UNIVERSITY HONORS PROGRAM

SENIOR PROJECT - APPROVAL

Name: Christopher P. Johnson

College: Mechanical, Aerospace, and Biomedical Eng. Department: Aerospace Engineering

Faculty Mentor: Dr. Robert Bond

PROJECT TITLE: Senior Design Project

I have reviewed this completed senior honors thesis with this student and certify that it is a project commensurate with honors level undergraduate research in this field.

Signed: ______________________, Faculty Mentor

Date: 5/3/05

Comments (Optional):

Senior Design Project

Christopher Buckley
Jonathan Ford
Christopher P. Johnson
Christopher R. Johnson
Daniel Pfeffer
Tyson Tucker

The University of Tennessee at Knoxville, Knoxville, TN, 37996

The main purpose of this paper is to present a 30 passenger regional jet that is to double as a commercial aircraft and a civil reserve fleet aircraft for national security and disaster purposes. The jet has to meet two different set of requirements, one for each role. Each role has a different affect on the design of the jet when it comes to take-off and landing distance, range, fuel weight, wing surface area and structural integrity. Several iterations were made for each different aspect of the plane until each condition was satisfied. The following paper analyzes and presents the steps taken to design such a jet, also known as the Diamondcutter.

Nomenclature

A = area[ft^2]
A_n = area of the nth stringer[ft^2]
α = angle of attack in radians
α_n = engine angle of attack in radians
AR = wing aspect ratio
B = wingspan[ft]
C_{Dbfus} = fuselage base-drag coefficient
C_{Dbn} = nacelle base-drag coefficient
C_{dc} = experimental steady state cross-flow drag coefficient of a circular cylinder
C_{Dflap} = coefficient of drag of the flaps
C_{Dint} = coefficient of drag from interference between different components
C_{DLfus} = coefficient of drag due to lift of the fuselage
C_{DLn} = coefficient of drag due to lift of the nacelle
C_{DLp} = coefficient of drag due to lift of the pylon
C_{DLw} = coefficient of drag due to lift of the wing
C_{Dofus} = zero lift coefficient of drag of the fuselage
C_{Don} = zero lift coefficient of drag of the nacelle
C_{Dop} = zero lift coefficient of drag of the pylon
C_{DoVT} = zero lift coefficient of drag of the vertical tail
C_{Dow} = zero lift coefficient of drag of the wing
Cf_{fus} = turbulent flat plate skin-friction coefficient of the fuselage
Cf_n = turbulent flat plate skin-friction coefficient of the nacelle
Cf_p = turbulent flat plate skin-friction coefficient of the pylon
Cf_{VT} = turbulent flat plate skin-friction coefficient of the vertical tail
Cfw = turbulent flat plate friction coefficient
C_{Lw} = wing lift coefficient
C_t = tip chord[ft]
C_r = root chord[ft]
C_x = chord as a function of spanwise location
d_f = maximum fuselage diameter in feet
d_n = maximum nacelle diameter in feet
$\Delta C_{dp\Lambda c/4=0}$ = the two-dimensional profile drag increment due to flaps
ΔC_{Lflap} = the incremental lift coefficient due to the flap

e = span efficiency factor
ϵ_t = wing twist angle in radians
Fa1 = interference factor for the wing and nacelle
Fa2 = interference factor for the fuselage and nacelle
Fw = fuel weight [lbs_f]
H_A = vertical reaction at A[lbs_f]
H_D = vertical reaction at D[lbs_f]
h_n = height of the nth stringer[ft]
I_{req} = required moment of inertia[ft^4]
I_{st} = stringer moment of inertia[ft^4]
I_{yc} = moment of inertia about the y axis[ft^4]
I_{xc} = moment of inertia about the x axis[ft^4]
$\Lambda c/4$ = wing quarter chord sweep angle in radians
Λ_{LE} = leading edge sweep
Λ_{TE} = trailing edge sweep
λ = taper ratio
l = length of wing[ft]
L' = airfoil thickness location parameter
lb = lift distribution for the back wing[lbs_f]
lf = lift distribution for the front wing[lbs_f]
l_f = fuselage length in feet
l_n = nacelle length in feet
K = empirical constant
η = ratio of the drag of a finite cylinder to the drag of an infinite cylinder
ωf = load distribution for the front wing[lbs_f]
ωb = load distribution for the back wing[lbs_f]
σ = stress [lbs]
σ_u = ultimate shear stress [lbs/ft^2]
E = modulus of elasticity
I = moment of inertia [ft^2]
M = internal bending moment[lbs*ft]
M_D = design load [lbs]
P_i = representative force
R_A = horizontal reaction at A[lbs_f]
R_D = horizontal reaction at A[lbs_f]
R_{wf} = wing/fuselage interference factor
R_{LS} = lifting surface correction factor
S = reference area in ft^2
S_{bfus} = fuselage base area in ft^2
S_{bn} = nacelle base area in ft^2
S_{fus} = fuselage base area in ft^2
S_n = maximum frontal area of the nacelle in ft^2
$Splf_{fus}$ = fuselage planform area in ft^2
$Splf_n$ = nacelle planform area in ft^2
S_{wetfus} = wetted area of the fuselage in ft^2
S_{wetn} = wetted area of the nacelle in ft^2
S_{wetp} = wetted area of the pylon in ft^2
S_{wetVT} = wetted area of the vertical tail in ft^2
S_{wetw} = wetted area of the wing in ft^2
S_{wf} = the flapped wing area in ft^2
t = thickness of the wing[ft]
t/c = thickness ratio at the mean geometric chord of the wing
t_{max} = max thickness of the wing [ft]
T_l = (22.5*SIN(Λ_{LE}))*22.5*0.5
T_l = (22.5*SIN(Λ_{LE}))*22.5*0.5

$Tank_w$ = Tank depth[ft]
$Tank_h$ = Tank height[ft]
V = internal shear stress[lbs]
v = induced drag factor due to linear twist
w = zero-lift drag factor due to linear twist
W_w = wing weight [lbs_f]
W_r = root wing weight [lbs_f]
W_x = wing weight as a function of spanwise location [lbs_f]
x_{fe} = spanwise location for free end analysis[ft]
z = spanwise location for energy analysis[ft]
A = area[ft^2]
A_n = area of the nth stringer[ft^2]
B = wingspan[ft]
C_t = tip chord[ft]
C_r = root chord[ft]
C_x = chord as a function of spanwise location
Wwing = wing weight [lbs_f]
Fw = fuel weight [lbs_f]
H_A = vertical reaction at A[lbs_f]
H_D = vertical reaction at D[lbs_f]
h_n = hieghth of the nth stringer[ft]
I_{req} = required moment of inertia[ft^4]
I_{st} = stringer moment of inertia[ft^4]
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Λ_{LE} = leading edge sweep
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ωf = load distribution for the front wing[lbs_f]
ωb = load distribution for the back wing[lbs_f]
σ = stress [lbs]
σ_u = ultimate shear stress [lbs/ft^2]
E = modulus of elasticity
I = moment of inertia [ft^2]
M = internal bending moment[lbs*ft]
M_D = design load [lbs]
P_i = representative force
R_A = horizontal reaction at A[lbs_f]
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t = thickness of the wing[ft]
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V = internal shear stress[lbs]
W_w = wing weight [lbs_f]
W_r = root wing weight [lbs_f]
W_x = wing weight as a function of spanwise location [lbs_f]
x_{fe} = spanwise location for free end analysis[ft]
z = spanwise location for energy analysis[ft]

Requirements

The purpose of this project was to design a 30 passenger regional jet with a nominal take-off and landing requirement of 3000 ft. The airplane was to have a block range of 1500 miles with reserves, and a minimum maximum speed of 400 knots. The mission profile was to include more than one take-off and landing, since outlying airports may not support refueling. Maximum landing weight was to be a minimum of 80% of the maximum take-off weight.

The secondary role for this jet was to serve in the civil reserve fleet, and be available during national security crises. The passenger accommodation would be removed and the aircraft would be outfitted as an air ambulance, a transport for fire fighters to remote locations and an emergency response vehicle for incidence of urban terrorism or natural disaster. For those missions, the aircraft would have to be able to transport half of its civilian payload at a range of 750 miles into make-shift landing zones which are cleared areas of at least 1400 ft without obstacles at the ends. Examples of this landing environment may be highways, a damaged airfield or even a barge.

Basic Dimensions

The following figure shows a layout of the plane drawn in Solid Edge. This figure gives the basic dimensions of the jet that were used throughout the project for various calculations.

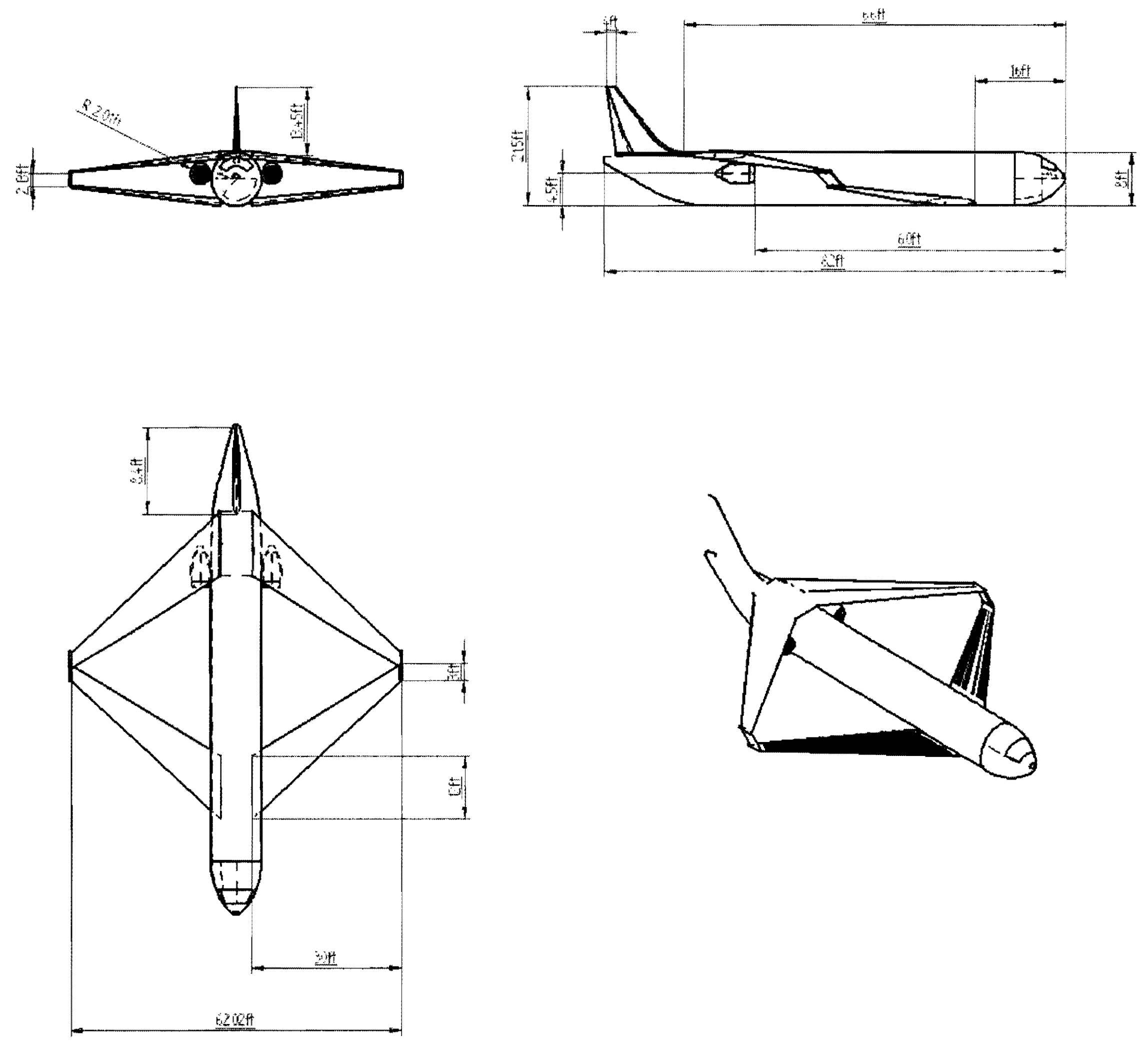

Figure 1 Basic Layout and Dimensions of the Jet

Solid Edge Report

One of the preliminary objectives of the design process was to manifest a schematic for the aircraft itself. A dimensioned 3-view general arrangement drawing would enhance the analysis of the aerodynamics and structure of the jet. To accomplish this task Solid Edge, a three dimensional modeling software, was utilized. Solid Edge provided the platform needed to draw the jet to scale with detail. During the design and analysis process the dimensions and placement each of the parts were constantly under adjustment. In order to compensate for this everything was defined dynamically so that if one aspect of the jet were to change the corresponding parts would adjust to balance.

The opening challenge was to represent the jet as accurately as possible, specifically with respect to the airfoil and internal structure. For the first run of the airfoil sketch the airfoil data sheet was utilized. A mesh was applied to the airfoil schematic so that the points could be pulled off of the drawing. The complete set of points were put into a spreadsheet and then exported to Solid Edge to deliver a partially accurate dimensioning of the airfoil. After continued research the contour data to the airfoil was located. This data was exported from Excel to Solid Edge which provided the precision desired.

The first cut of the wing geometry itself was based on principally research of similar jets. The wing span, chord root, and chord tip used were 60ft, 12 ft, and 3 ft respectively. For the box wing design the top and bottom wings had to have an anhedral and a dihedral respectively. This dimension was defined as a result of the arbitrary distance between wing tips and the fuselage diameter. The first approximation for the wing tip distance was 2 ft. In addition to these geometry specifications the distance between the wings was specified to achieve an indication of what the leading edge and trailing edge sweeps would be.

The fuselage was the next step. For our preliminary design the fuselage diameter was set at 10 ft. This was thought to be a sufficient cross-section to accommodate for the apparatus needed. In addition to the diameter the length was arbitrarily specified and set at 90 ft. After further analysis the fuselage was determined excessive in volume. A 20% reduction in diameter and a 10 ft reduction in length were performed.

The last part to be drawn was the tail.

Finally all of these individual parts had to be assembled to create the final product.

Weight Estimate

The weight of an airplane needs to be known to analyze its performance characteristics. However, knowing the real weight of the airplane can take many years of work to figure out. Therefore, an initial estimate of the weight needs to be solved before the airplane can be analyzed. This process can be done fairly quickly to give a close estimate of the overall weight of the airplane.

An airplane's weight is made up of many different components. The plane by itself takes up the majority of the weight, but the payload, crew, and fuel also take up a good percentage of the weight. The only known initial weights for the Diamond Cutter were its two man crew and thirty passenger payload. Therefore, the empty and fuel weights are needed to get the overall aircraft weight. However, they are both dependent on the total aircraft weight. Thus, an iteration process has to be used to estimate the overall weight.

Weight fractions of empty and fuel weight were used to make calculations less complicated. The total takeoff weight was found using equation (1).

$$W_o = W_{crew} + W_{payload} + \left(\frac{W_f}{W_o}\right)W_o + \left(\frac{W_e}{W_o}\right)W_o \tag{1}$$

Equation (1) was then simplified to solve for Wo, shown in equation (2).

$$W_o = \frac{W_{crew} + W_{payload}}{1-(W_f/W_o)-(W_e/W_o)} \tag{2}$$

The empty weight fraction was found using historical trends. Empty weight fractions vary from around 0.3 to 0.7 depending on the type of aircraft. Our aircraft was assumed to be a jet transport with fixed swept wings. The empty weight fraction was calculated from equation (3).

$$\frac{W_e}{W_o} = AW_o^C K_{vs} \tag{3}$$

The constants A and C were assumed to be 1.02 and -0.06. K_{vs} was assumed to be one since the wings are fixed.

The amount of fuel needed for our airplane was an extremely important aspect of our airplane. The fuel supply consists of three components: the mission fuel, the reserve fuel, and the trapped fuel. Fuel fraction was calculated from the mission profile of the airplane. A six percent increase in the mission fuel was added to compensate for the reserve and trapped fuel.

The mission profile of the airplane being design affects the weight of the aircraft greatly. An airplane made to travel far distances or flying for long periods of time will use more fuel, which means more weight. Also, the weight was going to be affected by the speed the aircraft travels at and how often the aircraft makes a landing and takeoff. The mission profile can be broken down into mission segments to better analyze the weight estimate. The Diamond Cutter was designed for two different missions. The first mission consisted of five mission segments: takeoff, climb, cruise, loiter, and land. The second mission had ten mission segments: takeoff, climb, cruise, loiter, land, and then repeated for the trip back. The two missions can be seen in Figure 2.

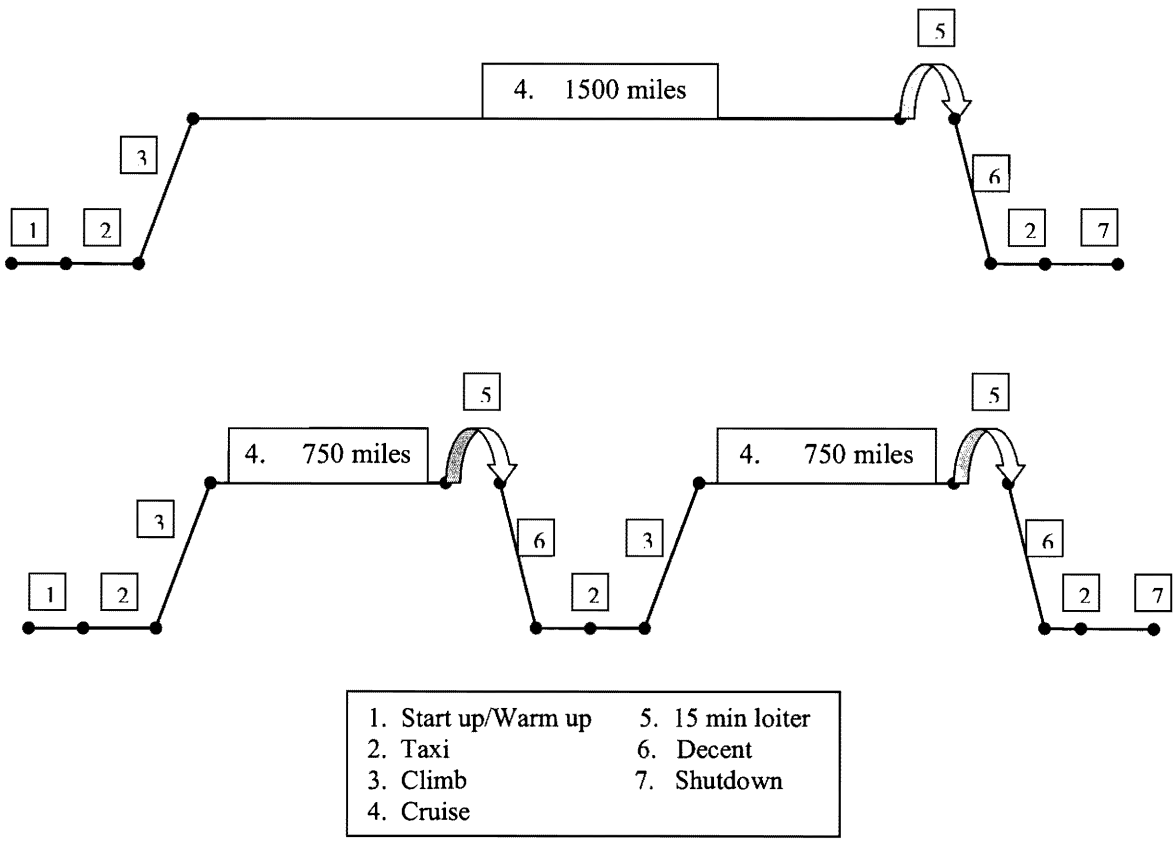

Figure 2 Mission profiles

Each mission was unique in how far and long the airplane travels. The airplane was designed for a cruise range of 1500 miles for mission 1 and 750 miles for mission 2.

Our airplane was modeled after known similar aircraft to get a fairly accurate initial weight estimate. Only the estimated dimensions and flight conditions of the aircraft are know at first. Therefore, several assumptions are made in figuring out the weight. The lift-to-drag ratio, specific fuel consumption, and wetted aspect ratio are all assumed to be close to that of similar aircraft. The accuracy of the weight estimate will be refined as more data from the engine and geometric characteristics are known.

During aircraft operations, our plane constantly loses weight by burning fuel. Each mission segment had an estimated weight fraction that consisted of the fuel weight used by that segment divided by the total takeoff weight. All the weight fractions for each mission were multiplied together to estimate the ratio of the aircraft weight.

The mission segment weight fractions were estimated from previous aircraft or calculated from performance equations. The warm-up, takeoff, and landing weight fractions were estimate from historical data shown in Table 1.

Table 1. Historical mission segment weight fraction.

Mission segment	(W_i/W_{i-1})
Warm-up and takeoff	0.970
Climb	0.985
Landing	0.995

The cruise mission segment was found using the Breguet range equation (4). The weight fraction for the cruise mission segment was found by equation (5).

$$R = \frac{V}{C}\frac{L}{D}\ln\left(\frac{W_{i-1}}{W_i}\right) \tag{4}$$

$$\frac{W_i}{W_{i-1}} = \exp\frac{-RC}{V(L/D)} \tag{5}$$

Our airplane was assumed to be a high-bypass turbofan with a specific fuel consumption of 0.5 hr^{-1} at cruise. The lift-to-drag ratio was assumed to be 13.9 for cruise. This estimation was made from our expected wetted aspect ratio of 1.2 and using historical trends found in Figure 3. Our L/D_{max} was estimated to be 16 from Figure 2. The lift-to-drag ratio for cruise was estimated to be 0.866 times L/D_{max}. The velocity and range differ depending on the mission.

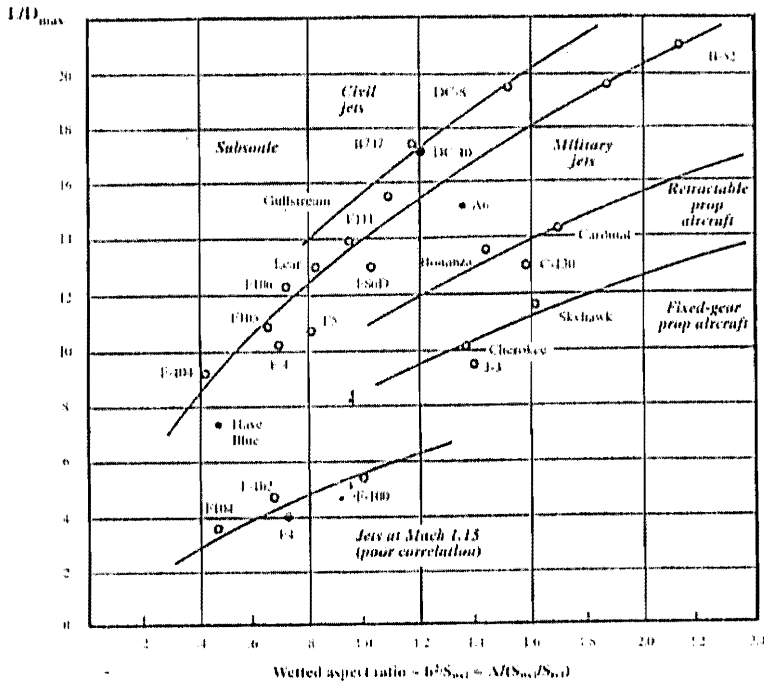

Figure 3 Maximum lift-to-drag trends

The loiter mission segment was found from the endurance equation (6). The weight fraction for loiter mission segment was found by equation (7).

$$E = \frac{L/D}{C}\ln\left(\frac{W_{i-1}}{W_i}\right) \tag{6}$$

$$\frac{W_i}{W_{i-1}} = \exp\frac{-EC}{L/D} \tag{7}$$

The specific fuel consumption at loiter for our plane was assumed to be 0.4 1/hr. The lift-to-drag ratio was assumed to be the same as the L/D_{max}.

Next, the fuel fraction estimation was calculated. This calculation was done with an added 6% more fuel to compensate for the reserved and trapped fuel. Equation (8) shows this calculation with x being the number of mission segments.

$$\frac{W_f}{W_o} = 1.06\left(1 - \frac{W_x}{W_o}\right) \tag{8}$$

Each mission segment was iterated using equation (2) until a total takeoff weight was found. The mission weight fractions for the first iteration done on our airplane can be seen in Figure 4.

Mission 1

1) Warm-up and takeoff	W1/Wo=	0.97
2) Climb	W2/W1=	0.985
3) Cruise	R=	1500
	SFC=	0.5
	V=	400
	L/D=	13.856
	W3/W2=	0.8891
4) Loiter	E=	15
	SFC=	0.4
	L/D=	16
	W4/W3=	0.9938
5) Land	W5/W4=	0.995

Mission 2

1) Takeoff	W6/W5=	0.97
2) Climb	W7/W6=	0.985
3) Cruise	R=	750
	SFC=	0.5
	V=	400
	L/D=	13.856
	W8/W7=	0.9429
4) Loiter	E=	45
	SFC=	0.4
	L/D=	16
	W9/W8=	0.9814
5) Land	W10/W9=	0.995
6) Takeoff	W6/W5=	0.97
7) Climb	W7/W6=	0.985
8) Cruise	R=	750
	SFC=	0.5
	V=	400
	L/D=	16
	W8/W7=	0.9504
9) Loiter	E=	45
	SFC=	0.4
	L/D=	16
	W9/W8=	0.9814
10) Land	W10/W9=	0.995

Figure 4 Weight fractions of each mission

The two different missions were examined and compared. The second mission produced the highest weights. Therefore, it was used in other calculations because the each mission has to be flown and the heaviest weight dominates the other one. The total takeoff weight for the first and second mission was estimated to be 23,582 and 33,691 lb.

Two more iterations were done when more information about our airplane was collected. Some of the main variables in calculating the weight were changed. Therefore, the weight of our airplane changed significantly. Our engine's specific fuel consumption for cruise and loiter was found to be 0.48 and 0.407 hr^{-1}. Then the L/D_{max} changed from 16 to 14.96 in the third iteration process. The comparison between the three iterations was shown in Table 3.

Table 2. Comparison of Weight Estimations

	1st iteration		2nd iteration		3rd iteration	
Weights (lb)	Mission 1	Mission 2	Mission 1	Mission 2	Mission 1	Mission 2
We	14,537	21,183	14,385	20,443	14,295	20183
Wf	5947	12,228	5806	11,514	3691	7740
Wo	26,245	39,171	25,952	37,716	23,582	33,684

Each iteration done produces a smaller weight. A smaller weight was good for making size and performance calculations. However, this was just estimation and was based on historical data, which our airplane was not particularly similar to other aircraft. These estimations did not take into affect our planes joint wing design. Therefore, a better estimation needs to be made to get more accurate results. A weight buildup was done to get a more accurate result.

Weight Buildup

The weight of the different components of our airplane was calculated to give a more accurate weight estimate. These calculations were done based the geometric characteristics of the airplane. The final weight buildup will add up every single part of the airplane, from the fuselage to a single bolt. However, for our calculations only the weights of the main components of the airplane were calculated to provide accurate results. The equations below show all of the main components of our aircraft and how they were calculated.

$$W_{wing} = 0.0051(W_{dg}N_z)^{0.557}S_w^{0.649}A^{0.5}(t/c)_{root}^{-0.4}(1+\lambda)^{0.1} \times (\cos\Lambda)^{-1.0}S_{csw}^{0.1}$$

$$W_{\substack{horizontal\\tail}} = 0.0379K_{uht}(1+F_w/B_h)^{-0.25}W_{dg}^{0.639}N_z^{0.10}S_{ht}^{0.75}L_t^{-1.0} \times K_y^{0.704}(\cos\Lambda_{ht})^{-1.0}A_h^{0.166}(1+S_e/S_{ht})^{0.1}$$

$$W_{\substack{vertical\\tail}} = 0.0026(1+H_t/H_v)^{0.225}W_{dg}^{0.556}N_z^{0.536}L_t^{-0.5}S_{vt}^{0.5}K_z^{0.875} \times (\cos\Lambda_{vt})^{-1}A_v^{0.35}(t/c)_{root}^{-0.5}$$

$$W_{fuselage} = 0.3280K_{door}K_{Lg}(W_{dg}N_z)^{0.5}L^{0.25}S_f^{0.302}(1+K_{ws})^{0.04}(L/D)^{0.10}$$

$$W_{\substack{main\ landing\\gear}} = 0.0106K_{mp}W_l^{0.888}N_l^{0.25}L_m^{0.4}N_{mw}^{0.321}N_{mss}^{-0.5}V_{stall}^{0.1}$$

$$W_{\substack{nose\ landing\\gear}} = 0.032K_{np}W_l^{0.646}N_l^{0.2}L_n^{0.5}N_{nw}^{0.45}$$

$$W_{\substack{nacelle\\group}} = 0.6724K_{ng}N_{Lt}^{0.10}N_w^{0.294}N_z^{0.119}W_{ec}^{0.611}N_{en}^{0.984}S_n^{0.224}$$

(includes air induction)

$$W_{\substack{engine\\controls}} = 5.0N_{en} + 0.80L_{ec}$$

$$W_{\substack{starter\\(pneumatic)}} = 49.19\left(\frac{N_{en}W_{en}}{1000}\right)^{0.541}$$

$$W_{\substack{fuel\\system}} = 2.405V_t^{0.606}(1+V_i/V_t)^{-1.0}(1+V_p/V_t)N_t^{0.5}$$

$$W_{\substack{flight\\controls}} = 145.9N_f^{0.554}(1+N_m/N_f)^{-1.0}S_{cs}^{0.20}(I_y \times 10^{-6})^{0.07}$$

$$W_{instruments} = 4.509K_rK_{tp}N_c^{0.541}N_{en}(L_f+B_w)^{0.5}$$

$$W_{hydraulics} = 0.2673N_f(L_f+B_w)^{0.937}$$

$$W_{electrical} = 7.291R_{kva}^{0.782}L_a^{0.346}N_{gen}^{0.10}$$

$$W_{avionics} = 1.73W_{uav}^{0.983}$$

$$W_{furnishings} = 0.0577N_c^{0.1}W_c^{0.393}S_f^{0.75}$$

$$W_{\substack{air\\conditioning}} = 62.36N_p^{0.25}(V_{pr}/1000)^{0.604}W_{uav}^{0.10}$$

$$W_{anti\text{-}ice} = 0.002W_{dg}$$

$$W_{\substack{handling\\gear}} = 3.0 \times 10^{-4}W_{dg}$$

W_{wing}=	12540.53	lb	$W_{instruments}$=	159.62	lb
$W_{vertical\ tail}$=	892.97	lb	$W_{hydraulics}$=	173.25	lb
$W_{fuselage}$=	5460.66	lb	$W_{electrical}$=	631.00	lb
$W_{main\ LG}$=	1104.01	lb	$W_{avionics}$=	624.99	lb
$W_{nose\ LG}$=	322.07	lb	$W_{furnishings}$=	115.56	lb
$W_{narcelle\ group}$=	55.41	lb	$W_{air\ conditioning}$=	482.43	lb
$W_{engine\ control}$=	50.00	lb	$W_{anti\text{-}ice}$=	76.00	lb
$W_{starter}$=	71.30	lb	$W_{handling\ gear}$=	11.40	lb
$W_{fuel\ system}$=	1134.99	lb	W_{engine}=	993	lb
$W_{flight\ controls}$=	172.21	lb			

All of the components were added together to produce the empty weight, which was calculated to be 25,071 lbs. The fuel weight needed for our plane was estimated to 10,000 lbs. This calculation was done using the fuel weight fraction of 0.2482, which was found previously. The initial fuel weight estimate was calculated to be 9935 lbs. This weight was rounded up to 10,000 to give a more realistic prediction and make other calculations easier. Therefore, once the fuel and empty are know, the total takeoff weight was found to be 40,831 lbs.

Center of Gravity

The center of gravity (c.g.) of the aircraft was found along side of the weight build up. Once the weight of each component was found, it was entered into a matlab code along with its respective distance from the nose. Matlab was then used to iterate to find the center of gravity of the entire aircraft. The final iteration done for the center of gravity along side the weight buildup gave a c.g. of 40.59ft. from the nose of the plane. Figure 5 shows a plot of the weights of the main components of the aircraft and their respective distance from the nose.

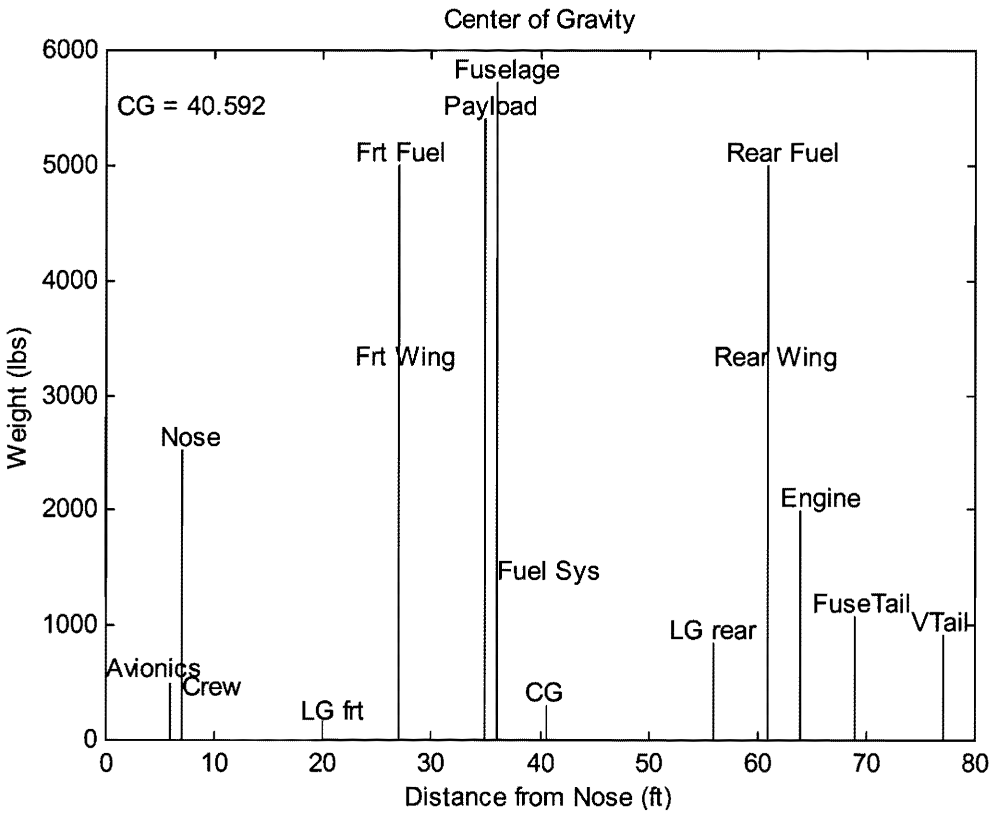

Figure 5 Component Weights for C.G. Estimate

<u>Drag Buildup</u>

The drag of our airplane was estimated to achieve a more accurate assessment of its performance characteristics. Each component of our airplane was analyzed and complied together to obtain our drag estimate. The equations below were used to calculate the drag for each component of our airplane.

$$C_D = C_{Do} + C_{DL}$$

$$C_{Dow} = (R_{wf})(R_{LS})(C_{fw})\{1 + L'(t/c) + 100(t/c)^4\}S_{wetw}/S$$

$$C_{DLw} = (C_{Lw})^2/\pi Ae + 2\pi C_{Lw}\varepsilon_t v + 4\pi 2(\varepsilon_t)^2 w$$

$$C_{Dofus} = R_{wf}C_{ffus}\{1 + 60/(l_f/d_f)^3 + 0.0025(l_f/d_f)\}S_{wetfus}/S + C_{Dbfus}$$

$$C_{DLfus} = 2\alpha^2 S_{bfus}/S + \eta c_{dc}|\alpha|^3 S_{plffus}/S$$

$$C_{DoVT} = (R_{wf})(R_{LS})(C_{fwVT})\{1 + L'(t/c) + 100(t/c)^4\}S_{wetVT}/S$$

$$C_{Don} = R_{wf}C_{fn}\{1 + 60/(l_n/d_n)^3 + 0.0025(l_n/d_n)\}S_{wetn}/S + C_{Dbn}$$

$$C_{DLn} = 2\alpha_n^2 S_{bn}/S + \eta c_{dc}|\alpha_n|^3 S_{plfn}/S$$

$$C_{Dop} = (R_{wf})(R_{LS})(C_{fp})\{1 + L'(t/c) + 100(t/c)^4\}S_{wetp}/S$$

$$C_{DLp} = (C_{Lp})^2/\pi Ae + 2\pi C_{Lp}\varepsilon_t v + 4\pi 2(\varepsilon_t)^2 w$$

$$C_{Dn\,\mathrm{int}} = F_{a1}(\Delta C_{Dn}/C_{Dn})C_{Dn} + F_{a2}\{(C_{Dn})' - 0.05\}(S_n/S)$$

$$C_{Dflap} = (C_{dp\Lambda c/4\,=\,0})(\cos\Lambda c/4)S_{wf}/S + K^2(\Delta C_{Lflap})^2\cos\Lambda c/4 + K\,\mathrm{int}\,\Delta C_{Dprofflap}$$

$$D = 0.5\rho V^2 S C_D$$

Our drag calculations were only done at cruise conditions at a speed of 400 knots. Drag was a contributing factor at other conditions. However, our aircraft will be operating majority at cruise conditions. Therefore, drag held the biggest factor at cruise conditions. The drag coefficient was estimated to be 0.0764. Therefore, the drag of our airplane was estimated to be 7671 lbs at cruise conditions.

Airfoil Selection

In the airfoil selection process, several important factors were considered key. As the aircraft must take off and land in a short runway, a high maximum lift coefficient was critical. This led our design team to the NACA 6-series airfoils whose design promotes higher lift coefficients and encourages laminar flow, which reduces the skin-friction drag over operating conditions. However, the NACA 6-series airfoils do have several drawbacks. They tend to create a high pitching moment, have sudden stall behavior, and with the laminar encouraging design, are susceptible towards skin roughness. After reviewing the various airfoils, the NACA 64_1-212 was chosen due to its high $C_L max$ of 1.6. This attribute may be seen below in Figure 6.a, which plots C_L as a function of angle of attack and C_D as a function of C_L. This airfoil has a location of minimum pressure at .4c, a thickness that is 12% of the chord, and a design lift coefficient of 0.2. The airfoil shape is plotted below in Figure 6.b. This design lift coefficient was especially attractive as our aircraft in cruise conditions of 400 knots (675.12 ft/s) at 30,000 ft will need to fly with a lift coefficient of 0.21. Therefore, the chosen airfoil design will encourage laminar flow at our aircraft's cruise operating conditions.

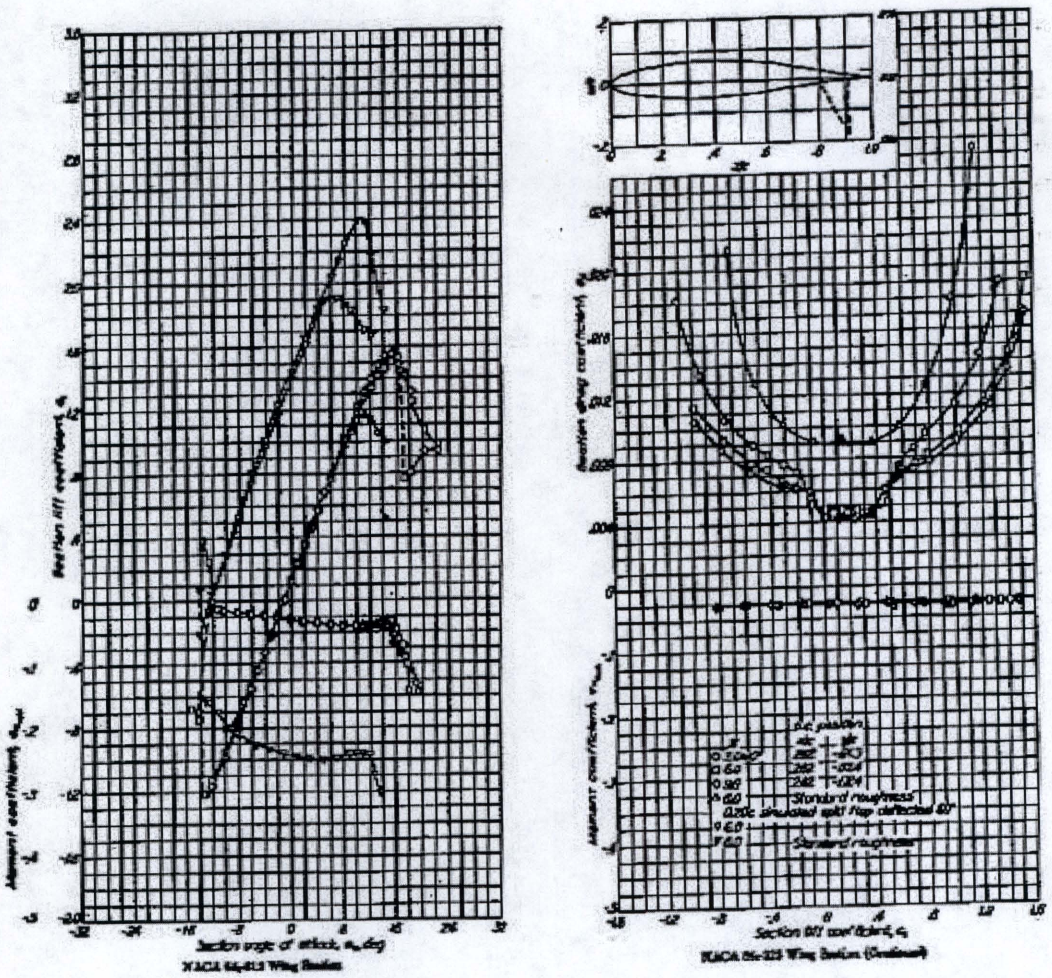

Figure 6.a C_L versus α and C_D versus C_L for the NACA 64_1-212 airfoil

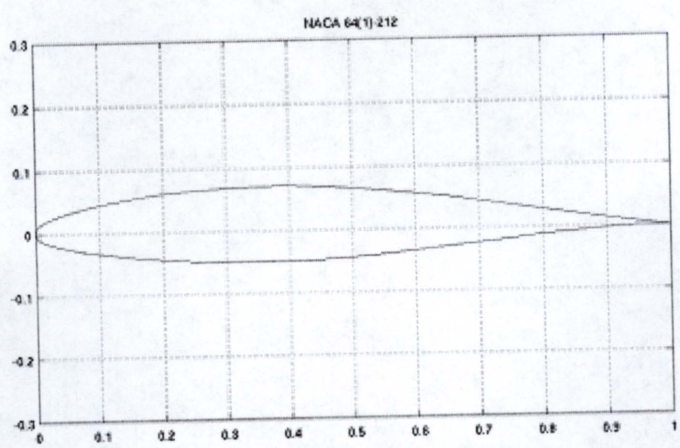

Figure 6.b NACA 64_1-212 airfoil design

Development of Vortex Lattice Code

In order to provide a first pass analysis of joined wing aerodynamics, a vortex lattice code was written using MATLAB to generate lift coefficients and a lift distribution across the wings. Several assumptions were made to simplify the initial code development. These included assuming that the wings were coplanar and had no dihedral. However, in actuality, the wings are joined at the tip with a dihedral and anhedral sufficient to allow the root to attach at the top and bottom of the fuselage. The subsequent code represents each wing as a flat surface with a finite number of discrete points and a horseshoe vortex system imposed on each element to form a panel. The number of panels in the spanwise direction is inputted as a variable while only one panel is used in the chordwise direction. It is assumed that the flow is tangent to the surface at each point and thus no flow separation effects are considered. Analysis of the trailing vortices upon leaving the surface of the wing is conducted by assuming linear theory, in which each trailing vortex extends parallel to the aircraft downstream to infinity. Using this simplified code, a joined wing system was analyzed for lift distribution comprised of two wings with a 60 ft wingspan, 12 ft root, 3 ft tip, and a 40.4° quarter-chord sweep for the front wing and a -40.4° quarter chord sweep for the aft wing. Figure 6 below demonstrates the plotted wing system with 30 spanwise panels.

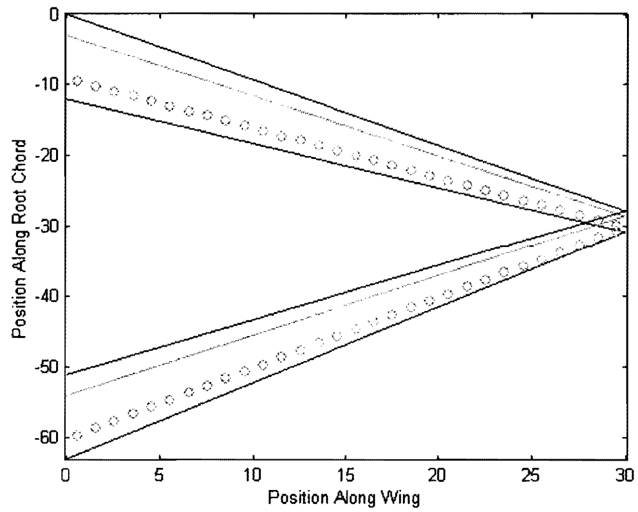

Fig. 7 Wing System Developed by 2-D Vortex Lattice Code

A major drawback of using this two dimensional analysis with this particular wing design is the coplanar overlapping of the wings at the tip. This generated large errors as the effects of the interference from each panel on the other where greatly exaggerated. A lift distribution for the aircraft wing design under cruise conditions of 30,000 ft at an airspeed of 400 knots (675 ft/s) and an angle of attack of 1.75° was then generated using the VLM code. These results are shown below in Figure 7. Further analysis of the wing design using the code yielded a lift curve slop ($C_{L\alpha}$) of 3.283 per radian (0.0573 per degree). A plot of the lift curve versus angle of attack is shown in Figure 8 below.

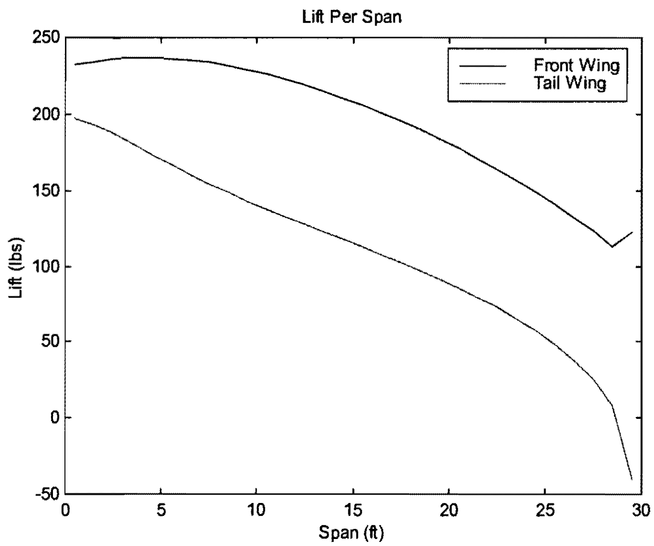

Fig. 8 Lift Distribution along each Wing Developed by 2-D Vortex Lattice Code

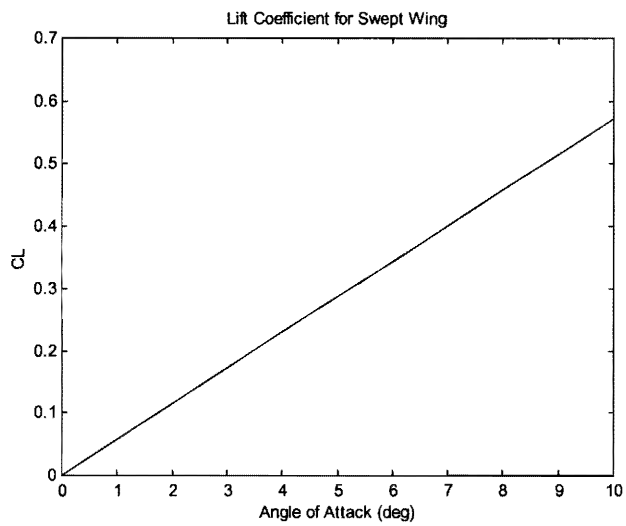

Fig. 9 Variance of Coefficient of Lift with Angle of Attack (α)

As the aircraft design utilized significant three dimensional properties such as highly swept wings with a large dihedral angle, and a spar joining the forward and aft wings' tips, it was concluded that a more in-depth aerodynamic study using a three dimensional analysis tool was critical for an accurate design. A search for methods to write a suitable program led the design team to a three-dimensional vortex lattice code named Tornado, written by Tomas Melin from the Royal Institute of Technology (KTH) for his master's thesis, "A Vortex Lattice MATLAB Implementation for Linear Aerodynamic Wing Applications." This MATLAB code uses a standard vortex lattice method with modifications to allow for a fully three dimensional analysis with trailing edge control surfaces and a trailing edge wake that follows the free-stream airflow. The most significant change is the alteration of the traditional horseshoe vortex into the vortex sling theory which uses a seven segment vortex line. Figure 9 below compares the two differing concepts. Wing designs may be inputted into Tornado by specifying sweep, dihedral, twist, taper, control surfaces, and camber. The program then solves for forces, moments, control derivatives, and aerodynamic coefficients.

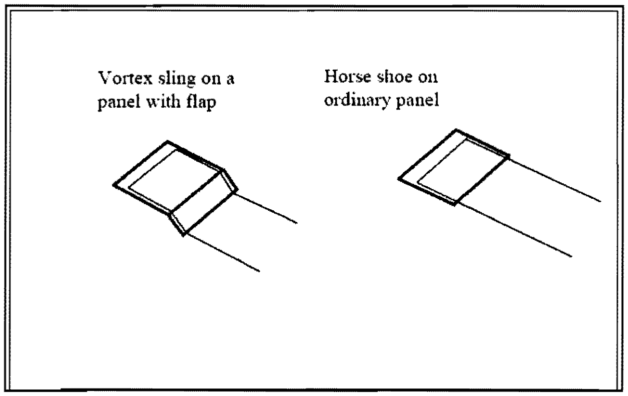

Fig. 10 Comparison of Vortex Sling and Horseshoe Vortex

Overall, the ability of the Tornado MATLAB program to provide accurate three dimensional analysis of the forces, moments, and coefficients with relative ease made it superior to the two dimensional VLM. Several studies were conducted to compare the two methods. Shown in Table 3 is a comparison of the two methods for cruise conditions. (30,000 ft altitude, 400 knots)

Table 3 Comparison of 2-D VLM and Tornado 3-D VLM for Cruise Conditions

	$C_{L\alpha}$	Lift α = 2 deg.
2-D VLM	3.283 per radian	41,860 lbs
Tornado	6.793 per radian	39,100 lbs

The wing design for the aircraft was then inputted into the Tornado program and analyzed for cruise conditions. Below in Figure 10 is a plot showing the three dimensional wing design with a coefficient of pressure change over the wings during cruise conditions. The need for a three dimensional analysis arose from the high anhedral and dihedral and the large offset between the wings that one may notice in the diagram below. In addition, Figure 11 below demonstrates a typical output for the forces, moments, and aerodynamic coefficients acting on the wing under cruise conditions.

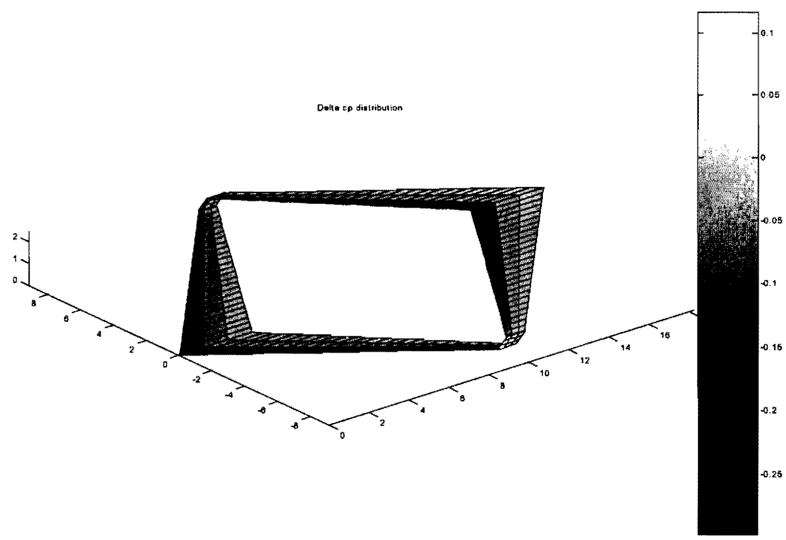

Fig. 11 Tornado Output: Coefficient of Pressure Change, Cruise Conditions

Tornado Computation Results

JID:	trial	Downwash matrix condition:	14.4514
Reference area:	41.8064		
Reference chord:	2.5603	Reference point pos:	0 0 0
Reference span:	18.288		

Net Wind Forces: (N)		Net Body Forces: (N)		Net Body Moments: (Nm)	
Drag:	226.1086	X:	-1278.6295	Roll:	6.3238e-011
Side:	-1.7084e-012	Y:	-1.7084e-012	Pitch:	-213710.7749
Lift:	49270.0025	Z:	49253.9275	Yaw:	-6.3629e-012

CL	0.12208	CZ	0.12204	Cm	-0.20683
CD	0.00056027	CX	-0.0031683	Cn	-8.6212e-019
CY	-4.2332e-018	CC	-4.2332e-018	Cl	8.5682e-018

STATE:

alpha:	1.75	P:	0		
beta:	0	Q:	0	Rudder setting [deg]:	0 0 0
Airspeed:	205.777	R:	0		
Density:	0.456				

Fig. 12 Tornado Data Output for Cruise Conditions

Additionally, the Tornado program solves for the local coefficient of lift (C_L) over the wing. Figures 13a and 13b below show C_L distributions for the first iteration of the front and back wings. Of importance is the non-elliptical shape of the front wing. An elliptical lift distribution will generally produce less induced drag and is a more efficient distribution. After this iteration, twisting was performed to give the front wing a more elliptical lift distribution. This design process will be described in more depth later. Additionally, the Figures 14a and 14b below demonstrate Tornado's ability to generate load distribution plots for the front and back wing respectively. These values were then used to structurally analyze the design of the wings.

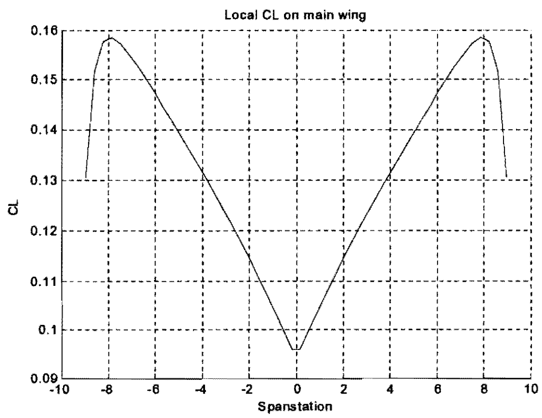

Fig. 13a Front Wing Local C_L Cruise Conditions

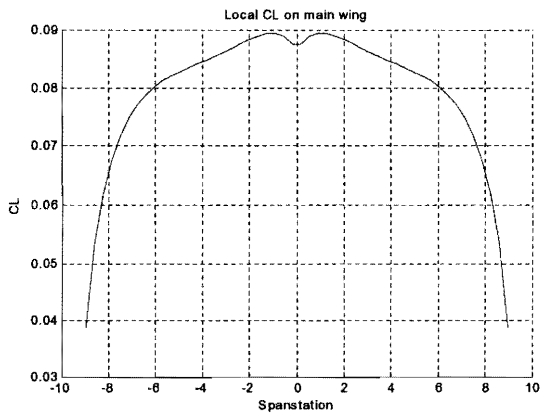

Fig. 13b Back Wing Local C_L Cruise Conditions

Total CL0.20904

Total CL0.20905

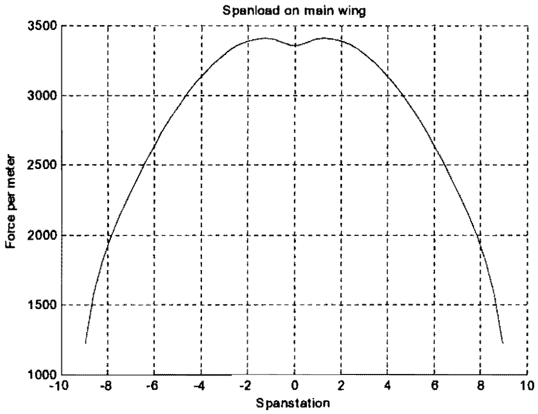

Fig. 14a Front Wing Spanload Cruise Conditions

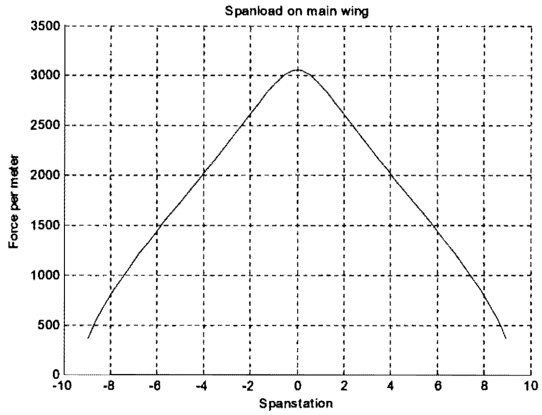

Fig. 14b Back Wing Spanload Cruise Conditions

Aerodynamic Center Calculation

The aerodynamic center (AC) or neutral-point (NP) is the point around which all the aerodynamic forces are balanced. For stability purposes, the aircraft should have an AC behind the CG. As this particular design in a non-traditional aircraft design, several methods were look at for calculating the AC and equation 9 below was chosen due to ease of conversion to a formula useful for our design. For the terminology in the equation, the front wing is denoted as the "wing" and the aft wing is denoted as the "stabilizer."

$$N_P = h_0 + \eta_s \times V_s \times (a_s/a_w)(1-(d\varepsilon/d\alpha)) \qquad (9)$$

$$\begin{aligned}
&where \quad N_P = Neutral\ Point\\
&h_0 = aerodynamic\ center\ of\ the\ wing,\ typically\ 0.25\\
&\eta_s = stabilizer\ efficiency, typically\ 0.6\\
&V_s = stabilizer\ volume\ coefficient\ (= 6.4)\\
&a_s = lift\ curve\ slope\ of\ stabilizer\\
&a_w = lift\ curve\ slope\ of\ wing\\
&d\varepsilon/d\alpha = change\ in\ stabilizer\ downwash\ angle\ vs.\\
&\qquad change\ in\ wing\ angle\ of\ attack\ (= 0.4)
\end{aligned}$$

The Tornado VLM program was used to determine the lift curve slope for the stabilizer and the wing. The front wing $C_{L\alpha}$ was found to be 4.003 per radian while the stabilizer $C_{L\alpha}$ was calculated to be 3.9697. From these values, an a_s/a_w value of 0.992 was computed. The stabilizer volume coefficient of 6.4 was determined using equation 10 below.

$$V_s = {S_s \times L_s}\Big/{S_w \times c} \qquad (10)$$

$$where\ S_s = stabilizer\ reference\ area\ (= 450\ ft^2)$$

$$S_w = wing\ reference\ area\ (= 450\ ft^2)$$

$$L_s = length\ between\ AC\ of\ wing\ and\ stabilizer\ (= 48\ ft)$$

$$c = length\ of\ chord\ at\ AC\ of\ wing\ (7.5\ ft)$$

From these values, a neutral point of 253.6% behind the leading edge of the wing was determined; this corresponds to a distance of 44.4 ft from the nose of the aircraft. As the center of gravity is 40.5 ft from the nose of the aircraft, the aircraft is stable. However, the moderate distance between the AC and the CG could potential act as a moment arm to yaw or pitch the craft during windy conditions. Thus further iterations may be necessary to reduce this relatively large distance, while still maintaining an aerodynamic center behind the center of gravity.

Vertical Tail Sizing

The sizing of the vertical tail vertical tail began by assuming from historical data that the tail volume ratio according to equation ***43049*** below is equal to 0.05.

$$Vol_{VT} = \frac{l_{VT} \times S_{VT}}{b \times S} \qquad (11)$$

$$where\ \ l_{VT} = length\ between\ c.g.\ of\ airplane\ and\ a.c.\ of\ vertical\ tail,$$

$$S_{VT} = Sideview\ area\ of\ vertical\ tail$$

$$b = wingspan\ (60\ ft)$$

$$S = Wing\ Planform\ Area\ (900\ ft)$$

As vertical tail size contributes significantly to the center of gravity location, an iterative process between the tail sizing and the center of gravity (c.g.) calculation had to be completed. For the first iteration, the vertical tail sizing assumed a c.g. location of 42.75 ft from the nose of the aircraft. Several additional assumptions were then made such as assuming the vertical tail root equal to 9 ft, the distance between the trailing edge of the root and the tail of the aircraft to be 1 ft, and the aerodynamic center of the vertical tail to arbitrarily be located 33.25 ft from the c.g. or 3 feet from the leading edge of the vertical tail. These dimensions are illustrated below in Figure 15a.

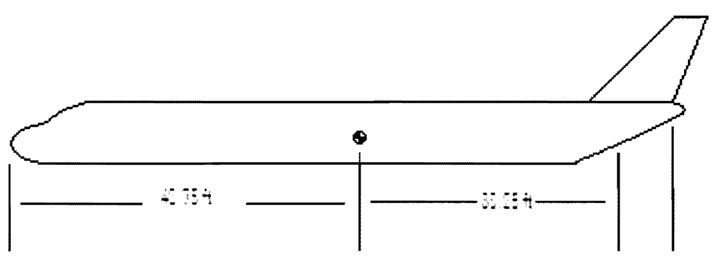

Figure 15a Dimensions used for vertical tail first iteration

Using these values, the side view area of the vertical tail was calculated to be 81.2 ft^2. Next, assuming an aspect ratio of 1.5 and a taper ratio of 0.4, the vertical tail height was calculated to be 11 ft, the root chord to be 10.5 ft, and the tip chord to be 4.2 ft. These dimensions were then inputted into the center of gravity formulation and a new center of gravity calculated.

After several iterations, the center of gravity moved to 40.5 ft and the distance between the c.g. and the a.c. of the vertical tail became 34 ft. These dimensions led to a minimum vertical tail surface area of 79.4 ft^2. The final vertical tail lengths of 8.4 ft for the chord foot, 4 ft for the chord tip, and 13.45 ft for the tail height were concluded upon. Additionally, 30 degrees was used for the leading edge sweep and a symmetrical NACA-0012 airfoil was used for the tail. Figure 15b below demonstrates the final calculated dimensions.

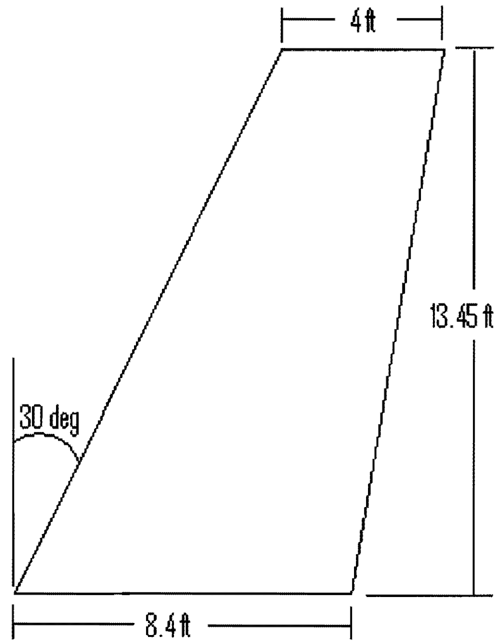

Figure 15b Dimensions used for vertical tail first iteration

Aft Wing Angle of Attack

In the analysis of the joined wing design, it was determined that the C_L lift distribution for the front wing was at a higher maximum value than that of the back wing. For example, before the effects of wing tip twisting to alter the lift distribution to a more efficient design, the C_{Lmax} for the front wing at an angle of attack of 1.75° at cruise conditions was 0.15, compared to a value of C_{Lmax} of 0.112 for the aft wing. This large discrepancy between the two values is mainly due to the effects of downwash on the aft wing and upwash from the aft wing on to the front wing. After determining that the wing tip would be twisted -1.0°, the front wing lift coefficient fell to 0.121. It was still decided that a study should be conducted to attempt to increase the angle of attack of the back wing, which would then act to increase the lift of the back wing. In the study, it was determined that the angle of attack of the aft wing would be limited in which values of C_{Lmax} for the aft wing were less than that of the front wing. This would ensure that the front wing of the aircraft would stall first, a nose down moment would then be created, and recovery attained. Should the rear wing stall fist, a nose up moment would be generated which would act to stall the front wing and create a potentially dangerous flight condition.

In attempting to implement the change of angle of attack of the aft wing in the Tornado MATLAB program, numerous difficulties arose. To circumnavigate these difficulties, it was necessary to analyze each wing separately and combine the data to predict the joined wing performance. First, the joined wing system was analyzed for a given angle of attack that was applied to both the front and rear wings. Then the front and back wing were analyzed individually and the data compared to determine the interference effects. After several studies, an empirical formula was developed to predict the performance of the joined wing system from individual wing analysis in the flight operating conditions of our design. Table 4a below demonstrates the development of this empirical formula for the lift and drag of the joined wing design. It was found that the total lift and C_L of the joined system is equal to the lift or C_L of the front wing plus 0.75 times the lift or C_L of the back wing. The total drag was found to equal that of the front wing plus 1.5 times the drag of the back wing.

Table 4a Development of Joined Wing Empirical Lift and Drag Formula

α = 1.75°	Joined Wings	Front Wing	Back Wing	lift = f + b drag = f + b	% error	lift = f + 3/4*b drag = f +3/2b	% error
Lift	80843.76	44020.94	49061.52	93082.47	-15.1387	80817.08	0.0330
Drag	603.19	179.53	281.47	461.00	23.5737	601.73	0.2417
CL	0.200	0.109	0.122	0.231	-15.1408	0.200	0.0312

A similar empirical method was used to determine equations to predict the C_{Lmax} and C_L for the joined wing system. The values for C_{Lmax} and C_L were first found for various angles of attack for the rear wing in a joined wing system and then these same terms were found for the solo back wing. By finding the ratio of the joined analysis values over the solo analysis values, an empirical constant may be approximated to predict the joined wing performance of the back wing from a solo back wing analysis. Table 4b below shows these empirical constants were found to be 0.69 for C_{Lmax} and 1.7 for C_L. Some error is introduced in that the empirical formulas were determined using a front and rear wing under the same angle of attack. Although in actuality, there will be changes to the empirical constants when the rear angle of attack is increased, this analysis will be a good prediction of actual performance.

Table 4b Development of Joined Wing Empirical C_{Lmax} and C_L Formula

	C_{Lmax}				C_L		
α	2°	3°	4°	α	2°	3°	4°
Joined, Back Wing	0.1020	0.1502	0.2000	Joined, Back Wing	0.2387	0.3569	0.4742
Solo, Back Wing	0.1450	0.2180	0.2900	Solo, Back Wing	0.1389	0.2080	0.2770
Joined/Solo	0.7034	0.6890	0.6897	Joined/Solo	1.7188	1.7154	1.7118
Empirical Value Joined Back Wing	0.1001	0.1504	0.2001	Empirical Value Joined Back Wing	0.2375	0.3557	0.4737
% error	1.912	-0.146	-0.050	% error	0.513	0.313	0.104

With these empirical formulas determined, the task of altering the rear wing angle of attack was begun by analyzing the back wing separately for various angles of attack and using the formulas to predict the back wing performance after incorporating interference effects from the joined system. The joined wing C_{Lmax} for the front wing was found to be 0.122. For the solo rear wing, values of angle of attack of 1.75, 2, 2.125, 2.25, 2.375, 2.5, and 3 degrees were analyzed and the empirical formula used to predict the joined wing C_{Lmax}. Table 4c below shows the results from this analysis. From this data, 2.25 degrees was chosen as an angle of attack that will increase the C_L of the rear wing from 0.127 to 0.1625, while still maintaining a C_{Lmax} below that of the front wing.

Table 4c Aft Wing C_{Lmax} for Solo and Joined Wing Prediction

	Angle of Attack	Max CL	Combined Max CL	% difference
Joined System, Front Wing	1.75	0.122	0.122	0.0000
Solo Rear Wing	1.75	0.127	0.0876	-0.0344
	2	0.145	0.1001	-0.0220
	2.125	0.154	0.1063	-0.0157
	2.25	0.1625	0.1121	-0.0099
	2.375	0.172	0.1187	-0.0033
	2.5	0.181	0.1249	0.0029

	3	0.218	0.1504	0.0284

Finally, the overall system performance was found by analyzing the back wing separately at 2.25 degrees angle of attack and using the empirical formulas to combine the performance values with that from the front wing at 1.75 degrees. As shown in Table 4d below, when compared to the joined wing analysis with both wings at 1.75 degrees, the altered rear wing angle of attack gives a predicted 12.5% increase in total lift. A large increase in drag was also found, however, this may be due to a larger induced drag and an error in the empirical evaluation of drag. A 46% increase in drag is not expected for the actual joined system.

Table 4d Evaluation of Joined System, Front Wing at 1.75° and Aft Wing at 2.25°

α (deg)	Joined 1.75	Front 1.75	Back 2.25	drag = f +3/2b lift, CL = f + 3/4*b	% difference
Lift (lbs)	17777.47	9494.42	14012.65	20003.91	-12.52
Drag (lbs)	131.83	37.79	103.15	192.52	-46.04
CL	0.1982	0.1058	0.1562	0.2230	-12.53

Wing Twisting

In order to provide a more elliptical lift distribution and reduce the induced drag on the wing, a study of the effects of twisting on the wing was conducted. The twist angle defined in this section is the angle between the tip chord of the wing and the root chord of the wing. An illustration of the wing tip being twisted down may be seen in Figure 16. It was thought that by washing out the angle of the wings, the sharp peaks of the untwisted wing shown in Figure 17 could be rounded and made more elliptical.

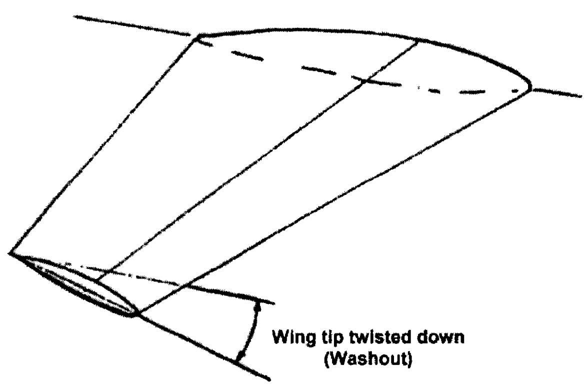

Figure 16 Wing Twisting Definition

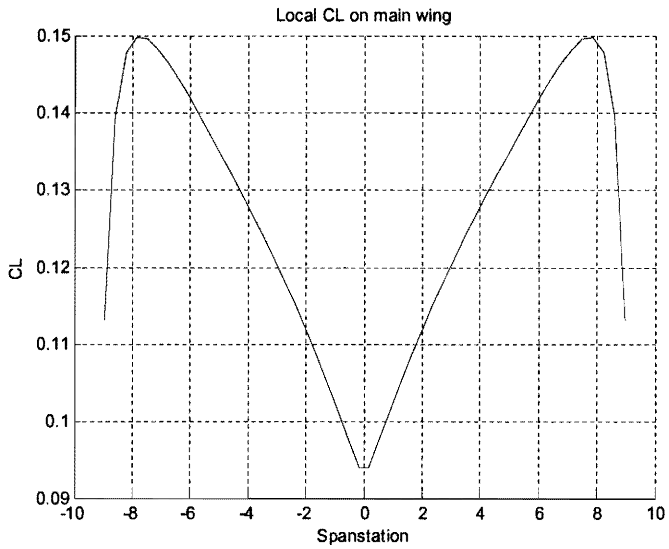

Fig. 17 Local C_L of Untwisted Wing

The wings were analyzed using the Tornado MATLAB program for an outer twist of 0, -0.75, -1.0, -1.25, -1.5 and -1.75 degrees. For each study, the lift and induced drag produced were recorded along with the maximum local C_L, the location of this maximum and the overall C_L of the wing. As demonstrated in Figures 18-21 below, as the twist value increases, the C_L distribution becomes more elliptical. However, the maximum value of C_L also decreases and the moves closer to the wing's root.

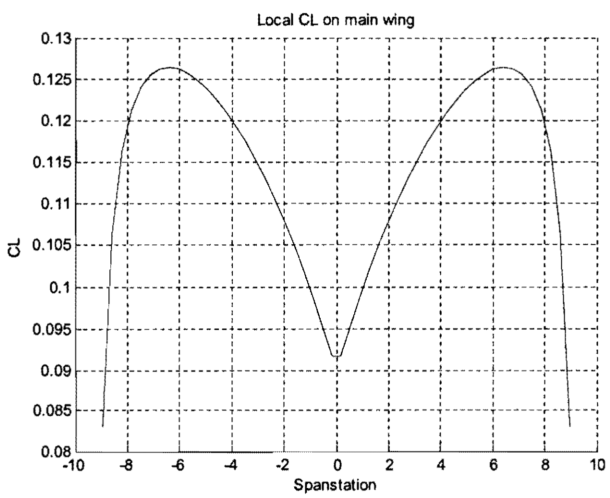

Fig. 18 Local CL, -0.75° Twist

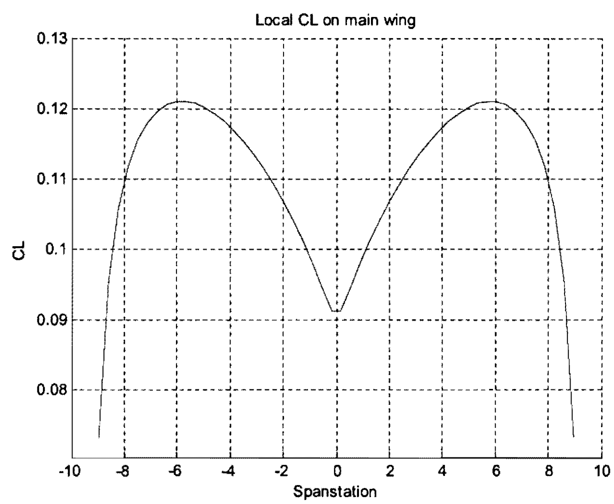

Fig. 19 Local CL , -1.0° Twist

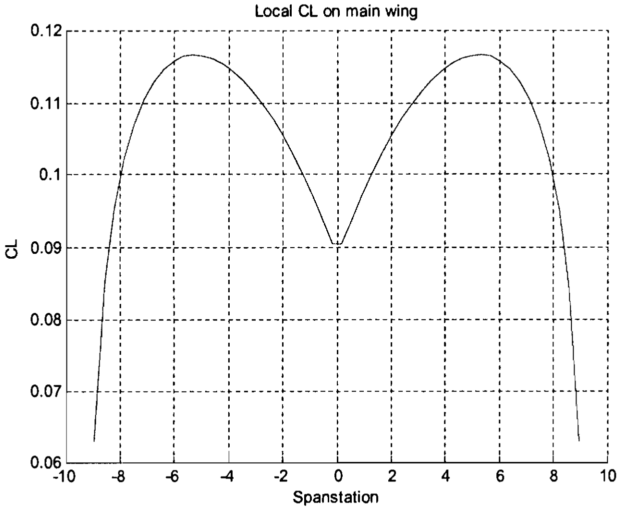

Fig. 20 Local CL , -1.25° Twist

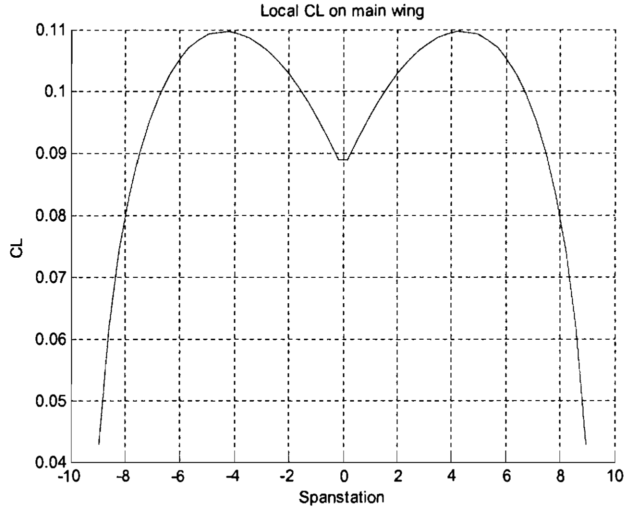

Fig. 21 Local CL, -1.75° Twist

From the data produced during the wing tip twisting study, shown in Table 5 below, a comparison of wing performance for the various twisting degrees was made. Although the larger twisting degrees produced less induced drag, they also had a lower maximum CL and overall CL. Thus a tradeoff between reduced drag and lift was shown by plotting the ratio of lift and induced drag versus maximum CL in Figure 22. In addition, the effect of twist on the maximum local CL was made in Figure 23. From these comparisons, it was concluded to use a wing tip twist of -1.0° for the front wing. This will allow the wing to act in a more effective and efficient manner with a more elliptical lift distribution, a reduction in induced drag and an increase in the ratio of lift/drag.

Table 5 Comparison of Wing Performance for Various Wing Tip Twists

Twist (deg)	0	-0.75	-1	-1.25	-1.5	-1.75
Max Local CL	0.15	0.127	0.121	0.117	0.112	0.11
Location of Max CL (y/c)	0.8	0.61	0.58	0.54	0.5	0.42
Lift (lbs)	10953.2	10078.0	9786.1	9494.4	9202.5	8910.9
Induced Drag (lbs)	50.26	42.15	39.90	37.89	36.10	34.55
L/Dv	217.9	239.1	245.2	250.6	254.9	257.9
CL	0.122	0.112	0.109	0.106	0.103	0.099

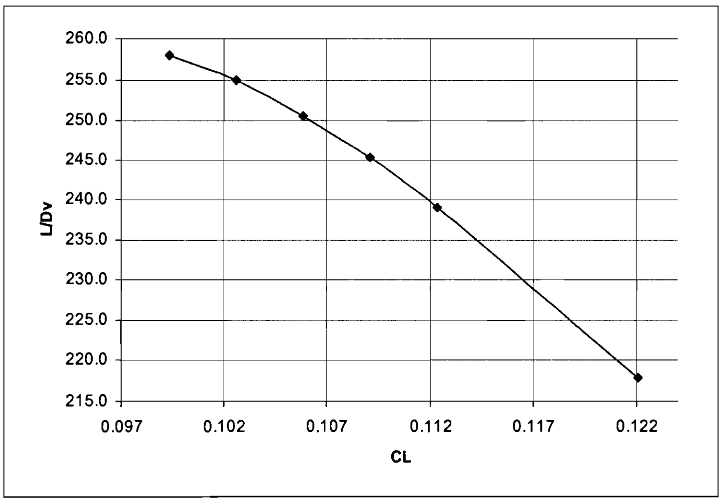

Fig. 22 Ratio of Lift to Induced Drag versus Coefficient of Lift

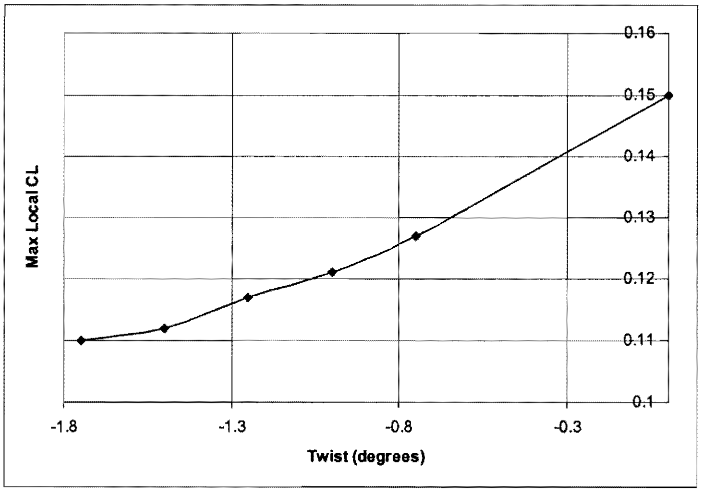

Fig. 23 Effect on Wing Tip Twist to Maximum Coefficient of Lift

<u>Control Surfaces</u>

In addition to the wing and tail surfaces, the aircraft needs some additional components that give the pilot the ability to control the direction of the plane. This is where the design of the control surfaces is important. A wing designed for efficient high-speed flight is often significantly different from one designed primarily for takeoff and landing. With respect to takeoff and landing performance the length of ground roll for both are strongly determined by the aircrafts stall speed. There are numerous methods to decreasing an aircraft's stall speed. The first is to increase the wing area. The problem with continually increasing the wing area is the fact that there are increased drag effects on the aircraft during cruise when there is an excess of wing area being drug through the air. It is also possible to reduce the stall speed on an aircraft by decreasing the aircrafts weight, or increasing the $C_{L\ mas}$. For the Diamond to achieve an optimized overall mission performance the method to use was to increase in the maximum lift coefficient. This inherently raises the wing loading (W/S), which then increases the lift to drag ratio during cruise conditions. Another positive byproduct of this is that it decreases the fuel that is used.

Due to the short takeoff and landing constraints, double-slotted type I flaps were selected versus the triple-slotted flaps. An illustration of the double-slotted flap and slat configuration can be seen in Fig. 24. Triple slotted flaps primarily are seen on large transport aircraft with very high wing loadings. In contrast the Diamond Cutter will have moderately low wing loading do to the high reference area.

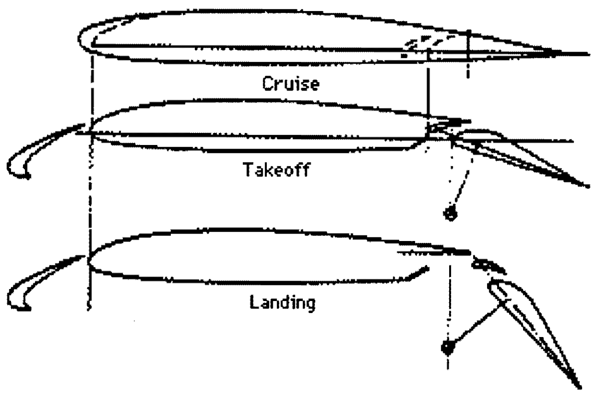

Figure 24: Double-Slotted Flap and Flap System

Often on transport aircraft, Krueger leading edge flaps are used on the inner section of the wing. Krueger flaps help to increase lift on the aircraft, but inherently impose a decrease in the stall angle. For the leading edge of the aircraft slats will be used to increase the stall angle, aiding the control of the aircraft at low speeds and high angles of attack.

The ailerons for the Diamond and are located on the tips of each wing. They are deflected in opposite directions (one goes trailing edge up, the other trailing edge down) to produce a change in the lift produced by each wing. On the wing with the aileron deflected downward, the lift increases whereas the lift decreases on the other wing whose aileron is deflected upward. The wing that produces more lift will roll upward causing the aircraft to go into a bank. The angle of deflection is usually considered positive when the aileron on the left wing deflects downward and that on the right wing deflects upward. The greater lift generated on the left wing causes the aircraft to roll to the right.

The elevators are located on the rear wing for the Diamond Cutter. On most typical aircraft these elevators will be found on the rear stabilizer, but since the Diamond has the rear wing the rear stabilizer wasn't necessary. The elevators can be deflected up or down to produce a change in the downforce produced by the horizontal tail. The angle of deflection is considered positive when the trailing edge of the elevator is deflected upward. Such a deflection increases the downforce produced by the rear wing causing the nose to pitch upward.

The decision was made that two setups, for the double-slotted flaps, would be used for the inner and middle parts of the wings. The inner portion of the wing would use more deflection and have a higher $C_{L\ max}$. The middle portion of the wing would use less flaps to reduce the drag and increase performance at near stall conditions.

The sizing of the control surfaces for the Diamond Cutter can be seen in Fig. 25.

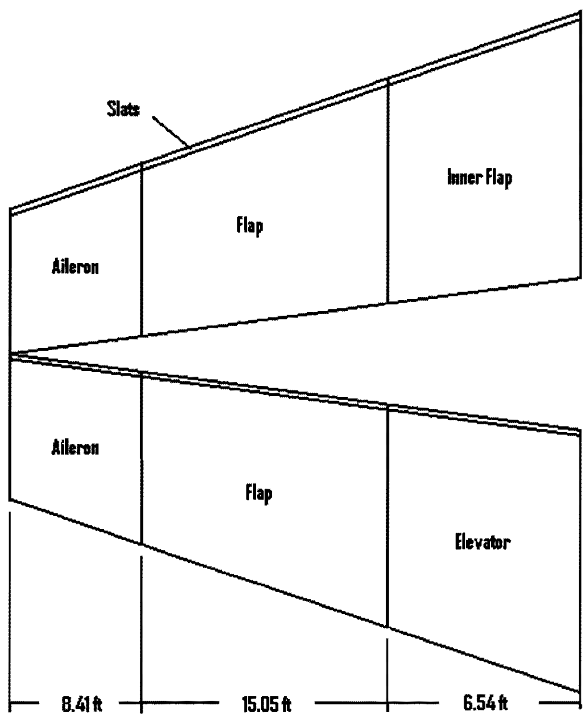

Figure 25: Control Surface Dimensioning

For the NACA 64-212 the ideal flapped percent of the chord length (c_f/c) for the main flap on the double slotted flap is 25%, and for the smaller portion of the flap (c_v/c) it is 7.5%. For take- off calculations the entire flaps are not deployed so a 15% in surface area of the flapped area was assumed to increase the value of C_l for take- off.

<u>Propulsion</u>

The first step in the propulsion system process for the jet was appropriate engine sizing. For our initial individual designs of thirty passenger planes last semester we arrived at the conclusion that the engines needed to be in the 5000 lb to 8000 lb thrust per engine range, giving about 10,000 lb to 16,000 lb of maximum thrust since the design called for two engines. Three companies were considered during the engine selection process: GE, Honeywell, and Pratt & Whitney.

The first engine that was considered was the GE CFE738. This engine is the power plant behind the Falcon 2000 business jet. At sea level this engine could produce up to 5,918 lbs of thrust per engine. This engine fit the requirements in most everyway but data on thrust as a function of Mach number and altitude was found to be insufficient making analysis of the engine very difficult.

The next company that was considered was Honeywell and their TFE731 series of engine. Specifically the 5BR, which is rated at approximately 4,743 lb of thrust at sea level. Fuel consumption was very good on this engine but the amount of thrust it produced proved to be lacking so it was eliminated from consideration.

Finally, Pratt & Whitney was contacted to obtain information on their PW300 series of engine. The PW305 was selected and a fact sheet was provided from Pratt & Whitney that provided all of the pertinent data, such as specific fuel consumption, thrust as a function of Mach number, and engine dry weight to name a few. This engine produces 5,225 lb of thrust at sea level and since the engine fit the requirements very well and sufficient data was readily available on it, the Pratt & Whitney PW305 was selected has the engine that would power the **Diamond Cutter**.

About half way through the semester when more in depth analysis of the **Diamond Cutter** took place, such as take-off and landing calculations and range it became evident that with the rigid constraints placed on the jet larger engines were needed to meet the criteria without problem. To solve this issue the Pratt & Whitney PW300 series was turned to again and this time the PW308B that produces 7,400 lb of thrust at sea level was selected as the final engine for the **Diamond Cutter**.

The PW308B has a dry weight of 993.0 lbs and its overall dimensions can be seen in Figure 24 below.

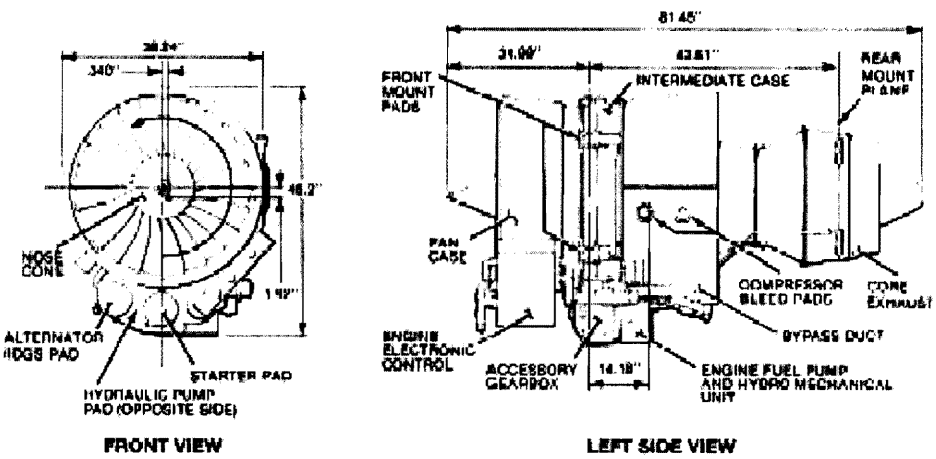

Figure 26 Schematic of Engine

The PW308B has a four stage compressor, with the first two stages incorporating variable guide vanes. This engine is said to be able to meet the needs of commercial aircraft for the next two decades. What makes it capable of doing this is its low weight, low fuel consumption, and low noise. A plot of thrust as a function of Mach number and altitude can be seen below in Figure 25.

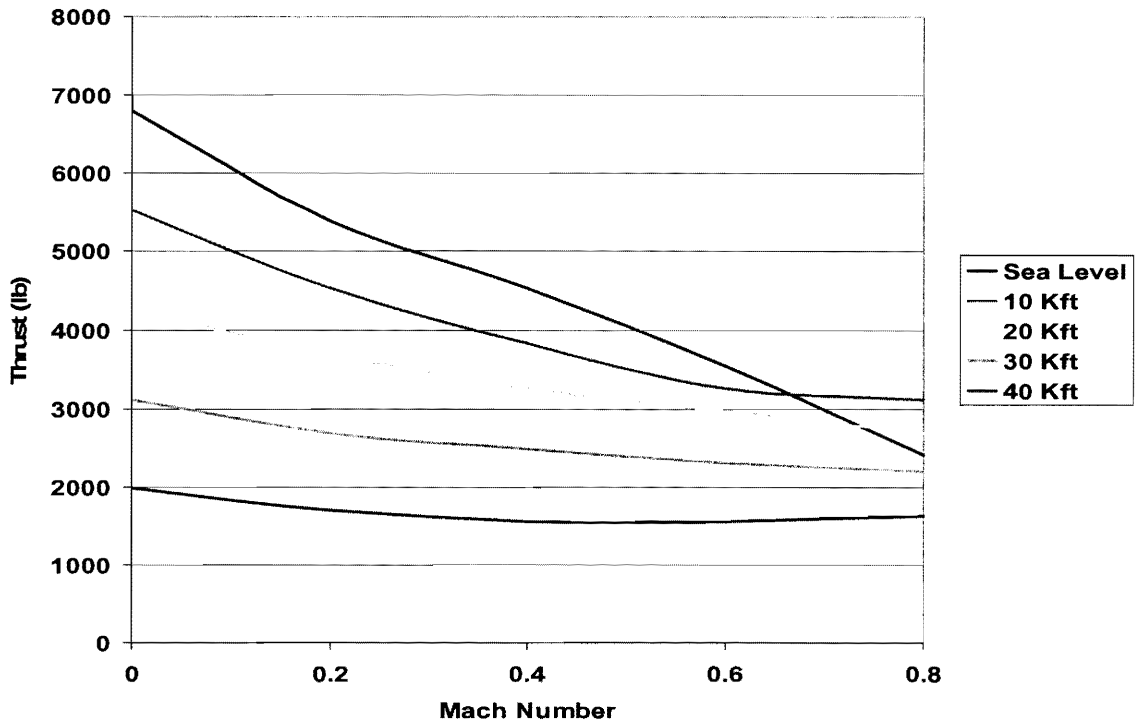

Figure 27 Thrust vs. Mach Number for Different Altitudes

The engine also has a good fuel consumption rate of 0.407 1/hr, which is need because of the range requirements place on the jet. The jet must be able to cruise for 1,500 miles without refueling and must take-off in a short range and in order to due this the plane must stay light. One way of keeping it light is to lessen the amount of fuel present at take-off, but this hinders the range so a strong furl consumption is required.

TAKEOFF

Short takeoff and landing were vital requirements to keep in mind when designing the aircraft. These two performance characteristics were essential when looking at the secondary role for this airplane. One of the pros to the joined-wing design was that it delivered the additional reference planform area that was necessary to achieve the short takeoff objective. The calculations of takeoff and landing both entail several variables and assumptions. For the takeoff analysis the lift off velocity was determined by assuming that it was 10% greater than the stall velocity. By using Newton's second law of motion, Eq. (12), the minimum takeoff distance could be determined.

$$F = m\frac{dv}{dt} \qquad (12)$$

Solving for the acceleration yielded Eq. (13).

$$\frac{dv}{dt} = \frac{T - D - \mu_r W}{W/g} \qquad (13)$$

The maximum allowable takeoff thrust for the PW308B engines used on the aircraft provided 7400 lb each. This value for thrust was held constant in the evaluation of acceleration. The value for drag although was not held constant because drag increases proportionally with velocity. To solve for the drag the drag polar, Eq. (14) was utilized.

$$C_D = C_{d_o} + KC_L^{\ 2} \qquad (14)$$

For the first run the values of $C_{d,o}$ and K were taken from the performance data sheet for the airfoil. The C_L originated from the airfoil data sheet where the airfoil was at a 2° incident angle. From this value the coefficient of drag was produced. This new coefficient was used in Eq. (##) below to solve for the drag.

$$D = \frac{1}{2} v \rho_\infty S C_L{}^2 \qquad (15)$$

A MATLAB m-file was written that provided an iterative process that numerically integrated the acceleration over the takeoff run. For each iteration the new drag was tabulated which provided a modified value for acceleration. This program runs until the takeoff velocity is finally achieved. While the loop is running to meet the velocity requirement the distance the plane is covering on the ground is being tabulated. After the loop constraint is satisfied the distance required is produced.

The procedure just described was a first run approximation for the ground roll required for takeoff. There is a significant factor that was neglected in the former analysis and had to be taken into account. This factor is the aircraft rotation during the takeoff. When this rotation that takes place it produces an extreme increase in the drag that the aircraft experiences. Therefore the drag is increased and coincidentally the ground roll is increased. The initial method to accommodate for this was to make time response estimation for the rotation. In the *Aircraft Performance* text the response time for a large aircraft was recorded to be 3 seconds. So to correct the ground roll calculation the time for take off for the first cut analysis was used and three seconds were taken off of that time.

To analyze the ground roll during rotation the program was set up to run identically as before up to 3 seconds early of the first run time. Once this point is reached the aircraft has begun its rotation. During the rotation there is a continual change in both the coefficients of drag and lift, and therefore there are changes in the aircraft's drag and lift. The program provides a loop that iterates through this process of change while increasing the coefficient of lift from its current value to the coefficient of lift that is required for take off. This increase is assumed to be linear over a period of three seconds. The result is a linear change in the coefficient of drag as well. This process of rotation increases the takeoff run slightly, because of the increased drag that is associated with it. The Matlab m-file that governs this takeoff analysis is attached in the appendix.

This takeoff analysis takes into account many of the aircrafts characteristics, but can still be improved upon. One thing that can be done to increase the accuracy of the ground roll approximation is to factor in the fuel burn of the aircraft during the takeoff run and during the taxing period. Taking this factor into account would minutely decrease the takeoff run, but would have an effect nonetheless.

Discuss the results from the program

Landing Analysis

The next task was to evaluate the aircraft's landing performance. This analysis shared a close to relation to that of the takeoff calculations. This particular problem was also solved by utilizing Matlab to solve for the ground roll that the plane would require to come to a complete stop. The specification for the aircraft design was that the jet would be able to nominally stop in a distance of 3000 ft for the primary role and a distance of 1400ft for the secondary role. The touch down velocity for the jet was assumed to be 15% greater than that of the stall speed. With this value at hand a program was written that would numerically integrate over the aircrafts deceleration performance to find the predicted ground roll. One notion that was factored in was the fact that the engines would have reverse thrust. The reverse thrust for the engines used on the jet was not specified so historical data from the Aircraft Performance text was utilized. From the text the historical data showed that a jet would typically use 40-50% of its maximum thrust as reverse thrust. In the program for this jet considering the reverse thrust for the PW308B was unknown a lower value of 35% maximum thrust was utilized. The m-file that was written to perform this landing ground roll analysis can be seen in the appendix.

Two plots were generated, similar to that of the takeoff analysis, to help illustrate the aircraft's performance under varying parameters. The first of the varying parameters was the aircraft's landing weight. To analyze the takeoff performance with the varying landing weight a wing reference area was held constant at 900 ft2. This plot can be seen below in Fig. 28.

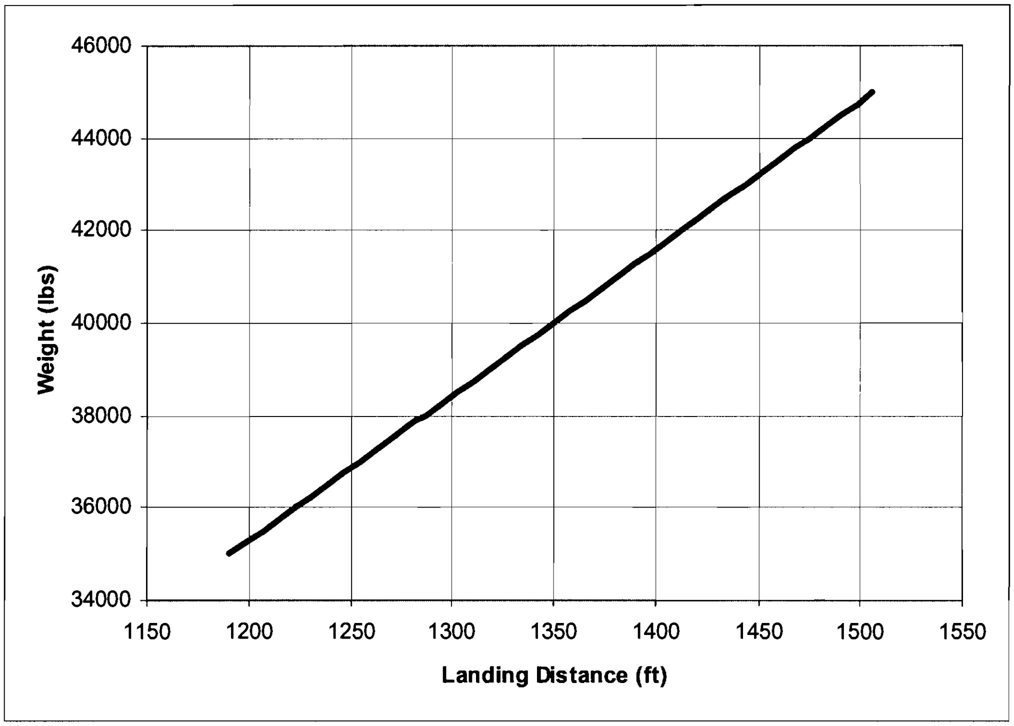

Figure 28: Weight of Aircraft vs. Landing Distance

As can be seen from the plot as long as the aircraft resides between the range of approximately 41,500 lbs and less the landing ground roll constraint will be reached. The same type of plot was generated for a varying wing reference area. To complete this plot the weight was held constant at 28,000 lbs while varying the wing area (figure 28).

Minimum Control Speed

The Minimum Control Speed, V_{MC}, is the airspeed at which the when the critical engine fails it is possible to maintain control of the aircraft with the engine still inoperative and maintain steady level flight at a speed that does not exceed 1.2 V_S and with a bank angle of no more than 5 degrees. Maximum available thrust at sea- level can be assumed for the calculations, which for the case of the PW308B was approximately 6,798 lb of thrust. To calculate this speed a couple parameters have to first be established. First set y equal to the distance from the center of the working engine to the centerline of the fuselage, then set x equal to the distance from the vertical tail to the center of gravity. Using these distances and a moment analysis the lift produce by the vertical tail, L_{VT}, can be found using the following equation where T is equal to thrust:

$$T^* y = L_{VT} * x \tag{16}$$

A visual of this set up found in Figure 29 can be found below:

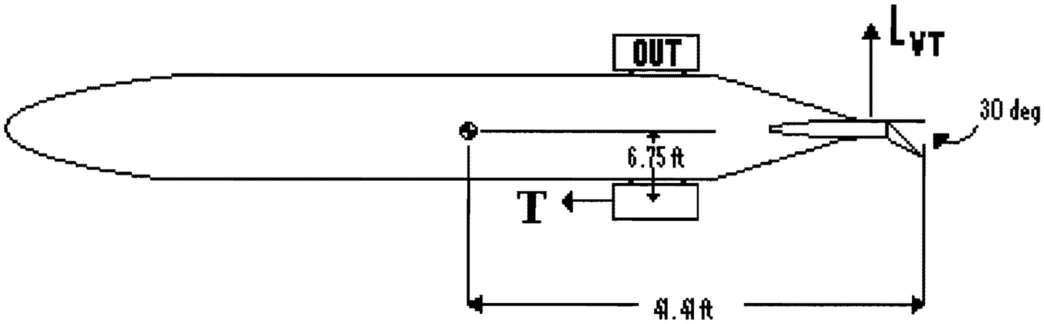

Figure 29 V_{MC} Analysis

Solving for LVT, the amount of lift needed for steady flight is 1,108 lbs. Using the equation for lift and solving for velocity:

$$\mathbf{L = \tfrac{1}{2} * \rho\, V^2 * s * C_L} \qquad (17)$$

$$V_{MC} = \sqrt{\frac{2 \times L}{\rho s C_L}} \qquad (18)$$

Where s is equal to the area of the vertical tail, which was found to be 83.39 ft^2 and C_L is the coefficient of lift produced by the tail with a 30 degree deflection which turned out to be approximately 0.70. Inserting in these values and solving for V_{MC}, the minimum control speed is found to be 74.9 knots or 126.4 ft/s. Comparing this to the stall speed of 154.7 ft/s , it can be seen that since the stall speed is higher than the minimum control speed, that the minimum control speed will just be taken as the stall speed.

<u>Structural Analysis</u>

To determine the lift distribution for the maximum loading scenario, the weight of the aircraft was multiplied by a limit load factor of 3.5 and a safety factor of 1.4. These values where chosen from typical values of general civil transport aircraft and used throughout the structural analysis. Next, the Tornado vortex lattice code was utilized to determine the lift distribution necessary to produce a lift equal to the maximum load.

A. <u>Free-End Analysis</u>

In order to get a first pass estimation of the inter shear and bending moment diagrams, the wings were modeled as two un-joined wings with the assumption of a free end for both wings. In addition, the wing weight and fuel weight were neglected. Using the lift distribution given from an aerodynamic analysis the internal shear stress and internal bending moments were derived using Eqs. (19) and (20) with x being the spanwise location on the wing measured from the wing root.

$$\mathrm{V} = \int \omega \mathrm{dx} \qquad (19)$$

$$\mathrm{M} = \int V \mathrm{dx} \qquad (20)$$

These equations were used to evaluate both the front (lower) and rear (higher) wings. Using MATLAB the resulting equations were plotted against span-wise location. Figures 30a and 30b show the shear distribution for the front and rear wings, respectively. The bending moment distributions for the front and rear wings are shown in figures 31 and 32. Table 6a shows the approximate equations used to make these plots.

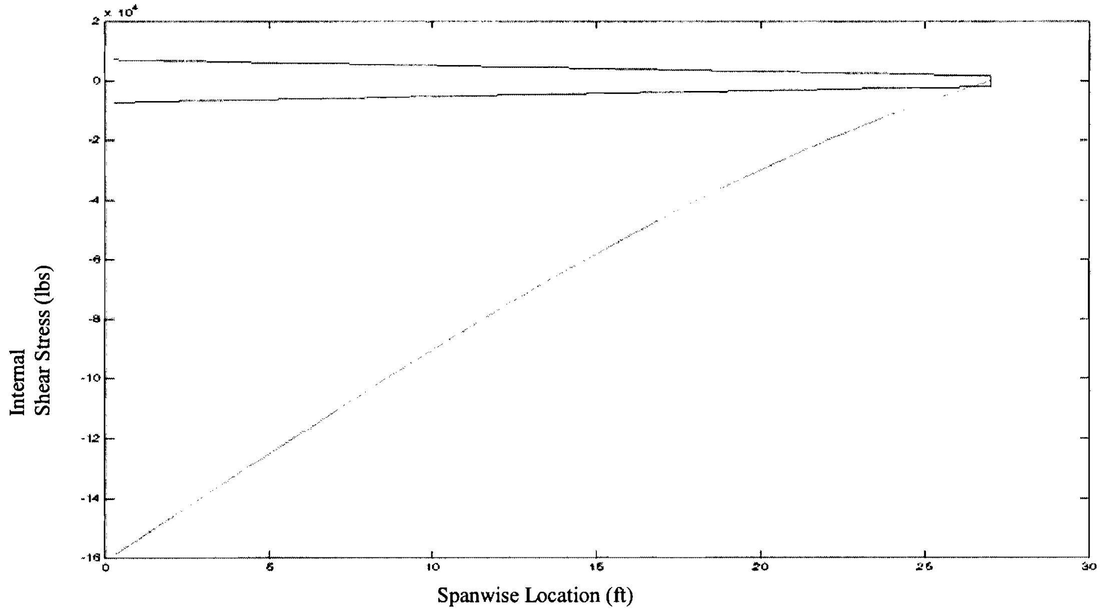

Fig. 30a: Internal Shear Stress Diagram for the Front Wing

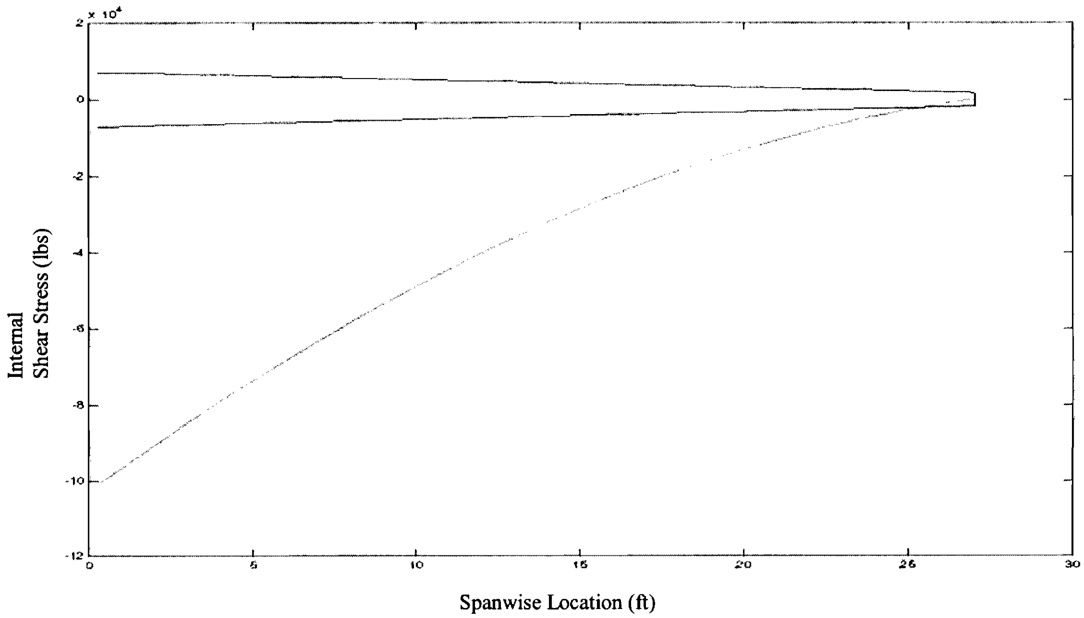

Fig. 30b: Internal Shear Stress Diagram for the Back Wing

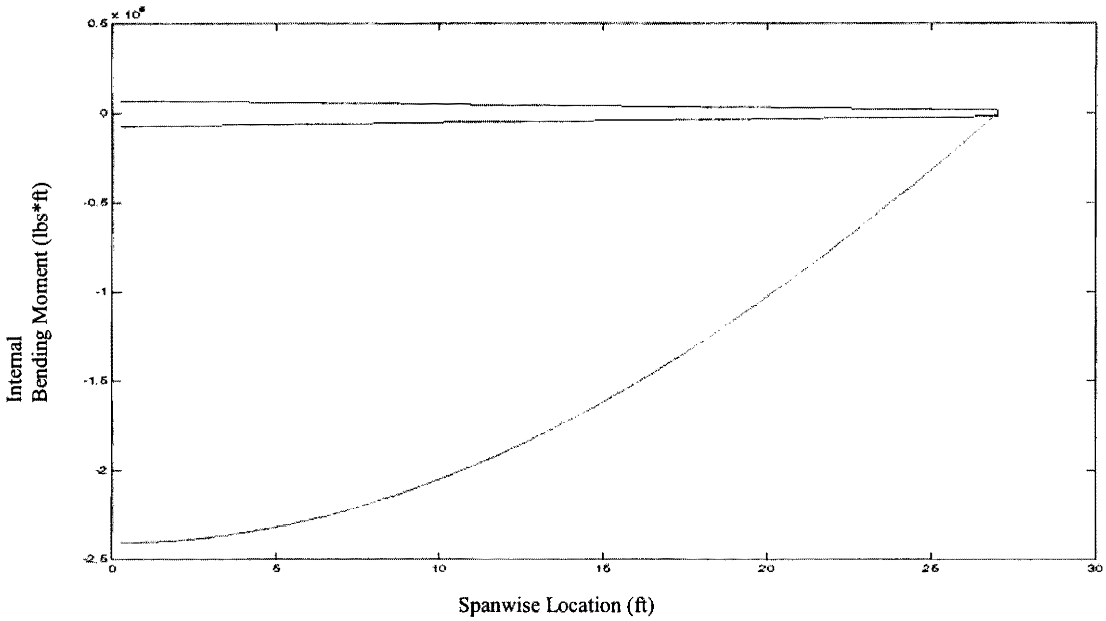

Fig. 31: Internal Bending Moment Diagram for the Front Wing

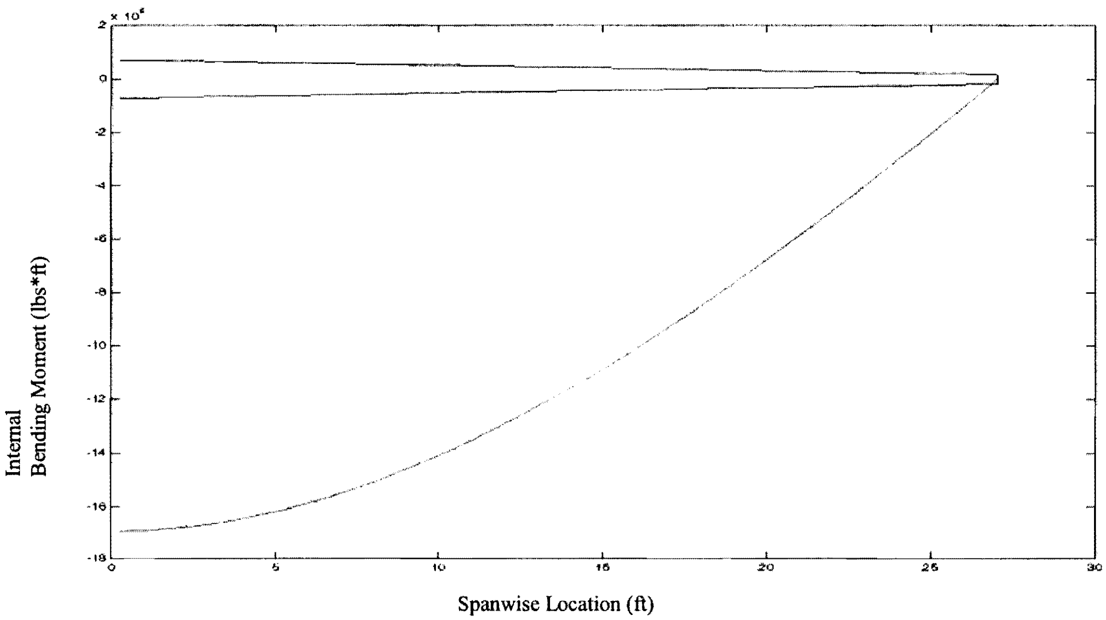

Fig. 32: Internal Bending Moment Diagram for the Back Wing

Table 6a: Lift, Shear, Bending Moment Equations for Free End Analysis

Lift	Equation
Front Wing	$\omega(x_{fe})$ = -.0156x_{fe} ^3 - 5.295x_{fe} ^2 + 10.208x_{fe} +7192.8
Rear Wing	$\omega(x_{fe})$ = - 1.2319x_{fe} ^2 - 153.81x_{fe} + 6175.1
Internal Shear Stress	
Front Wing	$V(x_{fe})$ = Vo1+(-.0039x_{fe}^4 –1.765x_{fe}^3 + 5.104x_{fe}^2 +7192.8x_{fe})
Vo1	-161113.3011 (lbs)

Rear Wing	$V(x_{fe}) = Vo2+(-.41063x_{fe}$^3 – 76.905$x_{fe}$^2 + 6175.1.8$x_{fe}$)
Vo2	-102501.4591 (lbs)
Internal Bending Moment	
Front Wing	$M(x_{fe})$ = Mo1+(-7.8E-4x_{fe}^5 - .44125x_{fe}^4 + 1.7013x_{fe}^3 + 3596.4x_{fe}^2)
Mo1	-2409572.45529 (lbs*ft)
Rear Wing	$M(x_{fe})$ = Mo2+(- .1026583x_{fe}^4 - 25.635x_{fe}^3 + 33087.55x_{fe}^2)
Mo2	-16556936.1614 (lbs*ft)

B. Wing, Fuel Weights, and Load Distribution

Before more structural analysis could be made, the wing weight and fuel weight could no longer be neglected. Using Equations (21), (22), and (23) the wing weight distribution was calculated using an initial value of 6270lbs for the weight of one wing. The resulting equation for the fuel weight is shown as equation (24)

$$W_r = 6W_w/(b(1+\lambda+\lambda^2)) \quad (21)$$
$$Cx = Cr[1-(2x_{fe}/b)(1-\lambda)] \quad (22)$$
$$W_x = W_r\,(Cy/Cr)^2 \quad (23)$$
$$W_x = 491.58[(12-.3x_{fe})/12]^2 \quad (24)$$

Next, an estimation of the fuel weight distribution was conducted. The first consideration for this calculation was the placement of the fuel tanks. Since all of the fuel will be stored in the wings, the tanks must be located between the main and rear spars. In order to compensate for the wing's thickness and chord change as the span location is increased, the fuel will be stored in an arrangement of tanks that decrease in size. The total amount of fuel to be carried during operation is 10,000 lbs. A fuel density of 6.7 lbs per gallon was used to convert this fuel weight to the total number of gallons needed, which is about 1493 gallons. This number was then converted to cubic feet using the Convert[1] program and resulted in a required fuel volume of about 200ft^3. Now that the required volume of the fuel tanks is known, the distance that the tanks extended into the wing was able to be determined. This was completed by setting the distance between each spar to 22.5 inches and having a trapezoidal prism shaped tank between each spar. From a top view of this shape, shown in Figure 31, the area was divided into two triangles and a rectangle Starting at the root of the wing, the rectangle's width and height of the tank was determined by Eq. (25) and (26), respectively.

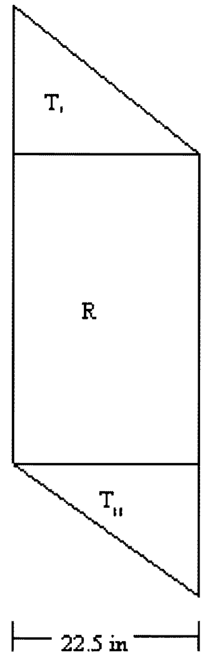

Fig. 33 Top View of Fuel Tank

$$Tank_w = (1/2)C_x \qquad (25)$$
$$Tank_h = t - 3/12 \qquad (26)$$

For the triangular sections the area of each was calculated by Eqs. (27) and (28).

$$T_I = (22.5*SIN(\Lambda_{LE}))*22.5*0.5 \qquad (27)$$
$$T_{II} = (22.5*SIN(\Lambda_{TE}))*22.5*0.5 \qquad (28)$$

These values were then added to the area of the rectangular section. The sum of these was then multiplied by the height of the tank, which resulted in the determination of the tank's volume. This value was then converted back to lbs of fuel and an iteration process was carried out until the total amount of fuel equaled 10000 lbs. The final result was that the fuel tanks would extend 14ft into the wing.

Now that the fuel and wing weight distributions were known, the final load distribution could be determine by adding the equations for each to the given lift distribution. Table 6b shows these equations and the resulting load distribution equations.

Table 6b: Lift, Fuel Weight, Wing Weight, and Load Distribution Equations

Lift Front Wing	$-0.0156\ x_{fe}$^3 - 5.295 x_{fe}^2 + 10.208 x_{fe} + 7192.8	
Lift Back Wing	-1.2319 x_{fe}^2 - 153.81 x_{fe} + 6175.1	
Wing Weight	0.3072 x_{fe}^2 - 24.579 x_{fe}^ + 491.58	
Fuel Weight	-23.711 x_{fe} + 503.55	for x<=14

Load Front	$-7.0016\ x_{fe}$^2 + 82.679 x_{fe} + 6135.5	
Load Back	$-2.2365\ x_{fe}$^2 - 89.495 x_{fe} + 5134.9	

C. Energy Analysis

Because the wings are not free ends a better method of analysis for the wing structures is required. After consulting with various texts and University of Tennessee graduate student Matthew Parsons, an energy method was determined to be the best way to evaluate the internal shear stress and internal bending moment of the joined wing design. Specifically, a manipulation of Castigliano's Second Theorem, Eq. (29), was implemented.

$$\Delta_i = \sigma C/\sigma P_i \qquad (29)$$

The original purpose of Castigliano's Second Theorem was to allow for the computation of a deflection at any point in a structure. However, this can be modified if the deflection at the selected point is known to yield the bending moment in that section of structure.

In order to quickly identify the loads, the wing loading equations were integrated and are shown in Table 7. Table 7 also displays the integral of the loading equation as a function of z and the centroid of the function

Table 7: Lift, Shear, Bending Moment Equations for Free End Analysis

	Equation/Value
Integral of Front Wing Loading	ω(z) = -2.27z^3+37.57z^2+6201.2z
Integral of Back Wing Loading	ω(z) = -.684z^3-48.5z^2+5200.5z
Evaluated for Front Wing	ω(z) = 158479.8
Evaluated for Rear Wing	ω(z) = 93866.4
Front Wing Centroid	.418z
Rear Wing Centroid	0.337z

Figure 34 shows a sketch of the structure that will be analyzed. From this vantage point the structure represents the port wing as seen from behind.

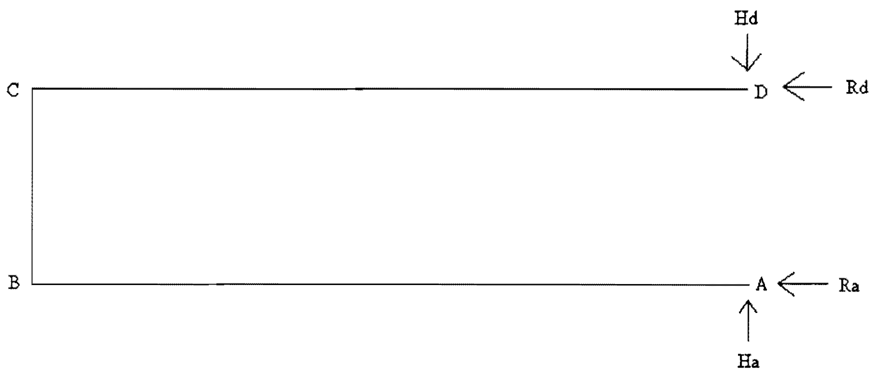

Fig. 34 Wing Structure with Reactions

The first step in determining the bending moment in the structure using this analytic method is to determine the reactions at A and D. To do this a sum of the moments about D was conducted. This resulted in a R_A of 216062, R_D of –216062 and a relationship between H_D and H_A that is shown as equation (30).

$$H_D = H_D + 158479.8 + 93866.4 \qquad (30)$$

Now that the reactions are defined and the load on the structure is known, Eq. (30) can be rearranged and then integrated to yield Eq. (31). Eq. (32) is a representation of Eq. (33) with the specific reactions and its accompanying deflections known.

$$\mathrm{C} = \int_L \int_0^M d\theta dM - P\Delta \qquad (31)$$

$$0 = \int_L \frac{M}{EI}\frac{\delta M}{\delta HA}\delta z \qquad (32)$$

Next, since the deflection at HA is known, the corresponding version of Eq. (33) is used to determine the moment in section AB, BC, and DC in terms of R_A, the integrated loads, and H_A. Eqs. (33)-(35) show the equations for each section.

$$M_{ab} = -H_a z + \int_0^z \omega_f (.5817) z\delta z + + \int_0^z \omega_b (.6622) z\delta z \qquad (33)$$

$$M_{bc} = R_a z - H_a z + \int_0^l \omega_f (.5817) l\delta z + + \int_0^l \omega_b (.6622) l\delta z \qquad (34)$$

$$M_{bc} = -H_d z = (-H_a + \int_0^{30} \omega_f \delta z + + \int_0^{30} \omega_b \delta z) z \qquad (35)$$

Then these expressions are substituted into the last form of Eq. (32) and evaluated. The result is a definition for HA and therefore a definition of HD. For a detailed derivation see the structural analysis hand calculations included in the appendix. Eq. (36) shows HA as a function of l, the length of the wing.

$$H_A = -1.3(l^3) - 9.06(l^2) + 7773.76(l) - 145668.9 \quad (36)$$

Plugging this value of HA into Eqs. (33)-(35) results in the equations for the internal bending moment for the wing. After which, the derivative of these equations was taken to yield the equations of the internal shear stress. Table 8 shows these resulting equations that were used to produce figures 33-38.

Table 8: Lift, Shear, Bending Moment Equations for Free End Analysis

	Internal Shear Stress	Internal Bending Moment
Front	-7.0944z^3 - 30.80z^2 + 14101.4z - 44289.9	-1.77z^4 - 10.27z^3 + 7050.7z^2 - 44289.9z
Spar	216002.2	216002.2z+2083789
Back	-11.82z^3 - 104.9z^2 + 22803.4z - 44289.9	-2.95z^4 - 34.953z^3 + 11401.7z^2 - 44289.9z

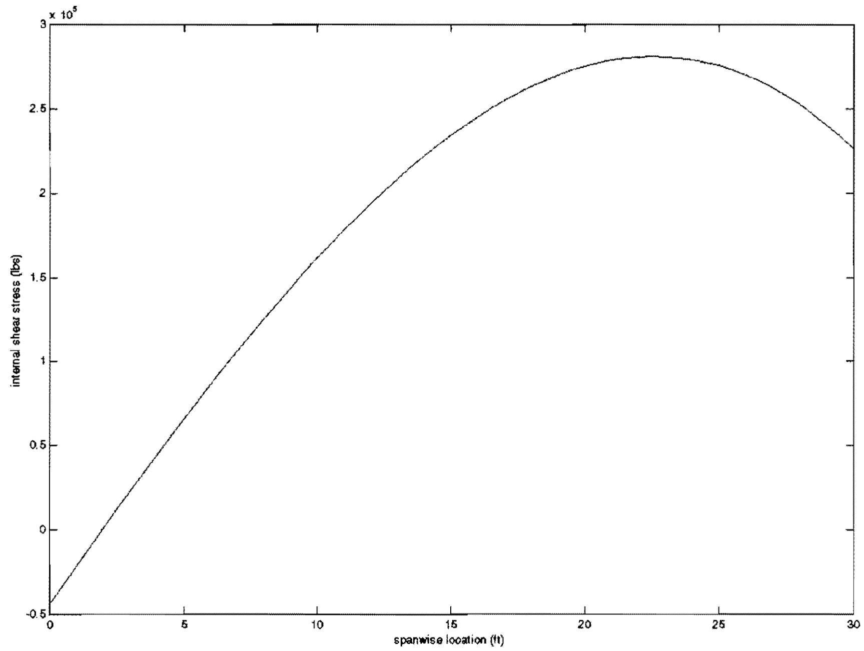

Fig. 35: Internal Shear Stress Diagram for the Front Wing

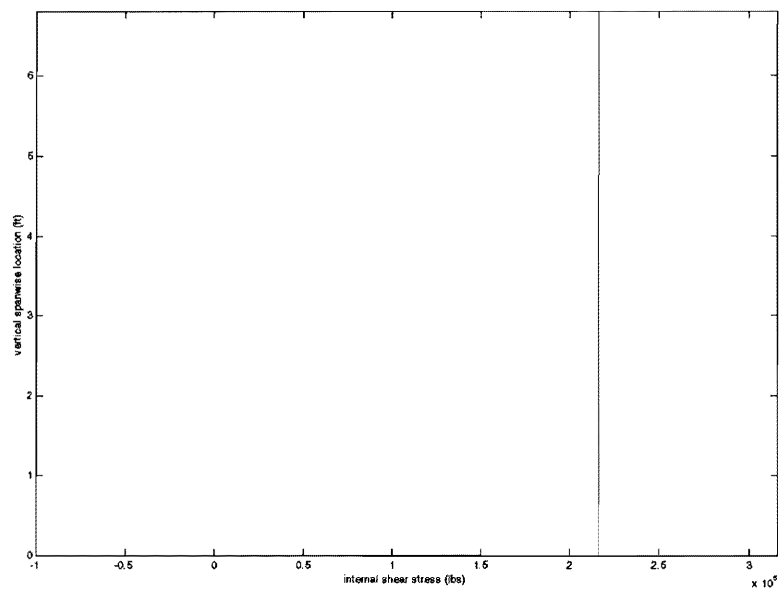

Fig. 36: Internal Shear Stress Diagram for the Spar

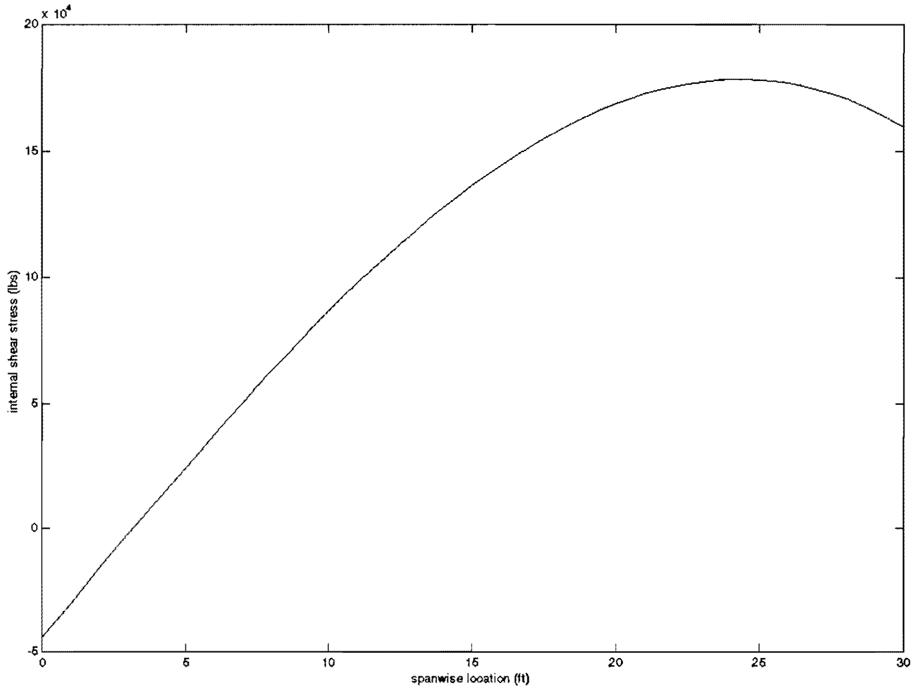

Fig. 37 Internal Shear Stress Diagram for the Back Wing

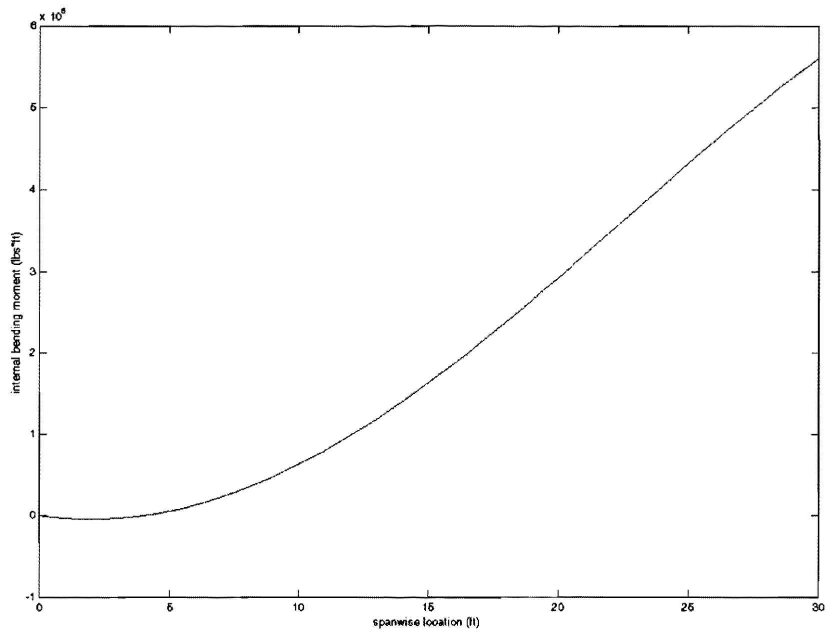

Fig. 38: Internal Bending Moment Diagram for the Front Wing

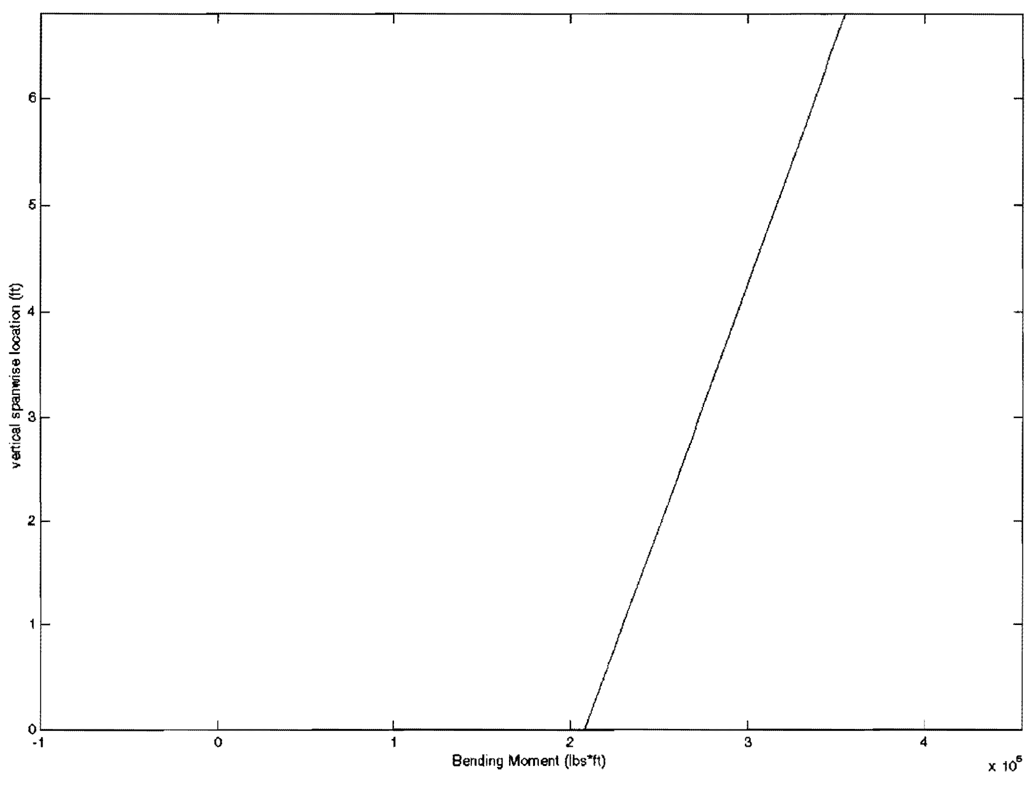

Fig. 39: Internal Bending Moment Diagram for the Spar

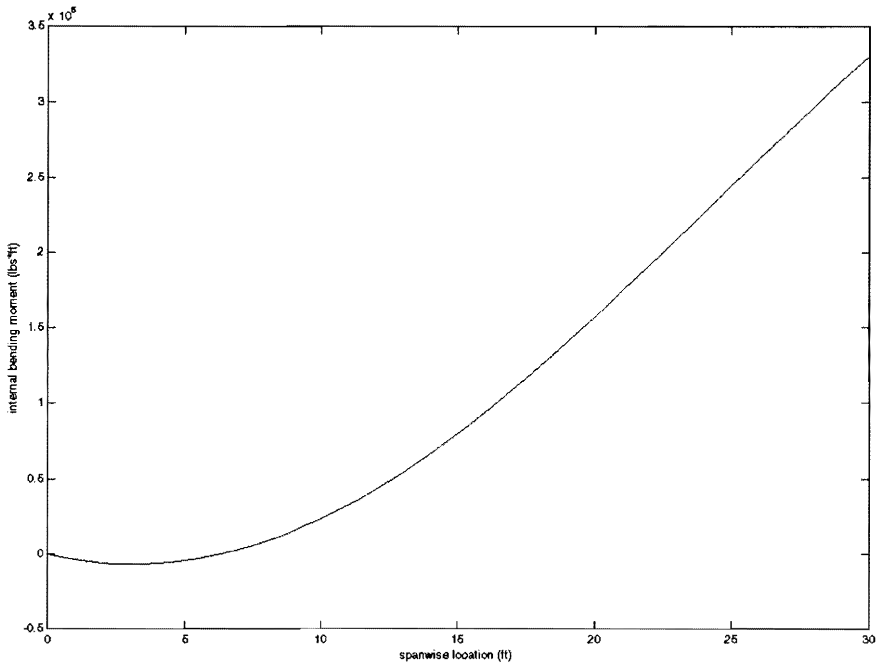

Fig. 40: Internal Bending Moment Diagram for the Back Wing

D. Material Selection

In order to select the proper material for the major load bearing members of the wing structure, the materials selected for similar aircraft in the past was heavily considered. The reason that historically chosen materials were given so much consideration is that these materials are readily available and their performance is well documented. Consequently, Aluminum alloy 2024-T4 was chosen as the material for the wing skins and Aluminum alloy 7075-T73 was chosen for the spars and stringers. Table 9 shows some important mechanical characteristics for these alloys.

Table 9: Properties of Aluminum 2024-T4 and Al 7075-T73

Alloy	Al 2024-T4	Al 7075-T73
Ultimate Strength Tension (ksi)	68	73.2
Ultimate Strength Shear (ksi)	41	43.5
Yield Strength Tension (ksi)	47	63.1
Elastic Moduli (x 10^6 psi)	10.6	10.4

E. Spar and Stringer Placement and Load Division

The locations of the main and rear spar were set at 25% and 70% of the chord to allow for a larger fuel tank system. The stringers begin at 15% of the chord and extend in 10in intervals to about the 50% spar. In addition to being located at the quarter chord, the main spar carries 55% of the load with the remaining load being divided as follows: the rear spar 20%, stringers 15%, and the wing skin 10%.

F. Component Design

Knowing the load division, the dimensions of the main and rear spars were determined by manual iteration with a MATLAB program that calculates the moment of inertia of the "I" beams. The total required moment of inertia was calculated by Eq. (36). Eqs. (37) and (38) represent the equations for the moment of inertia for an I beam where the variables s, d, b, h, and t are defined in Figure 39.

$$I_{req} = (M_D/\sigma_u)*(t_{max}/2) \qquad (36)$$

$$I_{xc}=((b*d\hat{}3)-(h\hat{}3)*(b-t))/12 \qquad (37)$$

$$I_{yc}=(2*s*(b\hat{}3)+h*(t\hat{}3))/12 \qquad (38)$$

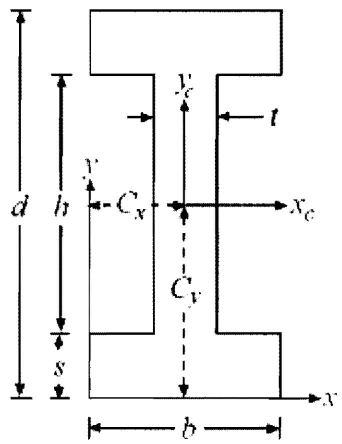

Fig. 41: I beam diagram and variables

The moment of inertia of the main spar was then set to be 55% of the total I_{req} and the moment of inertia of the rear spar was then set to 20% of I_{req}. To calculate the moment of inertia for the stringers, the stringer shape was neglected and Eq. (39) was used.

$$I_{st} = \Sigma A_n h_n^2 \qquad (39)$$

The number of stringers, ten, was determined by the location and spacing of the stringers. Table 10 shows the values for the variables shown in figure 41, which are the dimensions of the main and rear spars as well as the dimensions for the stringers.

Table 10: Necessary I_{req}, I_{xc}, I_{yc}, I_{main}, I_{rear}, and I_{st} values, Actual I_{req}, I_{xc}, I_{yc}, I_{main}, I_{rear}, and I_{st}, values, and corresponding dimensions for spars and stringers

Moments of Inertia	Required
Ireq	0.0019
Imain	0.0011
Irear	3.84E-04
Ist	2.88E-05
Main	**Calculated**
Ixc	0.013

Iyc	0.0011	
Rear		
Ixc	0.0031	
Iyc	4.00E-04	
Dimensions (ft)	**Main**	**Rear**
d	1.2	0.6
h	1.14	0.48
b	0.6	0.342
s	0.03	0.06
t	0.005833	0.005833

Wheel Sizing

The landing gear of the aircraft uses a tricycle configuration with each gear pod being comprised of a two-wheel landing gear system. A single landing gear pod is placed 20 ft from the nose in the center of the leading wing root and a pair of landing gear in pods is placed 36 feet further aft, 6 ft in front of the leading edge of the rear wing. This configuration will allow an even load distribution among the three landing gear pods and increase handling ease while on the ground. Analysis was accomplished to size the wheels required for each gear by using a center of gravity 38.5 ft from the nose of the aircraft and a conservative weight estimate of 41,000 lbs. A free body diagram of the landing gear system is shown below in Figure 40.

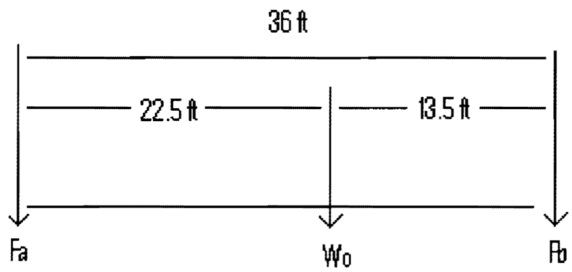

Figure 42 Free body of Landing Gear System

Solving for the forces in each wheel gives Fa = 15,375 lbs and Fb = 25,625 lbs. Next, following the format demonstrated on page 447 of Anderson's Aircraft Performance and Design, the diameter and width of each wheel in a two-wheel landing gear pod was found. For the equations below, F is the force load and W is the number of wheels distributing the load. For the forward landing gear, W = 2, while for the aft landing gear, W = 4 as there are two 2-wheel landing gear pods.

$$Diameter = 1.51 \times \left(\frac{F}{W}\right)^{0.349}$$

$$Width = 0.715 \times \left(\frac{F}{W}\right)^{0.312}$$

This analysis yielded a diameter dimension of 34.3 inches and width of 11.7 inches for the front wheel while the dimensions of each wheel in the back pods was calculated at 32.2 inch diameter and 11.1 inch width. Therefore, a diameter of 3 ft and a width of 1 ft is sufficient for use in both the front and back landing gear pods.

Table 11 Results from Wheel Dimension Analysis

	Diameter (inches)	Width (inches)
Forward Wheel	34.29	11.66
Aft Wheels	32.17	11.01

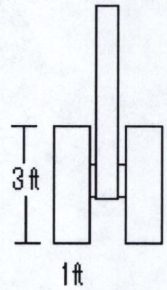

Figure 43 Dimensions of Landing Gear System

Interior Layout

After studying the interior layout of many planes of similar size and passenger loads, the basics of the seating arrangement for the DiamondCutter was established. The plane features ten rows of three seats with a forward lavatory and a rear galley. In addition, the aisle between the seats is six inches lower than the seat floor to allow for better movement of passengers while standing. Figures 44-46 show the interior layout of the plane as drawn in AutoCAD 2004.

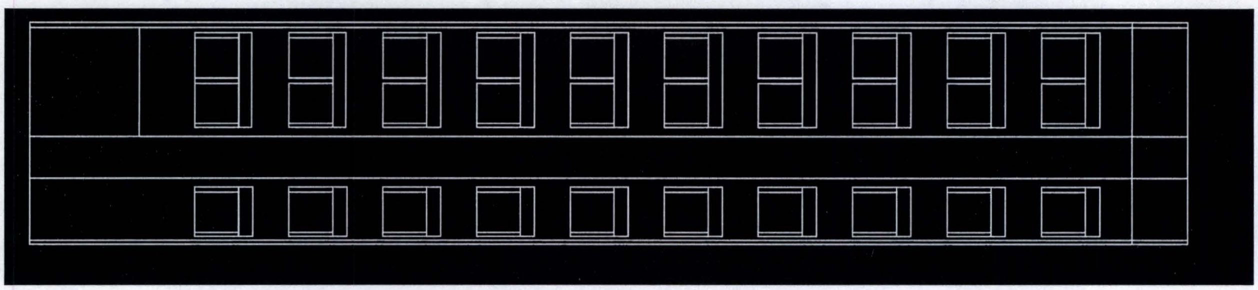

Fig 44: Top View of Interior Layout

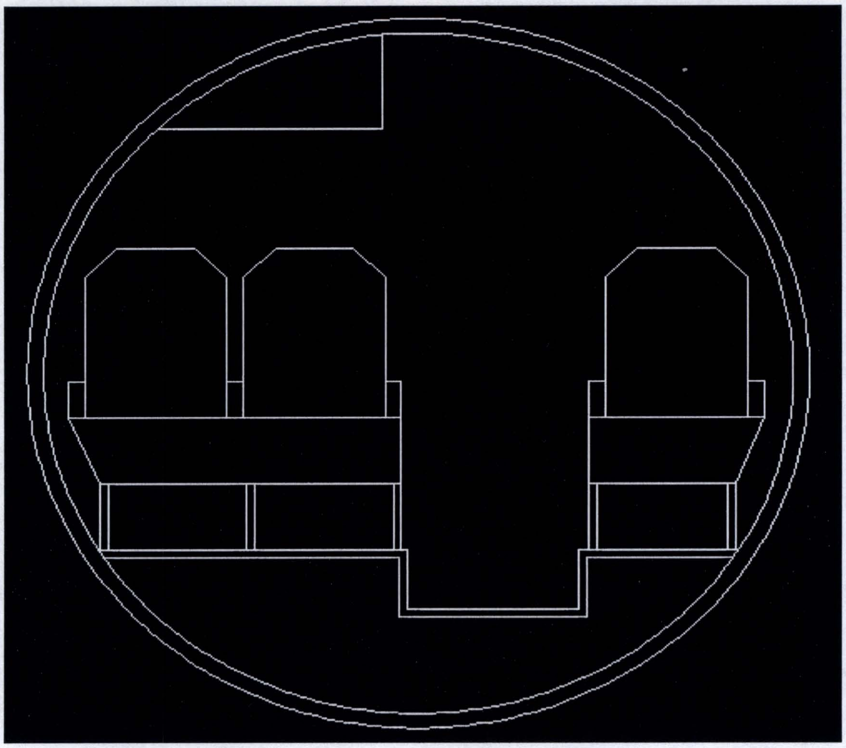

Fig 45: Cross Sectional View of the Passenger Compartment

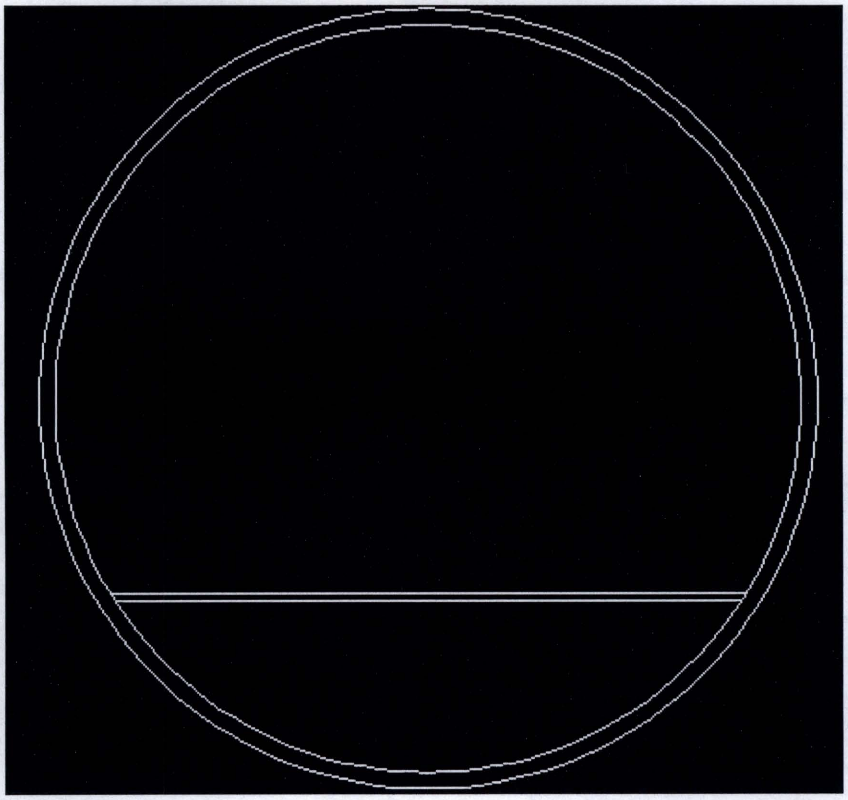

Fig 46: Cross Sectional View of the Baggage Compartment

Conclusion

In conclusion, it can be seen that the Diamond Cutter satisfies the requirements for both of the missions. The main components of the jet have been designed and shown on paper to fulfill what was asked of it. However, more work would need to be done before a full scale model of this plane would be ready to be built. This would include wind tunnel testing of a scale model along with a few smaller tasks such as deciding the placement of instruments, door placement and interior choices such as material, color, and so on.

References

[1]Anderson, J., *Aircraft Performance and Design*, WCB/McGraw-Hill, New York, 1999.

[2]Roskam, J., *Airplane Design I-VII*, DAR corporation, Lawrence, Kansas, 2000.

Appendix

Weight Estimate Iteration 1

WEIGHT ESTIMATE

Wcrew=	360
Wpayload=	5400

Empty weight (jet)

Wo=	26245
A=	1.02
C=	-0.06
Kvs=	1
We/Wo=	0.553933

Fuel Weight

SFCc=	0.5
SFCl=	0.4
L/D max=	16
L/D cruise=	13.856
L/D loiter=	16

Mission 1

1) Warm-up and takeoff	W1/Wo=	0.97	
2) Climb	W2/W1=	0.985	
3) Cruise	R=	1500 miles	7692000 feet
	SFC=	0.5 1/hr	
	V=	400 knots	460.3116 mph
	L/D=	13.856	
	W3/W2=	0.8891	
4) Loiter	E=	15 minutes	0.25 hours
	SFC=	0.4 1/hr	
	L/D=	16	
	W4/W3=	0.9938	
5) Land	W5/W4=	0.995	
6) Takeoff	W6/W5=	0.97	
7) Climb	W7/W6=	0.985	
8) Cruise	R=	750 miles	3846000 feet
	SFC=	0.5 1/hr	
	V=	400 ft/s	460.3116 mph
	L/D=	16	
	W8/W7=	0.95	
9) Loiter	E=	45 minutes	0.75 hours
	SFC=	0.4 1/hr	
	L/D=	16	

	W9/W8=	0.981
10) Land	W10/W9=	0.995

W5/Wo=	0.705498	27635
W10/W5=	0.786232	
Wf/Wo1=	0.312172	12228
Wf/Wo2=	0.226595	

Wo= 43018.88　　　Wo= 26244.78

Wo Guess	We/Wo	Wo
30000	0.549507	41642.18
40000	0.540103	38991.4
38000	0.541768	39435.82
39000	0.540924	39209.33
39171	0.540782	39171.47

Wo Guess	We/Wo	Wo
30000	0.549507	25725.9
25000	0.555551	26439.65
26000	0.554245	26282.12

We= 21182.97 lb
Wf= 12228.24 lb
Wo= 39171 lb

Mission 2

1) Takeoff	W6/W5=	0.97	
2) Climb	W7/W6=	0.985	
3) Cruise	R=	750 miles	3846000
	SFC=	0.5 1/hr	
	V=	400 ft/s	460.3116
	L/D=	13.856	
	W8/W7=	0.9429	
4) Loiter	E=	45 minutes	0.75
	SFC=	0.4 1/hr	
	L/D=	16	
	W9/W8=	0.9814	
5) Land	W10/W9=	0.995	
6) Takeoff	W6/W5=	0.97	
7) Climb	W7/W6=	0.985	
8) Cruise	R=	750 miles	3846000
	SFC=	0.5 1/hr	
	V=	400 ft/s	460.3116
	L/D=	16	
	W8/W7=	0.9504	
9) Loiter	E=	45 minutes	0.75
	SFC=	0.4 1/hr	
	L/D=	16	
	W9/W8=	0.9814	

10) Land	W10/W9=	0.995

Wo= 26245

mission 2

Wo guess	Wo
26000	26282
26200	26251
26250	26244
26245	

We= 14537
Wf= 5947
Wo= 26245

Weight Estimate Iteration 2

WEIGHT ESTIMATE

Wcrew= 360
Wpayload= 5400

Empty weight (jet)
Wo= 25951
A= 1.02
C= -0.06
Kvs= 1

We/Wo= 0.554308

Fuel Weight
SFCc= 0.48
SFCl= 0.407

L/D max= 16
L/D cruise= 13.856
L/D loiter= 16

Mission

1) Warm-up and takeoff	W1/Wo=	0.97
2) Climb	W2/W1=	0.985

3) Cruise	R=	1500 miles	7692000 feet
	SFC=	0.48 1/hr	
	V=	400 knots	460.3116 mph
	L/D=	13.856	
	W3/W2=	0.893252	
4) Loiter	E=	15 minutes	0.25 hours
	SFC=	0.407 1/hr	
	L/D=	16	
	W4/W3=	0.993661	
5) Land	W5/W4=	0.995	
6) Takeoff	W6/W5=	0.97	
7) Climb	W7/W6=	0.985	
8) Cruise	R=	750 miles	3846000 feet
	SFC=	0.48 1/hr	
	V=	400 ft/s	460.3116 mph
	L/D=	16	
	W8/W7=	0.952295	
9) Loiter	E=	45 minutes	0.75 hours
	SFC=	0.407 1/hr	
	L/D=	16	
	W9/W8=	0.981103	
10) Land	W10/W9=	0.995	

W5/Wo=	0.71201	26854.18	
W10/W5=	0.788923		
Wf/Wo1=	0.305269	11513.53	11600.22
Wf/Wo2=	0.223742	8502.192	

Wo=	41018.86	Wo=	25951.75

Wo Guess	We/Wo	Wo	Wo Guess	We/Wo	Wo
30000	0.549507	33568.56	30000	0.549507	25725.9
33000	0.546373	32966.55	25000	0.555551	26439.65
32960	0.546413	32974.05	26000	0.554245	26282.12
32970	0.546403	32972.18			
37716	0.542012	37716.26		0.554308	25951.75

We=	20442.51 lb	We=	14385.25 lb
Wf=	11513.61 lb	Wf=	5806.493 lb
Wo=	37716.26 lb	Wo=	25951.75 lb

Weight Estimate Iteration 3

WEIGHT ESTIMATE

Wcrew=	360		
Wpayload=	5400		
Empty weight (jet)			
Wo=	33684	20183.49	
A=	1.12		
C=	-0.06		
Kvs=	1		
We/Wo=	0.599201	23249	37925.07
Fuel Weight			
SFCc=	0.407		
SFCl=	0.407		
L/D max=	14.96		
L/D cruise=	12.95536		
L/D loiter=	14.96		

Mission 2

1) Warm-up and takeoff	W1/Wo=	0.97	
2) Climb	W2/W1=	0.985	
3) Cruise	R=	750 miles	3846000 feet
	SFC=	0.407 1/hr	
	V=	400 knots	460.3116 mph
	L/D=	12.96	
	W3/W2=	0.9501	
4) Loiter	E=	45 minutes	0.75 hours
	SFC=	0.407 1/hr	
	L/D=	14.96	
	W4/W3=	0.9798	
5) Land	W5/W4=	0.995	
6) Takeoff	W6/W5=	0.97	
7) Climb	W7/W6=	0.985	
8) Cruise	R=	750 miles	3846000 feet
	SFC=	0.407 1/hr	
	V=	400 ft/s	460.3116 mph
	L/D=	12.96	
	W8/W7=	0.9501	
9) Loiter	E=	45 minutes	0.75 hours
	SFC=	0.407 1/hr	
	L/D=	14.96	
	W9/W8=	0.9798	
10) Land	W10/W9=	0.995	

W10/Wo= 0.783212

Wf/Wo2= 0.229796 8916.072

Wo= 33683.57

Wo Guess	We/Wo	Wo
30000	0.549507	25212.25
25000		25897.4
32960		
32970		
33684	0.599201	33684

We= 20183.49 lb
Wf= 7740.339 lb
Wo= 33683.57 lb

Mission 1

1) Warm-up and takeoff	W1/Wo=	0.97	
2) Climb	W2/W1=	0.985	
3) Cruise	R=	1500 miles	7692000 feet
	SFC=	0.407 1/hr	
	V=	400 knots	460.3116 mph
	L/D=	12.96	
	W3/W2=	0.9027	
4) Loiter	E=	15 minutes	0.25 hours
	SFC=	0.407 1/hr	
	L/D=	14.96	
	W4/W3=	0.9932	
5) Land	W5/W4=	0.995	

38800

W10/Wo= 0.852349

Wf/Wo1= 0.15651 6072.603

Wo=	23578.68	
	23582.18	

Wo Guess	We/Wo	Wo
30000		25212.25
25000		25897.4
32960		
32970		
25779	0.554529	25778.96

We=	14295.2	lb
Wf=	3690.308	lb
Wo=	23582.18	lb

Weight Buildup

Weight Buildup

		Approx. factors			
S_{wing}=	495 ft^2	wing=	6 lb/ft^2	lavatories=	0.31 lb
$S_{fuselage}$=	2060.885 ft^2	fuselage=	3.5 lb/ft^2	MAC=	36.9 ft
S_{tail}=	321 ft^2	tail=	4 lb/ft^2		
TOGW=	33000 lb	land gear=	0.057 lb/ft^2		
Seats=	30	seats=	32 lb		
Instruments=	20	instruments=	1.5 lb		
Npass=	32				

Empty Weight				Input	
				$L_{fuselage}$=	82 ft
Wings=	11880.00 lb	Seats=	960	Lnose=	10 ft
Fnose=	7330.38 lb	Instruments=	30	Lmid=	52 ft
Fmid=	4574.16 lb	Lavatories=	31	Lrear=	20 ft
Frear=	2932.15 lb			$D_{fuselage}$=	8 ft
Tail=	1284.00 lb				
Land Gfront=	150.00				
Land Grear=	850.00 lb				
Engines=	993.00 lb				
Total=	29993.69 lb				

Total Weight	
Crew=	360 lb
Payload=	5400 lb
Fuel=	10000 lb

Total= 45753.69 lb

Approx. Location	
Wings=	14.76 ft

Input

A=	8.8	Aspect ratio	λ=	0.25	taper ratio
Bw1=	66 ft		Λ=	0.8248	wing sweep at 25% MAC
Bw2=	66 ft	wing span	(t/c)root=	0.15	thickness divide by chord
Ht/Hv=	0		Λvt=	0.8248	vertical wing sweep at 25%MAC
Iy=	0.000008		Av=	7	aspect ratio of vertical tail
K_{door}=	1.06		L/D=	14.96	
K_{Lg}=	1		Vstall=	169 ft/s	
Kmp=	1		flaps area=	100	
Kng=	1.017		elevator area=	75	
Knp=	1		aileron area=	90	
Kr=	1		n=	3.3	limit load factor
Ktp=	1				
Kws=	0.94231				
Kz=	20				
L=	52 ft				
La=	47				
Lec=	50				
Lf=	82				
Lm=	48				
Ln=	36				
Lt=	20				
Nc=	2				
Nen=	2				
Nf=	6				
Ngen=	2				
Nl=	4.5				
NLt=	0.5				
Nm=	1				
Nmss=	4				
Nmw=	4				
Nnw=	2				
Np=	32				
Nt=	4				
Nw=	0.333333				
Nz=	4.95				
Rkva=	50				
Scs=	265				
Scsw=	190				
Sf=	2060.885				

Sn=	0.166667
Svt=	321
Sw=	990
Vi=	1250
Vp=	0
Vpr=	2613.805
Vt=	10000 lb
Wc=	100 lb
Wdg=	38000 lb
Wec=	1168.944 lb
Wl=	38000 lb
Wuav=	400

W_{wing}=	12540.53 lb	Crew=	360 lb
$W_{vertical\ tail}$=	892.97 lb	Payload=	5400 lb
$W_{fuselage}$=	5460.66 lb	Fuel=	10000 lb
$W_{main\ LG}$=	1104.01 lb		
$W_{nose\ LG}$=	322.07 lb		
$W_{narcelle\ group}$=	55.41 lb		
$W_{engine\ conrol}$=	50.00 lb		
$W_{starter}$=	71.30 lb		
$W_{fuel\ system}$=	1134.99 lb		
$W_{flight\ controls}$=	172.21 lb		
$W_{instruments}$=	159.62 lb		
$W_{hydraulics}$=	173.25 lb		
$W_{electrical}$=	631.00 lb		
$W_{avionics}$=	624.99 lb		
$W_{furnishings}$=	115.56 lb		
$W_{air\ conditioning}$=	482.43 lb		
$W_{anti-ice}$=	76.00 lb		
$W_{handling\ gear}$=	11.40 lb		
W_{engine}=	993 lb		

Empty weight=	25071.41 lb
Total weight=	40831.41 lb

Drag Buildup

CD total	0.0763727	RWF	1	
Cdo	0.0449532	RLS	1	
		L'	2	
Component	CD	t/c	0.2	
CDL wing	0.0031892	Swet wing	2376	
CDo wing	0.0179325	S	495	
Cdofus	0.0058989	Cfwlam	0.001881	
Cdlfus	0.0001295	Cfwtur	0.0024	assume mac = 8
emp	0.0178863	Swetwinglam	23.6483	
Cdln	0.0000122	CLw	0.295444	
Cdon	0.0016311	A	8.8	
CDp	0.0016044	e	0.99	
CD flap	0.0165597	εt	0	chris may change
Cdintw	0.0029087	v	-0.0013	
Cdintf	0.0086201	w	0.0018	
		CL	0.295444	
D	7671.2617	CLc	0	
		Sc	0	
		Weight	40000	
		q	273.5144	

Cffus	0.0015	
lf	82	
df	8	
Swet fus	1800.202	est.
Cdbfus	0	
RNWlam	500000	cruise alt
α	0.05236	assumed 3 degrees
Sbfus	0	
η	0.69	
Cdc	1.2	
Splffus	539.2423	
density	0.000891	slugs/ft^3
velocity	675	ft/s
Cwlam	0.258564	wing reference length, laminar portion of wing pg 114
mu	3.11E-07	lbf*s/ft^2
lamba	10.25	

int		nacelle		flaps	
Fa1	1	S		Δcdprof flap	0.0126454
ΔCdn	1.77	Cffus	0.0012	swf	198.6
Cdn	0.0016	lf	7.9875	pΛc/4	0.06
Fa2	0.5	df	3.186667	cosΛc/4	0.5253
Sn	7.9756	Swet nac	69.84976	Δcdi flap	0.0018911
Cdn'	1.12	Cdbfus	0	K	0.3
		αn	0.05236	Clflap	0.2
		Splnar	25.4535	Δcdint flap	0.0020233
				Kint	0.4
				CD prof flap	0.0050582

horizontal tail		plyon	
Swet	400.32	Swet	14.68
cflam	0.001881	cflam	0.00176
L'	2	L'	1.2
t/c	0.2	t/c	0.375
S	83.4	S	26.68
Cftur	0.0024	Cftur	0.0018
Swetlam	8.745	Swetlam	8.97

Center of Gravity m.file

```
%Matlab code to calculate CG for Senior Design
%Jonathan Ford

clear all;

%center of gravity of each object distance from nose
dlgf = 20;
dlgr = 56;
dwingf = 27;
dwingr = 61;
dnose = 7;
dfuse = 36;
dfusetail = 69;
dengine = 64;
dtail = 77;
dcrew = 7;
dpayload = 35;
dfuelf = 27;
dfuelr = 61;
dfuelsystem = 36; %middle of fuselage, assumption
davionics = 6; %front of crew, instruments, etc....

%weight of each object
wlgf = 150;
wlgr = 850;
wwingf = 3226;
wwingr = 3226;
wnose = 2514.32;
wfuse = 5717.7;
wfusetail = 1073;
wengine = 1986;
wtail = 918.81;
wcrew = 360;
wpayload = 5400; %passenger weight
wfuelf = 5000; %fuel/2
wfuelr = 5000; %fuel/2
wfuelsystem = 1357.08;
wavionics = 500;

d1 = dlgf;
d2 = dlgr;
d3 = dwingf;
d4 = dwingr;
d5 = dnose;
d6 = dfuse;
d7 = dfusetail;
d8 = dengine;
d9 = dtail;
d10 = dcrew;
d11 = dpayload;
d12 = dfuelf;
d13 = dfuelr;
d14 = dfuelsystem;
d15 = davionics;
```

```
w1 = wlgf;
w2 = wlgr;
w3 = wwingf;
w4 = wwingr;
w5 = wnose;
w6 = wfuse;
w7 = wfusetail;
w8 = wengine;
w9 = wtail;
w10 = wcrew;
w11 = wpayload;
w12 = wfuelf;
w13 = wfuelr;
w14 = wfuelsystem;
w15 = wavionics;

weight = w1 + w2 + w3 + w4 + w5 + w6 + w7 + w8 + w9 + w10 + w11 + w12 + w13
+w14 +w15;

x = 0;

M = w1.*(x-d1) + w2.*(x-d2) + w3.*(x-d3) + w4.*(x-d4) + w5.*(x-d5) + w6.*(x-
d6) + w7.*(x-d7) + w8.*(x-d8) + w9.*(x-d9) + w10.*(x-d10) + w11.*(x-d11) +
w12.*(x-d12) + w13.*(x-d13) + w14.*(x-d14) + w15.*(x-d15);
while M<=0
   x = x + .001;
   M = w1.*(x-d1) + w2.*(x-d2) + w3.*(x-d3) + w4.*(x-d4) + w5.*(x-d5) +
w6.*(x-d6) + w7.*(x-d7) + w8.*(x-d8) + w9.*(x-d9) + w10.*(x-d10) + w11.*(x-
d11) + w12.*(x-d12) + w13.*(x-d13) + w14.*(x-d14) + w15.*(x-d15);
end

x
weight

d = [d1 d2 d3 d4 d5 d6 d7 d8 d9 d10 d11 d12 d13 d14 d15 x];
w = [w1 w2 w3 w4 w5 w6 w7 w8 w9 w10 w11 w12 w13 w14 w15 300];

bar(d,w,.7);
text(1,5500,['CG = ' num2str(x)]);
%text(1,5200,['Weight = ' num2str(weight)]);
text(dlgf-2,wlgf+100, ['LG frt']);
text(dlgr-4,wlgr+100, ['LG rear']);
text(dwingf-4,wwingf+100, ['Frt Wing']);
text(dwingr-5,wwingr+100, ['Rear Wing']);
text(dnose-2,wnose+100, ['Nose']);
text(dfuse-4,wfuse+100, ['Fuselage']);
text(dfusetail-4,wfusetail+100, ['FuseTail']);
text(dengine-2,wengine+100, ['Engine']);
text(dtail-3,wtail+100, ['VTail']);
text(dcrew,wcrew+100, ['Crew']);
text(dpayload-4,wpayload+100, ['Payload']);
text(dfuelf-4,wfuelf+100, ['Frt Fuel']);
text(dfuelr-4,wfuelr+100, ['Rear Fuel']);
text(x-2,400, ['CG']);
text(d14+.1,w14+100, ['Fuel Sys']);
text(d15-6,w15+120, ['Avionics']);
```

```
xlabel('Distance from Nose (ft)');
ylabel('Weight (lbs)');
title('Center of Gravity');
```

Take Off m.file

```
% Takeoff Calculation
clear all;
%   Inputs                   Units                Assumption or Note
% ---------------            -----              ----------------------
i        = 1;                                   % Counter 1
j        = 1;                                   % Counter 2
b        = 120;              % ft               % Wing span
Kuc      = 4.5e-5;                              % For moderate flap deflection
murbon   = 0.3;                                 % Dry Asphalt brakes on
murboff  = 0.04;                                % Dry Asphalt brakes off
W        = 40000;            % lbs              % Takeoff weight
S        = 900;              % ft^2             % Wing Planform Area
rhoinf   = 0.002369;         % slugss/ft^3      % Sea Level
CDo      = 0.0192;
CLmax    = 1.6;
CL       = 0.2;                                 % At 0 deg angle of attack
e        = 0.95;                                % Assumption
AR       = b^2/S;                               % Aspect Ratio
Engines = 1.1;                                    % Number of engines
T        = 7400*Engines;      % lbs              % Maximum Takeoff Thrust
pw308b
K        = 0.1062;
k1       = 0.0265;
h1       = 5.5;               % ft              % Average height of front wing
h2       = 8.5;               % ft              % Average height of rear wing
N        = 3;                 % s               % Time of rotation for large
aircraft

% Analysis
% -----------------------------------------------------------------------

i=1;
Vstall = sqrt((2/rhoinf)*(W/S)*(1/CLmax));
Vlo    = 1.1*Vstall;
Vinf   = 0.7*Vlo;
Kt     = (T/W - murboff);                       % 0.7  Vlo
%CD     = CDo + K*CL^2;

% Conversions
lb     = 4.448;              % N
ft     = 0.3048;             % m
lbm    = 0.4536;             % kg

Wm     = W*lb;
Sm     = S*ft^2;
mm     = W*lbm;
delCDo = (Wm/Sm)*Kuc*mm^-0.215;
G1     = (((16*h1)/b)^2)/(1+((16*h1)/b)^2);
G2     = (((16*h2)/b)^2)/(1+((16*h2)/b)^2);
G      = (G1 + G2)/2;
Ka     = (rhoinf/(2*(W/S)))*(CDo+delCDo+(k1+(G/(pi*e*AR)))*CL(1)^2-
murboff*CL);
%sg     = (1/(2*32.2*Ka))*log(1+(Ka/Kt)*Vlo^2);
CD     = CDo + delCDo + (k1 + (G/(pi*e*AR)))*CL(1)^2;
CLlo   = W/(0.5*rhoinf*Vlo^2*S);
```

```
%fprintf('\n   Iter  Position (ft)    Velocity (ft/s)    Acceleration
(ft/s^2)   Drag (lbs)');
%fprintf('\n   ----  ------------     ----------------   --------------------
----------')

% Calculation of takeoff time with no rotation included

CD(i)            = CDo + delCDo + (k1 + (G/(pi*e*AR))).*CL(i).^2;
%takeoff time(i)  =
takeofftime(Vlo,rhoinf,S,T,murboff,W,CDo,delCDo,k1,G,e,AR,CD);
%filler           = takeofftime(j);

% Including Rotation
%takeofftime2(i) =
takeofftime2(rhoinf,S,T,murboff,W,CLlo,N,filler,CDo,delCDo,k1,G,e,AR,CD,CL,z)
;

V(i)       = 0;
x(i)       = 0;
dt         = 0.1;

ii=1;
while V(ii) < Vlo
D(ii)     = 0.5*rhoinf*V(ii).^2*S*CD;
a(ii)     = (T-D(ii)-murboff*W)/(W/32.2);
%data     = [ii,x(ii),V(ii),a(ii),D(ii)];
%fprintf('\n  % 2.0f      %6.2f            %6.2f                  %6.2f
%6.2f',data);
V(ii+1)   = V(ii) + a(ii).*dt;
x(ii+1)   = x(ii) + ((V(ii) + V(ii+1))./2)*dt;
D(ii+1)   = D(ii);
a(ii+1)   = a(ii);
ii        = ii+1;
end

y        = ii/10;

i = 1;
V(i)       = 0;
x(i)       = 0;
a(i)       = 0;
D(i)       = 0;
dt         = 0.1;


for time = (0:.1:y-N)
        D(i)     = 0.5*rhoinf.*V(i).^2.*S.*CD(i);
        a(i)     = (T-D(i)-murboff*W)/(W/32.2);
        %data     = [i,x(i),V(i),a(i),D(i)];
        %fprintf('\n  % 2.0f      %6.2f            %6.2f                  %6.2f
%6.2f',data);
        V(i+1)   = V(i) + a(i).*dt;
        x(i+1)   = x(i) + ((V(i) + V(i+1))./2)*dt;
        D(i+1)   = D(i);
        a(i+1)   = a(i);
        CL(i+1)  = CL(i);
```

```
        CD(i+1) = CD(i);
        i=i+1;
end

while CL(i) < CLlo
    CD(i)     = CDo + delCDo + (k1 + (G/(pi*e*AR))).*CL(i).^2;
    D(i)      = 0.5*rhoinf.*V(i).^2.*S.*CD(i);
    a(i)      = (T-D(i)-murboff*W)/(W/32.2);
    %data      = [i,x(i),V(i),a(i),D(i)];
    %fprintf('\n   %2.0f      %6.2f            %6.2f                 %6.2f
%6.2f',data);
    V(i+1)    = V(i) + a(i).*dt;
    x(i+1)    = x(i) + ((V(i) + V(i+1))./2)*dt;
    D(i+1)    = D(i);
    a(i+1)    = a(i);
    CL(i+1)   = CL(i)+(CLlo-CL(1))/(N/dt);
    CD(i+1)   = CD(i);
    i=i+1;
end

if x(i-1) > 1400
    extra = x(i-1)-1400;
    fprintf('\n Takeoff distance is %6.2f ft so takeoff is %6.2f to
long.',x(i-1),extra)
end
if x(i-1) <= 1400
    spare = 1400 - x(i-1);
    fprintf('\n  Takeoff requirement satisfied. %6.2f  ft to spare!',spare)
end


% %figure (1)
% %clf reset;
% %subplot(2,2,1);
% %plot(x,V);
% xlabel('Position (ft)');
% ylabel('Velocity (ft/s)');
% title('Velocity vs. Position');
% axis([0,max(x),0,max(V)]);
% grid on
%
% subplot(2,2,2);
% plot(x,a);
% xlabel('Position (ft)');
% ylabel('Acceleration (ft/s^2)');
% title('Acceleration vs. Posistion');
% axis([0,max(x),min(a),max(a)]);
% grid on
%
% subplot(2,2,3);
% plot(x,D);
% xlabel('Position (ft)');
% ylabel('Drag (lbs)');
% title('Drag vs. Position');
% axis([0,max(x),0,max(D)]);
% grid on
% Iterations
```

```
tol = 0.1;
 %while abs(takeofftime2(j)-filler) > tol
%       filler              = takeofftime2(j);
%       takeofftime2(j+1) =
takeofftime2(rhoinf,S,T,murboff,W,CLlo,N,filler,CDo,delCDo,k1,G,e,AR,CD,CL);
%       j                   = j+1;
     %end

 % Final Takeosff approximation

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```

Landing m.file

```
% Landing Performance

clear all;

%   Inputs                  Units                Assumption or Note
% ---------------           -----              ----------------------
i        = 1;                                  % Counter 1
j        = 1;                                  % Counter 2
b        = 120;             % ft               % Wing span
Kuc      = 4.5e-5;                             % For moderate flap deflection
murbon   = 0.3;                                % Dry Asphalt brakes on
murboff  = 0.04;                               % Dry Asphalt brakes off
W        = 41486;           % lbs              % Takeoff weight
WL       = W-10000          % lbs              % Landing weight
S        = 900;             % ft^2             % Wing Planform Area
rhoinf   = 0.002369;        % slugss/ft^3      % Sea Level
CDo      = 0.0192;
CLmax    = 1.6;
CL       = 0.2;                                % At 0 deg angle of attack
e        = 0.95;                               % Assumption
AR       = b^2/S;                              % Aspect Ratio
Engines  = 2;                                  % Number of engines
T        = 7400*Engines;     % lbs             % Thrust
Trev     = 0.35*T*Engines;    % lbs             % Reverse Thrust
K        = 0.1062;
k1       = 0.0265;
h1       = 5.5;              % ft              % Average height of front wing
h2       = 8.5;              % ft              % Average height of rear wing
N        = 3;                % s               % Time of free roll for large
aircraft

% Analysis
% ----------------------------------------------------------------------

Vstall = sqrt((2/rhoinf)*(W/S)*(1/CLmax));
Vf     = 1.23*Vstall;
Vtd    = 1.15*Vstall;
R      = Vf^2/(0.2*32.2);
%Vapp   =...
%CD     =...
%D      = 0.5*rhoinf*Vapp^2*CD
%apangl = arcsin((D-T)/W);
apangl = 3;
hf     = R*(1-cos(apangl*(pi/180)));
sa     = (50-hf)/tan(apangl*(pi/180));
sf     = R*sin(apangl*(pi/180));

% Ground roll
Jt     = Trev/W + murbon;
% Conversions
lb     = 4.448;             % N
ft     = 0.3048;            % m
lbm    = 0.4536;            % kg

Wm     = W*lb;
```

```
Sm      = S*ft^2;
mm      = W*lbm;
delCDo = (Wm/Sm)*Kuc*mm^-0.215;
G1      = (((16*h1)/b)^2)/(1+((16*h1)/b)^2);
G2      = (((16*h2)/b)^2)/(1+((16*h2)/b)^2);
G       = (G1 + G2)/2;
CD      = CDo + delCDo + (k1 + (G/(pi*e*AR)))*CL^2;
Ja      = (rhoinf/(2*(W/S)))*(CDo + delCDo + (k1 + (G/(pi*e*AR)))*CL^2 -
murbon*CL);
sfr     = N*Vtd;
sg      = sfr + (1/(2*32.2*Ja))*log(1+(Ja/Jt)*Vtd^2);

total  = sg + sf + sa;

i = 1;
V(i)       = Vtd;
x(i)       = 0;
dt         = 0.1;

fprintf('\n   Iter  Position (ft)    Velocity (ft/s)    Acceleration (ft/s^2)
Drag (lbs)');
fprintf('\n   ----  ------------     ----------------   --------------------
----------');

while V(i) > 0;
    D(i)   = 0.5*rhoinf*V(i).^2.*CD*S;
    a(i)   = (D(i)+Trev+murboff*WL)/(WL/32.2);
    data   = [i,x(i),V(i),a(i),D(i)];
    fprintf('\n   %2.0f      %6.2f           %6.2f                %6.2f
%6.2f',data);
    x(i+1) = x(i) + V(i)*dt;
    V(i+1) = V(i) - a(i).*dt;
    a(i+1) = a(i);
    D(i+1) = D(i);
    i = i+1;
end
total2  =  x(i) + sf + sa;

fprintf('\n   The landing ground roll is %6.2f ft',x(i))
fprintf('\n   The total distance to land over a 50 ft obstabcle is %6.2f
ft',total2)

i    = 1;
x(i) = 0;
dx   = 0.001;
t    = (0:total*dx:total);

% while x(i) <= total
%     if x(i) <= sa
%         y(i)   = 50 - sin(apangl*(pi/180))*x(i);
%         x(i+1) = x(i) + total*dx;
%         y(i+1) = y(i);
%         i      = i+1;
%
%     elseif x(i) > sa
%         y(i)   = R*(1-cos(apangl*(pi/180)));
%         apangl = apangl - apangl.*dx;
```

```
%          x(i+1) = x(i) + total*dx;
%          y(i+1) = y(i);
%          i      = i+1;
%      else x(i) > sa + sf
%          y(i)   = 0;
%          x(i+1) = x(i) + total*dx;
%          y(i+1) = y(i);
%          i      = i+1;
%      end
% end


figure (1)
clf reset;
subplot(2,2,1);
plot(x,V)
title('Velocity vs. Landing Distance')
xlabel('Total Landing Distance (ft)')
ylabel('Velocity of Jet (ft/s)')
axis([0,max(x),0,Vtd])
grid on

subplot(2,2,2);
plot(x,a);
xlabel('Position (ft)');
ylabel('Acceleration (ft/s^2)');
title('Acceleration vs. Posistion');
axis([0,max(x),min(a),max(a)]);
grid on

subplot(2,2,3);
plot(x,D);
xlabel('Position (ft)');
ylabel('Drag (lbs)');
title('Drag vs. Position');
axis([0,max(x),0,max(D)]);
grid on


ÿ
```

V-n m.file

```
% V-n

clear all;

%   Inputs                    Units              Assumption or Note
% ---------------            -----            ----------------------
i            = 1;                               % Counter 1
b            = 120;            % ft             % Wing span
W            = 40000;          % lbs            % Takeoff weight
S            = 900;            % ft^2           % Wing Planform Area
rhoinf       = 0.002369;       % slugss/ft^3    % Sea Level
CLmax        = 1.6;
CLmaxn       = 1;
nmaxp        = 0;
nmaxn        = 0;
positive(i)  = 3.8;
negative(i)  = -1.8;
Vmax         = 550;

% Cornering Velocity
vcorn    = sqrt(((2*nmaxp)/(rhoinf*CLmax))*(W/S));

% V-n Diagram

% Need to define the ultimate load factor here
uppult = 5.5;
lowult = -3;

% Limit Load Factor
V1(i) = 0;
while V1(i) <= Vmax
    nmaxp(i)    = 0.5*rhoinf*V1(i)^2*(CLmax/(W/S));
    nmaxp(i+1) = nmaxp(i);
    V1(i+1)     = V1(i) + 1;

    if nmaxp(i) >= positive(1)
    positive(i+1)   = positive(1);
    else
    positive(i+1) = 0;
    r=i+1;
    end
    if nmaxp(i) >= uppult(1)
        uppult(i+1) = uppult(1);
    else
        uppult(i+1) = 0;
        v = i+1;
    end
i= i+1;
end
i = 1;
V2(i) = 0;
while V2(i) <= Vmax
    nmaxn(i)    = -0.5*rhoinf*V2(i)^2*(CLmaxn/(W/S));
    nmaxn(i+1) = nmaxn(i);
    V2(i+1)     = V2(i) + 1;
```

```
        if nmaxn(i) <= negative(1)
        negative(i+1)  = negative(1);
        else
        negative(i+1) = 0;
        s=i+1;
        end
        if nmaxn(i) <= lowult(1)
            lowult(i+1) = lowult(1);
        else
            lowult(i+1) = 0;
            p = i+1;
        end
        i           = i+1;
    end


    % References
    x1 = (0:.1:550);
    y1 = 0;
    x2 = 550;
    y2 = [nmaxp:.1:nmaxn];

    % Plot the V-n Diagram
    figure(1)
    clf reset
    plot(V1,nmaxp,'bl',V2,nmaxn,'bl',V1((r+1):max(V1)),positive((r+1):max(V1)),'g
    ',...
        V2((s+1):max(V2)),negative((s+1):max(V2)),'g',x1,y1,x2,y2,...

    V1((v+1):max(V1)),uppult((v+1):max(V1)),'r',V2((p+1):max(V2)),lowult((p+1):ma
    x(V2)),'r')
    title('V-n Diagram')
    xlabel('Velocity (ft/s)')
    ylabel('Load Factor, n')
    text(150,5,'Stall Area')
    text(150,-5,'Stall Area')
    text(450,4.5,'Limit Load')
    text(450,-2.3,'Limit Load')
    text(450,1,'Safe to Fly')ÿÿ
```

Range m.file

```
% Range

clear all;
rho = 0.00089068;                          % slugs/ft^3
S   = 900;                                 % ft^2
wf(1) = 10000;                             % lb
we = 36763-wf(1);                          % lb
V  = 675.12;                               % ft/s
Tr   = 2769.99 * 2 ;                       % lbf
CD  = Tr/ (.5 * rho * V^2 * S);            % Coefficient of Drag
TSFC = 0.407 ;                             % 1/hr
ct = TSFC/ 3600 ;                          % 1/s
mdotf = (ct * Tr) ;                        % lbf/s
dt = 600;                                  % Time
CL5_CDmax = 19.8489;
PA = 2200436;                              % Power Avail
PR = 744616.5;                             % Power Req'd
PA2= 3566779;                              % Power Avail
PR2= 1024946;                              % Power Req'd
wt=36763;                                  % Weight Total
h = 30000;                                 % Cruise Alt
TOF = 318 ;                                % lb of fuel
LOF = 67.19548789;                         % lb of fuel
LF  = 53;                                  % lb of fuel

tmax = 1800;
z = 2;
i = 1;
Vinf= 340;
RC = (PA - PR)/ wt
tcl = h / RC
tmin = tcl / 60
L_Dmax = 14.96 ;
climb_angle = asin(RC/Vinf) * (180/ pi)
climb_angle_max1 = asin( (Tr/wt) - (4 * 0.0192 * 0.04871555)^(1/2)) * (180 /
pi)
climb_angle_max = asin( (Tr/wt) - (1/ (L_Dmax)))* (180/ pi)

wfafterclimb = wf - mdotf * tcl;
wfacl = mdotf * tcl;
CLclimb  = wt/ (.5 * rho * V^2 * S);
    Rclimbft = (2/ct)* ( 2 / ( rho * S))^.5 * (CLclimb^ (.5)/ CD ) * ( wt^.5
-(we+ wfafterclimb) ^ .5);
    Rclimb = Rclimbft / 5280;
 wfforcruise = wfafterclimb - TOF - LOF - LF ;
while wfforcruise(i) >= 1500

i = 1;
for t= 0 : dt : tmax ;
   % w(i) = wf(i) + we ;
    w(i) = wfforcruise(i) + we;
    CL(i)  = w(i)/ (.5 * rho * V^2 * S);
    wfforcruise(i+1) = wfforcruise(i) - mdotf * dt;
    w(i+1) = wfforcruise(i+1) + we  ;
    i = i+1;
```

```
end

tmax = tmax + 600;
z = z + 1;
end
z;


    R = (2/ct)* ( 2 / ( rho * S))^.5 * (CL(i-1)^ (.5)/ CD ) * ( w(1)^.5 -
w(i) ^ .5);
    Rmax = (2/ct)* ( 2 / ( rho * S))^.5 * (CL5_CDmax) * ( w(1)^.5 - w(i) ^
.5)/5280;
    Rend = (R/5280) + Rclimb
```