ leads to

$$\begin{aligned} \underline{U}^T \underline{Y} &= \underline{U}^T \underline{K}\,\underline{M} \\ \underline{Y}^* &= \underline{U}^T \underline{K}\,\underline{M} \\ &= \underline{U}^T [\,\underline{U}\,\underline{\Sigma}\,\underline{V}^T\,]\,\underline{M} \end{aligned} \tag{4.6}$$

Replacing $\underline{M}$ in Equation 4.6 by $\underline{M}^*$ according to Equation 4.4 results in the process gain being post-multiplied by $\underline{V}$ and yields

$$\begin{aligned} \underline{Y}^* &= \underline{U}^T [\,\underline{U}\,\underline{\Sigma}\,\underline{V}^T\,]\,\underline{V}\,\underline{M}^* \\ &= \underline{\Sigma}\,\underline{M}^* \end{aligned} \tag{4.7}$$

Equation 4.7 shows the structured inputs, $\underline{M}^*$, and outputs, $\underline{Y}^*$, are completely decoupled at steady state. This is the fundamental reason for introducing the structured variables and underlies the formulation of the SVAC. As illustrated in Figure 4.1B, the structured outputs are compared to the desired structure set point, $\underline{R}^*$, which is formed by multiplying the physical variable set point by $\underline{U}^T$. The inputs to the diagonal controller are the structured error signals, $\underline{E}^*$, and the controller outputs are $\underline{M}^*$. This system may be rearranged , as shown in Figure 4.1C, by manipulating the block diagram so the error signals are calculated using the

non-transformed signals. The block diagram now looks like a typical multivariable control system block diagram. The structure of the SVAC is enclosed in the dashed box. The block labelled PI corresponds to a diagonal matrix with PI control action on the diagonal. The SVAC controller action is thus expressed as

$$\underline{M} = \underline{V}\ \mathrm{diag}(PI)\ \underline{U}^T\ \underline{E} \tag{4.8}$$

Multivariable Proportional Integral Tuning Regulator

A design method for on-line tuning of multivariable PI controllers was developed by Penttinen and Koivo (71,72) in 1980. The system is assumed to be linear, time invariant, and open loop stable. The following section is a summary of the controller development of the controller. A more detailed description of the multivariable PI regulator can be found in papers published by Penttinen and Koivo (71,72).

Consider a linear time invariant stable system

$$\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}\underline{U} \tag{4.9a}$$

$$\underline{Y} = \underline{C}\underline{X} \tag{4.9b}$$

$$\underline{E} = \underline{Y_r} - \underline{C}\underline{X} \tag{4.9c}$$

where $\underline{X}$ = state vector

$\underline{U}$ = input vector

$\underline{Y}$ = output vector

$\underline{A}$ = state matrix

$\underline{B}$ = input state matrix

$\underline{C}$ = output state matrix

$\underline{E}$ = error signal vector

$\underline{Y_r}$ = set point

The multivariable regulator splits the proportional and integral modes apart and has the following form

$$\underline{U} = \underline{PE} + \underline{KV} \tag{4.10}$$

where $\underline{E}$ = error signal vector

$\underline{V} = \int \underline{E}\, dt$ or $\dot{\underline{V}} = \underline{E}$

$\underline{P}$ = full constant matrix of proportional gains

$\underline{K}$ = full constant matrix of integral gains

A multivariable P-controller is of the form

$$\underline{U} = \underline{PE} \tag{4.11}$$

The main function of the proportional controller is to speed up the transient or to decouple the system during the transient. By applying the P-controller to Equation 4.9a , assuming that the initial condition is zero, results in

$$\dot{\underline{X}} = (\underline{A} - \underline{BPC}) \underline{X} + \underline{BPY_r} \qquad (4.12)$$

and after integrating the equation the following expression is obtained

$$\underline{X} = - (\underline{A} - \underline{BPC})^{-1} \underline{BPY_r} [1 - e^{(\underline{A}-\underline{BPC})t}] \qquad (4.13a)$$

Then, by substituting Equation 4.13a into 4.9b and expanding the exponential term into a Taylor series yields

$$\underline{Y} = \underline{CBPY_r}t + \underline{C} [\sum_{k=2}^{n} (\underline{A} - \underline{BPC})^{k-1} t^k/k!] \underline{BPY_r} \qquad (4.13b)$$

For small t, it is sufficient to choose matrix P so that the matrix CBP is a diagonal matrix with non-zero elements on the diagonal. This is done by letting

$$\underline{P} = \underline{Kp} \text{ diag } [p_1, p_2, \ldots] \qquad (4.14a)$$

where $\underline{Kp} = (\underline{CB})^{+}$ (4.14b)

pi = tuning parameters of the controller

By formulating equations analogous to Equations 4.9 to 4.14 only in the discrete or sampled data time domain, the following expression for $\underline{Y}(k)$ is obtained (72)

$$\underline{Y}(k) = \underline{C} \left(\sum_{i=0}^{k-1} (\underline{A} - \underline{BPC})^i \right) \underline{BPY_r} \qquad (4.15a)$$

$$= \underline{CBPY_r} + \underline{C} (\underline{A} - \underline{BPC}) \underline{BPY_r} + \ldots$$

$$+ \underline{C} (\underline{A} - \underline{BPC})^{k-1} \underline{BPY_r} \qquad \text{where } k \geq 1$$

This development allows the decoupling action to be designed for different times of the response when

$$k = 1, \quad \underline{Y}(1) = \underline{CBPY_r} \qquad (4.15b)$$

$$k = 2, \quad \underline{Y}(2) = (\underline{C} + \underline{C} (\underline{A} - \underline{BPC})) \underline{BPY_r} \qquad (4.15c)$$

$$k = 3, \quad \underline{Y}(3) = (\underline{C} + \underline{C} (\underline{A} - \underline{BPC}) \underline{B} + \underline{C} (\underline{A} - \underline{BPC})^2) \underline{BPY_r} \qquad (4.15d)$$

Therefore, the matrix $\underline{Kp}$ at various time can be obtained from inspection of Equation 4.15. For exmple when

$$k = 1, \quad \underline{Kp} = (\underline{CB})^+ \qquad (4.16a)$$

$$k = 2, \quad \underline{Kp} = ((\underline{C} + \underline{C} (\underline{A} - \underline{BPC})) \underline{B})^+ \qquad (4.16b)$$

$$k = 3, \quad \underline{Kp} = ((\underline{C} + \underline{C} (\underline{A} - \underline{BPC})) \underline{B} + \underline{C} (\underline{A} - \underline{BPC})^2) \underline{B})^+ \qquad (4.16c)$$

Comparing Equation 4.16a to 4.14b, it is shown that the matrix $\underline{P}$ is the same for both the continuous and discrete time domains at small time ($t \rightarrow 0$ or $k = 1$). Both Equations 4.16b and 4.16c contain $\underline{P}$, thus the proportional matrix $\underline{Kp}$ can not be calculated without knowing the values

of the pi tuning parameters. Peltomaa and Koivo (72) suggest that the terms containing P be assumed small then for k=2 and 3, $\underline{Kp}$ becomes

$$k = 2, \quad \underline{Kp} = (\underline{CB} + \underline{CAB})^{+} = ((\underline{C} + \underline{CA}) \underline{B})^{+} \tag{4.16d}$$

$$k = 3, \quad \underline{Kp} = (\underline{CB} + \underline{CAB} + \underline{CAAB})^{+}$$

$$= ((\underline{C} + \underline{CA} + \underline{CA^2}) \underline{B})^{+} \tag{4.16e}$$

The authors also state that "controller 4.16d is often more applicable than controller 4.16a, if the process is of high degree or otherwise difficult to control. If the matrix $(\underline{CB})^{+}$ does not exist, then of course we must use controller 4.16d."

The multivariable I-controller construction follows Davison's development in the continuous domain (73). The integral gain matrix is chosen such that the controller decouples the system at steady state. Davison shows that such a controller has the form

$$\underline{K} = \varepsilon \underline{KI} \tag{4.17}$$

where $\underline{KI} = (\underline{C} (\underline{I} - \underline{A})^{-1} \underline{B})^{+}$

ε = a scalar tuning parameter

Figure 4.2 illustrates the structure of the multivariable PI regulator for a 2x2 process.

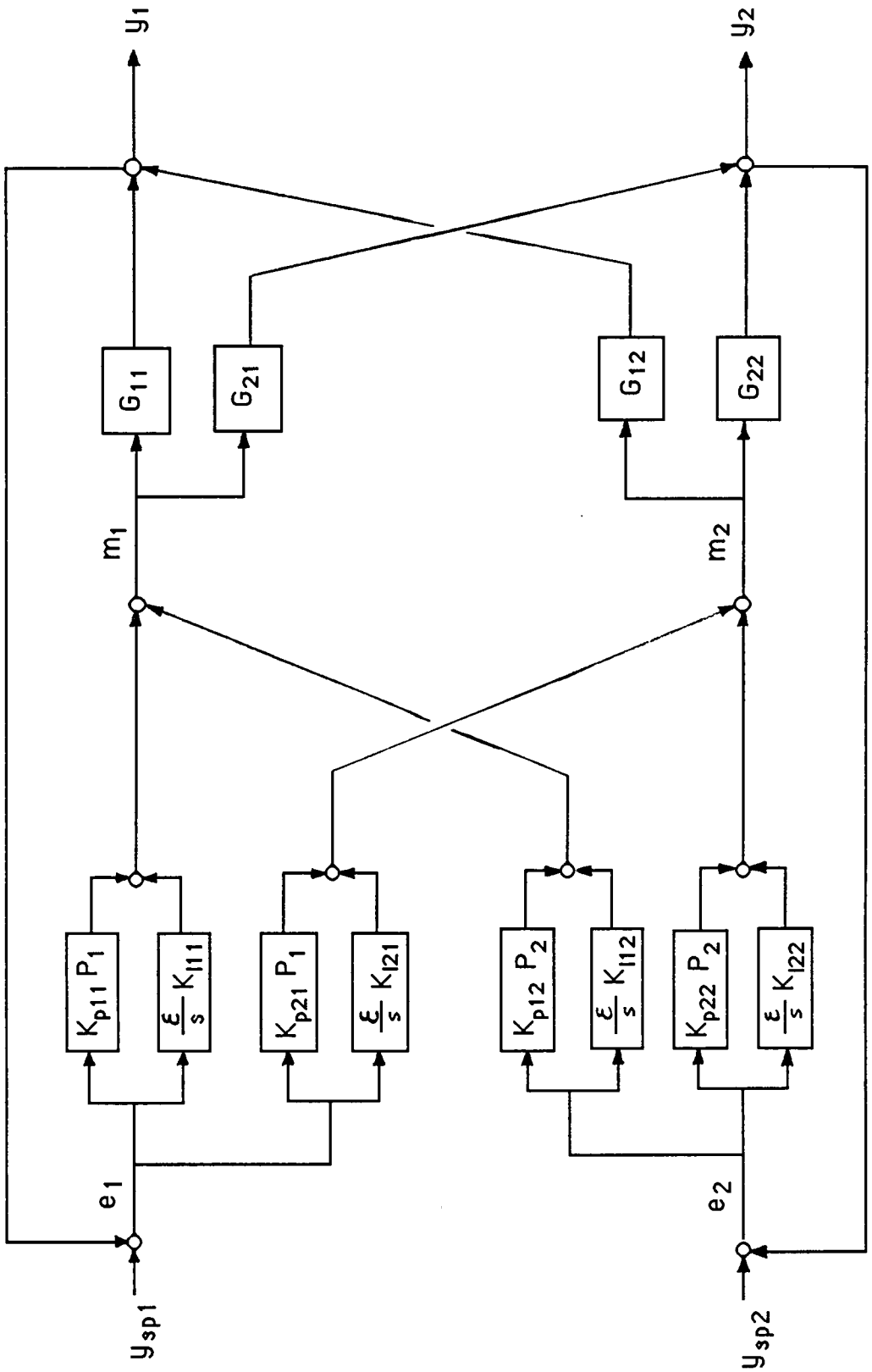


Figure 4.2. Multivariable proportional plus integral tuning regulator structure

Multivariable Single Input - Single Output Controller

MVSISO control technique is a design method composed of a series of single input - single output control loops. The most critical step of designing a MVSISO control system is to determine the best pairings between the controlled and manipulated variables. Improper pairings will give poor or unacceptable performance of the system. The criteria for selecting the best pairings is to minimize the interaction of the control loops. All the interaction measures discussed in Chapter 2 were developed to solve the pairing problem of multivariable control systems. Figure 4.3 shows the structure of MVSISO control of a 2x2 system for the two possible control configurations.

Comparison Of The Theoretical Basis Of The Studied Controllers

Based on the development and structure of multivariable single input - single output controller, multivariable PI tuning regulator, and singular value analysis controller discussed in previous sections, a comparision of the theoretical basis of the controllers is made. The MVSISO control technique is based on the steady state process gain, and the manipulated and controlled variables are paired according to RGA pairing rules to minimize interaction between loops. Similarly, the SVAC development is also based on the steady state process gain but it accounts for steady state interaction. In contrast to both MVSISO and SVAC, the MVPIR is developed from a dynamic state space model. The proportional and integral modes are designed to eliminate initial transient and steady state error respectively. Hence, in term of errors tracking, the

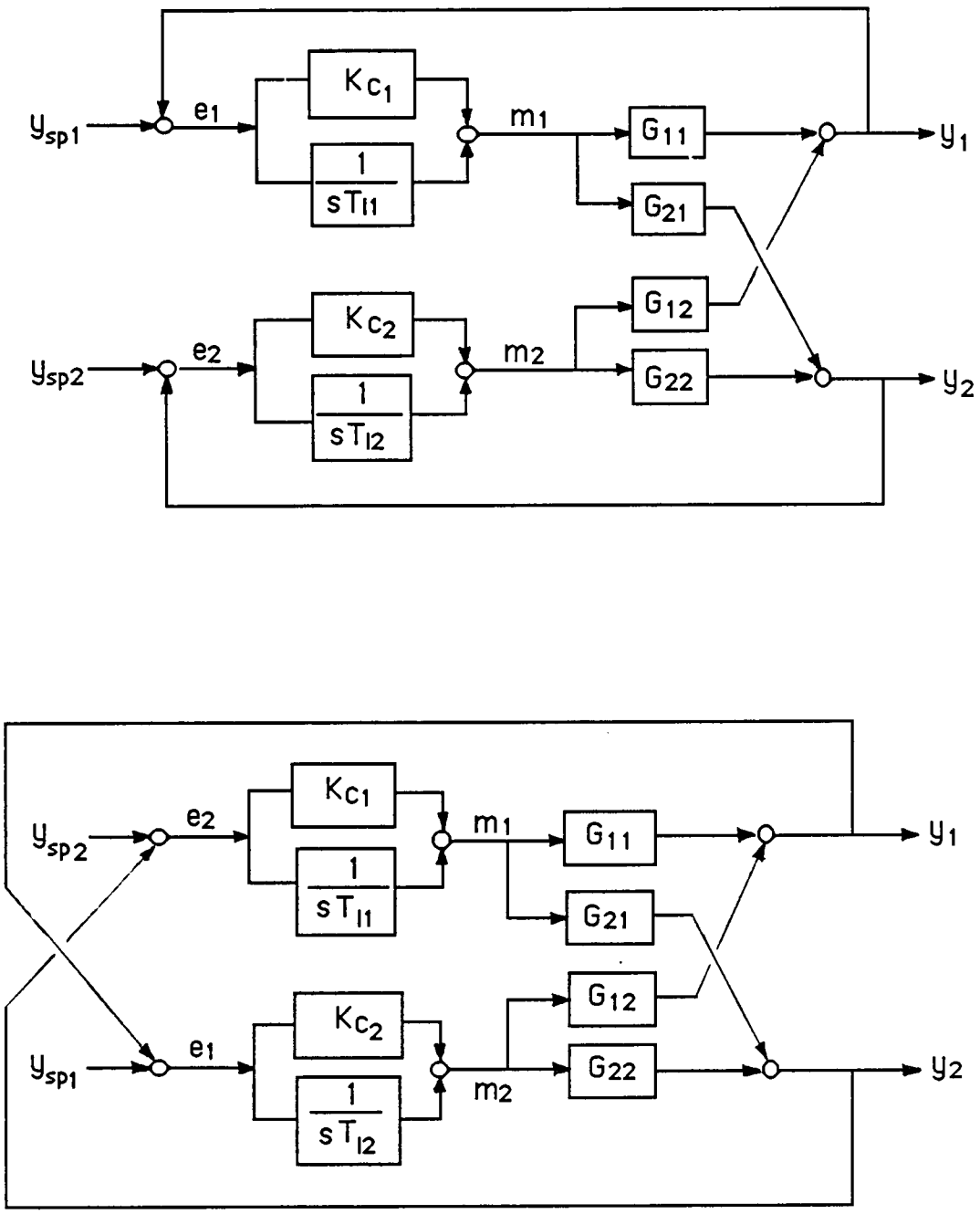


Figure 4.3. Multivariable single input - single output controller structure.

controllers may be ranked as best in the following order ; MVPIR, SVAC, MVSISO.

Although MVPIR is expected to control most of the processes successfully, it may have difficulty with complex transients, especially when the initial transient is different from later transients as in processes that exhibit sags. Both SVAC and MVSISO are also expected to encounter similar difficulty as MVPIR in sagging processes. Since SVAC only has the capability of controlling steady state interaction, it may fail to control a process when transient interaction is different from steady state interaction. Finally, the presence of steady state interaction, as well as transient interaction, between control loops will deteriorate performance of MVSISO controller. However, all the controllers are helped in overcoming their weaknesses by optimizing their tuning parameters.

Minimum Realization

Two ways of representing a dynamic system were mentioned earlier. In this work, the simulations are based on the state space model representations for the systems. Thus, the transformation from transfer functions to state space models is the first step required in this process. Then, the recursive equations are formed for the state space model and solved by matrix exponential method. Finally, the controller is implemented and the multivariable control systems are simulated.

Starting from the state space matrices, an unique corresponding transfer function matrix can be derived. However, starting from a transfer function matrix, the transformation to state space matrices is

not unique. This is primarly because more information is contained in the state space representation than in transfer function matrix. Consequently, there are many sets of state space equations that could be obtained from a transfer function matrix with different transformation methods. The minimum realization method is used in this work.

A procedure for obtaining a realization, state space model,for transfer function matrices without deadtime, $\underline{G}(s)$, is demonstrated in the following discussion. An example of minimum realization of a transfer function matrix is given in Appendix A. A more detailed description can be found in " Advanced Process Control " by Ray (37) and " Modern Control Theory " by Brogan (74). Consider a matrix $\underline{\emptyset}_n(s)$ which has the following properties :

1. Its dimensions are indentical to the transfer function matrix, $\underline{G}(s)$.
2. All but one of its elements are zero.
3. The non-zero element $g_{ij}(s)$ is in the same position as it occupied in $\underline{G}(s)$.

From the above properties of $\underline{\emptyset}_n(s)$, it is evident that

$$\underline{G}(s) = \underline{\emptyset}_1(s) + \underline{\emptyset}_2(s) + \cdots + \underline{\emptyset}_k(s) \qquad (4.18a)$$

or

$$\underline{G}(s) = \sum_{n=1}^{k} \underline{\Phi}_n(s) \qquad (4.18b)$$

where k is the total number of elements of $\underline{G}(s)$.

If each $\underline{\Phi}_n(s)$ matrix has real and distinct poles, then by partial fraction expansion, each $\underline{\Phi}_n(s)$ matrix can be expanded to form

$$\underline{\Phi}_n(s) = \sum_{j=1}^{P_n} \frac{\underline{M}_{nj}}{(s + \lambda_{nj})} \qquad n = 1, 2, \ldots k \qquad (4.19)$$

where $$\underline{M}_{nj} = \lim_{s \to -\lambda_{nj}} [\, (s + \lambda_{nj})\, \underline{\Phi}_n(s) \,] \qquad (4.20)$$

p_n = the number of poles of $\underline{\Phi}_n(s)$

In the case that $\underline{\Phi}_n(s)$ has real and repeated poles, $\underline{\Phi}_n(s)$ is approximated with two distinct poles so that this procedure is applicable. Substituting Equation 4.19 into 4.18a yields

$$\underline{G}(s) = \frac{\underline{M}_{11}}{(s+\lambda_{11})} + \frac{\underline{M}_{12}}{(s+\lambda_{12})} + \frac{\underline{M}_{1pi}}{(s+\lambda_{1pi})} + \frac{\underline{M}_{21}}{(s+\lambda_{21})}$$

$$\ldots + \frac{\underline{M}_{kpk}}{(s+\lambda_{kpk})} \qquad (4.21a)$$

or

$$\underline{G}(s) = \sum_{n=1}^{k} \sum_{j=1}^{p_n} \frac{\underline{M}_{nj}}{(s+\lambda_{nj})} \qquad (4.21b)$$

Recalling Equation 2.6 for $\underline{G}(s)$,

$$\underline{G}(s) = \underline{C}\,(s\underline{I} - \underline{A})^{-1}\,\underline{B}$$

If $\underline{A}$ is a diagonal matrix with the poles $-\lambda_i$ as its elements, then $(s\underline{I} - \underline{A})^{-1}$ will also be diagonal with its diagonal elements of the form

$$\frac{1}{(s+\lambda_i)} \qquad i = 1, 2, \ldots, p \qquad (4.22)$$

In addition, if $\underline{C}$ is a matrix made up of column vectors $\underline{c}_{ij}$ and $\underline{B}$ is composed of rows of $\underline{b}_{ij}^{T}$, then Equation 4.21b can be rewritten as

$$\underline{G}(s) = \sum_{i=1} \sum_{j=1} \frac{\underline{c}_{ij}\, \underline{b}_{ij}^{T}}{(s + \lambda_i)} \qquad (4.23)$$

If $\underline{M}_{ij}$ is rewritten as

$$\underline{M}_{ij}(s) = \sum \underline{c}_{ij}\, \underline{b}_{ij}^{T} \qquad i = 1, 2, \ldots, p \qquad (4.24)$$

Then, the vectors $\underline{c}_{ij}$ and $\underline{b}_{ij}^{T}$ will form the columns of $\underline{C}$ and rows of $\underline{B}$ respectively. Also, $\underline{A}$ is a matrix made up of the poles on its diagonal. The above stated procedures are illustrated by an example in Appendix A.

A system described by state space equations (Equations 4.9a and 4.9b) may alwalys be partitioned into four passible subsystems (75) :

1. A system which is controllable and observable.
2. A system which is observable but uncontrollable.
3. A system which is controllable but unobservable.
4. A system which is uncontrollable and unobservable.

By defination, a system is uncontrollable if there is no path from any input to one of the outputs. Similary, a system is unobservable if one of the state variable is not added to any of the system outputs. As a result, a dynamic system S is a "faithful" representation of a transfer function if and only if S is completely controllable and completely observable. The state space matrices (A(nxn), B(nxm), C(mxn)) obtained from minimum realization are controllable if

$$\underline{U} = [\underline{B}, \underline{AB}, \underline{A}^2\underline{B}, \ldots, \underline{A}^{n-1}\underline{B}] \quad (4.25)$$

has a rank of n and observable if

$$\underline{V} = [\underline{C}, \underline{AC}, \underline{A}^2\underline{C}, \ldots, \underline{A}^{n-1}\underline{C}] \quad (4.26)$$

has a rank of n. In addition, $\underline{U}$ and $\underline{V}$ are full rank matrices if their condition numbers are not equal to infinity.

Simulation Method

After the state space model is found, the system will be simulated. The state space model is a set of first order differential equations

$$\underline{X}(k+1) = \underline{A}\underline{X}(k) + \underline{B}\underline{U}(k) \quad (4.27)$$

and it may be integrated to give the analytical solution in discrete form

$$\underline{X}(k+1) = e^{\underline{A}T}\underline{X}(k) + \int_0^T e^{\underline{A}(T-t)}\,\underline{B}\underline{U}(t)\,dt \quad (4.28)$$

where T is the sample time and $e^{\underline{A}T}$ is the matrix exponential. If the input $\underline{U}$ is constant over the sample time, Equatiom 4.28 may be reduced to (76)

$$\underline{X}(k+1) = \underline{P}\underline{X}(k) + \underline{Q}\underline{U}(k) \tag{4.29}$$

where $$\underline{P} = e^{\underline{A}T} \tag{4.30}$$

$$\underline{Q} = \int_0^T e^{\underline{A}(T-t)}\,\underline{B}\,dt \tag{4.31}$$

If $\underline{A}$ is invertible, then

$$\underline{Q} = (\underline{P} - \underline{I})\,\underline{A}^{-1}\,\underline{B} \tag{4.32}$$

Numerous algorithms for computing the matrix exponential have been proposed. In this research, a Fortran program documented in Applied Control Laboratories Report 8601 (77), which is based on the method suggested by Ward (76) was applied for matrix exponential evaluation.

The method of Ward is directed at reducing the problems of numerical inaccuracies associated with large matrix norms. First, the infinity norm of a matrix is reduced by dividing the matrix by two until the norm is less than one. Then, Pade approximations are used to determine the matrix exponential. Once the exponential for the scaled matrix is found, the exponential for the original matrix is found by reversing the scaling procedures used to reduce the original matrix norm. This reversing procedure is called squaring. The discrete matrix $\underline{Q}$ is evaluated by using Equation 4.32. If $\underline{A}$ is singular, another method for

evaluating the integral must be used. One method requires the formation of the augmented matrix (78)

$$\underline{R} = \begin{bmatrix} \underline{A} & \underline{B} \\ 0 & 0 \end{bmatrix} \tag{4.33}$$

Then, the exponential of this matrix is computed by power series expansion to give

$$e^{\underline{R}} = \begin{bmatrix} \underline{P} & \underline{Q} \\ \underline{0} & \underline{I} \end{bmatrix} \tag{4.34}$$

Discrete matrices $\underline{P}$ and $\underline{Q}$ can be separated from the large matrix.

Optimal Tuning Procedure

In this research, single input - single output proportional plus integral controllers are implemented as the control elements for the MVSISO and part of the control structure for the SVAC. The velocity form of the PI controllers is one commonly used because it eliminates reset windup. In discrete form it is expressed as

$$m_{i+1} = m_i + Kc \left[(e_{i+1} - e_i) + (\Delta t/TI)\, e_{i+1} \right] \tag{4.35}$$

where Kc is the proportional gain and TI is the integral time. All the system simulations are in deviation variable form, so the initial valve position will be set to zero.

The control parameters are optimized by a pattern search technique, PATERN (79). There are three commonly used performance criteria ; the integral of the absolute error (IAE), integral of the square of the error (ISE), and integral of time multiplied by the absolute error (ITAE). The IAE criteria is used as performance index in this research. An initial guess is made for the control parameters. Then, the process is simulated and the IAE is calculated. Various sets of control parameters are tried until the optimum is found.

CHAPTER V

APPLICATION OF MULTIVARIABLE CONTROLLERS

The application of three multivariable controllers, namely the multivariable single input - single output or conventional PI controller (MVSISO), multivariable PI tuning regulator (MVPIR), and singular value analysis controller (SVAC) to several models will be compared. The models are taken from literature except that all the deadtime are negleted. Retaining the deadtime would only mask the comparisions since none of the design methods treats deadtime. First of all, the application methodology of the controllers requires the determination of design equations which have been discussed in the previous chapter. Then, a dynamic simulation is established for various step set point tests. Finally, the controllers are optimized or tuned to minimize the IAE by having a search routine drive the simulation. A schematic outlining this evaluation procedure is presented in Figure 5.1. Therefore, this chapter details simulation results of the controllers.

Selected Models Used In Studies

In order to evaluate the potential of the controllers, a simulation study is performed using the following models :

1. Model A : A 2x2 methanol - water distillation column by Wood and Berry (80) without delays.

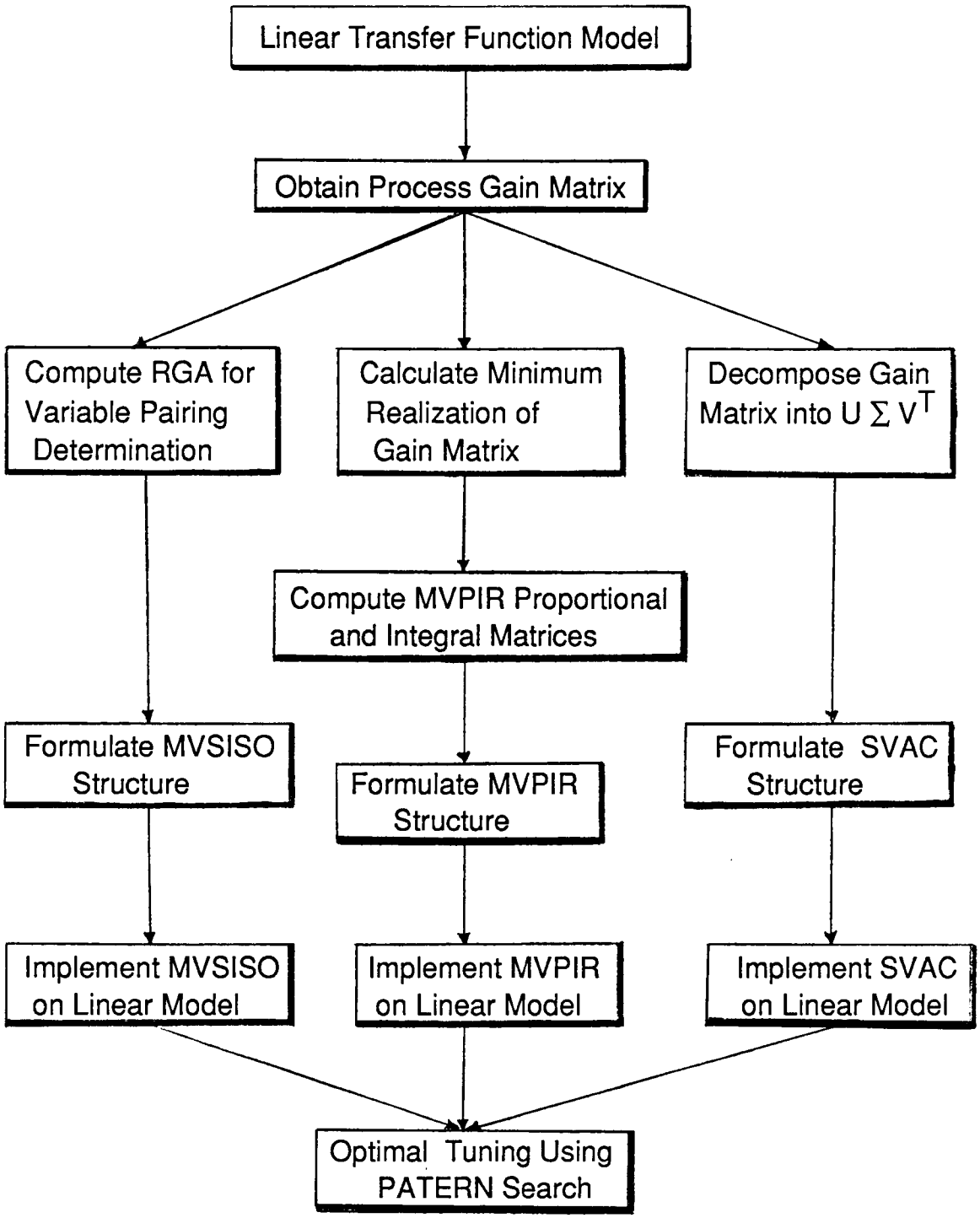


Figure 5.1. Evaluation procedure scheme.

2. Model B : A 2x2 methanol - ethanol distillation column by Weischedel and McAvoy (27) without delays.
3. Model C : A 3x3 ethanol - water distillation column by Ogunnaike and Ray (81) without delays.
4. Model D : A 3x3 benzene - toluene - xylene "light - out - first" distillation column by Alatiqi and Luyben (82) without delays.

The open loop transfer functions and state space models developed according to minimum realization of the original systems are given in Appendix B. The rank and condition number of controllability and observability matrices for each model are also given in Appendix B. Table 5.1 listed transfer function matrices of the models without deadtime. The associated process flowsheet, inputs and their constraints, and outputs are detailed below in the case study sections.

Before discussing the simulation results, it is important to note that the global optimum is not alwalys found by a direct search. If the surface is not unimodal as illustrated in Figure 5.2, the minimum found may be a local minimum. Consequently, the parameter searches used throughout this research are subject to this phenomenon. In order to reduce the possibility of finding a local minimum, several initial guesses for the search parameters are used. Although this procedure does increase the likelihood of finding the global optima, it does not gurantee that the global minimum will be found.

Table 5.1. Open loop transfer functions of selected models.

Model	Transfer Function
A	$\begin{bmatrix} \frac{12.8}{16.7s+1} & \frac{-18.9}{21.0s+1} \\ \frac{6.6}{10.9s+1} & \frac{-19.4}{14.4s+1} \end{bmatrix}$
B	$\begin{bmatrix} \frac{0.471}{(30.7s+1)^2} & \frac{-0.495}{(28.5s+1)^2} \\ \frac{0.749}{(57s+1)^2} & \frac{-0.832}{(50.5s+1)^2} \end{bmatrix}$
C	$\begin{bmatrix} \frac{0.66}{6.7s+1} & \frac{-0.61}{8.64s+1} & \frac{-0.0049}{9.06s+1} \\ \frac{1.11}{3.25s+1} & \frac{-2.36}{5.0s+1} & \frac{-0.012}{7.09s+1} \\ \frac{-34.68}{8.15s+1} & \frac{46.2}{10.9s+1} & \frac{0.87(11.61s+1)}{(3.89s+1)(18.8s+1)} \end{bmatrix}$
D	$\begin{bmatrix} \frac{2.7}{42s+1} & 0 & \frac{-0.89}{38s+1} \\ \frac{9.0}{(3.3s+1)^2} & \frac{2.05}{(38s+1)(25s+1)} & \frac{3.25}{110s+1} \\ \frac{-1.2}{(19s+1)^2} & \frac{-0.03}{(9.5s+1)^2} & \frac{-0.48}{(32s+1)^2} \end{bmatrix}$

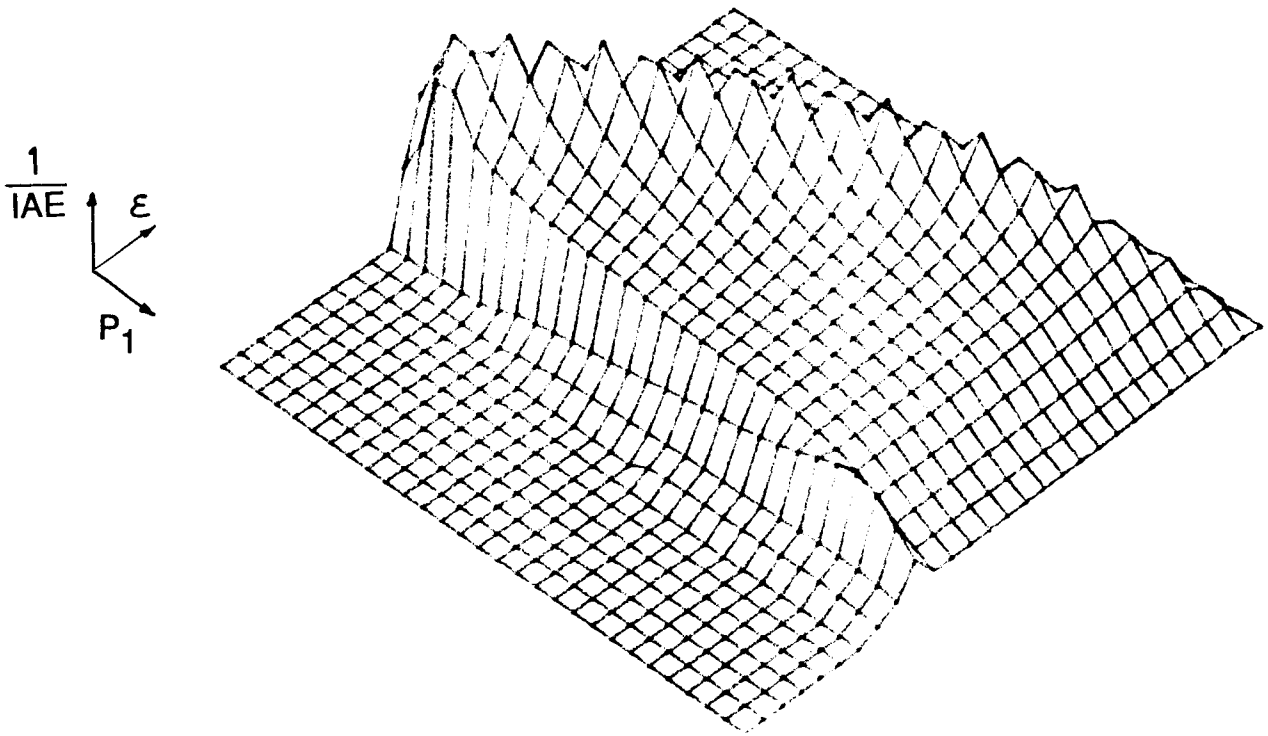


Figure 5.2. Non-unimodal search surface of Model B for MVPIR control system with P_2 fixed.

Multivariable Control Of Model A

The first application of the multivariable controllers is to the Wood and Berry without deadtime model which is considered to be an easily controllable system. One reason for using this example for the first study is that the MVSISO controller has been applied to the original model and reported in the literature. Thus, the present simulation can be "validated" by comparision to literature simulation. The methanol - water distillation column is a 9 in. diameter, 8 tray column equipped with a total condenser and a basket type reboiler. Figure 5.3 shows a schematic diagram of the column. The manipulated variables are reflux (R) and steam flow (S) while the controlled variables are composition of methanol in overhead (XD) and bottom product (XB). Steady state operating conditions and input constraints of the column are listed in Table 5.2. No operational constraints on the inputs are given by the reference so they were taken as ± 20% of the steady state operating values.

Table 5.3 summarizes the RGA, SVD, Niederlinski index, and MVPIR gain matrices of the Model A. The system has condition number of 7.48 and RGA of 2.01. The condition number indicates good process conditioning. The RGA is close to being in the ideal range of 0.67 to 1.5 given by McAvoy and certainly < 25 at which McAvoy states that other control strategies should be considered. The pairings obtained from RGA are R - XD and S - XB, while the SVD matrix $\underline{U}$ indicates the first output direction is made up of both outputs about equally, -0.75 and -0.67 as is the second output direction. Thus, the SVA does not indicate good SISO

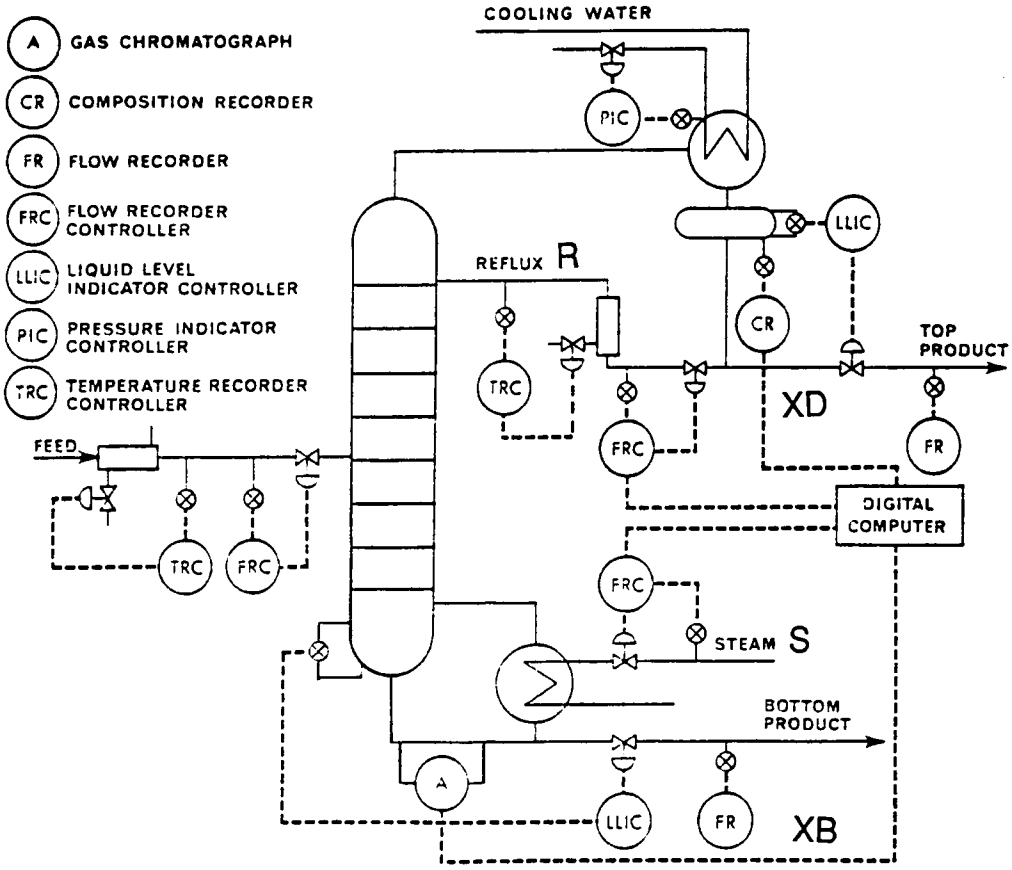


Figure 5.3. Schematic diagram of Wood and Berry column.

Table 5.2. Steady state operating conditions and input constraints of Wood and Berry model.

Steady State Operating Conditions :

Stream	Flow (lb/min)	Composition (wt. % methanol)
Overhead	1.18	96.0
Reflux	1.95	96.0
Bottoms	1.27	0.5
Feed	2.45	46.5
Steam	1.71	-

	Temperature (°F)
Reflux	151.7
Feed	168.0
Steam	233.0
Condensate	227.5
Plate 1	203.6
2	194.4
3	181.2
4	172.9
5	164.1
6	156.8
7	152.1
8	148.5
Condenser	143.9

System Constraints (as ± 20% from steady state) :

Manipulated Variables	Lower Constraint	Upper Constraint
Reflux flow rate	1.56 lb/min	2.34 lb/min
Reboiler steam flow rate	1.368 lb/min	2.052 lb/min

Table 5.3. Interaction analysis and design equations for Model A.

Steady State Gain Array

R	S	
12.8	-18.9	XD
6.6	-19.4	XB

Relative Gain Array

R	S	
2.0094	-1.0094	XD
-1.0094	2.0094	XB

Niederlinski index = 0.4977

Singular Value Decomposition

Singular values : 30.405 , 4.0645

V^T

R	S
-0.4585	-0.8887
0.8887	-0.4585

U

-0.7454	-0.6666	XD
-0.6666	0.7454	XB

Condition number = 7.48

Multivariable Tuning Regulator Matrices

$Kp = (CB)^+$		$KI = -CA^{-1}B$	
2.7625	-1.8455	0.1570	-0.1529
1.2416	-1.5717	0.0534	-0.1036

loop pairing in conflict with the RGA result. If the SVA pairing is done, anyhow, the largest magnitude element in the first column of $\underline{V}$ (or the first row of $\underline{V}^T$) , -0.89, and first column of $\underline{U}$, -0.75, suggests S - XD as a loop pair while the second columns (elements of 0.89 and 0.75) gives R - XB which is the opposite of the RGA pairing.

The RGA pairings are used for the MVSISO controller structure. There are known to give good results as shown in the literature reference for this model. The necessary condition for closed loop stability of the resulting pairings is confirmed by the Niederlinski index.

Performance of the multivariable controllers was evaluated by simulating the system for separate steps in each set point and a combination of a step in set point one followed by a step in set point two. The simulation used a sample time of 0.1 min and when the tuning was done for a step set point test for output two, the simulation ran for 300 samples. The simulation was checked by running the final result with a step size of 0.05 min which gave the same result as that ran at larger step of 0.1 min. The process was tuned minimizing the IAE of the error signals from both outputs with equal weights. The tuning parameters of the process using conventional PI, multivariable PI regulator and singular value analysis controller are summarized in Table 5.4. The dynamic response of the physical process variables to a step set point two change for the three different control schemes is presented in Figures 5.4, 5.5, and 5.6 respectively. Responses for other step tests are not shown because they are quite similar to Figures 5.4, 5.5, and 5.6. All the controllers respond about equally well and there is little difference

Table 5.4. IAE tuning of Model A.

Design Method	set Point	Tuning Parameters			IAE
	1	K_{c1} = -1.13	K_{c2} = -5.33		4.02
		T_{I1} = 0.067	T_{I2} = 0.064		
MVSISO	2	K_{c1} = 5.915	K_{c2} = -3.414		2.62
		T_{I1} = 0.0286	T_{I2} = 0.0399		
	1,2	K_{c1} = 7.874	K_{c2} = -7.645		6.17
		T_{I1} = 0.065	T_{I2} = 0.3480		
	1	P_1 = 1.06	P_2 = 9.6	$\mathcal{E}$ = 4.32	4.84
MVPIR	2	P_1 = 2.18	P_2 = 2.20	$\mathcal{E}$ = 41.28	2.57
	1,2	P_1 = 1.39	P_2 = 1.11	$\mathcal{E}$ = 6.30	10.13
	1	K_{c1} = 0.84	K_{c2} = 2.401		4.08
		T_{I1} = 0.076	T_{I2} = 0.1		
SVAC	2	K_{c1} = 6.89	K_{c2} = 11.01		2.56
		T_{I1} = 0.11	T_{I2} = 0.087		
	1,2	K_{c1} = 7.228	K_{c2} = 3.336		6.71
		T_{I1} = 0.1	T_{I2} = 0.1		

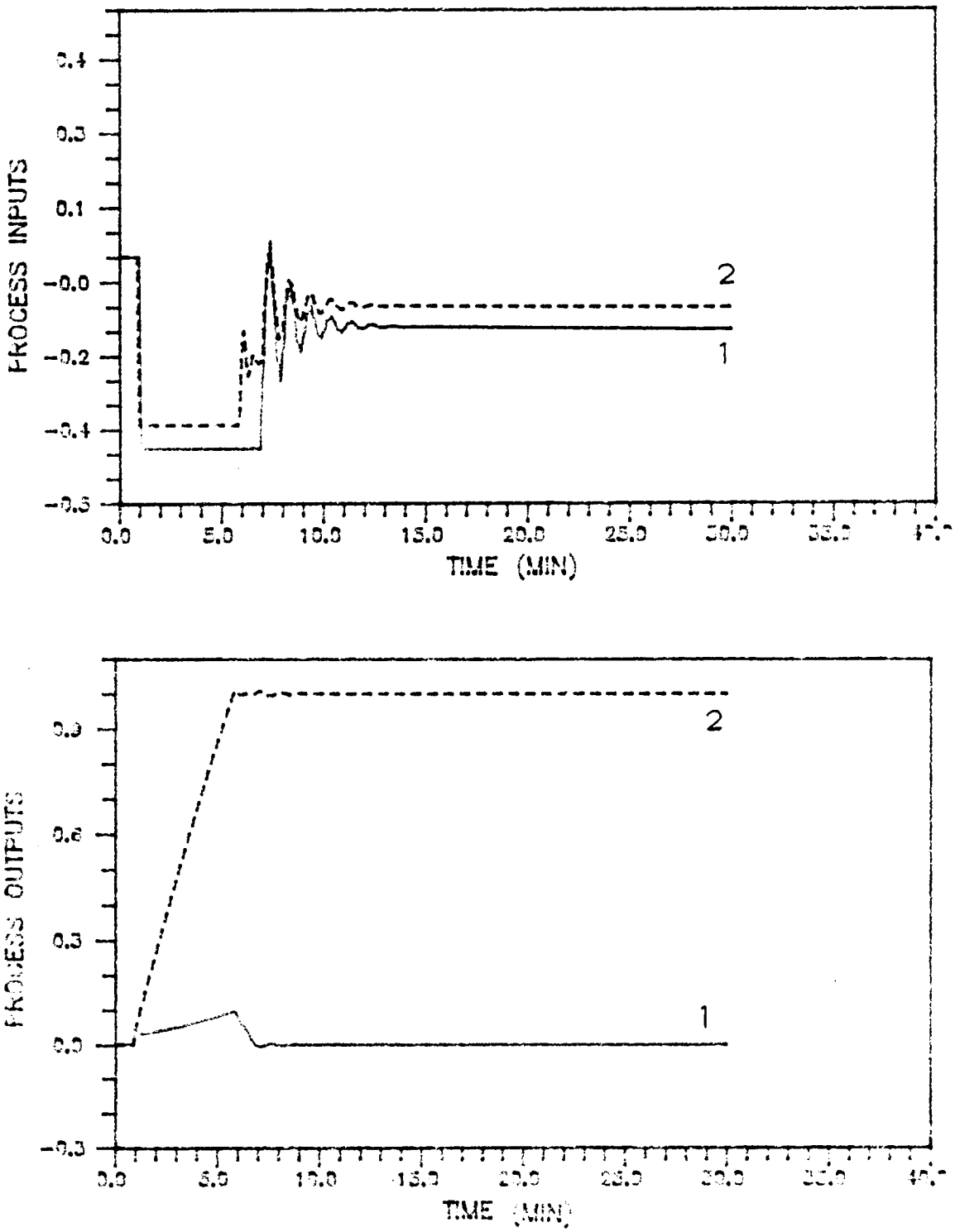


Figure 5.4. Model A response to a unit step in set point two using MVSISO.

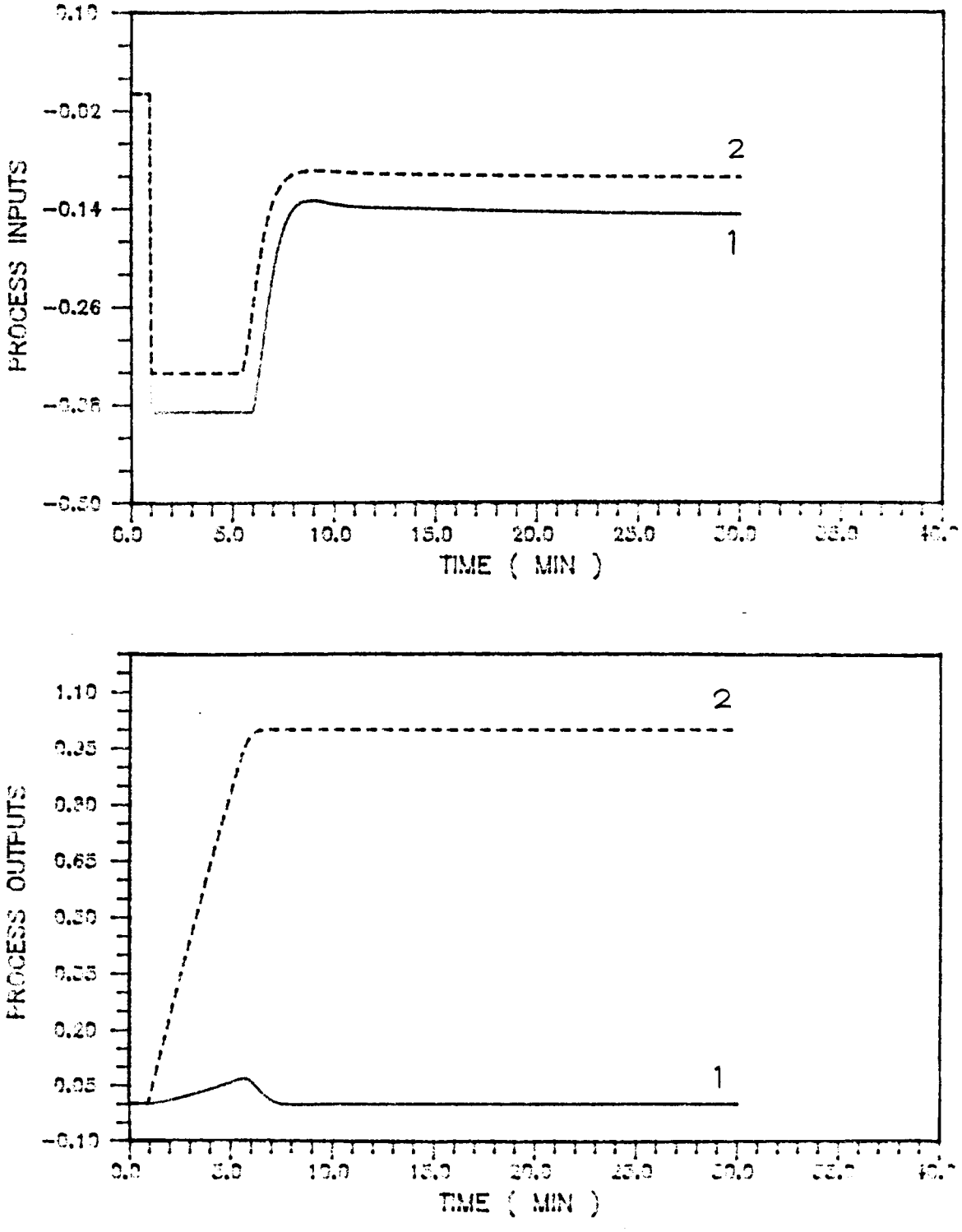


Figure 5.5. Model A response to a unit step in set point two using MVPIR.

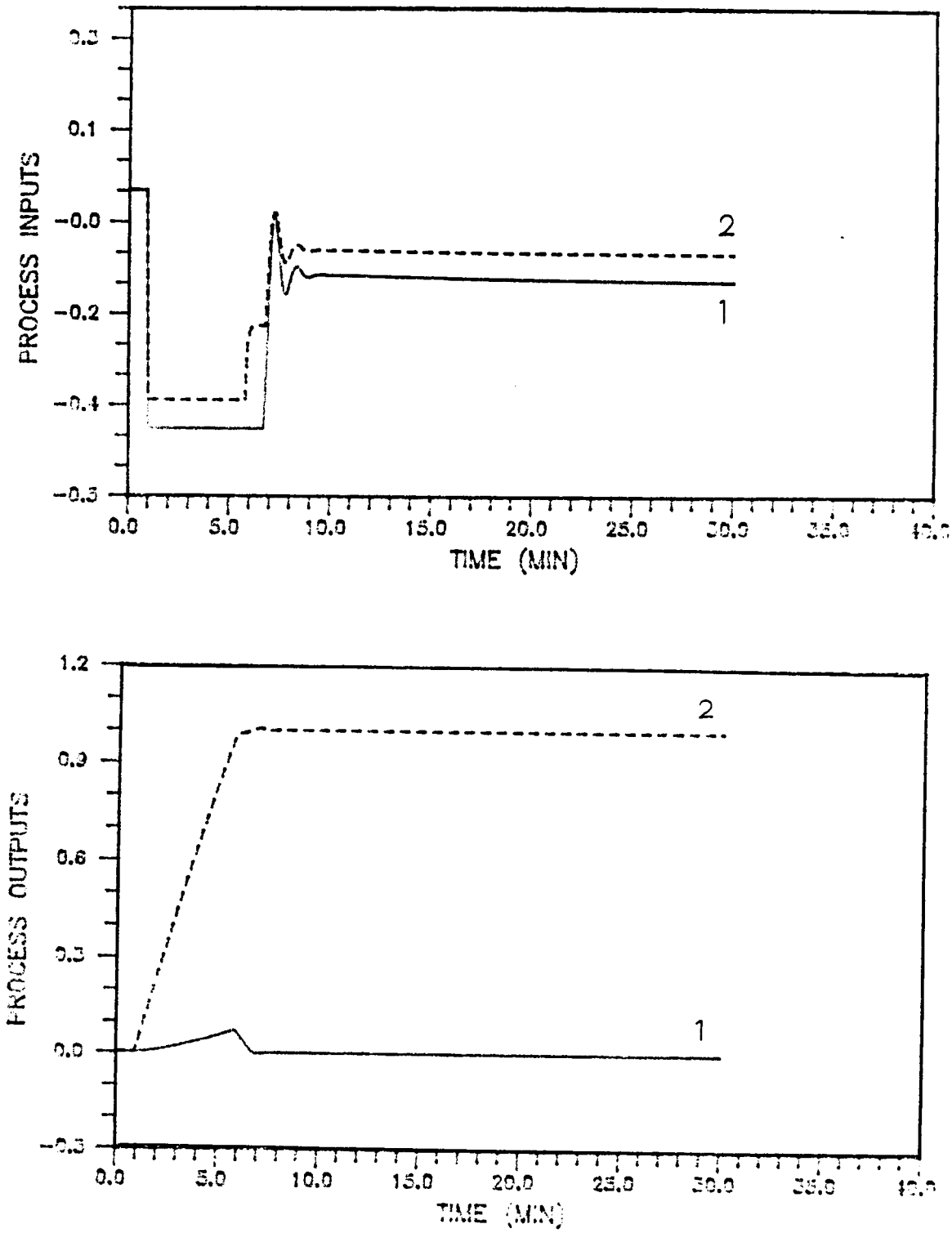


Figure 5.6. Model A response to a unit step in set point two using SVAC.

between the performance indices of the control systems. Among the controllers, the MVPI tuning regulator system would possibly be preferred because the inputs to the process are the smoothest. These results are expected as little interaction exists in the system.

Multivariable Control Of Model B

Since Weischedel and McAvoy presented this highly coupled methanol - ethanol column, called Column C in their paper , it has frequently been used as an example in process control design. The process flowsheet is similar to that of the Wood and Berry example. Steady state operating conditions and input constraints of the column are given in Table 5.5. The manipulated variables are reflux flow rate (R) and vapor flow rate from reboiler (V), while the controlled variables are composition of distillate (XD) and bottom product (XB). As input constraints of the system were not given in the literature, they are determined from steady state operating conditions. For this work, the input constraints are taken as ±20% from the steady state condition. Table 5.6 summarizes the results of the interaction analysis and design equations for the Model B. A RGA of 18,which is out of ideal range of 0.67 to 1.5 and near value of 25 where other control approaches are suggested, indicates that strong interactions exist between two loops. Similarly, a condition number of 81, greater than 50 as given by Moore, denotes that the process is poorly conditioned. If the pairing is made anyhow by using the positive elements, the RGA pairing has the distillate composition (XD) being controlled by manipulating reflux flow rate (R) and the bottom

Table 5.5. Steady state operating conditions and input constraints of Column C model.

Steady State Operating Conditions :

Mixture	methanol - ethanol
Product split	0.01 - 0.99
Vapor flow from reboiler	3.856 mol/min
Reflux flow rate	3.384 mol/min
Feed flow rate	1.50 mol/min
Mole fraction of methanol in feed	0.5

System Constraints (as ±20% from steady state) :

Manipulated Variables	Lower Constraint	Upper Constraint
Reflux flow rate	2.707 mol/min	4.061 mol/min
Vapor flow rate	3.085 mol/min	4.627 mol/min

Table 5.6. Interaction analysis and design equations for Model B.

Steady State Gain Array

R	V	
0.471	-0.495	XD
0.749	-0.832	XB

Relative Gain Array

R	V	
18.5572	-17.5572	XD
-17.5572	18.5572	XB

Niederlinski index = 0.0538

Singular Value Decomposition

Singular values : 1.3114 , 0.0161

V^T

R	V
0.6746	-0.7382
-0.7382	-0.6746

U

0.5209	-0.8536	XD
0.8536	0.5209	XB

Condition number = 81.44

Multivariable Tuning Regulator Matrices

$Kp = (CB + CAB)^+$		$KI = -CA^{-1}B$	
14470	-27030	41.5030	-24.6809
10225	-22165	37.4521	-23.4738

composition (XB) being controlled by adjusting vapor flow rate from the reboiler (V). From the result of SVD, the first column of $\underline{U}$ shows the first principle output direction depends somewhat more on XB while the second output direction depends more on XD. However, the column of $\underline{V}$ shows that the input directions are about equally dependent on both inputs (elements of 0.67 and 0.74). Therefore, the SVA also indicates there is not a good set of loop pairings. If the variables are paired anyhow by the largest elements, the resulting pairings, V - XB and R - XD, are the same as RGA pairings. As shown in Table 5.6, Niederlinski index of the resulting pairings is 0.0538 so the system satisfied the necessary condition for being closed loop stable.

Again, as for the Model A study, the optimum control parameters are found for separate steps in each set point and a combination of a step in set point one followed by a step in set point two. The optimal parameters for each case of set point tuning for Model B are presented in Table 5.7. By comparing the overall IAE, MVPIR has the smallest IAE while MVSISO has the largest IAE as expected. The IAE of SVAC control system is inconsistent, that is, this controller does well for set point one change but poorly for set point two change. The IAE of SVAC for a step in set point one and two are 0.17 and 0.41 respectively.

The responses for a step in set point one of 0.005 are shown in Figures 5.7, 5.8, and 5.9. Both MVPIR and SVAC control system show about three smooth cycles for the transient response. As expected the MVSISO control system produces an very oscillatory response indicating a lot of interaction.

Table 5.7. Tuning parameters of Model B.

Design Method	Set Point	Tuning Parameters			IAE
MVSISO	1	K_{c1}=22.52	K_{c2}=-89.90		0.4394
		T_{I1}=31.01	T_{I2}=80.1		
	2	K_{c1}=93.0	K_{c2}=-27.95		0.3800
		T_{I1}=79.50	T_{I2}=47.85		
	1,2	K_{c1}=68.0	K_{c2}=-34.0		0.8554
		T_{I1}=80.7	T_{I2}=44.95		
MVPIR	1	P_1=0.009	P_2=0.00289	ε=0.056	0.1803
	2	P_1=0.0412	P_2=0.00483	ε=0.0508	0.1990
	1,2	P_1=0.009	P_2=0.00369	ε=0.0552	0.4128
SVAC	1	K_{c1}=40.22	K_{c2}=188.13		0.1722
		T_{I1}=302.1	T_{I2}=58.68		
	2	K_{c1}=35.01	K_{c2}=45.51		0.4052
		T_{I1}=395.0	T_{I2}=43.70		
	1,2	K_{c1}=41.90	K_{c2}=69.50		0.6700
		T_{I1}=370.0	T_{I2}=45.0		

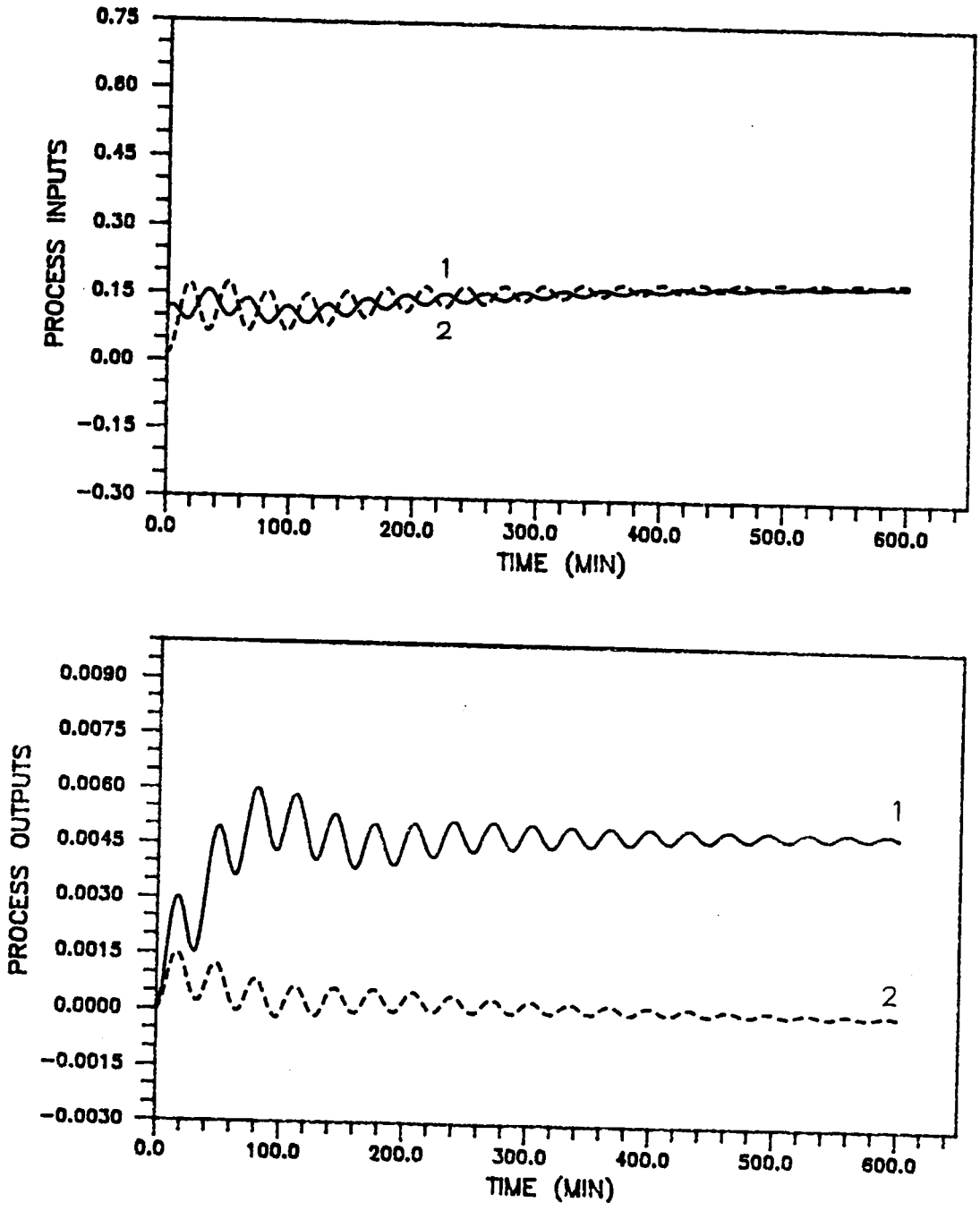


Figure 5.7. Response of Model B for step in set point one with MVSISO.

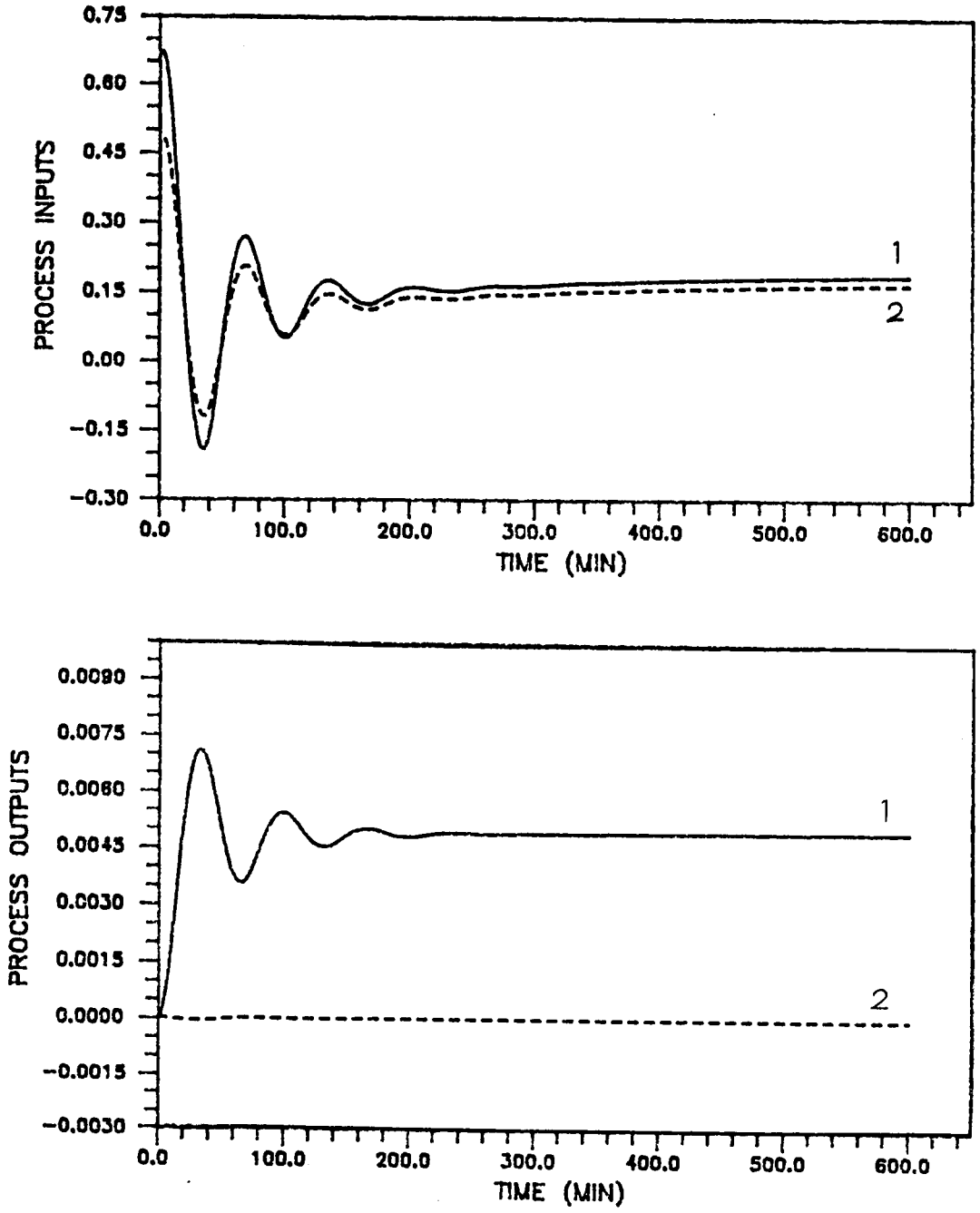


Figure 5.8. Response of Model B for step in set point one with MVPIR.

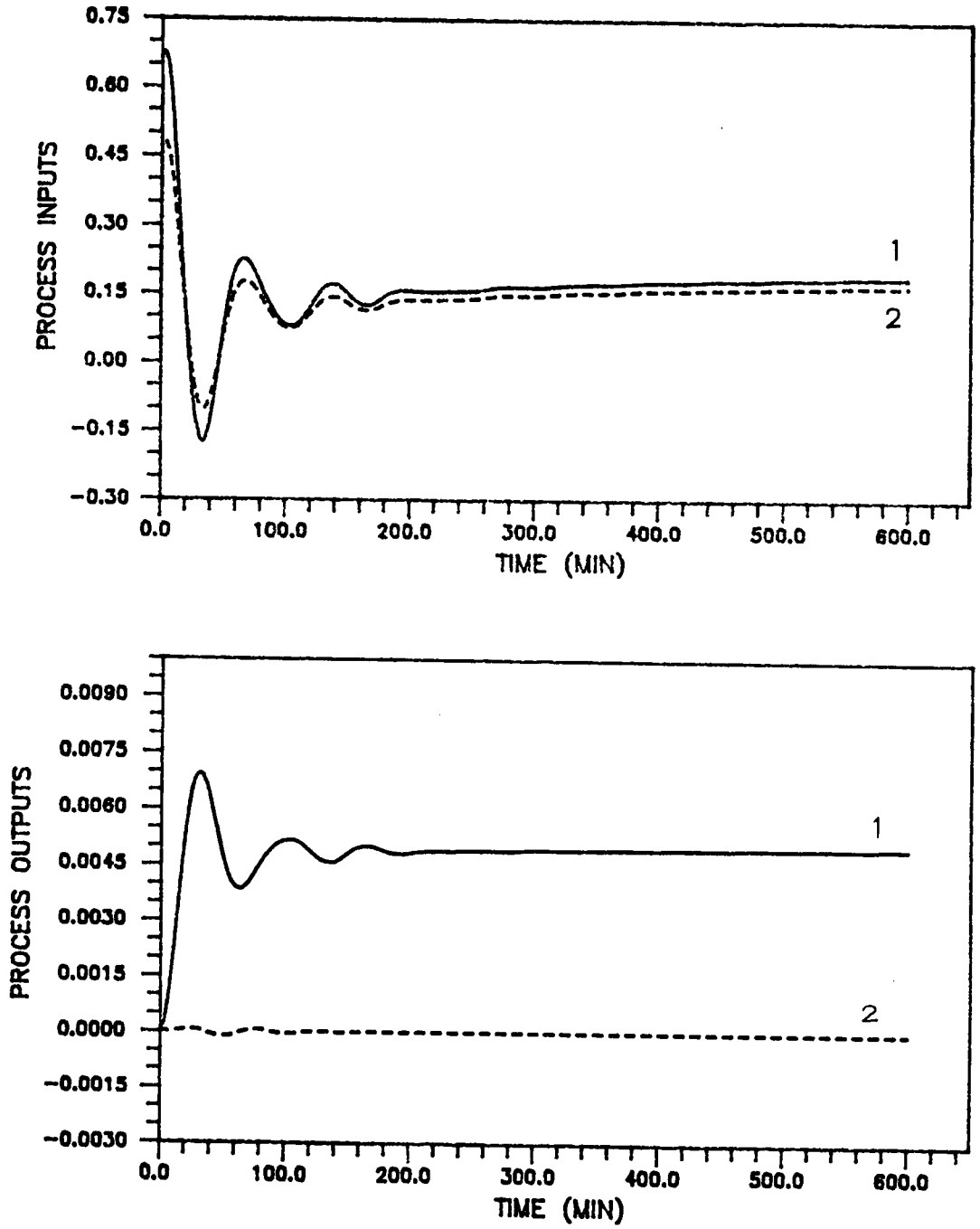


Figure 5.9. Response of Model B for step in set point one with SVAC.

For both the MVPIR and SVAC, the inputs to the process show essentially the similar trend. The manipulated process inputs required for steady state are 0.17 and 0.15 for the two input variables. Both MVPIR and SVAC generate a vigorous change in the inputs initially but both inputs move in the same direction. Although the magnitude of input changes for MVSISO is small compared to MVPIR and SVAC control schemes, both inputs are oscillatory and move in the opposite direction. This system has a RGA element of 18, an indication of considerable interaction between loops. Consequently, the interaction reduces control effectiveness of MVSISO control scheme.

Figures 5.10, 5.11, and 5.12 show the closed loop responses that are obtained when the system is subjected to a step in set point one at 1.0 minute followed by a step in set point two at 500 minute. The responses for a step in set point two look similar to that given in Figures 5.10 to 5.12 so they are not included. As can be seen by comparing the closed loop responses, MVPIR control system has the shortest transient. For both MVSISO and SVAC control systems, there is a relatively large difference between the proportional gains obtained when tuning for individual set point changes as tabulated in Table 5.7. When the step changes are made in sequence, the optimum parameters are in between the parameters of individual set point change. In addition, the IAE for the sequential set point changes is almost the direct sum of the IAE for each individually tuned loop. This indicates a relatively flat search function, as the tuning parameters are different.

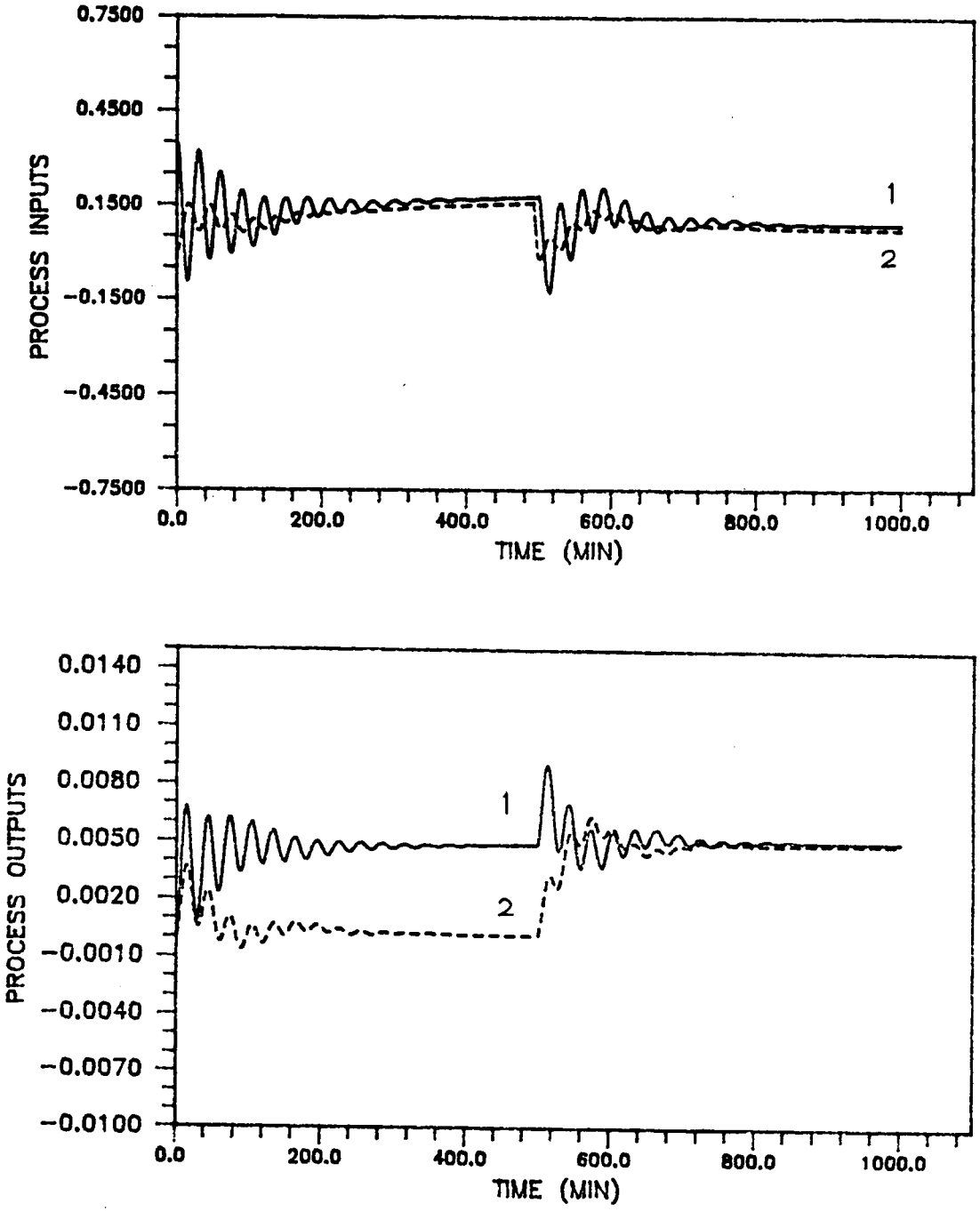


Figure 5.10. MVSISO control of Model B for step in set point one followed by step in set point two.

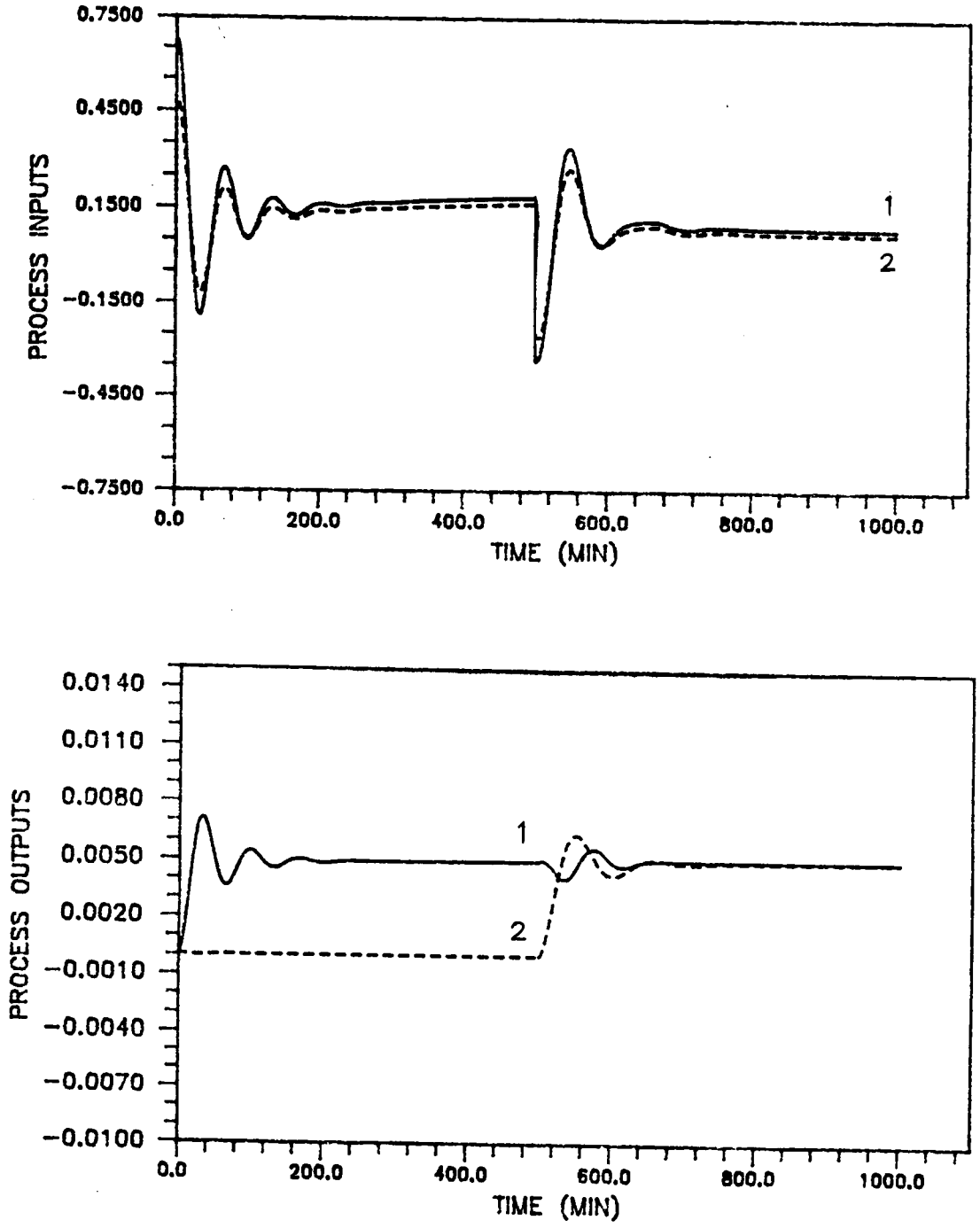


Figure 5.11. MVPIR control of Model B for step in set point one followed by step in set point two.

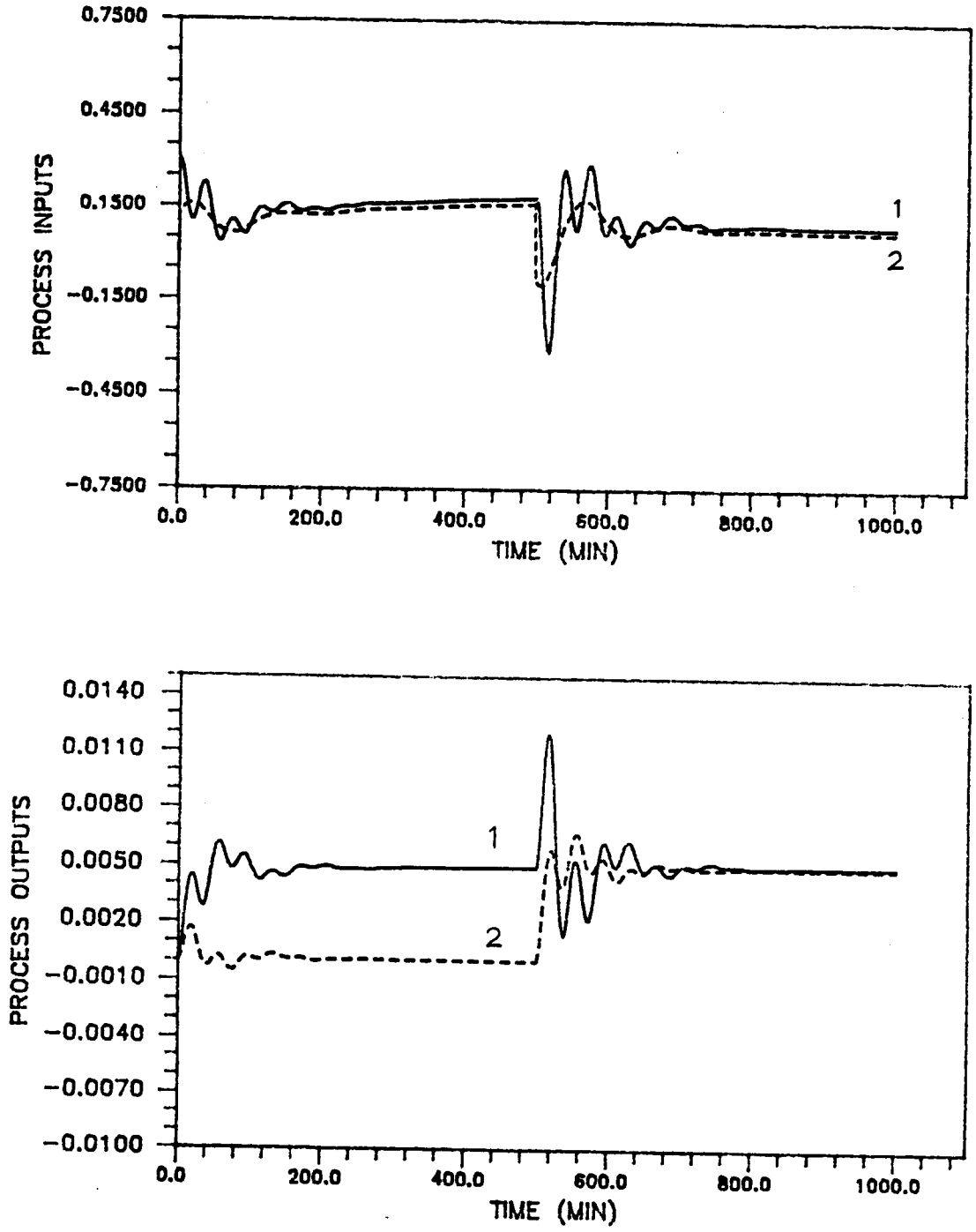


Figure 5.12. SVAC control of Model B for step in set point one followed by step in set point two.

As for the single set point changes, the oscillations in the process inputs of MVSISO are due to strong interaction between loops. This system has a condition number of 81, and the elements in the right singular vector are 0.67 and -0.74, which are about equal in magnitude but opposite signs. Thus, the process is more sensitive to differences in the inputs. This helps explain the longer transient of SVAC control system for a step in both set points in sequence than just a step in one set point.

Multivariable Control Of Model C

The ethanol - water distillation column is a 19 plate, 12 in. diameter copper column having variable feed and side stream draw off locations. A schemetic diagram and steady state operating conditions along with input constraints of the column are shown in Figure 5.13 and Table 5.8 respectively. The inputs and outputs of the system selected for study are :

R = reflux flow rate, gpm.

S = side stream flow rate, gpm.

P = reboiler steam pressure, kPa.

XD = ethanol mole fraction in overhead

XS = ethanol mole fraction in side stream

T_{19} = temperature of tray 19, °C.

The input constraints are taken from reference paper.

Results of SVA and RGA are listed in Table 5.9. As shown in Table 5.9, the main diagonal elements of RGA which are closest to 1.0 are 1.96, 1.89, and 1.52 which denote that there is little interaction between

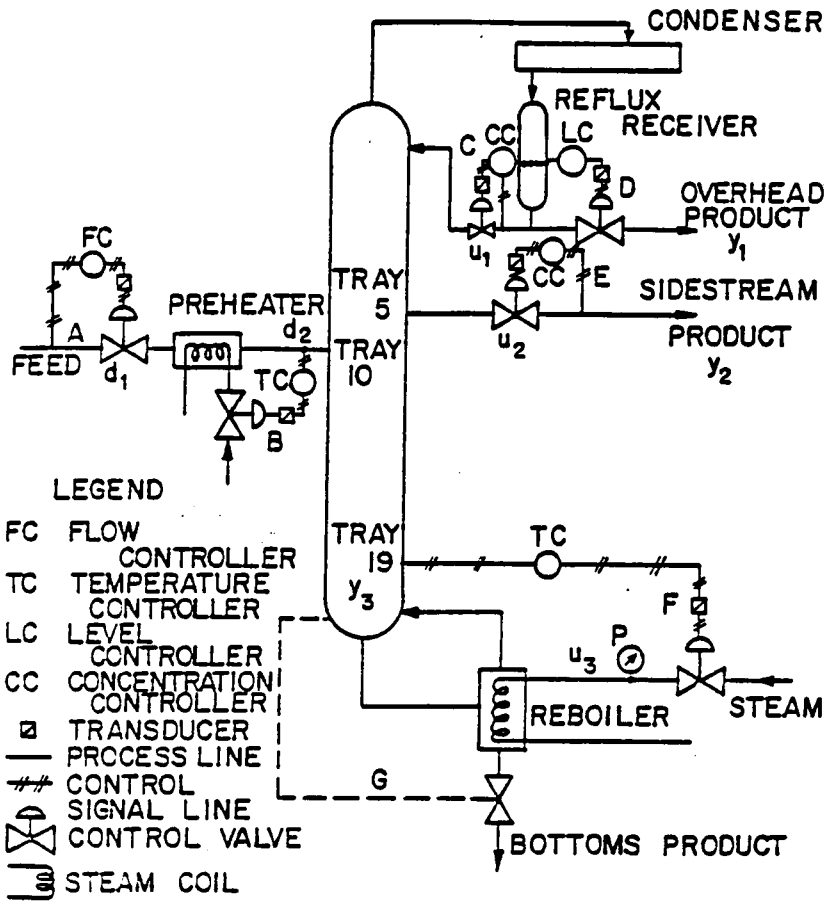


Figure 5.13. Schematic diagram of Ogunnaike and Ray experimental distillation column.

Table 5.8. Steady state operating conditions and input constraints of Ogunnaike and Ray column model.

Steady State Operating Conditions :

Reflux flow rate	0.18 gpm (0.01 L/s)
Side stream flow rate	0.046 gpm (0.003 L/s)
Reboiler steam pressure	20.0 psig (137.89 kPa)
Overhead ethanol mole fraction	0.7
Side stream ethanol mole fraction	0.52
Temperature on tray #19	92.0 °C

System Constraints (as given in the reference) :

Manipulated Variables	Lower Constraint	Upper Constraint
Reflux flow rate	0.068 gpm 0.004 L/s	0.245 gpm 0.02 L/s
Side stream flow rate	0.00694 gpm 0.00044 L/s	0.10 gpm 0.01 L/s
Reboiler steam pressure	15.6 psig 107.56 kPa	34.0 psig 234.4 kPa

Table 5.9. Interaction analysis and design equations for Model C.

Steady State Gain Array

R	S	P	
0.66	-0.61	-0.0049	XD
1.11	-2.36	-0.012	XS
-34.68	46.2	0.87	T_{19}

Relative Gain Array

R	S	P	
1.9618	-0.6648	-0.2970	XD
-0.6699	1.8921	-0.2222	XS
-0.2919	-0.2273	1.5192	T_{19}

Niederlinski index = 0.3818

Singular Value Decomposition

Singular values : 57.838, 0.5533, 0.0158

V^T

R	S	P
-0.6000	0.7989	0.0420
0.7999	0.6000	0.0127
0.0150	0.0413	0.9990

U

-0.0153	0.2918	0.9563	XD
-0.0442	-0.9557	0.2909	XS
0.9989	-0.0378	0.0275	T_{19}

Condition number = 3671

Multivariable Tuning Regulator Matrices

$Kp = (CB + CAB)^+$

24.2130	-3.4168	0.0589
13.9722	-4.9991	-0.0030
341.8272	60.6189	11.5007

$KI = -CA^{-1}B$

2.9328	-0.6273	0.0068
1.0853	-0.8089	-0.0052
59.3015	17.9844	1.7021

loops. However, the system has a condition number of 3671 most likely because the transfer functions have different units. After the system steady state gain matrix is scaled by nonlinear least squares search technique to minimize the condition number, the condition number is reduced to 7.4. The resulting pairings of RGA are R - XD, S - XS, and P - T_{19}. From the result of SVA, the first two columns of $\underline{V}$ show that the first input direction depends on both R and S as does the second input direction. This is an indication of pairing difficulty. However disregarding this, if the manipulated and controlled variables are paired by the largest elements in the principle input and output directions, the resulting pairings are, S - T_{19}, R - XS, and P - XD, do not agree with the RGA pairings. The column of $\underline{U}$ does indicate that the principle output directions are each mainly fixed by one of the process outputs, that is, the first, second and third directions are basically defined by T_{19}, XS, and XD respectively.

For this model, the system were simulated for separate steps in each set point and simultaneous steps in both set points. However, results for separate steps change are not included because they are quite similar to response of simultaneous steps change. The simulation used a sample time of 0.1 minute. The simulation ran for 500 samples with 2.5% and 4% step in set points one and two introduced simultaneously at the 10^{th} sample period.

Since process output one and two are mole fraction while output three is temperature in degree °C, the IAE is used as a performance index

with weighting factors of 100, 40, and 1 for corresponding error signals one, two, and three respectively. The weighting factors are evaluated from responses of open loop pulse testing. The average change of each process output due to pulse testing of each process input is obtained from the open loop responses given by Ogunnaike (75). The average changes for XD, XS, and T_{19} are 0.03, 0.0717, and 2.933 respectively. Then, the average output changes are normalized with respect to 2.933, and inverse of the normalized values are taken as the weighting factors. Table 5.10 summarizes the tuning results.

The process inputs and outputs for different control schemes are presented in Figure 5.14 to Figure 5.19. The MVPIR yields the smoothest system response while SVAC control system has the longest transient response. The transient of SVAC may be reduced by tuning the orthogonal matrices (54,55). Although all the control schemes show satisfactory control, the manipulated variables exhibit excessive fluctuations, which is an unexpected phenomenon. The larger amplitude fluctuation in the third output probably arises from its small weighting factor in the IAE. This oscillation could be also driving the oscillations in the other variable. One possible source for these high frequency inputs is the small integral time constant or integral tuning parameters, see Table 5.10. The rather unusual fluctuations could be reduced by increasing the parameters associated with integral mode. In addition, the fluctuations could also be filtered out by implementing a filter in the system, or they might be eliminated by input suppression where the maximum change of inputs over one sample time is constrainted. An example of MVSISO

Table 5.10. IAE tuning of Model C.

Design Method	Set Point	Tuning Parameters			IAE
MVSISO	1	K_{c1}=5.258	K_{c2}=-2.260	K_{c3}=2.198	11.861
		T_{I1}=0.021	T_{I2}=0.0053	T_{I3}=0.001	
	2	K_{c1}=1.466	K_{c2}=-3.231	K_{c3}=1.411	3.97
		T_{I1}= 0.072	T_{I2}=0.0049	T_{I3}=0.3124	
	1,2	K_{c1}=6.103	K_{c2}=-7.633	K_{c3}=7.977	5.203
		T_{I1}=0.004	T_{I2}=0.020	T_{I3}=0.0076	
MVPIR	1	P_1=0.677	P_2=0.336	P_3=14.54 $\mathcal{E}$=1.313	12.36
	2	P_1=16.04	P_2=0.263	P_3=14.73 $\mathcal{E}$=0.4353	4.29
	1,2	P_1=1.825	P_2=0.621	P_3=16.647 $\mathcal{E}$ = 17.146	5.95
SVAC	1	K_{c1}=0.77	K_{c2}=14.83	K_{c3}=4.37	13.47
		T_{I1}=0.014	T_{I2}=0.105	T_{I3}=0.0806	
	2	K_{c1}=2.19	K_{c2}=10.10	Kc3=22.0	4.326
		T_{I1}=0.10	T_{I2}=1.10	TI3=0.80	
	1,2	K_{c1}=0.402	K_{c2}=10.91	K_{c3}=6.143	7.051
		T_{I1}=0.0066	T_{I2}=0.190	T_{I3}=0.0017	

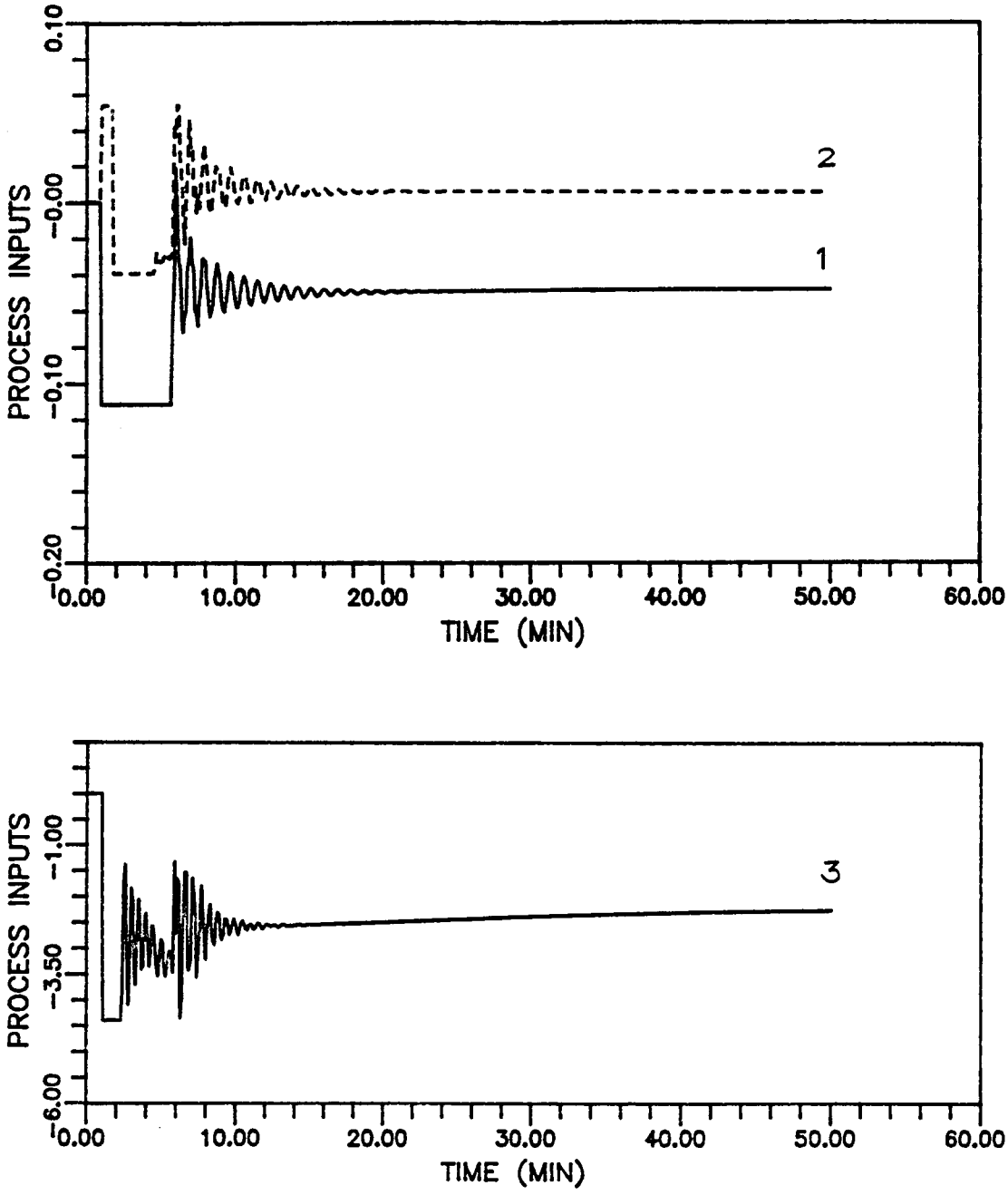


Figure 5.14. Process inputs for MVSISO to simultaneous steps in set points one and two for Model C.

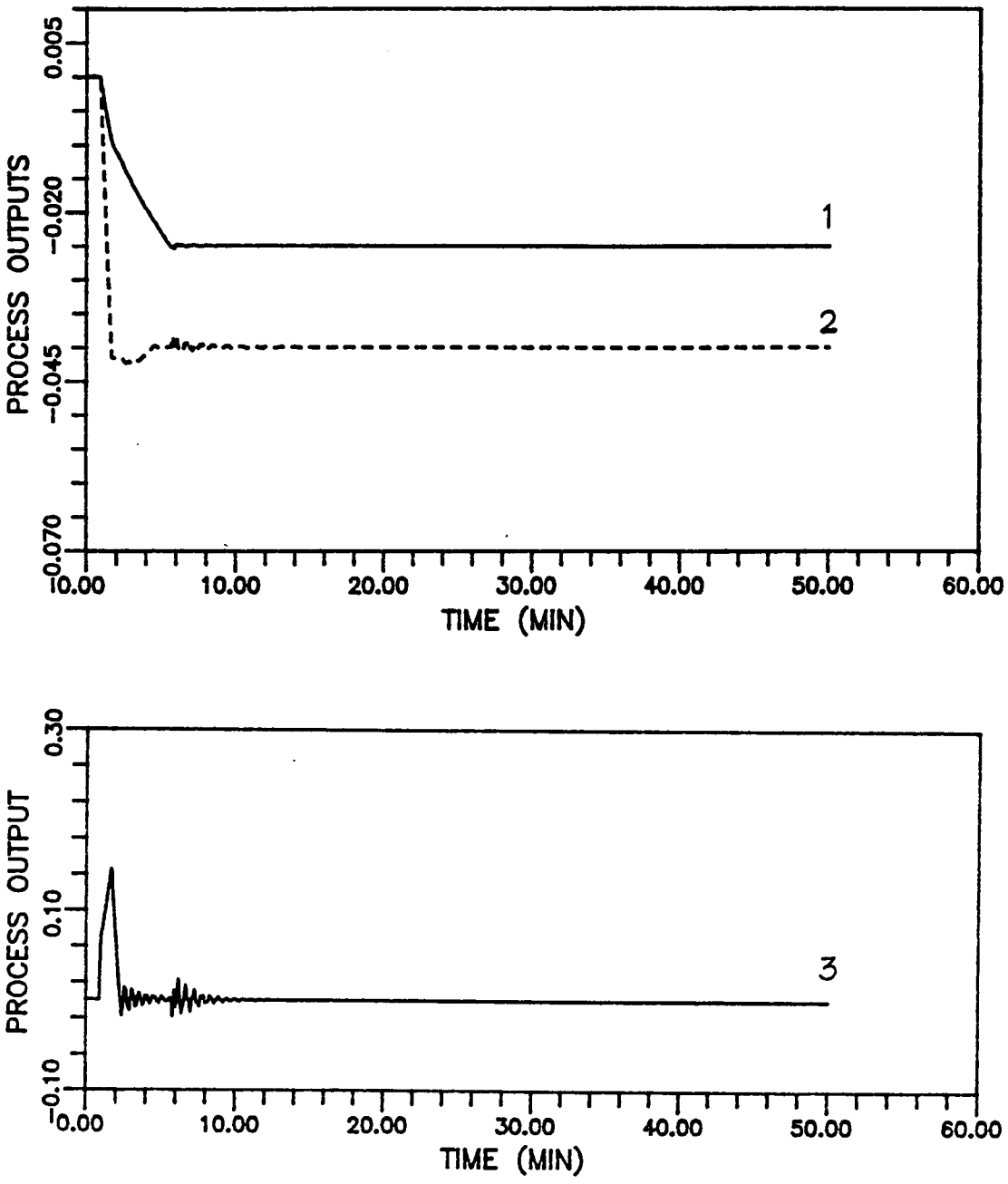
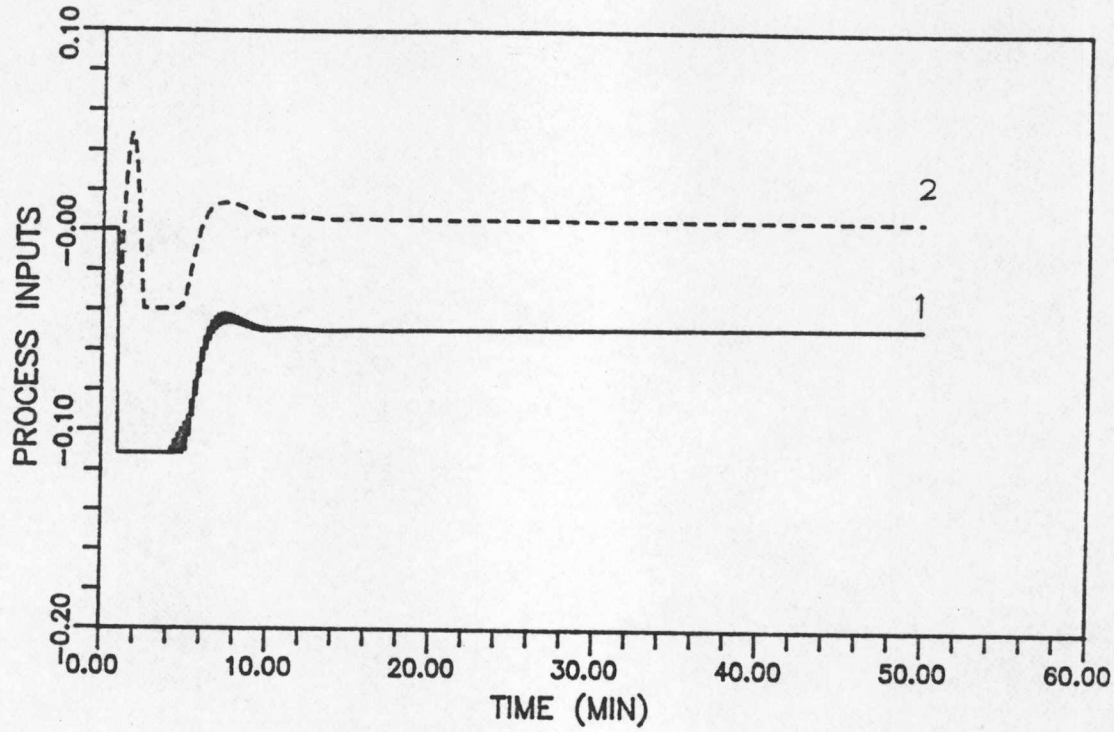


Figure 5.15. System response for MVSISO to simultaneous steps in set points one and two for Model C.



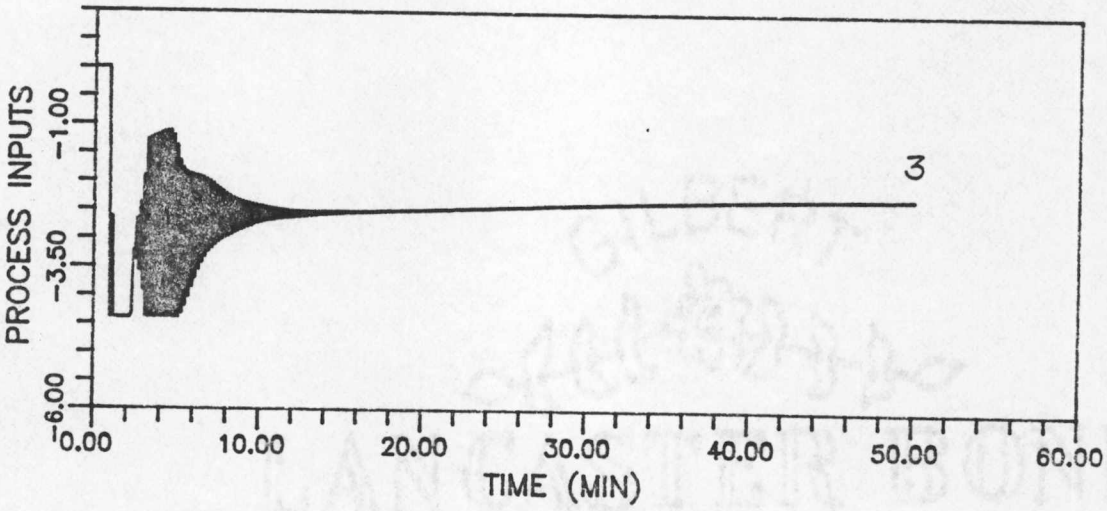


Figure 5.16. Process inputs for MVPIR to simultaneous steps in set points one and two for Model C.

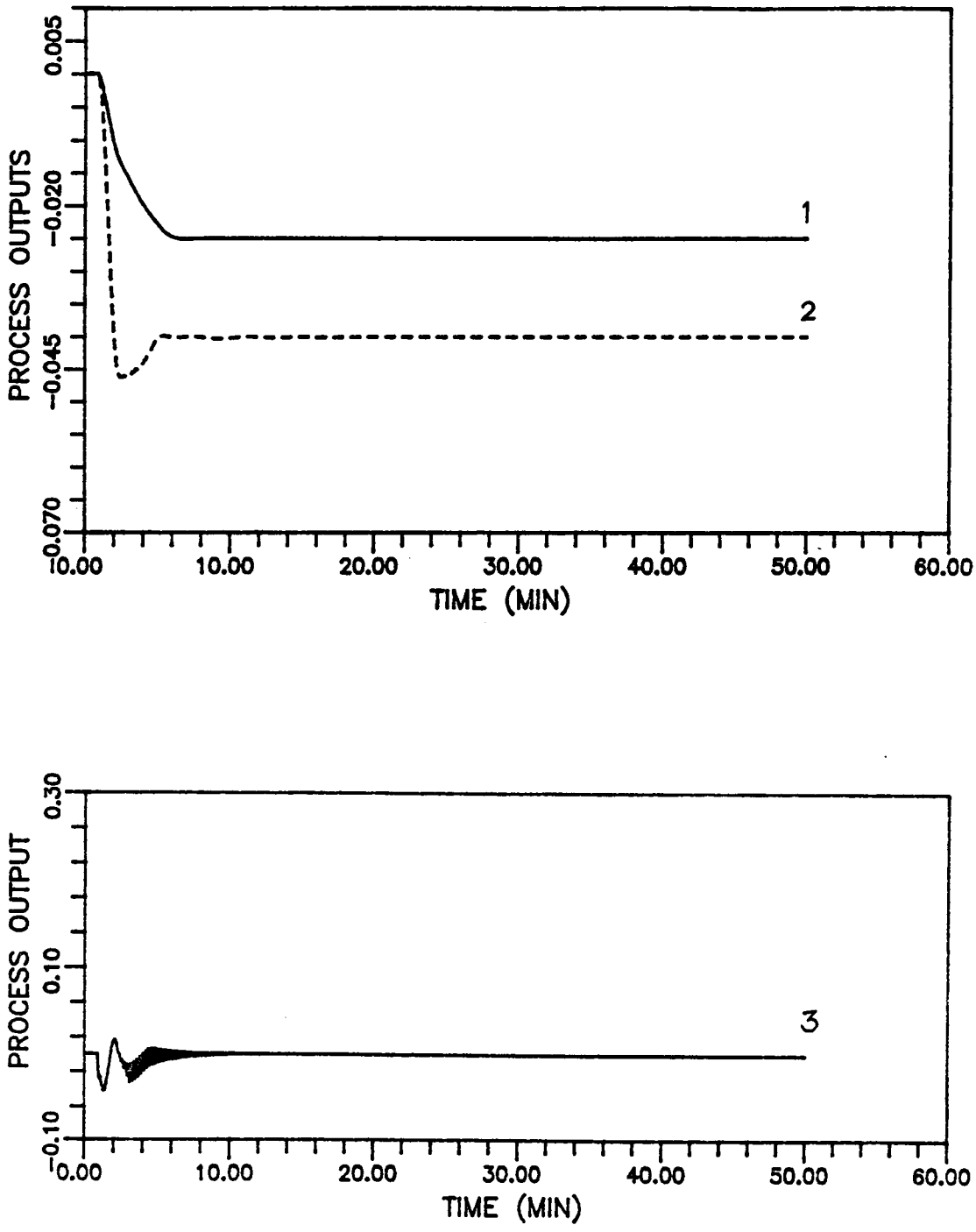


Figure 5.17. System response for MVPIR to simultaneous steps in set points one and two for Model C.

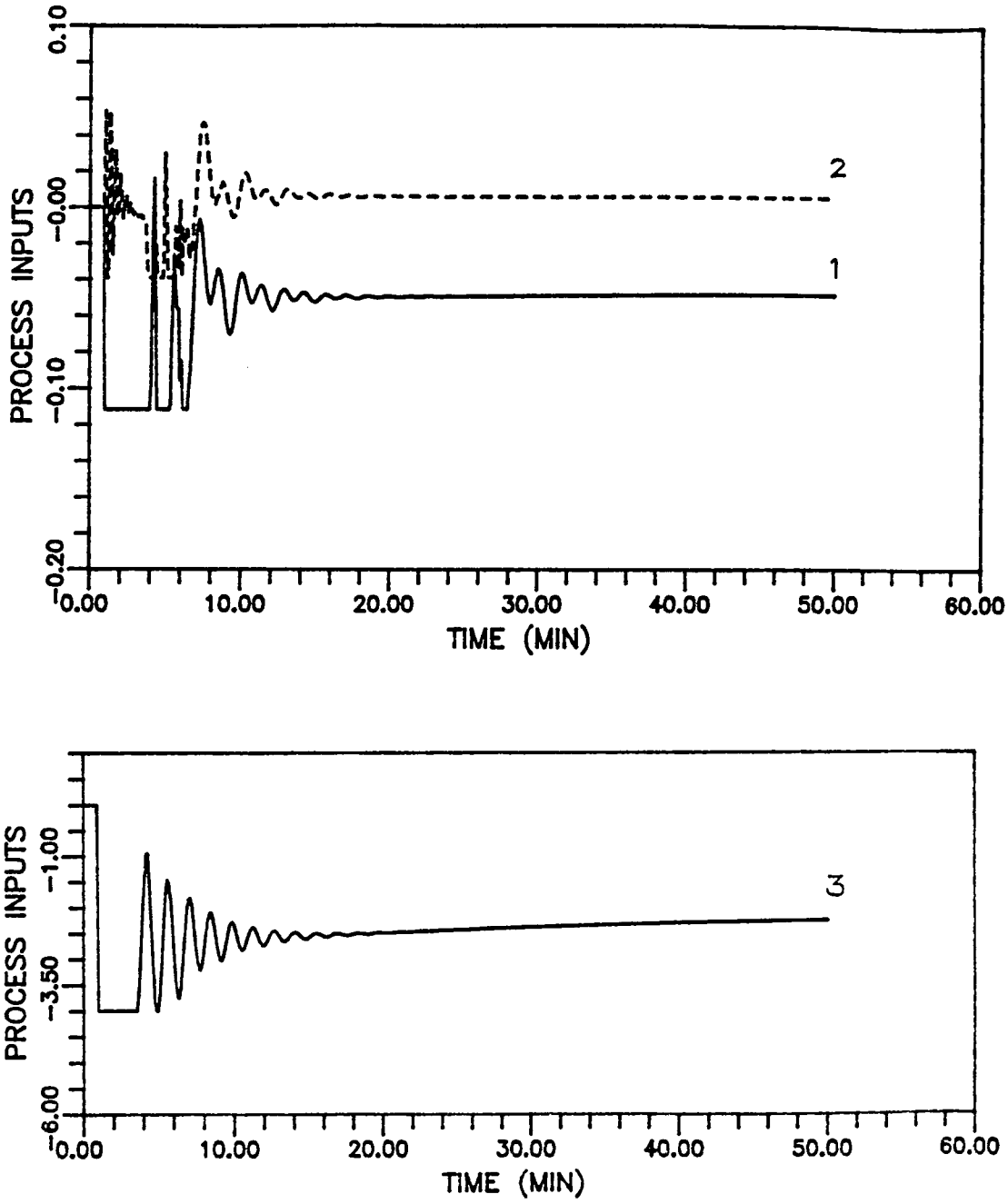


Figure 5.18. Process inputs for SVAC to simultaneous steps in set points one and two for Model C.

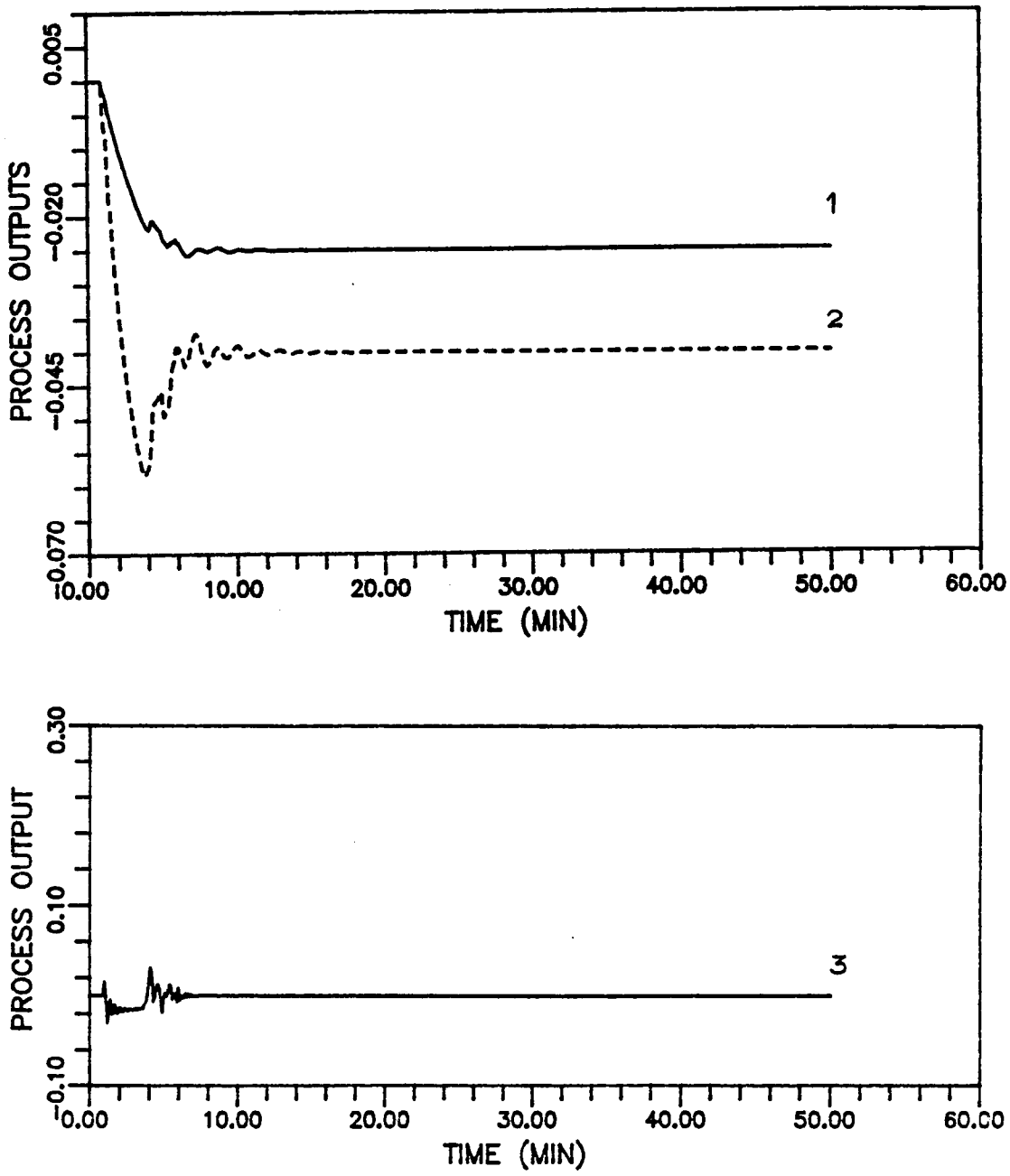


Figure 5.19. System response for SVAC to simultaneous steps in set points one and two for Model C.

control system with input suppression is presented in Figure 5.20 and 5.21. The input suppressions values used were 0.02, 0.01, and 2.0 respectively for process input one, two and three. Comparing Figures 5.14 and 5.15 with Figures 5.20 and 5.21, fluctuations are reduced while the controlled variables show transient behavior about twice as long and process output two shows considerable overshoot. Thus, the system response is more sluggish and overshoot results. The IAE for Figure 5.21 is 16.3 versus 5.2 of table 5.10. A compromise could be made between the high frequency fluctuations in Figure 5.14 and the longer transient shown in Figure 5.21 by adjusting the move suppression values.

Multivariable Control Of Model D

The last model examined in this research is a subsystem of the "light - out - first" distillation column by Alatiqi and Luyben without delays. The basic flowsheet for the "LOF" system is shown in Figure 5.22 and steady state operating conditions along with input constraints are listed in Table 5.11. Since no operational constraints on the inputs are given by the reference, they were defined as ± 20% of the steady state values. Manipulated variables and controlled variables for the system are :

R1 = reflux flow rate of first column, lb mole/hr.

R2 = reflux flow rate of second column, lb mole/hr.

ZF(2) = mole fraction of toluene in feed.

XD1(1) = mole fraction of benzene in distillate of first column.

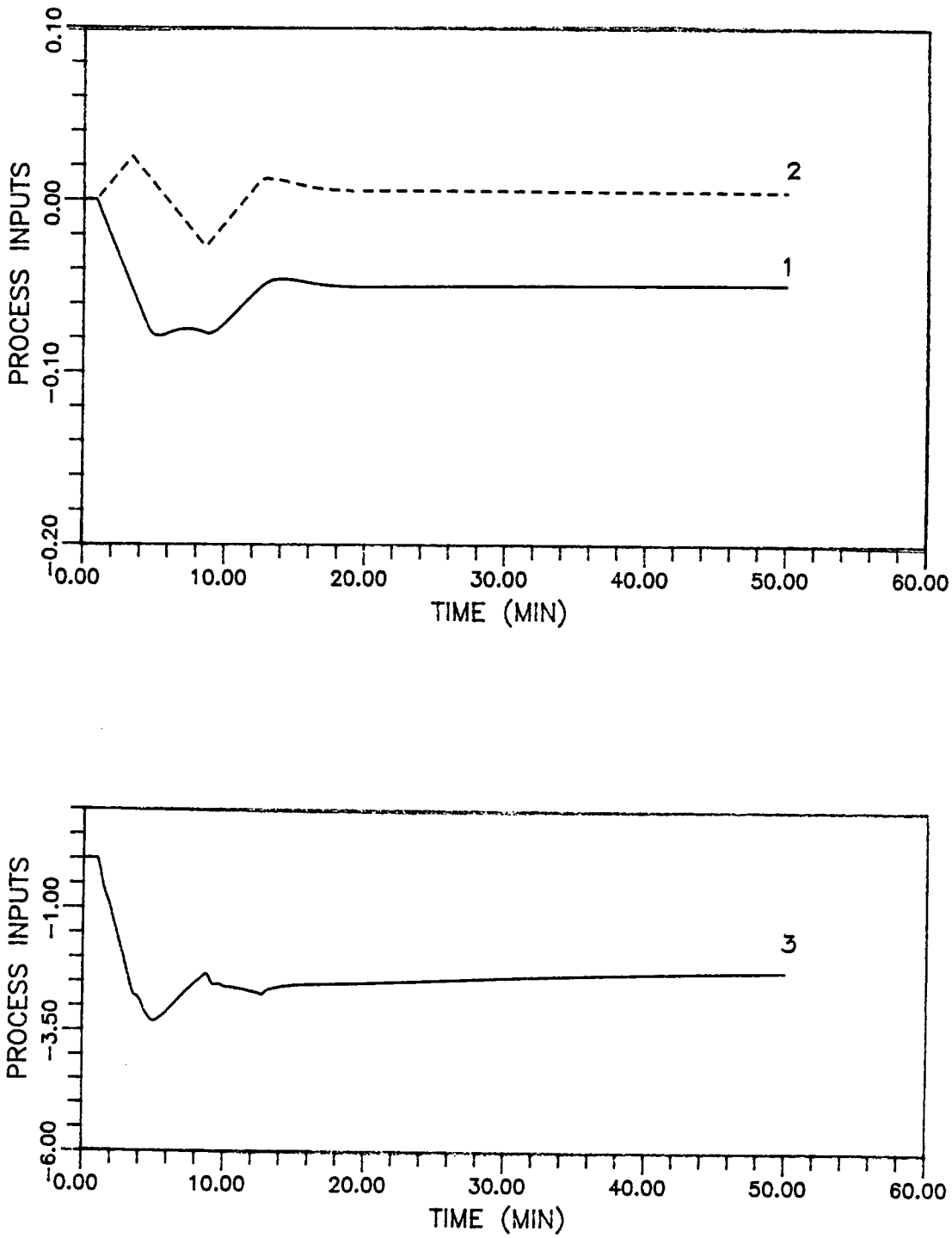


Figure 5.20. Process input for MVSISO to simultaneous steps in set points one and two for Model C with input suppression.

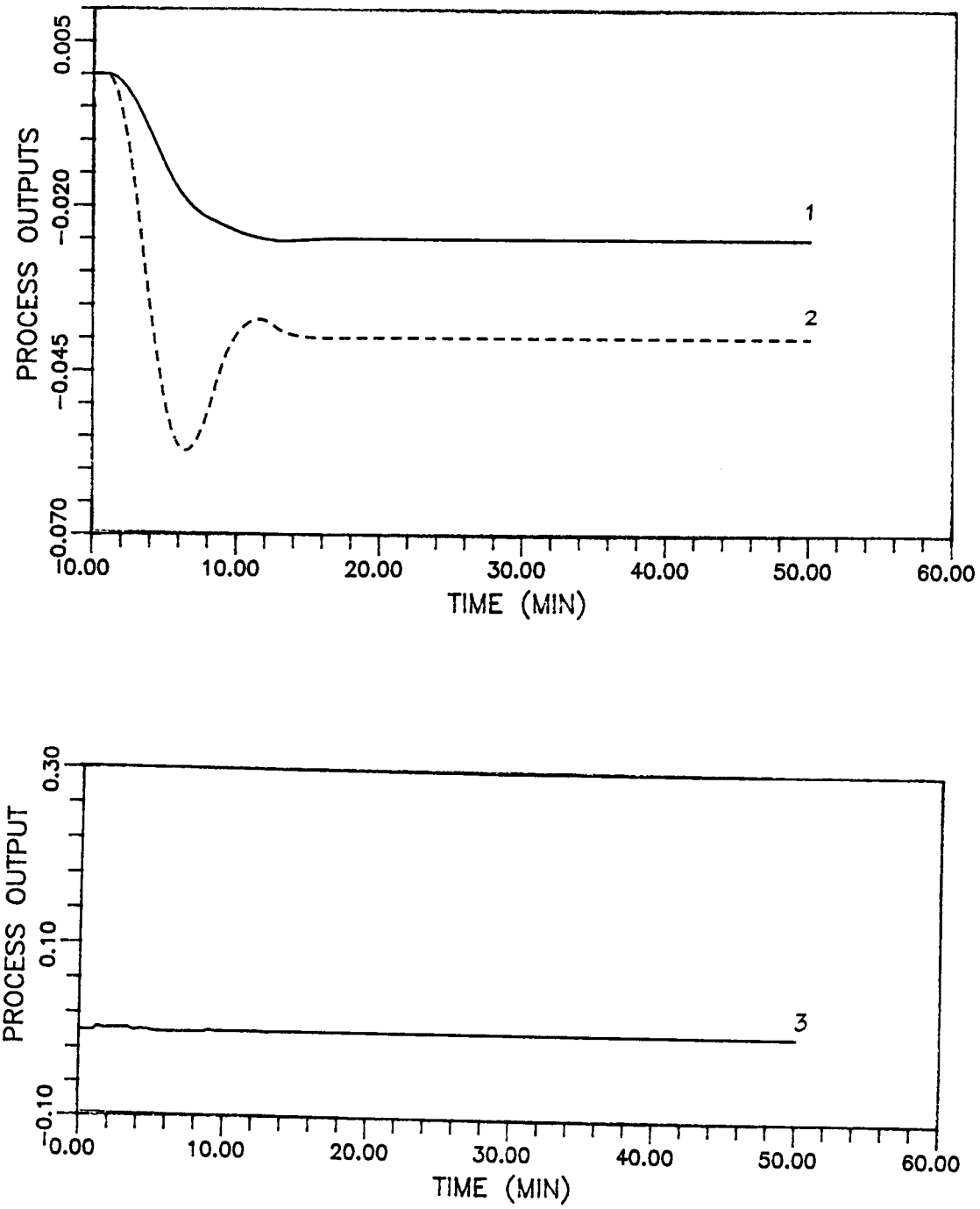


Figure 5.21. System response for MVSISO to simultaneous steps in set points one and two for Model C with input suppression.

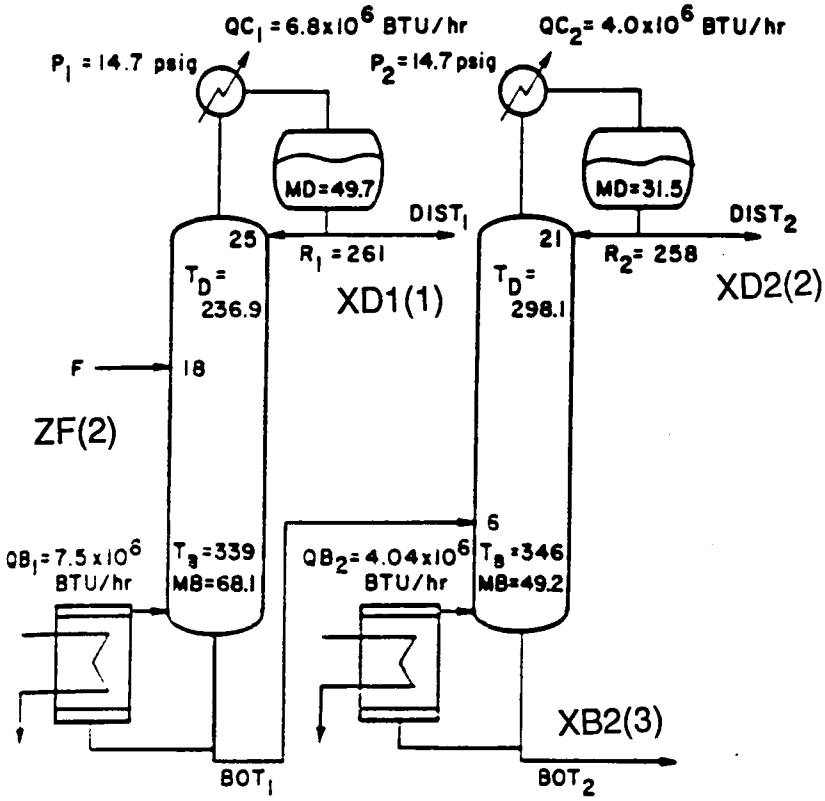


Figure 5.22. Schematic diagram of Alatiqi and Luyben "LOF" system.

Table 5.11. Steady state operating conditions and input constraints of Alatiqi and Luyben model.

Steady State Operating Conditions :

Feed flow rate	600 lb mol/hr
First LOF column reflux flow rate	261 lb mol/hr
Second LOF column reflux flow rate	258 lb mol/hr
Mole fraction of toluene in feed	0.1
Mole fraction of benzene in distillate of first column	0.95
Mole fraction of toluene in distillate of second column	0.90
Mole fraction of xylene in bottom of second column	0.95

System Constraints (as ± 20% from steady state) :

Manipulated Variables	Lower Constraint	Upper Constraint
Reflux flow rate of first column	208.8 lb mol/hr	313.2 lb mol/hr
Reflux flow rate of second column	206.4 lb mol/hr	309.6 lb mol/hr
Mole fraction of toluene in feed	0	0.3

XD2(2) = mole fraction of toluene in distillate of second column.

XB2(3) = mole fraction of xylene in bottom composition of second column.

Table 5.12 shows the results of SVD and RGA for Model D. The main diagonal elements of the RGA which are closest to +1.0 are 0.55, 1.12, and 0.61 which imply that strong interactions exist in loop one and three, where loop one and three, if done by RGA pairing anyway, are R1-XD1(1),and ZF(2) - XB2(2) respectively. Loop two would be R2 - XD2(2). SVA shows there should be good pairing as both the input and output principle directions are dominated by a single process input or output respectively. The pairings are R1 - XD2(2), ZF(2) - XD1(1), and R2 - XB2(2). The SVA pairing are completely different from the RGA pairings. The condition number of the system reduces from 41.7 to 3.87 after the steady state process gain matrix was scaled.

The system was simulated with a sample time of 0.1 hour for a 2% step change in process variable one, and the simulation ran for 700 samples. The system were not simulated for step changes in set point two and three because the RGA of loop two is 1.12, an indication of little interaction, while the result for step in set point three is predicted to be quite similar to the result of step change in set point one. The IAE tuned parameters are listed in Table 5.13, and the dynamic response for a step in set point one are illustrated in Figure 5.23, 5.24, and 5.25.

Figure 5.23 shows the closed loop responses that are obtained when the column is controlled by using two PI controllers in the conventional control configuration. As seen in Figure 5.23, the outputs of

Table 5.12. Interaction analysis and design equations for Model D.

Steady State Gain Array

R1	R2	ZF(2)	
2.7	0	-0.89	XD1(1)
9.0	2.05	3.25	XD2(2)
-1.2	-0.03	-0.48	XB2(3)

Relative Gain Array

R1	R2	ZF(2)	
0.5512	0.0000	0.4488	XD1(1)
-0.0553	1.1160	-0.0606	XD2(2)
0.5042	-0.1160	0.6118	XB2(2)

Niederlinski index = 1.6346

Singular Value Decomposition

Singular values : 10.116, 1.77, 0.242

V^T

R1	R2	ZF(2)
-0.9345	0.3414	-0.1007
-0.1963	-0.2563	0.9459
-0.2969	-0.9038	-0.3084

U

-0.2233	0.9747	-0.0105	XD1(1)
-0.9666	-0.2228	0.1264	XD2(2)
0.1255	-0.0180	0.9919	XB2(3)

Condition number = 41.72

Multivariable Tuning Regulator Matrices

$Kp = (CB)^+$

15.5555	12.5555	0.0000
0.0000	0.0000	0.0000
0.0000	33.8467	0.0000

$KI = -CA^{-1}B$

0.2042	-0.0061	-0.4200
-0.0969	0.5445	3.8663
-0.5043	-0.0187	-1.2746

Table 5.13. Tuning parameters for step change in set point one for Model D.

Design Method	Tuning Parameters			IAE
MVSISO	K_{c1}=104.6	K_{c2}=207.99	K_{c3}=-13.1	0.211
	T_{I1}=0.44	T_{I2}=229.85	T_{I3}=174.24	
MVPIR	P_1=0.642	P_2=19.76	P_3=0.01	0.181
	$\mathcal{E}$ =0.005			
SVAC	Kc_1=22.74	Kc_2=14.25	Kc_3=-0.072	0.11
	T_{I1}=0.49	T_{I2}=0.005	T_{I3}=0.135	

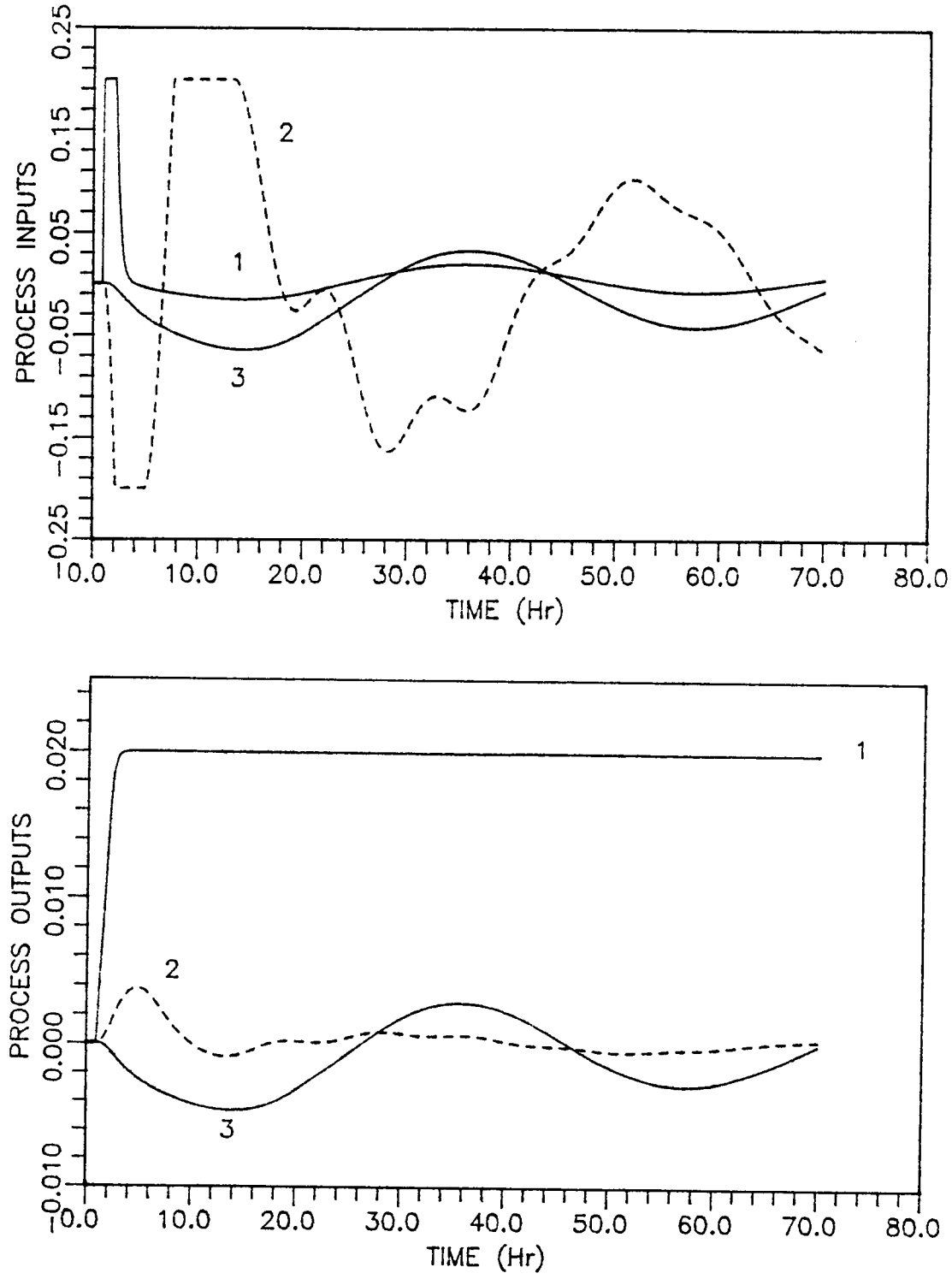


Figure 5.23. Response of Model D to a step in set point one with MVSISO.

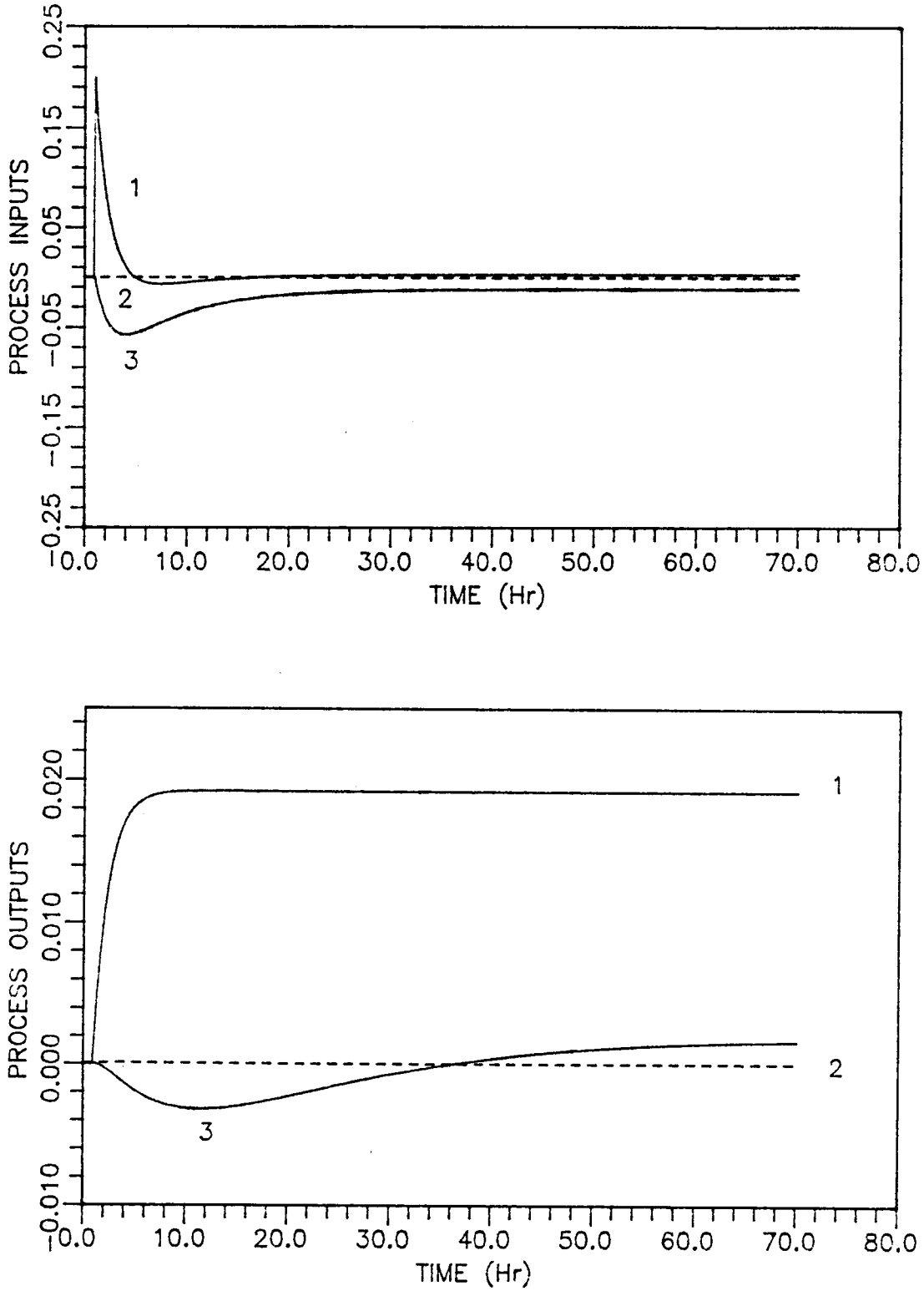


Figure 5.24. Response of Model D to a step in set point one with MVPIR.

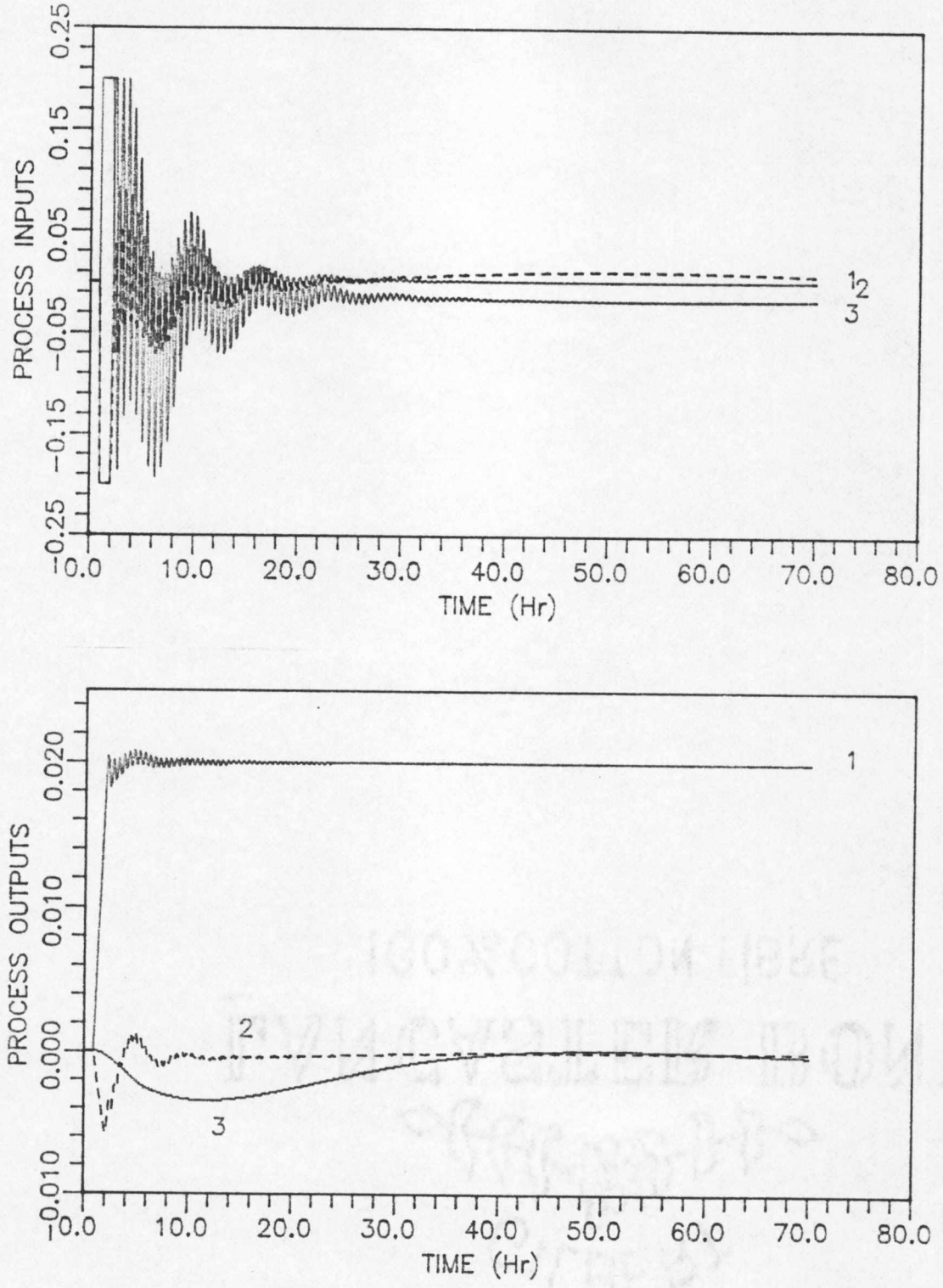


Figure 5.25. Response of Model D to a step in set point one with SVAC.

loop two and three are also influenced when output one is changed indicating considerable interaction. Since the elements in the first row of RGA are 0.55, 0.0, and 0.45, and the elements in the third row are 0.45, -0.06,and 0.61, the RGA interaction prediction is confirmed in this case and the poor performance of MVSISO is expected. For both MVSISO and MVPIR, a longer simulation time will make the control actions smaller, thus the response will be more sluggish. Figure 5.23 and 5.24 show that at the given ending time of 70 hours the IAE value is cut off (the errors are not zero), the IAE will be larger for longer simulation. Despite the considerable transient in process output three, the MVPIR control system completely decouples process output two. The response of SVAC in Figure 5.25 yields the lowest IAE. Besides that, SVAC is capable of eliminating all the steady state errors except a small lingering error in outputswithin 40 hours. However, the manipulated variables from the SVAC exhibit high frequency fluctuations. The fluctuations could be reduced by input suppression or changing the tuning criteria. As in previous model, input suppression means the maximum change of inputs over one sample time is constrainted. A better tuning criteria would be (83)

$$\text{COST} = \int_0^T |E|\, dt + \alpha \int_0^T (dm/dt)^2\, dt \qquad (5.1)$$

where E = error signal

m = process input

α = weighting factor

Table 5.14 listed the tuning parameters for Model D with input suppression and new tuning criteria. The dynamic responses for both cases are shown in Figures 5.26 and 5.27. Comparing Figures 5.25, 5.26, and 5.27, it can be seen that the high frequency fluctuations of the manipulated variables have reduced but the responses have longer transients and more overshoots. For this system, it is obvious that the SVAC with input move suppression or new tuning criteria and the MVPIR control system are superior to the MVSISO.

Summary

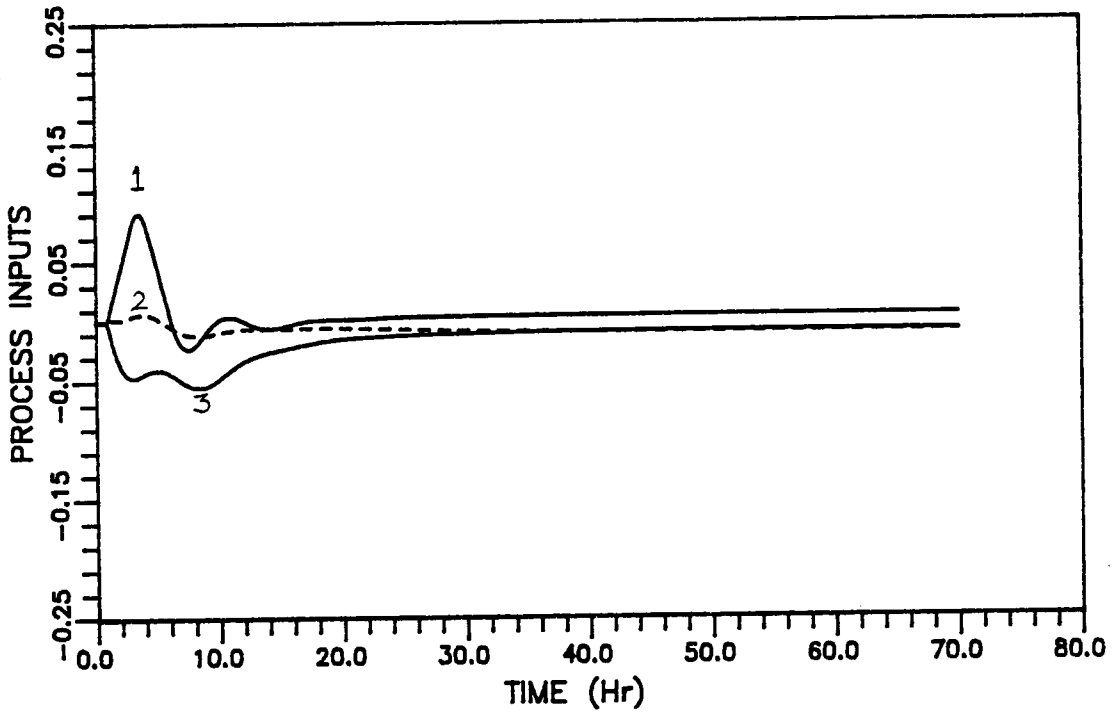
In this chapter, the application of the conventional PI controller, multivariable PI tuning regulator, and singular value analysis controller to several models were compared with respect to set point tracking. All the system were simulated for individual step set point or sequential step set point tests, and the controllers were optimized using PATERN search to minimize IAE.

Pairing between manipulated and controlled variables from RGA was seldom consistent with SVD pairing for the models used in this study. However, strong interaction between loops for both Model B and Model D predicted by RGA was shown in the simulation results.

A simulation study of four distillation columns indicates that the three control schemes under study could successfully control non-interacting systems. However, if considerable interactions exist between loops, the multivariable PI tuning regulator of Koivo has the best

Table 5.14. Comparison of SVAC tuning parameters for step change in set point one for Model D.

Criteria	Tuning Parameters			Cost
IAE	K_{c1}=22.74	K_{c2}=14.25	K_{c3}=0.072	0.11
	T_{I1}=0.49	T_{I2}=0.005	T_{I3}=0.135	
Input Suppression	K_{c1}=47.34	K_{c2}=6.46	K_{c3}=-0.212	0.144
	T_{I1}=3.06	T_{I2}=1.58	T_{I3}=0.85	
IAE and Input Penalty (α = 0.025)	K_{c1}=94.20	K_{c2}=10.03	K_{c3}=-0.129	0.11
	T_{I1}=1.26	T_{I2}=0.76	T_{I3}=0.131	



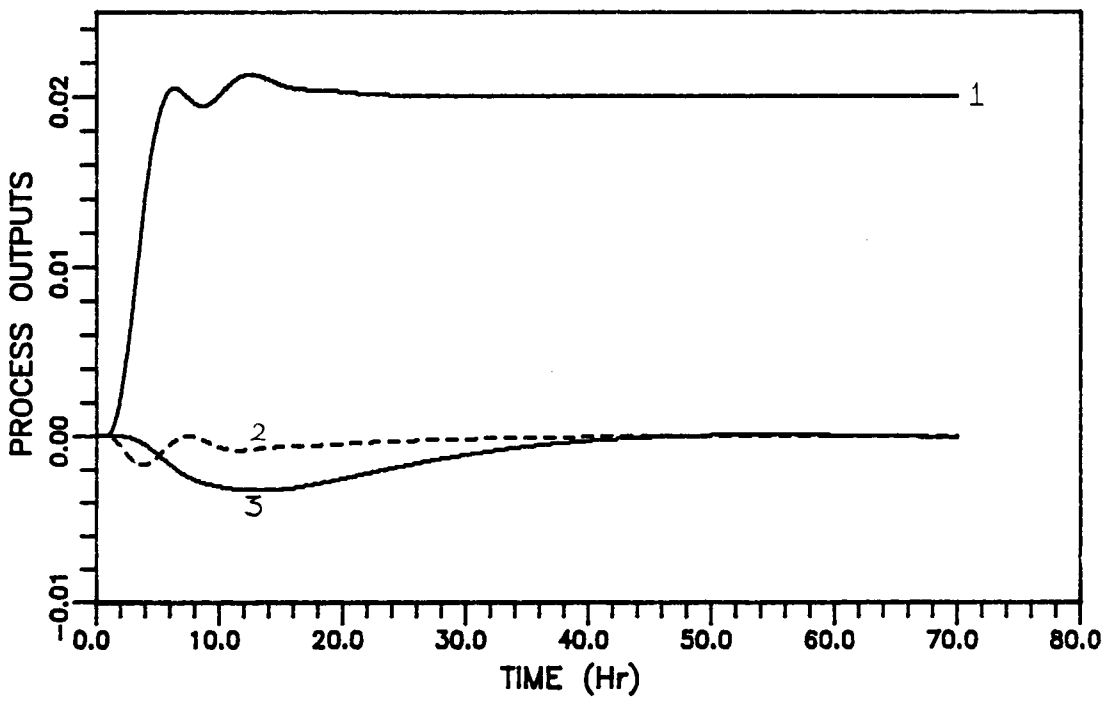


Figure 5.26. System response for SVAC to a step in set point one for Model D with input suppression.

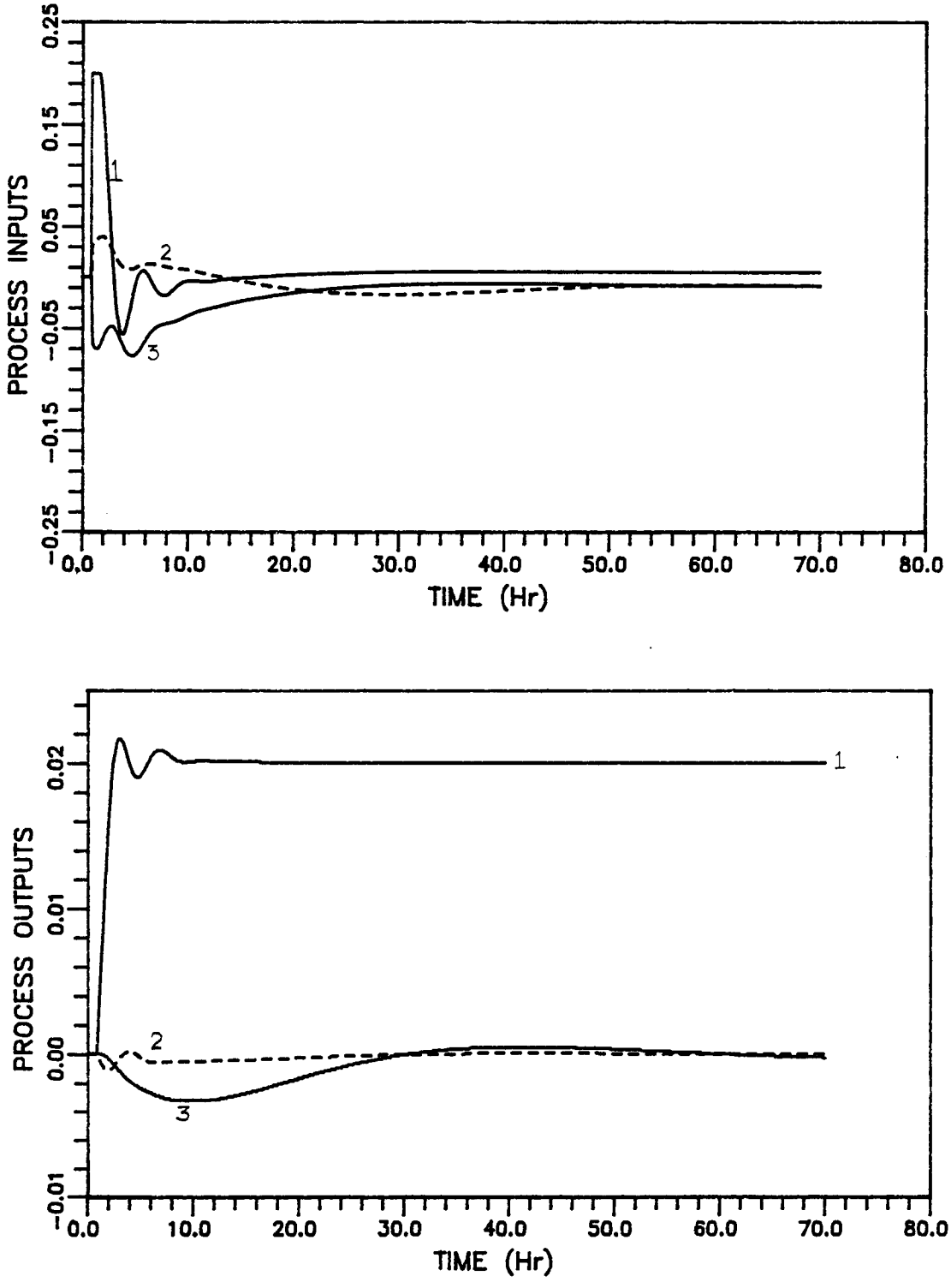


Figure 5.27. System response for SVAC to a step in set point one for Model D with new tuning criteria.

performance, followed by singular value analysis controller. The effectiveness of multivariable single input - single output control system is reduced as the interactions between loops increases. Contrarily, the stronger the interactions, the more useful the multivariable PI tuning regulator.

CHAPTER VI

CONCLUSIONS AND RECOMMEDATIONS

In this research, the performance of three multivariable controllers was evaluated via a simulation study of four example models which were literature distillation column transfer function models neglecting their deadtime. The examples were selected to exhibit various degrees of interaction. Comparision criteria used was optimal tuning for set point tracking obtained from IAE or IAE with a penalty for manipulated variable movement. This chapter will summarize results of this research and propose future work.

As expected the conventional MVSISO control scheme is only efficient for controlling systems without or with little interaction between loops. Although RGA and Niederlinski index are widely used to determine the pairing between manipulated and controlled variables and to indicate closed loop stability of the resulting pairing, the prediction of RGA is not consistent. Even though the SVA gives ambiguous pairing results in the context of the MVSISO control scheme, it is still helpful when used in conjunction with RGA. Moreover, MVSISO control method is only applicable to square process systems. Since only steady state information is employed in designing the control system, this technique does not provide any information regarding dynamic behavior of the control system.

The singular value analysis controller provides a promising approach since it addresses steady state interaction and is thus a truly multivariable controller. The controller is developed using information provided by singular value decomposition of the steady state process gain matrix. As a result, it does not contain any dynamic information.

Compare SVAC to MVSISO, the SVAC has yielded better control for both non-interacting and interacting systems except that it typically shows more oscillatory and longer transient responses than the MVPIR. However, the transient response could possibly be reduced by tuning the orthogonal matrices (54). Unlike MVSISO control, SVAC scheme is applicable to nonsquare process.

The multivariable PI tuning regulator appears to be superior to the conventional PI controller and often better than the singular value analysis controller. In contrast to the other two control designs, the MVPIR control system does include initial dynamic information in developing the system structure. Consequently, the MVPIR shows good transient behavior. In addition, it is simple because it has fewer tuning parameter. For a NxN system, there are N+1 tuning parameter for MVPIR while there are 2N tuning parameter for both MVSISO and SVAC.

Recommendations

The present research has explored only a very small portion of multivariable control. Still many questions can be raised based on the present work. These are listed below :

1. This research only used examples without dead time. The presence of dead time will complicate the control system but it is more realistic, thus a study including deadtime should be an obvious next step.
2. Traditionally, a nonsquare process transfer function matrix is reduced to a square matrix before analysis is undertaken. Since both MVPIR and SVAC are applicable to nonsquare process without any modification, the performance of these two controllers should be evaluated using nonsquare processes.
3. It is predicted that none of the control schemes analyzed in this research will provide satisfactory control when the multivariable controllers are implemented on a sagging process. Since the proportional gain matrix for the MVPIR is estimated from initial open loop response, which is the wrong way behavior part of a sagging process, it will be interesting to obtain the proportional matrix after the wrong way response and test the new estimation for dynamic interaction.
4. All the literature models analyzed in this research are linear and time invariant. However, most of the chemical process systems are nonlinear in nature. So, application of the control schemes to nonlinear processes could be a challenging and promising field to be developed.

5. System stability and controller robustness are critical factors in process control and design. Unfortunately, neither is emphasized or included in this work. The simulations of the tuned controller indicate the closed loop system are stable. It is strongly recommended that system stability and controller robustness be included in future research.
6. The MVPIRdeveloped by Koivo has only one parameter associated with the integral mode. This design method may be enhanced by increasing the number of I-mode parameters from one so that it has 2N tuning parameter for a NxN system instead of N+1 tuning parameters. The extended design may improve the performance of MVPIR for controlling system with wide spread time constants or with vary interaction during transients.
7. The number of terms taken or the time selected during the initial transient response to obtain the best MVPIR proportional gain matrix is ambiguous at this moment. The performance of the MVPIR is affected depending upon which part of the initial response is used; for example it various with the number of terms taken in forming the porprotional matrix. Guidelines to this problem would be an important addition to the design procedure. Such guidelines might be developed by analyzing this aspect in detail.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. MacFarlane, A. G. J., "A Survey of Some Recent Results in Linear Multivariable Feedback Theory," Automatica, 8, 455-492 (1972).

2. Doyle, J. C., and G. Stein, "Multivariable Feedback Design : Concepts for a Classical / Modern Synthesis," IEEE Trans. Auto. Control, AC-26, 4 (1981).

3. Mijares, G., J. D. Cole, N. W. Naugle, H. A. Preisig, and C. D. Holland, " A New Criterion for the Pairing of Control and manipulated Variable," AIChE J., 32 (9), 1439 (1986).

4. Bristol, E. H., "On a New Measure of Interaction for Multivariable Process Control," IEEE Trans. on Auto. Cont., AC-11, 133-136 (1966).

5. Grasdidier, P., M. Morari, and B. R. Holt, "Closed - Loop Properties from Steady State Gain Information," Ind. Eng. Chem. Fund., 24, 221-235 (1985).

6. McAvoy, T. J., Interaction Analysis : Principles and Application, Instruments Society of America, Research Triangle Park, North Carolina (1983).

7. Friedly, J. C., "Use of the Bristol Array in Designing Noninteracting Control Loops. A Limitation and Extension," Ind. Eng. Chem. Process Des. Dev., 23, 469-472 (1984).

8. Witcher, M. F., and T. J. McAvoy, "Interacting Control Systems : Steady State and Dynamic Measurement of Interaction," ISA Trans., 16 (3), 35 (1977).

9. Tung, L. S., and T. F. Edgar, "Analysis of Control Output Interaction in Dynamic Systems," AIChE J., 27 (4), 690 (1981).

10. Gagnepain, J. P., and D. E. Seborg, "An Analysis of Process Interaction with Application to Multiloop Control System Design," Ind. Eng. Chem. Process Des. Dev., (Nov 1986).

11. Jensen, N., D. G. Fisher, and S. L. Shah, "Interaction in Multivariable Process Control Systems," AIChE J., 32 (6), 959 (1986).

12. Roat, S. D., J. J. Downs, E. F. Vogal, and J. E. Doss, "The Integration of Rigorous Dynamic Modeling and Control System Synthesis for Distillation Columns: An Industrial Approach," Control System Engineering Department, Tennessee Eastman Company, Kingsport, Tennessee.

13. Johnson, C. R., "Hadamard Products of Matrices," Linear and Multilinear Algebra, 1, 295-307 (1974).

14. Boksenbom, A. S., and R. Hood, "General Algebraic Method Applied to Control Analysis of Complex Engine Types," Report NCA - TR - 980, National Advisory Committee for Aeronautics, Washington D. C., (1949).

15. Rosenbrock, H. H., "Design of Multivariable Control System Using the Invest Nyquist Array," Proc. IEE., 116, 1929-1936 (1969).

16. MacFarlane, A. G. J., and J. Belletrutti, "Characteristic Loci Techniques in Multivariable Control System Design," Proc IEE., 118, 1291-1297 (1971).

17. MacFarlane, A. G. J., and J. J. Belletratti, "The Characteristic Locus Design Method," Automatica, 9, 575-588 (1973).

18. Athans, M., and P. L. Falb, Optimal Control, McGraw Hill, New York (1986).

19. Rosenbrock, H. H., "Dinstinctive Problems of Process Control," Chem. Eng. Prog., 58 (9), 43-50 (1962).

20. Kalman, R. E., "Design of a Self - Optimizing Control System," Trans. ASME, 80, 468 (1958).

21. Rijnsdorp, J. E., and D. E. Seborg, "A Survey of Experimental Applications of Multivariable Control to Process Control Problem," Chemical Process Control, 72 (159), 112-123 (1976).

22. Ray, W. H., "Multivariable Process Control - A Survey," Computer and Chemical Engineering, 7 (4), 367-394 (1983).

23. Niederlinski, A., "A Heuristic Approach to the Design od Linear Multivariable Control Systems," Automatica, 7, 691 (1971).

24. Kavangh, R. S., "Noninteracting Control in Linear Multivariable Systems," Trans. AIEE, 74, 95-100 (1957).

25. Luyben, W. C., "Distillation Decoupling," AIChE J., 16 (2), (1970).

26. Zalkind, C. S., "Practical Approach to Noninteracting Control," ISA, 40 (89), 111 (1967).

27. Weischedel, K., and T. J. McAvoy, "Feasibility of Decouplinf in Conventionally Controlled Distillation Columns," Ind. Eng. Chem. Fund., 19, 379-384 (1980).

28. Siljak, D. D., "A Note on the Generalized Nyquist Criterion," IEEE Trans. Aut. Cont., AC-11, (April 1966).

29. McMorran, P. D., "Extension of the Inverse Nyquist Array," Electronics Letters, 6, 800-801 (1970).

30. McMorran, P. D., "Parameter Sensitivity and Inverse Nyquist Method," Proc. IEE, 118, 802 (1971).

31. Edgar, T. F., "Status of Design Methods for Multivariable Control," AIChE Syposium Series, Chem. Proc. Cont., 72 (159), (1976).

32. Rosenbrock, H. H., and P. A. Cook, "The Stability and the Eigenvalues of G(s)," Int. J. Control, 21, 99-104 (1975).

33. Edmunds, J., and B. Kouvaritakis, "Extensions of the Frame Alignment Technique and Their use in the Characteristic Locus Design Method," Int. J. Control, 29, 787-796 (1979).

34. Soliman, M. A., and W. H. Ray, "Optimal Control of Multivariable Systems with Pure Time Delays," Automatica, 7, 681, (1971).

35. Shih, P., and C. J. Chen, "On the Weighting Factors of the Quadratic Criterion in Optimal Control," Int J. Control, 19, 947 (1974).

36. Ray, W. H., and J. Szekely, Process Optimization, John Willey and Sons, New York (1973).

37. Ray, W. H., Advanced Process Control, McGraw Hill, New York (1981).

38. Novin - Hirbod, S., "Pole Assignment Using Proportional - Integral Output Feedback Control," Int. J. Control, 29, 1035 (1979).

39. Ramaz, K., and V. Gourishankar, "Algorithm for Closed Loop Pole Assignment With Constant Gain Output Feedback," Proc IEE, 122, 574 (1975).

40. Shapiro, E. Y., D. A. Fredricks, R. H. Rooney, and B. R. Barmish, "Pole Placement with Output Feedback," Joint Aut. Cont. Conf., WP1-C, (Aug. 1980).

41. Astrom, K. J., "Theory and Application of Adaptive Control - A Survey," Automatica, 19, 471 (1983).

42. Iserman, R., "Parameter Adaptive Control Algorithms - A tutorial," Automatica, 18, 513 (1982).

43. Wellstead, P. E., and P. M. Zanker, "Techniques of Self - Tuning," Optimal Cont. Appl. Meth., 3, 305 (1982).

44. Seborg, D. E., T. F. Edgar, and S. C. Shah, "Adaptive Control Strategies for Process Control : A Survey," AIChE J., 32 (6), (1986).

45. Mehra, R. K., A. Rault, and J. G. Reid, "Model Algorithmic Control : Theoretical Results on Robustness," Proceedings of Joint Auto. Control Conference, 387 (1979).

46. Mehra, R. K., R. Rouhani, and L. Praly, "New Theoretical Development in Multivariable Predictive Algorithmic Control," Proceedings of Joint Automatic Control Conference, F49-B (1980).

47. Mehra, R. K., R. Rouhani, J. Eterns, J. Richalet, and A. Raults, "Model Algorithmic Control : Review and Recent Development," Proceedings Chemical Process Control II Conference, (1981).

48. Cutler, C. R., "Dynamic Matrix Control of Imbalanced System," ISA Trans., 21 (1), (1982).

49. Cutler, C. R., and B. L. Ramaker, "Dynamic Matrix Control - A Computer Algorithm," 86th National Meeting AIChE (1979).

50. Economou, C., and M. Morari, "Internal Model Control : Multiloop Design," Ind. Eng. Chem. Process Des. Dev., 25, 411 (1986).

51. Garcia, C. E., and M. Morari, "Internal Model Control 1," Ind. Eng. Chem. Process Des. Dev., 21, 308 (1982).

52. _____, "Internal Model Control 2," Ind. Eng. Process Des. Dev., 24, 472 (1985a).

53. _____, "Internal Model Control 3," Ind. Eng. Process Des. Dev., 24, 484 (1985b).

54. Smith, C. R., "Multivariable Control Using Singular Value Analysis Processing," Ph. D. Dissertation, University of Tennessee, Knoxville, Tennessee (1981).

55. Gerstle, J. G., J. M. Keeton, et al., "Singular Value Decomposition Controller Design Using a Reduced Order Model," Department of Chemical Engineering, Applied Control Lab., University of Tennessee, Knoxville, Tennessee (1984).

56. Moore, C. F., "A Reliable Distillation Column Control Analysis Procedure for Using Initial Column Design," Annual Meeting of AIChE (1985).

57. Shimizu, K., and M. Matsubara, "Singular Value Analysis for the Robust Control System Design of Distillation System," J. Chem. Eng. Japan, 18 (6), 533 (1985).

58. Lau, H., J. Alvarez, and K. F. Jensen, "Synthesis of Control Structure by Singular Value Analysis Dynamic Measures of Sensitivity and Interaction," AIChE J., 31, 427 (1985).

69. Lau, H., and K. F. Jensen, "Evaluation of Changeover Control Policies by Singular Value Analysis : Effects of Scaling," AIChE J., 31, 135 (1985).

60. Smith, C. R., and D. D. Bruns, "Singular Value Analysis : A Geometrical Structure for Multivariable Process Control," AIChE National Meeting, Orlando, FL. (1982).

61. Noble, B., and J. W. Daniel, Applied Linear Algrebra, 2nd ED., Prentice Hall, Englewood Clifs, NJ. (1977).

62. Strang, G., Linear Algebra and Its application, Academic Press, New York (1976).

63. Bonvin, D., and D. A. Mellichamp, "A Scaling Procedure for the Structure and Interaction Analysis of Dynamic Models," AIChE J., 33 (2), 250-257 (1987).

64. Bauer, F. L., "Optimal Scaled Matrix," Numerical Mathematics, 5, 73-87 (1963).

65. Tomlin, J. A., "On Scaling Linear Programming Problems," Mathematical Programming Study, 4, 146-166 (1975).

66. Prasad, J., "Singular Value Analysis and Scaling : Application for Multivariable Process Design and Control," Ph. D. Dissertation, University of Tennessee, Knoxville, Tennessee (1984).

67. Dyer, C. W., "The Effects of Matrix Scaling on the Design and Control of Multivariable Processes," M. S. Thesis, University of Tennessee, Knoxville, Tennessee (1987).

68. IMSL, International Mathematical and Statistical Libraries, Inc., NBC Bldg., 7500 Bellaire Blvd., Houston, TX.

69. Smith, B. J., Matrix Eigensystem Routines - Eispac Guide, Springer - Verlag, New York (1970).

70. Dongarra, J. J., J. R. Bunch, C. B. Moler, and G. W. Stewart, Linpack User's Guide, Philadelphia, PA SIAM (1979).

71. Penttinen, J., and H. N. Koivo, "Multivariable Tuning Regulator for Unknown Systems," Automatica, 16, 393-398 (1980).

72. Peltomaa, A., and H. N. Koivo, "Tuning of a Multivariable Discrete Time PI Controller for Unknown Systems," Int J. Control, 38, 735-745 (1983).

73. Davision, E. J., "Multivariable Tuning Regulator : The Feedforward and Robust Control of a General Servo Mechanism Problem," IEEE Trans. Auto. Contr., AC-21, 35-47 (1976).

74. Brogan, W. L., Modern Control Theory, Quantum Publishers Inc., New York, (1974).

75. Ogunnaike, B. A., "Control Systems Design for Multivariable Systems with Multiple Time Delays," Ph.D. Dissertation, University of Wisconsin, Madison, Wisconsin (1981).

76. Ward, R. C., " Numerical Computation of the Matrix Exponential with Accuracy Estimation," SIAM J. Numer. Anal., 14, 600-610 (1977).

77. Bruns, D. D., J. G. Gerstle, and C. R. Smith, ACL Report 8601, Department of Chemical Engineering, University of Tennessee, Knoxville, TN. 37996-2200.

78. Lien, C. Y., "Linear Techniques for Dynamic Chemical Process Simulation and Direct Digital Control Algorithm Implementation," M. S. Thesis, University Of Tennessee, Knoxville, Tennessee (1986).

79. Moore, C. F., C. L. Smith, and P. W. Murill, "Multi-dimensional Optimization Using Patern Search," IBM Share Library, PID No. 6448 (1968).

80. Wood, R. K., and M. W. Berry, "Terminal Composition Control of a Binary Distillation Column," Chem. Eng. Sci., 28, 1707-1717 (1973).

81. Ogunnaike, B. A., J. P. Lemaire, M. Morari, and W. H. Ray, "Advanced Multivariable Control of a Pilot - Plant Distillation Column," AIChE J., 29, 632-640 (1983).

82. Alatiqi, I. M., and W. L. Luyben, "Control of a Complex Sidestream Column / Stripper Distillation Configuration," Ind. Eng. Chem. Process Des. Dev., 25, 762-767 (1986).

83. Smith, C. L., Digital Computer Process Control, International Textbook Co., Scranton, PA. (1972).

APPENDICES

APPENDIX A

AN EXAMPLE OF MINIMUM REALIZATION

Minimum Realization Of Model B

An outline of the method of minimum realization for a state space model from transfer function model was given in Chapter 4. This appendix illustrates the procedure by developing a state space model for Model B.

Consider the transfer function,

$$\underline{G}(S) = \begin{bmatrix} \dfrac{0.471}{(30.7S+1)^2} & \dfrac{-0.495}{(28.5S+1)^2} \\ \dfrac{0.749}{(57.0S+1)^2} & \dfrac{-0.832}{(50.5S+1)^2} \end{bmatrix} \qquad (A.1)$$

If one lets

$$\underline{\Phi}_1(S) = \begin{bmatrix} \dfrac{0.471}{(30.7S+1)^2} & 0 \\ 0 & 0 \end{bmatrix} ; \quad \underline{\Phi}_2(S) = \begin{bmatrix} 0 & \dfrac{-0.495}{(28.5S+1)^2} \\ 0 & 0 \end{bmatrix}$$

$$\underline{\Phi}_3(S) = \begin{bmatrix} 0 & 0 \\ \dfrac{0.749}{(57.0S+1)^2} & 0 \end{bmatrix} ; \quad \underline{\Phi}_4(S) = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{-0.832}{(50.5S+1)^2} \end{bmatrix} \qquad (A.2)$$

It is evident that

$$\underline{G}(S) = \underline{\Phi}_1(S) + \underline{\Phi}_2(S) + \underline{\Phi}_3(S) + \underline{\Phi}_4(S) \qquad (A.3)$$

is true. Now expand $\underline{\Phi}_i(s)$ by partial fraction expansion.

(I). The non-zero element of $\underline{\phi}_1(s)$ is

$$g_{11}(s) = \frac{0.471}{(30.7s + 1)^2}$$

Notice that the pole at 0.03257 repeated twice. In order to have a diagonal A matrix, $g_{11}(s)$ is approximated with two distinct poles, 0.03237 and 0.03277.

$$g_{11}(s) = \frac{0.471}{942.49\,(s + 0.03237)\,(s + 0.03277)}$$

so,

$$\underline{\phi}_1(S) = \frac{\underline{M}_1}{(S + 0.03237)} + \frac{\underline{M}_2}{(S + 0.03277)} \qquad (A.4)$$

where

$$\underline{M}_1 = \begin{bmatrix} 1.24935 & 0 \\ 0 & 0 \end{bmatrix} \; ; \quad \underline{M}_2 = \begin{bmatrix} -1.24935 & 0 \\ 0 & 0 \end{bmatrix}$$

The error in the characteristic equation introduced by the approximation of poles is 0.0002 as

$$(30.7S + 1)^2 = 942.49S^2 + 61.4S + 1$$

is compared to

$$942.49\,(S + 0.03237)\,(S + 0.03277) = 942.49S^2 + 61.4S + 0.9998$$

(II). The non-zero element of $\underline{\phi}_2(s)$ is

$$g_{12}(s) = \frac{-0.495}{(28.5s + 1)^2} = \frac{-0.495}{812.25\,(s + 0.03509)^2}$$

By similar approximation as used in (I),

$$g_{12}(s) = \frac{-0.495}{812.25\,(s + 0.03489)\,(s + 0.03529)}$$

and

$$\underline{\phi}_2(S) = \frac{\underline{M}_3}{(S + 0.03489)} + \frac{\underline{M}_4}{(S + 0.03529)} \qquad (A.5)$$

with

$$\underline{M}_3 = \begin{bmatrix} 0 & -1.52355 \\ 0 & 0 \end{bmatrix} \quad ; \quad \underline{M}_4 = \begin{bmatrix} 0 & 1.52355 \\ 0 & 0 \end{bmatrix}$$

(III). The nonzero element of $\underline{\phi}_3(s)$ is

$$g_{21}(s) = \frac{0.749}{(57.0s + 1)^2} = \frac{0.749}{3249\,(s + 0.01754)^2}$$

The denominator approximation leads to

$$g_{21}(s) = \frac{0.749}{3249\,(s + 001734)\,(s + 0.01774)}$$

and

$$\underline{\emptyset}_3(S) = \frac{\underline{M}_5}{(S + 0.01734)} + \frac{\underline{M}_6}{(S + 0.01774)} \qquad (A.6)$$

where

$$\underline{M}_5 = \begin{bmatrix} 0 & 0 \\ 0.57633 & 0 \end{bmatrix} \quad ; \quad \underline{M}_6 = \begin{bmatrix} 0 & 0 \\ -0.57633 & 0 \end{bmatrix}$$

(IV). The non-zero element of $\underline{\emptyset}_4(s)$ is

$$g_{22}(s) = \frac{-0.832}{(50.5s + 1)^2} = \frac{-0.832}{2550.25\,(s + 0.01980)^2}$$

and again approximation to give distinct poles will give

$$g_{22}(s) = \frac{-0.832}{2550.25\,(s + 0.01960)\,(s + 0.0020)}$$

so,

$$\underline{\emptyset}_4(S) = \frac{\underline{M}_7}{(S + 0.0196)} + \frac{\underline{M}_8}{(S + 0.0200)} \qquad (A.7)$$

and

$$\underline{M}_7 = \begin{bmatrix} 0 & 0 \\ 0 & -0.81561 \end{bmatrix} \quad ; \quad \underline{M}_8 = \begin{bmatrix} 0 & 0 \\ 0 & 0.81561 \end{bmatrix}$$

By re-expressing Equation A.3 in terms of $\underline{M}_i$,

$$\underline{G}(S) = \frac{\underline{M}_1}{(S + 0.03237)} + \frac{\underline{M}_2}{(S + 0.03277)} + \frac{\underline{M}_3}{(S + 0.03489)} + \frac{\underline{M}_4}{(S + 0.03529)} + \frac{\underline{M}_5}{(S + 0.01734)} + \frac{\underline{M}_6}{(S + 0.01774)} + \frac{\underline{M}_7}{(S + 0.01960)} + \frac{\underline{M}_8}{(S + 0.0200)} \qquad (A.8)$$

With each $\underline{M}_i$ matrix rewritten as a product of a column vector and a row vector will yield the following :

$$\underline{G}(S) = \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1.24935 & 0 \end{bmatrix}}{(S + 0.03237)} + \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1.24935 & 0 \end{bmatrix}}{(S + 0.03277)} + \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & -1.52355 \end{bmatrix}}{(S + 0.03489)} + \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1.52355 \end{bmatrix}}{(S + 0.03529)} +$$

$$\frac{\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0.57633 & 0\end{bmatrix}}{(S+0.01734)} + \frac{\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}-0.57633 & 0\end{bmatrix}}{(S+0.01774)} +$$

$$\frac{\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & -0.81561\end{bmatrix}}{(S+0.01960)} + \frac{\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & 0.81561\end{bmatrix}}{(S+0.0200)} \qquad (A.9)$$

Recalling Equation 4.23

$$\underline{G}(S) = \sum_{i=1}\sum_{j=1} \frac{\underline{c}_{ij}\,\underline{b}_{ij}^{T}}{(S+\lambda_i)}$$

and that $\underline{A}$ is a diagonal matrix with $-\lambda_i$ as its elements, $\underline{B}$ consists of rows of $\underline{b}_{ij}^{T}$, and $\underline{C}$ is a matrix made up of column vectors, $\underline{c}_{ij}$. Clearly, the $\underline{A}$, $\underline{B}$, and $\underline{C}$ state space matrices are

$$\underline{A} = \text{diag}\,[\,-0.03237, -0.03277, -0.03489, -0.03529, -0.01734, -0.01774, -0.01960, -0.0200\,]$$

$$\underline{B} = \begin{bmatrix} 1.24935 & 0 \\ -1.24935 & 0 \\ 0 & -1.52355 \\ 0 & 1.52355 \\ 0.57633 & 0 \\ -0.57633 & 0 \\ 0 & -0.81561 \\ 0 & 0.81561 \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

APPENDIX B

STATE SPACE MODELS

Wood and Berry Model

Transfer function :

$$\begin{bmatrix} \dfrac{12.8e^{-1.0s}}{16.7s+1} & \dfrac{-18.9e^{-3.0s}}{21.0s+1} \\ \dfrac{6.6e^{-7.0s}}{10.9s+1} & \dfrac{-19.4e^{-3.0s}}{14.4s+1} \end{bmatrix}$$

State space model :

$$A = \text{diag}\,[\,-0.05988, -0.04762, -0.09174, -0.06944\,]$$

$$B = \begin{bmatrix} 0.7664 & 0.0 \\ 0.0 & -0.9000 \\ 0.6055 & 0.0 \\ 0.0 & -1.3472 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

	Rank	Condition number
$\underline{U}=[\,\underline{B}, \underline{AB}, \underline{A^2B}, \underline{A^3B}\,]$	4	106.3
$V=[\,\underline{C}, \underline{AC}, \underline{A^2C}, \underline{A^3C}\,]$	4	162.96

Weischedel and McAvoy Column C

Transfer function :

$$\begin{bmatrix} \dfrac{0.471e^{-1.0s}}{(30.7s+1)(30.7s+1)} & \dfrac{-0.495e^{-2.0s}}{(28.5s+1)(28.5s+1)} \\ \dfrac{0.749e^{-1.7s}}{(57.0s+1)(57.0s+1)} & \dfrac{-0.832e^{-1.0s}}{(50.5s+1)(50.5s+1)} \end{bmatrix}$$

State space model :

$$A = \text{diag}\,[\,-0.03237, -0.03277, -0.03489, -0.03529, -0.01734, -0.01774, -0.01960, -0.02000\,]$$

$$B = \begin{bmatrix} 1.24935 & 0.0 \\ -1.24935 & 0.0 \\ 0.0 & -1.52355 \\ 0.0 & 1.52355 \\ 0.57633 & 0.0 \\ -0.57633 & 0.0 \\ 0.0 & -0.81561 \\ 0.0 & 0.81561 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

	Rank	Condition number
$\underline{U}=[\,\underline{B}, \underline{AB}, \ldots, \underline{A^7B}\,]$	8	7.307×10^7
$\underline{V}=[\,\underline{C}, \underline{AC}, \ldots, \underline{A^7C}\,]$	8	2.051×10^9

Ogunnaike and Ray Model

Transfer function :

$$\begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-1.0s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3.0s}}{5.0s+1} & \frac{-0.012e^{-1.25s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)e^{-1.0s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}$$

State space model :

$$A = \text{diag}\,[\,-0.1493, -0.1157, -0.1103, -0.3077, -0.2002, -0.1281, -0.1227, -0.0917, -0.2571, -0.0532\,]$$

$$B = \begin{bmatrix} 0.0984 & 0.0 & 0.0 \\ 0.0 & -0.0708 & 0.0 \\ 0.0 & 0.0 & -0.0005 \\ 0.3412 & 0.0 & 0.0 \\ 0.0 & -0.4718 & 0.0 \\ 0.0 & 0.0 & -0.0015 \\ -4.2552 & 0.0 & 0.0 \\ 0.0 & 4.2385 & 0.0 \\ 0.0 & 0.0 & 0.1157 \\ 0.0 & 0.0 & 0.0223 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

	Rank	Condition number
$\underline{U}=[\,\underline{B}, \underline{AB}, \ldots, \underline{A^9B}\,]$	10	5.6797×10^7
$\underline{V}=[\,\underline{C}, \underline{AC}, \ldots, \underline{A^9C}\,]$	10	1.4231×10^4

Alatiqi and Luyben Model

Transfer function :

$$\begin{bmatrix} \frac{2.7e^{-105s}}{42s+1} & 0 & \frac{-0.89e^{-24s}}{38s+1} \\ \frac{9.0e^{-23s}}{(33s+1)(33s+1)} & \frac{2.05e^{-3s}}{(38s+1)(25s+1)} & \frac{3.25e^{-24s}}{110s+1} \\ \frac{-1.2e^{-3.8s}}{(19s+1)(19s+1)} & \frac{-0.03e^{-4s}}{(9.5s+1)(9.5s+1)} & \frac{-0.48e^{-11s}}{(32s+1)(32s+1)} \end{bmatrix}$$

State space model :

A = diag [-0.02381, -0.02632, -0.03050, -0.03010, -0.02632,
-0.04000, -0.00909, -0.05243, -0.05283, -0.10546,
-0.10506, -0.03145, -0.03105]

$$B = \begin{bmatrix} 0.064286 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.023421 \\ -20.6612 & 0.0 & 0.0 \\ 20.6612 & 0.0 & 0.0 \\ 0.0 & 0.157741 & 0.0 \\ 0.0 & -0.157741 & 0.0 \\ 0.0 & 0.0 & 0.029545 \\ -8.31025 & 0.0 & 0.0 \\ 8.31025 & 0.0 & 0.0 \\ 0.0 & 0.83103 & 0.0 \\ 0.0 & -0.83103 & 0.0 \\ 0.0 & 0.0 & 1.17188 \\ 0.0 & 0.0 & -1.17188 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

	Rank	Condition number
$\underline{U}=[\,\underline{B},\,\underline{AB},\,\ldots,\,\underline{A}^{12}\underline{B}\,]$	13	$1.4095\text{x}10^{10}$
$\underline{V}=[\,\underline{C},\,\underline{AC},\,\ldots,\,\underline{A}^{12}\underline{C}\,]$	13	$1.0612\text{x}10^{10}$

APPENDIX C

COMPUTER PROGRAMS

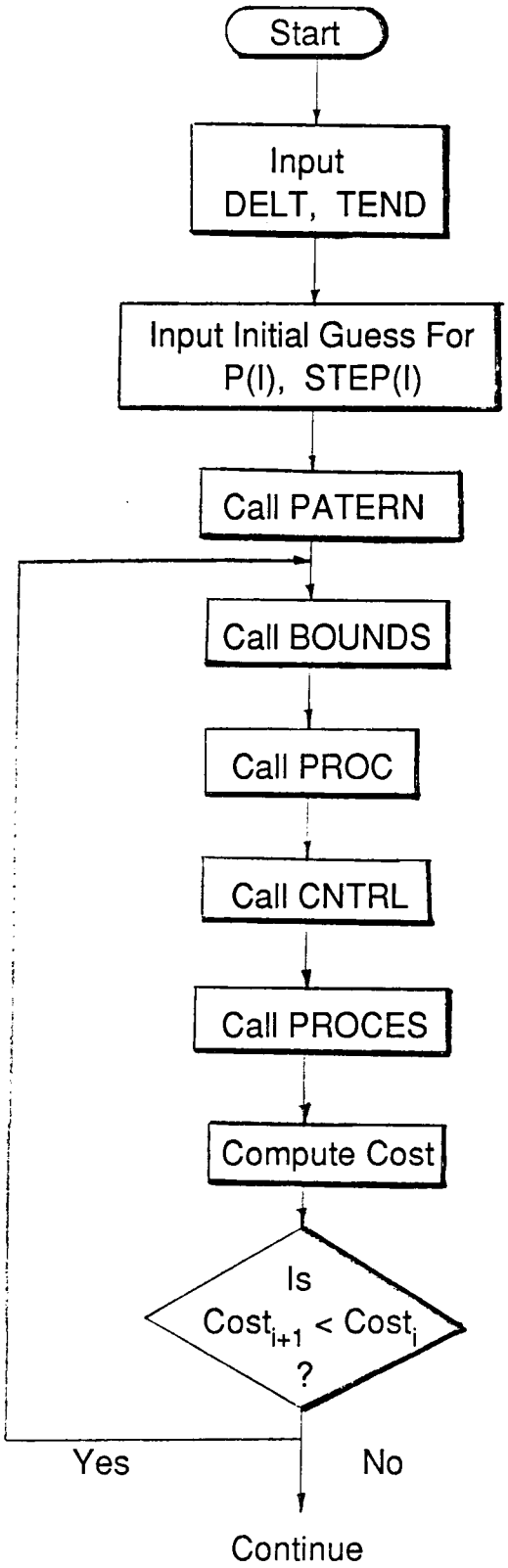
Definition of Variables :

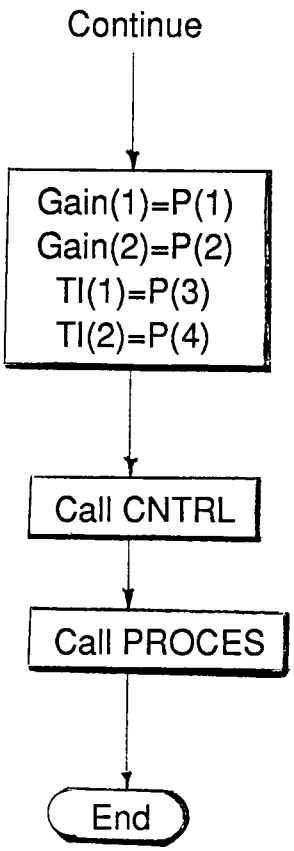
E = Error signal
M = Process input
N = Number of state variable
P = Search parameter
IN = Number of process input
NP = Number of search parameter
SP = Set point
TI = Parameter associated with integral mode
OUT = Number of process output
CONT = Process output
COST = Performance index, IAE
DELT = Sampling interval
GAIN = Parameter associated with proportional mode
STEP = Step size of search parameter
TEND = End of samples

Function of Subroutines :

PATERN = Multidimensional optimization
BOUNDS = Called by PATERN to check violation of constraints
PROC = Called by PATERN to calculate cost
PROCES = Called by PROC to iterate
$X(T+1) = PX(T) + QU(T)$
$Y(T+1) = CX(T+1)$
CNTRL = Called by PROC to compute controller output of selected control scheme
STATE = Called by Process to compute matrix exponential

Flow Chart :


Start
Input
DELT, TEND
Input Initial Guess For
P(I), STEP(I)
Call PATERN
Call BOUNDS
Call PROC
Call CNTRL
Call PROCES
Compute Cost
Is
Cost$_{i+1}$ < Cost$_i$
?
Yes
No
Continue


Continue
Gain(1)=P(1)
Gain(2)=P(2)
TI(1)=P(3)
TI(2)=P(4)
Call CNTRL
Call PROCES
End

```
C  PURPOSE: TO SIMULATE A PROCESS CONTROL SYSTEM WITH
C               CHOSEN CONTROL SCHEME.

      REAL P(4), STEP(4), M(2), CONT(2), E(2), X1(20)
      INTEGER IN, OUT, ICOUNT
      COMMON  GAIN(2), TI(2), SP(2), DELT, IN, OUT, N, TEND
      CHARACTER*20 FILENAME,OUTPUT
      TYPE *,'ENTER PROCESS OUTPUT FILENAME :'
      ACCEPT 2,FILENAME
2     FORMAT(A20)
      TYPE *,'ENTER DELT :'
      ACCEPT *,DELT
      TYPE *,'ENTER END OF SIMULATION TIME ,TEND :'
      ACCEPT *,TEND
      OPEN (UNIT=25,FILE=FILENAME,STATUS='NEW')
      NP=4
      N=4
      IN=2
      OUT=2
      DO 1 I=1,NP
         TYPE *,'INPUT INITIAL GUESS FOR P(',I,')'
         ACCEPT *,P(I)
         TYPE *,'INPUT INITIAL GUESS FOR STEP(',I,')'
         ACCEPT *,STEP(I)
1     CONTINUE
      NRED=3
      IO=2
      CALL PATERN(NP,P,STEP,NRED,IO,COST)
      WRITE(5,1100) (P(I),I=1,NP), COST
1100  FORMAT(' FINAL PARAMETERS: ',/,1X,6F15.7)
      DO 5 I=1,IN
5     GAIN(I)=0.0
      GAIN(1)=P(1)
      GAIN(2)=P(2)
      TI(1)=P(3)
      TI(2)=P(4)
      COST=0.0
      ICOUNT=0
      DO 10 I=1,IN
```

```
10        M(I)=0.0
          DO 20 J=1,OUT
             CONT(J)=0.0
             SP(J)=0.0
20        CONTINUE
          DO 21 J=1,N
21        X1(J)=0.0
          WRITE(25,*) '       SIMULATION OUTPUT FOR MVSISO '
          WRITE(25,*) '              Kc1 : ',P(1)
          WRITE(25,*) '              Kc2 : ',P(2)
          WRITE(25,*) '              TI1 : ',P(3)
          WRITE(25,*) '              TI2 : ',P(4)
          DO 100 I=0,TEND
             TIME=I*DELT
             IF( TIME .GE. 1.0 ) SP(1)=0.2
             IF( TIME .GE. 20.0 ) SP(2)=0.2
             CALL CNTRL(CONT,M,E,ICOUNT)
             CALL PROCES(CONT,M,X1,DELT)
             COST=COST+(ABS(SP(1)-CONT(1))+ABS(SP(2)-
     $              CONT(2)))*DELT
             WRITE(25,9) TIME,M(1),CONT(1),M(2),CONT(2)
             ICOUNT=2
100        CONTINUE
          WRITE(5,*) 'IAE :',COST
9         FORMAT (2X,5(G10.4,2X))
          STOP
          END

C-------SUBROUTINE NEEDED BY PATERN

          SUBROUTINE BOUNDS(P,IOUT)
          DIMENSION P(4)
          IOUT=0
          IF( P(1) .EQ. 0.0 ) IOUT=1
          IF( P(2) .EQ. 0.0 ) IOUT=1
          IF( P(3) .EQ. 0.0 ) IOUT=1
          IF( P(4) .EQ. 0.0 ) IOUT=1
          RETURN
          END
```

```
      SUBROUTINE PROC(P,COST)
        REAL P(4), M(2), CONT(2), E(2), X1(20)
        INTEGER IN, OUT, ICOUNT
        COMMON  GAIN(2), TI(2), SP(2), DELT, N, OUT, N, TEND
        DO 5 J1=1,IN
5       GAIN(J1)=0.0
        GAIN(1)=P(1)
        GAIN(2)=P(2)
        TI(1)=P(3)
        TI(2)=P(4)
        ICOUNT=0
        COST=0.0
        DO 10 J1=1,IN
10      M(J1)=0.0
        DO 20 J1=1,OUT
          CONT(J1)=0.0
20      SP(J1)=0.0
        DO 21 J=1,N
21      X1(J)=0.0
        DO 100 J1=0,TEND
          TIME=J1*DELT
          IF ( TIME .GE. 1.0) SP(1)=0.2
            IF ( TIME .GE. 20.0) SP(2)=0.2
          CALL CNTRL (CONT,M,E,ICOUNT)
          CALL PROCES (CONT,M,X1,DELT)
          COST=COST+(ABS(SP(1)-CONT(1))+ABS(SP(2)-
     $          CONT(2)))*DELT
          ICOUNT=2
100     CONTINUE
        RETURN
        END
```

```
C-------SUBROUTINE PROCES

      SUBROUTINE PROCES (CONT,M, X1, DELT)

C   PURPOSE: SIMULATION OF STATE SPACE MODELS USING MATRIX
C            EXPONENTIAL.

      REAL A(20,20), B(20,20), C(20,20)
      REAL P(20,20),Q(20,20),WRK(20,20)
      REAL X1(20),Y1(20),M(20),CONT(20)
      INTEGER IPVT(20),IN,OUT
      CHARACTER*20 FILENAME
      IF (IFLAG .GE. 1 ) GOTO 100
      IFLAG=1
      TYPE *,'ENTER DATA FILE FOR MATRIX A,B,C @ D :'
      ACCEPT 5,FILENAME
5     FORMAT(A20)
      OPEN (UNIT=20,FILE=FILENAME,STATUS='OLD')
C   SET ROW DIMENSION AS IT APPEARS IN THE ABOVE
C   DIMENSION STATEMENT
      IA=20
      TAU=DELT
C    READ THE NUMBER OF STATE VARIABLES
      READ (20,*) N
C    READ THE A MATRIX
      READ (20,*) ((A(I,J),J=1,N),I=1,N)
C    READ THE NUMBER OF INPUTS
      READ (20,*) IN
C    READ THE B MATRIX
      READ (20,*) ((B(I,J),J=1,IN),I=1,N)
C    READ THE NUMBER OF OUTPUTS
      READ (20,*) OUT
C    READ THE C MATRIX
      READ (20,*) ((C(I,J),J=1,N),I=1,OUT)
20    CONTINUE
      CALL SSPQ(A,N,IA,B,IN,IA,TAU,P,IA,Q,IA,WRK,IPVT)
```

```
C   BEGIN SIMULATION

100       DO 120 J1=1,N
             Y1(J1)=0.0
             DO 120 J2=1,N
120       Y1(J1)=Y1(J1)+P(J1,J2)*X1(J2)
          DO 130 J1=1,N
          DO 130 J2=1,IN
130       Y1(J1)=Y1(J1)+Q(J1,J2)*M(J2)
          DO 140 J1=1,OUT
             CONT(J1)=0.0
             DO 140 J2=1,N
140       CONT(J1)=CONT(J1)+C(J1,J2)*Y1(J2)
          DO 160 J1=1,N
160       X1(J1)=Y1(J1)
          RETURN
          END
```

```
C-------SUBROUTINE CNTRL FOR MVSISO

      SUBROUTINE CNTRL(CONT,M,E,ICOUNT)

C  PURPOSE: MULTIVARIABLE SINGLE INPUT - SINGLE OUTPUT
C           CONTROLLER

      REAL CONT(2), M(2), E(2), EM1(2)
      INTEGER IN,OUT,ICOUNT
      COMMON  GAIN(2), TI(2), SP(2), DELT, IN, OUT, N, TEND
300    IF (ICOUNT .GT. 1) GO TO 500
      DO 400 K=1,OUT
400    EM1(K)=0.0
500    DELTAT=DELT

C  CALCULATE THE ERROR SIGNAL

      DO 600 I=1,OUT
600    E(I)=SP(I)-CONT(I)

C  CALCULATE PI CONTROLLER OUTPUTS

      DO 900 I=1,IN
900    M(I)=M(I)+GAIN(I)*(E(I)-EM1(I)+E(I)*DELTAT/TI(I))

C  SHIFT THE ERROR SIGNALS

      DO 1000 I=1,OUT
1000   EM1(I)=E(I)

C  LIMIT THE OUTPUTS

       IF (M(1) .GE. 0.39) M(1)=0.39
       IF (M(1) .LE. -0.39) M(1)=-0.39
       IF (M(2) .GE. 0.342) M(2)=0.342
      IF (M(2) .LE. -0.342) M(2)=-0.342
      RETURN
      END
```

```
C------SUBROUTINE CNTRL FOR MVPIR

      SUBROUTINE CNTRL(CONT,M,E,ICOUNT)

C  PURPOSE: MULTIVARIABLE TUNING REGULATOR CONTROLLER

      REAL IMAT(2,2),PMAT(2,2),CONT(2),M(2),E(2)
      REAL EM1(2),MP(2),MI(2)
      INTEGER IN,OUT,ICOUNT
      COMMON  GAIN(2,2),EPS,SP(2),DELT,IN,OUT,N,NCOST
      CHARACTER*20 FILENAME
      IF (ICHK .GE. 1) GO TO 300
      TYPE *,'ENTER DATA FILE FOR KP AND KI'
      ACCEPT 5,FILENAME
5     FORMAT(A20)
      OPEN (UNIT=23,FILE=FILENAME,STATUS='OLD')
      DO 100 I=1,IN
100   READ(23,*) (PMAT(I,J),J=1,OUT)
      DO 200 I=1,IN
200   READ(23,*) (IMAT(I,J),J=1,OUT)
      ICHK=1
300   IF (ICOUNT .GT. 1) GO TO 500
      DO 400 K=1,OUT
400   EM1(K)=0.0
500   DELTAT=DELT

C  CALCULATE THE ERROR SIGNAL

      DO 600 I=1,OUT
600   E(I)=SP(I)-CONT(I)

C  FORMULATE THE P-PART

      DO 700 I=1,IN
         MP(I)=0.0
         DO 700 J=1,OUT
700   MP(I)=MP(I)+PMAT(I,J)*(E(J)-EM1(J))*GAIN(J,J)
```

```
C    FORMULATE THE I-PART

           DO 800 I=1,IN
              MI(I)=0.0
              DO 800 J=1,OUT
800        MI(I)=MI(I)+IMAT(I,J)*E(J)*DELTAT*EPS

C  ADD TO THE PREVIOUS OUTPUT

           DO 900 I=1,IN
900        M(I)=M(I)+MI(I)+MP(I)

C  SHIFT THE ERROR SIGNALS

           DO 1000 I=1,OUT
1000        EM1(I)=E(I)

C  LIMIT THE OUTPUTS

           IF (M(1) .GE. 0.39) M(1)=0.39
           IF (M(1) .LE. -0.39) M(1)=-0.39
           IF (M(2) .GE. 0.342) M(2)=0.342
           IF (M(2) .LE. -0.342) M(2)=-0.342
           RETURN
           END
```

```
C------SUBROUTINE CNTRL FOR SVAC

      SUBROUTINE CNTRL(CONT,M,E,ICOUNT)

C  PURPOSE: SINGULAR VALUE ANALYSIS CONTROLLER

      REAL UT(2,2),V(2,2),CONT(2),M(2),E(2),EM1(2),MP(2),EP(2)
      INTEGER IN,OUT,ICOUNT
      COMMON  GAIN(2,2),TI(2),SP(2),DELT,IN,OUT,N,TEND
      CHARACTER*20 FILENAME
      IF (ICHK .GE. 1) GO TO 300
      TYPE *,'ENTER DATA FILE FOR V AND UT'
      ACCEPT 5,FILENAME
5     FORMAT(A20)
      OPEN (UNIT=23,FILE=FILENAME,STATUS='OLD')
      DO 100 I=1,IN
100    READ(23,*) (V(I,J),J=1,OUT)
      DO 200 I=1,IN
200    READ(23,*) (UT(I,J),J=1,OUT)
      ICHK=1
300    IF (ICOUNT .GT. 1) GO TO 500
      DO 400 K=1,OUT
400    EM1(K)=0.0
500    DELTAT=DELT

C  CALCULATE THE ERROR SIGNAL

      DO 600 I=1,OUT
600    E(I)=SP(I)-CONT(I)

C  MULTIPLY ERROR SIGNAL BY UT

      DO 700 I=1,IN
         EP(I)=0.0
        DO 700 J=1,OUT
700    EP(I)=EP(I)+UT(I,J)*E(J)
```

```
C  CALCULATE THE CONTROLLER OUTPUT

          DO 800 I=1,IN
            MP(I)=0.0
800       MP(I)=GAIN(I,I)*((EP(I)-EM1(I))+(DELTAT*EP(I)/TI(I)))

C  MULTIPLY BY V

          DO 900 I=1,IN
          DO 900 J=1,IN
900       M(I)=M(I)+V(I,J)*MP(J)

C  SHIFT THE ERROR SIGNALS

         DO 1000 I=1,OUT
1000     EM1(I)=EP(I)

C  LIMIT THE OUTPUTS

       IF (M(1) .GE. 0.39) M(1)=0.39
       IF (M(1) .LE. -0.39) M(1)=-0.39
       IF (M(2) .GE. 0.342) M(2)=0.342
       IF (M(2) .LE. -0.342) M(2)=-0.342
       RETURN
       END
```

VITA

Gim Cheng Khor was born in Penang, Malaysia, on December 18, 1962. After graduating from Phor Tay High School, she came to United States to further her study and received her Bachelor of Science degree in March1985 from the University of Tennessee at Knoxville. In September 1985, she continued her graduate study at the same institution. She received her Master of Science degree in Chemical Engineering in August 1987. She is a member of Tau Beta Pi.