To the Graduate Council:

I am submitting herewith a thesis written by Shu-Hua Chang entitled "Nonlinear Feedback Control and Dead Time Compensation for Systems with Dead Time." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Chemical Engineering.

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The Graduate School

NONLINEAR FEEDBACK CONTROL AND DEAD TIME COMPENSATION FOR SYSTEMS WITH DEAD TIME

A Thesis

Presented for the

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Shu-Hua Chang

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DEDICATION

This thesis is dedicated to my parents, Tien-Yu Chang and Yuen-May Cheng Chang.

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ABSTRACT

A new class of relays, "negative bandwidth" relays, which have not been cited in previous literature are introduced and defined. Along with another new concept of "reduced dead time," these newly defined relays allow the design of a novel dead time compensation technique based on the linearized process model. Even when a significantly large dead time exists, this control design strategy provides a successful control implementation for the relay feedback control scheme.

The existence of considerable dead time in a control loop often prevents effective control from being achieved and even degrades the performance of the process. A perhaps surprising fact is that relay feedback control does not influence the dynamics of a process even in the presence of dead time. Thus the ability to easily compensate for dead time provides a strong motivation to investigate the use of the nonlinear feedback controller.

Previous studies have addressed another potential advantage of using relay with hysteresis feedback controllers instead of conventional linear controllers to maintain a process state near a steady state which is open loop unstable. When employing the design derived from the linearized model, in treating the actual nonlinear plant this design strategy gives good starting values for the

controller parameters. Subsequently, a successful control implementation may be achieved by on-line tuning or by relay modification based on the nonlinear simulation of the process without dead time.

Tsypkin's method provides an exact representation of the relay and has been shown to be superior to describing function analysis. Thus Tsypkin's method will be extended in this study. By following the procedures exploited for systems without dead time, a mathematical framework for dead time systems is developed to obtain the waveform characteristics and its stability. The new concepts of "reduced dead time" and "negative bandwidth relays" lead to a mathematical methodology based on Tsypkin's formalisms for a dead time compensation strategy which achieves a successful control scheme no matter how large the system dead time.

A nonisothermal CSTR which carries out a homogeneous, irreversible, first order, exothermal reaction serves as an example to demonstrate the concepts of the research.

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$\underline{\underline{A}}$	Matrix of the model linearized with respect to state variables
A_{ij}	Element of matrix $\underline{\underline{A}}$
B	Dimensionless heat transfer coefficient
$\underline{\underline{B}}$	Matrix of the model linearized with respect to control parameters
B_{ij}	Element of matrix $\underline{\underline{B}}$
C_A	Reactant concentration
C_{Af}	Reactant feed concentration
C_p	Specific heat
Da	Damkohler number $(=VK_o e^{-\gamma}/F)$
det	Determinant
E	Activation energy
F	Feed and effluent volumetric flow rate
G	Reactor transfer function
$\underline{\underline{G}}$	Reactor transfer function matrix
G_{ij}	Element of matrix $\underline{\underline{G}}$
$\mathcal{G}_z$	Z-transform of G(s)
$\mathcal{G}_{z,m}$	Advanced z-transform of G(s)
H_A	Overall heat-transfer coefficient times cooling surface area
Im[ ]	Imaginary part
j	$\sqrt{-1}$
K_o	Arrhenius constant
K	Gain constant

N	Function of relay element
p	Poles of reactor transfer function
R	Gas law constant
r	Rate of reaction
Re[ ]	Real part
s	Laplace transform parameter
T	Reactor temperature
T_c	Average coolant temperature
T_D	Dead time
T_{Dc}	Critical dead time
T_{Dr}	Reduced dead time
T_f	Feed stream temperature
T_o	Period of oscillation
t'	Time
tr	Trace of a matrix
u(t)	Dimensionless coolant temperature deviation
u_c	Control levels of symmetric relay with hysteresis
$\underline{U}$	Vector of input variables
$\mathcal{U}$	Laplace transform of u
$\mathscr{u}$	Dimensionless coolant temperature
V	Reactor volume
w	Dimensionless concentration deviation
$\mathscr{w}$	Dimensionless concentration
x	Error signal, input to control element
x_2	Bandwidth of relay controller
$x_{2,c}$	Bandwidth of compensated relay controller

$y(t)$	Dimensionless reactor temperature deviation
y_e	Maximum allowable deviation of controlled output from set point
$\underline{Y}$	Vector of state variable
Ψ	Laplace transform of y
$\mathcal{Y}$	Dimensionless reactor temperature
z	Z-transform parameter

GREEK SYMBOLS

α	Signal parameter
β	Dimensionless adiabatic temperature rise
γ	Dimensionless activation energy
δ	Fraction of y_e used to define the switching conditions
$(-\Delta H)$	Heat of reaction
θ	Reactor residence time
λ	Eigenvalue
$\Lambda(\Omega)$	Tsypkin's function
ρ	Density
Ω	Frequency of oscillation
Ω_o	Solution for frequency of oscillation

SUBSCRIPTS

g	Generation
r	Removal
ss	Steady state

SUPERSCRIPTS

*	Unstable steady state
•	First derivative

INTRODUCTION

Dead time, which is referred to by a variety of terms, including time delay, delay time, process lag and transportation lag, is commonly encountered in chemical engineering. The existence of significant dead time is a factor that causes controller design and process control to be more difficult for nearly any control strategy. The delays may arise due to the measurement elements, transmitters, actuation elements, digital sampling devices or lags directly associated with the process. The effect of any source of dead time is to delay the controller action which generally degrades the performance of the controller scheme, even causing stability or loss of control problems. Treatment of the dead time problem has led to an entire research area in process control called dead time compensation.

Another compelling reason to introduce dead time into a process model is that it is often desirable and practical to approximate a system of higher order by one of lower order with dead time [1, p. 8; 2, pp. 12-15]. In fact, the terminology, "applied process control" often carries the connotation of basing process identification, process modeling, controller design, controller tuning, etc. on the process characteristics being modelled or fitted to a first order transfer function with dead time (a linear first order ordinary differential equation with dead time) or perhaps a second order transfer function with dead time. In this case the transfer functions and dead times of all elements, such as, actuators and transmitters between the input test signal and the process response are associated with the "process" transfer function.

The treatment of dead time processes has been essentially limited to the use of linear controllers and linear compensators [2, pp. 227-255; 3; 4; 5; 6].

When a classical linear feedback control scheme is employed for control, the presence of dead time leads to a transcendental characteristic equation which contains the dead time term, $e^{-\tau s}$. This greatly enhances the difficulty of analysis, design and computation for processes with dead times. A 1948 translation of Oldenberg and Sartorius's textbook [1, pp. 215-218] shows that for periodic control or relay control under certain conditions, the dynamic behavior of linear process (e.g. the order of the characteristic equation, the eigenvalues, the gains, etc.) will not be influenced by the existence of dead time or the controller action on the system. This perhaps surprising fact gives an easier technique to circumvent the dead time problem. Consequently, when dealing with dead time, this astonishing discovery provides a potential advantage by using nonlinear control, in the form of a relay with hysteresis (on-off control), instead of a conventional linear feedback control scheme.

Application of relay with hysteresis feedback controllers to maintain process states near open loop unstable steady states have been developed and studied in detail by Bruns and Bailey for processes without dead time [7-9]. The result of such a control scheme is a stable, small amplitude oscillation in the neighborhood of the open loop unstable steady state. Their work addresses other potential advantages of using nonlinear control instead of linear control when the open loop steady state is unstable.

Their mathematical formulation of the problem serves as the starting basis for the present research. When a linearized representation of the process under study is employed, describing function analysis provides an analytical method of approximating the nonlinear controller [10; 11; 12; 13, pp. 403-431; 14]; more specifically, a harmonic linearization of the controller is used. When the nonlinearity is any type of relay, a technique known as Tsypkin's method provides an exact representation of the relay [7; 8; 10, pp. 185-199; 12, pp. 279-314]. Previous work [7-9] has shown the design and prediction capabilities of Tsypkin's method to be superior to describing function analysis, and thus, will be the method extended in this study.

The application of relay type control where the controller output has only a discrete number of outputs, requires a critical examination of the performance criterion which the controller should satisfy. As stated by Bruns and Baily [8]:

> Most conventional control strategies are based on a very restricted goal: keep the system at the chosen operation point. This requires at least local asymptotic stability of that state in the controlled process. It seems quite reasonable and consistent with current process practice to adopt a more general control objective: The controller should keep the process state within an adequately small neighborhood of the desired operating point. This property should be maintained to the greatest possible extent during and after upsets.
>
> This generalized criterion is met if the process state oscillates within a small neighborhood of the specified operating point, providing the oscillation is stable.

The major objectives of the present research are to:

a. extend the formalisms of Tsypkin's method to the design and analysis of relay feedback contollers to processes which contain dead times;

b. explore the potential advantages of using nonlinear feedback controllers in compensating for dead time;

c. establish a test to determine the stability of a limit cycle when one is predicted by Tsypkin's method for dead time processes, and

d. demonstrate the application of the newly derived results on an example.

A nonisothermal CSTR (continuous flow stirred tank reactor) which contains dead time and carries out a homogeneous, irreversible, first order, exothermal reaction will be the example used to investigate and demonstrate the concepts of the research. This two state variable reactor model has been a theme problem in chemical reactor theory and control studies since the early mathematical treatments of the area [15, 16]; also see references in Uppal, Ray and Poore [17]. In addition it is an example used in the recent studies of relay control without dead time [7, 9].

CHAPTER I

LITERATURE SURVEY

Steady State Multiplicity and Process Stability

The first paper discussing the multiple steady state phenomena and stability problem in a chemical reactor seems to have been done by Liljenroth in 1918 [18]. In Liljenroth's study, ammonia oxidation was used as an example to show that the curve of conversion of ammonia to nitric oxide with respect to temperature is in a sigmoidal shape. Moreover, he indicated that a linear relationship between burner temperature and conversion existed. Plotting these two curves, Liljenroth indicated the steady states of operation were located at the intersections. Unfortunately, this paper was not noted at that time.

In 1953 Van Heerden gave a paper based on a more mathematical analysis [19]. He viewed the system using an energy balance; it was shown that an autothermic process can be characterized by a diagram consisting of two curves showing the generation and the consumption of heat as a function of some reference temperatures. In that plot, which is now well known as the "Van Heerden Plot," the heat generation rate vs. temperature had a sigmoidal shape and the consumption rate with respect to temperature gave a straight line. Based on the relationship between the generation and removal rate curves at a steady state Van Heerden developed a criteria, called the "slope condition" for the steady state stability. The plot given by Van Heerden is similar to that obtained by Liljenroth experimentally. Again, there were three steady state solutions for the system. Two of which are stable, one at low temperature with low conversion and the other at high temperature with high conversion. The third solution at an intermediate temperature with intermediate conversion is an unstable steady state point.

However the so called "slope condition" derived and used by Van Heerden is now known to be a necessary and sufficient criteria for local asymptotic steady state stability only for a single dependent variable system. For all other systems (second and higher dependent variable systems) the slope condition is only a necessary condition for stability.

Furthermore, in 1958, Van Heerden [20] indicated that a necessary condition for the existence of multiplicity is the feedback of heat along the reaction path; this feedback effect may arise from convection (mixed reactor), conduction (flame) or heat exchange. He considered different cases including an ideal mixer and a plug flow reactor with longitudinal conduction of heat.

Since that time many papers have considered other systems. For instance, Matsuura and Kato [21] gave the stability analysis on the concentration effects under isothermal conditions in autocatalytic and heterogeneous catalytic reactions. Knorr and O'Driscoll [22] investigated the existence of multiple steady states of a free-radical polymerization reaction. In this polymerization reactor, it is desirable to operate at the intermediate steady state, since operation of the reactor at high level of conversion is nearly impossible due to the high viscosity, so called "gel effect", and operation at the low level of conversion does not give a product having the required properties. Moreover, Stephens [23] investigated the cause of disturbances encountered in a new methanol converter and traced the problem to steady state multiplicity. He also described the design of a stabilizing control scheme and a simple experimental optimization method.

Bilous and Amundson [24] and Aris and Amundson [15, 25] applied methods of nonlinear mechanics as developed and used by Poincare, Liapounov and Minorsky to give a more stringent mathematical criteria on the stability of steady states. This analysis is based on a linearized dynamic model around the steady state under study, thus it provides information on the behavior of the system only when it is sufficiently close to the steady state which is of interest. Therefore, this linearized stability is referred to as local asymptotic stability of a steady state to a sufficiently small perturbation. This stability is guaranteed if and only if the real parts of all eigenvalues of the coefficient matrix (Jacobian matrix) of linearized dynamic equations are negative (see Chapter II).

The application of these analysis was presented for an example which has become an intimate part of the literature on chemical reactor theory: the two state variable CSTR carrying out a homogeneous, first order, irreversible, exothermal reaction with external cooling.

Uppal, Ray and Poore [17] have presented an important paper with, to date, the most thorough classification and analysis of pathological behavior in the two variable CSTR noted above. They classified the possible dynamic behavior of the reactor according to a set of dimensionless system parameters. Plots in the parameter space are used to define the various steady state solution possibilities. In addition to the criteria for steady state stability presented by Aris and Amundson, analytic criteria derived with nonlinear mechanics and bifurcation theory are developed to predict the existence and stability characteristics of limit cycles as a function of the dimensionless parameters. Numerical examples are also given in the form of phase plane plots for each region of parameter space having a different character which provides a clearer insight into the dynamic behavior of the system.

The existence of multiplicity in a system may lead to unexpected pitfalls during the operation or control of the process, such as the occurance of a run away reaction. Thus it is desirable and necessary to determine under what conditions the multiplicity will appear and the behavior of the steady states.

Recently, abundant papers, analyzing multiplicity and stability in more complex systems, have been presented. For instance, the binary exothermic reaction in a non-adiabatic CSTR has been investigated by Lin [26]. Lumped-parameter systems in which two consecutive or two parallel reactions occur also have been studied in detail [27,28]. Moreover, Agrawal et al. [29] have given an analysis on the isothermal continuous stirred tank biological reactors. Recently, Schmidt et al. have provided a detailed study on both isothermal and nonisothermal solution homopolymerization and copolymerization in a CSTR [30, 31]. Additionally, the multiplicity and stability of steady states of two identical stirred tanks in sequence have been considered by Svoronos, Aris, and Stephanopoulos [32]. A large number of additional examples both in theoretical developments and experimental demonstrations can be found in a thorough review on this subject by Schmitz [33].

Examples of several systems exhibiting three steady states, one of which is necessarily unstable, exist. Unfortunately, as mentioned before, it is sometimes profitable to operate such a system at the intermediate unstable steady state. Operation at an unstable steady state obviously requires a control system and presents more difficulties than operation at a stable steady state. The first mathematical formulation for developing a control scheme for operation at an unstable steady state in chemical engineering literature was presented by Aris and Amundson in 1958 [15, 16]. Using the CSTR system which had been treated by Bilous and Amundson [24], Aris and Amundson showed with their mathematical analysis and extensive simulations that it is most feasible to use a proportional linear feedback control with a sufficiently high gain in adjusting the coolant flow rate based on the reactor temperature deviation. A potential problem for this conventional control framework may arise when a high gain is needed to achieve stability. The higher the controller gain the more severe is the requirement placed on the cooling system capacity. Therefore, a new control scheme arising in part to circumvent these problems has been developed.

Bruns, et al. have employed a new sophisticated control philosophy for the operation process without dead time near an unstable steady state employing a relay with hysteresis as a feedback controller [7-9]. Properly designed, this control scheme produces a stable, small amplitude oscillation in the neighborhood of the desired state. Systems with one variable and two variables have been studied in detail based on a mathematical analysis, formulated around Tsypkin's method.

Process Dead Time and Dead Time Compensation

Traditionally, dead time has played a role of "most difficult to incorporate" into process control analysis and controller design. Very often, the conventional three-mode PID controller (Proportional-Integral-Derivative controller) cannot overcome dead time effectively if a process is characterized by an "sufficiently large" dead time. When the dead time associated with a process is equal to or greater than the dominant time constant of the process, even the best PID controller fails to provide an effective control [34, 35]. Additionally, at best, the transcendental function $e^{-\tau s}$ can only be approximated by using a Pade' approximation [36, p .366]. This generates an infinite number of roots in the characteristic equation of the closed loop system, making it extremely difficult to analyze compared to the open loop system. Consequently, dead time is a formidable problem in process control theory and controller design.

In the late 1950's Smith developed a model based approach to better control single-input single-output (SISO) systems containing a single large dead time which has come to be known as "the Smith predictor [3, 4]." But this advanced strategy was not noted due to the lack of hardware for a pure dead time element until digital computers appeared in the market. A great number of papers have been presented to discuss the application and performance stability of this dead time compensator [37, 38, 39, 40, 41, 42]. Recently, Ogunnaike and Ray [43] have further extended the dead time compensator to deal with the multiple-input multiple-output (MIMO) systems containing multiple dead times.

This dead time compensation method starts with a removal of dead time from the control loop. This fictious process is used as a basis for determining the controller settings, the proportional band, the reset rate and derivative time to give the maximum speed of recovery from a disturbance, and the minimum offset within the limitations of stability. Next, applying the strategies developed by Smith requires the addition of a minor feedback loop around this conventional PID controller to construct a Smith predictor controller to compensate for dead time.

Oldenberg and Sartorius have demonstrated that for periodic control or relay control under certain conditions, the analysis of the dynamic behavior of the open loop system vs. the closed loop system will not be influenced by the presence of dead time [1, pp. 215-218]. This constrasts with the use of linear controllers where the open loop system has a polynomial characteristic equation and the closed loop system has a transcendental characteristic equation. When the input to the relay is periodic, by expressing the output of relay feedback control scheme with difference equations, Oldenberg and Sartorius proved that the order of characteristic equation will not be changed by the existence of dead time. This fact gives a strong motivation for investigating the use of a nonlinear control scheme in the form of a relay with hysteresis in feedback control loop.

CHAPTER II

THEORETICAL EXTENSION OF TSYPKIN'S METHOD TO DEAD TIME SYSTEMS

System Stability

It has been observed that the output of a system must take into account not only the initial conditions and the coefficients in the differential equations but also the nature of the inputs. The system output is often characterized in terms of the system stability.

The two most frequently employed definitions of stability of a dynamic system are:

1. Bounded-input bounded-output stability--For example, in the case of linear systems, a definition of stability often used is: A linear system is stable if for all bounded inputs the output is bounded.
2. Asymptotically stable--After a disturbance the free response of a dynamic system approaches its undisturbed or steady state conditions asympototically.

Stability of Systems Represented by Linear Ordinary Differential Equations

Stability under both of previous definitions for a linear system leads to the requirement that all the roots of the system characteristic equation must have negative real parts [44, pp. 356-361].

Consider a nth order linear system which is described by n ordinary differential equations

$$\underline{\dot{Y}} = \underline{\underline{A}}\underline{Y} \qquad (2.1)$$

where

$\underline{\dot{Y}}$ n x 1 vector containing state variable derivatives

$\underline{Y}$ n x 1 vector containing state variables

$\underline{\underline{A}}$ n x n Jacobian matrix of system

Let vector $\underline{Y}(t) = e^{\lambda t}\underline{\underline{C}}$ be a candidate solution of Equation (2.1) where λ is a number, and $\underline{\underline{C}}$ is a constant matrix. Differentiation of the candidate solution gives

$$\underline{\dot{Y}}(t) = \lambda e^{\lambda t}\underline{\underline{C}} = \lambda \underline{Y}(t) \qquad (2.2)$$

Substituting into Equation (2.1) yields

$$\lambda \underline{Y} = \underline{\underline{A}}\underline{Y} \qquad (2.3)$$

or, by rearrangement

$$(\lambda \underline{\underline{I}} - \underline{\underline{A}})\underline{Y} = 0 \qquad (2.4)$$

According to Cramer's theorem, this homogeneous system of linear equations has a nontrivial solution if and only if the corresponding determinant of the coefficients is zero [45, p. 352]

$$D(\lambda) = \det(\lambda \underline{\underline{I}} - \underline{\underline{A}}) = \begin{vmatrix} \lambda - A_{11} & \cdots & \cdots & -A_{1n} \\ -A_{21} & \lambda - A_{22} & \cdots & -A_{2n} \\ \vdots & & & \vdots \\ -A_{n1} & \cdots & \cdots & \lambda - A_{nn} \end{vmatrix} = 0 \quad (2.5)$$

Equation (2.5) is called the characteristic equation of the system corresponding to the matrix $\underline{\underline{A}}$. By developing $D(\lambda)$ we obtain a polynominal of nth degree in λ. The eigenvalues of a square matrix $\underline{\underline{A}}$ are the roots of the corresponding characteristic equation, and they play a significant role in determining the stability of the system. The stability can be assured if and only if the characteristic equation of system has roots whose real part are all negative; if any have a positive real part, the steady state solution is unstable. The critical case of a zero real part eigenvalue needs further discussion to determine the stability and is connected with the bifurcation problem [46, pp. 761-764]. For first, second or low order system the characteristic equation is low degree in λ, thus it is easy to find the eigenvalues. But for high order systems, the characteristic equation is of high degree in λ, and it is necessary to use numerical or graphical methods to extract the roots of the characteristic equation. These methods can become very laborious. Fortunately, there are many methods and criteria which can be used for the determination of stability without requiring the complete solution of the characteristic equation, for instance, Routh-Hurwitz criterion, Mikhailov criterion, Root locus method and Cauchy-Nyquist criterion, etc. [44, pp. 361-372; 47, pp. 144-147]

Stability Of Systems Represented by Nonlinear Ordinary Equations

The main difference between nonlinear and linear systems is that the principles of superposition and homogenity are not satisfied in the nonlinear case. Therefore, nonlinear models are much more difficult to analyze than linear ones. One feature of stability analysis which distinguishes linear systems from nonlinear systems is that linear systems have a single steady state, whereas, nonlinear system may have multiple steady states. Another feature is that a linear system is either stable or unstable in all of its state space, while a nonlinear system, which may contain several steady states, can be stable in some regions of the state space and unstable in others. A stable linear system returns to its steady state asymptotically after being perturbed, whereas, a perturbed nonlinear system, although initially at a locally stable steady state may return to the original steady state, may go to some other stable steady state, or may oscillate in a limit cycle around some steady states.

The method of stability analysis used for nonlinear systems in this work is based on the linearization of the system model. The nonlinear system or model is linearized by making a truncated Taylor series expansion around the steady state which is of interest. This perturbation method examines the stability of a system to a sufficiently small displacement about a given steady state point. Therefore, the nonlinear effects are neglected in the neighborhood of the steady state, that is, we linearize the differential equations around the desired steady state. Subsequently, linear stability analysis can be applied to the linearized model. Keep in mind that this linearized model only provides information on the dynamics and stability of the system in the small region about the steady state of interest, and therefore, is termed locally asymptotic stability. Another method to investigate the stability of nonlinear systems usually based on simulation studies of the nonlinear model is called the phase plane method. It considers the system in the large, thus, all the types of stability for a nonlinear system can be addressed. The phase plane analysis also can be used to determine the best start up to reach a given steady state or the fate of the system in the event of a runaway.

Transfer Functions

In chemical engineering, most systems are modelled by nonlinear equations. Often control system design procedures call for the system dynamic information in the form of transfer functions. As previously stated, in the study of control problems it is important to always keep in mind that when the linearized model is used it is only a local representation of the actual nonlinear process. The motivation for using the linearized model is to develop a desired characterization of the controlled process based on highly developed linear algebra analysis methods.

When the independent or input variables and a set of state variables (usually the dependent or output variables) have been identified, the system is linearized about the steady state values of these variables which is usually the desired operating point. In general, a multiple-input multiple-output system as sketched in Figure 2.1 may be described by:

$$\underline{Y} = \underline{\underline{A}}\,\underline{Y} + \underline{\underline{B}}\,\underline{U} \tag{2.6}$$

where

$\underline{Y}$ n x 1 vector of state variable derivatives

$\underline{Y}$ n x 1 vector of state variables

$\underline{U}$ m x 1 vector of input variables

$\underline{\underline{A}}$ n x n Jacobian matrix of state variable

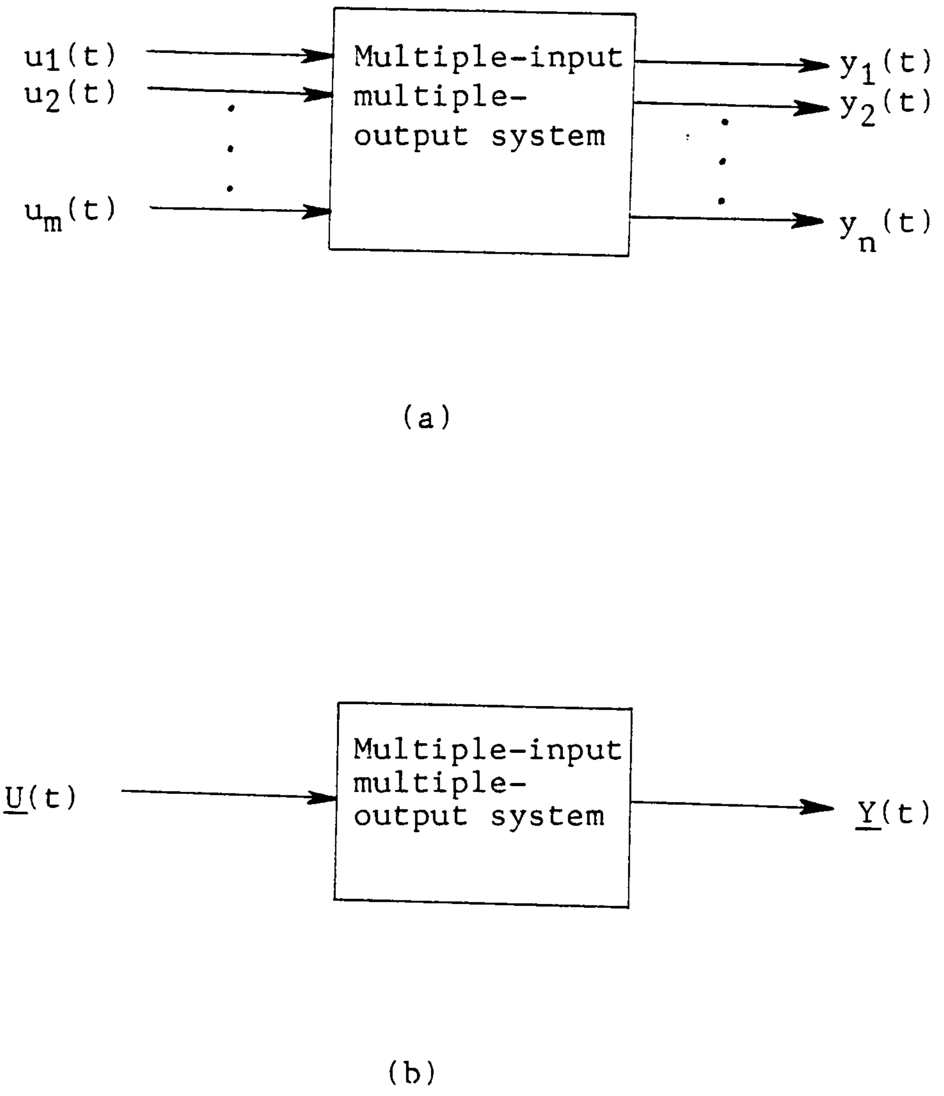


Figure 2.1. Block diagram representations of a multiple-input multiple-output system.

model linearization coefficients

$\underline{\underline{B}}$ n x m matrix of input variable model linearization coefficients

The vector U is often seperated into disturbance and manipulation variables once a control strategy is developed. Taking the Laplace transform of Equation (2.6), along with zero initial conditions, $\underline{Y}(0) = \underline{\underline{0}}$, leads to

$$s\dot{\underline{Y}}(s) = \underline{\underline{A}}\,\underline{Y}(s) + \underline{\underline{B}}\,\underline{U}(s) \tag{2.7}$$

Rearrangement of Equation (2.7) yields

$$\underline{Y}(s) = (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\,\underline{\underline{B}}\,\underline{U}(s) \tag{2.8}$$

or

$$\underline{Y}(s) = \frac{\mathrm{adj}(s\underline{\underline{I}} - \underline{\underline{A}})}{\det(s\underline{\underline{I}} - \underline{\underline{A}})}\,\underline{\underline{B}}\,\underline{U}(s) = \underline{\underline{G}}(s)\underline{U}(s) \tag{2.9}$$

Here matrix $\underline{\underline{G}}(s)$ is denoted as the transfer function matrix and expressed as:

$$\underline{\underline{G}}(s) = \begin{vmatrix} G_{11}(s) & \cdots & G_{1m}(s) \\ \vdots & & \vdots \\ G_{n1}(s) & \cdots & G_{nm}(s) \end{vmatrix} \tag{2.10}$$

An element G_{ij} is the transfer function relating the input variable u_j $(1 \leq j \leq m)$ to the process output variable x_i $(1 \leq i \leq n)$, and usually takes the form:

$$G_{ij}(s) = K_{ij} \frac{\prod_{g=1}^{a} (s + \mu_g)}{\prod_{h=1}^{n} (s + p_h)} \qquad a \leq n \qquad (2.11)$$

where $-\mu_g$ and $-p_h$ are the zeros and poles of the transfer function respectively, and K_{ij} is the residue at pole $-p_h$ or the transfer function gain constant.

Nonlinear Control Element

The system analyzed in this study is a nonlinear feedback control scheme as shown in Figure 2.2. Figure 2.2a contains the controlled process with the nonlinear control element and the nonlinear process. In this figure there are two types of nonlinearities, the "process nonlinearity" which cannot be removed as it is associated inherently with the process and the "control nonlinearity" that is introduced into the design of the process to achieve a special purpose.

The first step of the analysis procedures is to linearize the nonlinear system about the desired operating point. Figure 2.2b illustrates the resulting controlled linearized process to be analyzed. An approximate method

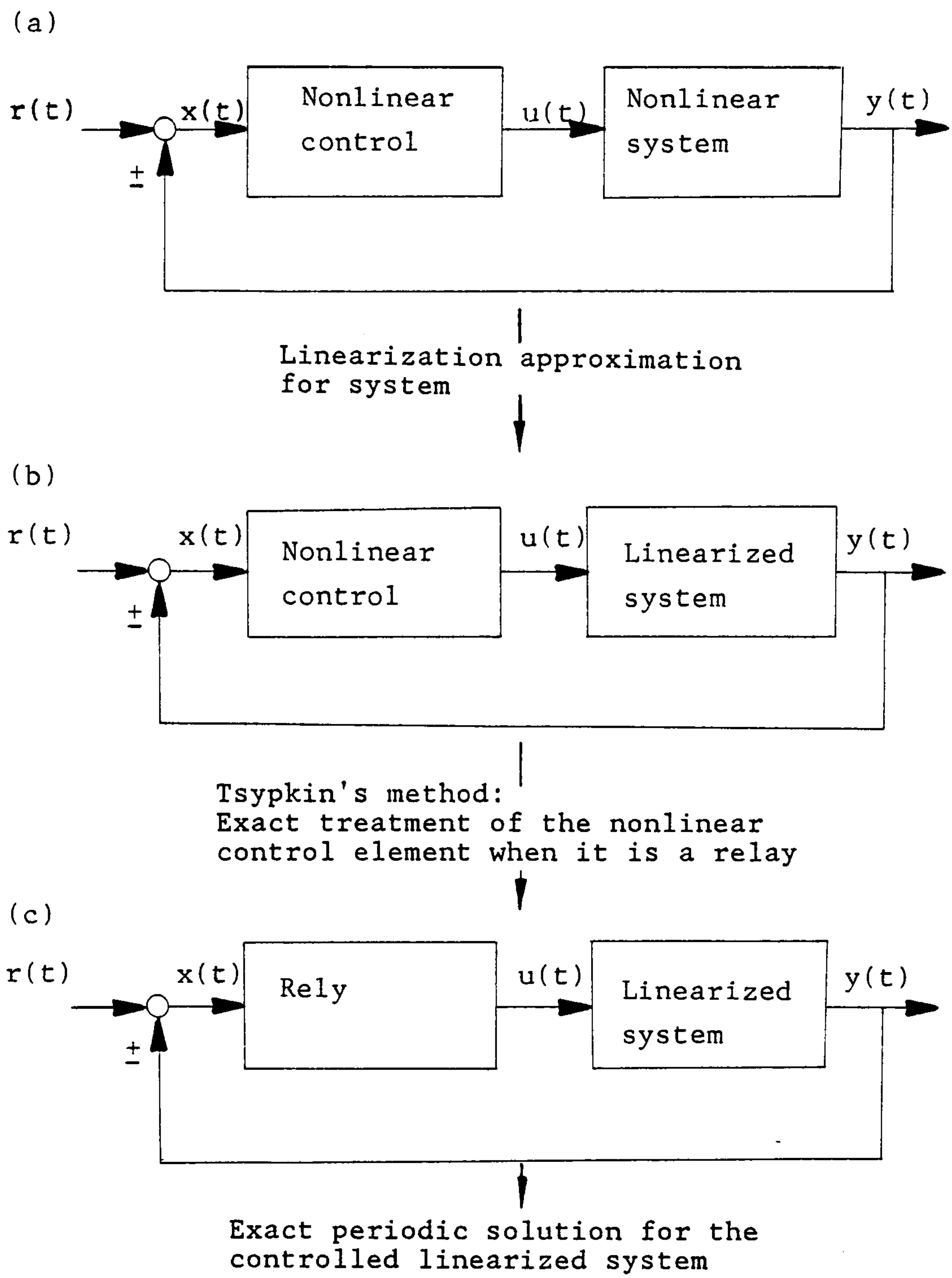
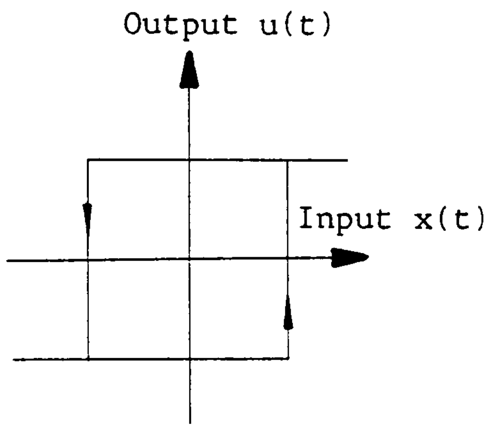


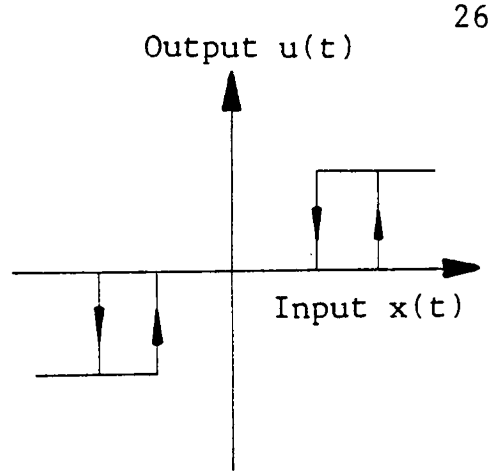
Figure 2.2. Diagram of analysis outline for a nonlinear feedback control scheme.

known as describing function theory can be utilized to solve this nonlinear feedback scheme. It provides an approximate representation of this nonlinear control element. However, when the nonlinear controller is some types of relay, another approach known as Tsypkin's method can be applied to the study of the feedback system with the linearized process. This procedure gives an exact representation of the relay controller. Hence, in this approach the only approximation is the linearization of the system to be controlled. Consequently, Tsypkin's method has been demonstrated to be superior to the describing function method when analyzing a nonlinear feedback control scheme containing a relay controller.

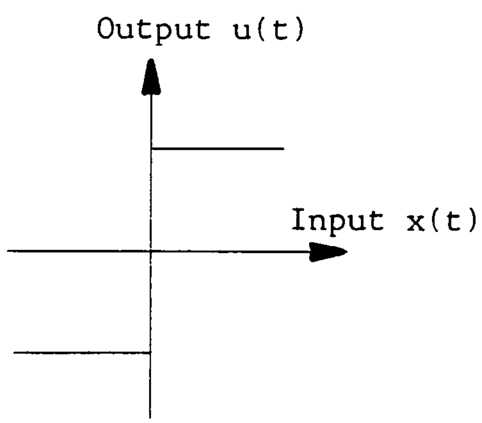
In general, the characteristics of a relay can have dead zones, hysteresis, or both. Several types of relays are shown in Figure 2.3. A relay with hysteresis is called a memory element because it is multiple valued for some values of input and the level of the output of the relay depends on the input history [12, pp. 195-209]. Memory elements can be divided into two classes, active elements and passive elements depending on the direction the memory characteristic is traversed [12, pp. 207-208]. For example, refer to the relays shown in Figure 2.4, the active relay is followed in a clockwise direction, while counter clockwise movement is obtained for the passive relay.



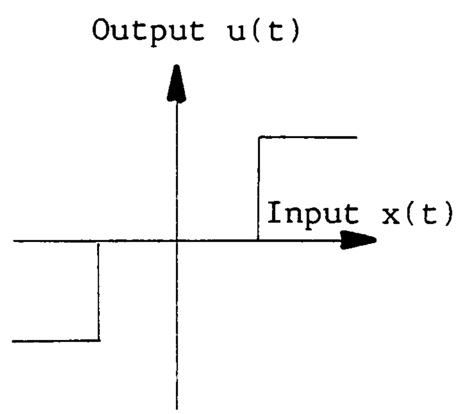
(a) A relay with hysteresis



(b) A relay with dead zones and hysteresis

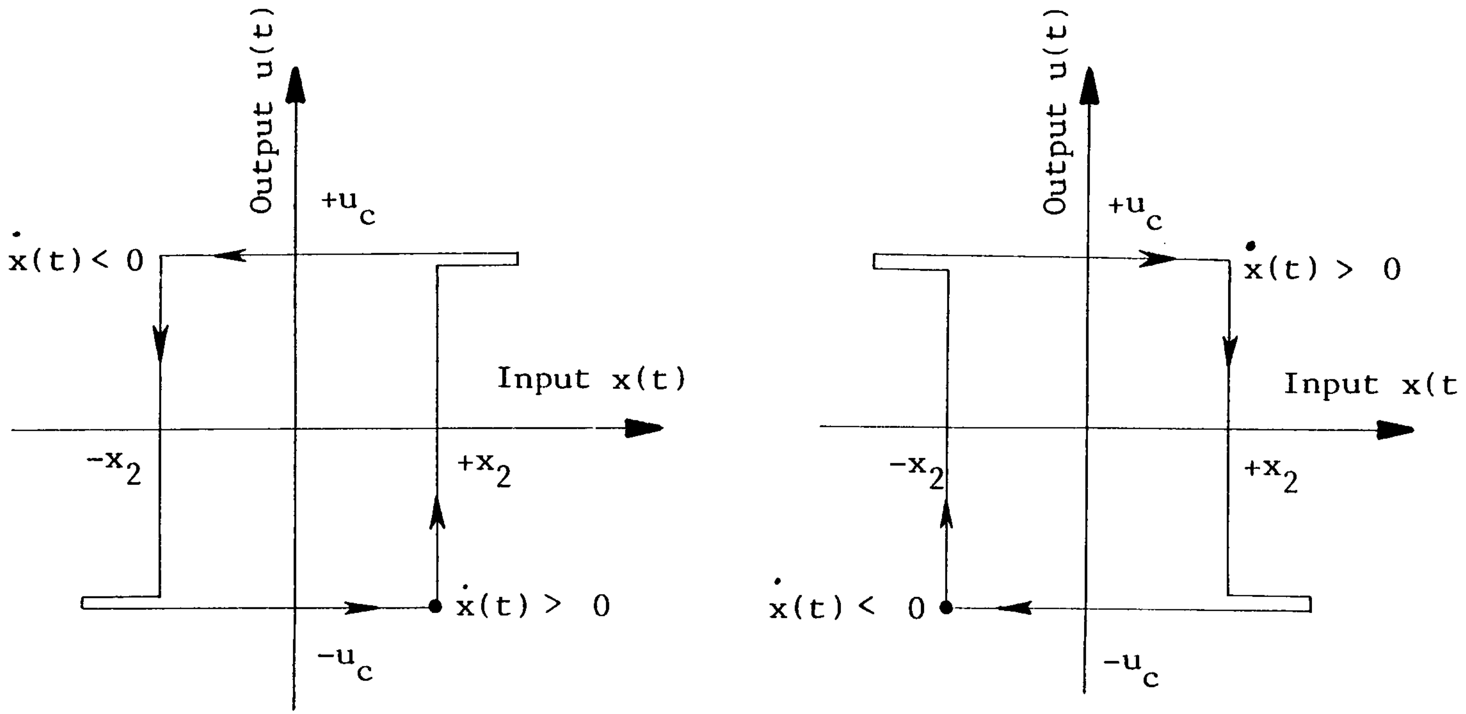


(c) An ideal relay



(d) A relay with dead zone

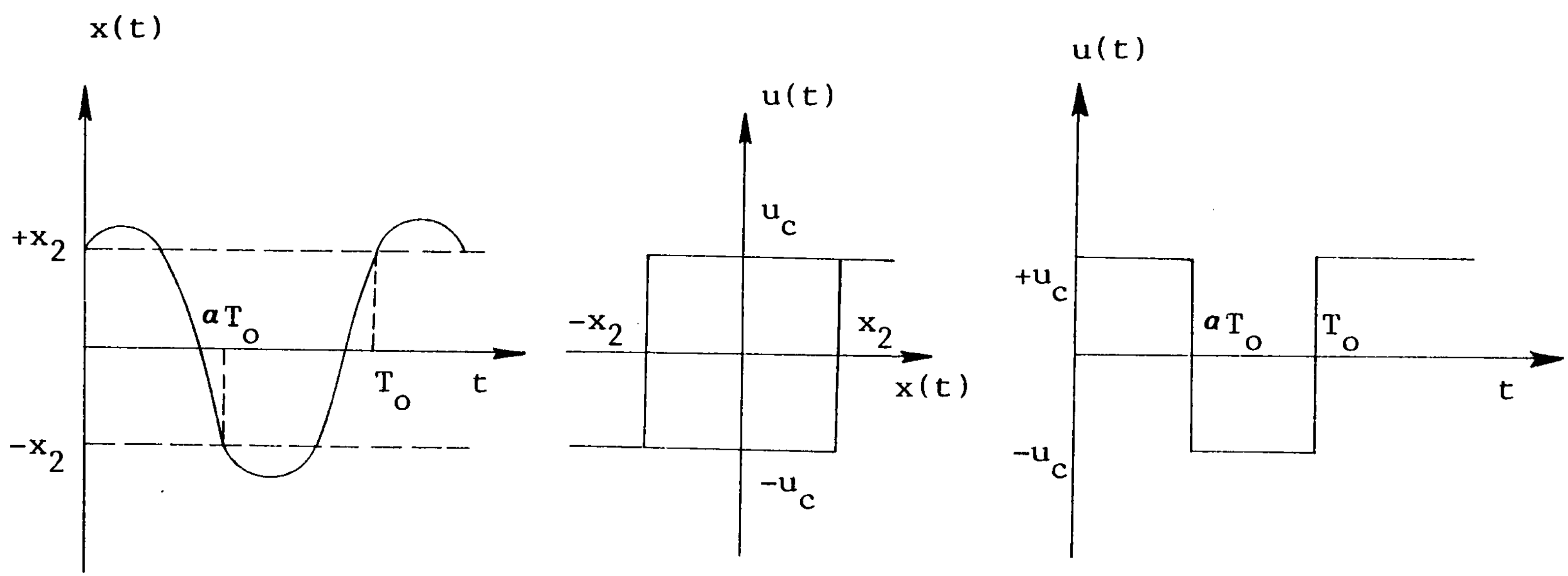
Figure 2.3. Several types of relays.



(a) Passive relay (b) Active relay

• -switch of the relay to a positive drive level which is used to define t = 0

Figure 2.4. Schematic representation of relays with hysteresis.



(c) Input-output relationship of passive relay

Figure 2.4 (continued)

The relay with hysteresis is the controller used in the nonlinear feedback control scheme studied in this research. Referring to Figure 2.4 the following notation is used:

$+u_c$ and $-u_c$	the two possible relay output values or drive levels
$-x_2$ and $+x_2$	relay input values at which the relay switches between drive levels
0 to $+x_2$	(or x_2) bandwidth of the relay
$\dot{x}(t)$	derivative of the input to the relay and its sign is shown at the input switching values for each relay
•	dot located on the relay where a switch to the positive drive level occurs, this reference point is used to define t = 0
o	open dot located on the relay to denote other times of importance in the analysis formalism
$\rightleftarrows$	double line outside the hysteresis of the relay is used to isolate the movement on the characteristic away from and back to the hysteresis region
T_o	period of the periodic input and output of the relay
a	fraction of the period of oscillation that the realy is at the positive drive level

Figure 2.4 and its notation are central to developing and understanding Tsypkin's method of analysis and the new extensions presented in this research. The notation of the double lines and indicated movement by the arrows outside the hysteresis region is new notation used in introducing some of the major new results from this study and will first be used in the next chapter.

Tsypkin's method of analysis is based on certain conditions which a relay input signal and output signal must fulfill if the relay is operative, i.e. changing drive levels, in a feedback control scheme as shown in Figure 2.2c. To present the first of these conditions, the switching condition, for this work, suppose a relay with hysteresis controller is used in a feedback control scheme as shown in Figure 2.2c. If the input to the relay controller, $x(t)$, is assumed to be periodic and of sufficient magnitude to cause the relay to switch between its drive levels it must fulfill switching conditions at the switching times. To mathematically state these conditions a time reference for the oscillation needs to be established. Time $\underline{t = 0}$ is defined as a time when the relay controller switches to the positive drive level. Consider the passive relay of Figure 2.4a and its input and output relationships of Figure 2.4c. If at $t = 0$, $x(0^-) = +x_2$, $x(0^-) > 0$ and $u(0) = -u_c$ (the dot (•) on the relay and at $t = 0$ on the input and output signals shown in Figure 2.4c, then at $t = \alpha T_o$ we must be arriving at $x(\alpha T_o) = -x_2$, $\dot{x}(\alpha T_o) < 0$ and $u(\alpha T_o) = +u_c$. Further at $t = T_o$, $x(T_o) = +x_2$, $\dot{x}(T_o) > 0$ and $u(T_o) = -u_c$. This sequence is repeated for every period of the oscillation and leads to the mathematic statement of the switching conditions for the passive relay with positive bandwidth:

$$x(nT_o^-) = x_2; \qquad \dot{x}(nT_o^-) > 0 \tag{2.12.a}$$

and

$$x((\alpha + n)T_o^-) = -x_2; \qquad \dot{x}((\alpha + n)T_o^-) < 0 \tag{2.12.b}$$

where

$$n = 0,1,2,3,\ldots$$

In a similar fashion, switching conditions for an active relay can be developed as:

$$x(nT_o^-) = -x_2; \qquad \dot{x}(nT_o^-) < 0 \tag{2.13.a}$$

and

$$x((\alpha + n)T_o^-) = x_2; \qquad \dot{x}((\alpha + n)T_o^-) > 0 \tag{2.13.b}$$

where

$$n = 0,1,2,3,\ldots$$

The notation of time equal to a specified time with a superscript minus, e.g. $t = t^-$, is needed to account for the discontinuities which occur in the relay's output at the switching times. The switching conditions are only <u>necessary</u> <u>conditions</u> for an oscillation to exist.

Additionally, it must be guarantee that the relay does not switch except at the assumed switching times. Again for the passive relay, when $u(t) = +u_c$, $x(t)$ must be greater than $-x_2$ for $0 < t < T_o$, $T_o < t < (\alpha + 1)T_o, \ldots,$ $nT_o < t < (\alpha + n)T_o$; during the other fraction of the cycle when the relay output $u(t) = -u_c$, $x(t)$ must be less than $+x_2$ until the next switching time, that is, for $T_o < t < T_o$, $(\alpha + 1)T_o < t < 2T_o, \ldots, (\alpha + n)T_o < t < (n + 1)T_o$. These conditions are called the continuity conditions. Thus, the continuity conditions for the passive relays with positive bandwidth are:

$$x(t) > -x_2; \qquad nT_o < t < (\alpha + n)T_o \tag{2.14.a}$$

and

$$x(t) < x_2; \qquad (\alpha + n)T_o < t < (n+1)T_o \tag{2.14.b}$$

where

$$n = 0,1,2,3,\ldots$$

and for the active with positive bandwidth relays are:

$$x(t) < x_2; \qquad nT_o < t < (\alpha + n)T_o \tag{2.15.a}$$

and

$$x(t) > -x_2; \qquad (\alpha + n)T_o < t < (n+1)T_o \tag{2.15.b}$$

where

$$n = 0,1,2,3,\ldots$$

The continuity conditions are <u>sufficient conditions</u> for a self-oscillation to exist in the controlled system.

The switching conditions and the continuity conditions taken together make up <u>necessary and sufficient conditions</u> for the existence of self-oscillation.

If the system is unforced (the set point is 0) and the relay is symmetric about the origin then α will be 1/2, and a symmetrical oscillation exists [12, p. 285]. All cases treated in the present work will have a set point of zero and the relay will be designed based on the linearized system leading to a symmetric relay, thus <u>in all developments below α will be taken to be 1/2.</u> with the parameter α specified only the first equation in Equations (2.12) to (2.15) need to be used in Tsypkin's analysis.

Extension of Tsypkin's Analysis to Systems Containing Dead Time

A summary of Tsypkin's analysis can be stated as [10, pp. 185-198; 13, pp. 455-463; 14, pp. 454-463; 48] : First, an oscillatory solution is assumed to exist in the system. Then utilizing the switching condition which is a necessary condition for the existence of self-oscillations, the frequencies of possible oscillations can be evaluated (Note multiple periodic solutions are possible). Next, the continuity condition, which is a sufficient condition to the existence of self-oscillations, is used to see if the solutions whose frequencies were determined by the switching condition are valid. Taken together the switching and continuity conditions are necessary and sufficient conditions. If a solution satisfies the switching and continuity conditions (necessary and sufficient conditions), the amplitude of oscillation can be calculated from the waveform needed to check the continuity condition. Finally, as in the case of steady state solutions, for the control and/or compensation strategy to be a success at least one periodic solution must be stable. Therefore the stability of the periodic solution needs to be determined.

Note that Tsypkin's analysis can be employed no matter if the process function is stable or unstable [7, pp. 113-114; 8].

The extension of Tsypkin's method to processes with dead time follows the formalisms of the developments for processes with no dead time [12, 7]. Therefore discussion of the ensuing derivations will be minimized and the reader is referred to the literature if additional details are needed. Additional complexities do arise when dead time is allowed to be part of the process transfer function. These new features will be treated in detail.

A partial check of the new results is provided by letting the dead time go to zero, in which case, the equations must be reduced to those for analysis with no dead time.

In the following treatment, notation for a total or complete transfer function is used:

$$G_T(s) = G_s(s)e^{-T_D s} \tag{2.16}$$

where

$G_T(s)$ total transfer function

$G_s(s)$ transfer function containing no dead time

T_D dead time

The process output y(t) will be discontinuous for the square wave input from the relay if the order of the denominator and numerator of $G_T(s)$ are the same. The derivative of the output y(t) will be discoutinuous if the order of the denominator is only greater than the numerator by one. Referring to Bergen [49], it is convenient to define a function $G_1(s)$ by:

$$G_s(s) = G_1(s) + G_s(\infty) \tag{2.17}$$

where

$$G_s(\infty) = \lim_{s \to \infty} G_s(s)$$

Note that $G_s(\infty)$ will be a constant. Utilizing this approach for the transfer function with dead time, i.e. substitute Equation (2.17) into (2.16), leads to

$$G_T(s) = [G_1(s) + G_s(\infty)]e^{-T_D s}$$

$$= G_1(s)e^{-T_D s} + G_s(\infty)e^{-T_D s}$$

and let

$$G_{T1}(s) = G_1(s)e^{-T_D s}$$

This formulation of $G_T(s)$ is illustrated in Figure 2.5.

$G_{T1}(s)$ and $G_1(s)$ are now transfer functions with at least one more pole than zeros. This means that the response of $G_{T1}(s)$ and $G_1(s)$ are continuous for impulse inputs as well as step inputs. $G_s(\infty)$ will be zero except in the case of the transfer function $G_s(s)$ having the same number of zeros as poles. Since $G_s(\infty)$ is a constant, the associated dead time just causes a delay in the output of relay as shown in Figure 2.5d. Therefore, the discontinuities that arise in the system output y(t) are contributed by $G_s(\infty)$ which is the same as for the system with no dead time. If $G_s(\infty)$ is a constant other than zero, the response of $G_s(\infty)$ to a step input will be a step, that is, the step response will have a

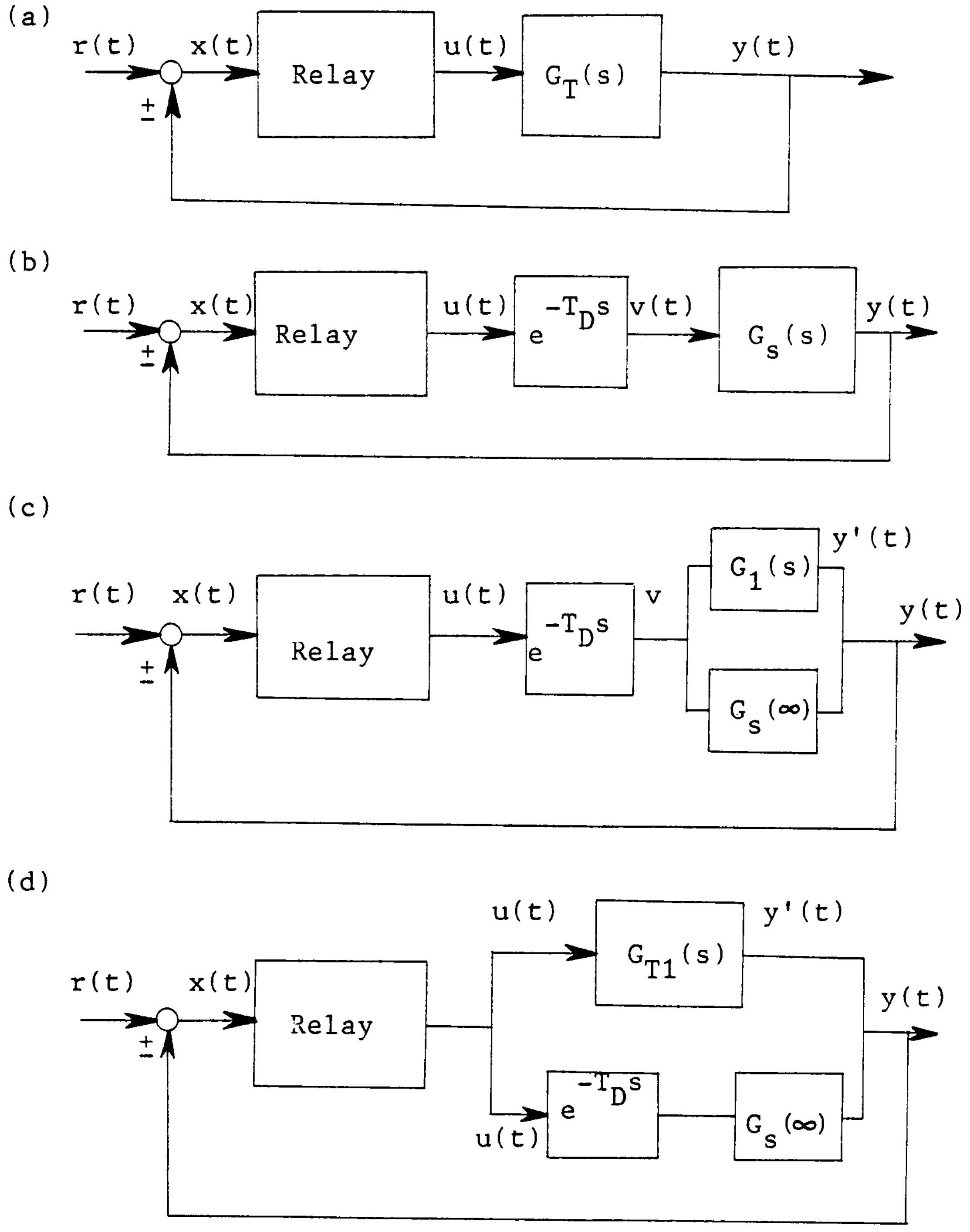


Figure 2.5. The relay controlled feedback system with the linear transfer function expressed as a low pass transfer function $G_{T1}(s)$ and a constant $G_s(\infty)$.

discontinuity. Likewise, the impulse response of $G_s(\infty)$ will be an impulse. The importance of these observations become obvious when it is recalled that the output of the relay is a square wave which is a series of steps. In addition, when the switching conditions are utilized in a negative feedback system requirements will be placed on the derivative of the transfer function output. When the derivative of the output, $\dot{y}(t)$, is taken it will involve taking derivatives of the input, u(t). As u(t) is a square wave, $\dot{u}(t)$ will be a series of impulses of alternating sign.

The output of the controller is a square wave of period T_o (see Figure 2.4c, page 28). Denoting its frequency by Ω_o, $\Omega_o = 2\pi/T_o$, the square wave can be expressed by a Fourier series, the result is

$$u(t) = \frac{4u_c}{\pi} \sum_{n\ \mathrm{odd}}^{\infty} \frac{1}{n} \sin n\Omega_o t \tag{2.19}$$

or in an equivalent form (recall $e^{\psi t} = \sin n\psi t + j\cos n\psi t$)

$$u(t) = \frac{4u_c}{\pi} \sum_{n\ \mathrm{odd}}^{\infty} \frac{1}{n} \mathrm{Im}\ e^{jn\Omega_o t} \tag{2.20}$$

where

$$n = 1,3,5,\ldots$$

The notation "odd" on the summation sign will be used to denote the sum is over odd values of n only.

No matter if the linear element is either stable or unstable, the steady state output resulting from a sine wave input is easily expressed in terms of the magnitude and phase angle of the linear element's transfer function [7, pp. 122-123]. Therefore as the superposition principle holds, the sum of the transfer function $G_{T1}(s)$ response from all sine wave inputs of Equations (2.19) and (2.20) gives

$$y(t) = \frac{4u_c}{\pi} \sum_{n\ odd}^{\infty} \frac{1}{n} \mathrm{Im}\left[G_1(jn\Omega_o)e^{jn\Omega_o(t - T_D)}\right] \tag{2.21}$$

where y (t) is the output from $G_{T1}(s)$ as shown in Figure 2.5. Since the order of the numerator of G_{T1} is at most the order of the denominator of G_{T1} for a real system, $G_s(\infty)$ is a constant. Thus, the waveform of oscillation y(t) and derivative $\dot{y}(t)$ can be written as:

$$y(t) = \frac{4u_c}{\pi} \sum_{n\ odd}^{\infty} \frac{1}{n} \mathrm{Im}\left[G_1(jn\Omega_o)e^{jn\Omega_o(t - T_D)}\right]$$

$$+ u(t - T_D)\, G_s(\infty) \tag{2.22}$$

$$\dot{y}(t) = \frac{4u_c\,\Omega_o}{\pi} \sum_{n\ odd}^{\infty} \mathrm{Re}\left[G_1(jn\Omega_o)e^{jn\Omega_o(t - T_D)}\right]$$

$$+ \dot{u}(t - T_D)G_s(\infty) \tag{2.23}$$

The Fourier series of a piecewise-continuous function converges to the midpoint of each finite discontinuity, Hence, this discontinuity must be accounted for at the switching time, $t = 1/2\ T_o = \pi/\Omega_o$,

$$y(\frac{\pi^-}{\Omega_o}) = -\frac{4u_c\Omega_o}{\pi}\sum_{n\ odd}^{\infty} Re\,[\,G_1(jn\Omega_o)e^{-jn\Omega_o T_D}\,] + u_c \lim_{s\to\infty} sG_1(s) \qquad (2.24)$$

$$y(\frac{\pi^+}{\Omega_o}) = -\frac{4u_c\Omega_o}{\pi}\sum_{n\ odd}^{\infty} Re\,[\,G_1(jn\Omega_o)e^{-jn\Omega_o T_D}\,] - u_c \lim_{s\to\infty} sG_1(s) \qquad (2.25)$$

Tsypkin defined a complex function which is called Tsypkin's function as :

$$\Lambda(\Omega) = \sum_{n\ odd}^{\infty} Re\ G_{T1}(jn\Omega) + j\sum_{n\ odd}^{\infty}\frac{1}{n}\, Im\ G_{T1}(jn\Omega) \qquad (2.26)$$

By employing Equation (2.12) or (2.13) with $a = 1/2$, the switching conditions, and rearranging Equations (2.22) and (2.24) the frequency Ω_o of the oscillation can be determined. For a passive element,

$$Im\,[\,\Lambda(\Omega_o)] = \sum_{n\ odd}^{\infty}\frac{1}{n} Im[G_1(jn\Omega_o)e^{-jn\Omega_o T_D}\,]$$

$$= \frac{\pi}{4}\left[G_s(\infty) - \frac{x_2}{u_c} \right] \tag{2.27}$$

$$\mathrm{Re}\,[\Lambda(\Omega_o)] = \sum_{n\ \mathrm{odd}}^{\infty} \mathrm{Re}[G_1(jn\Omega_o)e^{-jn\Omega_o T_D}]$$

$$< \frac{\pi}{4\Omega_o} \lim_{s\to\infty} sG_1(s) \tag{2.28}$$

and for an active element,

$$\mathrm{Im}\,[\Lambda(\Omega_o)] = \sum_{n\ \mathrm{odd}}^{\infty} \frac{1}{n} \mathrm{Im}[G_1(jn\Omega_o)e^{-jn\Omega_o T_D}]$$

$$= \frac{\pi}{4}\left[G_s(\infty) + \frac{x_2}{u_c}\right] \tag{2.29}$$

$$\mathrm{Re}\,[\Lambda(\Omega_o)] = \sum_{n\ \mathrm{odd}}^{\infty} \mathrm{Re}[G_1(jn\Omega_o)e^{-jn\Omega_o T_D}]$$

$$> \frac{\pi}{4\Omega_o} \lim_{s\to\infty} sG_1(s) \tag{2.30}$$

Here the transfer function $G_1(s)$ is derived by following the procedures stated previously and is represented by Equation (2.11). Further development of Tsypkin's analysis requires this transfer function to be expressed by a partial fraction expansion comprised of terms as:

$$\frac{1}{s+p},\quad \frac{s+\mu}{(s+p)^2},\quad \frac{(s+\mu_1)(s+\mu_2)}{(s+p)^3},\ \ldots$$

For the system containing zero dead time, i.e. $T_D = 0$ in

Equations (2.27-2.30), Table 2.1 developed by Gelb and Vander Velde [10] can be used to convey the summations of infinite series to closed forms. With the closed forms of Table 2.1, the solution for Ω_o which satisfies the switching condition can be found much faster and easier. Once a solution is found, Equations (2.14) and (2.15) the continuity conditions, must be checked by using Equation (2.22) to generate the waveform of oscillation. The appearance of the term $e^{jn\Omega_o t}$ in Equation (2.22) with T_D = 0 requires a different infinite series from those accounted in the switching conditions to be handled. Again, Gelb and Vander Velde [10] developed a table of closed forms for these special infinite series for use with continuity conditions (see Table 2.2).

Now let's return to systems containing dead time, T_D = 0. The switching conditions which can be used to solve the parameters of oscillation are expressed by Equation (2.27) or Equation (2.29) for passive and active elements, respectively. If we denote $(t - T_D)$ as t' in Equation (2.22), the form of the infinite series which are needed in the switching (Equations (2.27) and (2.29)) and continuity (Equation (2.22)) conditions for a dead time systems are identical to $e^{jn\Omega_o t}$ encounted in the continuity conditions for a zero dead time system except for the sign on the exponent. Consequently, the closed forms needed to analyze systems containing dead time can

Table 2.1. Closed forms of infinite series for use with Tsypkin's switching conditions of system without dead time.

$G_1(s)$	$\sum_{n=1,3,5,\ldots}^{\infty} \mathrm{Re}\,[G_1(jn\Omega_0)]$	$\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n}\mathrm{Im}\,[G_1(jn\Omega_0)]$
$\frac{1}{s+a}$	$\frac{\pi}{4\omega_0}\tanh\left(\frac{\pi a}{2\Omega_0}\right)$	$-\frac{\pi}{4a}\tanh\left(\frac{\pi a}{2\Omega_0}\right)$
$\frac{s+b}{(s+a)^2}$	$\frac{\pi}{8\Omega_0\cosh^2\left(\frac{\pi a}{2\Omega_0}\right)} \cdot \left[\sinh\left(\frac{\pi a}{\Omega_0}\right) - \frac{\pi}{\Omega_0}(b-a)\right]$	$-\frac{\pi}{8a^2\cosh^2\left(\frac{\pi a}{2\Omega_0}\right)} \cdot \left[b\sinh\left(\frac{\pi a}{\Omega_0}\right) - \frac{\pi a}{\Omega_0}(b-a)\right]$
$\frac{(s+b_1)(s+b_2)}{(s+a)^3}$	$\frac{\pi}{16\Omega_0\cosh^2\left(\frac{\pi a}{2\Omega_0}\right)}\left[2\sinh\left(\frac{\pi a}{\Omega_0}\right) - \frac{\pi^2}{\Omega_0^2}(a-b_1)(a-b_2)\tanh\left(\frac{\pi a}{2\Omega_0}\right) + \frac{2\pi}{\Omega_0}(2a-b_1-b_2)\right]$	$-\frac{\pi}{16a^3\cosh^2\left(\frac{\pi a}{2\Omega_0}\right)}\left[2b_1b_2\sinh\left(\frac{\pi a}{\Omega_0}\right) - \pi^2(a-b_1)(a-b_2)\tanh\left(\frac{\pi a}{2\Omega_0}\right) + 2\pi\left(\frac{a}{\Omega_0}\right)(a^2-b_1b_2)\right]$

* A useful relationship in applications of this table is $\frac{1}{(s+a)^2} = \lim_{b\to\infty}\left[\frac{1}{b}\,\frac{s+b}{(s+a)^2}\right]$.

Table 2.2. Summations of infinte series for use with the switching and continuity conditions of the system containing dead time.

(1) $\sum_{k\ \mathrm{odd}}^{\infty} \frac{\sin kx}{k} = \frac{\pi}{4}$ $0<x<\pi$

(2) $\sum_{k\ \mathrm{odd}}^{\infty} \frac{\sin kx}{k(k^2+a^2)} = f(x,a)$

(3) $\sum_{k\ \mathrm{odd}}^{\infty} \frac{\sin kx}{k(k^2+a_1^2)(k^2+a_2^2)} = \frac{f(x,a_1)-f(x,a_2)}{a_2^2-a_1^2}$

(4) $\sum_{k\ \mathrm{odd}}^{\infty} \frac{\cos kx}{k^2+a^2} = f_x(x,a)$

(5) $\sum_{k\ \mathrm{odd}}^{\infty} \frac{\cos kx}{(k^2+a_1^2)(k^2+a_2^2)} = \frac{f_x(x,a_1)-f_x(x,a_2)}{a_2^2-a_1^2}$

where $f(x,a) = \frac{\pi}{4}\,\frac{\cosh(\pi a/2) - \cosh(\pi a/2 - ax)}{a^2\cosh(\pi a/2)}$

$f_x(x,a) = \frac{\partial f}{\partial x}(x,a) = \frac{\pi}{4}\,\frac{\sinh(\pi a/2 - ax)}{a\cosh(\pi a/2)}$

be generated by employing Table 2.2. The closed forms are very important when translating the analysis equations into computer algorithms.

As an example suppose $G_s(s)$ has the form

$$G_s(s) = \frac{k}{s + p} \tag{2.31}$$

Then G which appears in Tsypkin's series is

$$G_T(s) = \frac{k}{s + p} e^{-T_D s} \tag{2.32}$$

Recalling the identity

$$e^{-j\psi} = \cos\psi - j\sin\psi \tag{2.33}$$

gives

$$G_T(jn\Omega_o) = \frac{k}{jn\Omega_o + p}(\cos n\Omega_o T_D - j\sin n\Omega_o T_D) \tag{2.34}$$

Rearranging $G_T(jn\Omega_o)$ in terms of real and imaginary parts yields

$$G_T(jn\Omega_o) = \frac{kp\cos n\Omega_o T_D - kn\Omega_o \sin n\Omega_o T_D}{p^2 + n^2\Omega_o^{\,2}}$$

$$+ j\,\frac{-kn\Omega_o \cos n\Omega_o T_D - kp\sin n\Omega_o T_D}{p^2 + n^2\Omega_o^{\,2}} \tag{2.35}$$

Consequently, Tsypkin's series can be expressed as:

$$\mathrm{Re}[\,\Lambda(\Omega_o)] = \sum_{n\,odd}^{\infty} \mathrm{Re}\; G_T(jn\Omega_o)$$

$$= \sum_{n\,odd}^{\infty} \frac{kp\cos n\Omega_o T_D - kn\Omega_o \sin n\Omega_o T_D}{p^2 + n^2\Omega_o^{\,2}}$$

$$= \frac{kp}{\Omega_o} \sum_{n\,odd}^{\infty} \frac{\cos n\Omega_o T_D}{(n^2 + (p/\Omega_o)^2)}$$

$$- \frac{k}{\Omega_o} \sum_{n\,odd}^{\infty} \frac{n\sin n\Omega_o T_D}{(n^2 + (p/\Omega_o)^2)} \tag{2.36}$$

and

$$\mathrm{Im}[\,\Lambda(\Omega_o)\,] = \sum_{n\,odd}^{\infty} \frac{1}{n}\, \mathrm{Im}\; G_T(jn\Omega_o)$$

$$= \sum_{n\,odd}^{\infty} \frac{1}{n} \left(\frac{-kn \cos n\Omega_o T_D - kp\sin n\Omega_o T_D}{p^2 + n^2\Omega_o^{\,2}} \right)$$

$$= -\frac{k}{\Omega_o} \sum_{n\,odd}^{\infty} \frac{\cos n\Omega_o T_D}{(n^2 + (p/\Omega_o)^2)}$$

$$- \frac{kp}{\Omega_o} \sum_{n\,odd}^{\infty} \frac{\sin n\Omega_o T_D}{n(n^2 + (p/\Omega_o)^2)} \tag{2.37}$$

Closed forms for all of the above infinite series are contained in Table 2.2 except for

$$\sum_{n\,odd}^{\infty} \frac{n\sin n\Omega_o T_D}{n^2 + (p/\Omega_o)^2}$$

in Equation (2.36). Consequently, additional development is needed to find the closed form of this infinite series. Observe that this summation which can be representented in the form

$$\sum_{nodd}^{\infty} \frac{n \sin nx}{n^2 + a^2} \tag{2.38}$$

where

$$a = p/\Omega_o$$

$$x = \Omega_o T_D$$

and can be obtained by differentiation with respect to x of entry 4 in Table 2.2. Thus

$$\frac{d}{dx} \sum_{nodd}^{\infty} \frac{\cos nx}{n^2 + a^2}$$

$$= \sum_{nodd}^{\infty} \frac{d}{dx} \frac{\cos nx}{n^2 + a^2}$$

$$= - \sum_{nodd}^{\infty} \frac{n \sin nx}{n^2 + a^2} \tag{2.39}$$

Therefore, by substitution of the closed form for entry 4 in step 2 below we obtain

$$\sum_{nodd}^{\infty} \frac{n \sin nx}{n^2 + a^2}$$

$$= - \frac{d}{dx} \sum_{nodd}^{\infty} \frac{\cos nx}{n^2 + a^2}$$

$$= - \frac{d}{dx} f_x(x,a)$$

$$= - f_{xx}(x,a) \tag{2.40}$$

From the definition of $f_x(x,a)$ in Table 2.2, $f_{xx}(x,a)$ can be obtained by differentiation of $f_x(x,a)$ with respect to x:

$$-f_{xx}(x,a) = \frac{d}{dx} f_x(x,a)$$

$$= - \frac{d}{dx}\left[\frac{\pi}{4} \frac{\sinh(\pi a/2 - ax)}{a\cosh(\pi a/2)}\right]$$

$$= \frac{\pi}{4} \frac{\cosh(\pi a/2 - ax)}{\cosh(\pi a/2)} \tag{2.41}$$

Employing Table 2.2 and Equations (2.40-2.41) allows Equations (2.36) and (2.37) to be written as

$Re[\Lambda(\Omega_o)]$

$$= \frac{kp}{\Omega_o^2} f_x(\Omega_o T_D, \frac{p}{\Omega_o}) - \frac{k}{\Omega_o}(-f_{xx}(\Omega_o T_D, \frac{p}{\Omega_o})) \tag{2.42}$$

$Im[\Lambda(\Omega_o)]$

$$= -\frac{k}{\Omega_o} f_x(\Omega_o T_D, \frac{p}{\Omega_o}) - \frac{kp}{\Omega_o^2} f(\Omega_o T_D, \frac{p}{\Omega_o}) \qquad (2.43)$$

where in order for the infinite series encountered in Equations (2.36) and (2.37) to be convergent and thus the closed forms in Equations (2.42) and (2.43) to be valid a restriction must be placed on x (see Equation (2.38) and Table 2.2 [7,10]) or T_D

$$0 < x < \pi$$

$$0 < T_D < T_o/2$$

where f, and f_x are defined in Table 2.2, page 43 and f_{xx} is defined by Equation (2.41). Expansion of Equations (2.42) and (2.43) by employing the definitions of f, f_x and f_{xx} yields

$$\mathrm{Re}[\Lambda(\Omega_o)]$$

$$= \frac{\pi k}{4\Omega_o} \frac{\sinh(\pi p/2\Omega_o - pT_D)}{\cosh(\pi p/2\Omega_o)}$$

$$- \frac{\pi k}{4\Omega_o} \frac{\cosh(\pi p/2\Omega_o - pT_D)}{\cosh(\pi p/2\Omega_o)} \qquad (2.44)$$

and

$$\mathrm{Im}[\Lambda(\Omega_o)]$$

$$= - \frac{\pi k}{4p} \frac{\sinh(\pi p/2\Omega_o - pT_D)}{\cosh(\pi p/2\Omega\ )}$$

$$- \frac{\pi k}{4p} \frac{\cosh(\pi p/2\Omega_o) - \cosh(\pi p/2\Omega_o - pT_D)}{\cosh(\pi p/2\Omega_o)} \quad (2.45)$$

Recalling

$$\sinh(x-y) = (\sinh x)(\cosh y) - (\cosh x)(\sinh y)$$

$$\cosh(x-y) = (\cosh x)(\cosh y) - (\sinh x)(\sinh y)$$

and

$$\sinh(a) = \frac{e^{a} - e^{-a}}{2}, \quad \cosh(a) = \frac{e^{a} + e^{-a}}{2}$$

allows compression of the two previous equations to

$$\mathrm{Re}[\Lambda(\Omega_o)]$$

$$= \frac{\pi k}{4\Omega_o} \left(\tanh\frac{\pi p}{2\Omega_o} - 1\right)e^{pT_D} \quad (2.46)$$

and

$$\mathrm{Im}[\Lambda(\Omega_o)]$$

$$= - \frac{\pi k}{4p}\left[\left(\tanh\frac{\pi p}{2\Omega_o} - 1\right)e^{pT_D} + 1\right] \quad (2.47)$$

With the switching conditions defined in Equations (2.27-2.30) and the Tsypkin's function for the example $G_s(s)$ of Equation (2.31) given by the previous two equations, the switching conditions for the example $G_s(s)$ can be written for a passive element as

$$-\frac{\pi k}{4p}\left[\left(\tanh\frac{\pi p}{2\Omega_o}-1\right)e^{pT_D}+1\right]$$

$$=\frac{\pi}{4}\left[G_s(\infty)-\frac{x_2}{u_c}\right] \tag{2.48}$$

$$\frac{\pi k}{4\Omega_o}\left(\tanh\frac{\pi p}{2\Omega_o}-1\right)e^{pT_D}$$

$$<\frac{\pi}{4\Omega_o}\lim_{s\to\infty} sG_1(s) \tag{2.49}$$

and for an active element as

$$-\frac{\pi k}{4p}\left[\left(\tanh\frac{\pi p}{2\Omega_o}-1\right)e^{pT_D}+1\right]$$

$$=\frac{\pi}{4}\left[G_s(\infty)+\frac{x_2}{u_c}\right] \tag{2.50}$$

$$\frac{\pi k}{4\Omega_o}\left(\tanh\frac{\pi p}{2\Omega_o}-1\right)e^{pT_D}$$

$$>\frac{\pi}{4\Omega_o}\lim_{s\to\infty} sG_1(s) \tag{2.51}$$

Moreover, the waveform of the oscillation, y(t), given by Equation (2.22) can be written in closed form for the first order transfer function containing dead time T_D of Equation (2.31). By realizing that $G_s(\infty) = 0$ and $G_1(\infty) = G_s(\infty)$ for this present example, the simplification of Equation (2.22) can be obtained

$$y(t) = \frac{4u_c}{\pi} \sum_{n\ odd}^{\infty} \frac{1}{n} \mathrm{Im}\,[\,G_1(jn\Omega_o)e^{jn\Omega_o(t - T_D)}\,] \qquad (2.52)$$

Substituting the expression of $G_1(jn\Omega_o)$ based on Equation (2.31), rearranging and grouping the imaginary part of Equation (2.52) give

$$y(t) = -\frac{4u_c k}{\Omega_o} \sum_{nodd}^{\infty} \frac{\cos n\Omega_o(t - T_D)}{(n^2 + (p/\Omega_o)^2)}$$

$$+ \frac{4u_c kp}{\Omega_o^2} \sum_{nodd}^{\infty} \frac{\sin n\Omega_o(t - T_D)}{n(n^2 + (p/\Omega_o)^2)} \qquad (2.53)$$

When converting these infinite summations to the closed forms in Equation (2.53) by employing Table 2.2, page 43, a restriction which is different from that encounted in the switching conditions is placed on time, t, instead of dead time, T_D, due to the different lumped parameters on the exponent. Comparison of the infinite summation in Equation (2.53) with those in Table 2.2, page 43 reveals that the lumped parameter $\Omega_o(t - T_D)$ in Equation (2.53) is identical to the x whose range is limited from zero to π in Table 2.2, page 43. Thus, a restriction is placed on t as

$$T_D < t < T_D + T_o/2 \tag{2.54}$$

Utilizing Table 2.2, page 43 allows Equation (2.53) to be expressed in the closed forms accompanying with a limited range of t stated in Equation (2.54)

$$y(t) = -\frac{ku_c}{p}\left[\left(\tanh\frac{\pi p}{2\Omega_o} + 1\right)e^{-P(t-T_D)} - 1\right] \tag{2.55}$$

The restriction on t in Equation (2.54) gives the fact that the waveform of oscillation expressed by Equation (2.55) is valid only when t is in the range from T_D to $T_D + T_o/2$.

Summary

In this chapter the extension of Tsypkin's method for the relay feedback control scheme on the system containing dead time has been established. Equations (2.22) and (2.27-2.30) serve as the starting point to give a quantative analysis of the characterization of the system. Moreover, the procedure of converting Tsypkin's equations into the closed forms is presented in detail via the example of a first order transfer function containing dead time (Equation (2.32)). Schemes for handling additional difficulties encountered in the switching conditions due to the existence of dead time are established.

However, it is noted that Table 2.2, page 43 for the closed form placed a restriction on T_D of the switching conditions, requiring T_D in the range from zero to half period of oscillation, $T_o/2$. This limitation will be removed in Chapter V and the case of significantly large dead time successfully addressed. This in part leads to the design strategies for dead time compensation. In treating the problems of large dead times, i.e. $T_D > T_o/2$, new concepts of "reduced dead time" and "negative bandwidth" relay which this research introduces and defines are needed.

Chapter IV details the negative bandwidth relay and states the appropriate switching and continuity conditions for these new relays. The additional detail added to the sketch of the classical positive bandwidth relay in Figure 2.4, pages 27-28 discussed at Equations (2.12-2.15) are very helpful in presenting the negative bandwidth relay.

Before proceeding to design strategies and dead time compensation, the central analysis equations, the switching conditions (Equations (2.27-2.28) for passive and Equations (2.29-3.30) for active relay) and the continuity conditions, Equation (2.22), are applied to the demonstration example of a CSTR.

CHAPTER III

APPLICATION AND EXAMPLE

Reactor Modeling and Analysis

The process chosen for application and discussion of the nonlinear feedback control scheme and development of design procedures is the externally cooled continuous flow stirred tank reactor (CSTR) shown in Figure 3.1. The CSTR is carrying out a homogeneous, first order, irreversible, exothermic reaction illustrated by the following equations:

$$A \xrightarrow{K} P \tag{3.1.a}$$

$$K = K_o e^{-E/RT} \tag{3.1.b}$$

$$r = KC_A \tag{3.1.c}$$

where

K_o Arrhenius constant

E activation energy

R gas constant

r rate of reaction

T temperature in the reactor

C_A concentration of reactant A in the reactor

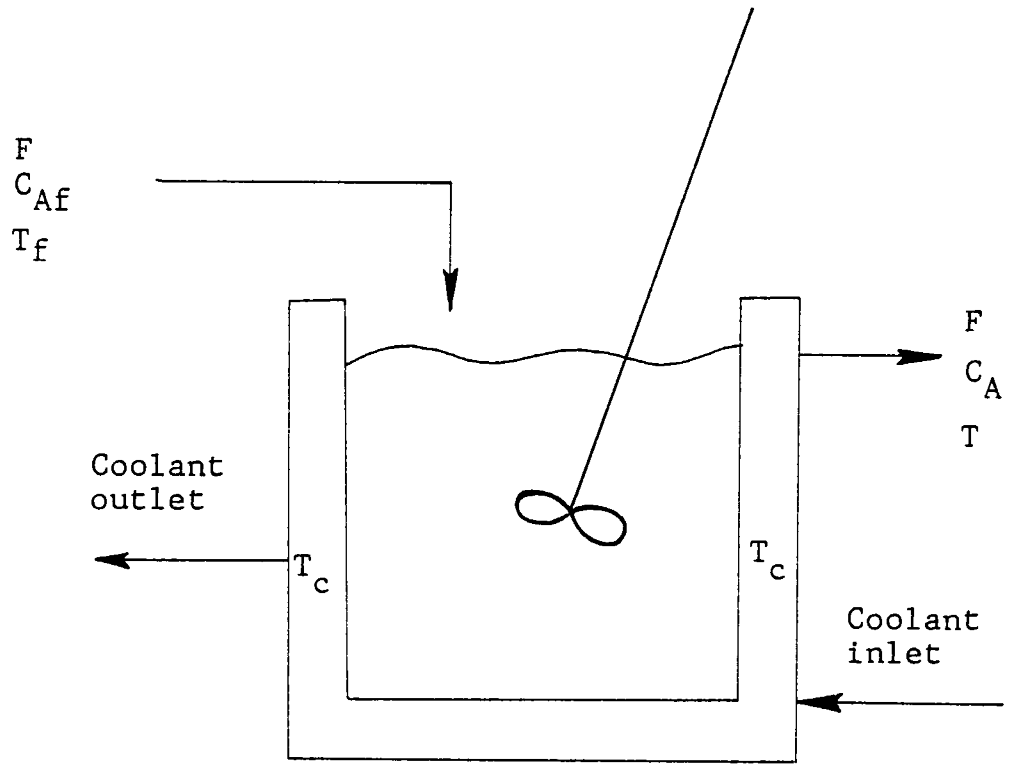


Figure 3.1. Schematic representation of an externally cooled CSTR.

The mass and energy balances for constant volume, density and heat capacity are:

$$V \frac{dC_A}{dt'} = F(C_{Af} - C_A) - VK_o e^{-E/RT} C_A \tag{3.2}$$

$$V\rho C_p \frac{dT}{dt'} = \rho C_p F(T_f - T) + V(-\Delta H)K_o e^{-E/RT} C_A$$

$$- H_A (T - T_c) \tag{3.3}$$

where

t'	time
V	reactor volume
ρ	density
$(-\Delta H)$	heat of reaction
C_{Af}	reactant feed concentration
T_c	average coolant temperature
F	volumetric flow rate
H_A	product of heat transfer coefficient and surface area in reactor
C_p	heat capacity
T_f	feed stream temperature

Introducing the dimensionless variables and parameters:

$$\theta = \frac{V}{F} \qquad t = \frac{t'}{\theta}$$

$$\mathcal{Y} = \frac{T}{T_f} \qquad \mathcal{W} = \frac{C_A}{C_{Af}}$$

$$\mathcal{U} = \frac{T_c}{T_f} \qquad \gamma = \frac{E}{RT_f}$$

$$B = \frac{H_A}{F\ C_p} \qquad \beta = \frac{(-\Delta H)C_{Af}}{C_p T_f}$$

$$Da = \frac{VK_o e^{-\gamma}}{F} \tag{3.4}$$

allows the model to be written in the dimensionless form:

$$\frac{d\mathcal{W}}{dt} = (1 - \mathcal{W}) - \mathcal{W} Da e^{-\gamma(1/\mathcal{Y} - 1)}$$

$$= f_1(\mathcal{W}, \mathcal{Y}, \mathcal{U}) \tag{3.5}$$

$$\frac{d\mathcal{Y}}{dt} = (1 - \mathcal{Y}) - B(\mathcal{Y} - \mathcal{U}) + Da\beta e^{-\gamma(1/\mathcal{Y} - 1)} \cdot \mathcal{W}$$

$$= f_2(\mathcal{W}, \mathcal{Y}, \mathcal{U}) \tag{3.6}$$

At steady state then, according to Equation (3.5)

$$\mathcal{W}_s = \frac{1}{1 + Da e^{-\gamma(1/\mathcal{Y}_s - 1)}} \tag{3.7}$$

here, the subscript s denotes the dimensionless concentration, $\mathcal{W}_s$, and dimensionless temperature, $\mathcal{Y}_s$, at steady state.

Equation (3.6) for steady state conditions can be rearranged to

$$-(1 - \mathcal{Y}_s) + B(\mathcal{Y}_s - \mathcal{U}_s) = Da\beta e^{-\gamma(1/\mathcal{Y}_s - 1)} \tag{3.8}$$

The left hand side of Equation (3.8) can be viewed as the total steady state rate of heat removal, Rr, by heating of the reactants and by the cooling system. The right hand side is then viewed as the steady state rate of heat generation, Rg, due to energy released by the reaction

$$R_r = -(1 - \mathcal{Y}_s) + B(\mathcal{Y}_s - \mathcal{U}_s) \tag{3.9}$$

$$R_g = Da\beta e^{-\gamma(1/\mathcal{Y}_s - 1)} \tag{3.10}$$

At steady state these two rates must be equal, i.e. for Equation (3.8) to be satisfied

$$R_r = R_g \qquad \text{at steady state}$$

Substitution of Equation (3.7) into Equation (3.10) gives

$$R_g = \frac{Da\beta e^{-\gamma(1/\mathcal{Y}_s - 1)}}{1 + Dae^{-\gamma(1/\mathcal{Y}_s - 1)}} \tag{3.11}$$

A plot of both Rr and Rg vs. $\mathcal{Y}$ illustrated in Figure 3.2 gives the so called "Van Heerden" plot [19,20]. It is obvious that R_r vs. $\mathcal{Y}$ is a straight line with the slope of (1 + B), while R_g vs. $\mathcal{Y}$ gives a sigmoid curve. The intersections of these two lines are steady state solutions. In general, there is an odd number of steady state solutions. Figure 3.2 shows a case of three steady states. After the steady state solutions for $\mathcal{Y}_s$ have been found, the corresponding values of $\mathcal{W}_s$ can be calculated from Equation (3.7).

Local Stability

The equations representing this two variable CSTR system are nonlinear. Treatment of this nonlinear system requires linearization around the desired steady state, and the linearized model gives an exact representation of the actual nonlinear system only in a sufficiently small region of this steady state. Thus analysis based on a linearized model can provide information on the dynamics of the nonlinear system only when it is sufficiently close to the steady state in question. Under this restriction, a steady state will be termed locally asymptotically

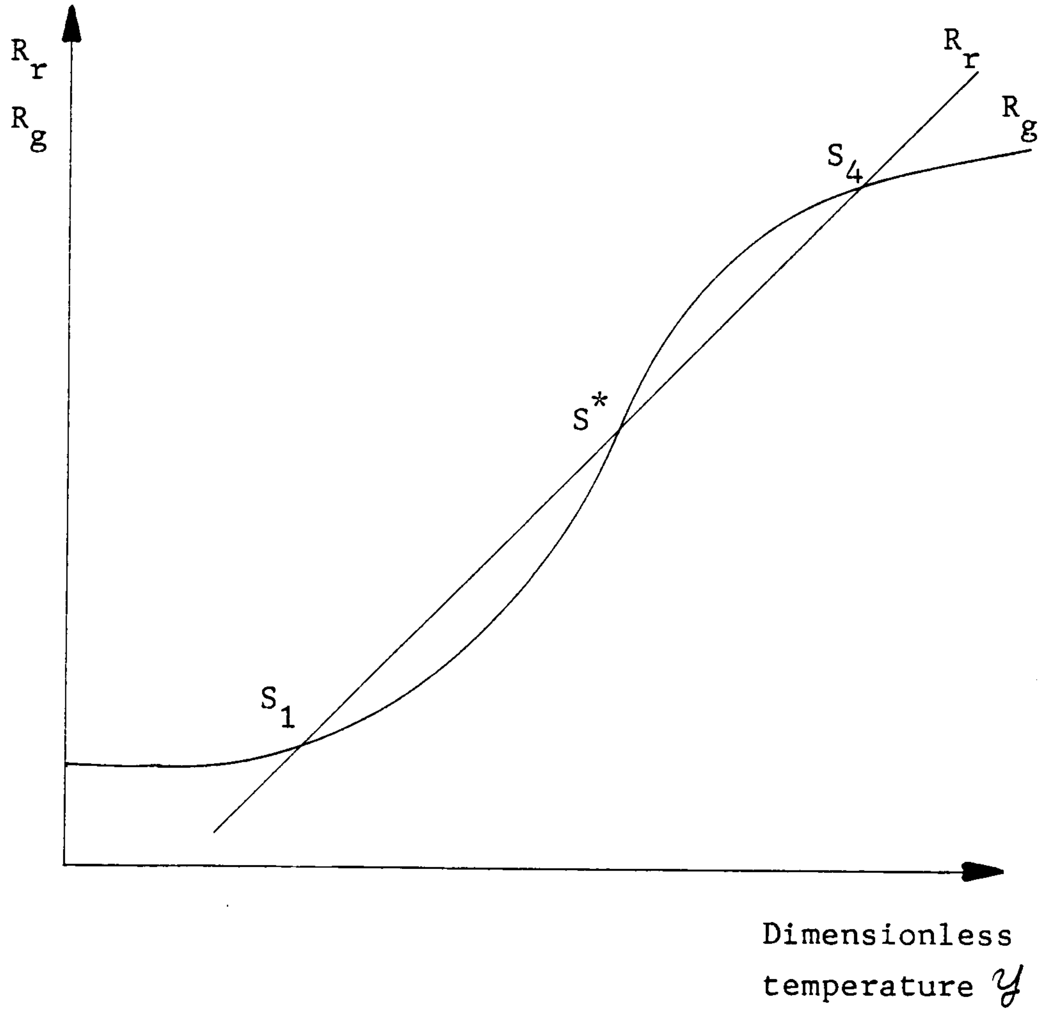


Figure 3.2. Van Heerden plot for the second order system with three steady state solutions.

stable if the system state returns to the steady state after being perturbed slightly. If the system leaves the neighborhood of the steady state after a small displacement, the steady state is locally asymptotically unstable.

As mentioned previously, Van Heerden gave a slope condition in his early paper [19] that states for a steady state to be stable:

$$\frac{dR_r}{d\mathcal{Y}} > \frac{dR_g}{d\mathcal{Y}} \tag{3.12}$$

and we will see later that it is only a necessary stability condition for second and higher order systems. Equations (3.5) and (3.6) can be linearized by the Taylor series expansion approximation around the steady state $(\mathcal{W}_s, \mathcal{Y}_s)$, truncating the series after the first order partial derivatives. Expressed in terms of deviation variables from the steady state the Jacobian matrix is obtained and Equations (3.5) and (3.6) can be written as

$$\begin{bmatrix} \dot{w} \\ \dot{y} \end{bmatrix} = \underline{\underline{A}} \begin{bmatrix} w \\ y \end{bmatrix} \tag{3.13}$$

where

$$w = \mathcal{W} - \mathcal{W}_s \tag{3.14.a}$$

$$y = \mathcal{Y} - \mathcal{Y}_s \tag{3.14.b}$$

$$\underline{\underline{A}} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

with

$$A_{11} = \left.\frac{\partial f_1}{\partial \mathcal{W}}\right|_{s.s.} = -1 - Da e^{-\gamma(1/\mathcal{Y}_s - 1)}$$

$$A_{12} = \left.\frac{\partial f_1}{\partial \mathcal{Y}}\right|_{s.s.} = -Da\mathcal{W}_s[e^{-\gamma(1/\mathcal{Y}_s - 1)}]/\mathcal{Y}_s^2$$

$$A_{21} = \left.\frac{\partial f_2}{\partial \mathcal{W}}\right|_{s.s.} = Da\beta e^{-\gamma(1/\mathcal{Y}_s - 1)}$$

$$A_{22} = \left.\frac{\partial f_2}{\partial \mathcal{Y}}\right|_{s.s.} = -1 - B + Da\gamma\beta\,\mathcal{W}_s[e^{-\gamma(1/\mathcal{Y}_s - 1)}]/\mathcal{Y}_s^2$$

With characteristic equation of the system determined by Equation (2.5), a steady state of the CSTR system will have local stability if and only if it meets the following two inequalities [50, p. 64]:

$$A_{11}A_{22} - A_{12}A_{21} > 0, \quad \text{or} \quad \det \underline{\underline{A}} > 0 \tag{3.15}$$

and

$$A_{11} + A_{22} < 0 \quad \text{or} \quad \text{tr}\,\underline{\underline{A}} < 0 \tag{3.16}$$

This criterion is satisfied if and only if all the real parts of eigenvalues are negative. Clearly, Equation (3.15) is equivalent to the "slope condition" which is merely a necessary condition for stability. Equation (3.16) is termed the "dynamic condition" which taken together with Equation (3.15) gives necessary and sufficient conditions for stability.

A unique steady state always fulfills the slope condition. When there are three steady states, the two extreme steady states satisfy the slope condition but may or may not meet the dynamic condition. The intermediate steady state is always unstable since the slope condition is always violated.

Nonlinear Control Policy

The relay is used in many control situations [10-14] ranging from simple furnace control where it is used merely because it is cheap and easy to apply to implementation of periodic control strategies [46] resulting from sophisticated analysis of nonlinear process models. The relay with hysteresis has been used to address the problem of control for open loop unstable systems [9]. This recent work provided the example CSTR selected for use in this research. Moreover, Matsubara and Onogi [51] have also recently used the relay with hysteresis for an open loop unstable reactor. In addition to stablizing the reactor, the periodic input to the reactor resulted in a more optimal operation of the reactor (better selectivity was achieved).

The initial design of a relay will be discussed in terms of an open loop unstable process. It is important to note that the same procedure could be used for any process, stable or unstable, even though the discussion is in terms of an open loop unstable example.

Suppose the tolerable dimensionless temperature away from the desired controlled temperature $\mathcal{Y}^*$ is y_e. Define $\mathcal{Y}_1$ and $\mathcal{Y}_4$ as :

$$\mathcal{Y}_1 = \mathcal{Y}^* - y_e \quad (3.17.a)$$

$$\mathcal{Y}_4 = \mathcal{Y}^* + y_e \quad (3.17.b)$$

Equations (3.9) and (3.10) give the fact that changing $\mathcal{U}$ only effects the intercept of the R_r curve. Following the sketch in Figure 3.3, let $\mathcal{U}$ be the value of the dimensionless coolant temperature T_c , so that S^* is an unstable steady state at the dimensionless temperature $\mathcal{Y}^*$. Denote $\mathcal{U}_1$ and $\mathcal{U}_4$ as other control levels such that S_1 and S_4 are the corresponding unstable points at the temperatures $\mathcal{Y}_1$ and $\mathcal{Y}_4$ respectively.

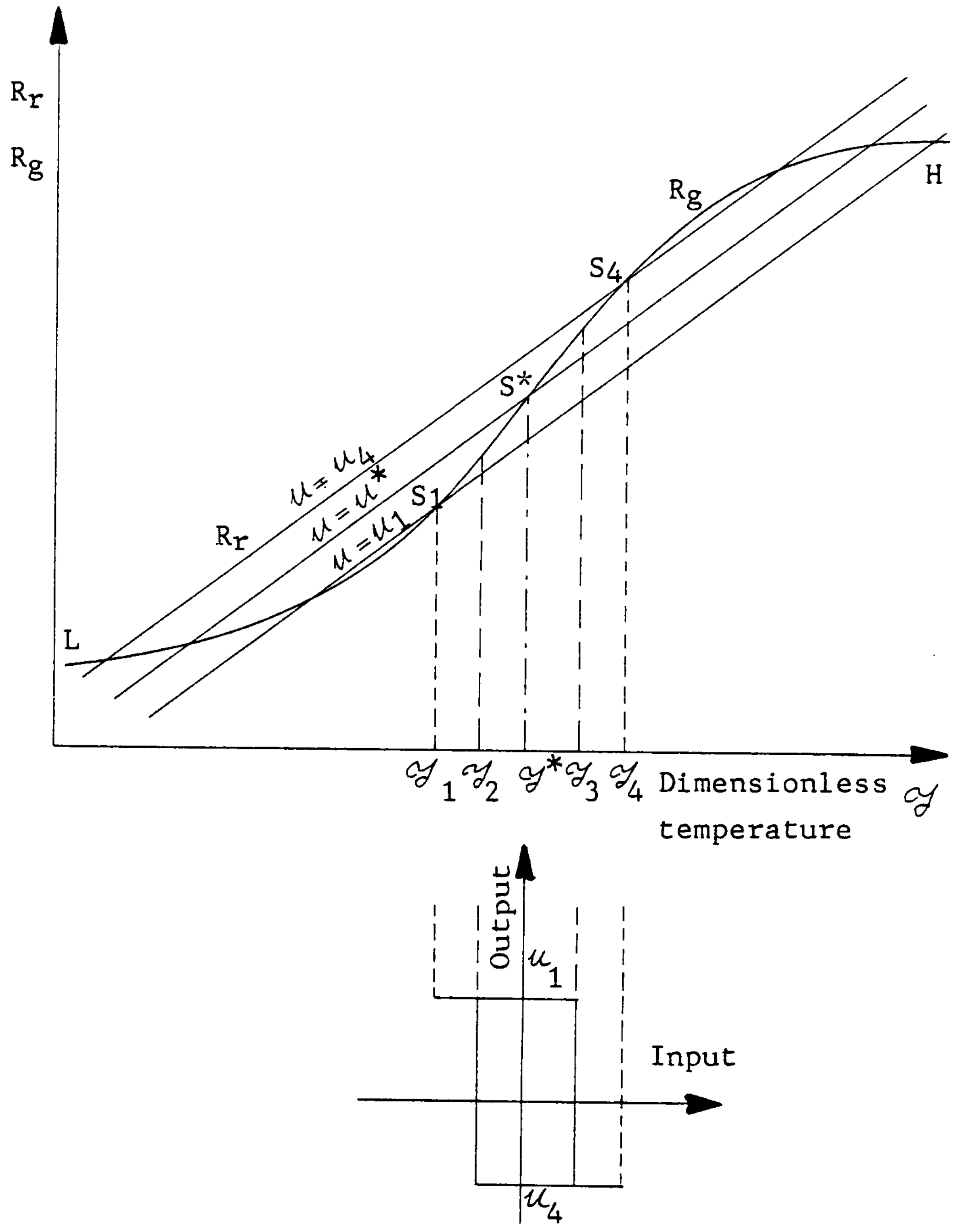


Figure 3.3. Pushing control policy on the two variable CSTR with nonlinear relay control element.

If only the control levels $\mathcal{U}_1$ and $\mathcal{U}_4$ are to be used in a scheme to maintain the system state near S^* some methods to select the control level is needed. Thus switching temperatures which determine the switching time between control level $\mathcal{U}_1$ and $\mathcal{U}_4$ are introduced. Define these temperatures $\mathcal{Y}_1$ and $\mathcal{Y}_3$ as

$$\mathcal{Y}_2 = \mathcal{Y}^* - \delta y_e \tag{3.18.a}$$

$$\mathcal{Y}_3 = \mathcal{Y}^* + \delta y_e \tag{3.18.b}$$

so that

$$\mathcal{Y}_1 < \mathcal{Y}_2 < \mathcal{Y} < \mathcal{Y}_3 < \mathcal{Y}_4 \tag{3.19}$$

which is illustrated in Figure 3.3.

Although the possible range of δ allowed by Equation (3.19) is $0 < \delta < 1$ the Van Heerden plot is not sufficient to determine if this will result in a stable oscillation. As a result, the δ value can't be determined without further analysis.

Referring to Figure 3.3 provides an illustration of a pushing control policy for operation near an unstable steady state [8]. Suppose the system state starts at S^* with $\mathcal{U} = \mathcal{U}_1$. As S^* is the unstable steady state, the temperature will rise and according to the control scheme, when $\mathcal{Y}_3$ is reached, the drive level is switched to

$\mu = \mu_4$. This will create a new unstable steady state S_4. Therefore, the system will be pushed by this unstable steady state toward to the lower temperature. When y_2 is met, again change the coolant temperature from μ_4 to μ_1. Repeating this procedure may possible generate a stable limit cycle around the unstable steady state S^*.

When there is only one steady state which is stable corresponding to each of the control levels μ_1 and μ_4, the control policy is termed pulling control (see Figure 3.4). Again, S^* is denoted as the desired unstable steady state with coolant temperature $\mu = \mu_1$. H and L are the stable steady state corresponding to the higher control level μ_1 and lower control level μ_4 at the temperature y_4 and y_1 respectively. Suppose the system state is at S^* with control level set at $\mu = \mu_1$. As H is the stable steady state it will pull the temperature up until y_3 is reached, then the coolant temperature will switch from μ_1 to μ_4. Now L is the new stable steady state, and it will pull the temperature down. As soon as y_2 is reached, the drive level will switch from μ_4 to μ_1. Repeating the previous procedures, a stable limit cycle around the unstable steady state exists.

The development of a pushing control policy can be pursued by introducing the deviation expressions in addition to Equations (3.14.a) and (3.14.b)

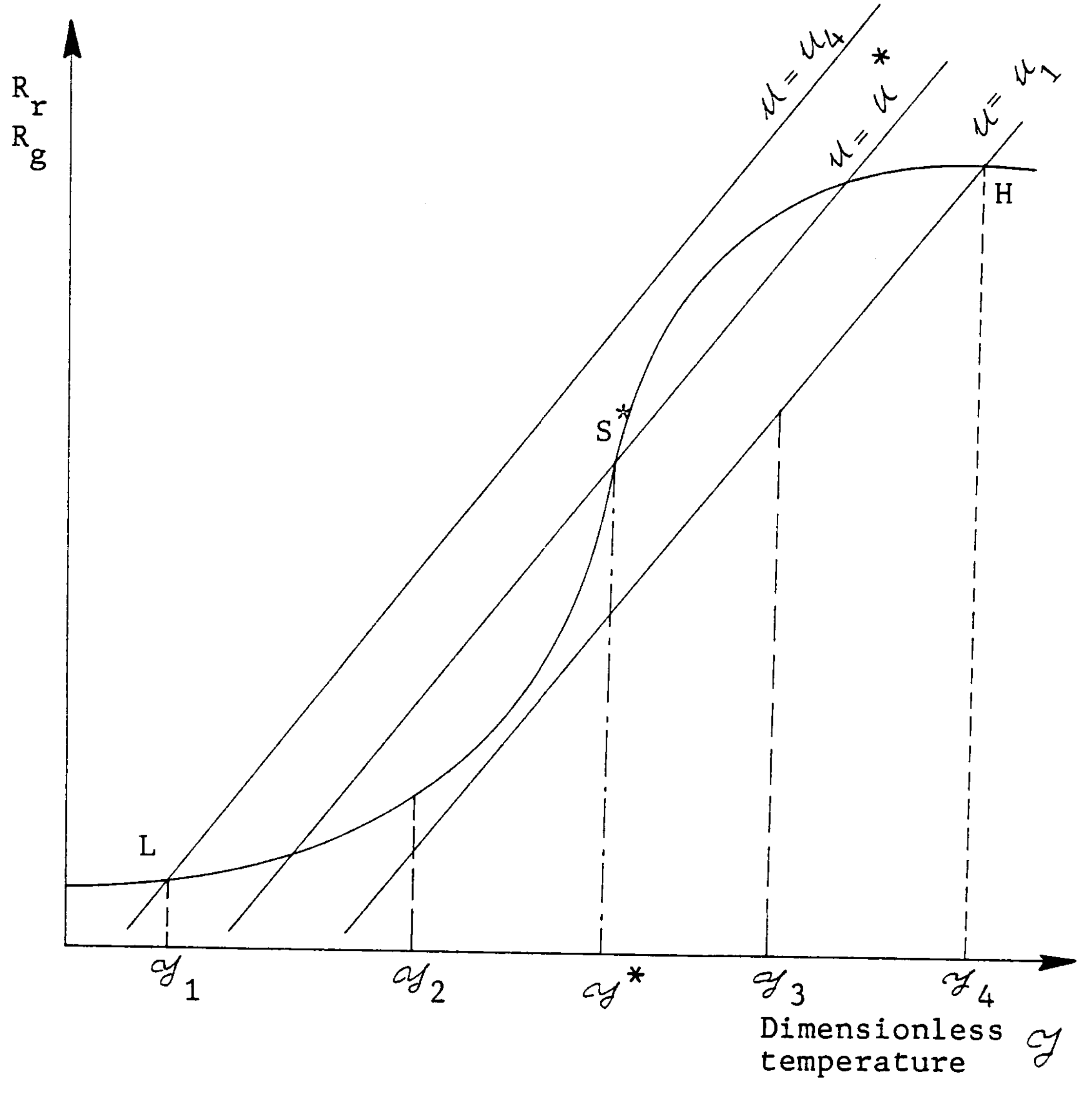


Figure 3.4. Another control policy--pulling control for the open loop unstable system.

$$y_i = \mathcal{Y}_i - \mathcal{Y}^* \tag{3.20.a}$$

$$u_i = \mu_i - \mu^* \tag{3.20.b}$$

$$u = \mu - \mu^* \tag{3.20.c}$$

Therefore, Equation (3.19) becomes

$$-y_e = y_1 \leq y_2 < 0 < y_3 \leq y_4 = +y_e \tag{3.21}$$

Then, a pushing control policy which will be employed next can be stated as:

$$u = u_1 \qquad y_1 < y \leq y_3 ; \quad \frac{dy}{dt} > 0, \; y_2 \leq y < y_3 \tag{3.22.a}$$

$$u = u_4 \qquad y_2 \leq y < y_4 ; \quad \frac{dy}{dt} < 0, \; y_2 < y \leq y_3 \tag{3.22.b}$$

The variable which will enter the controller as illustrated in Figure 3.5 is the error, x, and for a set point of $y = 0$ or $\mathcal{Y} = \mathcal{Y}^*$

$$x = -y$$

due to the negative feedback system. As a result, the pushing control policy restated in terms of x is

$$u = u_1, \qquad x_3 < x \leq x_1 ; \quad \frac{dx}{dt} < 0, \; x_3 < x < x_2 \tag{3.23.a}$$

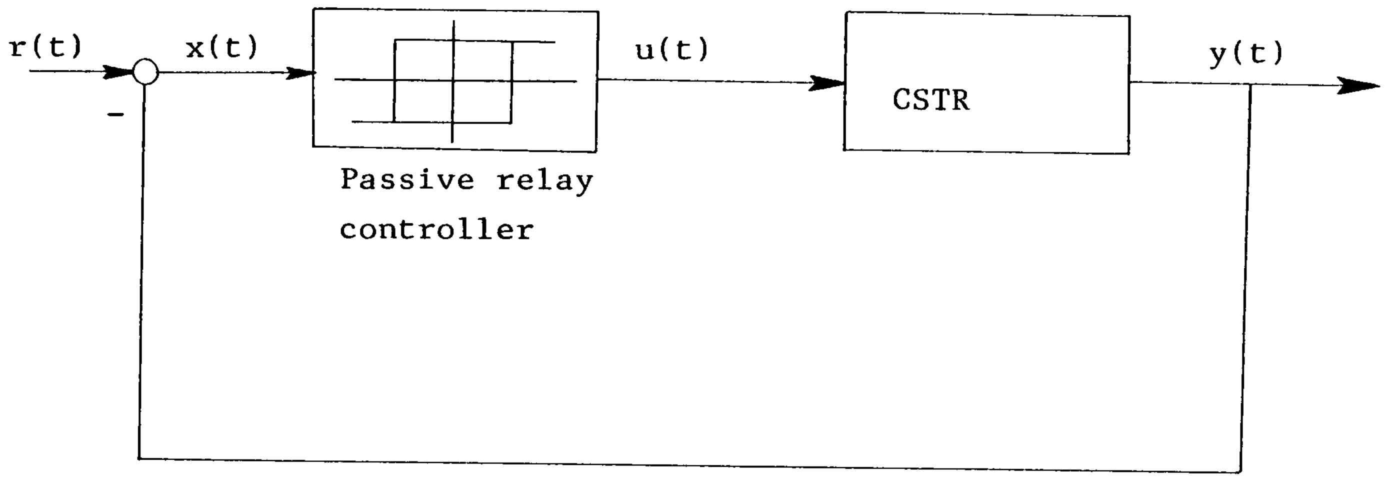


Figure 3.5. Relay feedback control block diagram for the nonlinear CSTR model.

$$u = u_4, \qquad x_4 \leqslant x < x_2; \frac{dx}{dt} > 0, \; x_3 \leqslant x < x_2 \qquad (3.23.b)$$

It is evident that the characteristics of a relay will fulfill the concept of a pushing control policy. If $|x_2| = |x_3|$ and $|u_1| = |u_4|$, then the relay will be symmetric. In the design of the relays the linearized model is used which results in a symmetric relay. In order for the control objective to be met, however, an oscillation must result from the control action and the oscillation must be stable.

The Existence of Dead Time

As noted previously, the existence of significant dead time can pose serious problems in process performance and plant operation. Unfortunately, the dead time problem is commonly encountered in chemical engineering processes.

A process instrument diagram for the block diagram shown in Figure 3.5 is sketched in Figure 3.6. First, there is a measurement element to record the temperature in the reactor and compare it with the set point temperature. Then an error signal is sent into the controller which will give a control action by generating a command to open and close the valves. There are two valves connected with reactor; one gives the higher drive level u_1 and the other provides the lower drive level u_4.

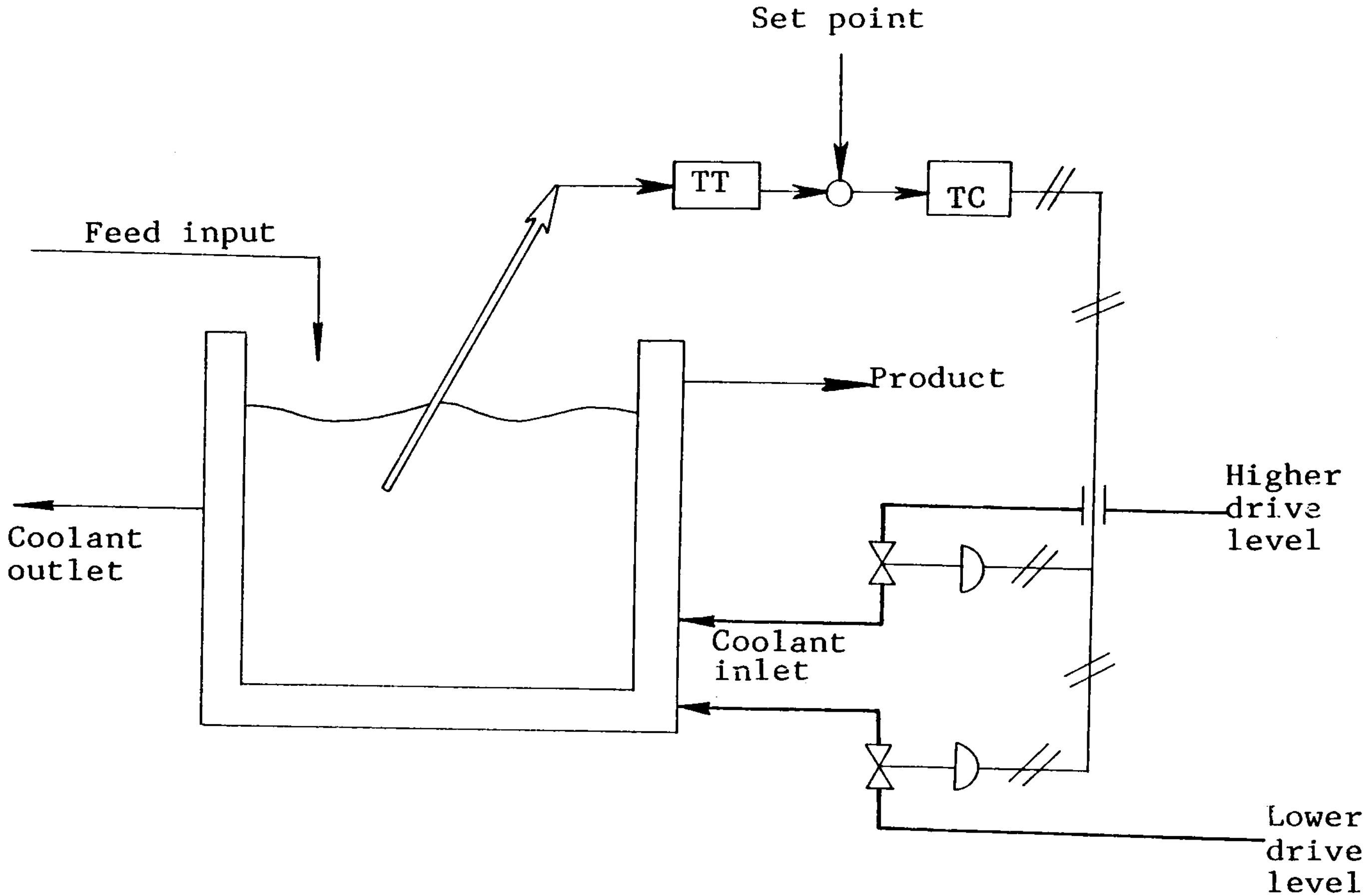


Figure 3.6. A process instrument diagram for the CSTR system.

The coolant temperature through out the reactor jacket will not be able to reach the desired value immediately because of the volume of the pipes between the valves and reactor, the volume of the reactor jacket, mixing in the pipes or jacket, heat capacity of the pipe and jacket walls, etc. Systems characteristic of this type are often lumped together and approximated by a dead time term. In these circumstances, the block diagram in Figure 3.5 has to be modified by adding the consideration of dead time and can be redrawn as in Figure 3.7.

Linearization

In order to investigate the effects of the controller on dynamics of the system, results from nonlinear mechanics will be used to give insight into the problem. Again, these results are based on the linearization of the process equations around the steady state of interest. This linearization is the same as that given in the section Local Stability except the manipulated variable used for control, μ , is retained in the linearization. If disturbance studies or load tuning were to be done on the linearized model the associated inputs would need to be kept in the model. Subsequently, the block diagram shown in Figure 3.7 is amenable to Tsypkin's analysis for characterization of the system after the nonlinear process is linearized. Linearization can be

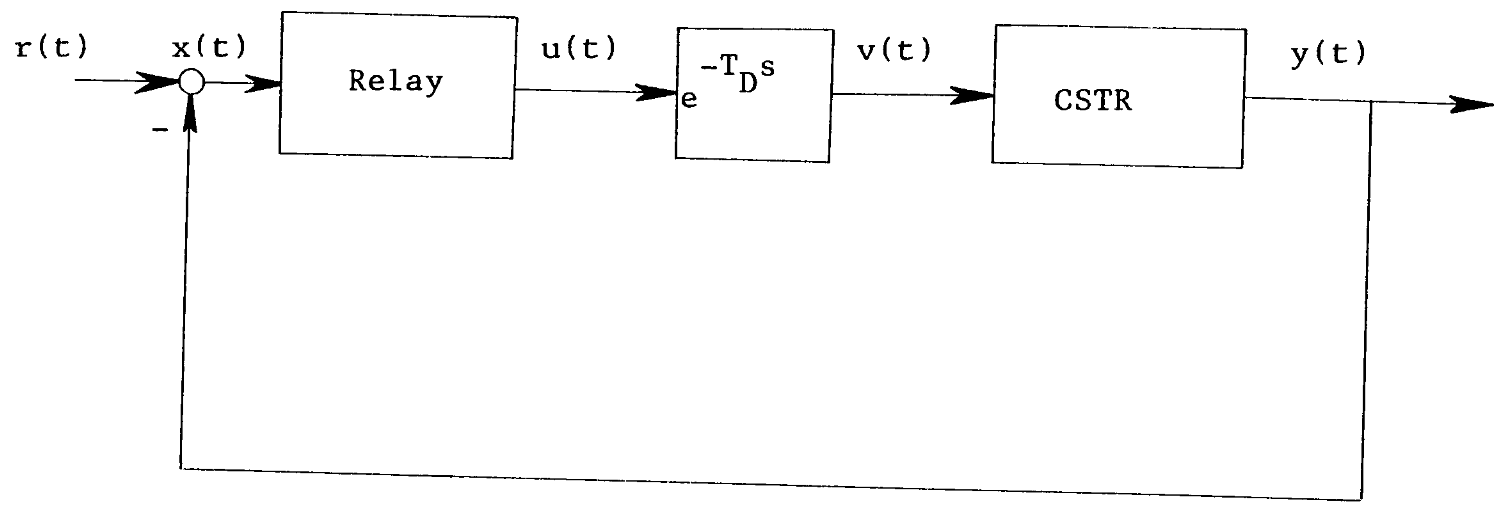


Figure 3.7. The overall block diagram for the nonlinear CSTR model with dead time.

obtained by Taylor series expansion around the unstable steady state $(\mathcal{W}^*, \mathcal{Y}^*, \mathcal{M}^*)$ which is the desired operating point, and truncating after the first order derivative terms.

$$f_1(\mathcal{W}, \mathcal{Y}, \mathcal{M}) = \frac{d\mathcal{W}}{dt} = f_1(\mathcal{W}^*, \mathcal{Y}^*, \mathcal{M}^*) + \left.\frac{\partial f_1}{\partial \mathcal{W}}\right|_{s.s}(\mathcal{W} - \mathcal{W}^*)$$

$$+ \left.\frac{\partial f_1}{\partial \mathcal{Y}}\right|_{s.s}(\mathcal{Y} - \mathcal{Y}^*) + \left.\frac{\partial f_1}{\partial \mathcal{M}}\right|_{s.s}(\mathcal{M} - \mathcal{M}^*) \quad (3.24)$$

$$f_2(\mathcal{W}, \mathcal{Y}, \mathcal{M}) = \frac{d\mathcal{Y}}{dt} = f_2(\mathcal{W}^*, \mathcal{Y}^*, \mathcal{M}^*) + \left.\frac{\partial f_2}{\partial \mathcal{W}}\right|_{s.s}(\mathcal{W} - \mathcal{W}^*)$$

$$+ \left.\frac{\partial f_2}{\partial \mathcal{Y}}\right|_{s.s}(\mathcal{Y} - \mathcal{Y}^*) + \left.\frac{\partial f_2}{\partial \mathcal{M}}\right|_{s.s}(\mathcal{M} - \mathcal{M}^*) \quad (3.25)$$

at steady state the derivatives of state variables with respect to time vanished, therefore,

$$f_1(\mathcal{W}^*, \mathcal{Y}^*, \mathcal{M}^*) = 0$$

$$f_2(\mathcal{W}^*, \mathcal{Y}^*, \mathcal{M}^*) = 0$$

Using the expressions of deviation variables which have been defined in Equations (3.14.a-3.14.b), we can rewrite Equations (3.24) and (3.25) as:

$$\frac{dw}{dt} = A_{11} w + A_{12} y + B_{11} u \tag{3.26}$$

$$\frac{dy}{dt} = A_{21} w + A_{22} y + B_{21} u \tag{3.27}$$

where :

$$A_{11} = \frac{\partial f_1}{\partial \mathcal{W}}\bigg|_{s.s.} = -1 - \mathrm{Da} e^{-\gamma(1/\mathcal{Y}_s - 1)}$$

$$A_{12} = \frac{\partial f_1}{\partial \mathcal{Y}}\bigg|_{s.s.} = -\mathrm{Da}\mathcal{W}_s\left[e^{-\gamma(1/\mathcal{Y}_s - 1)}\right]\Big/\mathcal{Y}_s^2$$

$$A_{21} = \frac{\partial f_2}{\partial \mathcal{W}}\bigg|_{s.s.} = \mathrm{Da}\beta e^{-\gamma(1/\mathcal{Y}_s - 1)}$$

$$A_{22} = \frac{\partial f_2}{\partial \mathcal{Y}}\bigg|_{s.s.} = -1 - B + \mathrm{Da}\beta\gamma\mathcal{W}_s\left[e^{-\gamma(1/\mathcal{Y}_s - 1)}\right]\Big/\mathcal{Y}_s^2$$

$$B_{11} = \frac{\partial f_1}{\partial \mu}\bigg|_{s.s.} = 0$$

$$B_{21} = \frac{\partial f_2}{\partial \mu}\bigg|_{s.s.} = B$$

Expressed in a state space or modern control theory form, the previous two equations are given in a more concise matrix form:

$$\dot{\underline{X}} = \underline{\underline{A}}\,\underline{X} + \underline{\underline{B}}\,\underline{U} \tag{3.28}$$

in which:

$$\underline{\dot{X}} = \begin{bmatrix} \frac{dw}{dt} \\ \frac{dy}{dt} \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} w \\ y \end{bmatrix}$$

$$\underline{U} = \begin{bmatrix} 0 \\ u \end{bmatrix}$$

$$\underline{\underline{A}} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$\underline{\underline{B}} = \begin{bmatrix} 0 \\ B_{21} \end{bmatrix}$$

At steady state, $\underline{\dot{X}} = 0$, noting $\underline{B} = 0$ one obtains from Equations (3.26) and (3.27)

$$w_s = - \frac{A_{12}}{A_{11}} y_s \tag{3.29}$$

$$u_s = - \frac{(A_{21} w_s + A_{22} y_s)}{B_{21}} \tag{3.30}$$

Substitution of Equation (3.29) into Equation (3.30) leads to the determination of higher drive level u_1 and lower drive level u_4 by substituting $y_s = -y_e$ and $+y_e$ respectively

$$u_s = \frac{(A_{12}A_{21} - A_{22}A_{11})y_s}{A_{11}B_{21}} \tag{3.31}$$

$$u_1 = u_s\Big|_{y_s = -y_e} \tag{3.32.a}$$

$$u_4 = -u_1 \tag{3.32.b}$$

Taking the Laplace transform of Equation (3.28) and finding the transfer function of reactor temperature to coolant temperature, $\Upsilon(s)/\,\mathrm{U}(s)$ which will be denoted as $G_s(s)$ we obtain

$$G_s(s) = \frac{B_{21}(s - A_{11})}{s^2 - (A_{11} + A_{22})s + (A_{11}A_{22} - A_{12}A_{21})} \tag{3.33}$$

In order to proceed further into application of Tsypkin's analysis it is convenient to represent $G_s(s)$ by partial fraction expansion.

$$G_s(s) = \frac{k_1}{s - \lambda_1} + \frac{k_2}{s - \lambda_2} \tag{3.34}$$

where λ_1 and λ_2 are the eigenvalues and k_1 and k_2 are the residues at these poles

$$\lambda_1 = (C + D)/2 \tag{3.35.a}$$

$$\lambda_2 = (C - D)/2 \tag{3.35.b}$$

where

$$C = A_{11} + A_{22}$$

$$D = \sqrt{(A_{11} + A_{22})^2 - 4(A_{11}A_{22} - A_{12}A_{21})}$$

$$k_1 = \frac{B_{21}(\lambda_1 - A_{11})}{\lambda_1 - \lambda_2} \tag{3.36.a}$$

$$k_2 = \frac{B_{21}(\ \ 2 - A_{11})}{\lambda_2 - \lambda_1} \tag{3.36.b}$$

With the transfer function $G_s(s)$, the block diagram in Figure 3.7 can be redrawn as in Figure 3.8. This formulation can now be treated exactly when Tsypkin's analysis is employed.

Tsypkin's Analysis

Once the partial fraction expansion of the transfer function is found by following the procedures detailed in Chapter II Tsypkin's equations can be written in closed form. In this case, $G_s(s)$ has two single poles as shown

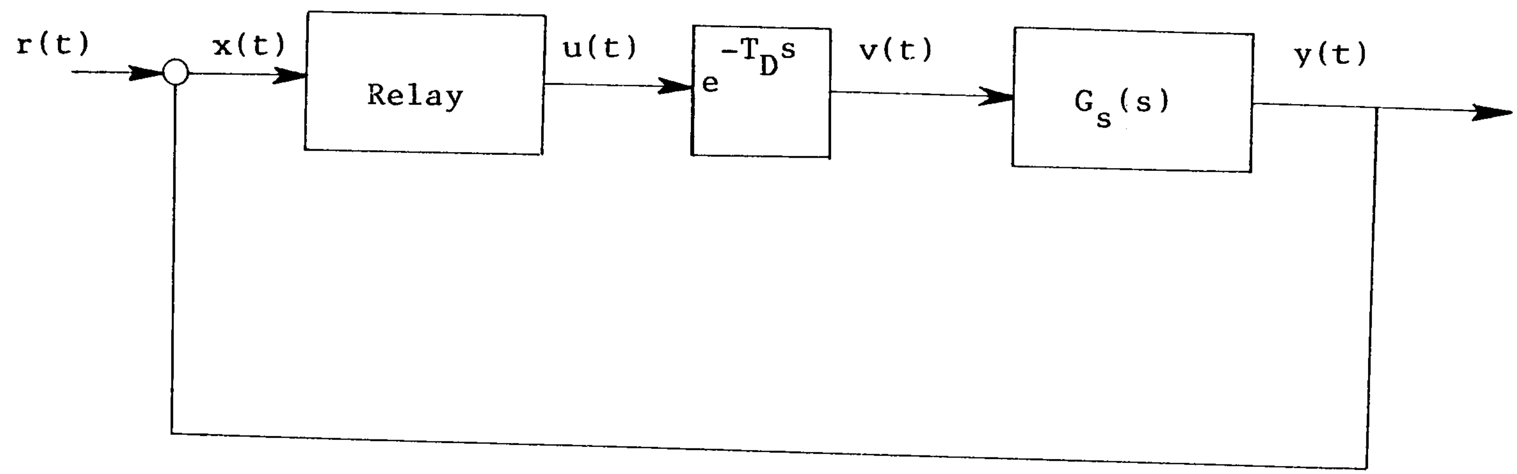


Figure 3.8. The overall block diagram for the linearized CSTR model with dead time.

in Equation (3.34). With the associated dead time, the total process transfer function $G_s(s)$ relating system output y(t) to manipulated variable u(t) is

$$G_s(s) = \frac{K_1 e^{-T_D s}}{s - \lambda_1} + \frac{K_2 e^{-T_D s}}{s - \lambda_2} \qquad (3.37)$$

By applying Equations (2.42) and (2.43) to each partial fraction, Tsypkin's function in closed form based on this particular transfer function $G_s(s)$ can be written as:

$$\mathrm{Re}[\Lambda(\Omega_o)]$$

$$= -\frac{k_1\lambda_1}{\Omega_o^2} f_x(\Omega_o T_D, \frac{\lambda_1}{\Omega_o}) - \frac{k_2\lambda_2}{\Omega_o^2} f_x(\Omega_o T_D, \frac{\lambda_2}{\Omega_o})$$

$$- \frac{k_1}{\Omega_o}(-f_{xx}(\Omega_o T_D, \frac{\lambda_1}{\Omega_o})) - \frac{k_2}{\Omega_o}(-f_{xx}(\Omega_o T_D, \frac{\lambda_2}{\Omega_o})) \qquad (3.38)$$

$$\mathrm{Im}[\Lambda(\Omega_o)]$$

$$= -\frac{k_1}{\Omega_o} f_x(\Omega_o T_D, \frac{\lambda_1}{\Omega_o}) - \frac{k_2}{\Omega_o} f_x(\Omega_o T_D, \frac{\lambda_2}{\Omega_o})$$

$$+ \frac{k_1\lambda_1}{\Omega_o^2}(f(\Omega_o T_D, \frac{\lambda_1}{\Omega_o})) + \frac{k_2\lambda_2}{\Omega_o}(f(\Omega_o T_D, \frac{\lambda_2}{\Omega_o})) \qquad (3.39)$$

here f, f_x and f_{xx} are defined in Table 2.2 and Equation (2.41).

According to the definitions of Equations (2.16-2.18) and transfer function $G_s(s)$ of Equation (3.36) for the present CSTR example

$$G_s(\infty) = 0 \tag{3.40.a}$$

$$G_1(s) = G_s(s) \tag{3.40.b}$$

$$\lim_{s \to \infty} sG_1(s) = B_{21} \tag{3.40.c}$$

simplifying the switching conditions in Equations (2.27-2.28) gives the following equations for a passive element

$$\mathrm{Im}\ [(\Omega_o)] = \sum_{n\ odd}^{\infty} \frac{1}{n} \mathrm{Im}\ G_{T1}(jn\Omega_o) = -\frac{\pi x_2}{4u_c} \tag{3.41}$$

$$\mathrm{Re}\ [(\Omega_o)] = \sum_{n\ odd}^{\infty} \mathrm{Re}\ G_{T1}(jn\Omega_o) < \frac{\pi B_{21}}{4\Omega_o} \tag{3.42}$$

and Equations (2.29-2.30) for an active element

$$\mathrm{Im}\ [(\Omega_o)] = \sum_{n\ odd}^{\infty} \frac{1}{n} \mathrm{Im}\ G_{T1}(jn\Omega_o) = +\frac{\pi x_2}{4u_c} \tag{3.43}$$

$$\mathrm{Re}\ [(\Omega_o)] = \sum_{n\ odd}^{\infty} \mathrm{Re}\ G_{T1}(jn\Omega_o) > \frac{\pi B_{21}}{4\Omega_o} \tag{3.44}$$

and the waveform of oscillation in Equation (2.22) can be simplified as:

$$y(t) = \frac{4u_c}{\Omega_o} \sum_{n\ odd}^{\infty} \frac{1}{n} \mathrm{Im}\left[G_1(jn\Omega_o)e^{jn\Omega_o(t-T_D)}\right] \qquad (3.45)$$

By applying the results in Table 2.2, page 43 and Equation (2.41), Tsypkin's function can be expressed as:

$$\mathrm{Im}[\Lambda(\Omega_o)] = \frac{-k_1}{\lambda_1}\left[e^{-\lambda_1 T_D}\left(\tanh\frac{\lambda_1}{2\Omega_o} + 1\right) - 1\right] \cdot$$

$$- \frac{k_2}{\lambda_2}\left[e^{-\lambda_2 T_D}\left(\tanh\frac{\lambda_2}{2\Omega_o} + 1\right) - 1\right] \qquad (3.46)$$

$$0 < T_D < T_o/2$$

$$\mathrm{Re}[\Lambda(\Omega_o)] = \frac{-k_1}{\Omega_o}\left[e^{-\lambda_1 T_D}\left(\tanh\frac{\lambda_1}{2\Omega_o} - 1\right)\right]$$

$$- \frac{k_2}{\Omega_o}\left[e^{-\lambda_2 T_D}\left(\tanh\frac{\lambda_2}{2\Omega_o} - 1\right)\right] \qquad (3.47)$$

$$0 < T_D < T_o/2$$

Then, the switching conditions expressed in Equations (3.41-3.44) can be rewritten for passive element as:

$$\frac{k_1}{\lambda_1}\left[e^{-\lambda_1 T_D}\left(\tanh\frac{\lambda_1}{2\Omega_o}+1\right)-1\right]$$

$$+\frac{k_2}{\lambda_2}\left[e^{-\lambda_2 T_D}\left(\tanh\frac{\lambda_2}{2\Omega_o}+1\right)-1\right]=\frac{x_2}{u_c} \qquad (3.48)$$

$$0 < T_D < T_o/2$$

$$\frac{k_1}{\Omega_o}e^{-\lambda_1 T_D}\left(\tanh\frac{\lambda_1}{2\Omega_o}-1\right)$$

$$+\frac{k_2}{\Omega_o}e^{-\lambda_2 T_D}\left(\tanh\frac{\lambda_2}{2\Omega_o}-1\right) > \frac{B_{21}}{\Omega_o} \qquad (3.49)$$

$$0 < T_D < T_o/2$$

and for an active element

$$\frac{k_1}{\lambda_1}\left[e^{-\lambda_1 T_D}\left(\tanh\frac{\lambda_1}{2\Omega_o}+1\right)-1\right]$$

$$+\frac{k_2}{\lambda_2}\left[e^{-\lambda_2 T_D}\left(\tanh\frac{\lambda_2}{2\Omega_o}+1\right)-1\right]=\frac{-x_2}{u_c} \qquad (3.50)$$

$$0 < T_D < T_o/2$$

$$\frac{k_1}{\Omega_o} e^{-\lambda_1 T_D} \left(\tanh \frac{\lambda_1}{2\Omega_o} - 1\right)$$

$$+ \frac{k_2}{\Omega_o} e^{-\lambda_2 T_D} \left(\tanh \frac{\lambda_2}{2\Omega_o} - 1\right) < \frac{B_{21}}{\Omega_o} \qquad (3.51)$$

$$0 < T_D < T_o/2$$

Referring to the derivation of Equation (2.53), the waveform of oscillation, needed to check the continuity condition can be expressed as:

$$y(t) = - \frac{k_1 u_c}{\lambda_1} \left[\left(\tanh \frac{\lambda_1}{2\Omega_o} - 1\right) e^{\lambda_1 (t - T_D)} + 1\right]$$

$$- \frac{k_2 u_c}{\lambda_2} \left[\left(\tanh \frac{\lambda_2}{2\Omega_o} - 1\right) e^{\lambda_2 (t - T_D)} + 1\right] \qquad (3.52)$$

$$T_D < t < T_D + \frac{T_o}{2}$$

Once the frequencies of oscillation have been determined by solving the switching condition equations, plugging these frequences into the Equation (3.52) provides the waveform of oscillation and thus the amplitude.

For Equations (3.48) and (3.50), the switching condition, if the frequency $\Omega_o \longrightarrow 0$, then the period $T_o \longrightarrow \infty$, and x_2 is the maximum value which allows a physical solution for the frequency of oscillation. If a large value for x_2 is used the frequency become negative or imaginary which says no oscillation exists in the closed loop system, taking $T_o \longrightarrow \infty$ in Equation (3.48) or (3.50) gives the maximum bandwidth

$$\frac{x_{2,max}}{u_c} = \frac{k_1}{\lambda_1}(2e^{-\lambda_1 T_D} - 1) - \frac{k_2}{\lambda_2} \tag{3.53}$$

From Equation (3.31) then,

$$\delta_{max} = \frac{x_{2,max}}{y_e} = \frac{(A_{22}A_{11} - A_{12}A_{21})}{A_{11}B_{21}} \frac{k_1}{\lambda_1}(2e^{-\lambda_1 T_D} - 1) - \frac{k_2}{\lambda_2} \tag{3.54}$$

Summary

The application of extended Tsypkin's method stated in Chapter II is presented via the example of a multiple steady state CSTR containing dead time. The procedures of linearization, transfer function development, control drive level determination and use of closed forms for specific infinite series are illustrated.

Development of Tsypkin's equations and translation from the infinite series in those equations to the closed forms yield an analytic means on the controller design and dead time compensation. Actually, the switching conditions which are written in the closed forms by employing Tables 2.1, and 2.2, page 43 give the relationship among the system parameters, i.e. the period of oscillation, bandwidth of controller, control drive level and dead time. Consequently, Equations (3.48-3.49) for passive relay or Equations (3.50-3.51) for active relay are the central analytic equations for controller design strategies and dead time compensation. Given three of the system parameters, the forth can be determined by substituting the values of known parameters and solving Equations (3.48) or (3.50) depending on what kind relay controller is being used.

However converting from infinite series in Tsypkin's equations to the more applicable closed forms places a restriction on dead time, ranging from zero to half period of oscillation. This restriction can be removed by introducing the new concepts of "negative bandwidth" relay and "reduced dead time" which will be stated in Chapter IV and V respectively.

Based on the mathematic framework stated in this chapter, the negative bandwidth controller can be defined mathematically and detail description will be given to these new relay controllers in chapter IV.

After demonstrating the controller design strategies of this dead time compensation by using the present CSTR example a general formula will be established which is applicable for any feedback control scheme containing symmetric relay controller and dead time.

CHAPTER IV

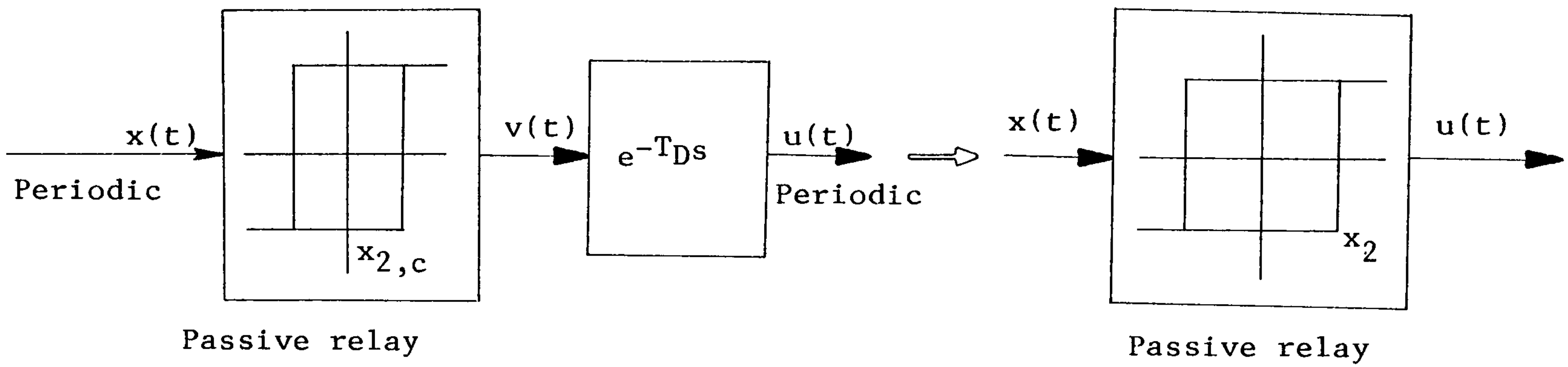
NEW RELAY: NEGATIVE BANDWIDTH RELAYS

Background and Introduction

Negative bandwidth relays for both passive and active memory elements are introduced and defined in this chapter. A literature survey has not been able to identify any type of relay with the new characteristics of the negative bandwidth relays. These novel relays arose in the present research as nearly all process control design for process with significant dead time is based on some strategy to compensate for the dead time. In addition, the need for a method to address the stability of periodic solutions for both positive and negative bandwidth relays when used as a feedback controller leads to a block diagram simplification that may result in a negative bandwidth relay.

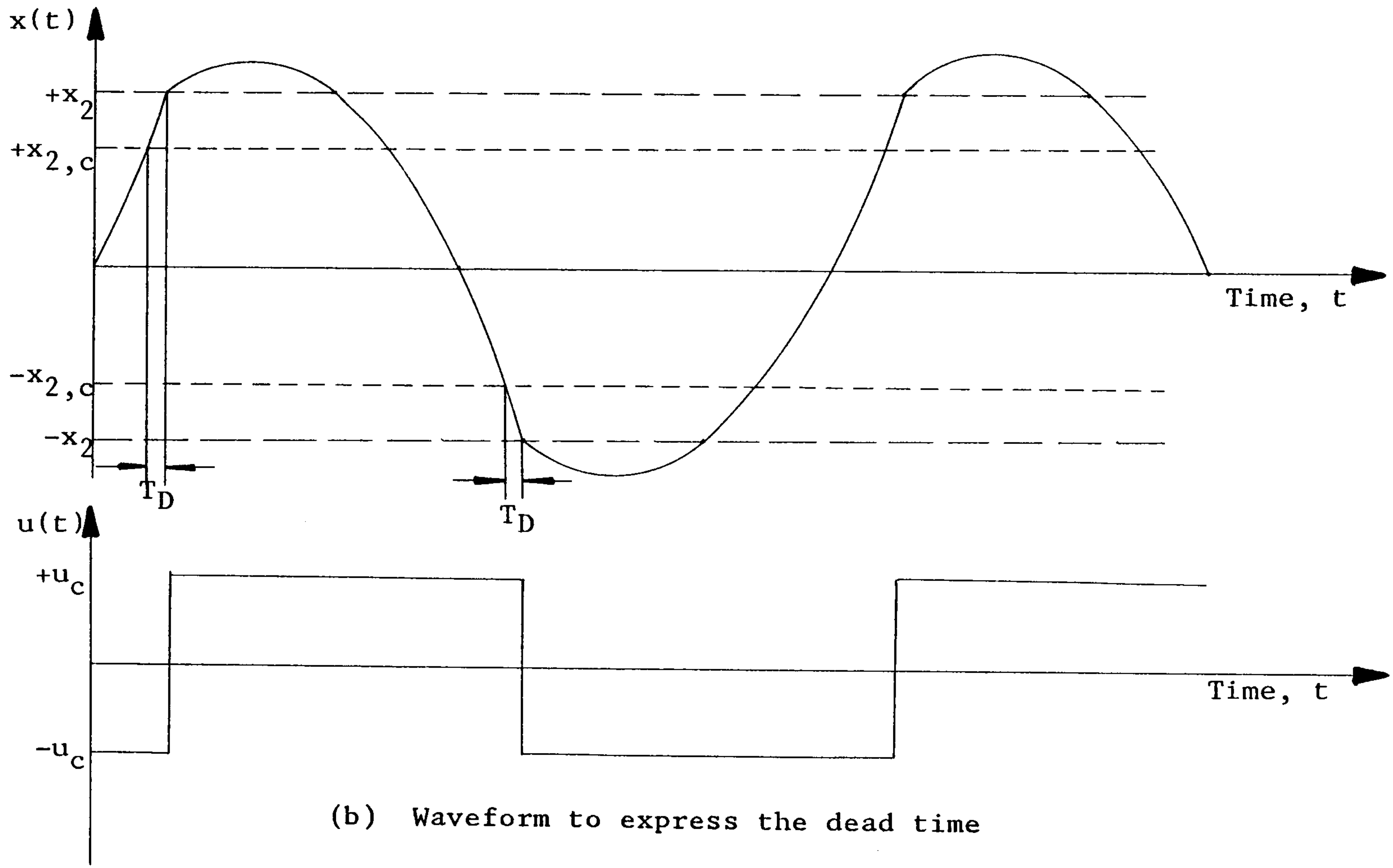
A brief background of the concepts that leads to the new relays is given first. This is followed by a formal definition of the negative bandwidth relays and the presentation of the switching and continuity conditions which a periodic input signal must satisfy in order to drive such a relay.

Consider a relay with hysteresis followed by a pure dead time element, as shown in Figure 4.1a. When the input to the relay is periodic and of sufficient amplitude (here amplitude is greater than x_2) an equivalent representation can be made in terms of a modified relay. This is also shown in Figure 4.1a. The modified relay to incorporate the effects of dead time has a larger bandwidth ($x_2 > x_{2,c}$). This physically makes sense as the dead time element will add to the time it takes for the output to switch from one drive level to the other. The relation between $x_{2,c}$ and x_2 or the change in bandwidth to make the two representations equivalent will depend on the actual shape of the periodic input and the dead time T_D. However, it must be emphasized that this equivalent representation holds only for oscillatory inputs to the relays [14, p. 212]. For example, consider the step response of the relay and dead time element vs. that of the modified relay with no dead time. For a step of sufficient (greater than x_2) amplitude, to cause both relays to change drive levels, the relay of bandwidth x_2 will switch immediately, whereas, the switch for the relay with the dead time element will be delayed by T_D. Moreover, when the input is periodic and a disturbance is added to the periodic input, the output of the hysteresis only element can respond immediately, but the hysteresis followed by the dead time element retains the dead time



(a) An equivalent representation of block diagram

Figure 4.1. A block diagram reduction for the combination of a relay and dead time element with a periodic input.



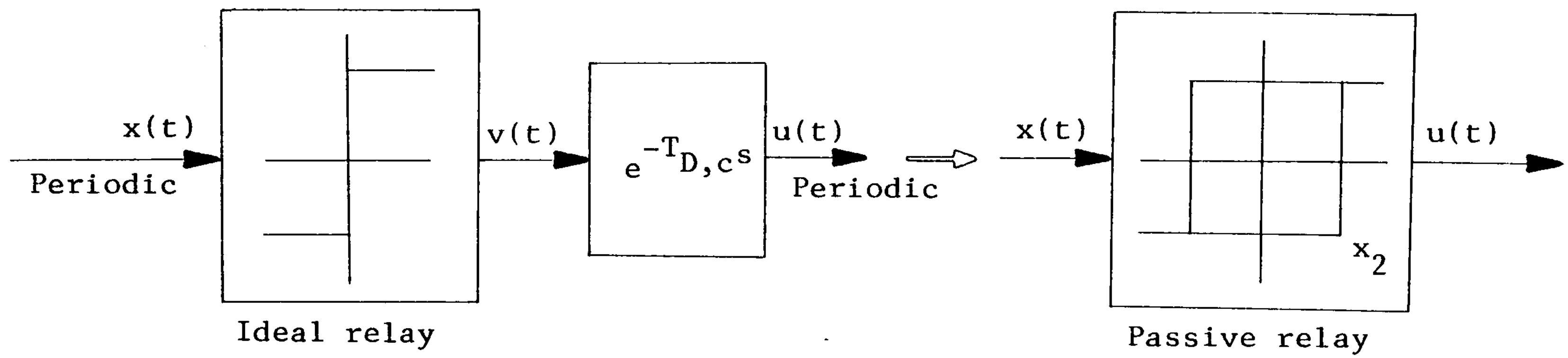
(b) Waveform to express the dead time

Figure 4.1 (continued)

characteristic or it suffers a "blind period." If this element is serving as a controller, it will fail to give control action instantly and that will mar the control performance when the disturbance is sufficient large. This will lead to another interesting topic when investigating the stability of the system.

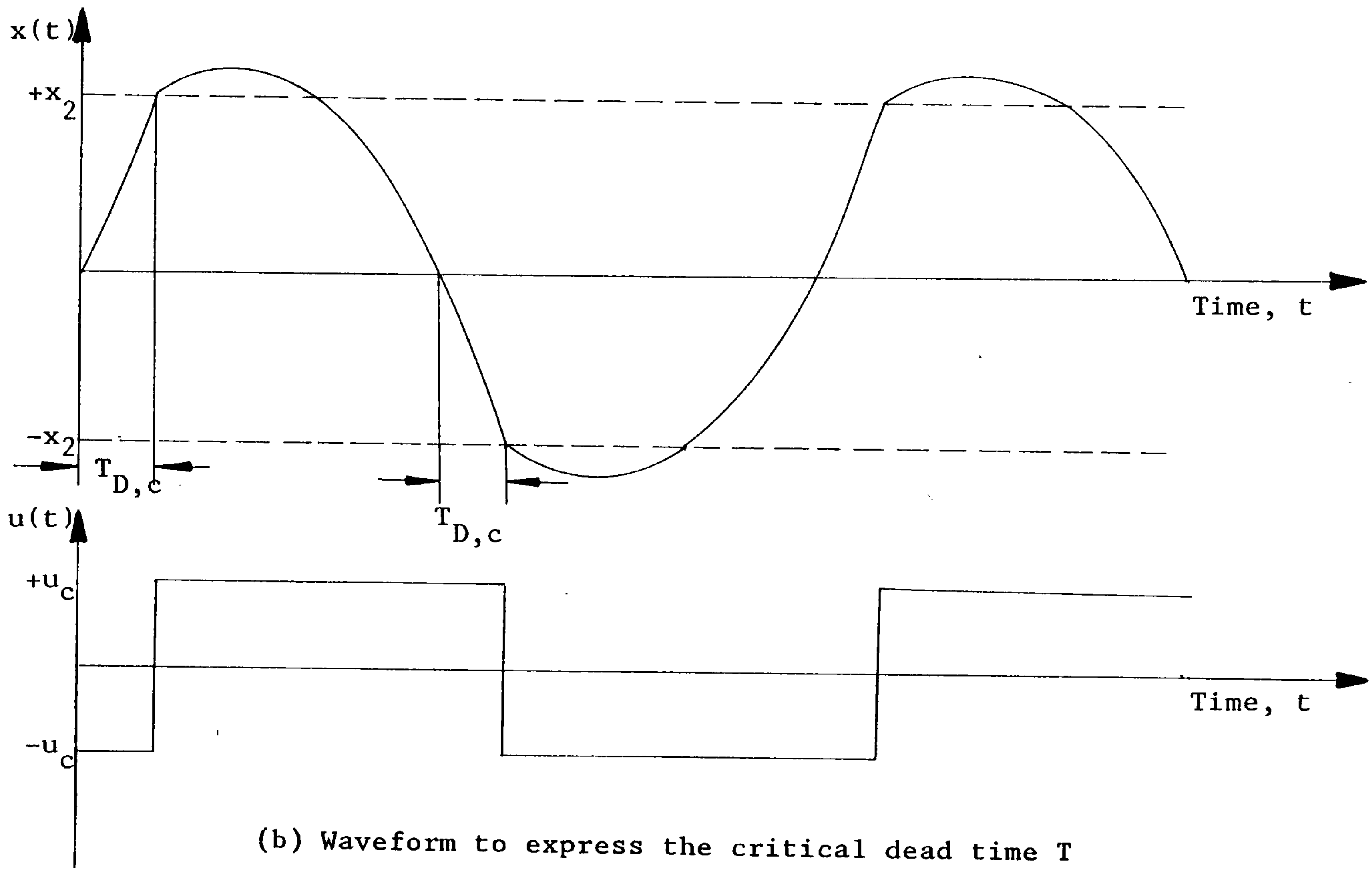
The critical dead time $T_{D,c}$, is introduced and defined as the delay associated with an ideal relay controller to give an equivalent switching time or equivalent output for a relay having bandwidth x_2 with zero dead time. This relationship can be illustrated clearly by the schematic representation in Figure 4.2 for a symmetric passive relay. If the relay switches at x = 0 the signal would need to be delayed for a time T_D to allow the periodic input to reach $x = x_2$ which is the desired switching time. In the case of the passive relay serving as a controller in a feedback system, which is sustaining an oscillation, by decreasing the bandwidth, x_2, of the passive relay to zero (thus we have the ideal relay) the maximum dead time that could be compensated for is $T_{D,c}$.

Since for the ideal relay there is no hysteresis, i.e. $x_2 = 0$, from Equation (3.48) or (3.50) we can obtain the critical dead time for a specified period of oscillation in the implicit form by substituting $x_2 = 0$



(a) An equivalent representation of block diagram

Figure 4.2. Relay with no dead time equivalent for an ideal relay plus critical dead time with a periodic input.



(b) Waveform to express the critical dead time T

Figure 4.2 (continued)

$$\frac{k_1}{\lambda_1}\left[e^{-\lambda_1 T_{D,c}}\left(\tanh\frac{\pi\lambda_1}{2\Omega_o}+1\right)-1\right]$$

$$+\frac{k_2}{\lambda_2}\left[e^{-\lambda_2 T_{D,c}}\left(\tanh\frac{\pi\lambda_2}{2\,\Omega_o}+1\right)-1\right]=0 \qquad (4.1)$$

Obviously, to maintain the period of oscillation unchanged, increasing dead time requires the narrowing of the relay controller bandwidth to allow more time for the switch in the relay output to effect the system output and thus compensate for the larger dead time. This can be illustrated in Figure 4.3 with a passive relay controller as an example.

Apparently, according to the definition, critical dead time is the maximum dead time the reduced bandwidth controller can handle for a given oscillation waveform. If the dead time is greater than the critical dead time, the on-off device has to switch even earlier, in other words, it needs to switch even before the error signal reaches the center axis. The bandwidth of the relay needs to be "less than zero"; indeed, in such a case to maintain a constant the original relay Equation (3.48) or (3.50) give a solution for x_2 which is negative. The relay having this particular characteristic is termed a relay with <u>negative</u> <u>bandwidth</u>.

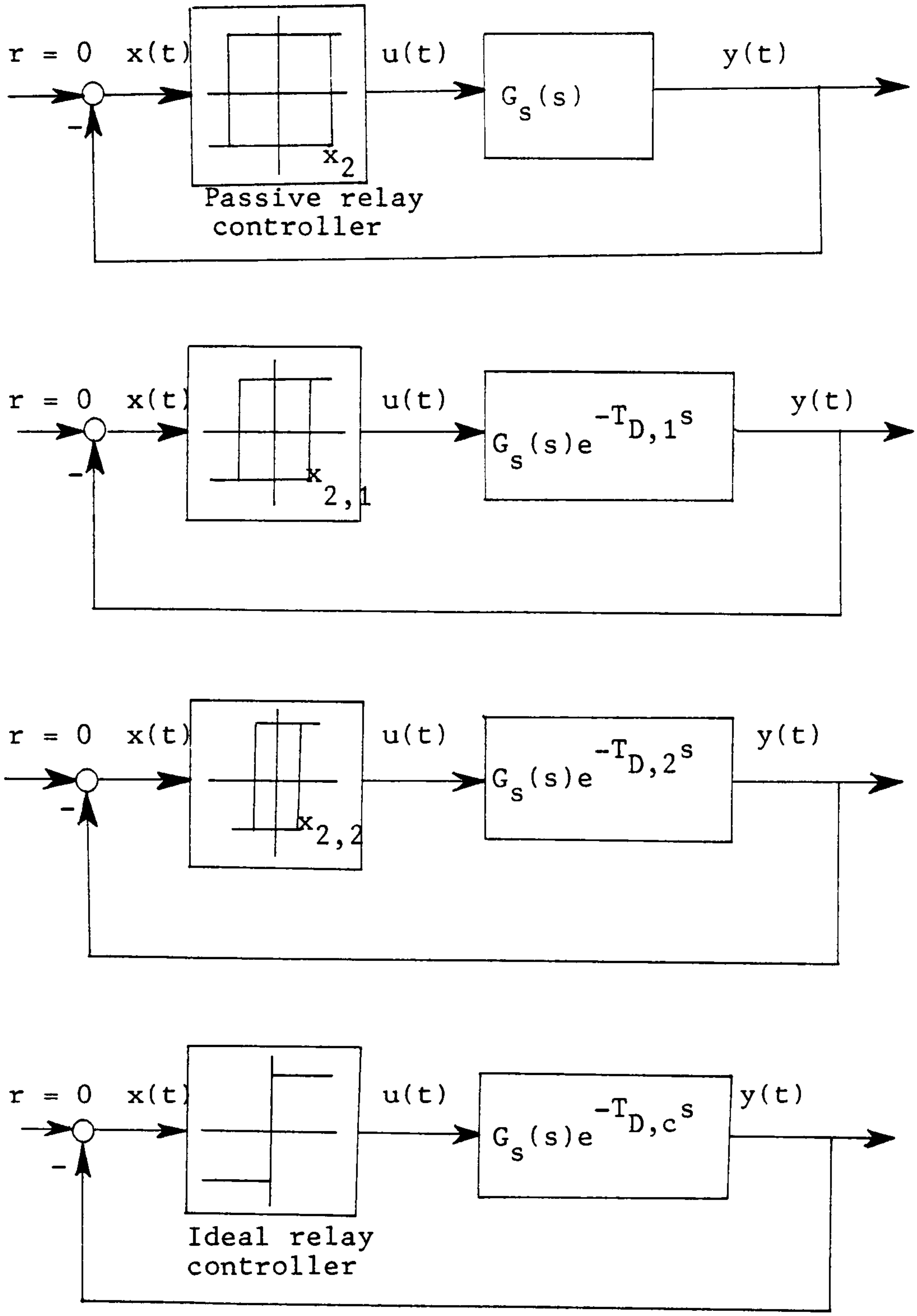
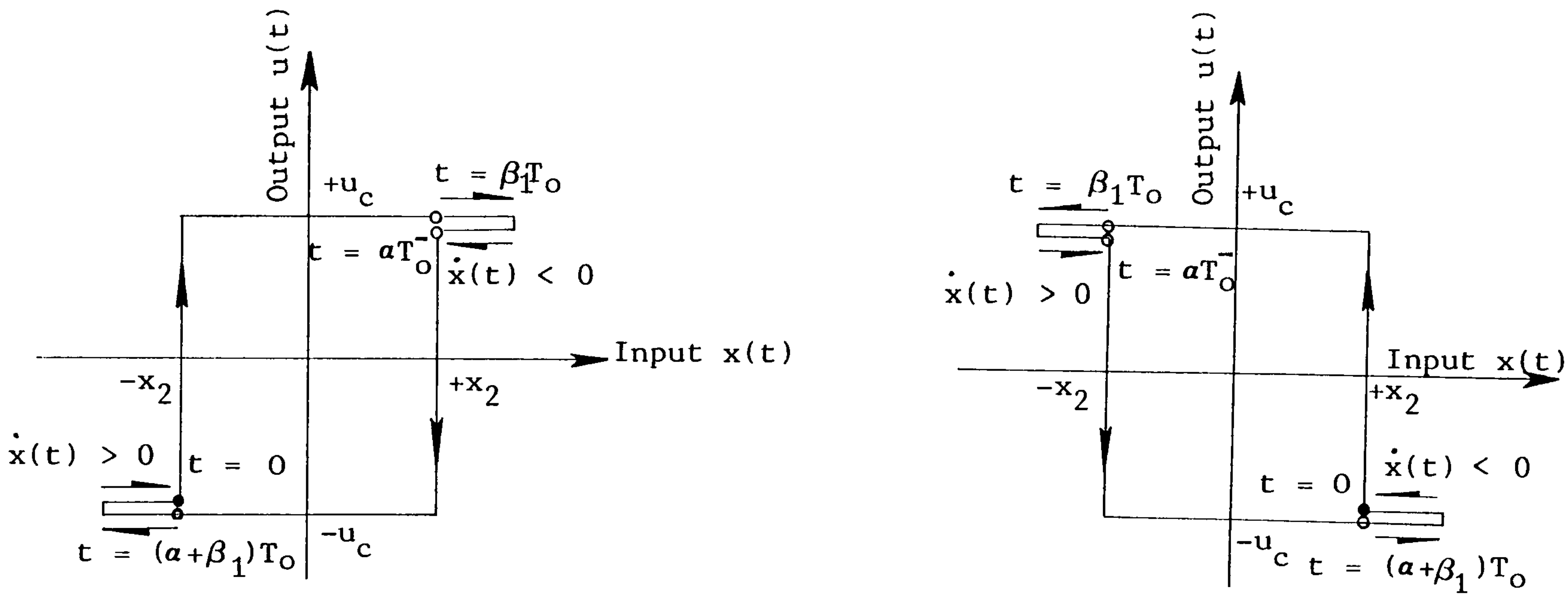


Figure 4.3. Schematic representation showing the effects of dead time on the relay, with $T_{D,1} < T_{D,2} < T_{D,c}$ and $x_2 > x_{2,1} > x_{2,2} > 0$.

Formal Definition of Negative Bandwidth Relays with Hysteresis

Here we define the useful negative bandwidth relays with hysteresis. They are presented schematically in Figure 4.4 where the notation in the figure is the same as in Figure 2.4, page 27-28 for the positive bandwidth relays. Again following past convention, time $t = 0$ is defined as a time when the relay switches to a positive drive level, that is, at $t = 0^-$ $u(0^-) = -u_c$ and at $t = 0^+$ $u(0^+) = +u_c$.

In contrast to relays with positive bandwidths, no overshoot (no movement out from and back to the hysteresis region) of the input signal is allowed after the relay switches. Additional contrast arises at the corners of the memory element where the relay switches drive levels, the negative bandwidth relays require the input signal to first pass or overshoot the switching value, i.e. $+x_2$ or $-x_2$, then return to the switching value before the drive level of the relay changes. In other words, the input signal must pass the switching value leaving the hysteresis region. The derivative of the input must then change so the signal returns toward the hysteresis region. When the input value reaches the edge of the hysteresis region the relay switches drive levels.



(a) Passive relay with negative bandwidth (b) Active relay with regative bandwidth

● -switch of the relay to a positive drive level which is used to define $t = 0$

O -notation to indicate other times used in stating the relay switch and continuity conditions; these times are given for $n = 0$ in equation 4.2 - 4.5

Figure 4.4. Two newly-defined relays with hysteresis.

In the limit of very small overshoot where the signal leaves and returns to the hysteresis region, the derivative of the input signal would be required to change sign <u>at</u> the switching value.

Switching and Continuity Conditions for Negative Bandwidth Relays

In this section the characteristics of these nearly defined relays are expressed mathematically. The more intricate character of negative bandwidth relays requires a more complex mathematical description compared to positive bandwidth relays. However implementation via a computer or processor algorithm is no problem.

Referring to Figure 4.4a for the passive element at $t = 0^-$, the input value approaches the hysteresis region where $x = -x_2$ from the left, thus $\dot{x}(0^-) > 0$. At $t = 0$, $x(0) = -x_2$ and the drive level of the relay changes discontinuously from $u(0^-) = -u_c$ to $u(0^+) = +u_c$. The value of x continues to increase and passes or overshoots the other switching point $+x_2$, without switching the drive level. After leaving the hysteresis range and returning, at $t = \alpha T_o$, x must be arriving at $x(\alpha T_o^-) = +x_2$ and $\dot{x}(\alpha T_o^-) < 0$. Instaneously, the drive level of relay changes from $u(\alpha T_o^-) = +u_c$ to $u(\alpha T_o^+) = -u_c$. This will drive the value of x to decrease and pass the switching point $-x_2$. Again the relay will not switch to positive drive level until the input signal leaves the hysteresis range and returns the switching point.

To cast the above movement around the relay into mathematical terms the following parameters are defined:

α fraction of the period that the relay is at the positive drive level

β_1 fraction of the period after a switch to the positive drive level that the input is within the hysteresis region

β_2 fraction of the period after a switch to the negative drive level that the input is within the hysteresis region

Note the α has the same definition in the switching and continuity conditions for positive bandwidth relays (Equations (2.12-2.15)). The parameters β_1 and β_2 are not needed for the positive bandwidth relays.

Again referring to Figure 4.4a the switching conditions for the passive relay with negative bandwidth can be expressed as:

$$x(nT_o^-) = -x_2 \; ; \; \dot{x}(nT_o^-) > 0 \tag{4.2.a}$$

$$x((\beta_1 + n)T_o) = x_2 \; ; \; \dot{x}((\beta_1 + n)T_o) > 0 \tag{4.2.b}$$

$$x((\alpha + n)T_o^-) = x_2 \; ; \; \dot{x}((\alpha + n)T_o^-) < 0 \tag{4.2.c}$$

$$x((\beta_2 + \alpha + n)T_o) = -x_2 \; ; \; \dot{x}((\beta_2 + \alpha + n)T_o) < 0 \tag{4.2.d}$$

where

$$n = 0,1,2,3.....$$

for the active relay with negative bandwidth

$$x(nT_o^-) = x_2 ; \dot{x}(nT_o^-) < 0 \qquad (4.3.a)$$

$$x((\beta_1 + n)T_o) = -x_2 ; \dot{x}((\beta_1 + n)T_o) < 0 \qquad (4.3.b)$$

$$x((\alpha + n)T_o^-) = -x_2 ; \dot{x}((\alpha + n)T_o^-) > 0 \qquad (4.3.c)$$

$$x((\beta_2 + \alpha + n)T_o) = x_2 ; \dot{x}((\beta_2 + \alpha + n)T_o) < 0 \qquad (4.3.d)$$

where

$$n = 0,1,2,3,....$$

Note that in Equation (4.2) the superscript minus on the time value indicating the value is approached below is only retained at the switching values where discontinuies occur. According the previous description, the continuity conditions of the passive negative bandwidth relays can be expressed mathematically as:

$$-x_2 < x(t) < x_2 ; nT_o < t < (\beta_1 + n)T_o \qquad (4.4.a)$$

$$x_2 < x(t); (\beta_1 + n)T_o < t < (\alpha + n)T_o \qquad (4.4.b)$$

$$-x_2 < x(t) < x_2 ; (\alpha + n)T_o < t < (\beta_2 + \alpha + n)T_o \qquad (4.4.c)$$

$$x(t) < -x_2 ; (\beta_2 + \alpha + n)T_o < t < (n + 1)T_o \qquad (4.4.d)$$

where

$$n = 0,1,2,3......$$

for active relay with negative bandwidth

$$-x_2 < x(t) < x_2 ;\ nT_o < t < (\beta_1 + n)T_o \qquad (4.5.a)$$

$$x(t) < -x_2 ;\ (\beta_1 + n)T_o < t < (\alpha + n)T_o \qquad (4.5.b)$$

$$-x_2 < x(t) < x_2 ;\ (\alpha + n)T_o < t < (\alpha + \beta_2 + n)T_o \qquad (4.5.c)$$

$$x_2 < x(t) ;\ (\alpha + \beta_2 + n)T_o < t < (n + 1)T_o \qquad (4.5.d)$$

where

$$n = 0,1,2,3,....$$

For a symmetric oscilltation (the set point is zero and the relay is symmetric) in a feedback control scheme $\alpha = 1/2$ and $\beta_1 = \beta_2$.

Finally, the relationship among the four kinds of relays with hysteresis is illustrated schematically in Figure 4.5.

Based on these derived equations a controller design strategy is established to compensate for dead time in next chapter and demonstrated with the example two variable CSTR.

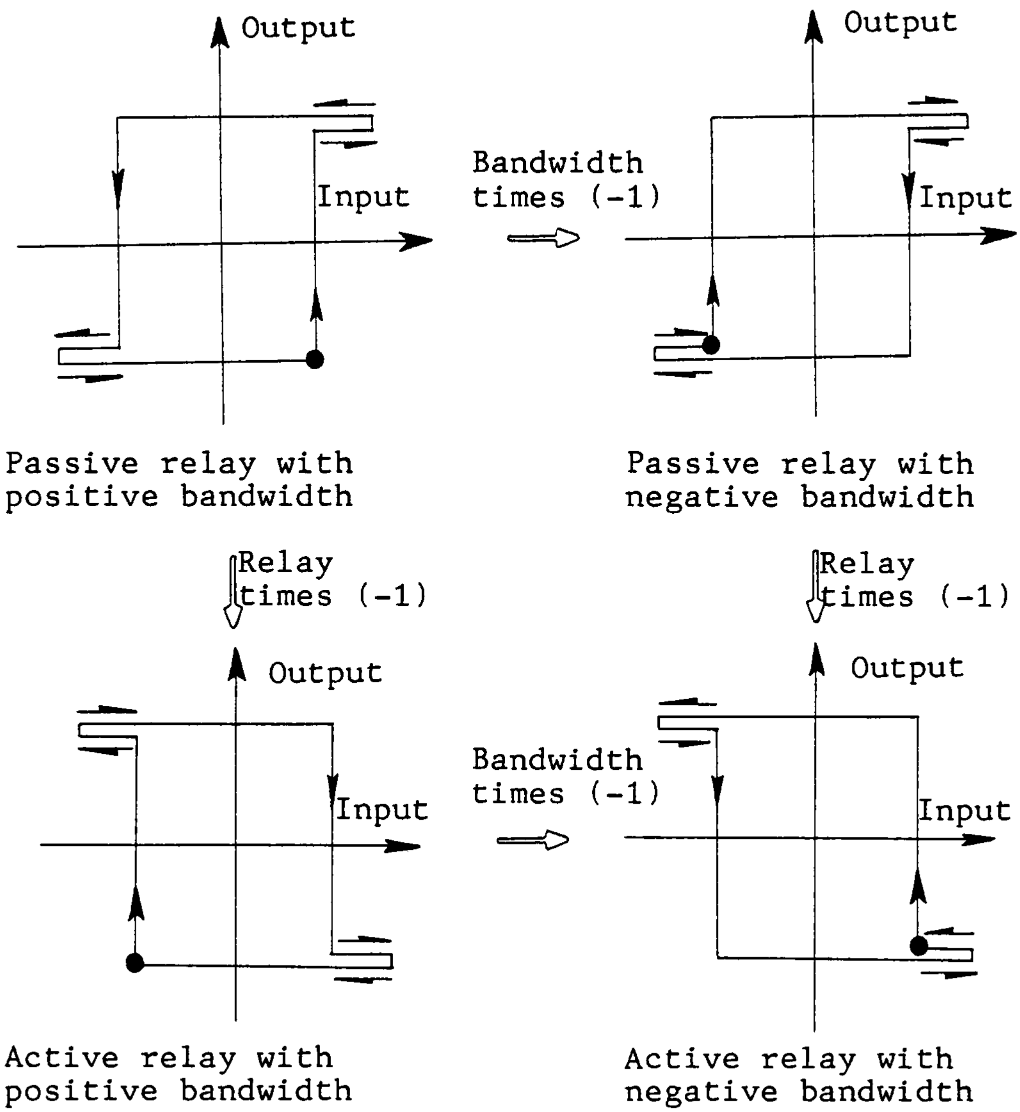


Figure 4.5. The relationship among four types of relays.

CHAPTER V

CONTROLLER ANALYSIS, DESIGN AND DEAD TIME COMPENSATION ON SYSTEMS WITH DEAD TIME

Controller Design Strategies on the Linearized CSTR System Containing Dead Time

The outline for the controller design on this specified linearized two variable CSTR system with dead time can be illustrated as in Figure 5.1. First, seperate the dead time from the process transfer function $G_s(s)$ and incorporate this dead time with, at this point in the analysis, an unknown relay controller which can be used to compensate for dead time effectively. Thus, this combination of unknown relay controller and dead time is an equivalent relay controller for the process transfer function with no dead time. For the CSTR example of Chapter III the equivalent relay is identical to a passive relay controller which, as mentioned before, can fulfill the pushing control policy and lead to a successful control scheme, maintaining system stable around the open loop unstable steady state.

The next question is how to determine the type and bandwidth of the unknown relay controller. Before proceeding, let's notice some useful properties of a periodic function. For a given period of oscillation T_o,

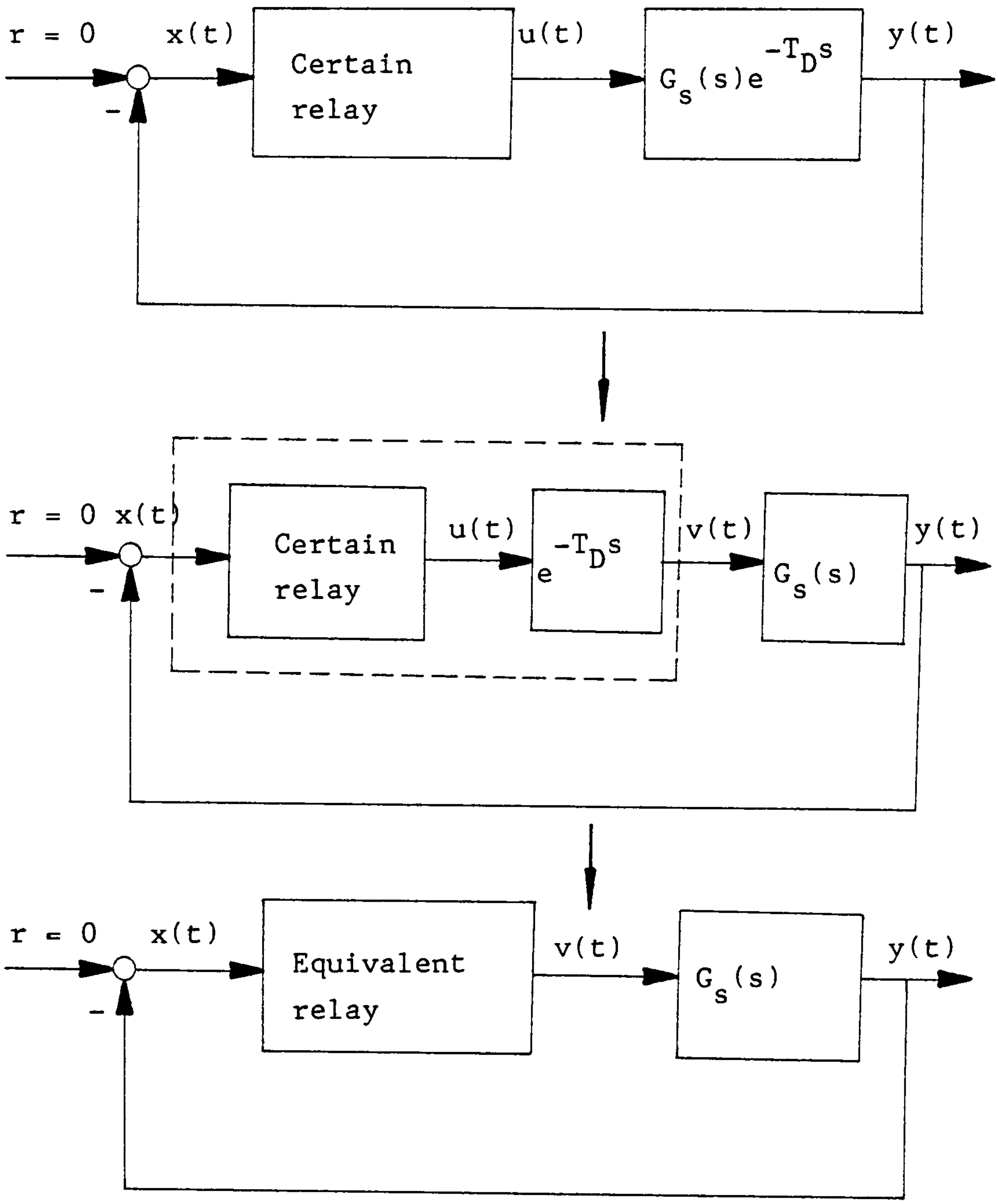


Figure 5.1. The diagram of outline for the controller design on systems containing dead time.

both symmetric passive and active relays exhibit the following properties with an integer k, and N denotes the relay function.

1. $N_{Passive}(t + kT_o) = N_{Passive}(t)$

2. $N_{Active}(t + kT_o) = N_{Active}(t)$

3. $N_{Passive}(t + (2k + 1)T_o) = N_{Passive}(t + T_o/2)$

$$= (-1) \times N_{Passive}(t)$$

$$= N_{Active}(t) \qquad (5.1)$$

Another major advantage of using relay controllers to compensate for dead time is contributed by these periodic properties which allow elimination of dead time efficiently by merely altering the type and bandwidth of the relays without changing the dynamic behavior of systems. Next, define the reduced dead time T_{Dr} as:

$$T_{Dr} = T_D - \left(\frac{T_o}{2}\right) n_r \qquad (5.2)$$

where n_r is the maximum integer so that

$$0 < T_{Dr} < T_o/2$$

As mentioned before, a passive relay can provide a successful control policy for the linearized CSTR system containing zero dead time, operating around the open loop unstable steady state. Consequently, the objective of this design is to identify an appropriate controller so that the combination of this controller and dead time is equivalent to a passive relay controller with the specified bandwidth x_2 determined by the design requirement.

In this design problem, x_2 which is always given by the required behavior of oscillation can determine the period of oscillation by substituting $T_D = 0$ and the value of x_2 into Equation (3.48). Keep in mind that Tsypkin's equations (Equations (3.48-3.51)) have a limitation on the dead time, ranging from zero to half period of oscillation. If the system dead time is greater than the half period, Equations (3.48-3.51) cannot be used. Consequently, in treating the large dead time problem, some modification is required to adapt the Tsypkin's equations. Using the periodic properties listed in Equation (5.1) and the definition of reduced dead time (Equation (5.2)) allows us to modify the switching conditions in terms of reduced dead time instead of system dead time. Utilizing this modification, no matter how large a dead time the system contains, a given period of oscillation (specified behavior of oscillation), for the system without dead time can be maintained by using a compensating relay. Once the reduced dead time is determined, Tsypkin equations can be ultilized to calculate the bandwidth of the compensating relay controller by substituting the period of oscillation, reduced dead time and the other fixed system parameters into the modified switching condition equations. Based on the previous statements, for a passive relay element, modified Equations of switching conditions from Equations (3.48-3.49) can be expressed as:

$$\frac{k_1}{\lambda_1}\left[e^{-\lambda_1 T_{Dr}}\left(\tanh\frac{\pi\lambda_1}{2\Omega_o} + 1\right) - 1\right] + \frac{k_2}{\lambda_2}\left[e^{-\lambda_2 T_{Dr}}\left(\tanh\frac{\pi\lambda_2}{2\Omega_o} + 1\right) - 1\right] = \frac{x_2}{u_c} \tag{5.3}$$

and

$$\frac{k_1}{\Omega_o} e^{-\lambda_1 T_{Dr}}\left(\tanh\frac{\pi\lambda_1}{2\Omega_o} - 1\right) + \frac{k_2}{\Omega_o} e^{-\lambda_2 T_{Dr}}\left(\tanh\frac{\pi\lambda_2}{2\Omega_o} - 1\right) > \frac{B_{21}}{\Omega_o} \tag{5.4}$$

Replacing T_D with T_{Dr} in Equations (3.50-3.51) gives the modified Equations of switching conditons for an active relay element:

$$\frac{k_1}{\lambda_1}\left[e^{-\lambda_1 T_{Dr}}\left(\tanh\frac{\pi\lambda_1}{2\Omega_o}+1\right)-1\right]$$

$$+\frac{k_2}{\lambda_2}\left[e^{-\lambda_2 T_{Dr}}\left(\tanh\frac{\pi\lambda_2}{2\Omega_o}+1\right)-1\right]=\frac{x_2}{u_c} \qquad (5.5)$$

and

$$\frac{k_1}{\Omega_o}\, e^{-\lambda_1 T_{Dr}}\left(\tanh\frac{\pi\lambda_1}{2\Omega_o}-1\right)$$

$$+\frac{k_2}{\Omega_o}\, e^{-\lambda_2 T_{Dr}}\left(\tanh\frac{\pi\lambda_2}{2\Omega_o}-1\right) < \frac{B_{21}}{\Omega_o} \qquad (5.6)$$

Note that the reduced dead time T_{Dr} is defined by Equation (5.2), ranging from zero to half period of oscillation, and the value of n determines the type of compensating relay controller. If n_r is even the compensating relay controller has the same character as the equivalent relay fixed by the process without dead time. Whereas, if n_r is odd a relay opposite to the equivalent relay is used in the control scheme. This is best addressed in terms of two cases, in both cases $x_{2,c}$ represents the bandwidth of the equivalent relay controller and x_2 is the exact (under no disturbances) switching point of relay input at which the oscillatory relay output switches. One case arises if n_r is odd ($n_r = 2k + 1$, $k = 0,1,2,3,\ldots$)

$$N_{Passive,x_2}(t)$$

$$=N_{Passive,x_{2,c}}(t + T_{Dr})$$

$$= N_{Passive,x_{2,c}}(t+T_D-T_o/2 -kT_o)$$

$$= N_{Passive,x_{2,c}}(t+T_D - T_o/2)$$

$$= N_{Active,x_{2,c}}(t + T_D) \tag{5.7}$$

The dead time here is composed of two parts; one is the reduced dead time T_{Dr} and the other is an odd number of half periods of oscillation. The later gives a phase lag to the oscillation. Consequently, an active relay which is a phase lead element is needed to compensate for this phase change (see Figures 5.2a and 5.2b). For these results, the active relay associated with system dead time T_D (Figure 5.2a) is equivalent to a combination of a passive relay with an unchanged bandwidth $x_{2,c}$ and reduced dead time T_{Dr} (Figure 5.2c). Finally a single passive

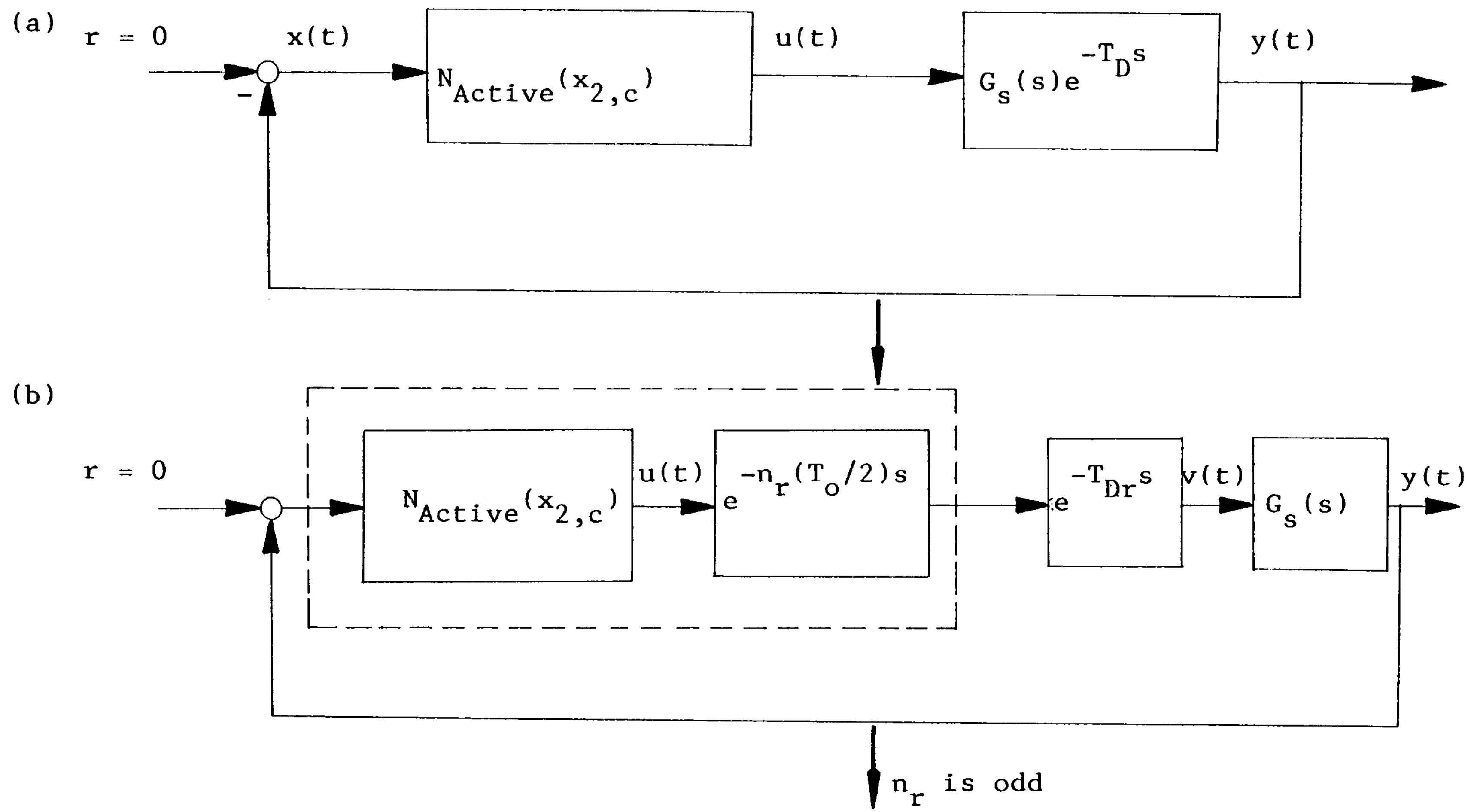


Figure 5.2. Dead time compensating active relay on the linearized CSTR model containing dead time which fits Equations (5.2) and gives an odd number of n_r.

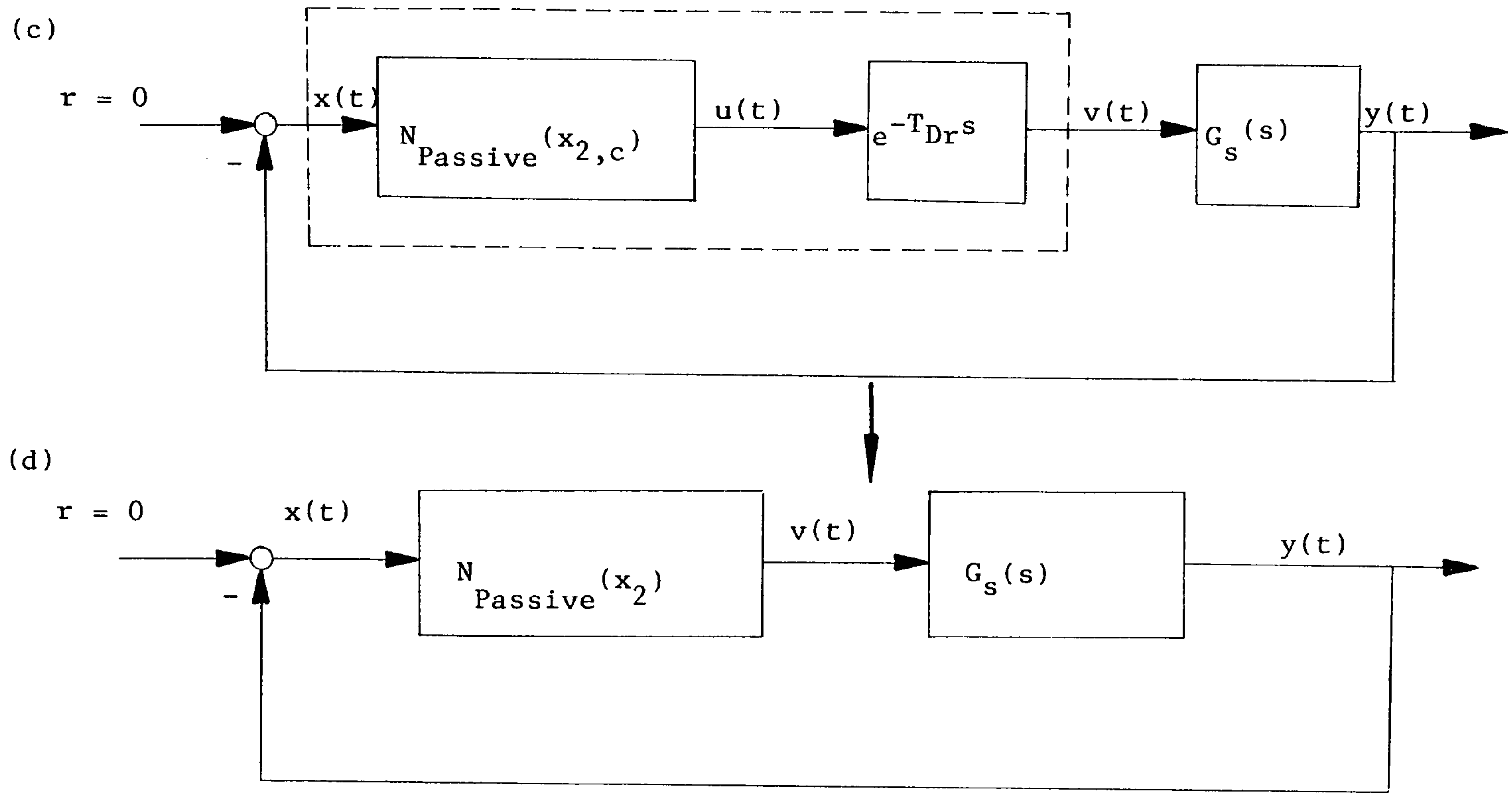


Figure 5.2. (continued)

relay with a larger bandwidth x_2 but without any dead time (Figure 5.2d) can replace this type of combination. Once x_2 is fixed to meet the design requirement, the period of oscillation T_o can be determined. Substituting this given period of oscillation and reduced dead time T_{Dr} into Equation (5.3) and solving for x_2 can give the desired bandwidth of the dead time compensating active relay controller (Figure 5.2a). If the reduced dead time T_{Dr} is greater than critical dead time $T_{D,c}$, a negative bandwidth active relay controller is expected. Otherwise a relay with positive bandwidth will be used to implement the control design.

The other case arises if n_r is even ($n_r = 2k$, $k = 0,1,2,3,\ldots$):

$$
\begin{aligned}
& N_{Passive,x_2}(t) \\
&= N_{Passive,x_{2,c}}(t + T_{Dr}) \\
&= N_{Passive,x_{2,c}}(t + T_D - kT_o) \\
&= N_{Passive,x_{2,c}}(t + T_D) \qquad (5.8)
\end{aligned}
$$

An integer number of oscillation period delays does not cause any phase change. Consequently, the original passive relay with dead time (Figure 5.3a) is equivalent to the same passive relay with reduced dead time (Figure 5.3b). In this case we don't need to change the type of relay. Next, the passive relay having bandwidth $x_{2,c}$ associated with reduced dead time (Figure 5.3c) is identical to another passive relay with a new bandwidth x_2 but no dead time(Figure 5.3d). Usually, the bandwidth x_2 is fixed by the design requirements, and it fixes the period of oscillation for the system without dead time. Consequently, to maintain the period of oscillation unchanged when there is dead time, the bandwidth of desired controller can be calculated by substituting the given period of oscillation and reduced dead time into Equation (5.3).

In summary, for the linearized two variable CSTR containing dead time, starting with the definition of reduced dead time, Equation (5.2), and then substitution of the reduced dead time and the system parameters into Equation (5.3) gives an appropriate bandwidth of controller $x_{2,c}$ which fulfills the control objective. Thus the four distinct possibilities that can arise are delineated as:

1. If n_r is odd and

(a) If $x_{2,c} > 0$, use active relay with positive bandwidth controller

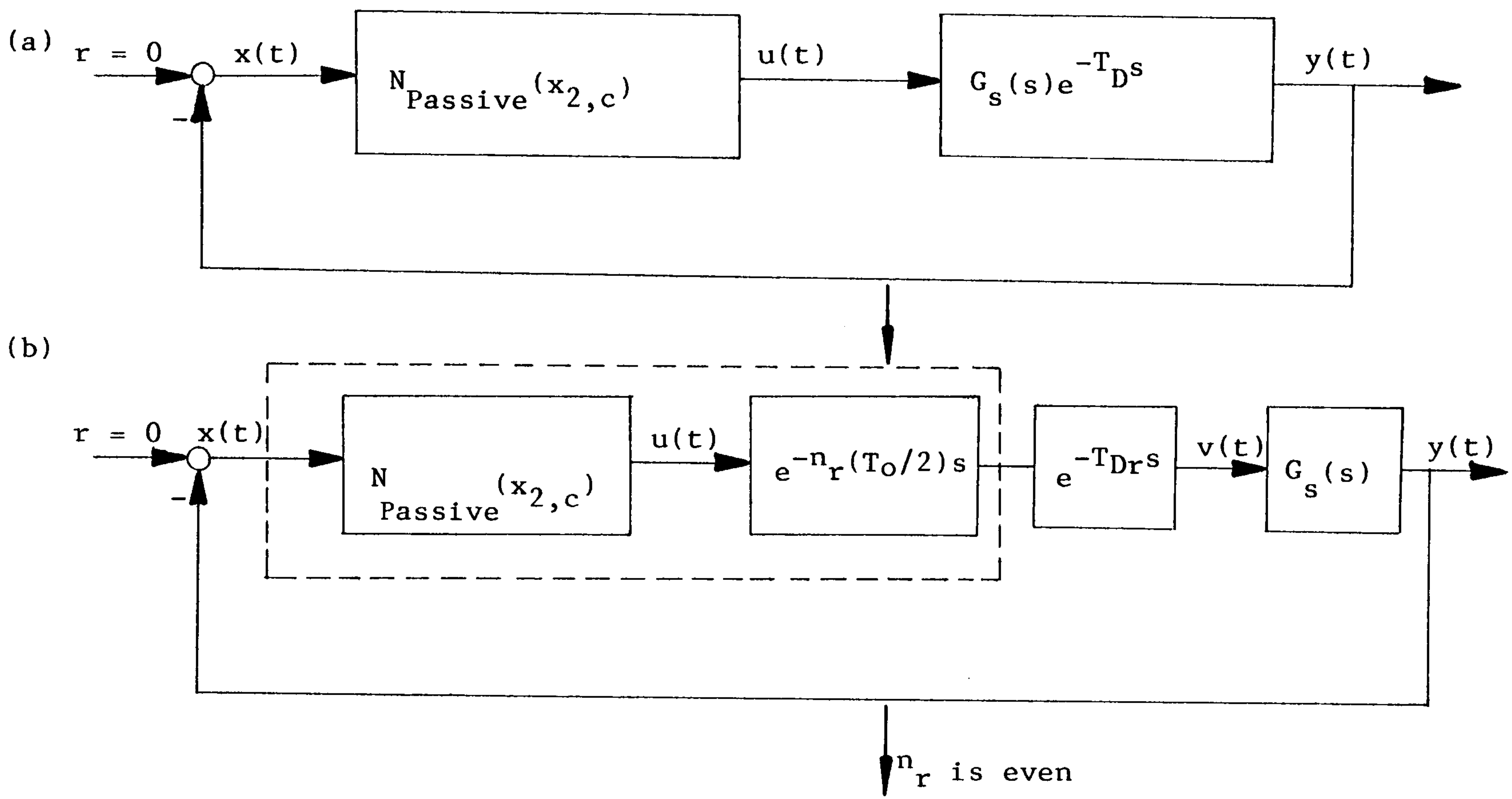


Figure 5.3. Dead time campensating relay on the linearized CSTR model containing dead time which fits Equation (5.2) and gives an even number of n_r.

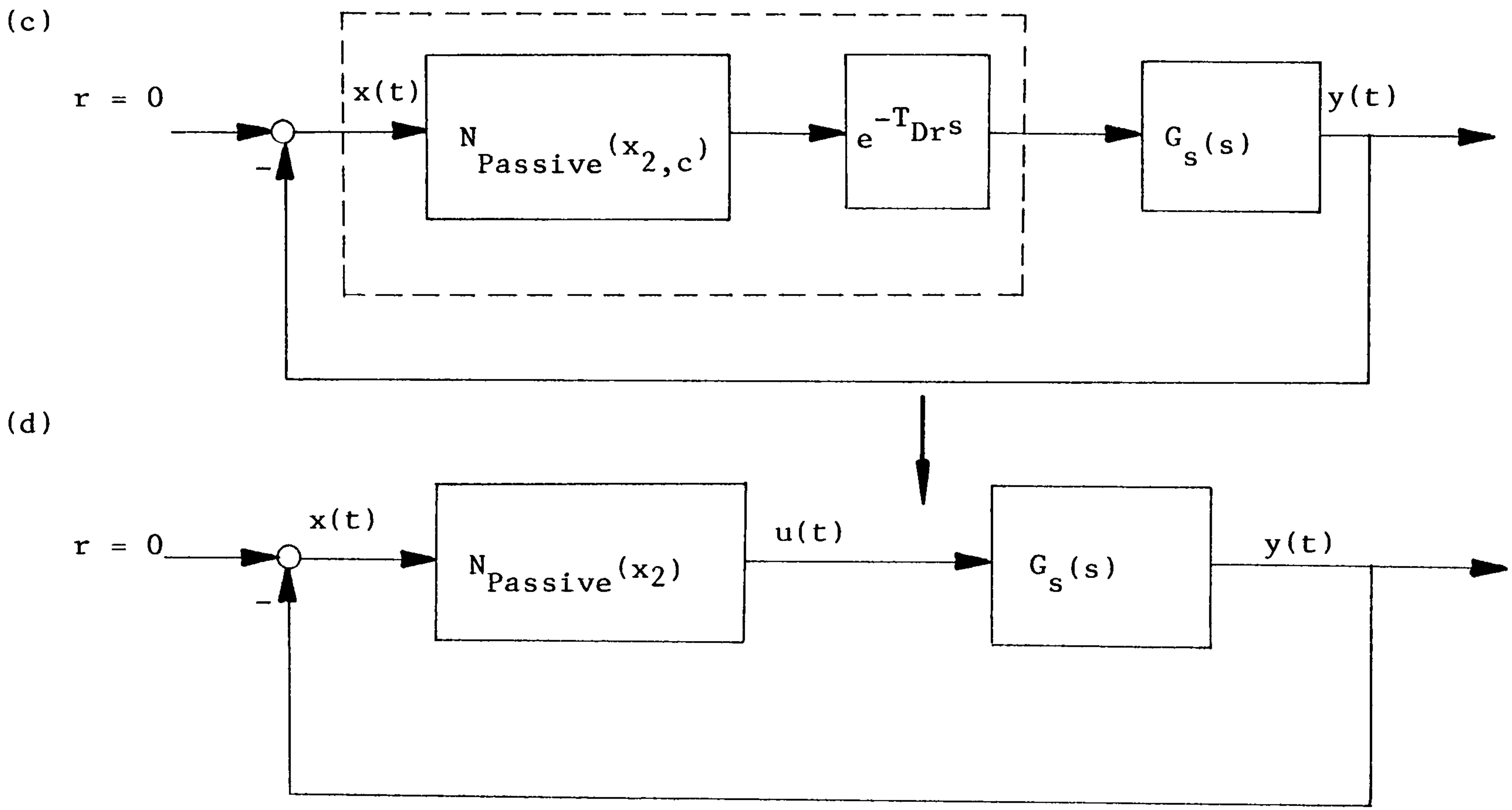


Figure 5.3. (continued)

(b) If $x_{2,c} < 0$, use active relay with negative bandwidth controller

2. If n_r is even and

(a) If $x_{2,c} > 0$, use passive relay with positive bandwidth controller

(b) If $x_{2,c} < 0$, use passive relay with negative bandwith controller

Numerical Example for CSTR

The numerical values of the system parameters in this example which have been used by others [7, 9, 24] are listed in Table 5.1, and the values of dimensionless parameters defined by the Equation (3.4) can be found in Table 5.2. The temperature and concentration of the unstable steady state are : T^*=322.98932 or $\mathcal{Y}^* = 1.10766$, and C_A^*=0.00258 or $\mathcal{W}^* = 0.51600$. The elements of the coefficient matrix of the linearized equations near the unstable steady state are evaluated by the Equations (3.26) and (3.27) and listed in Table 5.3, along with the eigenvalues of the system given by the Equations (3.35.a-3.35.b), and the constants of k_1 and k_2 calculated by Equations (3.36.a-3.36.b). If at least one eigenvalue has a positive real part the system is locally unstable. Specifying the tolerable dimensionless deviation y_e away from the desired operation point $\mathcal{Y}^*$ leads to the determination of the higher and lower drive levels which can

Table 5.1. The Numerical Values of the System Parameters.

System parameter	Value	Unit
density ρ	1.0	g/cc.
capacity, C_p	1.0	cal/(cc.)($^{\circ}$C)
flow rate, F	10.0	cc./sec
reactor volume, V	2,000.0	cc.
product of heat transfer coefficient and surface area, H_A	1.356	cal/($^{\circ}$C)(sec)
Arrhenius constant, K	7.86×10^{12}	sec
activation energy, E	22,500.0	Cal/gm-mole
reactant feed concentration, C_{Af}	0.005	gm-mole/liter
feed stream temperature, T_f	300.0	$^{\circ}$K
average coolant temperature, T_c	314.4	$^{\circ}$K

Table 5.2. The Values of Dimensionless Parameters.

Dimensionless parameter	Value
γ	37.74534
β	0.06366
B	0.13560
Da	0.16667

Table 5.3. The Values of Elements of Constant Matrix A, the Eigenvalues of System and the Constants of Partial Fraction Expansion of Transfer Function $G_s(s)$.

Parameters	Value
Element	
A_{11}	-1.9346
A_{12}	-15.704
A_{21}	0.15577
A_{22}	1.4817
B_{11}	0.0
B_{21}	0.1356
Eigenvalues	
λ_1	0.46026
λ_2	-0.91316
Constant of the partial fraction	
k_1	0.23645
k_2	-0.10085

fulfill the control design. The numerical values of dimensionless deviation coolant temperatures calculated by Equations (3.31), (3.32.a) and (3.32.b) based on allowable deviation of reactor temperature $\Delta\mathcal{Y} = 4\,^{o}C$ or dimensionless deviation reactor temperature $y_e = 0.013333$ are $u_C = u_1 = u_4 = 0.021366$, and the maximum possible bandwidth $x_{2,max} = 0.64612y_e = 0.0086147$. As a result, choose $x_2 = 0.5y_e = 0.0066665$ as the bandwidth of the original controller which can achieve the desired performance in the absence of dead time.

Without any dead time, i.e. $T_D = 0$, the period of oscillation T_o calculated by Equation (5.3) for the present example is 10.002 sec. If a dead time is added in this process, the design strategy is to compensate for this dead time by changing the bandwidth, and if necessary also changing the type of the relay controller without altering the dynamic behavior of the system. When the dead time is less than critical dead time, for an unchanged period of oscillation, as the system dead time increases, the controller bandwidth must decrease to compensate for this increasing dead time. A particular case is given by using an ideal relay controller when the dead time is equal to the critical dead time. If the dead time exceeds the critical dead time but is still less than half period of oscillation, the bandwidth is reduced to a negative value thus a negative bandwidth controller is used to compensate for dead time between $T_{D,c}$ and half of period. Therefore a negative value of x_2 will be obtained from Equation (5.3). When the dead time is over half period of oscillation, but still less than one period of oscillation, according to the definition of reduced dead time, $n_r = 1$ which is an odd number. Thus an active relay controller is necessary to replace the original passive relay controller so that the combination of active relay and the dead time of half period of oscillation is equal to a passive relay controller. Then this rearrangement will reduce the compensating controller to case 2(a) of page 118, in which only a passive relay controller along with a reduced dead time exists. Similarly, if the reduced dead time is greater than the critical dead time, a negative bandwidth will be expected (case 2(b) of page 118). Substitution of the unchanged period of oscillation and reduced dead time into Equation (5.3) gives the appropriate bandwidth of the compensating active relay controller $x_{2,c}$ which is to be used in the control network. The resulting values for the present example can be found in Table 5.4 based on different values of dead times. These results include all four possibilties of passive relay with positive bandwidth, passive relay with negative bandwidth, active relay with positive bandwidth and active relay with negative bandwidth (as listed on page 115).

Table 5.4. The Parameters of Compensating Controllers for the Systems Containing Different Values of Dead Time.

Case	Dead time T_D	n_r	Reduced dead time, T_{Dr}	Bandwidth of controller $\delta = x_{2,c}/y_e$
I	0.2	0		0.36914
II	2.1	0		-0.406020
III	5.2012	1	0.2002	0.369006
IV	6.8006	1	1.7996	-0.327561

For the dead time compensator design for the various dead times of Table 5.4, the waveform of the reactor's temperature output was calculated by using the closed form expressions for y(t) from Tsypkin's method given by Equation (3.52). Recalling that Tsypkin's method treats linearized processes under relay control exactly, simulations were carried out for the linearized model (Equations (3.26-3.27)) in corporating the appropriate dead time compensating relay controller equations into the simulation.

The waveforms given by Tsypkin's method and the numerical simulation are shown in Figures 5.4 to 5.7. The exact coincidence of the simulation curves and analytical solution shown in the figures for all cases demonstrates the validity of this concept of controller design strategy. Note also that curves in Figures 5.4 to 5.7 could be plotted on one curve which would fall on the curve for the process with no dead time using the passive relay with bandwidth being x_2. The seperate plots are kept here in testing the design procedure where the compensating controller is each of the four types of relay with hysteresis.

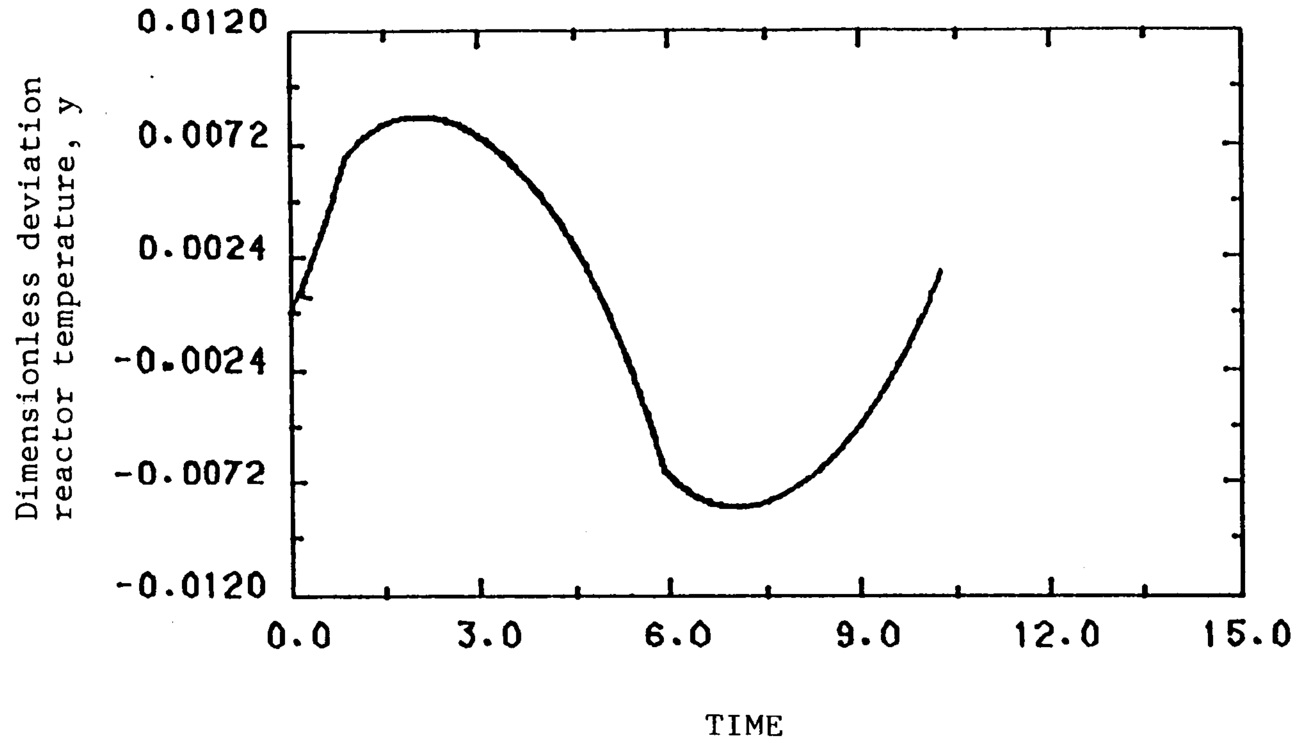


Figure 5.4. Comparison of exact waveform with the simulation for the linearized two variable CSTR system containing dead time $T_D = 0.2$ sec. and a passive relay controller with bandwidth $x_{2,C} = 0.36914 y_e$.

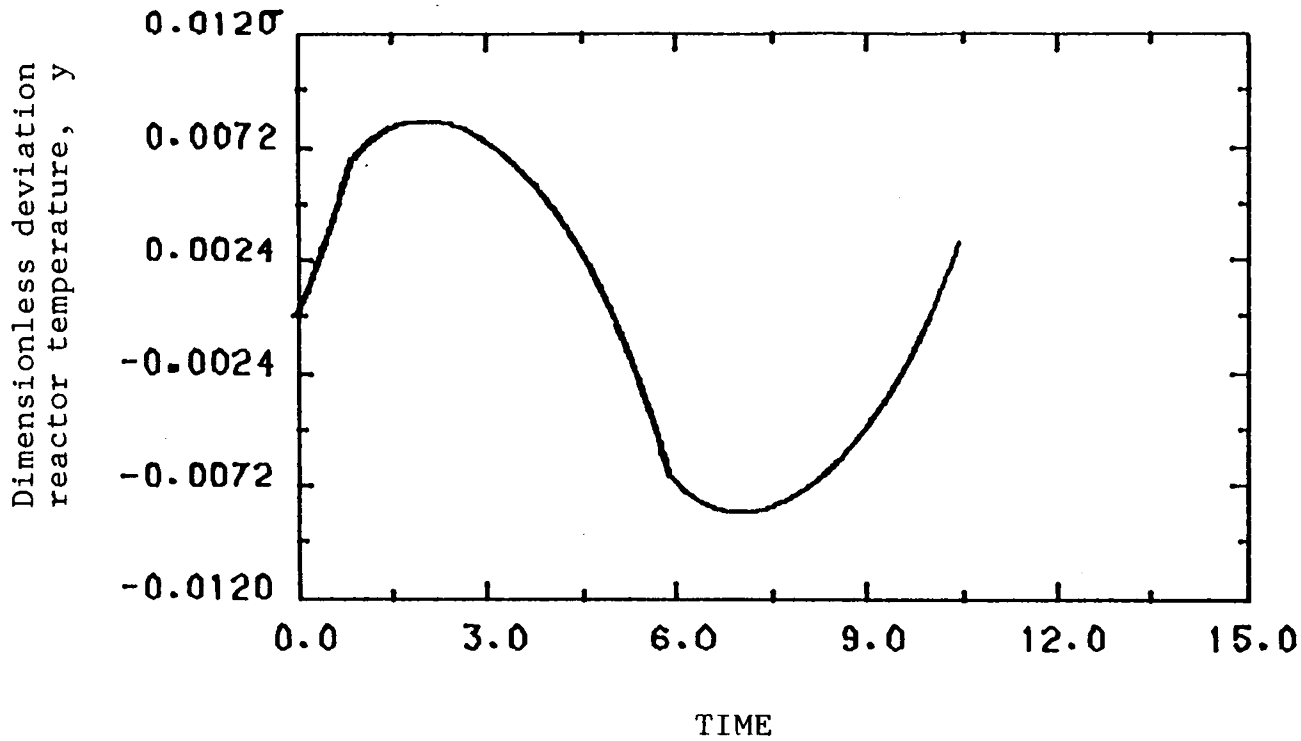


Figure 5.5. Comparison of exact waveform with the simulation for the linearized two variable CSTR system containing dead time $T_D = 2.1$ sec. and a passive relay controller with bandwidth $x_{2,C} = -0.40602y_e$.

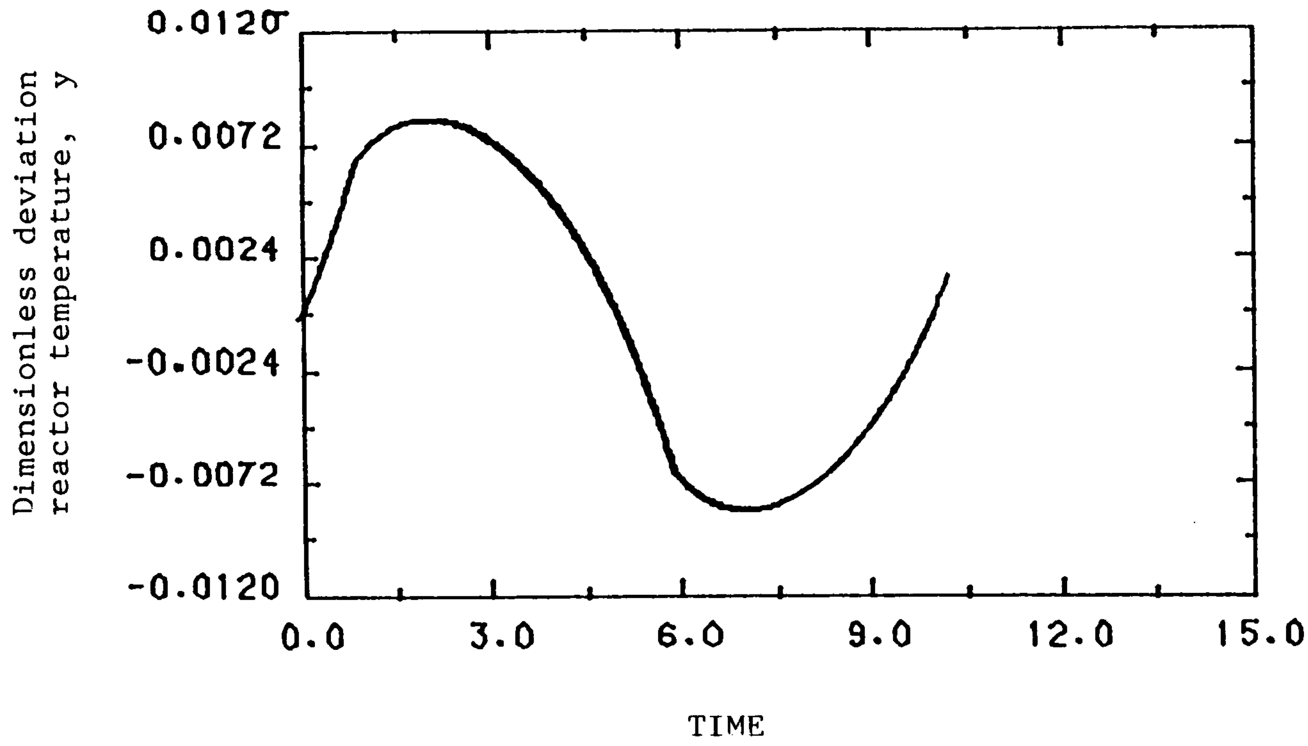


Figure 5.6. Comparison of exact waveform with the simulation for the linearized two variable CSTR system containing dead time $T_D = 5.2012$ sec. and an active relay controller with bandwidth $x_{2,C} = 0.369006 y_e$.

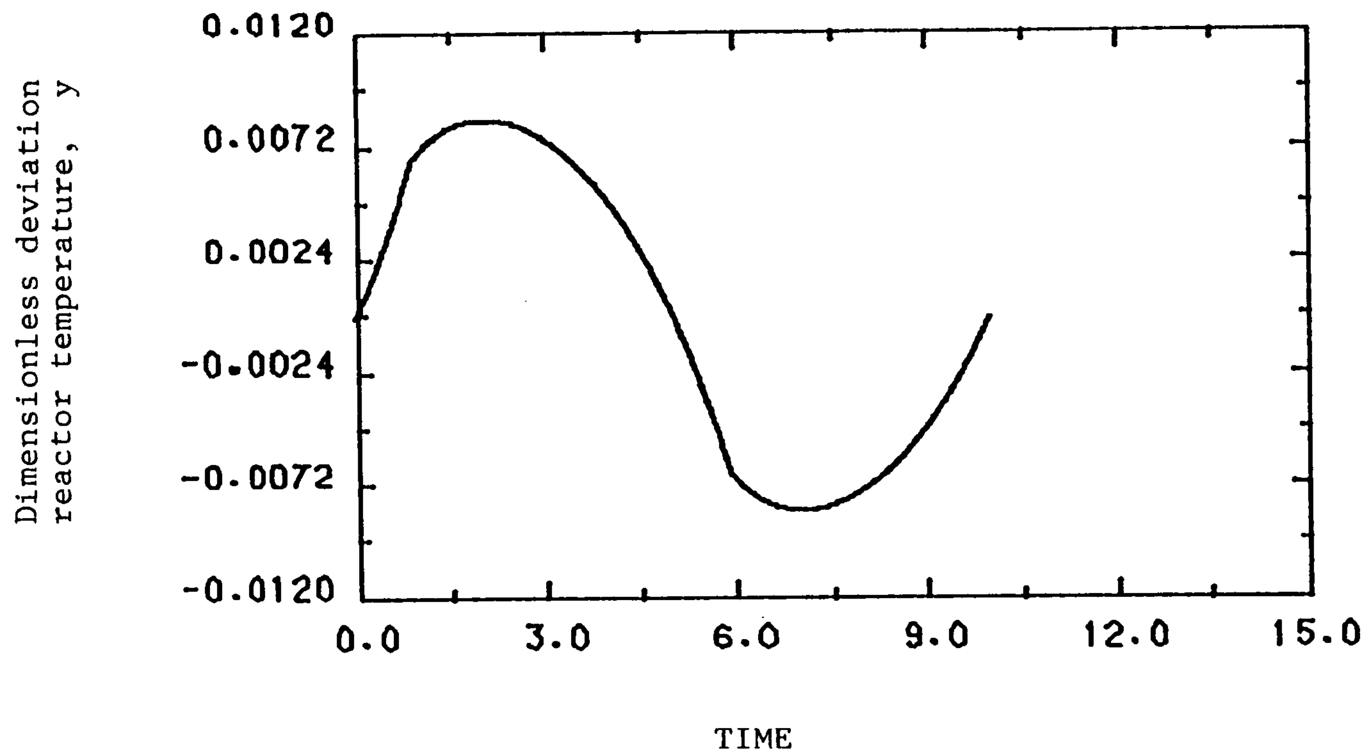


Figure 5.7. Comparison of exact waveform with the simulation for the linearized two variable CSTR system containing dead time $T_D = 6.8006$ sec. and an active relay controller with bandwidth $x_{2,C} = -0.327561 y_e$.

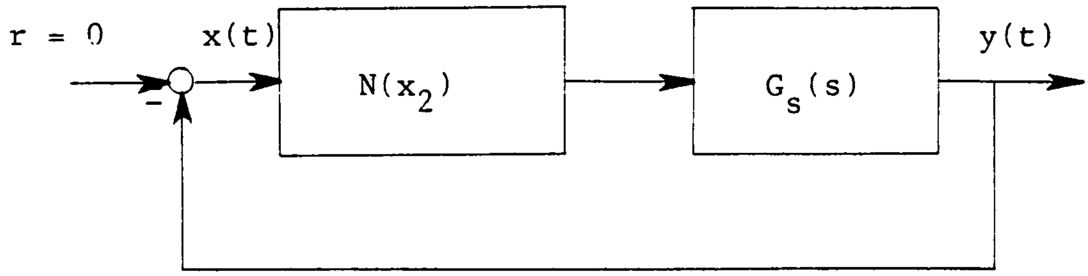
Simulation Technique

By utilizing UTCC Dec-10 system, a routine named DVERK (a differential equation solver in the IMSL package program based on a Runge-Kutta-Verner fifth and sixth order method) was employed to create a FORTRAN program for the purpose of simulating the relay feedback control scheme.

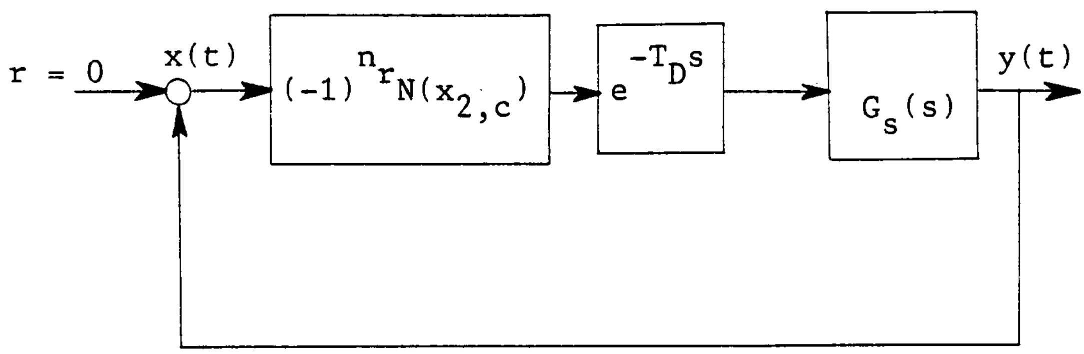
The integration output is discretized to have the relay switch at exactly the right time and to have the input from the controller to process delayed a time exactly equal to T_D. Special techniques were used in this discretization such that the end of an integration step was equal to these certain times. Moreover, the proper start up conditions which include the initial conditions and switch times of the relay until the simulation time is greater than T_D are required to achieve the control implementation. Errors in start up are equivalent to a disturbance which may impair the process performance. It is convenient to set the initial conditions of the system at the switching point of the compensating relay. Therefore, based on the waveform of the system without dead time, the appropriate initial conditions are determined by tracing back a time equal to the dead time from both switching points without dead time. These employed strategies are detailed in a seperate report [52].

General Design Formulation for Dead Time Compensation

Although this strategy was introduced by illustration with the accompanying CSTR example, it can be employed for any nonlinear feedback control loop containing a relay with hystereswas. Suppose for an arbitary system without dead time the relay with bandwidth being x_2 which is denoted by $N(x_2)$ can fulfill the control policy, as shown in Figure 5.8a. This specified bandwidth x_2 and process transfer function without dead time determines the closed loop periodic response characteristics, the period of oscillation, T_o, amplitude, etc. If system dead time T_D is now introduced, the previous design procedure allows the modified controller which can compensate for any value of dead time to be easily determined. Equations (2.27-2.28) and Equations (2.29-2.30) for passive and active element respectively serve as the starting basis for the derivation of the analytic dead time compensation equations. These analytic equations can be used to calculate the bandwidth of the compensating controller, $x_{2,c}$. Subsequently, the controller which can compensate for the dead time effectively is determined by $(-1)^{n_r}N(x_{2,c})$ which gives the same oscillatory output as the system without dead time (see Figure 5.8b). Note that the notation of (-1) to the



(a) The proper control scheme for the controlled system without dead time



(b) The controller design for the system containing dead time

Figure 5.8. Diagram of controller design for the general nonlinear feedback control scheme containing relay controller and dead time.

power n_r indicates, according to Equation (5.1), that if n_r is even the active-passive nature of the dead time compensating relay is the same as the relay specified in terms of the process without dead time. If n_r is odd the active-passive nature is changed.

Evaluation of a Specified Relay Controller On a Dead Time Process

Equations (5.3) and (5.4) give another useful application on evaluating the validity of a given relay with hysteresis feedback control scheme on a dead time system. For example, consider the two variable CSTR, as shown in Figure 3.8, page 80, with the given values of controller and system parameters. Substitution of the parameters (reduced dead time, bandwidth of controller and coolant dimensionless temperature deviation) into Equation (5.3) can answer the question "Does this design provide a performable result, that is, does a periodic closed loop solution exist." If a positive real value of period can be found from Equation (5.3) which also fulfills switching condition on the derivative, Equation (5.4), as well as, the continuity conditions, Equation (3.52), the given controller is viable. Otherwise, negative or imaginary values of the period reveal that the given control scheme is not practical. Unfortunately, as defined before, reduced dead times which appear in Equations (5.3-5.6) are determined by the period of oscillation which presently is an unknown value. Therefore, modification is needed in the analysis to give an applicable equation to avoid using reduced dead time, and keep the the presently unknown value of period of oscillation in the equations (5.3-5.7). This is again illustrated in terms of the example CSTR. Substituting for reduced dead time T_{Dr} as defined by Equation (5.2) into Equations (5.3) and (5.4) allows them to be expressed in terms of dead time T_D, period of oscillation T_o and an integer n_r.

$$\frac{k_1}{\lambda_1}\left[e^{-\lambda_1 T_D} e^{\lambda_1 n_r (T_o/2)} \left(\tanh \frac{\pi\lambda_1}{2\Omega_o} + 1\right) - 1\right]$$

$$+ \frac{k_2}{\lambda_2}\left[e^{-\lambda_2 T_D} e^{\lambda_2 n_r (T_o/2)} \left(\tanh \frac{\pi\lambda_2}{2\Omega_o} + 1\right) - 1\right] = \frac{x_2}{u_c} \quad (5.9)$$

$$\frac{k_1}{\Omega_o} e^{\lambda_1 T_D} e^{\lambda_1 n_r (T_o/2)} \left(\tanh \frac{\pi\lambda_1}{2\Omega_o} - 1\right)$$

$$+ \frac{k_2}{\Omega_o} e^{\lambda_2 T_D} e^{\lambda_2 n_r (T_o/2)} \left(\tanh \frac{\pi\lambda_2}{2\Omega_o} - 1\right) > \frac{B_{21}}{\Omega_o} \quad (5.10)$$

If the given relay is passive, it implies that n_r is an even number. Otherwise n_r is an odd number for an active relay. Equation (5.9) which must now be solved contains two unknown: n_r and T_o. The solutions of integer n_r and period of oscillation T_o can be determined by iteration, but obviously, is laborious. Another direct and easier method to find T_o and n_r is to use Tsypkin's plot [12]. From Equations (3.41-3.42), (3.46-3.47) and (5.2), Tsypkin's function can be expressed in terms of dead time, period of oscillation and integer n_r as follows:

$$\mathrm{Im}[\Lambda(\Omega_o)]$$

$$= \frac{-k_1\pi}{4\lambda_1}\left[e^{-\lambda_1 T_D} e^{\lambda_1 n_r (T_o/2)} \left(\tanh\frac{\pi\lambda_1}{2\Omega_o} + 1\right) - 1\right]$$

$$- \frac{k_2\pi}{4\lambda_2}\left[e^{-\lambda_2 T_D} e^{\lambda_2 n_r (T_o/2)} \left(\tanh\frac{\pi\lambda_2}{2\Omega_o} + 1\right) - 1\right] = \frac{-x_2\pi}{4u_c} \qquad (5.11)$$

$$\mathrm{Re}[\Lambda(\Omega_o)]$$

$$= \frac{-k_1\pi}{4\Omega_o} e^{\lambda_1 T_D} e^{\lambda_1 n_r (T_o/2)} \left(\tanh\frac{\pi\lambda_1}{2\Omega_o} - 1\right)$$

$$-\frac{\pi k_2}{4\,\Omega_o}\, e^{\lambda_2 T_D}\, e^{\lambda_2 n_r (T_o/2)} \left(\tanh\frac{\pi\lambda_2}{2\Omega_o} - 1\right) < \frac{\pi B_{21}}{4\Omega_o} \qquad (5.12)$$

Given values of CSTR parameters, including dead time, dimensionless coolant temperature deviation and bandwidth of controller, Tsypkin's locus is easily generated in the complex plane by Equations (5.11) and (5.12) for as a function of Ω_o based on different values of n_r. For the CSTR as the given relay is active $n_r = 1,3,5,\ldots$; otherwise, even numbers of n_r, $n_r = 0,2,4,\ldots$ need to be investigated if the given relay were passive. The right hand side of Equation (5.11) is plotted in the same diagram which is an horizontal line. The intersections of horizontal line (right hand side of Equation (5.11)) and one of the Tsypkin's locus corresponding to a particular value of n_r (left hand side of Equation (5.11)) give the possible solutions of period of oscillation and the value of n_r. The Tsypkin's loci of case III on page 119 based on different values of n_r are plotted in Figure 5.9. Since an active relay is used as a dead time compensator, odd numbers of n_r need to be investigated to find the possible solutions of period. Each locus corresponding to different values of n_r has two interesections with the horizontal line (right hand side of Equation (5.11)) in Figure 5.9. Once the possible

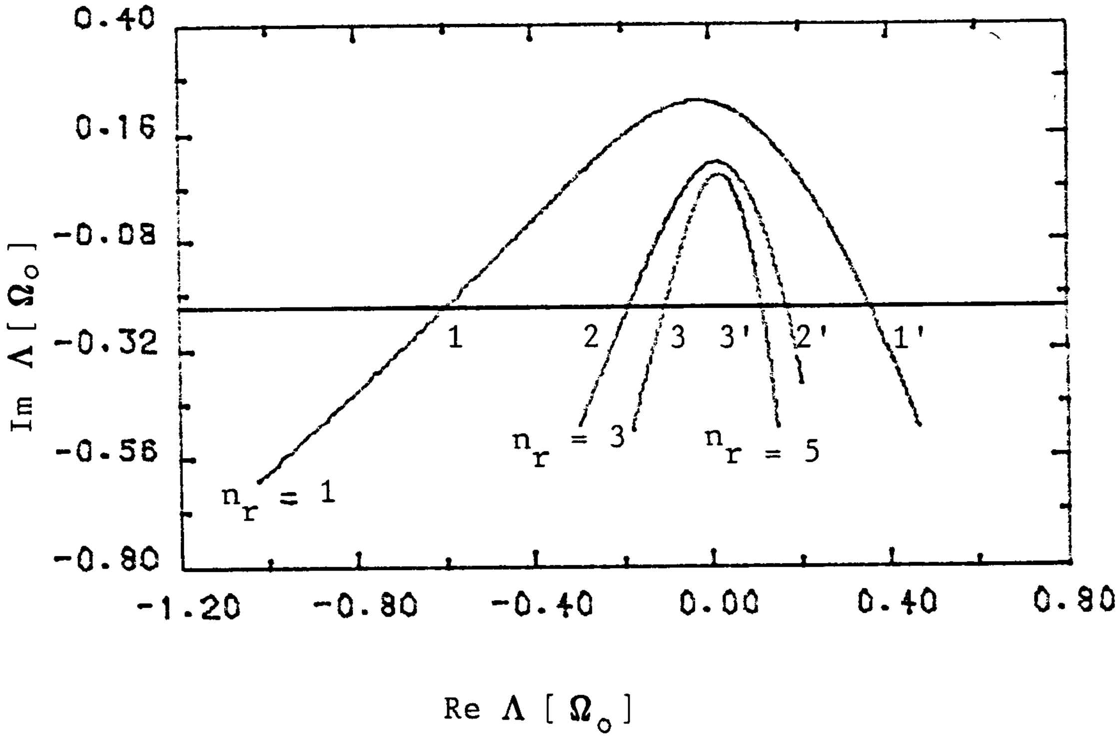


Figure 5.9. Tsypkin's locus for three different values of n_r.

periods of oscillation are determined from the frequencies at the interesections, corresponding reduced dead time can be calculated by Equation (5.2) and the results are listed in Table 5.5. All the interesections labelled as 2 in Figure 5.9 violate the another switching condition defined by Equation (5.12), and are the fault solutions of oscillation. Moreover, negative values of reduced dead time imply that the corresponding period is not valid periodic solution. Consequently, there is only one interesection labelled as 1 in Figure 5.9 of $n_r = 1$, T_o = 10.005 sec. which also fulfills the another switching condition (Equation (5.12)) is the possible solution to give a stable periodic oscillation.

If there is no valid periodic solution exists, it reveals this design is not practical. In the case that the given control system fails, previous sections provide a design technique to specify an appropriate relay controller.

Limit Cycle Stability

After the controller is designed and the characteristics of the oscillation determined, i.e. frequency and amplitude of the oscillation are found, the question coming up next will be whether these corresponding periodic solutions physical exist, in other words, whether the limit cycle solution is stable and will

Table 5.5. The Numerical Values of the Intersections on the Tsypkin's Plot in Figure 5.9.

Intersection points labelled in Figure 5.9	Period of oscillation T_D	Calculated reduced dead time T_{Dr}
1	10.00510	0.19865
2	3.76240	-0.44240
3	3.76439	-0.39847
1'	1.80344	
2'	2.28646	
3'	1.17136	

resume its periodic regime, after a sufficiently small disturbance.

The local stability for processes without dead time under this feedback system with a limit cycle solution can be addressed with an equivalent linear sampled-data system [12, pp. 309-311; 13, pp. 477-480]. This treatment of the oscillation stability has been highly developed, and has been employed to investigate the stability of this CSTR without dead time by Bruns [7]. As treated in Figure 4.1, page 91 a relay controller followed by a pure dead time element is equivalent to another relay having a larger bandwidth when the input is periodic and of sufficient magnitude. As the system containing dead time is reduced to a system without dead time by incorporating the dead time into the relay employing the equivalent block diagram relation (Figures 5.10a and 5.10b), the same analysis method can be used to investigate stability of the system containing dead time. Local stability of the periodic solution of frequency Ω_o is the same as the stability of the sampled-data feedback system with a sampling period, half of the oscillations period, or $\tau_s = \frac{\pi}{\Omega}$, a gain of $2u_c / |\dot{x}(\frac{\pi^-}{\Omega_o})|$ and a linear transfer function of $G_s(s)$. The scheme for this equivalent sampled data system is sketched in Figure 5.10c.

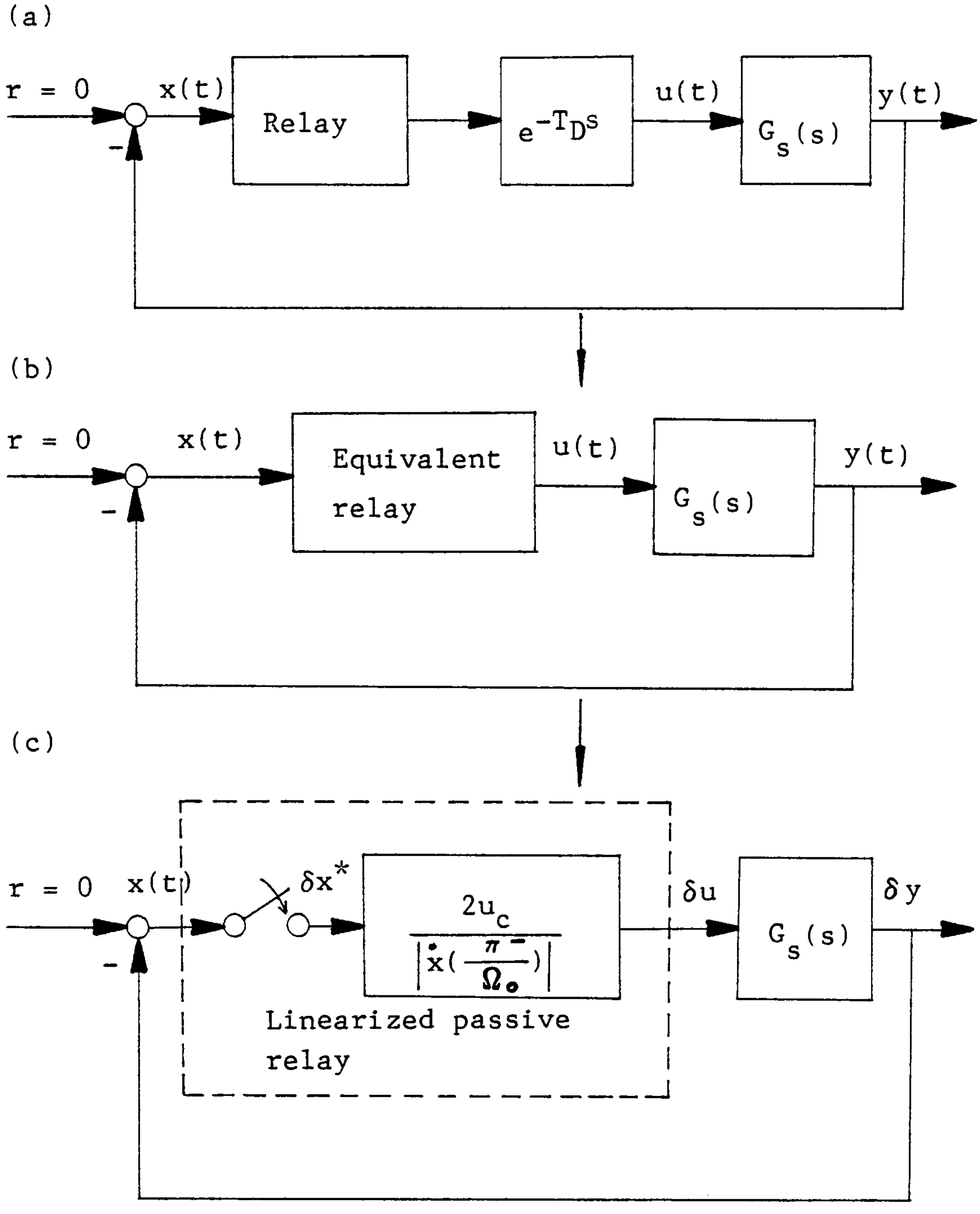


Figure 5.10. The equivalent sampled-data linear system to investigate the stability of relay with hysteresis on systems containing dead time.

Since the method of investigating the stability of sampled-data systems is well developed, the results for determining stability will only be stated: For a closed-loop sampled-data system in the form of Figure 5.10c, let the linear element of the system, $G_s(s)$, be given by an ordinary time-invariant linear differential equation whose states are completely controllable and observable with K_s the gain constant of the system and the sampling period, if

1. for all m, $\mathcal{G}_{z,m}(z,m)$ has the same set of poles as $\mathcal{G}_{z,m}(z,0)=\mathcal{G}_z(z)$, where $\mathcal{G}_{z,m}(z,m)$ and $\mathcal{G}_z(z)$ are the advanced z-transform and z-transform of $G_s(s)$ respectively, and if

2. all the zeros, z, of

   $1 + K_s\mathcal{G}_z(z) = 0$

   lie in $|z|<1$, the origin of the state space is stable. If any of the zeros lie in $|z|>1$ the origin will be unstable.

3. The periodic solution being investigated will be locally stable if previous statement are fulfilled except for a single zero at $z = -1$.

In this example, $z_1 = -1$ and $z_2 < 1$, therefore the periodic oscillation is locally stable.

Implementation of Controller Design on the Nonlinear CSTR

Simulation of the nonlinear system using the compensated relay based on the linearized model design for different values of dead times are shown in Figures 5.11-5.15. The corresponding linear simulation results were given in Figures 5.4-5.7, pages 126-129.

The simulation technique for nonlineaer CSTR is the same as for the linearized model except the nonlinear CSTR equations (Equations (3.2) and (3.3)) are used instead of the linearized equations (Equations (3.26) and (3.27)). The start up conditions used in the nonlinear simulation are determined from the linearized model waveform. Consequently, such start up conditions inevitably create errors or deviations from the nonlinear system oscillation. Satisfactory results only occur in the cases of zero and small dead time (Figures 5.11 and 5.12). If system contains small dead time, the errors due to the use of start up conditions determined by the linearized model waveform are insignificant and do not cause system to go out of control during start up. However, if the system contains considerable dead time, the errors resulting from employing the linearized model initial conditions will be "integrated" or build up during the dead time. These errors will cause the relay to switch incorrectly for the nonlinear model.

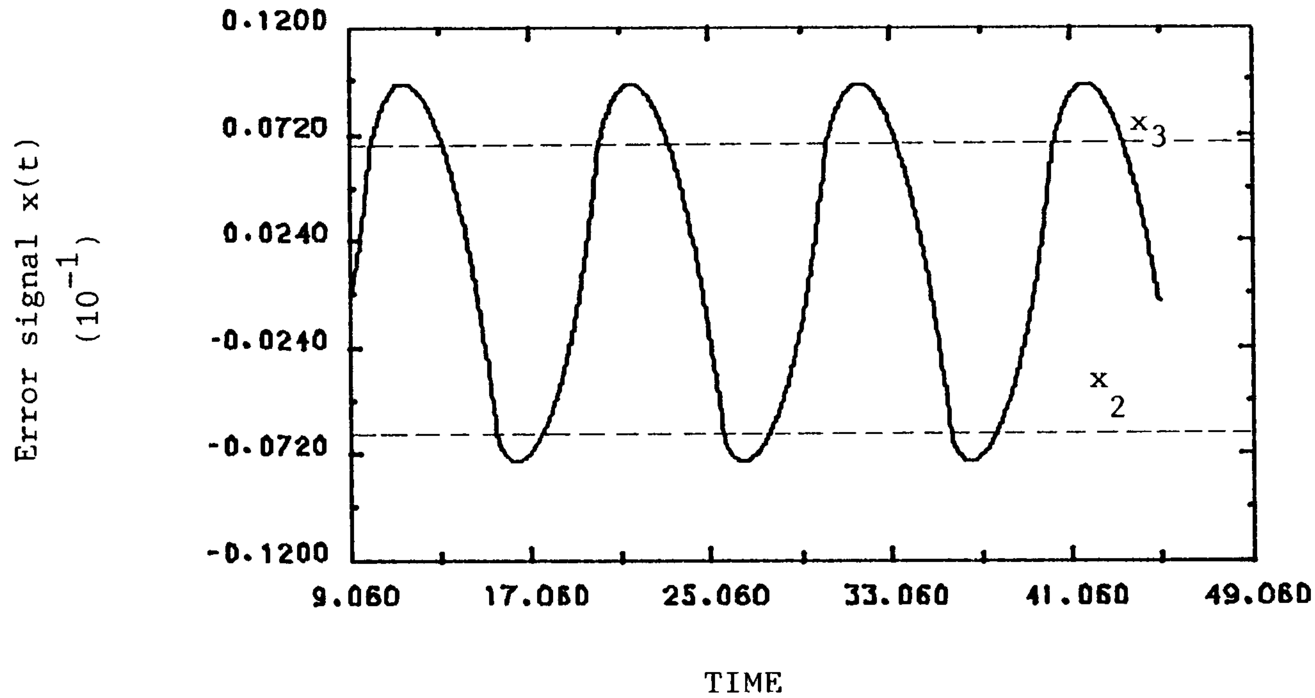


Figure 5.11. Simulation of the nonlinear feedback control scheme containing nonlinear CSTR process, zero dead time and a passive relay controller with bandwidth $x_{2,c} = 0.5y_e$.

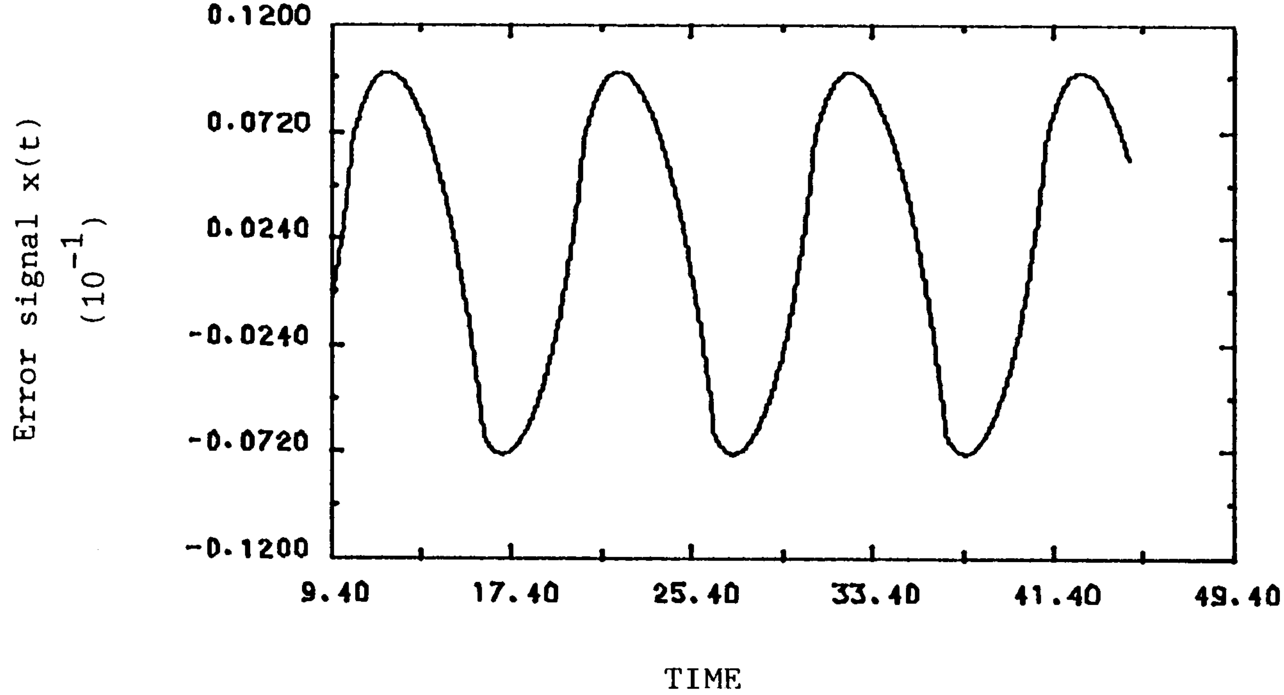


Figure 5.12. Simulation of the nonlinear feedback control scheme containing nonlinear CSTR process, dead time $T_D = 0.2$ sec., and a passive relay controller with bandwidth $x_{2,c} = 0.36914y_e$.

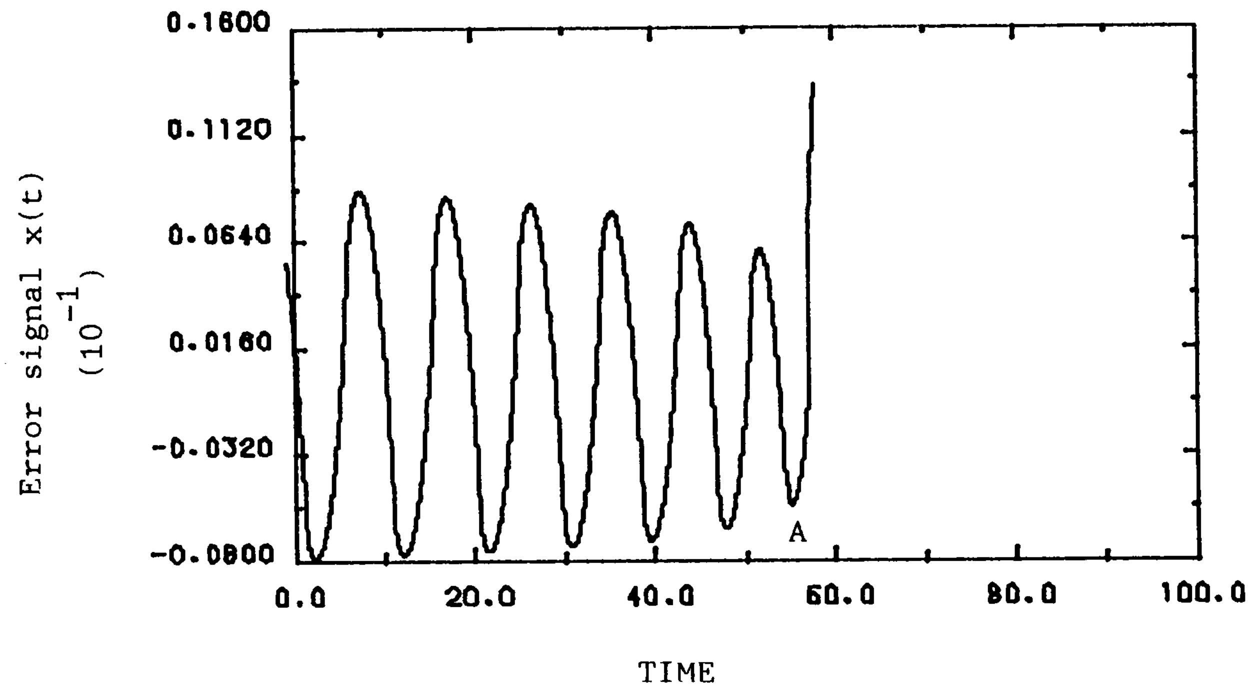


Figure 5.13. Simulation of the nonlinear feedback control scheme containing nonlinear CSTR process, dead time T_D = 2.1 sec., and a passive relay controller with bandwidth $x_{2,c} = -0.40602y_e$.

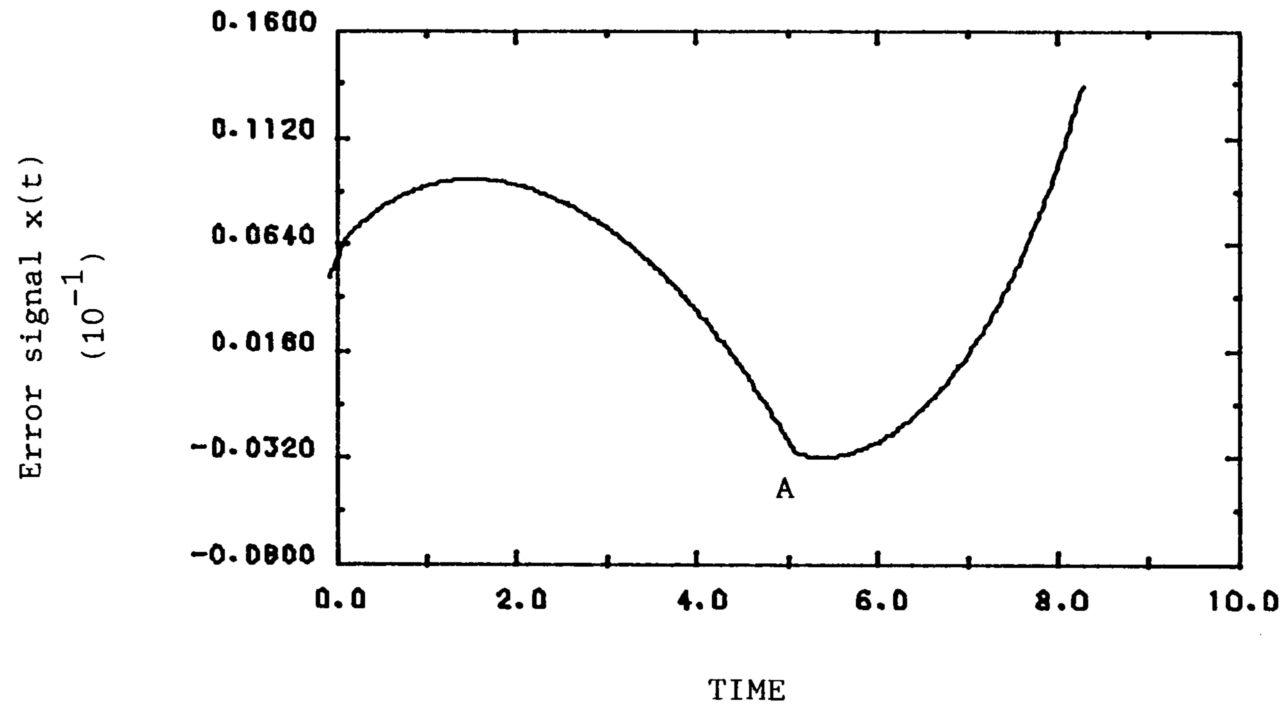


Figure 5.14. Simulation of the nonlinear feedback control scheme containing nonlinear CSTR process, dead time T_D = 5.2012 sec., and an active relay controller with bandwidth $x_{2,c} = 0.369006 y_e$.

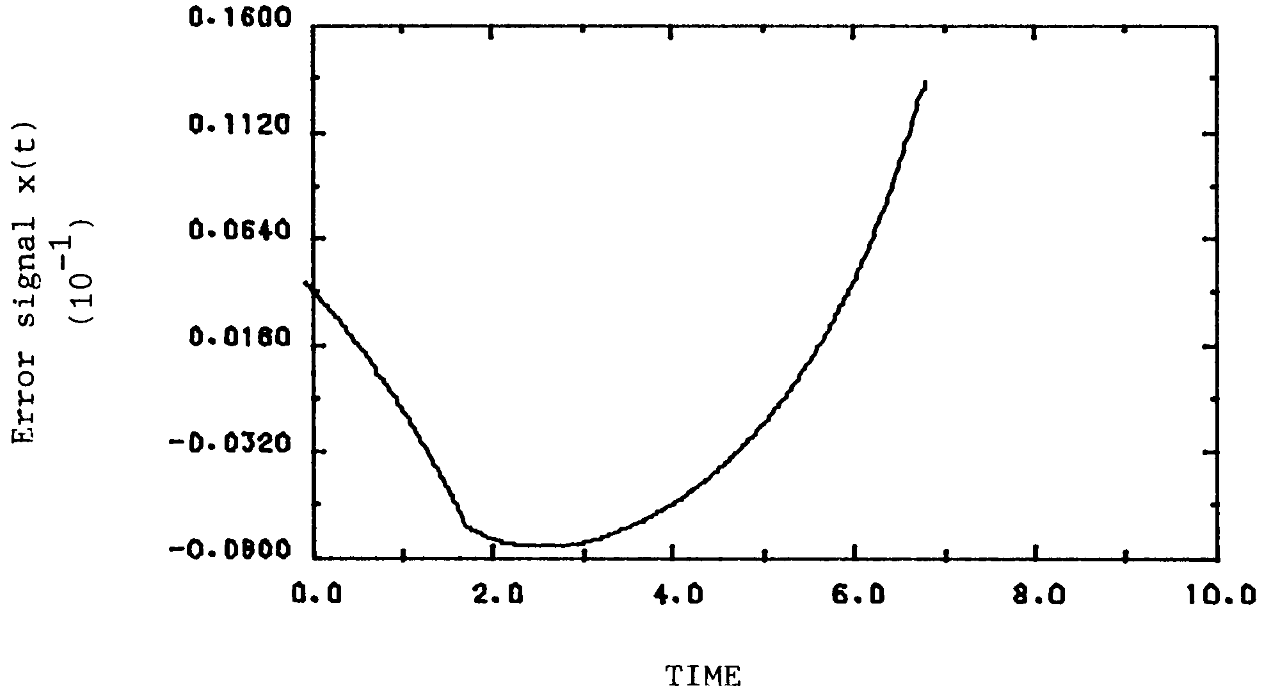


Figure 5.15. Simulation of the nonlinear feedback control scheme containing nonlinear CSTR process, dead time T_D = 6.8006 sec., and an active relay controller with bandwidth $x_{2,c} = -0.327561y_e$.

At point A of Figure 5.13-5.14, the output of the relay flips before the switching value, $x_{2,c}$ is reached. Consequently, the process output does not achieve the switching point, and this degrades the implementation of control.

Figure 5.15 shows another possibility leading to lose of control in the nonlinear simulation. In this case, as the lower part of the waveform has an overshoot that is comparatively small, it takes a shorter time than the dead time to reach the exact switch point. Therefore, the relay output will not switch at the proper point. Often it drives the system toward the stability boundary, the maximum allowable deviation from the desired operating point and not lose control, and eventually, lose control.

The inability of maintaining control of the nonlinear system when it contains larger dead times may arise from the mismatch between linearized model approximation and the nonlinear system. Utilizing the linear model and associated Tsypkin's analysis cannot take into account the modeling approximation due to the linearization of nonlinear system, the closed-loop performance of the nonlinear plant deteriorates from that predicted by the linearized model and often causes losing control. This may be due largely to the inproper programming for start up, i.e. the use of the linear simulation for start up procedures.

The comparision of oscillation curves between linear and nonlinear models for both the systems without and containing small dead time are given in Figures 5.16 and 5.17 respectively. The major difference that distinguishes the linear and the nonlinear simulations is that the linear simulation gives a symmetric oscillation, whereas, the nonlinear simulation has a nonsymmetric oscillation.

In order to deal with the real nonlinear system containing dead time, we may use some on-line tuning technique to adjust the designed relay based on the linearized model.

Another tchenique for controller tuning in the nonlinear system may be to keep the symmetric bandwidth and adjust the lower or higher drive level so that a more symmetric waveform may be obtained.

Alternatively, based on the nonsymmetric waveform of system without dead time, i.e. $|x_2| \neq |x_3|$, as shown in Figure 5.11, appropriate initial conditions can be determined by the same technique employed in the linear simulation. Likwise, tracing back a time equal to the dead time from both switching points without dead time labelled as x_2 and x_3 in Figure 5.11 leads to the use of a nonsymmetric bandwidth of relay, i.e. $\alpha = 1/2$, and the determination of appropriate initial conditions. This application of nonsymmetric bandwidth and use of the nonlinear waveform allows appropriate start up conditions for the nonlinear process.

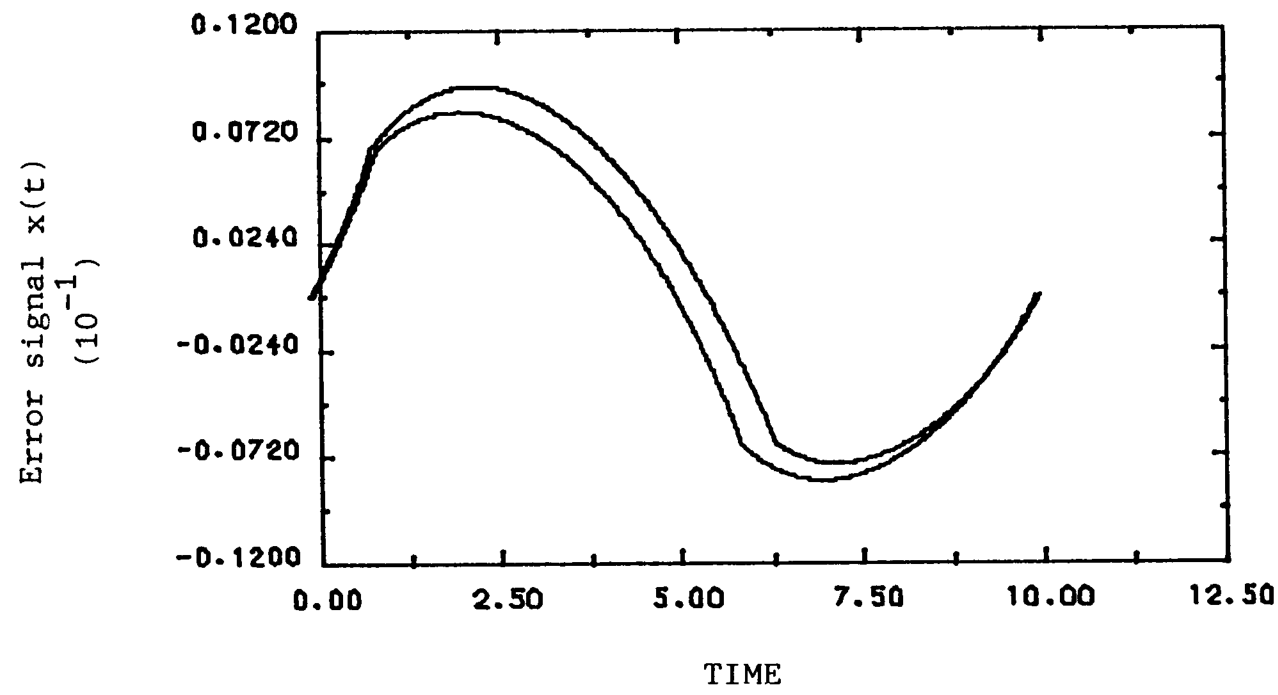


Figure 5.16. Comparison between linearized and nonlinear simulations on the two variable CSTR system containing zero dead time. A passive relay controller with bandwidth $x_{2,c} = 0.5y_e$ is used.

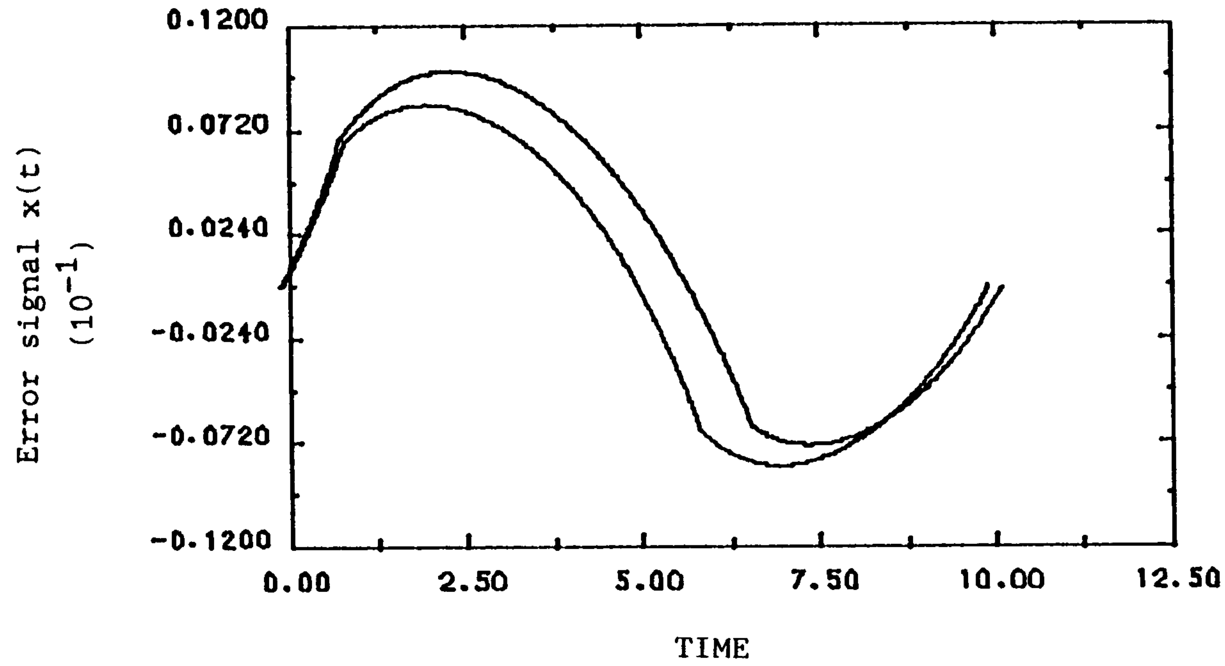


Figure 5.17. Comparison between linearized and nonlinear simulations on the two variable CSTR system containing dead time $T_D = 0.2$ sec. A passive relay controller with positive bandwidth $x_{2,c} = 0.36914y_e$ is used.

Conclusion and Future Study

This study herin provides an analytical means of designing a relay feedback control loop containing dead time. The straightforward extension of Tsypkin's analysis allows us to deal with dead time less than half period of oscillation (see Chapter II). However, by introducing and developing the new comcepts of "reduced dead time" (see Chapter V) and "negative bandwidth" relays (see Chapter IV), this restriction can be removed. Equations (2.27-2.28) or (2.29-2.30) depending on the type of relay are the basic equations to derive the analytic equations for compensating controller design. A CSTR system serves as an example for demonstration and discussion in this study.

The design strategies developed in this research start with finding the reduced dead time and the value of n defined by Equation (5.2) according to specified period of oscillation, T_o, which is typically fixed by the design requirement. The value of n_r can determine the type of relay which can fulfill the control implementation. Then an appropriate bandwidth of controller, $x_{2,c}$ can be calculated by substituting reduced dead time and period of oscillation into Equation (5.3) and solving for x_2 in the equation. After the parameters of system have been determined, an analytical solution of waveform can be expressed by Equation (3.52). The exactness between analytic solution given by Equation (3.52) and simulation of the linearized model as shown in Figures 5.4-5.7, pages 126-129 based on different values of dead times gives the demonstration of this design strategy and the validate of the equations which have been derived.

Additionally, this study provides another application on investigating the feasibility of a given control design. Plotting the Tsypkin's locus of system in the complex plane as shown in Figure 5.9, page 132 can facilitate finding the solution of oscillation, if one exists.

If the disturbance is sufficient small, for a periodic input, the representation of relay with dead time is identical to another relay without dead time. Moreover, the stability of this relay feedback control without dead time is equivalent to a linear sampled-data system. Consequently, the stability of dead time system can be examined easily by reducing the dead time system to a equivalent system without dead time and subsequently by employing sampled-data theory which has been highly developed, as shown in Figure 5.10, page 135.

The controller design strategy presented in this research can cope with any value of dead time for the linearized system as long as the system is equipped with a computer controller which can provide the memory of "prehistory" for a period of dead time before starts up.

Although this study has provided a novel technique to compensate for the dead time in a relay control scheme, further analysis is still needed to give a clearer insight into the analysis and application of this nonlinear relay system. Consequently, the areas for future study include:

1. Investigation of start up procedures when implementing a dead time compensator on the nonlinear system.

2. Further development of a relay compensator design based on the desired waveform of the nonlinear simulation without dead time.

3. Comparison of the relay compensation technique with other dead time techniques e.g. the Smith predictor [2, 3, 4].

4. Additional study of the stability analysis. In this work the equivalence under periodic input of a relay with dead time to a relay with larger bandwidth was used (see Figure 4.1, page 91). This effectively leads to a larger or smaller gain, K_s (see Figure 5.10, page 135), for the equivalent sampled-data system. Another approach would be to pursue the analysis retaining the process model with dead time and use sampled-data theory for processes with dead time.

Moreover, related to the stability analysis is the obvious observation that the sufficiently small deviation which the nonlinear system can encounter and return to its periodic steady state must be smaller as the dead time increases. The larger the dead time and thus the more relay switches during the "blind period" the larger effect a given disturbance can have. A study of this phenomena may provide a way to harden the control strategy or provide robustness to the controller.

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