To the Graduate Council:

I am submitting herewith a thesis written by Raed Hijer entitled "A Study of Local Observability and Its Usefulness." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

James C. Hung, Major Professor

We have read this thesis
and recommend its acceptance:

Don Hinton

Marshall Pace

Accepted for the Council:

The Graduate School

A Study of Local Observability and Its Usefulness

A Thesis

Presented for the

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Raed Hijer

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ABSTRACT

One of the major problems associated with any control system involving measurements is the *observability*. This problem can be looked upon either globally or locally. In the global sense, we are interested in exploring the observability characteristics of the system in general i.e., in the whole period of studying the system behavior. On the other hand, in the local sense, we might be interested in studying the observability effect on the system within a short period of time. Generally speaking, local observability concept is more practical in investigating the observability of control systems. This is due to the fact that some of the control systems are partially unobservable. That is, they may be unobservable only within a certain period of time. In this case, it is more practical to examine the local observability characteristics of such systems rather than their global observability characteristics.

A useful measure of the local observability is introduced by the concept of *the local observability matrix*. This matrix gives an indication of the degree of the system observability at each instant. The conditional number of the local observability matrix can be used as a measure of local observability.

In this study, a Kalman filter which is used in the Terrain-Aided Inertial Navigation System (INS) is considered as an application of the above measure. The standard deviations of the estimated parameters of the system were considered to represent the estimation error of these parameters. It turns out from the simulated results that when the local observability concept was employed in the case of an unobservable or partially observable systems, no substantial improvement in the convergence process of the estimation error was noticed. This is due to the fact that the adaptive concept for updating the system state parameters is imbeded in the structure of the Kalman filter. That is, the gain of Kalman filter is automatically reduced whenever the system is unobservable

at any instant. The simulation results also show that strong observability is associated with low navigation error and weak observability associated with high navigation error.

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CHAPTER 1

Introduction

Any control system that involves measurements in estimating its state parameters has the problem of observability. Therefore, the need arises for a measure that indicates the underlying system's observability. Several techniques were established to give an indication of such measure for time varying as well as time invariant systems. Additionally, these techniques explore the system's observability characteristics in both global and local sense. Local observability matrix, to be considered in this study, is one of these techniques which is used to investigate the local observability characteristics of the system. An example of a Kalman filter in Inertial Navigation System (INS) is then considered as an application of the above concept. This system involves measurement of its state parameters (location and velocity of a moving vehicle). Therefore, the observability is a major problem associated with this system.

In the INS, the error covariance matrix was chosen as a measure of how close the estimated state parameters are from their error corrupted measurements. It has been shown that these error values will grow with time during the period where the observability of the system is very low. This phenomenon was demonstrated by some examples. The behavior of the error values was studied for these examples under various observability levels.

Chapter 2 introduces the concept of local observability matrix and how it is utilized in a Kalman filtering scheme. It has been shown that no substantial improvement of the convergence of the error covariance values was achieved by utilizing the information provided by this matrix.

In Chapter 3, we briefly discuss the derivation of discrete Kalman filter equations and Kalman filtering algorithm to be used throughout the course of this

study.

In Chapter 4, a model for a terrain-aided inertial navigation system is developed and the operation of each component in this model is discussed. It also shows how all other input values needed for a Kalman filtering scheme are precomputed before starting the updating process of the state parameters by a Kalman filter. This chapter also shows how the concept of local observability matrix is used as a measure of the system observability.

Chapter 5 shows some simulated results for different paths of the aircraft with different levels of observability. A comparison is also made among these paths for the case when the concept of local observability matrix is employed to the system and when it is not. Finally, Chapter 6 summarizes the results of this study.

CHAPTER 2

System Observability

I Introduction

A discussion of observability requires that the concept of *measurement* be employed. Measurements are almost involved in every control system. At this point it suffices to say that measurements can indeed be made. They are denoted by $\underline{z}_k$ and are assumed linearly related to the discrete system state $\underline{x}_k$ by the observation matrix H_k , as

$$\underline{z}_k = H_k \underline{x}_k + \underline{v}_k \tag{1.1}$$

where $\underline{v}_k$ is the measurement noise. Given a sequence of measurements $\underline{z}_0, \underline{z}_1, \cdots, \underline{z}_k$, the *observability condition* defines our ability to determine $\underline{x}_0, \underline{x}_1, \cdots, \underline{x}_k$ from the measurements.

Consider the discrete deterministic, constant n^{th} order system [1][1] (here, a source-free system was considered to demonstrate the effect of observability on its states.)

$$\underline{x}_{k+1} = \Phi \underline{x}_k \tag{1.2}$$

with n scalar noise-free measurements

[1]A time-invariant discrete system is denoted by $\Phi_k = \Phi$, $H_k = H$ etc.

$$\underline{z}_k \;=\; H\;\underline{x}_k\;, \quad k \;=\; 0,\; 1,\; \cdots\;,\; n-1 \tag{1.3}$$

so that H is a constant, n-dimensional row vector. We may write

$$\begin{aligned} z_0 &= H\;\underline{x}_0 \\ z_1 &= H\;\underline{x}_1 \;=\; H\;\Phi\;\underline{x}_0 \\ &\cdot \\ &\cdot \\ &\cdot \\ z_{n-1} &= H\;\underline{x}_{n-1} \;=\; H\;\Phi^{n-1}\;\underline{x}_0 \end{aligned}$$

Therefore,

$$\begin{bmatrix} z_0 \\ z_1 \\ \vdots \\ z_{n-1} \end{bmatrix} = \begin{bmatrix} H \\ H\;\Phi \\ \vdots \\ H\;\Phi^{n-1} \end{bmatrix} \underline{x}_0 \;=\; M^T\;\underline{x}_0 \tag{1.4}$$

If $\underline{x}_0$ is to be determined uniquely, the matrix M^T (or equivalently M) must have an inverse – i.e., be nonsingular. Thus, the observability condition is that the matrix

$$M \;=\; \left[H^T \;\; \Phi^T H^T \;\; \cdots \;\; (\Phi^T)^{n-1} H^T\right] \tag{1.5}$$

be of a rank n. The same condition will apply when measurements are a *vector*, $\underline{z}$; extension to continuous systems is straightforward.

More complete statements of observability and the observability condition for continuous systems follows:

- A system is observable at time $t_1 > t_0$ if it is possible to determine the state $\underline{x}(t_0)$ by observing $\underline{z}(t)$ in the interval (t_0, t_1). If *all* states $\underline{x}(t)$ corresponding to all $\underline{z}(t)$ are observable, the system is *completely observable.*

- A continuous deterministic n^{th} order, constant coefficient, linear dynamic system is observable if, and only if, the matrix

$$M = \left[H^T \quad \Phi^T H^T \quad (\Phi^T)^2 H^T \quad \cdots \quad (\Phi^T)^{n-1} H^T\right] \tag{1.6}$$

  of a rank n.

The state vector equations, given by Eqs. (1.1) and (1.2), make evident connections between the internal variables (states) used to describe the system. This chapter introduces a fundamental concept dealing with the interactions of the output with these states. Roughly speaking, the property of complete observability implies that the output signal is influenced by each state variable in an independent fashion. Thus if a state vector equation possesses this property there are no state variables which are disconnected from the output.

II Observability of Time Varying Systems

The above concept of observability was dealing with the observability of time-invariant systems. This concept can be extended to the time varying systems as

follows:

Consider a discrete time-varying system given by the equations

$$\underline{x}_{k+1} = \Phi_k \, \underline{x}_k \tag{2.1}$$

with n scalar noise-free measurements

$$\underline{z}_k = H_k \, \underline{x}_k \tag{2.2}$$

where each component of the above equations is defined as earlier with the exception that the transition matrix , Φ_k , and the measurement matrix , H_k , are now time dependent matrices.

Following the same steps for the case of time invariant system, the observability matrix for the time-varying system is given by:

$$M_k^T = \begin{bmatrix} H_k \\ H_{k+1}\Phi_k \\ H_{k+2}\Phi_{k+1}\Phi_k \\ \vdots \\ H_{k+n-1}\Phi_{k+n-2}\ldots\Phi_k \end{bmatrix} \tag{2.3}$$

Hence, the observability condition for the underlying time-varying system is that the matrix M_k^T or equivalently M_k be of a rank n for all k . In addition, the closeness of M_k^T from being of rank n can be used as a measure of observability.

III Global Observability and Local Observability

So far the above concepts of observability are the traditional definitions of system observability in global sense, i.e., the system is considered to be observable if all the states were capable of exciting the output during the whole range of studying period of the system. However, for many reasons, one or more of the system states may not be observable during certain range of studying period. This will lead to a system which is partially observable, in other words, unobservable only within that range. The concept that deals with the observability of such kind of systems is what is called *the local observability concept.* In this concept, the observability characteristics of the system are investigated within different periods of time in an independent manner. This procedure allows us to explore the behavior of the system during different ranges of the studying process rather than giving a judgement of the observability of the system in the whole range.

IV Local Observability of the System and the Concept of Local Observability Matrix

From the past discussion, one can see that the local observability concept is more useful than the global observability concept in dealing with time-varying systems in general. Consequently, the need arises to have a measure of the observability of the systems in a local sense. Several techniques were established to give an indication of this measure.

It is well known that the observability of the system can be determined by using grammian matrix

$$V = \sum_{k=1}^{N} \Phi_k^T H_k^T H_k \Phi_k \tag{4.1}$$

where N is a finite time index indicating the time period over which the system activity is taking place [2, 3]. This grammian matrix is not convenient to use because of its summation nature. In addition, there are situations where a part of the system parameters may not be known, causing the evaluation of the grammian summation to be impossible.

In this study, a new observability measure called the "local observability matrix" is introduced for observability analysis. This observability measure will then be used to analyze the observability of a terrain-aided inertial navigation system. Simulation results are presented in Chapter 5 to verify the effectiveness of this measure.

A system represented by Eqs. (2.1) and (2.2) is said to be "locally observable" if the state $\underline{x}_j$ can be determined from the knowledge of $\underline{z}_j$ for $j = k$ to $k+n-1$, where n is the order of the system.

Define the "local observability matrix" M as

$$M = \begin{bmatrix} H_k \\ H_{k+1}\Phi_k \\ H_{k+2}\Phi_{k+1}\Phi_k \\ \vdots \\ H_{k+n-1}\Phi_{k+n-2}\cdots\Phi_k \end{bmatrix} \tag{4.2}$$

This matrix resembles the "observability matrix" for linear time-invariant systems in appearance, but is different in that Φ and H matrices are not constant [12.13].

Assertion: For a system represented by (2.1) and (2.2) to be locally observable over the time period from k to $k+n-1$, it is necessary and sufficient that the local observability matrix M for the period is of a rank n [4].

For a single output system the M matrix is square, therefore, the local observability requires that the M matrix be *nonsingular.*

V Usefulness of Observability in a Local Sense

As mentioned earlier, it is more practical to study the system observability characteristics in a local sense. This is mostly true for all control systems that involve measurements. In these type of systems, one or more of their states (parameters) might not be observable during the course of study. This might be caused by several sources such as environmental conditions, approximation involved in system modeling, defect in the measurement devices and others. All these factors may vary with time which could make the unobservable parameter(s) observable again. Hence, local observability concept is considered in this study to deal with such systems.

VI Conditional Number As a measure of System Observability

The analysis of the local observability matrix reveales the observability of the underlying system during a certain period of time. The *conditional number* of this matrix is chosen to be a measure of how close this matrix is from singularity, consequently, we can have an idea about the degree of the observability of the system at that point.

It has been shown in [12] that for linear systems this measure (i.e. condi-

tional number of the local observability matrix) is much more reliable and gives more indication about the nearness to singularity than quantities such as the determinant or the smallest pivot etc. The actual computation of the condition number $Cond(M)$ of a matrix M is as follows:

if m_j are the columns of the matrix M and $\tilde{m}_j$ are the columns of the matrix M^{-1}, then the condition number of the matrix M is given by

$$Cond(M) = \max_j \| m_j \| \cdot \max_j \| \tilde{m}_j \| \tag{6.1}$$

where $\| m_j \|$ is the norm of the jth column in M and $\| \tilde{m}_j \|$ is the norm of the jth column in M^{-1}. So if $Cond(M)$ is large, M is close to singular and vice versa.

VII An Application of the Local Observability Concept: Kalman Filtering

One of the examples that utilizes the local observability concept is the Kalman filter which is used in Inertial Navigation System (INS), to be discussed later. A successful use of a Kalman filter requires that the system be observable. Under this assumption and considering the noise associated with the system to be gaussian, a Kalman filter can then provide a minimum covariance estimate of the state parameters of the system. However, one or more of the system parameters may not be observable within some range of the studying period of the system. This causes erroneous results that might lead to the divergence of the error covariance elements when updating the state vector. Therefore, a measure of the system observability in a local sense is needed when studying this system. Local Observability Matrix, a suggested measure of the observability

of the system, is used to analyze the observability of the system throughout the course of study.

Computer simulation was conducted in this study to explore the usefulness of the local observability matrix when applied in INS. In the next chapter, the theory of Kalman filtering is briefly discussed to give a better understanding of this scheme.

CHAPTER 3

Kalman Filter Theory

I Introduction

This chapter is devoted to discuss the theory of Kalman filtering and to derive the equations of Kalman filter which is the main computational part of the INS. This chapter will also introduce an algorithm for updating the state vector of the underlying system. It will be shown later that, when the concept of local observability matrix [4] is applied to this algorithm, this technique gives less deviation of the error covariance values around their means. This will eventually result in the convergence of these values under the case of unobservable or partially observable system.

II Basic Concepts and General Background

The principal topic of this study is the estimation of a state vector from measurement data corrupted by noise. Optimal estimator that minimizes the mean square estimation error, in a well-defined statistical sense is considered here. Before embarking upon this course, we *briefly* provide some background on the subjects of the desirable *characteristics* of estimators in general.

An estimate , $\underline{\hat{x}}$, is the computed value of the quantity, $\underline{x}$, based on a set of measurements, z. An *unbiased* estimate is one whose expected value is the same as that of the quantity being estimated. A *minimum variance* (unbiased) estimate has the property that its error variance is less than or equal to that of any other unbiased estimate. A *consistent* estimate is one which converges to

the true value of $\underline{x}$, as the number of measurements increases. Thus, we shall look for unbiased, minimum variance, consistent estimator.

It is proven in [5] that, for a gaussian time-varying signal, the optimal (minimum mean square error) estimator is a linear estimator. Additionally, as a practical fact, most often all we know about the characterization of a given random process is its autocorrelation function. But there always exists a gaussian random process possessing the same autocorrelation function; we therefore might as well assume that the given random process is itself gaussian. That is, the two processes are indistinguishable from the standpoint of the amount of knowledge postulated. On the basis of these observations we are led to consider the optimal estimator as a linear operator in most applications. Next, we consider the recursive form of the linear estimator, which applies to gaussian random sequences.

III Recursive Filters

A recursive filter is one in which there is no need to store past measurements for the purpose of computing present estimates.

Consider a discrete system whose state at time t_k is denoted by $\underline{x}(t_k)$ or simply $\underline{x}_k$, where $\underline{w}_k$ is is a zero mean, white sequence of covariance $Q_k = E[\underline{w}_k \, \underline{w}_k^T]$,

$$\underline{x}_k = \Phi_{k-1} \, \underline{x}_{k-1} + \underline{w}_{k-1} \tag{3.1}$$

Measurements are taken as linear combinations of the system state variables, corrupted by uncorrelated noise. The measurement equation is written in vector-matrix notation as

$$\underline{z}_k \;=\; H_k\, \underline{x}_k \;+\; \underline{v}_k \tag{3.2}$$

where $\underline{z}_k$ is the set of l measurements at time t_k, namely, z_{1k}, z_{2k}, $\cdots$, z_{lk}, arranged in vector form

$$\underline{z}_k \;=\; \begin{bmatrix} z_{1k} \\ z_{2k} \\ \vdots \\ z_{lk} \end{bmatrix} \tag{3.3}$$

H_k is the measurement matrix at time t_k; It describes the linear combinations of state variables which comprise $\underline{z}_k$ in the absence of noise. The dimension of the measurement matrix is $l \times n$, corresponding to l -dimensioned measurements of an n -dimensioned state. The term $\underline{v}_k$ is a vector of random noise quantites (zero mean, covariance $R_k \;=\; E[\underline{v}_k\, \underline{v}_k^T]$) corrupting the measurements.

Given a *prior* estimate of the system state at time t_k denoted $\hat{\underline{x}}_k(-)$, we seek an updated estimate, $\hat{\underline{x}}_k(+)$, based on use of measurement, $\underline{z}_k$. In order to avoid a growing memory filter, this estimate is *sought* in the linear, recursive form[1]

$$\hat{\underline{x}}_k(+) \;=\; K_k'\, \hat{\underline{x}}_k(-) \;+\; K_k\, \underline{z}_k \tag{3.4}$$

where K_k' and K_k are the time-varying weighting matrices, as yet unspecified. Although the following derivation is for an assumed recursive, single-stage filter, the result has been shown to be a solution for a more general problem.

[1]Throughout this study, (-) and (+) are used to denote the times immediately *before* and immediately *after* a discrete measurement, respectively.

IV Discrete Kalman Filter

It is possible to derive the Kalman filter by optimizing the assumed form of the linear estimator. An equation for the *estimation error* after incorporation of the measurement can be obtained from Eq. (3.4) through substitution of the measurement equation [Eq. (3.2)] and the defining relations (tilde denotes the estimation error)

$$\begin{aligned} \underline{\hat{x}}_k(+) &= \underline{x}_k + \underline{\tilde{x}}_k(+) \\ \underline{\hat{x}}_k(-) &= \underline{x}_k + \underline{\tilde{x}}_k(-) \end{aligned} \tag{4.1}$$

The result is

$$\underline{\tilde{x}}_k(+) = [K'_k + K_k H_k - I]\, \underline{x}_k + K'_k\, \underline{\tilde{x}}_k(-) + K_k\, \underline{v}_k \tag{4.2}$$

By definition $E[\underline{v}_k] = \underline{0}$. Also, if $E[\underline{\tilde{x}}_k(-)] = \underline{0}$, this estimator will be *unbiased* (i.e., $E[\underline{\tilde{x}}_k(+)] = \underline{0}$) for any given state vector $\underline{x}_k$ only if the term in square brackets is zero. Thus, we *require*

$$K'_k = I - K_k H_k \tag{4.3}$$

and the estimator takes the form

$$\underline{\hat{x}}_k(+) = (I - K_k H_k)\, \underline{\hat{x}}_k(-) + K_k\, \underline{z}_k \tag{4.4}$$

or alternatively,

$$\underline{\hat{x}}_k(+) \;=\; \underline{\hat{x}}_k(-) \;+\; K_k\,[\underline{z}_k \;-\; H_k\;\underline{\hat{x}}_k(-)] \tag{4.5}$$

The corresponding estimation error is, from Eqs. (3.2), (4.1) and (4.5),

$$\underline{\tilde{x}}_k(+) \;=\; (I \;-\; K_k\;H_k)\;\underline{\tilde{x}}_k(-) \;+\; K_k\;\underline{v}_k \tag{4.6}$$

Error Covariance Update – Using (4.6) the change in the error covariance matrix when a measurement is employed can be derived. From the defintion

$$P_k(+) \;=\; E[\underline{\tilde{x}}_k(+)\;\underline{\tilde{x}}_k(+)^T] \tag{4.7}$$

Eq.(4.6) gives

$$\begin{aligned} P_k(+) \;&=\; E\{(I - K_k\;H_k)\;\underline{\tilde{x}}_k(-)\;[\underline{\tilde{x}}_k(-)^T(I - K_k\;H_k)^T + \underline{v}_k^T\;K_k^T] \\ &+\; K_k\;\underline{v}_k\;[\underline{\tilde{x}}_k(-)^T\;(I \;-\; K_k\;H_k)^T \;+\; \underline{v}_k^T\;K_k^T]\} \end{aligned} \tag{4.8}$$

By definition,

$$E[\underline{\tilde{x}}_k(-)\;\underline{\tilde{x}}_k(-)^T] \;=\; P_k(-) \tag{4.9}$$

$$E[\underline{v}_k\;\underline{v}_k^T] \;=\; R_k \tag{4.10}$$

and, as a result of measurement errors being uncorrelated,

$$E[\underline{\tilde{x}}_k(-)\;\underline{v}_k^T] \;=\; E[\underline{v}_k\;\underline{\tilde{x}}_k(-)^T] \;=\; 0 \tag{4.11}$$

Thus

$$P_k(+) \;=\; (I \;-\; K_k\, H_k)\, P_k(-)\, (I \;-\; K_k\, H_k)^T \;+\; K_k\, R_k\, K_k^T \qquad (4.12)$$

Optimum Choice of $\mathbf{K_k}$ – The criterion for choosing K_k is to minimize a weighted scalar sum of the diagonal elements of the error covariance matrix $P_k(+)$ because these elements represent the estimation error variances for the elements of the state vector being estimated. Thus, for the cost function, which is the function we need to minimize as the criterion of optimality for our estimator, we choose

$$J_k \;=\; E[\underline{\tilde{x}}_k(+)^T\; S\; \underline{\tilde{x}}_k(+)] \qquad (4.13)$$

where S is any positive semidefinite matrix. But it has been shown in [6] that the optimal estimate is independent of S; hence, we may as well choose $S \;=\; I$, yielding

$$J_k \;=\; trace\; [P_k(+)] \qquad (4.14)$$

This is equivallent to minimizing the *length* of the estimation error vector. To find the value if K_k which provides a minimum, it is necessary to take the partial derivative of J_k with respect to K_k and equate it to zero. Use is made of the relation for the partial derivative of the trace of the product of two matrices A and B (with B symmetric),

$$\frac{\partial}{\partial A}\left[trace\ (ABA^T)\right] \ = \ 2AB \tag{4.15}$$

From Eqs. (4.12) and (4.13) the result is

$$-\,2(I \ - \ K_k\ H_k)\ P_k(-)\ H_k^T \ + \ 2K_K\ R_k \ = \ 0 \tag{4.16}$$

Solving for K_k,

$$K_k \ = \ P_k(-)\ H_k^T\ [H_K\ P_k(-)\ H_k^T \ + \ R_k]^{-1} \tag{4.17}$$

which is referred to as the *Kalman gain matrix.* Examination of the Hessian of J_k reveals that this value of K_k does indeed *minimize* J_k [6].

Substitution of Eq. (4.15) into (4.12) gives, after some manipulation,

$$P_k(+) \ = \ P_k(-) - P_k(-)\ H_k^T\ [H_k P_k(-) H_K^T + R_k]^{-1}\ H_k P_k(-) \tag{4.18}$$

$$= \ [I \ - \ K_k\ H_k]\ P_k(-) \tag{4.19}$$

which is the optimized value of the updated estimation error covariance matrix.

Thus far we have described the discontinuous state estimate and error covariance matrix behavior *across* a measurement. The extrapolation of these quantities *between* measurements is

$$\underline{\hat{x}}_k(-) \ = \ \Phi_{k-1}\ \underline{\hat{x}}_{k-1}(+) \tag{4.20}$$

$$P_k(-) \ = \ \Phi_{k-1}\ P_{k-1}(+)\ \Phi_{k-1}^T \ + \ Q_{k-1} \tag{4.21}$$

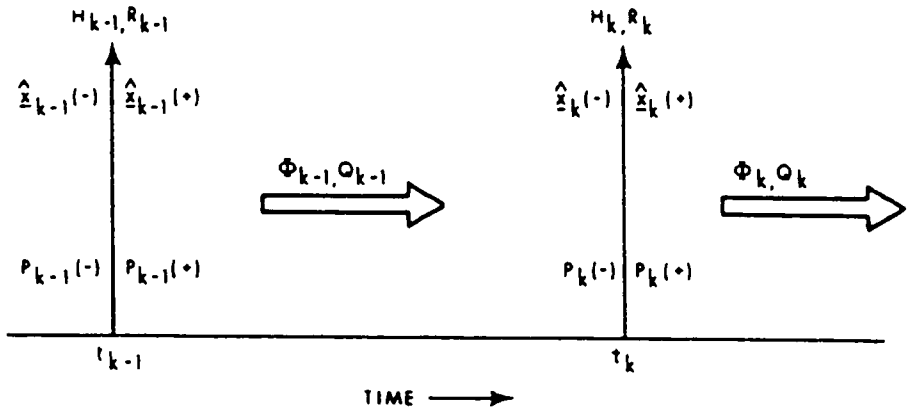


FIGURE 3-1

Discrete Kalman Filter Timing Diagram

see Fig. (3-1) for a "timing diagram" of the various quantities involved in the discrete optimal filter equations. The equations of the discrete Kalman filter are summarized in Table 3-1.

Figure (3-2) illustrates these equations in block diagram form. The Kalman filter to be *implemented* appears outside the dashed-line box. that appearing inside the box is simply a mathematical abstraction – a *model* of what we think the system and measurement processes are. It is to be noted here that *no* measurements were taken in this study, however, Kalman filtering approach still can be used to minimize the mean squared error of the estimated values. It is often said that the Kalman filter generates its own *error analysis.* Clearly, this refers to the computation of P_k, which provides an indication of the accuracy

of the estimate.

TABLE 3-1

Summary of Discrete Kalman Filter Equations

System Model	$\underline{x}_k = \Phi_{k-1}\underline{x}_{k-1} + \underline{w}_{k-1}, \quad \underline{w}_k \sim N(\underline{0}, Q_k)$
Measurement Model	$\underline{z}_k = H_k\ \underline{x}_k + \underline{v}_k, \quad \underline{v}_k \sim N(\underline{0}, R_k)$
Initial Conditions	$E[\underline{x}(0)] = \hat{\underline{x}}_0,\ E[(\underline{x}(0) - \hat{\underline{x}}_0)(\underline{x}(0) - \hat{\underline{x}}_0)^T] = P_0$
Other assumptions	$E[\underline{w}_k\ \underline{v}_j^T] = 0$ for all $j,\ k$
State Estimate Extrapolation	$\hat{\underline{x}}_k(-) = \Phi_{k-1}\hat{\underline{x}}_{k-1}(+)$
State Covariance Extrapolation	$P_k(-) = \Phi_{k-1}P_{k-1}(+)\Phi_{k-1}^T + Q_{k-1}$
State Estimate Update	$\hat{\underline{x}}_k(+) = \hat{\underline{x}}_k(-) + K_k[\underline{z}_k - H_k\hat{\underline{x}}_k(-)]$
State Covariance Update	$P_k(+) = [I - K_kH_k]P_k(-)$
Kalman Filter Matrix	$K_k = P_k(-)H_k^T[H_kP_k(-)H_K^T + R_k]^{-1}$

In the linear, discrete Kalman filter, calculations at the covariance level ultimately serve to provide K_k, which is then used in the calculation of the mean values (i.e., the estimate $\hat{\underline{x}}_k$). There is no feedback from the state equations to the covariance equations. This is illustrated in Figure (3-3), which is essentially a simplified computer flow diagram of the discrete Kalman filter.

A Simpler Form for $\mathbf{K_k}$ – There is a matrix inversion relationship which states that, for $P_k(+)$ as given in Eq. (4.16) , $P_k^{-1}(+)$ is expressible as:

$$P_k^{-1}(+) = P_k^{-1}(-) + H_k^T\ R_k^{-1}\ H_k \tag{4.22}$$

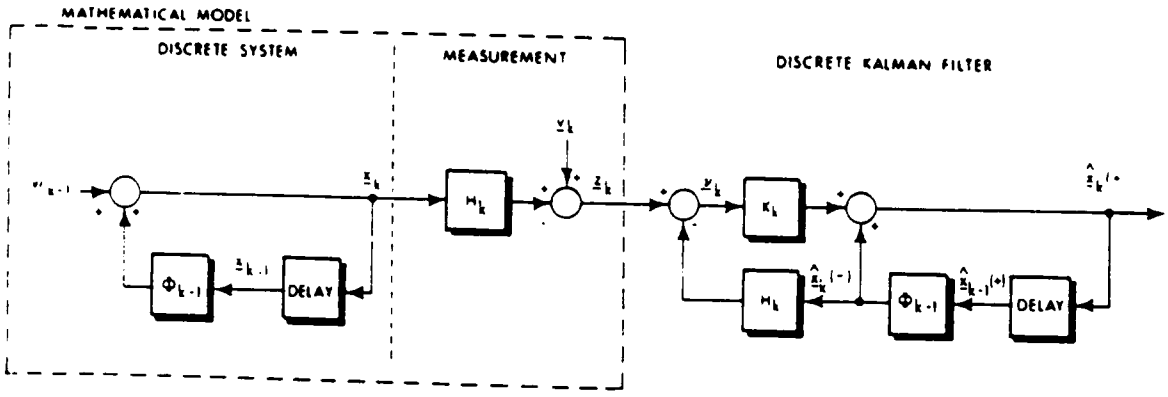


FIGURE 3-2

System Model and Discrete Kalman Filter

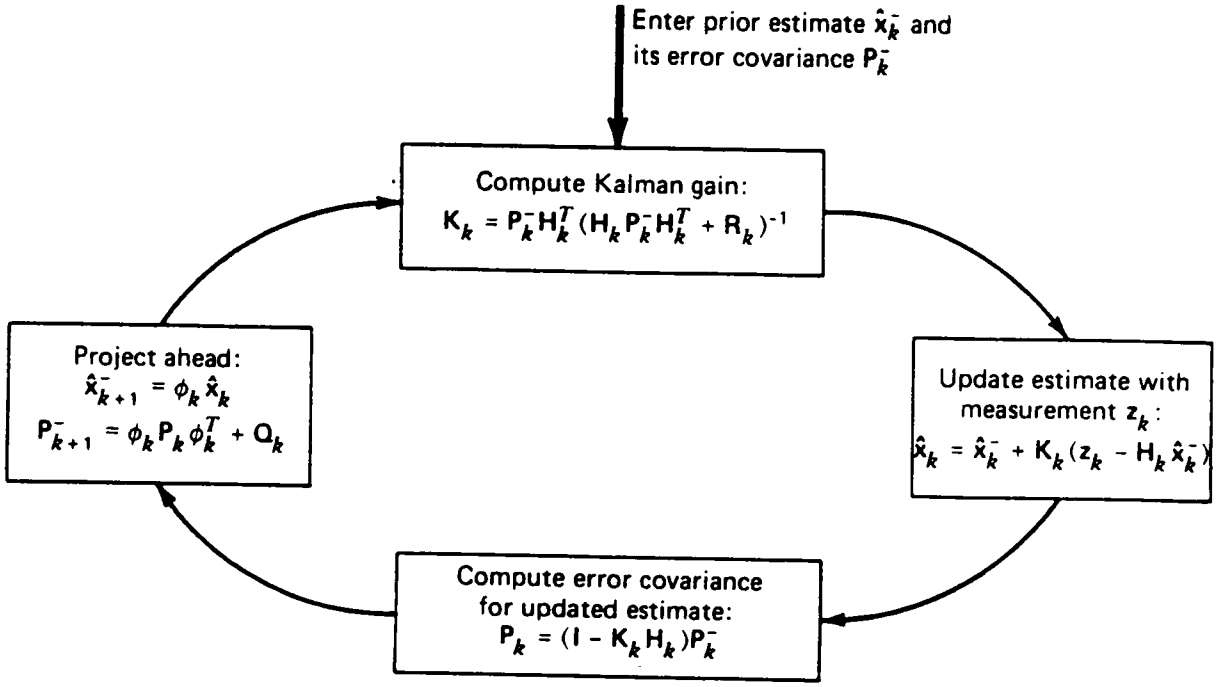


FIGURE 3-3

Discrete Kalman Filter Information Flow Diagram

This relationship can easily be verified by showing that $P_k(+)\ P_k^{-1}(+) = I$. We use this result to manipulate K_k as follows,

$$\begin{aligned} K_k &= [P_k(+)P_k^{-1}(+)]P_k(-)H_k^T[H_kP_k(-)\ H_k^T + R_k]^{-1} \\ &= P_k(+)\ [P_k^{-1}(-) + H_k^TR_k^{-1}H_k]P_k(-)H_K^T[H_kP_k(-)H_K^T + R_k]^{-1} \end{aligned}$$

Expanding and collecting terms yields

$$\begin{aligned} K_k &= P_k(+)H_k^T[I + R_k^{-1}H_kP_k(-)\ H_k^T]\ [H_kP_k(-)H_K^T + R_k]^{-1} \quad (4.23) \\ &= P_k(+)\ H_k^T\ R_k^{-1} \end{aligned}$$

which is the simpler form sought.

CHAPTER 4

Terrain-Aided Inertial Navigation System

I Introduction

An Inertial Navigation System is a device used to measure the orientation and position of a moving vehicle. It is widely used for navigation and guidance of aircrafts, missiles, ocean-vessels, satellites, and land vehicles. It is well known that an INS has an error which is time varying in nature. This error is mainly due to the drifts of gyros, biases of accelerometers, and the changes of their scale factors which is caused by the changing conditions within the aircraft and its environment: terrain variations and vehicle movement, during the measurement process. This error can be reduced by incorporating into the system one or more aiding sensors. Among the aiding sensors are odometers, altimeters, GPS (global positioning system) data, terrain profile data, and others. The choice of either one of these aiding sensors is application dependent. A navigation system using aiding sensors is often referred to as an "integrated navigation system." Such a system is a multiple measurements in nature, implying the existence of measurement redundancy. In this study a terrain profile data was used as an aiding sensor to the INS, as a result, the system is called a "terrain-aided inertial navigation system." Next, we consider a model of the INS and study each of its components.

II System Modeling

A model for a terrain-aided inertial navigation system is developed as depicted in Figure(4.1). The main component of the system is the Kalman filter which requires a set of input data such as the state transition and measurement equations and the error associated with these equations, and the terrain profile. These input data, along with the necessary initial conditions, should be supplemented to the Kalman filter in order to be able to construct the set of Kalman filter equations needed to start the navigation scheme.

As mentioned earlier, the INS is used to measure the location and speed of moving vehicle. This is done with the help of Kalman filter by updating the navigation state vector given by:

$$\underline{x} = [\ x \ \ y \ \ z \ \ v_x \ \ v_y\ ]^T \tag{2.1}$$

where x, y and z denote the aircraft position in three dimensional orthogonal coordinate system with the three axes pointed, respectively, eastward, northward and vertically upward, and v_x and v_y are velocities along the x and y directions, respectively.

In Figure(4-1) the system is modeled as a discrete time system having k as the time variable [7–11]. The system has INS as its main sensor and the radar altimeter as an aiding sensor. In addition, prerecorded topological data is stored on board which represents the terrain profile of each location as shown in Figures (4-2a) and (4-2b). A digital computer is used to execute a terrain-aided navigation scheme. In Figure(4-1) $\underline{x}(k)$ is the navigation state vector, measured by INS. It is corrected by the navigation scheme to generate an updated navigation state $\underline{\hat{x}}(k)$.

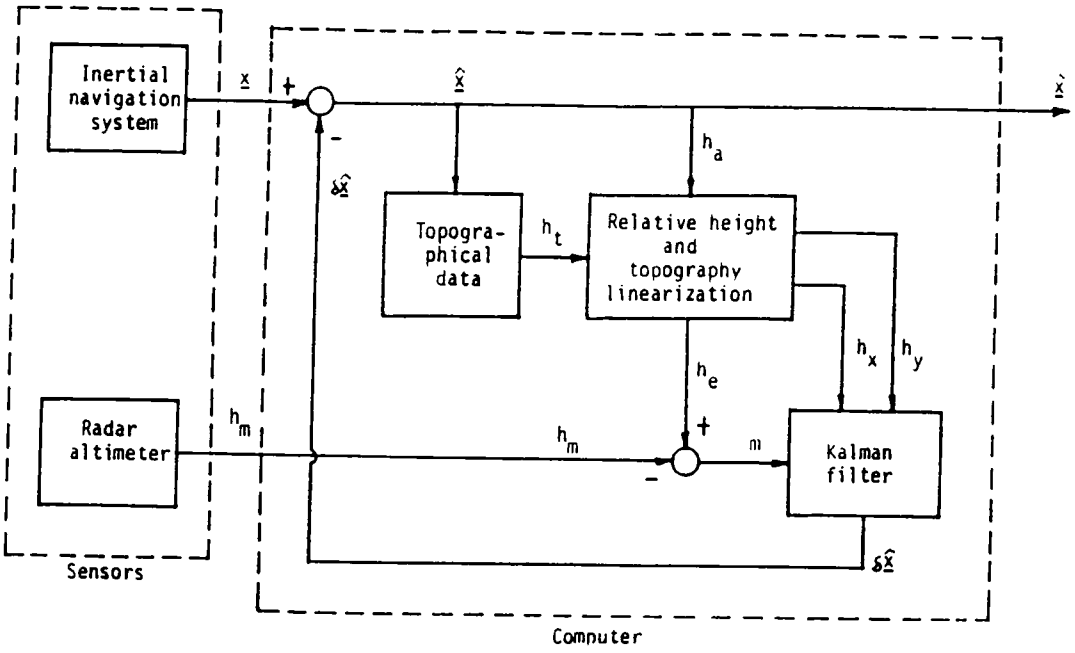
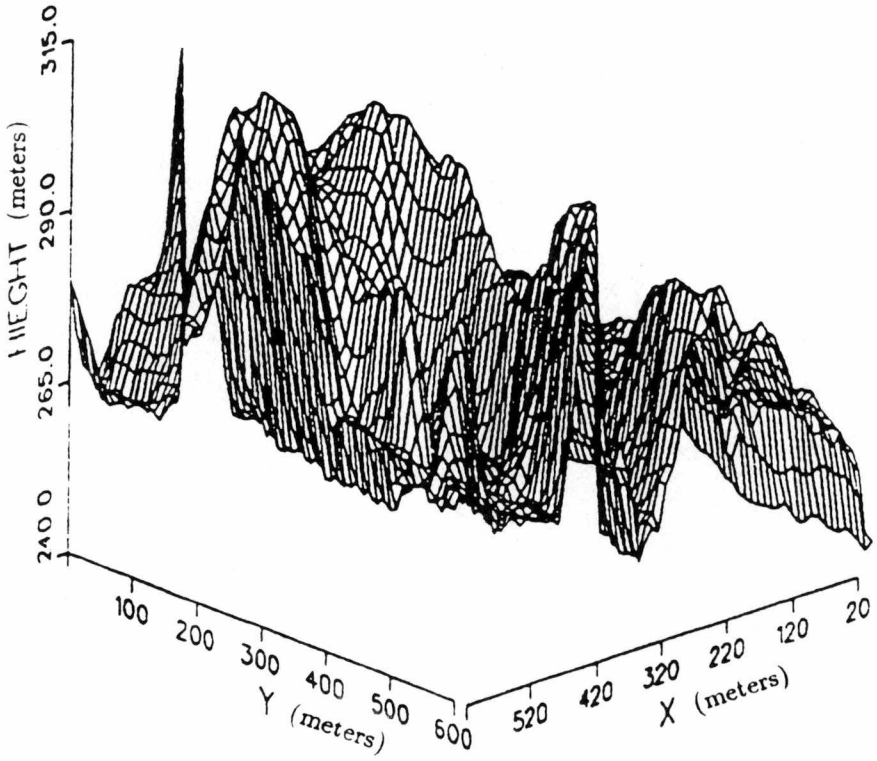


FIGURE 4-1

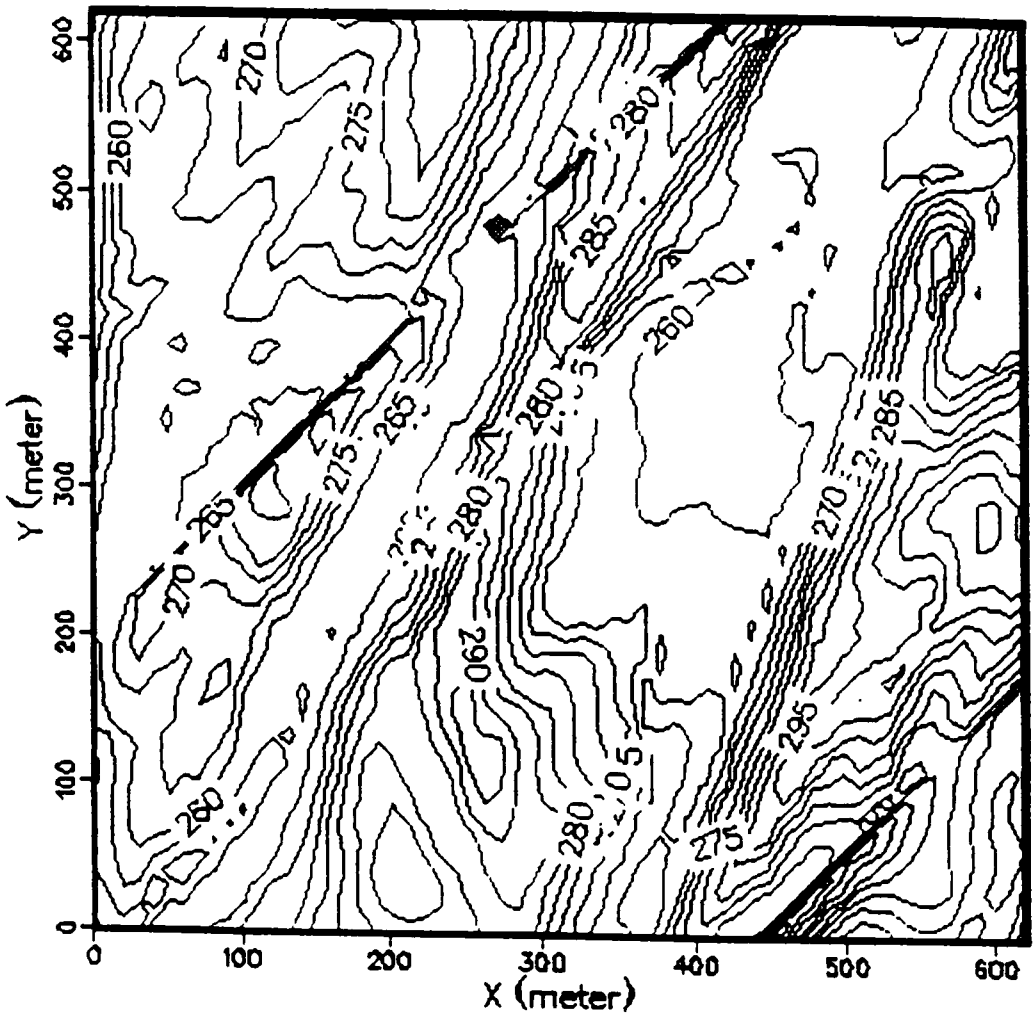
Block Diagram of a Terrain-Aided Inertial Navigation System



(a) 3D Perspective Plot

FIGURE 4-2

Terrain Profile



(b) Terrain Elevation Contour Map

FIGURE 4-2 (Continued)

The scheme computes a navigation state correction $\delta\hat{\underline{x}}(k)$ using the inputs from the INS, radar altimeter, and topologic data. The required computations include the estimation of the height $h_e(k)$ of the aircraft with respect to the ground, based on nonlinear relationship between the two, calculation of discrepancy $m(k)$ between $h_e(k)$ and the radar measured height $h_m(k)$, linearization of the nonlinear relationship between the height h and the navigation state $\underline{x}$, and performing Kalman filtering to generate the navigation state correction $\delta\hat{\underline{x}}(k)$. The output of the entire navigation system is the updated navigation state $\hat{\underline{x}}(k)$.

The state vector for the Kalman filter is chosen to be the error in $\underline{x}$, denoted by $\delta\underline{x}$. The associated state equation is

$$\delta\dot{\underline{x}} = A\,\delta\underline{x} + \underline{w} \tag{2.2}$$

where A is a constant system matrix given by

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.3}$$

and $\underline{w}$ is the process noise which is white. Discretizing (2.2) for a sufficiently small sampling period T gives

$$\delta\underline{x}(k+1) = F(T)\,\delta\underline{x}(k) + \underline{w}(k) \tag{2.4}$$

where the transition matrix $F(T)$ is given by

$$F(T) = \begin{bmatrix} 1 & 0 & 0 & T & 0 \\ 0 & 1 & 0 & 0 & T \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.5}$$

For a fixed sampling period T, $F(T)$ is a constant matrix.

Let the measurement be

$$m(k) = h_e(k) - h_m(k) \tag{2.6}$$

Note that

$$h_e(k) = h_a(k) - h_t(k) \tag{2.7}$$

where h_a and h_t are the absolute heights of the aircraft and the terrain, respectively, measured with respect to sea level. As mentioned earlier , h_t is a nonlinear function of position $\underline{x}$. Expand $h_t(k)$ into a Taylor series about the updated position $\underline{\hat{x}}$, and neglect the second and higher order terms.

$$[h_t(k)]_{\underline{x}} = [h_t(k)]_{\underline{\hat{x}}} + \left[\frac{\partial h_t(k)}{\partial \underline{x}(k)} \right]_{\underline{\hat{x}}} \delta \underline{x}(k) \tag{2.8}$$

where $h_t(k)$ is an abbreviation for $h_t[\underline{x}(k)]$ since h_t is a function of location $\underline{x}$.

Substituting Eq. (2.8) into Eq. (2.7) gives

$$m(k) \;=\; h_a(k) \;-\; [h_t(k)]_{\hat{\underline{x}}} \;-\; \left[\frac{\partial h_t(k)}{\partial \underline{x}(k)}\right]_{\hat{\underline{x}}} \delta\underline{x}(k) \;-\; h_m(k) \qquad (2.9)$$

In reality,

$$h_a(k) \;-\; [h_t(k)]_{\hat{\underline{x}}} \;-\; h_m(k) \simeq 0 \qquad (2.10)$$

Therefore Eq. (2.9) becomes

$$m(k) \;=\; H(k)\, \delta\underline{x}(k) \qquad (2.11)$$

where $H(k)$ is the measurement matrix given by

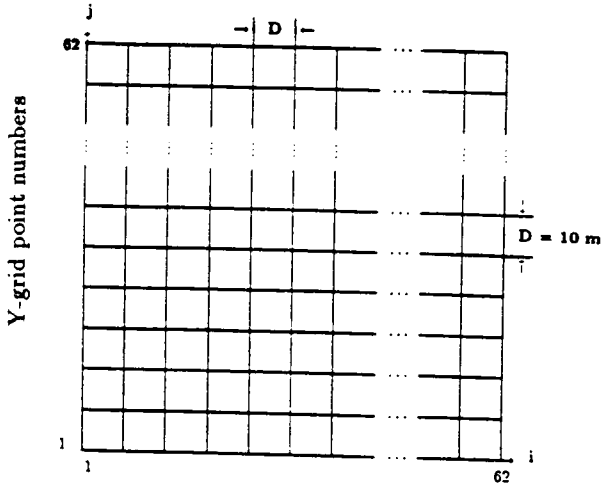
$$H(k) \;=\; -\left[\frac{\partial h_t(k)}{\partial \underline{x}(k)}\right]_{\hat{\underline{x}}} \;=\; -\left[\frac{\partial h_t}{\partial x}\;\frac{\partial h_t}{\partial y}\;\frac{\partial h_t}{\partial z}\;\frac{\partial h_t}{\partial v_x}\;\frac{\partial h_t}{\partial v_y}\right]_{\hat{\underline{x}}} \qquad (2.12)$$

Examining the above matrix, one sees that its third element is equal to 1 and its fourth and fifth elements are equal to zero, giving

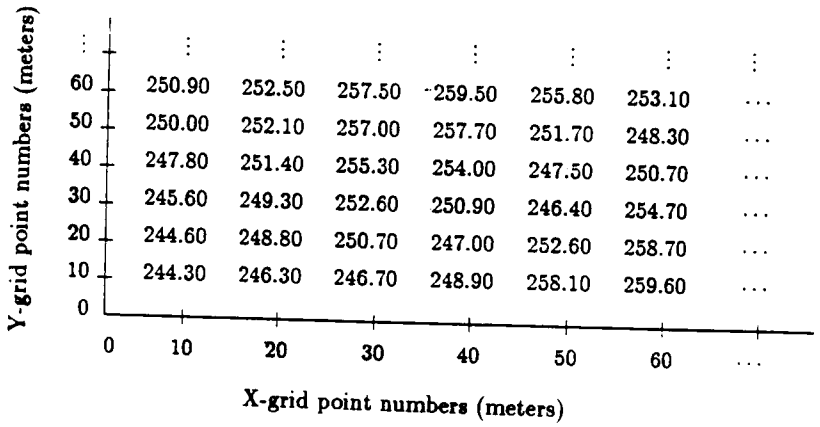
$$H(k) \;=\; [\,-h_x(k) \;\; -h_y(k) \;\; 1 \;\; 0 \;\; 0\,] \qquad (2.13)$$

where $h_x(k)$ and $h_y(k)$ are the slopes of terrain along the x and the y directions, respectively, evaluated at position $\hat{\underline{x}}$.

In order to be able to evaluate the components of the measurement matrix, namely h_x and h_y, the heights of the considered terrain area have to be digitized first. This could be represented by a grid as shown in Figure (4-3a) where each point in the grid represents the terrain height at a given coordinates



(a) A Grid for Digitizing the Terrain Heights



(b) Discrete Numbers Representing the Height of Terrain at Each Grid Point

FIGURE 4-3

Topological Data

of x and y values as shown in Figure (4-3b). Following this process, a simple linearization technique is applied to obatain the elements of $H(K)$ matrix. This is done by computing the slopes $h_x(k)$ and $h_y(k)$ directly from $h_t(i,j)$, the terrain height at a grid point (i,j) as follows:

$$\begin{aligned} h_x(k) &= [h_t(i+1,\, j) - h_t(i-1,\, j)] \,/\, (2D) \\ h_y(k) &= [h_t(i,\, j+1) - h_t(i,\, j-1)] \,/\, (2D) \end{aligned} \tag{2.14}$$

where D is the grid spacing on the digital map, see Figure (4-4).

At this point, one can write the system equations as follows:

$$\delta \underline{x}_{k+1} = \Phi_k \, \delta \underline{x}_k + \underline{w}_k \tag{2.15}$$

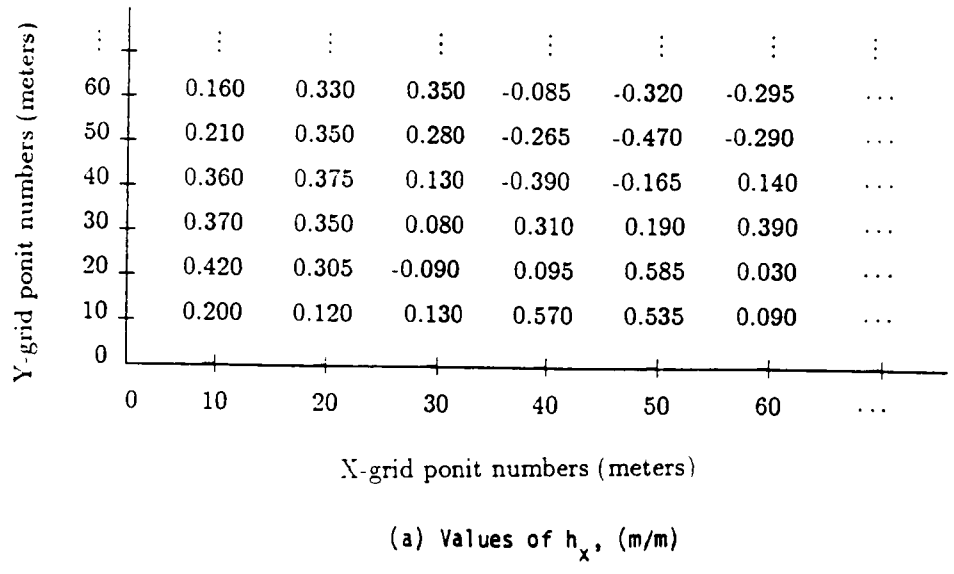
where $\delta \underline{x}_k$ is the navigation state correction vector and Φ_k is the transition matrix given by

$$\Phi_k = \begin{bmatrix} 1 & 0 & 0 & T & 0 \\ 0 & 1 & 0 & 0 & T \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.16}$$

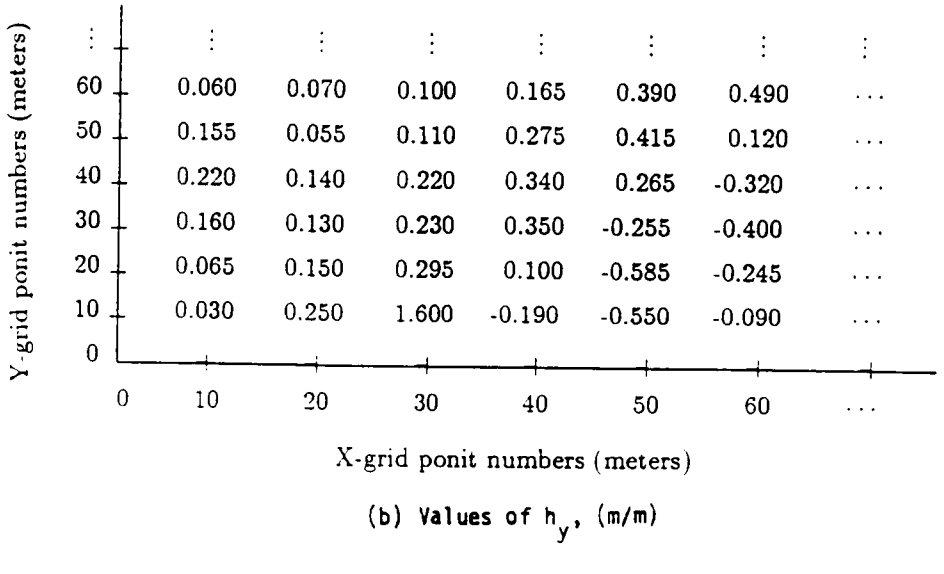
where T is the sampling period.

The measurement equation for the Kalman filter is given by

$$\underline{z}_k = H_k \, \delta \underline{x}_k + \underline{v}_k \tag{2.17}$$



(a) Values of h_x, (m/m)



(b) Values of h_y, (m/m)

FIGURE 4-4

Slope of Terrain along the x and the y Axes.

where H_k is the measurement matrix given by Eqs. (2.13) and (2.14)

After the above linearization process has been done, several paths for the aircraft could be chosen and the elements of the measurement matrix are evaluated in the above manner. Having established this matrix, along with the initial values for the error covariance matrix, measurement covariance matrix, and system disturbance matrix, an iterative cycle using Kalman filtering algorithm could be initiated.

III Observability Analysis

In order to utilize the local observability matrix in exploring the observability of the system, we seek a simpler form of this matrix. This is done by substituting Eqs. (2.2) and (2.4) into Eq. (2.7). In so doing, the local observabilty matrix is then obtained as follows

$$M = \begin{bmatrix} -h_x(k) & -h_y(k) & 1 & 0 & 0 \\ -h_x(k+1) & -h_y(k+1) & 1 & -h_x(k+1)T & -h_y(k+1)T \\ -h_x(k+2) & -h_y(k+2) & 1 & -2h_x(k+2)T & -2h_y(k+2)T \\ -h_x(k+3) & -h_y(k+3) & 1 & -3h_x(k+3)T & -3h_y(k+3)T \\ -h_x(k+4) & -h_y(k+4) & 1 & -4h_x(k+4)T & -4h_y(k+4)T \end{bmatrix} \quad (3.1)$$

The system is locally observable if, and only if, the matrix M is nonsingular. Eq.(3.1) above shows that the observability is highly dependent of terrain slopes h_x and h_y which vary with time from one point to another. So, the more the fluctuation in the terrain at that point, the higher the slope values will be and the matrix M will be far from singularity, or equivalently, the system will be more observable. On the other hand, if the fluctuation in the terrain

heights is not that significant, the corresponding slope values will be small and the matrix M will be close to singularity which yields to an unobservable system. Accordingly, there is a need to have a measure of the singularity of this matrix. This measure should indicate the degree of the system observability at each grid point. However, it should be emphasized here that there is no clear edge or fixed limit between the two cases of the system being observable and unobservable.

In this study the conditional number of the local observability matrix, given by Eq.(6.1) in Chapter, is applied to the local observability matrix given by Eq.(3.1) at each point of the flight path. Then the system is considered to be unobservable at that point if the value of the conditional number is very large. In summary, the lower the value of the conditionl number is, the more observable the system will be.

CHAPTER 5

Computer Simulation

I Introduction

This chapter is devoted to present the computer simulation of the Kalman filtering algorithm discussed in the previous two chapters. Two simulations were conducted to demonstrate the idea of how the overall system will behave for different flight paths with different levels of observability. In the first simulation, two paths were chosen with different values of terrain variations. The standard deviations of the error covariance matrix were then plotted for each path. This simulation was conducted to investigate the relative performance of the system under the two different cases of slope variations.

Comparison was also made between each component of the standard deviation of the error covariance elements for these two paths. At last, a set of plots was presented to show the slope variations as well as the conditional number for each of the two paths.

In the second simulation, a set of plots was presented where the information provided by the local observability matrix was utilized.

II Simulation Procedure: Practical Example

The simulation procedure was done on computer by software package written in FORTRAN language in the VAX mainframe. The listing of the program is attached in the Appendix.

The state space equations of the INS are:

$$\delta \underline{x}_{k+1} = \Phi_k \, \delta \underline{x}_k + \underline{w}_k \tag{2.1}$$

$$\underline{z}_k = H_k \, \delta \underline{x}_k + \underline{v}_k \tag{2.2}$$

The matrices used in the above two equations are the same as defined in the previous chapter.

An airplane flight over a mining area in Sichuan Province of China was chosen to the present simulation. Topological data at grid points are available for simulation. A simple linearization technique is used to obtain the elements of the measurement matrix H_k. This is done by computing the slope values $h_x(k)$ and $h_y(k)$ directly from $h_t(i, j)$, the terrain height at grid point (i, j) as illustrated by Equation (2.14) in the previous chapter.

In order to run the program to perform the simulation of Kalman filtering algorithm, several inputs have to be provided along with some initial conditions . These values were selected as follows:

N : number of parameters need to be estimated $(= 5)$

L : number of discrete points in the considered flight path $(= 30)$

D_s : grid spacing on the digital map $(= 10$ meter$)$

T_s : sampling period between two successive grid points $(= 5$ sec.$)$

r_0 : initial value for the measurement noise covariance matrix $(= 400 \ \text{m}^2)$

P_0 : $(N \times N)$ matrix represents the initial process error covariance matrix

$$P_0 = \begin{bmatrix} 10000\ m^2 & 0 & 0 & 0 & 0 \\ 0 & 10000\ m^2 & 0 & 0 & 0 \\ 0 & 0 & 10000\ m^2 & 0 & 0 \\ 0 & 0 & 0 & 1\ (\frac{m}{s})^2 & 0 \\ 0 & 0 & 0 & 0 & 1\ (\frac{m}{s})^2 \end{bmatrix}$$

Q_0 : $(N \times N)$ matrix represents the initial process noise covariance matrix

$$Q_0 = \begin{bmatrix} 100\ m^2 & 0 & 0 & 0 & 0 \\ 0 & 100\ m^2 & 0 & 0 & 0 \\ 0 & 0 & 100\ m^2 & 0 & 0 \\ 0 & 0 & 0 & 1\ (\frac{m}{s})^2 & 0 \\ 0 & 0 & 0 & 0 & 1\ (\frac{m}{s})^2 \end{bmatrix}$$

Φ_k : the state transition matrix correspondind to a sampling rate of 5 sec.

$$\Phi_k = \begin{bmatrix} 1 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

with this information the procedure of evaluating the relative performance of the Kalman filtering scheme under various observability conditions may be presented.

The conducted simulation was done for path aa' and path bb' shown in Figure 5–1 .

The plots will be devided into the following categories for convenience:

- The first set of plots (Figure 5–2 and 5–4) represents the standard deviations of the diagonal elements of the error covariance matrix. These values represent the estimation error in the location and the velocity of the movinig vehicle, namely, σ_x , σ_y , σ_z , σ_{v_x} and σ_{v_y} .

- The second set of plots (Figure 5–3 and 5–5) represents the conditional number of the local observability matrix. which gives an indication of how the system observability varies throughout the path of flight.

- The third set of plots (Figure 5–6 and 5–7) represents a comparison between each of the standard deviation values for path aa' and the corresponding one for path bb' .

- The fourth set of plots (Figure 5–8) represents a comparison between the slope values and the conditional numbers for these paths.

- The fifth set of plots (Figure 5–9, 5–10, 5–11 and 5–12) represents the standard deviations of path aa' when utilizing the concept of local observability matrix as a measure of the system observability.

When considered together, the above plots for these selected paths give the best indication of the overall performance of Kalman filter in the INS.

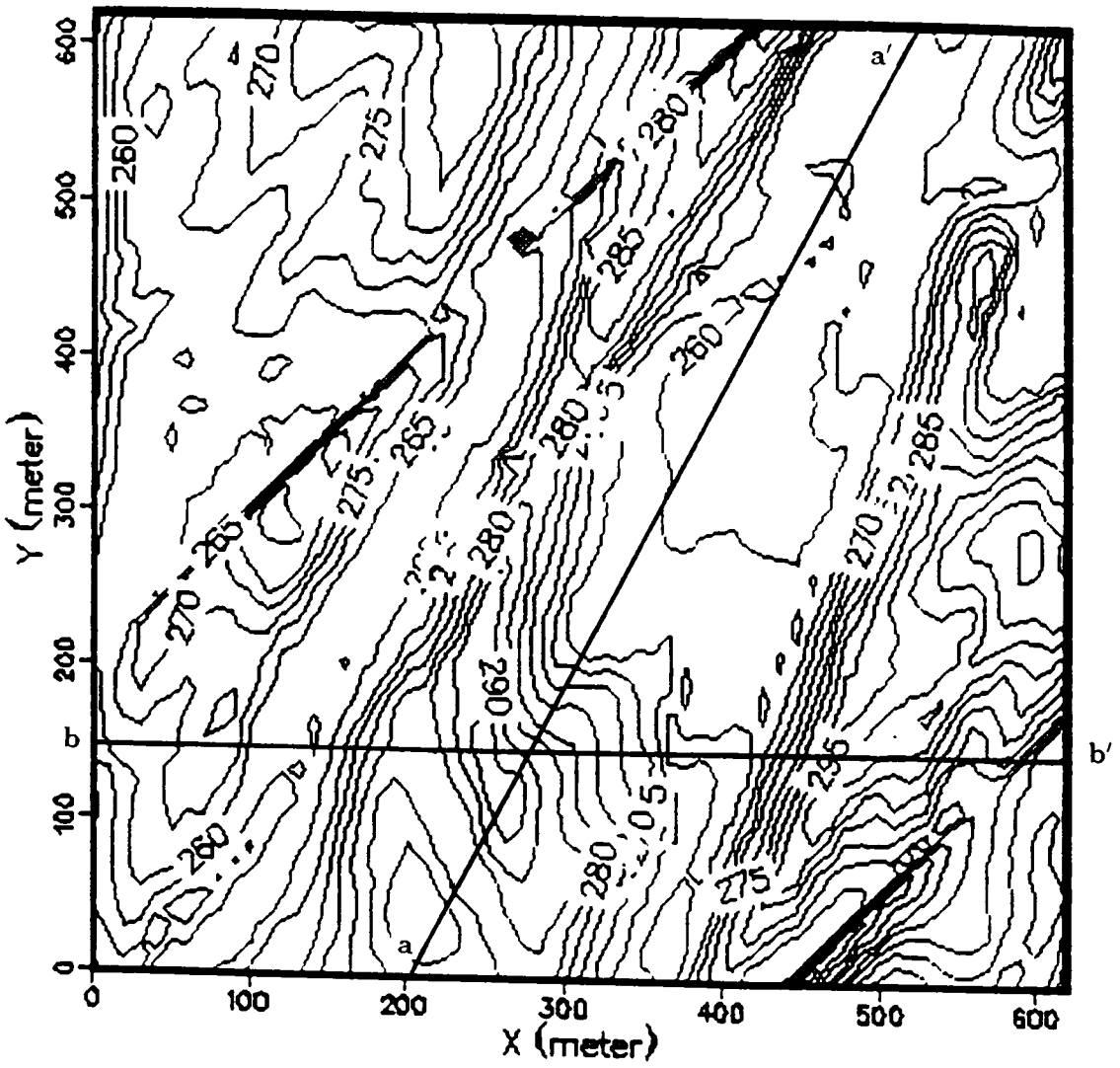


FIGURE 5-1

Terrain Elevation Contour Map with the Two Selected Paths, aa' and bb', for the Conducted Simulation.

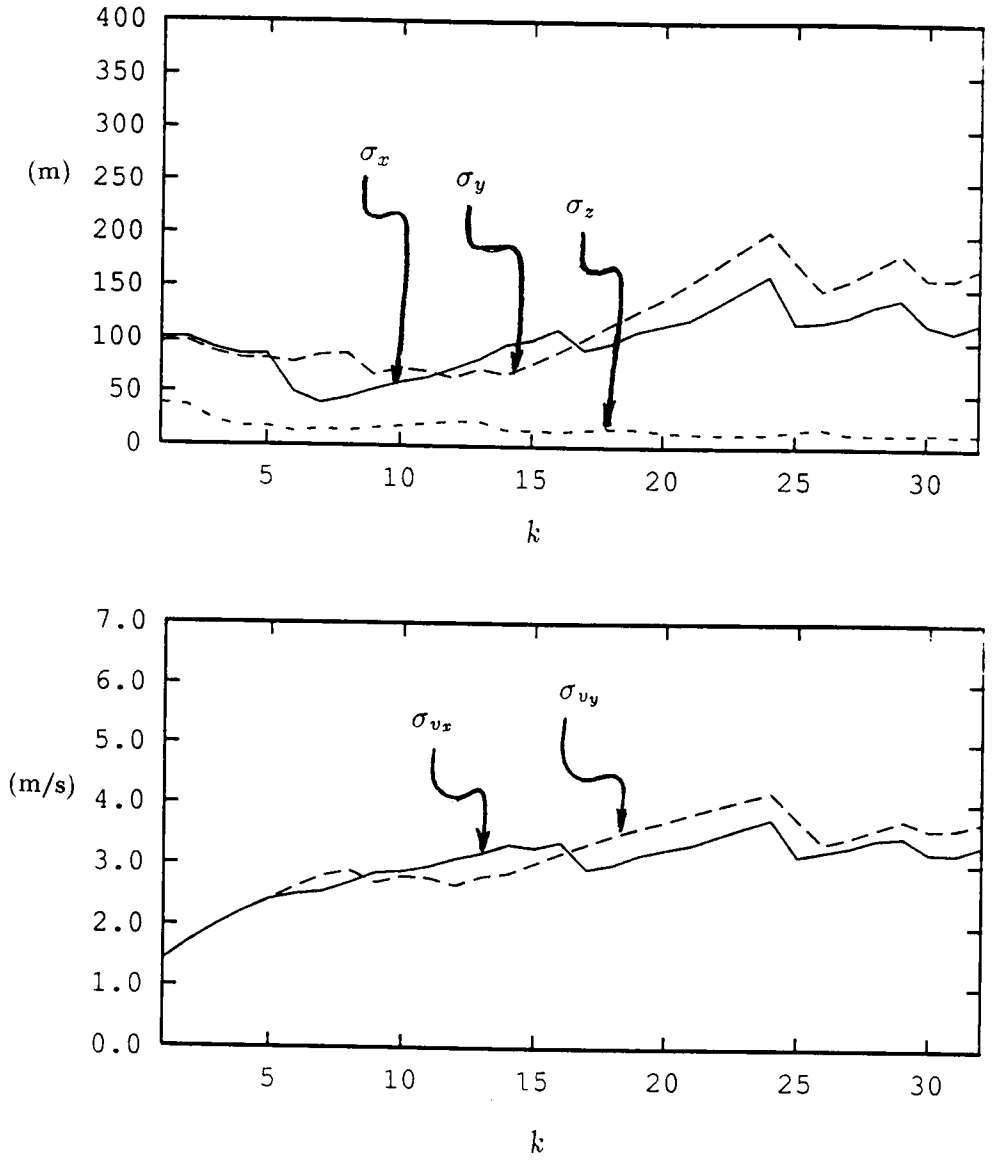


FIGURE 5-2

Standard Deviation of the Error Covariance Values
in the Location and the Velocity of
a Moving Vehicle for Path aa'

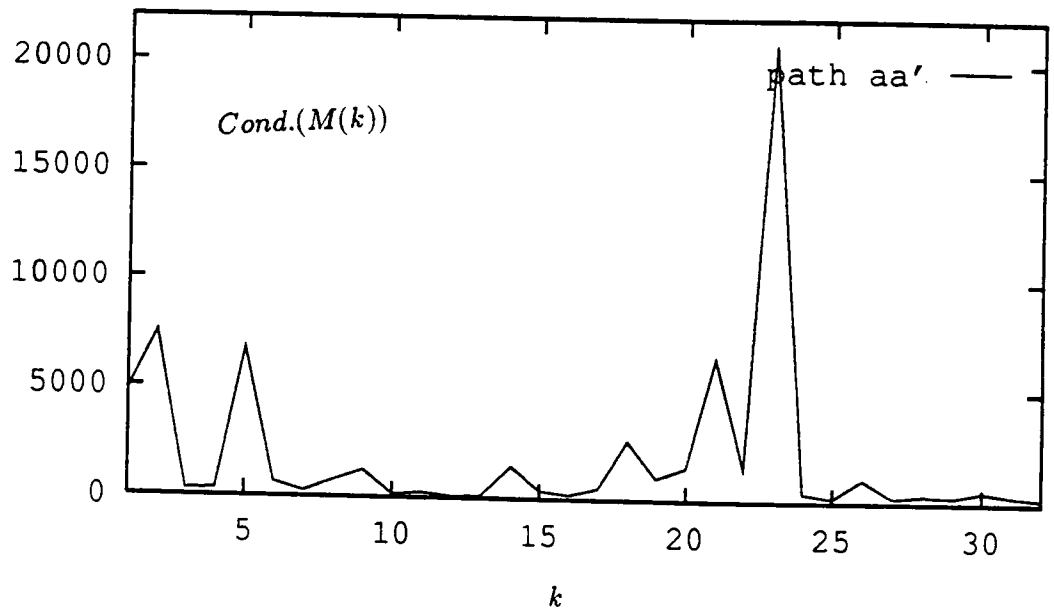


FIGURE 5-3

Conditional Number for Path aa'

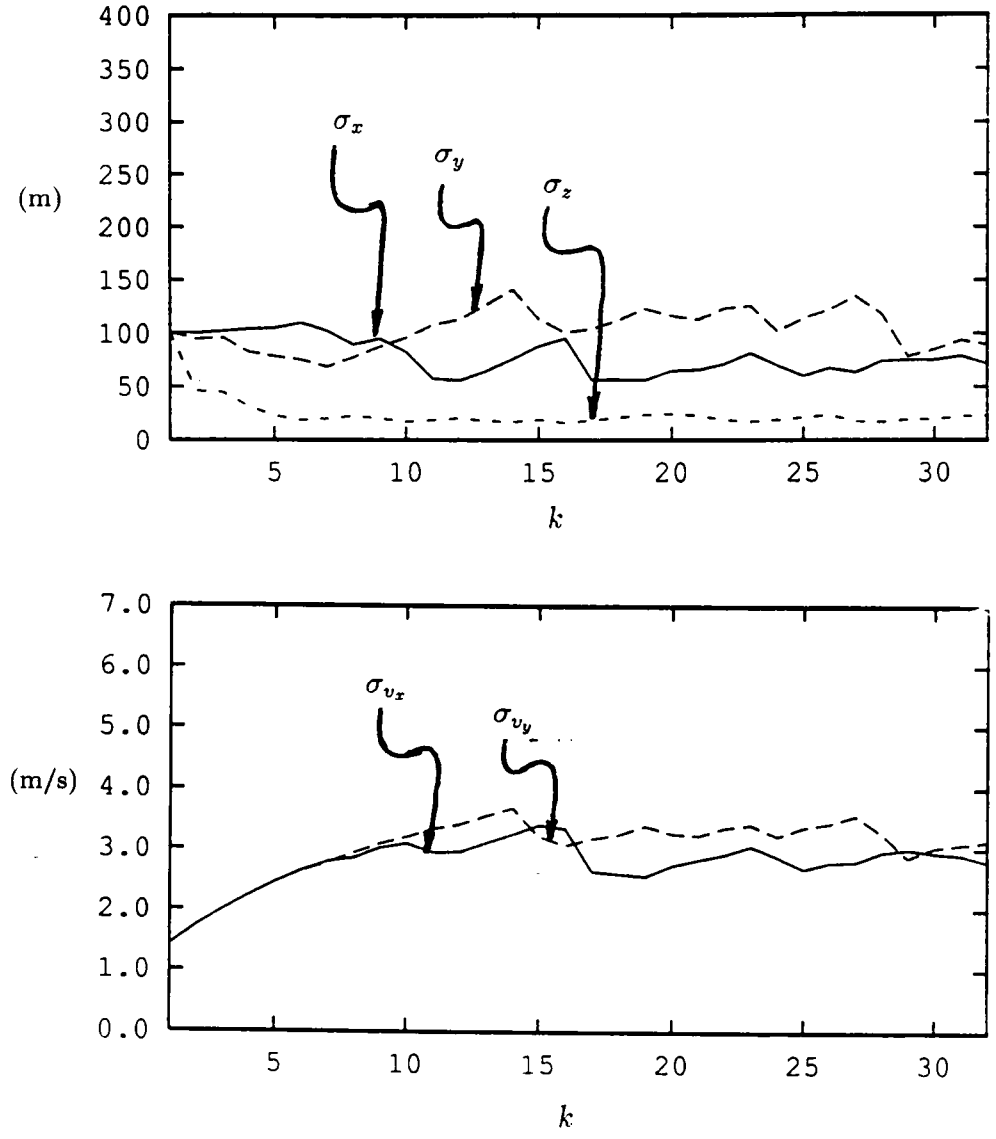


FIGURE 5-4

Standard Deviation of the Error Covariance Values
in the Location and the Velocity of
a Moving Vehicle for Path bb'

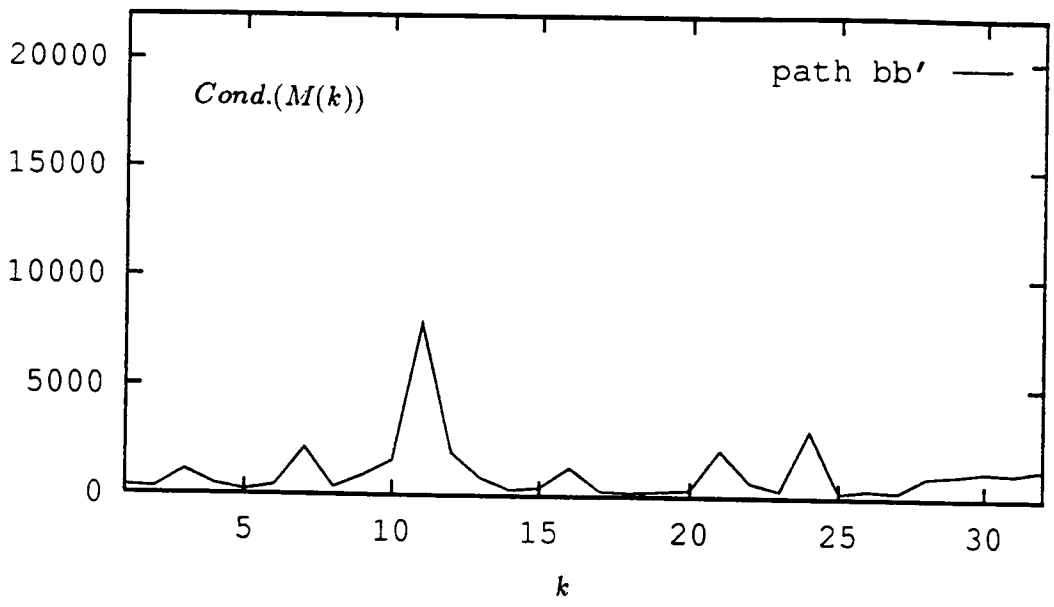


FIGURE 5-5

Conditional Number for Path bb'

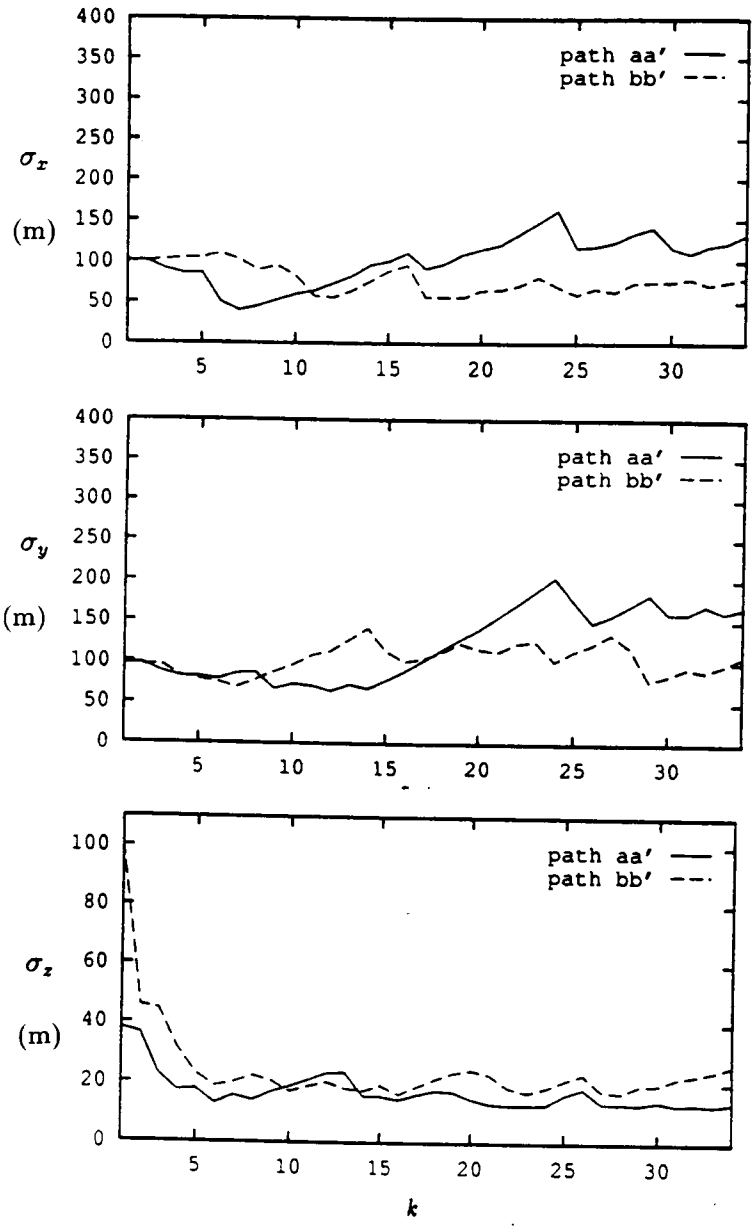
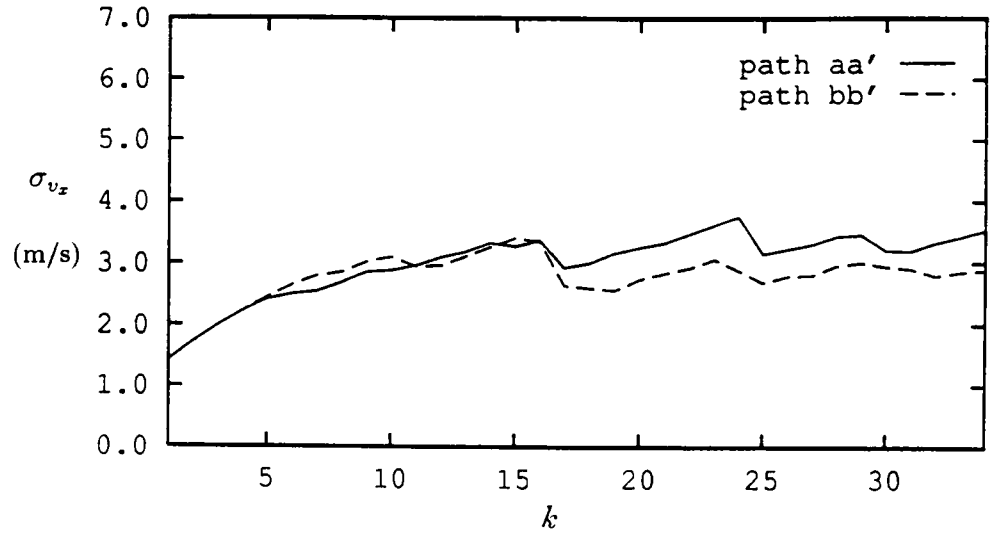


FIGURE 5-6

Comparison between the Standard Deviation of the Error Covariance Values in the Location of a Moving Vehicle for Path aa' and Path bb' .



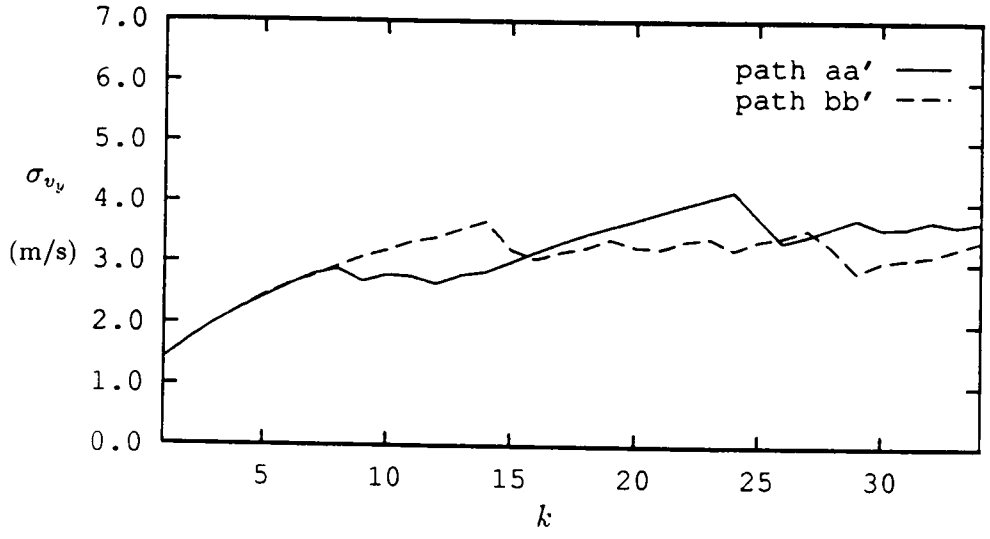


FIGURE 5-7

Comparison between the Standard Deviation of the Error Covariance Values in the Velocity of a Moving Vehicle for Path aa' and Path bb' .

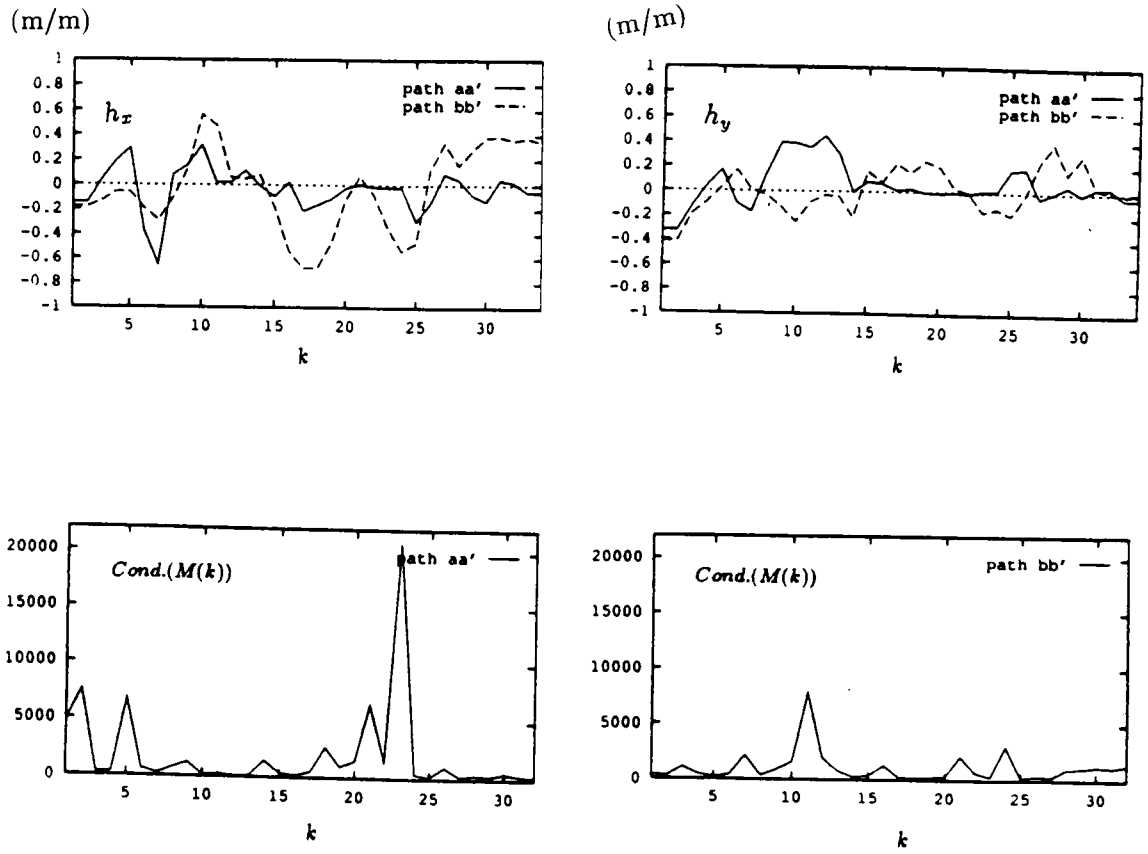


FIGURE 5-8

Comparison between the Slope Values Along the x and the y Directions for Path aa' and Path bb' and the Conditional Numbers of These Paths.

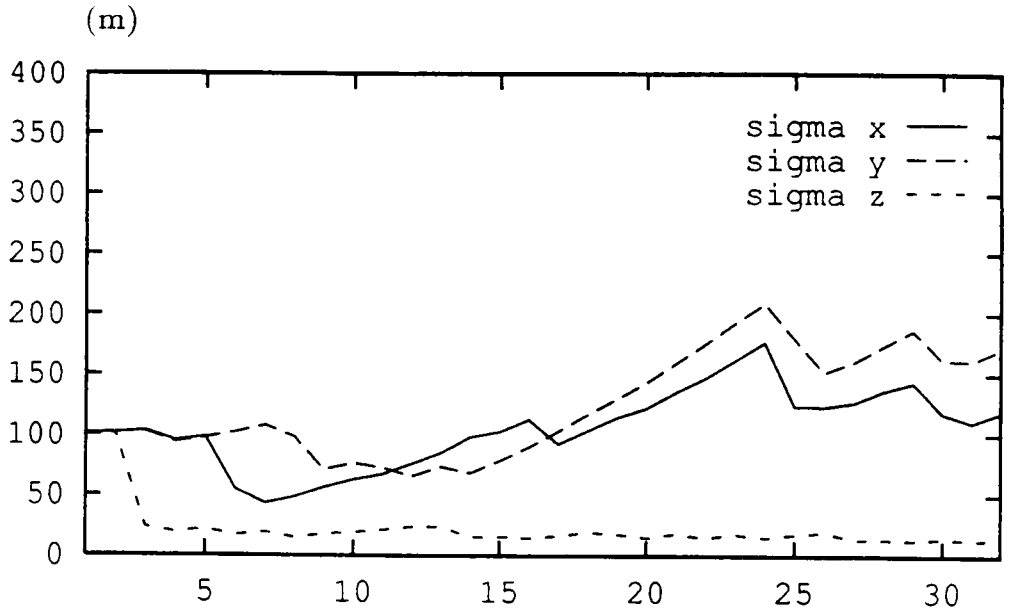


FIGURE 5-9

Standard Deviation of the Error Covariance Values in the Location of a Moving Vehicle for Path aa' When Utilizing the Information Provided by the Local Observability Matrix.

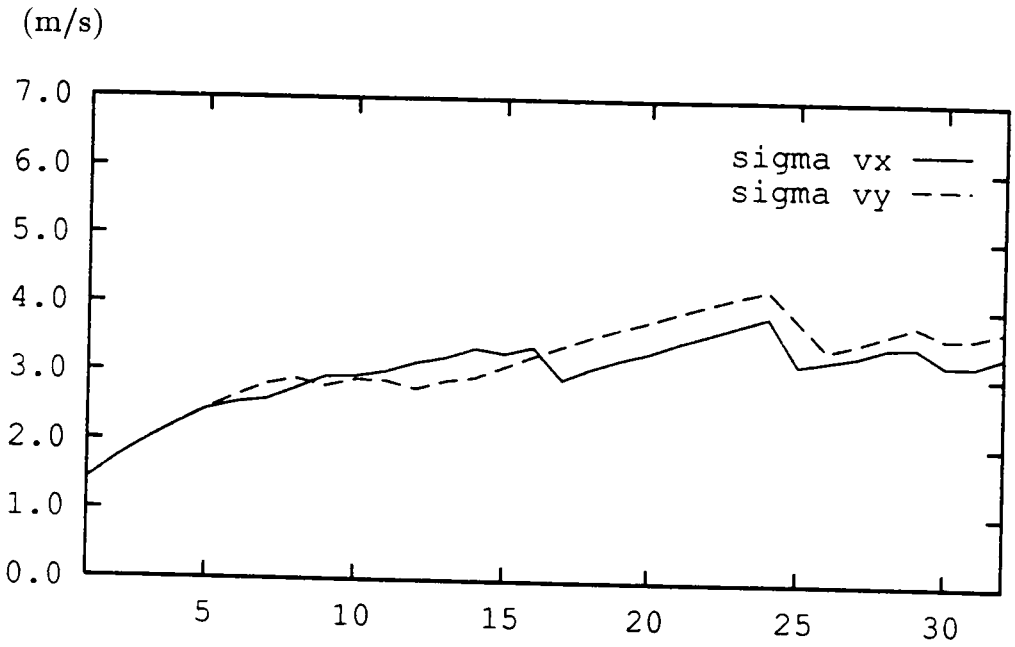


FIGURE 5-10

Standard Deviation of the Error Covariance Values in the Velocity of a Moving Vehicle for Path aa' When Utilizing the Information Provided by the Local Observability Matrix.

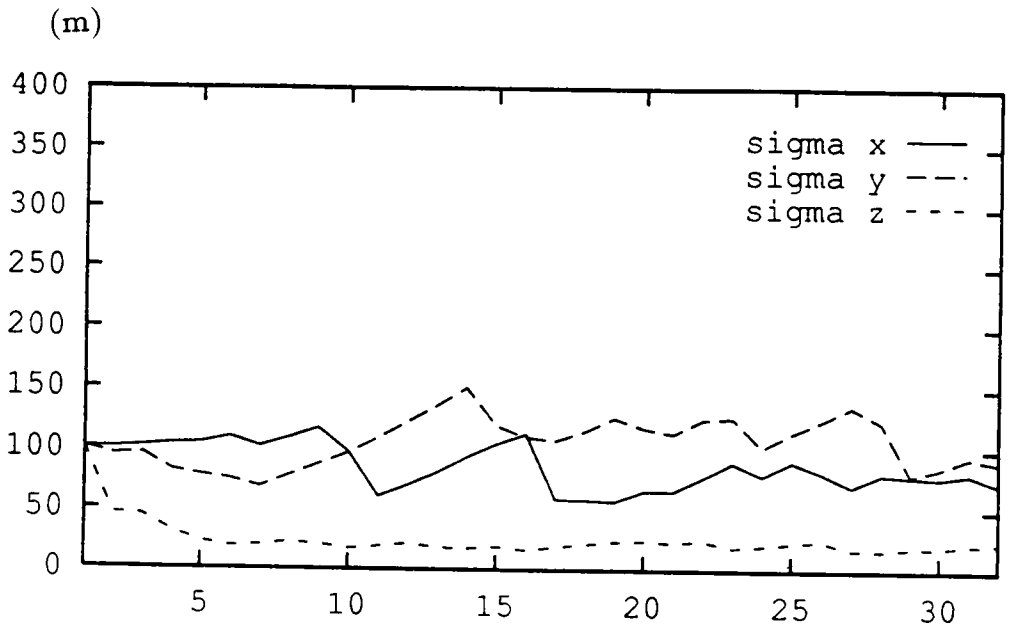


FIGURE 5-11

Standard Deviation of the Error Covariance Values in the Location of a Moving Vehicle for Path bb' When Utilizing the Information Provided by the Local Observability Matrix.

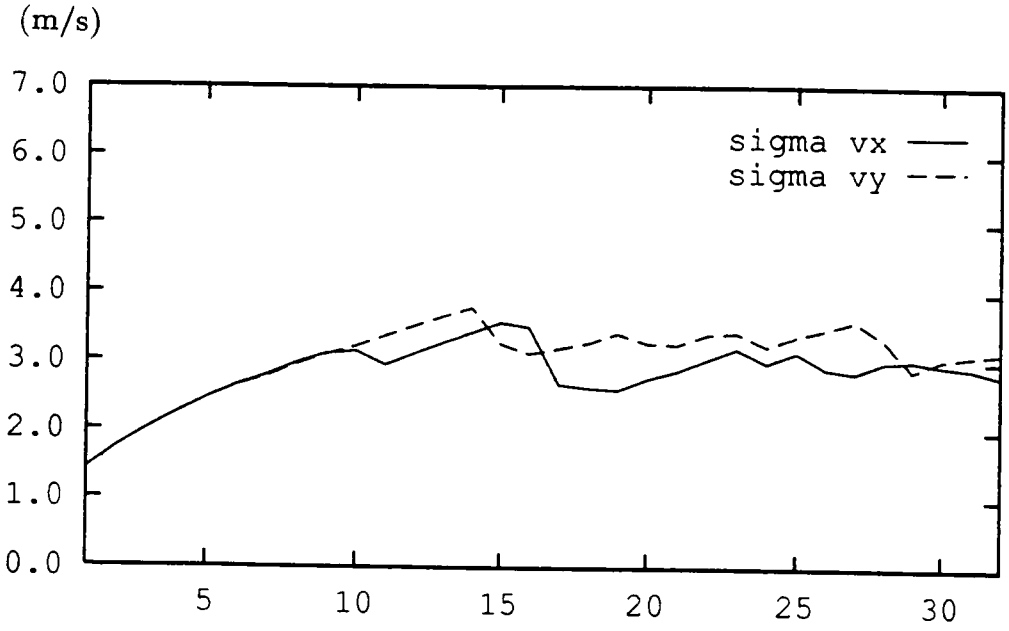


FIGURE 5-12

Standard Deviation of the Error Covariance Values in the Velocity of a Moving Vehicle for Path bb' When Utilizing the Information Provided by the Local Observability Matrix.

III Analysis of Simulation

From the conducted simulation illustrated by the attached figures, one can see (from Figure 5–6, 5–7 and 5–8) that the variation around the mean of the error values is higher for path aa' than that for path bb'. This is due to the fact that the fluctuations as well as the values of the slopes for path aa' are much smaller than those for path bb' which will result in a less observable system , i.e. higher conditional number of the local observability matrix, for path aa'.

In the set of plots represented by Figures 5–9, 5–10, 5–11 and 5–12, the concept of local observability matrix was employed. In this concept, the value of the measurement matrix, H_k , was set to $\underline{0}$ if the conditional number of the local observability matrix was very high at that point. A value of 1000 was chosen as an indication of the border between the system being observable and being unobservable i.e., the system is considered unobservable at that point if the conditional number of the local observability matrix exceeds 1000 at that particular instant.

One can see from Figure 5–9, 5–10, 5–11 and 5–12 that the fluctuation of the error covariance values around their means was substantially reduced. Eventually, this will lead the convergence of the error covariance values.

In order to appreciate this effect a set of measurements should be taken while the system is on operation then an on-line parameter estimation process can be done to evaluate the actual values of the system parameters.

CHAPTER 6

Conclusions

In this work we have presented the concept of local observability matrix and how it is utilized in Kalman filtering scheme to evaluate the accuracy of estimated values for the location and the velocity of a moving vehicle. This was tested against the case of unobservable or partially observable systems. In these systems the observability problem is caused primarily by a terrain which has a relatively *small* fluctuations or one which is almost *flat.* e.g., sea, desert, etc. Under this condition, updating process of the estimated parameters will no longer reduce their deviations from the true values. However, for the case of partially observable system, this problem is taken care of in the internal structure of Kalman filter. This amounts to the automatic reduction of the gain of Kalman filter whenever the system has low observability characteristics. Thus, the adaptive concept for updating the system state parameters is imbeded in the Kalman filter structure.

For the sake of illustrating the above idea, a terrain profile was chosen to conduct the computer simulation shown in this study. Several paths of an airplane flight were then chosen to investigate the *performance* of the system under each case. Path aa' and bb' were selected to give an example of different levels of terrain fluctuations. The relative performance of the system under was studied for each of these paths.

The simulation results show that high observability is indeed associated with low estimation error and low observability with high estimation error. One can see, from the plots presented in the previous chapter, that path aa' has *less* observability characteristics than path bb' . This was resulted in a continuous

increase of the error covariance values associated with path aa' when updating these values during the period where the system is unobservable as shown in Figures 5-4a and 5-4b .

In an attempt to improve the system performance for paths aa' and bb' local observability concept was employed. However, no substantial improvement in the variation of the error covariance values around their means was noticed, as shown in the simulated results presented in Figures 5-5 and 5-6 .

The results of this study indicate that the use of *Kalman filtering scheme* provides an effective approach to computing the location and velocity of a moving target prior to the launching process of inertially navigated weapon from the wing of an aircraft. This effect was supported by a real life example where simulation was conducted to illustrate the robustness of this technique.

Table 6–1 summarizes the whole results of this study

TABLE 6-1

Summary of the Results of This Study

	Terrain roughness	Condition number	Error Covariance	Local observability
Path bb'	severe	small	small	high
Path aa'	mild	large	large	low

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APPENDIX

Program Listing

```
implicit real*8 (a-h, o-z)
real  a(5,5),b(5,5),c(5,1),d(1,1),e(5,1),
     1  f(5,5),g(5,1),s(5,5),u(5,5),t(5,5),
     2 Q(5,5),Fi(5,5),temp2(5,5),temp3(5,5),
     3  Xp(5,5),X(5,5),P(5,5),Pp(5,5),hk(1,5),hkk(1,5),
     4 ht(62,62),H(62,5),M(5,5),temp(5,5),temp1(1,5),sigma(5),
     5 aj(5),ajt(5),Mt
integer ij(62,2)

open(4, file='topo.dat', status='old')
open(5, file='out1.dat', status='new')
open(6, file='out2.dat', status='new')
open(7, file='outb3.dat', status='new')
open(8, file='out4.dat', status='new')

c ------ initialization of parameters -------------
N = 5
L = 32
Ds = 1.0d+01
Ts = 5.0
r0 = 4.0d+02
print*, 'enter thresholding value for cond { M } '
read*, condt

c ------ ht; reading terrain data -----------------
```

```
do 10 i = 1, 62
  10 read(4, 100) (ht(i, j), j = 1, 62)
 100 format( (9(f6.2, 2x)) )
 400 format( (9(1x, f6.2, 2x)) )

c ------ Fi; transition matrix --------------------
call mtxide(Fi,N)
Fi(1,4) = Ts
Fi(2,5) = Ts
print*
write(5,*) ' Fi = '
do 60 i = 1, N
  60 write(5,200)(Fi(i,k),k=1,N)
 200 format( (5(f7.3, 2x)) )

c ------ P0; initial error covariance matrix ------
call mtxzro(P,N,N)
P(1,1) = 1.0d+04
P(2,2) = 1.0d+04
P(3,3) = 1.0d+04
P(4,4) = 1.0d+00
P(5,5) = 1.0d+00
write(5,*), ' (P(i,i), i= 1,', N, ' ) : '
write(5,500) (P(i,i), i = 1, N)
  500 format(1x, (5(d12.6, 2x)) )

c ------ Q; process noise covariance matrix -------
```

```
call mtxzro(Q,N,N)
do 20 i = 1, 3
  20 Q(i,i) = 1.0d+02
Q(4,4) = 1.0d+00
Q(5,5) = 1.0d+00
write(5,*) ' (Q(i,j), i= 1,', N, ', j = 1, ', N, ' ) : '
do 25 i = 1, N
  25 write(5,500) (Q(i,j), j = 1, N)

c ------ R; measurement noise covariance matrix ---
   R = r0

c ------ H; measurement matrix --------------------
call mtxzro(H,L,N)
ij(1,1) = 20
ij(1,2) = 1
ij(34,1) = 53
ij(34,2) = 62
do 40 k = 1, 32
ii = K + 20
jj = (62.0/33.0) * (ii - 20)
if(ii .le. 1) ii = 2
if(jj .le. 1) jj = 2
k1 = k + 1
ij(k1,1) = ii
ij(k1,2) = jj
hx = (ht(ii+1,jj) - ht(ii-1,jj)) / (2.0d+0*Ds)
```

```
hy = (ht(ii,jj+1) - ht(ii,jj-1)) / (2.0d+0*Ds)
  H(k1,1) = -hx
  H(k1,2) = -hy
  H(k1,3) = 1.0d+0
  40  continue
do 50 j = 1, 3              ! end ponits
   H(1,j) = H(2,j)             ! evaluation
  50 H(34,j) = H(33,j)
write(6,*) ' (H(i,j), i= 1,', L, ', j = 1, ',N, ' ) : '
do 55 k = 1, L
  55 write(6,600)k, (H(k,j), j = 1, N), (ij(k,nn), nn = 1, 2)

c ------------------------------------------------
c ------ kalman filter --------------------------
c ------------------------------------------------
do 11 k = 1, L
write(5, *) '------------------'
write(5, *) ' k = ', k
write(5, *) '------------------'
write(5, *)

c ---------- project ahead ...  Pp --------------
c ------------------------------------------------
call mtxtrp(Fi,f,N,N)
call mtxmul(P,f,s,N,N,N)
call mtxmul(Fi,s,a,N,N,N)          ! Pp to be used in the
call mtxadd(a,Q,Pp,N,N)            ! next iteration
```

```
write(5,*) ' (Pp(i,j), i= 1,', N, ', j = 1,', N, ' ) : '
do 70 i = 1, N
  70   write(5, 500) (Pp(i, j), j = 1, N)

c ------ calculation of Kalman Gain Matrix "G" ---
c ---------------------------------------------
do 30 i = 1, N
  30 hk(1, i) = H(k, i)
call mtxtrp(hk,e,1,N)
call mtxmul(Pp,e,c,N,N,1)
call mtxmul(hk,c,d,1,N,1)
a1 = d(1, 1)
e1 = a1 + R
f1 = 1.0d0 / e1
call mtxscl(c,g,N,1,f1)
write(5, *)
   write(5, *) ' Gain Matrix '
write(5, *) ' ========================='
write(5, *)
write(5,*) ' (G(i,1), i= 1,', N, ' ) : '
   write(5, 500) (G(j, 1), j = 1, N)
 300 format(2x, ' i = ', i3, ',', 2x, 5(f10.4, 2x))

c ---------------------------------------------
c ------ test for observability of the system ----
c ------ by local observability matrix M(k)   ----
c ---------------------------------------------
```

```
call mtxide(temp,N)
do 41 i = 1, N
  41  M(1, i) = H(k, i)
do 42 j = 1, N-1
call mtxmul(temp,Fi,temp2,N,N,N)
call mtxeq(temp2,temp,N,N)
do 43 i = 1, N
  43 hkk(1, i) = H(k+j, i)
call mtxmul(hkk,temp,temp1,1,N,N)
do 44 i = 1, N
  44 M(1+j, i) = temp1(1, i)
  42 continue
write(5, *)
write(5, *) ' local observability matrix'
write(5, *) ' ============================'
write(5, *)
write(5,*) ' (M(i,j), i= 1,', N, ', j = 1,', N, ' ) : '
do 45 i = 1, N
  45   write(5, 500) (M(i, j), j = 1, N)
call mtxinv(M,temp3,N,det)
do 47 j = 1, N
  aj(j) = 0.0d+00
  ajt(j) = 0.0d+00
do 47 i = 1, N
aj(j) = aj(j) + abs(M(i, j))
ajt(j) = ajt(j) + abs(temp3(i, j))
  47 continue
```

```
amx = aj(1)
atmx = ajt(1)
do 48 j = 2, N
if(aj(j) .gt. amx) amx = aj(j)
if(ajt(j) .gt. atmx) atmx = ajt(j)
  48 continue
cond_M = amx * atmx
write(5, *)
write(5, *) '========================================='
write(5, *) ' Cond. { M(k) } = ', cond_M
write(8, *) k, cond_M
write(5, *) '========================================='
write(5, *) ' det { M(k) } = ', det
iflag = 0
if (cond_M .gt. condt) iflag = 1
if (iflag .ne. 1) go to 3000
write(5, *) ' no updating for covariance matrix'
        do 81 i = 1, N
   81   hk(1,i) = 0.0
 3000 continue

c ------ updated error covariance matrix "P" -----
c ------------------------------------------------
call mtxmul(g,hk,t,N,1,N)
call mtxide(b,N)
call mtxsub(b,t,u,N,N)
call mtxmul(u,Pp,P,N,N,N)
```

```
 3010 continue
do 46 i = 1, N
  46 sigma(i) = p(i,i) ** 0.5
write(7,600) k, (sigma(i), i = 1, N)
 600 format(1x, i3, 2x, 5(f10.4, 2x), 2i3)
write(5, *)
write(5,*) ' (P(i,j), i= 1,', N, ', j = 1,', N, ' ) : '
do 80 i = 1, N
  80   write(5, 500) (P(i, j), j = 1, N)
   11 continue
stop
end


subroutine mtxzro(a,n1,n2)
c*******a=0; a is n1*n2.
real a(n1,n2)
do 10 i=1,n1
do 10 j=1,n2
 10 a(i,j)=0.0
return
end

subroutine mtxeq(a,b,n1,n2)
c*******b=a; both are n1*n2.
real a(n1,n2),b(n1,n2)
do 10 i=1,n1
```

```
do 10 j=1,n2
 10 b(i,j)=a(i,j)
return
end

subroutine mtxide(a,n)
c*******a is n*n
real a(n,n)
do 10 i=1,n
do 10 j=1,n
a(i,j)=0.0
 10 if (i.eq.j) a(i,j)=1.0
return
end

subroutine mtxneg(a,b,n1,n2)
c****** b=-a; both are n1*n2
real a(n1,n2),b(n1,n2)
do 10 i=1,n1
do 10 j=1,n1
 10 b(i,j)=-a(i,j)
return
end

subroutine mtxscl(a,b,n1,n2,scaler)
c****** b=a*scl; both are n1*n2
real a(n1,n2),b(n1,n2)
```

```
do 10 i=1,n1
do 10 j=1,n2
 10 b(i,j)=a(i,j)*scaler
return
end

subroutine mtxadd(a,b,c,n1,n2)
c****** c=a+b; all are n1*n2
real a(n1,n2),b(n1,n2),c(n1,n2)
do 10 i=1,n1
do 10 j=1,n2
 10 c(i,j)=a(i,j)+b(i,j)
return
end

subroutine mtxsub(a,b,c,n1,n2)
c****** c=a-b; all are n1*n2
real a(n1,n2),b(n1,n2),c(n1,n2)
do 10 i=1,n1
do 10 j=1,n2
 10 c(i,j)=a(i,j)-b(i,j)
return
end

subroutine mtxmul(a,b,c,n1,n2,n3)
c****** c=a*b; a is n1*n2, b is n2*n3, c is n1*n3
real a(n1,n2),b(n2,n3),c(n1,n3)
```

```
do 10 i=1,n1
do 10 j=1,n3
c(i,j)=0.0
do 10 k=1,n2
 10 c(i,j)=c(i,j)+a(i,k)*b(k,j)
return
end

subroutine mtxtrp(a,b,n1,n2)
c****** b= transpose of a; a is n1*n2 and b is n2*n1
real a(n1,n2),b(n2,n1)
do 10 j=1,n2
do 10 i=1,n1
 10 b(j,i)=a(i,j)
return
end

subroutine mtxinv(u,t,n,det)
c****** t= inverse of u; both are n*n
c det= determinant value
real u(n,n),t(n,n)
common hh(62,62)
do 1 i=1,n
do 1 j=1,n
  1 hh(i,j)=u(i,j)
do 2 i=1,n
do 2 j=1,n
```

```
t(i,j)=0.0
  2 if (i.eq.j) t(i,j)=1.0
det=1.0
do 9 i=1,n
m= i
do 3 j=i,n
  3 if (abs(hh(j,i)).gt.abs(hh(m,i))) m=j
if (m.eq.i) go to 6
do 4 j=i,n
s= hh(i,j)
hh(i,j)=hh(m,j)
  4 hh(m,j)=s
do 5 j=1,n
s=t(i,j)
t(i,j)=t(m,j)
  5 t(m,j)=s
det=-1.0*det
  6 det=hh(i,i)*det
if(det.eq.0) go to 100
s=1.0/hh(i,i)
do 7 j=1,n
if (j.gt.i) hh(i,j)=s*hh(i,j)
  7 t(i,j)=s*t(i,j)
do 9 j=1,n
if (j.eq.i) go to 9
s=hh(j,i)
do 8 k=1,n
```

```
if (k.gt.i) hh(j,k)=hh(j,k)-s*hh(i,k)
  8 t(j,k)=t(j,k)-s*t(i,k)
  9 continue
go to 200
 100 write(5,*) ' det = ', det
 200 continue
return
end
```

VITA

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