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I am submitting herewith a thesis written by Milton W. Davis, Jr. entitled "A Stage-By-Stage Compressor Modeling Technique for Single- and Dual-Spool Compression Systems." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

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A STAGE-BY-STAGE COMPRESSOR MODELING TECHNIQUE FOR
SINGLE- AND DUAL-SPOOL COMPRESSION SYSTEMS

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ABSTRACT

The objective of this effort was to extend a current one-dimensional, time-dependent, single-spool, stage-by-stage compressor modeling technique to include dual-spool compression systems. Prior to development of the dual-spool capability, a solution to a numerical instability problem which had existed in some previous compressor modeling efforts was developed by reformulating the modeling technique to use MacCormack's explicit differencing scheme and a method of characteristics boundary treatment to solve the set of differential equations. This investigation describes the numerical technique used to provide a more numerically stable model, incorporation of that numerical technique into an existing one-dimensional, single-spool compressor model, and the formulation of a dual-spool compression system model. The dual-spool model was applied to a current compression system and several types of planar destabilizing disturbances were studied to determine their effect on compression system stability. Model predictions were validated against experimental data whenever possible. This extension provides a tool for studying overall compression system and interspool reaction to time-variant planar disturbances and compression system throttling transients.

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NOMENCLATURE

A	Area
a	Acoustic velocity
CIVV	Compressor inlet variable vane, fan compressor
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
E	Energy function
e	Internal energy
F	Force of compressor blading and cases acting on fluid, including wall pressure area force
f	Friction factor
F	Force of compressor blading
f	Frequency
H	Total enthalpy
IMP	Impulse function
k	Coefficient of heat conductivity
M	Mach number
$\dot{M}$	Mass flow function based on total condition
MOC	Method of characteristics
N	Compressor rotor speed
P	Pressure
PR	Total pressure ratio
Q	Rate of heat addition to control volume
R	Gas constant
RCVV	Rear compressor variable vanes, high-pressure compressor

SW	Rate of shaft work
T	Temperature
TR	Total temperature ratio
t	Time
U	Generalized notation for Navier-Stokes equations
u	Axial velocity
V	Generalized notation for Navier-Stokes equations
v	Velocity component in direction of rotation
W	Mass flow rate
X	A point along the x-direction
x	Axial coordinate

Greek Symbols

α	Angle of attack
γ	Ratio of specific heats
ϵ	Energy term in Navier-Stokes equations
δ	Ratio of compressor entry total pressure to standard day, sea-level-static pressure
η_{ad}	Stage adiabatic efficiency
ρ	Density
σ	Normal stress
μ	First coefficient of viscosity
λ	Second coefficient of viscosity
ϕ	Flow coefficient
θ	Angular location, ratio of compressor inlet total temper- ature to standard day, sea-level-static temperature

ψ_1, ψ_2, ψ_3	Terms in compatibility equations which account for frictional and energy dissipation terms
ψ^P	Stage pressure coefficient
ψ^T	Stage temperature coefficient (stage loading parameter)
Δ	A difference

Superscripts

C	Corrector solution
P	Predictor solution
*	Pertaining to critical state

Subscripts

actual	Pertaining to actual measured value
avg	Average value
B	Pertaining to bleed flow
core	Pertaining to core engine/high-pressure compressor properties
design	Pertaining to design value
E1	Intersection of characteristic curve with previous time solution, exit boundary
E2	Intersection of streamline curve with previous time solution, exit boundary
entrance	Control volume entrance plane
fan	Pertaining to fan duct properties
I	Intersection of characteristic curve with previous time solution, inlet boundary

i	Spatial location
initial	Initial value
ISEN	Pertaining to isentropic flow conditions
max	Maximum value
min	Minimum value
new	New time
old	Old time
P1	Pseudoarea in the fan tip region
P2	Pseudoarea in the fan hub region
S	Static condition
stall	Value at stall
T	Total conditions
total	Pertaining to total geometric or thermodynamic properties
x	Axial component or direction

I. INTRODUCTION

An important subcomponent of the gas turbine engine is its compression system. It is the system performance which governs the other component performance. For most gas turbine engines, the compression system consists of one or more axial compressors aerodynamically coupled. These compressors may or may not be connected to the same shaft. An axial compressor consists of stages with a rotating component (rotor) and a stationary component (stator). The rotor is a series of airfoils which when rotated imparts kinetic energy to the fluid. The stator diffuses the flow and redirects the flow for the next row of rotors. Thus, it is the function of the compression system to increase the static pressure and density of the fluid. Without stable aerodynamic operation, the compression system cannot deliver the desired increase in static pressure and density.

Compressor performance is usually portrayed on a compressor "map." A compressor map gives compressor pressure ratio as a function of corrected airflow for a variety of constant corrected rotor speeds. In addition, the compressor nominal steady-state operating line and distortion-free compressor stability limit are also given. The stability limit refers to that locus of corrected rotor speed points for the minimum corrected flow rate for which stable operation of the compressor is obtained. For operation to the right of that critical point, the compressor runs in a stable manner, and to the left, the compressor is unstable. The location of the operating line is determined from the load imposed on the compressor from downstream

components. The compressor map is usually determined from experimental results. Because of test hardware and economical constraints, only a limited amount of experimental compressor performance and stability information can be obtained. In these cases, validated compressor mathematical models can be used to provide performance and stability information not obtained during experimental testing. In addition, compressor mathematical models also provide tools for studying effects of external disturbances on compressor performance and stability.

To provide an improved component simulation, a program was initiated to extend a current one-dimensional, time-dependent, single-spool compressor mathematical modeling technique to a dual-spool compressor model and to improve the numerical techniques employed to yield a more numerically stable model. This extension provides a tool for studying overall compression system reaction and spool interaction during time-variant planar disturbances.

Prior to describing the model, it may be helpful to review the types of compressor mathematical analysis tools and models available and how the present investigation complements previous works.

I.1 Previous Investigation

I.1.1 Early Analytical Methods

The earliest forms of compressor instability analysis dealt with the most violent form of compressor instability-surge. In works

by Hartog [1]¹ and Horlock [2], analysis of surge was performed by knowing the intersection of the pressure ratio/mass flow curves of the compressor and the throttle curve. The analysis indicated that the compressor/throttle combination where the slope of the compressor characteristic was greater than the throttle characteristic results in unstable compressor operation. This explanation provided an approximate analysis but predicted lower than experienced flow rate at surge.

While investigating axial compressor stability problems, Pearson and Bower [3] noted the discrepancy which existed between compressor experiments and the characteristic curve theory and that the characteristic theory did not account for the dynamic effects caused by volumetric capacitance, fluid column inertia, and flow resistance in the compressor and its inlet and exit ducting. Their approach was to represent the compressor by an electrical analogy with a set of time-dependent equations. Individual stage characteristics were coupled or "stacked" to form a representative multistage unit. Compressor instability was assumed to occur when the impedance of any given part of the circuit representing the compressor was zero. Positive circuit impedance indicated stable operation, while negative circuit impedance indicated unstable operation.

I.1.2 One-Dimensional Models

With the advent of the analog and digital computers, compressor performance and stability computations could be made by using

¹Numbers in brackets refer to similarly numbered references in the Bibliography.

numerical analysis techniques. One type of computer model of the compressor uses one-dimensional, lumped volume techniques. The time-dependent conservation laws of mass, momentum, and energy have been applied separately or in combination to the lumped volumes, thus accounting for the dynamic behavior of the system. The conservation laws are expressed with one-dimensional mathematics which only allows for spatially uniform flow to be considered. The stage or group of stages may be replaced by an actuator disc followed by a lumped volume equal to the stage, or group of stages. The required pressure and temperature rise across the actuator disc is usually supplied by a set of stage characteristics. The one-dimensional models are generally used to predict system instability for either undistorted flow or time-variant inflow distortions. Several examples are offered in the following paragraphs.

Gabriel, Wallner, and Lubick [4] developed a one-dimensional, 15-stage axial compressor model using only the conservation of mass principle. The compressor model was divided into four parts for which steady-state group characteristics were available. With low-frequency (less than 50 Hz) sinusoidal pressure variation applied to the inlet, the model predicted the experimentally observed stability limit. Because this model did not take into account momentum or energy transfer, the model could not simulate correct compressor behavior under high-frequency pressure variations.

Kuhlberg, Sheppard, and King [5] developed a one-dimensional model based on the conservation of mass and momentum. They investigated a continuity model (based only on the conservation of mass) and

found that this model was unsuited for determining dynamic compressor response over the stable operating range of the compression system. With the addition of the momentum equation, the model was able to overcome the problems associated with the mass continuity model. They also investigated a model based on the continuity, momentum, and energy equation and found that the model behaved in the same manner as the momentum-continuity model. Because of the complexity and lack of flexibility in the more complex model, they chose to use the momentum-continuity model for their analytical studies, which included surge line prediction with and without inlet planar pressure distortions. In a study of oscillatory inflow effects on compressor stability, their continuity-momentum model predicted the stability limit as observed experimentally up to a frequency of 60 Hz. Beyond that frequency, their model deviated from experimental results.

A one-dimensional model based on all three conservation principles was developed by Willoh and Seldner [6]. Their model assumes that each compressor stage (rotor, stator, and included volume) can be represented by a compressor element coupled to a lumped volume. The compressor element performance is obtained from steady-state stage characteristics. The gas dynamics of the following lumped volume are modeled through the application of continuity, momentum, and energy. Although the model uses the conservation principles, some simplifications were made. Convection terms in the momentum equation were neglected and the Mach number at the compressor element exit and in the following lumped volume were assumed the same. The later simplification resulted in dropping the distinction between static and

total pressure in the lumped volume element. This one-dimensional modeling technique was first implemented on an analog computer [6]. A simulation of the J85-13 compression system was constructed, and predictions of compressor stability limits were obtained and compared to experimental results. Willoh and Seldner found that the predicted stability limit agreed closely with the stall line of the actual jet engine compressor. Daniele, Blaha, and Seldner later implemented this technique on the digital computer [7]. In addition to the simplifications used in the analog computer simulation, the digital simulation included a simplification which eliminated the temperature dynamics in the lumped volume element. The results with this simplification were in agreement with those obtained by the analog simulation at corrected speeds of 80 percent and below. However, the results differed at the higher speeds.

I.1.3 Parallel Compressor Models

One-dimensional models are used primarily to predict the compressor stability limit with steady and time-variant uniform flow entering the compressor. Further information on previous one-dimensional efforts can be found in Reference [8]. Efforts have also been expended to predict the influence of external disturbances on compressor stability. One major effort has been termed as "parallel compressor theory."

Pearson described the parallel compressor theory for circumferential distortion in his 1963 work [9]. The theory divides the distorted compressor into sections, each section treated as an

individual compressor working in parallel with the other sections. The theory works with the following assumptions: (1) all parallel compressor sections discharge to the same static pressure; (2) there is no fluid dynamic connection between sections, i.e., crossflows are ignored; and (3) each section uses the undistorted stage characteristics. In its simplest form, the entire compressor is assumed to have reached instability when any one of the sectors reaches the stability limit.

In more complicated forms, provisions for approximating crossflows and determining the effect of unsteady aerodynamics on compressor behavior have been investigated. A compressor modeling technique based upon parallel compressor theory has been developed by Ward, which provides for circumferential crossflows between each parallel compressor segment [10]. The model is broken into control volumes based on rows (i.e., rotor row, stator row, and gaps between rotor and stator) in which the flow dynamics are represented by the one-dimensional, time-variant forms of the continuity, momentum, and energy equations. This model is capable of operating as a single-spool or dual-spool compression system. The fan-compressor coupling is obtained by splitting fan stage characteristics into inner-diameter and outer-diameter characteristics which then interact with the appropriate downstream condition. The model was able to predict the stability limit for uniform inlet, but it was unable to predict the compressor's response to circumferential distortion. Efforts by Mazzawy et al. [11, 12, 13] improved the modeling technique developed by Ward and incorporated such features as inlet flow angle variation,

fluid particle swirl, unsteady aerodynamics, and circumferential cross-flow through cavities. With these modeling improvements, Mazzawy was able to predict the correct trends when circumferential distortion was present.

A compressor modeling technique based on parallel compressor theory (known as the General Electric Dynamic Digital Blade Row Compression Component Stability Model) has also been developed by Tesch and Steenken [14]. Their technique is similar in concept to Ward's in that it uses one-dimensional, time-variant equations of continuity, momentum, and energy across each blade row control volume for each compressor sector. Stage characteristics are computed from given relative total pressure loss coefficient and deviation angle as a function of incidence angle. Crossflows between compressor sectors are ignored, and an accounting for effects of unsteady blade row aerodynamics was not made in their initial work. In later works [15, 16], Tesch, Moszee, and Steenken added the capability for circumferential crossflows' effects and unsteady blade row aerodynamics. The addition of unsteady blade aerodynamic effect led to a considerable gain in the ability of the model to predict experimentally observed compressor instability.

A modification by Hosny and Steenken to the General Electric model allowed the dynamic representation of a dual-spool, high bypass compression system [17]. The modification included the addition of compression system dual-path geometry, dual-spool matching requirements, and radial flow redistribution ahead of the fan bypass splitter. This model was based on a two-dimensional form of the conservation

principles which included the radial equilibrium equation. It was concluded that the model correctly simulated the flow interaction between the core and fan compression system while undergoing a throttling process.

I.1.4 Multidimensional Control Volume Model

While Mazzawy, Tesch, and others moved forward with the parallel compressor theory, Kimzey presented an improved modeling technique for computing the influence of external disturbances on turbine engine compressor stability [18]. In that work, Kimzey developed a one-dimensional, time-dependent, stage-by-stage compressor model for analysis of planar disturbances and a three-dimensional, time-dependent model for the analysis of distorted inflows including radial as well as circumferential distortion. These models use the nonlinear form of the conservation equations (mass, momentum, and energy). Compressor stage force and shaft work were determined from empirical stage characteristics with corrections made for unsteady aerodynamics. Using the one-dimensional formulation, three different single-spool compressors were modeled. These models were validated against experimental data and generally computed steady-state stability within 1 to 2 percent of that observed experimentally. Studies were conducted with these one-dimensional models to determine the effects of time-variant planar pressure oscillations and rapid inlet temperature ramps. The computations generally compared favorably to those results observed experimentally.

The concepts generated and validated in the development of Kimzey's one-dimensional model were extended to a three-dimensional model formulation to study the effects of distorted inlet flow. The compressor was divided circumferentially, radially, and axially to form a set of three-dimensional control volumes. The governing equations were developed based on three-dimensional, time-dependent conservation of mass, momentum, and energy. Empirical relationships were used to replace the circumferential and radial momentum equations, thus simplifying the model. In addition, the radial work distribution variations were approximated by modifying the average stage characteristics with characteristic total pressure and temperature profile corrections. One compressor was modeled using the three-dimensional technique and was subjected to varying degrees of radial pressure distortion, circumferential pressure distortion, and circumferential temperature distortion. Model predictions were generally in agreement with experimental results.

Chamblee applied the method to another compressor and introduced some refinements to improve the generality, accuracy, and numerical stability of the model [19]. These refinements included the ability to accommodate variable geometry features, specify gas specific heats as a function of temperature, and improve the numerical stability by reducing error buildup in the model calculations.

These two efforts demonstrated the validity of the one-dimensional and three-dimensional control volume modeling technique which allows for the study of time-variant planar and distorted

compressor inflows. Kimzey's work provided the basis for the subsequent work done by Chamblee and is the foundation for the present effort.

I.2 Purpose of the Present Investigation

The overall objective of this study was to extend a current one-dimensional, single-spool compressor modeling technique [18] to a dual-spool compression system model. This extension provides a tool for studying overall compression system and interspool reaction to time-variant planar disturbances and compression system throttling transients.

Prior to development of a dual-spool model, a shortcoming of the modeling technique had to be overcome. Some compressor models constructed according to the methods described by Kimzey have exhibited various levels of numerical instability--depending on compressor configuration or operating conditions [18, 19]. Previous solutions to this numerical instability problem have incorporated damping of the exponentially increasing, sinusoidal instabilities by increasing the friction in the inlet and exit ducting systems to unrealistic values. In addition to frictional damping, a dependent variable averaging/restart technique was also applied. These solutions present a problem in construction of a dual-spool model in that frictional damping not only damps unwanted perturbations but masks true physical behavior as well. In addition, the averaging/restart technique was dependent on the rate at which external disturbances were applied and on the averaging interval chosen. Thus,

a secondary but important objective of this study was to improve the numerical techniques employed, providing for a more numerically stable model.

This investigation describes the incorporation of numerical techniques into the modeling technique providing a numerically stable model and the construction of a one-dimensional, dual-spool, time-dependent compression system model. Model predictions are presented and compared with experimental results wherever possible.

II. ONE-DIMENSIONAL, TIME-DEPENDENT COMPRESSOR MODELING TECHNIQUE

The one-dimensional, time-dependent compressor modeling technique solves the nonlinear form of the conservation laws [18]. The models constructed from this technique can be used for the analysis of planar, transient, and dynamic effects on compressor operation and stability. A review of the governing equations, stage characteristic development, and method of solution is presented in this section.

II.1 Governing Equations

Illustrated in Fig. 1, Appendix,¹ is a representative single-spool compressor and ducting system which has been modeled. Included in this system is a portion of the compressor inlet and a portion of the combustor volume. The compressor and ducting system is modeled by an overall control volume shown in Fig. 1b. Acting on this fluid control volume is an axial force distribution, $F(x)$, due to the effects of the compressor blading and walls of the system. In addition, the rate of heat added to the fluid and shaft work done on the fluid are represented by distribution, $Q(x,t)$ and $SW(x)$, respectively. Mass transfer rate across boundaries other than the inlet or exit (such as the case of interstage bleed) is represented by the distribution $W_B(x,t)$. The time-dependent inlet boundary condition is the specification of total pressure and temperature.

¹All figures are presented in the Appendix.

The exit boundary condition is either the specification of static pressure or unity Mach number. The overall control volume is broken up into a set of elemental control volumes.

In the compressor section, an elemental control volume consists of a stator followed by a rotor, or vice versa, and associated volume representing a complete stage. This combination is dictated by the way the experimental data used for stage characteristic construction are obtained. The entrance and exit ducting sections of the overall control volume are divided into elemental control volumes to assure a frequency response as good as that in the compressor section (i.e., duct control volume lengths no longer than the longest compressor elemental control volume). The governing equations are derived by the application of mass, momentum, and energy conservation principles to the elemental control volume in Fig. 1c.

Applying the continuity principle to the elemental control volume yields

Continuity

$$\underbrace{W + \frac{\partial W}{\partial x} dx + W_B dx}_{\text{mass leaving control volume per unit time}} + \underbrace{\frac{\partial(\rho A dx)}{\partial t}}_{\text{time rate of increase of mass within the control volume}} = \underbrace{W}_{\text{mass entering control volume per unit of time}}, \quad (1)$$

which may be reduced to

$$\frac{\partial(\rho A)}{\partial t} = - \frac{\partial W}{\partial x} - W_B, \quad (2)$$

where W_B is the interstage bleed flow per distributed length per unit time.

Applying the conservation of momentum gives

Momentum

$$\begin{aligned}
 & \underbrace{Fdx + P_S A - \left[P_S A + \frac{\partial(P_S A)}{\partial x} dx \right] + P_S \left[A + \frac{\partial A}{\partial x} dx - A \right]}_{\text{axial forces acting upon the control volume}} \\
 & = \underbrace{\left[Wu + \frac{\partial(Wu)}{\partial x} dx \right]}_{\substack{\text{momentum leaving} \\ \text{control volume} \\ \text{per unit time}}} - \underbrace{Wu}_{\substack{\text{momentum entering} \\ \text{control volume} \\ \text{per unit time}}} + \underbrace{\frac{\partial}{\partial t} [\rho u A dx]}_{\substack{\text{time rate of} \\ \text{of increase} \\ \text{of momentum} \\ \text{within the} \\ \text{control volume}}} . \quad (3)
 \end{aligned}$$

total time rate of change of momentum

Equation (3) may be reduced to

$$\frac{\partial W}{\partial t} = - \frac{\partial(\text{IMP})}{\partial x} + F , \quad (4)$$

where

$$\text{IMP} = Wu + P_S A , \quad (5)$$

a momentum impulse term, and

$$F = F + P_S \frac{\partial A}{\partial x} , \quad (6)$$

an axial force distribution consisting of blade force and the force produced by the walls of the system.

Energy conservation yields

Energy

$$\underbrace{H + \frac{\partial H}{\partial x} dx}_{\substack{\text{enthalpy leaving} \\ \text{control volume} \\ \text{per unit time}}} + \underbrace{\frac{\partial}{\partial t} \left[\rho A \left(e + \frac{u^2}{2} \right) dx \right]}_{\substack{\text{time rate of increase} \\ \text{or energy within} \\ \text{control volume}}} = \underbrace{H}_{\substack{\text{enthalpy entering} \\ \text{control volume} \\ \text{per unit time}}} + \underbrace{SW dx}_{\substack{\text{rate of shaft work} \\ \text{done on fluid in} \\ \text{control volume}}} + \underbrace{Q dx}_{\substack{\text{rate of heat} \\ \text{added to} \\ \text{fluid in} \\ \text{control volume}}}, \quad (7)$$

which may be reduced to

$$\frac{\partial(EA)}{\partial t} = - \frac{\partial H}{\partial x} + SW + Q, \quad (8)$$

where

$$E = \rho \left(e + \frac{u^2}{2} \right) \quad (9)$$

and

$$H = c_p \cdot W \cdot T_T. \quad (10)$$

Equations (2), (4), and (8) represent the form of the nonlinear conservation equations used in the modeling technique.

Additional equations required include:

Equation of State

$$P_S = \rho R T_S. \quad (11)$$

Ratio of Specific Heats

$$\gamma = \frac{c_p}{c_v} = \text{constant} . \quad (12)$$

Definition of Mach Number

$$M = \frac{u}{a} , \quad (13)$$

where a is the acoustic velocity,

$$a = \sqrt{\gamma R T_S} . \quad (14)$$

Definition of Total Temperature

$$T_T = T_S \left[1 + \frac{\gamma-1}{2} M^2 \right] . \quad (15)$$

Definition of Total Pressure

$$P_T = P_S \left[1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{\gamma}{\gamma-1}} . \quad (16)$$

II.2 Compressor Stage Characteristics

The distributed force and shaft work terms in the momentum and energy equations, respectively, are supplied to the modeling technique by a set of experimentally derived stage characteristics. A stage flow coefficient (ϕ) is defined as

$$\phi = \frac{u}{v_\theta} , \quad (17)$$

where u is axial velocity and v_θ is the tangential velocity. The flow

coefficient is related to the blade angle of attack, α , through the stagger angle, λ , by the relationship

$$\phi = \cot(\lambda + \alpha) . \quad (18)$$

A decreasing flow coefficient means that the angle of attack is increasing, thus increasing the possibility of flow separation on the blade rows. A stage temperature coefficient (ψ^T) is defined as

$$\psi^T = \frac{c_p (TR-1)}{N^2/T_T} , \quad (19)$$

where TR is the stagnation or total temperature ratio, N is the rotor speed, and T_T is the stage inlet total temperature. Similarly, a stage pressure coefficient (ψ^P) is defined as

$$\psi^P = \frac{c_p \left[PR^{\frac{\gamma-1}{\gamma}} - 1 \right]}{N^2/T_T} , \quad (20)$$

where PR is the stage pressure ratio. The stage adiabatic efficiency is defined as

$$\eta_{ad} = \frac{\Delta h_{ISEN}}{\Delta h_{actual}} = \frac{PR^{\frac{\gamma-1}{\gamma}} - 1}{TR-1} \quad (21)$$

and can be defined as

$$\eta_{ad} = \frac{\psi^P}{\psi^T} . \quad (22)$$

A set of curves

$$\psi^T = \psi^T(\phi) , \psi^P = \psi^P(\phi) , \text{ and } \eta_{ad} = \eta_{ad}(\phi) \quad (23)$$

is a set of stage characteristics which fully define a compressor's stage performance. The set of equations represented by Eq. (23) is redundant; any two of the three variables are sufficient to compute the third.

Stage characteristics can be obtained either theoretically or through the use of experimental data. Since the form of the flow coefficient as defined in Eq. (17) does not lend itself to terms that are easily measured, a form based on measurable parameters (P_T , T_T , and flow) has been used for the compressor models reported herein.

$$\phi = \left\{ \left[\frac{W\sqrt{T_T}}{P_T A} \right] \left[\frac{(N/\sqrt{\theta})_{design}}{(N/\sqrt{\theta})_{actual}} \right] \right\} / \left\{ \frac{W^* \sqrt{T_T^*}}{P_T^* A^*} \right\} , \quad (24)$$

where * refers to the critical state, and

$$\frac{W^* \sqrt{T_T^*}}{P_T^* A^*} = 0.5318 . \quad (24a)$$

This expression can be derived by using the definition of flow coefficient (Eq. (17)), the steady-state flow relationship ($W = \rho AU$), the equation of state (Eq. (11)), and by nondimensionalizing the thermodynamic variables by the appropriate critical state variable (i.e., u^* , v^* , ρ^* , P_S^* , P_T^* , etc.). Two assumptions are also inherent in the definition of Eq. (24). The swirl velocity, v_θ , is equal to the rotational velocity at the mean geometric radius, and the corrected

speed corresponding to critical conditions is equal to design corrected speed. The form of flow coefficient expressed in Eq. (24) only differs from the definition by a constant.

The flow coefficient serves the same purpose in the compressor as angle of attack, α , does for aircraft wing dynamics. A decreasing flow coefficient means the angle of attack is increasing, thus increasing the possibility of flow separation. A typical set of stage characteristics is presented in Fig. 2. To the right of the stage peak pressure coefficient, the stage is unstalled (i.e., high ϕ and low α). To the left of the peak the stage is stalled and a loss of stage efficiency (Eq. (21)) can be inferred from the increase in stage temperature ratio as pressure ratio decreases.

II.3 Method of Solution

The compressor models are formulated as initial condition/ boundary value problems. With initial values for the dependent variables (pA , W , and EA) given at each calculating station, the specification of time-dependent boundary conditions, and the use of stage characteristics to get stage force and shaft work, the time derivatives can be calculated and then integrated to give the solution of the next time increment. A fourth-order Runge-Kutta numerical method was used to integrate the approximated differential equations. This formulation of the compressor modeling technique will be designated the Runge-Kutta formulation.

The one-dimensional, Runge-Kutta formulated modeling solution procedure is presented in Fig. 3. At time equal zero (Step A), initial

values for the dependent variables (density, airflow, and energy) are specified for all interior computational stations. Inlet and exit boundaries are also specified as functions of time to simulate the desired transient being studied. The computer simulation is started from a steady flow condition in which the initial and boundary conditions are representative of that condition. Thermodynamic and flow variables (pressure, temperature, velocity, enthalpy, etc.) as well as distributed forces and shaft work are calculated (Steps B and C) from the given dependent variables and stage geometry. The thermodynamic variables, flow variables, and distributed forces and shaft work are then used to calculate the right-hand side of Eqs. (2), (4), and (8) which are equal to the time derivatives of the dependent variables (Step D). The spatial derivatives, $\partial W/\partial x$, $\partial(\text{IMP})/\partial x$, and $\partial H/\partial x$, on the right-hand side of Eqs. (2), (4), and (8) are approximated by a noncentered, two-sided finite differencing technique. A fourth-order, single-step Runge-Kutta numerical scheme is used to integrate the dependent variable values to the next step in time. The sequence is repeated with the new values of the dependent variable replacing the initial conditions.

II.4 Numerical Instability Problems

The one-dimensional compressor models constructed according to the methods described in this section have exhibited various levels of numerical instability, even for stable flow situations. Previous solutions to this problem have incorporated damping of the exponentially increasing sinusoidal instabilities by increasing the

frictional coefficient in the inlet and exit ducting systems. However, this solution was not successful for all compressor conditions. In addition to frictional damping, a dependent variable averaging/restart technique has been applied to different compressor models with limited success [19]. However, the success of this technique was dependent on the rate at which external disturbances were applied and on the averaging interval chosen.

Further efforts have indicated that the instabilities might be coming from the forward and aft ducting system. A parametric study was performed to determine if there was some particular ducting configuration or frictional damping which would make the duct problem unconditionally stable. The duct configuration chosen consisted of a straight duct with constant cross-sectional area. The overall control volume was sliced into 19 elemental control volumes representing a previous compressor/ducting system. The duct was broken into three main sections (inlet, interior, and exit) in which each section consisted of more than one elemental control volume. The inlet sections consisted of those elemental control volumes representing the inlet ducting; the interior consisted of those elemental control volumes representing the compressor section; and the exit section consisted of those elemental control volumes representing the exit ducting of a previous system. The conservation equations were solved for each elemental control volume in the same manner as discussed in Section II.3. For the majority of the cases investigated, the interior was set equal to a volume set which represented the volume set of a particular compressor.

A variety of different inlet and exit volume sets and different amounts of frictional damping were investigated for their stabilizing influence. For each duct configuration selected, the model was supplied with initial conditions for each elemental control volume and overall control volume boundary conditions consistent with a steady-state flow condition. The model was then allowed to compute the time-dependent solution without outside perturbations being supplied. Thirty different cases were investigated which included the following variations: large volume inlet ducts, large volume exit ducts, small volume inlet ducts, small volume exit ducts, large frictional damping in exit ducts, large frictional damping in the first duct element, no frictional damping, and decreased integration time step. Of all the cases investigated, only three exhibited stable flow. The configuration that was the most stable when frictional damping was a realistic value, (i.e., friction factor, $f = 0.02$) included large inlet and exit duct volumes. Once the exit ducts were shortened in order to represent a nominal combustor volume, numerical stability could not be achieved without large amounts of frictional damping in those duct elements.

This investigation did not produce any viable solutions to the numerical instability problem associated with the Runge-Kutta formulation. Large volume ducts not only produce unrealistic lengths, but also reduce the frequency domain of the model. Large amounts of frictional damping in many duct elements not only damp unwanted perturbations, but mask true physical behavior as well.

A better solution to this numerical instability problem was necessary to allow realistic interaction between ducting and compressor sections of the model. Thus, the model was reformulated using a predictor/corrector numerical technique instead of the Runge-Kutta technique, in expectation that a more accurate solution to the differential equations would provide a numerically stable model.

III. MODEL REFORMULATION

The governing equations (Eqs. (2), (4), and (8)) of the compressor modeling technique are hyperbolic in nature. Thus, a numerical technique that has been applied successfully to hyperbolic equations was used in the model reformulation. The predictor/corrector method chosen was first employed by R. W. MacCormack in 1969 on work dealing with hypervelocity impact cratering [21]. In that work, MacCormack developed an explicit numerical method that has second-order accuracy in both time and space when applied to the time-dependent, compressible Navier-Stokes equations. Since that time, MacCormack, Hung, Paullay, Olson, McGowan, Cline [21, 22, 23, 24] and others have improved the technique and expanded its use to a variety of computation fluid dynamic problems. This method has since become known as the MacCormack second-order finite-difference scheme.

The compressor model is not only an initial value problem but a boundary problem as well. Because the treatment of the boundaries can be a cause of stability problems, new boundary formulations were employed. The numerical computational volume was divided into three areas; inlet, exit, and interior. MacCormack's scheme was applied to the interior; a method of characteristics (MOC) scheme was applied at the inlet; and either an MOC scheme for unchoked flow or an isentropic sonic nozzle scheme for choked flow was applied at the exit. The reformulated compressor model will be henceforth designated the MacCormack formulation.

III.1 MacCormack's Scheme and Application to the Modeling Technique

MacCormack's differencing scheme has in the past been typically applied to the Navier-Stokes equations. The basic concept will be presented using these equations, but will be specialized to the terms describing the compressor model (Eqs. (2), (4), and (8)). The unsteady compressible form of the Navier-Stokes equations for one-dimensional flow neglecting body forces and heat sources may be written in conservative form as

$$\frac{\partial U}{\partial t} + \frac{\partial V}{\partial x} = 0 , \quad (25)$$

where

$$U = \begin{Bmatrix} \rho \\ \rho u \\ e \end{Bmatrix} \quad \text{and} \quad V = \begin{Bmatrix} \rho u^2 \\ \rho u^2 + \sigma_x \\ (e + \sigma_x)u - k \frac{\partial \epsilon}{\partial x} \end{Bmatrix} ,$$

$$\sigma_x = P_S - (\lambda - 2\mu) \frac{\partial u}{\partial x} ,$$

$$\epsilon = \frac{e}{\rho} - \frac{u^2}{2} ,$$

k = coefficient of heat conductivity ,

λ and μ = coefficients of viscosity .

Applying the predictor portion of MacCormack's technique to Eq. (25) gives

$$U_i^{(P)} = U_i - \frac{\Delta t}{\Delta x_{i-1}} \left(V_i - V_{i-1} \right) , \quad (26)$$

where subscript i denotes a spatial location and superscript P denotes the predicted value. Correcting the predicted value then gives

$$U_i^{(C)} = 1/2 \left\{ U_i + U_i^{(P)} - \frac{\Delta t}{\Delta x_i} \left[V_{i+1}^{(P)} - V_i^{(P)} \right] \right\} . \quad (27)$$

The corrected value (denoted by superscript C) then becomes the value of the dependent variable for the next instant in time. Backward differences are used in the predictor step and forward differences in the corrector step. To avoid biasing the solution because of the order of differencing in the predictor/corrector steps, the method was modified to reverse the order of differencing every other time step. At the completion of the two-step process, first derivatives are approximated effectively by central differences.

Applying this method to the governing equations of the compressor model gives

Mass

Predictor:

$$\rho A_i^{(P)} = \rho A_i - \frac{\Delta t}{\Delta x_{i-1}} \left[W_i - W_{i-1} \right] - W_{B_{i-1}} \Delta t . \quad (28)$$

Corrector:

$$\rho A_i^{(C)} = 1/2 \left\{ \rho A_i + \rho A_i^{(P)} - \frac{\Delta t}{\Delta x_i} \left[W_{i+1}^{(P)} - W_i^{(P)} \right] - W_{B_i}^{(P)} \Delta t \right\} . \quad (29)$$

Momentum

Predictor:

$$W_i^{(P)} = W_i - \frac{\Delta t}{\Delta x_{i-1}} [IMP_i - IMP_{i-1}] + F_{i-1} \Delta t . \quad (30)$$

Corrector:

$$W_i^{(C)} = 1/2 \left\{ W_i + W_i^{(P)} - \frac{\Delta t}{\Delta x_i} [IMP_{i+1}^{(P)} - IMP_i^{(P)}] + F_i^{(P)} \Delta t \right\} . \quad (31)$$

Energy

Predictor:

$$EA_i^{(P)} = EA_i - \frac{\Delta t}{\Delta x_{i-1}} [H_i - H_{i-1}] + SW_{i-1} \Delta t + Q_{i-1} \Delta t . \quad (32)$$

Corrector:

$$EA_i^{(C)} = 1/2 \left\{ EA_i + EA_i^{(P)} - \frac{\Delta t}{\Delta x_i} [H_{i+1}^{(P)} - H_i^{(P)}] + SW_i^{(P)} \Delta t + Q_i^{(P)} \Delta t \right\} . \quad (33)$$

The bleed flow, the force of the compressor blading and casing, and the shaft work and heat transfer terms are distributions over a particular axial length. These distributed terms have to be consistent with the differencing equations for the equations to account for the correct mass, momentum, and energy transfer across each elemental control volume. When backward differencing is employed, the distributed term should then be associated with the upstream

calculating station. When forward differencing is employed, the term is associated with the calculating station of interest.

III.2 Boundary Treatment

In all numerical solutions to boundary value problems, the way the boundaries are treated can influence the stability and accuracy of the solution. As indicated by Moretti [25], a boundary value problem in which the permeable subsonic boundary has constant properties is an ill-posed problem. Moretti suggests that a stretching of the permeable boundary to infinity, where the velocity vanishes and other properties take on their stagnation values, is one way of having constant properties at the boundaries. Since one is limited to some finite domain, this approach has to be modified to some extent. Also, while working with numerical solutions to the Navier-Stokes equations in inlet regions, Olson et al. [23] found it necessary to allow the boundary conditions to become functions of time.

Abbett has performed a comparative study [26] of numerical techniques for computing the boundary flow conditions at solid walls and shock boundary points in supersonic, inviscid flow. This effort investigated 30 different calculation procedures for determining boundary flow conditions for four sample problems. The type of calculation procedures included: reflection, explicit one-sided difference, extrapolation, method of characteristics (at boundary points only), combined equations, and Euler predictor/simple wave corrector. The conclusions of the study were based on comparisons of each method with a pure method of characteristics (MOC) solution (i.e., all points,

interior as well as boundary points were calculated using MOC). When a full MOC was not utilized, Abbett concluded that the MOC boundary solution coupled with some high order differencing scheme for the interior points (such as MacCormack's method) gave the best results in calculating steady flow in supersonic flow fields. Abbett's conclusion is also valid for the compressor modeling technique, since both it and supersonic flow fields use hyperbolic equations to describe the fluid flow.

Cline [24], following the recommendations of Abbett, used the method of characteristics to transform the Navier-Stokes equations into a set of ordinary differential equations which were solved at the boundaries. The boundary treatment for the MacCormack compressor model reformulation as described in this study follows the treatment Cline used in his VNAP program.

Using the unsteady compressible form of the Navier-Stokes equations for two-dimensional flow without body forces and heat sources, Cline derived the following set of equations ([24], Appendix A) using the method of characteristics. These equations have been modified to treat one-dimensional flow only.

$$\underbrace{\frac{dP_S}{dt} - \rho a \frac{du}{dt} = a^2 \psi_1 - \rho a \psi_2 + \psi_3}_{\text{compatibility equation}} \quad \text{for} \quad \underbrace{\frac{dx}{dt} = u - a}_{\text{characteristic equation}}, \quad (34)$$

$$\underbrace{\frac{dP_S}{dt} + \rho a \frac{du}{dt} = a^2 \psi_1 + \rho a \psi_2 + \psi_3}_{\text{compatibility equation}} \quad \text{for} \quad \underbrace{\frac{dx}{dt} = u + a}_{\text{characteristic equation}}, \quad (35)$$

where

$$\psi_1 = 0 ,$$

$$\psi_2 = \frac{1}{\rho} \frac{\partial}{\partial x} \left[(\lambda + 2\mu) \frac{\partial u}{\partial x} \right] \text{ (momentum viscous damping terms) ,}$$

$$\psi_3 = (\gamma - 1) \left[(\lambda + 2\mu) \left(\frac{\partial u}{\partial x} \right)^2 + \frac{\partial}{\partial x} \left(k \frac{\partial T_S}{\partial x} \right) \right]$$

(energy dissipation and conduction terms) .

In the absence of any viscous forces or conduction heat transfer along the characteristic curves, the compatibility equations reduce to the following set of relationships:

$$dP_S - \rho a du = 0 \text{ for } \frac{dx}{dt} = u - a , \quad (36)$$

$$dP_S + \rho a du = 0 \text{ for } \frac{dx}{dt} = u + a . \quad (37)$$

III.2.1 Inlet Boundary Solution

The inlet boundary thermodynamic properties for any time can be calculated by specifying certain boundary conditions (P_T and T_T) and using the characteristic relationships of Eq. (36). The characteristic equation is first solved by approximating the total derivatives by differences

$$X_{\text{new}} - X_I = (u - a) \Delta t . \quad (38a)$$

Illustrated in Fig. 4 is the determination of the intersection of the characteristic curve (Point X_I) with the geometry of the previous time step (Line AB). The slope of the characteristic curve runs through the point of interest (i.e., Point C of Line CD) and a point which lies

along the geometry of the previous time step, where all thermodynamic properties are known at the calculation stations. The thermodynamic relationships at Point X_I can be determined by linear interpolation of the properties between Points A and B. Once this point is known, the compatability equation (Eq. (36)) can be approximated by differences

$$P_{S_{X_{\text{new}}}} - P_{S_{X_I}} = \rho a u_{X_{\text{new}}} - u_{X_I} \quad (38b)$$

An iterative technique is employed since there are one equation and two unknowns,

$$P_{S_{X_{\text{new}}}} \text{ and } U_{X_{\text{new}}}.$$

A guess at inlet Mach number is made, and along with the thermodynamic relationships

$$P_T = P_S \left[1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{\gamma}{\gamma-1}} \quad (39)$$

and

$$T_T = T_S \left[1 + \frac{\gamma-1}{2} M^2 \right], \quad (40)$$

Eq. (38) is solved using the interpolated values of density and acoustic velocity in the previous time frame. With P_T , T_T , and P_S known, a new value for inlet Mach number can be obtained and the process continued until Mach number does not change within a specified tolerance. To obtain a more accurate value of $P_{S_{\text{new}}}$, the above iterative technique is applied to Eq. (36), with density and acoustic

velocity now being an average between the interpolated value in the old time frame and the previous iterated value at the new time step. All thermodynamic properties are calculated using these relationships and the equation of state.

III.2.2 Exit Boundary Solution

For subsonic exit conditions, static pressure is specified as the exit boundary condition. A characteristic scheme, such as used in the inlet boundary scheme, can be employed. In addition to using the characteristic relationships of Eq. (37), a set of streamline equations is used:

$$dP_S - a^2 d\rho = 0 \text{ for } \frac{dx}{dt} = u . \quad (41)$$

A procedure similar to the inlet solution procedure is used with each compatibility equation solved along its characteristic curve or streamline curve, as illustrated in Fig. 5. With the specification of static pressure and the two compatibility equations, an iterative procedure is not needed. This type of boundary specification is used when exit static pressure variation is known (such as an exit pressure ramp leading to compressor surge).

For the case when an exit sonic condition is used simulating choking in the turbines, an imaginary, zero length convergent nozzle is employed as presented in Fig. 6. Flow within this nozzle has been assumed to be steady (i.e., at any time the steady-state flow equations govern the flow within the nozzle) and isentropic. Using the concept of mass flow function, $\dot{M}$, based on total conditions,

$$\dot{M} = \frac{W\sqrt{T_T}}{P_T A}, \quad (42)$$

the thermodynamic properties at the nozzle exit and inlet can be calculated. At sonic conditions the mass flow function has a constant value of 0.5318, which implies that the mass flow function at the nozzle inlet (Station N+1) also remains constant since the nozzle flow is governed by isentropic, steady-state equations. It is further assumed that isentropic flow exists in the preceding elemental control volume, thus specifying total pressure and temperature (P_T and T_T at Station N+1) to be equal to that at the previous calculating station (Station N). Boundary flow rate can then be calculated if the flow function at Station N+1 is specified based on the given area. This type of boundary condition allows compressor transients (such as rotor speed changes, inlet pressure and temperature perturbations) to take place without having to know how exit static pressure varies during those transients.

III.3 Model Solution Procedure

Figure 7 outlines the overall digital computer simulation procedure. At time equal zero (Step A), initial values of the dependent variables (mass, momentum, and energy) are specified for every control volume. The solution is started from a steady, uniform flow condition. With the specification of boundary conditions, all thermodynamic variables can be calculated (Step B) at the inlet, interior, and exit planes. Using the method of characteristics scheme or sonic exit

condition scheme, the inlet and exit conditions for the next step in time can be calculated (Step C).

Knowing all the interior thermodynamic properties from Step B, the force and work terms (Step D) necessary for the momentum and energy equations (Eqs. (4) and (8)) can be calculated for input into the predictor step of MacCormack's method (Step E). New values for force and work can be determined from the predicted solution (Step F) and then used in the corrector step of MacCormack's method (Step G). The new values of the dependent variables determined in Step G are then used to compute the thermodynamic properties (Step B) for that time step. The sequence is repeated with the boundary conditions changing in accordance with the specified event being simulated.

The momentum and energy equations (Eqs. (4) and (8)) require axial force and shaft work distributions representing the force of the compressor blading and casing on the fluid and the shaft work input per stage. These functions are constructed from the experimentally determined steady-state stage characteristics.

III.4 Stability Criteria and Frequency Disturbance Limitations

With any differencing scheme which approximates partial differential equations, some stability criteria must be set. The system of equations used by the model is hyperbolic in nature and represents the one-dimensional compressible flow equations with source terms. A stability criterion that has been used in the past with these equations is the Courant, Friedrichs, and Lewy (CFL) stability restriction [18]. The restriction states that the finite-difference

domain of influence must be at least as large as the physical domain of influence (i.e., a sound wave cannot travel more than one elemental control volume length in one time increment). Expressing this mathematically gives

$$\frac{(|u| + a)\Delta t}{\Delta x} \leq 1 . \quad (43)$$

For the maximum time step, Eq. (43) can be written as

$$\Delta t_{\max} = \frac{\Delta x_{\min}}{a + |u|_{\max}} . \quad (44)$$

For the high-pressure compressor, single-spool model (to be discussed in Section IV), the maximum time was based on $\Delta x_{\min} = 0.15$ ft, $a = 1,553$ ft/sec (static temperature of $1,000^{\circ}\text{R}$), and $u_{\max} = 600$ ft/sec (compressor exit velocity, subsonic). This gave a maximum time step of

$$\Delta t_{\max} \approx 7 \times 10^{-5} \text{ sec} .$$

The time step used for the high-pressure compressor, single-spool model was 5×10^{-5} sec.

For the dual-spool compression system model (to be described in Section V) the maximum time was based on $\Delta x_{\min} = 0.15$ ft, $a = 1,900$ ft/sec (static temperature of $1,500^{\circ}\text{R}$), and $u_{\max} = 625$ ft/sec. This gave a maximum time step of

$$\Delta t_{\max} \approx 6 \times 10^{-5} \text{ sec} .$$

The time step used for the dual-spool model was also 5×10^{-5} sec.

The model's response to some disturbance is dependent upon the elemental control volume length. Disturbance propagation through a control volume must be accomplished in a relatively short period of time to allow all the fluid to respond to the disturbance. According to Kimzey [18], maximum frequency response is limited by minimum disturbance wavelengths of $10 \Delta x_{\max}$. Thus, using Kimzey's definition,

$$f_{\max} = \frac{(u + a)_{\min}}{10 \Delta x_{\max}}, \quad (45)$$

the maximum frequency response for the high-pressure compressor, single-spool compressor model was estimated to be 630 Hz based on velocities encountered in the inlet ducting system and nominal duct lengths of 0.25 ft. According to Ward [10], a criterion for maximum frequency is that the propagation time through the control volume must be less than one-third the period of disturbance. Expressed mathematically this becomes

$$f_{\max} \leq \frac{a}{6\pi\Delta x},$$

where a and Δx are average quantities. Using this definition, the maximum frequency for the single-spool compressor model was estimated to be 385 Hz. Therefore, the maximum frequency probably lies somewhere in between these frequencies.

The maximum frequency response for the dual-spool compression system is fixed by a fan stage length of approximately 0.65 ft. Thus, maximum frequency response was estimated to be on the order of 300 Hz based on representative velocities in that particular stage using Kimzey's definition, and approximately 350 Hz using Ward's definition.

IV. SINGLE-SPOOL FORMULATION AND RESULTS

The reformulated compressor modeling technique was applied to a current 10-stage, high-pressure compressor as had previously been done with the Runge-Kutta formulated model. This high-pressure compressor was chosen because of the availability of accurate stage characteristics based on experimental data obtained during a test program at the Arnold Engineering Development Center (AEDC). A schematic of the elemental control volume makeup of the high-pressure compressor model is presented in Fig. 8.

IV.1 Steady-State Behavior

To determine the stability of the single-spool compressor model constructed with the MacCormack differencing scheme coupled with the method of characteristics boundary treatment, the model was first exercised in a steady-flow situation over a range of corrected rotor speeds from 77 to 102 percent of design. At each speed, a set of steady-state initial conditions was supplied without external disturbances applied to the boundaries. At five corrected speeds investigated (77, 82, 95, 98, and 102 percent), the model came to a steady-state solution after an initial start-up excursion. Typical behavior is shown in Fig. 9 for the 102-percent speed. At two corrected speeds (87 and 92 percent), the model exhibited a bounded instability with a frequency of approximately 250 Hz and an amplitude fluctuation of ± 0.06 percent of pressure ratio level (Fig. 10a) and a ± 0.14 -percent amplitude fluctuation in inlet duct Mach number (Fig. 10b).

Suspecting that the boundary calculations might be the cause of the bounded instability at the two corrected speeds, the boundaries were then held constant (i.e., P_T, T_T and Mach number specified at the inlet, and ρ, u , and P_S specified at the exit) at the steady-state initial condition. This produced no fluctuation in overall pressure ratio and indicated that the instabilities were indeed being produced by the boundary calculations. As was done with the Runge-Kutta formulation, a study was conducted to find a duct configuration or frictional damping technique which might minimize the bounded instability. Long inlet ducts which showed some success in the Runge-Kutta formulation only aggravated the instability problem. Decreasing the volume and thus the length of the first elemental control volume reduced the amplitude of the fluctuations but not the frequency (Fig. 11).

In addition to inlet duct volume reduction, frictional damping was also added to minimize the bounded instability. However, instead of applying frictional damping to many duct elements in the inlet or exit ducting system, it was applied only to the first inlet duct element (i.e., the same one which had been reduced in volume). It was determined that a friction factor, f , of 0.2 (10 times the friction factor in all other duct elements) not only reduced the amplitude of the instability, but the instability frequency was increased to a high rate ($>5,000$ Hz) (see Fig. 12). This high frequency could then be distinguished from wave phenomena frequencies which are on the order of 500 Hz or less.

The bounded instability as well as the start-up excursion is probably caused by the solution being reflected back on to itself at

the boundaries instead of some reflection and some transmission of the solution waves. The technique employed thus dampens out these reflected waves. A better solution may be to incorporate a technique which allows transmission of waves at the boundaries.

IV.2 Transient Behavior

To check out the transient behavior, a comparison of the model's computation of overall compressor performance to experimental results was conducted. The model was initially set at a constant corrected speed and allowed to come to a steady-state condition near its nominal operating point. Once stability had been assured, an increase in exit static pressure was programmed which increased the compressor pressure ratio at constant corrected speed until instability occurred. The exit pressure ramp was 50 psia/sec. Slower pressure ramps generally produced no difference in stability limits than those established at the 50 psia/sec rate. Thus, it was concluded that the results with this ramp rate could be compared to steady-state experimental results as a means of model validation. The experimental data were obtained during tests conducted at the AEDC of a core engine consisting of high-pressure compressor, combustor, and high-pressure turbine. The comparison was made over a range of corrected speeds of 77, 82, 87, 92, 95, 98, and 102 percent of design corrected speed.

The corrected speedlines as computed by the time-dependent compressor model are compared to steady-state experimental data in Fig. 13. The model predicted compressor surge within 1.3 percent in airflow and 1.7 percent in pressure ratio. At 95- and 98-percent

corrected speed, the model predicted compressor surge exactly as experienced experimentally. The worst prediction of compressor surge airflow occurred at 87-percent speed (1.3-percent error) and the worst prediction of compressor surge pressure ratio occurred at 77-percent speed (1.7-percent error).

In addition to overall performance, the model can predict stage performance and indicate which stage initiates compressor surge. Model prediction of stages in aerodynamic stall for the 102-percent corrected speed at time of compressor surge are indicated in Fig. 14. Stages 6, 7, and 8 are operating on the positive side of their characteristic, indicating that these stages may be aerodynamically stalled, causing a decrease in performance. Indicated in Fig. 15 is the airflow disturbance buildup in and around Stage 7 and the propagation forward and aft of Stage 7 prior to compressor surge.

To identify the stage which initiates compressor surge, static pressure behavior is a good indicator, experimentally as well as theoretically. A typical trace of high-response static pressure measurements during an actual compressor surge event is presented in Fig. 16. At time of compressor surge, the stage that initiates surge will indicate a sudden increase in stage entrance pressure and sudden decrease in exit pressure. This signature is caused by flow stagnation at the stage entrance and recirculation at the stage exit initiated by separation on the airfoils. The pressure signature is evident across the seventh stage of the high-pressure compressor (Fig. 16). Model prediction of static pressure at time of compressor surge at 102-percent corrected speed is presented in Fig. 17. The same pressure

signature as observed experimentally is evident across the seventh stage of the model and thus indicates that this stage initiated compressor surge.

A comparison of model predictions and experimental results of stages initiating compressor surge for all corrected speeds is presented in Table I. The experimental data indicate that Stages 7, 8, or 9 are generally identified as the critical stage. The model predicts that Stage 7 is generally the critical stage, with Stages 4, 6, and 8 in a highly loaded situation.

These results reveal that the reformulation of the compressor model provides a solution to the numerical instability problems previously encountered in the single-spool compressor formulation. The method is used as the basic approach for the formulation of a dual-spool compressor model.

Table I. Model Prediction of Stalling Stages at time of Compressor Surge and Stage Initiating Surge with Comparison to Experimental Results

Rotor Speed (%)	Stage Initiating Compressor Surge as Indicated by Experimental Data	Stalling Stages as Indicated by Time-Dependent Compressor Model	Stage Initiating Compressor Surge as Indicated by Time-Dependent Compressor Model
77	7 or 9	4, 7, 8	7
82	7 or 8	4, 5, 6, 7, 8	7
87	7 or 8	4, 6, 7, 8	7
92	7	4, 6, 7	7
95	6 or 7	4, 7, 8	7
98	7 or 9	8, 9	8
102	7 or 8	6, 7, 8	7

V. DUAL-SPOOL FORMULATION AND RESULTS

The reformulated compressor modeling technique has been extended to include dual-spool compression systems and has been applied to a 13-stage compression system. The 13-stage compression system consists of a three-stage axial flow fan compressor connected to a two-stage, low-pressure turbine and a 10-stage, high-pressure compressor connected to a two-stage, high-pressure turbine. This arrangement allows fan and compressor to operate at different speeds. Variable compressor inlet geometry is used on both the fan and high-pressure compressor and is scheduled as a function of airflow, which is determined from compressor inlet temperature and compressor rotor speed.

Stage characteristics necessary for model construction were based on experimental data. High-pressure compressor characteristics were the same as those used in the single-spool model. Fan characteristics were synthesized from fan rig data. Stage characteristics for both the fan and high-pressure compressor incorporate nominally scheduled variable geometry effects.

V.1 Dual-Spool Formulation

A schematic of the system modeled is presented in Fig. 18. Fan stages are represented by Volumes 5, 6, and 7, while high-pressure compressor stages are represented by Volumes 12 through 21. Upstream of the fan are four elemental control volumes representing approximately 1.5 ft of the inlet duct. Volumes 8 and 9 represent the space

prior to the fan splitter. Volumes 10 and 11 are inlet volumes to the high-pressure compressor and are physically separate from the fan duct (Volumes 27, 28, and 29). At the exit of the high-pressure compressor are four elemental control volumes representing approximately 0.6 ft of the combustor.

The model operates to a known compressor/fan speed ratio based on the nominally determined compression system operating line. Fan speed is furnished as an input and high-pressure compressor speed is calculated from the known speed relationship. Fixing the speed relationship to this particular schedule limits the model to rapid transients (less than 1.0 sec) where physical rotor speeds do not have enough time to respond to the changing environment. With a more sophisticated controls simulation, the dual-spool model could simulate other types of transients where rotor speeds could then respond.

Boundary conditions are specified at the inlet (P_T and T_T), the fan duct exit (P_S or choked nozzle), and the high-pressure compressor exit (P_S or choked nozzle). Initial values of the dependent variables (ρA , W , and EA) corresponding to a steady flow situation are specified at each calculating station to provide information to start the solution procedure. The solution procedure is the same as given in Fig. 7, with one exception. In the compression system, the fan airflow is split into two separate flow streams after compression has taken place in the fan. The flow splits by means of a splitter into a fan stream exhausting to the augmentor section and a core stream to be further compressed by the high-pressure compressor. These two streams are aerodynamically coupled (i.e., perturbations in

one stream are eventually felt by the other stream). A technique based on steady flow conditions and a pseudoflow area was used to allow the aerodynamic coupling. A schematic of the splitter area and the pseudoflow area technique is presented in Fig. 19.

Initially, airflow split is known as a function of the initial conditions. The calculating station upstream of the splitter plane is split into two pseudoflow areas (A_{p1} and A_{p2}) based on a proportional ratio of fan duct flow to the total flow at the splitter plane. An airflow is calculated at each pseudoarea (W_{p1} and W_{p2}) based on the instantaneous airflow at the upstream calculating station and the proportional split ratio determined at the splitter plane. Thermodynamic properties at the upstream calculating station are known based on the whole area and are used to calculate the pseudoarea properties (i.e., enthalpy, impulse, and force) needed for the MacCormack formulated difference equations (Eqs. (28) through (33)). If backward differencing is used at the calculating station just upstream of the splitter plane, the pseudoflow technique is not needed. If forward differencing is used, the properties at the upstream station are an average of the transmitted flow properties in the fan stream and core stream. At the splitter plane the reverse is true (i.e., forward differencing does not require the pseudoflow technique but backward differencing does). The process is repeated for each time calculation and the pseudoflow area split is recalculated based on the instantaneous flow split at the splitter plane. Using this pseudoarea technique, perturbations from either the fan or core stream then can

influence the upstream flow properties and allow interaction between the fan and core streams.

V.2 Dual-Spool Results

The dual-spool model was exercised to simulate several different types of planar disturbances that might be encountered by turbine engine compression systems. The fan component was loaded to surge by rapidly increasing the fan duct bypass pressure, thus simulating a "hard" augmentor light. The high-pressure compressor was also loaded to surge by a rapid increase in compressor exit pressure simulating a combustor lighting problem. Stability limits and system behavior for sinusoidal inlet total pressure perturbations were established for frequencies up to 300 Hz. Sinusoidal fan duct perturbations for frequencies up to 100 Hz were applied to the model to simulate augmentor rumble and its effects on compression system stability. Also investigated was the effect of rapid inlet temperature ramps (as might be experienced during armament firing) on compression system stability. Wherever possible, the results from these model studies have been compared to experimental data for validation purposes.

V.2.1 Fan Loading to Surge

The model was initially set at an operating condition for an altitude of 30,000 ft, Mach number 0.8, intermediate power ($P_T = 6.651$ psia, $T_T = 439.67^\circ\text{R}$, and 106-percent fan speed). The model was allowed to run at a steady-state condition to ensure numerical stability. Once stability had been assured, an increase in

fan duct exit static pressure was programmed at 20 psia/sec, which increased the fan pressure ratio at constant corrected speed until instability occurred. Time traces of compression system performance parameters during the simulated transient event are presented in Fig. 20. The fan component variables (speed, pressure ratio, and airflow) changed as expected during the fan loading transient. Pressure ratio increased, corrected flow decreased slightly, and speed remained constant. The high-pressure compressor did not move far from its initial operating point (i.e., speed, pressure ratio, and airflow changed very little). High-pressure compressor behavior was as expected since the model does not simulate the control logic. There was an increase in overall pressure ratio and a decrease in bypass ratio (ratio of fan duct flow to core flow). Compression system instability was indicated by the model instability occurring approximately 0.32 sec into the transient event.

A comparison of model results to those obtained during steady-state fan loading is presented in Fig. 21. The experimental data were obtained at 30,000 ft, Mach number 0.8 during a fan stability audit test program conducted at AEDC in 1979. Fan performance during the transient is compared on the fan map (Fig. 21a). A discrepancy in airflow between the model and data can be accounted for by the difference between experimental and model compressor inlet variable vane (CIVV) setting. The model does not have the direct capability to predict compressor operation off nominal CIVV schedule. However, information is available which can quantitize the effect in airflow for misscheduled CIVV's. According to test results conducted at AEDC,

a difference of five degrees more cambered in CIVV setting may decrease airflow as much as 3 percent at high corrected speeds and not affect airflow at low corrected speeds. Since the model CIVV setting is nominally 3.2 degrees more cambered than the experimental data CIVV setting, a decrease in airflow is expected. Both the model and the experimental data pressure ratio fell within 1.0 percent of the fan rig experimental surge line.

High-pressure compressor performance is compared to the same experimental data on the compressor map (Fig. 21b). Both the model and the data indicate that the high-pressure compressor operates near its nominal operating line, thus leaving adequate surge margin. Because the model does not interact with an active control (i.e., mechanical speed was constant) and the experimental data reflect the effects with an active control (mechanical speed increased), the high-pressure compressor performance is similar only in pressure ratio level and not in corrected flow extent.

Compression system bypass ratio is compared to experimental results in Fig. 21c. Again, the discrepancy in fan inlet airflow is a result of the different CIVV settings. Model initial bypass ratio was a little higher than experimental (0.73 versus 0.66), but both ended at a value of 0.42 at compression system surge.

Indicated in Fig. 22 is the static pressure variation at time of compression system instability. Using the signature of a rapid increase in stage entrance pressure and a decrease in stage exit pressure as the criteria for determining the stage initiating compression system surge, it can be noted that the model predicted the

first stage of the fan as the critical stage. The model's prediction of the stage initiating fan surge cannot be verified because experimental interstage data indicating critical stage at time of fan surge were not available. Indicated in Fig. 23 is the airflow variation at time of compression instability which shows a buildup of flow disturbance in the fan stages.

V.2.2 High-Pressure Compressor Loading to Surge

To simulate high-pressure compressor loading caused by a rapid pressure ramp in the combustor area, the model was initially set at an operating condition of 30,000 ft altitude, Mach number 0.8, 92-percent fan speed. The model was allowed to run at a steady-state condition to ensure numerical stability. Once stability had been assured, an increase in high-pressure compressor exit static pressure was programmed at 50 psia/sec, thus driving the high-pressure compressor up a corrected speedline until instability occurred. Time traces of compression system performance parameters during the transient event are presented in Fig. 24. The variations in high-pressure compressor performance were as expected during the loading process. Pressure ratio increased, airflow decreased, and speed remained constant. The fan did not move far from its initial operating point, but this was also expected since the model does not simulate the control logic. Overall compression system pressure ratio and system bypass ratio increased during the transient. Compression system instability was indicated by the model instability occurring approximately 0.52 sec into the transient event.

A comparison of model results to those obtained during experimental steady-state, high-pressure compressor loading by inbleeding is presented in Fig. 25. The data were obtained at 30,000 ft, Mach number 0.8 during a high-pressure compressor stability audit conducted at AEDC in 1979. High-pressure compressor performance during the transient is compared on a compressor map (Fig. 25a). As was the case with the fan loading, an airflow discrepancy between the model and experimental data exists, but can be accounted for by the difference in rear compressor variable vane (RCVV) setting. Experimental results indicate that for every degree more axial in RCVV setting, the airflow will increase 1 percent. Since the model RCVV setting is approximately three degrees more axial than the experimental data's setting and the model does not directly correct for off-schedule operation, a 3-percent increase in airflow is expected. The experimental surge line presented in Fig. 25a was obtained with a core engine (compressor-burner-turbine) test conducted at AEDC and is the same source of data as was used to construct the stage characteristics.

Fan performance is presented on a fan map and compared to the corresponding experimental data in Fig. 25b. Both the model and the experimental data indicate that the fan component operates near its nominal operating line, thus leaving approximately two-thirds of the initial surge margin available. As was the case during the fan loading transient, the model does not interact with an active control (mechanical speed was constant), while the experimental data reflect the effects with an active control (mechanical speed varied). Thus,

the fan performance is similar only in pressure ratio level and not in corrected airflow extent.

Compression system bypass ratio behavior is compared to experimental results in Fig. 25c. Model bypass ratio behavior was similar in trend to that experienced experimentally, but with a different corrected airflow extent because of the difference in RCVV settings.

The model predicted that the 11th stage of the compression system (eighth stage of the high-pressure compressor) was the stage which initiated compression system surge. When the high-pressure compressor model by itself was subjected to a similar pressure ramp, but at sea-level-static condition (Section IV), it indicated that the seventh stage of the high-pressure compressor was the critical stage. However, that case also predicted that the eighth stage was in a highly stalled condition. Thus, it may be concluded that the critical stage may change depending on flight conditions.

V.2.3 Oscillating Inflow Effects on Compression System Stability

To determine the effects of oscillating inflow on compression system stability, the model was subjected to inlet total pressure variations. The model was initially set at sea-level-static conditions, intermediate power ($P_T = 14.696$ psia, $T_T = 518.67^\circ\text{R}$, and 105-percent fan speed) and allowed to run at a steady-state condition to ensure numerical stability. Once stability had been assured, the inlet total pressure was varied sinusoidally with a linearly increasing amplitude until system instability occurred. Time traces of compression system performance parameters during a 20-Hz transient are presented in Fig. 26. Oscillatory inlet total pressure causes oscillation in all

flow properties. Corrected speeds oscillated because temperatures oscillate; the same situation applies with the flows. High-pressure compressor flow is initially near sinusoidal, but as stages are driven into and out of their stalled regions, the waveform becomes distorted (Fig. 26f). Overall pressure ratio oscillates over a wide range (14 to 45), while bypass ratio only ranges from 0.6 to 0.83.

Since with oscillating inflow the system does not operate along constant corrected speedlines, a simplified trace of compression system behavior is presented in Fig. 27. With linearly increasing total pressure fluctuations, the fan and the high-pressure compressor loop around the initial operating points. The fan and the high-pressure compressor can traverse their surge lines without surge occurring if the period of time in which they are above those limits is short enough. Once the stay time becomes of sufficient length, some stage or group of stages within the compression system will be in aerodynamic stall long enough to initiate compression system instability.

Presented in Fig. 28 are the individual stage flow coefficients' behavior during the 20-Hz oscillating inflow. Also indicated in the figure is the initial steady-state stall value of the flow coefficient. This serves to indicate the initial compression system loading. At onset of oscillatory flow, none of the stages was in a stalled condition. The stage flow coefficient traces do indicate that the instability was initiated in the fan component for the 20-Hz case. Further investigation into static pressure behavior at time of system

instability indicated that it was in fact the first stage of the fan which initiated compression system surge.

For comparison purposes, results from an experimental investigation and other mathematical model studies were examined for indications of system stability limit trends. An experimental investigation was conducted with a two-stage fan in which inlet total pressure was varied over frequencies ranging from 42 to 613 Hz [27]. A pulse generator was used which could vary the inlet pressure very nearly sinusoidally. Initially, fan operating conditions were set to a steady-state condition, at which time the generator varied the inlet pressure at some amplitude. If surge was not encountered, a new fan operating condition was set at a higher pressure ratio along the corrected speed-line. The stability limit in terms of magnitude of time-variant pressure distortion ($P_{Tmax} - P_{Tmin}/P_{Tavg}$) for two rotor-to-stator spacing fan rigs at 100-percent speed is presented in Fig. 29a. Compressor model studies with the J93 compressor simulation [28] and NACA-8 simulation [19] have also been conducted which establish stability limits.

The results of these studies are presented in Figs. 29b and 29c. The procedures with these studies were a little different in that an operating point was first established and then a linearly increasing sinusoidal inlet pressure variation was applied to the inlet until system instability was encountered. The stability limits (in terms of the magnitude of inlet pressure fluctuations) established with the two-stage experimental fan rig, the J93 compressor analog

simulation, and the NACA-8 compressor model were generally of the same trend.

A fundamental discussion of oscillatory inflow effects on a single stage taken from Reference [28] can help explain the nature of the compressor system stability limit. Initially, as the frequency increases, the amplitude level to cause instability decreases. Physically, the stage has a volume which requires a certain amount of time to respond to the pressure change. If the change is slow enough (low frequencies), the stage can follow the changes with very little lag. However, if the change is rapid (high frequencies), a lag in flow will occur across the stage volume. As the flow lag increases, flow coefficient oscillations become larger for any given inlet oscillation amplitude. This is caused by a phase lag and an amplitude attenuation across the stage volume.

Illustrated in Fig. 30 is a graphical representation of the amplitude attenuation and phase lag phenomenon across a single stage at high frequencies. With the case of two in-phase waves but with a stage output smaller in amplitude compared to the input, the ratio becomes oscillatory. Conversely, with two equal amplitude waves but of differing phase angles, the ratio is again oscillatory. As the frequency is greatly increased, amplitude attenuation approaches zero and phase angle approaches 90 degrees. This implies that at high frequencies, the volume is physically averaging out the oscillation, resulting in a steady flow output. In the case of a multistage compressor, the stages behind the stage receiving the high-frequency pulsation would see steady flow. Thus, the stage receiving the

pulsation is acting as a filter and shielding the following stages from oscillations.

Model predictions of the 13-stage compression system stability limits during planar oscillating inflow are presented in Fig. 31. The magnitude of the pressure fluctuations required to cause system instability followed the same trend established in the previous figures. The amplitude required for instability decreased up to a frequency of 100 Hz, slightly increased to a frequency of 200 Hz, and began to decrease again to a frequency of 300 Hz. At the lower frequencies (20 and 50 Hz), the first stage of the fan component initiated system instability. At a frequency of 100 Hz, the critical stage moved to the third stage of the high-pressure compressor. As the frequency increases above 150 Hz, the amplitude attenuation across the forward stages increases and protects the aft stages, allowing a stage farther forward to become critical. Pressure fluctuations with frequencies above 300 Hz were not applied to the model because of the frequency disturbance limitation discussed in Section III.4.

V.2.4 Oscillatory Fan Duct Pressure Fluctuation Effects on Compression System Stability

Augmentor rumble is a combustion instability that sets up pressure pulsations that can travel up a fan duct and affect compression system stability. To simulate this type of external disturbance, the model was subjected to sinusoidal pressure variation imposed on the fan duct exit boundary. The model was initially set at sea-level-static conditions, intermediate power, and allowed to run at a steady-state condition to ensure numerical stability. Once

stability had been assured, the fan duct exit static pressure was varied sinusoidally with a linearly increasing amplitude until system instability occurred.

Time traces of compression system performance parameters during a 50-Hz transient are presented in Fig. 32. Both the fan and the high-pressure compressor are influenced by the oscillatory fan duct flow. For small oscillation amplitudes, the compression system performance parameters (speed, pressure ratio, and airflow) oscillate sinusoidally. However, as the amplitude increases, these waveforms become distorted as stages or groups of stages move in and out of their stalled region. Flow breakdown appears to have occurred in the fan component as indicated by the rapidly diverging airflow (Fig. 32c). The investigation was conducted over a range of frequencies from 30 to 100 Hz. Oscillations of the compression system operating points are illustrated in Fig. 33. Both the fan and high-pressure compressor may traverse their steady-state surge lines prior to system instability. The component causing system instability cannot be determined from these figures, but these figures can serve as general indications of system behavior.

Indicated in Fig. 34 are the individual stage flow coefficients' behavior during the 50-Hz oscillatory transient. To indicate initial compression system loading, the initial steady-state stall value of flow coefficient is also given. None of the stages were in aerodynamic stall at time of transient initiation. What is indicated is the attenuation of the oscillation back through the fan and hardly any attenuation forward through the high-pressure

compressor. Therefore, the first stage of the fan is protected by the other fan stages and is not expected to be the critical stage initiating system instability. Further investigation into the static pressure behavior prior to system instability indicates that the second stage of the fan was the critical stage at all frequencies investigated.

Model predictions of the 13-stage compression system stability limits (magnitude of pressure fluctuation required to cause instability) during fan duct oscillatory static pressure sinusoidal variations occurring at sea-level-static, intermediate power, are presented in Fig. 35. Experimental data which describe compression system stability limits while undergoing similar transients were not available. In most testing instances, an augmentor rumble limit of some peak-to-peak variation is sufficient for the engine to be taken out of augmentation. This peak-to-peak limit is usually well under the limit for complete system flow breakdown. The model does indicate that there is a decrease in the pressure fluctuation required to cause instability as the frequency increases.

V.2.5 Rapid Inlet Temperature Ramps' Effects on Compression System Stability

Engine stability problems have occurred when exhaust products from rocket or gun firings are ingested. From previous investigations of this problem, it has been determined that the rapid inlet temperature increase is probably the predominant factor causing compression system instability in these situations. To simulate this temperature disturbance, the model was subjected to rapid inlet temperature ramps

ranging from 2,000 to 10,000°R/sec. The model was initially set at sea-level-static conditions at intermediate power and allowed to run at a steady-state condition to ensure numerical stability. Once stability had been assured, the system inlet temperature was increased linearly at a particular rate of increase until compression system instability was indicated.

System behavior during a typical temperature transient is indicated in Fig. 36. Fan-corrected airflow and speed decrease because of the increase in inlet temperature. Because of the rapid change in inlet temperature, an engine control would not be able to react and system transient behavior similar to the model's behavior would be evident. Pressure ratio does not have time to react to the change but an imbalance in forces among the stages is present. The overshoot of the stability limit indicates the front-to-rear airflow mismatch causing system instability. As the front of the fan moves toward the stability limit at a near constant pressure ratio, the rear stages have not seen as much a temperature change and thus would operate at a corrected flow corresponding to a stable condition. In fact, closer analysis using the pressure signature criteria indicates that the first or second stage of the fan is the stage initiating compression system instability. This is consistent with experimental results, which indicates the front of the compression system can no longer sustain its flow during the transient [8].

Model prediction of the 13-stage compression system stability limit (temperature rise for instability) and comparison to experimental results are presented in Fig. 37. The stability limit established at

three temperature ramp rates falls within or above the range of the experimental data, which consist of full-face and partial-face temperature ramps on three different compression systems (15-stage compressor, two-stage fan and seven-stage, high-pressure compressor, and TF30-P-6 compressor system) [8].

VI. SUMMARY OF RESULTS

An existing stage-by-stage compressor modeling technique has been modified to extend its capabilities to a dual-spool compression system. This dual-spool capability provides an analytical tool for analyzing dual-spool behavior under a variety of external disturbances. The following significant results have been drawn from this effort.

1. Some previous compressor models based on the one-dimensional, time-dependent, single-spool compressor modeling technique have exhibited various levels of numerical instability. Prior to development of a dual-spool model, this shortcoming had to be overcome.
 - a. The model was reformulated using MacCormack's second-order finite differencing scheme for interior points and a method of characteristics treatment for boundaries.
 - b. The reformulation was generally found to be a solution to the numerical instability problem. However, for a portion of the operating envelope the model exhibited a bounded instability during calculation of steady-state conditions. Incorporation of a single reduced initial volume element along with increased damping in that element produced a viable solution.
2. Exercise of the reformulated single-spool model in a time-variant mode and comparison with experimental data for

both compressor surge and critical stage identification revealed excellent results.

- a. The model predicted compressor surge exactly as observed experimentally at 95- and 98-percent corrected speeds and was within 1.7 percent and 0.5 percent in pressure ratio and corrected airflow, respectively, at the least accurate computation, 77-percent corrected speed.
 - b. The model predicts which stage initiates compressor surge by exhibiting the same static pressure signature across the critical stage as observed experimentally.
3. The reformulated compressor modeling technique was extended to include dual-spool compression systems and was applied to a current compression system.
4. To validate the model of the current compression system, the model was subjected to several different types of planar disturbances that might be encountered by turbine engine compression systems and compared against experimental results.
 - a. Fan Loading: At the one speed setting investigated, the model generally indicated the same fan and high-pressure compressor behavior as obtained experimentally when the fan was loaded to its surge point. An error in the prediction of airflow level can be accounted for by the difference in CIVV setting. High-pressure compressor behavior differences during fan loading are

- the result of an active control on the experimental engine and no control simulation included in the model.
- b. High-Pressure Compressor Loading: At the one speed setting investigated, the model generally indicated the same high-pressure compressor and fan behavior as experimental results when the high-pressure compressor was loaded to surge. An error in the prediction of high-pressure compressor airflow setting can also be attributed to a difference in RCVV settings. Fan behavior differences are the results of an active control on the experimental engine and no control simulation included in the model.
 - c. Oscillatory Inflow Loading to Surge: For one operating point, the model predicted instability limits (amount of pressure fluctuation amplitude required to cause instability) for frequencies ranging to 300 Hz. When compared to experimental results from a two-stage fan and other model results of a single-spool compressor system, the stability limit boundary generated by the dual-spool model was of the same general shape. The model indicates that the stage which initiates system instability moves from the first-stage fan at low frequencies to different stages in the forward part of the high-pressure compressor, or rear part of the fan at the higher frequencies.

- d. Oscillatory Fan Duct Exit Flow: At one speed setting, the model was subjected to sinusoidal oscillatory fan duct exit pressure with a linearly increasing amplitude until compression system instability occurred. The investigation was conducted over a range of frequencies from 30 to 100 Hz. A stability limit (magnitude of pressure fluctuation required to cause instability) was established for that one initial operating point. Experimental data describing system stability limits while under similar transients were not available.
- e. Rapid Inlet Temperature Ramps: Model prediction of the stability limit (temperature rise for instability) while experiencing a rapid rise in inlet temperature generally fell within the range of experimental results, as experienced by three different compression systems. The model predicted that the first or second stage of the fan was the critical stage initiating system instability. This is consistent with experimental results which indicate that the front of the compression system can no longer sustain its flow during the transient.

VII. RECOMMENDATIONS

The dual-spool model developed in this work provides an analytical tool for analyzing dual-spool behavior under a variety of external disturbances. However, improvements should be made and multi-dimensional extensions should be done to increase the value of this modeling technique.

1. Stage Characteristics: Stage characteristics are the heart of this modeling technique. Future experimental efforts should be directed to obtain high fidelity stage characteristics in the stalled as well as unstalled region. Theoretical methods for obtaining characteristics should be developed to augment the experimental information.
2. Post-Surge Capability: Techniques should be developed which allow the model to operate beyond the inception of system surge. With this capability, the model could begin to predict system behavior while undergoing surge loops, rotating stall, and stagnation stall.
3. Multidimensional Capability: The MacCormack reformulated single-spool modeling technique should be extended to two-dimensional and three-dimensional space. Once this has been accomplished, the single-spool, multidimensional model should be extended to include dual-spool compression systems.
4. Controls Simulation: A simulation of turbine engine control functions should be included in future compressor

modeling efforts. A control simulation will remove the fixed speed schedule and will provide for more compression system transients to be simulated.

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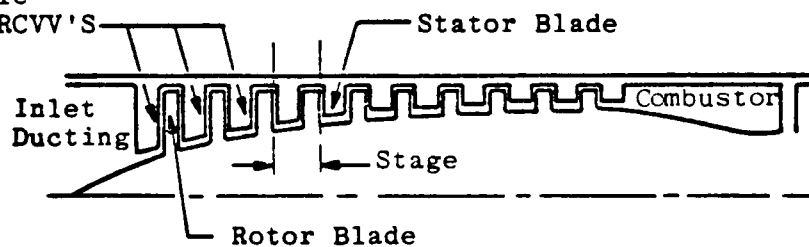
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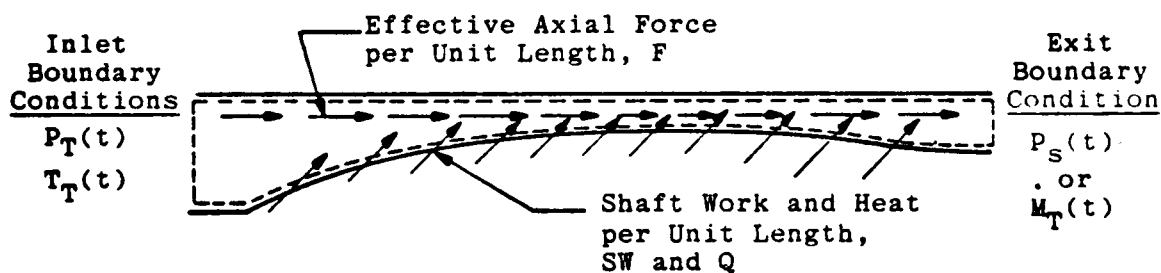
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APPENDIX

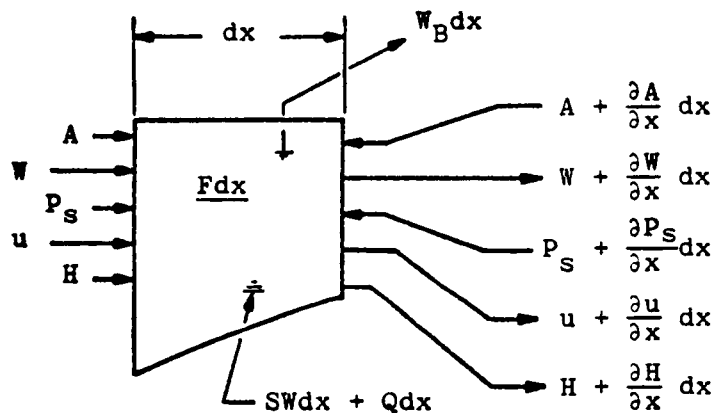
Rear Compressor
Variable
Vanes RCVV'S



a. Compressor and ducting system



b. Overall control volume



c. Elemental control volume

Figure 1. Physical compression system modeled and control volume concepts

$$\phi = \frac{U}{U_{wh}}$$

$$\psi^T = \frac{C_p (TR - 1)}{U_{wh}^2 / T}$$

$$\psi^P = \frac{C_p (PR - 1)^{\gamma-1/\gamma}}{U_{wh}^2 / T}$$

$$\eta_{ad} = \frac{\psi^P}{\psi^T}$$

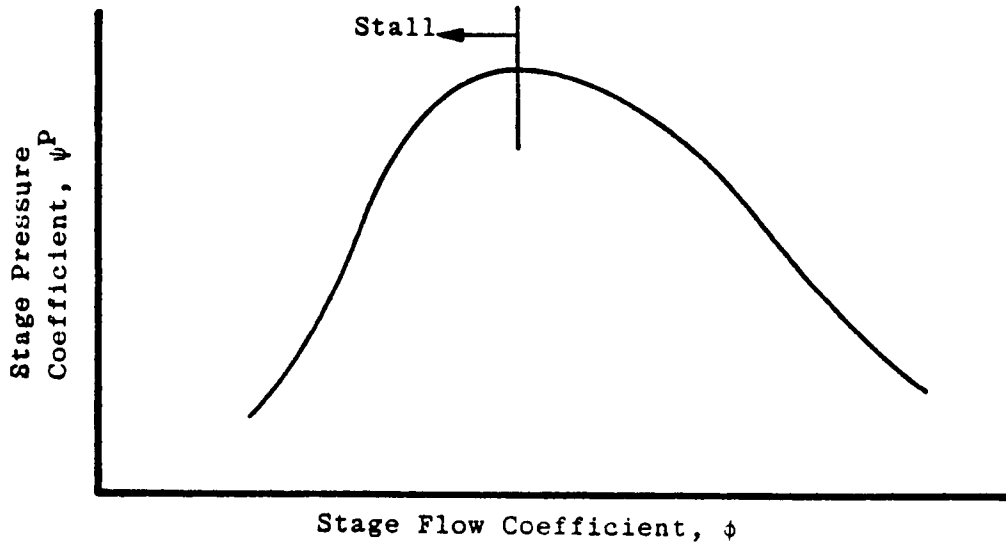
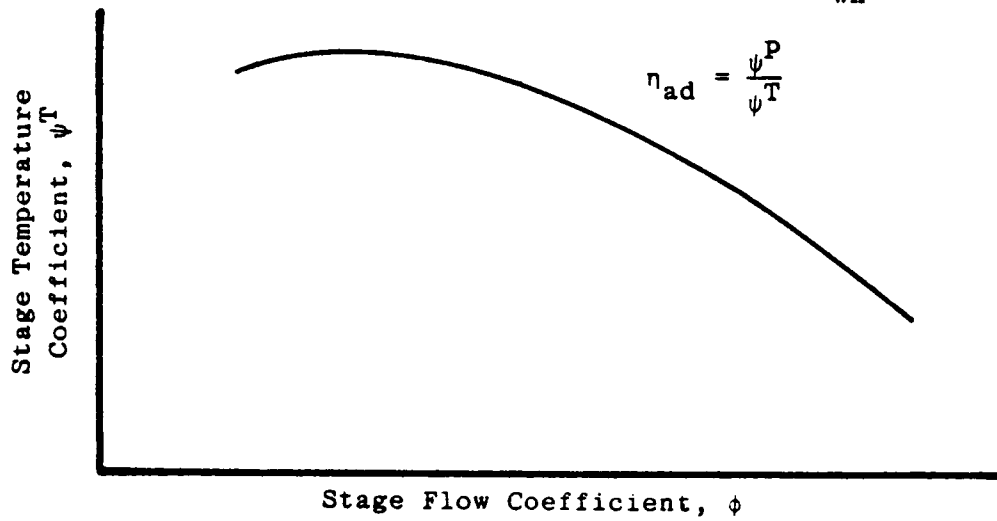


Figure 2. Typical temperature and pressure coefficients as a function of flow coefficient at constant rotational speed.

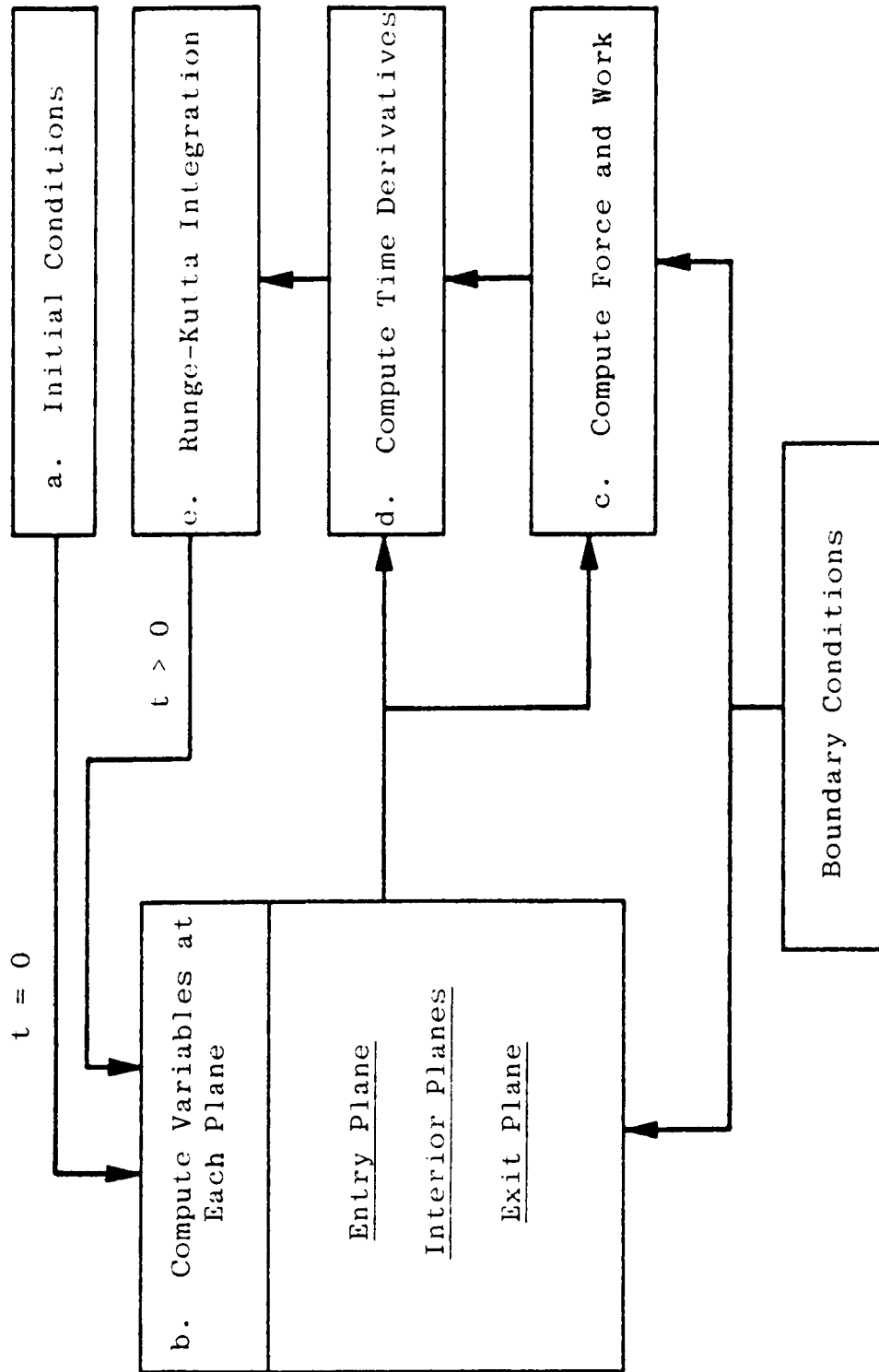


Figure 3. Time-dependent compressor model solution procedure, Runge-Kutta formulation.

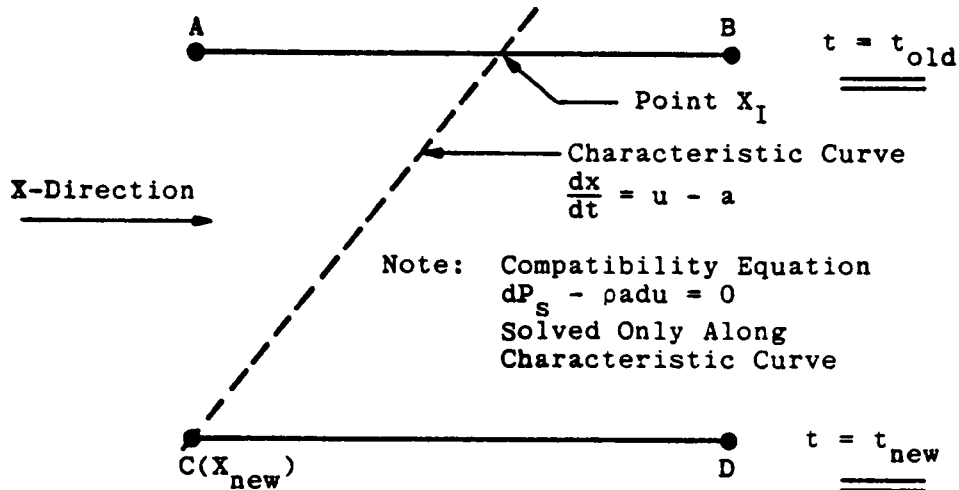


Figure 4. Schematic of inlet characteristic boundary scheme.

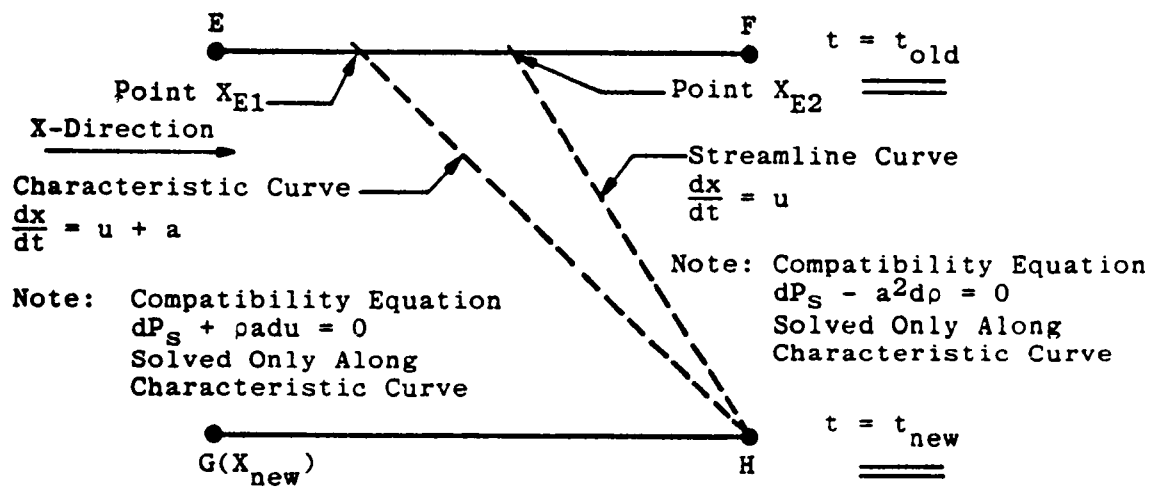


Figure 5. Schematic of exit characteristic boundary scheme.

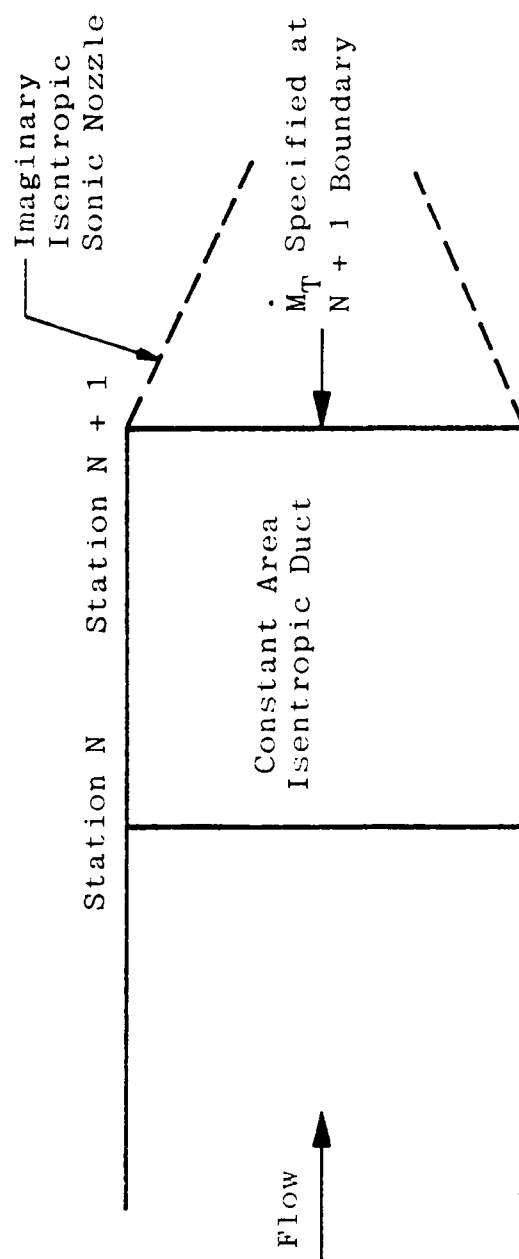


Figure 6. Sonic nozzle exit boundary condition.

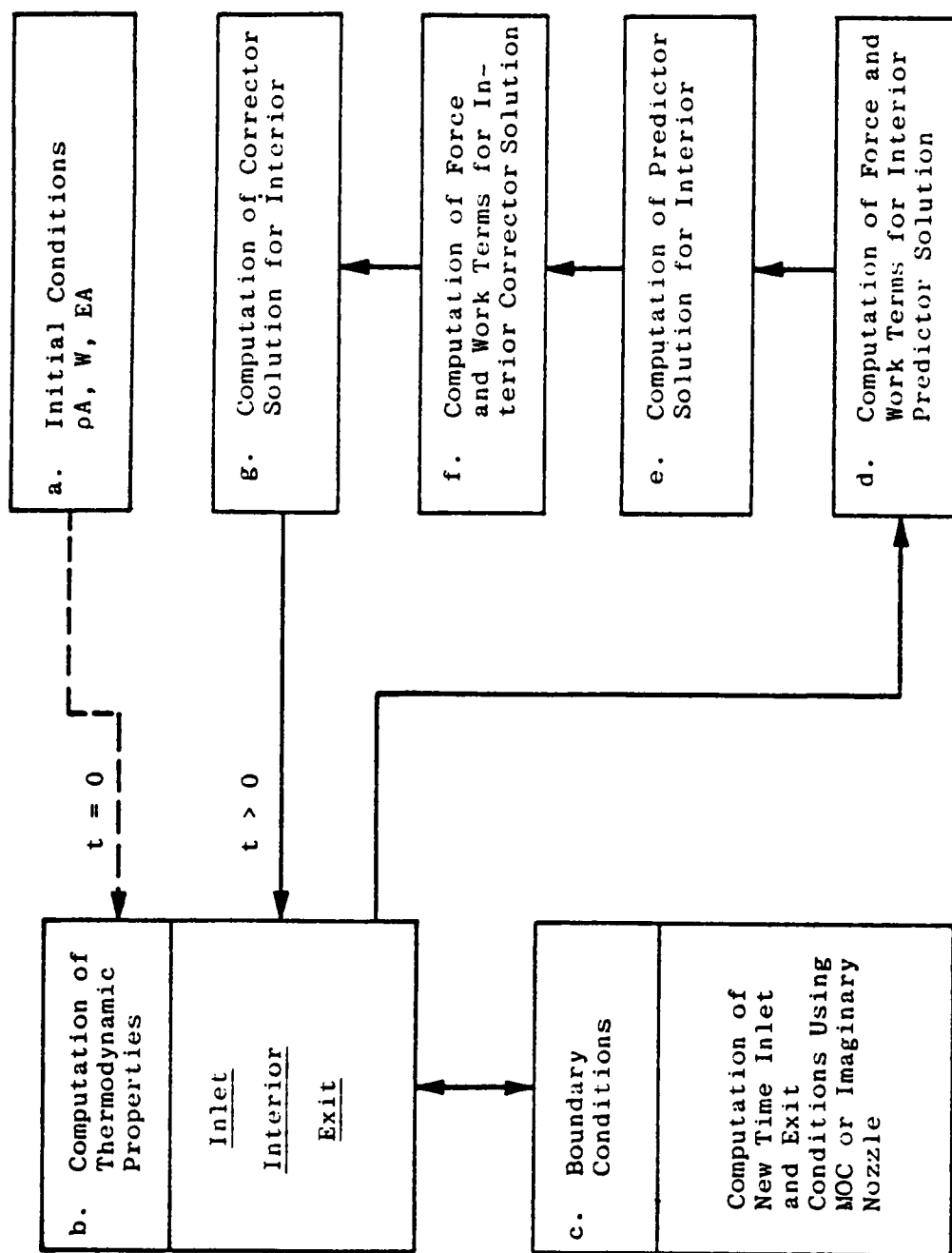


Figure 7. Time-dependent compressor model solution procedure, MacCormack formulation.

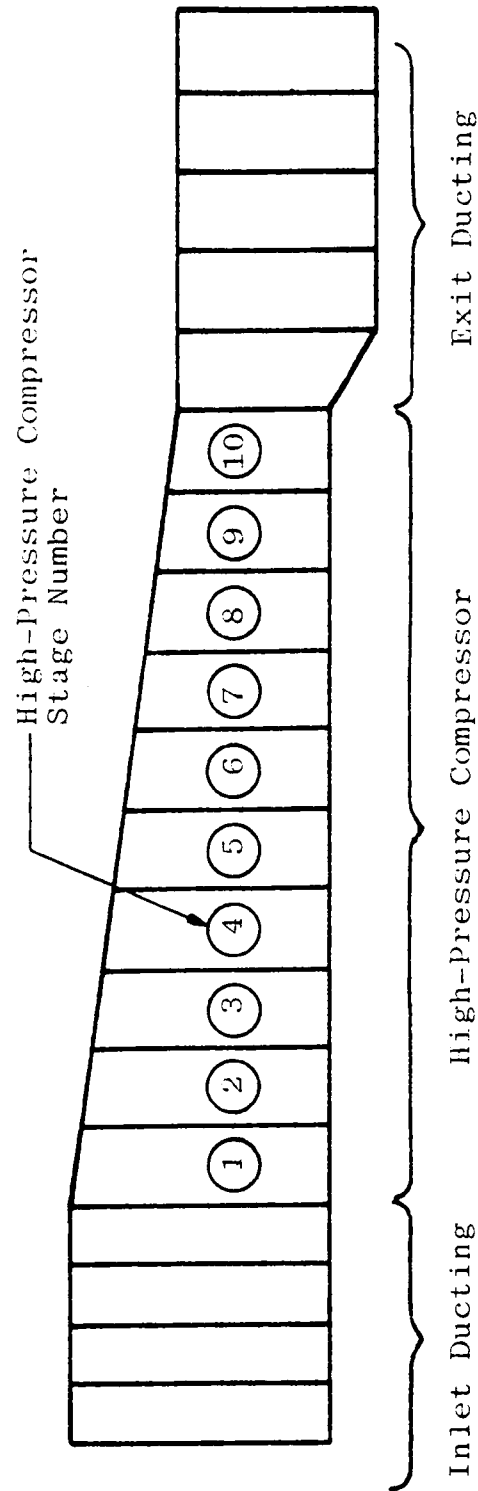


Figure 8. Schematic of the 10-stage, high-pressure compressor model.

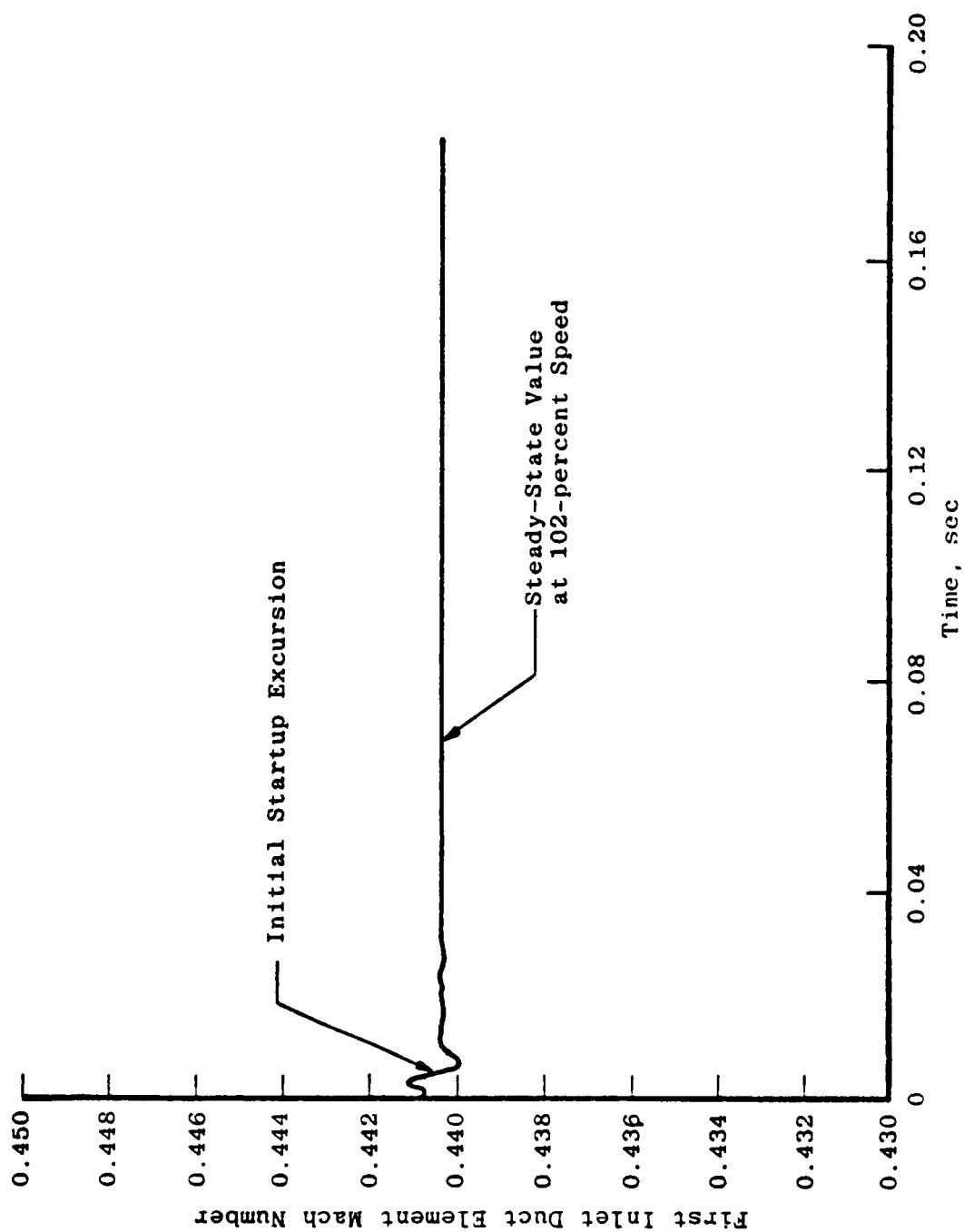
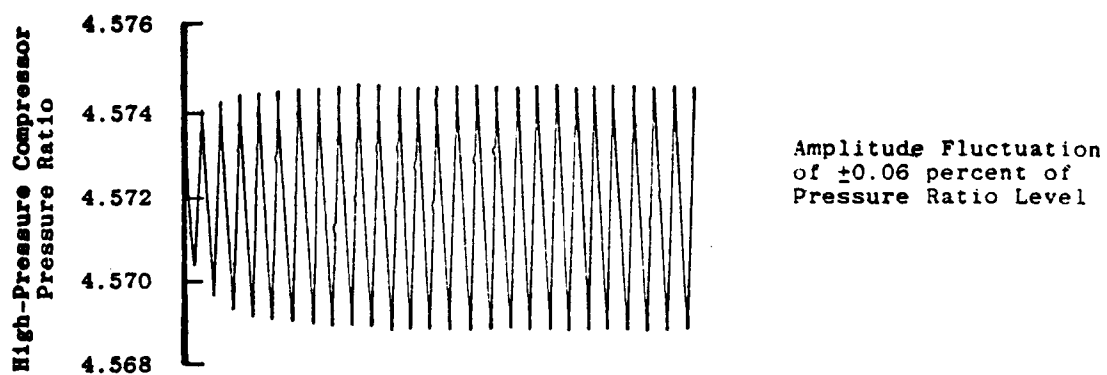
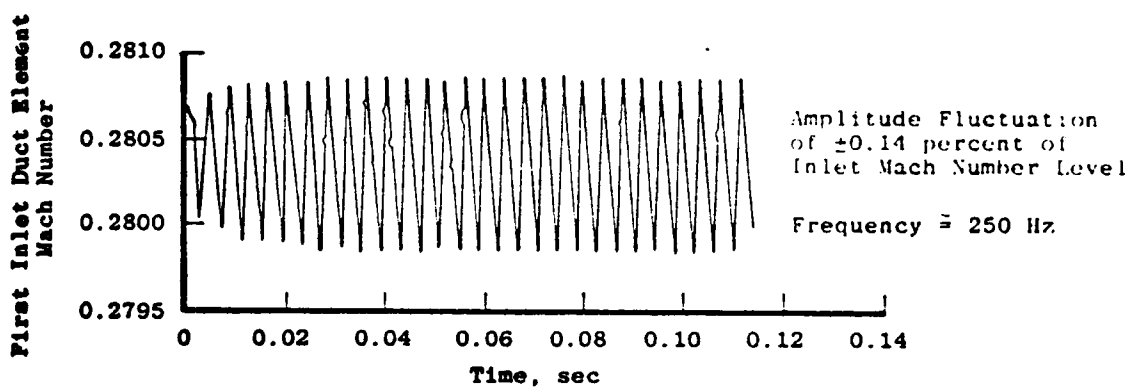


Figure 9. Typical inlet Mach number readjustment at model start-up and the obtaining of a steady solution.



a. Compressor pressure ratio



b. Compressor inlet duct Mach number

Figure 10. Occurrence of numerical instability in the MacCormack formulation, steady-state conditions.

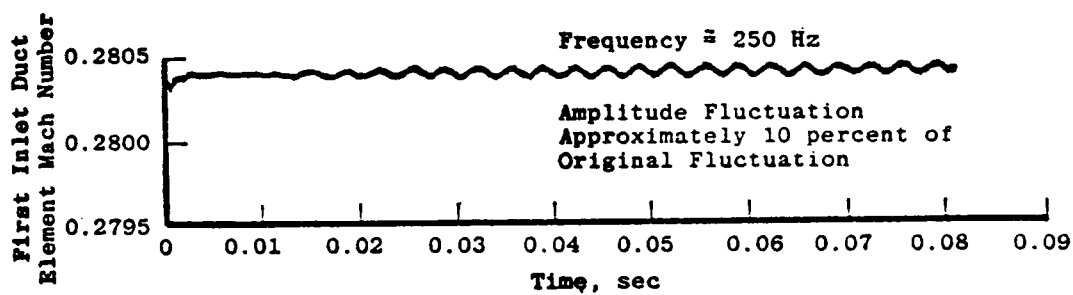


Figure 11. Effect of a 75-percent reduction in first inlet duct element length on numerical stability.

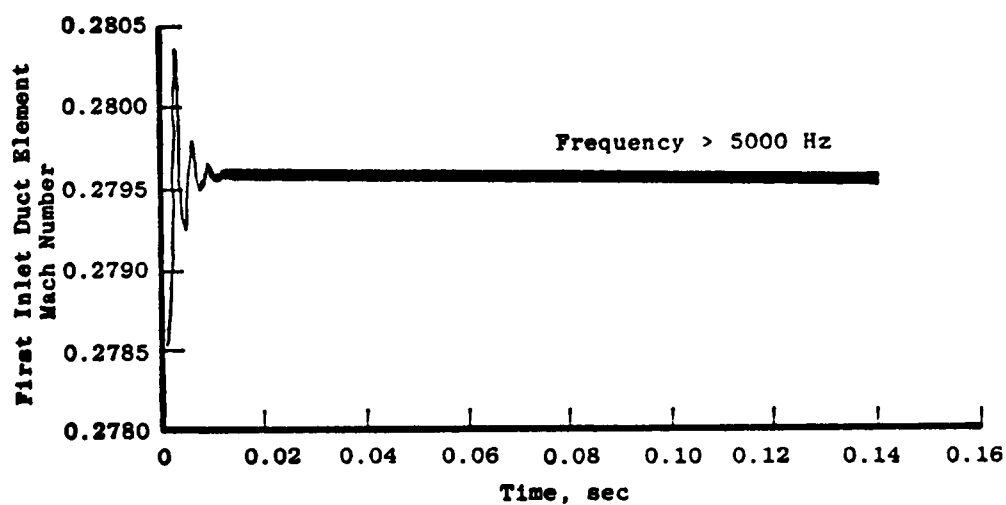
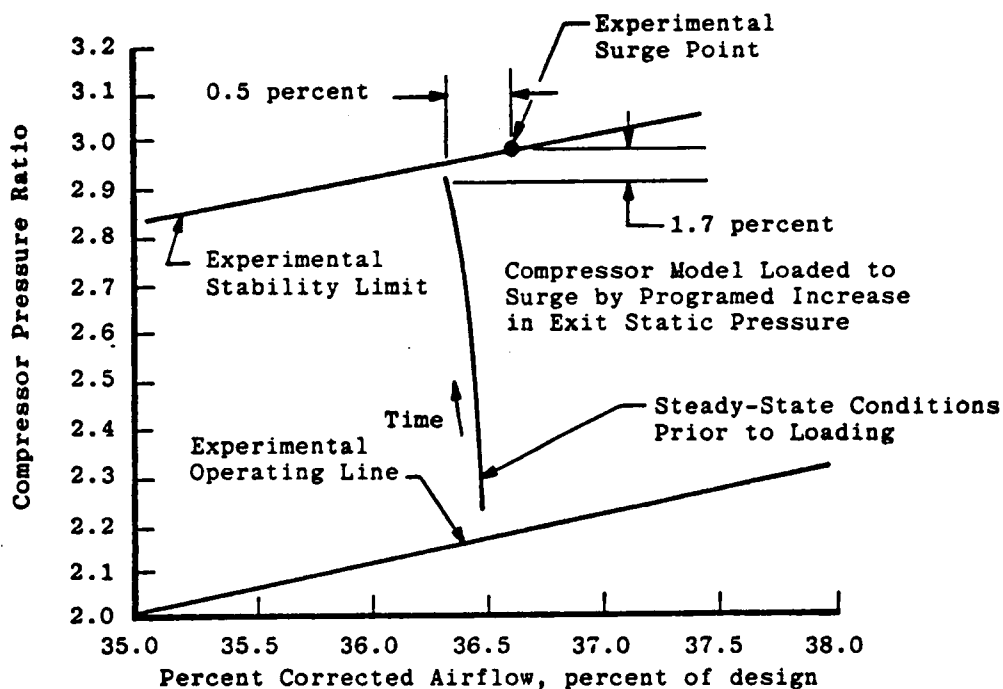
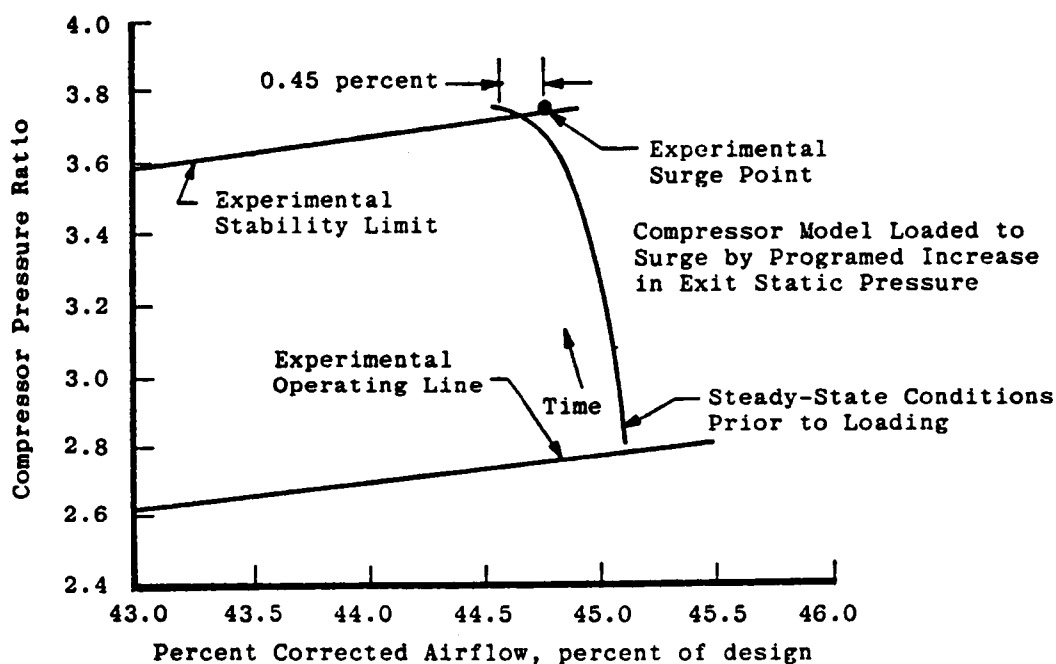


Figure 12. Effect of frictional damping in the first inlet duct element on numerical stability.

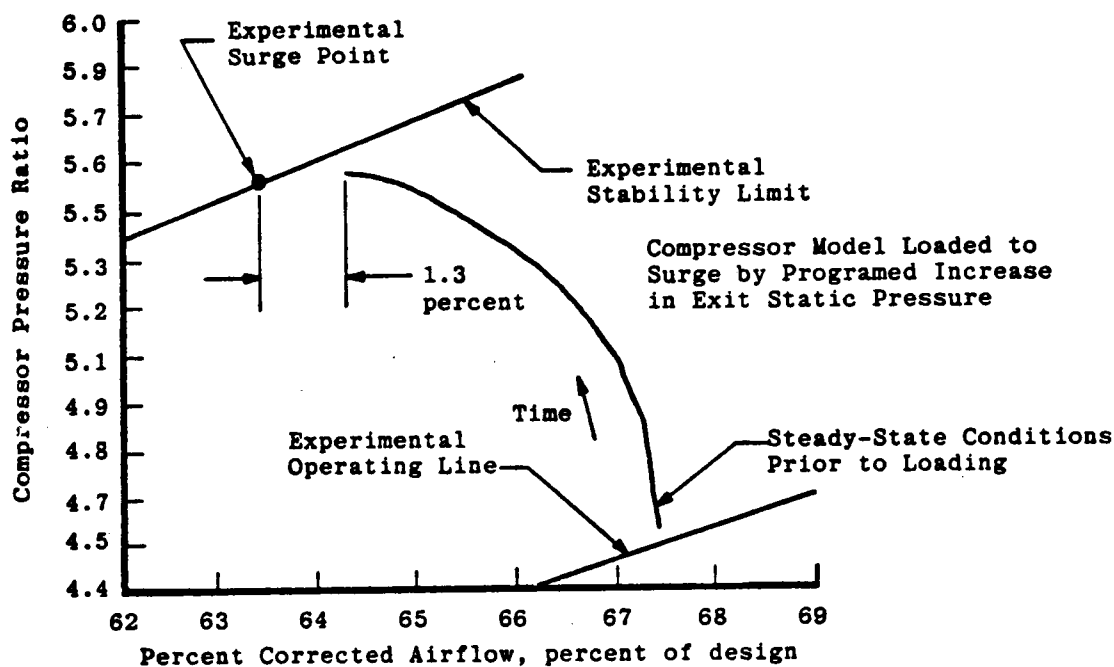


a. 77-percent corrected speed

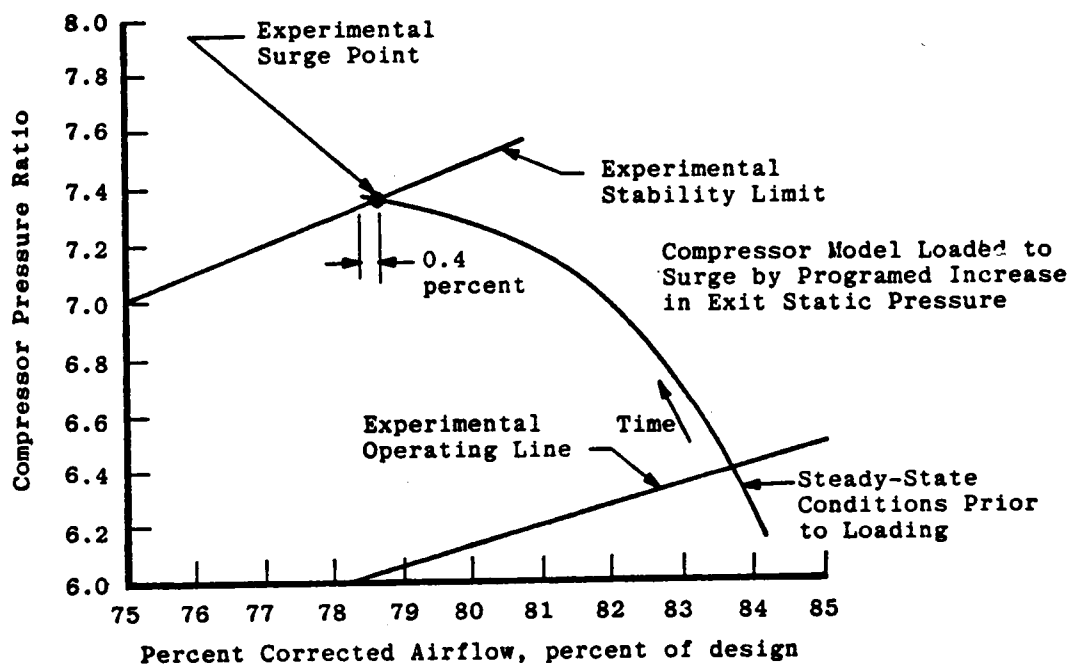


b. 82-percent corrected speed

Figure 13. Comparison of model predicted stability limits with experimental results (MacCormack formulation).

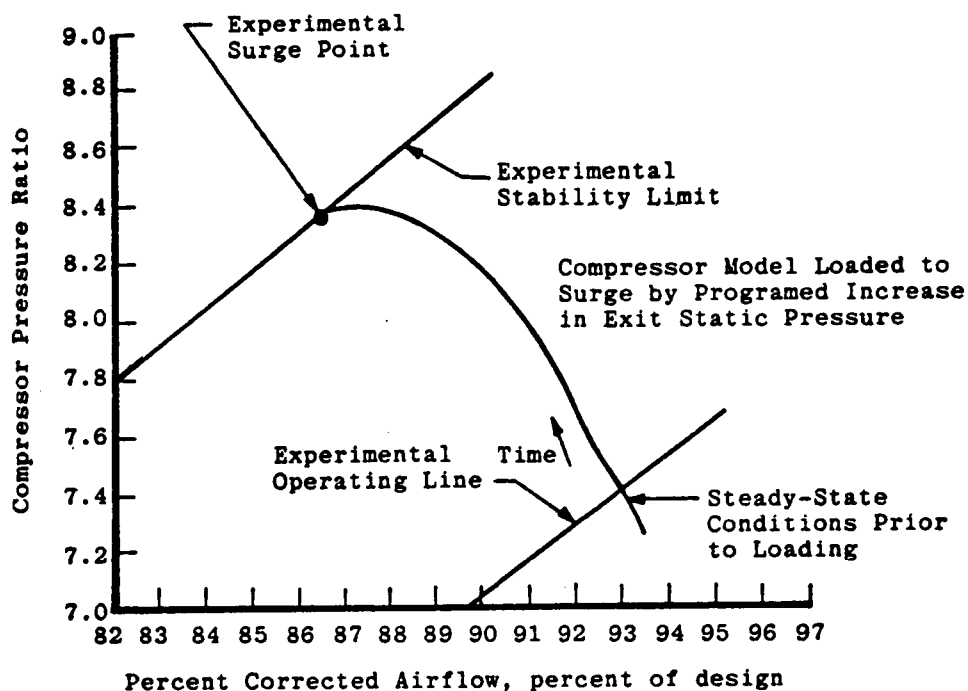


c. 87-percent corrected speed

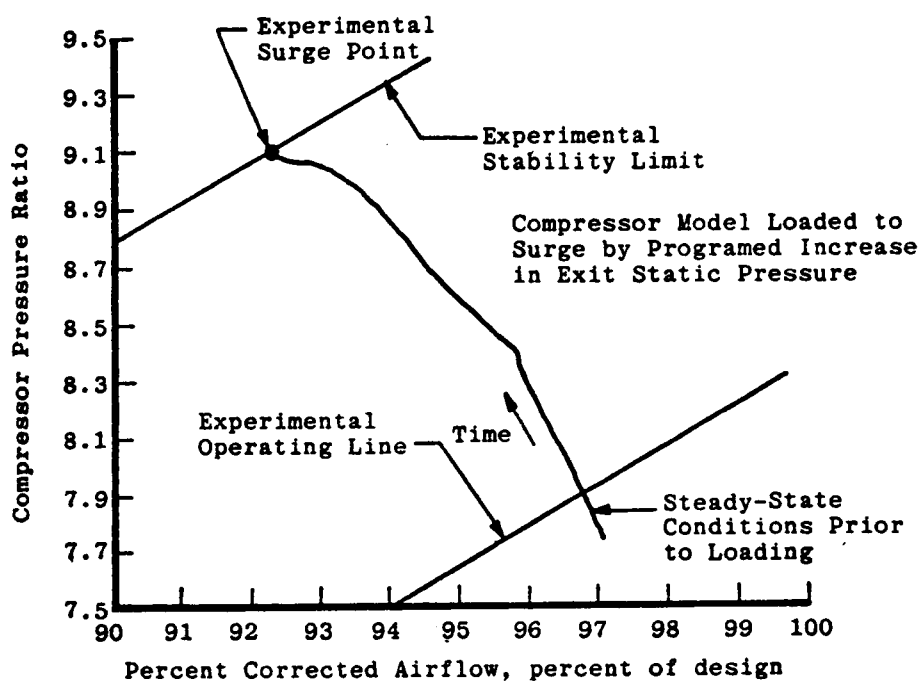


d. 92-percent corrected speed

Figure 13. (continued)

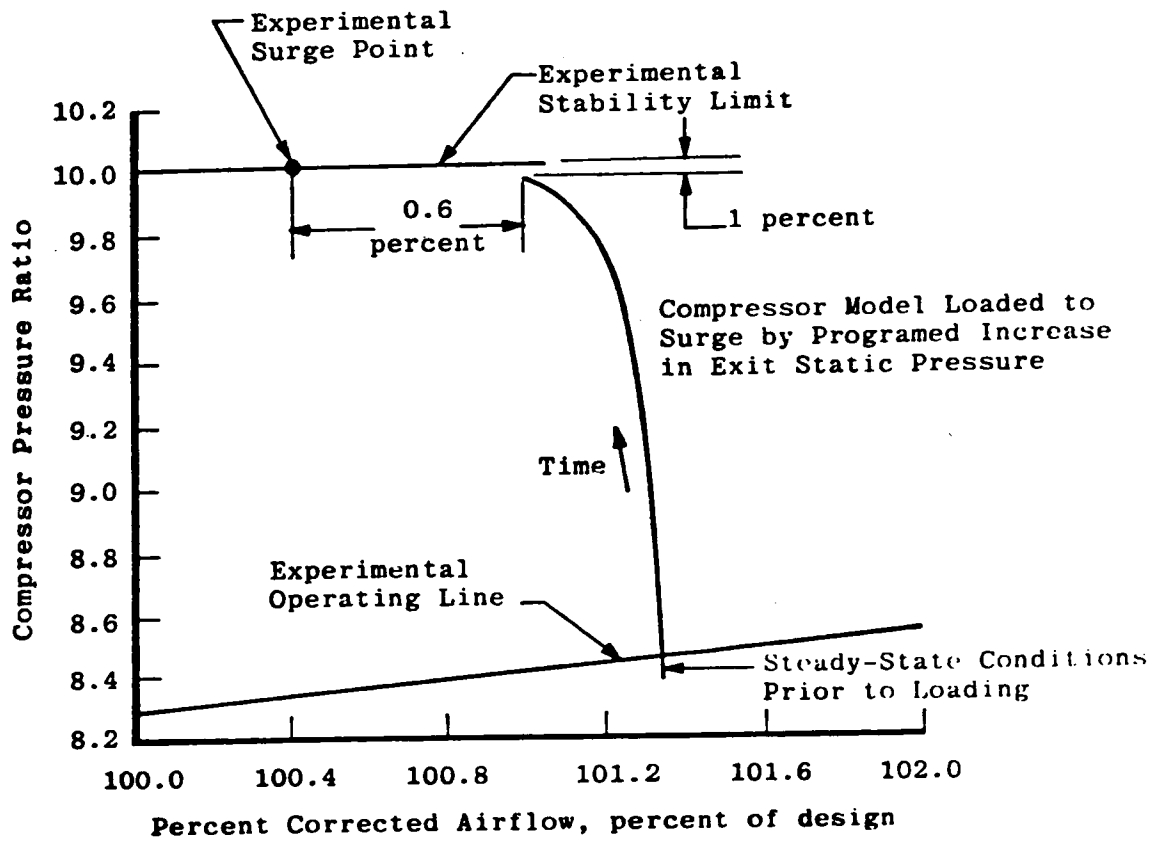


e. 95-percent corrected speed



f. 98-percent corrected speed

Figure 13. (continued)



g. 102-percent corrected speed

Figure 13. (continued)

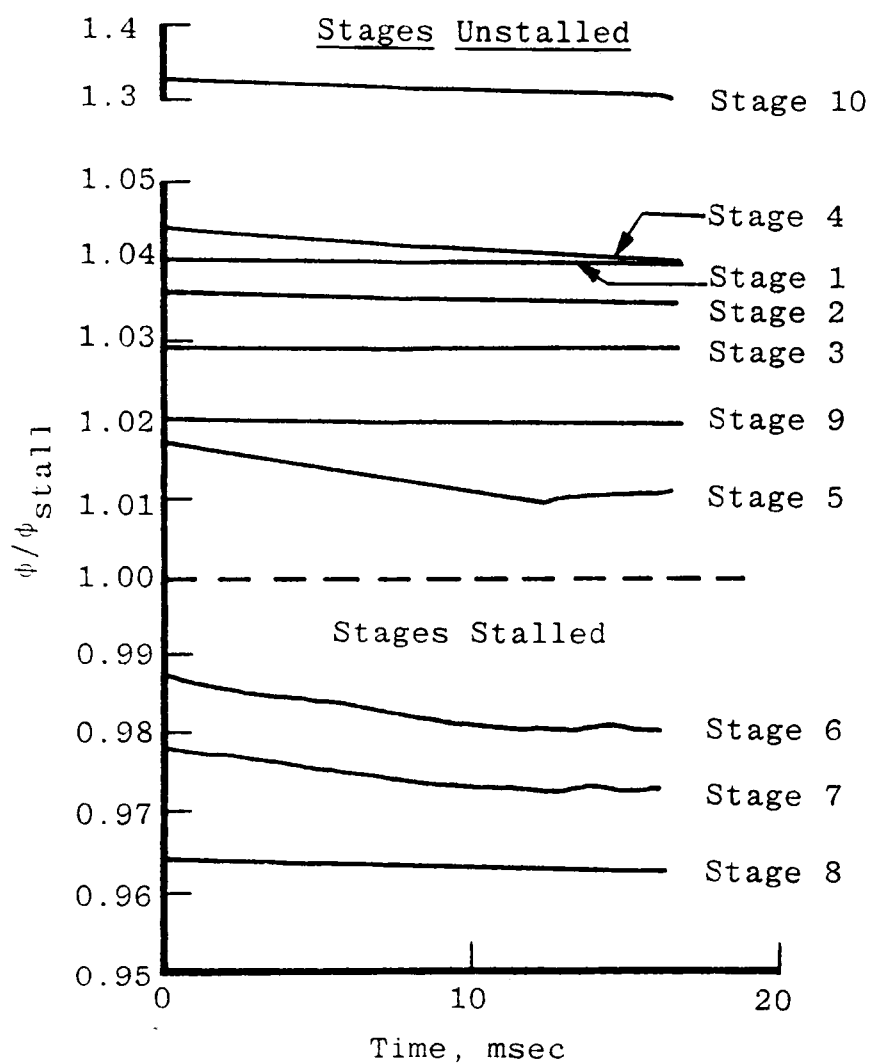


Figure 14. Model prediction of stages in aerodynamic stall at time of compressor surge, 102-percent corrected speed.

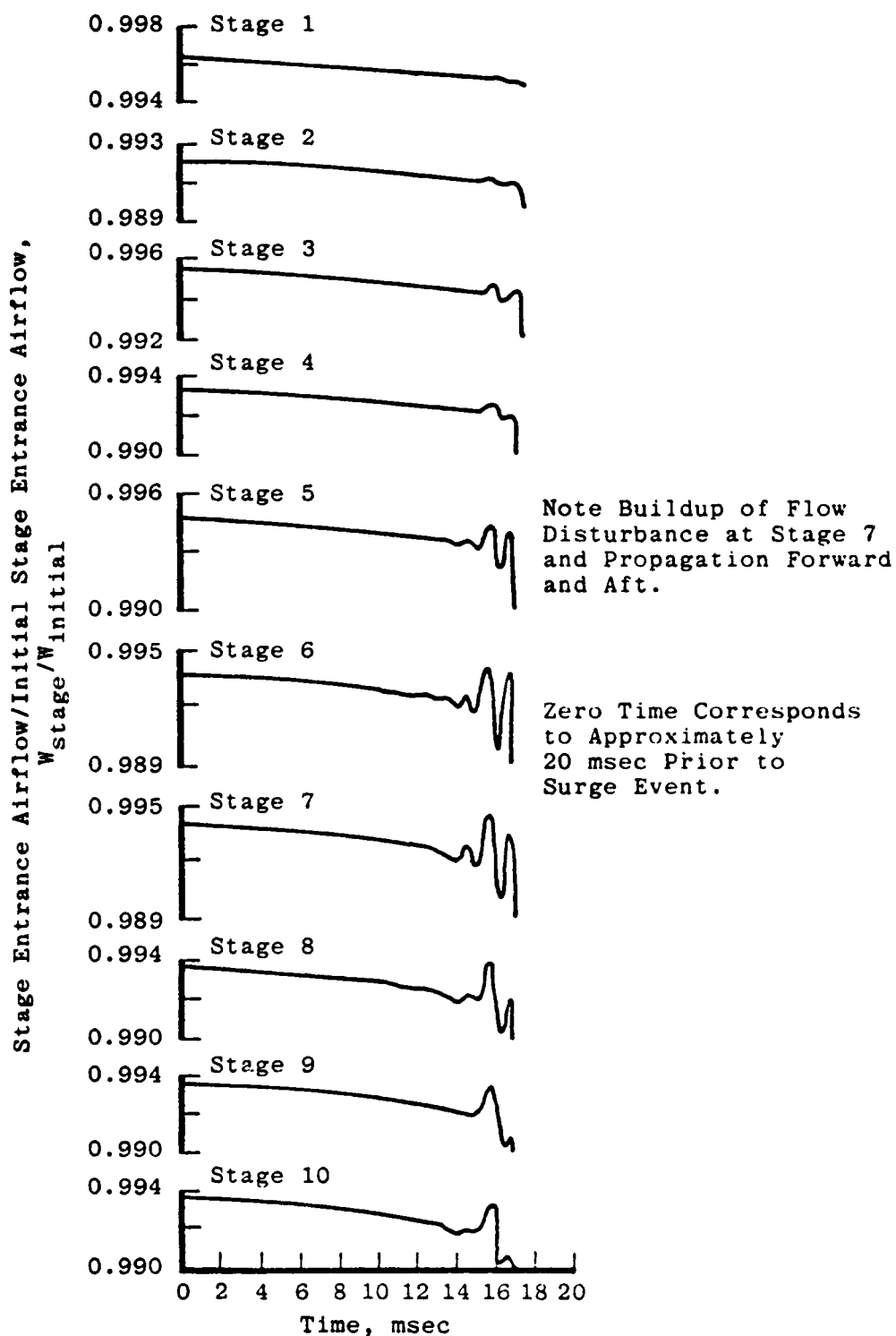


Figure 15. Model predicted airflow disturbance prior to compressor surge, 102-percent corrected speed.

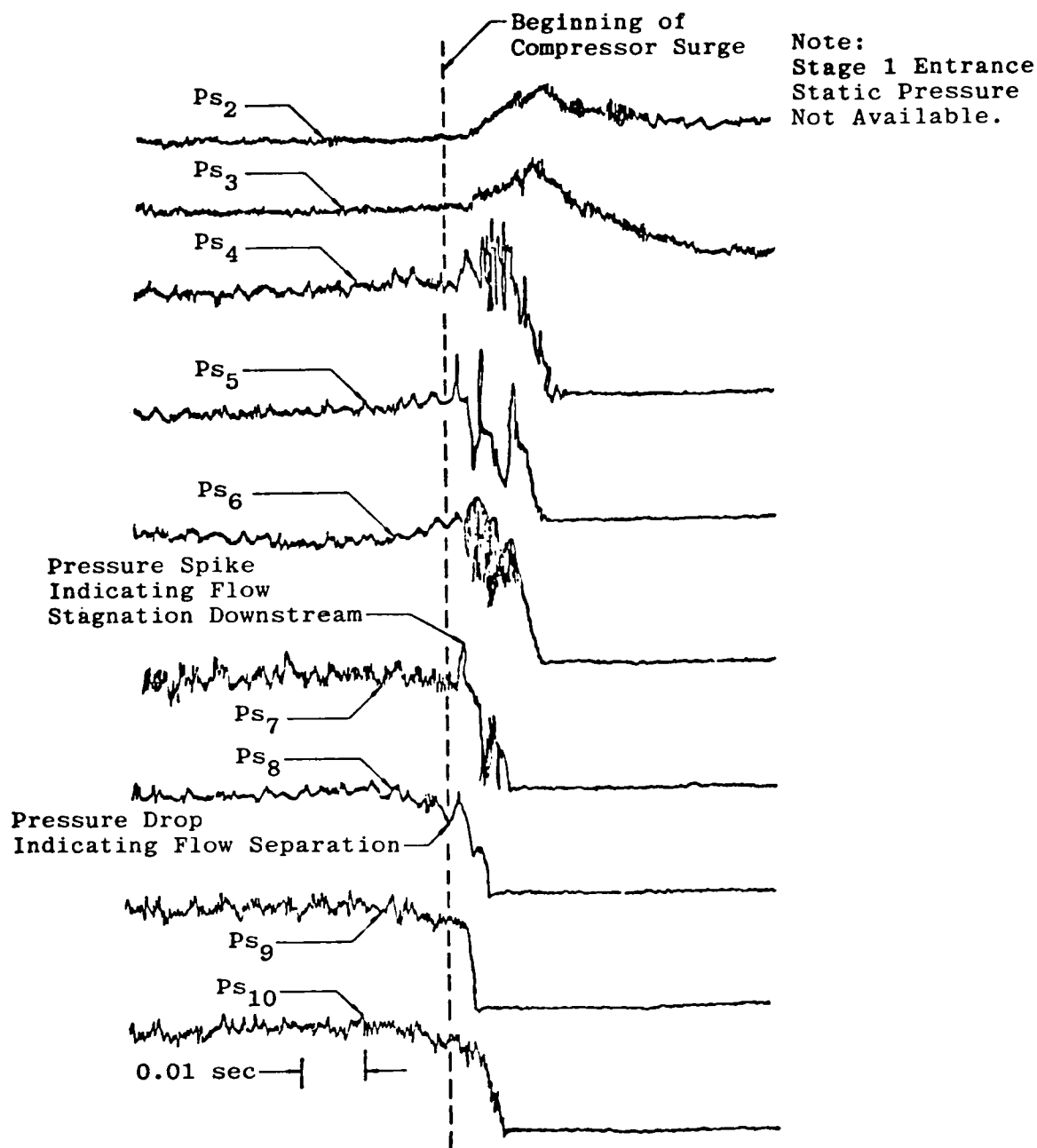


Figure 16. Typical time trace of the 10-stage, high-pressure compressor stage entrance static pressure at time of compressor surge.

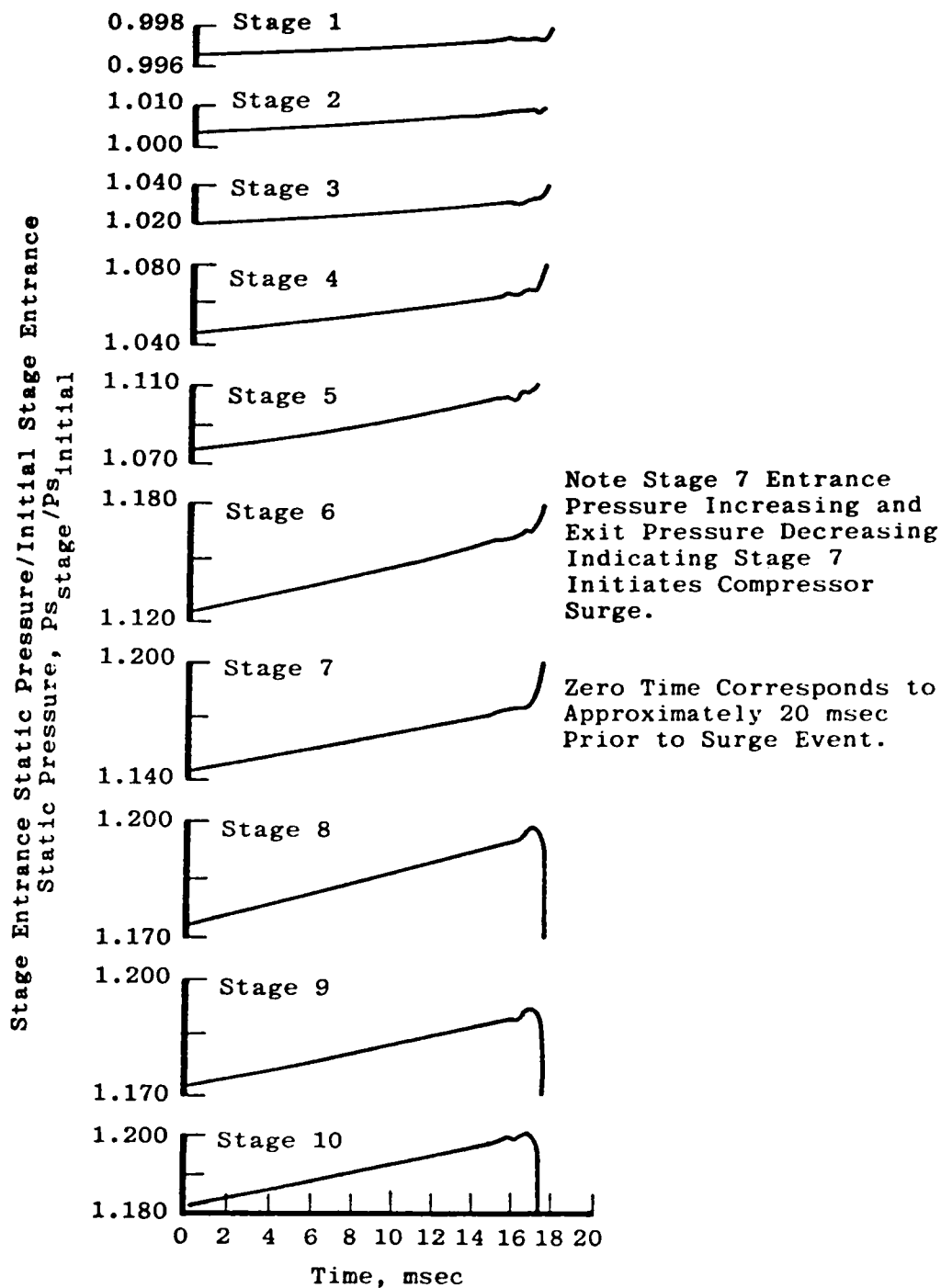


Figure 17. Model prediction of stage initiating compressor surge as identified by static pressure signature, 102-percent corrected speed.

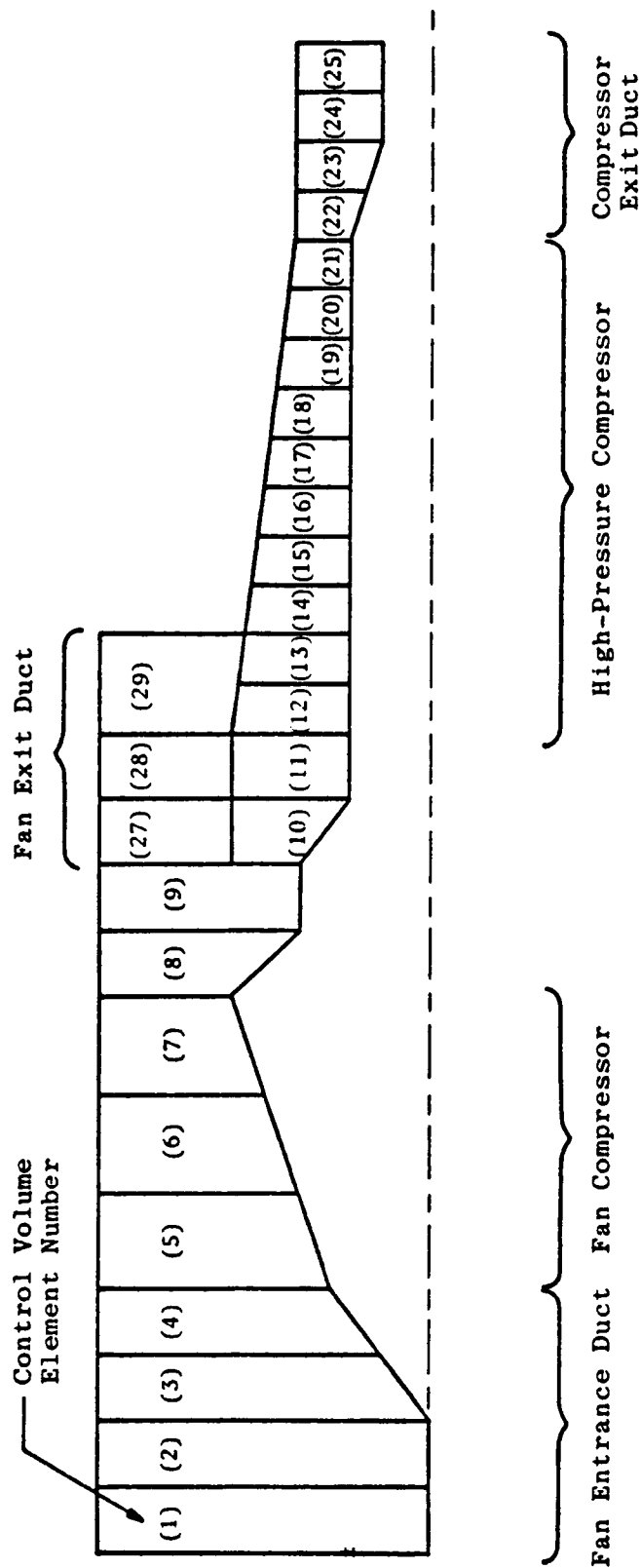


Figure 18. Schematic of the 13-stage, dual-spool compression system model.

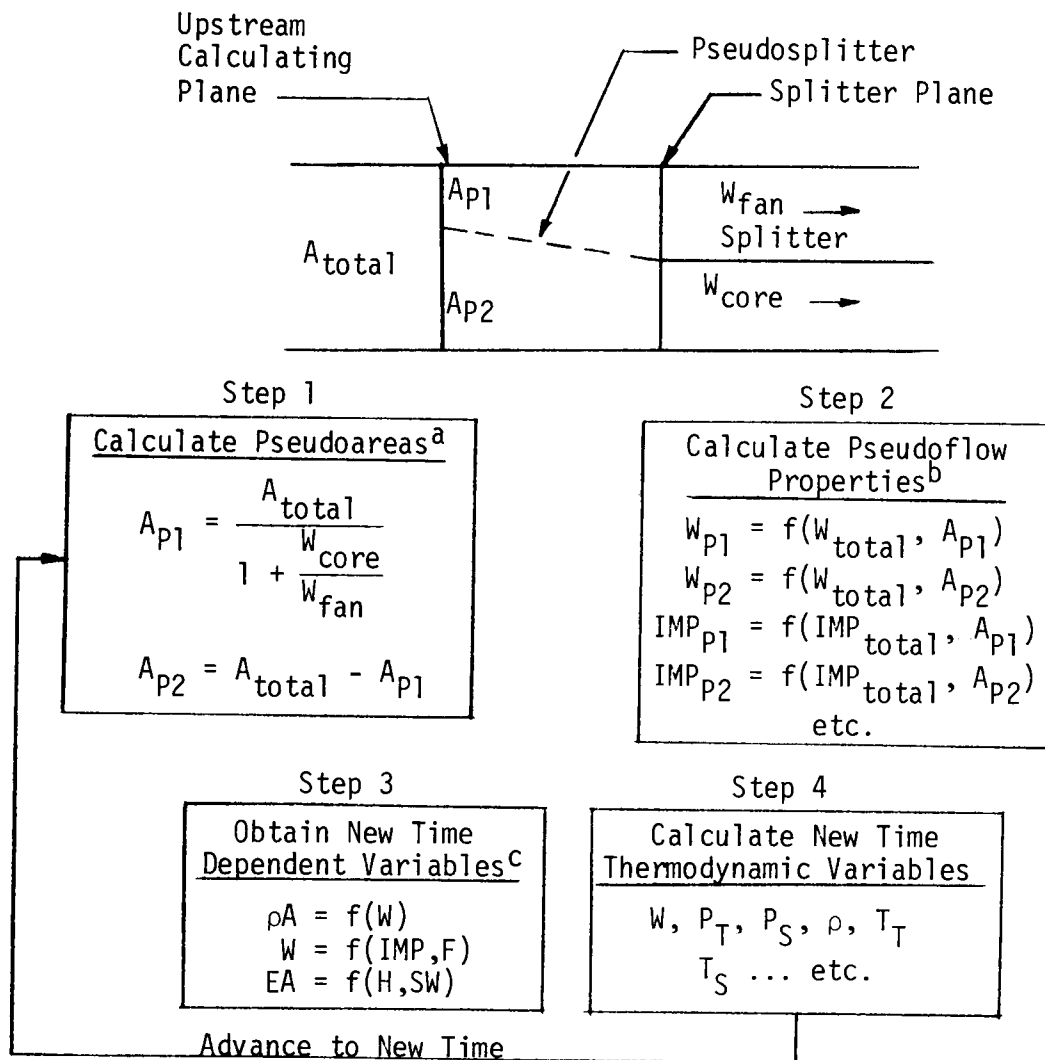


Figure 19. Bypass flow calculation procedure.

^a W_{fan} and W_{core} are known from previous time calculations.

^bAll thermodynamic properties at the upstream calculating station are known from previous time calculation.

^cNew time dependent variables at upstream calculating station are an average of the pseudoflow properties.

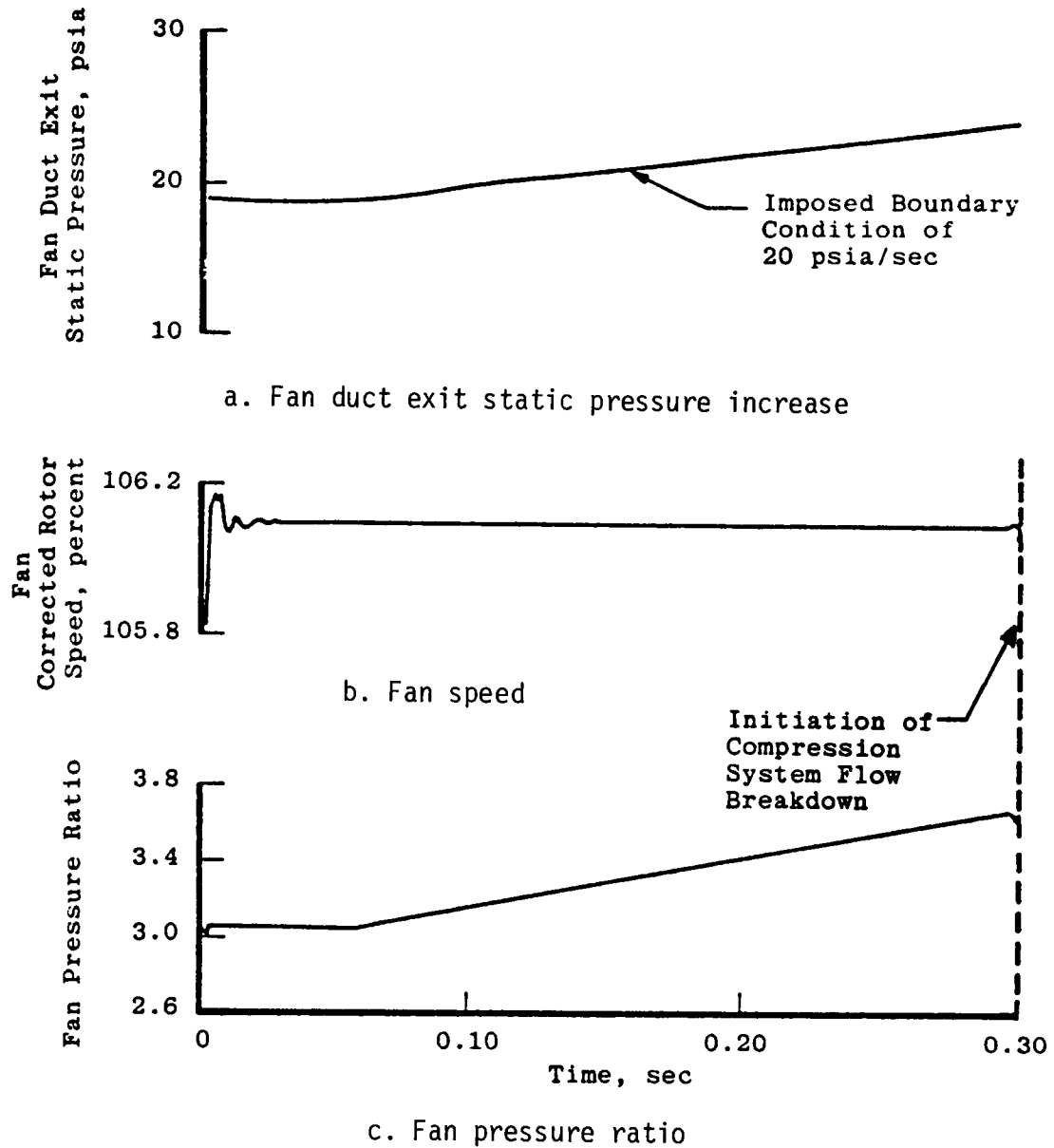
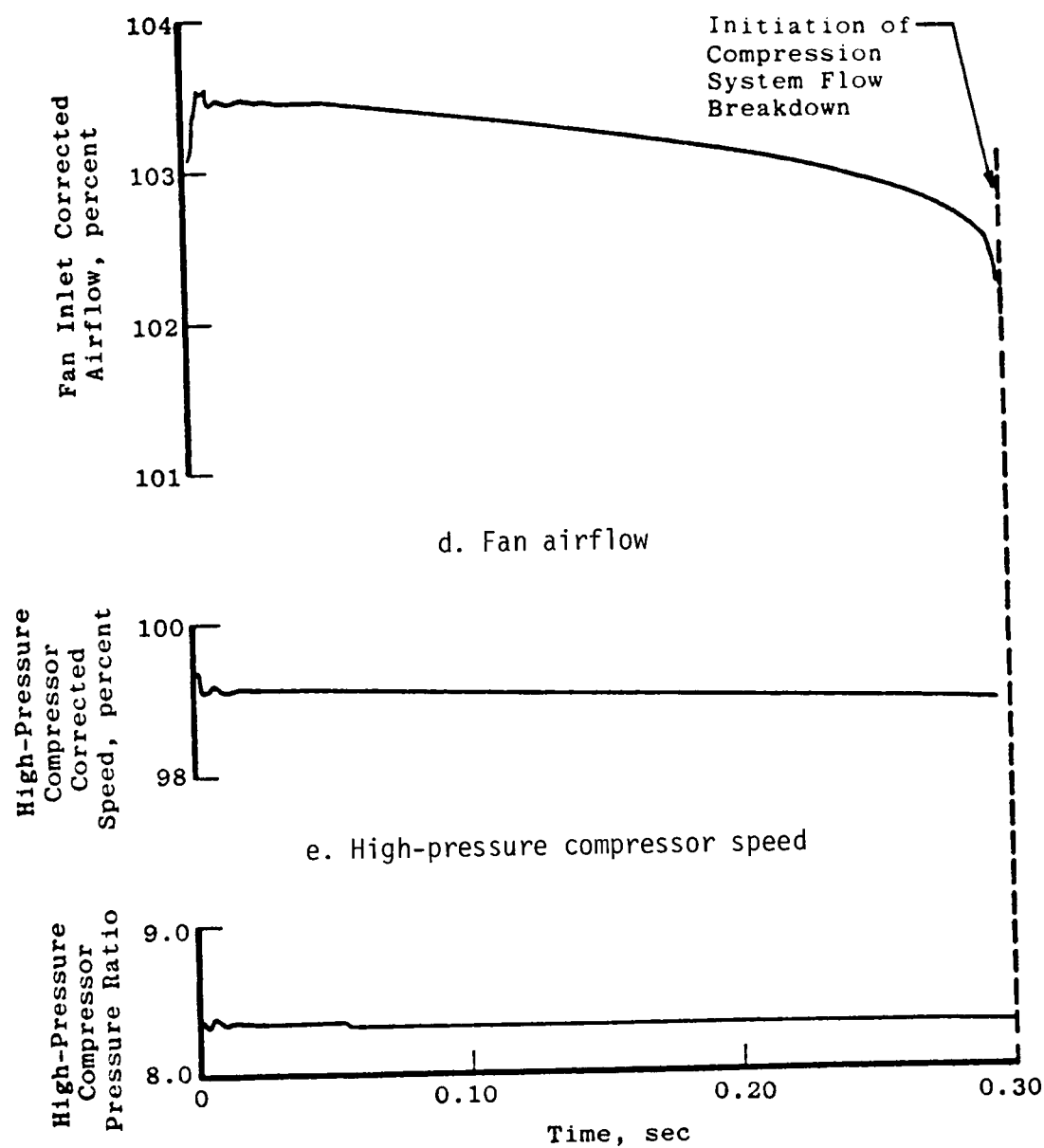


Figure 20. Dual-spool compression system model under fan loading at 30,000 ft, Mach number 0.8, intermediate power.



f. High-pressure compressor ratio

Figure 20. (continued)

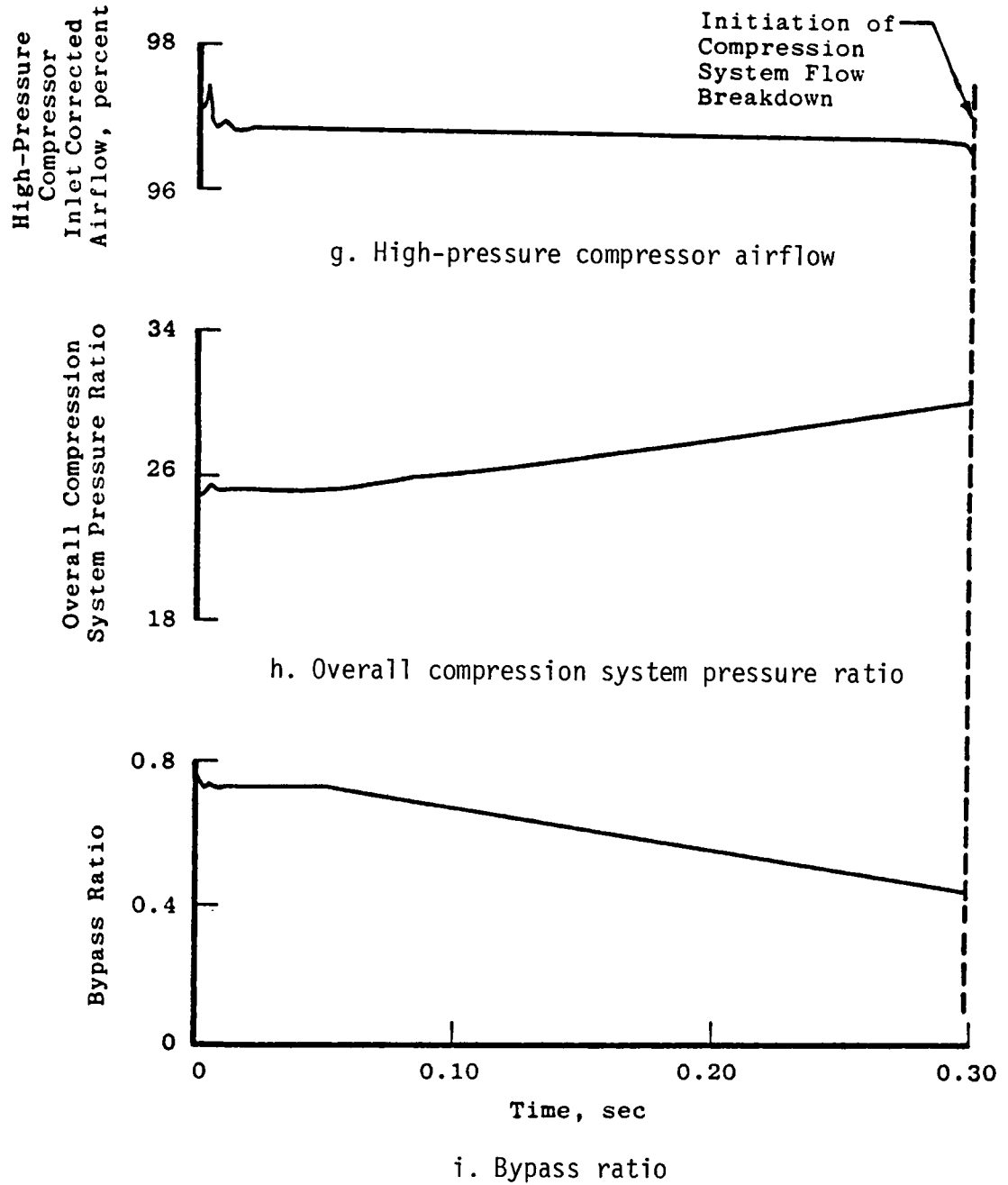
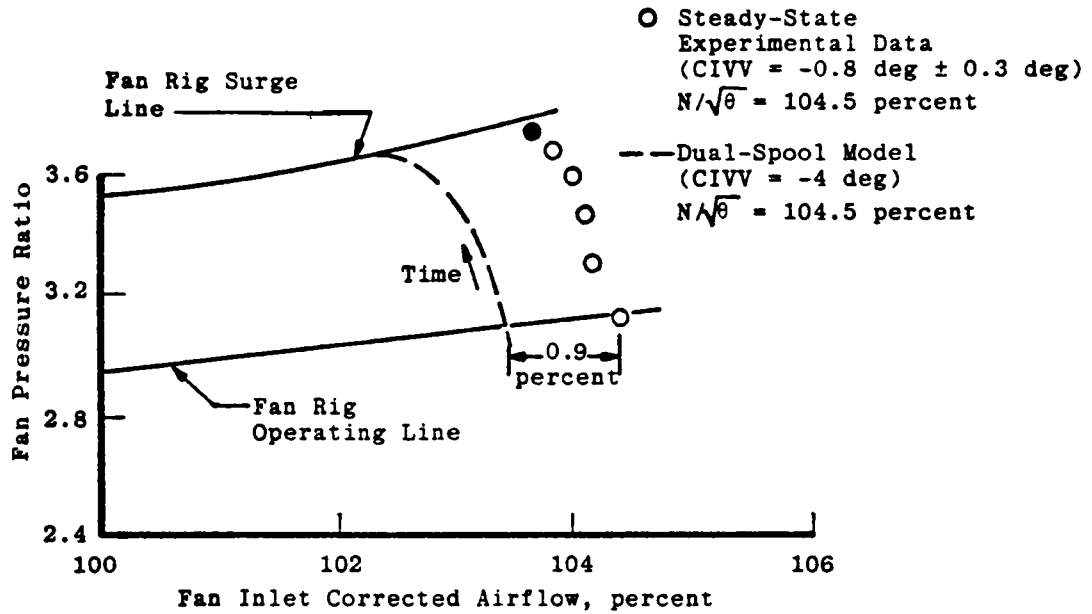
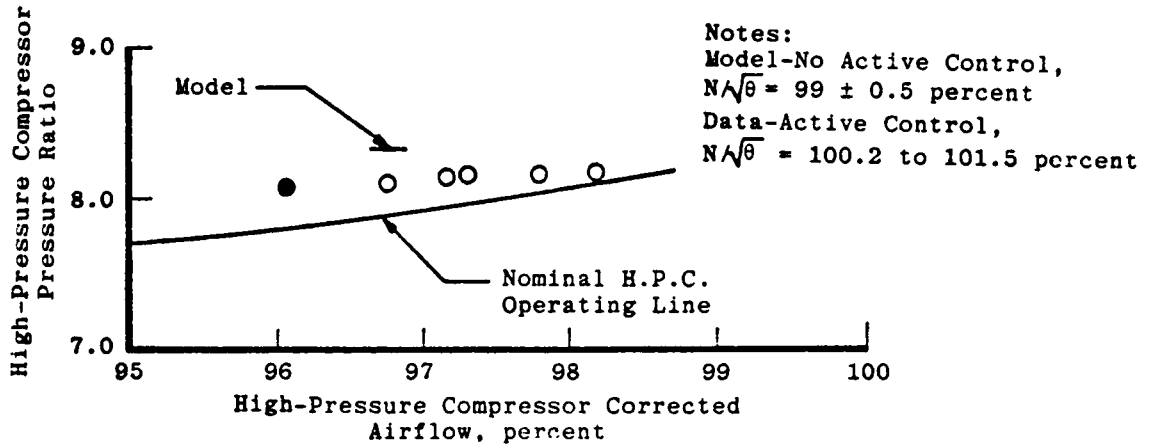


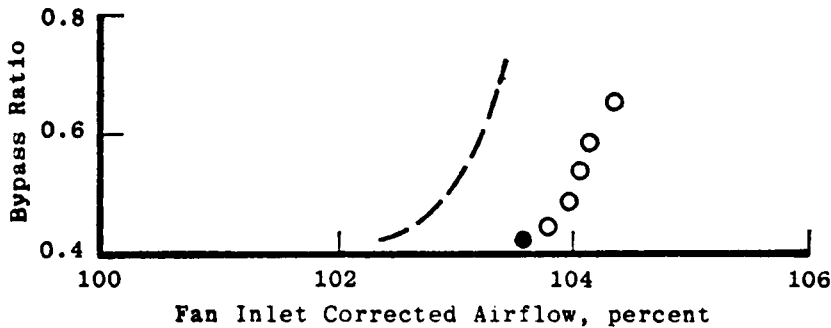
Figure 20. (continued)



a. Fan map



b. High-pressure compressor map



c. Bypass ratio variation

Figure 21. Comparison of model predicted compression system behavior during fan loading at 30,000 ft, Mach number 0.8, to experimental results.

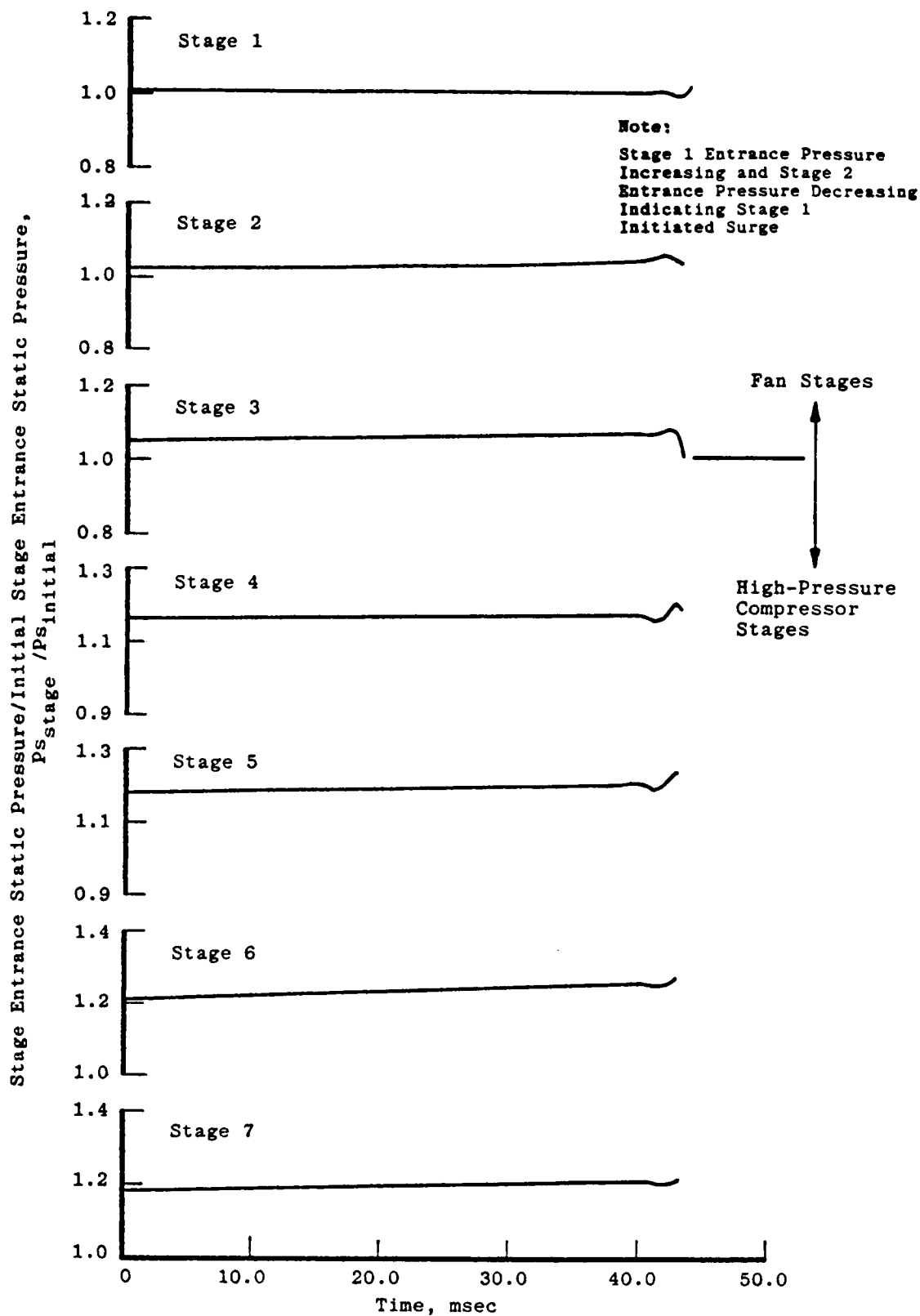


Figure 22. Model prediction of stage entrance static pressure signature at time of compression system surge during fan loading.

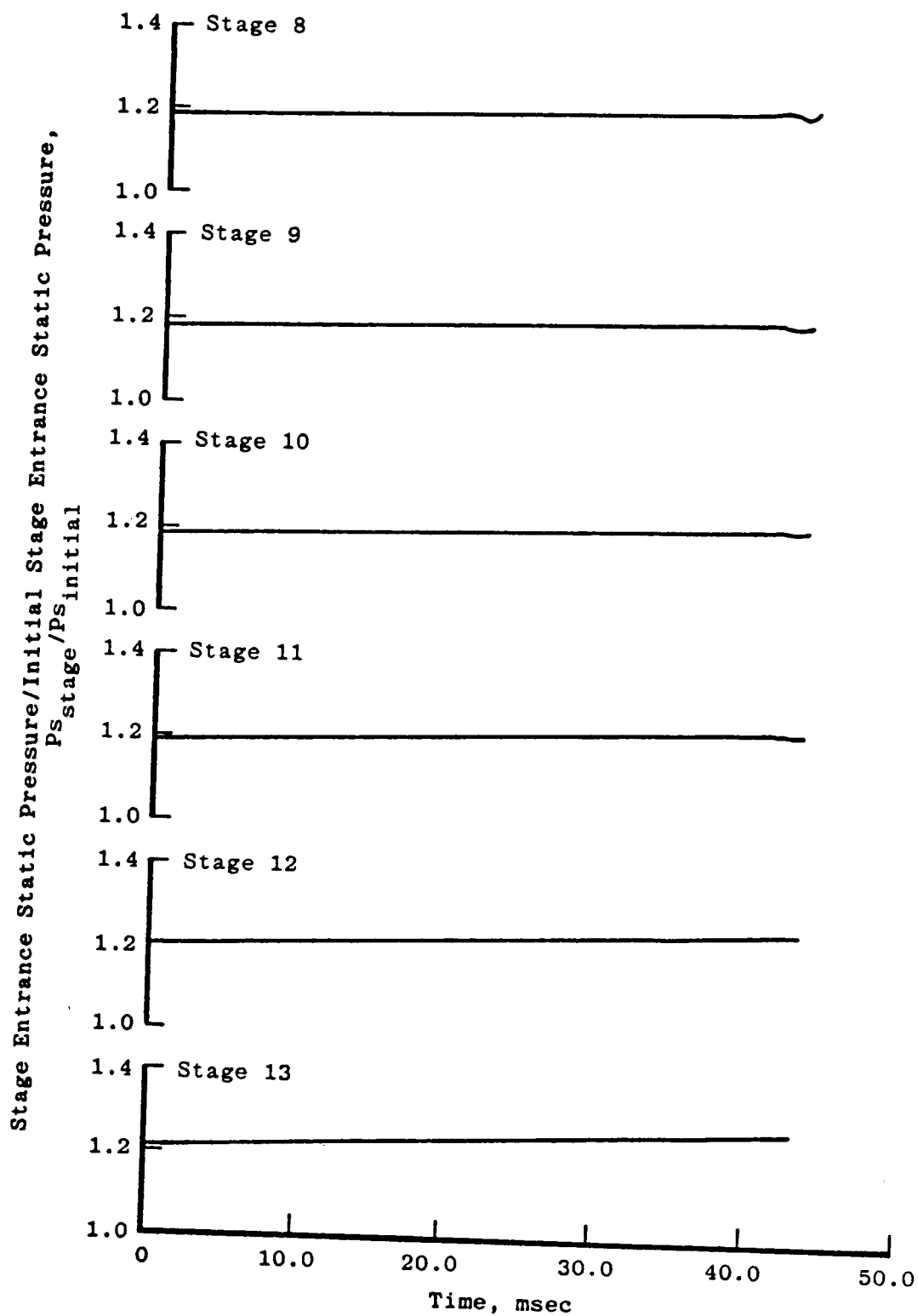


Figure 22. (continued)

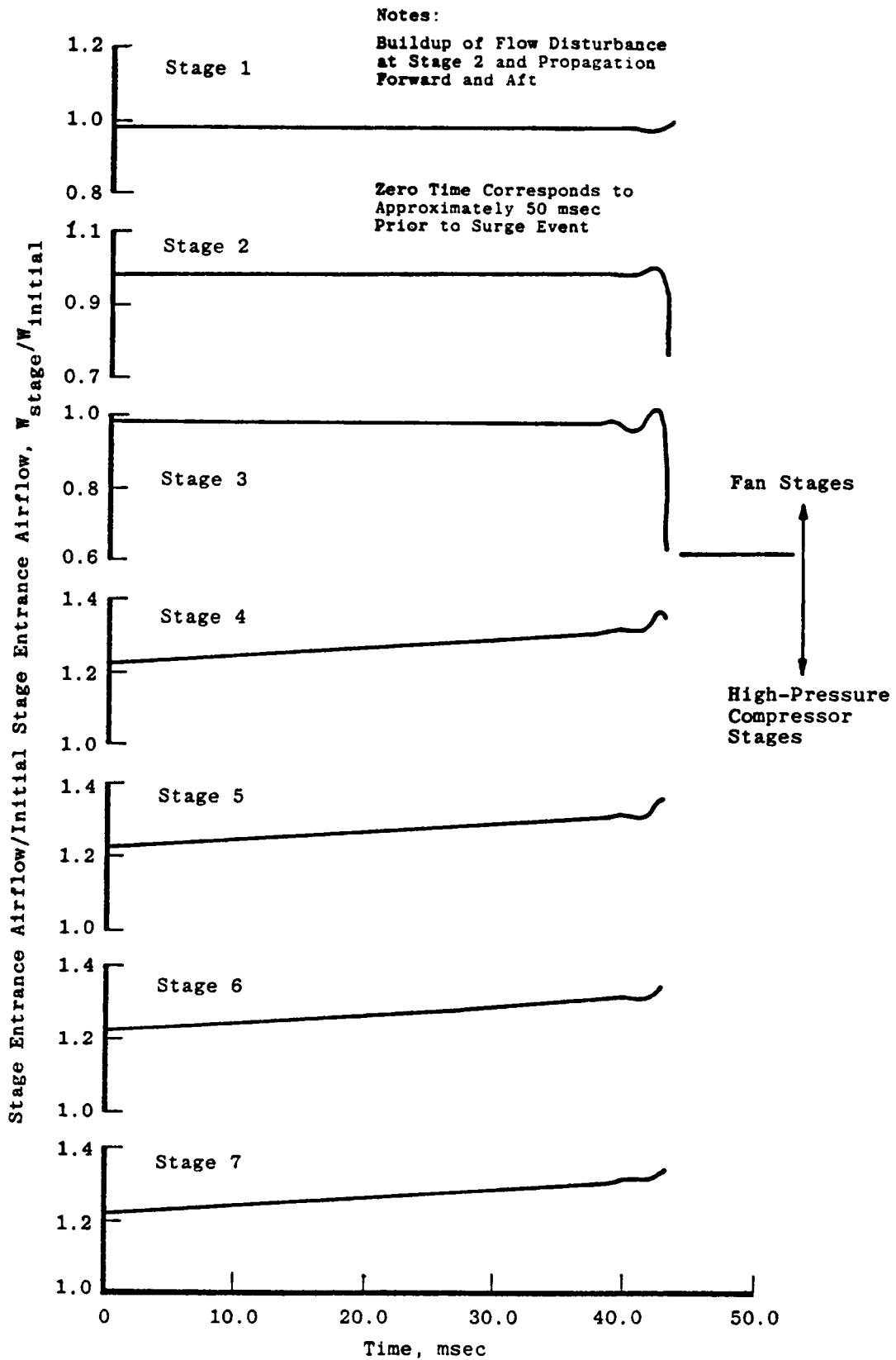


Figure 23. Model predicted airflow disturbance prior to fan surge.

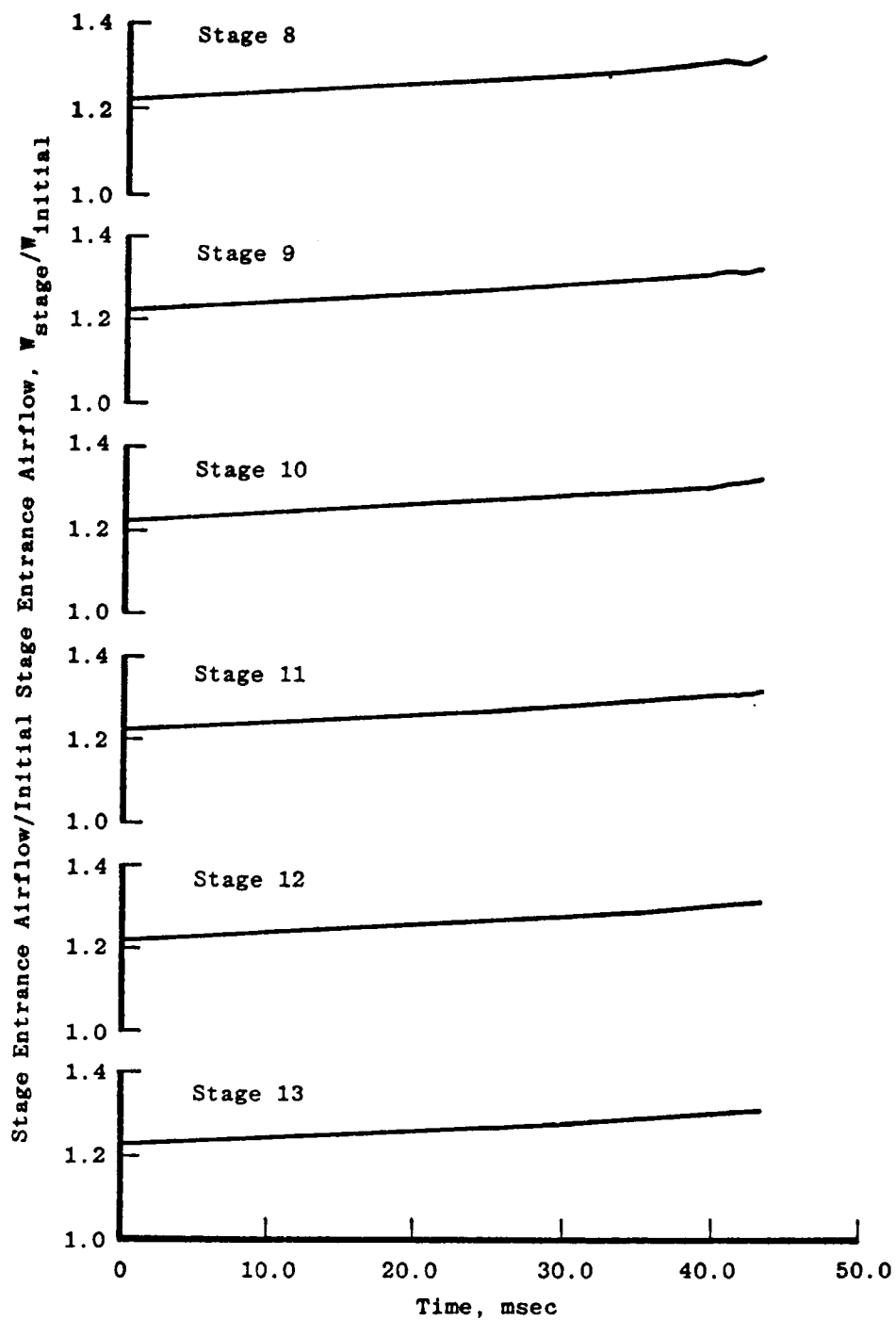
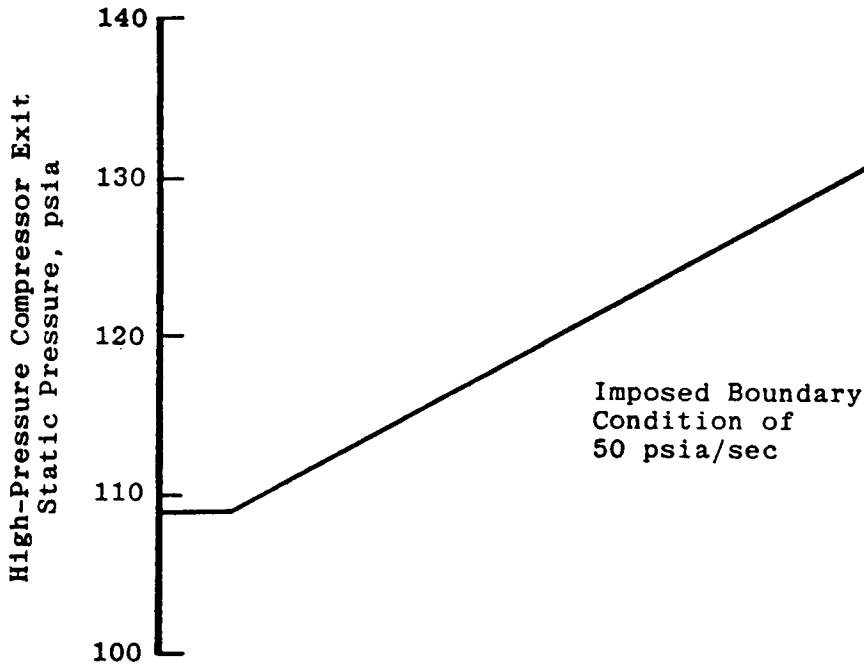
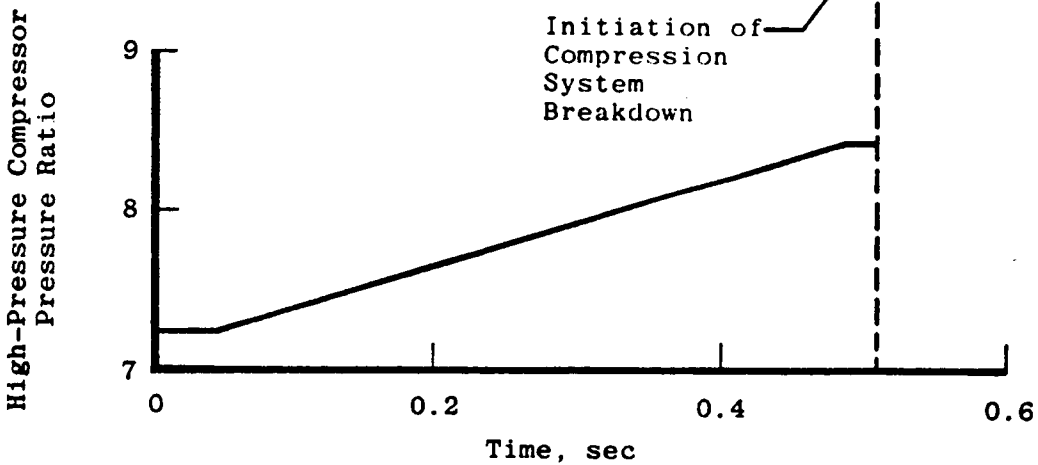
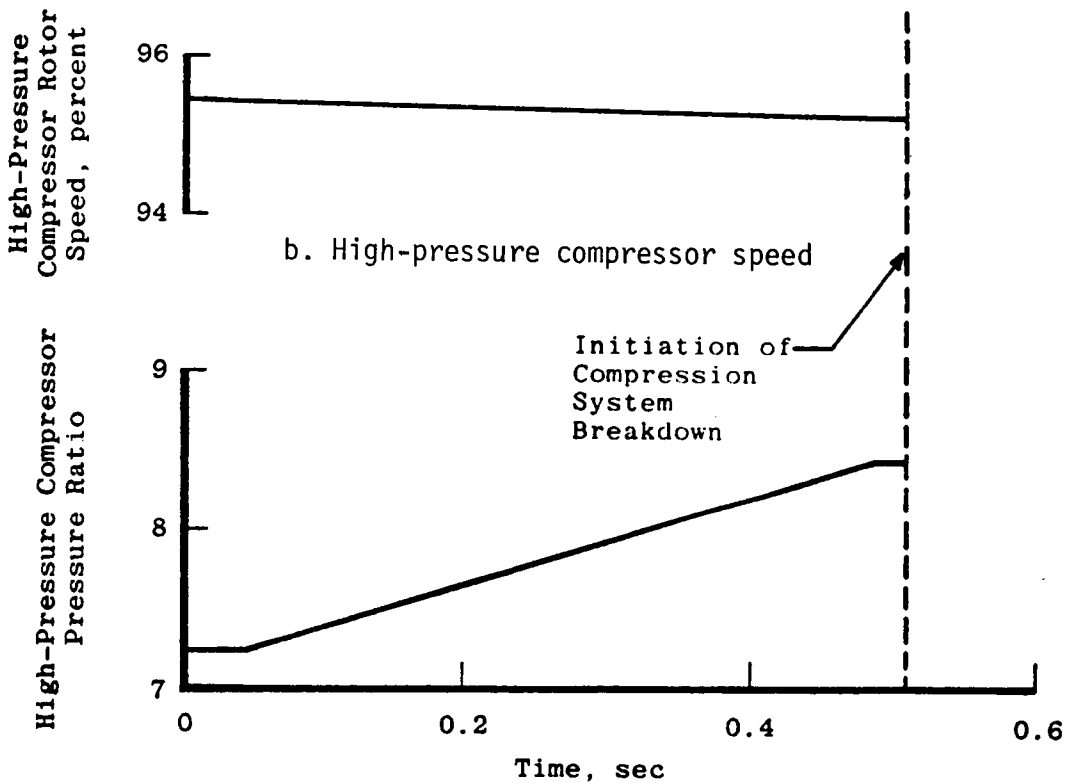


Figure 23. (continued)



a. High-pressure compressor exit static pressure increase



c. High-pressure compressor pressure ratio

Figure 24. Dual-spool compression system model under high-pressure compressor loading at 30,000 ft, Mach number 0.8, 92-percent fan speed.

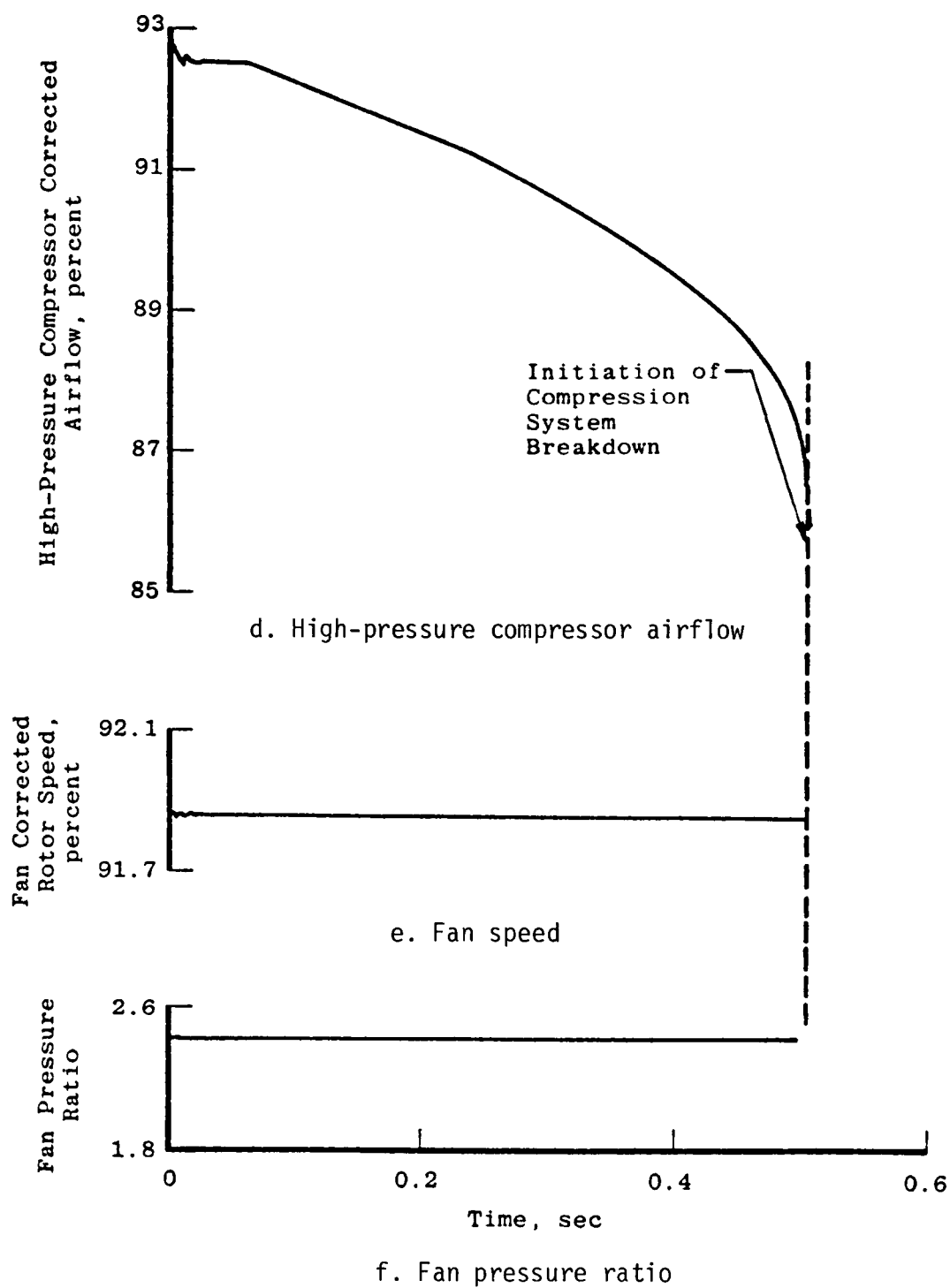
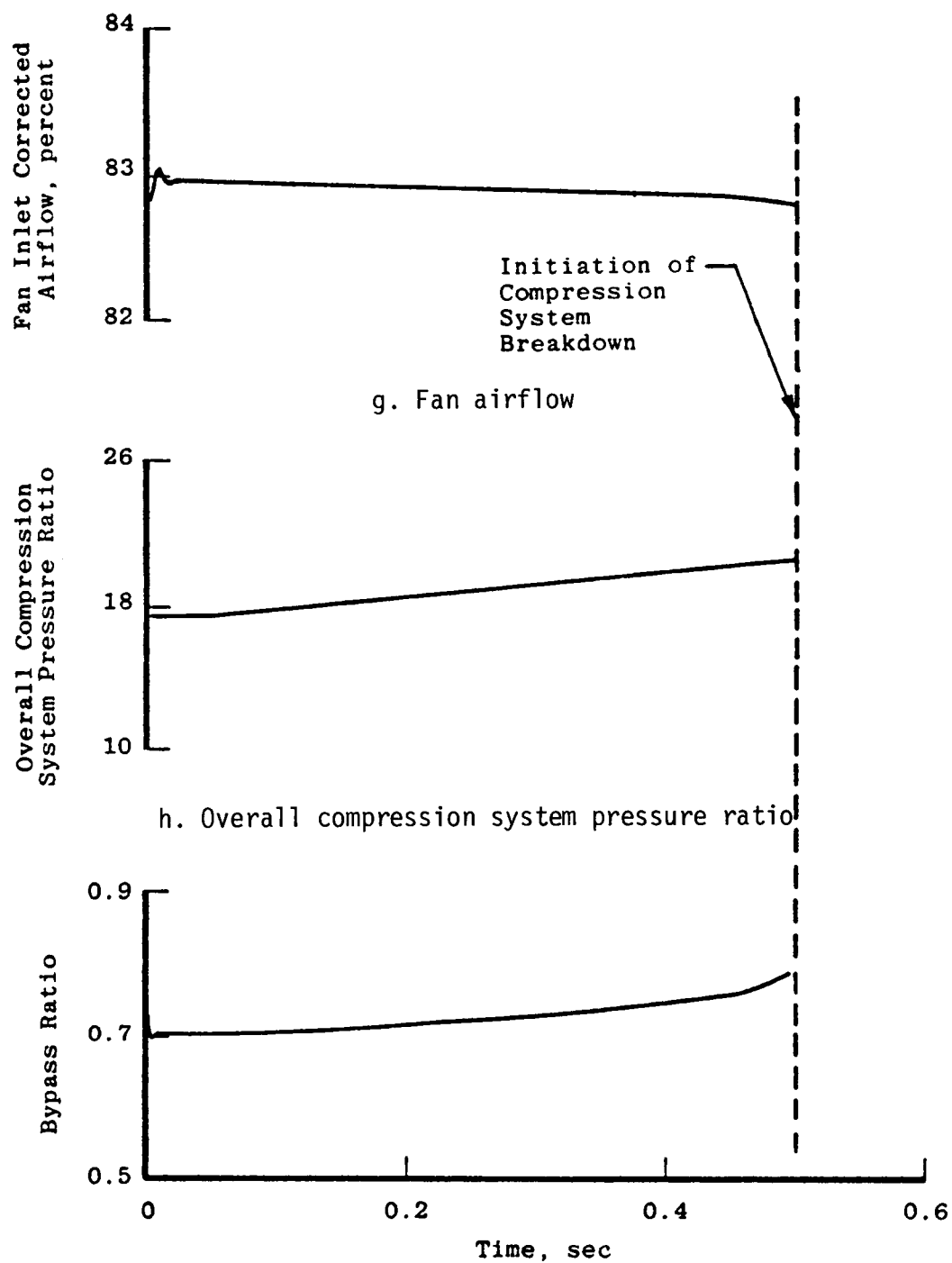


Figure 24. (continued)



i. Bypass ratio
Figure 24. (continued)

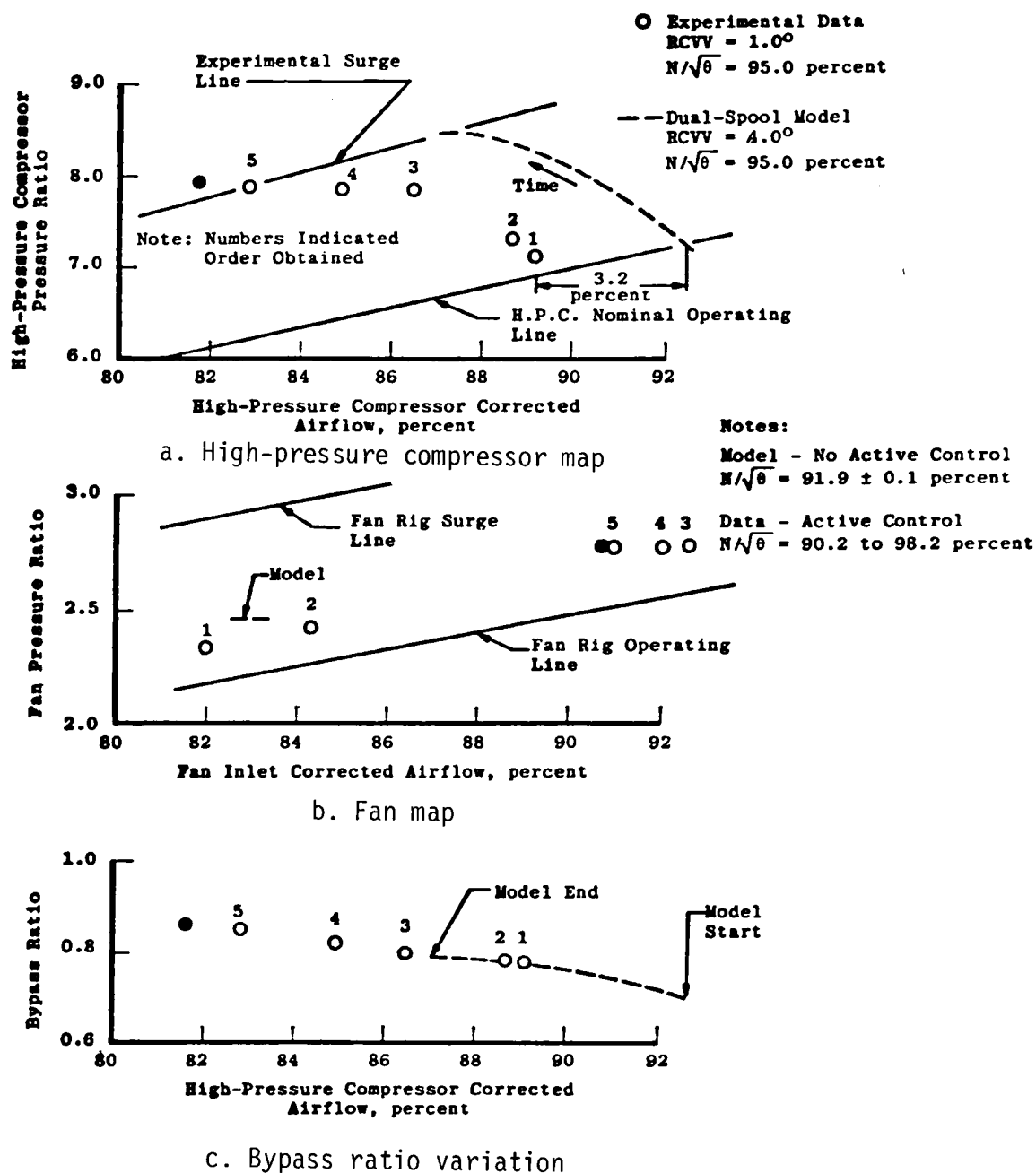
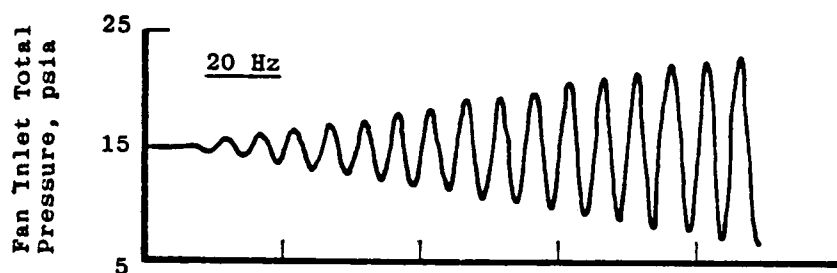
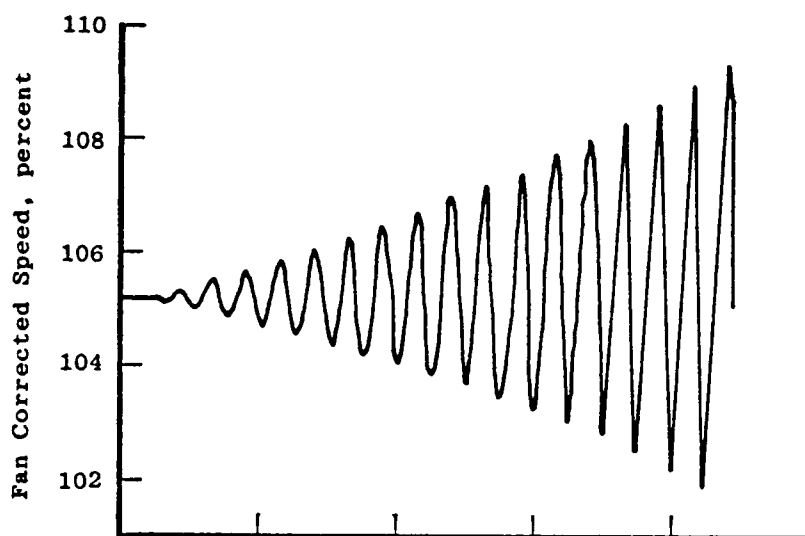


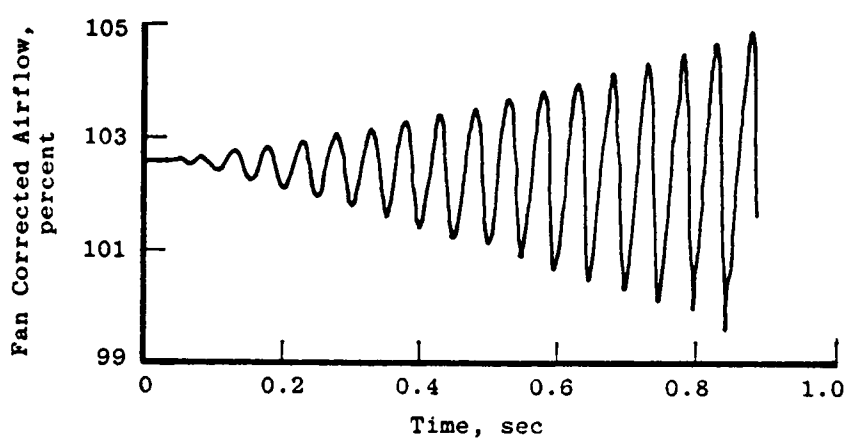
Figure 25. Comparison of model predicted compression system behavior during high-pressure compressor loading at 30,000 ft, Mach number 0.8, to experimental results.



a. Imposed inlet total pressure variation

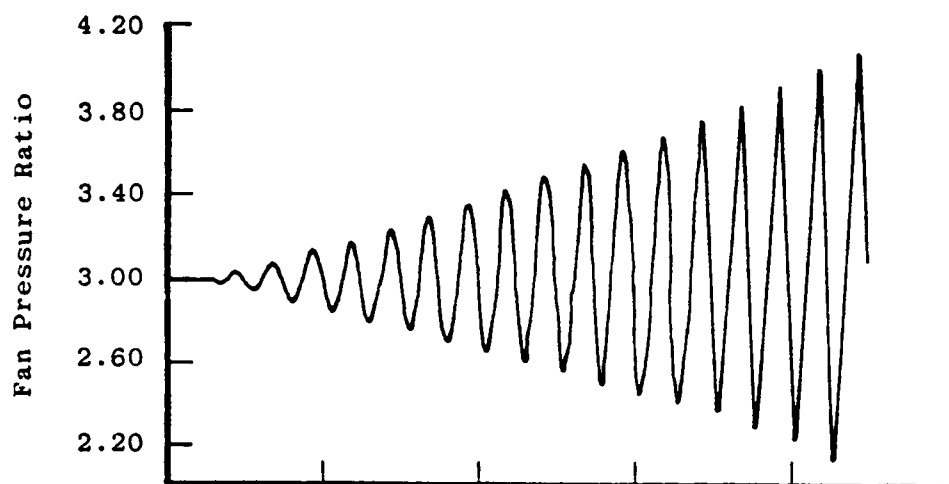


b. Fan corrected speed

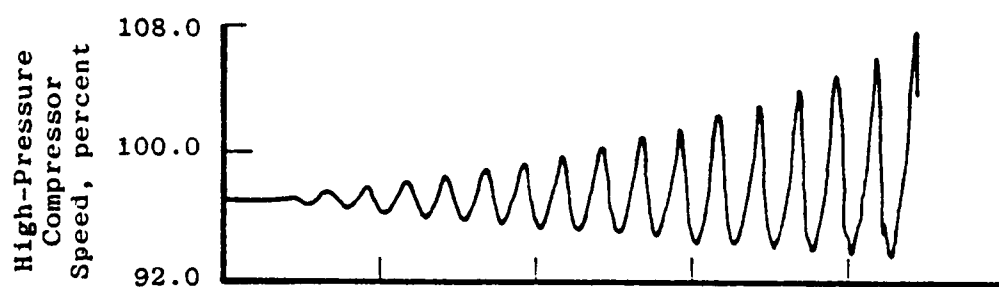


c. Fan corrected airflow

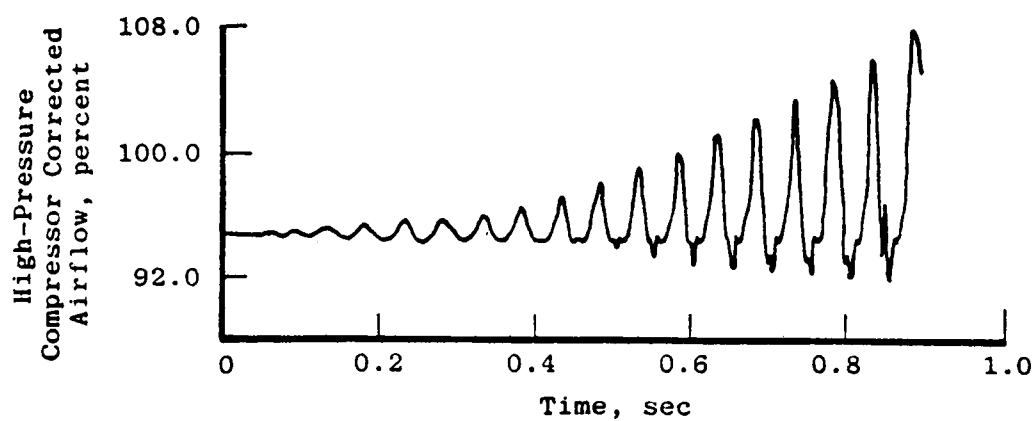
Figure 26. Time traces of compression system performance parameters during 20-Hz oscillatory inflow loading, sea-level-static conditions, intermediate power.



d. Fan pressure ratio

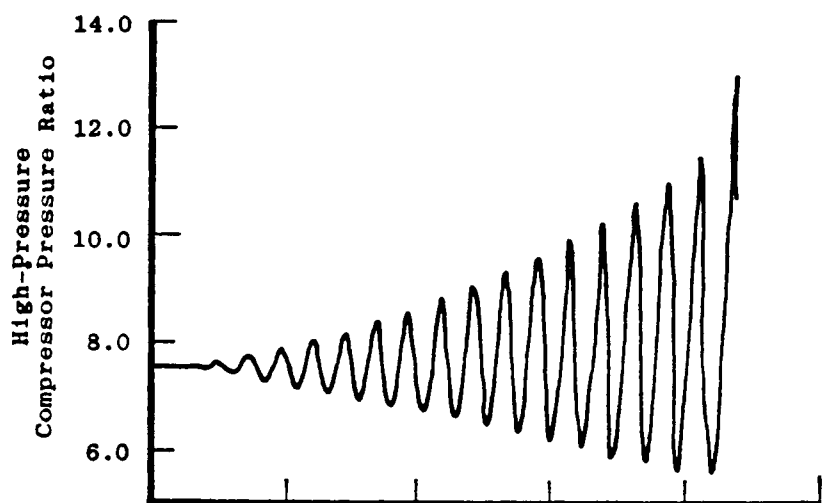


e. High-pressure compressor corrected speed

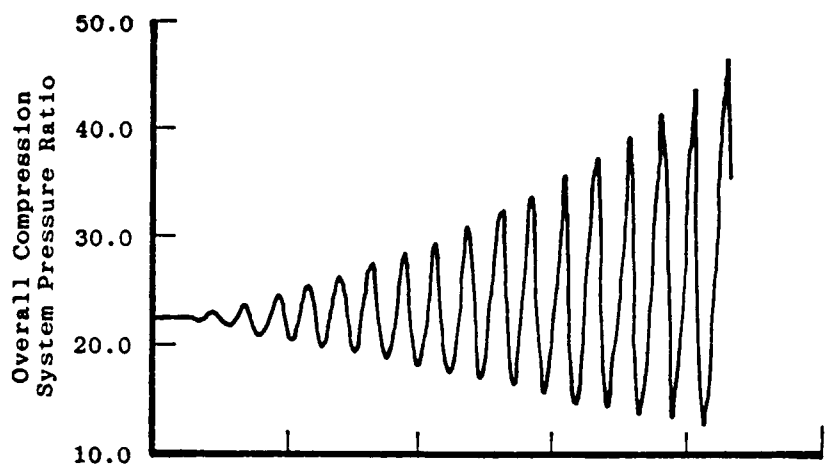


f. High-pressure compressor corrected airflow

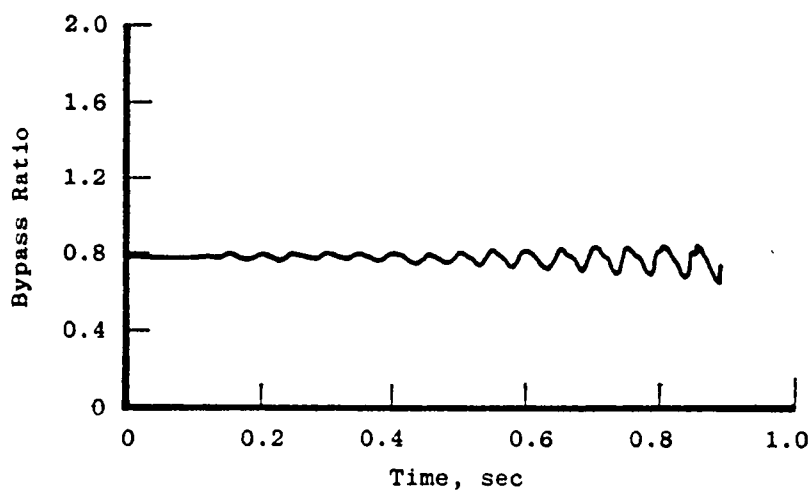
Figure 26. (continued)



g. High-pressure compressor pressure ratio

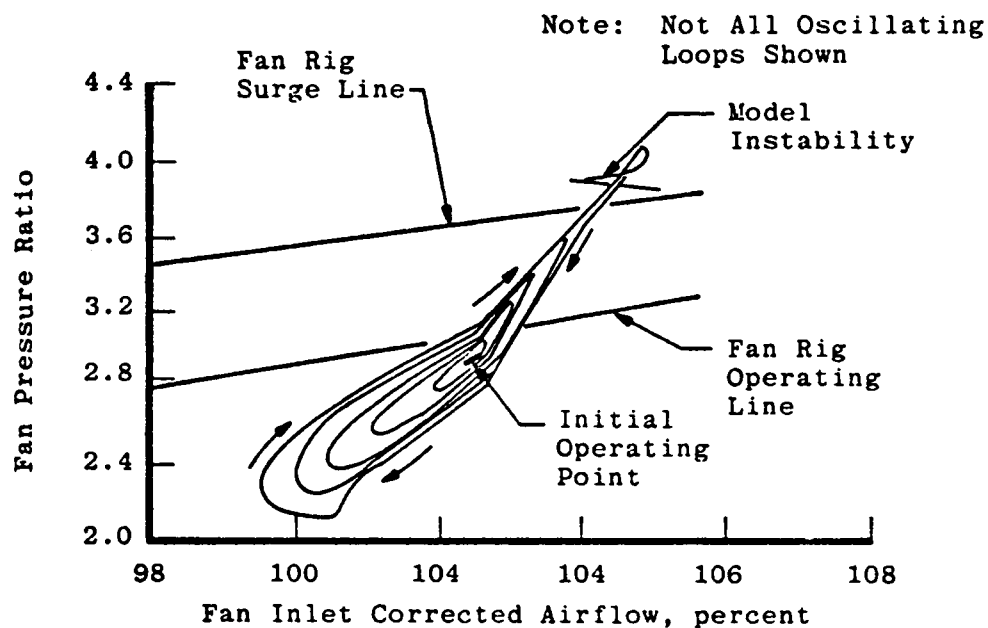


h. Overall compression system pressure ratio

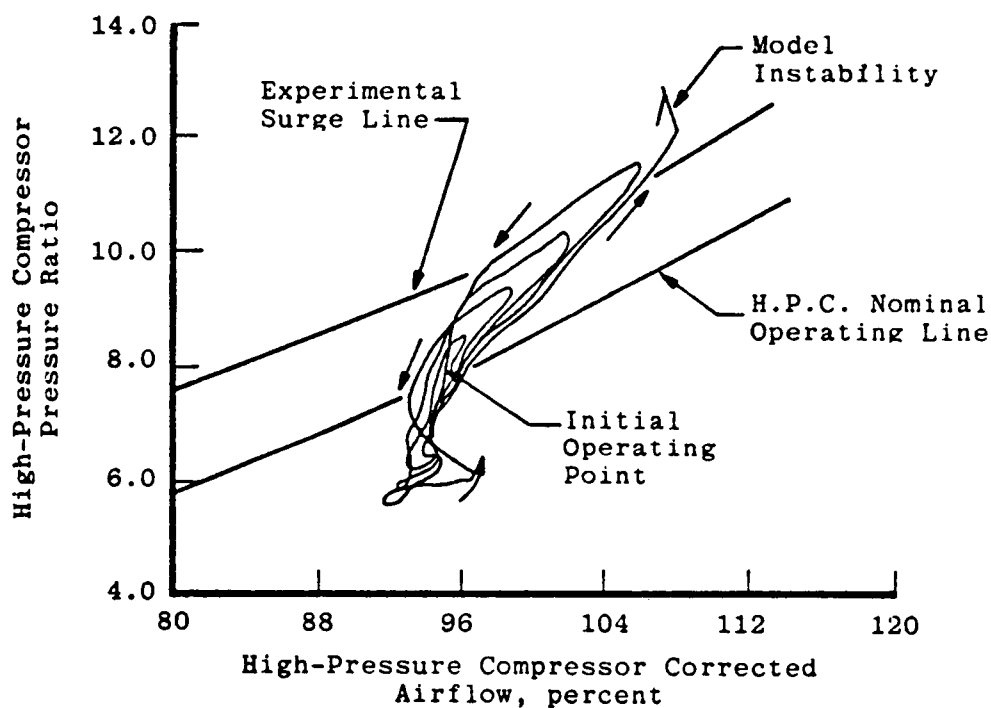


i. Bypass ratio

Figure 26. (continued)



a. Fan map



b. High-pressure compressor map

Figure 27. Model predicted oscillatory inflow effects on compression system operation, sea-level-static condition, intermediate power.

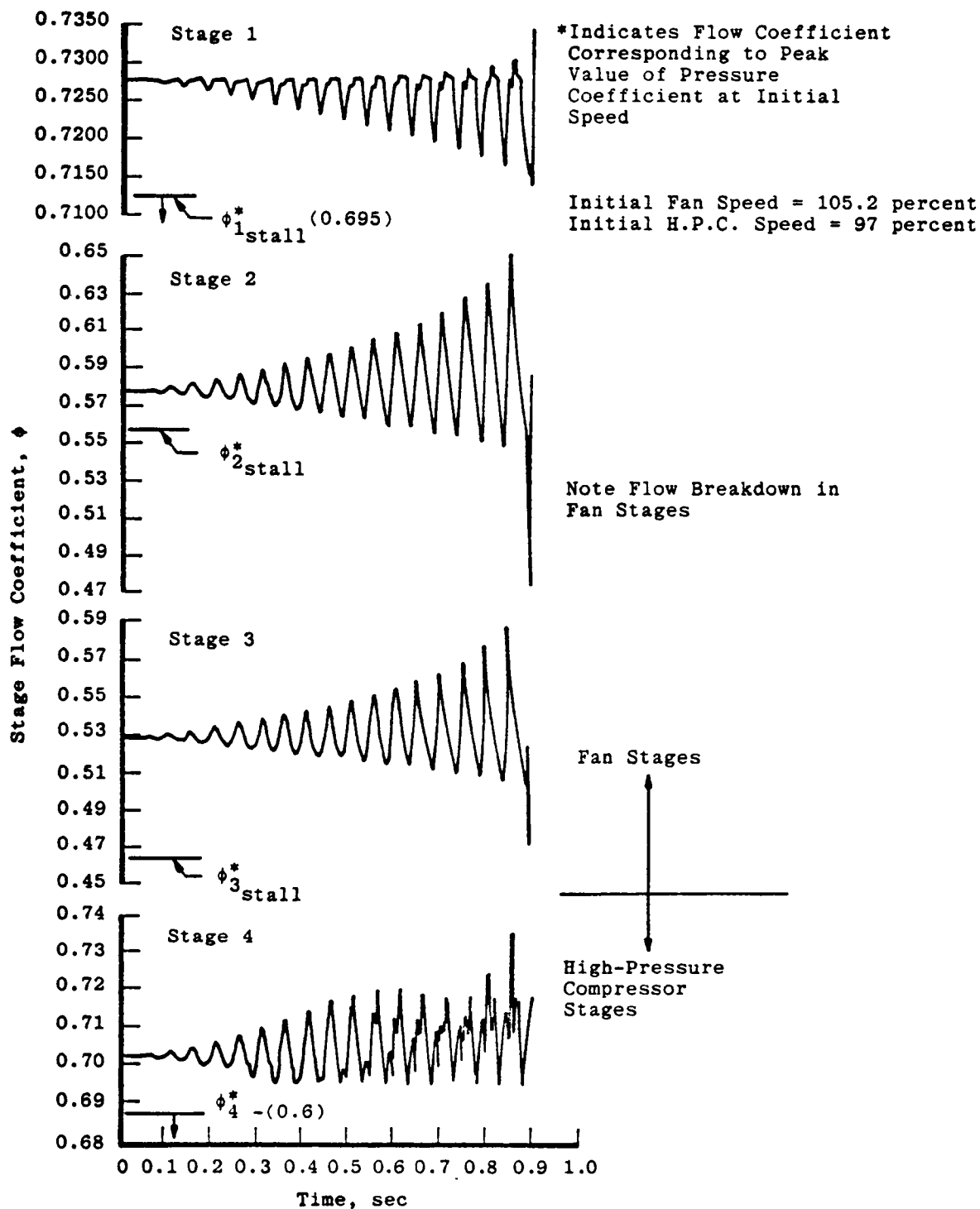


Figure 28. Effects of 20-Hz oscillating inflow loading on stage coefficient, sea-level-static conditions, intermediate power.

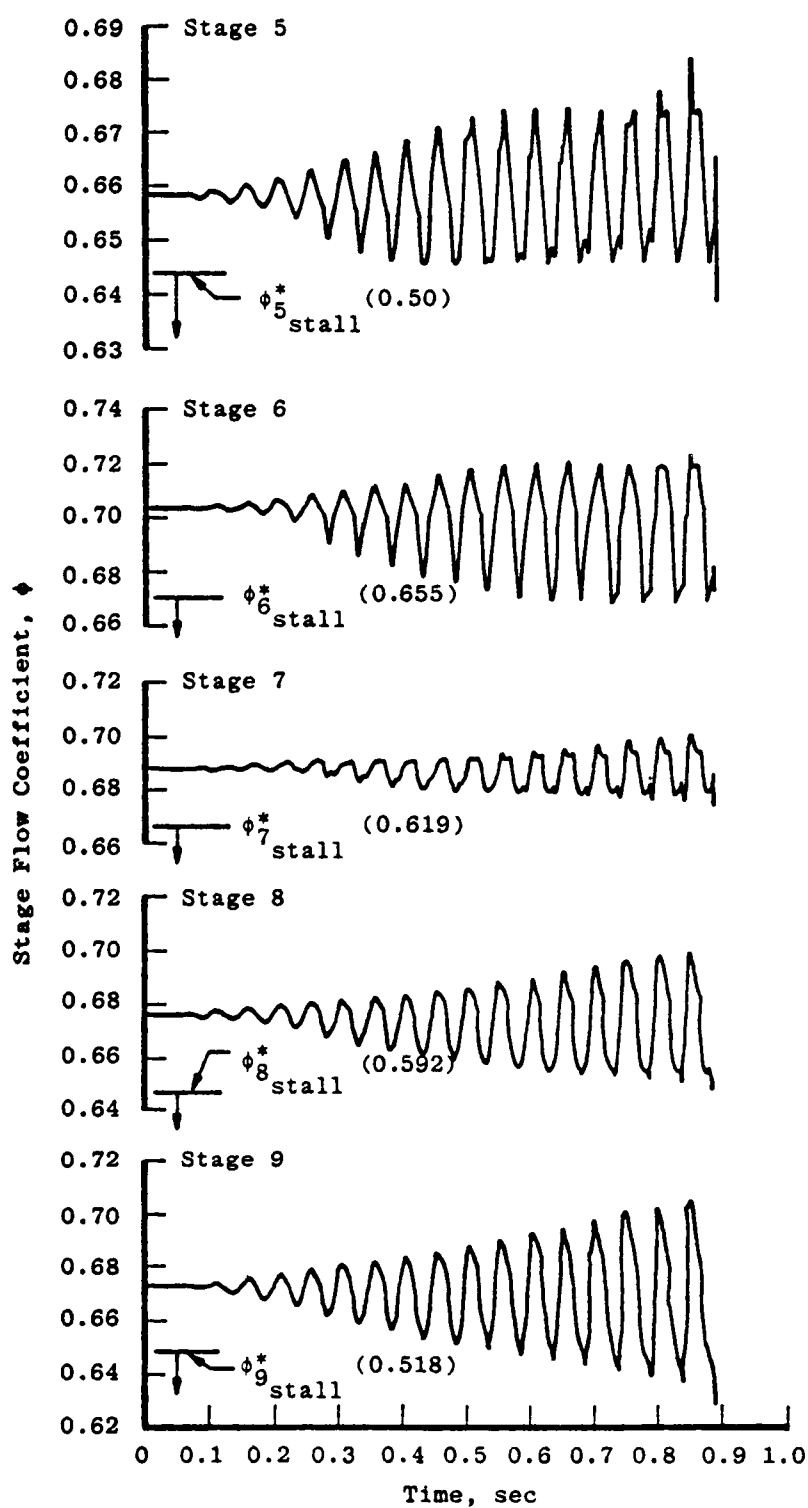


Figure 28. (continued)

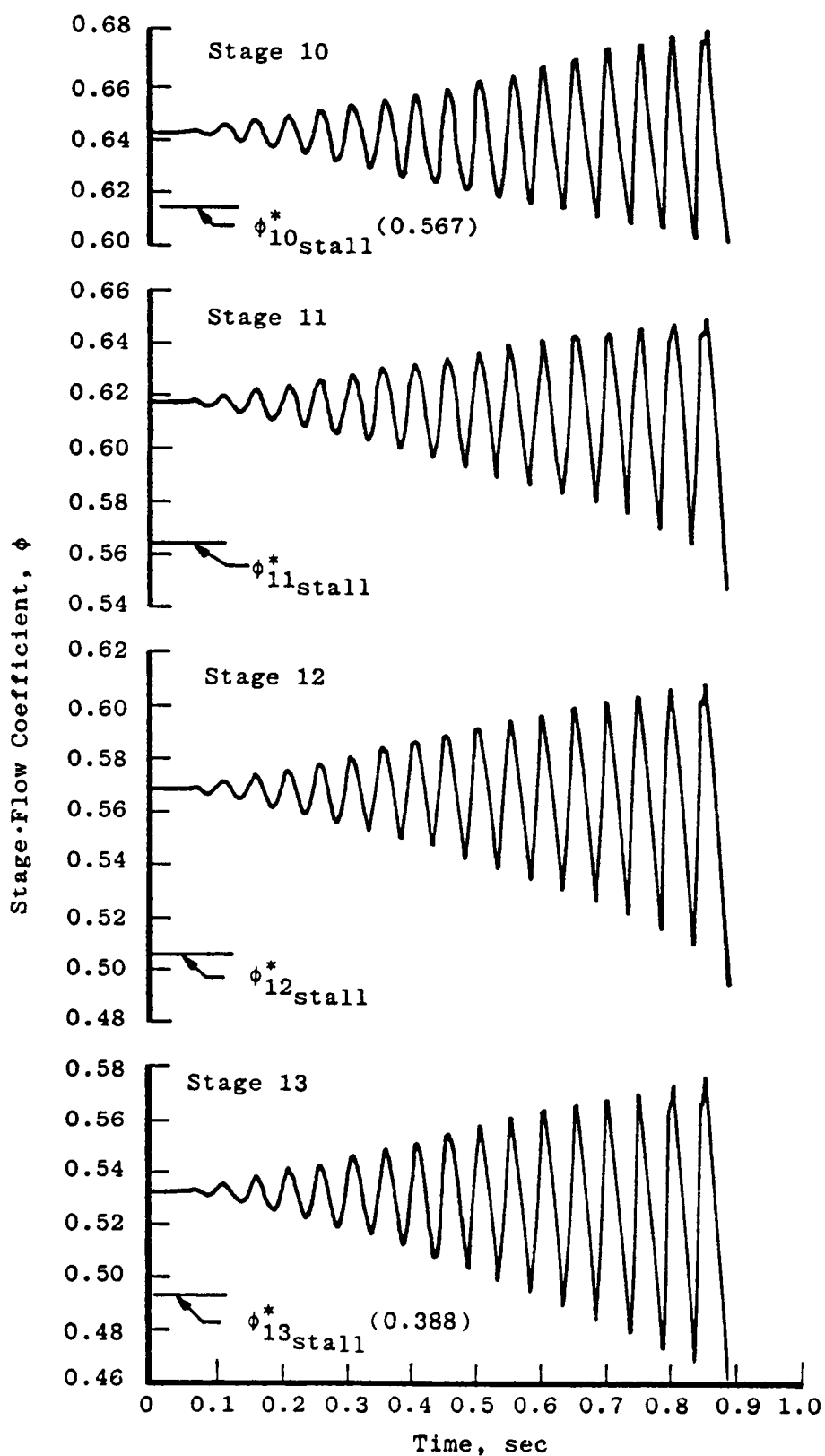


Figure 28. (continued)

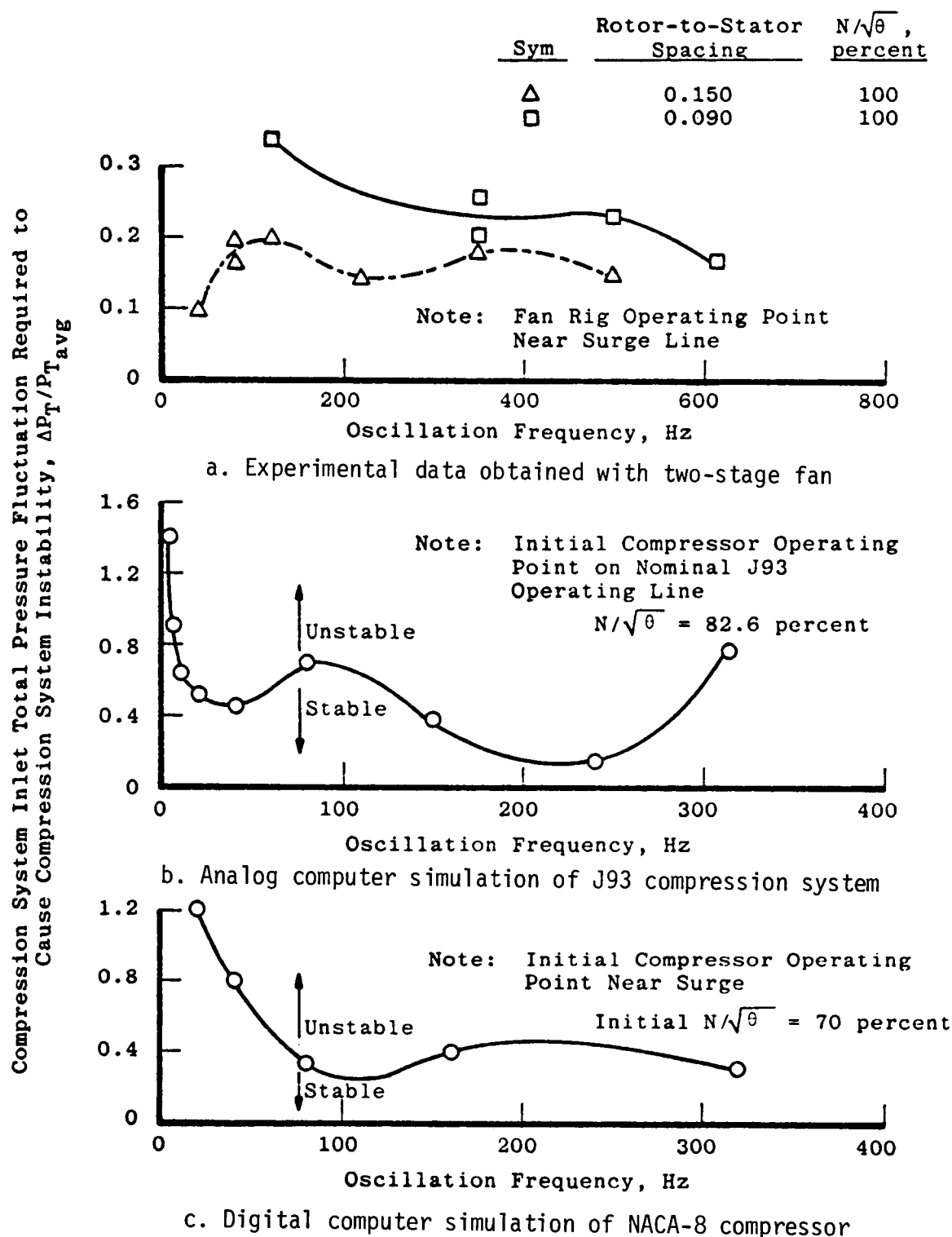
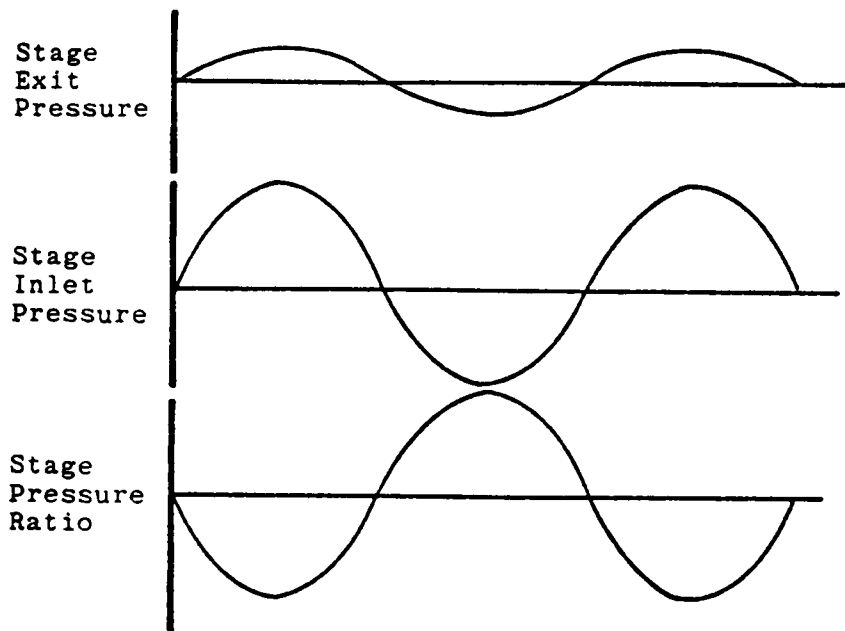
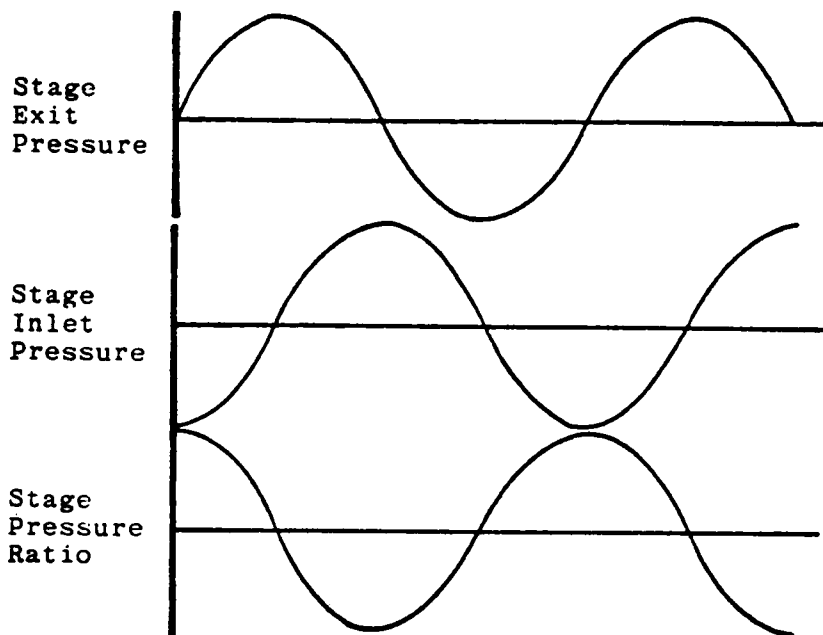


Figure 29. Compression systems stability limit trends for planar oscillating inflow.



a. Amplitude attenuation effect



b. Phase angle effect

Figure 30. Stage pressure ratio oscillation caused by either pressure amplitude attenuation or pressure phase shift between stages.

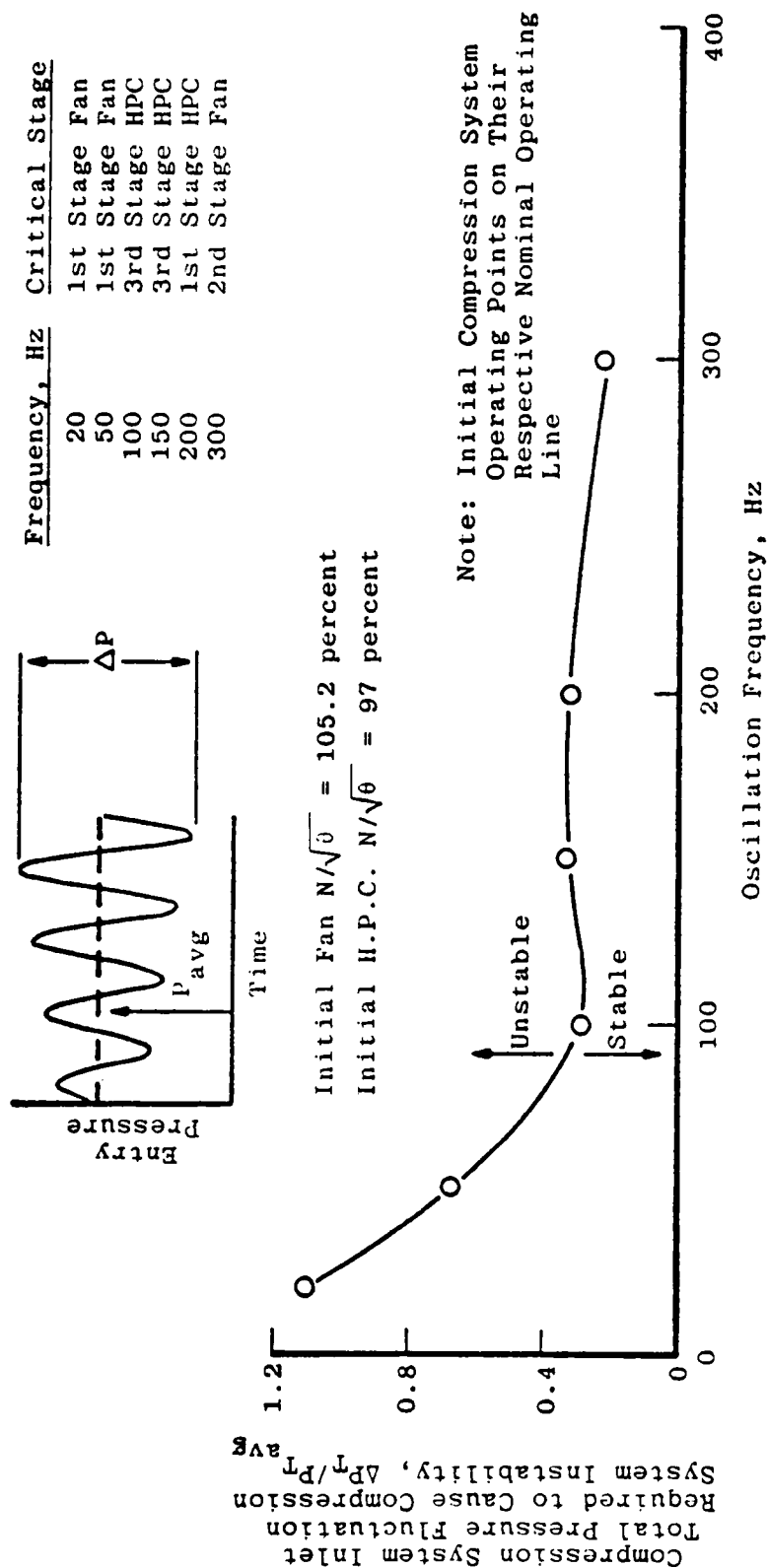
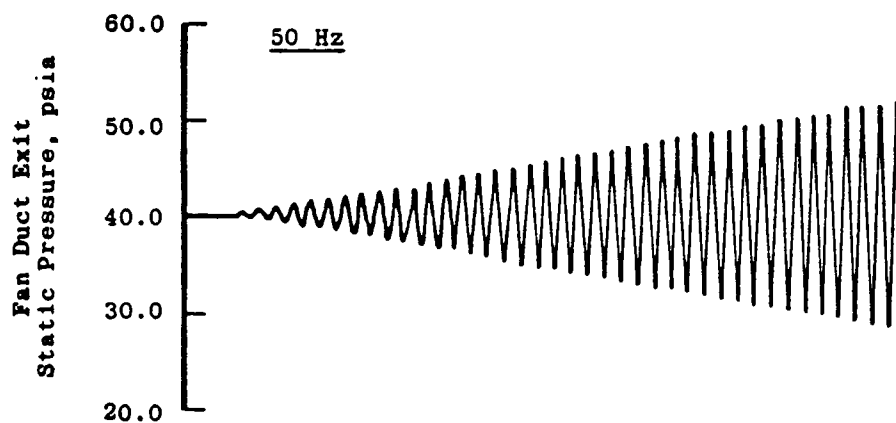
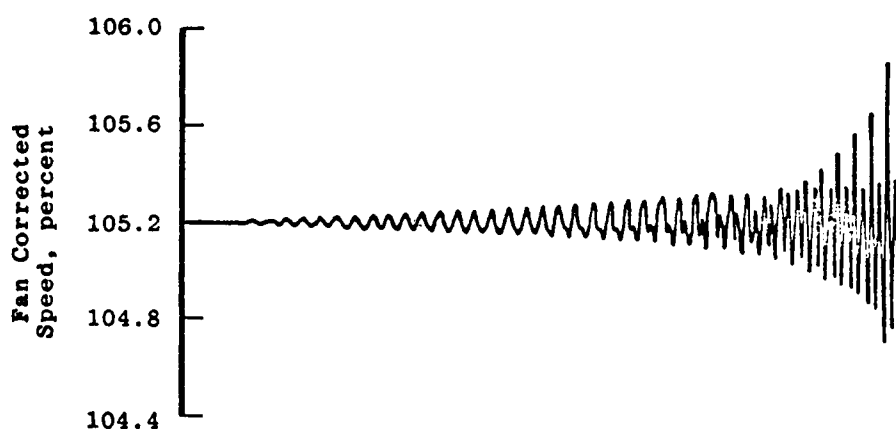


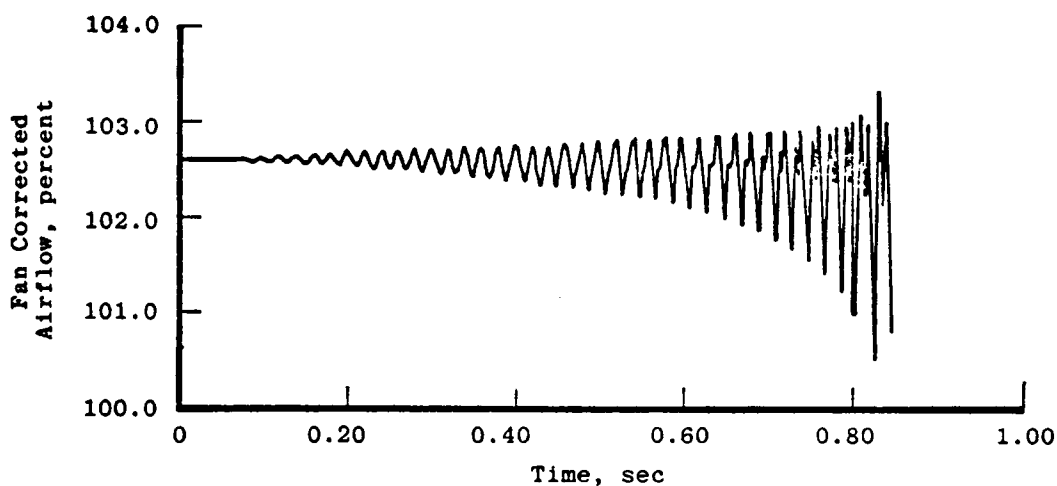
Figure 31. Model predicted stability limits for planar oscillating inflow.



a. Imposed fan duct exit static pressure variation

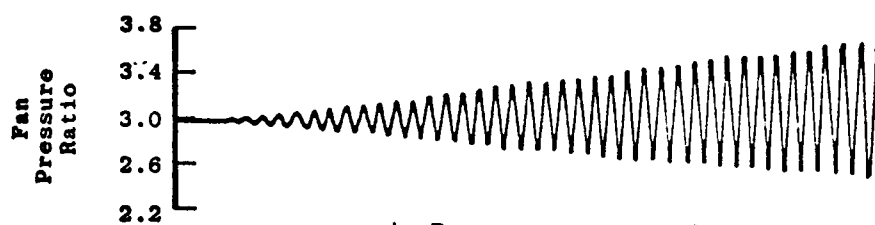


b. Fan corrected speed

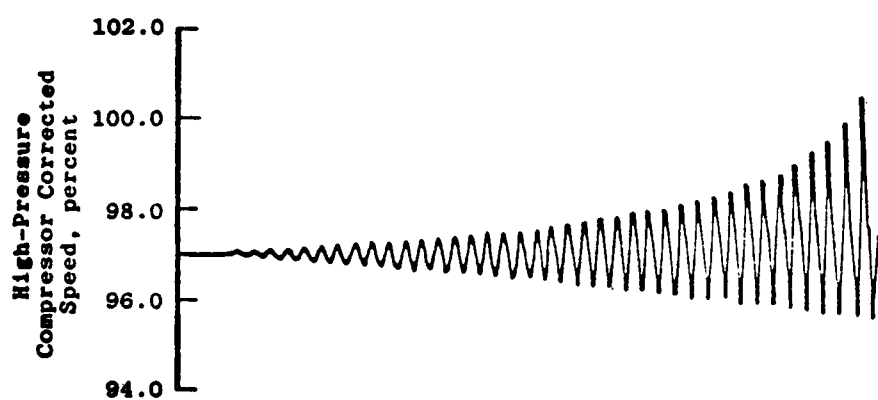


c. Fan corrected airflow

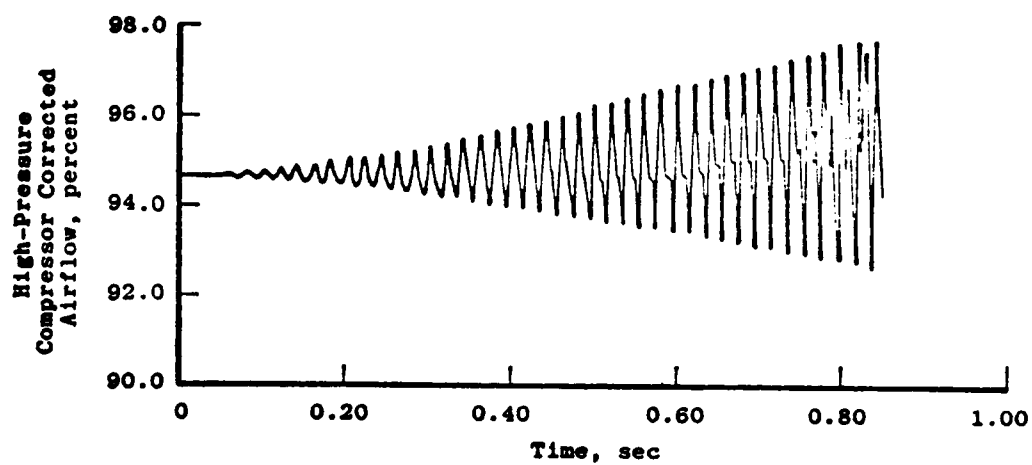
Figure 32. Time traces of compression system performance parameters during 50-Hz oscillatory fan duct exit pressure loading, sea-level-static conditions, intermediate power.



d. Fan pressure ratio

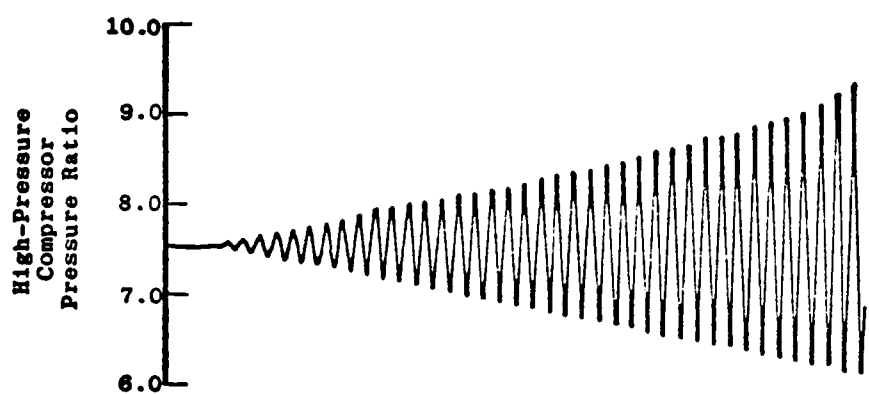


e. High-pressure compressor corrected speed

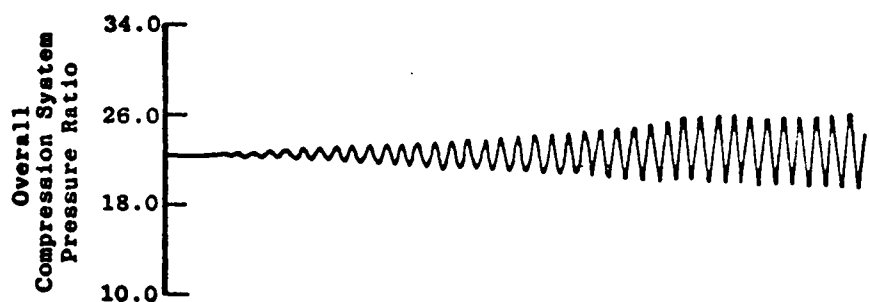


f. High-pressure compressor corrected airflow

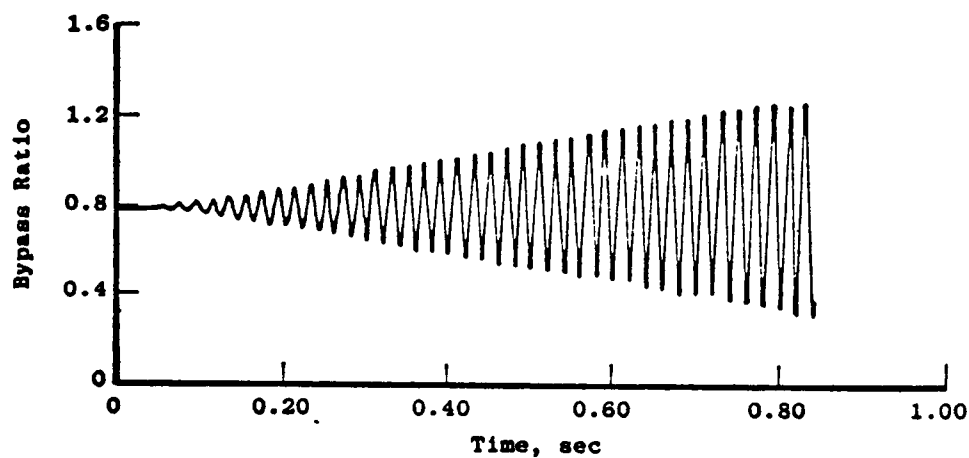
Figure 32. (continued)



g. High-pressure compressor pressure ratio

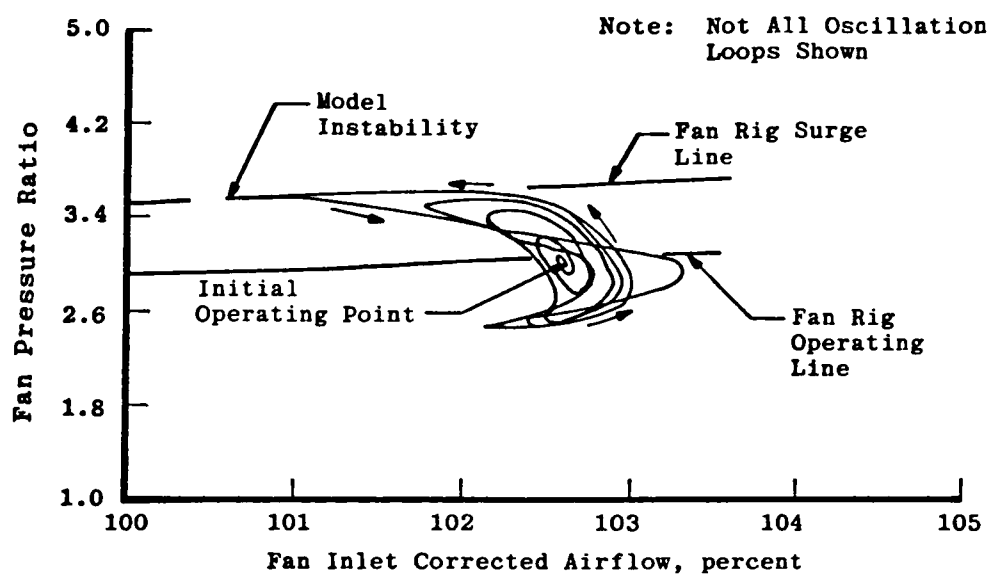


h. Overall compression system pressure ratio

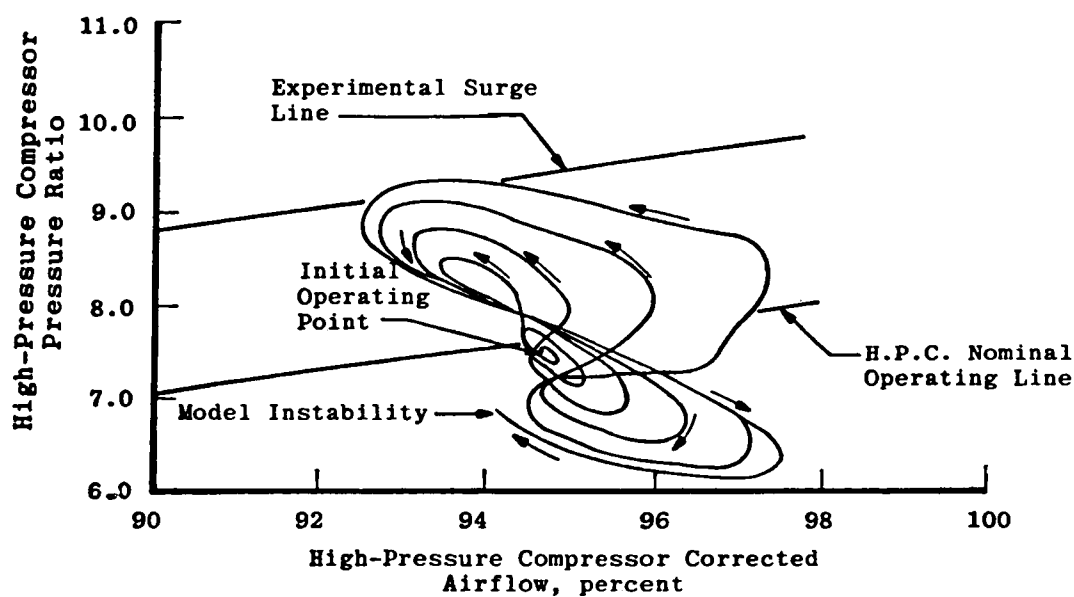


i. Bypass ratio

Figure 32. (continued)



a. Fan map



b. High-pressure compressor map

Figure 33. Model predicted oscillatory fan duct exit static pressure effects on compression system operation, sea-level-static conditions, intermediate power.

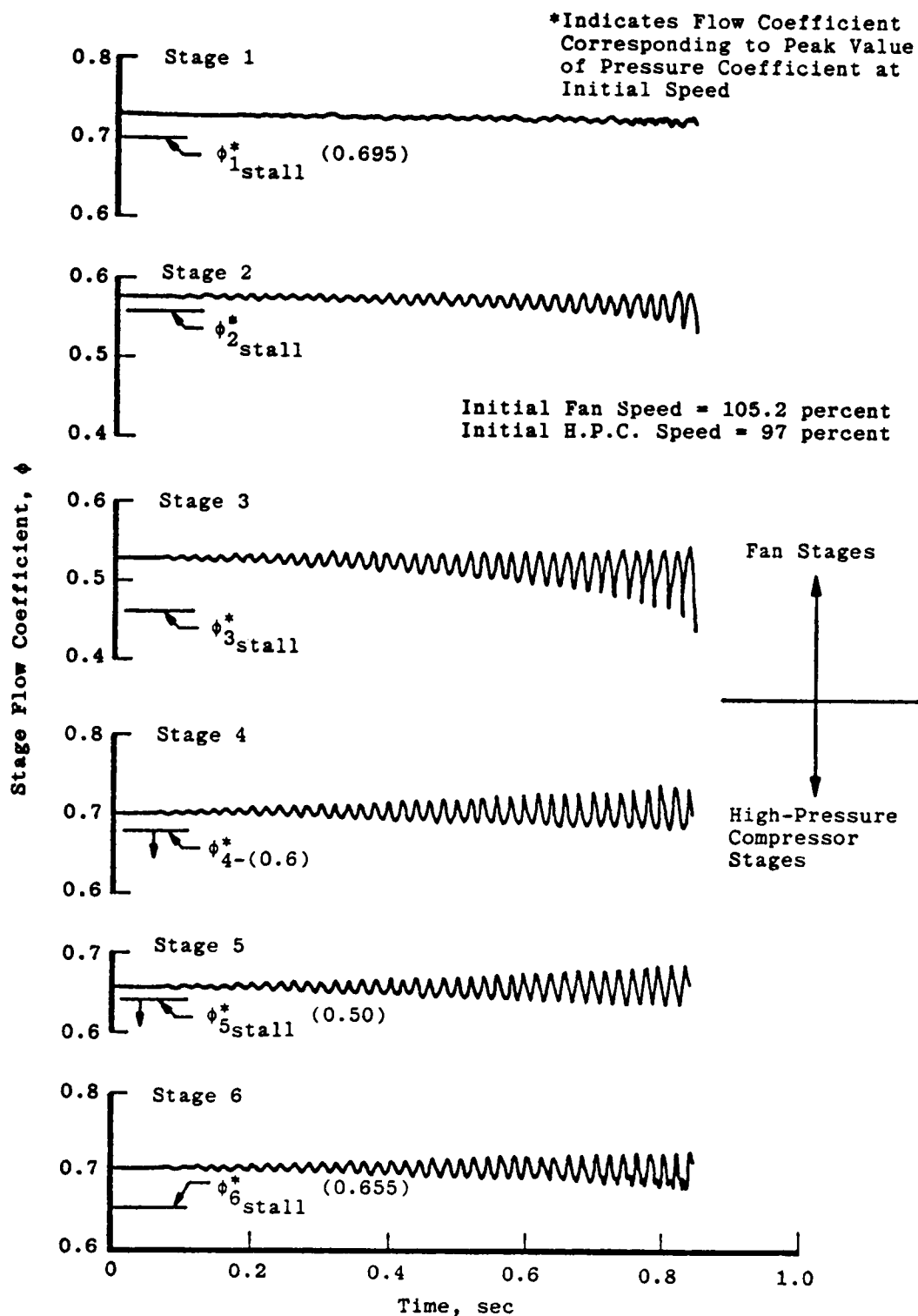


Figure 34. Model predicted effects of 50-Hz oscillating fan duct static pressure to stall on stage coefficient, sea-level-static conditions, intermediate power.

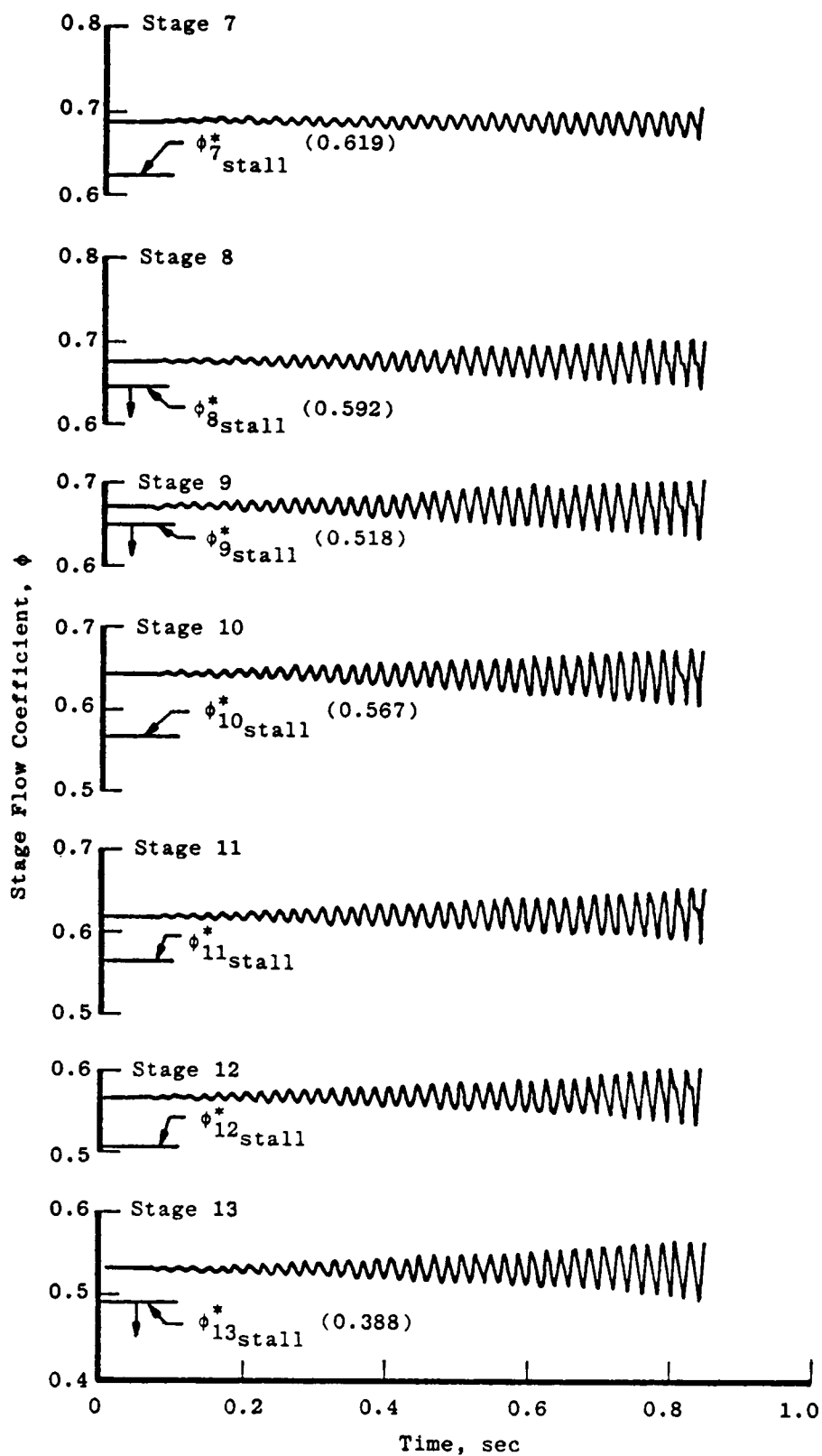


Figure 34. (continued)

- Notes: (1) Compression System Operating on Its Nominal Operating Line
 (2) Second-Stage Fan Initiated Compression System Instability at All Frequencies

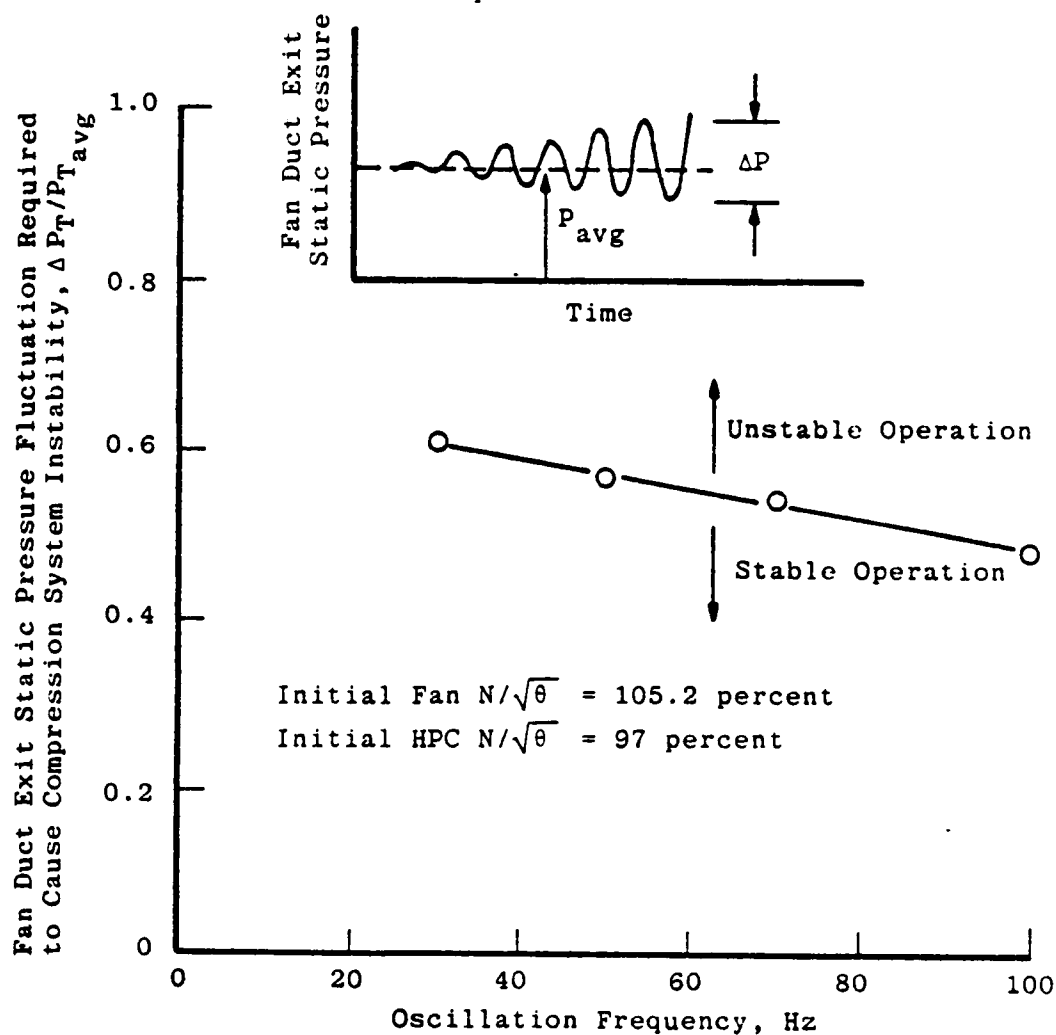


Figure 35. Model predicted stability limits for planar oscillating fan duct exit static pressure, sea-level-static conditions, intermediate power.

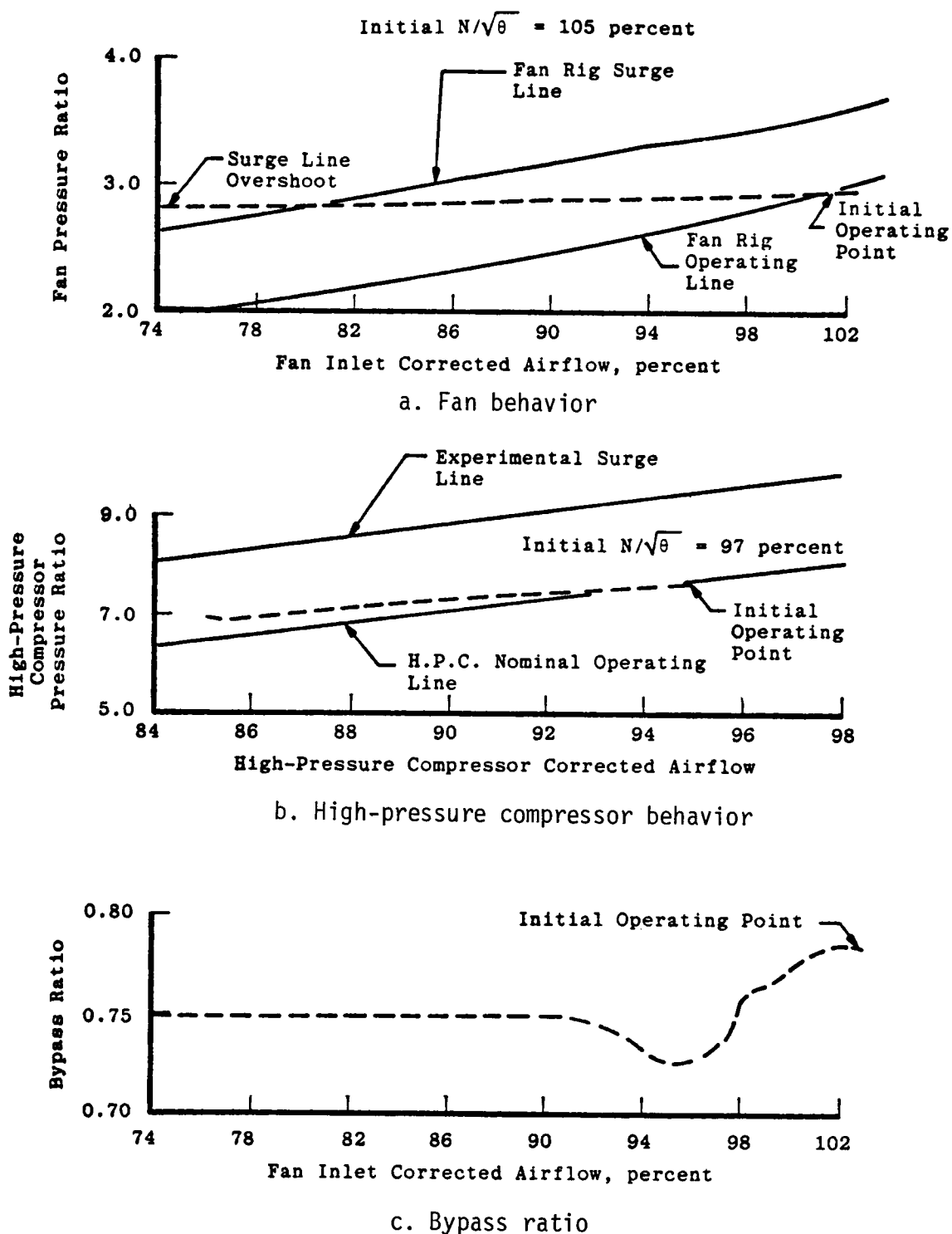


Figure 36. Model predicted effects of rapid inlet temperature ramps on compression system behavior, sea-level-static conditions, intermediate power.

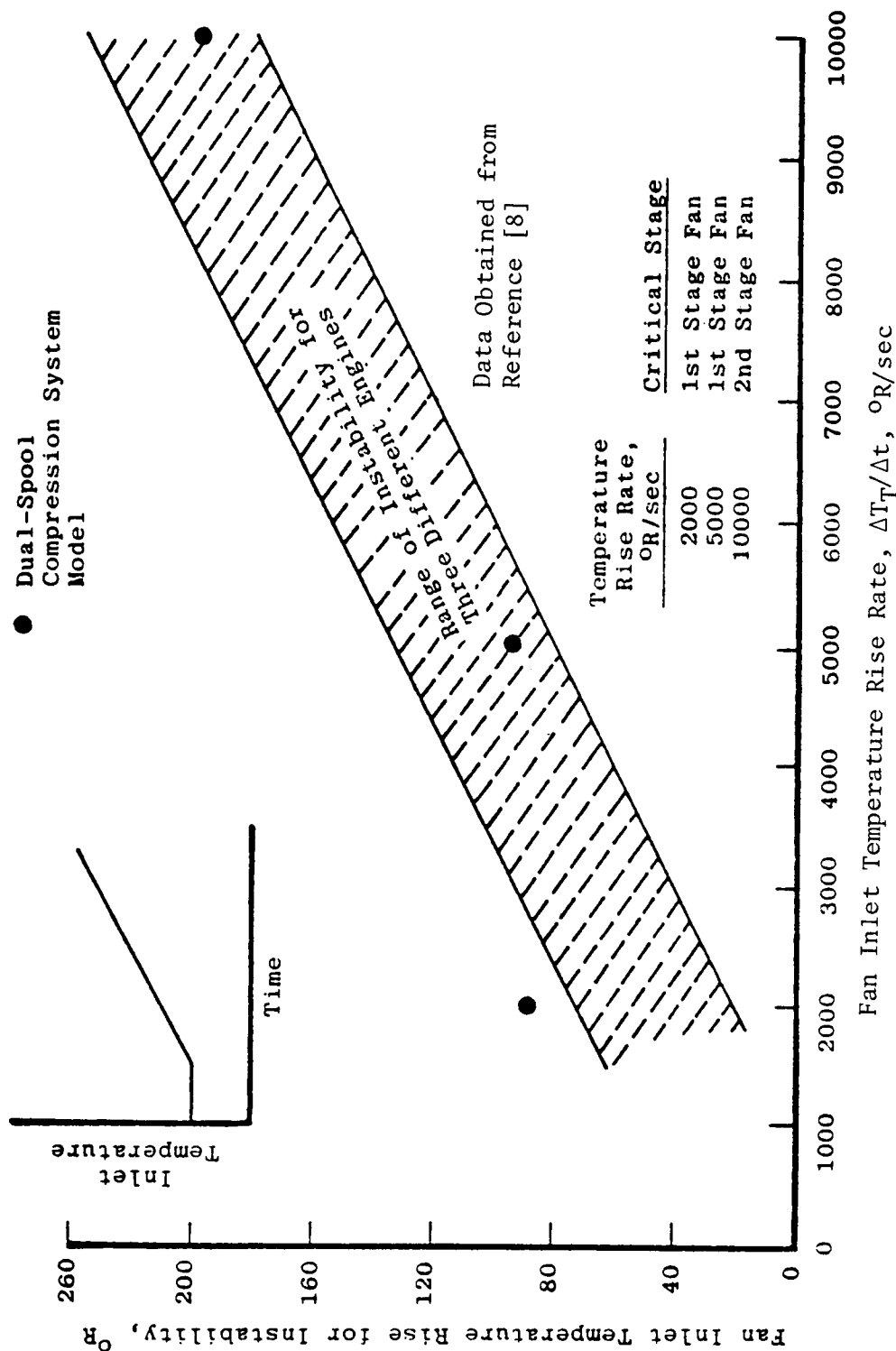


Figure 37. Model predicted stability limits for planar inlet temperature ramps, sea-level-static conditions, intermediate power.

VITA

Milton W. Davis, Jr. was born in Atlanta, Georgia, on June 21, 1948. He attended elementary school and high school in Tullahoma, Tennessee. In the fall of 1966 he entered Tennessee Technological University, receiving a Bachelor of Science degree in Engineering Science in June 1972. During the periods of fall 1967 to fall 1968 and fall 1969 and fall 1970, he participated in the cooperative engineering student program and worked for ARO, Inc., operating contractor for the Arnold Engineering Development Center.

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Mr. Davis is married to the former Virginia Ragan of Tullahoma, Tennessee, and they have two children, Christopher and Patrick.