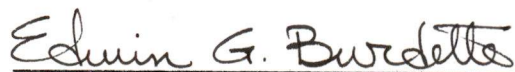


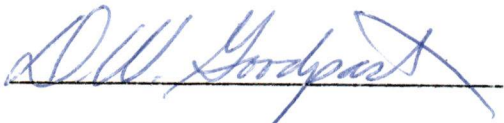
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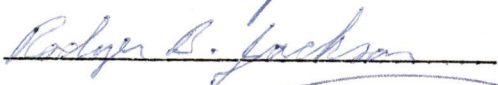
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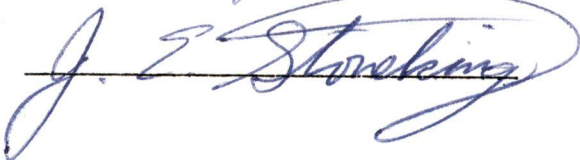


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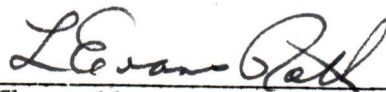
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SEISMIC DESIGN OF REINFORCED CONCRETE BUILDINGS:
AN INELASTIC RESPONSE SPECTRUM APPROACH

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Prospero Dionisio Bernal

August 1979

1308028

- A los que han caído luchando por la Patria, la Justicia y la Verdad.
- A los que dedican todos sus esfuerzos en la batalla contra el Hambre y la Ignorancia.
- A los compañeros del Frente Sandinista de Liberación Nacional, que en este Julio de 1979, con su victoria han llenado de esperanzas y optimismo el corazón de los que luchan.
- A todos ellos está dedicado mi esfuerzo.

El Autor

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I also wish to express my appreciation to my wife Evelyn; her understanding and support were and are immeasurably valuable.

ABSTRACT

The only rigorous technique for evaluating the response of an inelastic, multidegree of freedom (MDOF) structure to an arbitrarily defined ground motion is a step-by-step integration of the equations of motion. This procedure, while theoretically feasible, is extremely costly and impractical for normal design. The Inelastic Response Spectrum Approach (IRSA), consisting of the superposition of modal responses obtained from inelastic earthquake spectra, is not a rigorous analytical technique because it relies on the principle of superposition which does not apply past the elastic limit. However, the simplicity of the IRSA and its ability to explicitly account for all the important variables have made it a promising procedure in the search for an improved version of the presently applicable earthquake design approaches.

The primary purpose of this investigation was to test the applicability of the IRSA as a tool for the design of moment-resisting, reinforced concrete frames under seismic excitation. The applicability of this method can be measured by the accuracy (or lack of it) of the predicted inelastic demands when compared to those obtained from a rigorous, step-by-step, inelastic dynamic analysis. In order to test the applicability of the method, two frames were designed for various combinations of ground motions and design ductilities and then analyzed using DRAIN-2D, a computer program with inelastic dynamic analysis capabilities.

Another objective of this dissertation was to develop design procedures to insure that the inelastic action in the frames designed according to the IRSA would be restricted to the beams.

From the comparisons between predicted inelastic demands from the IRSA and those obtained from step-by-step inelastic dynamic analyses, it was found that the story drift ductility (defined as the ratio between the maximum story drift and the yield value) is in good agreement with the design ductility, μ , used to develop the inelastic response spectrum. It was also shown that the plastic hinge rotations in the beams can be calculated with sufficient accuracy from the structure's yield drifts (which depend on the elastic properties) and the design ductility, μ . Although not enough cases were considered to permit a definite conclusion, the results obtained indicate that, as one may expect, the accuracy of the inelastic demands predicted using the IRSA decreases as the design ductility increases.

In order to insure a strong column-weak beam behavior (i.e., to restrict the formation of plastic hinges to the beams) a special column design procedure was developed (the term, design procedure, is used here to refer to the determination of the critical column actions). This column design procedure was found to be efficient in preventing any significant yielding in the columns of the reinforced concrete moment resisting frames considered in this investigation.

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NOTATION

a	=	depth of equivalent rectangular stress block
a_o, a_1	=	proportionality factors
A_g	=	gross area of the cross section
A_s	=	area of steel
b	=	width of a section
c	=	damping coefficient
c	=	depth to the neutral axis
c_c	=	critical damping coefficient
c_n^*	=	modal damping coefficient in the n^{th} mode
$[c]$	=	damping matrix
C	=	seismic design coefficient in the SEAOC code
d	=	effective depth of member
D	=	dead load
e	=	eccentricity
E	=	earthquake
E_c	=	concrete modulus of elasticity
f	=	natural frequency
f	=	inertial loads
f	=	flexibility
f'_c	=	concrete compressive strength (6in x 12in cylinder)
f_y	=	yield strength of the steel
$\{f_n\}$	=	vector of inertial loads in the n^{th} mode
F_i	=	lateral force applied to level i ; (SEAOC code)
F_t	=	that portion of V considered concentrated at the top of the structure; (SEAOC code)

$F_{(t)}$	= dynamic load
F_x	= lateral force applied to level x ; (SEAOC code)
g	= acceleration of gravity
h	= story height
h_i, h_x	= height to level i or x ; (SEAOC code)
h_j	= height of story j (measured from ϕ to ϕ of beams)
h_n	= total height of the structure, measured from the base; (SEAOC code)
i	= integer
I	= importance factor; (SEAOC code)
I	= moment of inertia
I_c	= moment of inertia of a column
j	= integer
j	= moment arm coefficient
k	= spring constant, elastic slope of the resistance curve
k	= effective length factor for a column
kd	= depth to the neutral axis at yield
k_{ij}	= stiffness influence coefficient
k_n^*	= modal stiffness coefficient in the n^{th} mode
$[k]$	= stiffness matrix
K	= coefficient dependent on the type of construction; (SEAOC code)
ℓ_p	= equivalent plastic hinge length
ℓ_u	= unsupported length of a column
L	= beam span
L	= live load
L_r	= reduced live load
m	= integer
m	= mass

m_n^*	= modal mass coefficient in the n^{th} mode
$[m]$	= mass matrix
M	= moment
$M_{c(\text{SRSS})}$	= SRSS column moment
M_{d+1}	= moment induced by the dead plus live loading
M_{ec}	= maximum moment induced by the earthquake
M_{eb}	= earthquake moment in the beam; SRSS value
M_n	= nominal moment strength at a section
$M_{p-\Delta}$	= moments induced by P- Δ effect
M_u	= design moment
M_u^+	= positive design moment (sagging moment)
M_u^-	= negative design moment (hogging moment)
n	= integer
n	= modular ratio
N	= number of stories in a building; (SEAOC code)
N	= number of degrees of freedom in a MDOF system
P	= axial load
$P_{c(\text{SRSS})}$	= SRSS column axial load
P_{d+1}	= axial load from dead plus live loading
P_{ec}	= maximum earthquake induced axial load
P_j	= total gravity load above level j
$P_{p-\Delta}$	= axial load from P- Δ effect
P_u	= design axial load
q	= proportionality coefficient used in moment distribution for lateral loads
$\{q\}$	= vector of proportionality coefficients
r	= radius of gyration

$\{r\}$	=	vector of displacements due to a unit displacement of the ground
R	=	structural resistance; equal to k_u in the elastic range
R_y	=	yield resistance
S	=	soil factor; (SEAOC code)
S	=	lateral load
S_a	=	spectral acceleration
$S_{a,n}$	=	spectral acceleration for the n^{th} normal mode
t	=	height of a section
t_b	=	height of a beam cross-section
T	=	period of vibration
T_n	=	period of n th normal mode
u	=	relative displacement
u_m	=	maximum relative displacement
u_m^+	=	maximum positive relative displacement
u_m^-	=	maximum negative relative displacement
u_y	=	relative displacement at first yield
$\{u\}$	=	vector of relative displacements
u_n	=	relative displacement in the n^{th} mode
U	=	required strength
V	=	base shear; (SEAOC code)
w	=	weight
w_{d+1}	=	uniformly distributed gravity loading
w_i, w_x	=	that portion of W which is located at or is assigned to level i or x respectively; (SEAOC code)
W	=	total building weight; (SEAOC code)
x	=	ground displacement
x_m	=	maximum ground displacement

y	= total displacement
Y_n	= generalized displacement for n^{th} normal mode
Y	= vector of generalized coordinates
z	= distance from the critical section to the point of contraflexure
α	= load factor
β_1	= a factor which relates the neutral axis depth to the depth of the equivalent rectangular stress block
δ	= drift
δ_e	= SRSS drift
δ_n	= drift in the n^{th} mode
δ_y	= yield drifts
δ_1	= first order drift from P- Δ effect
Δ_u	= total drifts
ΔM_b	= change in the bottom column moment as the structure sways from δ_y to Δ_u
ΔM_t	= change in the top column moment as the structure sways from δ_y to Δ_u
$\Delta S_{p-\Delta}$	= increase in story shears due to the P- Δ effect
$\Delta\theta$	= rotation of the column tangent with respect to the chord as the structure sways from δ_y to Δ_u
ϵ_u	= ultimate strain in the concrete
λ	= stability coefficient
θ_{cu}	= slope between the column and its chord at maximum drift
θ_{cy}	= slope between the column and its chord at yield drift
θ_p	= plastic hinge rotation
θ_{pa}	= available plastic hinge rotation capacity
θ_{pp}	= predicted plastic hinge rotation
θ_t	= total beam rotation at maximum drift
μ	= design ductility

μ_s	=	story drift ductility
ξ	=	critical damping ratio
ξ_n	=	critical damping ratio in the n^{th} mode
ρ	=	percentage of steel
ρ_b	=	percentage of steel corresponding to balanced conditions
ρ_s	=	ratio of the volume of confining steel to the volume of the concrete core
ϕ	=	capacity reduction factor
ϕ_n	=	n^{th} mode shape
ϕ_{nx}	=	modal displacement (n^{th} mode, x position)
ϕ_n^T	=	transpose of the n^{th} mode shape vector
ϕ_u	=	ultimate curvature
ϕ_y	=	yield curvature
$[\Phi]$	=	mode shape matrix
Γ_n	=	participation factor for the n^{th} normal mode
ω	=	undamped natural circular frequency
ω	=	a parameter used in the calculation of M_n ; given by $\rho \frac{f_y}{f'_c}$
ω_n	=	undamped circular frequency of the n^{th} normal mode

CHAPTER I

INTRODUCTION

At the present time, most structures in seismic regions are designed for seismic effects using code-specified equivalent forces. These forces are intended to account for the stresses induced by the inertial forces generated in the event of an earthquake. The code-specified equivalent forces are, however, substantially smaller than those which would be generated if the structure were to remain elastic when subjected to an earthquake of even moderate intensity; thus, inelastic action is expected. Indeed, it is generally agreed that inelastic behavior is inevitable under the economic constraints of practical building design and that only high-performance structures, such as nuclear power plants, can be justifiably designed for the high forces required to ensure elastic behavior.

The consideration of post-elastic behavior makes seismic design basically different from that for other normally occurring loads such as dead load, live load and wind. The present seismic design codes deal with the structure's post-elastic behavior in an indirect manner through the use of various coefficients (1), but inelastic demands are never explicitly calculated in the design procedure.

The most widely used treatise in the field, the "Recommended Lateral Force Requirements and Commentary" (1), was developed and published by the Structural Engineers Association of California in 1959. Though this

document, the SEAOC code, has been updated almost annually in recent years, its essential basis is the state-of-the-art and technology of earthquake resistant design which was prevalent in the pre-1959 period. Much more is known today than what is presently reflected in the SEAOC code, or for that matter in any code. The Olive View Hospital failure during the San Fernando earthquake in 1971 and the collapse of the Charaima building during the Venezuelan earthquake of 1967 are only two of a long list of failures which were, not only predictable, but preventable under present available knowledge. Unfortunately, it is codes rather than available knowledge which seem to determine building design approaches and, in turn, properties of structures. The need for a dynamic, inelastic design code which allows greater utilization of available knowledge is generally recognized. The Inelastic Response Spectrum Approach (IRSA) combines the necessary simplicity required for an eventual incorporation into a code approach with the advantage that the inelastic demands are explicitly calculated, and therefore can be adjusted, during the design procedure.

I. DYNAMIC ANALYSIS

The response of a structure to a seismic excitation is a problem of structural dynamics. The dynamic analysis of linear structures subjected to ground motion excitation is well documented in the literature (2). One of the most commonly used methods of computing the dynamic response of a structure is the Modal Superposition Method. This method recognizes that, for an N degree of freedom elastic system vibrating within the

elastic range, there are N characteristic modes of vibration, and that the total response can be represented by the sum of the individual responses from each of the characteristic modes. The modal superposition time history method is time consuming and costly because the complete time history of every response parameter must be obtained in order to select the maximum values for design. As an alternative, the Response Spectrum Method can be used to solve the equations of motion; it provides an estimate of the maximum response of the structure and does not require the computation of the complete time history of the response.

II. RESPONSE SPECTRUM: ELASTIC AND INELASTIC

When a structure is subjected to ground motion in an earthquake, it responds in a vibratory fashion, the maximum response being dependent on the structure's natural period of vibration and the magnitude of the damping present. A response spectrum is simply a plot of the maximum responses attained by single-degree of freedom oscillators of different natural periods when subjected to the same ground motion. Figure 1* shows a typical response spectrum for various damping ratios.

Inelastic Spectrum

Figure 2 shows a single degree of freedom model responding inelastically to an arbitrarily specified ground motion. The ductility factor, μ , is given by the largest of:

*All figures are in Appendix B.

$$\mu = \frac{u_m^+}{u_y}$$

$$\mu = \frac{u_m^-}{u_y}$$

(1.1)

$$\mu = \frac{(u_m^+ + u_m^- - u_y)}{u_y} \quad (\text{To be explained in Chapter III.})$$

If the resistance, R_y , is selected (necessarily by trial and error) such that the computed ductility is equal to some pre-selected value and a plot is made, for example, of the maximum absolute acceleration for all the possible periods, the inelastic response spectrum curve for absolute acceleration for the ductility factor selected is generated. Figure 3 shows a qualitative acceleration spectrum for various ductility factors.

III. INELASTIC RESPONSE SPECTRUM AND MULTIDEGREE OF FREEDOM SYSTEMS

If the structure to be designed is a single-degree-of-freedom (SDOF) system, the use of an inelastic response spectrum such as the one in Figure 3 is directly applicable. For multidegree-of-freedom (MDOF) systems the use of a response spectrum approach relies on modal analysis. Modal analysis, while valid for linear systems, is not rigorously valid for inelastic structures because it is based on the principle of superposition which does not hold past the elastic limit (3). The only rigorously valid technique to evaluate the dynamic response of a nonlinear structure is a step-by-step integration of the equations of

motion, a procedure which is extremely costly and impractical for normal design. Thus, one is faced with the following situation: (1) Inelastic behavior is a necessity under typical economic design constraints, and (2) Inelastic response spectra are not rigorously applicable to multi-degree of freedom structures. Realizing that the whole subject is so complex that complete rigor is not possible (or for that matter necessary), one logically asks the following questions: How large are the errors introduced by using modal analysis in combination with an Inelastic Response Spectrum? How accurate are the predicted inelastic demands from an IRSA when compared to those obtained from a rigorous step-by-step integration? Can we devise methodology which, when used in conjunction with the IRSA, will insure a conservative but reasonable design?

IV. PURPOSE

The purposes of this dissertation are manifold. First, an investigation will be made of the applicability of an Inelastic Response Spectrum Approach as a tool for the design of structures under seismic excitation. The applicability of this method can be measured by the accuracy (or lack of it) of the predicted inelastic demands when compared to those obtained from a rigorous step-by-step, inelastic dynamic analysis. The second objective, which is very much interrelated with the first, is to develop design methodology to try to make the IRSA as useful as possible, and to insure that some basic design criteria essential in reinforced concrete seismic design is met. Finally, it is

expected that this research, which was carried out with the intention of remaining always practical, will contribute to the overall effort directed towards improving the present earthquake resistant design regulations.

V. APPROACH

In order to evaluate the applicability of the IRSA, a method to obtain rigorous, "exact" answers which can be used as reference values is needed. DRAIN-2D (4) is a computer program developed by Amin E. Kanaan and Graham H. Powell at the University of California at Berkeley which has the capability of performing step-by-step dynamic inelastic analysis of plane structures. This program is used in this research to obtain the "right answers" to which those from the IRSA will be compared.

Two structures are used in this dissertation to investigate the IRSA. The first structure, labeled in this work as FRAME 1, is a six story, single bay, reinforced concrete frame built in New Zealand, the data from which were obtained from Reference 5, and is described here in Figure 4. The second structure, FRAME 2, is a ten story, three bay, reinforced concrete frame which initially appeared in Reference 6, and is shown here in Figure 5.

VI. SCOPE

Even though some portions of this research are equally applicable to steel or reinforced concrete structures, this dissertation only deals with the application of the IRSA to reinforced concrete structures and,

in particular, to reinforced concrete, moment-resisting frames. No shearwalls or frame-shearwall combinations are included in this study.

Design Earthquake

The selection of the design earthquake is presently the subject of a number of research projects all over the world (7,8). The selection of a design earthquake is one of the steps with the highest degree of uncertainty in the overall procedure of earthquake resistant design. The ground motion experienced at a site is a function of, among other things, the type and characteristics of the source mechanism, the nature of the intervening geological structure, and the topographical and soil conditions near the site. Since this dissertation is intended to test a design procedure, a deterministic approach was taken. In other words, this dissertation begins after a design ground motion has been selected. Two different ground motions are used in the research presented here. The first ground motion, labeled GM1, is a short event obtained by taking the two seconds of strongest shaking in the El Centro Earthquake of December 30, 1934. Ground motion 2 (GM2), corresponds to the first six seconds of the El Centro Earthquake of May 1940. Tables 1* and 2 present the digitized data for both ground motions.

*All tables are in Appendix A.

CHAPTER II

CURRENT PRACTICE IN THE DESIGN FOR EARTHQUAKE MOTIONS

This chapter is not intended to be a report on "the state of the art" on the design for earthquake motions. Neither is it an attempt to provide an exhaustive list of all the methodologies or approaches that are currently used when designing earthquake resistant structures. Instead, the intention of this chapter is to describe, in general terms, the two broad categories in which any type of earthquake analysis falls: (a) a "code type" approach where the effect of the earthquake induced vibrations is simulated by "equivalent lateral forces" or (b) an approach which explicitly considers the dynamic characteristics of the problem. Section I of this chapter is dedicated to a brief presentation of the "Recommended Lateral Force Requirements" prepared by the Structural Engineers Association of California, because these recommendations are included in the Uniform Building Code (UBC) and are widely used in the United States and some other countries as well. Section II presents the mathematical formulation of the dynamic approach used in predicting the elastic response of structures to earthquake motions; and finally Section III is dedicated to a brief discussion of some approximate methods of nonlinear analysis.

I. RECOMMENDED LATERAL FORCE REQUIREMENTS (1975) (SEAOC CODE)

The SEAOC recommendations are intended to provide minimum standards to assure public safety. The design provisions in the SEAOC Code are

given in terms of a set of equivalent lateral forces which simulate the inertial loads generated by the earthquake-induced vibrations. It must be emphasized that the lateral forces prescribed by the SEAOC recommendations are not implied as the real forces in the event of an earthquake. The actual inertial forces would be several times higher if the structure were to remain elastic. The reasons for allowing lower values for design are several, the most important being the capacity of the structure to absorb energy through inelastic deformations.

Minimum Earthquake Forces

The minimum lateral seismic force for which a structure has to be designed according to the SEAOC Code, is given in terms of the design base shear, V , as follows:

$$V = ZIKCSW \quad (2.1)$$

where the terms of this expression are defined as follows:

Z is a seismic zoning factor, the value of which depends on the location of the structure. The value $Z=1$ corresponds to the areas of highest seismicity.

I is an importance factor taken as 1.5 for essential facilities. For all other structures I is taken as 1.0.

K is a numerical coefficient reflecting the influence of the type of construction on the damping, ductility and/or energy-absorption capacity of the structure. The numerical values of K for the various types of construction are given in Table 1-A in Reference 1.

C is a seismic response factor taken as:

$$C = \frac{1}{15\sqrt{T}} \quad (2.2)$$

where T is the natural period of vibration of the structure. The value of C need not exceed 0.12, or the product CS need not exceed 0.14. The period T can be established using the mass and stiffness properties of the structure or using the following formula

$$T = \frac{0.05 h_n}{D} \quad (2.3)$$

where h_n is the height of the building and D is the dimension in the direction being investigated, both in feet.

For buildings consisting of moment resisting space frames which resist 100% of the lateral load, T may be taken as:

$$T = 0.1 N \quad (2.4)$$

where N is the number of stories above the base.

S is a foundation factor accounting for the influence of the soil conditions. The value of S ranges from 1 to 1.5.

W is the dead load of the building plus 25% of the design live load.

Distribution of Lateral Seismic Force

The total lateral force V is to be distributed over the height of the structure according to the following formula:

$$V = F_t + \sum_{i=1}^n F_i \quad (2.5)$$

where F_t is given by

$$F_t = 0.07 TV \quad (2.6)$$

but F_t need not exceed $0.25 V$ and may be taken as zero if T is less than 0.7 seconds. The force F_t is applied at the top of the structure. The remaining portion of the total base shear V is distributed over the height of the structure in accordance with the formula

$$F_x = \frac{(V - F_t) w_x h_x}{\sum_{i=1}^n w_i h_i} \quad (2.7)$$

where F_x is the force at the level being considered, w_x is the total weight at level x and h_x is the height of level x above the base.

Figure 6 shows a typical distribution of equivalent lateral forces and story shears in a building with uniform mass distribution.

II. DYNAMIC APPROACH

A dynamic approach is an analysis which provides a rational basis for establishing equivalent static forces to simulate the conditions and stresses that will occur under complex earthquake ground motion.

Traditionally the dynamic approach had been limited to "unusual or complex structural configurations" for which the equivalent lateral forces prescribed by the SEAOC Code are not applicable (1). In recent years due to the increasing availability of computers, elastic dynamic analysis of important structures, whether irregular or not, are being

carried out more and more often. The mathematical formulation of the dynamic approach is presented in the following.

Equation of Motion

The analysis of structural response to earthquake motions requires the solution of the following equations:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = -[m]\{r\}\ddot{x} \quad (2.8)$$

where $[m]$, $[c]$, and $[k]$ are the mass, damping and stiffness matrices respectively. The vectors $\{\ddot{u}\}$, $\{\dot{u}\}$ and $\{u\}$ represent the relative accelerations, velocities and displacements, respectively, between the selected degrees of freedom and the ground. $\{r\}$ is the vector of displacements resulting from a unit displacement of the ground and $\ddot{x}$ is the time history of ground acceleration describing the earthquake. The number of equations in (2.8) is equal to the number of degrees of freedom considered in the analysis, which for regular multistory buildings is one per story (2).

Transformation to Normal Coordinates

Equation (2.8) describes the displaced position of an arbitrary multidegree of freedom system though N^* components of the displacement vector $\{u\}$. However, for dynamic analysis of linear systems, a much more useful representation of the displacements is provided by the free vibration mode shapes. These shapes constitute N independent displacement

*Where N is the number of degrees of freedom.

patterns, the amplitudes of which may serve as generalized coordinates to express any form of displacement (2). Consider, for example, the cantilever beam shown in Figure 7, for which the deflection can be described by three translational coordinates u_1 , u_2 and u_3 . Let it be assumed that the free vibration mode shapes had been obtained. It can be seen from the figure that any deflected shape can be expressed as a linear combination of the free vibration mode shapes; mathematically:

$$\{u\} = \phi_1 Y_1 + \phi_2 Y_2 + \phi_3 Y_3 \quad (2.9)$$

where ϕ_1 , ϕ_2 and ϕ_3 are the free vibration mode shapes and Y_1 , Y_2 and Y_3 are the modal amplitudes.

It is apparent that in general for any N-DOF system

$$\{u\} = [\phi]Y \quad (2.10)$$

where $[\phi]$ is the matrix of all the mode shapes and Y is the vector of modal amplitudes.

It can be seen from Equation (2.10) that the mode shape matrix $[\phi]$ serves to transform from the generalized coordinates Y to the geometric coordinates $\{u\}$.

The mode shapes are obtained by solving the following equation (2)

$$[k]\phi_n - [m]\omega_n^2 \phi_n = 0 \quad (2.11)$$

where ω_n is the natural circular frequency of the n^{th} mode. The mode

shapes obtained by solving Equation (2.11) are orthogonal with respect to the mass and the stiffness matrices (2); these properties can be stated mathematically as follows:

$$\phi_n^T[m]\phi_m = 0 \quad m \neq n \quad (2.12)$$

and

$$\phi_n^T[k]\phi_m = 0 \quad m \neq n \quad (2.13)$$

For $m = n$

$$\phi_n^T[m]\phi_n = m_n^* \quad (2.14)$$

and

$$\phi_n^T[k]\phi_n = k_n^* \quad (2.15)$$

where m_n^* and k_n^* are scalar quantities which are called the modal mass and stiffness for the n^{th} mode, respectively (2). If the mode shapes are to be orthogonal with respect to the damping matrix, it can be shown (2) that this matrix must be expressed as a linear combination of the mass and the stiffness matrices. This condition is stated in Equation (2.16)

$$[c] = a_0[m] + a_1[k] \quad (2.16)$$

in which a_0 and a_1 are arbitrary proportionality factors. If the damping matrix is of the form given by Equation (2.16) then:

$$\phi_n^T [c] \phi_m = 0 \quad m \neq n \quad (2.17)$$

For $m = n$

$$\phi_n^T [c] \phi_n = c_n^* \quad (2.18)$$

where c_n^* is the modal damping coefficient for the n^{th} mode.

If one now substitutes Equation (2.10) (and its derivatives) into (2.8) the result is:

$$[m][\phi]\ddot{Y} + [c][\phi]\dot{Y} + [k][\phi]Y = -[m]\{r\}\ddot{x} \quad (2.19)$$

premultiplying Equation (2.19) by ϕ_n^T gives

$$\phi_n^T [m][\phi]\ddot{Y} + \phi_n^T [c][\phi]\dot{Y} + \phi_n^T [k][\phi]Y = -\phi_n^T [m]\{r\}\ddot{x} \quad (2.20)$$

However, from the orthogonality conditions expressed in Equations (2.12) through (2.18), the matrix Equation (2.20) becomes

$$m_n^* \ddot{Y}_n + c_n^* \dot{Y}_n + k_n^* Y_n = -\phi_n^T [m]\{r\}\ddot{x} \quad (2.21)$$

which is the SDOF equation for the n^{th} mode. Dividing Equation (2.21) by m_n^* one gets

$$\ddot{Y}_n + \frac{c_n^*}{m_n^*} \dot{Y}_n + \frac{k_n^*}{m_n^*} Y_n = - \frac{\phi_n^T[m]\{r\}}{m_n^*} \ddot{x} \quad (2.22)$$

the term $\frac{\phi_n^T[m]\{r\}}{m_n^*}$ which can also be expressed as $\frac{\phi_n^T[m]\{r\}}{m_n^*}$ is usually referred to as the earthquake participation factor for the n^{th} mode and it's usually designated as Γ_n . Equation (2.22) can then be written as

$$\ddot{Y}_n + \frac{c_n^*}{m_n^*} \dot{Y}_n + \frac{k_n^*}{m_n^*} Y_n = -\Gamma_n \ddot{x} \quad (2.23)$$

It is convenient to express Equation (2.23) in a slightly different format. Premultiplying Equation (2.11) by ϕ_n^T one obtains

$$\phi_n^T[k]\phi_n = \phi_n^T[m]\omega_n^2\phi_n \quad (2.24)$$

or

$$k_n^* = m_n^*\omega_n^2$$

from which

$$\omega_n^2 = \frac{k_n^*}{m_n^*} \quad (2.25)$$

The following will not be demonstrated here but it can be shown that (2)

$$\frac{c_n^*}{m_n^*} = 2\xi_n\omega_n \quad (2.26)$$

where ξ_n is the damping ratio in the n^{th} mode.

Substituting Equations (2.25) and (2.26) into (2.23) one gets

$$\ddot{Y}_n + 2\xi_n\omega_n\dot{Y}_n + \omega_n^2 Y_n = -\Gamma_n \ddot{x} \quad (2.27)$$

The solution of Equation (2.27), which is a SDOF equation, yields the time history of the modal amplitude in the n^{th} mode for an arbitrarily specified ground motion $\ddot{x}$. The final solution for the relative displacement, u , can be then calculated from:

$$\{u\} = \sum_{n=1}^N \phi_n Y_n \quad (2.28)$$

The solution in modal coordinates through the use of Equations (2.27) and (2.28) presents two major advantages when compared with the direct integration of the equations in (2.8). The first advantage of using the modal formulation is that the equations to be solved are uncoupled, eliminating the need for any simultaneous equation solution. Second, and at least as important, is the fact that a very good approximation of the exact response can be obtained by only including the first few terms in the series of Equation (2.28). Thus, systems with dozens or even hundreds of degrees of freedom can be analyzed by only considering a small number of normal modes. For regular multistory buildings the first

three modes are sufficient to describe the displacements with acceptable accuracy. The axial loads, shears, and moments induced by the vibrations can be calculated from the relative displacements using standard procedures of structural analysis.

III. APPROXIMATE METHODS OF NONLINEAR ANALYSIS

In the preceding section a rigorous approach for the analysis of elastic systems subjected to arbitrarily defined ground motions was presented. Unfortunately, as was previously stated in Chapter I, the inertial forces generated by earthquakes of even moderate intensity on elastic structures are so large that it is not economically feasible to design for them; instead the designer counts on the energy absorbing capabilities of yielding structures to absorb a portion of the energy input by the earthquake. Since all the methodology developed in the previous section is only valid for elastic systems, and because rigorous, step-by-step integration of the equations of motion is as yet an impractical approach for large structures, some approximate inelastic dynamic analysis approaches have been proposed. One of them is the Inelastic Response Spectrum Approach which is the main subject of this dissertation and which will be discussed in detail in the following chapters. Another is the Ductility-Factor Method which will be briefly presented in the following.

Ductility-Factor Method

The ductility-factor method simply stipulates that the values of the moments shears and axial loads obtained from an elastic analysis such as

that described in Section II can be reduced by a factor, μ , which accounts for the fact that the structure is ductile and will respond inelastically. The way designers arrive at a value for a ductility factor varies from case to case. The value of 4 has been popular because dynamic analyses of structures subjected to the El Centro Earthquake have shown that the elastic forces generated in typical structures are about 4 times as large as the yield level provided by the equivalent lateral forces which the SEAOC Code would prescribe. In some cases the ductility factor, μ , has been varied, depending on the element to which it is being applied. For example, in the design of the Bonaventure Hotel in downtown Los Angeles, a value of 6 was used for the girders and 2 for the columns in an attempt to produce a strong column weak girder design. The structure is 37 stories high (9). It should be pointed out that the ductility factor, μ , refers to a displacement ductility, and even though related, it is not equal to the ratio of the ultimate curvature, ϕ_u , to the yield curvature ϕ_y . This distinction has not always been made when using the ductility-factor method (10).

CHAPTER III

THE INELASTIC RESPONSE SPECTRUM METHOD

The Response Spectrum Method of dynamic analysis consists of the superposition of the modal responses obtained from an earthquake spectra. Since the maximum values of the individual modes do not necessarily occur at the same time nor have the same sense, the exact way in which various modes combine cannot be precisely determined. In this dissertation, the usual procedure of combining the modal contributions using the square root of the sum of the squares (SRSS) was selected.

If the spectrum used in the analysis is an elastic one, the only source of inaccuracy in the Response Spectrum Method lies in the way the modal contributions are combined. However, if the selected spectrum is inelastic, the superposition of modal contributions is no longer theoretically correct. A discussion of the theoretical limitations of the Inelastic Response Spectrum Method is presented in Chapter I.

I. BASIC STEPS IN THE INELASTIC RESPONSE SPECTRUM METHOD

For a given building structure and a selected design earthquake, the basic steps in the Inelastic Response Spectrum Method are as follows:

- (1) Development of the Inelastic Response Spectrum
- (2) Selection of the design ductility
- (3) Calculation of modal shapes and natural frequencies
- (4) Evaluation of modal parameters and SRSS combinations

- (5) Gravity load analysis
- (6) Evaluation of P-Δ effects
- (7) Determination of load factors and combinations of loading cases
- (8) Strength design of the structural elements
- (9) Calculation of the inelastic deformation demands
- (10) Calculation of allowable values for the inelastic demands:
evaluation of the adequacy of the design

The development of Inelastic Response Spectra is covered in some detail in this chapter. Of the remaining steps outlined above, steps (2) through (8) will be discussed in Chapter IV while (9) and (10) are treated in Chapter V. Both Chapters IV and V deal with the specific case of moment-resisting, reinforced concrete frames.

II. INELASTIC RESPONSE SPECTRA

Support Motions

Figure 8(a) shows a single degree of freedom oscillator responding to an arbitrarily specified ground motion. From the figure the equation of motion is:

$$m\ddot{y} + c\dot{u} + R = 0 \quad (3.1)$$

where m = mass of the oscillator, c = damping coefficient, u = relative displacement between the mass and the ground, R = structural resistance, which is a function of the relative displacement, u , and of the previous time history of the response, Figure 8(b), y = total displacement of the

mass, and x = ground displacement. It is important to realize that of the three terms in Equation (3.1), the inertial term, $m\ddot{y}$, is dependent on the total displacement while the damping and the structural resistance terms are only dependent on the relative motion. It can be seen from Figure 8(a) that

$$y = x + u \quad (3.2)$$

Differentiating Equation (3.2) twice, one obtains

$$\ddot{y} = \ddot{x} + \ddot{u} \quad (3.3)$$

Substituting Equation (3.3) into (3.1) and rearranging, one gets:

$$m\ddot{u} + c\dot{u} + R = -m\ddot{x} \quad (3.4)$$

The expression in (3.4) is the equation which governs the behavior of a SDOF system subjected to an arbitrary ground motion acceleration $\ddot{x}$. The equation of motion of a SDOF system subjected to an arbitrary time varying load $F_{(t)}$ is (11)

$$m\ddot{u} + c\dot{u} + R = F_{(t)} \quad (3.5)$$

By comparing Equations (3.4) and (3.5) one can see that the effect of support motion on the relative displacements is the same as that of an equivalent load $F_{e(t)} = -m\ddot{x}$. In earthquake analysis, the minus sign on the right hand side of Equation (3.4) is meaningless and is usually

disregarded. In order to obtain a response spectrum for a particular ground motion $\ddot{x}$ one needs to solve Equation (3.4) repeatedly for as many values of the natural period, T (see Equation (3.10)), as needed to plot an accurate curve of the parameter selected. The resulting response spectrum is a plot of this parameter, typically acceleration, versus T . Of course, the time variation of $\ddot{x}$ in the case of earthquakes is such that one is forced to resort to numerical integration as the only feasible way of obtaining a solution.

Ductility Factor

The definition of ductility factor for single degree of freedom systems used here is given by Equation (1.1) and is illustrated in Figure 2. Equation (1.1) is repeated here as Equation (3.6) for convenience.

$$\text{The largest of } \left\{ \begin{array}{l} \mu = \frac{u_m^+}{u_y} \\ \mu = \frac{u_m^-}{u_y} \\ \mu = \frac{(u_m^+ + u_m^- - u_y)}{u_y} \end{array} \right. \quad (3.6)$$

Of the three terms which must be evaluated to obtain the ductility factor as defined by Equation (3.6), the first two are of standard use and need no further discussion. The third term can be labeled as "cyclic ductility" since it has no meaning unless at least one full cycle of

positive and negative yielding takes place during the response. It is the opinion of this writer that this term should always be included in the development of Inelastic Response Spectra for seismic motions (where the response is always cyclic); otherwise the actual inelastic demand on the structure may be significantly underestimated. The best measure of the inelastic demand imposed on a yielding structure (without considering cumulative effects) is the length of the yield plateau. Equation (3.6) insures that the largest yield plateau is used in computing the ductility factor. The following example illustrates how the inelastic demand on the structure can be underestimated unless the provision for cyclic effects is included.

Figure 9 shows a possible resistance-deformation response of a single degree of freedom elasto-plastic system. The following values are obtained from the figure:

$$u_m^+ = 4 \text{ in.}$$

$$u_m^- = 4 \text{ in.}$$

$$u_y = 1 \text{ in.}$$

The ductility factor without including the provision for cyclic effects is,

$$\mu = \frac{4}{1} = 4$$

However, when cyclic effects are included, the ductility factor is computed to be,

$$\mu = \frac{(4 + 4 - 1)}{1} = 7$$

As can be seen from Figure 9, the latter value accounts for the yield plateau that goes from 2 to -4 inches.

Discussion and Solution of Equation (3.4)

When the ground motion is zero, Equation (3.4) corresponds to a damped, free vibration. That value of the coefficient c which corresponds to the limiting case for periodic motion is called the critical damping coefficient, and has a magnitude which can be written as (12)

$$c_c = 2m\omega \quad (3.7)$$

in which ω is the circular frequency of undamped vibration, given by the equation (12)

$$\omega = \frac{k}{m} \quad (3.8)$$

where k = slope of the elastic portion of the load deflection curve for the structural spring, Figure 8(b).

The natural frequency f and the period T are readily determined from the relations (12)

$$f = \frac{\omega}{2\pi} \quad (3.9)$$

and

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad (3.10)$$

It is convenient to define the proportion of critical damping, ξ , as the ratio between c and its critical value c_c (12).

$$\xi = \frac{c}{c_c} = \frac{c}{2m\omega} = \frac{cT}{4\pi m} \quad (3.11)$$

It was stated earlier in this section that the solution of Equation (3.4) for any specified time history of ground motion acceleration, $\ddot{x}$, can be carried out using numerical integration. In this dissertation, Equation (3.4) was solved using a modified version of a computer program developed in Reference 13.

Development of Inelastic Spectra

The inelastic spectra developed in this dissertation are presented in terms of spectral accelerations, S_a , versus period. The spectral acceleration is that value of the acceleration which when multiplied by the mass of the oscillator, yields the maximum force which must be resisted by the structural spring. Since the maximum force resisted by the spring in the inelastic case is R_y , the spectral acceleration, S_a , is then given by

$$S_a = \frac{R_y}{m} \quad (3.12)$$

It should be noted that Equation (3.12) is only valid if the structural spring reaches its maximum resistance R_y . In the case of elastic behavior Equation (3.12) becomes

$$S_a = \frac{ku_m}{m} = \omega^2 u_m$$

It is important to note that the spectral acceleration, S_a , is attained at the time the structure reaches maximum response (characterized by maximum relative displacement). This can be seen from the following considerations; from Equation (3.1)

$$\ddot{y} = \frac{-(R + c\dot{u})}{m} \quad (3.13)$$

By definition when $u = u_m$, $\dot{u} = 0$ and $R = R_y$; so the expression above becomes (disregarding the minus sign)

$$\ddot{y} = \frac{R_y}{m} \quad (3.14)$$

Comparing Equations (3.12) and (3.14) one can see that the spectral acceleration is equal to the absolute acceleration $\ddot{y}$ at the time of maximum response. In general the maximum value of the absolute acceleration can be higher than the spectral acceleration, but the difference in the inertial forces is resisted by the damping mechanism.

In order to develop the Inelastic Response Spectra for the ground motions used in this dissertation (spectral acceleration vs. period), the following procedure was followed:

- (1) The first value of the period T , is selected.
- (2) The proportion of critical damping, ξ , and desired ductility, μ , are selected. (In this dissertation $\xi = 0.05$ for the first two modes and μ is either 1, 4 or 6.)
- (3) The mass, m , is fixed arbitrarily. ($10 \frac{\text{k-sec}^2}{\text{in}}$ was used herein)
- (4) The damping coefficient, c , is calculated for Equation (3.11) and the slope of the elastic portion of the load-deflection curve, k , is calculated from Equation (3.10).
- (5) An initial assumption on the value of R_y , the yield resistance, is made. The value of the yield displacement, u_y , is then computed as

$$u_y = \frac{R_y}{k} \quad (3.15)$$

- (6) Equation (3.4) is numerically integrated to obtain the time history of the relative displacement, u , for the duration of the earthquake. From this time history, the values of u_m^+ and u_m^- are obtained and the ductility, μ , is calculated from Equation (3.6).
- (7) If the computed ductility is not equal to the desired value selected in (2), the yield resistance, R_y , is modified and the integration is repeated until the ductility displayed is equal (within 1%) to the preselected value.
- (8) The value of the spectral acceleration, S_a , is then obtained by dividing the last selected value of R_y (for which convergence

was satisfactory) by the mass, m (which in this dissertation is fixed at $10 \frac{\text{k-sec}^2}{\text{in}}$), as indicated by Equation (3.12).

The steps just described define one point on the inelastic spectrum for the ductility selected; the complete spectrum is generated by repeating the procedure for different values of the period, T , until enough points are available to plot an accurate curve. Figures 10, 11 and 12 show the Inelastic Response Spectra for ground motions 1, 2, and 2A for ductilities of 4 and 6 and 5% of critical damping. Ground motions 1 and 2 were defined in Chapter I, Section IV. Their digitized data is presented in Tables 1 and 2 respectively. Ground motion 2A (GM2A) is the same as ground motion GM2 except that the accelerations were scaled up to give a peak ground acceleration of 0.5g. The elastic spectrum is also shown in the figures for comparison. Tables 3, 4 and 5 show the digitized data from where Figures 10, 11 and 12 were plotted.

CHAPTER IV

APPLICATION OF THE IRSA TO REINFORCED CONCRETE MOMENT RESISTING FRAMES

I. SELECTION OF THE DESIGN DUCTILITY

The ductility factor μ for a SDOF system is given by Equation (1.1) which in essence defines it as the ratio of the maximum displacement to the yield displacement. For multistory structures, which are MDOF systems, the term ductility factor has been used very loosely, meaning different things in different research publications (14, 15). In general, ductility is measured by the ratio of the ultimate to the yield value of a parameter which is related to structural deformations. The term ductility can, therefore, be used to refer to strain, displacement, curvature or rotation. Nevertheless, when using Inelastic Response Spectra for the analysis of MDOF systems, it is only the displacement ductility for these systems which can be related to the ductility used in developing the spectra. The curvature ductility or the rotational ductility required at plastic hinges are not equal to the μ value from the Response Spectrum. This is true because, once yielding has commenced in a frame, the deformations concentrate at the plastic hinge locations, thus making the rotational and the curvature ductilities higher than the displacement ductility, μ . The relationship between displacement ductility and plastic hinge rotations is discussed in Chapter V.

When using the IRSA, the designer needs to select a value for the design ductility μ in order to start the design process. The following factors need to be considered when selecting a value of μ to use in the design of reinforced concrete moment resisting frames.

Design Earthquake

The design earthquake has an influence on the selection of μ because, even though a certain amount of damage may not be permissible when designing for an earthquake with a recurrence interval of 25 years, the same amount may be acceptable if one is designing for the largest possible event, which may be an earthquake with a recurrence interval of 100 years or more.

Importance of the Structure

The importance of the structure has recently been included in many of the modern earthquake design codes (1, 16) through the use of an importance coefficient which increases the lateral design loads for essential facilities. When using the IRSA, the importance of the structure is taken into account in the selection of the design ductility. In the extreme case of nuclear power plants where the design criteria calls for elastic behavior, the ductility factor is taken as 1.0 which calls for an elastic analysis. For other cases such as hospitals or schools, the designer can adjust the design ductility to obtain a design which meets the performance and safety criteria specified for the particular case.

It is important to realize that the design ductility is simply the ratio of the maximum story drift expected, Δ_u , to the story yield drift, δ_y . The inelastic portion of the story drift is given by:

$$(\Delta_u - \delta_y) = (\mu - 1)\delta_y \quad (4.1)$$

As can be seen from Equation (4.1), the inelastic drift (which is shown in Chapter V to be directly related to the plastic hinge rotation) is directly proportional to the yield drift δ_y . This means that, when designed for the same ductility factor, flexible structures, which have larger values of δ_y , will undergo inelastic distortions that may be significantly higher than those for stiff structures. The previous statement leads to a very important conclusion which may be stated as follows: The ductility factor, μ , is not an absolute measure of the inelastic demands expected in a structure; the rigidity of the structure plays a role which is just as important as the design ductility in determining the severity of the inelastic action expected. For example, for a very flexible structure, a design using $\mu=4$ may imply such large values of maximum drifts that the resulting inelastic demands are unacceptable, while the same value of μ for a rather stiff structure may imply displacements and rotations which are still well within acceptable limits. This basic difference between flexible and stiff structures shows that the design ductility should not be a fixed value for any given system but that it should be a design parameter which can be adjusted during the design process. The erroneous idea that a certain value of ductility factor can be identified with a given damage level

is often used in defining design criteria (3, 16). In fact, as far as this writer knows, there has not been anything published yet that attempts to base design criteria on something more rational than some sort of table of correspondence between structural systems and allowable design ductilities (3, 16). It should be apparent from the previous discussion that this writer does not agree with such an approach and that, instead, he favors an approach in which the ductility factor is adjusted until the inelastic demands are in accordance with allowable values, which in turn are computed from the principles which govern the behavior of the structural elements in question. The material presented in Reference 17, although not in the earthquake area, is very closely related to the discussion presented in this section.

The difference in behavior between a flexible and a stiff design is illustrated in the following example. In this example, the structure shown in Figure 13(a) will first be designed for the minimum possible steel area in the column; this will yield the largest and therefore stiffest cross-section. After this design is complete, the column will be redesigned using the maximum possible steel area, that is, the minimum possible cross-section; the result this time will be a very flexible design. Both times the structure will be designed for GM2A using the Inelastic Response Spectrum in Figure 12 and a ductility of 4. As shown in Figure 13(a) the width of the section is kept constant at 12 in. in both designs.

The system in Figure 13(a) can be idealized as a SDOF system (11) in which

$$k = \frac{3E_c I}{h^3} = \frac{3 \times 3605 \times I}{(120)^3} = 0.006259I$$

where,

$$I = \frac{1}{12} \times 12 \times t^3 \quad (\text{neglecting the effect of cracking})$$

Then,

$$k = 0.006259t^3 \text{ k/in}$$

$$m = \frac{20}{386.4} = 0.05176 \frac{\text{k-sec}^2}{\text{in}} \quad (\text{neglecting the weight of the column itself})$$

Thus, from Equation (3.10),

$$T = 2\pi \sqrt{\frac{0.05176}{0.006259t^3}} = 18t^{-1.5}$$

Design of Stiff Structure $\rho = \rho_{\min}$

Try $t = 28 \text{ in.}$

$d = 25 \text{ in.}$

$$T = 18 \times 28^{-1.5} = 0.12 \text{ sec.}$$

From Figure 12,

$$S_a = 0.54 g$$

Thus, the maximum lateral load is

$$f = 0.54 \times 20 = 10.8^k$$

The design moment is then

$$M_u = 10.8 \times 120 = 1296 \text{ k-in}$$

The resisting moment ϕM_n is given by (18)

$$\phi M_n = \phi f'_c b d^2 \omega (1 - 0.59\omega)$$

where

$$\omega = \rho \frac{f_y}{f'_c} = \frac{200 \times 60}{60,000 \times 4} = 0.05$$

Substituting numerical values one obtains

$$\phi M_n = 0.9 \times 4 \times 12 \times d^2 \times 0.05 (1 - 0.59 \times 0.05) = 2.0963d^2$$

Since

$$\phi M_n = M_u$$

$$2.0963d^2 = 1296$$

or

$$d = 24.86 \text{ in} \approx 25 \text{ in assumed O.K.}$$

yield displacement

$$\delta_y = \frac{f h^3}{3 E_c I} = \frac{10.8 \times (120)^3}{3 \times 3605 \times 21952} = 0.0786 \text{ in}$$

The plastic hinge rotation is, as illustrated in Figure 13(b)

$$\theta_p = (\mu - 1) \frac{\delta_y}{h} = \frac{3 \times 0.0786}{120} = 0.001965 \text{ rad.}$$

Considered next is a design using $\rho = \rho_{\max}$ which, for earthquake design, is $0.5 \rho_b$.

Design of Flexible Structure $\rho = \rho_{\max}$

$$\rho_b = 0.0285 \quad (\text{from Table 1 of Reference 19})$$

so

$$\rho_{\max} = 0.5 \times 0.0285 = 0.01425$$

Try $t = 15 \text{ in.}$

$d = 12 \text{ in.}$

$$T = 18 \times 15^{-1.5} = 0.31 \text{ sec.}$$

From Figure 12

$$S_a = 0.45 g$$

Thus, the maximum lateral force is

$$f = 0.45 \times 20 = 9.0^k$$

The design moment is then

$$M_u = 9.0 \times 120 = 1080 \text{ k-in}$$

$$\omega = 0.01425 \times \frac{60}{4} = 0.21375$$

The resisting moment is

$$\phi M_n = 0.9 \times 4 \times 12 \times d^2 \times 0.21375 (1 - 0.59 \times 0.21375) = 8.07d^2$$

from which

$$8.07d^2 = 1080$$

or

$$d = 11.6 \text{ in.} \approx 12 \text{ in. assumed O.K.}$$

yield displacement

$$\delta_y = \frac{9.0 \times (120)^3}{3 \times 3605 \times 3375} = 0.4261 \text{ in}$$

The plastic hinge rotation is

$$\theta_p = \frac{3 \times 0.4261}{120} = 0.010652 \text{ rad.}$$

As can be seen from the previous calculations, the plastic hinge rotation required at the base of the column for the flexible design, 0.010652 rad, is 5.4 times the value required in the case of the stiff design, even though in both cases a design ductility of 4 was used.

II. CALCULATION OF MODAL SHAPES AND NATURAL FREQUENCIES

From a mathematical point of view, the calculation of modal shapes and natural frequencies is a standard eigenvalue problem. The equation from where the mode shapes and natural frequencies are computed was given in Chapter II as Equation (2.11) and is repeated here as Equation (4.2) for convenience.

$$[k]\phi_n - [m]\omega_n^2\phi_n = 0 \quad (4.2)$$

In practice the solution of Equation (4.2) for the mode shapes ϕ_n and natural frequencies ω_n can be carried out using several different analytical techniques ranging from a simple hand solution using an iterative Rayleigh-Ritz approach to a very sophisticated computer subspace iteration solution. The method selected in any particular case will depend basically on the importance of the problem and the facilities available to the designer. Reference 2 has an extensive coverage of the presently available methods used to solve Equation (4.2).

In order to determine the mode shapes and natural frequencies, the mass and the stiffness matrices must be calculated. The mass matrix, which is usually diagonal, can be readily assembled from the story masses; the only practical decision that the designer may have to take in obtaining these mass coefficients relates to what percentage of the live load should be included as contributing to story inertia. Reference 1 prescribes a minimum of 25% of the live load to be included when computing the story masses. Before the stiffness matrix can be

calculated, a number of practical decisions related to the modeling of the structure must be made; some of the most typical are listed below:

- (1) Selection of gross vs. cracked moment of inertias, or use of a more refined approach based on effective moments of inertia.
- (2) Selection of equivalent beam inertias to model in two dimensions the three dimensional behavior of the floor system.
- (3) Foundation modeling. Is the foundation going to be modeled as perfectly fixed or pinned; or is the actual situation of partial fixity going to be incorporated in the model?
- (4) Selection of clear distances versus centerline to centerline dimensions or, perhaps, inclusion of a more rigorous approach which accounts for the rigidity of the joints.

These are some of the decisions which must be made by the designer before the standard procedures of structural analysis can be used to define the stiffness matrix. The modeling criteria used in this dissertation when designing FRAME 1 and FRAME 2 are presented in Chapter VI.

III. EVALUATION OF MODAL PARAMETERS AND SRSS COMBINATIONS

Once the structure's modal shapes and natural frequencies have been determined (not necessarily all of them, since usually the first few modes will yield enough accuracy) the values of the spectral accelerations can be read from the applicable Inelastic Response Spectrum. These values will be labeled $S_{a,n}$; where S_a stands for spectral acceleration, and the subscript n refers to the mode to which it corresponds. With

the exception on how the value of $S_{a,n}$ is obtained (Chapter III) the method of computation of the modal parameters such as forces, moments, shears and axial loads is identical in the IRSA to a standard spectrum approach. One must keep in mind, however, that when using an elastic spectrum, the computed parameters are the actual forces, moments, shears and axial loads that the structure will experience. In the case when one is using an Inelastic Response Spectrum the purpose of the analysis is to determine the yield load level for which the structure will behave inelastically with a ductility equal to the preselected value.

Modal Forces

For the case of the SDOF system used to develop the inelastic spectral acceleration, it was stated in Chapter III that the maximum force developed for the particular ductility selected is given by

$$f = S_a m \quad (4.3)$$

For a MDOF system the equation which governs the response in each one of the modes was derived in Chapter II and is presented as Equation (2.21). By comparing the forcing functions (the right hand side term) between Equations (2.21) and (3.4) one can see that for the n^{th} mode of the MDOF system the equivalent spectral acceleration is:

$$\frac{S_{a,n}}{m} \phi_n^T [m] \{r\}$$

Since the geometric displacements are related to the modal amplitudes

through the mode shape vector, ϕ_n , the spectral accelerations at the various levels of the MDOF system in the n^{th} mode are

$$\frac{S_{a,n}}{m} \phi_n^T [m] \{r\} \phi_n$$

In order to obtain the forces one only needs to multiply these spectral accelerations by the masses at the different stories. Then,

$$\{f_n\} = \frac{S_{a,n}}{m} \phi_n^T [m] \{r\} \phi_n \quad (4.4)$$

Now, the value of the mass, m , of the SDOF equation in (3.4) must be equal to the modal mass m_n^* if the two equations, that is (2.21) and (3.4), are to have the same period of vibration. From Equation (2.14) the modal mass is given by

$$m_n^* = \phi_n^T [m] \phi_n \quad (4.5)$$

Substituting (4.5) into (4.4)

$$\{f_n\} = S_{a,n} \left(\frac{\phi_n^T [m] \{r\}}{\phi_n^T [m] \phi_n} \right) [m] \phi_n \quad (4.6)$$

The term in parentheses in Equation (4.6) was defined in Chapter II as the participation factor for n^{th} mode; for convenience its definition is repeated here as Equation (4.7).

$$\Gamma_n = \frac{\phi_n^T [m] \{r\}}{\phi_n^T [m] \phi_n} \quad (4.7)$$

Substituting (4.7) into (4.6) one gets

$$\{f_n\} = S_{a,n} \Gamma_n [m] \phi_n \quad (4.8)$$

Equation (4.8) gives the values of the maximum modal forces in the n^{th} normal mode. The values of the modal moments shears, and axial loads are determined from the modal forces using standard methods of structural analysis.

Modal Drifts

Drift is defined as the relative deflection between two consecutive story levels. The interstory deflection or story drift is an excellent measure of structural response because its value at any level is equal to the distortion induced in that story. The story drifts in a given mode could be computed by calculating the interstory deflections induced by the forces given by Equation (4.8) using standard methods for deflection calculations. However, as will be shown in the following, the modal drifts can be calculated directly. The deflections induced by the modal forces are obtained by multiplying Equation (4.8) by the inverse of the stiffness matrix (flexibility) as shown:

$$\{u_n\} = [k]^{-1} S_{a,n} \Gamma_n [m] \phi_n \quad (4.9)$$

By definition the drift at any level is $u_{n,x} - u_{n,(x-1)}$ where $u_{n,x}$ and $u_{n,(x-1)}$ are the deflections in mode n at levels x and $(x-1)$ respectively. Therefore,

$$\{\delta_n\} = [k]^{-1} S_{a,n} \Gamma_n [m] (\phi_{n,x} - \phi_{n,(x-1)}) \quad (4.10)$$

where $\{\delta_n\}$ is the vector of modal drifts. Now, from Equation (4.2)

$$[k] = [m] \omega_n^2$$

Inverting both sides

$$[k] = [m]^{-1} \frac{1}{\omega_n^2} \quad (4.11)$$

Substituting Equation (4.11) into (4.10) and noting that $[m]^{-1} [m] = [I]$ where $[I]$ is the identity matrix, the following is obtained:

$$\{\delta_n\} = \frac{S_{a,n} \Gamma_n}{\omega_n^2} (\phi_{n,x} - \phi_{n,(x-1)}) \quad (4.12)$$

Equation (4.12) gives an expression for the modal drifts, δ_n , in terms of the modal shape and frequency and the corresponding spectral acceleration.

SRSS Combinations

The modal parameters calculated as described in this section are the maximum values attained during the response in the particular mode

considered. The maximum total response cannot, in general, be obtained by merely adding the modal maxima because these maxima usually do not occur at the same time. In most cases when one mode reaches its maximum response, the other modal responses are less than their individual maxima. Therefore, although a direct addition of the modal maxima provides an upper bound to the total response, this approach is generally unduly conservative. A number of different formulas have been proposed to obtain a reasonable estimate of the maximum response. As was stated in Chapter III, the SRSS method was selected here to combine the modal response maximums. It should be noted that the SRSS combination must be applied to each parameter independently; that is, it is not correct to calculate the SRSS value for the modal forces and then assume that the moments, shears, and axial loads from these forces are equal to the SRSS values for these parameters.

IV. GRAVITY LOAD ANALYSIS

The calculation of the moments, shears, and axial loads induced by the gravity loading is a problem of structural analysis which is well known and need not be discussed herein. It is worth pointing out however, that the structural model used for gravity loading is, in general, different from the one used to determine the lateral load analysis of the structure (unless one uses a three dimensional model). This is because the assumptions used to account for the actual three dimensional (3-D) behavior in a two dimensional analysis do not hold for both types of loading. For example, gravity load analysis of reinforced concrete

frames is frequently carried out using the Equivalent Frame Method presented in Reference 20. In this method the 3-D effects are accounted for by using an equivalent reduced column stiffness; it can be shown (21) that this correction, although correct for gravity loading, is in the wrong direction for lateral loads. The general consensus at the present time is that the equivalent frame for lateral loads should be made up of the actual columns and an equivalent effective width to model the floor system as a beam; the various papers which deal with the subject of equivalent beam widths are summarized in Reference 22.

V. EVALUATION OF P- Δ EFFECTS

In a very simple form the P- Δ effect is illustrated in Figure 14(a). The cantilever column is loaded both vertically and horizontally at the top. Because of the horizontal displacement Δ , the vertical load P acts eccentrically, increasing the moments in the column as shown in Figure 14(b). As can be seen in Figure 14(c) the maximum moment, $Sh + P\Delta$, is also obtained if one increases the lateral load S by an amount equal to $\frac{P\Delta}{h}$. When the eccentricity of the P load is accounted for by increasing the lateral load S, the resulting moment diagram is a straight line which only approximates the true moment variation along the column. However, unless the column is extremely slender, this approximation is sufficiently accurate for design purposes. By analogy with the previous discussion, it may be seen that the P- Δ effects in multistory frames can be accounted for by increasing the story shears by an amount given by:

$$(\Delta S_{P-\Delta})_j = P_j \cdot \frac{\Delta_u}{h_j} \quad (4.13)$$

where P_j is the total gravity load assumed to be present during the earthquake above level j , h_j is the story height of level j , and Δ_u is the ultimate story drift expected at this level. The calculation of Δ_u in Equation (4.13) is discussed in the following.

Figure 15(a) shows the cantilever column previously shown in Figure 14. The maximum deflection induced by the earthquake, without considering any $P-\Delta$ effect, is given by $\mu\delta_e$, where δ_e is the "elastic" drift induced by the S loading (where S is the reduced lateral load obtained from the Inelastic Response Spectrum). Figure 15(b) shows the equivalent increase in the lateral load induced by the $P-\Delta$ effect. As can be seen in the figure, this increase in the lateral load induces an additional deflection Δ_1 , which in itself contributes to a further increase in the $P-\Delta$ effect, and so on. The total increase in the lateral load from the $P-\Delta$ effect is:

$$\Delta S = \frac{P}{h} (\delta_e \mu + \Delta_1 + \Delta_2 \dots + \Delta_n) \quad (4.14)$$

But

$$\Delta_1 = f \frac{P}{h} (\mu \delta_e) \quad (4.15)$$

where f is the flexibility of the column. Also,

$$\Delta_2 = f \frac{P}{h} \Delta_1 \quad (4.16)$$

Substituting Equation (4.15) into (4.16),

$$\Delta_2 = f^2 \left(\frac{P}{h}\right)^2 (\mu \delta_e) \quad (4.17)$$

Or, in general,

$$\Delta_n = f^n \left(\frac{P}{h}\right)^n (\mu \delta_e) \quad (4.18)$$

Substituting Equations (4.15), (4.16) and (4.17) into Equation (4.14),

$$\Delta S = \frac{P}{h} \mu \delta_e \left(1 + f\left(\frac{P}{h}\right) + f^2\left(\frac{P}{h}\right)^2 + \dots + f^n\left(\frac{P}{h}\right)^n\right) \quad (4.19)$$

or

$$\Delta S = \frac{P}{h} \mu \delta_e \left(1 + \sum_{n=1}^{\infty} f^n \left(\frac{P}{h}\right)^n\right) \quad (4.20)$$

In order for the summation inside the brackets to have a finite limit, the product $f \frac{P}{h}$ must be less than unity. This condition is a necessary condition for stability under the combined application of lateral and gravity loads. If one defines,

$$\lambda = 1 + \sum_{n=1}^{\infty} f^n \left(\frac{P}{h}\right)^n \quad (4.21)$$

Equation (4.20) may be written as,

$$\Delta S = \frac{P}{h} \mu \delta_e \lambda \quad (4.22)$$

It has been shown (23) that Equation (4.21) can be closely approximated by the following expression

$$\lambda \approx \frac{1}{1 - f \frac{P}{h}} = \frac{1}{1 - \frac{\Delta_1}{\mu \delta_e}} \quad (4.23)$$

Since Equation (4.23) is much simpler to evaluate than (4.21) it will be used when calculating λ in this dissertation. By analogy with the previous derivation for the simple cantilever column, the increase in the lateral shears due to the P- Δ effects in multistory frames is given by

$$\{(S_{P-\Delta})\} = \left\{ \frac{P_j}{h_j} \right\} \mu \{\delta_e\} \{\lambda\} \quad (4.24)$$

where P_j is the vector of the total gravity loads acting above each level, which is divided by the corresponding story heights; $\{\delta_e\}$ is the vector of the earthquake induced story drifts (given by the SRSS of $\{\delta_n\}$), and $\{\lambda\}$ is the vector of stability coefficients given by:

$$\{\lambda\} = \frac{\{1\}}{\{1\} - \frac{\{\delta_1\}}{\mu \{\delta_e\}}} \quad (4.25)$$

where $\{\delta_1\}$ is the vector of the first order drifts induced by the P- Δ effect. By comparing Equations (4.13) and (4.24) it may be seen that:

$$\{\Delta_u\} = \mu\{\delta_e\}\{\lambda\} \quad (4.26)$$

The story yield drifts δ_y are then given by:

$$\{\delta_y\} = \{\delta_e\}\{\lambda\} \quad (4.27)$$

The procedure to calculate the P- Δ effects in an arbitrary case can be summarized as follows:

- (a) The first order increase in the story shears are calculated from Equation (4.24) by taking the λ vector as a unit vector.
- (b) The vector of first order P- Δ drifts $\{\delta_1\}$ is computed from the first order shears calculated in (a).
- (c) The actual λ vector is then calculated from Equation (4.25).
- (d) The total increases in the story shears are obtained from Equation (4.24) using the actual λ vector. From these shears equivalent lateral loads can be computed. The P- Δ effect on the moments, shears, and axial loads is then calculated from these loads using any standard procedure of structural analysis.

In a well-designed frame the P- Δ induced forces should be small when compared to the forces induced by other loads. Reference 3 suggests that the P- Δ effect can be disregarded if the first order story shears calculated in (a) are less than 10% of the initial (SRSS) story shears. This simplification was not included in the design of the frames in Chapter VI.

It should be noted that the stability coefficients λ in Equation (4.25) depend on the structure's flexibility matrix $[f]$ which increases

after inelastic action commences; this refinement was not considered in this dissertation.

VI. DETERMINATION OF LOAD FACTORS AND COMBINATIONS OF LOADING CASES

As previously stated, the goal of the IRSA is to determine, for a particular structure and design earthquake, the yield load level for which the structure will behave inelastically with a ductility equal to a preselected value. The following combination of loads is consistent with the ideas just presented:

$$U = D + L_r + \alpha E \quad (4.28)$$

where U = ultimate strength capacity required, D = member force from dead load, L_r = member force from the reduced live load, E = member force from the Inelastic Spectrum (SRSS of the individual modal contributions), and α is a design criteria coefficient which is dependent on the parameter being considered (moment, axial load, etc.) and on the type of member being designed, e.g., beam or column. The value of L_r , the reduced live load in Equation (4.28) is theoretically equal to the value which is expected to be present at the time of the earthquake which is, of course, a matter of judgment. In this dissertation the full value of the specified live load was used in the design of the frames in Chapter VI. The coefficient α will be discussed in detail in the following section in connection with the strength design of the structural elements. All other load combinations not involving earthquake should

be satisfied as prescribed by the applicable code. The effects of vertical earthquake accelerations in design can be approximately accounted for by including a load combination such as:

$$U = \frac{2}{3} (D + L_r) + \alpha E \quad (4.29)$$

Since vertical accelerations were not included in the DRAIN-2D runs of the frames investigated in Chapter VI, Equation (4.29) was not included in their design.

Strong Column-Weak Beam

The design of reinforced concrete moment resisting frames in earthquake regions is governed by a design criterion which is usually referred to as the strong column-weak beam concept. This design approach aims at having all the inelastic action restricted to the beams; that is, plastic hinges, in so far as possible, should form in the beams rather than in the columns. There are at least four very good reasons why the inelastic demands should be limited to the beams:

(a) The presence of axial load significantly reduces the curvature ductility, $\frac{\phi_u}{\phi_y}$, of a section (18). Because the plastic hinge rotation capacity of a member is dependent on the $\frac{\phi_u}{\phi_y}$ ratio the presence of axial load drastically reduces the capacity to withstand inelastic rotations. In fact, if the axial load acting on the section is equal to or greater than the one corresponding to balanced conditions (20), the inelastic rotation capacity is reduced to practically zero. It follows then,

that it is much easier to provide plastic hinge rotation capacity in beams, where the axial load level is usually negligible, than in columns, members whose main function is to carry axial loads.

(b) In the case of plastic hinges forming in the columns, a single story column sidesway mechanism, as shown in Figure 16(b), may result. This mechanism is very inefficient in absorbing the earthquake energy input because only a few members (columns within the swaying story) absorb energy through inelastic deformation. If the inelastic action is restricted to the beams, it is not possible to obtain a collapse mechanism until all the beams form plastic hinges. It will also be necessary to develop a plastic hinge at the base of each of the first story columns. This situation is illustrated in Figure 16(c). For the same amount of inelastic energy absorption, the plastic hinge rotations required in the beam sidesway mechanism are significantly smaller than those in the column sidesway mechanism of Figure 16(b). Reference 18 has an extensive discussion of these two sidesway collapse mechanisms.

(c) In general, in a moment resisting frame a column is a much more important member than a beam. This statement can be justified if one considers the consequences of a column failure as compared to those of a beam failure. The failure of a column implies the failure of all the beams that frame into it, plus the very distinct possibility of a chain reaction type of failure of all the stories above the particular failed column. Conversely the failure of a beam is a rather localized phenomenon which does not have a pronounced effect on the overall stability of the structure.

(d) Finally, it is desirable to avoid plastic hinge formation in the columns because the straightening and repair of these members is difficult. The repair of beams is much easier and economical (24).

The parameter α in Equation (4.28) is intended to ensure that the strong column-weak beam criteria be satisfied; this is accomplished, as discussed later, by using a larger value of α for columns.

VII. STRENGTH DESIGN OF THE STRUCTURAL ELEMENTS

In this section, the design requirements used to ensure that the structure behaves according to the strong column-weak beam criteria are presented. This section is not intended to cover the principles of strength design; these principles are well established, and extensive coverage can be found elsewhere (18, 20).

Beams

The aim of the IRSA, as applied herein to reinforced concrete moment resisting frames, is to have all the beams form plastic hinges at both ends when the story drift reaches the yield value, as given by Equation (4.27). The moments acting at the ends of a given beam at the attainment of the yield drift are given by:

$$(M_u^+)_{\text{a}} = (M_{\text{eb}})_{\text{a}} + (M_{\text{p}-\Delta})_{\text{a}} - (M_{\text{d}+1})_{\text{a}} \quad (4.30)$$

and

$$(M_u^-)_{\text{b}} = (M_{\text{eb}})_{\text{b}} + (M_{\text{p}-\Delta})_{\text{b}} + (M_{\text{d}+1})_{\text{b}} \quad (4.31)$$

where M_u^+ is the positive design moment, M_u^- is the negative design moment, M_{eb} is the moment induced by the earthquake (SRSS of the modal contributions), $M_{p-\Delta}$ is the moment induced by the P- Δ effect, and M_{d+1} is the moment induced by the gravity loading (dead plus live loading). The subscripts a and b are used to designate the two ends of the beam. Since the moments M_{eb} and $M_{p-\Delta}$ can have either sign at any of the two beam ends, it is apparent that Equations (4.30) and (4.31) apply to each joint in the beam; that is, given the earthquake moment, the P- Δ moment, and the gravity load moment, the design beam moments at any section are:

$$M_u^+ = M_{eb} + M_{p-\Delta} - M_{d+1} \quad (4.32)$$

and

$$M_u^- = M_{eb} + M_{p-\Delta} + M_{d+1} \quad (4.33)$$

The signs have already been accounted for in Equations (4.32) and (4.33), the values of M_{eb} , $M_{p-\Delta}$ and M_{d+1} used in these equations should be taken as positive. It is apparent from an inspection of Equations (4.32) and (4.33) that the value of α in Equation (4.28) for the case of beams is taken as unity.

The design for shear forces is not treated here because DRAIN-2D, the program used to analyze the structures considered in this dissertation, does not have the capability to model any shear effects in reinforced concrete flexural members. References 18 and 24 have a

detailed coverage of the shear problem in the beams of moment resistant reinforced concrete frames designed for earthquake. The calculation of required and allowable plastic hinge rotations is presented in Chapter V.

Columns

The strong column-weak beam design concept aims at having plastic hinges form in the beams rather than in the columns. Some codes, for example the seismic provisions of ACI 318-77 (20), require that at beam column connections the sum of the moment strength of the columns should exceed the sum of the moment strengths of the beams along each principal plane at the connection. Section 2 of Reference 1 contains a similar statement. It has been shown (18, 24) and this writer has confirmed in this research, that the design limitation previously stated will not prevent the formation of plastic hinges in the columns. The primary reason why this requirement does not prevent column hinging is the fact that, after hinging of the beams, and as the structure deflects inelastically from its yield drift, δ_y , to its ultimate or maximum drift, Δ_u , the column inflection points depart significantly from mid-height, making one of the columns framing at the joint resist the major portion of the input moment from the beams. In some cases the inflection point moves all the way to the joint forcing the remaining column to resist the total input moment from the beams. In the extreme case when the point of inflection lies outside the column height, the strength of one column section needs to exceed the moment input from the beams. This very extreme situation of single curvature in a column is not very

common. It is believed that a reasonable upper bound for the earthquake moment induced in a column at any time during the response is given by:

$$M_{ce} \leq \sum M_{be} \quad (4.34)$$

where M_{ce} is the earthquake induced moment in the column and $\sum M_{be}$ is the total beam moment input at a beam-column connection obtained from the earthquake loading. Even though Equation (4.34) represents a "realistic" upper bound for the induced earthquake moment in a column, it is not always necessary to design the columns for this upper bound; actually, as the design ductility is reduced, the maximum earthquake moment should be closer and closer to the SRSS value obtained from the modal analysis. The reason for this is that the redistribution of column moments (migration of the inflection points) is dependent on the extent of the inelastic drift, which is itself dependent on the design ductility (and the flexibility of the structure as was discussed in Section I of this chapter). From an investigation carried out by this writer on a number of time histories of column moments in inelastic frames, it was found that another expression for the upper bound of the column earthquake moments is given by:

$$M_{ce} \leq \alpha M_{c(SRSS)} \quad (4.35)$$

where α was found to vary with the design ductility as follows:

$$\text{for } \mu = 4, \alpha = 1.7 \quad (4.36)$$

for $\mu = 6$, $\alpha = 1.85$

(4.37)

In Equation (4.35), $M_{c(SRSS)}$ is the SRSS column moment obtained from the modal analysis. In the event of an elastic design the value of $M_{c(SRSS)}$ is equal (within the accuracy of the SRSS combination) to the maximum earthquake moment M_{ce} ; thus, one concludes that for $\mu=1$, $\alpha=1$. The values of α given above are comparable to parameters used for a similar purpose elsewhere (6, 24). From the preceding discussion it follows that the maximum earthquake moment in a column can be taken to be the lesser value of those obtained from Equations (4.34) and (4.35). The use of Equations (4.34) and (4.35) to determine the maximum probable earthquake moment ensures a high degree of protection against any significant column hinging with the exception of the ground floor column at its connection with the foundation. At this point Equation (4.34) is obviously not applicable since there are no framing beams, and Equation (4.35) usually underpredicts the value of the moment required to maintain elasticity. The investigation carried out by this writer on the time histories of column moments showed that, in general, the value of α required for elasticity at the base is too high to be practical. In fact, it is widely recognized (5, 6, 24) that the formation of plastic hinges in the ground floor columns at the foundation level may not be avoided. It follows then that, for the case of fixed foundations, the ground column at its connection with the base should be carefully detailed to ensure effective concrete confinement; recommended procedures are given in References 18 and 24.

The calculation of the earthquake induced axial loads are much simpler than those required to obtain the maximum moments because the inelastic drift of the structure does not significantly affect the SRSS values obtained from the modal analysis. That is, the maximum earthquake axial load which can be either positive (compression) or negative (tension) is approximately given by:

$$P_{ec} = P_c(\text{SRSS}) \quad (4.38)$$

where P_{ec} is the maximum earthquake induced axial load and $P_c(\text{SRSS})$ is the SRSS value obtained from the modal analysis. The value of the maximum earthquake induced moment, given by the lesser value from Equations (4.34) and (4.35), is not necessarily attained at the same time that the axial load effect is at a maximum. If these maxima are considered to occur simultaneously for design purposes, as is the case here, an additional safety factor against column hinging is added to the overall procedure.

P-M Combinations for Design

Figure 17(a) shows a frame with gravity loading w_{d+1} applied to its beam elements. In Figure 17(b) the moment diagram resulting from this gravity loading is shown for a typical exterior column in column line C. When the structure is drifting to the right the earthquake induced moments, as shown in Figure 17(c) will be of the same sign as those induced by the gravity loads and the induced axial load will be compression. It follows then that one of the load combinations to be considered for exterior columns is:

$$P_u^{\max} = P_{ec} + P_{d+1}$$

4.39)

$$M_u^{\max} = M_{ec} + M_{d+1}$$

where P_{d+1} and M_{d+1} are the axial load and moment, respectively, induced by the gravity loads.

The P-M combination above will be referred to as combination A. When the frame is drifting to the left the signs on the earthquake effects shift and the resulting P-M combination, combination B, is:

$$P_u^{\min} = -P_{ec} + P_{d+1}$$

(4.40)

$$M_u^{\min} = -M_{ec} + M_{d+1}$$

Combination B needs to be checked because even though the load and the moment are reduced, the shape of the interaction diagram for reinforced concrete columns makes it still possible for this to be a critical combination. Figure 18 shows a qualitative moment-axial load interaction diagram illustrating how it is possible that combination B may control the design of the column. For interior columns the moment diagrams induced by the gravity loads may be such that the minimum axial load may combine with the maximum moment (this would be the case, for example, in Figure 17, if the columns of line B had a gravity load moment diagram with opposite sign to the one shown in Figure 17(b) but the structural properties were such that a drift to the right would

induce compression). Fortunately, the moments from gravity loads in interior columns are small as are the earthquake induced axial forces; this permits the use of the following additional P-M combination for interior columns without introducing excessive conservatism in those cases where the necessary conditions are not met.

$$\begin{aligned} P_u^{\min} &= -P_{ec} + P_{d+1} \\ M_u^{\max} &= M_{ec} + M_{d+1} \end{aligned} \tag{4.41}$$

Equation (4.31) is designated here as Combination C.

Slenderness Effects

For those cases where the P- Δ effect is not directly included in the analysis, the columns should be designed as compression members of an unbraced frame according to the procedure given in Reference 20. If the P- Δ is explicitly included as described in Section V of this chapter, then the secondary moments from the lateral deflections have been already accounted for, and the columns should be designed as compression members of a braced frame. The braced frame slenderness effect evaluation is necessary to account for any possible increase in the effective moments due to the elastic deflection between the column and its chord (recalling from Section V that the P- Δ effect as evaluated herein accounts for the relative displacement between the column ends but it does not account for the actual deflected shape of the column).

When the P- Δ effect is explicitly considered in the design, the P-M combinations given by Equations (4.39), (4.40) and (4.41) become:

$$\begin{aligned}
 (A) \quad P_u^{\max} &= P_{ec} + P_{p-\Delta} + P_{d+1} \\
 M_u^{\max} &= M_{ec} + M_{p-\Delta} + M_{d+1}
 \end{aligned}
 \tag{4.42}$$

$$\begin{aligned}
 (B) \quad P_u^{\min} &= -P_{ec} - P_{p-\Delta} + P_{d+1} \\
 M_u^{\min} &= -M_{ec} - M_{p-\Delta} + M_{d+1}
 \end{aligned}
 \tag{4.43}$$

and

$$\begin{aligned}
 (C) \quad P_u^{\min} &= -P_{ec} - P_{p-\Delta} + P_{d+1} \\
 M_u^{\max} &= M_{ec} + M_{p-\Delta} + M_{d+1}
 \end{aligned}
 \tag{4.44}$$

where $P_{p-\Delta}$ and $M_{p-\Delta}$ are the axial load and moment, respectively, induced by the $P-\Delta$ effect.

Summary

The equations developed in this section are intended to insure that the strong column-weak beam design criterion is satisfied. They are summarized here for easier reference.

(1) Beams

$$M_u^+ = M_{eb} + M_{p-\Delta} - M_{d+1} \tag{4.32}$$

$$M_u^- = M_{eb} + M_{p-\Delta} + M_{d+1} \tag{4.33}$$

(2) Columns

For exterior columns, the critical combination is either A or B.

$$\begin{aligned} (A) \quad P_u^{\max} &= P_{ec} + P_{p-\Delta} + P_{d+1} \\ M_u^{\max} &= M_{ec} + M_{p-\Delta} + M_{d+1} \end{aligned} \quad (4.42)$$

$$\begin{aligned} (B) \quad P_u^{\min} &= -P_{ec} - P_{p-\Delta} + P_{d+1} \\ M_u^{\min} &= -M_{ec} - M_{p-\Delta} + M_{d+1} \end{aligned} \quad (4.43)$$

For interior columns, the critical combination is either A or C, where

$$\begin{aligned} (C) \quad P_u^{\min} &= -P_{ec} - P_{p-\Delta} + P_{d+1} \\ M_u^{\max} &= M_{ec} + M_{p-\Delta} + M_{d+1} \end{aligned} \quad (4.44)$$

In Equations (4.42), (4.43) and (4.44), M_{ec} and P_{ec} are given by

$$M_{ec} = \alpha M_c(\text{SRSS}) \quad (4.35)$$

where α is as follows:

$$\begin{aligned} \text{for } \mu = 1 \quad \alpha &= 1 \\ \mu = 4 \quad \alpha &= 1.7 \end{aligned} \quad (4.36)$$

$$\mu = 6 \quad \alpha = 1.85 \quad (4.37)$$

Also,

$$M_{ec} \leq \Sigma M_{be} \quad (4.34)$$

and,

$$P_{ec} = P_c(\text{SRSS}) \quad (4.38)$$

CHAPTER V

INELASTIC DEFORMATIONS

It was shown in Chapter IV that the values of the elastic drifts in the various stories of a building frame are a function of the dynamic properties of the structure (mode shapes and natural frequencies) and the spectral accelerations obtained from the applicable Inelastic Response Spectrum. The value of the total drift, Δ_u , expected in a particular story is related to the yield drift, δ_y , through the design ductility factor μ . It should be noted that this relationship, that is, $\Delta_u = \delta_y \mu$ is the MDOF equivalent of the SDOF expression $u_m = u_y \mu$. Since the IRSA is not a rigorous procedure for MDOF systems (discussion in Chapter I, Section III), the expression relating total drift to yield or elastic drift, given below as Equation (5.1), is not a rigorous (derivable) equation.

$$\Delta_u = \mu \delta_y \quad (5.1)$$

Equation (5.1) is the mathematical expression of the basic assumption on which the use of the IRSA is dependent. As previously stated in Chapter I, one of the primary aims of this dissertation is to test the applicability of the extrapolation implied in Equation (5.1).

I. CALCULATION OF INELASTIC DEMANDS

From Equation (5.1) the maximum values of the total interstory drifts can be calculated (in the following, Equation (5.1) is assumed

applicable; its applicability is investigated in Chapter VI). In some cases, the values of Δ_u alone will suffice in rejecting a preliminary design; this will occur when some of the computed drifts are larger than some preselected value included in the design criteria. The recent comprehensive report prepared by the Applied Technology Council (ATC) on seismic design recommended a limiting value of $0.01h$ for the maximum story drift (25). However, in most cases in order to complete the evaluation of the design, the story drifts have to be related to the inelastic demands in the individual members making up the structure. According to the strong column-weak beam criteria specified in Section VI of Chapter IV, the columns (with an exception noted therein) should not yield, thus reducing the problem to relating the story drift to the inelastic demands on the beams.

Plastic Hinge Rotations-Beams

Even though there are several ways of expressing inelastic demands on beams (10), the plastic hinge rotation is probably the best of all of them because it is directly and easily relatable to the $M-\phi$ curve and the geometric properties of the beam cross-section. The following derivation is based on the assumption that the beams will yield at both ends simultaneously. If the procedure outlined in Section VII of Chapter IV is followed, this condition will be met.

Figure 19 shows a frame and a beam column subassemblage in its displaced position at maximum drift. The deflections are greatly exaggerated in order to clearly illustrate the geometric relationships. The plastic hinge rotation is by definition the change in the angle

between the beam and the column connection. For the sake of clarity, Figure 19 shows, in addition to the actual beam, the position that an elastic beam would have in the same displaced position; this way the plastic hinge rotation can be clearly visualized as the angle between the real beam and the fictitious elastic beam.

From Figure 19,

$$\theta_t = \theta_p + \theta_y$$

so

$$\theta_p = \theta_t - \theta_y \quad (5.2)$$

It can also be seen from Figure 19 that

$$\theta_t = \frac{\Delta_u}{h} - \theta_{cu} \quad (5.3)$$

where θ_{cu} is the angle between the column tangent and the chord at maximum drift, Δ_u . Figure 20 shows a sketch of the same subassemblage in Figure 19 except that the displacement is now only δ_y which corresponds to the onset of yielding. From Figure 20 one can see that,

$$\theta_y = \frac{\delta_y}{h} - \theta_{cy} \quad (5.4)$$

where θ_{cy} is the angle between the column tangent and the chord at a

drift equal to δ_y . Substituting both Equations (5.3) and (5.4) into (5.2) one obtains

$$\theta_p = \frac{\Delta_u}{h} - \theta_{cu} - \frac{\delta_y}{h} + \theta_{cy} \quad (5.5)$$

Now substituting (5.1) into (5.5) and rearranging:

$$\theta_p = (\mu-1) \frac{\delta_y}{h} - (\theta_{cu} - \theta_{cy}) \quad (5.6)$$

If one now defines

$$\Delta\theta = \theta_{cu} - \theta_{cy} \quad (5.7)$$

Equation (5.6) can be written as

$$\theta_p = (\mu-1) \frac{\delta_y}{h} - \Delta\theta \quad (5.8)$$

The value of $\Delta\theta$ depends on how much the moments at the ends of the column change as the structure deflects from its yield story drift, δ_y , to the maximum drift, Δ_u . In general, if the moment increments at the top and bottom of the column are designated as ΔM_t and ΔM_b respectively, the value of $\Delta\theta$ is given by:

$$\Delta\theta = \frac{h}{6 E_c I_c} (2 \Delta M_t - \Delta M_b) \quad (5.9)$$

Equation (5.9) is simply the expression for the slope of a member subjected to moments ΔM_t and ΔM_b at its ends. This equation can be easily derived using conjugate beam, moment area, or any other method for calculating structural deformations.

The values of ΔM_t and ΔM_b (which are taken as positive if counter-clockwise at the ends of the column) are dependent on the extremely complex distribution of moments in the inelastic structure. An investigation of the time history of column moments as the structure goes from δ_y to Δ_u for a number of frames using DRAIN-2D (4) did not yield any clues as to how the values of ΔM_t and ΔM_b could be determined because the results varied "all over the map." The investigation did, however, show that the error in Equation (5.8) if one simply takes $\Delta\theta$ as zero is acceptable and that the value thus obtained for θ_p is conservative (larger than the actual) because $\Delta\theta$ is normally positive. An exception to this behavior was found in the two lower stories of the frames investigated, where because of the abrupt discontinuity in the rigidity induced by the assumed fixed base, the value of $\Delta\theta$ was consistently negative. This behavior can be explained by recalling that in Section VII of Chapter IV it was stated that the ground column will usually yield at its connection with the base. This means that a reasonable value of ΔM_b can be calculated for the first story as the difference between the column base moment at first yielding of the beam given by

$$M = M_c(\text{SRSS}) \pm M_{d+1} \quad (5.10)$$

and the maximum column base movement is given by

$$M_u = \alpha M_{c(SRSS)} \pm M_{d+1} \quad (5.11)$$

The difference is

$$\Delta M_b = (\alpha - 1) M_{c(SRSS)} \quad (5.12)$$

If we now conservatively assume that ΔM_t is zero, $\Delta \theta$ for the first story can be computed from Equation (5.9) as

$$\Delta \theta = - \frac{h}{6 E_c I_c} (\alpha - 1) M_{c(SRSS)} \quad (5.13)$$

where it is recalled that $M_{c(SRSS)}$ is the SRSS column moment obtained from the modal analysis.

The effect of the base fixity extends into the second story also but in a much reduced intensity. From an analysis of the net effect of $\Delta \theta$ in the second story plastic hinge rotations, this writer concluded that its inclusion (for which values of ΔM_b and ΔM_t would have to be obtained) was not warranted; that is $\Delta \theta$ is taken as zero for the columns in the second story just as it is in all the remaining stories.

II. CALCULATION OF ALLOWABLE VALUES FOR THE INELASTIC DEMANDS: EVALUATION OF THE ADEQUACY OF THE DESIGN

The rotation capacity of any particular plastic hinge can be calculated from a knowledge of the moment-curvature relationship of the

critical section and the equivalent spread of plasticity ℓ_p . This section is not intended to cover the development of M- ϕ curves nor to discuss the theoretical considerations involved in the calculations of available rotation capacity; and adequate coverage of these subjects is presented in Reference 18. In the following, the equations necessary to calculate the available rotation capacity are given with some comments which are particularly relevant. The development of these equations can be found in Reference 18.

The available plastic hinge rotation to one side of the critical section (which for the cases of interest herein is the column face) can be written as:

$$\theta_{pa} = (\phi_u - \phi_y) \ell_p \quad (5.14)$$

where ϕ_u , ϕ_y and ℓ_p , which are the ultimate curvature, the yield curvature, and the equivalent length of the plastic hinge are given by:

$$\phi_u = \frac{\epsilon_u}{c} \quad (5.15)$$

$$\phi_y = \frac{\epsilon_y}{d - kd} \quad (5.16)$$

and

$$\ell_p = 0.5d + 0.05z \quad (5.17)$$

where ϵ_u is the ultimate compressive concrete strain, c is the depth to

the neutral axis at ultimate, ϵ_y is the yield strain of the steel, d is the effective depth of the section, kd is the depth to the neutral axis at yield, and z is the distance from the critical section to the inflection point. Equations (5.15) and (5.16) can be derived from first principles (18), but Equation (5.17), which gives the equivalent length of the plastic hinge, is empirical and was proposed in Reference 26. The value of ϵ_u in Equation (5.15) is also an empirically obtained parameter for which Reference 26 has suggested the following expression:

$$\epsilon_u = 0.003 + 0.02 \frac{b}{z} + 0.2 \rho_s \quad (5.18)$$

where b is the width of the section and ρ_s is the ratio of the volume of confining steel (includes stirrups and compression steel) to the volume of the concrete core. The available plastic hinge rotations can be calculated by substituting the appropriate values into Equation (5.14). It should be remembered that the higher the value of the tension steel ratio the lower the rotation capacity; therefore, Equations (5.15) and (5.16) should be evaluated for negative moment because it is for this direction that the top steel (which is the largest) is in tension in the region of a plastic hinge.

A design example illustrating the methodology presented in Chapters IV and V is included as Appendix C to this dissertation.

CHAPTER VI

DESIGN OF TWO REINFORCED CONCRETE MOMENT RESISTING FRAMES USING THE IRSA: COMPARISON OF PREDICTED INELASTIC DEMANDS WITH THOSE FROM RIGOROUS STEP-BY-STEP INELASTIC DYNAMIC ANALYSIS

The material presented in this chapter is the result of the work carried out by this writer in an attempt to answer the following two questions:

- (1) How accurate are the inelastic demands predicted using the IRSA when compared to results obtained from rigorous step-by-step inelastic dynamic analyses?
- (2) Are the design procedures developed in Chapter IV able to insure that the resulting frames behave according to the strong column-weak beam design criteria?

In an attempt to answer these questions two reinforced concrete moment resisting frames were designed using the IRSA for several combinations of ground motions and design ductilities. These frames were subsequently analyzed using DRAIN-2D, and inelastic dynamic analysis computer program described in Reference 4. The results of these analyses are presented and used in this chapter to answer, at least for the structures considered herein, the two questions posed above.

I. DESCRIPTION AND MODELING OF THE FRAMES

Figures 21 through 24 show the centerline-to-centerline dimensions, story weights, and overall member sizes for the two frames investigated in this chapter. In practice, the member sizes (which must be known for the earthquake analysis) are usually determined from a preliminary design for gravity loads or from experience with similar structures. The member sizes shown in Figures 22 and 24 for FRAMES 1 and 2 respectively, are the same as those used in previous investigations of these frames reported in References 5 and 6.

Modeling For Dynamic Analysis

The modeling of the frames for dynamic analysis is as follows:

- (1) Masses are lumped at floor levels, each level having one horizontal dynamic degree-of-freedom. The assumption of one dynamic degree-of-freedom per story implies no axial deformations in the girders.
- (2) Plastic hinges are assumed to form at the ends of the beams whenever the moment exceeds the yield level. For columns, yielding occurs when the P-M combination falls outside the load-moment interaction diagram specified in the design.
- (3) The damping assumed is of the viscous type with 5% damping ratio assumed in the first two modes. In general, the coefficients a_0 and a_1 in Equation (2.16) used to specify the damping matrix $[c]$ are calculated from the following equations (4).

$$a_0 = \frac{4\pi(T_i \xi_i - T_j \xi_j)}{(T_i^2 - T_j^2)} \quad (6.1)$$

and

$$a_1 = \frac{T_i T_j (T_i \xi_i - T_j \xi_j)}{(T_i^2 - T_j^2)}$$

Where T and ξ stand for natural period and critical damping ratio respectively. The subscripts i and j refer to modes i and j , respectively. In order to obtain 5% of critical damping in the first two modes:

$$\xi_1 = \xi_2 = 0.05 \quad (6.3)$$

Replacing the subscripts i and j by 1 and 2 and substituting Equation (6.3) into (6.1) and (6.2) one obtains:

$$a_0 = \frac{0.2\pi}{(T_1 + T_2)} \quad (6.4)$$

and

$$a_1 = \frac{0.05 T_1 T_2}{\pi(T_1 + T_2)} \quad (6.5)$$

Once the values of a_0 and a_1 are calculated from Equations (6.4) and (6.5), the variation of the critical damping ratio in the higher

modes is fixed. The critical damping ratio in any mode, once a_0 and a_1 have been calculated, is given by (4)

$$\xi_n = \frac{a_0 T_n}{4\pi} + \frac{a_1 \pi}{T_n} \quad (6.6)$$

- (4) The moments of inertia for FRAME 1 were taken from Figure 8 in Reference 5. For FRAME 2, the equivalent frame for lateral loads (or for dynamic analysis) was defined using gross moments of inertia for the beams and columns. The effect of the floor system was accounted for by including an equivalent flange width for the beams. This equivalent width was calculated using the procedure described in Reference 27. It was found when using this procedure that approximately 40% of the total bay width of 27 ft., that is 10.8 ft., is effective (as far as stiffness is concerned) for lateral loads. The moments of inertia, calculated using this 10.8 ft. flange width, are given in Table 6.

Modeling for Static Analysis

Since the effect of gravity loading is included in some of the designs investigated in this chapter, a model for gravity loads had to be defined for each one of the frames. For FRAME 1 the floor system effect was not considered in the design; that is, the model for both lateral and gravity loads was taken to be the same (as described in Figure 8 of Reference 5). For FRAME 2, "the equivalent frame method" described in Reference 20 was used to define the model used for gravity

loading analysis. The beams running perpendicular to the plane of the frame were assumed to have the same cross-section as the beams in the frame when defining the "equivalent frame" for FRAME 2.

Modeling Common to both Static and Dynamic Analysis

For FRAME 1 the model was defined using centerline-to-centerline dimensions. For FRAME 2 the joint size effect was accounted for by using rigid links to model the area within the beam-column connection. In those cases where the joint size effect is considered, the plastic hinge rotations as calculated from Equation (5.8) must be modified. This is because the hinge rotations are affected by the excentricities between the faces of the connections and the joints. Figure 25 shows, for a case where the joint size effect is considered, the incremental joint rotations as the structure drifts inelastically from δ_y to Δ_u . It should be noticed, by comparison with the derivation in Chapter V, that the term $\Delta\theta$ has already been taken as zero in Figure 25. The plastic hinge rotation is, from the figure, given by:

$$\theta_p = \frac{\delta_y}{h} (\mu - 1) + \frac{\Delta_v}{L - e_i - e_j} \quad (6.7)$$

where Δ_v is the relative vertical deflection between the two ends of the beam induced by the inelastic rotation of the joints; and e_i and e_j are the beam eccentricities (normally half of the column dimension) at joints i and j respectively. Also, from the figure,

$$\Delta_v = \frac{\delta_y}{h} (\mu - 1) (e_i + e_j) \quad (6.8)$$

Substituting Equation (6.8) into (6.7) and rearranging one obtains,

$$\theta_p = (\mu - 1) \frac{\delta y}{h} \left\{ 1 + \frac{(e_i + e_j)}{(L - e_i - e_j)} \right\} \quad (6.9)$$

If one defines

$$\Omega = 1 + \left(\frac{e_i + e_j}{L - e_i - e_j} \right) \quad (6.10)$$

Equation (6.9) can be written as:

$$\theta_p = (\mu - 1) \frac{\delta y}{h} \Omega \quad (6.11)$$

By analogy with the derivation in Chapter V the effect of any moment changes at the ends of the columns during the inelastic drift can be accounted for by including the term $\Delta\theta$ (given by Equation 5.9)).

Including this term for generality, Equation (6.11) becomes:

$$\theta_p = (\mu - 1) \frac{\delta y}{h} \Omega - \Delta\theta \quad (6.12)$$

As can be seen by comparing Equations (6.12) and (5.8) the effect of the joint size is reflected by the parameter Ω ; for the cases where the joint size is not considered, $\Omega = 1$ and Equations (6.12) and (5.8) are identical.

II. DESIGN OF THE FRAMES

The two frames* considered in this investigation were designed according to the criteria specified in Chapter IV. For beams the positive and negative yield moments were obtained from Equations (4.32) and (4.33). For columns the design consisted in determining an interaction diagram that satisfied the loading combinations obtained from Equations (4.42) and (4.43) for exterior columns, or the combinations from (4.42) and (4.44) for interior columns. The various member forces (from earthquake, P- Δ , and gravity) necessary for the design are given in the following paragraphs.

Earthquake Effect (From Inelastic Spectra)

Before one can obtain the solution for any particular ground motion and design ductility selected, it is necessary to solve for the natural periods and modal shapes for the structure being considered. Tables 7 and 8 show the complete set of natural periods and modal shapes for the two frames designed in this chapter. The next step that must be carried out, before one can perform a response spectrum solution is to obtain the spectral accelerations. For each ground motion considered, these spectral accelerations are a function of the natural period and the critical damping ratio in any given mode. Tables 9 and 10 show the damping ratios (obtained from Equation (6.6)) and the spectral accelerations for the various ground motions considered when designing FRAMES 1 and 2. The earthquake induced forces can now be calculated

*Material properties: $f'_c = 4000$ psi; Grade 60 steel; $E_c = 5000$ psi (FRAME 1) and 3605 ksi (FRAME 2).

for any combination of ground motion and design ductility using the procedure described in Chapter IV, Section III. Because the required calculations are too lengthy to do by hand, the computer program in Reference 28 (STRU DL) was used to obtain the modal forces. The results for the cases considered in this investigation are presented in Figures 26 through 30.

P- Δ Effects

The member forces induced by the P- Δ effect were not considered in the design of FRAME 1. For FRAME 2 the P- Δ effect was accounted for using the procedure described in Chapter IV, Section V. The calculation of the equivalent lateral forces used to simulate the P- Δ effects is presented in Table 11. The member forces induced by these loads are shown in Figure 31.

Gravity Loading

The moments and axial loads induced by the gravity loads in both frames are shown in Figures 32 and 33. It should be noticed from Figure 33 that the moments in the beams are not equal to the algebraic sum of the column moments. This apparent anomaly is due to the fact that, according to the equivalent frame method in Reference 20, the beam only takes a portion of the total negative bay moment. When applying the equivalent frame criterion to FRAME 2, it was found that the beams resisted 85% of the column strip moment. For the dimensions and rigidities of FRAME 2 it was also found that the column strip should be proportioned to take 75% of the total negative moment; so the beam

moments are only $0.85 \times 0.75 = 0.6375$ or 63.75% of the total negative moment. If the beam moments shown in Figure 33 are multiplied by $\frac{1}{0.6375}$ to obtain the total bay moments, it will be found that the balance between column and beam moments is restored.

III. DESCRIPTION OF ANALYSES PERFORMED

Cases Considered

The combination of ground motions and design ductilities considered in the design and subsequent analyses of FRAMES 1 and 2 are:

- (1) IFR1U4GM1
- (2) IFR1U6GM1
- (3) IFR1U4GM2
- (4) IFR1U6GM2
- (5) IFR2U6GM2A

The notation used to describe these cases is as follows. I stands for inelastic, the following three characters FR1 or FR2 are used to identify the frame being considered; U4 and U6 are used to represent the design ductility, and the rest of the characters are the name of the ground motion used.

Investigation of the Effect of Gravity Loads

The member forces induced by gravity loads were included in the design procedure developed in Chapter IV. The criterion used therein to combine the gravity loading effect with that from the earthquake was intended to insure that the inelastic demands would not be affected by the presence of gravity loads. Notice that the plastic hinge rotations,

as predicted from Equation (5.8), are independent of any effects induced by gravity loads. In order to evaluate the accuracy of the procedure used to keep gravity loads from affecting inelastic demands, a number of the basic cases previously outlined were designed and then analyzed with and without including the effect of gravity loads. The results are presented and discussed later in this chapter.

Investigation of the Effect of Overstrength

In many practical cases the beams in the frames designed using the IRSA will have strengths which are higher than those required from Equations (4.32) and (4.33). This will be the case when the sections are controlled by load combinations not involving earthquake or when the moments obtained from (4.32) and/or (4.33) are less than those provided using the minimum reinforcing steel ratio allowed, $\rho_{\min}$. The presence of over-strong beams will disrupt the behavior that the IRSA is trying to enforce; in order to evaluate this effect, some of the basic cases considered were designed and subsequently analyzed with and without including the $\rho_{\min}$ limitation.

Table 12 shows all the cases designed and analyzed in this chapter.

IV. RESULTS AND COMPARISONS

The applicability of the IRSA can be measured by the accuracy (or lack of it) of the predicted inelastic demands when compared to those obtained from a step-by-step inelastic dynamic analysis. The basic assumption on which the use of the IRSA is dependent was stated in Chapter V as Equation (5.1). As can be seen from this equation, it is

the story drift ductility which is directly relatable to the design ductility used in developing the inelastic spectra. The story drift ductility, μ_s , is defined as the ratio of the maximum story drift, Δ_u^* (obtained from the inelastic computer run) to the yield drift for the particular story considered. The comparisons between the design ductility, μ , and the story ductilities, μ_s , for the design cases herein investigated, are presented later in this section. Even though the drift in itself is an important parameter (usually limited to avoid excessive nonstructural damage (17)), the designer, from a structural viewpoint, is more interested in the plastic hinge rotations which are expected in the members of the frame. A procedure which can be used to calculate the plastic hinge rotations in the beams of a strong column-weak beam frame, was developed in Chapter V. It should be noted from Equation (5.8) that the validity of the predicted plastic hinge rotations is directly dependent on the inelastic drift and thus on the correlation between μ and μ_s . Tables 13 through 23 present the results of the inelastic dynamic analyses and the comparisons with the predicted values for the 11 cases described in Table 12. The following observations result from the analysis of the data presented in Tables 13 through 23.

Observations on the Design Ductility and Story Drift Ductilities

- (1) In general, the story drift ductility, μ_s , obtained from the inelastic step-by-step dynamic analyses is in good agreement

*Includes cyclic effects as explained in Chapter III, Section II.

with the design ductility, μ . Figures 34 through 38 show plots of μ and μ_s for the five different combinations of frame, ground motion, and design ductilities investigated (basic cases in Table 12).

- (2) From the point of view of design conservatism, it is important to note that the design ductility is, for all the cases investigated, higher than the average story drift ductility obtained from the inelastic dynamic analysis. The average over all the stories, of the ratio between story ductility and design ductility, is shown in Tables 13 to 23 for all the design cases considered.
- (3) The average value of the $\frac{\mu_s}{\mu}$ ratio varies from 0.68 to 0.98 for the cases investigated. If the cases including the $\rho_{\min}$ limitation are not considered (the prediction in the cases where $\rho_{\min}$ controls are nominal), the range is from 0.75 to 0.98. Considering the nature of the problem, this correlation between μ and μ_s is good.
- (4) Even though the average value of the $\frac{\mu_s}{\mu}$ ratio is always less than one, the story ductility is sometimes, for a particular story, higher than the design ductility. The maximum value of the $\frac{\mu_s}{\mu}$ ratio obtained at any story in all the cases considered is only 1.18. It is interesting to note that only 5 out of 72 $\frac{\mu_s}{\mu}$ ratios computed exceed the 1.10 level.
- (5) The overall average of the $\frac{\mu_s}{\mu}$ ratio for all the cases investigated is 0.87. If the cases which include the $\rho_{\min}$ limitation are excluded the overall average is 0.90.

- (6) It can be expected that the accuracy of the inelastic demands predicted using the IRSA will decrease as the design ductility increases. This reasoning is based on the fact that the spectrum approach is a rigorous procedure (within the accuracy of the modal combinations) for elastic behavior, but not analytically correct for inelastic response. It follows then that, as the design ductility departs more and more from the elastic, $\mu = 1$ value, the results from an IRSA would be less and less accurate. For the design cases considered in this investigation, the overall average of the $\frac{\mu_s}{\mu}$ ratio for those cases designed for a ductility of 4 is 0.91. For the cases designed for a ductility of 6 the overall average decreased to 0.86. As can be seen the trend is consistent with the previous reasoning, although not enough cases were considered here to permit a definite conclusion.

Observations on the Plastic Hinge Rotations

- (1) In general, the plastic hinge rotations predicted, θ_{pp} , are in good agreement with the actual hinge rotations, θ_p , obtained from the inelastic step-by-step dynamic analysis.
- (2) The average value of the ratio between the actual plastic hinge rotation and the predicted value, $\frac{\theta_p}{\theta_{pp}}$, is less than unity for all the cases investigated. This implies that the predicted hinge rotations, in the average, are conservative when compared to the actual values.

- (3) The average value of the $\frac{\theta_p}{\theta_{pp}}$ ratio varies from 0.6 to 0.98 for the cases investigated. If the cases including the $\rho_{\min}$ limitation are not considered (the predictions in the cases where $\rho_{\min}$ controls are nominal), the range is from 0.72 to 0.98. This agreement between the predicted and the actual values of the average plastic hinge rotation is good.
- (4) The highest value of the $\frac{\theta_p}{\theta_{pp}}$ ratio is 1.26. This value indicates that, in the worst case, the plastic hinge rotation was only underpredicted by 26%. The plastic hinge rotations available cannot usually be predicted within this accuracy.
- (5) The overall average of the $\frac{\theta_p}{\theta_{pp}}$ ratio for all the cases investigated is 0.85. If the cases which include the $\rho_{\min}$ limitation are excluded the overall average is 0.88.

Effect of Gravity Loads on the Inelastic Demands

Figures 39 through 41 show the effect of gravity loading on the story ductility for three cases designed and subsequently analyzed with and without including the effect of gravity loads. As can be seen from these figures, the effect of gravity on the story ductility varies from negligible to none. The plastic hinge rotations (which can be compared from Tables 13 and 17, 14 and 18 and 16 and 20) were also very little affected by the presence of gravity loads. These results indicate that if the member forces induced by gravity loading are combined with the earthquake effect as described in Chapter IV, the inelastic demands (which are computed to be independent of the gravity loading) will not be appreciably affected.

Effect of Beams with Higher Strengths than Required from the IRSA

Figures 42 and 43 show plots of the story ductilities for two cases which were designed and analyzed with and without including the $\rho_{\min}$ design limitation. As can be seen from the figures, the $\rho_{\min}$ limitation controlled the design only in the upper levels of the frames where the earthquake effect is small. In both cases investigated the story ductility was reduced in the stories controlled by $\rho_{\min}$ and increased in the stories below. The reduction of the story ductilities in those levels controlled by $\rho_{\min}$ is logical since these stories are stronger than required by the IRSA. The increase in some of the levels not controlled by $\rho_{\min}$ is due to the concentration of inelastic action in these now relatively weaker stories. The effect of the $\rho_{\min}$ limitation is more pronounced in the case of FRAME 1 (compare the results for FRAME 1 in Figure 42 with those for FRAME 2 in Figure 43) because consideration of $\rho_{\min}$ in the design of FRAME 1 had a much greater effect on member strengths provided than did the consideration of $\rho_{\min}$ in FRAME 2.

Calculation of Available Plastic Hinge Rotations

Table 24 shows a comparison between the maximum plastic hinge rotations obtained from the inelastic dynamic analyses and the minimum available calculated according to the procedure outlined in Chapter V, Section II. It should be noted that the minimum available hinge rotation capacity shown for any case in Table 24 may not correspond to the same beam that has the maximum required value. Since all the design cases have higher minimum values of plastic hinge rotation than the

absolute maximum value reported, it was not necessary to evaluate any more available plastic hinge rotations. Cases 1 through 4 are not shown in Table 24 because their maximum values of plastic hinge rotations are about the same as those in cases 5 through 8 and the available minimum values are much higher due to the smaller design moments (the design of cases 1 through 4 does not include gravity loads).

Behavior of the Frames

An investigation of the time history of the column actions for the 11 design cases considered in this chapter showed that the strong column-weak beam design criterion described in Chapter IV was always satisfied. The accuracy of the column "design procedure" developed in Chapter IV (the term "design procedure" is used here to refer to the determination of the critical column actions), at least for the cases herein investigated, was found to be excellent. Most of the columns reached the threshold of yielding; that is, the columns were just "strong enough" to remain elastic. No unnecessary conservatism was found when comparing the actual column actions with those called for in the design procedure developed in Chapter IV.

Summary

At the beginning of this chapter it was stated that the purpose of the investigation presented herein was to answer the following two questions:

- (1) How accurate are the inelastic demands predicted using the IRSA when compared to results obtained from rigorous step-by-step inelastic dynamic analyses?

- (2) Are the design procedures developed in Chapter IV able to insure that the resulting frames behave according to the strong column-weak beam design criterion?

Based on the results and observations herein presented these questions can be answered as follows:

- (1) the predicted inelastic demands using the IRSA are in good agreement with the values obtained from rigorous inelastic dynamic analysis.
- (2) The design procedures developed in Chapter IV are effective in preventing any significant column yielding, thus insuring a strong column-weak beam behavior for the frames.

It should be emphasized that there is neither an intention or a need to predict inelastic rotations and deflections with a high degree of precision. In order for the predictions of inelastic deformations obtained using the IRSA to be acceptable and useful from the viewpoint of design, they should be accurate within, say, 20 to 30%. The results of the investigation indicate that such an accuracy may reasonably be expected from the method.

CHAPTER VII

SUMMARY AND CONCLUSIONS

I. SUMMARY

The Applied Technology Council (3) in 1974 undertook a major effort to evaluate the potential use of the Inelastic Response Spectrum Approach (IRSA) to improve on present methods of seismic analysis for use in design. The method is not a rigorously correct one; thus, an evaluation of its accuracy requires that it be compared to results obtained from methods which are rigorously correct. The primary objective of this dissertation, consistent with the research needs identified in the Applied Technology Council's report (3), was to test the applicability of the IRSA as a tool for the analysis and design of reinforced concrete moment resisting frames subjected to seismic excitation. The results obtained using the IRSA were compared to results obtained using DRAIN-2D, a computer program with inelastic dynamic analysis capabilities. Two frames were designed for various combinations of ground motions and design ductilities and were then analyzed using DRAIN-2D. The results of these analyses and comparisons with the IRSA predictions are reported in Chapter VI.

Another objective of this dissertation was to develop design procedures which would insure that the inelastic action in the frames designed according to the IRSA would be restricted to the beams. The result of the investigation conducted with this aim is the column design

procedure developed in Chapter IV (given by Equations (4.35) through (4.37) and (4.42) through (4.44)).

Two different ground motions were used in this work; the first one, (GM1) is a short event obtained by taking two seconds of strong shaking from the El Centro Earthquake of December 30, 1934. Ground motion 2 (GM2) corresponds to the first six seconds of the El Centro Earthquake of May 1940. For the design of FRAME 2 the accelerations in GM2 were scaled up to give a peak ground acceleration of 0.5g. The development of inelastic response spectra is presented in Chapter III. Figures 10, 11 and 12 show the inelastic spectra for ground motions 1, 2 and 2A respectively.

An equation (Equation 5.8) relating the design ductility, μ , to the plastic hinge rotation expected in the beams was developed in Chapter V. This equation was later modified in Chapter VI to account for the effect of the finite size of the beam-column connection. The joint size correction (Ω in Equation (6.12)), even though small for the frames herein considered, is of major importance in frames with wide columns such as those considered in the analyses of coupled shear walls (29). The equations necessary to calculate the available plastic hinge rotations from the moment-curvature relationships are also presented in Chapter V.

A design example, illustrating the application of the IRSA as herein developed, is presented in Appendix C.

II. CONCLUSIONS

All comparisons upon which conclusions are based were limited to two reinforced concrete frames. These two frames were chosen to provide

results which are believed to be typical of results which would be
 s designed by the IRSA with the design criteria pre-
 wever, it should be noted here that further, similar
 er frames would be most useful in documenting

arison between predicted inelastic demands from the
 ined from step-by-step inelastic dynamic analyses,
 usions can be drawn:

drift ductility, μ_s , defined as the ratio between the
 ory drift and the yield value as obtained from
 4.27), is in good agreement with the design ductility,

designed according to the criteria described in this
 on, the plastic hinge rotations in the beams can be
 ith sufficient accuracy from Equation (5.8) (or
 5.12) if the joint size effect is included).

- (3) Although not enough cases were here considered to permit a
 definite conclusion, the results obtained indicated that, as one
 may expect, the accuracy of the IRSA decreases as the design
 ductility increases.
- (4) When gravity loading is included in the design as described in
 Chapter IV, the inelastic demands (which are computed to be
 independent of gravity loads) are not appreciably affected.
- (5) The effect of providing over-strength in some of the beams of
 the frame (for example through the inclusion of the ACI Code

freedom structures subjected to time
 varying loads, U.T.K., Fall 1977

requirements for $\rho_{\min}$) is to decrease the inelastic demands in the levels where these beams are located (typically the upper stories) and to increase them in some of the adjacent, now relatively weaker stories.

- (6) The design procedures developed in this dissertation were efficient in insuring a strong column-weak beam behavior for the frames herein considered.

The following findings are not directly the result of comparisons between DRAIN-2D and IRSA results; for this reason they are separated from the previous set of conclusions. The findings presented in the following are the result of the analytical studies and general research which the preparation of this dissertation required. Although these ideas have been previously discussed herein, the writer feels that their importance justifies their inclusion in this section.

- (1) The design ductility, μ , used in developing the inelastic response spectra is (within the accuracy of the IRSA) equal to the story drift ductility, μ . The curvature ductility, $\frac{\phi_u}{\phi_y}$, required at the critical sections, is not in general equal to the design ductility, μ , as it has been sometimes assumed (9, 15). The $\frac{\phi_u}{\phi_y}$ ratio at the critical sections can be calculated from the required plastic hinge rotations and the equivalent spread of plasticity using Equation (5.14).
- (2) The design ductility, μ , is not an absolute measure of the inelastic demands expected in a structure; the rigidity of the structure plays a role which is just as important as the design ductility in determining the severity of the inelastic action

expected. This can be seen by comparing the maximum plastic hinge rotations between cases 10 and 11. Both these cases were designed for a ductility of 6; however, as can be seen from Table 24 the maximum plastic hinge rotation in the more flexible FRAME 2 is 2.3 times as great as that in FRAME 1.

Based upon the work presented and discussed in this dissertation, it appears that the IRSA is potentially a very useful, practical tool for use in seismic design of reinforced concrete moment resisting frames. The application of the method begins with the design of the structure to behave in an efficient way from a seismic point of view and culminates in a comparison of required rotation capacities with rotation capacities available.

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APPENDICES

APPENDIX A

TABLES

Table 1

Digitized Data for Ground Motion 1 (GM1)

Time	Accel*	Time	Accel*	Time	Accel*
0.0	0.0	0.070	0.0117	0.120	0.0058
0.130	0.0316	0.160	0.0363	0.260	-0.0596
0.340	0.0241	0.350	0.0134	0.430	0.0035
0.470	0.0086	0.480	0.0086	0.550	0.0483
0.560	0.0303	0.590	0.0306	0.660	0.0291
0.670	0.0225	0.750	0.0946	0.860	0.0769
0.910	0.0025	0.990	-0.1100	1.000	-0.1064
1.150	-0.2551	1.370	0.1653	1.530	-0.0459
1.580	-0.0569	1.610	-0.0599	1.710	-0.0441
1.790	0.0621	1.870	-0.0383	1.930	-0.0497
2.000	0.0473	2.070	0.0	10.000	0.0

*in g units

Table 2

Digitized Data for Ground Motion 2 (GM2)

Time	Accel*	Time	Accel*	Time	Accel*
0.0	0.0	0.010	0.0108	0.040	0.0010
0.100	0.0159	0.160	-0.0001	0.220	0.0189
0.260	0.0001	0.290	0.0059	0.330	-0.0012
0.370	0.0200	0.430	-0.0237	0.470	0.0076
0.580	0.0425	0.620	0.0094	0.660	0.0138
0.720	-0.0256	0.790	-0.0568	0.870	-0.0343
0.940	-0.0603	1.000	-0.0789	1.060	-0.0666
1.090	-0.0429	1.170	0.0897	1.320	-0.1696
1.380	-0.0828	1.410	-0.0828	1.440	-0.0945
1.480	-0.0885	1.510	-0.1080	1.540	-0.1280
1.630	0.1144	1.700	0.2355	1.800	0.1428
1.860	0.1777	1.920	-0.2610	2.010	-0.3194
2.210	0.2952	2.270	0.2634	2.320	-0.2984
2.390	0.0054	2.450	0.2865	2.520	-0.0469
2.570	0.1516	2.650	0.2077	2.710	0.1087
2.770	-0.0325	2.890	0.1033	2.980	-0.0803
3.070	0.0520	3.130	-1.1547	3.210	0.0065
3.250	-0.2060	3.390	0.1927	3.420	-0.0937
3.530	0.1708	3.600	-0.0359	3.670	0.0365
3.740	-0.0736	3.830	0.0311	3.900	-0.1833
4.010	0.0227	4.060	-0.0435	4.110	0.0216
4.220	-0.1972	4.310	-0.1762	4.420	0.1460
4.470	-0.0047	4.620	0.2572	4.670	-0.2045
4.760	0.0608	4.830	-0.2733	4.970	0.1779
5.040	0.0301	5.110	0.2183	5.200	0.0267
5.230	0.1252	5.300	0.1290	5.330	0.1089
5.340	-0.0239	5.450	0.1723	5.510	-0.1021
5.610	0.0141	5.690	-0.1949	5.770	-0.0242
5.800	-0.0050	5.810	-0.0275	5.870	-0.0573
5.880	-0.0327	5.920	0.0216	5.980	0.0108
6.010	0.0235				

*in g units

Table 3

Digitized Spectra for Ground Motion 1 (GM1) $\xi = 0.05$

Period T (sec)	Spectral Acceleration (g units)		
	Elastic $\mu = 1$	$\mu = 4$	$\mu = 6$
0.04	0.2650	0.2240	0.2137
0.06	0.2650	0.2135	0.2044
0.08	0.2778	0.2035	0.1943
0.10	0.2468	0.2035	0.1920
0.15	0.2938	0.1900	0.1723
0.20	0.2505	0.1795	0.1603
0.30	0.3035	0.1680	0.1519
0.40	0.4008	0.1954	0.1645
0.50	0.5133	0.1700	0.1384
0.60	0.5005	0.1681	0.1321
0.70	0.4579	0.1586	0.1203
0.80	0.3963	0.1406	0.1000
0.90	0.3540	0.1207	0.0816
1.00	0.3091	0.1025	0.0689
1.20	0.2303	0.0749	0.0533
1.50	0.1507	0.0537	0.0392
2.00	0.0933	0.0350	0.0256
2.50	0.0690	0.0251	0.0187
3.00	0.0548	0.0192	0.0144
5.00	0.0275	0.0095	0.0068
10.00	0.0028	0.0026	0.0019

Table 4

Digitized Spectra for Ground Motion 2 (GM2) $\xi = 0.05$

Period T (sec)	Spectral Acceleration (g units)		
	Elastic $\mu = 1$	$\mu = 4$	$\mu = 6$
0.02	0.3413	0.2970	0.2930
0.04	0.3588	0.2858	0.2810
0.06	0.4014	0.2750	0.2671
0.08	0.5389	0.2900	0.2811
0.10	0.6028	0.3263	0.3020
0.15	0.7853	0.3667	0.3207
0.20	1.0166	0.3480	0.2817
0.30	0.7664	0.2920	0.2482
0.40	0.8929	0.2726	0.2000
0.50	1.0642	0.2153	0.1416
0.60	0.8219	0.1866	0.1397
0.70	0.5572	0.1721	0.1464
0.80	0.4327	0.1854	0.1234
0.90	0.5588	0.1323	0.1026
1.00	0.4880	0.1278	0.0948
1.20	0.2669	0.0890	0.0705
1.50	0.1699	0.0728	0.0564
2.00	0.1540	0.0603	0.0390
2.50	0.2483	0.0463	0.0347
3.00	0.1676	0.0405	0.0258
5.00	0.0559	0.0116	0.0072
10.00	0.0111	0.0030	0.0020

Table 5

Digitized Spectra for Ground Motion 2A (GM2A) $\xi = 0.05$

Period T (sec)	Spectral Acceleration (g units)		
	Elastic $\mu = 1$	$\mu = 4$	$\mu = 6$
0.02	0.5343	0.4649	0.4587
0.04	0.5617	0.4474	0.4400
0.06	0.6284	0.4305	0.4181
0.08	0.8436	0.4540	0.4400
0.10	0.9436	0.5108	0.4728
0.15	1.2293	0.5740	0.5020
0.20	1.5914	0.5448	0.4410
0.30	1.1997	0.4571	0.3855
0.40	1.3978	0.4267	0.3131
0.50	1.6659	0.3370	0.2217
0.60	1.2866	0.2921	0.2187
0.70	0.8723	0.2694	0.2292
0.80	0.6744	0.2902	0.1932
0.90	0.8748	0.2071	0.1606
1.00	0.7639	0.2001	0.1484
1.20	0.4178	0.1393	0.1104
1.50	0.2660	0.1140	0.0883
2.00	0.2411	0.0944	0.0611
2.50	0.3887	0.0725	0.0543
3.00	0.2624	0.0634	0.0404
5.00	0.0875	0.0182	0.0113
10.00	0.0174	0.0047	0.0031

Table 6

Equivalent Beam Inertias for Dynamic Analysis of FRAME 2

Beams at Level	Moment of Inertia (in. ⁴)
1	92993
2	92993
3	92993
4	92993
5	92993
6	92993
7	72927
8	72927
9	56113
10	42265

Table 7
Modal Shapes and Natural Periods for FRAME 1

	Mode	Period sec.	Modal Amplitudes					
			Story Level					
			1	2	3	4	5	6
FRAME 1	1	0.631	0.1424	0.3368	0.5501	0.7302	0.8927	1.0000
	2	0.220	-0.3850	-0.7581	-0.8342	-0.4879	0.2647	1.0000
	3	0.128	0.7477	1.0000	0.1007	-0.9480	-0.7556	1.0000
	4	0.090	0.6530	0.4144	-0.6844	-0.3932	1.0000	-0.5194
	5	0.066	0.8199	-0.1538	-0.8170	1.0000	-0.5583	0.1718
	6	0.050	-0.9593	1.0000	-0.6692	0.3315	-0.1053	0.0223

Table 8
Modal Shapes and Natural Periods for FRAME 2

Mode	Period sec.	Modal Amplitudes									
		Story Level									
		1	2	3	4	5	6	7	8	9	10
1	1.106	0.1369	0.2445	0.3527	0.4565	0.5600	0.6612	0.7640	0.8596	0.9452	1.0000
2	0.407	-0.3508	-0.5800	-0.7357	-0.7893	-0.7273	-0.5373	-0.2125	0.2151	0.6828	1.0000
3	0.229	-0.6106	-0.8673	-0.7917	-0.4013	0.1915	0.7449	1.0000	0.6644	-0.1901	-0.9714
4	0.154	0.7556	0.8140	0.2501	-0.5221	-0.9251	-0.5266	0.4436	1.0000	0.3887	-0.8250
5	0.112	0.8937	-0.5825	0.4635	0.9704	0.2319	-0.8859	-0.7704	0.6994	1.0000	-0.8287
6	0.087	0.7836	0.1511	-0.7873	-0.4008	0.7539	0.4796	-0.8343	-0.2369	1.0000	-0.5216
7	0.068	0.7770	-0.2554	-0.7899	0.4649	0.6771	-0.8475	-0.1649	1.0000	-0.9493	0.3587
8	0.056	0.7079	-0.5575	-0.3991	0.8667	-0.3408	-0.5198	1.0000	-0.9606	0.5773	-0.1784
9	0.047	0.6355	-0.7889	0.2020	0.5557	-0.9923	1.0000	-0.6992	0.4098	-0.1853	0.0493
10	0.038	0.5452	-0.9583	1.0000	-0.7975	0.5043	-0.2824	0.1295	0.0546	0.0195	-0.0042

FRAME 2

Table 9
Spectral Accelerations for FRAME 1

Mode	Period (see)	Damping ratio ξ	Spectral Acceleration g's					
			GM1			GM2		
			Elastic	$\mu=4$	$\mu=6$	Elastic	$\mu=4$	$\mu=6$
FRAME 1	1	0.631	0.4946	0.1654	0.1300	0.7426	0.1749	0.1425
	2	0.220	0.3275	0.1696	0.1505	0.7920	0.2983	0.2665
	3	0.128	0.2539	0.1916	0.1779	0.6170	0.3574	0.3053
	4	0.090	0.2634	0.1978	0.1860	0.5002	0.2970	0.2809
	5	0.066	0.2604	0.2027	0.1904	0.4222	0.2682	0.2550
	6	0.050	0.2618	0.2054	0.1942	0.3505	0.2727	0.2599

Table 10
Spectral Accelerations for FRAME 2

Mode	Period (sec)	Damping Ratio ξ	Spectral Accelerations, g's	
			Elastic	GM2A $\mu=6$
1	1.106	0.050	0.5961	0.1243
2	0.407	0.050	1.4061	0.2960
3	0.229	0.072	1.0491	0.3850
4	0.154	0.102	1.0118	0.4482
5	0.112	0.136	0.7617	0.4324
6	0.087	0.174	0.7187	0.3963
7	0.068	0.221	0.6158	0.3759
8	0.056	0.266	0.5504	0.3765
9	0.047	0.319	0.5495	0.3772
10	0.038	0.391	0.5607	0.3768

Table 11

Calculation of Equivalent Lateral Forces to Simulate the P-Δ Effect for FRAME 2, μ=6 and GM2A

Level	P _j (k)	δ _e (in.)	h _j (in.)	ΔS _{p-Δ} ^{*ψ} (k)	Equivalent [*] Forces (k)	δ ₁ (in.)	λ ^{**}	ΔS _{p-Δ} ^Ω (k)	Equivalent ^Ω Forces (k)
10	378.0	0.1480	144	2.33	2.33	0.0055	1.006	2.34	2.34
9	799.2	0.2199	144	7.32	4.99	0.0101	1.008	7.38	5.04
8	1220.4	0.2275	144	11.57	4.25	0.0122	1.009	11.67	4.29
7	1641.6	0.2288	144	15.65	4.08	0.0136	1.010	15.81	4.14
6	2062.8	0.2158	144	18.55	2.90	0.0137	1.011	18.75	2.94
5	2484.0	0.2158	144	22.34	3.79	0.0147	1.011	22.59	3.84
4	2905.2	0.2152	144	26.05	3.71	0.0157	1.012	26.36	3.77
3	3326.4	0.2265	144	31.39	5.34	0.0180	1.013	31.80	5.44
2	3747.6	0.2294	144	35.82	4.43	0.0194	1.014	36.32	4.52
1	4168.8	0.2982	192	38.85	3.03	0.0259	1.015	39.43	3.11

* {λ} = {1}

Ω Final Values

ψ From Equation (4.24)

** From Equation (4.25)

Table 12
Design Cases Used to Investigate the Applicability of the IRSA

Basic Denomination	Cases Investigated					
	Gravity Loading Not Included			Gravity Loading Included		
	ρ_{min} Limitation Not Included	ρ_{min} Limitation Included	ρ_{min} Limitation Not Included	ρ_{min} Limitation Included	ρ_{min} Limitation Not Included	ρ_{min} Limitation Included
IFR1U4GM1	1*			5		
IFR1U6GM1	2*			6		7
IFR1U4GM2	3*					
IFR1U6GM2	4*			8		10
IFR2U6GM2A				9*		11

* Referred to as the basic cases.

Table 13

Calculation of Inelastic Demands for Case 1. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Ductility μ	Story Ductility μ_s	μ_s / μ	δ_y / h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p / θ_{pp}
1	0.1234	0.5280	4.0	4.3	1.08	0.00077	0.00267	0.00316	1.18
2	0.1677	0.6808	4.0	4.1	1.03	0.00112	0.00336	0.00351	1.04
3	0.1833	0.6860	4.0	3.7	0.93	0.00122	0.00366	0.00311	0.85
4	0.1555	0.5737	4.0	3.7	0.93	0.00104	0.00312	0.00248	0.79
5	0.1429	0.4536	4.0	3.2	0.80	0.00095	0.00285	0.00184	0.65
6	0.0978	0.3827	4.0	3.9	0.98	0.00065	0.00195	0.00200	1.03

$$\Delta\theta = \frac{-0.7 \times 4551.8 \times 160}{6 \times 5000 \times 47200} = -0.00036 \quad \left(\frac{\mu_s}{\mu}\right)_{av} = 0.96 \quad \left(\frac{\theta_p}{\theta_{pp}}\right)_{av} = 0.92$$

*Obtained from DRAIN-2D

Table 14

Calculation of Inelastic Demands for Case 2. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Design Ductility μ	Story Ductility μ_s	μ_s μ	δ_y h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p θ_{pp}
1	0.0973	0.6282	6.0	6.5	1.08	0.00061	0.00339	0.00394	1.16
2	0.1320	0.7559	6.0	5.7	0.95	0.00088	0.00440	0.00427	0.97
3	0.1442	0.7699	6.0	5.3	0.88	0.00096	0.00480	0.00389	0.81
4	0.1225	0.6799	6.0	5.6	0.93	0.00082	0.00410	0.00337	0.82
5	0.1131	0.5709	6.0	5.0	0.83	0.00075	0.00375	0.00292	0.78
6	0.0783	0.5419	6.0	6.9	1.15	0.00052	0.00260	0.00322	1.24

$$\Delta\theta = \frac{-0.85 \times 3591 \times 160}{6 \times 5000 \times 47200} = -0.00034 \quad \left(\frac{\mu_s}{\mu}\right)_{av} = 0.97 \quad \left(\frac{\theta_p}{\theta_{pp}}\right)_{av} = 0.96$$

*Obtained from DRAIN-2D

Table 15

Calculation of Inelastic Demands for Case 3. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Design Ductility μ	Story Ductility μ_s	μ_s / μ	δ_y / h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p / θ_{pp}
1	0.1327	0.3717	4.0	2.8	0.70	0.00083	0.00288	0.00256	0.89
2	0.1787	0.6479	4.0	3.6	0.90	0.00119	0.00357	0.00358	1.00
3	0.1944	0.7547	4.0	3.9	0.98	0.00130	0.00390	0.00371	0.95
4	0.1662	0.6765	4.0	4.1	1.03	0.00111	0.00333	0.00302	0.91
5	0.1572	0.5111	4.0	3.3	0.83	0.00105	0.00315	0.00194	0.62
6	0.1137	0.3982	4.0	3.5	0.88	0.00076	0.00228	0.00201	0.88

$$\Delta\theta = \frac{-0.7 \times 4909.8 \times 160}{6 \times 5000 \times 47200} = -0.00039 \quad \mu_s / \mu_{av} = 0.89 \quad \theta_p / \theta_{pp} = 0.88$$

*Obtained from DRAIN-2D

Table 16
Calculation of Inelastic Demands for Case 4. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Ductility μ	Story Ductility μ_s	μ_s / μ	δ_y / h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p / θ_{pp}
1	0.1086	0.4243	6.0	3.9	0.65	0.00068	0.00378	0.00316	0.84
2	0.1459	0.7167	6.0	4.9	0.82	0.00097	0.00486	0.00418	0.86
3	0.1584	0.7840	6.0	4.9	0.82	0.00106	0.00528	0.00397	0.75
4	0.1357	0.6761	6.0	5.0	0.83	0.00090	0.00452	0.00321	0.71
5	0.1296	0.5125	6.0	4.0	0.67	0.00086	0.00432	0.00208	0.48
6	0.0949	0.4059	6.0	4.3	0.72	0.00063	0.00316	0.00223	0.71

$$\Delta\theta = \frac{-0.85 \times 4020.2 \times 160}{6 \times 5000 \times 47200} = -0.00039 \quad \mu_s / \mu = 0.75 \quad \theta_p / \theta_{pp} = 0.73$$

*Obtained from DRAIN-2D

Table 17

Calculation of Inelastic Demands for Case 5. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Design Ductility μ	Story Ductility μ_s	μ_s μ	δ_y h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p θ_{pp}
1	0.1234	0.5219	4.0	4.2	1.05	0.00077	0.00267	0.00315	1.18
2	0.1677	0.6794	4.0	4.1	1.03	0.00112	0.00336	0.00351	1.04
3	0.1833	0.6876	4.0	3.8	0.95	0.00122	0.00366	0.00312	0.85
4	0.1555	0.5796	4.0	3.7	0.93	0.00104	0.00312	0.00251	0.80
5	0.1429	0.4586	4.0	3.2	0.80	0.00095	0.00285	0.00188	0.66
6	0.0978	0.3889	4.0	4.0	1.00	0.00065	0.00195	0.00204	1.05

$$\Delta\theta = \frac{-0.7 \times 4551.8 \times 160}{6 \times 5000 \times 47200} = -0.00036 \quad \left(\frac{\mu_s}{\mu}\right)_{av} = 0.96 \quad \left(\frac{\theta_p}{\theta_{pp}}\right)_{av} = 0.93$$

*Obtained from DRAIN-2D

Table 18

Calculation of Inelastic Demands for Case 6. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Ductility μ	Story Ductility μ_s	μ_s μ	δ_y μ_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p θ_{pp}
1	0.0973	0.6044	6.0	6.2	1.03	0.00061	0.00339	0.00386	1.14
2	0.1320	0.7499	6.0	5.7	0.95	0.00088	0.00440	0.00428	0.97
3	0.1442	0.7757	6.0	5.4	0.90	0.00096	0.00480	0.00405	0.84
4	0.1225	0.6924	6.0	5.7	0.95	0.00082	0.00410	0.00353	0.86
5	0.1131	0.5884	6.0	5.2	0.87	0.00075	0.00375	0.00297	0.79
6	0.0783	0.5429	6.0	6.9	1.15	0.00052	0.00260	0.00328	1.26

$$\Delta\theta = \frac{-0.85 \times 3591 \times 160}{6 \times 5000 \times 47200} = -0.00034 \quad \frac{\mu_s}{\mu} = 0.98 \quad \frac{\theta_p}{\theta_{pp}} = 0.98$$

*Obtained from DRAIN-2D

Table 19

Calcualtion of Inelastic Demands for Case 7. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Design Ductility μ	Story Ductility μ_s	μ_s / μ	δ_y / h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p / θ_{pp}
1	0.1327	0.3667	4.0	2.8	0.70	0.00083	0.00288	0.00262	0.91
2	0.1787	0.6802	4.0	3.8	0.95	0.00119	0.00357	0.00384	1.08
3	0.1944	0.7849	4.0	4.0	1.00	00.13	0.00390	0.00372	0.95
4	0.1662	0.6624	4.0	4.0	1.00	0.00111	0.00333	0.00265	0.80
5	0.1572	0.4203	4.0	2.7	0.68	0.00105	0.00315	0.00131	0.42
6	0.1137	0.2280	4.0	2.0	0.50	0.00076	0.00228	0.00080	0.35

$$\Delta\theta = \frac{-0.7 \times 4909.8 \times 160}{6 \times 5000 \times 47200} = -0.00039 \quad \left(\frac{\mu_s}{\mu}\right)_{av} = 0.81 \quad \left(\frac{\theta_p}{\theta_{pp}}\right)_{av} = 0.75$$

*Obtained from DRAIN-2D

Table 20

Calculation of Inelastic Demands for Case 8. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Ductility μ	Story Ductility μ_s	μ_s / μ	δ_y / h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p / θ_{pp}
1	0.1086	0.4146	6.0	3.8	0.63	0.00068	0.00378	0.00316	0.84
2	0.1459	0.7172	6.0	4.9	0.82	0.00097	0.00486	0.00419	0.86
3	0.1584	0.7875	6.0	5.0	0.83	0.00106	0.00528	0.00398	0.75
4	0.1357	0.6779	6.0	5.0	0.83	0.00090	0.00452	0.00321	0.71
5	0.1296	0.5147	6.0	4.0	0.67	0.00086	0.00432	0.00208	0.48
6	0.0949	0.4058	6.0	4.3	0.72	0.00063	0.00316	0.00222	0.70

$$\Delta\theta = \frac{-0.85 \times 4020.2 \times 160}{6 \times 5000 \times 47200} = -0.00039 \quad \left(\frac{\mu_s}{\mu}\right)_{av} = 0.75 \quad \left(\frac{\theta_p}{\theta_{pp}}\right)_{av} = 0.72$$

*Obtained from DRAIN-2D

Table 21

Calculation of Inelastic Demands for Case 9. Comparison with DRAIN-2D Results

Story Level	Yield Drift δy (in.)	Maximum Drift Δu (in.)	Design Ductility μ	Story* Ductility μ_s	$\frac{\mu_s}{\mu}$	Ω^{**}	$\delta_h \left(\frac{Y}{h} \right) \Omega$	Plastic hinge Rotation θ_{pp} (rad)	Actual Ψ Hinge rotation θ_{pp} (rad)	$\frac{\theta_p}{\theta_{pp}}$
1	0.3027	1.1457	6.0	3.8	0.63	1.114	0.00176	0.00960	0.00769	0.80
2	0.2326	1.3457	6.0	5.8	0.97	1.114	0.00180	0.00900	0.00964	1.07
3	0.2294	1.5069	6.0	6.6	1.10	1.107	0.00176	0.00880	0.01009	1.15
4	0.2178	1.5355	6.0	7.1	1.18	1.107	0.00167	0.00835	0.00985	1.18
5	0.2182	1.4338	6.0	6.6	1.10	1.099	0.00167	0.00835	0.00861	1.03
6	0.2182	1.2634	6.0	5.8	0.97	1.099	0.00167	0.00835	0.00732	0.88
7	0.2311	1.1184	6.0	4.8	0.80	1.091	0.00175	0.00875	0.00679	0.78
8	0.2295	1.1348	6.0	4.9	0.82	1.109	0.00174	0.00870	0.00623	0.72
9	0.2217	1.0825	6.0	4.9	0.82	1.084	0.00167	0.00835	0.00616	0.74
10	0.1489	0.9246	6.0	6.2	1.03	1.084	0.00112	0.00560	0.00626	1.12

$$\Delta\theta = \frac{-0.85 \times 10351 \times 175}{6 \times 3605 \times 89430} = -0.00080$$

$$\left(\frac{\mu_s}{\mu} \right)_{av} = 0.94$$

$$\left(\frac{\theta_p}{\theta} \right)_{av} = 0.95$$

* Obtained from DRAIN-2D

** Average value for the story considered

 Ψ Maximum for the level considered

Table 22

Calculation of Inelastic Demands for Case 10. Comparison with DRAIN-2D Results

Story Level	Yield Drift δ_y (in.)	Maximum Drift* Δ_u (in.)	Design Ductility μ	Story Ductility μ_s	μ_s / μ	δ_y / h_j	Plastic Hinge Rotation Predicted θ_{pp} (rad)	Actual* Hinge Rotation θ_p (rad)	θ_p / θ_{pp}
1	0.1086	0.4955	6.0	4.6	0.77	0.00068	0.00378	0.00362	0.96
2	0.1459	0.7756	6.0	5.3	0.88	0.00097	0.00486	0.00450	0.93
3	0.1584	0.7983	6.0	5.0	0.83	0.00106	0.00528	0.00368	0.70
4	0.1357	0.6031	6.0	4.4	0.73	0.00090	0.00452	0.00220	0.49
5	0.1296	0.3855	6.0	3.0	0.50	0.00086	0.00432	0.00135	0.31
6	0.0949	0.2124	6.0	2.2	0.37	0.00063	0.00316	0.00066	0.21

$$\Delta\theta = \frac{-0.85 \times 4020.2 \times 160}{6 \times 5000 \times 47200} = -0.00039 \quad \left(\frac{\mu_s}{\mu} \right)_{av} = 0.68 \quad \left(\frac{\theta_p}{\theta_{pp}} \right)_{av} = 0.60$$

*Obtained from DRAIN-2D

Table 23

Calculation of Inelastic Demands for Case 11. Comparison with DRAIN-2D Results

Story Level	Yield Drift δy (in.)	Maximum Drift Δu (in.)	Design Ductility μ	Story* Ductility μ_s	$\frac{\mu_s}{\mu}$	Ω^{**}	$\delta \left(\frac{Y}{h} \right) \Omega$	Plastic hinge Rotation θ_{pp} (rad)	Actual Ψ^* Hinge rotation θ_{pp} (rad)	$\frac{\theta_p}{\theta_{pp}}$
1	0.3027	1.1617	6.0	3.8	0.63	1.114	0.00176	0.00960	0.00758	0.79
2	0.2326	1.3531	6.0	5.8	0.97	1.114	0.00180	0.00900	0.00951	1.06
3	0.2294	1.5150	6.0	6.6	1.10	1.107	0.00176	0.00880	0.01016	1.15
4	0.2178	1.5475	6.0	7.1	1.18	1.107	0.00167	0.00835	0.00998	1.20
5	0.2182	1.4520	6.0	6.7	1.12	1.099	0.00167	0.00835	0.00877	1.05
6	0.2182	1.2887	6.0	5.9	0.98	1.099	0.00167	0.00835	0.00756	0.91
7	0.2311	1.1435	6.0	4.9	0.82	1.091	0.00175	0.00875	0.00683	0.78
8	0.2295	1.1043	6.0	4.8	0.80	1.091	0.00174	0.00870	0.00608	0.70
9	0.2217	0.9878	6.0	4.5	0.75	1.084	0.00167	0.00835	0.00415	0.50
10	0.1489	0.6419	6.0	4.3	0.72	1.084	0.00112	0.00560	0.00389	0.69

$$\Delta \theta = \frac{-0.85 \times 10351 \times 175}{6 \times 3605 \times 89430} = -0.00080 \quad \left(\frac{\mu_s}{\mu} \right)_{av} = 0.91 \quad \left(\frac{\theta_p}{\theta_{pp}} \right)_{av} = 0.88$$

* Obtained from DRAIN-2D

** Average value for the story considered

Ψ Maximum for the level considered

Table 24

Comparison of Maximum Hinge Rotation with Minimum Available

Case	Maximum hinge Rotation at Level	$(\theta_p)_{\max}$ (rad)	$(\theta_{pa})_{\min}$ (rad)	Comments
5	2	0.00351	0.0116	ok
6	2	0.00428	>0.0116	ok
7	2	0.00384	0.0116	ok
8	2	0.00419	>0.0116	ok
9	3	0.01009	0.01538	ok
10	2	0.00450	>0.0116	ok
11	3	0.01016	0.01538	ok

APPENDIX B

FIGURES

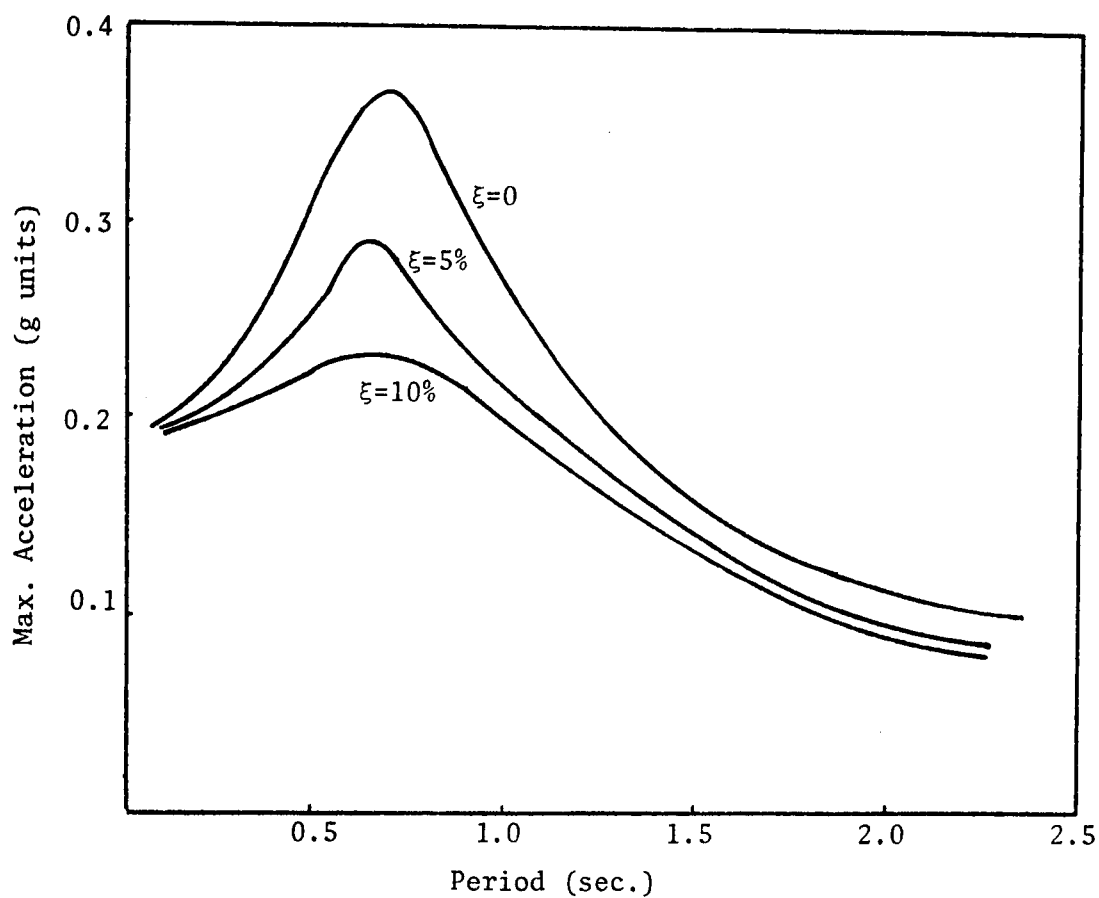


Figure 1. Response Spectrum for Typical Earthquake for Various Damping Ratios Expressed as a Percent of Critical Damping

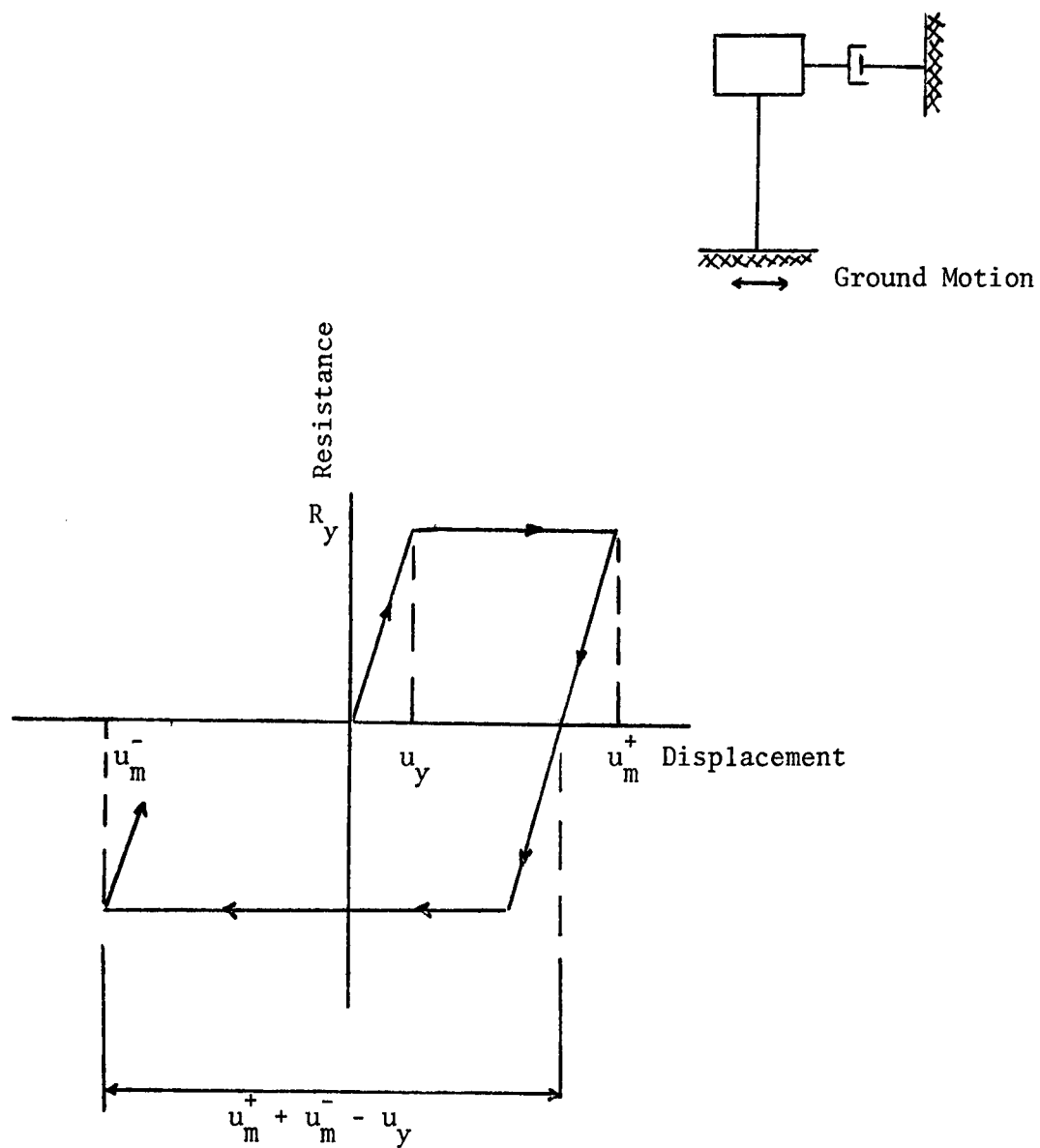


Figure 2. Elasto Plastic System Subjected to Ground Motion

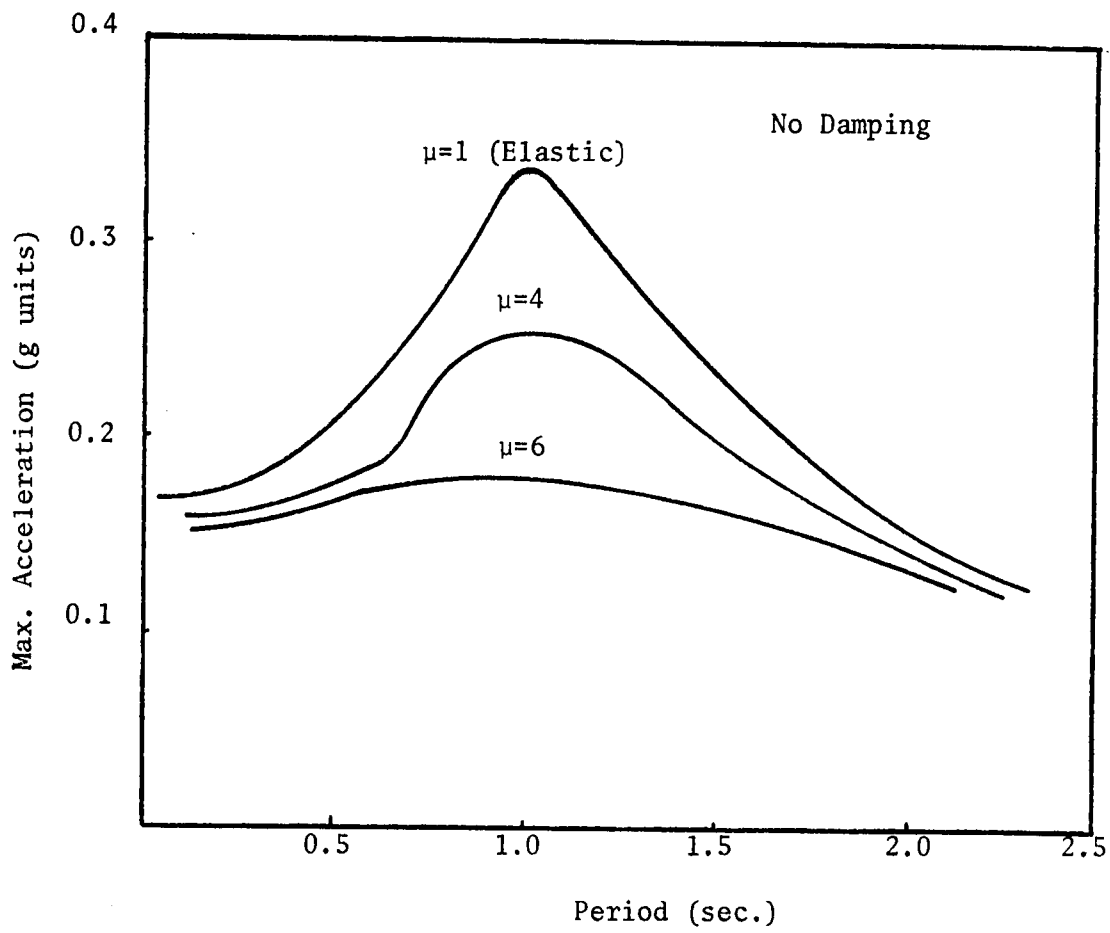


Figure 3. Inelastic Acceleration Spectra for Different Ductility Factors

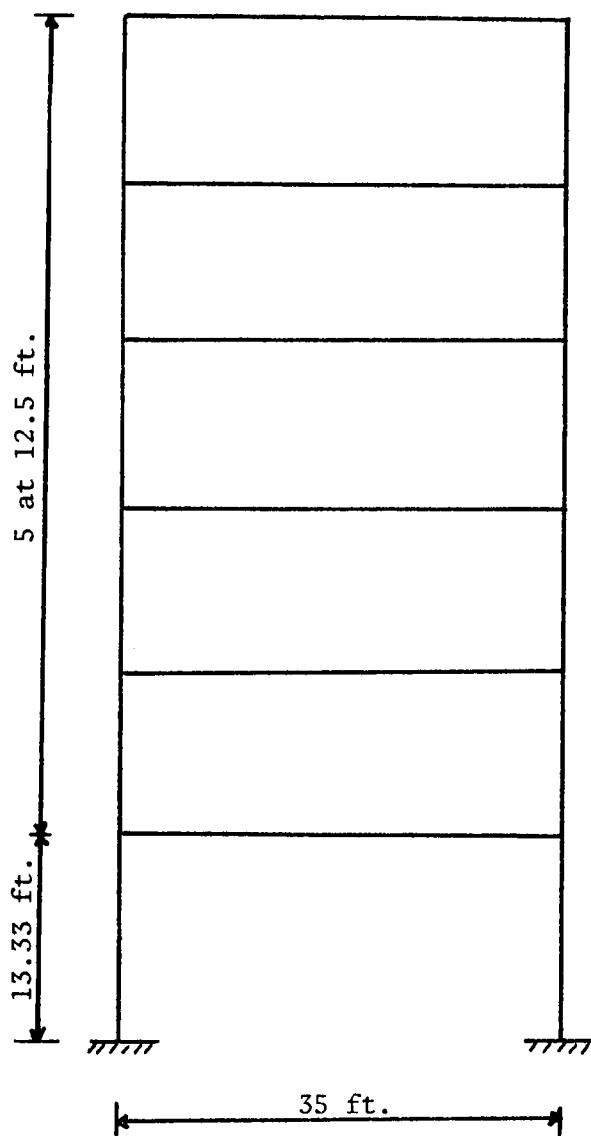


Figure 4. FRAME 1: Elevation Showing the Overall Geometry

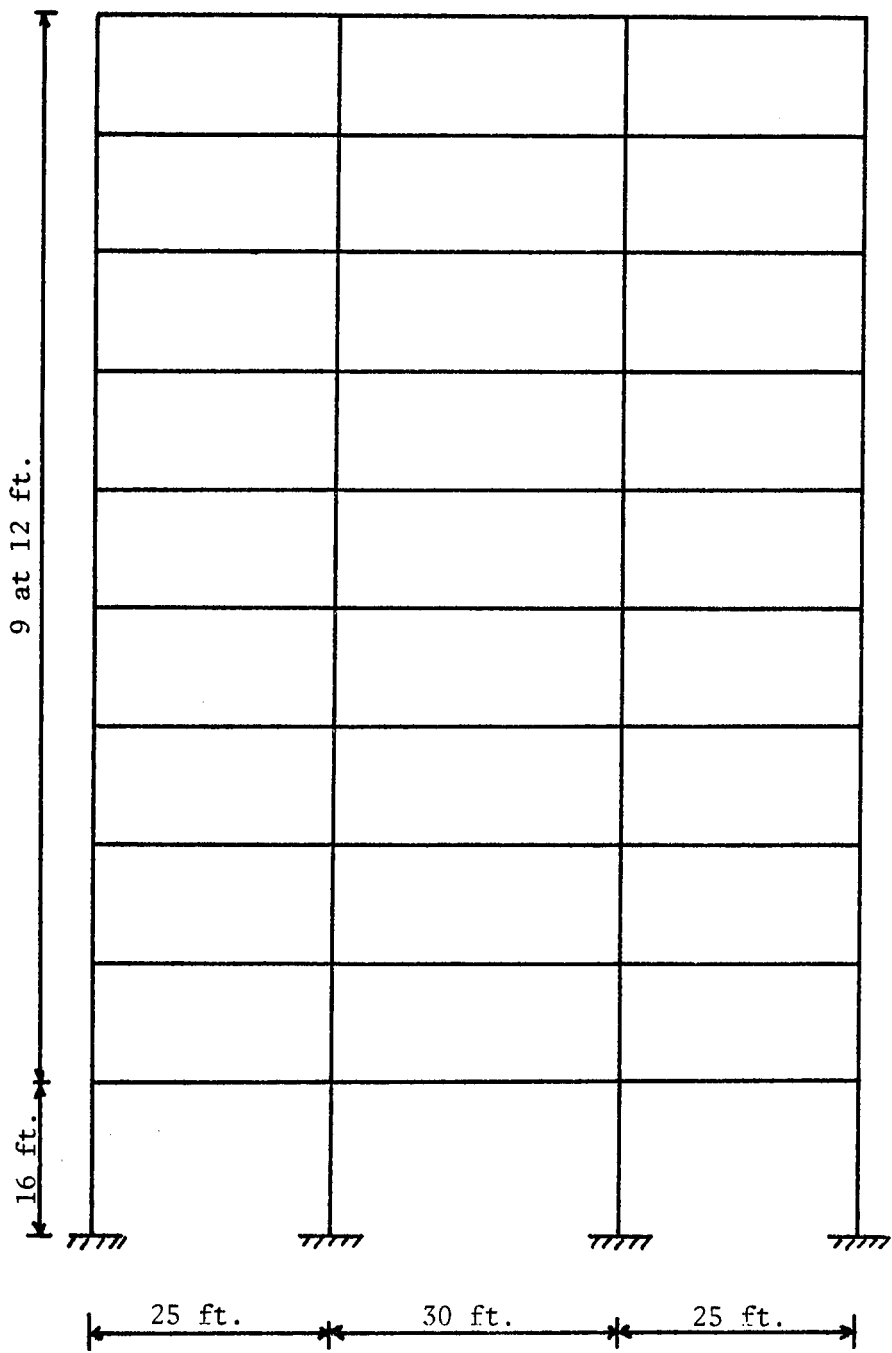


Figure 5. FRAME 2: Elevation Showing the Overall Geometry

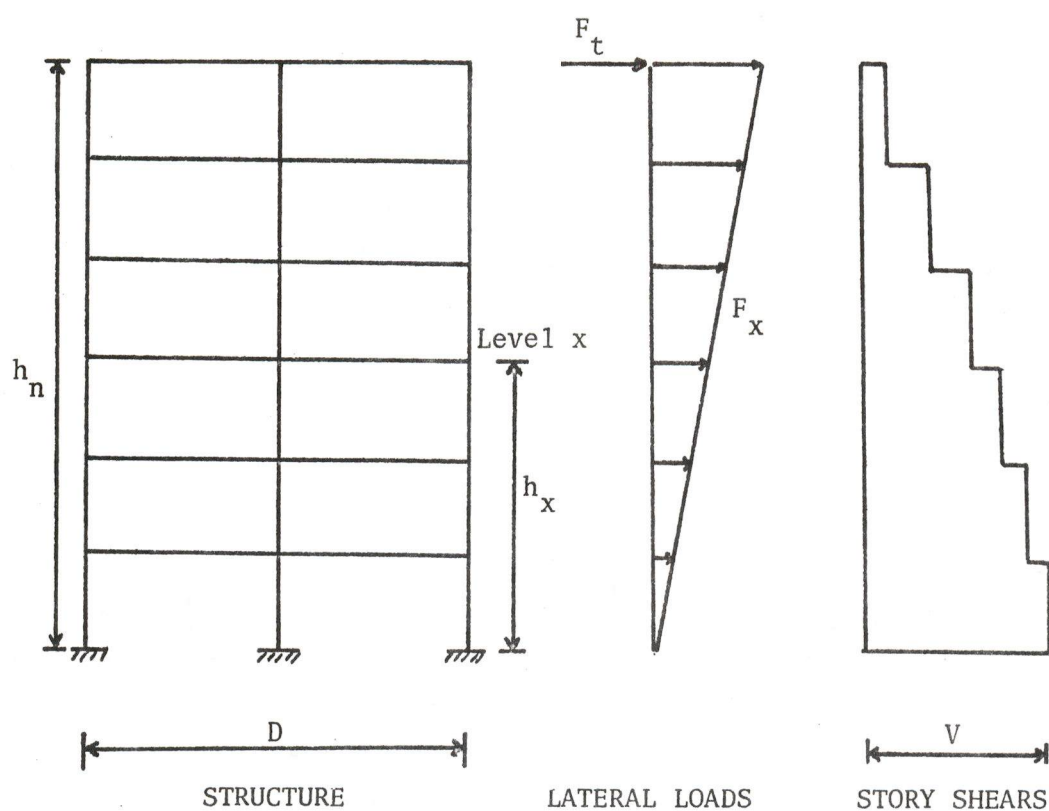
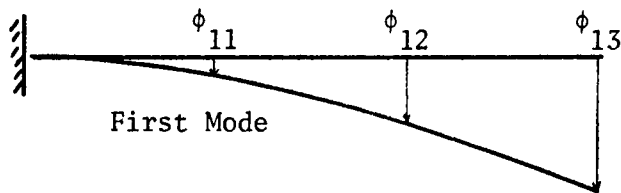
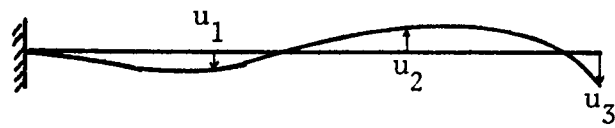
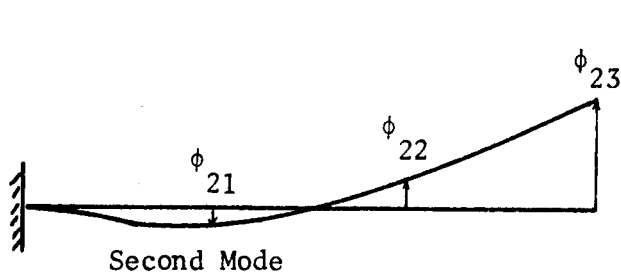


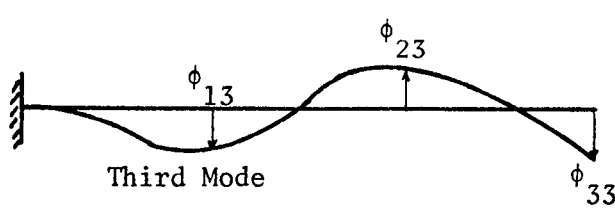
Figure 6. Typical Distribution of Equivalent Lateral Loads and Resulting Story Shears for a Building with Uniform Mass Distribution



$$\phi = \begin{Bmatrix} \phi_{11} \\ \phi_{12} \\ \phi_{13} \end{Bmatrix}$$

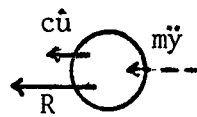
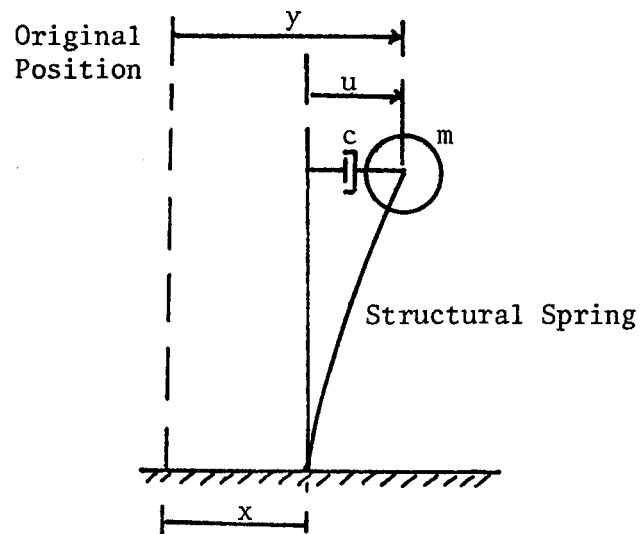


$$\phi = \begin{Bmatrix} \phi_{21} \\ \phi_{22} \\ \phi_{23} \end{Bmatrix}$$

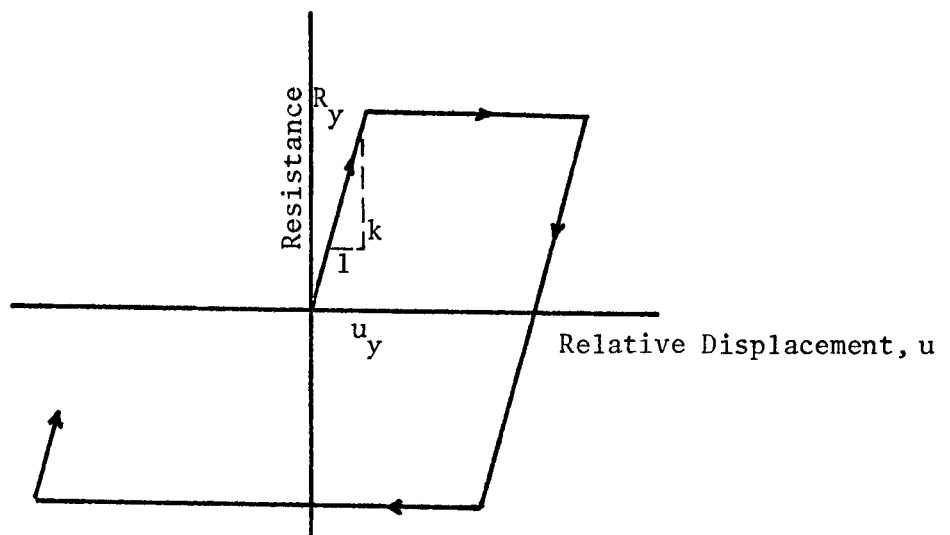


$$\phi = \begin{Bmatrix} \phi_{31} \\ \phi_{32} \\ \phi_{33} \end{Bmatrix}$$

Figure 7. Deflections as the Sum of Modal Components



(a)



(b)

Figure 8. a) Single Degree of Freedom System Subjected to Ground Motion
b) Load-Deflection Curve for the Structural Spring

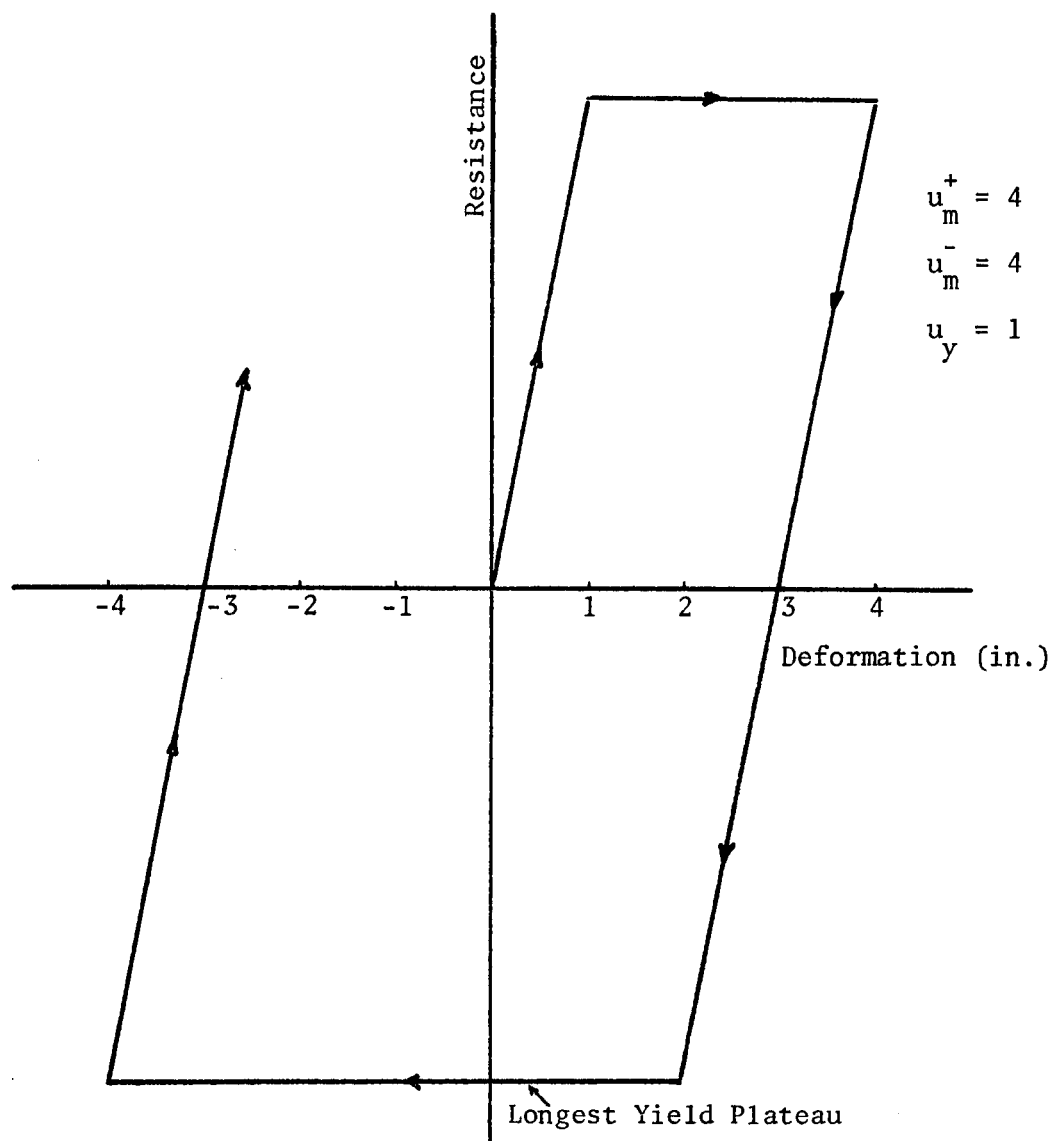


Figure 9. Single Degree of Freedom Response Used to Illustrate the Effects of Cyclic Behavior on the Definition of Ductility

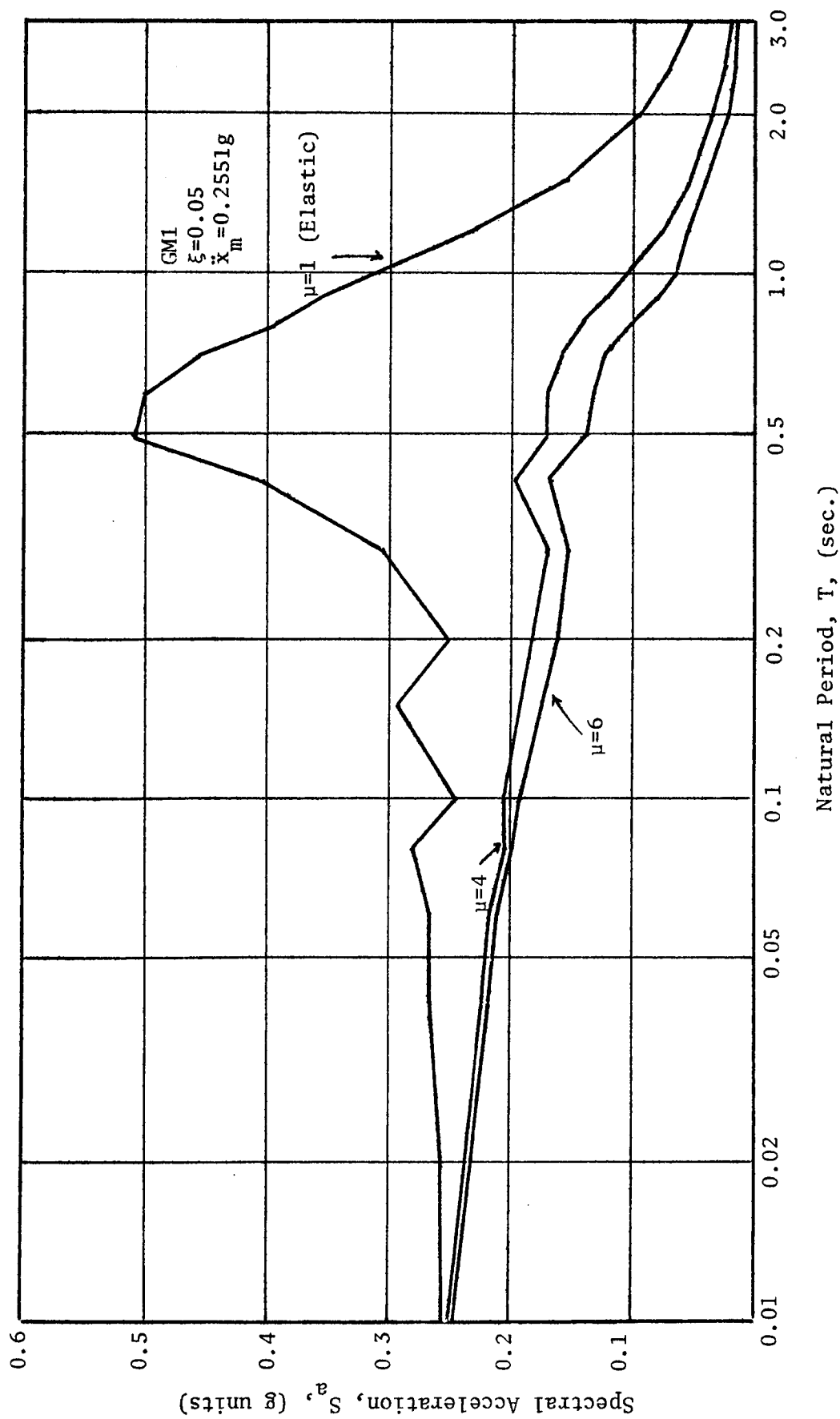


Figure 10. Inelastic Response Spectrum for GM1

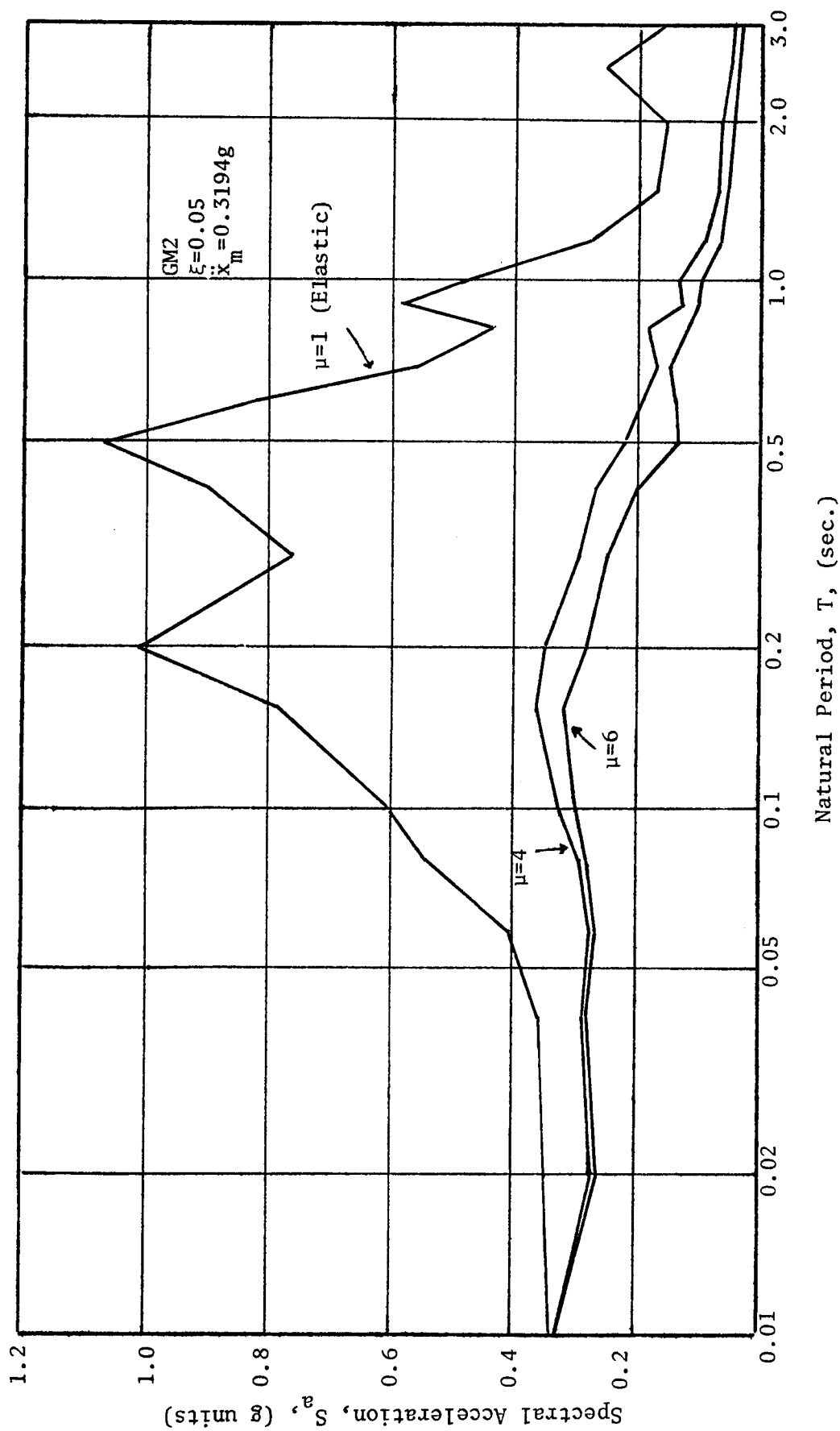


Figure 11. Inelastic Response Spectrum for GM2

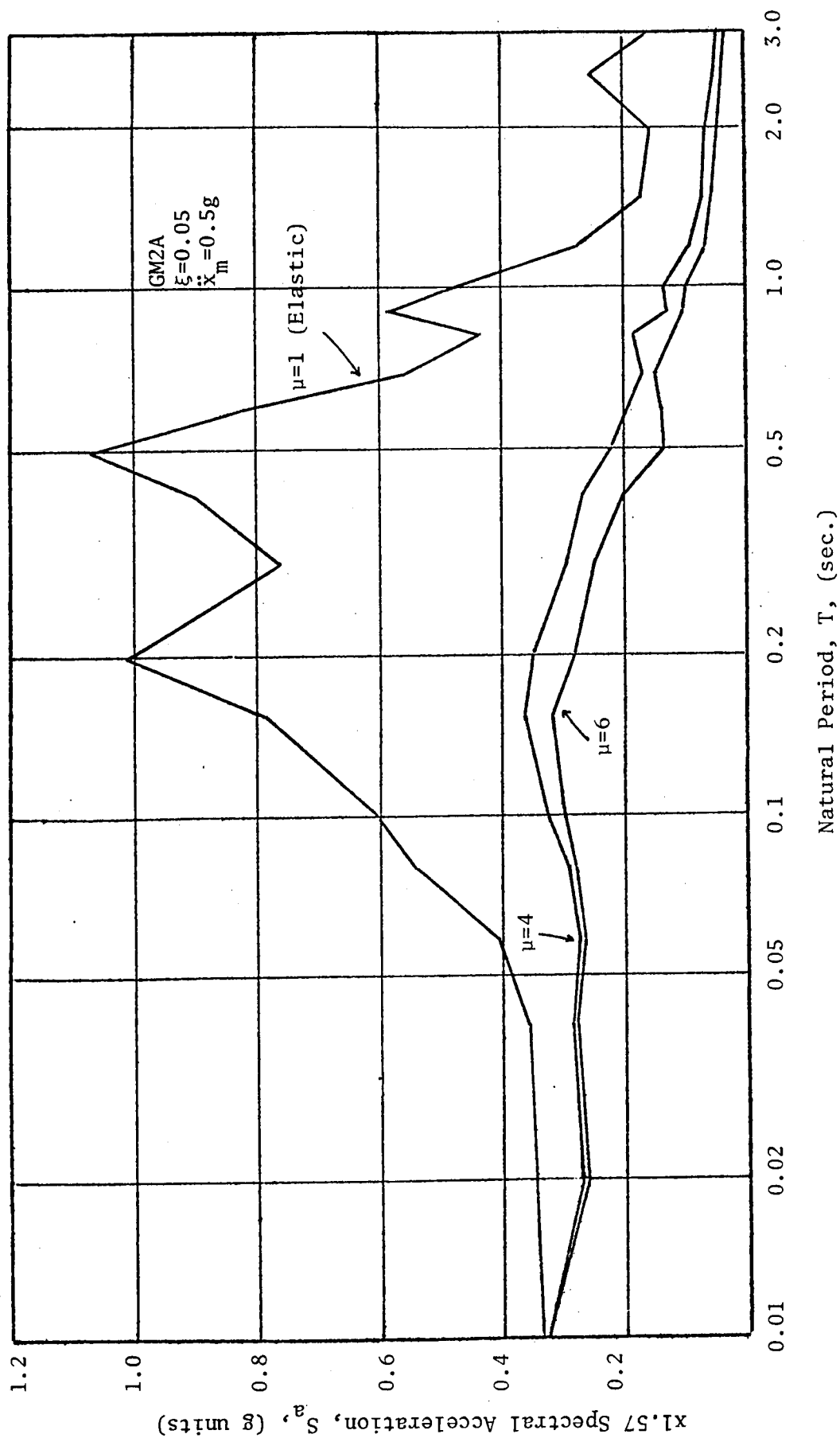


Figure 12. Inelastic Response Spectrum for GM2A

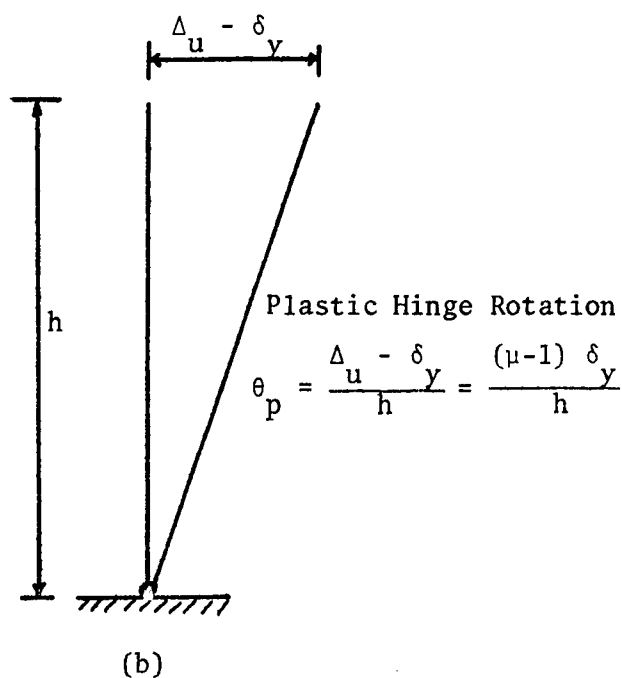
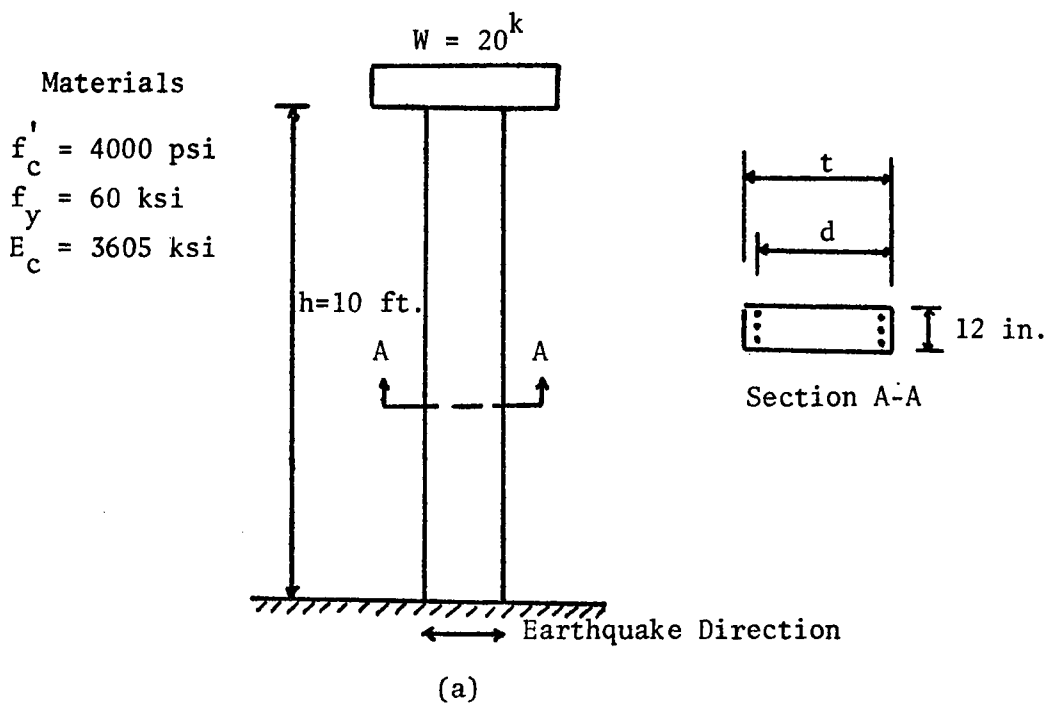


Figure 13. (a) Structure Used in the Example Illustrating the Difference Between Stiff and Flexible Designs
 (b) Plastic Hinge Rotation at the Base of the Column

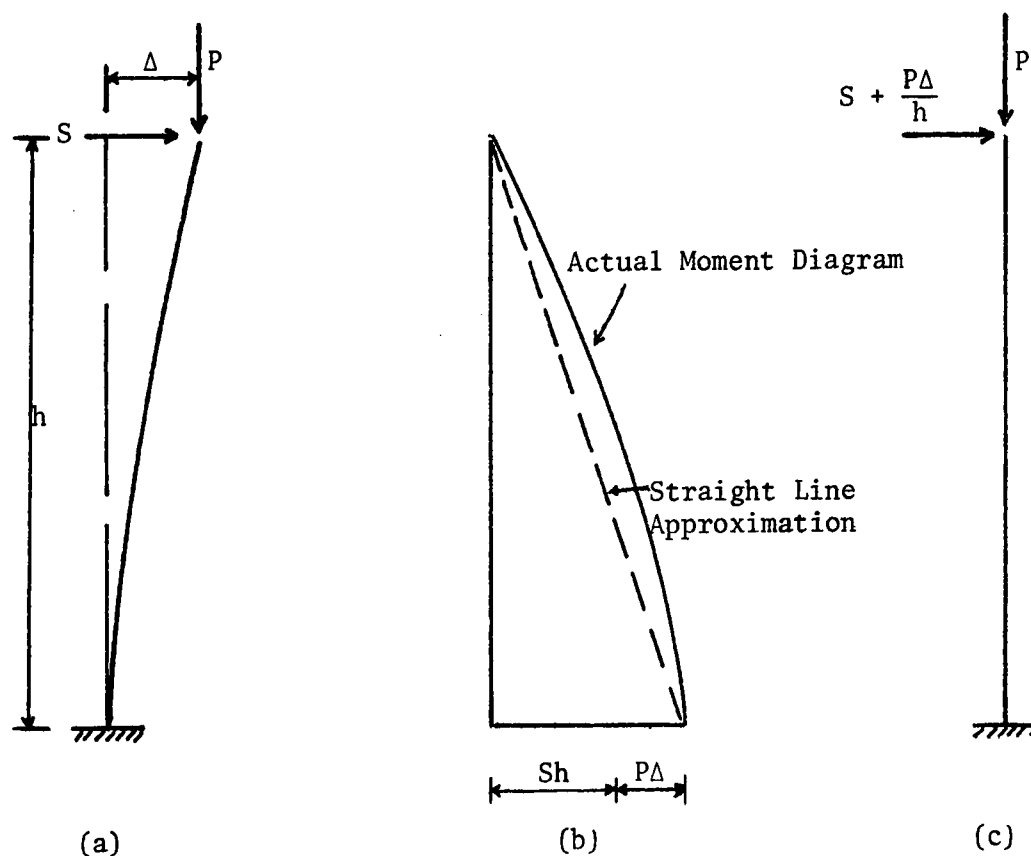


Figure 14. P-Δ Effect for Simple Column

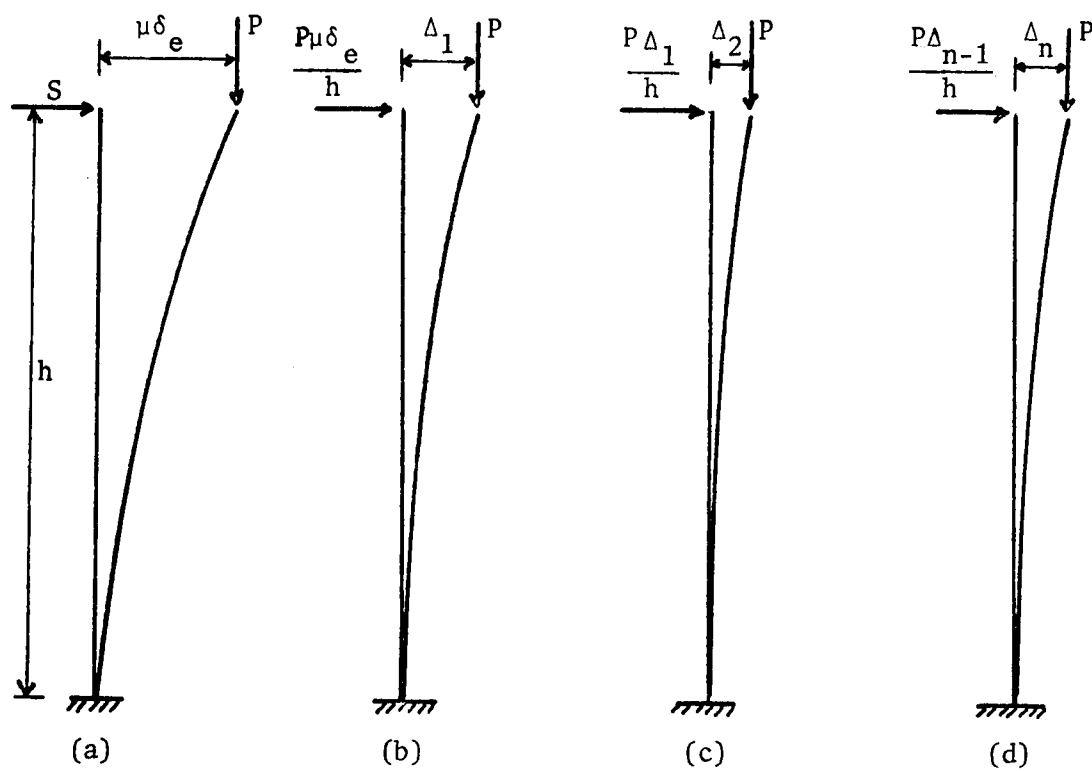


Figure 15. (a) Cantilever Column at Maximum Earthquake Drift (Without Considering the P - Δ Effect) (b), (c) and (d) Additional Drift Induced by P - Δ Effect

• Plastic Hinge

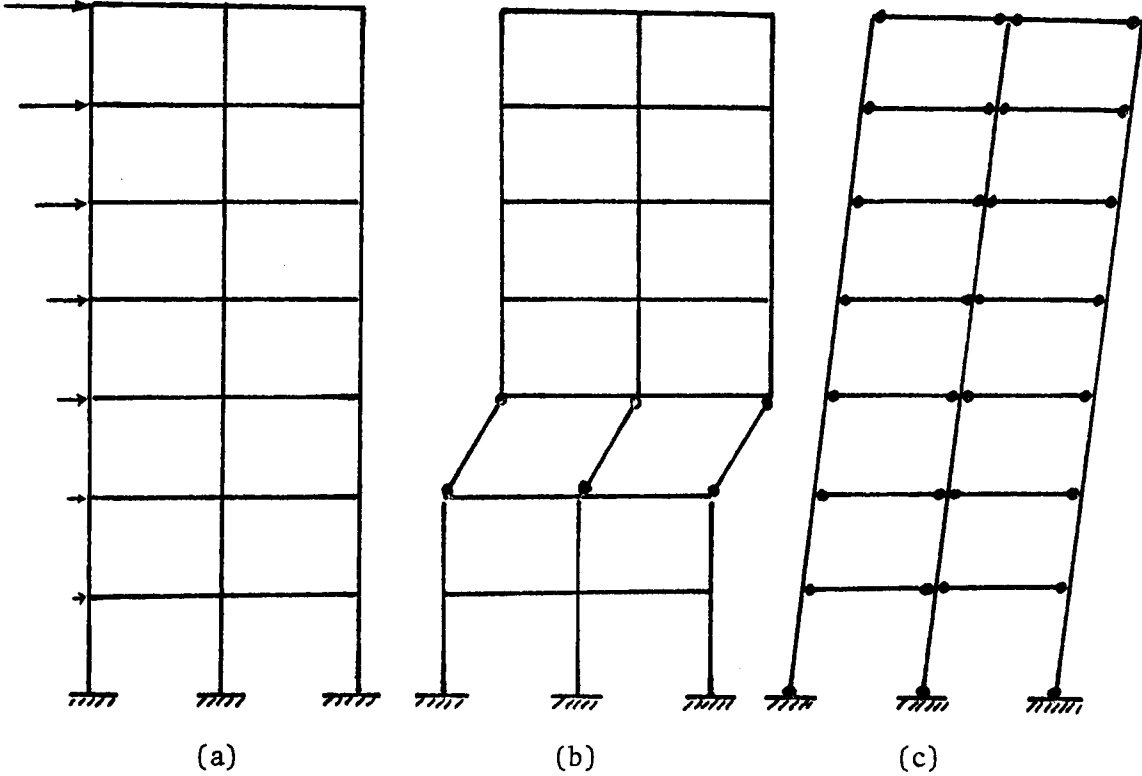
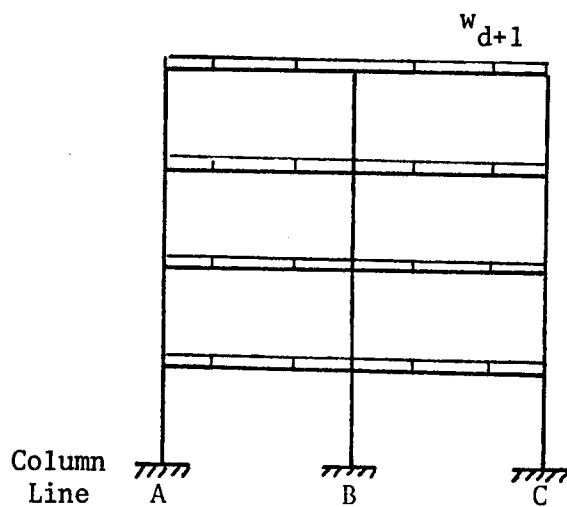
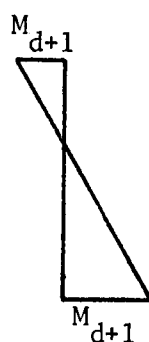


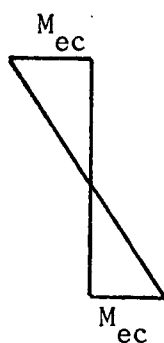
Figure 16. (a) Frame Considered (b) Single Story Column Sidesway Mechanism (c) Beam Sidesway Mechanism



(a)



(b)



(c)

Figure 17. (a) Frame with Gravity Load (b) Typical Moment Diagram for a Column of Line C Under Gravity Load (c) Earthquake Induced Moments in a Typical Column of Line C for a Drift to the Right

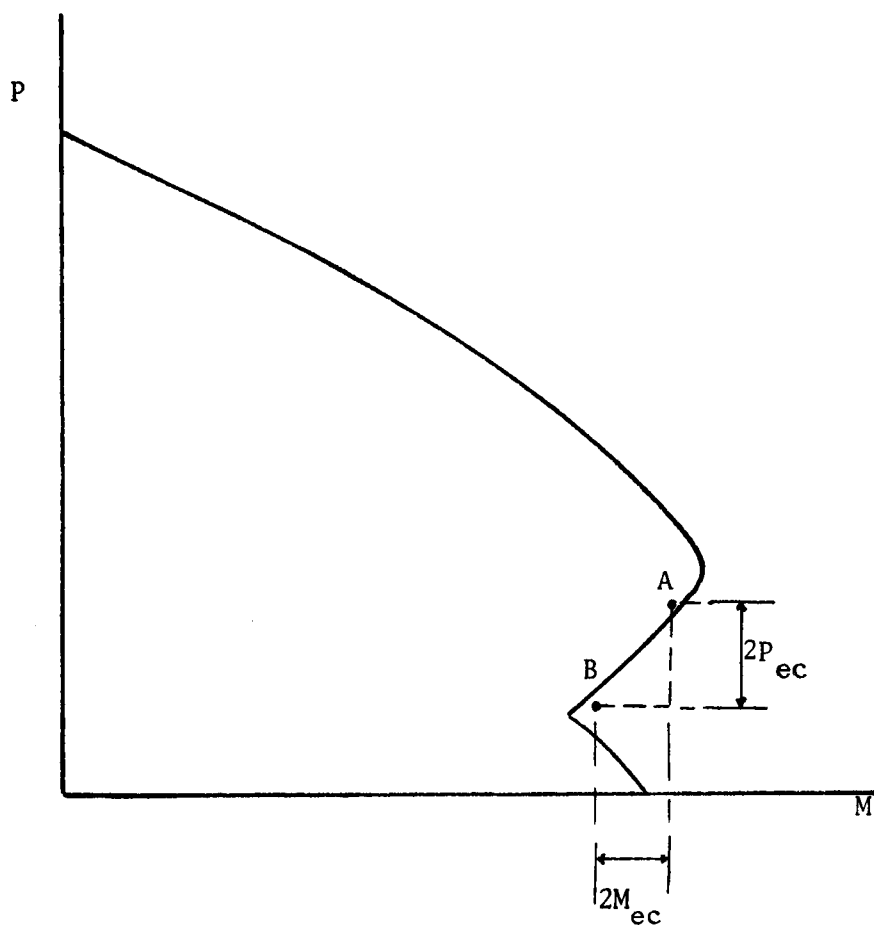


Figure 18. Moment-Axial Load Interaction Diagram for a Reinforced Concrete Column Illustrating the P-M Combinations A and B in a Case where B Controls

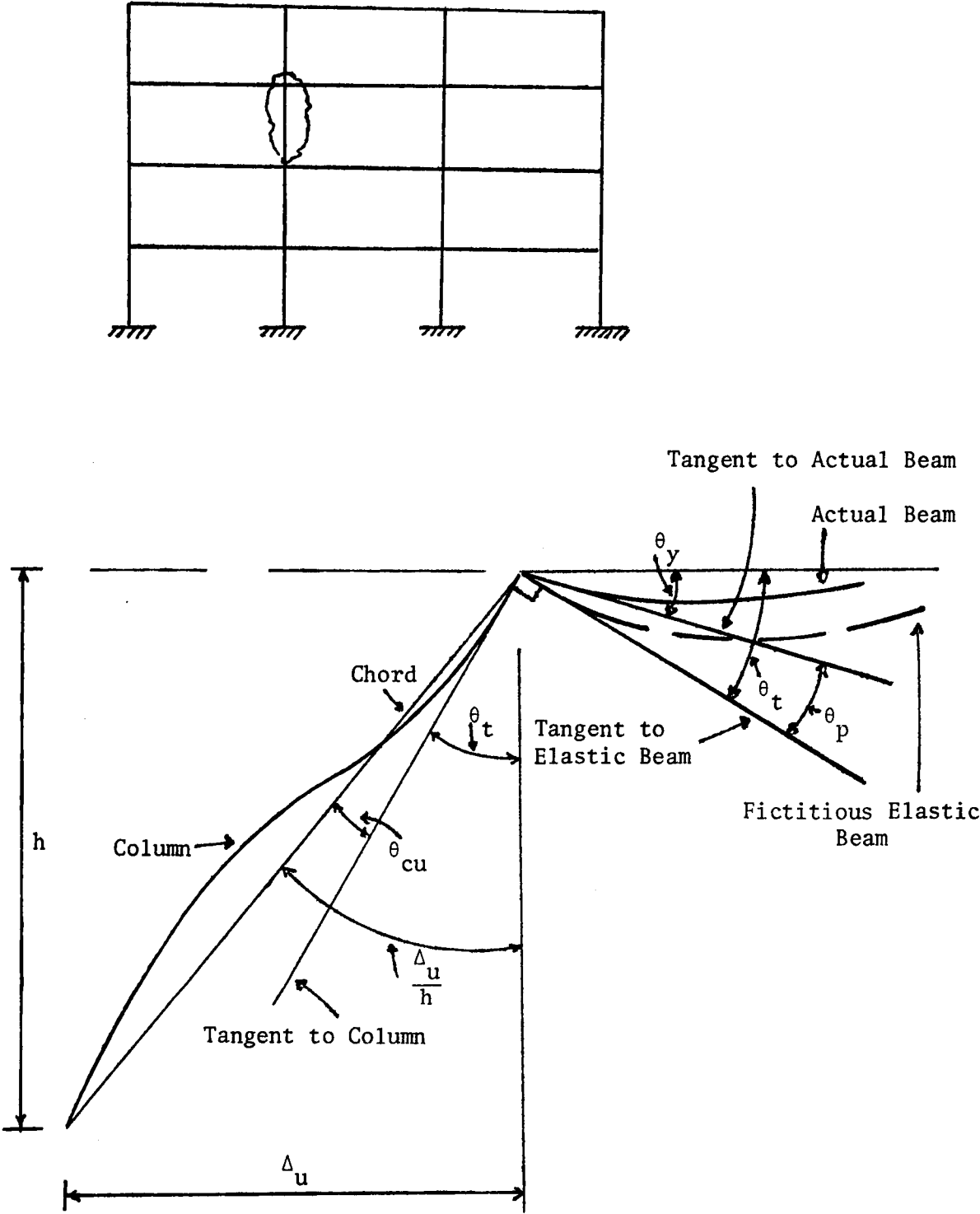


Figure 19. Relationship Between Story Drift and Plastic Hinge Rotations in the Beams

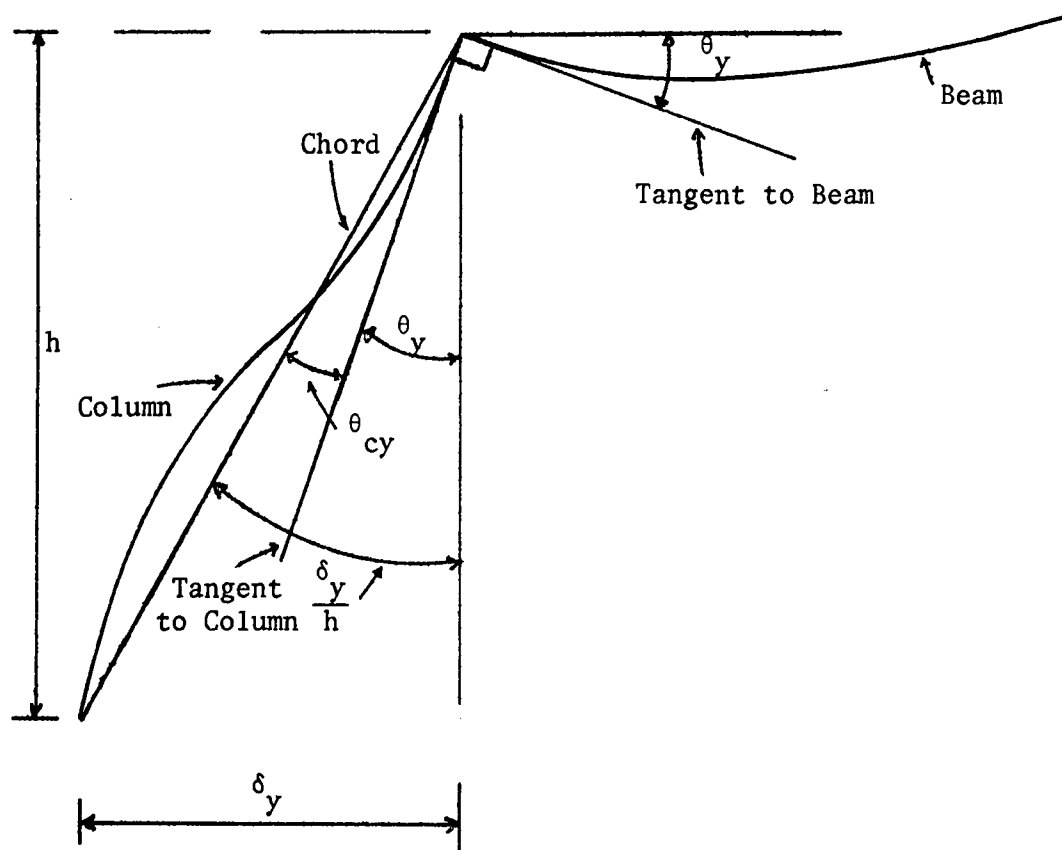


Figure 20. Relationship Between Yield Story Drift and Yield Rotation in the Beams

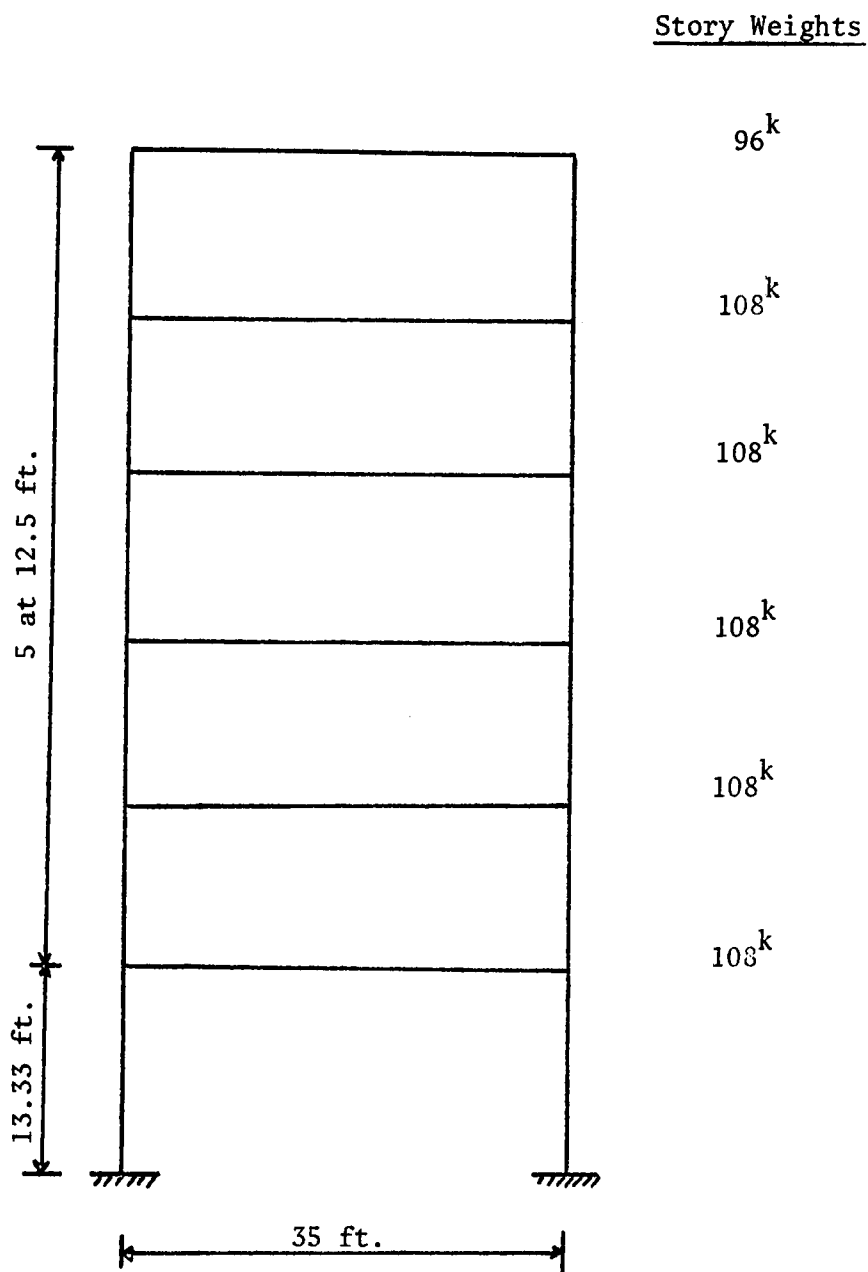


Figure 21. FRAME 1: Dimensions and Story Weights

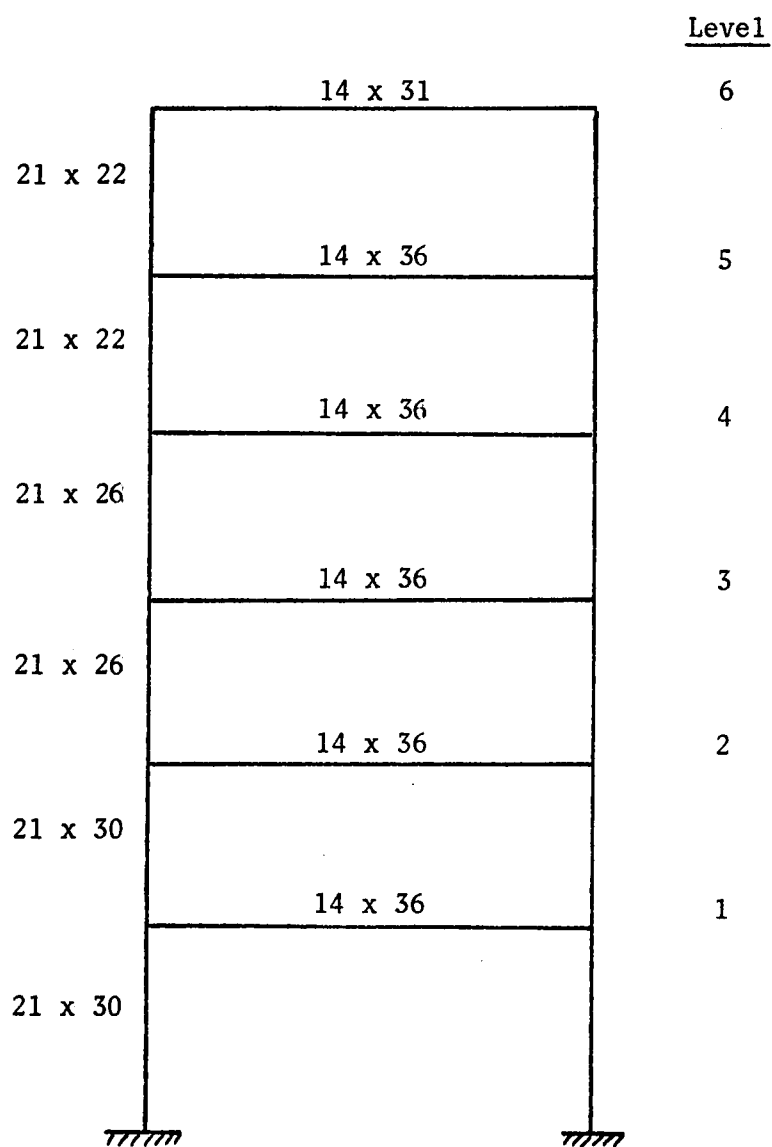


Figure 22. Member Sizes for FRAME 1

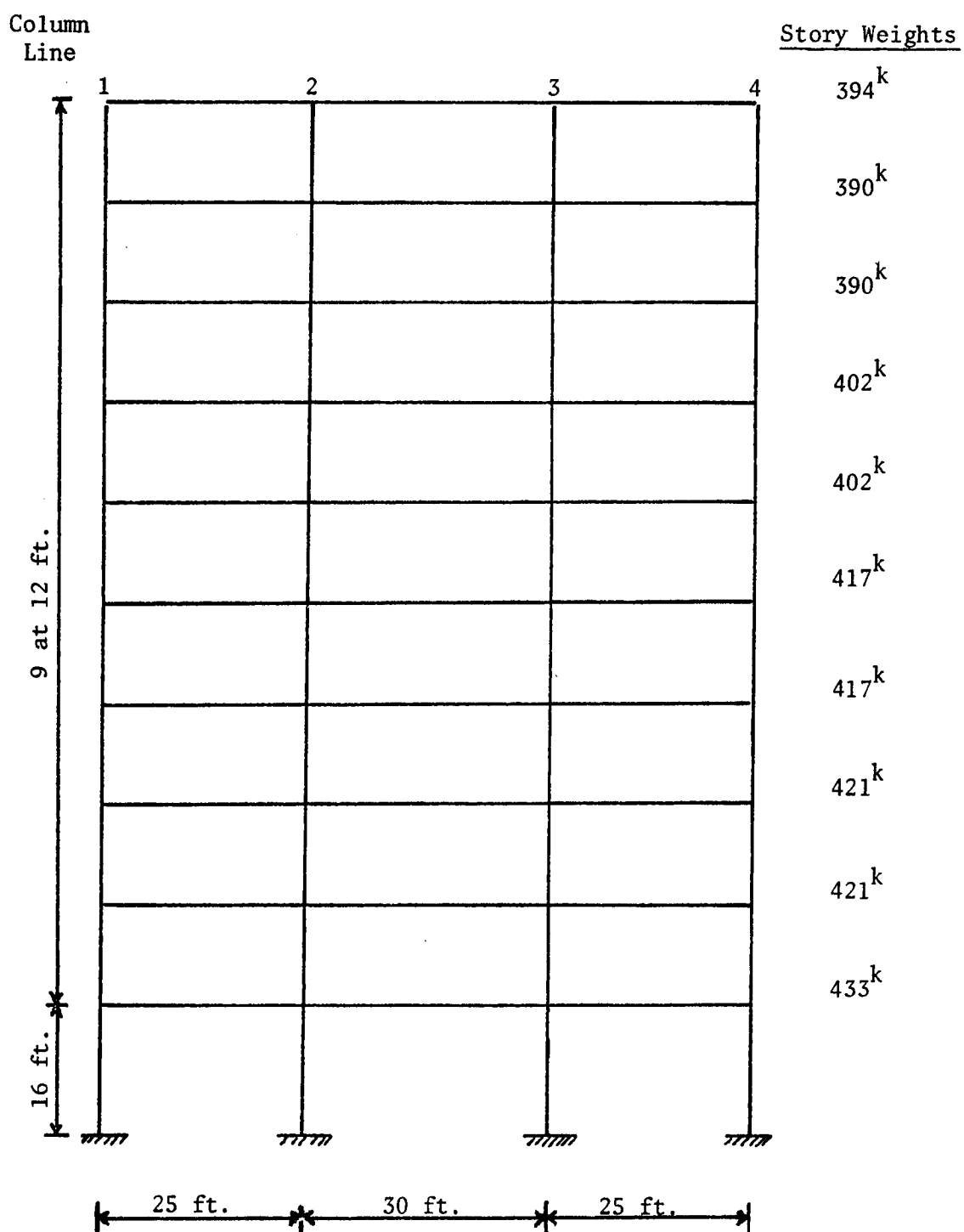


Figure 23. FRAME 2: Dimensions and Story Weights

			<u>Level</u>
	14 x 28	14 x 28	10
22 x 22	26 x 26 15 x 30	15 x 30	9
22 x 22	26 x 26 16 x 32	16 x 32	8
24 x 24	28 x 28 16 x 32	16 x 32	7
24 x 24	28 x 28 17 x 34	17 x 34	6
26 x 26	30 x 30 17 x 34	17 x 34	5
26 x 26	30 x 30 17 x 34	17 x 34	4
28 x 28	32 x 32 17 x 34	17 x 34	3
28 x 28	32 x 32 17 x 34	17 x 34	2
30 x 30	34 x 34 17 x 34	17 x 34	1
30 x 30	34 x 34		

- Notes: (1) Identical Frames Spaced Transversely Every 27 ft.
 (2) All Beam Sizes Include 6 in. of Floor Slab

Figure 24. Member Sizes for FRAME 2

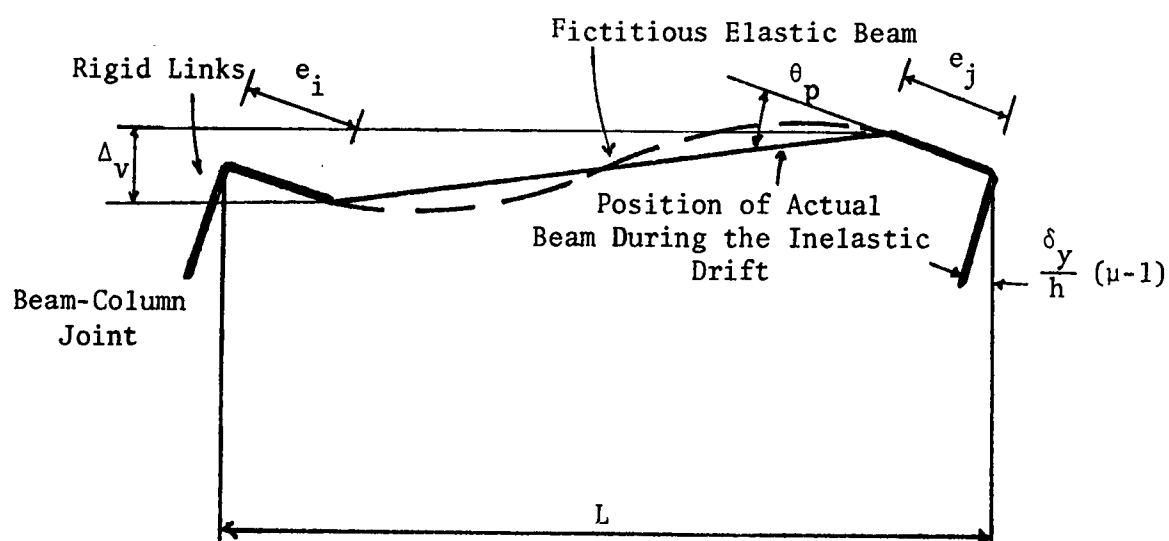


Figure 25. Effect of Joint Size on Plastic Hinge Rotation

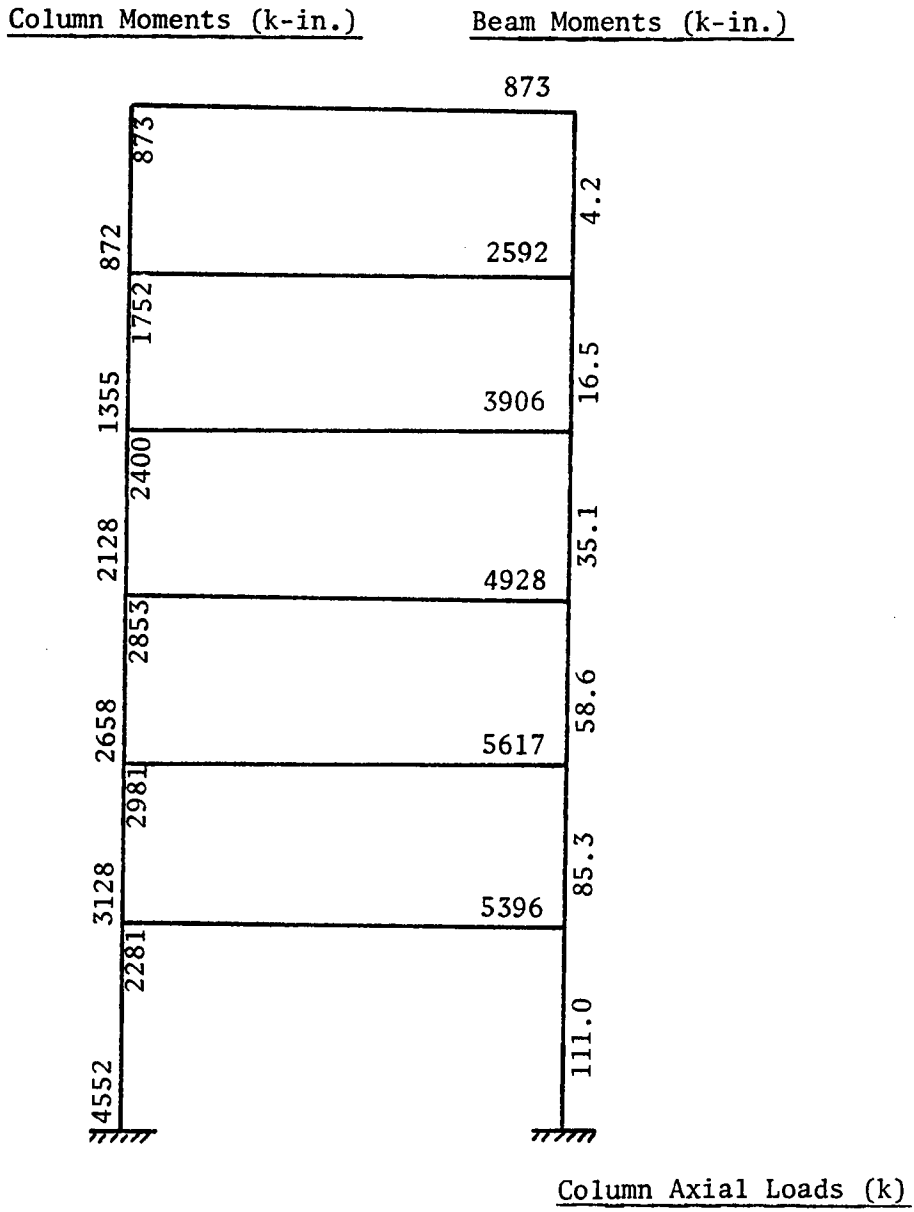


Figure 26. Response Spectrum Solution for FRAME 1; GM1, $\mu = 4$

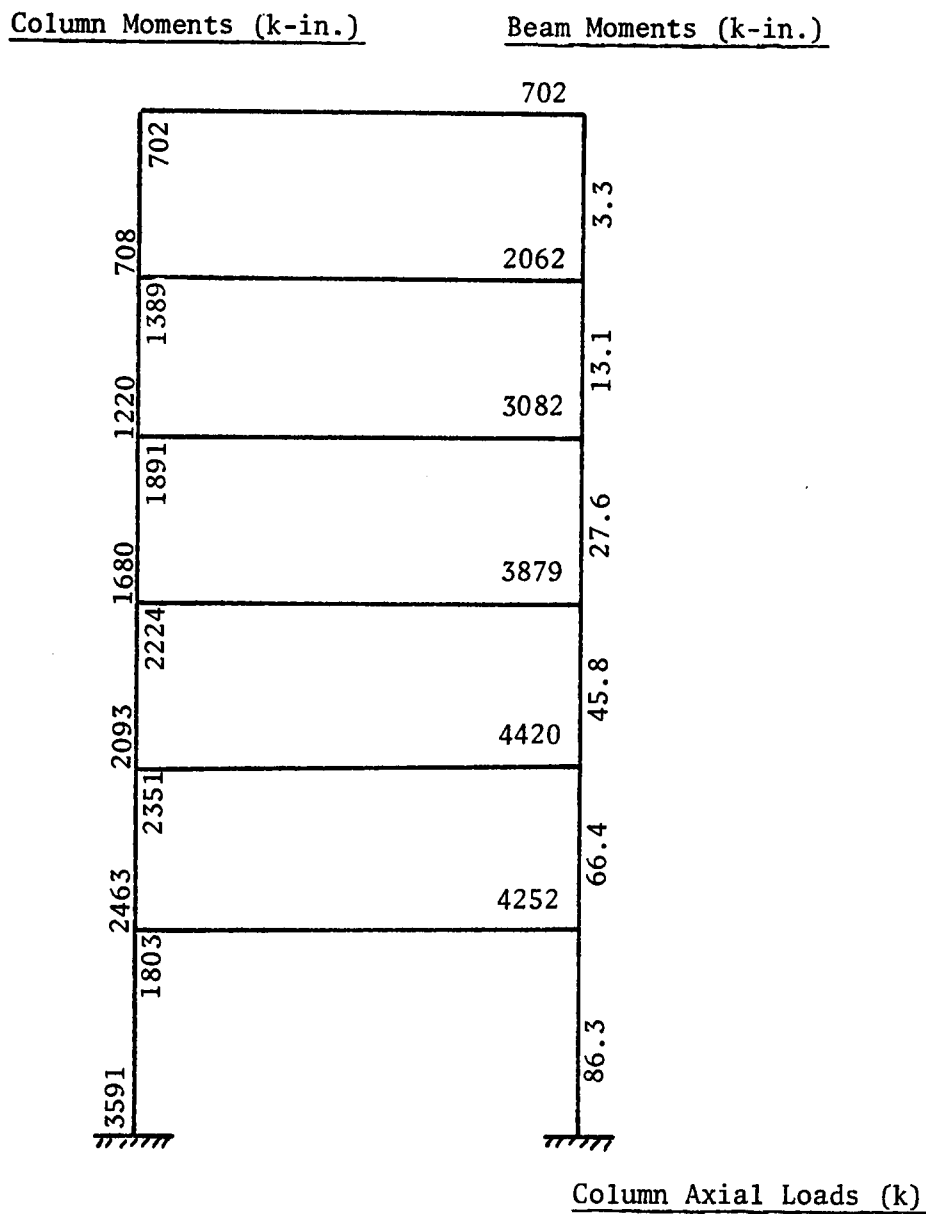


Figure 27. Response Spectrum Solution for FRAME 1; GM1, $\mu = 6$

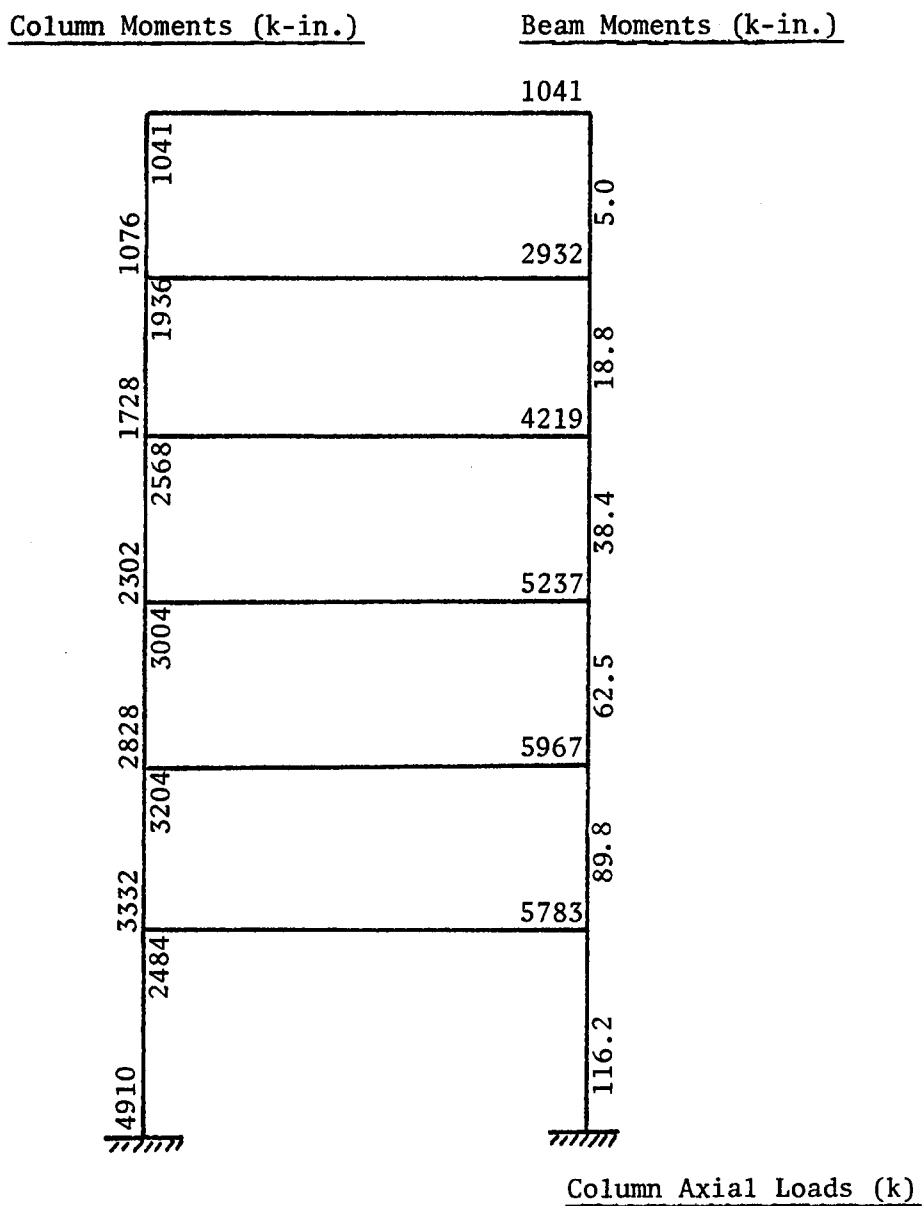


Figure 28. Response Spectrum Solution for FRAME 1; GM2, $\mu = 4$

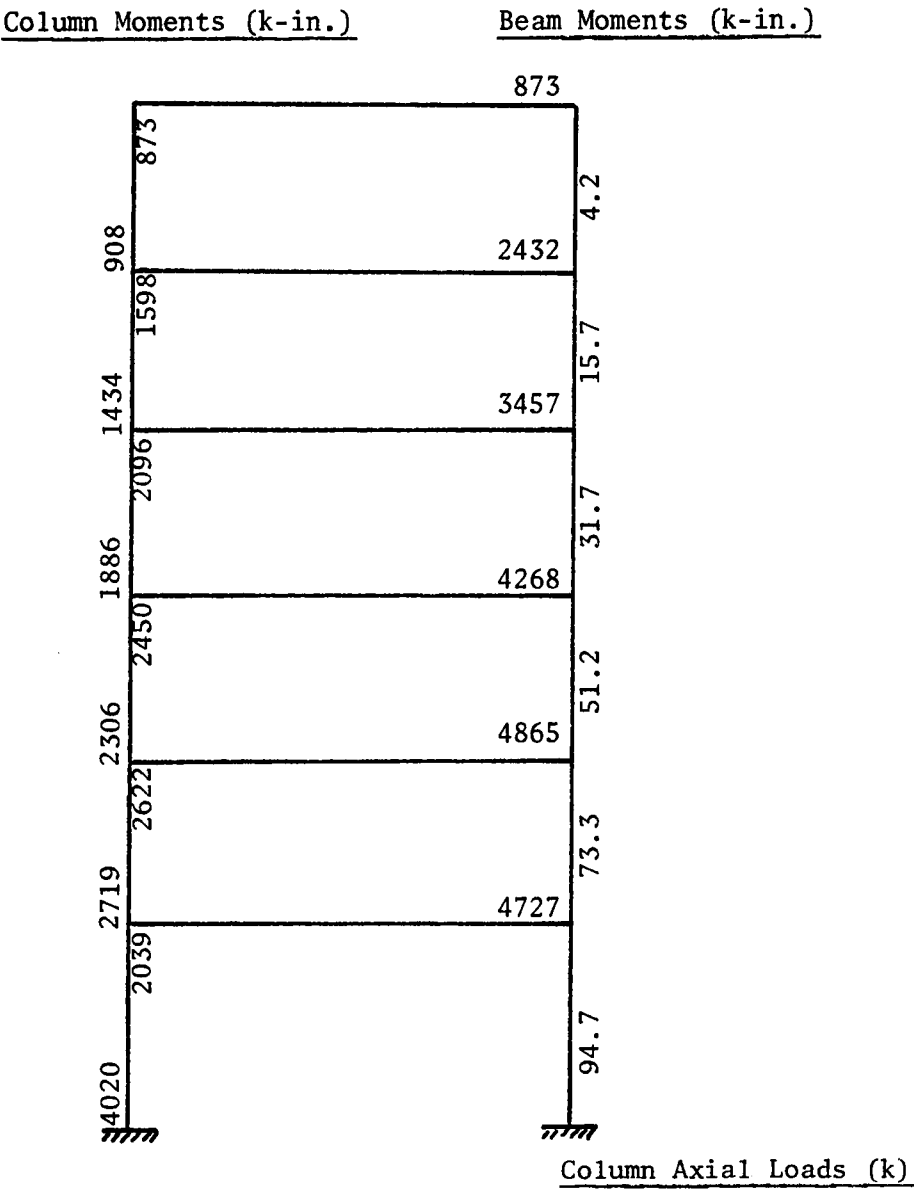


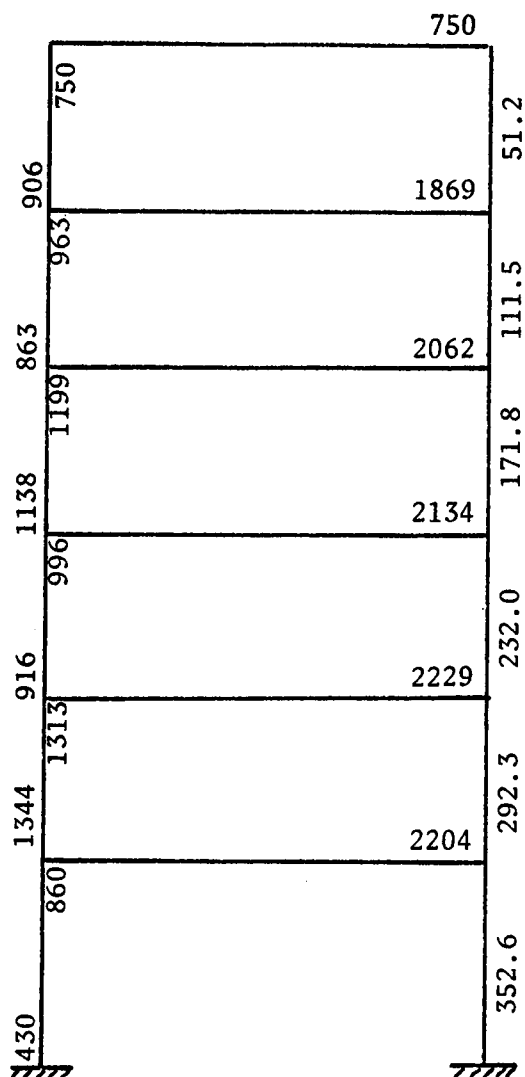
Figure 29. Response Spectrum Solution for FRAME 1; GM2, $\mu = 6$

<u>Column Moments (k-in.)</u>		<u>Beam Moments (k-in.)</u>			<u>Column Axial Loads (k)</u>	
		1676	1325	1346	<u>Line 1</u>	<u>Line 2</u>
10351	6491	17090	10734		448.5	147.1
3979	4359	6662	7268	7920	373.8	111.7
3849	3973	6658	6897	7488	307.5	82.9
3481	3653	6136	6405	7262	247.5	60.2
3183	3246	5796	5945	6977	192.5	41.9
3192	2941	5865	5367	6753	141.8	27.8
2561	2614	4764	4873	5747	98.3	16.7
2460	2127	4668	4190	5521	58.4	9.8
1338	1659	2850	3489	4657	30.3	6.9
610	1040	1312	2216	3626	9.7	3.2
				3110		
				3138		

Figure 30. Response Spectrum Solution for FRAME 2; GM2A, $\mu = 6$

<u>Column Moments (k-in.)</u>		<u>Beam Moments (k-in.)</u>		<u>Column Axial Loads (k)</u>	
				<u>Line 1</u>	<u>Line 2</u>
		68	22	26	
26					0.2
	44	158	98	101	0.2
96					0.8
	154	239	196	200	0.4
135					2.1
	278	320	341	338	0.4
201					4.4
	354	358	422	418	0.0
231					7.2
	440	446	534	530	0.9
293					10.8
	502	492	638	631	1.9
358					15.0
	564	552	762	753	3.4
433					20.0
	681	615	918	906	5.4
512					26.1
	759	680	1069	1055	8.1
599					33.2
	989				11.4
807					
	1331				
Column Line					
1	2	3	4		

Figure 31. P-Δ Effects for FRAME 2; GM2A, $\mu = 6$

Column Moments (k-in.)Beam Moments (k-in.)Loading

Uniform Load in all the beams except the roof = 2.09 k/ft.

Roof Load = 0.72 k/ft.

Column Axial Loads (k)

Figure 32. Gravity Loading Solution for FRAME 1

Loading

157

Uniform load in all the beams except the roof = 5.27. Roof Load = 4.73 k/ft.

Column Axial Loads (k)

<u>Column Moments (k-in.)</u>		<u>Beam Moments (k-in.)</u>			<u>Line 1</u>	<u>Line 2</u>
		2642	2004	1589		
448	903				56.9	132.1
2388	2241	807	835	2986	620.8	1253.0
1763	1877	823	860	2980	551.7	1111.5
2228	2298	1015	1064	2969	481.5	971.1
2048	2048	1008	1054	2955	410.0	832.0
2275	2314	1137	1133	2938	337.8	693.6
1895	1964	947	986	2951	266.5	554.3
2109	2113	1080	1091	2936	195.5	414.7
1521	1521	932	978	2948	126.3	273.3
2152	2492	1017	1001	2928	56.9	132.1
136	269				685.6	1398.8

Figure 33. Gravity Loading Solution For FRAME 2

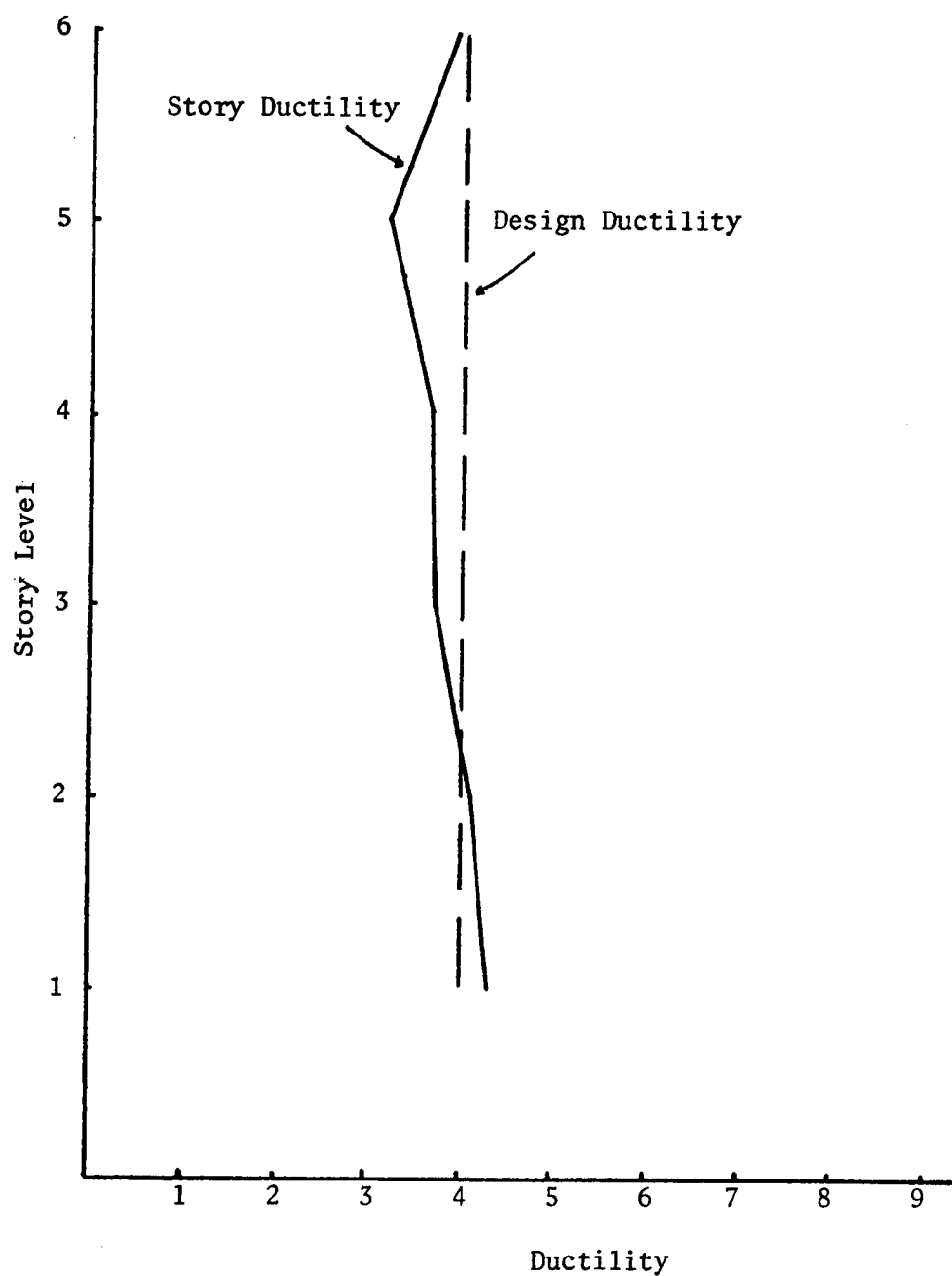


Figure 34: Case 1: Comparison of Design Ductility with Story Ductilities from DRAIN-2D (Values from Table 13)

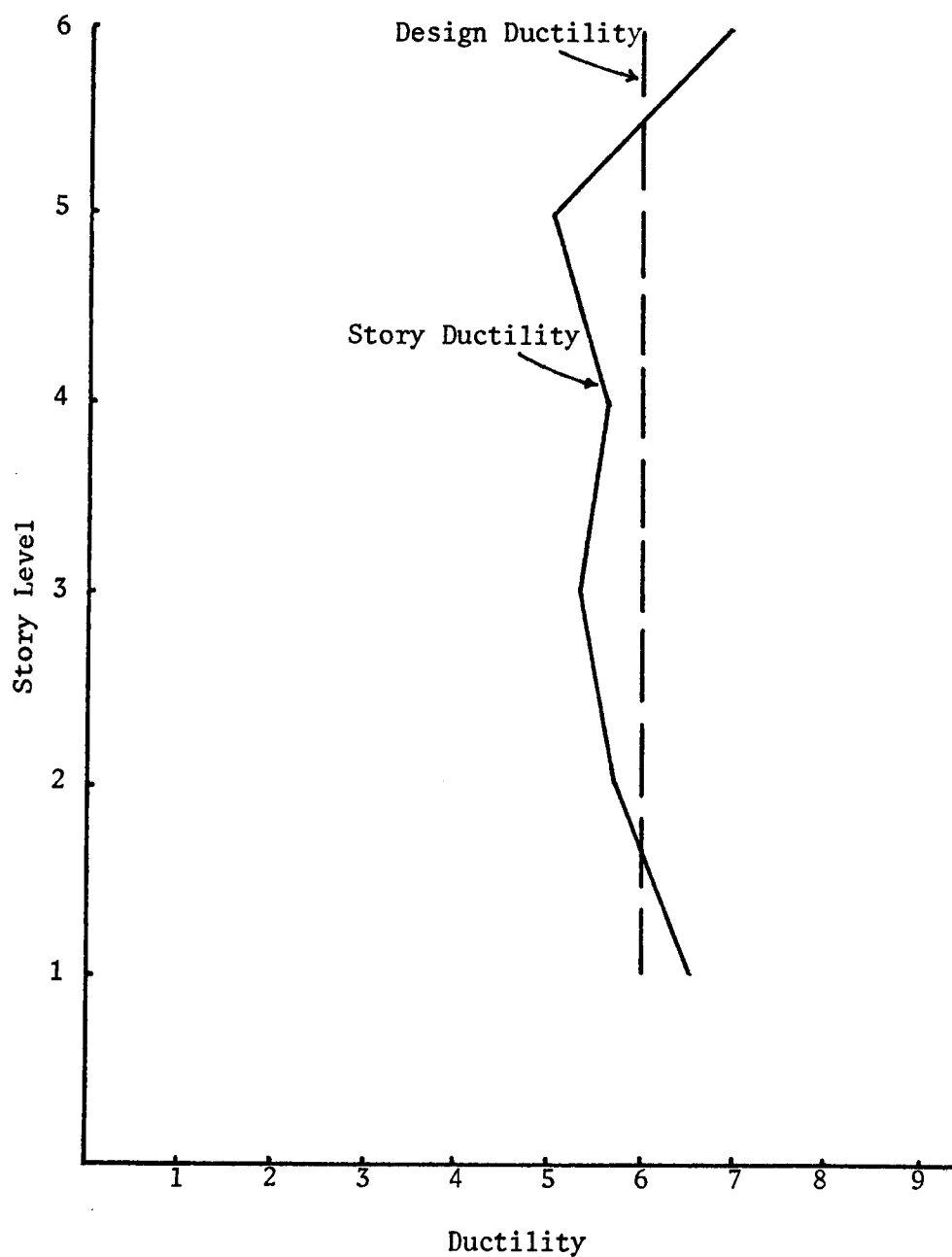


Figure 35. Case 2: Comparison of Design Ductility with Story Ductilities from DRAIN-2D (Values from Table 14)

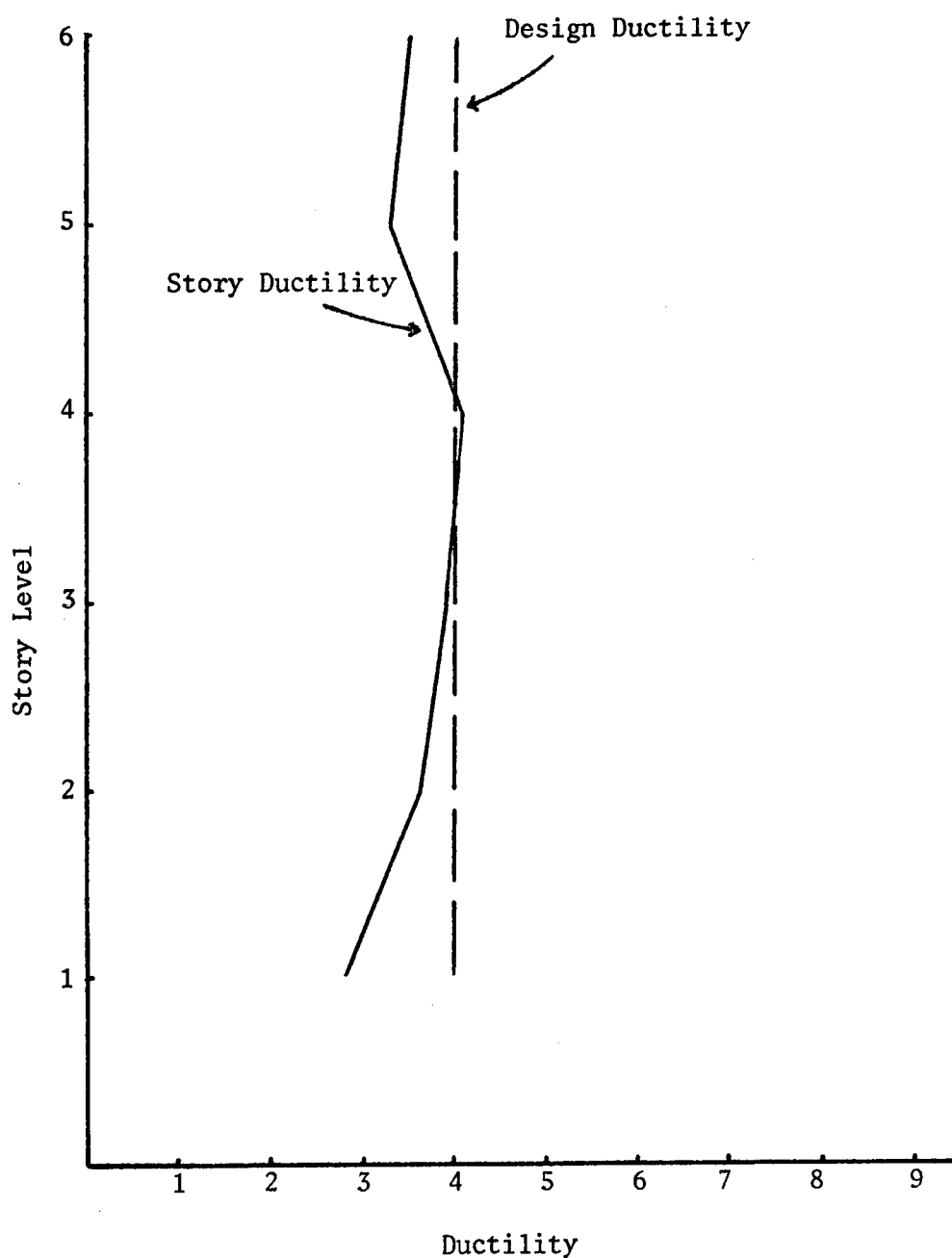


Figure 36. Case 3: Comparison of Design Ductility with Story Ductilities from DRAIN-2D (Values from Table 15)

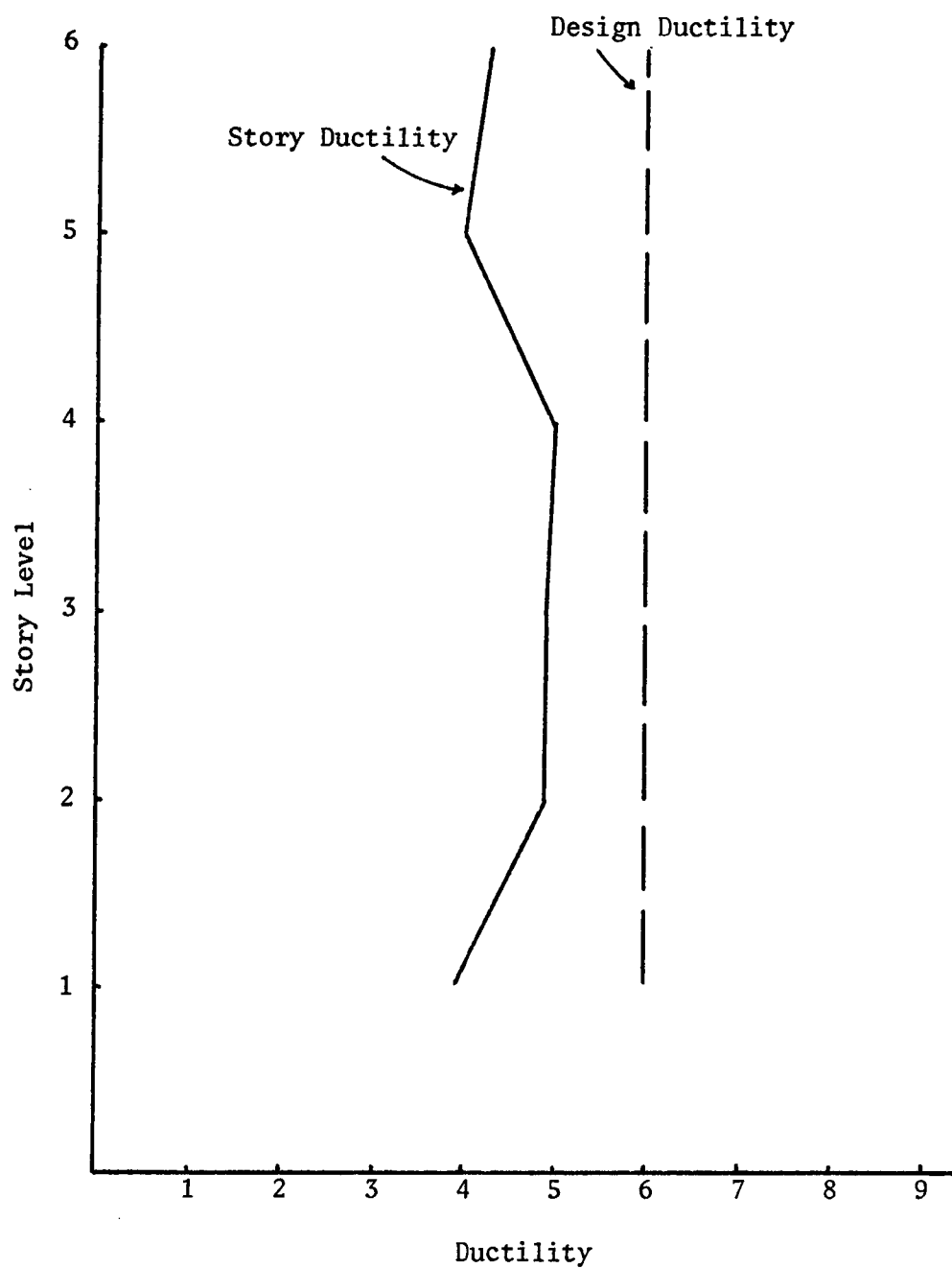


Figure 37. Case 4: Comparison of Design Ductility with Story Ductilities from DRAIN-2D (Values from Table 16)

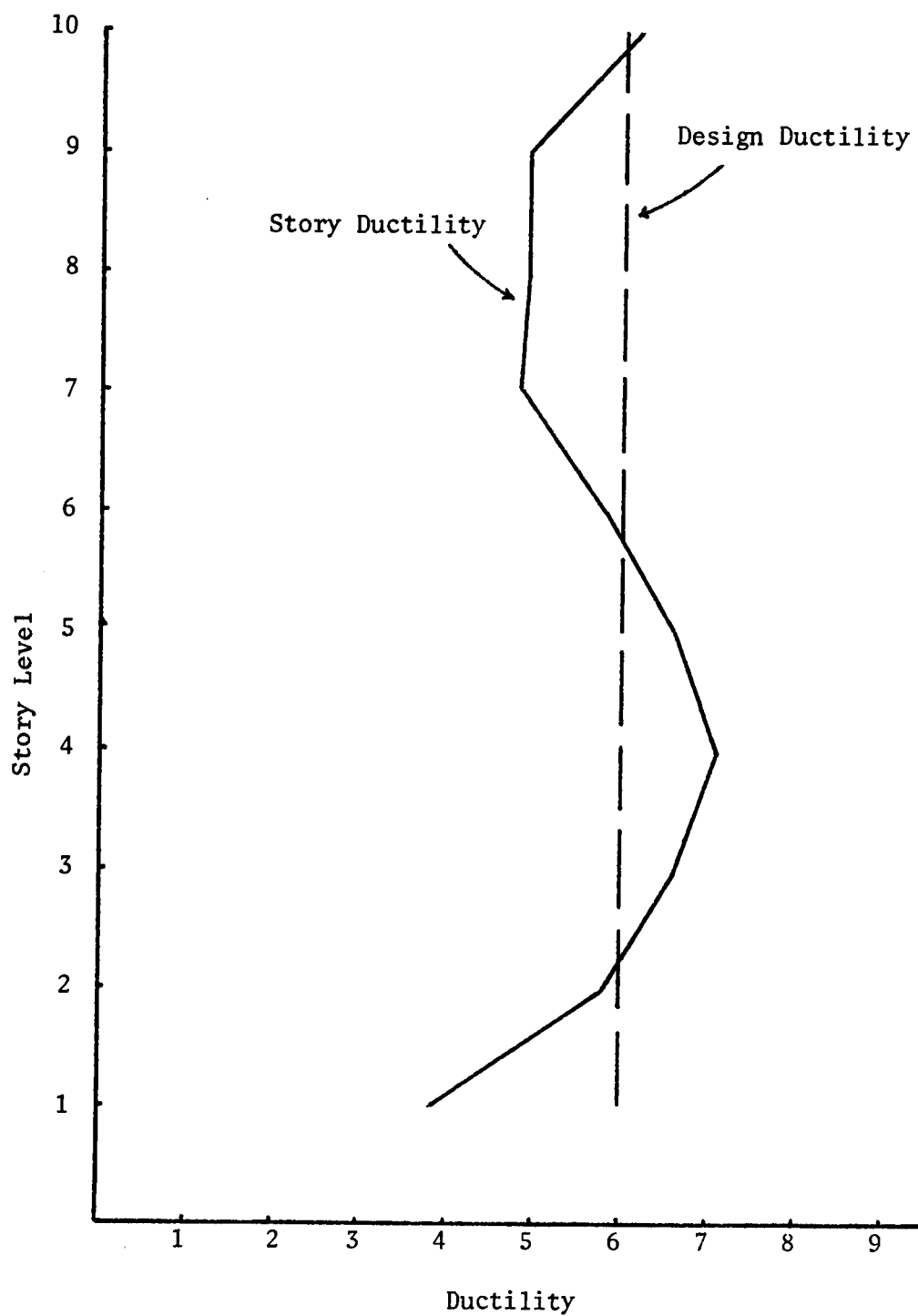


Figure 38. Case 9: Comparison of Design Ductility with Story Ductilities from DRAIN-2D (Values Plotted from Table 21)

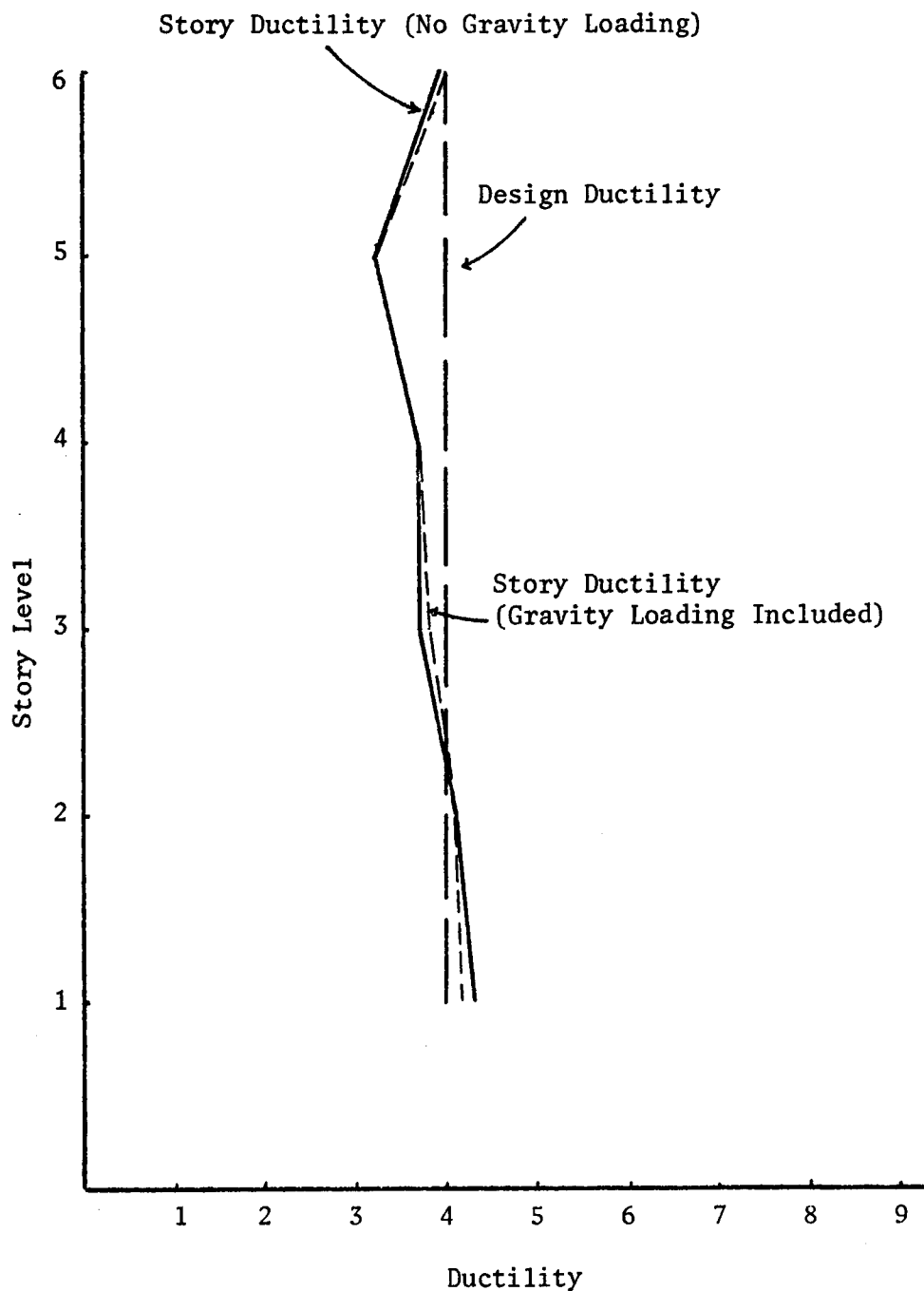


Figure 39. Effect of Gravity Loading on Story Ductility for IFR1U4GM1
(Curves Plotted from Tables 13 and 17)

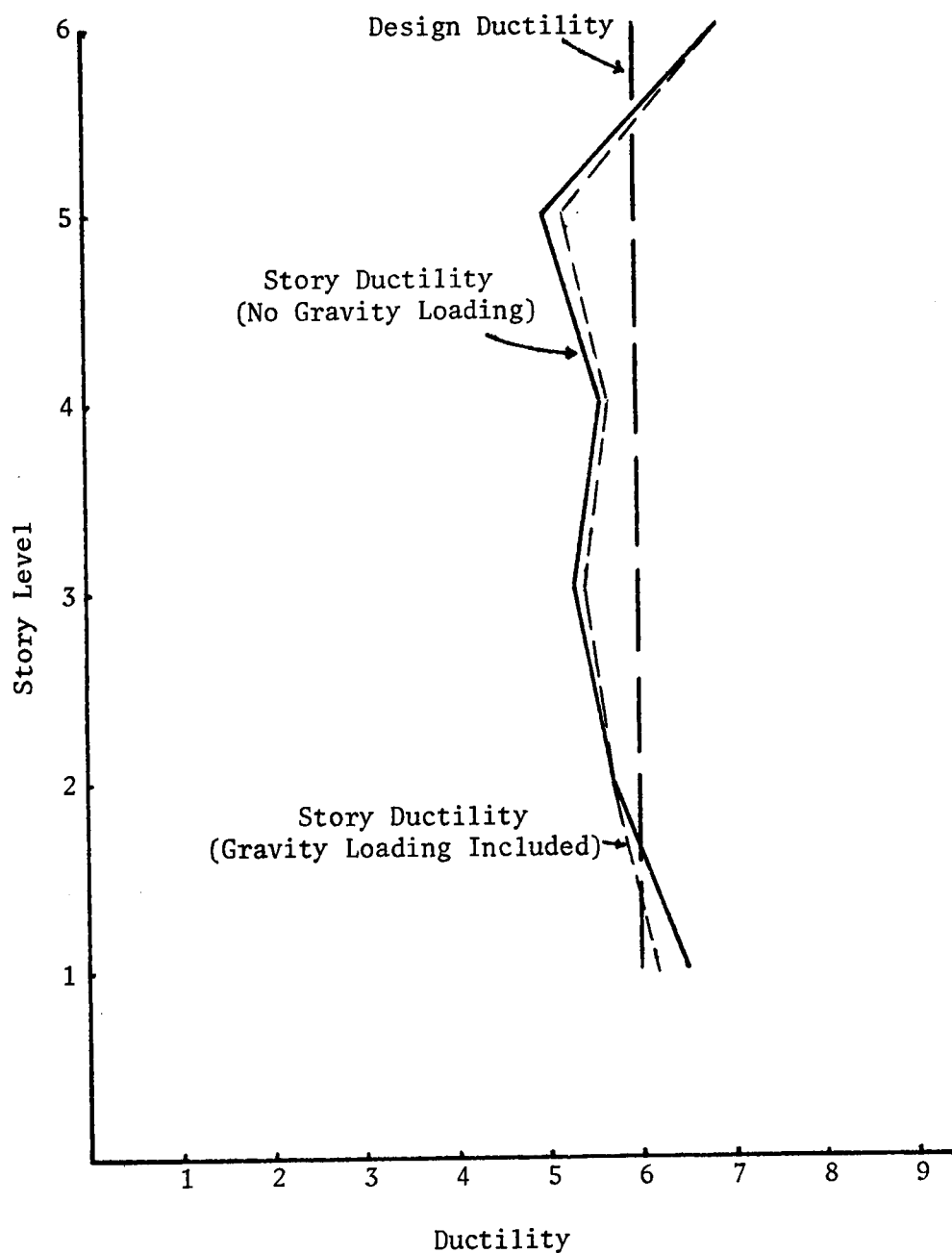


Figure 40. Effect of Gravity Loading on Story Ductility for IFR1U6GM1
(Curves Plotted from Tables 14 and 18)

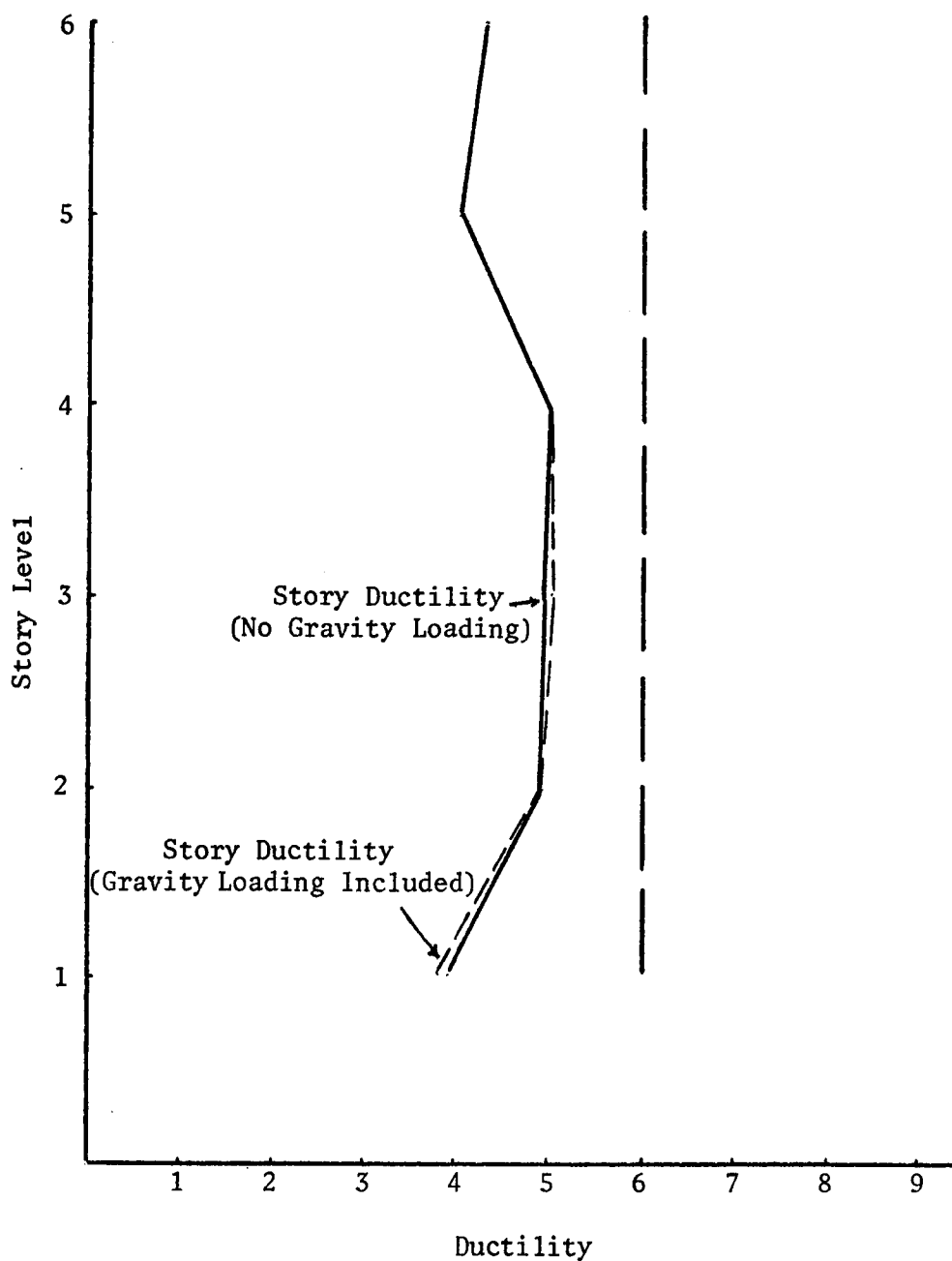


Figure 41. Effect of Gravity Loading on Story Ductility for IFR1U6GM2
(Curves Plotted from Tables 16 and 20)

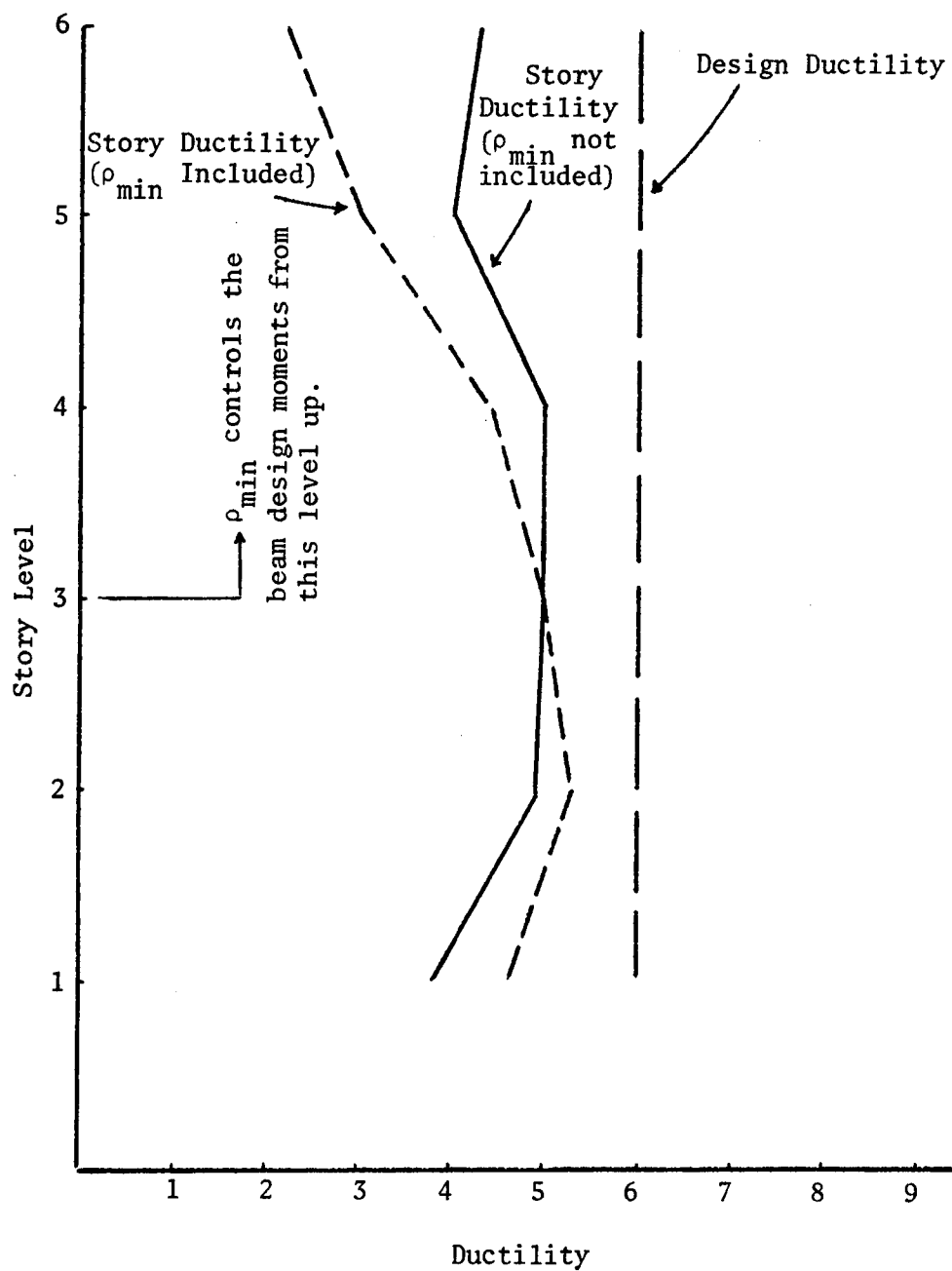


Figure 42. Effect of ρ_{min} on Story Ductility for IFR1U6GM2
(Curves Plotted from Tables 20 and 22)

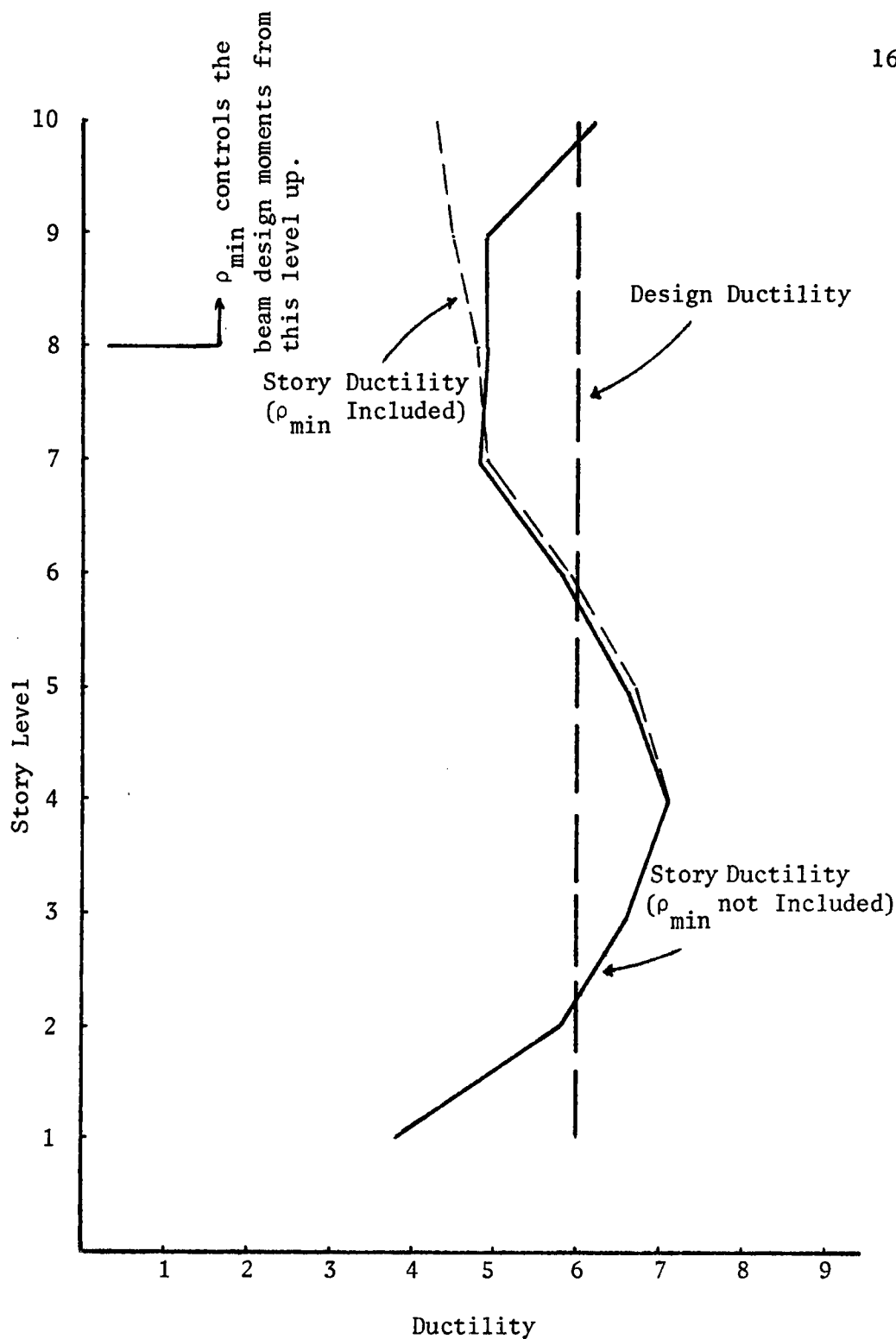


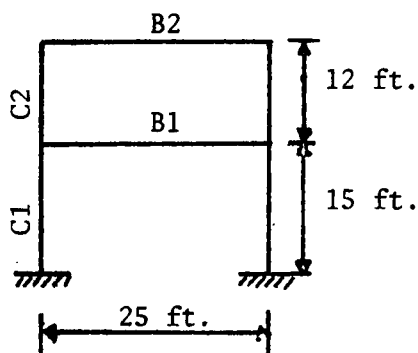
Figure 43. Effect of ρ_{min} on Story Ductility for IFR2U6GM2A
(Curves Plotted from Tables 21 and 23)

APPENDIX C

DESIGN EXAMPLE

DESIGN EXAMPLE

Level
2



B1 & B2 12" x 20"
C1 & C2 12" x 12"

$$\begin{aligned} f'_c &= 4000 \text{ psi} \\ f_y &= 60 \text{ ksi} \\ E_c &= 3605 \text{ ksi} \end{aligned}$$

D = 0.8 k/ft. on B1 and B2
L = 0.4 k/ft. on B1 and B2

Design Earthquake - GM2A - $\xi = 0.05$ (5% of Critical Damping)

For this reinforced concrete structure a ductility factor of 4 is reasonable. Say $\mu = 4$.

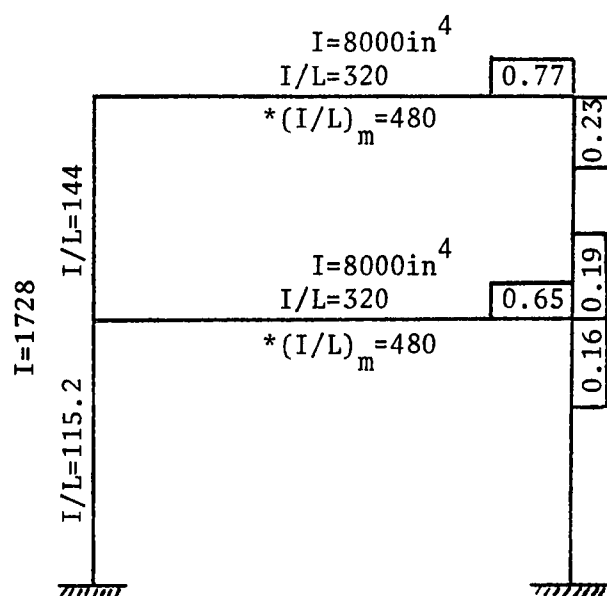
The story weights are computed from the full dead load plus 25% of the live load; thus

$$m = \frac{w}{g} = \frac{0.8 \times 25 + 0.25 \times 0.4 \times 25}{386.4} = 0.05823 \frac{\text{k-sec}^2}{\text{in.}}$$

The mass matrix is then,

$$[m] = \begin{bmatrix} 0.05823 & 0 \\ 0 & 0.05823 \end{bmatrix}$$

The stiffness is obtained by successive application of unit displacements at the selected degrees of freedom. The calculations below are self explanatory.

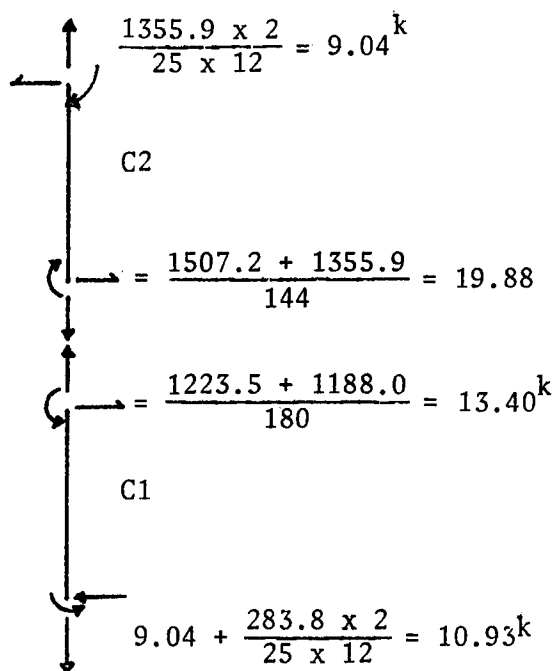
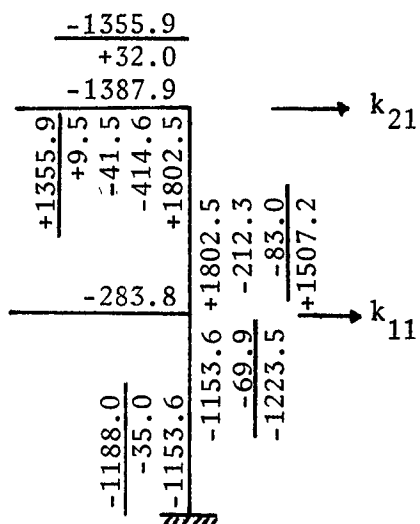


*Based on symmetry

Fixed-end moments due to $u_1 = 1$ in.

$$M_{c1} = \frac{6 \times E \times I \times 1}{(180)^2} = \frac{6 \times 3605 \times 1728}{(180)^2} = 1153.6 \text{ k-in.}$$

$$M_{c2} = \frac{6 \times 3605 \times 1728}{(144)^2} = 1802.5 \text{ k-in.}$$



$$k_{11} = 2 \times (19.88 + 13.40) = 66.56 \text{ k/in.}$$

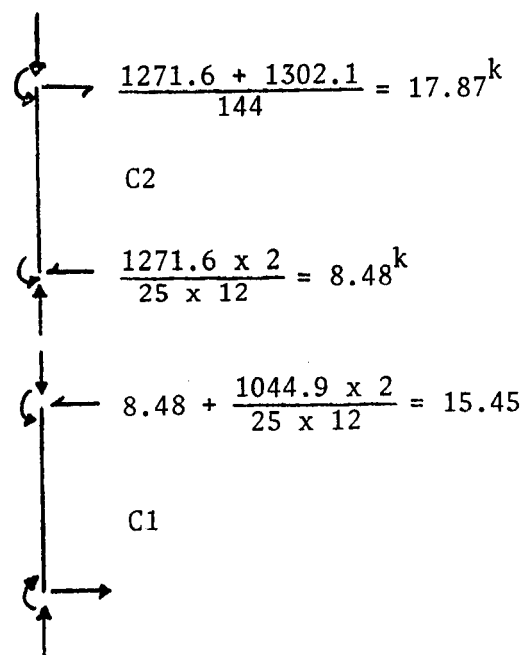
$$k_{21} = 2 \times 19.88 = -39.76 \text{ k/in.} \quad (- \text{ means opposite to the direction shown})$$

Fixed-end moments due to $u_2 = 1$ in.

$$M_{c1} = 0$$

$$M_{c2} = 1802.5 \text{ k-in.}$$

$$\begin{array}{r}
 +1271.6 \\
 -116.3 \\
 \hline
 +1387.9 \\
 \hline
 \begin{array}{r}
 -1271.6 \\
 -34.8 \\
 +151.1 \\
 +414.6 \\
 -1802.5 \\
 \hline
 \end{array}
 \end{array}$$



$$k_{22} = 2 \times 17.87 = 35.75 \text{ k/in.}$$

$$k_{12} = k_{21} \quad (\text{From Betti's Law})$$

Therefore the stiffness matrix is

$$[k] = \begin{bmatrix} 66.56 & -39.76 \\ -39.76 & 35.76 \end{bmatrix}$$

The mode shapes and natural frequencies are now determined from Equation (2.11).

$$\phi_n [k - m\omega_n^2] = 0$$

$$\begin{bmatrix} 66.56 - 0.05823 \omega_n^2 & -39.76 \\ -39.76 & 35.75 - 0.05823 \omega_n^2 \end{bmatrix} = 0$$

or

$$(66.56 - 0.05823 \omega_n^2) (35.75 - 0.05823 \omega_n^2) - (39.76)^2 = 0$$

Resolving the parenthesis one gets

$$0.00339 (\omega_n^2)^2 - 5.9575 \omega_n^2 + 798.66 = 0$$

for which the roots are

$$\omega_1^2 = 146.23$$

$$\omega_2^2 = 1611.14$$

or

$$\omega_1 = 12.09 \text{ rad/sec}$$

$$\omega_2 = 40.14 \text{ rad/sec}$$

The natural periods can now be obtained using Equation (3.10).

$$T_1 = \frac{2\pi}{\omega_1} = 0.52 \text{ sec.}$$

$$T_2 = \frac{2\pi}{\omega_2} = 0.16 \text{ sec.}$$

The corresponding mode shapes are obtained now by returning to Equation (2.11). Since the mode shapes have arbitrary amplitudes, one has to assign a value to one amplitude in the mode vector to fix the other values.

In the case set $\phi_{n2} = 1$.

First Mode Shape

$$\omega_1^2 = 146.23$$

$$\begin{bmatrix} 58.045 & -39.76 \\ -39.76 & 27.235 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

From which

$$58.045 \phi_{11} = 39.76$$

or

$$\phi_{11} = 0.685$$

So,

$$\phi_1 = \begin{Bmatrix} 0.685 \\ 1.000 \end{Bmatrix}$$

Second Mode Shape

$$\omega_2^2 = 1611.14$$

$$\begin{bmatrix} -27.257 & -39.76 \\ -39.76 & -58.067 \end{bmatrix} \begin{Bmatrix} \phi_{21} \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

From which

$$-27.257 \phi_{21} = 39.76$$

or

$$\phi_{21} = -1.459$$

So,

$$\phi_2 = \begin{Bmatrix} -1.459 \\ 1.000 \end{Bmatrix}$$

(3) Evaluation of Modal Parameters and SRSS Combination

From Figure 12 for $\mu = 4$

$$S_{a,1} = 0.32 \text{ g}$$

$$S_{a,2} = 0.57 \text{ g}$$

First Mode

Participation Factor; from Equation (4.7)

$$\Gamma_1 = \frac{0.685 \times 1.000 \times 0.05823 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 0.685 \\ 1.000 \end{Bmatrix}}{0.685 \times 1.000 \times 0.05823 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 0.685 \\ 1.000 \end{Bmatrix}} = \frac{0.0981176}{0.085553} = 1.147$$

The modal forces from Equation (4.8) are:

$$\begin{Bmatrix} f_{11} \\ f_{12} \end{Bmatrix} = 0.05823 \times 1.147 \times 0.32 \times 386.4 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 0.685 \\ 1.000 \end{Bmatrix} = \begin{Bmatrix} 5.66^k \\ 8.26^k \end{Bmatrix}$$

The values of the modal moments, shears and axial loads are easily obtained since the lateral sway analysis required for a moment distribution analysis has already been made when obtaining the stiffness matrix. It is, therefore, only necessary to calculate the correction factors, which are herein designated as q . The following equation is simply the matrix format for the simultaneous equations which must be solved to obtain the correction factors.

$$[k] \begin{Bmatrix} q \end{Bmatrix} = \begin{Bmatrix} f_n \end{Bmatrix}$$

or

$$q = [k]^{-1} \begin{Bmatrix} f_n \end{Bmatrix} \quad (6.1)$$

in this case

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \frac{1}{798.66} \begin{bmatrix} 35.75 & 39.76 & 5.66 \\ 39.76 & 66.56 & 8.26 \end{bmatrix} \begin{Bmatrix} 5.66 \\ 8.26 \end{Bmatrix}$$

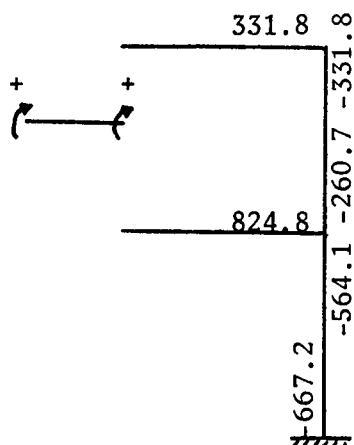
$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0.665 \\ 0.970 \end{Bmatrix}$$

For example, the modal moment in beam B1 is:

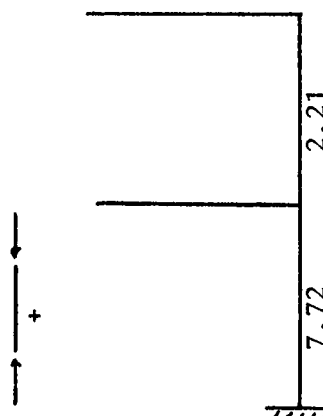
$$M_{B1} = 0.665 \times (-283.8) + 0.97 \times 1044.9 = 824.8 \text{ k-in.}$$

The complete set of modal moments and column axial loads are given in the following sketches. (Shears are not given because they will not be used in the design.)

First Mode Moments (k-in.)



First Mode Axial Loads (k)



First Mode-Drift Values

From Equation (4.12) the drifts are given by,

$$\begin{Bmatrix} \delta_n \end{Bmatrix} = \frac{S_{a,n} \Gamma_n}{\omega_n^2} \begin{Bmatrix} \phi_{nx} - \phi_{n(x-1)} \end{Bmatrix}$$

In this case,

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \frac{0.32 \times 386.4 \times 1.147}{146.23} \begin{Bmatrix} 0.685 \\ 0.315 \end{Bmatrix} = \begin{Bmatrix} 0.664 \\ 0.306 \end{Bmatrix}$$

Second Mode

Participation factor:

$$\Gamma_2 = \frac{-1.459 \times 1.000 \times 0.05823}{-1.459 \times 1.000 \times 0.05823} \frac{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} -1.459 \\ 1.000 \end{Bmatrix}} = \frac{-0.02673}{0.18218} = -0.1467$$

Modal forces:

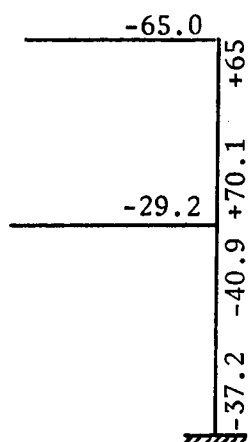
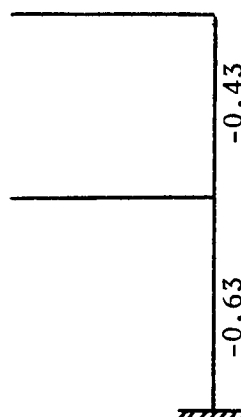
$$\begin{Bmatrix} f_{21} \\ f_{22} \end{Bmatrix} = 0.05823 \times (-0.1467) \times 0.57 \times 386.4 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} -1.459 \\ 1.000 \end{Bmatrix} = \begin{Bmatrix} 2.745^k \\ -1.880^k \end{Bmatrix}$$

Correction factors:

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \frac{1}{798.66} \begin{bmatrix} 35.75 & 39.79 \\ 39.76 & 66.56 \end{bmatrix} \begin{Bmatrix} 2.745 \\ -1.880 \end{Bmatrix}$$

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0.0292 \\ -0.0200 \end{Bmatrix}$$

Modal moments and axial loads:

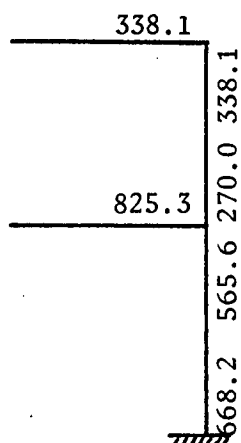
Second Mode Moments (k-in.)Second Mode Axial Loads (k)

Second Mode-Drift Values

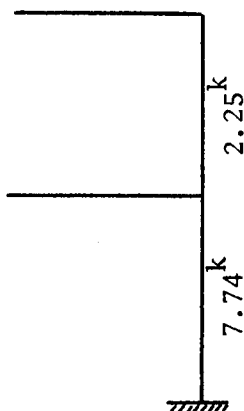
$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \frac{0.57 \times 386.4 \times (-0.1467)}{1611.14} \begin{Bmatrix} -1.459 \\ 2.459 \end{Bmatrix} = \begin{Bmatrix} 0.0293 \\ -0.0493 \end{Bmatrix}$$

SRSS COMBINATION

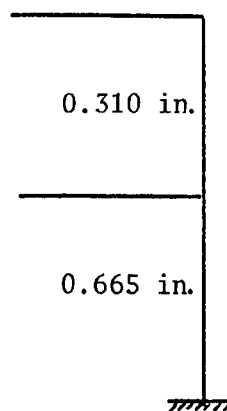
SRSS Moments*



SRSS Axial Loads*



SRSS Drifts*



*All these values can be plus or minus

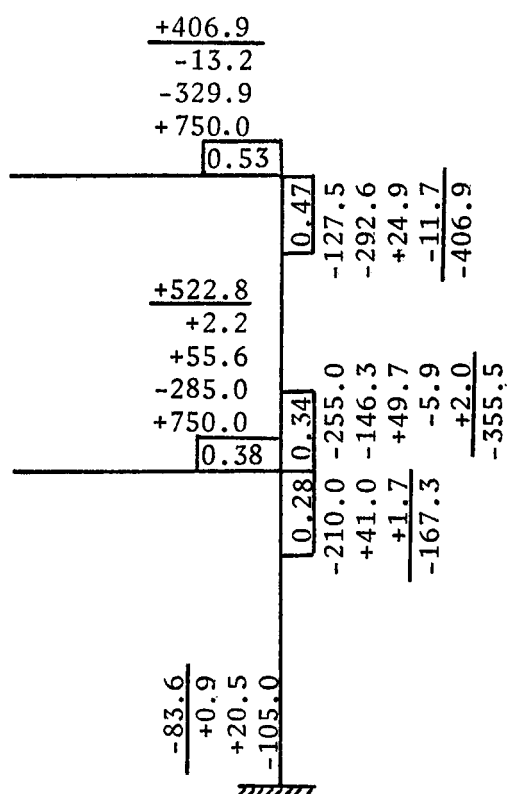
(4) Gravity Load Analysis

The modified I/L values for the beams used to account for symmetry in the gravity load analysis is $(I/L)_m = \frac{320}{2} = 160$.

The fixed end moments are:

$$M = \frac{1}{12} \times \frac{(0.8 + 0.4)}{12} \times (25 \times 12)^2 = 750 \text{ k-in.}$$

The moment distribution shown below is self explanatory.

Axial Loads

$$P_{c1} = \frac{1.2 \times 25}{2} = 15^k$$

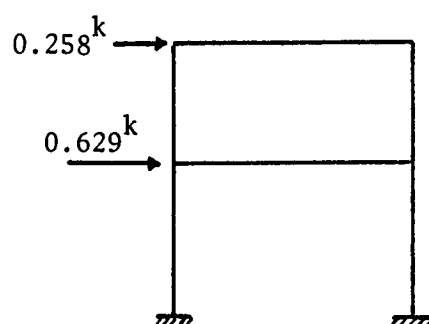
$$P_{c2} = 15 + 15 = 30^k$$

(5) P-Δ Effects

From Equation (4.24) taking the λ vector as unity

$$\begin{Bmatrix} (\Delta S_{P-\Delta}) \end{Bmatrix} = \begin{Bmatrix} 0.3333 \\ 0.2083 \end{Bmatrix} 4 \begin{Bmatrix} 0.665 \\ 0.310 \end{Bmatrix} = \begin{Bmatrix} 0.887 \\ 0.258 \end{Bmatrix}$$

These shear increases correspond to the following equivalent loads.



The vector of first order P-Δ drifts is calculated from these loads as follows:

$$\begin{Bmatrix} \delta_1 \end{Bmatrix} = \frac{1}{198.66} \begin{bmatrix} 35.75 & 39.76 \\ 39.76 & 66.56 \end{bmatrix} \begin{Bmatrix} 0.629 \\ 0.258 \end{Bmatrix} = \begin{Bmatrix} 0.041 \\ 0.0528 \end{Bmatrix}$$

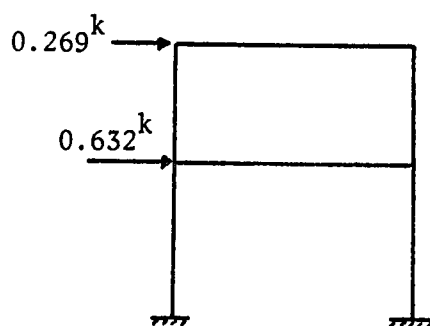
The actual λ vector is then calculated from Equation (4.25)

$$\begin{Bmatrix} \lambda \end{Bmatrix} = \frac{\begin{Bmatrix} 1 \end{Bmatrix}}{\begin{Bmatrix} 1 \end{Bmatrix} - \frac{\begin{Bmatrix} 0.041 \\ 0.0528 \end{Bmatrix}}{4 \begin{Bmatrix} 0.665 \\ 0.310 \end{Bmatrix}}} = \begin{Bmatrix} 1.0157 \\ 1.0445 \end{Bmatrix}$$

The final increase in the story shears are then obtained using the actual λ vector

$$\left\{ \begin{matrix} (\Delta S_{p-\Delta}) \end{matrix} \right\} = \begin{bmatrix} 0.887 \times 1.0157 \\ 0.258 \times 1.0445 \end{bmatrix} = \begin{bmatrix} 0.901 \\ 0.269 \end{bmatrix}$$

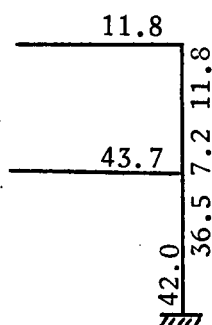
The loads corresponding to these shear increases are:



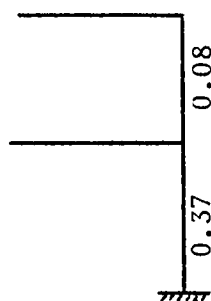
It is apparent from the value of these forces, that the P- Δ effect in this case is very small and in practice could be neglected. In this design example, the P- Δ effect is carried through for illustration purposes.

The moments and axial loads induced by the P- Δ effect as computed from the loads shown above are given the following sketches.

P- Δ Moments*



P- Δ Axial Loads*



*All these values can be plus or minus

(6) Load Factors and Combination of Loading Cases

Using the criteria specified in Sections VI and VII of Chapter IV one gets:

FOR THE BEAMS

Beam	M_{eb}	M_{d+1}	$M_{p-\Delta}$	M_u^-	M_u^+
B_1	825.3	522.8	43.7	1391.8	346.2
B_2	338.1	406.9	11.8	756.8	(-57)*

*No moment reversal expected, provide $\rho = \rho_{min}$.

FOR THE COLUMNS

Column	Joint	Moment				Axial		
		D+L	SRSS	P- Δ	M_{ec}	D+L	SRSS	P- Δ
C1	b	83.6	668.2	42.0	1135.9	30.0	7.74	0.37
	t	167.3	565.6	36.5	961.5			
C2	b	355.5	270.0	7.2	459.0	15.0	2.25	0.08
	t	406.9	338.1	11.8	338.1			

Column	Joint	Combinations*	
		A	B
C1	b	38.11 ^k	21.89 ^k
	t	1261.5 ^{k-in.}	1094.3 ^{k-in.}
C2	b	17.33 ^k	12.67 ^k
	t	821.7 ^{k-in.}	110.7 ^{k-in.}

*Combination C need not be checked because both columns are exterior columns.

(7) Strength Design of the Structural Elements

BEAM B1

$$M_u^- = 1391.8 \text{ k-in.}$$

$$M_u^+ = 346.2 \text{ k-in.}$$

Design for M_u^-

$$\text{Assume } j = 0.93 \quad A_s^- = \frac{1391.8}{0.9 \times 60 \times 0.93 \times 17.6} = 1.575 \text{ in}^2$$

and $d = 17.6 \text{ in.}$ check j ,

$$a = \frac{1.575 \times 60}{0.85 \times 4 \times 12} = 2.316 \text{ in.}$$

$$j = 1 - 0.5 \frac{a}{d} = 0.934 \quad \text{o.k.} \quad \begin{array}{l} \text{Use 2\#8 bars} \\ (A_s = 1.58 \text{ in}^2) \end{array}$$

Design for M_u^+

$$\rho_{\min} = \frac{200}{f_y} \quad (A_s)_{\min} = \frac{200}{f_y} bd = 0.704 \text{ in}^2$$

For which,

$$a = \frac{0.704 \times 60}{12 \times 0.85 \times 4} = 1.035 \text{ in.}$$

So,

$$(\phi M_n)_{\min} = 0.9 \times 60 \times 0.704 \times \left(17.6 - \frac{1.035}{2}\right) = 649.4 \text{ k-in} > M_u^+$$

so use $(A_s)_{\min}$ $\frac{\text{Use 2\#6 bars}}{(A_s = 0.88 \text{ in}^2)}$

BEAM B2

$$M_u^- = 756.8 \text{ k-in.}$$

$$M_u^+ - \text{No moment reversal expected, provide } (A_s)_{\min}$$

Design for M_u^-

Assume $j = 0.95$

$$A_s^- = \frac{756.8}{0.9 \times 0.95 \times 17.6 \times 60} = 0.838 \text{ in}^2$$

No need to check j $\frac{\text{Use 2\#6 bars}}{(A_s = 0.88 \text{ in}^2)} - \text{Top \& Bottom.}$

COLUMN C1

Slenderness Evaluation

P-Δ moments have been accounted for. The only effect to be considered herein is the column curvature. Using the ACI 318-77 procedure.

$$\ell_u = 180 - \frac{t_b}{2} = 170 \text{ in.}$$

$$r = 0.3 \times 12 = 3.6 \text{ in.}$$

Idealizing the column as pinned-fixed $k=0.7$. Then,

$$\frac{k\ell_u}{r} = \frac{0.7 \times 170}{3.6} = 33 < 34 \quad \text{No additional slenderness effects in Column C1.}$$

Since the axial load level is small ($< 0.1 f'_c A_g$) the critical combination is A so:

$$P_u = 38.11^k$$

$$M_u = 1261.5 \text{ k-in.}$$

From page 3-49 in Reference 30

$$\frac{\text{Use 6\#10 bars}}{(A_s = 7.62 \text{ in}^2)} \quad (\text{Two Faces})$$

COLUMN C2

$$\ell_u = 144 - t_b = 124 \text{ in.} \quad r = 3.6 \text{ in.} \quad \text{conservatively } k=1$$

$$\frac{k\ell_u}{r} = \frac{124 \times 1}{3.6} = 34.4 < 34 - (1.2 \frac{M_1}{M_2}) \quad \text{no additional slenderness effects.}$$

$$P_u = 17.35^k$$

$$M_u = 821.7^{\text{k-in.}}$$

From page 3-20 in Reference 30

$$\begin{array}{l} \text{Use 4\#9 bars} \\ (A_s = 4.00 \text{ in.}^2) \end{array}$$

(8) Calculation of the Inelastic Deformation Demands

From Equation (4.27) the yield drifts are given by:

$$\left\{ \delta_y \right\} = \left\{ \delta_e \right\} \left\{ \lambda \right\}$$

so

$$\delta_y = \begin{Bmatrix} 0.665 \\ 0.310 \end{Bmatrix} \begin{Bmatrix} 1.0157 \\ 1.0445 \end{Bmatrix} = \begin{Bmatrix} 0.675 \\ 0.324 \end{Bmatrix}$$

The plastic hinge rotations expected in the beams are computed from Equation (5.8) which is

$$\theta_p = (\mu - 1) \frac{\delta_y}{h} - \Delta\theta$$

BEAM B1

The term $\Delta\theta$ is evaluated from Equation (5.13)

$$\Delta\theta = \frac{-180 \times (1.7 - 1) \times 668.2}{6 \times 3605 \times 1728} = -0.00225 \text{ rad}$$

so

$$\theta_p = \frac{3 \times 0.675}{180} + 0.00225 = 0.01350 \text{ rad}$$

BEAM B2

$$\Delta\theta = 0$$

so

$$\theta_p = \frac{3 \times 0.324}{140} = 0.00675 \text{ rad}$$

(9) Calculation of Allowable Values of the Inelastic Demands: Evaluation of the Adequacy of the Design

The first place to check is Beam B1 since it has the highest inelastic demand and the highest steel ratio (which will lower the rotation capacity). If this beam is satisfactory, then B2 need not be checked.

BEAM B1

The distance from the critical section to the inflection point, z , is calculated as shown:

$$z = \frac{1391.8}{(1391.8 + 346.2)} \times 25 \times 12 = 240.2 \text{ in.}$$

taking ρ_s as zero; from Equation (5.18) one gets

$$\epsilon_u = 0.003 + \frac{0.02 \times 12}{240.2} = 0.004$$

For $f'_c = 4000$ psi $\beta_1 = 0.85$ (ACI 318-77). Neglecting the effect of the compression steel, the depth to the neutral axis at ultimate is,

$$c = \frac{a}{\beta_1} = \frac{2.316}{0.85} = 2.725 \text{ in.}$$

From Equation (5.15) the ultimate curvature is then,

$$\phi_u = \frac{0.004}{2.725} = 0.00147 \text{ in}^{-1}$$

The value of kd neglecting compression steel can be calculated from the well known expression

$$kd = d(\sqrt{\rho n^2 + 2\rho n} - \rho n)$$

in this case ρn is;

$$\rho n = \frac{1.58 \times 2900}{12 \times 17.6 \times 3605} = 0.06$$

so

$$kd = 17.6(\sqrt{(0.06)^2 + 0.12} - 0.06) = 5.13 \text{ in.}$$

From Equation (5.16)

$$\phi_y = \frac{0.00207}{(17.6 - 5.13)} = 0.000166 \text{ in}^{-1}$$

The equivalent length of the plastic hinge is given by Equation (5.17) which for the values herein yields.

$$\ell_p = 0.5 \times 20 + 0.05 \times 240.2 = 22 \text{ in.}$$

Substituting the values of ϕ_u , ϕ_y and ℓ_p into Equation (5.14) the available plastic hinge rotation for Beam B1 is computed to be:

$$\theta_{pa} = (0.00147 - 0.000166)22 = 0.02869 \text{ rad}$$

Since the value of the available rotation capacity θ_{pa} is larger than the required plastic hinge rotation, Beam B1 is satisfactory. As it was mentioned earlier, Beam B2 need not be checked because θ_{pa} for this beam will be larger and the required plastic hinge rotation is smaller than for Beam B1.

VITA

Prospero Dionisio Bernal was born in Santo Domingo, Dominican Republic on November 4, 1953. He started his undergraduate studies at the University Autonoma of Santo Domingo in July 1970. In August of 1973 he transferred to The University of Tennessee at Chattanooga where in May 1975 he was graduated with the degree of Bachelor of Science in Engineering. In September of 1975 he began studies towards a Master's degree in Civil Engineering at The University of Tennessee at Knoxville. This degree was awarded in December 1976. He continued his graduate studies in Civil Engineering and in the winter of 1978 was accepted as a candidate for the Doctor's degree by The University of Tennessee's Graduate School. He received the Doctor of Philosophy degree with a major in Civil Engineering in August of 1979.

The author will work for The University of Tennessee as a Visiting Assistant Professor in the Civil Engineering Department for one year after graduation.