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I am submitting herewith a thesis written by Dennis Duncan Earl entitled "Application of Fresnel Diffraction To Distance Metrology." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.

  
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# **APPLICATION OF FRESNEL DIFFRACTION TO DISTANCE METROLOGY**

A Thesis

Presented for the

Master of Science

Degree

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Dennis Duncan Earl

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## ABSTRACT

The use of Fresnel diffraction as a means for measuring absolute distances represents a unique approach to the science of non-contact distance metrology. Fresnel diffraction-based metrology measures the central intensity of a Fresnel diffraction pattern to determine the absolute distance to the source which is producing the pattern. This technique is presented at a time when traditional methods of non-contact distance measurement, particularly interferometric techniques and triangularization systems, are approaching a limit in their abilities. A Fresnel diffraction-based distance measurement system offers the potential for becoming the new standard in high accuracy, high versatility measurement tools. Because of the inherent simplicity involved in the extraction of distance information from a single Fresnel diffraction pattern, this technique is proving useful in a wide variety of applications.

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## LIST OF ABBREVIATIONS

|                                         |      |
|-----------------------------------------|------|
| Coordinate Measuring Machines . . . . . | CMM  |
| Helium Neon Laser . . . . .             | HeNe |
| Signal-to-Noise Ratio . . . . .         | SNR  |

## 1. Introduction

In 1818, A. J. Fresnel presented a prize winning essay on the diffraction of light to the French Académie des Sciences. His essay mathematically predicted the production and structure of optical diffraction patterns produced by various obstacles and apertures in both the near and far field. His conclusions were quickly challenged by an esteemed committee member and advocate of the corpuscular theory of light, S. D. Poisson, on the basis of absurdity. Poisson observed that Fresnel's theory predicted that light diffracted by a circular obstacle would produce a diffraction pattern with a bright central intensity in the shadow region of the obstacle. Since this seemed intuitively impossible, Poisson suggested that Fresnel's theory must be wrong and should be viewed as evidence of the inaccuracy of the wave theory of light.

Several years later, Poisson's "impossibility" was observed by the French physicist D. F. Arago and termed the Spot of Arago, more commonly known today as Poisson's Spot. This astounding observation would play a significant role in establishing the acceptance of the wave theory of light<sup>1</sup>.

Unfortunately, despite the significant role of Poisson's spot, its occurrence and other associated features of near field diffraction would remain principally an optical curiosity which only found application in displaying the wonderment of diffraction. As of this writing, the practical application of Fresnel diffraction theory is extremely limited, most likely due to its perceived mathematical complexity.

This document seeks to demonstrate that Fresnel diffraction can be used as a means of measuring absolute distances with high accuracy. Conditions will be shown to exist which result in the simplification of traditional Fresnel diffraction theory<sup>2</sup>, allowing for the practical application of this technique. What follows are the theoretical principles which this method is based upon, the experimental evidence which supports the application of Fresnel diffraction to distance measurement, the characterization of a prototype device based upon this method, and the projected advancement and potential application of this technique.

## **I. FRESNEL DIFFRACTION THEORY**

## 2. Traditional Development of Fresnel Diffraction Theory

### Introduction

Before an explanation of the application of Fresnel diffraction to distance measurement can be made, a basic understanding of Fresnel diffraction for simple apertures must be addressed. What follows is a derivation of the Fresnel diffraction integrals for an axially symmetric circular aperture. These integrals have been derived in terms of laboratory coordinates to better represent the experimental conditions which will be encountered later in this writing.

### Basic Fresnel Theory for a Circular Aperture

Monochromatic light emitted by a point source  $Q$  passes through a circular aperture and is analyzed at a distance  $L_{\text{observe}}$  and at a distance  $\rho$  off-axis (see Figure 1):

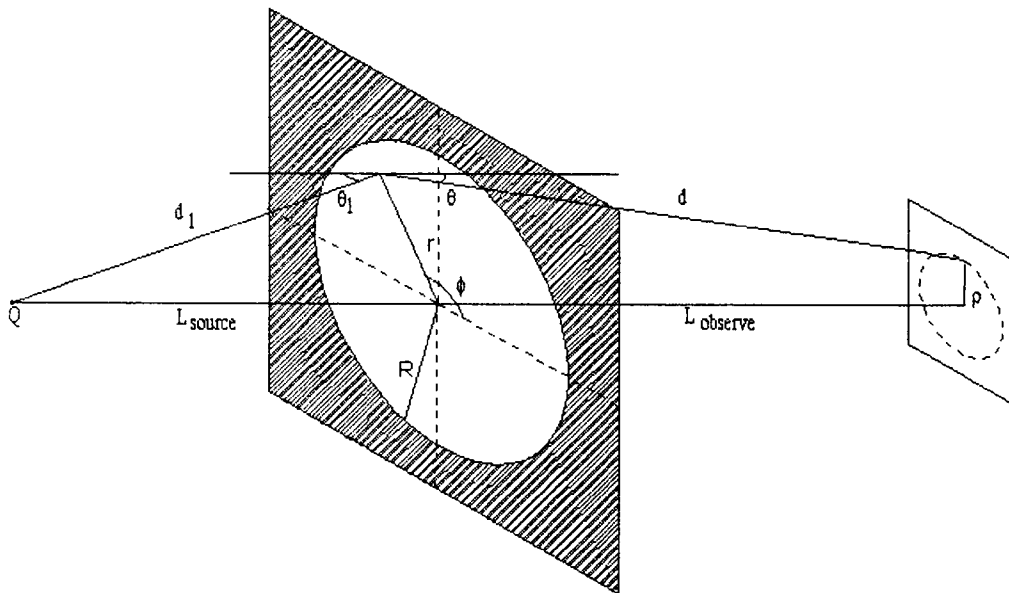


Figure 1: Circular Aperture With Defined Parameters

Huygen's principle states that the propagation of electromagnetic radiation can be modeled as the absorption and re-emission of an initial electromagnetic wave. Therefore, if we choose the initial wave emitted by point Q to be a spherical wave, then the amplitude at some distance  $r$  away from the source is given by the Equation 1:

$$\psi(r) = \frac{A_o}{r} e^{i(\vec{k} \cdot \vec{r})} \quad (\text{Equation 1})$$

Putting this equation in the nomenclature of Figure 1, we have the amplitude of the incident wave, in the plane of the pinhole, defined by Equation 2:

$$\psi(d_1) = \frac{A_o f_s}{d_1} e^{i(kd_1)} \quad (\text{Equation 2})$$

where  $f_s$  is the transmission function for the diffracting aperture. For the circular aperture, we will take  $f_s$  to be simply:

$$f_s = \begin{cases} 1 & \text{for } 0 \leq r \leq R \\ 0 & \text{for } r > R \end{cases}$$

Working in spherical coordinates with the origin at point Q and the  $\theta$  and  $\phi$  spherical component equal to zero, the gradient of Equation 2 is calculated to be Equation 3:

$$\nabla \psi = \frac{f_s A_o \vec{d}_1}{d_1^3} (ikd_1 - 1) e^{ikd_1} \quad (\text{Equation 3})$$

From the Helmholtz Equation (which can be solved using Green's Theorem; see Heckt<sup>3</sup>), the Kirchhoff integral theorem is deduced to Equation 4:

$$\psi_{POINT}(r) = \frac{1}{4\pi} \iint_{\substack{\text{Closed} \\ \text{Surface}}} \frac{1}{r^3} e^{kr} [(ikr - 1)\vec{r} \psi + r^2 \nabla \psi] \cdot \vec{n} dS \quad (\text{Equation 4})$$

Where  $\psi_{\text{point}}(\mathbf{r})$  is the amplitude measured at some arbitrary point located within a closed surface, and  $\vec{n}$  is the vector normal to the closed surface. For our case, the surface to be integrated over will be the pinhole area, which can be considered to be located on the closed surface of an infinite-radius sphere. It can be noted that the portion of the infinite-radius sphere occupied by the pinhole can be considered to be a planar disc and any points of observation will be contained within the infinite-radius sphere (which is centered to the right of the pinhole in Figure 1).

Substituting Equations 2 and 3 into Equation 4 gives Equation 5:

$$\psi(\rho) = \frac{A_o}{4\pi} \iint_{\text{surface}} f_s e^{ik(d+d_1)} \left[ \frac{\vec{d} \cdot \vec{n}}{d_1 d^3} (ikd - 1) - \frac{\vec{d}_1 \cdot \vec{n}}{d d_1^3} (ikd_1 - 1) \right] dS \quad (\text{Equation 5})$$

From Figure 1 we can deduce Equation 6:

$$\begin{aligned} \vec{d} \cdot \vec{n} &= d \cos \theta \\ \vec{d}_1 \cdot \vec{n} &= d_1 \cos \theta_1 \end{aligned} \quad (\text{Equation 6})$$

Therefore giving Equation 7:

$$\psi(\rho) = \frac{A_o}{4\pi} \iint_{\text{surface}} f_s e^{ik(d+d_1)} \left[ \frac{d \cos \theta}{d_1 d^3} (ikd - 1) + \frac{d_1 \cos \theta_1}{d d_1^3} (ikd_1 - 1) \right] dS \quad (\text{Equation 7})$$

Evaluating this equation further yields Equation 8:

$$\psi(\rho) = \frac{A_o}{4\pi} \iint_{\text{surface}} f_s e^{ik(d+d_1)} \left[ \frac{2ik}{dd_1} \left( \frac{\cos \theta + \cos \theta_1}{2} \right) - \left( \frac{d_1 \cos \theta + d \cos \theta_1}{d^2 d_1^2} \right) \right] dS \quad (\text{Equation 8})$$

Which further separates into Equation 9:

$$\psi(\rho) = \frac{ikA_o}{2\pi} \iint_{\text{surface}} \frac{f_s}{dd_1} e^{ik(d+d_1)} \left( \frac{\cos \theta + \cos \theta_1}{2} \right) dS - \frac{A_o}{2\pi} \iint_{\text{surface}} \frac{f_s}{d^2 d_1^2} e^{ik(d+d_1)} \left( \frac{d_1 \cos \theta + d \cos \theta_1}{2} \right) dS \quad (\text{Equation 9})$$

where the first surface integral represents the classical derivation of Fresnel diffraction (with the trigonometric factor defined as the obliquity factor), and the second surface integral is a second-order correction (nearly zero when  $d$  and  $d_1$  are much larger than the wavelength). Because we will be dealing with distances much greater than the wavelength of light, the second integral will be neglected in all subsequent calculations. The validity of such a dismissal can be demonstrated by dimensionally analyzing the ratio of the two integrands in Equation 9. This technique gives the approximate result:

$$Ratio \approx \frac{\lambda}{d}$$

where  $d$  is the smaller of the observation or source distance. Obviously, when  $d \gg \lambda$  then the second integral can be discarded. Consequently this gives Equation 10:

$$\psi(\rho) = \frac{ikA_o}{2\pi} \iint_{surface} \frac{f_s}{dd_1} e^{ik(d+d_1)} \left( \frac{\cos\theta + \cos\theta_1}{2} \right) dS \quad (Equation 10)$$

Using Euler's identity, Equation 10 can be separated into the following real and imaginary integrals as described in Equations 11 and 12:

$$\psi_{real}(\rho) = \frac{-kA_o}{2\pi} \iint_{surface} \frac{f_s}{dd_1} \left( \frac{\cos\theta + \cos\theta_1}{2} \right) \sin[k(d+d_1)] dS \quad (Equation 11)$$

$$\psi_{imaginary}(\rho) = \frac{ikA_o}{2\pi} \iint_{surface} \frac{f_s}{dd_1} \left( \frac{\cos\theta + \cos\theta_1}{2} \right) \cos[k(d+d_1)] dS \quad (Equation 12)$$

Continuing with a little bit of geometry, we derive Equation 13:

$$\begin{aligned}
 d_1 &= \sqrt{L_{source}^2 + r^2} \\
 d &= \sqrt{L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi} \\
 \cos\theta_1 &= \frac{L_{source}}{\sqrt{L_{source}^2 + r^2}} \\
 \cos\theta &= \frac{L_{observe}}{\sqrt{L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi}}
 \end{aligned} \tag{Equation 13}$$

Substituting these values into Equations 11 and 12 along with Equation 14:

$$\begin{aligned}
 dS &= r dr d\phi \\
 k &= \frac{2\pi}{\lambda}
 \end{aligned} \tag{Equation 14}$$

gives Equation 15:

$$\begin{aligned}
 \psi_{real}(\rho) &= \frac{-A_o}{2\lambda} \int_0^{2\pi} \int_0^R \frac{\left( L_{observe} \sqrt{L_{source}^2 + r^2} + L_{source} \sqrt{L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi} \right)}{\left( L_{source}^2 + r^2 \right) \left( L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi \right)} \\
 &\quad \cdot \sin \left[ \frac{2\pi}{\lambda} \left( \sqrt{L_{source}^2 + r^2} + \sqrt{L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi} \right) \right] r dr d\phi
 \end{aligned} \tag{Equation 15}$$

and Equation 16:

$$\begin{aligned}
 \psi_{imaginary}(\rho) &= \frac{A_o}{2\lambda} \int_0^{2\pi} \int_0^R \frac{\left( L_{observe} \sqrt{L_{source}^2 + r^2} + L_{source} \sqrt{L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi} \right)}{\left( L_{source}^2 + r^2 \right) \left( L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi \right)} \\
 &\quad \cdot \cos \left[ \frac{2\pi}{\lambda} \left( \sqrt{L_{source}^2 + r^2} + \sqrt{L_{observe}^2 + r^2 + \rho^2 - 2r\rho\sin\phi} \right) \right] r dr d\phi
 \end{aligned} \tag{Equation 16}$$

Therefore, the intensity measured at a point  $\rho$  off-axis is given by Equation 17:

$$I = \psi * \psi = \psi_{real}^2 + \psi_{imaginary}^2 \tag{Equation 17}$$

For the on-axis case (where  $\rho$  is zero), Equations 15 and 16 simplify into Equation 18:

$$\psi_{real}(0) = \frac{-\pi A_o}{\lambda} \int_0^R \frac{\left( L_{observe} \sqrt{L_{source}^2 + r^2} + L_{source} \sqrt{L_{observe}^2 + r^2} \right)}{\left( L_{source}^2 + r^2 \right) \left( L_{observe}^2 + r^2 \right)} \cdot \sin \left[ \frac{2\pi}{\lambda} \left( \sqrt{L_{source}^2 + r^2} + \sqrt{L_{observe}^2 + r^2} \right) \right] r dr$$

(Equation 18)

and Equation 19:

$$\psi_{imaginary}(0) = \frac{\pi A_o}{\lambda} \int_0^R \frac{\left( L_{observe} \sqrt{L_{source}^2 + r^2} + L_{source} \sqrt{L_{observe}^2 + r^2} \right)}{\left( L_{source}^2 + r^2 \right) \left( L_{observe}^2 + r^2 \right)} \cdot \cos \left[ \frac{2\pi}{\lambda} \left( \sqrt{L_{source}^2 + r^2} + \sqrt{L_{observe}^2 + r^2} \right) \right] r dr$$

(Equation 19)

Having established Equations 15-19, a numerical analysis of these integrals can now be performed which will provide greater insight into the intensity-to-distance relationship.

### **3. Numerical Analysis of Diffraction Integrals**

#### **Introduction**

A numerical analysis of the diffraction integrals, along with supporting experimental data, is presented below to establish the fundamental theoretical formulas needed to describe Fresnel diffraction-based metrology. These formulas detail the behavior of the near-field diffraction pattern as a function of source distance and observation distance.

A numerical analysis of Equations 15 and 16 will be performed to qualitatively describe the nature of a Fresnel diffraction pattern, while a numerical evaluation of Equations 18 and 19 will be performed to quantitatively reveal that the dependence of the pattern's central intensity on the source and observation distance

#### **Numerical Evaluation of Equations 15 and 16**

Equations 15 and 16 allow us to accurately calculate a diffraction pattern's intensity profile for a given source and observation distance. Sixth-order Newton-Cotes formula was performed on a 133 MHz pentium processor PC to precisely calculate Equations 15 and 16 in regions which were most likely to be encountered in the laboratory.

One of the most noticeable characteristics of Fresnel diffraction is the complexity of its patterns in the near field. As opposed to Fraunhofer diffraction theory (which predicts a very simple pattern which merely enlarges with distance), Fresnel diffraction

patterns come in a variety of assortments. In fact, Fresnel diffraction patterns, in the near field, are a lot like snow flakes: No two are ever the same.

To qualitatively analyze the ability of the theoretical model to predict what is physically observed, the pattern for a pinhole of radius 1mm (illuminated with a 632.8nm point source at a distance of 0.780 meters and observed at a distance of 0.289 meters) was calculated and is illustrated in Figure 2:

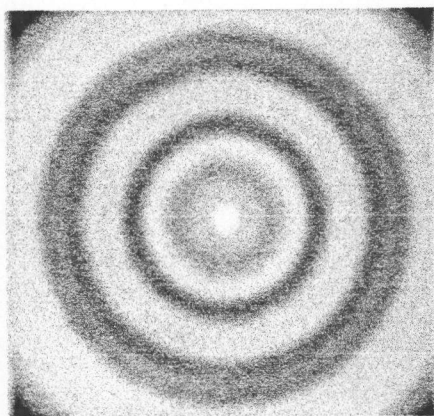


Figure 2 (Theoretically Generated Diffraction Pattern)

To compare this theoretical calculation with what is physically observed in the laboratory, a 1mW HeNe laser was used to illuminate a pinhole of 1mm ( $\pm 0.005\text{mm}$ ) radius. The laser light was first passed through a spatial filter (with a 5 micron aperture to produce a point source of monochromatic light) and then aligned axially with the pinhole a distance 0.780 meters ( $\pm 0.001\text{m}$ ) away. The pinhole was a precision laser-bored steel-film pinhole which had been black anodized on both sides to reduce the amount of scattered light.

An 8-bit 640x480 pixel CCD camera, with a sensing area of 6.40 x 4.80 mm, was positioned at an observation distance of 0.287 meter ( $\pm 0.001\text{m}$ ) from the pinhole. The

focusing lens normally attached to the CCD had been removed to permit true image capture(non-magnified) and reduce unwanted internal reflections. The CCD image was captured with a frame grabber board(Data Translations DT3851) which was specifically designed to suppress undesirable instrument noise. The image, once captured, was converted into a text file and the background noise(previously measured) was subtracted from the image (See Figure 3). A comparison of the theoretically predicted intensity profile(solid line) along with the experimentally acquired intensity profile(diamonds) is shown in Figure 4:

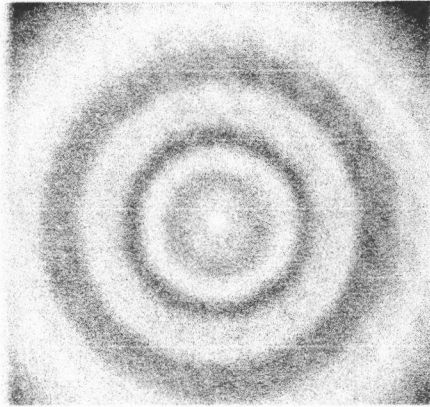


Figure 3 (Experimentally Acquired Diffraction Pattern)

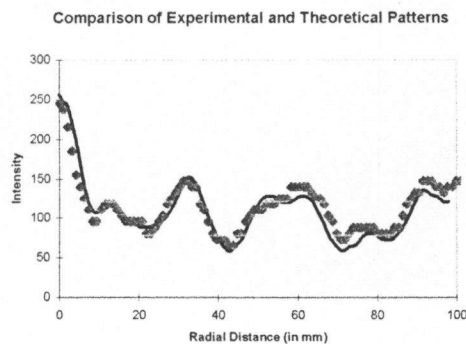


Figure 4 (Comparison of Experimental and Theoretical Intensity Profiles)

While visual comparison of Figures 2 and 3 qualitatively agree, Figure 4 reveals differences between the theoretically calculated and experimentally observed intensity patterns. These differences in the intensity profile (see Figure 5) are attributed to several different variables.

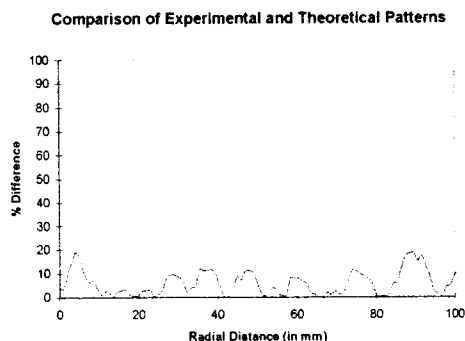


Figure 5 (Differences Between Experimental and Theoretical Intensity Profiles)

Most notably, the superposition of the desired diffraction pattern with an unwanted interference pattern, which can easily be observed in Figure 3, results in possible errors in the intensity profile of the image. Plus, the resolution of the CCD camera plays an important role in determining the accuracy of the pattern's intensity profile. For CCD's with large area pixels, the recorded intensities (actually the average intensity over the entire pixel area) will have a tendency to "smooth" the expected intensity profile.

The largest electronic errors can be attributed to electrical noise associated with the CCD. This noise, often referred to as "pixel jitter," is the result of thermal effects on the synchronization timing mechanism within the CCD camera. The result is a voltage measurement for a single pixel which will typically fluctuate by about 5% of its total signal. This noise can, however, be reduced by using a thermoelectrically cooled CCD in conjunction with a phase-locked loop circuit frame grabber board. While it was not

possible to acquire a thermoelectrically cooled CCD for this experiment, it was possible to acquire a frame grabber board, specifically designed to reduce pixel jitter, which contained a Digital Clock Sync Circuit experiencing half the pixel jitter of a Phase-Locked Loop Circuit board.

Additional sources of error include distortions in the source wavefront, non-circularity of the pinhole, scattered light, and high-frequency intensity fluctuations in the source. The effect of these errors, however, can be considered small ( $<1\%$  of the measured voltage).

### **Numerical Evaluation of Equations 18 and 19**

Equations 18 and 19 allow us to accurately calculate a diffraction pattern's central intensity for a variety of source and observation distances. Once again, sixth-order Newton-Cotes formula was used, with a 133 MHz pentium processor PC, to precisely calculate Equations 18 and 19 in regions which were most likely to be encountered in the laboratory.

It should be noted that an analysis of the Fresnel pattern central intensity as a function of observation distance is a classic textbook derivation. However, these derivations make the simplifying assumption that the source distance is equal to infinity. While this assumption simplifies the diffraction integrals and allows them to be solved analytically, the dependence of the diffraction pattern as a function of the source distance is lost. Unfortunately, it is this dependence on source distance which, as will be illustrated in section 5, makes Fresnel diffraction so applicable to distance metrology. Therefore, what

follows is a numerical analysis of the dependence of the central intensity on both the *source* and the observation distance.

### Variation With Observation Distance

Numerical integration of Equations 18 and 19 for a source distance of one meter, a wavelength equal to 632.8 nm, and a pinhole radius of 0.5mm is shown in Figure 6:

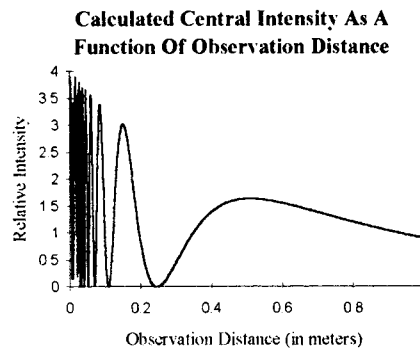


Figure 6 (Dependence of Central Intensity On Observation Distance)

As can be seen in Figure 6, the oscillation of the central intensity decreases in frequency as the observation distance is increased. Likewise, as the observation distance decreases the intensity increases. This is illustrated in Figure 7 (which is merely a magnification of Figure 6 in the region from 0.03 to 0.11 meters):

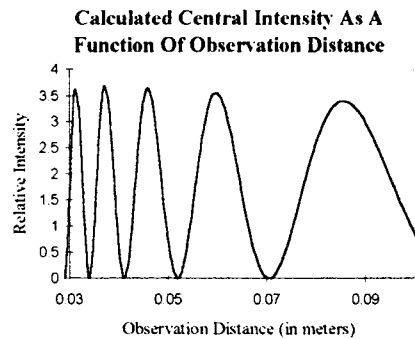


Figure 7 (Close-Up View of Central Intensity Behavior)

Based on the decreasing frequency with distance, as well as the decrease in average amplitude, Equation 20 is fitted to the numerically derived graphs:

$$I_{center} = \frac{4I_o}{(L_{observe} + L_{source})^2} \sin^2 \left[ \frac{R^2 \pi}{2\lambda} \left( \frac{1}{L_{observe}} + \frac{1}{L_{source}} \right) \right] \quad (\text{Equation 20})$$

Below in Figure 8 is a comparison of Equation 20 to Figure 7:

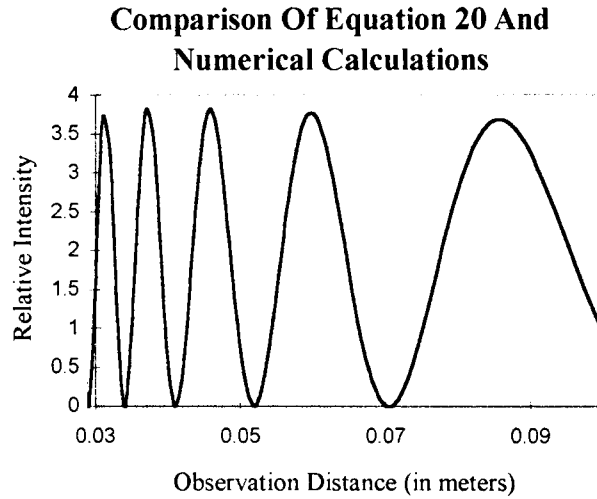


Figure 8 (Comparison of Fresnel Approximation Equation  
and Numerically Derived Data)

The fit of Equation 20 to the numerically derived data in Figure 7 agrees to one part in a million over a range from 0 to 2 meters (See Appendix B for more details). As will be discussed in Section 6, it will only be necessary to measure intensities to an accuracy of one part in ten thousand to produce a distance measurement device capable of micron accuracy. Consequently, Equation 20 supplies a theoretical approximation of Fresnel theory which will be sufficiently accurate to be used with a Fresnel-based distance measurement system.

This structure of Equation 20 is not surprising since the analytical solution to the simplified case (source distance equals infinity) is found to be<sup>4</sup>:

$$I_c = \frac{4I_0}{(L_{observe} + L_{source})^2} \sin^2 \left[ \frac{R^2 \pi}{2\lambda} \left( \frac{1}{L_{observe}} \right) \right] \quad (\text{Equation 21})$$

Where  $I_0$  is the total intensity of light passing through the pinhole. It is obvious that Equation 20 will reduce to Equation 21 when the source distance  $L_{source}$  goes to infinity.

#### Variation With Source Distance

It is evident from Equation 20 that the source and observation distances play completely equivalent roles in determining the central intensity. Therefore, performing the same calculations with the *observation distance* equal to one meter and the *source distance* varying, we get, in Figure 9:

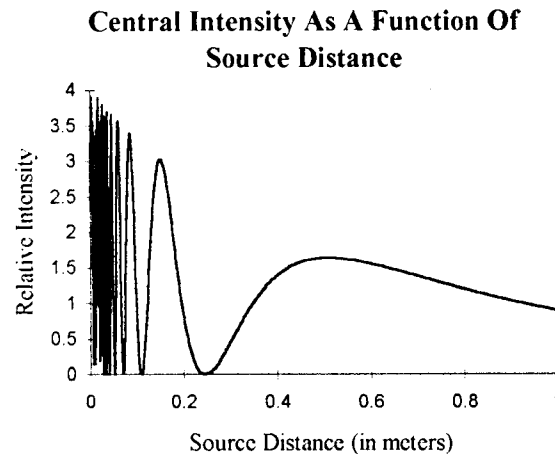


Figure 9 (Dependence of Central Intensity On Source Distance)

As before, magnifying a small region close to the pinhole illustrates the similar decrease in frequency with source distance (see Figure 10).

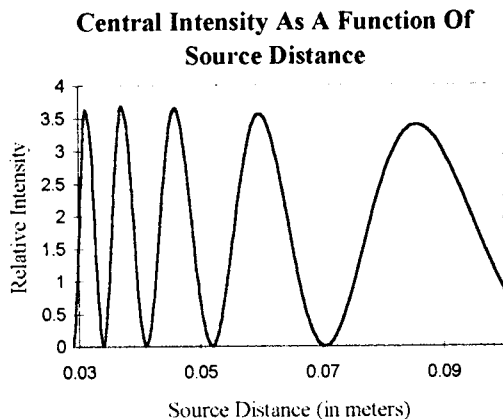


Figure 10 (Close-Up View of the Central Intensity Behavior with Source Distance)

Below, in Figure 11, is a fit of Equation 20 to Figure 10:

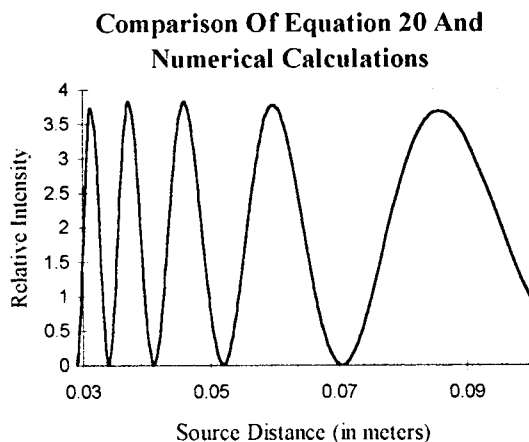


Figure 11 (Comparison of Fresnel Approximation Equation and Numerically Derived Data)

As before, the fit of Equation 20 to the numerically derived data was verified accurate to one part in a million. Consequently, Equation 20 supplies a sufficiently accurate approximation of Fresnel theory for the purposes of a Fresnel-based distance measurement system.

## 4. Experimental Verification of Numerical Analysis

### Introduction

In this section, the experimental data relating to the behavior of a Fresnel diffraction pattern's central intensity will be presented. The agreement with the numerical analysis prediction of Section 3 will be shown. At this point, the experimental data presented is only meant to offer rough verification of the qualitative nature of the central intensity oscillation. Rigorous experimental proof is presented later in Section Eight.

### Experimental Evidence

Theoretical calculations (described in Section 1) predicted that the central intensity of a Fresnel diffraction pattern should oscillate according to Equation 22 (which is a normalized version of Equation 20):

$$I_c = 4I_o \sin^2 \left[ \frac{R^2 \pi}{2\lambda} \left( \frac{1}{L_{source}} + \frac{1}{L_{observe}} \right) \right] \quad (\text{Equation 22})$$

To experimentally confirm this prediction, the simple experimental setup shown in Figure 12 was constructed:

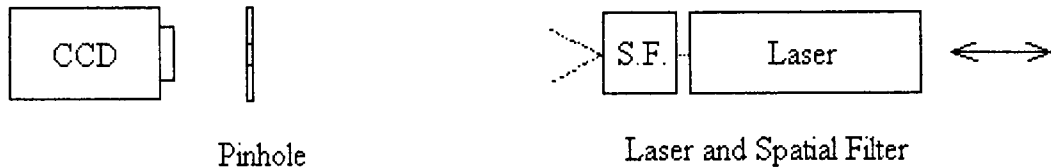


Figure 12 (Experimental Setup for the Determination of the Central Fringes Intensity Dependence with Source Distance )

This setup consisted of a 1 mW Helium-Neon laser and a spatial filter mounted onto a motor driven computer controlled translational stage. The laser beam was spatially filtered(20 micron pinhole) to produce a diverging point source as well as an optically clean wavefront. A 2mm diameter pinhole produced the diffraction pattern resulting from the point source, and a CCD camera (640 x 480 pixels) captured the resulting diffraction pattern. The CCD image was captured and saved with the help of a Frame Grabber board installed in a nearby computer.

Approximately 400 images were captured (at intervals of 5 millimeters) and the central intensity of each pattern was recorded over a range from .1 to 2 meters. The data points are shown in Figure 13:

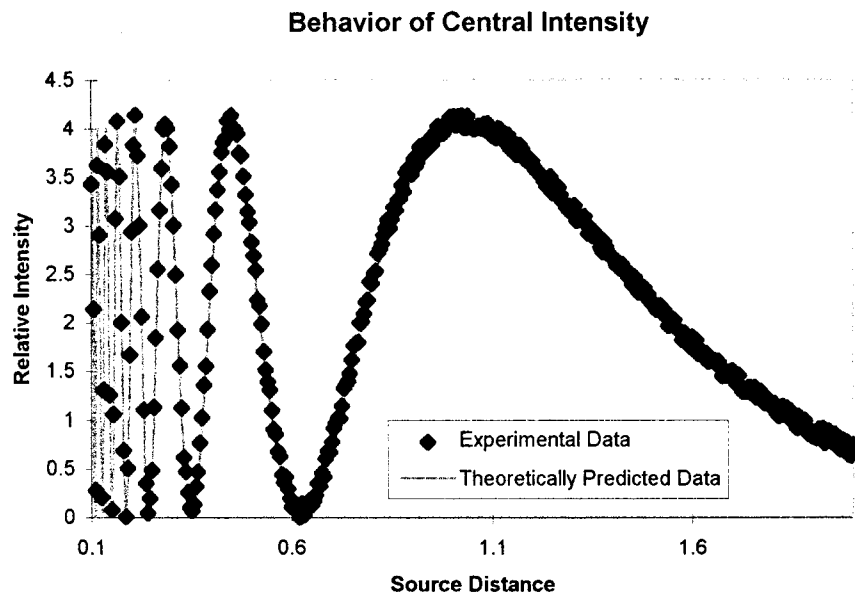
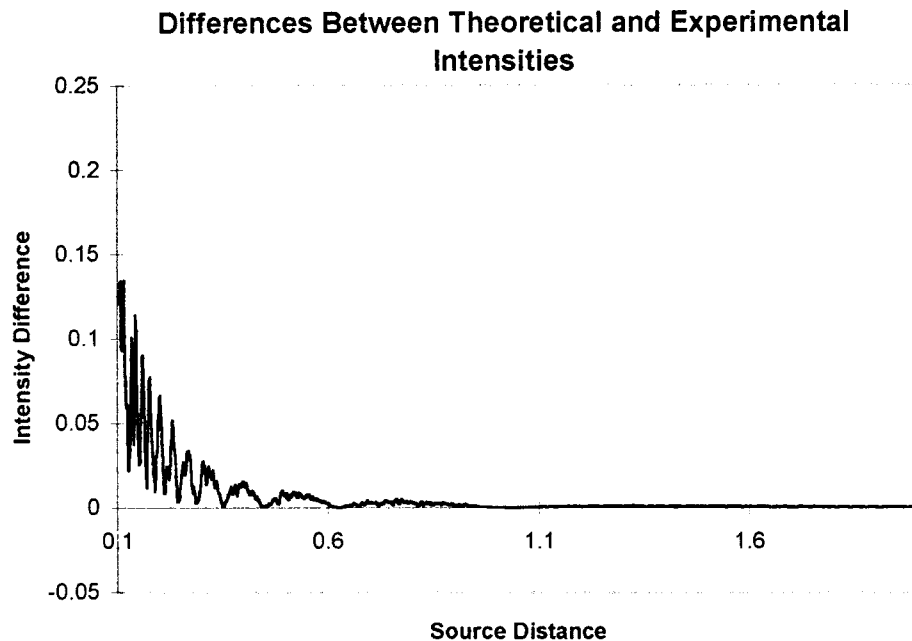


Figure 13 (Crude Experimental Verification of the Central Intensity's Behavior with Source Distance)

As predicted, the central intensity of the Fresnel diffraction pattern appears to oscillate according to Equation 22. Slight differences between the experimental data(diamonds) and the theoretically predicted curve(solid line) can be seen in the above figure. These differences (see Figure 14) are due to various factors which will be discussed in more detail in Section 6.



*Figure 14 (Difference Between Experimentally Acquired and Theoretically Predicted Central Intensity Values)*

For experimental verification of the behavior of the central intensity with respect to changing wavelength, observation distance, and pinhole radius, the reader is directed to the following papers<sup>4-8</sup>.

## **II. APPLICATION TO DISTANCE METROLOGY**

## 5. Application of Fresnel Diffraction to Distance Metrology

### Introduction

The preceding sections have detailed the variation of the Fresnel diffraction pattern as a function of distance. This section will now describe in some detail how these variations can be transformed into a practical and accurate measurement of absolute distance.

It is important to remember, when reading this section, that the distances being measured are between a pinhole and a “point source of light.” The point source of light may be achieved in any of several ways, such as the focal point of a tightly focused laser<sup>9-12</sup>, the active area of a non-coherent LED, or the radiance of light created when a focused laser beam hits a rough surface.

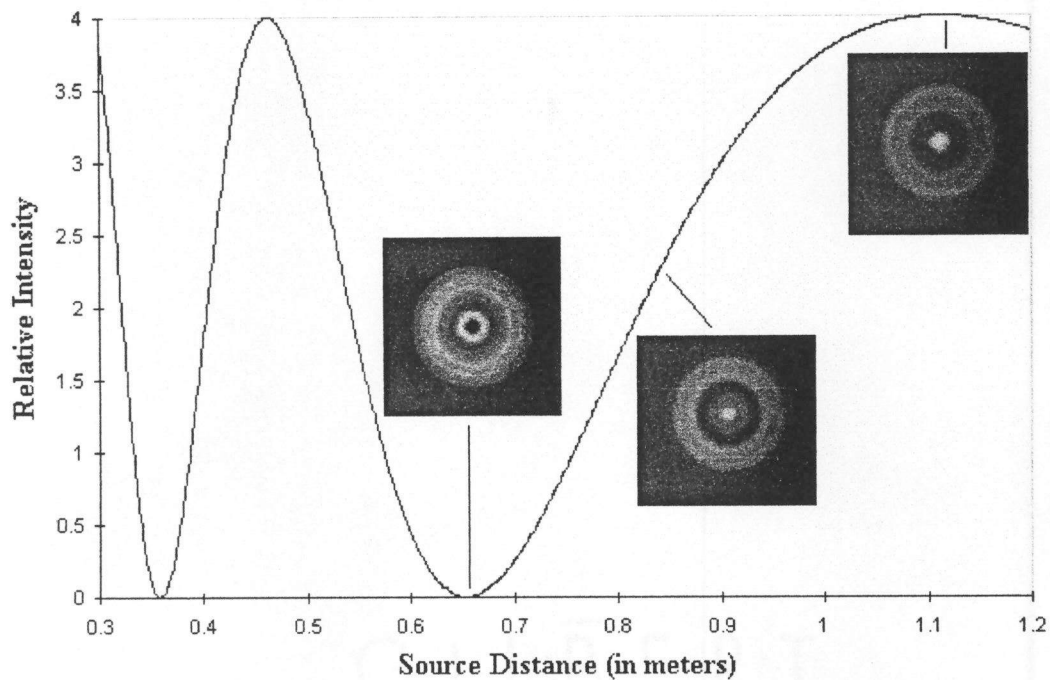
### Principle of Fresnel Diffraction-Based Distance Measurement

Previous sections have shown that the central intensity of a Fresnel diffraction pattern oscillates according to Equation 23:

$$I_{center} = 4I_o \sin^2 \left( \frac{R^2 \pi}{2\lambda} \left( \frac{1}{L_{observe}} + \frac{1}{L_{source}} \right) \right) \quad (Equation 23)$$

Therefore, if a point source is positioned a distance  $L_{source}$  away from a pinhole, then the patterns produced as the source approaches the pinhole will be similar to those illustrated in Figure 15 on the following page:

### Variation Of Central Intensity



*Figure 15 (Graphical Illustration of Central Intensity Dependence On Source Distance)*

Most notably, this figure demonstrates how the central intensity of the pattern changes as the source distance varies. **Because we can calculate what the central intensity should be for a given source distance (using Equation 23), we can conversely calculate what the source distance should be for a measured central intensity.** This is the basic principle behind Fresnel-diffraction-based distance measurement.

Pursuing this logic in more detail, assume that the measurement of a pattern's central intensity is taken and, therefore, there exists a source distance which has produced this pattern, as is illustrated in Figure 16:

Possible Distances Predicted By Central Intensity Equation

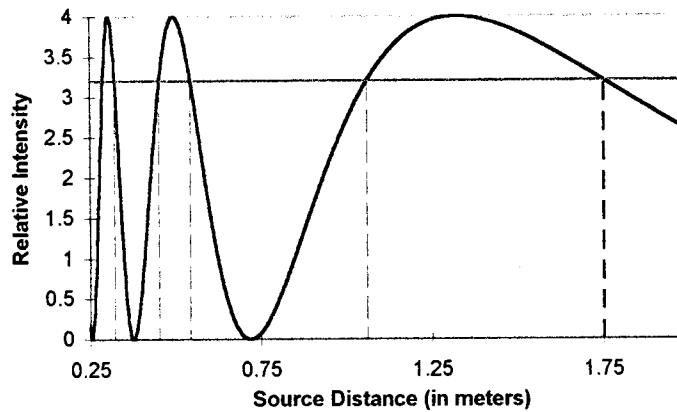


Figure 16 (Distance Ambiguity Associated With a Single Intensity Measurement)

In Figure 16, a relative intensity measurement of 3.2 (relative to the initial source intensity) has been taken from a diffraction pattern, and the possible source distances which could have produced the pattern are indicated by the dashed lines in the graph. These values are calculated by performing the inverse operation of Equation 23, giving Equation 24:

$$L_{source} = \frac{R^2 \pi L_{observe}}{2\lambda L \sin^{-1}\left(\sqrt{\frac{I_{center}}{4I_o}}\right) - R^2 \pi} \quad (\text{Equation 24})$$

Because the arcsine function has multiple solutions, so too does the equation describing the possible source distances. However, the multiple source distances which could have produced the central intensity value measured in Figure 16 can be reduced to only one possible distance if a crude knowledge of the potential source distance is known. For example, if we know the source distance to be at a point some distance between 1.5 and 2.0 meters, then we can resolve that the correct source distance which produced the

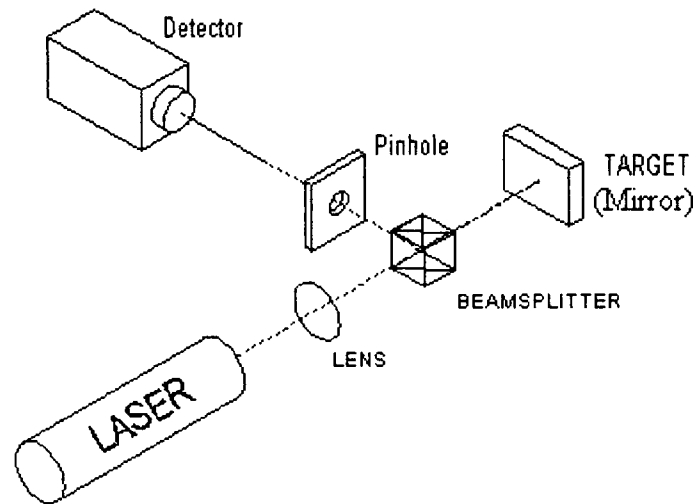
intensity measurement of 3.2 must be equal to 1.75 meters. It will be shown later that there exists other methods for resolving this ambiguity.

Using Fresnel diffraction for distance measurement offers several important benefits over traditional distance measurement devices:

- 1) Because of the unique nature of the central intensity's oscillation with distance, it is possible to calculate a source's *absolute* distance with the analysis of only a single pattern. Unlike interferometric systems which can measure only the *relative* distance which a target has moved and must continually monitor the intensity fluctuations of the required pattern, Fresnel diffraction-based measurements can be made in either a continuous or pulsed mode and are referenced relative to the measurement instrument.
- 2) Since the central intensity oscillates very rapidly with small changes in the source distance, the absolute distance of the source can be determined with high accuracy (single micron accuracy) over relatively long ranges (1 meter).
- 3) Since the measured central intensity is produced by single-beam diffraction (as opposed to two-beam interference), the requirements on beam alignment and source coherence are intrinsically easy to achieve. These two characteristics are traditionally difficult to realize with popular interferometric systems.
- 4) The point source needed to produce the measured diffraction patterns can be produced by a small focused monochromatic light source reflected from a reflective target, or it can be the result of light scattering from the surface of a diffuse target.

## Typical Distance Measurement Setups

To illustrate how Fresnel diffraction can be used to measure target distances, a typical measurement system setup is shown in Figure 17:



*Figure 17 (Typical Reflective Surface Distance Measurement Setup)*

As can be seen in Figure 17, a point source of light is focused through a beamsplitter and onto a mirror. The mirror then reflects the now-diverging light back through the beamsplitter and through a pinhole. The pinhole diffracts the light emanating from the source and produces a diffraction pattern, which is analyzed with the aid of either a single detector located at the center of the pattern or a CCD Camera. As the mirror moves closer to and farther away from the pinhole, it will locate the source (the focal point of the laser) at different distances away relative to the pinhole. Consequently, the variation in the central intensity measured by the detector will oscillate as given in Equation 1, with the source distance now being directly related to the mirror's distance away from the pinhole. In this setup the change in the diffraction pattern's central intensity can now be used to determine the distance of the mirror from the pinhole.

While the previous method is useful for measuring reflective surfaces, it is also possible to measure diffuse surfaces using the setup shown in Figure 18:

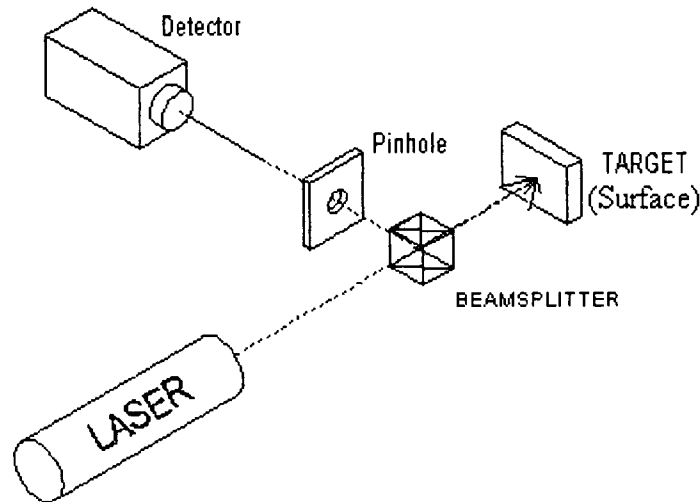


Figure 18 (Typical Rough Surface Distance Measurement Setup)

This setup differs from the reflective surface setup in that the location of the source is actually the target's surface itself.

While the above two diagrams are a simplification of the actual devices, they show the basics behind Fresnel diffraction-based distance metrology.

### **Resolution of Distance Ambiguity**

The multiple number of distances which can produce a single central intensity measurement is infinite. Consequently, to create a device which is capable of absolute distance measurements this ambiguity must be resolved to a single distance determination. Various techniques have been developed for this purpose. The simplest of these methods involves first measuring the approximate distance to the target and then

using this rough value to select the most probable distance determination.. Several methods exist for performing such approximate distance measurements:

1)  *$1/R^2$  Intensity Measurements* - A common technique (though not very accurate) for determining the distance of a surface, is to measure the drop in intensity between the emitted light and the reflected light. Obviously, as a target moves away from an illuminating source, the amount of scattered light returning to the source will decrease as  $1/(\text{source distance squared})$ . Therefore, if the initial distance and reflected intensity are known, then the distance of the target at varying distances can be measured through the measurement of subsequent reflected intensities.

2) *Dual Wavelength System* - A Fresnel diffraction-based measurement system which uses two alternating sources of differing wavelength will produce uniquely differing central intensities at various distances. This is illustrated in Figure 19:

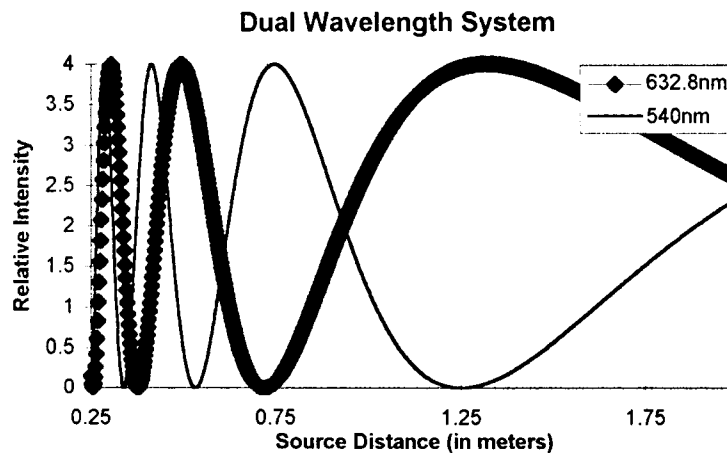


Figure 19 (Comparison of Dual Wavelength Behavior)

Therefore, over a given range, the two measurements of central intensity (two measurements corresponding to two different wavelengths) can adequately resolve any

distance ambiguity. This method is very accurate but, unfortunately, can be technically challenging.

3) *Commercial System* - Because a number of non-contact commercial systems exist which can measure the absolute distance of a target with low accuracy ( $< 5\text{mm}$ ), it is often useful to employ these systems for the removal of distance ambiguities. Such systems include ultrasonic positioners, triangularization systems, and enhanced imaging systems.

### **Explicit Description of Inversion of Equation 23**

Assuming the approximate distance of the target is known, and the central intensity of the associated Fresnel diffraction pattern has been measured, then it is possible to use Equation 24 to perform a high-accuracy calculation of the source's distance. However, because the arcsine function always gives us a value between  $-\pi/2$  and  $+\pi/2$ , regardless of the argument (for example:  $\arcsin[\sin(\pi/2)] = \arcsin[\sin(5\pi/2)] = \pi/2$ ), it is necessary to create an algorithm which will compensate for this problem. Therefore, when calculating a distance from a central intensity measurement, the following method should be used.

#### *Procedure for the Calculation of Distance Measurements Involving the Arcsine*

First, a variable  $K$  is defined by Equation 25:

$$K = \frac{\pi R^2}{2\lambda} \left( \frac{1}{L_{\text{observe}}} + \frac{1}{L_{\text{estimated\_distance}}} \right) \quad (\text{Equation 25})$$

Where  $K$  is essentially the argument of the sine function, which has appeared in several previous equations (e.g., Equations 20, 22, and 23). Next, the quadrant of  $K$  is found and denoted by  $V_o$  and described by Equation 26:

$$V_o = \left[ \text{int}\left(\frac{2K}{\pi}\right) \right] - 4 \text{int}\left[ \frac{\text{int}\left(\frac{2K}{\pi}\right)}{4} \right] + 1 \quad (\text{Equation 26})$$

Then the sine itself is found from the central intensity and denoted by the variable  $x$  and described by Equation 27:

$$x = -1^{\text{int}\left(\frac{V_o}{3}\right)} \sqrt{\frac{I_c}{4I_o}} \quad (\text{Equation 27})$$

Note that  $x$  is positive in the first and second quadrants and negative in the third and fourth quadrants, as expected. Continuing, calculation of the “true” arcsine is then defined by Equation 28:

$$\alpha \sin_{true}(x) = \arcsin(x) \cdot \left[ -1^{\text{int}[abs(2.5-V_o)]-1} \right] + \left[ \pi - \frac{\pi}{2} \text{int}\left(\frac{V_o}{3}\right) \right] \cdot \text{int}\left\{ \frac{\left[ \left( 1 + \text{int}\left(\frac{V_o}{3}\right) \right) K \right]}{\pi} + \frac{-1^{(V_o+1)} - 1}{2} \right\} \quad (\text{Equation 28})$$

Which can then be used to calculate the measured distance given by Equation 29:

$$L_{calculated} = \left[ \frac{2\lambda}{R^2 \pi} \arcsin(x) - \frac{1}{L_{observe}} \right]^{-1} \quad (\text{Equation 29})$$

## 6. Error Analysis

### Introduction

Potential errors which can be encountered in a Fresnel-diffraction-based-distance measurement system are the result of several contributing factors. These factors can be grouped into two main categories: *environmental instabilities errors* and *signal acquisition errors*. A qualitative and quantitative description of each error follows.

### Environmental Instabilities

The stability of the operating environment (i.e. temperature stability, mechanical rigidity, etc.) can have a measurable effect on the accuracy of any optical distance measurement system. For a Fresnel-diffraction-based system, errors in distance measurement are the result of one or more of the following factors:

- Fluctuations in source distance ( $L_{\text{source}}$ ) due to changes in the index of refraction of the transversing medium, resulting from changes in the outside temperature
- Fluctuations in the observation distance ( $L_{\text{observe}}$ ) due to changes in the index of refraction of the transversing medium (resulting from temperature fluctuations).
- Fluctuations in the observation distance ( $L_{\text{observe}}$ ) due to mechanical expansion within the mounting structure.
- Fluctuations in the pinhole radius ( $R$ ) due to temperature fluctuations.
- Fluctuations in the source wavelength ( $\lambda$ ).

### Fluctuations in Source-to-Pinhole Distance ( $L_{source}$ )

Temperature fluctuations in the medium between the source and pinhole will cause fluctuations in the overall measured optical path distance  $L_{measured}$ . As the temperature of the environment increases and decreases, so too does the refractive index of the media and its corresponding optical path length:

$$L_{measured} = L_{optical} = n_{medium} \cdot L_{source} \quad (Equation 30)$$

The refractive index of the medium will typically fluctuate as:

$$n(T) = n_o + kT \quad (Equation 31)$$

where  $T$  is the temperature in Celsius and  $k$  is a constant with units  $C^{-1}$ . Substituting Equation 31 into Equation 30 gives the measured path distance as a function of temperature and is given by Equation 32:

$$L_{measured}(T) = L_{source} [n_o + kT] \quad (Equation 32)$$

Where  $L_{source}$  is not the optical path distance but the actual physical distance. Consequently, fluctuations in the ambient-medium temperature ( $\Delta T$ ) result in fluctuations in the measured source-to-pinhole distance ( $\Delta L$ ) given by Equation 33:

$$\Delta L_{measured} = L_{source} [k(\Delta T)] \quad (Equation 33)$$

Therefore, in order to ensure micron accuracy at one meter in an air medium (where  $k \approx 1 \times 10^{-6}$  and  $n_o = 1.000291$ ), it is necessary to control or know the temperature of the ambient air to  $\pm 1^\circ$ .

### Fluctuations in Pinhole-to-Observation Distance ( $L_{observe}$ )

Equation 32 also holds for the pinhole-to-observation distance  $L$ . However, the effect of observation distance fluctuations on the final measured distance is considerably more complicated than before. As was described in Section 5, the source distance is determined through Equation 34:

$$L_{source} = \left\{ \left( \frac{2\lambda}{R^2\pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{center}}{4I_o}} \right) \right] - \frac{1}{L} \right\}^{-1} \quad (\text{Equation 34})$$

If the refractive index of the observation media  $n_{media}$  is fluctuating with temperature (as described in Equation 32) then Equation 34 becomes Equation 35:

$$L_{source}(T) = \left\{ \left( \frac{2\lambda}{R^2\pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{center}}{4I_o}} \right) \right] - \frac{1}{L_{observe}[n_o - kT]} \right\}^{-1} \quad (\text{Equation 35})$$

Therefore, differentiating Equation 35 gives Equation 36:

$$\frac{dL_{source}}{dT} = \left| \left\{ \left( \frac{2\lambda}{R^2\pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{center}}{4I_o}} \right) \right] - \frac{1}{L_{observe}[n_o - kT]} \right\}^{-2} \left\{ \frac{-k}{L_{observe}^2[n_o - kT]^2} \right\} \right|$$

(Equation 36)

Simplifying Equation 36 gives Equation 37:

$$\frac{dL_{source}}{dT} = \left| \frac{L_{source}^2 k}{L_{observe}^2 [n_o - kT]^2} \right| \quad (\text{Equation 37})$$

And consequently, Equation 38 is deduced:

$$\Delta L_{source} = \left| \frac{L_{source}^2 k}{L_{observe}^2 [n_o - kT]^2} \right| \Delta T \quad (\text{Equation 38})$$

where  $\Delta T$  is a fluctuation around a set temperature  $T$ . Therefore, if air is the medium between the pinhole and the detector (with  $k = 1 \times 10^{-6}$  and  $n_o = 1.000291$ ) then Equation 38 will become Equation 39:

$$\Delta L_{source} = \left| \frac{L_{source}^2 10^{-6}}{L_{observe}^2 [1.000291 - 10^{-6} T]^2} \right| \Delta T \quad (\text{Equation 39})$$

Below, in Figure 20, is a graph of Equation 39 for distances ranging from zero to two meters (with  $R = .001\text{m}$ ,  $\lambda = 632.8 \times 10^{-9} \text{ m}$ ,  $T = 23^\circ\text{C}$ , medium = air):

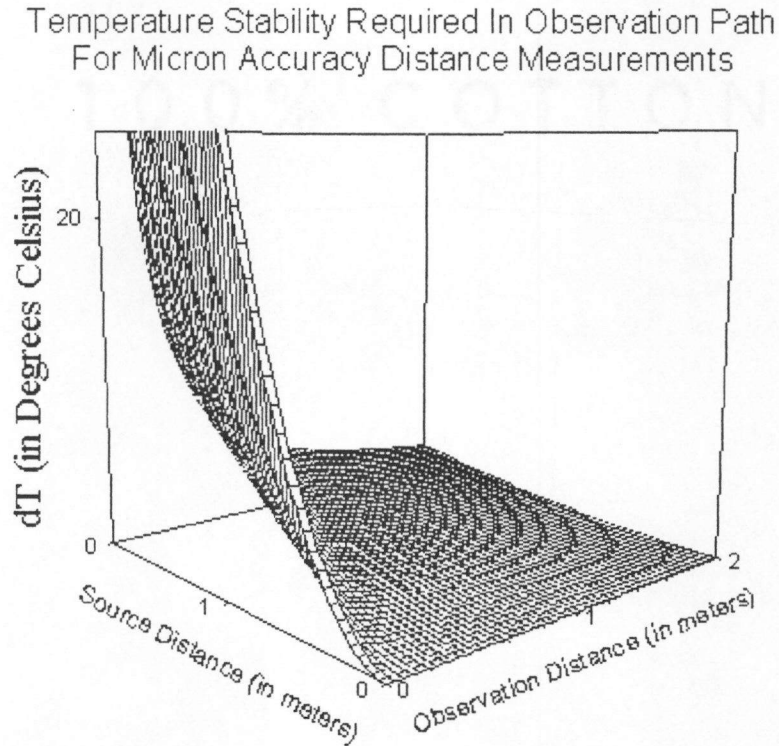


Figure 20 (Temperature Stability Requirements)

Obviously, as the observation distance increases and the source distance decreases, temperature fluctuations in the media between the pinhole and the detector produce less and less of an effect on the final distance measurement. For an observation distance of

one meter and a source distance of one meter at room temperature, the temperature of the air spanning the observation distance must be known to  $\pm 1^\circ \text{C}$  to ensure micron accuracy.

In addition to temperature fluctuations in the medium between the pinhole and the detector, there will also exist an error due to thermal expansions of the mechanical structure which aligns and mounts the pinhole and the detector. The effect of these expansions can be calculated by first approximating the thermal expansions with Equation 40:

$$L_{\text{observation}} = L_0(1 + \alpha\Delta T) \quad (\text{Equation 40})$$

Obviously, the linear coefficient of thermal expansion  $\alpha$  is analogous to the index of refraction constant  $k$  first introduced in Equation 31 (with  $n_0 = 1$ ). Therefore, Equation 38 can be easily modified to express the resulting change in the measured source distance as a function of the structural thermal expansions of the observation distance and is given by Equation 41:

$$\Delta L_{\text{source}} = \left| \frac{L_{\text{source}}^2 \alpha}{L_{\text{observe}}^2 [1 + \alpha T]^2} \right| \Delta T \quad (\text{Equation 41})$$

At an observation distance of one meter, a source distance of one meter, and an initial temperature at room temperature, the temperature of the materials in Table 1 on the following page must be known to the given tolerances to ensure micron accuracy:

**Table 1: Effects of Structural Expansion**

| Material  | Coefficient of Thermal Expansion | $\Delta T$                |
|-----------|----------------------------------|---------------------------|
| Aluminum  | $2.4 \times 10^{-5}$             | $\pm 0.042^\circ\text{C}$ |
| Steel     | $1.1 \times 10^{-5}$             | $\pm 0.1^\circ\text{C}$   |
| Invar     | $1 \times 10^{-6}$               | $\pm 1^\circ\text{C}$     |
| ULE Glass | $1 \times 10^{-9}$               | $\pm 1000^\circ\text{C}$  |

### Fluctuations in Pinhole Radius (R) With Temperature

Fluctuations in the pinhole diameter can occur as changes in the temperature cause thermal expansions and contraction in the pinhole material. Changes in the diameter of the pinhole with respect to temperature can be modeled by Equation 42:

$$R_{pinhole} = R_o(1 + \alpha T) \quad (\text{Equation 42})$$

Where  $R_o$  is the radius of the pinhole at zero degrees Celsius,  $T$  is equal to the current temperature, and  $\alpha$  is the coefficient of thermal expansion for the pinhole material.

Plugging this expression into our formula for the source distance we get Equation 43:

$$L_{source} = \left\{ \left( \frac{2\lambda}{R_o^2(1 + \alpha T)^2 \pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{center}}{4I_o}} \right) \right] - \frac{1}{L_{observe}} \right\}^{-1} \quad (\text{Equation 43})$$

Which can be differentiated with respect to  $T$  to give Equation 44:

$$\frac{dL_{source}}{dT} = \left| \left\{ \left( \frac{2\lambda}{R_o^2(1 + \alpha T)^2 \pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{center}}{4I_o}} \right) \right] - \frac{1}{L_{observe}} \right\}^{-2} \left\{ \left( \frac{4\alpha\lambda}{R_o^2(1 + \alpha T)^3 \pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{center}}{4I_o}} \right) \right] \right\} \right| \quad (\text{Equation 44})$$

Equation 44 can be simplified to become Equation 45:

$$\frac{dL_{source}}{dT} = \left[ \left( \frac{2\alpha}{(1+\alpha T)} \right) \left( \frac{L_{source}^2}{L_{observe}} + L_{source} \right) \right] \quad (Equation 45)$$

And therefore, Equation 46 can be deduced:

$$\Delta L_{source} = \left[ \left( \frac{2\alpha}{(1+\alpha T)} \right) \left( \frac{L_{source}^2}{L_{observe}} + L_{source} \right) \right] \Delta T \quad (Equation 46)$$

Assuming a fixed source distance of one meter and a fixed observation distance of one meter, then the errors due to a temperature change in a 1mm radius steel-film pinhole are shown in Figure 21:

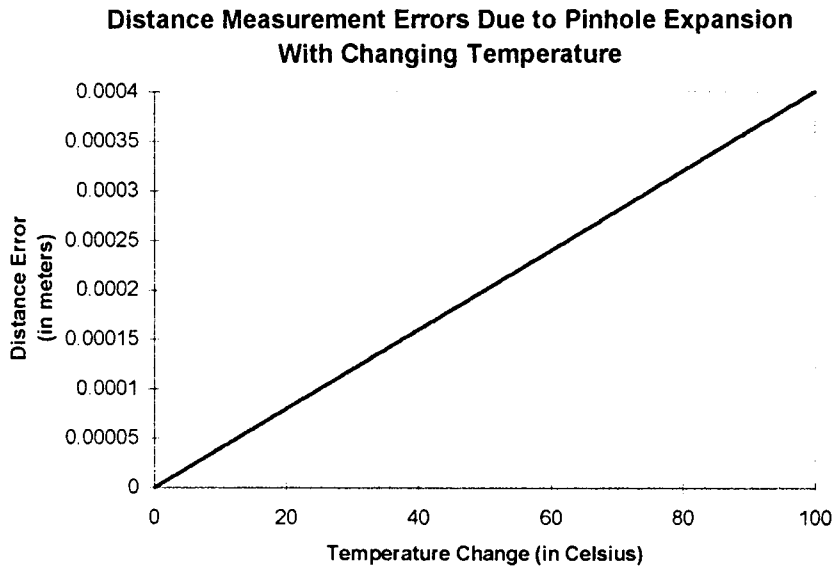


Figure 21 (Effect of Thermal Expansion in Pinhole)

Therefore, to ensure micron accuracy at these distances, the temperature of the steel-film pinhole must be known to  $\pm 0.3^\circ \text{C}$ . From Equation 46 we can note that the temperature

stability needed for micron accuracy is reduced as  $L_{\text{observe}}$  gets larger and  $L_{\text{source}}$  becomes smaller.

### Fluctuations in Source Wavelength ( $\lambda$ )

Fluctuations in the source wavelength can be caused by a number of factors depending on the type of source used (factors ranging from changes in operating temperature to changes in output intensity). Therefore, a treatment of the effects of wavelength fluctuations is necessary. As before, the measurement of source distance is given by Equation 47:

$$L_{\text{source}} = \left\{ \left( \frac{2\lambda}{R^2\pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{\text{center}}}{4I_o}} \right) \right] - \frac{1}{L_{\text{observe}}} \right\}^{-1} \quad (\text{Equation 47})$$

Which can be differentiated with respect the  $\lambda$  to give Equation 48:

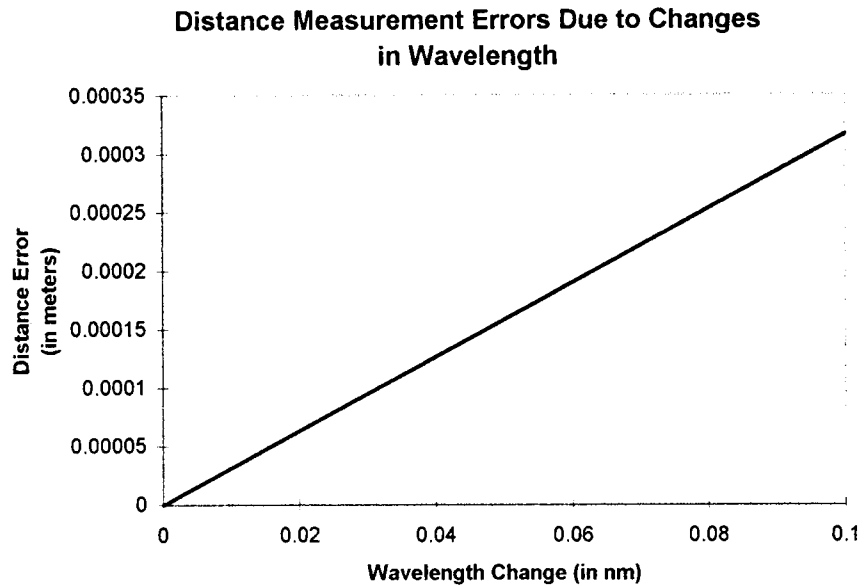
$$\frac{dL_{\text{source}}}{d\lambda} = \left| \left\{ \left( \frac{2\lambda}{R^2\pi} \right) \left[ \arcsin \left( \sqrt{\frac{I_{\text{center}}}{4I_o}} \right) \right] - \frac{1}{L_{\text{observe}}} \right\}^{-2} \left[ \left( \frac{2}{R^2\pi} \right) \arcsin \left( \sqrt{\frac{I_{\text{center}}}{4I_o}} \right) \right] \right|$$

(Equation 48)

Which simplifies to Equation 49:

$$\Delta L_{\text{source}} = \left| \left( \frac{1}{\lambda} \right) \left( \frac{L_{\text{source}}^2}{L_{\text{observe}}} + L_{\text{source}} \right) \right| \Delta \lambda \quad (\text{Equation 49})$$

The behavior of this error with increasing wavelength uncertainty is shown in Figure 22 on the following page (with  $R = .001\text{m}$ ,  $\lambda = 632.8 \times 10^{-9} \text{ m}$ ,  $L_{\text{observe}} = 1\text{m}$ , and  $L_{\text{source}} = 1 \text{ m}$ ).



*Figure 22 (Effects of Wavelength Instability)*

To ensure micron accuracy at these distance and source frequency, the wavelength of the source must be stabilized to  $\pm 0.0004$  nm (which is possible for most gas lasers). As is the case for the pinhole radius, the stabilization of the wavelength needed to ensure micron accuracy is reduced as  $L_{\text{observe}}$  gets larger and  $L_{\text{source}}$  becomes smaller.

### Signal Acquisition Errors

All measurements of distance made with a Fresnel-diffraction-based system are dependent on the accurate measurement of a Fresnel pattern's central intensity. Whatever instrument is used to make this measurement, either a single detector or a CCD array, the resolution of the analog-to-digital conversion plus the signal-to-noise ratio experienced will be the limiting factor in the accuracy of a distance measurement.

### A/D Conversion Limitations

If a Fresnel diffraction pattern is produced with a relative central intensity of say, .654321 (minimum intensity = 0, maximum intensity = 1), then an 8-bit detection system will record the intensity as  $.654 \pm .002$ , assuming no noise is encountered in the signal. This uncertainty ( $\pm .002$ ), which is inherent in the analog-to-digital conversion, will consequently produce an uncertainty in any calculation of distance.

To illustrate why this uncertainty is produced, Figure 23 illustrates the digitization of a known parabolic intensity profile:

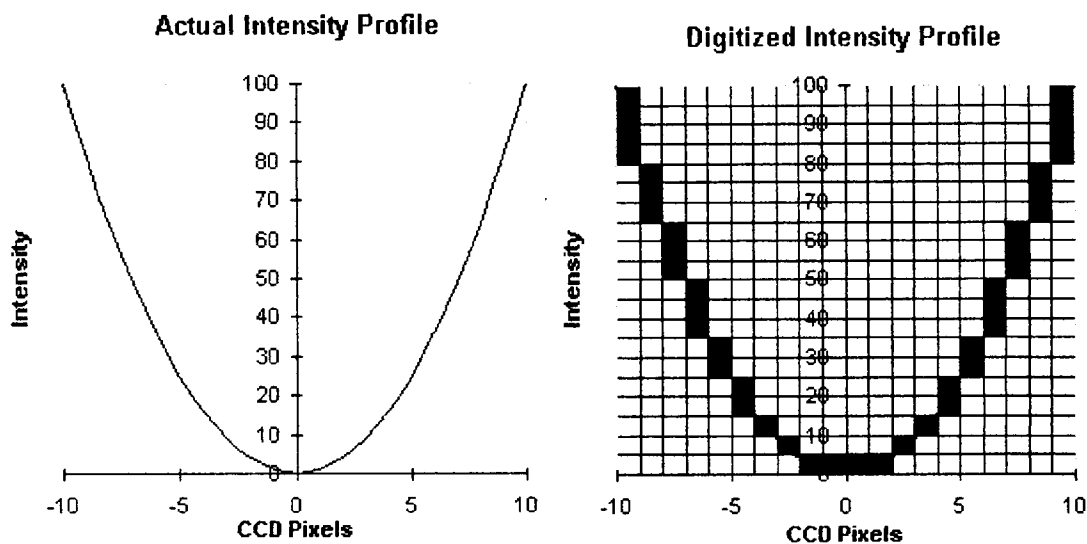


Figure 23 (Analog-to-Digital Conversion Errors)

Obviously, the resolution of the digitized picture is a function of both the analog-to-digital conversion resolution as well as the CCD pixel density (since the spatial resolution of a diffraction pattern can be increased through optical means, the latter dependence will not be treated). Analog-to-digital conversion systems are described in terms of bit precision and are commercially available as either 8-bit (256 intervals), 10-

bit (1024 intervals), 12-bit (4096 intervals), 14-bit (16,384 intervals), or 16-bit (65,536 intervals). However, as will be discussed later, the precision of the conversion system does not necessarily describe the accuracy with which a measurement can be made.

Assuming noise levels are not a factor, increasing the bit precision of an A/D conversion system will naturally produce a more accurate measurement of intensity and therefore give a more accurate distance measurement. Figures 24, 25, and 26 plot the uncertainty which results when distance measurements are made using an 8-bit, 12-bit, and 16-bit detection system. The optical parameters used were:  $R = .001\text{m}$ ,  $\lambda = 632.8 \times 10^{-9}\text{ m}$ , and  $L_{\text{observe}} = .1\text{m}$ .

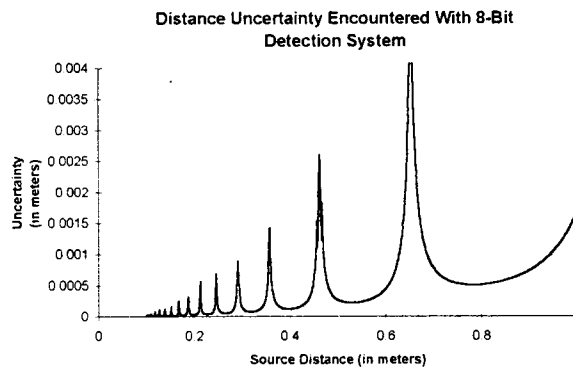


Figure 24 (8-Bit Detection System Errors)

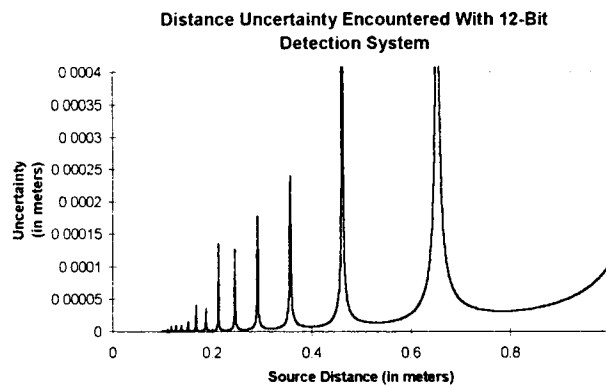
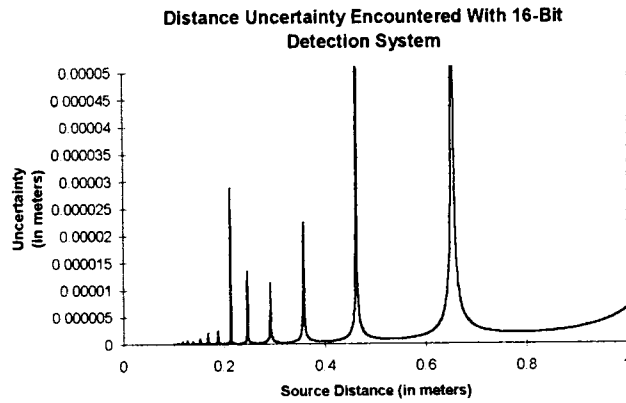


Figure 25 (12-Bit Detection System Errors)



*Figure 26 (16-Bit Detection System Errors)*

As can be seen in the above graphs, the majority of the uncertainty measurements fall below a certain level, with periodic “spikes” in the uncertainty occurring at the location of extrema in the pattern. If these “spikes” can be averted (which naturally occurs when using methods described in the previous section to remove ambiguities) then the uncertainties associated with a given bit precision are described by Table 2:

**Table 2: Bit Precision vs. Uncertainty**

| Bit Precision | Distance Uncertainty Over 1 meter Range |
|---------------|-----------------------------------------|
| 8             | < 2 mm                                  |
| 12            | < 100 microns                           |
| 16            | < 5 microns                             |

It should also be noted that increasing the pinhole radius will decrease the uncertainty, regardless of the bit precision. However, it has been experimentally noted that the most easily imaged patterns for a range of one meter is approximately  $R = 1\text{mm}$ .

### Signal-to-Noise Ratio

Regardless of how precisely a data acquisition system can resolve a measurement, all measurements experience some form of noise. This noise is typically the result of either unwanted outside or internal electrical interference or optical image degradation. The effects of noise on the determination of distance are qualitatively illustrated in Figure 27:

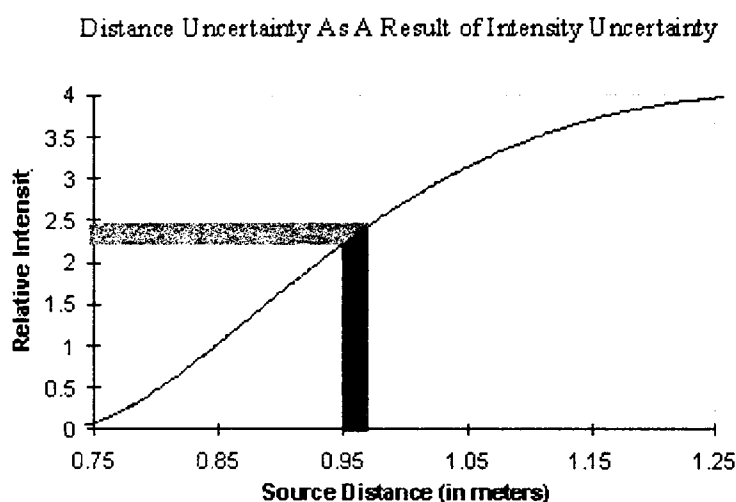


Figure 27 (Effects of Signal Uncertainty)

Unlike the static uncertainties associated with A/D conversions, the uncertainties due to noise are typically dependent upon the strength of the signal. Therefore, a signal-to-noise ratio is more convenient to use when discussing the effects of noise.

The effects of three signal-to-noise ratios (which give uncertainties similar to Figures 24, 25, and 26) are illustrated in Figures 28, 29, and 30 on the following page. As before, the optical parameters used were:  $R = .001\text{m}$ ,  $\lambda = 632.8 \times 10^{-9}\text{ m}$ , and  $L_{\text{observe}} = .1\text{m}$ .

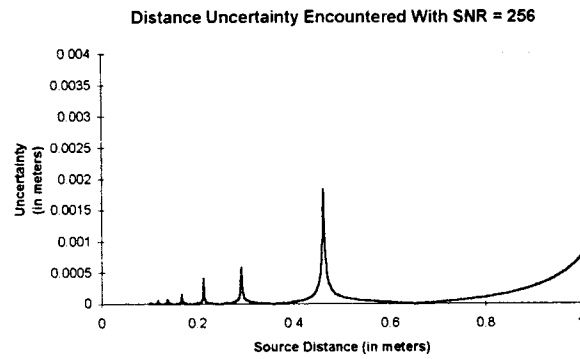


Figure 28 (Errors with SNR = 256)

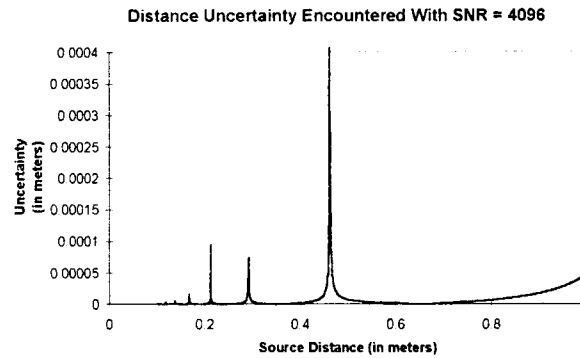


Figure 29 (Errors with SNR = 4096)

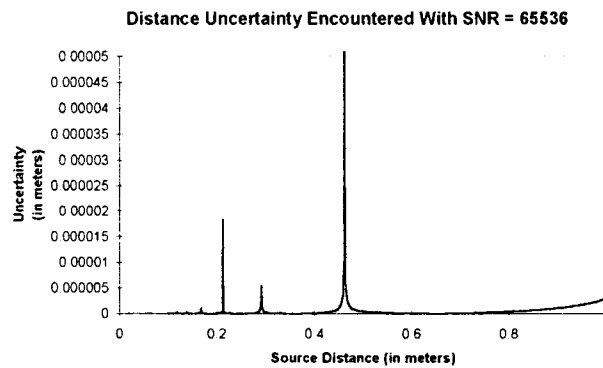


Figure 30 (Errors with SNR = 65536)

When these figures are compared to Figures 24, 25, and 26, it can be seen that the number of “spikes” in the noise uncertainty graphs are less by a factor of two. This is because the minima of the oscillating central intensity do not produce large errors since their intensity values are small. Likewise the average values for the uncertainties associated with noise are reduced as described in Table 3:

**Table 3: SNR vs. Uncertainty**

| SNR   | Distance Uncertainty |
|-------|----------------------|
| 256   | < 1 mm               |
| 4096  | < 50 microns         |
| 65536 | < 3 microns          |

Obviously, keeping the signal-to-noise ratios at this level is not a trivial matter.

## **7. Optimized Pattern Production and Analysis**

### **Introduction**

As detailed in the previous section, high accuracy distance measurements can only be obtained with high resolution and high accuracy measurements of the central intensity. Such measurements can be made through two separate techniques: 1) a single low noise detector can be located at the center of the Fresnel diffraction pattern and digitized with a 16-bit A/D board, or 2) an 8-bit CCD camera and frame grabber board can capture the entire diffraction pattern and image analysis algorithms can be used to produce a calculated 16-bit accuracy measurement of the central intensity. While both methods have been successfully used to measure distances with high accuracy, this writing will detail the development of the latter approach. Therefore, this section will describe the methods which are used to optimize the production, capture, and analysis of a Fresnel diffraction image.

### **Pattern Production**

The optical source used to produce a Fresnel diffraction pattern is often the limiting factor in determining the accuracy of subsequent distance measurements. Because a Fresnel diffraction pattern can be produced with almost any monochromatic source, the possibilities for pattern production techniques are nearly endless. However, there do

exist three basic sources which can be used for the production of Fresnel diffraction patterns and they are described here.

*1) Non-Coherent Monochromatic Source*

Passing light from a non-coherent monochromatic source through a pinhole is probably the simplest and cheapest method for producing a Fresnel diffraction pattern. However, to ensure a high quality Fresnel diffraction pattern suitable for distance measurement, several factors must be considered when using such a source. First, the bandwidth of the source must be significantly narrowed to reduce any “smearing” of the pattern and, consequently, unwanted loss in distance resolution (see Figure 31).

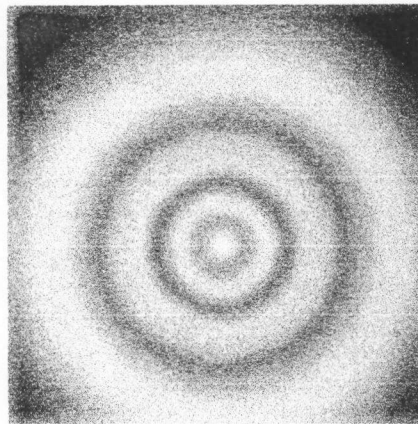


Figure 31 (Pattern Produced By a Monochromatic Source)

This can be accomplished by filtering the light with the aid of a monochromator or by using a solid state monochromatic light source which has a known bandwidth. Second, the light source must be small in diameter (see Section 9 for more details) to produce an accurate diffraction pattern. Unfortunately, these two requirements typically result in a light source which has too low of an intensity for useful measurements. However, some success has been observed using fiber optic transmitters as the source. These solid state

devices emit a high intensity narrow band of non-coherent light from a small active area. Regardless of various shortcomings, this potential light source has been mentioned due to its lack of interference effects, and hence, no Newton Rings or Speckle patterns are encountered with this source. This will be shown later to be a very important characteristic for diffuse surface measurements.

## *2) Coherent Continuous Monochromatic Source*

Focusing the light from a continuous wave laser through a pinhole produces some of the brightest and highest definition Fresnel diffraction patterns of any source. Typically a Helium-Neon laser can be commercially purchased which will give a high intensity monochromatic wave with a wavelength of  $632.8 \pm 0.0001$  nm. This narrow bandwidth combined with the high intensity, allows Fresnel diffraction patterns to be produced simply and efficiently. Unfortunately, it also means that the coherence length of the laser is very long, resulting in image degrading interference effects distorting the wanted diffraction pattern (see Figure 32).

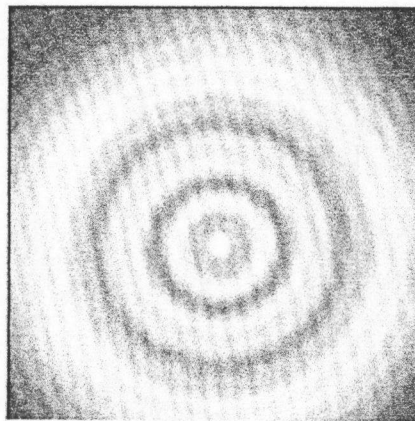


Figure 32 (Pattern Produced By a HeNe Laser Source)

When measuring distances from rough surfaces, these interference effects (speckle) can become completely debilitating to the system. However, this source does have many attractive characteristics which typically make it the best source for most distance measurements.

### *3) Quasi-Coherent Pulsed Monochromatic Sources*

These sources combine some of the good and some of the bad features from the two types of sources given above. Typically pulsed monochromatic sources display only temporal coherence (and not spatial coherence) therefore providing a narrow linewidth source which displays only a minimum of interference behavior (see Figure 33).

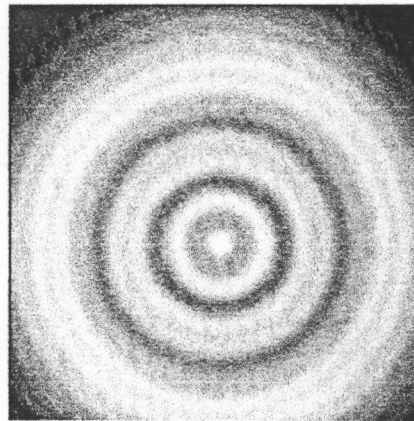


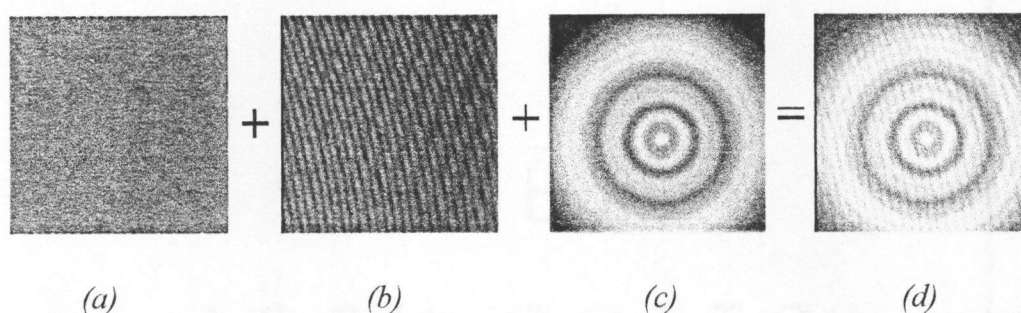
Figure 33 (Pattern Produced By a Pulsed Laser Source)

These sources appear to have much use in Fresnel based distance measurement but still need further study.

Due to factors of availability, intensity, simplicity, and cost, source two is the source which is currently used in the Fresnel diffraction-based distance measurement system.

## Anatomy of an Fresnel Diffraction Image

The Fresnel diffraction pattern produced with a Helium Neon laser and recorded with a CCD array typically looks very similar to Figure 34. If we were to break the image down into its fundamental parts, it would consist of the following three components: a static background intensity, a static background interference pattern, and a Fresnel diffraction pattern (see Figure 34).



*Figure 34 (Anatomy of a Fresnel Diffraction Image)*

The static background intensity (a) is a combination of stray light (i.e., from room lighting, stray reflections, etc.) and/or dark current in the CCD detectors (an electrical background signal which unavoidably exists in an uncooled detector due to thermal emissions). This background intensity measurement is typically only about 5% of the full measurable range of the CCD array and, therefore, is not very significant. Optical filters, light shields, and stabilized operating temperatures can aid in reducing the amplitude and instability of this static background intensity.

The static background interference pattern (b) is the result of interfering reflections within the optical components of the system. Components such as beamsplitters, lenses, and even the protective glass covering of the CCD array, create multiple reflections of

the coherent laser light which interfere to produce the static background interference pattern. While the intensity of this pattern is proportional to the intensity of the source, the position of the pattern is static (since it depends only on the positions of the fixed optical components within the system). This pattern is undesirable because of its distorting effect on the intensity profile of the wanted diffraction pattern.

Obviously, the diffraction pattern (c) is the pattern which is produced as the source light passes through a pinhole. Ideally this is the pattern which we would like to generate and examine.

If the static background intensity and the static background interference pattern are known, then it is possible to enhance an acquired image (d) so that it displays only the desired undistorted diffraction pattern. To produce such an image, it is first necessary to record each one of the components (a, b, & d) separately using the following method:

- 1) First, the laser source is turned off and the CCD image (static background intensity) is captured.
- 2) Second, the complete pattern (interference effects and all) is captured.
- 3) Third, the pinhole is removed and the laser light is allowed to hit the CCD undiffracted (static background interference pattern).

Once these images have been captured it is possible to subtract images one and three from image two to give the non-degraded diffraction pattern, which then gives the true measurement of the diffraction intensities.

Figure 35 details the information acquired in the previously described steps:

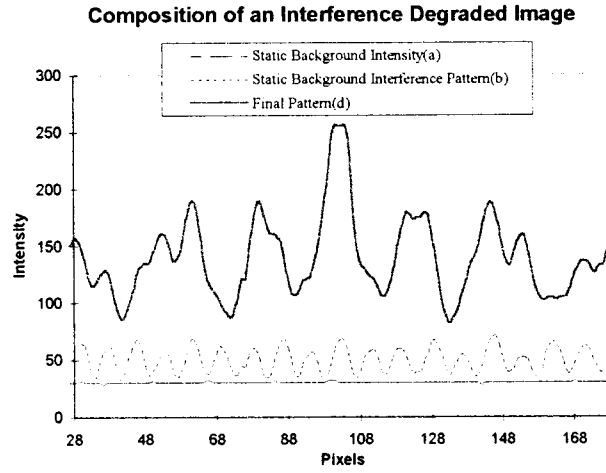


Figure 35 (Components of a Fresnel Diffraction Image)

By using Equation 50, the image acquired above can be accurately enhanced to its undisturbed intensity profile:

$$I_{enhanced} = I_{acquired} - I_{background} - \left( I_{acquired} (I_{interference} - I_{average}) \right) \quad (Equation 50)$$

Performing this calculation on the above figure gives the following intensity profile compared with its known correct profile (See Figure 36):

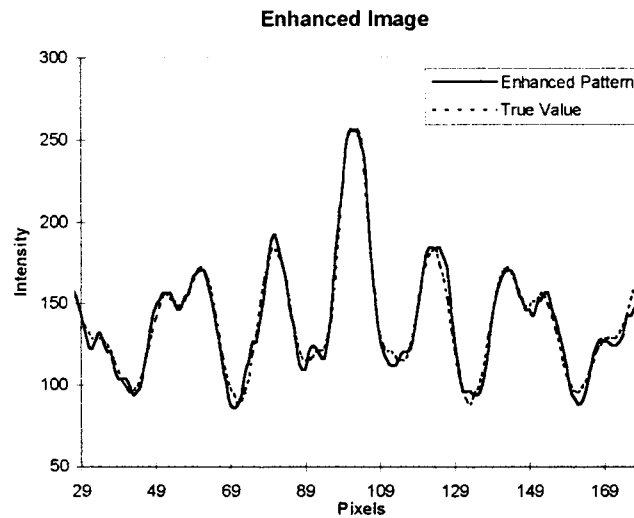


Figure 36 (Noise-Corrected Image)

### **Intensity Profile Measurements and Enhancement of the Central Intensity**

An accurate intensity profile of a diffraction pattern can be produced by first enhancing the image (as described previously) and then finding the center of the pattern. With the center of the pattern found, a radial average can be taken and the intensity as a function of radial distance plotted. This intensity plot can then be fitted with the appropriate polynomial to enhance the resolution of the central intensity measurement

#### *Location of the Center of a Fresnel Diffraction Pattern*

While the center of a Fresnel diffraction pattern is immediately obvious to the human eye, a computer has quite a bit more difficulty in arriving at that same location. To accomplish this task, a computer algorithm was devised which could accurately identify the center of any Fresnel diffraction pattern. This algorithm identified the central fringe location by best fitting a series of circular rings to the pattern. The algorithm is described in greater detail in Appendix C.

#### *Radial Intensity Profiling*

With the center of the pattern identified, an intensity profile is created by averaging radially around the image enhanced diffraction pattern. The average of the intensities is then divided by the average of all the pixel's intensity and these values are plotted versus the radial distance (in pixels).

### *Central Intensity Enhancement*

With the intensity profile of the pattern known, the central intensity can be resolved to greater detail by fitting the nearest ten pixels to a fourth order polynomial. Such curve fitting accurately predicts the true central intensity of the diffraction pattern to roughly 1 part in 20000.

Figure 37 shows a theoretically predicted intensity profile which has been purposely degraded with simulated noise and 8-bit digitization effects:

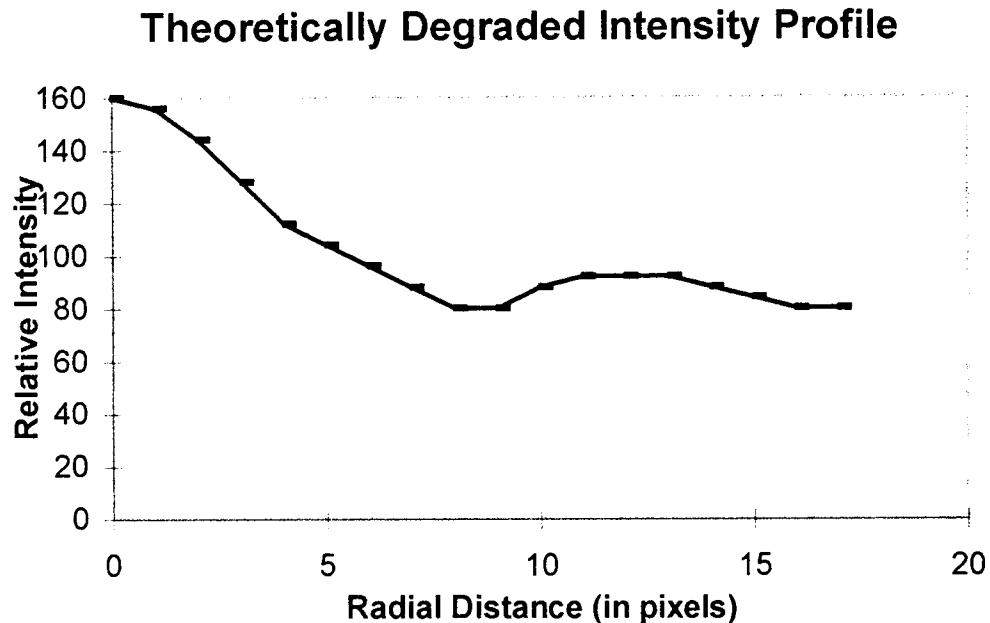
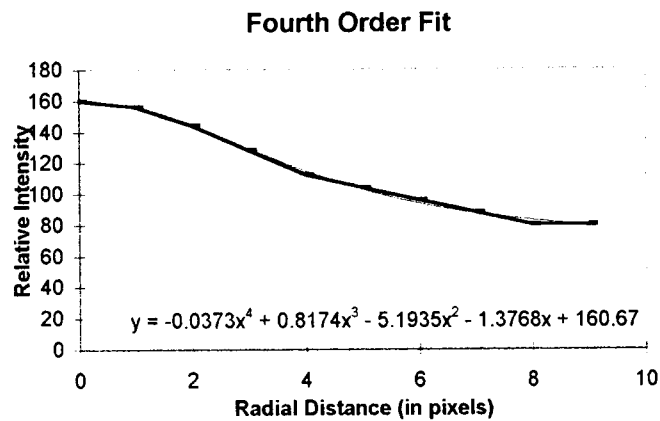


Figure 37 (Purposely Degraded Known Data)

Since this intensity plot is degraded from known data, the central intensity of this degraded image is known to be 160.65744.

Fitting the first ten points to a fourth-order polynomial gives a central intensity of 160.67 and is shown in Figure 38:



*Figure 38 (Fourth-Order Fit of Degraded Data)*

The difference between the actual and the fitted intensity, being a little better than one part in 10000, is roughly equivalent to a 16-bit detector response. The validity of a fourth-order fit over the first ten data points appears adequate to provide a good approximation of the central intensity when the optical noise evident in the image is at a minimum. Consequently, it is often useful to first perform a moving average over the surface of the image before a fit is attempted, if the optical noise is known to be large. Plus the weighting of each data point has proven another useful technique for enhancing the accuracy of the fourth order fit.

## 8. Experimental Data

### Introduction

In this section, the experimental setup, data, and data analysis detailing the response of a Fresnel diffraction-based distance measurement system is presented. The distance accuracy of the Fresnel diffraction-based distance measurement system is presented over a one meter range.

### Basis of Experiment

This experiment applies the approach and techniques detailed in Section 5 for the application of Fresnel diffraction to distance metrology. To summarize, the following formula for the behavior of a Fresnel diffraction pattern's central intensity:

$$I_c = 4I_o \sin^2 \left[ \frac{R^2 \pi}{2\lambda} \left( \frac{1}{L_{source}} + \frac{1}{L_{observe}} \right) \right] \quad (\text{Equation 51})$$

is inverted (according to Equation 28) to produce a measurement of the source's absolute distance. To perform these measurements the setup in Figure 39 was constructed:

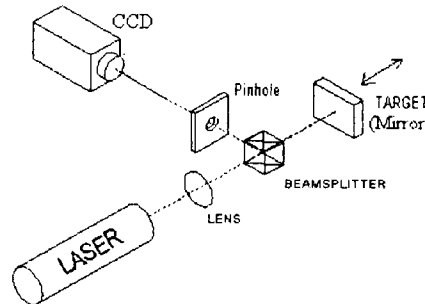


Figure 39 (Distance Metrology Setup)

The setup consisted of a Helium-Neon laser and a spatial filter mounted onto an anti-vibration optical table. The light emitted from the spatial filter was focused with a 0.05 meter fixed focal length lens through a beamsplitter and onto the surface of a high quality optical mirror (which is the target in this experiment). The mirror then reflected the light back through the beamsplitter, this time sending the light through a 1mm-radius pinhole and onto a CCD array. The diffraction image was captured and saved with the help of a Frame Grabber board installed in a nearby computer. A computer controlled translational stage allowed the position of the target to be known with micron accuracy. As the mirror moved away, the source distance (source distance = total reflected distance = distance from the focused point, to the mirror, plus the distance from the mirror to the pinhole) was expected to increase at twice the rate of the translational stage.

### **Detailed Description of Setup**

A 1mW Melles Griot HeNe laser was spatially filtered to create a five-micron-diameter point source, which would serve as the optical source, as well as would remove any unwanted interference patterns resulting from interference within the laser windows and the spatial filter's microscope objective. The laser was allowed to warm up for approximately two hours prior to taking any data (to ensure frequency and amplitude stabilization) and was originally calibrated at an operating wavelength of  $632.807 \pm 0.0001$  nm with 0.1% amplitude fluctuations under normal operating conditions.

A  $0.05 \pm 0.0001$  meter focal length best-form lens (designed for the 632.8 nm wavelength) was used to focus the light diverging from the spatial filter down into a

second small-diameter point. The focus of this lens passed the light through the nearby non-polarizing beamsplitter (see figure) and onto the surface of a high-reflectivity-front surface mirror (positioned at closest approach). The mirror reflected the focused light back along its original path, passing once again through the beamsplitter and traveling off at 90° toward a 2mm-diameter pinhole. The mirror was attached to a gimbal mount which allowed the precise retro-reflection of the beam. Both the best-form lens and the beamsplitter were coated with anti-reflection coatings, specifically designed for the 632.8 nm wavelength, offering reduced reflections of < 1%.

The  $0.002011 \pm 0.000001$  meter diameter pinhole was a precision laser-bored steel-film pinhole which had been black anodized on both sides to reduce unwanted reflections. The pinhole was mounted in a specifically designed holder (provided by the manufacturer) which allowed the pinhole to be easily positioned relative to the beamsplitter without compromising its orientation to the incoming beam. The angle of the pinhole's axis relative to the incoming beam was estimated to be less than one degree off parallel.

An 8-bit 640 x 480 pixel CCD camera was used to capture the diffraction pattern emerging from the 2mm pinhole. The surface of the CCD array was placed at an observation distance of  $0.289 \pm 0.001$  meters from the pinhole. The image of the diffraction pattern was allowed to fall directly onto the surface of the CCD detector array (pixel size = 10 microns by 10 microns) allowing the intensity profile and the physical dimensions of the pattern to be recorded. Unfortunately, the detector array was covered with a protective glass covering which was not coated with anti-reflection coatings.

Consequently, interference patterns resulting from the numerous internal reflections of the diffraction pattern would degrade the system's final image. The normal to the CCD surface was estimated to be parallel to the incoming diffraction pattern to less than one degree. The CCD was equipped with a variable gamma correction factor which was fixed at 1.0 for the experiment. The manual gain selection on the CCD was chosen to ensure that the pattern's intensity profile would remain within its full range. This meant that it was occasionally necessary to readjust the gain of the CCD as the intensity of the source fell off. When such situations occurred, the measurements of background intensity, static background interference patterns, and several other factors were rerecorded to ensure proper intensity measurements. The shutter speed for the CCD camera was set at 1/20000th of a second to reduce amplitude fluctuations in the pattern due to possible fluctuations in the source.

The CCD image was transferred through a coaxial cable to a frame grabber board (Data Translation's DT3851 board) located within a 133 MHz Packard Bell Pentium Processor PC. The frame grabber board was specifically designed for imaging applications and offered a feedback circuit which reduced pixel jitter in the CCD camera as well as reduced signal noise. The board was controlled with a program written in Microsoft's Visual Basic for Windows which allowed the immediate acquisition and analysis of data.

An Aerotech Unidex 11 translational stage, operating through the rotation of a Ballscrew Lead equipped with a rotary encoder, allowed the translational stage's platform (which held the target/mirror) to be positioned with an accuracy of  $\pm 2.5$  microns

(pre-calibrated by Aerotech). In addition, the translation stage was certified at less than five microns deviation from straightness and flatness. Because the full translational range of the platform was only six inches, it was necessary to move the entire translational stage several times. Fortunately, the multiple screws used to mount the stage to the optical table allowed the alignment to be preserved between moves. The translational stage was controlled through an RS232 communication port connected to the nearby Packard Bell PC. Consequently, it was possible to automate the movement of the translational stage with the acquisition of images and data. As data were taken, positioning of the target was done with minimal acceleration and velocity (50 microns per second). Once arriving at a new data position, the translational stage would idle for roughly one minute before any data were taken. This was done to ensure that all measurements would be taken with the system in equilibrium (with respect to temperature, vibrations, etc.). All components were mounted on an anti-vibration Newport optical table which provided a vibration damped steel surface as the test bed for the distance measurements. The table was then shielded with opaque partitions to reduce the amount of ambient lighting and air flow. Room lights and computer displays were turned off during all experiments.

A low temperature thermocouple was positioned within the setup to measure the temperature of the operating environment. This temperature was taken on several occasions to accommodate for any thermal fluctuations which might be experienced. Because the laboratory was already in a fairly controlled temperature environment, it was possible to maintain the temperature within the experiment to  $\pm 0.1^{\circ}$  C. When it was

necessary to adjust components within the setup, the system was allowed to thermally and vibrationally stabilize for a period of roughly thirty minutes before any data were taken.

It should be noted that many of these precautions turned out not to be necessary (at least not to the extreme described above) for micron-accuracy distance measurements. However, because this experiment was performed for the sole purpose of establishing the true accuracy of the system, the rigorous conditions were deemed necessary.

### **Experimental Data**

Twelve hundred data points were taken over a range from five centimeters to two meters. The data points were taken in intervals such that the number of data points taken per central-fringe “cycle” was constant (about 60 points per fringe). This allowed the full accuracy of the system to be examined at positions close to the pinhole (where the accuracy was estimated to be best) as well as at far away distances (where the accuracy was known to be reduced). While the source distance was translated over a two meter range, the actual translational stage passed through only a one meter range. The source distance referred to in the following data is the actual source distance (distance from the focus of the best-form lens to the pinhole) and not the target distance (distance from the mirror to the pinhole). This condition is purely an objective decision and does not affect the outcome of the data but needs to be specified.

All data points collected in this section were analyzed using the image-analysis techniques described earlier. For the resolution of distance ambiguities, the drop in the

intensity of the source with distance squared was used to determine the source's approximate location. Using the above conditions, the distances measured by the Fresnel-diffraction-based system versus the distances measured with the translational stage is shown in Figure 40:

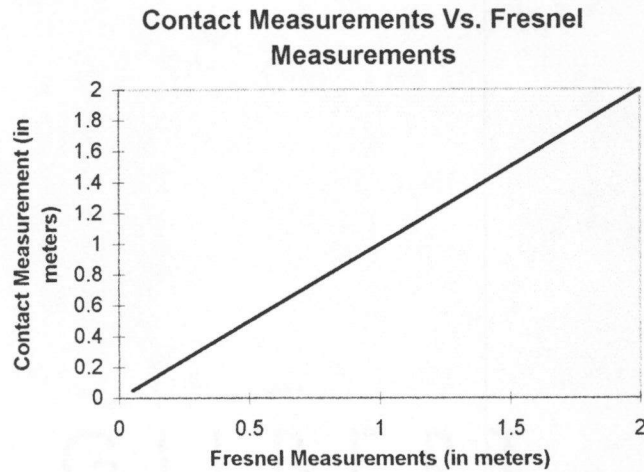


Figure 40 (Comparison of Non-Contact Vs. Contact Measurements)

Plotting the difference between these two measurements allows us to examine the accuracy of the system for the given range and is shown in Figure 41:

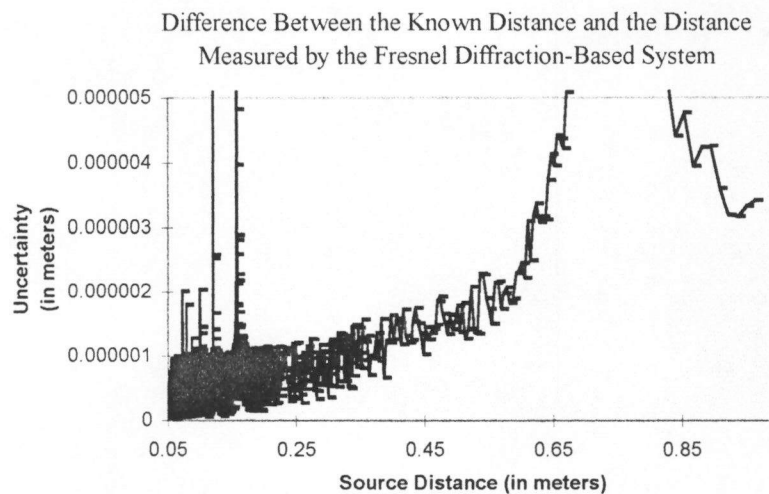


Figure 41 (Differences Between Fresnel-Based Distance Measurement and Contact-Based Distance Measurement)

Several spikes located near the beginning of the plot are the result of errors made in the determination of the center of the diffraction pattern resulting from corrupted image data and can be disregarded. The increasing distance uncertainty with source distance extends to a maximum height of 8 microns at approximately eighty centimeters. The uncertainty is the result of the expected uncertainty of the Fresnel-based distance measurement system as well as additional errors inherent in the translational stage at large distances. Obviously, errors at distances greater than one meter range eventually become larger than the single micron errors of Figure 41.

The errors due to temperature fluctuation ( $\pm 0.1^{\circ}\text{C}$ ) were calculated using the error-analysis formulas described in Section 6 and were calculated to be not greater than  $\pm 2$  microns over the one-meter distance range. Because the translational stage was known to have a distance error of  $\pm 2.5$  microns, it is possible that many of the errors displayed in Figure 41 are the result of errors in the translational stage combined with environmental errors. While a certified instrument was not available which could determine the distance of the translational stage with an accuracy better than  $\pm 2.5$  microns, the sensitivity of the Fresnel diffraction-based distance measurement system is adequately demonstrated and proven to be in the single-micron accuracy range over a distance of one meter.

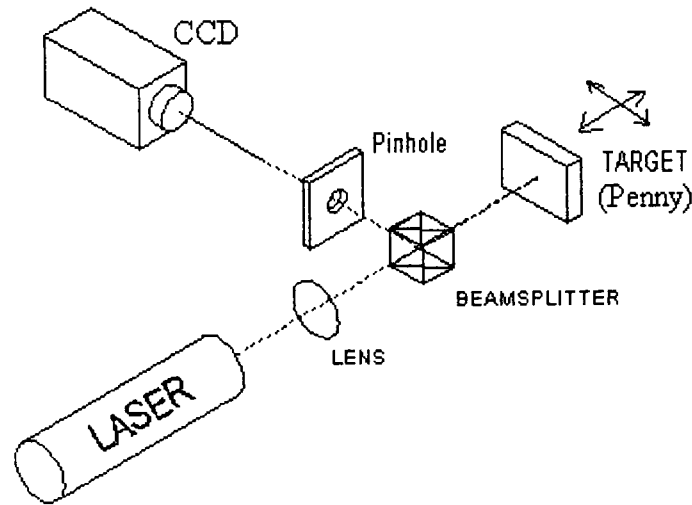
## **9. Future Research**

### **Introduction**

One of the most exciting applications of the Fresnel-based-distance measurement system is the possibility of using the system to accurately profile diffuse surfaces. Currently, a number of obstacles must first be overcome before this possibility can become a reality. While some of these obstacles have already been conquered, as will be seen in the data presented qualitatively in this section, much research is still needed before the high accuracy surface profiling of large area objects is possible. This section represents only the beginning of diffuse surface measurements using Fresnel-diffraction-based systems and it is provided to help identify the potential future applications of this technology.

### **Preliminary Diffuse Surface Measurements**

The surface profiling of non-reflective targets can be accomplished through the focusing of a laser beam onto the surface of a diffuse target and using the resulting scattered light as the source for a Fresnel-diffraction-based distance measurement device. Figure 42 represents one possible variation of this technique and was used to gather preliminary measurements made on a semi-diffuse target:



*Figure 42 (Experimental Setup for Diffuse Surface Profilometry)*

The diffuse target was translated along the x and y axis with a two-axis computer controlled translational stage which allowed the target to be positioned with micron accuracy. A 1mW HeNe laser was focused precisely onto the target's surface using a 0.05 centimeter best-form lens with anti-reflection coating. The surface of the target was relatively flat, not greatly exceeding the focused spot width. Tight focusing of the laser significantly reduced much of the speckle associated with diffuse surface reflections as well as produced a suitable point source for diffraction measurements. Reflections from the target surface were directed, with the aid of a beamsplitter, through a 2mm pinhole and into a 640 x 480 8-bit CCD camera. Distance measurements were performed on the acquired image as described in previous sections.

The data recorded from the x-y scan of the targets surface (a total of 40000 points) are shown in Figure 43:

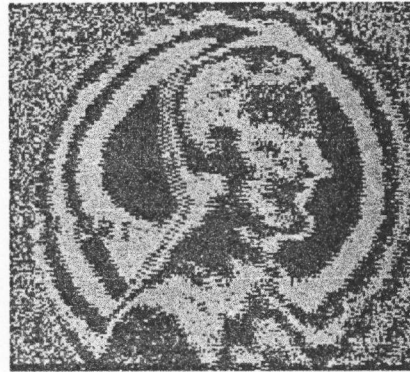


Figure 43 (Grey Scale Image of a Profiled U.S. Penny)

This grey scale representation of the various heights on the target is presented to qualitatively depict the ability of the Fresnel diffraction-based system to surface profile a diffuse target.

### **Future Research and Goals**

While the feasibility of diffuse surface profiling has been demonstrated, considerable research is still required before this application can become a reality. In addition, a study of the basic Fresnel diffraction processes must still be continued to better understand and enhance the current Fresnel-diffraction-based measurement system. In the near future, calibration techniques for the determination of variable parameters will be developed as well as faster algorithms for the determination of the corrected central fringe intensity. Continued research and application will hopefully establish this technique as a new and powerful method of precision distance metrology.

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## REFERENCES:

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## **APPENDICES**

## **Appendix A: Applications**

## **Appendix A: Applications**

### **Introduction**

One of the most exciting aspects of this research is its potential application to many areas of physics and engineering. What follows are a few brief descriptions of the current applications which are being sought for the Fresnel diffraction-based distance measurement system.

### **Coordinate Measuring Machines**

Coordinate measuring machines (or CMM's) are a common manufacturing tool required in the production of high quality machined parts. These machines are gaining wider usage as the tolerances on commercially machined parts are becoming tighter and tighter. Production of a hybrid CMM, using the Fresnel diffraction-based distance measurement system as the instrument's sensing probe, could produce the next generation of machine inspection systems. Potential capabilities include the inspection of parts while in production, high-accuracy high-speed three dimensional modeling, and user-friendly CMM operation.

### **Space-Based Scientific Measurement Tool**

The Fresnel diffraction-based distance measurement system is being considered for two unique space-based applications. NASA's next generation space telescopes could employ the Fresnel diffraction-based system to perform high accuracy non-contact

distance measurements on free-floating secondary and primary telescope mirrors. Plus, the SEE project, a joint effort between the University of Tennessee, ORNL, and NASA, is planning to use the Fresnel diffraction-based system to measure, with low-impact, the distances of free-floating gravitationally attracting spheres. This work is aimed at producing an accurate measurement of the gravitational constant  $G$  and requires a high-accuracy non-contact distance measurement system which can be operated in a pulsed mode.

### **Surface Profiler**

A wide variety of applications exist for the surface profiling of diffuse surfaces. Many applications have been considered ranging from the profiling of ground glass lenses to the profiling of detailed microchip circuits. While this area of application spans numerous various fields and technologies, the potential application of a Fresnel diffraction-based system are confined to those situations in which high accuracy surface profiling is needed at a relatively large standoff distance. Much interest has already been expressed by the steel industry for large scale surface profiling of hot mill rollers used in the production of rolled steel. This application utilizes the system's ability to make micron accuracy absolute distance measurements at standoff distances of one meter.

### **Vibration Analysis**

The analysis of small scale vibrations experienced by high speed rotors is another application of the Fresnel diffraction-based system which is currently being examined.

Such measurements would allow the instabilities experienced by high speed rotors, as a result of fatigue and wear, to be identified before destructive oscillations result. This application could potentially help create higher efficiency motors and flywheels.

## **Conclusions**

While much work is needed in the development of the Fresnel diffraction-based distance measurement system, the basic principles underlying its conception and workings are presented in this writing. These principles describe a new class of distance measurement systems with qualities useful to the physics, engineering, and manufacturing communities. Continued development of this technique should produce a distance measurement device which will make a valuable contribution to the science of distance measurement.

## **Appendix B: Fresnel Diffraction Modeling**

## Appendix B: Fresnel Diffraction Modeling

### Purpose

The Fresnel-diffraction-based distance measurement technique proposed in this writing is based upon the assumption that Equation 1B:

$$I_{center} = \frac{4I_o}{(L_{observe} + L_{source})^2} \sin^2 \left[ \frac{R^2 \pi}{2\lambda} \left( \frac{1}{L} + \frac{1}{L_1} \right) \right] \quad (Equation 1B)$$

accurately describes the dependence of a diffraction pattern's central fringe intensity with distance. As will be shown in this appendix, Equation 1B is an exact solution to a very simplified model of the Fresnel diffraction process. It is not certain, however, how well Equation 1B will agree with more advanced models of the Fresnel diffraction process. To study these potential inaccuracies, Equation 1B will be compared with the numerical results of five increasingly complex diffraction models. These five models will represent the four most significant physical processes which need be represented in any accurate diffraction calculations.

### MODEL #1: Simple Fresnel Integral

A simplified model of the Fresnel diffraction process by a circular aperture can be derived by assuming that the source and observation distances are large when compared to the pinhole radius. In this way, the angles  $\theta$  and  $\theta_1$  (illustrated previously in Figure 1) approach zero. Consequently, the obliquity factor  $\left( \frac{\cos \theta + \cos \theta_1}{2} \right)$  in Equation 10 can be

approximated as one. Plus, the term  $e^{ik(d+d_1)}$  in Equation 10 can be reduced to

$e^{ik(L_{source}+L_{observe})} e^{\frac{i\pi r^2}{\lambda} \left( \frac{1}{L_{source}} + \frac{1}{L_{observe}} \right)}$  through the following simplification of  $(d + d_1)$ :

$$\begin{aligned} d + d_1 &= \sqrt{L_{observe}^2 + r^2} + \sqrt{L_{source}^2 + r^2} \\ &= L_{observe} \sqrt{1 + \left( \frac{r}{L_{observe}} \right)^2} + L_{source} \sqrt{1 + \left( \frac{r}{L_{source}} \right)^2} \end{aligned}$$

and since:  $\sqrt{1+x^2} \approx 1 + \frac{1}{2}x^2$

$$\begin{aligned} d + d_1 &= L_{observe} \left( 1 + \frac{1}{2} \left( \frac{r}{L_{observe}} \right)^2 \right) + L_{source} \left( 1 + \frac{1}{2} \left( \frac{r}{L_{source}} \right)^2 \right) \\ &= (L_{observe} + L_{source}) \left( \frac{r^2}{2} \left( \frac{1}{L_{source}} + \frac{1}{L_{observe}} \right) \right) \end{aligned}$$

Consequently,

$$e^{ik(d+d_1)} = e^{\frac{2i\pi}{\lambda}(L_{observe}+L_{source})} e^{\frac{i\pi r^2}{\lambda} \left( \frac{1}{L_{source}} + \frac{1}{L_{observe}} \right)}$$

Combining these simplifications gives the diffraction integral Equation 2B:

$$\psi(\rho) = \frac{ikA_o}{2\pi} \iint_{surface} \frac{e^{\frac{i\pi r^2}{\lambda} \left( \frac{1}{L_{source}} + \frac{1}{L_{observe}} \right)}}{dd_1} d\tilde{S} \quad (Equation 2B)$$

where the expression  $e^{\frac{2i\pi}{\lambda}(L_{observe}+L_{source})}$  has been dropped since it is merely a phase factor.

This simplistic model is expected to correspond very closely to Equation 1B.

### MODEL #2: Included Obliquity Factor Integral

Model two consists of including the full obliquity factor term  $\left(\frac{\cos \theta + \cos \theta_1}{2}\right)$  while

using the simplified exponential argument  $e^{\frac{i\pi r^2}{\lambda} \left(\frac{1}{L_{source}} + \frac{1}{L_{observe}}\right)}$ . Therefore, the Fresnel diffraction integral becomes Equation 3B:

$$\psi(\rho) = \frac{ikA_o}{2\pi} \iint_{surface} \frac{e^{\frac{i\pi r^2}{\lambda} \left(\frac{1}{L_{source}} + \frac{1}{L_{observe}}\right)}}{dd_1} \left(\frac{\cos \theta + \cos \theta_1}{2}\right) d\vec{S} \quad (Equation 3B)$$

Numerically integrating Equation 3B is expected to produce only small changes to the measurements of model #1.

### MODEL #3: Full Exponential Integral

Another possible method to increase the accuracy of Model #1 would be to integrate Equation 2B with the full exponential term:  $e^{ik(d+d_1)}$ . Therefore, the Fresnel diffraction integral would become Equation 4B:

$$\psi(\rho) = \frac{ikA_o}{2\pi} \iint_{surface} \frac{e^{ik(d+d_1)}}{dd_1} d\vec{S} \quad (Equation 4B)$$

Numerically integrating Equation 4B is expected to provide a more accurate prediction than Model #3:

#### MODEL #4: Full Integral

A significantly more accurate model of the Fresnel diffraction process would involve the inclusion of both the obliquity factor  $\left(\frac{\cos \theta + \cos \theta_1}{2}\right)$  and the full exponential term.

This model more accurately describes the emission of an electromagnetic wave from a Huygen's wavelet and will give more realistic predictions of the Fresnel diffraction process. Therefore, the Fresnel integral will appear as Equation 5B:

$$\psi(\rho) = \frac{ikA_o}{2\pi} \iint_{\text{surface}} \frac{f_s}{dd_1} e^{ik(d+d_1)} \left(\frac{\cos \theta + \cos \theta_1}{2}\right) d\vec{S} \quad (\text{Equation 5B})$$

With this model, the calculated central fringe intensity will be very nearly exact.

#### MODEL #5: Comprehensive Fresnel Integral

Another model of the Fresnel diffraction process, which is increasing in complexity, involves including a higher order term generally dropped from calculations. Including this term the Fresnel integral becomes Equation 6B:

$$\psi(\rho) = \frac{ikA_o}{2\pi} \iint_{\text{surface}} \frac{f_s}{dd_1} e^{ik(d+d_1)} \left(\frac{\cos \theta + \cos \theta_1}{2}\right) d\vec{S} - \frac{A_o}{2\pi} \iint_{\text{surface}} \frac{f_s}{d^2 d_1^2} e^{ik(d+d_1)} \left(\frac{d_1 \cos \theta + d \cos \theta_1}{2}\right) d\vec{S}$$

Equation 6B

With this model, nearly all physical processes which are occurring have been taken into account.

### Analysis of the Accuracy of Each Model

The approximation equation given by Equation 1B can be compared with models one through five through a very simple comparison scheme. First, models one through five are restated below in Table 4:

**Table 4: Definition of Models**

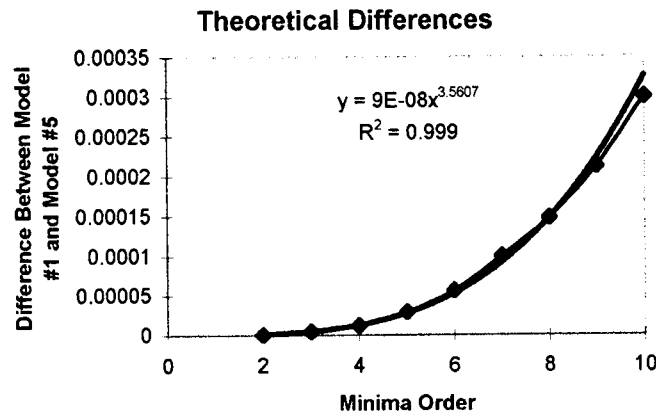
| Model No. | Obliquity Factor | Exponential            | Higher Order     |
|-----------|------------------|------------------------|------------------|
| 1         | Simplified (= 1) | Simplified (2nd Order) | Simplified (= 0) |
| 2         | Exact            | Simplified (2nd Order) | Simplified (= 0) |
| 3         | Simplified (= 1) | Exact                  | Simplified (= 0) |
| 4         | Exact            | Exact                  | Simplified (= 0) |
| 5         | Exact            | Exact                  | Exact            |

The minima predicted by each model can be compared against the minima predicted by Equation 1B. Obviously, Equation 1B will predict a minima when the argument of the sine is equal to a multiple of  $\pi$ . Therefore, we can compare how each model's minima predictions agree with the multiple of  $\pi$  prediction of Equation 1B. Table 5 lists the differences in the predicted minima location between each different model.

**Table 5: Location of Relative Minima**

| Minima Order | Model #1 | Model #2 | Model #3        | Model #4        | Model #5        |
|--------------|----------|----------|-----------------|-----------------|-----------------|
| 2            | $2\pi$   | $2\pi$   | $(2.000001)\pi$ | $(2.000001)\pi$ | $(2.000001)\pi$ |
| 3            | $3\pi$   | $3\pi$   | $(3.000005)\pi$ | $(3.000005)\pi$ | $(3.000005)\pi$ |
| 4            | $4\pi$   | $4\pi$   | $(4.000012)\pi$ | $(4.000012)\pi$ | $(4.000012)\pi$ |
| 5            | $5\pi$   | $5\pi$   | $(5.000029)\pi$ | $(5.000029)\pi$ | $(5.000029)\pi$ |
| 6            | $6\pi$   | $6\pi$   | $(6.000056)\pi$ | $(6.000056)\pi$ | $(6.000056)\pi$ |
| 7            | $7\pi$   | $7\pi$   | $(7.000101)\pi$ | $(7.000101)\pi$ | $(7.000101)\pi$ |
| 8            | $8\pi$   | $8\pi$   | $(8.000148)\pi$ | $(8.000148)\pi$ | $(8.000148)\pi$ |
| 9            | $9\pi$   | $9\pi$   | $(9.000212)\pi$ | $(9.000212)\pi$ | $(9.000212)\pi$ |
| 10           | $10\pi$  | $10\pi$  | $(10.00030)\pi$ | $(10.00030)\pi$ | $(10.00030)\pi$ |

The slight shifting in the location of the minima is illustrated below in Figure 1B:



*Figure 1B: Shifting of the Minima*

Obviously, it is possible to fit an equation to the shifting of the minima (most likely a fourth order or power fit). Because the behavior of the shift is rather predictable, it is possible that Equation 1B could be enhanced to provide a more accurate description of the central intensity behavior. However, this task will be left for future endeavors. The expression of Table 5 in terms of physical distances is calculated for an observation distance of one meter, a wavelength of 632.8nm, and a pinhole radius of one millimeter.

The results are listed in Table 6:

**Table 6: Location of Minima ( $L_{\text{observe}} = 1$  meter)**

| Minima Order | Model #1 | Model #2 | Model #3 | Model #4 | Model #5 |
|--------------|----------|----------|----------|----------|----------|
| 2            | .6530825 | .6530825 | .6530818 | .6530818 | .6530818 |
| 3            | .3575514 | .3575514 | .3575507 | .3575507 | .3575507 |
| 4            | .2461600 | .2461600 | .2461590 | .2461590 | .2461590 |
| 5            | .1876877 | .1876877 | .1876864 | .1876864 | .1876864 |
| 6            | .1516622 | .1516622 | .1516606 | .1516606 | .1516606 |
| 7            | .1272394 | .1272394 | .1272373 | .1272373 | .1272373 |
| 8            | .1095914 | .1095914 | .1095892 | .1095892 | .1095892 |
| 9            | .0962427 | .0962427 | .0962402 | .0962402 | .0962402 |
| 10           | .0857927 | .0857927 | .0857899 | .0857899 | .0857899 |

## **Conclusion**

The ability of Equation 1B to precisely predict a Fresnel diffraction pattern's central fringe intensity is sufficient for distance measurements requiring only single micron accuracy. This is clearly evident in Table 6 which does not display a distance error, due to theoretical inaccuracies, of greater than two microns. Obviously, to move to greater accuracy levels a new approximation formula will need to be developed or, more likely, Equation 1B must be revised slightly.

## **Appendix C: Central Fringe Location Algorithm**

## **Appendix C: Central Fringe Location Algorithm**

### **Algorithm Description**

While the center of a Fresnel diffraction pattern is immediately obvious to the human eye, a computer has quite a bit more difficulty in arriving at that same location. To accomplish this task, a computer algorithm<sup>9</sup> was devised which could accurately identify the center of any Fresnel diffraction pattern. The principle behind the algorithm is as follows:

#### Stage 1:

- 1) Eight imaginary horizontal lines are drawn across the imaged pattern at equally spaced intervals.
- 2) The intensity profile traced by each line is collected and the number of peaks profiled per line is calculated.
- 3) The line which crosses the most peaks is selected as the optimal position for analysing an outer diffraction ring.

#### Stage 2:

- 4) The most prominent diffraction ring encountered by the chosen line is profiled and a line which is tangent to the ring is calculated (based on a linear best fit of the surrounding pixel intensities)

- 5) An orthogonal line is then computed which must pass approximately through the center of the diffraction pattern.
- 6) A series of circles with increasing radii are produced in a direction orthogonal to the tangent line and “pinned” to the prominent peak of step 4.
- 7) The intensity values around the circle are collected and the radii with the minimal standard deviation is chosen.

Stage 3:

- 8) The chosen radii and circle are then rotated around the prominent peak of step 4 until the best location for minimizing the intensity deviation around the circle has been determined.
- 9) The radii is again fluctuated to search for a better fitting circle around this new location.
- 10) The improved circle is again rotated to best fit the circle.

Steps nine and ten can be repeated as many times as needed but one time has typically been found to be sufficient. The above algorithm was applied to 1200 various Fresnel diffraction pattern images, and it accurately predicted the central fringe's location every time. This algorithm was even capable of detecting the central fringe's location when only partial viewing of the pattern was possible. Further enhancements to this algorithm will focus on exploiting characteristics unique to a Fresnel Diffraction pattern<sup>10-13</sup>.

## VITA

Dennis Duncan Earl was born in Bremerhaven, Germany on February 16, 1972 to Dennis and Pamela Earl. Being born into a military family, he received much of his early education overseas attending DOD provided schools. After graduating from Rutledge High School, Rutledge, Tennessee in 1990, he attended the University of Tennessee at Knoxville, enrolled in the Engineering Physics Department. He graduated with a Bachelor of Science degree in Engineering Physics in December 1993.

Upon graduation, he entered the University of Tennessee graduate program in pursuit of a Master of Science degree in Physics. Newly married and working as a Graduate Research Assistant for Dr. Thomas Ferrell of the Health Research Division at Oak Ridge National Laboratory, he aided in the development and conception of several electro-optical bio-sensors. Eventually transferring to the Engineering Technology Division in pursuit of a full-time position, he continued research relating to the application of optical theory. In June of 1995 he invented the Fresnel Diffraction-based measurement technique which led to the topic of his Master's thesis. He graduated from the University of Tennessee with a Master of Science degree in Physics in August of 1997.