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MATHEMATICAL MODELLING FOR NONLINEAR FEEDBACK CONTROL
OF MULTI-INPUT/MULTI-OUTPUT CHEMICAL SYSTEMS
USING TSYPKIN'S ANALYTICAL APPROACH

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DEDICATION

This thesis is affectionately dedicated to my parents, wife,
and brother-in-law, Mr. A. U. Akpabio

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ABSTRACT

Several methods are available for mathematical prediction of the oscillatory characteristics of a system under nonlinear feedback control. Of these methods, Tsypkin's analysis which treats relay controller exactly has been found to offer superior predictions when applicable. These procedures, in particular Tsypkin's method, have essentially been presented for only single-input/single-output systems. Because Tsypkin's analysis has been successfully tested in several studies, and its applicability experimentally demonstrated for single-input/single-output processes, this thesis extends the mathematical formalisms to encompass multi-input/multi-output process utilizing relay as a control element.

This new theory has general application in nonlinear system analysis. The specific motivation in this research is the stabilization of chemical reaction systems at an unstable steady state condition where control may be desirable. For mathematical tractability and ease of demonstration, the classical literature example of the lumped-parameter nonisothermal process undergoing a first order exothermic reaction was chosen as a specific example. The reactor's concentration and temperature were the controlled output variables which were manipulated with the reactor's input flow rate and coolant temperature respectively. Treatment of this reactor model with relay

controllers gave birth to the new control theory for multi-input/multi-output control systems.

The prediction of the reactor's oscillatory response near an unstable steady state using two relays according to the new theory was compared to the exact response of the process via nonlinear simulation for several sets of relay design parameters. In all cases of control demonstration, sustained oscillation of all control loops of the system (a condition necessary for the application of the new theory) was possible only when the control bandwidth of each controller was restricted within some range of values. A failure to comply to this restriction always resulted in loss of control, and, consequently, in a break down of the new theory application.

The nonlinear simulation results compared well with the new theory results especially at small values of control bands where small amplitude oscillations of the system about an unstable steady state always resulted. In such control restriction, the system approximated a linear process, a condition exploited in the development of the new theory, the analysis and the simulation results agreed very well. But at large control bands, poor comparisons often resulted, an indication that the process was becoming more nonlinear.

Thus the new control theory is applicable for control of multivariable processes when restrictions are placed on control bands of each feed back loop, and when, with

reasonable choice of other control parameters, small amplitude oscillation of the system about an unstable steady state condition is ensured.

TABLE OF CONTENTS

CHAPTER		PAGE
1	LITERATURE SURVEY OF PREVIOUS STUDIES.....	1
	1.1 On Multiplicity of Steady States.....	1
	1.2 On Process Operation and Control.....	5
	1.3 On Control of Unstable Process Operation.....	7
2	THE CONCEPTS OF DECOUPLING, PAST AND PRESENT..	16
	2.1 Early Attempts at Decoupling and Design of MI/MO Processes.....	17
	2.2 Types of Interaction.....	20
	2.3 Some Decoupling Techniques.....	22
	Exact Decoupling Technique.....	22
	Approximate Decoupling Technique.....	28
	Ideal Decoupling Technique.....	29
	State Variable Feed Back Technique.....	35
3	ILLUSTRATIVE EXAMPLE SYSTEM.....	39
	3.1 System and Mathematical Model.....	39
	3.2 Linearization.....	44
	Open-Loop Transfer Functions.....	47
	3.3 Relative Gain.....	49
	Relative Gain of the Illustrative Example System.....	56
	3.4 The Parameters and Constants of the Example System.....	59
4	THE NOVEL CONTROL POLICY FOR CONTROL OF SINGLE-INPUT/SINGLE-OUTPUT PROCESSES.....	62
	4.1 Some Basic Definitions.....	63
	4.2 SI/SO Control Problem.....	67
	van Heerden Diagrams.....	67
	The Slope Condition for Steady State Stability.....	74
	The Control Problem.....	75
	4.3 Quantitative Prediction of Periodic Oscillations.....	84
	Relay as a Nonlinear Control Element...	86
	The General Periodicity Condition for Symmetric Relays.....	86
	The Switching Conditions.....	88
	The Continuity Conditions.....	89
	Tsytkin's Analysis for SI/SO Processes (Old Theory).....	90

5	THE NEW THEORY: DIRECT TREATMENT OF COUPLED MI/MO PROCESSES UNDER FEEDBACK RELAY CONTROL.....	100
5.1	Tsyarkin's Analysis for 2x2 Coupled Process for Cases Where all Loop Controllers are Switching.....	101
	Closed Forms.....	111
5.2	Generalization of Tsyarkin's Analysis for nxn Coupled Processes.....	121
5.3	Possible Switching Modes of the Control System's Relays.....	124
6	SI/SO AND MI/MO SYSTEMS CONTROL RESULTS FOR THE EXAMPLE REACTOR.....	129
6.1	SI/SO Control.....	130
	Design of Concentration Controlling Relay.....	130
	Design of Temperature Controlling Relay.....	134
	Control Procedure.....	136
	Error Evaluation.....	140
	Results and Discussion.....	141
6.2	Multivariable System Control.....	144
	Design of the Concentration Control Loop Relay.....	145
	The Control Procedure.....	155
	Results, Discussion and Recommendation	157
7	PROPOSED FUTURE STUDIES.....	164
	LIST OF REFERENCES CITED.....	165
	APPENDIX.....	174
	VITA.....	195

LIST OF TABLES

TABLE		PAGE
3.1	Illustrative System's Parameters and Constants.....	60
3.2	Illustrative System's Matrices, Eigenvalues and Steady State Solutions.....	61
4.1	Closed Forms of Infinite Series for Tsytkin's Switching Conditions.....	94
4.2	Summations of Infinite Series for Tsytkin's Continuity Conditions.....	94
6.1	Results of Simulation of the Nonlinear Reactor and from Tsytkin's Analysis of the Linearized Reactor Model for Concentration as the SI/SO System Output.....	143
6.2	Results of Simulation of Nonlinear Uncoupled Reactor and from Tsytkin's Analysis of the Linearized Reactor Model for Temperature as the the System's Output.....	144
6.3	Effect of Variation of the Concentration Relay's Bandwidths on the Coupled Loops' Response for $\sigma_1 = 0.7500$, $\sigma_2 = 0.7160$, $\delta_2 = 0.3300$ ($UC1_{ref} = 0.2000$, $UC2_{ref} = 0.0419$, $ye_{ref} = 0.0086$, $we_{ref} = 0.0200$).....	158
6.4	Effects of Variation of the Temperature Relay's Bandwidths on the Coupled Loops' Response for $\sigma_1 = 0.7500$, $\sigma_2 = 0.7160$, $\delta_1 = 1.1100$ ($UC1_{ref} = 0.2000$, $UC2_{ref} = 0.0419$, $ye_{ref} = 0.0086$ $we_{ref} = 0.0200$).....	159
6.5	Effects of Variation of the Concentration Relay's Drive Levels on the Coupled System's Response. $\sigma_2 = 0.7160$, $\delta_1 = 1.1100$, $\delta_2 = 0.3300$ ($UC1_{ref} = 0.2000$, $UC2_{ref} = 0.0419$, $ye_{ref} = 0.0086$, $we_{ref} = 0.0200$).....	160

6.6 Effects of Variation of the Temperature
Relay's Drive Levels on the Coupled System's
Response. $\sigma_1 = 0.7500$, $\delta_1 = 1.1100$, $\delta_2 = 0.3300$

($UC1_{ref} = 0.2000$, $UC2_{ref} = 0.0419$, $ye_{ref} = 0.0086$,

$we_{ref} = 0.0200$)..... 161

LIST OF FIGURES

FIGURE		PAGE
2.1	Open-loop Partially Coupled 4x4 Multivariable System Showing seven Disturbance Paths.....	21
2.2	A Completely Coupled Closed-loop Multivariable System Showing sixteen Disturbance Paths.....	23
2.3	Decoupling of an nxn Coupled Multivariable Process with n-th Order Exact Decoupler.....	24
2.4	A Matrix-vector Presentation of a Coupled Multivariable Process Under Negative Feed Back Control.....	30
2.5	A 2x2 Completely Coupled Multivariable System Decoupled with an Ideal Decoupler into two Independent Subsystems.....	32
3.1	A Schematic Sketch of the CSTR of the Illustrative Second Order System Described by Equations 3.2 and 3.3.....	41
3.2	An SI/SO Control Scheme for Control of Concentration According to Eqns. 4.30, 4.31...	48
3.3	An SI/SO Control Scheme for Control of Temperature.....	48
3.4	Feed Back Control Scheme for Control of Coupled 2x2 Multivariable System.....	50
4.1	van Heerden Diagram for the Second Order Process Obtained with Plot of R_C and R_T Versus u According to Eqns. 4.8 and 4.16.....	71
4.2	van Heerden Diagram for Second Order Process $R_g^T(y)$ and $R_r^T(u, y)$ Versus Temperature (eqn. 4.18, 4.19).....	73
4.3	Graphical Illustration of the Control Policy for 2nd Order System.....	77

4.4	Feedback Control Schemes for Control of the Unstable Reactor (Fig.3.1).....	82
4.5	Relationship Between Input $x(t)$ and Output $u(t)$ of a Passive Relay with a Drive Level $\pm u_c$ and Hysterisis $\pm d$	87
5.1	Relationship Between Concentration Loop Relay's Output and Temperature Loop Relay's Output.....	103
5.2	SI/SO Control System with a Constant Load y Resulting from 2x2 MI/MO System of Fig. 3.4 when only m_2 can Switch.....	126
6.1	van Heerden Diagrams for Design Strategy of SI/SO Relay for Concentration Control ($We_{\max} = w^{\text{lt}}_{\max} - w^*$, $We_{\min} = w^{\text{lt}}_{\min} - w^*$)	131
6.2	SI/SO Output of Temperature System of Figure 3.3 Resulting from Integrating Equations 3.5 and 3.6 for case of $\sigma_2=1.00$, $\delta_2=0.15$	137
6.3	SI/SO Output of Temperature System of Figure 3.3 Resulting from Integrating Eqns. 3.5 and 3.6 for case of $\sigma_2=0.90$ and $\delta_2=0.60$	138
6.4	SI/SO of Concentration System of Figure 3.2 Resulting from Integrating Eqns. 3.5 and 3.6 for case of $\sigma_1=0.95$ and $\delta_1=0.75$	139
6.5	van Heerden Diagrams for MI/MO Concentration Loop Relay Design for case of $\mathcal{F}^* = \mathcal{F} \pm 25\% \mathcal{F}^*$ $We_{\max}^{\text{lt}} = w^{\text{lt}}_{\max} - w^*$, $We_{\min}^{\text{lt}} = w^{\text{lt}}_{\min} - w^*$	147
6.6	van Heerden Diagrams for MI/MO Concentration Loop Relay Design for case of $\mathcal{F}^* = \mathcal{F} \pm 25\% \mathcal{F}^*$ $We_{\min}^{\text{ut}} = w^{\text{ut}}_{\min} - w^*$, $We_{\max}^{\text{ut}} = w^{\text{ut}}_{\max} - w^*$	148
6.7	Multiple van Heerden Diagrams for MI/MO Concentration Loop Relay Design for case of $\mathcal{F}^* = \mathcal{F}^* \pm 10\% \mathcal{F}^*$, $We_{\min}^{\text{ut}} = w^{\text{ut}}_{\min} - w^*$, $We_{\max}^{\text{ut}} = w^{\text{ut}}_{\max} - w^*$	153

6.8 Multiple van Heerden Diagrams for MI/MO
Concentration Loop Relay Design for case

$$\text{of } \mathcal{F} = \mathcal{F}^* + 10\% \mathcal{F}^*, \quad W_{\max}^{lt} = w_{\max}^{lt} - w^*, \quad W_{\min} \\ = w_{\max}^{lt} - w^* \dots\dots\dots$$

TABLE OF NOMENCLATURE

<i>R</i>	Reaction rate, 1/sec
R	Gas constant, cal/(K gm-mole)
E	Activation energy, cal/gm-mole
A	Heat transfer area, cm ²
<i>c_p</i>	Heat capacity of reactor solution, cal/cc K
k	Rate constant, 1/sec
<i>k₀</i>	Preexponential factor, 1/sec
V	Reactor volume, cc
C	Concentration, gm-mole/cc
T	temperature, K
<i>T_c</i>	Coolant temperature, K
U	Overall heat transfer coefficient, cal/(cm ² K sec)
-ΔH	Heat of reaction, cal/gm-mole
ρ	Density of reactor solution, gm-mole/cc
<i>F₀</i>	Feed flow rate, cc/sec
<i>C₀</i>	Feed concentration, gm-mole/cc
<i>T₀</i>	Feed temperature, K
β	Dimensionless adiabatic temperature rise $\frac{-\Delta H C_0}{\rho c_p T_0}$
ξ	Arrhenius number, E / R T ₀
θ	residence time, V / F
Da	Damkohler number , <i>k₀</i> θ
τ	Dimensionless time , $\frac{t}{\theta}$
γ	Dimensionless heat transfer coefficient, $\frac{UA}{F \rho c_p}$
<i>y</i>	Dimensionless temperature, T / T ₀
<i>ce</i>	Dimensionless concentration, C / C ₀

$\mathcal{U}$	Dimensionless coolant temperature, T / T_0
$\mathcal{F}$	Dimensionless flow rate, F / F_0
y	Small deviation in temperature from unstable steady state value
w	Small deviation in concentration from unstable steady state value
u	Small deviation in coolant temperature from unstable steady state value
f	Small deviation in flow rate from unstable steady state value
s	Laplace transform variable
$G(s)$	Open loop transfer function of linearized system
$G(\infty)$	Constant with which the process discontinuities are associated
$\underline{a}$	System's Jacobian matrix
$\underline{b}$	System's input matrix
Λ	Relative gain matrix
α_{ij}	Relative gain of i -th output manipulated with j -th input
$r_i(t)$	Command input (set point) of i -th control loop
$\bar{x}_i(t)$	Error input to nonlinearity element of i -th control loop
n_i	Nonlinear control element of i -th control loop
$u_i(t)$	Nonlinearity output of i -th control loop
R_g	Generation quantity
R_r	Removal quantity
d_i	Bandwidth of i -th control loop's relay

UC_i	Drive level of i -th control loop relay
Y_e	Small change in y
W_e	Small change in w
T_o	Period of system (all loop) oscillation
Ω	Frequency of system (all-loop) oscillation
A_i	Amplitude of i -th control loop's periodic output
ϕ	Phase shift among system's periodic outputs
t_ϕ	Shift in switching time between nonlinear control elements
α	Signal parameter
α_n	n -th partial fraction expansion coefficient of process transfer function
λ_n	n -th eigenvalue of linearized system
e	Relative error introduced by linearization approximation
N	Small neighbourhood about unstable steady state point
H	High temperature steady state
L	Low temperature steady state
M	Unstable steady state
$Im()$	Imaginary component of quantity in the bracket
$Re()$	Real component of quantity in the bracket

SUBSCRIPTS

Min	Minimum values only
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Max	Maximum values only
1	Pertaining to concentration
2	Pertaining to temperature
A ₁	Of concentration amplitude
A ₂	Of temperature amplitude
o	First oscillatory solution from Tsyarkin's analysis
<i>ut</i>	Of upper tangency
<i>lt</i>	Of lower tangency

SUPERSCRIPTS

sim	Of simulation
Tsyp	Of Tsyarkin's
*	Pertaining to unstable steady state
-	At minimum flow rate only
+	At maximum flow rate only

CHAPTER 1

LITERATURE SURVEY OF PREVIOUS STUDIES

1.1 ON MULTIPLICITY OF STEADY STATES

The discovery of the existence of multiplicity of steady states in a chemical system was first reported by Lijenroth (9) in 1918 while studying oxidation of ammonia using platinum gauze as catalyst. He was able to relate the percentage conversion of ammonia to the variations of the system's temperature with what he called a "conversion curve." In addition, Lijenroth observed a linear dependency of the conversion of ammonia with the gauze temperature by means of another curve termed a "heat of conversion line." The plots of these two curves as functions of the system's temperature produced intersections which represented the steady state conditions of the system. Although Lijenroth intuitively realized that not all of these states were stable, he was unable to experimentally establish such as a scientific fact.

Little or nothing was done to develop or substantiate Lijenroth's idea until about the middle of the 1940's when Wagner (10) rediscovered this pathological behaviour in a flow reactor. Again, in 1953, after intensive study of reactions in autothermic flow systems, van Heerden (11)

independently established a basis for steady state analysis. His approach, like Lijenroth's (and Wagner's) was a graphical presentation of the process, but with more mathematical flavour: van Heerden carried out both mass and energy balances on his system and extracted for the first time the mechanisms of heat generation and removal as functions of the system's temperature. Moreover, he (again, intuitively) noted the equality of these two quantities at the steady state condition of the process. With this important insight, it was logical for him to conclude that the intersections of his generation and removal curves gave the steady state temperatures.

From this study sprang the criteria of the "slope condition" for stability of a steady state which helped explain via physical reasoning the system's pathological behaviour at these conditions. The slope condition states that the slope of the removal curve must exceed that of the generation curve at a steady state for the state to be stable. Stated mathematically, the slope condition for steady state stability is

$$\frac{d\sigma_r}{dx} > \frac{d\sigma_g}{dx} \quad 1.1$$

where σ_r and σ_g are the removal and the generation quantities respectively, and x is the dependent variable maintained in the steady state analysis. That is, all other dependent variables are eliminated from the steady state equation.

van Heerden extended this analysis to a continuous stirred tank reactor (CSTR) and to a plug flow reactor (PFR) with axial temperature conduction (12).

The slope condition based only on intuitive arguments was treated as a necessary and sufficient condition for local steady state stability until Aris and Amundson (13-15) put the steady state analysis on a firm mathematical basis in 1958. In view of the misunderstanding of the slope condition still found in the literature today, it is important to emphasize that this condition is in general only a necessary condition for steady state stability. Only in the case of a one variable system (completely describable by one differential equation) does the slope condition become a necessary and sufficient condition for stability.

The development of this important analytic formalism of van Heerden's coupled with the resurgence of interest in the early 1950's in the nonlinear mechanics available in works such as those of Liapounov (82), Poincare (83), and Minorsky(84) not only served as an impetus prompting more investigations in this area of study but also lent greater insight into the problem. Thus in 1958, Aris and Amundson, in a series of now classical papers (13-15), using the available nonlinear mechanics tools, set the local stability analysis on a concrete mathematical foundation.

van Heerden's slope condition (11,12) indicated that in the case of multiplicity of steady states, only the intermediate states were unstable. Subsequent studies (14-16,85) proved that other states could be unstable. In particular, the phase plane diagrams of Aris and Amundson (13,14) showed that all the existing states could be unstable. Among the variations of and the departures from such observations have been the theoretical studies of the systems involving more than one phase, more than one reaction, or systems with reactors in series. Generally, the features studied are similar to the afore-mentioned earlier investigations, but the complexity of the system's equations often leads to an increase in the multiple steady states and additional complexity in the process pathology. Schmitz and Amundson (85) showed, for example, that a single exothermic reaction may give rise to as many as nine steady states when it occurs in two fluid phases with transfer resistance between the phases. In a similar situation involving multi-phase polymerization, Goldstein and Amundson (86) encountered up to twenty five different steady states.

These early expositions stemming from the works of van Heerden greatly broadened understanding of the dynamics and other pathological characteristics of chemical systems. This naturally paved way for a voluminous amount of studies on chemical system design, control and stability analysis. Most of these works were focused on mixed flow reactors which included theoretical (9-13,16,28,30) and experimental

(11,18-24,25,26) treatments.

The interest in chemical systems with multiple steady states has also been sparked , in part, by the growing awareness in the chemical industry of the desirability, and even the necessity, of stabilizing some processes at an unstable steady state condition and, in part, by the extensive researches by biologists and biophysicists into similar pathological behaviours in organic systems such as cells. Excellent reviews of the past and the current research activities have been given by Ray (5), Schmitz (17), and others (87-89).

1.2 ON PROCESS OPERATION AND CONTROL

The large amount of interest devoted to operation of processes at unstable steady state is not without some justification. Westerterp (27) carried out both experimental and theoretical studies of the synthesis of phthalic anhydride in a well-mixed reactor. This system is known to exhibit two locally asymptotically stable steady states and one unstable steady state. Westerterp's analysis emphasized the necessity of operating this system at the unstable steady state temperature. At the low temperature steady state, the conversion is too low for practical purposes. On the other hand, at the high temperature steady state, only degraded products result. Therefore, it is

imperative to carry out this synthesis at the intermediate unstable state.

Knorr and O'Driscoll (28) considered free-radical polymerization of styrene in isothermal CSTR. They showed that multiplicity of steady states could arise in this system due to viscosity or "gel" effects. At the high conversion steady state, this effect is so large that practical operation is impossible, while at the low conversion steady state, the conversion rate is too low. Once again, the operation of this system at the unstable state condition offers the best compromise.

Other interesting studies that deserve notice here are those of Schmeal and Barkelaw (95); Iscol (98); Elnashaie, and Merik and Ray (29); Andrews (96,97) and Sharma, Ding et al (21). For full discussion of these additional works, see (5,17,26,34,87-89).

The literature review would not be complete without mention of the big effort devoted to the area of process control called "periodic control." While research in this area has been mainly focused on improving process performance of stable systems through the use of periodic forcing inputs, the same implications surely hold for unstable processes. The emergence of these implications is shown in the work of Matsubara and Onogi (92). For additional information, the reader is referred to the recent review by Bailey (90).

1.3 ON CONTROL OF UNSTABLE PROCESS OPERATION

Much of the interest in the design and control of processes at unstable steady state conditions, as indicated above, has been spurred by the findings in numerous studies which support the desirability and sometime the necessity of process operation at this condition. The effort to solve this important problem has grown tremendously in the past few decades resulting in linear feed-back control strategies for lumped parameter reactors at unstable conditions. In the early works of Aris and Amundson (13-15), a method of design and control operation at unstable point was developed. Now, a wealth of theories spiced with the essential mathematical formalisms, for this control design has been amply furnished in the standard texts such as those of Aris (103) , Douglas (93), and Perlmutter (105) .

In his text, Aris gives an extensive treatment of the design aspect of the control problem for CSTR and tubular reactors. Dwelling upon these mathematical arguments of Aris', Bruns (34), and Paredes (26) outlined a linear control (captioned "Conventional Approach") at an unstable steady state. A successful implementation of this control strategy leads to a closed-loop system whose steady state is asymptotically stable for small deviations from the condition at which the control is desirable. In order for this to be possible, this linear control design may require drastic

changes in the process control parameter. Effecting such changes may require excessive equipment, heating or cooling capacity, large amount of recycle, and other extreme conditions. The consequences of the demands being difficult to meet are often the reason for rejection of open loop unstable processes as a candidate for the final design. This failure naturally necessitates some quests into other control and design alternatives, one of which is the so-called "Novel Control Philosophy" employing a nonlinear feed back control scheme for stabilizing the process at an unstable steady state condition. The related design procedure has been developed by Bruns and Bailey (7,36). Such alternatives often overcome the difficulties imposed by the Linear Approach. The novel control strategy is the basis of this thesis, and the argument which justifies its application in this work is given in chapter four.

The conventional approach, however, has been studied in several works. Root and Schmitz (25) reported operation of a loop reactor (a tubular reactor with recycle) at an unstable operating point for short periods of time by manually adjusting according to a "modified proportional" manipulation scheme. They also reported that their attempt at automatic linear feed-back control failed. This study, however, served as a precursor study to that of Chang and Schmitz discussed next.

Chang and Schmitz (19,20) used linear controllers (proportional and proportional-integral) for the exothermic liquid-phase reaction between sodium thiosulphate and hydrogen peroxide in a nonadiabatic CSTR. In this system, they found experimental conditions which result in three steady states, two of which are stable and one unstable; and conditions which result in a unique unstable state. For both of these types of unstable states, they achieved operation at the unstable point and obtained a very good agreement with theoretical predictions.

Literature is very rich in both the theoretical studies and experimental applications of linear control strategies. The reader is referred to (17,26,36) for further reading.

Although the conventional control strategies have apparently been successfully applied, there is motivation to implement new control policies. As mentioned before, these new approaches often overcome the problems which accompany application of linear methods. One such method (mentioned above) has been developed by Bruns and Bailey. Central to this new control strategy is the adoption of a less restrictive control objective. Most conventional control methods are based upon a very restrictive goal: keep the system at the chosen operating point. This requires at least a local asymptotic stability of the state in the controlled process (7,34,36). It seems quite reasonable and consistent with the current practice to adopt a more general

control objective: the controller should keep the process within an adequately small neighbourhood of the desired operating point. Pursuing this generalized control objective, a stable operation near an unstable steady state can be realized by using a relay with hysteresis as a feed-back controller to establish a stable, small amplitude oscillation in the neighbourhood of the operating point. The details of such control design are given in Chapter 4.

This novel control policy has been developed and expounded in several publications of Bruns et al. (7,34-36), and has recently been successfully implemented by Paredes (26) in his experimental study and control of a CSTR. Matsubara and Onogi (92), in studying the optimization of a certain chemical reactor, applied this control policy to stabilize operation near an unstable steady state of a process. Sharma (94), and Ding et al. (21), in a study of gas-liquid adiabatic CSTR showed theoretically and experimentally that this system for chlorination of liquid n-decane exhibits multiple steady states with varying liquid residence time. The method of controlling the process about the unstable steady state was achieved experimentally by adjusting an input between two levels to achieve an oscillatory response in the vicinity of the unstable steady state. This control scheme is similar to the novel control policy but with some differences. Such differences are discussed by Bruns (34,pp.28).

Thus nonlinear control of unstable processes has not been in extensive development and application in comparison with linear control operations. It is unfortunate also to note that all the theoretical and experimental studies (discussed above) on both linear and nonlinear control and design were, without an exception, devoted to process containing a single control loop. Nowhere in the chemical engineering literature has there been reported similar studies on processes containing multi-control loops. This thesis seeks to fill a part of this vacuum by modifying the novel control policy for the design and control of processes using multi-control loops which contain nonlinear controllers.

Throughout this thesis, single-input/single-output (SI/SO) process will be used to mean a process which contains a single control loop where one output (dependent variable) is measured and one input (independent variable) is manipulated. Likewise, multi-input/ multi-output (MI/MO) process will mean a process which contains two or more control loops where several outputs are measured and correspondingly several inputs are measured. The term "multivariable" control will also be used for MI/MO process control. It is appropriate to also note that a single variable process is one that can be described by a single differential equation and an n-variable process requires n differential equations to describe its state. The number of such differential equations is the "order" of the process.

As it is shown in the previous section, the control theory for the past few decades has undergone such a tremendous development that it is today outstanding among applied sciences. But most of these studies are directed at the SI/SO process control and design with the consequences that one of the major branches of the control theory, the control of MI/MO systems, is still at the embryonic state of its development. Moreover, many of the developments in MI/MO control are based on extensions of SI/SO control theories. The logical inconsistency in regarding problems in MI/MO processes as mere generalizations of similar problems of SI/SO systems is obvious enough: SI/SO systems are special cases of MI/MO processes, and not vice versa.

This and other factors had contributed to the unnecessary lag in the theoretical development and practical utilization of universal techniques for multivariable control and design worthy of extensive application in Chemical Process Industry. In fact, this majestic branch of applied science has historically lagged far behind the electronic- and aerospace-oriented industries in the application of multivariable control theory. Perhaps this lag was justified in view of the chemical processes in comparison to the speed of elements inherent in electronic systems. But as these techniques are accepted in the chemical industry and the benefit of the improved performance realized, this lag will certainly diminish.

One of the chief properties of an MI/MO process is interaction (or cross-coupling), a condition which occurs when there are inputs which simultaneously affect more than one output. Interaction, more than any other factors, presented the greatest difficulties to the early engineers working with multivariable control processes and, it can be said without qualms, that it was the chief contributor to the slow advancement in the multivariable control theory. In the early fifties, engineers realized that certain processes with multi-loop feed-back exhibited some characteristics which defied analysis of the MI/MO processes with the existing single-loop control practices. To bring a system back to a desirable operating level, adjustments had to be made on all the affected loops. This practice led to the notion that the controlled variables could be decoupled through relationships between several process inputs and outputs. Several approaches were suggested, and most were adopted for the design of these systems with the objective of avoiding interactions known to be prejudicial to the system's performance. Chapter 2 will look at some of these early developments which culminated in today's control and decoupling techniques a few of which are also given in Chapter 2.

The presence of interaction makes the control of MI/MO systems difficult and, perhaps in some cases, impossible. In the design of such a system, care must be taken to ensure that each loop is well compromised so that its action will

not adversely affect the remaining loops response. This is often difficult to achieve as even the simplest multivariable control design can be mathematically complicated. This difficulty has been handled and sometimes circumvented in several ways (see Chapter 2) each approach striving to break down the coupled system into independent subsystems each with a single output as a function of one and only one input. The subsystems can then be independently regulated with the SI/SO control strategies.

The principal aim of this research is to make the required theoretical extensions of the Novel Control Policy (see section 1.3) which has been presented for SI/SO process control to allow the direct analysis and design for processes under multivariable control. This type of control has hitherto not been present in the literature reviewed above. The essential feature will be the need to incorporate the effects of cross-coupling among the control loops. This is accomplished by employing the concept of Tsytkin's analysis for single control loop processes in the context of the more general MI/MO control problems. This has resulted in analysis equations that can be used for control design studies.

The focus of the research is on the control problems faced in stabilizing a process near an unstable operating point. The emphasis is further on the theoretical developments which must be made. Thus the classic

literature example of the exothermic reaction in a stirred tank has been utilized as the example in discussions and in illustration of applications.

Some initial investigation was directed at the interaction which would occur between nonlinear controllers and process decouplers. Of specific interest was the possibility of the nonlinear nature of such controllers being used to address some of the problems faced in decoupling such as decoupler saturation. This appeared to be a consideration of such complexity that it would justify a separate study. This initial effort resulted in the literature survey presented in Chapter 2. It is noted to the reader that the central part of this research can be followed without reading Chapter 2.

The fact remains that if a process is suitably decoupled, each control loop can be treated with SI/SO control design methods. In this context, Chapter 4 presents a theory for SI/SO process control with a relay as a control element. The results of such applications given in Chapter 6.

The SI/SO theory given in Chapter 4 is the starting point for the new theoretical extensions given in Chapter 5 for MI/MO control. The results from applications are also given in Chapter 6. Other interesting and relevant studies are suggested in Chapter 7.

CHAPTER 2

THE CONCEPTS OF DECOUPLING, PAST AND PRESENT

This research emphasizes the multivariable control problems and direct analysis accounting for all interactions in such control problems. It is also recognized that the inherent cross-coupling is often an undesirable property in such problems and the possibility utility that decoupling techniques might serve. To confine this work within the purview of control formalisms which may result in a new theory for multivariable process control, none of the decoupling techniques will be tested or applied. However, this chapter seeks to outline in a chronological fashion the efforts at decoupling of MI/MO systems. Slow and precarious though they were, these early attempts have today culminated in the classical MI/MO system decoupling and design techniques. As a spring-board for future continuation of this study, the second section of this chapter summarizes some of the most applied of these techniques and the extents and limits of their applicability.

Within the context of the control application of interest, the multivariable control of an unstable reactor, a point of concern with the utilization of various decoupling techniques is if their theoretical basis encompasses unstable processes. This question could not be adequately addressed within the confines of the following survey. The

theoretical restriction on analysis techniques are often not well stated beyond their original presentation. Consequently, the reader is cautioned in utilizing techniques which may require the system under study to be open loop stable although it is not explicitly stated.

2.1 EARLY ATTEMPTS AT DECOUPLING AND DESIGN OF MI/MO PROCESSES

Among the reported attempts at a design and decoupling of multivariable processes were those of Boksenbom and Hood (42) who, in the early fifties studied interactive effects in the turbopropello- engines and came to a solution which applied some feed-back control scheme to cancel cross-effects. Pevejsil and Fuchs (43) developed a synthesis method which relied on trial and error analog computer techniques. Amara (44,45), and Hsieh et al.(46) came closest to the actual control situation by extending the method of synthesizing a random input filter to multivariable coupled systems.

By this time, matrix notation had been introduced primarily for its value in organizing process state variables. But the first application of rigorous mathematical analysis for a basic understanding of the properties of multivariable systems was made by Kavanagh (40,41) who used matrix algebra to study some general

problems by obtaining controller characteristics on the basis of the knowledge of the closed loop behaviour of the systems. Freeman (47,48) treated the more realistic problems by synthesizing controllers by root-locus techniques and also by introducing the concept of output noninteraction. Goodman (50), and Chestnut and Meyer (51) developed some methods for determining the transfer function of MI/MO systems with random inputs. Kinnen and Liu (52) proposed designs of systems' transfer functions with root-loci. These root-locus techniques did provide a systematic application of the familiar graphical approaches when the design specification involved the system's transient interactions rather than noninteractions.

Most of these early attempts to design and decouple multivariable processes had a common set back: the so-called compensating devices became extremely complex for systems with large number of inputs and outputs. The introduction of the state variable notations gave the problem another useful mathematical twist, and helped in no small way to push the problem forward by providing the engineers a new effective tool. The earliest use of this technique was in the synthesis method developed in 1959 by Bertram (53). The general nature of the decoupling problem was highly advanced because of the application of this new technique (54,58,59-61). These ideas could now be extended for systems with fewer inputs than outputs by means of practical decoupling (50-59).

Morgan (62,63) utilized this state variable feed-back approach and made it possible to decouple a process by means of feed-back and feed-forward amplifiers. Rekasius (66) extended Morgan's results and outlined a trial and error procedure for specifying certain number of the system's poles while decoupling the system. But neither of these men gave a clear proof of sufficiency, and they did not consider the question of necessity for decouplability and controllability of multivariable systems. Falb et al. (54,55), and Wolovich (58) lent this problem a more mathematical touch by theoretically determining both the necessary and sufficient conditions for decoupling MI/MO systems by first deriving a class of matrices which characterized the processes.

Another useful approach for the design of MI/MO systems is derived from the canonical form presentation of the systems. Mesarovic (64,65) outlined a simple approach by first transforming a system to a V-canonical form in such a way that the cross-effects became internal feed-back signals which could be easily cancelled by addition of the corresponding external feed-back loops. Foster and Stevens (67,68) based their method on a modified V-canonical transformation which could be applied to systems with unequal number of inputs and outputs. This was accomplished by applying compensators in the form of feed-forward amplifiers and proportional plus derivative feed-back controllers.

Although the state space and canonical form techniques have had a major impact on the theoretical developments of multivariable system design and control, the frequency domain design technique is equally powerful if not a more effective mathematical tool. In the past ten years, there have been numerous attempts to extend this method to the design of MI/MO systems, especially in England where Rosenbrock, McFarlane, Mayne and their colleagues as well as students of the University of Manchester Institute of Sciences and Technology (UMIST) have proposed a variety of approaches to the problem. In his pioneering paper (71), Rosenbrock applied frequency domain technique for the MI/MO system design. Beside providing a useful theoretical basis, he described his design scheme based upon the Inverse Nyquist Array (INA). This is probably the most widely applied technique to date. Rosenbrock and his colleagues have published numerous papers (71-80) on the extensions and modifications of this approach for various design and control situations.

2.2 TYPES OF INTERACTION

Coupling appears in two forms: partial and complete coupling. In partial coupling, one or more manipulated variables (u_i) affect two or more controlled variables (y_i) while the rest of the manipulated variables influence only their respective controlled loops. Figure 2.1 shows partial

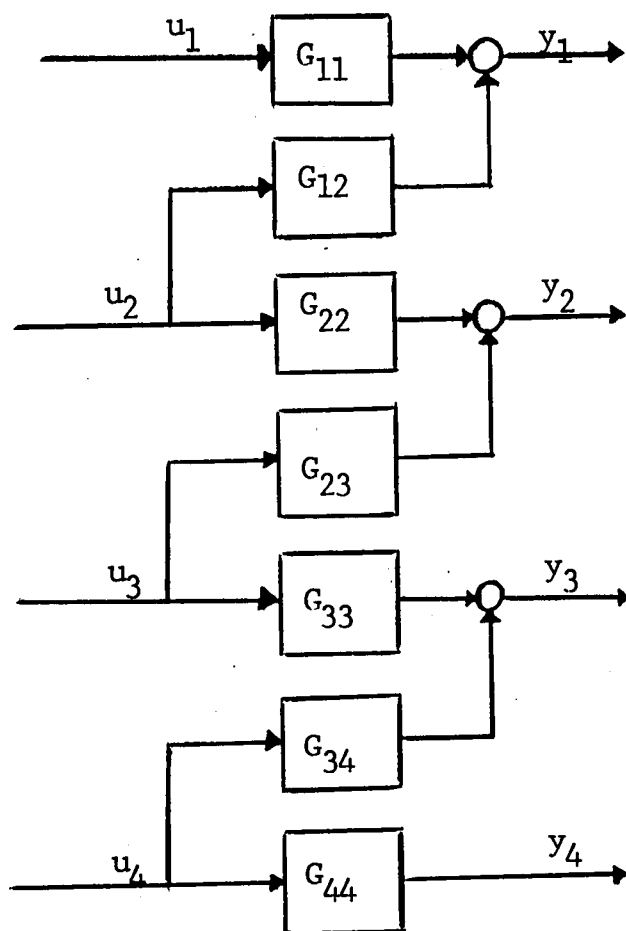


Figure 2.1: Open-loop partially coupled 4x4 multivariable system showing seven disturbance paths

coupling in a 4x4 open-loop MI/MO system. Here, the total number of disturbance paths is seven which is less than the total expected (sixteen are possible) in a completely coupled 4x4 system.

Complete coupling is said to exist in a system if each manipulated variable influences all the system's controlled loops. Such coupling forms additional closed loops so that in any $n \times n$ completely coupled system, the total number of closed loops is 2^n if all the disturbance paths are present. Fortunately, in many practical engineering systems, some of these paths are absent or sufficiently weak reducing the total and possibly resulting in only 2x2 coupled system. A completely 4x4 coupled multivariable process is shown in figure 2.2.

2.3 SOME DECOUPLING TECHNIQUES

Exact Decoupling Technique

The design of a controlled system based upon Exact Decoupling Technique (also called "Diagonization" approach) may be achieved through the following reasoning: consider a p -canonical model of a completely coupled $n \times n$ MI/MO process depicted in figure 2.3A where $\underline{G}$ and $\underline{D}$ are respectively the process and decoupling matrices evaluated in the Laplacian domain. The "pseudo-process" (figure 2.3B) resulting from

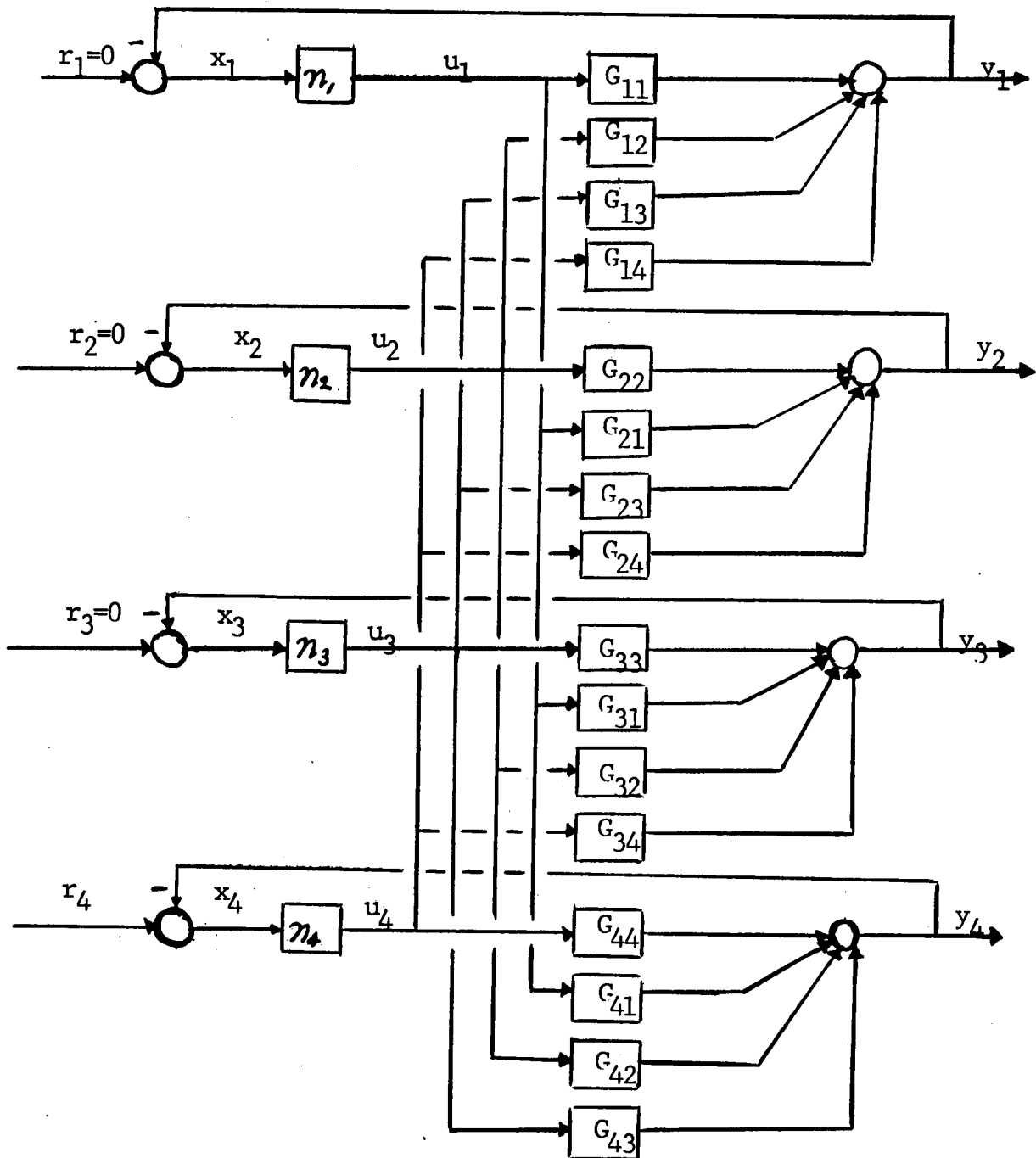
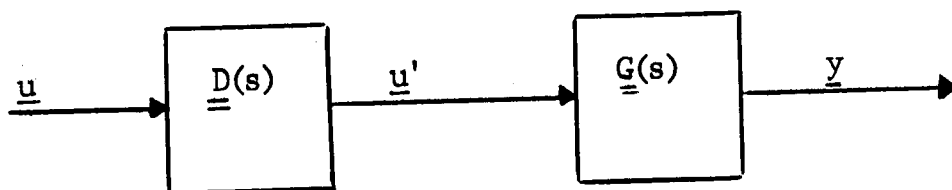
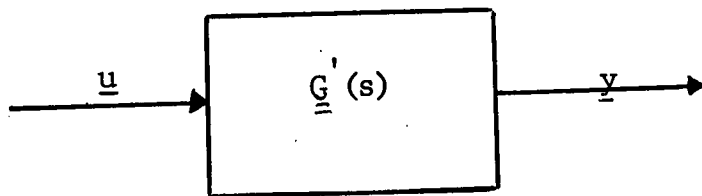


Figure 2.2 : A completely coupled closed-loop multivariable system showing sixteen disturbance paths



A): Coupled Process



B): Decoupled Process

Figure 2.3: Decoupling of an $n \times n$ Coupled Multivariable Process with n -th Order Exact Decoupler

the use of an n-th order decoupler is assumed to be completely decoupled so that the system can now be treated with any of the usual single-loop control approaches.

The mathematical treatment which may lead to successful design and implementation of this decoupler can be carried out in the following manner: for the nxn coupled process, let

$\underline{\underline{G}}$ = the open-loop transfer matrix

$\underline{\underline{D}}$ = nxn decoupling matrix

$\underline{\underline{u}}'$ = n-input vector to the decoupling matrix

$\underline{\underline{u}}$ = n-input vector to the open-loop transfer matrix

$\underline{\underline{y}}$ = n-output vector

From figure 2.3A, it is obvious that

$$\underline{\underline{u}}' = \underline{\underline{D}} \underline{\underline{u}} \quad 2.1$$

$$\text{and } \underline{\underline{y}} = \underline{\underline{G}} \underline{\underline{u}}' = \underline{\underline{G}} \underline{\underline{D}} \underline{\underline{u}} = \underline{\underline{G}}' \underline{\underline{u}} \quad 2.2$$

Let $\underline{\underline{G}}'$ be the pseudo-process transfer matrix which, according to equation 2.2, is the product of the process matrix ($\underline{\underline{G}}$) and the decoupling matrix ($\underline{\underline{D}}$). In the matrix-vector form, equation 2.2 may be written as

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11} & \cdot & \cdot & g_{1n} \\ \cdot & & & \cdot \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ g_{n1} & \cdot & \cdot & g_{nn} \end{bmatrix} \begin{bmatrix} d_{11} & \cdot & \cdot & d_{1n} \\ \cdot & & & \cdot \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ d_{n1} & \cdot & \cdot & d_{nn} \end{bmatrix} \begin{bmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{bmatrix} \quad 2.3$$

The decoupling requires that each column and row of G' contain one and only one nonzero element.

In order for the i -th input(u_i) to affect only the i -th output (y_i), G' must be a diagonal matrix in which all of the off principal diagonal elements are zero. For ease of demonstrating the development of this design technique, consider a 3x3 completely coupled system. Substituting 3 for n in eqn.2.3, the elements of G' are easily determined to be

$$g'_{11} = g_{11} d_{11} + g_{12} d_{21} + g_{13} d_{31} \quad 2.4$$

$$g'_{12} = g_{11} d_{12} + g_{12} d_{22} + g_{13} d_{32} \quad 2.5$$

$$g'_{13} = g_{11} d_{13} + g_{12} d_{23} + g_{13} d_{33} \quad 2.6$$

$$g'_{21} = g_{21} d_{11} + g_{22} d_{21} + g_{23} d_{31} \quad 2.7$$

$$g'_{22} = g_{21} d_{12} + g_{22} d_{22} + g_{23} d_{32} \quad 2.8$$

$$g'_{23} = g_{21} d_{13} + g_{22} d_{23} + g_{23} d_{33} \quad 2.9$$

$$g'_{31} = g_{31} d_{11} + g_{32} d_{21} + g_{33} d_{31} \quad 2.10$$

$$g'_{32} = g_{32} d_{12} + g_{32} d_{22} + g_{33} d_{32} \quad 2.11$$

$$g'_{33} = g_{33} d_{13} + g_{32} d_{23} + g_{33} d_{33} \quad 2.12$$

The requirement that the entries not on the principal diagonal be zero is met by setting equations 2.5, 2.6, 2.7, 2.9, 2.10 and 2.11 to be equal to zero. Eqns. 2.4-2.12 can further be simplified by setting the diagonal entries equal to unity. This latter simplification is justified by the fact that the primary objective of a great majority of the

chemical process applications is the regulator-type control, and this simplification is the maximum requirement to force the solutions to a form readily implemented. With these assumptions, eqns. 2.4-2.12 can be simultaneously solved to give

$$d_{11} = d_{22} = d_{33} = 1$$

$$d_{12} = \frac{g_{13}g_{32} - g_{12}g_{33}}{g_{11}g_{33} - g_{13}g_{31}}$$

$$d_{13} = \frac{g_{12}g_{23} - g_{13}g_{22}}{g_{11}g_{22} - g_{12}g_{21}}$$

$$d_{21} = \frac{g_{23}g_{31} - g_{21}g_{33}}{g_{22}g_{33} - g_{23}g_{32}}$$

$$d_{23} = \frac{g_{12}g_{23} - g_{13}g_{22}}{g_{11}g_{23} - g_{12}g_{21}}$$

$$d_{31} = \frac{g_{32}g_{21} - g_{22}g_{31}}{g_{22}g_{23} - g_{23}g_{32}}$$

$$d_{32} = \frac{g_{12}g_{31} - g_{11}g_{32}}{g_{11}g_{22} - g_{12}g_{21}}$$

The Exact Decoupling is readily implemented either in feed-forward or feed-back control schemes for processes composed entirely of the first order lags. That is, systems with open-loop transfer matrix entries as

$$g_{ij} = \frac{kp_{ij}}{1 + \tau_{ij}s} \quad 2.13$$

The technique can also be applied for systems with dead time

$$g_{ij} = \frac{kp_{ij}}{1 + \tau_{ij}s} e^{-\phi_{ij}s} \quad 2.14$$

When the Exact Decoupling is successfully implemented, the decoupling is exact. That is, the system is completely decoupled into independent subsystems.

Approximate Decoupling Technique

The implementation of the Exact Decoupling for a system composed entirely of the second order lags, that is, the systems whose transfer matrices contain entries of the form

$$g_{ij} = \frac{kp_{ij}}{(1 + \tau_{A_{ij}}s)(1 + \tau_{B_{ij}}s)} \quad 2.15$$

is difficult because of the complications following introduction of the additional parameters. The problem is circumvented by following the suggestion of Zalkind (101,102). This makes it possible to tune each controller to optimize the process performance. Moreover, this approach permits the measurement of the extent of

interaction. The technique also allows decoupling of selected outputs by adjusting the corresponding parameters in a column of $\underline{D}$. Thus to decouple y_i , one needs to adjust the control parameters in the i -th column of $\underline{D}$.

Because of the selective decoupling (as opposed to complete decoupling in the Exact Decoupling Technique), this technique is not exact. The residual interaction can be minimized by careful design and by correct pairing of the state variables (see section 3.3). The approximate decoupling approach is recommended for very strongly coupled systems and for those systems not easily amenable to the exact decoupling method.

Ideal Decoupling Technique

Consider a p -canonical model of an $n \times m$ completely coupled system under the feed-back control as depicted in figure 2.4. With

- $\underline{G}$ = $n \times n$ open-loop transfer matrix
- $\underline{G}^c$ = $n \times n$ control matrix
- $\underline{u}$ = n -input vector
- $\underline{y}$ = n -output vector
- $\underline{r}$ = n -set point vector

the Ideal Decoupling technique (69) can be developed for

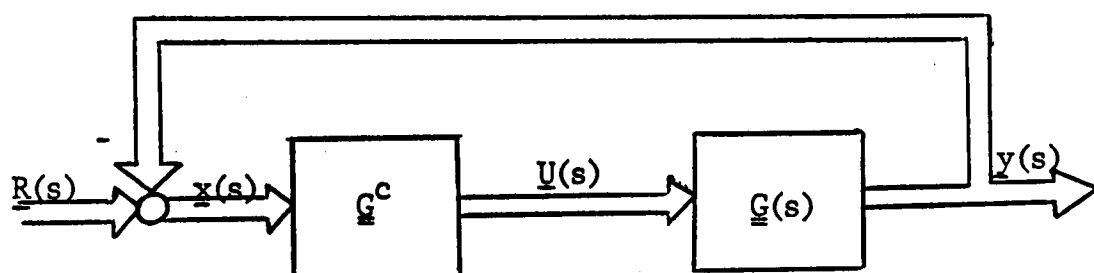


Figure 2.4: A matrix-vector presentation of a coupled multivariable process under negative feed back control

this process as follows: the output vector $\underline{y}$ for this system is determined to be

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ . \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11} & . & . & . & g_{1n} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ g_{n1} & . & . & . & g_{nn} \end{bmatrix} \begin{bmatrix} u_1 \\ . \\ . \\ . \\ . \\ u_n \end{bmatrix} \quad 2.16$$

The input vector is given as

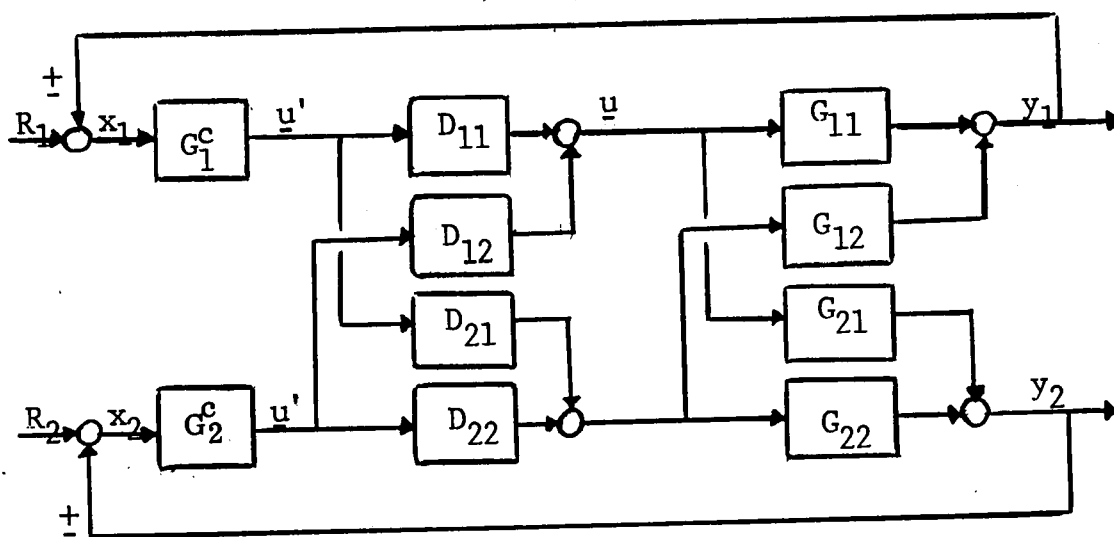
$$\begin{bmatrix} u_1 \\ . \\ . \\ . \\ . \\ u_n \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & . & . & . & 0 \\ 0 & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ 0 & 0 & . & . & . & g_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ . \\ . \\ . \\ . \\ x_n \end{bmatrix} \quad 2.17$$

Equations 2.16 and 2.17 may be rewritten in more condensed forms using notation as

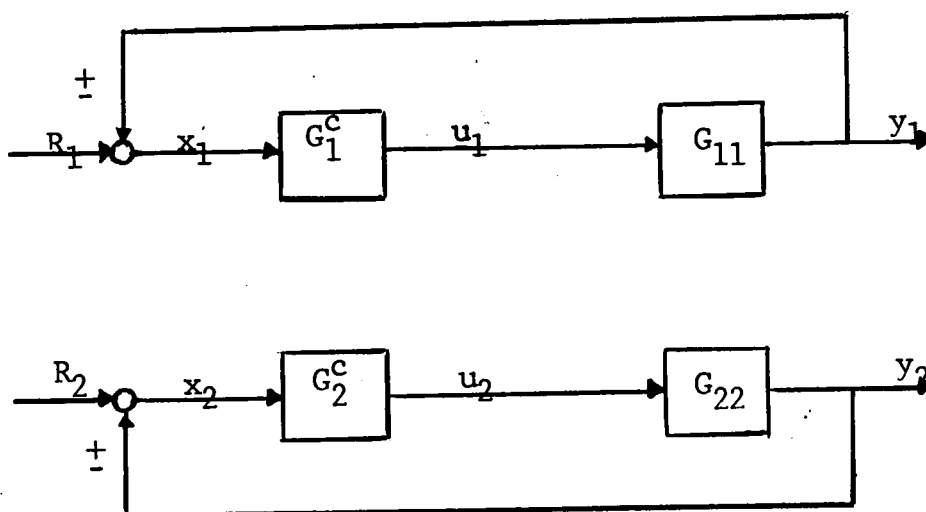
$$\underline{y} = \underline{G} \underline{u} = [\underline{I} + \underline{G}^c \underline{G}]^{-1} \underline{G}^c \underline{G} \underline{R} \quad 2.18$$

$$\underline{u} = \underline{G} \underline{x} = \underline{G}^c (\underline{R} - \underline{y}) \quad 2.19$$

To eliminate interactions between the control loops, the decoupling element $\underline{D}$ is added to the system as shown in figure 2.5A. Thus eqn. 2.18 becomes



A): Coupled 2x2 System



B): Decoupled 2x2 System

Figure 2.5: A 2x2 Completely Coupled Multivariable System Decoupled with an Ideal Decoupler into two Independent Subsystems

$$\underline{y} = [I + \underline{G}^c \underline{D} \underline{G}]^{-1} \underline{G}^c \underline{D} \underline{G} \underline{R} \quad 2.20$$

$$= \underline{\Phi} \underline{R} \quad 2.21$$

where Φ is the closed-loop transfer matrix. In noninteraction control the response of each loop must be the same as that which would be obtained when the other loop were on manual control (that is, fixing the other loop's manipulated variable). Thus the responses of the two loops in figure 2.5A are ideal. Consequently, the closed-loop transfer matrix can be easily evaluated as

$$\Phi = \begin{vmatrix} \frac{g_{11}^c g_{11}}{1 + g_{11}^c g_{11}} & 0 & \dots & 0 \\ 0 & & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \frac{g_{nn}^c g_{nn}}{1 + g_{nn}^c g_{nn}} \end{vmatrix} * \begin{vmatrix} \frac{1}{1 + g_{11}^c g_{11}} & 0 & \dots & 0 \\ 0 & & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{1 + g_{nn}^c g_{nn}} \end{vmatrix} \begin{vmatrix} g_{11}^c \dots 0 \\ 0 \dots 0 \\ \vdots \vdots \vdots \\ 0 \dots g_{nn}^c \end{vmatrix} \begin{vmatrix} g_n^0 \dots 0 \\ 0 \dots 0 \\ \vdots \vdots \vdots \\ 0 \dots g_n \end{vmatrix} \quad 2.22$$

Hence, from eqns. 2.20 and 2.21

$$\underline{\underline{G}} \underline{\underline{D}} = \begin{vmatrix} g_{11} & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & & & 0 \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ \cdot & & & & \cdot \\ 0 & \cdot & \cdot & \cdot & g_{nn} \end{vmatrix} \quad 2.23$$

That is,

$$\underline{\underline{D}} = (\underline{\underline{G}})^{-1} (\text{Diag. } \underline{\underline{G}}) \quad 2.24$$

That is, the decoupler matrix is the product of the inverse and the diagonal of $\underline{\underline{G}}$, the open loop transfer matrix of the system. Figure 2.5A shows the use of the decoupler in the process, while figure 2.5B shows the effect of such usage, a complete decoupling of the system.

Because of the involvement of the inverse matrix in this development, care must be taken to ensure that $\underline{\underline{G}}$ is a non-singular matrix. That is,

$$\det(\underline{\underline{G}}) \neq 0 \quad 2.25$$

One way of doing this is to use the right manipulated variable (u) to control the right output (y) by correct pairing of the state variables of the process of interest. One of the methods available for this purpose is the Relative Gain Technique whose development is given in Chapter 3.

The ideal decoupling elements, like those of the Exact and Approximate approaches are physically realizable, and with the commercial control implementation, can be implemented. The chief disadvantage of this technique is the development of unstable feed-back loops when the approach is applied for such processes as distillation column where the feed-back tends to be positive with the increasing purity (69).

State Variable Feed Back Technique

Suppose the process depicted in figure 2.4 is that of a coupled m -input/ n -output ($m \times n$) system. Using the state variable approach (4,ch.1), this system can be described as follows:

$$\dot{\underline{x}} = \underline{A} \underline{x} + \underline{B} \underline{u} \quad 2.26$$

$$\underline{y} = \underline{C} \underline{x} \quad 2.27$$

where

$$\begin{aligned}
 \underline{x} &= n\text{-state vector} \\
 \underline{u} &= m\text{-input vector} \\
 \underline{y} &= n\text{-output vector} \\
 \underline{A} &= nxn \text{ matrix} \\
 \underline{B} &= nxm \text{ matrix} \\
 \underline{C} &= mxn \text{ matrix}
 \end{aligned}$$

Suppose that introduction of matrices $\underline{F}$ (mxn), $\underline{G}$ (mxm) and a vector $\underline{w}$ (m) with the relationship

$$\underline{u} = \underline{F} \underline{x} + \underline{G} \underline{w} \quad 2.28$$

can decouple the process. The necessary and sufficient condition of decouplability for this process is given in the following developments: let

$$\begin{aligned}
 &\alpha_1, \alpha_2, \dots, \alpha_m \text{ be defined as} \\
 \alpha_i &= \min[j: c \underline{A} \underline{B} = 0, j = 0, 1, \dots, n-1] \quad 2.29
 \end{aligned}$$

where c is the i -th row of $\underline{C}$. For the cases where α_i equals to zero, α_i is given as

$$\alpha_i = n - 1 \quad \text{for all } i \quad 2.30$$

Falb et al.(54,55) have shown that if $\underline{F}$ and $\underline{G}$ are a pair of matrices which can decouple this system, then there must an $m \times m$ matrix β^* defined as

$$\beta^* = \begin{vmatrix} C_1 & A & B \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ C_m & A^m & B \end{vmatrix} \quad 2.31$$

such that $\det(\beta^*) \neq 0$. This nonsingularity of β^* is the necessary and sufficient condition for the existence of a decoupling pair of matrices $\underline{F}$ and $\underline{G}$. But the condition for this to be the right pair for a particular system is established by defining an $n \times m$ matrix

$$\phi^i(F) = \begin{vmatrix} C_i & (A + BF)^{n-1} B \\ C_i & (A + BF)^{n-2} B \\ \cdot & \\ \cdot & \\ \cdot & \\ C_i & (A + BF)^{i+1} B \\ C_i & (A + BF)^i B \\ 0 \end{vmatrix} \quad 2.32$$

Since ϕ^i is a function of only $\underline{F}$ (and not $\underline{G}$), it makes sense to speak of the class of the matrices $\underline{F}$ which can decouple

the system. It turns out that the only condition for $\underline{F}$ and $\underline{G}$ to be the right decoupling pair is that

$$\text{rank}(\phi^i(F)) = 1 \quad 2.33$$

for all i .

Thus the nonsingularity of ϕ^i is the necessary and sufficient condition for the existence of a decoupling pair F and G . Also, the necessary and sufficient condition for these to be the right pair is for the $\text{rank}(\phi^i(F))=1$.

The practical implementation of this decoupling technique has begun in the aerospace industry. But much remains to be done to transform it into a design technique applicable for chemical systems.

CHAPTER 3.

ILLUSTRATIVE EXAMPLE SYSTEM

The purpose of this chapter is to generate some of the mathematical analysis which will be utilized in subsequent chapters for developing a new theory for multivariable process control and for illustration of its application. For mathematical tractability and ease of demonstrating the derivation of this new control philosophy, the simplest multivariable system, a two-input/two-output(TI/TO) chemical coupled process which allows the existence of multiplicity of steady states for some set of parameters, is chosen. The modelling of the process is carried out in section 3.1. Linearization of the system about its unstable steady state is carried out in section 3.2, while the concept of Relative Gain, a powerful mathematical tool for pairing of input and output variables for control and decoupling purpose, is given in section 3.3. Finally, the values of the system's parameters as well as other design constants are summarized in section 3.4.

3.1 SYSTEM AND MATHEMATICAL MODEL

The multivariable process chosen for this study is a two variable chemical system: a cooled CSTR undergoing an exothermic, first order, irreversible reaction of the form



A schematic sketch of the reaction system is shown in figure 3.1. This system was first studied by Bilous and Amundson (16) to demonstrate the existence of multiple steady states and the process has come to represent a standard system to utilize in theoretical studies. The same process was later used by Bruns (7) to demonstrate the novel control policy for the system's operation at an unstable steady state.

The terminology two by two (2x2) will be used here to mean that of all the system's inputs or independent variables only two are specified as manipulated variables while all others are kept constant or perhaps considered as disturbance variables. Further, only two of the outputs or dependent variables are controlled variables. This terminology will be used with this understanding throughout this thesis. The choice of manipulated and controlled variables is dictated by the practical interest and the control requirement. The pairing of an input variable to a right output variable is of extreme importance. Relative Gain Technique (see section 3.3) may be employed for this purpose.

The rate of the above reaction (eqn.3.1) is a function of temperature and concentration, and it is given by the Arrhenius relation

$$R(T,C) = kC = k_0 C \exp(-E/RT) \quad 3.2$$

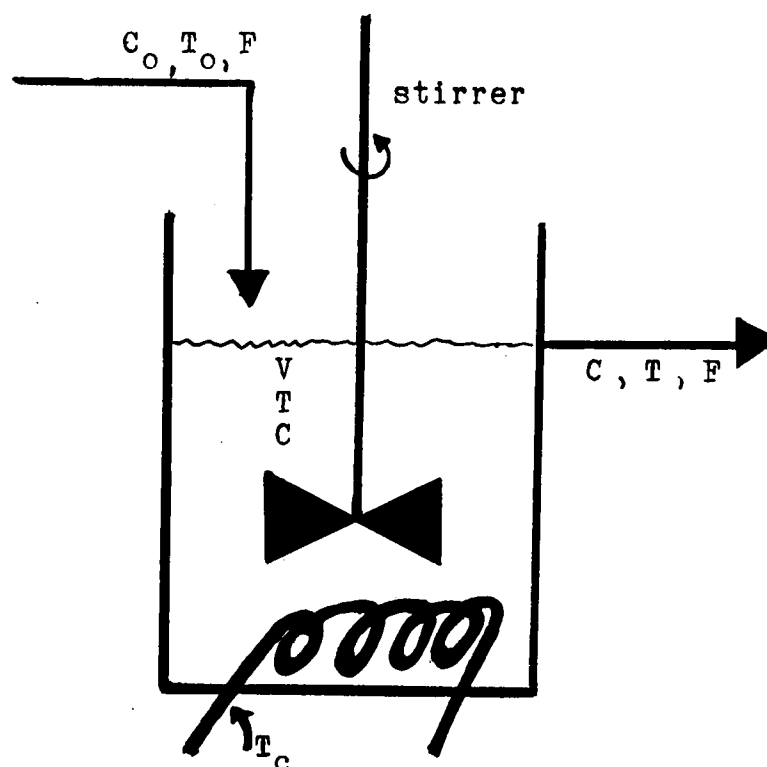


Figure 3.1: A schematic sketch of the CSTR of the illustrative second order system described by equations 3.2 and 3.3

where C = the reactant concentration, gm-mole/m³
 T = absolute temperature of system, K
 R = gas constant, Kcal/(K,Kg-mol)
 E = Activation energy, KCal/Kgm-mol
 k_0 = Arrhenius preexponential factor, 1/sec

In using this process, the following standard assumptions are made: that the reactor solution has constant density and that this solution is perfectly mixed. Also the reactor is of constant volume. These assumptions justify the lumped parameter description of the system which is soon to be given.

The controlled variables are the reactor's concentration C , and temperature T (note that this process has only two dependent variables). These variables are to be manipulated with the flow rate F and the coolant temperature T_c respectively. A classic study of a single loop control of this reactor (14) employed T_c to control T . Such a choice of the manipulating variables and their pairing with T and C will be considered in section 3.3. The condition about which the reactor is to be maintained is an unstable steady state.

The transient mass balance of this process is given as

$$\frac{dC}{d\tau} = \frac{F}{V} (C_0 - C) - k_0 C \exp(-E/RT) \quad 3.3$$

and the transient energy balance is

$$\frac{dT}{d\tau} = \frac{F}{V} (T_o - T) - \frac{UA}{\rho c_p V} (T - T_c) + \frac{(-\Delta H)k_o \exp(-E/RT)}{c_p \rho} C \quad 3.4$$

These equations may be simplified by introduction of the following dimensionless variables and parameters:

$$\beta = \frac{(-\Delta H)C_o}{\rho c_p T_o} \quad , \quad \text{dimensionless adiabatic temperature rise}$$

$$\xi = E/RT_o \quad , \quad \text{Arrhenius number}$$

$$Da = k_o \theta \quad \text{Damkohler number}$$

$$\gamma = \frac{UA}{F \rho c_p} \quad , \quad \text{dimensionless heat transfer coefficient}$$

$$\theta = V/F \quad , \quad \text{residence time}$$

$$\tau = t/\theta \quad , \quad \text{dimensionless time}$$

$$\mathcal{T} = T/T_o \quad , \quad \text{dimensionless temperature}$$

$$\mathcal{U} = T_c/T_o \quad , \quad \text{dimensionless coolant temperature}$$

$$\mathcal{C} = C/C_o \quad , \quad \text{dimensionless concentration}$$

$$\mathcal{F} = F/F_o \quad , \quad \text{dimensionless flow rate}$$

In terms of these dimensionless groups, eqns. 3.3 and 3.4 may be rewritten as

$$\begin{aligned} \frac{dw}{d\tau} &= f(1-w) - Da w \exp(-F/y) \\ &= f_1(F, w, y) \end{aligned} \quad 3.5$$

$$\begin{aligned} \frac{dy}{d\tau} &= f(1-y) - \gamma(y-w) + \beta Da w \exp(-F/y) \\ &= f_2(F, w, y, w) \end{aligned} \quad 3.6$$

3.2 LINEARIZATION

Equations 3.5 and 3.6 are nonlinear, first order differential equations which cannot be solved analytically. Having specified the manipulated parameters for this system, the system's equations are linearized about its unstable steady state condition treating the control parameters as variables in the linearization. The linearization procedure, it must be admitted, places limitations on analysis schemes as the linearized model gives an accurate account of the nonlinear system's response only in a sufficiently small neighbourhood of the reference point (unstable steady state). As pointed out in section 1.3, with this control analysis, only small amplitude oscillations of the process about the reference condition are anticipated. Hence, the restriction imposed by the linearization is expected to be of no major consequence. It is only expected that as the control states are moved toward the desired steady state, the linearized model is an increasingly accurate approximation (see Chapter 4).

Using the first order Taylor's series expansion, and assuming small deviation of the state variables from the reference condition, equations 3.5, and 3.6 are linearized to give

$$\frac{dw}{d\tau} = a_{11}w + a_{12}y + b_{11}f + b_{12}u \quad 3.7$$

$$\frac{dy}{d\tau} = a_{21}w + a_{22}y + b_{21}f + a_{22}u \quad 3.8$$

where

$$a_{11} = \left. \frac{\partial f_1}{\partial w} \right|_* = -\mathcal{F}^* - Da \exp(-\xi/y^*)$$

$$a_{12} = \left. \frac{\partial f_1}{\partial y} \right|_* = -\xi/y^{*2} Da w^* \exp(-\xi/y^*)$$

$$a_{21} = \left. \frac{\partial f_2}{\partial w} \right|_* = \beta Da \exp(-\xi/y^*)$$

$$a_{22} = \left. \frac{\partial f_2}{\partial y} \right|_* = -(\mathcal{F}^* + \gamma) + \frac{\xi}{y^{*2}} \beta Da w^* \exp(-\xi/y^*)$$

$$b_{11} = \left. \frac{\partial f_1}{\partial \mathcal{F}} \right|_* = 1 - w^*$$

$$b_{12} = \left. \frac{\partial f_1}{\partial u} \right|_* = 0$$

$$b_{21} = \left. \frac{\partial f_2}{\partial \mathcal{F}} \right|_* = 1 - y^*$$

$$b_{22} = \left. \frac{\partial f_2}{\partial u} \right|_* = \gamma$$

The notation (*) is used to signify the reference condition. In this study, the steady state condition of interest will be an unstable steady state. Equations 3.7, and 3.8 are

deviation equations in which all the indicated variables are deviations from the reference steady state. These are defined as

$$\begin{aligned} y &= \mathcal{Y} - \mathcal{Y}^* \\ w &= \mathcal{W} - \mathcal{W}^* \\ u &= \mathcal{U} - \mathcal{U}^* \\ f &= \mathcal{F} - \mathcal{F}^* \end{aligned} \quad 3.9$$

Because equations 3.7, and 3.8 are linear, they can be set in matrix-vector form as

$$\frac{d}{d\tau} \begin{pmatrix} w \\ y \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} w \\ y \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} f \\ u \end{bmatrix} \quad 3.10$$

or

$$\dot{\underline{V}} = \underline{a} \underline{V} + \underline{b} \underline{Z} \quad 3.11$$

where $\underline{V}$ = output vector

$\underline{Z}$ = input vector

$\underline{a}$ = Jacobian matrix of the process

$\underline{b}$ = open-loop input transfer matrix

($\dot{}$) = time derivative

From the Jacobian matrix " $\underline{a}$ ", the system's eigenvalues are evaluated to be

$$\lambda_{1,2} = \frac{1}{2} \left| (a_{11}a_{22}) \pm \sqrt{(a_{11}+a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})} \right| \quad 3.12$$

Open-loop Transfer Functions

Equation 3.11 may be Laplace-transformed to give

$$\underline{V}(s) = (s\underline{I} - \underline{a})^{-1} \underline{b} \underline{Z}(s) \quad 3.13$$

where $s = \frac{d(\)}{d\tau}$, the Laplace parameter
 $\underline{I} =$ Identity matrix

The open-loop transfer matrix $\underline{G}(s)$ of this coupled system are

$$\begin{aligned} \underline{G}(s) &= \underline{V}(s) \underline{Z}^{-1}(s) = (s\underline{I} - \underline{a})^{-1} \underline{b} \\ &= \frac{\text{adj}(s\underline{I} - \underline{a})}{|s\underline{I} - \underline{a}|} \underline{b} \end{aligned} \quad 3.14$$

Equation 3.14 can then be expanded to give

$$\begin{aligned} G_{11}(s) &= \frac{a_{12} b_{21} + b_{11}(s-a_{22})}{s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21}} \\ G_{12}(s) &= \frac{a_{12} b_{22} + b_{12}(s-a_{22})}{s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21}} \\ G_{21}(s) &= \frac{a_{21} b_{11} + b_{21}(s-a_{11})}{s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21}} \\ G_{22}(s) &= \frac{a_{21} b_{12} + b_{22}(s-a_{11})}{s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21}} \end{aligned} \quad 3.15$$

The denominator of these transfer functions define the system's characteristic equation

$$s^2 - (a_{11} + a_{22})s + (a_{11}a_{22} - a_{12}a_{21}) = 0 \quad 3.16$$

Figure 3.4 shows the description of the linearized reactor with the two proposed feed back control loops in terms of the linearized system's transfer functions. Note that $G_{21}(s)$ and $G_{12}(s)$ define the cross paths between the two control loops. Thus a change in control level of one loop is transferred via the respective path as a load disturbance to the other loop.

If there were no coupling in the system, we could think of the process as two SI/SO systems acting independently of one another. Thus each control loop could be studied individually. This would only be true if the disturbance paths between the two resulting loops were nonexistent, that is, $G_{12}(s) = G_{21}(s) = 0$.

The most that could be hoped for in a real process is that the cross effect is relatively weak compared to the direct path, that is, via $G_{11}(s)$ and $G_{22}(s)$. If the cross coupling is small enough than we would be able to justify analyzing the TI/TO systems as two independent SI/SO systems. Criteria for small enough gain matrix perhaps are available in the literature.

A more realistic approach would be the application of some decoupling as discussed in Chapter 2. A decoupler then has the task of compensating for the cross effects so that, ideally, these effects are exactly cancelled by decoupling compensation. Thus with such decoupler in place, the effects of the cross path transfer functions ($G_{12}(s)$ and $G_{21}(s)$) are ideally cancelled out. Each loop can then be analyzed independently using the theory for SI/SO processes (Chapter 4).

If in addition the decoupler is designed such that the open-loop characteristics are not modified, the example reactor with a perfect decoupler would then be simply described by retaining only $G_{11}(s)$ and $G_{22}(s)$ of equation 3.15.

It is again noted that the application of a decoupler technique must be based on theory valid for an unstable reactor if applied for our current example. Figures 3.2, and 3.3 show the feed back schemes for control of the concentration and temperature loops respectively under the assumption of perfect decoupling, while figure 3.4 shows the feedback control scheme for the coupled 2x2 system.

3.3 RELATIVE GAIN

Some of the decoupling techniques outlined in Chapter 2 require that the open-loop transfer matrix $G(s)$ be square. Naturally, the question arises: for an $m \times n$ MI/MO process for which $m \neq n$, how can the choice of input and

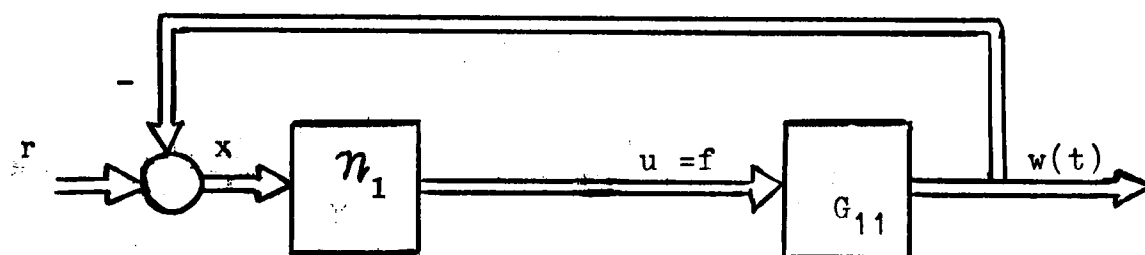


Figure 3.2: An SI/SO control scheme for control of concentration according to eqns. 4.30, 4.31

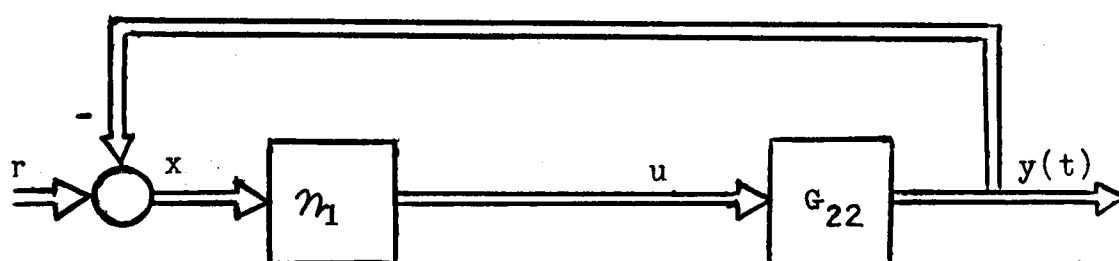


Figure 3.3: An SI/SO control scheme for control of temperature

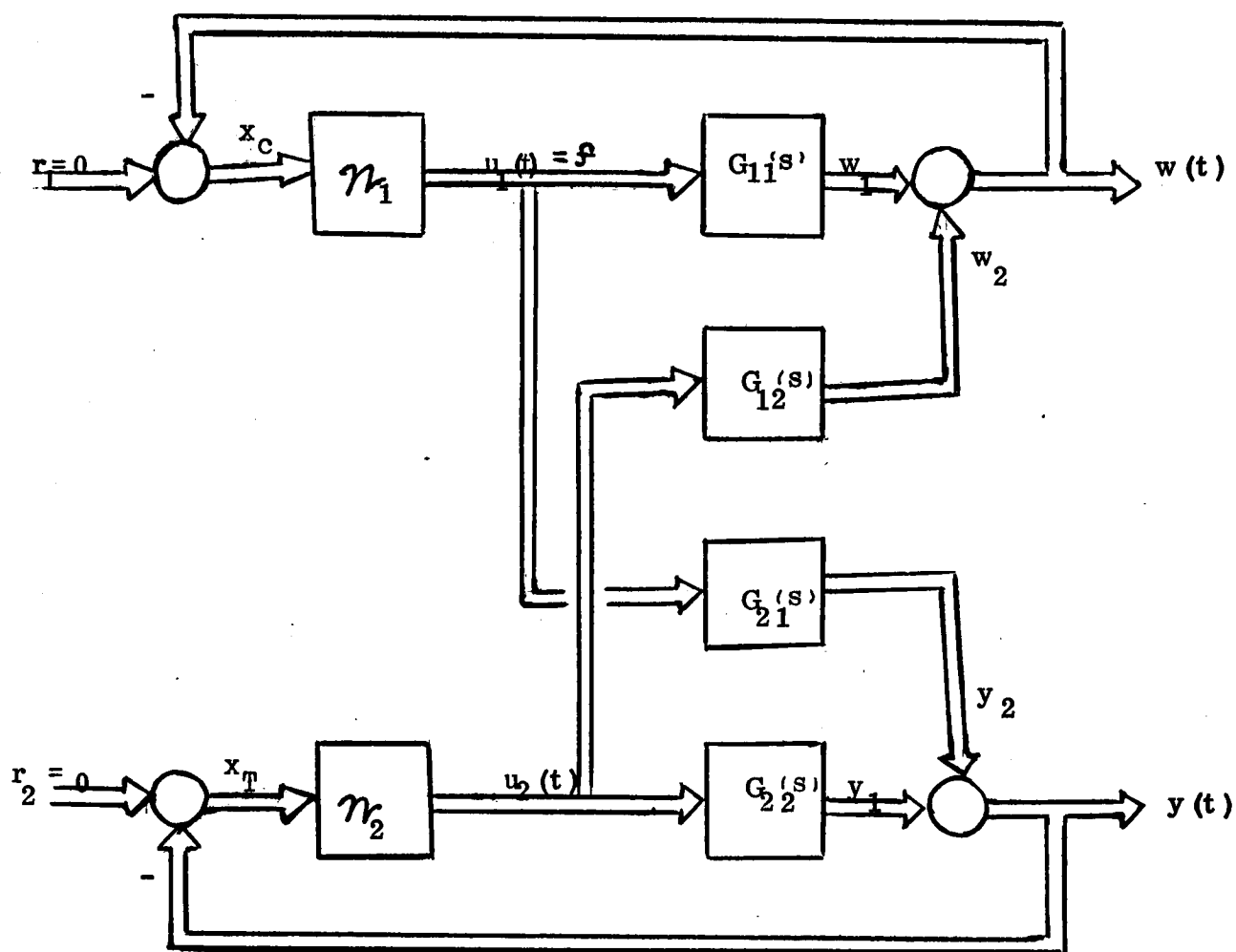


Figure 3.4: Feed back control scheme for control of coupled 2x2 multivariable system

output variables be made such that G is square and the control criteria met?. One simple answer is: if $n > m$, the process has more outputs than inputs. Hence, some of the outputs must be linked together within the system or some of the outputs are not to be measured or controlled. But should $m > n$, that is, the number of inputs is greater than that of outputs. Some of the inputs must be fixed (made constant) or ignored.

In order to determine which input variable should be manipulated and which should be fixed, one must be able to determine which input variable can best control a given output. This choice of the right control variables is of paramount importance in process control and much more so if decoupling of the various loops must come into the picture. A wrong choice of variables may render the system impossible to control and to decouple. In some control situations, such a choice may be obvious, but in majority of cases, it is far from being so. Even in a 2×2 system, the simplest MI/MO process, there is a 50 percent chance for poor selection. Such will eventually lead to disappointing results.

To eliminate the necessity of guess work and to provide the engineer with a readily available mathematical tool for a successful control system design, the idea of "Relative Gain", which was first introduced by Bristol (100) and expounded by Shinskey (1, 2) is given here.

Simply, a relative gain is a degree of influence a given manipulated variable has upon a particular controlled variable compared to other controlled variables. That is, it is an indicator as to how the manipulated and controlled variables should be paired to effect efficient control. To determine a relative gain, an open loop gain is first calculated with respect to a manipulated variable while all other manipulated variables are kept constant, and reevaluated with respect to a controlled variable with all other controlled variables held constant. If these two gain evaluations do not vary significantly in the two evaluations (that is, $\alpha_{ij} \approx 1$), then the interaction between direct and cross paths is negligible and, consequently, may be ignored in control design. Further this is an indication of the right pairing of manipulated and controlled variables. But should there be a sizeable difference in the two calculations, a strong interaction between direct and cross paths is indicated. This implies a wrong pairing of variables. In this case, alternative pairings are necessary, decoupling is required, or the coupling must be accounted for in the control system design.

Whether or not the relative gain technique is applicable for unstable processes is a question not answered in the current literature search. However, the concept of relative gain is utilized here for our unstable reactor with hope that such application is justifiable. More search of literature beyond the depth allowed by the inelastic scope

of this research, in terms of utilization of the concept for unstable processes, is necessary before one can make a concrete and reasonable conclusion about the range of applicability of the relative gain technique.

Mathematically, a relative gain is defined as

$$\alpha_{ij} = \frac{\left. \frac{\partial y_i}{\partial u_j} \right|_{u_{k \neq j}}}{\left. \frac{\partial y_i}{\partial u_j} \right|_{y_{k \neq i}}} \quad 3.17$$

If $\alpha_{ij} = 1$, a weak interaction between the cross and the direct paths is indicated. However, this also means that the chosen manipulated variable (u_j) is the right manipulated variable for the control of the output (y_i) in question. Hence, cross path effects or interactions may be safely ignored in any control design and other formalisms involving the choice of this pair of the variables.

If $\alpha_{ij} = 0$, then a change in u_j does not significantly influence y_i . That is the direct path interaction is negligible. This, of course, means that the chosen manipulated variable u_j is not effective in the control of the output y_i .

$\alpha_{ij} = \infty$ when $\partial y_i / \partial u_j$ with all $y_{k \neq i}$ held constant = 0. Thus $\alpha_{ij} = \infty$ suggests an inhibition arising through the cross path due to the changes in $u_{k \neq j}$ required to maintain all $y_{k \neq i}$ constant when a change in u_j is made to effect

y_i in the direct path. That is, for every change in y_i in the direct path as a result of changes in u_j , there must be some additional changes in $u_{k \neq j}$ to fix all other output variables ($y_{k \neq i}$). These other changes either increase significantly the direct path response of y_i or suppresses the response of y_i (in the direct path) to changes in u_j . This is an inhibition of the direct path responses, and it is an indication of a wrong pairing of variables.

To speed our understanding of the relative gain, it is advisable to array the gains determined for a given system in the form of matrix known as relative gain matrix

$$\Lambda = \begin{array}{c|cccc} & u_1 & . & . & u_n \\ y_1 & \alpha_{11} & . & . & \alpha_{1n} \\ . & . & & & . \\ . & . & & & . \\ . & . & & & . \\ . & . & & & . \\ y_n & \alpha_{n1} & . & . & \alpha_{nn} \end{array} \quad 3.18$$

where the u_i label the column and y_i the rows. An important property of this matrix is that for a given row and column, all its entries must sum up to 1. Therefore, for an $n \times n$ MI/MO process, $(n-1)$ such entries need be evaluated to completely determine the matrix.

It is important to note that, in a 2×2 system, for example, all the entries of a given row or column of Λ may lie in the interval $(0,1)$. Should any entry be greater than

1, then there must be a negative number in the same row or column. If Λ contains the same absolute number (0.5 in case of 2x2 process), a highly stable and equally coupled system is indicated (equally in the sense that it does not matter which manipulated variable is chosen to regulate which control variable, the system behaves in the same manner). In a given process, both the controllability and decouplability are greatly enhanced by a choice of controlled and manipulated variables whose corresponding entry in an Λ is closest to unity. Or if the entry greater than unity, it does not normally exceed 5 for a practical system (2).

Relative Gain of the Illustrative Example System

The relative gain of the example process can be easily determined in the following manner: consider the coupled process depicted in figure 3.4. The steady state relation of both loops are given as

$$w = K_{11} f + K_{12} u \quad 3.19$$

$$y = K_{21} f + K_{22} u \quad 3.20$$

where K_{ij} 's are the gains of the corresponding transfer functions from G_{ij} 's. Solving for u in equation 3.20 and substituting into equation 3.19, we get

$$w = K_{11}f + K_{12}\left(\frac{y - K_{21}f}{K_{22}}\right) \quad 3.21$$

The relative gain matrix for the process is of the form

$$\Lambda_{2 \times 2} = \begin{array}{c} \begin{array}{cc} & \begin{array}{c} f \\ u \end{array} \\ \begin{array}{c} w \\ y \end{array} & \begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix} \end{array} \quad 3.22$$

By the summation property of Λ , for this 2×2 process, we need evaluate only one entry of Λ to fix the entire matrix. Suppose α_{11} is the entry so determined, then eqn. 3.22 takes the form

$$\Lambda_{2 \times 2} = \begin{array}{c} \begin{array}{cc} & \begin{array}{c} f \\ u \end{array} \\ \begin{array}{c} w \\ y \end{array} & \begin{vmatrix} \alpha_{11} & 1 - \alpha_{11} \\ 1 - \alpha_{11} & \alpha_{11} \end{vmatrix} \end{array} \quad 3.23$$

From eqn. 3.18

$$\alpha_{11} = \frac{\frac{\partial w}{\partial f}|_u}{\frac{\partial w}{\partial f}|_y} \quad 3.24$$

From equation 3.19

$$\left. \frac{\delta w}{\delta f} \right|_u = K_{11} \quad 3.25$$

and from 3.22

$$\left. \frac{\delta w}{\delta f} \right|_y = \frac{K_{11}K_{22} - K_{12}K_{21}}{K_{22}} \quad 3.26$$

Therefore,

$$\alpha_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}} \quad 3.27$$

From eqn. 3.24

$$1 - \alpha_{11} = \frac{1}{1 - \frac{K_{11}K_{22}}{K_{12}K_{21}}} \quad 3.28$$

Substitution of K_{ij} 's from eqn. 3.15 into eqns. 3.27 and 3.28 and employing appropriate entries of the matrices $\underline{a}$ and $\underline{b}$ (see page 61) to obtain

$$\begin{array}{cc} & \begin{array}{c} f \\ u \end{array} \\ \begin{array}{c} w \\ y \end{array} & \left| \begin{array}{cc} -4.56 & 5.56 \\ 5.56 & -4.56 \end{array} \right| \end{array}$$

This result indicates that, for this process,

$\alpha_{11} < \alpha_{12}$ and $\alpha_{22} < \alpha_{21}$ and that $\alpha_{12} = \alpha_{21} = 5.56$.

Because a relative gain greater than 5 is not expected in a real system (2), and because the simulations of this process by Aris and Amundson (14) emphasize the desirability of regulating the reactor's temperature with the coolant input temperature, the above result seriously calls into question once more the wisdom of utility of relative technique for unstable process control design. Equally questionable is our present understanding of the significance of a negative relative gain. Thus the variable pairing that has already been successfully applied (14,16,34) will be adopted for the reactor temperature control loop: control of reactor temperature with the coolant temperature. The control pairing selected for the concentration control loop in this work is the control of reactor concentration with the flow rate.

3.4 THE PARAMETERS AND CONSTANTS OF THE EXAMPLE SYSTEM

The parameters and constants of the example process chosen for the numerical study utilizing the new control theory to be developed in this work are the same as those used by Bilous and Amundson (16). These quantities as well as the standard dimensionless groups are given in Table 3.1. In Table 3.2 are given the steady state solutions as well as the numerical values of the process matrices and vectors as developed in this chapter resulting from linearization of the system at an unstable steady state. Also given in Table 3.2 are the eigenvalues of the system.

TABLE 3.1: Illustrative system's parameters and constants

Constants and Parameters

$$\begin{aligned}
 V &= 2000.00 \text{ cc} \\
 A &= 3.39 * 10^6 \text{ cm}^2 \\
 k_0 &= 7.86 * 10^{12} \text{ sec}^{-1} \\
 -\Delta H &= 1 * 10^4 \text{ cal/gm-mole} \\
 c_p &= 1.0 \text{ cal/(cc K)} \\
 \rho &= 1.0 \text{ gm/cc} \\
 F_0 &= 10 \text{ cc/sec} \\
 T_c &= 314.4 \text{ K} \\
 T_0 &= 300 \text{ K} \\
 C_0 &= 0.005 \text{ gm-mole/cc} \\
 R &= 1.987 \text{ cal/(gm-mole K)} \\
 U &= 4 * 10^{-7} \text{ cal/(cm}^2 \text{ sec K)}
 \end{aligned}$$

Dimensionless groups

$$\begin{aligned}
 \beta &= 0.1667 \\
 \xi &= 37.745 \\
 \gamma &= 0.1356 \\
 Da &= 1.572 * 10^{15}
 \end{aligned}$$

TABLE 3.2: Illustrative System's Matrices, Eigenvalues and Steady State Solutions

$$\underline{\underline{a}} = \begin{array}{c} \text{Jacobian matrix} \\ \left[\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right] \end{array} \quad \begin{bmatrix} -1.936 & -15.74 \\ 0.1558 & 1.488 \end{bmatrix}$$

$$\underline{\underline{b}} = \begin{array}{c} \text{Input matrix} \\ \left[\begin{array}{cc} b_{11} & b_{12} \\ b_{21} & b_{22} \end{array} \right] \end{array} \quad \begin{bmatrix} 0.4850 & 0.0000 \\ -0.0770 & 0.1356 \end{bmatrix}$$

Eigenvalues

$$\lambda_1 = 0.4654$$

$$\lambda_2 = -0.9134$$

Steady state solutions

(dimensionless quantities)

$$M = (w, y) = (0.5150, 1.0770)$$

$$L = (w, y) = (0.8508, 1.0297)$$

$$H = (w, y) = (0.2022, 1.1233)$$

(deviation variables)

$$M = (w, y) = (0.0000, 0.0000)$$

$$L = (w, y) = (0.3358, -0.0473)$$

$$H = (w, y) = (-0.3128, 0.0463)$$

CHAPTER 4

THE NOVEL CONTROL POLICY FOR CONTROL OF SINGLE-INPUT/SINGLE-OUTPUT PROCESSES

The main purpose of this chapter is to demonstrate the analytical procedures that may lead to a successful application of the novel SI/SO control policy for the control of the unstable reactor taken as the example system. As emphasized in chapter one, the analysis of this control strategy is based upon Tsytkin's analytical technique and is developed only for SI/SO systems. Because this control policy has been successfully tested (7,34,36) and its applicability experimentally demonstrated (26), it is called an "old theory" in this work. Yet, it is the basis upon which the "new theory", soon to be developed, (see chapter five) for multivariable or multi-loop system control is founded.

To apply the novel control strategy to our example reactor, the control loops of this system are assumed to be completely decoupled so that such loops per se constitute separate and independent subsystems. That is, any loop activity such as a change in control level cannot be transmitted to the rest of the loops, there being no disturbance paths between them. Such subsystems for concentration and temperature control are depicted in figures 3.2 (page 50) and 3.3 (page 50) respectively. As indicated in these figures, it is further assumed that the

decoupler used does not alter the open loop transfer functions for the direct paths, that is, G_{11} and G_{22} remain as the process transfer functions in the two control loops. This would result with the Ideal or Exact Decoupler when D_{11} and D_{22} are taken to be unity(conditions for applying these to unstable processes need to be established).

The format of this chapter is as follows: section 4.1 provides a collection of definitions of terms repeatedly used in this thesis. In terms of the illustrative system described in Chapter 3, section 4.2 identifies the basic control problem and presents a design strategy for a solution of the problem , the process control at an unstable steady state. Finally, section 4.3 presents the novel control strategy as presently available in the literature for the quantitative prediction of the system's anticipated oscillation near the unstable steady state condition where the operation is desired.

4.1 SOME BASIC DEFINITIONS

A "linear" process is one whose performance obeys both the Superposition and the Homogeneity principles (8,31). The superposition principle states that the response of such a system to several external input signals acting together on the system is the sum of its responses to these inputs acting individually. Thus if a linear process is

represented by a linear operator and the input signals by $u(t)$, these principles can be stated mathematically as

$$\Psi\{cU(t)\} = c\Psi\{U(t)\} \quad 4.1$$

$$\Psi\{\sum_{k=1}^K u_k(t)\} = \sum_{k=1}^K \Psi\{u_k(t)\} \quad 4.2$$

$$k = 1, 2, \dots, K$$

and for all real constants c

The relation of eqn. 4.1 is the homogeneity principle. For any time invariant system, the superposition principle allows the system's behaviour to be determined for any input if the system's response is known for any other input.

A "nonlinear" process is one for which the superposition (and homogeneity) principle is not valid. It is this distinction between a linear and a nonlinear process which makes the term "stability" rather vague when used to describe nonlinear systems. A linear process is inherently stable or unstable due to the superposition principle. For a nonlinear system, however, it is necessary to put distinctions on the initial conditions etc. before making statements concerning stability of a certain steady state of the system. For detailed information on distinctions between linear and nonlinear processes, see(34,section 1.A). A nonlinearity is either single-valued (memoryless) or multi-valued (with memory). If the input to a nonlinearity is denoted with $x(t)$ and its output with $u(t)$, the input/output relationship of a memoryless type nonlinearity

is given as

$$u(t) = F(x(t)) \quad 4.3$$

That is, the output $u(t)$ bears a one-to-one relation to the input $x(t)$. In memory type nonlinearities, such relation depends, in some fashion, upon the input history. For many such elements, the input/output relationship is given as

$$u(t) = F(x(t), \text{sign } \dot{x}(t)) \quad 4.4$$

The common examples of memoryless nonlinearities include saturation, deadzone, and gain-changing. Some of the memory type nonlinearities are backlash, relays with hysteresis. A memory type nonlinearity is either passive or active (8, page 207). It is passive if, for one cycle of its periodic input, the characteristics of the element is traversed such that the nonzero area encircled is in a counter clock-wise direction. But should the nonzero area be traversed in a clock-wise direction, the nonlinear element is active. Nonlinear elements with passive memory always give rise to lagging phase shifts while active memory types always give rise to leading phase shifts. The relationship between memory nonlinearities of the same kind is given as

$$-n_{\text{PASSIVE}} = n_{\text{ACTIVE}} \quad 4.5$$

The term "steady state" as used in this thesis is meant to signify that the time rate of change of all state variables of a given process at a given point are identically zero.

"Multiplicity of steady states" implies the total number of steady states exhibited by a process for a given set of parameters is more than one. Consequently, a steady state solution is not unique. For lumped parameter processes (such as those treated in this work), multiplicity of steady states will arise only in nonlinear systems.

A "multivariable" process is one with more than one dependent variable. A block diagram of a multivariable process such as the one given in figure 3.4 (page 51) shows more than one feed back loop. Hence, this type of controlled system is known as a "multi-looped" or a "multi-input/multi-output" (MI/MO) process. When only a single control loop is on a system, the controlled system is called a "single-input/single-out" (SI/SO) process.

A "singlevariable" system is one with only one dependent variable.

The term "locally asymptotically stable" describes a steady state where perturbations within an arbitrarily small neighbourhood of the state die away asymptotically. A system operating at such a state responds to small disturbances such that it reestablishes itself at the steady

state. This condition is often loosely referred to as a "stable" steady state.

"Unstable" describes the condition of a system where even the smallest perturbation grows and eventually causes the system to "run away" from that steady state.

A steady state is said to be "globally stable" if the system returns to the state after being excited by an arbitrarily large disturbance.

4.2 SI/SO CONTROL PROBLEM

As in the literature, the control design for our illustrative system begins with van Heerden's diagrams.

van Heerden Diagrams

At steady state condition ($dy/d\tau$ and $dw/d\tau$ are equal to zero), equations 3.5 and 3.6 respectively become

$$f(1 - u) - wDa \exp(-E/y) = 0 \quad 4.6$$

$$f(1 - y) - f(y - u) + \beta Da w \exp(-E/y) = 0 \quad 4.7$$

A study of eqn.4.6 reveals two quantities, a concentration generation term

$$R_g^C(u, f) = f(1 - u) \quad 4.8$$

due to the process bulk flow, and a concentration removal term

$$R_r^C(u, f) = Da u \exp(-f/f) \quad 4.9$$

due to chemical reaction. At steady state, these two terms are equal

$$R_g^C(u, f) = R_r^C(u, f) \quad 4.10$$

Similarly, from eqn. 4.7, one can extract a temperature generation quantity

$$R_g^T(u, f) = \beta Da u \exp(-f/f) \quad 4.11$$

and a removal quantity

$$R_r^T(f, f, u) = -f(1 - f) + \delta(f - u) \quad 4.12$$

Also, at steady state, these are equal

$$R_r^T(f, f, u) = R_g^T(f, u) \quad 4.13$$

In the functional notation for the various generation and removal terms, the independent variables $\mathcal{F}$ and $\mathcal{U}$ are retained as these will be the manipulated variables in the control strategies. Recalling from chapter three, $\mathcal{F}$ will be used to control $\mathcal{U}$ and $\mathcal{U}$ to control y .

With the superscript (*) implying a steady state value, y^* can be evaluated analytically from eqns. 4.6 and 4.7 and substituted in eqn. 4.10 to eliminate y^* from the relation. This may be done by multiplying eqn 4.6 by β and subtracting the result from eqn. 4.7.

y^* is then easily obtained to be

$$y^*(\mathcal{U}, \mathcal{F}, \mathcal{U}) = \frac{\beta \mathcal{F}(1 - \mathcal{U}) + \mathcal{F} + \delta \mathcal{U}}{\mathcal{F} + \delta} \quad 4.14$$

Substitution of y^* into eqn. 4.10 yields the needed result

$$R_g^C(\mathcal{U}, \mathcal{F}) = R_r^C(\mathcal{U}, y^*) = R_r^C(\mathcal{U}, \mathcal{F}, \mathcal{U}) \quad 4.15$$

$$\text{where } R_g^C(\mathcal{F}, \mathcal{U}) = \mathcal{F}(1 - \mathcal{U}) \quad 4.8$$

and

$$R_r^C(\mathcal{U}, \mathcal{F}, \mathcal{U}) = Da\mathcal{U} \exp\left(\frac{-\beta(\mathcal{F} + \delta)}{\beta \mathcal{F}(1 - \mathcal{U}) + \mathcal{F} + \delta \mathcal{U}}\right) \quad 4.16$$

R_g^C being a function of u , f , and R_r^C a function of u , f , and u implies that a change in f (with u fixed) will produce a family of these curves. Note that R_g^C is a linear function of u whose slope is $-f$ and whose intercept is f . Plots of these curves at fixed values of $(f, u) = (f^*, u^*)$ give the so-called van Heerden's curves shown in figure 4.1.

van Heerden diagrams can also be drawn for temperature if the temperature generation and removal terms are put in the forms suitable for such plots. This is done by obtaining steady state (that is, with $dw/d\tau = 0$) solution of u from eqn. 4.6 and substituting this into eqn. 4.13. That is,

$$u^* = \frac{f}{f + Da \exp(-\theta/a_f)} \quad 4.17$$

When the above substitutions are carried out, eqn. 4.13 becomes

$$R_g^T(f) = R_r^T(f, u, f)$$

where

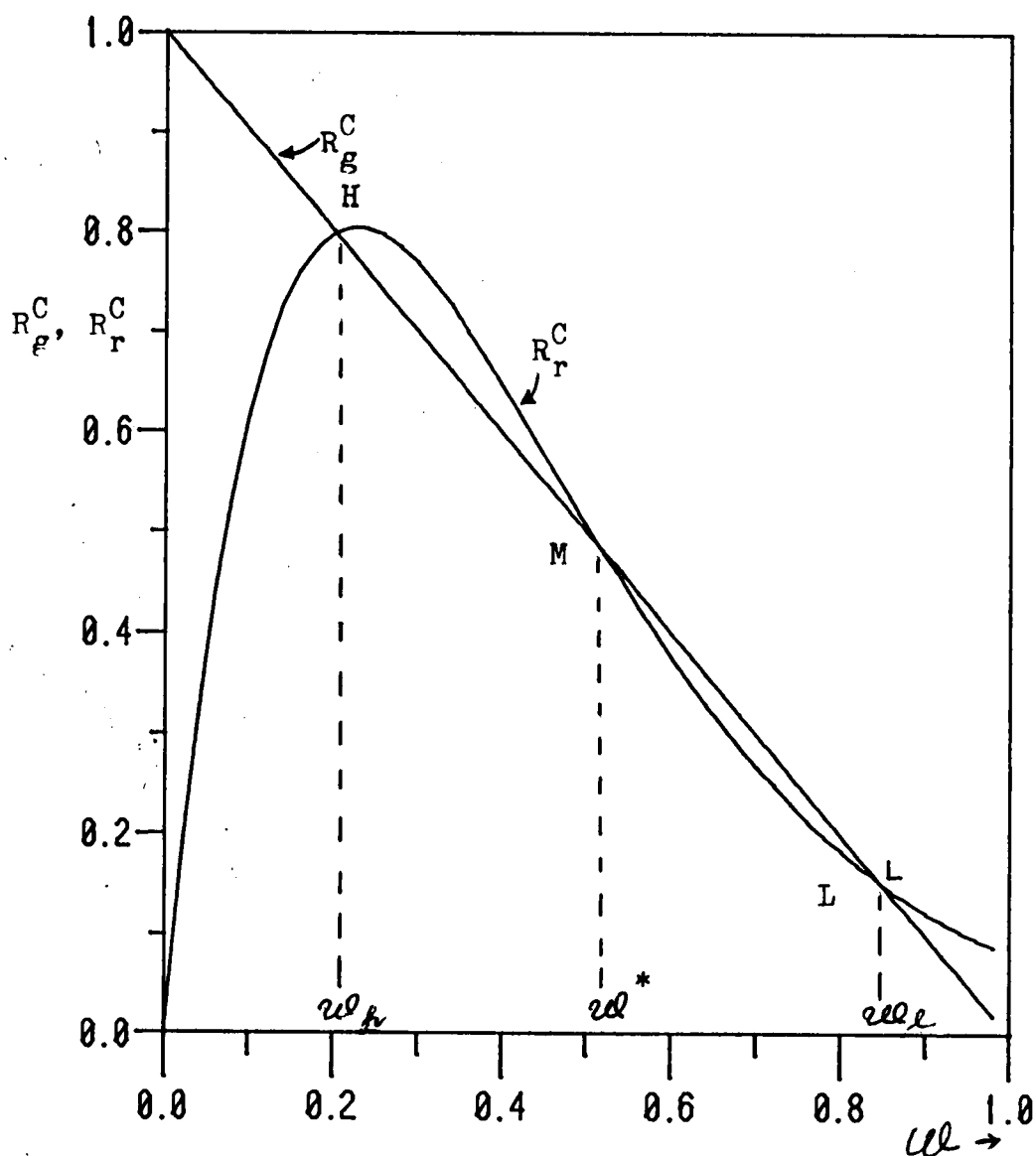


Figure 4.1: van Heerden diagram for the Second order process obtained with plot of R_g^C and R_r^C versus w according to eqns. 4.8 and 4.16

$$R_g^T(y) = \frac{\theta F Da \exp(-E/y)}{F + Da \exp(-E/y)} \quad 4.18$$

$$R_r^T(y, F, u) = y(F + \gamma) - (F + \gamma u) \quad 4.19$$

Note the dependency of R_g^T upon y only, and that of R_r^T upon y , F and u with F fixed at F^* , R_r^T is a linear function with slope of $(y^* + \gamma)$ and intercept of $-(F^* + \gamma u)$. Because only R_r^T depends upon u , a variation in the coolant temperature only produces a family of curves for R_r^T . Figure 4.2 is the van Heerden's diagram for coolant temperature and for flow rate values fixed at $(u, F) = (u^*, F^*)$. Note the familiar sigmoidal (s-shaped) form of the R_g^T curve.

The intersections H, M, L of the generation and the removal terms in both fig. 4.1 and 4.2 represent the steady state conditions giving steady state concentration and temperature respectively. See section 6.1 under "Results and Conclusion" for the values of the output variables (temperature and concentration) at the three steady states of the unstable reactor for the parametric values given in chapter three (the same solutions are also given in Table 3.1).

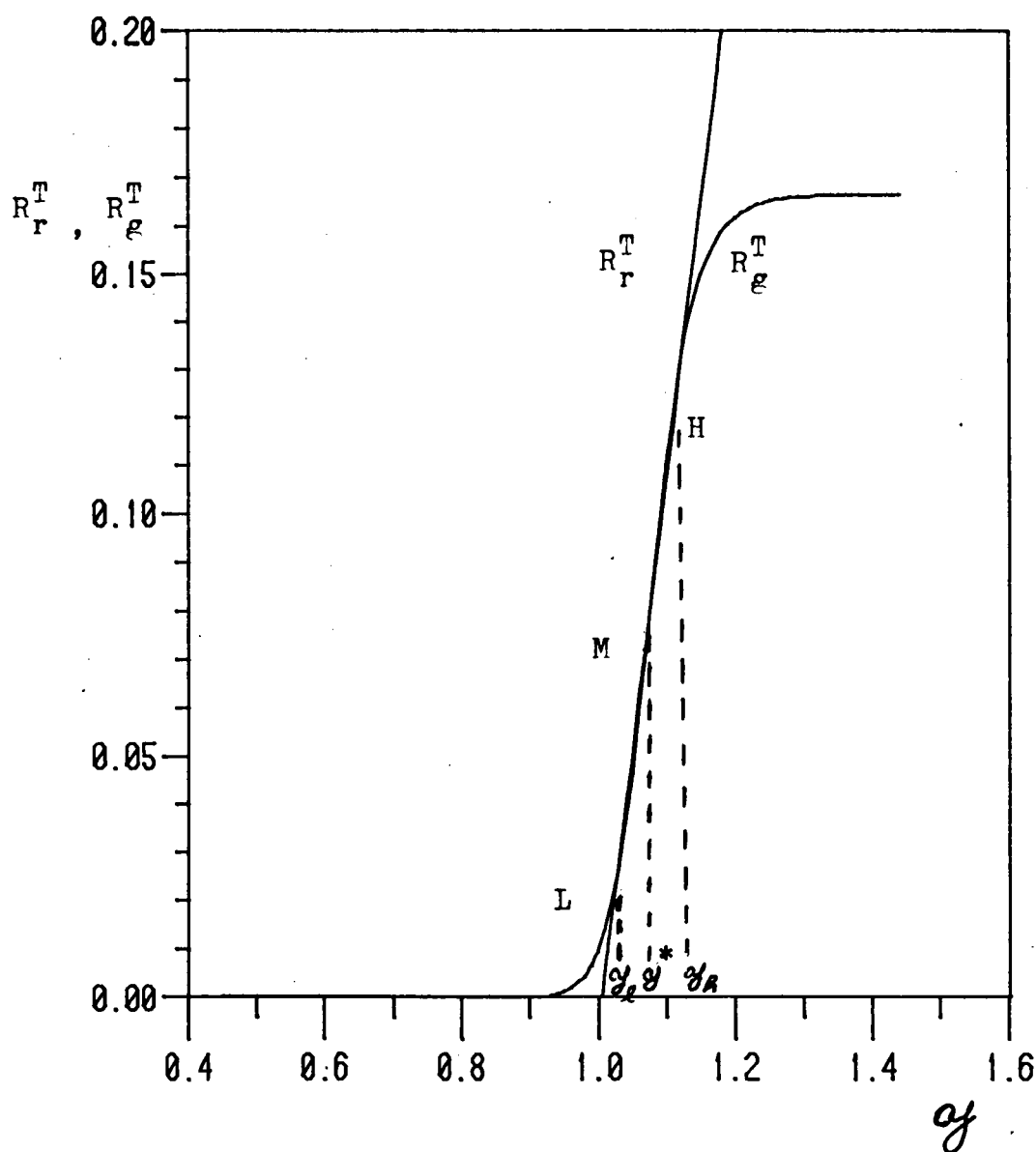


Figure 42: van Heerden Diagram for second order Process $R_g^T(y)$ and $R_r^T(y, y)$ versus Temperature (eqn. 4.18, 4.19)

The Slope Condition for Steady State Stability

Having drawn the van Heerden diagrams for our example system and identified the system's steady states, we can now proceed to decide the nature of these states in terms of their stability to small disturbances. The so-called van Heerden's slope condition for steady state stability spelt out in equation 1.1 provides only a necessary condition.

The slope condition for the reactor studied here is, in terms of concentration,

$$\frac{d R_r^C}{d u} > \frac{d R_g^C}{d u} \quad 4.20$$

or in terms of temperature

$$\frac{d R_r^T}{d y} > \frac{d R_g^T}{d y} \quad 4.21$$

Note that eqns. 4.20 and 4.21 are equivalent as they are the expressions of the same condition (the slope condition) only in different terms. It must be emphasized again that, for this second order process or any higher order systems, the slope condition is only a necessary condition for state stability for a reason which will be given shortly. It is only for first order systems that this condition serves as both a necessary and a sufficient condition for state

stability.

When the slope conditions given above are applied to the steady states H, M and L in figures 4.1 and 4.2, it is immediately evident that H and L satisfy this necessary condition. Thus to completely determine their steady state stability, additional analysis (such as finding the process eigenvalues from linearized model around the steady state of interest) must be undertaken. For the example reactor and parameters chosen, they are found to be stable. The reactor does not satisfy the slope condition at M, thus it is an unstable steady state. Here a disturbance, no matter how slight, grows and eventually causes the system to "run away" to a more stable condition, either H or L depending upon the disturbance. Condition M is rigorously defined as a locally asymptotically unstable steady state.

The Control Problem

In the present chapter, the unstable reactor of the illustrative example will be treated as an SI/SO process for the separate control of concentration and temperature by regulating the flow rate and the coolant temperature respectively in accordance with the control schemes depicted in figures 3.2 and 3.3 for the decoupled process. As stated in the introductory part of this chapter, under this condition, loop by loop design of the process for its control at the unstable steady state condition can be

carried out. This corresponds to maintaining the reactor at the condition M (see figure 4.1 (page 71), and figure 4.2 (page 73)). In Chapter 1 is also discussed the likely difficulties which may accompany any effort to effect a control of the system at this condition with any of the conventional (linear) controllers, and how Bruns and Bailey (7,34) engineered an alternate control policy (novel control policy) with which the process can be controlled at the unstable point. A successful implementation of this control strategy will produce about this point small amplitude oscillations of the process. This is often regarded as an acceptable alternative to absolute maintenance of the system at the unstable operating point.

Before discussion of this control approach, consideration is first given here to some design aspect of the process based upon the van Heerden diagrams of figures 4.1 and 4.2. To avoid unnecessary repetitions of the steps involved, attention is going to be focused on the concentration control. The same arguments apply for temperature control.

van Heerden diagrams are redrawn for the concentration at different levels of $\mathcal{F}(\mathcal{F}^+, \mathcal{F}^*, \mathcal{F}^-)$. According to eqns. 4.8 and 4.16, three families of the R_r^C , and R_g^C curves are produced. These are shown in figure 4.3. The control problem can be explained as follows: Suppose the control specifications are such that the concentration be maintained

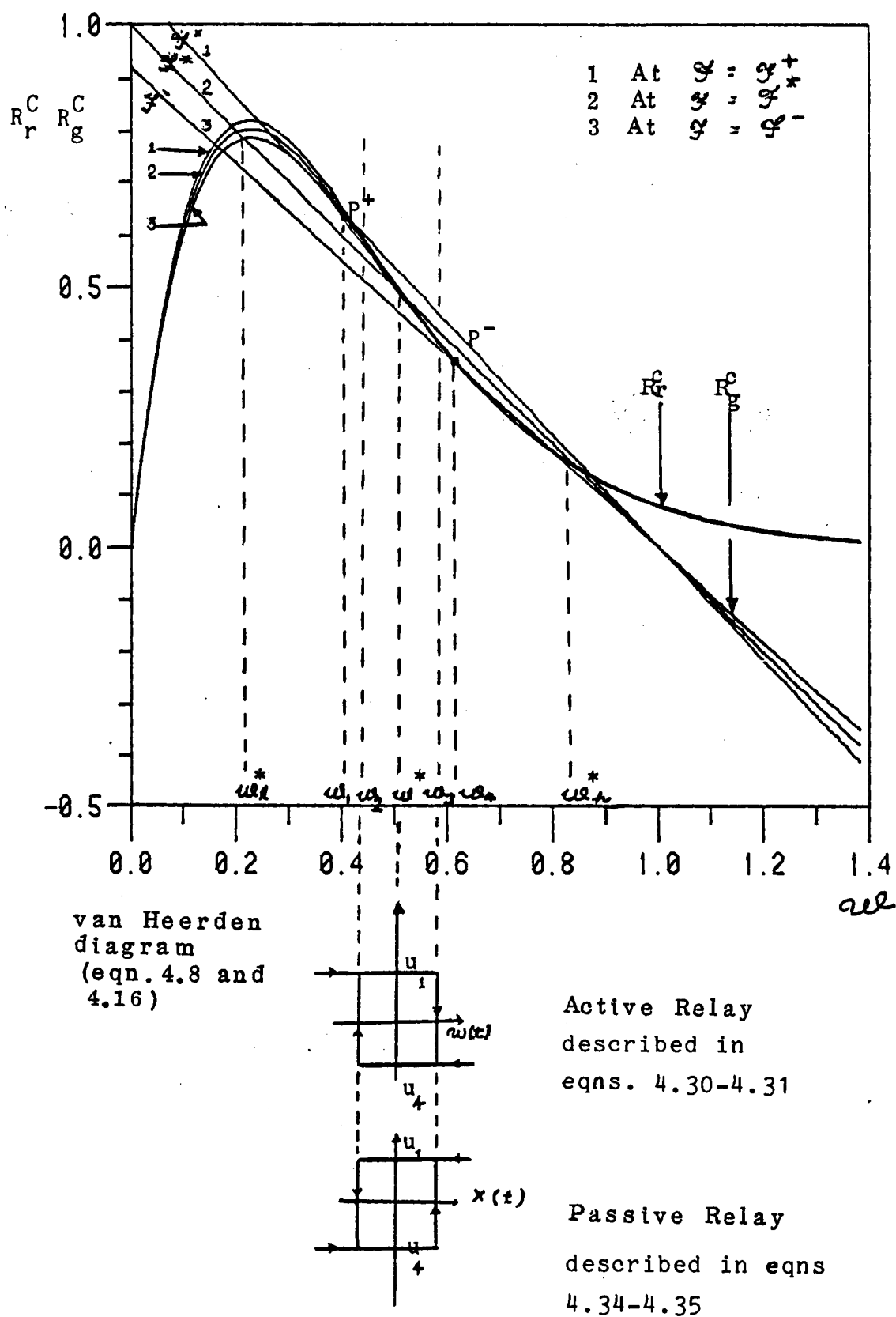


Figure 4.3: Graphical Illustration of the Control Policy for 2nd order system

within a small neighbourhood of M defined as

$$w_1 = w^* - We < w < w^* + We = w_4 \quad 4.22$$

where We is an allowable (small) error in the concentration from the unstable steady state. To satisfy this condition, two feeds g^+ , g^- could be chosen such that operation at these flow rates gives another set of unstable steady states as P^+ and P^- indicated in figure 4.3.

Next, the actual "switching concentrations" are chosen: These are w_2 and w_3 defined as

$$w_2 = w^* - \delta_c We \quad 4.23$$

$$w_3 = w^* + \delta_c We \quad 4.24$$

where δ_c is a design parameter that must be chosen such that the control specifications are met. The interval defined in eqns. 4.23 and 4.24 is traditionally known as the "bandwidth." When the concentration in the reactor alternately reaches these two concentrations, the control variable is switched between its two different levels (the controller is a relay).

To gain some physical insight into the nonlinear technique which has been proposed, figure 4.3 will be used assuming that the steady state relation plotted as R_r^C and R_g^C correctly indicate the dynamics of the reactor. This is

done fully realizing that the information in the plot is correct only at the steady states (that is, at the intersections of the curves). The fact that the dynamics are not necessarily correctly inferred in such a figure is the inherent reason why the van Heerden's slope condition is only a necessary condition for steady state stability for this second order (and higher order) process. The description of the nonlinear control action can be correctly presented for this second order system by using a phase plane presentation (see (34)).

With the above limitation in mind, the operation of the process may be explained as follows: suppose with $\mathcal{F} = \mathcal{F}^-$, w lies between w_2 and w_3 . Since $R_r^C > R_g^C$, $dw/d\tau < 0$. This implies that the concentration will decrease until it comes to a value w_2 where a controller switches the flow rate to level $\mathcal{F}^+$. Now, since $R_g^C > R_r^C$, $dw/d\tau > 0$. Hence, the concentration increases in value until it reaches w_3 where the controller switches to $\mathcal{F}^-$. The process is thus repeated ad infinitum. With proper control implementation, w is anticipated to become a stable periodic function of time. This will only be so if proper start up condition is used.

For second and higher order systems, the start up conditions will not be obvious. In general, the requirements can be stated as being sufficiently close to the stable periodic oscillations. In this work, when

simulation is used (see chapter 6) , the start up or initial values will be taken as the values of the desired operation point.

The motion of the process described above is in a sense "pushed" by the unstable steady states labelled P^+ and P^- in figure 4.3. Therefore, these states are known as the "pushing" states, and the control policy which can effect the process motion is known as a "pushing" control policy. Defining the following deviation variables

$$w = \mathcal{W} - \mathcal{W}^* \quad 4.25$$

$$w_i = \mathcal{W}_i - \mathcal{W}^* \quad 4.26$$

$$i = 1, 2, 3, 4$$

$$u = \mathcal{F} - \mathcal{F}^* \quad 4.27$$

$$u_1 = \mathcal{F}^+ - \mathcal{F}^* \quad 4.28$$

$$u_4 = \mathcal{F}^- - \mathcal{F}^* \quad 4.29$$

where it is noted that $\mathcal{F}^+$ and $\mathcal{F}^-$ will be calculated from the linearized reactor model so $u_1 = -u_4$. This is done in chapter 6 and leads to a symmetric relay with hysteresis.

The controller action may be described mathematically as

$$u = u_1, \quad w_1 < w < w_2; \text{ and } \frac{dw}{d\tau} > 0 \quad w_2 < w \leq w_3 \quad 4.30$$

$$u = u_4, \quad w_3 < w < w_4; \text{ and } \frac{dw}{d\tau} < 0 \quad w_2 \leq w < w_3 \quad 4.31$$

with control levels being changed when the concentration

alternately reaches w_2 and w_3 . It is important to note that this motion of the process will be realized only after the system's oscillation has been established with a proper start up procedure as stated above.

A nonlinear controller such as an active relay with hysteresis could be employed to effect the switching control described in equations 4.30 and 4.31. Such a control policy is illustrated in figure 4.4 where, here, the system is under a negative feed back control, and thus the required relay is a passive one. The system is unforced, that is, the set point $r(t)$ is zero in deviation variables. $x(t)$ is the error which, in a negative feed back control scheme, is defined as

$$x = -w = w^* - w \quad 4.32$$

$$x_i = -w = w^* - w_i \quad 4.33$$

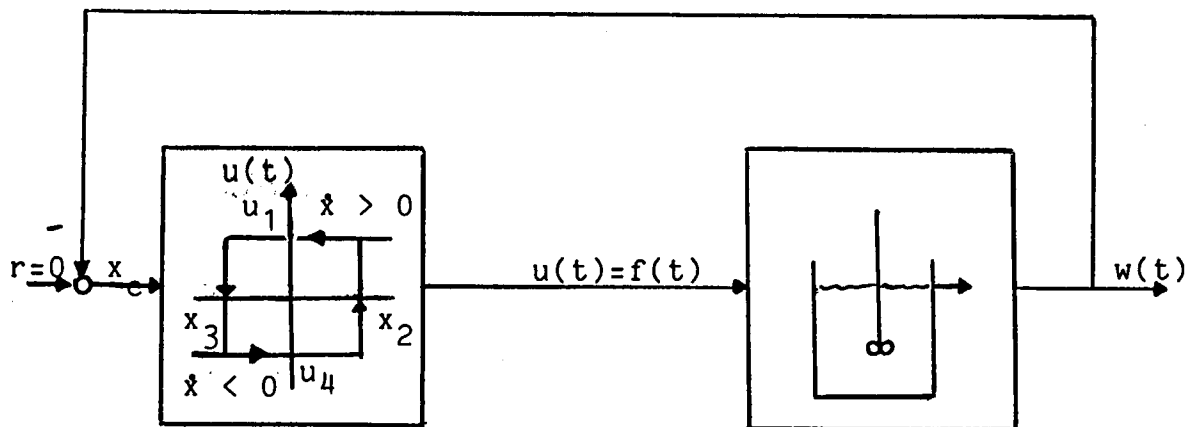
$$i = 1, 2, 3, 4$$

If the process is under a negative feed back control, the control policy of equation 4.30 and 4.31 for the passive relay with hysteresis is

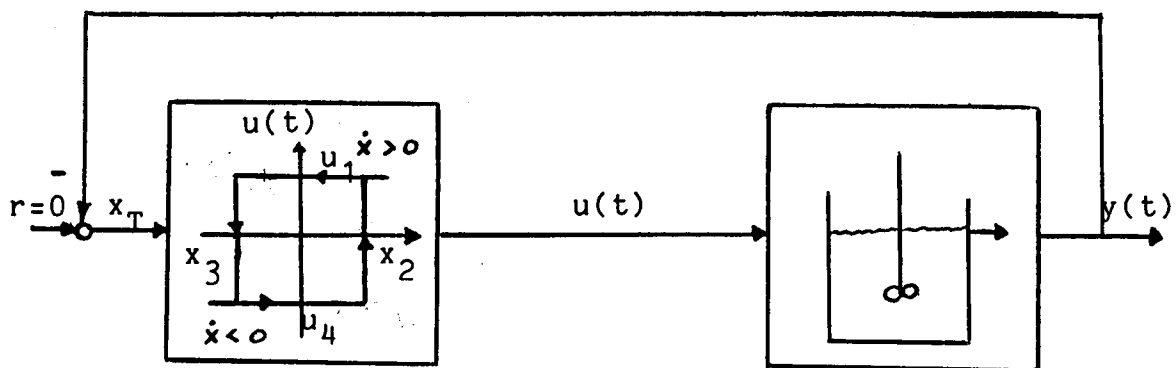
$$u = u_1 \quad x_2 < x < x_1 \text{ any } \dot{x} ; \quad x_3 \leq x < x_2 \text{ and } \dot{x} < 0 \quad 4.34$$

$$u = u_4 \quad x_4 < x < x_3 \text{ any } \dot{x} ; \quad x_3 < x \leq x_2 \text{ and } \dot{x} > 0 \quad 4.35$$

and the corresponding development for an active relay with hysteresis



A: A negative feedback control scheme with a passive relay with hysteresis for Concentration Control



B: A negative feedback control scheme with a passive relay with hysteresis for Temperature Control

Figure 4.4: Feedback control schemes for control of the unstable reactor (fig. 3.1).

would give

$$u = u_1 \quad x_4 < x < x_3 \quad \text{any } \dot{x} \quad ; \quad x_3 < x \leq x_2 \quad \text{and } \dot{x} > 0 \quad 4.36$$

$$u = u_4 \quad x_2 < x < x_1 \quad \text{any } \dot{x} \quad ; \quad x_2 < x \leq x_3 \quad \text{and } \dot{x} < 0 \quad 4.37$$

Figure 4.4A shows a negative feedback control scheme for the SI/SO concentration control loop suggested by the above analysis.

The above control action can be repeated for the temperature control by defining the following deviation variables

$$y = \mathcal{Y} - \mathcal{Y}^* \quad 4.38$$

$$y_i = \mathcal{Y}_i - \mathcal{Y}^* \quad 4.39$$

$$i = 1, 2, 3, 4$$

$$u = \mathcal{U} - \mathcal{U}^* \quad 4.40$$

$$u_1 = \mathcal{U}^+ - \mathcal{U}^* \quad 4.41$$

$$u_4 = \mathcal{U}^- - \mathcal{U}^* \quad 4.42$$

For the SI/SO temperature control loop, the control scheme suggested the use of a passive relay with hysteresis in a negative feedback control scheme shown in figure 4.4B. Detailed treatment of this control loop has been presented by Bruns and Bailey (7).

4.3 QUANTITATIVE PREDICTION OF PERIODIC OSCILLATIONS

Several approaches which are usually described as "Exact" or "Inexact" are available for the analytical investigations of the periodic oscillations of the system such as that described in section 4.2. The techniques have been basically developed in the electrical engineering literature where a linear process is nearly always used. Thus the terms "Exact" and "Inexact" refer to the treatment of the control element. Of the inexact methods, the Describing Function analyses have enjoyed immense popularity in application (8). Apart from the process linearization described in chapter three, the describing function approach also requires the so-called "equivalent" or "harmonic" linearization of the nonlinear control element. This and the assumption that the oscillation be sinusoidal which, in turn, requires the open-loop transfer function $G(s)$ to possess sufficient attenuation or filter properties, render these approaches inexact. Exact methods, on the other hand, do not necessitate any more approximation than that entailed in the process linearization. For detailed discussion and treatment of these approaches, see (8,31,33,34).

In section 4.2, a relay with hysteresis was chosen to carry out the necessary switchings for the system's control action described in eqns. 4.34 and 4.35. When the nonlinear control element in figure 3.4 is some kind of relay, Tsypkin's analytical technique, which has been

developed and elaborately advanced for application for SI/SO control, may be applied to predict nonlinear oscillations of the closed loop system. This analysis is an exact method. As presented in the readily available literature by Gelb and Velde (8), Gibson et al. (99), and in other texts (see (34, page 111-115)), this approach is presented in the unnecessary format of requiring the linear plant to be stable although the original formalism by Tsytkin (91) treated feed back systems with linear plants either as stable or unstable. All theory for the analysis of the stability of the resulting oscillation found in this literature has required the process transfer function to be of two or greater order where the order is defined as the number of system's poles minus zeros. That is, stability analysis, for predicted oscillation requires

$$\theta(G(s)) \geq 2$$

4.43

But the Tsytkin's treatment as to be presented in this study was modified by Bruns (34) for applications which encompass processes containing an unstable linear plant with $\theta(G(s)) \geq 0$.

To fully savour the spirit of formalisms and understand the underlying basis of the mathematical developments leading to the Tsytkin's approach, we digress at this point to take a closer look at relay as a nonlinear control element.

Relay as a Nonlinear Control Element

The popularity of relay as a nonlinear control element is widespread (4,8). A relay is not only simple in structure, rugged and reliable in performance, but is also relatively inexpensive. A relay is a power element as it is quite capable of delivering full power to a plant in a short time.

There are no restrictions on the periodic input forms to a relay beyond the requirement that such an input comply with the so-called periodicity conditions given below. This is so because a relay's output is always made of two or more constant levels (drive levels). For the relay control element employed in this research (symmetric relay with hysteresis), the output signal will always be a square wave. The input/output relationships of a passive relay with hysteresis are shown in figure 4.5.

The General Periodicity Condition for Symmetric Relays

When an SI/SO closed-loop system such as shown in figure 3.2 (page 50) or 3.3 (page 50) has as a controller a relay with hysteresis, certain conditions must be fulfilled before periodic oscillation of the system can be maintained (4,7,8,34). These are known as the "switching" and the "continuity" conditions which together are known as the periodicity conditions.

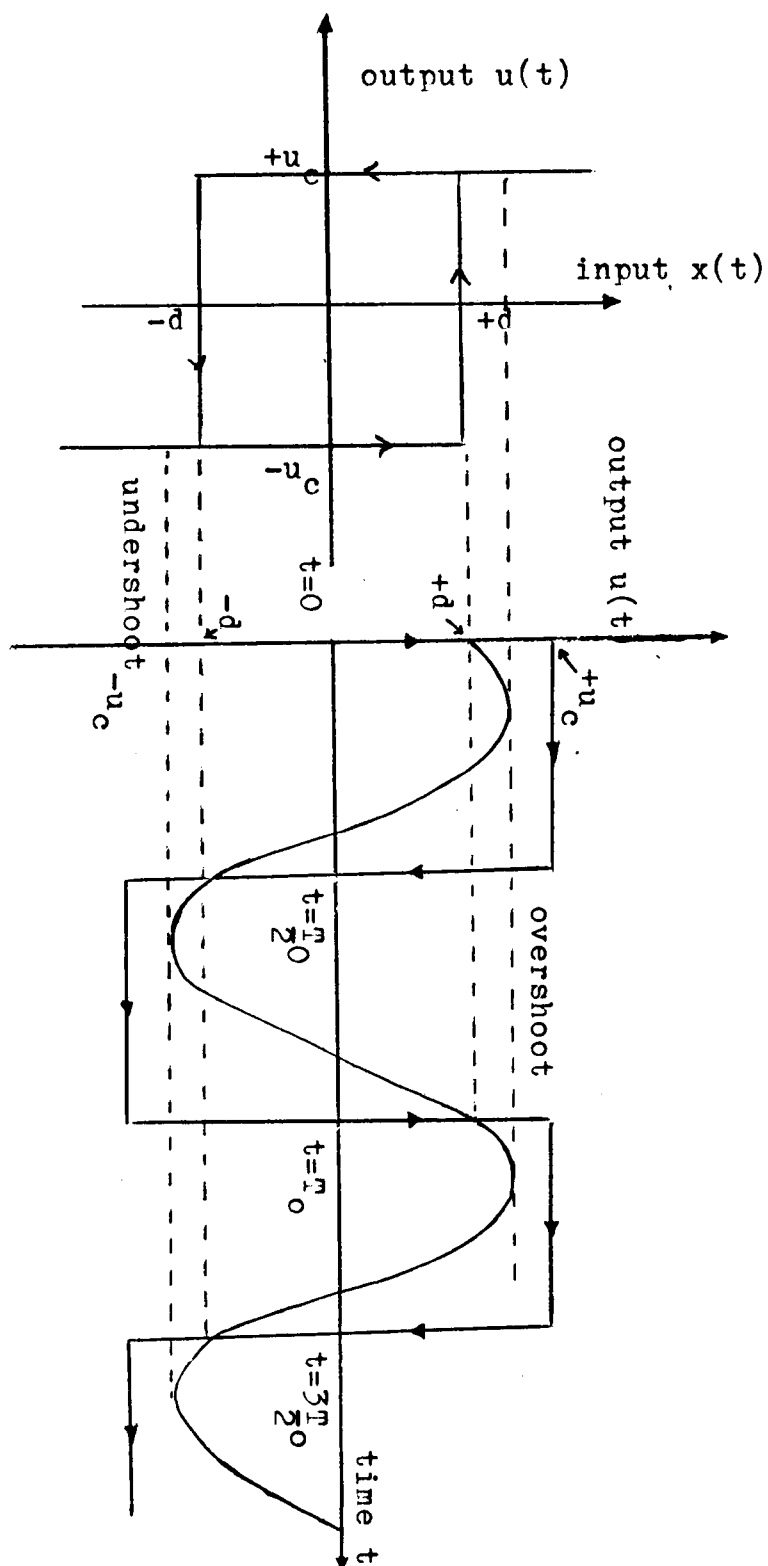


Figure 4.5: Relationship between input $x(t)$ and output $u(t)$ of a passive relay with a drive level $+u_c$ and hysteresis $+d$

Suppose a nonlinear control element such as indicated in figure 3.2 is a relay with an hysteresis of $\pm d$ and drive levels of $\pm u_c$. Suppose also that the relay is excited with a periodic function $f(\Omega_0 t)$ given as

$$x(t) = f(\Omega_0 t) \quad 4.43$$

where Ω_0 is the periodic input frequency. Hence, the period is

$$T_0 = \frac{2\pi}{\Omega_0} \quad 4.44$$

As stated above, the relay's output will be a square wave with the same frequency. These periodic input and output are given in figure 4.5. To fully understand the periodicity conditions given immediately below, the reader is advised to first study these figures and grasp the relationships between the input and output wave forms of the relay.

The Switching Conditions

Let $t=0$ be the time the relay switches to its positive drive level. Also, for a symmetric relay, the signal parameter $\alpha = 1/2$ which means the relay will spend half of each period at each drive level. This is true only for systems whose open-loop transfer function $G(s)$ has a pole at $s=0$ or for systems (such as the one used in this study) in which

the command input (set point) is zero.

The switching conditions for the existence of sustained oscillation in a closed loop system can be deduced from the observation of the periodic requirement of the periodic input $x(t)$ to the relay. For a symmetric relay with hysteresis, the conditions are given as

$$\begin{aligned}
 x \left| \frac{1}{2}(2n+1) \frac{2\pi}{\Omega_0} \right|^- &= -rd & r\dot{x} \left| \frac{1}{2}(2n+1) \frac{2\pi}{\Omega_0} \right|^- &< 0 & 4.45 \\
 n &= 0, 1, 2, \dots \\
 r &= +1 \text{ (passive relay)} \\
 &= -1 \text{ (active relay)}
 \end{aligned}$$

The Continuity Conditions

In addition to the switching conditions, it must be ascertained that the input signal $x(t)$ is not of a form that will cause the relay to switch at times other than $\frac{1}{2}(2n+1)T_0$ as stipulated in the switching conditions. The continuity conditions state this requirement. These conditions are given as

$$\begin{aligned}
 r \left| x(t) \right| &> -d & nT_0 &< t < \frac{(2n+1)}{2}T_0 & 4.46 \\
 n &= 0, 1, 2, \dots \\
 r &= +1 \text{ (passive relay)} \\
 &= -1 \text{ (active relay)}
 \end{aligned}$$

The switching and continuity conditions taken together constitute the necessary and sufficient condition for the existence of sustained self-oscillation in closed-loop systems.

Tsytkin's Analysis for SI/SO Processes (Old Theory)

The Tsytkin's method provides a scheme to check the switching conditions, that is, to find solutions that satisfy eqn. 4.45. Once these oscillation characteristics have been found, the amplitude of the process oscillation can be determined by developing expressions to check the continuity conditions (eqn. 4.46). The following Tsytkin's analysis will focus on the concentration control system of fig. 3.2 (page 50) and, at the end of this presentation, similar results will be made for the temperature control system presented in fig. 3.3 (page 50).

The presentation of the Tsytkin's analysis (4,34) that follows includes the following assumptions:

- (1) A solution or solutions for the periodic input $x(t)$ exist.
- (2) The relay
 - (a) is time invariant
 - (b) is symmetric, that is, $\alpha = 1/2$
 - (c) generates no subharmonics
- (3) The linear plant
 - (a) is time invariant

(b) operates at a fundamental or basic frequency of Ω_0 . That is

$G_{11}(s)$ generates no subharmonics

(4) The controlled system is unforced, and thus the periodic oscillation is symmetric.

For any periodic input $x(t)$, the nonlinearity output $f(t)$ (or $u(t)$ see figure 3.2 page 50) is a symmetric square wave. Since $f(t)$ is thus a periodic bounded function which, for a given period, has a finite number of local maxima and minima as well as a finite number of local points of discontinuity (see fig. 4.5, page 87), it satisfies the Dirichlet conditions. Consequently, $f(t)$ can be represented by a Fourier series. Because of the symmetry of $f(t)$, the constant Fourier series coefficient g is identically zero. Also, because the orientation of $f(t)$ with respect to the vertical axis is symmetric (see figure 4.5), $f(t)$ is an odd function of time. That is, $f(-t) = -f(t)$. Therefore, the cosine component of the series is zero. The Fourier series presentation of $f(t)$ is given as

$$f(t) = u_1(t) = \frac{4UC1}{\pi} \sum_{k=1}^{\infty} \frac{\sin(k \Omega_0 t)}{k} \quad 4.47$$

where $UC1$ is the concentration controlling relay's drive level. In the polar form, eqn. 4.47 becomes

$$u_1(t) = \frac{4UC1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im}(e^{jk\Omega_o t}) \quad 4.48$$

where the subscript "o" on the summation sign indicates that the summation is carried over odd integers only. That is $k = 1, 3, 5, \dots$. The system's output $w(t)$ and its derivative $\dot{w}(t)$ are not necessarily continuous (it would only be continuous if $\sigma(G(s)) \gg 2$). Hence, Bergen (38) defined the function $G_{11}(s)$ as

$$\hat{G}_{11}(s) = G_{11}(s) - G_{11}(\infty) \quad 4.49$$

$$\text{where } G_{11}(\infty) = \lim_{s \rightarrow \infty} G_{11}(s) \quad 4.50$$

is a constant term with which all discontinuities can be associated. This makes $\hat{G}_{11}(s)$ a low-pass transfer function. That is, $\hat{G}_{11}$ contains at least one more pole than zeros. It is emphasized that for processes such as the one used for this study where $\sigma(G(s)) = 1$, $G_{11}(\infty) = 0$, hence, $\hat{G}_{11}(s) = G_{11}(s)$.

With the above conditions, $w(t)$, the system's output is readily evaluated as

$$w(t) = \frac{4UC1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left[\hat{G}_{11}(jk\Omega_o) e^{jk\Omega_o t} \right] + f(t) G_{11}(\infty) \quad 4.51$$

and its time derivative as

$$\dot{w}(t) = \frac{4UC1\Omega_o}{\pi} \sum_{k=1}^{\infty} \operatorname{Re} \left(\hat{G}_{11}(jk\Omega_o) e^{jk\Omega_o t} \right) + G_{11}(\infty) \dot{f}(t) \quad 4.52$$

At the switching times $t = \frac{\pi}{\Omega_o}$, eqns. 4.51, 4.52 become

$$w(\frac{n}{\Omega_0})^- = -\frac{4UC1}{n} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im}(\hat{G}_{11}(jk\Omega_0)) + G_{11}(\infty)Uc1 \quad 4.53$$

$$\dot{w}(\frac{n}{\Omega_0})^- = -\frac{4UC1}{n} \Omega_0 \sum_{k=1}^{\infty} \operatorname{Re}(\hat{G}_{11}(jk\Omega_0)) + G_{11}(\infty) \dot{f}(\frac{n}{\Omega_0})^- \quad 4.54$$

where $s = (jk\Omega_0)$ has been substituted in the above eqns. At a point of discontinuity, the above series converge to the average value of the functions (34). Thus eqn. 4.54 is rewritten as

$$\dot{w}(\frac{n}{\Omega_0})^- = -\frac{4UC1}{n} \Omega_0 \sum_{k=1}^{\infty} \operatorname{Re}(\hat{G}_{11}(jk\Omega_0)) + UC1 \lim_{s \rightarrow \infty} s \hat{G}_{11}(s) \quad 4.55$$

The delta function due to $u(t)$ at $t = \frac{n}{\Omega_0}$ has been dropped out as this occurs before or after the switching time .

The quantitative prediction of the oscillation about the unstable steady state condition according to the Tsypkin's analysis reduces to one's ability to evaluate the infinite series given in eqns. 4.53 and 4.55. There are, at least, two approaches to the problem. One is a brute force technique in which the indicated summations could be carried out with a computer until some convergent criteria are met. Another approach is to obtain closed forms of the series. Gelb and Henrikson (8, page 194 - 195) have developed such closed forms for the type of series encountered in the Tsypkin's analysis. To obtain the closed forms, the series is replaced by an appropriate contour integral which is evaluated through application of the Cauchy Residue theory. The tables generated by Gelb and Henrikson are given as Tables 4.1 and 4.2 in this thesis.

Table 4.1: Closed forms of infinite series for Tsyarkin's switching conditions

$G_1(s)$	$\sum_{n=1,3,5,\dots}^{\infty} \operatorname{Re} [G_1(jn\Omega_0)]$	$\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \operatorname{Im} [G_1(jn\Omega_0)]$
$\frac{1}{s+a}$	$\frac{\pi}{4\omega_0} \tanh \left(\frac{\pi a}{2\Omega_0} \right)$	$-\frac{\pi}{4a} \tanh \left(\frac{\pi a}{2\Omega_0} \right)$
$\frac{s+b}{(s+a)^2}$	$\frac{\pi}{8\Omega_0 \cosh^2 \left(\frac{\pi a}{2\Omega_0} \right)} \cdot \left[\sinh \left(\frac{\pi a}{\Omega_0} \right) - \frac{\pi}{\Omega_0} (b-a) \right]$	$\frac{\pi}{8a^2 \cosh^2 \left(\frac{\pi a}{2\Omega_0} \right)} \cdot \left[b \sinh \left(\frac{\pi a}{\Omega_0} \right) - \frac{\pi a}{\Omega_0} (b-a) \right]$
$\frac{(s+b_1)(s+b_2)}{(s+a)^2}$	$\frac{\pi}{16\Omega_0 \cosh^2 \left(\frac{\pi a}{2\Omega_0} \right)} \left[2 \sinh \left(\frac{\pi a}{\Omega_0} \right) - \frac{\pi^2}{\Omega_0^2} (a-b_1)(a-b_2) \tanh \left(\frac{\pi a}{2\Omega_0} \right) + \frac{2\pi}{\Omega_0} (2a-b_1-b_2) \right]$	$-\frac{\pi}{16a^2 \cosh^2 \left(\frac{\pi a}{2\Omega_0} \right)} \left[2b_1 b_2 \sinh \left(\frac{\pi a}{\Omega_0} \right) - \pi^2 (a-b_1)(a-b_2) \tanh \left(\frac{\pi a}{2\Omega_0} \right) + 2\pi \left(\frac{a}{\Omega_0} \right) (a^2 - b_1 b_2) \right]$

* A useful relationship in applications of this table is $\frac{1}{(s+a)^2} = \lim_{b \rightarrow \infty} \left[\frac{1}{b} \frac{s+b}{(s+a)^2} \right]$.

Table 4.2: Summations of infinite series for Tsyarkin's continuity conditions

$$\begin{aligned}
 (1) \quad & \sum_{k \text{ odd}}^{\infty} \frac{\sin kx}{k} = \frac{\pi}{4} & 0 < x < \pi \\
 (2) \quad & \sum_{k \text{ odd}}^{\infty} \frac{\sin kx}{k(k^2 + a^2)} = f(x, a) \\
 (3) \quad & \sum_{k \text{ odd}}^{\infty} \frac{\sin kx}{k(k^2 + a_1^2)(k^2 + a_2^2)} = \frac{f(x, a_1) - f(x, a_2)}{a_1^2 - a_2^2} \\
 (4) \quad & \sum_{k \text{ odd}}^{\infty} \frac{\cos kx}{k^2 + a^2} = f_z(x, a) \\
 (5) \quad & \sum_{k \text{ odd}}^{\infty} \frac{\cos kx}{(k^2 + a_1^2)(k^2 + a_2^2)} = \frac{f_z(x, a_1) - f_z(x, a_2)}{a_1^2 - a_2^2}
 \end{aligned}$$

$$\text{where } f(x, a) = \frac{\pi \cosh(\pi a/2) - \cosh(\pi a/2 - ax)}{a^2 \cosh(\pi a/2)}$$

$$f_z(x, a) = \frac{\partial f}{\partial x}(x, a) = \frac{\pi \sinh(\pi a/2 - ax)}{a \cosh(\pi a/2)}$$

To obtain a closed form of the infinite series in the equations given above, $\hat{G}_{11}(s)$ must first be expanded as partial fraction. As the example system's eigenvalues are real and distinct, the partial fraction expansion has the form

$$\hat{G}_{11} = \frac{\alpha_1}{s - \lambda_1} + \frac{\alpha_2}{s - \lambda_2} \quad 4.56$$

where α 's are real constants and, as it may be recalled, λ_1 and λ_2 are the system's eigenvalues. Carrying out the indicated partial fraction expansion, we get

$$\alpha_1 = \frac{b_{11}(a_{22} - \lambda_1) - a_{12}b_{21}}{\lambda_2 - \lambda_1} \quad 4.57$$

$$\alpha_2 = \frac{b_{11}(a_{22} - \lambda_2) - a_{12}b_{21}}{\lambda_1 - \lambda_2} \quad 4.58$$

Hence, eqns. 4.53 and 4.55 become after substitution of the expansion for $\hat{G}_{11}$

$$\dot{w}\left(\frac{\Pi}{\Omega_0}\right)^- = -\frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \text{Im} \left(\frac{\alpha_1}{s - \lambda_1} + \frac{\alpha_2}{s - \lambda_2} \right) \quad 4.59$$

$$\dot{w}\left(\frac{\Pi}{\Omega_0}\right)^- = -\frac{4UC1\Omega_0}{\Pi} \sum_{k=1}^{\infty} \text{Re} \left(\frac{\alpha_1}{s - \lambda_1} + \frac{\alpha_2}{s - \lambda_2} \right) + UC1b_{11} \quad 4.60$$

Note that $G_{11}(\infty)=0$ has been incorporated in the above

equations. Recall that if $\sum_k a$ and $\sum_k b$ are absolutely convergent series, so also is $\sum_k (a+b)$, so that $\sum_k (a+b) = \sum_k a + \sum_k b$. This fact is used in writing 4.61-4.62.

The real and the imaginary terms in these equations are in the forms of the first entry in Table 4.1. Thus eqn. 4.59 and 4.60 respectively become

$$w\left(\frac{\pi}{\Omega_0}\right)^- = UC1 \left| \frac{\alpha_1}{\lambda_1} \tanh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) + \frac{\alpha_2}{\lambda_2} \tanh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) \right| \quad 4.61$$

$$\dot{w}\left(\frac{\pi}{\Omega_0}\right)^- = UC1 \left| \alpha_1 \tanh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) + \alpha_2 \tanh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) + b_{11} \right| \quad 4.62$$

The input to the nonlinearity (see fig. 3.2) is given as

$$x_c(t) = -w(t) \quad 4.63$$

and

$$\dot{x}_c(t) = -\dot{w}(t) \quad 4.64$$

By eqn. 4.45, the switching conditions are determined for this system to be

$$-\frac{\alpha_1}{\lambda_1} \tanh\frac{\pi\lambda_1}{2\Omega_0} - \frac{\alpha_2}{\lambda_2} \tanh\frac{\pi\lambda_2}{2\Omega_0} = -\frac{r_1^d}{UC1} \quad 4.65$$

$$-r_1(\alpha_1 \tanh\frac{\pi\lambda_1}{2\Omega_0} + \alpha_2 \tanh\frac{\pi\lambda_2}{2\Omega_0} + b_{11}) < 0 \quad 4.66$$

From eqn. 4.51, the closed form of the system's output signal is easily obtained by substituting the partial fraction for $\hat{G}(s)$ extracting the indicated Re and Im components and employing Table 4.2.

$$w(t) = - \frac{UC1 \alpha_1}{\lambda_1 \cosh\left(\frac{\pi \lambda_1}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi \lambda_1}{2\Omega_0} - \lambda_1 t\right) + \cosh\left(\frac{\pi \lambda_1}{2\Omega_0}\right) - \cosh\left(\frac{\pi \lambda_1}{2\Omega_0} - \lambda_1 t\right) \right| - \frac{UC1 \alpha_2}{\lambda_2 \cosh\left(\frac{\pi \lambda_2}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi \lambda_2}{2\Omega_0} - \lambda_2 t\right) + \cosh\left(\frac{\pi \lambda_2}{2\Omega_0}\right) - \cosh\left(\frac{\pi \lambda_2}{2\Omega_0} - \lambda_2 t\right) \right|$$

4.67

$$\text{for } 0 < t < \frac{\pi}{\Omega_0}$$

Eqn. 4.65 can be easily solved to obtain the frequency Ω_0 for the concentration control loop, the only unknown in this expression. The solution is then checked in the inequality (eqn. 4.66). If the inequality is satisfied, the solution is then submitted into eqn. 4.67 to obtain the periodic output wave whose form is checked in the continuity conditions (eqn. 4.46) to ascertain that such a periodic form complies with the restrictions imposed by the necessary and sufficient conditions. A failure of the periodic wave results in the rejection of the solution. The detailed solutions of the above equations are given in section 6.1.

The temperature control system presented in fig. 3.3 can be similarly treated by following the mathematical arguments given above for the concentration control system. Thus the switching conditions for this system can be easily determined to be

$$-\frac{\alpha_7}{\lambda_1} \tanh \frac{\pi \lambda_1}{2\Omega_0} - \frac{\alpha_8}{\lambda_2} \tanh \frac{\pi \lambda_2}{2\Omega_0} = -\frac{r_{d2}}{UC2} \quad 4.68$$

and

$$-r_2(\alpha_7 \tanh \frac{\pi \lambda_1}{2\Omega_0} + \alpha_8 \tanh \frac{\pi \lambda_2}{2\Omega_0} + b_{22}) < 0 \quad 4.69$$

The system output is also found in closed form to be

$$\begin{aligned} y(\tau) = & -\frac{UC2\alpha_7}{\lambda_1 \cosh(\frac{\pi \lambda_1}{2\Omega_0})} \left| \sinh\left(\frac{\pi \lambda_1}{2\Omega_0} - \lambda_1 \tau\right) \right. \\ & + \cosh\left(\frac{\pi \lambda_1}{2\Omega_0}\right) - \cosh\left(\frac{\pi \lambda_1}{2\Omega_0} - \lambda_1 \tau\right) \Big| \\ & - \frac{UC2\alpha_8}{\lambda_2 \cosh(\frac{\pi \lambda_2}{2\Omega_0})} \left| \sinh\left(\frac{\pi \lambda_2}{2\Omega_0} - \lambda_2 \tau\right) \right. \\ & + \cosh\left(\frac{\pi \lambda_2}{2\Omega_0}\right) - \cosh\left(\frac{\pi \lambda_2}{2\Omega_0} - \lambda_2 \tau\right) \Big| \quad 4.70 \\ & \quad \quad \quad 0 < \tau < \frac{\pi}{\Omega_0} \end{aligned}$$

These equations are treated as discussed above for the concentration control system to obtain the temperature control system's output wave forms which can be taken as the required solutions only when they conform with the necessary and sufficient periodicity conditions.

Finally, it should be noted that not only is a periodic solution required for prediction of successful control but a stable solution is also needed. Stability theory is available for SI/SO systems (4,7).

CHAPTER 5

THE NEW THEORY: DIRECT TREATMENT OF COUPLED MI/MO PROCESSES UNDER FEEDBACK RELAY CONTROL

The Tsytkin's theory presented in Chapter 4 for single loop process control has stood the test of time. It has not only been intensely studied but has been found, where applicable, to provide superior predictions of oscillations imposed upon unstable processes with relay controllers compared to predictions of other approximating methods such as inexact analysis (34). Naturally, such findings invoke the following questions: could the Tsytkin's analytical approach which holds so well for SI/SO systems be modified or extended for MI/MO systems with their notorious tendency to exhibit cross-effects? How much insight could such an extension lend into the nature and even magnitude of these effects, which probably, could result in some control design aimed at compensating for such effects?

The answers to the above questions are the nucleus of this research. This chapter will strive to answer the first question by extending the Tsytkin's analysis to multi-loop systems thus accounting for all interactions among the control loops. This is a new theory, and this thesis is alone for its coinage and proclamation. The second question will be addressed in Chapter 6 where analytical results

from the test of the new theory will be presented.

In section 5.1, the mathematical formalism for extending Tsyarkin's approach for MI/MO processes is illustrated with the 2x2 chemical system treated in chapter three. Then in section 5.2, the analysis is extended to encompass any coupled MI/MO system where all transfer functions are of order greater than or equal to two, that is, $\theta(G(s)) \geq 2$. In section 5.3, additional oscillatory modes of coupled loops and their effects upon the system's performance are discussed.

5.1 TSYPKIN'S ANALYSIS FOR 2x2 COUPLED PROCESS FOR CASES WHERE ALL LOOP CONTROLLERS ARE SWITCHING

Consider the closed loop process depicted in figure 3.4 (pp. 51). According to the mathematical modelling of the unstable CSTR which resulted in this control scheme, there are interactions between the two control loops through $G_{12}(s)$ and $G_{21}(s)$. Consequently, any change in the drive levels of a relay in either loop will serve as a load disturbance to the other loop through the respective interaction path. Thus a switch in the drive level of n_1 , for example, does not only pass through $G_{11}(s)$ to regulate $w(t)$ but is also fed through $G_{21}(s)$ to the temperature loop.

In addition to these interactions, there are other modes of operation that the coupled system can display. For instant, cases such as a failure of one or both of the controllers to switch are very possible and, as will be shown in section 5.3 are not uncommon. The qualitative discussion and theoretical developments in this section, however, are confined to most interesting of these loop activities, the case where all controllers do switch. Discussion of additional switching possibilities and how these modes might be analysed is given in section 5.3.

Suppose the controllers $\mathcal{N}_1$ and $\mathcal{N}_2$ in figure 3.4 are symmetric relays with the respective drive levels of $\pm UC_1$ and $\pm UC_2$, and contain hysteresis bands of $\pm d_1$ and $\pm d_2$. In the present situation where both relays do switch, time $t=0$ is defined with reference to the relay labelled "1". Following the classical Tsypkin's analysis (4,8,34), $t=0$ is defined as the time when this relay switches to its positive drive level ($\pm UC_1$). Recalling that the periodicity conditions imposed by a relay element formed the basis of the analysis in the SI/SO cases (see chapter four), it should be anticipated that in this 2x2 process, a set of periodicity conditions will be needed for each controller. Thus, the switching times for the second relay must be specified: the time at which this relay switches to its positive drive level with reference to $t = 0$ is denoted as t_ϕ (see figure 5.1). As discussed below, the oscillatory inputs to the relays are taken to have the same frequency.

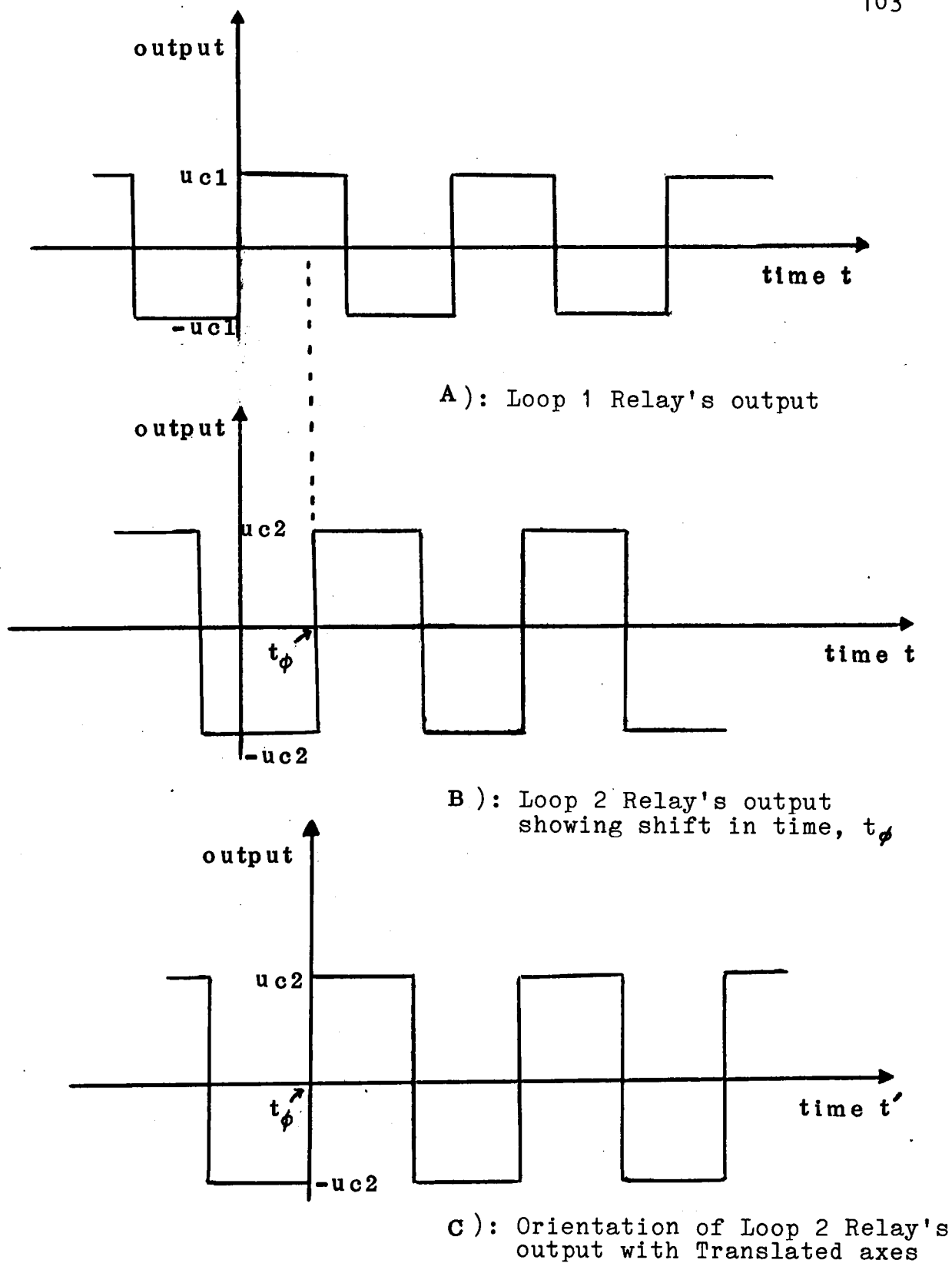


Figure 5.1: Relationship between Concentration Loop Relay's Output and Temperature Loop Relay's Output

That is $\Omega_{0C} = \Omega_{0T} = \Omega_0$ but they may be shifted in time t_ϕ .
Hence,

$$\phi = -t_\phi \Omega_0 \quad 5.1$$

where ϕ is a phase shift between the two periodic relay outputs. The convention of ϕ being negative if the second wave lags and ϕ being positive if the second wave leads is followed in writing the above equation.

The basic assumptions entailed in the Tsypkin's treatment for SI/SO systems in chapter four are retained in the following new theory. These assumptions restated in the current context are:

- (1) A solution (or solutions) for the periodic inputs $x_i(t)$ exist.
- (2) The relays
 - (a) are time invariant
 - (b) are symmetric
 - (c) generate no subharmonics
 - (d) are both switching
- (3) The linear plant
 - (a) is time invariant
 - (b) generates no subharmonics
- (4) The controlled system is unforced.

The following additional assumptions are also invoked on the nature of the periodic solutions for the inputs $x_i(t)$ (or on the outputs $w(t)$ and $y(t)$):

(5) The periodic solutions are all of the same frequency

(6) The periodic solutions are all symmetric, that is, $\alpha_i = 1/2$.

Criteria for the last two assumptions have not been developed. It is felt that these will usually hold for the linearized process description with exceptions possibly being found in special cases such as harmonic resonance.

New theoretical analysis is also required to address the stability of multivariable processes containing more than one relay. This would require considerable additional analysis and obtaining an analytical procedure is not assured. Pursuit of such an undertaking is clearly the making of an additional study. This remains an important question as stable periodic solutions are required to meet the objective of controlled operation in the neighbourhood of an unstable steady state condition. In this work (see chapter six), simulation of the unstable reactor is carried out to obtain comparisons with the analytical predictions. These will also serve to indicate the solutions stability.

With the above assumptions, the steps toward an analytical prediction of the coupled system's behaviour according to Tsytkin's method proceed from the proper understanding of the nature of both the nonlinearity inputs

$x_1(t)$ and outputs $u_1(t)$.

The nonlinearity outputs $u_1(t)$ and $u_2(t)$, as before, are symmetric square waves (figure 5.1). It is obvious that $u_1(t)$ is an odd function of time (see Chapter 4). Hence, its Fourier series representation is given as

$$u_1(t) = \frac{4UC_1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin(k \Omega_0 t) \quad 5.2$$

Again, the subscript "o" on the summation sign implies that the indicated summation is carried over odd integers only. On the other hand, the difference in the switching time of t_ϕ for the second relay renders the square wave $u_2(t)$ nonsymmetric with respect to its vertical axis. The straight forward application of Fourier series to this wave must, therefore, contain both the sine and cosine components. In terms of Heaviside function, the nonsymmetric wave is easily given as

$$u_2(t) = UC_2 \mu_{-1}(t - t_\phi) - 2UC_2 \mu_{-1}(t - \frac{1}{2}T_0 - t_\phi) + \dots \quad 5.3$$

where $\mu_{-1}(t - t_\phi)$ is a Heaviside function defined as

$$\mu_{-1}(t - t_\phi) = \begin{cases} 0 & t < t_\phi \\ 1 & t \geq t_\phi \end{cases} \quad 5.4$$

The mathematical representation of this nonsymmetric square wave is much less involved if the axes of $u_2(t)$ are translated as shown in figure 5.1C on the basis of a different time t' defined as

$$t' = t - t_\phi \quad 5.5$$

Eqn. 5.3 then becomes

$$u_2(t') = UC2 \mu_{-1}(t') - 2UC2 \mu_{-1}(t' - \frac{1}{2}T_0) + \dots \quad 5.6$$

and

$$\mu_{-1}(t') = \begin{cases} 0 & t' < 0 \\ 1 & t' \geq 0 \end{cases} \quad 5.7$$

$u_2(t')$ is now an odd function. That is

$$u_2(-t') = -u_2(t') \quad 5.8$$

The Fourier series of this wave in terms of t' is then given as

$$u_2(t') = \sum_{k=1}^{\infty} b_k \sin\left(\frac{\pi k t'}{\frac{1}{2}T_0}\right) \quad 5.9$$

$$\text{Where } b_k = \frac{2}{\frac{1}{2}T_0} \int_0^{\frac{1}{2}T_0} \sin\left(\frac{\pi k t'}{\frac{1}{2}T_0}\right) dt \quad 5.10$$

$$= \frac{2 \Omega_0 UC2}{\pi k \Omega_0} \cos(k \Omega_0 t') \Big|_0^{\pi/\Omega_0}$$

$$= \begin{cases} 0 & k \text{ even} \\ \frac{4UC2}{\pi k} & k \text{ odd} \end{cases} \quad 5.11$$

Substitution of b_k and $t' = t - t_\phi$ into eqn. 5.9 yields

$$u_2(t) = \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin(k \Omega_o (t - t_\phi)) \quad 5.12$$

$u_1(t)$ (eqn. 5.2) and $u_2(t)$ (eqn. 5.12) can be set in polar forms as

$$u_1(t) = \frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \text{Im} \left| e^{jk \Omega_o t} \right| \quad 5.13$$

$$u_2(t) = \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \text{Im} \left| e^{jk \Omega_o (t - t_\phi)} \right| \quad 5.14$$

From these equations, the loop outputs (see Figure 3.4) can be readily determined as

$$w_1(t) = \frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \text{Im} \left| \hat{G}_{11}(jk \Omega_o) e^{jk \Omega_o t} \right| \quad 5.15$$

$$+ f(t) G_{11}^{(\infty)}$$

$$w_2(t) = \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \text{Im} \left| \hat{G}_{12}(jk \Omega_o) e^{jk \Omega_o (t - t_\phi)} \right| + u_2(t) G_{12}^{(\infty)} \quad 5.16$$

and

$$y_1(t) = \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \text{Im} \left| \hat{G}_{22}(jk \Omega_o) e^{jk \Omega_o (t - t_\phi)} \right| \quad 5.17$$

$$+ u_2(t) G_{21}^{(\infty)}$$

$$y_2(t) = \frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{21}(jk\Omega_o) e^{jk\Omega_o t} \right| + f(t) G_{21}(\infty); \quad 5.18$$

where $G_{ij}(s) = \hat{G}_{ij}(s) + G_{ij}(\infty)$ as defined in chapter four.

For the specific example system under study, $G_{ij}(s) = \hat{G}_{ij}(s)$.

Hence, for this system, $\hat{G}_{ij}(s)$ will always be used in lieu of $G_{ij}(s)$. Thus the process outputs are given as

$$w(t) = w_1(t) + w_2(t) = \frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{11}(jk\Omega_o) e^{jk\Omega_o t} \right| + \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{12}(jk\Omega_o) e^{jk\Omega_o(t - t_\phi)} \right| \quad 5.19$$

and

$$y(t) = y_1(t) + y_2(t) = \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{22}(jk\Omega_o) e^{jk\Omega_o(t - t_\phi)} \right| + \frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{21}(jk\Omega_o) e^{jk\Omega_o t} \right| \quad 5.20$$

The respective time derivatives of these output signals are given as

$$\dot{w}(t) = \frac{4UC1\Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{11}(jk\Omega_o) e^{jk\Omega_o t} \right| + \frac{4UC2\Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{12}(jk\Omega_o) e^{jk\Omega_o(t - t_\phi)} \right| \quad 5.21$$

and

$$\dot{y}(t) = \frac{4UC1\Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{21}(jk\Omega_o) e^{jk\Omega_o t} \right| + \frac{4UC2\Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{22}(jk\Omega_o) e^{jk\Omega_o(t - t_\phi)} \right| \quad 5.22$$

From figure 5.1, it is apparent that the switching times of n_1 is $(\frac{\Pi}{\Omega_o})^-$ and that of n_2 is $(\frac{\Pi}{\Omega_o} - \frac{\phi}{\Omega_o})^-$. It is of paramount importance to note that the switching conditions

given in Chapter 4 (equation 4.45) can, in the present control situation, be applicable to only the concentration loop. To apply these conditions for the temperature loop, the switching time given as $\frac{\pi}{\Omega_o}$ in eqn. 4.45 must be replaced with the temperature loop relay's switch time given above. Thus, for the temperature loop relay, the switching conditions may be easily restated as

$$x \left[(2n + 1) \frac{\Pi}{\Omega_o} - \frac{\phi}{\Omega_o} \right]^- = -rd_2 \quad r \dot{x} \left| (2n + 1) \frac{\Pi}{\Omega_o} - \frac{\phi}{\Omega_o} \right|^- < 0 \quad 5.23$$

The continuity conditions (eqn.4.46) can be similarly restated for the temperature loop as

$$r \left[x(t) \right] > -d_2 \quad \left(n \frac{2\Pi}{\Omega_o} - \frac{\phi}{\Omega_o} \right) < t < \left| (2n + 1) \frac{\Pi}{\Omega_o} - \frac{\phi}{\Omega_o} \right| \quad 5.24$$

where $n=0,1,\dots$

$r = +1$ (for passive relay)

$= -1$ (for active relay)

At the switch times given above for the two relays eqns. 5.19-5.22 are written as

$$w \left(\frac{\Pi}{\Omega_o} \right)^- = - \frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{11}(jk\Omega_o) \right| - \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{12}(jk\Omega_o) e^{jk\phi} \right| \quad 5.25$$

$$\dot{w} \left(\frac{\Pi}{\Omega_o} \right)^- = - \frac{4UC1\Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{11}(jk\Omega_o) \right| - \frac{4UC2\Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{12}(jk\Omega_o) e^{jk\phi} \right| + UC1 \lim_{s \rightarrow \infty} s \hat{G}_{11}(s) \quad 5.26$$

$$y\left(\frac{\pi}{\Omega_0} - \frac{\phi}{\Omega_0}\right)^- = -\frac{4UC1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{21}(jk\Omega_0) e^{-jk\phi} \right| - \frac{4UC2}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{22}(jk\Omega_0) \right| \quad 5.27$$

$$\dot{y}\left(\frac{\pi}{\Omega_0} - \frac{\phi}{\Omega_0}\right)^- = -\frac{4UC1\Omega_0}{\pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{21}(jk\Omega_0) e^{-jk\phi} \right| - \frac{4UC2\Omega_0}{\pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{22}(jk\Omega_0) \right| + UC2 \lim_{s \rightarrow \infty} s \hat{G}_{22}(s) \quad 5.28$$

These series, as discussed in Chapter 4, converge to the midpoint of discontinuities, thus the need for terms as $uc1 \lim_{s \rightarrow \infty} s \hat{G}_{11}(s)$. In writing eqns. 5.26 and 5.28, the delta functions due to $\dot{f}\left(\frac{\pi}{\Omega_0}\right)$ and $\dot{u}_2\left(\frac{\pi}{\Omega_0} - \frac{\phi}{\Omega_0}\right)$ have been discarded as these occur after or before switching times. Further, $\dot{f}\left(\frac{\pi}{\Omega_0}\right)$ and $\dot{u}_2\left(\frac{\pi}{\Omega_0} - \frac{\phi}{\Omega_0}\right)$ are zero as f and u_2 are constant at these times, as well as before and after respectively.

Closed Forms

As in the analysis in Chapter 4, the infinite series above can be solved rather easily due to the availability of closed forms. Tables 4.1 and 4.2 developed by Gelb and Henrikson provide the needed closed forms. These tables were originally developed in the context of a stable transfer function $\hat{G}(s)$, but the mathematical procedure involved in generating the closed forms does not require a stable element (34). Therefore, these tables can also be used for unstable elements as well.

The procedure for generating the closed forms of eqns. 5.25-5.28 is the same as the one adopted in Chapter 4: the open-loop transfer functions are first expanded as partial fractions to give

$$\hat{G}_{11}(s) = \frac{\alpha_1}{s - \lambda_1} + \frac{\alpha_2}{s - \lambda_2} \quad 5.29$$

$$\hat{G}_{12}(s) = \frac{\alpha_3}{s - \lambda_1} + \frac{\alpha_4}{s - \lambda_2} \quad 5.30$$

$$\hat{G}_{21}(s) = \frac{\alpha_5}{s - \lambda_1} + \frac{\alpha_6}{s - \lambda_2} \quad 5.31$$

$$\hat{G}_{22}(s) = \frac{\alpha_7}{s - \lambda_1} + \frac{\alpha_8}{s - \lambda_2} \quad 5.32$$

where λ_1 and λ_2 , as it may be recalled, are the process eigenvalues and α 's are real constants determined to be

$$\begin{aligned} \alpha_1 &= \frac{b_{11}(a_{22} - \lambda_1) - a_{12}b_{21}}{\lambda_2 - \lambda_1} \\ \alpha_2 &= \frac{b_{11}(\lambda_2 - a_{22}) + a_{12}b_{21}}{\lambda_2 - \lambda_1} \\ \alpha_3 &= \frac{b_{12}(a_{22} - \lambda_1) - a_{12}b_{22}}{\lambda_2 - \lambda_1} \\ \alpha_4 &= \frac{b_{12}(\lambda_2 - a_{22}) + a_{12}b_{22}}{\lambda_2 - \lambda_1} \\ \alpha_5 &= \frac{b_{21}(a_{11} - \lambda_1) - a_{21}b_{11}}{\lambda_2 - \lambda_1} \\ \alpha_6 &= \frac{b_{21}(\lambda_2 - a_{11}) + a_{21}b_{11}}{\lambda_2 - \lambda_1} \\ \alpha_7 &= \frac{b_{22}(a_{11} - \lambda_1) - a_{21}b_{12}}{\lambda_2 - \lambda_1} \\ \alpha_8 &= \frac{b_{22}(\lambda_2 - a_{11}) + a_{21}b_{12}}{\lambda_2 - \lambda_1} \end{aligned}$$

Substituting the expanded $\hat{G}(s)$'s into eqn. 5.25, for instant, yields

$$y\left(\frac{\Pi}{\Omega_0} - \frac{\phi}{\Omega_0}\right) = -\frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \left| \frac{1}{k} \operatorname{Im} \left(\frac{\alpha_5}{jk\Omega_0 - \lambda_1} + \frac{\alpha_6}{jk\Omega_0 - \lambda_2} \right) * \right. \\ \left. (\cos k\phi - j \sin k\phi) \right| \\ - \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \left| \frac{1}{k} \operatorname{Im} \left(\frac{\alpha_7}{jk\Omega_0 - \lambda_1} + \frac{\alpha_8}{jk\Omega_0 - \lambda_2} \right) \right| \quad 5.33$$

Multiplying both the denominator and the numerator of the first term inside the first summation sign of eqn. 5.33 by the complex conjugate $-jk\Omega_0 - \lambda_1$, and taking the indicated imaginary component of the resulting term, we obtain after simplification

$$- \frac{4UC1}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left(\frac{\alpha_5}{jk\Omega_0 - \lambda_1} (\cos k\phi - j \sin k\phi) \right) \\ = \frac{4UC1}{\Pi} \alpha_5 \sum_{k=1}^{\infty} \left| \frac{\cos k\phi}{\Omega_0 \left| k^2 + \left(\frac{\lambda_1}{\Omega_0}\right)^2 \right|} - \frac{\lambda_1}{\Omega_0^2} \frac{\sin k\phi}{k \left| k^2 + \left(\frac{\lambda_1}{\Omega_0}\right)^2 \right|} \right| = z \quad 5.34$$

Using the second and the fourth terms of table 4.2, eqn. 5.34 is readily replaced with the closed forms of

$$z = \frac{UC1 \alpha_5}{\lambda_1 \cos \left(\frac{\Pi \lambda_1}{2\Omega_0} \right)} \left| \sinh \left(\frac{\Pi \lambda_1}{2\Omega_0} - \frac{\lambda_1 \phi}{\Omega_0} \right) \right. \\ \left. - \cosh \left(\frac{\Pi \lambda_1}{2\Omega_0} \right) + \cosh \left(\frac{\Pi \lambda_1}{2\Omega_0} - \frac{\lambda_1 \phi}{\Omega_0} \right) \right| \quad 5.35$$

The second term in the first summation of eqn. 5.33 can be similarly treated to give a form similar to that in eqn. 5.35. Because the series in the second summation of eqn. 5.33 do not contain transport lag terms, Table 4.1 (pp. 94) is used for evaluating the closed forms.

Substituting all the derived closed forms of the individual series in eqn. 5.33 yields

$$\begin{aligned}
 y\left(\frac{\pi}{\Omega_o} - \frac{\phi}{\Omega_o}\right)^- &= \frac{UC1\alpha_5}{\lambda_1 \cosh\left(\frac{\pi\lambda_1}{2\Omega_o}\right)} \left| \sinh\left(\frac{\pi\lambda_1}{2\Omega_o} - \frac{\lambda_1\phi}{\Omega_o}\right) \right. \\
 &\quad \left. - \cosh\left(\frac{\pi\lambda_1}{2\Omega_o}\right) + \cosh\left(\frac{\pi\lambda_1}{2\Omega_o} - \frac{\lambda_1\phi}{\Omega_o}\right) \right| \\
 &+ \frac{UC1\alpha_6}{\lambda_2 \cosh\left(\frac{\pi\lambda_2}{2\Omega_o}\right)} \left| \sinh\left(\frac{\pi\lambda_2}{2\Omega_o} - \frac{\lambda_2\phi}{\Omega_o}\right) \right. \\
 &\quad \left. - \cosh\left(\frac{\pi\lambda_2}{2\Omega_o}\right) + \cosh\left(\frac{\pi\lambda_2}{2\Omega_o} - \frac{\lambda_2\phi}{\Omega_o}\right) \right| \\
 &+ UC2 \left| \frac{\alpha_7}{\lambda_1} \tanh \frac{\pi\lambda_1}{2\Omega_o} + \frac{\alpha_8}{\lambda_2} \tanh \frac{\pi\lambda_2}{2\Omega_o} \right| \quad 5.36
 \end{aligned}$$

We can take exactly the same mathematical steps to obtain the closed forms for the infinite series in eqn. 5.28. But the first series is found to contain the term

$$\frac{k \sin k\phi}{k^2 + \left(\frac{\lambda_i}{\Omega_o}\right)^2} \quad (i = 1, 2)$$

which, as it is, has no closed form representation in either table 4.1 or 4.2. This difficulty is surmounted by creating a closed form for the above structure by partially differentiating entry four in table 4.2. Thus, borrowing the terminology from the table

$$\begin{aligned}
 -\sum_{k=1}^{\infty} \left(\frac{k \sin kx}{(k^2 + a^2)} \right) &= \frac{\partial_x^f(x, a)}{\partial x} \\
 &= -\frac{\pi}{4} \frac{\cosh\left(\frac{\pi a}{2} - ax\right)}{\cosh\left(\frac{\pi a}{2}\right)}
 \end{aligned} \tag{5.37}$$

with $\phi = x$, $\left(\frac{\lambda i}{\Omega_0}\right) = a$, eqn. 5.28 becomes

$$\begin{aligned}
 \dot{y}\left(\frac{\pi}{\Omega_0} - \frac{\phi}{\Omega_0}\right) = & \frac{UC1\alpha_5}{\cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) \right. \\
 & + \left. \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) \right| \\
 & + \frac{UC1\alpha_6}{\cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) \right. \\
 & + \left. \sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) \right| \\
 & + UC2 \left[\alpha_7 \tanh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) + \alpha_8 \tanh\left(\frac{\pi\lambda_2}{\Omega_0}\right) \right] \\
 & + UC2b_{22}
 \end{aligned} \tag{5.38}$$

Note that the periodic inputs $(x_c(t), x_T(t))$ and their derivatives are defined from the system's outputs as

$$x_c(t) = -w(t) \quad 5.39$$

$$\dot{x}_c(t) = -\dot{w}(t) \quad 5.40$$

and

$$x_T(t) = -y(t) \quad 5.41$$

$$\dot{x}_T(t) = -\dot{y}(t) \quad 5.42$$

The set points are zero as the system under study is unforced.

Using eqns. 5.41 and 5.42 to define the error inputs and applying the appropriate switching conditions to the two feed back loops of this system, the switching conditions equations (eqns. 5.23) for the temperature loop are easily determined as

$$\begin{aligned} & - \frac{UC1\alpha_5}{\lambda_1 \cosh(\frac{\pi\lambda_1}{2\Omega_0})} \left| \sinh(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}) - \cosh(\frac{\pi\lambda_1}{2\Omega_0}) \right. \\ & + \cosh(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}) \left| \right. \\ & - \frac{UC1\alpha_6}{\lambda_2 \cosh(\frac{\pi\lambda_2}{2\Omega_0})} \left| \sinh(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}) - \cosh(\frac{\pi\lambda_2}{2\Omega_0}) \right. \\ & + \cosh(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}) \left| \right. \\ & - UC2 \left| \frac{\alpha_7}{\lambda_1} \tanh(\frac{\pi\lambda_1}{2\Omega_0}) + \frac{\alpha_8}{\lambda_2} \tanh(\frac{\pi\lambda_2}{2\Omega_0}) \right| = -r_2 d_2 \quad 5.43 \end{aligned}$$

and

$$\begin{aligned}
 & \left\{ -\frac{UC1\alpha_5}{\cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) + \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) \right| \right. \\
 & - \frac{UC1\alpha_6}{\cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) + \sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) \right| \\
 & - UC2 \left| \alpha_7 \tanh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) + \alpha_8 \tanh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) \right| \\
 & \left. - UC2b_{22} \right\} r_2 < 0
 \end{aligned}
 \tag{5.44}$$

Beginning from eqns. 5.25 and 5.26 and following the above arguments used for the derivation of the temperature loop switching conditions, the switching conditions for the concentration loop can be similarly determined to be

$$\begin{aligned}
 & -\frac{UC2\alpha_3}{\lambda_1 \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) + \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) \right. \\
 & - \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) \left| - \frac{UC2\alpha_4}{\lambda_2 \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) \right. \right. \\
 & + \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) \left| \right. \\
 & - UC1 \left| \frac{\alpha_1}{\lambda_1} \tanh \frac{\pi\lambda_1}{2\Omega_0} + \frac{\alpha_2}{\lambda_2} \tanh \frac{\pi\lambda_2}{2\Omega_0} \right| = -r_1 d_1
 \end{aligned}
 \tag{5.45}$$

and

$$\begin{aligned}
 & \left\{ \frac{UC2\alpha_3}{\cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) - \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \frac{\lambda_1\phi}{\Omega_0}\right) \right| \right. \\
 & + \frac{UC2\alpha_4}{\cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) - \sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \frac{\lambda_2\phi}{\Omega_0}\right) \right| \\
 & - UC1 \left| \alpha_1 \tanh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) + \alpha_2 \tanh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) \right| \\
 & \left. - UC1b_{11} \right\} r_{11} < 0
 \end{aligned}
 \tag{5.46}$$

In order to ascertain that the system's response , and consequently the periodic inputs to nonlinearities, conform with the periodicity requirements (eqns. 5.41 - 4.44), it is necessary to set eqns. 5.19 and 5.20 into closed forms for easy solution:

$$\begin{aligned}
w(t) = & - \frac{UC1\alpha_1}{\lambda_1 \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1 t\right) \right. \\
& + \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1 t\right) \left| \right. \\
& - \frac{UC1\alpha_2}{\lambda_2 \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2 t\right) \right. \\
& + \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2 t\right) \left| \right. \\
& - \frac{UC2\alpha_3}{\lambda_1 \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1 \left(t + \frac{\phi}{\Omega_0}\right)\right) \right. \\
& + \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1 \left(t + \frac{\phi}{\Omega_0}\right)\right) \left| \right. \\
& - \frac{UC2\alpha_4}{\lambda_2 \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \left(\sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2 \left(t + \frac{\phi}{\Omega_0}\right)\right) \right. \right. \\
& + \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2 \left(t + \frac{\phi}{\Omega_0}\right)\right) \left. \right| \quad 5.47
\end{aligned}$$

for $0 < t < \frac{\pi}{\Omega_0}$

$$\begin{aligned}
y(\tau) = & - \frac{UC2\alpha_7}{\lambda_1 \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1\left(\tau + \frac{\phi}{\Omega_0}\right)\right) \right. \\
& + \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1\left(\tau + \frac{\phi}{\Omega_0}\right)\right) \left| \right. \\
& - \frac{UC2\alpha_8}{\lambda_2 \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2\left(\tau + \frac{\phi}{\Omega_0}\right)\right) \right. \\
& + \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2\left(\tau + \frac{\phi}{\Omega_0}\right)\right) \left. \right] \\
& - \frac{UC1\alpha_5}{\lambda_1 \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1\tau\right) \right. \\
& + \cosh\left(\frac{\pi\lambda_1}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_1}{2\Omega_0} - \lambda_1\tau\right) \left| \right. \\
& - \frac{UC1\alpha_6}{\lambda_2 \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right)} \left| \sinh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2\tau\right) \right. \\
& + \cosh\left(\frac{\pi\lambda_2}{2\Omega_0}\right) - \cosh\left(\frac{\pi\lambda_2}{2\Omega_0} - \lambda_2\tau\right) \left| \right.
\end{aligned}$$

If the oscillations exist for the feed back system of fig. 3.4, then real solutions of eqns. 5.43 and 5.45 are obtainable. These solutions are the frequency ω_0 and phase shift ϕ which then must be tested in the inequalities (eqns. 5.44 and 5.46). The solutions are substituted into eqns. 5.47 and 5.48 from which the time responses of the two loops are obtained. From such responses, the continuity conditions can be checked (eqn. 5.24), and additionally, the amplitudes A_1 and A_2 of the temperature and the concentration output wave forms respectively are easily obtained. An application of this procedure and the computer solutions are all given in Chapter 6.

5.2 GENERALIZATION OF TSYPKIN'S ANALYSIS FOR $n \times n$ COUPLED PROCESSES

With only a few modifications, the results presented in the previous section for the 2×2 process can be generalized for any $n \times n$ coupled system. These extensions are given here only for the case where all transfer functions describing the process are of order one or greater, that is, $\theta(G(s)) \geq 1$. Recall that under these conditions, $\hat{G}_{ij}(\infty) = 0$ so that $\hat{G}_{ij}(s) = G_{ij}(s)$. The needed modifications call for an accounting for the phase shifts between all inputs to the relay controllers. As defined below, this is done again with respect to the input to a relay denoted as relay "1".

Retaining all the assumptions made in section 5.1, we proceed as follows: the system contains n closed loops each of which is controlled with a symmetric relay with hysteresis so that there are $\eta_1, \eta_2, \dots, \eta_n$ such relays. Let $t = 0$, as before, when the first relay switches to its positive drive level, while the remaining relays switch to their respective positive drive levels at times $t_{\phi_2}, t_{\phi_3}, \dots, t_{\phi_n}$. Recall that t_{ϕ_i} can be either positive or negative.

The phase shifts for the periodic output from the relay in each loop with respect to the output from the first relay are defined as

$$t_{\phi_i} = -\frac{\phi_i}{\Omega_0} \quad i = 2, \dots, n \quad 5.49$$

$$t_{\phi_1} = 0 \quad 5.50$$

With the shifts in phase among the periodic wave forms of all the system's closed loops so defined, we can now proceed as previously to determine the loops' periodic outputs. Thus for i -th loop, the output signal is given as a double summation

$$y_i(t) = \sum_{J=1}^n \frac{4U_J}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{iJ} e^{jk \Omega_0 (t - t_{\phi_J})} \right| \quad 5.51$$

$$i = 1, 2, \dots, n$$

and

$$\dot{y}_i(t) = \sum_{J=1}^n \frac{4U_J \Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{iJ}(jk \Omega_o) e^{jk \Omega_o(t - t_{\phi J})} \right| \quad 5.52$$

$$i = 1, 2, \dots, n$$

The switching times for the i -th loop's relay is $(\frac{\pi}{\Omega_o} + t_{\phi i})^-$. At this time, eqn. 5.51 and 5.52 become

$$y_i(\frac{\pi}{\Omega_o} + t_{\phi i})^- = - \sum_{J=1}^n \frac{4U_J}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left| \hat{G}_{iJ}(jk \Omega_o) e^{jk(t_{\phi i} - t_{\phi J})} \right| \quad 5.53$$

$$i = 1, 2, \dots, n$$

$$\dot{y}_i(\frac{\pi}{\Omega_o} + t_{\phi i})^- = - \sum_{J=1}^n \left\{ \frac{U_J \Omega_o}{\Pi} \sum_{k=1}^{\infty} \operatorname{Re} \left| \hat{G}_{iJ}(jk \Omega_o) e^{jk(t_{\phi i} - t_{\phi J})} \right| \right\} \quad 5.54$$

$$+ u_i \lim_{s \rightarrow \infty} s G_{ii}(s)$$

These equations can then be cast in closed forms by using tables 4.1 and 4.2. Then the n sets of switching conditions for the process can be stated by applying the switching conditions as stated in eqn. (5.23) where it is recognized that there are now n relays. The solution of these eqns. yields the frequency Ω_o and the phase shifts ϕ_i . If no

valid solution is found (note that α_0 must be positive), no oscillation is predicted. The continuity conditions are then checked by writing equations 5.51 and 5.52 in closed forms via tables 4.1 and 4.2. These conditions are stated in eqns. (5.24) where, again, it is recognized that there are n relays. In checking the continuity conditions, the wave forms of the all outputs are found. Thus the amplitudes A_i ($i = 1, 2, \dots, n$) of all outputs can be determined.

5.3 POSSIBLE SWITCHING MODES OF THE CONTROL SYSTEM'S RELAYS

Analytical treatments of a multivariable process cannot be complete without some consideration given to every switching possibility which the controller of every loop can exhibit. In the case of the 2x2 system used in this study and the assumptions imposed (see section 5.2), the following switching activities of the two controllers (see figure 3.4) are possible:

- (1) Both relays switch.
- (2) Only $\mathcal{N}_2$ switches while $\mathcal{N}_1$ stays at either its positive or negative drive level.
- (3) Only $\mathcal{N}_1$ switches while $\mathcal{N}_2$ stays at either its positive or negative drive level.
- (4) Neither relay switches.

The switching of both relays are probably possible if the values of the control bandwidths of the relays are chosen within some operation intervals. For this reason, some preferential or selective switching of only one relay can be effected if that relay's bandwidth is chosen within the interval while that of the other controller is selected outside the interval.

Our immediate concern is to show what effects the above loop activities can have on overall process performance, and what modifications are necessary to adopt the analysis detailed in this study for the new control situation.

Of the switching possibilities listed above, only number one (the case where the two relays switch) is amenable to the analytical treatment detailed in section 5.1 for the MI/MO control. This approach, it must be emphasized, is not applicable to the loop operation (2) and (3) that are given above, as the process at these conditions is no longer an MI/MO. Consider, for example, case (2) in which $\mathcal{N}_2$ switches while $\mathcal{N}_1$ stays at its positive drive level. Then $u_1(t)$ in figure 3.4 is now a constant term $\hat{u}_1$ which when conveyed across $\hat{G}_2(s)$, yields $\bar{y}_2$ after sufficient time. $\bar{y}_2$ is, therefore, a constant load disturbance to the temperature loop (see figure 5.2). The overbar term is used to emphasize that $\bar{y}_2$ is a constant value. That is,

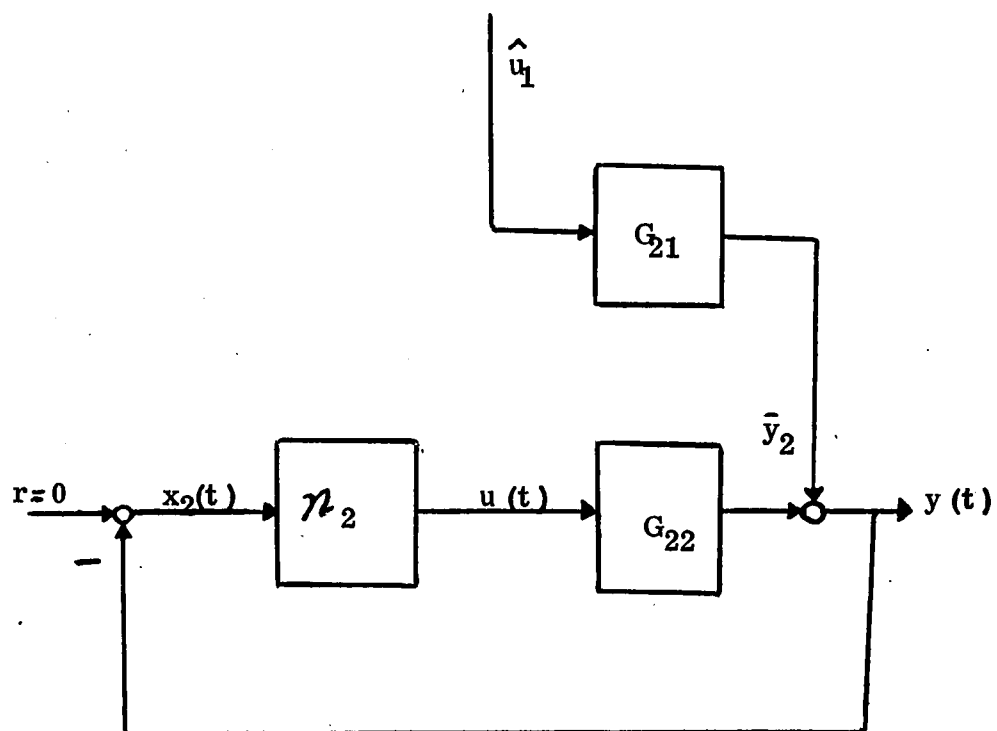


Figure 5.2: SI/SO control system with a constant load y_0 resulting from 2×2 MI/MO system of fig. 3.4 when only γ_2 can switch

$$\bar{y}_2 = \hat{u}_1 \hat{G}_{21}$$

5.55

where G_{21} is now a steady state transfer function. Therefore, the output signal of the temperature loop is given as

$$y(t) = \frac{4UC2}{\Pi} \sum_{k=1}^{\infty} \frac{1}{k} \operatorname{Im} \left[\hat{G}_{22}(jk\Omega_0) e^{jk\Omega_0 t} \right] + u_2(t) G_{22}(\infty) + \bar{y}_2 \quad 5.56$$

Except for the presence of the constant $\bar{y}_2$, eqn. 5.56 is the same as eqn. 4.54. Hence, the temperature control loop is an independent SI/SO system with a constant load of $\bar{y}_2$. The analytical technique developed in chapter four can be applied to this system with one modification: Recall that in defining the deviation variables (eqn. 3.9), the reference condition was an unstable steady state, that is, $u_1(t)=0$ or $\bar{y}_1=0$. Hence, the present system whose output signal is given in eqn. 5.56 cannot be used in that analysis without first "removing" the load $\bar{y}_2$. Such a removal can be easily accomplished if the process deviation variables are redefined such that $\bar{y}_2$, as a deviation variable, is identically zero. This is done by taking the reference condition about which the linearization is made as $(w^*, y, f, u) = (w^*, y^* + \bar{y}_2, f^*, u^*)$. That is, the deviation variables are

$$y = f - (f^* + \bar{y}_2)$$

$$w = (w - w^*)$$

$$\begin{aligned} u &= (\mathcal{U} - \mathcal{U}^*) \\ f &= (\mathcal{F} - \mathcal{F}^*) \end{aligned}$$

With the new deviation variables so defined, the SI/SO analysis can be applied to this process model. The same strategy is followed if $u_1(t) = -u_1$, noting that $\bar{y}_c$ will be a different constant.

An identical approach can be adopted for the treatment of the concentration loop should the switching activity number (3) take place (that is constant level of $\mathcal{N}_2$ while $\mathcal{N}_1$ switches). A new reference condition must be specified accounting for $\bar{x}_2$ in defining appropriate deviation variables before the resulting system can be considered as a candidate for the SI/SO analysis of Chapter 4.

The case of no switching as in number (4) indicates that no oscillation exists in the system. For the application of multi-relay control for an unstable process control system failure is indicated.

CHAPTER 6

SI/SO AND MI/MO SYSTEMS CONTROL RESULTS FOR THE EXAMPLE REACTOR

The new control theory proposed in this thesis for predicting oscillatory response of coupled MI/MO systems will now be applied to the example reactor modelled in Chapter 3 . The theory can be applied for control of any MI/MO process provided that all the interaction paths are properly accounted for in the mathematics involved, and that all the system's outputs are symmetric and oscillate at the same frequency. To establish a reasonable basis for comparison, the MI/MO system's results obtained from the new theory will be compared with those gotten by simulating the nonlinear process. Also presented in this chapter are the results of SI/SO system regulated in accordance with the control policy given in Chapter 4 . These results are valid if only one control loop is used or the system, if coupled, is suitably decoupled. The nonlinear control designs and the actual implementation of both the old and the new control policies are presented in sections 6.1 and 6.2 respectively. The control results as well as discussion and conclusions for these two control situations are given in the respective sections.

6.1 SI/SO CONTROL

It was shown in Chapter 4 how a relay with hysteresis could be used to effect the system's motion depicted in eqns. 4.30 and 4.31 by changing the control levels in compliance with the conditions given in eqns. 4.34-4.37. This section begins with the actual design of the controller to ensure its suitability for effecting the desirable system motion.

Design of Concentration Controlling Relay

It is possible to vary f in eqns. 4.8 and 4.16 such that at some maximum value (f_{max}) and minimum value (f_{min}), R_g^C lines are just tangent to the R_r^C curves as shown in figure 6.1. The corresponding concentration at these maximum and minimum flow rates are denoted as w^{ut} and w^{lc} respectively. Note that it is from these values in concentration that the maximum and the minimum (w_{max} , w_{min}) errors in concentration from the reference condition are defined. If flow rates outside the range of f_{max} and f_{min} are used, a unique steady state solution will result. w_{max} and w_{min} are used to denote the steady state minimum and maximum deviations in the concentration from w^* at the tangent points resulting from the flow rates f_{min} and f_{max} respectively. For the concentration deviations inside the range w_{min} to w_{max} , a flow rate can be found such that the chosen concentration will be an intermediate unstable steady state. Concentration

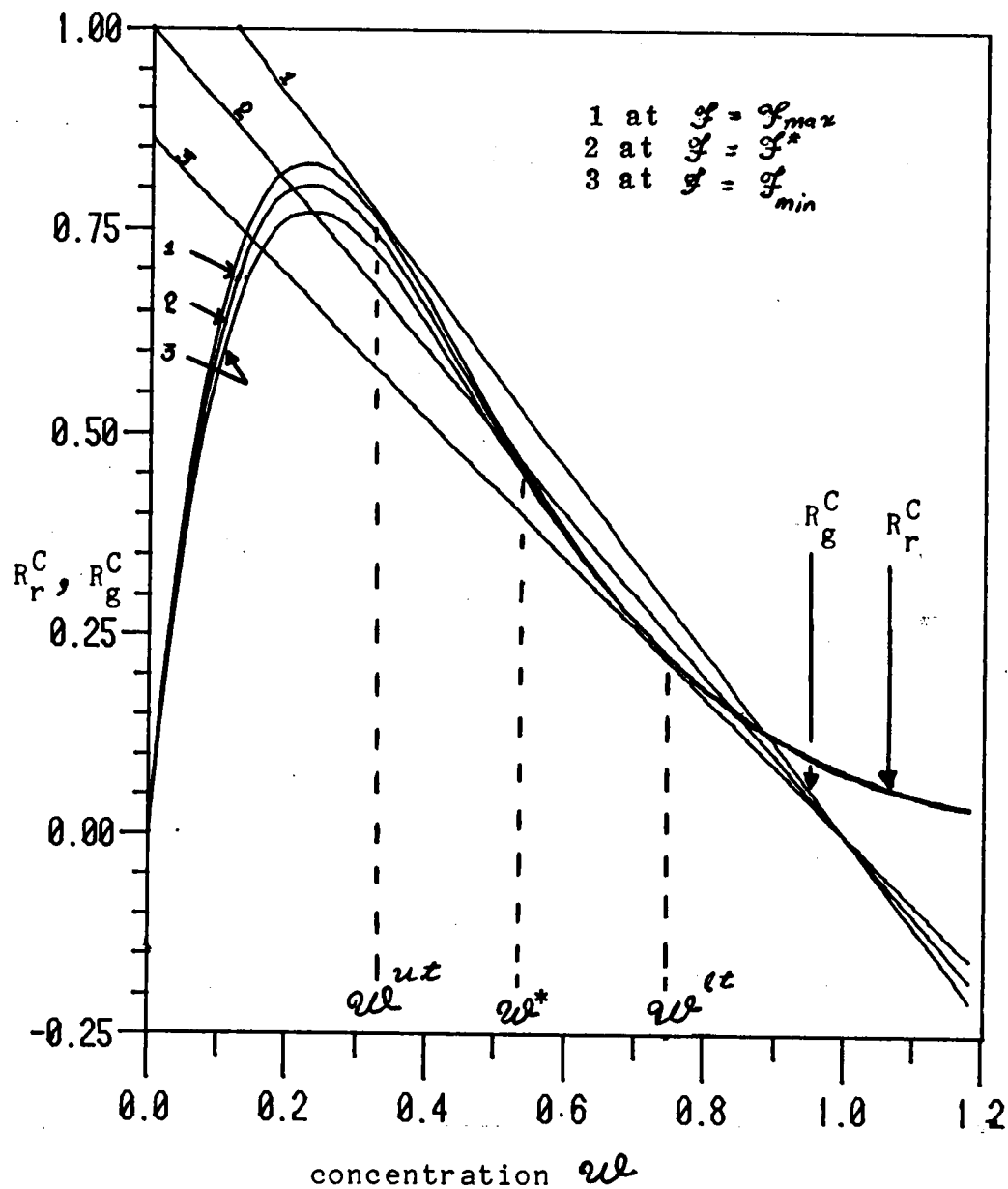


Figure 6.1: van Heerden diagrams for design strategy of SI/SO relay for concentration control
 ($w_{max} = w^{lt} - w^*$, $w_{min} = w^{ut} - w^*$)

deviations outside the range $w_{e_{max}}$ to $w_{e_{min}}$ cannot be made an intermediate unstable steady state by merely changing the flow rate. $w_{e_{max}}$ and $w_{e_{min}}$, therefore, establish the minimum and the maximum allowable error from w^* at which an intermediate unstable steady state can be found by adjusting the flow rate.

This analysis is not necessary in establishing the parameters defining a relay with hysteresis (see Chapter 4). However, if these limits are observed, multiple steady states will exist in the system for both relay drive levels, and is conceptually connoted as a pushing control policy by Bruns (34). These limits will be used in this section.

As the linearized reactor model will actually be used to calculate the flow rate or relay's drive levels needed to establish unstable steady states at $\pm w_e$, some allowable deviation error from w^* , $w_{e_{max}}$ is defined according to

$$w_{e_{max}} = \min[|w_{e_{max}}|, |w_{e_{min}}|] \quad 6.1$$

Note that $w_{e_{max}}$ and $w_{e_{min}}$ have been used to describe the deviations found from analysis of the nonlinear reactor and $w_{e_{max}}$ defined by eqn. 6.1 will be used with the linearized reactor equations.

Here it should be noted that a symmetric relay will result due to the unstable steady states ($\pm w_e$) being chosen symmetrically around w^* and using the linearized model to calculate the required flow rate ($\pm UC1$).

To establish the maximum drive levels $\pm UC1_{max}$ according to the linearized model and $w_{e_{max}}$, the linear differential equations 3.7 and 3.8 are used

$$\frac{dw}{d\tau} = a_{11} w + a_{12} y + b_{11} f + b_{12} u \quad 3.7$$

$$\frac{dy}{d\tau} = a_{21} w + a_{22} y + b_{21} f + b_{22} u \quad 3.8$$

At steady state condition, eqn. 3.7 is solved for f

$$f = UC1 = -\left(\frac{a_{11}w + a_{12}y + b_{12}u}{b_{11}}\right) \quad 6.2$$

y is obtained from eqn. 3.8 (with $\dot{y} = 0$)

$$y = -\left(\frac{a_{21}w + b_{21}f + b_{22}u}{a_{22}}\right) \quad 6.3$$

Upon substituting y into eqn. 6.2 and evaluating at $w = -w_{e_{max}}$, we get

$$f_{\max} = UC1_{\max} = \left[\frac{(a_{11}a_{22} - a_{12}a_{21})w}{(a_{12}b_{21} - a_{22}b_{11})} \right] w = -w_{e_{\max}} \quad 6.4$$

Thus these limits will be imposed on the drive levels of the concentration relay. To complete the specification of the relay, the actual drive levels used and the switching concentrations (setting the bandwidth of the relay) need to be chosen. Parameters used to define these are treated below.

Design of Temperature Controlling Relay

Repeating the above procedure for the temperature controlling relay, a $y_{e_{\max}}$ is determined to use in the linearized model

$$y_{e_{\max}} = \min[|y_{e_{\max}}|, |y_{e_{\min}}|] \quad 6.5$$

and the maximum drive levels +UC2 are determined to be

$$u_{\max} = UC2_{\max} = \left[\frac{(a_{12}a_{21} - a_{11}a_{22})y}{(a_{11}b_{22} - a_{12}b_{12})} \right] y = -y_{e_{\max}} \quad 6.6$$

$UC1_{\max}$ and $UC2_{\max}$ are the maximum control levels to which the reactor can be subjected and still can exhibit multiple steady states (in terms of the linearized model system) when the process is completely decoupled or when only

one of the control loops is implemented. $w_{e_{max}}$ and $y_{e_{max}}$ are the maximum deviations in the concentration and temperature respectively under these conditions. Normally, the reactor is kept at less extreme conditions as dictated by the control requirements, that is, the drive levels are set less than $UC1_{max}$ or less than $UC2_{max}$, and the bandwidths are inside $\pm w_{e_{max}}$ or $\pm y_{e_{max}}$ respectively. For this reason, it is appropriate to define the following control parameters:

$$\delta_1 = \frac{w}{w_{e_{max}}} \quad 6.7$$

$$\delta_2 = \frac{y}{y_{e_{max}}} \quad 6.8$$

$$\sigma_1 = \frac{UC1}{UC1_{max}} \quad 6.9$$

$$\sigma_2 = \frac{UC2}{UC2_{max}} \quad 6.10$$

Therefore, by using σ_1 and δ_1 for a control loop, where these values are limited to the range 0 to 1 to always assure that multiple steady states exist at all control levels, the relay is completely specified and the characteristics of the expected oscillation are determined with the analysis of chapter four. Thus these equations allow a design study to be made for the proposed control scheme.

Control Procedure

The unstable reactor whose dynamics is described in eqns. 3.5 and 3.6 was directly simulated on a DEC System-10 digital computer. This was done by integrating these nonlinear differential equations with a Fourth-order Runge-Kutta integration package with variable step size (106). The value of the manipulated variable was set by the appropriate nonlinear control policy. The two control loops described above were treated separately as SI/SO systems. The integration was discontinued once the loop's oscillatory response reached steady state. Such time responses are shown in figures 6.2 and 6.3 for the temperature control system and in figure 6.4 for the concentration control SI/SO system. The computer algorithm for the simulation procedure is given in the appendix.

Next, eqn. 4.65 and 4.66 or 4.68 and 4.69 were analytically solved on a computer to obtain the respective frequencies of Ω_{01} or Ω_{02} depending on the control loop being studied. The Interval-Halving technique (see appendix) was employed as there was one unknown in the respective eqns. 4.65 or 4.68. The respective frequencies were then submitted into eqns. 4.67 or 4.70, which were then solved analytically to obtain the concentration and the temperature amplitudes of A_1 and A_2 respectively. Both the simulation and the analytical results are compared in table 6.1 for concentration and tables 6.2 for temperature ,

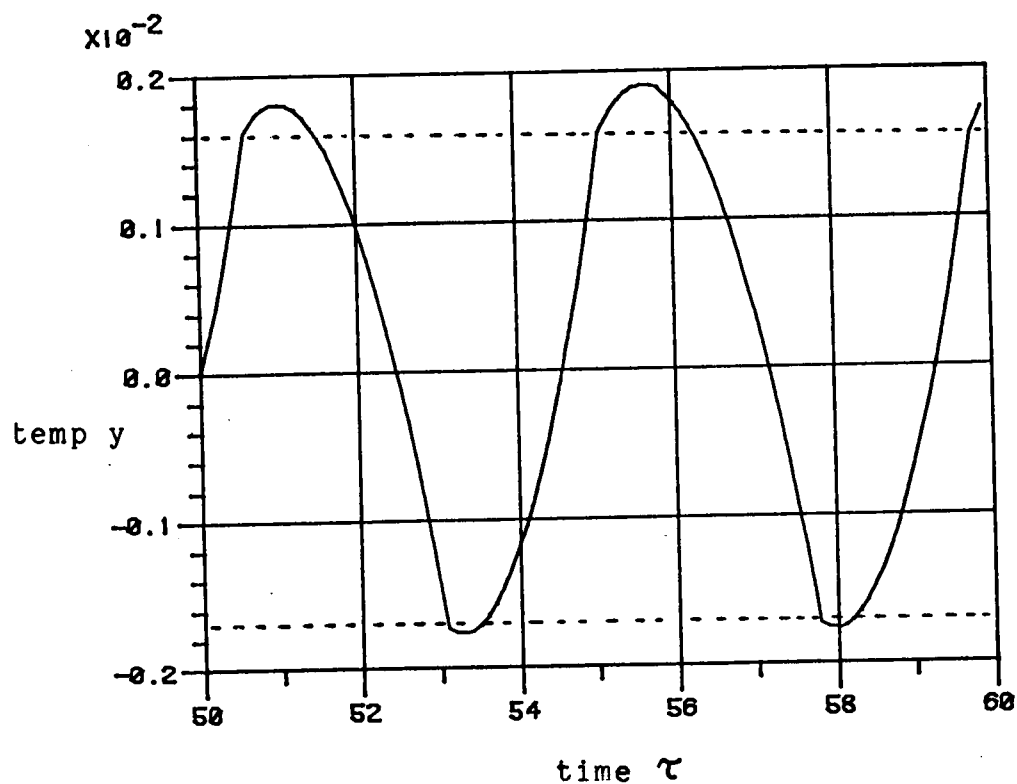


Figure 6.2: SI/SO output of temperature system of figure 3.3 resulting from integrating equations 3.5 and 3.6 for case of $\sigma_2 = 1.00$, $\delta_2 = 0.15$

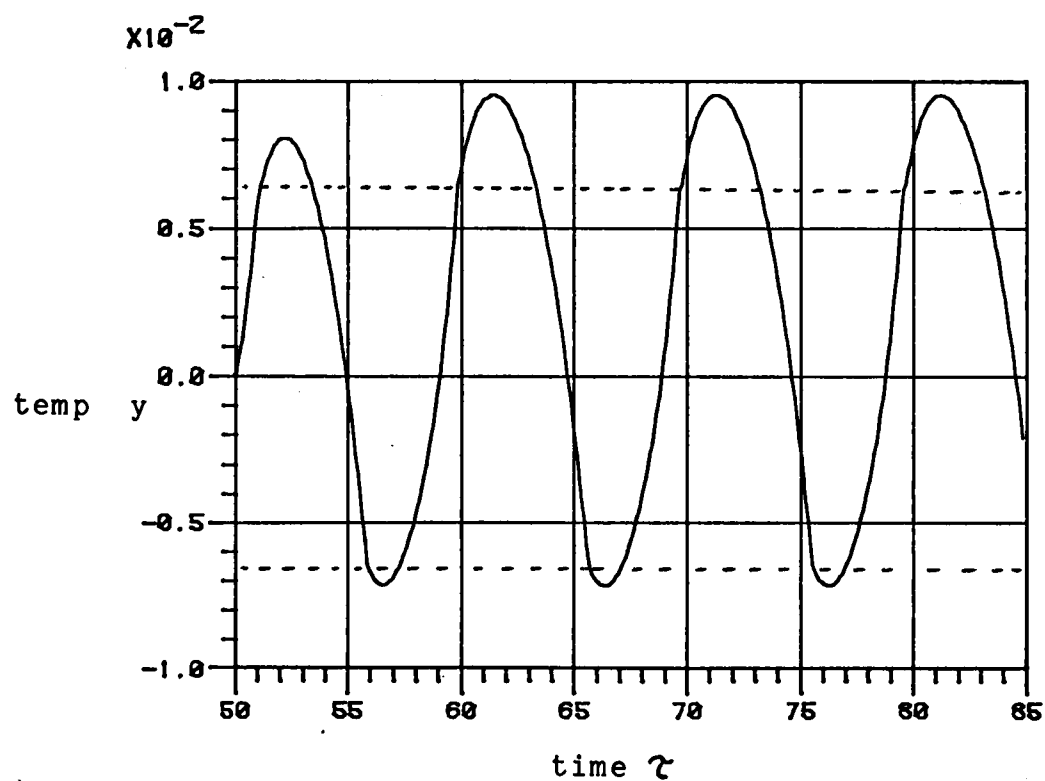


Figure 6.3: SI/SO output of temperature system of figure 3.3 resulting from integrating eqns. 3.5 and 3.6 for case of $\sigma_2 = 0.90$ and $\delta_2 = 0.60$

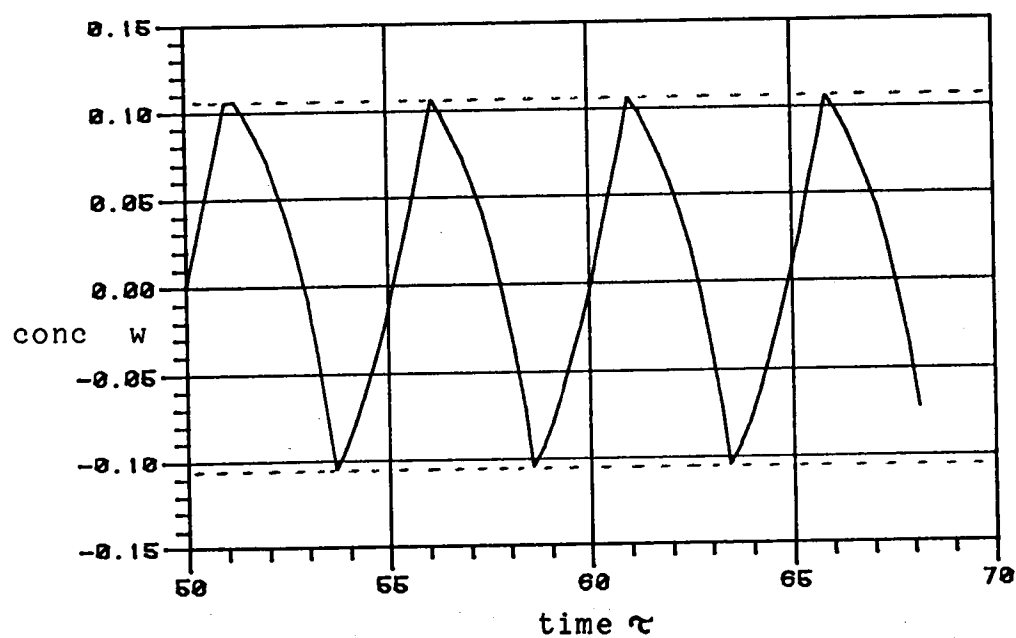


Figure 6.4: SI/SO of concentration system of figure 3.2 resulting from integrating Eqns . 3.5 and 3.6 for case of $\sigma_1 = 0.95$ and $\delta_1 = 0.75$

and these results will be discussed shortly.

Error Evaluation

A quick and reasonable comparison of the simulation and analysis results were carried out in terms of errors between the predicted and exact (simulated) values of frequency and amplitude. Because the Tsytkin's approach treats the relay exactly (by virtue of embodiment of the infinite series into its formalisms), any error which showed up in such comparisons were attributable to the linearization of the system (see Chapter 3).

The errors in the amplitudes and the frequencies were evaluated with respect to the simulation results as follows:

$$e_{A1} = \frac{A_1^{\text{Sim}} - A_1^{\text{tsyp}}}{A_1^{\text{Sim}}} \quad 6.11$$

$$e_{A2} = \frac{A_2^{\text{Sim}} - A_2^{\text{tsyp}}}{A_2^{\text{Sim}}} \quad 6.12$$

$$e_{\Omega 01} = \frac{\Omega_{01}^{\text{Sim}} - \Omega_{01}^{\text{tsyp}}}{\Omega_{01}^{\text{Sim}}} \quad 6.13$$

$$e_{\Omega 02} = \frac{\Omega_{02}^{\text{Sim}} - \Omega_{02}^{\text{tsyp}}}{\Omega_{02}^{\text{Sim}}} \quad 6.14$$

where A_i^{Sim} was evaluated by taking the arithmetic average of the absolute values of the maximum and minimum amplitudes of the periodic output when steady state is reached. These amplitudes will be different in the simulation due to the nonlinearity of the process.

Results and Discussion

The CSTR in this study was found to exhibit three steady states for the set of parametric values given in table 3.1 (page 60). In terms of the manipulated variables being at their reference values $(u, f) = (u^*, f^*)$, the steady states (in dimensionless quantities, see figure 4.1, page 71 or figure 4.2, page 73) were $L = (w, y) = (0.8508, 1.0297)$, $M = (0.5150, 1.0770)$, and $H = (0.2022, 1.1233)$ which corresponded to $L = (w, y) = (0.3358, -0.0473)$, $M = (0.0000, 0.0000)$, and $H = (-0.3128, 0.0463)$ in deviation variables. Moreover, the maximum allowable errors in concentration and temperature were found to be $w_{\text{max}} = 0.1850$ and $y_{\text{max}} = 0.0133$. These corresponded to errors of 0.000925 gm-mole/cc in concentration and 3.99 K in temperature from the unstable steady state. These errors represented 36% and 1.2% maximum deviations from the reference condition values C^* and T^* respectively. The maximum control levels according to eqns. 6.4 and 6.6 were also determined. These were $UC1_{\text{max}} = 0.1611$ and $UC2_{\text{max}} = 0.0216$ which corresponded to the largest allowable change of 1.611 cc/sec in the flow rate,

and 6.48 K in the coolant temperature from the reference unstable steady state values. The maximum errors corresponded to 16.1% and 2.1% respectively from the reference values F^* and T_C^* .

With the illustrative system treated as an SI/SO process for the independent control of the concentration and temperature in the fashion outlined in Chapter 4, the control results in tables 6.1 and 6.2 were obtained. The errors arising in the comparison of the analysis and the simulation results were due to the linearization process of the nonlinear reactor. It would, therefore, be anticipated that the larger the deviations of the concentration or the temperature from the reference condition, the more the departure of the process from linearity, and, consequently, the larger the errors. A glance at these tables shows this to be the case, as well as the fact that the errors introduced in linearization was quite small. This means that the Tsytkin's analysis prediction for these SI/SO processes was almost exact.

6.2 MULTIVARIABLE SYSTEM CONTROL

Now, suppose that both the concentration u and the temperature y are to be simultaneously regulated by manipulating the flow rate f and the coolant temperature u respectively. This is now a 2x2 system which is to be controlled according to the nonlinear feed back scheme depicted in figure 3.4 (pp. 51) and whose oscillatory

Table 6.2: Results of Simulation of Nonlinear Uncoupled Reactor
and from Tsytkin's Analysis of the Linearized Reactor
Model for Temperature as the System's Output

					$y_{e\max} = 0.0133$		$UC_2\max = 0.0216$		
					Tsytkin		Simulation		Error
σ_2	UC_2	δ_2	d_2	$e\Omega_{02}$	d_2	Ω_{02}	d_2	$e\Omega_{02}$	e_{A2}
1.00	0.0216	0.49	0.0065	0.6686	0.0116	0.6473	0.0108	- 0.03	0.07
1.00	0.0216	0.32	0.0043	1.1108	0.0072	1.0926	0.0069	- 0.02	- 0.04
1.00	0.0216	0.13	0.0017	2.7187	0.0027	2.6971	0.0027	0.00	0.00

characteristics are to be predicted according to the new theory developed in Chapter 5.

Design of the Concentration Control Loop Relay

Suppose a control restriction is placed on the process that a maximum allowable change of $\pm 25\%$ in the flow rate (from the value of a reference condition) can be made in adjusting $\mathcal{F}$ for control purposes. Restrictions of this type would depend upon the specific process being studied.

Once again, restrictions will be placed on the manipulated variables such that at any combination of drive levels, there are now two relays, multiple steady states may exist in the process. This might result in a class of controls termed "pushing control." This restriction is not a limitation on the analysis equations. Since the maximum deviation in flow rate has been fixed as given above, the limits on coolant temperature changes which can be made and still be able to retain multiplicity of steady states can now be obtained.

In using eqns. 4.8 and 4.16 to obtain the van Heerden diagrams in chapter 4, $\mathcal{F}$ was fixed at its reference feed value $\mathcal{F}^*$. But in the present control situation where changes in both the flow rate and the coolant temperature occur, these changes must be accounted for in establishing limits on T_C . Eqns. 4.8 and 4.16 are rewritten here for convenience

$$R_g^C(\mathcal{F}, \mathcal{U}) = \mathcal{F}(1 - \mathcal{U}) \quad 6.15$$

$$R_r^C(\mathcal{F}, \mathcal{U}, \mathcal{U}) = Da \mathcal{U} \exp\left[\frac{-\mathcal{F}(\mathcal{F} + \mathcal{J})}{\beta \mathcal{F}(1 - \mathcal{U}) + \mathcal{F} + \mathcal{J}\mathcal{U}}\right] \quad 6.16$$

Again, R_g^C is a linear function of $\mathcal{U}$ (when $\mathcal{F}$ is fixed) with slope of $-\mathcal{F}$ and with the intercept of $+\mathcal{F}$ while R_r^C is a nonlinear function. The procedure for obtaining the maximum allowable errors in the concentration is basically the same as that used in the previous section for the SI/SO cases: with $\mathcal{F} = \pm 0.25\mathcal{F}^* + \mathcal{F}^*$ submitted into eqns. 6.15 and 6.16, $\mathcal{U}$ in eqn. 6.16 is varied until the R_g^C line is just tangent to the R_r^C curve. Both the upper and the lower tangencies are obtainable with appropriate variations in $\mathcal{U}$. From eqn. 6.16, it is obvious that increase in $\mathcal{U}$ is required to obtain a lower tangency point while an upper tangency point is possible with decrease in $\mathcal{U}$.

Interval-halving computer technique was employed for determining these tangent points. The points were evaluated at the two levels of the flow rate: $\mathcal{F}^+ = \mathcal{F}^* + 0.25\mathcal{F}^*$, and $\mathcal{F}^- = -0.25\mathcal{F}^* + \mathcal{F}^*$. Thus for this system, four tangency points (instead of two in the case of SI/SO system) given in figures 6.5 and 6.6 were obtainable. These points are

$$\mathcal{U}_{\max}^{\mathcal{U}t} \text{ at } \mathcal{F} = \mathcal{F}^+ \text{ with } \mathcal{U} = \mathcal{U}_{\max}^+$$

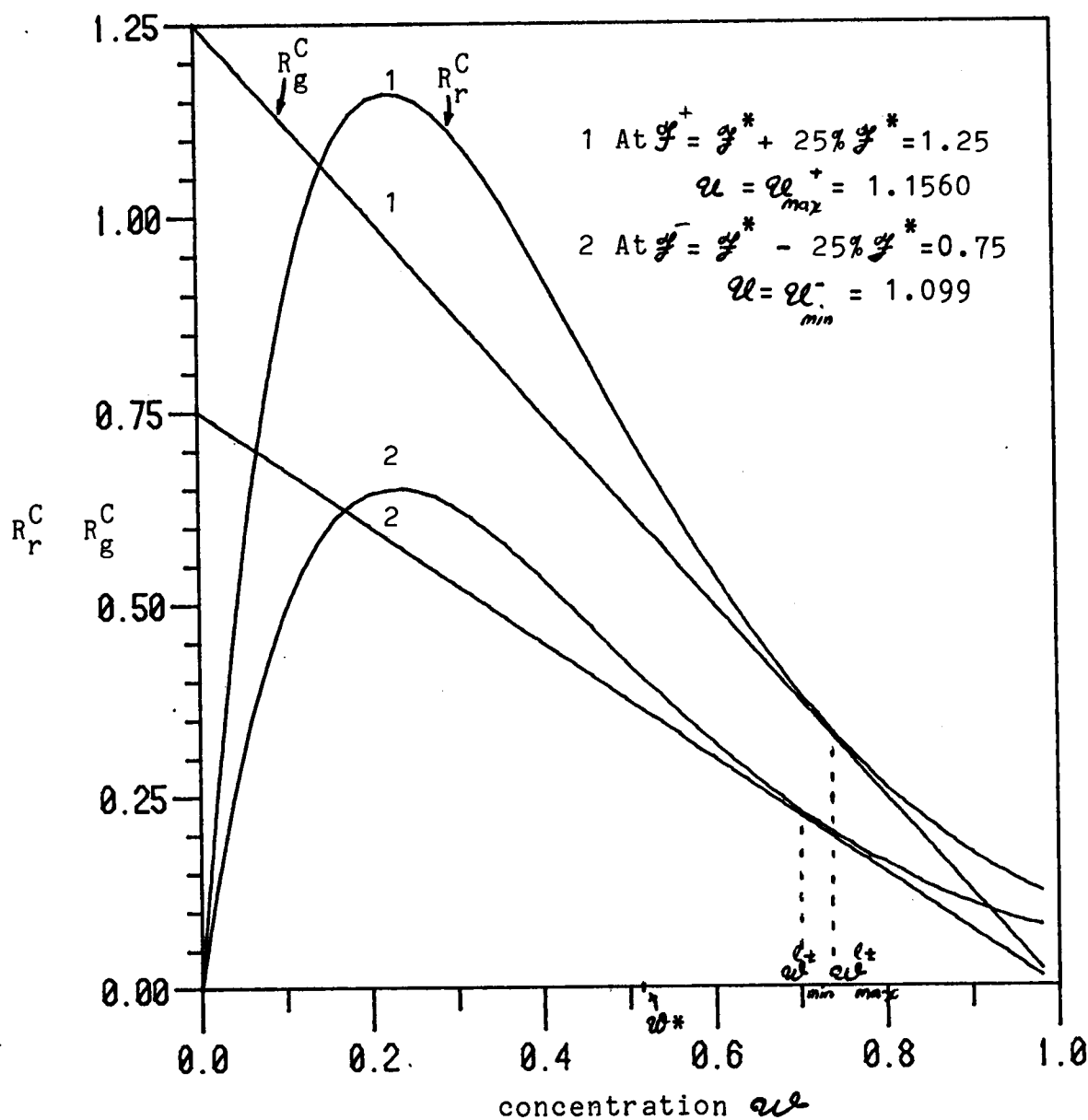


Figure 6.5: van Heerden diagrams for MI/MO concentration loop relay design for case of $F = F^* + 25\%F^*$
 $w_{\max}^L = w_{\max}^+ - w^{*-}$, $w_{\min}^L = w_{\min}^+ - w^*$

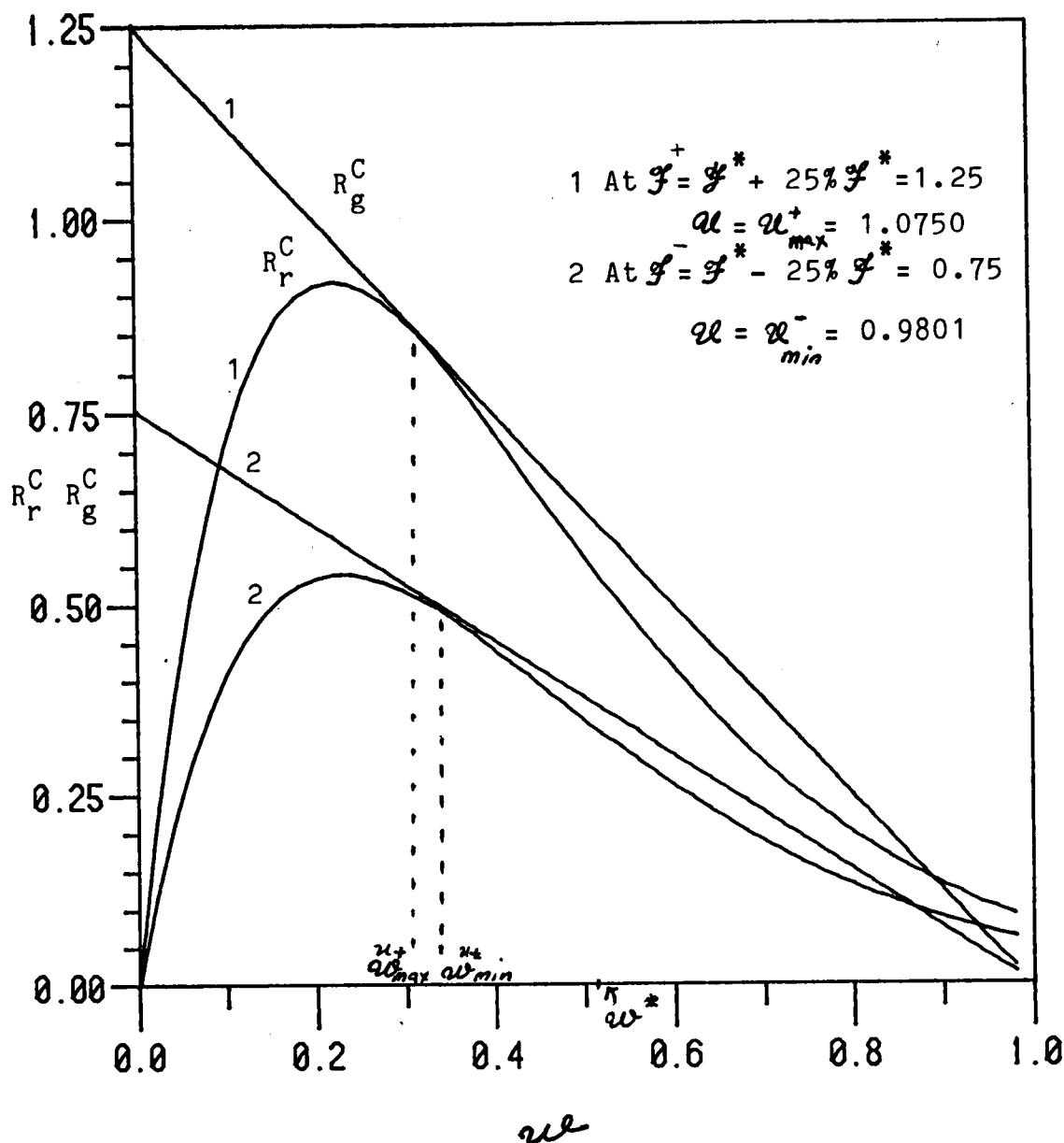


Figure 6.6: van Heerden diagrams for MI/MO concentration loop*relay design for case of $F = F^* + 25\%F^*$
 $u_{min}^+ = u_{min}^+ - w^*$, $u_{max}^+ = u_{max}^+ - w^*$

$$w_{min}^{ut} \text{ at } \mathcal{F} = \mathcal{F}^- \text{ with } \mathcal{U} = \mathcal{U}_{min}^-$$

$$w_{max}^{lt} \text{ at } \mathcal{F} = \mathcal{F}^+ \text{ with } \mathcal{U} = \mathcal{U}^+$$

$$w_{min}^{lt} \text{ at } \mathcal{F} = \mathcal{F}^- \text{ with } \mathcal{U} = \mathcal{U}_{min}^-$$

where the superscript "ut" indicates upper tangency point, and "lt" a lower tangency point, "+" a positive change in the flow rate, "-" a negative change in the flow rate. The subscript "max" indicates largest deviation from unstable steady state condition, and "min" implies least deviation from unstable steady state condition. The limits provided by these points define two closed intervals

$$N_C^+ = w_{max}^{ut} \leq w \leq w_{max}^{lt} \quad 6.17$$

$$N_C^- = w_{min}^{ut} \leq w \leq w_{min}^{lt} \quad 6.18$$

The intervals so gotten, according to figures 6.5 and 6.6, are such that

$$w^* \in N_C^+ \cap N_C^- \quad 6.19$$

Such intersections can also be determined for other variables. Thus for the temperature $\mathcal{Y}$ and the coolant temperature $\mathcal{U}$, the respective intersections are given as

$$\mathcal{Y}^* \in N_T^+ \cap N_T^- \quad 6.20A$$

$$\mathcal{U}^* \in N_{T_C}^+ \cap N_{T_C}^- \quad 6.21A$$

where, again, * signify the reference condition values, and the subscripts T and T_C represent the temperature and the coolant temperature respectively. It is important to note that, for any choice of maximum change in the flow rate (such as $\pm 25\%$ used presently), the conditions expressed by eqns. 6.19-6.21 must all be satisfied to ensure the resulting control design results in an application of the "pushing control" strategy. Note also that the intersections (eqns. 6.18-6.21) may be null. For this system with the parameters of table 3.1 and the 25% maximum allowable change in the flow rate, the interval N_C^+ and N_C^- envelop the unstable steady state (see figures 6.5 and 6.6).

A close study of figures 6.5 and 6.6 reveals that, for the maximum change of $\pm 25\%$ in the flow rate, $N_C^+ = [0.3194, 0.7300]$, and $N_C^- = [0.3300, 0.7091]$. Similarly, $N_{T_C}^+ = [1.0750, 1.1560]$, $N_{T_C}^- = [0.9801, 1.0990]$. From eqn. 3.6 (page 44), setting $\dot{y} = 0$ to obtain the steady state temperature given the steady state concentration, the corresponding intervals for the temperature are easily determined to be $N_T^+ = [1.0531, 1.0948]$, $N_T^- = [1.0411, 1.0843]$. The respective intersections according to eqns. 6.19A-6.21A are

$$N_C^+ \cap N_C^- = [0.3300, 0.7091]$$

$$N_{Tc}^+ \cap N_{Tc}^- = [1.0750, 1.0990]$$

and

$$N_T^+ \cap N_T^- = [1.0531, 1.084]$$

Recalling that $y^* = 1.0770$, $w^* = 0.5150$ and that $u^* = 1.048$, it is obvious that

$$y^* \in N_T^+ \cap N_T^- \quad 6.20B$$

$$u^* \notin N_{Tc}^+ \cap N_{Tc}^- \quad 6.21B$$

The failure of the condition of eqn. 6.21 to be satisfied disqualifies the choice of the maximum flow rate of 25% as a suitable candidate resulting in a pushing control design with a symmetric relay around the reference condition as such a symmetry cannot be established. What eqn. 6.21B implies is that there is no symmetry in the temperature controlling relay with respect to its drive level.

van Heerden analysis for the control design of this system at other choices of maximum flow rates gives similar results with only slight variations. Choice of 10% deviation in y , for instant, satisfies all the conditions of eqns 6.19-6.21, but the maximum drive level ($UC2_{max}$), obtained from the procedure analogous to those of section

6.1, is only a fraction of 1%. Reasonable control probably could not be carried out with such an insignificant change in the coolant temperature. Figures 6.7 and 6.8 are the van Heerden diagrams of the system at 10% change in flow rate.

From the above results, it is concluded that a control design based upon the restriction of always having multiple steady states for all combinations of drive levels cannot be achieved in this system. Hence, in this research, controller design was started by picking reasonable changes in f and u and calculating from the linearized equations (eqns. 3.7 and 3.8 setting $\dot{w} = \dot{y} = 0$) the corresponding steady state values of the concentration and the temperature. These were taken as reference errors in these variables to be used in defining the normalized relay design parameters σ_1 and δ_1 . The picked values of the flow rate was $\pm 20\%$ from f^* , and of the coolant temperature $\pm 4\%$ from T_C^* which gave $UC1_{ref} = 0.2000$, $UC2_{ref} = 0.0419$ respectively. These corresponded to 0.001 gm-mole/cc and 12.57K respectively. The reference values for the bandwidths of each controller were determined in the following manner:

$$w = -\left[\frac{a_{12}y + b_{11}f}{a_{11}} \right] \quad 6.22$$

and from eqn. 3.8 (with $\dot{y} = 0$)

$$y = -\left[\frac{a_{21}w + b_{21}f + b_{22}u}{a_{22}} \right] \quad 6.23$$

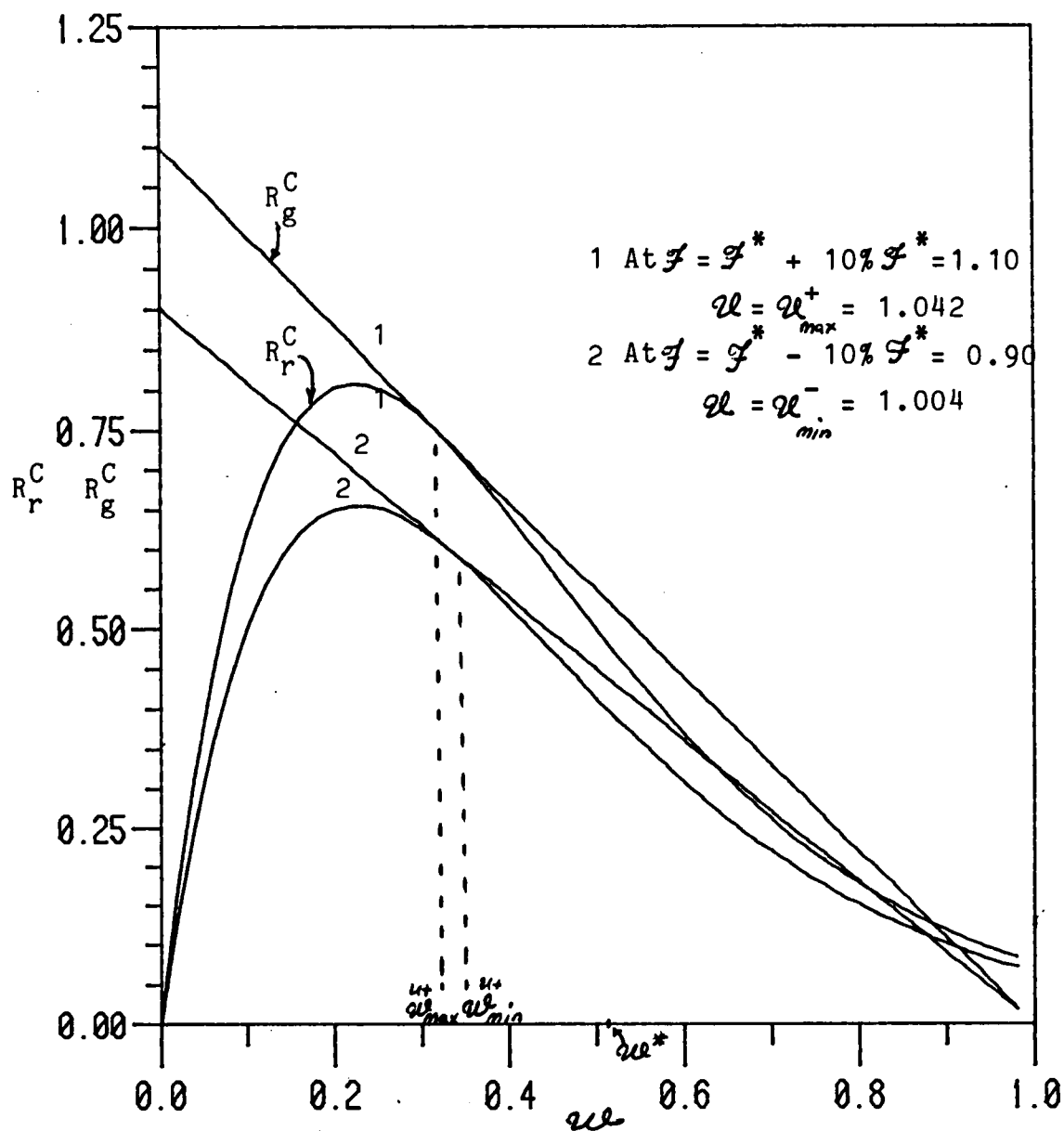


Figure 6.7 : Multiple van Heerden diagrams for MI/MO concentration loop relay design for case of $F = F^* \pm 10\% F^*$,
 $We_{min}^{u_l} = u_l^{u_l} - u_l^*$, $We_{max}^{u_l} = u_l^{u_l} - u_l^*$

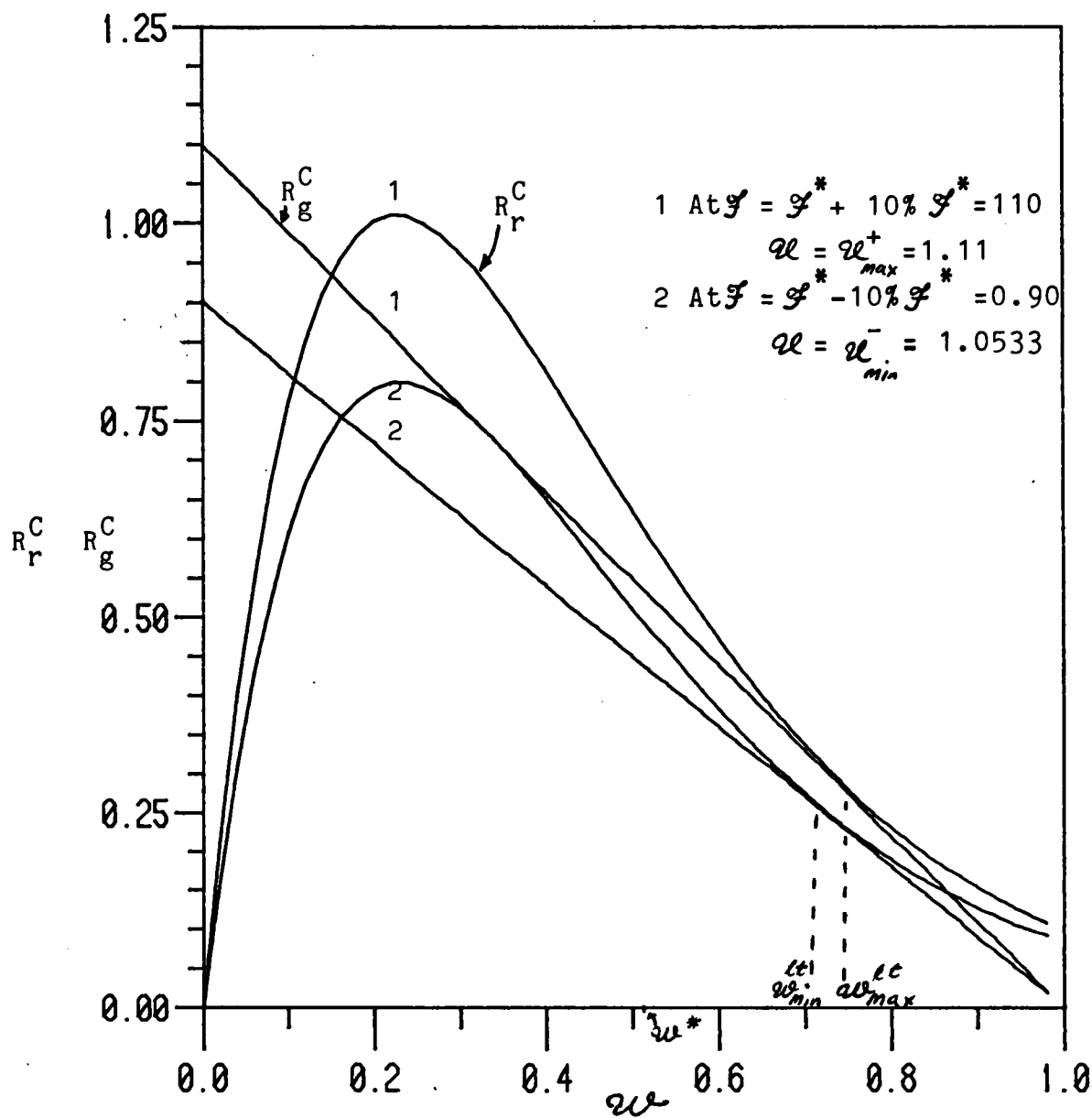


Figure 6.8: Multiple van Heerden diagrams for MI/MO concentration loop*relay design for case of $F = F^* \pm 10\% F^*$,
 $We_{max}^{lt} = u_{max}^{lt} - u^*$, $We_{min}^{lt} = u_{min}^{lt} - u^*$

Substituting y from eqn. 6.23 into eqn 6.22 yields

$$w = w_{\text{ref}} = - \left[\frac{(a_{12}b_{21} - a_{22}b_{11})f + a_{12}b_{22}u}{a_{11}a_{22} - a_{12}a_{21}} \right] \begin{matrix} f = +UC1_{\text{ref}} \\ u = +UC2_{\text{ref}} \end{matrix} \quad 6.24$$

Also, substituting w from eqn. 6.22 into eqn. 6.23 gives

$$y = y_{\text{ref}} = \left[\frac{(a_{21}b_{11} - a_{11}b_{21})f - a_{11}a_{22}u}{a_{11}a_{22} - a_{12}a_{21}} \right] \begin{matrix} f = +UC1_{\text{ref}} \\ u = +UC2_{\text{ref}} \end{matrix} \quad 6.25$$

Positive changes of the relays' drive levels, changes in flow rate (+20%) and in coolant temperature (+4%), were chosen for substitution into eqns. 6.24 and 6.24 simply because these positive changes in the manipulated variables gave the most reasonable values of the calculated reference half bandwidths in terms of their deviations from the unstable steady state condition. The reference half bandwidths determined from eqns. 6.24 and 6.25 were $w_{\text{ref}} = 0.02000$ and $y_{\text{ref}} = 0.0086$ respectively.

The Control Procedure

Basically, the control procedure was like that given for SI/SO control in the previous section. The parameter definition for σ_i and δ_i given in eqns. 6.7 - 6.10 were again used. Thus the system's responses were simulated by integrating simultaneously eqns. 3.5 and 3.6 with Runge-Kutta integration routine as before but now with both control loops regulated simultaneously with the respective

relays. If the system showed sustained oscillation, definite frequency Ω_0 and amplitudes A_1 resulted when a steady state was reached. Under such circumstance, the steady state periodic output frequencies of both control loops were always equal, that is, $\Omega_{01} = \Omega_{02} = \Omega_0$. This, at least for this system, makes the assumption given in chapter five concerning such equality reasonable.

The mathematical prediction of the coupled system's response according to the new control theory given in chapter five was obtained by simultaneous solutions of eqns. 5.43 and 5.45 for frequency Ω_0 and shift in phase ϕ . The solutions of these equations was carried out with PATTERN computational program written by Moore et al.(104) for multi-dimensional optimization. The solutions so obtained were tested in the inequalities (eqns. 5.44 and 5.46). Again, only the solutions which satisfied the inequalities were accepted. For this system and for the choice of the control levels given above, the solutions so obtained were always unique and they always satisfied the inequalities. These solutions (that is, Ω_0 and ϕ) were submitted into eqns. 5.47 and 5.48 which were then analytically solved (see appendix for algorithm) to check the continuity conditions given in eqn. 5.24. Also from the analytical solutions the output amplitudes A_1 and A_2 were obtained. The nonlinear simulation and the analysis results are compared in tables 6.3-6.6 at the indicated control levels.

Finding a physically realizable solution for Ω_0 (this must be real and positive) was an indication that the control was successful with both relay switching. If no solution was found, cases 2 and 3 of section 5.3 must be considered. If no solution was found for these cases, the controlled process was unstable.

Results, Discussion and Recommendation

The steady states M, H, L were the same as those given in section 6.1. As stated above, the maximum change in flow rate was taken as 20% from F^* , and of coolant temperature as 4% from T_C^* . These gave the drive levels of ± 0.2000 and ± 0.0419 respectively which corresponded to flow rate of ± 0.2000 cc / sec and the coolant temperature of ± 12.57 K. The reference half bandwidth of each of the controllers was also evaluated. This was ± 0.0200 for the concentration controlling relay and ± 0.0086 for the temperature controlling relay. These values corresponded to 0.0001 gm-mole/cc and 2.58K or errors of 3.9% and 0.8% respectively. The simulation and the analysis results at different control levels are compared in tables 6.3-6.6.

To obtain comparisons between theoretical predictions and simulation of the coupled MI/MO system's behaviour, only one of the relay design parameters (σ_i, δ_i) defined in eqn. 6.7 -6.10 was changed at a time while all others were fixed at constant values. Tables 6.3 summarises the

Table 6.3: Effects of Variation of the Concentration Relay's Bandwidths on the Coupled Loops' Response for

$$\sigma_1 = 0.7500, \sigma_2 = 0.7160, \delta_2 = 0.3300 \text{ (UC1}_{\text{ref}} = 0.2000,$$

$$\text{UC2}_{\text{ref}} = 0.0419, \text{ye}_{\text{ref}} = 0.0086, \text{we}_{\text{ref}} = 0.0200).$$

	Case I, $\delta_1 = 1.4805$			Case II, $\delta_1 = 1.1100$			Case III, $\delta_1 = 0.7400$		
	Simulation	Tsypkin	% Error	Simulation	Tsypkin	% Error	Simulation	Tsypkin	% Error
A1	0.0296	0.0296	0.00	0.0223	0.0222	0.00	0.0148	0.0148	0.00
A2	0.0033	0.0029	12.35	0.0032	0.0028	10.69	0.0030	0.0029	5.63
Ω_0	5.2883	3.6815	30.38	6.2811	5.3222	15.27	9.6371	8.7838	8.85
$t\phi$	- 0.0178	- 0.0151	14.91	0.0584	0.0532	8.98	0.1311	0.1201	8.39
ϕ	0.0941	0.0558	40.76	- 0.3668	- 0.2829	22.87	- 1.2634	- 1.0549	16.49

Table 6.4: Effects of Variation of the Temperature Relay's Bandwidths on the Coupled Loops' Response for

$\sigma_1 = 0.7500$, $\sigma_2 = 0.7160$, $\delta_1 = 1.1100$ ($UC1_{ref} = 0.2000$,
 $UC2_{ref} = 0.0419$, $ye_{ref} = 0.0086$, $we_{ref} = 0.0200$).

	Case I, $\delta_2 = 0.4401$			Case II, $\delta_2 = 0.3300$			Case III, $\delta_2 = 0.2200$		
	Simulation	Tsypkin	% Error	Simulation	Tsypkin	% Error	Simulation	Tsypkin	% Error
A1	0.0223	0.0023	0.00	0.0223	0.223	0.00	0.0223	0.0223	0.00
A2	0.0048	0.0039	17.81	0.0032	0.0028	10.69	0.0022	0.0031	6.00
Ω_0	7.1482	5.8809	17.73	6.2811	5.3222	15.27	5.3522	4.9138	8.19
$t\phi$	0.1540	0.1385	10.05	0.0584	0.0532	8.98	- 0.0352	- 0.0318	9.54
ϕ	- 1.1010	- 0.8147	26.00	- 0.3668	- 0.2829	22.87	0.1882	0.1563	16.93

Table 6.5: Effects of Variation of the Concentration Relay's Drive Levels on the Coupled System's.

$$\sigma_2 = 0.7160, \delta_1 = 1.1100, \delta_2 = 0.3300 \text{ (UC1}_{\text{ref}} = 0.2000,$$

$$\text{UC2}_{\text{ref}} = 0.0419, \text{ye}_{\text{ref}} = 0.0086, \text{we}_{\text{ref}} = 0.0200).$$

	Case I, $\sigma_1 = 1.00$			Case II, $\sigma_1 = 0.75$			Case III, $\sigma_1 = 0.50$		
	Simulation	Tsyppkin	% Error	Simulation	Tsyppkin	% Error	Simulation	Tsyppkin	% Error
A1	0.0223	0.0223	0.00	0.0223	0.0223	0.00	0.0223	0.0223	0.00
A2	0.0036	0.0031	13.53	0.0032	0.0028	10.69	0.0030	0.0029	5.32
Ω_0	8.5522	6.8921	19.41	6.2811	5.3222	15.27	4.8521	4.2723	11.95
$t\phi$	0.0206	0.0187	9.34	0.0584	0.05315	8.98	0.1782	0.1693	5.02
ϕ	- 0.1765	- 0.1289	26.95	- 0.3668	- 0.2829	22.87	- 0.8646	- 0.7231	16.37

Table 6.6: Effects of Variation of the Temperature Relay's Drive Levels on the Coupled System's Response.

$$\sigma_1 = 0.7500, \delta_1 = 1.1100, \delta_2 = 0.3300 \text{ (UC1}_{\text{ref}} = 0.2000,$$

$$\text{UC2}_{\text{ref}} = 0.0419, \text{ye}_{\text{ref}} = 0.0086, \text{we}_{\text{ref}} = 0.0200).$$

	Case I, $\sigma_2 = 0.9547$			Case II, $\delta_2 = 0.7160$			Case III, $\sigma_2 = 0.4773$		
	Simulation	Tsypkin	% Error	Simulation	Tsypkin	% Error	Simulation	Tsypkin	% Error
A1	0.0223	0.0223	0.00	0.0223	0.0223	0.00	0.0223	0.0223	0.00
A2	0.0038	0.0031	18.37	0.0032	0.0028	10.69	0.0031	0.0028	8.39
Ω_0	7.0124	5.7096	18.58	6.2811	5.3222	15.27	5.8948	5.1217	13.11
$t\phi$	0.1232	0.0922	25.19	0.0584	0.0532	8.98	0.0159	0.0154	2.10
ϕ	- 0.8639	- 0.5263	39.09	- 0.3688	- 0.2829	22.87	- 0.0926	- 0.0788	14.96

system's response under the variations in δ_1 the concentration bandwidths, while table 6.4 gives such responses under variation of δ_2 . Tables 6.5 and 6.6 show the coupled system's responses at different values of σ_1 and σ_2 respectively. As before, a comparison between the theoretical analysis and the simulation results is provided by the magnitudes of the errors between the theoretical and simulated frequency ω_0 , phase shift, ϕ and the amplitudes A_i which are collected in the tables.

Comparisons between the analysis and the nonlinear simulation results follow the same general trend in all the tables: the smaller the relay controller's drive levels and bandwidths (σ_i , δ_i), the smaller the errors. This was to be expected for as the relay parameters become smaller, the deviations from the unstable steady state become smaller and, consequently, the process increasingly behaves in a linear fashion. As evidenced by the results of tables 6.3-6.6, these errors are generally less than 30% at all values of relay parameters studied. The largest errors in the comparisons arise in the case of phase shifts. The correct phase relation, lead or lag, was predicted for all cases studied. Yet, like other oscillatory characteristics, the direction of changes in the phase shift with changes in σ_i or δ_i were quite predictable with the associated errors being increasingly small as the process became more linear.

Judging from the error magnitudes, predictions of this process performance are reasonable. It is, therefore, concluded that the new control theory developed in this study is capable of a reliable prediction of an oscillatory response of the MI/MO illustrative system at an unstable steady state condition. Because the theory presupposes linear process model, such predictions at small control levels (small deviations of control and manipulated variables from reference condition at which control is effected) are exact or nearly so, and this theory is highly recommended for application under such control conditions.

CHAPTER 7

PROPOSED FUTURE STUDIES

A number of interesting potential areas of future research arise from the study presented in this thesis. In particular, the question of carrying out an experiment on a practical chemical multivariable system to test the applicability of the new theory proposed in this study, is of considerable interest. Other tempting and highly recommended areas of studies which could follow this research include

- (a) The stability analysis of the multivariable system oscillatory responses when regulated with relays in accordance with the new theory for MI/MO processes presented in Chapter 5.
- (b) The establishment of theoretical conditions under which all periodic outputs of an MI/MO process can oscillate at the same frequency, and in which such oscillations are symmetric.
- (c) Device of a nonlinear decoupler with which a coupled multi-loop system can be decoupled.

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APPENDIX

```

C.....
C    THE PROGRAM REF. NAME:CVAN2.FOR
C    PROGRAM DATE: DEC./79
C    PROGRAMMER:OHA/79
C.....
C    THE PURPOSE
C    .....
C    THIS INTERVAL-HALVING TECHNIQUE GENERATES DATA
C    FILE FOR PLOTTING MULTIPLE VAN HEERDEN DIAGRAMS
C    OF FIGURES 6.1,6.5-6.8 FOR THE PURPOSE OF DESIGN
C    OF THE CONCENTRATION CONTROLLING RELAY. THIS
C    PROGRAM CAN BE MODIFIED TO PLOT FIGURE 4.1 BY
C    RETAINING ONLY THE FIRST TWO FUNCTIONS DEFINED
C    BELOW (THAT IS, GEN(X), REM(X)).
C.....
C    REAL*8 IFILE
C    REAL LAMBDA
C    REAL MU
C    REAL F
C    REAL MF
C    FUNCTION DEFINITION. RETAIN ONLY GEN(X) AND REM(X)
C    FOR MAKING PLOTS OF FIG. 4.1
C    ( NOTE THAT: F=F, MF=F, PF=F, F=F, PF=F, MF=F)
C
C    REM(X)=DA*X*EXP(-E*(F+LAMBDA)/(BETA*F*(1-X)+
1  F+LAMBDA*U))
C    GEN(X)=-F*(X-1)
C    REM2(X)=DA*X*EXP(-E*(CF+LAMBDA)/(BETA*CF*(1-X)+
1  CF+LAMBDA*PU))
C    GEN2(X)=-CF*(X-1)
C    REM3(X)=DA*X*EXP(-E*(MF+LAMBDA)/(BETA*MF*(1-X)+
1  MF+LAMBDA*PMU))
C    GEN3(X)=-MF*(X-1)
C
C    OPEN(UNIT=21,FILE=IFILE,ACCESS='SEQOUT')
C    WRITE(5,1)
1  FORMAT(3X,'PLOTTER DATA FILE NAME:',$,)
C    READ(5,2)
2  FORMAT(A10)
C
C    READING DIMENSIONLESS TERMS
C
C    BETA=0.1667
C    DA=1.572E15
C    LAMBDA=0.1356
C    E=37.745
C    READING FLOW RATES FOR REF. STATE AND FOR
C    UPPER AND LOWER TANGENCY POINTS
C    F=1.0
C    CF=1.250
C    MF=0.750
C    READING COOLANT TEMP. FOR UPPER AND LOWER

```

C TANGENCY POINTS

PU=1.156
PMU=1.0199
F=1.0
U=1.048

C
C

YI=0.0
YF=1.0
DELTAY=0.02
Y=YI

C
C

10

RG=GEN(Y)
RR=REM(Y)
RG2=GEN2(Y)
RR2=REM2(Y)
RG3=GEN3(Y)
RR3=REM3(Y)
TYPE5,Y, RG, RR, RG2, RR2, RG3, RR3
WRITE(21)Y, RG, RR, RG2, RR2, RG3, RR3
5 FORMAT(5X, 7F10.5)

Y=Y+DELTAY
IF(Y.GT.YF)STOP
GO TO 10
CLOSE(UNIT=21, FILE=IFILE, ACCESS='SEQOUT')
STOP
END

```

C.....
C      PROGRAM REF. NAME: VTEM.FOR
C      PROGRAM DATE: MARCH/1979
C      PROGRAMMER: OHA/79
C.....
C      THE PURPOSE
C      .....
C      THIS INTERVAL-HALVING TECHNIQUE IS FOR GENERATING
C      DATA FILE FOR PLOTTING VAN HEERDEN DIAGRAM FOR
C      TEMPERATURE (FIG.4.2) FROM EQNS.4.18 AND 4.19
C.....
C      REAL*8 IFILE
C      REAL LAMBDA
C.....FUNCTIONS DEFINITION FOR GENERATION AND REMOVAL
C.....TERMS
C      GEN(X)=BETA*DA*EXP(-E/X)*(F/(F+DA*EXP(-E/X)))
C      REM(X)=X*(F+LAMBDA)-(F+LAMBDA*U)
C      OPEN(UNIT=21,FILE=IFILE,ACCESS='SEQOUT')
C      WRITE(5,1)
1      FORMAT(3X,'PLOTTER DATA FILE NAME:',$,)
C      READ(5,2)
2      FORMAT(A10)
C.....READING DIMENSIONLESS QUANTITIES
C
C      BETA=0.1667
C      DA=1.572E15
C      LAMBDA=0.1356
C      E=37.745
C.....F=1==FLOW RATE AT UNSTABLE STEADY STATE
C
C      F=1.0
C      U=1.048
C      YI=0.45
C.....YF===FINAL VALUE OF TEMP. AT WHICH COMPUTATION ENDS
C      YF=1.0
C
C      DELTAY===INCREMENTS IN TEMP.
C
C      DELTAY=0.01
C      Y=YI
10     RG=GEN(Y)
C      RR=REM(Y)
C      TYPE5,Y, RG, RR
C      WRITE(21)Y, RG, RR
5      FORMAT(5X, 3F10.5)
C      Y=Y+DELTAY
C      IF(Y.GT.YF)STOP
C      GO TO 10
C      CLOSE(UNIT=21,FILE=IFILE,ACCESS='SEQOUT')
C      STOP
C      END

```

```

C.....
C    PROGRAM REF. NAME: HALF.FOR
C    PROGRAM DATE: JANUARY, 1979
C    PROGRAMMER: OKON/79
C.....
C    PURPOSE
C    .....
C    THIS IS INTERVAL-HALVING TECHNIQUE FOR SOLVING EQN
C    4.68 AND 4.69 FOR THE FREQUENCY (SI/SO TSYPKIN'S
C    METHOD)FOR TEMP. SI/SO SYSTEM OF FIG. 3.2
C.....
C    UC1 AND UC2 ARE CONC. RELAY'S AND TEMP. RELAY'S
C    DRIVE LEVELS RESPECTIVELY.
C.....
C    REM(X)=(-ALPHA7/A1)*TANH(PIE*A1/(2*X))
1    -(ALPHA8/A2)*TANH(PIE*A2/(2*X))-DELTA2/UC2
C    GEN(X)=0.0
C    A1=0.4609
C    A2=-.921
C    PIE=3.141593
C    UC1=0.146
C    UC2=0.02052
C    A11=-1.936
C    A12=-15.74
C    A21=0.1559
C    A22=1.488
C    B11=0.4757
C    B12=0.0
C    B21=0.0
C    B22=0.1356
C    DIV=A1-A2
C    ALPHA7=B22*(A11-A1)/DIV
C    ALPHA8=B22*(A2-A11)/DIV
C    SIGNP=1
C    SIGNM=1
C    DELWO=0.05
C    DELTA2=0.00648
C    N=0
C    WO=10
100   CALREM=REM(WO)
C    CALGEN=GEN(WO)
C    N=N+1
C    IF(N.GT.200000)STOP
C    IF(ABS(CALGEN-CALREM).LT.0.00001) GO TO 15
C    IF(CALREM-CALGEN) 10,10,12
12    IF(SIGNP.GT.0) GO TO 13
C    DELWO=DELWO/2
13    WO=WO+DELWO
C    SIGNM=-1
C    GO TO 100
10    IF(SIGNM.GT.0) GO TO 14
C    DELWO=DELWO/2
14    WO=WO-DELWO
C    SIGNP=-1

```

```
GO TO 100
15  CALWO=WO
    CONST=ALPHA7*TANH(PIE*A1/(2*CALWO))+ALPHA8*TANH(PIE*A
2    1  /(2*CALWO))+B22
    IF(CONST.GT.0) GO TO 200
    TYPE 3,CALWO
3    FORMAT(10X,'THE VALUE OF FREQUENCY IS WO=',F10.5 )
200  CONTINUE
    STOP
    END
```

```

C.....
C      PROGRAM REF. NAME: TEM.FOR
C      PROGRAM DATE: MAY/1979
C      THE PROGRAMMER: OHA/79
C.....
C      THE PURPOSE
C      .....
C      THIS PROGRAM IS TO DETERMINE THE TIME RESPONSE OF
C      TEMP. SI/SO SYSTEM ACCORDING TO EQN. 4.70
C.....
C
C      WO =  FREQ
C      WO=0.61436
C      PIE=3.141593
C
C      RO1 AND RO2 ARE THE PROCESS EIGENVALUES
C
C      RO1=0.4609
C      RO2=-0.921
C
C      UC1 AND UC2 ARE CONC. AND TEMP. RELAY DRIVE LEVELS
C      RESPECTIVELY
C
C      UC1 IS CONC. RELAY DRIVE LEVEL, AND UC2 IS TEMP.
C      RELAY DRIVE LEVEL
C
C      UC1=0.146
C      UC2=0.02052
C      READING ENTRIES OF JACOBIAN AND INPUT MATRICES
C      (SEE TABLE 3.2)
C
C      A11=-1.936
C      A12=-15.74
C      A21=0.1559
C      A22=1.488
C      B11=0.4757
C      B12=0.0
C      B21=0.0
C      B22=0.1356
C      DIV=RO2-RO1
C      NC=30
C      T=0.0
C      DELTAT=PIE/NC
C      N=0
C.....READING COEFF. OF PARTIAL FRAC. EXP.
C      ALPHA1=B11*(A22-RO1)/DIV
C      ALPHA2=B11*(RO2-A22)/DIV
C      ALPHA7=B22*(A11-RO1)/DIV
C      ALPHA8=B22*(RO2-A11)/DIV
C      TYPE 5
5      FORMAT(7X,'TEMP==')
10      Y=-(UC2*ALPHA7/RO1)*((SINH(PIE*RO1/(2*WO))-RO1*T))/(

```

```

1  COSH(PIE*RO1/(2*WO)))-(UC2*ALPHA7/RO1)*((COSH(PIE
1  *RO1/(2*WO))-COSH(PIE*RO1/(2*WO)-RO1*T))/(COSH(PIE*
1  RO1/(2*WO)))-(UC2*ALPHA8/RO2)*((SINH(PIE*RO2/(2*WO)
1  -RO2*T)) /(COSH(PIE*RO2/(2*WO)))-(UC2*ALPHA8/RO2)*
1  ((COSH(PIE*RO2/(2*WO))-COSH(PIE*RO2/(2*WO)-RO2*T))
1  /(COSH(PIE*RO2/(2*WO))))
    IF(T.GT.PIE) STOP
    TYPE 11,T,Y
11  FORMAT(5X,2F20.10)
    N=N+1
    IF(N.GT.100) GO TO 12
    T=T+DELTAT
    GO TO 10
12  CONTINUE
    STOP
    END

```

```

C.....
C      PROGRAM REF. NAME: PROG3.FOR
C      PROGRAM DATE: MARCH/1979
C      THE PROGRAMMER: OHA/79
C.....
C      THE PURPOSE
C      .....
C
C      THIS PROGRAM IS FOR SOLVING THE DIFFERENTIAL EQNS.
C      OF EQNS. 3.5,3.6. THAT IS, FOR SIMULATION OF SI/SO
C      SYSTEM GIVEN IN FIG. 3.3.
C.....
C      EXTERNAL FCT,OUTP
C      DIMENSION PRMT(6),Y(2),DERY(2),AUX(8,2)
C      REAL*8 IFILE
C      COMMON/COMCT/N
C      COMMON/COMOUP/IOUTP
C      COMMON/COMREL/U,D1,UC,EPSI1,FINALT,STEP,IAP,ISWIT,UCC
C      COMMON/COMFCT/DA,EP,ALAMB,BETA
C
C      OPEN(UNIT=21,FILE=IFILE,ACCESS='SEQOUT')
C
C      WRITE(5,1000)
1000  FORMAT( 3X,'PLOTTER DATA FILE NAME: ', $)
C      READ(5,1500)IFILE
1500  FORMAT( A10)
C
C      N=0
C      IAP=1
C      IPER = NUMBER OF PERIODS
C
C      IPER=5
C      NC=0
C      IOUTP=0
C      ISWIT=0
C      NDIM=2
C      DA=1.5720E15
C      READING OF THE PROCESS DIMENSIONLESS QUANTITIES
C
C      EP=37.745
C      BETA=0.1667
C      ALAMB=0.1356
C      Y(1)=0.5166
C      Y(1), Y(2) ARE THE UNSTABLE STEADY CONDITION VALUES
OF
C      CONC. AND TEMP. RESPECTIVELY
C
C      Y(2)=1.077
C      D1 = TEMP. CONTROLLING RELAY DRIVE LEVEL
C
C      D1=0.0216
C      U=1.048
C      EPSI1 DEFINES LIMITS OF RELAY'S BANDWIDTHS
C      EPSI1=0.00521

```

```

C      RKGS SUBROUTINE PARAMETERS:
C      PRMT(1)=INITIAL VALUE OF INDEP. VARIABLE
C      PRMT(2)=FINAL VALUE OF INDEP. VARIABLE
C      PRMT(3)=ERROR CRITERION
C      PRMT(4)=ERROR CRITERION
C      PRMT(5)=PROG. CONTROL PARAM.
C
C      PRMT(1)=0.0
C      PRMT(2)=100.0
C      PRMT(3)=0.1
C      PRMT(4)=1.0E-06
C      PRMT(5)=0.0
C      PRMT(6)=1.0
C      FINALT=PRMT(2)
C      STEP=PRMT(3)
C.....
C
10      CALL RELAY(PRMT,X,Y,DERY)
C      N=N+1
C      IOUTP=0
C      DERY(1)=0.5
C      DERY(2)=0.5
C      CALL RKGS(PRMT,X,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
C      IF(ISWIT.GT.IPER*2) GO TO 15
C      GO TO 10
15      CONTINUE
C      CLOSE(UNIT=21,FILE=IFILE,ACCESS='SEQOUT')
C      STOP
C      END
C.....
C      SUBROUTINE RELAY(PRMT,X,Y,DERY)
C      DIMENSION PRMT(6),Y(2),DERY(2)
C      COMMON/COMCT/N
C      COMMON/COMREL/U,D1,UC,EPSI1,FINALT,STEP,IAP,ISWIT,UCC
C      HERE THE CONTROLLER IS DELIBERATELY SET AT POSITIVE
C      DRIVE LEVEL AT THE INITIAL TIME
C
C      IF(NC.EQ.3) PRMT(6)=1.0
C      IF(PRMT(6).LT.0.0) GO TO 10
C      UC=D1+U
C      IBACK CONTROLS SYSTEM'S DIRECTION:
C      IBACK = 1 FOR TIME DOMAIN INTEGRATION
C
C      IBACK=-1
C      PRMT(6)=1.0
C      UCC=EPSI1
C      PRMT(2)=FINALT
C      PRMT(3)=STEP
C      PRMT(5)=0.0
C      IF(N.GT.1) PRMT(1)=Y(2)
C      IF(N.GT.1) Y(2)=X
C      N=2
C      NC=0

```

```

      RETURN
C      .....
C      BACK INTEGRATION TO THE SWITCHING POINT
10     IF(IBACK.GT.0) GO TO 20
        IBACK=1
        ISWIT=ISWIT+1
        ISIGN=(-1)**(ISWIT+1)
        PRMT(1)=Y(2)
        IF(ISIGN.GT.0) PRMT(2)=1.077+EPSI1
        IF(ISIGN.LT.0) PRMT(2)=1.077-EPSI1
        PRMT(3)=(PRMT(2)-PRMT(1))
        PRMT(5)=0.0
        Y(2)=X
        NC=NC+1
        RETURN
C      HERE THE RELAY SWITCHES TO NEGATIVE DRIVE LEVEL
C
20     IBACK=-1
        UC=U-D1
        UCC=-EPSI1
        PRMT(1)=Y(2)
        PRMT(2)=FINALT
        PRMT(3)=STEP
        PRMT(5)=0.0
        PRMT(6)=1.0
        Y(2)=X
        NC=NC+1
        RETURN
C      END.
C      .....

      SUBROUTINE FCT(X,Y,DERY,PRMT)
      DIMENSION PRMT(6),Y(2),DERY(2),Z(4)
      COMMON/COMREL/U,D1,UC,EPSI1,FINALT,STEP,IAP,ISWIT,UCC
      COMMON/COMFCT/DA,EP,ALAMB,BETA
      IF(PRMT(6).LT.0.0) GO TO 10
      HERE, X = TIME
C
C
      Z(1)=Y(1)
      Z(2)=Y(2)
      DERY(1)=1.0-Z(1)-DA*Z(1)*EXP(-EP/Z(2))
      DERY(2)=1.0-Z(2)-ALAMB*(Z(2)-UC)+BETA*DA*
1     Z(1)*EXP(-EP/Z(2))
      RETURN
C
C
10     Z(1)=Y(1)
      Z(2)=X
      DERY(1)=(1.0-Z(1)-DA*Z(1)*EXP(-EP/Z(2)))/(1.0
1     -Z(2)-ALAMB*(Z(2)-UC)+BETA*DA*Z(1)*EXP(-EP/Z(2)))
C
C
      HERE, X = TEMPERATURE
C
      DERY(2)=1.0/(1.0-Z(2)-ALAMB*(Z(2)-UC)+BETA*DA*Z(1))

```

```

1  *EXP(-EP/Z(2)))
END

```

```

C.....
C
SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
DIMENSION PRMT(6),Y(2),DERY(2)
COMMON/COMOUP/IOUTP
COMMON/COMREL/U,D1,UC,EPSI1,FINALT,STEP,IAP,ISWIT,UCC
IF(IOUTP.EQ.1) GO TO 10
IF(PRMT(6).LT.0.0) GO TO 100
IOUTP=1
SY1=Y(1)
SY2=Y(2)
NS=0
5  TYPE 5,X,Y(1),Y(2),UC,DERY(1),DERY(2),IHLF
10  FORMAT(3X,6F10.6,3X,I3)
IF(PRMT(6).LT.0.0) GO TO 100
CALL SWITCH(PRMT,Y)
IF(PRMT(5).GT.0.0) GO TO 15
RETURN
15  TYPE 5,X,Y(1),Y(2),UC,DERY(1),DERY(2),IHLF
WRITE(21)X,Y
NS=0
RETURN
100 TYPE 101
101 FORMAT(10X,'BACK INTEGRATION',/)
TYPE 5,X,Y(1),Y(2),UC,DERY(1),DERY(2),IHLF
NS=0
RETURN
END

```

```

C.....
C
SUBROUTINE SWITCH(PRMT,Y)
DIMENSION PRMT(6),Y(2)
COMMON/COMREL/U,D1,UC,EPSI1,FINALT,STEP,IAP,ISWIT,UCC
C
C
EPSIU=1.077+EPSI1
IF(Y(2).GT.EPSIU.AND.(-UCC*IAP).LT.0.0)
1  PRMT(6)=-1
IF(Y(2).LT.(1.077-EPSI1).AND.(-UCC*IAP).GT.0.0)
1  PRMT(6)=-1
IF(Y(2).GT.EPSIU.AND.(-UCC*IAP).LT.0.0)
1  PRMT(5)=10.0
IF(Y(2).LT.(1.077-EPSI1).AND.(-UCC*IAP).GT.0.0)
1  PRMT(5)=10.0
RETURN
END

```

```

C.....
C      PROGRAM REF. NAME: OUTM.FOR
C      PROGRAM DATE: JAN/1980
C      THE PROGRAMMER: OHA/80
C.....

C      THE PURPOSE
C      .....
C      THIS PROGRAM IS FOR DETERMINING TIME RESPONSE OF THE
C      COUPLED SYSTEM (FIG. 3.4) ACCORDING TO ANALYSIS
C      GIVEN IN SECTION 5.1 USING EQNS. 5.47 AND 5.48
C.....

C      TEMPERATURE LOOP

C      TEM1(X,Y,Z)=(-UC2*ALPHA7/(A1*COSH(PIE*A1/(2*Z))))*
1      (SINH(PIE*A1/(2*Z))-A1*(Y+X/Z))+COSH(PIE*A1/(2*Z))-
1      COSH(PIE*A1/(2*Z)-A1*(Y+X/Z)))

C
C
C      TEM2(X,Y,Z)=(-UC2*ALPHA8/(A2*COSH(PIE*A2/(2*Z))))*
1      (SINH(PIE*A2/(2*Z))-A2*(Y+X/Z))+COSH(PIE*A2/(2*Z))-
1      COSH(PIE*A2/(2*Z)-A2*(Y+X/Z)))

C
C
C      TEM3(Y,Z)=(-UC1*ALPHA5/(A1*COSH(PIE*A1/(2*Z))))*
1      (SINH(PIE*A1/(2*Z))-A1*Y)+COSH(PIE*A1/(2*Z))
1      -COSH(PIE*A1/(2*Z)-A1*Y))

C
C
C      TEM4(Y,Z)=(-UC1*ALPHA6/(A2*COSH(PIE*A2/(2*Z))))*
1      (SINH(PIE*A2/(2*Z))-A2*Y)+COSH(PIE*A2/(2*Z))-
1      COSH(PIE*A2/(2*Z)-A2*Y))

C
C
C      CONCENTRATION LOOP

C      CON1(Y,Z)=(-UC1*ALPHA1/(A1*COSH(PIE*A1/(2*Z))))*
1      (SINH(PIE*A1/(2*Z))-A1*Y)+COSH(PIE*A1/(2*Z))-
1      COSH(PIE*A1/(2*Z)-A1*Y))

C
C
C      CON2(Y,Z)=(-UC1*ALPHA2/(A2*COSH(PIE*A2/(2*Z))))*
1      (SINH(PIE*A2/(2*Z))-A2*Y)+COSH(PIE*A2/(2*Z))-
1      COSH(PIE*A2/(2*Z)-A2*Y))

C
C
C      CON3(X,Y,Z)=(-UC2*ALPHA3/(A1*COSH(PIE*A1/(2*Z))))*
1      (SINH(PIE*A1/(2*Z))-A1*(Y+X/Z))+COSH(PIE*A1/(2*Z))-
1      COSH(PIE*A1/(2*Z)-A1*(Y+X/Z)))

```

```

1 CON4(X,Y,Z)=(-UC2*ALPHA4/(A2*COSH(PIE*A2/(2*Z))))*
1 (SINH(PIE*A2/(2*Z))-A2*(Y+X/Z))+COSH(PIE*A2/(2*Z))-
1 COSH(PIE*A2/(2*Z)-A2*(Y+X/Z)))

```

C
C
C
C
C
C

```

DELTA1=D1, HALF-BANDWIDTH OF CONC. RELAY
DELTA2=D2, HALF-BANDWIDTH OF TEMP. RELAY
DELTA1=0.0098000
DELTA2=0.0032250
R01=0.4654
R02=-0.9134
A1=R01
A2=R02
PIE=3.141593
N=0
NC=20
DELTAT=PIE/NC

```

C
C
C
C

```

PHI= PHASE SHIFT
WO= FREQUENCY

```

```

PHI=-1.14599
WO=3.04786
A11=-1.936
A12=-15.74
A21=0.1559
A22=1.488
B11=0.4850
B12=0.0
B21=-0.076
B22=0.1356
DIV=R02-R01
UC1=0.075
UC2=0.01575

```

C
C
C

ALPHA'S ARE THE COEFF. OF PARTIAL FRAC. EXP.

```

ALPHA1=(B11*(A22-R01)-A12*B21)/DIV
ALPHA2=(B11*(R02-A22)+A12*B21)/DIV
ALPHA3=(B12*(A22-R01)-A12*B22)/DIV
ALPHA4=(B12*(R02-A22)+A12*B22)/DIV
ALPHA5=(B21*(A11-R01)-A21*B11)/DIV
ALPHA6=(B21*(R02-A11)+A21*B11)/DIV
ALPHA7=(B22*(A11-R01)-A21*B12)/DIV
ALPHA8=(B22*(R02-A11)+A21*B12)/DIV
TS=PIE/WO
TPHI=PHI/WO

```

C
C

```

T=0.0
TYPE 1, TS, TPHI, UC1, UC2, DELTA1, DELTA2

```

```

1      FORMAT(10X,'CONC.SWITCH TIME=',F7.4,2X,',','TPHI=',
1      F7.4,'UC1==',2X,F7.4,2X,'UC2==',F7.4,10X,
1      '//,10X,'DELTA1==',F7.4,2X,'DELTA2==',F7.4,//,
1      2X,'DELTA2==',F7.4,/)
      TYPE 5
5      FORMAT(15X,'TIME',15X,'TEMP.',13X,'CONC.')
```

10 RSST=TEM1(PHI,T,WO)+TEM2(PHI,T,WO)+TEM3(T,WO)+
1 TEM4(T,WO)
 RSSC=CON1(T,WO)+CON2(T,WO)+CON3(PHI,T,WO)+CON4(
1 PHI,T,WO)
 IF(T.EQ.TPHI) GO TO 17
 IF(T.GT.TS) GO TO 12
 TYPE 11,T,RSST,RSSC
11 FORMAT(4X,3F20.10)
 IF(T.EQ.TS) GO TO 14
 T=T+DELTAT
 GO TO 10
12 T=TS
 GO TO 10
14 T=TPHI
 GO TO 10
17 type 16,rsst
16 FORMAT(10X,'***TEMP. AT INITIAL SWITCH
1 TIME===',F10.6)
 CONTINUE
 END

```

C.....
C   PROGRAM REF. NAME: PROG5.FOR
C   PROGRAM DATE: JULY/1979
C   THE PROGRAMMER: OHA/79
C.....
C   THE PURPOSE:
C   .....
C
C   THIS PROGRAM IS INTENDED FOR SIMULTANEOUS
C   SOLUTION OF THE DIFFERENTIAL EQNS. GIVEN IN
C   EQNS. 3.5 AND 3.6. THAT IS, SIMULATION OF THE
C   THE NONLINEARIZED CHEMICAL REACTOR (FIG. 3.1)
C   FOR THE PURPOSE OF CONTROL AND STUDY OF INTER-
C   ACTION. THE LOOP OPERATION MAY BE EXPLAINED AS
C   FOLLOWS: THE REAL TIME DOMAIN INTEGRATION
C   PROCEEDS BY SIMULTANEOUSLY INTEGRATING EQNS.
C   3.5 AND 3.6 UNTIL EITHER LOOP'S RESPONSE IS
C   SUCH THAT THE LOOP'S CONTROLLER'S BANDWIDTH
C   IS EXCEEDED. AT THIS POINT, THE FORWARD INTEG-
C   RATION IS DISCONTINUED, AND BACKWARD INEGRAT-
C   ION BEGINS IN THE TEMP. OR CONC. DOMAIN DEPENDING
C   UPON WHICH OF THESE DEPENDENT VARIABLES STOPPED
C   THE FORWARD INTEGRATION UNTIL AN EXACT SWITCH
C   POINT (LIMIT OF BANDWIDTH) IS REACHED. THEN
C   THE FORWARD INTEGRATION OF BOTH LOOPS PROCEEDS
C   AT THIS POINT UNTIL EITHER OF THE LOOP'S
C   RESPONSES GOES BEYOND THE LOOP'S CONTROL
C   BANDWIDTH. HERE AGAIN THE FORWARD INTEGRATION
C   STOPS IN LIEU OF THE BACKWARD INTEGRATION IN
C   AN APPROPRIATE DOMAIN UNTIL THE SWITCH POINT
C   IS REACHED ONCE MORE. WHEN CONTROL
C   IS ACHIEVED, THIS PROCESS CAN CONTINUE AD
C   INFINITUM.
C
C   THE SYSTEM'S SPECIFICATION
C   .....
C
C   ALL VARIABLES DIMENSIONLESS
C   PRMT(6),(7),(8),(9) ARE CONTROL PARAMETERS
C   DA,EP,BETA,ALAMB ARE SYSTEM PARAMETERS
C   AT TIME=T=0, THE CONCENTRATION LOOP RELAY IS AT
C   ITS POSITIVE DRIVE LEVEL, WHILE THE TEMP. LOOP
C   DOES SO AFTER, SOME TIME LAPSE.
C   REFERENCE STATE : UNSTABLE STEADY STATE
C   .....
C   CONTROL ELEMENT SPECS:
C   IAP=1 FOR PASSIVE ELEMENT
C   =-1 FOR ACTIVE ELEMENT
C   EPSI=CONTROL BANDWIDTH
C   UC=DRIVE LEVEL
C
C   PRMT(6) AND PRMT(7) ARE THE TEMP. AND CONC.
C   CONTROL PARAM.ESPECTIVELY: THESE ARE >0 DURING
C   TIME - DOMAIN (OR FORWARD) INTEG., BUT < 0 WHEN

```

```

C      THE INDEPENDENT VARIABLE CHANGES TO CONC. OR TEMP.
C.....
C
      EXTERNAL FCT,OUTP
      DIMENSION PRMT(9),Y(2),DERY(2),AUX(8,2)
      COMMON/COMOUP/IOUTP
      COMMON/COMREL/UYSS,UWSS,DY,DW,UCY,UCW,EPSIY,EPSIW,
1  FINALT,STEPT,IAPY,IAPW,ISWITY,ISWITW,UCCY,UCCW,
1  YSS,WSS
C.....

      IAPY=-1
      IAPW=1
      NY=0
      NW=0
C      IPER=THE NUMBER OF DESIRABLE PERIODS
      IPER=6
      IOUTP=0
      ISWITY=0
      ISWITW=0
      NDIM=2
C      Y(1), AND Y(2) ARE THE UNSTABLE STEADY STATE VALUES
C      OF CONC. AND TEMP.

      Y(1)=0.515
      Y(2)=1.077
      YSS=Y(2)
      WSS=Y(1)
C      DY AND DW DEFINE LIMITS OF CONC. AND TEMP.
C      RELAYS' DRIVE LEVELS RESPECTIVELY
      DY=-0.0285
      DW=0.1654
      UYSS=1.048
      UWSS=1.00
C      EPSIY AND EPSIW DEFINE THE LIMITS OF THE CONC. AND
C      TEMP. RELAYS' BANDWIDTHS RESPECTIVELY.
      EPSIY=0.0128
      EPSIW=0.1654
      PRMT(1)=0.0
      PRMT(2)=100.0
      PRMT(3)=0.1
      PRMT(4)=1.0E-05
      PRMT(5)=0.0
      PRMT(6)=1.0
      PRMT(7)=1.0
      PRMT(9)=0.0
      PRMT(8)=0.0
      FINALT=PRMT(2)
      STEPT=PRMT(3)
      NCY=0
      NCW=0
      ISWITY=0
      ISWITW=0
      TYPE 3

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```

3      FORMAT(/,6X,'YSS=1.077',4X,'WSS=0.515',4X,'UYSS=1.04
1      2X,'UWSS=1.0',/)
      TYPE 1000,DW,DY,EPSIW,EPSIY
1000   FORMAT(6X,'UC1=',F5.3,2X,'UC2=',F5.3,2X,'DELTA1=',
1      F5.3,2X,'DELTA2=',F5.3,/)
      TYPE 2
2      FORMAT(6X,'TIME',5X,'CONC',4X,'TEMP',5X,'UCW',7X,
1      'UCY',5X,'DERY(1)',2X,'DERY(2)',4X,'IHLF',/)
10     CALL RELAY(PRMT,X,Y,DERY)
      NY=NY+1
      NW=NW+1
      IOUTP=0
      DERY(1)=0.5
      DERY(2)=0.5
      CALL RKGS(PRMT,X,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
      IF(ISWITY.GT.IPER*2) GO TO 15
      GO TO 10
15     CONTINUE
      END
C.....

      SUBROUTINE RELAY(PRMT,X,Y,DERY)
      DIMENSION PRMT(9),Y(2),DERY(2)
      COMMON/COMREL/UYSS,UWSS,DY,DW,UCY,UCW,EPSIY,EPSIW,
1      FINALT,STEPT,IAPY,IAPW,ISWITY,ISWITW,UCCY,UCCW,
1      YSS,WSS
      IF(NCY.EQ.3) PRMT(6)=1.0
      IF(NCW.EQ.3) PRMT(7)=1.0
      IF(PRMT(6).LT.0.0.OR.PRMT(7).LT.0.0) GO TO 5
C      HERE FOR INTEGRATION AT POSITIVE DRIVE LEVELS
C      FOR BOTH RELAYS
C
      IF(PRMT(8).GT.0.0.AND.IBACK.GT.0) PRMT(1)=Y(2)
      IF(PRMT(9).LT.0.0.AND.JBACK.GT.0) PRMT(1)=Y(1)
      IF(PRMT(8).GT.0.0.AND.IBACK.GT.0) Y(2)=X
      IF(PRMT(9).LT.0.0.AND.JBACK.GT.0) Y(1)=X
      PRMT(2)=FINALT
      PRMT(3)=STEPT
      PRMT(5)=0.0
      IF(PRMT(8).GE.0.0.AND.IBACK.GE.0) UCY=UYSS+DY
      IF(PRMT(9).LE.0.0.AND.JBACK.GE.0) UCW=UWSS+DW
      IF(PRMT(9).LE.0.0.AND.JBACK.GE.0) UCCW=EPSIW
      IF(PRMT(8).GE.0.0.AND.IBACK.GE.0) UCCY=EPSIY
      IF(PRMT(8).GE.0.0.AND.IBACK.GE.0) IBACK=-1
      IF(PRMT(9).LE.0.0.AND.JBACK.GE.0) JBACK=-1
      IF(NCY.EQ.3) NCY=0
      IF(NCW.EQ.3) NCW=0
      RETURN
C.....
C      TEMPERATURE(ONLY) BACK INTEGRATION
C      HERE,X=TEMP,Y(2)=TIME

```

```

5      IF(PRMT(7).LT.0.0) GO TO 30
      IF(IBACK.GT.0) GO TO 20
      IBACK=1
      ISWITY=ISWITY+1
      ISIGNY=IAPY*((-1)**(ISWITY))
      PRMT(1)=Y(2)
      IF(ISIGNY.GT.0) PRMT(2)=YSS-EPSIY
      IF(ISIGNY.LT.0) PRMT(2)=YSS+EPSIY
      PRMT(3)=PRMT(2)-PRMT(1)
      PRMT(5)=0.0
      Y(2)=X
      NCY=NCY+1
      RETURN

```

```

C      TEMP CONTROLLING RELAY AT NEGATIVE DRIVE LEVEL
C

```

```

20     IBACK=-1
      UCY=UYSS-DY
      UCCY=-EPSIY
      PRMT(1)=Y(2)
      PRMT(2)=FINALT
      PRMT(3)=STEPT
      PRMT(5)=0.0
      PRMT(6)=1.0
      PRMT(8)=1.0
      Y(2)=X
      NCY=NCY+1
      RETURN

```

```

C      .....
C      CONC.(ONLY) BACK INTEGRATION
C

```

```

30     IF(JBACK.GT.0) GO TO 35
      JBACK=1
      ISWITW=ISWITW+1
      ISIGNW=IAPW*((-1)**(ISWITW+1))
      PRMT(1)=Y(1)
      IF(ISIGNW.GT.0) PRMT(2)=WSS+EPSIW
      IF(ISIGNW.LT.0) PRMT(2)=WSS-EPSIW
      PRMT(3)=PRMT(2)-PRMT(1)
      PRMT(5)=0.0
      Y(1)=X
      NCW=NCW+1
      RETURN

```

```

C      .....
C      CONC. CONTROLLING RELAY AT NEGATIVE DRIVE LEVEL
C

```

```

35     JBACK=-1
      UCW=UWSS-DW
      UCCW=-EPSIW
      PRMT(1)=Y(1)
      PRMT(2)=FINALT
      PRMT(3)=STEPT
      PRMT(5)=0.0
      PRMT(7)=1.0

```

```

PRMT(9)=-1.0
Y(1)=X
NCW=NCW+1
RETURN
END

```

```

C.....
SUBROUTINE FCT(X,Y,DERY,PRMT)
  DIMENSION PRMT(9),Y(2),DERY(2)
  COMMON/COMREL/UYSS,UWSS,DY,DW,UCY,UCW,EPSIY,EPSIW,
1  FINALT,STEPT,IAPY,IAPW,ISWITY,ISWITW,UCCY,UCCW,
1  YSS,WSS
  C  READING OF THE SYSTEM'S DIMENSIONLESS VARIABLES
  C
  DA=1.572E15
  EP=37.745
  BETA=0.1667
  ALAMB=0.1356
  IF(PRMT(6).LT.0.0) GO TO 5
  IF(PRMT(7).LT.0.0) GO TO 10

  C  HERE X=TIME
  C
  C
  DERY(1)=UCW*(1-Y(1))-DA*Y(1)*EXP(-EP/Y(2))
  DERY(2)=UCW*(1-Y(2))-ALAMB*(Y(2)-UCY)+BETA*DA
1  *Y(1)*EXP(-EP/Y(2))
  RETURN

  C  HERE X=TEMPERATURE
  C
  C
  5  DERY(1)=(UCW*(1-Y(1))-DA*Y(1)*EXP(-EP/Y(2)))/(
1  UCW*(1-Y(2))-ALAMB*(Y(2)-UCY)+BETA*DA*Y(1)
1  *EXP(-EP/Y(2)))
  DERY(2)=1/(UCW*(1-Y(2))-ALAMB*(Y(2)-UCY)+BETA
1  *DA*Y(1)*EXP(-EP/Y(2)))

  C  HERE X=CONCENTRATION
  RETURN
10  DERY(1)=1/(UCW*(1-Y(1))-DA*Y(1)*EXP(-EP/Y(2)))
  DERY(2)=(UCW*(1-Y(2))-ALAMB*(Y(2)-UCY)+BETA*DA
1  *Y(1)*EXP(-EP/Y(2)))/(UCW*(1-Y(1))-DA*Y(1)*
1  EXP(-EP/Y(2)))
  END

C.....
C
SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
  DIMENSION Y(2),PRMT(8),DERY(2)
  COMMON/COMOUP/IOUTP
  COMMON/COMREL/UYSS,UWSS,DY,DW,UCY,UCW,EPSIY,EPSIW,
1  FINALT,STEPT,IAPY,IAPW,ISWITY,ISWITW,UCCY,UCCW,
1  YSS,WSS
  IF(PRMT(6).LT.0.0) GO TO 10
  IF(PRMT(7).LT.0.0) GO TO 20
  TYPE 5,X,Y(1),Y(2),UCY,UCW,DERY(1),DERY(2),IHLF
5  FORMAT(2X,7F9.5,5X,I2)

```

```
CALL SWITCH(PRMT,Y)
RETURN
```

```
C
10 TYPE 11
11 FORMAT(7X,'TEMPERATURE BACK INTEGRATION',/)
   TYPE 5,X,Y(1),Y(2),UCY,UCW,DERY(1),DERY(2),IHLF
   RETURN
```

```
C
20 TYPE 21
21 FORMAT(7X,'CONCENTRATION BACK INTEGRATION',/)
   TYPE 5,X,Y(1),Y(2),UCY,UCW,DERY(1),DERY(2),IHLF
   RETURN
END
```

```
C.....
C
SUBROUTINE SWITCH(PRMT,Y)
  DIMENSION PRMT(9),Y(2)
  COMMON/COMREL/UYSS,UWSS,DY,DW,UCY,UCW,EPSIW,EPSIW,
1  FINALT,STEPT,IAPY,IAPW,ISWITY,ISWITW,UCCY,UCCW,
1  YSS,WSS
  IF(Y(1).GT.(WSS+EPSIW).AND.(-UCCW*IAPW).LT.0.0)
1  PRMT(7)=-1
  IF(Y(1).LT.(WSS-EPSIW).AND.(-UCCW*IAPW).GT.0.0)
1  PRMT(7)=-1
  IF(Y(2).GT.(YSS+EPSIW).AND.(-UCCY*IAPY).LT.0.0)
1  PRMT(6)=-1
  IF(Y(2).LT.(YSS-EPSIW).AND.(-UCCY*IAPY).GT.0.0)
1  PRMT(6)=-1
  IF(Y(1).GT.(WSS+EPSIW).AND.(-UCCW*IAPW).LT.0.0)
1  PRMT(5)=1.0
  IF(Y(1).LT.(WSS-EPSIW).AND.(-UCCW*IAPW).GT.0.0)
1  PRMT(5)=10.0
  IF(Y(2).GT.(YSS+EPSIW).AND.(-UCCY*IAPY).LT.0.0)
1  PRMT(5)=10.0
  IF(Y(2).LT.(YSS-EPSIW).AND.(-UCCY*IAPY).GT.0.0)
1  PRMT(5)=10.0
  RETURN
END
```

VITA

Okon Hanson Akpan was born in Uyo, Nigeria on May 31, 1953. He attended Nung Ukana Elementary School , and later, Lutheran High School, Obot Idim where he graduated with West African School Certificate in December, 1968. After this, he pursued pre-medical studies in Hope Waddell Training Institute, Calabar and obtained his HSC (University of Cambridge, England) in December, 1970. He taught physics and chemistry in Itam Secondary School, Nigeria from January, 1971 to December, 1973.

Okon came to this country, USA, in January, 1974 and attended Maryville College, Tennessee where he graduated with a B.A. degree in mathematics in December, 1976. In January of the following year, he began his studies in Chemical Engineering in the University of Tennessee, Knoxville. He received the Masters of Science degree with a major in Chemical Engineering in June, 1980.

Okon is married to the former Miss Victoria Ekpe Okon of Uyo, Nigeria, and has a daughter Ituen Okon Akpan.