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I am submitting herewith a thesis written by Terry W. Skiles entitled "Turbine Engine Flowpath Averaging Techniques." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

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TURBINE ENGINE FLOWPATH AVERAGING TECHNIQUES

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Terry W. Skiles

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ABSTRACT

Methods of determining the one-dimensional or mean value of a measured flow property were investigated. This investigation was partially accomplished by a literature review of previous investigations and current flowpath averaging practices. Two basic methods for determining the mean value of a measured flow property were identified. Comparison of these flowpath averaging methods for typical turbine engine flows was conducted. Recommendations for further flowpath averaging considerations are offered.

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NOMENCLATURE

A	Area
a	Sonic velocity
B	Bias
BPR	By-pass ratio
CD	Convergent-divergent
CP	Specific heat at constant pressure
dA	Incremental area
ETAB	Burner efficiency
ETAC	Compressor efficiency
ETAM	Mechanical efficiency
FAR	Fuel-air ratio
FHV	Fuel heating value
FN	Engine net thrust
H	Total enthalpy
HAE	Augmenter exit total enthalpy
HC	Engine core total enthalpy
HD	By-pass duct total enthalpy
HFA	Augmenter fuel enthalpy
HPC	High-pressure compressor
LPC	Low-pressure compressor
LPT	Low-pressure turbine
M	Mach number
$\dot{m}$	Mass flow function, $\frac{W \times \sqrt{T}}{PS \times A}$
N	Compressor rotor speed

n	Number of individual areas
P	Total pressure
PS	Static pressure
R	Radius; gas constant
S	Precision error
SFC	Specific fuel consumption
t_{95}	Student's "t" distribution
T	Total temperature
TD	Temperature deviation
TS	Static temperature
U	Uncertainty
V	Velocity
VAE	Augmenter exit velocity
VCE	Core exit velocity
VDE	By-pass duct exit velocity
VNE	Nozzle exit velocity
W	Mass flow rate
WAC	Engine core mass flow rate
WAD	By-pass duct mass flow rate
WAE	Augmenter exit gas flow rate
WAT	Augmenter total mass flow rate
WAI	Augmenter inlet gas flow rate
WF	Main burner fuel flow
WFA	Augmenter fuel flow

Subscripts

A	Augmenter
AREA AVG	Average value calculated by area weighted method
AVG, AV	Average value
c	Compressor
CALC	Calculated value
DELTA	One percent change in averaged measured parameter
HUB	Compressor hub radius
i	Individual value
isen	Isentropic
loc	Local
MASS AVG	Average value calculated by mass weighted method
MAX	Maximum value
meas	Measured value
MIN	Minimum value
NOMINAL	Nominal value
PROBE	Value of individual probe on measurement rake
REF	Reference value
t	Turbine
TIP	Compressor tip radius
TOTAL	Total value
X	Coordinate variable
0	Freestream
2	Low pressure compressor inlet
25	Low pressure compressor exit, high pressure compressor inlet

3	High pressure compressor exit
4	Burner exit
5	Low pressure turbine exit, core exit, by-pass duct exit
7	Nozzle inlet
9	Nozzle exit

Greek Symbols

γ	Ratio of specific heats
ΔP	Change in total pressure
η	Efficiency
ϕ	Parameter representation
ρ	Density
θ	Ratio of compressor inlet total temperature to standard day, sea-level-static temperature

CHAPTER I

INTRODUCTION

The gas turbine engine is composed of different components arranged in series and/or parallel and, from a performance point of view, is amenable to one-dimensional flow analysis. Each engine component exerts changes on the gas as it flows through the engine creating changes in the flow properties. The changes exerted on the gas by each engine component must be known in order to determine performance of engine components and engine systems. Engine performance is typically determined using one-dimensional analysis based on averaged measured values and mean calculated properties. For example, the turbine engine test analysis standard data reduction program used at the Arnold Engineering Development Center, Engine Test Facility, utilizes performance equations based on one-dimensional, steady-state equations. The assumption of one-dimensional flow is a simplification generally applied to turbine engine flow processes with the coordinate variable in the direction of flow (1).[1] At any given cross-section fluid properties are considered constant and, therefore, vary only with axial position. Equations based on one-dimensional flow do not precisely describe the fluid motion; however, the one-dimensional flow assumption has been found to be a good approximation to many actual flows when considering overall performance.

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[1]Numbers in parentheses refer to similarly numbered references in the Bibliography.

The one-dimensional approximation represents the oldest and simplest approximation of the equations of gas dynamics. It can be applied to the study of the flow in ducts of small divergence and curvature. The consequence of the small divergence is that a flow configuration can exist where the velocities are nearly parallel indicating the velocities have small oblique angles to the axis. In this case, the velocity components normal to the axis are small compared to the axial components and, if the diameter of the duct is small compared with the curvature of the axial line, the transverse gradients of pressure can be neglected. Thus, the pressure can be assumed to be a constant and a function only of the axial coordinate.

The assumption of small obliquity of the velocities is fundamental in the one-dimensional treatment. The fact that in ducts the flow is completely enclosed by solid boundaries represents a great advantage for the one-dimensional treatment because the amount of fluid involved is well defined. It is further assumed that not only the relative obliquity of the velocities is small, but also that the magnitude is constant at each cross section. Similarly, for small curvatures, not only the pressure at each point is the same, but the temperature and any other parameter related to the physical conditions are also distributed uniformly.

It is realized that while the obliquity assumption can be very closely satisfied in practical cases, the uniformity assumption never represents the actual physical conditions. The velocity is never uniform in the duct because it must always vanish (i.e., the no-slip condition) at the boundaries. Similarly, heat or mass exchanges can only be present if a transverse gradient of temperature or mass exists. It is a basic assumption that if proper "averaged" values of the flow and thermodynamic parameters are defined, the actual nonuniform flow can be approximately replaced by an equivalent uniform flow characterized by values of the velocity, temperature, and other physical parameters equal to these "averaged" values. The important consequence of the uniformity assumption is that changes of the physical quantities take place only in the axial direction, so that each quantity is a function of a single spatial coordinate defined along the axial line. Henceforth, the term "one-dimensional flow" is used in the above sense (2).

The definition of "averaged" values for a nonuniform flow represents a replacement of the varying parameters over a cross-section by "averaged" parameters which are uniformly distributed. Considering a duct cross-section in which the flow is nonuniform, determination of the "averaged" parameters by application of the conservation laws of mass, momentum, and energy from a multidimensional to one-dimensional flow can be accomplished, with reference to Figure 1,[2] as follows:

Conservation of Mass

$$\int_{\text{Nonuniform Flow}} \rho V_x dA = \int_{\text{Uniform Flow}} \rho V_x dA = W_{TOTAL} \qquad [1]$$

[2]All figures appear in Appendix A.

Conservation of Energy

$$\underset{\text{Nonuniform Flow}}{\int} \rho V_x H dA = \underset{\text{Uniform Flow}}{\int} \rho V_x H dA \qquad [2]$$

$$\underset{\text{Nonuniform Flow}}{\int} \rho cp T V_x dA = \underset{\text{Uniform Flow}}{\int} \rho cp T V_x dA = cp W_{Total} T_{\substack{Mass\\Avg}} \qquad [3]$$

Conservation of Momentum

$$\underset{\text{Nonuniform Flow}}{\int} \rho V_x V_x dA + \underset{\text{Nonuniform Flow}}{\int} ps\ dA = \underset{\text{Uniform Flow}}{\int} \rho V_x V_x dA + \underset{\text{Uniform Flow}}{\int} PS\ dA$$

$$= W_{Total} V_{x_{\substack{Mass\\Avg}}} + PS_{\substack{Area\\Avg}} A \qquad [4]$$

The nonuniform flow is assumed to mix to uniformity in a constant area duct in the absence of heat transfer, work transfer, or skin friction. Mixing losses are inherently contained in the average flow value.

Reduction of the more complicated flow to such an assumed one-dimensional flow is a common simplification to analysis of ducted flow systems and, therefore, readily finds application to overall performance analysis of gas turbine engines (3).

Furthermore, the assumption of one-dimensional flow greatly simplifies the basic equations and allows simplification of data reduction computer programs including mathematical simulations. The information required for performance determination is usually obtained at the component interfaces of the engine system. Averaged parameters are determined from measured distributions, which can be significantly nonuniform.

No universally accepted practice for averaging thermodynamic and flow properties across engine flowpaths, commonly called flowpath averaging techniques, currently exists in the aerospace turbine engine community. Identification and evaluation of flow averaging practices are needed to determine component and engine performance, increase fidelity of component and engine simulation and provide a basis for comparison of data from the various test facilities.

The specific objectives of this report were to identify existing flowpath averaging practices and numerically compare some of the more common flowpath averaging techniques.

CHAPTER II

FLOWPATH AVERAGING PRACTICES

II.1 LITERATURE REVIEW

Several investigations of flowpath averaging techniques have been accomplished previously for different classes of flow. However, no previous investigation has directly addressed engine system averaging techniques. Tyler (4) presented a method of determining "suitable mean values" of flow parameters to be used when one-dimensional flow methods are applied to a circular duct in which a boundary layer exists. Results indicate that differences between the one-dimensional method considering the existence of a boundary layer and the one-dimensional method neglecting the boundary layer tends to increase with Mach number and boundary layer.

Wyatt (5) developed a method of determining a uniform flow by equating the mass flow and momentum of the uniform flow to the mass flow and momentum of the real flow in a constant-area duct without wall friction. In addition, Wyatt numerically investigated conventional averaging methods by considering several velocity gradients confined to subsonic compressible flow with constant static pressure and total temperature. Comparison of conventional averaging methods and the mass-momentum method developed during Wyatt's investigation concluded that conventional averaging methods can cause errors in calculated uniform flow properties while uniform flow properties determined from the mass-momentum method are without error.

Livesey et al (6 and 7) considered the problem of the definition of mean values of flow quantities for use in the application of one-dimensional gas dynamics to nonuniform flows. A method of determining mean values of flow parameters was developed based on the equivalence of a uniform flow and a nonuniform flow utilizing ethalpy flux, entropy flux and mass flow rate. The development represented a replacement of the varying parameters over a cross-section by mean parameters which are uniformly distributed. This was thought of as the creation of an equivalent stream which flows through the same duct arrangement but with uniform flow parameters. Livesey's development results in a set of mean flow property definitions including velocity, density, static and total temperature, and total pressure.

Traupel and Dzung (8) have each developed a method of flow parameter determination based on the solution of physical and thermodynamic relations for a thermally perfect, one-dimensional, compressible fluid. Traupel utilizes the definitions of flow velocity, temperature based on the enthalpy definition, entropy, and pressure as a function of enthalpy and entropy to determine average flow parameters. Dzung utilizes the conservation equations of continuity, moment of momentum, momentum, and energy and a basic relation of enthalpy as a function of gamma, average pressure, and average density to determine average flow parameters.

Decher (9) investigated the roles of mass averaged total temperature, total pressure, and exit static pressure nonuniform profiles on nozzle performance. Values for the average total pressure and total temperature which result from conservation of mass and

and total enthalpy were of interest because these values can be used in the simple one-dimensional forms of the nozzle flow equations. During this investigation, Decher developed equations for the equivalent mass averaged total pressure and total temperature of nozzle flow based on the conservation of mass and total enthalpy by considering nozzle flow to consist of a large number of noninteracting stream tubes. Results indicate mass averaged temperatures and pressures and nozzle performance are influenced by nonuniform profiles of temperature and pressure.

Duchar and Tabakoff (10) analytically investigated several methods of averaging total pressure and temperature profiles to arrive at a single, one-dimensional value at the entrance to jet engine exhaust nozzles using a concept of conservation of ideal available thrust. In addition, a brief analysis of profile effects on converging-diverging (CD) nozzle performance was conducted. Results indicate pressure profiles should be mass weighted and temperature profiles should be "thrust" weighted for analyzing nozzle performance, and total pressure and temperature profiles have a negligible effect on CD nozzle performance.

II.2 CURRENT FLOWPATH AVERAGING PRACTICES

The design of a turbine engine system requires the use of analyses which can relate changes in component configuration to changes in engine performance. In the design phase of turbomachinery components, flowpath property averaging methods usually use mass averaging. In actual practice, data generated from both component analysis and previous experimental results are stored in the form of tables and coefficients of aerothermodynamic properties of air and combustion products, aerothermodynamic performance of the various engine components as a function of the controlling independent variable, power losses by auxiliary units and bleed flows, and data that relates geometry changes to engine performance. The data used to generate the performance tables and coefficients are flow averaged in accordance with the particular component and engine system flow averaging test practices of each testing establishment.

The selection of a flowpath property averaging method for the turbomachinery component test phase is usually mass averaging which is consistent with the design phase. Engine system flowpath property averaging methods usually use area averaging. Combination of components in order to yield engine system performance is confined to mathematical simulations in the form of engine balancing techniques as opposed to engine system performance calculations utilizing measured parameters and one-dimensional flow assumptions.

Information on engine flowpath averaging practices were obtained (11, 12, and 13) from the following test establishments:

1. Arnold Engineering Development Center (AEDC)
2. National Aeronautics and Space Administration--
   Lewis Research Center (NASA--Lewis)
3. National Gas Turbine Establishment (NGTE)
4. Centre d'Essais des Propulseurs (CEPr)

Flowpath averaging techniques of the major testing establishments include various methods of parameter averaging. Current practices of each of the contributing test establishments are presented in Tables 1 thru 5.[3] The determination of a mean value, in general, falls into three categories; (1) some form of averaging of measured flowpath properties such as area or mass averaging, (2) energy or work balances across a component or engine system such as the determination of burner exit temperature from an energy balance across the gas turbine main combustor, and (3) solution of a system of conservation equations such as the DZUNG average (8). The category investigated for this report is the averaging of measured flowpath properties since this category is considered the most important. In defining a method of determining the mean value of a measured flow property, two basic methods stand out. These are area weighted and mass weighted methods. Combinations of both are used to evaluate engine performance parameters. Measured pressures and temperatures used to compute engine system performance are usually area averaged, although, mass averaged values are also used. Calculation of engine system performance often requires determination of thermodyanmic flow properties (14), such as enthalpy for component efficiency, in addition to pressures and temperatures. References 15 through 22 describe some of the experimental and flowpath averaging practices used in component and engine system performance testing.

Instrumentation is an important consideration in selecting a flowpath averaging technique. Some form of averaging of measured flowpath properties is usually performed when instrument rakes are

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[3]All tables appear in Appendix B.

located at engine measurement stations. Inlet and exit pressure and temperature are usually measured at engine component interfaces or measurement stations (Figure 2) using fixed rakes spaced in circumferential locations. The number of measurements made at each station can vary depending on the objectives of the test under consideration. Wall static pressures can be located in a circumferential position at the station inlet and exit. The general practice of pressure and temperature rakes is to locate measurement probes on centers of equal areas.

Energy or work balances and simultaneous solution of conservation and state equations are usually performed when specific or limited instrumentation is available.

II.3 COMPARISON OF FLOWPATH AVERAGING TECHNIQUES

Different techniques of determining the averaged value of a measured flow property were reviewed. The most commonly used techniques are area weighted and mass weighted. The derivation of an area weighted parameter and mass weighted parameter are presented in Appendix C.

An averaged parameter obtained by the area weighted method is calculated by a summation of each individual measured parameter multiplied by the individual cross-sectional area corresponding to the individual measured parameter and divided by the total cross-sectional area (Figure 2).

$$\phi_{AREA\ AVG} = \frac{\Sigma\ \phi_i\ A_i}{A_{TOTAL}} \qquad [5]$$

An averaged parameter obtained by the mass weighted method is calculated by a summation of each individual measured parameter multiplied by the individual mass flow corresponding to the individual measured parameter and divided by the total mass flow.

$$\phi_{MASS\ AVG} = \frac{\Sigma\ \phi_i\ W_i}{W_{TOTAL_{meas}}} \qquad [6]$$

To mass average test data, individual mass flow is calculated. In order to calculate this individual mass flow, a local static flow property, usually static pressure, is required.

As a result of the standard practice of using measurements located on centers of equal cross-sectional areas, a method is required to determine the individual mass flow corresponding to the individual measured parameter. One method of determining the individual mass flow utilizes the mass flow function. The local mass flow function is defined by

$$\dot{m} = \frac{W\sqrt{T}}{PS\ A}\ , \qquad [7]$$

which is easily shown to depend only on the local Mach number

$$\dot{m} = \sqrt{\frac{\gamma}{R}}\ M\sqrt{1 + \frac{\gamma-1}{2}M^2}\ . \qquad [8]$$

Rearranging the mass flow function equation to obtain an individual mass flow results in

$$W_i = \frac{\dot{m}_i\ PS\ A_i}{\sqrt{T_i}}\ , \qquad [9]$$

where $\dot{m}_i$ is based on the individual total-to-static pressure ratio, A_i is the individual cross-sectional area and, T_i is the individual total temperature. Generally, the static pressure is considered constant across a cross-sectional area (4, 6 and 10).

In order to determine the individual mass flow, the static pressure must be known. In instances where the static pressure is not measured, a value for static pressure is assumed and individual mass flows are calculated. In order to determine if the proper value of static pressure has been assumed, the mass flow function for the total bulk mass flow is introduced,

$$\dot{m} = \frac{W_{TOTAL}\sqrt{T_{MASS\ AVG}}}{PS \quad A_{TOTAL}} \, . \qquad [10]$$

Rearranging to obtain the total bulk mass flow results in

$$W_{TOTAL} = \frac{\dot{m}\ PS\ A_{TOTAL}}{\sqrt{T_{MASS\ AVG}}} \qquad [11]$$

where $\dot{m}$ is based on the mass averaged total-to-static pressure ratio, A_{TOTAL} is the total cross-sectional area and, $T_{MASS\ AVG}$ is the mass averaged total temperature. When the calculated total mass flow equals the measured total bulk mass flow, the proper static pressure is determined.

The iteration process involved with the assumption of a static pressure and measured total bulk mass flow results in a static pressure, mass weighted total pressure and mass weighted total temperature which is a consistent set of parameters that results in an equivalent bulk mass flow. This set of parameters does not imply that the true static pressure has been determined, since a different set of parameters may result in a different static pressure. However, what is strived for is a consistent set of parameters.

Comparison of the basic flow averaging methods was accomplished using the high-pressure compressor (HPC) component of a current engine system. The HPC component was used because the nonuniform flowfield patterns generated are representative of engine interface nonuniform flowfield distributions for normal operating flight conditions of current engine systems. A measure of the nonuniformity of the flowfield is the distortion index. A baseline compressor distortion pattern was obtained in terms of the measured parameters of total pressure and total temperature (Figure 3). Data for the generation of the baseline compressor distortion pattern were obtained during sea-level-static testing at the AEDC. The measured parameters were determined from fixed rakes having five measurement probes located on centers of equal area (Figure 2). Total pressure distribution (isobars) and total temperature distribution (isotherms) at the inlet and exit of the representative HPC are presented in Figures 4 and 5. The total pressure and total temperature values are normalized by the area integrated values of pressure and temperature, respectively. The area integrated value is obtained from the baseline distortion patterns by summing the local areas associated with each distortion index.

Radially averaged circumferential total pressure and total temperature profiles at the inlet and exit of the representative HPC obtained using the area and mass averaging methods are compared in Figure 6. Actual total pressure and total temperature values were obtained from the baseline distortion patterns by superimposing a hypothetical five-probe rake (probes distributed on equal areas) at each successive circumferential location for which there were no measurements. Averaged values of total pressure and total temperature were obtained using the two averaging methods. The averaged values were referred to an area integrated value obtained from the baseline distortion patterns. Figure 6 indicates that the circumferential location and number of measurement rakes can influence the averaged value as much as 1.3 percent by misplacement of one or more rakes. For example, a single temperature rake located at 180 degrees at the HPC inlet is 1.3 percent below the integrated value using the mass weighted method. Two rakes, one located at zero degrees and the other located at 225 degrees, are 0.22 percent (typically, $TD_{225}-TD_{0}=0.32-0.10=0.22$) of the integrated value for the mass weighted method and 0.08 percent for the area weighted method.

Distortion index is a measure of the combination of circumferential and radial distortions and can be used as a measure of power level. Total pressure and total temperature distortion at the inlet and exit of the representative HPC as a function of corrected rotor speed is presented in Figure 7. The total pressure and total temperature distortion is normalized by either the area or mass averaged values of pressure and temperature. No discernible difference is noticed by using the mass averaged or area averaged values of pressure and temperature in the calculation of the distortion index. Figure 7 indicates the change in distortion is a function of engine power level.

Radial total pressure and total temperature profiles at the inlet and exit of the representative HPC showing the effect of corrected rotor speed is presented in Figure 8. The total pressure and total temperature profiles are normalized by either the area or mass averaged values of pressure and temperature. Again, no discernible difference is noticed by using the mass averaged or area averaged values of pressure and temperature in the distortion index calculation. Figure 8 shows that the overall radial pressure profile increases with corrected rotor speed at the inlet and remains approximately the same at the exit. The temperature profiles, in terms of corrected rotor speed, remain approximately the same at the inlet and exit.

Total pressure and total temperature comparisons of the basic averaging methods as a function of corrected rotor speed are presented in Figure 9. The mass averaged and area averaged values of total pressure and total temperature are within 0.6 of one percent of each other at the inlet and exit. The area averaged total pressure and total temperature values are, in general, greater than the mass averaged total pressure and total temperature at the inlet with no observable trend at the exit.

II.4 PERFORMANCE CHANGE AND MEASUREMENT UNCERTAINTY

When dealing with gas turbine engines, it is desirable to know how engine performance changes with variation of parameters used to calculate engine performance. Basic engine performance equations were derived following the methods presented in reference 23 for a mixed-flow turbofan engine (Appendix C). Changes of engine performance due to variations of total pressure and temperature were determined by application of the basic performance equations. Nominal engine performance was obtained by application of the basic performance equations utilizing data obtained during sea-level-static testing of a current engine system at the AEDC. Total pressure and temperature were independently varied one percent above and below the nominal value at the engine inlet, HPC inlet, and HPC exit and engine performance obtained. This simulates the effect of applying different averaging methods to the same measured data at the same engine station and determines the impact on basic engine performance. Engine performance changes due to a one percent variation of total pressure or temperature at the representative engine interfaces are less than one percent. These engine performance changes are presented in Table 6.

All measurements contain measurement errors. These errors are the differences between the measurements and the true value. Measurement uncertainty is defined as the maximum error which might reasonably be expected. Uncertainty assessment consists of an audit of the random (Precision) and fixed (Bias) errors which result from the measurement of a value. Precision error is the random error observed in a set of repeated measurements. Bias is the difference between the average of all possible measured values and the true value. For simplicity, a single number (a combination of bias and precision error) is needed to express a reasonable limit for error. This single number, defined as the measurement uncertainty, must have a simple interpretation (the largest error reasonably expected) and be useful without complex explanation. Measurement uncertainty for a single measurement is defined as

$$\pm U = \pm (B + t_{95} S), \quad [12]$$

where B is the upper limit of the bias error from the true value, S is the precision error, which is an estimate of the true standard deviation of repeated values of the measurement, and t_{95} is the student "t" statistical parameter at the 95 percent confidence level (24). The bias and precision errors utilized for the evaluation of uncertainty values were obtained during testing of a state-of-the art turbofan engine at AEDC.

Flowpath averaged parameters are not single measurements but are determined as a function of several individual measurements. To assess the measurement uncertainty of flowpath averaged parameters, it is necessary to propagate individual measurement uncertainties through a function that relates the flowpath averaged parameters and the individual measurements. The error propagation is approximated with a first order Taylor's series method. Measurement uncertainties were obtained for the mass averaged and area averaged methods utilizing the same bias and precision errors. Measurement uncertainties for the mass weighted method were higher than for the area weighted method. The mass weighted temperature and pressure uncertainties were as much as 2.2° R and 1.02 psia greater than the area weighted temperature and pressure uncertainties. Uncertainty results are presented in Table 7.

CHAPTER III

CONCLUSIONS AND RECOMMENDATIONS

The overall objective of this report was to investigate methods of determining the one-dimensional or mean value of a measured flow property. Specific efforts were a literature review of previous investigations and current flowpath averaging practices and a numerical comparison of fundamental flowpath averaging techniques. The following conclusions have been drawn from these efforts:

1. Several investigations of flowpath averaging techniques for different classes of flow have been accomplished previously, but the subject of engine flowpath averaging of measured flow properties has not been reported.
2. Instrumentation measurements can influence the method used to obtain an average flow property value; i.e., some form of averaging of measured flowpath properties, energy or work balances, or simultaneous solution of conservation and state equations. The method chosen often depends on the number and placement of instrumentation measurements.
3. Area weighted and mass weighted types of flowpath averaging are generally performed for component and engine system testing for averaging values of measured parameters.
4. The mass weighted method of flowpath averaging is within 0.5 percent of the area weighted method for the average

total pressure and within 0.6 percent of the area weighted method for the average total temperature for normal operating flight conditions of current engine systems.

5. Uncertainty calculations utilizing the method presented in this investigation, indicate that mass averaging methods increase the uncertainty level of the averaged flow properties.

6. Utilizing basic engine performance equations, a one percent variation of measurement station total pressure and temperature at the conditions investigated results in engine performance changes of less than one percent.

Conclusions 4, 5, and 6 indicate that the change in engine performance obtained by using mass weighted or area weighted methods to determine engine performance would be expected to be within 0.5 percent.

Two basic methods of measured flowpath property averaging have been considered in this investigation. These are mass weighted and area weighted methods. In consideration of the general desire to satisfy the basic conservation laws, a proper definition of measured flowpath property averaging is the mass averaging method. The ease of calculation, general acceptance, and the closeness to the mass averaging method indicate the area weighted method can be used in many applications of engine performance determination. It is recommended, for proper application of conservation principles, that mass weighted averaging of measured flowpath properties be considered in the determination of gas turbine engine performance.

There are additional areas of averaged flow property determination which should be considered.

1. Boundary layer effects: The change in the value of an averaged parameter considering the effects of boundary layer should be investigated.
2. Balancing techniques: Energy and work balances are currently used in determination of component interface average properties. An effort should be made to compare differences between mass and area averaging type calculations and balancing calculations.
3. Systems of equations: Investigation of the determination of mean values of a flowpath property by simultaneous solution of conservation and state equations should be made and compared to mass and area averaging type calculations.

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APPENDIXES

APPENDIX A

FIGURES

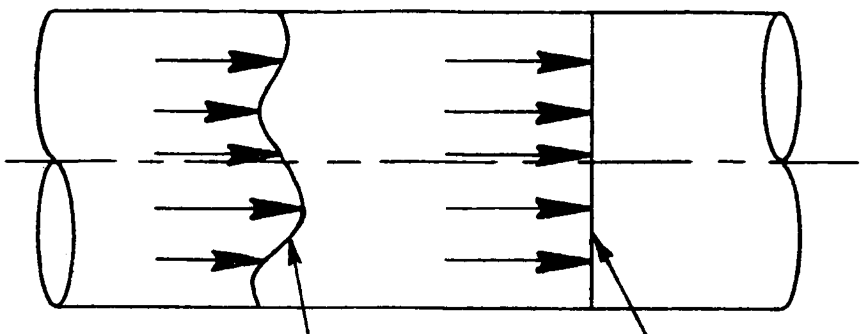
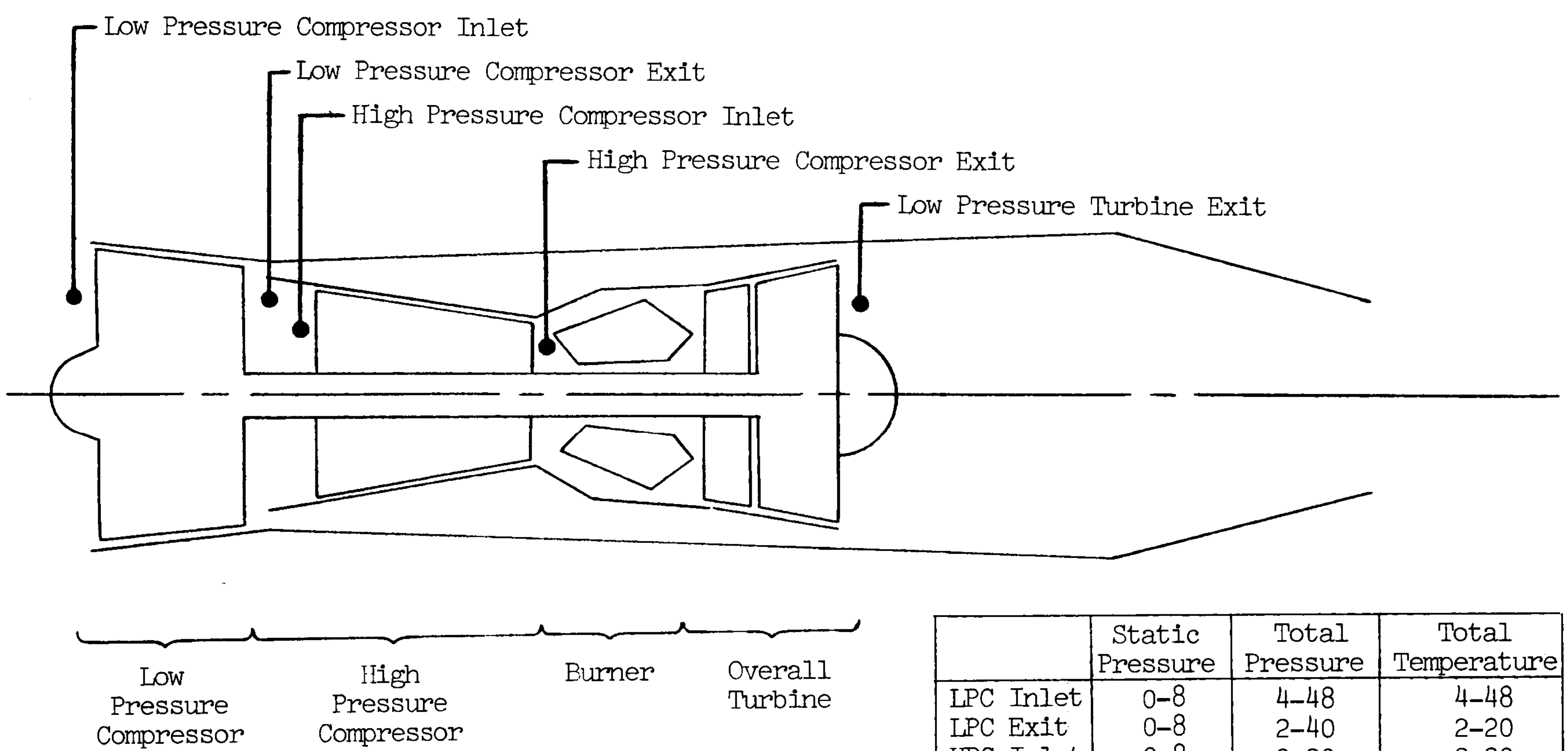


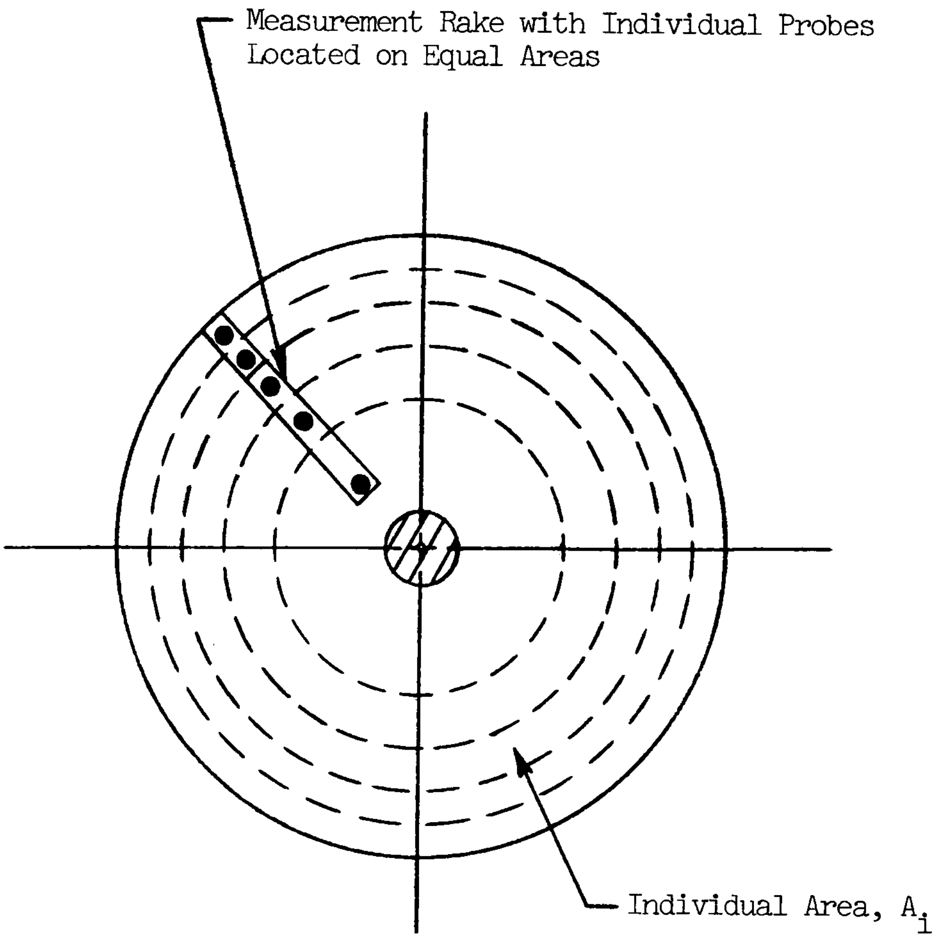
Figure 1. Representation of uniform profile development in a constant area duct.



	Static Pressure	Total Pressure	Total Temperature
LPC Inlet	0-8	4-48	4-48
LPC Exit	0-8	2-40	2-20
HPC Inlet	0-8	2-20	2-20
HPC Exit	0-4	2-20	2-20
LPT Exit	0-4	0-20	0-20

A. Measurement Stations and Number of Parameters Measured

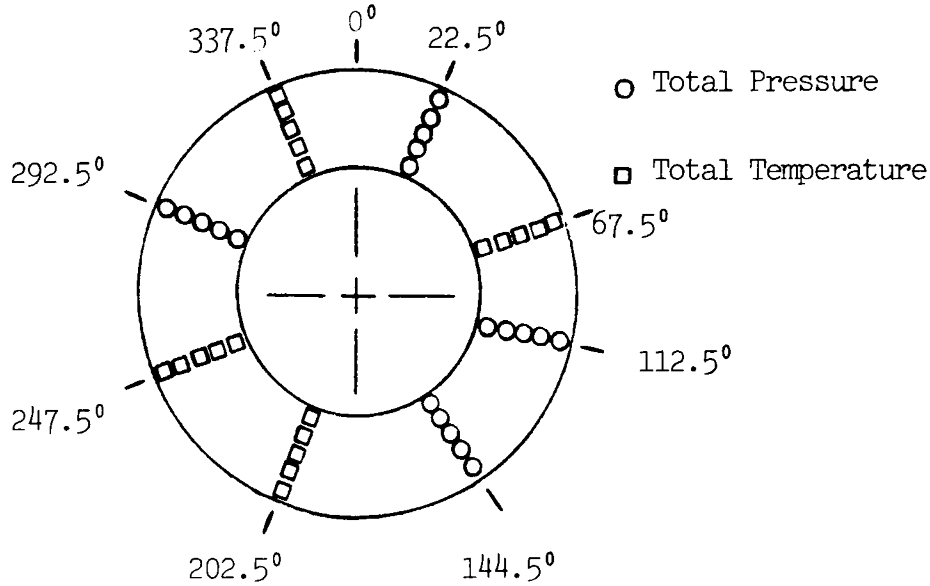
Figure 2. Representative engine system.



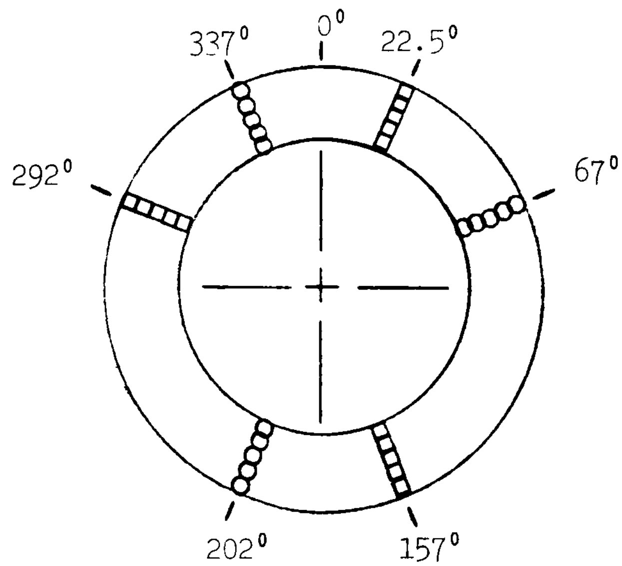
$A_{Total} = \Sigma A_i$

B. Parameter Location on Individual Cross-Sectional Areas

Figure 2. (Continued)

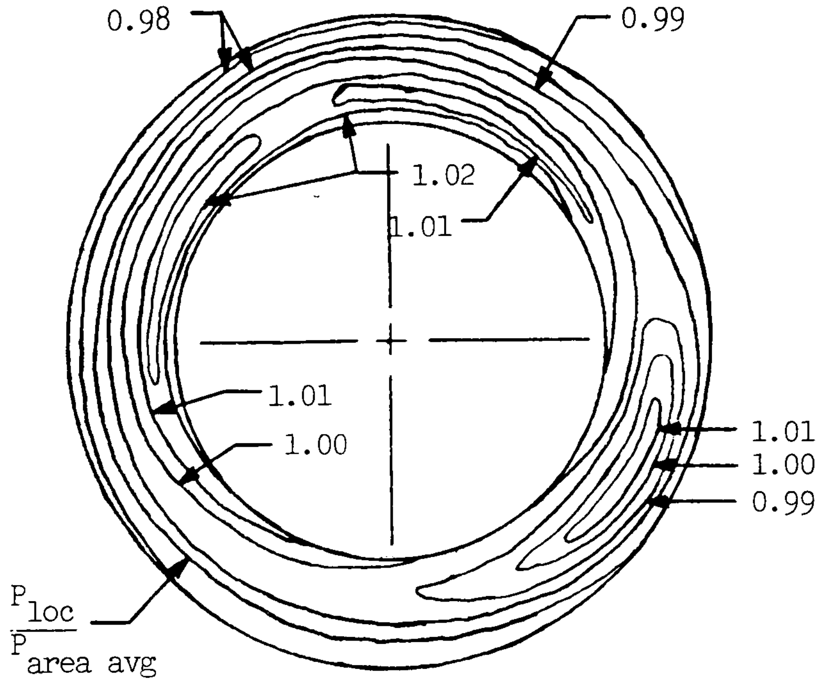


A. High Pressure Compressor Inlet

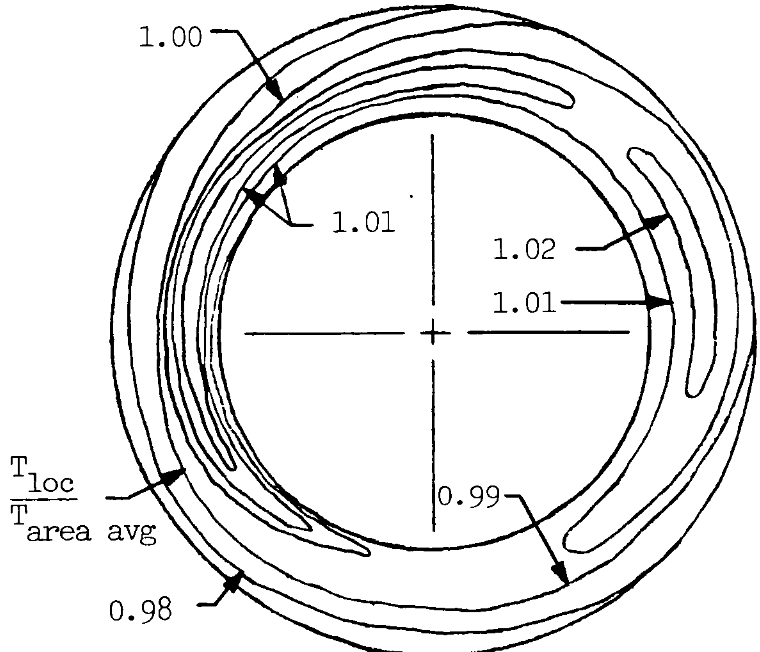


B. High Pressure Compressor Exit

Figure 3. Schematic of compressor instrumentation used for flow averaging evaluation.

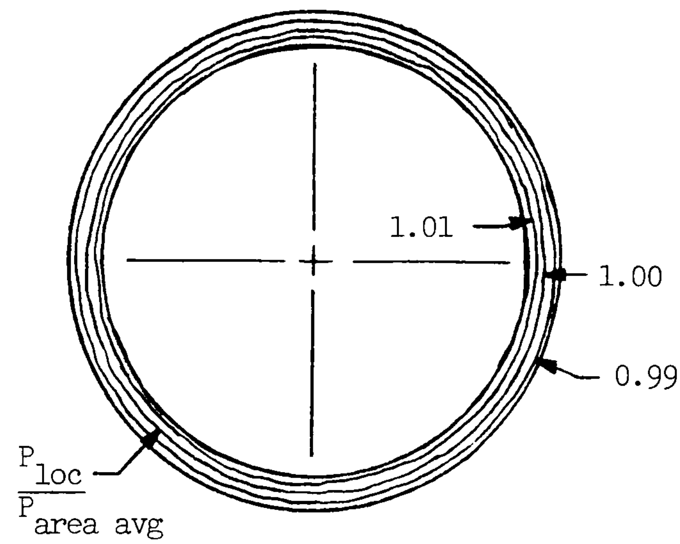


A. Total Pressure Distribution (Isobars)

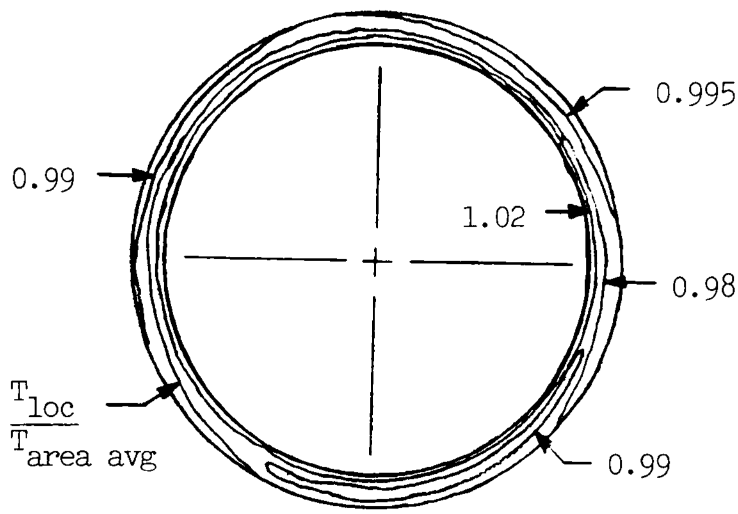


B. Total Temperature Distribution (Isotherms)

Figure 4. Baseline compressor inlet distortion pattern for flow averaging evaluation.

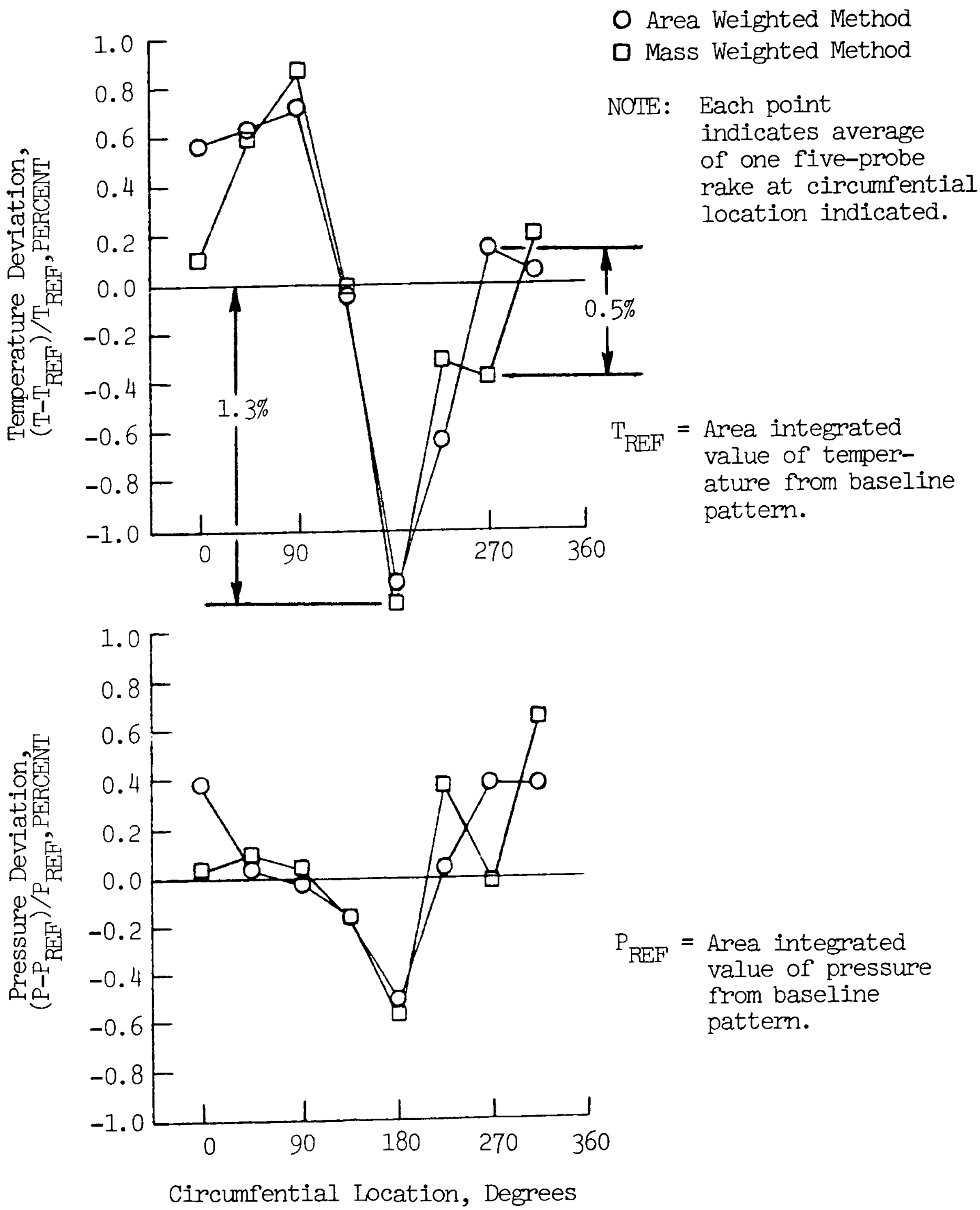


A. Total Pressure Distribution (Isobars)



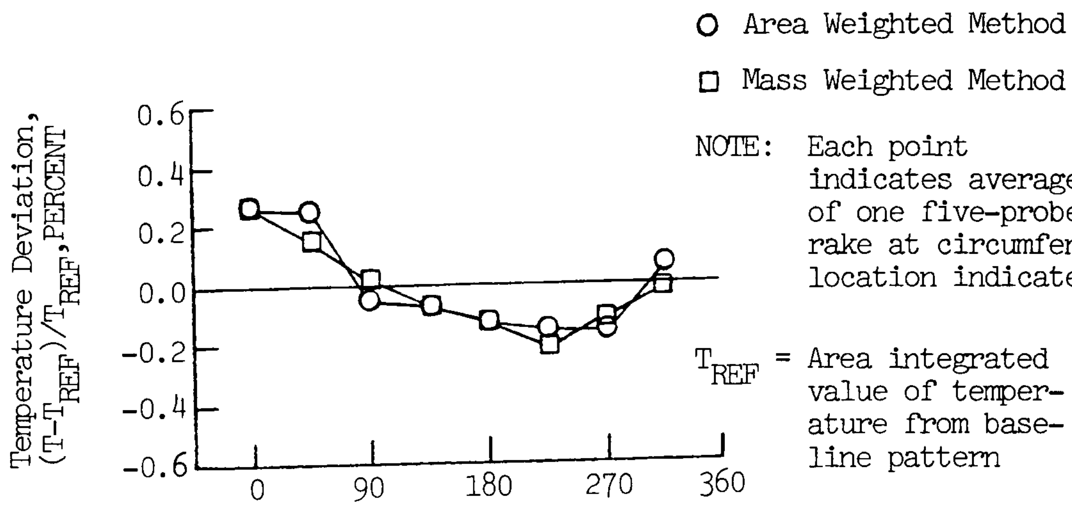
B. Total Temperature Distribution (Isotherms)

Figure 5. Baseline compressor exit distortion pattern for flow averaging evaluation.



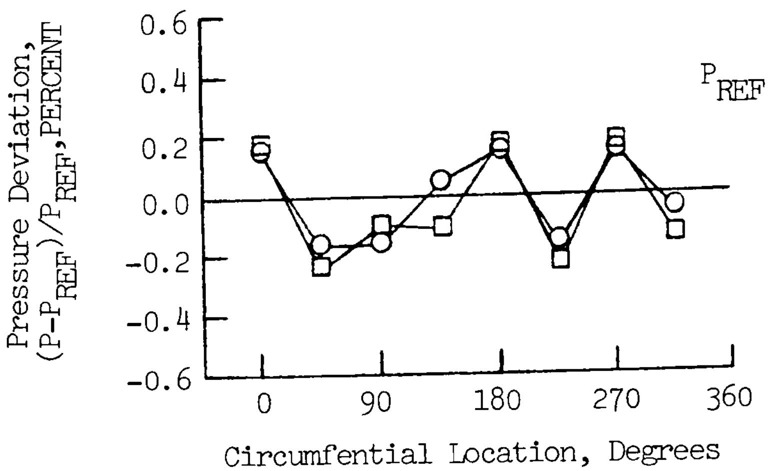
A. High Pressure Compressor Inlet

Figure 6. Comparison of area weighted and mass weighted averaging methods for baseline pattern.



NOTE: Each point indicates average of one five-probe rake at circumfential location indicated.

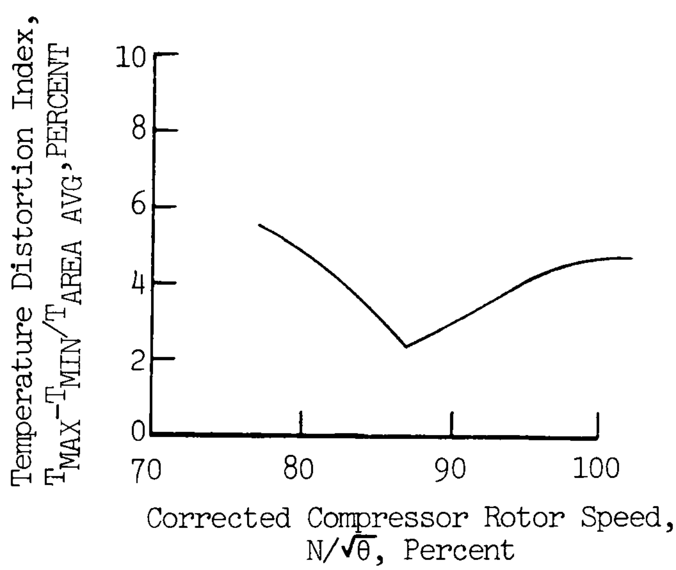
T_{REF} = Area integrated value of temperature from baseline pattern

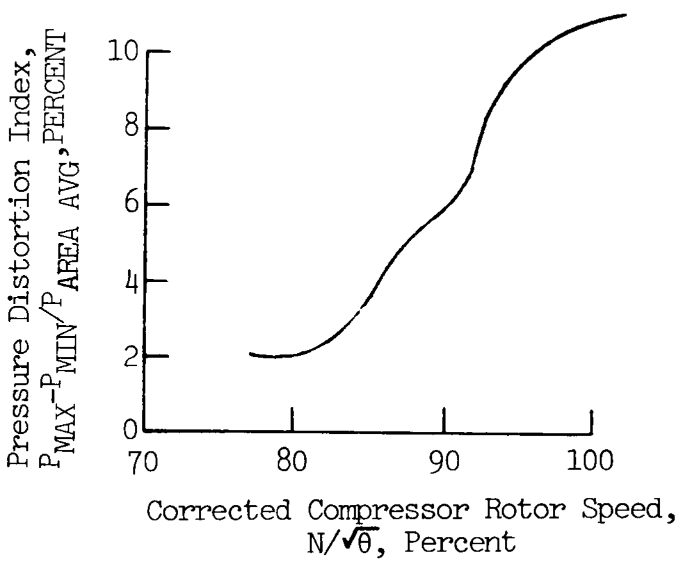


P_{REF} = Area integrated value of pressure from baseline pattern.

B. High Pressure Compressor Exit

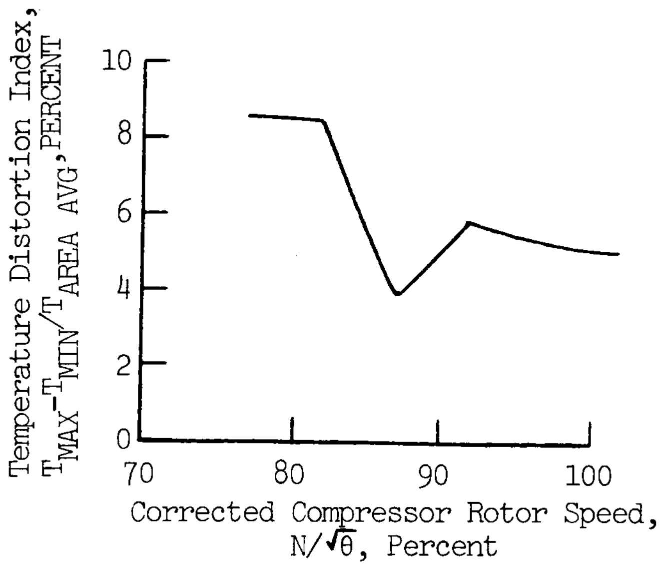
Figure 6. (Continued)

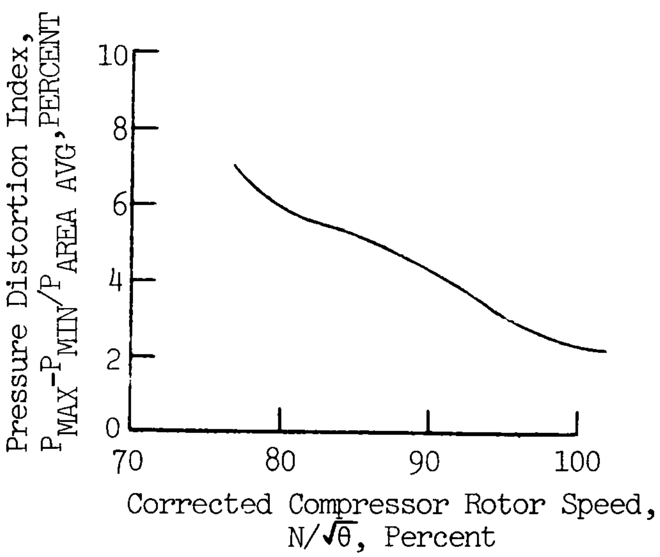




A. High Pressure Compressor Inlet

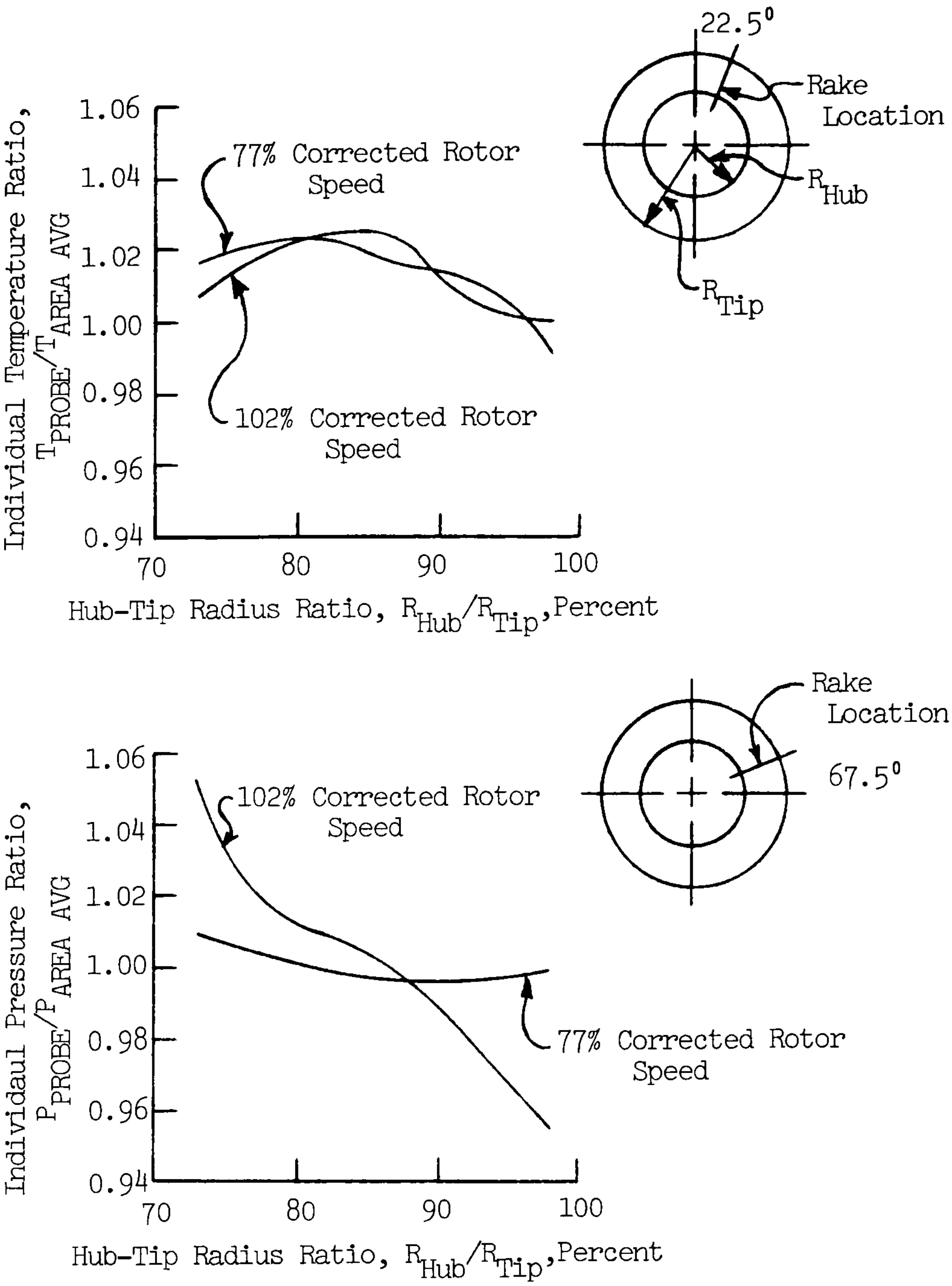
Figure 7. Distortion index as a function of corrected rotor speed.





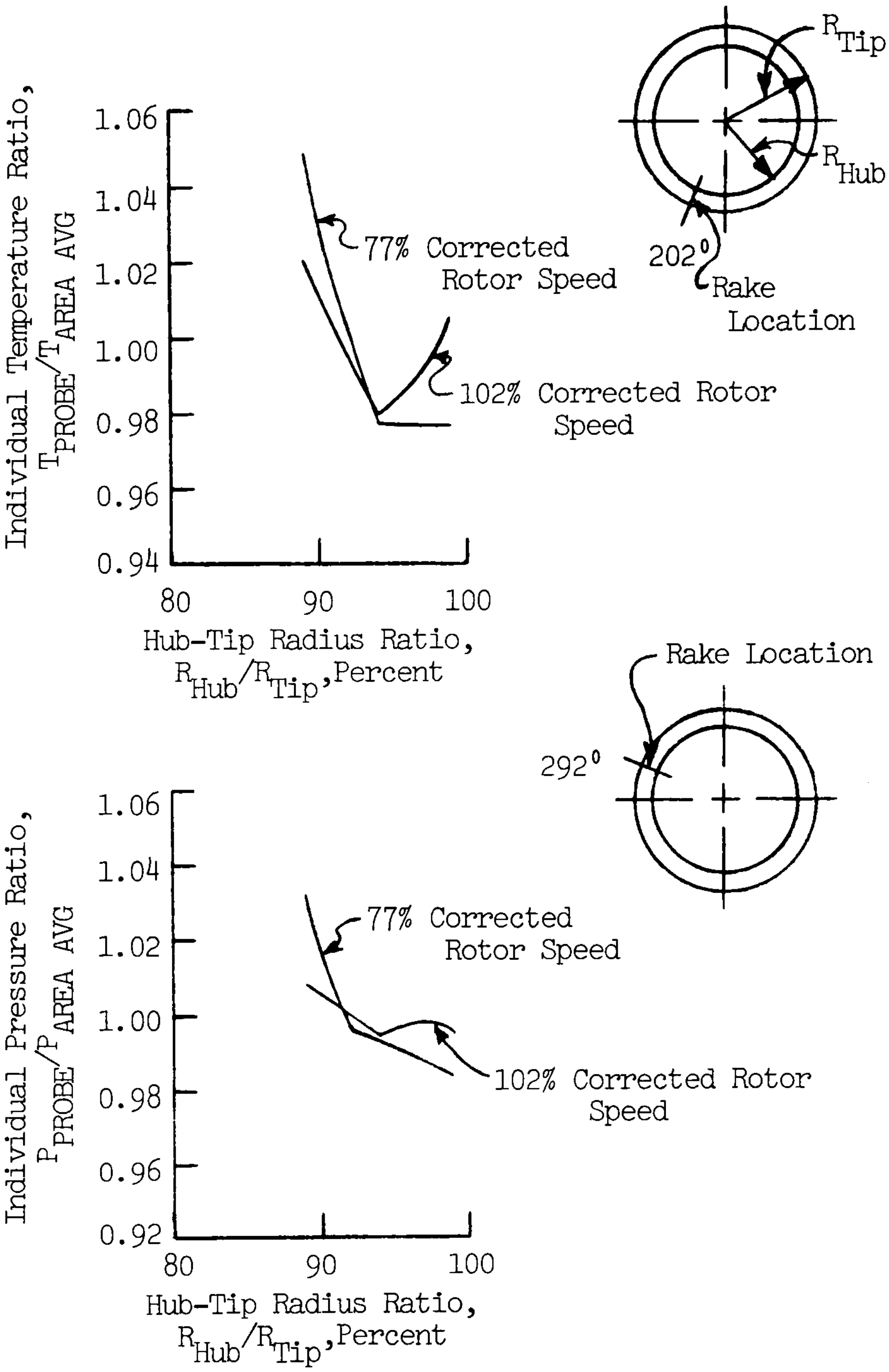
B. High Pressure Compressor Exit

Figure 7. (Continued)



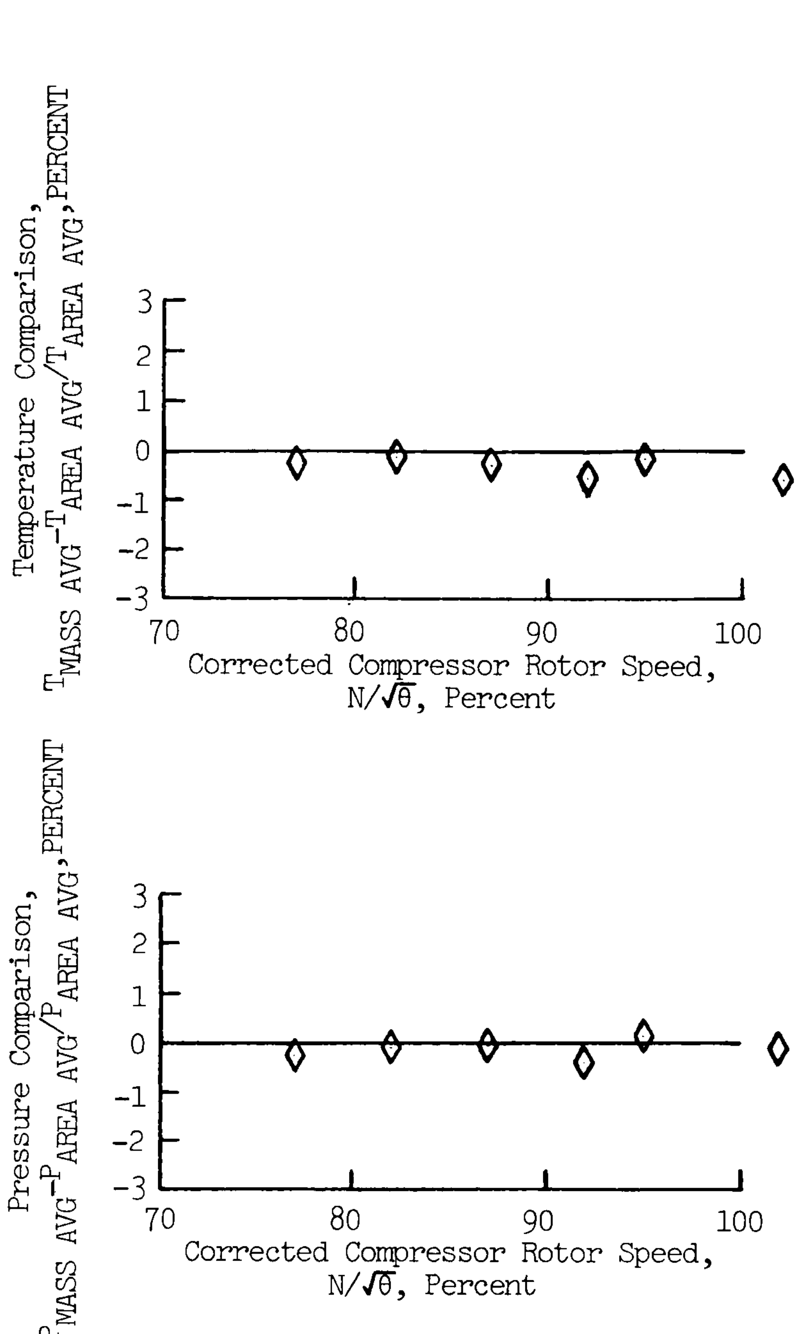
A. High Pressure Compressor Inlet

Figure 8. Effect of corrected rotor speed on compressor radial distortion.



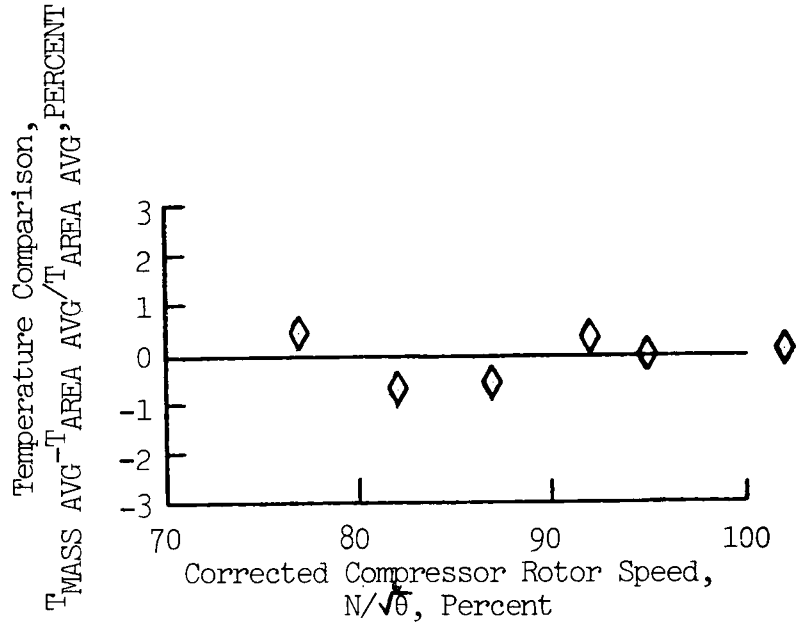
B. High Pressure Compressor Exit

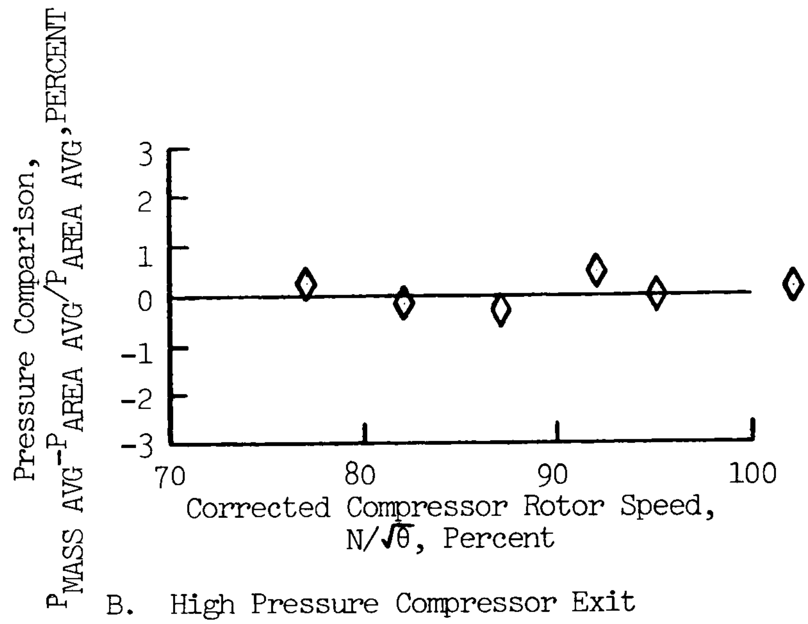
Figure 8. (Continued)



A. High Pressure Compressor Inlet

Figure 9. Comparison of flow averaging techniques.





B. High Pressure Compressor Exit

Figure 9. (Continued)

APPENDIX B

TABLES

TABLE 1

ENGINE SYSTEM FLOWPATH AVERAGING PRACTICES FOR COMPRESSORS

Organization	Performance Parameters	Inlet Parameters			Outlet Parameters		
		Measured	Averaged		Measured	Averaged	
Arnold Engineering Development Center	Efficiency	P	P_{AV}	Area AVG	P	P_{AV}	Area AVG
	Pressure Ratio	T	T_{AV}		T	T_{AV}	
	Temperature Ratio	PS (wall)	PS_{AV}	Numeric AVG	PS (wall)	PS_{AV}	Numeric AVG
		Airflow					
Centre d'Essais Des Propulseurs	Efficiency	P	P_{AV}	Area AVG	P	P_{AV}	Area AVG
	Pressure Ratio	T	T_{AV}	Enthalpy Mass AVG	T	T_{AV}	Enthalpy Mass AVG
		Airflow					
National Gas Turbine Establishment	Efficiency	P	P_{AV}	Area AVG	P	P_{AV}	Area AVG
	Pressure Ratio	T	T_{AV}	Numeric AVG	T	T_{AV}	Numeric AVG
	Temperature Ratio	PS (wall)	PS_{AV}		PS (wall)	PS_{AV}	
		Airflow					
National Aeronautics and Space Administration (Lewis Research Center)	Efficiency	P	P_{AV}	Area AVG	P	P_{AV}	Area AVG
		T	T_{AV}		T	T_{AV}	
		Airflow					

TABLE 2

ENGINE SYSTEM FLOWPATH AVERAGING PRACTICES FOR COMBUSTORS

Organization	Performance Parameters	Inlet Parameters		Outlet Parameters	
		Measured	Averaged	Measured	Averaged
Arnold Engineering Development Center	Efficiency	P	P_{AV} } Area AVG		P_{AV} (1)
		T	T_{AV} } Area AVG		H_{AV} (2)
	Burner Loss	PS (wall)	PS_{AV} } Numeric AVG		T_{AV} } $f(H_{AV}, P_{AV}, FAR)$ (3)
		Airflow (4)			
		Fuel Flow			

(1) Burner pressure is estimated using a numeric average of combustor chamber static pressure and estimated burner Mach number.

(2) Burner enthalpy is estimated using compressor mass flow minus turbine cooling and leakage, compressor discharge enthalpy, measured fuel flow and combustion efficiency.

(3) Temperature obtained from thermodynamic tables for specified enthalpy, pressure and fuel-air ratio.

(4) Core flow is calculated using a high-pressure turbine flow function or from a compressor-turbine work balance.

TABLE 2 (CONTINUED)

Organization	Performance Parameters	Inlet Parameters		Outlet Parameters	
		Measured	Averaged	Measured	Averaged
National Gas Turbine Establishment	Outlet Gas Temperature	P	P_{AV} } Area AVG		P_{AV} (5)
		PS (wall)			
		Airflow(4)	PS_{AV} } Numeric AVG		T_{AV} (5)
		Fuel Flow			
National Aeronautics and Space Administration (Lewis Research Center)	Burner Loss	P	P_{AV} } Area AVG		T_{AV} (3)
		T	T_{AV} } Area AVG		
		Airflow(4)			
		Fuel Flow			

(3) Temperature obtained from thermodynamic tables for specified enthalpy, pressure and fuel-air ratio.

(4) Core flow is calculated using a high-pressure turbine flow function or from a compressor-turbine work balance.

(5) Calculated using airflow, fuel flow and combustion efficiency.

TABLE 3

ENGINE SYSTEM FLOWPATH AVERAGING PRACTICES FOR TURBINES

Organization	Performance Parameters		Inlet Parameters Measured	Inlet Parameters Averaged	Outlet Parameters Measured	Outlet Parameters Averaged
Arnold Engineering Development Center	Efficiency	H		P_{AV}(1)	T	T_{AV} } Area AVG
				H_{AV}(2)	P	P_{AV} } Area AVG
	Pressure Ratio	P		T_{AV}} $f(H_{AV}, P_{AV}, FAR)$(3)	PS (wall)	PS_{AV}} Numeric AVG
	Temperature Ratio	L	T	T_{AV} } Area AVG	T	T_{AV} } Area AVG
			P	P_{AV} } Area AVG	P	P_{AV} } Area AVG
		P	PS (wall)	PS_{AV}} Numeric AVG		

(1) Burner pressure is estimated using a numeric average of combustor chamber static pressure and estimated burner Mach number.

(2) Burner enthalpy is estimated using compressor mass flow minus turbine cooling and leakage, compressor discharge enthalpy, measured fuel flow and combustion efficiency.

(3) Temperature obtained from thermodynamic tables for specified enthalpy, pressure and fuel-air ratio.

TABLE 3 (CONTINUED)

Organization	Performance Parameters	Inlet Parameters Measured	Inlet Parameters Averaged	Outlet Parameters Measured	Outlet Parameters Averaged
Centre d'Essais Des Propulseurs	Efficiency	P	P_{AV} } Area AVG	P	P_{AV} } Area AVG
	Pressure Ratio	T	T_{AV} } Enthalpy Mass AVG	T	T_{AV} } Enthalpy Mass AVG
	Temperature Ratio				
National Gas Turbine Establishment	Efficiency	HP	$P_{AV}^{(4)}$	P	P_{AV} } Area AVG
	Pressure Ratio		$T_{AV}^{(4)}$	T	T_{AV} } Numeric AVG
	Temperature Ratio			PS (wall)	PS_{AV} } Numeric AVG
		LP	P_{AV} } Area AVG		
		P	T_{AV} } Numeric AVG	P	P_{AV} } Area AVG
		T	PS_{AV} } Numeric AVG	T	T_{AV} } Numeric AVG
		PS (wall)		PS (wall)	PS_{AV} } Numeric AVG

(4) Calculated using airflow, fuel flow and combustion efficiency.

TABLE 3 (CONTINUED)

Organization	Performance Parameters	Inlet Parameter		Outlet Parameters	
		Measured	Averaged	Measured	Averaged
National Aeronautics and Space Administration (Lewis Research Center)	Efficiency	P T	P_{AV}, T_{AV} } Area AVG	P T	P_{AV}, T_{AV} } Area AVG

TABLE 4

ENGINE SYSTEM FLOWPATH AVERAGING PRACTICES FOR NOZZLES

Organization	Performance Parameters	Inlet Parameters Measured	Inlet Parameters Averaged	Outlet Parameters Measured	Outlet Parameters Averaged
Arnold Engineering Development Center	Flow Coefficient	P (no reheat)	P_{AV}} Area AVG	PS (atmos) PS}	Numeric AVG
	Thrust Coefficient		$T_{AV}^{(1,2,3)}$	Thrust	
National Gas Turbine Establishment	Flow Coefficient	PS (wall)	PS_{AV}}Numeric AVG	PS (atmos) PS}	Numeric AVG
	Thrust Coefficient			Thrust	

(1) $T_{AV} = (VAE)^2/(2\gamma\, R/1-\gamma)\left\{1 - \frac{P_{AV}(\text{reheat})}{PS_{AV}}^{\frac{\gamma-1}{\gamma}}\right\}$, then calculate thrust based on T_{AV}; continue interaction until set tolerance not exceeded.

(2) Augmenter energy balance: (WAE) (HAE) = (WAI) (HAE) + WFA { HFA + (η_A) (FHV) } .

(3) Temperature obtained from thermodynamic tables for specified enthalpy, pressure and fuel-air ratio.

TABLE 5

ENGINE SYSTEM FLOWPATH AVERAGING PRACTICES FOR AUGMENTERS

Organization	Performance Parameters		Inlet Parameters Measured		Inlet Parameters Averaged	Outlet Parameters Measured	Outlet Parameters Averaged
Arnold Engineering Development Center	Efficiency	CORE	P	P_{AV}	Area AVG	P(no reheat)	P_{AV}(no reheat) Area AVG
			T	T_{AV}			P_{AV}(reheat)$^{(1)}$
		BY-PASS	P	P_{AV}	Area AVG		$T_{AV}^{(2,3,4)}$
	Burner Loss		T	T_{AV}			

(1) $P_{AV}(\text{reheat}) = P_{AV}(\text{reheat}) \left[\left\{ \frac{P_{AV(\text{reheat})}}{P_{AV}(\text{no reheat})} \right\}_{CALC} - \Delta P(\text{drag}) \right]$, where $\left\{ \frac{P(\text{reheat})}{P(\text{no reheat})} \right\}_{CALC}$ is evaluated using Rayleigh line heat addition.

(2) $T_{AV} = (VAE)^2/(2\gamma\ R/1-\gamma) \left\{ 1 - \frac{P_{AV}(\text{reheat})}{PS_{AV}} \ \frac{\gamma-1}{\gamma} \right\}$, then calculate thrust based on T_{AV}; continue interaction until set tolerance not exceeded.

(3) Augmenter energy balance: (WAE) (HAE) = (WAI) (HAE) + WFA {HFA + (η_A) (FHV)}.

(4) Temperature obtained from thermodynamic tables for specified enthalpy, pressure and fuel-air ratio.

TABLE 5 (CONTINUED)

Organization	Performance Parameters		Inlet Parameters Measured	Inlet Parameters Averaged		Outlet Parameters Measured	Outlet Parameters Averaged
		MIXED		P_{AV}			
		MIXED		H_{AV} (5)			
		MIXED	Fuel Flow	$T_{AV} f(P_{AV}, H_{AV}, FAR)$ (4)			
National Gas Turbine Establishment	Efficiency	CORE	P	P_{AV}	Area AVG	P	P_{AV} (6)
		CORE	T	T_{AV}	Numeric AVG		
		CORE	PS (wall)	PS_{AV}	Numeric AVG		
		CORE	Fuel Flow			T	T_{AV} (7,8)

(4) Temperature obtained from thermodynamic tables for specified enthalpy, pressure and fuel-air ratio.

(5) $H_{AV} = \{(WAC)\ (HC) + (WAD)\ (HD)\}/WAT$.

(6) Calculated using static pressure and mass flow.

(7) Calculated using thrust and mass flow.

(8) Calculated using total pressure flow area and mass flow.

TABLE 5 (CONTINUED)

Organization	Performance Parameters	Inlet Parameters: Measured	Inlet Parameters: Averaged		Outlet Parameters: Measured	Outlet Parameters: Averaged	
National Gas Turbine Establishment	Outlet Gas Temperature	BYPASS: P	P_{AV}	Area AVG	PS (wall)	PS_{AV}	Numeric AVG
		BYPASS: T	T_{AV}	Numeric AVG			
		BYPASS / MIXED: PS (wall)	PS_{AV}	Numeric AVG			
		MIXED: P	P_{AV} (9)				
		MIXED: T	T_{AV} (9)				
National Aeronautics and Space Administration (Lewis Research Center)	Efficiency	Airflow			P	T_{AV} (10)	
		Fuel Flow	T_{AV}	Area AVG			
		T			Thrust	P_{AV}	Area AVG

(9) Calculated using airflow, fuel flow and combustion efficiency.

(10) T_{AV} is calculated using measured thrust, exit total pressure, measured airflow and measured fuel flow.

TABLE 6

EFFECT ON ENGINE PERFORMANCE OF A ONE PERCENT VARIATION OF TOTAL PRESSURE AND TEMPERATURE

	Engine Inlet		HPC Inlet		HPC Exit	
	P	T	P	T	P	T
	±1%	±1%	±1%	±1%	±1%	±1%
Engine Net Thrust Change*	±0.78	±0.10	±0.50	±0.04	±0.04	±0.14
Specific Fuel Consumption Change**	±0.79	±0.10	±0.50	±0.04	±0.27	±0.88

$$* \quad \frac{\left(\frac{FN}{W_{TOTAL}}\right)_{DELTA} - \left(\frac{FN}{W_{TOTAL}}\right)_{NOMINAL}}{\left(\frac{FN}{W_{TOTAL}}\right)_{NOMINAL}} \times 100$$

$$** \quad \frac{SFC_{DELTA} - SFC_{NOMINAL}}{SFC_{NOMINAL}} \times 100$$

TABLE 7

ESTIMATED MEASUREMENT UNCERTAINTY

Parameter Designation	Precision Index	Bias	Uncertainty Area Weighted	Uncertainty Mass Weighted
High-Pressure Compressor Inlet Temperature	$\pm 0.6^{o}R$	$\pm 0.9^{o}R$	$\pm 0.5^{o}R$	$\pm 2.7^{o}R$
High-Pressure Compressor Exit Temperature	$\pm 0.6^{o}R$	$\pm$0.25 percent of reading	$\pm 0.8^{o}R$	$\pm 2.5^{o}R$
High-Pressure Compressor Inlet Pressure	$\pm$0.15 percent of reading	$\pm$0.2 percent of reading	$\pm$0.02 psia	$\pm$0.14 psia
High-Pressure Compressor Exit Pressure	$\pm$0.15 percent of reading	$\pm$0.2 percent of reading	$\pm$0.12 psia	$\pm$1.14 psia

APPENDIX C

DERIVATIONS

1. Derivation of expression for the area weighted method,

$$\phi_{AREA\ AVG} = \frac{\Sigma\ \phi_i\ A_i}{A_{TOTAL}} . \qquad [1]$$

Using total pressure as an example,

$$P_{AREA\ AVG} = \frac{\Sigma\ P_i\ A_i}{A_{TOTAL}}$$

$$A_{TOTAL} = nA_i \qquad [2]$$

$$P_{AREA\ AVG} = \frac{A_i\ \Sigma\ P_i}{nA_i}$$

$$P_{AREA\ AVG} = \frac{\Sigma\ P_i}{n}$$

2. Derivation of expression for the mass weighted method,

$$\phi_{MASS\ AVG} = \frac{\Sigma\ \phi_i\ W_i}{W_{TOTAL_{meas}}} , \qquad [3]$$

where

$$W_i = \frac{\dot{m}_i\ PS_i\ A_i}{\sqrt{T_i}} . \qquad [4]$$

The individual mass flow function, $\dot{m}_i$, is based on the individual total pressure and static pressure ratio. Using total pressure as an example,

$$P_{MASS\ AVG} = \frac{\Sigma\ P_i\ W_i}{W_{TOTAL_{meas}}} \qquad [5]$$

The assumption is made that $PS_i = PS_{AVG}$.

$$P_{MASS\ AVG} = \frac{\Sigma\ P_i\ \dfrac{\dot{m}_i\ PS_i\ A_i}{\sqrt{T_i}}}{W_{TOTAL_{meas}}} \qquad [6]$$

In order to determine the individual mass flow, the static pressure must be known. In instances where the static pressure is not measured, a value for static pressure is assumed and individual mass flows are calculated. In order to determine if the proper value of static pressure has been assumed, the mass flow function for the total bulk mass flow is used,

$$\dot{m} = \frac{W_{TOTAL_{meas}} \sqrt{T_{MASS\ AVG}}}{PS_{AVG} \quad A_{TOTAL}}, \qquad [7]$$

where $\dot{m}$ is based on the mass averaged total pressure and static pressure ratio. When the calculated total mass flow equals the measured total bulk mass flow, the proper static pressure, and hence, the total mass weighted pressure and total mass weighted temperature, is determined.

3. Derivation of basic performance equations for a turbofan engine,

$$FN = \left[(WAC+WAD) + WF\right]VNE-(WAC+WAD)V_o + A_9(PS_9-PS_o) \qquad [8]$$

Dividing by total mass flow, multiplying the first term by $\frac{V_o}{WAC}\Big/\frac{V_o}{WAC}$, using the Mach number definition in the first and second terms, and expanding the third term results in:

$$\frac{FN}{WAC+WAD} = a_o \left[1 + \frac{FAR}{1 + BPR}\right] M_o \frac{VNE}{V_o} - a_o M_o$$

$$+ \frac{A_9\ PS_9}{WAC+WAD}\left(1 - \frac{PS_o}{P_2}\ \frac{P_2}{P_{25}}\ \frac{P_{25}}{P_5}\ \frac{P_5}{P_7}\ \frac{P_7}{P_9}\ \frac{P_9}{PS_9}\right). \qquad [9]$$

Ratio of Mach numbers

$$\frac{M_o^{\ 2}}{M_9^{\ 2}} = \frac{V_o^{\ 2}\ \gamma_t\ R\ TS_9}{VNE^2 \gamma_c\ R\ TS_o}. \qquad [10]$$

Cancellation of like terms and rearrangement results in:

$$M_o \left(\frac{VNE}{V_o}\right) = M_9 \left[\frac{\gamma_t}{\gamma_c}\ \frac{TS_9}{TS_o}\right]^{\frac{1}{2}} . \qquad [11]$$

Temperature ratio

$$\frac{T_7}{T_5} = \frac{T_9}{T_5} = \frac{T_9}{TS_9}\ \frac{TS_9}{TS_o}\ \frac{TS_o}{T_5}. \qquad [12]$$

Using the Mach number definition and rearrangement results in:

$$\frac{TS_9}{TS_o} = \frac{T_7}{T_5}\,\frac{T_5}{TS_o}\,\frac{1}{1+\frac{\gamma_t-1}{2}M_9^2}\ . \qquad [13]$$

Mixing process (constant area duct):

$$\frac{P_{25}}{PS_{25}} = \left[1 + \frac{\gamma_c-1}{2}M_{25}^2\right]^{\frac{\gamma_c}{\gamma_c-1}} \qquad [14]$$

$$\frac{P_5}{PS_5} = \left[1 + \frac{\gamma_t-1}{2}M_5^2\right]^{\frac{\gamma_t}{\gamma_t-1}}\ . \qquad [15]$$

Since the static pressure at the primary and secondary streams are equal, $PS_{25} = PS_5$,

$$\frac{P_{25}}{\left[1+\frac{\gamma_c-1}{2}M_{25}^2\right]^{\frac{\gamma_c}{\gamma_c-1}}} = \frac{P_5}{\left[1+\frac{\gamma_t-1}{2}M_5^2\right]^{\frac{\gamma_t}{\gamma_t-1}}}\ . \qquad [16]$$

Rearrangement results in:

$$M_{25} = \left\{\frac{2}{\gamma_c-1}\left[\left\{\frac{P_{25}}{P_5}\left[1+\frac{\gamma_t-1}{2}M_5^2\right]^{\frac{\gamma_t}{\gamma_t-1}}\right\}^{\frac{\gamma_c-1}{\gamma_c}} - 1\right]\right\}^{\frac{1}{2}}\ . \qquad [17]$$

Enthalpy balance across the mixing duct,

$$(WAC+WF)HC_5 + WAD\ HD = (WAC+WAD+WF)HAE. \qquad [18]$$

Using the total enthalpy definition, dividing by WAC CP_C T_5, and rearrangement results in:

$$\frac{T_7}{T_5} = \frac{(1 + FAR)\dfrac{CP_t}{CP_c} + BPR\ \dfrac{T_{25}}{T_5}}{\dfrac{CP_t}{CP_c}(1 + BPR + FAR)}. \qquad [19]$$

Mass conservation across the mixing duct using the continuity equation in the form

$$PA = W(T)^{\frac{1}{2}}\,\frac{1}{M}\left(\frac{R}{\gamma}\right)^{\frac{1}{2}}\left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}},$$

and applying it across the mixing duct,

$$\frac{P_7A_7}{P_5A_5} = \frac{(WAC+WAD+WF)}{(WAC+WF)}\left(\frac{T_7}{T_5}\right)^{\frac{1}{2}}\frac{M_5}{M_7}\,\frac{\left(\dfrac{R}{\gamma_t}\right)^{\frac{1}{2}}}{\left(\dfrac{R}{\gamma_t}\right)^{\frac{1}{2}}}\;\frac{\left[1 + \dfrac{\gamma_t-1}{2}M_7^2\right]^{\frac{\gamma_t+1}{2(\gamma_t-1)}}}{\left[1 + \dfrac{\gamma_t-1}{2}M_5^2\right]^{\frac{\gamma_t+1}{2(\gamma_t-1)}}}. \qquad [20]$$

Cancelation of like terms and application of previously developed equation results in:

$$\frac{P_7}{P_5} = \frac{(1 + BPR + FAR)}{(1 + FAR)} \left[\frac{\frac{CP_t}{CP_c} + BPR \frac{T_{25}}{T_5}}{\frac{CP_t}{CP_c} (1 + BPR)} \right]^{\frac{1}{2}} \frac{M_5}{M_7} \frac{A_5}{A_7}$$

$$\left[\frac{1 + \frac{\gamma_t - 1}{2} M_7^2}{1 + \frac{\gamma_t - 1}{2} M_5^2} \right]^{\frac{\gamma_t + 1}{2(\gamma_t - 1)}} . \qquad [21]$$

Momentum balance across the mixing duct,

$$(WAC+WF)VCE+PS_5A_5 + WAD\ VDE + PS_{25}A_{25} = WAE\ VAE$$

$$+ PS_7A_7 . \qquad [22]$$

Using the perfect gas equation of state, the Mach number definition, multiplying by $\frac{\gamma_t}{\gamma_t}$, and rearrangement results in:

$$PS_7 = \frac{PS_5A_5(\gamma_t M_5^2 + 1) + PS_{25}A_{25}(\gamma_c M_{25}^2 + 1)}{A_7(\gamma_t M_7^2 + 1)} . \qquad [23]$$

Combining the previously developed equations for $\frac{P_7}{P_5}$, PS_7, and the isentropic pressure ratio,

$$\frac{P}{PS} = \left[1 + \frac{\gamma - 1}{2} M^2 \right]^{\frac{\gamma}{\gamma - 1}} \qquad [24]$$

results in:

$$\frac{\left[1 + \frac{\gamma_t - 1}{2} M_7^2\right]^{\frac{1}{2}} M_7}{(\gamma_t M_7^2 + 1)} = (1 + BPR + FAR)$$

$$\frac{\left[\frac{\frac{CP_t}{CP_c}(1+BPR)\frac{T_{25}}{T_5}}{\frac{CP_t}{CP_c}(1 + BPR)}\right]^{\frac{1}{2}} \left[1 + \frac{\gamma_t - 1}{2} M_5^2\right]^{\frac{1}{2}} M_5 \; A_5}{(1 + FAR) \left[A_5(\gamma_t M_5^2 + 1) + A_{25}(\gamma_c M_{25}^2 + 1)\right]} , \qquad [25]$$

which can be solved iteratively for M_7.

Enthalpy balance across the main burner,

$$\text{WAC } HC_3 + \text{ETAB WF FHV} = (\text{WAC+WF}) \; HC_4 . \qquad [26]$$

Using the total enthalpy definition, dividing by WAC, CP_c, TS_o, and rearrangement results in:

$$FAR = \frac{\left[\left(\frac{CP_t}{CP_c} \; \frac{T_4}{TS_o}\right) - \left(\frac{T_2}{TS_o} \; \frac{T_3}{T_{25}} \; \frac{T_{25}}{T_2}\right)\right]}{\left[\left(\frac{ETAB \; FHV}{CP_c \; TS_o}\right) - \left(\frac{CP_t}{CP_c} \; \frac{T_4}{TS_o}\right)\right]} . \qquad [27]$$

Mass flow at nozzle exit,

$$WAE = WAC + WAD + WF = \rho_9 A_9 \, VNE. \qquad [28]$$

Multiplying by $\frac{M_o^2}{V_o \, WAC} \Big/ \frac{M_o^2}{V_o \, WAC}$, using the perfect gas equation of state, the Mach number definition, and rearrangement results in:

$$\frac{A_9 \, PS_9}{WAC+WAD} = \left[1 + \frac{FAR}{(1 + BPR)}\right] \frac{V_o}{M_o} \frac{1}{\gamma_c} \frac{1}{M_o \frac{VNE}{V_o}} \frac{TS_9}{TS_o} . \qquad [29]$$

Combining the previously developed equations for $M_o \left(\frac{VNE}{V_o}\right)$, $\frac{T_7}{T_5}$, and $\frac{A_9 PS_9}{WAC+WAD}$ results in:

$$\frac{A_9 PS_9}{WAC+WAD} = \left[1 + \frac{FAR}{(1 + BPR)}\right] \frac{a_o}{\gamma_c} \left\{ \frac{\left[(1 + FAR) \frac{CP_t}{CP_c} + BPR\right.}{\left[\frac{CP_t}{CP_c} (1 + BPR + FAR)\right]} \right.$$

$$\left. \frac{\left.\frac{T_{25}}{T_5}\right] \frac{T_5}{TS_o}}{\left[1 + \frac{\gamma_t - 1}{2} M_9^2\right]} \right\}^{\frac{1}{2}} \frac{1}{M_9 \left(\frac{\gamma_t}{\gamma_c}\right)^{\frac{1}{2}}} . \qquad [30]$$

Power balance,

$$WAC \, CP_c (T_{25} - T_2) + WAD \, CP_c (T_{25} - T_2) + WAC \, CP_c \, (T_3 - T_{25})$$

$$= ETAM(WAC+WAD+WF) \, CP_t \, (T_4 - T_5). \qquad [31]$$

Dividing by WAC, CP_c, TS_o, and rearrangement results in:

$$\frac{T_5}{T_4} = 1 - \frac{\frac{T_2}{TS_o}\left(\frac{T_3}{T_{25}}\frac{T_{25}}{T_2} - 1\right) + BPR\,\frac{T_2}{TS_o}\left(\frac{T_{25}}{T_2} - 1\right)}{ETAM\,\frac{CP_t}{CP_c}\,(1 + BPR + FAR)\,\frac{T_4}{TS_o}} \,. \qquad [32]$$

Compressor efficiency,

$$ETAC = \frac{T_{3_{isen}} - T_{25}}{T_3 - T_{25}} \,. \qquad [33]$$

Using the isentropic relation,

$$\left(\frac{P_3}{P_{25}}\right)^{\frac{\gamma_c - 1}{\gamma_c}} = \frac{T_{3_{isen}}}{T_{25}} \,, \qquad [34]$$

and rearrangement results in:

$$\frac{T_3}{T_{25}} = \left[\left\{\left(\frac{P_3}{P_{25}}\right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right\}\frac{1}{ETAC}\right] + 1. \qquad [35]$$

Specific fuel consumption,

$$SFC = \frac{WF}{FN} \,. \qquad [36]$$

Multiplying by

$$\frac{1}{(WAC+WAD)WAC} \Bigg/ \frac{1}{(WAC+WAD)WAC} \qquad [37]$$

and rearrangement results in:

$$SFC = \frac{\left[\dfrac{FAR}{(1 + BPR)}\right]}{\left[\dfrac{FN}{(WAC + WAD)}\right]} \; . \qquad [38]$$

VITA

Terry W. Skiles was born in Martin, Kentucky, on March 3, 1948. He attended elementary schools in Hamilton County, Tennessee, and was graduated from Red Bank High School in June, 1966. That fall he entered Indiana Institute of Technology, and in September, 1970, he received a Bachelor of Science degree in Aerospace Engineering.

He accepted a position with ARO, Inc., in July, 1972, and continued study, on a part-time basis, at the University of Tennessee Space Institute. In August, 1981, he was awarded the Master of Science degree in Mechanical Engineering.

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