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I am submitting herewith a thesis written by Hugh Jay Robinson III entitled "A General Mathematical Model of Dispersed Single Phase Induction Generators Employed Within a Three-Phase Distribution System." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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A GENERAL MATHEMATICAL MODEL OF DISPERSED SINGLE PHASE INDUCTION GENERATORS EMPLOYED WITHIN A THREE-PHASE DISTRIBUTION SYSTEM

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Hugh Jay Robinson III

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DEDICATION

To my mother, who has been fighting valiantly against cancer for a number of years only to enter the business world for the first time and achieve substantial success. Hos ego versiculos feci, tulit alter honores. . . .

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ABSTRACT

In this study, a mathematical model is constructed of single-phase induction generators interacting with an adjacent three-phase distribution system. It is shown that this model is valid for describing the joint operation of an induction generator and distribution system. Model performance is illustrated initially by digital simulation of the distribution system acting alone. Subsequent simulations include the operation of the single-phase induction generator and distribution feeder collectively. These cases are performed with and without mutual coupling between phases of the distribution system. Power system faults and circuit breaker responses are also represented.

TABLE OF CONTENTS

CHAPTER	PAGE
1. INTRODUCTION	1
2. SINGLE-PHASE INDUCTION GENERATOR	4
2.1 Mathematical Model	4
2.2 Characteristics of Single-Phase Operation	7
3. DISTRIBUTION SYSTEM MODEL	10
3.1 Formulation of Differential Voltage Equations	13
3.2 Formulation of Differential Current Equations	17
3.3 The Generalized Mathematical Model	20
4. SIMULATION OF THE GENERALIZED MODEL	22
4.1 General Flow Chart	22
4.2 Simulation of Torque, Fault(s), and Circuit Breakers	25
4.3 Subroutine RUNGE	31
4.4 Cases to Be Simulated	34
4.5 Results of Computer Simulation	35
5. CONCLUSIONS	63
LIST OF REFERENCES	64
APPENDICES	66
VITA	100

LIST OF FIGURES

FIGURE		PAGE
3-1.	Distribution System Circuit Model	11
3-2.	Current Nodes of Figure 3.1	12
4-1.	General Flow Chart	23
4-2.	Flow Chart of Subroutine RUNGE	33
4-3.	Case #1: Bus #1 Voltages	36
4-4.	Case #1: Bus #2 Voltages	37
4-5.	Case #2: Bus #1 Voltages	38
4-6.	Case #2: Bus #2 Voltages	39
4-7.	Case #2: Bus #1 Voltages--Breaker Reclosure	40
4-8.	Case #2: Bus #2 Voltages--Breaker Reclosure	41
4-9.	Case #2: Line Currents to Bus #1	42
4-10.	Case #2: Line Currents to Bus #2	43
4-11.	Case #2: Line Currents to Bus #1--Breaker Reclosure	44
4-12.	Case #2: Line Currents to Bus #2--Breaker Reclosure	45
4-13.	Case #2: Armature, Direct, and Quadrature Currents of Induction Generator	46
4-14.	Case #3: Armature, Direct, and Quadrature Currents of Induction Generator	47
4-15.	Case #3: Bus #1 Voltages	48
4-16.	Case #3: Bus #2 Voltages	49
4-17.	Case #3: Bus #1 Voltages--Breaker Reclosure	50
4-18.	Case #3: Bus #2 Voltages--Breaker Reclosure	51
4-19.	Case #3: Line Current to Bus #1	52
4-20.	Case #3: Line Current to Bus #2	53

LIST OF FIGURES

(Continued)

FIGURE		PAGE
4-21.	Case #3: Line Current to Bus #1--Breaker Reclosure . .	54
4-22.	Case #3: Line Current to Bus #2--Breaker Reclosure . .	55
4-23.	Case #2: SLIP	56
4-24.	Case #3: SLIP	57
4-25.	Case #2: Transients in SLIP	58
4-26.	Case #3: Transients in SLIP	59

CHAPTER 1

INTRODUCTION

Throughout the past few years, the induction generator has been accepted, and recognized, as an effective rotary converter.[1,2] If the popularity of the induction generator increases, especially in single-phase applications, questions may arise concerning the dynamic interaction of this type of generator with the existing power system. Remarkably, little information is available about the dynamic and steady-state characteristics of the single-phase induction generator. Emphasis, invariably, is directed toward the steady-state induction motor.[3,4,5] Presently, research is being conducted on the application of both the three-phase and single-phase induction generator to alternate energy sources. However, most of this research activity is primarily empirical, with little, if any, attention being allocated toward theoretical implications.[6,7,8]

This particular study attempts to investigate the theoretical aspects of the single-phase induction generator interacting with a three-phase distribution system. Although much is known about the single-phase induction motor, and the induction generator is virtually identical to the motor in construction, distinctive features may be recognized after a complete investigation.

The objectives of this study are:

1. To obtain, from the existing literature, a valid mathematical model of the single-phase induction generator.

2. To derive a mathematical model of a three-phase distribution system which involves all mutual coupling between phases.
3. To combine the mathematical models of the single-phase induction generator and distribution system into one generalized model which is capable of describing the composite system.
4. To test the validity of this generalized mathematical model by simulating selected cases on the digital computer.

Once these objectives are achieved, a means of examining single-phase induction generator operation and its consequences on the neighboring utility will become available.

In this study, appropriate assumptions will be made when needed and/or required. For instance, because of the voltage limitations of single-phase operation, transmission systems are ignored. An infinite bus is assumed to exist within a reasonable distance from the induction generator bus. This assumption automatically presumes that the transmission voltage of the utility grid cannot change.

The essence of this project is the manner in which the above generalized model is simulated. The digital computer has been chosen for this endeavor. The particular method of simulation will apply the fourth-order RUNGE-KUTTA with a fixed increment of numerical approximation to the solutions of the pertinent differential equations. The versatility of the collective model may be enhanced by a variable integration procedure; however, initially, emphasis should be directed toward the quality and accuracy of the models being simulated. After a successful series of simulations have been performed, fine adjustments and revisions are in order. In fact, they should be made; but to do so before the generalized model has been thoroughly tested may be premature. For this reason, sophisticated techniques which correlate accuracy with economy have been, temporarily, placed aside.

In a purely theoretical fashion, a single-phase induction generator will be placed on different phases of the distribution system at diverse locations. Mechanical torque will be applied to the rotor of the induction machine in such a manner as to produce generator action. All pertinent variables of the induction generator and distribution system will then be calculated, simulated, and stored on magnetic tape for future study. Comparisons will be made to regular power system variables simulated without the influence of the induction generator. Power system faults will also be considered and simulated, including circuit breaker response.

CHAPTER 2

SINGLE-PHASE INDUCTION GENERATOR

In order to arrive at a generalized mathematical model which incorporates a number of induction generators interacting with an adjacent utility system, a mathematical model of the induction generator should be obtained and tested thoroughly. This may be accomplished by decoupling the induction machine from the utility's distribution system analytically; i.e., by utilizing the infinite bus format and ignoring all mutual coupling between the other phases, the transmission lines, and the phase employing the induction generator. This infinite bus will be capable of supplying all voltage and power requirements of the induction machine, with no subsequent effect whatsoever on the existing distribution system.

2.1 MATHEMATICAL MODEL

The theoretical equations for a single-phase induction machine are:

$$
\begin{aligned}
V_a^s &= R^s i_a^s + L^s \frac{d}{dt}(i_a^s) + L^{sr}\frac{d}{dt}(i_d^r) \\
0 &= L^{sr}\frac{d}{dt}(i_a^s) + R^r i_d^r + L^r\frac{d}{dt}(i_d^r) + \nu\omega_m^r L^r i_q^r \qquad (2\text{-}1) \\
0 &= -\nu\omega_m^r L^{sr} i_a^s - \nu\omega_m^r L^r i_d^r + R^r i_q^r + L^r \frac{d}{dt}(i_q^r)
\end{aligned}
$$

A derivation of Equation (2-1), based upon the information provided in Reference 3, is found in Appendix A, as are all mathematical symbols.

In addition to the above equations, two other mathematical relationships may be defined for any single-phase induction machine:

$$\text{Electromagnetic Torque:} \quad T_{em} = \nu L^{sr} i_a^s i_q^r$$
$$\text{Angular Acceleration:} \quad \frac{d}{dt}(\omega_m^r) = (T_{pm} - T_{em})/J \qquad (2\text{-}2)$$

where, as before, all mathematical symbols are defined in Appendix A.

Finally, the slip of an induction machine is defined as:

$$\text{Slip} = s \equiv \frac{\omega^s - \nu\omega_m^r}{\omega^s}$$

and for generator action, $s < 0$.

In matrix notation, the linearized equations become:

$$\begin{bmatrix} V_a^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} L^s & L^{sr} & 0 \\ L^{sr} & L^r & 0 \\ 0 & 0 & L^r \end{bmatrix} \begin{bmatrix} pi_a^s \\ pi_d^r \\ pi_q^r \end{bmatrix} + \begin{bmatrix} R^s & 0 & 0 \\ 0 & R^r & \nu\omega_m^r L^r \\ -\nu\omega_m^r L^{sr} & -\nu\omega_m^r L^r & R^r \end{bmatrix} \begin{bmatrix} i_a^s \\ i_d^r \\ i_q^r \end{bmatrix} \qquad (2\text{-}3)$$

where the operator p has been substitued for the differential operator $\frac{d}{dt}$.

In state variable format, Equation (2-3) becomes:

$$[V_t] = [A]p[I_g] + [B][I_g] \qquad (2\text{-}4)$$

with the variables in Equation (2-4) defined in a corresponding manner.

Equation (2-4) implies that

$$p[I_g] = -[A]^{-1}[B][I_g] + [A]^{-1}[V_t]. \tag{2-5}$$

Since matrix [A] is symmetric, $[A]^{-1}$ is also symmetric, and

$$[A]^{-1} = [A]^c/\det[A].$$

Thus,

$$[A]^{-1} = \begin{bmatrix} L^r/\alpha & -L^{sr}/\alpha & 0 \\ -L^{sr}/\alpha & L^s/\alpha & 0 \\ 0 & 0 & 1/L^r \end{bmatrix}$$

where $\alpha = L^sL^r - (L^{sr})^2$.

Carrying out all matrix operations required by Equation (2-5) yields:

$$p\begin{bmatrix} i_a^s \\ i_d^r \\ i_q^r \end{bmatrix} = \begin{bmatrix} -L^rR^s/\alpha & R^rL^{sr}/\alpha & \nu\omega_m^rL^rL^{sr}/\alpha \\ R^sL^{sr}/\alpha & -L^sR^r/\alpha & -L^s\nu\omega_m^rL^r/\alpha \\ \nu\omega_m^rL^{sr}/L^r & \nu\omega_m^r & -R^r/L^r \end{bmatrix}\begin{bmatrix} i_a^s \\ i_d^r \\ i_q^r \end{bmatrix}$$
$$+ \begin{bmatrix} L^r/\alpha & -L^{sr}/\alpha & 0 \\ -L^{sr}/\alpha & L^s/\alpha & 0 \\ 0 & 0 & 1/L^r \end{bmatrix}\begin{bmatrix} V_a^s \\ 0 \\ 0 \end{bmatrix} \tag{2-6}$$

Now, Equation (2-2) reduces to

$$p\omega_m^r = [T_{pm} - \nu L^{sr} i_a^s i_q^r]/J. \tag{2-7}$$

Equations (2-6) and (2-7) together form the non-linear defining equations for the single-phase induction generator.

2.2 CHARACTERISTICS OF SINGLE-PHASE OPERATION

The following assumptions will be adhered to throughout this project:

1. All induction machines will have an armature voltage rating of 220 volts.
2. This armature voltage of 220 volts is the secondary voltage of a single-phase distribution transformer.
3. Power system frequency is held constant.

With the above assumptions in mind, the following observations will be made: The contribution of the induction machine to the total line current of any phase of the existing distribution system is small. From Assumption 2, the implied turns ratio of the distribution transformer is 60:1. Therefore, approximately 1/60th of the armature current of the induction generator is transformed to the power system side. For example, if an armature current of 60 amps should exist in the induction machine, which may be considered as being high for a single-phase machine, only one ampere of current would result in the primary winding of the distribution transformer.

Furthermore, for a constant rotor velocity, Equation (2-2) implies that

$$p\omega_m^r = 0 = (T_{pm} - T_{em})/J$$

or

$$T_{pm} = T_{em}.$$

However, it is recognized that for the induction generator to remain stable the above relation between prime mover torque and electromagnetic torque must always be eventually satisfied. This merely insists that any accelerating torque on the rotor of the induction machine be of a temporary nature and of such duration and magnitude that the stability limit of the particular induction generator is not reached. At any rate, whenever this relationship between torques is satisfied, slip cannot change. Therefore, the real power output of the induction generator cannot change. Under stable operating conditions, the adjacent utility grid must absorb the effect of any subsequent reduction in induction generator load. The power system must also supply any increase in load adjacent to the induction generator bus. Under certain conditions, such as very light loading, the net direction of the line current of the phase interfaced with the induction generator may be subject to change.

The above conclusions regarding the characteristics of the single-phase induction generator are quite important in fault analysis. The induction generator is somewhat different than the induction motor under fault conditions. With breaker action, the rotor velocity of the induction generator will increase drastically. Totally different transient phenomena should occur upon breaker reclosure. After breaker reclosure,

T_{em} must be greater than T_{pm} temporarily in order for stability to prevail.

CHAPTER 3

DISTRIBUTION SYSTEM MODEL

A general model of a distribution system will now be derived which includes all mutual coupling between phases. The differential equations to be derived are based upon the three-dimensional circuit model depicted in Figure 3.1. Artificial admittances/conductances have been included in the circuit diagram as a means of subjecting the distribution system to all pertinent power system faults. These elements are drawn as variable resistors in Figure 3.1. In Figure 3.2, the currents associated with any power system fault are denoted by the subscript F, followed by the phase symbol and bus number. For example,

I_{FA1} => Fault current, phase A to ground--bus #1,

and

I_{FCA2} => Fault current, phase C to phase A--bus #2.

Mutual capacitances are depicted in Figure 3.1 by traditional capacitive elements between the distribution phases. Figure 3.2 symbolizes the displacement current through these capacitances according to the phases and bus number. That is,

I_{AB1} => Displacement current from phase A to phase B--bus #1.

Mutual inductances are indicated in the circuit diagram by an M drawn between the appropriate phases. Distribution line currents are symbolized in Figure 3.2 by the particular phase and direction number, if applicable:

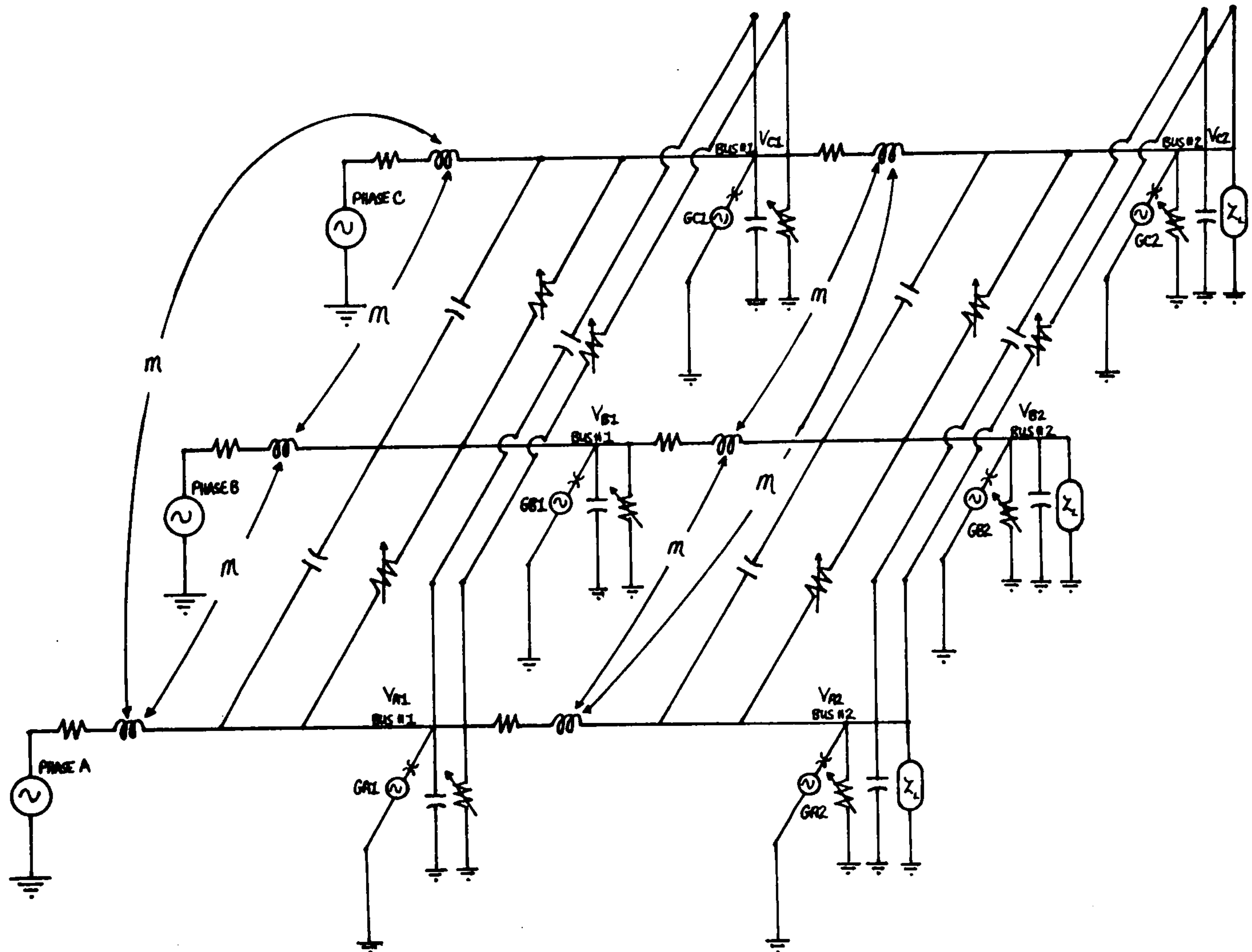

Figure 3-1. Distribution System Circuit Model

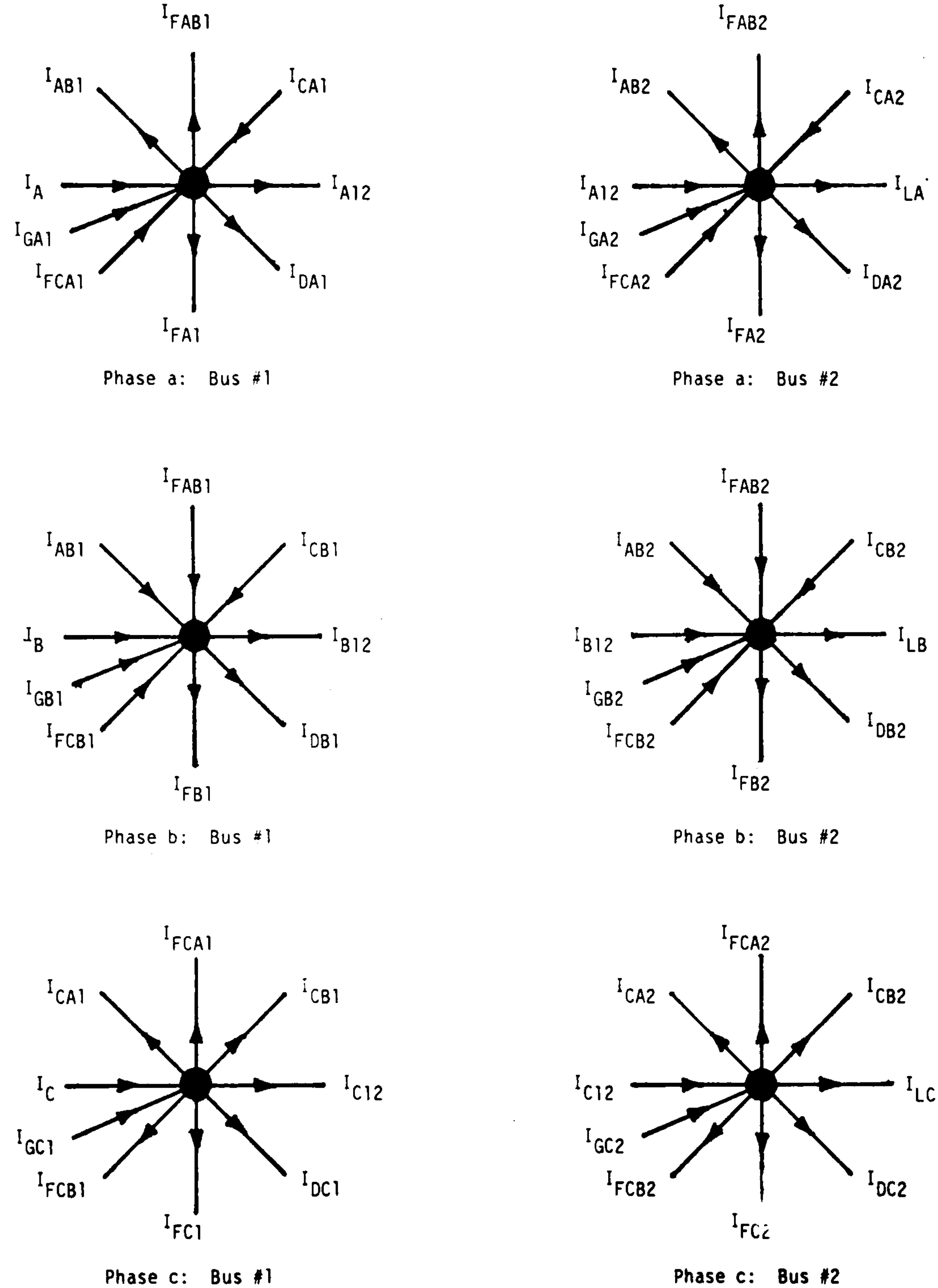

Figure 3-2. Current Nodes of Figure 3.1

I_A = Total line current, phase A, from the infinite bus to bus #1.

Similarly,

I_{A12} = Line current, phase A, from bus #1 to bus #2.

Finally, single-phase induction generators are labeled in Figure 3.1 according to phase and bus number. Load currents, all of which originate from bus #2, are classified by an L and phase designation. For instance,

I_{GA1} = Stator current of induction generator on phase A, bus #1, transformed to power system side.

I_{LA} = Load current phase A.

It should be noted that Figure 3.1 represents a three-wire grounded distribution system, whereas in practice, most distribution systems consist of a four-wire configuration. The only difference between a three-wire and a four-wire system is the addition of a neutral conductor, with this neutral being solidly grounded at prescribed locations along the distribution circuit. Although the presence of a neutral wire presumes current flow and, hence, influences the other wires of the system as far as inductive coupling is concerned, its presence will be ignored in this study. Furthermore, all distribution transformers are assumed to be ideal within this three-wire grounded system.

3.1 FORMULATION OF DIFFERENTIAL VOLTAGE EQUATIONS

By applying Kirchhoff's current law to the nodes of Figure 3.2, the following set of equations emerge:

$$
\begin{aligned}
&I_A + I_{CA1} + I_{GA1} + I_{FCA1} = I_{AB1} + I_{FAB1} + I_{A12} + I_{DA1} + I_{FA1} \\
&I_{A12} + I_{CA2} + I_{GA2} + I_{FCA2} = I_{AB2} + I_{FAB2} + I_{LA} + I_{DA2} + I_{FA2} \\
&I_B + I_{AB1} + I_{FAB1} + I_{CB1} + I_{GB1} + I_{FCB1} = I_{B12} + I_{DB1} + I_{FB1} \\
&I_{B12} + I_{AB2} + I_{FAB2} + I_{CB2} + I_{GB2} + I_{FCB2} = I_{LB} + I_{DB2} + I_{FB2} \\
&I_C + I_{GC1} = I_{CA1} + I_{FCA1} + I_{CB1} + I_{C12} + I_{DC1} + I_{FC1} + I_{FCB1} \\
&I_{C12} + I_{GC2} = I_{CA2} + I_{FCA2} + I_{CB2} + I_{LC} + I_{DC2} + I_{FC2} + I_{FCB2}.
\end{aligned}
\tag{3-1}
$$

However, the following relationships exist between the phase currents and phase voltages:

$$
\begin{aligned}
I_{CA1} &= C_{ac1} pV_{CA1} = C_{ac1}(pV_{C1} - pV_{A1}) \\
I_{FCA1} &= Y_{ca1}(V_{C1} - V_{A1}) \\
I_{AB1} &= C_{ab1} pV_{AB1} = C_{ab1}(pV_{A1} - pV_{B1}) \\
I_{FAB1} &= Y_{ab1}(V_{A1} - V_{B1}) \\
I_{DA1} &= C_{ag1} pV_{A1} \; p \\
I_{FA1} &= Y_{ag1} V_{A1} \\
I_{CA2} &= C_{ac2} pV_{CA2} = C_{ac2}(pV_{C2} - pV_{A2}) \\
I_{FCA2} &= Y_{ca2}(V_{C2} - V_{A2}) \\
I_{AB2} &= C_{ab2} pV_{AB2} = C_{ab2}(pV_{A2} - pV_{B2}) \\
I_{FAB2} &= Y_{ab2}(V_{A2} - V_{B2}) \\
I_{LA} &= Y_{LA} V_{A2} \\
I_{DA2} &= C_{ag2} pV_{A2} \\
I_{FA2} &= Y_{ag2} V_{A2}
\end{aligned}
$$

$$
\begin{aligned}
I_{CB1} &= C_{cb1}pV_{CB1} = C_{cb1}(pV_{C1} - pV_{B1}) \\
I_{FCB1} &= Y_{cb1}(V_{C1} - V_{B1}) \\
I_{DB1} &= C_{bg1}pV_{B1} \\
I_{FB1} &= Y_{bg1}V_{B1} \\
I_{CB2} &= C_{cb2}pV_{CB2} = C_{cb2}(pV_{C2} - pV_{B2}) \\
I_{FCB2} &= Y_{cb2}(V_{C2} - V_{B2}) \\
I_{DB2} &= C_{bg2}pV_{B2} \\
I_{FB2} &= Y_{bg2}V_{B2} \\
I_{LB} &= Y_{LB}V_{B2} \\
I_{DC1} &= C_{cg1}pV_{C1} \\
I_{FC1} &= Y_{cg1}V_{C1} \\
I_{LC} &= Y_{LC}V_{C2} \\
I_{DC2} &= C_{cg2}pV_{C2} \\
I_{FC2} &= Y_{cg2}V_{C2}
\end{aligned}
\qquad (3\text{-}2)
$$

where, as before, p has been substituted for the differential operator $\frac{d}{dt}$. Also, all mathematical symbols in the above two sets of equations, and in all figures and equations to follow, are defined in the appendixes.

In classical power system analysis, the following set of identities are assumed:

$$C_{ag1} = C_{ag2} = C_{ag}$$

$$C_{bg1} = C_{bg2} = C_{bg}$$

$$
\begin{aligned}
C_{cg1} &= C_{cg2} = C_{cg} \\
C_{ag} &= C_{bg} = C_{cg} \\
C_{ab1} &= C_{ab2} = C_{ab} \\
C_{bc1} &= C_{bc2} = C_{bc} \\
C_{ac1} &= C_{ac2} = C_{ac} \\
C_{ab} &= C_{bc}.
\end{aligned} \tag{3-3}
$$

It will be shown later that all of the identities of Equation Set (3-3), except $C_{ag} = C_{bg} = C_{cg}$, are indeed valid due to the chosen configuration of the power lines. Generally, $C_{ag} \neq C_{bg} \neq C_{cg}$, in the sense that C_{bg} is actually less than C_{ag} and C_{cg}. However, $C_{ag} = C_{cg}$, and the error involved by assuming complete symmetry is negligible, considering the magnitude of the voltages involved.

Substituting Equation (3-2) into Equation (3-1) and simplifying, the following set of simultaneous differential equations is realized:

$$
\begin{aligned}
-X_1PV_{A1} + X_2PV_{B1} + X_3PV_{C1} &= a_1V_{A1} - Y_{ab1}V_{B1} - Y_{ac1}V_{C1} + I_{A12} \\
&\quad - I_A - I_{GA1} \\
-X_1PV_{A2} + X_2PV_{B2} + X_3PV_{C2} &= a_2V_{A2} - Y_{ab2}V_{B2} - Y_{ac2}V_{C2} - I_{A12} - I_{GA2} \\
X_2PV_{A1} - X_4PV_{B1} + X_2PV_{C1} &= -Y_{ab1}V_{A1} + b_1V_{B1} - Y_{cb1}V_{C1} + I_{B12} \\
&\quad - I_B - I_{GB1} \\
X_2PV_{A2} - X_4PV_{B2} + X_2PV_{C2} &= -Y_{ab2}V_{A2} + b_2V_{B2} - Y_{cb2}V_{C2} - I_{B12} - I_{GB2} \\
-X_3PV_{A1} - X_2PV_{B1} + X_1PV_{C1} &= Y_{ca1}V_{A1} + Y_{cb1}V_{B1} - C_1V_{C1} - I_{C12} \\
&\quad + I_C + I_{GC1} \\
-X_3PV_{A2} - X_2PV_{B2} + X_1PV_{C2} &= Y_{ca2}V_{A2} + Y_{cb2}V_{B2} - C_2V_{C2} + I_{C12} + I_{GC2}
\end{aligned} \tag{3-4}
$$

In matrix format, these equations become

$$[X]p[V] = [Y][V] + [D][I] + [E][I_G]$$

such that

$$p[V] = [X]^{-1}[Y][V] + [X]^{-1}[D][I] + [X]^{-1}[E][I_G] \tag{3-5}$$

and is the matrix equation that describes all differential voltages of the distribution system, containing up to six induction generators, to be simulated later. An exact derivation of Equation (3-5) is found in Appendix B. It will be seen in Appendix B that the elements of the matrices in Equation (3-5) have been left in generalized form; i.e, as mathematical variables, since these matrices reflect the state of the power system.

3.2 FORMULATION OF DIFFERENTIAL CURRENT EQUATIONS

Again, referring to Figure 3.1 on page 11 and applying Kirchhoff's voltage law to each phase between appropriate buses, the following equations may be written:

$$\begin{aligned}
\Delta V_A &= Z_{AA}I_A + Z_{AB}I_B + Z_{AC}I_C \\
\Delta V_{A1} &= Z_{AA1}I_{A12} + Z_{AB1}I_{B12} + Z_{AC1}I_{C12} \\
\Delta V_B &= Z_{AB}I_A + Z_{BB}I_B + Z_{BC}I_C \\
\Delta V_{B1} &= Z_{AB1}I_{A12} + Z_{BB1}I_{B12} + Z_{BC1}I_{C12} \\
\Delta V_C &= Z_{AC}I_A + Z_{BC}I_B + Z_{CC}I_C \\
\Delta V_{C1} &= Z_{AC1}I_{A12} + Z_{BC1}I_{B12} + Z_{CC1}I_{C12}
\end{aligned} \tag{3-6}$$

where

$$\Delta V_A = V_A - V_{A1}$$

$$\Delta V_{A1} = V_{A1} - V_{A2}$$

$$\Delta V_B = V_B - V_{B1}$$

$$\Delta V_{B1} = V_{B1} - V_{B2}$$

$$\Delta V_C = V_C - V_{C1}$$

$$\Delta V_{C1} = V_{C1} - V_{C2}$$

from line symmetry

$$Z_{AA} = Z_{AA1}$$

$$Z_{BB} = Z_{BB1}$$

$$Z_{CC} = Z_{CC1}$$

$$Z_{AB} = Z_{AB1}$$

$$Z_{BC} = Z_{BC1}$$

$$Z_{AC} = Z_{AC1}.$$

Following the methodology described by Anderson[9] and assuming earth return current, the above impedances will be defined as follows:

$$\begin{aligned} Z_{AA} &= (r_a + r_d) + L_{AA}p \\ Z_{BB} &= (r_b + r_d) + L_{BB}p \\ Z_{CC} &= (r_c + r_d) + L_{CC}p \\ Z_{AB} &= r_d + L_{AB}p \end{aligned} \tag{3-7}$$

$$Z_{BC} = r_d + L_{BC}p$$

$$Z_{AC} = r_d + L_{AC}p.$$

For,

$$\begin{aligned}
L_{AA} &= K\ell n(D_e/D_{sa}) \\
L_{BB} &= K\ell n(D_e/D_{sb}) \\
L_{CC} &= K\ell n(D_e/D_{sc}) \\
L_{AB} &= K\ell n(D_e/D_{AB}) \\
L_{BC} &= K\ell n(D_e/D_{BC}) \\
L_{AC} &= K\ell n(D_e/D_{AC}).
\end{aligned} \tag{3-8}$$

When Equation (3-7) is substituted into Equation (3-6), the following set of differential equations arise:

$$\begin{aligned}
V_A - V_{A1} &= (r_a + r_d + L_{AA}P)I_A + (r_d + L_{AB}P)I_B + (r_d + L_{AC}P)I_C \\
V_{A1} - V_{A2} &= (r_a + r_d + L_{AA}P)I_{A12} + (r_d + L_{AB}P)I_{B12} + (r_d + L_{AC}P)I_{C12} \\
V_B - V_{B1} &= (r_d + L_{AB}P)I_A + (r_a + r_d + L_{BB}P)I_B + (r_d + L_{BC}P)I_C \\
V_{B1} - V_{B2} &= (r_d + L_{AB}P)I_{A12} + (r_a + r_d + L_{BB}P)I_{B12} + (r_d + L_{BC}P)I_{C12} \\
V_C - V_{C1} &= (r_d + L_{AC}P)I_A + (r_d + L_{BC}P)I_B + (r_a + r_d + L_{CC}P)I_C \\
V_{C1} - V_{C2} &= (r_d + L_{AC}P)I_{A12} + (r_d + L_{BC}P)I_{B12} + (r_a + r_d + L_{CC}P)I_{C12}
\end{aligned} \tag{3-9}$$

which may be expressed in matrix notation as

$$[F][V] + [V_f] = [R][I] + [L]p[I] \tag{3-10}$$

$\therefore$

$$p[I] = -[L]^{-1}[R][I] + [L]^{-1}[F][V] + [L]^{-1}[V_f]. \tag{3-11}$$

Equation (3-11) describes, completely, all of the differential currents associated with the distribution system chosen for simulation purposes. The missing steps between Equations (3-9) and (3-10) are elaborated upon in Appendix B. A quick glance at Appendix B should disclose that, unlike the differential voltage matrix equation, the elements of the above matrices in Equation (3-11) are independent of the particular state of the power system. Once the parameters of the distribution line are determined, $[L]^{-1}$ remains unchanged throughout all events to be simulated later.

3.3 THE GENERALIZED MATHEMATICAL MODEL

The mathematical model of a three-phase distribution system employing single-phase induction generators merely requires a mathematical summary of all equations up to this point. This summary involves

$$p[I_g] = -[A]^{-1}[B][I_g] + [A]^{-1}[V_t] \tag{2-5}$$

$$p\omega_m^r = (T_{pm} - \nu L^{sr} i^s i_q^r)/J \tag{2-7}$$

$$p[V] = [X]^{-1}[Y][V] + [X]^{-1}[D][I] + [X]^{-1}[E][I_g] \tag{3-5}$$

$$p[I] = -[L]^{-1}[R][I] + [L]^{-1}[F][V] + [L]^{-1}[V_f]. \tag{3-11}$$

The three matrix equations above actually envelop 36 non-linear simultaneous differential equations. Equation (2-5) may include up to 20 differential equations alone.

A generalized mathematical model was chosen because of the innate capacity associated with such a model to describe many circumstances with little modification. For instance, this model will handle from one to six induction generators, in any particular sequence, on any particular phase of the distribution system. However, each phase is limited to two induction generators at any one time. On the other hand, capacitor banks may be added without any mathematical addendum. Single-phase operation, without any mutual coupling whatsoever, is also possible. Finally, this model is not necessarily restricted to induction generators and a fixed power system load.

CHAPTER 4

SIMULATION OF THE GENERALIZED MODEL

4.1 GENERAL FLOW CHART

The mathematical model derived in Chapter 3 consists of 36 non-linear differential equations. The quantity of these differential equations actively employed depends on the nature of the case to be simulated. For example, a case involving two induction generators on each of the three phases of the distribution system would require the full complement of differential equations. All other cases would require less.

Fortunately, a general FORTRAN program for solving differential equations based upon the fourth-order RUNGE KUTTA method is available. This program was specifically designed to accommodate N simultaneous differential equations and has been tested thoroughly. Only slight modifications have been needed to adapt the original program to the total mathematical model of Chapter 3. A complete listing of the program is found in Appendix D.

In order to avoid editing the computer program according to the case to be simulated, the general flow chart depicted in Figure 4.1 will be used. The following sequences are symbolized in Figure 4.1:

Block 1: Data card is read which lists the primary variables of the computer program,

NV = total number of differential equations potentially utilized--always 36;

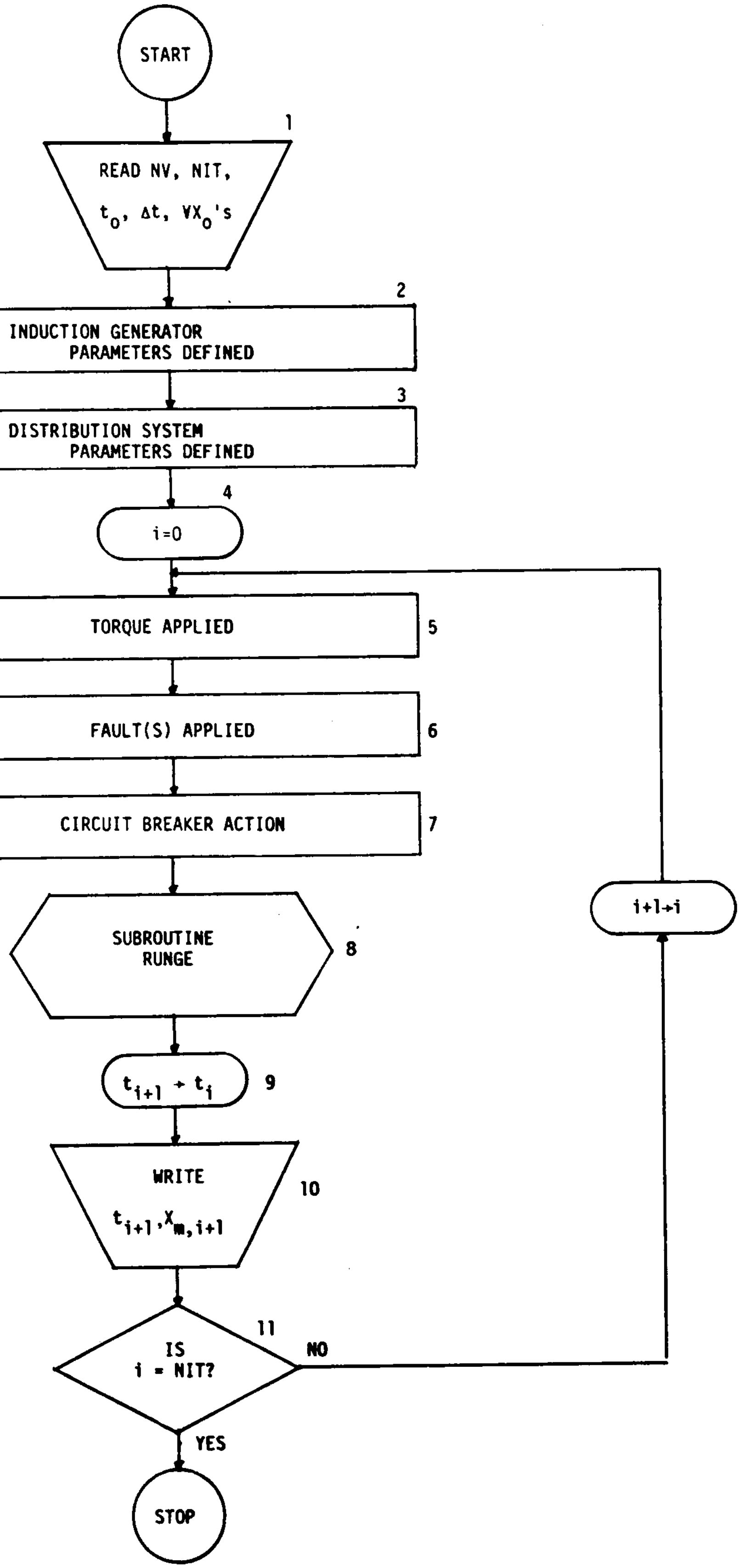

Figure 4-1. General Flow Chart

NIT = total number of time increments;

t_0 = starting time;

Δt = increment of time.

Furthermore, all initial conditions of all variables are read.

Block 2: Induction generator(s) parameters defined and/or calculated.

Block 3: Distribution system parameters defined and/or calculated which are independent of the state of the power system.

Block 4: Counter is set to zero; $i = 0 \Rightarrow t = 0$.

Block 5: Mechanical torque is applied as a function of time.

Block 6: Power system faults are applied/terminated as a function of time.

Block 7: Circuit breaker response is considered.

Block 8: Control is transferred to subroutine which approximates the values of all pertinent variables via the use of a function subprogram.

Block 9: After all applicable differential equations are approximated, the time counter is incremented by an amount Δt.

Block 10: All variables are written on magnetic tape for t_{i+1}; i.e., $\forall X_{m,i+1}$; $0 < M \leq NV$, and $i + 1 \leq NIT$.

Block 11: A check is performed to ascertain if NIT has been satisfied. If not, the counter is boosted by one increment; otherwise, all operations cease.

Since the heart of the computer simulation resides within the blocks numbered 5 through 8, these particular blocks will be examined in greater detail.

4.2 SIMULATION OF TORQUE, FAULT(S), AND CIRCUIT BREAKERS

4.2.1 Torque

Initially, mechanical torque will be simulated as a ramp function. This type of simulation is achieved as follows: Mathematically, a ramp function may be expressed as

$$f(t) = At + f(t_o), \text{ for } t \leq t_i$$

$$f(t) = \text{constant}, \quad \text{for } t > t_i.$$

Now letting

$t_n = t_o + \Sigma\Delta t_i$ = total accumulation of time

t_a = time at which full torque should be applied, T_{pm}

TM = A = rate of torque increase, $\Delta T_{pm}/\Delta t$, for $t \leq t_a$.

Then

$$T_{pm}(t_n) = (TM)t_n + T_{pm}(t_o), \text{ for } t_n \leq t_a$$

$$T_{pm}(t_n) = T_{pm}, \qquad \text{for } t_n > t_a$$

which can be described easily in FORTRAN language.

4.2.2 Fault(s)

For the application of fault(s), the following definitions will be made:

t_f = time of fault

t_{fd} = time of fault duration

t_{bf} = time before fault

t_{af} = time after fault.

These mathematical relationships then arise as a consequence:

$$t_{bf} = t_n - t_f \tag{4-1}$$

$$t_{af} = t_{bf} - t_{fd} \tag{4-2}$$

where, as before, $t_n = t_o + \Sigma\Delta t_i$.

Since $t_n \geq 0$, Equations (4-1) and (4-2) may be scrutinized in this fashion:

For $0 \leq t_n \leq t_f$: Interval of time before fault,

$$-t_f \leq t_{bf} \leq 0$$

$$-(t_f + t_{fd}) \leq t_{af} \leq -t_{fd}.$$

For $t_f < t_n \leq t_{fd} + t_f$: Fault period,

$$0 < t_{bf} \leq t_{fd}$$

$$-t_{fd} < t_{af} \leq 0.$$

Therefore, during the interval of time before any power system fault,

$$t_{bf} \leq 0 \text{ and}$$

$$t_{af} < 0,$$

both of which are true, simultaneously, only within this period of time. Likewise, during the fault period

$$t_{bf} > 0 \text{ and}$$

$$t_{af} \leq 0.$$

The above restrictions may be written as:

$$\text{IF}(t_{bf} \leq 0 \ \& \ t_{af} < 0)Y = Y_{bf}$$

$$\text{IF}(t_{bf} > 0 \ \& \ t_{af} \leq 0)Y = Y_{f}$$

where

Y => appropriate admittance

Y_{bf} => admittance before fault

Y_{f} => fault admittance.

Subscripts may be added to identify any one of the six sections of the distribution system and/or induction generator buses. As an example, suppose a three-phase fault occurs at bus number 1. From Appendix A, the following admittances are affected:

$$Y_{ab1}$$ Y_{bc1}?

$$Y_{ac1}.$$

Therefore, for a given t_f, t_{fd}, and Y_f, this fault may be simulated as

$$IF(t_{bf} \leq 0 \;\&\; t_{af} < 0) Y_{ab1} = Y_{ac1} = Y_{bf}$$

$$IF(t_{bf} > 0 \;\&\; t_{af} \leq 0) Y_{ab1} = Y_{ac1} = Y_{f}.$$

4.2.3 Circuit Breaker Response

For circuit breaker action, another mathematical relationship may be defined based upon the following:

t_{bd} = time of breaker duration; breaker is opened

t_{br} = time of breaker reclosure

such that

$$t_{br} = t_{af} - t_{bd}$$

which implies that the breaker trips at t_{af}. However, breakers attempt to trip at zero current and not at some predetermined time. To accommodate this, a modified version of the epsilon/delta approach will be used. Let

$$t_q = t_f + t_{fd} - t_\varepsilon \text{ and}$$

$$t_{qq} = t_q + t_{bd} + t_\varepsilon$$

where

t_q => guess time to the left of breaker time interval

t_{qq} => upper bound of breaker time interval

t_ε => some small arbitrary interval of time wherein the computer will compare the magnitude of the fault current to some desired tolerance level.

Then,

$$\text{For } t_q + t_\varepsilon < t_n \le t_{qq}\text{:} \quad \text{Breaker response period,}$$

$$-t_{bd} < t_{br} \le 0$$

$$t_{fd} < t_{bf} \le t_{fd} + t_{bd}$$

$$0 < t_{af} \le t_{bd}$$

$$\therefore \quad t_{br} \le 0$$

$$t_{bf} > 0$$

$$t_{af} > 0.$$

$$\text{For } t_n > t_{qq}\text{:} \quad \text{Time of breaker reclosure,}$$

$$t_{br} > 0$$

$$t_{bf} > t_{bd} + t_{fd}$$

$$t_{af} > t_{bd}.$$

Again, all of these restrictions apply only within these prescribed intervals of time.

Now let

$|\varepsilon'|$ = magnitude of the fault current at which the circuit breaker is allowed to trip--somewhere close to zero current.

Then, by incorporating all of the above restrictions, the following logical expression describes how the circuit breaker will trip:

$$\text{IF}(t_n \ge t_q \ \& \ t_n \le t_{qq} \ \& \ -\varepsilon' \le I_f \le \varepsilon')\text{TRIP CIRCUIT BREAKER.}$$

Similarly, to reclose the breaker,

$$\text{IF}(t_{af} > t_{bd})\text{RECLOSE CIRCUIT BREAKER.}$$

The only problem remaining is the manner in which the computer simulates the tripping and reclosing of the breaker. Thus far, it knows when to do it but not how to do it. A common parameter, S, will be used in the general program and function subprogram as a means of solving this problem. The value of this parameter will depend on the outcome of the above logical expressions. In turn, certain functions in the subprogram will change accordingly. That is,

$$\text{IF}(t_n \geq t_q \;\&\; t_n \leq t_{qq} \;\&\; -\varepsilon' \leq I_f \leq \varepsilon')S = 0.$$

$$\text{IF}(t_{af} > t_{bd})S = 1.$$

When the requirements of the first logical expression are met, the fault current is forced to zero by S being employed as a function multiplier. For instance,

$$\frac{dX(N)}{dt} = S * \frac{dX(N)}{dt} = 0$$

$$X(N) = S * X(N) = 0$$

where

$\frac{dX(N)}{dt}$ => the appropriate differential equation

$X(N)$ => the integrated approximation of the particular function--in this case, fault current.

For breaker reclosure, $S \equiv 1$, and

$$\frac{dX(N)}{dt} = S * \frac{dX(N)}{dt} = \frac{dX(N)}{dt}$$

$$X(N) = S * X(N) = X(N)$$

thereby proffering the capability of simulating circuit breaker responses.

4.3 SUBROUTINE RUNGE

The classical fourth-order RUNGE KUTTA numerical approximation method consists of the following equation:

$$X_{i+1} = X_i + 1/6k_o + 1/3k_1 + 1/3k_2 + 1/6k_3 \tag{4-3}$$

where

$$k_o = f(t_i, X_i)\Delta t$$

$$k_1 = f(t_i + 1/2\Delta t, X_i + 1/2k_o)\Delta t$$

$$k_2 = f(t_i + 1/2\Delta t, X_i + 1/2k_1)\Delta t$$

$$k_3 = f(t_i + \Delta t, X_i + k_2)\Delta t.$$

For M simultaneous differential equations, Equation (4-3) becomes

$$X_{m,i+1} = X_{m,i} + 1/6k_{m,o} + 1/3k_{m,1} + 1/3k_{m,2} + 1/6k_{m,3} \tag{4-4}$$

where M varies from 1 to NV, or

$$1 \leq M \leq NV \Rightarrow 1 \leq M \leq 36 \text{ and}$$

$$i \ni 0 \leq i \leq NIT.$$

Equation (4-4) is estimated in Block 8 of the general flow chart depicted in Figure 4.1. Figure 4.2 describes the procedures leading up to the approximate solution of Equation (4-9). The subroutine RUNGE follows these procedures throughout the domains of M and i.

In Figure 4.2, the following methodology is realized in subroutine RUNGE:

Block a:	For a given i from the general flow chart, M is set to 1.
Block b:	$k_{m,o}$ for the particular i and M is computed.
Block c:	Subsidiary counter, ℓ, is set to 2.
Blocks d-e:	$k_{m,\ell-1}$ for the particular i, M, and ℓ is computed.
Block f:	Check is performed to see if $\ell = 3$. If $\ell \neq 3$, the ℓ counter is incremented by 1 and control transfers back to Block d. If $\ell = 3$, this $\Rightarrow$ $k_{m,1}$ and $k_{m,2}$ have been computed.
Blocks g-h:	$k_{m,3}$ for the particular i and M is computed.
Block i:	$X_{m,i+1}$ is estimated.
Block j:	Check is performed to see if M = NV has been satisfied. If not, M is incremented by 1 and control returns to Block b. Otherwise, control is transferred to Block 9 in the general flow chart.

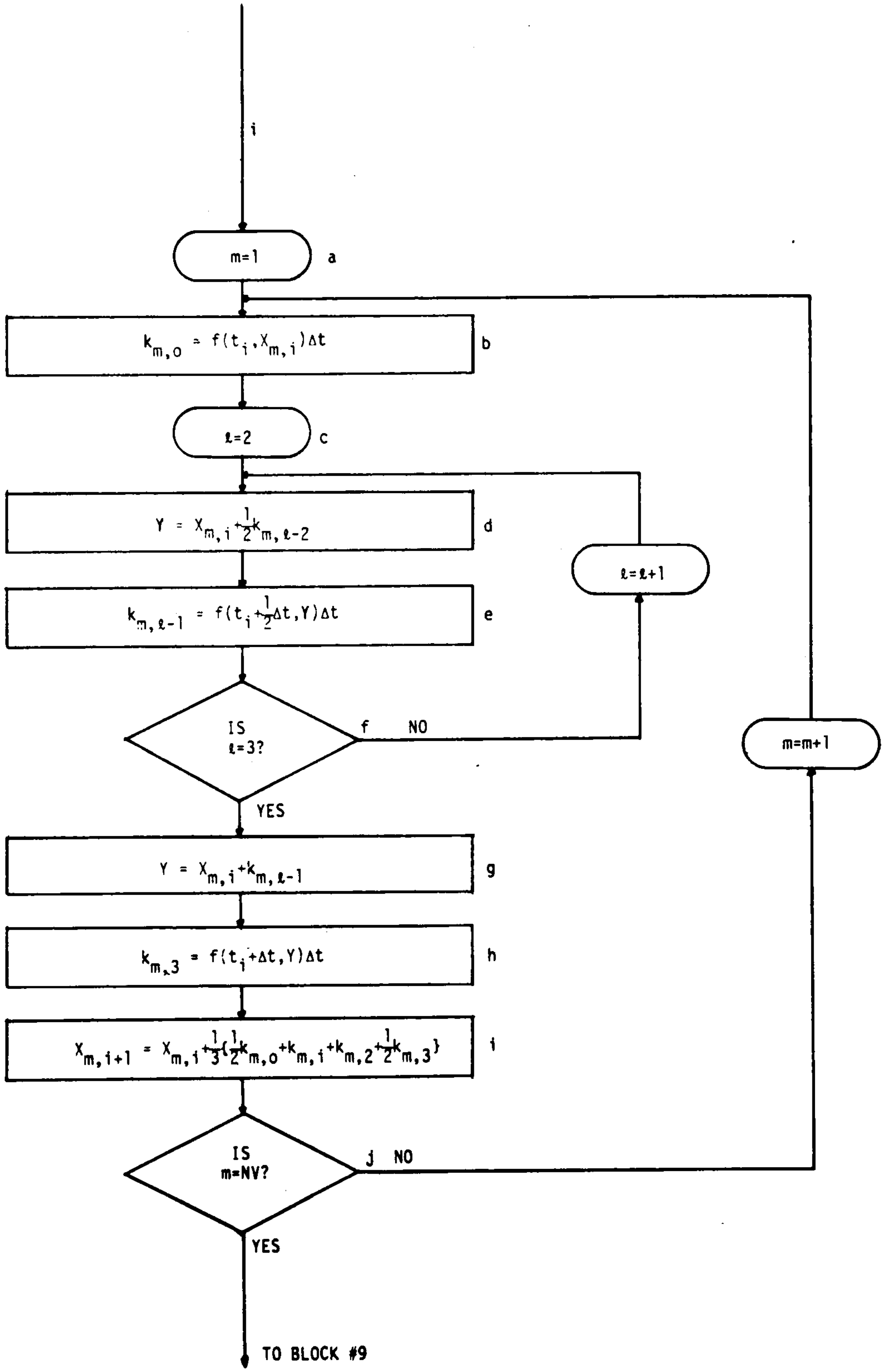

Figure 4-2. Flow Chart of Subroutine RUNGE

4.4 CASES TO BE SIMULATED

Three cases will be simulated on the digital computer as a means of testing the generalized mathematical model of Chapter 3. All power system variables will be calculated and stored on magnetic tape. Selected variables will be plotted.

4.4.1 Preliminary Simulation

Case 1 will be a simulation of the three-phase distribution model. This involves simulating the electrical tie-in between the infinite bus and distribution system. No induction generators are involved whatsoever. A unity power factor load is assumed to exist at bus number 2. The purpose of this simulation is to obtain steady-state values of power system variables which will be used as initial conditions for the induction generator cases which follow. This is tantamount to saying that prior to any induction generator activity, the power system is electrically active. All mutual coupling between the phases of the distribution system is considered in Case 1.

4.4.2 Composite System Simulation

A single-phase induction generator will be connected to bus number 1 on phase A of the distribution system. At $t = 0^-$, the rotor velocity of the induction generator is assumed to be 377 radians/sec. At $t = 0^+$, a mechanical torque is applied to the rotor of the induction generator which is, simultaneously, attached to the three-phase distribution system. Approximately 1/10 of a second later, a single-phase to ground fault is applied at the bus of the induction generator. The duration of this fault will be at least three cycles. A single-phase circuit breaker will clear this fault at a zero current crossing and will remain opened for a maximum of three additional cycles. Breaker reclosure occurs at this time.

Case #2 will include all mutual coupling between phases. Case #3 is similar to Case #2 except for the inclusion of mutual coupling and corresponding initial conditions. Appropriate observations will then be made regarding these two cases.

4.5 RESULTS OF COMPUTER SIMULATION

The results of computer simulation will be found in Figures 4.3-4.26. All computer plots are labeled according to case and variables. As denoted previously, the entire FORTRAN program utilized in this study is found in Appendix D.

4.5.1 Distribution System Tie-In

Figures 4.3 and 4.4 describe the transient response of the three-phase distribution system. Figure 4.3 is a computer plot of the voltages at bus #1 originally represented in Figure 3.1 on page 11. Likewise, Figure 4.4 is a plot of the voltages at bus #2. It will be noted from these plots that all transients essentially die out within 4 msec. Furthermore, the waveform of the transient at bus #2 differs from that at bus #1, implying that system impedance as seen from bus #2 is different than at bus #1. It will be seen later that all power system transients appear to be more severe in nature at bus #2.

4.5.2 Representative Plots

Figures 4.5-4.26 are plots of distribution system voltages and currents for Cases #2 and #3. Induction generator currents are plotted

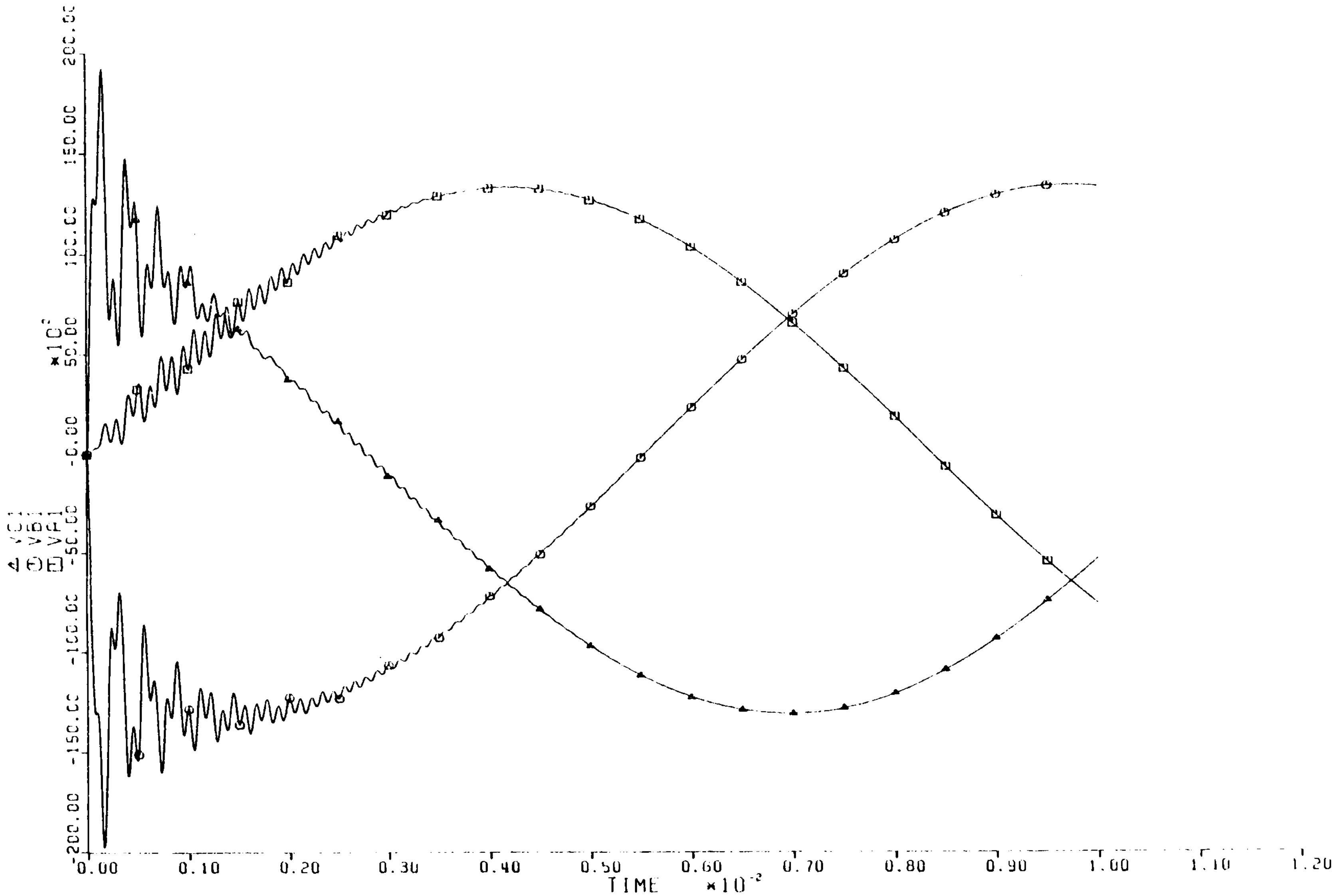

Figure 4.3. Case #1: Bus #1 Voltages

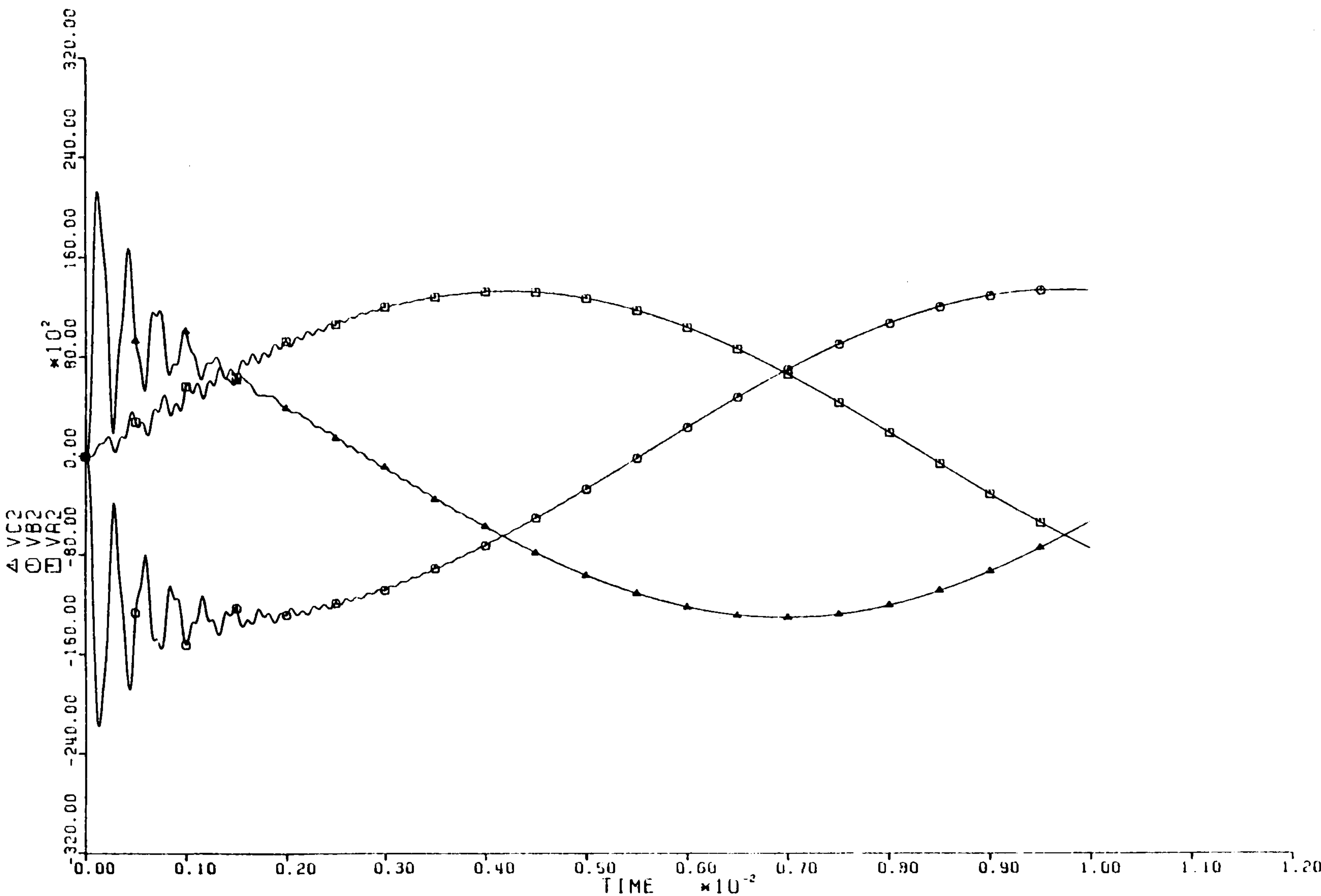

Figure 4.4. Case #1: Bus #2 Voltages

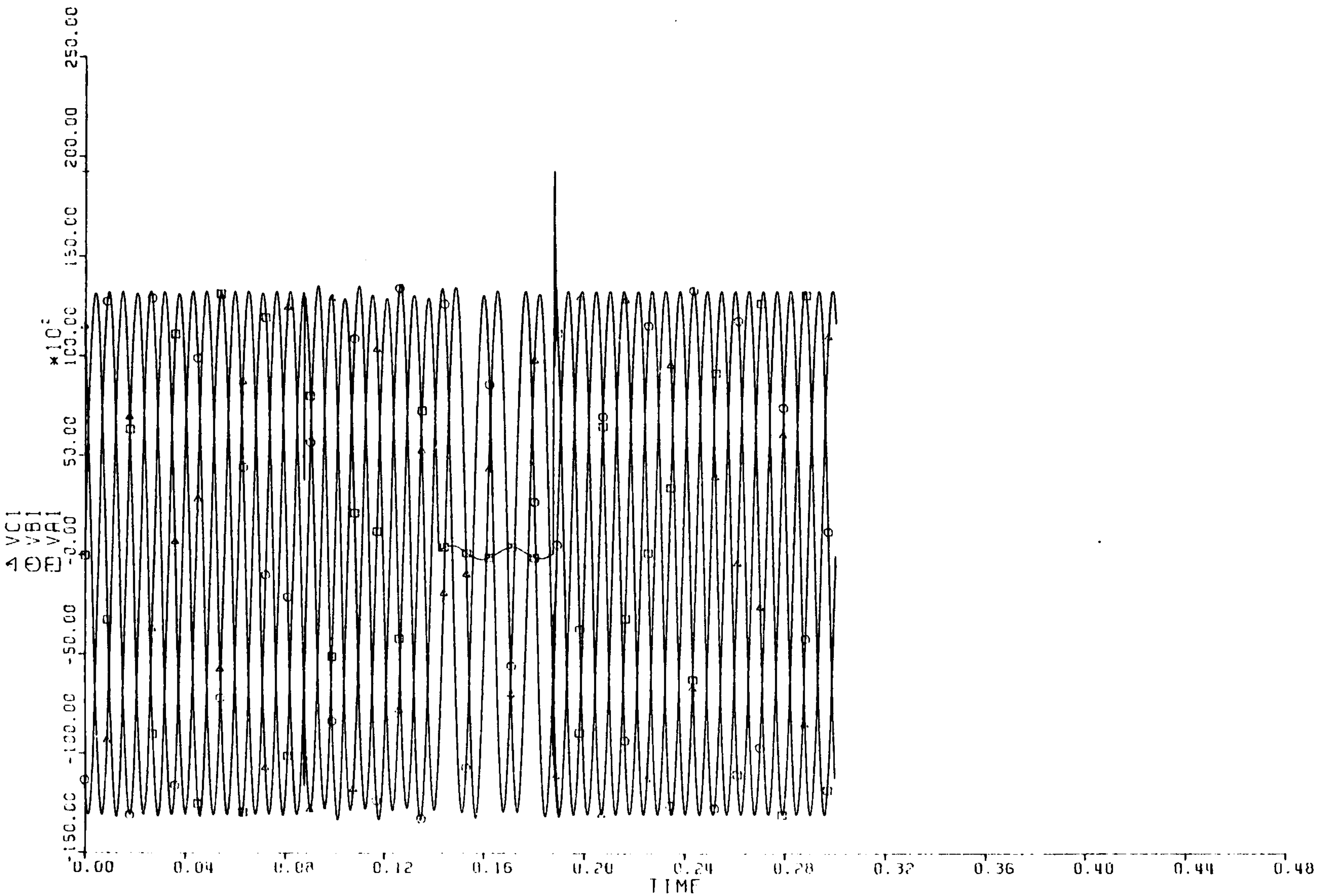

Figure 4.5. Case #2: Bus #1 Voltages

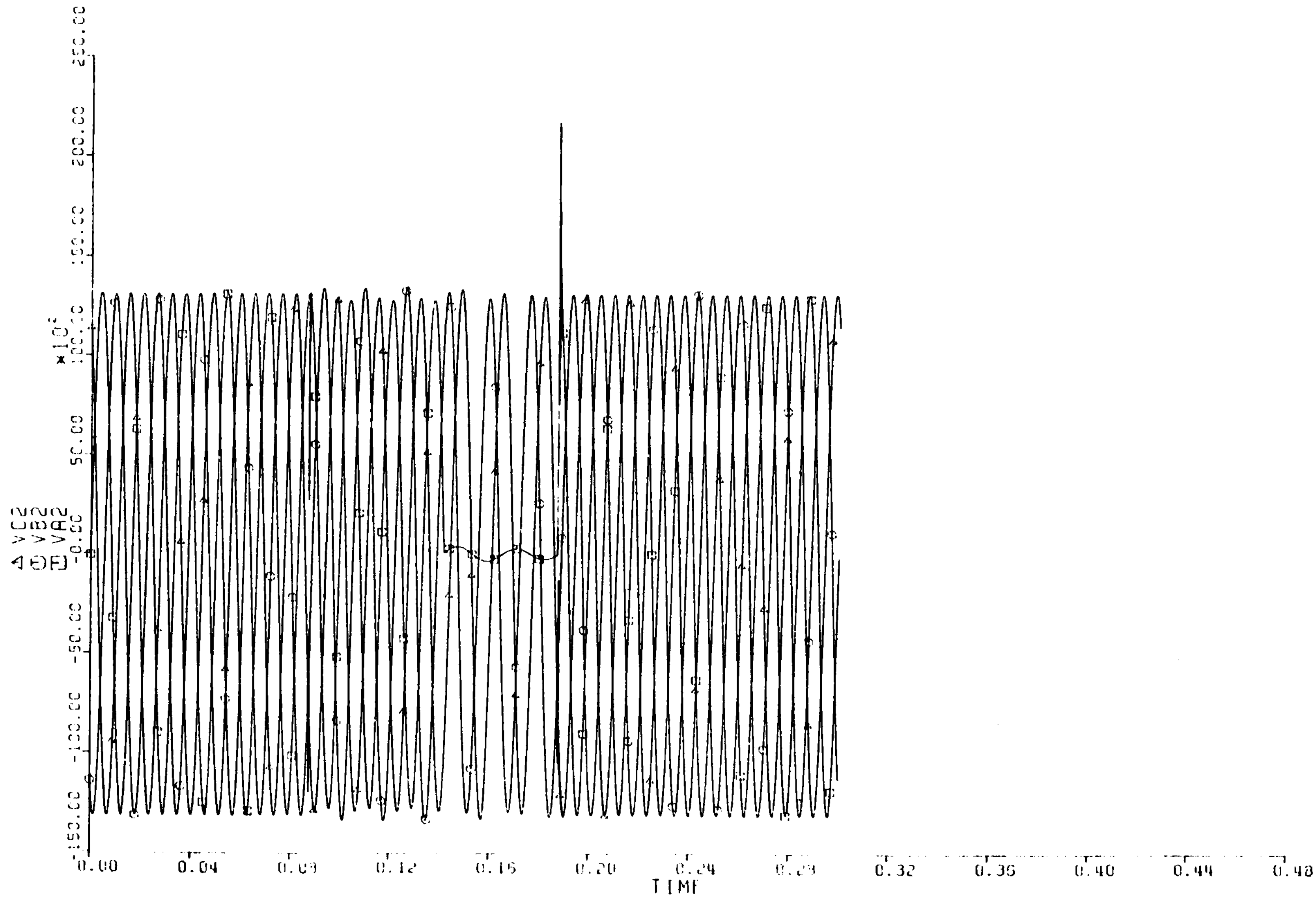

Figure 4.6. Case #2: Bus #2 Voltages

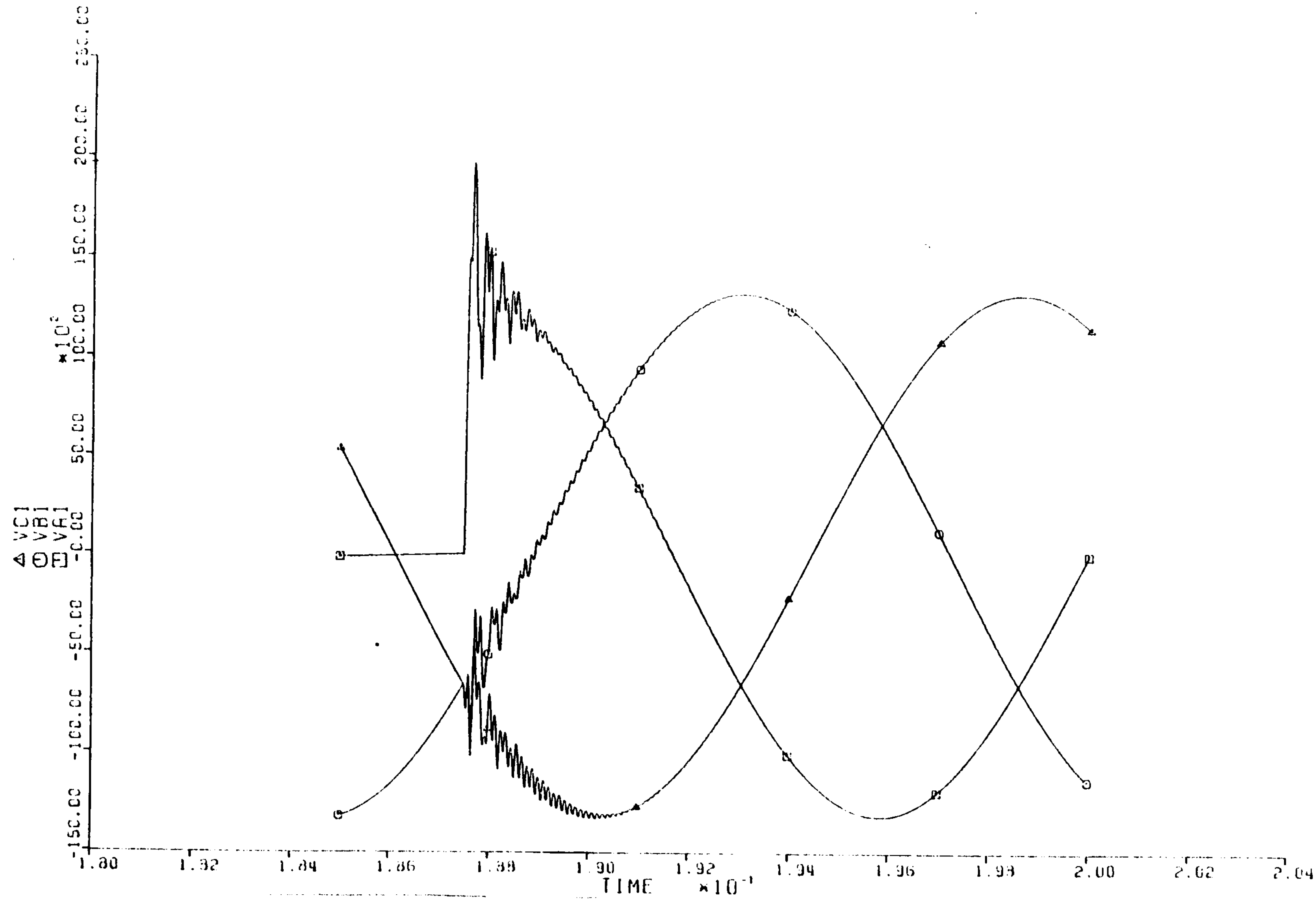

Figure 4.7. Case #2: Bus #1 Voltages--Breaker Reclosure

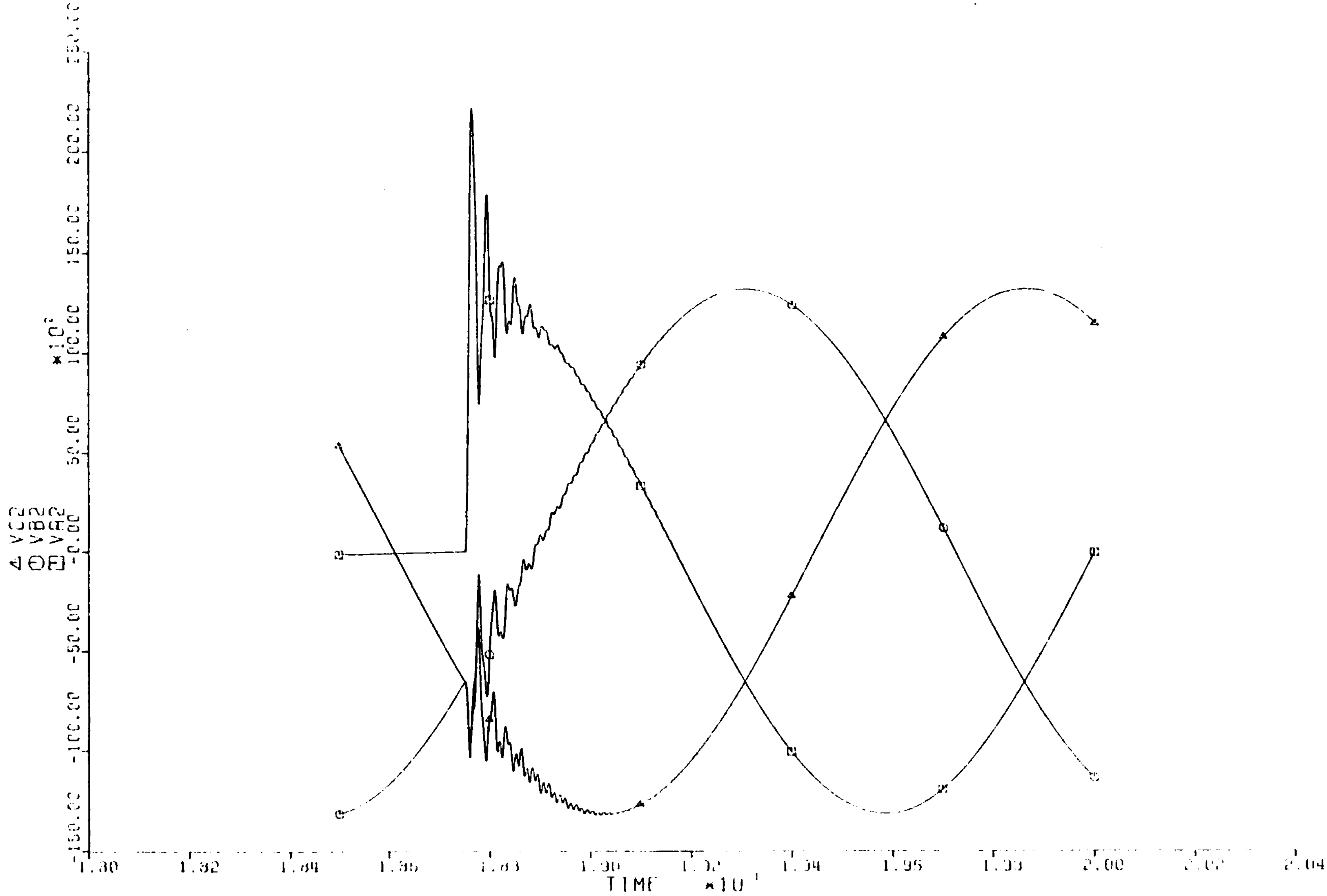

Figure 4.8. Case #2: Bus #2 Voltages--Breaker Reclosure

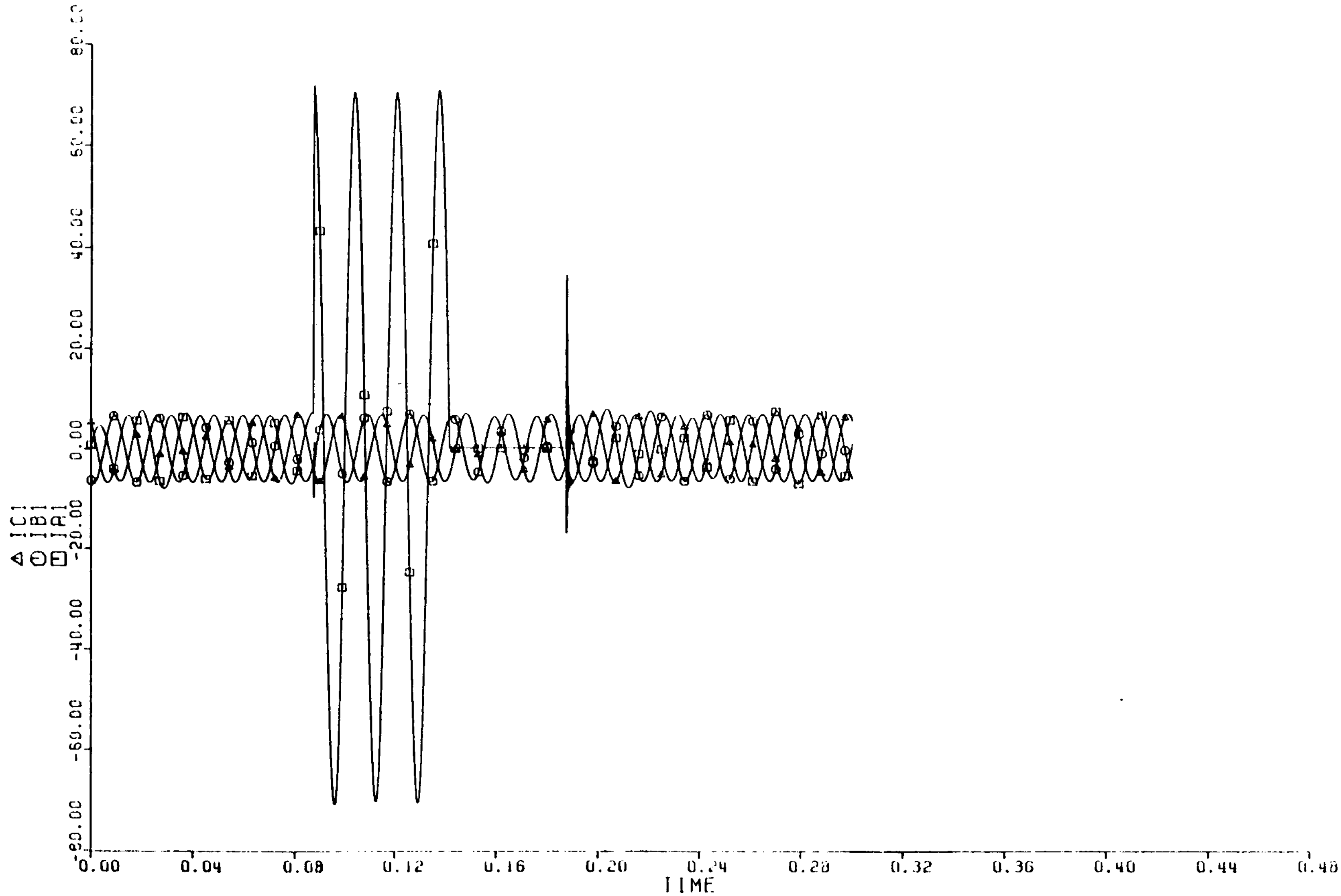

Figure 4.9. Case #2: Line Currents to Bus #1

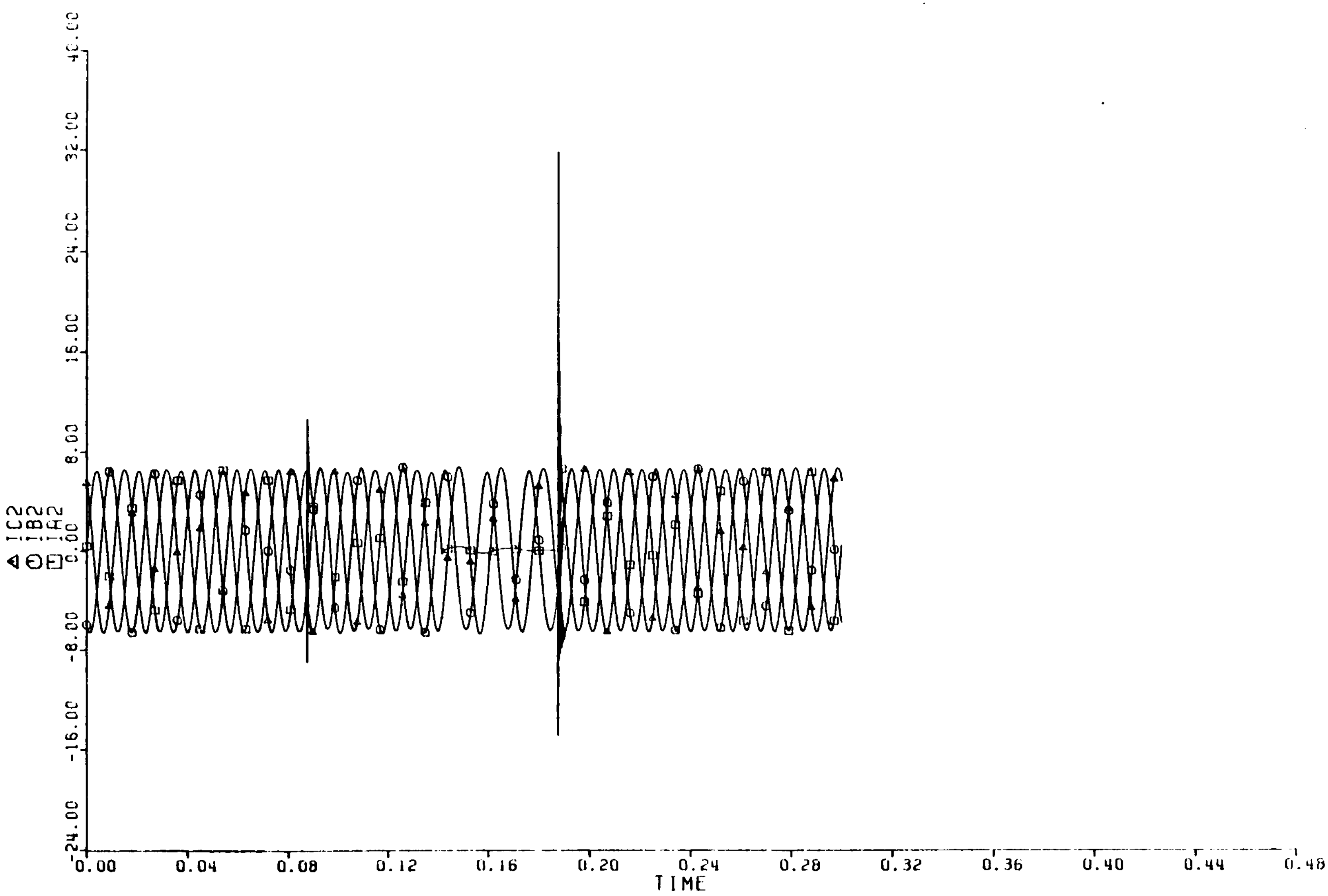

Figure 4.10. Case #2: Line Currents to Bus #2

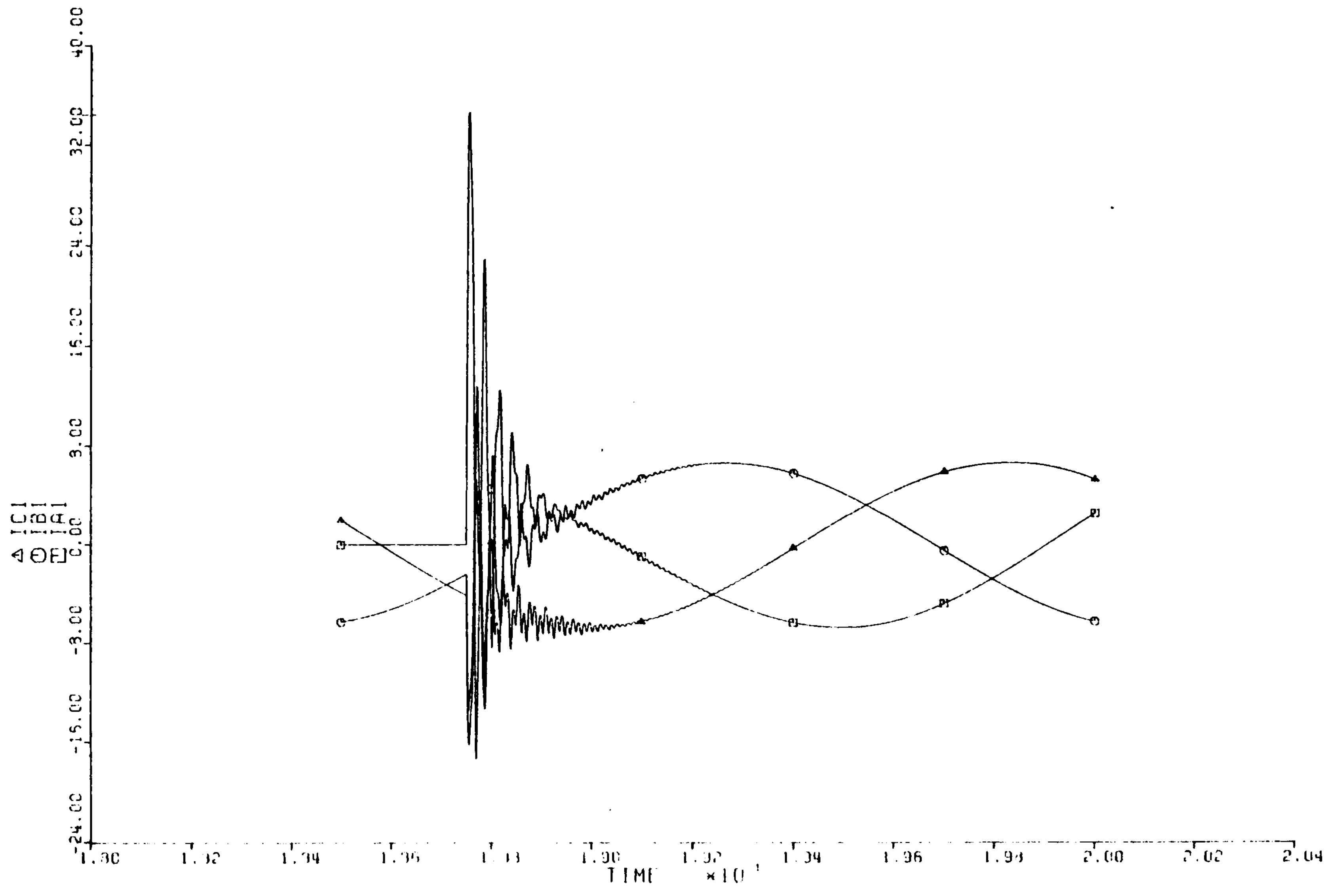

Figure 4.11. Case #2: Line Currents to Bus #1--Breaker Reclosure

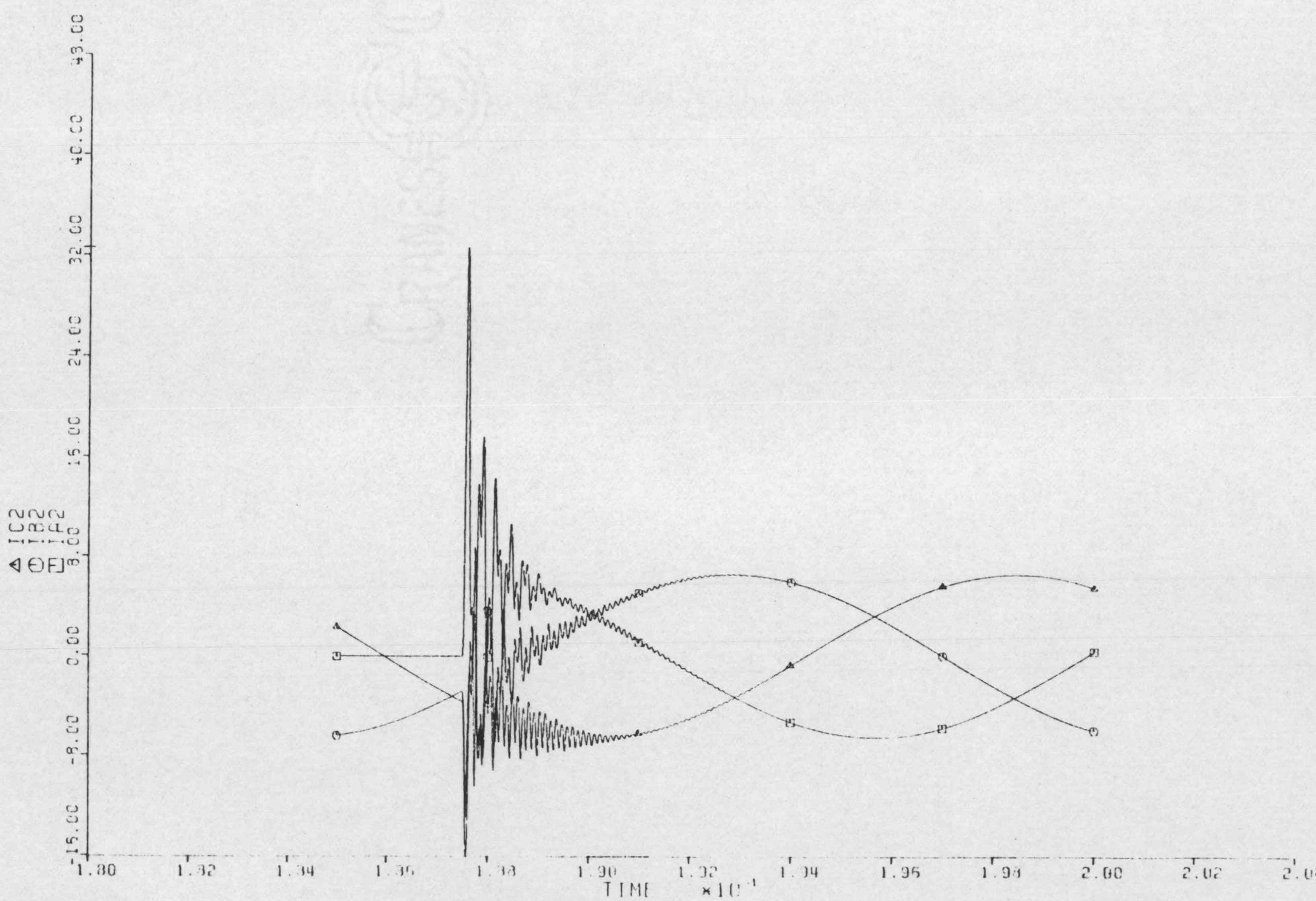

Figure 4.12. Case #2: Line Currents to Bus #2--Breaker Reclosure

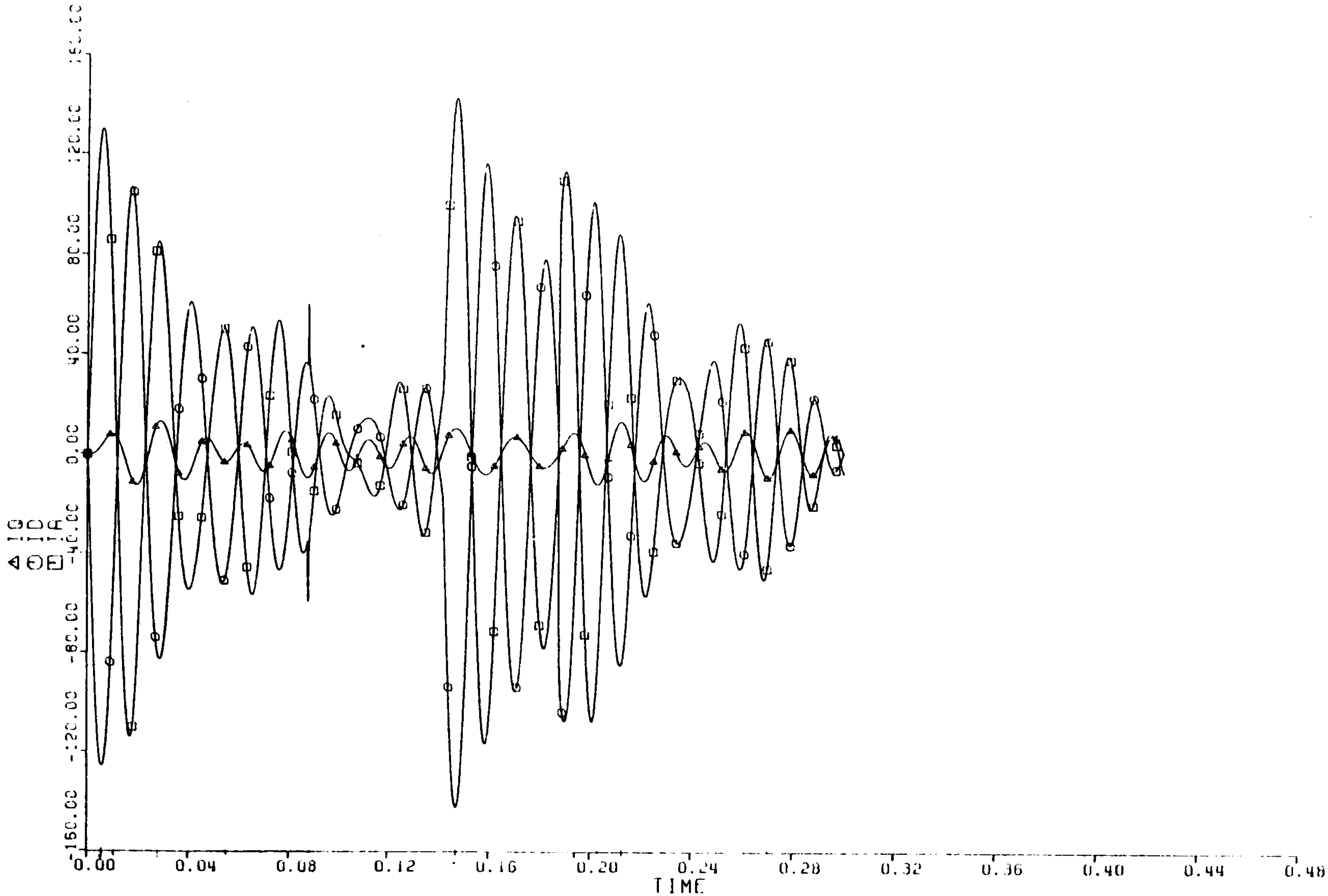

Figure 4.13. Case #2: Armature, Direct, and Quadrature Currents of Induction Generator

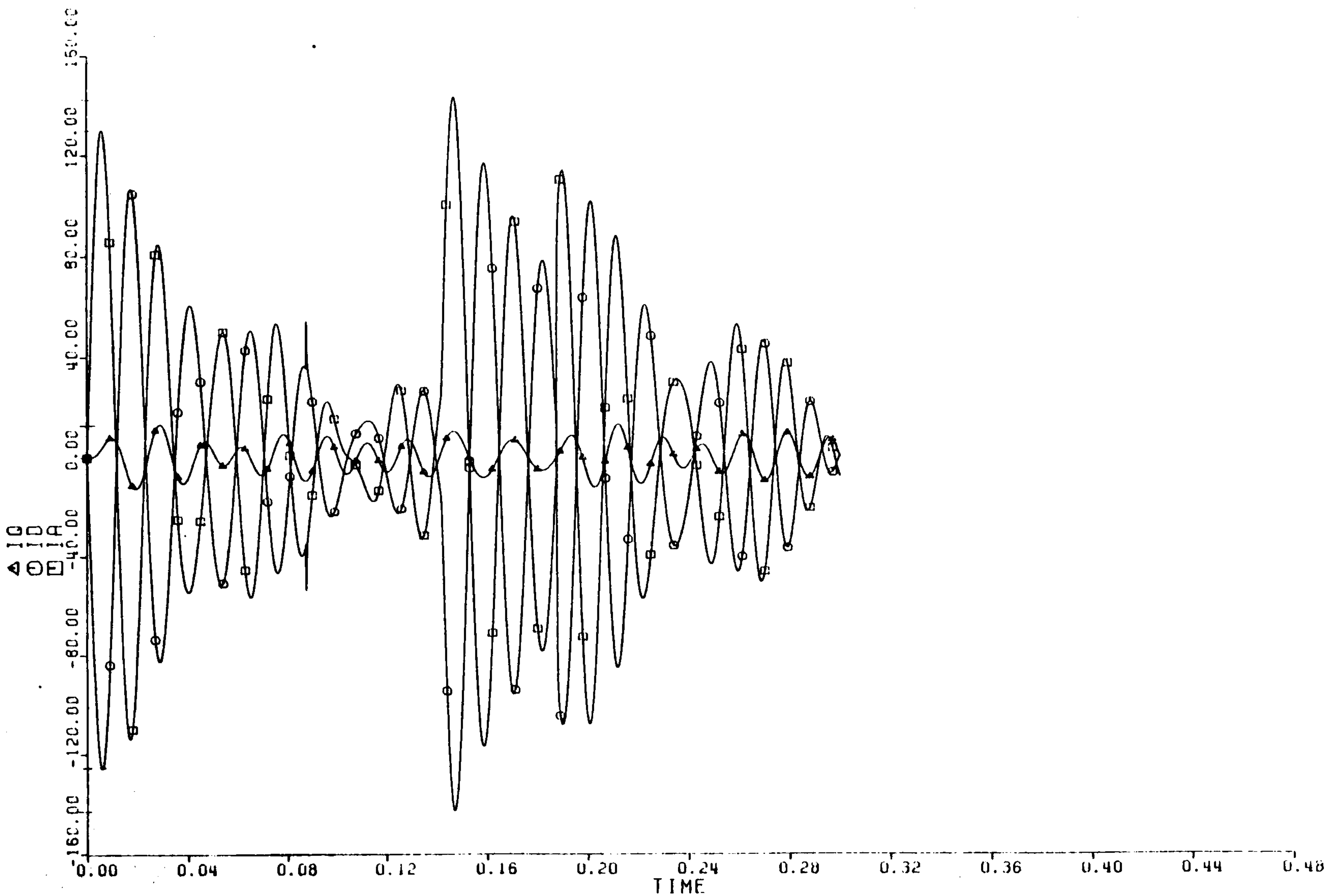

Figure 4.14. Case #3: Armature, Direct, and Quadrature Currents of Induction Generator

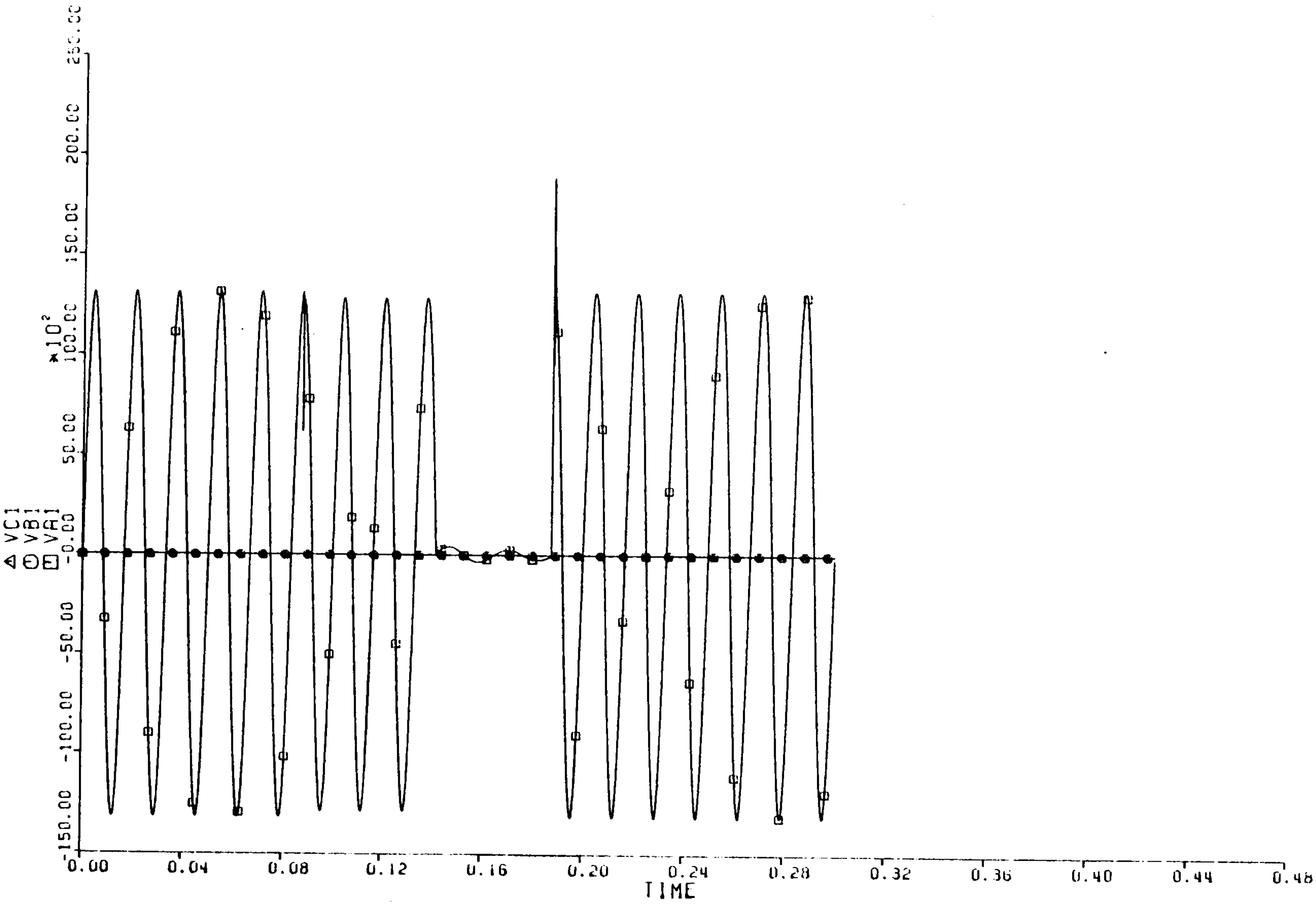

Figure 4.15. Case #3: Bus #1 Voltages

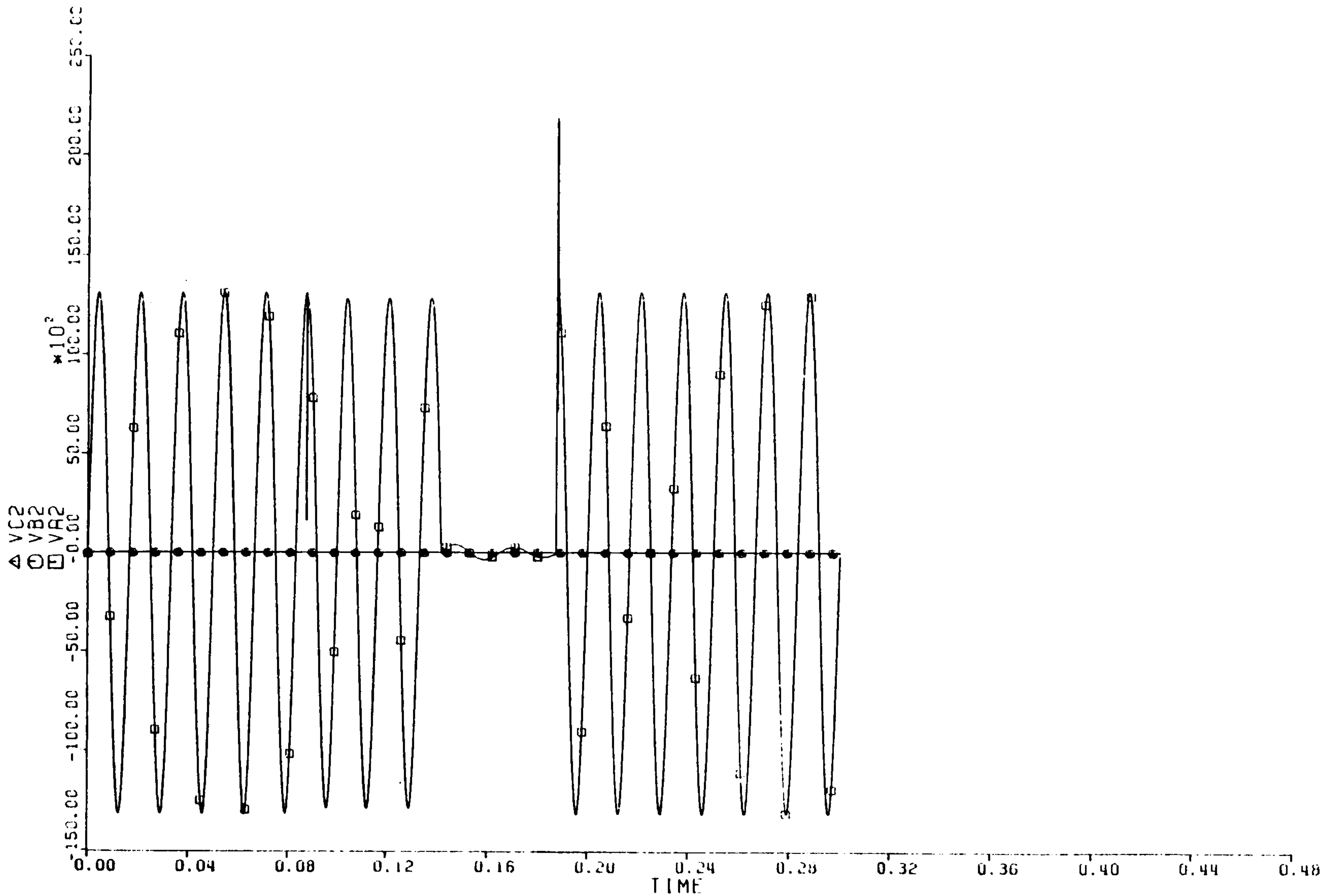

Figure 4.16. Case #3: Bus #2 Voltages

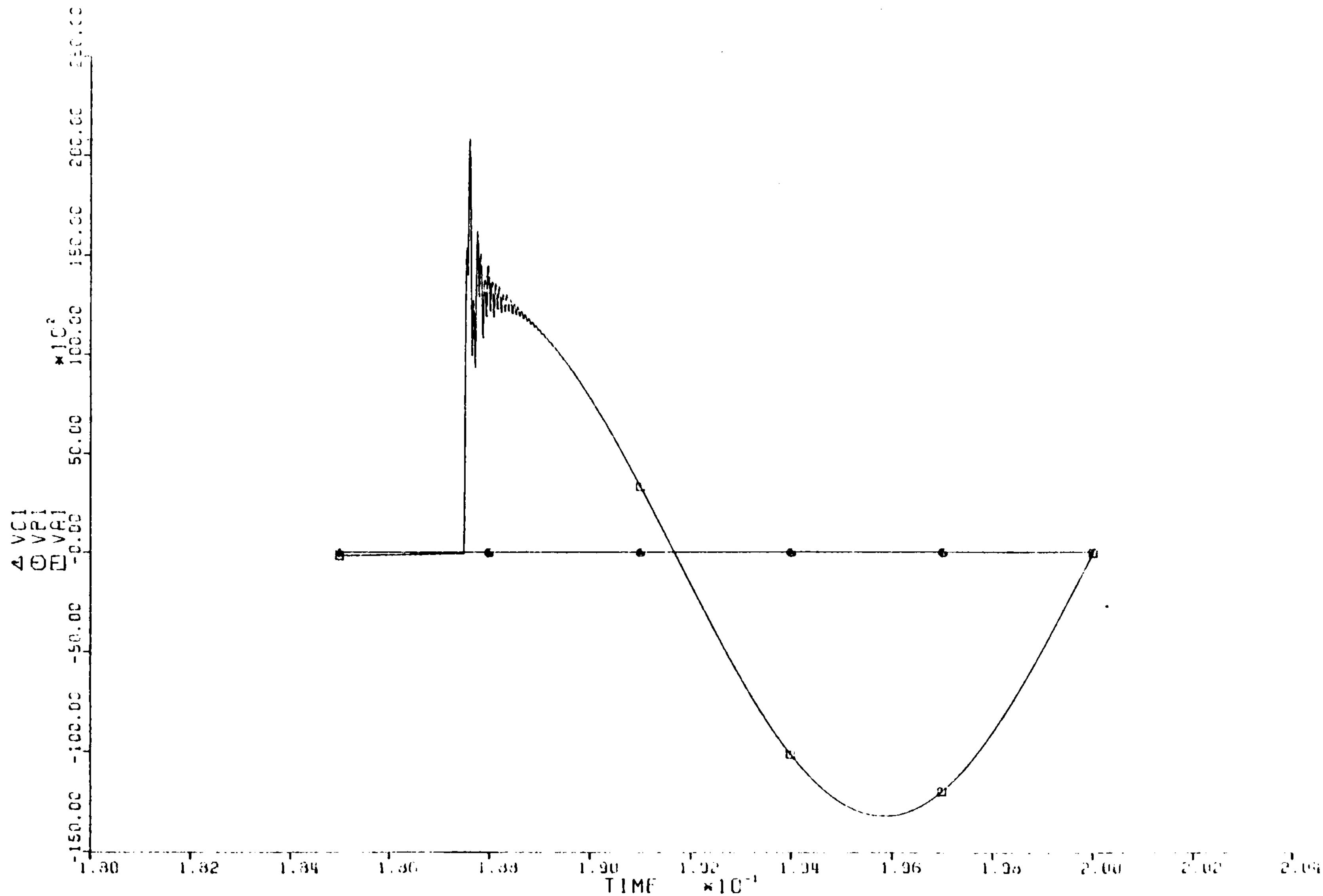

Figure 4.17. Case #3: Bus #1 Voltages--Breaker Reclosure

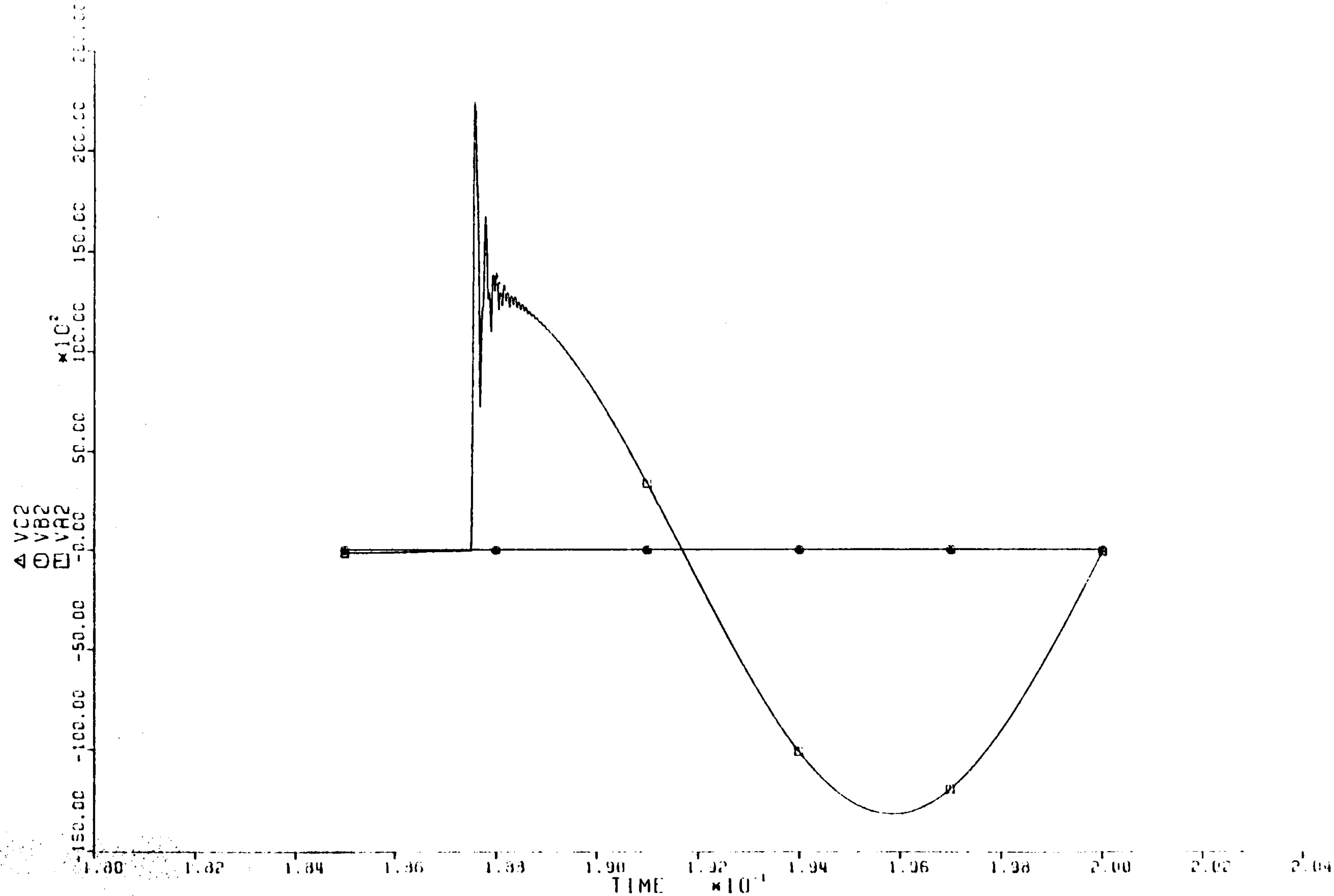

Figure 4.18. Case #3: Bus #2 Voltages--Breaker Reclosure

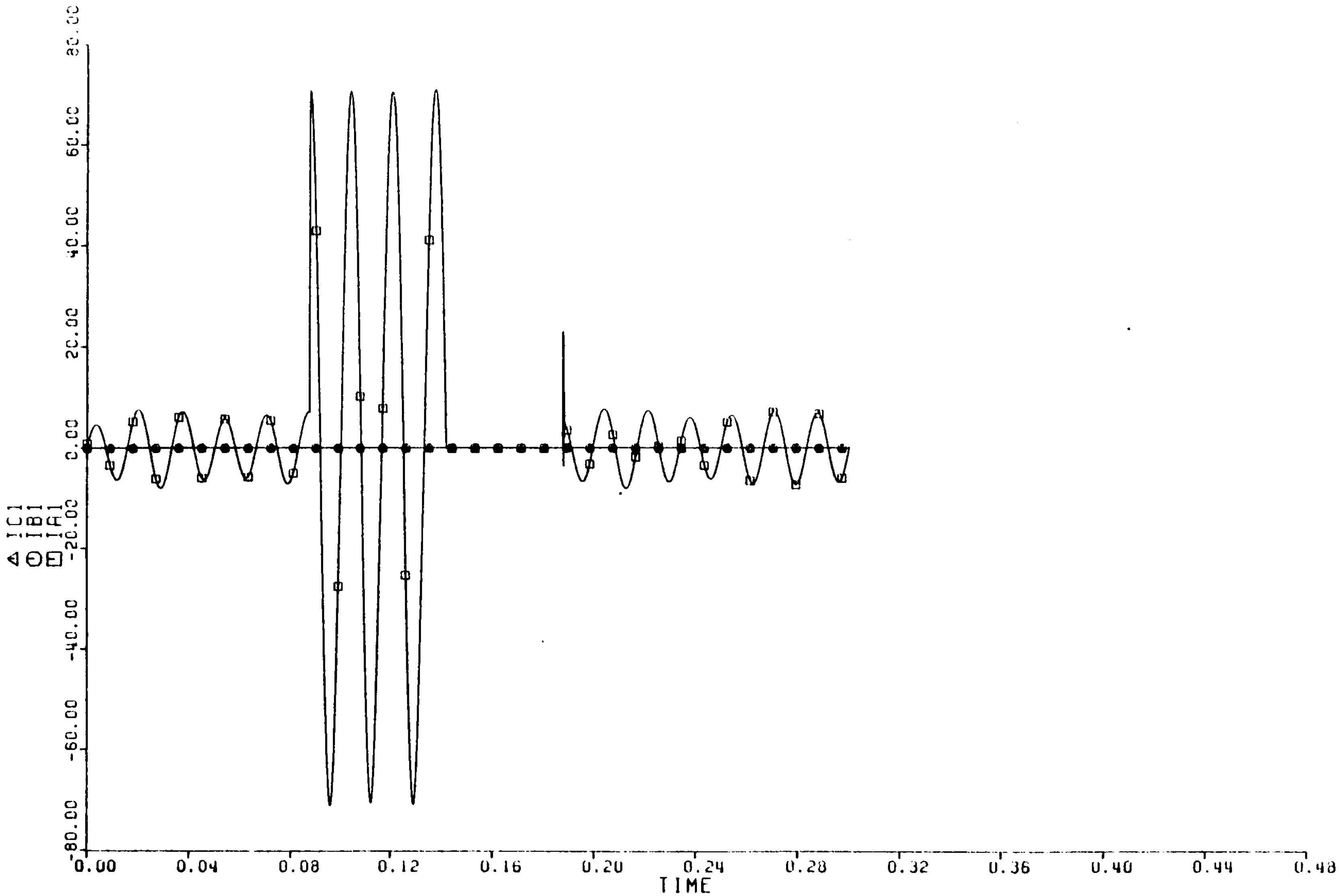

Figure 4.19. Case #3: Line Current to Bus #1

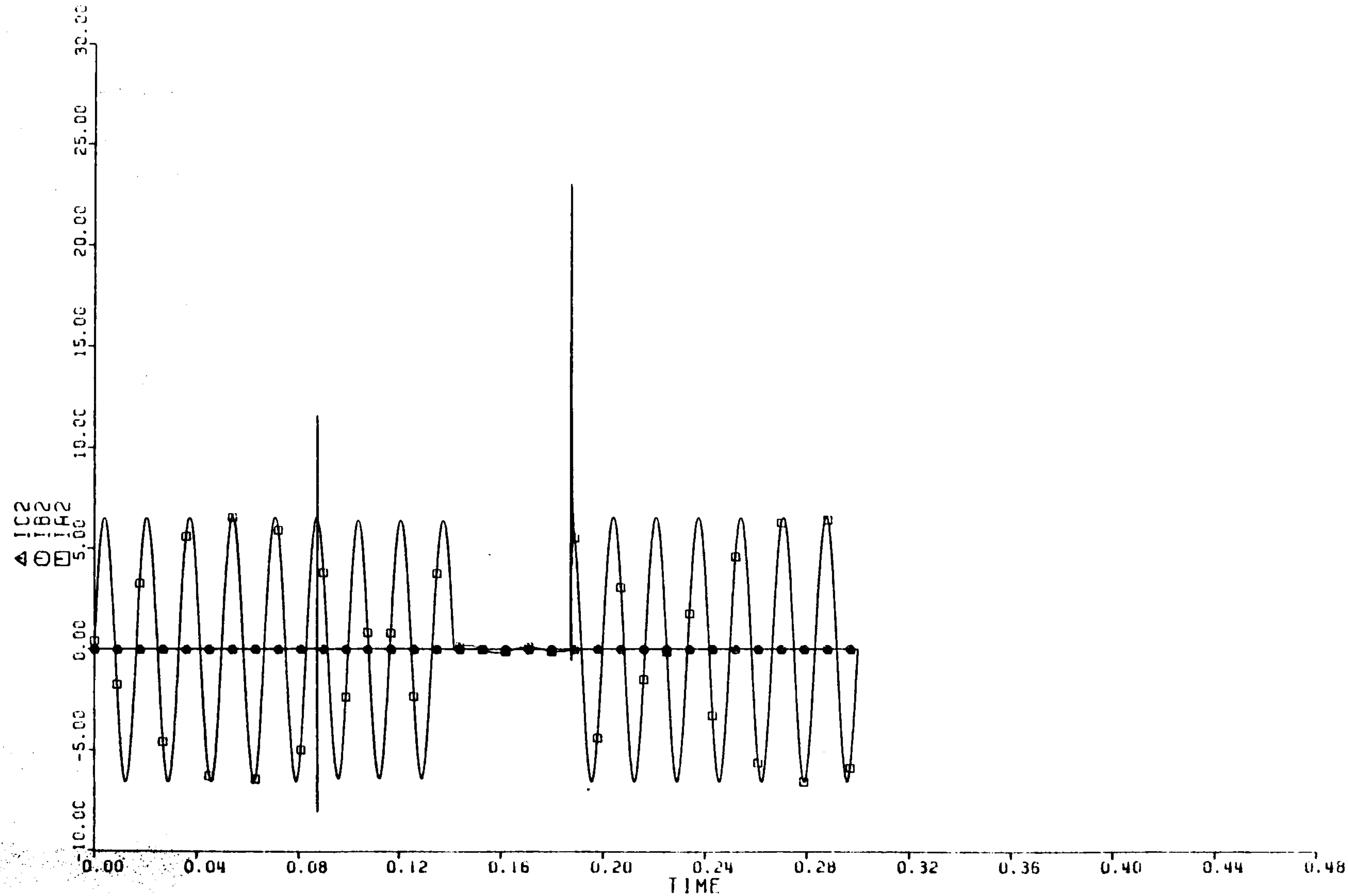

Figure 4.20. Case #3: Line Current to Bus #2

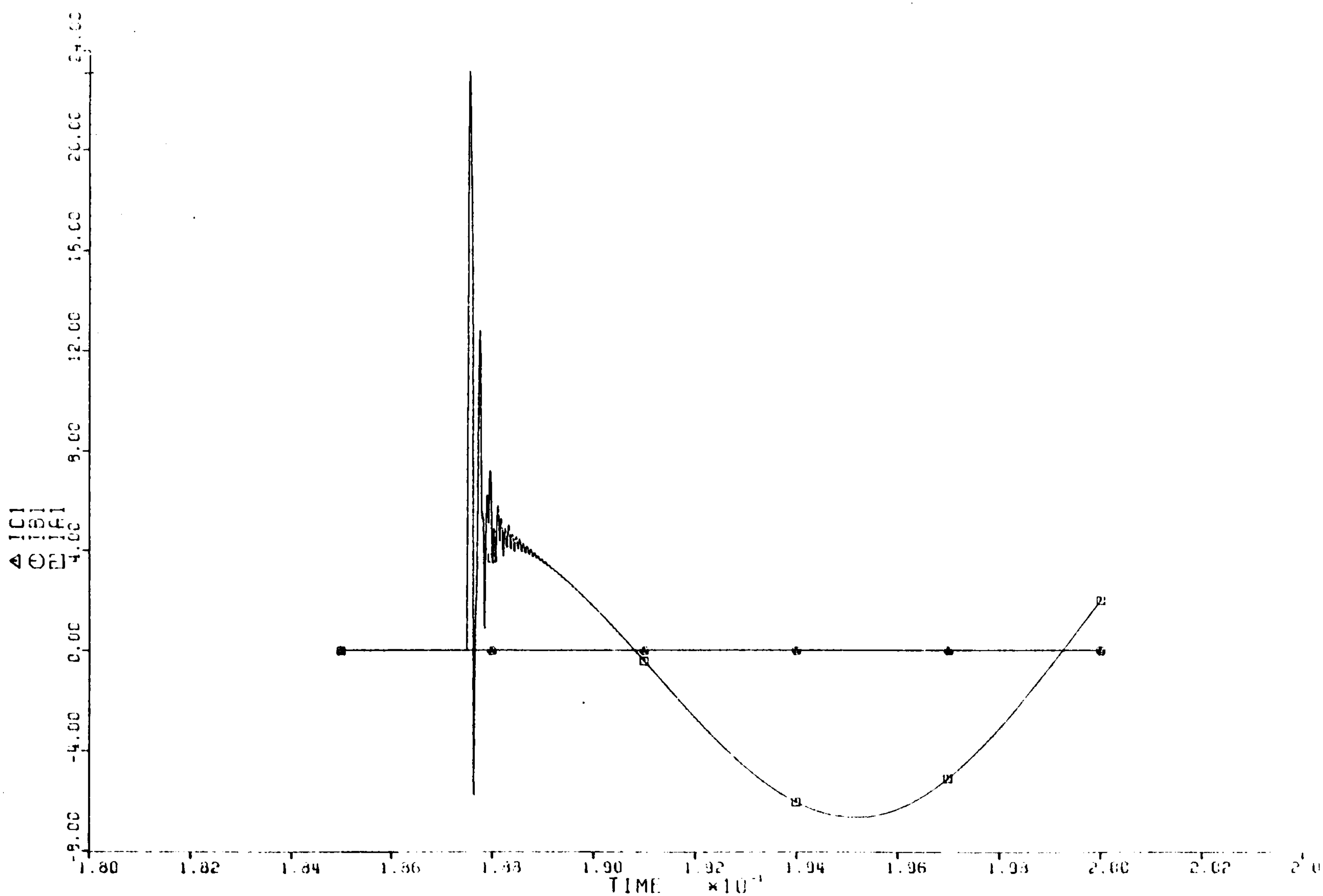

Figure 4.21. Case #3: Line Current to Bus #1--Breaker Reclosure

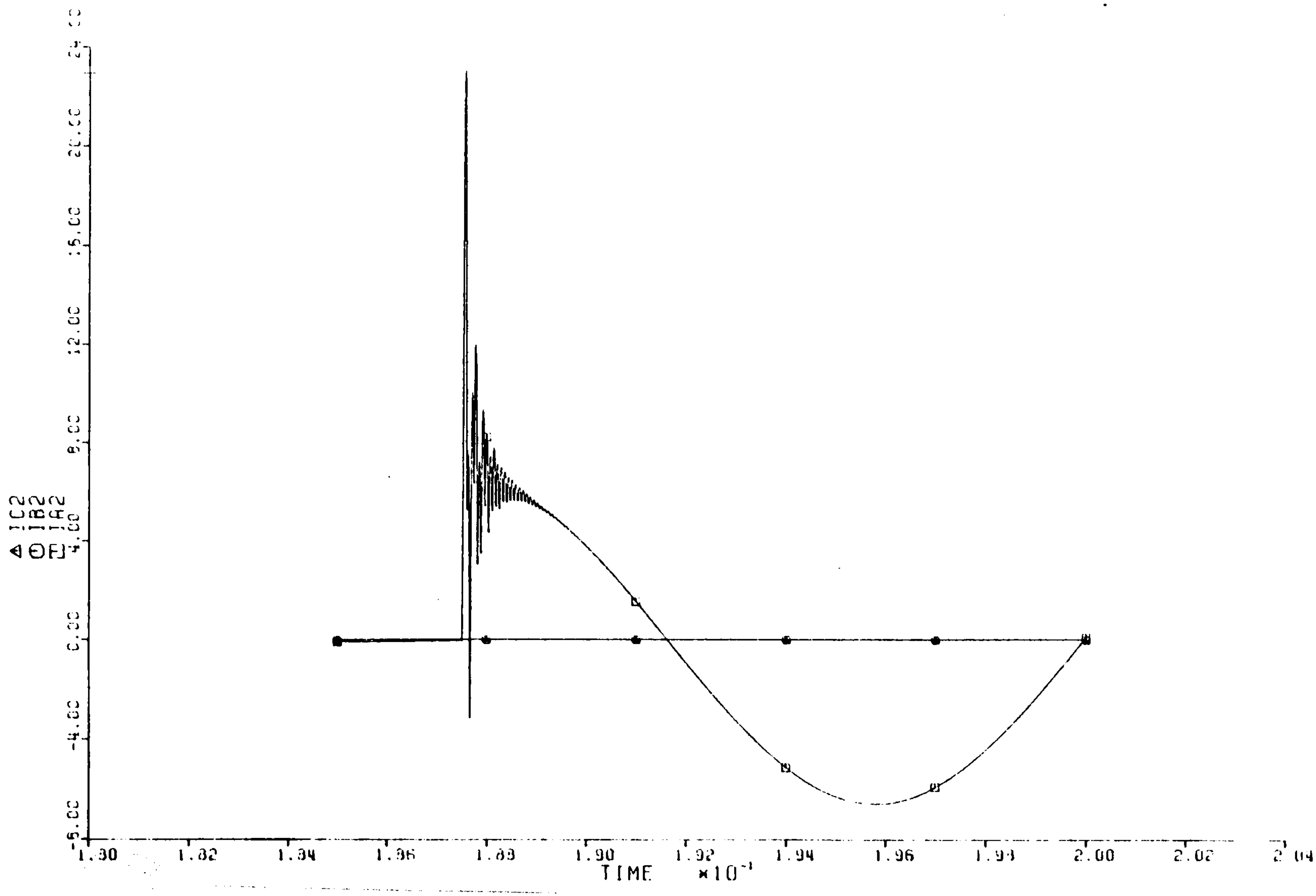

Figure 4.22. Case #3: Line Current to Bus #2--Breaker Reclosure

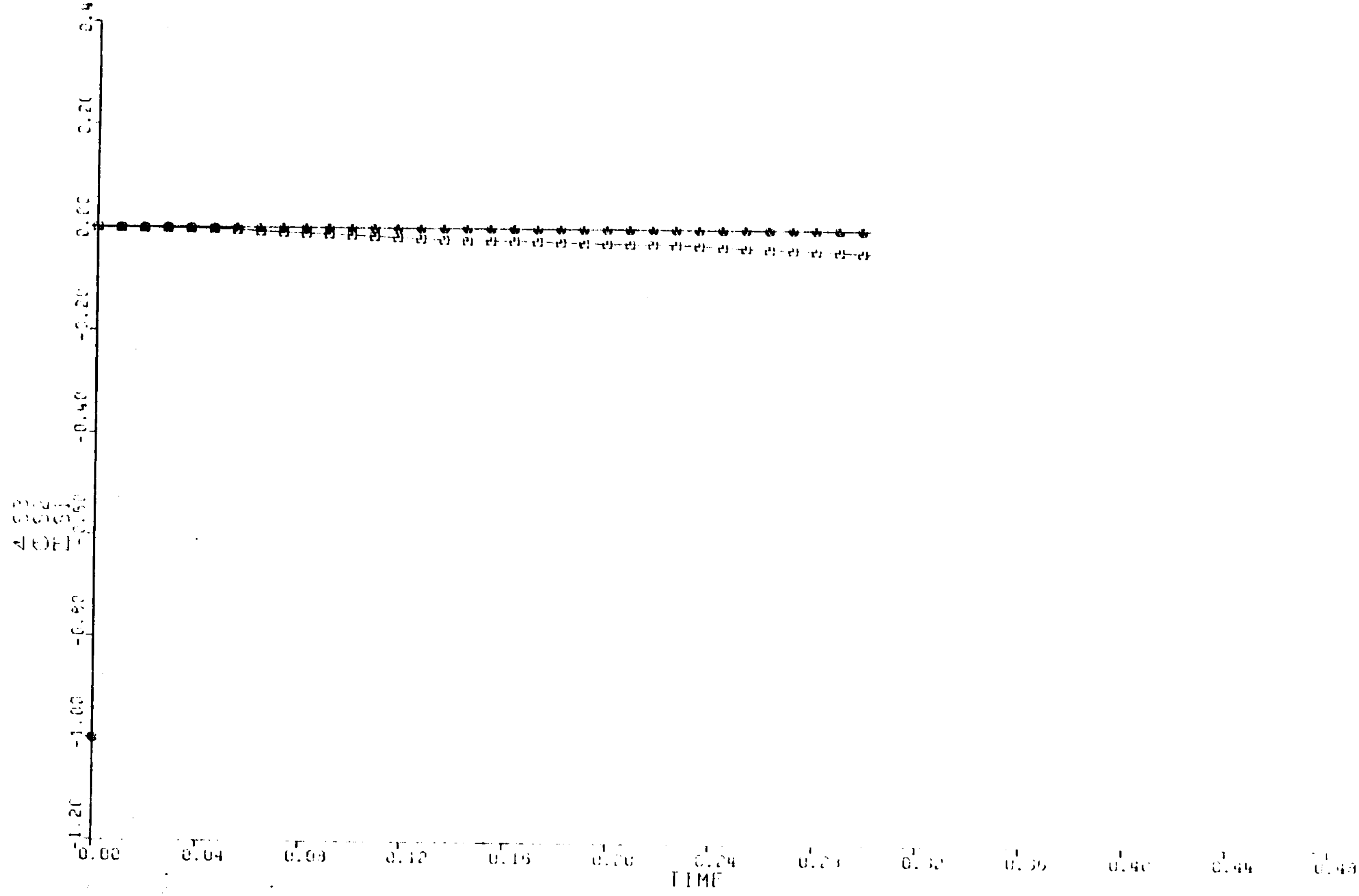

Figure 4.23. Case #2: SLIP

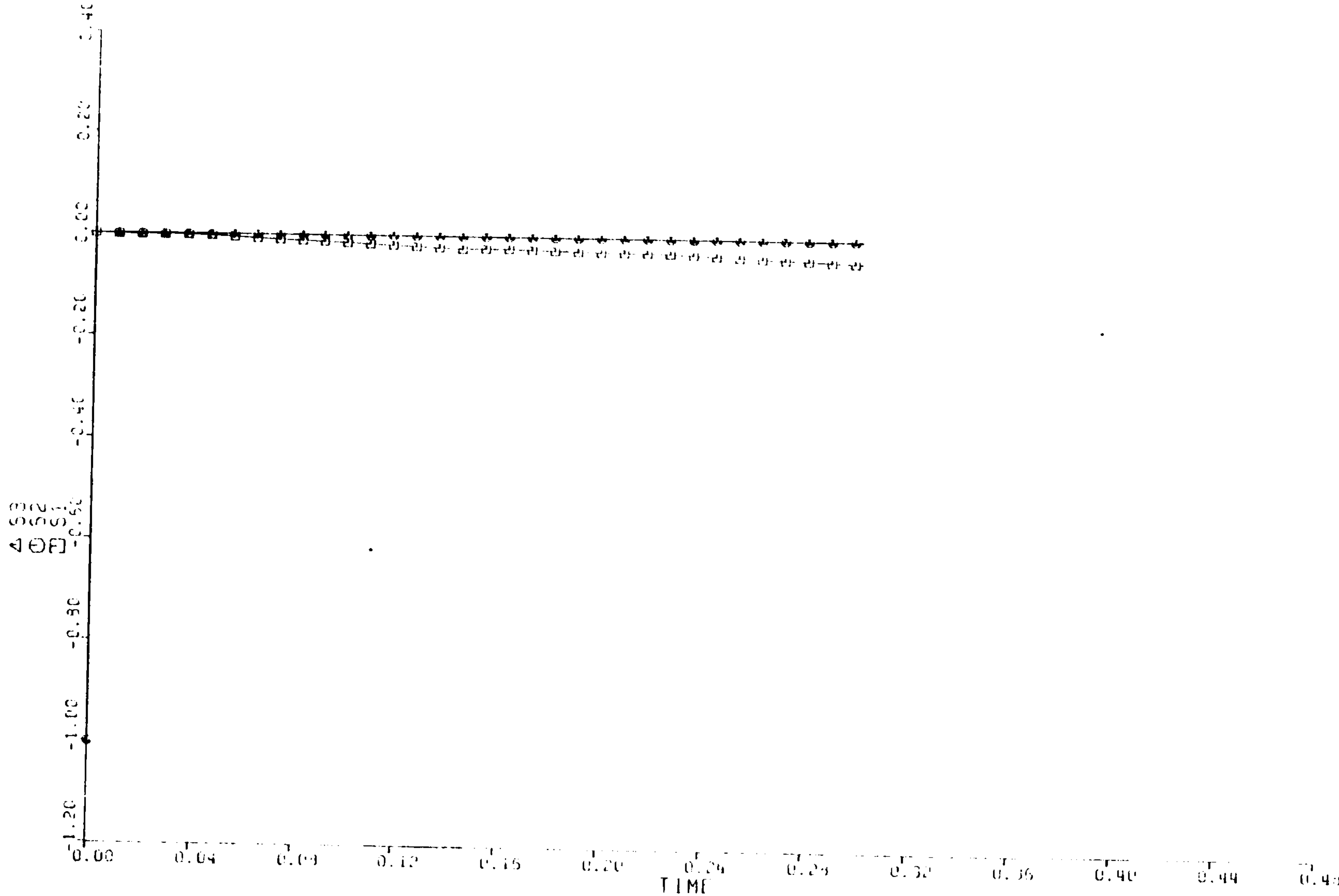

Figure 4.24. Case #3: SLIP

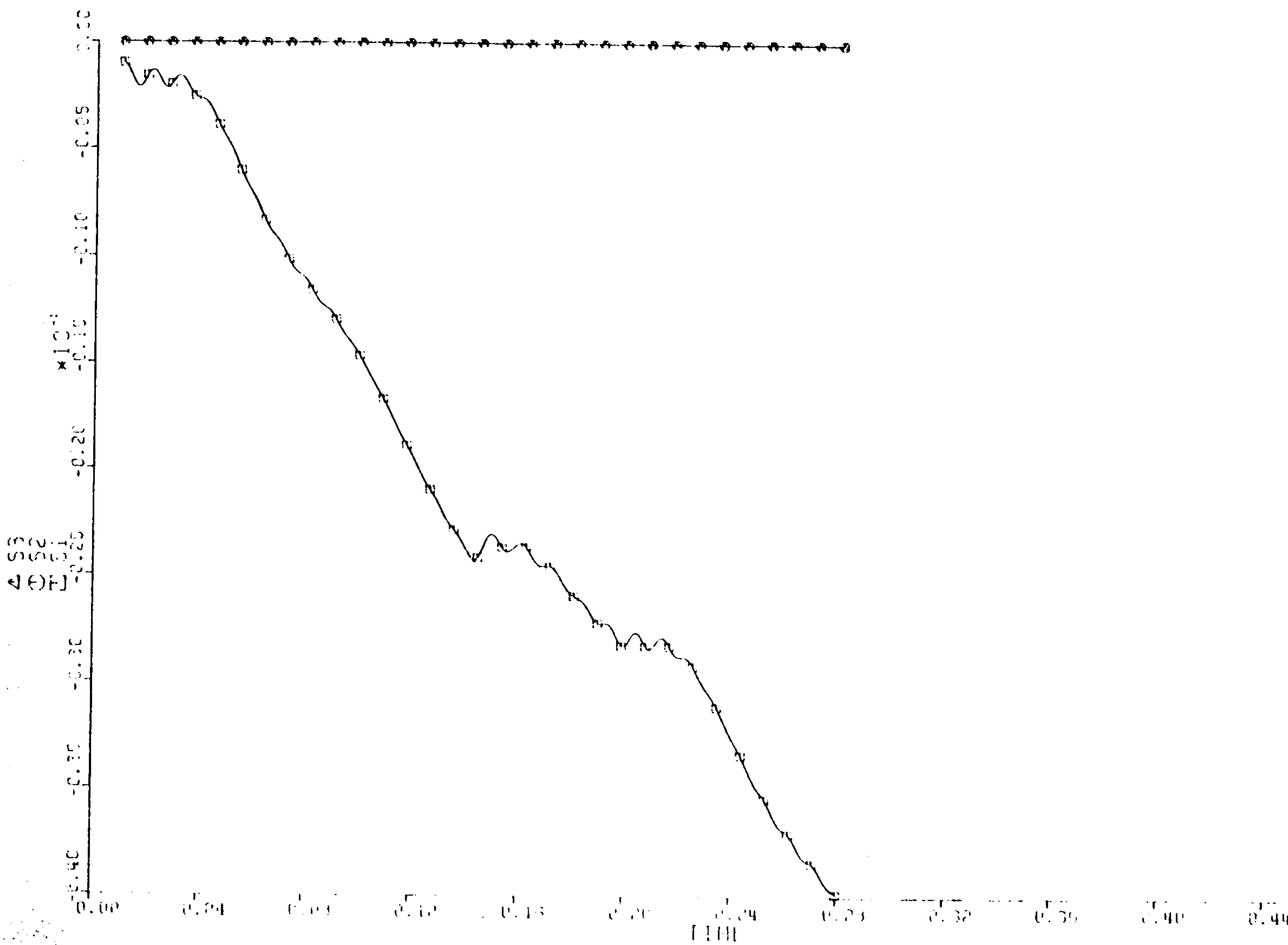

Figure 4.25. Case #2: Transients in SLIP

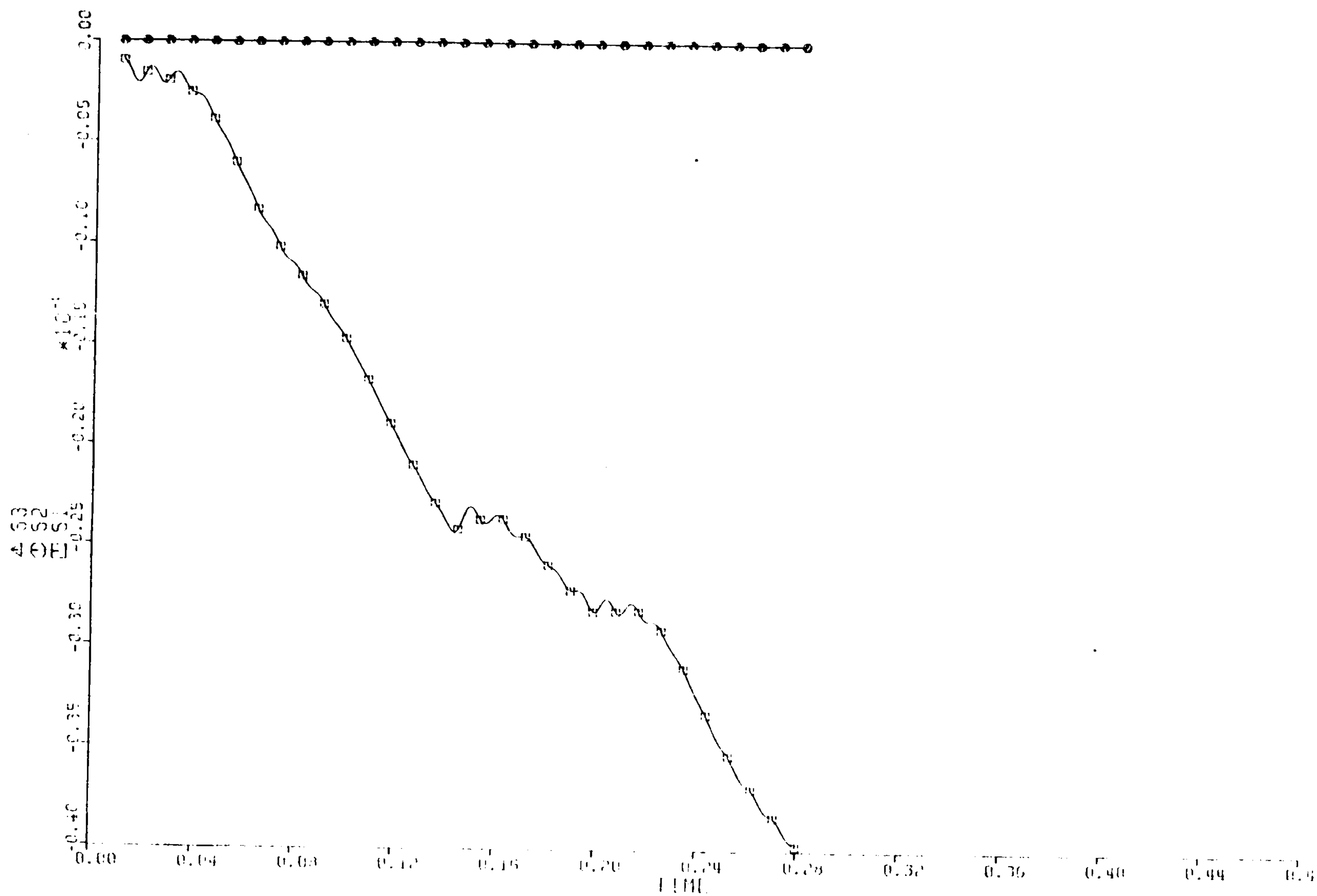

Figure 4.26. Case #3: Transients in SLIP

in Figures 4.13 and 4.14. The slip of the induction generator is depicted in Figures 4.23 and 4.24. Certain observations will now be made concerning these figures and the system variables they represent:

Case #2: Induction Generator on Phase A, Bus #1, All Mutual Coupling Considered

1. At the time of the fault (87.5 msec), voltages on phases A and C drop in magnitude. This magnitude is less than the voltage magnitude on phase B throughout the fault period. Figures 4.5 and 4.6 support this.
2. Figures 4.7 and 4.8, which are plots of the voltage transients at breaker reclosure, imply that the recovery voltage at bus #2 is somewhat larger than at bus #1. Furthermore, these transients differ in waveform and in frequency.
3. Figures 4.9 and 4.10, distribution line currents to bus #1, show a slight pulsation in phase A current before and after the fault period. At breaker reclosure, again, the transients appear to be worse at bus #2. Figures 4.11 and 4.12 are plots of these transients.
4. Figure 4.13 is a plot of the induction generator currents.

IA -- Armature Current

ID -- Direct Axis Current

IQ -- Quadrature Current.

Should these variables be compared to Figure 4.14, Case #3, it will be seen that they are virtually identical except for the transient spike at the time the fault occurs.

Case #3: Induction Generator on Phase A, Bus #1, Without Mutual Coupling

1. Figures 4.15 and 4.16, bus voltages, show that the recovery voltage at bus #2 is substantially higher than at bus #1, or than in Case #2.
2. Figures 4.17 and 4.18, transients of bus voltages at breaker reclosure, show the different fluctuations in voltage. The damping present at bus #2 predominates.
3. Figure 4.19 is a better illustration of the slight pulsation in line current mentioned earlier in Case #2. Figure 4.20 shows the sensitivity of bus #2 to any abrupt power system change. Figures 4.21 and 4.22 depict the transients in line current. The classical "ringing" characteristic of single-phase current adjustment is pictured. Should these plots be compared to the ones shown in Figures 4.11 and 4.12, obvious differences emerge, notably the initial transients of Cases #2 and the transient decay time.

Slip: Cases #2 and #3

Figures 4.23 and 4.24 show that the slip of the single-phase induction generator is independent of mutual coupling between distribution phases. In both cases, slip appears to be identical. Figures 4.25 and

4.26 describe the transients in slip for both cases. It should be noted that the oscillations in slip occur when the induction generator is connected to the distribution system; when the circuit breaker clears the fault at 0.137 seconds; and when the breaker recloses at 0.187 seconds.

CHAPTER 5

CONCLUSIONS

In this study, a mathematical model has been derived for a three-phase distribution system with uniform air gap single-phase induction generators. The purpose of this composite model was to develop a means of simulating single-phase induction generators interacting with an adjacent power system. Although the composite mathematical model derived in this study may not be referred to as an exact model, the results of computer simulation indicate that it is capable of describing realistic situations involving the interfacing of the induction generator with a three-phase distribution system.

However, future study is required. In this particular study, certain simplifications were allowed in order to facilitate computer simulation. For instance, only the single-phase to ground fault was considered. This fault was of brief duration. Breaker response has been defined to be clean and efficient; no recovery voltages, or arcing problems, were considered. A reduction in computer time could be achieved by using a variable step integration procedure in the simulation.

LIST OF REFERENCES

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1. Special Issue on Wind Machines. Solar Engineering, June 1978.

2. Koyakumaru, Y. "An Analysis of Load Characteristics of Self-Excited, Single Phase Induction Generator." Elect. Engr. in Japan, Vol. 94, pp. 27-36, 1975.

3. Nasar, S. A. Electromagnetic Energy Conversion Devices and Systems. Prentice-Hall, Inc., 1970.

4. Fitzgerald, A. C., Kingsley, C., and Kusko, A. Electric Machinery. 3rd ed., McGraw-Hill, 1971.

5. Matsch, L. W. Electromagnetic and Electromechanical Machines. 2nd ed., Thomas Y. Crowell Co., Inc., 1977.

6. Jayadev, T. S. "Windmills Stage a Comeback." IEEE Spectrum, November 1976.

7. __________. "Novel Electric Generation Schemes for Wind Power Plants." Proc. Second Workshop on WEC Systems, June 9-11, 1975, pp. 298-306, NSF-RA-N-75-050.

8. Hinrichsen, E. N. Induction and Synchronous Machines for Vertical Axis Wind Turbines. Final Report, SAND 79-7017.

9. Anderson, P. M. Analysis of Faulted Power Systems. Iowa State University Press, 1973.

APPENDICES

APPENDIX A

DERIVATION OF dq TRANSFORMATION AND DEFINITIONS OF MATHEMATICAL SYMBOLS

A.1 dq TRANSFORMATION OF INDUCTION GENERATOR EQUATIONS

For a uniform air gap two-phase machine, the general equations of motion are:[3]

$$\left[\begin{array}{c} \nu_{ab}^{s} \\ \hline \nu_{ab}^{r} \end{array}\right] = \left[\begin{array}{c:c} R^{s} + pL^{ss} & pL^{sr} \\ \hline pL^{rs} & R^{r} + pL^{rr} \end{array}\right] \left[\begin{array}{c} i_{ab}^{s} \\ \hline i_{ab}^{r} \end{array}\right] \tag{A.1-1}$$

$$T_{em} = -\frac{1}{2} \left[\begin{array}{c:c} i_{ab}^{s} & i_{ab}^{r} \end{array}\right] \frac{\partial}{\partial\theta} \left[\begin{array}{c:c} L^{ss} & L^{sr} \\ \hline L^{rs} & L^{rr} \end{array}\right] \left[\begin{array}{c} i_{ab}^{s} \\ \hline i_{ab}^{r} \end{array}\right] \tag{A.1-2}$$

where

$$\nu_{ab}^{s} = \begin{bmatrix} \nu_a^s \\ \nu_b^s \end{bmatrix}; \quad \begin{array}{l} \nu_a^s \Rightarrow \text{Stator voltage, phase a} \\ \nu_b^s \Rightarrow \text{Stator voltage, phase b} \end{array}$$

$$\nu_{ab}^{r} = \begin{bmatrix} \nu_a^r \\ \nu_b^r \end{bmatrix}; \quad \begin{array}{l} \nu_a^r \Rightarrow \text{Rotor voltage, phase a} \\ \nu_b^r \Rightarrow \text{Rotor voltage, phase b} \end{array}$$

$$i^s_{ab} = \begin{bmatrix} i^s_a \\ i^s_b \end{bmatrix}; \quad \begin{aligned} i^s_a &= \text{Stator current, phase a} \\ i^s_b &= \text{Stator current, phase b} \end{aligned}$$

$$i^r_{ab} = \begin{bmatrix} i^r_a \\ i^r_b \end{bmatrix}; \quad \begin{aligned} i^r_a &= \text{Rotor current, phase a} \\ i^r_b &= \text{Rotor current, phase b} \end{aligned}$$

and

$$R^S = \begin{bmatrix} R^S & 0 \\ 0 & R^S \end{bmatrix}; \quad R^r = \begin{bmatrix} R^r & 0 \\ 0 & R^r \end{bmatrix}; \quad L^{ss} = \begin{bmatrix} L^s & 0 \\ 0 & L^s \end{bmatrix}; \quad L^{rr} = \begin{bmatrix} L^r & 0 \\ 0 & L^r \end{bmatrix}.$$

$$L^{sr} = \begin{bmatrix} L^{sr} \cos \nu\theta & -L^{sr} \sin \nu\theta \\ L^{sr} \sin \nu\theta & L^{sr} \cos \nu\theta \end{bmatrix}; \; L^{rs} = (L^{sr})^T, \; \theta = \omega^r_m t + \delta.$$

Equation (A.1-1) may be written as two equations:

$$[\nu^s_{ab}] = \{[R^S] + p[L^{ss}]\}[i^s_{ab}] + p[L^{sr}][i^r_{ab}] \qquad \text{(A.1-3)}$$

$$[\nu^r_{ab}] = p[L^{rs}][i^s_{ab}] + \{[R^r] + p[L^{rr}]\}[i^r_{ab}] \qquad \text{(A.1-4)}$$

Now it should be noted that the matrices $[L^{sr}]$ and $[L^{rs}]$ are time varying, since $\theta = \theta(t)$. Furthermore, all voltage and current vectors vary with time. Therefore,

$$p[L^{sr}][i_{ab}^{r}] = \{p[L^{sr}]\}[i_{ab}^{r}] + [L^{sr}]\{p[i_{ab}^{r}]\}$$

$$p[L^{rs}][i_{ab}^{s}] = \{p[L^{rs}]\}[i_{ab}^{s}] + [L^{rs}]\{p[i_{ab}^{s}]\}, \text{ etc.}$$

For a dq transformation:

$$\begin{matrix} [\nu_{ab}^{r}] = [S_{dq}][\nu_{dq}^{r}] \\ [i_{ab}^{r}] = [S_{dq}][i_{dq}^{r}] \end{matrix} \quad \text{for } [S_{dq}] = \begin{bmatrix} \cos\nu\theta & \sin\nu\theta \\ -\sin\nu\theta & \cos\nu\theta \end{bmatrix} = \text{transformation matrix.}$$

Now it can be shown that

$$[L^{sr}] = L^{sr}[S_{dq}]^{-1} \Rightarrow p[L^{sr}] = L^{sr}p[S_{dq}]^{-1} = L^{sr}p[S_{dq}]^{T}$$

and

$$[L^{rs}] = [L^{sr}]^{T} = L^{sr}\left[[S_{dq}]^{-1}\right]^{T} = L^{sr}[S_{dq}]$$

where

L^{sr} => A constant term and not a submatrice.

L^{sr} -- Mutual inductance between stator and rotor.

Based upon the above:

$$p\{[L^{sr}][i_{ab}^{r}]\} = p\{L^{sr}[S_{dq}]^{-1}[S_{dq}][i_{dq}^{r}]\} = L^{sr}[1]p[i_{dq}^{r}]$$

$$p\{[L^{rs}][i_{ab}^{r}]\} = L^{sr}\{p[S_{dq}]\}[i_{ab}^{s}] + L^{sr}[S_{dq}]\{p[i_{ab}^{s}]\}$$

$$= L^{sr}[a][i_{ab}^{s}] + L^{sr}[S_{dq}]p[i_{ab}^{s}]$$

for

$$[1] = \text{Identity matrix}$$

and

$$[a] \equiv \begin{bmatrix} -\nu \sin \nu\theta p\theta & \nu \cos \nu\theta p\theta \\ -\nu \cos \nu\theta p\theta & -\nu \sin \nu\theta p\theta \end{bmatrix} = \begin{bmatrix} -\nu\omega_m^r \sin \nu\theta & \nu\omega_m^r \cos \nu\theta \\ -\nu\omega_m^r \cos \nu\theta & -\nu\omega_m^r \sin \nu\theta \end{bmatrix}$$

$$\because \quad p\theta = \omega_m^r.$$

Similarly,

$$p[L^{rr}][i_{ab}^r] = p\{[L^{rr}][S_{dq}]\,[i_{dq}^r]\} = [L^{rr}]\{p[S_{dq}]\}[i_{dq}^r] + [L^{rr}][S_{dq}]\{p[i_{dq}^r]\}$$

$$\because \quad p[L^{rr}] = \text{Null matrix}$$

$\therefore$

$$p\{[L^{rr}][S_{dq}][i_{dq}^r]\} = [L^{rr}][a][i_{dq}^r] + [L^{rr}][S_{dq}]p[i_{dq}^r]$$

Finally, Equations (A.1-3) and (A.1-4) become

$$[\nu_{ab}^s] = \{[R^s] + [L^{ss}]p\}[i_{ab}^s] + L^{sr}p[i_{dq}^r] \qquad \text{(A.1-5)}$$

$$\begin{aligned}[S_{dq}][\nu_{dq}^r] = {} & L^{sr}[a][i_{ab}^s] + L^{sr}[S_{dq}]p[i_{ab}^s] + [R^r][S_{dq}][i_{dq}^r] \\ & + [L^{rr}][a][i_{dq}^r] + [L^{rr}][S_{dq}]p[i_{dq}^r]. \end{aligned} \qquad \text{(A.1-6)}$$

Premultiplying Equation (A.1-6) by $[S_{dq}]^{-1}$ gives

$$[v_{dq}^{r}] = L^{sr}[b][i_{ab}^{s}] + [L^{sr}]p[i_{ab}^{s}] + [R^{r}][i_{dq}^{r}] + [L^{rr}][b][i_{dq}^{r}] + [L^{rr}]p[i_{dq}^{r}]$$

for

$$[b] = [S_{dq}]^{-1}[a] = \begin{bmatrix} 0 & v\omega_{m}^{r} \\ -v\omega_{m}^{r} & 0 \end{bmatrix}.$$

Therefore, Equations (A.1-5) and (A.1-6) become matrix Equation (A.1-7):

$$\begin{bmatrix} v_{a}^{s} \\ v_{b}^{s} \\ v_{d}^{r} \\ v_{q}^{r} \end{bmatrix} = \begin{bmatrix} (R^{s} + L^{s}p) & 0 & L^{sr}p & 0 \\ 0 & (R^{s} + L^{s}p) & 0 & L^{sr}p \\ L^{sr}p & v\omega_{m}^{r}L^{sr} & (R^{r} + L^{r}p) & v\omega_{m}^{r}L^{r} \\ -v\omega_{m}^{r}L^{sr} & L^{sr}p & -v\omega_{m}^{r}L^{r} & (R^{r} + L^{r}p) \end{bmatrix} \begin{bmatrix} i_{a}^{s} \\ i_{b}^{s} \\ i_{d}^{r} \\ i_{q}^{r} \end{bmatrix} \qquad (A.1\text{-}7)$$

For a single-phase induction machine,

$$v_{b}^{s} = i_{b}^{s} = 0 \quad \text{and} \quad v_{d}^{r} = v_{q}^{r} = 0$$

and Equation (A.1-7) simplifies to

$$\begin{bmatrix} v_{a}^{s} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (R^{s} + L^{s}p) & L^{sr}p & 0 \\ L^{sr}p & (R^{r} + L^{r}p) & v\omega_{m}^{r}L^{r} \\ -v\omega_{m}^{r}L^{sr} & -v\omega_{m}^{r}L^{r} & (R^{r} + L^{r}p) \end{bmatrix} \begin{bmatrix} i_{a}^{s} \\ i_{d}^{r} \\ i_{q}^{r} \end{bmatrix}$$

which is identical to Equation set (2-1).

Through similar techniques, Equation (A.1-2) transforms to

$$T_{em} = \nu L^{sr} i_q^r i_a^s$$

so that

$$p\omega_m^r = [T_{pm} - \nu L^{sr} i_a^s i_q^r]/J$$

which is Equation (2-7).

Furthermore, as $p \to j\omega^s$ and $p\omega_m^r \to 0$, Equation set (2-1) describes the steady-state model of the induction machine as well. In this study, ν has been assumed to equal 1. However, Equation set (2-1) implies that the frequency of the rotor currents depends upon the stator frequency and the mechanical speed of the rotor,[3] or

$$\omega^r = \omega^s - \nu\omega_m^r$$

$$\Rightarrow \quad f^r = f^s - \nu f_m^r.$$

Therefore, the relationship between stator and rotor frequencies involves ν, which further implies that

$$s = \frac{\omega^s - \nu\omega_m^r}{\omega^s}.$$

A.2 INDUCTION GENERATOR/MACHINE

V_a^s -- Stator voltage phase a

R^s -- Stator resistance

R^r -- Rotor resistance

L^s -- Stator inductance

L^r -- Rotor inductance

L^{sr} -- Mutual inductance between stator and rotor

i_a^s -- Stator, armature, current

i_d^r -- Rotor current projected along direct axis

i_q^r -- Rotor current projected along quadrature axis

ω_m^r -- Rotor velocity of induction machine in radians/sec.

ν -- Number of pairs of poles

T_{em} -- Electromagnetic torque

T_{pm} -- Prime mover torque

s -- Slip

J -- Inertia constant of induction machine under consideration in $Kg \cdot m^2$

ω^s -- Equivalent angular velocity of power system voltages and currents in radians/sec.

I_g -- Generator currents, collectively, on secondary side of transformer

V_t -- Terminal voltage of induction generator

I_G -- Armature current of induction generator referred to primary side of transformer

A.3 POWER SYSTEM

I_A -- Phase A current from infinite bus to bus #1

I_{AB1} -- Displacement current between phases A and B, bus #1

I_{FAB1} -- Fault current between phases A and B, bus #1

I_{CA1} -- Displacement current between phases A and C, bus #1

I_{FCA1} -- Fault current between phases A and C, bus #1

I_{DA1} -- Displacement current phase A to ground, bus #1

I_{FA1} -- Fault current phase A to ground, bus #1

I_{GA1} -- Induction generator armature current, phase A, bus #1

I_{A12} -- Phase A current from bus #1 to bus #2

I_{AB2} -- Displacement current between phases A and B, bus #2

I_{FAB2} -- Fault current between phases A and B, bus #2

I_{CA2} -- Displacement current between phases A and C, bus #2

I_{FCA2} -- Fault current between phases A and C, bus #2

I_{DA2} -- Displacement current phase A to ground, bus #2

I_{FA2} -- Fault current phase A to ground, bus #2

I_{GA2} -- Induction generator armature current, phase A, bus #2

I_{LA} -- Phase A load current

I_B -- Phase B current from infinite bus to bus #1

I_{CB1} -- Displacement current between phases B and C, bus #1

I_{FCB1} -- Fault current between phases B and C, bus #1

I_{B12} -- Phase B current from bus #1 to bus #2

I_{DB1} -- Displacement current phase B to ground, bus #1

I_{FB1} -- Fault current phase B to ground, bus #1

I_{GB1} -- Induction generator armature current, phase B, bus #1

I_{CB2} -- Displacement current between phases B and C, bus #2

I_{FCB2} -- Fault current between phases B and C, bus #2

I_{DB2} -- Displacement current phase B to ground, bus #2

I_{FB2} -- Fault current phase B to ground, bus #2

I_{GB2} -- Induction generator current, phase B, bus #2

I_{LB} -- Phase B load current

I_C -- Phase C current from infinite bus to bus #1

I_{DC1} -- Displacement current phase C, bus #1

I_{FC1} -- Fault current phase C to ground, bus #1

I_{GC1} -- Induction generator current, phase C, bus #1

I_{C12} -- Phase C current from bus #1 to bus #2

I_{DC2} -- Displacement current phase C, bus #2

I_{FC2} -- Fault current phase C to ground, bus #2

I_{GC2} -- Induction generator current, phase C, bus #2

I_{LC} -- Phase C load current

V_A -- Infinite bus voltage, phase A

V_{A1} -- Phase A voltage, bus #1

V_{A2} -- Phase A voltage, bus #2

V_B -- Infinite bus voltage, phase B

V_{B1} -- Phase B voltage, bus #1

V_{B2} -- Phase B voltage, bus #2

V_C -- Infinite bus voltage, phase c

V_{C1} -- Phase C voltage, bus #1

V_{C2} -- Phase C voltage, bus #2

C_{ag} -- Capacitance, phase A to ground

C_{ab} -- Capacitance, phase A to phase B

C_{ac} -- Capacitance, phase A to phase C

C_{bg} -- Capacitance, phase B to ground

C_{bc} -- Capacitance, phase B to phase C

C_{cg} -- Capacitance, phase C to ground

L_{AA} -- Self-inductance, phase A

L_{AB} -- Mutual inductance, phases A and B

L_{AC} -- Mutual inductance, phases A and C

L_{BB} -- Self-inductance, phase B

L_{BC} -- Mutual inductance, phases B and C

L_{CC} -- Self-inductance, phase C

Y_{ag1} -- Admittance, phase A to ground, bus #1

Y_{ab1} -- Admittance, phase A to phase B, bus #1

Y_{ac1} -- Admittance, phase A to phase C, bus #1

Y_{ag2} -- Admittance, phase A to ground, bus #2

Y_{ab2} -- Admittance, phase A to phase B, bus #2

Y_{ac2} -- Admittance, phase A to phase C, bus #2

Y_{LA} -- Load admittance, phase A

Y_{bg1} -- Admittance, phase B to ground, bus #1

Y_{bc1} -- Admittance, phase B to phase C, bus #1

Y_{bg2} -- Admittance, phase B to ground, bus #2

Y_{bc2} -- Admittance, phase B to phase C, bus #2

Y_{LB} -- Load admittance, phase B

Y_{cg1} -- Admittance, phase C to ground, bus #1

Y_{cg2} -- Admittance, phase C to ground, bus #2

Y_{LC} -- Load admittance, phase C

ε -- Permittivity of free space

r_a -- Resistance, phase A conductor

r_b -- Resistance, phase B conductor

r_c -- Resistance, phase C conductor

r_d -- Resistance of the earth at a frequency of 60 Hz

D_e -- Equivalent depth of earth return current

D_{sa} -- Geometric mean radius at frequency of 60 Hz, phase A

D_{sb} -- Geometric mean radius at frequency of 60 Hz, phase B

D_{sc} -- Geometric mean radius at frequency of 60 Hz, phase C

D_{AB} -- Distance between conductors on phases A and B

D_{BC} -- Distance between conductors on phases B and C

D_{AC} -- Distance between conductors on phases A and C

K -- Inductance multiplying constant derived from permeability considerations

H_A -- Geometric mean distance between phase A conductor and its image

H_{AB} -- Geometric mean distance between phase A conductor and the image of phase B conductor

H_{AC} -- Geometric mean distance between phase A conductor and the image of phase C conductor

RAD_a -- Radius of conductor on phase A

RAD_b -- Radius of conductor on phase B

RAD_c -- Radius of conductor on phase C

V_f -- Voltage forcing function; infinite bus voltage

q -- Charge distribution

Z_{AA} -- Equivalent line impedance of phase A

Z_{AB} -- Equivalent mutual impedance between phases A and B

Z_{AC} -- Equivalent mutual impedance between phases A and C

Z_{BB} -- Equivalent line impedance of phase B

Z_{BC} -- Equivalent mutual impedance between phases B and C

Z_{CC} -- Equivalent line impedance of phase C

APPENDIX B

DERIVATION OF DIFFERENTIAL MATRIX EQUATIONS

B.1 DIFFERENTIAL VOLTAGE MATRIX EQUATION

After substitution of Equation (3-2) into Equation (3-1) and after utilizing the identities in Equation (3-3), the following differential equations are obtained to form Equation (B.1-1):

$$
\begin{aligned}
-(C_{ac} + C_{ab} + C_{ag})pV_{A1} + C_{ab}pV_{B1} + C_{ac}pV_{C1} &= (Y_{ac1} + Y_{ab1} + Y_{ag1})V_{A1} \\
&\quad - Y_{ab1}V_{B1} - Y_{ac1}V_{C1} + I_{A12} \\
&\quad - I_{A} - I_{GA1} \\
-(C_{ac} + C_{ab} + C_{ag})pV_{A2} + C_{ab}pV_{B2} + C_{ac}pV_{C2} &= (Y_{ac2} + Y_{ab2} + Y_{ag2} + Y_{LA})V_{A2} \\
&\quad - Y_{ab2}V_{B2} - Y_{ac2}V_{C2} - I_{A12} \\
&\quad - I_{GA2} \\
C_{ab}pV_{A1} - (2C_{ab} + C_{ag})pV_{B1} + C_{ab}pV_{c1} &= -Y_{ab1}V_{A1} + (Y_{ab1} + Y_{cb1} + Y_{bg1})V_{B1} \\
&\quad - Y_{cb1}V_{C1} + I_{B12} - I_{B} - I_{GB1} \\
C_{ab}pV_{A2} - (2C_{ab} + C_{ag})pV_{B2} + C_{ab}pV_{C2} &= - Y_{ab2}V_{A2} + (Y_{ab2} + Y_{cb2} + Y_{LB} \\
&\quad + Y_{bg2})V_{B2} - Y_{cb2}V_{C2} - I_{B12} - I_{GB2} \\
-C_{ac}pV_{A1} - C_{ab}pV_{B1} + (C_{ac} + C_{ab} + C_{ag})pV_{C1} &= Y_{ca1}V_{A1} + Y_{cb1}V_{B1} - (Y_{ca1} \\
&\quad + Y_{cg1} + Y_{cb1})V_{C1} - I_{C12} \\
&\quad + I_{C} + I_{CG1} \\
-C_{ac}pV_{A2} - C_{ab}pV_{B2} + (C_{ac} + C_{ab} + C_{ag})pV_{C2} &= Y_{ca2}V_{A2} + Y_{cb2}V_{B2} - (Y_{ca2} \\
&\quad + Y_{cg2} + Y_{cb2})V_{C2} + I_{C12} + I_{GC2}.
\end{aligned}
$$

By letting

$$X_1 = C_{ac} + C_{ab} + C_{ag}$$

$$X_2 = C_{ab} = C_{bc}$$

$$X_3 = C_{ac}$$

$$X_4 = C_{ag} + 2C_{ab}$$

$$a_1 = Y_{ac1} + Y_{ab1} + Y_{ag1}$$

$$a_2 = Y_{ac2} + Y_{ab2} + Y_{ag2} + Y_{LA}$$

$$b_1 = Y_{ab1} + Y_{cb1} + Y_{bg1}$$

$$b_2 = Y_{ab2} + Y_{cb2} + Y_{LB} + Y_{bg2}$$

$$c_1 = Y_{ca1} + Y_{cg1} + Y_{cb1}$$

$$c_2 = Y_{ca2} + Y_{cg2} + Y_{LC} + Y_{cb2}$$

Equation (3-4) is defined. Equation (3-5) describes the solution appertaining to the differential voltages; namely,

$$p[V] = [X]^{-1}[Y][V] + [X]^{-1}[D][I] + [X]^{-1}[E][I_G]$$

where

$$[X]^{-1} = \begin{bmatrix} -X_1 & 0 & X_2 & 0 & X_3 & 0 \\ 0 & -X_1 & 0 & X_2 & 0 & X_3 \\ X_2 & 0 & -X_4 & 0 & X_2 & 0 \\ 0 & X_2 & 0 & -X_4 & 0 & X_2 \\ -X_3 & 0 & -X_2 & 0 & X_1 & 0 \\ 0 & -X_3 & 0 & -X_2 & 0 & X_1 \end{bmatrix}^{-1} ;$$

$$[Y] = \begin{bmatrix} a_1 & 0 & -Y_{ab1} & 0 & -Y_{ac1} & 0 \\ 0 & a_2 & 0 & -Y_{ab2} & 0 & -Y_{ac2} \\ -Y_{ab1} & 0 & b_1 & 0 & -Y_{bc1} & 0 \\ 0 & -Y_{ab2} & 0 & b_2 & 0 & -Y_{bc2} \\ Y_{ac1} & 0 & Y_{bc1} & 0 & -c_1 & 0 \\ 0 & Y_{ac2} & 0 & Y_{bc2} & 0 & -c_2 \end{bmatrix} .$$

$$[D] = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} ; \; [E] = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} .$$

$$p[V] = p\begin{bmatrix} V_{A1} \\ V_{A2} \\ V_{B1} \\ V_{B2} \\ V_{C1} \\ V_{C2} \end{bmatrix} \; ; \quad [V] = \begin{bmatrix} V_{A1} \\ V_{A2} \\ V_{B1} \\ V_{B2} \\ V_{C1} \\ V_{C2} \end{bmatrix} \; ; \quad [I] = \begin{bmatrix} I_{A} \\ I_{A12} \\ I_{B} \\ I_{B12} \\ I_{C} \\ I_{C12} \end{bmatrix} \; ; \quad [I_G] = \begin{bmatrix} I_{GA1} \\ I_{GA2} \\ I_{GB1} \\ I_{GB2} \\ I_{GC1} \\ I_{GC2} \end{bmatrix} .$$

Albeit the digital computer may be used in finding $[X]^{-1}$, the matrix product, $[X]^{-1}[Y]$, will ultimately become a function of the state of the power system; i.e., the elements of matrix [Y], all of which are admittances, are subject to sudden change during faults on the distribution system or during load changes. For this reason, the inverse of [X] will be computed manually so that all elements of the resulting matrices shall remain in general form.

Let

$$[X] = \left[\begin{array}{c:c} B' & C' \\ \hdashline D' & E' \end{array}\right]$$

where

$$B' = \begin{bmatrix} -X_1 & 0 & X_2 \\ 0 & -X_1 & 0 \\ X_2 & 0 & -X_4 \end{bmatrix} \; ; \quad C' = \begin{bmatrix} 0 & X_3 & 0 \\ X_2 & 0 & X_3 \\ 0 & X_2 & 0 \end{bmatrix}$$

$$D' = \begin{bmatrix} 0 & X_2 & 0 \\ -X_3 & 0 & -X_2 \\ 0 & -X_3 & 0 \end{bmatrix} ; \quad E' = \begin{bmatrix} -X_4 & 0 & X_2 \\ 0 & X_1 & 0 \\ -X_2 & 0 & X_1 \end{bmatrix}$$

Then it can be shown that

$$[X]^{-1} = \left[\begin{array}{c|c} X' & Y' \\ \hline Z' & U' \end{array}\right]$$

For: $X' = [B' - C'E'^{-1}D']^{-1}$ $\quad U' = [E' - D'B'^{-1}C']^{-1}$

$Y' = -B'^{-1}C'U'$ $\quad Z' = -E'^{-1}D'X'$

since B'^{-1} and E'^{-1} both exist.

Once these matrix operations are performed, the sub-matrices of $[X]^{-1}$ become

$$X' = \begin{bmatrix} -\rho/n & 0 & X_2/\sigma \\ 0 & -\rho/n & 0 \\ X_2/\sigma & 0 & (X_1 - X_3)/\sigma \end{bmatrix} \quad U' = \begin{bmatrix} (X_1 - X_3/\sigma & 0 & -X_2/\sigma \\ 0 & \rho/n & 0 \\ X_2/\sigma & 0 & \rho/n \end{bmatrix}$$

$$Y' = \begin{bmatrix} 0 & -(\sigma - \rho)/n & 0 \\ X_2/\sigma & 0 & -(\sigma - \rho)/n \\ 0 & -X_2/\sigma & 0 \end{bmatrix} \quad Z' = \begin{bmatrix} 0 & X_2/\sigma & 0 \\ (\sigma - \rho)/n & 0 & X_2/\sigma \\ 0 & (\sigma - \rho)/n & 0 \end{bmatrix}$$

where

$$\rho = X_2^2 - X_1X_4 \ ; \quad \sigma = X_2^2 + X_3X_4 + \rho \ ; \quad n = \sigma(X_1 + X_3).$$

Substituting the results of the $[X]^{-1}$ computation into Equation (3-41) and carrying out the matrix multiplications will yield the following:

$$[X]^{-1}[Y] = \begin{bmatrix} DA & 0 & DD & 0 & DG & 0 \\ 0 & DC & 0 & DF & 0 & DI \\ DB & 0 & DE & 0 & DH & 0 \\ 0 & DK & 0 & DN & 0 & DR \\ DJ & 0 & DM & 0 & DQ & 0 \\ 0 & DL & 0 & DP & 0 & DS \end{bmatrix}$$

where

$$DA = \{-\rho a_1 - X_2Y_{ab1}(X_1 + X_3) - Y_{ac1}(\sigma - \rho)\}/n;$$

$$DB = \{X_2a_1 - Y_{ab1}(X_1 - X_3) - Y_{ac1}X_2\}/\sigma;$$

$$DC = \{-Y_{ab2}X_2(X_1 + X_3) - Y_{ac2}(\sigma - \rho) - \rho a_2\}/n;$$

$$DD = \{\rho Y_{ab1} + X_2b_1(X_1 + X_3) - Y_{bc1}(\sigma - \rho)\}/n;$$

$$DE = \{-X_2Y_{ab1} + b_1(X_1 - X_3) - Y_{bc1}X_2\}/\sigma;$$

$$DF = \{\rho Y_{ab2} + X_2b_2(X_1 + X_3) - Y_{bc2}(\sigma - \rho)\}/n;$$

$$DG = \{\rho Y_{ac1} - X_2Y_{bc1}(X_1 + X_3) + C_1(\sigma - \rho)\}/n;$$

$$DH = \{-X_2Y_{ac1} - Y_{bc1}(X_1 - X_3) + X_2C_1\}/\sigma;$$

$$DI = \{\rho Y_{ac2} - X_2Y_{cb2}(X_1 + X_3) + C_2(\sigma - \rho)\}/n;$$

$$DJ = \{a_1(\sigma - \rho) - X_2Y_{ab1}(X_1 + X_3) + \rho Y_{ac1}\}/n;$$

$$DK = \{X_2a_2 - Y_{ab2}(X_1 - X_3) - X_2Y_{ac2}\}/\sigma;$$

$$DL = \{a_2(\sigma - \rho) - Y_{ab2}X_2(X_1 + X_3) + \rho Y_{ac2}\}/n;$$

$$DM = \{-Y_{ab1}(\sigma - \rho) + X_2b_1(X_1 + X_3) + \rho Y_{bc1}\}/n;$$

$$DN = \{-X_2Y_{ab2} + b_2(X_1 - X_3) - X_2Y_{bc2}\}/\sigma;$$

$$DP = \{-Y_{ab2}(\sigma - \rho) + b_2X_2(X_1 + X_3) + \rho Y_{bc2}\}/n;$$

$$DQ = \{-Y_{ac1}(\sigma - \rho) - X_2Y_{bc1}(X_1 + X_3) - \rho C_1\}/n;$$

$$DR = \{-X_2Y_{ac2} - Y_{bc2}(X_1 - X_3) + C_2X_2\}/\sigma;$$

$$DS = \{-Y_{ac2}(\sigma - \rho) - Y_{bc2}X_2(X_1 + X_3) - \rho C_2\}/n.$$

and

$$[X]^{-1}[D] =$$

$$\begin{bmatrix} \rho/n & -\rho/n & -X_2/\sigma & X_2/\sigma & -(\sigma - \rho)/n & (\sigma - \rho)/n \\ 0 & \rho/n & 0 & -X_2/\sigma & 0 & -(\sigma - \rho)/n \\ -X_2/\sigma & X_2/\sigma & -(X_1 - X_3)/\sigma & (X_1 - X_3)/\sigma & -X_2/\sigma & X_2/\sigma \\ 0 & -X_2/\sigma & 0 & -(X_1 - X_3)/\sigma & 0 & -X_2/\sigma \\ -(\sigma - \rho)/n & (\sigma - \rho)/n & -X_2/\sigma & X_2/\sigma & \rho/n & -\rho/n \\ 0 & -(\sigma - \rho)/n & 0 & -X_2/\sigma & 0 & \rho/n \end{bmatrix}$$

$$[X]^{-1}[E] =$$

$$\begin{bmatrix} \rho/n & 0 & -X_2/\sigma & 0 & -(\sigma - \rho)/n & 0 \\ 0 & \rho/n & 0 & -X_2/\sigma & 0 & -(\sigma - \rho)/n \\ -X_2/\sigma & 0 & -(X_1 - X_3)/\sigma & 0 & -X_2/\sigma & 0 \\ 0 & -X_2/\sigma & 0 & -(X_1 - X_3)/\sigma & 0 & -X_2/\sigma \\ -(\sigma - \rho)/n & 0 & -X_2/\sigma & 0 & \rho/n & 0 \\ 0 & -(\sigma - \rho)/n & 0 & -X_2/\sigma & 0 & \rho/n \end{bmatrix}$$

B.2 DIFFERENTIAL CURRENT MATRIX EQUATION

It is assumed that all phases are composed of identical conductors and that the alignment of these conductors is such that an abc orientation ensues. Furthermore, the distribution line is untransposed throughout its length, and the spacing of the conductors is completely symmetrical with respect to phase B. With this in mind, Equation (3-8) may be simplified.

$$\because \quad D_{sa} = D_{sb} = D_{sc} \text{ and}$$

$$\& \quad D_{AB} = D_{BC}.$$

It follows that

$$L_{AA} = L_{BB} = L_{CC};\ L_{AB} = L_{BC}.$$

Also,

$$r_a = r_b = r_c.$$

So, letting

$$R = (r_a + r_d) = (r_b + r_d) = (r_c + r_d),$$

Equation (3-9) becomes matrix Equation (3-10):

$$[F][V] + [V_f] = [R][I] + [L]p[I]$$

where

$$[F] = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}; \quad [R] = \begin{bmatrix} R & 0 & r_d & 0 & rd & 0 \\ 0 & R & 0 & r_d & 0 & r_d \\ r_d & 0 & R & 0 & r_d & 0 \\ 0 & r_d & 0 & R & 0 & r_d \\ r_d & 0 & r_d & 0 & R & 0 \\ 0 & r_d & 0 & r_d & 0 & R \end{bmatrix};$$

$$[L] = \begin{bmatrix} L_{AA} & 0 & L_{AB} & 0 & L_{AC} & 0 \\ 0 & L_{AA} & 0 & L_{AB} & 0 & L_{AC} \\ L_{AB} & 0 & L_{AA} & 0 & L_{AB} & 0 \\ 0 & L_{AB} & 0 & L_{AA} & 0 & L_{AB} \\ L_{AC} & 0 & L_{AB} & 0 & L_{AA} & 0 \\ 0 & L_{AC} & 0 & L_{AB} & 0 & L_{AA} \end{bmatrix};$$

$$[V] = \begin{bmatrix} V_{A1} \\ V_{A2} \\ V_{B1} \\ V_{B2} \\ V_{C1} \\ V_{C2} \end{bmatrix}; \quad [V_f] = \begin{bmatrix} V_A \\ 0 \\ V_B \\ 0 \\ V_C \\ 0 \end{bmatrix}; \quad [I] = \begin{bmatrix} I_A \\ I_{A12} \\ I_B \\ I_{B12} \\ I_C \\ I_{C12} \end{bmatrix}; \quad p[I] = p \begin{bmatrix} I_A \\ I_{A12} \\ I_B \\ I_{B12} \\ I_C \\ I_{C12} \end{bmatrix}$$

APPENDIX C

CALCULATION OF INDUCTANCES AND CAPACITANCES

For all of the calculations which follow, the following data will be used considering the characteristics of average damp earth and the information provided in Reference 9.

Conductor: #1/ϕACSR, 6 strands per conductor

$r_a = r_b = r_c = 0.888\ \Omega/\text{mile}$

$r_d = 0.09528\ \Omega/\text{mile}$ @ 60 cps

$D_e = 2790$ feet

$RAD_a = RAD_b = RAD_c = 0.0166$ feet

$D_{sa} = D_{sb} = D_{sc} = 0.00446$ feet

$K = 0.3219 \times 10^{-3}/\text{mile}.$

Typical spacing of conductors in distribution systems is approximately 18 inches. With this in mind, and the appropriate definitions in Appendix A, the following quantities are computed:

$D_{AB} = D_{BC} = 1.5$ feet

$D_{AC} = 3.0$ feet

$H_A = H_B = H_C = 80.0$ feet

$H_{AB} = H_{BC} = 80.0141$ feet

$H_{AC} = 80.0562$ feet.

From page 19, the self and mutual inductances are calculated to be:

$$L_{AA} = 4.2962 \text{ mH/mile}$$

$$L_{AB} = 2.4234 \text{ mH/mile}$$

$$L_{AC} = 2.2002 \text{ mH/mile.}$$

The capacitances of the three-phase system with earth return current are determined from the relationship[9]

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} P_{AA} & P_{AB} & P_{AC} \\ P_{BA} & P_{BB} & P_{BC} \\ P_{CA} & P_{CB} & P_{CC} \end{bmatrix} \begin{bmatrix} q_A \\ q_B \\ q_C \end{bmatrix} \text{ volts/meter,}$$

or

$$[V_f] = [P][q] \qquad \text{(C-1)}$$

since the definition of capacitance implies that

$$[C][V_f] = [q]. \qquad \text{(C-2)}$$

Then Equation (C-1) is valid if and only if

$$[V_f] = [P][q] = [C]^{-1}[q]$$

$$\therefore$$

$$[C]^{-1} = [P].$$

The previous assumptions concerning the symmetry of the distribution lines produce the following identities:

$$P_{AA} = P_{BB} = P_{CC}$$

$$P_{BA} = P_{AB}$$

$$P_{CA} = P_{AC}$$

$$P_{BC} = P_{CB}$$

since the matrix [P] is defined in such a way that

$$P_{AA} = \frac{1}{2\pi\varepsilon} \ell n \; (H_A/RAD_a) \quad MF^{-1} \text{ mile}$$

$$P_{AB} = \frac{1}{2\pi\varepsilon} \ell n \; (H_{AB}/D_{AB}) \quad MF^{-1} \text{ mile}$$

$$P_{AC} = \frac{1}{2\pi\varepsilon} \ell n \; (H_{AC}/D_{AC}) \quad MF^{-1} \text{ mile.}$$

Then, after utilizing the appropriate parameters, matrix [P] becomes

$$[P] = \begin{bmatrix} 94.85 & 44.48 & 36.73 \\ 44.48 & 94.85 & 44.48 \\ 36.73 & 44.48 & 94.85 \end{bmatrix} MF^{-1} \text{ mile}$$

which requires that

$$[C] = \begin{bmatrix} 14.17 & -5.23 & -3.04 \\ -5.23 & 15.44 & -5.23 \\ -3.04 & -5.23 & 14.17 \end{bmatrix} \text{nf/mile}$$

or

$$C_{AA} = C_{CC} = 14.17 \text{ nf/mile}$$

$$C_{BB} = 15.44 \text{ nf/mile}$$

$$C_{ab} = C_{bc} = 5.23 \text{ nf/mile}$$

$$C_{ac} = 3.04 \text{ nf/mile}$$

and

$$C_{ag} \equiv C_{AA} + C_{ab} + C_{ac} = 5.90 \text{ nf/mile}$$

$$C_{bg} \equiv C_{BB} + C_{ab} + C_{bc} = 4.98 \text{ nf/mile}$$

$$C_{cg} \equiv C_{CC} + C_{ac} + C_{bc} = 5.90 \text{ nf/mile.}$$

APPENDIX D

```
      DIMENSION X(40),NUM(40),F(40)
      COMMON /DATA/RS,RSL,RR,RL,SL,XTWO,XTHREE
      COMMON /PARAM/A,C,U,Z,XONE,RHO,DELLT,UNU,DA,DB,DC,DD,DE,DF,DG,DH,
     1DI,DJ,DK,DL,DM,DN,DP,DQ,DR,DS
      COMMON /INP/TPMT,S
      COMMON /FFF/F,W
C     INDUCTION GENERATOR ON PHASE A,BUS#1
C     GENERATOR STARTED UP...THEN FAULTED...BREAKER ACTION OCCURS
C     CASE#1...ALL MUTUAL COUPLING CONSIDERED
C     CASE#2 IS IDENTICAL TO CASE#1 EXCEPT AS NOTED
C     MISCELLANEOUS CONSTANTS
      W=377.0
      C=-1.0
      U=1.0
      TPM=0.0
      TA=0.0002
      TM=250000.0
      TF=0.0875
      TFD=0.05
      TBD=0.05
      EPS=0.001
      EPSS=0.50
      EPSSS=C*EPSS
C     INDUCTION GENERATOR PARAMETERS
      Z=0.5840
      RS=1.1581
      RR=1.1581
      RSL=0.0819
      RL=0.0821
      SL=0.0821
      A=(SL*RL)-(RSL*RSL)
```

```
C   DISTRIBUTION LINE PARAMETERS
      CAG=29.50E-09
C   FOR CASE#2,CP=CPP=0.0
      CP=25.55E-09
      CPP=15.0E-09
      RBF=1.0E-12
      RAF=0.005
      YATWO=1.0E-12
C   FOR CASE#2,ALL YB'S,YC'S,YAB'S,YAC'S,YCB'S ARE ZERO
      YBONE=1.0E-12
      YBTWO=1.0E-12
      YCONE=1.0E-12
      YCTWO=1.0E-12
      YABONE=1.0E-12
      YABTWO=1.0E-12
      YACONE=1.0E-12
      YACTWO=1.0E-12
      YCBONE=1.0E-12
      YCBTWO=1.0E-12
      YLA=5.0E-04
      YLB=5.0E-04
      YLC=5.0E-04
      XONE=CAG+CP+CPP
      XTWO=CP
      XTHREE=CPP
      XFOUR=CAG+2*CP
    1 FORMAT (I4,I5,3F10.5)
    2 FORMAT (5F14.5)
      READ 1,  NV,NIT,T,DLT,IPRINT
      READ 2, (X(I),I=1,NV)
C   INITIALIZATION AND I.C. VALUES WRITTEN ON TAPE
      TN=T
      WRITE(8,178)(X(I),I=1,30)
      NPRINT=IPRINT/DLT+0.500
      DO 6 J=1,NIT
```

```
C   RAMP FUNCTION TO APPLY TORQUE
      IF(TN.LE.TA)TPMT=TM*TN
      IF(TN.GT.TA)TPMT=50.0
C   APPLICATION OF FAULT AND BREAKER SIMULATION
      BF=TN-TF
      AF=BF-TFD
      BR=AF-TBD
      QT=TF+TFD-EPS
      QTT=QT+EPS+TBD
      IF(TN.GE.0)S=0
      IF((BF.LE.0).AND.(AF.LT.0))YAONE=RBF
      IF((BF.GT.0).AND.(AF.LE.0))YAONE=RAF
      IF((TN.GE.QT).AND.(TN.LE.QTT).AND.(EPSSS.LE.X(11)).AND.(X(11).LE.
     1EPSS))S=S*TPM
      IF(AF.GT.TBD)YAONE=RBF
      AONE=YAONE+YABONE+YACONE
      ATWO=YATWO+YABTWO+YACTWO+YLA
      BONE=YBONE+YABONE+YCBONE
      BTWO=YBTWO+YABTWO+YCBTWO+YLB
      CONE=YCONE+YACONE+YCBONE
      CTWO=YCTWO+YACTWO+YCBTWO+YLC
      RHO=(XTWO*XTWO)-XONE*XFOUR
      DELLT=(XTWO*XTWO)+(XTHREE*XFOUR)+RHO
      UNU=DELLT*(XONE+XTHREE)
C   DIFFERENTIAL VOLTAGE MATRIX ELEMENTS
      DA=-(RHO*AONE+XTWO*YABONE*(XONE+XTHREE)+YACONE*(DELLT-RHO))/UNU
      DB=(XTWO*AONE-YABONE*(XONE-XTHREE)-YACONE*XTWO)/DELLT
      DC=-(YABTWO*XTWO*(XONE+XTHREE)+YACTWO*(DELLT-RHO)+RHO*ATWO)/UNU
      DD=(RHO*YABONE+XTWO*BONE*(XONE+XTHREE)-YCBONE*(DELLT-RHO))/UNU
      DE=(-XTWO*YABONE+BONE*(XONE-XTHREE)-YCBONE*XTWO)/DELLT
      DF=(RHO*YABTWO+XTWO*BTWO*(XONE+XTHREE)-YCBTWO*(DELLT-RHO))/UNU
      DG=(RHO*YACONE-XTWO*YCBONE*(XONE+XTHREE)+CONE*(DELLT-RHO))/UNU
      DH=-(XTWO*YACONE+YCBONE*(XONE-XTHREE)-XTWO*CONE)/DELLT
      DI=(RHO*YACTWO-XTWO*YCBTWO*(XONE+XTHREE)+CTWO*(DELLT-RHO))/UNU
      DJ=(AONE*(DELLT-RHO)-XTWO*YABONE*(XONE+XTHREE)+RHO*YACONE)/UNU
      DK=(XTWO*ATWO-YABTWO*(XONE-XTHREE)-XTWO*YACTWO)/DELLT
```

```
      DL=(ATWO*(DELLT-RHO)-YABTWO*XTWO*(XONE+XTHREE)+YACTWO*RHO)/UNU
      DM=(-YABONE*(DELLT-RHO)+XTWO*BONE*(XONE+XTHREE)+RHO*YCBONE)/UNU
      DN=(-XTWO*YABTWO+BTWO*(XONE-XTHREE)-XTWO*YCBTWO)/DELLT
      DP=(-YABTWO*(DELLT-RHO)+BTWO*XTWO*(XONE+XTHREE)+YCBTWO*RHO)/UNU
      DQ=(-YACONE*(DELLT-RHO)-XTWO*YCBONE*(XONE+XTHREE)-CONE*RHO)/UNU
      DR=(-XTWO*YACTWO-YCBTWO*(XONE-XTHREE)+CTWO*XTWO)/DELLT
      DS=(-YACTWO*(DELLT-RHO)-YCBTWO*XTWO*(XONE+XTHREE)-CTWO*RHO)/UNU
      CALL RUNGE  (TN,DLT,X,NV)
178   FORMAT(160X,30F16.7)
      X(11)=X(11)*S
      TN=T+FLOAT(J)*DLT
      SLIPA=(W-X(4))/W
      SLIPB=(W-X(20))/W
      SLIPC=(W-X(24))/W
      SLIPAA=(W-X(28))/W
      SLIPBB=(W-X(32))/W
      SLIPCC=(W-X(36))/W
      VA=13200.0*SIN(W*(TN+0.10))
      VA=VA*S
      VB=13200.0*SIN(W*(TN+0.10)+4.1888)
      VC=13200.0*SIN(W*(TN+0.10)+2.0944)
      WRITE(8,179)TN,VA,VB,VC,SLIPA,
     1SLIPB,SLIPC,SLIPAA,SLIPBB,SLIPCC,(X(I),I=1,30)
179   FORMAT(40F16.7)
5     TN=T+FLOAT(J)*DLT
    6 CONTINUE
      STOP
      END
```

```
      SUBROUTINE RUNGE(T,DLT,X,NV)
      DIMENSION XN(40),X(40),TAY(4,40)
      DO  1 M=1,NV
    1 TAY(1,M)=FCN(T,X,M)*DLT
      DO 20 L=2,3
      DO 2 M=1,NV
    2 XN(M)=X(M)+TAY(L-1,M)/2.
      DO 20 N=1,NV
   20 TAY(L,N)=FCN(T+DLT/2.,XN,N) *DLT
       DO 30 N=1,NV
   30 XN(N)=X(N)+TAY(3,N)
      DO 3 M=1,NV
    3 TAY(4,M)=FCN(T+DLT    ,XN,M) *DLT
      DO 4 M=1,NV
      XN(M)=X(M)+(TAY(1,M)+2*TAY(2,M)+2.*TAY(3,M)+TAY(4,M))/6.
    4 X(M)=XN(M)
      RETURN
      END
```

```
      FUNCTION FCN(T,Y,K)
      DIMENSION F(40),Y(40)
      COMMON /DATA/RS,RSL,RR,RL,SL,XTWO,XTHREE
      COMMON /PARAM/A,C,U,Z,XONE,RHO,DELLT,UNU,DA,DB,DC,DD,DE,DF,DG,DH,
     1DI,DJ,DK,DL,DM,DN,DP,DQ,DR,DS
      COMMON /INP/TPMT,S
      COMMON /FFF/F,W
      VA=13200.0*SIN(W*(T+0.10))
      VA=VA*S
      VB=13200.0*SIN(W*(T+0.10)+4.1888)
      VC=13200.0*SIN(W*(T+0.10)+2.0944)
      F(1)=(Y(5)/60-RS*Y(1)+(RSL*RR*Y(2))/RL+(U*Y(4)*RSL*Y(3)))*RL/A
      F(2)=((C*RSL*Y(5)/60)+(RSL*RS*Y(1))-(RR*SL*Y(2))-(U*Y(4)*SL*RL*Y(3
     1))))/A
      F(3)=(U*Y(4)*RSL*Y(1))/RL+(U*Y(4)*Y(2))-(RR*Y(3))/RL
      F(4)=(TPMT-(RSL*Y(1)*Y(3))))/Z
      F(5)=DA*Y(5)+DD*Y(7)+DG*Y(9)+(Y(11)*RHO)/UNU-(Y(12)*RHO)/UNU-(Y(13
     1)*XTWO)/DELLT+(Y(14)*XTWO)/DELLT-(DELLT-RHO)*Y(15)/UNU+(DELLT-RHO)
     1*Y(16)/UNU+(RHO/UNU)*(Y(1)/60)-(XTWO/DELLT)*(Y(17)/60)-(DELLT-RHO)
     1*Y(21)/(60*UNU)
      F(6)=DC*Y(6)+DF*Y(8)+DI*Y(10)+(RHO/UNU)*Y(12)-(XTWO/DELLT)*Y(14)-
     1(DELLT-RHO)*Y(16)/UNU+(RHO/UNU)*Y(25)/60-(XTWO/DELLT)*(Y(29)/60)-(
     1DELLT-RHO)*Y(33)/(60*UNU)
C   FOR CASE#2,F(7)=F(8)=F(9)=F(10)=0.0
      F(7)=DB*Y(5)+DE*Y(7)+DH*Y(9)-(XTWO/DELLT)*Y(11)+(XTWO/DELLT)*Y(12)
     1-(XONE-XTHREE)*Y(13)/DELLT+(XONE-XTHREE)*Y(14)/DELLT-(XTWO/DELLT)*
     1Y(15)+(XTWO/DELLT)*Y(16)-(XTWO/DELLT)*Y(1)/60-(XONE-XTHREE)*Y(17
     1)/(DELLT*60)-(XTWO/DELLT)*Y(21)/60
      F(8)=DK*Y(6)+DN*Y(8)+DR*Y(10)-(XTWO/DELLT)*Y(12)-(XONE-XTHREE)*Y(1
     14)/DELLT-(XTWO/DELLT)*Y(16)-(XTWO/DELLT)*(Y(25)/60)-(XONE-XTHREE)*
     1Y(29)/(DELLT*60)-(XTWO/DELLT)*(Y(33)/60)
      F(9)=DJ*Y(5)+DM*Y(7)+DQ*Y(9)-(DELLT-RHO)*Y(11)/UNU+(DELLT-RHO)*Y(1
     12)/UNU-(XTWO/DELLT)*Y(13)+(XTWO/DELLT)*Y(14)+(RHO/UNU)*Y(15)-(RHO/
     1UNU)*Y(16)-(DELLT-RHO)*(Y(1)/(UNU*60))-(XTWO/DELLT)*(Y(17)/60)+(RH
     1O/UNU)*(Y(21)/60)
      F(10)=DL*Y(6)+DP*Y(8)+DS*Y(10)-(DELLT-RHO)*Y(12)/UNU-(XTWO/DELLT)*
```

```
     1Y(14)+(RHO/UNU)*Y(16)-(DELLT-RHO)*Y(25)/(UNU*60)-(XTWO/DELLT)*(Y(2
     19)/60)+(RHO/UNU)*(Y(33)/60)
      F(11)=-390.60*Y(11)+148.46*Y(13)+101.07*Y(15)-85.68*Y(5)+36.14*Y(7
     1)+25.43*Y(9)+85.68*VA-36.14*VB-25.43*VC
      F(11)=F(11)*S
      F(12)=-390.59*Y(12)+148.46*Y(14)+101.07*Y(16)+85.68*Y(5)-85.68*Y(6
     1)-36.14*Y(7)+36.14*Y(8)-25.43*Y(9)+25.43*Y(10)
C   FOR CASE#2,F(13)=F(14)=F(15)=F(16)=0.0
      F(13)=149.90*Y(11)-423.22*Y(13)+149.90*Y(15)+36.14*Y(5)-93.37*Y(7
     1)+36.14*Y(9)-36.14*VA+93.37*VB-36.14*VC
      F(14)=149.90*Y(12)-423.22*Y(14)+149.90*Y(16)-36.14*Y(5)+36.14*Y(6
     1)+93.37*Y(7)-93.37*Y(8)-36.14*Y(9)+36.14*Y(10)
      F(15)=101.07*Y(11)+148.46*Y(13)-390.60*Y(15)+25.43*Y(5)+36.14*Y(7)
     1-85.68*Y(9)-25.43*VA-36.14*VB+85.68*VC
      F(16)=101.07*Y(12)+148.46*Y(14)-390.60*Y(16)-25.43*Y(5)+25.43*Y(6
     1)-36.14*Y(7)+36.14*Y(8)+85.68*Y(9)-85.68*Y(10)
      F(17)=0.0
      F(18)=0.0
      F(19)=0.0
      F(20)=0.0
      F(21)=0.0
      F(22)=0.0
      F(23)=0.0
      F(24)=0.0
      F(25)=0.0
      F(26)=0.0
      F(27)=0.0
      F(28)=0.0
      F(29)=0.0
      F(30)=0.0
      F(31)=0.0
      F(32)=0.0
      F(33)=0.0
      F(34)=0.0
      F(35)=0.0
      F(36)=0.0
      FCN=F(K)
      RETURN
      END
```

VITA

Hugh Jay Robinson III was born in Birmingham, Alabama, on November 3, 1947. He graduated from Berry High School in that same metropolitan area in May 1965. Following a tour of duty in the United States Navy he attended colleges in the State of California and later performed independent study in philosophy, history, and physics. He commenced his study in engineering in 1975 and graduated with honors from the School of Engineering at The University of Alabama, Birmingham, in September 1977.

After working as an engineer in industry, Mr. Robinson entered the graduate school of The University of Tennessee, Knoxville, on a teaching and research assistantship in March 1978. He received the Master of Science degree in Electrical Engineering in March 1980.

The author is a member of Tau Beta Pi, National Society of Professional Engineers, and the Institute of Electrical and Electronics Engineers.