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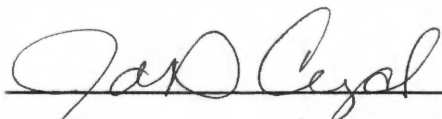
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EFFECTS OF COEFFICIENT ERRORS  
ON THE PERFORMANCE PREDICTIONS OF  
A TYPICAL FIGHTER AT MEDIUM ALTITUDE

A Thesis  
Presented for the  
Master of Science  
Degree  
The University of Tennessee, Knoxville

John D. Cloyd

December, 1988

## DEDICATION

This thesis is dedicated to four very special people. To my parents, whose steadfast support enabled me to realize my dreams, first in cockpit of the premier fighter aircraft of the time, and now in the pursuit of the knowledge of aerodynamics. To my wife, Hsiu Hua, whose encouragement and tolerance through the long hours make this accomplishment as much hers as mine. And to my daughter, Kristy, who managed to keep me smiling no matter how frustrating the problem had become. It is their presence in my life that gives meaning to anything I endeavor to accomplish.

I also dedicate this work to the memory of Thomas David Cloyd, Maud Hixson Cloyd, John Baker Sheats, and Grace Napps Sheats.

## ACKNOWLEDGEMENTS

I wish to express my gratitude to those who have greatly assisted me in the preparation of this thesis. I am particularly indebted to Dr. A. D. Vakili, my major professor, whose advice and suggestions made this work far better than it would otherwise have been.

A special thank you to Dr. C. F. Lo whose efforts on my behalf enabled me to pursue this investigation. Also, thanks are due to Dr. E. M. Kraft and R. L. Parker, Jr. without whose support this work could not have been accomplished in a timely manner.

Special appreciation is extended to Daryl Sinclair who helped nurture the investigation through its infancy, and to Clay Bearden for his assistance with the Tekplot computer program. I am especially grateful to Suzanne Foster for the hours she spent in preparation of the text and tables.

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## ABSTRACT

Performance prediction is one of the primary uses of wind tunnel data. Uncertainties in the tunnel data are therefore directly related to errors in performance predictions. The objective of this study was to examine the effects of aerodynamic coefficient errors on the point performance predictions for a typical fighter at medium altitude and subsonic velocity. The point performance parameters were formulated as functions of the aerodynamic coefficients. A computer program was written to calculate the selected performance parameters for 121 cases, each case representing a different combination of coefficient errors. The result was the correlation of the percentage error in the point performance parameters as a function of the percent errors in the aerodynamic coefficients. The use of these techniques would result in more efficient testing through the identification of regimes where test data accuracy is critical for accurate performance prediction.

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## NOMENCLATURE

|                |                                                                  |
|----------------|------------------------------------------------------------------|
| $a$            | Local sonic velocity, ft/sec                                     |
| $a_c$          | Radial acceleration (horizontal turn), ft/sec <sup>2</sup>       |
| AOA            | Angle of Attack, deg                                             |
| $a_r$          | Acceleration perpendicular to local horizon, ft/sec <sup>2</sup> |
| $a_\theta$     | Acceleration parallel to local horizon, ft/sec <sup>2</sup>      |
| $c$            | Thrust specific fuel consumption, lbm/lbf-sec                    |
| $C_D$          | Coefficient of drag                                              |
| $\Delta C_D$   | Increment in coefficient of drag                                 |
| $C_L$          | Coefficient of lift                                              |
| $\Delta C_L$   | Increment in coefficient of lift                                 |
| CPU            | Central processing unit                                          |
| $D$            | Total drag, lbf                                                  |
| $E$            | Total energy, ft lbf                                             |
| $E_{a_\theta}$ | Error in horizontal acceleration, percent                        |
| $E_{C_D}$      | Error in coefficient of drag, percent                            |
| $E_s$          | Specific range, ft/lbm fuel                                      |
| $E_T$          | Error in thrust, percent                                         |
| $F_c$          | Centrifugal force (horizontal turn), lbf                         |
| $g$            | Local gravitational acceleration, ft/sec <sup>2</sup>            |
| $g_0$          | Sea level gravitational acceleration, 32.174 ft/sec <sup>2</sup> |
| $h$            | Height above sea level, ft                                       |
| $L$            | Total lift, lbf                                                  |

|                  |                                                            |
|------------------|------------------------------------------------------------|
| m                | Mass, lbm                                                  |
| M                | Mach number                                                |
| MSL              | Mean sea level                                             |
| n                | Load factor, g's                                           |
| p                | Local atmospheric pressure, lbf/ft <sup>2</sup>            |
| PERCAL           | Point performance computer program                         |
| P <sub>s</sub>   | Specific excess power, ft/sec                              |
| q                | dynamic pressure (=0.5 $\rho V^2$ ), lbf/ft <sup>2</sup>   |
| R                | Distance traveled, ft                                      |
| r <sub>e</sub>   | Radius of the earth, 20890584.0 ft                         |
| R <sub>s</sub>   | Specific range, ft/lbm fuel                                |
| r <sub>t</sub>   | Turn radius, ft                                            |
| S                | Reference area, ft <sup>2</sup>                            |
| T                | Net thrust, lbf                                            |
| T <sub>req</sub> | Thrust required, lbf                                       |
| TSFC             | Thrust specific fuel consumption, lbm/lbf-sec              |
| V                | Velocity along flight path, ft/sec                         |
| V <sub>n</sub>   | Velocity normal to flight path, ft/sec                     |
| V <sub>R</sub>   | Velocity perpendicular to local horizon, ft/sec            |
| V <sub>θ</sub>   | Velocity parallel to local horizon, ft/sec                 |
| W                | Weight corrected for altitude above sea level, lbf         |
| W <sub>0</sub>   | Sea level weight, lbf                                      |
| W <sub>f</sub>   | Fuel consumed, lbm                                         |
| α                | Angle between flight path and fuselage reference line, deg |

|              |                                                  |
|--------------|--------------------------------------------------|
| $\gamma$     | Angle between flight path and local horizon, deg |
| $\gamma_s$   | Ratio of specific heats, 1.4 for air             |
| $\phi$       | Bank angle, deg                                  |
| $\dot{\psi}$ | Turn rate, deg/sec                               |

## CHAPTER I

### INTRODUCTION

#### A. BACKGROUND

Considerable attention has been focused on establishing the data accuracy requirements associated with wind tunnel testing. In Reference 1, Steinle and Stanewsky suggest coefficient accuracy requirements of 0.01 for  $\Delta C_L$  and 0.0001 for  $\Delta C_D$ . The accuracy requirements for extrapolation of wind tunnel data to the flight condition was specifically addressed by AGARD Working Group 09 on boundary layer simulation and control. For fighter aircraft the Group's conclusions were stated by Haines (Reference 2) as a  $\Delta C_L$  of 0.02 for buffet onset, a  $\Delta C_D$  of 0.0002 for cruise and loiter, and a  $\Delta C_D$  of 0.0005 for maneuver and high speed dash. These goals serve as impetus for continued improvements in testing equipment and methodology.

In contrast to the establishment of test data accuracy goals, very little has been published detailing the coefficient errors associated with testing in a particular wind tunnel. This is partly attributable to difficulty in establishing the relative magnitude of the errors associated with the multitude of sources (balance, humidity, wall interference, etc.), as the errors may vary considerably with

configuration and flow field from test to test. Whorric and Hobbs (Reference 3) found the total uncertainties for the aerodynamic coefficients in the Arnold Engineering Development Center (AEDC) Aerodynamic Wind Tunnel (4T) and Propulsion Wind Tunnel (16T) to be significantly outside the Reference 1 and 2 requirements. Reference 3 establishes a  $\Delta C_L$  range from 0.012 to 0.035 (3 to 16 percent nominal lift), and a  $\Delta C_D$  range from 0.002 to 0.004 (3 to 14 percent nominal drag) for typical fighter and transport configurations.

With the lack of documentation on the error levels associated with specific wind tunnels its no wonder that works relating wind tunnel error levels to performance predictions are rare. A general criteria for relating coefficient accuracies to allowable errors in design and performance parameters for supersonic missiles is given in Reference 4. Though best suited for conceptual and preliminary design, the Reference 4 criteria would produce a usable error band on performance predictions given the wind tunnel coefficient errors. Unfortunately a comparable work pertaining to aircraft was not found.

## B. STATEMENT OF THE PROBLEM

For research purposes, the accuracy of wind tunnel data should be such that the data can be used with the confidence that inaccuracies will not compromise the research objectives. The objective of the study documented in Reference 3 was to prioritize the uncertainty

sources in tunnel test in order to concentrate resources on those area where higher measurement accuracies will yield the most significant improvement in the data. Also, by establishing the data uncertainties, the research objectives may be set more accurately.

Wind tunnel data is used by the manufacturer to achieve the most accurate performance prediction obtainable within cost constraints. In this respect, uncertainties in the tunnel data are directly related to uncertainty in the predicted performance.

The objective of this study was to determine the effects of uncertainties in wind tunnel test data on the point performance predictions for a typical fighter at medium altitude and subsonic velocity.

### C. APPROACH TO INVESTIGATION

A five step process was used to determine the effect of uncertainties in aerodynamic coefficients on the point performance predictions for a typical fighter:

- 1) specific point performance parameters were chosen which encompassed the spectrum of capabilities of a typical fighter at medium altitude,
- 2) the equations of the selected parameters were formulated in terms of the aerodynamic coefficients and thrust model,

- 3) a computer program was written which calculates the selected performance parameters from an input aerodynamic and thrust models,
- 4) an aerodynamic model which closely approximates a typical fighter at 30,000 feet mean sea level (MSL) was developed from a combination of wind tunnel, flight test, and flight manual data,
- 5) the aerodynamic model was input into the computer program, varying the coefficients of lift and drag from -20 to +20 percent, with the percentage error of the selected performance parameters as a function of percentage error of the coefficients of lift and drag noted.

#### D. ORGANIZATION OF THESIS

In Chapter II the basic relationships upon which the point performance parameters are based will be developed. The equations used in the computer program to calculate the selected performance parameters will be derived.

Chapter III will discuss the methodology used to develop the aerodynamic and thrust model used for input to the computer program. The coefficients of lift and drag as functions of Mach number and angle-of-attack, thrust available as a function of mach number, and

thrust specific fuel consumption as function of Mach number and thrust level will be presented.

In Chapter IV the effects of errors in the coefficients of lift and drag on the selected performance parameters will be examined. The percent error in each performance parameter as a function of percent errors in the aerodynamic coefficients will be shown.

Chapter V contains the conclusions drawn from the investigation and recommendations for further improvements and implementations of the point performance techniques presented.

## CHAPTER II

### POINT PERFORMANCE PARAMETER

#### A. DEFINITION AND ASSUMPTIONS

Point performance is defined as the aircraft performance at a given weight, altitude, and throttle setting. The equations for the parameters of interest are derived from the basic relationships in the horizontal and vertical planes. Throughout the determination of the point performance parameters the following assumptions are employed:

- 1) the aircraft is considered as a point mass,
- 2) the thrust vector incidence angle is zero,
- 3) thrust acts down the chord line,
- 4) atmospheric properties are known functions of altitude.

The point performance equations and parameters have been extensively treated in the technical literature on aircraft design and performance. The real value of point performance techniques is that they reduce a problem with a large number of parameters to reasonably simple expressions. Although the assumptions made will lead to some degree of inaccuracy, the results are no less meaningful as they quantify the relationship between the primary parameters with reasonable accuracy.

## B. BASIC RELATIONS

In the next three sections the basic relationships in the vertical and horizontal planes, and energy state approximation will be derived. These relationships form the foundation of the point performance parameters.

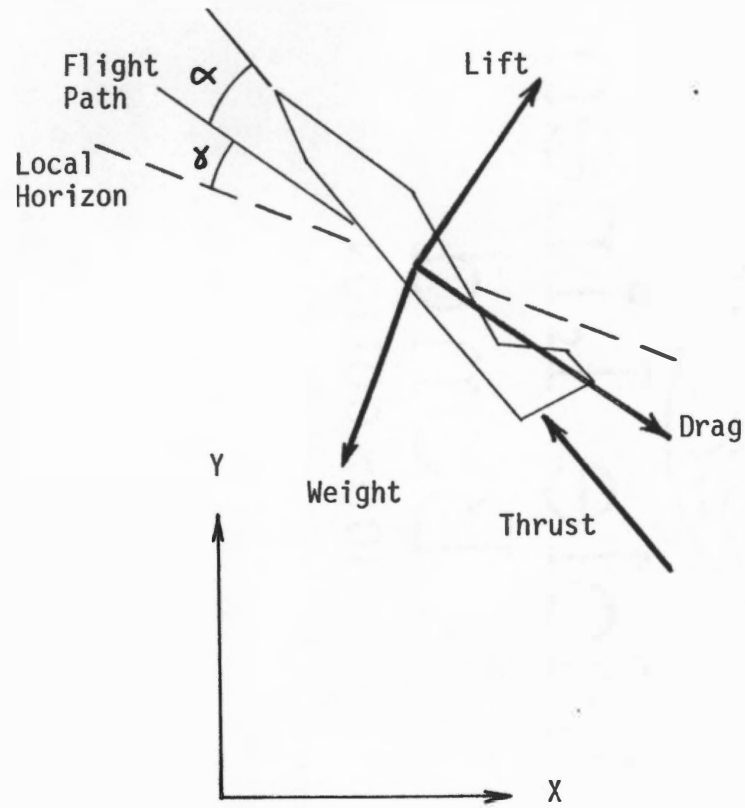
### 1. THE VERTICAL PLANE

The basic relations in the vertical plane are derived from the free body diagram as shown in Figure 1. Equating forces parallel and normal to the flight path:

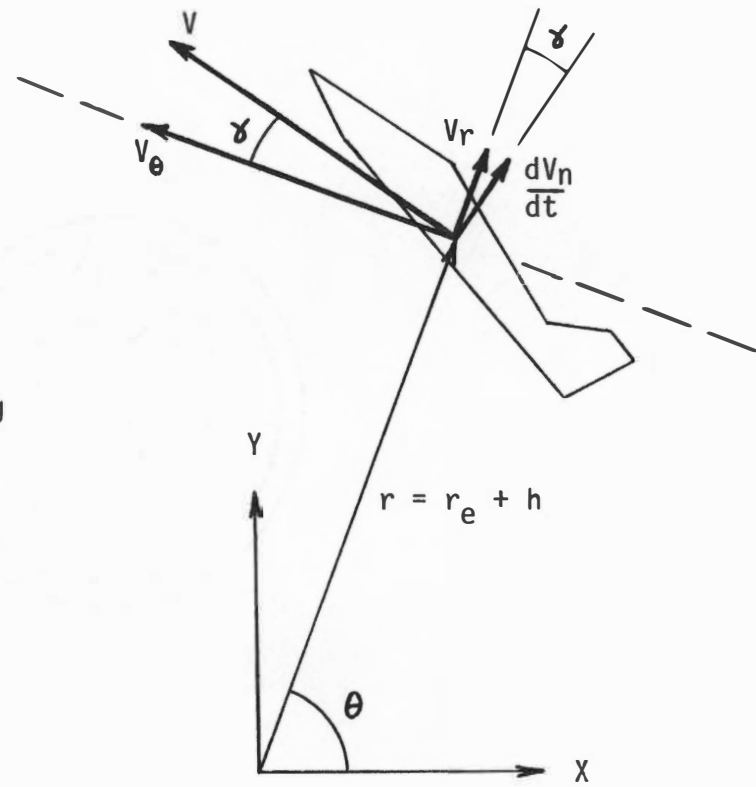
$$T \cos \alpha - D - W \sin \gamma = m \frac{dV}{dt} \quad (1)$$

$$T \sin \alpha + L - W \cos \gamma = m \frac{dV_n}{dt} \quad (2)$$

The weight in Eq. (1) and Eq. (2) is calculated from the sea level weight ( $W_0$ ), sea level gravitational acceleration ( $g_0$ ), and the local acceleration of gravity ( $g$ ):



A. Forces



B. Velocities

Figure 1. Basic Relations in the Vertical Plane

$$g = g_o \left[ \frac{r_e^2}{(r_e + h)^2} \right]$$

$$W = \frac{W_o r_e^2}{(r_e + h)^2} \quad (3)$$

where  $r_e$  = the radius of the earth, and

$h$  = the altitude above sea level.

From the derivation in Appendix A, the acceleration normal to the flight path ( $dV_n/dt$ ) is:

$$\frac{dV_n}{dt} = V \frac{d\gamma}{dt} - \frac{V^2 \cos \gamma}{(r_e + h)} \quad (4)$$

Substituting from Eq. (3) and Eq. (4) into Eq. (1) and Eq. (2) yields the general equations for forces parallel and normal to the flight path in the vertical plane:

$$T \cos \alpha - D - \frac{W_o r_e^2 \sin \gamma}{(r_e + h)^2} = \frac{W_o}{g_o} \frac{dV}{dt} \quad (5)$$

$$T \sin \alpha + L - \frac{W_o r_e^2 \cos \gamma}{(r_e + h)^2} = \frac{W_o}{g_o} \left[ V \frac{d\gamma}{dt} - \frac{V^2 \cos \gamma}{(r_e + h)} \right] \quad (6)$$

## 2. THE HORIZONTAL PLANE

The Basic relations in the horizontal plane are derived from the free body diagram as shown in Figure 2. The turn is considered to be level, steady, and coordinated (zero side forces). Equating forces in the horizontal and vertical:

$$L = W / \cos\phi \quad (7)$$

$$L \sin\phi = F_c \quad (8)$$

The centrifugal force in Eq. (8) is equal to the product of the aircraft mass and the radial acceleration ( $a_c$ ):

$$F_c = \frac{W}{g} a_c \quad (9)$$

The turn rate ( $\dot{\Psi}$ ) and radial acceleration are given by:

$$\dot{\Psi} = \frac{V}{r_t} \quad (10)$$

and,

$$a_c = r_t \dot{\Psi}^2 = \frac{V^2}{r_t}$$

where  $r_t$  = the turn radius.

Substituting into Eq. (9):

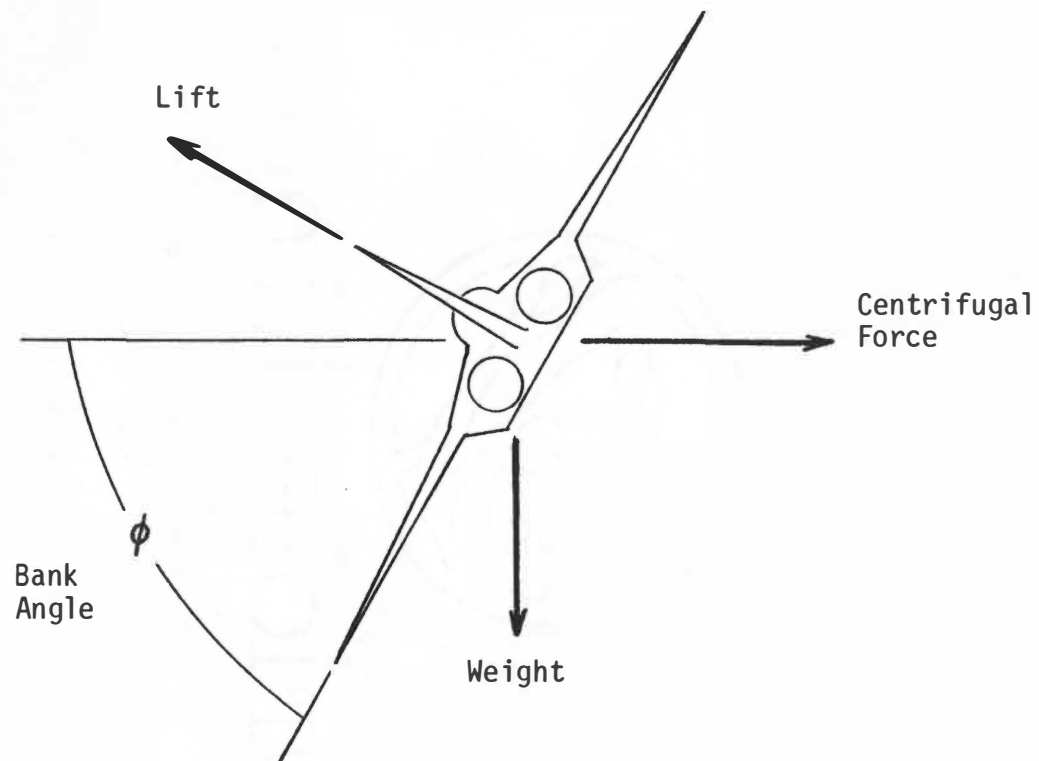


Figure 2. Basic Relations in the Horizontal Plane

$$F_c = \frac{W}{g} \frac{V^2}{r_t} \quad (11)$$

The load factor ( $n$ ) is defined as lift divided by weight:

$$n = \frac{L}{W} = \frac{1}{\cos\phi}$$

Substituting  $F_c$  from Eq. (11) into Eq. (8) and using  $\tan\phi = \sqrt{n^2 - 1}$  yields the basic equation for the turn radius:

$$r_t = \frac{V^2}{g\sqrt{n^2 - 1}} \quad (12)$$

Substituting for  $r_t$  in Eq. (10) gives the basic relation for the turn rate:

$$\dot{\psi} = \frac{g\sqrt{n^2 - 1}}{V} \quad (13)$$

### 3. ENERGY STATE APPROXIMATION

Assuming rotation to be zero, the aircraft's total energy at a specified altitude and velocity is:

$$E = Wh + \frac{WV^2}{2g}$$

Dividing through by weight and differentiating with respect to time gives the expression for the time rate of change of specific energy (referred to as specific excess power) with units of feet per second.

$$P_s = \frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt} \quad (14)$$

The  $P_s$  value represents the aircraft's capability to accelerate and/or climb. In level flight ( $dh/dt=0$ )  $P_s$  may be calculated by substituting for  $dV/dt$  in Eq. (1), multiplying through by  $V/W$ , and rearranging to yield:

$$P_s = \frac{V(T \cos \alpha - D)}{W} \quad (15)$$

### C. POINT PERFORMANCE PARAMETERS

To determine the point performance parameters from the basic relations the following aircraft aerodynamic and thrust data at the desired altitude is assumed known:

- 1) the coefficients of lift and drag as a function of Mach number and angle of attack,

- 2) the net thrust at the desired throttle setting as a function of the Mach number,
- 3) the thrust specific fuel consumption as a function of the Mach number and the throttle setting, and
- 4) the basic aircraft data (sea level weight, reference area, etc.).

The point performance methods in this chapter are a compilation of the techniques detailed in References 5, 6, 7, 8, 9, 10 and 11. The techniques in these references assume that the angle of attack is small enough such that the cosine equals one and the sine equals zero. With the coefficient of lift as a function of Mach number and angle of attack known, this assumption may be eliminated by the technique detailed in the following section.

## 1. MAXIMUM HORIZONTAL FLIGHT SPEED

As shown in Figure 3, the maximum horizontal flight speed is obtained from the intersection of the thrust available and thrust required curves. The net thrust available curve is assumed to be known. In order to calculate the thrust required at each velocity, the level nonaccelerating flight angle of attack at each velocity must be determined. With the coefficient of drag as a function of Mach number and angle of attack known, the thrust required at each velocity may be calculated.

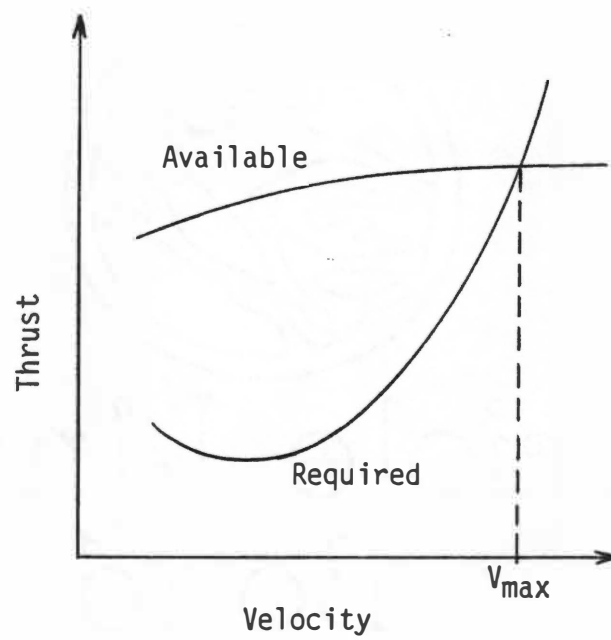


Figure 3. Thrust Available and Required

To determine the level nonaccelerating angle of attack an equation is derived which expresses the level flight coefficient of lift as a function of known parameters and angle of attack. From Eq. (6) with  $\gamma=0$  and  $d\gamma/dt=0$ :

$$L = W \left[ 1 - \frac{V^2}{g(r_e + h)} \right] - T \sin \alpha$$

Substituting into the definition of coefficient of lift the desired expression for the level flight coefficient of lift is obtained.

$$C_{L_{lvl\ flt}} = \frac{2 \left\{ W \left[ 1 - \frac{V^2}{g(r_e + h)} \right] - T \sin \alpha \right\}}{\gamma_s \rho M^2 S} \quad (16)$$

A unique level flight coefficient of lift (and corresponding angle of attack) at each velocity is determined by the intersection of the known coefficient of lift versus angle of attack curve at that velocity with the curve generated by Eq. (16).

With the level flight angle of attack and therefore coefficient of drag known the thrust required at each velocity is calculated from the basic relations and the definition of the coefficient of drag. From Eq. (1):

$$T = D / \cos \alpha$$

or

$$T_{reg} = (C_D \gamma_s p M^2 S) / (2 \cos \alpha) \quad (17)$$

## 2. HORIZONTAL ACCELERATION

The level flight horizontal acceleration ( $dV/dt$ ) is calculated by equating the right hand sides of Eq. (14) and Eq. (15) with  $dh/dt=0$ . Multiplying through by  $V/g$  yields:

$$\frac{dV}{dt} = \frac{g(T \cos \alpha - D)}{W} \quad (18)$$

## 3. MAXIMUM CLIMB ANGLE AND RATE

From Figure 1B the aircraft's rate of climb is given by  $dh/dt=V \sin \gamma$ . Holding the velocity constant in Eq. (14) yields:

$$P_s = \frac{dh}{dt} = V \sin \gamma$$

therefore:

$$\sin \gamma = \frac{P_s}{V} \quad (19)$$

For  $0 < \gamma < \pi/2$  the value of  $P_s/V$  which maximizes  $\sin \gamma$  also maximizes  $\tan \gamma$ . Therefore the maximum climb angle ( $\gamma_{max}$ ), and the corresponding velocity ( $V_A$ ), may be obtained from Figure 4 as the angle between the abscissa and a line through the origin tangent to the  $P_s$  versus velocity curve.

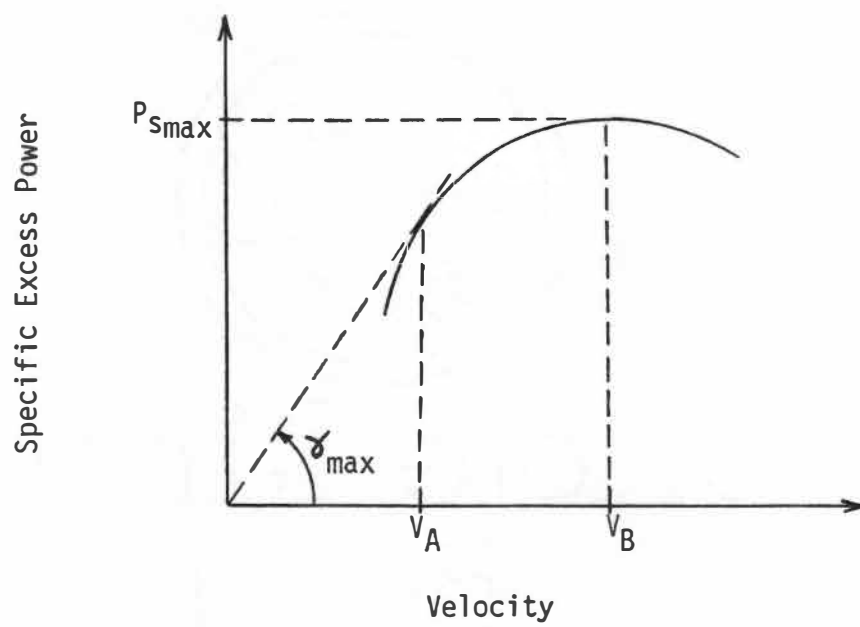


Figure 4. Specific Excess Power versus Velocity

Figure 4 is also used to obtain the maximum climb rate at constant velocity. The maximum climb rate  $(dh/dt)_{\max}$  equals  $P_{S\max}$  and occurs at velocity  $V_B$ .

#### 4. LEVEL FLIGHT STALL MACH

The equation for the level flight stall Mach is obtained by setting  $C_L = C_{L\max}$  in Eq. (16) and solving for Mach number:

$$M_{Stall} = \left[ \frac{2(W - T \sin \alpha)}{\gamma_s \rho S C_{L_{\max}} + \frac{2W a^2}{g(r_e + h)}} \right]^{\frac{1}{2}} \quad (20)$$

The level flight stall Mach is the Mach number at which the angle of attack required for level flight is equal to the angle of attack which produces  $C_{L\max}$ .  $M_{stall}$  is calculated as a function of Mach number and the intersection of that curve with a straight line through the origin with a slope of one yields the level flight stall Mach.

#### 5. BEST INSTANTANEOUS TURN

The maximum instantaneous turn rate and minimum turn radius are achieved at the lowest flight velocity where the structurally limiting load factor can be obtained. The weight in Eq. (20) is replaced by the product of the limiting load factor and the weight. An iterative

technique is used to find the lowest flight velocity for which Eq. (20) is satisfied. The turn radius is then calculated from Eq. (12) and the turn rate from Eq. (13).

## 6. MAXIMUM RANGE AND ENDURANCE

Specific range ( $R_s$ ) is the distance traveled divided by the weight of fuel consumed:

$$R_s = \frac{dR}{dW_f} = \frac{dR}{dt} \frac{dt}{dW_f} = \frac{V}{dW_f/dt} \quad (21)$$

The fuel flow rate ( $dW_f/dt$ ) can be expressed as the product of the engine thrust ( $T$ ) and the thrust specific fuel consumption ( $c$ ). Substituting into Eq. (21):

$$R_s = V/Tc \quad (22)$$

The maximum range Mach number is obtained by plotting specific range versus Mach number as shown in Figure 5.

Specific endurance ( $E_s$ ) is the flight time divided by the weight of fuel consumed. Employing the same methods used in obtaining Eq. (22):

$$E_s = 1/Tc \quad (23)$$

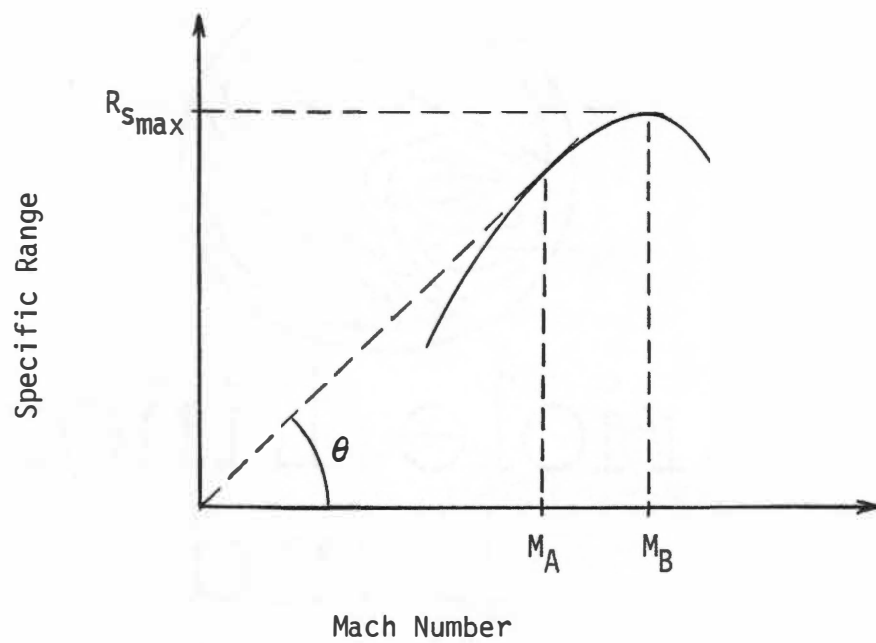


Figure 5. Specific Range versus Mach Number

The maximum endurance Mach number is also obtained from Figure 5. Noting that  $\tan \theta = R_S/V = 1/T_c = E_S$ , the maximum endurance Mach number ( $M_A$ ) is obtained by constructing a line through the origin tangent to the  $R_S$  versus Mach number curve as shown.

#### D. POINT PERFORMANCE COMPUTER PROGRAM

A Fortran computer program (PERCAL) was written to calculate the point performance parameters using the techniques of this chapter and the known aerodynamic coefficients and thrust data. The program employs a cubic spline routine for interpolating and uses the coefficients of the cubic equation on the appropriate interval to algebraically evaluate derivatives, maximum values, and curve intersections. The program has the capability to perturb the input lift and drag coefficients singularly or in combination in order to observe the effect of the perturbations on the performance parameters.

## CHAPTER III

### AIRCRAFT MODEL

#### A. LIFT COEFFICIENTS

The low angle of attack (AOA) coefficient of lift data is given in Table 1. This data is the measured coefficients taken on a 0.05-scale model of a fighter aircraft in the AEDC tunnel 4T. The high AOA data is given in Table 2. The Table 2 data is the measured coefficients, corrected for cavity and base pressure and weight tares, taken on a 0.047-scale model of the same fighter in the AEDC tunnel 16T. Both sets of data were used to obtain  $C_L$  data over a full range of AOA.

Using the wind tunnel data,  $C_L$  versus Mach number curves were constructed at two AOAs in the low AOA range. Assuming a linear relationship,  $C_L$  versus Mach number curves were calculated at specific AOA intervals throughout the low AOA regime. In the region where the high and low AOA data overlapped,  $C_L$  versus Mach number curves were constructed as shown in Figure 6. In the high AOA regime  $C_L$  versus Mach number curves were constructed at selected AOAs from the high AOA data. Then  $C_L$  versus Mach number curves at specific intervals through the high AOA regime were calculated iteratively by cross comparison of  $C_L$  versus Mach number at constant AOA with  $C_L$  versus AOA at constant Mach. The resulting data (Table 3) was used as the lift coefficient

Table 1. Low AOA Reference Lift Coefficient Data

| AOA              | MACH NUMBER |        |        |        |        |        |        |         |        |
|------------------|-------------|--------|--------|--------|--------|--------|--------|---------|--------|
|                  | LOW         | →      | →      | →      | →      | →      | →      | →       | HIGH   |
| LOW              | -0.105      | -0.115 | -0.110 | -0.120 | -0.125 | -0.140 | -0.185 | ----    | -0.135 |
| ↓                | 0.030       | 0.030  | 0.035  | 0.035  | 0.040  | 0.035  | 0.000  | - 0.010 | 0.005  |
| ↓                | 0.170       | 0.180  | 0.185  | 0.190  | 0.205  | 0.205  | 0.205  | 0.180   | 0.155  |
| ↓                | 0.300       | 0.320  | 0.330  | 0.350  | 0.375  | 0.390  | 0.395  | 0.340   | 0.310  |
| ↓                | 0.435       | 0.470  | 0.490  | 0.515  | 0.555  | 0.565  | 0.580  | 0.505   | 0.470  |
| ↓                | 0.585       | 0.620  | 0.650  | 0.680  | 0.720  | 0.735  | 0.750  | 0.680   | 0.620  |
| ↓                | 0.715       | 0.745  | 0.770  | 0.795  | 0.830  | 0.875  | 0.905  | 0.840   | 0.775  |
| ↓                | 0.825       | 0.855  | 0.870  | 0.900  | 0.925  | 0.945  | 1.035  | 0.990   | 0.900  |
| ↓                | 0.940       | 0.970  | .990   | 1.010  | 1.040  | 1.060  | 1.145  | 1.110   | 1.030  |
| ↓                | 1.035       | 1.075  | 1.095  | 1.115  | 1.150  | 1.170  | 1.250  | 1.220   | 1.150  |
| ↓                | 1.135       | 1.170  | 1.185  | 1.210  | 1.235  | 1.255  | 1.350  | 1.330   | 1.250  |
| MID              | 1.230       | 1.260  | 1.270  | 1.295  | 1.320  | 1.335  | 1.450  | 1.435   | 1.350  |
| LIFT COEFFICIENT |             |        |        |        |        |        |        |         |        |

Table 2. High AOA Reference Lift Coefficient Data

| AOA              | MACH NUMBER |        |        |        |        |
|------------------|-------------|--------|--------|--------|--------|
|                  | LOW         | →      | →      | →      | HIGH   |
| MID              | 0.6810      | 0.6976 | 0.7269 | 0.7689 | 0.8992 |
| ↓                | 0.7992      | 0.8270 | 0.8760 | 0.9123 | 1.0046 |
| ↓                | -----       | 0.9416 | 0.9928 | 1.0315 | 1.1200 |
| ↓                | 0.9675      | 0.9987 | 1.0427 | -----  | -----  |
| ↓                | 1.0181      | 1.0510 | 1.1058 | 1.1533 | 1.2601 |
| ↓                | 1.1134      | 1.1405 | 1.1964 | 1.2559 | 1.3653 |
| ↓                | 1.2080      | 1.2397 | 1.2862 | 1.3522 | 1.4901 |
| ↓                | 1.2926      | 1.3062 | 1.3577 | 1.4424 | 1.5590 |
| ↓                | 1.3511      | 1.3728 | 1.4220 | 1.5079 | 1.6200 |
| ↓                | 1.4007      | 1.4269 | 1.4763 | 1.5553 | 1.6929 |
| ↓                | 1.4334      | 1.4730 | 1.5094 | 1.5958 | 1.7354 |
| ↓                | 1.4727      | 1.5144 | 1.5398 | 1.6151 | 1.7623 |
| ↓                | 1.5071      | 1.5344 | 1.5540 | 1.6280 | 1.7865 |
| ↓                | 1.5324      | 1.5556 | 1.5574 | 1.6322 | 1.7971 |
| ↓                | 1.5438      | 1.5477 | 1.5611 | 1.6454 | 1.8009 |
| ↓                | 1.5333      | 1.5368 | 1.5609 | 1.6589 | 1.8094 |
| ↓                | 1.5021      | 1.5145 | 1.5530 | 1.6532 | 1.7993 |
| HIGH             | 1.4591      | 1.4871 | 1.5360 | 1.6301 | 1.7569 |
| LIFT COEFFICIENT |             |        |        |        |        |

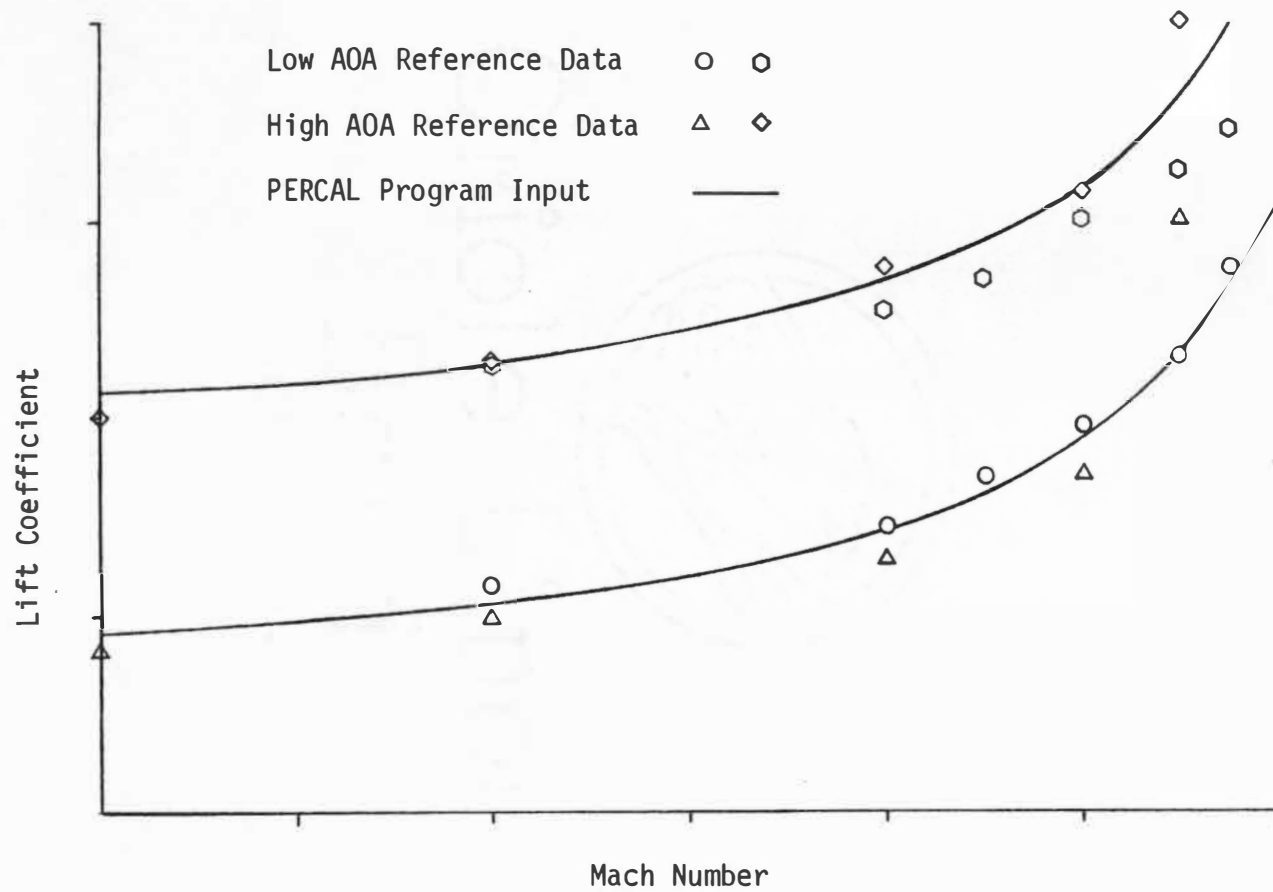


Figure 6. Lift Coefficient versus Mach Number

Table 3. PERCAL Computer Program Lift Coefficient Data

| AOA              | MACH NUMBER |        |        |        |        |        |        |        |
|------------------|-------------|--------|--------|--------|--------|--------|--------|--------|
|                  | LOW         | →      | →      | →      | →      | →      | →      | →      |
| LOW              | -0.111      | -0.111 | -0.111 | -0.111 | -0.111 | -0.111 | -0.111 | -0.111 |
| ↓                | 0.022       | 0.022  | 0.023  | 0.023  | 0.024  | 0.024  | 0.025  | 0.026  |
| ↓                | 0.155       | 0.155  | 0.156  | 0.157  | 0.158  | 0.159  | 0.161  | 0.163  |
| ↓                | 0.288       | 0.288  | 0.290  | 0.291  | 0.293  | 0.295  | 0.297  | 0.300  |
| ↓                | 0.421       | 0.422  | 0.424  | 0.426  | 0.428  | 0.430  | 0.433  | 0.437  |
| ↓                | 0.554       | 0.555  | 0.557  | 0.559  | 0.562  | 0.565  | 0.569  | 0.574  |
| ↓                | 0.687       | 0.688  | 0.691  | 0.694  | 0.697  | 0.700  | 0.705  | 0.711  |
| ↓                | 0.806       | 0.808  | 0.812  | 0.814  | 0.817  | 0.822  | 0.827  | 0.835  |
| ↓                | 0.915       | 0.916  | 0.920  | 0.924  | 0.930  | 0.935  | 0.942  | 0.951  |
| ↓                | 1.018       | 1.020  | 1.024  | 1.027  | 1.032  | 1.038  | 1.046  | 1.056  |
| ↓                | 1.113       | 1.114  | 1.119  | 1.123  | 1.129  | 1.136  | 1.145  | 1.155  |
| ↓                | 1.201       | 1.202  | 1.206  | 1.210  | 1.215  | 1.222  | 1.231  | 1.242  |
| ↓                | 1.277       | 1.278  | 1.282  | 1.286  | 1.292  | 1.299  | 1.309  | 1.321  |
| ↓                | 1.343       | 1.344  | 1.349  | 1.354  | 1.360  | 1.368  | 1.378  | 1.389  |
| ↓                | 1.401       | 1.402  | 1.408  | 1.412  | 1.419  | 1.427  | 1.437  | 1.450  |
| ↓                | 1.450       | 1.451  | 1.456  | 1.462  | 1.468  | 1.476  | 1.486  | 1.498  |
| ↓                | 1.487       | 1.488  | 1.494  | 1.499  | 1.506  | 1.514  | 1.523  | 1.535  |
| ↓                | 1.514       | 1.515  | 1.521  | 1.526  | 1.533  | 1.541  | 1.551  | 1.564  |
| ↓                | 1.532       | 1.533  | 1.538  | 1.542  | 1.549  | 1.557  | 1.569  | 1.583  |
| ↓                | 1.532       | 1.534  | 1.543  | 1.549  | 1.556  | 1.564  | 1.574  | 1.584  |
| ↓                | 1.524       | 1.526  | 1.533  | 1.538  | 1.544  | 1.551  | 1.559  | 1.569  |
| ↓                | 1.492       | 1.494  | 1.502  | 1.508  | 1.515  | 1.523  | 1.532  | 1.543  |
| HIGH             | 1.452       | 1.454  | 1.459  | 1.463  | 1.468  | 1.475  | 1.484  | 1.494  |
| LIFT COEFFICIENT |             |        |        |        |        |        |        |        |

Table 3. Concluded

| AOA              | MACH NUMBER |        |        |        |        |        |        |
|------------------|-------------|--------|--------|--------|--------|--------|--------|
|                  | →           | →      | →      | →      | →      | →      | HIGH   |
| LOW              | -0.109      | -0.107 | -0.103 | -0.099 | -0.092 | -0.092 | -0.108 |
| ↓                | 0.029       | 0.032  | 0.038  | 0.044  | 0.054  | 0.062  | 0.061  |
| ↓                | 0.167       | 0.172  | 0.179  | 0.188  | 0.201  | 0.215  | 0.231  |
| ↓                | 0.305       | 0.311  | 0.320  | 0.331  | 0.347  | 0.369  | 0.400  |
| ↓                | 0.443       | 0.450  | 0.461  | 0.474  | 0.493  | 0.523  | 0.569  |
| ↓                | 0.581       | 0.590  | 0.602  | 0.618  | 0.640  | 0.676  | 0.739  |
| ↓                | 0.719       | 0.729  | 0.743  | 0.761  | 0.786  | 0.830  | 0.908  |
| ↓                | 0.844       | 0.856  | 0.872  | 0.890  | 0.915  | 0.960  | 1.049  |
| ↓                | 0.961       | 0.975  | 0.991  | 1.010  | 1.037  | 1.084  | 1.173  |
| ↓                | 1.068       | 1.082  | 1.100  | 1.121  | 1.150  | 1.198  | 1.285  |
| ↓                | 1.167       | 1.182  | 1.199  | 1.221  | 1.252  | 1.303  | 1.377  |
| ↓                | 1.255       | 1.271  | 1.292  | 1.316  | 1.349  | 1.396  | 1.457  |
| ↓                | 1.336       | 1.354  | 1.376  | 1.400  | 1.427  | 1.467  | 1.525  |
| ↓                | 1.404       | 1.423  | 1.446  | 1.471  | 1.500  | 1.535  | 1.576  |
| ↓                | 1.465       | 1.483  | 1.504  | 1.528  | 1.556  | 1.584  | 1.615  |
| ↓                | 1.513       | 1.531  | 1.552  | 1.575  | 1.598  | 1.623  | 1.650  |
| ↓                | 1.550       | 1.569  | 1.588  | 1.609  | 1.628  | 1.647  | 1.666  |
| ↓                | 1.579       | 1.595  | 1.612  | 1.629  | 1.646  | 1.662  | 1.679  |
| ↓                | 1.597       | 1.612  | 1.626  | 1.640  | 1.655  | 1.669  | 1.684  |
| ↓                | 1.595       | 1.607  | 1.620  | 1.634  | 1.648  | 1.663  | 1.679  |
| ↓                | 1.580       | 1.592  | 1.606  | 1.620  | 1.636  | 1.653  | 1.670  |
| ↓                | 1.555       | 1.562  | 1.585  | 1.601  | 1.619  | 1.637  | 1.656  |
| HIGH             | 1.506       | 1.520  | 1.537  | 1.558  | 1.581  | 1.607  | 1.636  |
| LIFT COEFFICIENT |             |        |        |        |        |        |        |

model for input into the PERCAL computer program.

The Table 3 data compares favorable with both the high and low AOA reference data, covers the entire subsonic range, provides lift coefficients at small Mach number intervals, and is considerably smoother than the reference data. The latter two conditions prevent the generation of erroneous local maxima or minima by the computer program spline fit routine.

#### B. DRAG COEFFICIENTS

Coefficient of drag data was obtained from flight test data of the selected fighter. The data was obtained on a clean configuration at 30,000 feet MSL using maximum thrust accelerations and maximum thrust accelerating and decelerating turns. The flight test low  $C_L$  drag polar is shown in Figure 7 and the high  $C_L$  drag polar is shown in Figure 8.

Drag coefficients for input into the PERCAL computer program were obtained from Figures 7 and 8 using the Table 3 lift coefficients. The resulting  $C_D$  model is given in Table 4.

#### C. THRUST AVAILABLE AND FUEL CONSUMPTION

The total net thrust available was obtained from flight test data and is shown in Figure 9. The data used was for the selected fighter with engines operating at military power at 30,000 feet MSL. Specific range as a function of Mach number and aircraft gross weight was

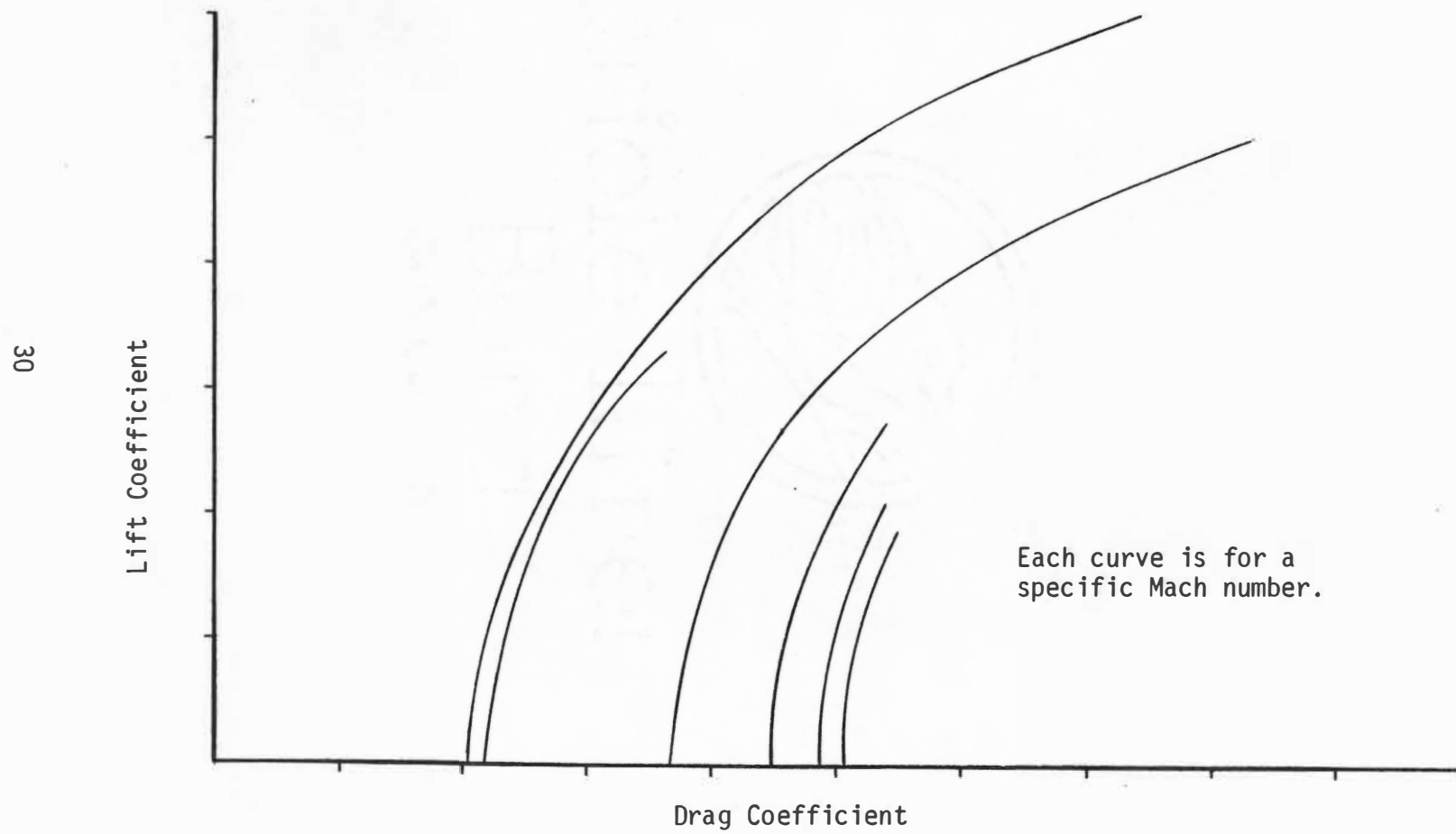


Figure 7. Flight Test Low  $C_L$  Drag Polar

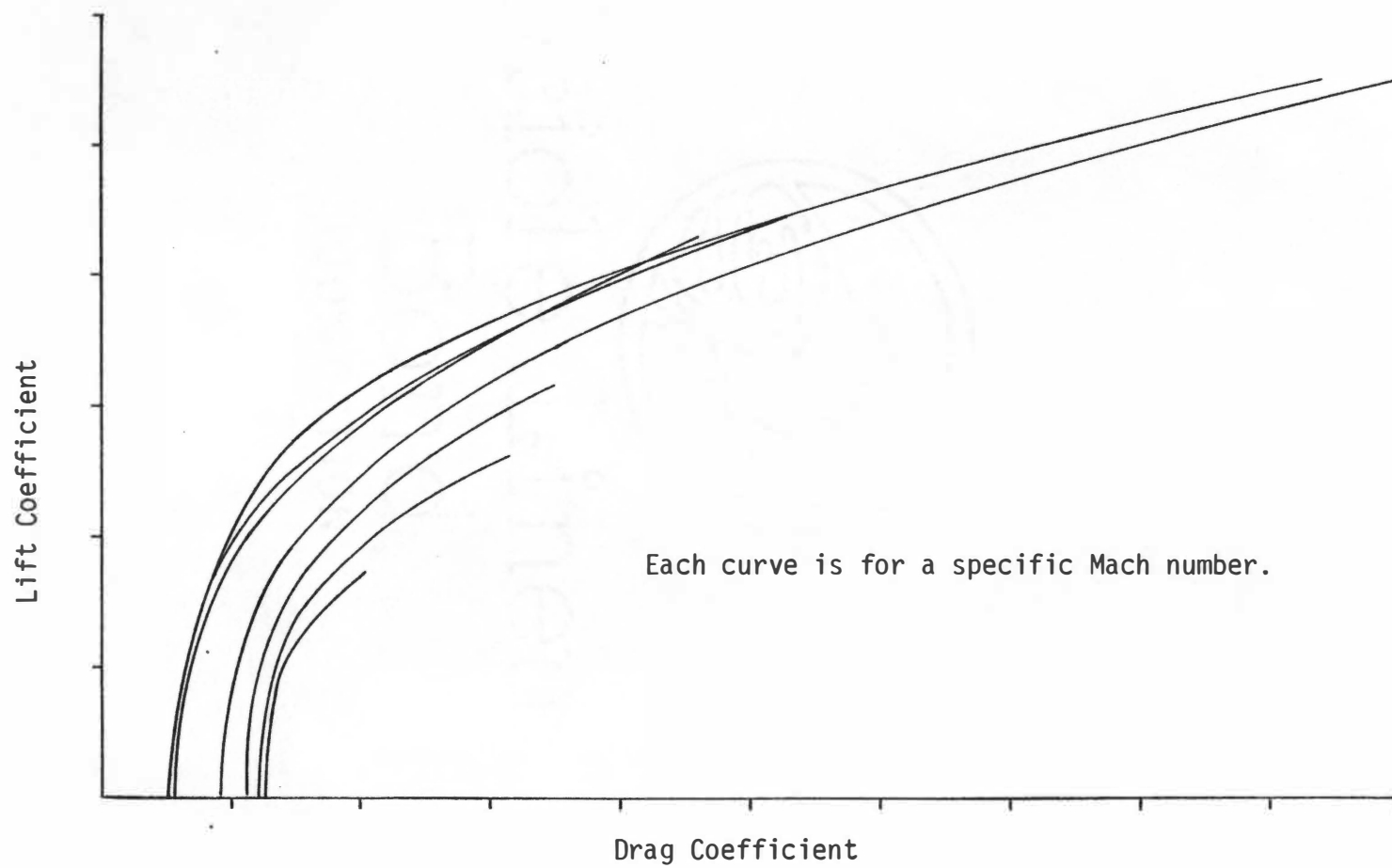


Figure 8. Flight Test High  $C_L$  Drag Polar

Table 4. PERCAL Computer Program Drag Coefficient Data

| AOA              | MACH NUMBER |        |        |        |        |
|------------------|-------------|--------|--------|--------|--------|
|                  | LOW         | →      | →      | →      | →      |
| LOW              | 0.0222      | 0.0222 | 0.0222 | 0.0222 | 0.0222 |
| ↓                | 0.0208      | 0.0208 | 0.0208 | 0.0208 | 0.0208 |
| ↓                | 0.0236      | 0.0236 | 0.0237 | 0.0237 | 0.0237 |
| ↓                | 0.0308      | 0.0308 | 0.0309 | 0.0310 | 0.0311 |
| ↓                | 0.0423      | 0.0424 | 0.0426 | 0.0429 | 0.0431 |
| ↓                | 0.0624      | 0.0627 | 0.0632 | 0.0637 | 0.0645 |
| ↓                | 0.1044      | 0.1048 | 0.1060 | 0.1072 | 0.1083 |
| ↓                | 0.1635      | 0.1645 | 0.1665 | 0.1675 | 0.1690 |
| ↓                | 0.2289      | 0.2295 | 0.2323 | 0.2351 | 0.2392 |
| ↓                | 0.3068      | 0.3085 | 0.3119 | 0.3145 | 0.3187 |
| ↓                | 0.3876      | 0.3884 | 0.3927 | 0.3961 | 0.4012 |
| ↓                | 0.4624      | 0.4632 | 0.4666 | 0.4700 | 0.4743 |
| ↓                | 0.5270      | 0.5278 | 0.5312 | 0.5346 | 0.5397 |
| ↓                | 0.5831      | 0.5839 | 0.5882 | 0.5924 | 0.5975 |
| ↓                | 0.6324      | 0.6332 | 0.6383 | 0.6417 | 0.6477 |
| ↓                | 0.6740      | 0.6749 | 0.6791 | 0.6842 | 0.6893 |
| ↓                | 0.7055      | 0.7063 | 0.7114 | 0.7157 | 0.7216 |
| ↓                | 0.7284      | 0.7293 | 0.7344 | 0.7386 | 0.7446 |
| ↓                | 0.7437      | 0.7446 | 0.7488 | 0.7522 | 0.7582 |
| ↓                | 0.7437      | 0.7454 | 0.7531 | 0.7582 | 0.7641 |
| ↓                | 0.7369      | 0.7386 | 0.7446 | 0.7488 | 0.7539 |
| ↓                | 0.7097      | 0.7114 | 0.7182 | 0.7233 | 0.7293 |
| HIGH             | 0.6757      | 0.6774 | 0.6817 | 0.6851 | 0.6893 |
| DRAG COEFFICIENT |             |        |        |        |        |

Table 4. Continued

| AOA              | MACH NUMBER |        |        |        |        |
|------------------|-------------|--------|--------|--------|--------|
|                  | →           | →      | →      | →      | →      |
| LOW              | 0.0222      | 0.0222 | 0.0222 | 0.0221 | 0.0220 |
| ↓                | 0.0208      | 0.0208 | 0.0208 | 0.0208 | 0.0209 |
| ↓                | 0.0238      | 0.0239 | 0.0239 | 0.0241 | 0.0243 |
| ↓                | 0.0312      | 0.0314 | 0.0317 | 0.0320 | 0.0325 |
| ↓                | 0.0433      | 0.0436 | 0.0440 | 0.0446 | 0.0453 |
| ↓                | 0.0653      | 0.0663 | 0.0676 | 0.0694 | 0.0717 |
| ↓                | 0.1095      | 0.1117 | 0.1143 | 0.1179 | 0.1223 |
| ↓                | 0.1715      | 0.1740 | 0.1780 | 0.1825 | 0.1895 |
| ↓                | 0.2427      | 0.2475 | 0.2538 | 0.2615 | 0.2723 |
| ↓                | 0.3238      | 0.3306 | 0.3391 | 0.3493 | 0.3612 |
| ↓                | 0.4071      | 0.4148 | 0.4233 | 0.4335 | 0.4462 |
| ↓                | 0.4802      | 0.4879 | 0.4972 | 0.5083 | 0.5219 |
| ↓                | 0.5457      | 0.5542 | 0.5644 | 0.5771 | 0.5924 |
| ↓                | 0.6043      | 0.6128 | 0.6222 | 0.6349 | 0.6511 |
| ↓                | 0.6545      | 0.6630 | 0.6740 | 0.6868 | 0.7021 |
| ↓                | 0.6961      | 0.7046 | 0.7148 | 0.7276 | 0.7429 |
| ↓                | 0.7284      | 0.7361 | 0.7463 | 0.7590 | 0.7752 |
| ↓                | 0.7514      | 0.7599 | 0.7709 | 0.7837 | 0.7973 |
| ↓                | 0.7650      | 0.7752 | 0.7871 | 0.7990 | 0.8117 |
| ↓                | 0.7709      | 0.7794 | 0.7879 | 0.7973 | 0.8075 |
| ↓                | 0.7599      | 0.7667 | 0.7752 | 0.7845 | 0.7947 |
| ↓                | 0.7361      | 0.7437 | 0.7531 | 0.7633 | 0.7692 |
| HIGH             | 0.6953      | 0.7029 | 0.7114 | 0.7216 | 0.7335 |
| DRAG COEFFICIENT |             |        |        |        |        |

Table 4. Concluded

| AOA              | MACH NUMBER |        |        |        |        |
|------------------|-------------|--------|--------|--------|--------|
|                  | →           | →      | →      | →      | HIGH   |
| LOW              | 0.0218      | 0.0218 | 0.0220 | 0.0232 | 0.0383 |
| ↓                | 0.0209      | 0.0210 | 0.0213 | 0.0226 | 0.0374 |
| ↓                | 0.0246      | 0.0250 | 0.0256 | 0.0279 | 0.0435 |
| ↓                | 0.0332      | 0.0341 | 0.0356 | 0.0410 | 0.0610 |
| ↓                | 0.0465      | 0.0482 | 0.0577 | 0.0654 | 0.1005 |
| ↓                | 0.0749      | 0.0800 | 0.0995 | 0.1135 | 0.1625 |
| ↓                | 0.1284      | 0.1379 | 0.1590 | 0.1805 | 0.2570 |
| ↓                | 0.2000      | 0.2119 | 0.2280 | 0.2600 | 0.3585 |
| ↓                | 0.2846      | 0.3000 | 0.3230 | 0.3629 | 0.4570 |
| ↓                | 0.3764      | 0.3944 | 0.4190 | 0.4598 | 0.5461 |
| ↓                | 0.4607      | 0.4794 | 0.5057 | 0.5491 | 0.6192 |
| ↓                | 0.5397      | 0.5601 | 0.5882 | 0.6281 | 0.6828 |
| ↓                | 0.6111      | 0.6315 | 0.6545 | 0.6885 | 0.7369 |
| ↓                | 0.6706      | 0.6919 | 0.7165 | 0.7463 | 0.7774 |
| ↓                | 0.7199      | 0.7403 | 0.7641 | 0.7879 | 0.8084 |
| ↓                | 0.7607      | 0.7803 | 0.7998 | 0.8211 | 0.8363 |
| ↓                | 0.7913      | 0.8092 | 0.8253 | 0.8415 | 0.8490 |
| ↓                | 0.8117      | 0.8262 | 0.8406 | 0.8542 | 0.8593 |
| ↓                | 0.8236      | 0.8355 | 0.8483 | 0.8602 | 0.8633 |
| ↓                | 0.8185      | 0.8304 | 0.8423 | 0.8551 | 0.8593 |
| ↓                | 0.8066      | 0.8185 | 0.8321 | 0.8466 | 0.8522 |
| ↓                | 0.7888      | 0.8024 | 0.8177 | 0.8330 | 0.8410 |
| HIGH             | 0.7480      | 0.7658 | 0.7854 | 0.8075 | 0.8251 |
| DRAG COEFFICIENT |             |        |        |        |        |

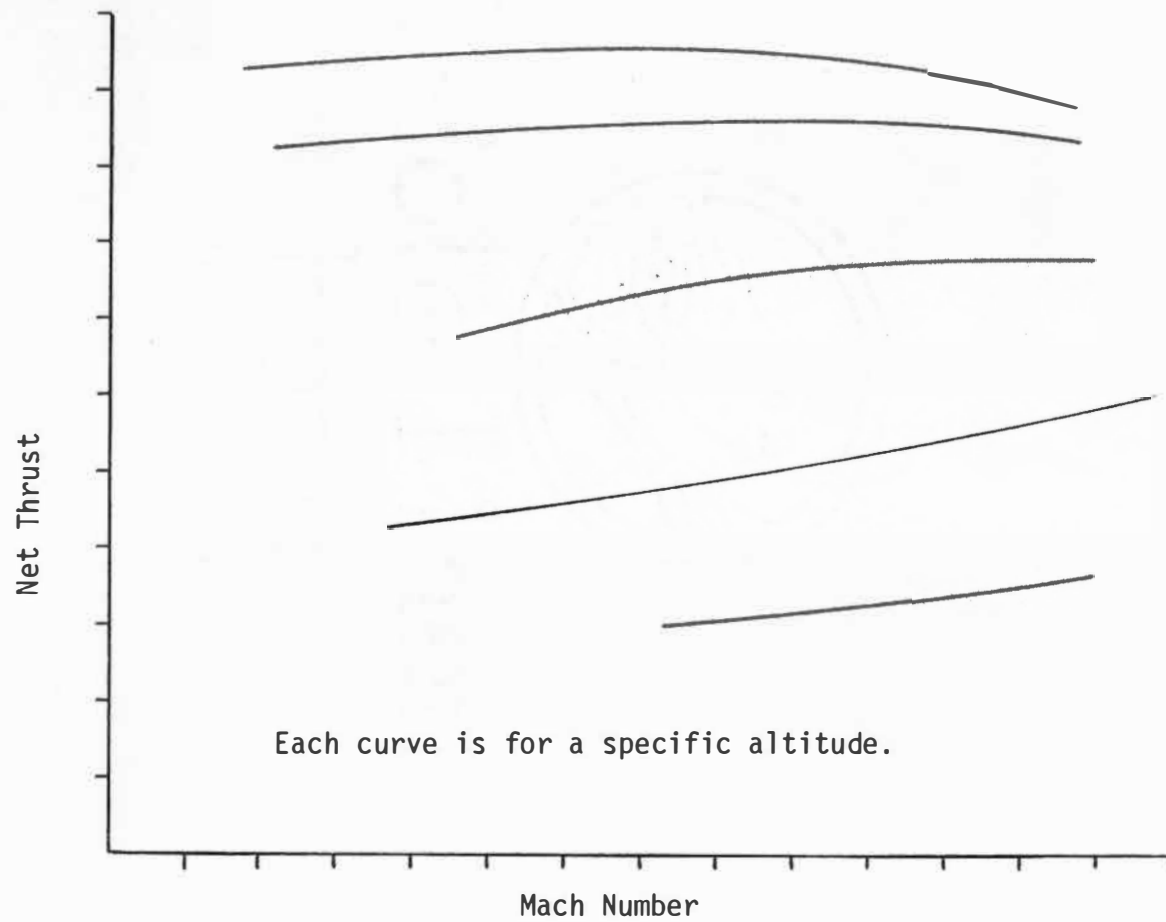


Figure 9. Net Thrust Available

obtained from the selected fighter flight manual and is shown in Figure 10.

Thrust specific fuel consumption (TSFC) as a function of thrust level and Mach number was calculated to model the Figure 10 specific ranges. From Eq. (17) the thrust required at small Mach number intervals for a range of aircraft gross weights was found. The corresponding specific ranges for these weights and Mach numbers were obtained from Figure 10. The TFSCs were then calculated from Eq. (22). The results are given in Table 5.

The Table 3  $C_L$  model, Table 4  $C_D$  model, Figure 9 thrust model, and Table 5 thrust specific fuel consumption model comprise the aircraft model input into the PERCAL computer program. This is the baseline (zero perturbation) model used to calculate the 0 percent error case for each of the point performance parameters.

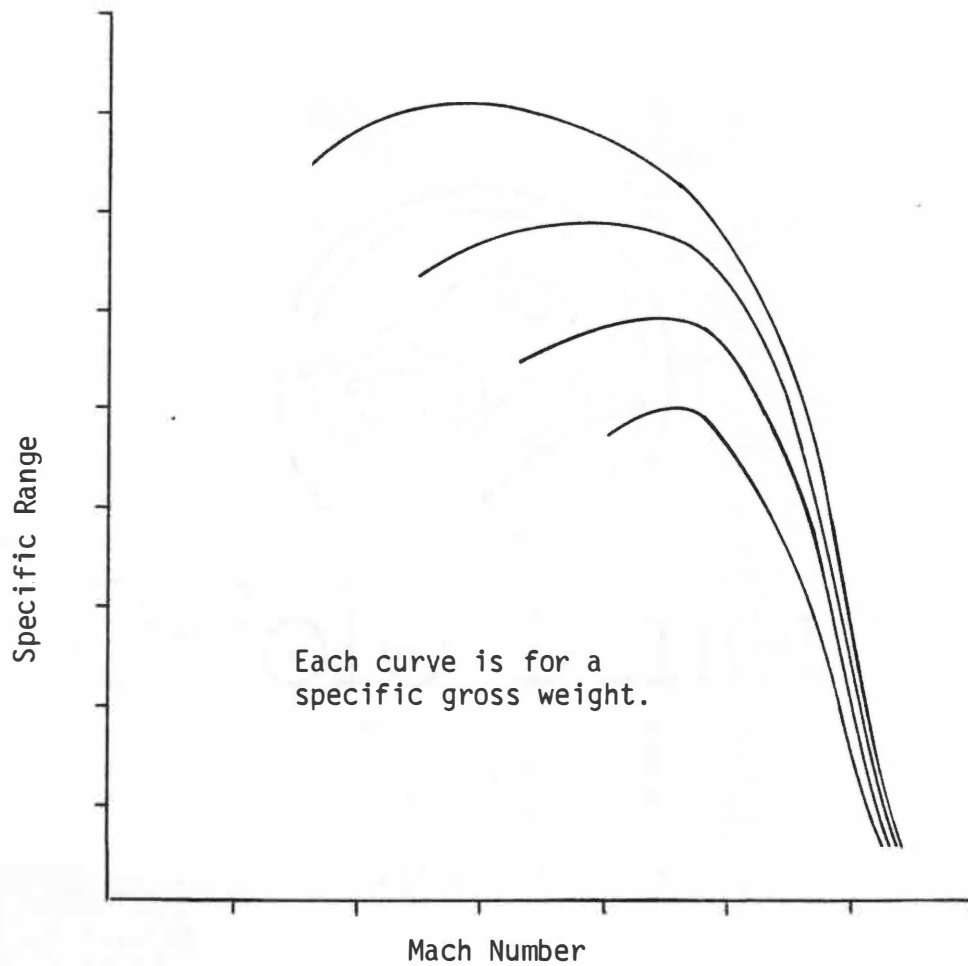


Figure 10. Specific Range

Table 5. PERCAL Computer Program Thrust Specific Fuel Consumption Data

| MACH | AIRCRAFT WEIGHT |       |        |       |        |       |         |       |
|------|-----------------|-------|--------|-------|--------|-------|---------|-------|
|      | LIGHT           | →     | →      | →     | →      | →     | →       | HEAVY |
| LOW  | 3023.0          | 0.958 | 4775.0 | 0.928 | 8136.0 | 0.862 | 12984.0 | 0.890 |
| ↓    | 2992.0          | 0.966 | 4443.0 | 0.925 | 7243.0 | 0.863 | 11518.0 | 0.866 |
| ↓    | 3007.0          | 0.960 | 4261.0 | 0.925 | 6519.0 | 0.863 | 10206.0 | 0.903 |
| ↓    | 3037.0          | 0.950 | 4146.0 | 0.923 | 5988.0 | 0.877 | 9110.0  | 0.918 |
| ↓    | 3070.0          | 0.956 | 4055.0 | 0.930 | 5635.0 | 0.888 | 8223.0  | 0.948 |
| ↓    | 3115.0          | 0.961 | 4002.0 | 0.933 | 5394.0 | 0.897 | 7557.0  | 0.966 |
| ↓    | 3186.0          | 0.961 | 4002.0 | 0.929 | 5193.0 | 0.911 | 7069.0  | 0.978 |
| ↓    | 3276.0          | 0.956 | 4039.0 | 0.920 | 5040.0 | 0.923 | 6712.0  | 0.977 |
| ↓    | 3372.0          | 0.954 | 4083.0 | 0.916 | 4968.0 | 0.928 | 6426.0  | 0.979 |
| ↓    | 3469.0          | 0.954 | 4125.0 | 0.922 | 4978.0 | 0.921 | 6186.0  | 0.978 |
| ↓    | 3568.0          | 0.957 | 4181.0 | 0.927 | 5024.0 | 0.909 | 6026.0  | 0.972 |
| ↓    | 3674.0          | 0.958 | 4266.0 | 0.927 | 5061.0 | 0.902 | 5973.0  | 0.956 |
| ↓    | 3789.0          | 0.959 | 4374.0 | 0.923 | 5095.0 | 0.900 | 5993.0  | 0.935 |
|      | Treq.           | TSFC  | Treq.  | TSFC  | Treq.  | TSFC  | Treq.   | TSFC  |

Table 5. Concluded

| MACH | AIRCRAFT WEIGHT |       |         |       |         |       |         |       |
|------|-----------------|-------|---------|-------|---------|-------|---------|-------|
|      | LIGHT           | →     | →       | →     | →       | →     | →       | HEAVY |
| ↓    | 3913.0          | 0.961 | 4487.0  | 0.919 | 5152.0  | 0.904 | 6028.0  | 0.919 |
| ↓    | 4447.0          | 0.961 | 4596.0  | 0.919 | 5238.0  | 0.902 | 6052.0  | 0.904 |
| ↓    | 4189.0          | 0.961 | 4703.0  | 0.922 | 5338.0  | 0.899 | 6083.0  | 0.899 |
| ↓    | 4340.0          | 0.959 | 4811.0  | 0.929 | 5437.0  | 0.899 | 6139.0  | 0.894 |
| ↓    | 4511.0          | 0.957 | 4936.0  | 0.930 | 5543.0  | 0.902 | 6225.0  | 0.893 |
| ↓    | 4711.0          | 0.951 | 5097.0  | 0.930 | 5671.0  | 0.904 | 6342.0  | 0.889 |
| ↓    | 4917.0          | 0.964 | 5278.0  | 0.929 | 5807.0  | 0.910 | 6466.0  | 0.903 |
| ↓    | 5059.0          | 0.973 | 5411.0  | 0.949 | 5895.0  | 0.936 | 6548.0  | 0.937 |
| ↓    | 5132.0          | 1.020 | 5491.0  | 0.993 | 5943.0  | 0.991 | 6594.0  | 0.993 |
| ↓    | 5451.0          | 0.991 | 5823.0  | 1.012 | 6255.0  | 1.017 | 6896.0  | 1.027 |
| ↓    | 6454.0          | 0.963 | 6829.0  | 0.952 | 7251.0  | 0.963 | 7854.0  | 0.989 |
| ↓    | 8212.0          | 0.868 | 8577.0  | 0.868 | 8993.0  | 0.879 | 9526.0  | 0.925 |
| HIGH | 10342.0         | 0.908 | 10690.0 | 0.848 | 11101.0 | 0.863 | 11547.0 | 0.911 |
|      | Treq.           | TSFC  | Treq.   | TSFC  | Treq.   | TSFC  | Treq.   | TSFC  |

## CHAPTER IV

### EFFECT OF COEFFICIENT ERRORS

#### A. PERFORMANCE PARAMETER PERCENT ERROR CALCULATION

The PERCAL program was used to generate a value for each point performance parameter for 121 cases. Each case corresponded to a particular combination of errors in coefficients of lift and drag; each coefficient being perturbed by  $\pm 20$ ,  $\pm 10$ ,  $\pm 5$ ,  $\pm 2$ ,  $\pm 1$ , or 0 percent. The percent error in the point performance parameters for each case was calculated using:

$$\% Error = \left( \frac{PVALUE}{NVALUE} - 1.0 \right) 100 \quad (24)$$

where PVALUE = the parameter value for the specific case, and

NVALUE = 0 percent error value.

A positive percent error corresponds to a parameter value greater than the 0 percent error case.

Using the PERCAL program the maximum horizontal flight speed for the 0 percent error case was found to be 1010.84 ft/sec. For the case where the coefficients of lift and drag were both increased by 5 percent the maximum horizontal flight speed was calculated to be 1004.87 ft/sec. From Eq. (24) this yields the percent error in the maximum horizontal flight speed for this case as:

$$V_{max} \text{ error} = \left( \frac{1004.87}{1010.84} - 1.0 \right) 100 = -0.591\%$$

The minus sign indicating the velocity for this case is less than that of the 0 percent error case. Likewise a percent error was calculated for each of the nine selected point performance parameters for each of the 121 combinations of errors in the coefficients of lift and drag. The resulting percent errors for the 121 cases were input into the AEDC Tekplot computer program for graphic analysis. The output plots (Figures 11 through 25) represent the percent error on the ordinate versus the percent error in the coefficient of lift or drag on the abscissa. The individual curves are for a constant percent error of the coefficient not used on the abscissa. The vertical dashed lines indicate the maximum  $C_L$  and  $C_D$  errors as established by Reference 3.

If each of the various aerodynamic, thrust, and performance parameters used in Chapter II to calculate point performance could be expressed as an equation, then an expression relating the error in the performance parameter to errors in each of the variables could be obtained using a Taylor series expansion. Of the nine selected performance parameters only the horizontal acceleration may be directly expanded using the Taylor series. Series expansion of the other eight parameters would be possible only by simplifying the aircraft model to a generic form.

Assuming the angle of attack to be zero, the level flight horizontal acceleration is given from Eq. (18):

$$a_{\theta} = \frac{dV}{dt} = \frac{g(T-D)}{W} \quad (25)$$

Expanding, using the Taylor series in thrust and drag:

$$\Delta a_{\theta} = \Delta T \left( \frac{g}{W} \right) - \Delta D \left( \frac{g}{W} \right) \quad (26)$$

From Eq. (24) the error in the horizontal acceleration ( $E_{a_{\theta}}$ ) may be expressed as :

$$E_{a_{\theta}} = \left( \frac{\Delta a_{\theta}}{a_{\theta}} \right) 100$$

Substituting for  $\Delta a_{\theta}$  from Eq. (26) and  $a_{\theta}$  from Eq. (25) yields:

$$\frac{E_{a_{\theta}}}{100} = \frac{\Delta T}{(T-D)} - \frac{\Delta D}{(T-D)} \quad (27)$$

The right hand side of Eq. (27) may be expressed in terms of the error in thrust ( $E_T$ ) and the error in the coefficient of drag ( $E_{C_D}$ ) by using  $D=C_D q S$  and the error equations:

$$E_T = \left( \frac{\Delta T}{T} \right) 100 \quad \text{and} \quad E_{C_D} = \left( \frac{\Delta C_D}{C_D} \right) 100$$

Substituting into Eq. (27) yields:

$$E_{a_\theta} = \left\{ \left[ \frac{1}{\left( \frac{T}{C_D q S} - 1 \right)} \right] + 1 \right\} E_T - \left[ \frac{1}{\left( \frac{T}{C_D q S} - 1 \right)} \right] E_{C_D} \quad (28)$$

Using the 0 percent error level flight drag coefficient and the military power setting thrust available for 0.9 Mach, Eq. (28) reduces to:

$$E_{a_\theta} = 1.934 E_T - 0.934 E_{C_D} \quad (29)$$

Assuming a zero thrust error, Eq. (29) agrees exactly with PERCAL computer program generated error curve (Figure 13) for a 0 percent error in  $C_L$ .

Although a lift term does not expressly appear in the equation for the level flight horizontal acceleration, an error in the lift coefficient will affect the prediction of the horizontal acceleration as shown in Figure 13. An error in the lift coefficient will cause a redefinition of the level flight angle of attack and drag. Therefore an error in the prediction of horizontal acceleration is induced by the deviation from the 0 percent error case of the level flight drag.

Eq. (28) is a general expression of the relationship of the variables involved, and application is not restricted to a particular aircraft. It is readily observed from Eq. (28) that the results of Figure 13 can be generalized provided the thrust to drag ratio remains fixed. Similar criteria for the generalization of the other performance parameters is not available (as previously discussed) due to the inability to relate the constituent variables through precise equations.

#### B. MAXIMUM HORIZONTAL FLIGHT SPEED

The effect of coefficient errors on the maximum horizontal flight speed is shown in Figures 11 and 12. The dominate effect is that of an error in  $C_D$ . Approximately a 10 percent error in  $C_L$  is required to produce the same magnitude effect as a 1 percent error in  $C_D$ . Errors which decrease  $C_D$  produce slightly larger effects than errors which increase  $C_D$ .

Errors in  $C_L$  have very little effect on the maximum horizontal flight speed. For a given error in  $C_D$ , a 20 percent error in  $C_L$  will yield less than a 0.5 percent change in the maximum flight speed. The effect of errors which decrease  $C_L$  is additive to the effect of errors which increase  $C_D$ .

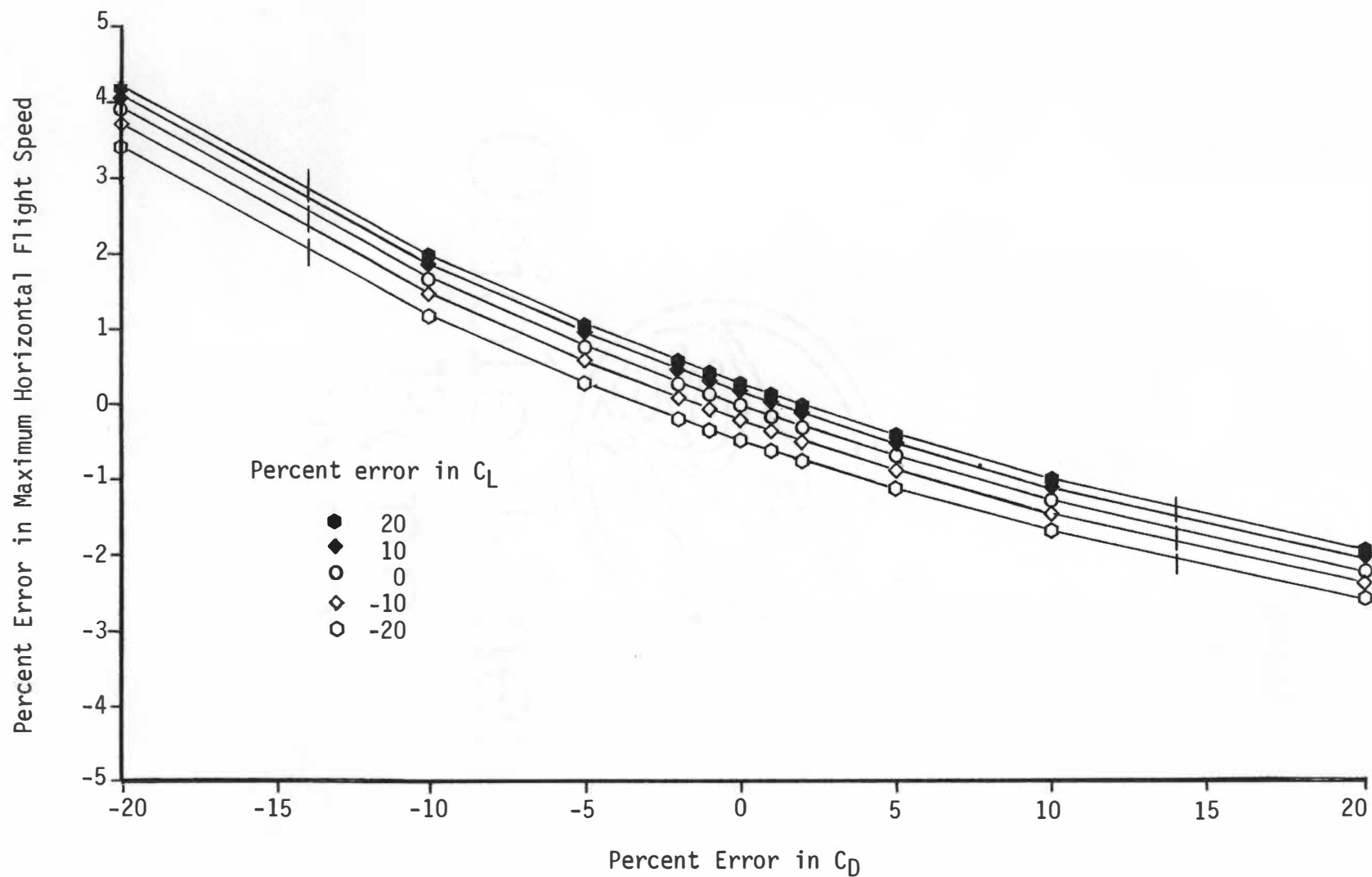


Figure 11. Effect of  $C_D$  Error on Maximum Horizontal Flight Speed

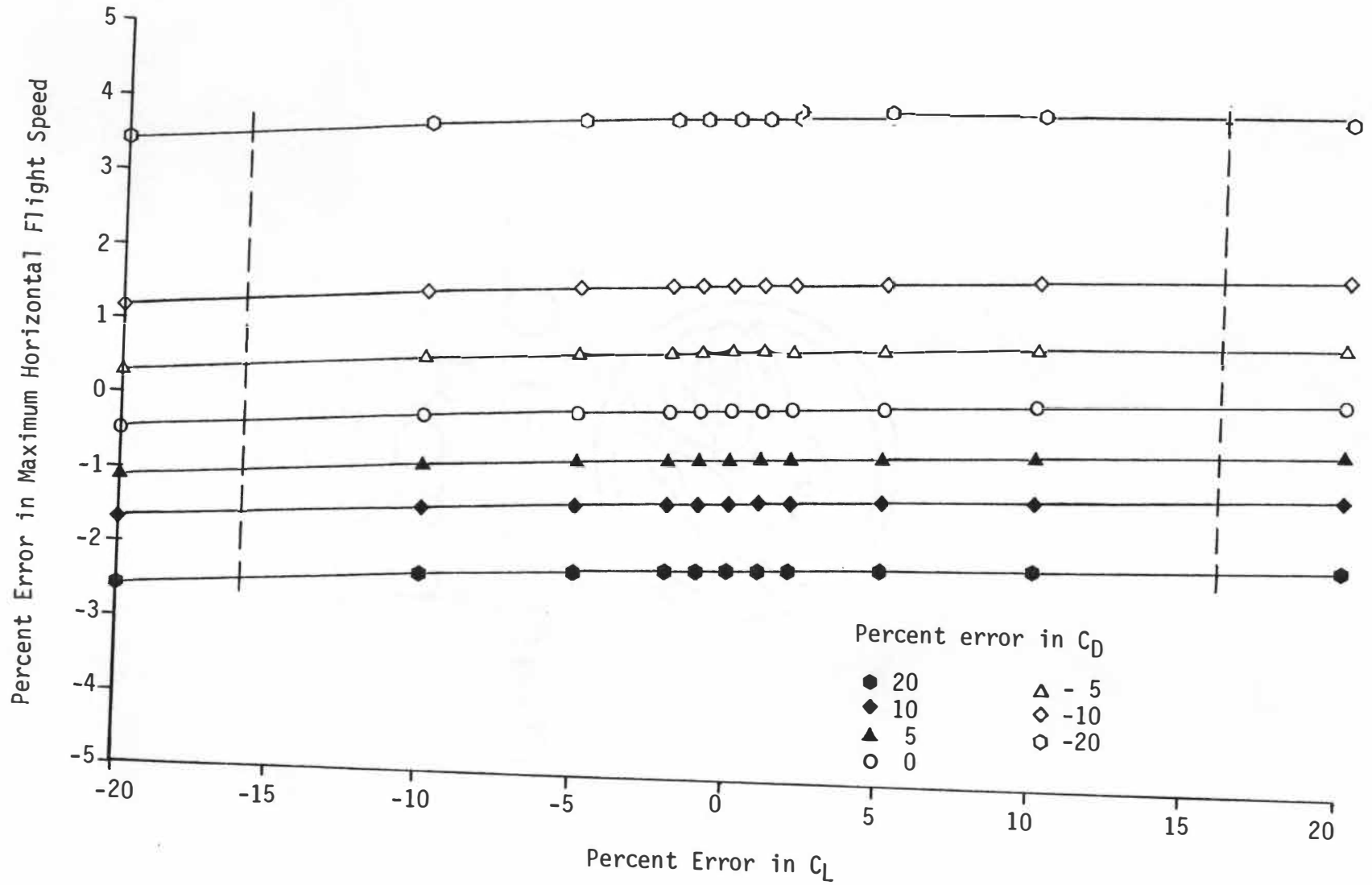


Figure 12. Effect of  $C_L$  Error on Maximum Horizontal Flight Speed

### C. HORIZONTAL ACCELERATION

Figures 13 and 14 show the effects of errors in  $C_D$  and  $C_L$  on 0.9 Mach horizontal acceleration. The effects of errors in  $C_D$  are approximately two to four times greater than the effects of errors in  $C_L$ . Each 1 percent error in  $C_D$  produces approximately a 1 percent error in the horizontal acceleration.

Each 1 percent error in  $C_L$  yields an error in the horizontal acceleration of 0.25 to 0.5 percent. The effect of errors in  $C_L$  is noticeably greater if the errors decrease  $C_L$  by more than 10 percent. The effect of errors which produce a decrease in  $C_L$  is additive to the effect of errors which increase  $C_D$ .

### D. MAXIMUM CLIMB ANGLE

The effects of errors in  $C_D$  and  $C_L$  on the maximum angle of climb is shown in Figures 15 and 16. The effect of a 1 percent error in  $C_D$  is three to four times greater than the effect of a 1 percent error in  $C_L$ . Each 1 percent error in  $C_D$  produces slightly less than a 1 percent error in the maximum climb angle.

A 1 percent error in  $C_L$  yields a 0.25 to 0.35 percent error in the maximum climb angle. The effect of an error in  $C_L$  is more pronounced if the error decreases  $C_L$  by more than 10 percent. Errors which

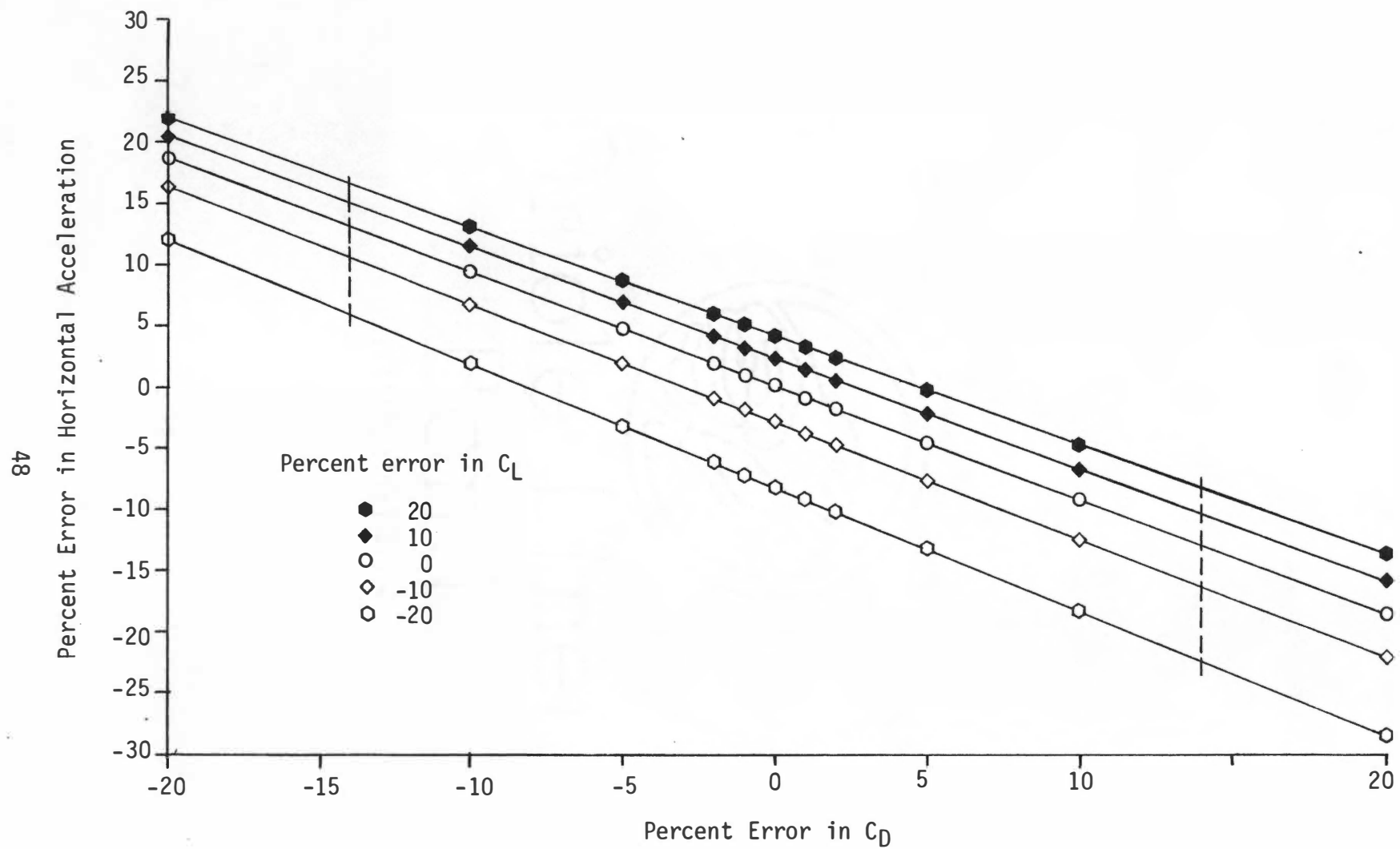


Figure 13. Effect of  $C_D$  Error on Horizontal Acceleration

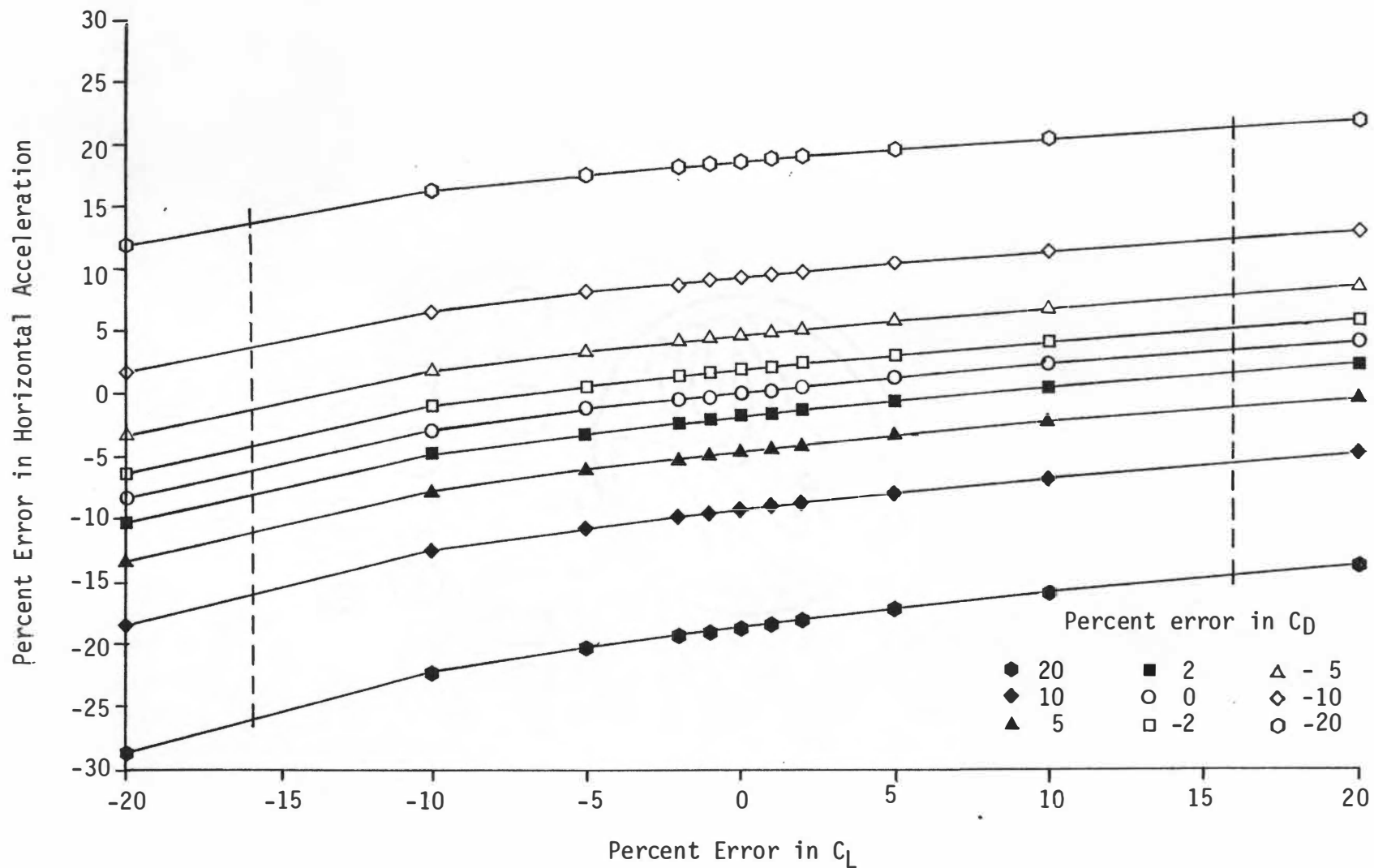


Figure 14. Effect of  $C_L$  Error on Horizontal Acceleration

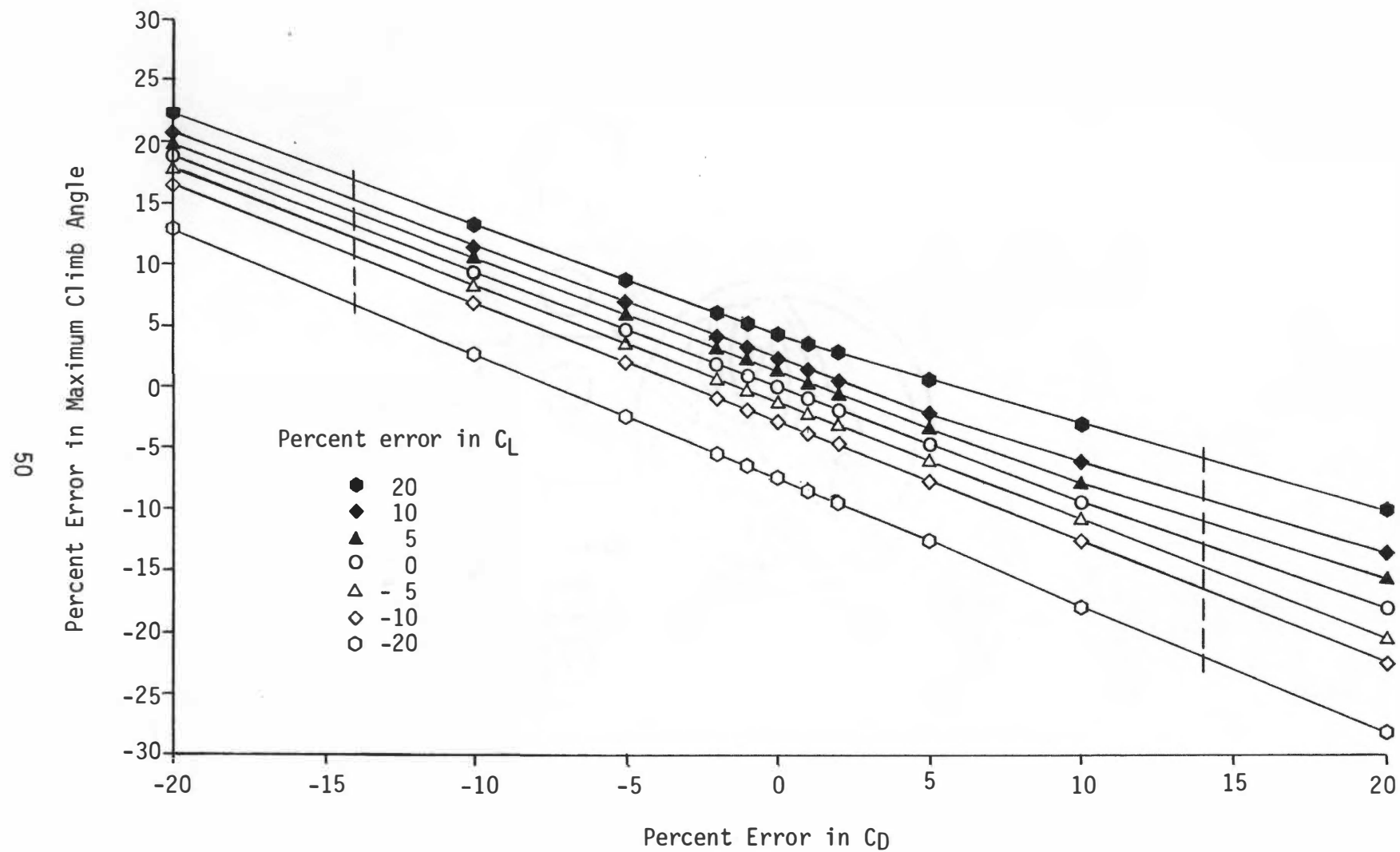


Figure 15. Effect of  $C_D$  Error on Maximum Climb Angle

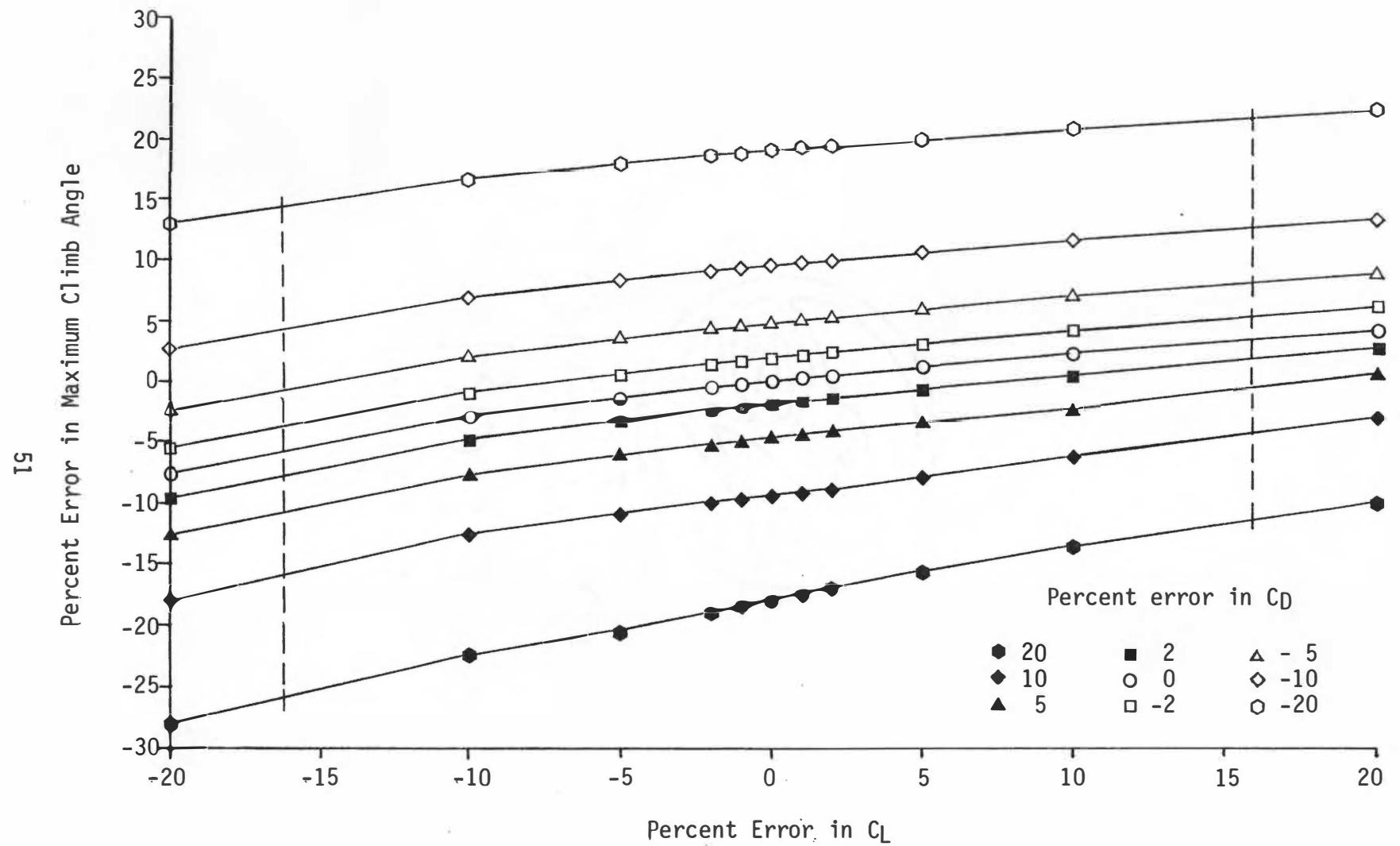


Figure 16. Effect of  $C_L$  Error on Maximum Climb Angle

decrease  $C_L$  or increase  $C_D$  result in a decrease in the maximum climb angle.

#### E. MAXIMUM CLIMB RATE

Figures 17 and 18 show the effects of errors in  $C_L$  and  $C_D$  on the maximum rate of climb. Errors in  $C_D$  produce an effect approximately four times greater than errors in  $C_L$ . Each 1 percent error in  $C_D$  produces approximately a 1 percent error in the maximum climb rate. An error which causes an increase in  $C_D$  will result in a decrease in the maximum climb rate.

Each 1 percent error in  $C_L$  yields an error in the maximum rate of climb of 0.2 to 0.3 percent. The effect of errors in  $C_L$  is noticeably greater if the errors result in a decrease in  $C_L$  of 10 percent or more. Errors which decrease  $C_L$  yield a decrease in the maximum rate of climb.

#### F. LEVEL FLIGHT STALL VELOCITY

From Eq. (20) it is noted that the level flight stall velocity is not a function of  $C_D$ . Figure 19 shows the effects of errors in  $C_L$  on the stall velocity. Each 1 percent error in  $C_L$  yields approximately a 0.5 percent error in the level flight stall velocity. The effect of an error which decreases  $C_L$  produces an increase in the level flight stall velocity.

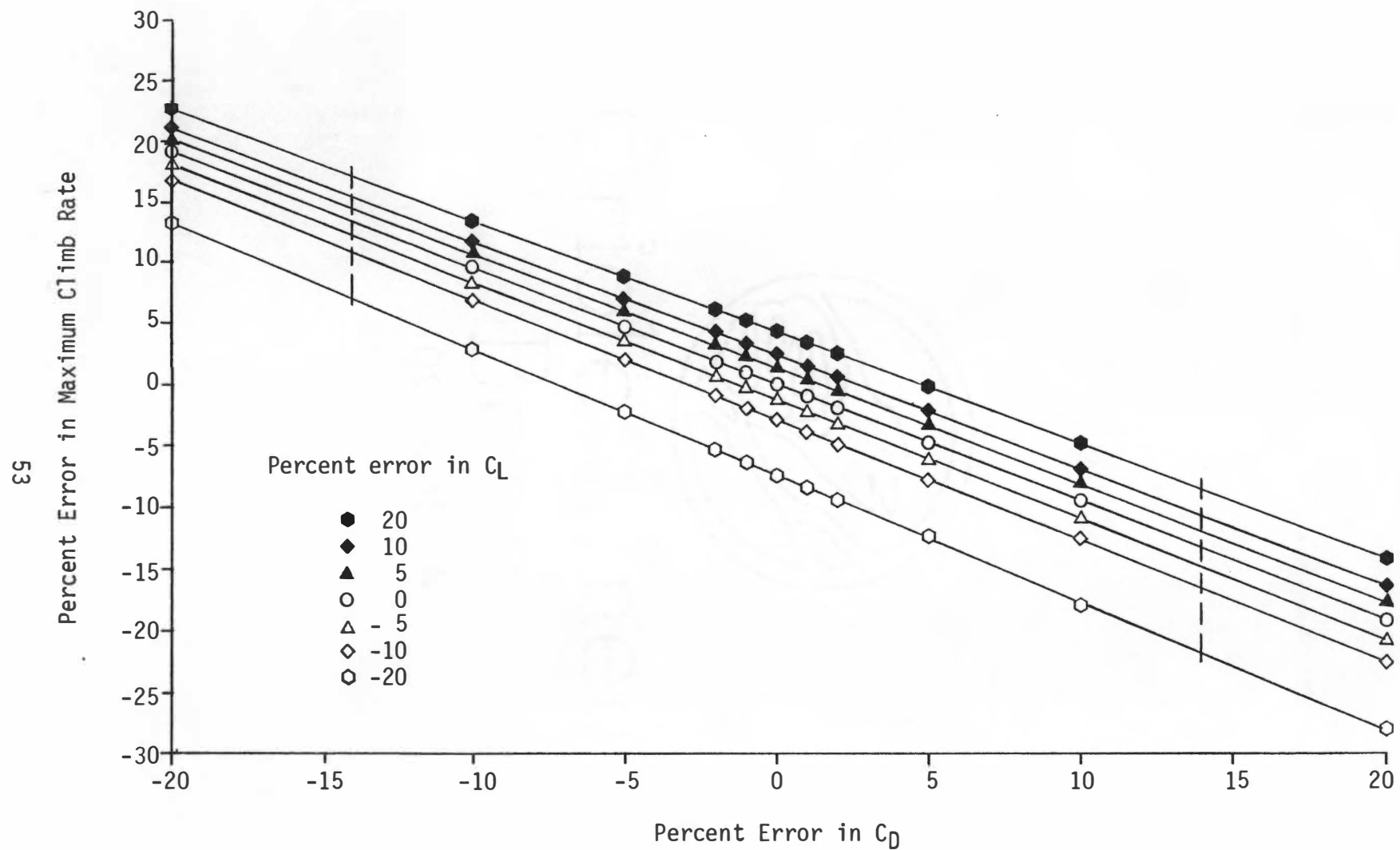


Figure 17. Effect of  $C_D$  Error on Maximum Climb Rate

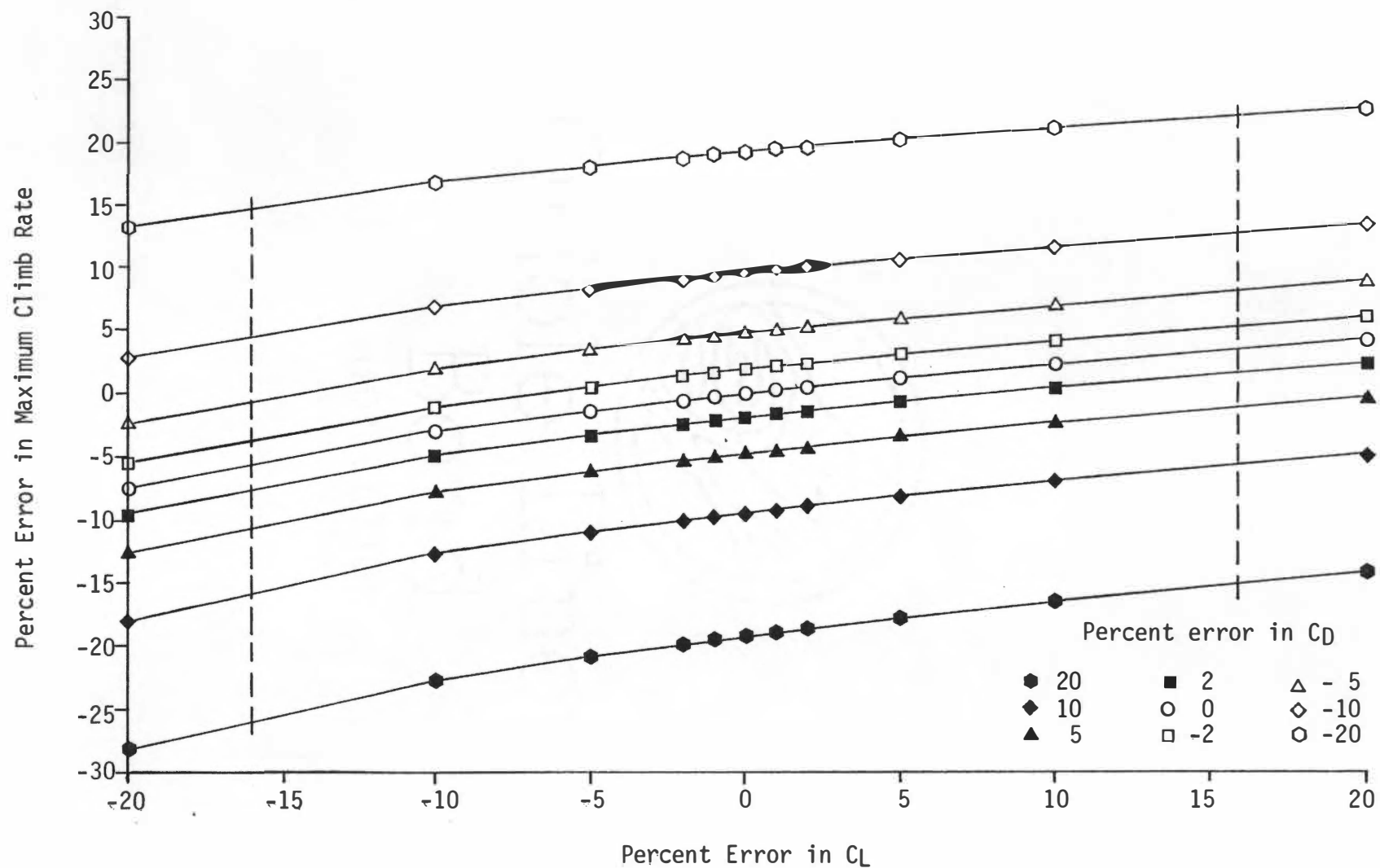


Figure 18. Effect of  $C_L$  Error on Maximum Climb Rate

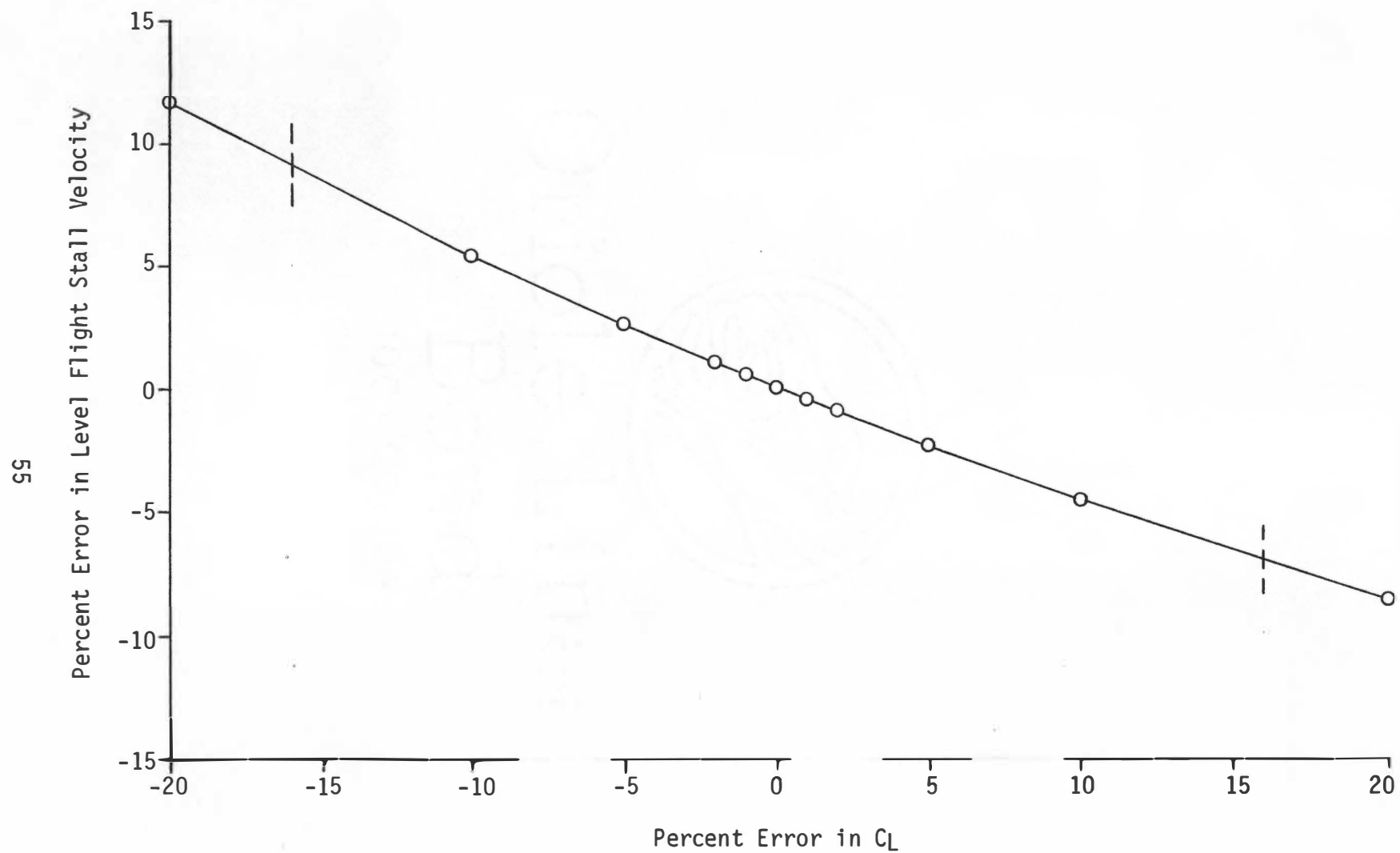


Figure 19. Effect of  $C_L$  Error on Level Flight Stall Velocity

#### G. MINIMUM TURN RADIUS

From Eq. (12) and Eq. (20) it is seen that the instantaneous minimum turn radius is not a function of  $C_D$ . The effects of errors in  $C_L$  on the minimum turn radius is shown in Figure 20. Each 1 percent error in  $C_L$  yields approximately a 1 percent error in the minimum turn radius. The effect of errors which decrease  $C_L$  produce an increase in the minimum turn radius.

#### H. MAXIMUM TURN RATE

As with the minimum turn radius, the instantaneous maximum turn rate is not a function of  $C_D$ . Figure 21 shows the effects of errors in  $C_L$  on the maximum turn rate. Each 1 percent error in  $C_L$  yields approximately a 0.5 percent error in the maximum turn rate. An error which causes a decrease in  $C_L$  produces a decrease in the instantaneous maximum turn rate.

#### I. MAXIMUM SPECIFIC RANGE

Figures 22 and 23 show the effect of errors in  $C_D$  and  $C_L$  on the maximum specific range. The effect on the maximum specific range of errors in  $C_D$  is approximately twice that of errors in  $C_L$ . Each 1

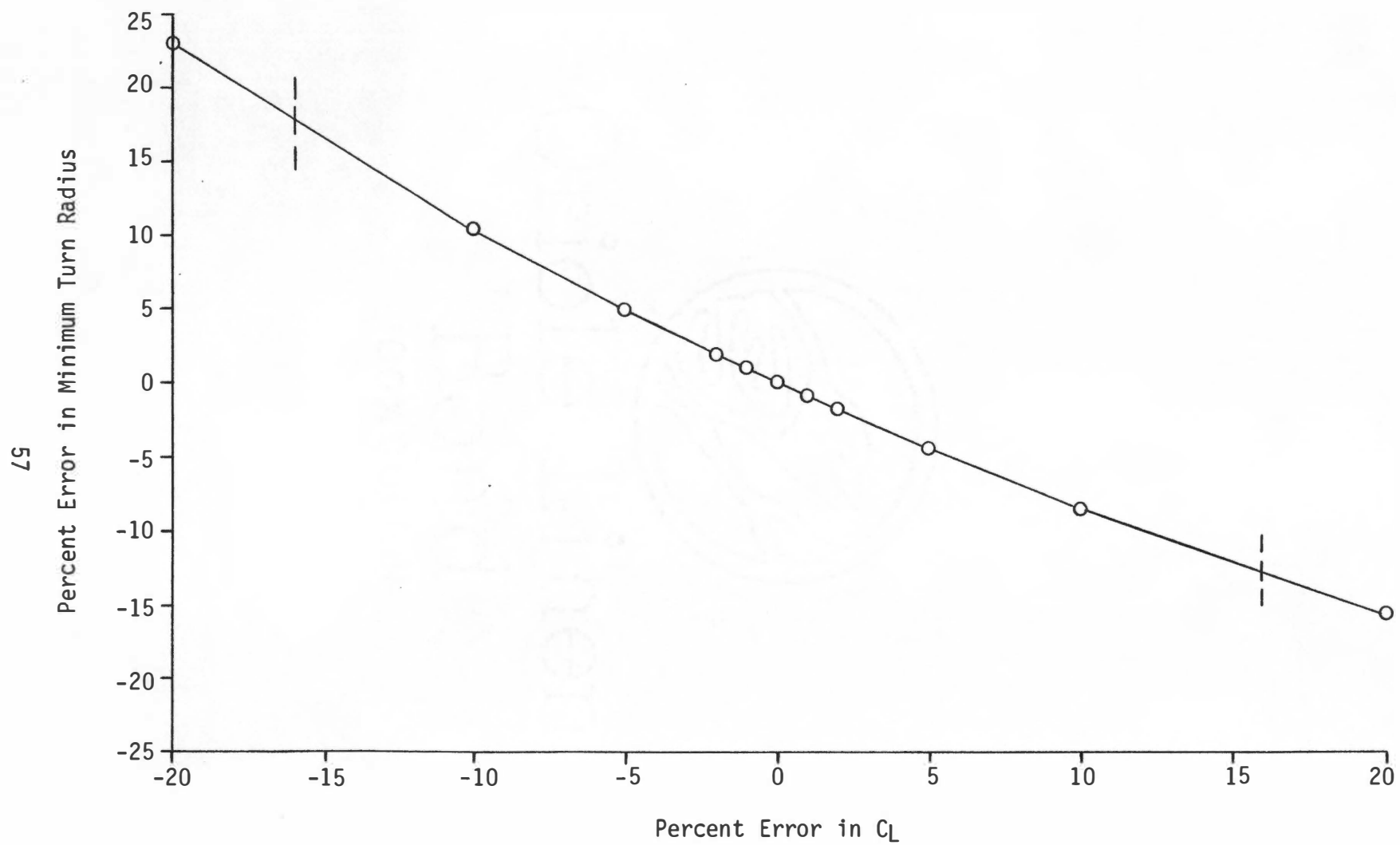


Figure 20. Effect of  $C_L$  Error on Minimum Turn Radius

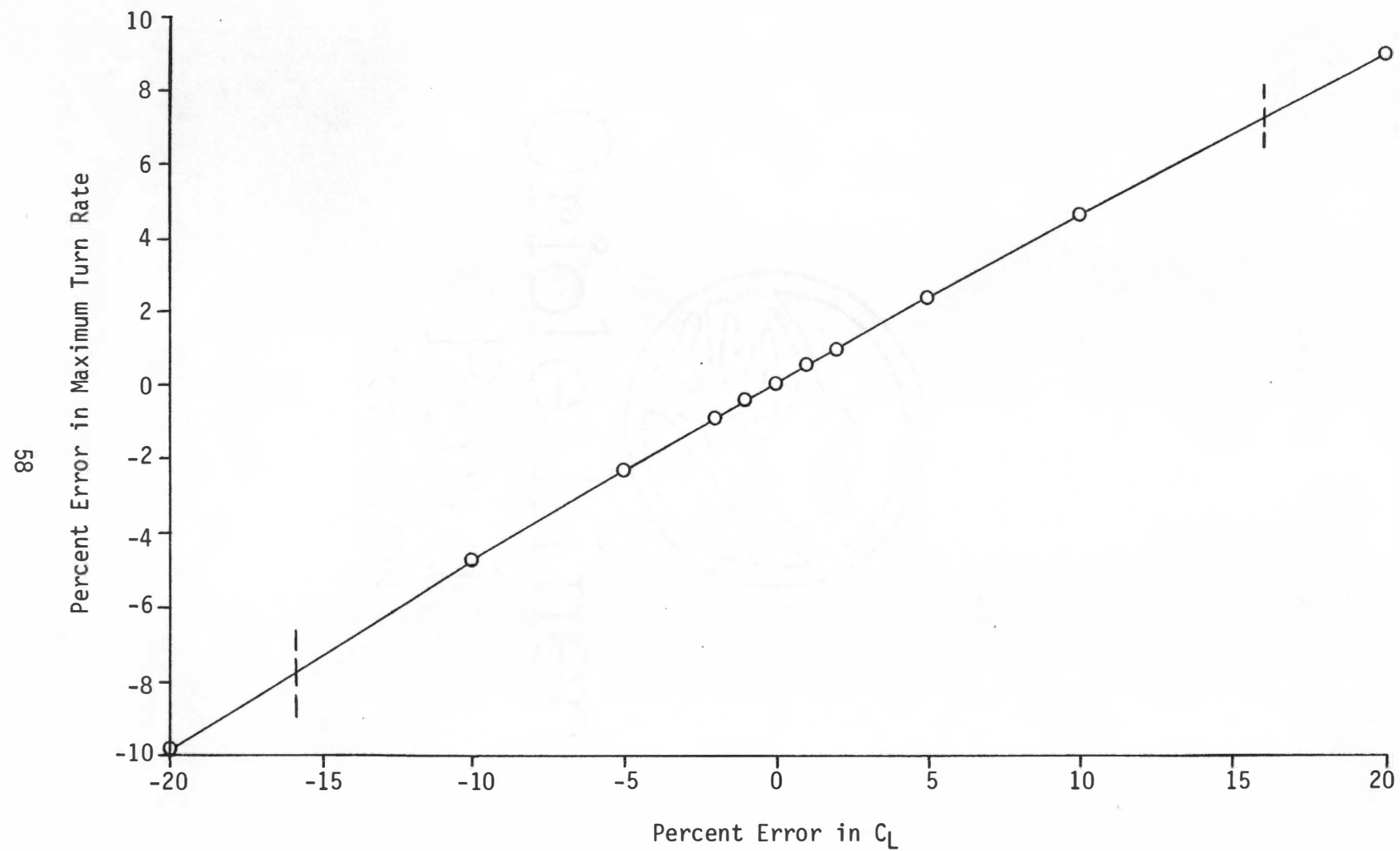


Figure 21. Effect of  $C_L$  Error on Maximum Turn Rate

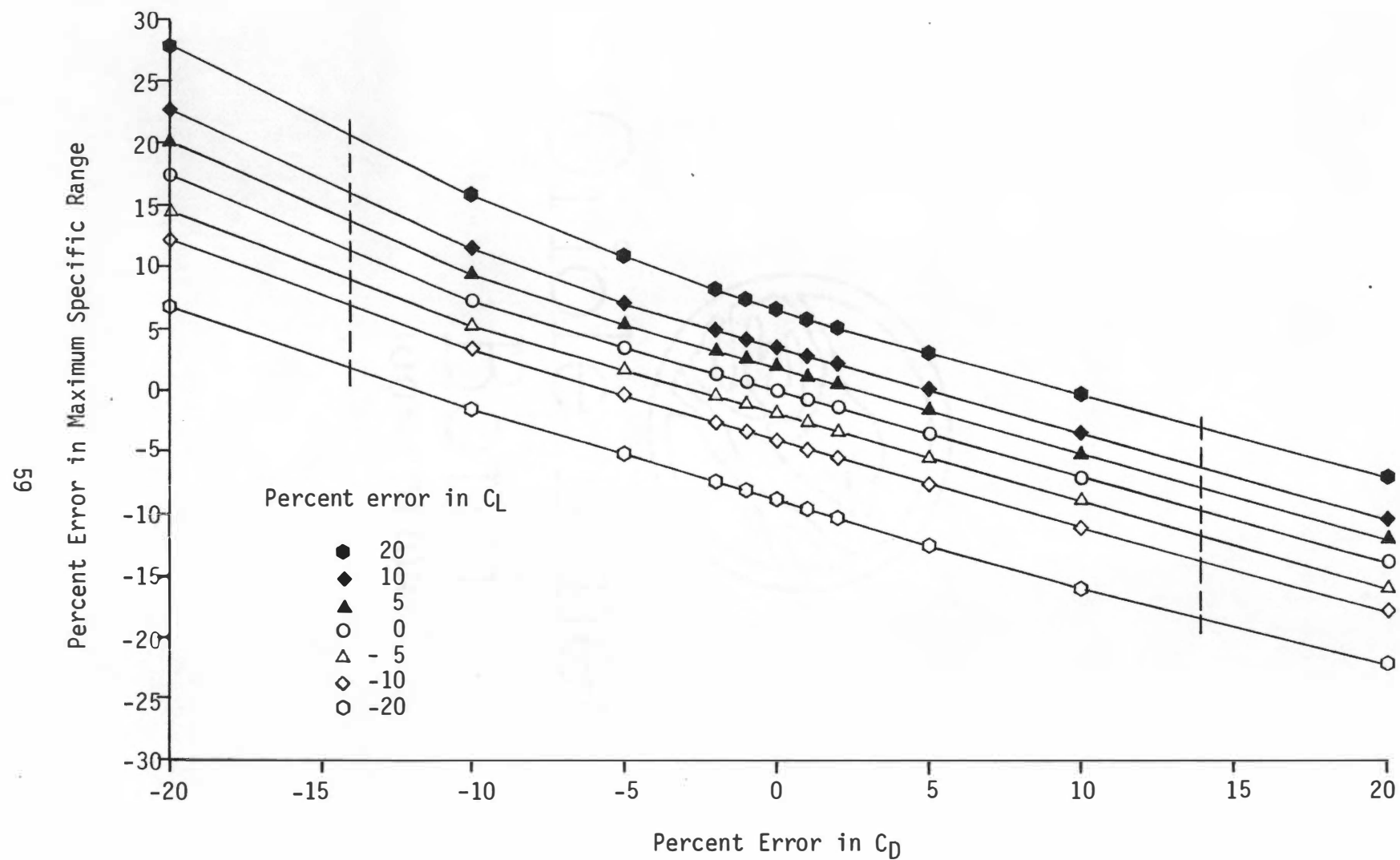


Figure 22. Effect of  $C_D$  Error on Maximum Specific Range

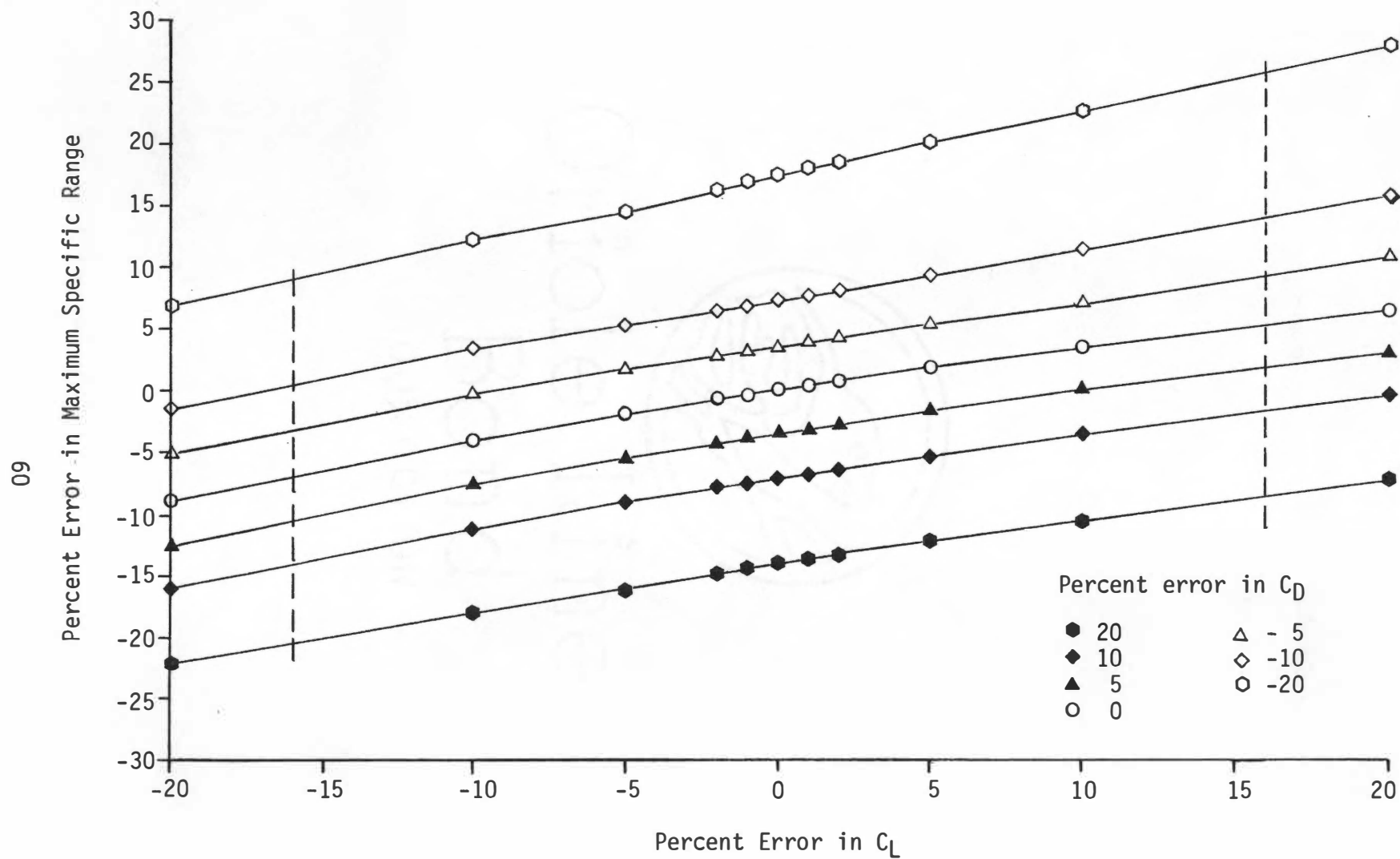


Figure 23. Effect of  $C_L$  Error on Maximum Specific Range

percent error in  $C_D$  produces approximately a 0.8 percent error in the maximum specific range. Errors which increase  $C_D$  yield a decrease in the maximum specific range.

Each 1 percent error in  $C_L$  produces approximately a 0.4 percent error in the maximum specific range. The maximum specific range is decreased by errors which decrease  $C_L$ .

#### J. MAXIMUM SPECIFIC ENDURANCE

The effect of errors in  $C_D$  and  $C_L$  on the maximum specific endurance is shown in Figures 24 and 25. The effect of errors in  $C_D$  and  $C_L$  is approximately the same, each 1 percent error in  $C_D$  or  $C_L$  producing approximately a 1 percent error in the maximum specific endurance. The maximum specific endurance is increased by errors which decrease  $C_D$  or increase  $C_L$ .

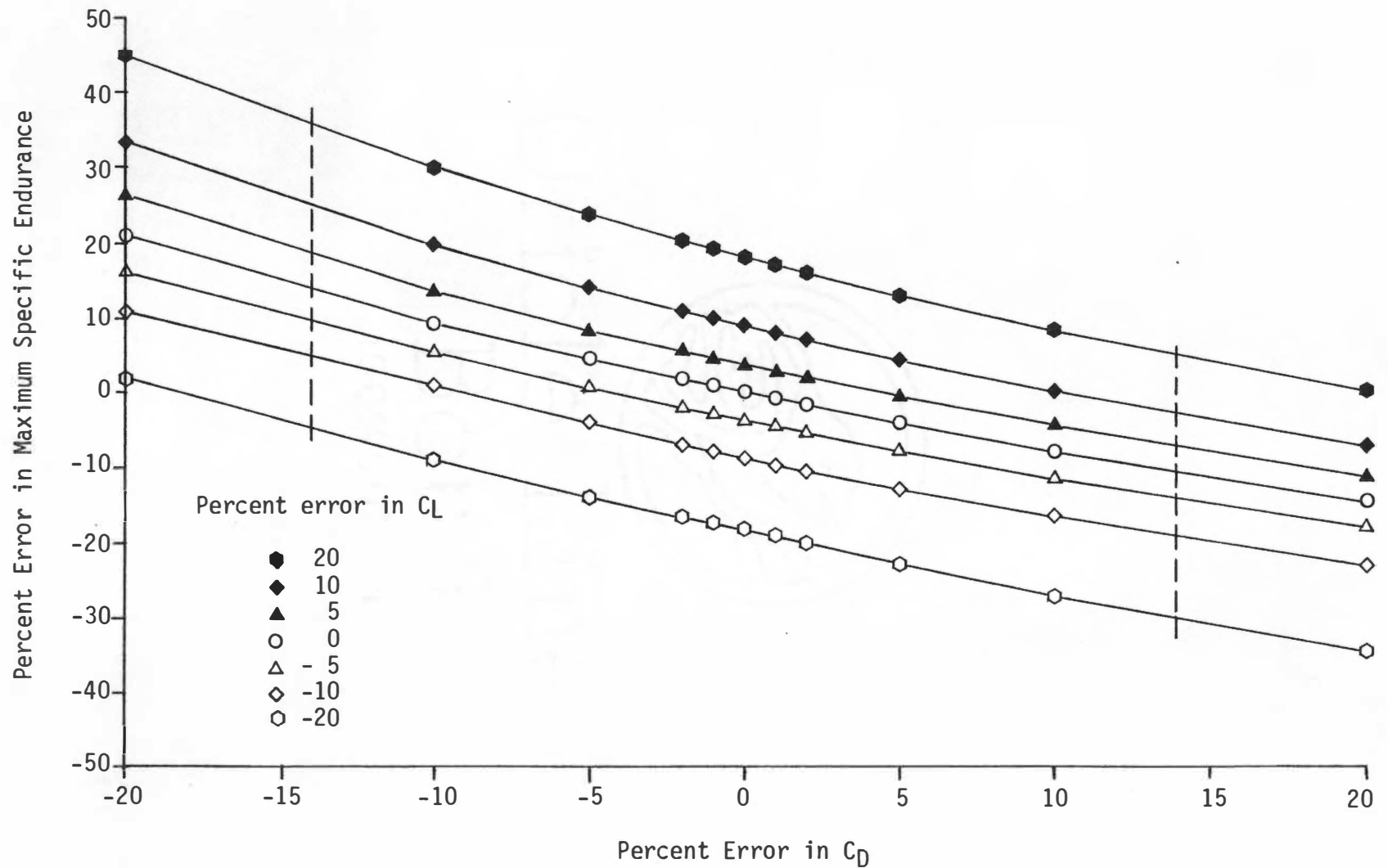


Figure 24. Effect of  $C_D$  Error on Maximum Specific Endurance

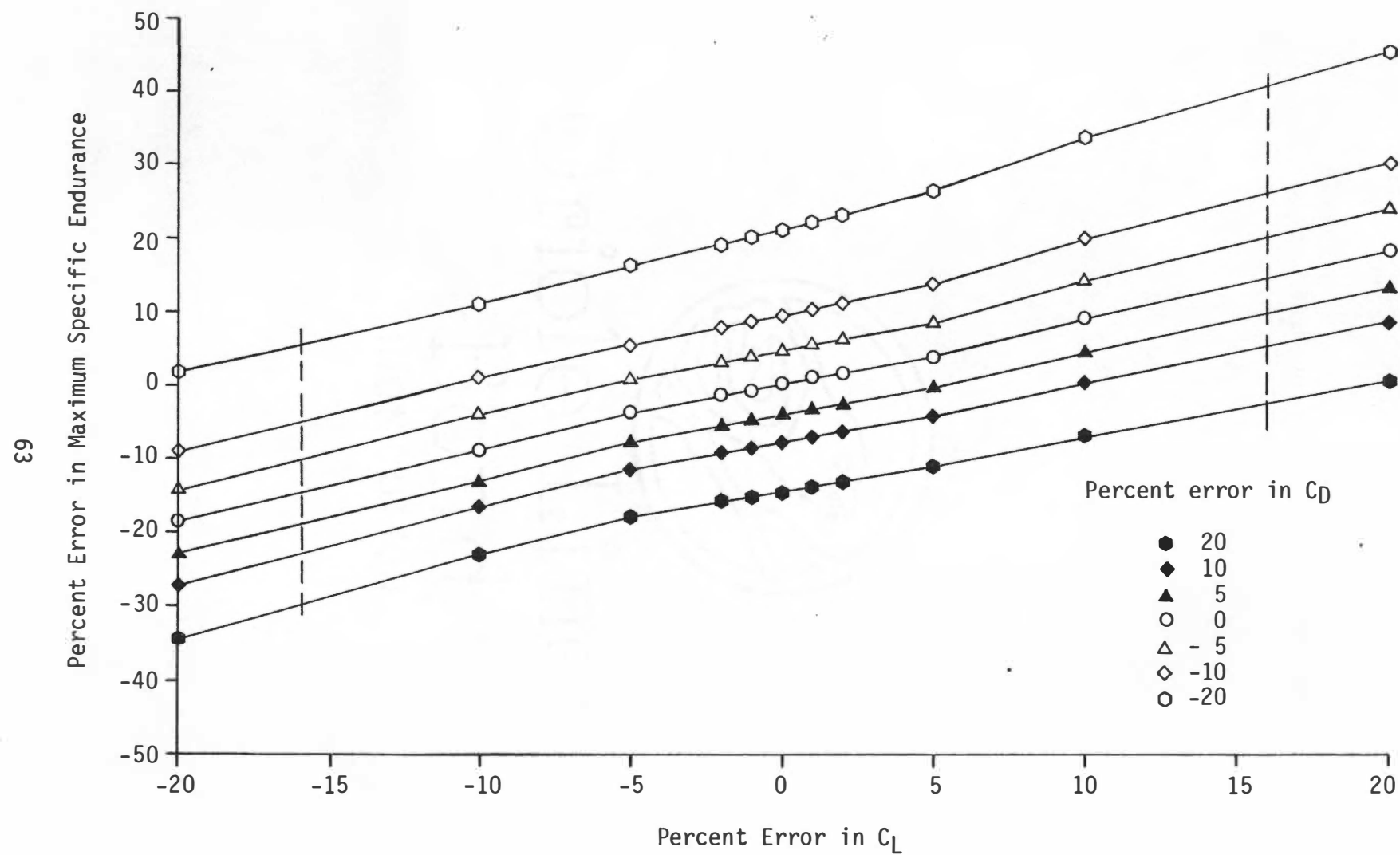


Figure 25. Effect of  $C_L$  Error on Maximum Specific Endurance

## CHAPTER V

### RESULTS AND RECOMMENDATIONS

#### A. METHODOLOGY

The point performance techniques were found to be good vehicles for this error analysis. The performance parameters calculated were easily programmed using simple iterative techniques when necessary, an "off the shelf" cubic spline routine, and algebraic manipulations of the cubic spline coefficients. The PERCAL computer program required less than five seconds of CPU time to run on the AEDC Amdahl 5860 computer.

When doing computer performance calculations from input data points particular attention must be given to the cubic spline curve fits. If experimental data is used without smoothing, the inherent scatter, though quite small, may generate false local maxima and minima. To obtain acceptable spline fits may require data be concentrated in regions where a cubic equation is not likely to yield a good fit. For example, in the lift curve the cubic spline must be closely monitored over the linear portion and the transition region from linear to nonlinear. It was found by trial and error that the spline routine used in the PERCAL program required coefficient data at intervals of 0.05 Mach or less to produce consistently reliable

results. The interval at which coefficient data is required may well be a function of the particular type of spline fit used. A different spline routine (Akima for example) may allow the data intervals to be larger. Regardless of the routine selected, the user must be aware of the characteristics of the type of spline employed.

The coefficients of lift, drag, and thrust parameters input into the computer program produced a 0 percent error performance model that very closely approximated a medium weight fighter at 30,000 feet MSL. From the authors years of experience in fighter aircraft the calculated best instantaneous turn performance is slightly better than that of a production aircraft. This is most likely because of a higher  $C_{L_{max}}$  as a result of the  $C_L$  data being taken at a horizontal stabilator setting of zero rather than deflected to produce the necessary angle of attack. This error is quite small, approximately 5 percent or less, and should have no significant effect on the error levels presented. The accuracy of the other performance parameters is within the differences between individual aircraft.

## B. PERFORMANCE PARAMETER ERROR ANALYSIS

The relationship between the percent error in a given point performance parameter and the percent error in  $C_L$  or  $C_D$  was found to be approximately linear through the -20 to +20 percent coefficient error range. This was as expected since the point performance parameters are

based on first order terms only. In the performance parameters that are functions of both  $C_L$  and  $C_D$  the effect of errors in  $C_D$  was at least twice the effect of errors in  $C_L$  except for maximum specific endurance where the effect of errors for both coefficients was approximately equal. Errors which increased  $C_L$  or decreased  $C_D$  enhanced all performance parameters while decreasing  $C_L$  or increasing  $C_D$  was detrimental to all performance parameters. Of the performance parameters calculated, the maximum horizontal flight speed was found to be the least sensitive to errors in either  $C_L$  or  $C_D$ . This is attributable to the analysis being restricted to the military power setting thereby giving the model insufficient thrust to overcome transonic drag.

The horizontal acceleration, maximum climb angle and climb rate, are all basically functions of specific excess power. The effects of errors in  $C_D$  for these parameters was two to four times the effect of errors in  $C_L$ . Each 1 percent error  $C_D$  resulted in approximately a 1 percent error in the performance parameter.

Level flight stall speed and best instantaneous turn radius and rate are functions of  $C_L$  only. Each 1 percent error in  $C_L$  yielded a 0.5 to 1.0 percent error in those parameters.

Maximum specific range and endurance are significantly effected by errors in both  $C_L$  and  $C_D$ . For specific range the effect of errors in  $C_D$  is twice the effect of errors in  $C_L$  while for specific endurance the effect of errors in  $C_L$  and  $C_D$  is approximately equal. Each 1 percent

error in  $C_D$  produces approximately a 1 percent error in specific range and endurance.

Table 6 summarizes the results of the investigation. The approximate effects (in percent) on each of the selected performance parameters for each 1 percent error in the coefficients of drag and lift are given.

The relative importance of the selected performance parameters is dependent on the aircraft/mission combination. An interceptor on a lane or point defense mission would require a good maximum climb rate (assuming a scramble launch) to intercept altitude, and good specific endurance to maintain time on station once established in an orbit. In a strictly air-to-air mode a premium is placed on acceleration to gain energy for maneuvering, and a maximum turn rate to position the nose for weapons employment. A close air support fighter would emphasize specific endurance for time on station, and good turn rate and radius to position for weapons delivery and for defense against surface-to-air threats. Maximum specific range is desirable for fighters on deep interdiction strikes and for their air-to-air escorts.

### C. RECOMMENDATIONS

Despite the relative simplicity and ease of calculation, the point performance parameters were found to yield quite accurate results.

Table 6. Summary of the Effects of Coefficient Errors

| Parameter                        | Effect (%) for each 1% error in: |             |
|----------------------------------|----------------------------------|-------------|
|                                  | $C_D$                            | $C_L$       |
| Maximum Level Flight Speed       | 0.18                             | 0.025       |
| 0.9 Mach Horizontal Acceleration | 1.0                              | 0.25 - 0.5  |
| Maximum Climb Angle              | 0.9                              | 0.25 - 0.35 |
| Maximum Climb Rate               | 1.0                              | 0.2 - 0.3   |
| Level Flight Stall Speed         | ----                             | 0.5         |
| Minimum Turn Radius              | -----                            | 1.0         |
| Maximum Turn Rate                | ----                             | 0.5         |
| Maximum Specific Range           | 0.8                              | 0.4         |
| Maximum Specific Endurance       | 1.0                              | 1.0         |

Therefore the following extensions and applications of these techniques should be considered:

- 1) An analysis of this type should be performed prior to wind tunnel testing of a proposed aircraft. The design phase lift curve, drag polar, and thrust estimates should be sufficiently accurate to establish where testing inaccuracies would lead to misconceptions about the proposed designs performance. Knowing the areas where testing should be concentrated would improve test efficiency and reduce costs.
- 2) A computer program similar to PERCAL should be available either on line during test data acquisition or easily accessible off line in order to rapidly evaluate the data being taken. If the data yields gross performance parameter deviations from those expected further investigation into the data accuracy would be warranted.
- 3) An extension of the point performance techniques to path performance should be pursued. Path performance would have the advantage of identifying performance parameters and associated accuracies critical to the particular aircraft/mission combination. For example the performance parameters and accuracies required for a given aircraft might be substantially different for a high altitude intercept mission as opposed to a low level surface attack mission. Path performance techniques used in conjunction with testing

as suggested in parts 1 and 2 of this section would be valuable tools in aircraft design and design analysis.

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## BIBLIOGRAPHY

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## APPENDIX

## APPENDIX A

### DERIVATION OF ACCELERATION NORMAL TO FLIGHT PATH

From Figure 1B:

$$V_r = \frac{dr}{dt} = V \sin \gamma$$

$$V_\theta = r \frac{d\theta}{dt} = V \cos \gamma$$

$$\frac{d^2 r}{dt^2} = V \cos \gamma \frac{d\gamma}{dt} + \sin \gamma \frac{dV}{dt}$$

$$\frac{d^2 \theta}{dt^2} = \frac{d}{dt} \left( \frac{V \cos \gamma}{r} \right) = \frac{\cos \gamma}{r} \frac{dV}{dt} - \frac{V \sin \gamma}{r} \frac{d\gamma}{dt} - \frac{V^2 \sin \gamma \cos \gamma}{r^2}$$

$$X = r \cos \theta$$

$$Y = r \sin \theta$$

$$\frac{dX}{dt} = \cos \theta \frac{dr}{dt} - r \sin \theta \frac{d\theta}{dt}$$

$$\frac{d^2 X}{dt^2} = \cos \theta \frac{d^2 r}{dt^2} - 2 \sin \theta \frac{dr}{dt} \frac{d\theta}{dt} - r \cos \theta \left( \frac{d\theta}{dt} \right)^2 - r \sin \theta \frac{d^2 \theta}{dt^2}$$

$$\frac{dY}{dt} = r \cos \theta \frac{d\theta}{dt} + \sin \theta \frac{dr}{dt}$$

$$\frac{d^2Y}{dt^2} = \sin \theta \frac{d^2r}{dt^2} + 2 \cos \theta \frac{dr}{dt} \frac{d\theta}{dt} - r \sin \theta \left( \frac{d\theta}{dt} \right)^2 + r \cos \theta \frac{d^2\theta}{dt^2}$$

Resolve  $\frac{d^2X}{dt^2}$  and  $\frac{d^2Y}{dt^2}$  into components in the  $r$  and  $\theta$  directions:

$$\ddot{X}_r = \left( \frac{d^2X}{dt^2} \right)_{r \text{ dir}} = \cos \theta \frac{d^2X}{dt^2}$$

$$\ddot{X}_\theta = \left( \frac{d^2X}{dt^2} \right)_{\theta \text{ dir}} = -\sin \theta \frac{d^2X}{dt^2}$$

$$\ddot{Y}_r = \left( \frac{d^2Y}{dt^2} \right)_{r \text{ dir}} = \sin \theta \frac{d^2Y}{dt^2}$$

$$\ddot{Y}_\theta = \left( \frac{d^2Y}{dt^2} \right)_{\theta \text{ dir}} = \cos \theta \frac{d^2Y}{dt^2}$$

Calculate radial acceleration ( $a_r$ ) and angular acceleration ( $a_\theta$ ):

$$a_r = \ddot{X}_r + \ddot{Y}_r = \cos \theta \frac{d^2X}{dt^2} + \sin \theta \frac{d^2Y}{dt^2}$$

$$a_r = \cos^2 \theta \frac{d^2 r}{dt^2} - 2 \sin \theta \cos \theta \frac{dr}{dt} \frac{d\theta}{dt} - r \cos^2 \theta \left( \frac{d\theta}{dt} \right)^2 - r \sin \theta \cos \theta \frac{d^2 \theta}{dt^2}$$

$$+ \sin^2 \theta \frac{d^2 r}{dt^2} + 2 \sin \theta \cos \theta \frac{dr}{dt} \frac{d\theta}{dt} - r \sin^2 \theta \left( \frac{d\theta}{dt} \right)^2 + r \sin \theta \cos \theta \frac{d^2 \theta}{dt^2}$$

$$a_r = \frac{d^2 r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2$$

$$a_\theta = \ddot{X}_\theta + \ddot{Y}_\theta = -\sin \theta \frac{d^2 X}{dt^2} + \cos \theta \frac{d^2 Y}{dt^2}$$

$$a_\theta = -\sin \theta \cos \theta \frac{d^2 r}{dt^2} + 2 \sin^2 \theta \frac{dr}{dt} \frac{d\theta}{dt} + r \sin \theta \cos \theta \left( \frac{d\theta}{dt} \right)^2 + r \sin^2 \theta \frac{d^2 \theta}{dt^2}$$

$$+ \sin \theta \cos \theta \frac{d^2 r}{dt^2} + 2 \cos^2 \theta \frac{dr}{dt} \frac{d\theta}{dt} - r \sin \theta \cos \theta \left( \frac{d\theta}{dt} \right)^2 + r \cos^2 \theta \frac{d^2 \theta}{dt^2}$$

$$a_\theta = 2 \frac{dr}{dt} \frac{d\theta}{dt} + r \frac{d^2 \theta}{dt^2}$$

Calculate acceleration normal to flight path ( $dV_n/dt$ ):

$$\frac{dV_n}{dt} = a_r \cos \gamma - a_\theta \sin \gamma$$

$$\frac{dV_n}{dt} = \cos \gamma \frac{d^2 r}{dt^2} - r \cos \gamma \left( \frac{d\theta}{dt} \right)^2 - 2 \sin \gamma \frac{dr}{dt} \frac{d\theta}{dt} - r \sin \gamma \frac{d^2 \theta}{dt^2}$$

$$\frac{dV_n}{dt} = V \cos^2 \gamma \frac{d\gamma}{dt} + \sin \gamma \cos \gamma \frac{dV}{dt} - \frac{v^2}{r} \cos^3 \gamma - \frac{2V^2}{r} \sin^2 \gamma \cos \gamma$$

$$- \sin \gamma \cos \gamma \frac{dV}{dt} + V \sin^2 \gamma \frac{d\gamma}{dt} + \frac{V^2}{r} \sin^2 \gamma \cos \gamma$$

$$\frac{dV_n}{dt} = V \frac{d\gamma}{dt} - \frac{V^2 \cos \gamma}{r}$$

## VITA

John David Cloyd was born in Knoxville, Tennessee on September 25, 1949. He attended elementary and high school in Jefferson City, Tennessee, graduating from Jefferson High School in 1967. Following one year at Carson-Newman College he enrolled at the University of Tennessee, Knoxville, receiving his Bachelor of Science in Aerospace Engineering in 1972.

From 1973 through 1985 he served in the United States Air Force as a pilot and instructor pilot in tactical fighters, accumulating 2000 hours of flight time in that capacity. In 1986 he enrolled at the University of Tennessee Space Institute to begin work toward a Master of Science degree. In August 1987 he began work as an Aerospace Engineer for Calspan Corporation at Arnold Engineering and Development Center in the Technology and Analysis Branch. He is currently working on a project researching the internal carriage and release of aircraft stores and on a project to develop new technology for the simulation of external viscous flow.