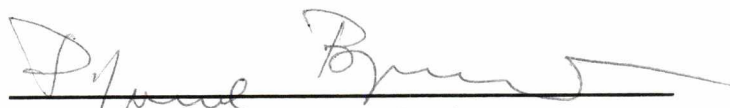
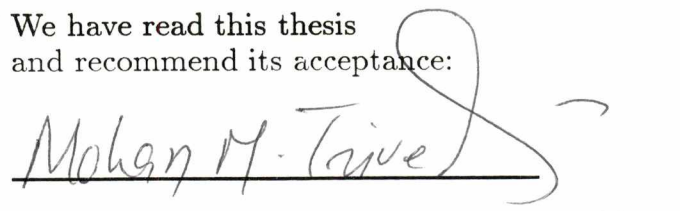


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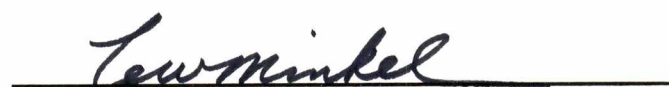
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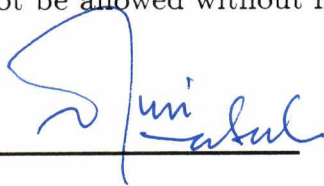

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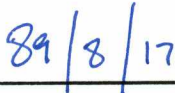
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FUZZY PYRAMID LINKING

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Nabeel W. H. Sufi

December 1989

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Abstract

An iterative image segmentation algorithm utilizing image pyramids with fuzzy links is presented. The concept of a membership function is used to establish links between nodes on adjacent levels of the pyramid. The base of the pyramid is the image to be segmented. Each subsequent level of the pyramid is derived by convolving the preceding level of the pyramid with a Gaussian mask in an overlapped manner and has half the spatial resolution of its predecessor. Nodes at adjacent levels of the pyramid are linked in a bottom-up manner using a membership function which is chosen to yield the desired segmentation result. The pyramid is recomputed based on the previous links and a new set of links is defined. This iterative process culminates when the links reach a steady state. Segmentation is achieved in one top down pass, whereby nodes on a higher resolution level are replaced by nodes from the immediately adjacent lower resolution level above based on the strength of the links. The fuzzy pyramid linking algorithm is a generalization of the classical 'hard' pyramid linking algorithm, and can be tuned to produce the desired segmentation results. While the fuzzy pyramid linking procedure is computationally more demanding, it greatly improves the quality of the segmentation and is guaranteed to converge in a finite number of iterations.

TABLE OF CONTENTS

CHAPTER		PAGE
1. INTRODUCTION		1
2. IMAGE PYRAMIDS		5
2.1 The Gaussian Image Pyramid		6
2.2 Linking		11
2.3 Inter-Level Relationships		12
2.4 Hard Linking		13
2.5 Fuzzy Pyramid Linking		18
3. CONVERGENCE		24
3.1 Hard Pyramid Linking		25
3.1.1 ISODATA Clustering Algorithm		29
3.2 Fuzzy Pyramid Linking		31
4. RESULTS		35
4.1 Segmentation - Hard Pyramid Linking		36
4.2 Segmentation - Fuzzy Pyramid Linking		39
4.2.1 Membership Functions		41
4.2.2 Segmentation Parameters - Alpha, Gamma, and Tau		47
5. CONCLUSIONS		61

CHAPTER	PAGE
BIBLIOGRAPHY	63
VITA	67

LIST OF FIGURES

FIGURE	PAGE
2.1 Gaussian Image Pyramid.	7
2.2 The Assignment of Kernel Coefficients.	10
2.3 The Kernel $\mathbf{W}$ for $a = 0.125$ and $b = 0.375$	10
2.4 Son Nodes Generating a Father Node.	15
2.5 Candidate Father Nodes for a Son Node.	15
2.6 Hard Linking.	16
2.7 Membership Function.	20
2.8 Fuzzy Linking.	20
3.1 1-D Representation of a Two Level Pyramid	26
3.2 Link Assignment - Iteration 1	27
3.3 Link Assignment - Iteration 2	27
3.4 Segmentation by Hard Linking	28
3.5 Fuzzy Links in a Two Level Pyramid	32
4.1 Segmentation of Test Images 1-3 using Hard Linking Algorithm.	38
4.2 Thresholding of Images Segmented using Hard Linking Algorithm.	40
4.3 Linear-like Membership Function.	43
4.4 Sigmoid-like Membership Function.	44
4.5 Step Membership Function.	44
4.6 Effect of Membership Functions on Segmentation of Test Image 4.	48

FIGURE	PAGE
4.7 Effect of Membership Functions on Segmentation of Test Image 5.	49
4.8 Effect of Membership Functions on Segmentation of Test Image 6.	50
4.9 Effect of Parameter α on Segmentation of Test Image 1.	52
4.10 Effect of Parameter α on Segmentation of Test Image 2.	53
4.11 Effect of Parameter α on Segmentation of Test Image 3.	54
4.12 Effect of Parameter γ on Segmentation of Test Image 1.	55
4.13 Effect of Parameter γ on Segmentation of Test Image 2.	56
4.14 Effect of Parameter γ on Segmentation of Test Image 3.	57
4.15 Effect of Parameter τ on Segmentation of Test Image 1.	58
4.16 Effect of Parameter τ on Segmentation of Test Image 2.	59
4.17 Effect of Parameter τ on Segmentation of Test Image 3.	60

CHAPTER 1

INTRODUCTION

The process of segmenting images representing surface quality for identifying anomalies is often complicated by the presence of texture. Ideally, the segmentation is expected to yield two primary regions, defects and background. When the surfaces are characterized by complex textural patterns, such as wood grain in the case of wood, the segmentation often results in the misinterpretation of the textural pattern as defect. The two most common methodologies used to achieve segmentation are edge detection and region growing. Edge detection is used to find the boundary between two adjacent regions. Region growing involves starting with small uniform areas and expanding them outwards until they cannot be expanded any further without violating the uniformity constraint.

Pyramid linking, introduced in 1979 by Burt [6], is a multiresolution approach to region growing. An image pyramid consists of an original sample (image) at the base of the pyramid representing the highest spatial frequency. Each subsequent level is obtained by reduction of the preceding level such that each spatial dimension is halved. Thus, for each level above the base of the pyramid, the spatial frequency bandwidth is halved with respect to the image at the base level. The highest level in the pyramid represents the steady state (DC) component of the spectrum. Burt's reduction scheme is based on spatial convolution using a

four by four kernel, which approximates Gaussian-like low pass filters [7], to minimize the aliasing during the reduction process. The hierarchical nature of the pyramid data structure allows for the connectivity of information between the varying levels of resolution within the image pyramid through a process known as linking. The segmentation of images is based on the bottom-up linking of nodes at adjacent levels of the pyramid based on their similarity and subject to spatial linking constraints. Specifically, a node at a given level of the pyramid is linked to the most similar node at the adjacent higher level, subject to spatial linking constraints, with all other candidates being ignored. Once the links have propagated to the top of the pyramid, all levels of the pyramid, with the exception of the base level, are redefined based on the links. The bottom-up linking process is repeated until a stable set of links has been established. Segmentation is achieved in one top-down pass whereby every son node is replaced by the father node that is identified as the maximal link. This technique, originally introduced by Burt, has been studied by several authors, the most notable being Rosenfeld [18,19,17,9,2,15,16,13], Antonisse [1], Tanimoto [22,23], Meer [14], and Cibulskis [8].

The pyramid linking approach was part of a study of texture based detection of surface defects by Brzakovic et al. [4,5,3] for detection of defects in wood images. The detection of surface defects in wood is an interesting class of problems which has been investigated by several authors including Connors [10]. In this study, initially, Burt's pyramid segmentation approach, which will be referred to as the hard pyramid linking algorithm, was implemented. After evaluating the

segmentation of wood images containing defects it was concluded that the pyramid linking algorithm was too sensitive to the naturally occurring wood grain in the detection of defects in wood samples. Specifically, the segmentation achieved using the hard pyramid linking algorithm identified the wood grain as a separate region, which was not always distinguishable from the defect. This shortcoming of the hard pyramid linking algorithm was attributed to the binary linking scheme used, i.e., a son node is linked to only one of the four candidate father nodes. With this motivation and the awareness of a weighted pyramid linking approach proposed by Baugher [2], a fuzzy pyramid linking algorithm was identified in collaboration with Satori-Angus [20]. The term 'fuzzy' is used since the concept of Membership functions, from fuzzy set theory, is used in this algorithm. The fuzzy pyramid linking algorithm, which is a generalization of the hard pyramid linking algorithm, is guaranteed to converge in a finite number of iterations. The fuzzy pyramid linking algorithm was used for the segmentation of images of wood samples. Based on the results it can be concluded that the fuzzy pyramid algorithm provides a distinct improvement in the quality of segmentation over the hard pyramid linking algorithm.

The thesis describes the iterative fuzzy pyramid linking algorithm for the segmentation of wood images and provides an understanding of the segmentation parameters and their influence on the performance of the algorithm. A discussion of the convergence of the fuzzy pyramid linking algorithm is also provided.

This thesis is presented in four logical units. The first unit, Chapter 2, identifies the concept of image pyramids, their creation, and the linking schemes.

Section 2.1 defines the procedure used to obtain the Gaussian image pyramid. Section 2.2 identifies the concept of linking of nodes at two adjacent levels of the pyramid. Section 2.3 presents the inter-level relationships that define the spatial linking constraints for each node at a given level in the image pyramid. The hard pyramid linking algorithm is defined in Section 2.4. The main contribution of this Chapter is the definition of the fuzzy pyramid linking algorithm which is presented in Section 2.5.

Chapter 3 discusses the convergence of the hard Pyramid linking algorithm from two perspectives. Section 3.1 presents the convergence of the hard pyramid linking algorithm as a process of minimizing the mean square difference between the father nodes and the son nodes. Section 3.2 outlines the proof of convergence based on the Isodata clustering algorithm. The main contribution of this Chapter, Section 3.3, is the proof of convergence for the fuzzy pyramid linking algorithm which is shown to be a generalization of the hard pyramid linking algorithm.

Chapter 4 presents the discussion of the results of segmentation using the hard pyramid linking algorithm in Section 4.1. The main contribution of this Chapter is the quantitative and qualitative analysis of the membership functions used for the fuzzy pyramid linking algorithm presented in Section 4.2. The effect of the segmentation parameters is discussed in Section 4.3.

The last unit, Chapter 5, presents the conclusions of this thesis based on the results presented in Chapter 4.

CHAPTER 2

IMAGE PYRAMIDS

An image pyramid is a data structure in which the input image forms the base of the pyramid. Each subsequent level of the pyramid is formed by an averaging of the image at the level immediately below such that the spatial resolution is halved along each dimension. One of the key advantages of using an image pyramid is that it facilitates the use of results from the analysis at lower resolutions to guide the analysis at higher resolutions, also referred to in literature as "coarse-fine" search [22,6,7].

The simplest scheme for creating an image pyramid is to use 2×2 window averaging in a non-overlapped manner with all elements being equally weighted. The convolution of an image with a 2×2 averaging kernel results in an image with each dimension halved and thus a reduction in the number of pixels by a factor of four. If the original image (array), I_0 , is of dimensions $2^n \times 2^n$ then each subsequent level of the pyramid, $I_1, \dots, I_n$, is of dimensions $2^{n-l} \times 2^{n-l}$ where l represents the level within the image pyramid. The apex of the image pyramid, at the extreme, can be a 1×1 array represented by I_n . However, if desired, a pyramid may be truncated at a level prior to the 1×1 level. More generally, image pyramids are truncated at a level where there are sufficient number of nodes to represent the regions that may exist within an image.

The averaging process used to generate the levels within a pyramid can destroy some essential information, resulting in the smearing of spots and edges, thereby creating a distortion of the features that exist within an image, as shown by Tanimoto in [23]. One scheme that minimizes the distortion is the overlapping Gaussian weighted averaging technique. The image pyramid thus created is known as a Gaussian pyramid and is shown in Figure 2.1. This technique was initially proposed by Burt [6] and has since been investigated by various authors, the most notable being Rosenfeld [21,15,18], Antonisse [1], Tanimoto [22,23], Cibulskis and Dyer [8], Meer [14]. The Gaussian pyramid shown in Figure 2.1 created using a 4×4 window with a fifty percent overlap, as proposed originally by Burt, is introduced in the following section.

2.1 The Gaussian Image Pyramid

The Gaussian pyramid is a sequence of images with the original image at the base of the pyramid followed by reduced resolution images that are obtained by convolving the image at the level below with a 4×4 Gaussian mask. Let $I_l(i, j)$ index a pixel, or node, in an image at level l of the image pyramid and at position (i, j) ; where i is the row index and j is the column index. Thus $I_0(i, j)$, for $i = 0, \dots, N$ and $j = 0, \dots, N$, represents all nodes at the level zero, the base of the image pyramid. Each subsequent level of the image pyramid, $I_1, \dots, I_n$, is obtained from the image at the level below by an overlapping averaging method. Each node of the array I_1 is obtained by a weighted average of I_0 nodes within

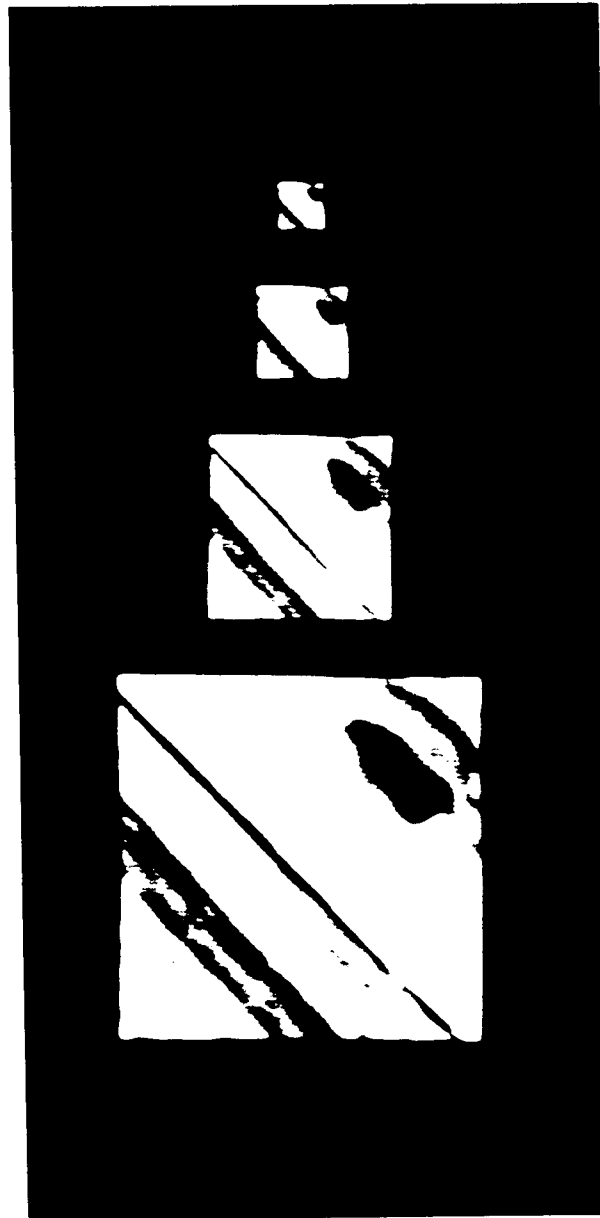


Figure 2.1: Gaussian Image Pyramid.

a 4×4 window given by

$$I_l(i, j) = \sum_{m=0}^{K-1} \sum_{n=0}^{K-1} w(m, n) I_{l-1}(2i + m, 2j + n) \quad \text{for } 0 \leq i, j < (N/2) - 1 \quad (2.1)$$

where $w(m, n)$ represents the discrete function for the Gaussian kernel, also known as the *generating kernel* of the pyramid, while K represents the size of the kernel, and N represents the size of the image $I_l(i, j)$, i.e., $N \times N$. The image I_{l-1} , which is convolved with the generating kernel to produce I_l , is first padded with two rows and two columns to produce the required neighbors for the outlying nodes. The coefficients of the Gaussian generating kernel are chosen based on the following spatial domain constraints:

1. Normalization:

$$\sum_{m=0}^{K-1} \sum_{n=0}^{K-1} w(m, n) = 1 \quad (2.2)$$

2. Symmetry:

$$\begin{aligned} w(m, n) &= w(K - 1 - m, n) = w(m, K - 1 - n) = \\ &w(K - 1 - m, K - 1 - n) \quad \text{for all } m, n \end{aligned} \quad (2.3)$$

3. Unimodality:

$$0 \leq w(m, n) \leq w(p, q) \quad \text{for } m \leq p < K/2 \quad \text{and } n \leq q < K/2 \quad (2.4)$$

4. Equal contribution to the next level:

$$\sum_{i=0}^{K-1} \sum_{j=0}^{K-1} w(m + 2i, n + 2j) = 1/4 \quad \text{for } m, n = 0, 1 \quad (2.5)$$

5. Separability:

$$w(m, n) = \hat{w}(m)\hat{w}(n). \quad (2.6)$$

For a 4×4 , kernel Equation (2.2) implies that the sum of the sixteen coefficients add up to one. The combination of the symmetry constraint of Equation (2.3) and the separability constraint of Equation (2.6), yield a quadratic relationship between the two unknowns a and b for the generating kernel, given by

$$4b^2 + 8ab + 4a^2 = 1.0 \quad (2.7)$$

where b^2 is assigned to the center elements of the kernel and ab is assigned to the peripheral elements, not including the four corner elements, which are assigned the value of a^2 . The assignment is illustrated in Figure 2.2. The unknowns a and b can be chosen based on the solution for Equation (2.7), for which one solution is

$$a + b = 0.5 \quad (2.8)$$

and subject to the unimodality constraint that implies $b > a$. The shape of the generating kernel when a and b are chosen to have values 0.125 and 0.375 respectively, as shown in Figure 2.3, closely approximates the Gaussian probability density function. The image pyramid thus created is known as the Gaussian pyramid and may be viewed as a multiresolution low-pass filter, as shown in [6,4].

It should be noted that the kernel coefficients can be defined by

$$\mathbf{W} = \mathbf{k}^T \mathbf{k} \quad (2.9)$$

where $\mathbf{k} = \{a, b, b, a\}$. The convolution of the kernel $\mathbf{W}$ with an image can

	a	b	b	a
a	a^2	ab	ab	a^2
b	ab	b^2	b^2	ab
b	ab	b^2	b^2	ab
a	a^2	ab	ab	a^2

Figure 2.2: The Assignment of Kernel Coefficients.

0.015625	0.045	0.045	0.015625
0.045	0.140625	0.140625	0.045
0.045	0.140625	0.140625	0.045
0.015625	0.045	0.045	0.015625

Figure 2.3: The Kernel $\mathbf{W}$ for $a = 0.125$ and $b = 0.375$.

be implemented in two successive passes where the first pass is used for row multiplications and the next pass is used for column multiplications. If the pattern of weights is traced from any level l node, it can be shown [6] that there is always an equivalent weighting function W_l which could have been applied directly to the image I_0 to obtain the image I_l .

2.2 Linking

Linking is the process by which the pixels or the nodes belonging to a given level of the pyramid are connected with nodes at adjacent levels. Since each level is created by convolving the 4×4 kernel W with the preceding level of the pyramid, there exists a spatial relationship between the nodes at two adjacent levels. A node at the lower resolution has an array of 4×4 nodes at the immediately adjacent higher resolution level to which it can trace its origin. For the purpose of linking, the node at the lower resolution can be referred to as a *father* node while the 4×4 array of nodes at the immediately adjacent higher resolution level can be referred to as the candidate *son* nodes. A son node is linked to a father node based on a comparison of the property vector $\mathbf{x} = [x_1, x_2, \dots, x_m]^T$ associated with the son node, to that of a father node. Based on the linking scheme, links are established for all son nodes in the pyramid starting with the base of the pyramid. Once the linking arrangement within the pyramid has stabilized, image segmentation can be achieved by mapping the lower resolution image at the apex of the pyramid onto the original image at the base of the pyramid by follow-

ing the linking paths through the intermediate levels. This form of segmentation by hierarchical region growing results in a number of segments totaling no more than the total number of nodes at the apex of the pyramid provided that there are no constraints limiting the propagation of the apex nodes to the base of the pyramid.

Various schemes can be used for linking nodes at adjacent levels of the pyramid, the most obvious scheme being one where a son node is linked to the father node that is most similar based on the property vector $\mathbf{x}$. This scheme was originally proposed by Burt [6] and will be referred to as the hard linking technique. This linking scheme, as well as a fuzzy linking scheme that is similar to one proposed by Baugh [2], will be introduced in this chapter after an elaboration of the relationships that exist within the pyramid.

2.3 Inter-Level Relationships

The manner in which subsequent levels of the pyramid are created from the original image defines certain relationships between the nodes on two adjacent levels in the pyramid. These relationships form the basis for the hierarchical manner of analysis whereby the information from a lower resolution is used to guide the analysis at the higher resolution. Some terminology is introduced here that will be used in the following chapters. Given two adjacent levels l and $l + 1$ from within a pyramid of n levels, where $n = 0$ is the base of the image pyramid, the $2^{n-l} \times 2^{n-l}$ nodes on level l are the son nodes of the $2^{n-(l+1)} \times 2^{n-(l+1)}$ father

nodes on level $l + 1$.

For the Gaussian image pyramid described, each father node at level l , for $l > 0$, has a 4 by 4 array of candidate son nodes on level $l - 1$, as shown in Figure 2.4. The father node is really a weighted average of the sixteen son nodes. Node $I_l(i, j)$, for example, has 16 candidate son nodes denoted by $I_{l-1}(i', j')$, where

$$\begin{aligned} i' &= 2i - 2, 2i - 1, 2i, 2i + 1 \\ j' &= 2j - 2, 2j - 1, 2j, 2j + 1. \end{aligned} \quad (2.10)$$

Conversely, for each son node at level l below the apex of the pyramid, there exists a 2 by 2 array of candidate father nodes as shown in Figure 2.5. For node $I_l(i, j)$ there are four candidate father nodes denoted by $I_{l+1}(i'', j'')$, where

$$\begin{aligned} i'' &= i/2, i/2 + 1 \\ j'' &= j/2, j/2 + 1. \end{aligned} \quad (2.11)$$

Equation 2.11 represents spatial linking constraints for each son node in the image pyramid. It should be noted that for nodes along the image border the missing relatives are defined by extending the image beyond the border. This is done by reflecting the border rows as well as columns of the image outwards.

2.4 Hard Linking

The hard pyramid linking procedure was introduced by Burt in 1979 [7]. Since then several variations of the linking procedure have been proposed by authors

such as Narayanan et al. [15], Cibulskis and Dyer [8], Silberberg et al. [21], Antonisse [1], and others. A generalization of the hard pyramid linking approach was introduced by Baugh [2] and a new generalization is presented in Section 2.6 as the fuzzy pyramid linking approach. Initially, however, Burt's algorithm is introduced as the hard pyramid linking algorithm, mainly to distinguish it from the proposed generalization which utilizes a concept from fuzzy sets and is therefore referred as the fuzzy pyramid linking algorithm.

Once the image pyramid has been created, property vectors denoted by $\mathbf{x}$ of dimension m are computed for each node in the pyramid. Next, every son node, starting from the base of the pyramid, is linked to the candidate father node with the most similar property vector, with ties being resolved randomly. Similarity is deduced by comparing the property vector, $\|\mathbf{x}\|$, representing the son node to that of the father node i.e., $\|\mathbf{x}_{son} - \mathbf{x}_{father}\|$. Thus, every son node is linked to only the one father node that is most similar and is referred to as the *maximal link*. A one dimensional case is depicted in Figure 2.6. The following data structures are defined for the linking and the ensuing iterative process:

- $I_l(i, j)$: intensity value associated with each node within the image pyramid;
- $\mathbf{x}_l(i, j)$: the local image property vector associated with the node $I_l(i, j)$;
- $s_l(i, j)$: sum of the intensity values of the son nodes identified as maximal links to the father node. This data structure is initialized for level $l = 0$ as follows:

$$s_0(i, j) = I_0(i, j);$$

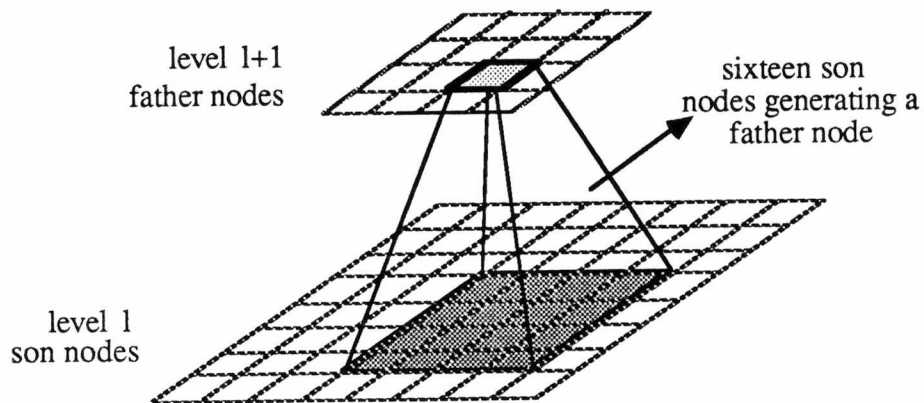


Figure 2.4: Son Nodes Generating a Father Node.

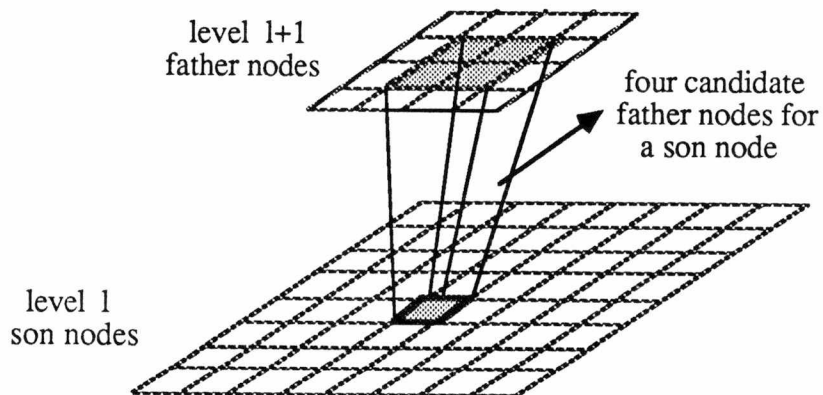


Figure 2.5: Candidate Father Nodes for a Son Node.

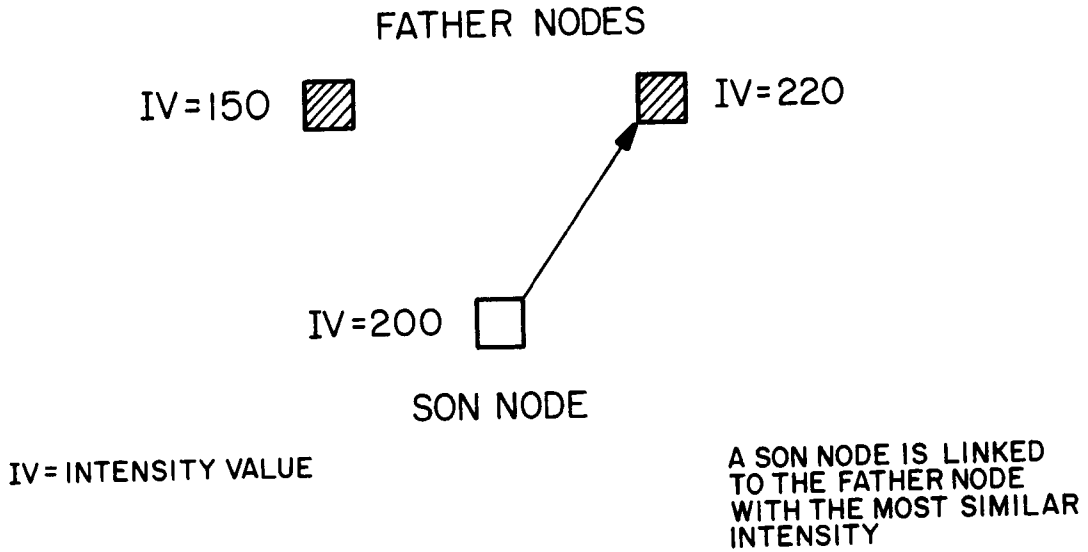


Figure 2.6: Hard Linking.

- $c_l(i, j)$: a counter, keeping count of the number of sons that are identified as maximal links to the father node. This data structure is initialized for level $l = 0$ as follows:

$$c_0(i, j) = 1;$$

- $n_l(i, j)$: a pointer to the father node at level $l + 1$ which is defined as a maximal link for the son node $I_l(i, j)$.

With the property vectors $\mathbf{x}_l(i, j)$ computed for every node within the pyramid the iterations proceed in the following manner:

1. For each level l from 1 to n set

$$c_l(i, j) = \sum_{i'j'} c_{l-1}(i', j')$$

and

$$s_l(i, j) = \sum_{i'j'} s_{l-1}(i', j')$$

where the summation is over all the sons that are identified as maximal links to the son nodes at the level below in the linking process.

2. For each node at level l , for $0 \leq l < n$, the pointer $n_l(i, j)$ points to the father node that represents the maximal link. When there is more than one candidate father node for the maximal link, which does not include the father node established as the maximal link in the previous iteration, the tie is broken randomly. However, if the candidate father nodes that are tied include the father node that was identified as a maximal link in the preceding iteration, then the tie is broken in favor of that candidate father node. This arbitration is added to prevent numerical cycling.
3. Once the links have propagated to the top of the pyramid, every node, except for those at level 0, is recomputed in the following manner:

$$I_l(i, j) = s_l(i, j)/c_l(i, j) \quad \text{for } c_l(i, j) > 0.$$

This procedure is repeated until a stable set of links has been established. A set of links is considered to be stable if every link in the set is maintained in two successive iterations. In the case that links change in two successive iterations, the property vectors $\mathbf{x}_l(i, j)$ are recomputed for every node in the redefined pyramid and the linking procedure is repeated for the new image pyramid.

Once convergence has been achieved, the segmentation occurs in one top down pass in which every son node is replaced by the father node which is defined as its maximal link. The results of segmentation using the hard linking approach are discussed in Chapter 4. It should be noted that in the hard linking procedure, as described above, a son node is forced to link to one of its candidate father nodes without regard to the degree of similarity. There are some alternatives to the forced linking, the simplest being the establishment of a *maximum dissimilarity threshold* such that a son to father link is not established unless the dissimilarity falls below the maximum dissimilarity threshold. These and other variations have been investigated by several authors [18,19,17,21,23,1,9,2] and it is in light of these variations that the proposed fuzzy linking algorithm was devised.

2.5 Fuzzy Pyramid Linking

The notion of inexactness based on the theory of fuzzy sets was introduced by Zadeh in 1965 [26]. Fuzziness is a type of imprecision that stems from the grouping of elements into classes that do not have sharply defined boundaries. Fuzzy sets have been used in mathematical models [25] of empirical phenomena to define the ambiguity that exists in nature. These applications of fuzzy sets are based on the fuzzy algebra [27] associated with fuzzy sets that has been steadily formalizing since the introduction of the fuzzy concepts by Zadeh. Fuzzy clustering techniques have been used for the segmentation of images based on a multi-resolution approach by Trivedi [24]. In the proposed linking procedure,

fuzzy set concepts are used to avoid the binary approach of creating links between a son node and its candidate father nodes, similar to Burt's hard linking algorithm. In the hard linking algorithm a son node is linked to the most similar candidate father node based on the property vectors of the son and the father node, thus establishing only one link referred to as the maximal link. In the fuzzy linking algorithm a son node is linked to all four candidate father nodes. Each link has a membership grade associated with it that defines the strength of the link. The strength of the link is based on the Euclidean distance between the property vector of the son node and that of the candidate father node. In fuzzy set terminology the strength represents the grade of membership in the set. The membership grade (strength or weight) is assigned based on a membership function. The choice of the membership function is subjective and may be unique for the particular problem at hand. The membership function used for this case is a sigmoid-like function, shown in Figure 2.7 that yields the best results in this study, as discussed in Chapter 4. The linking procedure is depicted in Figure 2.8.

The membership function **LS** (link strength) is defined as

$$LS(u; \alpha, \beta, \gamma) = S(u; \alpha, \beta, \gamma) \quad (2.12)$$

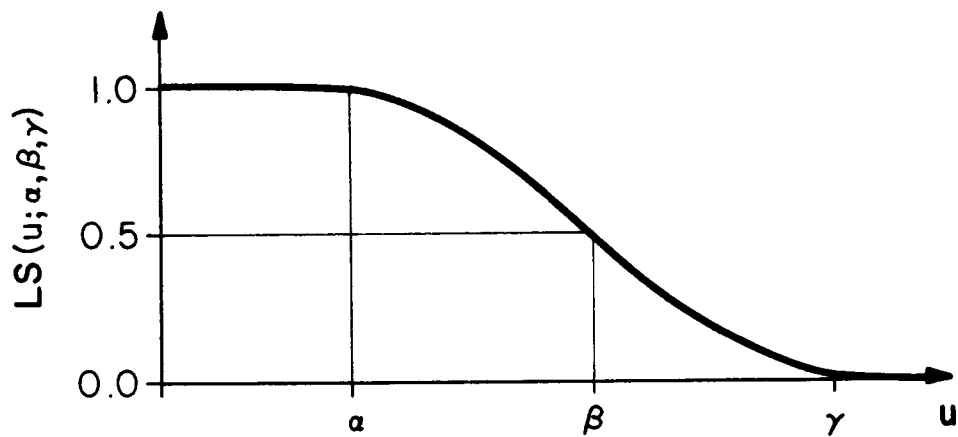


Figure 2.7: Membership Function.

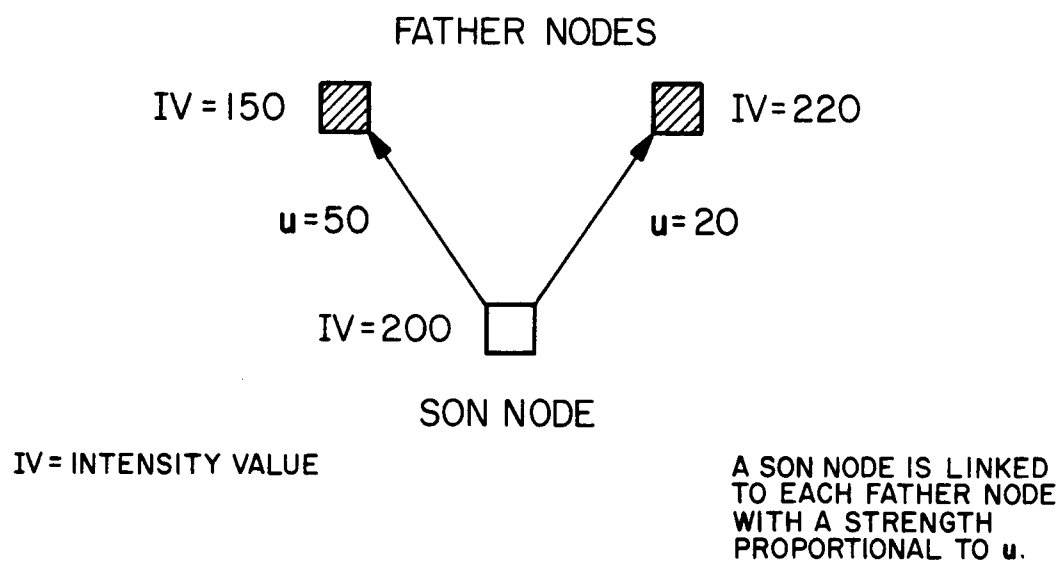


Figure 2.8: Fuzzy Linking.

where

$$S(u; \alpha, \beta, \gamma) = \begin{cases} 1 & \text{for } u \leq \alpha \\ 1 - 2 \left(\frac{u-\alpha}{\gamma-\alpha} \right)^2 & \text{for } \alpha \leq u \leq \beta \\ 2 \left(\frac{u-\gamma}{\gamma-\alpha} \right)^2 & \text{for } \beta \leq u \leq \gamma \\ 0 & \text{for } u \geq \gamma \end{cases} \quad (2.13)$$

and $u = \| \mathbf{x}_{son} - \mathbf{x}_{father} \|$ is norm of the difference in the property vectors between a son node and a candidate father node; α represents the property vector difference below which the link strength assigned is 1; γ is the property vector difference above which the link strength assigned is 0; $\beta = \frac{\alpha+\gamma}{2}$ is the crossover point at which $LS(u; \alpha, \beta, \gamma) = 0.5$. The parameters α and β are chosen by the user and have a very profound effect on the quality of segmentation. This effect is discussed in detail in the results which are presented in Chapter 4.

As in the hard pyramid linking algorithm, the property vectors associated with each node in the pyramid are computed first; the linking process then begins with the nodes at the base of the pyramid and ends with the nodes at the level immediately below the apex. The following data structures are defined for the linking and the ensuing iterative process:

- $I_l(i, j)$: intensity value associated with each node within the image pyramid;
- $\mathbf{x}_l(i, j)$: the local image property vector associated with the node $I_l(i, j)$ which is computed over the son nodes that are linked to the father node and is weighted with respect to link strength values associated with the son node;

- $s_l(i, j)$: sum of the intensity values of the son nodes linked to the father node, weighted with respect to link strength values associated with the son.

This data structure is initialized for level $l = 0$ as follows:

$$s_0(i, j) = I_0(i, j);$$

- $n_l(i, j)$: a pointer to the node's father, in the level above, that has the largest link strength value; and
- $w_l(i, j)$: the sum of strength values of the son nodes linked to a father node.

This data structure is initialized for level $l = 0$ as follows:

$$w_0(i, j) = 1.$$

The iterations proceed in the following manner:

1. For each level l from 1 to N set

$$w_l(i, j) = \sum_{i', j'} w_{l-1}(i', j') LS(u; \alpha, \beta, \gamma)$$

and

$$s_l(i, j) = \sum_{i', j'} s_{l-1}(i', j') LS(u; \alpha, \beta, \gamma)$$

where $LS(u; \alpha, \beta, \gamma)$ is the strength of the link between node (i, j) at level l and it's son (i', j') at level $l - 1$, and the summation is performed over all sons of the node.

2. For each node at level l , for $0 \leq l < N$, the pointer $n_l(i, j)$ points to the father node at level $l + 1$ that has the largest link strength value among

the four candidate father nodes. If two or more fathers have the same link strength value then a link is chosen randomly. A link is considered to have been reassigned only if the father node associated with the largest strength value changes from one iteration to another.

3. Once the links have propagated to the top of the pyramid, every node, except those at level 0, is recomputed in the following manner:

$$I_l(i, j) = s_l(i, j) / w_l(i, j) \quad \text{for } w_l(i, j) > 0.$$

This procedure is repeated until a stable set of links have been achieved. Segmentation can be achieved by replacing son nodes with the father node that is identified as the strongest link. It should be noted that in this scheme a criteria of maximum dissimilarity can be used in deciding whether or not to replace a son node with a candidate father node. The results of the segmentation achieved using this scheme are discussed in Chapter 4.

CHAPTER 3

CONVERGENCE

The stability of an algorithm based on an iterative process, such as the pyramid linking procedure, is an important consideration in evaluating the performance of the algorithm. The convergence of the Hard Pyramid Linking algorithm has been studied by two groups of authors, Cibulskis and Dyer [8], and Kasif and Rosenfeld [13], with both studies concluding that the linking procedure utilized in the hard pyramid Linking algorithm is guaranteed to yield a stable set of links in a finite number of iterations. Cibulskis and Dyer maintain that the iterative hard pyramid linking algorithm, whereby father nodes are replaced by the average of the son nodes linked to that father node, after each iteration of linking, is equivalent to minimizing the sum of the square distances between father nodes and son nodes. In the following section it will be shown that the average or the mean value does minimize the sum of square difference function. Kasif and Rosenfeld draw an analogy between the pyramid linking process and clustering. The proof is based on the fact that the hard linking procedure is a special case of the ISODATA¹ algorithm. The ISODATA Clustering algorithm can be found in any of several texts on pattern recognition, such as the one by Tou and Gonzalez

¹The term "ISODATA" is derived from *Iterative Self-Organizing Data Analysis Techniques A*, with the A being added to make the word pronounceable.

[12]. These proofs are presented in the following sections followed by a proof for the fuzzy pyramid linking algorithm.

3.1 Hard Pyramid Linking

In order to simplify the proof of convergence of the Hard Pyramid Linking algorithm a two-level pyramid is defined with each level having nodes defined along one spatial dimension. Let nodes $S_1, S_2, \dots, S_8$ represent the son nodes at level 0, the base of the 1-D pyramid. Let nodes $F_1, \dots, F_4$ represent the father nodes which are derived from the son nodes using overlapped Gaussian averaging as described in Chapter 2. In the hard linking case, a son node is linked to the candidate father node that is most similar with respect to a property value, which in the simplest case, can be the intensity value. Figure 3.1 depicts the father and son nodes at initialization in a one dimensional representation of a two level pyramid, with the encircled values representing the intensity values of the nodes. Figures 3.2 and 3.3 depict the iterative linking procedure during which the father nodes progress toward a stable state. At the end of the first iteration each father node is reassigned an intensity which is an average of the intensities of the son nodes linked to that particular father node, as shown in Figure 3.3. At the conclusion of the second iteration it is noted that the son nodes are linked to the identical father nodes that they were linked to in iteration 1. The links have not changed and the linking process can be considered to have converged to a stable set of links. Figure 3.4 shows the resulting segmentation achieved by replacing

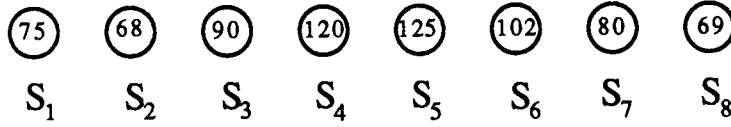


Figure 3.1: 1-D Representation of a Two Level Pyramid

the intensities of the son nodes by the intensities of the father nodes that they are linked to. This test case is a simplification of the hard pyramid linking algorithm that will be used to demonstrate the convergence of the algorithm.

The hard pyramid linking algorithm is thus a procedure whereby the sum of the mean square differences between the father nodes and the son nodes is minimized iteratively, subject to spatial linking constraints. The mean square difference function is defined as

$$V_c = \sum_{i=1}^N \sum_{j=1}^{M_i} (F_i - S_j)^2 \quad (3.1)$$

where S_{M_i} are son nodes linked to the father node represented by F_i , N is an integer value representing the number of father nodes, M_i is an integer value representing the number of son nodes linked to the father node F_i , and c is the

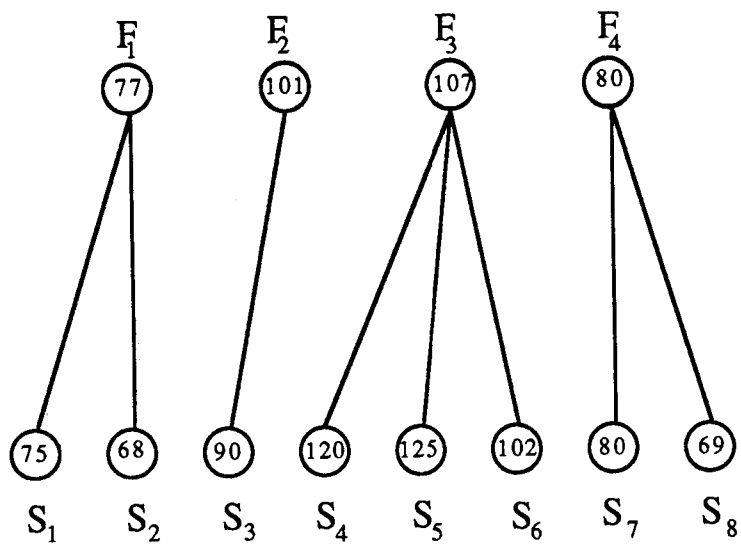


Figure 3.2: Link Assignment - Iteration 1

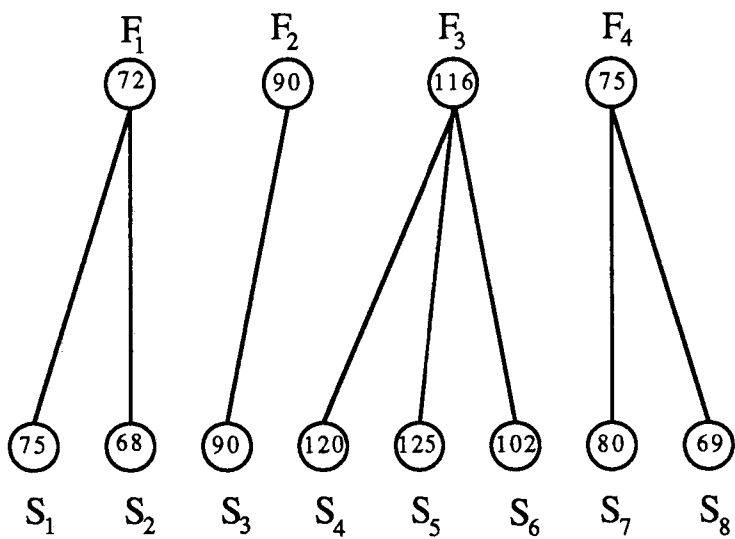


Figure 3.3: Link Assignment - Iteration 2

Original	75	68	90	120	125	102	80	69
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Segmented	72	72	90	116	116	116	75	75

Figure 3.4: Segmentation by Hard Linking

iteration count. Equation (3.1) can be expanded for the test case represented in Figures 3.2, 3.3, and 3.4 as

$$V_c = \sum_{j=1}^{M_1} (F_1 - S_j)^2 + \sum_{j=1}^{M_2} (F_2 - S_j)^2 + \sum_{j=1}^{M_3} (F_3 - S_j)^2 + \sum_{j=1}^{M_4} (F_4 - S_j)^2 \quad (3.2)$$

$$V_c = \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 + \mathcal{F}_4 \quad (3.3)$$

where

$$\mathcal{F}_i = \sum_{j=1}^{M_i} (F_i - S_j)^2 \quad (3.4)$$

From Equations (3.2), (3.3), and (3.4) it can be seen that the function V_c , which is minimized in the iterative hard linking algorithm, can be expanded in to a summation of the mean square difference functions for each father node in the pyramid. During the iterative linking process a son node S_a linked to a father

node F_a may cease to do so and, link to a father node S_b if and only if

$$|F_a - S_b| > |F_a - S_a| \quad (3.5)$$

provided that the spatial linking constraints are not violated. Minimization of the function V_c is thus dependent on the minimization of each $\mathcal{F}_i$ identified in Equation (3.3). The minimization of the function $\mathcal{F}_i$ can be evaluated by taking the partial derivative of $\mathcal{F}_i$ with respect to F_i and equating the resulting expression to zero

$$\frac{\partial \mathcal{F}_i}{\partial F_i} = \sum_{j=1}^{M_i} -2(F_i - S_j) = 0 \quad (3.6)$$

$$\sum_{j=1}^{M_i} F_i = \sum_{j=1}^{M_i} S_j \quad (3.7)$$

$$F_i = \frac{1}{M_i} \sum_{j=1}^{M_i} S_j. \quad (3.8)$$

The minimization of $\mathcal{F}_i$ requires that the father nodes be updated by the average of the son nodes linked to the father node. This is exactly the procedure used to update the father nodes in the hard pyramid linking algorithm. As a result, the hard pyramid linking algorithm is guaranteed to converge in a finite number of iterations.

3.1.1 ISODATA Clustering Algorithm

Kasif and Rosenfeld [13] have shown that the iterative pyramid linking procedure is a simplified version of the ISODATA clustering algorithm, where the son nodes represent sample points and the father nodes represent cluster centers. Let $S = \{z_1, z_2, \dots, z_n\}$ be a set of real numbers that represents the son nodes on a

given level, which could be a property vector or, in the simplest case, a scalar representing intensity. Let $S_1^{(0)}, S_2^{(0)}, \dots, S_m^{(0)}$ be an arbitrary initial partition of S and let $\mu_i^{(0)}$ be the mean of the z 's in $S_i^{(0)}$ for $1 \leq i \leq m$. It should be noted that in the ISODATA clustering algorithm the mean $\mu_i^{(0)}$ of each set $S_i^{(0)}$ represents the cluster center. The linking of son nodes to a father node can thus be viewed as a clustering process. With each iteration the partitions are modified by the shifting of the nodes represented by z from one subset to another. The shifting is subject to the constraint that z can shift from $S_i^{(0)}$ to $S_j^{(0)}$ only if z is closer to $\mu_j^{(0)}$ than to $\mu_i^{(0)}$. This is exactly the constraint used in the ISODATA algorithm to assign sample points to the cluster centers. The shifting of membership results in a new partition set $S_1^{(1)}, S_2^{(1)}, \dots, S_m^{(1)}$ with new means $\mu_1^{(1)}, \mu_2^{(1)}, \dots, \mu_m^{(1)}$ where some of the subsets may be empty and have undefined means. The iterative linking procedure results in redefinition of the son nodes which corresponds to the redefinition of cluster centers in the ISODATA algorithm. In the pyramid linking procedure a pre-defined set of relationships exist and a son node can be assigned to only one of the four candidate father nodes and is thus a special case of the Isodata algorithm.

A cost function

$$F \equiv \sum_{i=1}^m \sum_{z \in S_i} (z - \mu_i)^2 \quad (3.9)$$

is defined to prove the convergence of the iterative pyramid linking algorithm.

At a given iteration k let T_{ij} represent the set of z 's that shift from S_i to S_j .

Then,

$$\sum_{z \in S_1^{k-1}} (z - \mu_i^{(k-1)})^2 \geq \sum_{z \in S_1^k} (z - \mu_i^{(k)})^2 \quad (3.10)$$

Therefore,

$$F^{k-1} \geq F^k \quad (3.11)$$

In general $F^{(0)}, F^{(1)}$ is strictly decreasing. Since there are a finite number of partitions of z 's the sequence will terminate in a finite number of iterations at which point,

$$F^{(m)} = F^{(m-1)}. \quad (3.12)$$

3.2 Fuzzy Pyramid Linking

The convergence of the Fuzzy Pyramid Linking procedure can be proved by considering the linking process in detail. A pyramid consisting of two levels with the fuzzy linking assignment is shown in Figure 3.5. For simplification each level is represented by nodes along one dimension only. The nodes $S_1, S_2, \dots, S_8$ represent the son nodes at the base of the pyramid. The nodes $F_1, \dots, F_4$ are the father nodes at level one to which the son nodes are to be linked. In the Fuzzy linking procedure a son node is linked to each of the candidate father nodes with the strength of the link being defined by a membership function. The father node to which a son node is linked with the largest weighting i.e., the maximal link, is noted. The links and the weight assignment are shown in Figure 3.5, where the weight w_{11} defines the strength of the link between the son node S_1 and the father node F_1 and the w_{21} defines the strength of the link between the son node

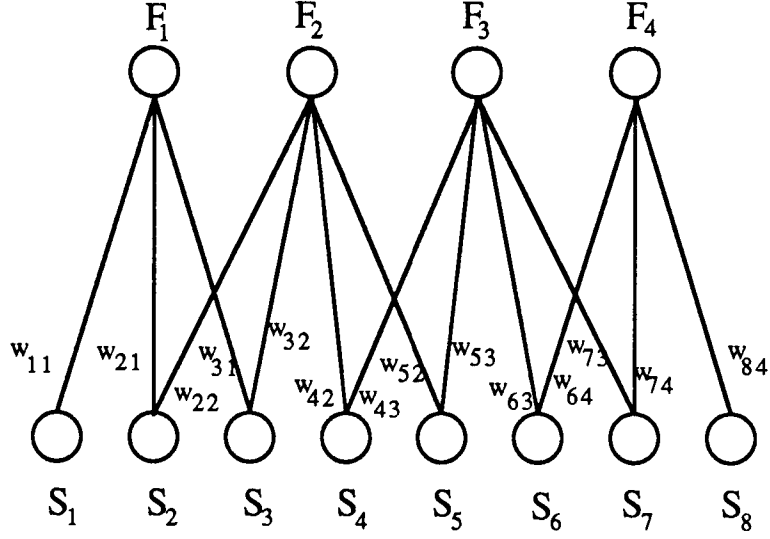


Figure 3.5: Fuzzy Links in a Two Level Pyramid

S_2 and the father node F'_1 . Once the links have been defined between the two levels, the father nodes are recomputed yielding F'_1, F'_2, F'_3, F'_4 , which are defined as

$$F'_1 = \frac{w_{11}}{W_1} S_1 + \frac{w_{21}}{W_1} S_2 + \frac{w_{31}}{W_1} S_3 \quad (3.13)$$

$$F'_2 = \frac{w_{22}}{W_2} S_2 + \frac{w_{32}}{W_2} S_3 + \frac{w_{42}}{W_2} S_4 + \frac{w_{52}}{W_2} S_5 \quad (3.14)$$

$$F'_3 = \frac{w_{43}}{W_3} S_4 + \frac{w_{53}}{W_3} S_5 + \frac{w_{63}}{W_3} S_6 + \frac{w_{73}}{W_3} S_7 \quad (3.15)$$

$$F'_4 = \frac{w_{64}}{W_4} S_6 + \frac{w_{74}}{W_4} S_7 + \frac{w_{84}}{W_4} S_8 \quad (3.16)$$

where,

$$W_1 = w_{11} + w_{21} + w_{31} \quad (3.17)$$

$$W_2 = w_{22} + w_{32} + w_{42} + w_{52} \quad (3.18)$$

$$W_3 = w_{43} + w_{53} + w_{63} + w_{73} \quad (3.19)$$

$$W_1 = w_{64} + w_{74} + w_{84} \quad (3.20)$$

Once the father nodes F_1, F_2, F_3, F_4 , have been recomputed the links are reassigned. A link is considered to have changed if a maximal link changes. Thus if F_1 is initially defined as the maximal link for S_1 , but, after recomputing the father nodes F'_2 is defined as the maximal link for S_1 , then a change is considered to have occurred in the link assignment and the father nodes are recomputed at least one more time. However, if all previous maximal links are maintained when the links are reassigned, then the linking algorithm is considered to have converged. A significant point in evaluating the convergence of the fuzzy pyramid linking algorithm is that while a son node is linked to all candidate father nodes the basis of change is only a change in the maximal link, which is exactly the case for the hard pyramid linking algorithm.

The basis of convergence of the fuzzy pyramid linking algorithm is therefore the minimization of the mean square difference function between the father nodes and the son nodes, as is the case for the hard pyramid linking algorithm. The mean square difference function between the father nodes and the son nodes, defined by Equation (3.1) for the hard pyramid linking case, also applies for the fuzzy case. The mean square difference was shown to be minimized by the expression given by Equation (3.8), which is of the form

$$F_i = \lambda_1 S_1 + \lambda_2 S_2 + \cdots + \lambda_{M_i} S_{M_i} \quad (3.21)$$

where λ represents the contribution of all son nodes linked to the father node F_i

such that

$$\lambda_1 = \lambda_2 = \cdots = \lambda_{M_i} = \frac{1}{M_i} \quad (3.22)$$

$$\lambda_1 + \lambda_2 + \cdots + \lambda_{M_i} = 1. \quad (3.23)$$

Equations (3.13), (3.14), (3.15), and (3.16) represent the formulation used to update the father nodes in the Fuzzy Pyramid Linking algorithm. The expression equivalent to Equation (3.21) for the fuzzy pyramid Linking algorithm which is used for updating the father nodes at the beginning of each iteration can be written as

$$F'_i = \xi_1 S_1 + \xi_2 S_2 + \cdots + \xi_{M_i} S_{M_i} \quad (3.24)$$

for which the normalization conditions equivalent to Equation (3.23)

$$\xi_1 + \xi_2 + \cdots + \xi_{M_i} = 1 \quad (3.25)$$

hold true. However, The equal weighting condition represented by Equation (3.24) cannot be guaranteed. The fuzzy pyramid Linking algorithm, thus, differs from the hard pyramid linking algorithm in the manner in which it updates the father nodes. Instead of updating a father node by an equally weighted average of the son nodes for which the father node is identified as a maximal link, in the fuzzy pyramid linking algorithm the father nodes are replaced by a weighted average of all the son nodes that are linked to it. Hence, the fuzzy pyramid linking algorithm is a generalization of the hard pyramid linking algorithm and is guaranteed to converge in a finite number of iterations.

CHAPTER 4

RESULTS

In this section the quality of segmentation achieved using the fuzzy pyramid linking algorithm is compared to the segmentation achieved using the hard pyramid linking algorithm. While the quality of segmentation of the two algorithms is discussed qualitatively and quantitatively, other issues such as the selection of a membership function and the affect of the segmentation parameters parameters on the quality of segmentation will also be discussed. It should be noted that in the hard pyramid linking algorithm a son node is linked only to the most similar father node whereas in the fuzzy pyramid linking algorithm a son node is linked to all four candidate father nodes. The fuzzy pyramid linking algorithm thus requires more memory, as well as numerical operations, than the hard pyramid linking algorithm.

The hard pyramid linking algorithm and the fuzzy pyramid linking algorithm were implemented in FORTRAN on a Vax 11/780. The images presented in the following sections are 128×128 in size, and were reproduced using a half-toning technique known as dithering.

4.1 Segmentation - Hard Pyramid Linking

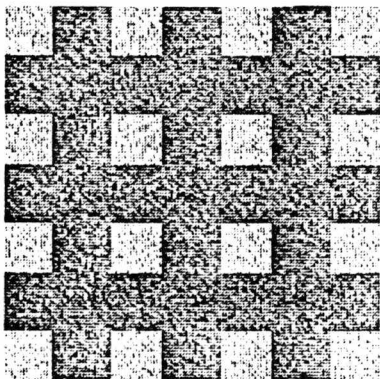
The input to the hard pyramid linking algorithm procedure is a Gaussian image pyramid with the original image at the base of the pyramid. Subsequent to the initialization, two parameters are needed as input before the algorithm can produce a segmented image. One of these parameters is the maximum number of iterations allowed. While the hard pyramid linking algorithm is guaranteed to converge in a finite number of iterations it may not always be practical to let the algorithm reach convergence. Experimental results have shown that, unless the input image contains a high degree of textural micropatterns or noise, the algorithm converges within ten iterations. The second parameter needed as input is the level of the pyramid chosen as the root level for the segmentation procedure. This level determines the maximum number of segments that may exist in the segmented image. For example, if the segmentation begins from a 2×2 array, a maximum of four segments may exist in the final segmented image. Similarly, if the segmentation begins from a 4×4 array, a maximum of sixteen segments may exist. Both parameters may be preselected based on apriori knowledge about the number of distinct segments that exist in the image and the complexity of the scene. For the results shown in Figure 4.1, the maximum number of iterations allowed was ten and a 2×2 array was used as the root level for the segmentation procedure. Three test images used to demonstrate the quality of segmentation achieved by the hard pyramid linking algorithm are depicted in Figure 4.1. Test Image 1 is a checkerboard like image which was degraded by the addition of

Gaussian noise. Test Image 2 is an image of two pieces of wood placed diagonally against each other with the top piece depicting a defect known as a knot and the lower piece exhibiting a defect known as mineral streaks. The boundary between the two pieces is also prominent and is of interest in the segmentation. Test Image 3 is an image of a piece of metal exhibiting two vertical scorch marks. The metal image was taken with a single light source placed to the right of the sample with camera placed away from the specular angle to avoid hot spots.

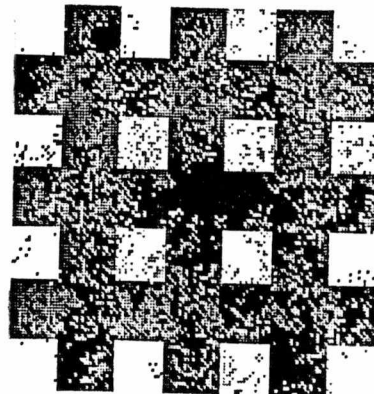
For the checkerboard-like image (Test Image 1), Figure 4.1, it can be seen that the hard pyramid linking algorithm defines three distinct regions in the segmented image with two regions stemming from the black and white checkerboard. The third region, which is of an intermediate intensity between the black and white squares represents some of the noise added to the checkerboard-like image. It should also be noted that in the segmented image the boundary between the black and white squares is sharp while some residue of the noise, that was added, can be seen. The segmentation of the wood image (Test Image 2) yields the three prominent features which are the knot, the split between the wood pieces, and the mineral streaks. The segmentation also yields several smaller regions associated with the wood grain patterns that occur naturally in wood. The overall segmentation, in particular the split between the two pieces of wood, is corrupted by the smaller regions associated with the wood grain.

The segmentation of the metal image (Test Image 3) yields four distinct regions that partially distinguish the scorch marks from the normal background of the metal surface. The four segments more strongly identify the lighting

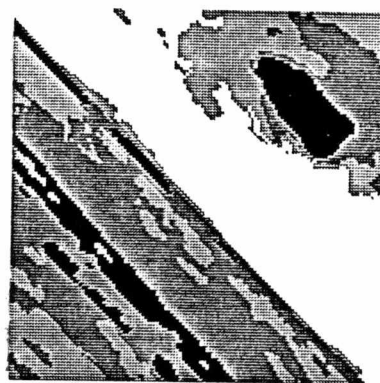
Test Image 1



Output Images



Test Image 2



Test Image 3

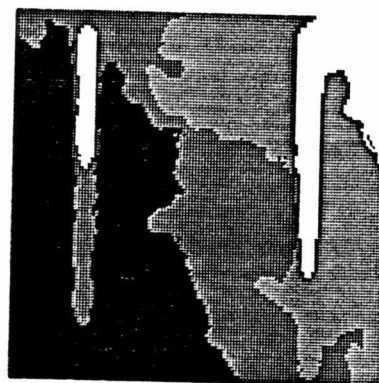
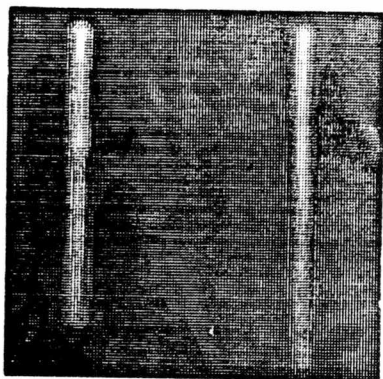


Figure 4.1: Segmentation of Test Images 1-3 using Hard Linking Algorithm.

gradient across the metal surface with the scorch marks identified as two or more segments.

The results indicate that for a given image the hard pyramid linking algorithm defines distinct segments with crisp boundaries for the more prominent features in the image. Smaller features, such as noise or microtextural patterns, also tend to appear as isolated segments, which can interfere with the segments associated with the more prominent features in an image. This can be understood in terms of the binary linking nature of the hard pyramid linking algorithm whereby a son node is linked to the most similar father node based on a property such as intensity. In doing so, the smaller segments, such as noise, are preserved since they link to the father node that best represents them at the level above. In some cases the smaller segments can be removed by a simple thresholding that makes the smaller segments part of a larger segment, however, this is not always the case. Figure 4.2 shows that while a simple thresholding is sufficient to suppress the noise in the segmented checkerboard-like image, it is not sufficient for isolating the prominent features in the wood and metal images. It is in this light that the fuzzy pyramid linking algorithm was conceived, the results for which are presented in the following sections.

4.2 Segmentation - Fuzzy Pyramid Linking

The input to fuzzy pyramid linking procedure consists of the the Gaussian image pyramid, the maximum number of iterations allowed, and the level from

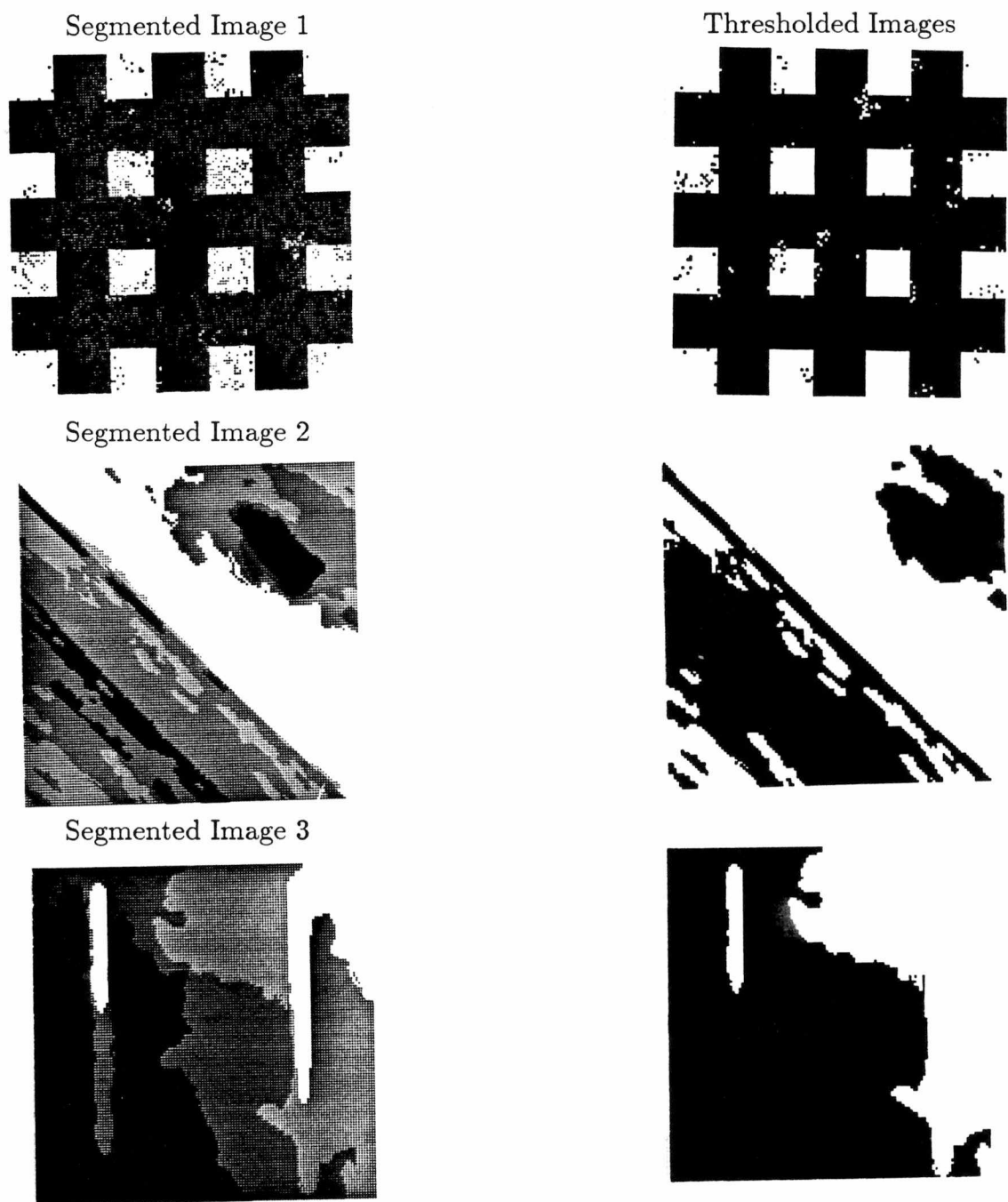


Figure 4.2: Thresholding of Images Segmented using Hard Linking Algorithm.

which the segmentation is to begin. These parameters are identical to those required for the hard pyramid linking algorithm. Three additional parameters are required which are α , γ , and, τ . The parameters α and γ are the critical points that define the membership function and are described in detail in Chapter 2. The third parameter, τ represents the minimum threshold link strength and is used to decide whether a son node can be replaced by a father node in the segmentation procedure. In the fuzzy pyramid linking algorithm the root level alone does not determine the maximum number of segments that may exist in the segmented image. The number of segments in the final segmented image also depends on the parameter τ since the links with a strength value below the minimum threshold strength do not propagate down to the base of the pyramid.

The performance of the Fuzzy Segmentation algorithm depends very heavily upon the membership function which is defined by the parameters α and γ . In the following sections the merits of various membership functions are discussed followed by an investigation of the effect each of the three parameters α , γ and τ have on the segmentation.

4.2.1 Membership Functions

In fuzzy set terminology a membership function defines the grade of membership in a fuzzy set. In the fuzzy pyramid linking algorithm the membership function defines the weight assigned to the links between son nodes and father nodes. The weight assigned to a link not only represents the strength of the son node to father node relationship, it also distinguishes one son node to a candidate

father node link from a link to a different candidate father node. The weight assigned can be based on similarity or dissimilarity between a given son node and a candidate father node based on a particular property which, in the simplest case, can be the intensity. Given a son node with four candidate father nodes, the membership function defines the weight associated with the link between the son node and each father node. Three monotonically decreasing functions were investigated as membership functions for the fuzzy pyramid linking algorithm which include a linear-like function, a sigmoid-like function, and a step function. These membership functions are defined as a function of the absolute difference in intensity between a son node and a candidate father node denoted by u . The linear-like membership function is defined in terms of the critical points α and γ and is shown in Figure 4.3. The Linear-like Link Strength function is given by

$$LILS(u; \alpha, \beta, \gamma) = \begin{cases} 1 & \text{for } u \leq \alpha \\ \frac{1}{\gamma - \alpha} (\gamma - u) & \text{for } \alpha \leq u \leq \gamma \\ 0 & \text{for } u \geq \gamma \end{cases} \quad (4.1)$$

The Sigmoid-like membership function is a quadratic function that is piecewise continuous as shown in Figure 4.4. It is also defined as a function of the absolute difference between a son node and a candidate father node, u , with the critical points α and γ

$$SILS(u; \alpha, \beta, \gamma) = \begin{cases} 1 & \text{for } u \leq \alpha \\ 1 - 2 \left(\frac{u - \alpha}{\gamma - \alpha} \right)^2 & \text{for } \alpha \leq u \leq \beta \\ 2 \left(\frac{u - \gamma}{\gamma - \alpha} \right)^2 & \text{for } \beta \leq u \leq \gamma \\ 0 & \text{for } u \geq \gamma \end{cases} \quad (4.2)$$

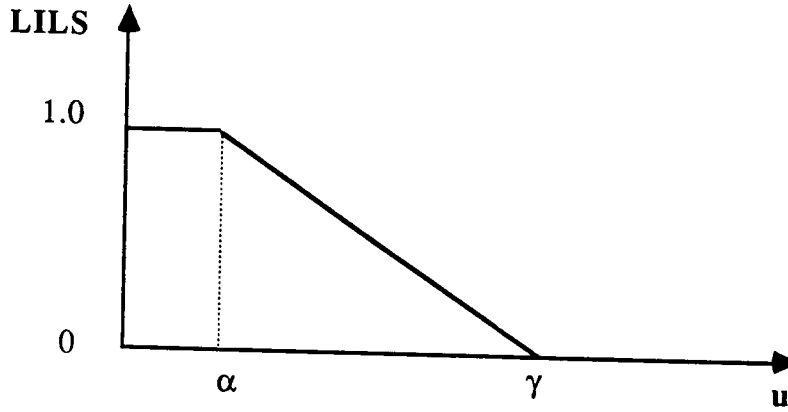


Figure 4.3: Linear-like Membership Function.

where $\beta = \frac{\alpha+\gamma}{2}$ is the crossover point. Like its counterparts, the Step membership function, shown in Figure 4.5 is a function of u with only one critical point α and is defined as

$$STLS(u; \alpha) = \begin{cases} 1 & \text{for } u \leq \alpha \\ 0 & \text{for } u > \alpha \end{cases} \quad (4.3)$$

Figures 4.6, 4.7, and 4.8 show the effect of the membership function on the quality of segmentation of Test Images 4, 5, and 6. Test Image 4 is an image of three strips of wood placed side by side, with each strip exhibiting a defect known as a worm hole. Test Image 5 is also an image of three strips of wood placed side by side with two pieces depicting worm holes while the third piece exhibiting some wood rot. Test Image 6 is an artificial image which was created

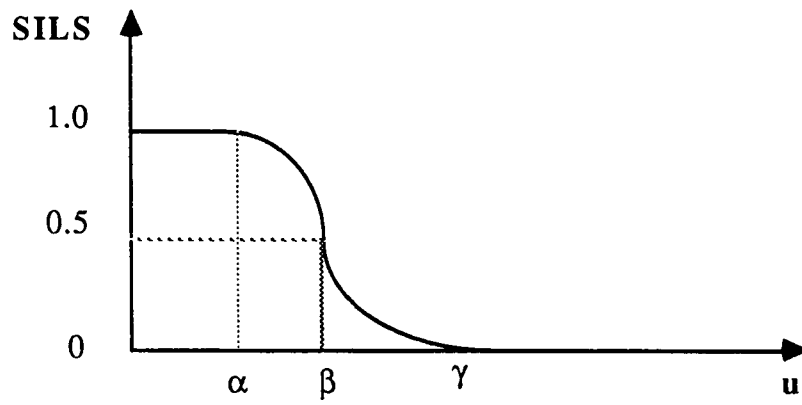


Figure 4.4: Sigmoid-like Membership Function.

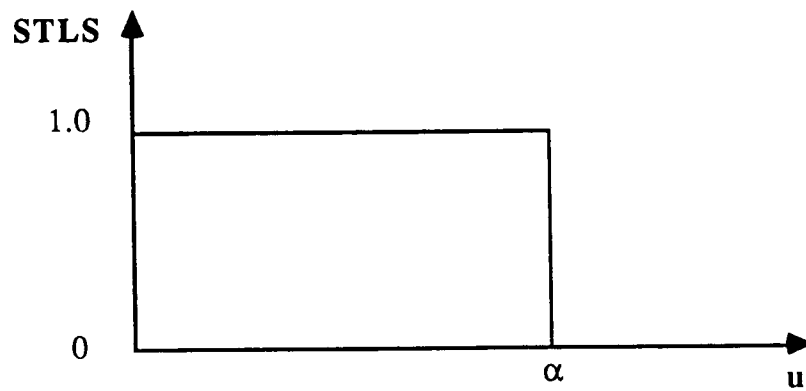


Figure 4.5: Step Membership Function.

by adding Gaussian noise to an image of a square box of uniform intensity having a dark border also of uniform intensity. While the results shown in Figure 4.6, Figure 4.7, and Figure 4.8 clearly indicate that the sigmoid-like membership function yields the best overall performance, a metric FR (Fisher Ratio) was used to quantify the quality of segmentation associated with each membership function.

The metric FR , which is based upon a one-dimensional version of the Fisher Criterion Function [11], measures the separation between the regions identified as background and defect. The Fisher Ratio is defined as

$$FR = \frac{|\mu_d - \mu_b|^2}{\sigma_d^2 + \sigma_b^2} \quad (4.4)$$

where the subscripts b and d refer to background and defect pixels respectively, with μ_i being the mean of class i given as

$$\mu_i = \frac{1}{N_i} \sum_{x \in \{class\ i\}} x \quad (4.5)$$

where N_i is the number of samples in class i , and the scatter, σ_i^2 , within the class is given by

$$\sigma_i^2 = \sum_{x \in \{class\ i\}} (x - \mu_i)^2 \quad (4.6)$$

The metric FR is tabulated in Table 4.1 for each of the three membership functions used for the segmentation of Test Images 4, 5, and 6, for which the results are shown in Figures 4.6, 4.7, and 4.8. In this case the calculation was based on regions that were identified as defect and background.

Table 4.1: Fisher Ratios for Membership Functions

<i>Membership Function</i>	<i>Fisher Ratio $\times 10^{-3}$ (FR)</i>		
	Test Image 4	Test Image 5	Test Image 6
SILS	3.88	4.09	0.63
LILS	1.98	2.27	0.23
STLS	1.53	1.84	0.55

From Table 4.1 it can be seen that the Fisher Ratio is highest for the sigmoid-like membership function for all three test images. The Fisher Ratio is the lowest for the step-like membership function except for Test Image 6, where the linear-like step function has the lowest Fisher Ratio. For Test Images 4, 5, and 6, the sigmoid-like membership function yields a segmentation, with the separation between the defect and background region, almost two times better than that yielded by the linear-like membership function based on the Fisher Ratio. The linear-like membership function in general yields a better separation between defect and background than the step membership function, as seen in Figures 4.6 and 4.7. The results of Test Image 6 in Figure 4.8 show a better segmentation, qualitatively, for the linear-like membership function, however, the Fisher Ratio does not reflect that. A further measure of the quality of segmentation can be identified using the measures of missed detection and false detection of defect pixels. From the segmentation of Test Images 4, 5, and 6 it can be seen that the measure of false detections is a more meaningful than the measure of missed

detections. The false detections, which are tabulated below for Test Images 4 and 5, are defined as the percentage of background pixels that are incorrectly identified as defects.

Table 4.2: False detections for Membership Functions

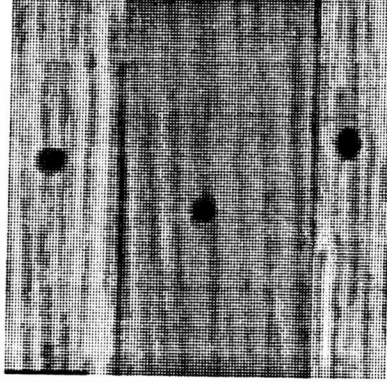
<i>Membership Function</i>	<i>Percentage of False Detections</i>	
	Test Image 4	Test Image 5
SILS	1.2	2.0
LILS	5.7	10.1
STLS	7.2	11.7

The measures of separation between the background and defect populations using Fisher ratio and the percentage of false detections show that the sigmoid like membership function yields the highest quality of segmentation.

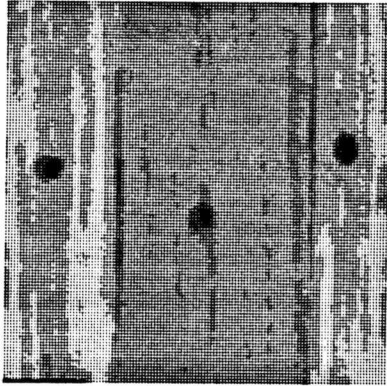
4.2.2 Segmentation Parameters - Alpha, Gamma, and Tau

In this section the effect of the parameters α , γ , and τ on the quality of segmentation is presented. The sigmoid-like membership function was used for all experimentation. The parameter α represents the maximum absolute difference between a son node and its father node below which the link is assigned a strength value of 1.0. This parameter was varied between 2.0 and 20.0 as shown in Figures 4.9, 4.10, and 4.11. From the results it can be seen that the quality of segmentation is best for the lower values, in this case 2.0.

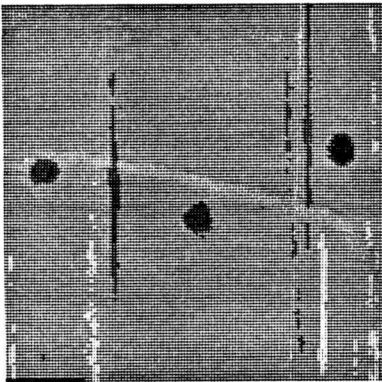
Test Image 4



LLS: $\alpha = 15, \gamma = 160, \tau = 0.9$



SILS: $\alpha = 15, \gamma = 160, \tau = 0.9$



STLS: $\alpha = 15.0, \tau = 0.0$

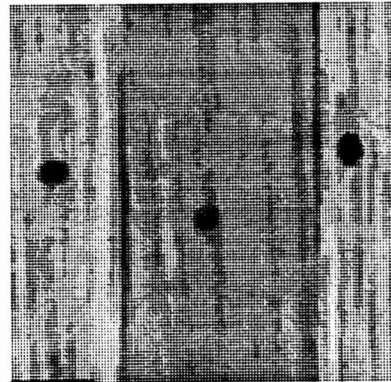
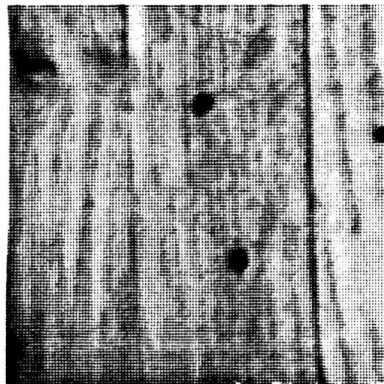


Figure 4.6: Effect of Membership Functions on Segmentation of Test Image 4.

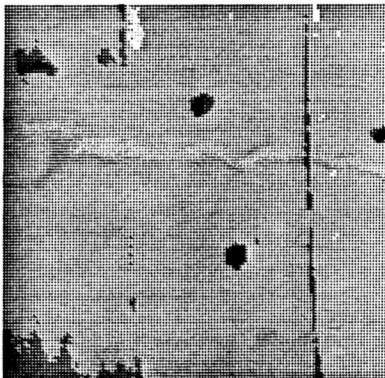
Test Image 5



LLS: $\alpha = 15, \gamma = 160, \tau = 0.9$



SILS: $\alpha = 15, \gamma = 160, \tau = 0.9$

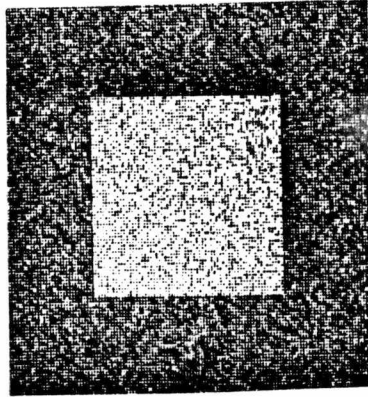


STLS: $\alpha = 15.0, \tau = 0.0$

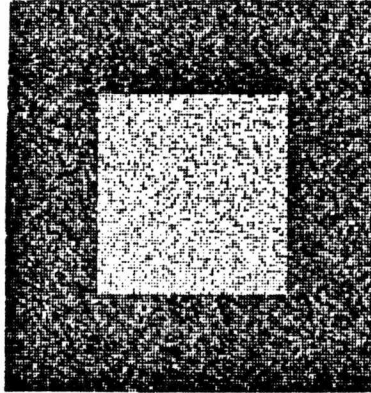


Figure 4.7: Effect of Membership Functions on Segmentation of Test Image 5.

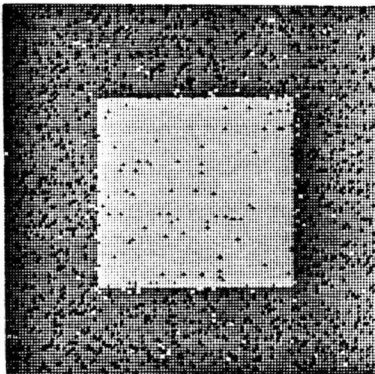
Test Image 6



LLS: $\alpha = 10, \gamma = 130, \tau = 0.2$



SILS: $\alpha = 10, \gamma = 130, \tau = 0.2$



STLS: $\alpha = 10.0, \tau = 0.0$

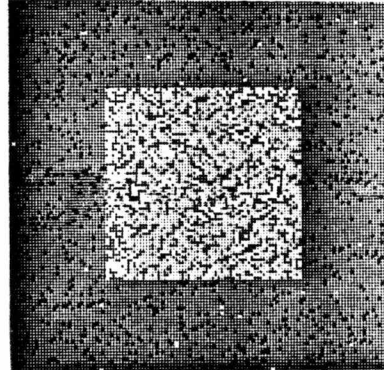


Figure 4.8: Effect of Membership Functions on Segmentation of Test Image 6.

The parameter γ represents the absolute difference between a son node and its father node, above which the link is assigned a strength value of 0.0. This parameter was varied from very low values to the mean intensity value of the image, as shown in Figures 4.12, 4.13, and 4.14. The results indicate that for higher values of γ the segmentation shows a higher degree of smoothing. As a result, smaller segments representing noise or textural patterns are averaged out. In general the mean value is a good choice for the parameter γ .

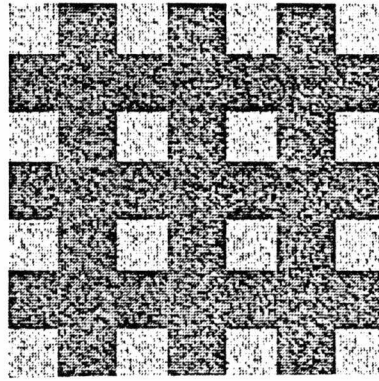
The parameter τ represents the minimum strength a son to father link must have in order for the son node to be replaced by the father node in the segmentation procedure. This parameter was varied between 0.5 and 0.9 as shown in Figures 4.15, 4.16, and 4.17. The results show that a τ of 0.5 is sufficient for isolating the more prominent features in the image, however, a larger value is required for the segmented image to include features that have a lower contrast and, as a result, are less prominent features.

Test Image 1

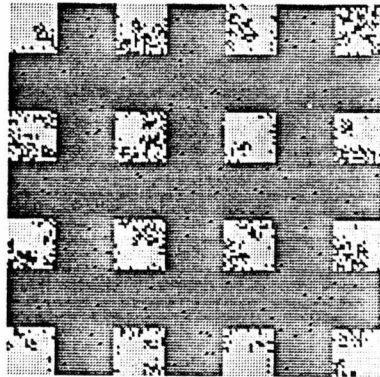
Mean = 160.0

$\gamma = 160.0$

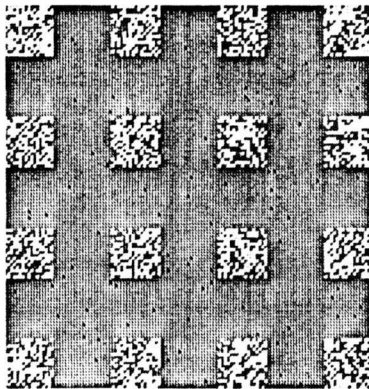
$\tau = 0.5$



$\alpha = 2.0$



$\alpha = 10.0$



$\alpha = 20.0$

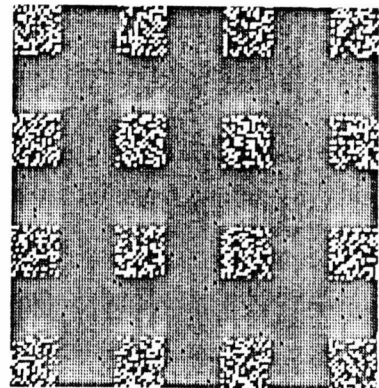
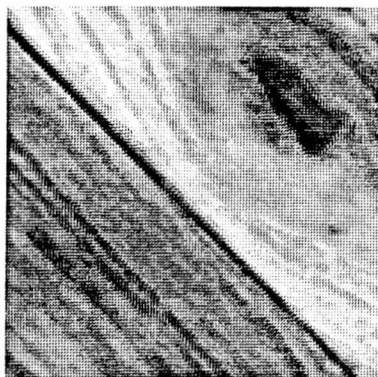


Figure 4.9: Effect of Parameter α on Segmentation of Test Image 1.

Test Image 2

Mean = 173.0



$\gamma = 170.0$

$\tau = 0.5$

$\alpha = 2.0$



$\alpha = 10.0$



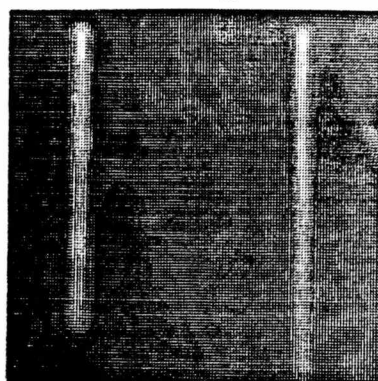
$\alpha = 20.0$



Figure 4.10: Effect of Parameter α on Segmentation of Test Image 2.

Test Image 3

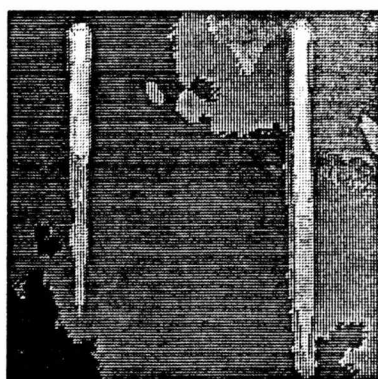
Mean = 60.9



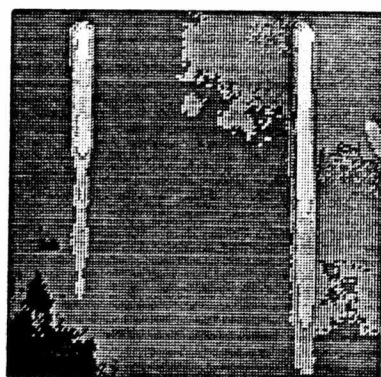
$\gamma = 60.0$

$\tau = 0.5$

$\alpha = 2.0$



$\alpha = 10.0$



$\alpha = 20.0$

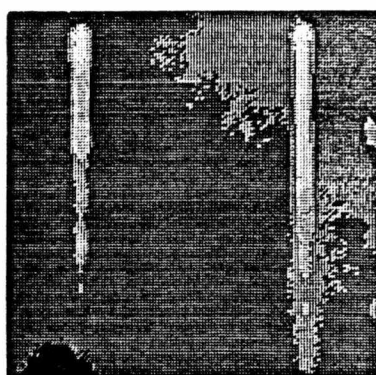


Figure 4.11: Effect of Parameter α on Segmentation of Test Image 3.

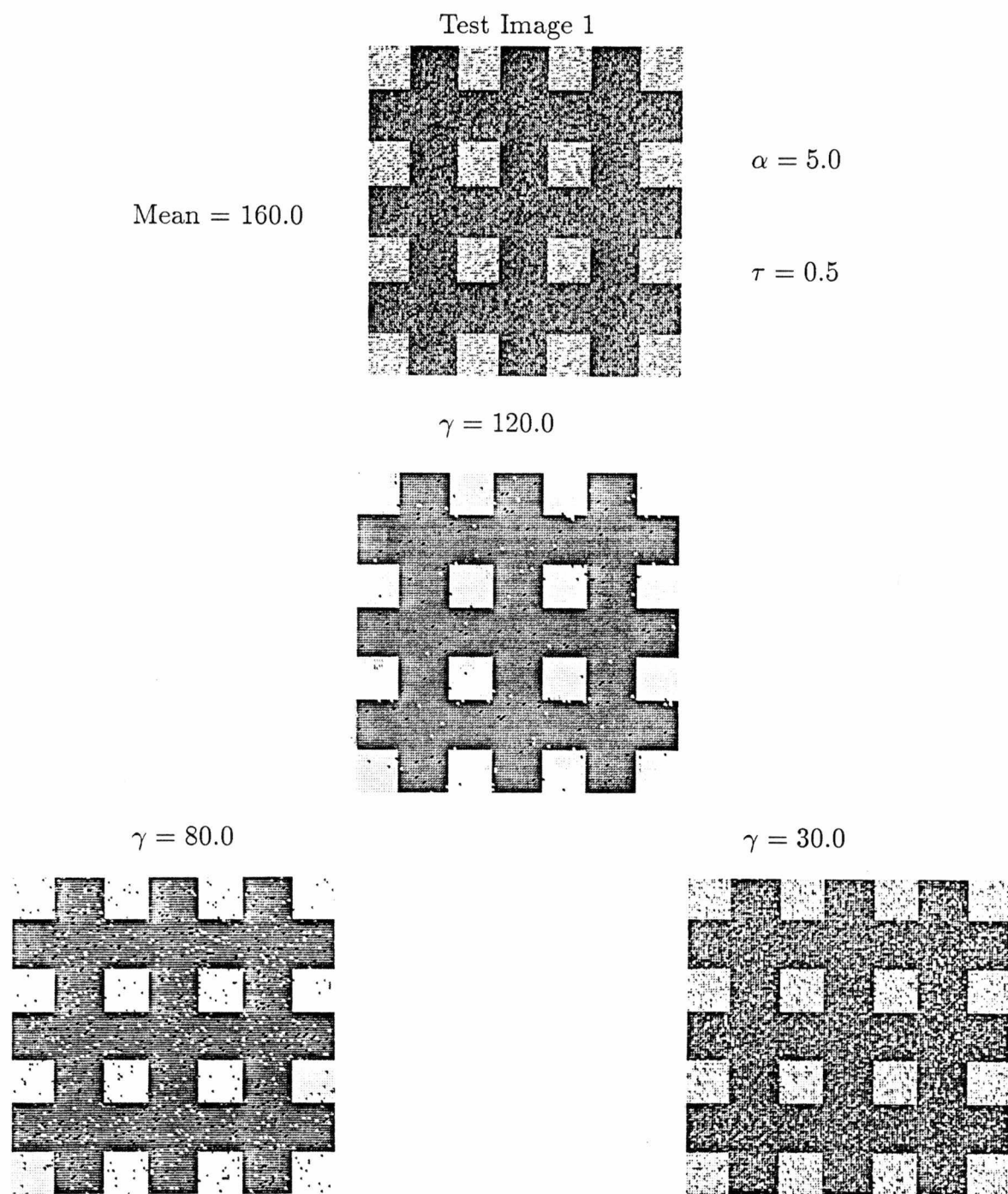
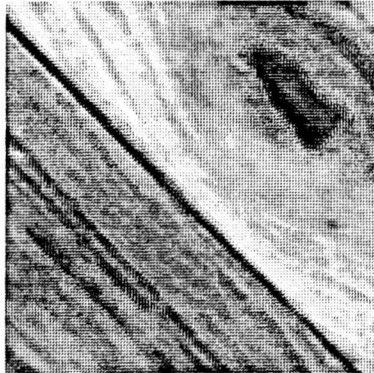


Figure 4.12: Effect of Parameter γ on Segmentation of Test Image 1.

Test Image 2

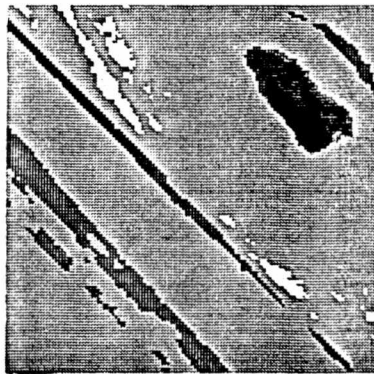
Mean = 173.0



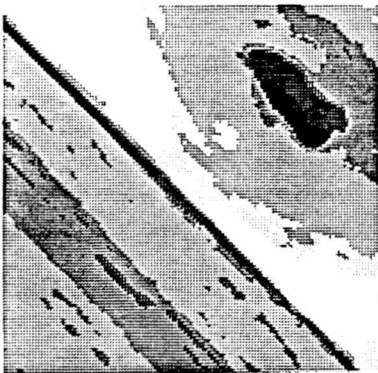
$\alpha = 5.0$

$\tau = 0.5$

$\gamma = 130.0$



$\gamma = 80.0$



$\gamma = 30.0$



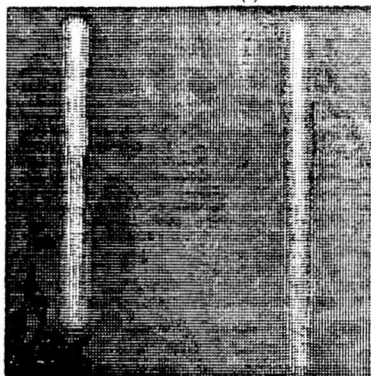
Figure 4.13: Effect of Parameter γ on Segmentation of Test Image 2.

Test Image 3

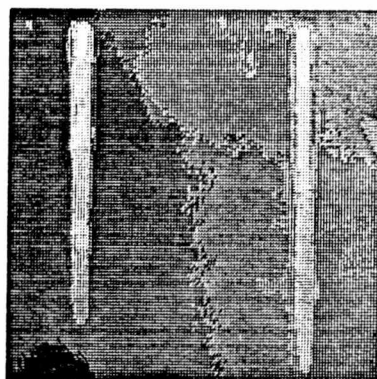
Mean = 60.9

$\alpha = 5.0$

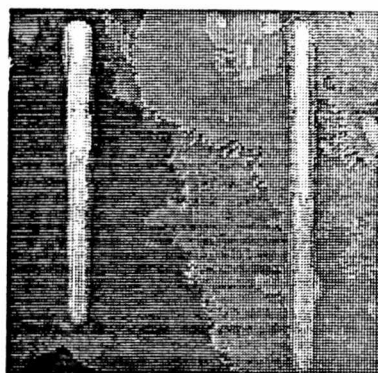
$\tau = 0.5$



$\gamma = 40.0$



$\gamma = 30.0$



$\gamma = 20.0$

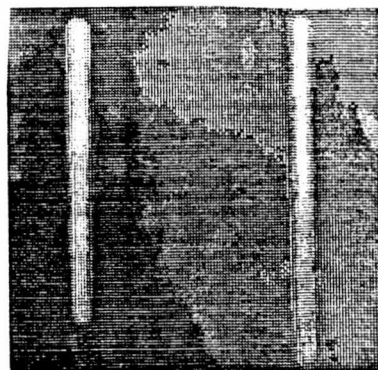


Figure 4.14: Effect of Parameter γ on Segmentation of Test Image 3.

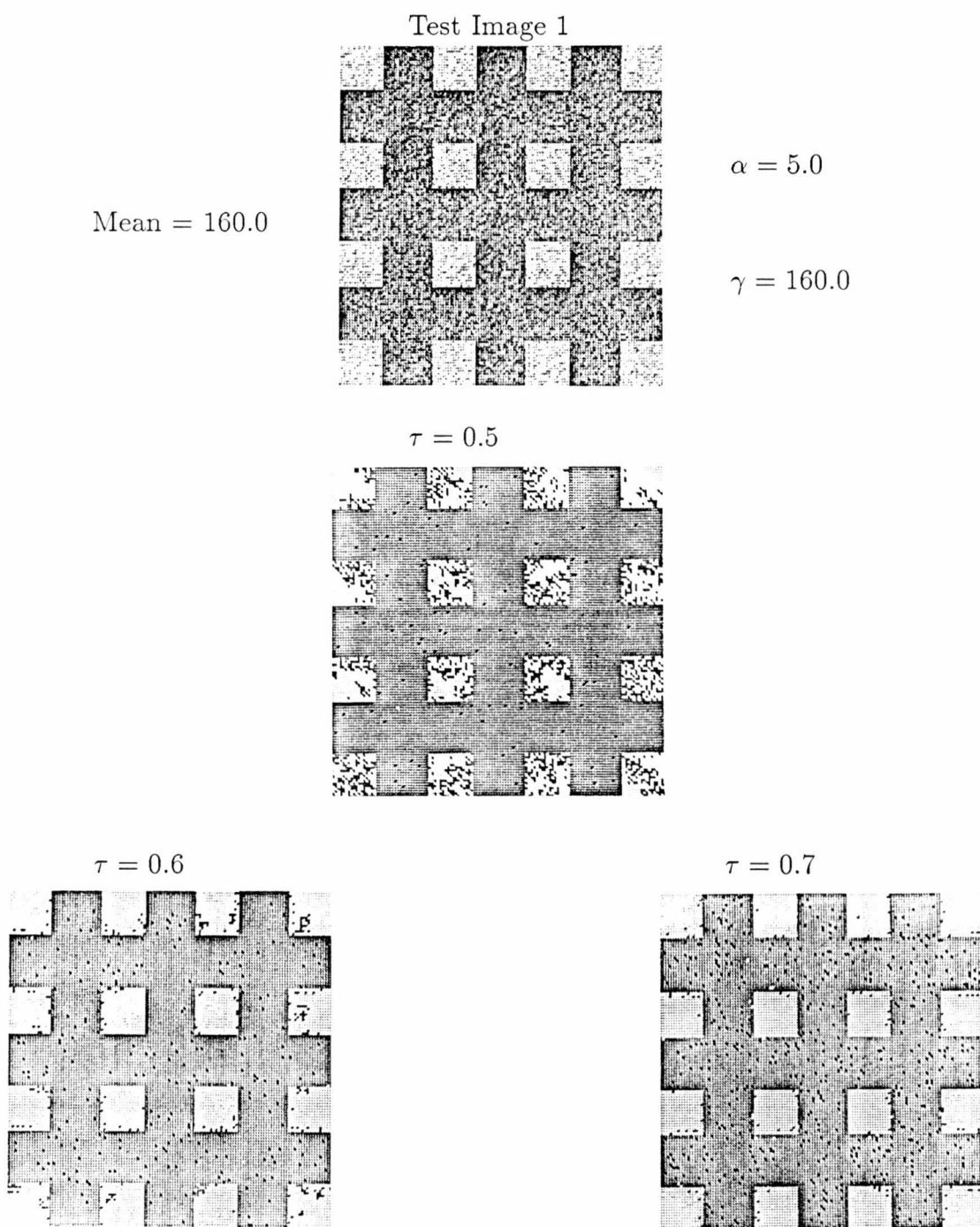


Figure 4.15: Effect of Parameter τ on Segmentation of Test Image 1.

Test Image 2

Mean = 173.0



$\alpha = 5.0$

$\gamma = 170.0$

$\tau = 0.4$



$\tau = 0.5$



$\tau = 0.6$

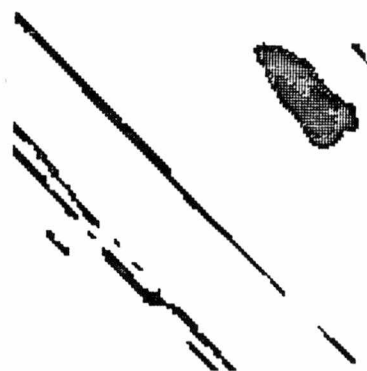
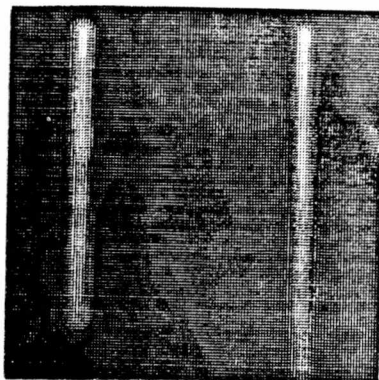


Figure 4.16: Effect of Parameter τ on Segmentation of Test Image 2.

Test Image 3

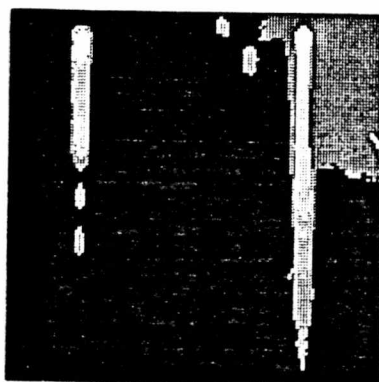
Mean = 60.9



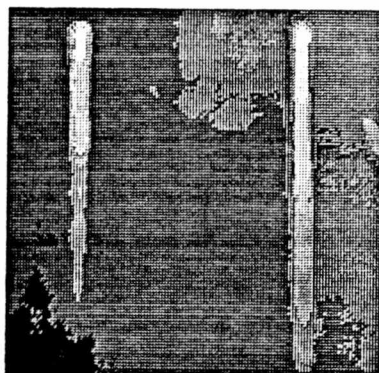
$\alpha = 5.0$

$\gamma = 160.0$

$\tau = 0.0$



$\tau = 0.5$



$\tau = 0.9$

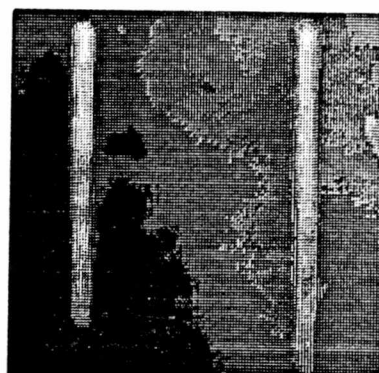


Figure 4.17: Effect of Parameter τ on Segmentation of Test Image 3.

CHAPTER 5

CONCLUSIONS

The segmentation of wood images using the hard pyramid linking algorithm successfully identifies the more prominent features as isolated segments with crisp boundaries. However, low contrast features such as textural patterns and smaller features such as noise tend to diminish the quality of segmentation. The vulnerability to noise and low contrast features is inherent in the linking scheme used by the hard pyramid linking algorithm. In the hard pyramid linking algorithm a son node is linked only to the most similar candidate father node. All other candidate nodes are ignored. Son nodes representing noise and textural patterns are thus linked to the father nodes that best represent them at the adjacent, lower resolution, level of the pyramid. Consequently, some of the noise and textural patterns creep up into the higher levels of the pyramids and are distinguishable in the final segmented image. The textural patterns and noise can be eliminated in some cases by applying a single threshold to the segmented image, however, this is not always the case.

The fuzzy pyramid linking algorithm, in comparison, successfully identifies the more prominent features as isolated segments and reduces the variance within a region. Lower contrast features such as texture and smaller features such as noise are often eliminated by a judicious choice of segmentation parameters.

The fuzzy pyramid linking algorithm uses a weighted linking scheme whereby a son node is linked to each of the four candidate father nodes with a weight corresponding to the membership function used such that the more similar a father node the larger the weight assigned to the link. The father nodes are redefined at the end of each iteration by a weighted average of not only the most similar son nodes, but also the less similar father nodes. This produces an averaging effect that eliminates noise and reduces the variance within a region. The lower contrast features are eliminated by allowing only a portion of the links to propagate through to the base of the pyramid.

The iterative linking process for both the hard pyramid linking algorithm and the fuzzy pyramid linking algorithm is a minimization of the mean square difference between the son nodes and the father nodes. Since the father nodes are replaced by the average of the son nodes that are linked to them, in the hard pyramid linking algorithm as well as the fuzzy pyramid linking algorithm, the algorithms are guaranteed to converge in a finite number of iterations.

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VITA

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