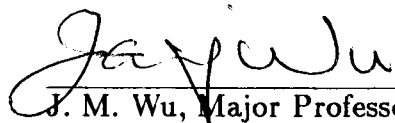
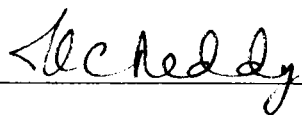
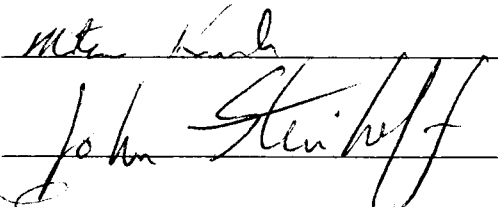
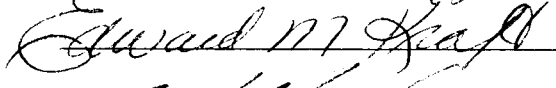
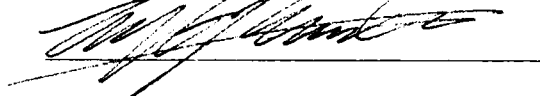


To the Graduate Council:

I am submitting herewith a dissertation written by P. Sundaram entitled "Studies on Unsteady Vortex Motions Including Thermo-Fluid Interactions". I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy with a major in Aerospace Engineering.


J. M. Wu, Major Professor

We have read this dissertation
and recommend its acceptance:

Accepted for the Council:


Vice Provost
and Dean of the Graduate School

STUDIES ON UNSTEADY VORTEX MOTIONS INCLUDING
THERMO-FLUID INTERACTIONS

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

P. Sundaram

March 1987

DEDICATION

This work is dedicated to my father, late Mr. S. Pichuraman, who always was an inspiration to all of us for his simplicity; and to my mother, Mrs. P. Unnamalai, for her awe-inspiring capacity for analysis.

ACKNOWLEDGMENTS

The author gratefully acknowledges the immense help, constant encouragement and guidance, and friendly advice and treatment that he enjoyed over the years from Dr. James M. Wu, supervisor and chairman of the doctoral committee. The author expresses his solemn gratitude for the invaluable help and guidance offered by Dr. Mitsuru Kurosaka throughout the course of the work, and also to Dr. Trevor Moulden for the stimulating discussions and the diligent perusal of the manuscript. Solemn appreciations are also offered to the other members of the committee, Dr. K. C. Reddy, Dr. Edward Kraft and Dr. John Steinhoff for their help and advice.

Deep sense of gratitude are also expressed to Dr. S. N. Chaudhuri for providing the author an opportunity to pursue his research and for his invaluable help and advice during the early years of his work at UTSI.

Special thanks also belong to the author's brother Dr. V. P. Srimi, for his constant encouragement and to his mother and other members of the family for their patience and their encouragement.

Sincere thanks are also expressed to the UTSI administration for their financial support and for permission to the extensive use of the computer facilities; to Dr. A. D. Vakili for several useful conversations; to many colleagues without naming individuals for their encouragement; to the UTSI laboratory personnel for their help in conducting experiments; and last but not the least, to the UTSI library personnel, particularly to Mrs. Mary Lo, for her cheerful and tireless help towards finding numerous technical literature.

ABSTRACT

An inviscid unsteady vortex dynamics study is presented here to study the flow details in several free and bounded shear flows. The specific flows considered here are the starting vortex behind sharp edges, the vortex street behind bluff body wakes, the plane mixing layers between two streams and the wall bounded vortex street. Using the unsteady vortex dynamics models developed for each of these flows and through the Lagrangian dynamics computational approach, the streakline and pathline pattern have been computed for these flows. From the combined study of the computed streakline and pathline pattern, various flow visualization details and the illusions created by the streaklines have been identified and explained.

With the accuracy of the unsteady vortex model for the vortex street wake having been verified by the good agreement obtained between the computed and flow visualization results, the model has been used to investigate the different flow interactions present in this complex flow. The first case chosen for analysis is the thermo-fluid interaction that exists in this flow. It has been shown, through the computed instantaneous total temperature, that the bluff body wake exhibits regions of separated energy. Further, the well known phenomenon of Eckert-Weise effect, which predicts a cold near wake, has been obtained by temporally averaging the instantaneous total temperature, thus proving that the cause of this phenomenon is the unsteady vortex motion in the wake. In a similar manner, the familiar result of a loss in total pressure in the near wake can be interpreted to result from the time averaged total pressure separation phenomenon.

The second case considered here for the vortex interaction study is the interference caused by the channel walls in a wall bounded vortex street wake. From

the computed streakline pattern and also through the flow visualization studies, an interesting vortex dynamics result that forces the flow from both sides of the wake to interchange their sides has been obtained. This result explains the mechanism for the static temperature reversal between the two sides of a wall bounded wake that was observed by Minchin. Further, the present computational study highlights the important inviscid entrainment effects of the convecting vortices by showing that the eruption-like appearance of the wall layer streaklines observed in the flow visualization pictures is predominantly due to the inviscid entrainment effects and not merely caused by the viscous effects.

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LIST OF SYMBOLS

a	distance between the vortices of same strength in a vortex street
h	lateral distance between the two rows of vortices in a vortex street
k	trochoidal pathline parameter
p	static pressure
S	Strouhal number
t	time
t	static temperature
$\vec{u}$	velocity vector
w	complex potential
$\vec{x}$	position vector
z	complex quantity ($x + iy$)
Pr	Prandtl number
Re	Reynolds number
P_t	total pressure
r_0	initial radius of the vortex core
S_0	original strength of the evolving vortex
t_0	release time of the fluid particle
T_t	total temperature
u_c	velocity of the vortex cylinder
U_m	velocity of the mixing layer
U_s	relative velocity between a vortex and free stream
u_v	velocity of an isolated convecting vortex
U_{st}	velocity of the vortex street
U_{st}^b	velocity of the wall bounded vortex street

u_θ	tangential velocity
U_∞	free stream velocity
y_0	center of the isolated convecting vortex
α	wave length of the trochoidal particle path
γ	ratio of specific heats
Γ	circulation strength of the vortex
λ	street spacing ratio
ν	kinematic viscosity
ρ	density
τ	vortex shedding period
Φ	dissipation function
ψ	stream function
ω	angular velocity
c_p	specific heat at constant pressure
$P_{t\infty}$	free stream total pressure
$T_{t\infty}$	free stream total temperature
ρ_0	distance between the vortex center and the particle release point

CHAPTER I

INTRODUCTION

In any complex shear flow, the fundamental kinematical event is the unsteady motion of large scale vortices. Visual examples showing the presence of unsteady vortex motion in flows are the starting vortex behind sharp edges, the vortex street formed in the wake of bluff bodies, and the mixing layer between two parallel streams of unequal velocities. The essential sources of these vortices in flows where a solid-fluid interface is present are the diffusion of vorticity from the surface of the body and the flow separation; whereas, in free shear flows where discontinuities in flow properties are present, the vortices are generated due to the Kelvin-Helmholtz and Taylor instabilities. While these vortices are in their formative stages, they are not only influenced by the convective forces present in the local flow but also by the viscous forces exhibited by the shear layer. These rolled up large scale vortices, having concentrated viscous cores, convect downstream such that their dynamics essentially follow the inviscid laws of Biot-Savart. The dynamical interactions between these vortices result in their orbiting around each other, pairing with one another, and eventually amalgamating (with their pairs) altogether. The analysis of the unsteady vortex motion in these flows is essential in order to understand the dynamics of the flow.

The similarity principle of aerodynamic testing applicable for a stationary body in a moving stream is not directly applicable to a moving body in a coordinate system fixed with the body, particularly at points in the flow field. This is due to the fact that the work and consequently the total pressure changes are inadequately accounted for at these points. A similar case exists when the flow

over the body is unsteady (due to unsteady separation) while the body itself is not moving. The fundamental differences between steady and unsteady flows can cause several illusions while observing the streakline pattern in unsteady vortex flows. The realization of the differences between steady and unsteady flows can help us comprehend phenomena like 'aerodynamic cooling' and the existence of 'negative entropy' regions in a flow field.

The identification of the presence of large scale vortical structures in a complex vortex flow have bolstered the development of analytical techniques for the flow field analysis from the viewpoint of unsteady vortex interactions. The semi-Lagrangian method of investigating the dynamics of vortices in an unsteady flow with respect to a frame of reference translating along with a vortex indeed alleviates the complexity in the analysis; however, it hides the presence of several salient flow features otherwise conspicuous if analyzed using an inertial frame. The approach to choose an inertial frame of reference requires the knowledge of the vortex 'celerity', which in general is a non-linear function of time and space. Under simplifying assumptions, reasonable approximations to this quantity could be found and the overall flow field might then be iteratively computed.

Flow visualization studies have long been acknowledged as the preliminary step towards understanding the underlying physics in complex unsteady flows. In the visual study of any unsteady flow, the two important aspects of the flow traces are the streaklines and pathlines. Each of them describes a definite aspect of the flow. The streaklines in some sense identify the phase averaged values of the flow parameters at a particular point and at a specific instant of time; whereas the pathlines yield the instantaneous description of the physical properties of each flow marker at any particular time. The streakline pattern might identify definite

regions of vorticity and could aid in the understanding of the global features of the flow. On the other hand, pathlines reflect the dynamical force balance in the flow. The combined analysis of streaklines and pathlines is imperative in order to make a correct interpretation of the mechanisms involved in any complex flow.

In the present investigation, dynamical models have been presented for the study of unsteady flows wherein large scale vortex motion are prevalent. In order to understand the extent of the influence of vortex dynamics in the flow field, different classes of two-dimensional unsteady vortex flows have been examined. The present methodology deviates from the usual kinematical approach taken for the similar class of problems and uses inviscid incompressible vortex dynamics models. Since the viscous effects are not included in the computations, the regions within the vortex cores (where the effects of viscosity would be important, requiring the detailed numerical solutions of the Navier-Stokes equations) have been carefully identified and omitted from the region of specific interest. But, in specific cases, the effects of viscosity have been included as a higher order influence through the approximation that the vortices obey the familiar Lamb's exponential decay law.

Simple dynamical models involving the unsteady motion of finite core vortices are developed in Chapter III. Specific cases included are the time evolutions of a single vortex, infinite vortices in a vortex street wake, and the vortices in a mixing layer between two parallel streams of unequal velocity. These models have then been used to compute the pathline and streakline pattern in each of these flows as described in Chapter III. Also shown therein are the good comparisons of the computed flow streaklines with the experimental flow visualization results. In Chapter IV, the intricate and subtle features of these flows have been highlighted and the various wrong notions that could be accrued by a naive observation of

the streakline pictures alone are pointed out and the importance of the combined study of both streakline and pathline traces are emphasized.

With the overall flow dynamics in these vortex flows verified to be well represented by these dynamical models, the important kinematical events that result as a consequence of the unsteady vortex motion in the wake flow have subsequently been investigated using these models. As a first example, the dynamical evidence for the phenomenon of 'aerodynamic cooling' in the near wake of a bluff body has been studied. Experimental recordings of the 'negative recovery factor' identifying this phenomenon had been reported more than forty years ago by Eckert and Weise. To study this phenomenon in an 'unsteady vortex motion' perspective, the problem has been divided into two parts. In the first part, the motion of the fluid particle has been determined through the long range influence of the vortex motion; while in the second part, the flow energy at a point has been represented by the unsteady energy equation. The assumption that enables the above approximation is that the fluid compressibility has a negligible influence on the dynamics of the fluid particle.

The 'aerodynamic cooling' phenomenon has been discovered, through numerical computations, to be due to the time averaged effect of the instantaneous energy separation that occurs in the wake of a bluff body. This energy separation has indeed been elsewhere verified to be present in the wake of a circular cylinder through the numerical solution of the Navier-Stokes equations and also through the aspiration probe measurements in the wake. Using similar arguments, the dynamical reason for the loss of total pressure in the wake of a bluff body can be explained to be the time averaged effect of the instantaneous total pressure separation in the wake. The dynamical explanations for the cause of the occurrence

together with the computational results of the total temperature traces along the pathlines and the instantaneous distribution of total temperatures in the wake are presented in Chapter V. Also presented in Chapter V are the time averaged results, computed using the present vortex dynamics approach, of the total temperatures in the wake. The results clearly show the presence of a cold wake, thus demonstrating that the cause of the 'aerodynamic cooling' phenomenon is the unsteady vortex motion.

As a second example to illustrate the kinematical features of the unsteady vortex motion, another interesting phenomenon where a static temperature interchange occurs between the two sides of a wall in a wall bounded wake behind a circular cylinder has been considered. The dynamical evidence of this phenomenon, based on the present numerical computations are discussed in detail in Chapter VI. Also discussed therein are the theoretical arguments based on the unsteady boundary layer development due to the convected two-dimensional vortices moving close to a solid wall. Experimental results, through flow visualization pictures, showing the exact details of the flow and proving that the dynamics of the unsteady vortex motion is the only cause for this phenomenon is also presented in Chapter VI. Although a video movie was also taken for this flow in a water tunnel, no still pictures are presented here. The flow visualization pictures were taken for the wake behind a circular cylinder in the UTSI water tunnel using the Laser Induced Fluorescence technique. The exact reason for the flow cross-over observed in the flow visualization experiments are not clear and are considered to be future work in this area.

During the course of the present numerical experiments for the wall bounded vortex street problem, streaklines corresponding to the particles that were released

close to the walls resembled those of the turbulent boundary layer bursting process. Since the present computations are essentially inviscid, the bursting-like eruptions appear only due to the entrainment effect of the unsteady vortex motion. Careful examination of these streaklines to ascertain the influence of viscosity in these flows needs to be reiterated. However, a detailed study of the inviscid effects through the unsteady vortex motion close to the wall can itself provide enormous information about the overall flow structure, as demonstrated in the present work.

The fact that the above studies have significance in coherent structures and turbulence production mechanisms is clear. It would be an interesting future research topic to include additional details into the present model and compute the flow field with more rigor.

CHAPTER II

TECHNICAL BACKGROUND

The identification of various flow structures in a complex vortex flow can best be carried out by flow visualization experiments. Flow patterns like streamlines, streaklines, pathlines, timelines, etc., and important flow parameters like density distributions and stress distributions can all be visualized and photographed using this versatile tool. Also, the salient flow features such as flow separation and reattachment, shock waves, and regions of vortex flows can be easily diagnosed using flow visualization techniques. Although flow visualization results are technically qualitative, the information contained in them is not only crucial for the physical understanding of these flows but also provides the foundations for the development of mathematical models for such flows. Detailed explanations showing the differences between streaklines and pathlines are best described in the movies of Kline (1969).

A *streamline* is an Eulerian description of the flow and is expressed in the symmetric differential form as

$$\frac{dx}{u(\bar{x}, t)} = \frac{dy}{v(\bar{x}, t)} = \frac{dz}{w(\bar{x}, t)}$$

where $\bar{u}(\bar{x}, t) = (u, v, w)$ are the velocities along the coordinate directions x, y , and z represented by the vector $\bar{x}$. The streamline pattern is obtained when a lot of fine tracer particles are uniformly suspended in a flow and their motion is photographed with a proper short exposure time. By tracing on the picture continuous lines that have the direction of the streaks at every point portrays the streamlines for either steady or unsteady flow. In a steady flow, the equilibrium flow properties are preserved along streamlines. In an unsteady non-turbulent flow with periodic

vortex shedding, the instantaneous streamline patterns for each vortex shedding cycle will have the same qualitative features as the phase averaged results of the corresponding steady turbulent flow. The topology of the instantaneous streamline patterns in an unsteady vortex flow exhibit critical points. In two-dimensional incompressible flow, the only allowable critical points that do not violate flow continuity are 'centers' and 'saddles'. Perry, Chong & Tam (1982) explain that these critical points, to first order, translate with a uniform velocity equivalent to the velocity of the pressure extrema. Coles (1984) and Hussain (1984) point out that in most free shear layers, the vortex stretching and the consequent turbulent production occur near these 'saddles'.

A *streakline*, represented by the equation

$$\bar{u} = \bar{q}[f(\bar{x}, t_0), g(\bar{x}, t_0), h(\bar{x}, t_0), t]$$

is the instantaneous locus of all fluid particles which have passed through a given point in the flow field. Streaklines can be observed by continuously injecting tracer materials into the flow at selected positions. Streaklines also indicate the approximate regions of concentrated vorticity. Extreme caution has to be exercised while interpreting the physical mechanisms underlying the flows based on the streakline pattern alone. We will return to these points later on in the following chapter.

A *Pathline*, defined by the equation

$$\frac{\partial \bar{x}}{\partial t} = \bar{u}(\bar{x}, t)$$

is the path of a particular fluid particle in the flow field. It is a Lagrangian description of fluid motion. The particle path represents the instantaneous behavior of the flow properties at a given point lying on the locus of the particle. In other

words, it is the path along which the flow history has been carried along as the particle moves. The tangent drawn at any point of the pathline curve yields the direction of motion of the particle at that instant. A way of recording the particle paths is to suspend a small number of fine tracer particles in the fluid and photograph their motion with a long exposure time.

Even though all the three flow lines described above are identical in a steady flow, they are completely different in an unsteady flow. This observation is vividly demonstrated in the movies by Kline (1968). Galilean transformations do exist to compute the other flow lines when one of them is known. In the present work the pathlines are first computed through the dynamical force balance and the other two lines are later reconstructed from the pathlines.

Besides these three basic flow traces, there is one other important flow line for unsteady flow, called the *timeline*, which indicates the local velocity profile at each point in the flow field. It is the curve formed by fluid particles which have initiated simultaneously from a straight line across the flow. These lines are obtained by hydrogen bubble, smoke wire and other techniques.

Theoretical studies on the motion of two-dimensional, time-dependent unsteady vortices were initiated by the important work of Prandtl (1922). In his spiral model for this unsteady vortex motion, he considered that the supply of vorticity to the core of the unsteady vortex is through a vortex sheet connecting the parent body to the core. Hall (1961) presented a theoretical study of the vortex core structure and found approximate solutions for the velocity and pressure distributions by dividing the flow into an inviscid convective outer part and a diffusive sub-core that obeys a boundary-layer type approximation. Later, Stewartson & Hall (1963) presented a singular perturbation solution to the same problem and

defined appropriate inner and outer variables for the matching process. Brown (1965) included compressibility effects into the analysis of the core structure and obtained solutions for this problem that comprised of an incompressible outer part and a compressible inner part. These theories for the vortex core have viscous effects included in a fragmentary fashion only. Recently, Kurosaka (1986) obtained a complete solution for the vortex problem throughout its life span by solving the axi-symmetric Navier-Stokes equations. With the inclusion of the vorticity fed to the core during its formation stages through an appropriate source term, (this source term being abruptly removed when the vortex detaches itself from the mother body) he showed that the solution asymptotically attains the proper Lamb-Oseen type viscous vortex behavior farther downstream.

Another well-known flow geometry that produces vortices periodically is the circular cylinder. Von Kármán (1911), with his insight, presented a simple inviscid model for the vortex street observed downstream of the cylinder and also presented a stability theory for the street and predicted the drag of the cylinder fairly accurately. This amazingly simple treatment of such a complex flow phenomenon, which shows excellent agreement with experiments, has been investigated even today in the efforts to understand the complex flow structures observed in the wake through the coherent structure viewpoint. The present study falls into this category and extends this approach to include additional details as will be discussed in the following chapters.

At a Reynolds number large enough for the vortex street to develop in the wake of a bluff body, the boundary layers separate from the upper and lower surfaces of the body and roll up in the form of vortices which are then convected downstream in the wake. Fage & Johansen (1927) observed that the ratio of the

width of the street to the distance between the adjacent vortices in the same row of a vortex street increases with the downstream distance behind the body. He explained that this is due to the fact that the height of the street increases faster than the distance between the vortices as the vortices travel downstream. They estimated that only about 60% of the vorticity discharged from the surfaces of the cylinder is present in the large individual eddies of the street.

The stability of the vortex street came under serious investigation after several observations where the vortex street ratios different from the Kármán's value of 0.281 were observed in various flows. Birkhoff (1953) proposed that the mean lateral and longitudinal spacing in a vortex street are 'dynamical invariants' and that all spacing ratios in an inviscid flow are stable. Wille (1960) criticized Birkhoff's solution by indicating that if the 'invariance theorem' were to be true, it should apply to symmetrical vortex streets as well and pointed out that there were no flows where such symmetrical streets were observed. Kochin, Kiebel & Roze (1964) reiterated that any array of vortices is unstable to any order of perturbations higher than the first and established that the Kármán's street arrangement is particularly unstable to any three-dimensional disturbance. Later, Kronauer (1964) proposed a new criterion for the stability of the street which states that for a given vortex velocity, the vortex street adjusts itself into the configuration giving minimum drag coefficient. Further studies in this area receded after the complex flow interaction mechanisms in these flows were revealed by Taneda (1965) who experimentally generated a nearly symmetrical vortex street by oscillating a cylinder, thus questioning Wille's criticism.

Early experimental investigations by various researchers on cylinder wakes were directed towards the frequency of vortex shedding. Roshko (1953) determined

that the transition process from a vortex street to a turbulent wake occurred in the Reynolds number range of 150 to 300. He observed that the vortex shedding was highly irregular in this region. Sato & Kuriki (1961) experimentally observed three distinct ranges during this transition process; a linear range, a non-linear range, and a three-dimensional range. They interpreted the velocity perturbations in each of these ranges through suitable vortex street models.

Sreenivasan (1985) explained this transition process from the 'strange attractor' theory perspective by discovering, through the energy spectrum measurements in the wake of a circular cylinder, that successive chaos and order with increasing Reynolds number indeed occurs in this regime. Later, Sirovich (1985) showed that such effects could be attributed to the vortex shedding process which acts as a driving mechanism and contended that the excitation of additional frequencies are likely to bring on chaotic behavior. Recently, Cantwell & Coles (1983) carried out a thorough investigation of the cylinder near wake using a flying hot wire technique and concluded that the dominant topological feature of the near wake is the formation and evolution of saddle points which play a dominant role in the turbulence production and entrainment mechanisms.

The presence of a Kármán vortex street behind a circular cylinder even at transonic Mach numbers have been photographed using spark shadowgraph technique by Dymet, Gryson and Ducruet (1980) behind a circular cylinder and by Heinemann, Lawaczeck & Butefisch (1975) in the flow behind turbine blades.

The first detailed numerical work on the vortex shedding process was carried out by Abernathy & Kronauer (1962) when they studied the vortex street formation process in an incompressible inviscid flow. By considering the nonlinear interactions of two infinite vortex sheets, initially a fixed distance apart, they were

able to simulate the rolling up process and identify the reason for the loss of over 50% of the strength of each wake vortex downstream compared to their values at the formation stage. Subsequently Chorin (1973) presented a unique method of solving the Navier-Stokes equations by modeling the convective terms of the equations by discrete vortex filaments and the diffusive term by a random walk method. This model yielded a good description of the cylinder wake. Later, several investigators (see for example; Clements 1973, Kuwahara & Takami 1973, Sarpkaya 1975, Stansby 1980) adapted discrete vortex methods and obtained approximate models for the vortex wake simulations. Recently, Saffman & Schatzman (1982) presented a global analysis of the wake by solving the Euler's inviscid conservation equations while accounting for the finite core size of the vortices and were able to predict the wake parameters accurately.

It is well known that the presence of a wall is very important for turbulence production. It is also well established that the inner region of a turbulent boundary layer stays in a quiescent state for a sufficiently long time and that this quiescent state is abruptly terminated by the periodic violent eruptions of the wall layer fluid into the outer layer by an event usually referred to as bursting. This bursting phenomenon is the mechanism by which the turbulent boundary layer maintains itself, as new vorticity is continuously generated near the wall by the local pressure gradient. Kline *et al.* (1967) photographed these wall streaks by the hydrogen bubble flow visualization technique. These streaks were later detected in the hot-wire signals by Rao, Narasimha & Badri Narayanan (1971). Extensive studies of the outer layer by Falco (1977) revealed the presence of vortical structures of small scale as well as large scale. Willmarth (1975) proposed a simple inviscid two-dimensional vortex model for this process and predicted the wall pressure

fluctuations with good qualitative agreement to experimental results. Extensive experimental studies of the outer layer by Falco (1977) revealed the presence of vortical structures of small as well as large scale. Detailed descriptions of the high and low speed streaks during the various stages of their life are modeled by Beljaars, Krishna Prasad & Devries (1981) and by Perry & Chong (1982).

Because of the non-linear dynamical interactions present in the near wall region, the bursting process and the large scale vortex motion in the outer layer are complementary to each other. Blackwelder (1983) has proposed that there is a similarity between the turbulent boundary layer bursting and the transition problem. By measuring the conditional averaged streamwise velocity profiles, he showed that the boundary layer profiles are highly inflexional both at, and before, bursting. Doligalski, Smith & Walker (1980) showed that the wall layer bursting is indeed due to the large scale vortex motion in the outer region of the boundary layer and conjectured that the eruption of the wall layer fluid creates a secondary vortex structure close to the wall as the primary vortex passes by. Since the convecting line vortex generally induces a region of adverse pressure gradient near the wall, the strong viscous-inviscid interaction produces unsteady separation. Doligalski & Walker (1984) carried out a detailed unsteady boundary layer analysis of this problem for various fractional convection velocities, αU_∞ , of the vortex, where U_∞ is the free stream flow velocity, and concluded that for $\alpha < 0.75$, a rapid lift-up of the boundary layer streamlines occurs which results in strong boundary layer growth in the region behind the vortex, and for $\alpha \geq 0.75$ they observed that the boundary layer development is more gradual.

A simple way to produce convecting nearly two-dimensional line vortices in close proximity to a wall is by the flow over a bluff body which is confined between

two walls. Since the generated vortices are naturally immersed in the channel boundary layer, it is an interesting example for the studies in order to understand the physical mechanisms related to the bursting phenomenon.

The circular cylinder located close to a wall has been of interest for quite some time. The early interest in this study was in relation to the wind tunnel wall interference correction when testing bluff bodies. Glauert (1928) and Rosenhead (1929) derived approximate relations for the cylinder drag and showed that the presence of the channel slightly changes the vortex street spacing. Bearman & Zdravkovich (1978) found that the cylinder vortex shedding frequency increases as the cylinder approaches close to the wall. Kiya & Arie (1980) studied the vortex street formation behind a circular cylinder immersed in a sheared flow using a discrete vortex method and concluded that the background vorticity from the wall boundary layer aids the shed vortices from the cylinder of the same direction of circulation and vice versa. They also showed that the double row of vortices shed from the cylinder would eventually degenerate into a mixing-layer-like coherent structure vortex pattern when the cylinder is brought very close to the wall. Recently, Davis, Moore & Purtell (1984) solved the two-dimensional Navier-Stokes equations while studying the confined flow behind rectangular cylinders and obtained solutions which consisted of regions with moving circulation between the wake and the walls.

Mixing layers are the other important free shear layer that have been extensively studied in the last decade. Subsequent to the observation of the organized motion by Brown & Roshko (1974) in a mixing layer formed between two fluids of different densities, several interesting studies on mixing layers have since been conducted to study the coherent structures present in these flows. As explained by

Birkhoff (1962), the shear layer between two streams moving parallel to each other with unequal velocities are triggered by the Kelvin-Helmholtz instability and roll up into a large scale vortex pattern. Later studies on mixing layers were especially directed towards their downstream growth mechanisms. Although vortex pairing and the subsequent coalescence of these vortices play an important role in the growth rate of a mixing layer, the effect of the evolution of the primary vortices in the lateral direction should not be ignored, as clearly pointed out by Moore and Saffman (1975). The fact that the pairing process is independent of Reynolds number was established by Dimotakis and Brown (1978). Further investigations by Bernal (1981) and Jimnez, Cogollos, & Bernal (1985) revealed the presence of secondary streamwise structures that enhance the shear layer fluid mixing during the mixing transition process. Recently, Ashurst (1979) discovered the presence of coherent structures in the flame fronts of combustion flames as well.

A detailed discussion of the vortex dynamics in mixing layers through numerical solutions of the Navier-Stokes equations has been presented in a series of papers by Corcos and co-workers: two-dimensional motion by Corcos & Sherman (1984) followed by the description of the evolution of three-dimensional instability by Corcos & Lin (1984) and finally the presence of longitudinal vortices by Lin & Corcos (1984). Several investigators proposed (see for example Acton 1976, Kransy 1983) discrete vortex models for this problem and were able to model the vortex pairing process to some extent of rigor.

With this brief overall review of the earlier work in the general area of unsteady vortex motion, the different shear layers that are of specific interest to us are analyzed in the following chapters using the ideas of dynamical models developed for the individual problems.

CHAPTER III

FLOW LINES IN UNSTEADY VORTEX FLOWS

A. Single Vortex Motion

Following the descriptions in Chapter II of the different models developed by earlier researchers for the structure of the vortex cores, a study is presented here to investigate the overall influence of their unsteady motion upon the flow field. The periodic shedding of the vortices from a body and their later convection downstream implies that the particle path, obtained by the application of the force balance at a point, should also show a similar periodic behavior. Preston (1961) indicated that the passing of a vortex in the neighborhood a point imposes a pulse-like pressure disturbance at that point, which, as Abramovich (1979) explains, mimics the flow past an oscillating airfoil. One of the effective ways to simulate a periodic disturbance at a point is through the trochoidal wave model, developed by Milne-Thompson (1960).

In this section, an unsteady vortex model for a vortex convecting with a uniform velocity relative to a free stream is presented. The proposed dynamical model for an isolated vortex motion assumes a forced vortex type of behavior at points inside the domain of the vortex core and a free vortex like characteristics in the region outside of it. For the two-dimensional flow under consideration, the vortex is represented by a rotating cylinder whose axis lies perpendicular to the plane of the flow. It can be shown that the vortex (cylinder) induces considerable pressure and velocity fluctuations only when its convection velocity is not equal to the local flow velocity and the angular velocity of the rotating cylinder differs from the local flow vorticity respectively. By using this dynamical

model, the streakline and pathline patterns for the vortex flow are computed. Qualitative comparisons are made between the computed streakline pattern and the flow visualization results.

Dynamical Model

Consider that a particle Q be situated on the y axis and also on the boundary of a circular cylinder whose center lies on the y axis. Assume that the cylinder is slipping at a uniform velocity u_c parallel to the x axis, and simultaneously rotating at a steady angular velocity ω . The simple sketch of Figure (III-1) shows the configuration and the related quantities.

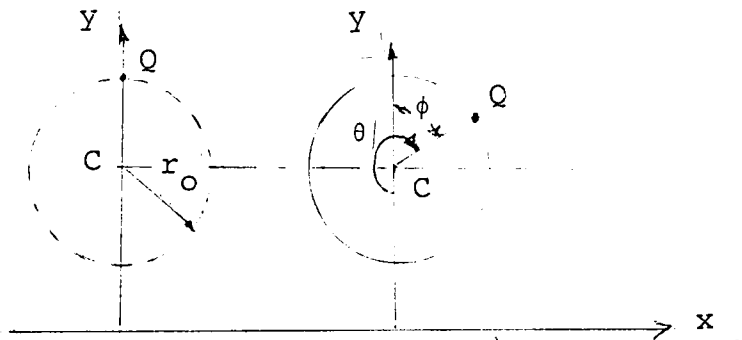


Figure (III-1) Sketch showing the single vortex model configuration details.

Observing from a cylinder fixed coordinate system, the position of the particle at time 't' is given by,

$$x = r_0 \sin \omega t, \quad y = r_0 \cos \omega t$$

$$r_0 \omega = u_\theta = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Transforming to the laboratory coordinate system, the particle path equation be-

comes:

$$x = u_c t + r_0 \sin \frac{u_\theta t}{r_0}, \quad y = y_0 + r_0 \cos \frac{u_\theta t}{r_0} \quad (\text{III-1}).$$

which could be rewritten, upon the substitution:

$$\theta = (u_\theta t / r_0) + \pi, \quad (x / r_0) = X, \quad (y / r_0 - y_0 / r_0 + u_c / u_\theta) = Y, \quad (u_c / u_\theta) = a, \quad b = 1$$

$$X = a\theta - b \sin \theta$$

$$Y = a - b \cos \theta \quad (\text{III-2}).$$

Except for the difference in the location of the origin, the above two equations are the parametric equations for trochoids.

$$\begin{aligned} \text{If } \frac{u_c}{u_\theta} &> 1, \quad \text{a curtate trochoid} \\ \frac{u_c}{u_\theta} &= 1, \quad \text{a cycloid} \\ \frac{u_c}{u_\theta} &< 1, \quad \text{a prolate trochoid.} \end{aligned}$$

The particle path equation. Equation (III-1), is identical to that for a single vortex motion at a velocity u_v relative to a free stream as shown in Appendix A. The quantity u_θ here corresponds to the parameter $(\Gamma / 2\pi r_0)$ of Appendix A, where Γ is the circulation of the vortex. A more general case of a particle being released at a time t_0 when the center of the cylinder lies at an arbitrary x position $u_c t_0$ has also been derived in Appendix A.

It has thus been shown that the present model yields a good description of the flow in the neighborhood of a vortex. This model should adequately represent the unsteady pressure and velocity disturbances caused at a point due to the convective motion of the vortex. The present model should particularly be useful when interpreting the flow around a vortex through the dynamical considerations only. This model could be a viable alternative to the discrete vortex method.

Computed Flow Lines

To examine the validity of this model, some results of streakline pattern computed using this model are compared with those obtained from the flow visualization pictures for the starting vortex motion. Figure (III-2) outlines the pathlines pattern for various $k = u_c/u_\theta$ values. It shows the different kinds of trochoidal pathlines corresponding to various release times, t_0 , of the particles. The wavelengths of these trochoidal pathlines vary according to $\alpha = 2\pi k\rho_0$, where ρ_0 is the distance between the vortex center and the particle release point.

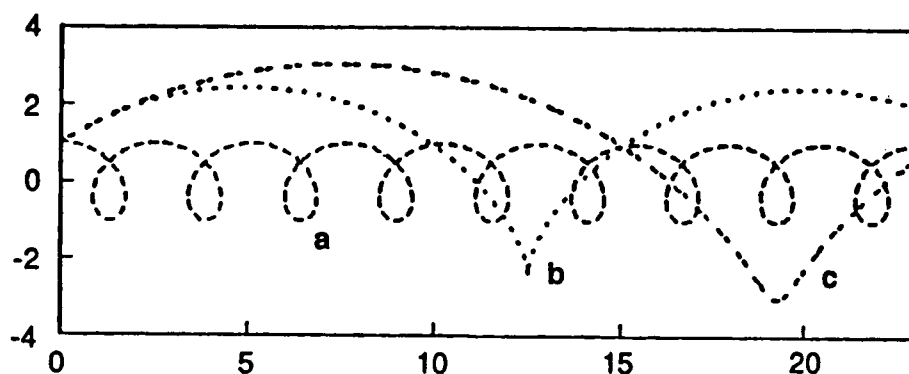


Figure (III-2) Pathlines pattern for various $k = \frac{u_c}{u_\theta}$ values.

The streaklines corresponding to them are shown in Figure (III-3). It is to be noted that the nearly concentric circular pattern of the computed streaklines close to the vortex core is similar to the one considered by Küchemann & Weber (1967) in their spiral model for the core. The relative positions of streaklines and pathlines at the same instant of time are superposed in Figure (III-4). Pullin & Perry (1980) have photographed the streaklines for the starting vortex formed at

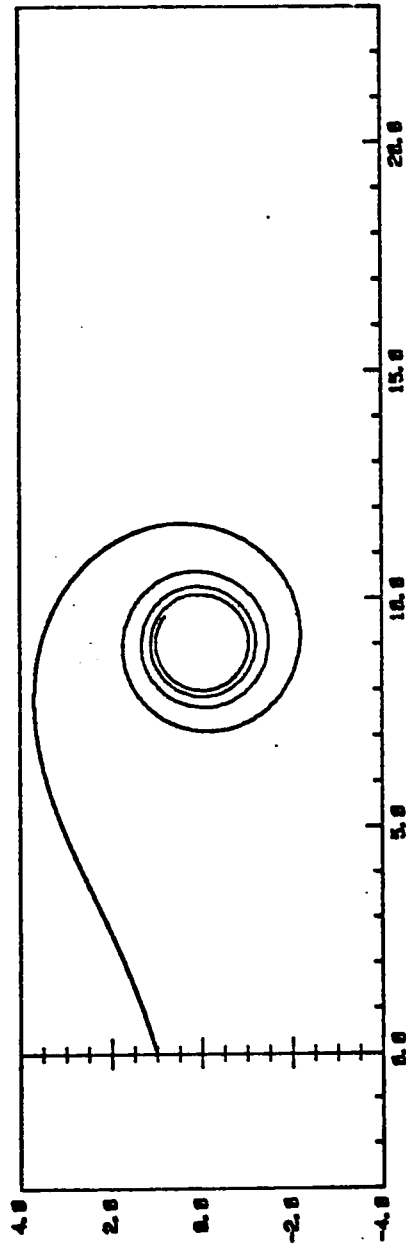


Figure (III-3) Streakline pattern for an isolated vortex motion.

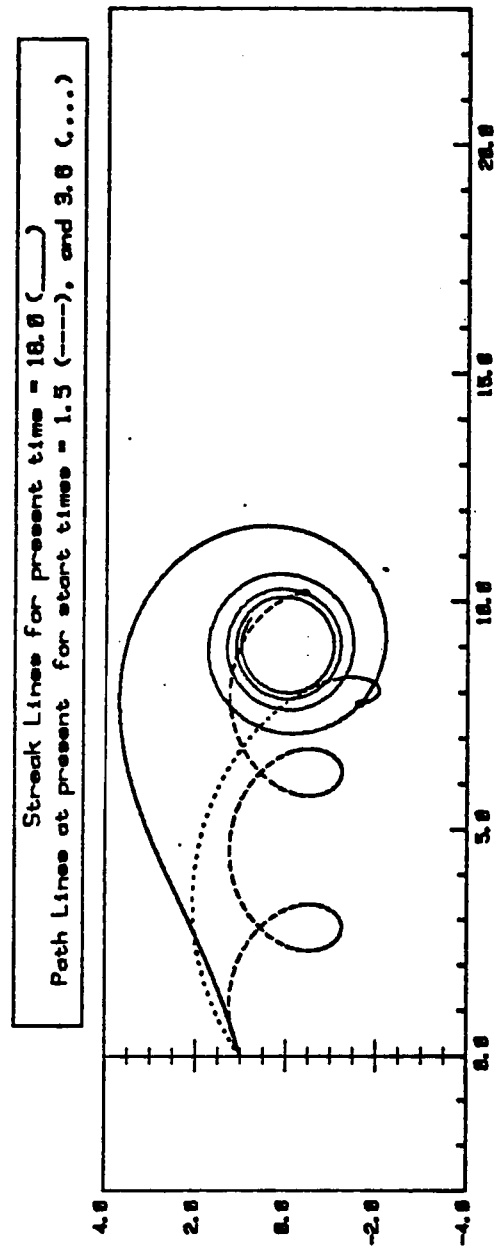


Figure (III-4) Superimposed streakline and pathline for a single vortex motion.

the edge of a two-dimensional wedge in a water tunnel. Figure (III-5) presents one of their streakline pictures showing the unsteady starting vortex at that instant. It can thus be seen that a good representation of the unsteady starting vortices can be made using the simple model presented in this section.

At a higher level in the hierarchy of the unsteady vortex model is the inclusion of the time evolution properties of the vortex, particularly during their formation stages. A model similar to the one described above with the vortex strength varying linearly with time (see Kurosaka, 1986) with α as the proportionality constant could be considered. The pathlines in this case are given by the relation,

$$\frac{dz}{dt} = u_v - i \left(\frac{s_0}{2\alpha} \right) (z - u_v t), \quad \text{for } \sqrt{(x - u_v t)^2 + y^2} < \alpha t$$

$$\frac{dz}{dt} = u_v - i \left(\frac{s_0}{2\alpha} \right) \frac{(\alpha t)^2}{(x - u_v t)^2 + y^2} (z - u_v t), \quad \text{for } \sqrt{(x - u_v t)^2 + y^2} > \alpha t.$$

where s_0 is the original strength of the evolving vortex and u_v is the vortex convection velocity. The streaklines and pathlines are superimposed one over the other for a typical evolving vortex as shown in Figure (III-6), (the evolution parameters of the vortex are indicated in the figure). It can be clearly observed that the effect of the vortex evolution has indeed improved the computed streakline pattern, particularly near the vortex core in comparison with the experimental results of Figure (III-5).

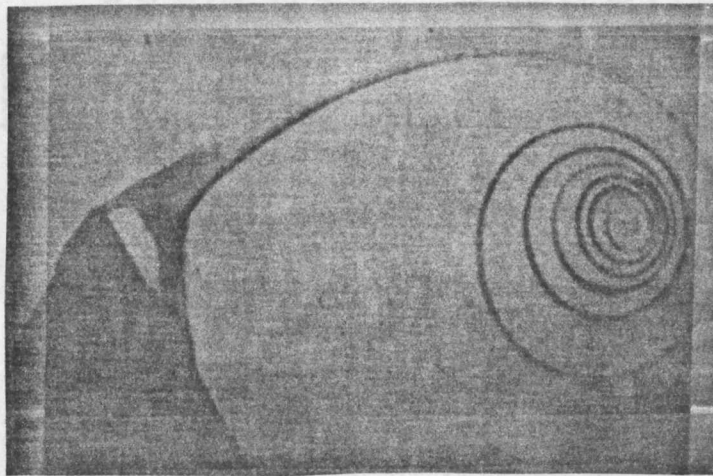


Figure (III-5) Photograph showing the starting vortex behind a wedge
(Ref. Perry & Pullin, 1980)

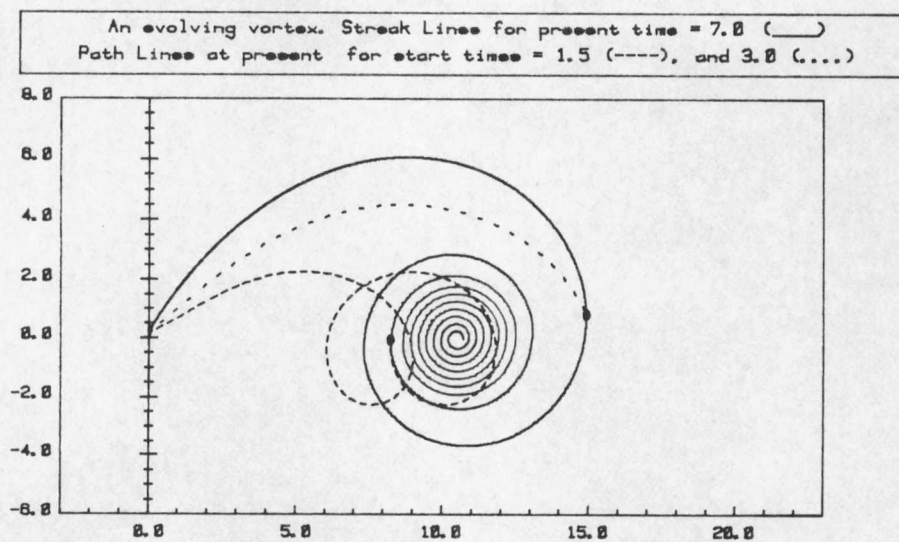


Figure (III-6) Streaklines superimposed on pathlines for an evolving vortex.

B. Vortex Street Wake

Even though an excellent literature is already available for the vortex street wake problem, as reviewed in Chapter II, the methods of analysis so far studied have not incorporated the unsteady nature of the entire flow. Most of the earlier analyses have essentially treated the problem as a steady one by suitably transforming the equations to a coordinate system that moves with the vortex. Although this transformation helped to circumvent the mathematical complexities involved in the analytical studies, the results obtained applying them failed to reveal the important flow details associated with the unsteady flows. With the advent of the renewed interests in the unsteady driven separated flows, as reviewed by Reynolds & Carr (1985), it is necessary to understand the effects of the unsteady mechanism that dictate the flow. For this purpose, an inertial frame of reference fixed with the laboratory coordinates has been chosen. Of particular interest in this study are the flow lines and observation as to how truly the flow visualization pictures can be simulated by the proposed unsteady vortex dynamics models. With this motivation, the classical Kármán vortex street model for the wake behind a bluff body has been chosen for the present study.

Inviscid Vortex Model

The complex potential for the Kármán's infinite double row of staggered vortices when superposed on a uniform onset flow U_∞ can be written (see Milne-Thompson 1960, *ibid*) in a vortex fixed coordinate system as,

$$w(z) = U_\infty z + \frac{i\Gamma}{2\pi} \ln \sin \left[\frac{\pi}{a} \left(z - \frac{ih}{2} \right) \right] - \frac{i\Gamma}{2\pi} \ln \sin \left[\frac{\pi}{a} \left(z - \frac{a}{2} - \frac{ih}{2} \right) \right]$$

where Γ and $-\Gamma$ are the circulation of each vortex in the top and bottom rows: a and h are the longitudinal spacing between the vortices of the same row and lateral

distance between opposite rows respectively. The vortex street can be shown to move with a uniform velocity,

$$U_{st} = \frac{\Gamma}{2\pi a} \tanh \left(\frac{\pi h}{a} \right)$$

Since it is intended to study the streakline pattern in a laboratory coordinate system, it is necessary to change the equations to this inertial frame of reference by superposing the vortex velocity distance, $U_{st}t$. The detailed derivations of the particle path is outlined in Appendix B; only the final expressions are given here. After proper non-dimensionalization of the equations, the particle path equation can be written as,

$$\frac{dz}{dt} = \frac{U_\infty}{U_{st}} \left\{ \frac{1}{\frac{h}{a}} - \frac{i\Gamma}{2hU_\infty} \left[\cot \left[\frac{\pi h}{a} \left(\bar{z} - \frac{t}{\frac{h}{a}} + \frac{i}{2} \right) \right] - \cot \left[\frac{\pi h}{a} \left(\bar{z} - \frac{t}{\frac{h}{a}} - \frac{i}{2} - \frac{1}{2\frac{h}{a}} \right) \right] \right] \right\}$$

and the vortex street velocity is once again written in the laboratory coordinate system as,

$$\frac{U_{st}}{U_\infty} = 1 - \frac{\Gamma}{hU_\infty} \left(\frac{h}{a} \right) \tanh \left(\frac{\pi h}{a} \right)$$

The unknown parameters in these equations are the vortex spacing ratio, (h/a) , and the non-dimensional circulation, (Γ/hU_∞) . These values could be obtained from experimental data. Alternately, Salley (1975) proposed a novel scheme to determine their values based on wake momentum consideration. In particular, to calculate the circulation of the vortices shed behind a circular cylinder, the method requires the knowledge of the Strouhal number, S , and drag coefficient, c_d , from experiments, or otherwise, at any Reynolds number of the flow. Using his scheme, the non-dimensional circulation can be determined from the relation:

$$\frac{\Gamma}{hU_\infty} = \frac{1}{\left(\frac{h}{a} \right)} - \frac{\left(\frac{h}{a} \right) + \left[\left(\frac{h}{a} \right)^2 - \frac{c_d}{\left(\frac{a}{d} \right)} \left\{ \frac{1}{\pi} - 2\frac{h}{a} \tanh \left(\frac{\pi h}{a} \right) \right\} \right]^{\frac{1}{2}}}{\frac{1}{\pi} - 2\left(\frac{h}{a} \right) \tanh \left(\frac{\pi h}{a} \right)}$$

where the cubic equation for the ratio (a/d) , d being the diameter of the cylinder, is:

$$A\left(\frac{a}{d}\right)^3 + B\left(\frac{a}{d}\right)^2 + C\left(\frac{a}{d}\right) + c_d = 0$$

with

$$\begin{aligned} A &= S^2 \left[8\left(\frac{h}{a}\right) \coth\left(\frac{\pi h}{a}\right) - \frac{4}{\pi} \coth^2\left(\frac{\pi h}{a}\right) \right] \\ B &= S \left[\frac{8}{\pi} \coth^2\left(\frac{\pi h}{a}\right) - 12\left(\frac{h}{a}\right) \coth\left(\frac{\pi h}{a}\right) \right] \\ C &= \left[\frac{4}{\pi} \coth^2\left(\frac{\pi h}{a}\right) - 4\left(\frac{h}{a}\right) \coth\left(\frac{\pi h}{a}\right) \right] \end{aligned}$$

In our computations, the values for the parameters (Γ/hU_∞) and (h/a) have been obtained from the experimental values of Griffin & Ramberg (1975) who quoted these values to be 2.234 and 0.18 respectively for a Reynolds number of 500.

The initial conditions correspond to the point at which the fluid particles are released. In all our studies, the computations are started when one of the vortices that is numbered '0' is located on the y -axis and the particles are always released from the y -axis. To obtain a smooth trace of the streaklines, we used a sixth order Runge-Kutta-Fehlberg scheme (1967) with error control and kept the time lapse between the successive particle release to be sufficiently small. The solution procedure is once again highlighted in Appendix B.

Flow Lines Computations

The computed pathlines shown in Figure (III-7) once again indicate the trochoidal pattern observed in the previous section for the case of the starting vortex motion. It is easy to realize that by just looking at the pathline pattern, there is no clue about the presence of any vortices and other well known flow features of a wake behind the cylinder. However, pathlines represent the dynamic force balance acting on a particle and have an important significance as will be highlighted later.

Figures (III-8a) and (III-8b) show the pathline pattern of particles released from various locations on the y -axis at a particular start time. In all the pathline and streakline pictures, the vortices are numbered in the order in which they enter the region of interest as they traverse from left to right with the street velocity, U_{st} , the number zero corresponds to the one which was located on the y -axis at the start of the numerical experiments.

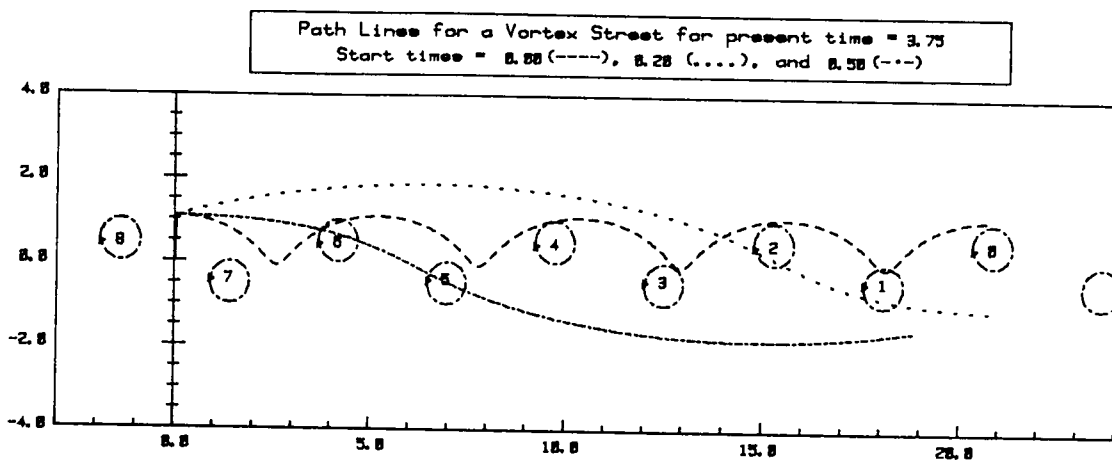
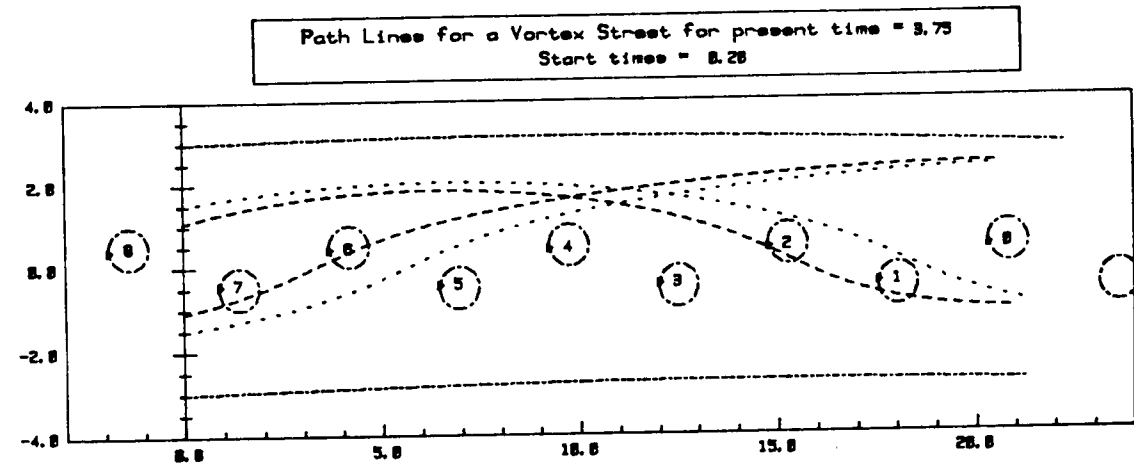


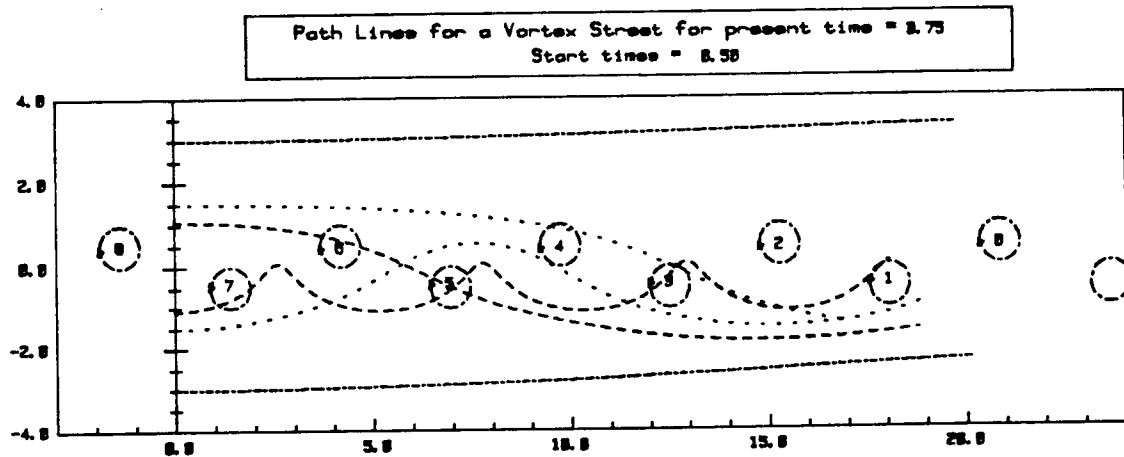
Figure (III-7) Pathline pattern at the present time $t = 3.75 t_0 = 0.0, 0.2, \text{ and } 0.5$.

In addition to the pathlines belonging to the trochoidal type, there is another one that corresponds to the class that is categorized as the cross-over or the degenerate-trochoid type. In this case, the particle that is released from one side of the street travels across the street and arrives at the other side (see Figure (III-7)). Due to the periodicity of the basic flow field, this particle must, further downstream return through the street to its starting side. In a real flow, where the vortex strength is decreasing due to viscous diffusion, such a cross-over process may degenerate further such that a permanent reversal takes place. A schematic of such a particle trajectory has been indicated on Figure (III-9).

The flow visualization picture shown in Figure (III-10), taken from the UTSI water tunnel, proves the fact that such pathlines indeed exist in a wake flow. Although this picture shows the streakline, since a particle is present on the side opposite to the side from which it was injected implies that it has in fact gone through the wake center line.



(a) $t_0 = 0.2$,



(b) $t_0 = 0.5$.

Figure (III-8) particles released from $y = 1.05$ at different start times:

(a) $t_0 = 0.2$, (b) $t_0 = 0.5$.

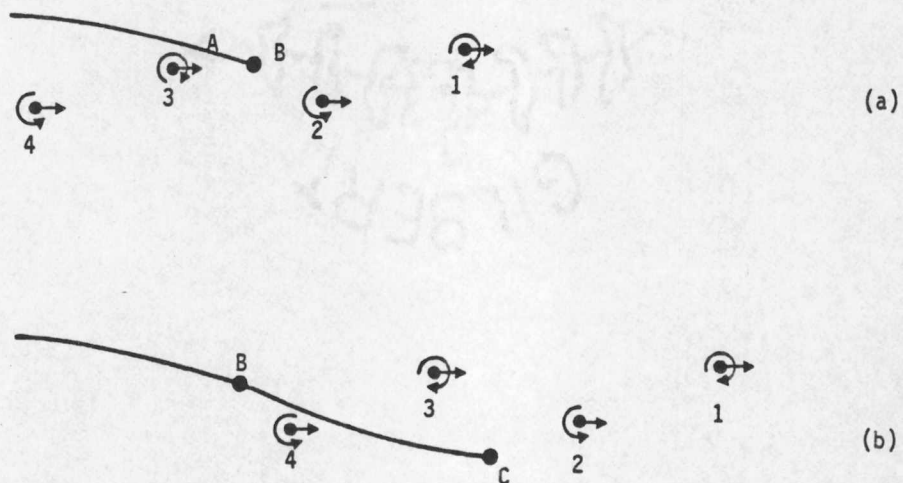


Figure (III-9) Cross-over pathlines in a vortex street.
 (a) entry of a fluid particle
 (b) exit

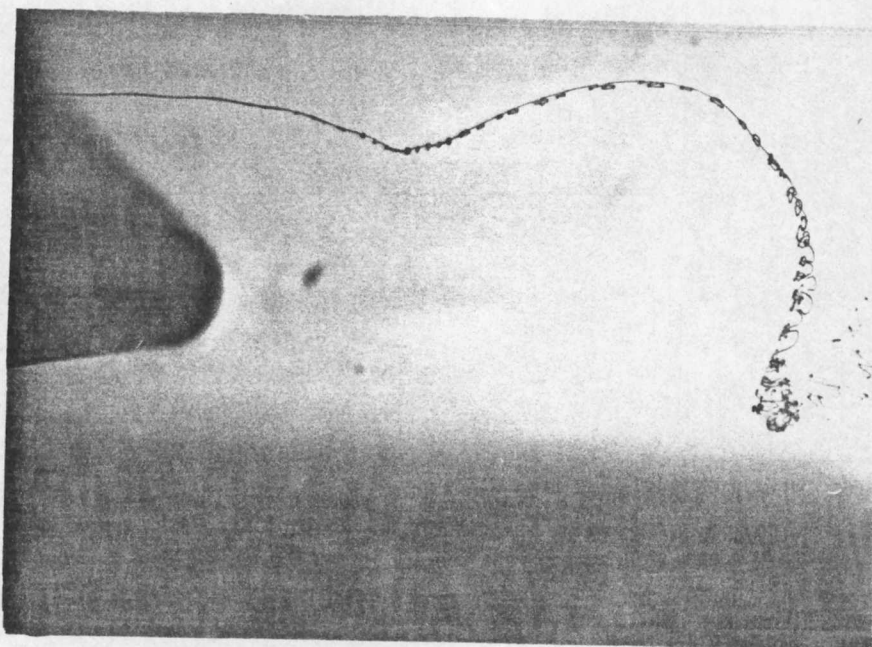


Figure (III-10) Dye lines showing cross-over pathline; photograph obtained from UTSI water tunnel.

To understand the dynamical reasons for the presence of the cross-over type pathlines in a real wake flow, the following explanations are appropriate. The initial entrainment of the particle into the street, as shown in Figure (III-9a), is caused by the slight imbalance between the vertical components of velocity induced by the vortices 1 and 3 on fluid particles such as B. Once at B, the dynamics due to vortices 2 and 3 forces the particle through the street. Soon after, the particle is pushed out of the street by a similar unbalance after the particle reaches the lower row of vortices, approaching C, near the midway between the vortices 2 and 4 as shown in Figure (III-9b). The importance of this pathline will be highlighted during the discussions on total temperature separation in Chapter V and again in Chapter VI while studying the effects of a channel wall in a wall bounded wake flow. In the case of a wall bounded vortex street model, due to the interference caused by the presence of channel walls, the existence of these types of pathlines will become more apparent. The different families of pathline curves discussed here have also been obtained by Davis & Moore (1982) in their numerical simulation of the vortex shedding process behind a rectangular cylinder.

The streakline patterns have been reconstructed from the pathline by joining the present location of each particle which are continuously released from the same starting point. Figure (III-11) shows the streakline pattern at time $t=3.75$ for various starting points which looks very similar to the flow visualization picture in the wake of a circular cylinder, as shown in Figure (III-12), which has been obtained from Cornish (1982). It is observed that the simple inviscid unsteady model of the vortex street shows good comparison and highlights the intricate details of the flow. This proves that the flow field results can be effectively simulated to sufficient accuracy by the inviscid unsteady vortex model itself.

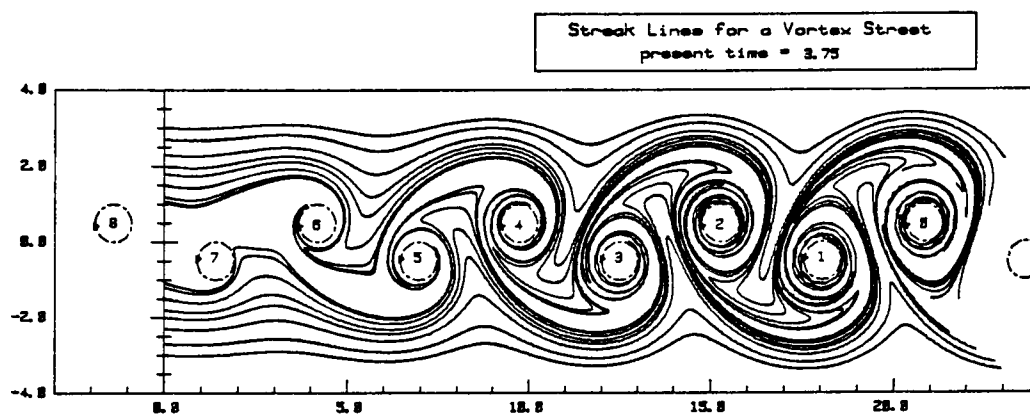


Figure (III-11) Numerical flow visualization of the vortex street wake showing streaklines at the present time $t = 3.75$.



Figure (III-12) Smoke flow visualization photograph (Ref. Cornish (1982))

In addition to the pathline and streakline patterns, it is also necessary to compute the instantaneous streamline pattern because of their importance regarding the singularities in the topology (see by Perry, Chong & Lim 1982, *ibid*) which has significance in the turbulence production mechanism. In the case of a Kármán vortex street, the instantaneous streamlines at time t can be obtained for constant values of the stream function ψ . The stream function equation in the laboratory coordinate system can be derived as:

$$\psi = U_{\infty}y + \frac{\Gamma}{4\pi} \ln \left[\frac{\cosh\left\{\frac{2\pi}{a}\left(y - \frac{h}{2}\right)\right\} - \cos\left\{\frac{2\pi}{a}(x - U_{st}t)\right\}}{\cosh\left\{\frac{2\pi}{a}\left(y + \frac{h}{2}\right)\right\} + \cos\left\{\frac{2\pi}{a}(x - U_{st}t)\right\}} \right]$$

Figure (III-13) shows the instantaneous streamlines for the vortex street which illustrates the presence of saddles and centers. It can be remarked that the vortex fixed coordinate system also shows a similar streamline pattern.

Viscous Correction

To incorporate the viscous diffusion effects on the vortex street, a procedure similar to the one proposed by Schaefer & Eskinazi (1956) in which the strength of each vortex in the vortex street is multiplied by the Lamb-Oseen exponential decay solution,

$$\Gamma = \Gamma_0 \left[1 - \exp\left(\frac{-r^2}{4\nu t}\right) \right]$$

has been chosen. In this relation, Γ is the strength of the vortex at time t , whose initial strength at time $t = 0$ was Γ_0 ; r is the distance between the point of interest and the vortex center; and ν is the kinematic viscosity. To obtain the particle path equation with viscous effects, a semi-infinite street of vortices with an exponential decay law have been added as they cross the y -axis and another semi-infinite row of constant strength vortices are subtracted from the original infinite Kármán vortex street.

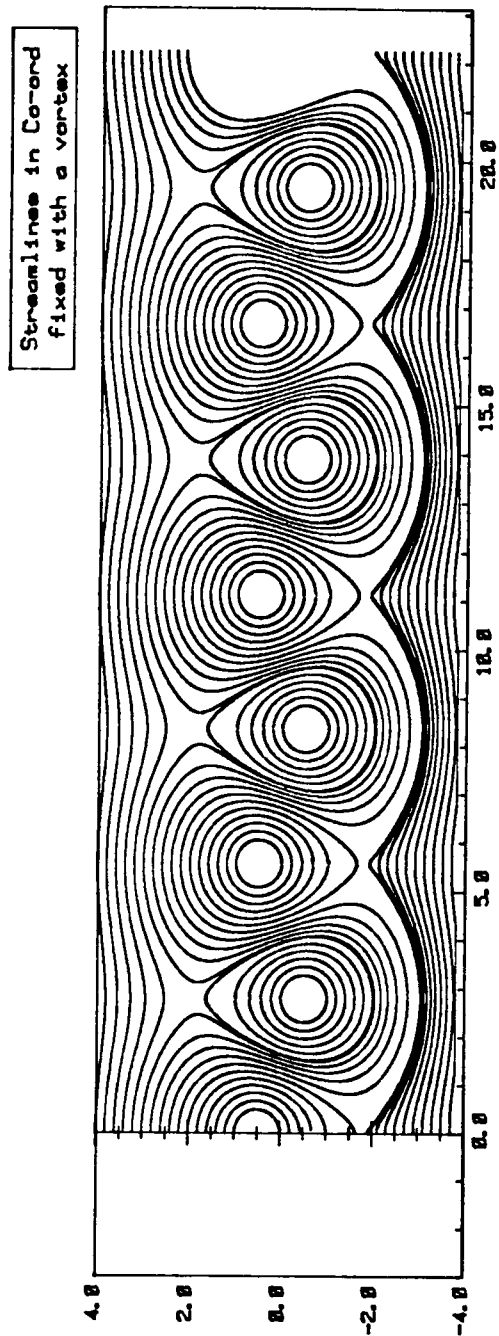


Figure (III-13) Instantaneous streamline pattern for the vortex street.

Following the same procedure as discussed in Appendix B, and after some algebra, and by introducing another non-dimensional variable $\nu^* = (\nu/U_{st}h)$, the particle path equation is derived as:

$$\begin{aligned} \frac{dz}{dt} = & \frac{U_\infty}{U_{st}} \left\{ \frac{1}{\lambda} - \frac{i}{2} \frac{\Gamma}{hU_\infty} \left[\cot \left[\frac{\pi h}{a} \left(\bar{Z} - \frac{t}{\lambda} + \frac{i}{2} \right) \right] - \cot \left[\frac{\pi h}{a} \left(\bar{Z} - \frac{t}{\lambda} - \frac{i}{2} - \frac{1}{2\lambda} \right) \right] \right] \right. \\ & - \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \frac{[1 - \exp\{-|Z^+|^2\lambda/4\nu\tau\}]}{Z^+} \\ & - \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \sum_{n=1}^{\infty} \frac{[1 - \exp\{-|Z^+ - \frac{n}{\lambda}|^2\lambda/[4\nu(n+\tau)]\}]}{(Z^+ - \frac{n}{\lambda})} \\ & + \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \frac{[1 - \exp\{-|Z^-|^2\lambda/4\nu\tau_1\}]}{Z^-} \\ & + \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \sum_{n=1}^{\infty} \frac{[1 - \exp\{-|Z^- - \frac{n-1/2}{\lambda}|^2\lambda/[4\nu(n+\tau_1)]\}]}{(Z^- - \frac{n-1/2}{\lambda})} \\ & + \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \frac{1}{Z^+} + \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \sum_{n=1}^{\infty} \frac{1}{(Z^+ - \frac{n}{\lambda})} \\ & \left. - \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \frac{1}{Z^-} - \frac{i}{2\pi\lambda} \left(\frac{\Gamma}{hU_\infty} \right) \sum_{n=1}^{\infty} \frac{1}{(Z^- - \frac{n-1/2}{\lambda})} \right\} \end{aligned}$$

where h/a has been replaced by λ for compactness; and

$$\begin{aligned} Z^+ &= \left(Z - \frac{\tau}{\lambda} - \frac{i}{2} \right) \\ Z^- &= \left(Z - \frac{\tau_1}{\lambda} - \frac{i}{2} \right) \end{aligned}$$

with $\tau(0,1) = [t - int(t)]$ and $\tau_1 = \max(0, \tau - 1/2)$. In the above equation, the first three terms on the right hand side are identical to the infinite Kármán vortex street, the fourth and fifth terms represent the viscous vortex terms added for the top row, and sixth and seventh are the viscous vortex terms added to the bottom row; the constant strength vortices corresponding to these four terms are subtracted out in the next four terms.

In this model, no account for the effects of viscous diffusion on the street velocity or the spacing ratio has been included because they are considered to be higher order effects on the basis of the conclusions by Hooker (1934) through his experimental and theoretical considerations of the amount of back flow in the vortex street due to viscous diffusion. As seen from Figure (III-14) the effect of viscosity is not significant.

Thus, it has been demonstrated that the vortex street model with the inclusion of the unsteady effects is accurate for the unsteady wake study of the overall effects of the flow.

C. Plane Mixing Layer

Vortex Model and Streakline Computation

To investigate the primary effects of a mixing layer on the overall flow field, an infinite row of vortices moving at a uniform velocity U_m is considered. This is equivalent to introducing a uniform infinite row of vortices of constant strength Γ into a uniform stream flow with velocity U_m . The complex potential for this flow at time $t = 0$ takes the form,

$$w(z) = \frac{i\Gamma}{2\pi} \ln \left[\sin \frac{\pi z}{a} \right]$$

where a is the distance between each vortex. It is important to note that the velocities induced by these vortices at large distances above and below the plane mixing layer will respectively be $(\Gamma/2a)$ and $-(\Gamma/2a)$, so that a non-uniform free stream velocity profile naturally results. With the introduction of an equivalent free stream, U_m , the velocities at the far field boundaries above and below the mixing layer becomes $U_m + (\Gamma/2a)$ and $U_m - (\Gamma/2a)$.

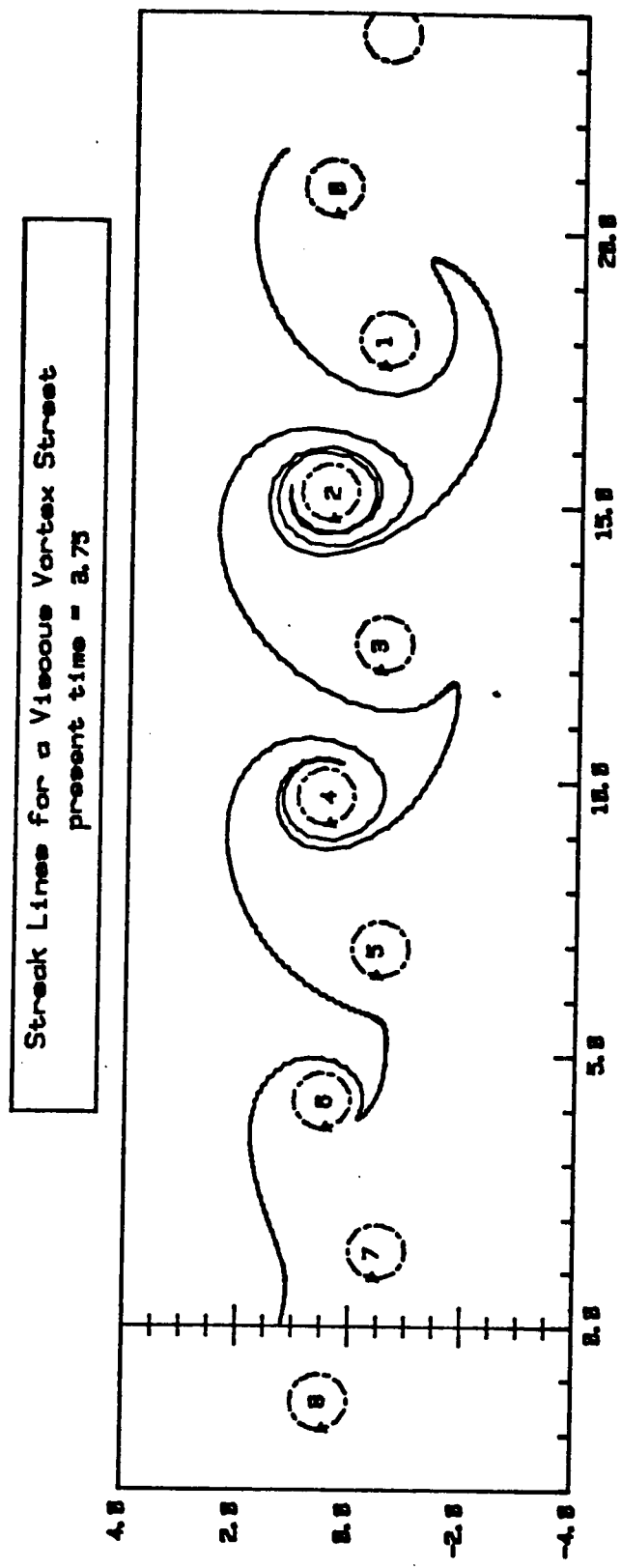


Figure (III-14) Streaklines showing the effects of viscous diffusion of the vortex street vortices.

This is in agreement with the general observation that the mixing layer moves with the average velocity of the top and bottom streams. In other words, the mixing layer developed by the two shearing streams is approximately simulated by the above vortex model.

At an arbitrary time t , the complex potential for this inviscid model becomes,

$$w(z) = U_m z + \frac{i\Gamma}{2\pi} \ln \left[\sin \frac{\pi}{a} (z - U_m t) \right]$$

Thus the particle path equation can be derived as,

$$\frac{dz}{dt} = U_m - \frac{i\Gamma}{2a} \cot \left[\frac{\pi}{a} (\bar{z} - U_m t) \right]$$

which could be written in terms of non-dimensional variables as,

$$\frac{dz}{dt} = 1 - \frac{i\Gamma}{2aU_m} \cot [\pi(\bar{z} - t)]$$

The streakline pattern for this plane mixing layer is shown in Figure (III-15) which looks similar to that of Figure (III-16), the photograph taken by Roberts, F. A., Dimotakis, P. E. & Roshko, A. (see Van Dyke, 1982).

In retrospect, it has been shown that for most free shear flows it is necessary to incorporate the unsteady motion of the vortices in the appropriate vortex models in order to adequately represent the long range effects of the flow. The specific details of the intricate structures of the streakline pattern are elaborated in the next Chapter.

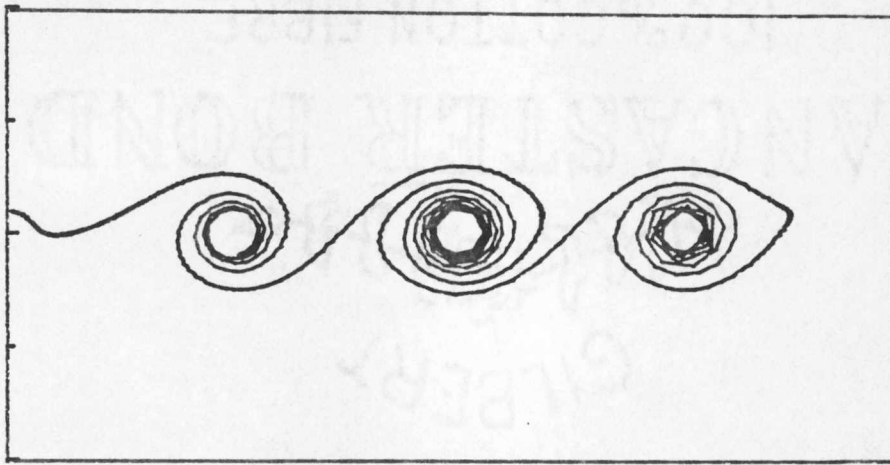


Figure (III-15) Computed streakline for a mixing layer simulated by infinite row of vortices with unsteady motion.

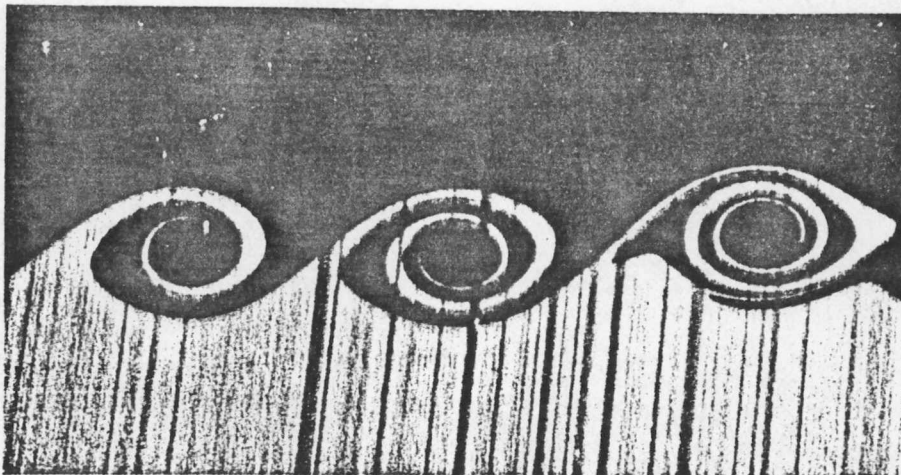


Figure (III-16) Photograph of mixing layer flow visualization experiment (Ref. Van Dyke (1982))

CHAPTER IV

ILLUSORY APPEARANCE OF STREAKLINES

In this Chapter, the discussion on the streaklines and pathlines for different flows are continued and the subtle features of the unsteady flow patterns observed in free shear flows are explained to highlight the illusion created by them. Often useful, sometimes indispensable and always aesthetically appealing as pictures are, the interpretation of streaklines, the flow lines most commonly used in unsteady flow visualization, demands a vigilant caution. The content of this Chapter can also be found in Kurosaka & Sundaram (1986).

A strong caveat against the deceptive illusion created by the streaklines in unsteady flows was voiced by Hama (1962), who numerically treated the problem of the critical layer in a sheared flow and showed that the curling streaklines observed in this flow gives a visual impression of the presence of vortices at regions where there were not any. Despite this early warning and also the remarks of caution by Williams & Hama (1982) indicating the false appearances of the vortex pairing process in a laminar mixing layer, considerable amount of confusion and misinterpretations of the physics behind the flow still prevail in the area of unsteady vortex flows. This Chapter highlights the importance of these illusions and explains the underlying reasons for their occurrences.

Examples of Illusory appearance

Three streaklines displayed in Figure (IV-1) correspond to the snapshots of fluid particles continuously released from a fixed point: they appear to convey somehow an impression that either the spiral size or its strength is increasing with time. They are, however, none other than streaklines winding around a classical

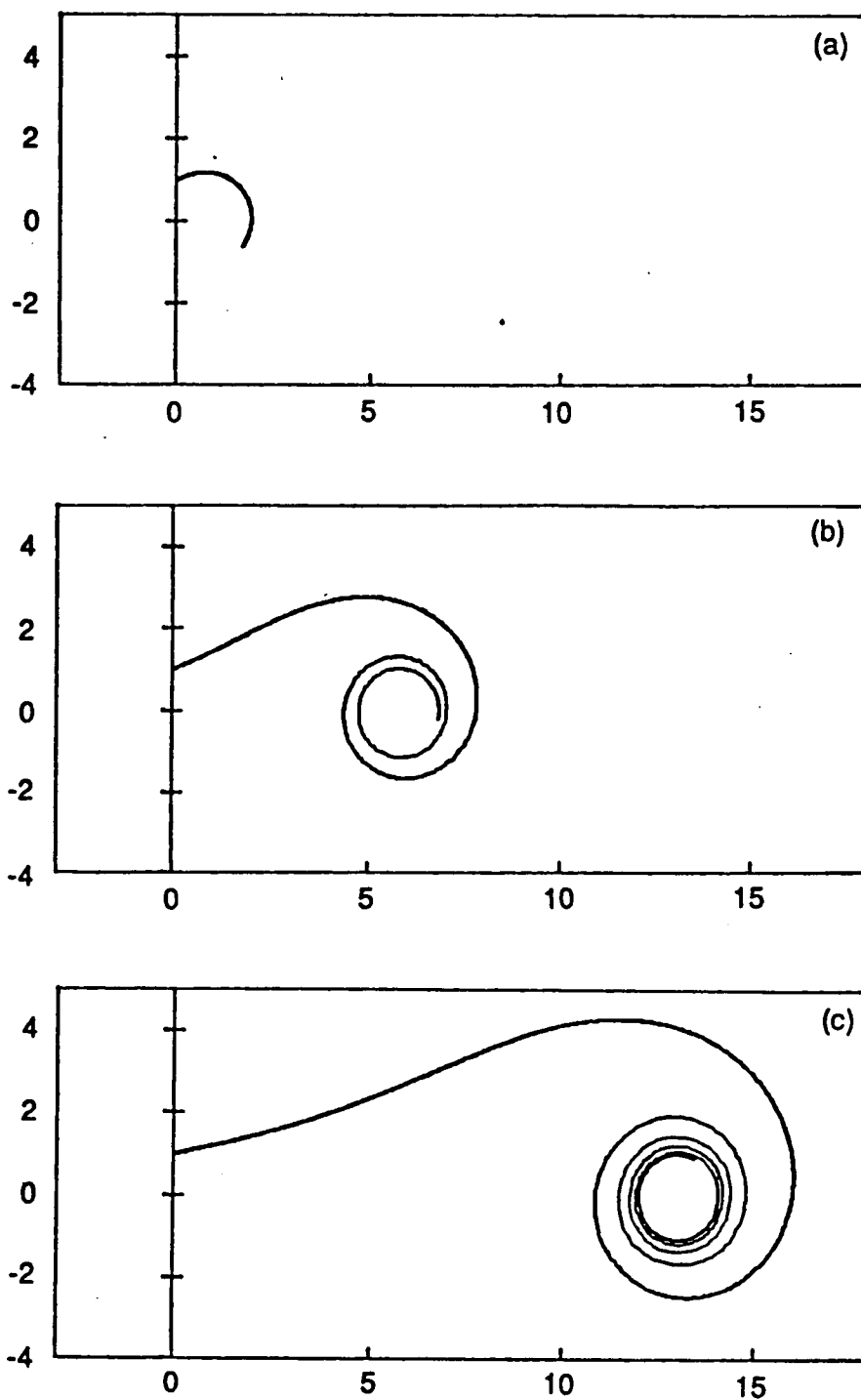


Figure (IV-1). Streaklines for a Potential Vortex.

(a) $t = 0.7$, (b) $t = 4.5$, (c) $t = 10.0$;

$u_v = 1.3$, $\Gamma = 20.0$, $Y_0 = 1.0$

potential vortex of constant strength, being convected from left to the right as studied in Section A of Chapter III.

In spite of the appearances of growth of the streaklines, the corresponding pathlines trace precisely periodic curves called trochoids. In Figure (IV-2), these three kinds of pathlines traced by the three particles are superimposed on the streaklines of Figure (IV-1); dots in the streaklines represent positions occupied by the particles at the time corresponding to **a**, **b**, and **c**, respectively (the similar superposition of **b** and **c** is not shown for Figure (IV-1a) because of the fact that the particles **b** and **c** are released later than the corresponding time).

The reason for the false impression of an apparent growth in this vortex of constant strength is as follows. At $t = 0$, where the vortex center is just passing the origin and the injection of the particles at y_0 begins, the velocity induced by the vortex on the first particle is in the x direction only. As soon as the vortex passes the origin, however, it starts to induce the y component of the velocity, as shown in Figure (IV-3), upon the subsequently released particles and fling them upwards. This, together with the increasing number of turns around the vortex, creates the misleading deception. Even near the core of the vortex, the pathlines still remain as trochoids and this is an answer to another false impression: near the central portion of a tightening spiral vortex the fluid particle between adjacent turns, their radial distance becoming shorter and shorter, is seemingly squeezed out but does not have any place to go in a two-dimensional flow.

Physics of the Turning Points

The illusory appearance of growth persists even for the classical infinite Kármán vortex street of constant strength, as observed in Figure (IV-4) where the snap-

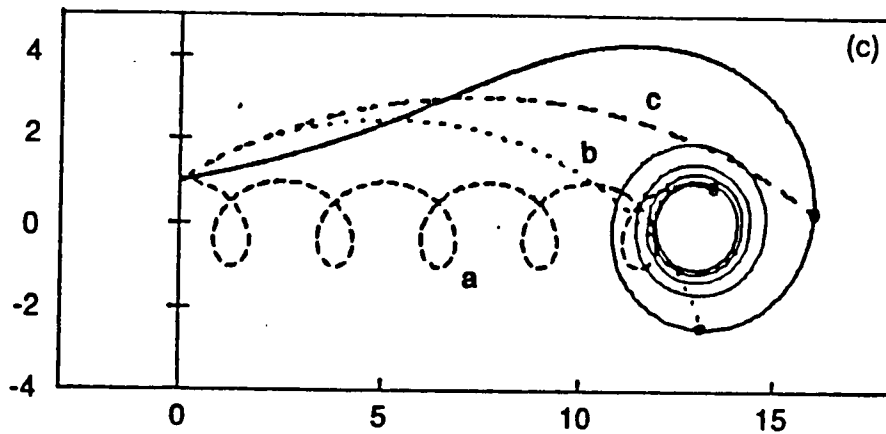
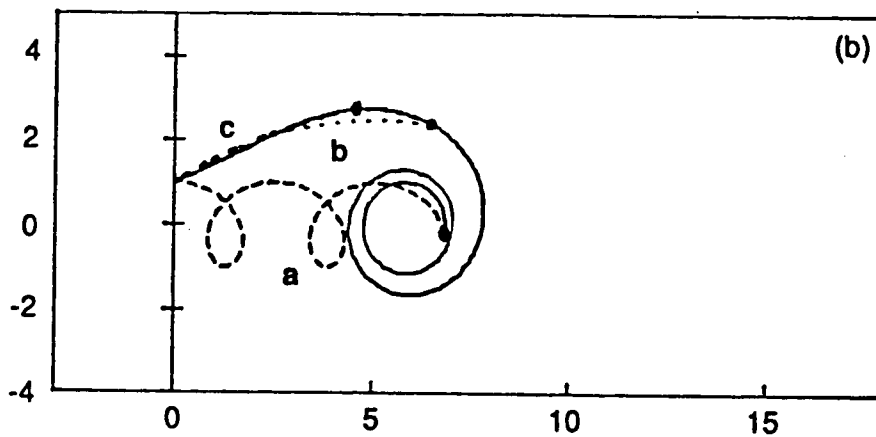
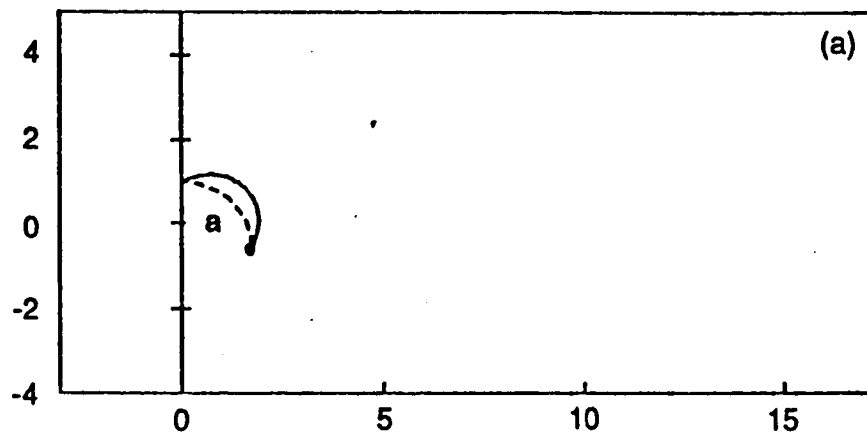


Figure (IV-2). Superposition of pathlines upon streaklines of Figure 1; a (prolate) corresponds to $t_0 = 0.0$, b (cycloid) to 1.75 ; and c (curtate) to $t_0 = 2.2$.

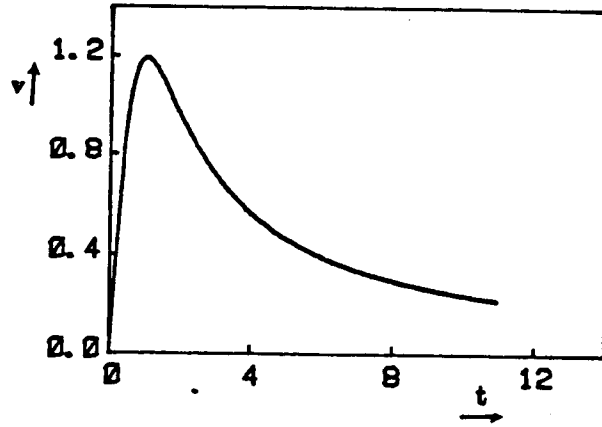


Figure (IV-3). y component of the fluid velocity, v , at the particle release point corresponding to figures 1 and 3.

shots of streaklines, shown as solid lines, are superposed with pathlines. Figure (IV-4) also reveals an additional illusion around what may be called 'turning points': the coiling pattern around the vortex 2 of (b), shown as a close-up view in Figure (IV-5), appears to unwind near the turning point T. In reality, however, the fluid particles do not reverse their course near T. Instead, as shown in Figure (IV-6), where the x and y positions are plotted against t_0 , the turning point is simply the one stationary with regard to the release time of the particles:

$$\frac{\partial x}{\partial t_0} = 0 \quad \text{and} \quad \frac{\partial y}{\partial t_0} = 0 \quad \text{simultaneously.} \quad (\text{IV-1})$$

If we denote the distance from a point on the pathline, (x, y) , to the vortex center (x_c, y_c) under consideration (vortex 2 of Figure (IV-5), for instance) is denoted by

$$\rho_0 = [(x(t_0, t) - x_c(t))^2 + (y(t_0, t) - y_c(t))^2]^{1/2}, \quad (\text{IV-2})$$

then (IV-1) leads to

$$\frac{\partial \rho_0}{\partial t_0} = 0. \quad (\text{IV-3})$$

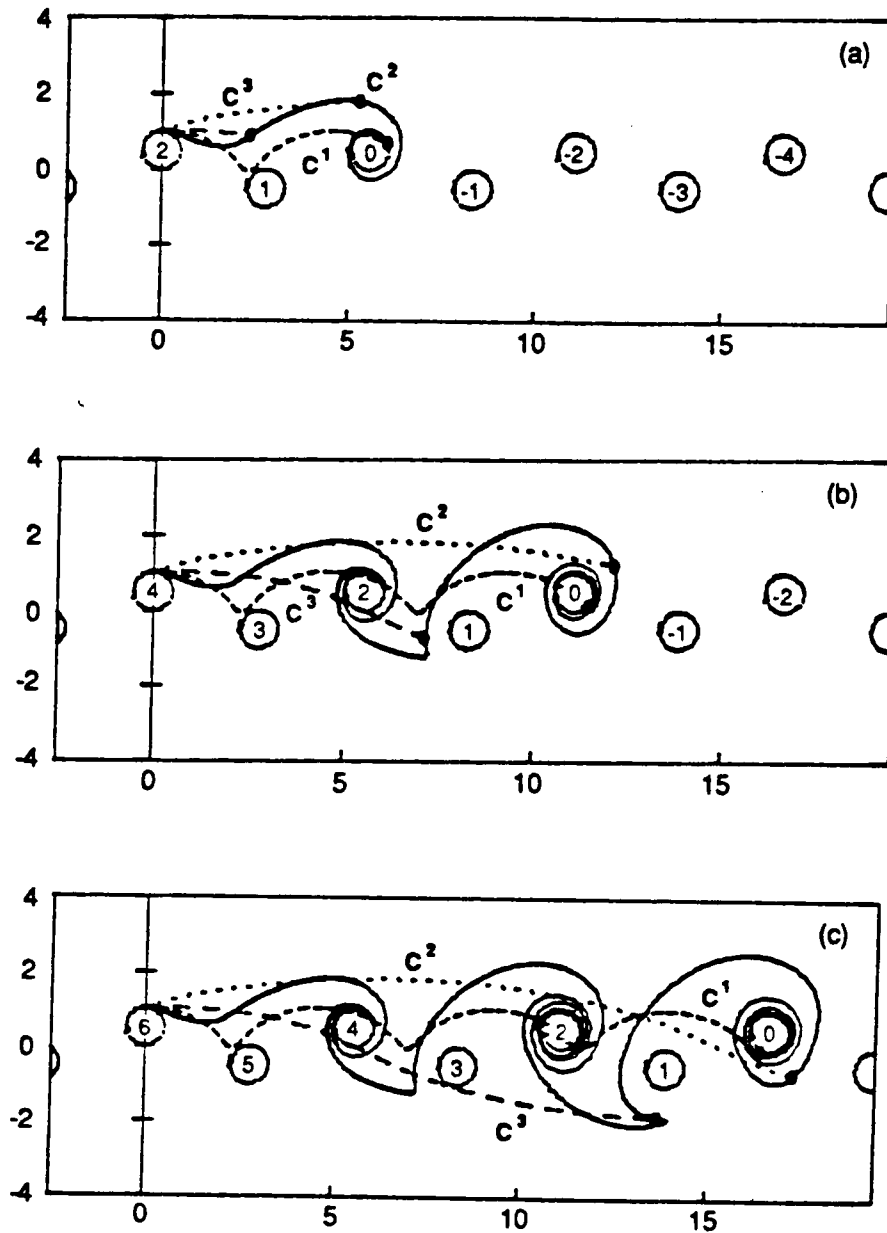


Figure (IV-4). Superposition of pathlines upon streaklines for the infinite Kármán Vortex Street;

(a) $t = 1.0$, (b) $t = 2.0$, (c) $t = 3.0$. $\frac{\Gamma}{hu_{\infty}} = 2.234$, $\frac{h}{a} = 0.180$, $t_0 = 0.0$, $y_0 = 1.05$, $h = 1.0$.

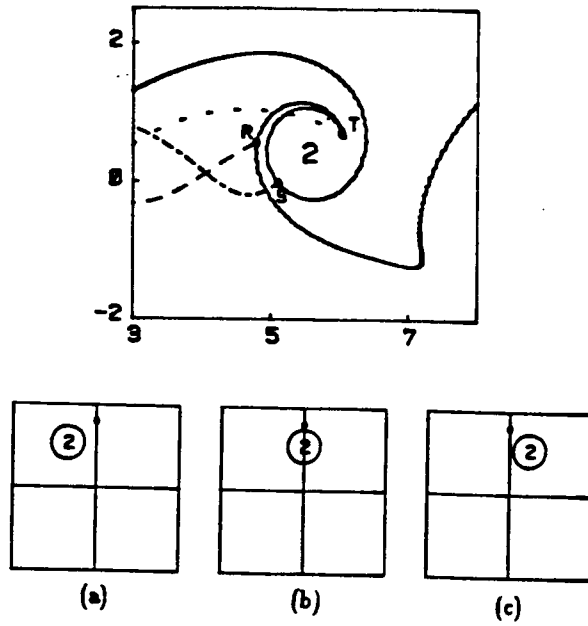


Figure (IV-5). Closeup of 'turning point'; particle T is released just when the vortex 2 is passing the y axis at $t_0 = 1.0$ and particles R and S are released slightly before and after the vortex 2 passes the y axis respectively.

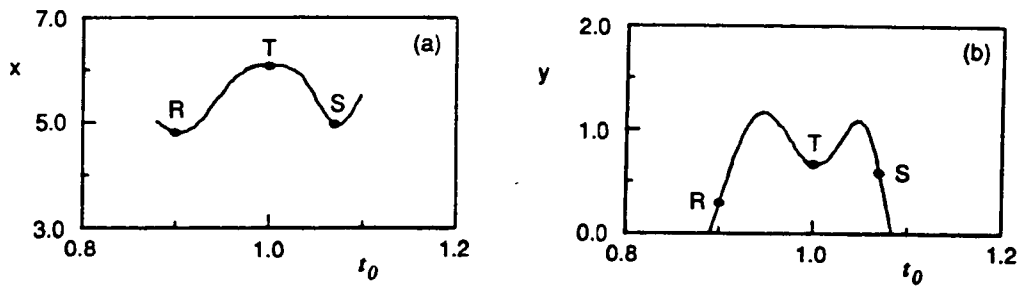


Figure (IV-6). Variation of x and y positions around the 'turning point' T of figures (IV-5).

(a) x - position, (b) y -position

Since for points near the vortex 2 the ρ_0 value remains unchanged, this implies that the turning point corresponds to the situation where at the time of the particle-release, the distance between the particle release point and the vortex center becomes a minimum. Indeed, as shown in the inset of Figure (IV-5), when the center of the vortex 2 is just passing the y axis, the particle corresponding to the turning point is released at y_0 ; thereafter that particle always constitutes the turning point around the vortex 2. Due to the present manner of particle release, there is no turning point around the vortex 0 in Figure (IV-5) and this is the reason for the absence of the turning point for an isolated vortex of Figure (IV-1). Appendix C contains the mathematical proof for the absence of the turning point in the case of an isolated vortex motion.

Figure (IV-4c) also displays 'turning points' belonging to a different family, observable around the lower row of vortices but still along the same streakline consisting of particles released at $y_0 = 1.05$. These 'turning points' are indexed as T_1 in Figure (IV-7). Similar sets of turning points exist along another streakline when the particles are released at the opposite position $y_0 = -1.05$. The flow visualization picture in Figure (IV-8), taken from Perry, Chong, & Lim (1982) also indicates these family of 'turning points'. It is necessary to point out that they made an erroneous interpretation in the paper which said that the presence of dye, near a turning point, from the opposite side to the release side is due to buoyancy. In reality, these turning points correspond to the particle release time whence a vortex belonging to the opposite row is closest to the released particle (note that this is slightly in error due to the fact that the turning point in this case is not sharp). The x and y variations with respect to t_0 corresponding to this turning point is given in Figure (IV-9).

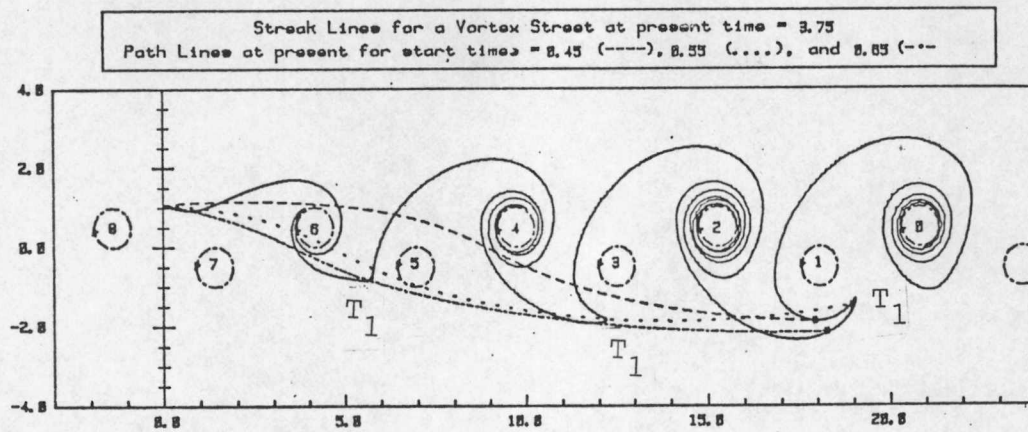


Figure (IV-7). Computed results showing 'turning point' of a different family, T_1 . Also superposed are the pathlines for particles released close to the turning point.

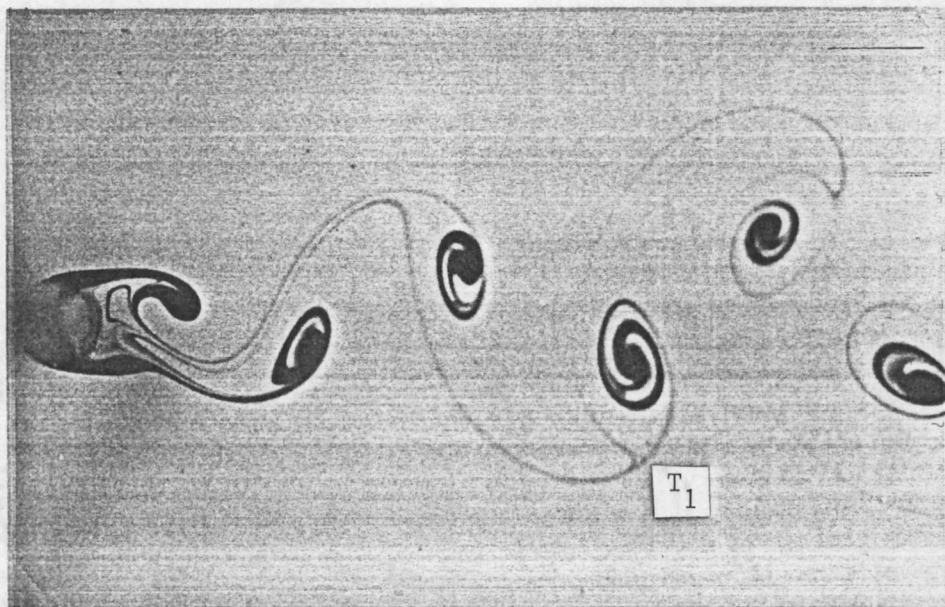


Figure (IV-8). Flow visualization picture showing the turning point of the family discussed in figure (IV-7).

Ref: Perry, Chong & Tam (1982).

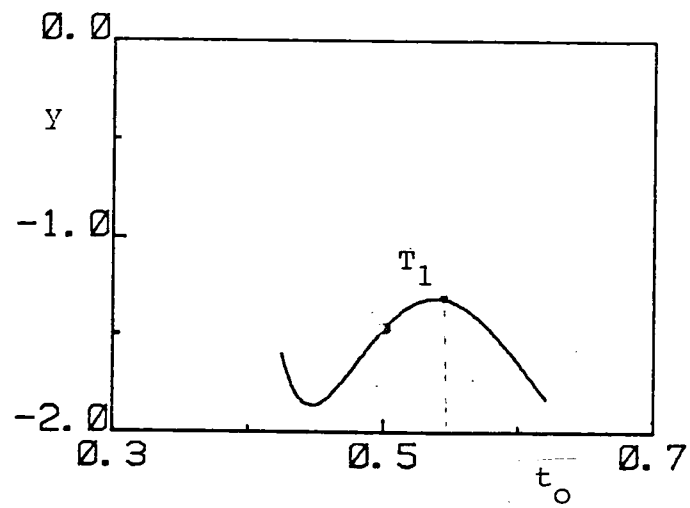
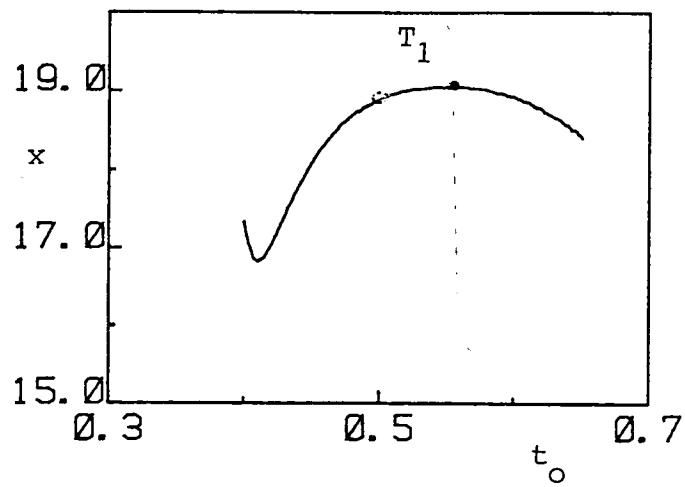


Figure (IV-9). Variation of x and y positions around the turning point T_1 of figure (IV-7).

The combined pattern of two such streaklines, corresponding to the present time $t = 5.2$, is shown as a close-up in Figure (IV-10). In its overall appearance, it is not unlike Figure (IV-11). The photograph of Figure (IV-11) was taken in the UTSI water tunnel using the technique of laser-induced fluorescence of dye injected from the leeward surface of a circular cylinder. Details regarding the UTSI water tunnel facility and the flow visualization technique are discussed by Wu, Vakili, & Yu (1986).

Around the 'turning points', the flow does not physically reverse the direction, despite the visual impression to the contrary. Seeing should not always be believing. .

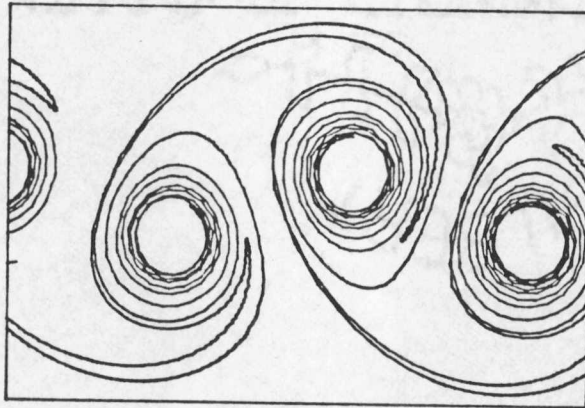


Figure (IV-10) Combined picture of two streaklines.

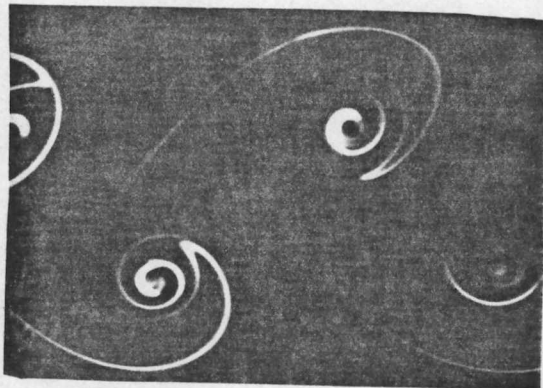


Figure (IV-11). Photo of streakline. Reynolds number based on the cylinder diameter = 380.

CHAPTER V

ENERGY SEPARATION IN UNSTEADY VORTEX WAKES

When a bluff body is immersed in a uniform cross-flow, it is a well known fact that there is a vortex street formed in the wake of a bluff body. In addition, it has been known through experimental observations that the total temperature in the wake becomes substantially lower than the free stream value, as manifested by the fact that the recovery factor near the rearmost surface of the bluff body takes even negative values at high subsonic free stream Mach numbers. This 'aerodynamic cooling' phenomenon is known as the Eckert-Weise effect. In a recent article, Eckert (1984) re-kindled this problem and explained the interesting flow phenomena that occur in this and similar flows where energy separation occurs.

In this chapter, it will be shown that the presence of a vortex street results in an energy separation in the wake. Detailed computational results for the total temperature are presented that elucidate the dynamical reasons for its occurrence. It will also be shown that when time averaged, the separated energy in the wake results in the wake with an energy defect. As a continuation of the previous chapters, the method of approach to study this problem is once again based on the unsteady vortex dynamics of a single vortex and the Kármán vortex street models. Thus, through the numerical solution of the dynamical model equations for the total temperatures, it has been conclusively proven here that the primary reason for the energy separation that occurs in the wake is the unsteady pressure variations, which in turn is caused by the unsteady vortex motion.

A. Total Temperature Computation

Physical Model

The essential dynamical interpretations to explain this phenomenon are contained in the frictionless form of the energy equation [Liepmann & Roshko (1957)].

$$c_p \frac{DT_t}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t} \quad (v-1)$$

where $\frac{D}{Dt}$ is the substantial derivative. T_t is the total temperature, p is the static pressure, and t is the time. Since the flow is unsteady, to compute the total temperature at a point, it is necessary that the integration be carried out along a material line. The left hand side of Equation (v-1) involves the computation of the variation of total temperature along the particle path. Since the particle paths of the single vortex and the vortex street models have already been computed in the previous chapters, they have been utilized here to compute the total temperature distribution along the particle path. The unsteady pressure term on the right hand side has been evaluated through the generalized form of the Bernoulli's equation. Detailed derivations, using the complex potentials, for obtaining the unsteady pressure for the vortex street model in incompressible flow have been included in Appendix D.

At this point, it has been assumed that the dynamics of a fluid particle in a vortex dominated flow is not significantly altered by relaxing the flow incompressibility restriction, thereby facilitating the direct use of the particle path equations derived in Chapter III. This assumption is validated by the presence of vortex streets behind bluff bodies and bluff trailing edges as shown in the shadowgraph pictures of Dymet. Gryson & Ducruet (1980, *ibid*) and Heinemann, Lawaczeck, & Butefisch (1975, *ibid*). Thus, the study of the dynamical mechanisms involved

in the total temperature distribution in a vortex street has here been divided into two parts. In the first part, the position of the fluid particle at any instant of time has been obtained by using the particle path equation; and in the second, the energy equation has been integrated along the particle path up to the present time to obtain the total temperature at the present particle location. Before proceeding to discuss the results for the vortex street wake, it is easier to discuss the dynamical mechanism for the total temperature separation by analyzing the unsteady motion of a single vortex.

Isolated Vortex

It has been shown in Chapter III that the pathlines for the single vortex motion can be represented by trochoidal curves. Although they take various trochoidal shapes, the application of Equation (v-1) to any of these brings forth a common feature to all: the separation of T_t around vortices. Taking for example a curtate trochoid as a pathline, a fluid particle which momentarily occupies the 3 o'clock position (see Figure V-1), towards which the vortex center is approaching, undergoes acceleration. This acceleration causes the pressure to fall off in time, which implies that $\frac{\partial p}{\partial t} < 0$, and hence the total temperature falls with time along the particle path according to equation (v-1). On the other hand, at the 9 o'clock position, from which the vortex center is receding, the particle undergoes conflicting velocity due to the forward moving vortex and rearward directed vortex angular velocity; which means that $\frac{\partial p}{\partial t} > 0$ and $\frac{DT_t}{Dt} < 0$. At the 6 or 12 o'clock position, the unsteady pressure neither rises nor falls: i.e., $\frac{\partial p}{\partial t} = 0$. As a result, $\frac{DT_t}{Dt} = 0$. In fact, the minimum and maximum $\frac{\partial p}{\partial t}$ gradient or $\frac{\partial^2 p}{\partial t^2}$ occurs near the 12 o'clock and 6 o'clock positions respectively, because of the two influences from u_θ and u_v , where u_θ is the angular velocity at a particular radius of the vortex

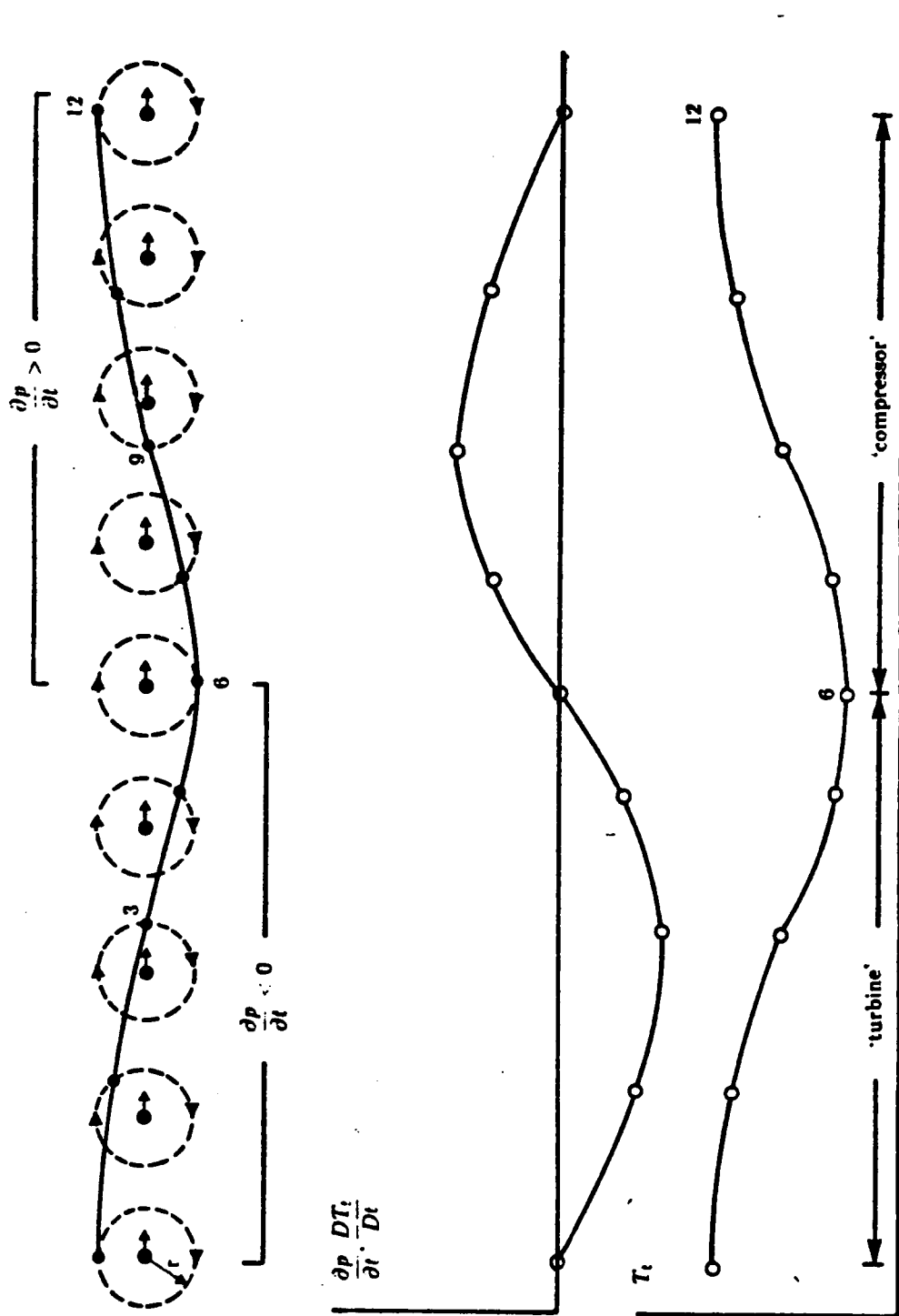


Figure (V-1) Variation of T_t along a curvilinear trochoidal pathline.

and u_v is the vortex convection velocity. From these arguments, it can be concluded that T_t undergoes a cyclic change of rise and fall due to the dynamical contributions from the translational and rotational motion associated with the vortex. In a vortex fixed coordinate system (either rotating or translating) it is not possible to observe these dynamical changes in T_t .

To the extent that T_t decreases or increases during each half cycle as the particle goes around the circumference of a vortex, the half cycle during which the particle experiences supporting influence from both the rotational and translational motion of the vortex (in the 12 - 3 - 6 positions of Figure V-1) may be called the 'turbine cycle' and the other half cycle when the particle experiences conflicting contributions from the rotational and translational motion of the vortex (6 - 9 - 12) as the 'compressor cycle'.

These results also hold for other trochoidal pathlines, cycloidal or prolate type, because of the fact that the nature of the pathline is determined by the dynamical balance at any particle position between the rotational and translational velocity contributions from the vortex as it convects and is independent of the particle release time (it was shown in Chapter III that the different trochoidal pathlines result only because of the variation in the particle release times).

Vortex Street Wake

Since it was shown in Chapter III that the pathlines for a single vortex motion and the Kármán vortex street are similar, at least where the trochoidal nature of the curves are concerned, the discussion on the total temperature separation for the single vortex motion explained above is applicable to the vortex street as well. The major difference between the two being that the T_t separation becomes

directionally oriented in the vicinity of any vortex. Cold spots with lower T_t values compared to the far upstream T_t are present in the neighborhood of the wake centerline; while hot spots with local T_t higher than the far upstream T_t value occurs on the outer edges of the wake. This segregation happens because of the opposite circulation associated with the staggered double row of the street vortices. A plot showing the T_t variations along a pathline at three different start times of 0.0, 0.2, and 0.5 are shown in the pathline curve of Figure (V-2) in the color pictures showing the details regarding the temperatures.

The variation of T_t along the cross-over type pathline (the cross-over pathline was discussed in Chapter III) have also been included in Figure (V-2). For the variation of T_t along its initial phase of entry into the vortex street, the fact that the fluid particle becomes entrained implies that vortex 3 exerts stronger influence than that of vortex 1 in Figure (V-3). Thus, for the fluid particle B, the effect of the approaching vortex 3 is similar to that of the 3 o'clock position for a trochoidal pathline, i.e., T_t falls. The opposite process occurs for position C where T_t rises. In its course along the crossover pathline, T_t at a point initially experiences a fall followed by a rise, with its lowest value attained at or near the wake centerline.

A complete picture of the instantaneous total temperature distribution obtained from the streakline computation at a time $t = 3.75$ is qualitatively presented in Figure (V-3) and a detailed representation of the variation of the total temperature is given in the color graphics picture of Figure (V-4). In the color graphic pictures, all the particles were released with a dimensionless total temperatures value of zero. These results show very good comparison with the large eddy simulation of the Navier-Stokes equations as described in Kurosaka *et al.* (1986).

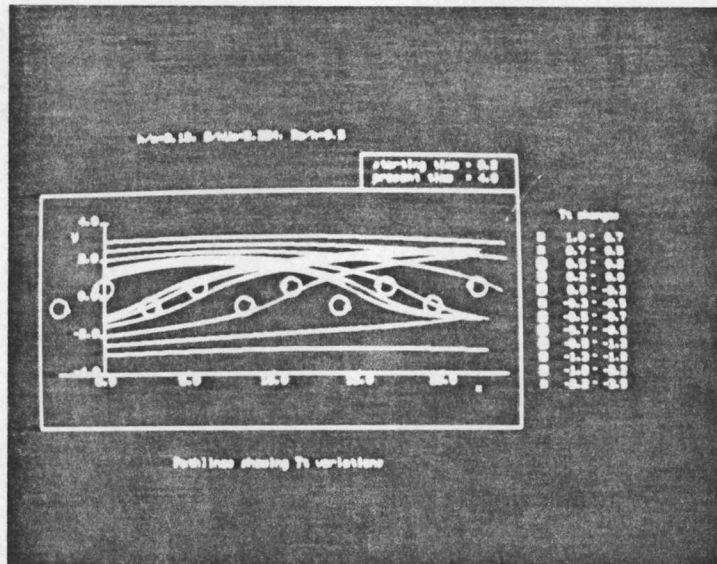


Figure (V-2) Color graphics showing detailed variation of T_t along particle paths.

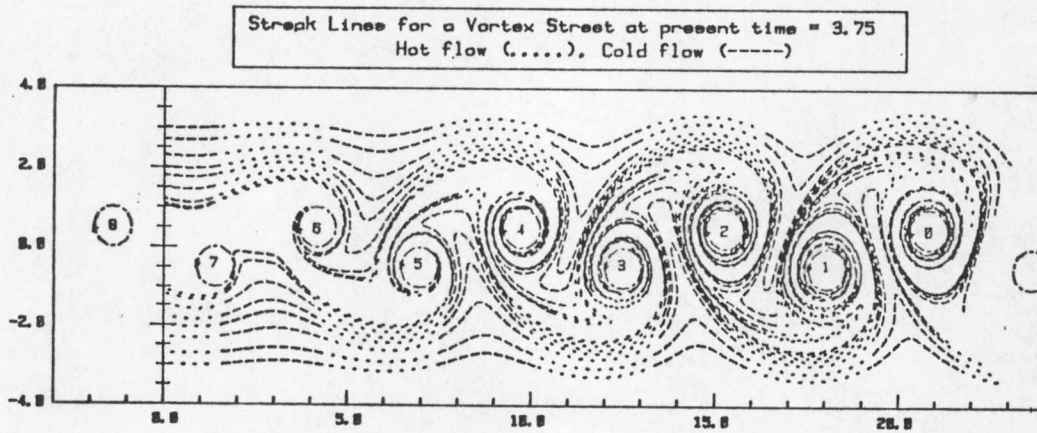


Figure (V-3) Qualitative variation of T_t along streaklines. This is also the instantaneous T_t distribution in the flow field at $t = 3.75$.

Temporal Averaging

With the interesting phenomenon of total temperature separation adequately explained, the temporal average of these instantaneous T_t distribution in the flow is considered next. The lowered value of T_t at such points as A tends to cancel the higher T_t associated with the other fluid particle located at the 12 o'clock position (see Figure V-4) along the trochoidal pathline around the vortex 3. This is the reason why the time-averaged T_t distribution does not show the presence of excess T_t that was observed in the instantaneous results.

To compute the temporal average of T_t , the following sequence of steps are carried out. After dividing the near wake region into a fine mesh of rectangular cells, a T_t value of zero has been assigned initially to each cell corner. With the points that lie along the particle path having been determined already in Chapter III,

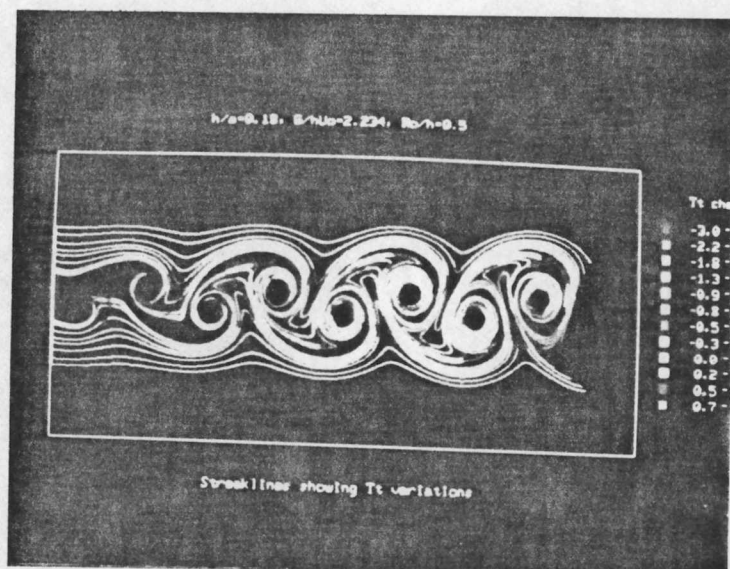


Figure (V-4) Color picture showing the total temperature separation in the wake showing hot and cold regions.

the T_t at each point along its pathline as the particle traverses has been computed using the relations given in Appendix D. Three point central differencing for the first and second derivatives and three point mixed forward and backward differencing for the cross derivatives of the T_t variations along the pathline have been computed for each interior point of the particle path. These are then used to compute the T_t values at the cell corners adjacent to the instantaneous position of the particle. Thus, as the particle traverses along its path, the indices of the cells closest to the position of the particle, the T_t values at each cell corner, and the time of arrival of the particle to that point have all been computed and stored. The above sequence of steps have been carried out for each fluid particle released. Thus, a set of entries (T_t, t) at each cell corner has been obtained. This set has then been reshuffled such that the 't' in the set are in ascending order. Assuming that the T_t value was zero until the first particle arrived at each cell and that the T_t value remains constant between successive arrival of particles carrying different T_t values into a cell, the time averaged T_t distribution has been computed at each cell corner.

As seen from the computed results, in the time averaged sense, the separated energy results in only the cold wake, thus establishing the fact that the Eckert-Weise effect is a consequence of time averaging the instantaneous energy separation.

B. Total Pressure Distribution

For an incompressible flow, under the same condition where Equation (V-1) holds, the internal energy does not vary along pathlines. Then Equation (V-1) reduces to

$$\frac{DP_t}{Dt} = \frac{\partial p}{\partial t} \quad (\text{V-2})$$

where P_t is the total pressure. This equation resembles Equation (V-1) for the total temperature distribution. Consequently, the arguments previously presented pertaining to the separation of total temperature once again apply. Hence, for flows at low Mach numbers, the instantaneous total pressure measurements should show similar behavior as that of total temperature discussed earlier. Eliminating the unsteady pressure term from equation (v-1) and (v-2), the energy equation can be written as

$$\rho c_p \frac{DT_t}{Dt} = \frac{DP_t}{Dt}$$

which can again be rewritten as

$$\frac{\rho c_p T_{t\infty}}{P_{t\infty}} \left(\frac{T_t}{T_{t\infty}} - 1 \right) = \frac{P_t}{P_{t\infty}} - 1$$

Thus the the instantaneous distribution of $P_t/P_{t\infty}$ should be an almost fascimile of those of $T_t/T_{t\infty}$ except for the scaling factor $\gamma/(\gamma - 1)$. This fact can also be verified from the experimental results of Kurosaka *et. al.* (1986, *ibid*).

The long familiar fact that a defect in the total pressure exists in the wake behind a bluff body may now be reinterpreted as the superposition of the instantaneous spatially separated total pressure field in a manner similar to that of the time-averaged Eckert-Weise effect; the difference between the time-averaged P_t and T_t should reflect the dissipative effects and should be connected with the entropy rise.

C. Real Flow Considerations

Having established that the unsteady motion is the cause of the energy separation, through a simple dynamical model, it is appropriate to investigate the additional influences caused by the details of the real flow compared to the idealized model presented here. The first parameter to be investigated is the celerity of the street vortices in contrast to the constant value assumed in the dynamical model. To study this, it is logical to consider a coordinate system fixed with one of the vortices that convects with a velocity $u_v(t)$ in the x -direction as the local arbitrary reference coordinate system and derive the relations with respect to it. Writing

$$x_v = x - \int_0^t u_v(t) dt$$

$$y_v = y$$

$$\tau = t$$

the frictionless form of the energy equation becomes

$$c_p \frac{DT_t}{Dt} = \frac{1}{\rho} \left(\frac{\partial p}{\partial \tau} - u_v(t) \frac{\partial p}{\partial x_v} \right)$$

where $\frac{\partial p}{\partial \tau}$ and $\frac{\partial p}{\partial x_v}$ represent the temporal and spatial changes in pressure with respect to the moving vortex coordinate system. Considering that the vortex strength does not vary, $\frac{\partial p}{\partial \tau} = 0$, the rate of change of T_t along pathlines would depend only on the second term on the right hand side of Equation (V-2). In reality, $\frac{\partial p}{\partial \tau} < 0$ in the near wake for an evolving vortex even when $u_v \neq 0$; and hence, when taking into account the actual non-uniform velocity of the vortex, it is evident that cold spots become colder.

As a second perturbation, it is important to consider the viscous effects in the model and discuss the consequences. The general form of the energy equation

written with the non-dimensional variables is of the form:

$$\frac{DT_t^*}{Dt^*} = \frac{1}{\rho^*} \frac{\partial p^*}{\partial t^*} + \frac{1}{Re} \left(\frac{\tau U_\infty}{d} \right) \left[\Phi^* + \frac{1}{Pr} \nabla^{*2} T^* + \vec{u}^* \cdot \vec{F}_f^* \right], \quad (8)$$

where Pr is the Prandtl number, Re the Reynolds number based on the diameter of the cylinder d ; the starred quantities denote their dimensionless ones; T_t and t are non-dimensionalized by U_∞^2/c_p in the same way as shown in Appendix D; p by $\rho_\infty U_\infty^2$; ρ by ρ_∞ ; t by τ , the time required for a fluid particle to travel across one full wavelength for a trochoidal pathline or to traverse the wake for a cross-over pathline; Φ , the dissipation function by $\mu U_\infty^2/d^2$; $\vec{F}_f$, non-dimensionalized by $\mu U_\infty/d^2$, is $\vec{F}_f = \text{div}[\mu(\text{def}\vec{u})] + \text{grad}(\lambda \text{div}\vec{u})$, where def is the deformation tensor, and λ is the second coefficient of viscosity.

The time-scale, τ , is of the order of the vortex shedding period and an estimate yields $\frac{\tau U_\infty}{d} \sim 5$; thus for a flow with a high Reynolds number, the second term on the right hand side may be discarded. Thus, as far as the dynamics of the changes in the instantaneous total temperatures are concerned, dissipative effects are of negligible importance. However, they of course exert a cumulatively decided effect upon the far downstream T_t .

Equation (v-1) can therefore be used not only for the qualitative explanation of T_t separation, as discussed in the previous section, but also for the *quantitative* calculation of instantaneous T_t separation in the near wake, without resorting to the full Navier-Stokes equations.

By making use of the fact that the total temperature separation occurs in the wake due to the presence of the vortex street, a vorticity meter can be developed based on the measurement of a total temperature survey only. This technique is much simpler and faster compared to the method based on a laser flow field survey, particularly for complex flow field measurements.

CHAPTER VI

VORTEX STREET WITH WALL INTERFERENCE

A. Physical Model of the Flow

When a vortex is confined between solid walls on either side, it is easy to visualize that there exists an infinite image vortex system in the lateral direction to satisfy the inviscid boundary condition on the wall. Also, when the flow in the wake is represented by a vortex street, the application of the solid wall boundary condition results in a doubly infinite image vortex system with the vortex strengths being $\pm\Gamma$. These image vortices are located at various points, in the entire xy plane, depending upon the spacing ratio of the vortex street and the channel width. A sketch of the vortex street and the image vortex system along with the detailed derivation of the equations for the trajectory of a particle in the laboratory coordinate system are given in Appendix E. The particle path equation for this problem that has been derived is:

$$\begin{aligned} \frac{dz}{dt} = \frac{U_\infty}{U_{st}^b \lambda} \left\{ 1 - \left(\frac{\Gamma}{h U_\infty} \right) \left(\frac{h}{2c} \right) \left[\sum_{j=1}^{\infty} 2q_1^{2j-1} \left(\sin 2\bar{\xi} \left\{ \frac{q_1 \bar{\vartheta}_2 - \bar{\vartheta}_1}{\bar{\vartheta}_1 \bar{\vartheta}_2} \right\} \right. \right. \right. \\ \left. \left. \left. + \sin 2\bar{\eta} \left\{ \frac{q_1 \bar{\vartheta}_4 - \bar{\vartheta}_3}{\bar{\vartheta}_3 \bar{\vartheta}_4} \right\} \right) + \frac{1}{2} (\tan \bar{\eta} + \cot \bar{\xi}) \right] \right\} \end{aligned} \quad (VI-1)$$

with street height ratio, $\lambda = (h/a)$, and wall height ratio, $e = (c/h)$, and c is the width of the channel. The ϑ functions are defined as:

$$\begin{aligned} \bar{\vartheta}_1 &= (1 - 2q_1^{2j} \cos 2\bar{\xi} - q_1^{4j}) \\ \bar{\vartheta}_2 &= (1 - 2q_1^{2j-1} \cos 2\bar{\xi} + q_1^{4j-2}) \\ \bar{\vartheta}_3 &= (1 + 2q_1^{2j} \cos 2\bar{\eta} + q_1^{4j}) \\ \bar{\vartheta}_4 &= (1 - 2q_1^{2j-1} \cos 2\bar{\eta} + q_1^{4j-2}) \end{aligned}$$

and the quantities with an over bar represent complex conjugates; further,

$$\xi = \frac{\pi}{\tau}(\mu_1 - U_{st}^b t), \quad \eta = \frac{\pi}{\tau}(\mu_1 - U_{st}^b t), \quad \lambda = \frac{2\pi h}{\tau}, \quad \mu_1 = z - i\frac{h}{2}, \quad \mu_2 = z + i\frac{h}{2}$$

$$q_1 = \exp(-\frac{\pi a}{2c}), \quad \text{and} \quad \tau = 2ic.$$

The ratio of the wall bounded vortex street velocity, U_{st}^b , to the free stream velocity, U_∞ , takes the form:

$$\frac{U_{st}^b}{U_\infty} = 1 - \left(\frac{\Gamma}{hU_\infty} \right) \left(\frac{\lambda}{2} \right) \left\{ \tanh(\pi\lambda) - \sum_{r=1}^{\infty} \left[\frac{4\exp(-2\pi r\lambda e)}{1 - \{-\exp(-2\pi\lambda e)\}^r} \sinh(2\pi r\lambda) \right] \right\}$$

All the vortex street parameters are set to their values corresponding to the unbounded vortex street case assumed in Chapter III. Although the consideration of uniform free stream is not quite correct for a channel flow where the boundary layer exists on both walls, it is a reasonable approximation for a fully developed turbulent flow in the channel.

As a continuation of the methodology adapted in the earlier chapters, the pathline and streakline patterns for this wall bounded vortex street problem has been computed based on the above unsteady vortex dynamics model. Dye particles were continuously released from different y locations from the $x = 0$ plane with varying start times. The released particles traverse along a path that is determined by the force balance due to the doubly infinite vortex system. The solution procedure is once again as discussed in Appendix B.

Computation of the Flow Lines

Figure (VI-1) shows the pathline patterns at the present time $t_1 = 3.75$ for the particles released at start times 0.0, 0.1, and 0.2. (the time period between successive vortices of identical circulation passing through a particular point is unity). The channel width for this case is $3h$, where h is the height of the vortex

street. The pathlines once again resemble the familiar trochoidal curves that have been discussed in the earlier chapters. Figure (VI-2) shows the streakline pattern for a particle released close to a wall which indicates the entrainment effect of the convecting vortices. Caution needs to be exercised because these streaklines could be misjudged as 'hairpin vortices' that occur in a turbulent boundary layer.

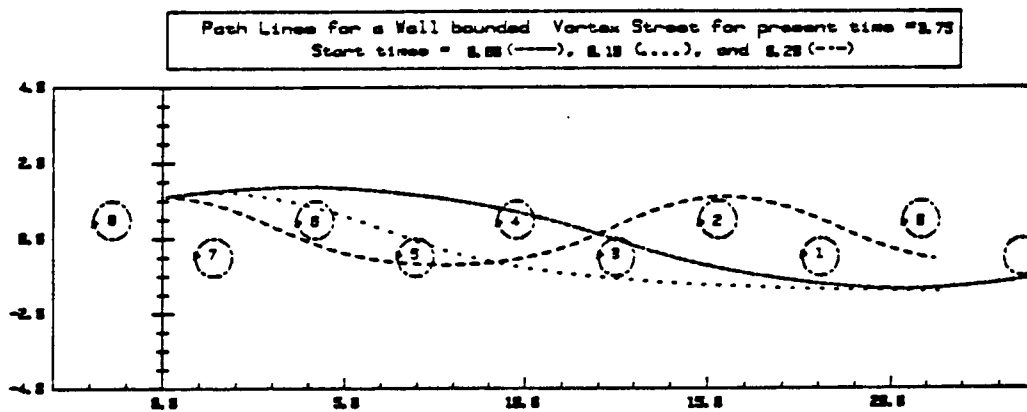


Figure (VI-1) Path lines for a wall bounded vortex street; $c/a = 3.0$.

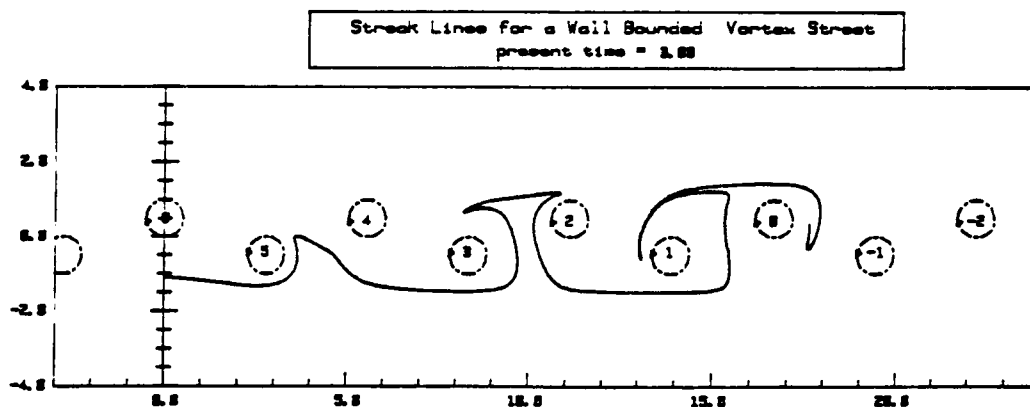


Figure (VI-2) Streakline of the dye released at $y = -1.1$ for a wall bounded vortex street. $c/a = 3.0$.

A similar study where a vortex that convects in the boundary layer of a solid wall was studied in detail by Doligalski, Smith & Walker (1980). They presented flow visualization pictures and the related vortex dynamics models and concluded, based on the streakline observations, that the unsteady boundary layer due to the convecting vortex results in the eruption of wall layer fluid. On this basis, they argued that a secondary vortex of the same or opposite circulation depending upon the celerity of the vortex is always generated. From the inviscid streakline computations presented in Figure (VI-2) to Figure (VI-4) and also verified by

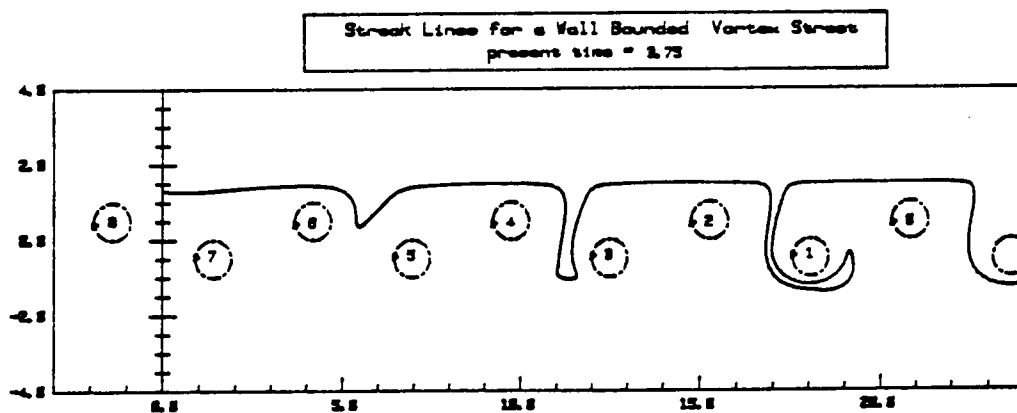


Figure (VI-3) Streaklines corresponding to particles released close to the wall at $y = 1.3$.

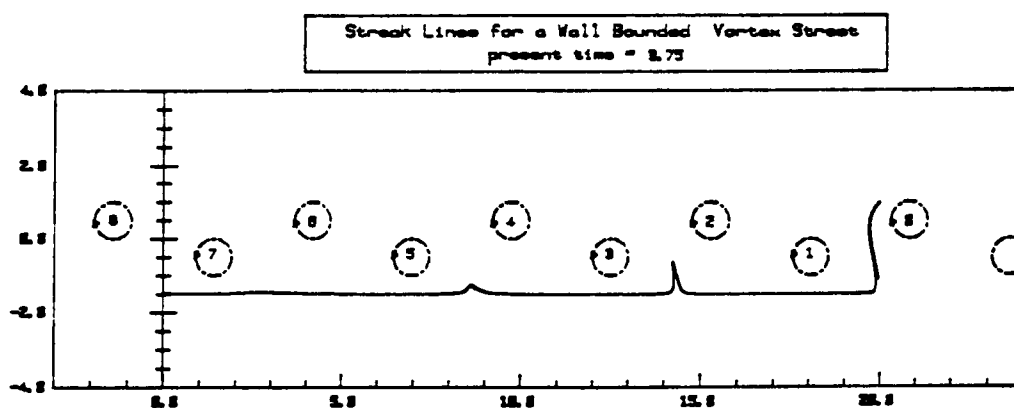


Figure (VI-4) Streakline for same case as Figure (VI-3) but the dye release point is changed to $y = -1.5$.

the flow visualization movie (described later), it is clear that the eruption like appearance of the near wall streaklines is merely due to the inviscid entrainment effect of the vortices and that no secondary vortex as described by Doligalski, Smith & Walker is present in the flow. Although the importance of the unsteady boundary layer is not disputed in this context, it is however reiterated that utmost care needs to be taken while interpreting the physical mechanisms based on the streakline observation alone.

B. Flow Visualization Experiment

The photograph of Figure (VI-5) shows the streakline pattern for the wall bounded wake flow behind a circular cylinder using the laser induced fluorescent technique. The photograph was taken from the UTSI water tunnel facility with dye injection from the leeward surface of the cylinder. The comparison of the flow

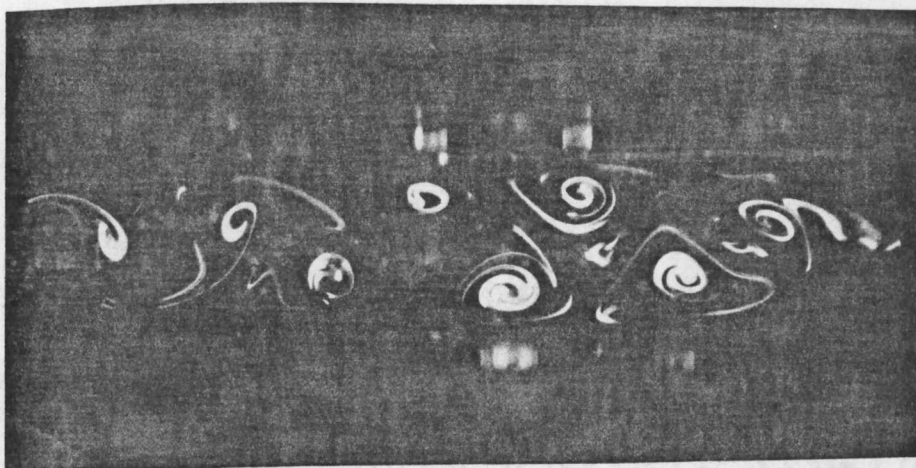


Figure (VI-5) Photograph of the streakline pattern. $c/a = 6.0$

visualization picture and the computed streakline pattern of Figure (VI-6) is fairly good.

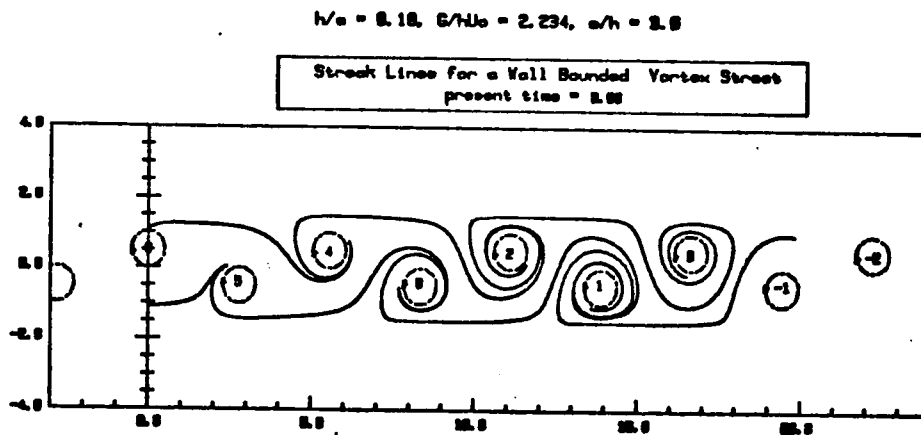


Figure (VI-6) Combined streakline pattern for a wall bounded vortex street. Dye release point is $y = \pm 1.1$.

A real time movie of the flow was also recorded. In the movie, dye was injected from the channel wall at various downstream stations in addition to the dye release from the surface of the cylinder. The purpose of this was to establish that the eruption of the streaklines close to the wall is indeed due to the convecting wake vortices and not merely because of the interference between the near wake and the channel walls. The real time movie was recorded for various tunnel velocities and gap ratios. From a detailed study of the movie the following important observations have been made:

(1) The nearly periodic eruption of the wall layer fluid occurs at several downstream stations in the wake, as also shown in the streakline computational results of Figure (VI-2) to Figure (VI-4).

(2) The erupted streaklines do not indicate the presence of any secondary vortices, but merely represent the entrainment effect due to the wake vortices shed from the cylinder which the erupted streaklines embrace as they traverse downstream.

(3) The most important observation was that the vortices switched over and reversed the sides from which they originally emanated from the surface of the cylinder. Although this phenomenon is not obvious from the still photograph shown in Figure (VI-5), it can be construed, by studying the direction of circulation of the vortices in the photograph, that they indeed have switched sides. Although the vortices that crossed over to the opposite side tend to switch back to their respective original sides, the complex interactions between the vortices prevented the occurrence of this switch-back. The downstream location where the cross-over occurred appeared to be a function of both the tunnel velocity and the channel gap ratio.

When the channel width increases, the lateral push that the vortices experience due to the entrained fluid mass reduces. The pathlines for start times 0.1, 0.2, and 0.5 for the increased channel width are shown in Figure (VI-7). As shown in the streakline plot of Figure (VI-8), the effect of the channel wall is clearly decreased and the streaklines resemble more closely the case when the walls are absent. Figure (VI-9) once again shows the streaklines for the dye released close to the wall for this wall gap ratio which also indicates that the violent eruption-like entrainment behavior has diminished because of the increased channel width.

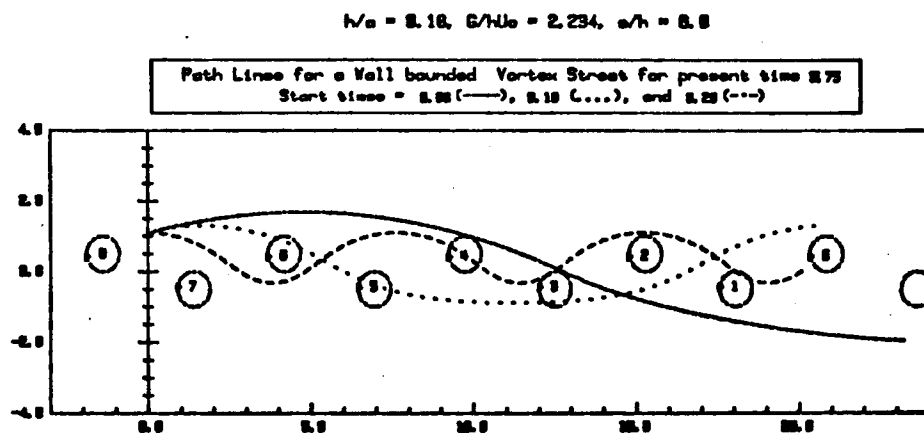


Figure (VI-7) Pathline pattern for particles released for different start times.
 $c/a = 6.0$.

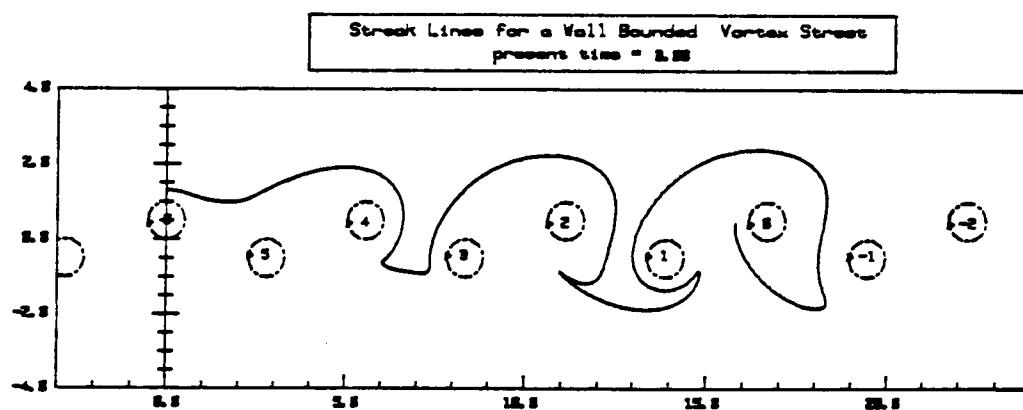


Figure (VI-8) Streaklines for a wall bounded vortex street with the walls taken farther apart. Note that the effect of wall is reduced.

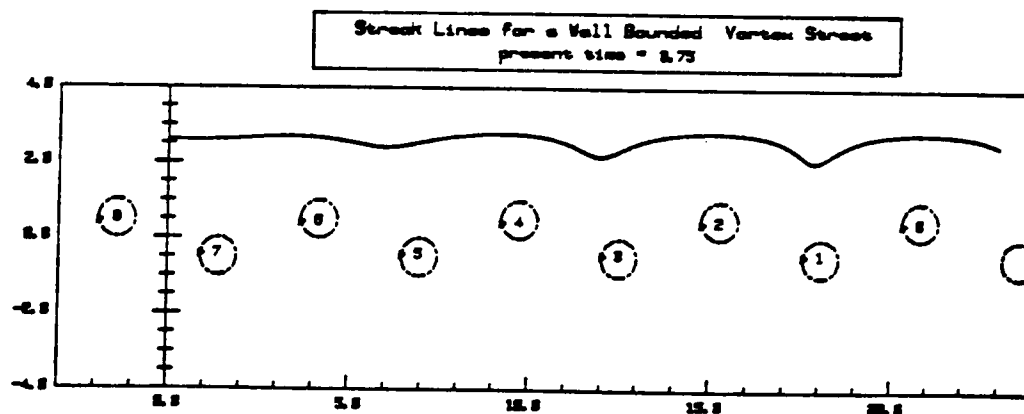


Figure (VI-9) Streaklines for a wall bounded vortex street for particles released close to the wall. $c/a = 6.0$.

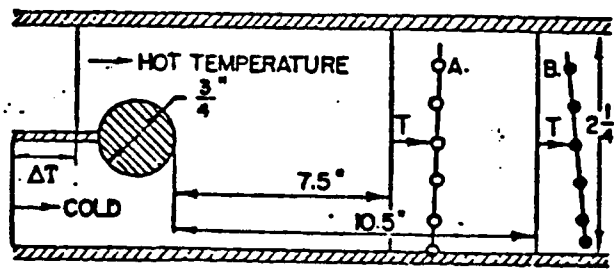


Figure (VI-10). Flow over a circular cylinder with confining walls showing static temperature reversal (Ref. Minchin (1951)).

C. Static Temperature Cross-Over

In this connection an interesting phenomenon was observed by Minchin (1951) while experimentally studying some combustion related problems. He introduced hot and cold air separated by a splitter plate as the upstream onset flow for a wall bounded wake behind a circular cylinder (see Figure VI-10) and observed that the static temperature distribution a few diameters downstream of the cylinder wake was nearly uniform, with its magnitude being nearly equal to the mean of the upstream values of the static temperature. Further downstream, and to everyone's surprise, the temperature profile appeared to be nearly anti-symmetric compared to the distribution upstream of the cylinder. This phenomenon remained unsolved for several years as it was revisited once again by Hawthorne (1957).

Dynamical evidences to show why this temperature reversal takes place is once again contained in the video movie of the flow visualization experiment explained earlier. This peculiar phenomenon is also evident in Figure (VI-10) taken from the computational results of Davis, Moore & Purtell (1984), for a channel width of $4b$ where b is the height of the square prism. It is interesting to note that their results reproduced the essentially single row of vortices of opposite circulation being aligned with the center line of the channel [see Figure (VI-11)] as was seen in the video pictures with increasing channel width. Keeping the channel width constant, when the tunnel velocity was increased, thus increasing the Reynolds number of the flow, the video movie showed that the longitudinal station where the vortices switched from one side of the channel to the other side moved further downstream. This can be explained as being due to the reduced entrainment when the width of the channel is increased.

Thus, the mechanism for the temperature reversal can be attributed to the fact that the entire flow from one side of the channel moves to the other side. This is an enhanced global influence of the cross-over pathlines described in Chapter III which in turn is due to the increased entrainment effect of the convecting vortices moving close to the solid wall. The region where the flow appeared to be temporarily mixed, as indicated by the uniform temperature distribution across the channel width as reported by Minchin (1951) is just the region where the hot and cold flows from either sides temporarily cross over to the opposite sides. Further downstream, the temperature profile shows the cross-over behavior as clearly observed in the video. The position where the switch over occurred may be comprehended to increase with the Reynolds number based on the width of the channel.

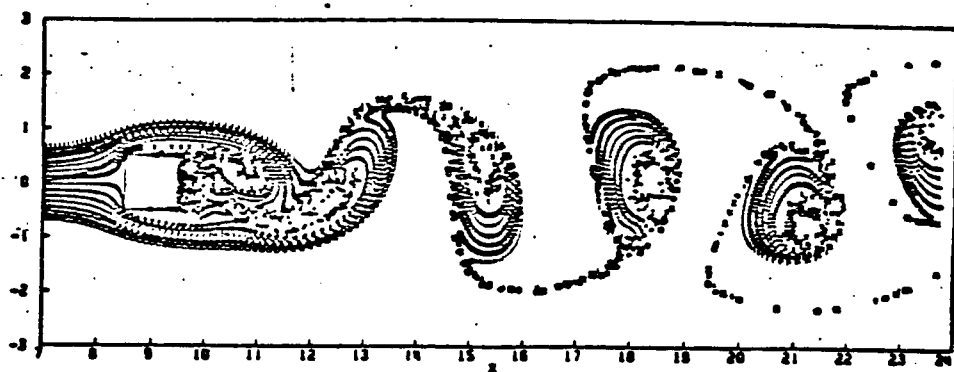


Figure (VI-11) Navier-Stokes simulation of flow behind a square prism by Davis & Moore (1984) showing nearly straight row of opposite vortices along the centerline of the channel when $\frac{h}{a} = 6.0$.

The effect of introducing a non-uniform free stream profile into the computations to simulate the laminar flow case should change the above computed streakline and pathline pattern only slightly. The present model, however, cannot account for this fact.

The detailed mechanism of why the vortices switch over in the near wake when the Reynolds number based on the channel width is small is quite complicated. As observed from the movie, the vortices switched over immediately after they were shed from the cylinder. A reason for this could be due to the 'Coanda effect', which needs further investigation.

The study reported here has important significance in relation to the large scale vortex motion, particularly with reference to the bursting phenomenon and the turbulence production mechanisms close to the wall. It appears that further insight into these complex flows could be achieved by considering the flow to be essentially inviscid while taking into account the unsteady motion of the shear layer vortices. Additional details to incorporate the influence of the real flows could be added as a perturbation to the inviscid incompressible model presented here.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATION

Using Lagrangian dynamics, several models have been presented in this study to analyze the detailed mechanisms of unsteady vortex motion in shear layers. Through these simple dynamical models, it has been shown here that the essential flow details in the specific cases including the starting vortex, the vortex street wake, and the mixing layer could be extracted. The streakline and pathline computations for these flows using these unsteady vortex models have shown close agreement with the flow visualization results. The study also highlighted the importance of the consideration of an inertial frame of reference while using these models to obtain the important unsteady flow information in these flows.

Several visual illusions created by the streaklines in these shear flows have been identified using these models. Through careful numerical studies using these models, the physical reasons why such deceptive pictures arise have been explained. This fact stresses the importance of studying both pathlines and streaklines simultaneously to make any valid judgment of the mechanisms involved in a vortex dominated flow. The difficulty in experimentally visualizing pathlines in unsteady flows and the ease with which we can obtain pathlines using simple dynamical models for these flows makes the present approach very promising for understanding the qualitative behavior of complex vortex dominated flow problems. In addition, the singularity problem and the necessity of an arbitrarily chosen criterion to amalgamate the vortices that are the bottlenecks in the well known discrete vortex method have been overcome in the present approach of using the trochoidal model for the pathlines. Further, it has been shown that the effect of evolving and

decaying properties of the vortices can be easily implemented to account for the vorticity fed into the core by the feeder sheet.

With these simple models for these vortex flows and also their computed pathline pattern, the energy equation has been integrated along the particle path. Using the computational study for the case of a vortex street, it has been discovered that the instantaneous total temperature distribution in the wake, obtained from the total temperature computation for the streaklines in the vortex street wake, exhibits a total temperature separation phenomenon. It has also been conclusively identified from the present computations that the exact cause for this phenomenon to occur is the presence of the vortex street wake behind the bluff body. Also, the total temperature distribution in the wake shows good qualitative comparison with experimental results and the Navier-Stokes solution using large eddy simulation for the flow behind a circular cylinder.

The Eckert-Weise effect of the cold wake observed behind a bluff body has once again been identified here to be the result of the time averaging process of the total temperature distribution in the wake. Since the energy equation can also be written in a similar form in terms of either the total pressure or the total temperature, it can be stated that an identical total pressure separation phenomenon occurs in the wake of a bluff body. Thus, the well known fact that a total pressure loss occurs in the wake of a bluff body should in fact be interpreted as the time averaged outcome of the instantaneous total pressure separation phenomenon in the wake. Overall, a case where strong fluid-thermal interaction occurs in the flow has here been investigated in detail.

Using the result that the total temperature distribution around a convecting vortex exhibits hot and cold regions due to the unsteady vortex motion, a novel

method to identify the presence of vortices and their strength in a complex shear flow has been indicated based on the total temperature measurement at a point. Since the total temperature measurement can be done at a very fast rate and with much less probe interference, this method should be ideal for tracking vortices in complex internal and external flows where large scale vortex motion is present. Particular application for this being the turbomachinery flows where vortex street is known to be present even at high transonic Mach numbers.

The inclusion of viscous effects into the present model with the incorporation of comprehensive viscous models proposed by researchers in this area could be implemented. This would bring in the concept of instantaneous local negative entropy regions in a flow (this has been observed in the wake) through merely dynamical reasons. This is an important recommendation for future work in this area. To include the compressibility effects in the model, a small inner core region of the vortex where the compressibility effects are important in an otherwise incompressible dynamical model associated with the vortex can be contemplated.

Another example for the fluid interaction studied here is the vortex interactions due to wall interference. The problem investigated here is the wall bounded vortex street wake. There are several fundamental flow problems where this type of interaction takes place. In an effort to understand the dynamics of the wake vortices, the analysis was restricted to the overall understanding of the modification of the streakline and pathline patterns. The kinematical changes to influence the vortex dynamics have not been implemented in the present study. Although the analysis presented here is very naive, it was possible to clarify the important effects of the presence of wall to the extent that the entrainment property of the vortices have been shown to be considerably enhanced. To supplement the anal-

ysis to assist the intricate details of the flow field, a real time movie of the flow behind a circular cylinder in a water tunnel has also been recorded. An interesting observation in the present inviscid analysis is that the eruption like appearance of the wall layer fluid in a flow visualization picture does not necessarily mean that secondary vortices exist near the wall. Hence, care needs to be exercised to evaluate the details of the inviscid flow close to the wall to determine the importance of the entrainment effect of the vortices in addition to the effects of viscosity.

From the present pathline computations, it has been evaluated that when a bluff body is tested in a wind tunnel, corrections for the wake blockage effects have to be incorporated even for a 10% blockage by the model. The movie taken for the wall bounded vortex street problem indicated the dynamical reasons for the static temperature cross-over in a wall bounded flow measured by Minchin to be the actual crossing over of the flow itself from one side to the other.

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APPENDIXES

APPENDIX A

PARTICLE PATH FOR SINGLE VORTEX MOTION

In this appendix, the necessary equations for computing the particle path of a single isolated vortex moving relative to a free stream flow will be derived. Also shown will be how this problem could be modeled, within the potential flow assumptions, by the motion of a circular cylinder rotating about its axis. It will also be shown that the particle trajectory for this case are nothing but trochoids.

A fluid particle path under the influence of an isolated single vortex moving relative to a free stream moving with a velocity u_v can be computed using the inviscid model described in Chapter III. The complex potential for this case is given by,

$$w(z) = u_v z + \frac{i\Gamma}{2\pi} \ln(z - z_0)$$

$$\frac{dw}{dz} = \frac{dz}{dt} = u_v - \frac{i\Gamma}{2\pi} \frac{1}{\bar{z} - \bar{z}_0}$$

Since $(dz_0/dt) = u_v$ we can write,

$$\frac{d}{dt}(z - z_0) = -\frac{i\Gamma}{2\pi} \frac{1}{(\bar{z} - \bar{z}_0)}$$

letting $(z - z_0) = \rho(t)e^{i\theta(t)}$,

$$\frac{d\rho}{dt} + i\rho(t)\frac{d\theta}{dt} = -\frac{i\Gamma}{2\pi} \frac{1}{\rho(t)}$$

comparing real and imaginary parts, we obtain,

$$z - z_0 = \rho_0 \exp\left[i\left\{-\frac{i\Gamma}{2\pi} \frac{(t - t_0)}{\rho_0^2} - \theta_0^*\right\}\right], \quad \text{where } \theta_0^* = 0 \text{ at } t = t_0$$

Identifying $(\Gamma/2\pi\rho_0^2) = \omega$, we can derive

$$z - z_0 = \rho_0 e^{i[-\omega(t-t_0) + \theta_0^*]}, \quad \text{where } z_0 = u_v t + \frac{h}{2}i, \quad \theta_0^* = \frac{\pi}{2} + \theta_0$$

For a particle released from $y = \rho + \frac{h}{2}$ at $t = 0$,

$$\theta_0 = \tan^{-1}\left(\frac{u_v t_0}{\rho}\right), \quad \rho_0 = \sqrt{\rho^2 + (u_v t_0)^2},$$

which results:

$$z - z_0 = \rho_0 [-\sin\{\theta_0 - \omega(t - t_0)\} + i \cos\{\theta_0 - \omega(t - t_0)\}]$$

Thus, the trajectory of a particle released from a point P becomes,

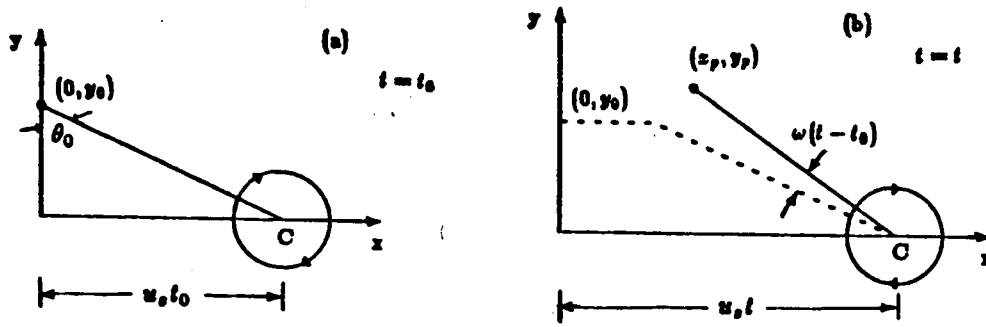
$$\begin{aligned} x &= u_v t + \rho_0 \sin\{\omega(t - t_0) - \theta_0\} \\ y &= \frac{h}{2} + \rho_0 \cos\{\omega(t - t_0) - \theta_0\} \end{aligned} \tag{A-1}$$

It will next be shown that by using the dynamical model discussed in Chapter III, a particle under the influence of the forces due to a rotating cylinder (which is also simultaneously slipping) the same particle path equations can be obtained. In other words, the particle path equations of (A-1), for an isolated vortex motion is the same as that of a trochoidal pathline curve.

Returning to the cylinder model discussed in Chapter III, it is necessary to consider the case when the fluid particle release time is t_0 at which the cylinder is situated at a distance $u_v t_0$ from the y axis. From Figure (A-1) we can see that at time t ,

$$\begin{aligned} x &= u_v t - \rho_0 \sin\{\omega(t - t_0) - \theta_0\} \\ y &= \frac{h}{2} - \rho_0 \cos\{\omega(t - t_0) - \theta_0\} \end{aligned}$$

which is identical to (A-1).



To find ω , a change to the cylinder fixed coordinate system is appropriate.

$$\left(\frac{dx}{dt}\right)_{cyl} = \rho_0 \omega \cos[\omega(t - t_0) - \theta_0]$$

$$\left(\frac{dy}{dt}\right)_{cyl} = -\rho_0 \omega \sin[\omega(t - t_0) - \theta_0]$$

$$\rho_0 \omega = u_\theta = \sqrt{\left(\frac{dx}{dt}\right)_{cyl}^2 + \left(\frac{dy}{dt}\right)_{cyl}^2}$$

Setting $(u_\theta/\rho_0)(t - t_0) - \theta_0 = \phi$, and $t = t_0 + (\phi - \theta_0)(\rho_0/u_\theta)$ Hence, the particle path equation becomes

$$x = u_c \left[t_0 + \frac{(\phi - \theta_0)\rho_0}{u_\theta} \right] - \rho_0 \sin \phi$$

$$y = y_0 - \rho_0 \cos \phi$$

which is represented as,

$$X = \frac{u_c}{u_\theta} \phi + \sin \phi$$

$$Y = \cos \phi$$

Thus except for the difference in the origin of the coordinate system, the above

equations are the equations for a trochoid.

$$\begin{aligned} \text{If } \frac{u_c}{u_\theta} &> 1, & \text{a curtate,} \\ \frac{u_c}{u_\theta} &= 1, & \text{a cycloid,} \\ \text{and } \frac{u_c}{u_\theta} &< 1, & \text{a prolate.} \end{aligned}$$

For a potential vortex,

$$u_\theta = \rho_0 \omega = \frac{\Gamma}{2\pi\rho_0}, \quad \text{or} \quad \omega = \frac{\Gamma}{2\pi\rho_0^2}$$

Hence, the trajectory is

- i. curtate trochoid if the particle is released far away from the center or within the vortex core where u_θ is small,
- ii. cycloid if the particle is released from the vortex inviscid boundary,
- iii. prolate trochoid if the particle is released from near the vortex core where u_θ is large.

APPENDIX B

PARTICLE TRAJECTORY FOR A VORTEX STREET

Consider an infinite Kármán vortex street consisting of two rows of vortices, each vortex of constant circulation strength Γ , with the horizontal and vertical spacing between the vortices being 'a' and 'h' respectively, translating in the positive x direction with a uniform velocity U_{st} , as shown in Figure (B-1). Superimposing a uniform flow U_∞ , the complex potential for the unsteady flow at an arbitrary time 't' that has elapsed since a vortex of the top row swam past the y axis in a laboratory (inertial) coordinate system could be derived as,

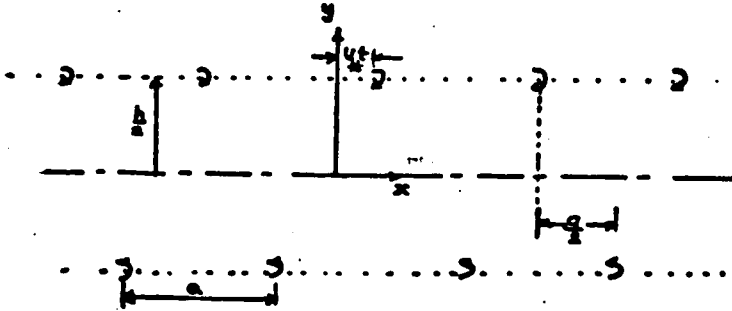
$$w(z) = U_\infty z + i \frac{\Gamma}{2\pi} \ln \left\{ \sin \left[\frac{\pi}{a} \left(z - U_{st}t - i \frac{h}{2} \right) \right] \right\} - i \frac{\Gamma}{2\pi} \ln \left\{ \sin \left[\frac{\pi}{a} \left(z - U_{st}t - \frac{a}{2} + i \frac{h}{2} \right) \right] \right\} \quad (\text{B-1})$$

where,

$$z = x - iy, \quad \frac{dw(z)}{dz} = u - iv, \quad \text{and} \quad \frac{dz}{dt} = u + iv$$

and the vortex street velocity is given by the relation,

$$U_{st} = U_\infty - \frac{\Gamma}{2a} \tanh \left(\frac{\pi h}{a} \right) \quad (\text{B-2})$$



Differentiating Equation (B-1) with respect to z and taking complex conjugates, we can obtain the particle path equation as,

$$\begin{aligned} \frac{dz}{dt} = & U_{\infty} - i \frac{\Gamma}{2a} \cot \left[\frac{\pi}{a} \left(\bar{z} - U_{st}t + i \frac{h}{2} \right) \right] \\ & + i \frac{\Gamma}{2a} \cot \left[\frac{\pi}{a} \left(\bar{z} - U_{st}t - \frac{a}{2} - i \frac{h}{2} \right) \right] \end{aligned} \quad (\text{B-3})$$

where the 'bar' quantities represent complex conjugates.

Introducing dimensionless quantities,

$$z^* = z/h, \quad t^* = t/\tau$$

where $\tau = (a/U_{st})$ is the period of the vortex motion. Substituting the above quantities into (AII-E3) and dropping '*' everywhere for brevity, we obtain,

$$\begin{aligned} \frac{dz}{dt} = & \frac{U_{\infty}}{U_{st}} \left\{ \frac{1}{(h/a)} - i \frac{\Gamma}{2hU_{\infty}} \cot \left[\frac{\pi h}{a} \left(\bar{z} - \frac{t}{(h/a)} + \frac{i}{2} \right) \right] \right. \\ & \left. + i \frac{\Gamma}{2hU_{\infty}} \cot \left[\frac{\pi h}{a} \left(\bar{z} - \frac{t}{(h/a)} - \frac{1}{2(h/a)} - \frac{i}{2} \right) \right] \right\} \end{aligned} \quad (\text{AII-E4})$$

Equation (B-4) is the equation of the pathlines for an infinite Kármán vortex street. This is a non-autonomous first order ordinary differential equation with complex quantities. It is prudent to solve the above equation in complex form. A sixth order Runge-Kutta-Fehlberg algorithm with seventh order error criterion has been utilized to control the step size. The seventh order formula can be derived as,

$$y_{n+1} = y_n - \frac{11}{864} k_0 - \frac{1771561}{6289920} k_3 - \frac{32}{105} k_4 - \frac{243}{2560} k_5 - \frac{16807}{74880} k_6 - \frac{11}{270} k_8 - \frac{11}{270} k_9$$

where,

$$k_l = \Delta t_n f(y_n - Y_l, t_n - \tau_l \Delta t_n), \quad l = 0, 1, 2, \dots, 9$$

Table (B-1) gives the magnitudes of τ_l and Y_l for various l values.

Table: (B-1). Coefficients of the sixth order Fehlberg formula.

l	τ_l	Y_l
0	0	0
1	$\frac{2}{33}$	$\frac{2}{33}k_0$
2	$\frac{4}{33}$	$\frac{4}{33}k_1$
3	$\frac{2}{11}$	$\frac{1}{22}k_0 + \frac{3}{22}k_2$
4	$\frac{1}{2}$	$\frac{43}{64}k_0 - \frac{165}{64}k_2 + \frac{77}{32}k_3$
5	$\frac{2}{3}$	$-\frac{2383}{486}k_0 + \frac{1067}{54}k_2 - \frac{26312}{1701}k_3 + \frac{2176}{1701}k_4$
6	$\frac{6}{7}$	$\frac{10077}{4802}k_0 - \frac{5643}{686}k_2 + \frac{116259}{16807}k_3 - \frac{6240}{16807}k_4 + \frac{1053}{2401}k_5$
7	1	$-\frac{733}{176}k_0 + \frac{141}{8}k_2 - \frac{335763}{23296}k_3 + \frac{216}{77}k_4 - \frac{4617}{2816}k_5 + \frac{7203}{9152}k_6$
8	0	$\frac{15}{352}k_0 - \frac{5445}{46592}k_3 + \frac{18}{77}k_4 - \frac{1215}{5632}k_5 + \frac{1029}{18304}k_6$
9	1	$-\frac{1833}{352}k_0 + \frac{141}{8}k_2 - \frac{51237}{3584}k_3 + \frac{18}{7}k_4 - \frac{729}{512}k_5 + \frac{1029}{1408}k_6 + k_8$

The residual ' R ' between the sixth and seventh order term is given as,

$$R = \left(\frac{11}{864} - \frac{77}{1440} \right) k_0 + \frac{11}{270} (k_8 - k_9 - k_7) / \Delta t$$

Defining a control parameter δ based on a specified tolerance ϵ for both real and imaginary parts,

$$\delta = \left(\frac{\epsilon}{2R} \right)^{\frac{1}{6}}$$

the time step can be controlled by the following criteria,

$$\Delta t_n = \begin{cases} 0.1\Delta t_n & \text{if } \delta \leq 0.1, \\ 4\Delta t_n & \text{if } \delta > 4, \\ \delta\Delta t_n & \text{otherwise.} \end{cases}$$

Because of the extremely large numbers of the right hand side function evaluations involved in computing the streakline pattern, it is imperative to simplify the right hand side function analytically to reduce the complex number library function evaluations. Hence, the particle path equation, Equation (B-4), can be rewritten as,

$$\frac{dz}{dt} = \frac{U_\infty}{U_{st}} \left\{ \frac{1}{(h/a)} - i \frac{\Gamma}{2hU_\infty} \left[\frac{\Lambda_1}{\Lambda_2} - \frac{\Lambda_3}{\Lambda_4} \right] \right\} \quad (\text{AII-E5})$$

with the functions $\Lambda_1, \Lambda_2, \Lambda_3$, and Λ_4 are defined as,

$$\Lambda_1(z, t) = \sin A \cosh C - i \cos A \sinh C$$

$$\Lambda_2(z, t) = - [\cos A \cosh C + i \sin A \sinh C]$$

$$\Lambda_3(z, t) = \cos A \cosh B + i \cos A \sinh B$$

$$\Lambda_4(z, t) = \sin A \cosh B - i \cos A \sinh B$$

$$\text{and } A = \pi \left(x - \frac{t}{(h/a)} \right), \quad B = \frac{\pi}{(h/a)} \left(y + \frac{1}{2} \right), \quad C = \frac{\pi}{(h/a)} \left(y - \frac{1}{2} \right)$$

APPENDIX C

ABSENCE OF TURNING POINT IN A SINGLE VORTEX

In this appendix it is demonstrated that the streakline pattern for a single vortex motion does not have any turning point. The single vortex motion is described by trochoidal pathline equation as shown in Appendix A. Taking Equation (A-1)

$$\begin{aligned}x(t_0, t) &= u_v t + \rho_0 \sin [\omega(t - t_0) - \theta_0] \\y(t_0, t) &= \frac{h}{2} + \rho_0 \cos [\omega(t - t_0) - \theta_0]\end{aligned}$$

where

$$\omega = \frac{\Gamma}{2\pi\rho_0^2}, \quad \rho_0 = \sqrt{\rho^2 + (u_v t_0)^2}, \quad \text{and} \quad \tan \theta_0 = \frac{u_v t_0}{\rho}$$

Differentiating the above quantities with respect to t_0 ,

$$\begin{aligned}\frac{\partial x}{\partial t_0} &= \frac{\partial \rho_0}{\partial t_0} \sin [\omega(t - t_0) - \theta_0] + \rho_0 \cos [\omega(t - t_0) - \theta_0] \left[\frac{\partial \omega}{\partial t_0} t - \omega - \frac{\partial \theta_0}{\partial t_0} \right] \\ \frac{\partial y}{\partial t_0} &= \frac{\partial \rho_0}{\partial t_0} \cos [\omega(t - t_0) - \theta_0] - \rho_0 \sin [\omega(t - t_0) - \theta_0] \left[\frac{\partial \omega}{\partial t_0} t - \omega - \frac{\partial \theta_0}{\partial t_0} \right] \\ \frac{\partial \omega}{\partial t_0} &= -\frac{\Gamma u_v^2 t_0}{\pi \rho_0^4}; \quad \frac{\partial \rho_0}{\partial t_0} = \frac{u_v^2 t_0}{\rho_0}; \quad \frac{\partial \theta_0}{\partial t_0} = \frac{u_v \rho}{\rho_0^2}.\end{aligned}$$

The above equations for the derivatives of x and y are rewritten as:

$$\begin{aligned}\frac{\partial x}{\partial t_0} &= \frac{u_v^2 t_0}{\rho_0} \sin [\omega(t - t_0) - \theta_0] + \rho_0 \cos [\omega(t - t_0) - \theta_0] \left[-\frac{\Gamma u_v^2 t_0}{\pi \rho_0^4} - \omega - \frac{u_v \rho}{\rho_0^2} \right] \\ \frac{\partial y}{\partial t_0} &= \frac{u_v^2 t_0}{\rho_0} \cos [\omega(t - t_0) - \theta_0] - \rho_0 \sin [\omega(t - t_0) - \theta_0] \left[-\frac{\Gamma u_v^2 t_0}{\pi \rho_0^4} - \omega - \frac{u_v \rho}{\rho_0^2} \right].\end{aligned}$$

The criterion for turning point, $\frac{\partial x}{\partial t_0} = 0$ & $\frac{\partial y}{\partial t_0} = 0$, results:

$$\rho_0^2 \left[\frac{\Gamma u_v^2 t_0}{\pi \rho_0^4} - \omega + \frac{u_v \rho}{\rho_0^2} \right]^2 = -\left(\frac{u_v t_0}{\rho_0} \right)^2$$

which cannot be satisfied by any real values of t_0 . This implies that the streakline pattern for a single vortex motion does not exhibit any turning points.

APPENDIX D

TEMPERATURE DISTRIBUTION IN A VORTEX STREET

In this appendix the analytical expressions for the total temperature variations at different points around the vortex street is derived. The energy equation for an incompressible adiabatic gas for unsteady motion in the absence of any body forces can be written as,

$$c_p \frac{DT_t}{Dt} = \frac{1}{\rho} \frac{\partial p}{\partial t} \quad (D-1).$$

In the above equation T_t is the total temperature; c_p, ρ, p , and t are the specific heat at constant pressure, density, static pressure, and time respectively; and $\frac{D}{Dt}$ denotes total derivative. The individual terms for the present vortex street problem based on the particle path equations obtained in Appendix B. The unsteady pressure term can thus be seen to contribute to the non-uniform distribution of total temperature near a vortex.

The Bernoulli equation for the unsteady flow is written as,

$$\frac{p}{\rho} - \frac{1}{2} q^2 - \frac{\partial p}{\partial t} = \frac{p_\infty}{\rho} - \frac{1}{2} u_\infty^2 \quad (D-2)$$

where q is the resultant velocity. Differentiating (D-2) with respect to t ,

$$\frac{1}{\rho} \frac{\partial p}{\partial t} = -\frac{1}{2} \frac{\partial q^2}{\partial t} - \frac{\partial^2 \phi}{\partial t^2} \quad (D-3)$$

With $w = \phi + i\xi$ as the complex potential. the relation between q and w is

$$q^2 = \frac{dw}{dz} \frac{\overline{dw}}{\overline{dz}}$$

Hence,

$$\frac{\partial q^2}{\partial t} = \frac{\partial}{\partial t} \left(\frac{dw}{dz} \right) \frac{\overline{dw}}{\overline{dz}} - \frac{dw}{dz} \frac{\partial}{\partial t} \left(\frac{\overline{dw}}{\overline{dz}} \right)$$

Differentiating Equation (B-3) with respect to the variable t , and denoting

$$\Theta_1 = \frac{\pi}{a} \left(z - U_{st} - i \frac{h}{2} \right)$$

$$\Theta_2 = \frac{\pi}{a} \left(z - U_{st} - \frac{a}{2} + i \frac{h}{2} \right)$$

$$\frac{\partial}{\partial t} \left(\frac{dw}{dz} \right) = i \frac{\Gamma}{2\pi} \left(\frac{\pi}{a} \right)^2 U_{st} \left[\frac{1}{\sin^2 \Theta_1} - \frac{1}{\sin^2 \Theta_2} \right]$$

can be obtained. Hence,

$$\frac{\partial q^2}{\partial t} = \Omega + \bar{\Omega}$$

where,

$$\Omega = i \frac{\Gamma}{2\pi} \left(\frac{\pi}{a} \right)^2 U_{st} \left[\frac{1}{\sin^2 \Theta_1} - \frac{1}{\sin^2 \Theta_2} \right] \left\{ U_{\infty} - i \frac{\Gamma}{2a} (\cot \bar{\Theta}_1 - \cot \bar{\Theta}_2) \right\}$$

Differentiating Equation (B-1) twice with respect to t ,

$$\frac{\partial^2 w}{\partial t^2} = -i \frac{\Gamma U_{st}^2}{2} \left[\frac{1}{\sin^2 \Theta_1} - \frac{1}{\sin^2 \Theta_2} \right]$$

Since $\phi = \frac{w - \bar{w}}{2}$, this implies

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{1}{2} \left[\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 \bar{w}}{\partial t^2} \right]$$

Again,

$$\frac{1}{\rho} \frac{\partial p}{\partial t} = - \left(\frac{1}{2} \right) \left(\frac{\Gamma}{2hU_{\infty}} \right) \pi \left(\frac{h}{a^2} \right) u_{\infty}^3 (\Sigma - \bar{\Sigma}) \quad (D-4)$$

with $\Sigma(t, z)$ defined as,

$$\Sigma = i \left(\frac{U_{st}}{U_{\infty}} \right) \left[\frac{1}{\sin^2 \Theta_1} - \frac{1}{\sin^2 \Theta_2} \right] \left[1 - i \frac{\Gamma}{2hU_{\infty}} \left(\frac{h}{a} \right) (\cot \bar{\Theta}_1 - \cot \bar{\Theta}_2) - \left(\frac{U_{st}}{U_{\infty}} \right) \right]$$

Using Equation (D-4), the energy equation can be rewritten in the form,

$$c_p \frac{DT_t}{Dt} = -\left(\frac{1}{2}\right) \left(\frac{\Gamma}{2hU_\infty}\right) \pi \left(\frac{h}{a^2}\right) u_\infty^3 (\Sigma + \bar{\Sigma}) \quad (D-5)$$

Introducing non-dimensional variables,

$$t^* = \frac{t}{\tau} = \frac{tU_{st}}{a}, \quad \text{and} \quad z^* = \frac{z}{h}$$

and defining the non-dimensional total temperature as,

$$T_t^* = \frac{2c_p U_{st}}{\left(\frac{\Gamma}{2hU_\infty}\right) \pi \frac{h}{a} u_\infty^3} T_t$$

and letting $\Sigma^* = \Sigma(t^*, z^*)$, and $\bar{\Sigma}^* = \bar{\Sigma}(t^*, z^*)$, the final form of the energy equation is:

$$\frac{DT_t^*}{Dt^*} = -(\Sigma^* + \bar{\Sigma}^*) \quad (D-6).$$

Integrating the above equation along the pathline, the change in the total temperature between two positions over a certain time is given by the relation,

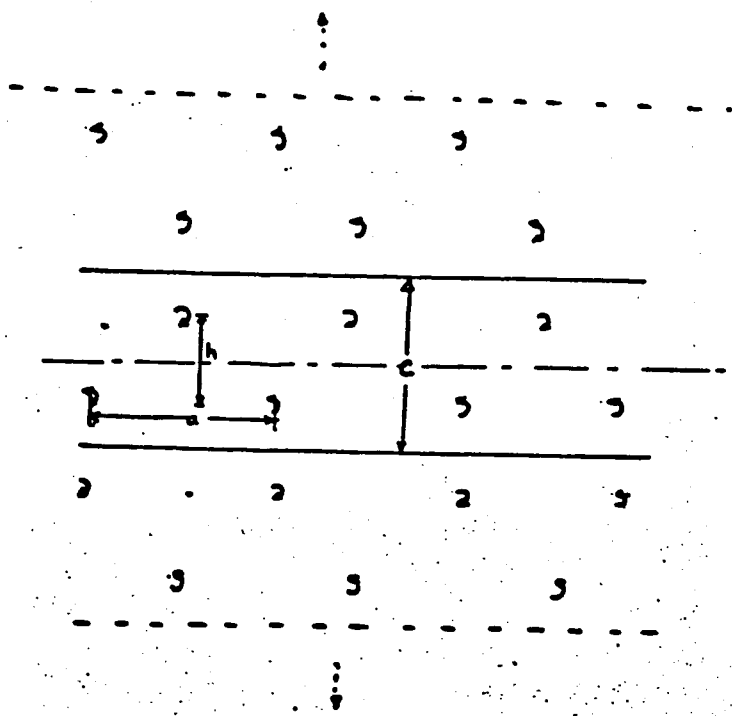
$$\Delta T_t^* = T_t^* - T_{t_0}^* = - \int_{t_0^*}^{t^*} (\Sigma^* + \bar{\Sigma}^*) Dt^* \quad (D-7)$$

By fixing the total temperature of a particle at the release point, its magnitude at any other point (the coordinate of which has been determined by solving Equation (B-5) of the particle path) along its trajectory can be computed from the above integral.

APPENDIX E

PARTICLE PATH IN A WALL BOUNDED WAKE

Consider the infinite Kármán vortex street as shown in Appendix B be situated between two walls at $y = \pm \frac{c}{2}$. As a first step in the consideration of wall effects, the impermeability condition has to be satisfied. This in turn brings the infinite image vortices for each vortex inside the wall as shown in Fig. (A4-F1). There are positive vortices at $(na, \frac{h}{2})$ and negative vortices at $(ma - \frac{a}{2}, -\frac{h}{2})$, where m and n assume all positive and negative integral values.



The complex potential for the wall bounded vortex street could then be represented as:

$$w = -\frac{i\kappa}{2\pi} \ln f(z)$$

where the function $f(z)$ has simple zeros at the positive vortices and simple Apoles at the negative vortices. Neglecting additive constants, the complex potential for a positive vortex of strength Γ at the point $(ra, \frac{h}{2})$ between the barriers $y = \pm \frac{c}{2}$ is,

$$w(z) = \frac{i\kappa}{2\pi} \ln \left\{ \frac{\prod_{n=-\infty}^{n=\infty} [z - ra - i(2nc + \frac{h}{2})]}{\prod_{m=-\infty}^{m=\infty} [z - ra - i(2mc + c - \frac{h}{2})]} \right\}$$

$$= \frac{i\kappa}{2\pi} \ln \left\{ \frac{(z - ra - i\frac{h}{2}) \prod_{n=-\infty}^{n=\infty} [(z - ra - i\frac{h}{2})^2 + 4n^2c^2]}{\prod_{m=-\infty}^{m=\infty} [(z - ra + i\frac{h}{2})^2 + (2m+1)^2c^2]} \right\}$$

Using the product relation for hyperbolic function, (ref. Milne-Thompson, pp. 374),

$$\sinh(\pi x) = \pi x (1 + x^2) (1 + \frac{x^2}{2^2}) \dots (1 + \frac{x^2}{n^2}) \dots$$

the complex potential can be derived as:

$$w = \frac{i\kappa}{2\pi} \ln \left\{ \frac{\sinh \left[(z - ra - i\frac{h}{2}) \frac{\pi}{2c} \right]}{\cosh \left[(z - ra + i\frac{h}{2}) \frac{\pi}{2c} \right]} \right\}$$

Hence, the complex potential for the case of an infinite Kármán vortex street bounded between the channel walls becomes,

$$w = \frac{i\kappa}{2\pi} \ln \left\{ \frac{\prod_{r=-\infty}^{r=\infty} \sinh \left[(z - ra - i\frac{h}{2}) \frac{\pi}{2c} \right] \prod_{s=-\infty}^{s=\infty} \cosh \left[(z - sa + \frac{a}{2} - i\frac{h}{2}) \frac{\pi}{2c} \right]}{\prod_{r=-\infty}^{r=\infty} \cosh \left[(z - ra - i\frac{h}{2}) \frac{\pi}{2c} \right] \prod_{s=-\infty}^{s=\infty} \sinh \left[(z - sa + \frac{a}{2} + i\frac{h}{2}) \frac{\pi}{2c} \right]} \right\}$$

where the summation over r represents the contributions from all the positive vortices and that over the s represent the contributions from all the negative vortices. Replacing the infinite sum as semi-infinite sum and using the product formula for the hyperbolic functions, and setting $\mu_1 = (z - i\frac{h}{2})$ and $\mu_2 = (z + i\frac{h}{2})$, the above equation becomes:

$$w = \frac{i\kappa}{2\pi} \ln \left\{ \frac{\sinh \frac{\mu_1 \pi}{2c} \prod_{r=1}^{\infty} \left[\cosh \frac{ra\pi}{c} - \cosh \frac{\mu_1 \pi}{c} \right] \prod_{s=1}^{\infty} \left[\cosh \frac{(2s-1)a\pi}{2c} + \cosh \frac{\mu_1 \pi}{c} \right]}{\cosh \frac{\mu_2 \pi}{2c} \prod_{r=1}^{\infty} \left[\cosh \frac{ra\pi}{c} + \cosh \frac{\mu_2 \pi}{c} \right] \prod_{s=1}^{\infty} \left[\cosh \frac{(2s-1)a\pi}{2c} + \cosh \frac{\mu_2 \pi}{c} \right]} \right\}$$

Writing in terms of the exponential functions and letting

$$q_1 = \exp\left(-\frac{\pi a}{2c}\right), \quad \text{and} \quad \tau = 2ic$$

it becomes:

$$w = \frac{i\kappa}{2\pi} \ln \frac{\sin \frac{\mu_1 \pi}{\tau} \prod_{r=1}^{\infty} \left[1 - 2q_1^{2r} \cos \frac{2\mu_1 \pi}{\tau} + q_1^{4r} \right] \prod_{s=1}^{\infty} \left[1 + 2q_1^{2s-1} \cos \frac{2\mu_1 \pi}{\tau} + q_1^{4s-2} \right]}{\cos \frac{\mu_2 \pi}{\tau} \prod_{r=1}^{\infty} \left[1 + 2q_1^{2r} \cos \frac{2\mu_2 \pi}{\tau} + q_1^{4r} \right] \prod_{s=1}^{\infty} \left[1 - 2q_1^{2s-1} \cos \frac{2\mu_2 \pi}{\tau} + q_1^{4s-2} \right]}$$

This is the complex potential function for a wall bounded vortex street in the coordinate system moving with a vortex.

To find the complex potential in a laboratory coordinate system, it is necessary to displace the coordinates by the quantity, $-U_{st}^b t$, to all the z positions in the above equation. Here U_{st}^b is the velocity of the wall bounded vortex street in the positive x direction. Using the substitution,

$$\xi = \frac{\pi}{\tau}(\mu_1 - U_{st}^b t), \quad \eta = \frac{\pi}{\tau}(\mu_2 - U_{st}^b t), \quad \text{and} \quad \mu_1 = z - \frac{ih}{2}, \quad \mu_2 = z + \frac{ih}{2}$$

it becomes,

$$w = \frac{i\kappa}{2\pi} \ln \left\{ \frac{\sin \xi \prod_{r=1}^{\infty} (1 - 2q_1^{2r} \cos 2\xi - q_1^{4r}) \prod_{s=1}^{\infty} (1 - 2q_1^{2s-1} \cos 2\xi + q_1^{4s-2})}{\cos \eta \prod_{r=1}^{\infty} (1 + 2q_1^{2r} \cos 2\eta + q_1^{4r}) \prod_{s=1}^{\infty} (1 - 2q_1^{2s-1} \cos 2\eta + q_1^{4s-2})} \right\}$$

Adding the onset flow complex potential, $U_{\infty} z$, and differentiating with respect to z and non-dimensionalizing in the same manner as shown in Appendix B, we

derive, after some algebra, the particle path equation as,

$$\begin{aligned} \frac{dz}{dt} = \frac{U_\infty}{U_{st}^b \lambda} \left\{ 1 + \left(\frac{\Gamma}{h U_\infty} \right) \left(\frac{h}{2c} \right) \left[\sum_{j=1}^{\infty} 2q_1^{2j-1} \left(\sin 2\bar{\xi} \left\{ \frac{q_1 \bar{\vartheta}_2 - \bar{\vartheta}_1}{\bar{\vartheta}_1 \bar{\vartheta}_2} \right\} \right. \right. \right. \\ \left. \left. \left. + \sin 2\bar{\eta} \left\{ \frac{q_1 \bar{\vartheta}_4 - \bar{\vartheta}_3}{\bar{\vartheta}_3 \bar{\vartheta}_4} \right\} \right) \right] + \frac{1}{2} (\tan \bar{\eta} + \cot \bar{\xi}) \right\} \end{aligned} \quad (E-1)$$

where,

$$\bar{\vartheta}_1 = (1 - 2q_1^{2j} \cos 2\bar{\xi} + q_1^{4j})$$

$$\bar{\vartheta}_2 = (1 + 2q_1^{2j-1} \cos 2\bar{\xi} + q_1^{4j-2})$$

$$\bar{\vartheta}_3 = (1 + 2q_1^{2j} \cos 2\bar{\eta} + q_1^{4j})$$

$$\bar{\vartheta}_4 = (1 - 2q_1^{2j-1} \cos 2\bar{\eta} + q_1^{4j-2})$$

and the quantities with an over bar represent complex conjugates. The ratio of the vortex street velocity to the free stream velocity can be dericed as,

$$\frac{U_{st}^b}{U_\infty} = 1 - \left(\frac{\Gamma}{h U_\infty} \right) \left(\frac{\lambda}{2} \right) \left\{ \tanh(\pi \lambda) - \sum_{r=1}^{\infty} \left[\frac{4 \exp(-2\pi r \lambda e)}{1 - \{-\exp(-2\pi \lambda e)\}^r} \sinh(2\pi r \lambda) \right] \right\}$$

with street height ratio, $\lambda = \frac{h}{a}$, and wall height ratio, $e = \frac{c}{h}$.

VITA

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