LONG TRANSMISSION LINE

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I. PREFACE

Transmission lines vary in many ways. They may carry direct or alternating current. Alternating current has many different conditions under which it can be transmitted by overhead conductors. It can be single-phase, double-phase, of three-phase, with any frequency. Another large factor in a transmission line is the nature of the power to be delivered and the nature of the power which can be generated to the greatest advantages under given conditions. In some cases the power factor is not a determining factor while in others it is an important factor. Some demands are for a constant voltage with varing load,etc., while others have constant loads through out the day.

From the above and other similar variations in different lines, it would be impossible to cover all types of long transmission lines,with details of each, in one thesis.

In this country three-phase alternating current is used in most cases to transmit electrical power by overhead conductors, therefore a line of this type was chosen. The transmission line chosen for this thesis was the Cheoah, Alcoa Line (North Carolina and Tennessee) of the Knoxville Power Company.

It is the object of this thesis to develop and study this line with a view to give a complete and accurate calculations. The following are to be studied and calculated:- comparison of copper and aluminum conductors, size of conductor to be used, spacing of the conductors, line constants, performance of line under different loads, and sags and tension in different spans.

II. GIVEN DATA.*

Knoxville Power Company

Cheoah (North Carolina, U.S.A.)	Alcoa Line (Tennessee, U.S.A.)
Alternating Current, Three-Phase.	60 cycle.
Power Factor at receiver end	one
Voltage at receiver end	150,000 volts
Maximum Load (constant thruout day)	55,000 kw.
Minimum Load (" " ")	20,000 kw.
Neutral grounded at generator end.	
Length of Line	25 miles (40.2 km)
Normal Span	800 feet (244 m)
Maximum Span	1295 feet (395 m)
Maximum Span (Special)	5,010 feet (1527 m)

* Data obtained from "Aluminum Conductors, Typical Installations", by the Aluminum Company of America.

III. CHARACTERISTICS OF LINE.

This line has several striking characteristics. The power supplied is used by an aluminum company with runs with a constant load throughout the day. In different seasons the load varies but this variation can be easily taken care of at the generator because it is known when a change of load is to occur. Another characteristic is that the power used has a power factor of one (1), therefore there is no reactive current at the receiver end.

Present plans are to tie other lines into this line which explains the high voltage used for a comparative short line. This high voltage explains the low line losses.

The span of 5,010 feet has a special designed conductor which is not considered in this thesis. The calculations are based on the asumption that this portion of the line has the same size conductor as the rest. This assumption has little or no effect on the results of the calculations.

The line is subject to very little wind. The effect of this is neglected in calculations for sags and tensions. The temperature ranges are not very great. Allowing a considerable margin, the temperature is taken from 30° fahrenheit to 120° fahrenheit. Made the assumption that the line may be covered with ½ inch of ice at 30°F.

IV. CONDUCTORS.

1. Comparison of copper and aluminum conductors.

Aluminum wire has a conductivsly of 60 % on the base that copper has 100% conductivily. The weight of an aluminum conductor is 50% of that of a copper conductor of equal resistance. The diameter of an alumimum is 1.26 times that of copper of same conductivily.

Aluminum has a disadvantage of being easily scratched and cut, readily attacked by alkaline substances, and difficulty of sodering.

In short spans the difference in sag of aluminum conductors is often considerable between summer and winter temperatures, but in long spans the difference in sag due to temperature changes is very small. On spans of moderate length the sag of aluminum conductors are about 30% or 35% greater than copper conductors. Around a span of 1000 feet the sag will be approximately the same.

The comparison of the cost of the two conductors will not be made because in different sections of the country the cost will not be the same, and because the cost will vary according to demand and supply.

If aluminum conductors have steel cores, the strength will be increased, which will overcome many of the disadvantages of aluminum conductors. The Aluminum Cable Steel Reinforced (A.C.S.R.) is a conductor of the above type.

The following table gives a comparison of copper conductors and Aluminum Cables Steel Reinforced.

Resistance, ohms per 1000 feet		0.0648	0.1026	0.1639	0.2610	0.4150
B. & S.	Copper	3/0	1/0	2	4	6
Gage No.	A.C.S.R.	(266,800 c.m.)	3/0	1/0	2	4
Weight,	Copper	508.0	319.5	200.9	126.4	79.5
lb.per 1000 ft.	A.C.S.R.	343.0	232.5	147.0	92.4	58.0
Diameter,	Copper	0.410	0.325	0.258	0.204	0.162
inch	A.C.S.R.	0.633	0.501	0.398	0.316	0.250
Ultimate Str'th	Copper	6,580	4,558	3,127	1,967	1,237
lbs.	A.C.S.R.	9,385	6,660	4,200	2,660	1,665

From the above table it can be seen that conductors of the same resistances, the weight of A.C.S.R. is 70% of copper conductor, the diameter of A.C.S.R. is 1.5 times that of copper conductor, and the ultimate strength of A.C.S.R. is 1.4 times that of copper conductor. The smaller weight and larger strength are advantages. The only disadvantage of a larger diameter is the wind effect, and load due to snow and ice, are greater.

The conductor chosen for this line was the A.C.S.R. because of the advantages shown above, and because the writer was interested more in aluminum conductors than in copper conductors.

2. Size of conductor.

P = power of line, in watts
E = voltage between conductors in volts
e = voltage to neutral in volts
I = current in amperes.
(p.f.) = cos θ = power factor.

$P_r = \sqrt{3}\, E_r I_r \cos\theta_r$

$I_r = \frac{P_r}{\sqrt{3}\, E_r \cos\theta_r}$

$= \frac{55{,}000{,}000}{\sqrt{3} \times 150{,}000 \times 1}$

= 211.7 amperes.

$E_r = \sqrt{3}\, e_r$

$e_r = \frac{E_r}{\sqrt{3}} = \frac{150{,}000}{\sqrt{3}}$ = 86,600 volts.

Assume voltage drop due to resistance is 10% or 15,000 volts therefore the voltage drop in one conductor due to resistance is 8,660 volts.

Resistance = $\frac{\text{voltage drop}}{\text{current}} = \frac{8660}{211.7}$

= 4.0907 ohohms per conductor per 25 miles
= 0.163628 ohms " " " 1 mile
= 0.0310 ohms " " " 1000 ft.

From table of Aluminum Cable Steel Reinforced (A.C.S.R.), page 113 in "Aluminum Conductors, Typical Installations" selected a conductor with nearest resistance to that found above, which was as follows:-

A.C.S.R. - Aluminum area 500,000 c.m. or 0.3927 sq.in.
Resistance 0.0347 ohms per 1000 ft.
Diameter 0.904 inch

3. Arrangement of Conductors.

This transmission line has a three-phase system. Assuming the three phases are balanced, and using the voltage to the neutral, and using in the line constants the inductance and condensance to the neutral, the performance of the transmission line may be found by analyzing one phase to the neutral, because the system is balanced.

"In order that the three phases shall have identical performance curves the electrical constants must be the same: that is, 1. The three conductors must be of the same length and cross-section and be of the same material so that the resistances shall be equal. 2. The inductance per conductor to neutral must be the same for the three phases. 3. The condensance per conductor to neutral must be the same for the three phases."

The first is easily and always met in a line.

In order that the inductance and condensance of the three phases be equal the conductors must be properly transposed. This may be done by having two complete transpositions which divide the line into three sections of equal length. (Figure 1).

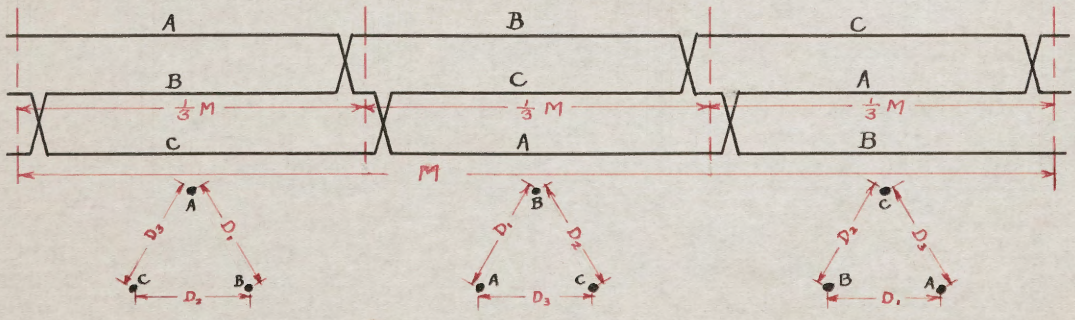


Figure 1.

The inductance is effected by the distance between the conductors, so in order that only one calculation be necessary, it is desirable to find the equivalent equidistant spacing (D), which will give the same inductance per conductor for the whole line as that given by the sum of the inductances of each conductor for one-third of the length. As shown in Figure 1, D_1 = distance between first and second conductors, D_2 = distance between second and third conductors, and D_3 = distance between third and first conductors. D = equivalent equidistant spacing, in inches.

The normal span in this transmission line is 800 feet, therefore the spacing of the conductors is based on this value. From Fig.39, in "Overhead Electric Power Transmission", for a span of 800 feet, the equivalent equidistant spacing (D) is about 180 inches. A small variation in D will have very little effect on performance of the line.

For this line took D = 180 inches.

$$D^3 = D_1 \times D_2 \times D_3 \quad \underline{\text{or}} \quad D = (D_1 \times D_2 \times D_3)^{\frac{1}{3}}$$

V. LINE CONSTANTS.

4. Resistance.

r = resistance of each conductor, in ohms.
ρ = resistance of each conductor per foot, in ohms per foot.
l = length of conductor, in feet.

$\rho = 0.0000347$ ohms per ft.

$r = l \times \rho = 132{,}000 \times 0.0000347$

$= 4.580$ ohms.

5. Inductance.

L = inductance, in henrys.
l = length of line, in miles.
D = equivalent equidistant spacing between conductors, in inches.
d = diameter of conductor, in inches.

$$L = l(0.00074 \log_{10}\frac{2D}{d} + 0.0000805)$$

$$= 25\ (0.00074 \log_{10}\frac{2 \times 180}{0.904} + 0.0000805)$$

$$= 25\ (0.00074 \times 2.60013 + 0.0000805)$$

$$= 25\ (0.0019241 + 0.0000805)$$

$$= 25 \times 0.0020046$$

$$= 0.050 \text{ henry.}$$

6. Reactance.

x = reactance, in ohms.
f = frequency.

$$x = 2\pi f L$$

$$= 2 \times 3.1416 \times 60 \times 0.050$$

$$= 18.850 \text{ ohms.}$$

7. Impedance.

z = impedance, in ohms.

$$z = (r^2 + x^2)^{\frac{1}{2}}$$

$$= (\overline{4.580}^2 + \overline{18.850}^2)^{\frac{1}{2}} = (20.9764 + 355.3225)^{\frac{1}{2}}$$

$$= (376.2989)^{\frac{1}{2}}$$

$$= 19.398 \text{ ohms.}$$

8. Condensance.

C = condensance, in farads.
Other units same as for inductance.

$$C = 1 \times \frac{0.0388 \times 10^{-6}}{\log_{10} \frac{2D}{d}}$$

$$= 24 \times \frac{0.0388 \times 10^{-6}}{\log_{10} \frac{2 \times 180}{0.904}} = 24 \times \frac{0.0388 \times 10^{-6}}{2.60013}$$

$$= 0.3581 \times 10^{-6} \text{ farads.}$$

9. Shunted line susceptance.

b = shunted line susceptance, in mhos.

$$b = 2 \pi f C$$

$$= 2 \times 3.1416 \times 60 \times 0.3581 \times 10^{-6}$$

$$= 0.13500 \times 10^{-3} \text{ mhos.}$$

10. Summary.

Resistance - - - - - - - - -	4.580 ohms
Inductance - - - - - - - - -	0.050 henry
Reactance - - - - - - - - -	18.850 ohms
Impedance - - - - - - - - -	19.398 ohms
Condensance - - - - - - - -	0.3581 x 10^{-6} farads
Shunted line susceptance -	0.13500 x 10^{-3} mhos.

VI. CONSTANTS OF LINE EQUATIONS.

11. α

$$\alpha = \left[\tfrac{1}{2} b (z - x)\right]^{\frac{1}{2}}$$

$$= \left[\tfrac{1}{2} \times 0.1350 \times 10^{-3} (19.398 - 18.850)\right]^{\frac{1}{2}}$$

$$= (\tfrac{1}{2} \times 0.1350 \times 10^{-3} \times 0.548)^{\frac{1}{2}} = (0.00003699)^{\frac{1}{2}}$$

$$= 0.006082 \text{ radian.}$$

$$\alpha^2 = 0.000036992$$

$$1 + \frac{\alpha^2}{2} = 1.000018496$$

12. β

$$\beta = \left[\tfrac{1}{2} b (z + x)\right]^{\frac{1}{2}}$$

$$= \left[\tfrac{1}{2} \times 0.13500 \times 10^{-3} (19.398 + 18.850)\right]^{\frac{1}{2}}$$

$$= (\tfrac{1}{2} \times 0.1350 \times 10^{-3} \times 38.248)^{\frac{1}{2}} = (0.00258174)^{\frac{1}{2}}$$

$$= 0.050811 \text{ radian.}$$

$$\beta^\circ = 0.050811 \times 57.3 = 2.9114703^\circ$$

$$= 2^\circ\ 54'\ 41''$$

$$\sin\beta = 0.05079$$

$$\cos\beta = 0.99871$$

$$\beta \sin\beta = 0.002581$$

$$\beta \cos\beta = 0.050745$$

13. $a_1 - j a_2$

$$a_1 - j a_2 = (1 - \tfrac{\alpha^2}{2}) \cos\beta - j \alpha \sin\beta$$

$$= 1.000018496 \times 0.99871 - j (0.006082 \times 0.05079)$$

$$= 0.998728 - j\ 0.00030890$$

14. $c_1 - j\,c_2$

$$c_1 - j\,c_2 = \frac{1}{z}\left[\alpha\beta\cos\beta - \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\,\frac{1}{z}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta + \alpha^2\cos\beta\right]$$

$$= \frac{1}{19.398}\left[(0.006082 \times 0.050745) - (0.006082 \times 1.000018496 \times 0.05079)\right] - j\,\frac{1}{19.398}\left[(0.050811 \times 1.000018496 \times 0.05079) + (0.000036992 \times 0.99871)\right]$$

$$= \frac{1}{19.398}\,(0.000308631090 - 0.000308905714144) - j\,\frac{1}{19.398}\,(0.00258104773817 6 + 0.000036944280 32)$$

$$= \frac{1}{19.398}\,(-0.000000274623414 4) - j\,\frac{1}{19.398}\,(0.00261799201849 6)$$

$$= -0.00000001416 - j\,0.00013496$$

15. $d_1 - j\,d_2$

$$d_1 - j\,d_2 = \frac{1}{b}\left[\alpha\beta\cos\beta + \alpha\left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right] - j\,\frac{1}{b}\left[\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta - \alpha^2\cos\beta\right]$$

$$= \frac{1}{0.1350 \times 10^{-3}}(0.000308631090 + 0.000308905714144) - j\,\frac{1}{0.1350 \times 10^{-3}}(0.00258104773817 6 - 0.000036944280 32)$$

$$= \frac{1000}{0.1350}\,(0.000617536804144) - j\,\frac{1000}{0.1350}\,(0.00254410345785 6)$$

$$= 4.5743 - j\,18.845$$

16. Summary.

$$a_1 - j\,a_2 = 0.998728 - j\,0.00030890$$

$$c_1 - j\,c_2 = -0.00000001416 - j\,0.00013496$$

$$d_1 - j\,d_2 = 4.5743 - j\,18.845$$

VII. PERFORMANCE OF LINE

17. Equations.

E = voltage between conductors, in volts.
e = voltage to neutral in volts.
$\dot{e}$ = vector voltage
e' = power voltage in volts
e" = reactive voltage in volts
I = current in amperes
$\dot{I}$ = vector current
i' = power current in amperes
i" = reactive current in amperes
P = power of line in watts
p = power of each conductor in watts
Q = power of line in volt amperes
q = power of each conductor in v.a.
p.f. = cosθ = power factor
Note:- subscript r denotes at receiver end
subscript g denotes at generator end

$$e'_r = \frac{E_r}{\sqrt{3}} \qquad \#1$$

$$i'_r = I_r = \frac{P_r}{\sqrt{3}\, E_r \cos\theta_r} \qquad \#2$$

$$\dot{I}_g = i'_g + j\, i''_g = (i'_r + j\, i''_r)(a_1 - j\, a_2) + e'_r\,(c_1 - j\, c_2) \qquad \#3$$

$$\dot{e}_g = e'_g + j\, e''_g = e'_r(a_1 - j\, a_2) + (i'_r + j\, i''_r)(d_1 - j\, d_2) \qquad \#4$$

$$I_g = \left[(i'_g)^2 + (i''_g)^2\right]^{\frac{1}{2}} \qquad \#5$$

$$e_g = \left[(e'_g) + (e''_g)^2\right]^{\frac{1}{2}} \qquad \#6$$

$$E_g = e_g \times \sqrt{3} \qquad \#7$$

$$p_g = e'_g\, i'_g + e''_g\, i''_g \qquad \#8$$

$$P_g = p_g \times 3 \qquad \#9$$

$$q_g = e_g\, I_g \qquad \#10$$

$$Q_g = q_g \times 3 \qquad \#11$$

$$(p.f.)_g = p_g \div q_g \qquad \#12$$

$$\text{Line loss} = P_g - P_r \qquad \#13$$

$$\%\ \text{line loss} = \frac{P_g - P_r}{P_g} \qquad \#14$$

Note :- these equations are used for all the calculations of the performance of the line under different loads, when used they are numbered corresponding to the above.

18. Load of 20,000 kw.

P_r = 20,000,000 watts
E_r = 150,000 volts
i''_r = 0
$(p.f.)_r$ = 1

$$e'_r = \frac{150,000}{\sqrt{3}} \qquad \#1$$

$$= 86,600 \text{ volts.}$$

$$i'_r = I_r = \frac{20,000,000}{\sqrt{3} \times 150,000 \times 1} \qquad \#2$$

$$= 76.98 \text{ amperes}$$

$$\dot{I}_g = (76.98 + j\,0)(a_1 - j\,a_2) + 86,600(c_1 - j\,c_2) \qquad \#3$$

$$= (76.88 - j\,0.02) + (0 - j\,1169)$$

$$= 76.88 - j\,11.71$$

$$\dot{e}_g = 86,600(a_1 - j\,a_2) + (76.98 - j\,0)(d_1 - j\,d_2) \qquad \#4$$

$$= (86,490 - j\,3) + (352 - j\,1451)$$

$$= 86,842 - j\,1454$$

$$I_g = \left[(76.88)^2 + (11.71)^2\right]^{\frac{1}{2}} \qquad \#5$$

$$= (5910.5344 + 137.1241)^{\frac{1}{2}} = (6047.6585)^{\frac{1}{2}}$$

$$= 77.77 \text{ amperes.}$$

$$e_g = \left[(86,842)^2 + (1454)^2\right]^{\frac{1}{2}} \qquad \#6$$

$$= (7,541,532,964 + 2,114,116)^{\frac{1}{2}} = (7,543,647,080)^{\frac{1}{2}}$$

$$= 86,854 \text{ volts.}$$

$$E_g = 86,854 \times \sqrt{3} \qquad \#7$$

$$= 150,431 \text{ volts.}$$

$$p_g = (86,842 \times 76.88) + (1454 \times 11.71) \qquad \#8$$

$$= 6676,400 + 17,000$$

$$= 6,693,400 \text{ watts.}$$

$$P_g = 6,693,400 \times 3 \qquad \#9$$

$$= 20,080,000 \text{ watts.}$$

q_g = 86,854 x 77.77 #10

= 6,754,600 volt amperes.

Q_g = 6,754,600 x 3 #11

= 20,264 kv.a.

$(p.f.)_g$ = 6,693,400 ÷ 6,754,600 #12

= 0.991

Line loss = 20,080 - 20,000 #13

= 80 kw.

% line loss = $\frac{80}{20,080}$ #14

= 0.40 %

19. Load of 30,000 kw.

P_r = 30,000 kw.
E_r = 150,000 volts.
i''_r = 0
$(p.f.)_r$ = 1

e'_r = $\frac{150,000}{\sqrt{3}}$ #1

= 86,600 volts.

$i'_r = I_r$ = $\frac{30,000,000}{\sqrt{3} \times 150,000 \times 1}$ #2

= 115.47 amperes.

$\dot{I}_g$ = (115.47 + j 0)(a_1 - j a_2) + 86,600(c_1 - j c_2) #3

= (115.32 - j 0.04) + (0 - j 11.69)

= 115.32 - j 11.73

$\dot{e}_g$ = 86,600 (a_1 - j a_2) + (115.47 + j 0)(d_1 - j d_2) #4

= (86,490 - j 3) + (528 - j 2176)

= 87,018 - j 2179

I_g = $\left[(115.32)^2 + (11.73)^2\right]^{\frac{1}{2}}$ #5

= $(13,298.7024 + 137.5929)^{\frac{1}{2}}$ = $(13,436.2953)^{\frac{1}{2}}$

= 115.92 amperes.

$e_g = [(87,018)^2 + (2179)^2]^{\frac{1}{2}}$ #6

$= (7,572,132,324 + 4,748,041)^{\frac{1}{2}} = (7,576,880,365)^{\frac{1}{2}}$

$= 87,045$ volts.

$E_g = 87,045 \times \sqrt{3}$ #7

$= 150,762$ volts.

$p_g = (87,018 \times 115.32) + (2179 \times 11.73)$ #8

$= 10,034,900 + 25,600$

$= 10,060,500$ watts.

$P_g = 10,060,500 \times 3$ #9

$= 30,182$ kw.

$q_g = 87,045 \times 115.92$ #10

$= 10,090,300$ volt amperes.

$Q_g = 10,090,300 \times 3$ #11

$= 30,271$ kv.a.

$(p.f.)_g = 10,060,500 \div 10,090,300$ #12

$= 0.997$

Line loss $= 30,182 - 30,000$ #13

$= 182$ kw.

% line loss $= \frac{182}{30,182}$ #14

$= 0.60$ %

20. Load of 40,000 kw.

$P_r = 40{,}000$ kw.
$E_r = 150{,}000$ volts.
$i_r'' = 0$
$(p.f.)_r = 1$

$$e_r' = \frac{150{,}000}{\sqrt{3}} \qquad \#1$$

$= 86{,}600$ volts.

$$i_r' = I_r = \frac{40{,}000{,}000}{\sqrt{3} \times 150{,}000 \times 1} \qquad \#2$$

$= 153.96$ amperes

$$\dot{I}_g = (153.96 + j\,0)(a_1 - j\,a_2) + 86{,}600\,(c_1 - j\,c_2) \qquad \#3$$

$$= (153.76 - j\,0.05) + (0 - j\,11.69)$$

$$= 153.76 - j\,11.74$$

$$\dot{e}_g = 86{,}600\,(a_1 - j\,a_2) + (153.96 + j\,0)(d_1 - j\,d_2) \qquad \#4$$

$$= (86{,}490 - j\,3) + (704 - j\,2901)$$

$$= 87{,}194 - j\,2904$$

$$I_g = \left[(153.76)^2 + (11.74)^2\right]^{\frac{1}{2}} \qquad \#5$$

$$= (23{,}642.1376 + 137.8276)^{\frac{1}{2}} = (23{,}779.9652)^{\frac{1}{2}}$$

$= 154.21$ amperes.

$$e_g = \left[(87{,}194)^2 + (2904)^2\right]^{\frac{1}{2}} \qquad \#6$$

$$= (7{,}602{,}793{,}636 + 8{,}433{,}216)^{\frac{1}{2}} = (7{,}611{,}226{,}852)^{\frac{1}{2}}$$

$= 87{,}242$ volts.

$$E_g = 87{,}242 \times \sqrt{3} \qquad \#7$$

$= 151{,}103$ volts.

$$P_g = (87{,}194 \times 153.76) + (2904 \times 11.74) \qquad \#8$$

$$= 13{,}406{,}900 + 34{,}100$$

$= 13{,}441{,}000$ watts.

$$P_g = 13{,}441{,}000 \times 3 \qquad \#9$$

$= 40{,}323$ kw.

q_g = 87,242 x 154.21 #10

= 13,453,600 volt amperes.

Q_g = 13,453,600 x 3 #11

= 40,361 kv.a.

$(p.f.)_g$ = 13,441,000 ÷ 13,453,600 #12

= 0.999

Line loss = 40,323 - 40,000 #13

= 323 kw.

% line loss = $\frac{323}{40,323}$ #14

= 0.80 %

21. Load of 50,000 kw.

P_r = 50,000 kw.
E_r = 150,000 volts.
i''_r = 0
$(p.f.)_r$ = 1

$e'_r = \frac{150,000}{\sqrt{3}}$ #1

= 86,600 volts.

$i'_r = I_r = \frac{50,000,000}{\sqrt{3} \times 150,000 \times 1}$ #2

= 192.44 amperes.

$\dot{I}_g$ = (192.44 + j 0)(a_1 - j a_2) + 86,600 (c_1 - j c_2) #3

= (192.21 - j 0.06) + (0 - j 11.69)

= 192.21 - j 11.75

$\dot{e}_g$ = 86,600 (a_1 - j a_2) + (192.44 + j 0)(d_1 - j d_2) #4

= (86,490 - j 3) - (880 - j 3627)

= 87,370 - j 3630

$I_g = \left[(192.21)^2 + (11.75)^2\right]^{\frac{1}{2}}$ #5

$= (36{,}944.6841 + 138.0625)^{\frac{1}{2}} = (37{,}082.7466)^{\frac{1}{2}}$

$= 192.57$ amperes.

$e_g = \left[(87{,}370)^2 + (3630)^2\right]^{\frac{1}{2}}$ #6

$= (7{,}633{,}516{,}900 + 13{,}176{,}900)^{\frac{1}{2}} = (7{,}646{,}693{,}800)^{\frac{1}{2}}$

$= 87{,}445$ volts.

$E_g = 87{,}445 \times \sqrt{3}$ #7

$= 151{,}455$ volts.

$p_g = (87{,}370 \times 192.21) + (3630 \times 11.75)$ #8

$= 16{,}793{,}400 + 42{,}700$

$= 16{,}836{,}100$ watts.

$P_g = 16{,}836{,}100 \times 3$ #9

$= 50{,}508$ kw.

$q_g = 87{,}445 \times 192.57$ #10

$= 16{,}839{,}300$ volt amperes.

$Q_g = 16{,}839{,}300 \times 3$ #11

$= 50{,}518$ kv.a.

$(p.f.)_g = 16{,}836{,}100 \div 16{,}839{,}300$ #12

$= 0.9999$

Line loss $= 50{,}508 - 50{,}000$ #13

$= 508$ kw.

$\%$ line loss $= \frac{508}{50{,}508}$ #14

$= 1.01\ \%$

22. Load of 55,000 kw.

$P_r = 55{,}000$ kw.
$E_r = 150{,}000$ volts.
$i''_r = 0$
$(p.f.)_r = 1$

$$e'_r = \frac{150{,}000}{\sqrt{3}} \qquad \#1$$

$= 86{,}600$ volts.

$$i'_r = I_r = \frac{55{,}000{,}000}{\sqrt{3} \times 150{,}000 \times 1} \qquad \#2$$

$= 211.69$ amperes.

$$\bar{I}_g = (211.69 + j\,0)(a_1 - j\,a_2) + 86{,}600\,(c_1 - j\,c_2) \qquad \#3$$

$= (211.43 - j\,0.07) + (0 - j\,11.69)$

$= 211.43 - j\,11.76$

$$\bar{e}_g = 86{,}600\,(a_1 - j\,a_2) + (211.69 + j\,0)(d_1 - j\,d_2) \qquad \#4$$

$= (86{,}490 - j\,3) + (968 - j\,3989)$

$= 87{,}458 - j\,3992$

$$I_g = \left[(211.43)^2 + (11.76)^2\right]^{\frac{1}{2}} \qquad \#5$$

$= (44{,}702.6449 + 138.2976)^{\frac{1}{2}} = (44{,}840.9425)^{\frac{1}{2}}$

$= 211.76$ amperes.

$$e_g = \left[(87{,}458)^2 + (3992)^2\right]^{\frac{1}{2}} \qquad \#6$$

$= (7{,}648{,}901{,}764 + 15{,}936{,}064)^{\frac{1}{2}} = (7{,}664{,}837{,}828)^{\frac{1}{2}}$

$= 87{,}549$ volts.

$$E_g = 87{,}549 \times \sqrt{3} \qquad \#7$$

$= 151{,}635$ volts.

$$p_g = (87{,}458 \times 211.43) + (3992 \times 11.76) \qquad \#8$$

$= 18{,}491{,}200 + 47{,}000$

$= 18{,}538{,}200$ watts.

$$P_g = 18{,}538{,}200 \times 3 \qquad \#9$$

$= 55{,}615$ kw.

q_g = 87,549 x 211.76 #10

= 18,539,400 volt amperes.

Q_g = 18,539,400 x 3 #11

= 55,618 kv.a.

$(p.f.)_g$ = 18,538,200 ÷ 18,539,400 #12

= 0.9999

Line loss = 55,615 - 55,000 #13

= 615 kw.

% line loss = $\frac{615}{55,615}$ #14

= 1.11 %

23. Load of 60,000 kw.

P_r = 60,000 kw.
E_r = 150,000 volts
i''_r = 0
$(p.f.)_r$ = 1

$e'_r = \frac{150,000}{\sqrt{3}}$ #1

= 86,600 volts.

$i'_r = I_r = \frac{60,000,000}{\sqrt{3} \times 150,000 \times 1}$ #2

= 230.94 amperes.

$\dot{I}_g$ = (230.94 + j 0)(a_1 - j a_2) + 86,600 (c_1 - j c_2) #3

= (230.65 - j 0.07) + (0 - j 11.69)

= 230.65 - j 11.76

$\dot{e}_g$ = 86,600 (a_1 - j a_2) + (230.94 + j 0)(d_1 - j d_2) #4

= (86,490 - j 3) + (1056 - j 4352)

= 87,546 - j 4355

$I_g = \left[(230.65)^2 + (11.76)^2\right]^{\frac{1}{2}}$ #5

$= (53,199.4225 + 138.2976)^{\frac{1}{2}} = (53,337.7201)^{\frac{1}{2}}$

$= 230.95$ amperes.

$e_g = \left[(87,546)^2 + (4355)^2\right]^{\frac{1}{2}}$ #6

$= (7,664,302,116 + 18,966,025)^{\frac{1}{2}} = (7,683,268,141)^{\frac{1}{2}}$

$= 87,654$ volts.

$E_g = 87,654 \times \sqrt{3}$ #7

$= 151,817$ volts.

$p_g = (87,546 \times 230.65) + (4355 \times 11.76)$ #8

$= 20,192,500 + 51,200$

$= 20,243,700$ watts.

$P_g = 20,243,700 \times 3$ #9

$= 60,731$ kw.

$q_g = 87,654 \times 230.95$ #10

$= 20,243,700$ volt amperes.

$Q_g = 20,243,700 \times 3$ #11

$= 60,731$ kv.a.

$(p.f.)_g = 20,243,700 \div 20,243,700$ #12

$= 1$

Line loss $= 60,731 - 60,000$ #13

$= 731$ kw.

% line loss $= \frac{731}{60,731}$ #14

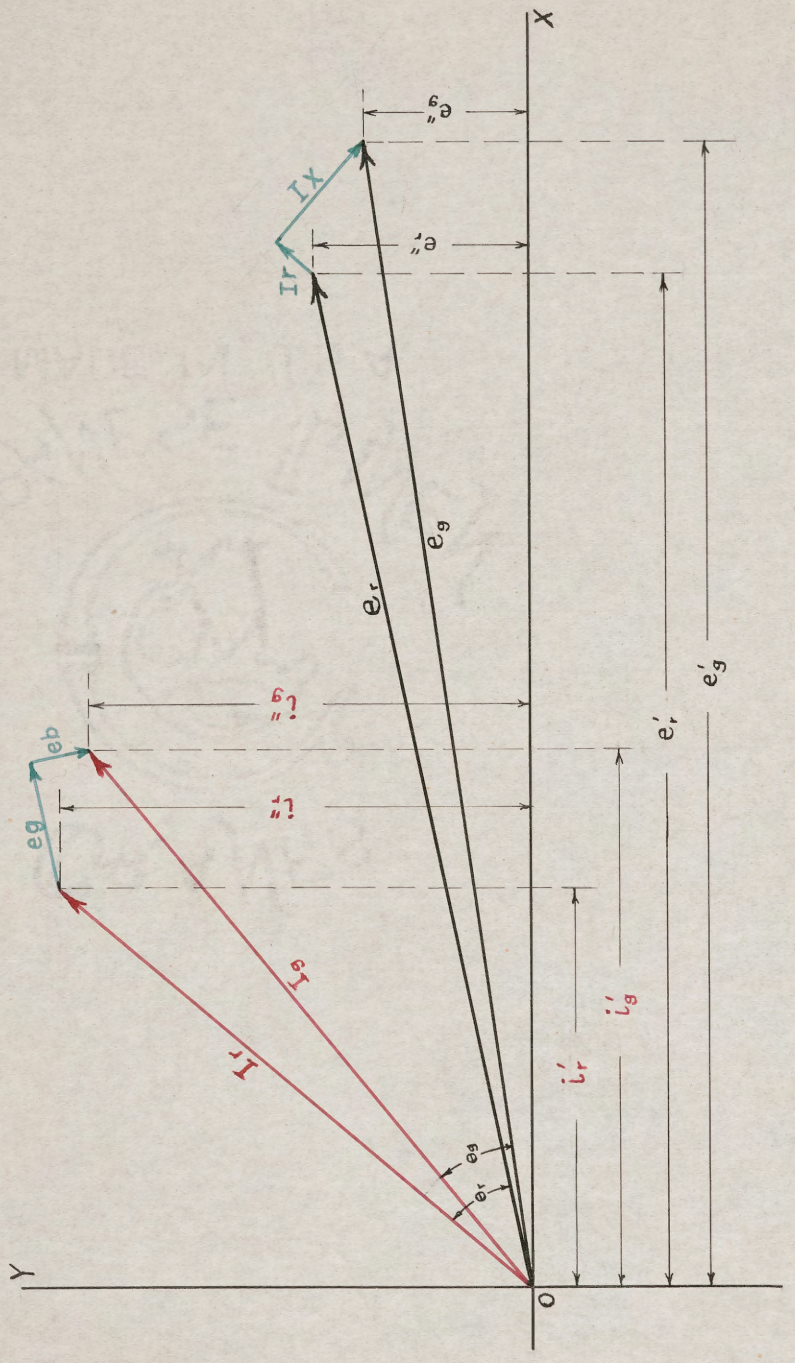
$= 1.20\ \%$

24. Summary.

E_r = 150,000 volts.				i''_r = 0		
e_r = 86,600 volts.				$(p.f.)_r$ = 1		
$P_r = Q_r$. kw. kv.a.	20,000	30,000	40,000	50,000	55,000	60,000
$p_r = q_r$. kw. kv.a.	6,667	10,000	13,333	16,667	18,333	20,000
$i'_r = I_r$ amp.	76.98	115.47	153.96	192.44	211.69	230.94
i'_g ... amp.	76.88	115.32	153.76	192.21	211.43	230.65
i''_g ... amp.	11.71	11.73	11.74	11.75	11.76	11.76
I_g ... amp.	77.77	115.92	154.21	192.57	211.76	230.95
e'_g .. volts	86,842	87,018	87,194	87,370	87,458	87,546
e''_g .. volts	1,454	2,179	2,904	3,630	3,992	4,355
e_g .. volts	86,854	87,045	87,242	87,445	87,549	87,654
E_g .. volts	150,431	150,762	151,103	151,455	151,635	151,817
p_g .... kw.	6,693	10,061	13,441	16,836	18,538	20,244
P_g .... kw.	20,080	30,182	40,323	50,508	55,615	60,731
q_g .. kv.a.	6,755	10,090	13,454	16,839	18,539	20,244
Q_g .. kv.a.	20,264	30,271	40,361	50,518	55,618	60,731
$(p.f.)_g$	0.991	0.997	0.999	0.9999	0.9999	1.0
Line loss kw.	80	182	323	508	615	731
% line loss	0.40	0.60	0.80	1.01	1.11	1.20

Curves for above on pages 35, 36, and 37.

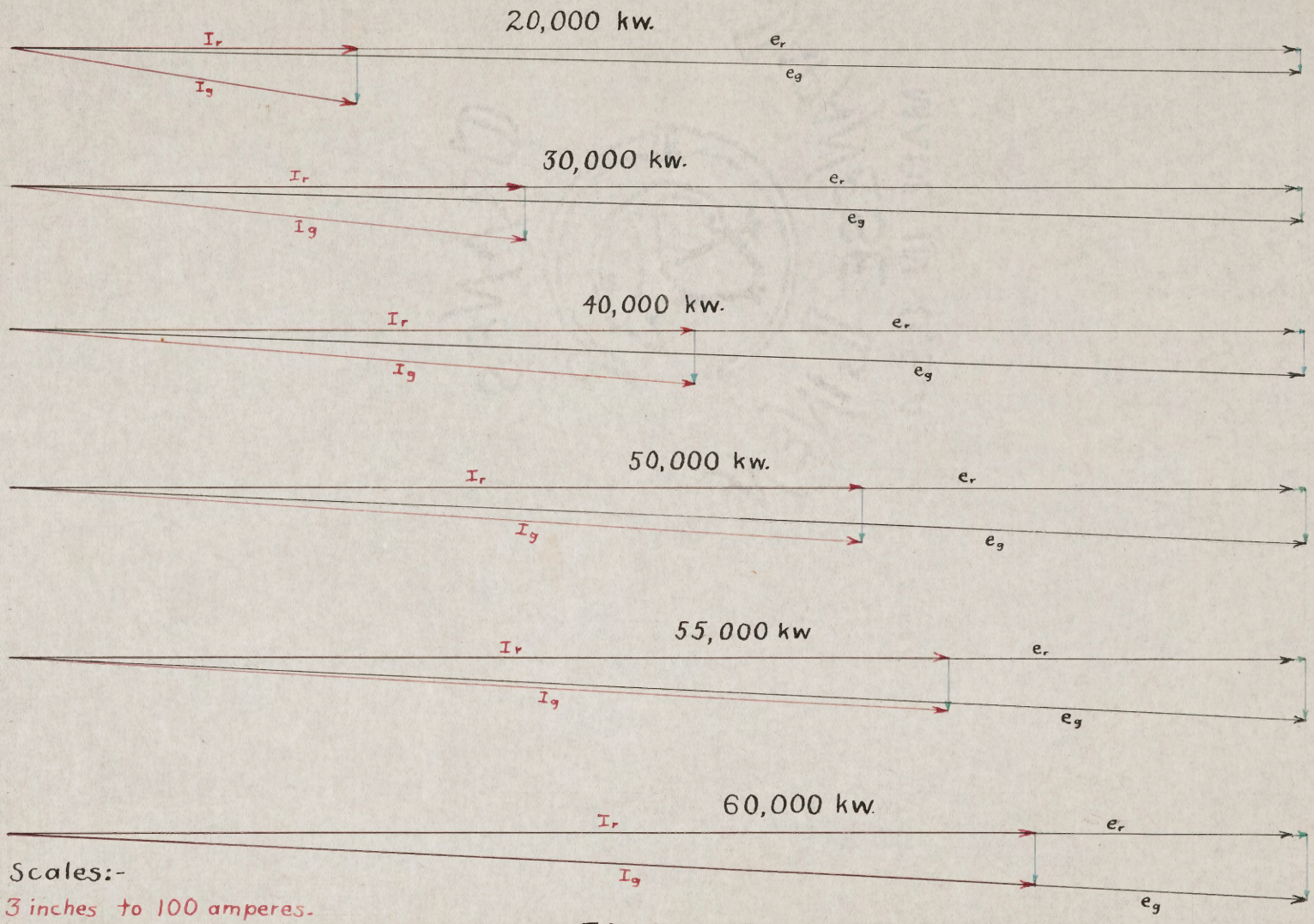
25. Vector Diagrams.



Elemental Vector Diagram
of Currents and Voltages

Figure 2.

Vector Diagrams



Scales:-

3 inches to 100 amperes.

1 inch to 10,000 volts.

Figure 3.

VIII. SAGS AND TENSIONS.

26. Given conditions.

Normal span ..... 800 feet

Maximum span ..... 1295 feet

No wind loads to be considered.

Load of ½" inch ice at 30° F.

Temperature range:- 30° F. to 120° F.

Conductor:- A.C.S.R. - 500,000 c.m.

Aluminum area ... 500,000 c.m. or 0.3927 sq.in.

Diameter ..... 0.904 inch

Elactic limit ... 17,400 lbs.

Ultimate Strength 24,080 lbs.

Weight ..... 783.0 lbs. per 1000 feet.

Dead load ..... 0.783 lbs. per linear foot.

Dead load + ½" ice .. 1.635 lbs. per linear foot.

Coefficient of expansion due to rise in temperature

77×10^{-7} ft.per ft.per degree F.*

Coefficient of expansion due to ½" ice load

0.000477 ft. per ft.*

* These coefficients were calculated from data given in "Transmission Towers" by American Bridge Company.

27. Formulas and Symbols.

L = length of span, in feet
L_c = length of cable, in feet
L'_c = $L_c - L$
l = change in length of cable due to change in temperature.
l' = change in length of cable due to ½" ice load.
S = sag of cable at center of span, in feet.
H = tension in cable at center of span, which is the horizontal component of the tension at any point, in pounds.
W = weight of cable, in pounds per linear foot.
t = temperature, in degrees fahrenheit
t' = change in temperature, in degrees fahrenheit
a = coefficient of expansion due to change in temperature, in ft. per ft. per degree F.
k = coefficient of expansion due to ½" ice load, in ft. per ft.

$$H = \frac{W L^2}{8 S} \qquad \#15$$

$$S = \frac{W L^2}{8 H} \qquad \#16$$

$$S = \sqrt{(L_c - L)\frac{3L}{8}} \text{ or } \sqrt{L'_c \frac{3L}{8}} \qquad \#17$$

$$L_c = L + \frac{8 S^2}{3 L} \qquad \#18$$

$$L'_c = \frac{8 S^2}{3 L} \qquad \#19$$

$$l = L a t' \qquad \#20$$

$$l' = L k \qquad \#21$$

28. Method of calculation.

The maximum tension occurs at 30°F. with ½" ice. This tension is taken constant for different spans. The tension at 120°F. should be about constant for different spans.

The first step is to assume a tension at 30°F. with ½" ice, and from this calculate the tension at 120°F. for different spans. If the tensions found are about the same for all spans at 120°F., the assumption for tension at 30°F. with ½" ice is correct.

After this maximum tension is found, the sag and tension can be found by the formula on preceeding page for the different conditions of the line.

In this line, calculated the sag and tension at 30°F. with ½" ice, 30°F. with no ice, 80°F., and 120°F. for spans of 600 feet, 900 feet, and 1300 feet, which cover practically all conditions to which the line is subject to.

29. Calculation of sags and tensions.

After many assumptions and calculations (which are not given here because of their length and unimportance), found the tension at 30°F. with ½" ice, which would give about a constant tension at 120°F. for different spans, to be 7,600 pounds.

(A). Span of 600 feet.

t = 30°F., ½" ice W = 1.635 lb./ft.

H = 7,600 lbs.

$S = \frac{1.635 \times (600)^2}{8 \times 7{,}600} = 9.68$ ft. #16

$L_c' = \frac{8 \times (9.68)^2}{3 \times 600} = 0.412$ ft. #19

t = 30°F., no ice. W = 0.783 lb./ft.

l' = 600 x 0.000477 = 0.286 ft. #21

$L_c' = 0.412 - 0.286 = 0.126$ ft.

$S = \sqrt{0.126 \frac{3 \times 600}{8}} = 5.33$ ft. #17

$H = \frac{0.783 \times (600)^2}{8 \times 5.33} = 6{,}610$ lbs. #15

t = 80°F. t' = 50° W = 0.783 lb./ft.

$l = 600 \times 77 \times 10^{-7} \times 50 = 0.231$ ft. #20

$L_c' = 0.126 + 0.231 = 0.357$ ft.

$S = \sqrt{0.357 \frac{3 \times 600}{8}} = 8.96$ ft. #17

$H = \frac{0.783 \times (600)^2}{8 \times 8.96} = 3{,}930$ lbs. #15

t = 120°F. t' = 90° W = 0.783 lb./ft.

$l = 600 \times 77 \times 10^{-7} \times 90 = 0.416$ ft. #20

$L_c' = 0.126 + 0.416 = 0.542$ ft.

$S = \sqrt{0.542 \frac{3 \times 600}{8}} = 11.0$ ft. #17

$H = \frac{0.783 \times (600)^2}{8 \times 11.0} = 3200$ lbs. #15

(B). Span of 900 feet.

t = 30°F., ½" ice W = 1.635 lb./ft.

H = 7600 lbs.

$S = \frac{1.635 \times (900)^2}{8 \times 7600} = 21.75$ ft. #16

$L'_c = \frac{8 \times (21.75)^2}{3 \times 900} = 1.40$ ft. #19

t = 30°F., no ice W = 0.783 lb./ft.

l' = 900 x 0.000477 = 0.429 ft. #21

L'_c = 1.400 - 0.429 = 0.971 ft.

$S = \sqrt{0.971 \frac{3 \times 900}{8}} = 18.11$ ft. #17

$H = \frac{0.783 \times (900)^2}{8 \times 18.11} = 4375$ lbs. #15

t = 80°F. t' = 50° W = 0.783 lb./ft.

$l = 900 \times 77 \times 10^{-7} \times 50 = 0.347$ ft #20

L'_c = 0.971 + 0.347 = 1.318 ft

$S = \sqrt{1.318 \frac{3 \times 900}{8}} = 21.09$ ft #17

$H = \frac{0.783 \times (900)^2}{8 \times 21.09} = 3760$ lbs. #15

t = 120°F. t' = 90° W = 0.783 lb./ft.

$l = 900 \times 77 \times 10^{-7} \times 90 = 0.624$ ft. #20

L'_c = 0.971 + 0.624 = 1.595 ft.

$S = \sqrt{1.595 \frac{3 \times 900}{8}} = 23.20$ ft. #17

$H = \frac{0.783 \times (900)^2}{8 \times 23.20} = 3410$ lbs. #15

(C). Span of 1300 feet.

t = 30°F., ½" ice W = 1.635 lb./ft.

H = 7600 lbs.

$S = \frac{1.635 \times (1300)^2}{8 \times 7600} = 45.5$ ft. #16

$L_c' = \frac{8 \times (45.5)^2}{3 \times 1300} = 4.26$ ft. #19

t = 30°F., no ice W = 0.783 lb./ft.

l' = 1300 x 0.000477 = 0.620 ft. #21

$L_c' = 4.26 - 0.620 = 3.64$ ft.

$S = \sqrt{3.64 \frac{3 \times 1300}{8}} = 42.13$ ft. #17

$H = \frac{0.783 \times (1300)^2}{8 \times 42.13} = 3920$ lbs. #15

t = 80°F. t' = 50° W = 0.783 lb./ft.

$l = 1300 \times 77 \times 10^{-7} \times 50 = 0.500$ ft #20

$L_c' = 3.640 + 0.500 = 4.14$ ft.

$S = \sqrt{4.14 \frac{3 \times 1300}{8}} = 44.92$ ft. #17

$H = \frac{0.783 \times (1300)^2}{8 \times 44.92} = 3680$ lbs. #15

t = 120°F. t' = 90° W = 0.783 lb./ft.

$l = 1300 \times 77 \times 10^{-7} \times 90 = 0.90$ ft. #20

$L_c' = 3.64 + 0.90 = 4.54$ ft.

$S = \sqrt{4.54 \frac{3 \times 1300}{8}} = 47.03$ ft. #17

$H = \frac{0.783 \times (1300)^2}{8 \times 47.03} = 3520$ lbs. #15

30. Summary.

Normal Span 800 feet. Maximum Span 1295 feet.

Maximum Tension at 30° F., with ½" ice, 7,600 pounds.

Temp.	Span ft.	SAG, feet 600	900	1300	TENSION, lb. 600	900	1300
30° F., ½" ice		9.68	21.75	45.50	7600	7600	7600
30° F., no ice		5.33	18.11	42.13	6610	4375	3920
80° F., no ice		8.96	21.09	44.92	3930	3760	3680
120° F., no ice		11.00	23.20	47.03	3200	3410	3520

Curves for above on page 38.

IX. RESULTS.

Cheoah - Alcoa Line (North Carolina, Tennessee)
of the Knoxville Power Company.

Alternating Current, Three-Phase, 60 Cycles.
Maximum Load 55,000 kw. Minimum Load 20,000 kw.
Length of Line 25 miles.
Normal Span 800 feet. Maximum Span 1295 feet.

31. Conductors:-

Aluminum Cable Steel Reinforced - 500,000 c.m.
Equivalent equidistant spacing (D) 180 inches.

32. Line Constants:-

Resistance (r)	4.580 ohms
Inductance (L)	0.050 henry
Reactance (x)	18.850 ohms
Impedance (z)	19.398 ohms
Condensance (C)	0.3581×10^{-6} farads
Shunted line susceptance (b) . .	0.13500×10^{-3} mhos.

33.Performance of Line:-

E_r = 150,000 volts.				$(p.f.)_r$ = 1		
$P_r = Q_r$ kw. kv.a.	20,000	30,000	40,000	50,000	55,000	60,000
I_r .... amp.	76.98	115.47	153.96	192.44	211.69	230.94
I_g .... amp.	77.77	115.92	154.21	192.57	211.76	230.95
E_g ... volts	150,431	150,762	151,103	151,455	151,635	151,817
P_g ..... kw.	20,080	30,182	40,323	50,508	55,615	60,731
Q_g ... kv.a.	20,264	30,271	40,361	50,518	55,618	60,731
$(p.f.)_g$	0.991	0.997	0.999	0.9999	0.9999	1.0
Line Loss kw.	80	182	323	508	615	731
% Line Loss.	0.40	0.60	0.80	1.01	1.11	1.20

Curves for above on following pages.

FormE-5

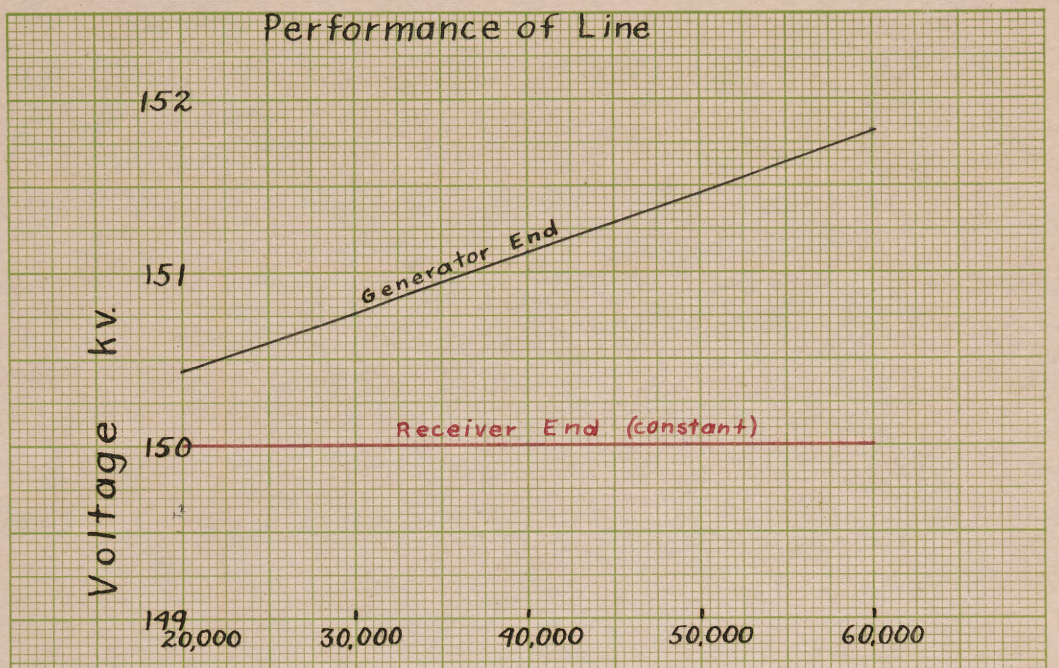


Figure 4

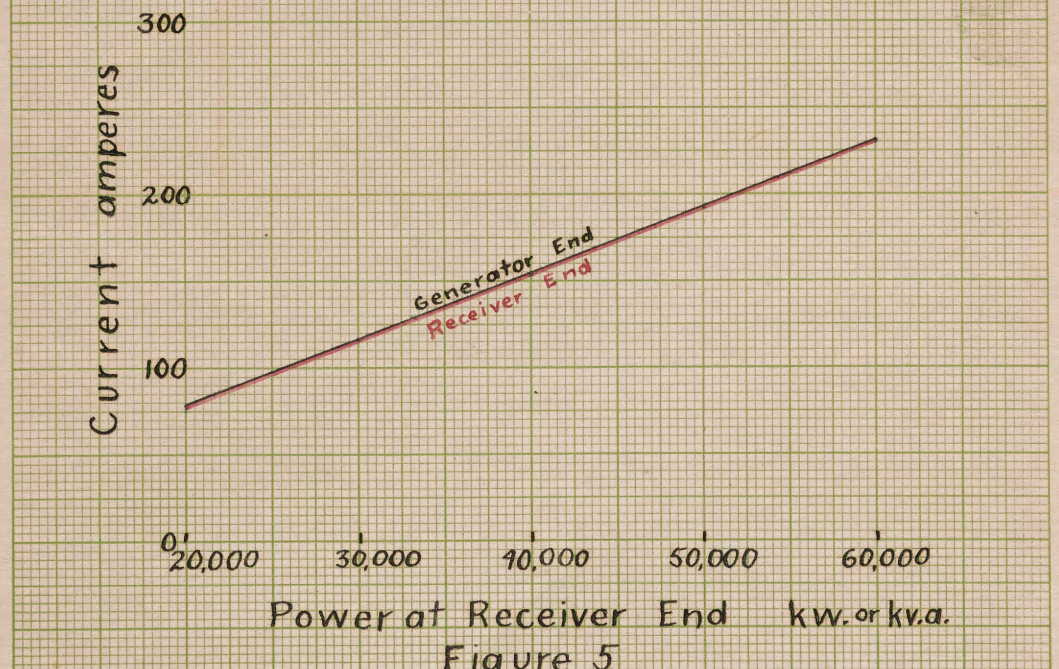


Figure 5

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FormE-5

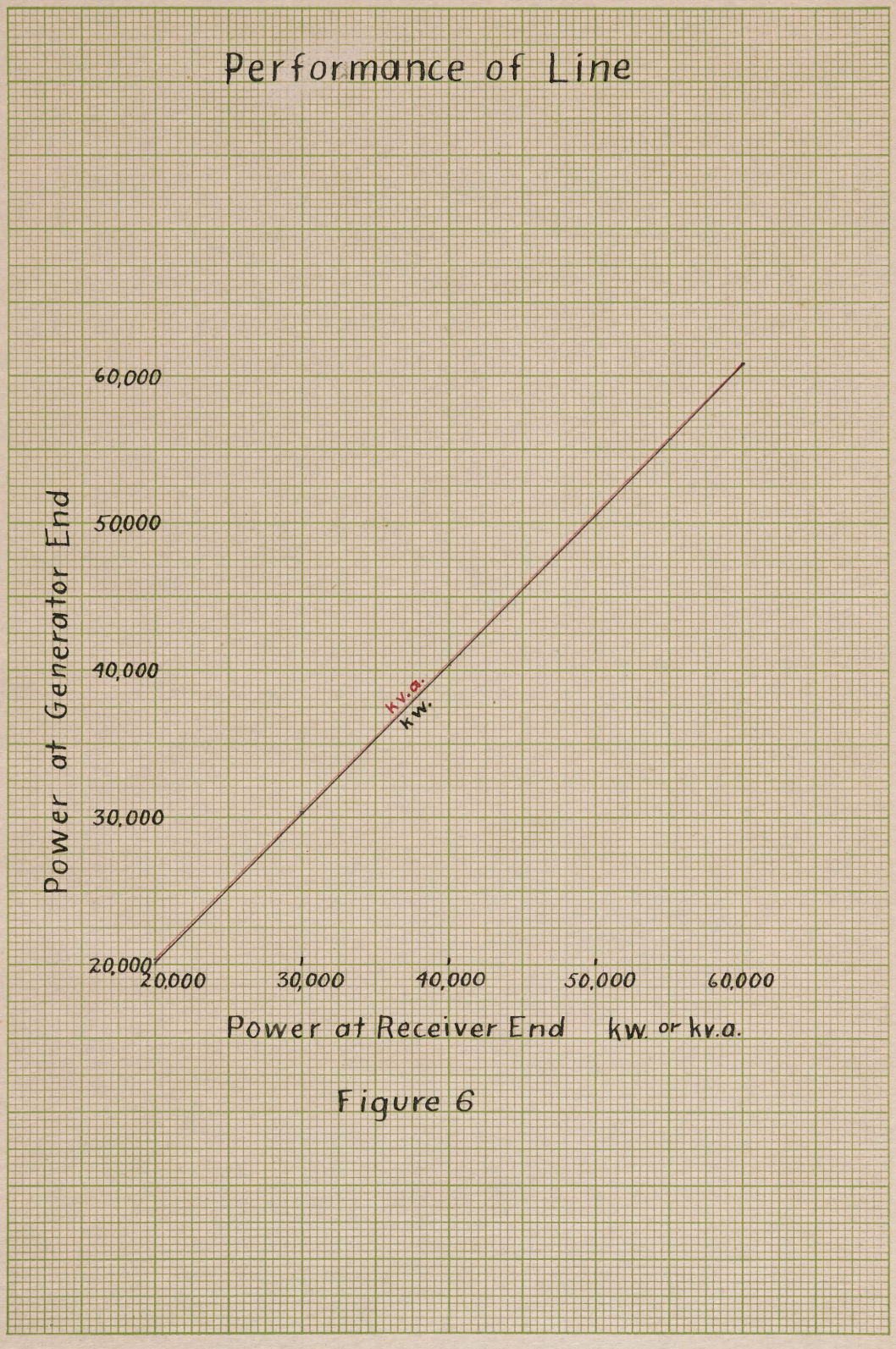


Figure 6

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FormE-5

Performance of Line

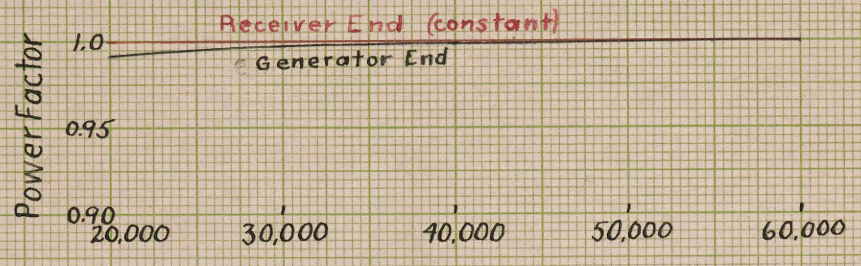


Figure 7

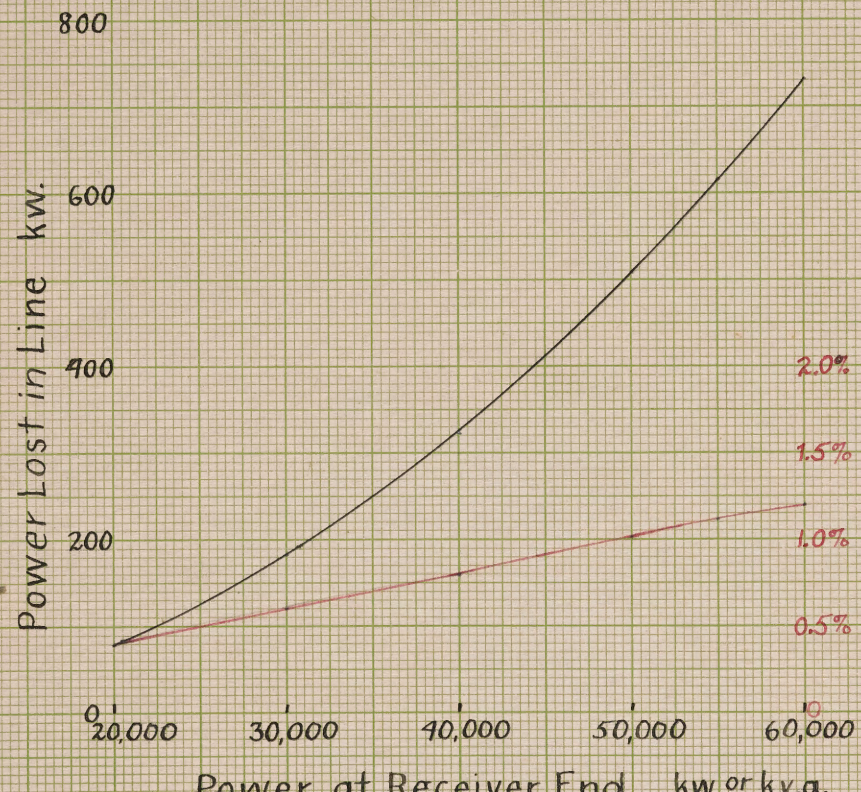


Figure 8

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34. Sags and Tensions.

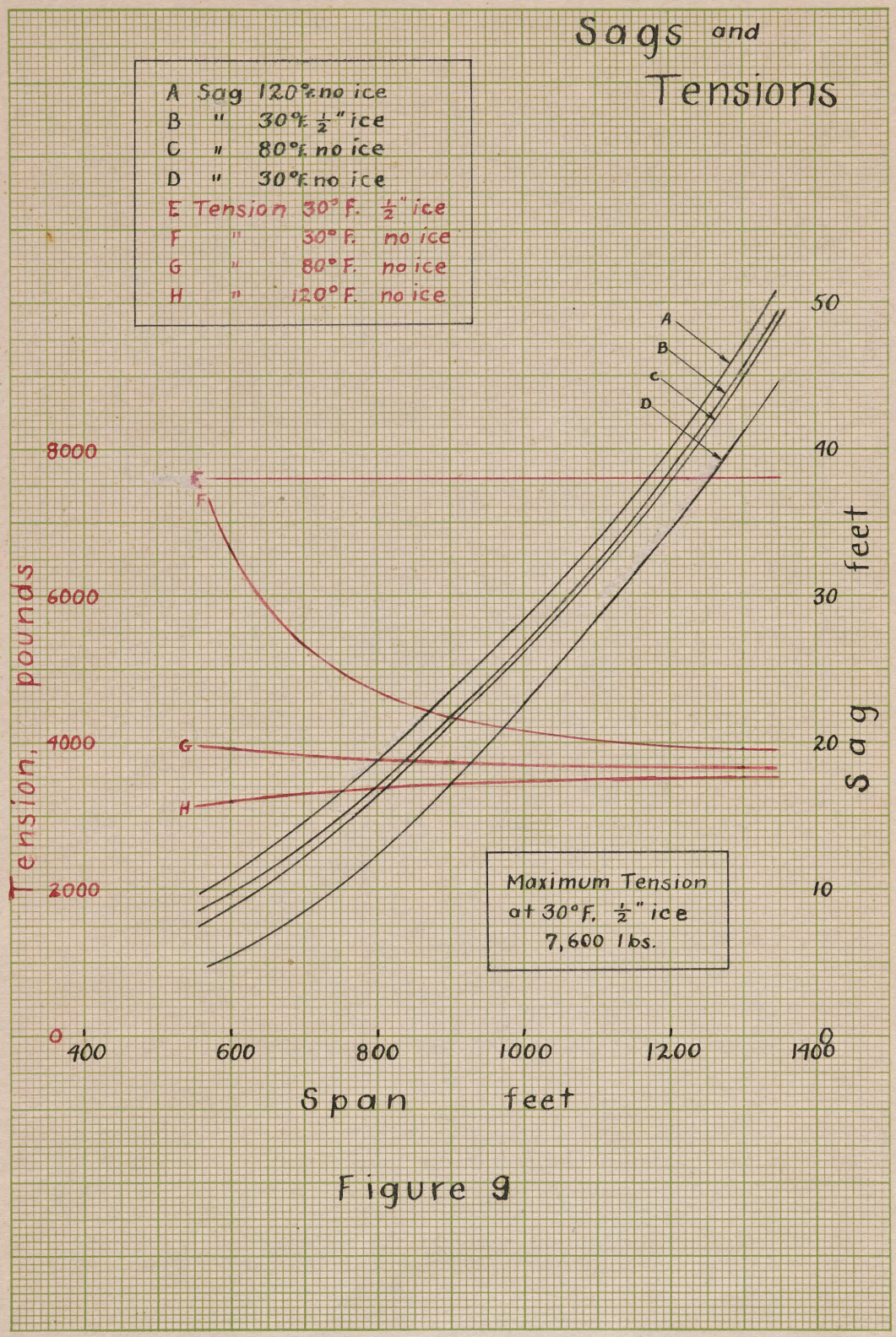


Figure 9

Published by The H. Cole Co., Columbus, Ohio, No. 290 G.

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by Alfred Still

Transmission Line Formulas

by H. B. Dwight

Transmission Towers

by American Bridge Company.

XI. APPENDIX.

35. Symbols for Electrical Calculations.

$a_1 - j\, a_2$ = line constant

α = line cinstant

b = shunted kine susceptance, in mhos.

β = line constant

C = condensance, in farads

$c_1 - j\, c_2$ = line constant

D = equivalent equidistant spacing between conductors, in inches

$d_1 - j\, d_2$ = line constant

d = diameter of conductor, in inches

E = voltage between conductors, in volts or

e = voltage to neutral, in volts

$\dot{e}$ = vector voltage

e' = power voltage, in volts

e" = reactive voltage, in volts

f = frequency

I = current, in amperes

$\dot{I}$ = vector current

i' = power current, in amperes

i" = reactive current, in amperes

L = inductance, in henrys

l = length of conductor

P = power of line, in watts or kw.

p = power of each conductor, in watts or kw.

p.f. = power factor

Q = power of line, in volt amperes or kv.a.

q = power of each conductor, in volt amperes or kv.a.

r = resistance of each conductor, in ohms.

ρ = resistance of each conductor per foot, in ohms per ft.

x = reactance, in ohms

z = impedance, in ohms

Note:- subscript r denotes value at receiver end
subscript g denotes value at generator end

36. Symbols for Sag and Tension Calculations.

a = coefficient of expansion due to change in temperature, in ft. per ft. per degree F.

H = tension in cable at center of span, which is the horizontal component of the tension at any point, in pounds

k = coefficient of expansion due to $\frac{1}{2}$" ice load, in ft. per ft.

L = length of span, in feet

L_c = length of cable, in feet

L_c' = $L_c - L$ in feet

l = change in length of cable due to change in temperature, in feet.

l' = change in length of cable due to $\frac{1}{2}$" ice load, in ft.

S = sag of cable at center of span, in feet

t = temperature, in degrees fahrenheit

t' = change in temperature, in degrees F.

W = weight of cable, in pounds per linear foot

XII. APPROVAL

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prerequisite to the degree for which it has been submitted. It is to be understood that by this approval the undersigned does not necessarily endorse or approve any statement made, opinions expresses, or conclusions drawn therein, but approves the thesis only for the purpose for which it is submitted.

Name Chas A Perkins

Title Professor
Electrical Engineering.

This thesis submitted May 18, 1925

by C. Wilson House.