THE DESIGN AND CONSTRUCTION OF A
CURRENT TRANSFORMER
TESTING SET

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INTRODUCTION.

In any electrical field, but particularly in distribution work, it is necessary to know the accuracy of current transformers used in metering and relay work. The accuracy test by the absolute method requires laboratory equipment which is too expensive for the average utility, or company to purchase. In addition, the time required for testing by this method is prohibitive, where there are a large number of transformers to be tested. Hence, several methods have been devised for making comparison tests with transformers which have already been calibrated by the absolute method.

The two methods in use employing standard transformers are known as the deflection method and as the null method. The deflection method depends entirely upon the accuracy of the wattmeters used, whereas the null method depends only on a circut of calibrated impedance and mutual inductance. The obvious advantage of the null method can be seen from the fact that the accuracy of the wattmeters used in the deflection method may vary over a wide range; and since sensitivity is a prequisite, damaged bearings can easily cause a large error in the readings. The calibration of the impedances, and mutual inductance used in the null method, is for all practical purposes permanent.

PURPOSE OF THIS THESIS.

A current transformer testing set utilizing the null method was constructed by the authors, E. Dempsey Jones and W. David Dishner, for the following reasons; primarily they were interested in electrical metering, having been associated with the meter department of the Tennessee Electric Power Company, and the East Tennessee Light and Power Company respectively; and secondarily they desired to produce something which would be of permanent benefit to the University of Tennessee by the addition of this set to their labaratory equipment.

ACKNOWLEDGEMENTS.

The authors wish to acknowledge the assistance of the following men, who made possible the completion of their work: Prof. J.G. Tarboux of the University; and particularly, Mr. Eugene Smith of the Tennessee Electric Power, for without his assistance we would not have been able to have calibrated our instrument, not to have completed it within the required time limit. Also we wish to acknowledge the courtesy of Mr. C.A. Duke of the Tennessee Electric Power Co., in granting us permission to use the electrical laboratory in Chattanooga.

THE THEORY OF THE CURRENT TRANSFORMER TESTING SET..

The first current transformer testing set utilizing the null method, was designed by Dr. Francis B. Silsbee of the Bureau of Standards. His circuit is shown in figure # 1, which also shows the location of the transformer being tested, the standard transformer, and the balancing circuit. If the ratio and the phase angle of the two transformers were identical, the current I_s would be equal to I_x, hence there would be no current flowing through the balancing circuit. But, if their ratio and phase angle are not identical, there will be a difference of current (ΔI(which will flow through the balancing circuit. This current is the vector difference of I_s and I_x. For determining the difference in ratio and phase angle of the two transformers, the balancing circuit is adjusted so that no current flows through the detector, and hence the voltage drops across the detector is zero. This is accomplished by adjusting the mutual inductance for balancing the 90° component of the differential current, (the secondary voltage of the mutual inductance being 90° out of phase with I_s) and adjusting the slide wire resistance for the in-phase component. It should be obvious that when there is no current through the detector circuit, the two transformers are exactly balanced and that the amount of resistance and inductance required to produce this balance is proportional to the difference in ratio and phase angle of the transformers being tested. For instance, if the transformers were identical, the inductance and resistance would be set at their zero points respectively. The reader should keep in mind the fact that the balancing circuit carries the difference of the two transformer currents, hence if there is no difference there can be no voltage across the detector.

The detector used is a dynamo-meter type wattmeter. The zero of the meter is set so that it corresponds to the point of zero mutual inductance between the potential and current coils. This setting prevents any error ~~from~~ arising, due to an induced voltage in the detector circuit. The exciting current for the wattmeter is supplied by two voltages 90° apart. These voltages are so adjusted that one voltage is exactly in phase with I_s and the other voltage at 90° to I_s. Hence the wattmeter is supplied from the 90° phase when adjusting the 90° component of ΔI and from the other phase when adjusting the in-phase component of ΔI.

The original Silsbee circuit was not used in this set, due to the difficulty in obtaining good contact with the resistance slider. The circuit used was the same as that used by Leeds and Northrup in their Silsbee set. This circuit was designed to overcome the difficulties due to the variable contact resistance. In this circuit, the sliding contact is used, but the contact is so arranged by balancing the circuit on either side of it, that there is no error involved due to its resistance. The diagram of the circuit used is shown in figure #2. The mathematical proof of this circuit, and the derivation of the equations giving the difference in ratio and phase angle in terms of the impedances and inductance is given on pages II - VII

CONSTRUCTION OF THE SET.

The actual circuit used in the construction of the set is shown in figure # 3. The switches shown are used in checking the polarity of the transformers, and in adjusting the exciting current of the wattmeter so that one phase of the quadrature voltages is exactly 90° out of phase with I_s. This eliminates error due to mutual inductance in the wattmeter, in addition to SUPPLYING the CORRECT PHASE ANGLE ~~polarity~~ needed for balancing the differential current. For further information as to the operation of the set, the reader should consult the directions within the set itself, where a complete discussion is given with rewpect to the procedure which should be followed in making a test.

The mutual inductance of the set was designed and built according to the method specified by Brooks, in the Bureau of the Standard's Bulletin # S-290. The amount of inductance for which it was designed was approximately 800 micro-henries according to the recommendation of Dr. Silsbee in the Bureau of Dtandard's Bulletin # 309. The values of the impedances were determined by trial and error methods. Circuits with different values of impedances were tried until those values were found which would give the best divisions for the scales within the desired range. Referring to figure # 2, the approximate values for the circuit constants are: R - 12 ohms, Z_1 and Z_4 about 1.5 ohms each,* and Z_2 and Z_3 about 18 ohms* each. These

*See next page.

values were never checked accurately, since the set was calibrated by comparison with a standard Leeds and Northrup Silsbee set.

For any further information with restect to this set, see the instructions within the set, or the references in the bibliography.

* These values refer only to the resistance of the circuits. The coils were not wound non-inductively, hence it was necessary to consider them as impedances in the discussion. The actual reactance of the coils is very low in comparison with the resistance.

ACCURACY OF SET.

This set should give fairly accurate results for two reasons: first, it was calibrated by comparison with a standard commercial set, and therefore would be subject only to the errors of this set.

Second, the set does not determine the transformer accuracy in absolute terms, but in differences of ratio and phase angle. For this reason an error in adjustment does not make as great an absolute error, when applied to the difference in ratio. For instance suppose two transformers differ by 1% in ratio, then an error of 1% in adjustment would result in only 0.01% error expressed in absolute units. Hence its accuracy is well within the limits required in commercial practice.

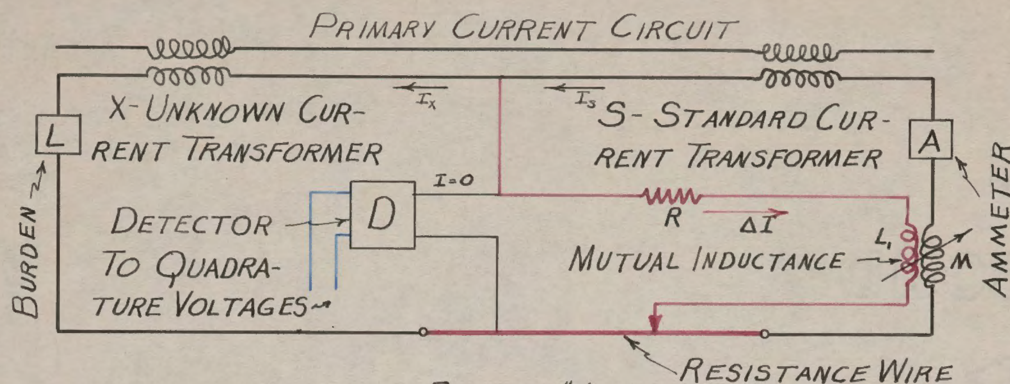

FIGURE #1

ORIGINAL SILSBEE CIRCUIT

NOTE: BALANCING CIRCUIT IN RED. FOR SYMBOLS IN DIAGRAMS FOLLOWING REFER TO DIAGRAM ABOVE

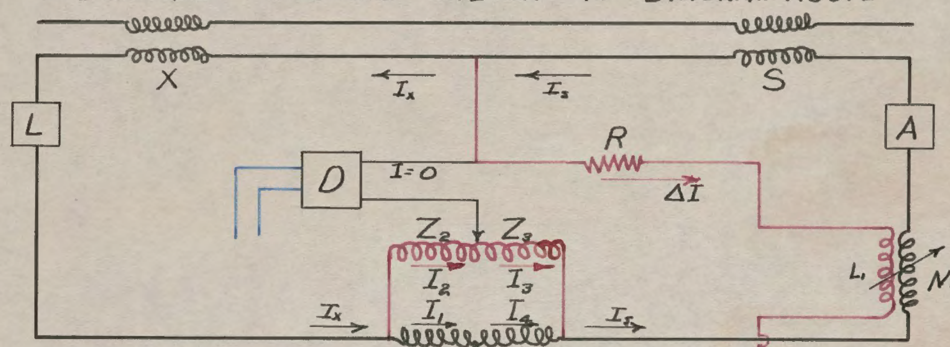

FIGURE #2

CIRCUIT DIAGRAM OF TESTING SET USED IN DERIVATION OF EQUATIONS

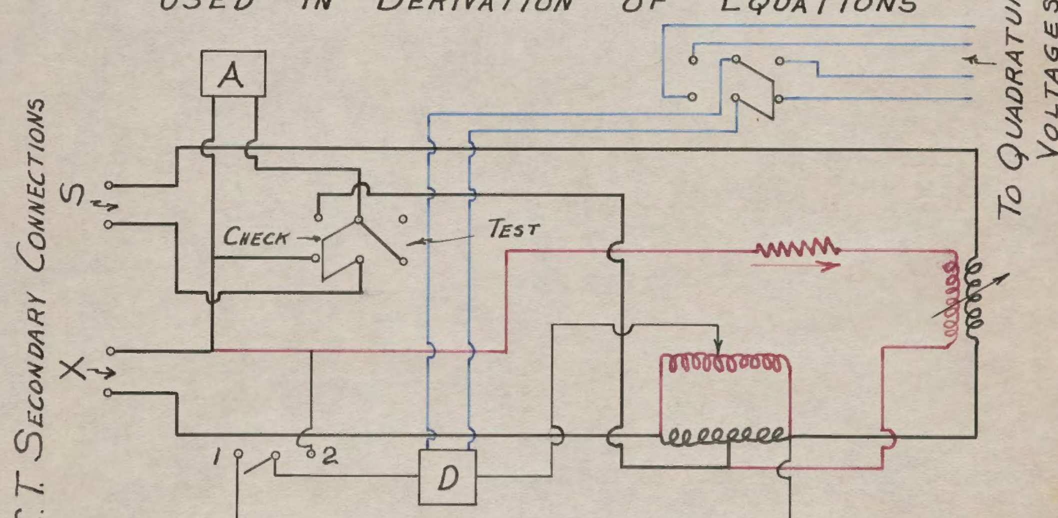

FIGURE #3

DIAGRAM OF THE ACTUAL CIRCUITS OF TESTING SET

Deriviation of the Equations Expressing the Ratio and Difference in Phase Angles of the Standard Current Transformer, and the Current Transformer Tested.

NOTE: Refer to previous discussion and circuit diagram for all symbols and subscripts.

CURRENT EQUATIONS.

(1) $I_s = I_x + \Delta I$

(2) $I_x = I_1 + I_2$

(3) $I_2 = I_3$

(4) $I_s = I_3 + I_4$

(5) $I_4 = I_1 + \Delta I$

VOLTAGE DROP EQUATIONS.

I. $\Delta I(R + jwL_1) + jI_s wM + I_4 Z_4 = I_3 Z_3$

II. $I_2(Z_2 + Z_3) = I_1 Z_1 + I_4 Z_4$

Let: $R + jwL_1 = k$

Then from I:

$$\Delta Ik + jI_s wM + I_4 Z_4 = I_3 Z_3$$

From (4) $I_3 = I_s - I_4$

Then

$$\Delta Ik + jI_s wM + I_4 Z_4 = I_s Z_3 - I_4 Z_3$$

$$\Delta Ik + jI_s wM + I_4 (Z_3 + Z_4) = I_s Z_3 \qquad \text{(A)}$$

From Equation II above:

$$I_2 (Z_2 + Z_3) = I_1 Z_1 + I_4 Z_4$$

(3) $$I_2 = I_3$$

(4) $$I_3 = I_s - I_4 = I_2$$

(2) $$I_1 = I_x - I_2 = I_x - I_s + I_4$$

Substituting in II. above:

$$(I_s - I_4)(Z_2 + Z_3) = I_x Z_1 + I_4 Z_1 + I_4 Z_4 - I_s Z_1$$

$$I_s Z_2 + I_s Z_3 - I_4 Z_2 - I_4 Z_3 = I_x Z_1 - I_s Z_1 + I_4 Z_4 + I_4 Z_1$$

$$I_4 (Z_1 + Z_2 + Z_3 + Z_4) = I_s (Z_1 + Z_2 + Z_3) - I_x Z_1$$

$$I_4 \; \frac{I_s (Z_1 + Z_2 + Z_3) - I_x Z_1}{Z_1 + Z_2 + Z_3 + Z_4} \qquad \text{(B)}$$

Substituting from above (B) = (A)

$$\Delta Ik + jI_s wM + (Z_3 + Z_4) \left[\frac{I_s (Z_1 + Z_2 + Z_3) - I_x Z_1}{Z_1 + Z_2 + Z_3 + Z_4} \right] = I_s Z_3$$

Let:

$$Z_3 + Z_4 = Zb = a$$

$$Z_1 + Z_2 + Z_3 = b$$

$$Z_1 + Z_2 + Z_3 + Z_4 = Z_1 + Z_2 + Z_b = c$$

$$\Delta Ik + jI_s wM + \frac{a}{c}(I_s b - I_x Z_1) = I_s Z_3$$

From (1): $\Delta I = I_s - I_x$

Substituting:

$$I_s k - I_x k + jI_s wM + I_s \frac{ab}{c} - I_x Z_1 \frac{a}{c} = I_s Z_3$$

$$I_s\left(k + jwM + \frac{ab}{c} - Z_3\right) = I_x\left(k + \frac{a}{c} Z_1\right)$$

$$\frac{I_s}{I_x} \quad \frac{k + \frac{a}{c} Z_1}{k + jwM - \frac{ab}{c} - Z_3} \quad \frac{ck + aZ_1}{ck + jwMc - ab - Z_3 c} \qquad \text{(C)}$$

The ratio of a transformer is expressed in terms of the ratio of the primary current to the secondary current. Using the primary current as a reference, The ratio of the standard transformer is:

$$R_s = \frac{I_p}{I_s \angle \alpha_s}$$

Similiarly the ratio of the unknown transformer is:

$$R_x = \frac{I_p}{I_x \angle \alpha_x}$$

Then since the primary current is each transformer is the same:

$$\frac{R_x}{I_x \angle \alpha_x} = \frac{R_s}{I_s \angle \alpha_s}$$

$$\frac{R_s}{R_x} = \frac{I_s \angle \alpha_s}{I_x \angle \alpha_x} = \frac{I_s}{I_x} \angle \alpha_s - \alpha_x$$

$$\frac{R_s}{R_x} \operatorname{Cos}(\alpha_x - \alpha_s) + j \operatorname{Sin}(\alpha_x - \alpha_s) = \frac{I_s}{I_x} \qquad \text{(D)}$$

Then Evaluating the constants in (C) in complex form:

$$ck + a_1 Z_1$$

Real terms:

$$R(r_1 + r_2 + r_b) - wL_1 (X_1 + X_2 + X_b) + r_1 r_b + X_1 X_b$$

Imaginary terms:

$$jR(X_1 + X_2 + X_b) + jwL_1 (r_1 + r_2 + r_b) + jX_b r_1 + jr_b X_1$$

Let the above real terms be equal to n, and the imaginary terms equal to j0.

Evaluating this factor:

$$ck + jwMc - ab - Z_3 c$$

Real terms:

$$R(r_1 + r_2 + r_b) - wL_1 (X_1 + X_2 + X_b)$$

$$-wM(X_1 + X_2 + X_b) + r_1 (r_3 + r_b) - X_1 (X_3 + X_b)$$

$$+ r_2 (r_3 + r_b) - X_2 (X_3 + X_b) + 2r_3 r_b - 2X_3 X_b$$

Imaginary terms:

$$jR(X_1 + X_2 + X_b) + jwL_1 (r_1 + r_2 + r_b)$$

$$jwM(r_1 + r_2 + r_b) + jr_1 (X_3 + X_b) + jr_2 (X_3 + X_b)$$

$$+jX_1(r_3+r_b)+jX_2(r_3+r_b)+2jX_b r_3+2jX_3 r_b$$

Again let the real terms be equal to P, and the imaginary terms equal to jQ:

Then from (C) above:

$$\frac{I_s}{I_x}=\frac{n+jQ}{P+jQ}=\left(\frac{n^2+o^2}{P^2\quad Q^2}\right)^{\frac{1}{2}}\underline{|\beta}$$

Substituting in (D):

$$\frac{R_s}{R_x}\left[\operatorname{Cos}(\alpha_x-\alpha_s)+j\operatorname{Sin}(\alpha_x-\alpha_s)\right]=$$

$$\left(\frac{n^2+o^2}{P^2+Q^2}\right)\left(\operatorname{Cos}\beta+j\operatorname{Sin}\beta\right)\qquad\text{(E)}$$

Then the above expression is an identity such as:

$$a\underline{|\alpha}=b\underline{|\beta}$$

then $a=b$

and $\alpha=\beta$

Then from (E) above:

$$\left(\frac{R_s}{R_x}\right)^2\operatorname{Cos}^2(\alpha_x-\alpha_s)=\left(\left(\frac{n^2+o^2}{P^2+Q^2}\right)^{\frac{1}{2}}\right)^2\operatorname{Cos}^2\beta$$

$$\left(\frac{R_s}{R_x}\right)^2\operatorname{Sin}^2(\alpha_x-\alpha_s)=\left(\left(\frac{n^2+o^2}{P^2+Q^2}\right)^{\frac{1}{2}}\right)^2\operatorname{Sin}^2\beta$$

Solving the above equations simultaneously:

$$\frac{R_s}{R_x}=\left(\frac{n^2+o^2}{P^2+Q^2}\right)^{\frac{1}{2}}\qquad\text{(F)}$$

This equation gives the difference in ratio of the standard and test transformers. The difference in phase angle of the transformers, $(\alpha_x - \alpha_s)$, may be found by rewriting equation (E) in the following form:

$$\frac{R_s}{R_x} \angle \alpha_x - \alpha_s = \left(\frac{N^2 + O^2}{P^2 \quad Q^2}\right)^{1/2} \angle \beta \qquad \text{(E)}$$

Therefore:

$$\alpha_x - \alpha_s = \beta$$

and:

$$\beta = \mathrm{Tan}^{-1}\frac{O}{N} - \mathrm{Tan}^{-1}\frac{Q}{P} \qquad \text{(G)}$$

From the equations above (F and G) the difference in ratio and phase angle of the current transformer used as a standard and the current transformer being tested can be found, provided that the constants of the test circuit are known.

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APPROVAL.

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prequisite to the degree for which it has been submitted. It is to be understood that by this approval, the undersigned does not necessarily endorse or approve any statement made, but approves the thesis only for the purpose for which it is submitted.

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