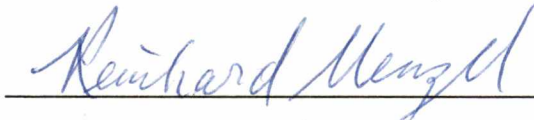


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EVALUATION OF A GEOMETRICAL OPTICS SOLID ANGLE TRANSFER
FUNCTION FOR A DEFOCUSED, FORWARD-SCATTERED
LIGHT COLLECTING OPTICAL SYSTEM

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

David George Ménard
December 1985

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DEDICATION

IN MEMORY

of

a once fluid human being

crammed and distorted

by

the classical mess

ACKNOWLEDGMENTS

The research presented in this thesis would not have been possible without the help and encouragement of some very special people.

A very special thanks goes out to Mr. Horace T. Castillo and Mrs. Rhea A. Ménard Castillo.

Honorable commendations go out to Dr. Mike Farmer, Dr. Reinhard Menzel, and Dr. Horace Carter.

A deep sense of appreciation goes to Dr. Arthur Mason and Mrs. Helen Mason.

Other thanks go out to Bettie Roberts, Mary-Francis Pomykal, Mary Lo, Colonel Edgar Masters, Callie Taylor, Ruth White, Anne Upchurch, and Anthony Saladino.

Above and beyond all else, THE BUSINESS thanks Tom Hood, a truly honorable person and Renaissance Man; and the Author acknowledges that the Grass grows in the Wood.

ABSTRACT

Because of the ambiguity in the trajectory of a particle embedded in a flow across the probe volume of a Laser Doppler Velocimeter, scattered light intensity measurements as a function of the particle size become difficult to interpret. By choosing the proper combination of photo-detector aperture setting, aperture magnification, and collecting lens focal number, a reduction in the field of view of the collection optics can be obtained such that only centrally localized scattered light from the probe volume gets detected. Hence, by applying appropriate optical design techniques, partial elimination of the trajectory ambiguity can be achieved, resulting in a higher level of confidence in the interpretation of the recorded data.

This study is the quantitative analysis of the scattered light collecting performance of a lens-aperture optical system, as used in a general applications Laser Doppler Velocimeter. Different ray trace analysis techniques are employed to support a common model for a defocused, forward-scattered light collecting optical system. A solid angle ratio function is developed on the basis of geometrical optics and allows for all calculations to be performed in the image of the aperture space. The solid angle ratio

makes the computations easier by eliminating the need to perform lens transformations. The results are used to plot normalized solid collection angle contours as a function of the point-source scattering position in the image of the aperture space. These contours are the graphical representations of a general functional expression which has been defined as the geometrical optics solid angle transfer function.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
II. GEOMETRICAL VIGNETTING OF A DEFOCUSED, FORWARD-SCATTERED LIGHT COLLECTING OPTICAL SYSTEM	12
III. PROBE VOLUME, SAMPLE VOLUME AND FOCAL VOLUME	17
IV. SOLID ANGLE RATIO.	28
V. CIRCULAR LENS AND CIRCULAR APERTURE.	37
VI. CIRCULAR LENS AND RECTANGULAR APERTURE	55
VII. SOLID ANGLE TRANSFER FUNCTION.	98
VIII. EXPERIMENTAL AND THEORETICAL RESULTS	111
IX. CONCLUSION	130
BIBLIOGRAPHY	135
VITA	150

LIST OF FIGURES

FIGURE		PAGE
1.	Representation of the Trajectory Ambiguity of a Particle Embedded in a Flow Across the Probe Volume of a Laser Doppler Velocimeter	3
2.	Linear Lens-Aperture Arrangement of the Collection Optics of a Dual-Scatter LDV-System.	6
3.	Linear Lens-Aperture Arrangement of the Collection Optics of a Mono-Scatter LDV-System.	7
4.	Representation of the Superposition of the Probe and the Sample Volumes.	8
5.	Detail Representation of the Focal Region	19
6.	Geometrical Representation of the Case of the Circular Lens and the Circular Aperture	25
7.	Geometrical Representation of the Case of the Circular Lens and the Rectangular Aperture.	26
8.	Representation of the Scattering Solid Angles (Major and Minor), Entrance Pupil Solid Angle, and Effective Solid Collection Angle	29
9.	Representation of the Paraxial and Marginal Sampling Solid Angles (Major and Minor)	31
10.	Ray Trace Diagram of a Linear Lens-Aperture Arrangement for a Forward-Scattered Light Collecting Optical System	34
11.	Ray Trace Diagram of the Circular Lens and the Circular Aperture for a Forward-Scattered Light Collecting Optical System	38
12.	Representation of the Circle of the Image of the Aperture (CIA) and the Circle of the Lens Interception (CLI)	40

FIGURE	PAGE
13. Representation of a General Ray Trace Analysis of an Arbitrary Scatter Point in the Near-Scatter Region; Denoted as the Point (a), $(r, -z)$, and of an Arbitrary Scatter Point in the Far-Scatter Region; Denoted as the Point (b), (r, z)	42
14. Representation of the Maximum and Minimum Radial Coordinates for the Distance Between the Farthest and Nearest Points of the CLI and the Geometrical Center of the Sample Volume, Respectively.	43
15. Representation of the Four Distinctive Scattered Light Collecting Regions for the Case of the Circular Lens and the Circular Photo-detector Aperture	46
16. Ray Trace Diagram of the Circular Lens and the Rectangular Aperture Arrangement for a Forward-Scattered Light Collecting Optical System.	56
17. Representation of the Rectangle of the Image of the Aperture (RIA) and the Circle of the Lens Interception (CLI)	59
18. Geometrical Optics Ray Trace Analysis of the Optical Superpositioning of the Source Blur Spot Function and the Collection Optics Sampling Function, Using the Convolution Theorem	61
19. Representation of the Two Specialized Cases for the Numerical Integration of the Convolution Integrals in the Treatment of the Circular Lens and the Rectangular Photo-detector Aperture	68
20. Representation of a General Ray Trace Analysis of Four Arbitrary Scatter Points in the Near-Scatter Region, Denoted as the Point (a) $(-x, 0, -z)$ or $(0, -y, -z)$, and the Point (c) $(x, 0, -z)$ or $(0, y, -z)$, and of Four Arbitrary Scatter Points in the Far-Scatter Region; Denoted as the Point (b) $(x, 0, z)$ or $(0, y, z)$ and the Point (d) $(-x, 0, z)$ or $(0, -y, z)$. . .	73

FIGURE	PAGE
21. Representation of the Four Distinctive Scattered Light Collecting Regions for the Case of the Circular Lens and the Rectangular Photo-detector Aperture	78
22. Representation of the Three Kinds of Transverse Scanning Situations	87
23. Diagram Representing the Well Defined Character and Discrete Nature of the Superpositioning of the CIA and CLI, as Described in Brayton's Treatment	103
24. Diagram Representing the Ill-Defined Character and Non-Discrete Nature of the Superpositioning of the RIA and CLI, as Described in the Author's Treatment.	106
25. Diagram Representing the Pinhole as a Point-Source Scatterer and the Photo-detector Apparatus Used in the Experimental Setup to Scatter-Scan the Upper-Half Marginal Octants of the Image of the Aperture Space.	113
26. Diagram of the Theoretical Results Representing the Contour Plots of the Solid Angle Ratio.	117
27. Graphical Representation of the Experimental and Theoretical Results for the Case of the Circular Lens/Circular Aperture	120
28. Graphical Representation of the Theoretical Results for the Case of the Circular Lens and the Rectangular Photo-detector Aperture	124
29. Graphical Representation of the Experimental and Theoretical Results for the Case of the Circular Lens/Rectangular Aperture.	127

NOMENCLATURE

- α = size parameter
 β = diffraction-limited blur spot diameter
 β_{LA} = longitudinal spherical aberration blur spot diameter
 δ = diffraction focal shift
 τ = longitudinal spherical aberration
 λ = wavelength of the incident light
 $\mu(t)$ = on-off condition function
 π = transcendental number (3.141592654...)
 τ = dummy variable of integration
 $\Omega_{eff},$
 $\Delta\Omega_{rel},$
 $(\Delta\Omega_{rel})_{max}$ = effective solid collection angle, relative and maximum relative errors for the effective solid collection angle, respectively
 Ω_{eff}^1 = effective solid collection angle for region one (No. 1)
 Ω_{eff}^2 = effective solid collection angle for region two (No. 2)
 Ω_{eff}^3 = effective solid collection angle for region three (No. 3)
 Ω_{eff}^4 = effective solid collection angle for region four (No. 4)
 Ω_{true} = true solid collection angle
 Ω_1, Ω_1^* = major sampling solid angles for the image of the aperture space and the aperture space, respectively
 Ω_2, Ω_2^* = minor sampling solid angles for the image of the aperture space and the aperture space, respectively

A = integration control function

$A^{(on)}$ = integration control function for on-axis scanning

$A^{(Near)}$ = integration control function for near off-axis scanning

$A_1^{(Near)}, A_2^{(Near)}$ = integration control functions for near off-axis scanning, associated with the intersection points, No. 1 and No. 2, of the geometrical intersection of the RIA and the CLI, respectively

$A^{(Far)}$ = integration control function for far off-axis scanning

$A_1^{(Far)}, A_{1*}^{(Far)}, A_2^{(Far)}, A_{2*}^{(Far)}$ = integration control functions for far off-axis scanning, associated with the intersection points, No. 1, No. 1*, No. 2, and No. 2*, of the geometrical intersection of the RIA and the CLI, respectively

A_{12} = area of two circular segments connected base-to-base

a_p = half-width of the rectangular image of the photo-detector aperture, and half-width of the rectangular-base prism with the edge of its apex along the paraxial scattering line of the rectangular double-prism sample volume

a_p^* = half-width of the rectangular photo-detector aperture

b_c = radius of the circular image of the photo-detector aperture, and base radius of the right circular cone with its apex at the paraxial scattering point of the double-cone sample volume

b_c^* = radius of the circular photo-detector aperture

b_p = half-length of the rectangular image of the photo-detector aperture, and half-length of the rectangular-base prism with the edge of its apex along the paraxial scattering line of the rectangular double-prism sample volume

b_p^* = half-length of the rectangular photo-detector aperture

$C(x)$ = one-dimensional convolution function as a function of the rectangular coordinate, x

$C(x,y)$ = two-dimensional convolution function as a function of the rectangular coordinates, x and y

$C(t)$ = one-dimensional convolution function as a function of the scanning coordinate, t

C_o = integration constant

$\text{Circ}[K(x^2, y^2)]$ = circular function representing the CIA

$\text{Circ}[J(x, x^2, y^2)]$ = circular function representing the CLI

D = particle diameter

$(ef/\#)_L$ = effective focal number of the collecting lens

$(ef/\#)_{S'}$ = effective aperture numbers of the circular and rectangular images of the circular and rectangular photo-detector apertures, respectively

f = focal length of the collecting lens

$(f/\#)$ = focal number of a lens working at infinite conjugates

$h(t)$ = blur spot function

h_c = axial-height length from the geometrical center to the paraxial scattering point of the sample volume for the case of the circular lens and the circular aperture, and height of the right circular cone with its apex at the paraxial scattering point of the double-cone sample volume

h_p = axial-height length from the geometrical center to the paraxial scattering point of the sample volume for the case of the circular lens and the rectangular aperture, and height of the rectangular-base prism with the edge of its apex along the paraxial scattering line of the rectangular double-prism sample volume

M_L = longitudinal magnification

M_T = transverse magnification

- n = index of refraction of the medium of light propagation
 n_p = index of refraction of the particle
 R = radius of the CLI
 $R(t)$ = functional representation of the semi-RIA
 $R(t-\tau)$ = shifting function representation of the semi-RIA
 r = point-source scatterer's r-radial distance from the optical axis
 r_L = radius of the collecting lens
 $r_{\max}$ = farthest CLI-point along the r-axis from the geometrical center of the sample volume
 $r_{\min}$ = nearest CLI-point along the r-axis from the geometrical center of the sample volume
 r_1', r_1'' = far-scatter location (r, z) and near-scatter location $(r, -z)$ to the farthest CLI-point, respectively
 r_2', r_2'' = far-scatter location (r, z) and near-scatter location $(r, -z)$ to the nearest CLI-point, respectively
 S = r-radial distance from the geometrical center of the sample volume to the center of the CLI
 $S(t)$ = functional representation of the semi-CLI
 t = scanning coordinate
 u, v, w = coefficients of the quadratic equation, $ux^2 + vx + w$
 x = point-source scatterer's x-rectangular distance from the optical axis
 $x(t)$ = sampling function
 $x_{\max}$ = farthest CLI-point along the x-axis from the geometrical center of the sample volume
 $x_{\min}$ = nearest CLI-point along the x-axis from the geometrical center of the sample volume

x_1', x_1'' = far-scatter location $(x, 0, z)$ and near-scatter location $(x, 0, -z)$ to the farthest CLI-point, respectively

x_1^*, x_1^{**} = far-scatter location $(-x, 0, z)$ and near-scatter location $(-x, 0, -z)$ to the farthest CLI-point, respectively

x_2', x_2'' = far-scatter location $(x, 0, z)$ and near-scatter location $(x, 0, -z)$ to the nearest CLI-point, respectively

x_2^*, x_2^{**} = far-scatter location $(-x, 0, z)$ and near-scatter location $(-x, 0, -z)$ to the nearest CLI-point, respectively

Y = y-rectangular distance from the geometrical center of the sample volume to the center of the CLI

y = point-source scatterer's y-rectangular distance from the optical axis

$y_{\max}$ = farthest CLI-point along the y-axis from the geometrical center of the sample volume

$y_{\min}$ = nearest CLI-point along the y-axis from the geometrical center of the sample volume

y_1', y_1'' = far-scatter location $(0, y, z)$ and near-scatter location $(0, y, -z)$ to the farthest CLI-point, respectively

y_1^*, y_1^{**} = far-scatter location $(0, -y, z)$ and near-scatter location $(0, -y, -z)$ to the farthest CLI-point, respectively

y_2', y_2'' = far-scatter location $(0, y, z)$ and near-scatter location $(0, y, -z)$ to the nearest CLI-point, respectively

y_2^*, y_2^{**} = far-scatter location $(0, -y, z)$ and near-scatter location $(0, -y, -z)$ to the nearest CLI-point, respectively

z = point-source scatterer's z-axial distance from the geometrical center of the sample volume

$z_L, \Delta z_L$ = image of the aperture-to-collecting lens distance and its associated sample volume axial increment, respectively

$z_K, \Delta z_K$ = aperture-to-collecting lens distance and its
associated optical transfer sample volume axial
increment, respectively

CHAPTER I

INTRODUCTION

Laser Doppler Anemometry (LDA) effectively came into being in the early 1970s with the advent of self-aligning laser interferometers. About that time interests shifted from doppler shift measurements to combined measurements of particle size, number density, and velocity. A group of interferometric instruments called Laser Doppler Velocimeters (LDV) evolved during the last decade into effective diagnostic research tools.

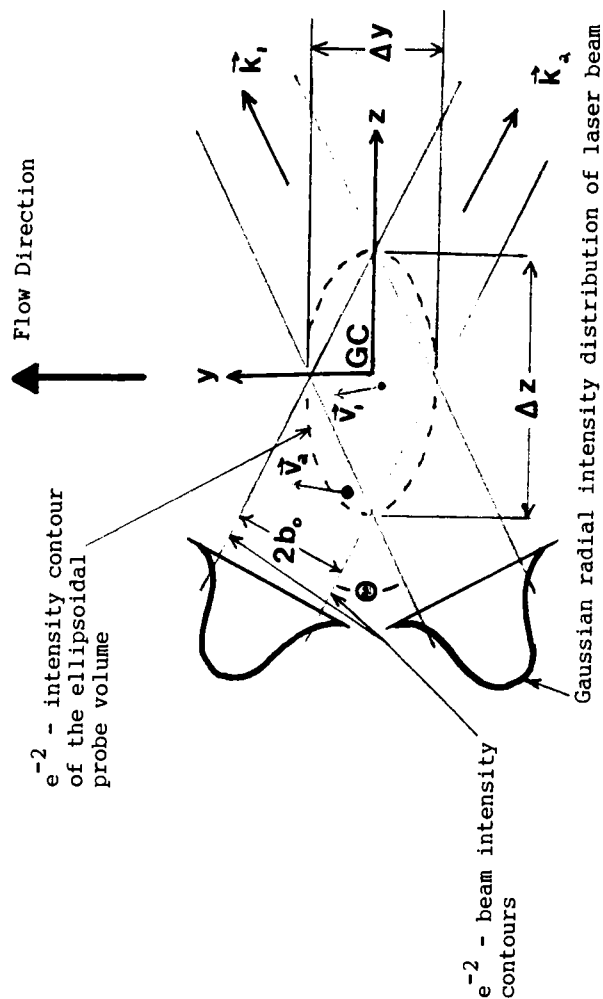
By design, an LDV-system is functionally dependent on the apertures and stops making up the optical collection system. Additionally, the performance of an LDV-system is limited by the signal-to-noise ratio, the probe volume size-to-particle size ratio, the probe volume size-to-sample volume size ratio, and the scattering solid angle-to-collection optics solid angle ratio.

Because of the ambiguity in the trajectory of a particle embedded in a flow across the probe volume of a Laser Doppler Velocimeter, scattered light intensity measurements as a function of the particle size become difficult to interpret.

The purpose of the present investigation is to use a probe volume-sample volume differentiation as a basis for a simple analysis for dealing with particle-sizing measurements. It is an attempt to eliminate the ambiguity in the scattering information that arises from indistinguishable signals resulting from small particles traversing near the geometrical center, and of large particles grazing the outer regions of the probe volume (see Figure 1). The results of the investigation partially eliminate the trajectory ambiguity.

Brayton, Kalb, and Crosswy [1]¹ show that the Gaussian intensity distribution of the illuminating laser beams and the scattered light collection efficiency of the collection optics contribute to the spatial variation of the detected signal. The photo-detector measures the scattered intensity which is a function of the point-source scattering position in the image of the aperture space. Holve and Self [2-3] state that the scattered signal is trajectory-dependent. Chigier, Ungut, and Atakan [4] state that the signal amplitude can be used as a measure of the particle size, provided that a separate indication of the transverse position of the particle within the probe volume exists. The signal characteristics, registered by the photo-detector, depend upon the geometry of the collection optics.

¹Numbers in brackets refer to similarly numbered references in the Bibliography.



GC = geometrical center of the probe volume.

θ = angular separation of the laser beams.

b_0 = radius of laser beam.

Δy = width of the probe volume.

Δz = length of the probe volume.

$\vec{k}_1, \vec{k}_2$ = propagation vectors of beam # 1 and # 2, respectively.

$\vec{v}_1, \vec{v}_2$ = velocity vectors of a small particle traversing near the geometrical center and of a large particle grazing the outer regions of the probe volume, respectively.

Figure 1. Representation of the Trajectory Ambiguity of a Particle Embedded in a Flow Across the Probe Volume of a Laser Doppler Velocimeter.

One type of analysis which has been paid widespread attention is the optical superpositioning of the sample volume with the probe volume. When the image of the photodetector aperture is superimposed onto the probe volume, the collecting lens-aperture combination can be made to act as a spatial filter controlling the optical volume sampling capability of the LDV-system.

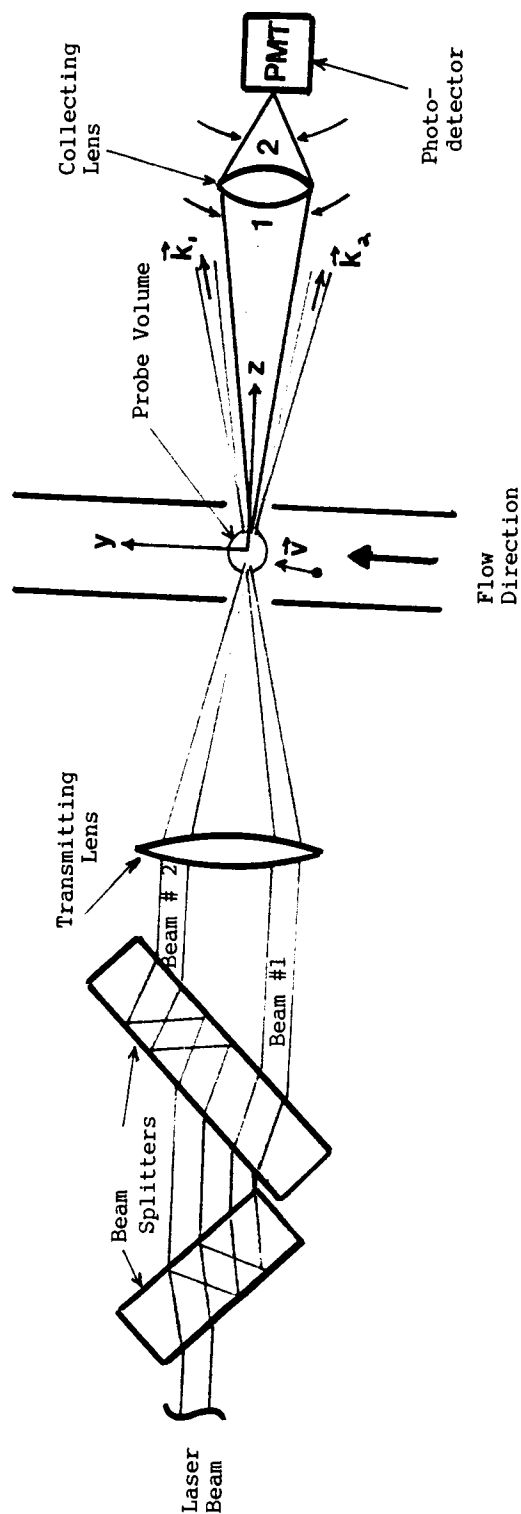
Gouesbet and Grehan [5] state that simultaneous velocimetry and sizing of single particles can be achieved by superimposing on the sizing probe (probe volume) a classical LDV control volume (sample volume). Avoidance of the particle trajectory ambiguity is achieved by correcting for the nonuniform Gaussian intensity distribution of the probe volume. The objective is to reduce the field of view of the collection optics such that only centrally localized scattered light, within an approximately uniform energy field of the probe volume, gets detected.

Farmer [6] introduces the concept of sample space and sets forth an analysis of the functional behavior of the sampling characteristics of LDV-systems designed to measure particle size and/or velocity. In general, the sample space is different in concept than the probe volume.

A character differentiation is made between the probe volume and the sample volume. The probe volume originates from some form of transmitted laser light illuminating a localized region of the particle flow. In the case

of the dual-scatter LDV-system, the probe volume is formed from the overlapping region of two identical laser beams (see Figure 2). In the case of the mono-scatter LDV-system, the probe volume is defined simply by a localized segment of a single laser beam (see Figure 3). The probe volume is also identified with the scattering volume. On the other hand, the sample volume arises from the collection optics geometry. In the case of a forward-scattered light collecting optical system, the sample volume is defined by the backward extension of all the scattered rays that are collected by the lens and transmitted through the edge of the photo-detector aperture. The sample volume is formed by drawing marginal and oblique ray traces from the edge of the clear aperture of the collecting lens and through the edge of the image of the photo-detector aperture (see Figures 3 and 4). The sample volume is also identified with the measurement volume and, under appropriate circumstances, the optical transfer of the sample volume is identified with the focal volume of the collection optics.

The study focuses on the geometrical characteristics and optical sampling properties of a lens-aperture collection optics of an LDV-system, and develops a quantitative analysis of the performance of such a system by introducing a general functional expression defined as the geometrical optics solid angle transfer function.



- 1 = scattering solid collection angle.
 2 = transmitting solid collection angle.
 $\vec{V}$ = velocity vector of a particle embedded in a flow across the probe volume.
 $\vec{K}_1$ = propagation vector of beam # 1.
 $\vec{K}_a$ = propagation vector of beam # 2.

Figure 2. Linear Lens-Aperture Arrangement of the Collection Optics of a Dual-Scatter LDV-System.

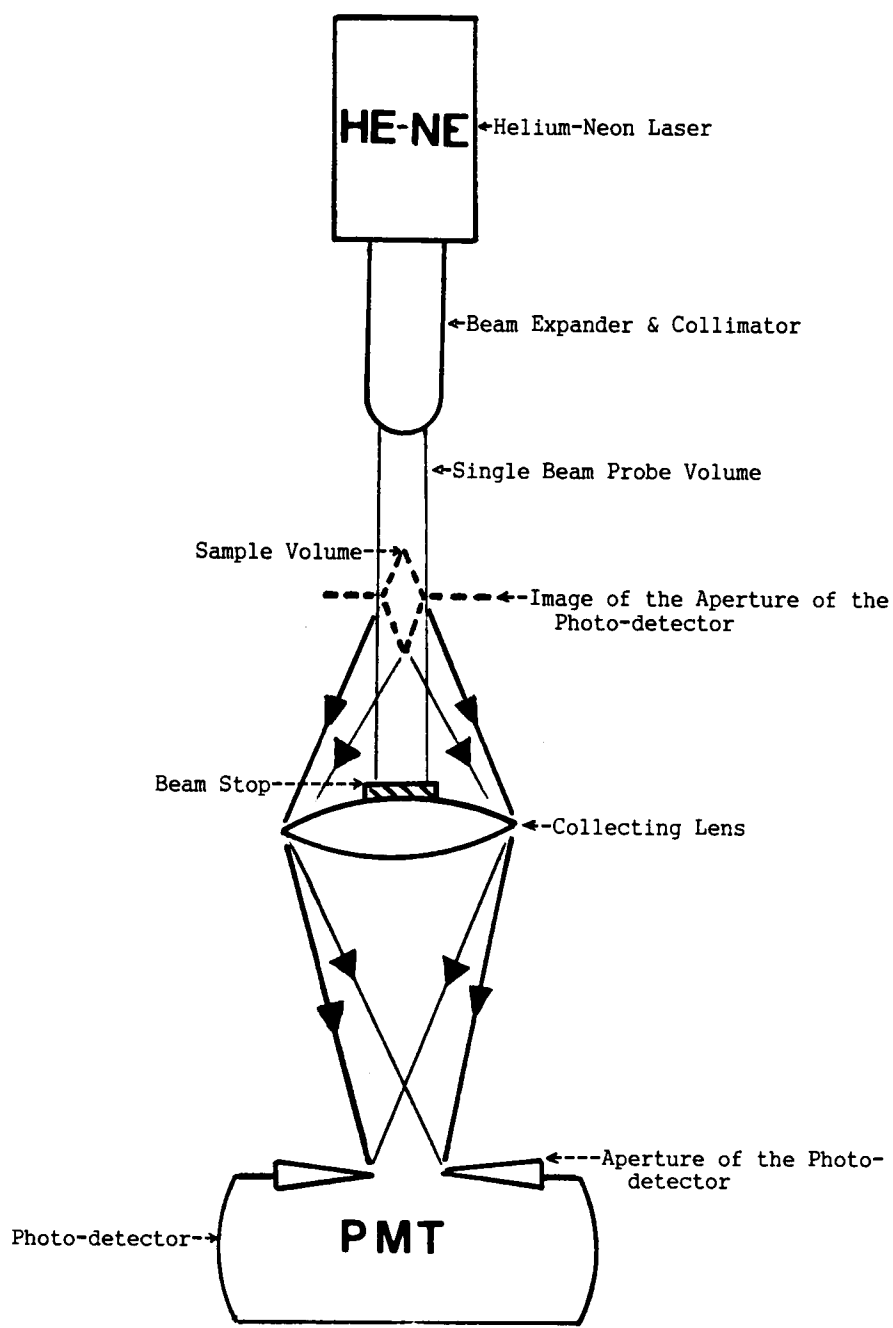


Figure 3. Linear Lens-Aperture Arrangement of the Collection Optics of a Mono-Scatter LDV-system.

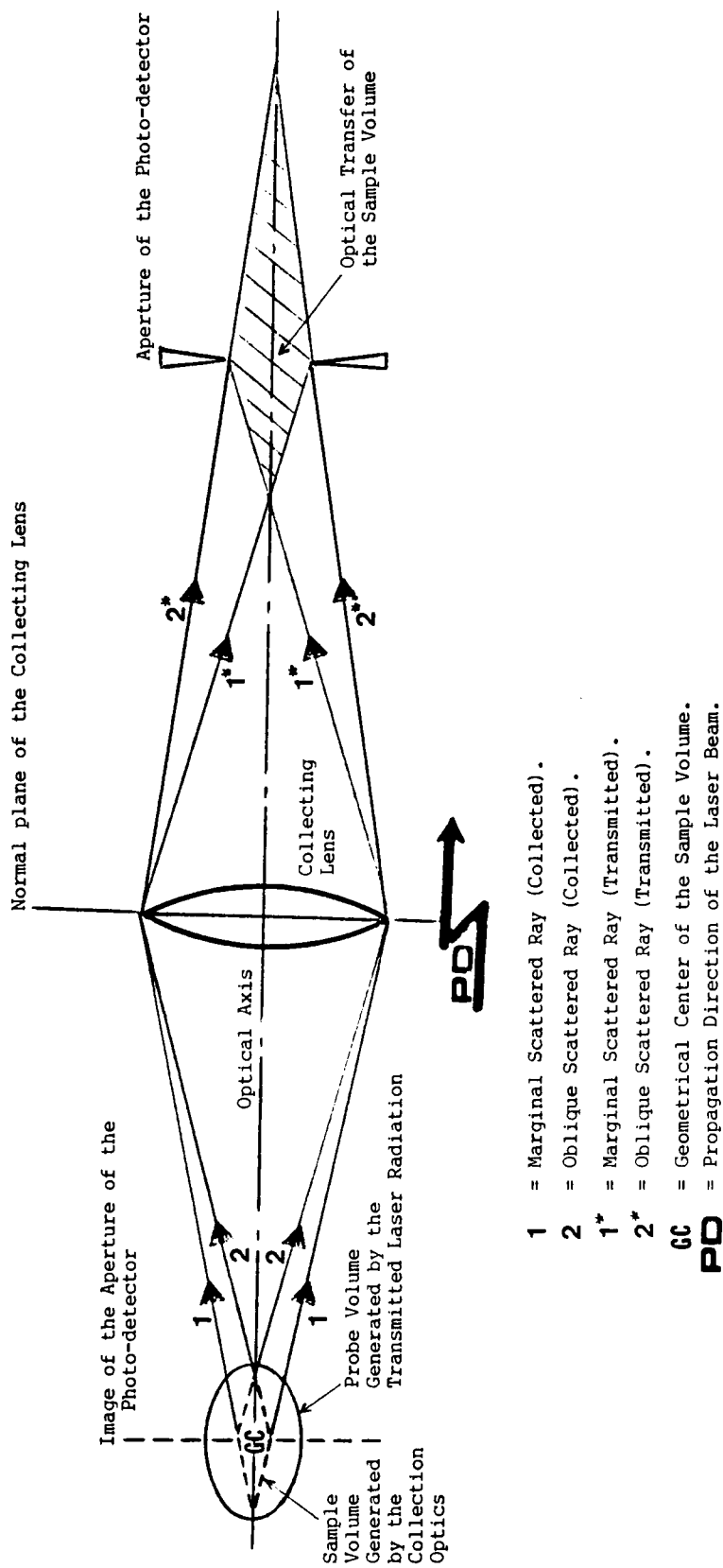


Figure 4. Representation of the Superposition of the Probe and the Sample Volumes.

Chapter II presents the preliminary considerations for a geometrical optics ray trace analysis of an LDV-system set-up in the forward light scattering mode. It also outlines the theory of a defocused optical system and the associated vignetting of the collection optics solid angle. The defocusing characteristics of an LDV-system result from the trajectory ambiguity of a particle as it traverses the probe volume.

Chapter III details the analysis necessary in determining the physical dimensions of the focal volume. The focal volume is used to set a lower-limit upon the size of the optical transfer of the sample volume. It also develops different sample volume geometries for two analytical cases of interest.

Chapter IV develops the concept of solid angle ratios needed to describe the characteristics of vignetting within a defocused optical system. It is shown that this concept allows for all calculations to be performed in the image of the aperture space where the sample volume is defined. Computations are made easier by eliminating the need to perform lens transformations.

Chapter V outlines Brayton's method for treating the case of the circular lens and the circular photo-detector aperture. This approach makes direct use of the solid angle ratio. The analysis will show that the image of the aperture space is made up of four distinctive scattered

light collecting regions, each with its own form of analytical solution.

Chapter VI presents the author's method for treating the case of the circular lens and the rectangular photo-detector aperture. This approach makes use of the solid angle ratio in a different manner than in Brayton's treatment. Because of the incompatible geometry of the optical elements and their lack of rotational symmetry about the optical axis, the ray trace analysis cannot proceed by way of analytical geometry. Using optical transfer theory, the convolution techniques can be used to perform an investigation equivalent to the ray trace analysis for this case. From this, the concept of the solid angle ratio evolves into the geometrical optics solid angle transfer function. The analysis shows that two specialized cases exist in solving the convolution integrals, and their combined solutions make up four distinctive scattered light collecting regions within the image of the aperture space.

Chapter VII discusses the theoretical and physical significance of the geometrical optics solid angle transfer function.

Chapter VIII describes the experimental set-up used to measure the amount of light scattered by a point-source located at different positions within the probe volume, and the experimental procedures used in scanning the different image of the photo-detector aperture geometries.

It also presents the experimental results which were obtained in the laboratory and compares them with the theoretical computations. Sources of error and other experimentally related problems are discussed.

Chapter IX summarizes the study conclusions. It shows that the solid angle transfer function is a more general form of the solid angle ratio of probe and sample volumes.

CHAPTER II

GEOMETRICAL VIGNETTING OF A DEFOCUSED, FORWARD-SCATTERED LIGHT COLLECTING OPTICAL SYSTEM

This chapter presents the preliminary considerations for a geometrical optics ray trace analysis of an LDV-system set-up in the forward light scattering mode. It is shown that the domain of applicability of such an analysis is confined to limited ranges in its system's parameters. It also outlines the theory needed to understand the behavior of a defocused optical system. The theory shows that geometrical vignetting of the light scattered from an out-of-focus point-source scatterer results relative to the detection field-of-view of the collection optics.

For scattering off a particle of a size of 20 wavelengths or more, it is possible to distinguish between the rays incident on the particle and the rays passing along the particle. When the particle is much larger than the wavelength, the incident beam of light can be described as individual rays pursuing their own path. This is known as the "principle of ray-localization" [7]. When the size parameter approaches infinity, geometrical ray-localization leads to a separation of the diffracted, refracted, and

reflected optical fields. This parameter is used in scattered light particle-sizing measurements, and can be written as:

$$\alpha = \frac{n_p D}{\lambda}$$

Hodkinson and Greenleaves [8] state that for particles larger than a few wavelengths, the integrated effects of the light scattered off them can be approximated as a combination of classical diffraction and of geometrical optics transmission and reflection.

Ungut, Grehan, and Gouesbet [9] state that calculating the scattered light properties of particles in an LDV-system can be made easier by using geometrical optics when the light is collected in the forward observation mode; as opposed to collecting the scattered light in the backward observation mode.

The domain of applicability of a geometrical optics light scattering theory is confined to the near forward directions (0-20°) for transparent spherical particles of diameters greater than several microns.

Geometrical optics can be used with fair accuracy for forward direction light scattering calculations related to particle sizing and characterization. Moreover, these computations are relatively easy to perform (i.e., fast

and inexpensive) especially in nonideal situations such as with aspherical particles and nonuniform illumination.

The basic measurement parameter is the intensity of the light scattered by a point-source located at different positions within the probe volume. Polarization is neglected in this treatment (i.e., the scattered field is treated as incoherent light). Phase effects of the various components (i.e., diffraction, refraction, and reflection) are excluded from this geometrical optics analysis.

The optical collection system is assumed to be isoplanatic; that is, it obeys the "Staeble-Lihotzky condition" [10]. An isoplanatic region of such a system is that portion of space over which the system's point spread function remains unchanged. Since an unchanging point spread function implies that the optical collecting and transmitting characteristics of the system are independent of any wave optical effects, the transition to a ray optics treatment is justified. Hence, the analysis is based on the geometrical optics concept of the limitation of rays by apertures or stops.

The defocusing characteristics of an LDV-system are an outcome of the trajectory ambiguity of a particle as it traverses the probe volume. The geometrical analysis of a defocused optical system is developed from the Abbe theory of stops and apertures. Ideally, the optical

system is designed such that the in-focus pencils of rays produce light spots of infinitesimal area. The out-of-focus pencils of rays produce light spots of finite area, i.e., blur regions. The effects of the light spots on the imaging characteristics of the system depend upon the angular dimensions of the pencils. In turn, these dimensions depend upon the aperture stop of the system, or equivalently, upon the pupils of the system.

A schematic of the collection optics of a monoscatter LDV-system consisting of a linear lens-aperture arrangement is considered in Figure 3 (page 7). The lens-aperture set-up consists of a collecting lens and a photo-detector aperture. In this case, the photo-detector aperture defines both the aperture stop and the exit pupil. The image of the photo-detector aperture defines the entrance pupil. The collecting lens defines the field stop, entrance window, and exit window.

Since the collecting lens is identified with the field stop, the field-of-view of the collection optics is controlled by the clear aperture of the lens. Dimensional relationships between the image of the photo-detector aperture and the collecting lens determine the effective field-of-view and the amount of vignetting. Levine [11] describes the characteristics of vignetting within an optical system. The vignetting of a lens is the reduction

in the light intensity of off-axis points in the receiving plane of the detector. This reduction is due to the blocking of the transmitted light when the mounting of an optical element interferes with some of the rays. Levine defines a vignetting factor which is the ratio of the flux through the entrance pupil alone when the lens has been removed. Aref'ev et al. [12] state that the illumination nonuniformity associated with a lens-aperture system is described as the vignetting of light by the transmitting source and the receiving aperture. Computations of the amount of light flux that passes through and is blocked by one of more vignetting apertures have been reported by Grigor'ev, Pavlova, and Sheinis [13].

Vignetting can also result from the defocusing of a lens-aperture optical system. Shul'man [14] related the nonuniformity of the exit pupil illumination to the defocusing of the optical system. He defines focusing as when all points of the source and all points at the edge of the field-of-view are imaged by light rays that are not intercepted by the mounting of the photo-detector aperture. This implies that defocusing occurs when an illuminated transition region appears at the edge of the angular field-of-view, due to the partial vignetting of the ray bundles. Therefore, vignetting occurs when the light spot becomes large enough or when it moves far enough off-axis so that some of the rays strike the mounting of the photo-detector aperture.

CHAPTER III

PROBE VOLUME, SAMPLE VOLUME AND FOCAL VOLUME

This chapter develops the physical dimensions of the focal volume. By approximate formulation, there exists a focal volume associated with the focusing properties of a diffraction-limited lens in the presence of axial aberrations of various orders. The focal volume is used to set a lower-limiting value upon the size of the optical transfer of the sample volume. It also develops different sample volume geometries for two analytical cases of interest.

The purpose of developing this focal volume formulation is to determine the smallest photo-detector aperture that will collect all of the light in the image of a point-source scatterer, when the point-source is located at the center of the image of the photo-detector aperture. The physical dimensions of the focal volume are determined by the longitudinal spherical aberrations.

The focal point of a real optical imaging system is not really a point, but a volume. The focal volume arises from deviations of the light rays from the point of the best diffraction focus. This point is the spot where the light intensity is a maximum and its location lies on the

optical axis midway between the extreme marginal and paraxial foci. The ray deviations are due to the point-spreading nature of the diffraction-limited lens and its geometrical optics aberrations. They can be calculated from diffraction theory and from a series expansion of the optical paths. The expansion is performed about the reference focal point. This point is taken to be the center of the reference sphere and is assumed to lie in the vicinity of the best diffraction focus.

For an aberration-free system, the diffraction-limited blur spot (i.e., Airy Disk) would represent the cross-sectional area of the diffraction-limited focal volume. However, when the lens-aberrations are much larger than the diffraction effects, the location on the optical axis of the best geometric focus is where the size of the blur spot is a minimum. The best geometric focus does not coincide with the location of the best diffraction focus; it lies shifted towards the marginal focus relative to the best diffraction focus (see Figure 5). In the case of the best geometric focus, the minimum blur spot diameter (i.e., the diameter of the Circle of Least Confusion) represents the largest cross sectional dimension of the geometric focal volume.

The importance of the blur spot diameter to the collection optics of an LDV-system is developed as follows.

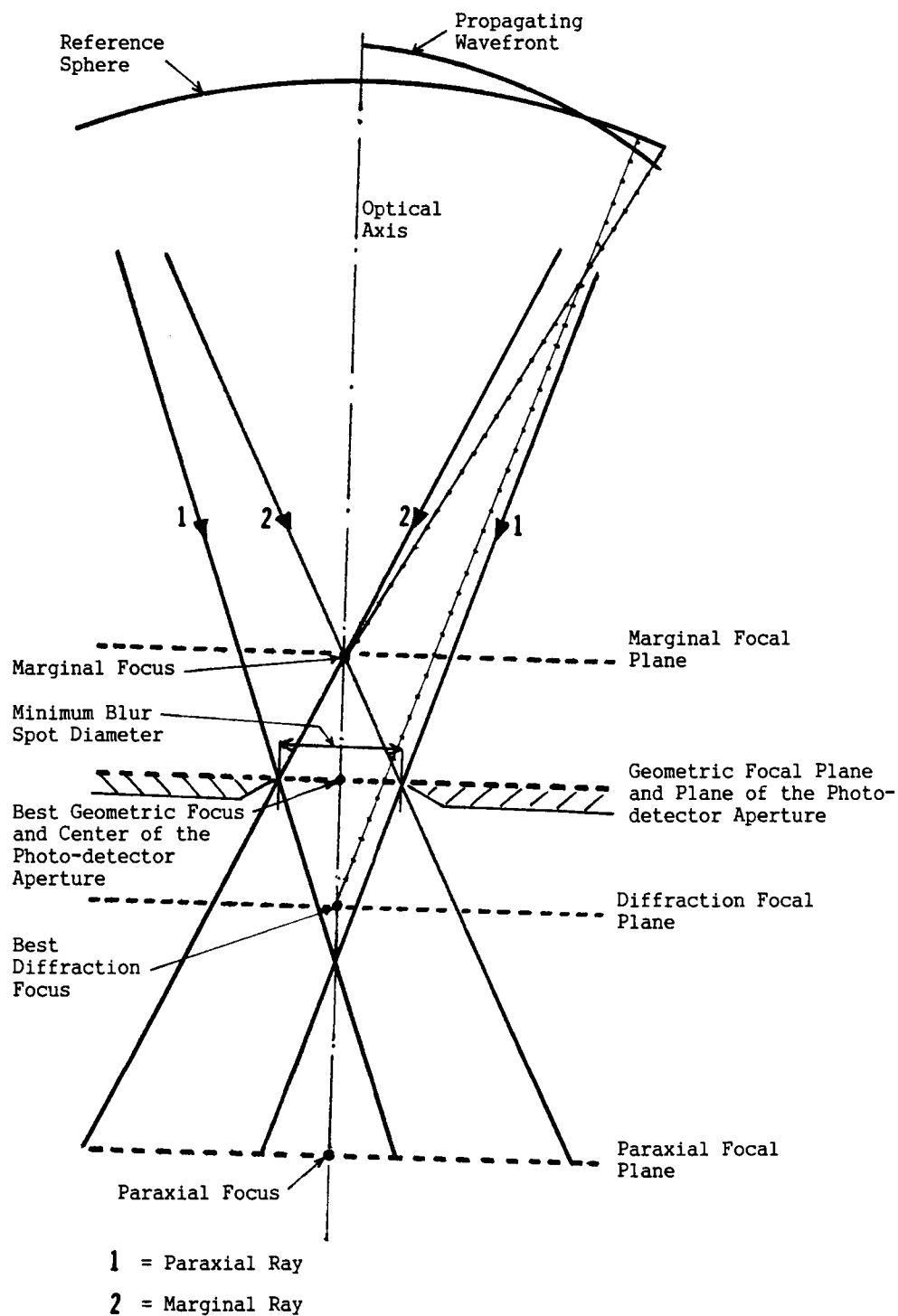


Figure 5. Detail Representation of the Focal Region.

Since the basic measurement parameter is the scattered light intensity and not the image quality, the choice of the point of the best focus is taken to be the minimum blur spot position (i.e., the point of the best geometric focus). This is the same choice as that of Warren J. Smith [17]. He states that the minimum blur spot position is seldom chosen as the point of the best focus in optical systems for visual or photographic work. But it is frequently chosen in systems which utilize photo-detectors since, in most cases, the purpose is to determine the smallest detector that will gather all of the energy in the in-focus image.

Because any real optical imaging system is affected by axial aberrations of various orders, the longitudinal dimension of the focal volume will be much larger than the focal range of the system. The focal range is equal to twice the distance of the diffraction focal shift. The diffraction focal shift is the distance from the paraxial focus to the point of the best diffraction focus. The diffraction focal shift is used as a variable in the theoretical design of the optical system and all coefficients of the power series are related to it. By applying Raleigh's quarter wavelength limit to an aberration-free system [15], the diffraction focal shift can be derived. The diffraction focal shift can be written as:

$$\delta = \frac{2 \lambda (f/\#)^2}{n} \quad (2)$$

The size of the diffraction-limited blur spot, based on the Raleigh limit, is determined by the focal number and the diffraction focal shift, and can be written as:

$$\beta = \frac{\delta}{(f/\#)} \quad (3)$$

The power series expansion has been performed by Ta-Hang [16], dealing with the location of the best diffraction focus of an axial pencil of rays in the presence of longitudinal spherical aberrations of various orders. In a well-designed system, where the axial spherical aberration is controllable up to second order (i.e., the zonal aberration is controllable), Ta-Hang shows that the maximum allowable longitudinal spherical aberration is 16 times the focal range of the system. This aberrational expression can be written as:

$$\zeta = \frac{64\lambda (f/\#)^2}{n} \quad (4)$$

The size of the longitudinal spherical aberration blur spot is determined by the focal number and the longitudinal spherical aberration, and can be written as:

$$\beta_{LA} = \frac{\zeta}{(f/\#)} \quad (5)$$

Hence, the smallest dimension of the photo-detector aperture must be chosen to be greater or equal to the diameter of this minimum blur spot (i.e., the longitudinal

spherical aberration blur spot). This choice assures that the light, scattered from the geometrical center of the sample volume and collected by the lens, gets detected without any vignetting.

The control in an LDV-system of the location of the measurement volume (i.e., the sample volume) rests mainly in the position of the lens system because the lens position determines the location of the superposition of the sample volume on the probe volume. Moreover, the collecting lens determines the superpositioning of the geometric focal volume onto the optical transfer of the sample volume. The collecting lens is most effective in transmitting the scattered rays to the photo-detector aperture, when it images the light from the geometrical center of the sample volume to the geometric focal volume. The effective focal plane of the collecting lens passes through the center of the photo-detector aperture and is assumed to be normal to the optical axis. The effective focal plane is identified with the plane of the photo-detector aperture. The center of the geometric focal volume (i.e., the point of the best geometric focus) coincides with the center of the photo-detector aperture. The largest cross-sectional area of the sample volume is the geometrical image of the photo-detector aperture as seen by the scattering volume (i.e., the probe volume) (see Figure 3, page 7). The sample volume is bounded by the marginal and oblique ray traces

of the image of the photo-detector aperture (see Figure 4, page 8). The optical transfer of the sample volume is formed from the light rays, scattered from the sample volume and collected by the lens, that get transmitted through the photo-detector aperture. The optical transfer of the sample volume is bounded by the marginal and oblique ray traces of the photo-detector aperture (see Figure 4, page 8). Since the largest cross-sectional area of the optical transfer of the sample volume is the photo-detector aperture, the geometric focal volume sets a lower-limiting value upon the size of the optical transfer of the sample volume. Under the condition of minimal photo-detector aperture setting, the optical transfer of the sample volume can be identified with the geometric focal volume of the collection optics. For a system's magnification of unity, the optical transfer of the sample volume is the same size as the sample volume; and the geometric focal volume also sets a lower-limiting value upon the size of the sample volume.

There are two analytical cases of interest, forming different sample volume geometries. These are:

1. The Circular Lens and the Circular Aperture
2. The Circular Lens and the Rectangular Aperture

First, consider the case of the circular lens and the circular aperture. The central cross section of the sample volume is formed by the image of the circular

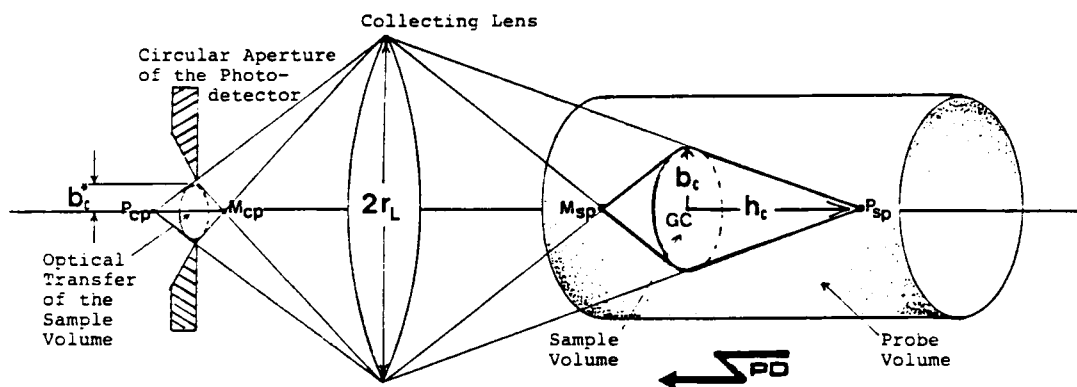
aperture of the photo-detector. It consists of two right circular cones, connected base-to-base, centered about the geometrical center of the probe volume (see Figure 6).

This specific sample volume forms a double-cone geometry. The axial-height length of the sample volume for the case of the circular lens and the circular aperture is defined as the height of the right circular cone, while the radius of the image of the photo-detector aperture is defined as the base radius of the right circular cone. The base radius and height of the right circular cone with its apex at the paraxial scattering point are given, respectively, as:

$$b_c = M_T b_c^* \quad (6)$$

$$h_c = M_T 2b_c^* (ef/\#)_L \quad (7)$$

Second, consider the case of the circular lens and the rectangular aperture. The central cross section of the sample volume is formed by the image of the rectangular aperture of the photo-detector. It consists of two rectangular-base prisms, connected base-to-base and having wedge-shaped ends, centered about the geometrical center of the probe volume (see Figure 7). This specific sample volume forms a rectangular double-prism geometry. The axial-height length of the sample volume for the case of the circular lens and the rectangular aperture is defined as



P_{cp} = Paraxial Collecting Point.
M_{cp} = Marginal Collecting Point.
P_{sp} = Paraxial Scattering Point.
M_{sp} = Marginal Scattering Point.

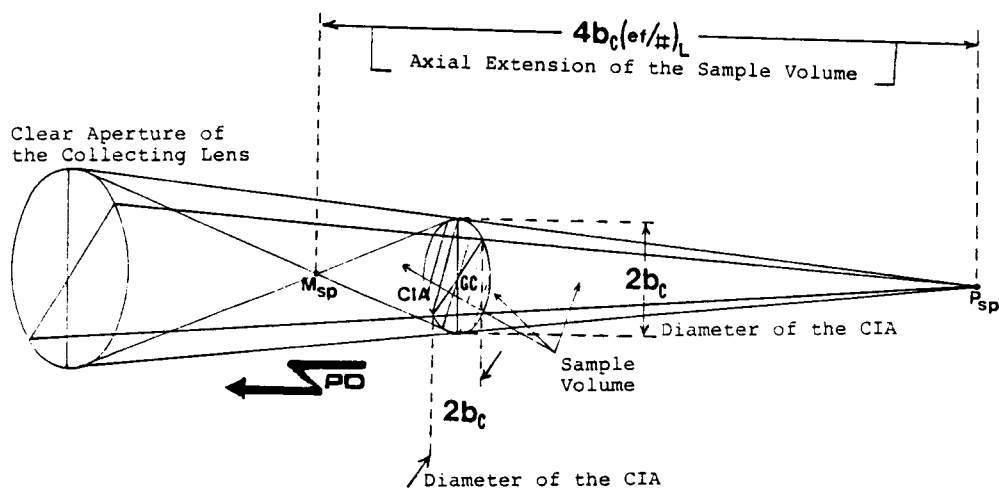


Figure 6. Geometrical Representation of the Case of the Circular Lens and the Circular Aperture.

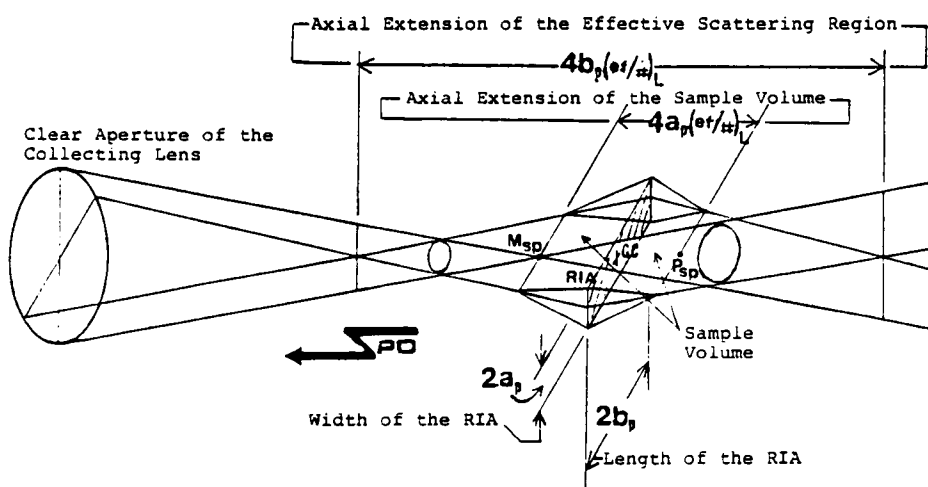
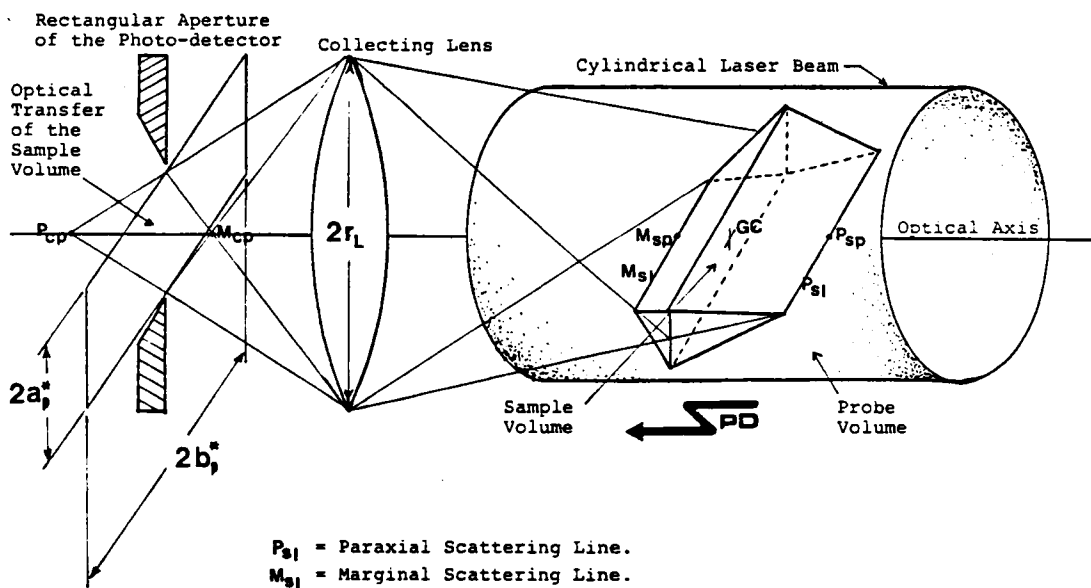


Figure 7. Geometrical Representation of the Case of the Circular Lens and the Rectangular Aperture.

the height of the rectangular-base prism, while the width of the image of the photo-detector aperture is defined as the width of the rectangular-base prism. The width, length, and height of the rectangular-base prism with the edge of its apex along the paraxial scattering line are given, respectively, as:

$$2a_p = M_T 2a_p^* \quad (8)$$

$$2b_p = M_T 2b_p^* \quad (9)$$

$$h_p = M_T 2a_p^* (ef/\#)_L \quad (10)$$

CHAPTER IV

SOLID ANGLE RATIO

This chapter develops the concept of solid angle ratios needed to describe the characteristics of vignetting within a defocused optical system. It shows that all calculations can be performed in the image of the aperture space where the sample volume is defined. Computations are made easier by eliminating the need to perform lens transformations.

The solid angle ratio is developed to evaluate the light collection performance of a lens-aperture imaging system. The concept of the solid angle ratio evolves out of Levine's vignetting factor [11]. This factor was defined previously in terms of a ratio of fluxes; it is also the ratio of the lens area to the area of the entrance pupil. In order to develop the concept of the solid angle ratio and of its computational applicability within the image of the aperture space, it is convenient to introduce the scattering solid angle and the sampling solid angle.

The scattering solid angle is divided into major and minor categories (see Figure 8). The major scattering solid angle is subtended in the image of the aperture space from the point-source scatterer to the clear aperture

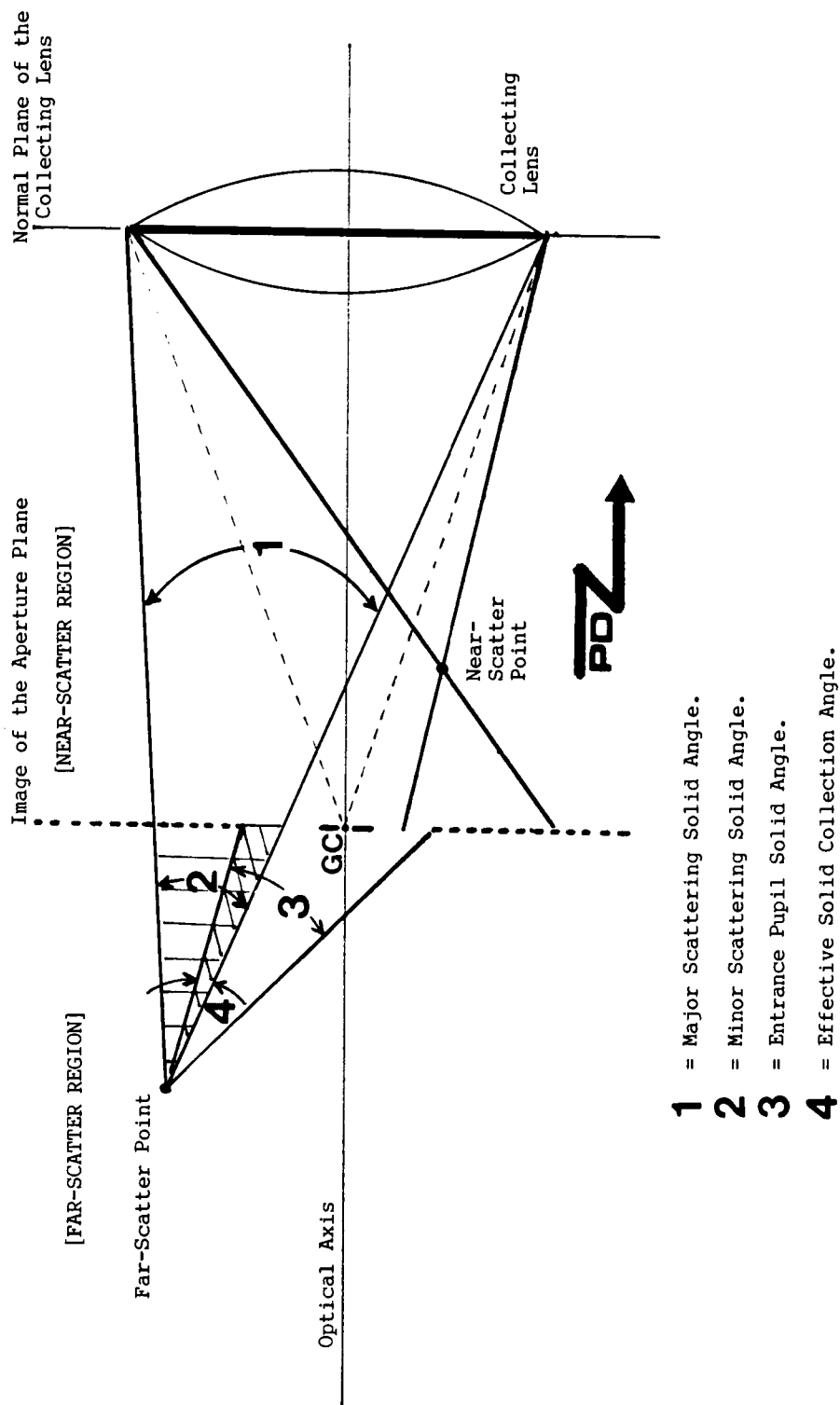


Figure 8. Representation of the Scattering Solid Angles (Major and Minor), Entrance Pupil Solid Angle, and Effective Solid Collection Angle.

margin of the collecting lens. The minor scattering solid angle is formed from the major scattering solid angle and the cross section of the image of the aperture plane through it. The minor scattering solid angle is the solid angle subtended in the image of the aperture space from the point-source scatterer to a particular area of the image of the aperture plane as dictated by the major scattering solid angle.

Similarly, the sampling solid angle is divided into major and minor categories (see Figure 9). The major sampling solid angle is subtended in the image of the aperture space from either of the two axial extremities of the sample volume to the margin of the collecting lens, depending on the location of the scatterer relative to the image of the aperture and the collecting lens. If the scatterer is located between the plane of the image of the aperture and the normal plane of the collecting lens, the major sampling solid angle will be subtended from the marginal point of the sample volume. This portion of the image of the aperture space will be denoted as the Near-Scatter Region. If the scatterer is located beyond the image of the aperture plane and the plane of the collecting lens, the major sampling solid angle will be subtended from the paraxial point of the sample volume. This portion of the image of the aperture space will be denoted as the Far-Scatter Region. The minor sampling solid angle is formed

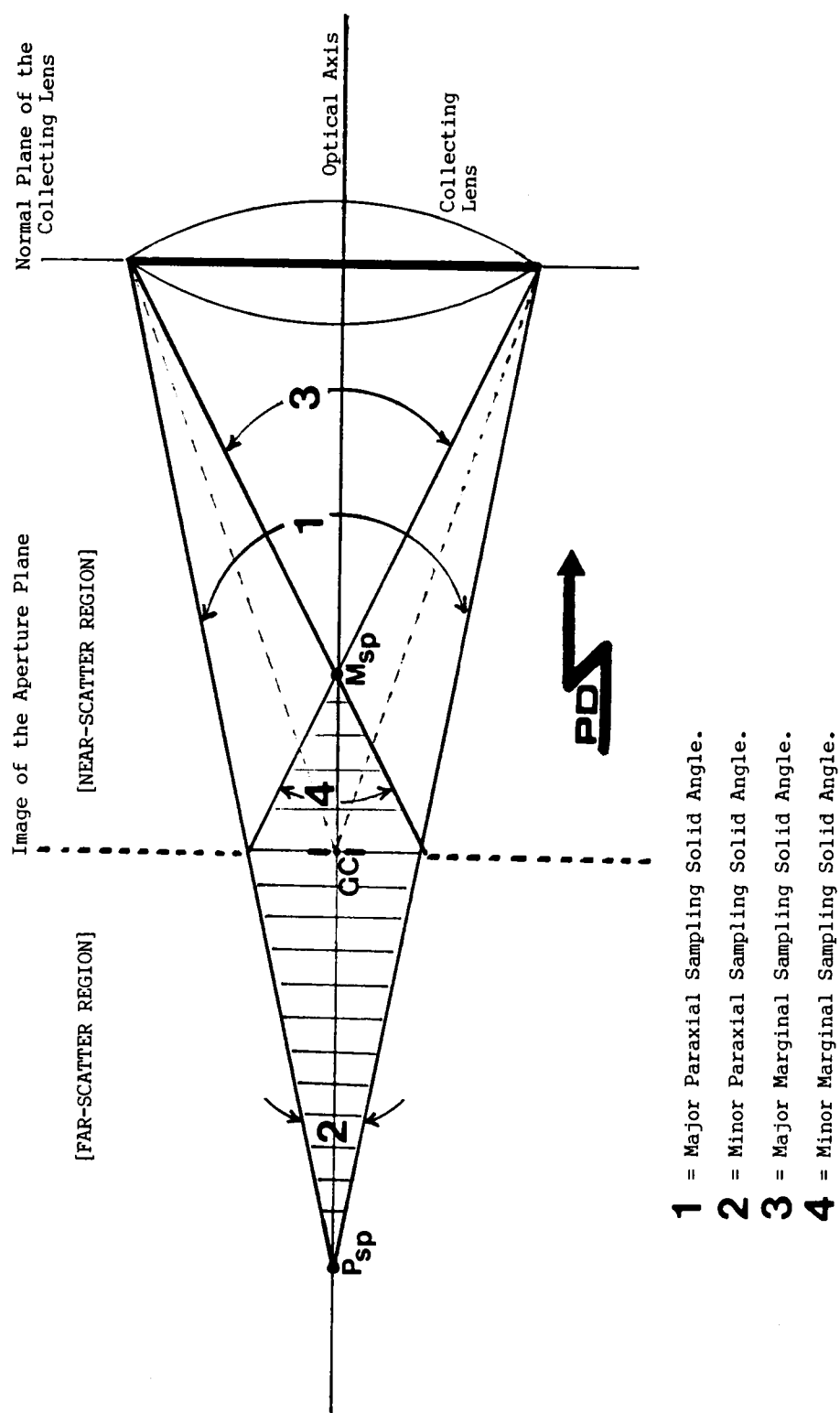


Figure 9. Representation of the Paraxial and Marginal Sampling Solid Angles (Major and Minor).

from the major sampling solid angle and the cross section of the image of the aperture plane through it. The minor sampling solid angle is the solid angle subtended in the image of the aperture space from either of the two axial extremities of the sample volume to the entrance pupil of the optical system.

For the Far-Scatter Region, both of these minor solid angles (scattering and sampling) are clearly formed from their respective major counterparts. However, for the Near-Scatter Region, it is necessary to envision the minor solid angles (scattering and sampling) as extending backwards to the plane of the image of the aperture.

The solid angle ratio is defined as the ratio of the effective solid collection angle to the minor scattering solid angle. The effective solid collection angle consists of the common portion of solid angle shared by the minor scattering solid angle and the solid angle subtended by the point-source scatterer to the entrance pupil.

It can be demonstrated that all scattered light collecting calculations involving the solid angle ratio can be performed in the image of the aperture space. This is done by showing the optical equivalence that exists between the major and minor sampling solid angles of the image of the aperture space and the major and minor conjugate sampling solid angles of the aperture space. If the optical system forms a conjugate sampling solid angle

(major or minor) in the aperture space which is geometrically similar to the corresponding sampling solid angle (major or minor) in the image of the aperture space, then the sampling solid angles (major or minor) of both these spaces are optically equivalent.

Consider the ray trace diagram of a linear lens-aperture arrangement for a forward-scattered light collecting optical system (see Figure 10). Let Δz_L and Δz_K be proportional to Δz_L^a or Δz_L^b and Δz_K^a or Δz_K^b , respectively. The ratios of the major and minor sampling solid angles for both the image of the aperture space and the aperture space can be written, respectively, as:

$$\frac{\Omega_1}{\Omega_2} = \frac{\pi r_L^2}{(z_L \pm \Delta z_L)^2} \cdot \frac{\Delta z_L^2}{\pi b_c^2} \quad (11)$$

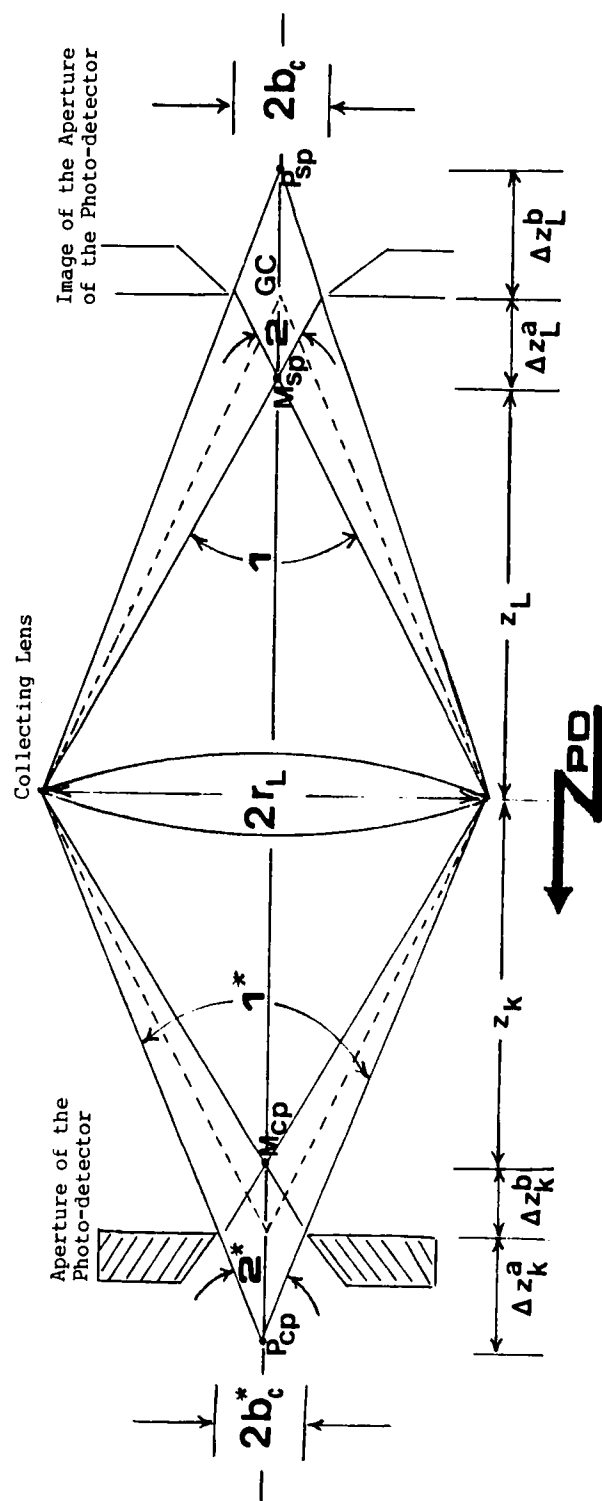
and

$$\frac{\Omega_1^*}{\Omega_2^*} = \frac{\pi r_L^2}{(z_K \mp \Delta z_K)^2} \cdot \frac{\Delta z_K^2}{\pi b_c^{*2}} \quad (12)$$

The relationship of these ratios is:

$$\frac{\Omega_1}{\Omega_2} = \frac{\Omega_1^*}{\Omega_2^*} \cdot \left\{ \frac{M_L}{M_T} \right\}^2 \cdot \left\{ \frac{z_K \mp \Delta z_K}{z_L \pm \Delta z_L} \right\}^2 \quad (13)$$

Expand $(z_L \pm \Delta z_L)^{-2}$ into a binomial series and retain only the first-order term in the expansion. This gives:



$1, 2$ = Major and Minor Marginal Sampling Solid Angles for the Image of the Aperture Space, Respectively.

$1^*, 2^*$ = Major and Minor Paraxial Sampling Solid Angles for the Aperture Space, Respectively.

Figure 10. Ray Trace Diagram of a Linear Lens-Aperture Arrangement for a Forward-Scattered Light Collecting Optical System.

Eq. (19) shows that the ratio of the minor sampling solid angles is equal to the ratio of the major sampling solid angles. This indicates that the scattered light collecting performance of such a system can be analyzed effectively from the image of the aperture space. This allows all calculations to be performed from a common origin located at the geometrical center of the sample volume. The quantitative analysis is made easier because it eliminates the need to perform lens transformations.

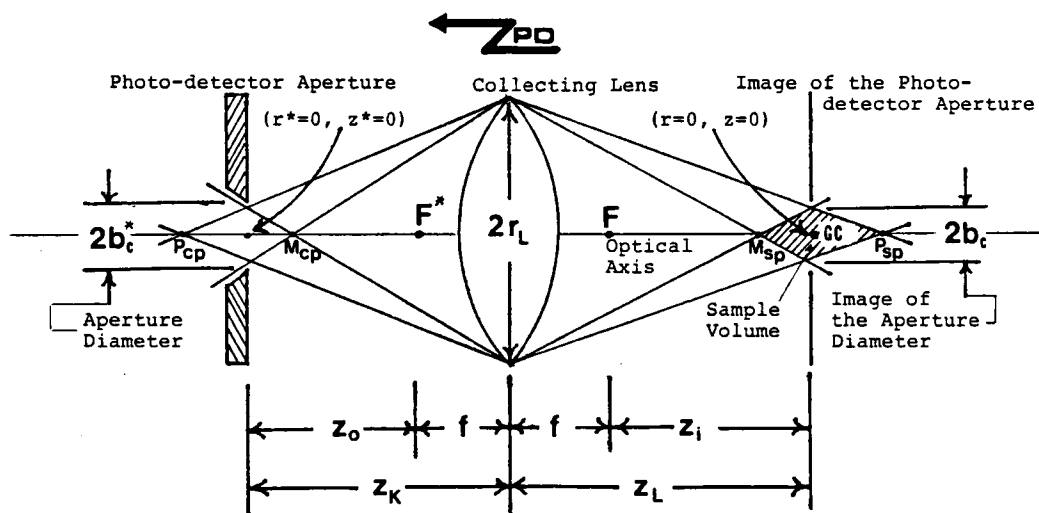
CHAPTER V

CIRCULAR LENS AND CIRCULAR APERTURE

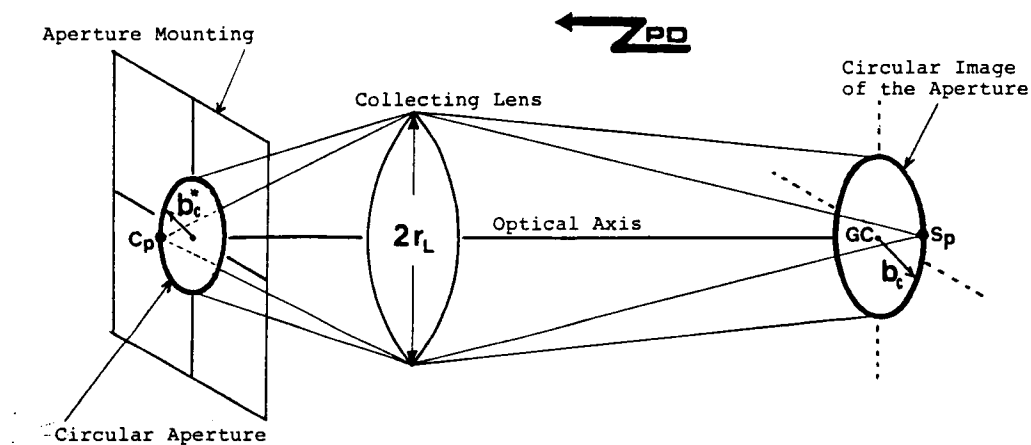
This chapter outlines the method used by Brayton [18] in the analysis of a circular lens and a circular aperture for an optical system used to collect radiation from a point-source of light. Brayton's results are presented in a more generalized form and used as a basis for comparison with the author's analysis of a circular lens and a rectangular aperture. Brayton's analysis can be stated in terms of the solid angle ratio.

Consider the ray trace diagram of a circular lens and a circular aperture arrangement for a forward-scattered light collecting optical system (see Figure 11). The collecting lens and photo-detector aperture are collinearly positioned on the optical axis in a forward location from the probe volume. The results of ray tracing and analytical geometry give information about the effective solid collection angle subtended by a point-source scatterer in the image of the aperture space.

A functional relationship is obtained between the effective solid collection angle and the spatial position of the point-source scatterer by introducing a collection criterion. This collection criterion is used to distinguish



F, F^* = Focal Points of the Collecting Lens in the Aperture Space and the Image of the Aperture Space, Respectively.



S_p = Scattering Point in the Image of the Aperture Space.
 C_p = Collection Point in the Aperture Space for the Optical Transfer of the Scattered Light from the Image of the Aperture Space.

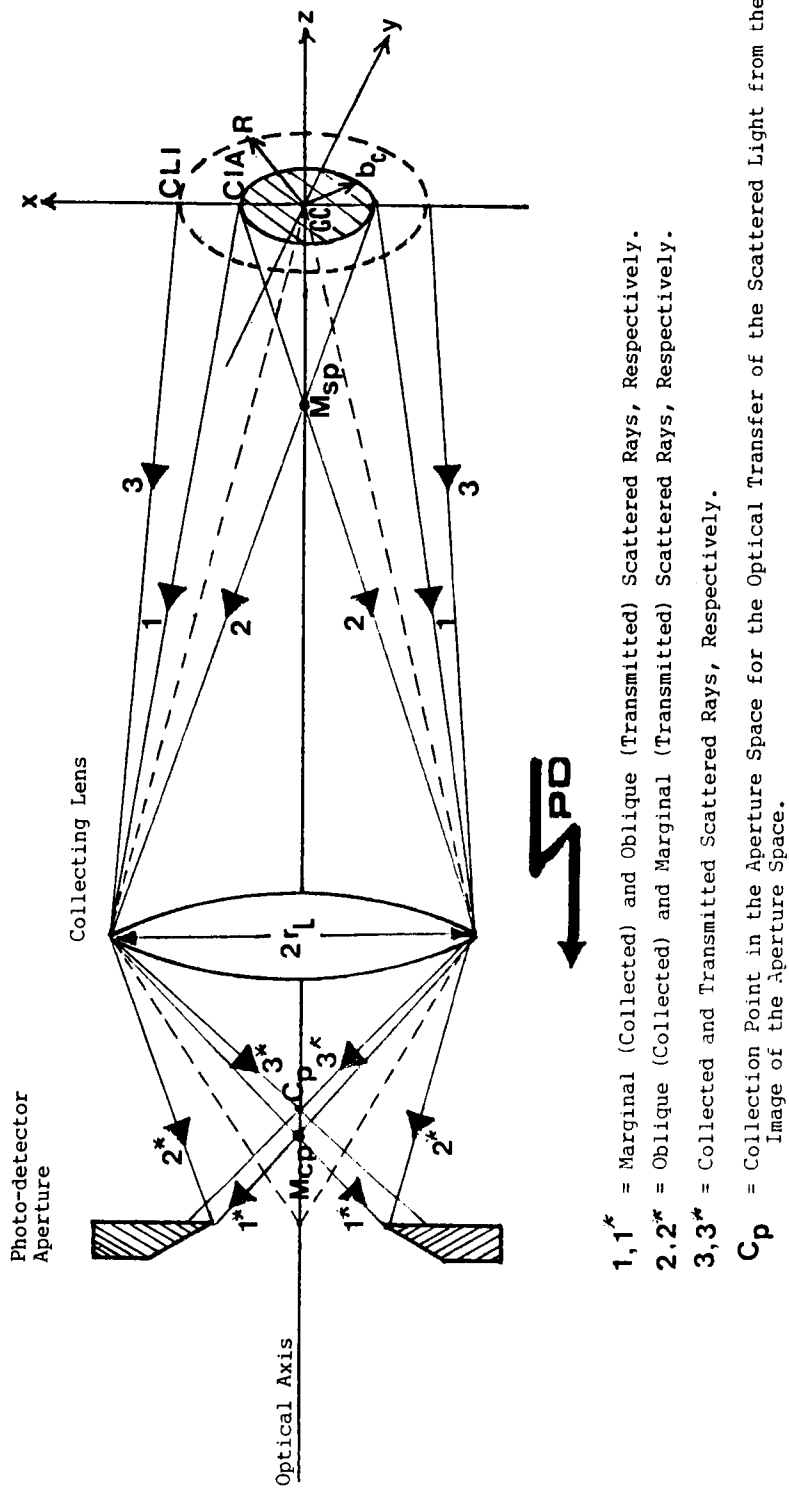
Figure 11. Ray Trace Diagram of the Circular Lens and the Circular Aperture for a Forward-Scattered Light Collecting Optical System.

between the rays that get detected and the ones that do not. The rays that get detected are the ones that pass through the exit pupil of the optical system. The rays that do not get detected are the ones that strike the mount of the photo-detector aperture. The collection criterion is a tool for distinguishing the rays in an optical system that are mechanically interrupted, and is a fundamental part in the development of the solid angle ratio.

The collection criterion defines two quantities which are to be used in the process of ray identification (see Figure 12). The first one is the circle of the image of the aperture, denoted as the CIA. The second one is the circle of the lens interception, denoted as the CLI. The CIA represents the largest fixed area of cross section of the sample volume. This cross-sectional region also represents the area of the entrance pupil. The CLI is a function of the position of the point-source scatterer in the image of the aperture space. The CLI represents the largest non-fixed area of cross section of the minor scattering solid angle at the plane of the image of the aperture. The common area of overlap of the CIA and the CLI is used in the development of the solid angle ratio.

For developing the solid angle ratio, assume:

1. A real image of the photo-detector aperture, i.e., $z_K > f$; where f = focal length of the collecting lens



- $1, 1^*$ = Marginal (Collected) and Oblique (Transmitted) Scattered Rays, Respectively.
 $2, 2^*$ = Oblique (Collected) and Marginal (Transmitted) Scattered Rays, Respectively.
 $3, 3^*$ = Collected and Transmitted Scattered Rays, Respectively.

C_p = Collection Point in the Aperture Space for the Optical Transfer of the Scattered Light from the Image of the Aperture Space.

Figure 12. Representation of the Circle of the Aperture (CIA) and the Circle of the Lens Interception (CLI).

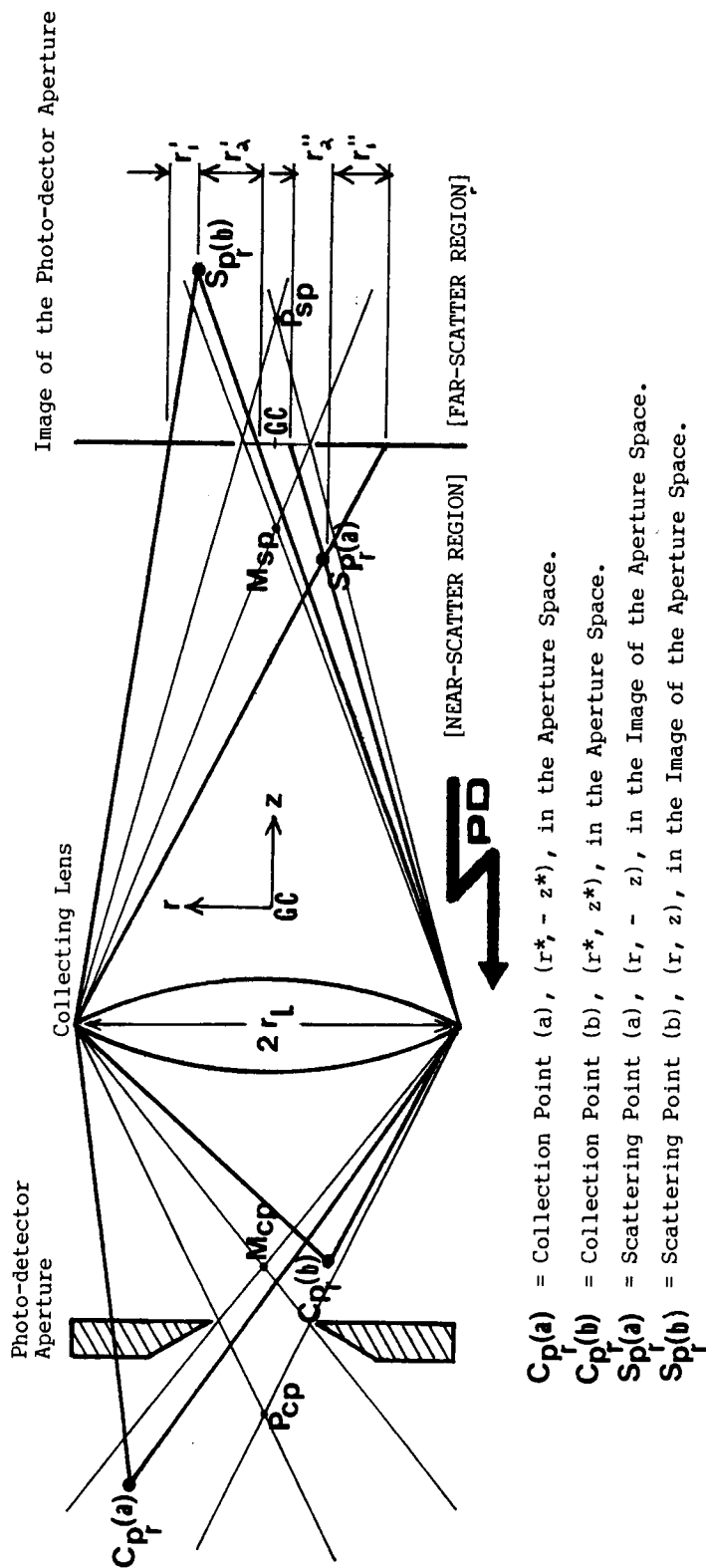
2. The thin lens approximation for the collecting lens
3. The small angle approximation, i.e., $z_L \gg r_L$
4. The radial coordinate needs only to be considered because the angular coordinate vanishes by way of the rotational symmetry of the system.

Also, introduce the effective solid collection angle given by:

$$\Omega_{\text{eff}} = \frac{A_{\text{eff}}}{z^2} \quad (20)$$

In order to find out if there exist any differences in the values of the effective solid collection angles, for points on either side of the image of the aperture plane, a set of two calculations is worked out.

Maximum and minimum expressions are introduced for the radial coordinate. These expressions are used to specify the location of the extreme boundaries of the CLI relative to the geometrical center of the sample volume. These boundary values are necessary to completely position the CIA-geometry relative to the CLI-geometry. The maximum and minimum radial distances to the farthest and nearest points of the CLI are given, respectively, as (see Figures 13 and 14) [18]:



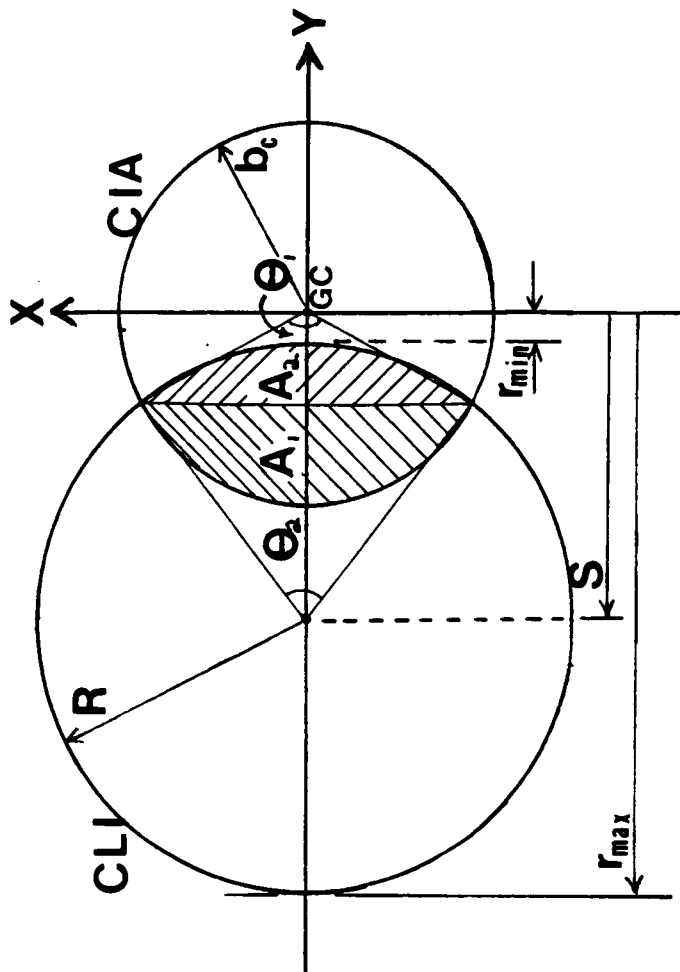
$C_p(a)$ = Collection Point (a), $(r^*, -z^*)$, in the Aperture Space.

$C_p(b)$ = Collection Point (b), (r^*, z^*) , in the Aperture Space.

$S_p(a)$ = Scattering Point (a), $(r, -z)$, in the Image of the Aperture Space.

$S_p(b)$ = Scattering Point (b), (r, z) , in the Image of the Aperture Space.

Figure 13. Representation of a General Ray Trace Analysis of an Arbitrary Scatter Point in the Near-Scatter Region; Denoted as the Point (a), $(r, -z)$, and of an Arbitrary Scatter Point in the Far-Scatter Region; Denoted as the Point (b), (r, z) .



θ_1, θ_2 = Intersection Angles of the CIA and CLI as viewed from the CIA and CLI, Respectively.

A_1, A_2 = Areas of the Circular Segments as Viewed from the CIA and CLI, Respectively.

Figure 14. Representation of the Maximum and Minimum Radial Coordinates for the Distance between the Farthest and Nearest Points of the CLI and the Geometrical Center of the Sample Volume, Respectively.

$$r_{\max} = \begin{cases} r + r'_1 \\ r + r''_1 \end{cases} \quad (21)$$

$$= r + \frac{|z| (r_L - rz/|z|)}{z_L + z} \quad (22)$$

and,

$$r_{\min} = \begin{cases} r - r'_2 \\ r - r''_2 \end{cases} \quad (23)$$

$$= r - \frac{|z| (r_L + rz/|z|)}{z_L + z} \quad (24)$$

The results for this set of calculations show negligible differences in the values of the effective solid collection angles for either the far-scatter location (r, z) or the near-scatter location $(r, -z)$.

The radial distance from the geometrical center of the sample volume to the center of the CLI is given by:

$$S = \frac{r_{\max} + r_{\min}}{2} \quad (25)$$

$$= \frac{rz_L}{z_L + z} \quad (26)$$

The radius of the CLI is given by:

$$R = r_{\max} - S \quad (27)$$

$$R = \frac{r_L |z|}{z_L + z} \quad (28)$$

The image of the aperture space is made up of four distinctive scattered light collecting regions, each with its own form of analytical solution for the effective solid collection angle (see Figure 15).

Region one is mathematically defined by the inequality:

$$S + R < b_c \quad (29)$$

This region is characterized by the total projection of the CLI into the CIA. All of the scattered light that is collected by the lens gets transmitted through the circular aperture of the photo-detector without any blockage by the photo-detector aperture mounting. This region is also identified with the sample volume of the optical system. The effective solid collection angle for this region is given by:

$$\Omega_{\text{eff}}^{(1)} = \frac{r_L^2}{(z_L + z)^2} \quad (30)$$

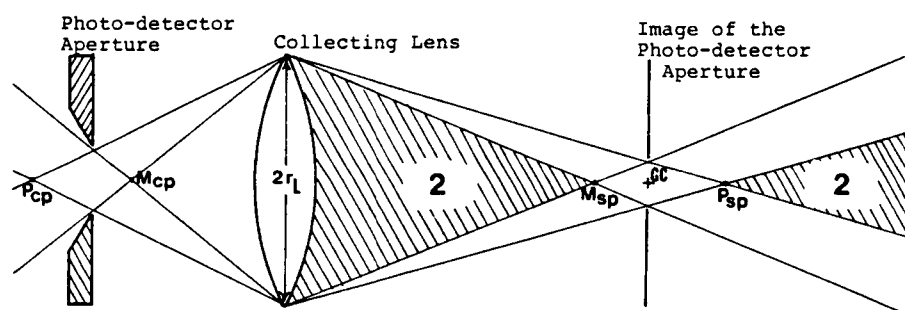
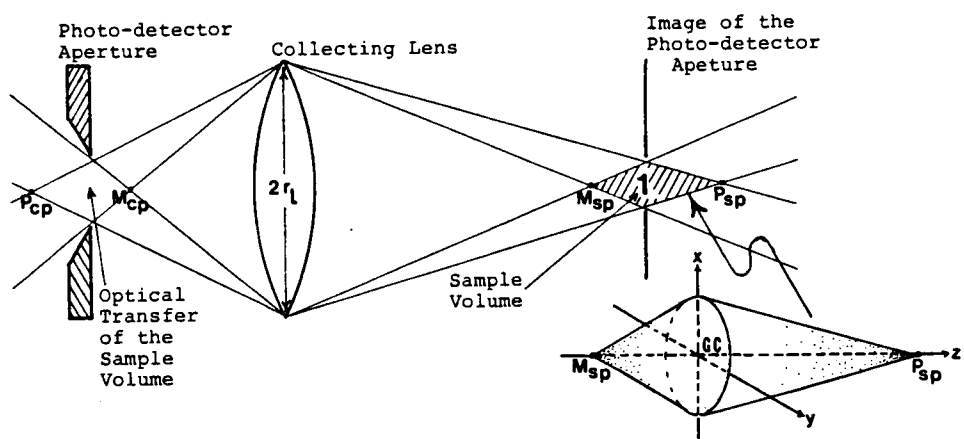


Figure 15. Representation of the Four Distinctive Scattered Light Collecting Regions for the Case of the Circular Lens and the Circular Photo-detector Aperture.

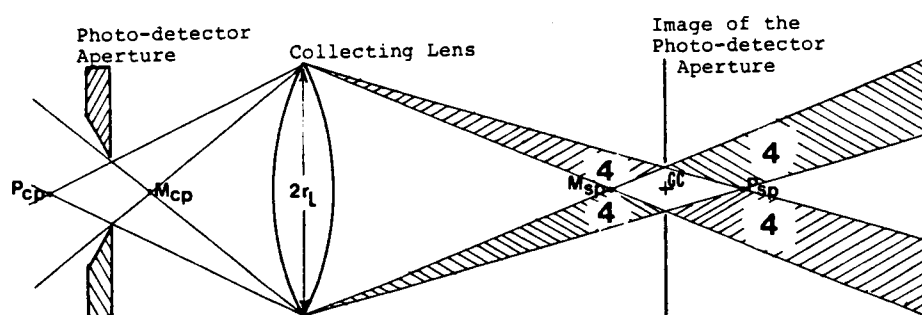
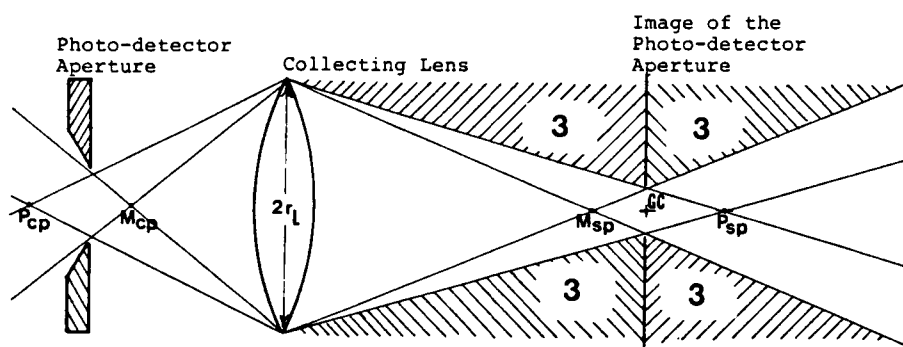


Figure 15. (Continued)

Region two is mathematically defined by the inequality:

$$S + b_c < R \quad (31)$$

This region is characterized by the complete covering over of the CIA by the CLI. Not all of the scattered light that is collected by the lens gets transmitted through the aperture of the photo-detector. Some of the light gets blocked by the photo-detector aperture mounting. The effective solid collection angle for this region is given by:

$$\Omega_{\text{eff}}^{(2)} = \frac{\pi b_c^2}{z^2} \quad (32)$$

Region three is mathematically defined by the inequality:

$$S > R + b_c \quad (33)$$

This region is characterized by the total projection of the CLI outside the fixed area of the CIA. None of the scattered light that is collected by the lens gets transmitted through the photo-detector aperture. All of the light gets mechanically interrupted by the photo-detector aperture mounting. The effective solid collection angle for this region is given by:

$$\Omega_{\text{eff}}^{(3)} = 0 \quad (34)$$

Region four is mathematically defined by the inequality:

$$|b_c - R| < S < b_c + R \quad (35)$$

This region is characterized by the partial projection of the CLI onto the CIA. The projection of the CLI covers only the outer circumferential part of the CIA. Not all of the scattered light that is collected by the lens gets transmitted through the aperture of the photo-detector. Some of the light gets blocked by the photo-detector aperture mounting. The effective solid collection angle for this region is given by [18]:

$$\begin{aligned} \Omega_{\text{eff}}^{(4)} = & \frac{b_c^2}{z^2} \cos^{-1} \left\{ \frac{b_c^2 (z_L + z)^2 - z^2 r_L^2 + r^2 z_L^2}{2 r z_L b_c (z_L + z)} \right\} \\ & + \frac{r_L^2}{(z_L + z)^2} \cos^{-1} \left\{ \frac{z^2 r_L^2 - b_c^2 (z_L + z)^2 + r^2 z_L^2}{2 r z_L |z| r_L} \right\} \\ & - \frac{r_L r z_L}{|z| (z_L + z)^2} \left[1 - \left\{ \frac{z^2 r_L^2 - b_c^2 (z_L + z)^2 + r^2 z_L^2}{2 r z_L |z| r_L} \right\}^2 \right]^{1/2} \end{aligned} \quad (36)$$

The solid angle ratios are formed from the normalization of the effective solid collection angles relative to the minor scattering solid angles. The reason for normalizing relative to the minor scattering solid angle is that the minor scattering solid angle represents the maximum amount of light that can be detected by the collection optics of the system. Geometrically, the optical system performs at its highest level when the effective solid collection angle is equal to the minor scattering solid angle. This optimal situation is achieved for the case of region one where it defines the sample volume.

The following parameters are useful for the optical system's characterization:

1. The effective focal number of the collecting lens
2. The effective aperture number of the image of the photo-detector aperture

The effective focal number of the collecting lens represents the average amount of light collected by the lens when no vignetting is present. The effective focal number of the collecting lens is equal to the distance between the collecting lens and the geometrical center of the sample volume divided by the diameter of the collecting lens, and can be written as:

$$(ef/\#)_L = \frac{z_L}{2r_L} \quad (37)$$

The effective aperture number of the image of the photo-detector aperture represents the amount of light received by the entrance pupil of the system when vignetting is present. The effective aperture number of the image of the photo-detector aperture is equal to the distance between the point-source scatterer and the geometrical center of the sample volume divided by the diameter of the entrance pupil, and can be written as:

$$(ef/\#)_S = \frac{z}{2b_c} \quad (38)$$

The image of the photo-detector aperture is characterized in terms of an effective aperture number, instead of an effective focal number, because the effective aperture number is more generally applicable to different geometries of the image of the photo-detector aperture. The effective focal number is related to the collecting lens and the amount of energy that is gathered within a given solid angle or light-collection cone. The effective aperture number is related to the entrance pupil of the optical system and the angular dimension that is most representative of the light sampling performance of the image of the photo-detector aperture.

Next, consider determining the maximum relative error introduced in the solid collection angles by the small angle

approximation. The assumption $r_L \ll z_L$ causes the values of the effective solid collection angles to be larger than the true solid collection angles. This error can become significant for a point-source scatterer located in region four because of its large inclination angle with respect to the z-axis. The relative error for the effective solid collection angle can be defined as:

$$\Delta\Omega_{\text{rel}} = \frac{\Omega_{\text{eff}} - \Omega_{\text{true}}}{\Omega_{\text{true}}} \quad (39)$$

In order to determine the maximum relative error for the effective solid collection angle, it is necessary to minimize the true solid collection angle. The minimum value of the true solid collection angle can be determined by the maximum distance of the point-source scatterer from an element of area of the geometrical intersection of the CIA and the CLI, and the maximum angle of inclination of this element of area with the optical axis. The results of these calculations are:

$$(\Delta\Omega_{\text{rel}})_{\text{max}} = \left[1 + \left(\frac{r_L + b_c}{z_L} \right)^2 \right]^{3/2} - 1 \quad (40)$$

Using $r_L \gg b_c$ and Eq. (37) then,

$$(\Delta\Omega_{\text{rel}})_{\text{max}} = \left[\frac{4(ef/\#)_L^2 + 1}{4(ef/\#)_L^2} \right]^{3/2} - 1 \quad (41)$$

The maximum relative error for the effective solid collection angle is used to explain certain discrepancies that occur in the contour plots of the normalized effective solid collection angles (i.e., the solid angle ratios) which are presented in the experimental results.

The results for the four distinctive scattered light collecting regions for the case of the circular lens and the circular aperture are summarized in Table 1. The approximate solid angle ratios are compared with the exact mathematical description for each region. The results are written in terms of the assumption that $z \ll z_L$. They will be used to plot normalized solid collection angle contours as a function of the point-source scattering position in the image of the aperture space.

Table 1. "Tabulation of the Approximate Solid Angle Ratio."

Region Number	Exact Mathematical Description of Region	Approximate Solid Angle Ratio Within the Given Region
1	$\frac{rz_L + r_L z }{z_L + z} < b_c$	1
2	$\frac{rz_L}{z_L + z} + b_c < \frac{r_L z }{z_L + z}$	$\left[\frac{(ef/\#)_L}{(ef/\#)_S} \right]^2$
3	$b_c + \frac{r_L z }{z_L + z} < \frac{rz_L}{z_L + z}$	0
4	$\left \frac{r_L z }{z_L + z} - b_c \right < \frac{rz_L}{z_L + z}$	$\left[\frac{(ef/\#)_L}{(ef/\#)_S} \right]^2 \cdot \frac{1}{\pi} \cos^{-1} \left\{ \frac{b_c}{2r} \left(1 - \left[\frac{(ef/\#)_S}{(ef/\#)_L} \right]^2 \right) + \frac{r}{2b_c} \right\} +$ $\frac{1}{\pi} \cos^{-1} \left\{ \frac{b_c}{2r} \left[\frac{(ef/\#)_S^2 - (ef/\#)_L^2}{(ef/\#)_S (ef/\#)_L} + \frac{r}{2b_c} \left[\frac{(ef/\#)_L}{(ef/\#)_S} \right] \right\} -$ $\frac{1}{\pi} \cdot \frac{r}{b_c} \left[\frac{(ef/\#)_L}{(ef/\#)_S} \right] \cdot \left[1 - \left\{ \frac{b_c}{2r} \left[\frac{(ef/\#)_S^2 - (ef/\#)_L^2}{(ef/\#)_S (ef/\#)_L} + \frac{r}{2b_c} \left[\frac{(ef/\#)_L}{(ef/\#)_S} \right] \right\}^2 \right]$

CHAPTER VI

CIRCULAR LENS AND RECTANGULAR APERTURE

This chapter outlines the treatment of the circular lens and the rectangular photo-detector aperture. It shows that the following approach makes use of the solid angle ratio in a different manner than in Brayton's treatment. The ray trace analysis is performed by using the convolution theorem which, in contrast to the geometrical symmetry of the problem in Chapter V, is necessary because of the incompatible geometries of the optical elements and their lack of rotational symmetry about the optical axis.

Consider the ray trace diagram of a circular lens and a rectangular aperture arrangement for a forward-scattered light collecting optical system (see Figure 16). As before, the collecting lens and photo-detector aperture are collinearly positioned on the optical axis in a forward location from the probe volume. As a result of the dissimilar geometries, a general type of geometrical optics ray trace analysis cannot be performed by way of the techniques of analytical geometry.

In Brayton's treatment, a point-by-point, static superpositioning of the CLI onto the CIA determined the effective solid collection angles or solid angle ratios.

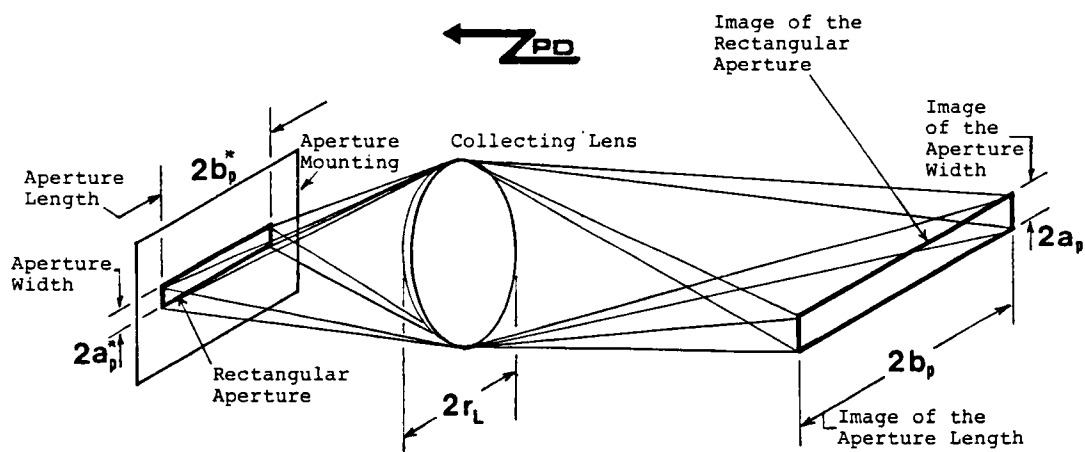
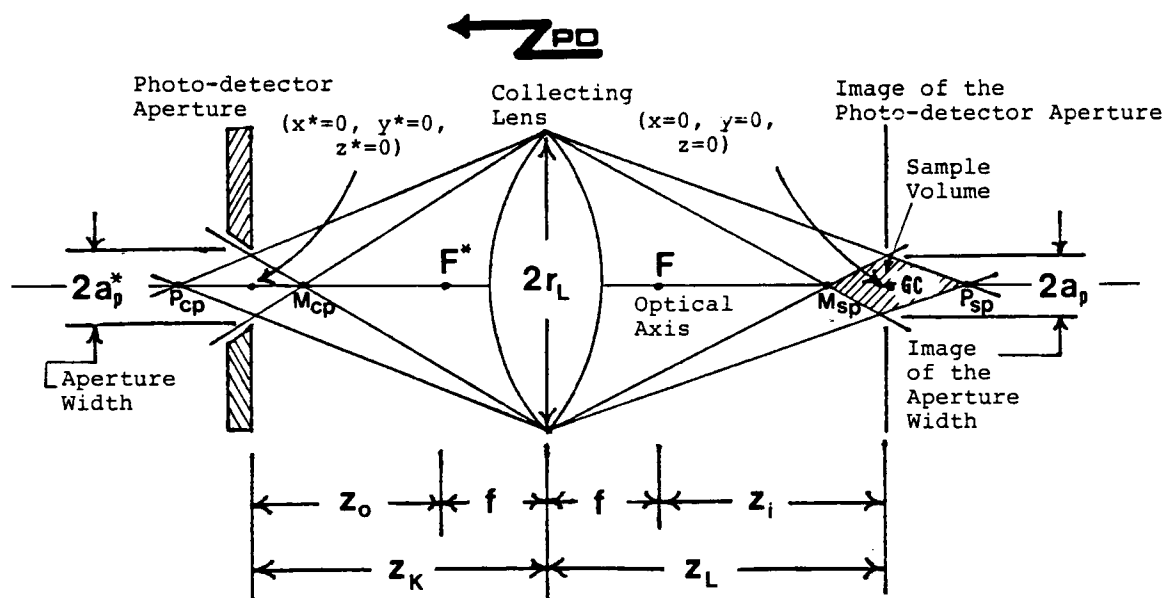


Figure 16. Ray Trace Diagram of the Circular Lens and the Rectangular Aperture Arrangement for a Forward-Scattered Light Collecting Optical System.

Only four general types of superpositionings were necessary to specify completely the scattered light collecting properties of the optical system. In the present treatment, no such general types of superpositionings can be specified. The aperture of the photo-detector is no longer made up of a smooth curve with a constant radius of curvature, but consists of straight edges forming sharp corners. It becomes impossible to calculate general analytical expressions for the common area of overlap of a circle with a rectangle, where the radius of the circle can take on any value between zero and infinity.

Because optical superpositioning of the sample volume with the probe volume is the major concept in this study, a general type of geometrical optics ray trace analysis can be performed by the convolution of two functions. An expression for the statistical light collecting performance of the optical system is obtained from this convolving process. This expression represents a functional extension of the solid angle ratio into a more general form of a normalized collection optics solid angle. This expression will be defined as the geometrical optics solid angle transfer function.

The convolution function occurs in the response of an optical collection system when the point spread function of the image of the point-source scatterer is superimposed

onto the aperture function of the photo-detector aperture. The out-of-focus image of a point-source scatterer detected by a scattered light collecting optical system appears to show that the rays originate from an extended source in the plane of the image of the photo-detector aperture. If the cross-sectional scattering area of this extended source is the blur spot function, and the cross-sectional sampling area of the image of the photo-detector aperture is the sampling function, the amount of scattered light that gets detected is equal to the combination of these two generating functions. The convolution of the source blur spot function, $h(t)$, with the collection optics sampling function, $x(t)$, can be written as:

$$C(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \quad (42)$$

The collection criterion remains essentially the same as it was defined in Chapter V. The CIA is replaced by the rectangle of the image of the aperture, denoted as the RIA, while the CLI remains unchanged (see Figure 17). In this treatment, the RIA and CLI are identified as the sampling and blur spot function, respectively.

The geometrical optics ray traces are still used to identify certain distinctive scattered light collecting regions. However, the projection of the CLI onto the fixed

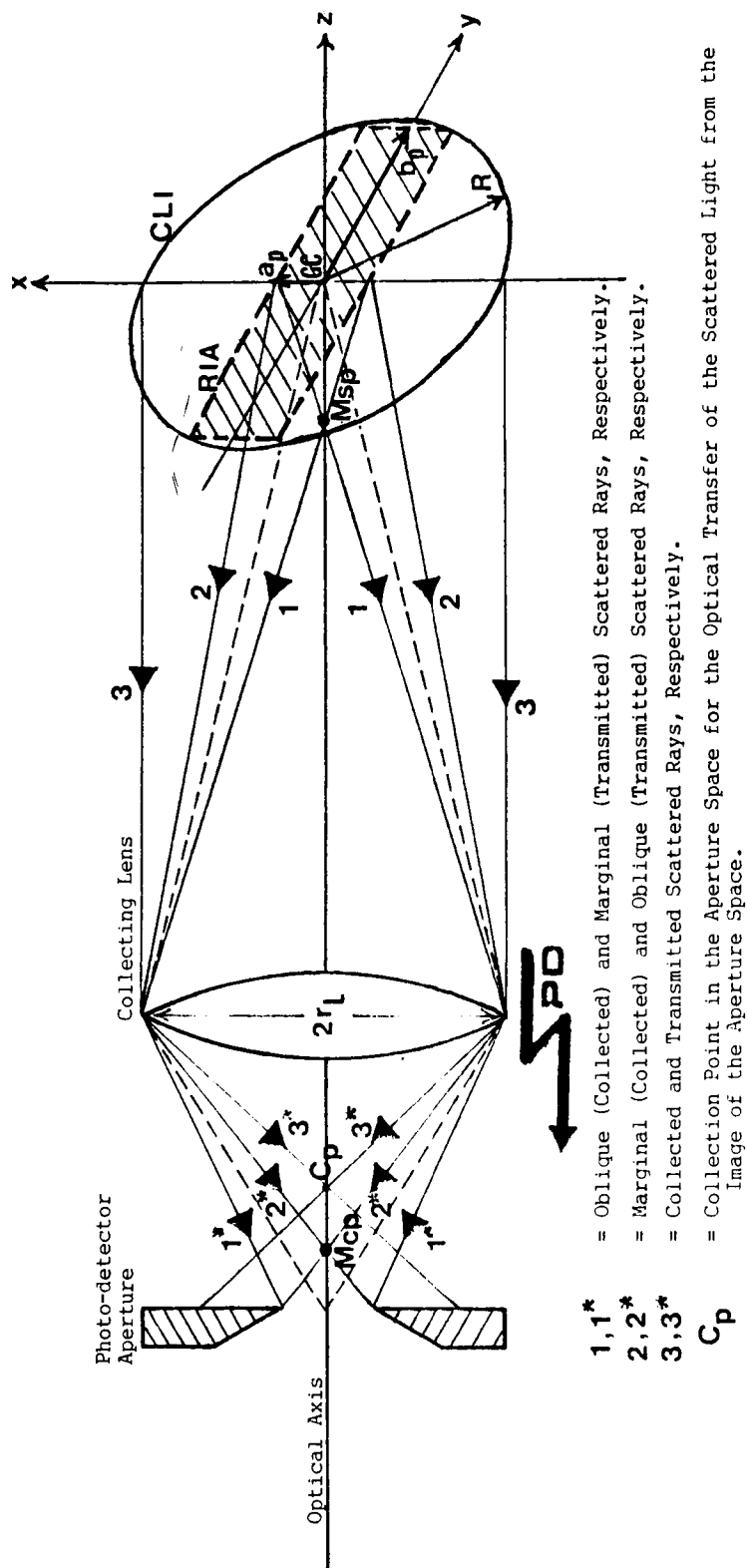


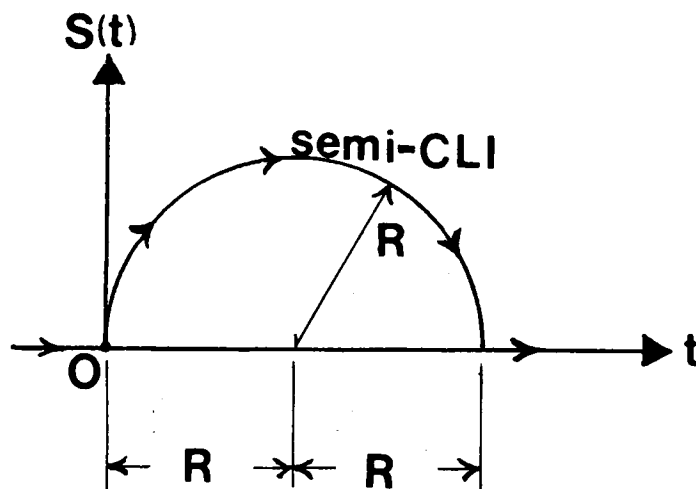
Figure 17. Representation of the Rectangle of the Image of the Aperture (RIA) and the Circle of the Lens Interception (CLI).

area of the RIA is treated differently in determining a quantity analogous to the solid angle ratio. A dynamic superpositioning of the CLI onto the RIA determines the effective solid collection angles or solid angle transfer functions. The cross-sectional area of the minor scattering solid angle, formed at the image of the aperture plane, is represented by the blur spot function. The cross-sectional area of the minor sampling solid angle, formed at the image of the aperture plane, is represented by the sampling function. The blur spot function is made to scan transversely across the sampling function.

The convolution is an integration process in which the integrand is the product of two generating functions. In this treatment of the circular lens and the rectangular photo-detector aperture, the sampling function is represented by a rectangular function and is denoted as the semi-RIA, while the blur spot function is represented by a semi-circular function and is denoted as the semi-CLI. Hence, the calculations proceed as follows (see Figure 18):

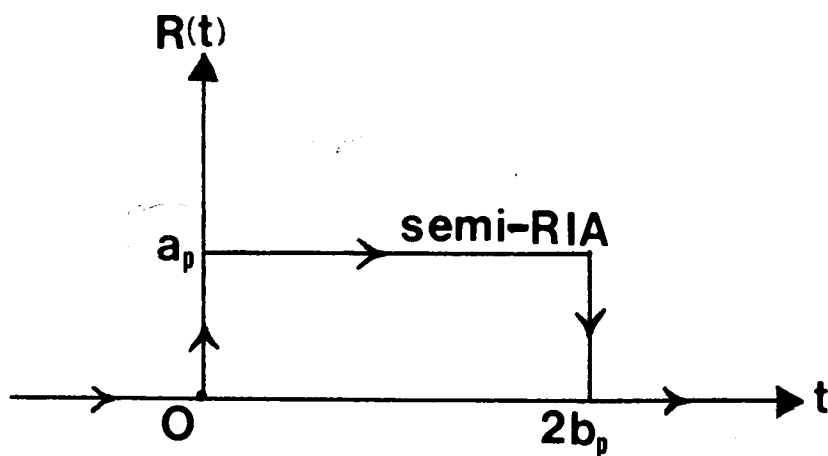
semi-RIA: rectangular function

$$R(t) = A[\mu(t) - \mu(t-2b_p)] \quad (43)$$



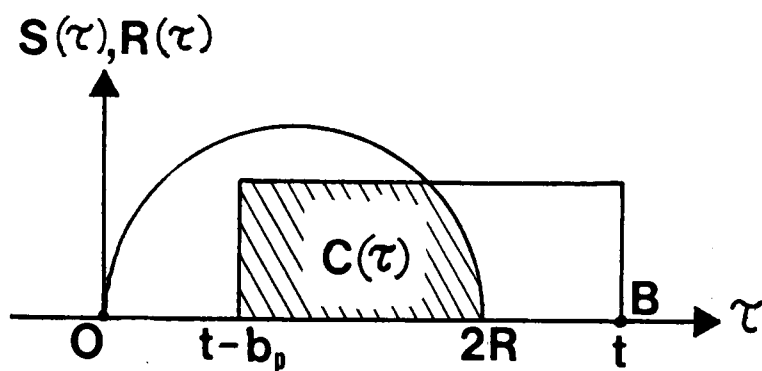
- (a) Representation of the Semi-circular Function as a Model for the Source Blur Spot Function, Denoted by the Semi-CLI.

Figure 18. Geometrical Optics Ray Trace Analysis of the Optical Superpositioning of the Source Blur Spot Function and the Collection Optics Sampling Function, Using the Convolution Theorem.



- (b) Representation of the Rectangular Function as a Model for the Collection Optics Sampling Function, Denoted by the Semi-RIA.

Figure 18. (Continued).



- (c) Representation of the Convolution of the Semi-circular and Rectangular Functions as a Model for Determining the Common Area of Overlap between the Semi-CLI and the Semi-RIA.

Figure 18. (Continued).

semi-CLI: semi-circular function

$$S(t) = \sqrt{R^2 - (t-R)^2} [\mu(t) - \mu(t-2R)] \quad (44)$$

Note that the integration control function plays a major role in the computational aspects of the solid angle transfer function. The integration control function acts as an indicator of the type of superpositioning that takes place between the semi-CLI and the semi-RIA for particular ranges along the scanning axis (t-axis). Once the type of superpositioning has been determined, this control function acts as an activator of the type of integration that is necessary in computing the common area of overlap between the semi-CLI and the semi-RIA. There are only two types of evaluation of the convolution: (1) integration along a circular arc and (2) integration along a straight line that is parallel to the scanning axis. The integration control function is related to the height of the rectangular function and can be expressed as:

$$A = A(\text{semi-RIA}) \quad (45)$$

For the purpose of the convolution calculations, the width (or height) of the semi-RIA will be made to vary between the values of zero and one, while the radius of the

semi-CLI will have the fixed value of unity. Of course, in the actual optical system, it is the blur spot function, i.e., CLI, which varies while the sampling function, i.e., RIA, remains constant in its value and fixed about the origin. The true value for the radius of the semi-CLI is given by Eq. (44), while the true value for the width (or height) of the semi-RIA is equal to a_p .

In the convolution integral one of the two generating functions is folded back and then shifted. For many optical problems, the generating functions are symmetric and so it does not matter which one gets chosen as the shifting function. Here, the rectangular function gets folded back and moved across the semi-circular function.

The origin of the scanning coordinate is not coincident with the (x,y,z) -origin that is located at the geometrical center of the sample volume. The origin of the scanning coordinate is positioned at the boundary point $(0, -b_p, 0)$. The scanning coordinate locates the leading edge of the semi-RIA (denoted by the point B in Figure 18). Because the position of the point-source scatterer in the image of the aperture space is taken relative to the geometrical center of the RIA (i.e., the sample volume), and since the position of the point-source is directly linked to the CLI, the scanning coordinate, t , can be expressed as:

$$t = \frac{r_L |z| + z_L |y|}{z_L + z} + b_p \quad (46)$$

The application of the convolution integral gives the following results:

$$C(t) = \int_{-\infty}^{\infty} R(t-\tau) S(\tau) d\tau \quad (47)$$

then:

$$A(\tau, R) = A \sqrt{R^2 - (\tau - R)^2} \quad (48)$$

and

$$\begin{aligned} C(t) = & \int_{-\infty}^{\infty} A(\tau, R) \mu(t-\tau) \mu(\tau) d\tau \\ & - \int_{-\infty}^{\infty} A(\tau, R) \mu(t-\tau) \mu(\tau-2R) d\tau \\ & - \int_{-\infty}^{\infty} A(\tau, R) \mu(t-2b_p-\tau) \mu(\tau) d\tau \\ & + \int_{-\infty}^{\infty} A(\tau, R) \mu(t-2b_p-\tau) \mu(\tau-2R) d\tau \end{aligned} \quad (49)$$

and the on-off condition function is defined as:

$$\mu(t) = \begin{cases} 1 & , \quad t \geq 0 \\ 0 & , \quad t < 0 \end{cases} \quad (50)$$

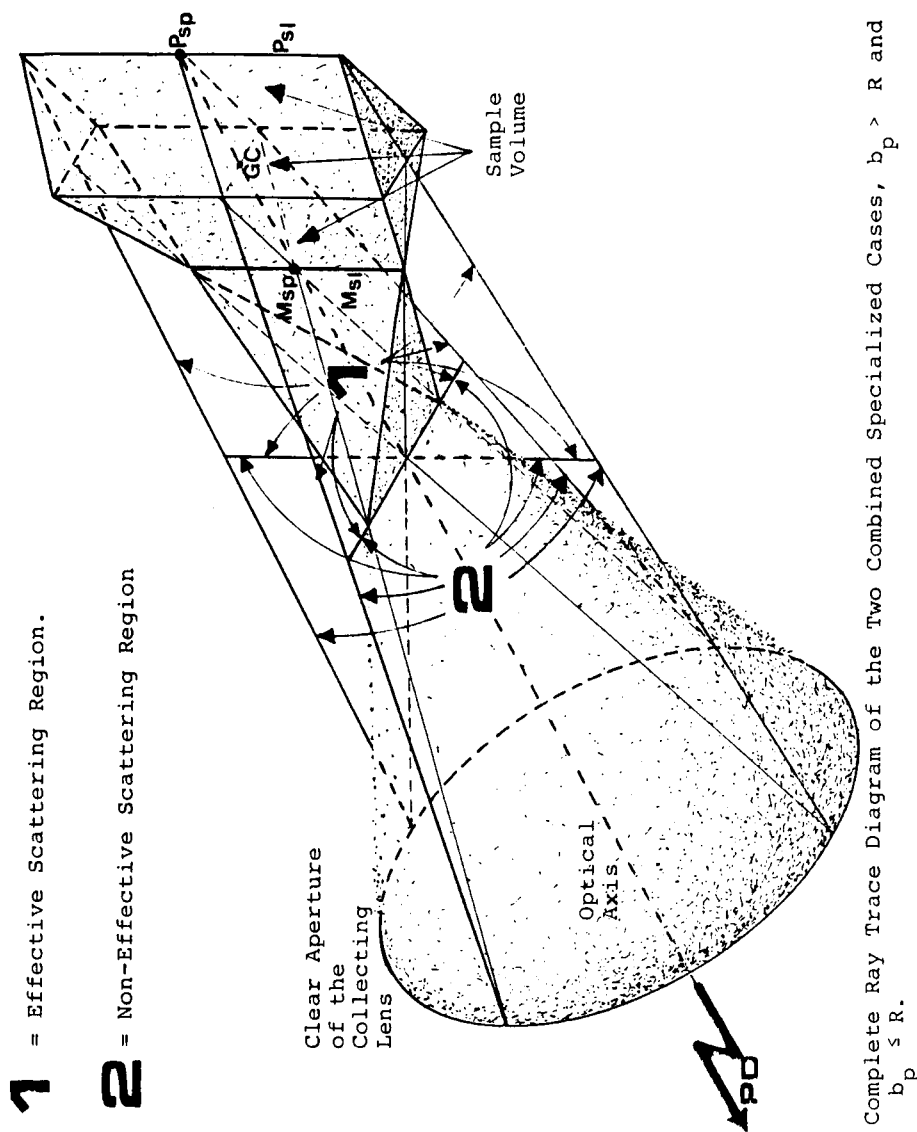
As a result of the definition of the on-off condition function, Eq. (50), it is possible to eliminate the infinite limits of integration:

$$\begin{aligned}
 C(t) = & \int_0^t A(\tau, R) d\tau \\
 & - \int_{2R}^t A(\tau, R) d\tau \\
 & + \int_{2R}^{t-2b_p} A(\tau, R) d\tau \\
 & - \int_0^{t-2b_p} A(\tau, R) d\tau
 \end{aligned} \tag{51}$$

The four integrals making up the convolution function, $C(t)$, contribute nonzero values only within specific ranges along the scanning axis (t -axis). Furthermore, this function breaks down into two specialized cases (see Figure 19):

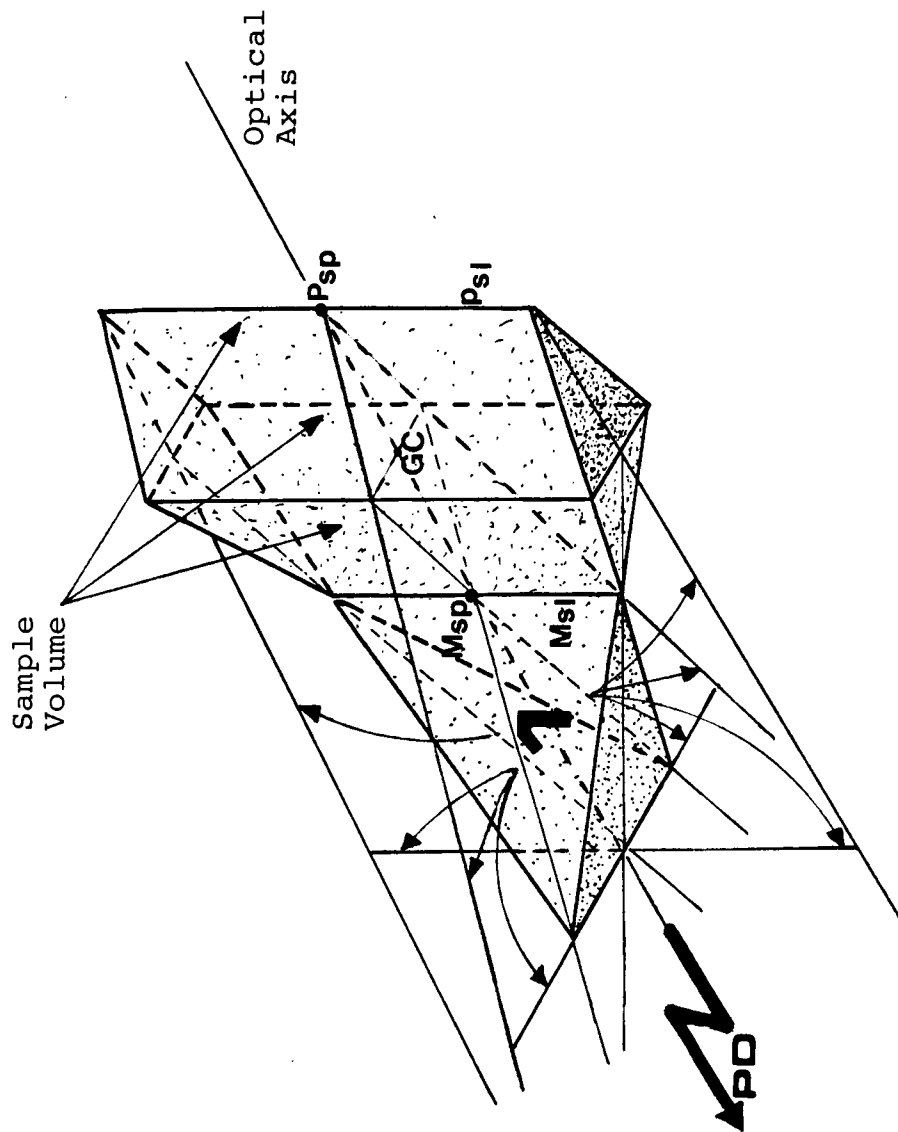
1. $b_p > R$
2. $b_p \leq R$

It is not necessary to treat both cases. Because the width of the image of the photo-detector aperture is taken to be very narrow as compared to its length (i.e., $a_p \ll b_p$): Case No. 1, $b_p > R$, is the only valuable



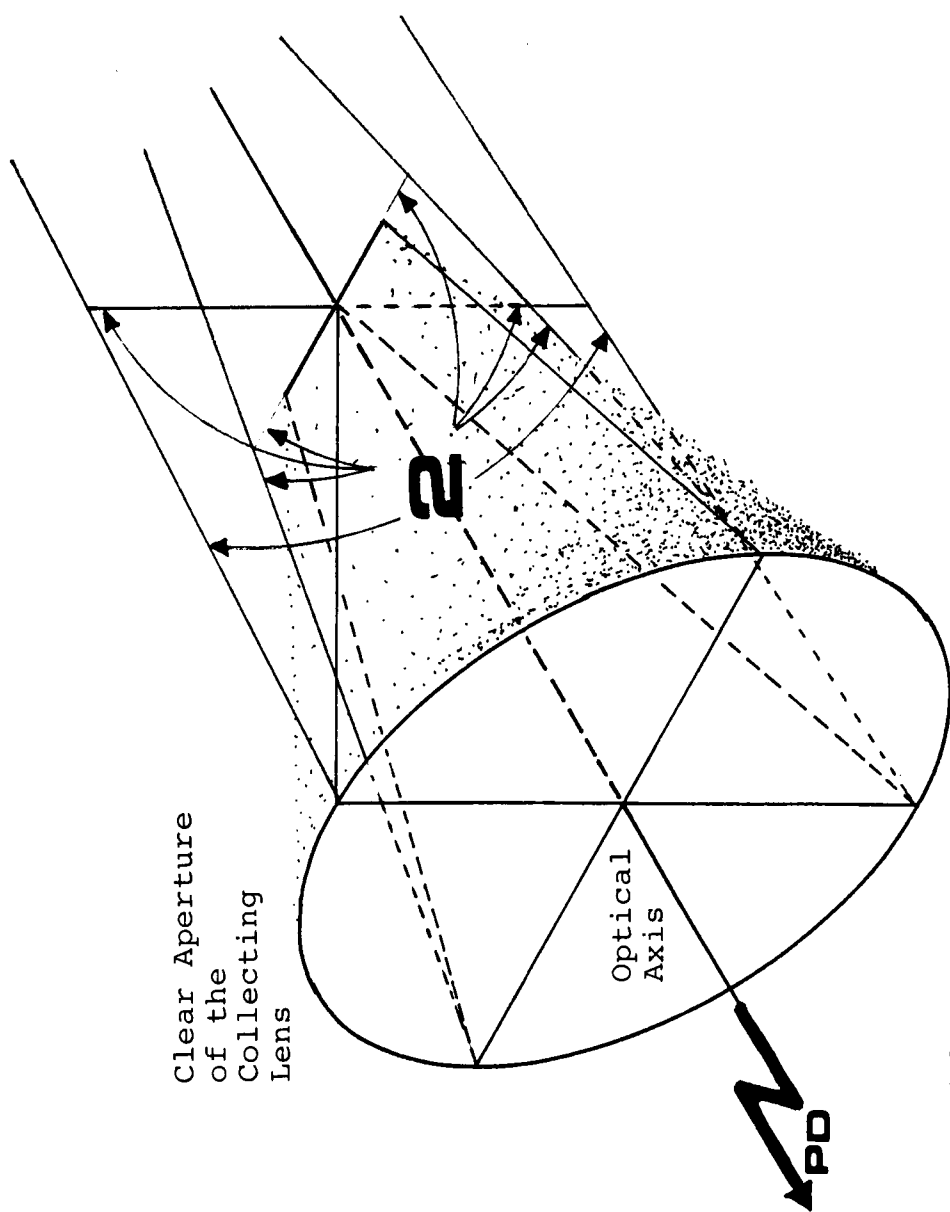
(a) Complete Ray Trace Diagram of the Two Combined Specialized Cases, $b_p > R$ and $b_p \leq R$.

Figure 19. Representation of the Two Specialized Cases for the Numerical Integration of the Convolution Integrals in the Treatment of the Circular Lens and the Rectangular Photo-detector Aperture.



(b) The Specialized Case of $b_p > R$.

Figure 19. (Continued).



(c) The Specialized Case of $b_p \leq R$.

Figure 19. (Continued).

computation of the solid angle transfer function. Case No. 2, $b_p \leq R$, is not considered because the computations yield values for the resulting transfer functions which are very small.

For Case No. 1, $b_p > R$, there results:

$$C_T = C_1(t) + C_2(t) + C_3(t) + C_4(t) + C_5(t) \quad (52a)$$

$$C_1(t) = 0 \quad t \leq 0 \quad (52b)$$

$$C_2(t) = \int_0^t A(\tau, R) d\tau \quad 0 < t \leq 2R \quad (52c)$$

$$C_3(t) = C_2(t) - \int_{2R}^t A(\tau, R) d\tau \quad 2R < t \leq 2b_p \quad (52d)$$

$$C_4(t) = C_3(t) - \int_0^{t-2b_p} A(\tau, R) d\tau \quad 2b_p < t \leq 2R+2b_p \quad (52e)$$

$$C_5(t) = 0 \quad 2R + 2b_p < t \quad (52f)$$

These integral expressions form the convolution transfer function, C_T . This function is the analogue of the common area of overlap of the CIA and the CLI, i.e., A_{eff} , described in Chapter V. The effective solid collection angle is given by:

$$\Omega_{eff} = \frac{C_T}{Z^2} \quad (53)$$

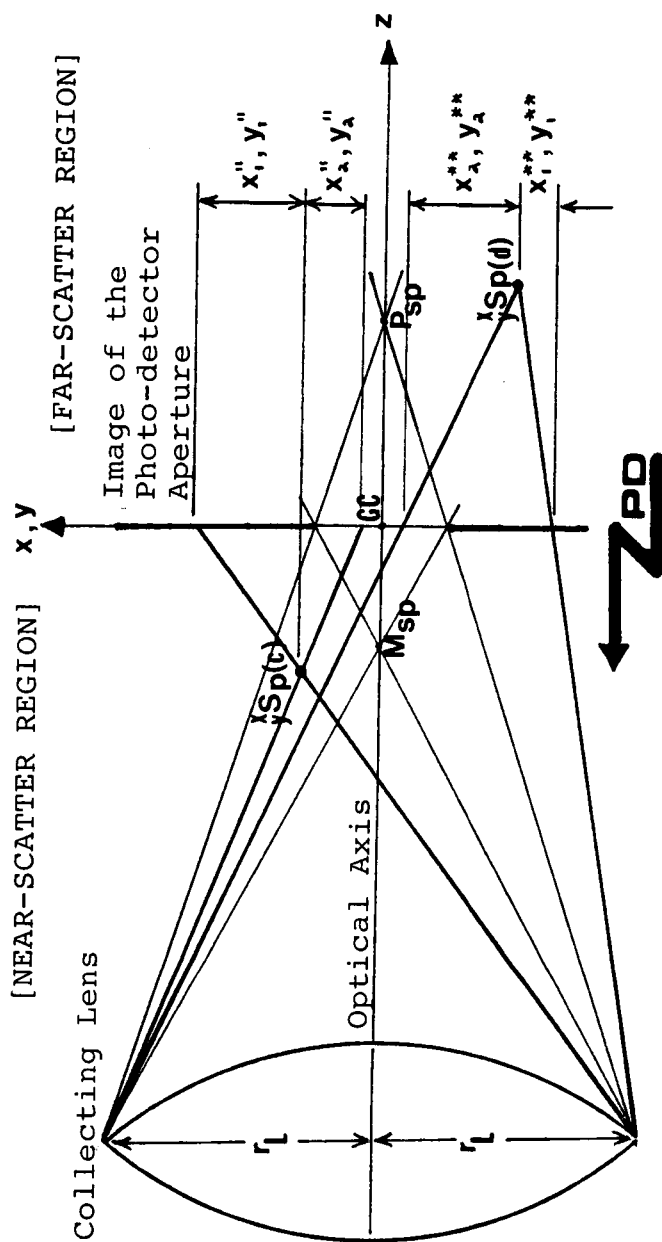
As before, for developing the solid angle transfer function, assume:

1. A real image of the photo-detector aperture,
i.e., $z_K > f$
2. The thin lens approximation for the collecting
lens
3. The small angle approximation, i.e., $z_L \gg r_L$

While in the treatment of Chapter V, the angular coordinate vanished by way of the rotational symmetry of the system, all three rectangular coordinates need to be considered in the present treatment because of the rectangular geometry relative to the optical axis.

In order to find out if there exist any differences in the values of the effective solid collection angles for points on either side of the image of the aperture plane, a set of eight calculations is worked out. This is necessary because of the lack of symmetry of the system.

Maximum and minimum expressions for the x-,y-coordinates locate the extreme boundaries of the CLI relative to the geometrical center of the sample volume. These boundary values completely position the CLI-geometry relative to the RIA-geometry. The maximum and minimum x-dimensions to the farthest and nearest points of the CLI for the four locations $(x,0,z)$, $(-x,0,z)$, $(x,0,-z)$, and $(-x,0,-z)$ are given, respectively, as (see Figure 20):



$x_{sp}(c)$ = Scattering Point (c), $(x, o, -z)$ or $(o, y, -z)$, in the Image of the Aperture Space.

$x_{sp}(d)$ = Scattering Point (d), $(-x, o, z)$ or $(o, -y, z)$, in the Image of the Aperture Space.

Figure 20. (Continued)

$$x_{\max} = \begin{cases} x + x_1' \\ -x + x_1^* \\ x + x_1'' \\ -x + x_1^{**} \end{cases} \quad (54)$$

$$= |x| + |z| \cdot \left[\frac{r_L - |x|z/|z|}{z_L + z} \right] \quad (55)$$

$$x_{\min} = \begin{cases} x - x_2' \\ -x - x_2^* \\ x - x_2'' \\ -x - x_2^{**} \end{cases} \quad (56)$$

$$= |x| - |z| \cdot \left[\frac{r_L + |x|z/|z|}{z_L + z} \right] \quad (57)$$

Similarly, the maximum and minimum y-dimensions to the farthest and nearest points of the CLI for the four locations $(0, y, z)$, $(0, -y, z)$, $(0, y, -z)$, and $(0, -y, -z)$ are given, respectively, as:

$$y_{\max} = \begin{cases} y + y_1' \\ -y + y_1'' \\ y + y_1^{**} \\ -y + y_1^* \end{cases} \quad (58)$$

$$= |y| + |z| \cdot \left[\frac{r_L - |y|z/|z|}{z_L + z} \right] \quad (59)$$

and,

$$y_{\min} = \begin{cases} y - y_2' \\ -y - y_2'' \\ y - y_2^{**} \\ -y - y_2^* \end{cases} \quad (60)$$

$$= |y| - |z| \cdot \left[\frac{r_L + |y|z/|z|}{z_L + z} \right] \quad (61)$$

The results for this set of calculations show negligible differences in the values of the effective solid collection angles for either the four far-scatter locations $(x, 0, z)$, $(-x, 0, z)$, $(0, y, z)$, and $(0, -y, z)$ or the four near-scatter locations $(x, 0, -z)$, $(-x, 0, -z)$, $(0, y, -z)$, and $(0, -y, -z)$.

The x -, y -distances from the center of the CLI to the geometrical center of the sample volume are given, respectively, as:

$$x = \frac{x_{\max} + x_{\min}}{2} \quad (62)$$

$$= \frac{|x|z_L}{z_L + z} \quad (63)$$

and,

$$y = \frac{y_{\max} + y_{\min}}{2} \quad (64)$$

$$y = \frac{|y|z_L}{z_L + z} \quad (65)$$

The radius of the CLI remains the same as defined by Eq. (28) of Chapter V.

Just as in Brayton's treatment, the image of the aperture space is made up of four distinctive scattered light collecting regions, each with its own numerical solution for the effective solid collection angle (see Figure 21).

Region No. 1 is characterized by the total projection of the CLI into the RIA. All of the scattered light that is collected by the lens gets transmitted without any interruption through the rectangular aperture of the photodetector. This region forms the sample volume of the optical system and is mathematically defined by the inequalities:

$$\left| \frac{|y|z_L}{z_L + z} \right| < b_p - \frac{r_L|z|}{z_L + z} \quad (66)$$

and

$$\left| \frac{|x|z_L}{z_L + z} \right| < a_p - \frac{r_L|z|}{z_L + z} \quad (67)$$

The effective solid collection angle for this region is given by:

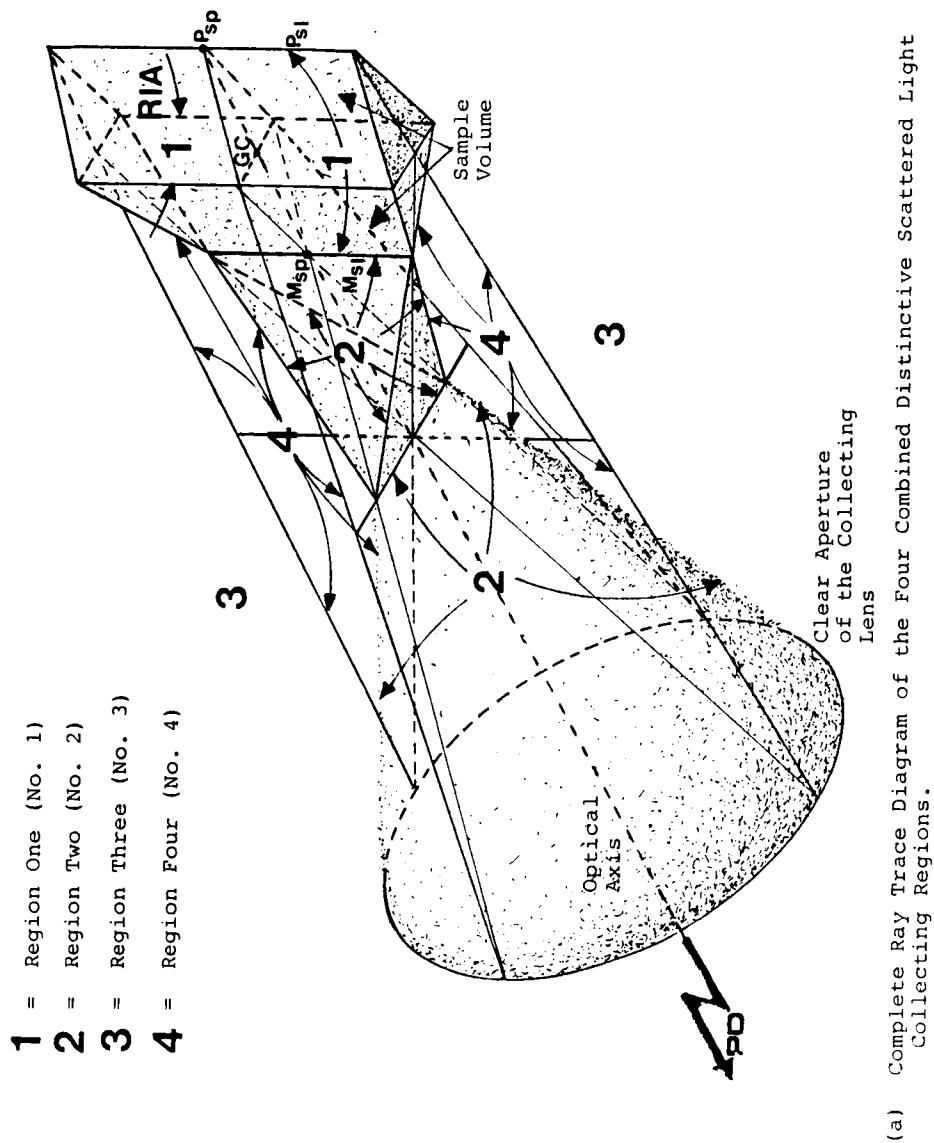
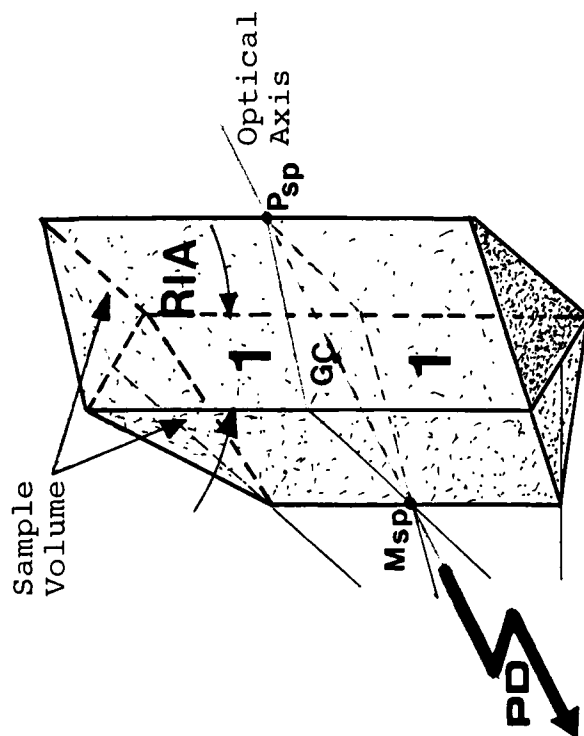
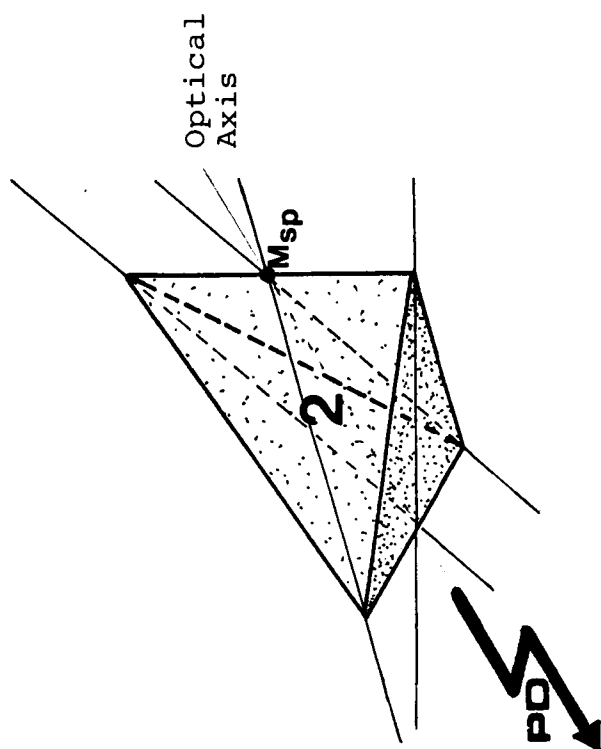


Figure 21. Representation of the Four Distinctive Scattered Light Collecting Regions for the Case of the Circular Lens and the Rectangular Photo-detector Aperture.



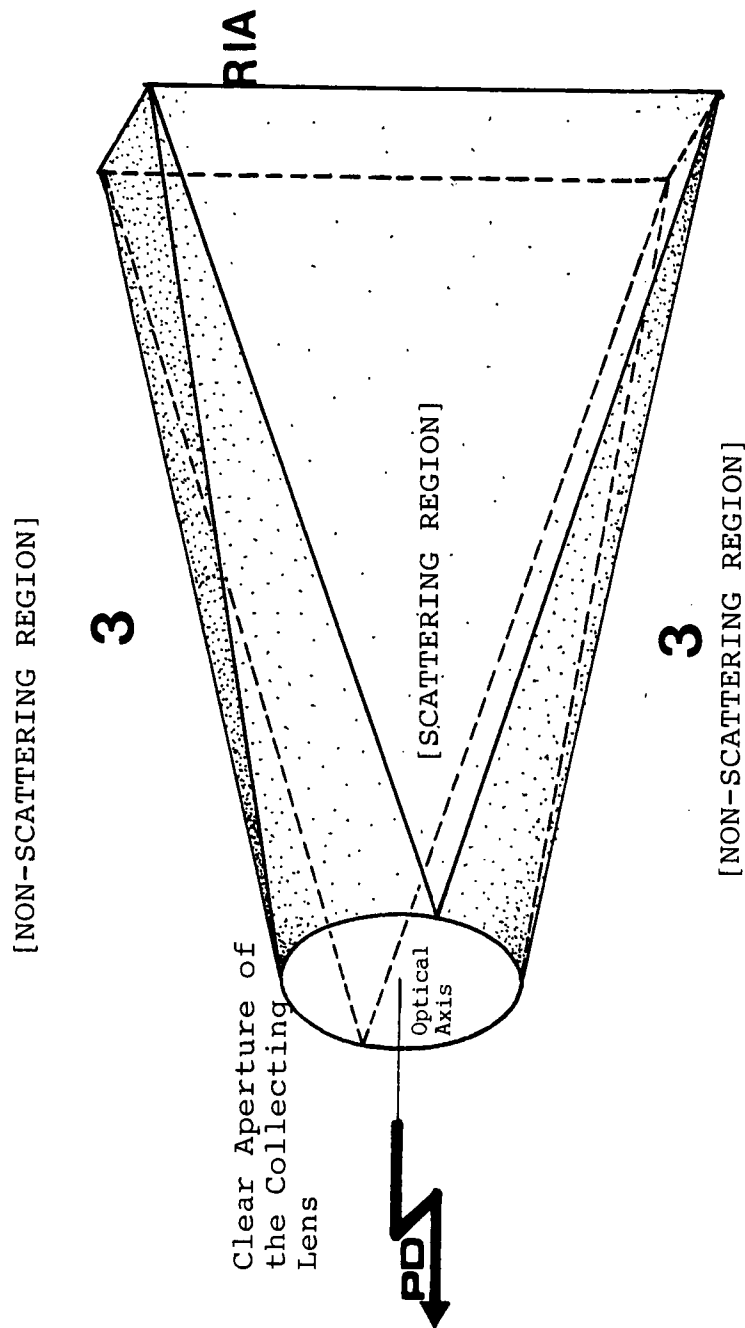
(b) Region One.

Figure 21. (Continued)



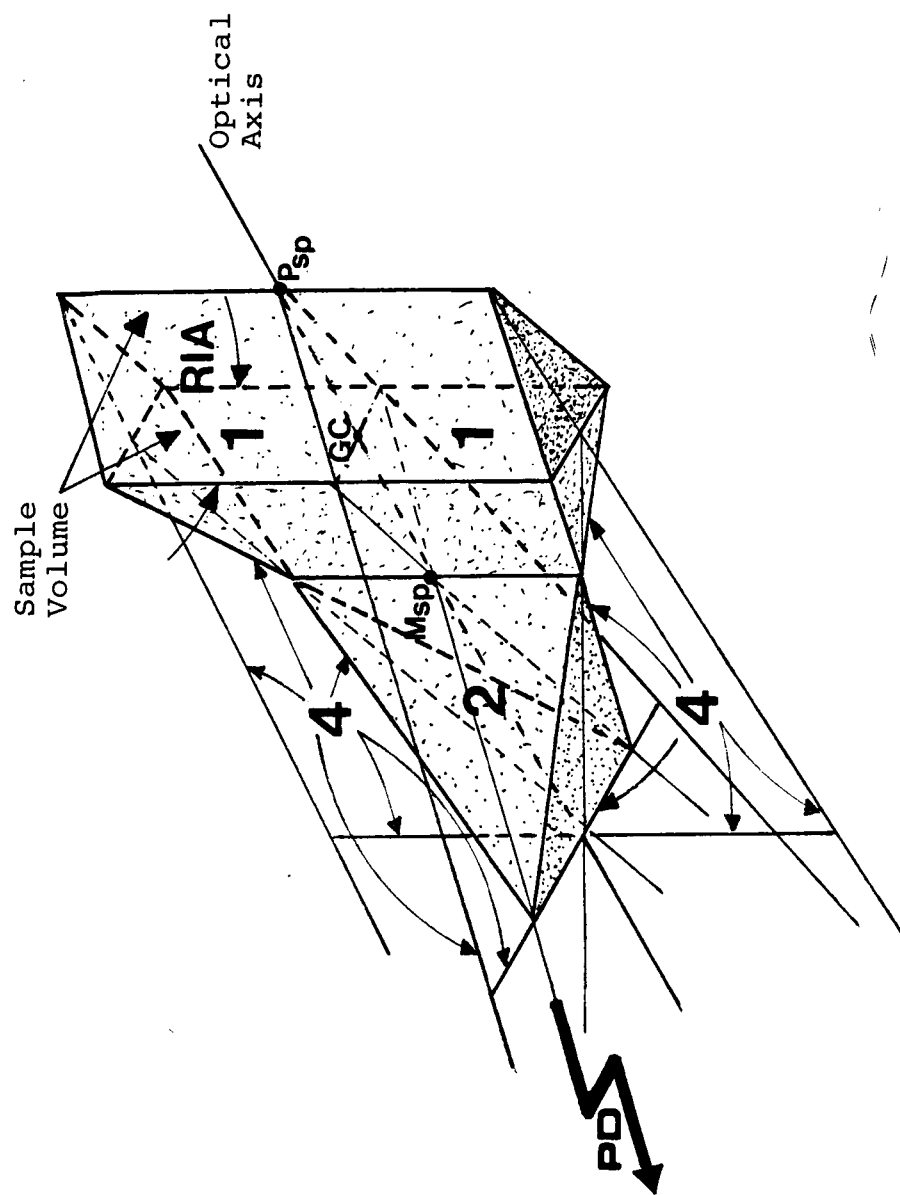
(c) Region Two.

Figure 21. (Continued)



(d) Region Three (Note that only the Near-Scatter Region is Shown).

Figure 21. (Continued)



(e) Region Four.

Figure 21. (Continued)

$$\Omega_{\text{eff}}^{(1)} = \frac{r_L^2}{\pi(z_L + z)^2} \quad (68)$$

Region No. 2 is characterized by the partial projection of the CLI onto the RIA. Not all of the scattered light that is collected by the lens gets transmitted through the photo-detector aperture. Some of the light rays are physically blocked by the photo-detector aperture mounting. This region is mathematically defined by the inequalities:

$$\left| \frac{|y|z_L}{z_L + z} \right| < b_p - \frac{r_L|z|}{z_L + z} \quad (69)$$

and

$$\left| \frac{|x|z_L}{z_L + z} \right| < a_p + \frac{r_L|z|}{z_L + z} \quad (70)$$

The effective solid collection angle for this region is defined by:

$$\Omega_{\text{eff}}^{(2)} = \frac{r_L|z|}{z^2(z_L + z)} \left\{ \int_0^t A(\tau, z) d\tau - \int_{2R}^t A(\tau, z) d\tau \right\} \quad (71)$$

with

$$A(\tau, z) = A \sqrt{\frac{2\tau(z_L + z)}{r_L|z|} - \frac{\tau^2(z_L + z)^2}{r_L^2 z^2}} \quad (72)$$

Region No. 3 is characterized by the total projection of the CLI outside of the RIA. None of the scattered light

that is collected by the lens gets transmitted through the photo-detector aperture. All of the light rays strike the mounting of the photo-detector aperture. This region is mathematically defined by the inequalities:

$$\left| \frac{|y| z_L}{z_L + z} \right| > b_p + \frac{r_L |z|}{z_L + z} \quad (73)$$

and

$$\left| \frac{|x| z_L}{z_L + z} \right| > a_p + \frac{r_L |z|}{z_L + z} \quad (74)$$

The effective solid collection angle is defined by:

$$\Omega_{\text{eff}}^{(3)} = 0 \quad (75)$$

Region No. 4 is characterized by the partial projection of the CLI onto the RIA. Not all of the scattered light that is collected by the lens is transmitted through the photo-detector aperture. Some of the light rays are mechanically interrupted by the photo-detector aperture mounting. This region is composed of two sections, depending on if the CLI-scan is entering or leaving the RIA. Region No. 4 for the entrance-scan is mathematically defined by the inequalities:

$$-\frac{r_L |z|}{z_L + z} - b_p < \left| \frac{|y| z_L}{z_L + z} \right| < \frac{r_L |z|}{z_L + z} - b_p \quad (76)$$

and

$$\left| \frac{|x| z_L}{z_L + z} \right| < a_p + \frac{r_L |z|}{z_L + z} \quad (77)$$

The effective solid collection angle is defined by:

$$\Omega_{\text{eff}}^{(4\text{-In})} = \frac{r_L |z|}{z^2 (z_L + z)} \int_0^t A(\tau, z) d\tau \quad (78)$$

Region No. 4 for the exit-scan is mathematically defined by the inequalities:

$$-\frac{r_L |z|}{z_L + z} + b_p < \left| \frac{|y| z_L}{z_L + z} \right| < \frac{r_L |z|}{z_L + z} + b_p \quad (79)$$

and

$$\left| \frac{|x| z_L}{z_L + z} \right| < a_p + \frac{r_L |z|}{z_L + z} \quad (80)$$

The effective solid collection angle is defined by:

$$\Omega_{\text{eff}}^{(4\text{-Out})} = \frac{r_L |z|}{z^2 (z_L + z)} \left\{ \int_0^t A(\tau, z) d\tau - \int_{2R}^t A(\tau, z) d\tau - \int_0^{t-2b_p} A(\tau, z) d\tau \right\} \quad (81)$$

A Fortran computer program was developed numerically to solve the integral expressions of the specialized case

$b_p > R$. The computer program was structured to treat three kinds of transverse scanning situations. These scanning situations are differentiated from each other by the position chosen for the center of the CLI relative to the scanning axis and the boundaries of the RIA. These situations are (see Figure 22):

1. On-axis scanning
2. Near off-axis scanning
3. Far off-axis scanning

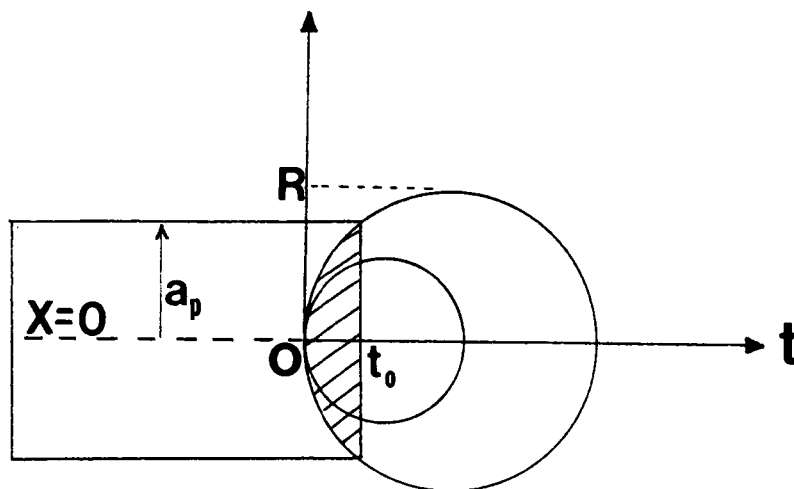
On-axis scanning describes a symmetric scanning defined by $X=0$, $0 < R < b_p$. In terms of the real optical system, the blur spot function (CLI) is positioned symmetrically about the t -axis as it scans across the sampling function (RIA). The center of the CLI moves along the scanning axis. The convolution transfer function is:

$$C_T^{(on)} = 2 \int_{-\infty}^{\infty} A^{(on)} \cdot C(\tau, R) d\tau \quad (82)$$

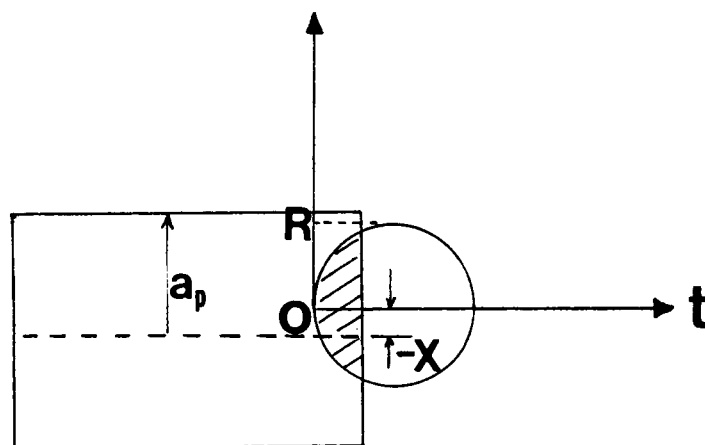
with

$$A^{(on)} = \begin{cases} 1 & \text{for } a_p > C(t, R) \\ \frac{C(t_0, R)}{C(t, R)} & \text{for } a_p < C(t, R) \end{cases} \quad (83)$$

$$C(t, R) \equiv C(\tau, R) \quad (84)$$

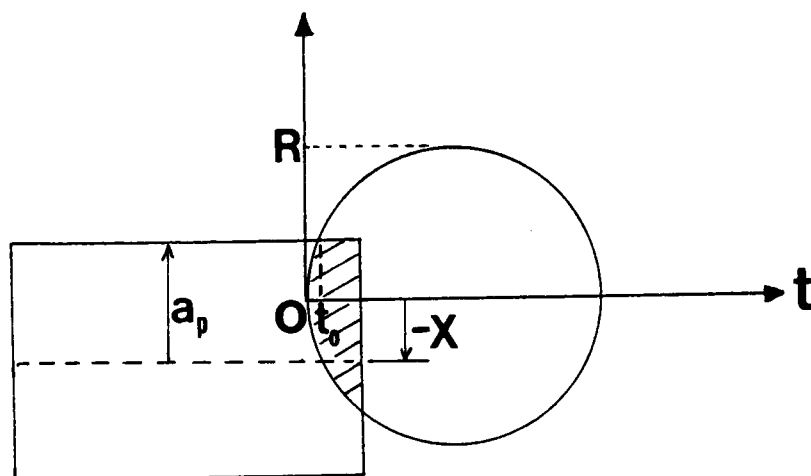


(a) On-Axis Scanning is Defined by $X = 0$, $0 < R < b_p$.

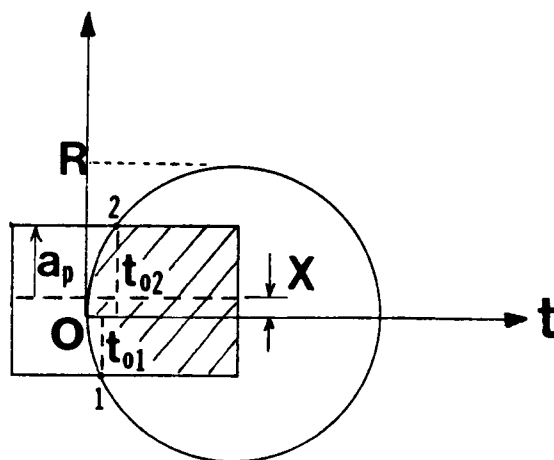


(b) Near Off-Axis Scanning is Defined by: [1]
 $|X| + R < a_p$ and $0 < R < a_p$.

Figure 22. Representation of the Three Kinds of Transverse Scanning Situations.

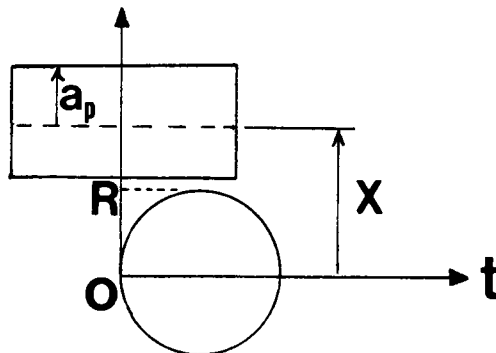


(c) [2] $|X| - R < a_p$, $|X| < a_p$, and $0 < R < 2a_p$.

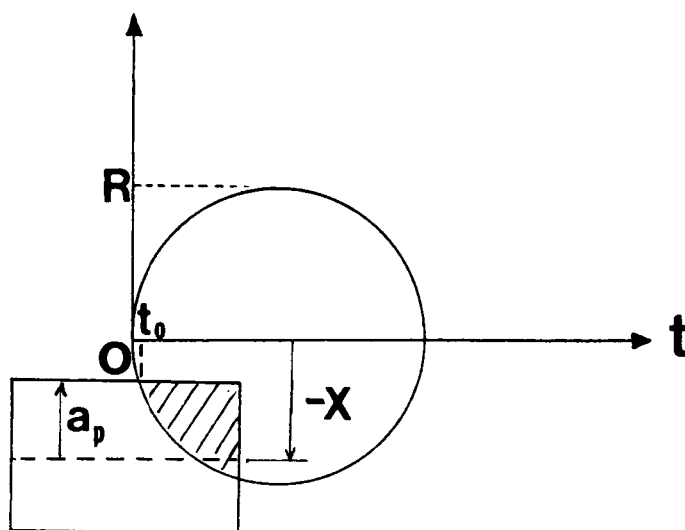


(d) [3] $|X| - R < a_p$, $|X| < a_p$, and $2a_p < R < b_p$.

Figure 22. (Continued).

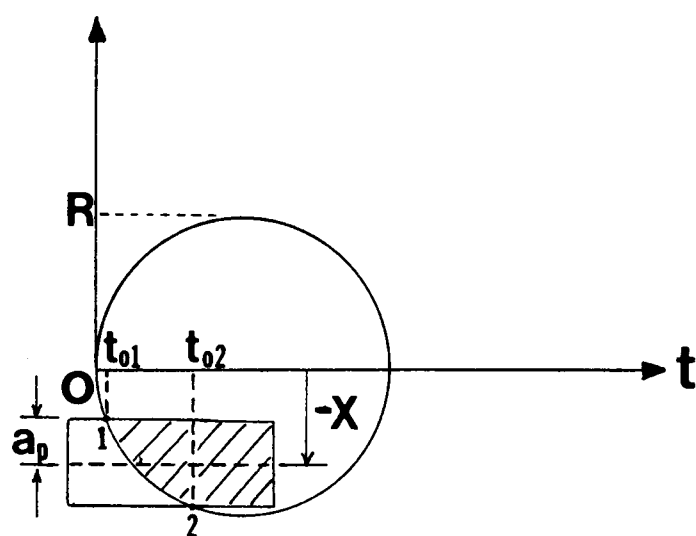


- (e) Far Off-Axis Scanning is Defined by: [1]
 $|X| - R > a_p$ and $0 < R < b_p$.



- (f) [2] $|X| - R < a_p$, $|X| > a_p$, and $0 < R < 2a_p$.

Figure 22. (Continued).



(g) [3] $|x| - R < a_p$, $|x| > a_p$, and $2a_p < R < b_p$.

Figure 22. (Continued).

$$C(t, R) = \sqrt{R^2 - (t - R)^2} \quad (85)$$

and

$$C(t_0, R) = \sqrt{R^2 - (t_0 - R)^2} \quad (86)$$

Near off-axis scanning is asymmetric. The center of the CLI is shifted away from the t -axis by an amount less than the half-width of the RIA. There are three overlapping patterns within the computational framework. The computations are performed by drawing a straight line through the center of the CLI, parallel to the t -axis, and then treating the resulting upper and lower circular segments as two separate convolution problems. If $|X| + R < a_p$ and $0 < R < a_p$, then the convolution transfer function is:

$$C_T^{(\text{Near})} = 2 \int_{-\infty}^{\infty} C(\tau, R) d\tau \quad (87)$$

If $|X| - R < a_p$, $|X| < a_p$, and $0 < R < 2a_p$, then the convolution transfer function is applied in a two-fold manner:

$$C_T^{(\text{Near})} = \int_{-\infty}^{\infty} C(\tau, R) d\tau + \int_{-\infty}^{\infty} A^{(\text{Near})} \cdot C(\tau, R) d\tau \quad (88)$$

with

$$A^{(\text{Near})} = \begin{cases} 1 & \text{for } a_p - |X| > C(t, R) \\ \frac{C(t_0, R)}{C(t, R)} & \text{for } a_p - |X| < C(t, R) \end{cases} \quad (89)$$

If $|X| - R a_p$, $|X| < a_p$, and $2a_p < R < b_p$, then the convolution transfer function is again applied in a two-fold manner:

$$C_T^{(\text{Near})} = \int_{-\infty}^{\infty} A_1^{(\text{Near})} \cdot C(\tau, R) d\tau + \int_{-\infty}^{\infty} A_2^{(\text{Near})} \cdot C(\tau, R) d\tau \quad (90)$$

$$A_1^{(\text{Near})} = \begin{cases} 1 & \text{for } a_p - |X| > C(t, R) \\ \frac{C(t_{01}, R)}{C(t, R)} & \text{for } a_p - |X| < C(t, R) \end{cases} \quad (91)$$

$$A_2^{(\text{Near})} = \begin{cases} 1 & \text{for } a_p + |X| > C(t, R) \\ \frac{C(t_{02}, R)}{C(t, R)} & \text{for } a_p + |X| < C(t, R) \end{cases} \quad (92)$$

with

$$C(t_{01}, R) = \sqrt{R^2 - (t_{01} - R)^2} \quad (93)$$

and

$$C(t_{02}, R) = \sqrt{R^2 - (t_{02} - R)^2} \quad (94)$$

Far off-axis scanning is another asymmetric scanning situation. The center of the CLI is shifted away from the t -axis by an amount greater than the half-width of the RIA. Again, there are three general overlapping patterns. If $|X| - R > a_p$ and $0 < R < b_p$, then the convolution transfer function is equal to zero, i.e., there is no overlapping of the CLI and the RIA:

$$C_T^{(\text{far})} = 0 \quad (95)$$

If $|X| - R < a_p$, $|X| > a_p$, and $0 < R < 2a_p$, then the convolution transfer function is applied twice in the following manner:

$$C_T^{(\text{Far})} = \int_{-\infty}^{\infty} A_1^{(\text{Far})} \cdot C(\tau, R) d\tau - \int_{-\infty}^{\infty} A_2^{(\text{Far})} \cdot C(\tau, R) d\tau \quad (96)$$

with

$$A_1^{(\text{Far})} = \begin{cases} 0 & \text{for } |X| - a_p > C(t, R) \\ 1 & \text{for } |X| - a_p < C(t, R) \end{cases} \quad (97)$$

$$A_2^{(\text{Far})} = \begin{cases} 0 & \text{for } |X| - a_p > C(t, R) \\ \frac{C(t_{02}, R)}{C(t, R)} & \text{for } |X| - a_p < C(t, R) \end{cases} \quad (98)$$

If $|X| - R < a_p$, $|X| > a_p$, and $2a_p < R < b_p$, then the convolution transfer function is again applied twice in the following manner:

$$C_T^{(\text{Far})} = \int_{-\infty}^{\infty} A_{1*}^{(\text{Far})} \cdot C(\tau, R) d\tau - \int_{-\infty}^{\infty} A_{2*}^{(\text{Far})} \cdot C(\tau, R) d\tau \quad (99)$$

with

$$A_{1*}^{(\text{Far})} = \begin{cases} 0 & \text{for } |X| - a_p > C(t, R) \\ 1 & \text{for } |X| - a_p < C(t, R) \\ \frac{C(t_{01}, R)}{C(t, R)} & \text{for } |X| + a_p < C(t, R) \end{cases} \quad (100)$$

$$A_{2*}^{(\text{Far})} = \begin{cases} 0 & \text{for } |X| - a_p > C(t, R) \\ \frac{C(t_{02}, R)}{C(t, R)} & \text{for } |X| - a_p < C(t, R) \end{cases} \quad (101)$$

As before, the solid angle transfer function is formed from the normalization of the effective solid collection angles relative to the minor scattering solid angles. While the effective focal number of the collecting lens remains the same as defined in Eq. (37), the effective aperture number of the image of the photo-detector aperture

must be redefined for the case of the circular lens and the rectangular aperture. In the case of the circular lens and the circular aperture, it was relatively easy to define the aperture number because of the circular symmetry of the image of the photo-detector aperture. Only one variable (i.e., r -coordinate) was needed to completely define the opening of the entrance pupil. In the case of the circular lens and the rectangular aperture, it takes two variables (i.e., x - and y -coordinates) to define the opening of the entrance pupil. The y -coordinate is associated with the length of the rectangular image of the photo-detector aperture and also represents the scanning axis. The x -coordinate is associated with the width of the rectangular image of the photo-detector aperture. Because the treatment in this chapter considers only the specialized case $b_p > R$ with $b_p \gg a_p$, the effective sampling capacity of the system is primarily determined by the width of the image of the photo-detector aperture. For this reason, the effective aperture number of the image of the photo-detector aperture, for the rectangular case, is specifically expressed in terms of the aperture width as:

$$(ef/\#)_{S*} = \frac{z}{2a_p} \quad (102)$$

Finally, the maximum relative error, introduced in the solid collection angles by the small angle approximation, is defined in a similar manner as described in Chapter V, and its value remains the same as given by Eq. (41).

The results for the four distinctive scattered light collecting regions for the case of the circular lens and the rectangular aperture are summarized in Table 2. The approximate solid angle transfer functions are compared with the exact mathematical description for each region. The results are written in terms of the assumption that $z \ll z_L$. They will be used to plot normalized solid collection angle contours as a function of the point-source scattering position in the image of the aperture space.

Table 2. "Tabulation of the Approximate Solid Angle Transfer Function."

Region Number	Exact Mathematical Description of Region	Approximate Solid Angle Transfer Function Within the Given Region
1	$\left \frac{y z_L}{z_L+z} \right < b_p - \frac{r_L z }{z_L+z},$ $\left \frac{x z_L}{z_L+z} \right < a_p - \frac{r_L z }{z_L+z},$ $b_p \gg a_p > \frac{r_L z }{z_L+z}$	1
2	$\left \frac{y z_L}{z_L+z} \right < b_p - \frac{r_L z }{z_L+z},$ $\left \frac{x z_L}{z_L+z} \right < a_p + \frac{r_L z }{z_L+z},$ $b_p > \frac{r_L z }{z_L+z}$	$\frac{1}{\pi a_p} \cdot \left[\frac{(ef/\theta)_L}{(ef/\theta)_{S^*}} \right]^2 \cdot \left\{ \int_0^{y_c(+)} A_\tau d\tau - \int_0^{y_c(+)} A_\tau d\tau \right\}$ $2a_p \frac{(ef/\theta)_{S^*}}{(ef/\theta)_L}$
3	$\left \frac{y z_L}{z_L+z} \right > b_p + \frac{r_L z }{z_L+z},$ $\left \frac{x z_L}{z_L+z} \right > a_p + \frac{r_L z }{z_L+z},$ $b_p > \frac{r_L z }{z_L+z}$	0
4 (Enter)	$-\frac{r_L z }{z_L+z} - b_p < \left \frac{y z_L}{z_L+z} \right < -\frac{r_L z }{z_L+z} + b_p,$ $\left \frac{x z_L}{z_L+z} \right < a_p + \frac{r_L z }{z_L+z},$ $b_p > \frac{r_L z }{z_L+z}$	$\frac{1}{\pi a_p} \cdot \left[\frac{(ef/\theta)_L}{(ef/\theta)_{S^*}} \right]^2 \cdot \left\{ \int_0^{y_c(+)} A_\tau d\tau \right\}$
4 (Exit)	$-\frac{r_L z }{z_L+z} + b_p < \left \frac{y z_L}{z_L+z} \right < -\frac{r_L z }{z_L+z} + b_p,$ $\left \frac{x z_L}{z_L+z} \right < a_p + \frac{r_L z }{z_L+z},$ $b_p > \frac{r_L z }{z_L+z}$	$\frac{1}{\pi a_p} \cdot \left[\frac{(ef/\theta)_L}{(ef/\theta)_{S^*}} \right]^2 \cdot \left\{ \int_0^{y_c(+)} A_\tau d\tau - \int_0^{y_c(+)} A_\tau d\tau - \int_0^{y_c(-)} A_\tau d\tau \right\}$ $2a_p \frac{(ef/\theta)_{S^*}}{(ef/\theta)_L}$
NOTE: $A_\tau = A \sqrt{\frac{2\tau}{a_p} \left[\frac{(ef/\theta)_{S^*}}{(ef/\theta)_L} \right] - \frac{\tau^2}{a_p^2}}, y_c(+) = y + a_p \left[\frac{(ef/\theta)_{S^*}}{(ef/\theta)_L} + \frac{b_p}{a_p} \right], y_c(-) = y + a_p \left[\frac{(ef/\theta)_{S^*}}{(ef/\theta)_L} - \frac{b_p}{a_p} \right]$		

CHAPTER VII

SOLID ANGLE TRANSFER FUNCTION

This chapter discusses the theoretical and physical significances of the geometrical optics solid angle transfer function. It shows that the convolution techniques are more generally applicable in solving problems dealing with the geometrical transfer of light collected by a lens-aperture optical system. It also shows that there exists a common geometrical transfer relationship evolving out of the derivations which result in the same mathematical form of the geometrical optics solid angle transfer function for the cases of the Circular Lens/Circular Aperture and the Circular Lens/Rectangular Aperture.

The solid angle transfer function is conceptually no different than the solid angle ratio developed in Chapters IV and V. Both the solid angle ratio and solid angle transfer function evolve out of the vignetting factor described in Chapter II, and represent a common geometrical quantity denoted as the solid angle transfer. While the vignetting factor ascribes a single point value to the optical system, the solid angle transfer is given in a functional form. The solid angle transfers function evaluates the scattered light collecting performance

of the system at every point in the image of the aperture space surrounding the sample volume or entrance pupil. The normalized values of the solid angle transfer function at these points represent the optical transfer of the effective solid collection angles from the image of the aperture space to the aperture space.

The solid angle transfer function differs from the solid angle ratio in the computational flexibility that it offers. Hecht [19] points out that the convolution techniques used in Chapter VI are more general applicable procedures, as compared to analytical geometric techniques, in solving optical problems dealing with the superpositioning of the image of an optical element and the cross-sectional area of a scattering solid angle. Hecht shows that the domain of applicability of the convolution integral encompasses that of analytical geometry. Geometrically, the convolution in one dimension is the area of overlap under the product of two functions, whereas, the convolution in two dimensions is the volume resulting from the product of the surface encompassing the region of overlap of two functions.

The convolution of two circle functions is also equivalent to Brayton's method for treating the case of the circular lens and the circular aperture, i.e.,

$$C(x,y) = \iint_{-\infty}^{\infty} \text{Circ}[K(x^2,y^2)] \cdot \text{Circ}[J(x,x^2,y^2)] dx dy \quad (103)$$

with

$$\text{Circ}[K(x^2,y^2)] = \begin{cases} 0 & \text{for } K(x^2,y^2) > b_c \\ 1 & \text{for } K(x^2,y^2) \leq b_c \end{cases} \quad (104)$$

$$\text{Circ}[J(x,x^2,y^2)] = \begin{cases} 0 & \text{for } J(x,x^2,y^2) > R \\ 1 & \text{for } J(x,x^2,y^2) \leq R \end{cases} \quad (105)$$

and

$$K(x^2,y^2) = \sqrt{x^2 + y^2} \quad (106)$$

$$J(x,x^2,y^2) = \sqrt{(x-S)^2 + y^2} \quad (107)$$

This convolution can be rewritten in terms of indefinite integrals with specific limits of integration.

$$C(x,y) = 2 \int_{S-R}^P \int_0^{I(x,x^2)} dy dx + 2 \int_P^C \int_0^{L(x^2)} dy dx \quad (108)$$

with

$$P = \frac{b_c^2 + S^2 - R^2}{2S} \quad (109)$$

$$I(x,x^2) = \sqrt{R^2 - (x - S)^2} \quad (110)$$

$$L(x^2) = \sqrt{b_c^2 - x^2} \quad (111)$$

The solutions to these integrals are identical to the ones derived in Brayton's treatment for the case of region four, as given by Eq. (36).

The general form of the indefinite integral with its solution for the optical convolution solid angle transfer problem is,

$$C(x) = \int Q(x^2, x) dx \quad (112)$$

$$= \left[\frac{2ux + v}{4u} \right] Q(x^2, x) + \left[\frac{v^2 - 4uw}{i8u\sqrt{u}} \right] \sin^{-1} \left\{ \frac{2ux + v}{\sqrt{v^2 - 4uw}} \right\} + C_o$$

with

$$Q(x^2, x) = \sqrt{ux^2 + vx + w} \quad (114)$$

The physical significance in the form of this solid angle transfer solution is strictly attributed to the mechanical interruption of the transmitted light rays that occurs at the photo-detector. If ray blockage due to the photo-detector aperture mounting occurs, the cross-sectional area of the collection optics solid angle transfer will always exhibit some form of circular segmented geometry.

In Brayton's treatment, this circular segmented geometry results from the common area of overlap of the CIA and the CLI, when the circular area of the solid collection angle of the scattered light superimposes onto the circular image of the photo-detector aperture. The common area of overlap of the CIA and the CLI exhibits complete circularity for the case of total overlap, or circular segmentation for the case of partial overlap. In Brayton's analysis, this circular segmentation forms two circular segments connected base-to-base. The general solution for the area of a circular segment is similar to that of Eq. (113) with $C_0 = \pi/2$, and the combined general solution for the area of the two circular segments connected base-to-base, as in the case of the common area of overlap of the CIA and the CLI, is given by:

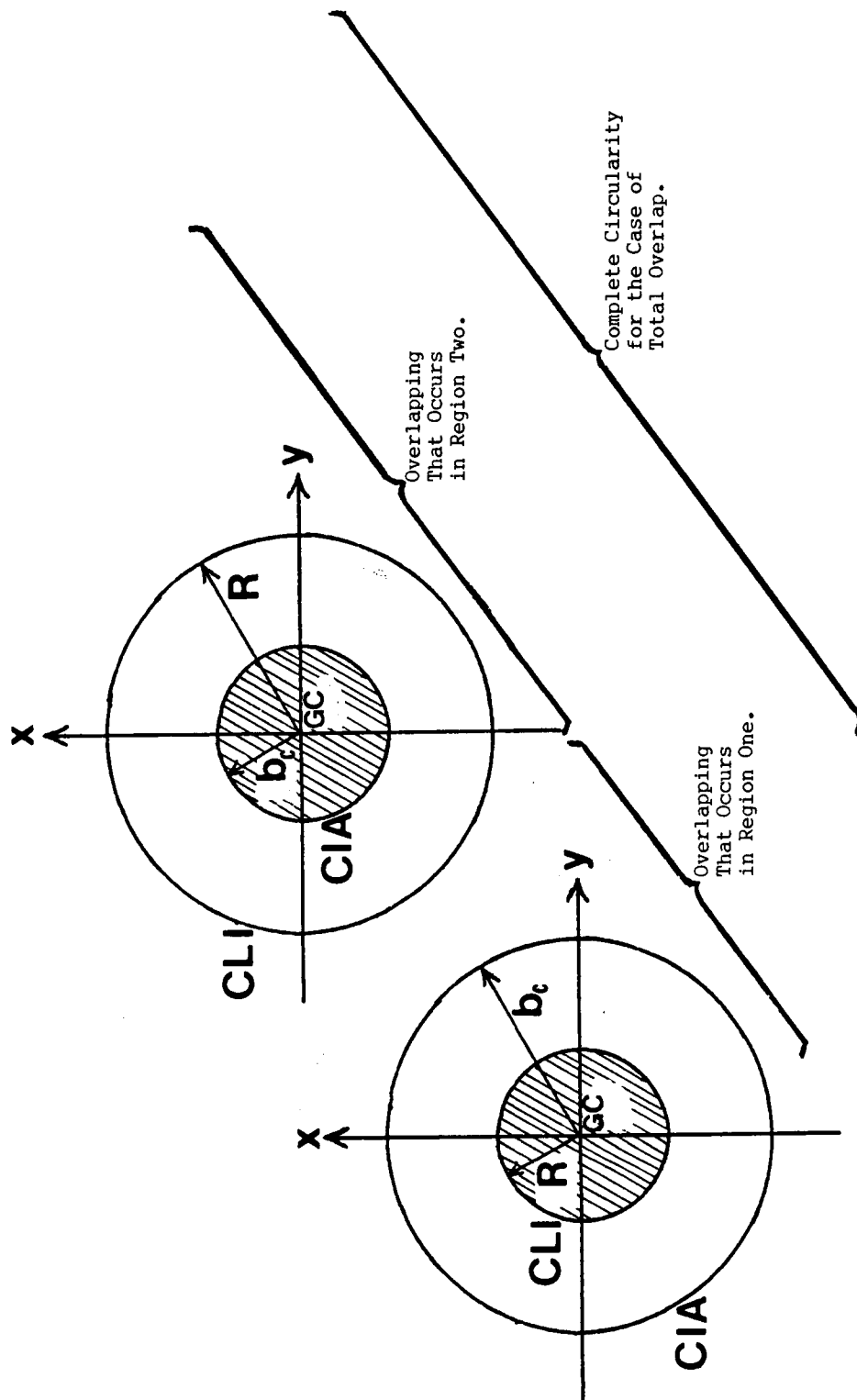
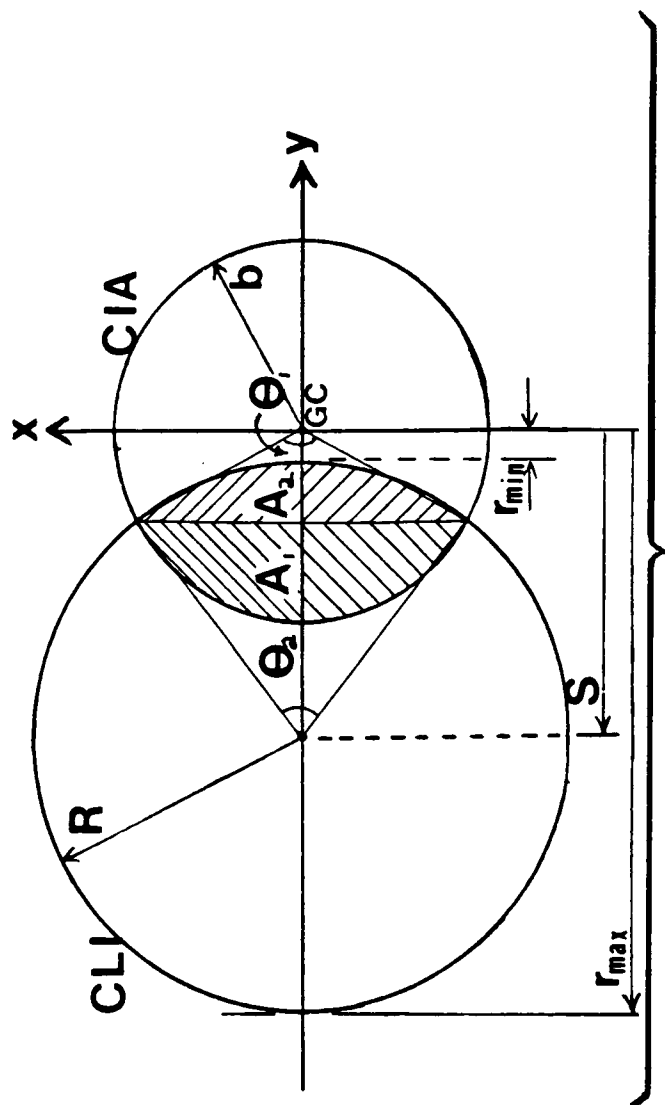


Figure 23. Diagram Representing the Well Defined Character and Discrete Nature of the Superpositioning of the CIA and CLI, as Described in Brayton's Treatment.



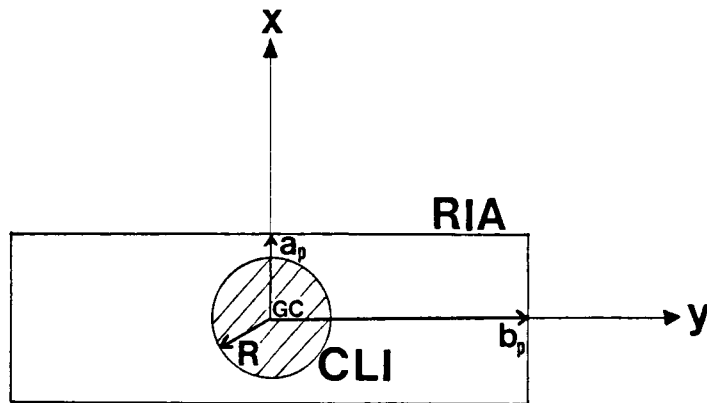
Circular Segmentation for the Case of Partial Overlap and Overlapping that Occurs in Region Four.

Figure 23. (Continued).

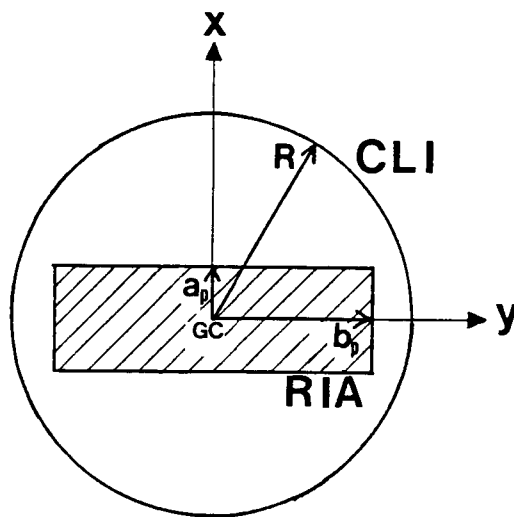
$$\begin{aligned}
 A_{12} = & \quad b_c^2 \cos^{-1} \left[\frac{b_c^2 - R^2 + S^2}{2Sb_c} \right] \\
 & + R^2 \cos^{-1} \left[\frac{R^2 - b_c^2 + S^2}{2SR} \right] \\
 & - SR \left[1 - \left(R^2 - b_c^2 + S^2 \right)^2 \right]^{1/2}
 \end{aligned} \tag{115}$$

For the special case of the circular lens and the circular aperture, it is easy to develop an analytical expression for the solid collection angle of the scattered light because of the well defined character and discrete nature of the superpositioning of the CIA and CLI. The resulting expression is the solid angle ratio, and its simplicity of application is exhibited in the four distinctive scattered light collecting regions that it identifies with the use of analytical geometry.

In the author's treatment, this circular segmented geometry results from the common area of overlap of the RIA and the CLI, when the circular cross-sectional area of the solid collection angle of the scattered light superimposes onto the rectangular image of the photo-detector aperture. The common area of overlap of the RIA and the CLI exhibits complete circularity or complete rectangularity for the case of total overlap, or circular segmentation for the case of partial overlap (see Figure 24,

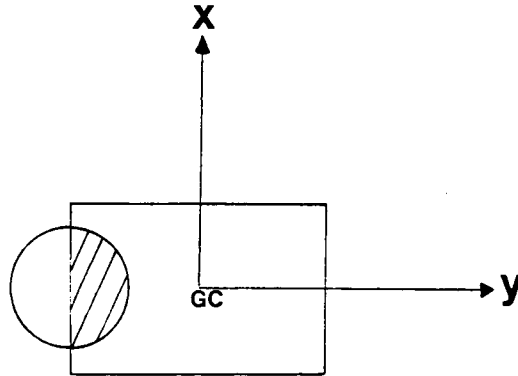


Complete Circularity for the Case of Total Overlap and Overlapping that Occurs in Region One of the Effective Scattering Region.



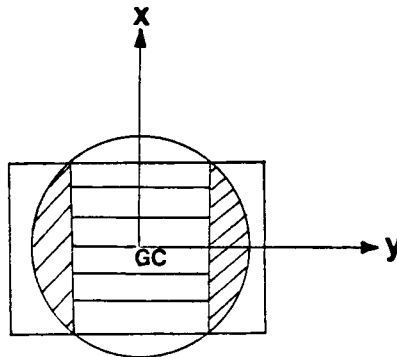
Complete Rectangularity of the Case of Total Overlap and Overlapping that Occurs in Region Four of the Non-Effective Scattering Region.

Figure 24. Diagram Representing the Ill-Defined Character and Non-Discrete Nature of the Superpositioning of the RIA and CLI, as Described in the Author's Treatment.

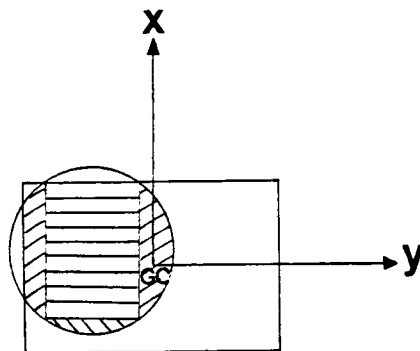


Far Off-Axis Overlapping that Occurs in Region Four of the Effective Scattering Region.

Three Examples of
the Circular
Segmentation
for the Case
of Partial
Overlap.



Overlapping that Occurs in Region Two of the Effective Scattering Region.



Near Off-Axis Overlapping that Occurs in Region Four of the Effective Scattering Region.

Figure 24. (Continued).

In the author's analysis, this circular segmentation takes on a variety of different forms (e.g. one circular segment, two unconnected circular segments, two adjacent circular segments and one adjacent modified form of a circular segment, and so forth). In many cases, this circular segmentation is accompanied by an adjoining common rectangular area of overlap of the RIA and the CLI. In the general development, it is not possible to perform the analysis on a case-by-case basis, as is done in Brayton's treatment, because there are too many cases to be analyzed. The general solution for the total common area of overlap of the RIA and the CLI is not geometrically simple, because it is not possible to develop a single expression, like the solid angle ratio, which identifies distinctive scattered light collecting regions. The domain of applicability of the solid angle ratio is constrained by geometry. The general solution for the total common area of overlap of the RIA and the CLI is at best numerical, and consists of a combination of conceptually simpler solutions, like that of Eq. (113), combined with specific solutions to common rectangular overlapping areas.

For the case of the circular lens and the rectangular aperture, it is possible to develop a numerically functional expression for the solid collection angle of the scattered light because the optical superpositioning of the sample

volume with the probe volume is conceptually compatible with the techniques of convolution. The resulting expression is the geometrical optics solid angle transfer function and it, too, is able to identify four distinctive scattered light collecting regions with the use of appropriate numerical integration techniques.

The theoretical domain of the solid angle transfer function can be extended into the contemporary foundations of the standard geometrical optics transfer functions. Direct evidence to support this conjecture is given by Richmond B. McQuistan [20]. He shows that the optical transfer properties of a scattered light collection optics can be determined from the convolution of the system's point spread and aperture stop functions. McQuistan's treatment develops the system's optical response to an infinite sequence of periodical scattered light signals, where the spatial period of the signals is greater than the scanning length of the aperture. This optical response is given in terms of a fourier series with its coefficients determined by the fourier transform of the convolution function taken over one signal period. McQuistan's analysis directly identifies the convolution function with the solid angle transfer function. His analysis treats the case of the rectangular aperture, and develops an optical response function that is equivalent to the geometrical optics transfer function for a

square-wave target used as the test object. Similar results can also be obtained for the case of the circular aperture. From these results and other related considerations, there exists a theoretical connection between the standard geometric optical transfer functions and the solid angle transfer functions. The solid angle transfer function is the zeroth order approximation of an optical transfer series expansion, where the first and second order approximations are the geometrical optics and diffraction optics transfer functions, respectively. However, further research will be necessary in order to confirm this supposition.

CHAPTER VIII

EXPERIMENTAL AND THEORETICAL RESULTS

This chapter describes the experimental setup used to measure the amount of light scattered by a point-source located at different positions within the probe volume. It describes the experimental procedures used in scanning the different images of the photo-detector aperture geometries. It also presents the experimental results which were obtained in the laboratory and compares them with the theoretical computations. Sources of error and other experimentally related problems are discussed.

A mono-scatter LDV-system was designed to function with an expanded laser beam having a radial Gaussian intensity profile. The geometrical characteristics of the probe volume were directly identified with those of the laser beam. The beam was expanded and collimated by passing it through a beam collimator.

A Spectra-Physics Stabilite Model 124-B Helium-Neon Laser with a maximum rating of 35 mW at a wavelength of $0.6328 \mu\text{m}$ was used. Connected to the end of the laser was a spatial filter. Connected to one end of the spatial filter was a collimator with a 50-mm clear aperture and a 200-mm effective focal length.

The emerging Gaussian plane wave laser beam was 2 inches in diameter. The light intensity of this beam dropped off to a normalized value of 3.35×10^{-4} at its one-inch radius.

The beam was incident on a large beam-stop whose function was to block the forward propagation of the beam. At the center of the beam-stop, a 50- μm pinhole aperture was used to simulate a point-source scatterer. Only the light passing through the pinhole was allowed to be gathered by the collecting lens positioned between the pinhole aperture and the photo-detector (see Figure 25). The beam-stop pinhole aperture was mounted to three micrometer tables allowing adjustment in the x,y,z-directions.

The laser light emerged from the pinhole as diverging spherical waves which were intercepted by the clear aperture of the collecting lens (see Figure 25).

The collecting lens consisted of a thick cornea-shaped doublet having an aperture diameter of 4.55 inches and an effective focal length of 9.50 inches.

The medium of propagation of the light was air and it is assumed that its index of refraction is approximately equal to 1.00.

The entire optical setup was designed to function at unit magnification in order to minimize aberrational

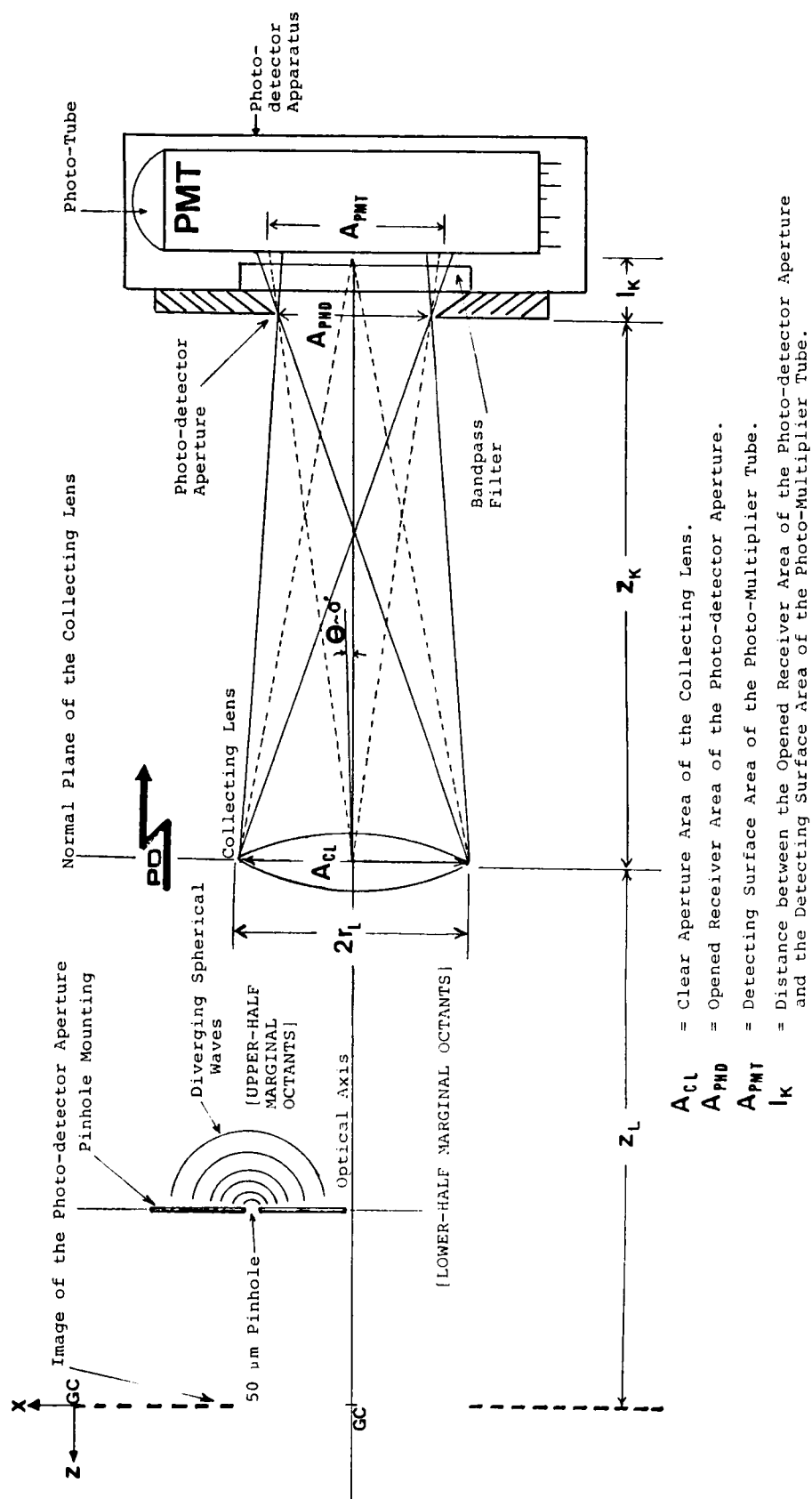


Figure 25. Diagram Representing the Pinhole as a Point-Source Scatterer and the Photo-detector Apparatus Used in the Experimental Setup to Scatter-Scan the Upper-Half Marginal Octants of the Image of the Aperture Space.

effects. Consequently, the collecting lens was positioned at such a point along the optical axis that it was a distance of two focal lengths from either the photo-detector aperture or the image of the photo-detector aperture.

The effective focal number of the collecting lens was equal to 4.18.

A narrow bandpass filter with a $10 \text{ \AA}$ pass band for a wavelength of $0.6328 \text{ }\mu\text{m}$ was placed in front of the phototube in order to remove background laboratory lighting. From Eq. (5), the longitudinal spherical aberration blur spot diameter was equal to $169.28 \text{ }\mu\text{m}$. The blur spot diameter sets a lower-limit upon the size of the photo-detector aperture. There were two types of removable apertures used in the experiment; one consisted of a $350\text{-}\mu\text{m}$ -diameter circular aperture (see Figure 11, page 38) while the other consisted of a $200\text{-}\mu\text{m}$ -wide by $5,000\text{-}\mu\text{m}$ -long rectangular aperture (see Figure 16, page 56).

For the case of the rectangular photo-detector aperture, the amplified signal was sent to and recorded on a strip chart recorder. The peak signal-to-noise ratio for the rectangular aperture arrangement was of the order of 72.

For the case of the circular photo-detector aperture,

the amplified signal was also recorded by a strip chart recorder. The peak signal-to-noise ratio for the circular aperture arrangement was of the order of 25.

The experimental procedures used in scanning the image of the aperture space of either optical arrangements were as follows:

The beam scan can be described as a probing process in which the scattered light, emanating from the pinhole, is collected by a lens and transmitted to a photo-detector. The pinhole simulates a point-source scatterer at different points in the probe volume. The scanning pinhole can be viewed by the photo-detector as scatter-scanning the image of the aperture space.

All scatter-scans were conducted in the upper-half marginal octants of the image of the aperture space (see Figure 25). The marginal octants (upper- and lower-halves are the regions of space between the image of the aperture plane and the normal plane of the collecting lens. These scatter-scans were performed in the xy-plane at different points along the optical axis (z-axis). The scatter-scanning process consisted initially in placing the pinhole on the optical axis (on-axis), and subsequently, at different heights or x-axis levels above the optical axis (off-axis), as it was made to move parallel to the y-axis across the effective scattering regions of the

image of the aperture space. This process was repeated for many different xy-planes at various z-locations along the optical axis.

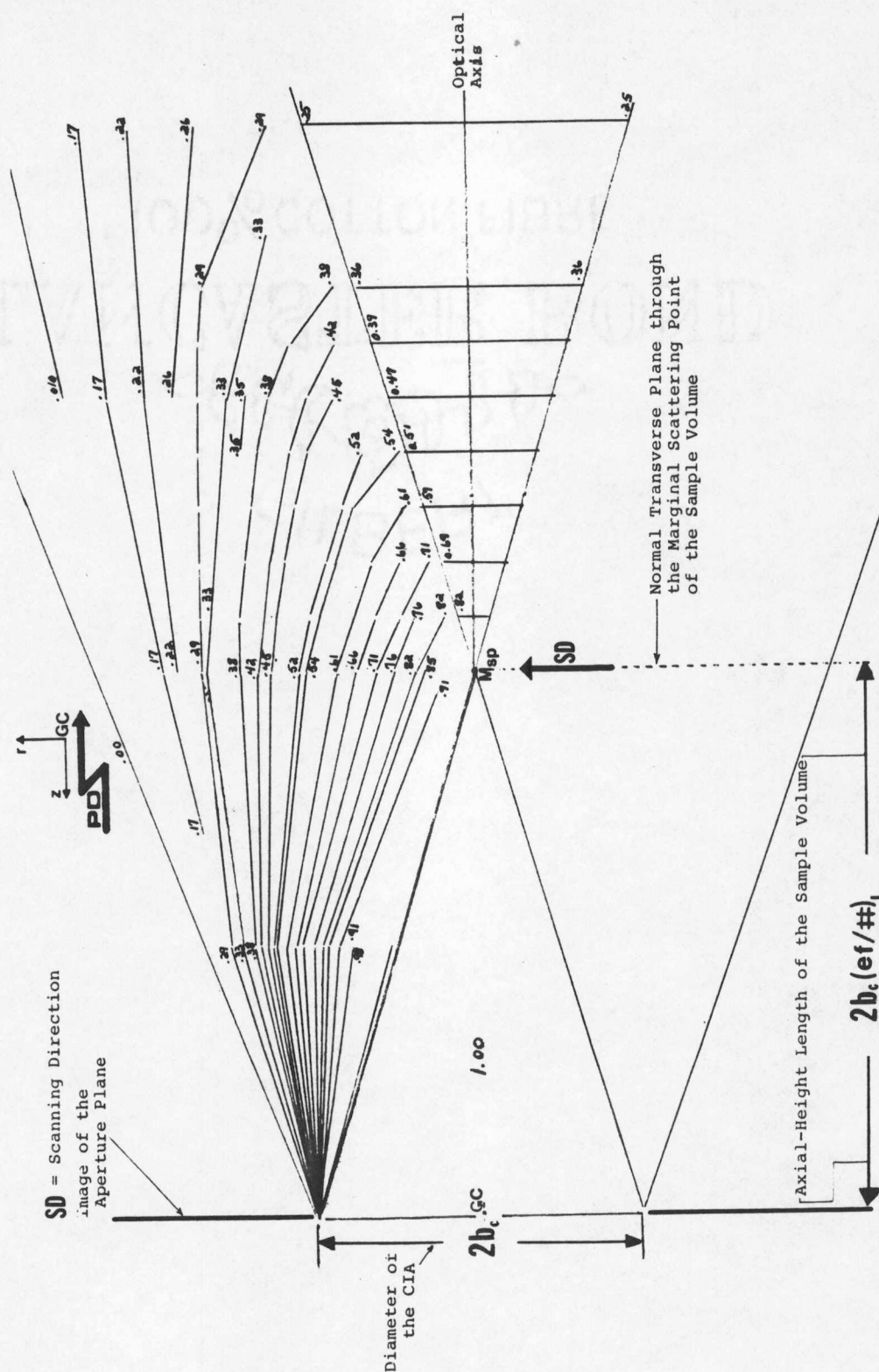
These experimental procedures were performed for the optical arrangements of the Circular Lens/Circular Aperture and Circular Lens/Rectangular Aperture. An intensity contour mapping for each image of the aperture space was recorded. Each mapping depicts the graphical representation of the normalized effective solid collection angles as a function of the pinhole positions in their respective image of the aperture space.

The experimental and theoretical results are presented with the assumption that the sample volume is symmetric about the image of the aperture plane. This is done for the cases of the Circular Lens/Circular Aperture and the Circular Lens/Rectangular Aperture.

The theoretical and experimental results for the case of the circular lens and the circular aperture are presented as follows.

The diameter of the circular aperture was set at the value of 350 μm and satisfied the requirements set forth in Chapter III.

First, the theoretical computations of the solid angle ratio, as described in Chapter IV and derived in Chapter V, are presented in Figure 26. This diagram



Graphical Representation of the Theoretical Results for the Case of the Circular Lens and the Circular Photo-detector Aperture.

Figure 26. Diagram of the Theoretical Results Representing the Contour Plots of the Solid Angle Ratio.

represents the contour plots of the normalized effective solid collection angles as a function of the point-source scattering positions in the image of the aperture space. Because of the symmetry about the optical axis and the assumed symmetry about the image of the aperture plane, the contours are plotted for only one octant of the image of the aperture space. The diagram offers a graphical view of the sample volume surrounded by contour surfaces. The geometry of the sample volume is conical, having a radial-base diameter of $2b_c$ equal to $350 \mu\text{m}$ and an axial-height length of $2b_c(ef/\#)_L$ equal to $1,463 \mu\text{m}$. The value of the solid angle ratio at any point within the sample volume is equal to unity. The shapes of the surrounding contours consist of either on-axis flat surfaces or off-axis curved surfaces, having relative solid angle ratio values ranging between zero and one (see Figure 26). Due to the assumption made in the tabulation of the approximate solid angle ratio, $z \ll z_L$, and to the small angle approximation, $z_L \gg r_L$, the off-axis values of the solid angle ratio can be appreciably different than their adjoining on-axis counterparts. For example, the on-axis value of the solid angle ratio drops off to 0.25 at a distance of $4b_c(ef/\#)_L$ from the geometrical center of the sample volume, while its adjoining off-axis value is 0.29, a difference of 16%. The difference between

the on-axis and off-axis values of the solid angle ratio increases with increasing distance away from the sample volume. This difference is mostly due to the assumption $z \ll z_L$. The maximum relative error introduced by the small angle approximation is equal to 2.15%. For this example, the error introduced by the assumption $z \ll z_L$ is 13.85%. Closer to the sample volume, this difference becomes negligible.

Second, the experimental results are compared to the theoretical computations in Figure 27. This graph represents the solid angle ratio as a function of a dimensionless quantity defined by r/b_c . The particular graph is a plot of the experimental and theoretical values of the solid angle ratio taken in the normal transverse plane through the marginal scattering point of the sample volume (see Figure 26), where $(ef/\#)_L$ is equal to $(ef/\#)_S$. Figure 27 is chosen as the most representative graph because it characterizes the behavior of the solid angle ratio in any other normal transverse plane through the sample volume. It also allows for representative sampling of the solid angle ratio in the region of the image of the aperture space lying outside of the sample volume.

The comparison between the experimental and theoretical results is good considering the error

- ① = Experimental Results.
 ② = Theoretical Results
 ③ = Boundary of the Geometrical Shadow of the Image of the Photo-detector Aperture

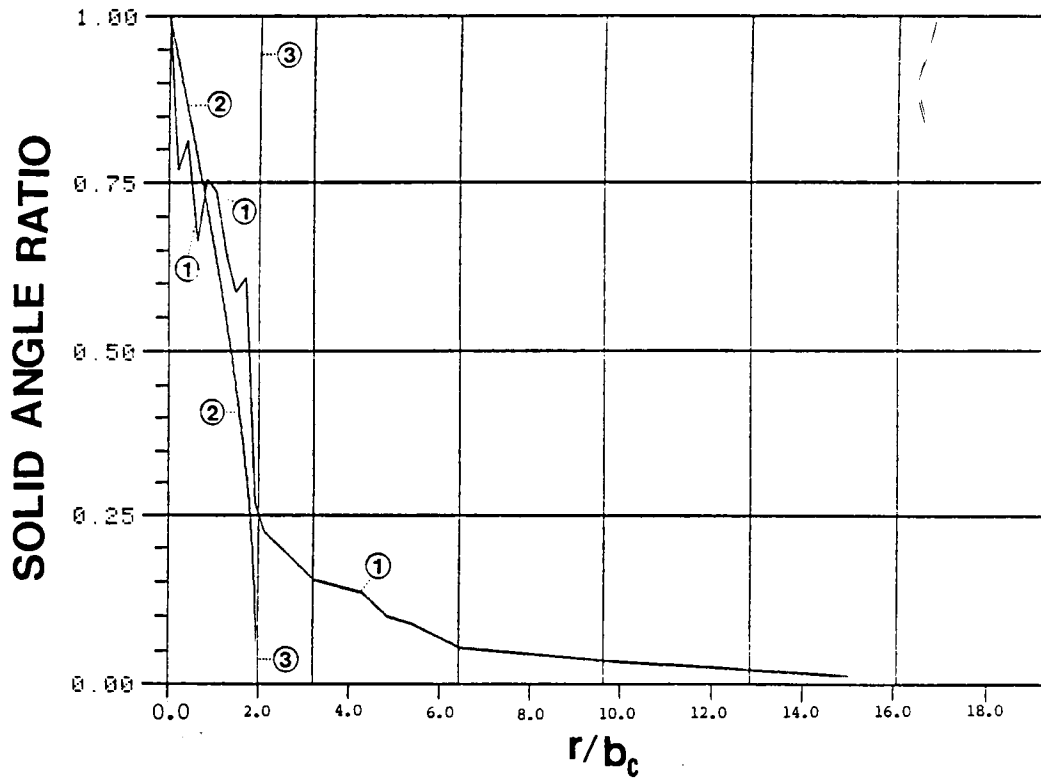


Figure 27. Graphical Representation of the Experimental and Theoretical Results for the Case of the Circular Lens/Circular Aperture.

introduced by the diffractive and aberrational effects arising from the propagation of the laser light through the circular pinhole and the defocusing properties of the collecting lens. The region of good correlation between the experimental and theoretical data lies within the illuminated region predicted by geometrical optics. The region of good correlation is shown in Figure 27 for the abscissa values between 0.0 and 2.0. The theoretical data is plotted for the region of the image of the aperture space between the marginal scattering point (abscissa value of 0.0) and the point of the geometrical shadow of the image of the photo-detector aperture (abscissa value of 2.0). The geometrical shadow is taken to be the point on the abscissa directly above the 0.25 intensity level of the experimental data [10]. The following two observations are made for the region of good correlation. A central concentration of the transmitted light about the center of the photo-detector aperture is due to the focusing properties of the diffraction-limited collecting lens. This central concentration of energy is seen in the steepness of the slope of the experimental curve as compared to the theoretical curve. A strong oscillatory behavior of the energy intensity across the photo-detector aperture is due to diffraction from the edge of the pinhole opening. The graph in Figure 27 shows that some of the

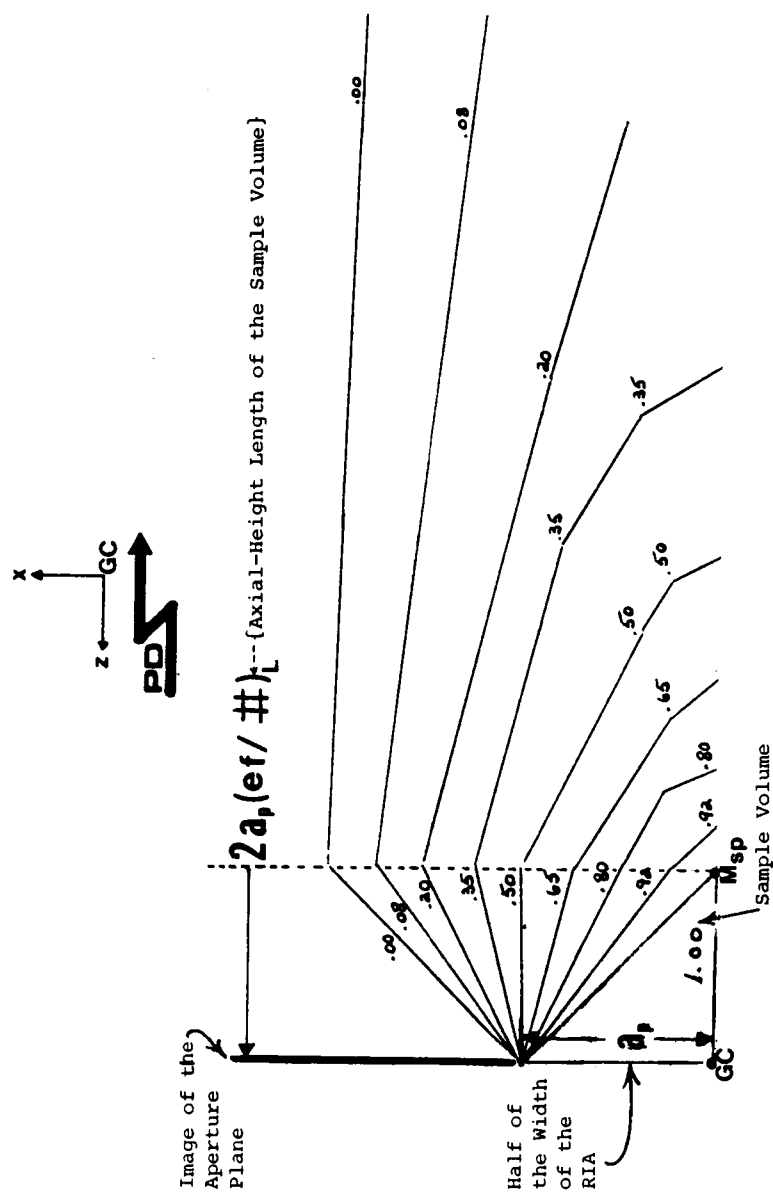
energy is transferred well into the geometrical shadow region [10], and results from pinhole edge diffraction effects. The energy transfer into the geometrical shadow diminishes the correlation between the experimental and theoretical data. This region is shown in Figure 27 for abscissa values greater than 2.0. The theoretical development is not intended for transverse planes that do not pass through the sample volume. However, there is still a good peak intensity prediction within a few axial-height lengths in the near vicinity of the sample volume. As the distance along the optical axis increases to approximately ten axial-height lengths, the theoretical data begins to predict a peak intensity that is double that of the experimental data. In this circular aperture image region strong diffractive and aberrational effects due to the combined action of the pinhole and the collecting lens take over; Diffraction and Aberration cause the image of the pinhole to defocus at the photo-detector aperture as the pinhole moves away from the in-focus image of the photo-detector aperture. It is the defocusing of the image of the pinhole that causes the measured peak intensity to be one-half of the theoretical value predicted by geometrical optics. These diffractive and aberrational effects were not considered in the theoretical analysis. The aim of this study was to determine the

range of applicability and good correlation of geometrical optics to this application.

The theoretical and experimental results for the case of the circular lens and the rectangular aperture are presented as follows.

The width of the rectangular aperture was set at the value of 200 μm and satisfied the requirements set forth in Chapter III.

First, the theoretical computations of the solid angle transfer function, as derived in Chapter VI, are presented as solid angle transfer contour plots in the following diagrams (see Figure 28 (a) and (b)). Contrary to the circularly symmetrical arrangement of the circular lens and the circular aperture, there is a lack of symmetry about the optical axis for this arrangement of the circular lens and the rectangular aperture; and two contour diagrams are necessary to describe the results. The geometry of the sample volume is approximately that of a rectangular parallelopiped, having a transverse-base width of $2a_p$ equal to 200 μm , a transverse-base length of $2b_p$ equal to 5,000 μm , and an axial-height length of $2a_p(\text{ef}/\#)_L$ equal to 836 μm . As in the circular aperture case, the value of the solid angle transfer function at any point within the sample volume is equal to unity. The shapes of all the surrounding contours consist of curved surfaces,



(a) Diagram of the Theoretical Results Representing the Solid Angle Transfer Function for the xz-plane at $x = 0$.

Figure 28. Graphical Representation of the Theoretical Results for the Case of the Circular Lens and the Rectangular Photo-detector Aperture.

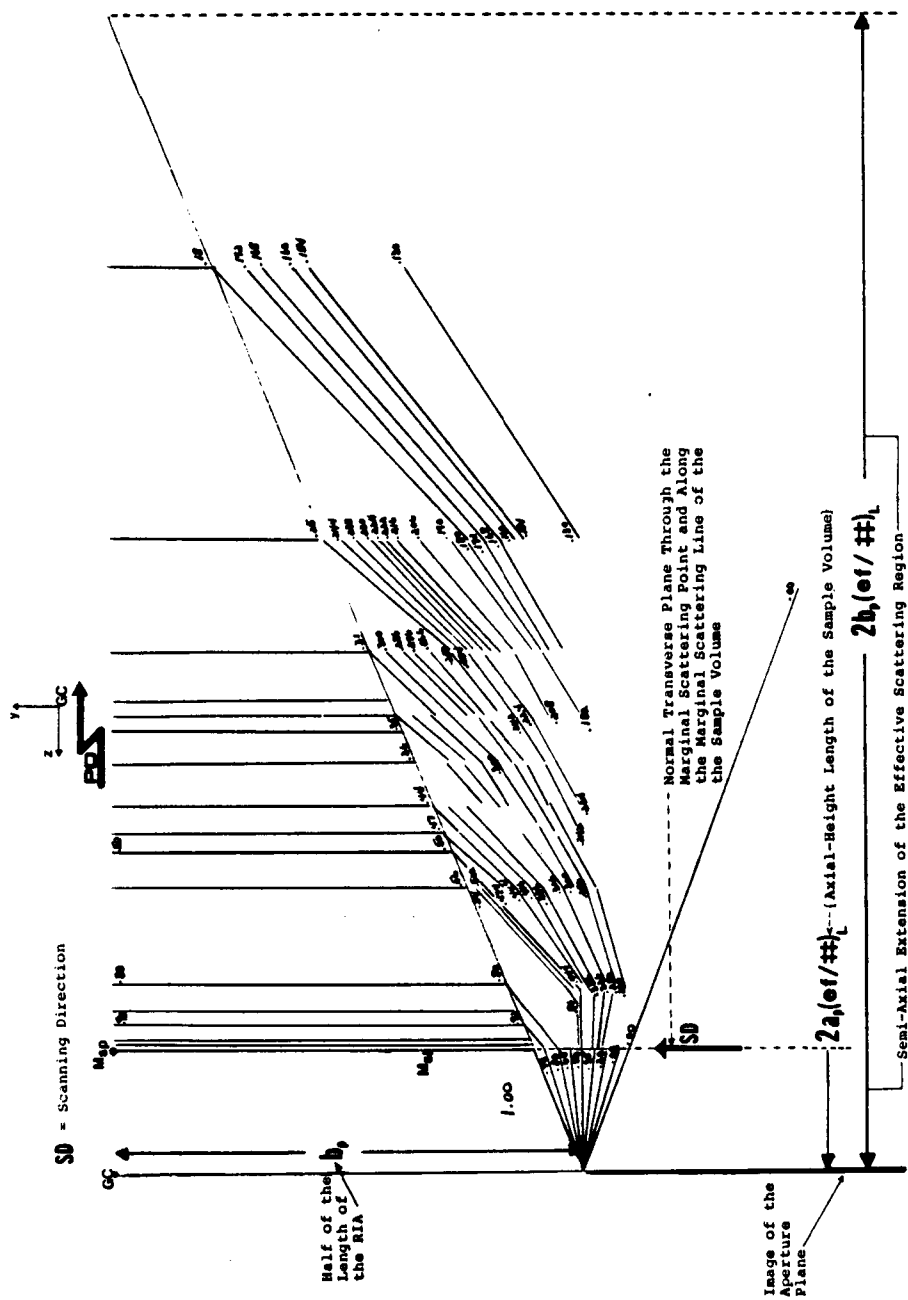


Figure 28. (Continued)

having relative transfer values ranging between zero and one (see Figure 28 (a) and (b)). Figure 28 (a) shows the contour plots for the xz-plane at y equal to zero, while Figure 28 (b) shows the contour plots for the yz-plane at x equal to zero.

Second, the experimental results are compared to the theoretical computations in Figure 29. This graph represents the solid angle transfer function as a function of a dimensionless quantity defined by t/a_p , which is also equal to $|y|/a_p + (ef/\#)_{s*} / (ef/\#)_L + b_p/a_p$. The graph is a plot of the experimental and theoretical values of the solid angle transfer function taken in the normal transverse plane through the marginal point of the sample volume, where $(ef/\#)_L$ is equal to $(ef/\#)_{s*}$. Figure 29 is chosen as the most representative graph because it characterizes the behavior of the solid angle transfer function in any other normal transverse plane through the sample volume. It also allows for representative sampling of the solid angle transfer function in the region of the image of the aperture space lying outside of the sample volume.

The comparison between the experimental and theoretical results is good; when error, associated with the diffractive and aberrational effects arising from the propagation of the laser light through the circular

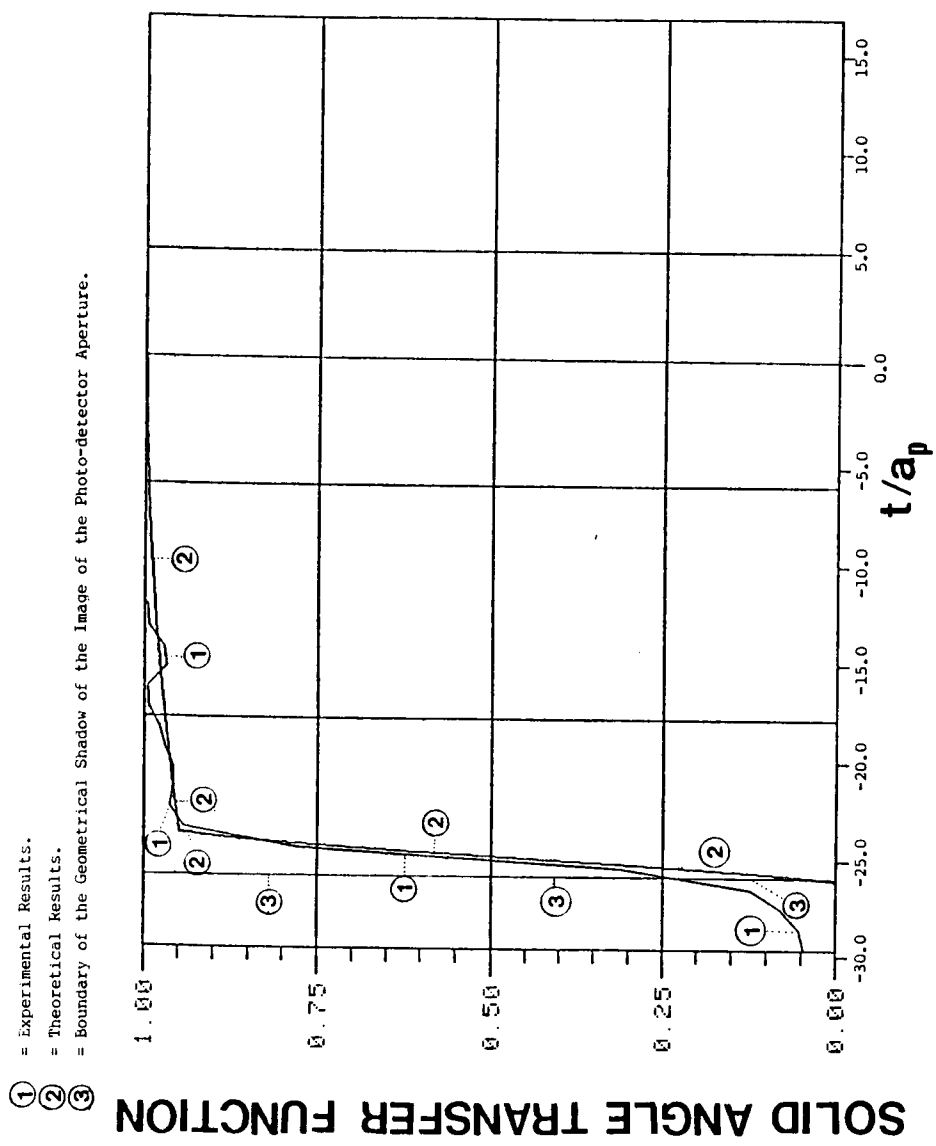


Figure 29. Graphical Representation of the Experimental and Theoretical Results for the Case of the Circular Lens/Rectangular Aperture.

pinhole and the defocusing properties of the collecting lens, is considered. The region of good correlation between the experimental and theoretical data lies within the illuminated region predicted by geometrical optics. The region of good correlation is shown in Figure 29 for the abscissa values between 0.0 and -26.0. The geometrical shadow is the point on the abscissa directly above the 0.25 intensity level of the experimental data [10]. The observations that are made for the good correlation region of the circular lens and the circular aperture also apply for the good correlation region of the circular lens and the rectangular aperture. The graph in Figure 29 shows that some of the energy is transferred well into the geometrical shadow region [10], and results from pinhole edge diffraction effects. The energy transfer into the geometrical shadow decreases the correlation between the experimental and theoretical data. Theoretical arguments developed in this study break-down when comparative graphs are examined for other normal transverse planes that do not pass through the sample volume. Finally, it should be noted that as the distance along the optical axis increases to approximately twenty axial-height lengths, the theoretical data begins to predict a peak intensity that is double that of the experimental data. In this rectangular aperture image region, strong diffractive and aberrational effects due to the combined action of

the pinhole and the collecting lens take over; Diffraction and Aberration cause the image of the pinhole to defocus at the photo-detector aperture as the pinhole moves away from the in-focus image of the photo-detector aperture. It is the defocusing of the image of the pinhole that causes the measured peak intensity to be one-half of the theoretical value predicted by geometrical optics.

CHAPTER IX

CONCLUSION

This chapter summarizes study results. It discusses the extent, limitations, and applications of the geometrical optics solid angle transfer function as a descriptive parameter for particle-sizing measurements in Laser Doppler Velocimetry.

The main idea of the study was that the lens-aperture combination of the collection optics can be made to act as a spatial filter controlling the optical sampling capability of the LDV-system, when the image of the photo-detector aperture is superimposed on the probe volume. The objective of the spatial filtering technique is to reduce the field of view of the collection optics such that only scattered light from near the center of the probe volume gets detected.

A partial elimination of the trajectory ambiguity of the particle location within the volume flow across the probe volume of an LDV-system results using aperture limiting techniques. A probe volume-sample volume differentiation was used as basis for a simple analysis for dealing with particle-sizing measurements. The analysis allows a partial elimination in the ambiguity of the scattering information that arises from otherwise

indistinguishable signals produced by (1) small particles traversing near the geometrical center, and (2) large particles grazing the outer regions of the probe volume.

The LDV-system was modeled as a defocused, scattered light collecting optical system. Geometrical optics indicates that the reduction in the field of view of the instrument can be analyzed by limiting the scattered light rays using apertures and/or stops. The applicability of the geometrical optics ray trace analysis is in the forward light scattering region. Vignetting estimates for a defocused optical system formed the foundation of this model. It was shown that the mechanical interruption of the transmitted radiation otherwise incident upon the photo-detector aperture, can be modeled using a solid angle ratio concept. The solid angle ratio was shown to evolve out of a vignetting factor that was defined as the ratio of the area of the collecting lens to the area of the image of the photo-detector aperture which formed the entrance pupil of the optical system. While the vignetting factor ascribes a point value to the optical system, the solid angle ratio evaluates the scattered light collection performance of the system at all points in the image of the aperture space surrounding the sample volume. The solid angle ratio divides the amount of energy, collected by the lens, by that part of the collected energy that passes through

the entrance pupil. It was shown that all solid angle calculations can be performed in the image of the aperture space, thereby eliminating the necessity of lens transformation calculations.

There were two analytical cases in this study, the Circular Lens/Circular Aperture and the Circular Lens/Rectangular Aperture geometries. In the former case, the solid angle ratio was obtained using geometrical arguments. In the latter case, the more complicated geometry required that the solid angle ratio be obtained using the convolution theorem. It was shown that the solid angle transfer function is a generalized form of the solid angle ratio in which the convolution integral includes all geometries. Solutions of the convolution integral show the same mathematical form of the solid angle transfer function for both geometrical cases.

The potential applicability of the solid angle transfer function to LDV-experiments lies within a limited range of the system's design parameters. In the design of a scattered light collecting system, the photo-detector aperture must be chosen as small as possible so that spatial filtering within the probe volume is achieved without having to rely on a high power of reduction of the image of the aperture. A low power of reduction of the image of the aperture is desired to

avoid unnecessary increased levels in the aberrational effects. The photo-detector aperture must be large enough to collect all of the light, transmitted by the collecting lens, that is scattered from a region of the image of the aperture space near the geometrical center of the sample volume. For an $f/4$ collecting lens and from Eq. (5), the smallest dimension of the photo-detector aperture is approximately $200\text{ }\mu\text{m}$. For a low power of reduction of the image of the aperture, the largest dimension of the photo-detector aperture can be approximated at $400\text{ }\mu\text{m}$.

Only the scattered light within the sample volume and a region of the image of the aperture space near the sample volume can be predicted by the solid angle transfer function. This region is approximately one to one-and-one-half times the size of the sample volume. For regions further away, the theory ceases to correlate with the experimental data. Diffractive and aberrational effects of the pinhole and the collecting lens become dominant. These have the effect of defocusing the image of the pinhole at the photo-detector aperture.

Diffractive and aberrational effects were not considered in this analysis. The aim of this study was to determine the range of applicability of geometrical optics applied to a defocused, forward-scattered light

collecting optical system. Further research work can extend the applicability of the solid angle transfer function to include diffractive and aberrational effects with wave optics replacing ray optics; the resulting solid angle transfer would become the diffraction optics solid angle transfer function. By including diffractive and aberrational effects, the accuracy of the solid angle transfer functions, for regions of the image of the aperture space several times the size of the sample volume, should substantially improve.

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