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To the Graduate Council:

I am submitting herewith a thesis written by Wallace Samuel Prescottt entitled "Experimental Investigation of Secondary Stresses in Gusset Plates." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Civil Engineering.

A. T. Granger
Major Professor

We have read this thesis
and recommend its acceptance:

Charles R. Walker
E. J. Fabian

Accepted for the Council:

E. J. Fabian
Dean of the Graduate School

EXPERIMENTAL INVESTIGATION OF SECONDARY
STRESSES IN GUSSET PLATES

33

A THESIS

Submitted to
The Graduate Council
of
The University of Tennessee
in
Partial Fulfillment of the Requirements
for the degree of
Master of Science

by

Wallace Samuel Prescott

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CHAPTER I

INTRODUCTION

Preliminary Statement

Prior to 1912 a majority of steel bridge trusses built in the United States were pin-connected. Since that time American practice has changed so that practically all bridge truss members are now connected by riveting. In very large bridge structures a few special connections are still made with pins.¹

Riveted joints for trusses gained rapidly in popularity because structures so connected are credited with being more rigid, less subject to wear and resulting vibrations, more durable and generally safer than pin-connected structures.²

This evolution in structural practice introduced new problems in structural design. In riveted construction, all truss members meeting at a joint are riveted or bolted to a plate (or plates) commonly called gusset plates. Since these plates are an integral part of the structure, their design demands the same consideration as is given to the design of the main members, even

¹Clifford D. Williams and Ernest C. Harris, Structural Design in Metals (New York: The Ronald Press Company, 1949), p. 39.

²Thomas Clark Shedd, Structural Design in Steel (New York: John Wiley and Sons, Inc., 1934) p. 264.

though this problem does not always receive the attention which it deserves. The substitution of rigid riveted joints for pin connections raised another problem in that it increased the magnitude and therefore the importance of secondary stresses, secondary stresses being here defined as those stresses resulting from flexural moments produced in the plane of the truss by the resistance of the joints to free rotation. This problem was not entirely new since secondary stresses are not altogether absent in pin-connected structures. The pins are never absolutely frictionless and therefore offer some rotational restraint to the members which they connect.

The design of gusset plates has followed the same general pattern for many years. The usual practice is to make the size of the gusset plate, in the plane of the truss, as large as necessary for proper placing of all required rivets, and then to select a tentative thickness in accordance with average values used for similar trusses.³ The stresses are then analyzed by passing various sections through the plate and solving for direct, bending, and shearing stresses. Waddell suggested essentially this same procedure in Bridge Engineering in 1916.⁴ As recently as 1949,

³Ibid., p. 349.

⁴J. A. L. Waddell, Bridge Engineering Vol. I (New York: John Wiley and Sons, Inc., 1916) p. 520.

Williams and Harris advocated the method outlined above, even though they did state that it is recognized that the gusset plate does not act in the same manner as the theoretical beam for which the flexure formula is developed.⁵

Even though the complexity of gusset plate stresses has long been recognized or at least suspected, a survey of published material indicates that only in recent years has any appreciable research been done on this problem.

The need of definite information regarding stresses in gusset plates led to a program of research at the University of Tennessee, beginning in 1941 with an investigation of gusset plates by the photoelastic method. This study was performed for a Master's thesis by F. J. Perna.⁶ Although some information was derived from this work, the main conclusion reached was that additional investigation was desirable.

This led to further testing by J. A. Sandel and R. E. Whitmore, their work being done as the basis for Master's theses. Sandel investigated the stresses in a bakelite model of a gusset plate by the photoelastic method,⁷ while Whitmore did research,

⁵Williams and Harris, op. cit., p. 210.

⁶F. J. Perna, "Photoelastic Stress Analysis With Special Reference to Stresses in Gusset Plates" (Unpublished Master's Thesis, University of Tennessee, 1941)

⁷J. A. Sandel, "Photoelastic Analysis of Gusset Plates", (Unpublished Master's Thesis, University of Tennessee, 1950)

involving a larger aluminum model, employing brittle coatings and strain gages.⁸

In 1952 further refinements were made in the model used by Whitmore and additional tests were made, the results of which are published in a bulletin entitled, "Experimental Investigation of Stresses in Gusset Plates".⁹

Purpose

Although the work described in the preceding paragraphs was carried out in great detail and valuable information concerning gusset plate stresses was established, the investigation was not complete; to cite one instance, only the effects of direct or axial stress in the truss members were considered.

Since it is definitely known that all trussed structures with riveted joints are subjected to secondary stresses, it seemed desirable to investigate the effects of such stresses in gusset plates. As previously stated, the term "secondary stresses" as used herein means those stresses resulting from flexural moments produced in the plane of the truss by the resistance of the joints

⁸R. E. Whitmore, "Experimental Investigation of Stresses in Gusset Plates", (Unpublished Master's Thesis, University of Tennessee, 1950)

⁹R. E. Whitmore, Experimental Investigation of Stresses in Gusset Plates (Knoxville, Tennessee: Engineering Experiment Station, The University of Tennessee, 1952)

to free rotation. Therefore, the purpose of this investigation is to study the magnitude and distribution of secondary stresses in gusset plates. Specifically, the purpose is to:

- a. Determine the general secondary stress distribution in gusset plates,
- b. Determine the maximum intensity of secondary stress and its location,
- c. Develop, if possible, a satisfactory method of determining the gusset plate stresses from calculated secondary shears, moments and axial loads,
- d. Compare the maximum intensity and location of secondary stresses with the intensity and location of primary stresses.

Scepe of Investigation

As in the case of previous studies, in which only the effect of direct stress was considered, it is proposed to limit this investigation to one particular type of gusset plate. It is realized that supplementary investigations dealing with other types of plates and other types of trusses would be desirable. Especially would it be desirable to test models of top chord joints and models of joints in which the ratio of the depths of the chords to the panel lengths is greater than that for the model chosen. However, in order to make such tests, much more time and

expense would be involved than is available for the present study.

Even though this investigation is of limited extent, it is felt that some results may be obtained which will have general application to the design of gusset plates.

CHAPTER II

DESIGN AND CONSTRUCTION OF THE MODEL AND LOADING SYSTEM

Selection of Prototype

Since the present study was to be limited to one particular type of gusset plate, it was desired to select one that is representative of a large and important group of plates. This suggested a joint from a common type of truss, with several members meeting at the joint, but with no chord splice in the joint.

Joint L_2 of a through Warren type highway bridge was selected since this type of truss is used almost exclusively in modern bridge design and the gusset plates at this joint represent a typical case.

The original aluminum model of the gusset plates used by Whitmore¹ was to be used in this experiment. This model represented a prototype of joint L_2 from a through Warren type highway bridge truss, having a span of 296 feet. The use of this same prototype would have been desirable in the present study, but unfortunately the complete design of this prototype was not available. However, a design of a similar highway truss with a 308 foot span was available and was used as the prototype, since it was necessary to

¹R.E. Whitmore, Experimental Investigation of Stresses in Gusset Plates, loc. cit.

have all the members sectioned before calculating the truss deflections, from which secondary effects are calculated. This substitution should have little effect on the final results of the investigation, since the proportions involved were quite similar.

The prototype dimensions and the sections used for the truss members are shown in Figure I on page 9.

Calculation of Secondary Effects in Prototype

In order to know what effects would have to be reproduced in the model, the prototype was first analyzed for secondary effects. From the original design the total axial stress, consisting of dead load stress, live load stress and impact, was known for each member in the truss. However, these values did not represent simultaneous values of stress because different positions of live load were used to produce the maximum live load stress in the various members. Simultaneous values of stress are necessary for the computation of deflections in a truss, so only the dead load stresses were used for this study since the dead load stresses are all developed at the same time.

The deformations of the individual truss members were computed from the familiar expression $\frac{SL}{AE}$ and from these deformations a Williot diagram was constructed. For the Williot construction, L_5 was assumed to be fixed in position and U_5L_5 fixed in direction. In order to check the correctness of the Williot diagram, the total

NOTES:

Live Load: H-20 AASHO

Rivets: $\frac{7}{8}$ " ϕ except use $\frac{3}{4}$ " ϕ through flanges of 10" $\mathbb{E}$.

Lacing and tie plates:

1. All web members except those having solid transverse plates to have $2\frac{3}{4} \times \frac{3}{8}$ single lacing @ 60° with common rivets and end tie $\mathbb{E}$ s $15 \times \frac{5}{16}$.
2. All top chords to have $2\frac{3}{4} \times \frac{9}{16}$ double lacing @ 45° with common rivets and end tie $\mathbb{E}$ s $30 \times \frac{1}{2}$.
3. End posts to have $3\frac{1}{2} \times \frac{5}{8}$ double lacing @ 45° with separate rivets and end tie $\mathbb{E}$ s $30 \times \frac{1}{2}$.
4. Members L_0L_2 to be laced same as web members.
5. All other bottom chords to have end tie $\mathbb{E}$ s $15 \times \frac{5}{16}$ and intermediate tie $\mathbb{E}$ s about 3 ft. c. to c.

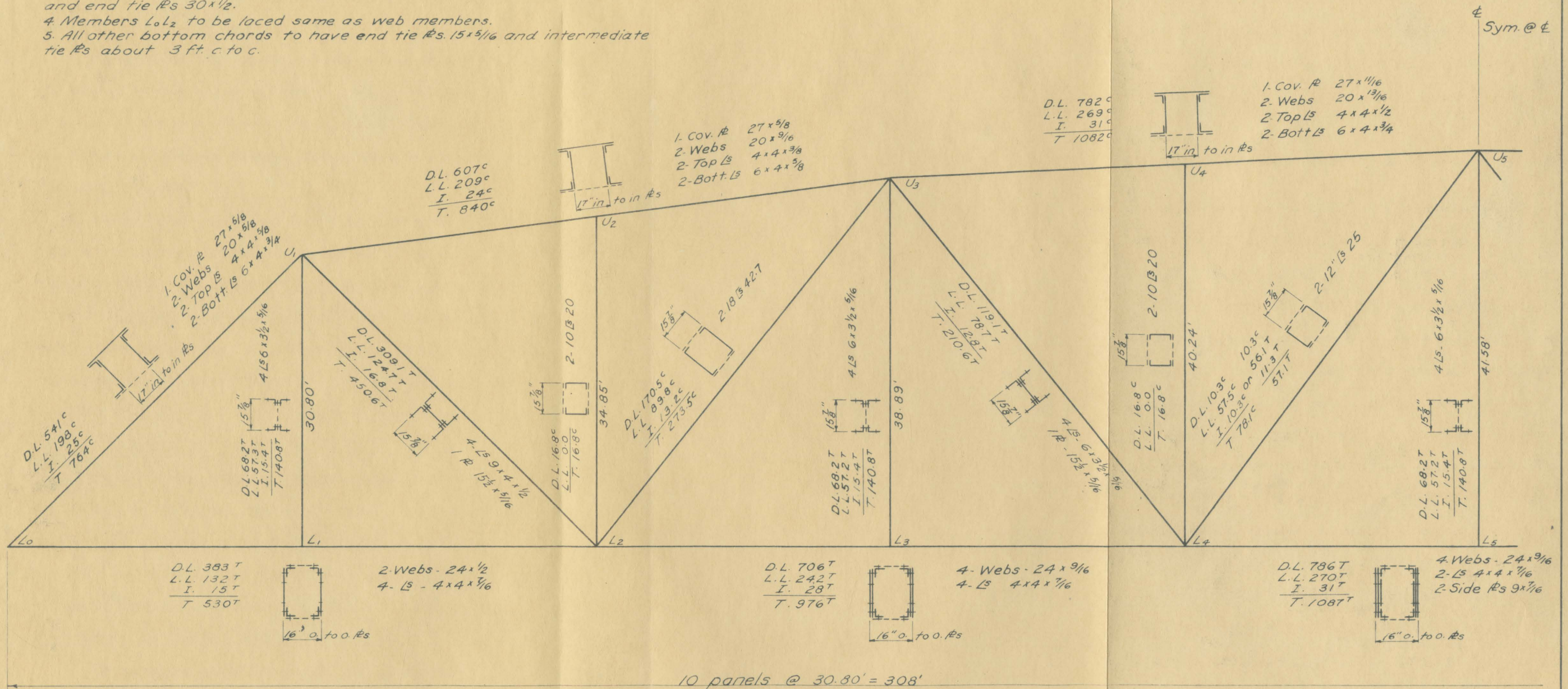


FIGURE 1. TRUSS SECTIONS AND DIMENSIONS

vertical deflection of L_5 was computed by the method of virtual work. The graphical solution and the analytical solution agreed very closely. The Williot diagram and accompanying information are shown in Figure 2, page 11.

From the Williot diagram the deflection, perpendicular to the axis of the member, of one end of a member relative to the other end of the same member could be scaled. These values for the various members are recorded in the table in Figure 2 under the column heading " Δ ".

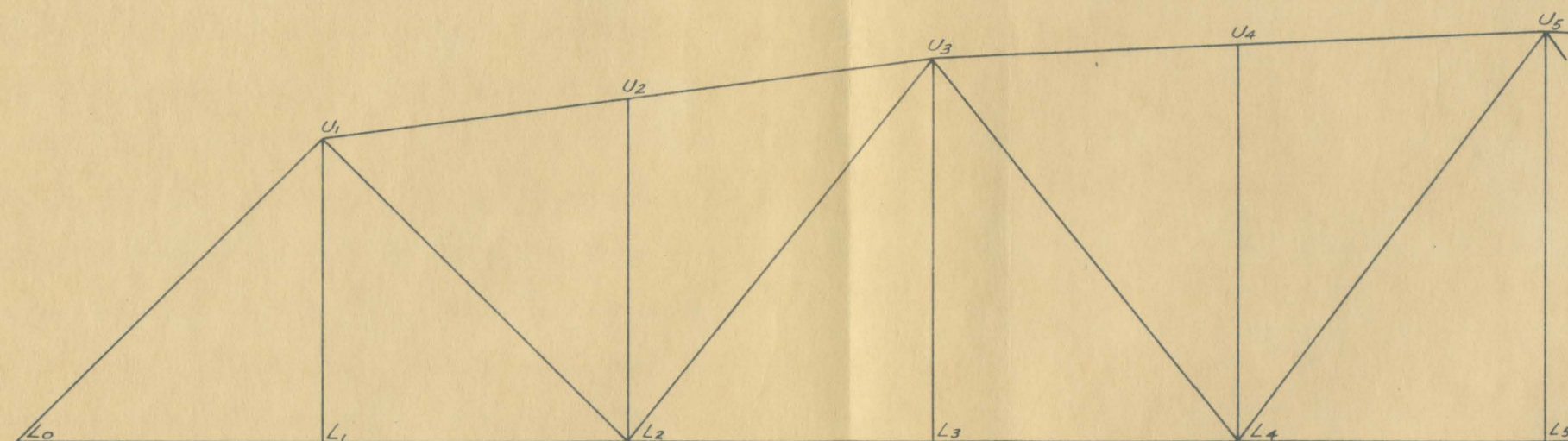
If one end of a straight member is allowed to deflect relative to the other end of the member and if the ends are fixed against rotation, end moments will be developed, the value of which can be found from the expression $M = \frac{6EI\Delta}{L^2}$. This expression can be derived very readily from the moment area theorems. Using this expression and the scaled values of Δ , the fixed-end moments resulting from the truss deflections were computed and recorded in the last column in the table of Figure 2.

These fixed-end moments represent a state of unbalance at the joints, so the joints must rotate until a state of balance is reached, and the final moments represent the resulting secondary moments. In this study the final end moments were obtained by the process of moment distribution. The results of this moment distribution are shown in Figure 3, page 13.

The resulting secondary moments are accompanied by shears.

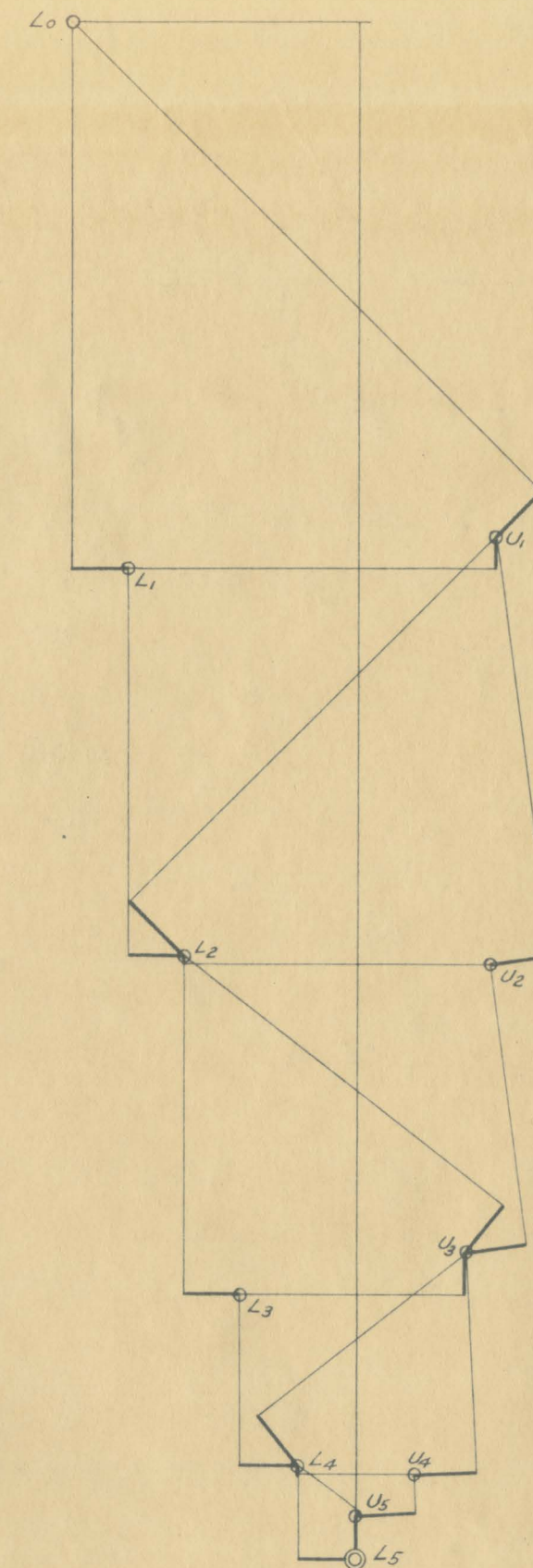
Location	Member	S Kips	L"	A in ²	$\frac{SL}{AE}$ "	Δ "	$\psi = \frac{\Delta}{L}$	I in ⁴	$K = \frac{I}{L}$ in ³	Moment @ the joints kips ft.
Bottom Chord	L ₀ L ₁	+383	369.6	37.24	+0.127	1.235	0.00335	2800	7.58	381
	L ₁ L ₂	+383	369.6	37.24	+0.127	0.875	0.00237	2800	7.58	270
	L ₂ L ₃	+706	369.6	67.24	+0.129	0.763	0.00207	4240	11.48	356
	L ₃ L ₄	+706	369.6	67.24	+0.129	0.390	0.00106	4240	11.48	183
	L ₄ L ₅	+786	369.6	75.12	+0.129	0.205	0.00056	4309	11.66	98
Top Chord	L ₀ U ₁	-541	522.7	64.97	-0.145	1.495	0.00286	4388	8.40	360
	U ₁ U ₂	-607	372.8	56.82	-0.133	0.955	0.00256	3866	10.35	398
	U ₂ U ₃	-607	372.8	56.82	-0.133	0.635	0.00170	3866	10.35	264
	U ₃ U ₄	-782	370.0	72.44	-0.133	0.495	0.00134	4794	12.95	260
	U ₄ U ₅	-782	370.0	72.44	-0.133	0.085	0.00023	4794	12.95	45
Diagonals	U ₁ L ₂	+309	522.7	29.84	+0.181	1.165	0.00223	514	0.98	33
	L ₂ U ₃	-171	595.3	24.96	-0.136	0.915	0.00154	1098	1.84	43
	U ₃ L ₄	+119	595.3	16.32	+0.145	0.595	0.00100	98	0.17	3
	L ₄ U ₅	-10.3	620.9	14.64	-0.146	0.170	0.00027	287	0.46	2
Verticals	U ₁ L ₁	+68.2	369.6	11.48	+0.073	0.825	0.00224	109	0.30	10
	U ₂ L ₂	-16.8	418.1	11.72	-0.020	0.685	0.00164	157	0.38	9
	U ₃ L ₃	+68.2	466.7	11.48	+0.093	0.505	0.00108	109	0.23	4
	U ₄ L ₄	-16.8	482.8	11.72	-0.023	0.262	0.00054	157	0.33	3
	U ₅ L ₅	+68.2	499.0	11.48	+0.099	0.000	0.00000	109	0.22	0

$$M = \frac{6EI\Delta}{L^2} = 6EK\psi \text{ in. Kips} = \frac{6EK\psi}{12} \text{ ft Kips} = 15,000 K\psi$$



TRUSS OUTLINE

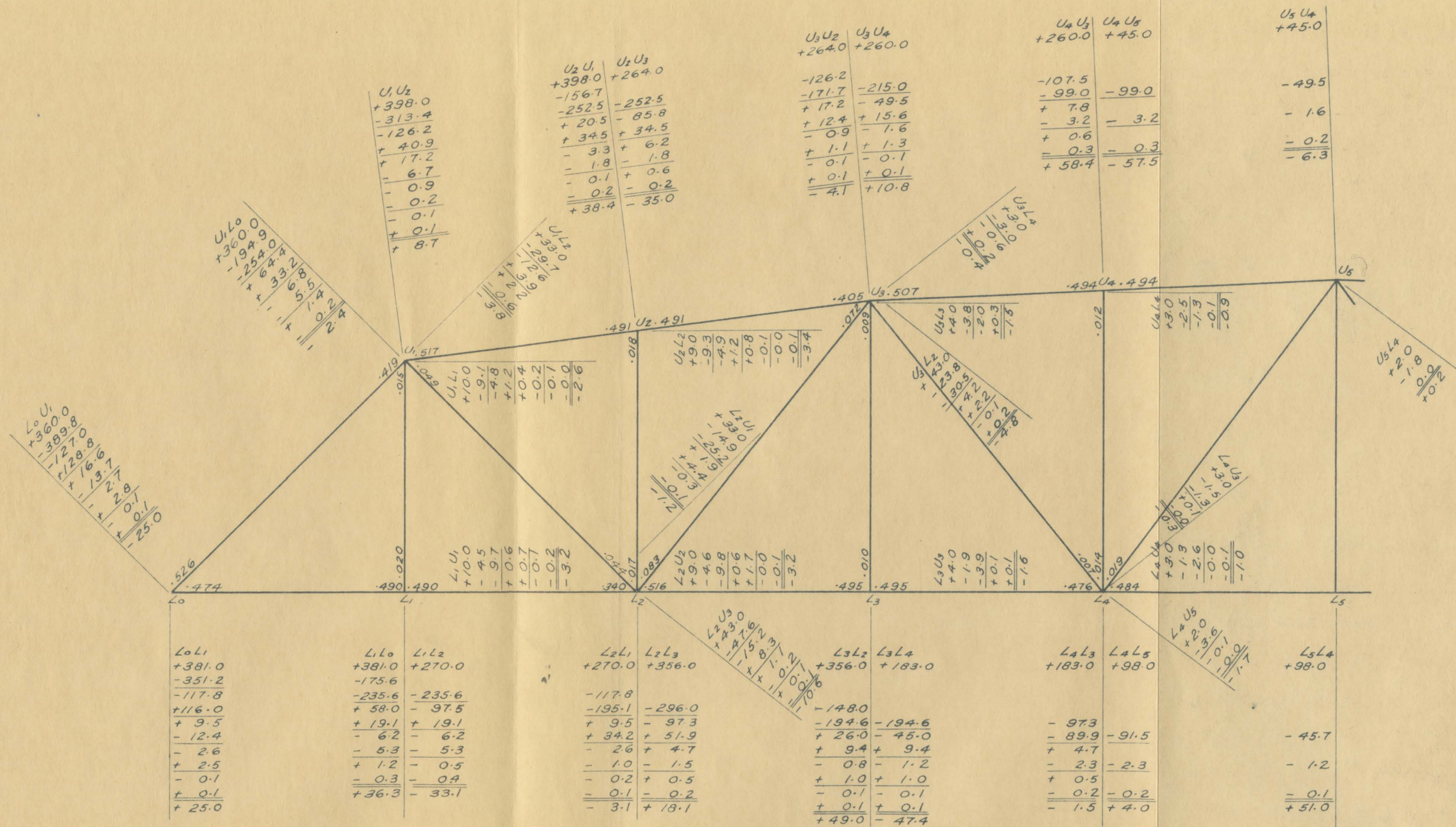
FIGURE 2. TRUSS DEFLECTIONS AND FIXED-END MOMENTS

WILLIOT DIAGRAM
Scale: 1" = 0.40"

These shears induce forces at the joints which cause additional axial stress in the truss members. The forces at the joints and the resulting axial stresses in the truss members are shown in Figure 4, page 14. These calculations were carried only far enough to include joint L_2 , which was the joint under consideration. If these additional axial stresses are appreciable, they have to be added to the original axial loads; then new deflections are computed, and the process described above is repeated. In the case under consideration, the axial loads resulting from secondary moments were so small relative to the original values that no redistribution was made.

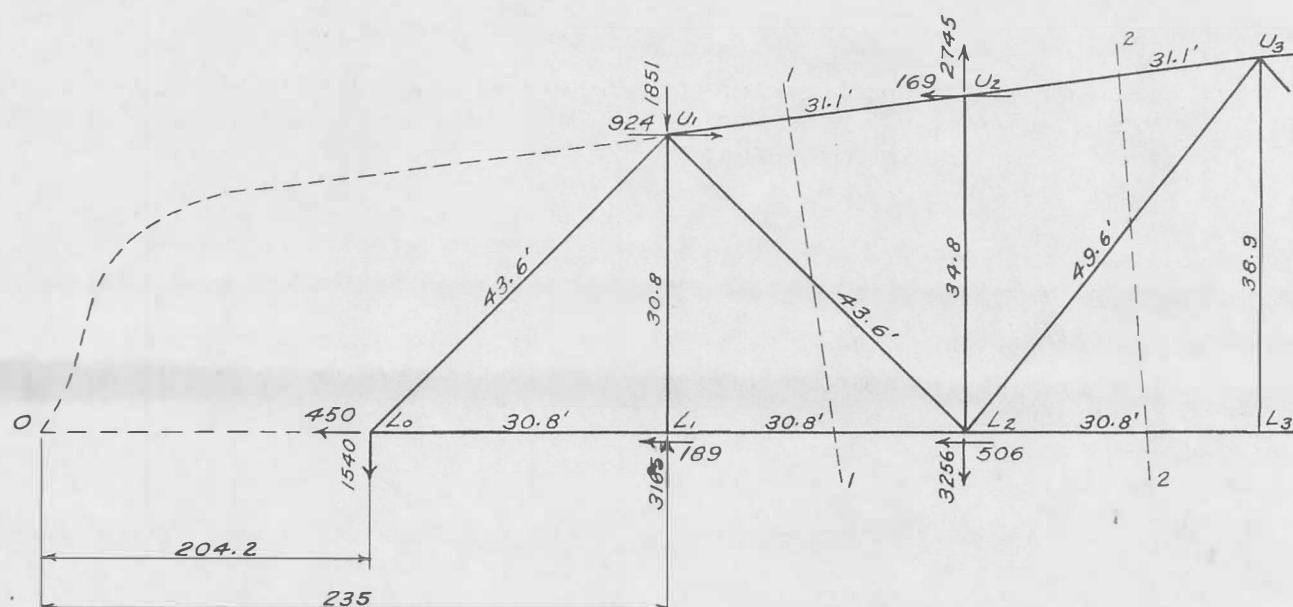
Selection of the Model

The original aluminum model used by Whitmore in his investigation had been preserved and the strain gages were still in place. Upon examination these gages appeared to be in good working condition. After some consideration it was concluded that this model might be adapted to the secondary stress study even though some changes in the original construction would have been desirable. For instance, if the members meeting at the joint had been long enough, they could have been loaded at the inflection points with shears only, since the moment is zero at these points. However, it was decided that necessary alterations could be made in much less time than would be required to build a new model. Also, the use of the existing



Moments are in kips ft.

FIGURE 3. SECONDARY MOMENTS BY MOMENT DISTRIBUTION



$L_1 L_2$ - Section 1-1

$\sum M_{U_1} = 0$

$$1540 \times 30.8 = 47500$$

$$(450 + 189) \times 30.8 = 19850$$

$$27650 \div 30.8 = -896 \#$$

$U_1 L_2$ - Section 1-1

$\sum M_{L_0} = 0$

$$924 \times 30.8 = 28,460$$

$$1851 \times 235 = 435,000$$

$$1540 \times 204.2 = 315,000$$

$$778,460$$

$$3165 \times 235 = 744,000$$

$$34,460 \div 265.8 = 130$$

$$130 \times \frac{43.6}{30.8} = -185 \#$$

$U_2 L_2 = +2745 \#$ (by inspection)

$L_2 L_3$ - (Section 2-2)

$\sum M_{U_2} = 0$

$$(450 + 189 + 506) \times 38.9 = 44,600$$

$$3165 \times 61.6 = 195,000$$

$$2745 \times 30.8 = 84,500$$

$$169 \times 4.04 = 680$$

$$324,780$$

$$1540 \times 92.4 = 142,300$$

$$1851 \times 61.6 = 114,000$$

$$3256 \times 30.8 = 100,300$$

$$924 \times 8.08 = 7460$$

$$364,060$$

$$324,780$$

$$39,280 \div 38.9 = -1020 \#$$

$L_2 U_3$ - Section 2-2

$\sum M_{L_2} = 0$

$$169 \times 34.8 = 5,880$$

$$(3165 - 1851) \times 235 = 309,000$$

$$314,880$$

$$(3256 - 2745) \times 265.8 = 136,000$$

$$924 \times 30.8 = 28,500$$

$$1540 \times 204.2 = 314,500$$

$$479,000$$

$$314,000$$

$$164,120$$

$$164,120 \div 265.8 = 620$$

$$620 \times \frac{49.6}{38.9} = +790 \#$$

FIGURE 4. AXIAL LOADS RESULTING FROM SECONDARY MOMENTS

strain gages and model material would represent a considerable reduction in the expense of the investigation. In view of these facts, the existing model was adopted for the present study.

The shape and size of this model are shown in Figure 5, page 16. At a scale ratio of 1:4, the model represents a prototype of two plates 66" x 50" x $\frac{1}{8}$ ". An examination of the detail of joint L_2 of the prototype (see Figure 6, page 17) indicates that the plates at this joint are only slightly larger than those given by the dimensions above. Therefore, the scale ratio was chosen as 1:4.

The exact shape of the members had been thought to be relatively unimportant for the first study and was judged to be unimportant for the present problem, especially since the depth ratios used were close to the proportions indicated by the scale ratio. Solid rectangular sections had been selected for the members to simplify constructing and testing the model.

The members were attached to the plates by a simple, symmetrical arrangement of tight-fitting pins and bolts, which assured unity of action of the two separate plates and simulated the action of the rivets in the prototype. The use of some bolts assured contact between the plates and allowed some of the load to be carried by friction.

A number of factors entered into the selection of the original model material. Since the experimental method to be used

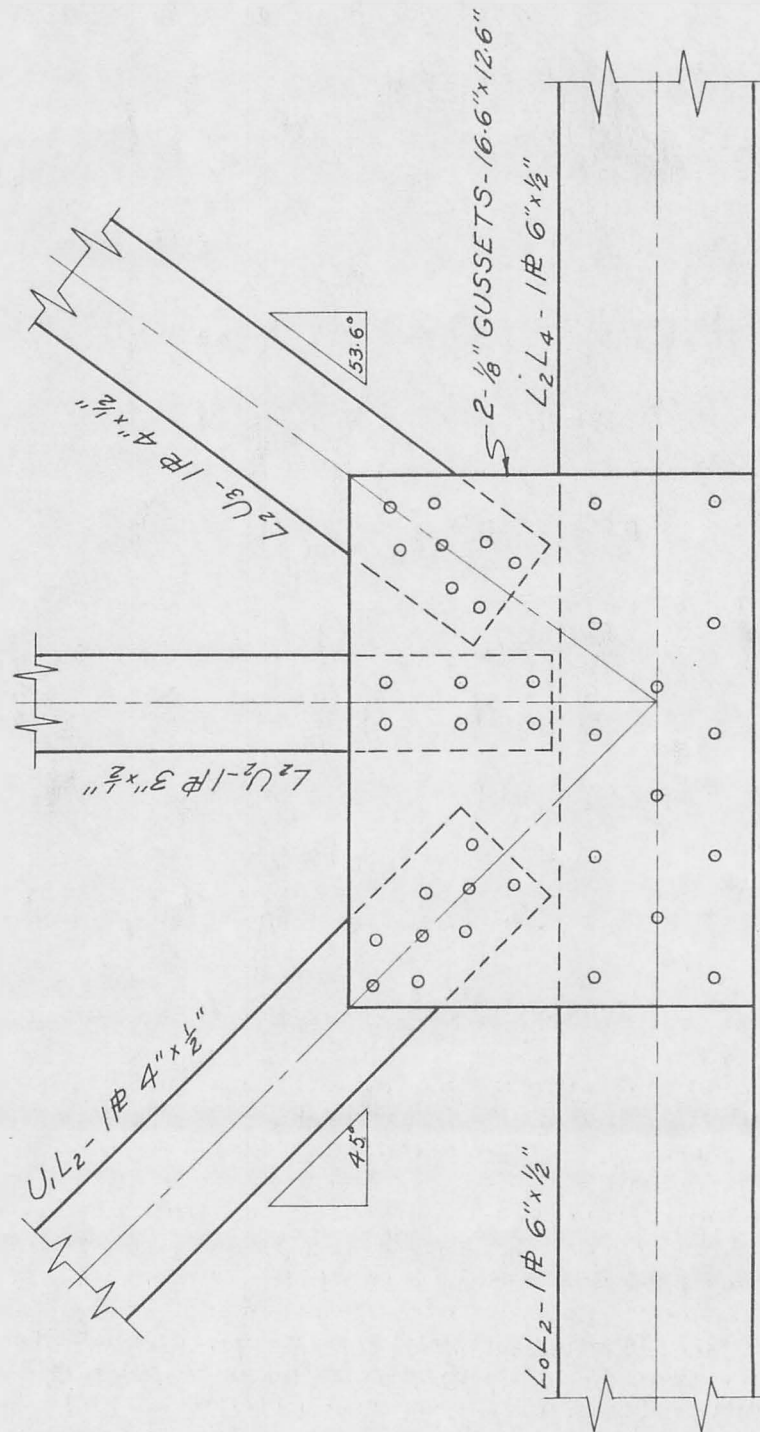


FIGURE 5. THE ALUMINUM MODEL

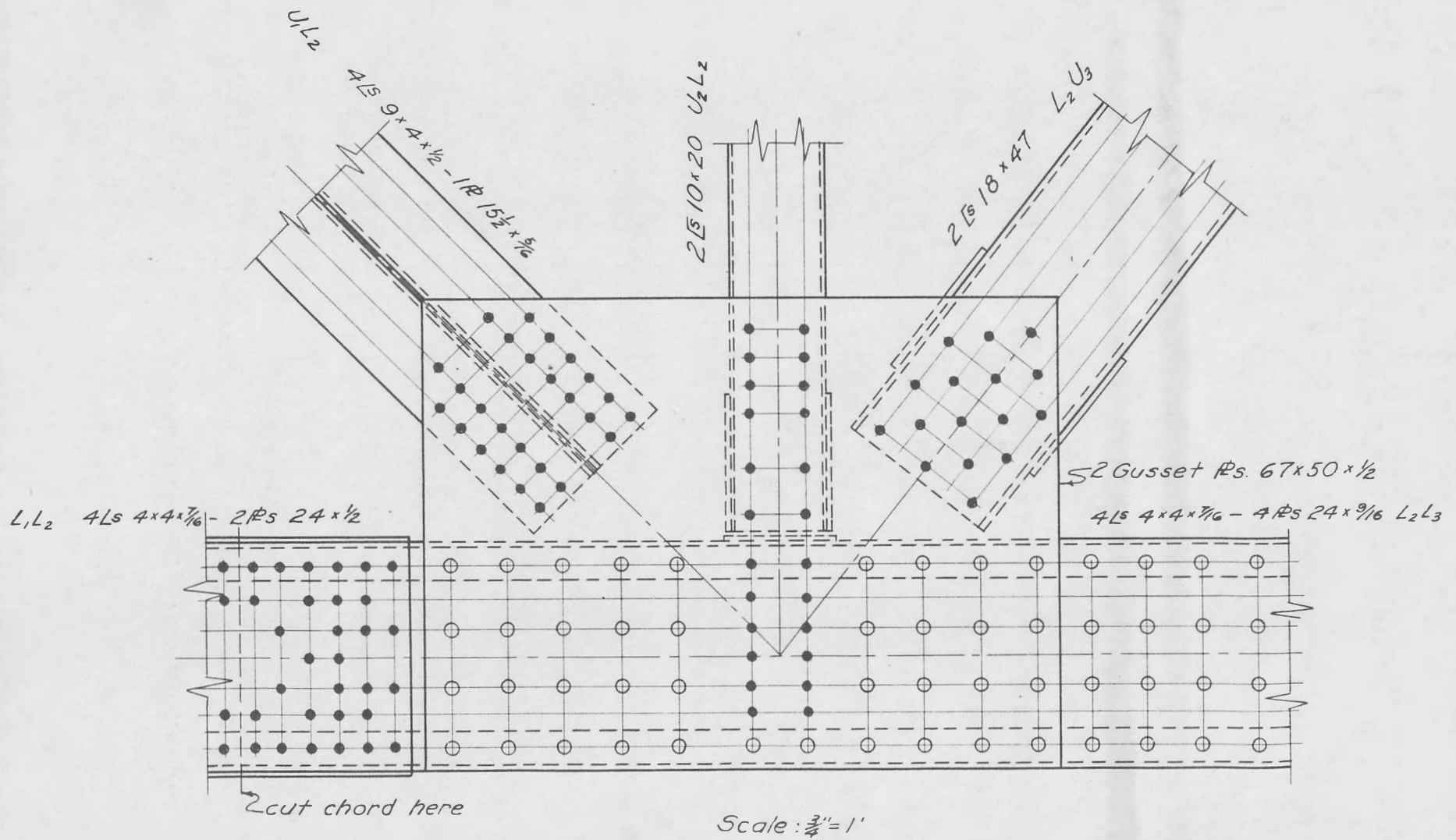


FIGURE 6. JOINT L_2 OF THE PROTOTYPE

was strain measuring, strains of large magnitude were desired, thus requiring a material with a low modulus of elasticity. Also it was desired that the material be of high strength, light in weight, ductile, and in general possessed of properties similar to those of structural steel. These factors indicated that a high strength aluminum alloy would be the most suitable material. Subsequently, the aluminum alloy selected was 61 ST, which has the following physical and mechanical properties:²

Modulus of elasticity -----	10,000,000 lbs. per sq. in.
Modulus of rigidity -----	3,800,000 lbs. per sq. in.
Poisson's ratio -----	0.33
Yield strength (tension and compression) -----	39,000 lbs. per sq. in.
Ultimate tensile strength -	45,000 lbs. per sq. in.
Per cent elongation in two inches -	15%
Yield strength (shear) ----	26,000 lbs. per sq. in.
Ultimate strength (shear) -	30,000 lbs. per sq. in.
Weight -----	0.98 lbs. per cubic in.

Alloys:

Copper -----	0.25 per cent
Silicon -----	0.6 per cent
Magnesium --	1.0 per cent
Chromium ----	0.25 per cent

The plates, members and pins were all of the same alloy, while the bolts were tight-fitting aircraft bolts, exact composition unknown.

Strain Measuring Devices and Equipment

In this investigation, the only method of measuring strains

²Ibid, p. 5

was by the use of SR-4 wire bonded strain gages. In order to obtain a complete picture of the state of stress within the loaded plates, the determination of the principal stresses and their directions (p , q , and θ) was necessary. Except at the edges of the plates, strain rosettes were used for this determination. Of several available types of rosettes, a simple 45-degree rectangular rosette (type AR-1) with three gages at 45 degrees with one another was selected, because for a given gage length, this type covers a minimum area.

Less expensive single gages (type A-1) were used along the periphery of the plates, since along the edges, the direction of the principal stresses, and the magnitude of one of the stresses, are known. When this much information is available, the use of costly rosettes is unnecessary.

Nineteen AR-1 and eleven A-1 gages were cemented to each plate. For the original study, the locations had been well scattered, but more closely spaced in what were expected to be critical regions. Also, an effort was made to place the gages with their axes approximately in the direction of the principal stresses. The direction of the principal stresses in the present investigation could not be accurately predicted, but it was felt that the location of the gages was satisfactory in the critical regions. The location and direction of the gages are shown in Figure 7, page 20.

There was an exact duplication of the strain gage pattern on each of the two plates, so that at each gage location, two independent gage readings would be available. By this arrangement,

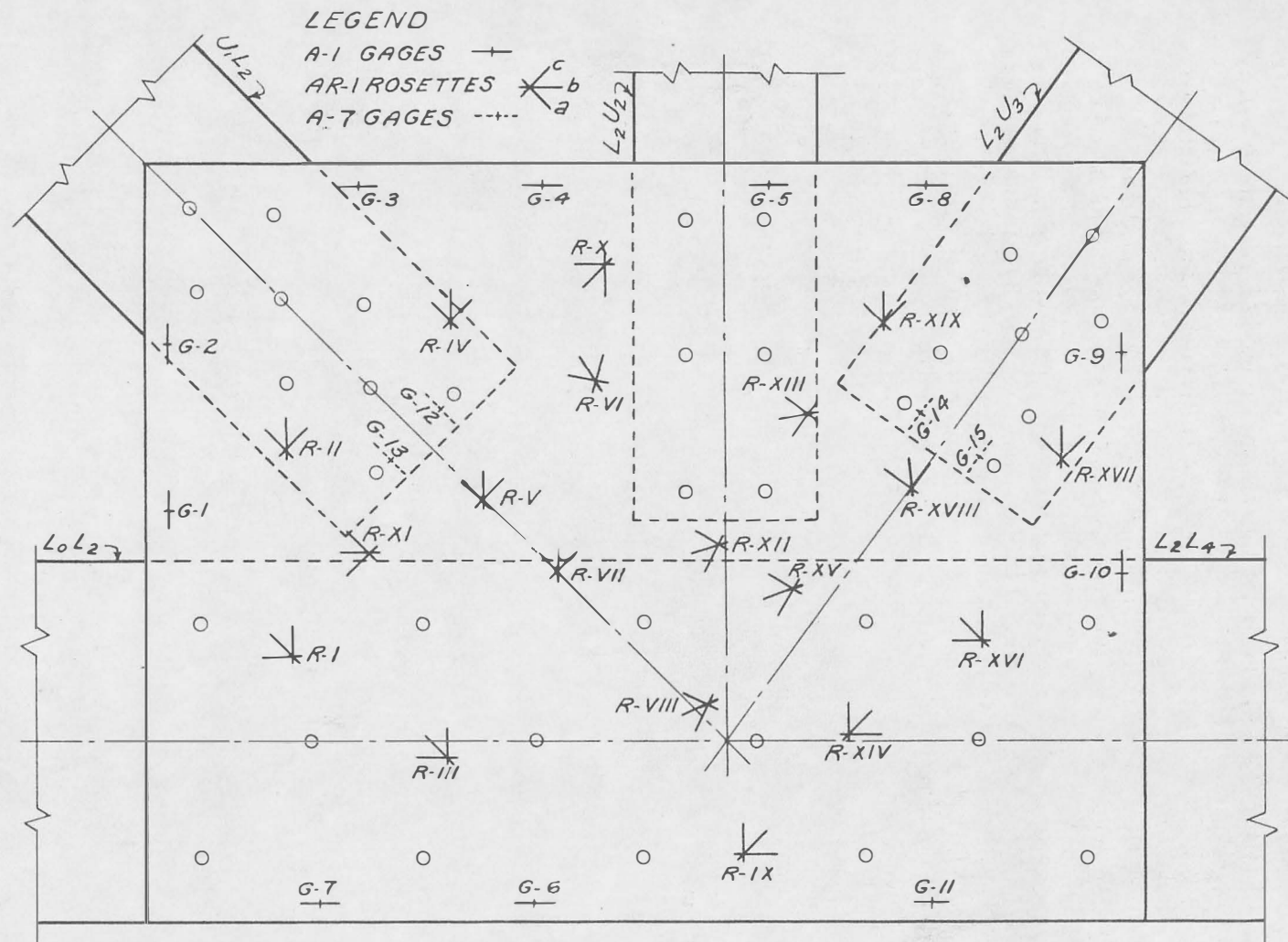


FIGURE 7. LOCATION, DIRECTION, AND IDENTIFICATION OF STRAIN GAGES

the two readings could be averaged, thus eliminating the effect of bending (perpendicular to the plane of the truss) on the stresses and strains in the plates.

On one plate only, four small gages (type A-7) were placed adjacent to the rivet holes at the ends of the diagonals in order to attempt to determine the effect of stress concentration near the holes in the plate.

Flexible, stranded copper wire was used to connect the gages to the indicator used to measure the strains. In order that each gage could be connected permanently throughout the series of tests, each gage was connected to a switch. The switches used were single-circuit with twenty-four positions. As there were 152 gages altogether, seven switches were required. The switches were connected through a common terminal to a type "K" portable strain indicator.³

The Loading System

After some consideration, it was decided that the original loading frame could also be used for the present study by the addition of necessary parts. The frame, made up of $2\frac{1}{2}$ -inch standard steel pipe, reinforced with steel plates $3\frac{1}{2}$ inches by $\frac{1}{2}$ inch, was rectangular and large enough to accommodate the model.

In the first study, only direct stress was applied to the

³Ibid, p.6

members and the loading was further simplified by making the stress in member L_2U_2 zero. This member in a Warren truss normally carries a relatively small load and to neglect this load seemed reasonable.

The manner in which the tension and compression forces were applied to the members should be apparent from Figure 8, page 23, which shows the original model and loading frame before any additions or alterations were made. The model was attached to the frame by specially built clevises, one at each end of the member.

Tension or compression was applied by tightening or loosening the appropriate nuts so as to move the clevises in the proper direction.

It was realized in the beginning that the loading of the model for the secondary stress investigation was going to be much more complex than the loading for the direct stress study, because each member was to be subjected not only to an axial load but also to a moment and a shear.

At first it was thought that the moments should be applied as couples and the shears as transverse forces. The axial forces could be developed by the original arrangement of clevises. However, this meant that four forces would have to be produced and measured for each member; two for the couple, one for the shear, and one for the axial load. Subsequently, it was decided that the loading could be simplified by applying the moment as an eccentric load, using a force equal to the desired axial load and adjusting the moment arm to give the desired moment. By adopting this scheme of loading, the number of forces could be reduced from a total of twenty to a total

Photograph of Original Model

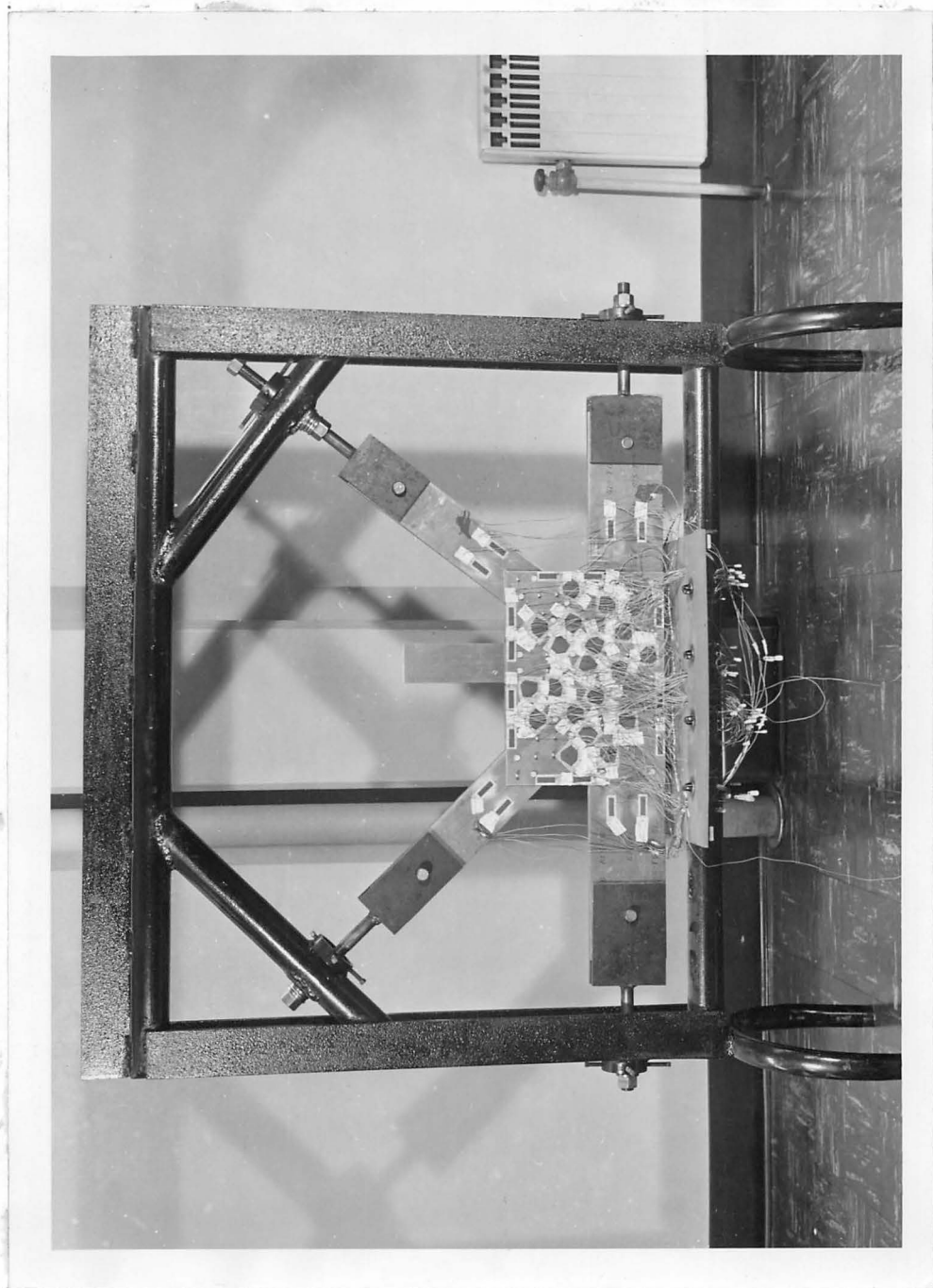


Figure 8. The Original Model and Loading Frame

of ten.

The next consideration was the manner in which forces could be applied. It was necessary that the forces be applied in such a way that their magnitude could be accurately determined. Spring balances with clevises were considered, but this arrangement would require so much space that the existing frame could not be conveniently used.

After considering other schemes for measuring the loads on the model, the use of strain gages seemed to offer the best solution. Small aluminum members with strain gages attached could be arranged in the proper position and direction of the desired forces, the force being developed by means of a clevis arrangement.

With this general scheme of loading in mind, a large scale drawing of the existing model and loading frame was made. Some adjustments were then made in the loading plan so that the model would be disturbed as little as possible and the existing frame could be utilized.

Near the end of each member, two two inch wide steel bars perpendicular to the member (one on each side of the member) were attached by means of two bolts, the bolts being placed as near the edges of the member as possible. A thickness of $\frac{1}{4}$ inch was used for the bars. In regions where stress was high, the bars were reinforced by the addition of another $2" \times \frac{1}{4}"$ bar, thus giving an effective bar $2" \times \frac{1}{2}"$ in size. The bars were made of sufficient

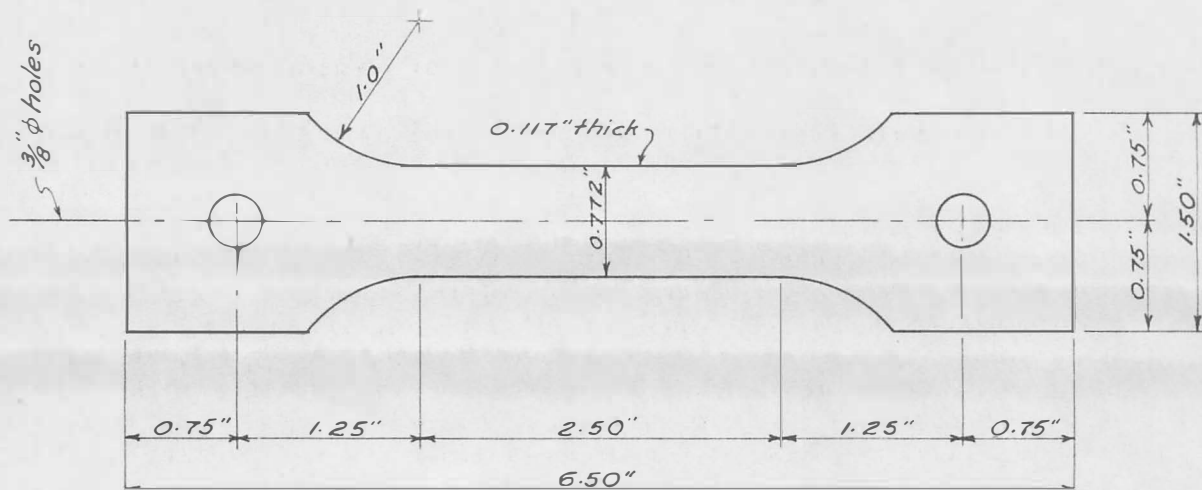
length to provide the desired moment arms. At the ends of these bars, one end of the aluminum loading strip was attached by means of a round pin. The other end of the loading strip was attached to a small clevis by means of a pin. The clevis was in turn connected by various arrangements to the loading frame. The same general arrangement was used to develop the shearing forces, except that for these loads, the loading strips were parallel to the attached bars, making the shear act at right angles to the member. In each case the loading strip was located in the plane of the members so that no bending in a plane perpendicular to the plane of the gusset plates would be produced. The whole system was devised in such a manner that the loading strips would always be loaded in tension, since they were to be made of thin material and would buckle if subjected to compression. The strips were made of 63S-T5 aluminum alloy, having a modulus of elasticity of 10,000,000 and a yield strength of 30,000 psi.

At the mid-length of the strip a type A-1 strain gage was placed on each side. Each of the gages was attached to the same type of switch as was used for the other gages.

The method of loading should be apparent from an examination of Figure 9, page 26.

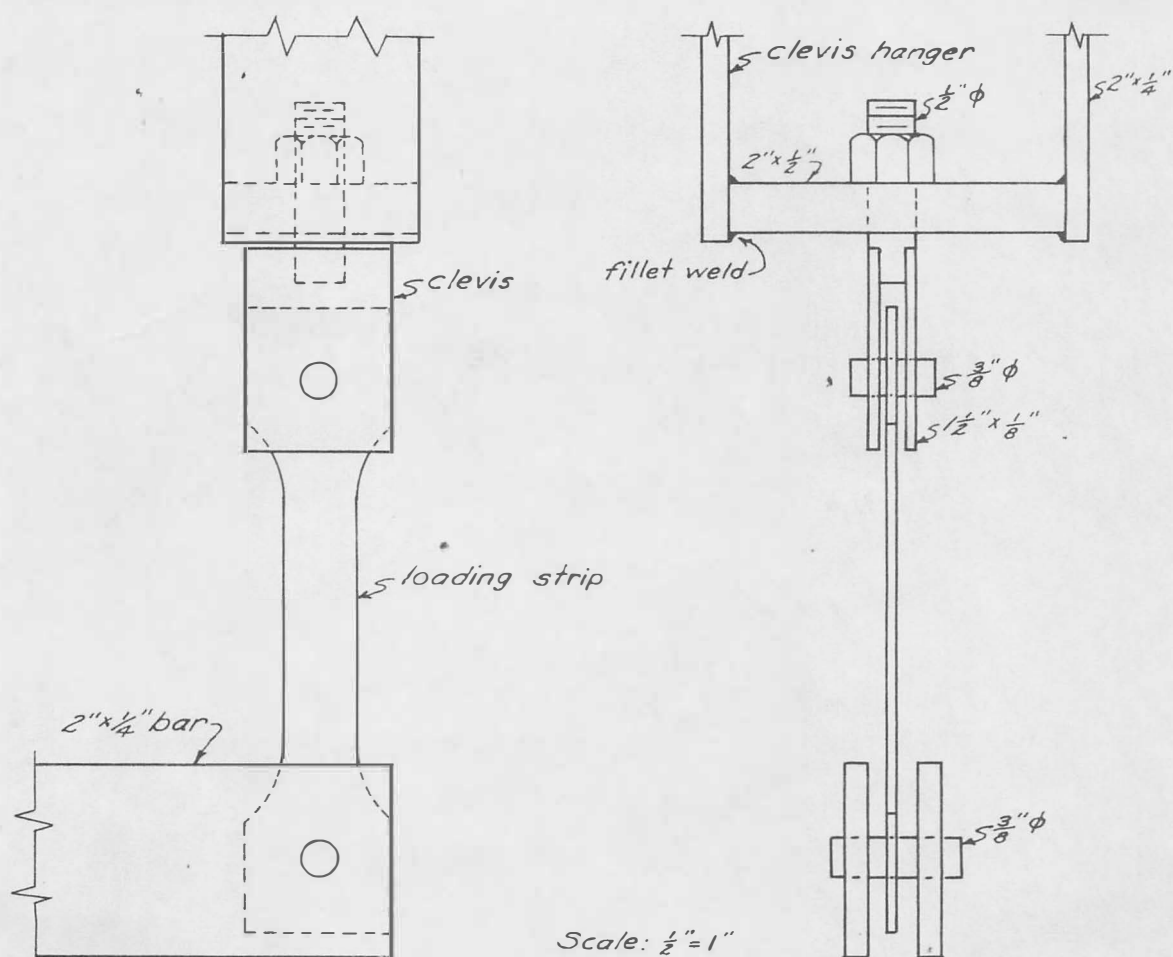
The details of a typical loading strip and clevis arrangement are shown in Figure 10, page 27.

In Figure 11, page 28 is shown a general view of the final arrangement.



THE LOADING STRIP

Scale: $\frac{3}{4}$ " = 1"



Scale: $\frac{1}{2}$ " = 1"

FIGURE 10. TYPICAL LOADING STRIP WITH CLEVIS

Photograph of Final Arrangement

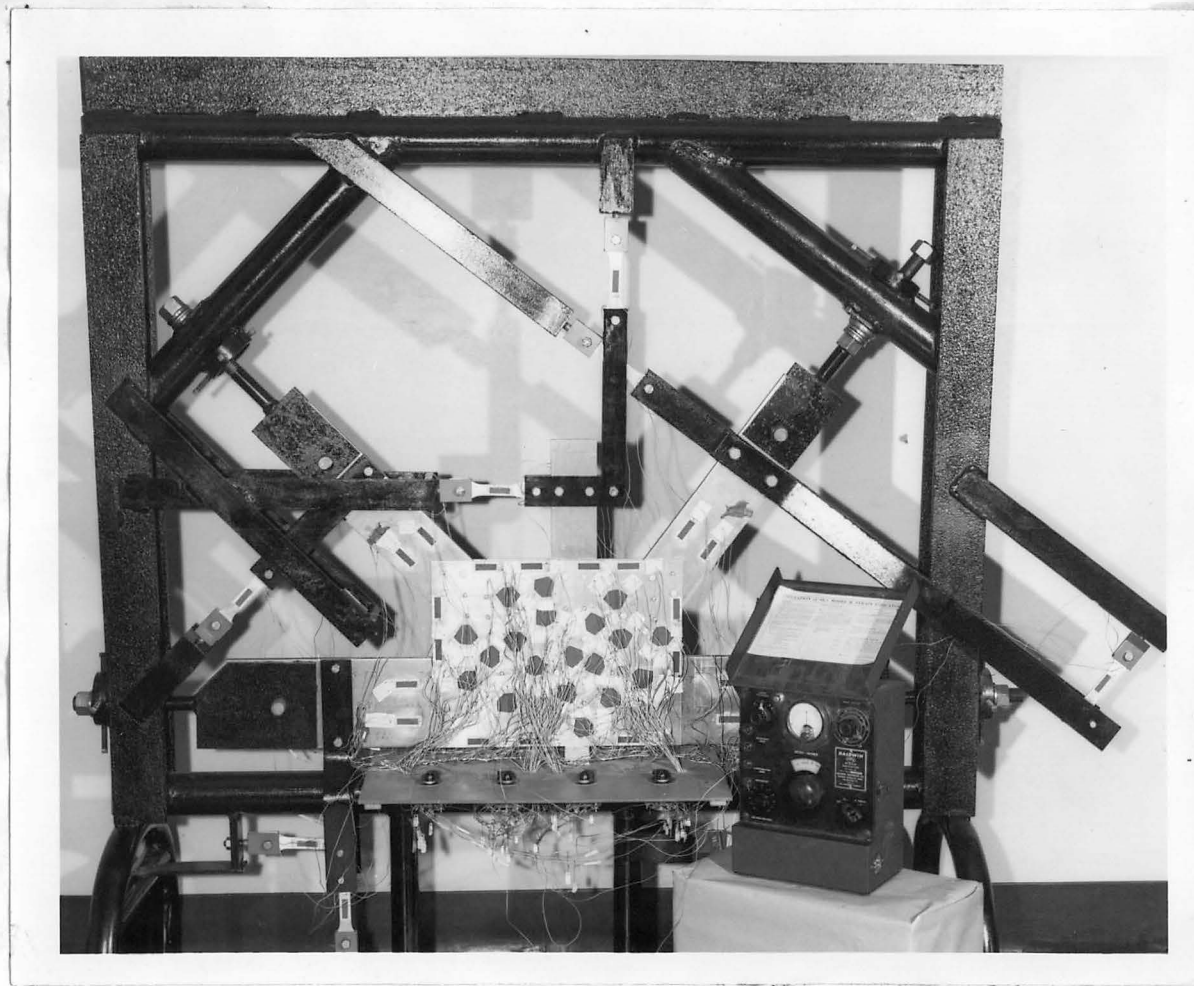


Figure 11. Final Arrangement of Model and Loading System

CHAPTER III

LOADING THE MODEL AND MEASURING STRAINS

Characteristics of the Loads

In any model analysis, a definite relationship between the stresses in the model and the stresses in the prototype must be established. After this decision is made, the magnitude of the loads to be applied to the model can be calculated. For the problem at hand, it was decided to make the stresses in the model equal to the stresses in the prototype.

The scale ratio of the model to the prototype was 1:4, meaning that any linear dimension of the model was $\frac{1}{4}$ that of the prototype. With this ratio of linear dimensions, the area of any section of the model was $\frac{1}{16}$ that of the corresponding section of the prototype and the section modulus of any section of the model was $\frac{1}{64}$ that of the corresponding section of the prototype. In order to produce equal stresses in the model and prototype, axial loads and shears had to have a ratio of 1:16, and moments a ratio of 1:64.

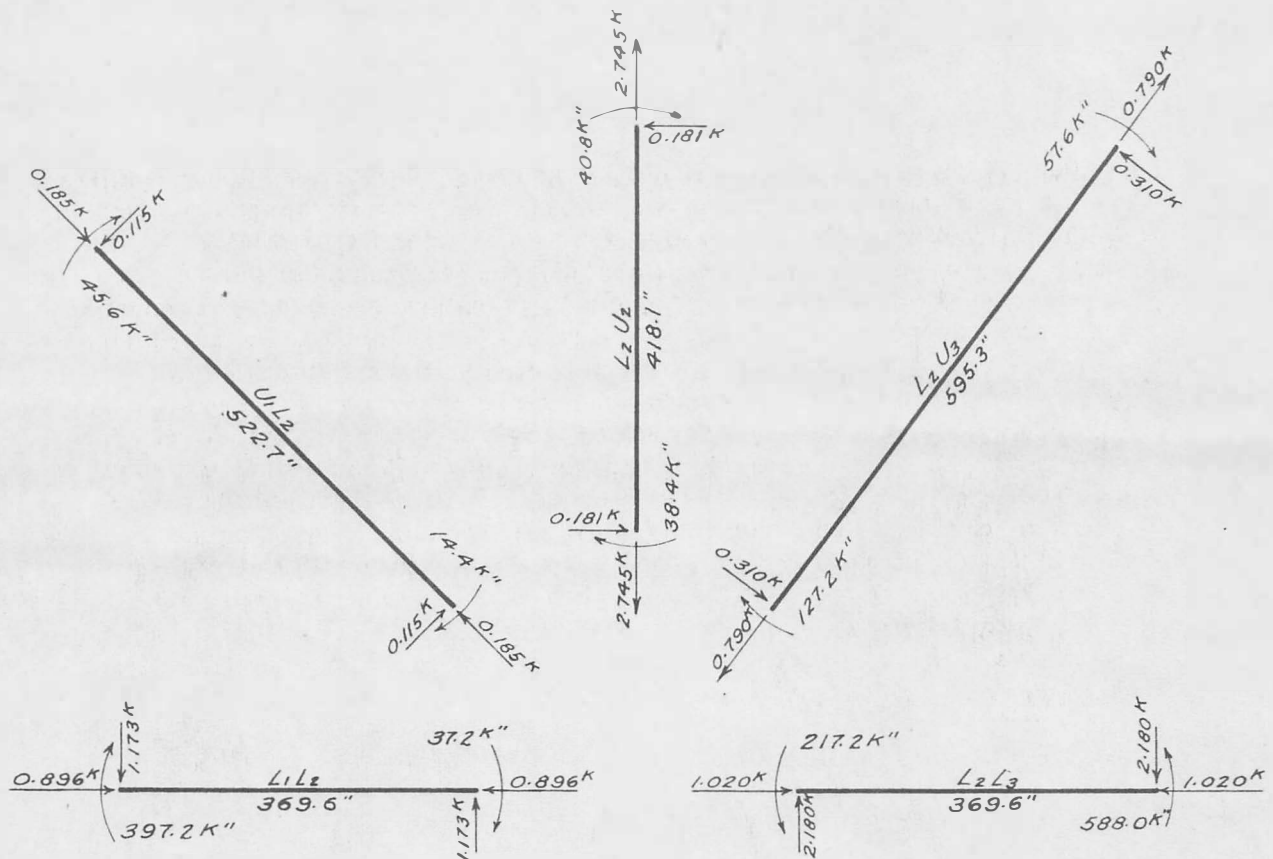
From the above proportions, the axial loads, shears, and moments to be applied to the model were computed using the necessary information from the prototype analysis. The axial load and shear were constant for any particular member and the moment used was that existing at the point where the moment was to be applied. Free

body diagrams for each member of the prototype meeting at joint L_2 are shown in Figure 12 and the corresponding diagrams for the model are shown in the same Figure.

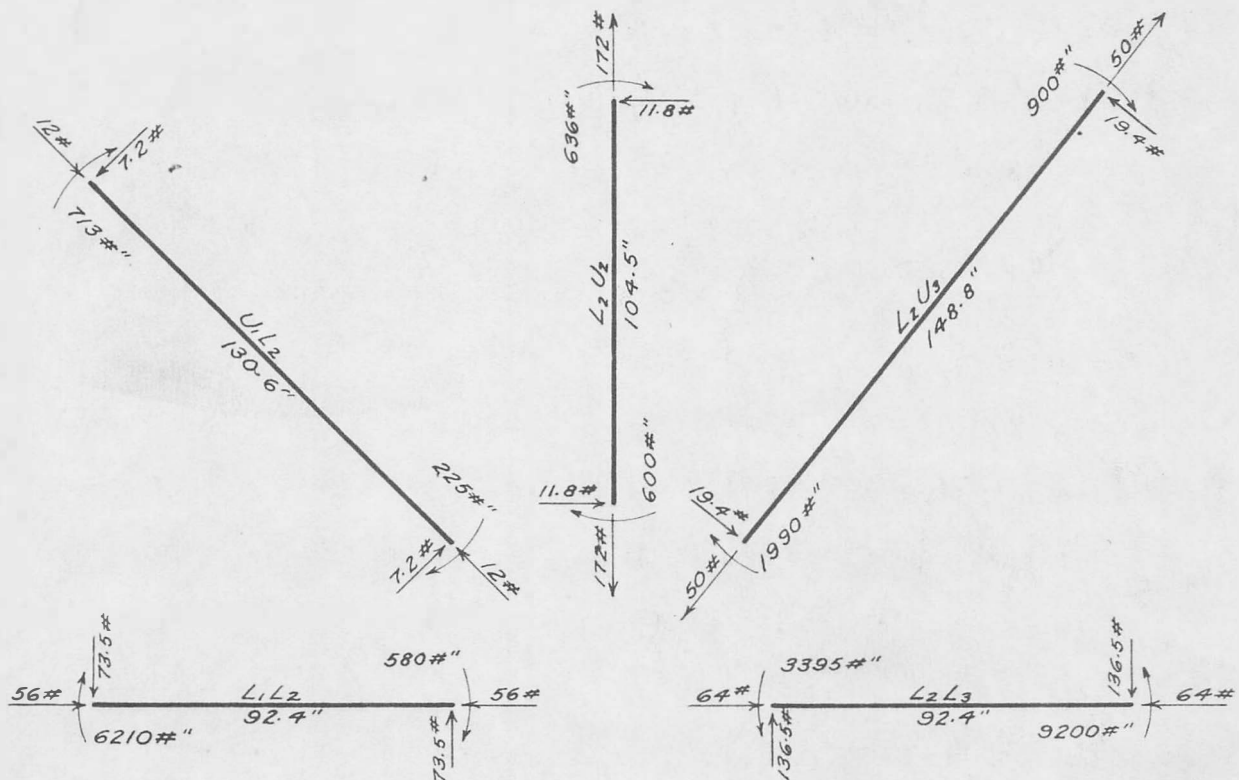
In Figure 13 is shown the theoretical loading of the model. However, due to construction difficulties, the achievement of this exact theoretical loading was not possible. Space was limited and clearances between the various parts of the loading system had to be maintained. Also, the thin narrow loading strips required tensile loads if buckling was to be prevented. Consequently, the actual loading used differed somewhat from the theoretical loading but it is believed that the differences produce no serious discrepancies in the analysis.

The actual loading is also shown in Figure 13. A comparison of the two loadings shows a close relationship between the theoretical loading and actual loading of the web members. The moments and shears in the chord members are close to the theoretical values. The only considerable discrepancy is in the axial loading of the chord members. However, an examination of the results of Whitmore's¹ investigations indicates that the axial load in the chord members has little effect on the gusset plate stresses. In view of these facts, it is believed that the actual loading produced stresses in the plates that would closely resemble those produced by the

¹R. E. Whitmore, Experimental Investigation of Stresses in Gusset Plates, loc. cit.

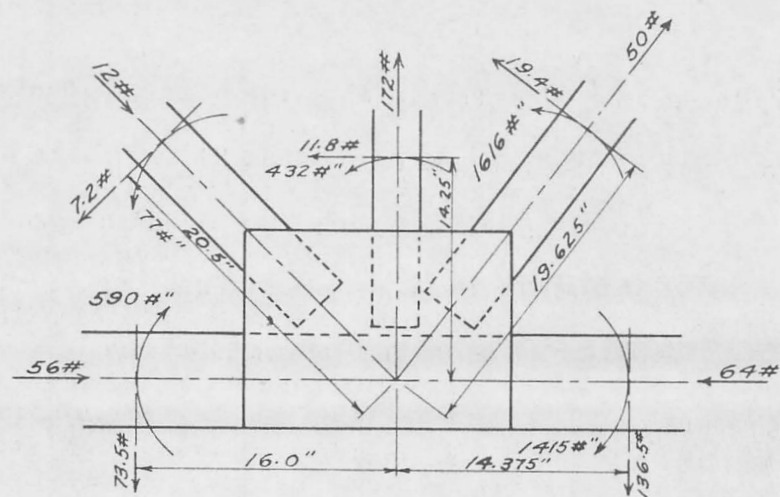


FREE-BODY DIAGRAMS OF PROTOTYPE MEMBERS

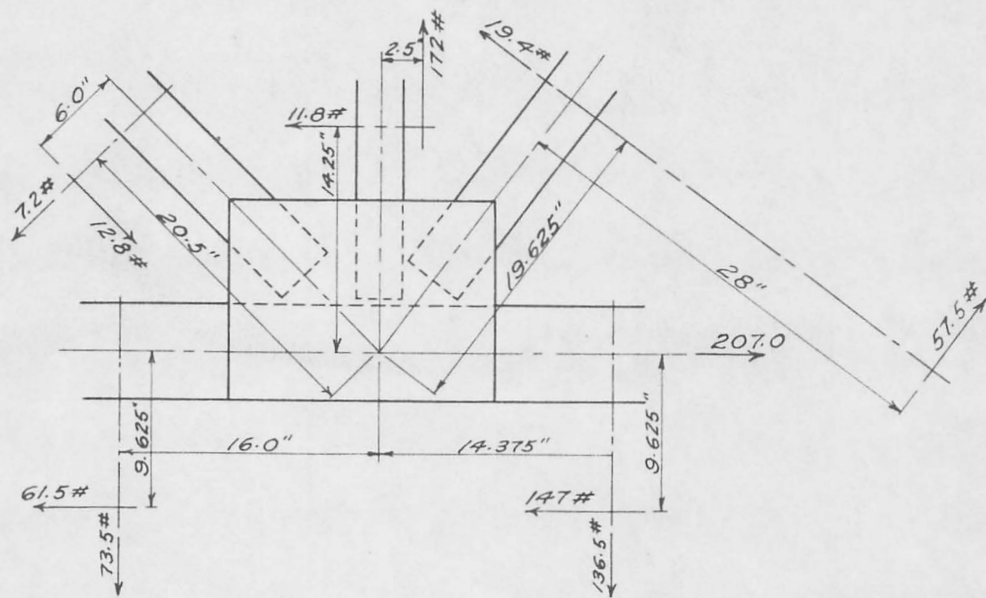


FREE-BODY DIAGRAMS OF MODEL MEMBERS

FIGURE 12. RELATIONSHIP OF PROTOTYPE LOADING TO MODEL LOADING



THEORETICAL LOADING



ACTUAL LOADING

FIGURE 13. RELATIONSHIP BETWEEN ACTUAL AND THEORETICAL LOADING

theoretical loading.

Application of the Loads

As indicated in Chapter II, the loads on the members were produced by a clevis and pin arrangement.

The only axial loads applied to the model were those resulting from secondary effects. Therefore, the large clevises at the ends of the web members were completely loosened in order to prevent any modification of the axial forces during the application of the secondary loads. A small amount of tension was placed on the chord members to hold the model in place until the secondary loading could be applied.

The nuts on the clevises attached to the loading strips were then tightened. This developed tension in the loading strips, thus applying the secondary loads.

Measurement of the Loads

The axial load to be applied to each loading strip had been determined as shown in Figure 13. In order to produce strains sufficiently large to measure, it was decided to make the loads applied to the model four times as large as the calculated loads. To measure these loads, two SR-4 type A-1 gages were cemented on the central axis of the loading member, one gage being placed on each face. Two gages were used since some bending of the loading strips was expected,

the bending resulting from possible initial crookedness of the strips and imperfect pin fits. With this arrangement, the two gage readings could be averaged and the axial strain determined.

From the desired loads, the modulus of elasticity, and the cross-sectional area of the loading strips, the necessary axial strains were computed. The strips used had a cross-sectional area of 0.0905 square inches. The required strain in inches per inch was computed as follows:

$\epsilon = \frac{S}{E} = \frac{P}{AE} = \frac{P}{0.0905 \times 10 \times 10^6}$, from which the strain in micro-inches per inch = $\frac{P}{0.905}$ where P is the desired load. (A micro-inch is one-millionth of an inch.)

The exact predetermined gage readings could not be obtained simultaneously since the average of two readings had to be used and it was possible to make only one reading at a time. Also, in adjusting the gage readings for one loading strip, readings for the other nine strips were somewhat affected. However, it was possible to get two complete loadings which were sufficiently exact to be regarded as satisfactory.

The data and results for two laboratory tests are given in Tables I and II.

Measuring the Strains in the Gusset Plate

The nineteen AR-1 rosettes and eleven A-1 gages had been cemented to each plate following the recommended procedure. Each

TABLE I

COMPARISON OF CALCULATED AND ACTUAL STRAIN
READINGS FOR LOAD MEASURING GAGES
FOR TEST NUMBER ONE

Load Position	Load Magnitude Pounds	Cal. Strain Micro $"/"$	Gage Number	Gage Reading	Ave. Gage Reading Micro $"/"$
Moment on L_2L_3	230	254	1	210	260
			2	310	
Shear on L_2L_3	78	86	3	50	80
			4	110	
Moment on L_2U_2	688	760	5	760	780
			6	800	
Shear on L_2U_2	47	52	7	80	60
			8	40	
Moment on L_2U_1	51	57	9	25	55
			10	85	
Shear on L_2U_1	29	32	11	40	30
			12	20	
Moment on L_2L_1	246	262	13	200	247
			14	295	
Shear on L_2L_1	284	314	15	250	340
			16	430	
Moment on L_2L_3	587	650	17	840	645
			18	450	
Shear on L_2L_3	546	605	19	720	600
			20	480	

TABLE II

COMPARISON OF CALCULATED AND ACTUAL STRAIN
READINGS FOR LOAD MEASURING GAGES
FOR TEST NUMBER TWO

Load Position	Load Magnitude Pounds	Cal. Strain Micro μ / μ	Gage Number	Gage Reading	Ave. Gage Reading Micro μ / μ
Moment on L_2L_3	230	254	1	200	248
			2	295	
Shear on L_2L_3	78	86	3	90	95
			4	100	
Moment on L_2U_2	688	760	5	705	753
			6	800	
Shear on L_2U_2	47	52	7	65	57
			8	50	
Moment on L_2U_1	51	57	9	20	55
			10	90	
Shear on L_2U_1	29	32	11	40	30
			12	20	
Moment on L_2L_1	246	262	13	180	267
			14	355	
Shear on L_2L_1	284	314	15	240	317
			16	395	
Moment on L_2L_3	587	650	17	585	650
			18	715	
Shear on L_2L_3	546	605	19	690	605
			20	520	

gage was identified by number and the exact location and alignment determined. This information is itemized in Table III, together with the gage factor for each gage.

Each gage was connected to the strain indicator through the switching arrangement and a "no load" indicator reading taken for each gage. After the loads had been applied to the members, the indicator readings were again recorded for each gage. The strain for each gage was obtained by subtracting the "no load" from the loaded indicator readings, the units being micro-inches per inch.

After all gage readings had been made and recorded, the loads were completely removed. Then the procedure described in the preceding paragraph was repeated, thus giving two independent readings. The differences between the larger strain readings for the two tests were relatively small. The smaller readings did not agree as well as might be desired. However, the reproducibility of the larger strains indicated that the results were, in general, satisfactory and could be used to calculate stresses with sufficient accuracy. The smaller strains were so small that a considerable percentage of error in them might be expected and is of no practical consequence.

A summary of the gage readings is given in Table IV. The data are the averages of corresponding gages on both plates for two laboratory tests.

In order to increase the strains in the plates, a third

loading was made using loads six times as great as the calculated loads. The two previous tests were made under loads four times as great as the calculated values. However, the smaller strains from this test were still somewhat erratic, and so it was decided to base the conclusions on the results of the first two tests. The larger strains did prove to be approximately one and one half times greater than the average of the larger strains from the first two tests. A comparison of some of the readings from the third test with the average readings of the first two tests are shown in Table V.

TABLE III

LOCATION, DIRECTION, AND GAGE FACTOR OF
GUSSET PLATE GAGES

Gage Number	Coordinates ^a		Direction ^b (degrees)	Gage Factor ^c	
	X (inches)	Y (inches)		Front Plate	Rear Plate
G-1	0.38	6.75	90	2.07	2.03
G-2	0.38	9.53	90	2.07	2.03
G-3	3.56	12.16	0	2.07	2.03
G-4	6.61	12.16	0	2.07	2.03
G-5	10.40	12.16	0	2.07	2.03
G-6	6.46	0.38	0	2.07	2.03
G-7	2.90	0.38	0	2.07	2.03
G-8	13.02	12.16	0	2.07	2.03
G-9	16.22	9.38	90	2.07	2.03
G-10	16.22	5.77	90	2.07	2.03
G-11	13.04	0.38	0	2.07	2.03
R-I	2.46	4.41	90	2.07	2.03
R-II	2.30	7.78	47	2.07	2.03
R-III	5.08	2.82	92	2.07	2.03
R-IV	5.08	9.88	44½	2.07	2.03
R-V	5.59	6.96	45	2.07	2.03
R-VI	7.46	8.95	76	2.07	2.03
R-VII	6.86	5.67	60½	2.07	2.03
R-VIII	8.80	3.61	154	2.07	2.03
R-IX	9.94	1.11	0	2.07	2.03
R-X	7.67	10.89	179	2.03	2.03
R-XI	3.69	6.10	135	2.03	2.03
R-XII	9.52	6.13	148	2.03	2.03
R-XIII	11.00	8.32	143½	2.03	2.03
R-XIV	11.71	3.09	0	2.07	2.03
R-XV	10.63	5.54	148½	2.07	2.03
R-XVI	13.86	4.68	90	2.07	2.03
R-XVII	15.20	7.58	53½	2.07	2.03
R-XVIII	12.66	7.09	53½	2.07	2.03
R-XIX	12.24	9.74	53½	2.07	2.03

^aOrigin of coordinate axes is the lower left hand corner of gusset plate.

^bAngle given is from horizontal counterclockwise to the "a" component of the rosette gages.

^cThe gage factor is set directly on the indicator and does not enter into any calculations.

TABLE IV

SUMMARY OF STRAINS FROM READINGS OF GAGES
ON THE MODEL FOR TESTS ONE AND TWO

Gage Number		Ave. Strain Both Plates Test #1 Micro "/"	Ave. Strain Both Plates Test #2 Micro "/"	Ave. Strain Both Plates Tests 1 & 2 Micro "/"
G-1		-53	-55	-54
G-2		-67	-63	-65
G-3		32	30	31
G-4		-25	-37	-31
G-5		-108	-128	-118
G-6		10	12	11
G-7		-12	-22	-17
G-8		-265	-297	-281
G-9		335	347	341
G-10		192	210	201
G-11		40	38	39
R-I	a	-55	-55	-55
	b	-28	-35	-31
	c	25	21	23
R-II	a	-7	-17	-12
	b	-33	-40	-36
	c	-10	-12	-11
R-III	a	-18	-27	-22
	b	-7	-10	-9
	c	15	10	12
R-IV	a	-63	-67	-65
	b	-7	-10	-8
	c	50	48	49
R-V	a	-33	-47	-40
	b	-35	-45	-40
	c	25	22	23
R-VI	a	-35	-42	-38
	b	65	45	55
	c	42	35	37

TABLE IV

SUMMARY OF STRAINS FROM READINGS OF GAGES
ON THE MODEL FOR TESTS ONE AND TWO
(continued)

Gage Number		Ave. Strain Both Plates Test #1 Micro μ/μ	Ave. Strain Both Plates Test #2 Micro μ/μ	Ave. Strain Both Plates Tests 1 & 2 Micro μ/μ
R-VII	a	-32	-30	-31
	b	-15	-18	-17
	c	37	42	40
R-VIII	a	17	10	14
	b	25	20	23
	c	5	-10	-3
R-IX	a	15	5	10
	b	10	-2	4
	c	-2	-15	-8
R-X	a	-23	-37	-30
	b	-52	-65	-59
	c	-5	-2	-4
R-XI	a	-5	-5	-5
	b	25	20	23
	c	-5	-10	-8
R-XII	a	57	56	57
	b	35	30	33
	c	-15	-21	-18
R-XIII	a	130	137	134
	b	10	12	11
	c	-90	-98	-94
R-XIV	a	32	30	31
	b	30	30	30
	c	12	9	10
R-XV	a	57	52	55
	b	42	42	42
	c	12	-15	-8

TABLE IV

SUMMARY OF STRAINS FROM READINGS OF GAGES
ON THE MODEL FOR TESTS ONE AND TWO
(continued)

Gage Number		Ave. Strain Both Plates	Ave. Strain Both Plates	Ave. Strain Both Plates
		Test #1 Micro μ / μ	Test #2 Micro μ / μ	Tests 1 & 2 Micro μ / μ
R-XVI	a	90	85	88
	b	0	-10	-5
	c	-12	-12	-12
R-XVII	a	180	182	181
	b	158	167	162
	c	-17	-30	-24
R-XVIII	a	7	7	7
	b	-20	-25	-23
	c	52	42	47
R-XIX	a	-150	-145	-148
	b	67	74	71
	c	167	167	167

TABLE V

COMPARISON OF SOME RESULTS OF
TEST 3 WITH TESTS 1 AND 2

Gage Number		Ave. Strain Both Plates Tests #1 & 2 Micro " / "	$1\frac{1}{2}$ Times Strain Both Plates Tests #1 & 2 Micro " / "	Ave. Strain Both Plates Test #3 Micro " / "
G-5		-118	-177	-154
G-8		-281	-421	-413
G-9		341	510	470
G-10		201	302	316
R-IV	a	-65	-97	-83
	b	-8	-12	-9
	c	49	69	64
R-VI	a	-38	-57	-91
	b	55	82	75
	c	37	56	62
R-XIII	a	134	201	200
	b	11	23	42
	c	-94	-140	-133
R-XIX	a	-148	-222	-213
	b	71	107	112
	c	167	250	216

CHAPTER IV

CALCULATION OF UNIT STRESSES FROM UNIT STRAINS

Calculation of Principal Strains

The measured strains, summarized at the end of Chapter III, are in themselves of little value except to give a general indication of stress distribution. The primary interest is in the actual value of stress. The usual conversion of unit strains to unit stresses is by calculating principal strains and then converting the principal strains to principal stresses.

Principal stresses are important because they represent the maximum and minimum values of stress that exist at a point and also because they afford one of the most convenient methods of completely describing the stress situation at a point. If the principal stresses are known, the stresses on all other planes through the point may be determined. The maximum shearing stress may be readily calculated by taking one half the difference of the principal stresses.

The principal strains at any point may be evaluated from two orthogonal normal strains and the accompanying shearing strain.¹ However, this is not a practical procedure because strain-measuring

¹Glenn Murphy, Advanced Mechanics of Materials (New York: McGraw-Hill Book Co., Inc., 1946) p. 47.

equipment has been designed to measure normal rather than shearing strains.

From the mathematical standpoint, principal strains can be readily evaluated in terms of three normal strains. Various analytical, semi-graphical and graphical methods have been developed to accomplish this purpose.

The principal strains at the interior points of the gusset plate had to be evaluated in terms of three measured normal strains. To illustrate the procedure, an analytical and a graphical solution are given below. The analytical solution, shown in Figure 14, is taken from one of the methods outlined by Murphy.² In Figure 15 is shown the graphical solution. This solution, which is essentially a Mohr circle construction, is taken from a suggestion by F.G. Tatnall.³

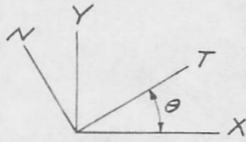
Principal strains and their directions for each rosette were determined graphically and the analytical solution was used to make spot checks of the graphical results. The values of the principal strains and their directions are recorded in Table VI.

Calculation of Principal Stresses From Principal Strains

Since the aluminum used in this study can be considered

²Ibid., p. 48.

³F. G. Tatnall, "Concerning the Subject of Testing", SR-4 News Letter, II: 1-8, March 1948.



Basic Equations

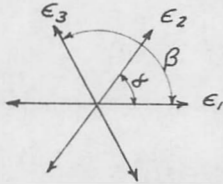
$$\epsilon_x = \epsilon_x \cos^2 \theta + \epsilon_y \sin^2 \theta - \tau_{xy} \sin \theta \cos \theta \quad (1)$$

$$\epsilon_y = \epsilon_x \sin^2 \theta + \epsilon_y \cos^2 \theta + \tau_{xy} \sin \theta \cos \theta \quad (2)$$

$$\epsilon_x + \epsilon_y = \epsilon_x + \epsilon_y \quad (3)$$

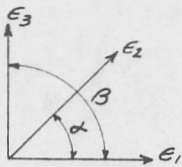
$$\epsilon_{u,v} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \sqrt{(\epsilon_x - \epsilon_y)^2 + \tau_{xy}^2} \quad (4)$$

$$\tan 2\theta = - \frac{\tau_{xy}}{\epsilon_x - \epsilon_y} \quad (5)$$



General case of three normal strains

Let the x direction be taken as the direction of ϵ_1 . Then from basic equation (1) above,
 $\epsilon_2 = \epsilon_x \cos^2 \alpha + \epsilon_y \sin^2 \alpha - \tau_{xy} \sin \alpha \cos \alpha$ (6)
 $\epsilon_3 = \epsilon_x \cos^2 \beta + \epsilon_y \sin^2 \beta - \tau_{xy} \sin \beta \cos \beta$ (7)
 Solve 6 and 7 simultaneously for ϵ_y and τ_{xy} . Then solve for principal strains from 4. Evaluate direction from 5.



Special Rosette

For the special rosette used, $\alpha = 45^\circ$, $\beta = 90^\circ$, $\epsilon_1 = \epsilon_x$, and $\epsilon_3 = \epsilon_y$.

$$\epsilon_2 = \epsilon_1 \cos^2 45 + \epsilon_3 \sin^2 45 - \tau_{xy} \sin 45 \cos 45$$

$$\epsilon_2 = \epsilon_1 (.5) + \epsilon_3 (.5) - \tau_{xy} (.5)$$

$$2\epsilon_2 = \epsilon_1 + \epsilon_3 - \tau_{xy}$$

$$\tau_{xy} = \epsilon_1 - 2\epsilon_2 + \epsilon_3$$

Example: Gage RXII

$$\epsilon_1 = 57, \epsilon_2 = 33, \epsilon_3 = -18 \text{ micro"/"}$$

$$\epsilon_x = 57, \epsilon_y = -18 \quad \& \quad \tau_{xy} = 57 - 2(33) + (-18) = -27$$

$$\epsilon_{u,v} = \frac{57-18}{2} \pm \frac{1}{2} \sqrt{(57+18)^2 + (-27)^2} = \frac{39}{2} \pm \frac{79.5}{2} = 19.5 \pm 39.8$$

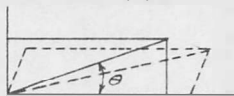
$$\epsilon_u = 19.5 + 39.8 = +59.3 \text{ micro"/"}$$

$$\epsilon_v = 19.5 - 39.8 = -20.3 \text{ micro"/"}$$

$$\tan 2\theta = - \frac{(-27)}{57+18} = 0.36$$

$$2\theta = 19.8^\circ, \theta = 9.9^\circ \text{ or } 99.9^\circ$$

$\theta = 9.9^\circ$ by inspection of given strains applied to elemental block.



$$S_u = E \frac{(\epsilon_u + \mu \epsilon_v)}{1 - \mu^2}$$

$$S_v = E \frac{(\epsilon_v + \mu \epsilon_u)}{1 - \mu^2}$$

$$p = S_u = \frac{10[59 + 0.33(-20)]}{1 - (.33)^2} = 587 \text{ psi.}$$

$$q = S_v = \frac{10[-20 + 0.33(59)]}{1 - (.33)^2} = -6 \text{ psi.}$$

$$T = S_{max} = \frac{1}{2} [587 - (-6)] = 297 \text{ psi.}$$

FIGURE 14. ANALYTICAL SOLUTION OF PRINCIPAL STRAINS AND STRESSES

Gage R XII

$$\epsilon_1 = 57, \epsilon_2 = 33, \epsilon_3 = -18 \text{ micro "/>$$

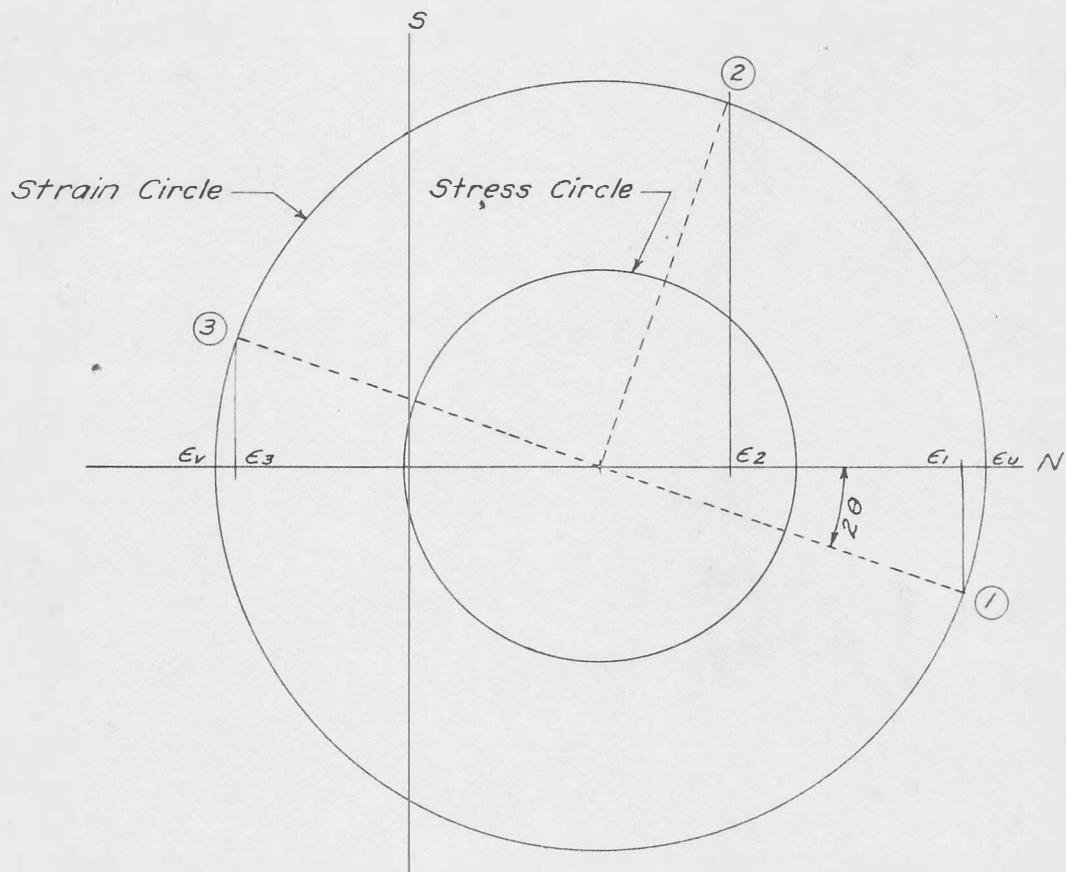
Scales:

$$\text{Strain circle} - 1'' = 20 \text{ micro "/>$$

$$\text{Stress circle} - \text{Radius} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{1}{2} \text{ radius of strain circle}$$

$$1'' = 20 \left(\frac{E}{1 - \mu} \right) = 20 \times 10 = 300 \text{ #/sq"}$$

Mohr's Circle:



Results:

$$\epsilon_u = +59.5 \text{ micro "/>$$

$$p = 1.98 \times 300 = 592 \text{ psi}, \quad q = -0.02 \times 300 = -6 \text{ psi.}$$

$$2\theta = 19.5^\circ \quad \theta = 9.8^\circ$$

$$\tau = \frac{1}{2} [592 - (-6)] = 299 \text{ psi.}$$

FIGURE 15. GRAPHICAL SOLUTION OF PRINCIPAL STRAINS AND STRESSES

isotropic, the directions of the principal strains and principal stresses are coincident.

The relationship between principal stresses and principal strains has been definitely established by the theory of elasticity. It is known that:

$$p = \sigma_{\max.} = \frac{E}{1 - \nu^2} (\epsilon_{\max.} \text{ plus } \nu \epsilon_{\min.})$$

$$q = \sigma_{\min.} = \frac{E}{1 - \nu^2} (\epsilon_{\min.} \text{ plus } \nu \epsilon_{\max.})$$

in which

σ = unit normal stress

ϵ = unit normal strain

ν = Poisson's ratio

E = modulus of elasticity.

Therefore, with the principal strains and the elastic constants known, the calculation of principal stresses is a simple matter. In Figure 14 is shown an example of principal stresses calculated from principal strains.

Tatnall's suggestion⁴ also includes a method of graphically determining the principal stresses. Using the construction from which principal strains were determined, the stress circle is plotted with the same center by merely changing the scale of

⁴Ibid., p. 5.

the radius by the factor $\frac{1-\nu}{1+\nu}$. Whatever scale is used for strain must be multiplied by $\frac{E}{1-\nu}$ to determine the stress. This method is illustrated in Figure 15.⁵

In this study the graphical method was used for principal stress determination, the analytical method being used to check some of the graphical results.

The principal stresses are recorded with the principal strains in Table VI.

Calculation of Maximum Shearing Stresses

The maximum shearing stress at a point is equal to one half of the difference of the principal stresses at that point. Since p and q had previously been calculated for each gage location, the calculation of maximum shearing stresses was a simple matter.

These values of each gage are also tabulated in Table VI.

Model Stresses Corresponding to Prototype Stresses

The values of stress recorded in Table VI are for loads four times as great as the loads required to produce the same stresses in the model as would exist in the prototype. In order to obtain the correct model and prototype stresses, the values of

⁵R. E. Whitmore, Experimental Investigation of Stresses in Gusset Plates, Lec. cit.

stress in Table VI are divided by four, and the resulting values are recorded in Table VII.

The directions, of course, are unchanged but are listed again in Table VII for convenience in observing the true stress situation in the plates.

TABLE VI

PRINCIPAL STRAINS AND STRESSES, THEIR DIRECTIONS
AND MAXIMUM SHEARING STRESSES^a

Gage Number	Principal Strains Micro $\frac{1}{2}$ "		Principal Stresses (lb per sq in)		Max. Shear Stresses (lb per sq in)	Dir. ^b Degrees
	max.	min.	p	q		
G-1		-54	0	-540	270	0
G-2		-65	0	-650	325	0
G-3	31		310	0	155	0
G-4		-31	0	-310	155	90
G-5		-118	0	-1180	590	90
G-6	11		110	0	55	0
G-7		-17	0	-170	85	90
G-8		-281	0	-2810	1405	90
G-9	341		3410	0	1705	90
G-10	201		2010	0	1005	90
G-11	39		390	0	195	0
R-I	26	-58	75	-555	315	10 $\frac{1}{2}$
R-II	13	-36	12	-355	184	1 $\frac{1}{2}$
R-III	13	-23	58	-207	132	9
R-IV	49	-65	297	-545	421	134 $\frac{1}{2}$
R-V	36	-53	204	-455	330	167 $\frac{1}{2}$
R-VI	66	-68	495	-505	500	138
R-VII	46	-37	375	-240	307	165 $\frac{1}{2}$
R-VIII	25	-14	225	-60	142	6
R-IX	11	-8	83	-55	69	170 $\frac{1}{2}$
R-X	27	-61	75	-585	330	125 $\frac{1}{2}$
R-XI	23	-36	125	-318	226	178 $\frac{1}{2}$
R-XII	59	-20	590	0	295	158
R-XIII	139	-99	1185	-590	887	139
R-XIV	35	6	427	96	165	21
R-XV	60	-13	625	87	269	164
R-XVI	104	-28	1070	78	496	70
R-XVII	211	-53	2170	192	989	73
R-XVIII	80	-27	810	0	405	177 $\frac{1}{2}$
R-IX	180	-160	1420	-1110	1265	133

^aValues given are average of both plates.

^bAngle given is from horizontal counterclockwise to "p" direction.

TABLE VII

PRINCIPAL STRESSES AND MAXIMUM SHEARING STRESSES
 RESULTING FROM LOADS PRODUCING EQUAL
 LOADS IN MODEL AND PROTOTYPE

Gage Number	Principal Stresses (lb per sq in)		Max. Shear Stress (lb per sq in)	Direction ^a (degrees)
	p	q		
G-1	0	-135	68	0
G-2	0	-162	81	0
G-3	78	0	39	0
G-4	0	-78	39	90
G-5	0	-295	148	90
G-6	23	0	14	0
G-7	0	-43	22	90
G-8	0	-702	351	90
G-9	850	0	425	90
G-10	502	0	251	90
G-11	98	0	49	0
R-I	19	-139	79	10 $\frac{1}{2}$
R-II	4	-89	46	1 $\frac{1}{2}$
R-III	15	-52	33	9
R-IV	74	-137	105	134 $\frac{1}{2}$
R-V	51	-114	83	167 $\frac{1}{2}$
R-VI	124	-126	125	138
R-VII	94	-60	77	165 $\frac{1}{2}$
R-VIII	56	-15	36	6
R-IX	21	-14	17	170 $\frac{1}{2}$
R-X	19	-146	83	125 $\frac{1}{2}$
R-XI	31	-80	55	178 $\frac{1}{2}$
R-XII	147	0	74	158
R-XIII	296	-148	222	139
R-XIV	107	24	41	21
R-XV	156	22	67	164
R-XVI	268	20	124	70
R-XVII	542	48	247	73
R-XVIII	203	0	101	177 $\frac{1}{2}$
R-XIX	354	-280	316	133

^aAngle given is from horizontal counterclockwise to "p" direction.

CHAPTER V

DISTRIBUTION OF STRESSES IN THE GUSSET PLATE

Principal Stress Directions And Stress Contours

The principal stresses and their directions, tabulated at the end of Chapter IV, give an adequate description of the state of stress at the various points in the gusset plate where the strains were measured.

In many cases, the magnitude and location of the maximum stresses are the only items of primary interest, while in other cases it is desirable to have a picture of the variation of principal stresses throughout the stressed member. This picture may be prepared by calculating the principal stresses at a number of points and plotting stress contours or lines that pass through points of equal principal stress.

The stress distribution may be further clarified by sketching a set of lines, each connecting points having identical directions of principal stress. These lines are called isoclinics. Another method of depicting the directions of the principal stresses is by drawing stress trajectories, which are lines so drawn that their tangents at every point are in the direction of the principal stress. The stress trajectories will consist of two sets of orthogonal lines, one set representing the directions of the "p" stresses and the other

set representing the directions of the "q" stresses.¹

To illustrate the findings of this study with regard to the stress distribution in the gusset plate, the following figures have been prepared:

- (a) Directions of the principal stresses, Figure 16,
- (b) Contours of maximum tensile stress, Figure 17,
- (c) Contours of maximum compressive stress, Figure 18,
- (d) Contours of maximum shearing stress, Figure 19.

Contours for stresses less than 100 pounds per square inch were not plotted, since these smaller stresses were calculated from strain readings so small as to make their accuracy questionable. The sketches of the stress contours were made without considering the "stress raising" effect of the rivet holes. Also the contours should be considered as approximate due to the limited number of points available for the interpolation.

Even though the above limitations do apply, it is felt that the contours as sketched give a reliable indication of the stress distribution in the plate and afford a means of visualizing the location of regions of maximum stress.

Due to the complexity of the stress pattern and insufficient coverage of the plate with strain gages, stress trajectories and isoclinics were not sketched. The directions of the principal stresses at the location of strain gages are shown in Figure 16.

¹Murphy, *op. cit.*, p. 36.

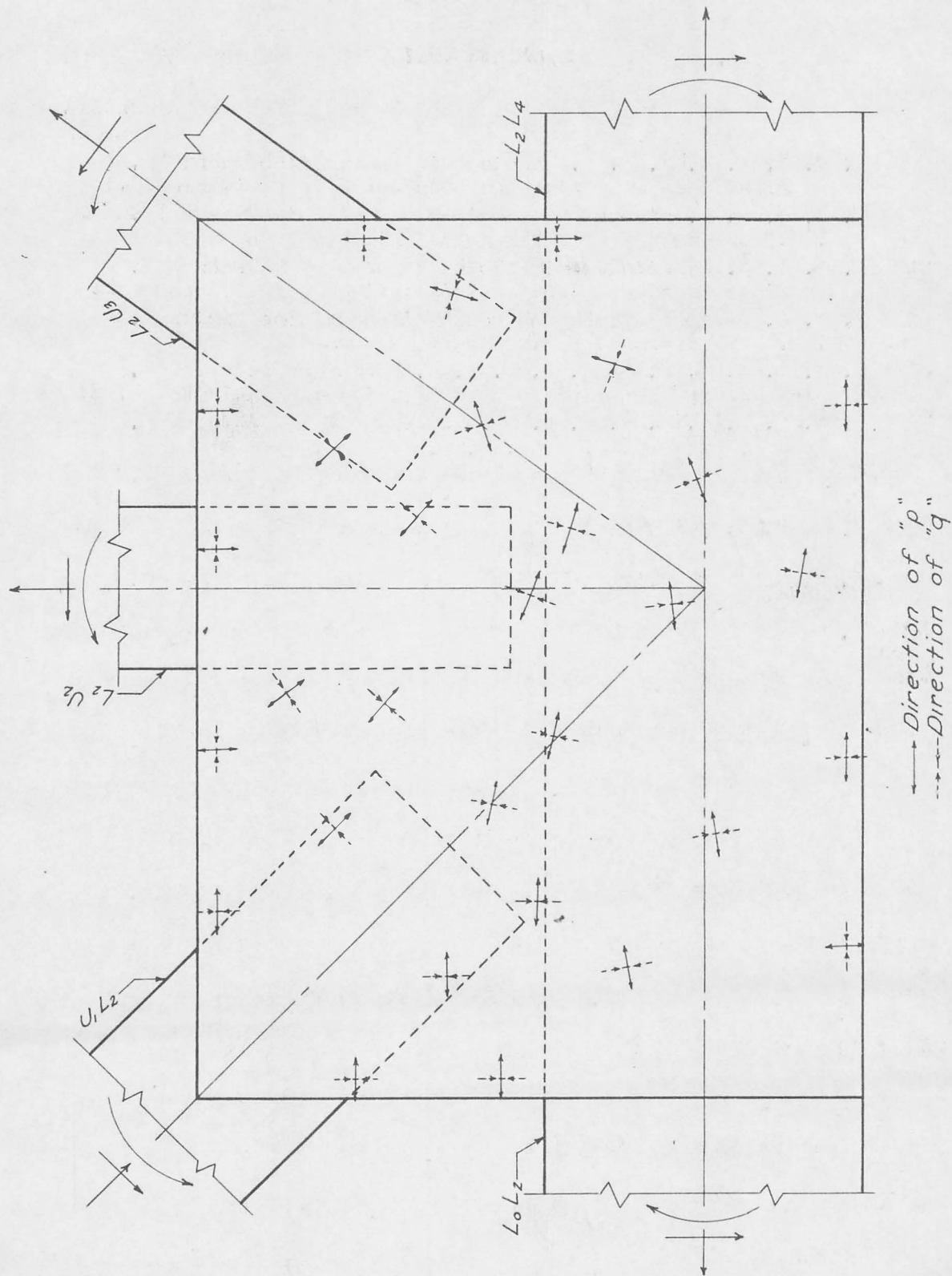


FIGURE 16. DIRECTIONS OF THE PRINCIPAL STRESSES

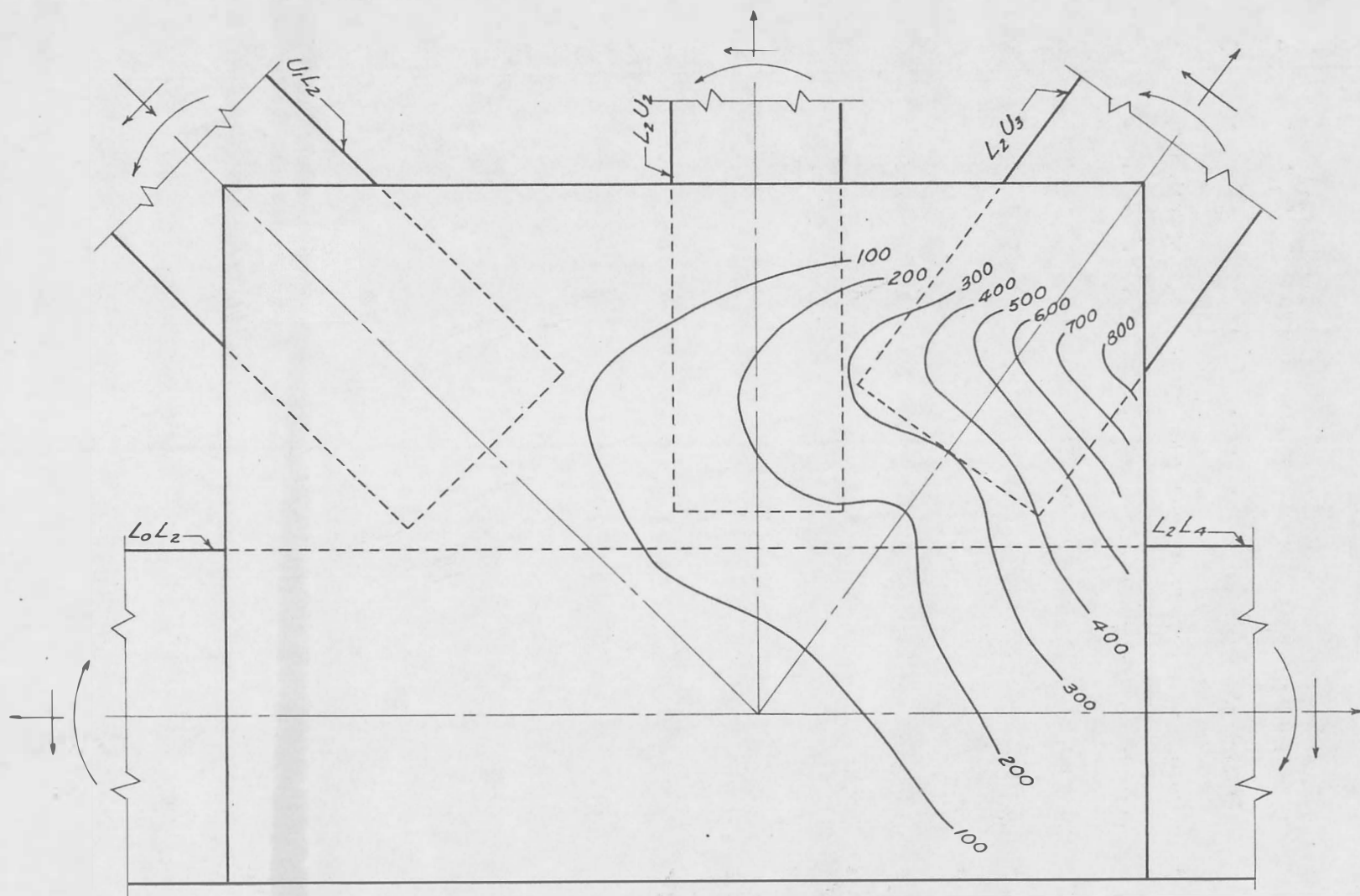


FIGURE 17. CONTOURS OF MAXIMUM TENSILE (p) STRESSES IN POUNDS PER SQUARE INCH

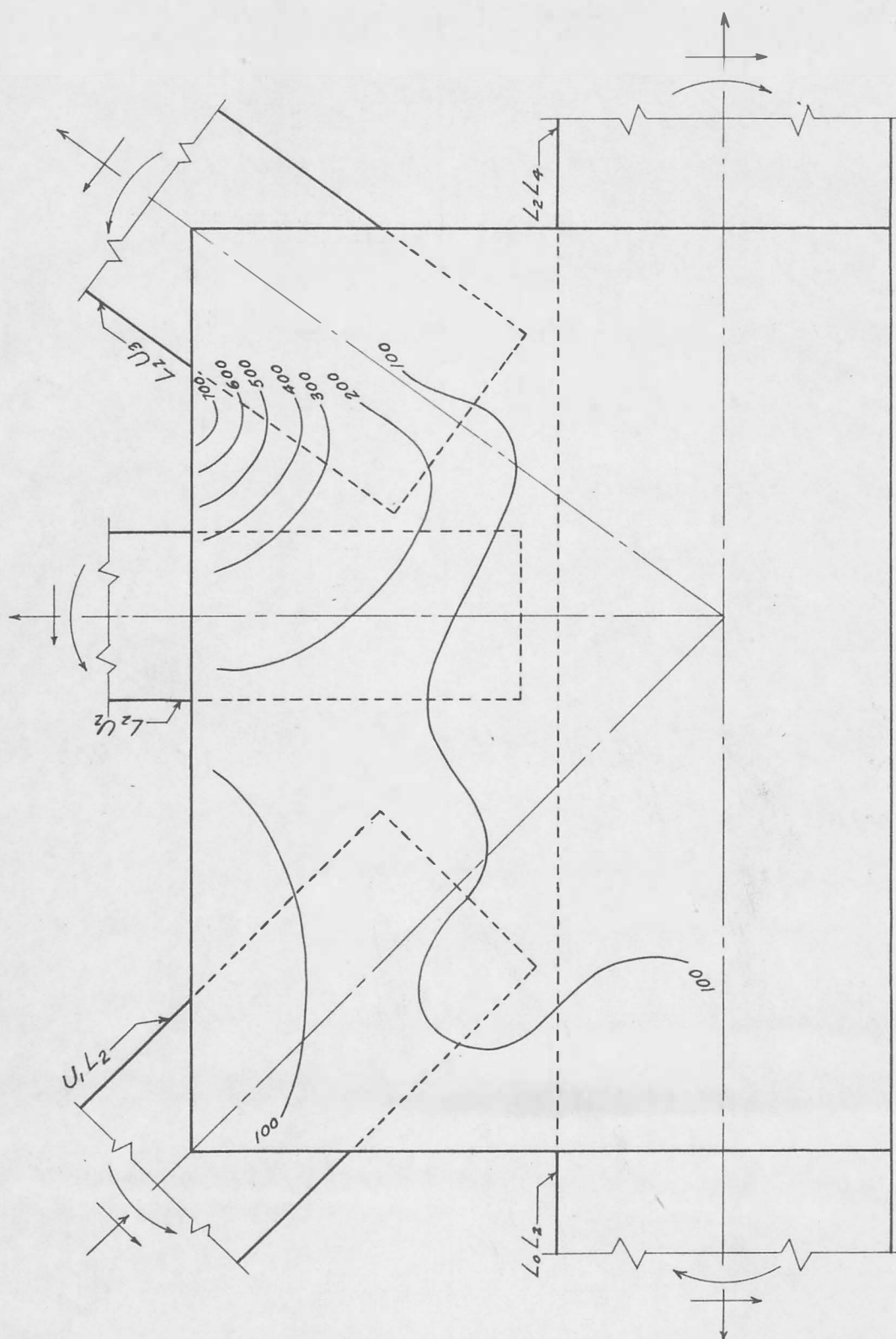


FIGURE 18. CONTOURS OF MAXIMUM COMPRESSIVE (q) STRESSES
IN POUNDS PER SQUARE INCH

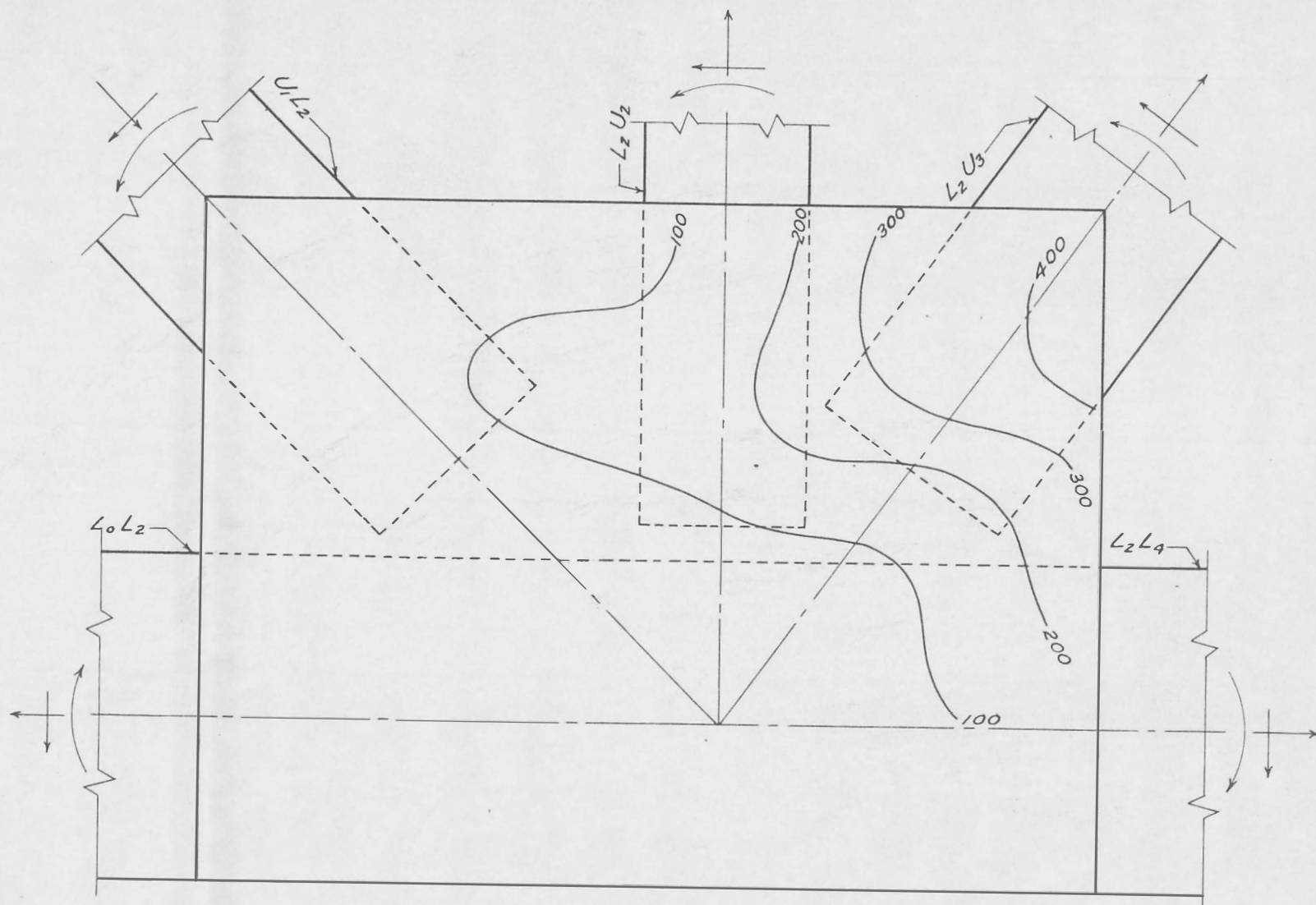


FIGURE 19. CONTOURS OF MAXIMUM SHEARING (τ) STRESSES IN POUNDS PER SQUARE INCH

A photoelastic study would have afforded an excellent means of obtaining the stress trajectories and isoclinics. However, limitations of time prevented making such a study.

Combination of Primary and Secondary Stresses

Frequently, secondary stresses are neglected in the design of trusses unless the depths of the members are large relative to the lengths. The depth to length ratio of the chord members is ordinarily greater than that of the web members. In general, if this ratio is less than one to ten for the chords, secondary stresses are disregarded in the truss design.

The ratio of depth to length of the chord members of the prototype used in this study was less than one to ten. In order to see if the secondary stresses would have any effect upon the design of the gusset plates, the combination of primary stresses and secondary stresses was studied.

The magnitude and location of primary stresses were estimated from Whitmore's experiments.² This estimate was made by assuming that the loads on the model should be $1/16$ of the loads on the prototype in order to produce equal stresses in the model and prototype. (This ratio gives equal stresses in the model and prototype. See page 29.) The ratio of $1/16$ of the prototype load to the load

²R. E. Whitmore, Experimental Investigation of Stresses in Gusset Plates, loc. cit.

used by Whitmore was calculated. The primary stresses of the original experiment were then multiplied by this factor to arrive at the value of primary stress that should exist simultaneously with the secondary stresses resulting from the loading used in the present study. The primary stresses resulting from dead load alone were used since the secondary stresses were based on dead loads only.

The primary stresses determined by the method outlined in the previous paragraph are to be considered somewhat approximate since the ratio of the loads on the web members for the model was not quite the same as existed in the prototype. However, the maximum primary tensile stress was estimated from the load on U_1L_2 and the maximum primary compressive stress was estimated from the load on L_2U_3 . It is believed that by this procedure a reasonable estimate of the correct primary stresses was obtained.

An inspection of the primary tensile stress contours and the secondary tensile stress contours clearly indicated that the maximum tensile primary stress and the maximum tensile secondary stress occurred in entirely different regions of the gusset plate and could at no point combine to give a stress greater than the maximum primary stress.

The maximum shearing stress from the primary effects and from the secondary effects also occurred in different regions, and no critical combination of the two was indicated.

The maximum primary compressive stress and the maximum second-

ary compressive stress did occur in the same general region and a more detailed study was made of their combinations.

The secondary stresses were known at the various strain gage locations and the primary stresses could be obtained by the process previously outlined. However, the primary and secondary stresses did not necessarily have the same direction and, therefore, could not be combined algebraically. The measured strains for both primary and secondary effects were in the same direction and could be combined algebraically.

The primary strains were modified to make them consistent with the $1/16$ of the prototype loading and these strains were then combined with the secondary strains. From these combined strains, new principal stresses were calculated. In order to compare the primary and secondary compressive stresses and to show their combined values, Table VIII was prepared. In Figure 20 are shown sample calculations of the combined values of primary and secondary stresses.

An examination of this table indicates that the combination of primary and secondary stress at no point gives a compressive stress greater than the maximum compressive stress resulting from the primary loading alone.

Analytical Check of Stress On One Section of the Gusset Plate

In the design of gusset plates, it would be highly desirable to be able to compute the maximum secondary stress in the plate from

TABLE VIII

COMPARISON AND COMBINATION OF
PRIMARY AND SECONDARY STRESSES

Location (Strain Gage)	Primary Stress ^a (lb. per sq.in)	Secondary Stress (lb. per sq.in)	Comb. Primary And Secondary (lb. per sq.in)
G-8	-3580	-702	-4282
G-9	-6350	850	-5500
G-10	-2360	502	-1858
R-XIII	-4750	-148	-4900
R-XVII	-5250	48	-4650
R-XVIII	-6250	0	-6250
R-XIX	-4375	-280	-4500

^aThe absolute maximum compressive stress for primary loads was recorded at the location of strain gage G-9.

Combination of primary and secondary stresses at the edges of the gusset plate:

$S = E(\epsilon_p + \epsilon_s)$, where ϵ_p = strain due to primary loads and ϵ_s = strain due to secondary loads.

Examples:

Location (strain gage)	ϵ_p micro"/"	ϵ_s micro"/"	total ϵ micro"/"	combined stress (lb. per sq. in.)
G-8	-358	-70	-428	-4280
G-9	-635	+85	-550	-5500

Combination of primary and secondary stresses at interior points of gusset plate:

Example:

Location (strain gage)	ϵ_p micro"/"	ϵ_s micro"/"	total ϵ micro"/"
R XVII a	-456	+45	-411
b	-391	+40	-351
c	+130	-6	+124

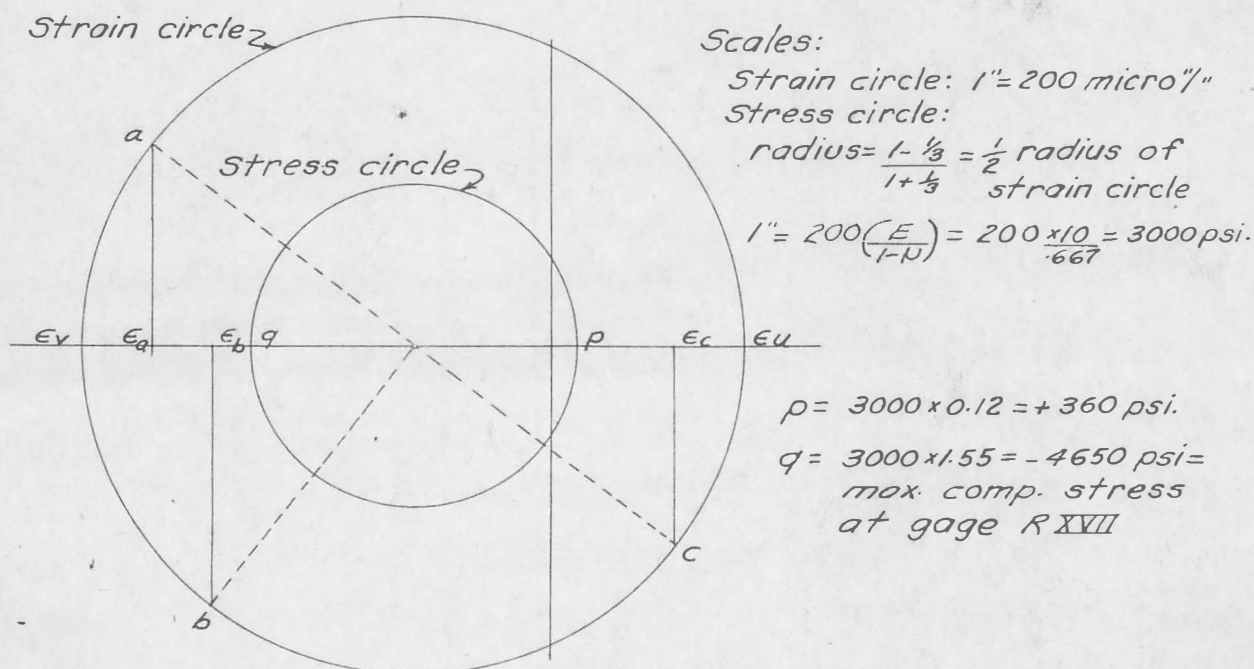


FIGURE 20. SAMPLE CALCULATIONS OF COMBINED VALUES OF PRIMARY AND SECONDARY STRESSES

the calculated secondary moments, shears and axial forces.

An examination of the results of the present study, and of previous studies, indicates that a complex stress situation exists throughout the gusset plate. The effects of moments, shears, and axial loads on the various members overlap to an unknown extent. There are localized disturbances caused by the rivets and rivet holes. The exact effect of friction between the members and the plates is unknown. It also is evident that the applied forces are too near the stressed section for St. Venant's principle to apply. Therefore, the ordinary beam formula will not apply.

In view of the above facts, an attempt at a mathematical analysis of the stresses on any given section seems almost useless. However, it was decided to check one section of the gusset plate to see if any correlation between analytical and experimental results could be established.

A section passing through strain gage G-9 was chosen since the maximum secondary stress was found to be at this point. The section was taken perpendicular to member L_2U_3 . The loading which is that resulting from secondary effects only, is the loading used in tests one and two. It will be remembered that this loading produced secondary stresses four times as great as the true prototype secondary stresses.

In the analysis, certain assumptions were made. Friction

was neglected and the pins were assumed to share direct loads equally. The pins in this region were subjected to two direct loads. One direct load was the axial force and the other was that resulting from transforming the eccentric shear load into a central load and a couple. Two moments acted on the pin group, one being the moment applied to L_2U_3 and the other resulting from the eccentricity of the shear load. The load on any rivet was computed from the following expression: $F = \frac{P_1}{N} + \frac{P_2}{N} + \frac{Tc}{\sum d^2}$, where

P_1 = direct load due to the axial force

P_2 = direct load due to the shear force

N = total number of pins in the group

T = total moment on the pin group

c = distance from centroid of group to the pin for which the load is desired

$\sum d^2$ = polar moment of inertia of the group.

After the loads on the rivets were computed, it was assumed that the cut section was loaded by the rivet loads on one side of the section. The stresses on the section were then computed from the familiar expression, $S = \frac{P}{A} \pm \frac{Mc}{I}$. Since the maximum stress occurred at an inclined edge, the computed normal stress was multiplied by secant θ . Of course, there is no specific evidence that this is a correct factor, but it is generally believed that the stress along an inclined edge is higher than the normal stress computed for a given section.

This complete analysis is shown in Figure 21. An examination of the results discloses that the computed stress on the tension side is about 25% less than the value arrived at by experiment. It is possible that the local effect of the pin load near the point in question raises the stress somewhat. On the compression side of the section, the computed compressive stress was about 14% greater than the experimental value. However, at this point it is possible that the effect of the loads on L_2U_2 is putting tension in the plate and that the compression is reduced considerably.

This analysis is included as a matter of interest only and it is not intended to indicate that the method is of any appreciable significance in the design of gusset plates.

Pin	Dist. from c.g.	d^2
a	1.6	2.56
b	2.4	5.76
c	1.6	2.56
d	0.3	0.09
e	1.15	1.32
f	1.15	1.32
g	2.0	4.00
h	2.0	4.00
$\Sigma d^2 =$		21.61

Section

Gage G-8

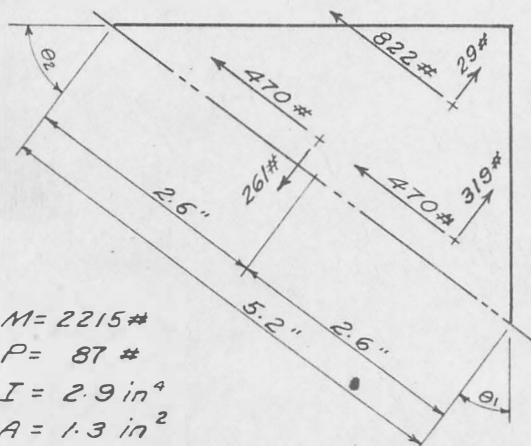
 $L_2 U_3$

Gage G-9

$$S_c = \left(\frac{P}{A} - \frac{Mc}{I} \right) \sec \theta_2$$

$$S_c = \left(\frac{87}{1.3} - \frac{2215 \times 2.6}{2.9} \right) 1.68 = 3200 \text{ psi.}$$

Experimental: 2810 psi comp.



$$M = 2215 \#$$

$$P = 87 \#$$

$$I = 2.9 \text{ in}^4$$

$$A = 1.3 \text{ in}^2$$

$$S_t = \left(\frac{P}{A} + \frac{Mc}{I} \right) \sec \theta_1$$

$$S_t = \left(\frac{87}{1.3} + \frac{2215 \times 2.6}{2.9} \right) 1.25 = 2570 \text{ psi. tension}$$

Experimental: 3410 psi. tension

$$T = 230 \times 28 + 77.6 \times 11.12 = 7315 \#$$

$$F_{Tb} = \frac{7315 \times 2.4}{21.61} = 812 \#$$

$$F_{Ta} = \frac{7315 \times 1.6}{21.61} = 540 \#$$

$$F_{Tc} = \frac{7315 \times 1.6}{21.61} = 540 \#$$

$$F_1 = \frac{77.6}{8} = 9.7 \#$$

$$F_2 = \frac{230}{8} = 28.8 \#$$

FIGURE 21. ANALYTICAL CHECK OF STRESS ON ONE SECTION

CHAPTER VI

FINDINGS AND CONCLUSIONS

An examination of the secondary stress contours for the gusset plate tested discloses that the maximum tensile stress, maximum compressive stress, and maximum shearing stress all occur in the quadrant of the plate to which member L_2U_3 is connected. The maximum tensile stress of 850 psi. occurred near the intersection of the right edge of the gusset plate and the lower edge of member L_2U_3 . The maximum compressive stress of 702 psi. occurred near the intersection of the top edge of the gusset plate and the upper edge of member L_2U_3 . The maximum shearing stress of 425 psi. was located at the same point as the maximum tensile stress.

The stresses appear to result largely from moment since the axial loads and shears are small as compared to the moments.

An inspection of the secondary moments and the loads applied to the various members leads to the conclusion that it is reasonable to expect that the maximum secondary stresses will occur in the region of the gusset plate to which member L_2U_3 connects. The secondary moment at the end of this member is about three and one third times the moment at the ends of the other two web members meeting at joint L_2 .

While it is not reasonable to assume that this proportion of secondary moments will exist for the web members of all Warren

trusses, it does seem reasonable to assume that the secondary moment will generally be larger in L_2U_3 than it will be in the other two web members meeting at joint L_2 . Member L_2U_3 is a heavy compression member while U_1L_2 is a tension member and U_2L_2 is a relatively light compression member. This means that L_2U_3 will usually have a greater stiffness than the other two members and therefore will take a greater part of any moment thrown into the joint than will either of the other two members.

A section passing through the intersections of the edges of the gusset plate and the edges of member L_2U_3 appears to be the critical section for secondary stresses. While this is true for the model tested, it may not be regarded as a general conclusion since the stresses on any given section will vary with the number and spacing of rivets connecting the member to the gusset plate. The number and spacing of the bolts and pins in the model did not closely resemble the number and spacing of rivets in the prototype. Even though the position of the critical section may be estimated, the beam formula does not furnish a reliable method for computing the stresses on this section.

In the gusset plate the maximum secondary tensile stress amounted to approximately eight per cent of the maximum primary tensile stress. These maximum stresses occurred in different regions of the plate and at no point combined to give a tensile stress greater than the maximum primary stress.

The maximum secondary compressive stress was approximately

eleven per cent of the maximum primary compressive stress. The primary and secondary compressive stresses occurred in the same general region but a detailed study of points in the region of high stress revealed that at no point did the primary and secondary stress combine to give a compressive stress greater than the maximum primary compressive stress.

The maximum primary shearing stress and the maximum secondary shearing stress occurred in different regions and did not give a critical combination.

The location and magnitude of the primary stresses were estimated from the results of Whitmore's experiments.¹

Undoubtedly, further testing is necessary before generalized conclusions in regard to secondary stresses in gusset plates may be drawn. The testing of joints representing prototypes having deeper, stiffer members would be highly desirable since the magnitude of secondary effects increases with the stiffness of the members. The testing of top chord joints in which chord members are spliced would also be desirable.

In drawing conclusions from the results of the experiments performed for this study, it must be kept in mind that they may not be applied to joints which differ appreciably in construction and loading from the joint tested. However, for joints of similar construction and loading, the results should be useful in predicting

¹R. E. Whitmore, Experimental Investigation of Stresses in Gusset Plates, loc. cit.

the location of maximum secondary stresses and in estimating the magnitude of the secondary stresses in terms of the primary stresses.

It would certainly appear from the results of this investigation that for trusses of the general proportions considered, secondary stresses in the bottom chord gusset plates are not likely to be critical; in fact, it appears that secondary effects are likely to be much more pronounced in the main members than in the gusset plates.

It is further hoped that this investigation may stimulate additional work in this field and that eventually enough information will be accumulated to enable simple, safe, and economical design of all types of gusset plates.

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