THE CIRCLE DIAGRAM OF THE SYNCHRONOUS MOTOR

by

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INTRODUCTION.

A circle diagram presents one of the most useful methods for determining the characteristic curves and efficiencies of a synchronous motor at any operating field current. This thesis deals with the circle diagram primarily and has one real purpose,namely: To show how actual test curves of synchronous motors compare with curves obtained from the circle diagram.

For several years,circle diagrams have been in use in connection with transmission systems but application to synchronous motors is a comparatively recent procedure.

GENERAL THEORY.

Fundamental Circuit Equation

"All electrical circuits receive power at one end and deliver power at the other." This can be represented diagrammatically thus:

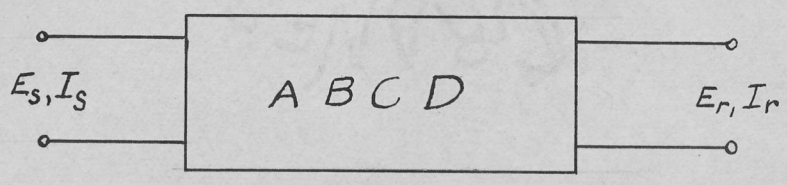


Fig.1.

Where E and I are sending voltage and current respectively,and E and I are receiver voltage and current respectively.

The diagram of Figure 1 may beexpressed symbolically as follows:

$$E_S = AE_r + BI_r \qquad (1)$$

$$I_S = CE_r + DI_r \qquad (2)$$

where A,B,C,and D are certain "circuit constants."

From equation (2)

$$I_r = \frac{I_s - CE_r}{D} \qquad (3)$$

Substituting equation (3) in equation (1) results in

$$E_s = AE_r + \frac{BI_s - BCE_r}{D} \qquad \text{or}$$

$$E_s = \frac{(AD - BC)\,E_r + BI_s}{D}$$

$$DE_s = (AD - BC)\,E_r + BI_s$$

$$(AD - BC)\,E_r = DE_s - BI_s \qquad \text{from which}$$

$$E_r = \frac{DE_s - BI_s}{AD - BC}\ ; \text{ or } E_r = HE_s - KI_s$$

where $H = \frac{D}{AD - BC}$ and $K = \frac{B}{AD - BC}$

The synchronous motor contains two separate sources of power input to the motor, namely: alternating current source to the stator and direct current to the field circuit. Since there are two separate sources of power, it is difficult to represent the synchronous motor by a simple electric circuit. The equivalent circuit diagram may be obtained, however if the field copper losses are neglected:

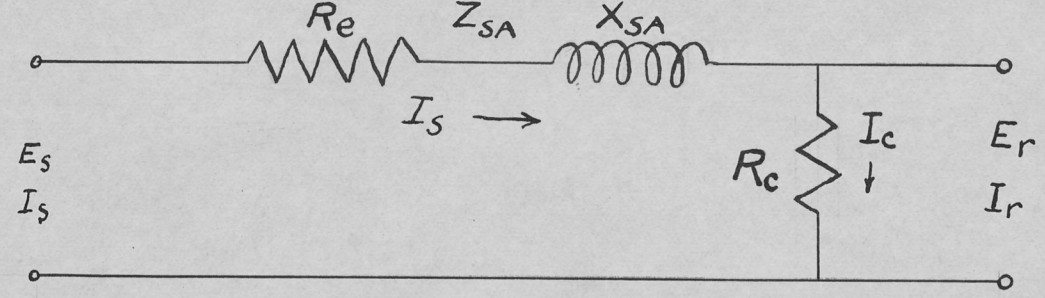


Fig.2. Equivalent Circle Diagram.

$$E_S = E_r + I_S Z_{SA} \qquad \text{(for A.C.Motors)}$$

where Z_{SA} = Re + jX sa

Re = effective A.C.resistance per phase.

X sa = Synchronous reactance (AIEE) per phase.

Rc in the above equivalent diagram represents the core loss in the synchronous motor and is located across the receiver terminals, being so located because the core loss is a function of the back voltage of the motor.

An equivalent circuit for the synchronous motor must fulfill the following conditions:

1. The voltage equation; $E_S = E_r + I_S Z_{SA}$
2. The losses:
   a. Copper loss of armature $I_S^2 R_e$ Watts/phase.
   b. Core loss which is function of flux.
      (1) Hysteresis
      (2) Eddy Current
   c. Mechanical loss and field copper loss- readily taken into account in plotting of circle diagram.
3. Provide for useful power output-

   Power output = $E_r I_r \cos \theta_r$

Circle Diagram Theory.

The fundamental equation used in obtaining the circle diagram is the following:

$E_S = E_r + I_S Z_{SA}$

but from the equivalent circle diagram Fig.2,

$I_S = I_r + I_c$

also

$$I_c = \frac{E_r}{R_c}$$

Therefore $I_S = I_r + \frac{E_r}{C}$

Substituting this value in the fundamental equation results- $E_S = E_r + I_r Z_{SA} + \frac{E_r Z_{SA}}{R_c}$

or collecting terms-

$$E_S = \left(1 + \frac{Z_{SA}}{R_c}\right) E_r + I_r (Z_{SA})$$

The above equation may be simplified by substituting

$$A = 1 + \frac{Z_{SA}}{R_c} \quad ; \quad B = Z_{SA}$$

thus obtaining the fundamental circuit equation, viz:

$$\underline{E}_S = \underline{A}\underline{E}_r + \underline{B}\underline{I}_r$$

The constants $\underline{A}$ and $\underline{B}$ in the above equation are simply operators which change the magnitude and position of vectors to which they belong.

Circle Diagrams to a Volt-Ampere Scale.

Given the constants in their complex form, namely:

$$A = 1 + \frac{Z_{SA}}{R_c} = a_1 + ja_2$$

$$B = Z_{SA} = b_1 + jb_2$$

$$H = 1 = h_1 + jh_2$$

$$K = Z_{SA} = k_1 + jk_2 = b_1 + jb_2$$

From the above constants, the location of the direction of sending and receiving circles' center is found by means of the angles between the vectors thus:

Receiver circles

$$\alpha = \tan^{-1} \frac{a_2}{a_1}$$

$$\beta = \tan^{-1} \frac{b_2}{b_1}$$

Sending circles

$$\mathcal{T} = \tan^{-1} \frac{h_2}{h_1} = 0$$

$$\Delta = \tan^{-1} \frac{k_2}{k_1} = \tan^{-1} \frac{X_{SA}}{R_e}$$

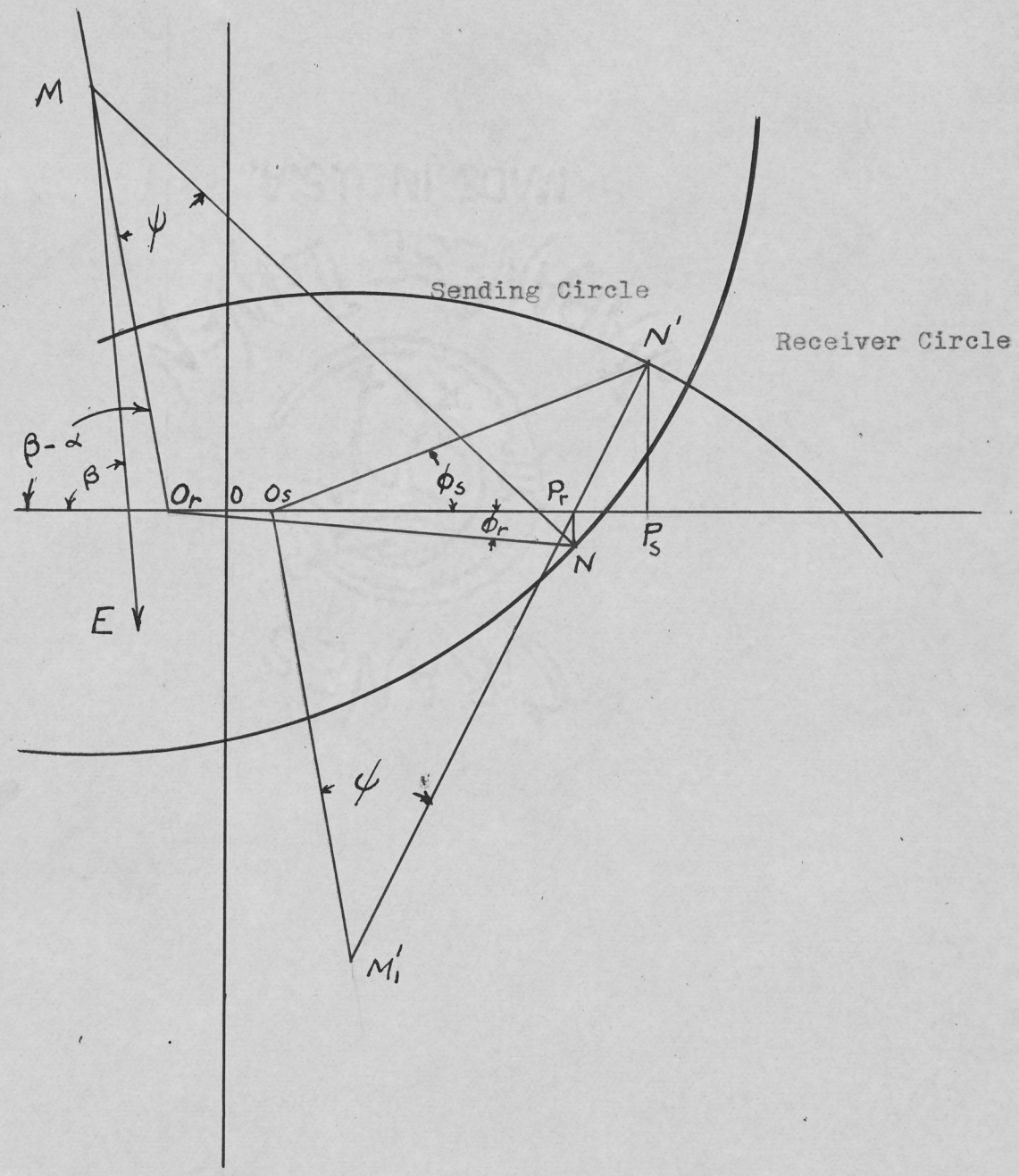


Fig.3.
Phase Circle Diagram of Synchronous Motor

To obtain Figure 3, it is necessary to determine the following quantities for the sending and receiving circles, thus:

$MO_r = \frac{AE_r^2}{Z_{SA}}$ $\qquad$ $M'O_s = \frac{E_s^2}{Z_{SA}}$

$OrN = E_r I_r$ $\qquad$ $ON' = E_s I_s$

$MN = \frac{E_s E_r}{Z_{SA}}$ $\qquad$ $M'N' = \frac{E_s E_r}{Z_{SA}}$

$OO_r = \frac{\text{Total Mechanical Losses}}{3}$ $\qquad$ $OO_s = \frac{I_f^2 R_f}{3}$

Specifications of Synchronous Motor.

The synchronous motor used to obtain the necessary data for plotting the circle diagram and load tests is a 15.5 K.V.A. synchronous motor with added squirrel cage bars for starting purposes. The motor also had these additional characteristics:

Power factor	90%
Speed	1200 R.P.M.
Rated Voltage	220 Volts.
Number Phases	1, 3, or 6
Manufacturer	Westinghouse
Field Exc. Amperes	7.8
Field Exc. Volts	125
Serial Number	8098345

Connection Diagram

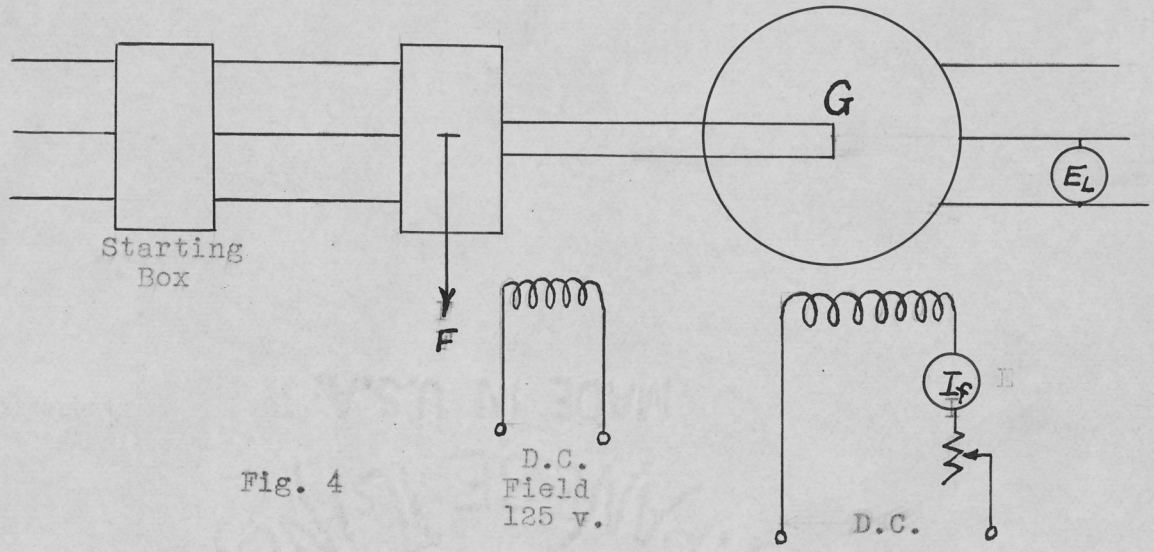


Fig. 4

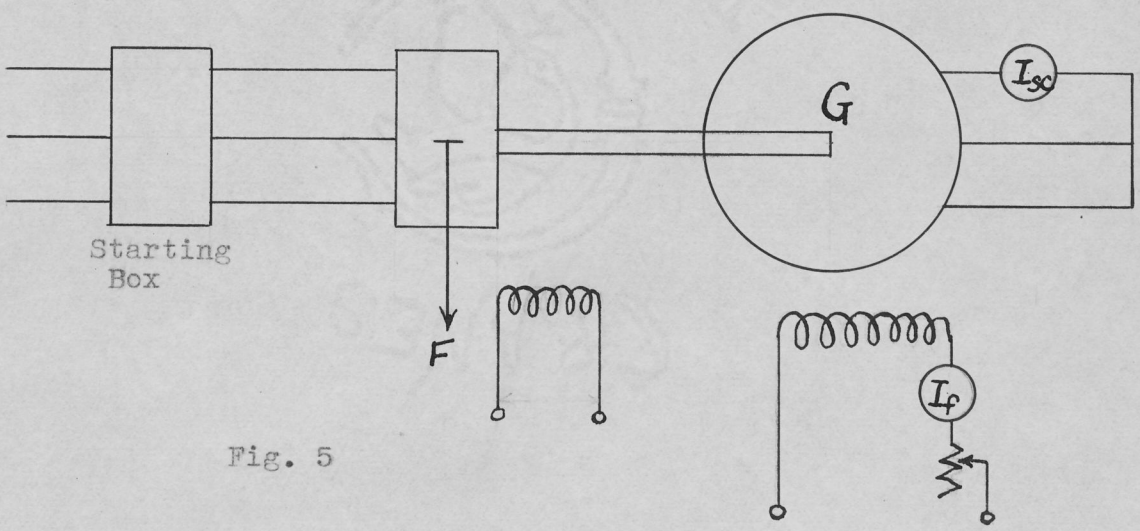


Fig. 5

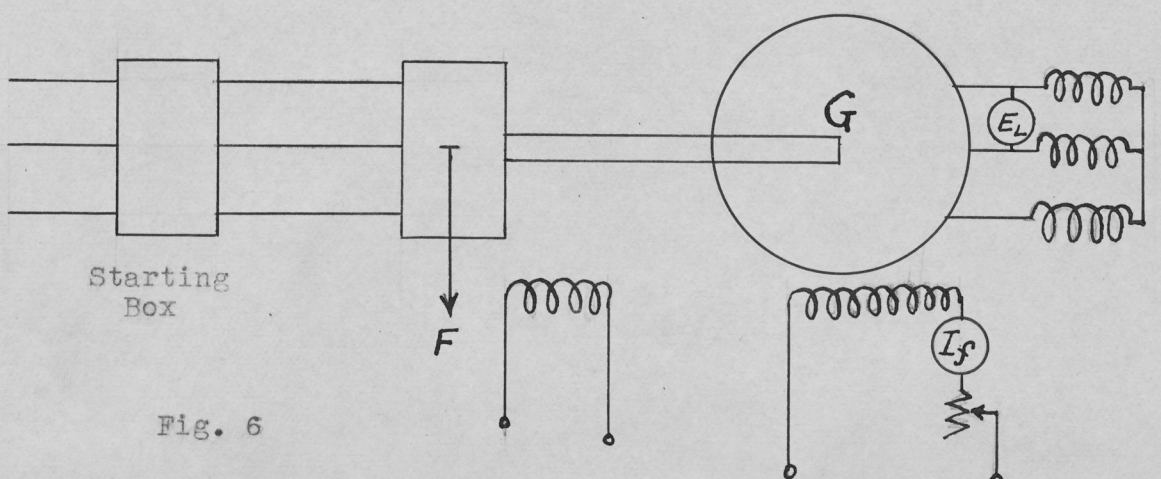


Fig. 6

Method of Performance.

In Run 1,the motor is operated as generator at no load for variable field currents. From this run,the no load saturation curve (Fig.7) and also the field resistance and the no load losses are obtained. (See Fig.4 for connection diagram.)

Run 2 is ths short circuit test of the given motor. The armature is short circuited and for each different field current,the corresponding armature short circuit is recorded. A curve is now plotted for the armature current against field current.(Fig.5 for connection)

Run 3 is a load run at 0% power factor lagging. The load is balanced for all three phases. In this test the field current necessary to deliver full load armature current at rated line voltage is recorded. (Fig.6 for connections.)

Run 4 is made to determine the Direct Current resistance between each pair of Armature windings by the voltmeter-ammeter method. From these resistances obtained,phase resistance will be computed.

Run 5. In this run,data is taken to determine the "V" curves of the motor. V curves are merely curves which show how the power factor and armature current vary with the field current. Run 5 is performed for zero,1/3,2/3 and full load outputs. (Fig.8 for V curves)

Run 6. The purpose of this run is to determine the load characteristics of the synchronous motor. The motor is loaded to its full load value and field current adjusted. The field current is kept constant and the armature current is gradually

decreased. The following readings are taken: output,field current,field voltage,input power,armature voltage,and current and phase angle.

Runs 7 and 8 are merely duplications of Run 6 but at power factor loads of 85% lagging and leading respectively.

TEST NO. 1 NO LOAD TEST

f	I_f	E_f	F	E_L	R_f	P_o	E_r	P_o
60	0.7	0	4# 3	0	0.00	520	00.0	173
60	1.1	9	4- 4	61	8.18	536	35.2	178.5
60	1.5	13	4-04	87	8.67	536	30.2	178.5
60	2.1	18	4-05	120	8.58	552	69.2	184
60	2.5	21.5	4-07	139	8.60	584	80.1	195
60	3.0	26	4-09	163	8.66	616	94.0	205.5
60	3.5	30.5	4-11	185	8.71	648	106.5	216
60	4.0	35.5	4-13	204	8.88	680	117.5	226
60	4.5	40	4-15	222	8.89	712	127.9	237.5
60	5.0	45	5-01	237	9.00	744	136.6	248
60	5.5	49.4	5-03	250	9.01	776.	144.1	258.5
60	6.0	55	5-05	260	9.16	808	149.9	269.5
60	6.5	60	5-07	270	9.23	840	155.8	280
60	7.0	65	5-09	278	9.28	872	160.3	290.5
60	7.5	71	5-11	286	9.46	904	165.0	301
60	8.0	76	5-12	292	9.50	920	168.3	306.5
60.	8.5	82	5-13	299	9.63	936	172.4	312

Average -- 8.97

Test No. 2 SHORT CIRCUIT TEST

f	I_f	I	Force	P_s / phase
60	0.8	6.7	4 # 5 oz	181
60	1.1	9.7	4 6	187
60	1.5	13.4	4 8	197
60	2.0	18.0	4 12	218
60	2.5	22.4	5 00	245
60	3.0	27.4	5 07	277
60	3.5	31.8	5 14	314
60	4.0	36.3	6 07	362
60	4.5	41.1	7 00	410
60	5.0	45.5	7 11	470
60	5.5	49.5	8 06	530

Test No. 3 ZERO % P.F. LAGGING FULL LOAD

SATURATION CURVE

I_f	V_L	I	V_p
7.8	165	40	95
9.3	208	40	120
10.6	240	40	138

LOAD RUNS

NO LOAD RUN(Labratory Data)

I_f	E_f	I_a	E_a	P.A.	P.F.
2.9	111.5	15.0	228	-80	0.174
3.5	111.0	10.0	228	-75	0.259
4.0	111.0	7.0	227	-65	0.423
4.6	111.0	5.0	229	-40	0.766
5.0	111.0	5.0	230	1	0.999
5.6	110.0	8.0	230	40	0.766
6.0	109.5	10.5	229	54	0.589
6.5	109.5	14.5	230	62	0.470
7.0	109.0	17.5-	230	66	0.407
7.5	108.5	21.0	232	70	0.342
8.0	108.5	24.5	232	74	0.275
8.5	108.0	27.2	232	74	0.275
9.1	108.0	31.0	232	77	0.275

LOAD RUNS

ONE THIRD FULL LOAD

I_f	E_f	I_a	E_a	P.A	P.F.	Input	Efficiency
2.7	106.0	23.0	214	-45	0.707	6020	85.2
3.6	106.0	18.5	214	-29	0.875	6020	85.2
4.0	105.5	17.5	216	-17	0.956	6250	82.0
4.5	105.0	17.0	217	-5	0.996	6370	80.5
5.1	105.0	17.5	218	10	0.985	6520	78.6
5.5	105.0	18.0	218	20	0.940	6400	80.0
6.1	105.0	21.0	218	32	0.848	6740	76.0
6.5	104.8	23.5	217	39	0.777	6870	74.7
7.0	104.5	26.0	218	45	0.707	6940	74.0
7.5	104.0	29.2	218	50	0.643	7100	72.4
8.1	104.5	32.0	218	55	0.574	6950	74.0
8.5	105.0	33.5	220	56	0.560	7150	71.8
8.9	105.0	36.0	220	59	0.520	7150	71.8

Constant Output of 5140 Watts

Dynamometer Reading 22 # 4 oz.

Tare Reading 2 # 2.5 oz.

LOAD RUNS

TWO THIRDS FULL LOAD

I_f	E_f	I_a	E_a	P.A.	P.F.	Input	Efficiency
3.1	117.0	39.5	216	-40	0.76	11,300	91.3
3.5	116.5	35.0	218	-33	0.84	11,100	93.0
4.1	116.5	32.0	218	-24	0.91	11,000	93.6
4.5	116.0	30.5	219	-16	0.96	11,100	93.0
5.0	116.0	29.7	221	-9	0.98	11,100	93.0
5.6	116.5	29.5	221	3	0.99	11,300	91.3
6.1	116.5	29.9	224	10	0.99	11,500	89.7
6.5	116.5	30.5	224	16	0.96	11,350	91.0
7.0	116.0	32.0	224	24	0.91	11,300	91.3
7.5	116.0	34.0	225	30	0.87	11,500	89.7
8.0	116.0	36.0	224	35	0.82	11,450	90.0
8.4	115.5	37.5	225	38	0.79	11,500	89.7
9.0	115.0	41.0	225	42	0.74	11,900	86.6
9.6	115.0	44.0	223	46	0.70	11,800	87.4

Constants for this run :

Output is 10,300

Dynamometer Reading is 42 # 8 oz.

LOAD RUNS

FULL LOAD

I_f	E_f	I_a	E_a	P.A.	P.F.	Input	Efficiency
5.1	130.5	49.0	220	-21	0.93	17,400	89.2
5.6	130.5	47.0	221	-14	0.97	17,480	88.9
5.9	130.0	46.0	221	-8	0.99	17,400	89.2
6.2	118.5	45.0	211	0	1.00	16,500	94.0
7.0	118.0	45.0	211	11	0.98	16,100	96.2
7.6	117.5	45.5	211	18	0.95	15,650	99.0
8.0	117.0	47.0	212	23	0.92	15,900	97.5
8.3	114.5	52.0	210	23	0.92	17,400	89.2
9.0	114.0	54.0	212	27	0.89	17,650	88.0

Constants for this runs :

Output is 15,500 Watts

Dynamometer Reading is 62 # 5 oz.

Tare Reading is 2 # 2.5 oz.

Wattmeter Reading is 7.0 x 2000

LOAD RUNS

RUN # 6

E_a	I_a	E_f	I_f	P.A.	W_p *	Force @
222	45.0	112	6.4	0	7.10	62 # 13 oz.
222	41.0	112	6.4	4	6.30	55- 14
223	35.0	112	6.4	10	5.29	47 - 5
224	31.0	112	6.4	14	4.36	40 - 15
224	29.0	112	6.4	21	3.82	35 - 2
224	26.0	112	6.4	25	3.24	30 - 3
226	23.0	112	6.4	30	2.63	25- 2
226	21.0	112	6.4	35	2.05	20 - 3
228	19.0	112	6.4	40	1.50	15 - 7
228	16.0	112	6.4	48	0.85	11 - 0

* K is 2000

@ Tare Weight is 2 # 2.5 oz.

Run # 7

E_a	I_a	E_f	I_f	P.A.	W_p *	Force @
212	56.0	121	4.7	-32	7.14	51 # 0 oz.
216	39.0	121	4.7	-20	5.75	43- 8
220	32.5	121	4.7	-15	4.88	37- 6
222	28.0	121	4.7	-11	4.14	30- 11
220	23.4	121	4.7	-8	3.33	24- 6
225	19.0	121	4.7	-7	2.58	18- 4
226	15.0	121	4.7	-6	1.85	13- 5
224	13.0	121	4.7	-6	1.20	8- 14
228	6.0	121	4.7	-5	0.62	5- 9

LOAD RUNS

RUN # 8

Ea	Ia	Ef	If	P.A.	Wp *	Force @
222	54.0	131	9.5	32	7.13	62 # 9 oz
224	51.0	131	9.5	36	6.30	55- 11
226	47.5	131	9.5	40	5.39	48- 3
229	45.0	131	9.5	43	4.75	42- 14
229-	40.5	131	9.5	44	3.92	35- 15
232	38.0	131	9.5	50	3.00	28- 4
232	37.5	131	9.5	58	2.38	23- 2
232	35.0	131	9.5	64	1.68	17- 1
234	33.5	131	9.5	68	1.02	12- 1
234	33.0	131	9.5	70	0.70	9- 9

* K is 2000

@ Tare Weight is 2 # 2.5 oz.

CALCULATIONS

E_r	P_c	$3\ E_r^2$	R_c	I_{sc}	$3\ I_{sc}^2$
65	15	12660	843	18	1080
94	28	26440	944	27	2430
117	50	41100	821	36	3880
127	62	48400	780	41	5040
136	73	55500	760	45	6100
143	85	61400	722	50	7500

$$R_c = \frac{3\ E_r^2}{P_c} \qquad R_e = \frac{P_e}{3\ I_{sc}^2}$$

$$\beta = \tan^{-1}\frac{X_{sa}}{R_e} \qquad \tan^{-1}\frac{3.7}{0.046} = 89^\circ 15'$$

$$MO_r = \frac{AE_r^2}{Z_{sa}} \qquad \frac{(1)(65)^2}{3.7} = 1140$$

$$MN = \frac{E_s\ E_r}{Z_{sa}} = \frac{(127)(65)}{3.7} = 2230$$

$$M'\ O_s = \frac{E_s^2}{Z_{sa}} = \frac{(127)^2}{3.7} = 4350$$

$O\ O_r$ = MEchanical Loss per Phase $= \frac{175}{3} = 87$ Watts

$O\ O_s$ = Copper Loss per Phase $= \frac{I_f^2\ R_f}{3} = \frac{(2)^2(8.97)}{3} = 12$

CALCULATIONS

I_f	X_{sa}	E_r	R_c	P_c	R_e
2.0	3.70	65	5060	45	0.042
3.0	3.70	94	4420	100	0.041
4.0	3.65	117	4110	193	0.050
4.5	3.52	127	4025	240	0.048
5.0	3.28	136	3960	300	0.049
5.5	2.90	143	3610	360	0.048
				Average--------	0.046

I_f	Z_{sa}	A
2.0	$0.046 + j3.7 = 3.7 \angle 89°15'$	$1 + j\ 0.00073$
3.0	$0.046 + j\ 3.7 = 3.7 \angle 89°15'$	$1 + j\ 0.00084$
4.0	$0.046 + j\ 3.65 = 3.65 \angle 89°10'$	$1 + j\ 0.00089$
4.5	$0.046 + j\ 3.52 = 3.52 \angle 89°10'$	$1 + j\ 0.00088$
5.0	$0.046 + j\ 3.28 = 3.28 \angle 89°10'$	$1 + j\ 0.00083$
5.5	$0.046 + j\ 2.90 = 2.90 \angle 89°05'$	$1 + j\ 0.00080$

I_F	β	α
2.0	89°15′	5′
3.0	89°15′	5′
4.0	89°15′	5′
5.0	89°10′	5′
5.5	89°10′	5′

$B = Z_{sa}$; $H = 1$; $K = Z_{sa}$ $\beta = \Delta$

CALCULATIONS

I_f	MO_r	MN	$M'O_s$	OO_r	OO_s
2.0	1140	2230	4350	87	12
3.0	2390	3220	4350	87	27
4.0	3750	4070	4420	87	48
4.5	4580	4580	4580	87	61
5.0	5640	5270	4920	87	77
5.5	7050	6260	5560	87	91
6.0	9100	7700	6500	87	108
6.5	10800	8870	7270	87	126
7.0	12800	10150	8070	87	146
7.5	14900	11500	8860	87	168
8.0	16800	12700	9660	87	192
8.5	19000	14100	10500	87	216
9.0	22100	16100	11800	87	242

JAPANESE LINES
MADE IN U.S.A.

CIRCLE DIAGRAM DATA

FULL LOAD

I_f	$E_s I_s$	I_s	θ_s	Cos θ_s	Output	Input	Efficiency
5.5	5800	45.7	-22° 00′	0.927	5170	5450	95.0
6.0	5440	42.8	-10° 00′	0.985	5170	5530	93.6
6.5	5560	43.7	-3° 00′	0.998	5170	5710	90.5
7.0	5380	42.6	6° 40′	0.993	5170	5520	93.7
7.5	5420	42.6	14° 30′	0.968	5170	5440	95.2
8.0	5600	44.1	20° 30′	0.937	5170	5410	95.6
8.5	5900	46.4	26° 30′	0.895	5170/	5440	95.2
9.0	6360	50.0	33° 20′	0.835	5170	5480	94.4

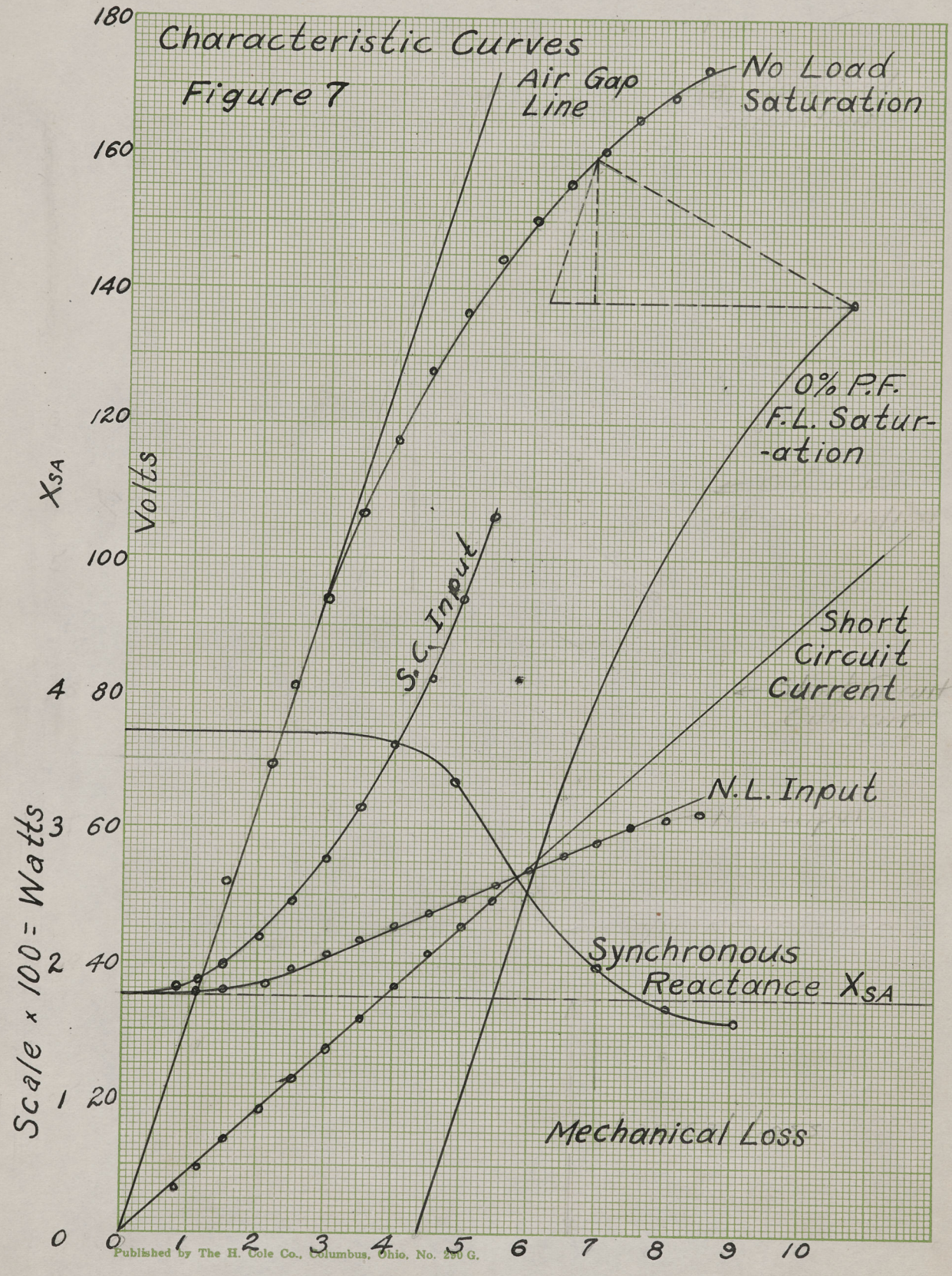
TWO - THIRDS FULL LOAD

I_f	$E_s I_s$	I_s	θ_s	Cos θ_s	Output	Input	Efficiency
4.0	4370	33.4	-35° 00′	0.819	3450	3660	94.3
4.5	3950	31.1	-25° 00′	0.906	3450	3660	94.3
5.0	3700	29.1	-16° 00′	0.961	3450	3650	94.6
5.5	3600	28.3	-6° 00′	0.994	3450	3640	94.8
6.0	3600	28.3	5° 00′	0.996	3450	3770	91.5
6.5	3680	29.0	14° 40′	0.967	3450	3720	92.8
7.0	4040	31.8	20° 00′	0.940	3450	4000	86.3
7.5	4480	35.3	24° 40′	0.909	3450	4260	81.0
8.0	4600	36.2	33° 00′	0.839	3450	4030	85.7
8.5	4740	37.3	42° 30′	0.737	3450	3670	94.20
9.0	5280	41.5	48° 30′	0.663	3450	3690	93.5

CIRCLE DIAGRAM DATA

ONE-THIRDS FULL LOAD OUTPUT

I_f	$E_s I_s$	I_s	θ_s	Cos θ_s	Output	Input	Efficiency
2.0	3480	27.4	$-59^\circ 30'$	0.507	1720	1860	92.5
3.0	2400	18.9	$-43^\circ 40'$	0.723	1720	1840	93.5
4.0	1900	14.9.	$-23^\circ 30'$	0.917	1720	1850	93.1
4.5	1800	14.1	$-10^\circ 30'$	0.983	1720	1860	92.5
5.0	1800	14.1	$2^\circ 00'$	0.999	1720	1900	90.7
5.5	1850	14.5	$14^\circ 00'$	0.970	1720	1880	91.5
6.0	2220	17.5	$19^\circ 00'$	0.945	1720	2590	66.5
6.5	2240	17.6	$38^\circ 00'$	0.788	1720	2000	86.0
7.0	2620	19.8	$46^\circ 40'$	0.686	1720	1980	87.0
7.5	3060	24.1	$52^\circ 30'$	0.609	1720	2070	83.1
8.0	3480	27.4	$56^\circ 40'$	0.549	1720	2090	82.9
8.5	3940	31.0	$61^\circ 30'$	0.477	1720	2060	83.6
9.0	4600	36.2	$66^\circ 30'$	0.399	1720	2000	86.0


Characteristic Curves
Figure 7
Air Gap Line
No Load Saturation
0% P.F. F.L. Saturation
S.C. Input
Short Circuit Current
N.L. Input
Synchronous Reactance X_{SA}
Mechanical Loss
Volts
X_{SA}
Scale x 100 = Watts
180
160
140
120
100
80
60
40
20
0
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
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FIELD CURRENT - I_x

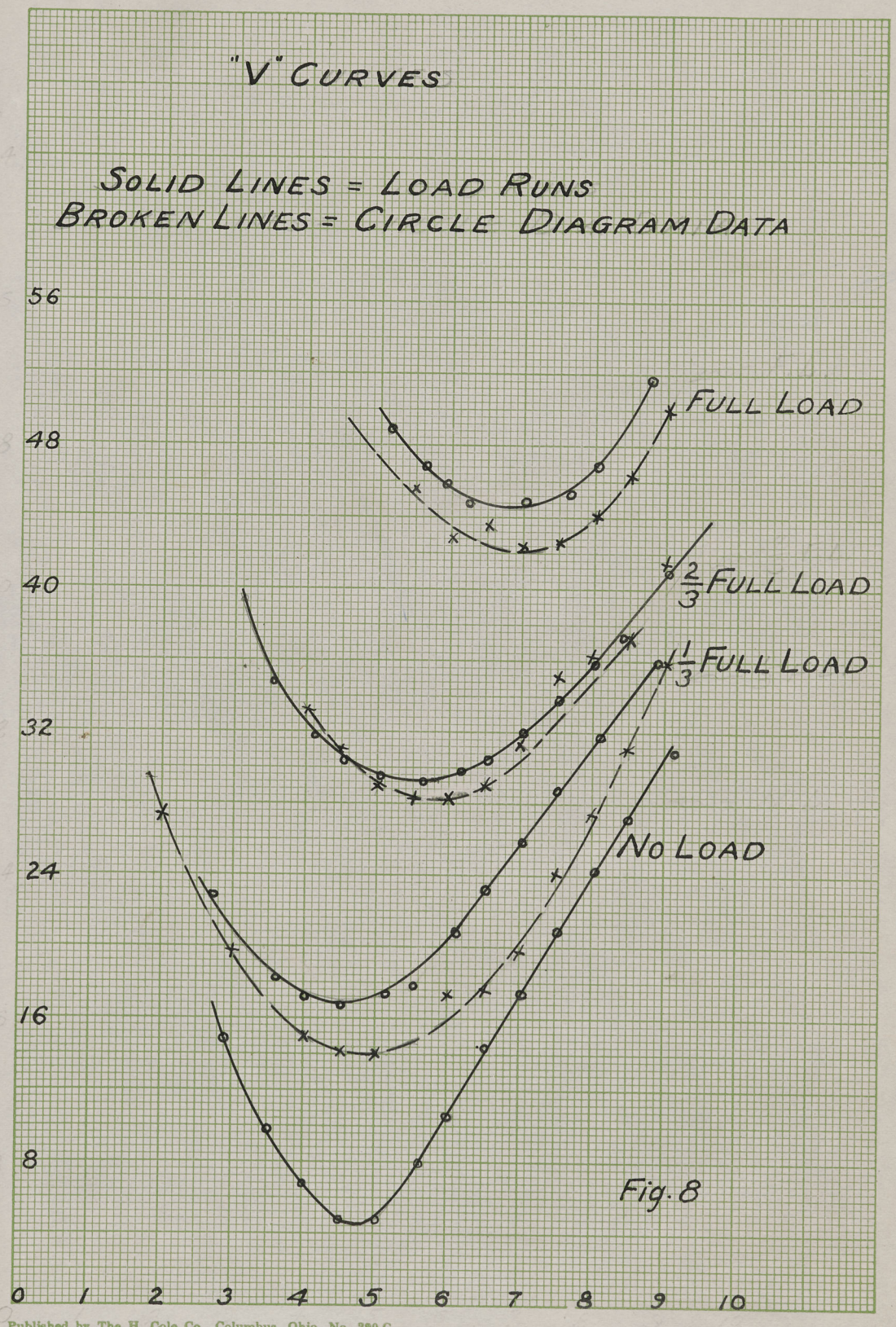
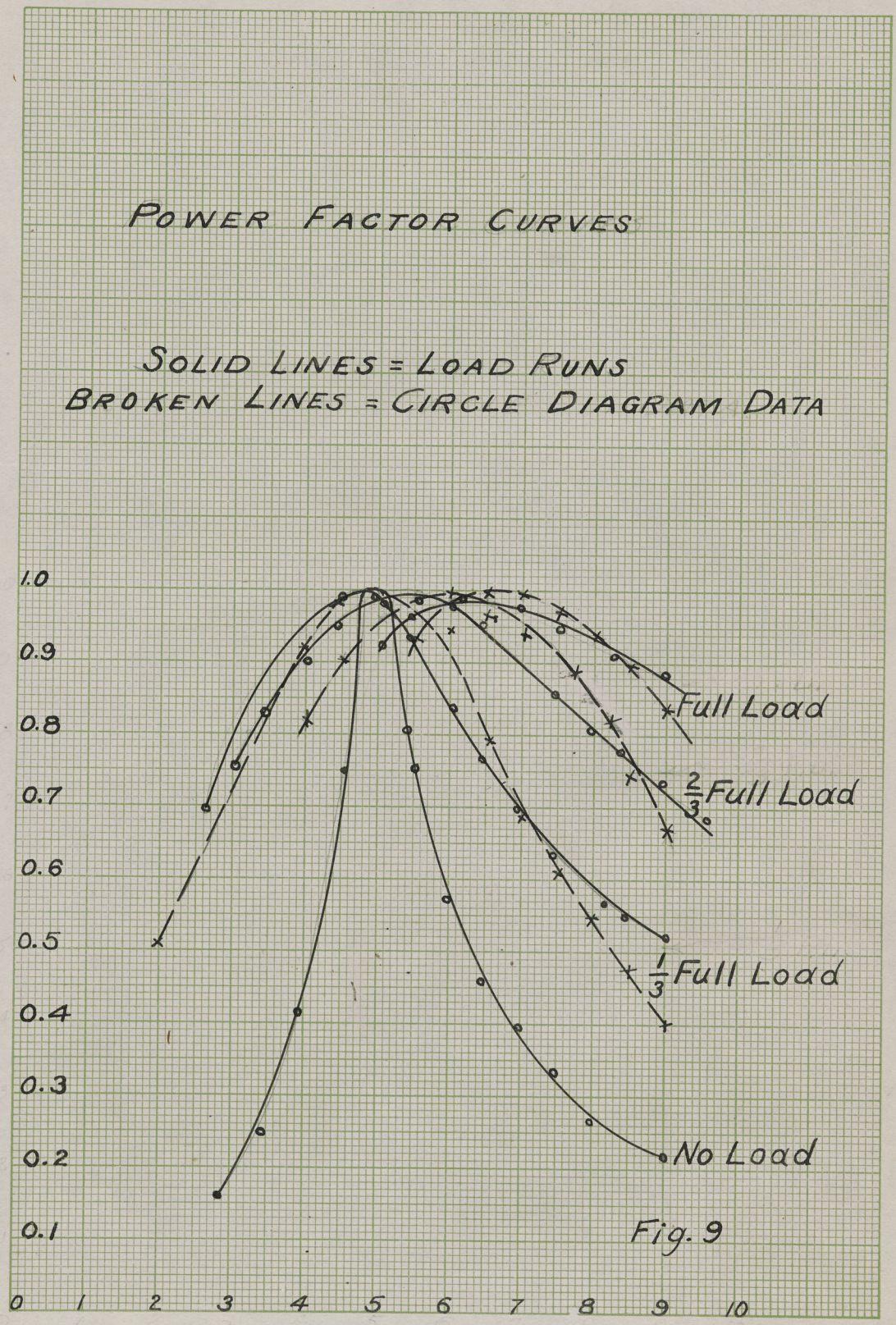


Fig. 8

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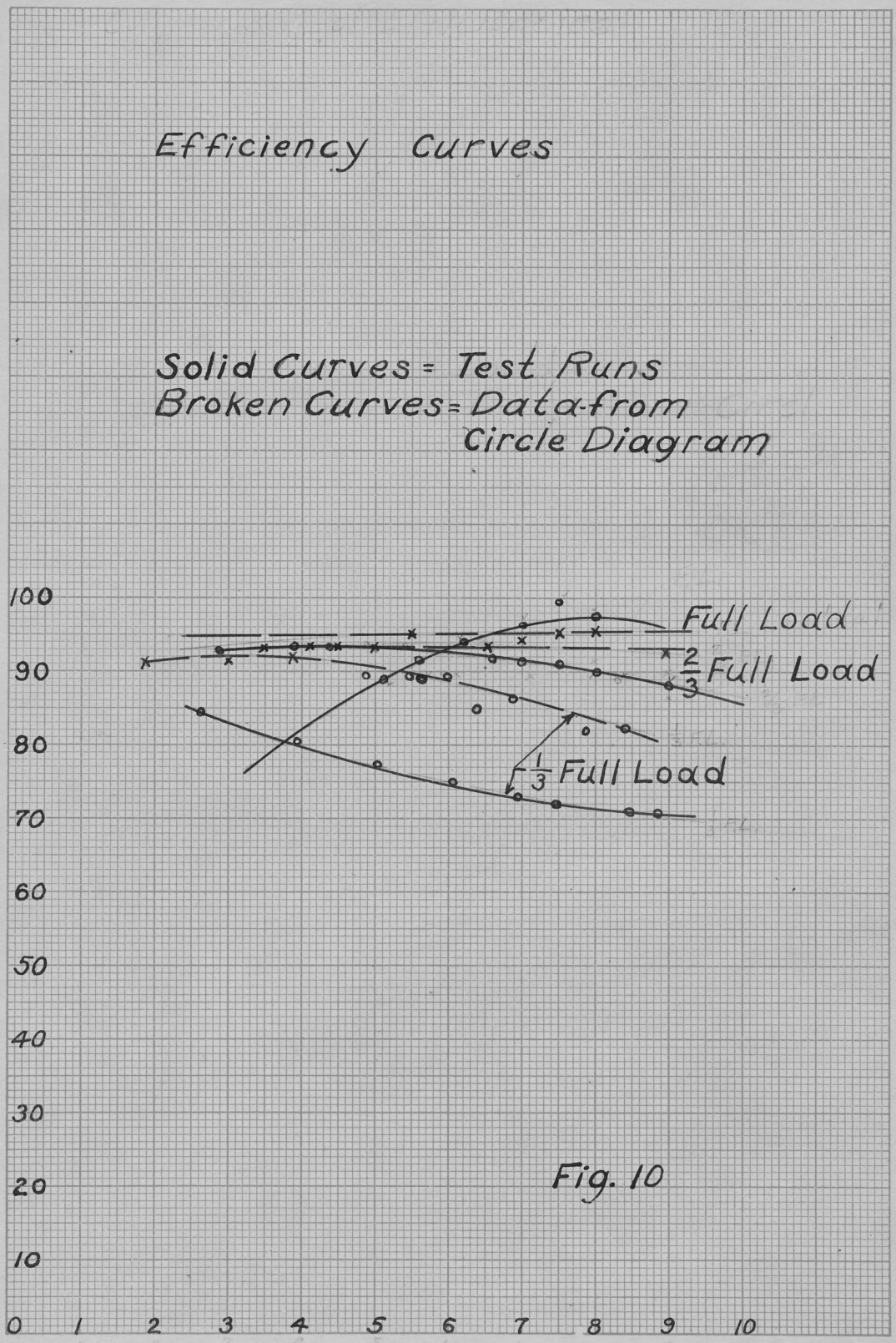
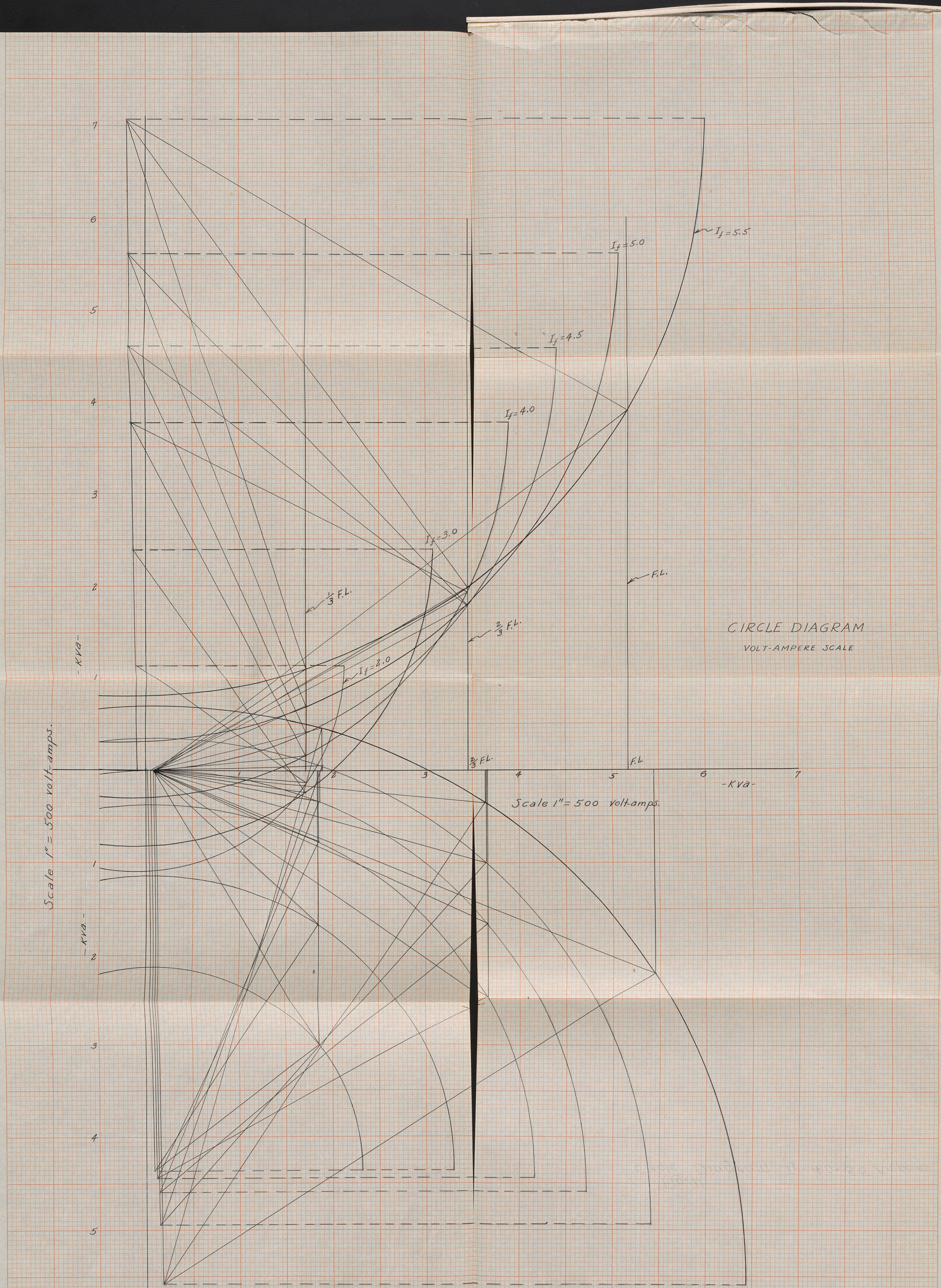


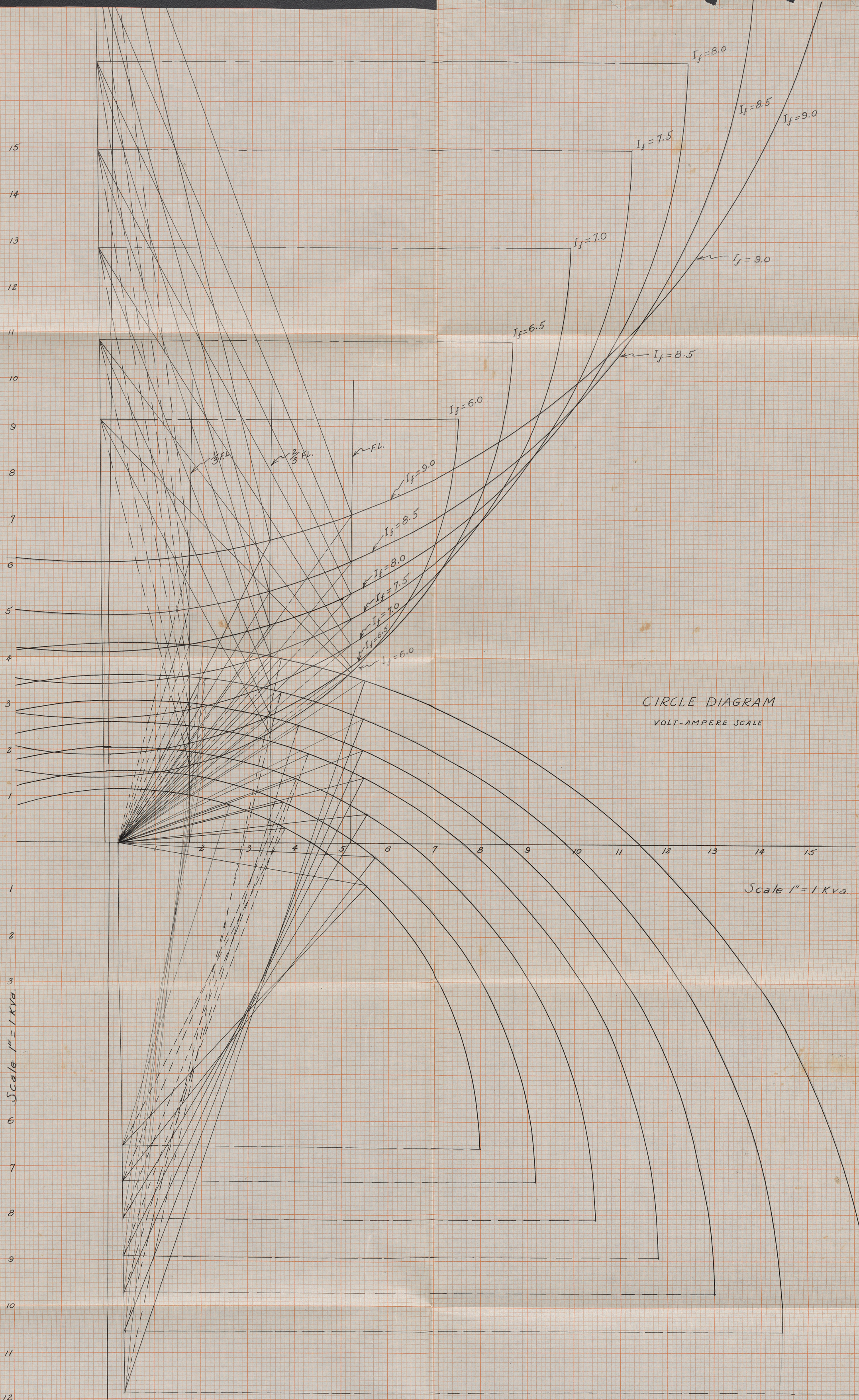
Fig. 10

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7
6
5
4
3
2
1
$I_f = 5.5$
$I_f = 5.0$
$I_f = 4.5$
$I_f = 4.0$
$I_f = 3.0$
$I_f = 2.0$
F.L.
2/3 F.L.
1/3 F.L.
-Kva-
Scale 1" = 500 volt-amps.
CIRCLE DIAGRAM
VOLT-AMPERE SCALE

CIRCLE DIAGRAM

VOLT-AMPERE SCALE



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APPROVAL.

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prerequisite to the degree for which it has been submitted. It is to be understood that by this approval the undersigned does not necessarily endorse or approve any statements made, opinions expressed, or conclusions drawn therein, but approves the thesis only for the purpose for which it is submitted.

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