To the Graduate Council:

I am submitting herewith a thesis written by Walter J. Eaton entitled "A Digital Filter Design Procedure with Application Program in APL." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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A DIGITAL FILTER DESIGN PROCEDURE
WITH APPLICATION PROGRAM IN APL

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Walter James Eaton

March 1987

Dedicated to my mother and

the memory of my father

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ABSTRACT

This study develops a fully documented computer program in APL for the synthesis of transfer functions and state-space realizations of digital filters configured in cascaded second-order subsections. The program designs Butterworth, Chebyshev, and Elliptic filters of the low-pass, high-pass, band-pass, and band-stop types. The realizations are designed to be computationally efficient with low roundoff noise.

The study essentially spans the complete development of a cascade digital filter by working through the following steps:

1. design parameters,
2. normalized analog transfer function,
3. denormalized analog transfer function,
4. digital transfer function,
5. realization coefficients.

Each step is examined in detail and at each point in the mathematical development, the program functions that perform the calculations at that point are identified. In addition, program function flow charts, detailed logic flow charts, and a glossary of terms are provided to complete the program documentation.

The mathematical development begins with descriptions and definitions of the Butterworth, Chebyshev, and Elliptic normalized analog low-pass filters. A standard form, for use throughout the study, is then established for the transfer functions of these filters. Next, procedures are established for calculating the coefficients of the

filters. A review is then made of computation parameters that can be used in the transformation from analog to digital transfer functions. These parameters, and the bilinear transformation, are then employed to generate low-pass and high-pass digital transfer functions with the desired cutoff frequencies and attenuation levels. The transformations to digital band-pass and band-stop transfer functions are handled in a similar manner. The transfer function development concludes with a general equation consisting of a constant times a ratio of products of second-order polynomials with real-valued coefficients.

The realization portion of the mathematical development begins with a brief review of the state-space concept. This is followed by procedures for applying the standard form equations in this study to second-order filter structures that employ state-space equations. Two of these structures have appeared only recently in the literature. The procedure used for selecting the appropriate filter structure for each second-order subsection of the transfer function is also relatively new and comes from the same source.

Numerical examples are provided to demonstrate the operation of the program. These examples are taken from the sources used in the mathematical development. Detailed comparisons are made between the results here and those of the original sources.

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CHAPTER I

INTRODUCTION

Digital filters have become increasingly more popular as the price of microprocessor chips has gone down. This is a natural state of affairs. Analog filter characteristics are component dependent so each of these filters is unique and can require adjustments for fine tuning. On the other hand, digital filters can be produced day after day with essentially identical characteristics. This kind of predictability is quite valuable so the interest in digital filters has naturally increased as these filters have become more cost effective.

There are two basic steps in the design of digital filters. The first step is to generate a digital transfer function (Z-transform) that acceptably represents the desired filter characteristics. This step is referred to as the approximation step. The second step is to determine exactly how this transfer function will be implemented in real life. This second step is referred to as the realization step.

There is a considerable body of literature dealing with the problem of generating a filter Z-transform and the corresponding set of coefficients for the filter realization. It is the goal of this paper to add to this literature by providing a detailed mathematical review of equations that can be used for digital filter design, and to provide a fully documented computer program that applies these equations, integrating the approximation and realization operations. The first step in this procedure is to review the basic definitions of

Butterworth, Chebyshev, and Elliptic normalized low-pass analog filters in [1, pp. 100-125]. The next step is to write general equations for the transfer functions of these filters in normalized low-pass form and to examine how their coefficients can be accurately determined. Then the general procedure outlined in [1, pp. 196-207] is used to develop denormalized low-pass, high-pass, band-pass, and band-stop filters from the original normalized filters. The next step is to rewrite the denormalized transfer functions in the form used in [2, pp. 178-180] so that procedures in [2] can be applied to find realizations with low fixed-point arithmetic roundoff noise. The procedure outlined in [2, pp. 191-193], whereby the appropriate realization structure is determined based on the pole angles, is then applied to obtain the low-roundoff-noise realization. All of these steps are finally integrated and documented in an APL computer program for designing digital filters.

Chapters II through VIII present the mathematical development outlined above. Appendices A through G deal with the computer program, its documentation, and sample program runs.

Throughout the text and appendices, references are made to a program and to specific functions within the program. Unless otherwise noted, these references pertain to the APL program titled DFD (Digital Filter Design) that appears in Appendix G. A function name listed beside an equation number in the text refers to the function where that particular equation is calculated for the specific purpose discussed at that point in the text. For example, the function name

PSCP1 appears beside equation number (60B), page 23, indicating that a value for the variable K_A is calculated in function PSCP1. Multiple function names beside an equation number means that a value is calculated for the variable in each of the functions listed. For example, the function names BLCC, CLCC, and ELCC appear beside the low-pass part of equation number (61A), page 23, indicating that a low-pass value for the variable K is calculated in each of these three functions. Frequently the value calculated for the variable will depend on which function is run. This in turn depends on the input parameters that are selected. The input parameters most likely to influence function selection are Design Type (DT) and Filter Type (FT).

CHAPTER II

NORMALIZED ANALOG FILTER TRANSFER FUNCTION DEVELOPMENT

Introduction

The fundamental concepts of obtaining normalized analog low-pass filter transfer functions are well documented. The Butterworth, Chebyshev, and Elliptic designs are reviewed thoroughly in [1, pp. 100-125]. The following descriptions and basic definitions follow that review. Digital transfer functions will be obtained later from these analog transfer functions.

Design Descriptions

The Butterworth design can be described as having a monotonically increasing attenuation in both the passband and stopband. Increasing the order of the filter will increase the asymtotic slope of the attenuation at the cutoff frequency. This design tends to produce a relatively slow transition from the passband to the stopband with a very flat passband. A more rapid transition can be achieved with the Chebyshev design which produces an attenuation that oscillates between zero and a maximum value, A_p, in the passband with a stopband characteristic which is similar to the Butterworth design. The Elliptic design produces an even more rapid transition by using a Chebyshev type passband with a stopband attenuation that oscillates between infinity and a minimum value, A_s. It should be noted that the Elliptic design is the most efficient of the three. For a given order of filter the Elliptic design produces the narrowest transition from passband to stopband.

Design Definitions

The attenuation of a Butterworth normalized low-pass filter is [1, p. 105]

$$A(\omega) = 10 \log(1 + \omega^{2N}) \ , \qquad (1)$$

where ω is the frequency in rad/sec, $A(\omega)$ is the attenuation in dB, and N is the order of the filter.

The Butterworth normalized low-pass transfer function that has the attenuation shown in (1) is

$$H_N(S) = \frac{1}{\prod_{k=1}^{N} (S - R_k)} \ , \qquad (2)$$

where R_k represents the $k = 1,2,3,\ldots,N$ complex roots of the transfer function denominator. Once the roots are calculated, $H_N(S)$ is completely determined. A procedure for finding the values of R_k appears in Chapter III.

In the Chebyshev case, the normalized low-pass filter attenuation is [1, p. 110]

$$A(\omega) = 10 \log[1 + (10^{0.1A_p} - 1)T_N^2(\omega)] \ , \qquad (3)$$

where A_p is the maximum attenuation in the passband and

$$T_N(\omega) = \begin{cases} \cos[N \cos^{-1}(\omega)] & \text{for } |\omega| \le 1 \\ \cosh[N \cosh^{-1}(\omega)] & \text{for } |\omega| > 1 \end{cases} \ . \qquad (4)$$

The Chebyshev normalized low-pass transfer function that has this attenuation is

$$H_N(S) = \frac{G_1}{\prod_{k=1}^{N} (S - R_k)} \quad , \tag{5}$$

where

$$G_1 = \begin{cases} \prod_{k=1}^{N} (-R_k) & \text{for odd } N \\ \\ 10^{-0.05A_p} \prod_{k=1}^{N} (-R_k) & \text{for even } N \end{cases} \quad . \tag{6}$$

As with the Butterworth case, the complex roots, R_k, must be calculated first since $H_N(S)$ and G_1 depend on these values. The procedure for this appears in Chapter III.

The Elliptic case is somewhat more complicated. The easiest way to deal with the attenuation is to define the maximum attenuation in the passband (A_p) and the minimum attenuation in the stopband (A_s) rather than defining the attenuation as a function of ω. If N and A_p are specified, the stopband attenuation is [1, p. 125]

$$A_s = 10 \log \left(\frac{10^{0.1A_p} - 1}{16q^N} + 1 \right) , \tag{7}$$

where q is the modular constant that appears in the mathematical development of the Elliptic filter.

The Elliptic normalized low-pass transfer function that exhibits these attenuation characteristics is

$$H_N(S) = \frac{G_1}{D_0(S)} \prod_{i=1}^{N_2} \left(\frac{S^2 + NA_{0i}}{S^2 + NB_{1i}\,S + NB_{0i}} \right), \tag{8}$$

where

$$N_2 = \begin{cases} \dfrac{N-1}{2} & \text{for odd } N \\ \\ \dfrac{N}{2} & \text{for even } N \end{cases}, \tag{9}$$

$$D_0(S) = \begin{cases} S + \sigma_0 & \text{for odd } N \\ 1 & \text{for even } N \end{cases}, \tag{10}$$

$$G_1 = \begin{cases} \sigma_0 \displaystyle\prod_{i=1}^{N_2} \frac{NB_{0i}}{NA_{0i}} & \text{for odd } N \\ \\ 10^{-0.05A_p} \displaystyle\prod_{i=1}^{N_2} \frac{NB_{0i}}{NA_{0i}} & \text{for even } N \end{cases}. \qquad \text{BNC} \tag{11}$$

The N in the coefficients of (8) indicates a normalized transfer function. Note that $D_0(S)$ represents the first-order term in the denominator of $H_N(S)$. Consequently $-\sigma_0$ represents a pole in the complex S-plane at $S = \sigma + j\omega$ where $\sigma = -\sigma_0$ and $\omega = 0$. Note also that the expressions for $H_N(S)$ and G_1 in the elliptic case are in terms of real-valued polynomial coefficients rather than the roots that appear in the Butterworth and Chebyshev cases. This difference is a result of the mathematical development in the Elliptic case which is quite different from the development in the other two cases. Chapter III discusses how to determine the values for NA_{0i}, NB_{1i}, and NB_{0i}.

In general, the Butterworth, Chebyshev, and Elliptic normalized transfer functions considered above can all be written in terms of second-order polynomials with real-valued coefficients as

$$H_N(S) = G_1 \prod_{i=\ell}^{N_2} \left(\frac{NA_{2i}\ S^2 + NA_{1i}\ S + NA_{0i}}{NB_{2i}\ S^2 + NB_{1i}\ S + NB_{0i}} \right) , \tag{12}$$

where $\ell = 0$ for odd N and $\ell = 1$ for even N.

G_1 is a constant and N_2 is the total number of second-order terms, or sections, in the filter. N_2 is found from N which is the order of the filter by

$$N_2 = \begin{cases} \dfrac{N-1}{2} & \text{for odd N} \\ \dfrac{N}{2} & \text{for even N} \end{cases} . \tag{13}$$

The total number of filter terms is then

$$N_T = \begin{cases} 1 + N_2 & \text{for odd N} \\ N_2 & \text{for even N} \end{cases} . \tag{14}$$

Transfer Function Standard Form

In the Butterworth case the normalized transfer function of (2) can be expressed as

$$H_N(S) = \frac{1}{(S - R_0) \prod_{i=1}^{N_2} [(S - R_i)(S - R_i^*)]} , \tag{15A}$$

where R_i^* is the complex conjugate of R_i and the first denominator term appears only when N is odd. Converting the terms to polynomials gives

$$H_N(S) = \frac{1}{(S + 1) \prod_{i=1}^{N_2} (S^2 + NB_{1i}\ S + 1)} , \tag{15B}$$

where the first denominator term appears only when N is odd. Thus, when expressed in the general form of (12), (15B) becomes

$$H_N(S) = \frac{G_1 \prod_{i=1}^{N_T} [NA_{2i}\, S^2 + NA_{1i}\, S + NA_{0i}]}{\prod_{i=\ell}^{N_2} [NB_{2i}\, S^2 + NB_{1i}\, S + NB_{0i}]}\,, \tag{16A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases}, \tag{16B}$$

$$G_1 = 1 \tag{16C}$$

$$\begin{aligned} NA_{2i} &= 0 \\ NA_{1i} &= 0 \\ NA_{0i} &= 1 \end{aligned} \qquad i = 1,2,3,\ldots,N_T, \tag{16D}$$

$$\begin{aligned} NB_{20} &= 0 \\ NB_{10} &= 1 \\ NB_{00} &= 1 \text{ as discussed in Chapter III} \end{aligned} \qquad \text{for odd } N, \tag{16E}$$

$$\begin{aligned} NB_{2i} &= 1 \\ NB_{1i} &\text{ will be defined in (24E)} \\ NB_{0i} &= 1 \text{ as discussed in Chapter III} \end{aligned} \qquad i = 1,2,3,\ldots,N_2. \tag{16F}$$

See function BNC.

Equation (16A) is modified slightly from (12) so that the numerator and denominator can be manipulated separately. The form of (16A) will be considered a standard form in this paper. Note that the product of terms in the denominator of (16A) follows the sequence from ℓ to N_2. This is done for the convenience of having the first second-order term identified by i = 1 for both odd and even N. This approach is taken in the program DFD in Appendix G. The program calculates the i = 0 terms for odd N in one subroutine, and the 1 to N_2 terms for both odd and even N in another subroutine. As an alternative, these same terms could be written in a sequence from 1 to N_T for both odd and even N as is done in the numerator of (16A). This approach is taken in the development of the general equation (169) in Chapter VII. The ℓ to N_2 sequence appears in the text and in the program whenever the first term is defined differently than the other terms in the sequence, and this difference must be dealt with in the calculations.

In the Chebyshev case the normalized transfer function of (5) becomes

$$H_N(S) = \frac{10^{-0.05A_p} \prod_{k=1}^{N} (-R_k)}{(S - R_0) \prod_{i=1}^{N_2} [(S - R_i)(S - R_i^*)]} , \tag{17A}$$

where the first numerator term appears only when N is even, and the first denominator term appears only when N is odd. Expressing the terms as polynomials gives

$$H_N(S) = \frac{10^{-0.05A_p} \prod_{k=1}^{N} (-R_k)}{(S + NB_{00}) \prod_{i=1}^{N_2} (S^2 + NB_{1i}\, S + NB_{0i})} . \tag{17B}$$

Here again the first numerator term appears only when N is even, and the first denominator term appears only when N is odd.

R_k is the k^{th} root of the denominator of (5) as k goes from 1 to N for odd or even N. Note that each second-order term in (17A) has two roots that are complex conjugates. These roots are multiplied together to get NB_{0i} since

$$(S - R_i)(S - R_i^*) = S^2 - (R_i + R_i^*)\, S + R_iR_i^* \quad , \tag{18A}$$

$$= S^2 + NB_{1i}\, S + NB_{0i} \ , \tag{18B}$$

$$\therefore\ NB_{1i} = -2\mathrm{Re}[R_i] \ , \tag{18C}$$

$$NB_{0i} = |R_i|^2 \ . \tag{18D}$$

Thus, from (6), G_1 can be defined in terms of NB_{0i} as

$$G_1 = \begin{cases} \prod_{k=1}^{N} (-R_k) = \prod_{i=0}^{N_2} (NB_{0i}) & \text{for odd } N \\ 10^{-0.05A_p} \prod_{k=1}^{N} (-R_k) = 10^{-0.05A_p} \prod_{i=1}^{N_2} (NB_{0i}) & \text{for even } N \end{cases} \ . \tag{19}$$

See function CNC.

Note that the definitions for G_1 in (19) take into account all of the roots of the denominator for both odd and even N.

The normalized Chebyshev transfer function in (17) can be expressed in the same standard form as the Butterworth filter giving

$$H_N(S) = \frac{G_1 \prod_{i=1}^{N_T} [NA_{2i}\, S^2 + NA_{1i}\, S + NA_{0i}]}{\prod_{i=\ell}^{N_2} [NB_{2i}\, S^2 + NB_{1i}\, S + NB_{0i}]} \ , \tag{20A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases} \ , \tag{20B}$$

G_1 is defined in (19), (20C)

$$\begin{aligned} NA_{2i} &= 0 \\ NA_{1i} &= 0 \\ NA_{0i} &= 1 \end{aligned} \qquad i = 1,2,3,\ldots,N_T, \tag{20D}$$

$$\begin{aligned} NB_{20} &= 0 \\ NB_{10} &= 1 \end{aligned} \qquad \text{for odd } N, \tag{20E}$$

NB_{00} will be defined in (35A) and (35B)

$NB_{2i} = 1$ $\qquad\qquad i = 1,2,3,\ldots,N_2.$ (20F)
NB_{1i} will be defined in (36A)
NB_{0i} will be defined in (37)

See function CNC.

In the Elliptic case the normalized transfer function is, from (8),

$$H_N(S) = G_1 \left(\frac{1}{S + NB_{00}}\right) \prod_{i=1}^{N_2} \left[\frac{(S^2 + NA_{0i})}{(S^2 + NB_{1i}\ S + NB_{0i})}\right], \tag{21}$$

where the first-order term appears only when N is odd. The constant G_1 can be defined as

$$G_1 = \begin{cases} \prod_{i=0}^{N_2} \dfrac{NB_{0i}}{NA_{0i}} & \text{for odd } N \\ 10^{-0.05A_p} \prod_{i=1}^{N_2} \dfrac{NB_{0i}}{NA_{0i}} & \text{for even } N \end{cases}. \qquad \text{ENC} \tag{22}$$

The normalized Elliptic transfer function in (21) can now be expressed in the same standard form as the Butterworth and Chebyshev transfer functions.

$$H_N(S) = \frac{G_1 \prod_{i=\ell}^{N_2} [NA_{2i}\ S^2 + NA_{1i}\ S + NA_{0i}]}{\prod_{i=\ell}^{N_2} [NB_{2i}\ S^2 + NB_{1i}\ S + NB_{0i}]}, \tag{23A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases}, \tag{23B}$$

G_1 is defined in (22), (23C)

$NA_{21} = 0$ for odd N, (23D)
$NA_{11} = 0$
$NA_{01} = 1$

$NA_{2i} = 1$ $i = 1,2,3,\ldots,N_2$, (23E)
$NA_{1i} = 0$
NA_{0i} will be defined in (51)

$NB_{21} = 0$ for odd N, (23F)
$NB_{11} = 1$
NB_{01} will be defined in (54)

$NB_{2i} = 1$ $i = 1,2,3,\ldots,N_2$. (23G)
NB_{1i} will be defined in (52)
NB_{0i} will be defined in (53)

See function ENC.

CHAPTER III

NORMALIZED ANALOG FILTER TRANSFER FUNCTION COEFFICIENTS

Butterworth

In the Butterworth case the poles of the normalized transfer function are located on the unit circle in the complex S-plane. The pole locations are a function of the factors of the denominator which will appear in pairs of the form $S - (\sigma + j\omega)$ and $S - (\sigma - j\omega)$ giving second-order terms. The first-order term, which appears when N is odd, will be $S - 1$. The second-order terms will be of the form

$$[NB_{2i}\, S^2 + NB_{1i}\, S + NB_{0i}] = [S - (\sigma_i + j\omega_i)]\,[S - (\sigma_i - j\omega_i)] \qquad (24A)$$

$$= S^2 - (2\sigma_i - j\omega_i + j\omega_i)\, S + (\sigma_i + j\omega_i)(\sigma_i - j\omega_i) \qquad (24B)$$

$$= S^2 - 2\sigma_i\, S + (\sigma_i{}^2 + \omega_i{}^2) \qquad (24C)$$

$$= S^2 - 2\sigma_i\, S + 1\ , \qquad (24D)$$

$$\therefore\ NB_{1i} = -2\sigma_i\ , \qquad \text{PSCP2} \qquad (24E)$$

$$NB_{0i} = (\sigma_i{}^2 + \omega_i{}^2) = 1\ . \qquad \text{PSCP2} \qquad (24F)$$

Note that $(\sigma_i{}^2 + \omega_i{}^2) = 1$ since the poles are all on the unit circle. The poles are located at

$$P_m = e^{j(N + 2m - 1)\pi/2N} = e^{jPA_m} \qquad \text{for } m = 1,2,3,\ldots,2N, \qquad (25)$$

where

$$PA_m = (N + 2m - 1)\pi/2N \qquad (26)$$

is the pole angle. The pole locations for N = 7 are shown in Figure 1, which also shows the relationships among P_m, R_k, and R_i.

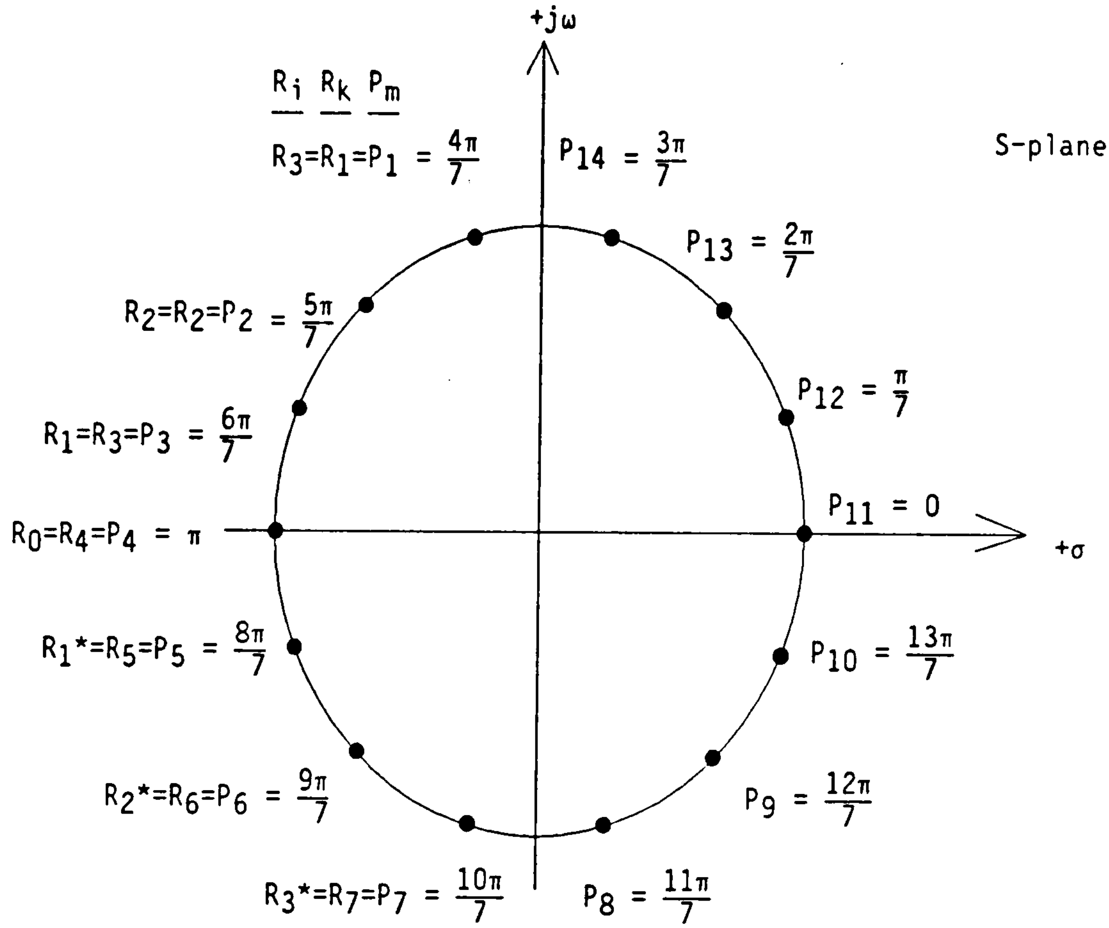

Figure 1. Pole Locations for Normalized Butterworth Filter with N = 7.

Only the poles in the left half of the S-plane are of interest since these poles represent a stable system. The first-order pole is P_4 at $\sigma = -1$ and $\omega = 0$. This pole is the same as root R_0 in (15A). The poles with the next largest real part magnitude are P_3 and P_5 where $P_5 = P_3{}^*$. These poles are the same as roots R_1 and $R_1{}^*$ in (15A). The next poles in the sequence are P_2 and P_6 which are R_2 and $R_2{}^*$ in (15A). And the last poles are P_1 and P_7 which are R_3 and $R_3{}^*$ in (15A).

With the R_i roots oriented in this manner, the NB_{1i} coefficients in the second-order terms in (15B) and (24A) will be arranged according to descending real part magnitude. Thus

$$NB_{1i} = -2\sigma_i = -2\cos(PA_i) = -2\cos(PA_m) \qquad \text{BNC} \qquad (27)$$

for $i = 1,2,3,\ldots,N_2$ and $m = N_2 + 1 - i$,

where PA_i is the subsection pole angle derived from PA_m by the relationship between i and m shown in (27). For N = 7, i = 1,2,3 and m = 3,2,1 as shown in Figure 1.

Chebyshev

In the Chebyshev case the poles of the normalized transfer function are located on an ellipse in the complex S-plane. The denominator factors will again be $S - (\sigma + j\omega)$ and $S - (\sigma - j\omega)$ for the second-order terms. Also, $NB_{1i} = -2\sigma_i$ and $NB_{0i} = (\sigma_i^2 + \omega_i^2)$ as before. However, in this case [1, p. 112]

$$\sigma_m = \sinh\left[\left(\frac{1}{N}\right)\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]\sin\left(\frac{(2m-1)\pi}{2N}\right) \quad , \qquad (28A)$$

$$\omega_m = \cosh\left[\left(\frac{1}{N}\right)\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]\cos\left(\frac{(2m-1)\pi}{2N}\right) \quad , \qquad (28B)$$

where $\epsilon = \sqrt{10^{0.1Ap} - 1}$. To simplify the computations the following constants can be defined:

$$\text{Chebyshev Constant \#1} = CC1 = \left(\frac{1}{N}\right)\sinh^{-1}\left(\frac{1}{\epsilon}\right) , \qquad \text{CNC} \qquad (29)$$

$$\text{Chebyshev Constant \#2} = CC2 = \sinh(CC1) , \qquad \text{CNC} \qquad (30)$$

$$\text{Chebyshev Constant \#3} = CC3 = \cosh(CC1) . \qquad \text{CNC} \qquad (31)$$

In the computation of σ_m and ω_m the value $\frac{(2m - 1)\pi}{2N}$ can be called the Computation Angle CA_m. The actual pole angle PA_m can be found from σ_m and ω_m

$$CA_m = \frac{(2m - 1)\pi}{2N} \quad , \qquad \text{CNC} \quad (32)$$

$$PA_m = \tan^{-1}\left(\frac{\omega_m}{\sigma_m}\right) . \qquad (33)$$

So now (28A) and (28B) become

$$\sigma_m = CC2 \ \sin(CA_m) \ , \qquad \text{CNC} \quad (34A)$$

$$\omega_m = CC3 \ \cos(CA_m) \ . \qquad \text{CNC} \quad (34B)$$

If N = 7 the pole locations and values of CA_m will be oriented as shown in Figure 2. As in Figure 1, the relationships among P_m, R_k, and R_i are also shown.

Note that the first order pole (for odd N, i = 0, and $m = 2N - (N_2 - i) = 11$) is at $P_{11} = \sigma_{11} + j\omega_{11}$ where σ_{11} and ω_{11} are computed from CA_{11}

$$\sigma_{11} = CC2 \ \sin(CA_{11}) = CC2 \ \sin\left(\frac{21\pi}{14}\right) = CC2 \ (-1) = -CC2 \quad , \qquad (35A)$$

$$\omega_{11} = CC3 \ \cos(CA_{11}) = CC3 \ \cos\left(\frac{21\pi}{14}\right) = CC3 \ (0) = 0 \quad . \qquad (35B)$$

As in the Butterworth case, the NB_{1i} coefficients for the second-order terms are found by employing the relationship between R_i and P_m. This results in

$$NB_{1i} = -2\sigma_i = -2\sigma_m = -2 \ CC2 \ \sin(CA_m) \quad , \qquad \text{CNC} \quad (36A)$$

for $i = 1,2,3,\ldots,N_2$ and $m = 2N - (N_2 - i)$.

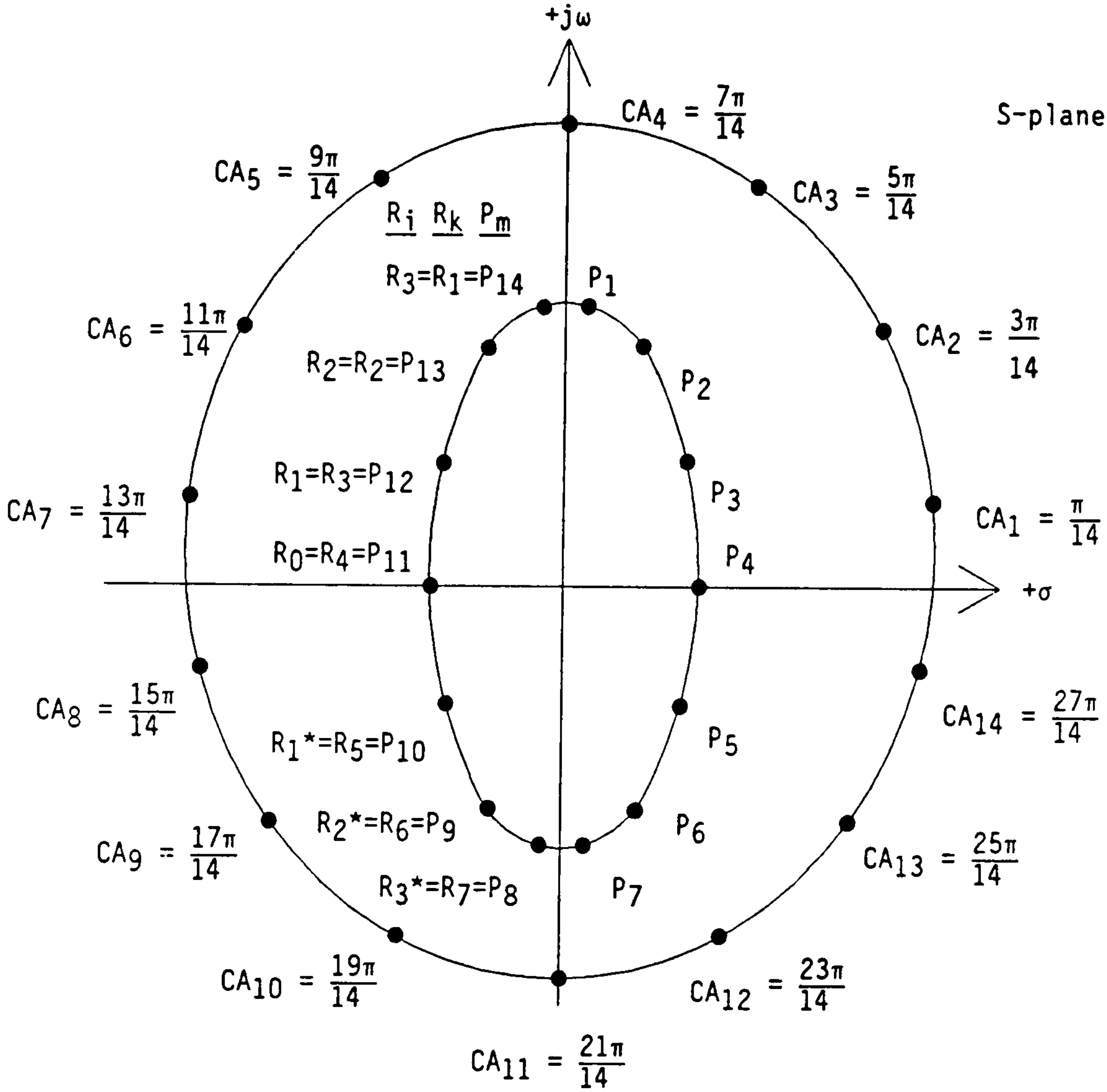

Figure 2. Pole Locations for Normalized Chebyshev Filter with N = 7.

The subsection pole angle will be

$$PA_i = PA_m = \tan^{-1}\left(\frac{\omega_m}{\sigma_m}\right), \qquad \text{SPAT} \quad (36B)$$

where i and m are the same as in (36A). For N = 7, i = 1,2,3 and m = 12,13,14 as shown in Figure 2. The NB_{0i} terms are found from

$$NB_{0i} = \sigma_i 2 + \omega_i 2 = \sigma_m 2 + \omega_m 2 \quad , \qquad \text{CNC} \quad (37)$$

where i and m are the same as in (36A) and (36B)

Elliptic

The normalized Elliptic coefficients are not computed directly from the pole locations. Instead they are computed from a sequence of formulas that result from a fairly lengthy mathematical development. A good review of this development is in [1, pp. 114, 126, 127]. The sequence of formulas is as follows:

$$k = \omega_{pb}/\omega_{sb} \quad \text{Where } \omega_{pb} = \text{Passband Freq. and } \omega_{sb} = \text{Stopband Freq.,} \qquad \text{ECP} \quad (38)$$

$$k' = \sqrt{1 - k^2} \, , \qquad \text{ECP} \quad (39)$$

$$q_o = \frac{1}{2} \; \frac{1 - \sqrt{k'}}{1 + \sqrt{k'}} \, , \qquad \text{ECP} \quad (40)$$

$$q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13} \, , \qquad \text{ECP} \quad (41)$$

$$D = \frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1} \quad \text{where } A_p = \text{Passband Attenuation and } A_s = \text{Stopband Attenuation,} \qquad \text{LHCP, PSCP1} \quad (42)$$

$$N_E = \frac{\log(16D)}{\log(1/q)} \qquad \text{ECP} \quad (43)$$

N = Smallest integer value larger than N_E , NCP (44)

$$\Lambda = \frac{1}{2N} \ln\left(\frac{10^{0.05A_p} + 1}{10^{0.05A_p} - 1}\right), \qquad \text{ENC} \quad (45)$$

$$\sigma_0 = \left[\frac{2q^{1/4} \sum_{m=o}^{\infty} (-1)^m q^{m(m+1)} \sinh[(2m+1)\Lambda]}{1 + 2\sum_{m=1}^{\infty} (-1)^m q^{m^2} \cosh(2m\Lambda)}\right], \qquad \text{ENC} \quad (46)$$

$$W = \sqrt{(1 + k\,\sigma_0^2)\left(1 + \frac{\sigma_0^2}{k}\right)}, \qquad \text{ENC} \quad (47)$$

$$\Omega_i = \frac{2q^{1/4} \sum_{m=o}^{\infty} (-1)^m q^{m(m+1)} \sin\left(\frac{(2m+1)\pi\mu}{N}\right)}{1 + 2\sum_{m=1}^{\infty} (-1)^m q^{m^2} \cos\left(\frac{2m\pi\mu}{N}\right)}, \qquad \text{ENC} \quad (48)$$

$$\text{where } \mu = \begin{cases} i \text{ for odd } N \\ i - 1/2 \text{ for even } N \end{cases} \quad i = 1,2,3,\ldots,N_2, \qquad \text{ENC} \quad (49)$$

$$V_i = \sqrt{(1 - k\Omega_i^2)\left(1 - \frac{\Omega_i^2}{k}\right)}. \qquad \text{ENC} \quad (50)$$

These relationships can be used to find the coefficients of $H_N(S)$ in (21)

$$NA_{0i} = \frac{1}{\Omega_i^2}, \qquad \text{ENC} \quad (51)$$

$$NB_{1i} = \frac{2\sigma_0 V_i}{1 + \sigma_0^2 \Omega_i^2}, \qquad \text{ENC} \quad (52)$$

$$NB_{0i} = \frac{(\sigma_0 V_i)^2 + (\Omega_i W)^2}{(1 + \sigma_0^2 \Omega_i^2)^2}, \qquad \text{ENC} \quad (53)$$

$$NB_{00} = \sigma_0 \text{ for the first-order term} . \qquad \text{ENC} \quad (54)$$

The summations in the numerator and denominator of σ_0 and Ω_i converge rapidly. The values of these summations were calculated for the filter in sample run #1 (run number RN = 1) in Appendix B of this paper. These values were accurate to 18 significant digits using only 4 terms in the summation. The approach taken in the program DFD in Appendix G is to use 11 terms which has been found to be acceptable for all situations encountered in practice.

CHAPTER IV

PRELIMINARY COMPUTATION PARAMETERS FOR DIGITAL FILTER TRANSFER FUNCTIONS

At this point the normalized analog low-pass transfer functions are complete and ready for use. The next step is the procedure outlined in [1, pp. 196-207] for transforming the normalized low-pass filter to a denormalized low-pass (LP), high-pass (HP), band-pass (BP), or band-stop (BS) filter. This chapter discusses the parameters which will be required as inputs to the program to specify the normalized filter and the transformation.

In general, the input parameters which are needed are:

ω_{pb} = Passband Frequency for LP and HP, (55A)

ω_{pb1} = Passband Frequency #1 for BP and BS, (55B)

ω_{pb2} = Passband Frequency #2 for BP and BS, (55C)

A_p = Maximum Passband Attenuation, (56)

ω_{sb} = Stopband Frequency for LP and HP, (57A)

ω_{sb1} = Stopband Frequency #1 for BP and BS, (57B)

ω_{sb2} = Stopband Frequency #2 for BP and BS, (57C)

A_s = Minimum Stopband Attenuation, (58)

$F_s = \frac{1}{T}$ = Sampling Frequency. (59)

See function IPMC.

Note that the frequency variables in (55), (57), and (59) can be entered into the program DFD with units that are multiples of hertz or radians/second in function IPMC. However, the frequencies in (55) and

(57) are all converted to multiples of radians/second and (59) is converted to a multiple of hertz in functions LHCP and PSCP1.

Then the following intermediate parameters are calculated for use in the transformation process of the next chapter:

$$K_0 = \frac{\tan\,(\omega_{pb}\;T/2)}{\tan\,(\omega_{sb}\;T/2)}\,, \qquad \text{LHCP} \qquad (60A)$$

$$K_A = \tan\left(\frac{\omega_{pb2}\;T}{2}\right) - \tan\left(\frac{\omega_{pb1}\;T}{2}\right), \qquad \text{PSCP1} \qquad (60B)$$

$$K_B = \tan\left(\frac{\omega_{pb1}\;T}{2}\right)\tan\left(\frac{\omega_{pb2}\;T}{2}\right), \qquad \text{PSCP1} \qquad (60C)$$

$$K_C = \tan\left(\frac{\omega_{sb1}\;T}{2}\right)\tan\left(\frac{\omega_{sb2}\;T}{2}\right), \qquad \text{PSCP1} \qquad (60D)$$

$$K_1 = \frac{K_A\,\tan(\omega_{sb2}\;T/2)}{K_B - \tan^2(\omega_{sb1}\;T/2)}\,, \qquad \text{PSCP1} \qquad (60E)$$

$$K_2 = \frac{K_A\,\tan(\omega_{sb2}\;T/2)}{\tan^2(\omega_{sb2}\;T/2) - K_B}\,, \qquad \text{PSCP1} \qquad (60F)$$

$$K = \begin{cases} K_0 \text{ for LP} & \text{BLCC, CLCC, ELCC} \\ 1/K_0 \text{ for HP} & \text{BHCC, CHCC, EHCC} \end{cases}, \qquad (61A)$$

$$K = \begin{cases} K_1 \text{ if } K_C \geq K_B \\ K_2 \text{ if } K_C < K_B \end{cases} \quad \text{for BP,} \qquad \text{PK10} \qquad (61B)$$

$$K = \begin{cases} 1/K_2 \text{ if } K_C \geq K_B \\ 1/K_1 \text{ if } K_C < K_B \end{cases} \quad \text{for BS,} \qquad \text{SK10} \qquad (61C)$$

$$D = \frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1} \quad \text{[same as (42)]}\,, \qquad \text{LHCP, PSCP1} \qquad (62)$$

$$N_E = \frac{\log(D)}{2\,\log(1/K)} \quad \text{(Butterworth),} \qquad \text{BCP} \qquad (63)$$

$$N_E = \frac{\cosh^{-1}(\sqrt{D}\,)}{\cosh^{-1}(1/K)} \qquad \text{(Chebyshev)}\ , \qquad \text{CCP} \qquad (64)$$

$$N_E = \frac{\log(16D)}{\log(1/q)} \qquad \text{[Elliptic, same as (43)]}\ , \qquad \text{ECP} \qquad (65)$$

$$N = \text{Smallest integer value larger than } N_E\ , \qquad \text{NCP} \qquad (66)$$

$$\omega_p = (10^{0.1A_p} - 1)^{1/2N} \qquad \text{(Butterworth)}\ , \qquad \text{BCP} \qquad (67)$$

$$\omega_p = 1 \qquad \text{(Chebyshev)}\ , \qquad \text{CCP} \qquad (68)$$

$$\omega_p = \sqrt{K} \qquad \text{(Elliptic)}\ , \qquad \text{ECP} \qquad (69)$$

$$\lambda = \begin{cases} \dfrac{\omega_p T}{2\tan(\omega_{pb}\,T/2)} & \text{for LP} \qquad \text{LCP} \\[2ex] \dfrac{2\omega_p \tan(\omega_{pb}\,T/2)}{T} & \text{for HP} \qquad \text{HCP} \end{cases} \ . \qquad (70)$$

CHAPTER V

DIGITAL LOW-PASS AND HIGH-PASS TRANSFER FUNCTIONS

Introduction

To get a digital LP or HP transfer function from a normalized LP transfer function requires making a substitution for the variable S. The substitutions in this chapter and the next are all based on the bilinear transform with prewarping of the cutoff frequencies [1, pp. 180-185]. The substitution will depend on whether the digital transfer function is to be LP or HP. The two cases are

$$S = \begin{cases} \lambda \left(\frac{2}{T}\right) \left[\frac{(Z-1)}{(Z+1)}\right] & \text{for LP} \\ \frac{\lambda}{\left[\left(\frac{2}{T}\right)\left(\frac{Z-1}{Z+1}\right)\right]} = \lambda \left(\frac{T}{2}\right)\left(\frac{Z+1}{Z-1}\right) & \text{for HP} \end{cases}, \tag{71}$$

where λ, from (70), is called the prewarping factor and $S = \frac{2}{T}\frac{(Z-1)}{(Z+1)}$ is the bilinear transformation.

To simplify the equations a constant can be defined

$$K_T = \begin{cases} \lambda \; \frac{2}{T} & \text{for LP} \\ \lambda \; \frac{T}{2} & \text{for HP} \end{cases}. \qquad \text{BCLHHZ} \tag{72}$$

Butterworth

Butterworth Low-Pass

For the Butterworth LP case and odd N the normalized LP transfer function is

$$H_N(S) = \frac{G_1}{(S + NB_{00}) \prod_{i=1}^{N_2} (S^2 + NB_{1i}\, S + NB_{0i})}, \tag{73}$$

which is a modified version of (16A). This equation could be simplified since $NB_{0i} = 1$ for all $i = 0,1,2,\ldots,N_2$ but the more general expression will be helpful later.

To convert to a digital LP transfer function, substitute

$$S = K_T \left(\frac{Z - 1}{Z + 1}\right) \tag{74}$$

into (73) which results in

$$H_{LP}(Z) = \frac{G_1}{\left[K_T\left(\frac{Z-1}{Z+1}\right) + NB_{00}\right] \prod_{i=1}^{N_2} \left[\left(K_T\left(\frac{Z-1}{Z+1}\right)\right)^2 + NB_{1i}\left(K_T\left(\frac{Z-1}{Z+1}\right)\right) + NB_{0i}\right]}, \tag{75A}$$

$$H_{LP}(Z) = \frac{G_1\,(Z + 1)^N}{\left[K_T\,(Z - 1) + NB_{00}\,(Z + 1)\right] \prod_{i=1}^{N_2} \left[K_T^2\,(Z - 1)^2 + NB_{1i}K_T\,(Z - 1)(Z + 1) + NB_{0i}\,(Z + 1)^2\right]}. \tag{75B}$$

Note that $(Z + 1)^N$ can be expanded to $(Z + 1)(Z^2 + 2Z + 1)^{N_2}$ for purposes of symmetry with the denominator. Continuing the reconfiguration gives

(75C)

$$H_{LP}(Z) = \frac{G_1\ (Z + 1)(Z^2 + 2Z + 1)^{N_2}}{\left[(K_T + NB_{00})Z + (NB_{00} - K_T)\right] \prod_{i=1}^{N_2} \left[(K_T^2 + NB_{1i}K_T + NB_{0i})Z^2 + (2NB_{0i} - 2\ K_T^2)Z + (K_T^2 - NB_{1i}K_T + NB_{0i})\right]},$$

(75D)

$$H_{LP}(Z) = \frac{G_1 \left(\frac{1}{K_T + NB_{00}}\right) \prod_{i=1}^{N_2} \left[\frac{1}{K_T^2 + NB_{1i}K_T + NB_{0i}}\right] [(Z + 1)(Z^2 + 2Z + 1)^{N_2}]}{\left[Z + \left(\frac{NB_{00} - K_2}{K_T + NB_{00}}\right)\right] \prod_{i=1}^{N_2} \left[Z^2 + 2\left(\frac{NB_{0i} - K_T^2}{K_T^2 + NB_{1i}K_T + NB_{0i}}\right)Z + \left(\frac{K_T^2 + NB_{0i} - NB_{1i}K_T}{K_T^2 + NB_{1i}K_T + NB_{0i}}\right)\right]} .$$

For even N delete the factor $\frac{1}{K_T + NB_{00}}$ and (Z + 1) from the numerator, and delete the first-order Z term from the denominator.

For computation purposes the constants in the numerator of $H_{LP}(Z)$ can be redefined. The results are

$G_1 = 1$ as defined in (16C), (76)

$$G_2 = \begin{cases} \dfrac{1}{K_T + NB_{00}} & \text{for odd } N \\ 1 & \text{for even } N \end{cases}, \qquad \text{BCLHHZ} \quad (77)$$

$$G_3 = \prod_{i=1}^{N_2} \frac{1}{K_T^2 + NB_{1i}K_T + NB_{0i}} . \qquad \text{LHHZD} \quad (78)$$

Using these constants and converting (76D) to standard form gives

$$H_{LP}(Z) = \frac{G_1 G_2 G_3 \prod_{i=\ell}^{N_2} \left[DA_{2i}\, Z^2 + DA_{1i}\, Z + DA_{0i} \right]}{\prod_{i=\ell}^{N_2} \left[DB_{2i}\, Z^2 + DB_{1i}\, Z + DB_{0i} \right]}, \qquad (79A)$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases}, \qquad (79B)$$

G_1, G_2, and G_3 are defined in (76), (77), and (78) respectively, (79C)

$DA_{20} = 0$ for odd N, (79D)
$DA_{10} = 1$
$DA_{00} = 1$

$DA_{2i} = 1$ $i = 1,2,3,\ldots,N_2$, (79E)
$DA_{1i} = 2$
$DA_{0i} = 1$

$DB_{20} = 0$ for odd N, (79F)
$DB_{10} = 1$
DB_{00} is defined in (75D)

$DB_{2i} = 1$ $i = 1,2,3,\ldots,N_2$. (79G)
DB_{1i} is defined in (75D)
DB_{0i} is defined in (75D)

See functions BCLHHZ and LHHZD.

The D in the coefficients of (79A) indicates a digital transfer function.

Butterworth High-Pass

For the Butterworth HP case the development is the same but the substitution for S is

$$S = K_T\left(\frac{Z+1}{Z-1}\right). \tag{80}$$

So (73) becomes

$$H_{HP}(Z) = \frac{G_1\left(\dfrac{1}{K_T + NB_{00}}\right)\prod_{i=1}^{N_2}\left[\dfrac{1}{K_T^2 + NB_{1i}K_T + NB_{0i}}\right][(Z-1)(Z^2-2Z+1)^{N_2}]}{\left[Z + \left(\dfrac{K_T - NB_{00}}{K_T + NB_{00}}\right)\right]\prod_{i=1}^{N_2}\left[Z^2 + 2\left(\dfrac{K_T^2 - NB_{0i}}{K_T^2 + NB_{1i}K_T + NB_{0i}}\right)Z + \left(\dfrac{K_2^2 + NB_{0i} - NB_{1i}K_T}{K_T^2 + NB_{1i}K_T + NB_{0i}}\right)\right]}. \tag{81}$$

As before, for even N delete the $\dfrac{1}{K_T + NB_{00}}$ factor and (Z - 1) from the numerator and delete the first-order Z term from the denominator. Expressing (81) in standard form results in

$$H_{HP}(Z) = \frac{G_1G_2G_3\prod_{i=\ell}^{N_2}\left[DA_{2i}\,Z^2 + DA_{1i}\,Z + DA_{0i}\right]}{\prod_{i=\ell}^{N_2}\left[DB_{2i}\,Z^2 + DB_{1i}\,Z + DB_{0i}\right]}, \tag{82A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd N} \\ 1 \text{ for even N} \end{cases}, \tag{82B}$$

G_1, G_2, and G_3 are defined in (76), (77), and (78) respectively, (82C)

$DA_{20} = 0$ for odd N, (82D)
$DA_{10} = 1$
$DA_{00} = -1$

$DA_{2i} = 1$ $i = 1,2,3,\ldots,N_2$, (82E)
$DA_{1i} = -2$
$DA_{0i} = 1$

$DB_{20} = 0$ for odd N, (82F)
$DB_{10} = 1$
DB_{00} is defined in (81)

$DB_{2i} = 1$ $i = 1,2,3,\ldots,N_2$. (82G)
DB_{1i} is defined in (81)
DB_{0i} is defined in (81)

See functions BCLHHZ and LHHZD.

Butterworth Combined Low-Pass and High-Pass

The general equation for the Butterworth LP and HP digital $H_{LH}(Z)$ transfer function when N is odd becomes

$$H_{LH}(Z) = \frac{G_1 G_2 G_3 \, (Z + DA_{00}) \prod_{i=1}^{N_2} (Z^2 + DA_{1i}\, Z + 1)}{(Z + DB_{00}) \prod_{i=1}^{N_2} (Z^2 + DB_{1i}\, Z + DB_{0i})}, \tag{83}$$

where the D indicates digital coefficients. Note that the LP case requires $DA_{00} = +1$ and $DA_{1i} = +2$ for $i = 1,2,3,\ldots,N_2$; and the HP case requires $DA_{00} = -1$ and $DA_{1i} = -2$ for $i = 1,2,3,\ldots,N_2$. The digital B coefficients can be computed from (75D) and (81). For even N delete the two first-order Z terms. The general equation in (83) can be expressed in standard form

$$H_{LH}(Z) = \frac{G_1G_2G_3 \prod_{i=\ell}^{N_2} \left[DA_{2i}\ Z^2 + DA_{1i}\ Z + DA_{0i}\right]}{\prod_{i=\ell}^{N_2} \left[DB_{2i}\ Z^2 + DB_{1i}\ Z + DB_{0i}\right]} , \tag{84}$$

where the constants and coefficients are defined in (79) and (82).

Chebyshev

In the Chebyshev case the development of H(Z) is the same as in the Butterworth case. So the entire Butterworth development from the normalized H(S) in (73) to the combined LP and HP H(Z) in (84) applies to the Chebyshev design as well. The numerical results are different because the normalized numerator constant (G_1) is defined in (16C) for Butterworth and (19) for Chebyshev. Also, for Butterworth NB_{1i} is defined in (27) and $NB_{0i} = 1$, while for Chebyshev NB_{1i} and NB_{0i} are found from (36A) and (37) respectively.

Elliptic

For the Elliptic case the denominator of H(Z) is computed in the same manner as in the Butterworth and Chebyshev H(Z) equations, (75D) and (81). This means that G_2 and G_3 will still be computed by equations (77) and (78) respectively since G_2 and G_3 are functions of the denominator transformations. G_1 in the numerator is defined in (22). However the numerator of H(Z) is complicated by the second-order terms in the numerator of the normalized $H_N(S)$ in (21)

$$\prod_{i=1}^{N_2} (S^2 + NA_{0i}) . \tag{85}$$

These terms must also be transformed by the substitution shown in equation (71).

Elliptic Low-Pass

In the LP case the numerator second-order terms become

$$\prod_{i=1}^{N_2} (S^2 + NA_{0i}) = \prod_{i=1}^{N_2} \left[\frac{K_T^2\ (Z-1)^2}{(Z+1)^2} + NA_{0i} \right] , \qquad (86A)$$

$$= \prod_{i=1}^{N_2} \left[\left(K_T^2 (Z-1)^2 + NA_{0i}\ (Z+1)^2 \right) \left(\frac{1}{(Z+1)^2} \right) \right] , \qquad (86B)$$

$$= \left(\frac{1}{(Z+1)^2} \right)^{N_2} \prod_{i=1}^{N_2} \left[K_T^2\ (Z^2 - 2Z + 1) + NA_{0i}\ (Z^2 + 2Z + 1) \right] , \qquad (86C)$$

$$(86D)$$

$$= \left(\frac{1}{(Z^2 + 2Z + 1)} \right)^{N_2} \prod_{i=1}^{N_2} \left[(K_T^2 + NA_{0i}) Z^2 + 2(NA_{0i} - K_T^2) Z + (K_T^2 + NA_{0i}) \right] ,$$

$$(86E)$$

$$= \left(\frac{1}{(Z^2 + 2Z + 1)} \right)^{N_2} \prod_{i=1}^{N_2} (K_T^2 + NA_{0i}) \prod_{i=\ell}^{N_2} \left[Z^2 + \left(\frac{2(NA_{0i} - K_T^2)}{(K_T^2 + NA_{0i})} \right) Z + 1 \right] .$$

The first term in this equation will be canceled by the last numerator term in equation (75D) when the Elliptic numerator terms (86E) become additional numerator factors in equation (75D) for H(Z). So the LP Elliptic H(Z) becomes

$$H_{LP}(Z) = \frac{G_1G_2G_3\,(Z+1)\prod_{i=1}^{N_2}(K_T^2 + NA_{0i})\prod_{i=\ell}^{N_2}\left[Z^2 + \left(\frac{2(NA_{0i} - K_T^2)}{(K_T^2 + NA_{0i})}\right)Z + 1\right]}{(Z + DB_{00})\prod_{i=1}^{N_2}(Z^2 + DB_{1i}\,Z + DB_{0i})} \quad (87)$$

For computation purposes the constant G_4 can be defined as

$$G_4 = \prod_{i=1}^{N_2}(K_T^2 + NA_{0i}) \, . \qquad \text{ELHHZ} \quad (88)$$

Expressing (87) in standard form gives

$$H_{LP}(Z) = \frac{G_1G_2G_3G_4 \prod_{i=\ell}^{N_2}\left[DA_{2i}\,Z^2 + DA_{1i}\,Z + DA_{0i}\right]}{\prod_{i=\ell}^{N_2}\left[DB_{2i}\,Z^2 + DB_{1i}\,Z + DB_{0i}\right]} \, , \qquad (89A)$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases} \, , \qquad (89B)$$

G_1, G_2, G_3, and G_4 are defined in (22), (77), (78), and (88) respectively, (89C)

$DA_{20} = 0$ for odd N, (89D)
$DA_{10} = 1$
$DA_{00} = 1$

$DA_{2i} = 1$ $i = 1,2,3,\ldots,N_2$, (89E)
DA_{1i} is defined in (87)
$DA_{0i} = 1$

$DB_{20} = 0$ for odd N, (89F)
$DB_{10} = 1$
DB_{00} is defined in (75D)

$DB_{2i} = 1$ $i = 1,2,3,\ldots,N_2.$ (89G)
DB_{1i} is defined in (75D)
DB_{0i} is defined in (75D)

See functions ELHHZ and LHHZD.

Note that the DB terms in (89F) and (89G) are the same as those in (79F) and (79G) respectively.

Elliptic High-Pass

The development for the HP case is similar and results in

$$H_{HP}(Z) = \frac{G_1G_2G_3G_4\,(Z-1)\prod_{i=1}^{N_2}(K_T^2 + NA_{0i})\prod_{i=1}^{N_2}\left[Z^2 + \left(\frac{2(K_T^2 - NA_{0i})}{(K_T^2 + NA_{0i})}\right)Z + 1\right]}{(Z + DB_{00})\prod_{i=1}^{N_2}(Z^2 + DB_{1i}\,Z + DB_{0i})}. \tag{90}$$

Using the standard form, $H_{HP}(Z)$ in (90) becomes

$$H_{HP}(Z) = \frac{G_1G_2G_3G_4\prod_{i=\ell}^{N_2}\left[DA_{2i}\,Z^2 + DA_{1i}\,Z + DA_{0i}\right]}{\prod_{i=\ell}^{N_2}\left[DB_{2i}\,Z^2 + DB_{1i}\,Z + DB_{0i}\right]}, \tag{91A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases}, \tag{91B}$$

G_1, G_2, G_3, and G_4 are defined in (22), (77), (78), and (88) respectively, (91C)

$DA_{20} = 0$ for odd N, (91D)
$DA_{10} = 1$
$DA_{00} = -1$

$DA_{2i} = 1$ $i = 1,2,3,\ldots,N_2$, (91E)
DA_{1i} is defined in (90)
$DA_{0i} = 1$

$DB_{20} = 0$ for odd N, (91F)
$DB_{10} = 1$
DB_{00} is defined in (75D)

$DB_{2i} = 1$ $i = 1,2,3,\ldots,N_2$. (91G)
DB_{1i} is defined in (75D)
DB_{0i} is defined in (75D)

See functions ELHHZ and LHHZD.

Here again the DB terms remain the same as in (79F) and (79G).

Elliptic Combined Low-Pass and High-Pass

The general equation for the Elliptic LP and HP H(Z) transform becomes

$$H_{LH}(Z) = \frac{G_1G_2G_3G_4\ (Z + DA_{00}) \prod_{i=1}^{N_2} (Z^2 + DA_{1i}\ Z + 1)}{(Z + DB_{00}) \prod_{i=1}^{N_2} (Z^2 + DB_{1i}\ Z + DB_{0i})}, \qquad (92)$$

where, as with the Butterworth and Chebyshev equations, the LP case requires $DA_{00} = +1$ and the HP case requires $DA_{00} = -1$. Equations (87) and (90) differ from equations (75D) and (81) in the values of DA_{1i} since the Elliptic case requires

$$DA_{1i} = \begin{cases} \dfrac{2(NA_{0i} - K_T^2)}{(K_T^2 + NA_{0i})} & \text{for LP and } i = 1,2,3,\ldots,N_2 \\[2ex] \dfrac{2(K_T^2 - NA_{0i})}{(K_T^2 + NA_{0i})} & \text{for HP and } i = 1,2,3,\ldots,N_2 \end{cases} \quad , \qquad (93)$$

and the Butterworth and Chebyshev cases require $DA_{1i} = +2$ for LP and $DA_{1i} = -2$ for HP and $i = 1,2,3,\ldots,N_2$. The DB coefficients in the denominator of (87) and (90) can be computed from equations (75D) and (81). And for even N the first-order Z terms are deleted as before.

In standard form (92) becomes

$$H_{LH}(Z) = \frac{G_1 G_2 G_3 G_4 \prod_{i=\ell}^{N_2} \left[DA_{2i}\, Z^2 + DA_{1i}\, Z + DA_{0i} \right]}{\prod_{i=\ell}^{N_2} \left[DB_{2i}\, Z^2 + DB_{1i}\, Z + DB_{0i} \right]} \quad , \qquad (94)$$

where the constants and coefficients are defined in (89) and (91).

General Low-Pass and High-Pass Equation

At this point the elliptic constant can be defined as $G_4 = 1$ for the Butterworth and Chebyshev cases. This allows (92) to become the general case equation for H(Z) for all three design types. The requirements for DA_{0i} and the first-order Z terms are the same for all three types, and the DA_{1i} terms are defined in previous equations for all three types. The individual constants in (92) can be combined into a total constant

$$G_{LH} = G_1 G_2 G_3 G_4 \ . \qquad (95)$$

Equation (92) then becomes the general LP and HP H(Z) transfer function

$$H_{LH}(Z) = \frac{G_{LH}\ (Z + DA_{00}) \prod_{i=1}^{N_2} (Z^2 + DA_{1i}\ Z + 1)}{(Z + DB_{00}) \prod_{i=1}^{N_2} (Z^2 + DB_{1i}\ Z + DB_{0i})}\ , \tag{96}$$

which can be expressed in standard form

$$H_{LH}(Z) = \frac{G_{LH} \prod_{i=\ell}^{N_2} [DA_{2i}\ Z^2 + DA_{1i}\ Z + DA_{0i}]}{\prod_{i=\ell}^{N_2} [DB_{2i}\ Z^2 + DB_{1i}\ Z + DB_{0i}]}\ , \tag{97}$$

where the constant and coefficients are previously defined for each case.

CHAPTER VI

DIGITAL BAND-PASS AND BAND-STOP TRANSFER FUNCTIONS

Introduction

To generate a digital band-pass (BP) or band-stop (BS) transfer function from a normalized LP transfer function requires making a substitution for the variable S as was done in the LP and HP cases. Due to the complexity of the substitution it is prudent to break the procedure into two steps. First the filter type is changed from analog normalized filter to analog BP or BS filter, then the change from analog filter to digital filter is made.

Butterworth

The Butterworth procedure begins with the normalized LP transfer function in (73) which is repeated here

$$H_N(S) = \frac{G_1}{(S + NB_{00}) \prod_{i=1}^{N_2} (S^2 + NB_{1i}\, S + NB_{0i})} \quad . \tag{98}$$

The first denominator term appears only when N is odd. In a similar manner to (15A), (98) can be rearranged to show the denominator roots R_0, R_i, and R_i* where R_0 is real and R_i* is the complex conjugate of R_i. This gives

$$H_N(S) = \frac{G_1}{(S - R_0) \prod_{i=1}^{N_2} \left[(S - R_i)(S - Ri*)\right]} \quad , \tag{99A}$$

where

$$R_i = \sigma + j\omega \text{ and } R_i^* = \sigma - j\omega \ . \tag{99B}$$

Butterworth Band-Pass

To convert (98) to a BP transfer function substitute

$$S = \frac{1}{B}\left(S' + \frac{\omega_o^2}{S'}\right) \tag{100}$$

into (99) which becomes

$$H_{BP}(S') = \frac{G_1}{\left[\frac{1}{B}\left(S' + \frac{\omega_o^2}{S'}\right) - R_0\right] \prod_{i=1}^{N_2} \left[\left(\frac{1}{B}\left(S' + \frac{\omega_o^2}{S'}\right) - R_i\right) \left(\frac{1}{B}\left(S' + \frac{\omega_o^2}{S'}\right) - R_i^*\right)\right]} \ , \tag{101A}$$

$$H_{BP}(S') = \frac{B^{N_T}\, G_1\, S'^{N_T}}{\left[S'^2 - BR_0\, S' + \omega_o^2\right] \prod_{i=1}^{N_2} \left[\left(S'^2 - BR_i\, S' + \omega_o^2\right) \left(S'^2 - BR_i^*\, S' + \omega_o^2\right)\right]} \ , \tag{101B}$$

where

$$B = \frac{2K_A}{T\omega_p} \ , \qquad \text{PCP} \tag{101C}$$

$$\omega_o = \frac{2\sqrt{K_B}}{T} \ , \qquad \text{PCP SCP} \tag{101D}$$

K_A, K_B, T, and ω_p are defined in Chapter IV, and N_T is the number of terms, all second-order. Note that each root in the parent $H_N(S)$ produces a second-order term in $H_{BP}(S')$. So the total number of terms is

$$N_T = \begin{cases} 1 + 2N_2 & \text{for odd N} \\ 2N_2 & \text{for even N} \end{cases} . \tag{101E}$$

Consequently, for $H_{BP}(S')$,

$$N_T = N \tag{101F}$$

which is the order of the parent low-pass filter. The order of the $H_B(S')$ transform is

$$N_B = 2N_T = 2N \tag{101G}$$

since there are two roots for each root in the parent low-pass filter.

Since R_0 and ω_0 are real the first term in the denominator of (101B) can be handled easily. However BR_i and BR_i^* are complex. The quadratic equation can be used to separate the roots of these terms. Starting with the terms with BR_i as a coefficient the roots are

$$R_i' = \frac{BR_i \pm \sqrt{(BR_i)^2 - 4\omega_o^2}}{2} , \tag{102A}$$

$$R_i' = R_{1i}', R_{2i}' . \tag{102B}$$

The terms with coefficient BR_1^* have roots

$$R_i'^* = \frac{BR_i^* \pm \sqrt{(BR_i^*)^2 - 4\omega_o^2}}{2} , \tag{103A}$$

$$R_i'^* = R_{3i}', R_{4i}'. \tag{103B}$$

To see how these four roots are related, (102A) is expanded to show real and imaginary parts. Define

$$BR_i = B(\sigma + j\omega) = B\sigma + jB\omega = \sigma_a + j\omega_a \; . \qquad \text{PCP} \qquad (104)$$

This gives

$$R_i' = \frac{\sigma_a + j\omega_a \pm \sqrt{(\sigma_a + j\omega_a)^2 - 4\omega_o^2}}{2} \; , \qquad (105A)$$

$$R_i' = \frac{\sigma_a}{2} + \frac{j\omega_a}{2} \pm \sqrt{\frac{\sigma_a^2 + j2\omega_a\sigma_a - \omega_a^2 - 4\omega_o^2}{4}} \; , \qquad (105B)$$

$$R_i' = \frac{\sigma_a}{2} + \frac{j\omega_a}{2} \pm \sqrt{\frac{\sigma_a^2 - \omega_a^2 - 4\omega_o^2}{4} + \frac{j\omega_a\sigma_a}{2}} \; , \qquad (105C)$$

$$R_i' = \sigma_a' + j\omega_a' \pm \sqrt{\sigma_b' + j\omega_b'} \; , \qquad \text{PSCP3} \qquad (105D)$$

$$R_i' = \sigma_a' + j\omega_a' \pm \sqrt{r\angle\theta} \; , \qquad (105E)$$

where

$$r = \sqrt{\sigma_b'^2 + \omega_b'^2} \; , \qquad \text{PSCP3} \qquad (105F)$$

$$\theta = \begin{cases} \arctan\left(\dfrac{\omega_b'}{\sigma_b'}\right) & \text{for } \sigma_b' \geq 0 \\ \arctan\left(\dfrac{\omega_b'}{\sigma_b'}\right) + \pi & \text{for } \sigma_b' < 0 \end{cases} \; . \qquad \text{PSCP3} \qquad (105G)$$

Note that the arctan function is multi-valued and the computer will select the angle θ in the range $(-\pi/2) < \theta < (\pi/2)$. If the vector $r\angle\theta$ is known to be in the left half of the S-plane ($\sigma_b < 0$) then π must be added to θ to move the vector to the correct quadrant. This becomes important later when θ is used in $\sin(\theta)$ and $\cos(\theta)$ applications.

Continuing

$$R_i' = \sigma_a' + j\omega_a' \pm r'\angle\theta' \;, \qquad (105H)$$

where

$$r' = \sqrt{r} \;, \qquad \text{PSCP3} \quad (105I)$$

$$\theta' = \frac{\theta}{2} \,. \qquad \text{PSCP3} \quad (105J)$$

Finally

$$R_i' = \sigma_a' + j\omega_a' \pm (\sigma_c' + j\omega_c') \;, \qquad (105K)$$

where

$$\sigma_c' = r' \cos(\theta), \qquad \text{PSCP3} \quad (105L)$$

$$\omega_c' = r' \sin(\theta). \qquad \text{PSCP3} \quad (105M)$$

Likewise, (103A) is expanded to

$$R_i'^* = \frac{\sigma_a - j\omega_a \pm \sqrt{(\sigma_a - j\omega_a)^2 - 4\omega_o^2}}{2} \;, \qquad (106A)$$

$$R_i'^* = \frac{\sigma_a}{2} - \frac{j\omega_a}{2} \pm \sqrt{\frac{\sigma_a^2 - j2\omega_a\sigma_a - \omega_a^2 - 4\omega_o^2}{4}} \;, \qquad (106B)$$

$$R_i'^* = \frac{\sigma_a}{2} - \frac{j\omega_a}{2} \pm \sqrt{\frac{\sigma_a^2 - \omega_a^2 - 4\omega_o^2}{4} - \frac{j\omega_a\sigma_a}{2}} \;, \qquad (106C)$$

$$R_i'^* = \sigma_a' - j\omega_a' \pm \sqrt{\sigma_b' - j\omega_b'} \;, \qquad (106D)$$

$$R_i'^* = \sigma_a' - j\omega_a' \pm \sqrt{r\angle{-\theta}} \;, \qquad (106E)$$

$$R_i'^* = \sigma_a' - j\omega_a' \pm r'\angle{-\theta'} \;, \qquad (106F)$$

$$R_i'^* = \sigma_a' - j\omega_a' \pm (\sigma_c' - j\omega_c') \,. \qquad (106G)$$

The two roots in (105K) can be listed separately as they were in (102B),

$$R_i' = (\sigma_a' + \sigma_c') + j(\omega_a' + \omega_c'),\ (\sigma_a' - \sigma_c') + j(\omega_a' - \omega_c') \ . \tag{107}$$

Similarly, the roots in (106G) can be separated as in (103B),

$$R_i'^{*} = (\sigma_a' + \sigma_c') + j(-\omega_a' - \omega_c'),\ (\sigma_a' - \sigma_c') + j(-\omega_a' + \omega_c') \ . \tag{108}$$

From (102B), (103B), (107), and (108) it can be seen that

$$R_{3i}' = R_{1i}'^{*} \ , \tag{109A}$$

$$R_{4i}' = R_{2i}'^{*} \ . \tag{109B}$$

This can be expressed in a simplified form

$$R_{1i}' = \sigma_{di} + j\omega_{di} \ , \tag{110A}$$

$$R_{3i}' = \sigma_{di} - j\omega_{di} \ , \tag{110B}$$

$$R_{2i}' = \sigma_{ei} + j\omega_{ei} \ , \tag{110C}$$

$$R_{4i}' = \sigma_{ei} - j\omega_{ei} \ , \tag{110D}$$

where

$$\sigma_d = (\sigma_a' + \sigma_c') \ , \qquad \text{PSCP3} \tag{110E}$$

etc.

These roots can be reqrouped as conjugate pairs to provide real coefficients in the denominator terms of (101B). So (101B) becomes

$$H_{BP}(S') = \frac{B^{N_T} G_1 S'^{N_T}}{\left[S'^2 - BR_0 S' + \omega_o^2\right] \prod_{i=1}^{N_2} \left[\left((S' - R_{1i}')(S' - R_{3i}')\right)\left((S' - R_{2i}')(S' - R_{4i}')\right)\right]}, \tag{111A}$$

$$H_{BP}(S') = \frac{B^{N_T} G_1 S'^{N_T}}{\left[S'^2 - BR_0 S' + \omega_o^2\right] \prod_{i=1}^{N_2} \left[\left([S' - (\sigma_{di} + j\omega_{di})][S' - (\sigma_{di} - j\omega_{di})]\right)\left([S' - (\sigma_{ei} + j\omega_{ei})]\;[S' - (\sigma_{ei} - j\omega_{ei})]\right)\right]}, \tag{111B}$$

$$H_{BP}(S') = \frac{B^{N_T} G_1 S'^{N_T}}{\left[S'^2 - BR_0 S' + \omega_o^2\right] \prod_{i=1}^{N_2} \left[\left(S'^2 - (2\sigma_{di})S' + (\sigma_{di}^2 + \omega_{di}^2)\right)\left(S'^2 - (2\sigma_{ei})S' + (\sigma_{ei}2 + \omega_{ei}^2)\right)\right]}. \tag{111C}$$

Defining the denominator coefficients as BB_{1i} (Band-pass B_{1i}) and BB_{0i} (Band-pass B_{0i}) gives

$$H_{BP}(S') = \frac{(BS')^{N_T} G_1}{\prod_{i=1}^{N_T} \left[S'^2 + BB_{1i} S' + BB_{0i}\right]} \tag{112}$$

for the Butterworth band-pass $H(S')$. Recall that for $H_{BP}(S')$, $N_T = N$ where N is the number of roots in the normalized parent transfer function. So both odd and even cases are accounted for by $i = 1,2,3,\ldots,N_T$.

The standard form of (112) is

$$H_{BP}(S') = \frac{G_1 \prod_{i=1}^{N_T} \left[BA_{2i}\, S'^2 + BA_{1i}\, S' + BA_{0i}\right]}{\prod_{i=\ell}^{2N_2} \left[BB_{2i}\, S'^2 + BB_{1i}\, S' + BB_{0i}\right]}, \tag{113A}$$

where

$$\ell = \begin{cases} 0 \text{ for even } N \\ 1 \text{ for even } N \end{cases}, \tag{113B}$$

$N_T = N$ from (101F), (113C)

$G_1 = 1$ from (16C), (113D)

$$\begin{aligned} BA_{2i} &= 0 \\ BA_{1i} &= B \\ BA_{0i} &= 0 \end{aligned} \qquad i = 1,2,3,\ldots,N_T, \tag{113E}$$

$$\begin{aligned} BB_{20} &= 1 \\ BB_{10} &= -BR_0 \\ BB_{00} &= \omega_0 2 \end{aligned} \qquad \text{for odd N,} \tag{113F}$$

$$\begin{aligned} &BB_{2i} = 1 \\ &BB_{1i} \text{ is defined in (111C)} \\ &BB_{0i} \text{ is defined in (111C)} \end{aligned} \qquad i = 1,2,3,\ldots,2N_2. \tag{113G}$$

See functions PCP and PSCP3.

As in (16A), the numerator of (113A) follows the sequence from 1 to N_T since these terms all have the same definition. The denominator follows the sequence from ℓ to $2N_2$ since the first term for odd N is defined differently than the other terms in the sequence, and is handled by a separate subroutine in the program DFD. This allows the 1 to $2N_2$ terms to be calculated by the same subroutine for both odd and even N.

The next step is to convert $H_{BP}(S')$ to $H_{BP}(Z)$ by substituting

$$S' = \frac{2}{T}\ \frac{Z - 1}{Z + 1} \tag{114}$$

into (112) which results in

$$H_{BP}(Z) = \frac{B^{N_T}\ G_1 \left[\frac{2}{T}\left(\frac{Z - 1}{Z + 1}\right)\right]^{N_T}}{\prod_{i=1}^{N_T} \left[\left(\frac{2}{T}\left(\frac{Z - 1}{Z + 1}\right)\right)^2 + BB_{1i}\left(\frac{2}{T}\left(\frac{Z - 1}{Z + 1}\right)\right) + BB_{0i}\right]}, \tag{115A}$$

$$H_{BP}(Z) = \frac{B^{N_T}\ G_1 \left(\frac{2}{T}\right)^{N_T} \left(\frac{(Z - 1)^{N_T}}{(Z + 1)^{N_T}}\right)}{\prod_{i=1}^{N_T} \left[\left(\frac{2}{T}\right)^2 \left(\frac{(Z - 1)^2}{(Z + 1)^2}\right) + BB_{1i}\left(\frac{2}{T}\right)\left(\frac{(Z - 1)}{(Z + 1)}\right) + BB_{0i}\right]}, \tag{115B}$$

(115C)

$$H_{BP}(Z) = \frac{(Z + 1)^{2N_T}\ B^{N_T}\ G_1 \left(\frac{2}{T}\right)^{N_T} \left(\frac{(Z - 1)^{N_T}}{(Z + 1)^{N_T}}\right)}{\prod_{i=1}^{N_T} \left[\left(\frac{2}{T}\right)^2 (Z - 1)^2 + BB_{1i}\left(\frac{2}{T}\right)(Z - 1)(Z + 1) + BB_{0i}\ (Z + 1)^2\right]},$$

(115D)

$$H_{BP}(Z) = \frac{(Z+1)^{N_T}\, B^{N_T}\, G_1 \left(\frac{2}{T}\right)^{N_T} (Z-1)^{N_T}}{\prod_{i=1}^{N_T} \left[{\frac{2}{T}}^{2} (Z^2 - 2Z + 1) + BB_{1i} \left(\frac{2}{T}\right) (Z^2 - 1) + BB_{0i} (Z^2 + 2Z + 1) \right]} ,$$

(115E)

$$H_{BP}(Z) = \frac{B^{N_T}\, G_1 \left(\frac{2}{T}\right)^{N_T} \left[(Z+1)^{N_T} (Z-1)^{N_T} \right]}{\prod_{i=1}^{N_T} \left[\left(\frac{4}{T^2} + BB_{1i} \left(\frac{2}{T}\right) + BB_{0i} \right) Z^2 + \left(\frac{-8}{T^2} + 2BB_{0i} \right) Z + \left(\frac{4}{T^2} - BB_{1i} \left(\frac{2}{T}\right) + BB_{0i} \right) \right]} ,$$

(115F)

$$H_{BP}(Z) = \frac{\prod_{i=1}^{N_T} \left[\dfrac{1}{\frac{4}{T^2} + BB_{1i} \left(\frac{2}{T}\right) + BB_{0i}} \right] \left[B^{N_T}\, G_1 \left(\frac{2}{T}\right)^{N_T} \right] \left[(Z^2 - 1)^{N_T} \right]}{\prod_{i=1}^{N_T} \left[Z^2 + \left(\dfrac{\frac{-8}{T^2} + 2BB_{0i}}{\frac{4}{T^2} + BB_1\left(\frac{2}{T}\right) + BB_{0i}} \right) Z + \left(\dfrac{\frac{4}{T^2} - BB_{1i}\left(\frac{2}{T}\right) + BB_{0i}}{\frac{4}{T^2} + BB_{1i}\left(\frac{2}{T}\right) + BB_{0i}} \right) \right]} .$$

So the general equation for the Butterworth BP digital transfer function becomes

$$H_{BP}(Z) = \frac{G_{BP}\, (Z^2 - 1)^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\, Z + DB_{0i})} , \qquad (116)$$

The DB coefficients and the constant G_{BP} can be calculated from the expanded version of $H_{BP}(Z)$ in (115F).

Equation (116) can be expressed in the standard form

$$H_{BP}(Z) = \frac{G_{BP} \prod_{i=1}^{N_T} \left[DA_{2i}\ Z^2 + DA_{1i}\ Z + DA_{0i} \right]}{\prod_{i=1}^{N_T} \left[DB_{2i}\ Z^2 + DB_{1i}\ Z + DB_{0i} \right]}\ , \qquad (117A)$$

where

$$N_T = N \text{ from (101F)}, \qquad (117B)$$

$$G_{BP} = \prod_{i=1}^{N_T} \left[\frac{1}{\frac{4}{T^2} + BB_{1i}\left(\frac{2}{T}\right) + BB_{0i}} \right] \left[B^{N_T}\ G_1 \left(\frac{2}{T}\right)^{N_T} \right] \qquad (117C)$$

$$\begin{aligned} DA_{2i} &= 1 \\ DA_{1i} &= 0 \\ DA_{0i} &= -1 \end{aligned} \qquad i = 1,2,3,\ldots,N_T, \qquad (117D)$$

$$\begin{aligned} DB_{2i} &= 1 \\ DB_{1i} &\text{ is defined in (115F)} \\ DB_{0i} &\text{ is defined in (115F)} \end{aligned} \qquad i = 1,2,3,\ldots,N_T. \qquad (117E)$$

See functions PSHZD and BCPSHZ.

Butterworth Band-Stop

For the band-stop case, the substitution into (99A) is

$$S = \frac{BS'}{S'^2 + \omega_o^2}\ . \qquad (118)$$

This gives

$$H_{BS}(S') = \frac{G_1}{\left[\dfrac{BS'}{S'^2 + \omega_o^2} - R_0\right] \prod_{i=1}^{N_2} \left[\left(\dfrac{BS'}{S'^2 + \omega_o^2} - R_i\right)\left(\dfrac{BS'}{S'^2 + \omega_o^2} - R_i^*\right)\right]}, \tag{119A}$$

$$H_{BS}(S') = \frac{G_1}{\left[\dfrac{BS' - R_0\, S'^2 - R_0\omega_o^2}{S'^2 + \omega_o^2}\right] \prod_{i=1}^{N_2} \left[\left(\dfrac{BS' - R_i\, S'^2 - R_i\omega_o^2}{S'^2 + \omega_o^2}\right)\left(\dfrac{BS' - R_i^*\, S'^2 - R_i^*\omega_o^2}{S'^2 + \omega_o^2}\right)\right]}, \tag{119B}$$

$$H_{BS}(S') = \frac{G_1\, (S'^2 + \omega_o^2)^{N_T}}{(-R_0\, S'^2 + BS' - R_0\omega_o^2) \prod_{i=1}^{N_2} \left[(-R_i\, S'^2 + BS' - R_i\omega_o^2)(-R_i^*\, S'^2 + BS' - R_i^*\omega_o^2)\right]}, \tag{119C}$$

$$H_{BS}(S') = \frac{\left[\prod_{i=1}^{N_T} \left(\dfrac{1}{-R_i}\right)\right] G_1\, (S'^2 + \omega_o^2)^{N_T}}{\left[S'^2 + \dfrac{B}{-R_0}\, S' + \omega_o^2\right] \prod_{i=1}^{N_2} \left[\left(S'^2 + \left(\dfrac{B}{-R_i}\right) S' + \omega_o^2\right)\left(S'^2 + \left(\dfrac{B}{-R_i^*}\right) S' + \omega_o^2\right)\right]}, \tag{119D}$$

where

$$B = \frac{2K_A\omega_p}{T} , \qquad \text{SCP} \quad (119E)$$

K_A, ω_p, and T are defined in Chapter IV, N_T is defined in (101F), and ω_0 is from (101D). So the R_i roots where $i = 1,2,3,\ldots,N_T$ in the numerator of (119D) are the same roots as R_0, R_i, and R_i^* where $i = 1,2,3,\ldots,N_2$ in the denominator of (99A). As in (101B), the first term in the denominator of (119D) has all real coefficients. The remaining terms have complex coefficients due to R_i and R_i^*. These terms can be handled by defining

$$\frac{B}{R_i} = \frac{B}{\sigma + j\omega} = \frac{B}{|R_i| \angle\Psi} = R_S\angle{-\Psi} , \qquad (120A)$$

where

$$|R_i| = \sqrt{\sigma^2 + \omega^2} , \qquad (120B)$$

$$\Psi = \arctan \frac{\omega}{\sigma} + \pi , \qquad \text{SCP} \quad (120C)$$

$$R_S = \frac{B}{|R_i|} . \qquad \text{SCP} \quad (120D)$$

Note that σ is known to be a negative number since the roots are in the left half of the S-plane. So π must be added to Ψ to move Ψ to the left half of the S-plane as was done with θ in (105G).

The complex number $R_S\angle{-\Psi}$ can be separated into its real and imaginary components, $\sigma_a + j\omega_a$, where

$$\sigma_a = R_S \cos(-\Psi) , \qquad \text{SCP} \quad (120E)$$

$$\omega_a = R_S \sin(-\Psi) . \qquad \text{SCP} \quad (120F)$$

This is equivalent to defining $BR_i = \sigma_a + j\omega_a$ in (104) in the BP case. The values for σ_a and ω_a in (120E) and (120F) can be inserted into (105A) to produce band-stop coefficients BB_{1i} and BB_{0i} by the same procedure used for the band-pass case. So the configuration of the denominator of $H_{BS}(S')$ will be the same as $H_{BP}(S')$ in (112). This means that (119D) will become

$$H_{BS}(S') = \frac{\prod_{i=1}^{N_T} \left(\frac{1}{-R_i}\right) G_1 \; (S' + \omega_o{}^2)^{N_T}}{\prod_{i=1}^{N_T} (S'^2 + BB_{1i}\; S' + BB_{0i})} \; . \tag{121}$$

The standard form of (121) is

$$H_{BS}(S') = \frac{G_5 G_1 \prod_{i=1}^{N_T} \left[BA_{2i}\; S'^2 + BA_{1i}\; S' + BA_{0i}\right]}{\prod_{i=\ell}^{2N_2} \left[BB_{2i}\; S'^2 + BB_{1i}\; S' + BB_{0i}\right]} \; , \tag{122A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases} , \tag{122B}$$

$$N_T = N \text{ as in the BP case,} \tag{122C}$$

$$G_1 = 1 \text{ from (16C),} \tag{122D}$$

$$G_5 = \prod_{i=1}^{N_T} \left(\frac{1}{-R_i}\right) = \frac{1}{-R_o} \prod_{i=1}^{N_2} \left(\frac{1}{(-R_i)(-R_i{}^*)}\right) = \prod_{i=1}^{N_T} \left(\frac{1}{NB_{0i}}\right), \tag{122E}$$

$$\left.\begin{aligned} BA_{2i} &= 1 \\ BA_{1i} &= 0 \\ BA_{0i} &= \omega_o{}^2 \end{aligned}\right. \qquad i = 1,2,3,\ldots,N_T, \tag{122F}$$

$BB_{20} = 1$ for odd N, (122G)
$BB_{10} = B/-R_0$
$BB_{00} = \omega_o 2$

$BB_{2i} = 1$ $i = 1,2,3,\ldots,N_T$. (122H)
BB_{1i} is defined in (111C)
BB_{0i} is defined in (111C)

See functions SCP and PSCP3.

As previously noted, the values of BB_{1i} and BB_{0i} in (122H) are different from those in (113G) because the definitions of σ_a and ω_a come from (104) in the BP case, and from (120E) and (120F) in the BS case. This will result in different values for the coefficients in (111C).

The next step is to convert $H_{BS}(S')$ to $H_{BS}(Z)$ by substituting (114) into (121). The denominator will be the same as in (115F). However, the numerator will be different due to the change from S'^{N_T} in the numerator of (112) to $(S'^2 + \omega_o 2)^{N_T}$ in the numerator of (121). The result is

(123A)

$$H_{BS}(Z) = \frac{\prod_{i=1}^{N_T}\left(\dfrac{1}{\dfrac{4}{T^2} + BB_{1i}\ \dfrac{2}{T} + BB_{0i}}\right) G_5\ G_1\ (Z+1)^{2N_T} \left[\dfrac{2}{T}\left(\dfrac{Z-1}{Z+1}\right)^2 + \omega_o{}^2\right]^{N_T}}{\prod_{i=1}^{N_T}\left[Z^2 + \dfrac{\dfrac{-8}{T^2} + 2BB_{0i}}{\dfrac{4}{T^2} + BB_1\left(\dfrac{2}{T}\right) + BB_{0i}}\ Z + \dfrac{\dfrac{4}{T^2} - BB_{1i}\ \dfrac{2}{T} + BB_{0i}}{\dfrac{4}{T^2} + BB_{1i}\left(\dfrac{2}{T}\right) + BB_{0i}}\right]},$$

$$H_{BS}(Z) = \frac{G_{BS1}\ (Z+1)^{2N_T} \left[\left(\frac{2}{T}\right)^2 \left(\frac{(Z-1)^2}{(Z+1)^2}\right) + \omega_o^2 \right]^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})}, \tag{123B}$$

$$H_{BS}(Z) = \frac{G_{BS1}\ (Z+1)^{2N_T} \left(\frac{1}{Z+1}\right)^{2N_T} \left[\left(\frac{2}{T}\right)^2 (Z-1)^2 + \omega_o^2\ (Z+1)^2 \right]^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})}, \tag{123C}$$

$$H_{BS}(Z) = \frac{G_{BS1} \left[\left(\frac{4}{T^2}\right) (Z^2 - 2Z + 1) + \omega_o^2 (Z^2 + 2Z + 1) \right]^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})}, \tag{123D}$$

$$H_{BS}(Z) = \frac{G_{BS1} \left[\left(\frac{4}{T^2} + \omega_o^2\right) Z^2 + \left(2\omega_o^2 - \frac{8}{T^2}\right) Z + \left(\frac{4}{T^2} + \omega_o^2\right) \right]^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})}, \tag{123E}$$

$$H_{BS}(Z) = \frac{G_{BS1} \left(\frac{4}{T^2} + \omega_o^2\right)^{N_T} \left[Z^2 + \left(\frac{2\omega_o^2 - \frac{8}{T^2}}{\frac{4}{T^2} + \omega_o^2} \right) Z + 1 \right]^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})}, \tag{123F}$$

$$H_{BS}(Z) = \frac{G_{BS1}\ G_{BS2}\ (Z^2 + DA_1\ Z + 1)^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})} \quad . \tag{123G}$$

So the general equation for the Butterworth BS digital transform becomes

$$H_{BS}(Z) = \frac{G_{BS}\ (Z^2 + DA_1\ Z + 1)^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})} \quad , \tag{124}$$

where the constant and coefficients can be determined from (123A). Expressing (124) in the standard form results in

$$H_{BS}(Z) = \frac{G_{BS} \prod_{i=1}^{N_T} \left[DA_{2i}\ Z^2 + DA_{1i}\ Z + DA_{0i}\right]}{\prod_{i=1}^{N_T} \left[DB_{2i}\ Z^2 + DB_{1i}\ Z + DB_{0i}\right]} \quad , \tag{125A}$$

where

$$N_T = N \text{ as in the BP case,} \tag{125B}$$

$$G_{BS} = G_{BS1}\ G_{BS2} \ , \tag{125C}$$

$$G_{BS1} = \prod_{i=1}^{N_T} \left(\frac{1}{\frac{4}{T^2} + BB_{1i} \left(\frac{2}{T}\right) + BB_{0i}} \right) G_5 G_1 \ , \tag{125D}$$

$$G_{BS2} = \left(\frac{4}{T^2} + \omega_o^2 \right)^{N_T} \quad . \tag{125E}$$

$$\left.\begin{aligned} &DA_{2i} = 1 \\ &DA_{1i} \text{ is defined in (123F)} \\ &DA_{0i} = 1 \end{aligned}\right. \qquad i = 1,2,3,\ldots,N_T, \tag{125F}$$

$DB_{2i} = 1$ $\qquad i = 1,2,3,\ldots,N_T.$ (125G)
DB_{1i} is defined in (115F) and (123A)
DB_{0i} is defined in (115F) and (123A)

See functions PSHZD and BCPSHZ.

Butterworth Combined Band-Pass and Band-Stop

Comparing (116) to (124) indicates that a single equation can be generated for both the BP and BS Butterworth digital transforms. The generalized equation is

$$H_B(Z) = \frac{G_B\ (Z^2 + DA_1\ Z + DA_0)^{N_T}}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})}, \tag{126}$$

where the subscript B indicates either a BP or BS transform. The standard form for $H_B(Z)$ is

$$H_B(Z) = \frac{G_B \prod_{i=1}^{N_T} \left[DA_{2i}\ Z^2 + DA_{1i}\ Z + DA_{0i}\right]}{\prod_{i=1}^{N_T} \left[DB_{2i}\ Z^2 + DB_{1i}\ Z + DB_{0i}\right]}, \tag{127}$$

where the constant and coefficients for the band-pass case can be calculated from (117), and for the band-stop case from (125).

Chebyshev

It became evident in the development of low-pass and high-pass H(Z) transforms that both the normalized Butterworth H(S) and the normalized Chebyshev H(S) could be represented by (73) and then transformed to H(Z) by substitution. The only difference in the final H(Z) in (84) was the numerical value of the constant and the coefficients.

This is also true with the development of the band-pass and band-stop H(Z) for the same reason. The configuration of the terms with variables is essentially the same for both designs. So the entire Butterworth development from the normalized H(S) in (98) to the general BP and BS H(Z) in (127) applies to the Chebyshev design as well. Of course the numerical value of the constant and the coefficients will be different.

Elliptic

In the development of low-pass and high-pass H(Z) transfer functions it was noted that the normalized Elliptic H(S) differed from the normalized Butterworth and Chebyshev H(S) due to the second-order terms in the numerator as shown in (85) and repeated here

$$\prod_{i=1}^{N_2} (S^2 + NA_{0i}) \ . \tag{128}$$

As in the LP and HP cases, these terms must be transformed by substitution to generate BP and BS Elliptic H(Z) transfer functions. The terms in (128) can be rearranged to show the individual roots

$$\prod_{i=1}^{N_2} \Big[(S - R_i)(S - R_i^*) \Big] \ , \tag{129A}$$

$$\prod_{i=1}^{N_2} \Big[(S - j\omega_{fi})(S + j\omega_{fi}) \Big] \ . \qquad \begin{matrix} \text{EPCP} \\ \text{ESCP} \end{matrix} \tag{129B}$$

Note that R_i and R_i^* in (129A) have no real part since $NA_{1i} = 0$ in (128).

Elliptic Band-Pass

For the band-pass case, (100) can be substituted into (129B) to convert these numerator terms into functions of S'

$$\prod_{i=1}^{N_2} \left[\left(\frac{1}{B} \left(S' + \frac{\omega_o^2}{S'} \right) - j\omega_{fi} \right) \left(\frac{1}{B} \left(S' + \frac{\omega_o^2}{S'} \right) + j\omega_{fi} \right) \right] , \tag{130A}$$

$$\prod_{i=1}^{N_2} \left[\left(\frac{S'}{B} + \frac{\omega_o^2}{BS'} - j\omega_{fi} \right) \left(\frac{S'}{B} + \frac{\omega_o^2}{BS'} + j\omega_{fi} \right) \right] , \tag{130B}$$

$$\frac{1}{(BS')^{2N_2}} \prod_{i=1}^{N_2} \left[\left(\frac{S'^2}{B} + \frac{\omega_o^2}{B} - (j\omega_{fi})S' \right) \left(\frac{S'^2}{B} + \frac{\omega_o^2}{B} + (j\omega_{fi})S' \right) \right] , \tag{130C}$$

$$\frac{1}{(BS')^{2N_2}} \prod_{i=1}^{N_2} \left[[S'^2 + \omega_o^2 - (Bj\omega_{fi})S'] \; [S'^2 + \omega_o^2 + (Bj\omega_{fi})S'] \right] , \tag{130D}$$

$$\frac{1}{(BS')^{2N_2}} \prod_{i=1}^{N_2} \left[[S'^2 - (Bj\omega_{fi})S' + \omega_o^2] \; [S'^2 + (Bj\omega_{fi})S' + \omega_o^2] \right] . \tag{130E}$$

The roots of (130E) can be separated in a similar manner to the roots of the denominator in (101B). This will produce equations similar to (102) and (103)

$$R_i' = \frac{Bj\omega_{fi} \pm \sqrt{-B^2\omega_{fi}^2 - 4\omega_o^2}}{2} , \tag{131A}$$

$$R_i' = R_{1i}', R_{2i}' , \tag{131B}$$

$$R_i'^* = \frac{-Bj\omega_{fi} \pm \sqrt{-B^2\omega_{fi}^2 - 4\omega_o^2}}{2} , \tag{132A}$$

$$R_i'^* = R_{3i}', R_{4i}' . \tag{132B}$$

Note that the radical contains only negative numbers. The first term under the radical is negative since B^2 and ω_{fi}^2 are positive and

$(Bj\omega_{fi})^2 = j^2B^2\omega_{fi}^2 = -B^2\omega_{fi}^2$. The second term is negative since ω_o^2 is positive. This means that R_i' and $R_i'^*$ can be expressed as

$$R_i' = j\omega_{gi} \pm j\omega_{hi} = j\omega_{gi} + j\omega_{hi},\ j\omega_{gi} - j\omega_{hi} \ , \tag{133}$$

$$R_i'^* = -j\omega_{gi} \pm j\omega_{hi} = -j\omega_{gi} + j\omega_{hi},\ -j\omega_{gi} - j\omega_{hi} \ . \tag{134}$$

where

$$\omega_{gi} = \frac{B\omega_{fi}}{2} \ , \qquad \text{EPCP ESCP} \tag{135A}$$

$$\omega_{hi} = \frac{\sqrt{\left| -B\omega_{fi}^2 - 4\omega_o \right|}}{2} \ . \qquad \text{EPCP ESCP} \tag{135B}$$

These roots can be regrouped as

$$R_i' = j(\omega_{gi} + \omega_{hi}),\ j(\omega_{gi} - \omega_{hi}) \ , \tag{136}$$

$$R_i'^* = -j(\omega_{gi} - \omega_{hi}),\ -j(\omega_{gi} + \omega_{hi}) \ . \tag{137}$$

From (131B), (132B), (136), and (137) it can be seen that

$$R_{4i}' = R_{1i}'^* \ , \tag{138}$$

$$R_{3i}' = R_{2i}'^* \ . \tag{139}$$

These roots can be regrouped as conjugate pairs to provide real coefficients in (130E) which becomes

$$\frac{1}{(BS')^{2N_2}} \prod_{i=1}^{N_2} \left[(S' - R_{1i}')(S' - R_{4i}') \quad (S' - R_{2i}')(S' - R_{3i}') \right] , \tag{140A}$$

$$\frac{1}{(BS')^{2N_2}} \prod_{i=1}^{N_2} \left[S'^2 + (\omega_{gi} + \omega_{hi})^2 \quad S'^2 + (\omega_{gi} - \omega_{hi})^2 \right] , \tag{140B}$$

$$\frac{1}{(BS')^{2N_2}} \prod_{i=1}^{2N_2} \left[S'^2 + BA_{0i} \right] \ . \qquad \text{EPCP} \tag{140C}$$

When these additional numerator terms are included in the Butterworth $H_{BP}(S')$ in (112) the result is the Elliptic $H_{BP}(S')$

$$H_{BP}(S') = \frac{(BS')\ G_1 \prod_{i=1}^{2N_2} \left[BA_{2i}\ S'^2 + BA_{1i}\ S' + BA_{0i}\right]}{\prod_{i=1}^{N_T} \left[BB_{2i}\ S'^2 + BB_{1i}\ S' + BB_{0i}\right]}, \qquad (141)$$

where the first numerator term appears only when N is odd.

The standard form of (141) is

$$H_{BP}(S') = \frac{G_1 \prod_{i=\ell}^{2N_2} \left[BA_{2i}\ S'^2 + BA_{1i}\ S' + BA_{0i}\right]}{\prod_{i=\ell}^{2N_2} \left[BB_{2i}\ S'^2 + BB_{1i}\ S' + BB_{0i}\right]}, \qquad (142A)$$

where

$$\ell = \begin{cases} 0 \text{ for odd N} \\ 1 \text{ for even N} \end{cases}, \qquad (142B)$$

$BA_{20} = 0$ for odd N, (142C)
$BA_{10} = B$
$BA_{00} = 0$

$BA_{2i} = 1$ $i = 1,2,3,\ldots,2N_2$, (142D)
$BA_{1i} = 0$
BA_{0i} is defined in (140B)

$BB_{20} = 1$ for odd N, (142E)
$BB_{10} = -BRO$
$BB_{00} = \omega_o 2$

$BB_{2i} = 1$ $i = 1,2,3,\ldots,2N_2$. (142F)
BB_{1i} is defined in (111C)
BB_{0i} is defined in (111C)

See functions EPCP, PSCP3, and EPSCP.

Since (142A) is a band-pass equation, the BB_{1i} and BB_{0i} coefficients in (142F) are taken from values in (111C) that are derived from the definitions of σ_a and ω_a in (104) in a similar manner to the coefficients in (113G). Recall that the BB_{1i} and BB_{0i} coefficients in (122H), which are band-stop coefficients, are dependent on values in (111C) that are derived from definitions of σ_a and ω_a in (120E) and (120F).

The next step is to convert the Elliptic $H_{BP}(S')$ in (141) to $H_{BP}(Z)$ by substituting (114) into (141). Note that the difference between (112) and (141) is the single (BS′) term and the second-order terms in the numerator of (141). Substituting (114) into the numerator terms of (141) results in

$$B\left[\frac{2}{T}\left(\frac{Z-1}{Z+1}\right)\right] G_1 \prod_{i=1}^{2N_2} \left[\left(\frac{2}{T}\left(\frac{Z-1}{Z+1}\right)\right)^2 + BA_{0i}\right] , \tag{143A}$$

$$\left[\frac{2B}{T}\left(\frac{Z-1}{Z+1}\right)\right] G_1 \prod_{i=1}^{2N_2} \left[\left(\frac{2}{T}\right)^2 \left(\frac{(Z-1)^2}{(Z+1)^2}\right) + BA_{0i}\right] , \tag{143B}$$

$$\left[\frac{2B}{T}\left(\frac{Z-1}{Z+1}\right)\right] G_1 \left(\frac{1}{(Z+1)^{4N_2}}\right) \prod_{i=1}^{2N_2} \left[\left(\frac{2}{T}\right)^2 (Z-1)^2 + BA_{0i}\,(Z+1)^2\right] , \tag{143C}$$

$$\left[\frac{2B}{T}\left(\frac{Z-1}{Z+1}\right)\right] G_1 \left(\frac{1}{(Z+1)^{4N_2}}\right) \prod_{i=1}^{2N_2} \left[\left(\frac{2}{T}\right)^2 Z^2 - 2\left(\frac{2}{T}\right)^2 Z + \left(\frac{2}{T}\right)^2 + BA_{0i}\,Z^2 + 2BA_{0i}\,Z + BA_{0i}\right] , \tag{143D}$$

$$\left[\frac{2B}{T}\left(\frac{Z-1}{Z+1}\right)\right] G_1 \left(\frac{1}{(Z+1)^{4N_2}}\right) \prod_{i=1}^{2N_2} \left[\left(\frac{4}{T^2} + BA_{0i}\right) Z^2 + \left(2BA_{0i} - \frac{8}{T^2}\right) Z + \left(\frac{4}{T^2} + BA_{0i}\right)\right] , \tag{143E}$$

$$\left[\frac{2B}{T}\left(\frac{Z-1}{Z+1}\right)\right] G_1\left(\frac{1}{(Z+1)^{4N_2}}\right)\prod_{i=1}^{2N_2}\left(\frac{4}{T^2}+BA_{0i}\right) \tag{143F}$$

$$\prod_{i=1}^{2N_2}\left[Z^2+\left(\frac{2BA_{0i}-\frac{8}{T^2}}{\frac{4}{T^2}+BA_{0i}}\right)Z+1\right] .$$

The denominator of (141) will follow the same development as (112) as shown in (115A) through (115F). So the denominator of (141) becomes

$$\frac{(Z+1)^{2N_T}\prod_{i=1}^{N_T}\left[\frac{1}{\frac{4}{T^2}+BB_{1i}\left(\frac{2}{T}\right)+BB_{0i}}\right]}{\prod_{i=1}^{N_T}\left[Z^2+\left(\frac{\frac{-8}{T^2}+2BB_{0i}}{\frac{4}{T^2}+BB_{1i}\left(\frac{2}{T}\right)+BB_{0i}}\right)Z+\left(\frac{\frac{4}{T^2}-BB_{1i}\left(\frac{2}{T}\right)+BB_{0i}}{\frac{4}{T^2}+BB_{1i}\left(\frac{2}{T}\right)+BB_{0i}}\right)\right]} . \tag{144}$$

Multiplying (143F) by (144) gives a complete equation for the Elliptical BP digital transform

(145)

$$H_{BP}(Z) = \frac{\left(\dfrac{\dfrac{2B}{T}\Big[(Z - 1)(Z + 1)\Big]}{\dfrac{4}{T^2} + BB_{10}\left(\dfrac{2}{T}\right) + BB_{00}}\right) G_1 \prod_{i=1}^{2N_2}\left[\dfrac{\dfrac{4}{T^2} + BA_{0i}}{\dfrac{4}{T^2} + BB_{1i}\left(\dfrac{2}{T}\right) + BB_{0i}}\right] \prod_{i=1}^{2N_2}\left[Z^2 + \left(\dfrac{2BA_{0i} - \dfrac{8}{T^2}}{\dfrac{4}{T^2} + BA_{0i}}\right) Z + 1\right]}{\prod_{i=1}^{N_T}\left[Z^2 + \left(\dfrac{\dfrac{-8}{T^2} + 2BB_{0i}}{\dfrac{4}{T^2} + BB_1\left(\dfrac{2}{T}\right) + BB_{0i}}\right) Z + \left(\dfrac{\dfrac{4}{T^2} - BB_{1i}\left(\dfrac{2}{T}\right) + BB_{0i}}{\dfrac{4}{T^2} + BB_{1i}\left(\dfrac{2}{T}\right) + BB_{0i}}\right)\right]} ,$$

where the first numerator term appears only when N is odd. When N is even, the first term in (143F) disappears and the (Z + 1) terms in (143F) and (144) cancel since $2N_T = 4N_2$. Also, the N_T factors in the constant term in (144) are all accounted for in the $2N_2$ factors in the constant in (145). When N is odd, $2N_T = 4N_2 + 2$ and the first term in (143F) appears. So there is one (Z + 1) term that does not cancel. Also, since $N_T = 2N_2 + 1$ there is an additional constant term in (144) that appears as the denominator of the first numerator term in (145). This allows the first numerator term in (145) to account for all the additional factors that appear when N_T is odd.

So the general equation for the Elliptic BP digital transfer function becomes

$$H_{BP}(Z) = \frac{G_{BP1}\,(Z^2 - 1)\; G_{BP2} \prod_{i=1}^{2N_2} \left[Z^2 + DA_{1i}\, Z + 1\right]}{\prod_{i=1}^{N_T} \left[Z^2 + DB_{1i}\, Z + DB_{0i}\right]}\,, \tag{146}$$

where $G_{BP1}\,(Z^2 - 1)$ appears only when N is odd.

The standard form for (146) is

$$H_{BP}(Z) = \frac{G_{BP} \prod_{i=\ell}^{2N_2} \left[DA_{2i}\, Z^2 - DA_{1i}\, Z + DA_{0i}\right]}{\prod_{i=1}^{N_T} \left[DB_{2i}\, Z^2 - DB_{1i}\, Z + DB_{0i}\right]}\,, \tag{147A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd N} \\ 1 \text{ for even N} \end{cases}, \tag{147B}$$

$$G_{BP} = G_{BP1}\, G_{BP2}, \tag{147C}$$

$$G_{BP1} = \begin{cases} \dfrac{\frac{2B}{T}}{\frac{4}{T^2} + BB_{10}\left(\frac{2}{T}\right) + BB_{00}} & \text{for odd N} \\ 1 & \text{for even N} \end{cases}, \tag{147D}$$

$$G_{BP2} = G_1 \prod_{i=1}^{2N_2} \left[\frac{\frac{4}{T^2} + BA_{0i}}{\frac{4}{T^2} + BB_{1i}\left(\frac{2}{T}\right) + BB_{0i}}\right] \quad \text{for both odd and even N,} \tag{147E}$$

$$\begin{aligned} DA_{20} &= 1 \\ DA_{10} &= 0 \\ DA_{00} &= -1 \end{aligned} \quad \text{for odd N,} \tag{147F}$$

$DA_{2i} = 1$ $\qquad$ $i = 1,2,3,\ldots,2N_2,$ $\qquad$ (147G)
DA_{1i} is defined in (145)
$DA_{0i} = 1$

$DB_{2i} = 1$ $\qquad$ $i = 1,2,3,\ldots,N_T.$ $\qquad$ (147H)
DB_{1i} is defined in (145)
DB_{0i} is defined in (145)

See functions PSHZD, EPHZ, and EPSHZ.

Elliptic Band-Stop

In the Elliptic band-stop case, as in the Elliptic band-pass case, the numerator second-order terms in the normalized H(S) must be transformed by substitution. This procedure begins by substituting (118) into (129B) which gives the following numerator terms for $H_{BS}(S')$

$$\prod_{i=1}^{N_2}\left[\left(\frac{BS'}{S'^2 + \omega_o^2} - j\omega_{fi}\right)\left(\frac{BS'}{S'^2 + \omega_o^2} + j\omega_{fi}\right)\right], \tag{148A}$$

$$\prod_{i=1}^{N_2}\left[\left(\frac{BS' - j\omega_{fi}S'^2 - j\omega_{fi}\omega_o^2}{S'^2 + \omega_o^2}\right)\left(\frac{BS' + j\omega_{fi}S'^2 + j\omega_{fi}\omega_o^2}{S'^2 + \omega_o^2}\right)\right], \tag{148B}$$

$$\left(\frac{1}{(S'^2 + \omega_o^2)^{2N_2}}\right)\prod_{i=1}^{N_2}\left[(-j\omega_{fi}S'^2 + BS' - j\omega_{fi}\omega_o^2)(+j\omega_{fi}S'^2 + BS' + j\omega_{fi}\omega_o^2)\right], \tag{148C}$$

$$\left(\frac{\prod_{i=1}^{N_2}\omega_{fi}^2}{(S'^2 + \omega_o^2)^{2N_2}}\right)\prod_{i=1}^{N_2}\left[\left(S'^2 + \left(\frac{B}{-j\omega_{fi}}\right)S' + \omega_o^2\right)\left(S'^2 + \left(\frac{B}{-j\omega_{fi}}\right)S' + \omega_o^2\right)\right]. \tag{148D}$$

Separating the roots as in (131) and (132) gives

$$R_i' = \frac{\dfrac{B}{j\omega_{fi}} \pm \sqrt{\dfrac{B^2}{-\omega_{fi}^2} - 4\omega_o^2}}{2}\ , \tag{149A}$$

$$R_i' = R_{1i}',\ R_{2i}', \tag{149B}$$

$$R_i'^* = \frac{\dfrac{-B}{j\omega_{fi}} \pm \sqrt{\dfrac{B^2}{-\omega_{fi}^2} - 4\omega_o^2}}{2}\ , \tag{150A}$$

$$R_i'^* = R_{3i}',\ R_{4i}'\ . \tag{150B}$$

These roots can be expressed as

$$R_i' = j\omega_{gi} \pm j\omega_{hi} = j\omega_{gi} + j\omega_{hi},\ j\omega_{gi} - j\omega_{hi}\ , \tag{151}$$

$$R_i'^* = -j\omega_{gi} \pm j\omega_{hi} = -j\omega_{gi} + j\omega_{hi},\ -j\omega_{gi} - j\omega_{hi}\ , \tag{152}$$

in a similar manner to (133) and (134). Rearranging terms gives

$$R_i' = j(\omega_{gi} + \omega_{hi}),\ j(\omega_{gi} - \omega_{hi})\ , \qquad \text{ESCP} \tag{153}$$

$$R_i'^* = -j(\omega_{gi} - \omega_{hi}),\ -j(\omega_{gi} + \omega_{hi})\ , \qquad \text{ESCP} \tag{154}$$

where ω_{hi} and ω_{gi} have different values in (153) and (154) than they do in (136) and (137) due to different values of the coefficients in (148D) and (130E).

Continuing the procedure, the numerator roots are now regrouped into conjugate pairs to provide real coefficients in (148D) which becomes

$$\left(\frac{\prod_{i=1}^{N_2} \omega_i^2}{(S'^2 + \omega_o^2)^{2N_2}}\right) \prod_{i=1}^{N_2} \left[\Big((S' - R_{1i}')(S' - R_{4i}')\Big)\Big((S' - R_{2i}')(S' - R_{3i}')\Big)\right]\ , \tag{155A}$$

$$\left(\frac{\prod_{i=1}^{N_2} \omega_i^2}{(S'^2 + \omega_0^2)^{2N_2}}\right) \prod_{i=1}^{N_2} \left[\left(S'^2 + (\omega_{gi} + \omega_{hi})^2\right)\left(S'^2 + (\omega_{gi} - \omega_{hi})^2\right)\right] , \tag{155B}$$

It should be noted that (129B) was used as the starting point in the Elliptic band-stop procedure to generate (148A) and ultimately (155A). This was done to show that the numerator roots of (155A) are imaginary and are derived in a similar manner to the numerator roots of (140A) in the band-pass case. If the starting point is (129A) instead of (129B) then the first term in (155B) will change to

$$\frac{\prod_{i=1}^{N_2} (\omega_i^2)}{(S'^2 + \omega_0^2)^{2N_2}} = \frac{\prod_{i=1}^{N_2} (R_i^2)}{(S'^2 + \omega_0^2)^{2N_2}} = \frac{\prod_{i=1}^{2N_2} (-R_i)}{(S'^2 + \omega_0^2)^{2N_2}} . \tag{156}$$

Substituting (156) into (155B) and generalizing coefficients produces

$$\left(\frac{\prod_{i=1}^{2N_2} (-R_i)}{(S'^2 + \omega_0^2)^{2N_2}}\right) \prod_{i=1}^{2N_2} (S'^2 + BA_{0i}) . \qquad \text{ESCP} \tag{157}$$

Factoring these additional numerator terms into the Butterworth $H_{BS}(S')$ in (121) gives the Elliptic $H_{BS}(S')$ as

$$H_{BS}(S') = \frac{\left(\dfrac{S'^2 + \omega_0 2}{-R_0}\right) G_1 \prod_{i=1}^{2N_2} (S'^2 + BA_{0i})}{\prod_{i=1}^{N_T} (S'^2 + BB_{1i}\, S' + BB_{0i})} , \tag{158}$$

where the first numerator term appears only when N is odd.

The standard form of (158) is

$$H_{BS}(S') = \frac{G_6 G_1 \prod_{i=\ell}^{2N_2} \left[BA_{2i}\ S'^2 + BA_{1i}\ S + BA_{0i}\right]}{\prod_{i=\ell}^{2N_2} \left[BB_{2i}\ S'^2 + BB_{1i}\ S + BB_{0i}\right]}\ , \tag{159A}$$

where

$$\ell = \begin{cases} 0 \text{ for odd } N \\ 1 \text{ for even } N \end{cases}, \tag{159B}$$

G_1 is defined in (22), (159C)

$$G_6 = \begin{cases} \dfrac{1}{-R_0} & \text{for odd } N \\ 1 & \text{for even } N \end{cases}, \tag{159D}$$

$BA_{20} = 1$ for odd N, (159E)
$BA_{10} = 0$
$BA_{00} = \omega_o^2$

$BA_{2i} = 1$ $i = 1,2,3,\ldots,2N_2$, (159F)
$BA_{1i} = 0$
BA_{0i} is defined in (155B)

$BB_{20} = 1$ for odd N, (159G)
$BB_{10} = B/-R_0$
$BB_{00} = \omega_o 2$

$BB_{2i} = 1$ $i = 1,2,3,\ldots,2N_2$. (159H)
BB_{1i} is defined in (111C)
BB_{0i} is defined in (111C)

See functions ESCP, PSCP3, and EPSCP.

Note that the band-stop coefficients in (159H), like those in (122H), are dependent on values in (111C) that are derived from definitions of σ_a and ω_a in (120E) and (120F).

Comparing (158) to (141) reveals that the difference is the substitution of

$$\left(\frac{S'^2 + \omega_o^2}{-R_0}\right) \tag{160}$$

in (158) for (BS′) in (141). Substituting (114) into (160) gives

$$\frac{1}{-R_0}\left[\left(\frac{2}{T}\left(\frac{Z-1}{Z+1}\right)\right)^2 + \omega_o^2\right] , \tag{161A}$$

$$\frac{1}{-R_0}\left[\frac{1}{(Z+1)^2}\right]\left[\frac{2}{T}(Z-1)^2 + \omega_o^2 (Z+1)^2\right] , \tag{161B}$$

$$\frac{1}{-R_0}\left[\frac{1}{(Z+1)^2}\right]\left[\frac{2}{T}(Z^2 - 2Z + 1) + \omega_o^2 (Z^2 + 2Z + 1)\right] , \tag{161C}$$

$$\frac{1}{-R_0}\left[\frac{1}{(Z+1)^2}\right]\left[\left(\frac{2}{T} + \omega_o^2\right) Z^2 + \left(2\omega_o^2 - \frac{4}{T}\right) Z + \left(\frac{2}{T} + \omega_o^2\right)\right] , \tag{161D}$$

$$\frac{1}{-R_0}\left[\frac{\frac{2}{T} + \omega_o^2}{(Z+1)^2}\right]\left[Z^2 + \left(\frac{2\omega_o^2 - \frac{4}{T}}{\frac{2}{T} + \omega_o^2}\right) Z + 1\right] . \tag{161E}$$

Substituting (161E) for the first term in (143F) gives

$$\frac{1}{-R_0}\left[\frac{\frac{2}{T} + \omega_o^2}{(Z+1)^2}\right]\left[Z^2 + \left(\frac{2\omega_o^2 - \frac{4}{T}}{\frac{2}{T} + \omega_o^2}\right) Z + 1\right] G_1 \left[\frac{1}{(Z+1)^{4N_2}}\right]$$
$$\prod_{i=1}^{2N_2}\left(\frac{4}{T^2} + BA_{0i}\right) \prod_{i=1}^{2N_2}\left[Z^2 + \left(\frac{2BA_{0i} - \frac{8}{T^2}}{\frac{4}{T^2} + BA_{0i}}\right) Z + 1\right] . \tag{162}$$

Multiplying (162) by (144) gives a complete equation for the Elliptical BS transfer function

$$\left[\left(\frac{\frac{1}{-R_0}\left(\frac{2}{T} + \omega_o^2\right)}{\frac{4}{T^2} + BB_{10}\left(\frac{2}{T}\right) + BB_{00}}\right)\left(Z^2 + \left(\frac{2\omega_o^2 - \frac{4}{T}}{\frac{2}{T} + w_o}\right)Z + 1\right)\right] G_1 \tag{163}$$

$$H_{BS}(Z) = \frac{\prod_{i=1}^{2N_2}\left[\frac{\frac{4}{T^2} + BA_{0i}}{\frac{4}{T^2} + BB_{1i}\left(\frac{2}{T}\right) + BB_{0i}}\right] \prod_{i=1}^{2N_2}\left[Z^2 + \left(\frac{2BA_{0i} - \frac{8}{T^2}}{\frac{4}{T^2} + BA_{0i}}\right)Z + 1\right]}{\prod_{i=1}^{N_T}\left[Z^2 + \left(\frac{\frac{-8}{T^2} + 2BB_{0i}}{\frac{4}{T^2} + BB_1\left(\frac{2}{T}\right) + BB_{0i}}\right)Z + \left(\frac{\frac{4}{T^2} - BB_{1i}\left(\frac{2}{T}\right) + BB_{0i}}{\frac{4}{T^2} + BB_{1i}\left(\frac{2}{T}\right) + BB_{0i}}\right)\right]} ,$$

where the first numerator term appears only when N is odd.

Substituting generalized coefficients gives

$$H_{BS}(Z) = \frac{G_{BS1}\ (Z^2 + DA_{10}\ Z + 1)\ G_{BS2} \prod_{i=1}^{2N_2} (Z^2 + DA_{1i}\ Z + 1)}{\prod_{i=1}^{N_T} (Z^2 + DB_{1i}\ Z + DB_{0i})} . \tag{164}$$

Expressing (164) in standard form results in

$$H_{BS}(Z) = \frac{G_{BS} \prod_{i=1}^{N_T} [DA_{2i}\ Z^2 + DA_{1i}\ Z + DA_{0i}]}{\prod_{i=1}^{N_T} [DB_{2i}\ Z^2 + DB_{1i}\ Z + DB_{0i}]} , \tag{165A}$$

where

$$G_{BS} = G_{BS1}\ G_{BS2} , \tag{165B}$$

$$G_{BS1} = \begin{cases} \dfrac{\dfrac{1}{R_0}\left(\dfrac{2}{T} + \omega_o{}^2\right)}{\dfrac{4}{T^2} + BB_{10}\left(\dfrac{2}{T}\right) + BB_{00}} & \text{for odd } N \\ 1 & \text{for even } N \end{cases}, \tag{165C}$$

$$G_{BS2} = G_1 \prod_{i=1}^{2N_2} \left[\frac{\dfrac{4}{T^2} + BA_{0i}}{\dfrac{4}{T^2} + BB_{1i}\left(\dfrac{2}{T}\right) + BB_{0i}}\right] \quad \text{for both odd and even } N, \tag{165D}$$

$DA_{2i} = 1$ $\quad i = 1,2,3,\ldots,N_T,$ (165E)
DA_{1i} is defined in (163)
$DA_{0i} = 1$

$DB_{2i} = 1$ $\quad i = 1,2,3,\ldots,N_T.$ (165F)
DB_{1i} is defined in (163)
DB_{0i} is defined in (163)

See functions PSHZD, ESHZ, and EPSHZ.

Note that the constant G_6 defined as $1/R_0$ for odd N in (159D) is incorporated into G_{BS1} in (165C).

Elliptic Combined Band-Pass and Band-Stop

Comparing (146) and (164) indicates that a single equation can be generated for both the BP and BS Elliptic transfer functions. The combined equation is

$$H_B(Z) = \frac{G_B \prod_{i=1}^{N_T} [Z^2 + DA_{1i}\, Z + DA_{0i}]}{\prod_{i=1}^{N_T} [Z^2 + DB_{1i}\, Z + DB_{0i}]}, \tag{166}$$

where the subscript B indicates either a BP or BS transfer function. The standard form for $H_B(Z)$ is

$$H_B(Z) = \frac{G_B \prod_{i=1}^{N_T} [DA_{2i}\, Z^2 + DA_{1i}\, Z + DA_{0i}]}{\prod_{i=1}^{N_T} [DB_{2i}\, Z^2 + DB_{1i}\, Z + DB_{0i}]} \,, \tag{167}$$

where the constant and coefficients for the band-pass case can be calculated from (147), and for the band-stop case from (165).

General Band-Pass and Band-Stop Equation

Comparing (167) with (127) indicates that the general Elliptic BS H(Z) can be used to represent the Butterworth, Chebyshev, and Elliptic H(Z) transfer functions of both the band-pass and band-stop types. So the general equation for all of these transfer functions is

$$H_B(Z) = \frac{G_B \prod_{i=1}^{N_T} [DA_{2i}\, Z^2 + DA_{1i}\, Z + DA_{0}]}{\prod_{i=1}^{N_T} [DB_{2i}\, Z^2 + DB_{1i}\, Z + DB_{0i}]} \,, \tag{168}$$

where the subscript B indicates either a band-pass or band-stop filter. The numerical value of the constant and coefficients can be determined from the expanded equation for each case.

CHAPTER VII

GENERAL EQUATION

Comparing (168) to (97) indicates that a general equation can be written for all Butterworth, Chebyshev, and Elliptic H(Z) transfer functions for all low-pass, high-pass, band-pass, and band-stop filter types. The standard form for this general equation is

$$H(Z) = \frac{G \prod_{i=1}^{N_T} \left[DA_{2i}\, Z^2 + DA_{1i}\, Z + DA_{0i} \right]}{\prod_{i=1}^{N_T} \left[DB_{2i}\, Z^2 + DB_{1i}\, Z + DB_{0i} \right]}, \tag{169}$$

where the numerical value of the general constant G and the coefficients can be determined from the standard form equations for each case that appear in Chapters V and VI. The equation numbers for these equations are listed in the order of their development in Table 1.

Table 1. Filter Types and their Digital Transfer Function Equation Numbers

Filter Type	Equation Number
Butterworth LP	(79)
Butterworth HP	(82)
Butterworth Combined LP and HP	(84)
Chebyshev LP	(79)[1]
Chebyshev HP	(82)[1]
Chebyshev Combined LP and HP	(84)[1]
Elliptic LP	(89)
Elliptic HP	(91)
Elliptic Combined LP and HP	(94)
General LP and HP	(97)
Butterworth BP	(117)
Butterworth BS	(125)
Butterworth Combined BP and BS	(127)
Chebyshev BP	(117)[1]
Chebyshev BS	(125)[1]
Chebyshev Combined BP and BS	(127)[1]
Elliptic BP	(147)
Elliptic BS	(165)
Elliptic Combined BP and BS	(167)
General BP and BS	(168)
General Equation	(169)

[1]The Butterworth and Chebyshev designs use the same equations but the coefficients have different values. See the Chebyshev sections of the text.

CHAPTER VIII

REALIZATIONS

Configuration of H(Z)

In order to make use of the results in [2] and those developed here it is necessary at this point to examine the relationship between the transfer function representation used here and that used in [2]. This begins by removing the first-order term from (96) and expressing it as

$$H_0(Z) = G_0 \left[\frac{Z + DA_{00}}{Z + DB_{00}}\right] , \tag{170A}$$

where G_0 is used here to refer to the first-order factor in G_{LH}. By comparison, the same first-order term is expressed in [2, p. 178] as

$$H_0(Z) = \frac{\alpha_{20}Z^{-1}}{1 + \beta_{20}Z^{-1}} + d_o , \tag{170B}$$

where subscripts have been added for comparison purposes. This can be rearranged as

$$H_0(Z) = \frac{\alpha_{20}}{Z + \beta_{20}} + d_o . \tag{170C}$$

Finding a common denominator and separating d gives

$$H_0(Z) = d_o \left[\frac{Z + \left(\frac{\alpha_{20}}{d_o} + \beta_{20}\right)}{Z + \beta_{20}}\right] . \tag{171}$$

Since (170A) is equivalent to (171), the coefficients have the following relationships

$$d_o = G_0 \ , \tag{172A}$$

$$\beta_{20} = DB_{00} \ , \tag{172B}$$

$$DA_{00} = \frac{\alpha_{20}}{d_o} + \beta_{20} = \frac{\alpha_{20}}{G_0} + DB_{00} \ , \tag{172C}$$

$$\therefore \alpha_{20} = (DA_{00} - DB_{00})\ G_0 \ . \tag{172D}$$

In a similar manner, the second-order terms can be removed from (96) and expressed as

$$H_i(Z) = G_i \left[\frac{Z^2 + DA_{1i}\ Z + 1}{Z^2 + DB_{1i}\ Z + DB_{0i}}\right] , \tag{173}$$

where G_i is the factor in the constant G associated with the i^{th} subsection of H(Z). By comparison, the same second-order terms are expressed in [2, p. 180] as

$$H_i(Z) = \frac{\alpha_{1i}\ Z^{-1} + \alpha_{2i}\ Z^{-2}}{1 + \beta_{1i}\ Z^{-1} + \beta_{2i}\ Z^{-2}} + d_i \ , \tag{174}$$

where the i subscripts are added to indicate subsections. Equation (174) can be rearranged as

$$H_i(Z) = \frac{\alpha_{1i}\ Z + \alpha_{2i}}{Z^2 + \beta_{1i}\ Z + \beta_{2i}} + d_i \ . \tag{175}$$

Finding a common denominator and separating d_i gives

$$H_i(Z) = d_i \left[\frac{Z^2 + \left(\frac{\alpha_{1i}}{d_i} + \beta_{1i}\right) Z + \left(\frac{\alpha_{2i}}{d_i} + \beta_{2i}\right)}{Z^2 + \beta_{1i}\ Z + \beta_{2i}}\right] . \tag{176}$$

Since (173) is equivalent to (176), the coefficients have the following relationships

$$d_i = G_i \ , \tag{177A}$$

$$\beta_{1i} = DB_{1i} \ , \tag{177B}$$

$$\beta_{2i} = DB_{0i} \ , \tag{177C}$$

$$DA_{1i} = \frac{\alpha_{1i}}{d_i} + \beta_{1i} = \frac{\alpha_{1i}}{G_i} + DB_{1i} \ , \tag{177D}$$

$$\therefore \ \alpha_{1i} = (DA_{1i} - DB_{1i}) \ G_i \ , \tag{177E}$$

$$DA_{0i} = 1 = \frac{\alpha_{2i}}{d_i} + \beta_{2i} = \frac{\alpha_{2i}}{G_i} + DB_{0i} \ , \tag{177F}$$

$$\therefore \ \alpha_{2i} = (1 - DB_{0i}) \ G_i \ . \tag{177G}$$

The relationships in (172) and (177) allow the realization procedures developed for (170B) and (174) to be directly applied to (170A) and (173). The realization procedures are used to produce appropriate coefficients for the state-space structures to be discussed later.

Cascade and Parallel Realizations

A digital filter can be realized as either the parallel or cascade connection of subfilters. Either of these realizations has the advantage of reducing the problem of realizing the overall filter to that of realizing the individual subfilters.

A cascade realization is generated by applying a realization procedure to a transfer function that is expressed as a product of terms. Typically the terms are second-order except for the single first-order term that appears when the order of the filter is odd,

$$H(Z) = \prod_{i=1}^{N_T} H_i(Z) \ . \tag{178}$$

The general equation (169) is a transfer function of this type.

A parallel realization is generated by applying a realization procedure to a transfer function that has been expanded into partial fractions and is expressed as a sum of terms. As in the cascade realization, the terms of the transfer function are typically second-order except for the single first-order term that appears when the order of the filter is odd,

$$H(Z) = \sum_{i=1}^{N_T} H_i(Z) \ . \tag{179}$$

Each configuration has its advantages and disadvantages. Only the cascade configuration is examined in this paper because (169) is already in this form.

Reducing Cascade Multiply Operations

In the development of H(Z) in (169) a variety of multipliers were combined to produce the general multiplier G. The numerical value of G is dependent on the various multipliers that were combined from the expanded versions of H(Z). Consequently G is dependent on the design type, filter type, and specific value of N_T.

To understand the role of these multipliers it is necessary to understand that the Z-transform is a linear operator. This means that:

$$\text{If } Z[x(kt)] = X(Z) \text{ then } Z[c\, x(kt)] = c\, X(Z)\ , \tag{180}$$

where c is a constant.

So all of the multipliers that are combined into G in (169) are essentially level matching operators. By combining them into the single multiplier G, they can be factored into the cascade realization process as a single multiply operation. This will reduce the total number of multiply operations that take place in the realization. When the number of multiplies is reduced the filter becomes simpler and more cost effective. However, if G is an unwieldy (very large or small) number, it can be distributed evenly among the subsections as an alternative.

The concept of a common multiplier can be incorporated into (170B) and (174). In these equations the d and d_i terms become the scalar d when the state-space coefficients are calculated in [2]. It is possible to rearrange (170B) and (174) so that the values in positions d_0 and d_i become $d_0' = d_i' = 1$. Using $d_0' = d_i' = 1$ for the state-space scalar d means that multiplies will not be required to determine these values. Of course multipliers will appear at the beginning of each subsection $H_i(Z)$ but these multipliers [d_0 in (181) and d_i in (183)] can be combined in a cascade realization due to (180).

The procedure to make as many d_i's = 1 as possible is as follows. Rearrange the first-order term in (170C) so that the value d_0 is moved algebraically out in front of the function. This means that the value in position d_0 in (170C) will become $d_0' = 1$.

$$H_0(Z) = d_o \left[\frac{\frac{\alpha_{20}}{d_o}}{Z + \beta_{20}} + 1\right] = d_o \left[\frac{\alpha_{20}'}{Z + \beta_{20}} + 1\right] = d_o \left[\frac{\alpha_{20}'}{Z + \beta_{20}} + d_o'\right] . \tag{181}$$

This means that the relationships in (172) become

$$d_o = G_0 , \tag{182A}$$

$$d_o' = 1 , \tag{182B}$$

$$\beta_{20} = DB_{00} , \qquad \text{THZSC} \tag{182C}$$

$$\therefore \alpha_{20}' = \frac{\alpha_{20}}{d_o} = \frac{\alpha_{20}}{G_0} = DA_{00} - DB_{00} . \qquad \text{THZSC} \tag{182D}$$

Similarly, the second-order term (175) can be rearranged as

$$H_i(Z) = d_i \left[\frac{\left(\frac{\alpha_{1i}}{d_i}\right)Z + \frac{\alpha_{2i}}{d_i}}{Z^2 + \beta_{1i} Z + \beta_{2i}} + 1\right] = d_i \left[\frac{\alpha_{1i}' Z + \alpha_{2i}'}{Z^2 + \beta_{1i} Z + \beta_{2i}} + 1\right] \tag{183}$$

$$= d_i \left[\frac{\alpha_{1i}' Z + \alpha_{2i}'}{Z^2 + \beta_{1i} Z + \beta_{2i}} + d_i'\right] .$$

This means that the relationships in (177) become

$$d_i = G_i , \tag{184A}$$

$$d_i' = 1 , \tag{184B}$$

$$\beta_{1i} = DB_{1i} , \qquad \text{THZSC} \tag{184C}$$

$$\beta_{2i} = DB_{0i} , \qquad \text{THZSC} \tag{184D}$$

$$\therefore \alpha_{1i}' = \frac{\alpha_{1i}}{d_i} = \frac{\alpha_{1i}}{G_i} = DA_{1i} - DB_{1i} , \qquad \text{THZSC} \tag{184E}$$

$$\alpha_{2i}' = \frac{\alpha_{2i}}{d_i} = \frac{\alpha_{2i}}{G_i} = 1 - DB_{0i} . \qquad \text{THZSC} \tag{184F}$$

To summarize, H(Z) can be configured in a variety of ways to meet different objectives. The configuration in (169) is convenient for seeing the subsection relationships easily. Using the relationships in (172) and (177) to convert $H_i(Z)$ to (170B) and (174) allows realization procedures developed for (170B) and (174) to be used with (169). Using (182) and (184) to convert $H_i(Z)$ to (181) and (183) allows the same realization procedures to be used, with the additional benefit that the level matching d multipliers are separated from the filtering procedure and can be combined into one multiplier in a cascade realization.

The combined multipliers are shown as G_{LH} in (97), G_B in (168), and G in (169). These combined multipliers can be expressed as

$$d_c = \prod_{i=1}^{N_T} d_i, \tag{185}$$

when the subsections of H(Z) in (97), (168), and (169) are expressed in the form of (181) and (183). H(Z) then becomes

$$H(Z) = d_c \prod_{i=1}^{N_T} \left[\frac{\alpha_{1i}' Z + \alpha_{2i}'}{Z^2 + \beta_{1i} Z + \beta_{2i}} + 1\right] . \tag{186}$$

This form of H(Z) will be used in the realization procedure in this paper.

Selecting a Realization

There is a wide variety of realization types that can be produced to realize the same H(Z) transfer function. These types will vary greatly in such characteristics as roundoff noise gain with fixed-point arithmatic and number of multiplies. Considerable effort has been invested lately in developing methods for selecting a realization type to meet specific needs. Of particular interest are types that offer a reasonable trade-off between roundoff noise gain and number of multiplies. A thorough analysis of this problem is available in [2]. This analysis includes two new second-order state-space realizations and a procedure for selecting from these new realizations and the direct (canonic) form realization [1, p. 77]. This procedure and these realization types will be employed in this paper.

Before beginning an examination of realization types it is appropriate to review the concept of state-space equations. These equations are quite useful in the realization of digital filters. A digital filter is a rule for transforming an input number sequence, u(k), to an output number sequence, y(k) [2]. A Z-transform can be taken of these two functions of k. The ratio of these two transforms is the transfer function of the digital filter

$$H(Z) = \frac{Y(Z)}{U(Z)} \; . \tag{187}$$

Equation (169) is such a transfer function. If H(Z) is a rational function of Z, which (169) is, then y(k) can be related to u(k) by the state-space equations

$$\underline{x}(k + 1) = A\, x(k) + \underline{b}\, u(k) \; , \tag{188A}$$

$$y(k) = \underline{c}^t\, \underline{x}(k) + d\, u(k) \; , \tag{188B}$$

where the superscript t indicates the transpose of a matrix. The vector $\underline{x}$ is a finite-dimensional vector containing the filter state variables. The matrix A, the vectors b and c, and the scalar d are constants. A more detailed version of this summary is available in [2]. A comprehensive development of the state-space concept is available in [1].

The state-space equations in (188) can be used to implement a filter. This is done by calculating (188B), where $\underline{x}(k)$ contains the necessary past information, and then calculating (188A). The next calculation of (188B) will be shifted in time so that the new value of $\underline{x}(k)$ will be the old value of $\underline{x}(k + 1)$ from (188A). The repeated calculation of (188B) followed by (188A) constitutes an implementation of the desired filter.

Figure 3 shows a typical flow graph of a second-order state-space structure. Figure 4 shows a hardware configuration of the same structure with shift registers (D-flipflops), summing junctions, and multipliers. Each D-block in Figure 4 represents a group of D-flipflops acting in parallel as a unit delay to handle the time shift of the digital word from $\underline{x}(k + 1)$ to $\underline{x}(k)$ for each period T of the sampling frequency.

In a cascade configuration these second-order subsections would be connected in sequence, output to input. In a parallel configuration all of the inputs would be tied together and all of the outputs would be applied to a summing junction. Of course the Z-transform and the realization coefficients for a parallel structure would look quite different from those calculated for a cascade structure.

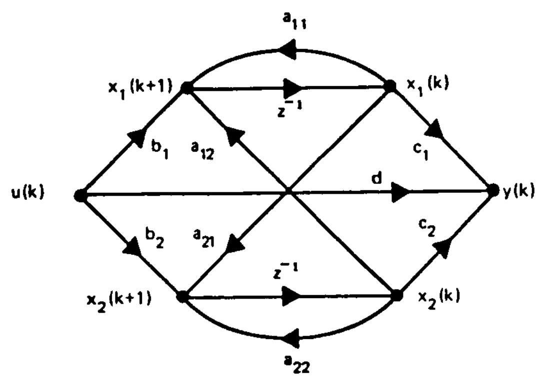

Figure 3. Typical Flow Graph of a Second-Order State-Space Structure.

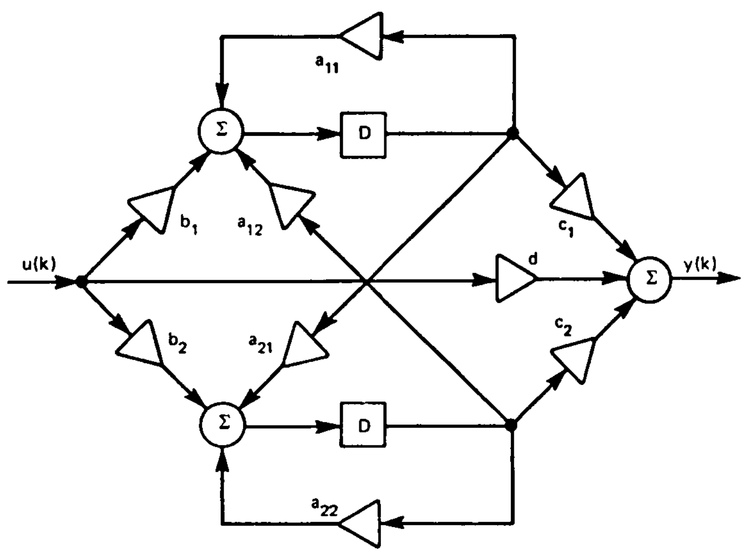

Figure 4. Typical Hardware Configuration of a Second-Order State-Space Structure.

The general state-space equation shown in (188) can be modified to move the scalar multiplier d outside of the function. This is accomplished by defining $d' = 1$ as was done in the transfer functions of (181) and (183),

$$\underline{x}(k+1) = A\,\underline{x}(k) + \underline{b}\,u(k)\ , \tag{189A}$$

$$y(k) = d_i\,[\underline{c}'^t\,\underline{x}(k) + d_i'\,u(k)] = d_i\,[\underline{c}'^t\,\underline{x}(k) + u(k)]\ . \tag{189B}$$

The realization procedure discussed earlier begins with a first-order section of H(Z). In the case of a first-order section, such as (170B), a scaled first-order state-space realization [2, p. 179] will be

$$a = -\beta_{20}\ , \tag{190A}$$

$$b = \sqrt{1 - \beta_{20}^2}\ , \tag{190B}$$

$$c = \frac{\alpha_{20}}{b}\ , \tag{190C}$$

$$d = d_o\ , \tag{190D}$$

where a, b, and c are scaler equivalents of the matrix A and vectors $\underline{b}$ and $\underline{c}$. The scalar d is equal to d_o in (170B) through (172).

Or, using the configuration of H(Z) in (181), the coefficients will be

$$a = -\beta_{20}\ , \qquad \text{SRC} \tag{191A}$$

$$b = \sqrt{1 - \beta_{20}^2}\ , \qquad \text{SRC} \tag{191B}$$

$$c' = \frac{\alpha_{20}'}{b}\ , \qquad \text{SRC} \tag{191C}$$

$d' = 1$, SRC (191D)

$d = G_0$. SRC (191E)

Note that in (190) the value of c depends on the value of α. This means that the value of c will change when α changes to α'. This change is identified in (189B) and (191C) by the use of c' instead of c.

For a scaled first-order section there is only one choice for {a, b, c, d} that realizes $H_i(Z)$ [2].

In the case of a second-order section there are an infinite number of choices for $\{A, \underline{b}, \underline{c}\}$, all of which represent the same $H_i(Z)$. The situation is examined in detail in [2]. That examination concludes with the following procedural recommendation.

Classify each second-order subsection of H(Z), as in (174), by its pole angle ϕ. The type of realization which should be used is then:

1. Type I when ϕ is nearest to 0°, SPAT (192A)
2. Direct Type when ϕ is nearest to 90°, SPAT (192B)
3. Type II when ϕ is nearest to 180°. SPAT (192C)

Note that the $r = 1$ circle in the Z-plane corresponds to the $\sigma = 0$ line in the S-plane. Both $\sigma \leq 1$ and $r \leq 1$ represent stable systems. So in the Z-plane the value of the pole angle does not indicate stability, as it does in the S-plane.

The three types will have scaled second-order state-space matrix A coefficients [2, pp. 192-193] as follows:

Type I

$$a_{11} = -(\beta_1 + 1) \ , \tag{193A}$$

$$a_{12} = \sqrt{2 + \beta_1\left[\frac{\beta_2\ (\beta_1 + 2) - \beta_1}{\beta_2 + 1}\right]} \ , \tag{193B}$$

$$a_{21} = \frac{-(1 + \beta_2 + \beta_1)}{a_{12}} \ , \tag{193C}$$

$$a_{22} = 1 \ , \tag{193D}$$

Direct Type

$$a_{11} = 0 \ , \tag{194A}$$

$$a_{12} = 1 \ , \tag{194B}$$

$$a_{21} = -\beta_2 \ , \tag{194C}$$

$$a_{22} = -\beta_1 \ , \tag{194D}$$

Type II

$$a_{11} = -(\beta_1 - 1) \ , \tag{195A}$$

$$a_{12} = \sqrt{2 + \beta_1\left[\frac{\beta_2\ (\beta_1 - 2) - \beta_1}{\beta_2 + 1}\right]} \ , \tag{195B}$$

$$a_{21} = \frac{-(1 + \beta_2 - \beta_1)}{a_{12}} \ , \tag{195C}$$

$$a_{22} = -1 \ . \tag{195D}$$

See function SRC.

All three types will have the same coefficients for vectors $\underline{b}$ and $\underline{c}$ and scalar d

$$b_1 = 0 \ , \tag{196A}$$

$$b_2 = \frac{1}{a_{12}} \sqrt{\left(\frac{1 - \beta_2}{1 + \beta_2}\right)\left[(\beta_2 + 1)^2 - \beta_1{}^2\right]} \ , \tag{196B}$$

$$c_1 = \frac{\alpha_1 a_{11} + \alpha_2}{a_{12} b_2} \ , \tag{197A}$$

$$c_2 = \frac{\alpha_1}{b_2} \ , \tag{197B}$$

$$d = d_i \ . \tag{198}$$

See function SRC.

Using the configuration of H(Z) in (183) will result in the same values for the elements of matrix A and vector $\underline{b}$ with d = 1. However, the elements of vector $\underline{c}$ will change since they vary with α which changes to α' in (183). The result is

$$c_1{}' = \frac{\alpha_1{}' \ a_{11} + \alpha_2{}'}{a_{12} b_2} \ , \tag{199A}$$

$$c_2{}' = \frac{\alpha_1{}'}{b_2} \ , \tag{199B}$$

$$d_i = G_i \ , \tag{200}$$

$$d_i{}' = 1 \ . \tag{201}$$

See function SRC.

The structure in (194) is the direct-form canonical structure that has been in use for a number of years, though in a somewhat indiscriminate manner. The structures in (193) and (195), as well as the selection process in (192), were first presented in [2]. Note that the realization coefficients in (193) through (201) are put into final matrix form in function RCMC in program DFD.

To make the selection in (192) requires determining the value of the pole angle ϕ for each second-order subsection of H(Z). The procedure is to express the denominator of (173) in terms of its complex roots

$$Z^2 + DB_{1i}\, Z + DB_{0i} = [Z - (\sigma_{zi} + j\omega_{zi})]\,[Z - (\sigma_{zi} - j\omega_{zi})], \quad (202A)$$

$$= Z2 - 2\sigma_{zi}\, Z + (\sigma_{zi}{}^2 + \omega_{zi}{}^2)\ . \quad (202B)$$

From this the following relationships result:

$$DB_{1i} = -2\sigma_{zi}\ , \quad (203A)$$

$$DB_{0i} = \sigma_{zi}{}^2 + \omega_{zi}{}^2\ , \quad (203B)$$

$$\therefore\ \sigma_{zi} = \frac{-DB_{1i}}{2}\ , \qquad \text{SPAT} \quad (203C)$$

$$\omega_{zi} = \pm\sqrt{DB_{0i} - \sigma_{zi}{}^2}\ . \qquad \text{SPAT} \quad (203D)$$

These relationships can be used to find

$$\phi_i = \begin{cases} \arctan\left(\dfrac{|\omega_{zi}|}{\sigma_{zi}}\right) & \text{for } \sigma_{zi} \geq 0 \\ \arctan\left(\dfrac{|\omega_{zi}|}{\sigma_{zi}}\right) + \pi & \text{for } \sigma_{zi} < 0 \end{cases} \ . \qquad \text{SPAT} \quad (204)$$

Note that the absolute value of ω_{zi} in (203D) is used in (204) to give a pole angle in the range $0 < \phi_i < \pi$ to identify the subsection type. Of course there is a conjugate root with a pole angle of $-\phi_i$ in each second-order subsection.

There is a situation where the second-order subsections may not have complex roots, and the pole angle approach is not reliable.

This situation occurs when a parent low-pass Z-transform with odd N is expanded to a band-pass or band-stop transform. In that case the first-order section of the low-pass transform is expanded into a second-order section that typically has real roots. These roots may include one positive root and one negative root, as is the case in run number 6 in Appendix B. In such a case one root will be Type I and the other will be Type II. Thus the subsection pole angle classification procedure in (192) is not applicable. Each second-order section of the low-pass transform is expanded into two second-order sections with complex roots. This expansion is shown in detail in (98) through (111C) for the Butterworth band-pass case. When $H_{BP}(S')$ is converted from (111B) to (111C) the complex conjugate pairs that appear as roots in the denominator of (111B) are converted to real coefficients of S' in the denominator of (111C). Notice that the first denominator term in (111B) is handled separately since it could have real roots, depending on the relationship between BR_0 and $\omega_0{}^2$. This distinction continues through the development of $H_{BP}(Z)$ in (117A) where the first term $(i = 1)$ could have real roots. Similarly for other BP and BS transforms.

The result is that a criterion other than subsection pole angle must be used to select realization coefficient types for the subsections that cannot use the pole angle approach. One approach is to calculate the noise gain for each type of realization and select the type with the lowest figure. This approach is used for all second-order subsections expanded from normalized first-order subsections in the program DFD in Appendix G. The noise gain approach could be used for all of the subsections in the filter if desired. The advantage of using the pole angle approach for subsections with complex roots is that the realization type can be selected before the realization coefficients are calculated. So it is not necessary to calculate coefficients that are not used, as is done in comparing the noise gain of Type I and Type II realizations.

The procedure selected for determining noise gain for Type I and Type II realizations comes from equation (314) in [2, p. 202].

$$\beta_1 = -(a_{11} + a_{22}) \,, \tag{205A}$$

$$\beta_2 = a_{11}a_{22} - a_{12}a_{21} \,, \tag{205B}$$

$$p = (\beta_2 - 1)[(\beta_2 + 1)^2 - \beta_1{}^2] \,, \tag{205C}$$

$$q = a_{21}c_2 - a_{22}c_1 \,, \tag{205D}$$

$$r = a_{12}c_1 - a_{11}c_2 \,, \tag{205E}$$

$$W_{11} = \frac{2c_1q\beta_1 - (c_1{}^2 + q^2)(\beta_2 + 1)}{p} \,, \tag{205F}$$

$$W_{22} = \frac{2c_2r\beta_1 - (c_2{}^2 + r^2)(\beta_2 + 1)}{p} \tag{205G}$$

$$G = W_{11} + W_{22} \,, \tag{205H}$$

See function NGCT12.

where the a_{xx} terms come from (193) or (195), and the c_x terms come from (197). For the Direct Type realization, there is no calculation required for x_1 since $a_{11} = 0$, $a_{12} = 1$, and $b_1 = 0$. So the roundoff associated with computing x_1 disappears and $G = W_{22}$ [3]. In this case the noise gain can be more easily determined from equation (39) in [3, p. 38]

$$\lambda = \frac{\beta_1}{\beta_2 + 1} \quad , \tag{206A}$$

$$W_{22} = \frac{\alpha_1{}^2 + \alpha_2{}^2 - 2\lambda\alpha_1\alpha_2}{(1 - \beta_2{}^2)(1 - \lambda^2)^2} \quad , \tag{206B}$$

$$G = W_{22} \ , \tag{206C}$$

See function NGCDT.

where α_1, α_2, β_1, and β_2 come from α_{1i}, α_{2i}, β_{1i}, and β_{2i} in (177), or their equivalents $\alpha_{1i}{}'$, $\alpha_{2i}{}'$, β_{1i}, and β_{2i}, in (184).

These relationships are used in the program DFD in Appendix G to select the realization type with the lowest noise gain for the first subsection of BP or BS transfer functions expanded from odd-order parent LP transfer functions.

Realization Procedure

This completes the information necessary to write a program that will generate state-space structure coefficients from an H(Z) transfer function. The program in Appendix G of this paper applies this information by following the sequence of instructions shown below.

1. Use (204) to determine ϕ_i for the second-order subsections of H(Z), where i = 1,2,3,...,N_2 for LP or HP H(Z); and i = 1,2,3,...,N_T for BP or BS H(Z) except for the first subsection of a BP or BS H(Z) expanded from a parent LP H(Z) with odd N. In that case use (205) and (206) to determine the realization type with lowest noise gain.

2. Use (192) to determine the type of each second-order subsection of H(Z) except as noted above.
3. Use (182) to rearrange the first-order subsection of H(Z) from (170A) to (181).
4. Use (184) to rearrange the second-order subsections of H(Z) from (173) to (183).
5. Use (191) to calculate the state-space coefficients for (181).
6. Use the appropriate selection of (193), (194), or (195) in conjunction with (196), (199), (200), and (185) to calculate the state-space coefficients for (183).
7. Print the results after the printout of the input parameters and H(Z).

CHAPTER IX

SUMMARY AND CONCLUSIONS

This paper essentially spanned the complete development of a cascade digital filter by working through the following steps:

1. design parameters,
2. normalized analog transfer function,
3. denormalized analog transfer function,
4. digital transfer function,
5. realization coefficients.

Each step was examined in considerable detail and a computer program was provided to calculate digital transfer functions and realization coefficients for the desired cascade filters. Since much of this paper was of a tutorial nature, the careful step-by-step development of the equations and the detailed documentation of the program constituted two of the primary objectives of the paper. In Chapter II and subsequent chapters, the specific program functions were identified that perform the mathematical calculations being discussed. This feature, along with the function flow charts and logic flow charts in the appendices, should make the job of understanding the program much easier and thus help to meet the tutorial goal of the paper.

The mathematical development began in Chapter II with descriptions and definitions of the Butterworth, Chebyshev, and Elliptic normalized low-pass filters. This is reasonable since these filter transfer functions have a variety of desirable features that are well documented. The remainder of Chapter II dealt with organizing these transforms and their coefficients into a standardized format which was used to simplify the programming task.

Chapter III examined the transfer function coefficients themselves. The defining equations of the transfer functions were used to establish a procedure for calculating the coefficients of the Butterworth and Chebyshev filters. The Elliptic case is more complicated and a procedure outlined in [1] was used for these coefficients.

Chapter IV was concerned with the preliminary computation parameters that appear in [1]. These parameters essentially translate filter requirements into variables that can be directly utilized in the transformation from analog to digital transfer function.

The mathematical development of the low-pass and high-pass digital transfer functions appeared in Chapter V. The bilinear transformation was employed for this development. Here again the goal was a standardized format to facilitate programming.

Chapter VI dealt with the transformation to band-pass and band-stop digital transfer functions. This transformation is frequently dealt with in a cursory manner in the literature but is seldom examined in detail. The mathematical details were included here because they give the reader some confidence in the final results, and they provided readily available programming steps.

Chapter VII showed that all of the digital transfer functions developed in Chapters V and VI can be provided by a single general equation in the standard form developed in Chapter II. Table 1 summarized the development sequence of the standard form equations.

Some general notes about realizations were provided in Chapter VIII. Also provided were the relationships between the standardized transfer function format developed here and the one used in [2]. This allows translation from one to the other as needed. An examination of the role of subsection multipliers was also included since these multipliers can be manipulated to advantage in a cascade realization. Chapter VIII also gave a quick review of the state-space concept which is fundamental to the realization procedure developed in [2]. This procedure was applied to the standardized transfer function format developed here. The transfer function development procedures in Chapters II through VII and the realization procedures in Chapter VIII were used in a fairly direct manner in the cascade filter design program DFD in Appendix G. As mentioned earlier, this direct relationship was identified throughout the text and was an important part of the tutorial goal.

There are limitations in this development and in the resultant program. One limitation is that only three design types were included. Another limitation is the inclusion of only one technique for computing realization coefficient values. Another is the absence of performance characteristics for different orderings of the cascaded subsections. A major limitation is the exclusion of parallel configurations and their performance characteristics. These limitations, and others, could be removed by a follow-up to this project. This would give the program more versatility and would be a useful area for future work.

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A

DESCRIPTION OF PROGRAM

Operation

The program DFD listed in Appendix G is a digital filter design program that incorporates the mathematical development presented in the text of this paper. The program is interactive with the user and selects the series of functions that will run based on user input. The sample program runs in Appendix B demonstrate proper operation of the program.

The user initializes the program by entering DFD on the terminal. The procedure for running the program is explained in a help file accessed by entering H on the terminal after the program is initialized. The help file is reprinted here for convenience.

---HELP FILE---

FOR DETAILED INFORMATION ON THIS PROGRAM SEE:

A DIGITAL FILTER DESIGN PROCEDURE
WITH APPLICATION PROGRAM IN APL

A THESIS PRESENTED FOR THE
MASTER OF SCIENCE DEGREE
THE UNIVERSITY OF TENNESSEE, KNOXVILLE

WALTER J. EATON
MARCH 1987

THIS PROGRAM (CALLED DIGITAL FILTER DESIGN) IS INITIALIZED BY ENTERING DFD ON THE TERMINAL. THE PROGRAM PROMPTS THE USER FOR INPUT PARAMETERS FOR THE DESIGN OF DIGITAL FILTERS. THE PROGRAM PRODUCES Z-TRANSFORM COEFFICIENTS AND STATE-SPACE REALIZATION COEFFICIENTS. IF H IS ENTERED, THIS FILE IS DISPLAYED. THE USER CAN ENTER Q FOR QUIT, AND THE PROGRAM WILL END WITHOUT MAKING A RUN. IF THE USER ENTERS V FOR VERIFICATION, THE EXISTING LIST OF INPUT PARAMETERS WILL BE DISPLAYED. THE USER CAN ALSO ENTER OK TO INDICATE THAT THE PARAMETERS ARE CORRECT AND A PROGRAM RUN IS DESIRED.

THE USER CAN CHANGE THE VALUE OF ANY OF THE INPUT PARAMETERS WHEN THE PROGRAM PROMPTS FOR A PARAMETER TO BE CHANGED. IF THE USER ENTERS DT TO CHANGE THE DESIGN TYPE, THE PROGRAM WILL RESPOND WITH THE PROMPT

```
ENTER:     B FOR BUTTERWORTH
           C FOR CHEBYSHEV
           E FOR ELLIPTIC
```

AND THEN IT WILL WAIT FOR A USER INPUT. SIMILARLY, IF THE USER ENTERS FT TO CHANGE THE FILTER TYPE, THE PROGRAM WILL RESPOND WITH THE PROMPT

```
ENTER:     LP FOR LOW-PASS
           HP FOR HIGH-PASS
           BP FOR BAND-PASS
           BS FOR BAND-STOP
```

AND AGAIN IT WILL WAIT FOR A RESPONSE. THE UNIT OF FREQUENCY IS SELECTED IN THE SAME MANNER. THE USER ENTERS UF AND THE PROGRAM RESPONDS WITH THE PROMPT

ENTER: H FOR HERTZ
KH FOR KILOHERTZ
MH FOR MEGAHERTZ
GH FOR GIGAHERTZ
R FOR RADIANS/SEC
KR FOR KILORADIANS/SEC
MR FOR MEGARADIANS/SEC
GR FOR GIGARADIANS/SEC

AND IT WAITS FOR A RESPONSE.

THE INPUTS THAT ARE REQUESTED WILL DEPEND ON THE FILTER TYPE SELECTED. ALL OF THE VALID FREQUENCY AND ATTENUATION INPUTS FOR EACH FILTER TYPE ARE SHOWN BELOW.

LOW-PASS

PBF = PASSBAND FREQUENCY
SBF = STOPBAND FREQUENCY
PA = PASSBAND ATTENUATION
SA = STOPBAND ATTENUATION
SF = SAMPLING FREQUENCY

HIGH-PASS

SBF = STOPBAND FREQUENCY
PBF = PASSBAND FREQUENCY
SA = STOPBAND ATTENUATION
PA = PASSBAND ATTENUATION
SF = SAMPLING FREQUENCY

BAND-PASS

SBF1 = STOPBAND FREQUENCY #1
PBF1 = PASSBAND FREQUENCY #1
PBF2 = PASSBAND FREQUENCY #2
SBF2 = STOPBAND FREQUENCY #2
SA = STOPBAND ATTENUATION
PA = PASSBAND ATTENUATION
SF = SAMPLING FREQUENCY

BAND-STOP

PBF1 = PASSBAND FREQUENCY #1
SBF1 = STOPBAND FREQUENCY #1
SBF2 = STOPBAND FREQUENCY #2
PBF2 = PASSBAND FREQUENCY #2
PA = PASSBAND ATTENUATION
SA = STOPBAND ATTENUATION
SF = SAMPLING FREQUENCY

IN EACH CASE THE FOLLOWING CONDITIONS MUST BE MET.

1. THE CUTOFF FREQUENCIES (PBF, SBF, ETC.) MUST HAVE ASCENDING VALUES IN THE ORDER SHOWN ON THE CRT. THE CORRECT ORDER OF FREQUENCIES WILL BE DISPLAYED FOR THE FILTER TYPE SELECTED.

2. THE SAMPLING FREQUENCY (SF) MUST BE AT LEAST 2.001 TIMES THE LARGEST CUTOFF FREQUENCY. THIS PREVENTS MATHEMATICAL PROBLEMS THAT OCCUR IF THE SAMPLING FREQUENCY IS EXACTLY EQUAL TO THE NYQUIST VALUE OF 2.000 TIMES THE LARGEST CUTOFF FREQUENCY.

3. THE STOPBAND ATTENUATION (SA) MUST BE GREATER THAN THE PASSBAND ATTENUATION (PA).

IF THESE CONDITIONS ARE VIOLATED, APPROPRIATE ERROR MESSAGES WILL BE DISPLAYED. ANY OF THESE PARAMETERS CAN BE CHANGED BE ENTERING THE APPROPRIATE ACRONYM LISTED ABOVE WHEN THE PROGRAM PROMPTS FOR A PARAMETER TO BE CHANGED.

NOTE THAT A RUN NUMBER APPEARS WHEN THE INPUT PARAMETERS ARE VERIFIED OR WHEN A PROGRAM RUN IS MADE. THIS NUMBER IS CONSIDERED TO BE AN INPUT PARAMETER AND CAN BE CHANGED BY ENTERING RN WHEN THE PROGRAM PROMPTS FOR A PARAMETER TO BE CHANGED. THE PROGRAM AUTOMATICALLY INCREMENTS THE RUN NUMBER EACH TIME THE PROGRAM IS RUN. SETTING THE RUN NUMBER TO 1, 11, OR 101 WILL RESULT IN A SERIES OF PROGRAM RUNS WITH RUN NUMBERS 1,2,3,...; 11,12,13,...; OR 101,102, 103,... . THIS IS HELPFUL IN KEEPING TRACK OF A SERIES OF RELATED PROGRAM RUNS. IF THE RUN NUMBER IS CHANGED TO 0, ALL USER INFORMATION WILL BE CLEARED FROM THE INPUT PARAMETER REGISTERS AND THE INPUT PARAMETERS WILL BE SET TO THE DEFAULT VALUES THAT APPEAR IN FUNCTION IIP. ALSO, NO RUN WILL BE MADE AND THE PROGRAM WILL END.

WHEN THE USER ENTERS OK THE PROGRAM PROCESSES THE INPUT INFORMATION AND DISPLAYS THE PROGRAM OUTPUT. THIS INCLUDES THE INPUT PARAMETERS, THE Z-TRANSFORM COEFFICIENTS, AND THE STATE-SPACE REALIZATION COEFFICIENTS FOR THE SELECTED FILTER. WHEN THIS IS COMPLETED THE PROGRAM PROMPTS FOR ANOTHER RUN. IF THE USER ENTERS Y THE PROGRAM WILL INITIALIZE, THE RUN NUMBER WILL BE INCREMENTED, AND ANOTHER RUN CAN BE MADE. IF THE USER ENTERS N THE PROGRAM WILL END.

---END OF HELP FILE---

General Information

It should be noted that when the program calculates coefficients for the LP and HP Z-transform, the first order terms are identified by setting the subscript i to i = 0. This is done to identify the first-order terms since they are calculated by a different procedure than the one used for the second-order terms. When an even-order filter is being run the i = 0 coefficients are set to the "null" value in the program. The second-order terms begin with i = 1 for both odd-order and even-order filters. The BP and BS Z-transforms are handled the same way. The first term when N is odd is identified by setting i = 0 and is calculated separately. This is a second-order term expanded from the first-order term in the parent normalized LP Z-transform. The remaining second-order terms begin with i = 1 for both odd-order and even-order filters.

Another characteristic of the program is that all significant variables in the program are global variables. This means that they maintain their values after the program has run. So the user can enter a variable such as TYPE for example, and the program will display the value of that variable. In the case of TYPE the value will be a matrix (N_T by 2) of realization types for the transfer function subsections of the filter that was run. Each row of the matrix will have a two character type identification selected from FO (First Order), T1 (Type I), DT (Direct Type), and T2 (Type II). The variable TYPE does not appear as part of the output information but might be of interest to the user. Global variables allow the user to write additional programming that employs variables that are not part of the output.

As mentioned in the help file, there are eight selections in the UF (Unit of Frequency) menu. All eight will give correct results in the output. Two of them (radians/sec and hertz) also give correct results in the intermediate variables such as T (period of the sampling frequency) and ω_{pb}. The remaining menu selections generate intermediate variables that are numerically correct but must have the correct units assigned to them. For example, if kilohertz is chosen as the unit of frequency then T is in milliseconds and ω_{pb} is in kiloradians. This characteristic of the program must be taken into account if the intermediate variables are used in additional programming. Another aspect of the unit of frequency is that it may have an impact on whether or not the range limits of the computer are exceeded. For example, a cutoff frequency selected as 5000 Hz might cause an overflow condition. But the same filter with the same cutoff frequency selected as 5 KHz might run with no problem. In other words the choice of units can be used to scale the magnitude of the variables as needed.

APPENDIX B

SAMPLE PROGRAM RUNS WITH NOTES

General Information

A number of sample runs of the program DFD in Appendix G appear at the end of this appendix. These program runs are identified by a run number (RN) that appears as the first variable listed in the printout.

The next two variables listed are the integer order of the filter (N), and the exact (with fraction) order of the filter (NE). In the case of BP or BS filters the values of N refer to the parent filter and are labeled accordingly (NP and NPE). The order of the BP or BS filter (NB) is then given. The purpose of providing NE is to allow the program user to evaluate the trade-off between a change in an input parameter value and a change in the order of the filter. For example, if NE is 5.01 then N will be 6. This means that a slight, possibly negligible, change in an input parameter might change NE to 4.99 and N to 5. So a significant reduction in the order of the filter might be obtained at the bargain price of only a trivial change in the input parameters.

The next variables listed in the printout are the input parameters as selected by the user. The unit of frequency (UF) selected by the user appears after each frequency variable.

The next part of the printout is the digital Z-transform. It should be noted that a term such as (Z2 + 2Z + 1)3 in the printout means $(Z^2 + 2Z + 1)^3$ in the format of the standard equations in the text.

The state-space realization coefficients appear in the last part of the printout. The combined d_c term (DC in DFD) comes from (185). The a_{xx} terms (A11, A12, A21, and A22 in DFD) come from either (193), (194), or (195) depending on the subsection type. The b_x terms (B1 and B2 in DFD) are from (196). The c_x terms (C1 and C2 in DFD) are from (199). Since the d_i terms in (200) are combined into d_c in (185), the subsection d terms (D in DFD) become 1 as shown in (201).

Specific Examples

The following discussions refer to specific sample runs. Each discussion is identified by the run number (RN) that appears as the first variable in the sample run printouts at the end of this appendix. The input parameters for the sample runs were taken from examples in [1] and [2] for comparison purposes. Note that examples 8.1, 8.2, and 8.3 in [1, pp. 208-214] do not include realization coefficients. So run numbers 2 through 6, which are based on these examples, have comparisons of Z-transforms only. Note also that examples 8.1 through 8.3 encompass all three filter types: Butterworth, Chebyshev, and Elliptic.

Run Number 1

The input parameters for RN = 1 are the same as those in the example in [2, p. 204]. The filter is specified in [2, p. 43] as a second-order Butterworth high-pass filter with a cutoff frequency of 250 Hz. This means that the passband has a 3 dB cutoff frequency (PBF) of 0.25 KHz as shown in run number 1. The sampling frequency (SF) is specified as 50 KHz. In the program run the stopband cutoff frequency (SBF) was selected as 0.19 KHz and the stopband attenuation (SA) was selected as 6 dB to cause the filter to be second-order. The Z-transform for this filter is given in equation (36) in [2, p. 43] as

$$H(Z) = 0.97803048 + \frac{-0.04344583\ Z^{-1} + 0.04250161Z^{-2}}{1 - 1.95557824Z^{-1} + 0.95654368Z^{-2}} \quad . \qquad (207A)$$

This can be rearranged as

$$H(Z) = (0.97803048)\ \frac{Z^2 - 2Z + 1}{Z^2 - 1.95557824\ Z + 0.95654368} \qquad (207B)$$

which agrees with the results in RN = 1 to four significant digits. The realization coefficients in [2, p. 204] are given as

$$A = \begin{bmatrix} 0.95557824 & 0.05400308 \\ -0.01787743 & 1 \end{bmatrix} , \qquad (208A)$$

$$\underline{b} = [\quad 0 \qquad 0.16960206] , \qquad (208B)$$

$$\underline{c}^t = [\ 0.10762209\ -0.25616336] . \qquad (208C)$$

Note that the realization coefficient d that appears in (188B) is that which appears in (174). The constant in (207A) gives

$$d = 0.97803048 . \qquad (208D)$$

These results agree with the results in RN = 1 to two significant digits.

The low correlation in the results is due to differences in the calculation precision of the two programs. This can be verified by inserting the coefficients from H(Z) in (207A) directly into the realization part of the program DFD in Appendix G after run number 1 is made. H(Z) in (207A) is in the form of (174) with α_1, α_2, β_1, β_2, and d being the transfer function coefficients. These coefficients must be modified by (184) to change (207A) to the form of (183) since the program is based on the second-order form of (183). This means that α_1 in (207A) must be divided by d in (207A) to generate α_1' in (183) as described in (184E). Also, α_2 in (207A) must be divided by d in (207A) to generate α_2' in (183) as described in (184F). Note that d in (183) and (184) has the same numerical value as d in (174) and (207A) since d in (183) was generated by moving d in (174) outside the brackets as a common multiplier. The values of α_1', α_2', β_1, and β_2 are then defined as the values of variables AL1P, AL2P, BE1, and BE2 respectively in the program. This is done by entering AL1P_ .NG0.04442176, etc. on the terminal. Note that these definitions are normally made by the program in function THZSC. The next step is to run functions SRC, RCMC, and RMC since these are the relevant functions that follow THZSC in Figure 8. Then the matrix RM is called up which gives the results for the realization coefficients. The matrix RM is defined in the glossary in Appendix F as the Realization Matrix generated in function RMC. The results are

$$d_c = 0.97805625 \ , \tag{209A}$$

$$A = \begin{bmatrix} 0.95557824 & 0.0540031187 \\ -0.017877486 & 1 \end{bmatrix} , \tag{209B}$$

$$\underline{b} = [\quad 0 \qquad 0.169602282] \ , \tag{209C}$$

$$\underline{c}'^{t} = [0.11040048 \qquad -0.2619172] \ , \tag{209D}$$

$$d = 1 \ , \tag{209E}$$

where d_c is the combined product of all the d terms in H(Z). There is only one d term in (207A) so d_c (defined as DC in the program) has only one component. Note that the results in (209) have four significant digit agreement with the results in (208), while the coefficients that were inserted had eight significant digit agreement. Since the program in [2] and the program here use the same equations for these particular calculations, the conclusion can be drawn that the low correlation in the output realization coefficients is caused by differences in the calculation precision of the two programs in determining the realization coefficients. There are probably differences in precision in the transfer function calculations as well but these calculations do not use the same equations in [2] as in the program here so the conclusion is not so obvious.

Run Number 2

This run has the same input parameters as Example 8.1 part A in [1, pp. 208-211]. In that example the Z-transform is calculated as

$$H(Z) = \frac{(1.0)\,[(0.368384\ Z - 0.368384)\,(0.148945\ Z^2 - 0.297889\ Z + 0.148945)\,(0.200026\ Z^2 - 0.400052\ Z + 0.200026)]}{[(Z + 0.263231)\,(Z^2 + 0.577816\ Z + 0.173594)\,(Z^2 + 0.775981\ Z + 0.576084)]} \tag{210A}$$

which can be written as

$$H(Z) = \frac{(0.010975)\,[(Z - 1)\,(Z^2 - 2Z + 1)]^2}{[(Z + 0.263231)\,(Z^2 + 0.577816\ Z + 0.173594)\,(Z^2 + 0.775981\ Z + 0.576084)]} \tag{210B}$$

which agrees with the results in RN = 2.

Run Number 3

The input parameters here are the same as those in Example 8.1 part B in [1, pp. 208-211]. That example has an output Z-transform given as

$$H(Z) = \frac{(0.891251)\,[(0.0512326\ Z^2 - 0.102465\ Z + 0.0512321)\,(0.183159\ Z^2 - 0.366318\ Z + 0.183159)]}{[(Z^2 + 1.31014\ Z + 0.515070)\,(Z^2 + 1.06398\ Z + 0.796619)]} \tag{211A}$$

which can be regrouped to give

$$H(Z) = \frac{(0.00836324)\,(Z^2 - 2Z + 1)^2}{[(Z^2 + 1.31014\ Z + 0.515070)\,(Z^2 + 1.06398\ Z + 0.796619)]} \tag{211B}$$

which agrees with the results in RN = 3.

Run Number 4

This run concludes Example 8.1 in [1, pp. 208-211] with part C. The output Z-transform for that example is

$$H(Z) = \frac{(1.0)\ [(0.204648\ Z - 0.204648)\ (0.206292\ Z^2 - 0.277126\ Z + 0.206292)]}{[(Z + 0.590704)\ (Z^2 + 0.985428\ Z + 0.675139)]} \tag{212A}$$

This can be rearranged as

$$\frac{(0.0422172)\ [(Z - 1)\ (Z^2 - 1.34337\ Z + 1)]}{[(Z + 0.590704)\ (Z2 + 0.985428\ Z + 0.675138)]} \tag{212B}$$

which agrees with the results in RN = 4.

<u>Run Number 5</u>

Example 8.2 in [1, pp. 211-213] is examined in RN = 5. That example has an output Z-transform calculated as

$$H(Z) = \frac{\begin{array}{l}(0.00293912)\ [(1.40266\ Z^2 - 0.0102157\ Z - 1.40266) \\ \quad (0.826391\ Z^2 - 1.32443\ Z + 0.826391) \\ \quad (1.07347\ Z^2 - 0.631800\ Z + 1.07347) \\ \quad (0.941754\ Z^2 - 1.25358\ Z + 0.941754)]\end{array}}{\begin{array}{l}[(Z^2 - 0.888660\ Z + 0.926867) \\ \ (Z^2 - 1.04661\ Z + 0.930606) \\ \ (Z^2 - 0.804891\ Z + 0.973854) \\ \ (Z^2 - 1.16031\ Z + 0.976782)]\end{array}} . \tag{213A}$$

Rearrangement of the coefficients gives

$$H(Z) = \frac{\begin{array}{l}(0.00344415)\ [(Z^2 - 0.00728309\ Z + 1) \\ \quad (Z^2 - 1.60267\ Z + 1) \\ \quad (Z^2 - 0.588558\ Z + 1) \\ \quad (Z^2 - 1.33111\ Z + 1)]\end{array}}{\begin{array}{l}[(Z^2 - 0.888660\ Z + 0.926867 \\ \ (Z^2 - 1.04661\ Z + 0.930606 \\ \ (Z^2 - 0.804891\ Z + 0.973854) \\ \ (Z^2 - 1.16031\ Z + 0.976782)]\end{array}} \tag{213B}$$

which agrees with the results in RN = 5 to four significant digits.

Run Number 6

In this run the input parameters are taken from Example 8.3 in [1, pp. 213-214]. The Z-transform calculated in that example is

$$H(Z) = \frac{\begin{array}{l}[(0.485564\ Z^2 - 0.472249\ Z + 0.485564)\\ (0.529076\ Z^2 - 0.514569\ Z + 0.529076)\\ (1.06121\ Z^2 - 1.03211\ Z + 1.06121)\\ (0.718033\ Z^2 - 0.698344\ Z + 0.718033)\\ (1.13666\ Z^2 - 1.10549\ Z + 1.13666)]\end{array}}{\begin{array}{l}[(Z^2 - 0.472249\ Z - 0.0288728)\\ (Z^2 + 0.0502889\ Z + 0.623010)\\ (Z^2 - 1.40016\ Z + 0.754357)\\ (Z^2 - 0.217511\ Z + 0.916899)\\ (Z^2 - 1.43593\ Z + 0.942893)]\end{array}} \qquad (214A)$$

This can be rearranged as

$$H(Z) = \frac{(0.222506)\ \begin{array}{l}[(Z^2 - 0.972578\ Z + 1)\\ (Z^2 - 0.972580\ Z + 1)\\ (Z^2 - 0.972578\ Z + 1)\\ (Z^2 - 0.972579\ Z + 1)\\ (Z^2 - 0.972578\ Z + 1)]\end{array}}{\begin{array}{l}[(Z^2 - 0.472249\ Z - 0.0288728)\\ (Z^2 + 0.0502889\ Z + 0.623010)\\ (Z^2 - 1.40016\ Z + 0.754357)\\ (Z^2 - 0.217511\ Z + 0.916899)\\ (Z^2 - 1.43593\ Z + 0.942893)]\end{array}} \qquad (214B)$$

which agrees with the results in RN = 6 to five significant digits.

Printouts

The following printouts show the results of program run numbers 1 through 6 as discussed above.

--------------------DIGITAL FILTER DESIGN--------------------

RUN NUMBER	RN = 1
ORDER OF FILTER	N = 2
EXACT ORDER OF FILTER	NE = 1.99844329

---INPUT PARAMETERS---

DESIGN TYPE	DT = B	(BUTTERWORTH)
FILTER TYPE	FT = HP	(HIGH-PASS)
STOPBAND FREQUENCY	SBF = 0.19 KHZ	
PASSBAND FREQUENCY	PBF = 0.25 KHZ	
STOPBAND ATTENUATION	SA = 6 DB	
PASSBAND ATTENUATION	PA = 3 DB	
SAMPLING FREQUENCY	SF = 50 KHZ	

---DIGITAL TRANSFORM---

H(Z) = (G) (NUMERATOR/DENOMINATOR)

G = 0.978056254

NUMERATOR =

(Z2 - 2 Z + 1)

DENOMINATOR =

(Z2 - 1.95563092 Z + 0.956594095)

---STATE-SPACE REALIZATION COEFFICIENTS---

DC = G = 0.978056254

A11	A12	A21	A22
0.955630923	0.0539395143	-0.0178565178	1

B1	B2	C1	C2	D
0	0.16950434	0.10996884	-0.261757764	1

--------------------DIGITAL FILTER DESIGN--------------------

RUN NUMBER	RN = 2
ORDER OF FILTER	N = 5
EXACT ORDER OF FILTER	NE = 4.34274155

---INPUT PARAMETERS---

DESIGN TYPE	DT = B	(BUTTERWORTH)
FILTER TYPE	FT = HP	(HIGH-PASS)
STOPBAND FREQUENCY	SBF = 1.5 RAD/SEC	
PASSBAND FREQUENCY	PBF = 3.5 RAD/SEC	
STOPBAND ATTENUATION	SA = 45 DB	
PASSBAND ATTENUATION	PA = 1 DB	
SAMPLING FREQUENCY	SF = 10 RAD/SEC	

---DIGITAL TRANSFORM---

H(Z) = (G) (NUMERATOR/DENOMINATOR)

G = 0.0109751825

NUMERATOR =

(Z - 1)
(Z2 - 2 Z + 1)2

DENOMINATOR =

(Z + 0.263231168)
(Z2 + 0.577815601 Z + 0.173593583)
(Z2 + 0.775980575 Z + 0.576083827)

---STATE-SPACE REALIZATION COEFFICIENTS---

DC = G = 0.0109751825

A11	A12	A21	A22
-0.263231168	0	0	0
0	1	-0.173593583	-0.577815601
0	1	-0.576083827	-0.775980575

B1	B2	C1	C2	D
0.964732788	0	-1.30941042	0	1
0	0.857183916	0.964094638	-3.00730748	1
0	0.711455857	0.595843255	-3.90183108	1

---DIGITAL FILTER DESIGN---

RUN NUMBER	RN = 3
ORDER OF FILTER	N = 4
EXACT ORDER OF FILTER	NE = 3.23529731

---INPUT PARAMETERS---

DESIGN TYPE	DT = C	(CHEBYSHEV)
FILTER TYPE	FT = HP	(HIGH-PASS)
STOPBAND FREQUENCY	SBF = 1.5 RAD/SEC	
PASSBAND FREQUENCY	PBF = 3.5 RAD/SEC	
STOPBAND ATTENUATION	SA = 45 DB	
PASSBAND ATTENUATION	PA = 1 DB	
SAMPLING FREQUENCY	SF = 10 RAD/SEC	

---DIGITAL TRANSFORM---

H(Z) = (G) (NUMERATOR/DENOMINATOR)

G = 0.00836323956

NUMERATOR =

(Z2 - 2 Z + 1)2

DENOMINATOR =

(Z2 + 1.31014021 Z + 0.515070441)
(Z2 + 1.06398297 Z + 0.796619353)

---STATE-SPACE REALIZATION COEFFICIENTS---

DC = G = 0.00836323956

A11	A12	A21	A22
-2.31014021	1.53016731	-1.846341	1
0	1	-0.796619353	-1.06398297

B1	B2	C1	C2	D
0	0.281327658	18.8901878	-11.7661386	1
0	0.487079398	0.417551322	- 6.29052056	1

--------------------DIGITAL FILTER DESIGN--------------------

RUN NUMBER	RN = 4
ORDER OF FILTER	N = 3
EXACT ORDER OF FILTER	NE = 2.66519368

---INPUT PARAMETERS---

DESIGN TYPE	DT = E	(ELLIPTIC)
FILTER TYPE	FT = HP	(HIGH-PASS)
STOPBAND FREQUENCY	SBF = 1.5 RAD/SEC	
PASSBAND FREQUENCY	PBF = 3.5 RAD/SEC	
STOPBAND ATTENUATION	SA = 45 DB	
PASSBAND ATTENUATION	PA = 1 DB	
SAMPLING FREQUENCY	SF = 10 RAD/SEC	

---DIGITAL TRANSFORM---

H(Z) = (G) (NUMERATOR/DENOMINATOR)

G = 0.0422173097

NUMERATOR =

(Z - 1)
(Z2 - 1.34336542 Z + 1)

DENOMINATOR =

(Z + 0.590703877)
(Z2 + 0.985428112 Z + 0.675138496)

---STATE-SPACE REALIZATION COEFFICIENTS---

DC = G = 0.0422173097

A11	A12	A21	A22
-0.590703877	0	0	0
0	1	-0.675138496	-0.985428112

B1	B2	C1	C2	D
0.806888425	0	-1.971405	0	1
0	0.596546382	0.544570403	-3.9037929	1

--------------------DIGITAL FILTER DESIGN--------------------

RUN NUMBER	RN = 5
ORDER OF PARENT LP FILTER	NP = 4
EXACT ORDER OF PARENT LP FILTER	NPE = 3.67112078
ORDER OF BP OR BS FILTER	NB = 8

---INPUT PARAMETERS---

DESIGN TYPE	DT = E (ELLIPTIC)
FILTER TYPE	FT = BP (BAND-PASS)
STOPBAND FREQUENCY #1	SBF1 = 0.8 KRAD/SEC
PASSBAND FREQUENCY #1	PBF1 = 0.9 KRAD/SEC
PASSBAND FREQUENCY #2	PBF2 = 1.1 KRAD/SEC
STOPBAND FREQUENCY #2	SBF2 = 1.2 KRAD/SEC
STOPBAND ATTENUATION	SA = 45 DB
PASSBAND ATTENUATION	PA = 1 DB
SAMPLING FREQUENCY	SF = 6 KRAD/SEC

---DIGITAL TRANSFORM---

H(Z) = (G) (NUMERATOR/DENOMINATOR)

G = 0.00344415294

NUMERATOR =

(Z2 - 0.00728353692 Z + 1)
(Z2 - 1.60266767 Z + 1)
(Z2 - 0.588557671 Z + 1)
(Z2 - 1.33111582 Z + 1)

DENOMINATOR =

(Z2 - 0.88866037 Z + 0.926866771)
(Z2 - 1.04660582 Z + 0.930605671)
(Z2 - 0.804891913 Z + 0.973853913)
(Z2 - 1.16030895 Z + 0.976782432)

---STATE-SPACE REALIZATION COEFFICIENTS---

DC = G = 0.00344415294

A11	A12	A21	A22
0	1	-0.926866771	0.88866037
0	1	-0.930605671	1.04660582
0	1	-0.973853913	0.804891913
0	1	-0.976782432	1.16030895

B1	B2	C1	C2	D
0	0.333083593	0.219564189	2.64611302	1
0	0.307571534	0.225620127	-1.80791063	1
0	0.207429471	0.126048084	1.04292915	1
0	0.173445707	0.133860725	-0.984785802	1

-------------------DIGITAL FILTER DESIGN-------------------

RUN NUMBER	RN = 6
ORDER OF PARENT LP FILTER	NP = 5
EXACT ORDER OF PARENT LP FILTER	NPE = 4.7016682
ORDER OF BP OR BS FILTER	NB = 10

---INPUT PARAMETERS---

DESIGN TYPE	DT = C	(CHEBYSHEV)
FILTER TYPE	FT = BS	(BAND-STOP)
PASSBAND FREQUENCY #1	PBF1 = 0.35 KRAD/SEC	
STOPBAND FREQUENCY #1	SBF1 = 0.43 KRAD/SEC	
STOPBAND FREQUENCY #2	SBF2 = 0.6 KRAD/SEC	
PASSBAND FREQUENCY #2	PBF2 = 0.7 KRAD/SEC	
PASSBAND ATTENUATION	PA = 0.5 DB	
STOPBAND ATTENUATION	SA = 40 DB	
SAMPLING FREQUENCY	SF = 3 KRAD/SEC	

---DIGITAL TRANSFORM---

H(Z) = (G) (NUMERATOR/DENOMINATOR)

G = 0.222505171

NUMERATOR =

(Z2 - 0.972579302 Z + 1)5

DENOMINATOR =

(Z2 - 0.472249108 Z - 0.028872799)
(Z2 - 1.40016295 Z + 0.754356982)
(Z2 + 0.0502888132 Z + 0.623010029)
(Z2 - 1.43592555 Z + 0.94289273)
(Z2 - 0.217511019 Z + 0.916899366)

---STATE-SPACE REALIZATION COEFFICIENTS---

DC = G = 0.222505171

A11	A12	A21	A22
0	1	0.028872799	0.472249108
0.400162955	0.722070235	-0.490525728	1
0	1	-0.623010029	-0.0502888132
0.435925548	0.738698697	-0.686297653	1
0	1	-0.916899366	0.217511019

B1	B2	C1	C2	D
0	0.873433383	1.17796368	-0.572831545	1
0	0.547773526	1.05363726	0.780584735	1
0	0.78183827	0.482184085	-1.30828607	1
0	0.303757	1.15467545	1.52538459	1
0	0.396540722	0.209563935	-1.90413806	1

APPENDIX C

THE APL LANGUAGE

Each programming language has its advantages and disadvantages. The primary advantage of APL is the matrix manipulation operators that are imbedded in the language. These operators are known as "primitive functions" [4, p. 31] because they are already written and ready for use. An example of an APL matrix operator can be found in line [16] of the function PSCP2 in Appendix G. In that situation a vector, or one dimensional array, of unknown size (S1G2) is being subtracted from another vector of the same size (NBOL) to produce a third vector of the same size (W2). This means that each element of S1G2 must be subtracted from the corresponding element of NBOL. The only programming necessary to accomplish this in APL is the statement W2_NBOL-S1G2. It makes no difference if the variables are vectors (one dimensional arrays), matrices (two dimensional arrays), or arrays of many dimensions. The problem can still be solved with the same one line program. This is a simple example but the operators are available for one line solutions to some fairly complex matrix manipulations. These operators become quite attractive when the alternative is to write a subroutine to perform each one of these operations.

Only a small portion of the power of the APL language is used in the program DFD in Appendix G. This is mainly due to a desire for simplified step-by-step procedures that can be followed easily by a reader with a minimal knowledge of APL. Also, there are frequent constructions of character matrix outputs which are done one line at a time in loops. Straight numerical calculations would not be handled in this manner. Of course the program could be condensed considerably by anyone familiar with APL, but it would become correspondingly more cryptic to the general reader which would defeat the purpose of this paper.

A good review of the fundamentals of the APL language can be found in [4, pp. 21-238]. This reference contains more than enough information to follow the program here.

APPENDIX D

PROGRAM FUNCTION FLOW CHARTS

Figure 5 through Figure 16 indicate the functions that are called up by the program DFD in Appendix G depending on the filter type and design type specified in the input parameters. The number and letter that appear above each function refer to the level and group that contain the function in the program listing in Appendix G. The function flow charts indicate that each program run begins by calling up the Group 1A function DFD which initializes the program. The function DFD is a Level 1 function in the hierarchy and will call up the Level 2 functions from left to right. Some of the Level 2 functions will call up Level 3 functions from left to right as shown. Similarly for Level 4 and Level 5. In each case the path lines in the figure indicate which functions are called up by a particular function and in what order they are called up. See the logic flow charts in Appendix E for detailed information on how the particular paths are chosen. The functions in Level 6 of the program listing do not appear in this appendix since they are generic math and logic functions that are for general use throughout the program.

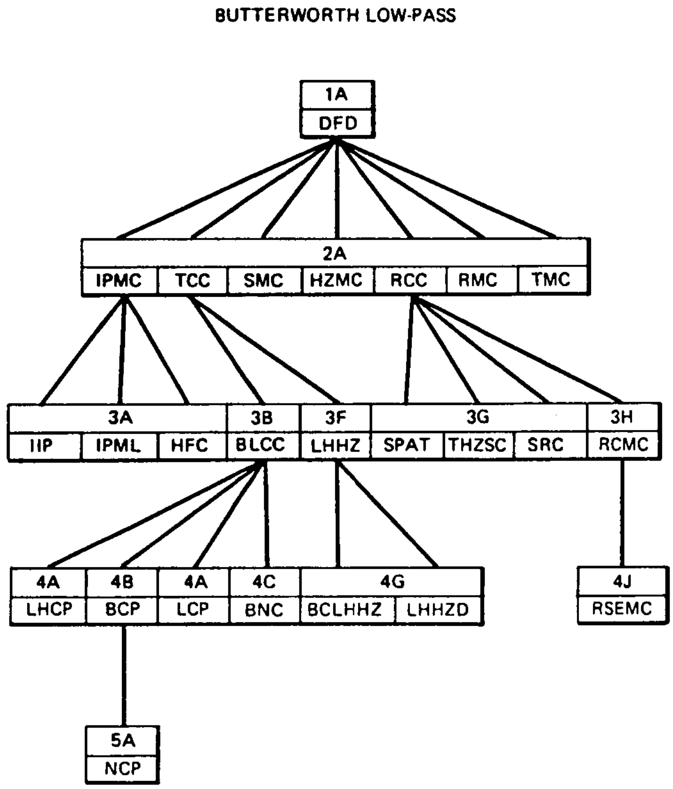

Figure 5. Butterworth Low-Pass Function Flow Chart.

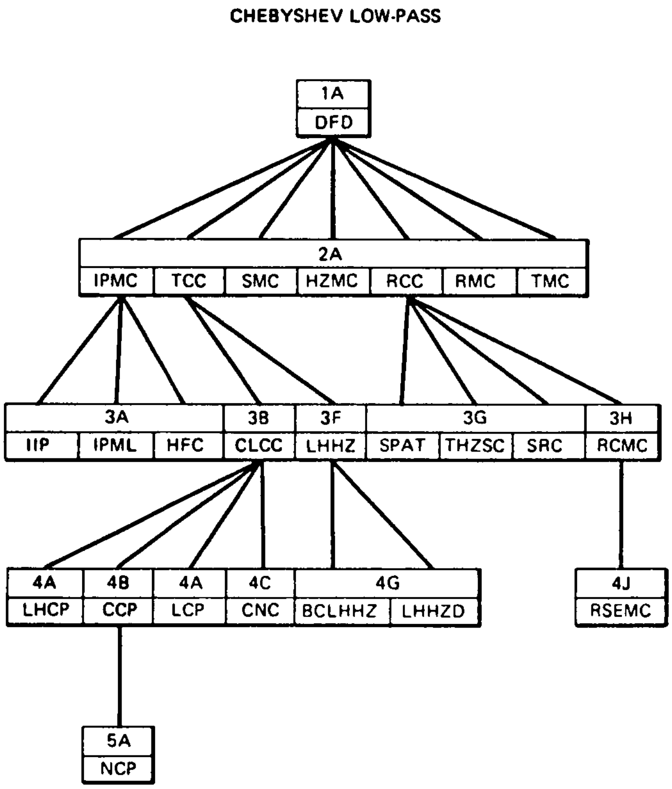

Figure 6. Chebyshev Low-Pass Function Flow Chart.

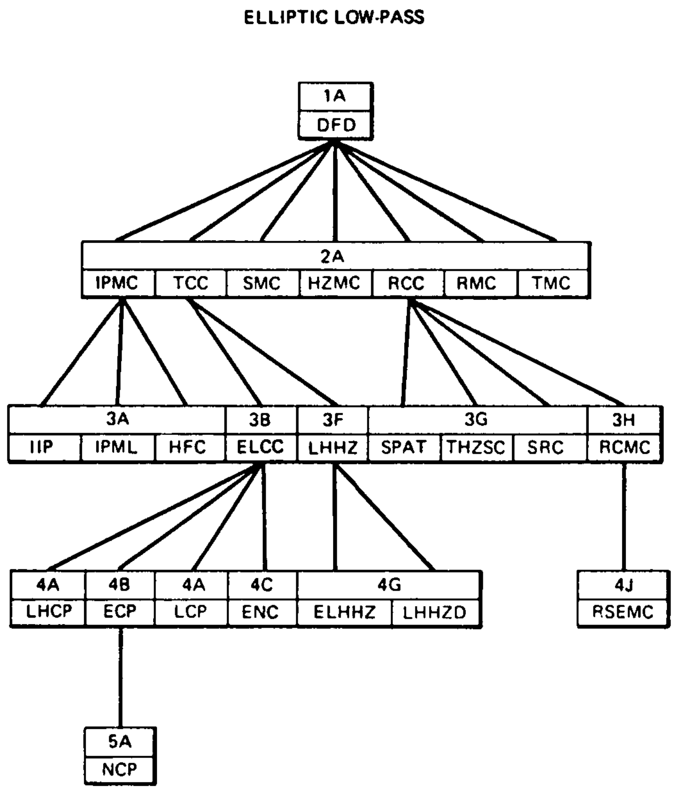

Figure 7. Elliptic Low-Pass Function Flow Chart.

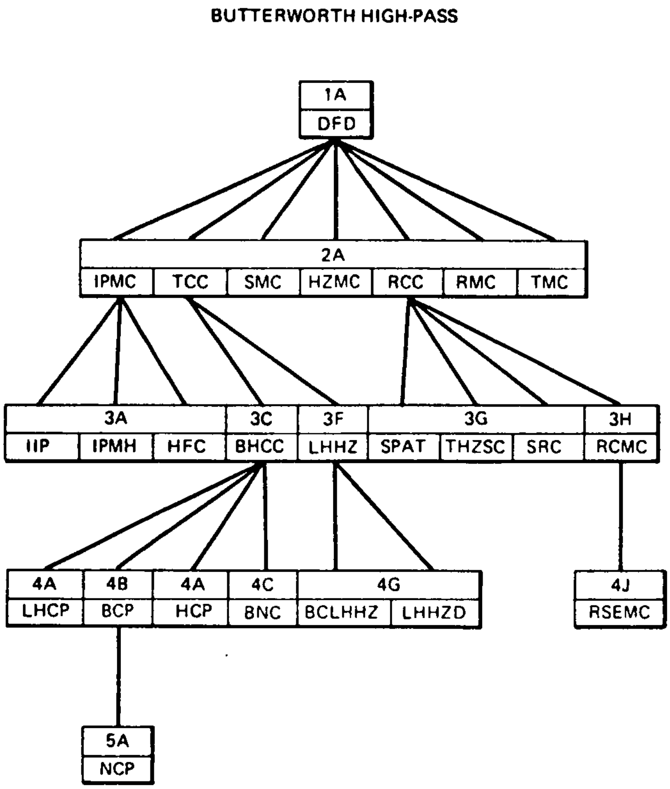

Figure 8. Butterworth High-Pass Function Flow Chart.

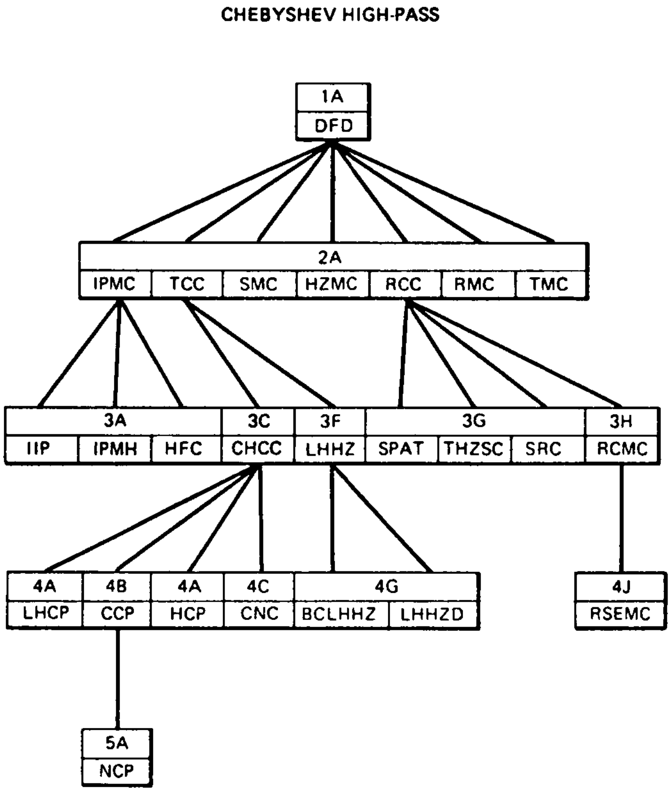

Figure 9. Chebyshev High-Pass Function Flow Chart.

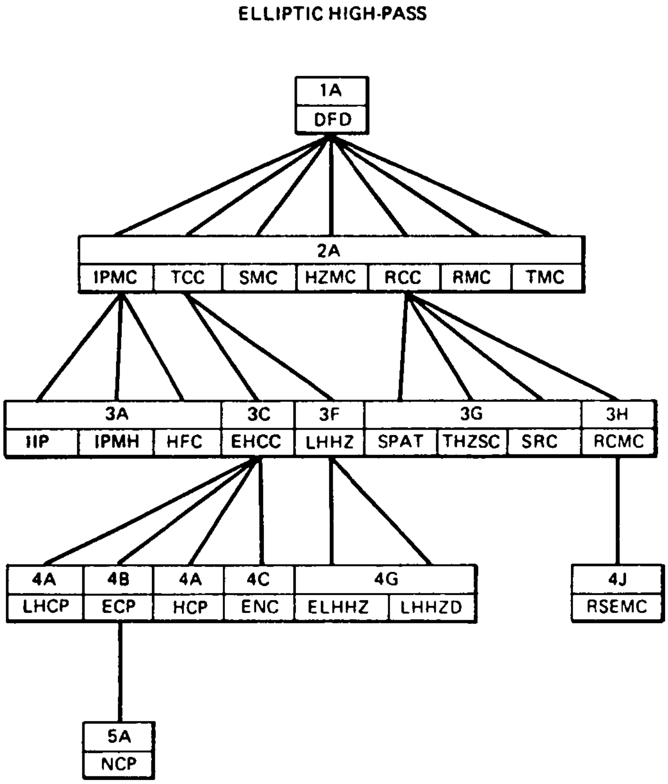

Figure 10. Elliptic High-Pass Function Flow Chart.

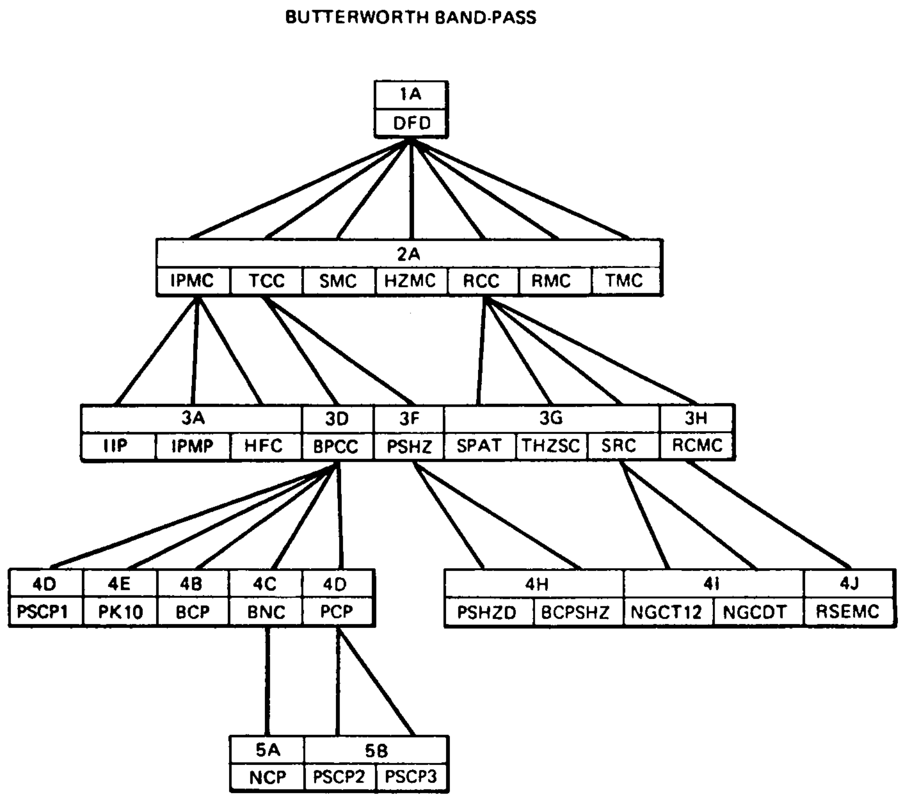

Figure 11. Butterworth Band-Pass Function Flow Chart.

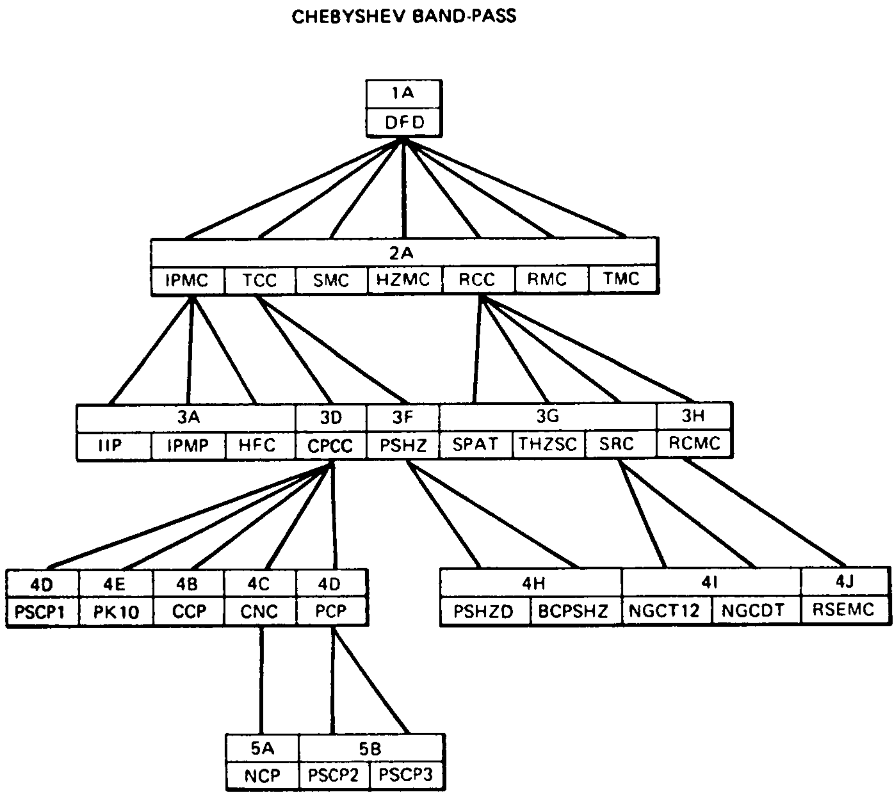

Figure 12. Chebyshev Band-Pass Function Flow Chart.

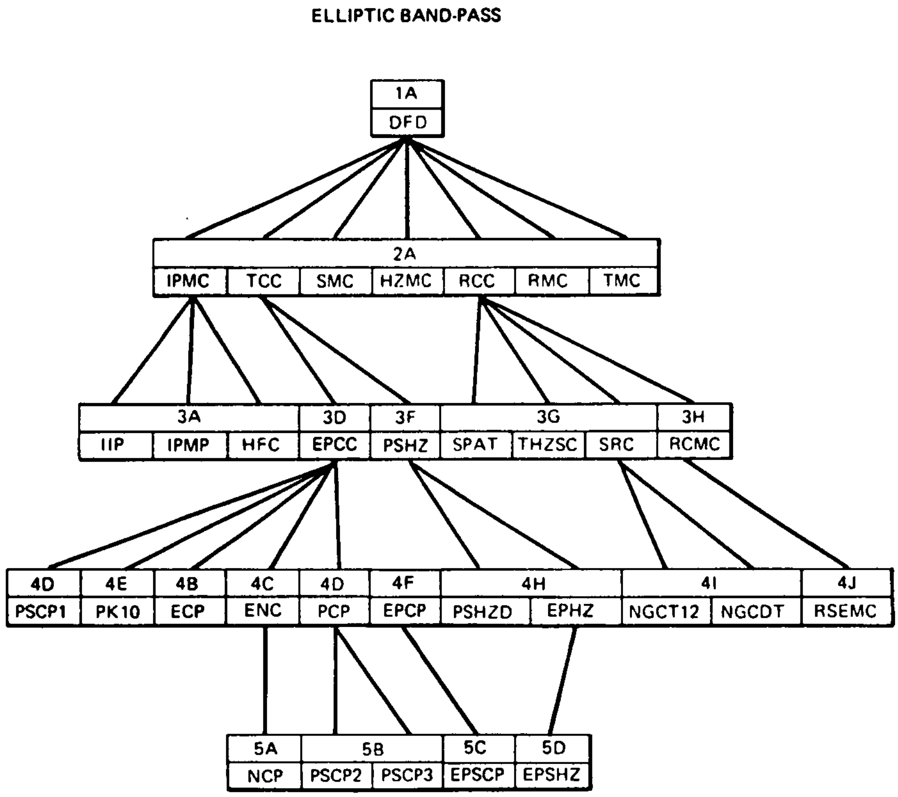

Figure 13. Elliptic Band-Pass Function Flow Chart.

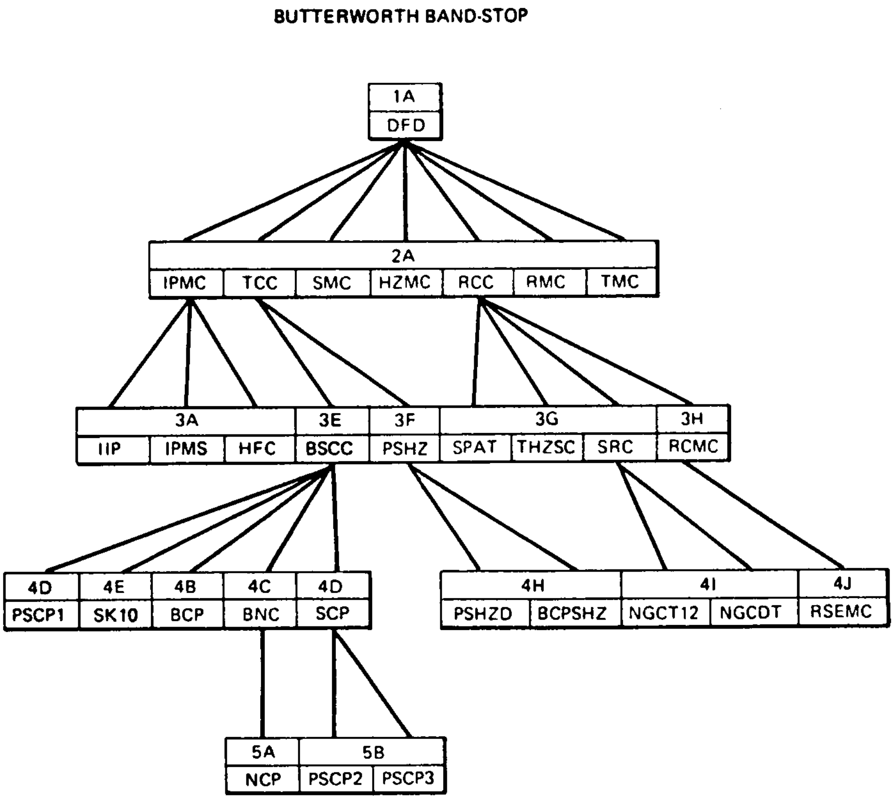

Figure 14. Butterworth Band-Stop Function Flow Chart.

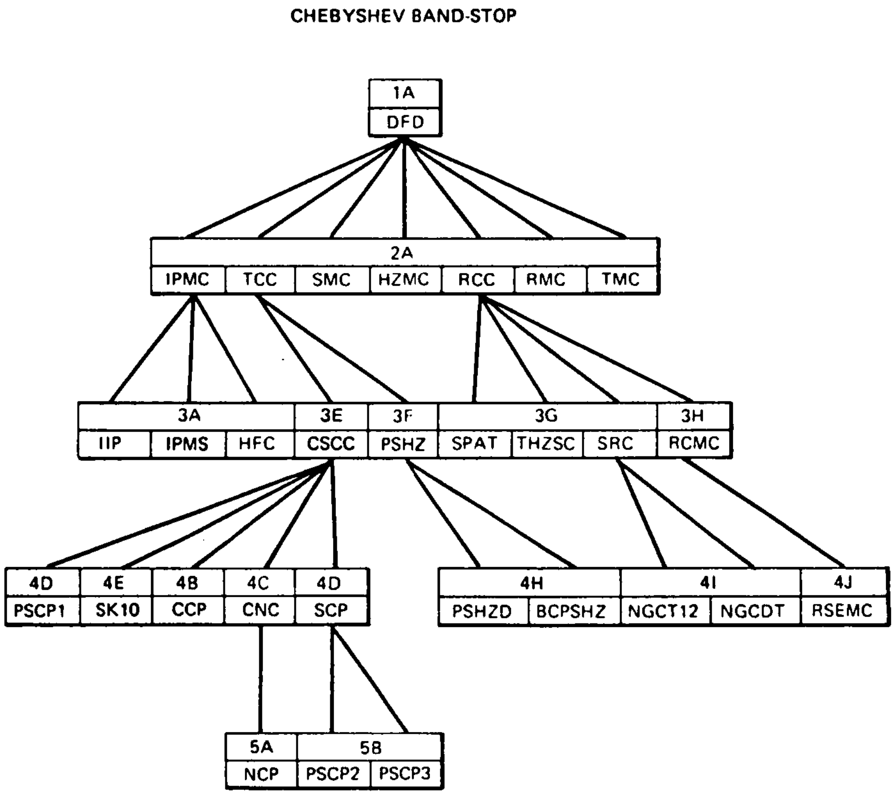

Figure 15. Chebyshev Band-Stop Function Flow Chart.

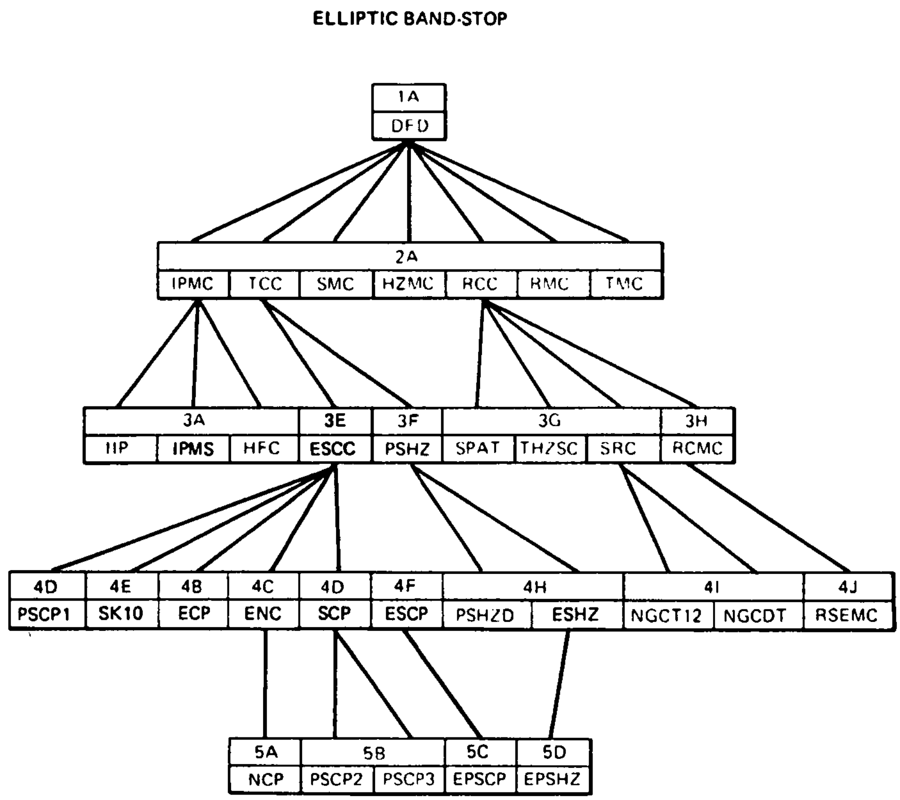

Figure 16. Elliptic Band-Stop Function Flow Chart.

APPENDIX E

LOGIC FLOW CHARTS

The flow charts in Figure 17 through Figure 79 show the operation of the program DFD in Appendix G in typical logic flow chart format. The functions that appear in the flow charts are in the same order as in Level 1 through Level 5 of the program listing in Appendix G. The general purpose math and logic functions in Level 6 of the program listing are not included. The logic flow charts have the following characteristics.

1. Functions that are called up as subroutines are shown inside rectangles.
2. Operations that do not require calling up another function are described beside blank rectangles.
3. Decision branches are shown by diamonds.
4. Labels are indicated by dots with the label names beside them.
5. Outputs to the CRT are shown by parallelograms.

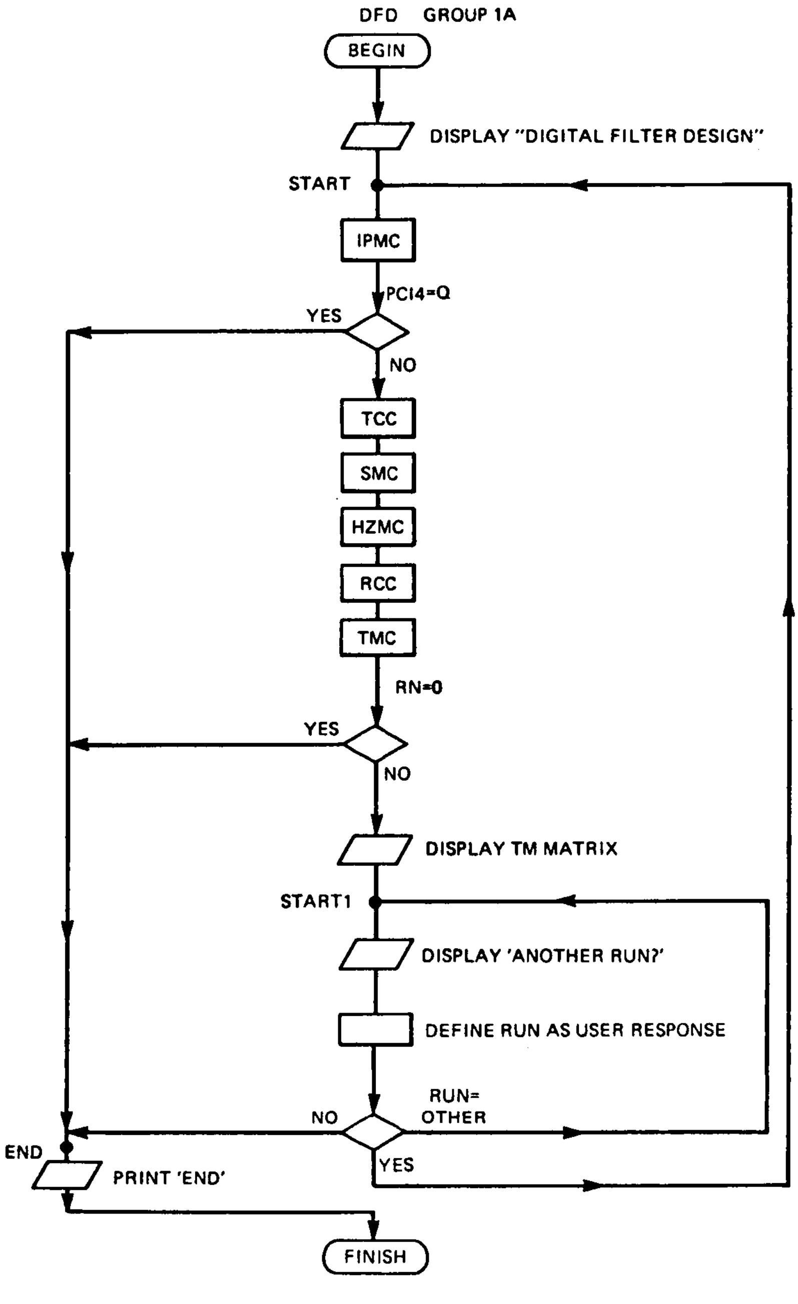

Figure 17. DFD (Digital Filter Design) Logic Flow Chart.

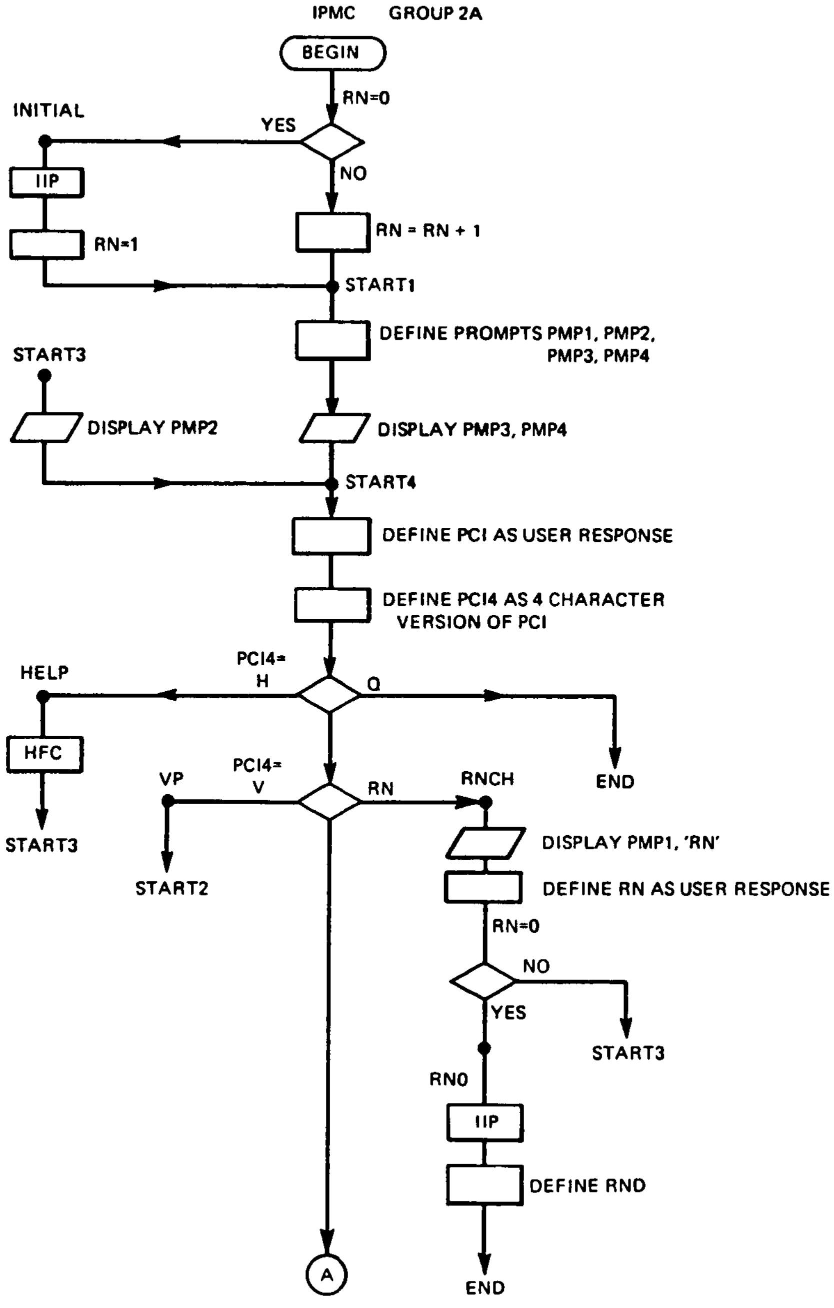

Figure 18. IPMC (Input Parameter Matrix Construction) Logic Flow Chart.

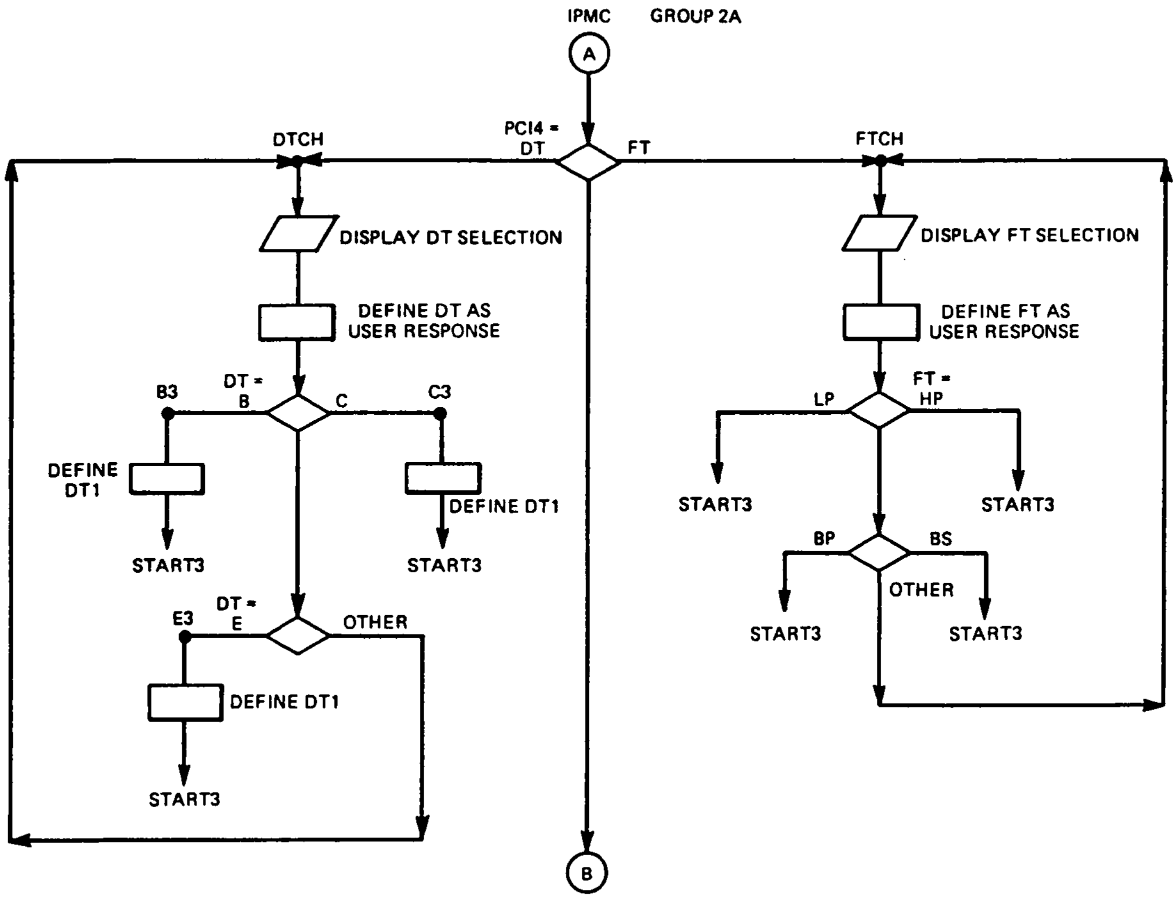

Figure 18. (Continued)

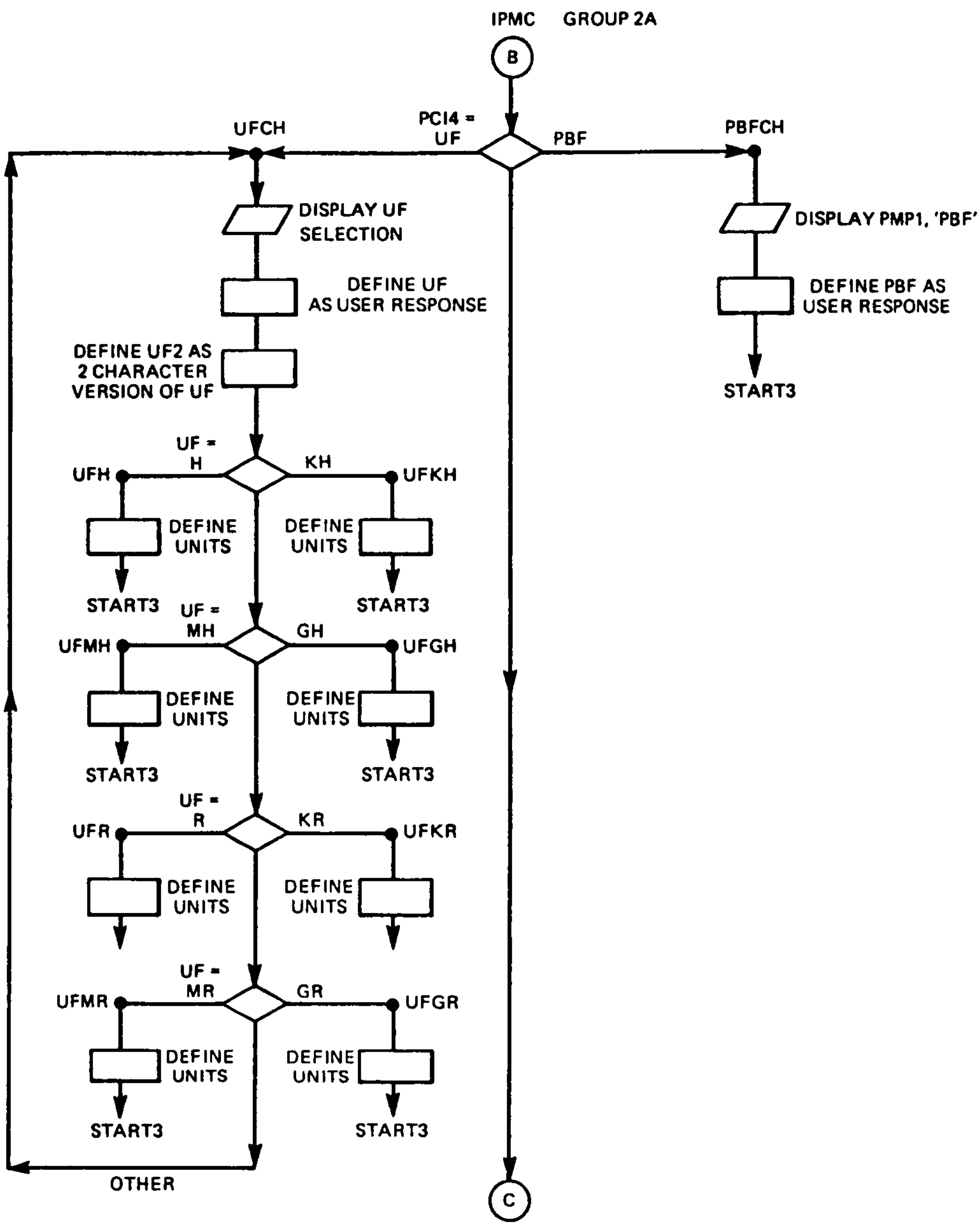

Figure 18. (Continued)

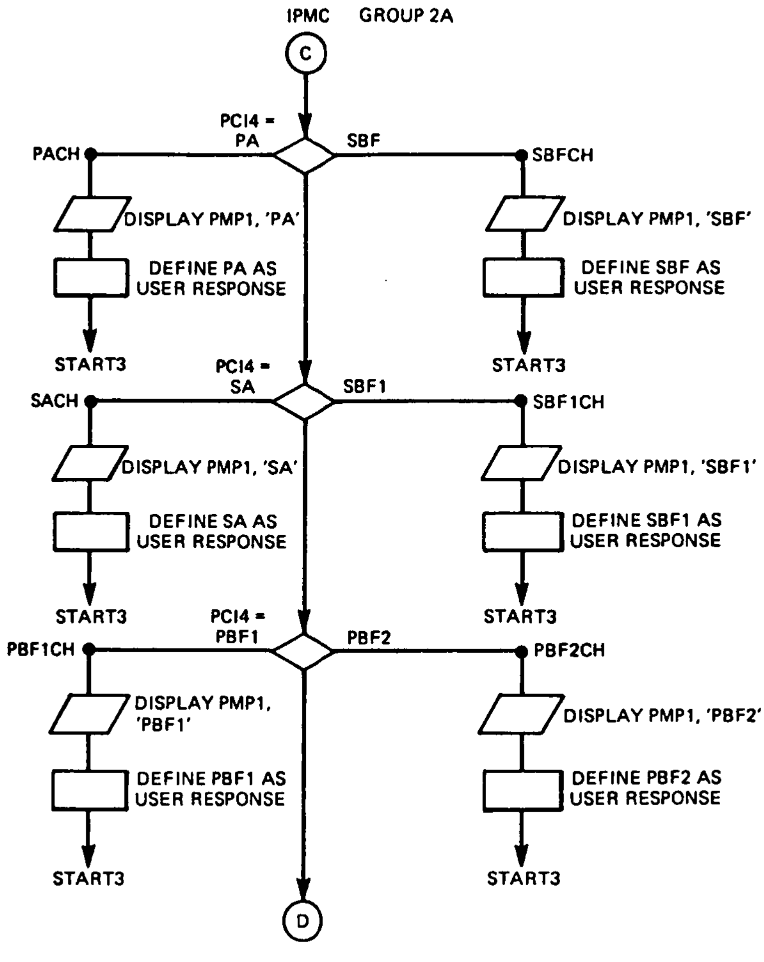

Figure 18. (Continued)

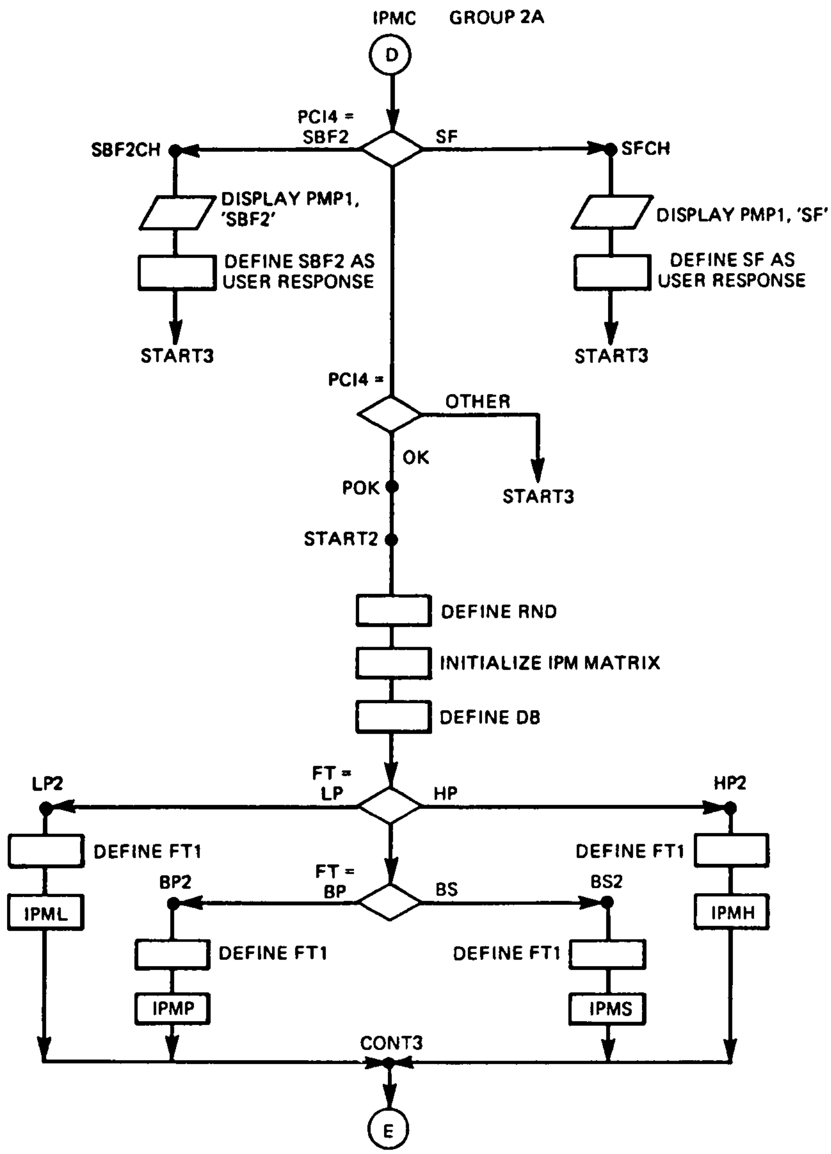

Figure 18. (Continued)

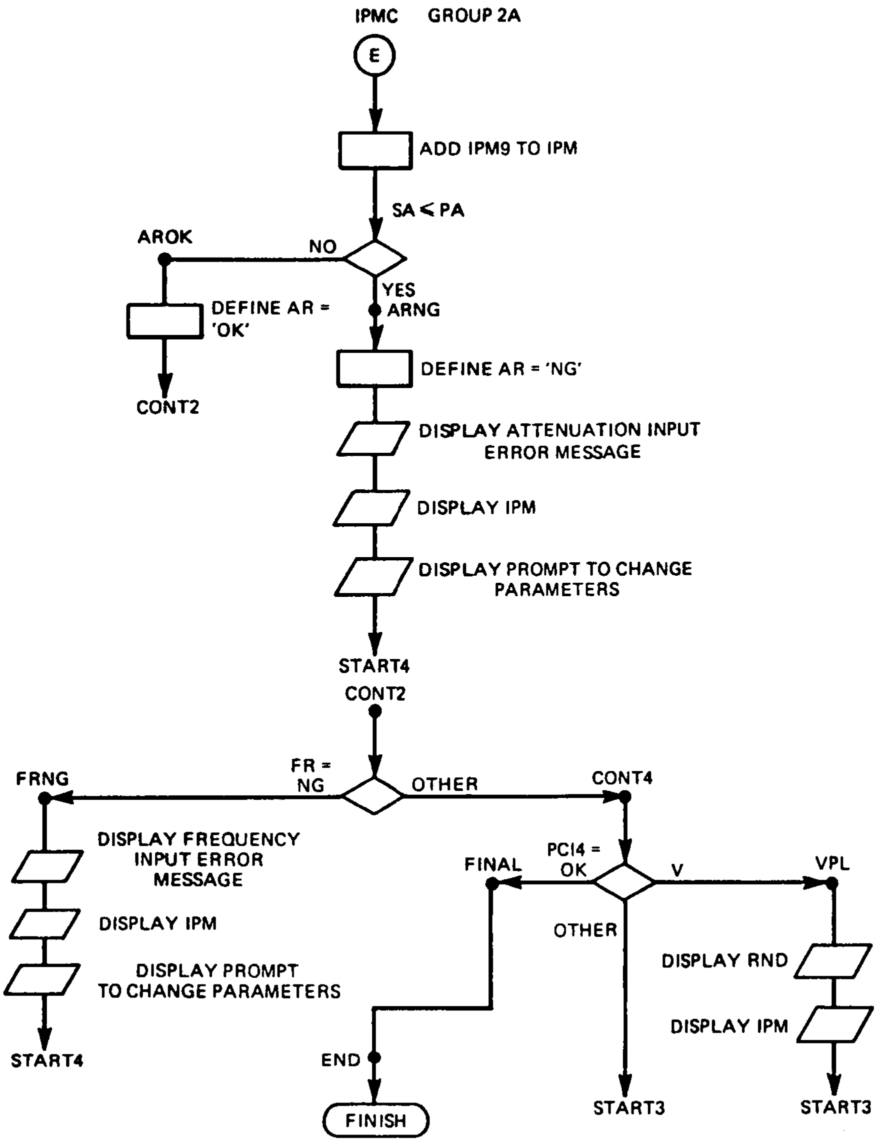

Figure 18. (Continued)

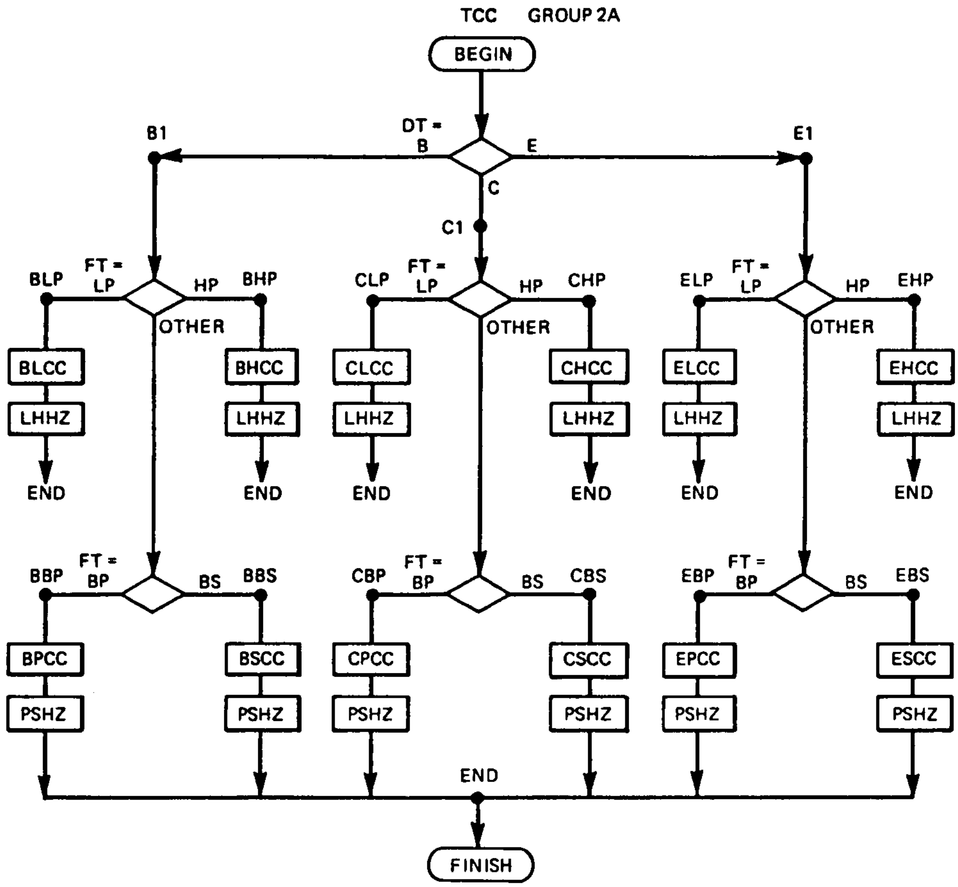

Figure 19. TCC (Transform Computation Control) Logic Flow Chart.

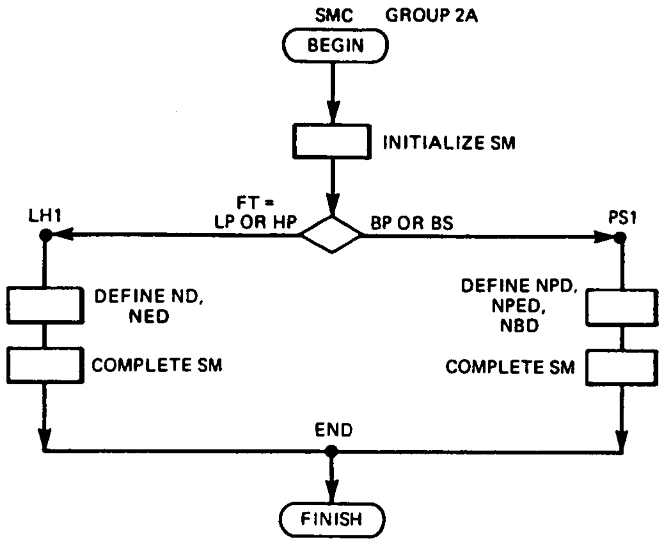

Figure 20. SMC (Status Matrix Construction) Logic Flow Chart.

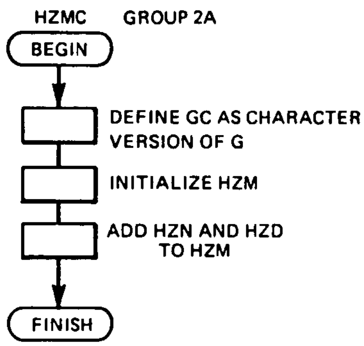

Figure 21. HZMC [H(Z) Matrix Construction] Logic Flow Chart.

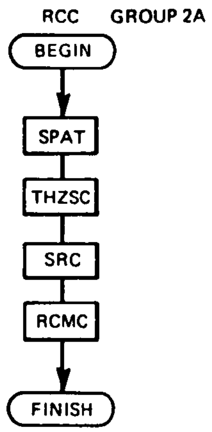

Figure 22. RCC (Realization Computation Control) Logic Flow Chart.

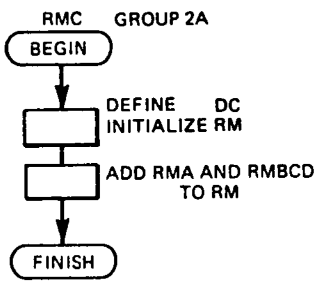

Figure 23. RMC (Realization Matrix Construction) Logic Flow Chart.

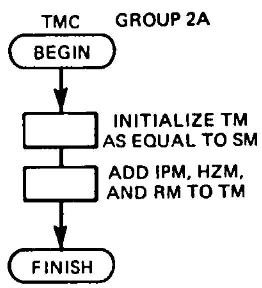

Figure 24. TMC (Total Matrix Construction) Logic Flow Chart.

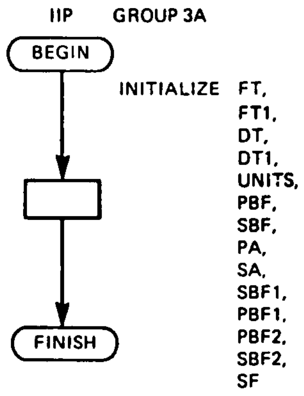

Figure 25. IIP (Initialize Input Parameters) Logic Flow Chart.

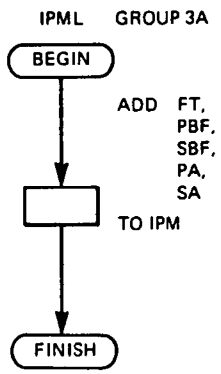

Figure 26. IPML (Input Parameter Matrix Low-Pass) Logic Flow Chart.

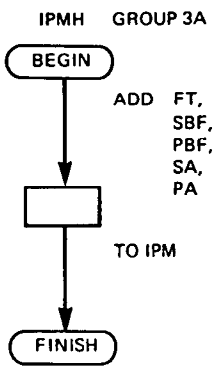

Figure 27. IPMH (Input Parameter Matrix High-Pass) Logic Flow Chart.

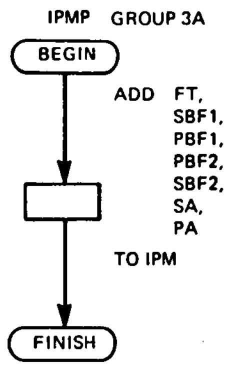

Figure 28. IPMP (Input Parameter Matrix Band-Pass) Logic Flow Chart.

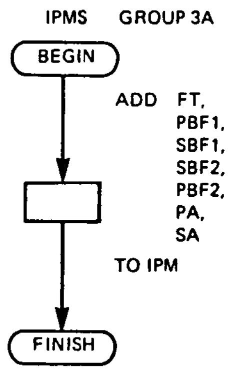

Figure 29. IPMS (Input Parameter Matrix Band-Stop) Logic Flow Chart.

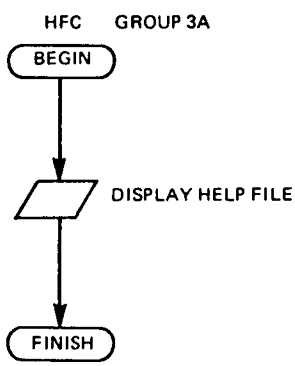

Figure 30. HFC (Help File Construction) Logic Flow Chart.

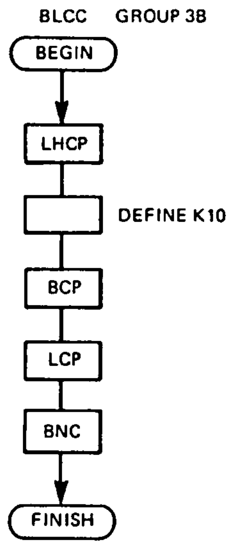

Figure 31. BLCC (Butterworth Low-Pass Computation Control) Logic Flow Chart.

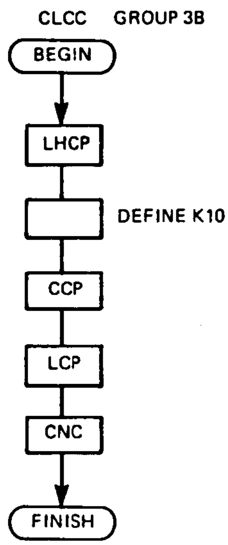

Figure 32. CLCC (Chebyshev Low-Pass Computation Control) Logic Flow Chart.

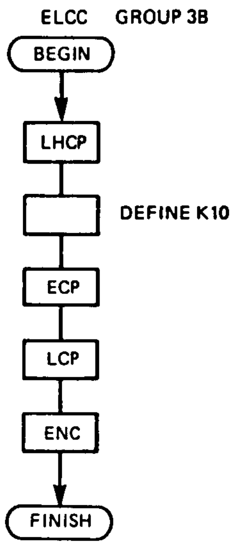

Figure 33. ELCC (Elliptic Low-Pass Computation Control) Logic Flow Chart.

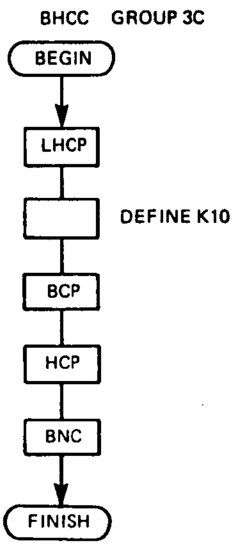

Figure 34. BHCC (Butterworth High-Pass Computation Control) Logic Flow Chart.

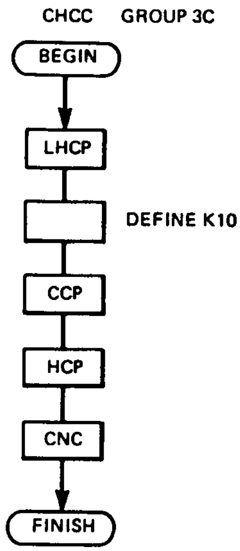

Figure 35. CHCC (Chebyshev High-Pass Computation Control) Logic Flow Chart.

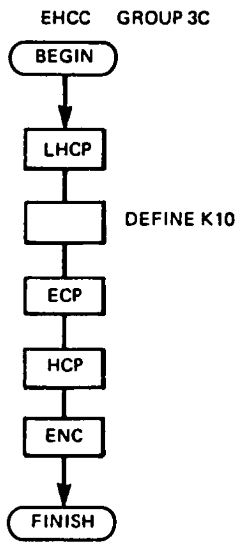

Figure 36. EHCC (Elliptic High-Pass Computation Control) Logic Flow Chart.

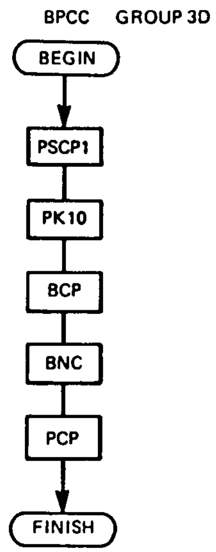

Figure 37. BPCC (Butterworth Band-Pass Computation Control) Logic Flow Chart.

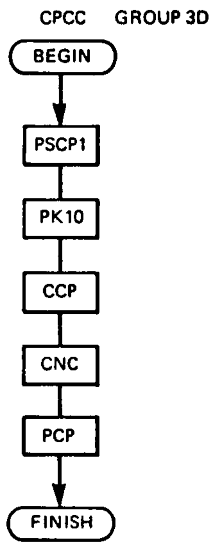

Figure 38. CPCC (Chebyshev Band-Pass Computation Control) Logic Flow Chart.

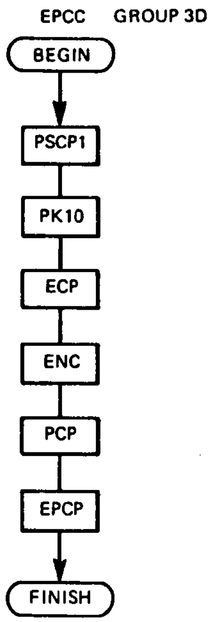

Figure 39. EPCC (Elliptic Band-Pass Computation Control) Logic Flow Chart.

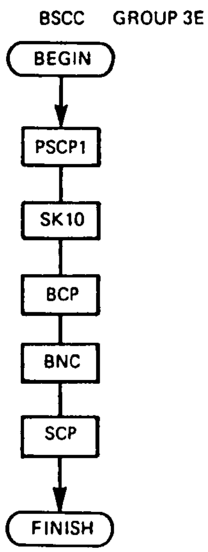

Figure 40. BSCC (Butterworth Band-Stop Computation Control) Logic Flow Chart.

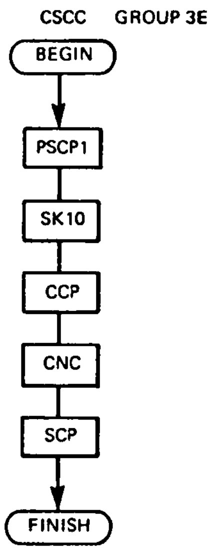

Figure 41. CSCC (Chebyshev Band-Stop Computation Control) Logic Flow Chart.

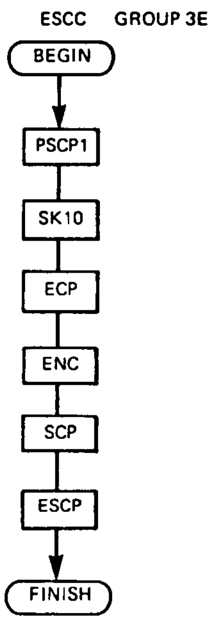

Figure 42. ESCC (Elliptic Band-Stop Computation Control) Logic Flow Chart.

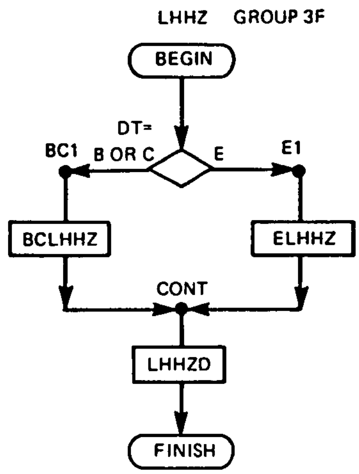

Figure 43. LHHZ [Low-High-Pass H(Z)] Logic Flow Chart.

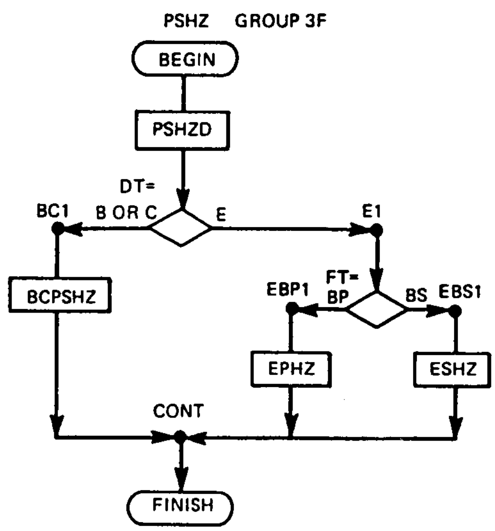

Figure 44. PSHZ [Band-Pass-Stop H(Z)] Logic Flow Chart.

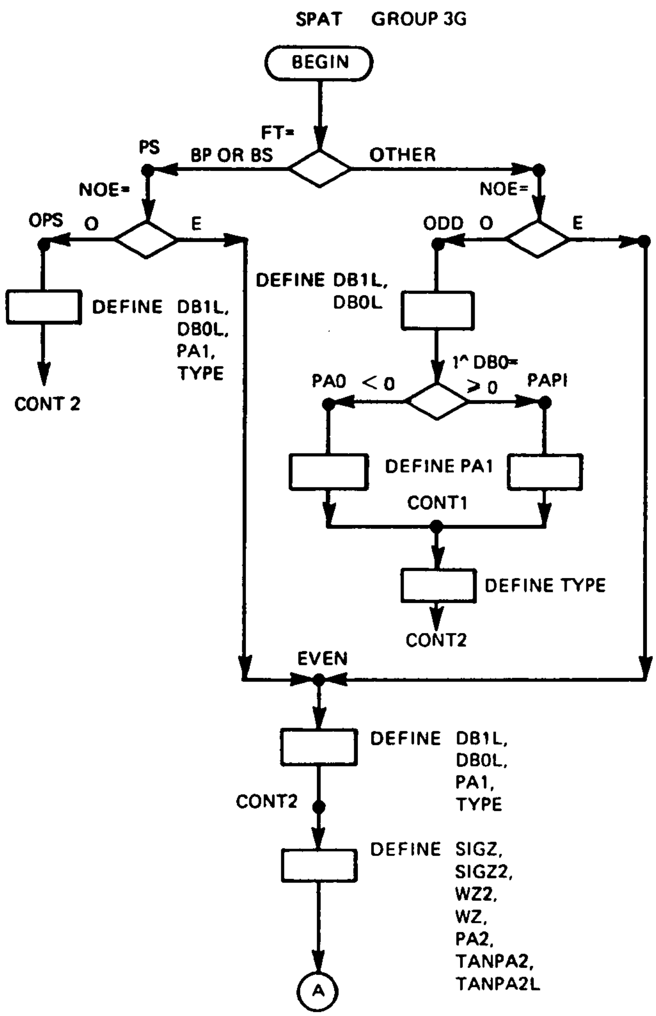

Figure 45. SPAT (Subsection Pole Angle and Type) Logic Flow Chart.

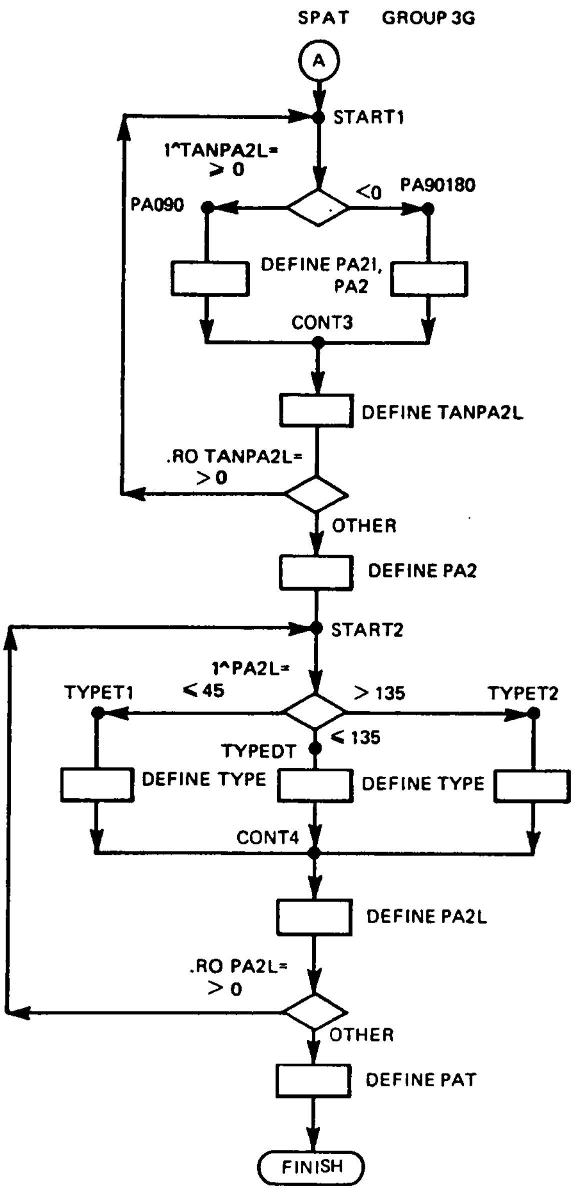

Figure 45. (Continued)

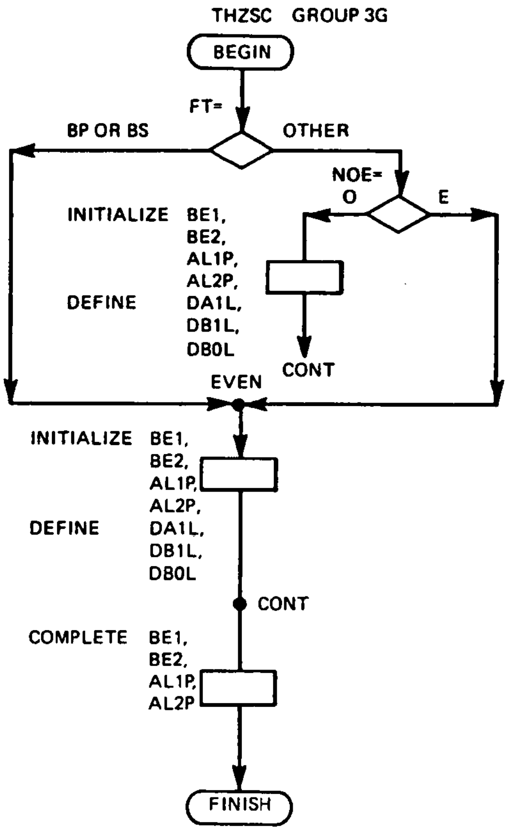

Figure 46. THZSC [Translation of H(Z) Subsection Coefficients] Logic Flow Chart.

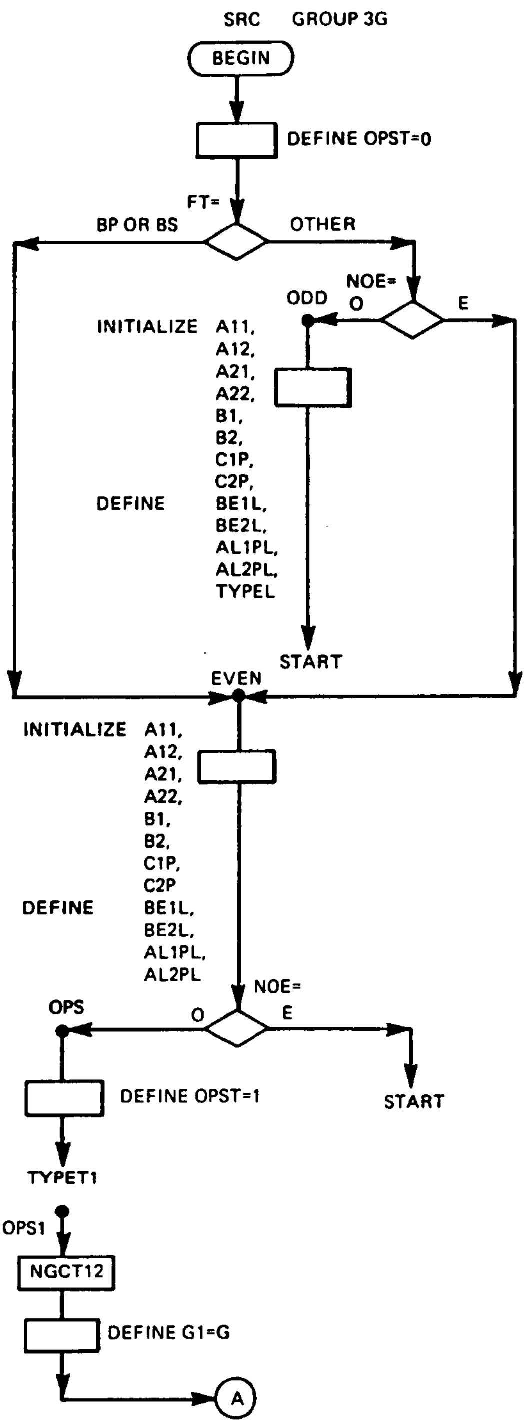

Figure 47. SRC (Subsection Realization Coefficients) Logic Flow Chart.

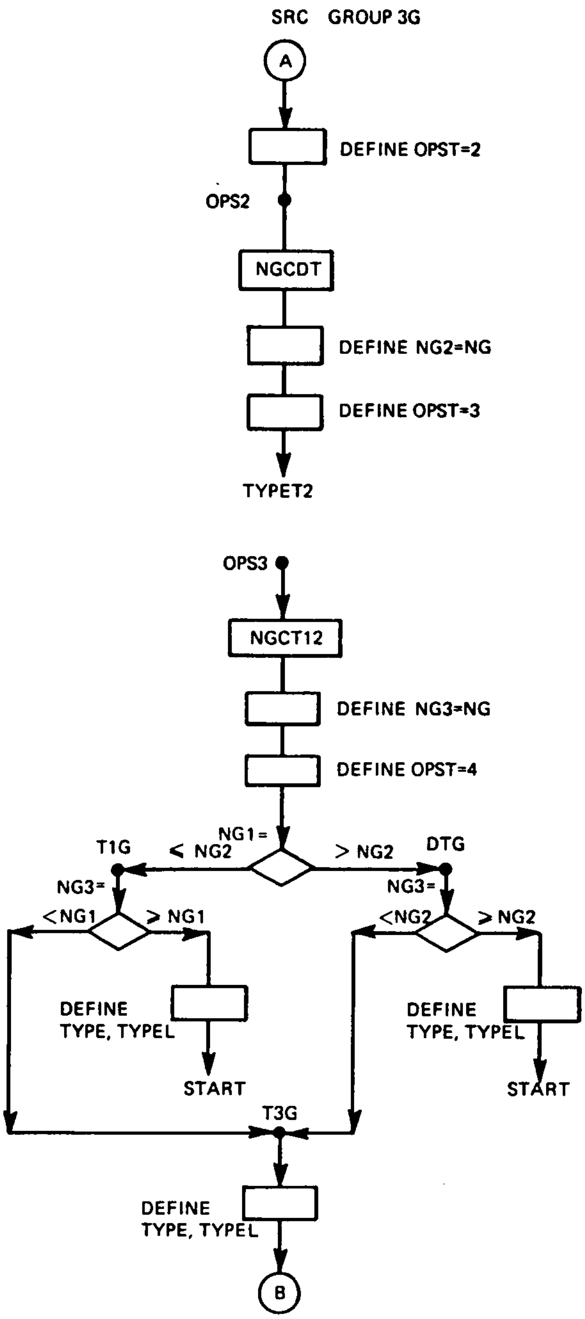

Figure 47. (Continued)

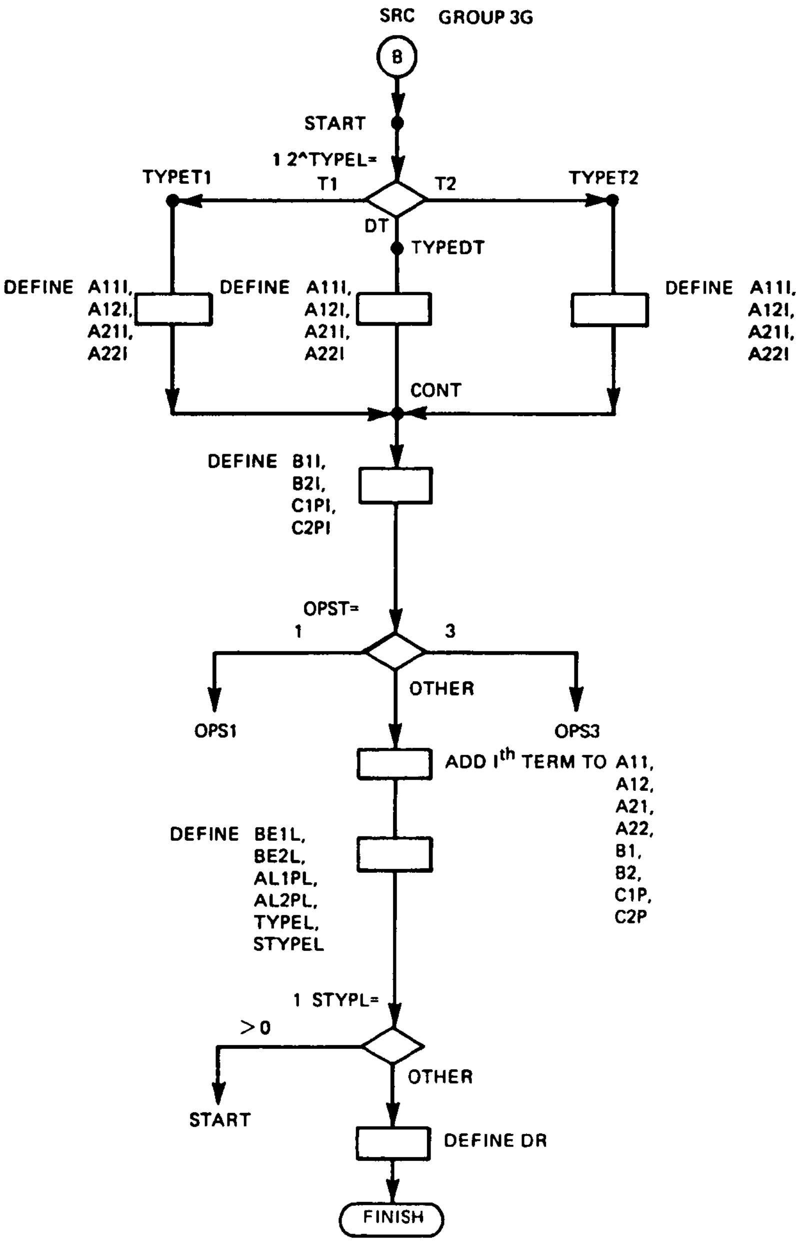

Figure 47. (Continued)

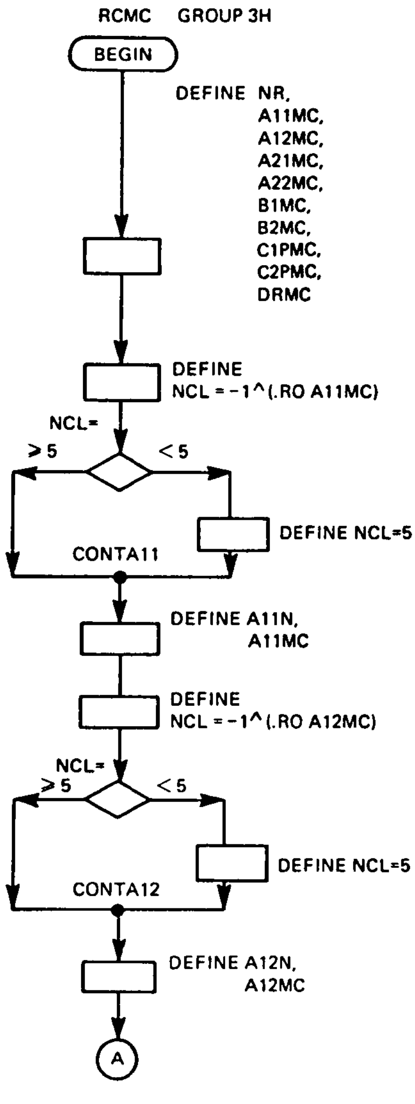

Figure 48. RCMC (Realization Coefficient Matrix Construction) Logic Flow Chart.

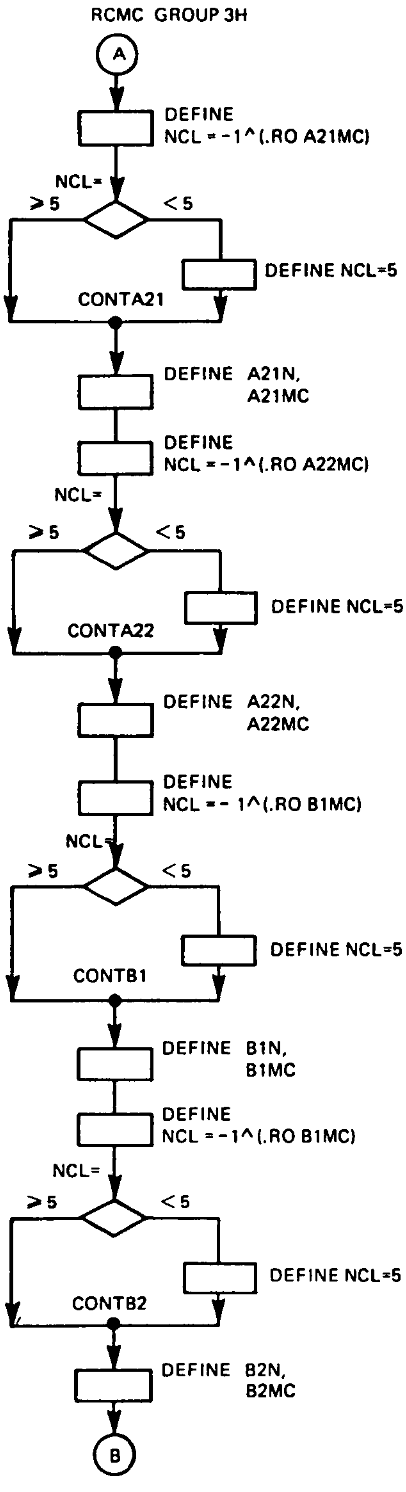

Figure 48. (Continued)

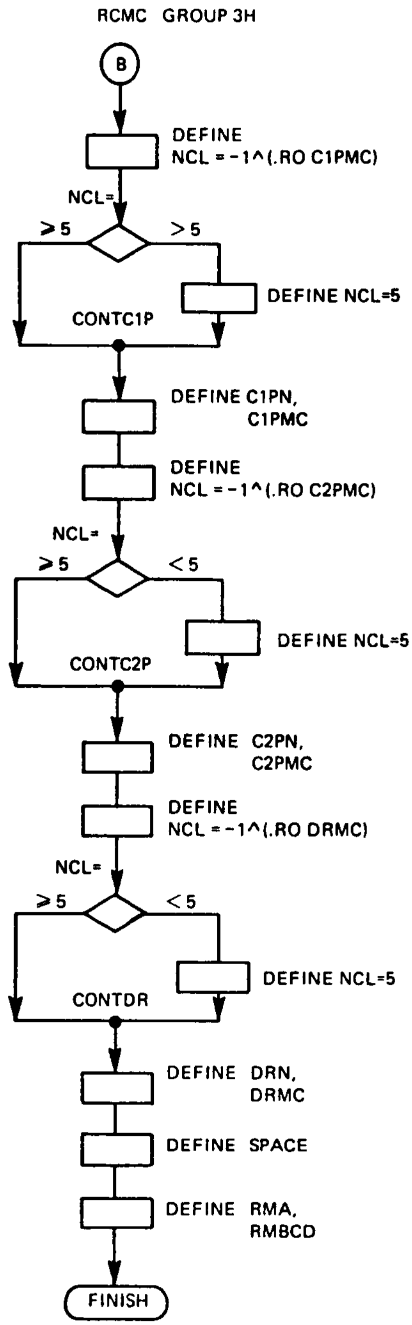

Figure 48. (Continued)

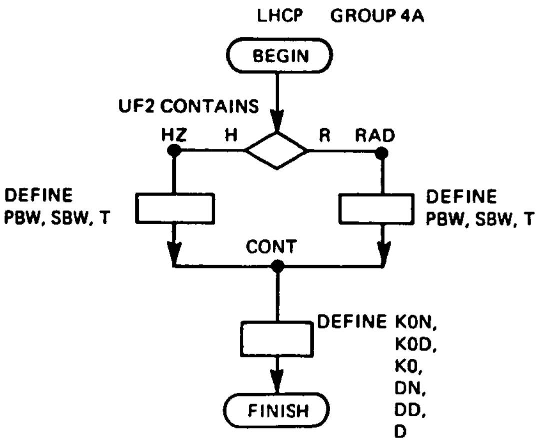

Figure 49. LHCP (Low-High-Pass Computation Parameters) Logic Flow Chart.

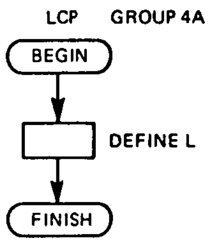

Figure 50. LCP (Low-Pass Computation Parameters) Logic Flow Chart.

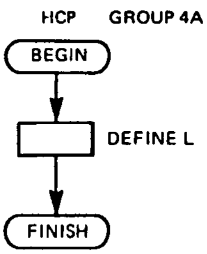

Figure 51. HCP (High-Pass Computation Parameters) Logic Flow Chart.

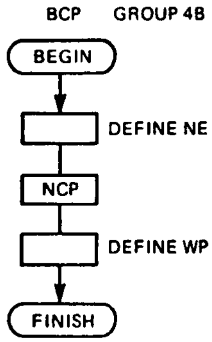

Figure 52. BCP (Butterworth Computation Parameters) Logic Flow Chart.

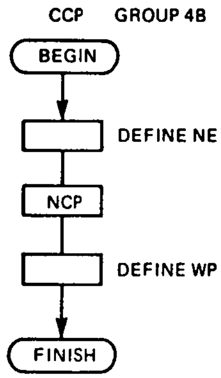

Figure 53. CCP (Chebyshev Computation Parameters) Logic Flow Chart.

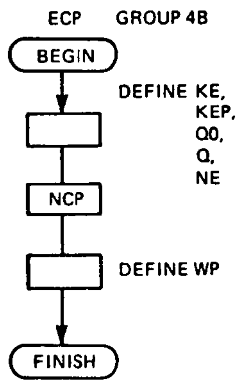

Figure 54. ECP (Elliptic Computation Parameters) Logic Flow Chart.

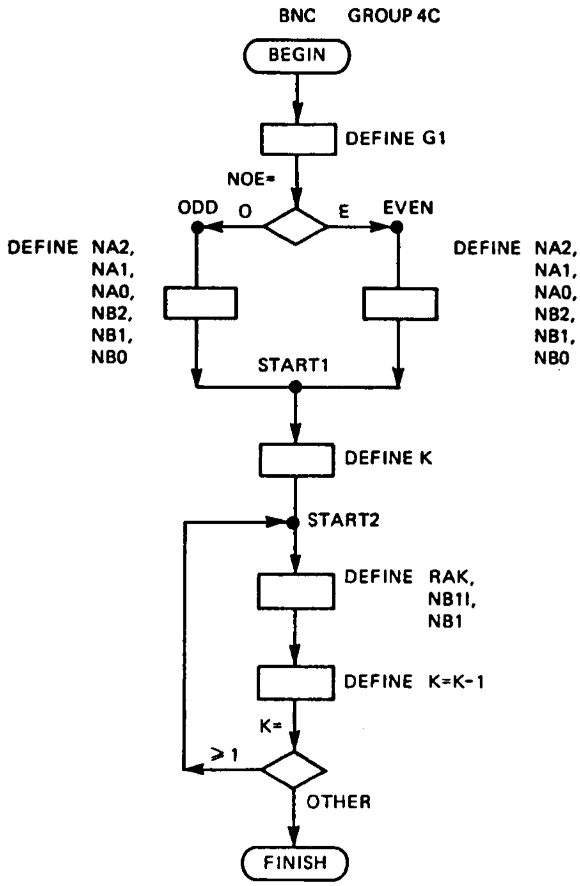

Figure 55. BNC (Butterworth Normalized Computations) Logic Flow Chart.

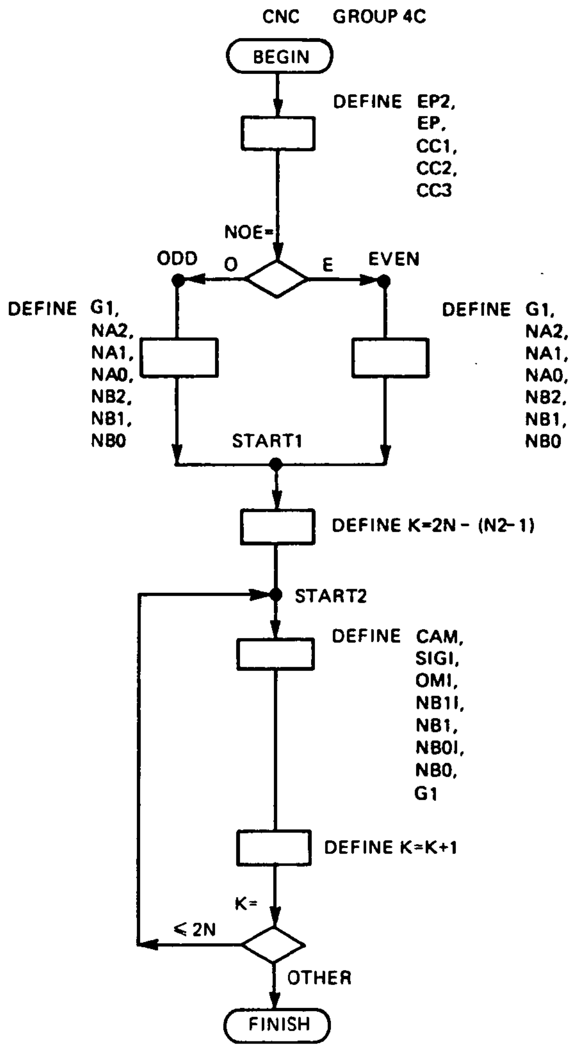

Figure 56. CNC (Chebyshev Normalized Computations) Logic Flow Chart.

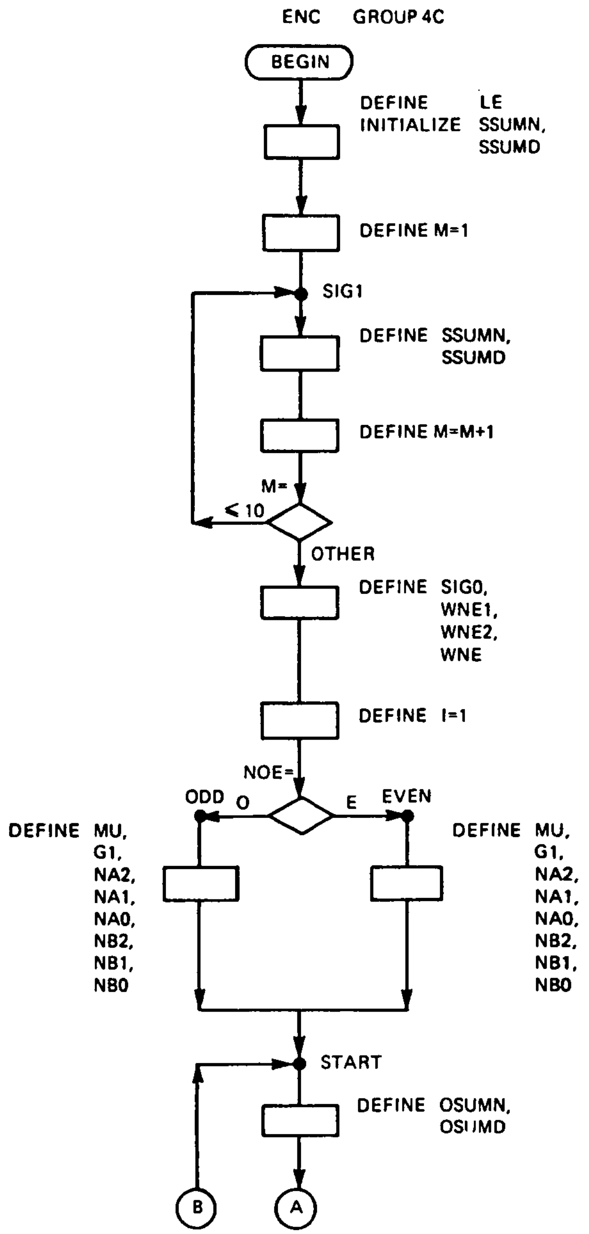

Figure 57. ENC (Elliptic Normalized Computations) Logic Flow Chart.

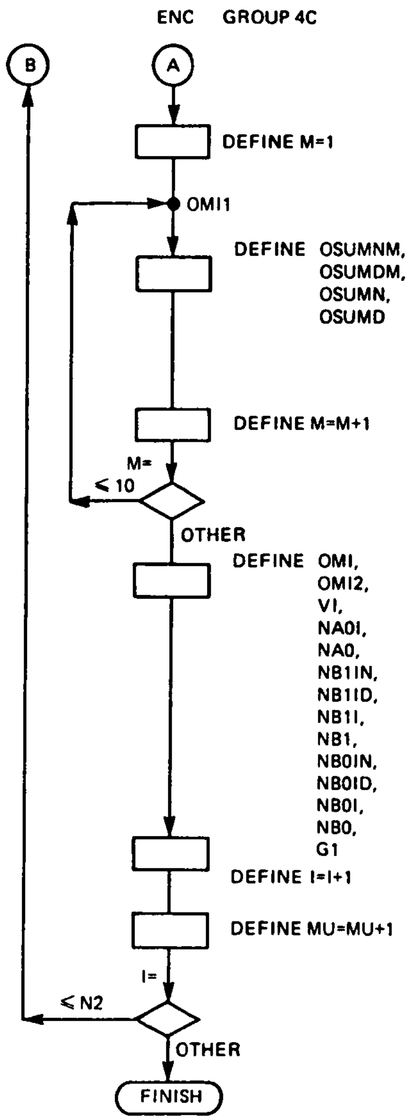

Figure 57. (Continued)

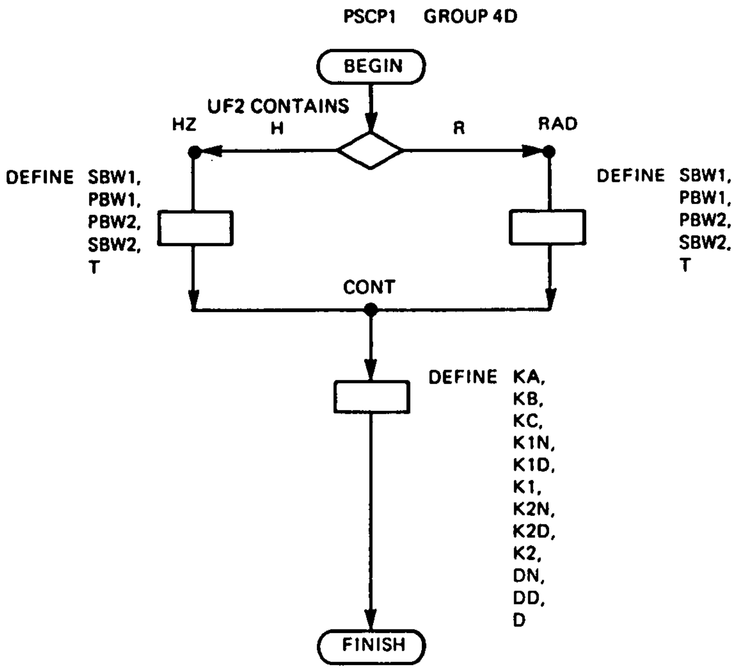

Figure 58. PSCP1 (Band-Pass-Stop Computation Parameters #1) Logic Flow Chart.

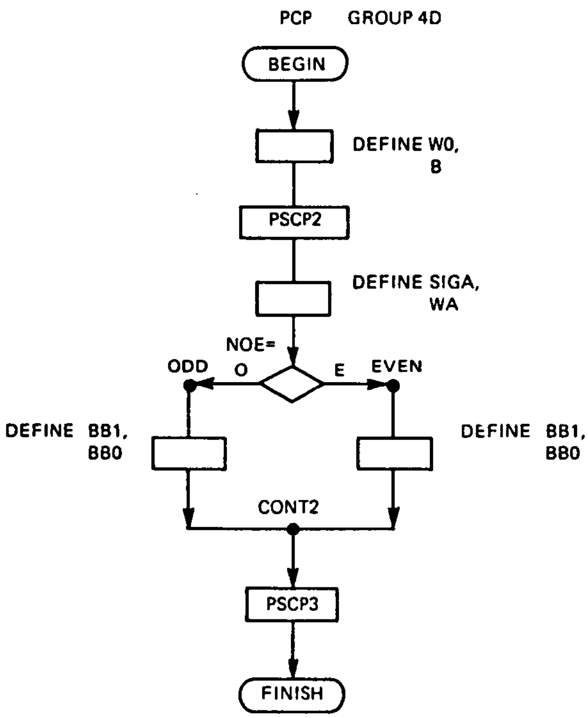

Figure 59. PCP (Band-Pass Computation Parameters) Logic Flow Chart.

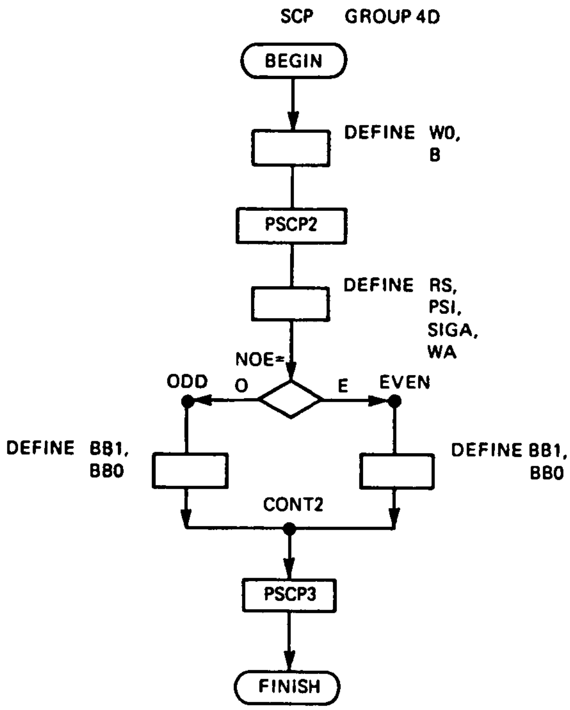

Figure 60. SCP (Band-Stop Computation Parameters) Logic Flow Chart.

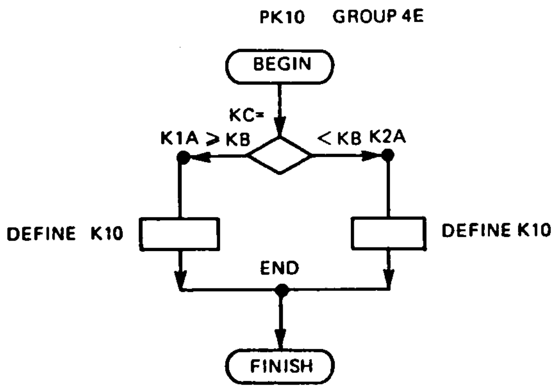

Figure 61. PK10 (Band-Pass K10) Logic Flow Chart.

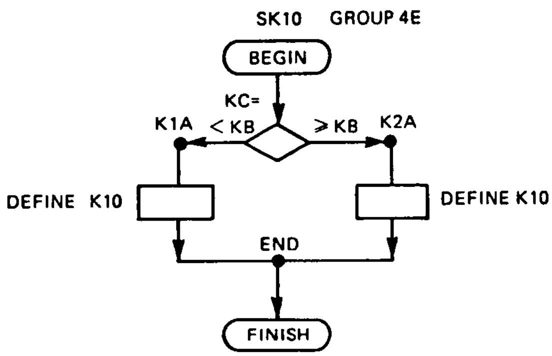

Figure 62. SK10 (Band-Stop K10) Logic Flow Chart.

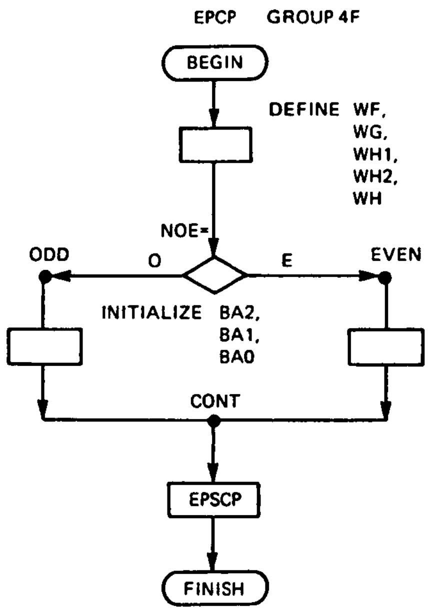

Figure 63. EPCP (Elliptic Band-Pass Computation Parameters) Logic Flow Chart.

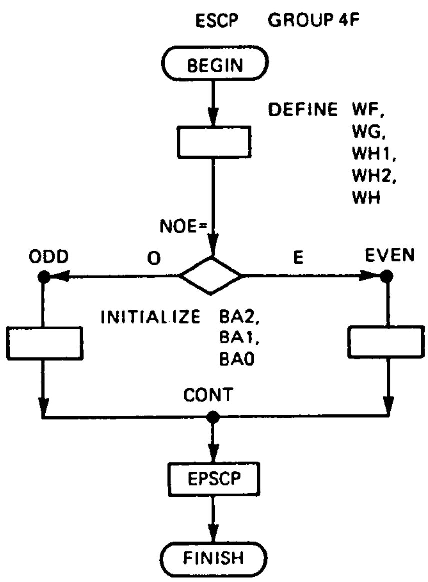

Figure 64. ESCP (Elliptic Band-Stop Computation Parameters) Logic Flow Chart.

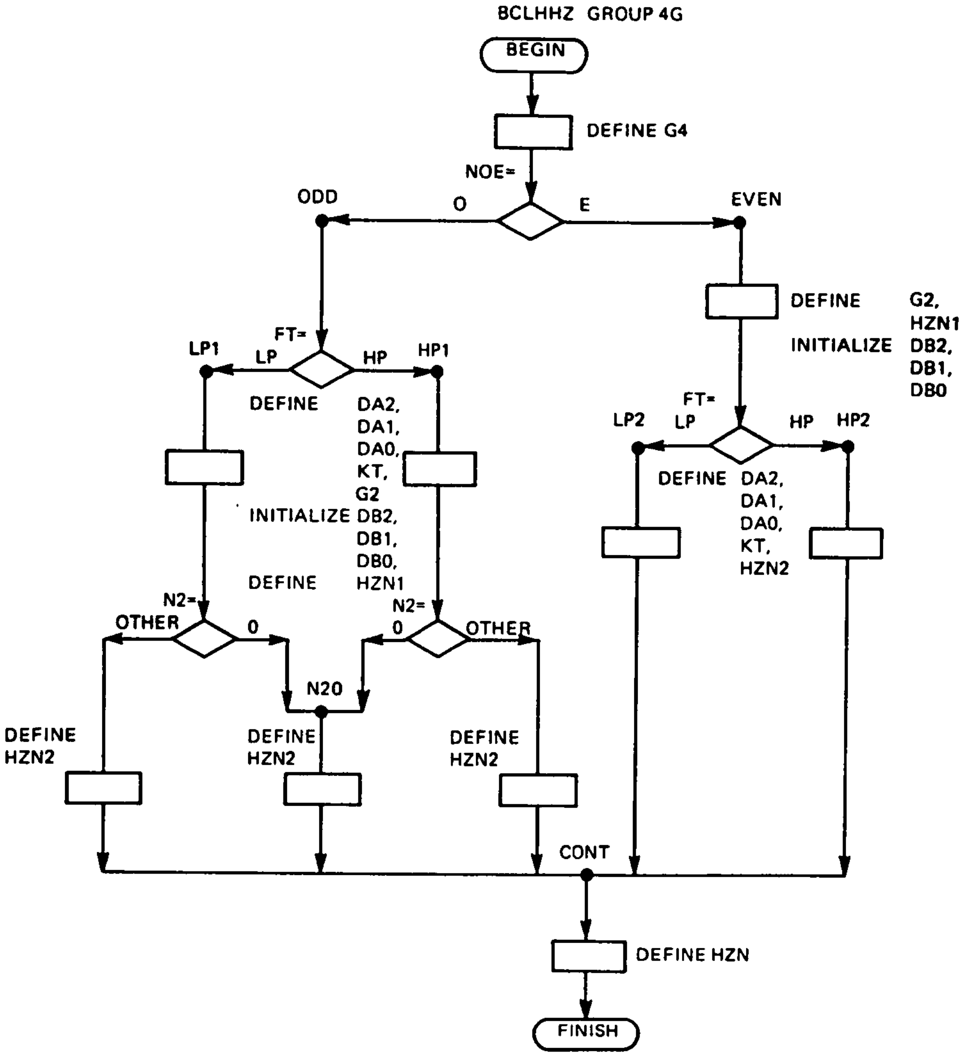

Figure 65. BCLHHZ [Butterworth Chebyshev Low-High-Pass H(Z)] Logic Flow Chart.

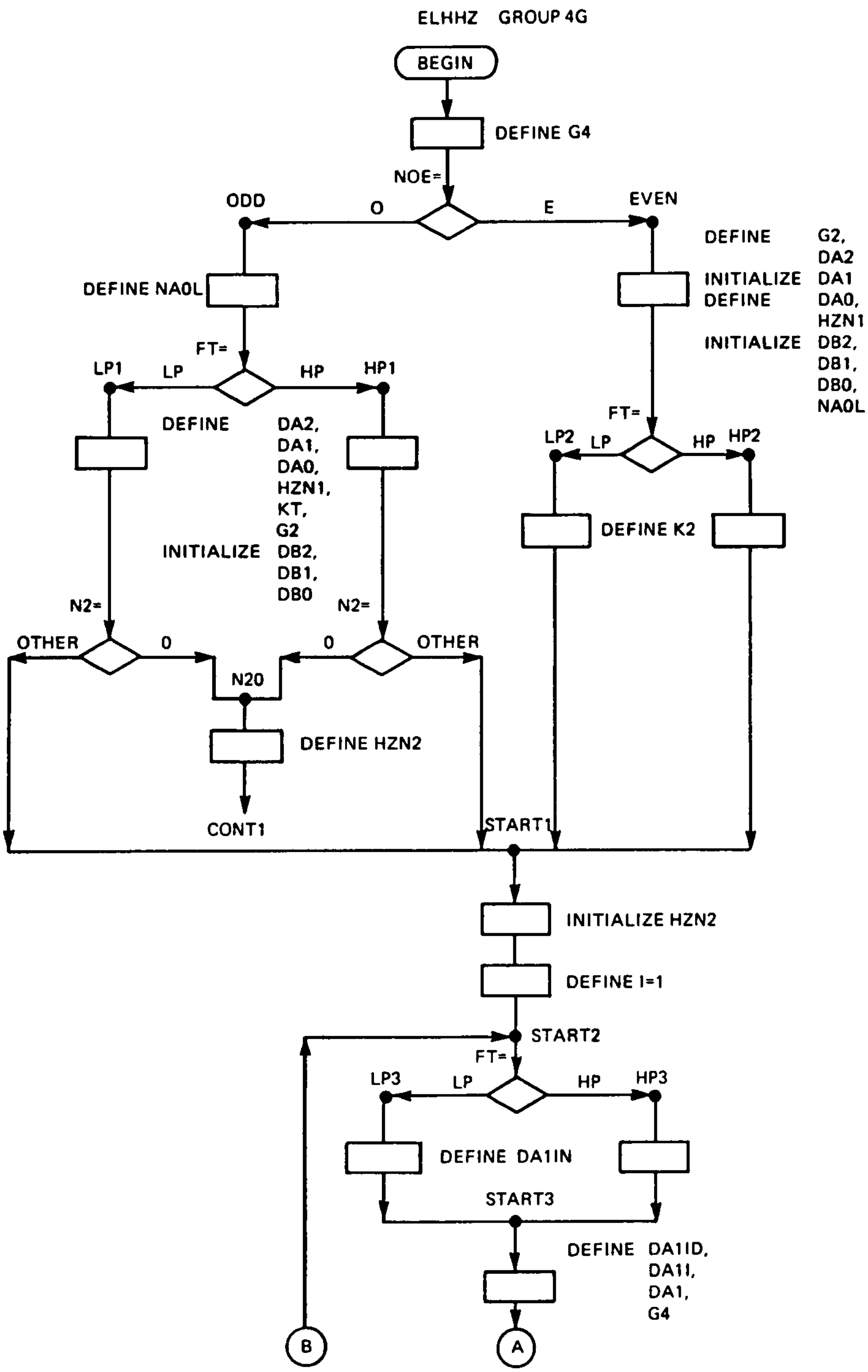

Figure 66. ELHHZ [Elliptic Low-High-Pass H(Z)] Logic Flow Chart.

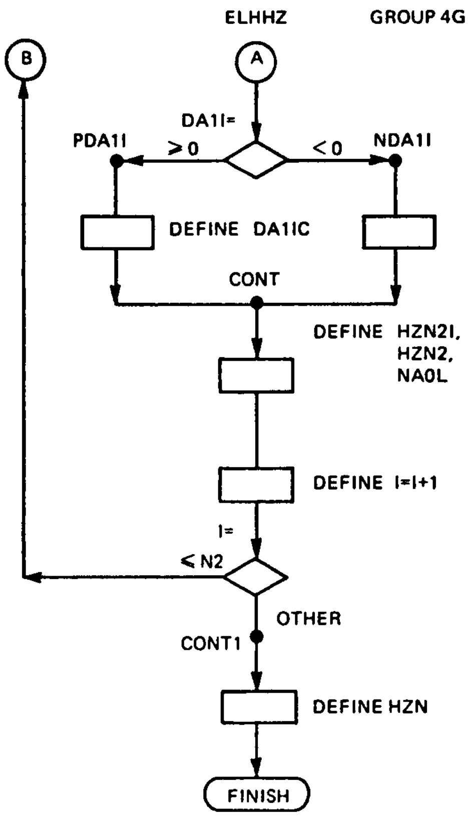

Figure 66. (Continued)

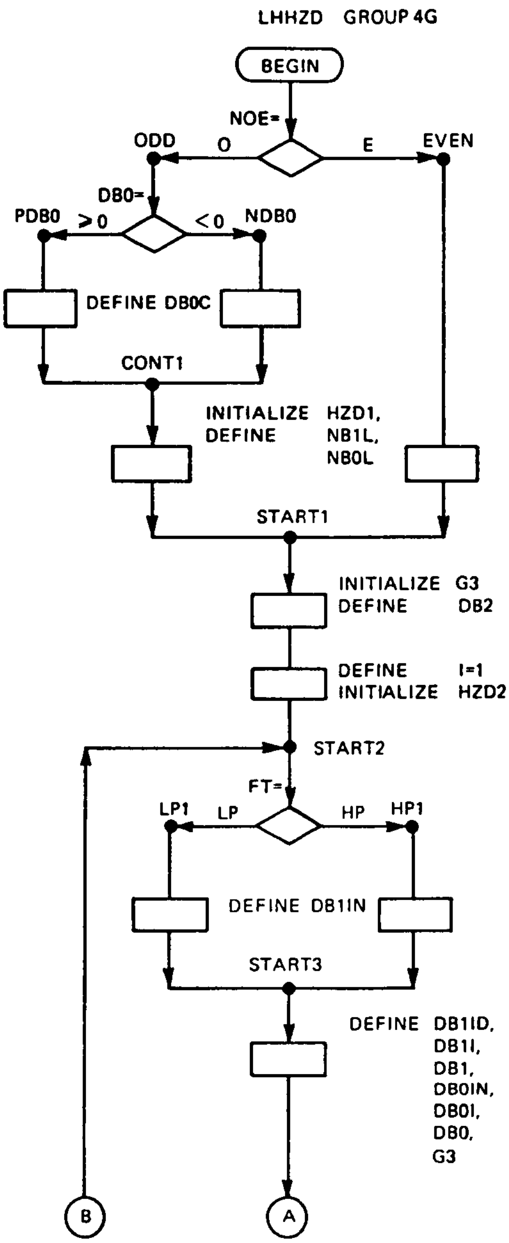

Figure 67. LHHZD [Low-High-Pass H(Z) Denominator] Logic Flow Chart.

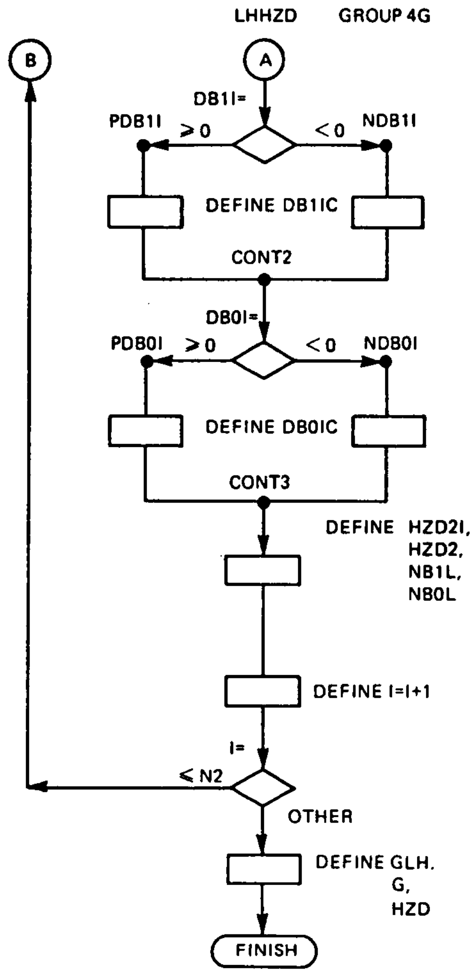

Figure 67. (Continued)

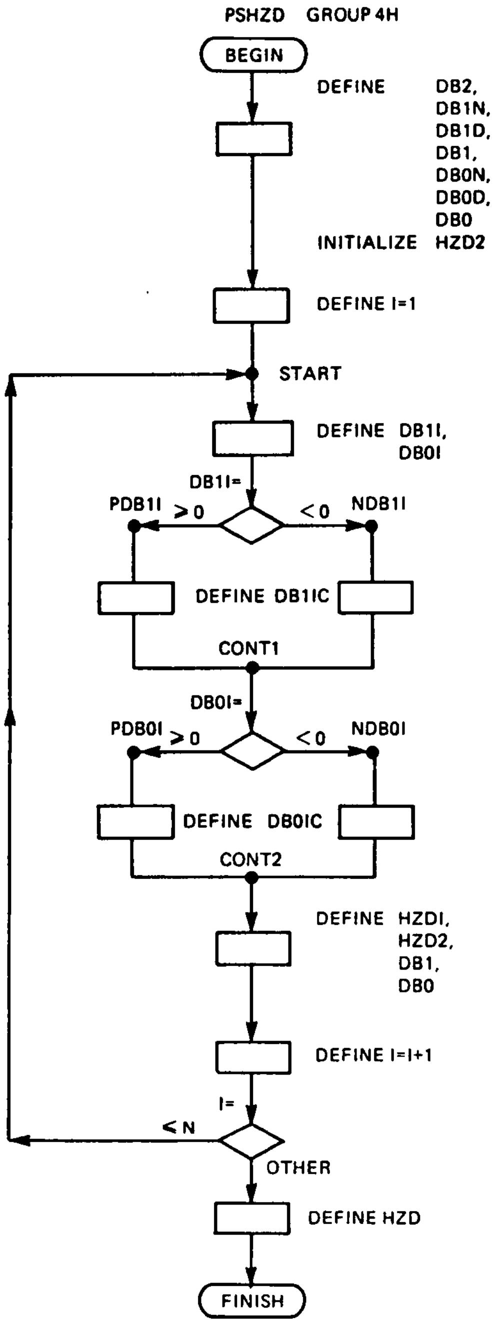

Figure 68. PSHZD [Band-Pass-Stop H(Z) Denominator] Logic Flow Chart.

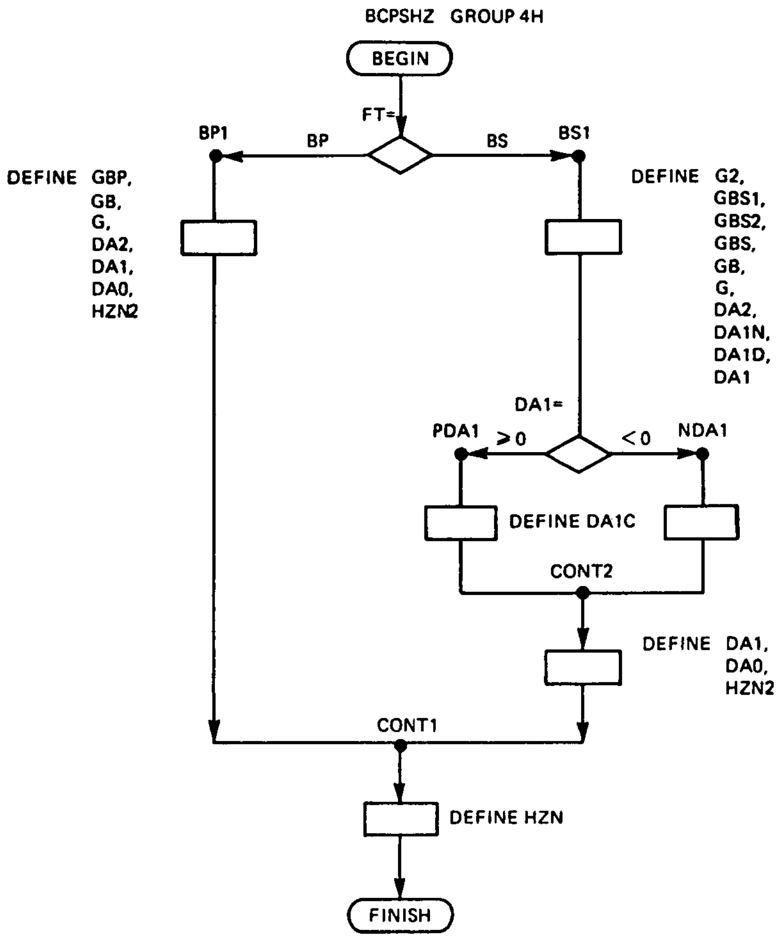

Figure 69. BCPSHZ [Butterworth Chebyshev Band-Pass-Stop H(Z)] Logic Flow Chart.

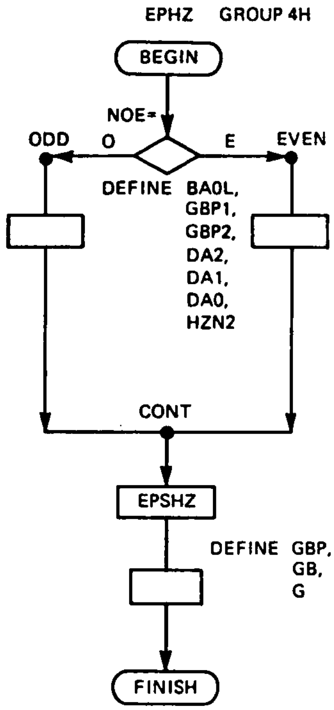

Figure 70. EPHZ [Elliptic Band-Pass H(Z)] Logic Flow Chart.

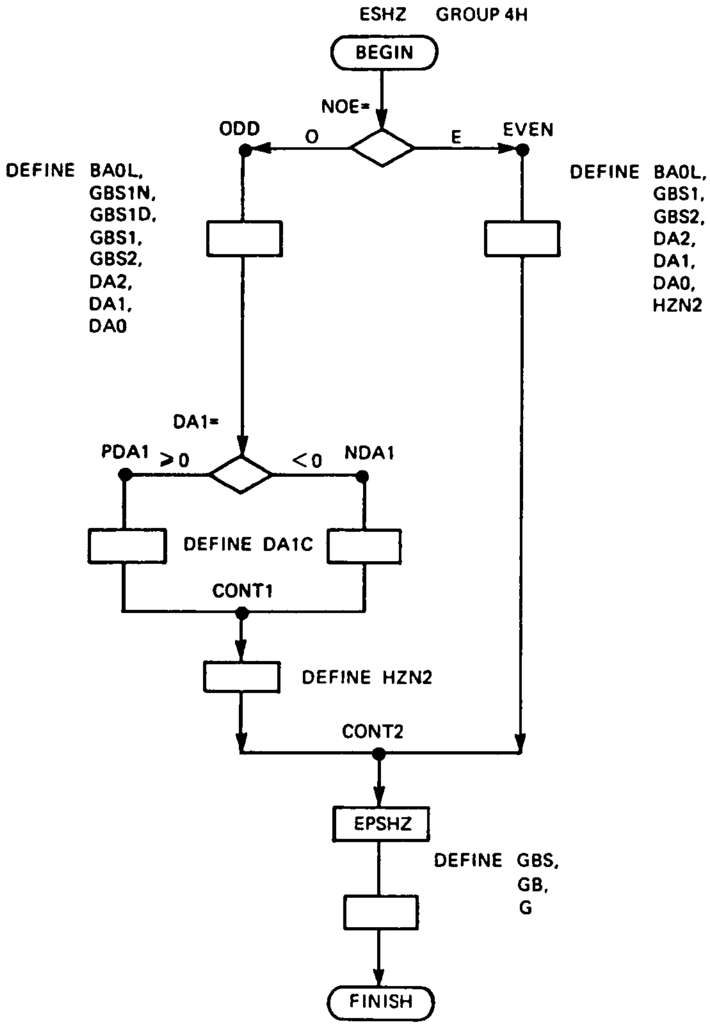

Figure 71. ESHZ [Elliptic Band-Stop H(Z)] Logic Flow Chart.

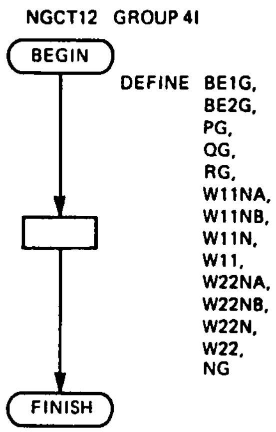

Figure 72. NGCT12 (Noise Gain Computation for Types I and II Realizations) Logic Flow Chart.

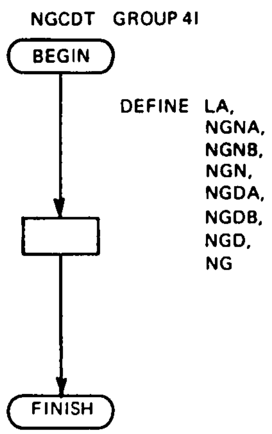

Figure 73. NGCDT (Noise Gain Computation for Direct Type Realization) Logic Flow Chart.

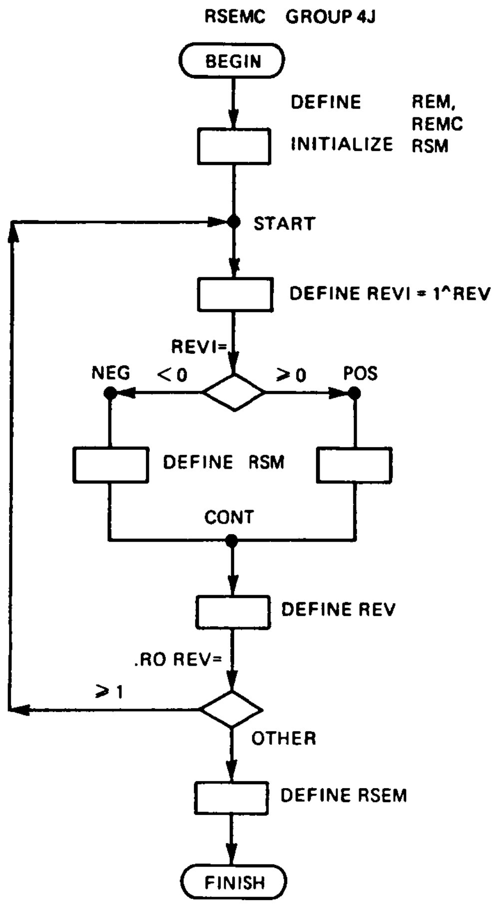

Figure 74. RSEMC (Realization Signed Element Matrix Construction) Logic Flow Chart.

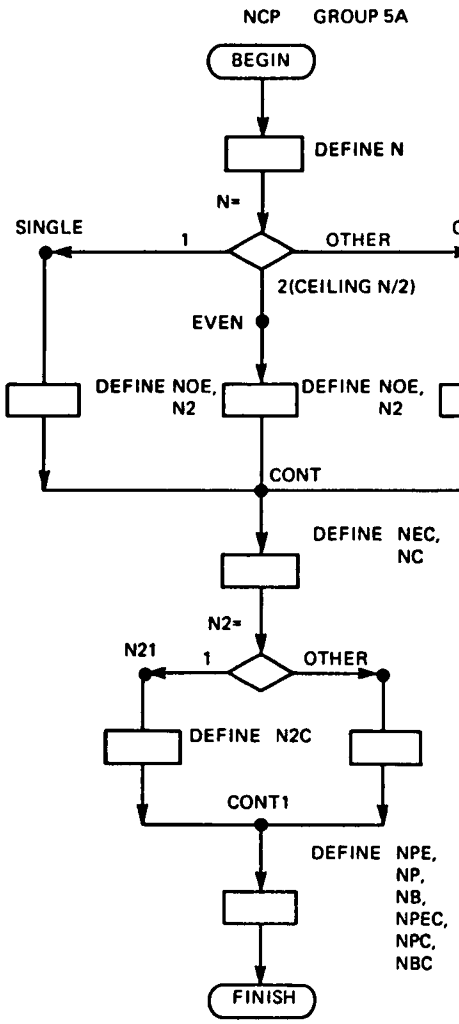

Figure 75. NCP (N Computation Parameters) Logic Flow Chart.

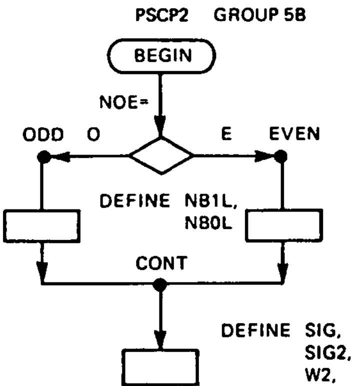

Figure 76. PSCP2 (Band-Pass-Stop Computation Parameters #2) Logic Flow Chart.

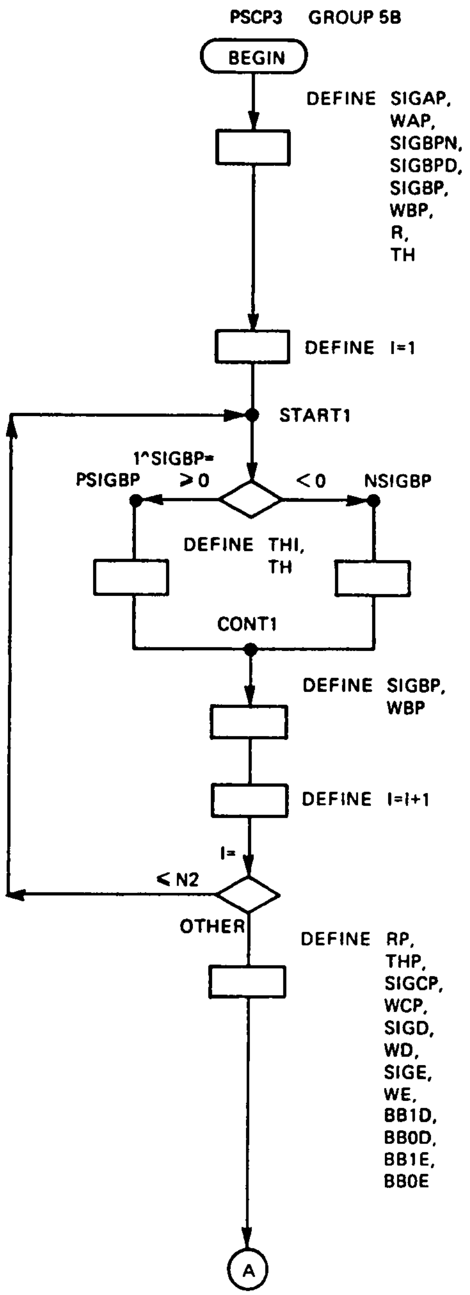

Figure 77. PSCP3 (Band-Pass-Stop Computation Parameters #3) Logic Flow Chart.

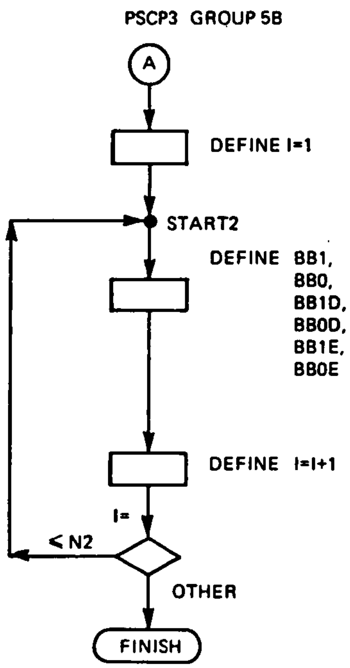

Figure 77. (Continued)

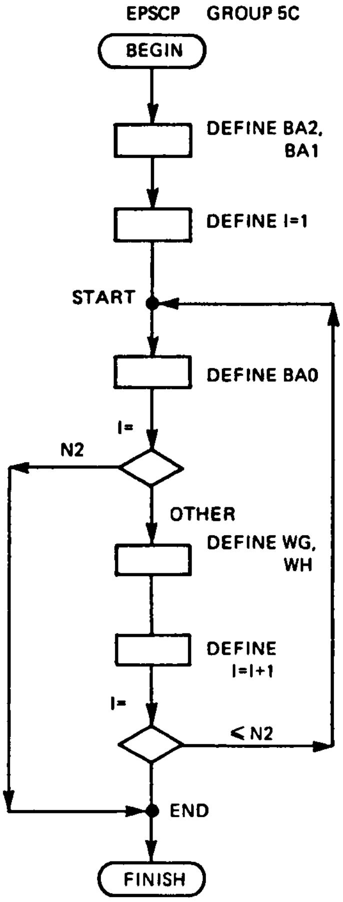

Figure 78. EPSCP (Elliptic Band-Pass-Stop Conputation Parameters) Logic Flow Chart.

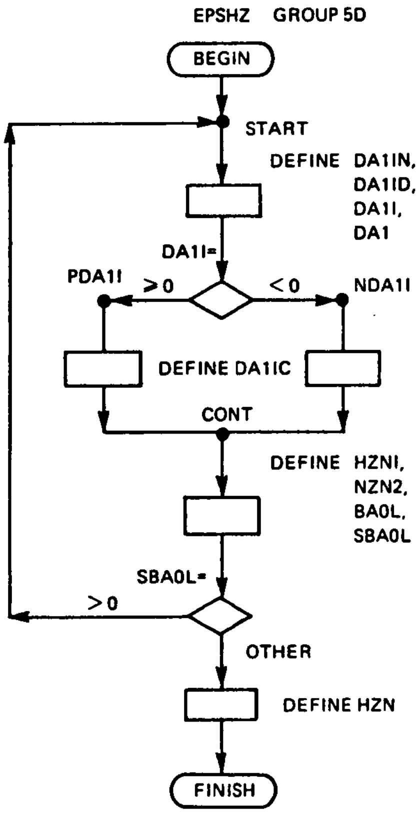

Figure 79. EPSHZ [Elliptic Band-Pass-Stop H(Z)] Logic Flow Chart.

APPENDIX F

PROGRAM GLOSSARY OF TERMS

The glossary of terms in Table 1 includes all of the functions, labels, and variables that appear in the program DFD in Appendix G. All three types of terms are listed together alphabetically. The definition of each term has four parts. First the term is identified as a function, label, or variable. A description of the term follows the type identification. The description normally indicates in capital letters how the acronym terms were derived. If the term is a mathematical variable, the description normally begins with the version of the variable that appears in the text. If the term appears in the equations in the text, the first equation where the term appears is listed after the description. The program function where the term first appears is listed after the equation number. In some cases, such as the label START, the function where the term first appears is of little interest since the term is used independently in each function. However, in most cases the first appearance of a term is where the term is defined for the first time and is a good place to see how the term is used.

Table 2. Program Glossary of Terms

Term	Type	Description	Eqn.	Function
A11	Variable	a_{11}	(193A)	SRC
A11I	Variable	A11 Ith iteration	(193A)	SRC
A11MC	Variable	A11 Matrix as Characters	--	RCMC
A11N	Variable	A11 Name	--	RCMC
A12	Variable	a_{12}	(193B)	SRC
A12I	Variable	A12 Ith iteration	(193B)	SRC
A12IA	Variable	A12 Ith iteration, part A	(193B)	SRC
A12IAD	Variable	A12 Ith iteration, part A Denominator	(193B)	SRC
A12IAN	Variable	A12 Ith iteration, part A Numerator	(193B)	SRC
A12IB	Variable	A12 Ith iteration, part B	(193B)	SRC
A12MC	Variable	A12 Matrix as Characters	--	RCMC
A12N	Variable	A12 Name	--	RCMC
A21	Variable	a_{21}	(193C)	SRC
A21I	Variable	A21 Ith iteration	(193C)	SRC
A21IN	Variable	A21 Ith iteration Numerator	(193C)	SRC
A21MC	Variable	A21 Matrix as Characters	--	RCMC
A21N	Variable	A21 Name	--	RCMC
A22	Variable	a_{22}	(193D)	SRC
A22I	Variable	A22 Ith iteration	(193D)	SRC
A22MC	Variable	A22 Matrix as Characters	--	RCMC
A22N	Variable	A22 Name	--	RCMC
AL1P	Variable	α_1' ALpha 1 Prime	(183)	THZSC

Table 2 Continued

Term	Type	Description	Eqn.	Function
AL1PL	Variable	AL1P, Local	--	SRC
AL2P	Variable	α_2' ALpha 2 Prime	(181)	THZSC
AL2PL	Variable	AL2P, Local	--	SRC
AR	Label	Attenuation Relationship subroutine	--	IPMC
ARNG	Label	Attenuation Relationship No Good subroutine	--	IPMC
AROK	Label	Attenuation Relationship OK subroutine	--	IPMC
B	Variable	B	(101C)	PCP
B1	Variable	b_1	(196A)	SRC
B1I	Variable	B1 Ith iteration	(196A)	SRC
B1MC	Variable	B1 Matrix as Characters	--	RCMC
B1N	Variable	B1 Name	--	RCMC
B2	Variable	b_2	(196B)	SRC
B2I	Variable	B2 Ith iteration	(196B)	SRC
B2IA	Variable	B2 Ith iteration, part A	(196B)	SRC
B2IB	Variable	B2 Ith iteration, part B	(196B)	SRC
B2IC	Variable	B2 Ith iteration, part C	(196B)	SRC
B2MC	Variable	B2 Matrix as Characters	--	RCMC
B2N	Variable	B2 Name	--	RCMC
B3	Label	Butterworth subroutine	--	IPMC
BAO	Variable	BA_0	(141)	EPCP
BAOL	Variable	BAO, Local	--	EPHZ

Table 2 Continued

Term	Type	Description	Eqn.	Function
BA1	Variable	BA_1	(141)	EPCP
BA2	Variable	BA_2	(141)	EPCP
BB0	Variable	BB_0	(111C)	PCP
BB0D	Variable	BB_{0d}	(111C)	PSCP3
BB0E	Variable	BB_{0e}	(111C)	PSCP3
BB1	Variable	BB_1	(111C)	PCP
BB1D	Variable	BB_{1d}	(111C)	PSCP3
BB1E	Variable	BB_{1e}	(111C)	PSCP3
BBP	Label	Butterworth Band-Pass subroutine	--	TCC
BBS	Label	Butterworth Band-Stop subroutine	--	TCC
BC1	Label	Butterworth Chebyshev subroutine	--	LHHZ
BCLHHZ	Function	Butterworth Chebyshev Low-High-pass H(Z)	--	LHHZ
BCP	Function	Butterworth Computation Parameters	--	BLCC
BCPSHZ	Function	Butterworth Chebyshev band-Pass-Stop H(Z)	--	PSHZ
BE1	Variable	β_1 BEta 1	(183)	THZSC
BE1G	Variable	β_1 BEta 1, Gain	(205A)	NGCT12
BE1L	Variable	BE1, Local	--	SRC
BE2	Variable	β_2 BEta 2	(182C)	THZSC
BE2G	Variable	β_2 BEta 2, Gain	(205G)	NGCT12
BE2L	Variable	BE2, Local	--	SRC
BHCC	Function	Butterworth High-pass Computation Control	--	TCC

Table 2 Continued

Term	Type	Description	Eqn.	Function
BHP	Label	Butterworth High-Pass subroutine	--	TCC
BLCC	Function	Butterworth Low-pass Computation Control	--	TCC
BLP	Label	Butterworth Low-Pass subroutine	--	TCC
BNC	Function	Butterworth Normalized Computations	--	BLCC
BP1	Label	Band-Pass subroutine	--	BCPSHZ
BP2	Label	Band-Pass subroutine	--	IPMC
BPCC	Function	Butterworth band-Pass Computation Control	--	TCC
BS1	Label	Band-Stop subroutine	--	BCPSHZ
BS2	Label	Band-Stop subroutine	--	IPMC
BSCC	Function	Butterworth band-Stop Computation Control	--	TCC
C1P	Variable	c_1'	(199A)	SRC
C1PI	Variable	C1P Ith iteration	(199A)	SRC
C1PID	Variable	C1P Ith iteration Denominator	(199A)	SRC
C1PIN	Variable	C1P Ith iteration Numerator	(199A)	SRC
C1PMC	Variable	C1P Matrix as Characters	--	RCMC
C1PN	Variable	C1P Name	--	RCMC
C2P	Variable	c_2'	(199B)	SRC
C2PI	Variable	C2P Ith iteration	(199B)	SRC
C2PMC	Variable	C2P Matrix as Characters	--	RCMC
C2PN	Variable	C2P Name	--	RCMC
C3	Label	Chebyshev subroutine	--	IPMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
CAM	Variable	CA_m Computation Angle Mth iteration	(31)	CNC
CBP	Label	Chebyshev Band-Pass subroutine	--	TCC
CBS	Label	Chebyshev Band-Stop subroutine	--	TCC
CC1	Variable	Chebyshev Constant #1	(28)	CNC
CC2	Variable	Chebyshev Constant #2	(29)	CNC
CC3	Variable	Chebyshev Constant #3	(30)	CNC
CCP	Function	Chebyshev Computation Parameters	--	CLCC
CHCC	Function	Chebyshev High-pass Computation Control	--	TCC
CHP	Label	Chebyshev High-pass subroutine	--	TCC
CLCC	Function	Chebyshev Low-pass Computation Control	--	TCC
CLP	Label	Chebyshev Low-Pass subroutine	--	TCC
CNC	Function	Chebyshev Normalized Computations	--	CLCC
CONT	Label	CONTinue the function	--	LHHZ
CONT1	Label	CONTinue the function	--	SPAT
CONT2	Label	CONTinue the function	--	IPMC
CONT3	Label	CONTinue the function	--	IPMC
CONT4	Label	CONTinue the function	--	IPMC
CONTA11	Label	CONTinue A11 subroutine	--	RCMC
CONTA12	Label	CONTinue A12 subroutine	--	RCMC
CONTA21	Label	CONTinue A21 subroutine	--	RCMC
CONTA22	Label	CONTinue A22 subroutine	--	RCMC
CONTB1	Label	CONTinue B1 subroutine	--	RCMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
CONTB2	Label	CONTinue B2 subroutine	--	RCMC
CONTC1P	Label	CONTinue C1P subroutine	--	RCMC
CONTC2P	Label	CONTinue C2P subroutine	--	RCMC
CONTDR	Label	CONTinue DR subroutine	--	RCMC
CPCC	Function	Chebyshev band-Pass Computation Control	--	TCC
CSCC	Function	Chebyshev band-Stop Computation Control	--	TCC
D	Variable	D	(62)	LHCP
DA0	Variable	DA_0	(79A)	BCLHHZ
DA1	Variable	DA_1	(79A)	BCLHHZ
DA1C	Variable	DA1, Character version	--	BCPSHZ
DA1D	Variable	DA1 Denominator	(123G)	BCPSHZ
DA1I	Variable	DA1 Ith iteration	(89A)	ELHHZ
DA1IC	Variable	DA1I, Character version	--	ELHHZ
DA1ID	Variable	DA1I Denominator	(89A)	ELHHZ
DA1IN	Variable	DA1I Numerator	(89A)	ELHHZ
DA1L	Variable	DA1, Local	--	THZSC
DA1N	Variable	DA1 Numerator	(123G)	BCPSHZ
DA2	Variable	DA_2	(79A)	BCLHHZ
DB	Variable	Character vector to add ´DB´ to IPM	--	IPMC
DB0	Variable	DB_0	(79A)	BCLHHZ
DB0C	Variable	DB0, Character version	--	LHHZD

Table 2 Continued

Term	Type	Description	Eqn.	Function
DBOD	Variable	DBO Denominator	--	PSHZD
DBOI	Variable	DB_{0i}	(79A)	LHHZD
DBOIC	Variable	DBOI, Character version	--	LHHZD
DBOIN	Variable	DBOI Numerator	(79A)	LHHZD
DBOL	Variable	DBO, Local	--	SPAT
DBON	Variable	DBO Numerator	(117A)	PSHZD
DB1	Variable	DB_1	(79A)	BCLHHZ
DB1D	Variable	DB1 Denominator	--	PSHZD
DB1I	Variable	DB_{1i}	(79A)	LHHZD
DB1IC	Variable	DB1I, Character version	--	LHHZD
DB1ID	Variable	DB1I Denominator	(79A)	LHHZD
DB1IN	Variable	DB1I Numerator	(79A)	LHHZD
DB1L	Variable	DB1, Local	--	SPAT
DB1N	Variable	DB1 Numerator	(117A)	PSHZD
DB2	Variable	DB_2	(79A)	BCLHHZ
DD	Variable	D Denominator	(62)	LHCP
DFD	Function	Digital Filter Design	--	--
DN	Variable	D Numerator	(62)	LHCP
DR	Variable	D Realization	--	SRC
DRMC	Variable	DR Matrix as Characters	--	RCMC
DRN	Variable	DR Name	--	RCMC
DT	Variable	Design Type	--	IPMC
DT1	Variable	Design Type, expanded expression	--	IPMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
DTCH	Label	Design Type CHange subroutine	--	IPMC
DTG	Label	Direct Type realization Gain subroutine	--	SRC
E3	Label	Elliptic subroutine	--	IPMC
EBP	Label	Elliptic Band-Pass subroutine	--	TCC
EBP1	Label	Elliptic Band-Pass subroutine	--	PSHZ
EBS	Label	Elliptic Band-Stop subroutine	--	TCC
EBS1	Label	Elliptic Band-Stop subroutine	--	PSHZ
ECP	Function	Elliptic Computation Parameters	--	ELCC
EHCC	Function	Elliptic High-Pass Computation Control	--	TCC
EHP	Label	Elliptic High-Pass subroutine	--	TCC
ELCC	Function	Elliptic Low-Pass Computation Control	--	TCC
ELHHZ	Function	Elliptic Low-High-Pass H(Z)	--	LHHZ
ELP	Label	Elliptic Low-Pass subroutine	--	TCC
ENC	Function	Elliptic Normalized Computations	--	ELCC
END	Label	ENDs a function	--	DFD
EP	Variable	ϵ Epsilon	(26)	CNC
EP2	Variable	ϵ^2 EPsilon squared	(26)	CND
EPCC	Function	Elliptic band-Pass Computation Control	--	TCC
EPCP	Function	Elliptic band-Pass Computation Parameters	--	EPCC
EPHZ	Function	Elliptic band-Pass H(Z)	--	PSHZ

Table 2 Continued

Term	Type	Description	Eqn.	Function
EPSCP	Function	Elliptic band-Pass-Stop Computation Parameters	--	EPCP
EPSHZ	Function	Elliptic band-Pass-Stop Computation Control	--	EPHZ
ESCC	Function	Elliptic band-Stop Computation Control	--	TCC
ESCP	Function	Elliptic band-Stop Computation Parameters	--	ESCC
ESHZ	Function	Elliptic band-Stop H(Z)	--	PSHZ
EVEN	Label	EVEN-order subroutine	--	SPAT
FINAL	Label	FINAL entry in display	--	IPMC
FR	Variable	Frequency Response	--	IPML
FRNG	Label	Frequency Response No Good subroutine	--	IPMC
FT	Variable	Filter Type	--	IPMC
FT1	Variable	Filter Type, expanded expression	--	IIP
FTCH	Label	Filter Type Change subroutine	--	IPMC
G	Variable	G	(169)	LHHZD
G1	Variable	G_1	(11)	BNC
G2	Variable	G_2	(77)	BCLHHZ
G3	Variable	G_3	(78)	LHHZD
G4	Variable	G_4	(95)	ELHHZ
GB	Variable	G_B	(126)	BCPSHZ
GBP	Variable	G_{BP}	(116)	BCPSHZ
GBP1	Variable	G_{BP1}	(146)	EPHZ

Table 2 Continued

Term	Type	Description	Eqn.	Function
GBP2	Variable	G_{BP2}	(146)	EPHZ
GBS	Variable	G_{BS}	(124B)	BCPSHZ
GBS1	Variable	GBS part 1	(124C)	BCPSHZ
GBS1D	Variable	GBS1 Denominator	(164)	ESHZ
GBS1N	Variable	GBS1 Numerator	(164)	ESHZ
GBS2	Variable	GBS part 2	(124D)	BCPSHZ
GC	Variable	G as Characters	--	HZMC
HCP	Function	High-pass Computation Parameters	--	BHCC
HELP	Label	HELP subroutine	--	IPMC
HFC	Function	Help File Construction	--	IPMC
HP1	Label	High-Pass subroutine	--	BCLHHZ
HP2	Label	High-Pass subroutine	--	IPMC
HP3	Label	High-Pass subroutine	--	ELHHZ
HZ	Label	HertZ subroutine	--	LHCP
HZD	Variable	H(Z) Denominator	--	LHHZD
HZD1	Variable	H(Z) Denominator 1st-order	--	LHHZD
HZD2	Variable	H(Z) Denominator 2nd-order	--	LHHZD
HZD2I	Variable	H(Z) Denominator 2nd-order Ith iteration	--	LHHZD
HZDI	Variable	H(Z) Denominator Ith iteration	--	PSHZD
HZM	Variable	H(Z) Matrix	--	HZMC
HZMC	Function	H(Z) Matrix Construction	--	DFD
HZMT	Variable	H(Z) Matrix Title	--	HZMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
HZN	Variable	H(Z) Numerator	--	BCLHHZ
HZN1	Variable	H(Z) Numerator 1st-order	(75B)	BCLHHZ
HZN2	Variable	H(Z) Numerator 2nd-order	(75B)	BCLHHZ
HZN2I	Variable	HZN2 Ith iteration	(89A)	ELHHZ
HZNI	Variable	HZN Ith iteration	--	EPSHZ
I	Variable	accumulation counter in loop	--	ENC
IIP	Function	Initialize Input Parameters	--	IPMC
INITIAL	Label	INITIALize variables subroutine	--	IPMC
IPM	Variable	Input Parameter Matrix	--	IPMC
IPM1	Variable	Input Parameter Matrix, Line ID #1	--	IPMC
IPM2	Variable	Input Parameter Matrix, Line ID #2	--	IPML
IPM3	Variable	Input Parameter Matrix, Line ID #3	--	IPML
IPM4	Variable	Input Parameter Matrix, Line ID #4	--	IPML
IPM5	Variable	Input Parameter Matrix, Line ID #5	--	IPML
IPM6	Variable	Input Parameter Matrix, Line ID #6	--	IPML
IPM7	Variable	Input Parameter Matrix, Line ID #7	--	IPMP
IPM8	Variable	Input Parameter Matrix, Line ID #8	--	IPMP
IPM9	Variable	Input Parameter Matrix, Line ID #9	--	IPMC
IPMC	Function	Input Parameter Matrix Construction	--	DFD
IPMH	Function	Input Parameter Matrix High-pass	--	IPMC
IPML	Function	Input Parameter Matrix Low-pass	--	IPMC
IPMP	Function	Input Parameter Matrix band-Pass	--	IPMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
IPMS	Function	Input Parameter Matrix band-Stop	--	IPMC
IPMT	Variable	Input Parameter Matrix Title	--	IPMC
K	Variable	accumulation counter in loop	--	BNC
K0	Variable	K_0	(60A)	LHCP
K0D	Variable	K0 Denominator	(60A)	LHCP
K0N	Variable	K0 Numerator	(60A)	LHCP
K1	Variable	K_1	(60E)	PSCP1
K10	Variable	K from [1, pp. 196-207]	(61)	BLCC
K1A	Label	K1 subroutine	(61B	PK10
K1D	Variable	K1 Denominator	(60E)	PSCP1
K1N	Variable	K1 Numerator	(60E)	PSCP1
K2	Variable	K_2	(60F)	PSCP1
K2A	Label	K2 subroutine	(61B)	PK10
K2D	Variable	K2 Denominator	(60F)	PSCP1
K2N	Variable	K2 Numerator	(60F)	PSCP1
KA	Variable	K_A	(60B)	PSCP1
KB	Variable	K_B	(60C)	PSCP1
KC	Variable	K_C	(60D)	PSCP1
KE	Variable	k Elliptic	(38)	ECP
KEP	Variable	k′, k Elliptic Prime	(39)	ECP
KT	Variable	K_T	(72)	BCLHHZ
L	Variable	λ Lambda	(70)	LCP
LCP	Function	Low-pass Computation Parameters	--	BLCC

Table 2 Continued

Term	Type	Description	Eqn.	Function
LE	Variable	Λ Lambda Elliptic	(45)	ENC
LED	Variable	LE Denominator	(45)	ENC
LEN	Variable	LE Numerator	(45)	ENC
LG	Variable	λ Lambda Gain	(206A)	NGCDT
LH1	Label	Low-High-pass subroutine	--	SMC
LHCP	Function	Low-High-pass Computation Parameters	--	BLCC
LHHZ	Function	Low-High-pass H(Z)	--	TCC
LHHZD	Function	Low-High-pass H(Z) Denominator	--	LHHZ
LP1	Label	Low-Pass subroutine	--	BCLHHZ
LP2	Label	Low-Pass subroutine	--	IPMC
LP3	Label	Low-Pass subroutine	--	ELHHZ
M	Variable	accumulation counter in loop	--	ENC
MU	Variable	μ MU	(49)	ENC
N	Variable	N, order of normalized low-pass filter	--	NCP
N2	Variable	Number of 2nd-order subsections	--	NCP
N20	Label	N2 = 0 subroutine	--	BCLHHZ
N2C	Variable	N2, Character version	--	NCP
NA0	Variable	NA_0	(16A)	BNC
NA0I	Variable	NA0 Ith subsection	(51)	ENC
NA0L	Variable	NA0, Local	--	ELHHZ
NA1	Variable	NA_1	(16A)	BNC
NA2	Variable	NA_2	(16A)	BNC

Table 2 Continued

Term	Type	Description	Eqn.	Function
NB	Variable	N Band-pass-stop	(101G)	NCP
NB0	Variable	NB_0	(16A)	BNC
NB0I	Variable	NB0 Ith subsection	(37)	CNC
NB0ID	Variable	NB0I Denominator	(53)	ENC
NB0IN	Variable	NB0I Numerator	(53)	ENC
NB0L	Variable	NB0, Local	--	LHHZD
NB1	Variable	NB_1	(16D)	BNC
NB1I	Variable	NB1 Ith subsection	(25)	BNC
NB1ID	Variable	NB1I Denominator	(52)	ENC
NB1IN	Variable	NB1I Numerator	(52)	ENC
NB1L	Variable	NB1, Local	--	LHHZD
NB2	Variable	NB_2	(16A)	BNC
NBC	Variable	NB, Character version	--	NBC
NBD	Variable	N Band-pass-stop Display	--	SMC
NC	Variable	N, Character version	--	NCP
NCL	Variable	N Character List for realization	--	RCMC
NCP	Function	N Computation Parameters	--	BCP
ND	Variable	N Display	--	SMC
NDA1	Label	Negative DA1 subroutine	--	BCPSHZ
NDA1I	Label	Negative DA1I subroutine	--	ELHHZ
NDB0	Label	Negative DB0 subroutine	--	LHHZD
NDB0I	Label	Negative DB0I subroutine	--	LHHZD
NDB1I	Label	Negative DB1I subroutine	--	LHHZD

Table 2 Continued

Term	Type	Description	Eqn.	Function
NE	Variable	N Exact	(63)	BCP
NEC	Variable	NE, Character version	--	NCP
NED	Variable	N Exact Display	--	SMC
NEG	Label	NEGative REVI subroutine	--	RSEMC
NG	Variable	Noise Gain	(206B)	NGCT12
NG1	Variable	Noise Gain of type I realization	--	SRC
NG2	Variable	Noise Gain of direct type realization	--	SRC
NG3	Variable	Noise Gain of type II realization	--	SRC
NGCDT	Function	Noise Gain Computation for Direct Type realizations	--	SRC
NGCT12	Function	Noise Gain Computation for Types I and II realizations	--	SRC
NGD	Variable	NG Denominator	(206B)	NGCDT
NGDA	Variable	NG Denominator, part A	(206B)	NGCDT
NGDB	Variable	NG Denominator, part B	(206B)	NGCDT
NGN	Variable	NG Numerator	(206B)	NGCDT
NGNA	Variable	NG Numerator, part A	(206B)	NGCDT
NGNB	Variable	NG Numerator, part B	(206B)	NGCDT
NOE	Variable	N Odd or Even	--	NCP
NP	Variable	N Parent	--	NCP
NPC	Variable	NP, Character version	--	NCP
NPD	Variable	N Parent Display	--	SMC
NPE	Variable	N Parent Exact	--	NCP

Table 2 Continued

Term	Type	Description	Eqn.	Function
NPEC	Variable	NPE, Character version	--	NCP
NPED	Variable	N Parent Exact Display	--	SMC
NR	Variable	N Realization	--	RCMC
NSIGBP	Variable	Negative SIGBP subroutine	--	PSCP3
ODD	Label	Odd-order subroutine	--	SPAT
OMI	Variable	ω_i OMega Ith subsection	(37)	CNC
OMI1	Label	OMI subroutine	--	ENC
OMI2	Variable	$\Omega_i{}^2$ OMega Ith subsection squared	(50)	ENC
OPS	Label	Odd-order band-Pass-Stop subroutine	--	SPAT
OPS1	Label	OPST = 1 subroutine	--	SRC
OPS2	Label	OPST = 2 subroutine	--	SRC
OPS3	Label	OPST = 3 subroutine	--	SRC
OPST	Variable	Odd Pass-Stop Test	--	SRC
OSUMD	Variable	Omega SUMmation Denonimator	(48)	ENC
OSUMDM	Variable	OSUMD Mth iteration	(48)	ENC
OSUMN	Variable	Omega SUMmation Numerator	(48)	ENC
OSUMNM	Variable	OSUMN Mth iteration	(48)	ENC
PA	Variable	Pass-band Attenuation	(56)	IPMC
PA0	Label	Pole Angle PA1 = 0 subroutine	--	SPAT
PA090	Label	Pole Angle between 0° and 90° subroutine	--	SPAT
PA1	Variable	Pole Angle of 1st-order subsection	--	SPAT

Table 2 Continued

Term	Type	Description	Eqn.	Function
PA2	Variable	Pole Angle of 2nd-order subsection	--	SPAT
PA2I	Variable	Pole Angle of 2nd-order subsection Ith iteration	(204)	SPAT
PA2L	Variable	PA2, Local	--	SPAT
PA90180	Label	Pole Angle between 90° and 180° subroutine	--	SPAT
PACH	Label	Pass-band Attenuation CHange subroutine	--	IPMC
PAPI	Label	Pole Angle PA1 = PI subroutine	--	SPAT
PAT	Variable	Pole Angle Total matrix	--	SPAT
PBF	Variable	Pass-Band Frequency	--	IPMC
PFB1	Variable	Pass-Band Frequency #1	--	IPMC
PBF1CH	Label	Pass-Band Frequency #1 CHange subroutine	--	IPMC
PBF2	Variable	Pass-Band Frequency #2	--	IPMC
PBF2CH	Label	Pass-Band Frequency #2 CHange subroutine	--	IPMC
PBFCH	Label	Pass-Band Frequency CHange subroutine	--	IPMC
PBW	Variable	ω_{pb}, Pass-Band	(55A)	LHCP
PBW1	Variable	ω_{pb1}, Pass-Band ω #1	(55B)	PSCP1
PBW2	Variable	ω_{pb2}, Pass-Band ω #2	(55C)	PSCP1
PCI	Variable	Parameter Change Input	--	IPMC
PCI4	Variable	Parameter Change Input, 4 characters	--	IPMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
PCP	Function	band-Pass Computation Parameters	--	BPCC
PDA1	Label	Positive DA1 subroutine	--	BCPSHZ
PDA1I	Label	Positive DA1I subroutine	--	ELHHZ
PDB0	Label	Positive DB0 subroutine	--	LHHZD
PDB0I	Label	Positive DB0I subroutine	--	LHHZD
PDB1I	Label	Positive DB1I subroutine	--	LHHZD
PG	Variable	P, Gain	(205C)	NGCT12
PK10	Function	band-Pass K10	(61B)	BPCC
PMP1	Variable	Prompt #1, 'ENTER THE NEW VALUE FOR'	--	IPMC
PMP2	Variable	Prompt #2, 'ENTER H,Q,V,OK, OR AN INPUT PARAMETER TO BE CHANGED.'	--	IPMC
PMP3	Variable	Prompt #3, 'ENTER H FOR HELP OR Q FOR QUIT. OTHERWISE ENTER V, OK,'	--	IPMC
PMP4	Variable	Prompt #4, 'OR AN INPUT PARAMETER TO BE CHANGED.'	--	IPMC
POK	Label	Parameters OK subroutine	--	IPMC
POS	Label	POSitive REVI subroutine	--	RSEMC
PS	Label	band-Pass-Stop subroutine	--	SPAT
PS1	Label	band-Pass-Stop subroutine	--	SMC
PSCP1	Function	band-Pass-Stop Computation Parameters #1	--	BPCC
PSCP2	Function	band-Pass-Stop Computation Parameters #2	--	PCP
PSCP3	Function	band-Pass-Stop Computation Parameters #3	--	PCP

Table 2 Continued

Term	Type	Description	Eqn.	Function
PSHZ	Function	band-Pass-Stop H(Z)	--	TCC
PSHZD	Function	band-Pass-Stop H(Z) Denominator	--	PSHZ
PSI	Variable	Ψ PSI	(120C)	SCP
PSIGBP	Label	Positive SIGBP subroutine	--	PSCP3
Q	Variable	q	(41)	ECP
Q0	Variable	q_o	(40)	ECP
QG	Variable	q, Gain	(205D)	NGCT12
R	Variable	r	(105F)	PSCP3
RAD	Label	RADians/sec subroutine	--	LHCP
RAK	Variable	RA_k Root Angle Kth iteration	(25)	BNC
RCC	Function	Realization Computation Control	--	DFD
RCMC	Function	Realization Coefficient Matrix Construction	--	RCC
REM	Variable	Realization Element Matrix	--	RSEMC
REMC	Variable	REM as Characters	--	RSEMC
REV	Variable	Realization Element Vector	--	RSEMC
REVI	Variable	REV Ith iteration	--	RSEMC
RG	Variable	r, Gain	(205E)	NGCT12
RM	Variable	Realization Matrix	--	RMC
RM2	Variable	Realization Matrix, part 2	--	RMC
RMA	Variable	Realization Matrix, A terms	--	RCMC
RMBCD	Variable	Realization Matrix, B C D terms	--	RCMC
RMC	Function	Realization Matrix Construction	--	DFD

Table 2 Continued

Term	Type	Description	Eqn.	Function
RMT	Variable	Realization Matrix Title	--	RMC
RN	Variable	Run Number	--	IPMC
RNO	Label	Run Number RN = 0 subroutine	--	IPMC
RNCH	Label	Run Number CHange subroutine	--	IPMC
RND	Variable	Run Number Display	--	IPMC
RP	Variable	r'	(105I)	PSCP3
RSEM	Variable	Realization Signed Element Matrix	--	RSEMC
RSEMC	Function	Realization Signed Element Matrix Construction	--	RCMC
RSM	Variable	Realization Signed Matrix	--	RSEMC
SA	Variable	Stop-band Attenuation	(58)	IPMC
SACH	Label	Stop-band Attenuation CHange subroutine	--	IPMC
SBAOL	Variable	Shape of BAOL	--	EPSHZ
SBF	Variable	Stop-Band Frequency	--	IPMC
SBF1	Variable	Stop-Band Frequency #1	--	IPMC
SBF1CH	Label	Stop-Band Frequency #1 CHange subroutine	--	IPMC
SBF2	Variable	Stop-Band Frequency #2	--	IPMC
SBF2CH	Label	Stop-Band Frequency #2 CHange subroutine	--	IPMC
SBFCH	Label	Stop-Band Frequency CHange subroutine	--	IPMC
SBW	Variable	ω_{sb}, Stop-Band ω	(57A)	LHCP
SBW1	Variable	ω_{sb1}, Stop-Band ω #1	(57B)	PSCP1

Table 2 Continued

Term	Type	Description	Eqn.	Function
SBW2	Variable	ω_{sb2}, Stop-Band ω #2	(57C)	PSCP1
SCP	Function	band-Stop Computation Parameters	--	BSCC
SF	Variable	F_s Sampling Frequency	(59)	IPMC
SFCH	Label	Sampling Frequency CHange subroutine	--	IPMC
SIG	Variable	σ SIGma	(24E)	PSCP2
SIGO	Variable	σ_o SIGma O	(46)	ENC
SIG1	Label	SIGma subroutine	--	ENC
SIG2	Variable	σ^2 SIGma squared	(24F)	PSCP2
SIGA	Variable	σ_a SIGma A	(104)	PCP
SIGAP	Variable	$\sigma_a{}'$ SIGma A Prime	(105D)	PSCP3
SIGBP	Variable	$\sigma_b{}'$ SIGma B Prime	(105D)	PSCP3
SIGBPD	Variable	SIGBP Denominator	(105D)	PSCP3
SIGBPN	Variable	SIGBP Numerator	(105D)	PSCP3
SIGCP	Variable	$\sigma_c{}'$ SIGma C Prime	(106G)	PSCP3
SIGD	Variable	σ_d SIGma D	(110A)	PSCP3
SIGE	Variable	σ_e SIGma E	(110C)	PSCP3
SIGI	Variable	σ_i SIGma Ith subsection	(37)	CNC
SIGZ	Variable	σ_z SIGma Z	(203C)	SPAT
SIGZ2	Variable	$\sigma_z{}^2$ SIGZ squared	(203B)	SPAT
SINGLE	Label	SINGLE-order filter subroutine	--	NCP
SK10	Function	band-Stop K10	(61C)	BSCC
SM	Variable	Status Matrix	--	SMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
SMC	Function	Status Matrix Construction	--	DFD
SMT	Variable	Status Matrix Title	--	SMC
SPACE	Variable	SPACE matrix	--	RCMC
SPAT	Function	Subsection Pole Angle and Type	--	RCC
SRC	Function	Subsection Realization Coefficients	--	RCC
SSUMD	Variable	SIGma SUMmation Denominator	(46)	ENC
SSUMDM	Variable	SSUMD Mth iteration	(46)	ENC
SSUMN	Variable	SIGma SUMmation Numerator	(46)	ENC
SSUMNM	Variable	SSUMN Mth iteration	(46)	ENC
START	Label	Starts a general subroutine	--	DFD
START1	Label	Starts a general subroutine	--	IPMC
START2	Label	Starts a general subroutine	--	IPMC
START3	Label	Starts a general subroutine	--	IPMC
START4	Label	Starts a general subroutine	--	IPMC
STYPEL	Variable	Shape of TYPE Local	--	SRC
T	Variable	T, period of sampling frequency	(59)	LHCP
T1G	Label	Type I realization Gain, using G1 subroutine	--	SRC
T3G	Label	Type II realization Gain, using G3 subroutine	--	SRC
TANPA2	Variable	TANgent of PA2	(204)	SPAT
TANPA2L	Variable	TANgent of PA2, Local	--	SPAT
TCC	Function	Transform Computation Control	--	DFD

Table 2 Continued

Term	Type	Description	Eqn.	Function
TH	Variable	θ THeta	(105G)	PSCP3
THI	Variable	θ_i THeta Ith subsection	--	PSCP3
THP	Variable	θ' THeta Prime	(105J)	PSCP3
THZSC	Function	Translation of H(Z) Subsection Coefficients	--	RCC
TM	Variable	Total Matrix	--	TMC
TMC	Function	Total Matrix Construction	--	DFD
TME	Variable	Total Matrix End	--	TMC
TYPE	Variable	TYPE of subsection based on pole angle	--	SPAT
TYPEDT	Label	TYPE = DT subroutine	(192B)	SPAT
TYPEL	Variable	TYPE, Local	--	SRC
TYPET1	Label	TYPE = T1 subroutine	(192A)	SPAT
TYPET2	Label	TYPE = T2 subroutine	(192C)	SPAT
UF	Variable	Unit of Frequency	--	IPMC
UF2	Variable	Unit of Frequency, 2 characters	--	IPMC
UFCH	Label	Unit of Frequency CHange subroutine	--	IPMC
UFGH	Label	Unit of Frequency, GigaHertz subroutine	--	IPMC
UFGR	Label	Unit of Frequency, GigaRadians/sec subroutine	--	IPMC
UFH	Label	Unit of Frequency, Hertz subroutine	--	IPMC
UFKH	Label	Unit of Frequency, KiloHertz subroutine	--	IPMC

Table 2 Continued

Term	Type	Description	Eqn.	Function
UFKR	Label	Unit of Frequency, KiloRadians/sec subroutine	--	IPMC
UFMH	Label	Unit of Frequency, MegaHertz subroutine	--	IPMC
UFMR	Label	Unit of Frequency, MegaRadians/sec subroutine	--	IPMC
UFR	Label	Unit of Frequency, Radians/sec	--	IPMC
UNITS	Variable	UNITS, expanded expression	--	IPMC
VI	Variable	V_i	(50)	ENC
VP	Label	Verify Parameters subroutine	--	IPMC
VPL	Label	Verify Parameters List subroutine	--	IPMC
W	Variable	ω	(24F)	PSCP2
WO	Variable	ω_o	(101D)	PSCP1
W11	Variable	W_{11}	(205F)	NGCT12
W11N	Variable	W11 Numerator	(205F)	NGCT12
W11NA	Variable	W11 Numerator, part A	(205F)	NGCT12
W11NB	Variable	W11 Numerator, part B	(205F)	NGCT12
W2	Variable	ω^2	(24F)	PSCP2
W22	Variable	W_{22}	(205G)	NGCT12
W22N	Variable	W22 Numerator	(205G)	NGCT12
W22NA	Variable	W22 Numerator, part A	(205G)	NGCT12
W22NB	Variable	W22 Numerator, part B	(205G)	NGCT12
WA	Variable	ω_a	(104)	PCB
WAP	Variable	ω_a' WA Prime	(105D)	PSCP3

Table 2 Continued

Term	Type	Description	Eqn.	Function
WBP	Variable	$\omega_b{}'$ WB Prime	(105D)	PSCP3
WCP	Variable	$\omega_c{}'$ WC Prime	(106G)	PSCP3
WD	Variable	ω_d	(110A)	PSCP3
WE	Variable	ω_e	(110C)	PSCP3
WE1	Variable	WE part 1	(47)	ENC
WE2	Variable	WE part 2	(47)	ENC
WF	Variable	ω_f	(129B)	EPCP
WG	Variable	ω_g	(135A)	EPCP
WH	Variable	ω_h	(135B)	EPCP
WH1	Variable	WH Part 1	(135B)	EPCP
WH2	Variable	WH Part 2	(135B)	EPCP
WNE	Variable	W Normalized Elliptic	(47)	ENC
WP	Variable	ω_p	(65)	BCP
WZ	Variable	ω_z	(203D)	SPAT
WZ2	Variable	$\omega_z{}^2$	(203B)	SPAT

APPENDIX G

PROGRAM LISTING

The program in this appendix is titled DFD (Digital Filter Design) and was written in the APL language on a DEC (Digital Equipment Corporation) PDP-10 computer and then transferred to a DEC VAX-cluster system. Since APL files are not stored in standard ASCII format, the program was converted to an ASCII file called DFD.AAS by using the APL command OUTPUT, specifying DFD as the filename, and listing the functions by name in the format .DL function name [.BX] .DL. The extension .AAS was added by the OUTPUT command. The file DFD.AAS contains only the sequence of functions used in the program. This file was then copied to another ASCII file called DFD.TXT. This file was then edited to include the title, index, group headers, and form feeds that appear with the program listing in this appendix. The only file needed to run the program is the DFD.AAS file. This file can be loaded into a computer and moved to an APL workspace by using the APL command INPUT and specifying DFD when prompted for a file name. The program can be initialized by defining the run number as zero (enter RN_0) before calling up the program as described in Appendix A. All other variables are initialized within the program.

```
* * * * * * * * * * * * * * *
*            DFD              *
*  (DIGITAL FILTER DESIGN)    *
* * * * * * * * * * * * * * *
```

PROGRAM LISTING

INDEX

GROUP	FUNCTION	DESCRIPTION
1A	DFD	DIGITAL FILTER DESIGN
2A	IPMC	INPUT PARAMETER MATRIX CONSTRUCTION
	TCC	TRANSFORM COMPUTATION CONTROL
	SMC	STATUS MATRIX CONSTRUCTION
	HZMC	H(Z) MATRIX CONSTRUCTION
	RCC	REALIZATION COMPUTATION CONTROL
	RMC	REALIZATION MATRIX CONSTRUCTION
	TMC	TOTAL MATRIX CONSTRUCTION
3A	IIP	INITIALIZE INPUT PARAMETERS
	IPML	INPUT PARAMETER MATRIX LOW-PASS
	IPMH	INPUT PARAMETER MATRIX HIGH-PASS
	IPMP	INPUT PARAMETER MATRIX BAND-PASS
	IPMS	INPUT PARAMETER MATRIX BAND-STOP
	HFC	HELP FILE CONSTRUCTION
3B	BLCC	BUTTERWORTH LOW-PASS COMPUTATION CONTROL
	CLCC	CHEBYSHEV LOW-PASS COMPUTATION CONTROL
	ELCC	ELLIPTIC LOW-PASS COMPUTATION CONTROL
3C	BHCC	BUTTERWORTH HIGH-PASS COMPUTATION CONTROL
	CHCC	CHEBYSHEV HIGH-PASS COMPUTATION CONTROL
	EHCC	ELLIPTIC HIGH-PASS COMPUTATION CONTROL
3D	BPCC	BUTTERWORTH BAND-PASS COMPUTATION CONTROL
	CPCC	CHEBYSHEV BAND-PASS COMPUTATION CONTROL
	EPCC	ELLIPTIC BAND-PASS COMPUTATION CONTROL
3E	BSCC	BUTTERWORTH BAND-STOP COMPUTATION CONTROL
	CSCC	CHEBYSHEV BAND-STOP COMPUTATION CONTROL
	ESCC	ELLIPTIC BAND-STOP COMPUTATION CONTROL
3F	LHHZ	LOW-HIGH-PASS H(Z)
	PSHZ	BAND-PASS-STOP H(Z)

INDEX CONTINUED

INDEX CONTINUED

FUNCTION LISTINGS

------------------------------ GROUP 1A ------------------------------

DFD

```
      .DL DFD
[1]     "
[2]     "---DIGITAL FILTER DESIGN---
[3]     "
[4]     .BX_' '
[5]     .BX_'DIGITAL FILTER DESIGN'
[6]     .BX_' '
[7]    START:
[8]     IPMC
[9]     .GO END IF 4=(+/PCI4='Q    ')
[10]    TCC
[11]    SMC
[12]    HZMC
[13]    RCC
[14]    RMC
[15]    TMC
[16]    .GO END IF RN=0
[17]    .BX_TM
[18]    .BX_' '
[19]    .BX_' '
[20]   START1:
[21]    .BX_' '
[22]    .BX_'ANOTHER RUN? (Y OR N)'
[23]    .BX_' '
[24]    RUN_.QQ
[25]    .GO START IF RUN='Y'
[26]    .GO END IF RUN='N'
[27]    .GO START1
[28]   END:
[29]    .BX_' '
[30]    .BX_' '
[31]    .BX_'END'
      .DL
```

----------------------------- GROUP 2A -----------------------------

IPMC TCC SMC HZMC RCC RMC TMC

```
      .DL IPMC
[1]    "
[2]    "---INPUT PARAMETER MATRIX CONSTRUCTION---
[3]    "
[4]    .GO INITIAL IF RN=0
[5]    RN_RN+1
[6]    .GO START1
[7]    INITIAL:IIP
[8]    RN_1
[9]   START1:
[10]   PCI4_'NEW '
[11]   PMP1_'ENTER THE NEW VALUE FOR '
[12]   PMP2_'ENTER H,Q,V,OK, OR AN INPUT PARAMETER TO BE CHANGED.'
[13]   PMP3_'ENTER H FOR HELP OR Q FOR QUIT.  OTHERWISE ENTER V,OK,'
[14]   PMP4_'OR AN INPUT PARAMETER TO BE CHANGED.'
[15]   .BX_' '
[16]   .BX_PMP3
[17]   .BX_PMP4
[18]   .GO START4
[19]  START2:
[20]   RND_'RUN NUMBER                              RN = ',.FM RN
[21]   IPM_ 1 80 .RO ' '
[22]   IPMT_'                   ---INPUT PARAMETERS---'
[23]   IPMT_ 1 80 .RO 80^IPMT
[24]   IPM_IPM,[1]IPMT
[25]   IPM_IPM,[1]1 80 .RO ' '
[26]   IPM1_'DESIGN TYPE                              DT = ',DT1
[27]   IPM_IPM,[1]1 80 .RO 80^IPM1
[28]   DB_' DB'
[29]   .GO LP2 IF 2=(+/FT='LP')
[30]   .GO HP2 IF 2=(+/FT='HP')
[31]   .GO BP2 IF 2=(+/FT='BP')
[32]   .GO BS2 IF 2=(+/FT='BS')
[33]  LP2:FT1_'LP    (LOW-PASS)'
[34]   IPML
[35]   .GO CONT3
[36]  HP2:FT1_'HP    (HIGH-PASS)'
[37]   IPMH
[38]   .GO CONT3
[39]  BP2:FT1_'BP    (BAND-PASS)'
[40]   IPMP
[41]   .GO CONT3
```

```
[42]   BS2:FT1_'BS   (BAND-STOP)'
[43]    IPMS
[44]   CONT3:
[45]    IPM9_'SAMPLING FREQUENCY                       SF = ',.FM SF
[46]    IPM9_IPM9,UNITS
[47]    IPM_IPM,[1]1 80 .RO 80^IPM9
[48]    .GO ARNG IF SA .LE PA
[49]   AROK:
[50]    AR_'OK'
[51]    .GO CONT2
[52]   ARNG: AR_'NG'
[53]    .BX_' '
[54]    .BX_'                *** ATTENUATION INPUT ERROR ***'
[55]    .BX_'THE STOPBAND ATTENUATION (SA) MUST BE GREATER THAN THE'
[56]    .BX_'PASSBAND ATTENUATION (PA).'
[57]    .BX_' '
[58]    .BX_IPM
[59]    .BX_' '
[60]    .BX_' '
[61]    .BX_'ENTER H,V,Q, OR AN INPUT PARAMETER TO BE CHANGED.'
[62]    .GO START4
[63]   CONT2:
[64]    .GO FRNG IF FR = 'NG'
[65]    .GO CONT4
[66]   FRNG:
[67]    .BX_' '
[68]    .BX_'                *** FREQUENCY INPUT ERROR ***'
[69]    .BX_'THE CUTOFF FREQUENCIES (PBF,SBF, ETC.) MUST HAVE ASCENDING'
[70]    .BX_'VALUES IN THE ORDER SHOWN.  THE SAMPLING FREQUENCY (SF) MUST'
[71]    .BX_'BE AT LEAST 2.001 TIMES THE LARGEST CUTOFF FREQUENCY.'
[72]    .BX_' '
[73]    .BX_IPM
[74]    .BX_' '
[75]    .BX_' '
[76]    .BX_'ENTER H,V,Q, OR AN INPUT PARAMETER TO BE CHANGED.'
[77]    .GO START4
[78]   CONT4:
[79]    .GO FINAL IF 4=(+/PCI4='OK  ')
[80]    .GO VPL IF 4=(+/PCI4='V   ')
[81]   START3:
[82]    .BX_' '
[83]    .BX_PMP2
```

```
[84]   START4:
[85]    .BX_' '
[86]    PCI_.QQ
[87]    .BX_' '
[88]    PCI4_4^PCI
[89]    .GO HELP IF 4=(+/PCI4='H   ')
[90]    .GO END IF 4=(+/PCI4='Q   ')
[91]    .GO VP IF 4=(+/PCI4='V   ')
[92]    .GO RNCH IF 4=(+/PCI4='RN  ')
[93]    .GO DTCH IF 4=(+/PCI4='DT  ')
[94]    .GO FTCH IF 4=(+/PCI4='FT  ')
[95]    .GO UFCH IF 4=(+/PCI4='UF  ')
[96]    .GO PBFCH IF 4=(+/PCI4='PBF ')
[97]    .GO PACH IF 4=(+/PCI4='PA  ')
[98]    .GO SBFCH IF 4=(+/PCI4='SBF ')
[99]    .GO SACH IF 4=(+/PCI4='SA  ')
[100]   .GO SBF1CH IF 4=(+/PCI4='SBF1')
[101]   .GO PBF1CH IF 4=(+/PCI4='PBF1')
[102]   .GO PBF2CH IF 4=(+/PCI4='PBF2')
[103]   .GO SBF2CH IF 4=(+/PCI4='SBF2')
[104]   .GO SFCH IF 4=(+/PCI4='SF  ')
[105]   .GO POK IF 4=(+/PCI4='OK  ')
[106]   .GO START3
[107]  HELP:HFC
[108]   .GO START3
[109]  VP:.GO START2
[110]  RNCH:.BX_PMP1,'RN'
[111]   RN_.BX
[112]   .GO RN0 IF RN=0
[113]   .GO START3
[114]  RN0:IIP
[115]   RND_'RUN NUMBER                          RN = ',.FM RN
[116]   .GO END
[117]  DTCH:
[118]   .BX_'ENTER:     B FOR BUTTERWORTH'
[119]   .BX_'           C FOR CHEBYSHEV'
[120]   .BX_'           E FOR ELLIPTIC'
[121]   .BX_' '
[122]   DT_.QQ
[123]   .BX_' '
[124]   .GO B3 IF DT='B'
[125]   .GO C3 IF DT='C'
[126]   .GO E3 IF DT='E'
[127]   .GO DTCH
```

```
B3:DT1_'B    (BUTTERWORTH)'
 .GO START3
C3:DT1_'C    (CHEBYSHEV)'
 .GO START3
E3:DT1_'E    (ELLIPTIC)'
 .GO START3
UFCH:
 .BX_'ENTER:     H  FOR HERTZ'
 .BX_'           KH FOR KILOHERTZ'
 .BX_'           MH FOR MEGAHERTZ'
 .BX_'           GH FOR GIGAHERTZ'
 .BX_'           R  FOR RADIANS/SEC'
 .BX_'           KR FOR KILORADIANS/SEC'
 .BX_'           MR FOR MEGARADIANS/SEC'
 .BX_'           GR FOR GIGARADIANS/SEC'
 .BX_' '
 UF_.QQ
 UF2_2^UF
 .GO UFH IF 2=(+/UF2='H ')
 .GO UFKH IF 2=(+/UF2='KH')
 .GO UFMH IF 2=(+/UF2='MH')
 .GO UFGH IF 2=(+/UF2='GH')
 .GO UFR IF 2=(+/UF2='R ')
 .GO UFKR IF 2=(+/UF2='KR')
 .GO UFMR IF 2=(+/UF2='MR')
 .GO UFGR IF 2=(+/UF2='GR')
 .GO UFCH
UFH:UNITS_' HZ'
 .GO START3
UFKH:UNITS_' KHZ'
 .GO START3
UFMH:UNITS_' MHZ'
 .GO START3
UFGH:UNITS_' GHZ'
 .GO START3
UFR:UNITS_' RAD/SEC'
 .GO START3
UFKR:UNITS_' KRAD/SEC'
 .GO START3
UFMR:UNITS_' MRAD/SEC'
 .GO START3
UFGR:UNITS_' GRAD/SEC'
 .GO START3
```

```
FTCH:
 .BX_'ENTER:      LP FOR LOW-PASS'
 .BX_'            HP FOR HIGH-PASS'
 .BX_'            BP FOR BAND-PASS'
 .BX_'            BS FOR BAND-STOP'
 .BX_' '
 FT_.QQ
 .BX_' '
 .GO START3 IF FT='LP'
 .GO START3 IF FT='HP'
 .GO START3 IF FT='BP'
 .GO START3 IF FT='BS'
 .GO FTCH
PBFCH:.BX_PMP1,'PBF'
 PBF_.BX
 .GO START3
PACH:.BX_PMP1,'PA'
 PA_.BX
 .GO START3
SBFCH:.BX_PMP1,'SBF'
 SBF_.BX
 .GO START3
SACH:.BX_PMP1,'SA'
 SA_.BX
 .GO START3
SBF1CH:.BX_PMP1,'SBF1'
 SBF1_.BX
 .GO START3
PBF1CH:.BX_PMP1,'PBF1'
 PBF1_.BX
 .GO START3
PBF2CH:.BX_PMP1,'PBF2'
 PBF2_.BX
 .GO START3
SBF2CH:.BX_PMP1,'SBF2'
 SBF2_.BX
 .GO START3
SFCH:.BX_PMP1,'SF'
 SF_.BX
 .GO START3
POK:
 .GO START2
VPL:.BX_RND
 .BX_IPM
 .GO START3
FINAL:.BX_' '
END:
.DL
```

```
      .DL TCC
[1]    "
[2]    "---TRANSFORM COMPUTATION CONTROL---
[3]    "
[4]    .GO B1 IF DT='B'
[5]    .GO C1 IF DT='C'
[6]    .GO E1 IF DT='E'
[7]   B1:
[8]    .GO BLP IF 2=(+/FT='LP')
[9]    .GO BHP IF 2=(+/FT='HP')
[10]   .GO BBP IF 2=(+/FT='BP')
[11]   .GO BBS IF 2=(+/FT='BS')
[12]  BLP:
[13]   BLCC
[14]   LHHZ
[15]   .GO END
[16]  BHP:
[17]   BHCC
[18]   LHHZ
[19]   .GO END
[20]  BBP:
[21]   BPCC
[22]   PSHZ
[23]   .GO END
[24]  BBS:
[25]   BSCC
[26]   PSHZ
[27]   .GO END
[28]  C1:
[29]   .GO CLP IF 2=(+/FT='LP')
[30]   .GO CHP IF 2=(+/FT='HP')
[31]   .GO CBP IF 2=(+/FT='BP')
[32]   .GO CBS IF 2=(+/FT='BS')
[33]  CLP:
[34]   CLCC
[35]   LHHZ
[36]   .GO END
[37]  CHP:
[38]   CHCC
[39]   LHHZ
[40]   .GO END
[41]  CBP:
[42]   CPCC
[43]   PSHZ
[44]   .GO END
[45]  CBS:
[46]   CSCC
[47]   PSHZ
[48]   .GO END
```

```
E1:
  .GO ELP IF 2=(+/FT='LP')
  .GO EHP IF 2=(+/FT='HP')
  .GO EBP IF 2=(+/FT='BP')
  .GO EBS IF 2=(+/FT='BS')
ELP:
 ELCC
 LHHZ
  .GO END
EHP:
 EHCC
 LHHZ
  .GO END
EBP:
 EPCC
 PSHZ
  .GO END
EBS:
 ESCC
 PSHZ
END:
.DL


.DL SMC
  "
  "---STATUS MATRIX CONSTRUCTION---
  "
  SM_ 2 80 .RO ' '
  SMT_'--------------------DIGITAL FILTER DESIGN--------------------'
  SMT_ 1 80 .RO 80^SMT
  SM_SM,[1]SMT
  SM_SM,[1]1 80 .RO ' '
  RND_ 1 80 .RO 80^RND
  SM_SM,[1]RND
  .GO LH1 IF 2=(+/FT='LP')
  .GO LH1 IF 2=(+/FT='HP')
  .GO PS1 IF 2=(+/FT='BP')
  .GO PS1 IF 2=(+/FT='BS')
LH1:
 ND_'ORDER OF FILTER                         N = ',NC
 ND_ 1 80 .RO 80^ND
 SM_SM,[1]ND
 NED_'EXACT ORDER OF FILTER                  NE = ',NEC
 NED_ 1 80 .RO 80^NED
 SM_SM,[1]NED
 .GO END
```

```
[23]  PS1:
[24]   NPD_'ORDER OF PARENT LP FILTER         NP = ',NPC
[25]   NPD_ 1 80 .RO 80^NPD
[26]   SM_SM,[1]NPD
[27]   NPED_'EXACT ORDER OF PARENT LP FILTER   NPE = ',NPEC
[28]   NPED_ 1 80 .RO 80^NPED
[29]   SM_SM,[1]NPED
[30]   NBD_'ORDER OF BP OR BS FILTER          NB = ',NBC
[31]   NBD_ 1 80 .RO 80^NBD
[32]   SM_SM,[1]NBD
[33]  END:
      .DL


      .DL HZMC
[1]    ''
[2]    '---H(Z) MATRIX CONSTRUCTION---
[3]    ''
[4]    GC_.FM G
[5]    HZM_ 1 80 .RO ' '
[6]    HZMT_'                    ---DIGITAL TRANSFORM---'
[7]    HZMT_ 1 80 .RO 80^HZMT
[8]    HZM_HZM,[1]HZMT
[9]    HZM_HZM,[1]1 80 .RO ' '
[10]   HZM_HZM,[1]1 80 .RO 80^'H(Z) = (G)(NUMERATOR/DENOMINATOR)'
[11]   HZM_HZM,[1]1 80 .RO ' '
[12]   HZM_HZM,[1]1 80 .RO 80^'G = ',GC
[13]   HZM_HZM,[1]HZN
[14]   HZM_HZM,[1]HZD
[15]   HZM_HZM,[1]1 80 .RO ' '
      .DL


     .DL RCC
[1]    ''
[2]    '---REALIZATION COMPUTATION CONTROL---
[3]    ''
[4]    SPAT
[5]    THZSC
[6]    SRC
[7]    RCMC
     .DL
```

```
.DL RMC
"
"---REALIZATION MATRIX CONSTRUCTION---
"
DC_G
DCC_.FM DC
RM_ 1 80 .RO ' '
RMT_'            ---STATE-SPACE REALIZATION COEFFICIENTS---'
RMT_ 1 80 .RO 80^RMT
RM_RM,[1]RMT
RM_RM,[1]1 80 .RO ' '
RM2_'DC = G = ',DCC
RM2_ 1 80 .RO 80^RM2
RM_RM,[1]RM2
RM_RM,[1]1 80 .RO ' '
RM_RM,[1]RMA
RM_RM,[1]1 80 .RO ' '
RM_RM,[1]RMBCD
RM_RM,[1]1 80 .RO ' '
.DL
```

```
.DL TMC
"
"---TOTAL MATRIX CONSTRUCTION---
TM_SM
TM_TM,[1]IPM
TM_TM,[1]HZM
TM_TM,[1]RM
TME_25.RO ' '
TME_TME,10.RO '-'
TME_ 1 80 .RO 80^TME
TM_TM,[1]TME
TM_TM,[1]2 80 .RO ' '
.DL
```

---------------------------- GROUP 3A ----------------------------

IIP IPML IPMH IPMP IPMS HFC

```
      .DL IIP
[1]    "
[2]    "---INITIALIZE INPUT PARAMETERS---
[3]    "
[4]    FT_'HP'
[5]    FT1_'HP    (HIGH-PASS)'
[6]    DT_'B'
[7]    DT1_'B     (BUTTERWORTH)'
[8]    UNITS_' KHZ'
[9]    SBF_0.19
[10]   PBF_0.25
[11]   SA_6
[12]   PA_3
[13]   SBF1_1
[14]   PBF1_2
[15]   PBF2_3
[16]   SBF2_4
[17]   SF_50
      .DL


      .DL IPML
[1]    "
[2]    "---INPUT PARAMETER MATRIX LOW-PASS---
[3]    "
[4]    IPM2_'FILTER TYPE                        FT = ',FT1
[5]    IPM_IPM,[1] 1 80 .RO 80^IPM2
[6]    IPM3_'PASSBAND FREQUENCY                PBF = ',.FM PBF
[7]    IPM3_IPM3,UNITS
[8]    IPM_IPM,[1]1 80 .RO 80^IPM3
[9]    IPM4_'STOPBAND FREQUENCY                SBF = ',.FM SBF
[10]   IPM4_IPM4,UNITS
[11]   IPM_IPM,[1]1 80 .RO 80^IPM4
[12]   IPM5_'PASSBAND ATTENUATION               PA = ',.FM PA
[13]   IPM5_IPM5,DB
[14]   IPM_IPM,[1]1 80 .RO 80^IPM5
[15]   IPM6_'STOPBAND ATTENUATION               SA = ',.FM SA
[16]   IPM6_IPM6,DB
[17]   IPM_IPM,[1]1 80 .RO 80^IPM6
[18]   .GO FRNG IF PBF .GE SBF
[19]   .GO FRNG IF SF < (2.001#SBF)
[20]   FR_'OK'
[21]   .GO END
[22]  FRNG: FR_'NG'
[23]  END:
      .DL
```

```
.DL IPMH
 "
 "---INPUT PARAMETER MATRIX HIGH-PASS---
 "
 IPM2_'FILTER TYPE                          FT = ',FT1
 IPM_IPM,[1]1 80 .RO 80^IPM2
 IPM3_'STOPBAND FREQUENCY                  SBF = ',.FM SBF
 IPM3_IPM3,UNITS
 IPM_IPM,[1]1 80 .RO 80^IPM3
 IPM4_'PASSBAND FREQUENCY                  PBF = ',.FM PBF
 IPM4_IPM4,UNITS
 IPM_IPM,[1]1 80 .RO 80^IPM4
 IPM5_'STOPBAND ATTENUATION                 SA = ',.FM SA
 IPM5_IPM5,DB
 IPM_IPM,[1]1 80 .RO 80^IPM5
 IPM6_'PASSBAND ATTENUATION                 PA = ',.FM PA
 IPM6_IPM6,DB
 IPM_IPM,[1]1 80 .RO 80^IPM6
 .GO FRNG IF SBF .GE PBF
 .GO FRNG IF SF < (2.001#PBF)
 FR_'OK'
 .GO END
FRNG: FR_'NG'
END:
.DL
```

```
.DL IPMP
 "
 "---INPUT PARAMETER MATRIX BAND-PASS---
 "
 IPM2_'FILTER TYPE                          FT = ',FT1
 IPM_IPM,[1]1 80 .RO 80^IPM2
 IPM3_'STOPBAND FREQUENCY #1              SBF1 = ',.FM SBF1
 IPM3_IPM3,UNITS
 IPM_IPM,[1]1 80 .RO 80^IPM3
 IPM4_'PASSBAND FREQUENCY #1              PBF1 = ',.FM PBF1
 IPM4_IPM4,UNITS
 IPM_IPM,[1]1 80 .RO 80^IPM4
 IPM5_'PASSBAND FREQUENCY #2              PBF2 = ',.FM PBF2
 IPM5_IPM5,UNITS
 IPM_IPM,[1]1 80 .RO 80^IPM5
 IPM6_'STOPBAND FREQUENCY #2              SBF2 = ',.FM SBF2
 IPM6_IPM6,UNITS
 IPM_IPM,[1]1 80 .RO 80^IPM6
 IPM7_'STOPBAND ATTENUATION                 SA = ',.FM SA
 IPM7_IPM7,DB
 IPM_IPM,[1]1 80 .RO 80^IPM7
 IPM8_'PASSBAND ATTENUATION                 PA = ',.FM PA
 IPM8_IPM8,DB
 IPM_IPM,[1]1 80 .RO 80^IPM8
```

```
[24]   .GO FRNG IF SBF1 .GE PBF1
[25]   .GO FRNG IF PBF1 .GE PBF2
[26]   .GO FRNG IF PBF2 .GE SBF2
[27]   .GO FRNG IF SF < (2.001#SBF2)
[28]   FR_'OK'
[29]   .GO END
[30]  FRNG: FR_'NG'
[31]  END:
      .DL


      .DL IPMS
[1]    "
[2]    "---INPUT PARAMETER MATRIX BAND-STOP---
[3]    "
[4]    IPM2_'FILTER TYPE                         FT = ',FT1
[5]    IPM_IPM,[1]1 80 .RO 80^IPM2
[6]    IPM3_'PASSBAND FREQUENCY #1             PBF1 = ',.FM PBF1
[7]    IPM3_IPM3,UNITS
[8]    IPM_IPM,[1]1 80 .RO 80^IPM3
[9]    IPM4_'STOPBAND FREQUENCY #1             SBF1 = ',.FM SBF1
[10]   IPM4_IPM4,UNITS
[11]   IPM_IPM,[1]1 80 .RO 80^IPM4
[12]   IPM5_'STOPBAND FREQUENCY #2             SBF2 = ',.FM SBF2
[13]   IPM5_IPM5,UNITS
[14]   IPM_IPM,[1]1 80 .RO 80^IPM5
[15]   IPM6_'PASSBAND FREQUENCY #2             PBF2 = ',.FM PBF2
[16]   IPM6_IPM6,UNITS
[17]   IPM_IPM,[1]1 80 .RO 80^IPM6
[18]   IPM7_'PASSBAND ATTENUATION                PA = ',.FM PA
[19]   IPM7_IPM7,DB
[20]   IPM_IPM,[1]1 80 .RO 80^IPM7
[21]   IPM8_'STOPBAND ATTENUATION                SA = ',.FM SA
[22]   IPM8_IPM8,DB
[23]   IPM_IPM,[1]1 80 .RO 80^IPM8
[24]   .GO FRNG IF PBF1 .GE SBF1
[25]   .GO FRNG IF SBF1 .GE SBF2
[26]   .GO FRNG IF SBF2 .GE PBF2
[27]   .GO FRNG IF SF < (2.001#PBF2)
[28]   FR_'OK'
[29]   .GO END
[30]  FRNG: FR_'NG'
[31]  END:
      .DL
```

```
      .DL HFC
[1]    "
[2]    "---HELP FILE CONSTRUCTION---
[3]    "
[4]    .BX_' '
[5]    .BX_'                   ---HELP FILE---'
[6]    .BX_' '
[7]    .BX_'FOR DETAILED INFORMATION ON THIS PROGRAM SEE:'
[8]    .BX_' '
[9]    .BX_'             A DIGITAL FILTER DESIGN PROCEDURE'
[10]   .BX_'              WITH APPLICATION PROGRAM IN APL'
[11]   .BX_' '
[12]   .BX_'                A THESIS PRESENTED FOR THE'
[13]   .BX_'                  MASTER OF SCIENCE DEGREE'
[14]   .BX_'           THE UNIVERSITY OF TENNESSEE, KNOXVILLE'
[15]   .BX_' '
[16]   .BX_'                     WALTER J. EATON'
[17]   .BX_'                       MARCH 1987'
[18]   .BX_' '
[19]   .BX_'THIS PROGRAM (CALLED DIGITAL FILTER DESIGN) IS INITIALIZED BY'
[20]   .BX_'ENTERING DFD ON THE TERMINAL.  THE PROGRAM PROMPTS THE USER'
[21]   .BX_'FOR INPUT PARAMETERS FOR THE DESIGN OF DIGITAL FILTERS.  THE'
[22]   .BX_'PROGRAM PRODUCES Z-TRANSFORM COEFFICIENTS AND STATE-SPACE'
[23]   .BX_'REALIZATION COEFFICIENTS.  IF H IS ENTERED, THIS FILE IS'
[24]   .BX_'DISPLAYED.  THE USER CAN ENTER Q FOR QUIT, AND THE PROGRAM'
[25]   .BX_'WILL END WITHOUT MAKING A RUN.  IF THE USER ENTERS V FOR'
[26]   .BX_'VERIFICATION, THE EXISTING LIST OF INPUT PARAMETERS WILL BE'
[27]   .BX_'DISPLAYED.  THE USER CAN ALSO ENTER OK TO INDICATE THAT THE'
[28]   .BX_'PARAMETERS ARE CORRECT AND A PROGRAM RUN IS DESIRED.'
[29]   .BX_' '
[30]   .BX_'THE USER CAN CHANGE THE VALUE OF ANY OF THE INPUT PARAMETERS'
[31]   .BX_'WHEN THE PROGRAM PROMPTS FOR A PARAMETER TO BE CHANGED.  IF'
[32]   .BX_'THE USER ENTERS DT TO CHANGE THE DESIGN TYPE, THE PROGRAM'
[33]   .BX_'WILL RESPOND WITH THE PROMPT'
[34]   .BX_' '
[35]   .BX_'ENTER:     B FOR BUTTERWORTH'
[36]   .BX_'           C FOR CHEBYSHEV'
[37]   .BX_'           E FOR ELLIPTIC'
[38]   .BX_' '
[39]   .BX_'AND THEN IT WILL WAIT FOR A USER INPUT.  SIMILARLY, IF THE'
[40]   .BX_'USER ENTERS FT TO CHANGE THE FILTER TYPE, THE PROGRAM WILL'
[41]   .BX_'RESPOND WITH THE PROMPT'
[42]   .BX_' '
[43]   .BX_'ENTER:     LP FOR LOW-PASS'
[44]   .BX_'           HP FOR HIGH-PASS'
[45]   .BX_'           BP FOR BAND-PASS'
[46]   .BX_'           BS FOR BAND-STOP'
[47]   .BX_' '
[48]   .BX_'AND AGAIN IT WILL WAIT FOR A RESPONSE.  THE UNIT OF FREQUENCY  '
[49]   .BX_'IS SELECTED IN THE SAME MANNER.  THE USER ENTERS UF AND THE'
[50]   .BX_'PROGRAM RESPONDS WITH THE PROMPT'
[51]   .BX_' '
```

```
.BX_'ENTER:      H  FOR HERTZ'
.BX_'            KH FOR KILOHERTZ'
.BX_'            MH FOR MEGAHERTZ'
.BX_'            GH FOR GIGAHERTZ'
.BX_'            R  FOR RADIANS/SEC'
.BX_'            KR FOR KILORADIANS/SEC'
.BX_'            MR FOR MEGARADIANS/SEC'
.BX_'            GR FOR GIGARADIANS/SEC'
.BX_' '
.BX_'AND IT WAITS FOR A RESPONSE.'
.BX_' '
.BX_'THE INPUTS THAT ARE REQUESTED WILL DEPEND ON THE FILTER'
.BX_'TYPE SELECTED.  ALL OF THE VALID FREQUENCY AND ATTENUATION'
.BX_'INPUTS FOR EACH FILTER TYPE ARE SHOWN BELOW.'
.BX_' '
.BX_'          LOW-PASS                              HIGH-PASS'
.BX_' '
.BX_' PBF = PASSBAND FREQUENCY            SBF = STOPBAND FREQUENCY'
.BX_' SBF = STOPBAND FREQUENCY            PBF = PASSBAND FREQUENCY'
.BX_'  PA = PASSBAND ATTENUATION           SA = STOPBAND ATTENUATION'
.BX_'  SA = STOPBAND ATTENUATION           PA = PASSBAND ATTENUATION'
.BX_'  SF = SAMPLING FREQUENCY             SF = SAMPLING FREQUENCY'
.BX_' '
.BX_' '
.BX_'          BAND-PASS                             BAND-STOP'
.BX_' '
.BX_'SBF1 = STOPBAND FREQUENCY #1       PBF1 = PASSBAND FREQUENCY #1'
.BX_'PBF1 = PASSBAND FREQUENCY #1       SBF1 = STOPBAND FREQUENCY #1'
.BX_'PBF2 = PASSBAND FREQUENCY #2       SBF2 = STOPBAND FREQUENCY #2'
.BX_'SBF2 = STOPBAND FREQUENCY #2       PBF2 = PASSBAND FREQUENCY #2'
.BX_'  SA = STOPBAND ATTENUATION          PA = PASSBAND ATTENUATION'
.BX_'  PA = PASSBAND ATTENUATION          SA = STOPBAND ATTENUATION'
.BX_'  SF = SAMPLING FREQUENCY            SF = SAMPLING FREQUENCY'
.BX_' '
.BX_' '
.BX_'IN EACH CASE THE FOLLOWING CONDITIONS MUST BE MET.'
.BX_' '
.BX_'    1. THE CUTOFF FREQUENCIES (PBF, SBF, ETC.) MUST HAVE '
.BX_'       ASCENDING VALUES IN THE ORDER SHOWN ON THE CRT.'
.BX_'       THE CORRECT ORDER OF FREQUENCIES WILL BE DISPLAYED'
.BX_'       FOR THE FILTER TYPE SELECTED.'
.BX_' '
.BX_'    2. THE SAMPLING FREQUENCY (SF) MUST BE AT LEAST 2.001'
.BX_'       TIMES THE LARGEST CUTOFF FREQUENCY.  THIS PREVENTS'
.BX_'       MATHEMATICAL PROBLEMS THAT OCCUR IF THE SAMPLING'
.BX_'       FREQUENCY IS EXACTLY EQUAL TO THE NYQUIST VALUE OF'
.BX_'       2.000 TIMES THE LARGEST CUTOFF FREQUENCY.'
.BX_' '
.BX_'    3. THE STOPBAND ATTENUATION (SA) MUST BE GREATER THAN'
.BX_'       THE PASSBAND ATTENUATION (PA).'
.BX_' '
```

```
.BX_'IF THESE CONDITIONS ARE VIOLATED, APPROPRIATE ERROR'
.BX_'MESSAGES WILL BE DISPLAYED.  ANY OF THESE PARAMETERS CAN'
.BX_'BE CHANGED BE ENTERING THE APPROPRIATE ACRONYM LISTED'
.BX_'ABOVE WHEN THE PROGRAM PROMPTS FOR A PARAMETER TO BE'
.BX_'CHANGED.'
.BX_' '
.BX_'NOTE THAT A RUN NUMBER APPEARS WHEN THE INPUT PARAMETERS'
.BX_'ARE VERIFIED OR WHEN A PROGRAM RUN IS MADE.  THIS NUMBER IS'
.BX_'CONSIDERED TO BE AN INPUT PARAMETER AND CAN BE CHANGED BY'
.BX_'ENTERING RN WHEN THE PROGRAM PROMPTS FOR A PARAMETER TO BE'
.BX_'CHANGED.  THE PROGRAM AUTOMATICALLY INCREMENTS THE RUN'
.BX_'NUMBER EACH TIME THE PROGRAM IS RUN.  SETTING THE RUN'
.BX_'NUMBER TO 1, 11, OR 101 WILL RESULT IN A SERIES OF PROGRAM'
.BX_'RUNS WITH RUN NUMBERS 1,2,3,...; 11,12,13,...; OR 101,102,'
.BX_'103,... .  THIS IS HELPFUL IN KEEPING TRACK OF A SERIES OF'
.BX_'RELATED PROGRAM RUNS.  IF THE RUN NUMBER IS CHANGED TO 0,'
.BX_'ALL USER INFORMATION WILL BE CLEARED FROM THE INPUT'
.BX_'PARAMETER REGISTERS AND THE INPUT PARAMETERS WILL BE SET'
.BX_'TO THE DEFAULT VALUES THAT APPEAR IN FUNCTION IIP.  ALSO,'
.BX_'NO RUN WILL BE MADE AND THE PROGRAM WILL END.'
.BX_' '
.BX_'WHEN THE USER ENTERS OK THE PROGRAM PROCESSES THE INPUT'
.BX_'INFORMATION AND DISPLAYS THE PROGRAM OUTPUT.  THIS INCLUDES'
.BX_'THE INPUT PARAMETERS, THE Z-TRANSFORM COEFFICIENTS, AND THE'
.BX_'STATE-SPACE REALIZATION COEFFICIENTS FOR THE SELECTED '
.BX_'FILTER.  WHEN THIS IS COMPLETED THE PROGRAM PROMPTS FOR'
.BX_'ANOTHER RUN.  IF THE USER ENTERS Y THE PROGRAM WILL'
.BX_'INITIALIZE, THE RUN NUMBER WILL BE INCREMENTED, AND ANOTHER'
.BX_'RUN CAN BE MADE.  IF THE USER ENTERS N THE PROGRAM WILL END.'
.BX_' '
.BX_'                    ---END OF HELP FILE---'
.DL
```

------------------------------- GROUP 3B -------------------------------

BLCC CLCC ELCC

```
     .DL BLCC
[1]   "
[2]   "---BUTTERWORTH LOW-PASS COMPUTATION CONTROL---
[3]   "
[4]   LHCP
[5]   K10_K0
[6]   BCP
[7]   LCP
[8]   BNC
     .DL
```

```
     .DL CLCC
[1]   "
[2]   "---CHEBYSHEV LOW-PASS COMPUTATION CONTROL---
[3]   "
[4]   LHCP
[5]   K10_K0
[6]   CCP
[7]   LCP
[8]   CNC
     .DL
```

```
     .DL ELCC
[1]   "
[2]   "---ELLIPTIC LOW-PASS COMPUTATION CONTROL---
[3]   "
[4]   LHCP
[5]   K10_K0
[6]   ECP
[7]   LCP
[8]   ENC
     .DL
```

------------------------------ GROUP 3C ------------------------------

BHCC CHCC EHCC

```
     .DL BHCC
[1]   "
[2]   "---BUTTERWORTH HIGH-PASS COMPUTATION CONTROL---
[3]   "
[4]   LHCP
[5]   K10_1%K0
[6]   BCP
[7]   HCP
[8]   BNC
     .DL

     .DL CHCC
[1]   "
[2]   "---CHEBYSHEV HIGH-PASS COMPUTATION CONTROL---
[3]   "
[4]   LHCP
[5]   K10_1%K0
[6]   CCP
[7]   HCP
[8]   CNC
     .DL

     .DL EHCC
[1]   "
[2]   "---ELLIPTIC HIGH-PASS COMPUTATION CONTROL---
[3]   "
[4]   LHCP
[5]   K10_1%K0
[6]   ECP
[7]   HCP
[8]   ENC
     .DL
```

---------------------------- GROUP 3D ----------------------------

BPCC CPCC EPCC

```
    ∇BPCC
[1]  "
[2]  "---BUTTERWORTH BAND-PASS COMPUTATION CONTROL---
[3]  "
[4]  PSCP1
[5]  PK10
[6]  BCP
[7]  BNC
[8]  PCP
    ∇
```

```
    ∇CPCC
[1]  "
[2]  "---CHEBYSHEV BAND-PASS COMPUTATION CONTROL---
[3]  "
[4]  PSCP1
[5]  PK10
[6]  CCP
[7]  CNC
[8]  PCP
    ∇
```

```
    ∇EPCC
[1]  "
[2]  "---ELLIPTIC BAND-PASS COMPUTATION CONTROL---
[3]  "
[4]  PSCP1
[5]  PK10
[6]  ECP
[7]  ENC
[8]  PCP
[9]  EPCP
    ∇
```

```
------------------------------ GROUP 3E ------------------------------

                    BSCC CSCC ESCC

     .DL BSCC
[1]   "
[2]   "---BUTTERWORTH BAND-STOP COMPUTATION CONTROL---
[3]   "
[4]   PSCP1
[5]   SK10
[6]   BCP
[7]   BNC
[8]   SCP
     .DL

     .DL CSCC
[1]   "
[2]   "---CHEBYSHEV BAND-STOP COMPUTATION CONTROL---
[3]   "
[4]   PSCP1
[5]   SK10
[6]   CCP
[7]   CNC
[8]   SCP
     .DL

     .DL ESCC
[1]   "
[2]   "---ELLIPTIC BAND-STOP COMPUTATION CONTROL---
[3]   "
[4]   PSCP1
[5]   SK10
[6]   ECP
[7]   ENC
[8]   SCP
[9]   ESCP
     .DL
```

------------------------------ GROUP 3F ------------------------------

LHHZ PSHZ

```
      .DL LHHZ
[1]    "
[2]    "---LOW-HIGH-PASS H(Z)---
[3]    "
[4]    .GO BC1 IF DT='B'
[5]    .GO BC1 IF DT='C'
[6]    .GO E1 IF DT='E'
[7]   BC1:
[8]    BCLHHZ
[9]    .GO CONT
[10]  E1:
[11]   ELHHZ
[12]  CONT:
[13]   LHHZD
      .DL


      .DL PSHZ
[1]    "
[2]    "---BAND-PASS-STOP H(Z)---
[3]    "
[4]    PSHZD
[5]    .GO BC1 IF DT='B'
[6]    .GO BC1 IF DT='C'
[7]    .GO E1 IF DT='E'
[8]   BC1:
[9]    BCPSHZ
[10]   .GO CONT
[11]  E1:
[12]   .GO EBP1 IF 2=(+/FT='BP')
[13]   .GO EBS1 IF 2=(+/FT='BS')
[14]  EBP1:
[15]   EPHZ
[16]   .GO CONT
[17]  EBS1:
[18]   ESHZ
[19]  CONT:
      .DL
```

------------------------------ GROUP 3G ------------------------------

SPAT THZSC SRC

```
.DL SPAT
 "
 "---SUBSECTION POLE ANGLE & TYPE---
 "
 .GO PS IF 2=(+/FT='BP')
 .GO PS IF 2=(+/FT='BS')
 .GO ODD IF NOE='O'
 .GO EVEN IF NOE='E'
PS:
 .GO OPS IF NOE='O'
 .GO EVEN IF NOE='E'
OPS:
 DB1L_1.DA DB1
 DB0L_1.DA DB0
 PA1_.IO 0
 TYPE_0 2 .RO 'NONE'
 .GO CONT2
ODD:
 DB1L_1.DA DB1
 DB0L_1.DA DB0
 .GO PA0 IF(1^DB0)<0
 .GO PAPI IF(1^DB0).GE 0
PA0:PA1_0
 .GO CONT1
PAPI:PA1_180
CONT1:
 TYPE_ 1 2 .RO 'FO'
 .GO CONT2
EVEN:
 DB1L_DB1
 DB0L_DB0
 PA1_.IO 0
 TYPE_ 0 2 .RO 'NONE'
CONT2:
 SIGZ_.NG1#(DB1L%2)
 SIGZ2_SIGZ*2
 WZ2_DB0L-SIGZ2
 WZ_WZ2*0.5
 PA2_.IO 0
 TANPA2_(WZ%SIGZ)
 TANPA2L_TANPA2
```

```
[41] START1:
[42]  .GO PA090 IF(1^TANPA2L).GE 0
[43]  .GO PA90180 IF(1^TANPA2L)<0
[44] PA090:
[45]  PA2I_ARCTAN(1^TANPA2L)
[46]  PA2I_(180%PI)#PA2I
[47]  PA2_PA2,PA2I
[48]  .GO CONT3
[49] PA90180:
[50]  PA2I_ARCTAN(1^TANPA2L)
[51]  PA2I_((180%PI)#PA2I)+180
[52]  PA2_PA2,PA2I
[53] CONT3:
[54]  TANPA2L_1.DA TANPA2L
[55]  .GO START1 IF(.RO TANPA2L)>0
[56]  PA2L_PA2
[57] START2:
[58]  .GO TYPET1 IF(1^PA2L).LE 45
[59]  .GO TYPEDT IF(1^PA2L).LE 135
[60]  .GO TYPET2 IF(1^PA2L)>135
[61] TYPET1:TYPE_TYPE,[1]1 2 .RO 'T1'
[62]  .GO CONT4
[63] TYPEDT:TYPE_TYPE,[1]1 2 .RO 'DT'
[64]  .GO CONT4
[65] TYPET2:TYPE_TYPE,[1]1 2 .RO 'T2'
[66] CONT4:
[67]  PA2L_1.DA PA2L
[68]  .GO START2 IF(.RO PA2L)>0
[69]  PAT_PA1,PA2
     .DL
```

```
      .DL THZSC
[1]    "
[2]    "---TRANSLATION OF H(Z) SUBSECTION COEFFICIENTS---
[3]    "
[4]    .GO EVEN IF 2=(+/FT='BP')
[5]    .GO EVEN IF 2=(+/FT='BS')
[6]    .GO ODD IF NOE='O'
[7]    .GO EVEN IF NOE='E'
[8]   ODD:
[9]    BE1_0
[10]   BE2_1^DBO
[11]   AL1P_0
[12]   AL2P_(1^DA0)-(1^DBO)
[13]   DA1L_1.DA DA1
[14]   DB1L_1.DA DB1
[15]   DB0L_1.DA DBO
[16]   .GO CONT
[17]  EVEN:
[18]   BE1_.IO 0
[19]   BE2_.IO 0
[20]   AL1P_.IO 0
[21]   AL2P_.IO 0
[22]   DA1L_DA1
[23]   DB1L_DB1
[24]   DB0L_DBO
[25]  CONT:
[26]   BE1_BE1,DB1L
[27]   BE2_BE2,DB0L
[28]   AL1P_AL1P,(DA1L-DB1L)
[29]   AL2P_AL2P,(1-DB0L)
      .DL
```

```
.DL SRC
 "
 "---SUBSECTION REALIZATION COEFFICIENTS---
 "
 OPST_0
 .GO EVEN IF 2=(+/FT='BP')
 .GO EVEN IF 2=(+/FT='BS')
 .GO ODD IF NOE='O'
 .GO EVEN IF NOE='E'
ODD:
 A11_.NG1#(1^BE2)
 A12_0
 A21_0
 A22_0
 B1_(1-((1^BE2)*2))*0.5
 B2_0
 C1P_(1^AL2P)%B1
 C2P_0
 BE1L_1.DA BE1
 BE2L_1.DA BE2
 AL1PL_1.DA AL1P
 AL2PL_1.DA AL2P
 TYPEL_1 0 .DA TYPE
 .GO START
EVEN:
 A11_.IO 0
 A12_.IO 0
 A21_.IO 0
 A22_.IO 0
 B1_.IO 0
 B2_.IO 0
 C1P_.IO 0
 C2P_.IO 0
 BE1L_BE1
 BE2L_BE2
 AL1PL_AL1P
 AL2PL_AL2P
 TYPEL_TYPE
 .GO OPS IF NOE='O'
 .GO START IF NOE='E'
OPS:
 OPST_1
 .GO TYPET1
OPS1:
 NGCT12
 NG1_NG
 OPST_2
```

```
OPS2:
 NGCDT
 NG2_NG
 OPST_3
 .GO TYPET2
OPS3:
 NGCT12
 NG3_NG
 OPST_4
 .GO T1G IF NG1.LE NG2
 .GO DTG IF NG2 < NG1
T1G:
 .GO T3G IF NG3 < NG1
 TYPE_'T1',[1] TYPE
 TYPEL_'T1',[1] TYPEL
 .GO START
DTG:
 .GO T3G IF NG3 < NG2
 TYPE_'DT',[1] TYPE
 TYPEL_'DT',[1] TYPEL
 .GO START
T3G:
 TYPE_'T2',[1] TYPE
 TYPEL_'T2',[1] TYPEL
START:
 .GO TYPET1 IF(2.RO (1 2 ^TYPEL))='T1'
 .GO TYPEDT IF(2.RO (1 2 ^TYPEL))='DT'
 .GO TYPET2 IF(2.RO (1 2 ^TYPEL))='T2'
TYPET1:
 A11I_.NG1#((1^BE1L)+1)
 A12IAN_((1^BE2L)#((1^BE1L)+2))-(1^BE1L)
 A12IAD_(1^BE2L)+1
 A12IA_A12IAN%A12IAD
 A12IB_2+((1^BE1L)#A12IA)
 A12I_A12IB*0.5
 A21IN_.NG1#(1+(1^BE2L)+(1^BE1L))
 A21I_A21IN%A12I
 A22I_1
 .GO CONT
TYPEDT:
 A11I_0
 A12I_1
 A21I_.NG1#(1^BE2L)
 A22I_.NG1#(1^BE1L)
 .GO CONT
```

```
TYPET2:
 A11I_.NG1#((1^BE1L)-1)
 A12IAN_((1^BE2L)#((1^BE1L)-2))-(1^BE1L)
 A12IAD_(1^BE2L)+1
 A12IA_A12IAN%A12IAD
 A12IB_2+((1^BE1L)#A12IA)
 A12I_A12IB*0.5
 A21IN_.NG1#(1+(1^BE2L)-(1^BE1L))
 A21I_A21IN%A12I
 A22I_.NG1
CONT:
 B1I_0
 B2IA_(((1^BE2L)+1)*2)-((1^BE1L)*2)
 B2IB_(1-(1^BE2L))%(1+(1^BE2L))
 B2IC_(B2IB#B2IA)*0.5
 B2I_(1%(1^A12I))#B2IC
 C1PIN_((1^AL1PL)#A11I)+(1^AL2PL)
 C1PID_A12I#B2I
 C1PI_C1PIN%C1PID
 C2PI_(1^AL1PL)%B2I
 .GO OPS1 IF OPST=1
 .GO OPS3 IF OPST=3
 A11_A11,A11I
 A12_A12,A12I
 A21_A21,A21I
 A22_A22,A22I
 B1_B1,B1I
 B2_B2,B2I
 C1P_C1P,C1PI
 C2P_C2P,C2PI
 BE1L_1.DA BE1L
 BE2L_1.DA BE2L
 AL1PL_1.DA AL1PL
 AL2PL_1.DA AL2PL
 TYPEL_ 1 0 .DA TYPEL
 STYPEL_.RO TYPEL
 .GO START IF(1^STYPEL)>0
 DR_(.RO A11).RO 1
.DL
```

------------------------------ GROUP 3H ------------------------------

RCMC

```
      .DL RCMC
[1]    "
[2]    "---REALIZATION COEFFICIENT MATRIX CONSTRUCTION---
[3]    "
[4]    NR_.RO A11
[5]    A11MC_RSEMC A11
[6]    A12MC_RSEMC A12
[7]    A21MC_RSEMC A21
[8]    A22MC_RSEMC A22
[9]    B1MC_RSEMC B1
[10]   B2MC_RSEMC B2
[11]   C1PMC_RSEMC C1P
[12]   C2PMC_RSEMC C2P
[13]   DRMC_RSEMC DR
[14]   NCL_.NG1^(.RO A11MC)
[15]   .GO CONTA11 IF NCL.GE 5
[16]   NCL_5
[17]  CONTA11:
[18]   A11N_(1,NCL).RO NCL^' A11'
[19]   A11MC_(NR,NCL)^A11MC
[20]   A11MC_A11N,[1]A11MC
[21]   NCL_.NG1^(.RO A12MC)
[22]   .GO CONTA12 IF NCL.GE 5
[23]   NCL_5
[24]  CONTA12:
[25]   A12N_(1,NCL).RO NCL^' A12'
[26]   A12MC_(NR,NCL)^A12MC
[27]   A12MC_A12N,[1]A12MC
[28]   NCL_.NG1^(.RO A21MC)
[29]   .GO CONTA21 IF NCL.GE 5
[30]   NCL_5
[31]  CONTA21:
[32]   A21N_(1,NCL).RO NCL^' A21'
[33]   A21MC_(NR,NCL)^A21MC
[34]   A21MC_A21N,[1]A21MC
[35]   NCL_.NG1^(.RO A22MC)
[36]   .GO CONTA22 IF NCL.GE 5
[37]   NCL_5
[38]  CONTA22:
[39]   A22N_(1,NCL).RO NCL^' A22'
[40]   A22MC_(NR,NCL)^A22MC
[41]   A22MC_A22N,[1]A22MC
[42]   NCL_.NG1^(.RO B1MC)
[43]   .GO CONTB1 IF NCL.GE 5
[44]   NCL_5
```

```
[45]  CONTB1:
[46]   B1N_(1,NCL).RO NCL^' B1'
[47]   B1MC_(NR,NCL)^B1MC
[48]   B1MC_B1N,[1]B1MC
[49]   NCL_.NG1^(.RO B2MC)
[50]   .GO CONTB2 IF NCL.GE 5
[51]   NCL_5
[52]  CONTB2:
[53]   B2N_(1,NCL).RO NCL^' B2'
[54]   B2MC_(NR,NCL)^B2MC
[55]   B2MC_B2N,[1]B2MC
[56]   NCL_.NG1^(.RO C1PMC)
[57]   .GO CONTC1P IF NCL.GE 5
[58]   NCL_5
[59]  CONTC1P:
[60]   C1PN_(1,NCL).RO NCL^' C1'
[61]   C1PMC_(NR,NCL)^C1PMC
[62]   C1PMC_C1PN,[1]C1PMC
[63]   NCL_.NG1^(.RO C2PMC)
[64]   .GO CONTC2P IF NCL.GE 5
[65]   NCL_5
[66]  CONTC2P:
[67]   C2PN_(1,NCL).RO NCL^' C2'
[68]   C2PMC_(NR,NCL)^C2PMC
[69]   C2PMC_C2PN,[1]C2PMC
[70]   NCL_.NG1^(.RO DRMC)
[71]   .GO CONTDR IF NCL.GE 5
[72]   NCL_5
[73]  CONTDR:
[74]   DRN_(1,NCL).RO NCL^' D'
[75]   DRMC_(NR,NCL)^DRMC
[76]   DRMC_DRN,[1]DRMC
[77]   SPACE_((NR+1),2)^' '
[78]   RMA_A11MC,[2]SPACE
[79]   RMA_RMA,[2]A12MC
[80]   RMA_RMA,[2]SPACE
[81]   RMA_RMA,[2]A21MC
[82]   RMA_RMA,[2]SPACE
[83]   RMA_RMA,[2]A22MC
[84]   RMA_((NR+1),80)^RMA
[85]   RMBCD_B1MC,[2]SPACE
[86]   RMBCD_RMBCD,[2]B2MC
[87]   RMBCD_RMBCD,[2]SPACE
[88]   RMBCD_RMBCD,[2]C1PMC
[89]   RMBCD_RMBCD,[2]SPACE
[90]   RMBCD_RMBCD,[2]C2PMC
[91]   RMBCD_RMBCD,[2]SPACE
[92]   RMBCD_RMBCD,[2]DRMC
[93]   RMBCD_((NR+1),80)^RMBCD
      .DL
```

------------------------------ GROUP 4A ------------------------------

LHCP LCP HCP

```
      .DL LHCP
[1]     "
[2]     "---LOW-HIGH-PASS COMPUTATION PARAMETERS---
[3]     "
[4]     .GO HZ IF 1=(+/UF2='H')
[5]     .GO RAD IF 1=(+/UF2='R')
[6]    HZ:
[7]     PBW_2#PI#PBF
[8]     SBW_2#PI#SBF
[9]     T_1%SF
[10]    .GO CONT
[11]   RAD:
[12]    PBW_PBF
[13]    SBW_SBF
[14]    T_1%(SF%(2#PI))
[15]   CONT:
[16]    KON_TAN(PBW#(T%2))
[17]    KOD_TAN(SBW#(T%2))
[18]    KO_KON%KOD
[19]    DN_(10*(0.1#SA))-1
[20]    DD_(10*(0.1#PA))-1
[21]    D_DN%DD
      .DL
```

```
     .DL LCP
[1]    "
[2]    "---LOW-PASS COMPUTATION PARAMETERS---
[3]    "
[4]    L_(WP#T)%(2#KON)
     .DL
```

```
     .DL HCP
[1]    "
[2]    "---HIGH-PASS COMPUTATION PARAMETERS---
[3]    "
[4]    L_(2#WP#KON)%T
     .DL
```

---------------------------- GROUP 4B ------------------------------

BCP CCP ECP

```
     .DL BCP
[1]   "
[2]   "---BUTTERWORTH COMPUTATION PARAMETERS---
[3]   "
[4]   NE_(LOG D)%(2#LOG(1%K10))
[5]   NCP
[6]   WP_((10*(0.1#PA))-1)*(1%(2#N))
     .DL

     .DL CCP
[1]   "
[2]   "---CHEBYSHEV COMPUTATION PARAMETERS---
[3]   "
[4]   NE_(ARCCOSH(D*0.5))%(ARCCOSH(1%K10))
[5]   NCP
[6]   WP_1
     .DL

     .DL ECP
[1]   "
[2]   "---ELLIPTIC COMPUTATION PARAMETERS---
[3]   "
[4]   KE_K10
[5]   KEP_(1-KE*2)*0.5
[6]   Q0_0.5#((1-KEP*0.5)%(1+KEP*0.5))
[7]   Q_Q0+(2#Q0*5)+(15#Q0*9)+(150#Q0*13)
[8]   NE_(LOG(16#D))%(LOG(1%Q))
[9]   NCP
[10]  WP_K10*0.5
     .DL
```

------------------------------ GROUP 4C ------------------------------

BNC CNC ENC

```
     .DL BNC
      "
      "---BUTTERWORTH NORMALIZED COMPUTATIONS---
      "
      G1_1
      .GO ODD IF NOE='O'
      .GO EVEN IF NOE='E'
     ODD:
      NA2_(1+N2).RO 0
      NA1_(1+N2).RO 0
      NA0_(1+N2).RO 1
      NB2_0,N2.RO 1
      NB1_1
      NB0_(1+N2).RO 1
      .GO START1
     EVEN:
      NA2_N2.RO 0
      NA1_N2.RO 0
      NA0_N2.RO 1
      NB2_N2.RO 1
      NB1_.IO 0
      NB0_N2.RO 1
     START1:
      K_N2
     START2:
      RAK_PI#(N+(2#K)-1)%(2#N)
      NB1I_.NG2#COS(RAK)
      NB1_NB1,NB1I
      K_K-1
      .GO START2 IF K.GE 1
     .DL
```

```
     .DL CNC
      "
      "---CHEBYSHEV NORMALIZED COMPUTATIONS---
      "
      EP2_(10*(0.1#PA))-1
      EP_EP2*0.5
      CC1_(1%N)#ARCSINH(1%EP)
      CC2_SINH CC1
      CC3_COSH CC1
      .GO ODD IF NOE='O'
      .GO EVEN IF NOE='E'
```

```
ODD:
 G1_CC2
 NA2_(1+N2).RO 0
 NA1_(1+N2).RO 0
 NA0_(1+N2).RO 1
 NB2_0,N2.RO 1
 NB1_1
 NB0_CC2
 .GO START1
EVEN:
 G1_10*(.NG0.05#PA)
 NA2_N2.RO 0
 NA1_N2.RO 0
 NA0_N2.RO 1
 NB2_N2.RO 1
 NB1_.IO 0
 NB0_.IO 0
START1:
 M_(2#N)-(N2-1)
START2:
 CAM_PI#((2#M)-1)%(2#N)
 SIGI_CC2#SIN(CAM)
 OMI_CC3#COS(CAM)
 NB1I_.NG2#SIGI
 NB1_NB1,NB1I
 NB0I_(SIGI*2)+(OMI*2)
 NB0_NB0,NB0I
 G1_G1#NB0I
 M_M+1
 .GO START2 IF M.LE (2#N)
.DL


.DL ENC
 "
 "---ELLIPTIC NORMALIZED COMPUTATIONS---
 "
 LEN_(10*(0.05#PA))+1
 LED_(10*(0.05#PA))-1
 LE_(1%(2#N))#LN(LEN%LED)
 SSUMN_SINH(LE)
 SSUMD_0
 M_1
SIG1:
 SSUMNM_((.NG1)*M)#(Q*(M#(M+1)))#SINH(((2#M)+1)#LE)
 SSUMDM_((.NG1)*M)#(Q*(M*2))#COSH(2#M#LE)
 SSUMN_SSUMN+SSUMNM
 SSUMD_SSUMD+SSUMDM
 M_M+1
 .GO SIG1 IF M.LE 10
 SIGO_(2#(Q*0.25)#SSUMN)%(1+(2#SSUMD))
 WNE1_1+(KE#(SIGO*2))
 WNE2_1+((SIGO*2)%KE)
 WNE_(WNE1#WNE2)*0.5
```

```
  I_1
  .GO ODD IF NOE='O'
  .GO EVEN IF NOE='E'
 ODD:
  MU_1
  G1_SIGO
  NA2_0,N2.RO 1
  NA1_(1+N2).RO 0
  NA0_1
  NB2_0,N2.RO 1
  NB1_1
  NB0_SIGO
  .GO START
 EVEN:
  MU_0.5
  G1_10*(.NG0.05#PA)
  NA2_N2.RO 1
  NA1_N2.RO 0
  NA0_.IO 0
  NB2_N2.RO 1
  NB1_.IO 0
  NB0_.IO 0
 START:
  OSUMN_SIN(PI#MU%N)
  OSUMD_0
  M_1
 OMI1:
  OSUMNM_((.NG1)*M)#(Q*(M#(M+1)))#SIN(((2#M)+1)#PI#MU%N)
  OSUMDM_((.NG1)*M)#(Q*(M*2))#COS(2#M#PI#MU%N)
  OSUMN_OSUMN+OSUMNM
  OSUMD_OSUMD+OSUMDM
  M_M+1
  .GO OMI1 IF M.LE 10
  OMI_(2#(Q*0.25)#OSUMN)%(1+(2#OSUMD))
  OMI2_OMI*2
  VI_((1-KE#OMI2)#(1-(OMI2%KE)))*0.5
  NA0I_1%OMI2
  NA0_NA0,NA0I
  NB1IN_2#SIGO#VI
  NB1ID_1+(SIGO*2)#(OMI*2)
  NB1I_NB1IN%NB1ID
  NB1_NB1,NB1I
  NB0IN_((SIGO#VI)*2)+((OMI#WNE)*2)
  NB0ID_NB1ID*2
  NB0I_NB0IN%NB0ID
  NB0_NB0,NB0I
  G1_G1#(NB0I%NA0I)
  I_I+1
  MU_MU+1
  .GO START IF I.LE N2
.DL
```

----------------------------- GROUP 4D -----------------------------

PSCP1 PCP SCP

```
 .DL PSCP1
  "
  "---BAND-PASS-STOP COMPUTATION PARAMETERS #1---
  "
  .GO HZ IF 1=(+/UF2='H')
  .GO RAD IF 1=(+/UF2='R')
 HZ:
  SBW1_2#PI#SBF1
  PBW1_2#PI#PBF1
  PBW2_2#PI#PBF2
  SBW2_2#PI#SBF2
  T_1%SF
  .GO CONT
 RAD:
  SBW1_SBF1
  PBW1_PBF1
  PBW2_PBF2
  SBW2_SBF2
  T_1%(SF%(2#PI))
 CONT:
  KA_(TAN(PBW2#T%2))-(TAN(PBW1#T%2))
  KB_(TAN(PBW1#T%2))#(TAN(PBW2#T%2))
  KC_(TAN(SBW1#T%2))#(TAN(SBW2#T%2))
  K1N_KA#TAN(SBW1#T%2)
  K1D_KB-(TAN(SBW1#T%2))*2
  K1_K1N%K1D
  K2N_KA#TAN(SBW2#T%2)
  K2D_((TAN(SBW2#T%2))*2)-KB
  K2_K2N%K2D
  DN_(10*(0.1#SA))-1
  DD_(10*(0.1#PA))-1
  D_DN%DD
 .DL
```

```
.DL PCP
 "
 "---BAND-PASS COMPUTATION PARAMETERS---
 "
 WO_2#(KB*0.5)%T
 B_(2#KA)%(T#WP)
 PSCP2
 SIGA_B#SIG
 WA_B#W
 .GO ODD IF NOE='O'
 .GO EVEN IF NOE='E'
ODD:
 BB1_B#(1^NBO)
 BBO_WO*2
 .GO CONT
EVEN:
 BB1_.IO 0
 BBO_.IO 0
CONT:
 PSCP3
.DL
```

```
.DL SCP
 "
 "---BAND-STOP COMPUTATION PARAMETERS---
 "
 WO_2#(KB*0.5)%T
 B_(2#KA#WP)%T
 PSCP2
 RS_B%(((SIG*2)+(W*2))*0.5)
 PSI_(ARCTAN(W%SIG))+PI
 SIGA_RS#COS(-PSI)
 WA_RS#SIN(-PSI)
 .GO ODD IF NOE='O'
 .GO EVEN IF NOE='E'
ODD:
 BB1_B%(1^NBO)
 BBO_WO*2
 .GO CONT
EVEN:
 BB1_.IO 0
 BBO_.IO 0
CONT:
 PSCP3
.DL
```

------------------------------ GROUP 4E ------------------------------

PK10 SK10

```
    .DL PK10
[1]  "
[2]  "---BAND-PASS K10---
[3]  "
[4]  .GO K1A IF KC.GE KB
[5]  .GO K2A IF KC<KB
[6] K1A:K10_K1
[7]  .GO END
[8] K2A:K10_K2
[9] END:
    .DL
```

```
    .DL SK10
[1]  "
[2]  "---BAND-STOP K10---
[3]  "
[4]  .GO K1A IF KC<KB
[5]  .GO K2A IF KC.GE KB
[6] K1A:K10_1%K1
[7]  .GO END
[8] K2A:K10_1%K2
[9] END:
    .DL
```

------------------------------ GROUP 4F ------------------------------

EPCP ESCP

```
      .DL EPCP
[1]    "
[2]    "---ELLIPTIC BAND-PASS COMPUTATION PARAMETERS---
[3]    "
[4]    WF_NA0*0.5
[5]    WG_B#WF%2
[6]    WH1_.NG1#(B*2)#(WF*2)
[7]    WH2_4#(W0*2)
[8]    WH_((| (WH1-WH2))*0.5)%2
[9]    .GO ODD IF NOE='O'
[10]   .GO EVEN IF NOE='E'
[11]  ODD:
[12]   BA2_0
[13]   BA1_B
[14]   BA0_0
[15]   .GO CONT
[16]  EVEN:
[17]   BA2_.IO 0
[18]   BA1_.IO 0
[19]   BA0_.IO 0
[20]  CONT:
[21]   EPSCP
      .DL
```

```
      .DL ESCP
[1]    "
[2]    "---ELLIPTIC BAND-STOP COMPUTATION PARAMETERS---
[3]    "
[4]    WF_NA0*0.5
[5]    WG_B%(2#WF)
[6]    WH1_.NG1#(B*2)%(WF*2)
[7]    WH2_4#(W0*2)
[8]    WH_((| (WH1-WH2))*0.5)%2
[9]    .GO ODD IF NOE='O'
[10]   .GO EVEN IF NOE='E'
[11]  ODD:
[12]   BA2_1
[13]   BA1_0
[14]   BA0_W0*2
[15]   .GO CONT
[16]  EVEN:
[17]   BA2_.IO 0
[18]   BA1_.IO 0
[19]   BA0_.IO 0
[20]  CONT:
[21]   EPSCP
      .DL
```

--------------------------- GROUP 4G ---------------------------

BCLHHZ ELHHZ LHHZD

```
.DL BCLHHZ
 "
 "---BUTTERWORTH CHEBYSHEV LOW-HIGH-PASS H(Z)---
 "
 G4_1
 .GO ODD IF NOE='O'
 .GO EVEN IF NOE='E'
ODD:
 .GO LP1 IF 2=(+/FT='LP')
 .GO HP1 IF 2=(+/FT='HP')
LP1:
 DA2_0,N2.RO 1
 DA1_1,N2.RO 2
 DA0_(1+N2).RO 1
 KT_L#(2%T)
 G2_1%(KT+(1^NB0))
 DB2_0
 DB1_1
 DB0_((1^NB0)-KT)%(KT+(1^NB0))
 HZN1_ 1 80 .RO 80^'(Z + 1)'
 .GO N20 IF N2=0
 HZN2_'(Z2 + 2 Z + 1)',N2C
 HZN2_ 1 80 .RO 80^HZN2
 .GO CONT
HP1:
 DA2_0,N2.RO 1
 DA1_1,N2.RO .NG2
 DA0_.NG1,N2.RO 1
 KT_L#(T%2)
 G2_1%(KT+(1^NB0))
 DB2_0
 DB1_1
 DB0_(KT-(1^NB0))%(KT+(1^NB0))
 HZN1_ 1 80 .RO 80^'(Z - 1)'
 .GO N20 IF N2=0
 HZN2_'(Z2 - 2 Z + 1)',N2C
 HZN2_ 1 80 .RO 80^HZN2
 .GO CONT
N20:
 HZN2_0 80 .RO 'NONE'
 .GO CONT
EVEN:
 G2_1
 HZN1_ 0 80 .RO 'NONE'
 DB2_.IO 0
 DB1_.IO 0
 DB0_.IO 0
 .GO LP2 IF 2=(+/FT='LP')
 .GO HP2 IF 2=(+/FT='HP')
```

```
 LP2:
  DA2_N2.RO 1
  DA1_N2.RO 2
  DA0_N2.RO 1
  KT_L#(2%T)
  HZN2_'(Z2 + 2 Z + 1)',N2C
  HZN2_ 1 80 .RO 80^HZN2
  .GO CONT
 HP2:
  DA2_N2.RO 1
  DA1_N2.RO .NG2
  DA0_N2.RO 1
  KT_L#(T%2)
  HZN2_'(Z2 - 2 Z + 1)',N2C
  HZN2_ 1 80 .RO 80^HZN2
 CONT:
  HZN_ 1 80 .RO ' '
  HZN_HZN,[1]1 80 .RO 80^'NUMERATOR ='
  HZN_HZN,[1]1 80 .RO ' '
  HZN_HZN,[1]HZN1
  HZN_HZN,[1]HZN2
 .DL


 .DL ELHHZ
   ''
   '---ELLIPTIC LOW-HIGH-PASS H(Z)---
   ''
  G4_1
  .GO ODD IF NOE='O'
  .GO EVEN IF NOE='E'
 ODD:
  NAOL_1.DA NAO
  .GO LP1 IF 2=(+/FT='LP')
  .GO HP1 IF 2=(+/FT='HP')
 LP1:
  DA2_0,N2.RO 1
  DA1_1
  DA0_1,N2.RO 1
  HZN1_ 1 80 .RO 80^'(Z + 1)'
  KT_L#(2%T)
  G2_1%(KT+(1^NBO))
  DB2_0
  DB1_1
  DB0_((1^NBO)-KT)%(KT+(1^NBO))
  .GO N20 IF N2=0
  .GO START1
```

```
[23]  HP1:
[24]   DA2_0,N2.RO 1
[25]   DA1_1
[26]   DA0_.NG1,N2.RO 1
[27]   HZN1_ 1 80 .RO 80^'(Z - 1)'
[28]   KT_L#(T%2)
[29]   G2_1%(KT+(1^NB0))
[30]   DB2_0
[31]   DB1_1
[32]   DB0_(KT-(1^NB0))%(KT+(1^NB0))
[33]   .GO N20 IF N2=0
[34]   .GO START1
[35]  N20:
[36]   HZN2_0 80 .RO 'NONE'
[37]   .GO CONT1
[38]  EVEN:
[39]   G2_1
[40]   DA2_N2.RO 1
[41]   DA1_.IO 0
[42]   DA0_N2.RO 1
[43]   HZN1_ 0 80 .RO 'NONE'
[44]   DB2_.IO 0
[45]   DB1_.IO 0
[46]   DB0_.IO 0
[47]   NA0L_NA0
[48]   .GO LP2 IF 2=(+/FT='LP')
[49]   .GO HP2 IF 2=(+/FT='HP')
[50]  LP2:
[51]   KT_L#(2%T)
[52]   .GO START1
[53]  HP2:
[54]   KT_L#(T%2)
[55]  START1:
[56]   HZN2_ 0 80 .RO 'INITIALIZE'
[57]   I_1
[58]  START2:
[59]   .GO LP3 IF 2=(+/FT='LP')
[60]   .GO HP3 IF 2=(+/FT='HP')
[61]  LP3:
[62]   DA1IN_2#((1^NA0L)-(KT*2))
[63]   .GO START3
[64]  HP3:
[65]   DA1IN_2#((KT*2)-(1^NA0L))
[66]  START3:
[67]   DA1ID_(KT*2)+(1^NA0L)
[68]   DA1I_DA1IN%DA1ID
[69]   DA1_DA1,DA1I
[70]   G4_G4#DA1ID
[71]   .GO PDA1I IF DA1I.GE 0
[72]   .GO NDA1I IF DA1I<0
```

```
 PDA1I:
  DA1IC_'+ ',.FM DA1I
  .GO CONT
 NDA1I:
  DA1IC_'- ',.FM (| DA1I)
 CONT:
  HZN2I_'(Z2 ',DA1IC,' Z + 1)'
  HZN2I_ 1 80 .RO 80^HZN2I
  HZN2_HZN2,[1]HZN2I
  NA0L_1.DA NA0L
  I_I+1
  .GO START2 IF I.LE N2
 CONT1:
  HZN_ 1 80 .RO ' '
  HZN_HZN,[1]1 80 .RO 80^'NUMERATOR ='
  HZN_HZN,[1]1 80 .RO ' '
  HZN_HZN,[1]HZN1
  HZN_HZN,[1]HZN2
 .DL


 .DL LHHZD
  "
  "---LOW-HIGH-PASS H(Z) DENOMINATOR---
  "
  .GO ODD IF NOE='O'
  .GO EVEN IF NOE='E'
 ODD:
  .GO PDB0 IF DB0.GE 0
  .GO NDB0 IF DB0<0
 PDB0:
  DB0C_'+ ',.FM DB0
  .GO CONT1
 NDB0:
  DB0C_'- ',.FM (| DB0)
 CONT1:
  HZD1_'(Z ',DB0C,')'
  HZD1_ 1 80 .RO 80^HZD1
  NB1L_1.DA NB1
  NB0L_1.DA NB0
  .GO START1
 EVEN:
  HZD1_ 0 80 .RO 'INITIALIZE'
  NB1L_NB1
  NB0L_NB0
 START1:
  G3_1
  DB2_DB2,N2.RO 1
  I_1
  HZD2_ 0 80 .RO 'INITIALIZE'
 START2:
  .GO LP1 IF 2=(+/FT='LP')
  .GO HP1 IF 2=(+/FT='HP')
```

```
LP1:
 DB1IN_2#((1^NB0L)-(KT*2))
 .GO START3
HP1:
 DB1IN_2#((KT*2)-(1^NB0L))
START3:
 DB1ID_(KT*2)+((1^NB1L)#KT)+(1^NB0L)
 DB1I_DB1IN%DB1ID
 DB1_DB1,DB1I
 DB0IN_(KT*2)+(1^NB0L)-((1^NB1L)#KT)
 DB0I_DB0IN%DB1ID
 DB0_DB0,DB0I
 G3_G3#(1%DB1ID)
 .GO PDB1I IF DB1I.GE 0
 .GO NDB1I IF DB1I<0
PDB1I:
 DB1IC_'+ ',.FM DB1I
 .GO CONT2
NDB1I:
 DB1IC_'- ',.FM (| DB1I)
CONT2:
 .GO PDB0I IF DB0I.GE 0
 .GO NDB0I IF DB0I<0
PDB0I:
 DB0IC_'+ ',.FM DB0I
 .GO CONT3
NDB0I:
 DB0IC_'- ',.FM (| DB0I)
CONT3:
 HZD2I_'(Z2 ',DB1IC,' Z ',DB0IC,')'
 HZD2I_ 1 80 .RO 80^HZD2I
 HZD2_HZD2,[1]HZD2I
 NB1L_1.DA NB1L
 NB0L_1.DA NB0L
 I_I+1
 .GO START2 IF I.LE N2
 GLH_G1#G2#G3#G4
 G_GLH
 HZD_ 1 80 .RO ' '
 HZD_HZD,[1]1 80 .RO 80^'DENOMINATOR ='
 HZD_HZD,[1]1 80 .RO ' '
 HZD_HZD,[1]HZD1
 HZD_HZD,[1]HZD2
.DL
```

------------------------------ GROUP 4H ------------------------------

PSHZD BCPSHZ EPHZ ESHZ

```
.DL PSHZD
 "
 "---BAND-PASS-STOP H(Z) DENOMINATOR---
 "
 DB2_N.RO 1
 DB1N_(.NG8%(T*2))+(2#BB0)
 DB1D_(4%(T*2))+(BB1#(2%T))+BB0
 DB1_DB1N%DB1D
 DB0N_((4%(T*2))-(BB1#(2%T)))+BB0
 DB0D_DB1D
 DB0_DB0N%DB0D
 HZD2_ 0 80 .RO 'INITIALIZE'
 I_1
START:
 DB1I_1^DB1
 DB0I_1^DB0
 .GO PDB1I IF DB1I.GE 0
 .GO NDB1I IF DB1I<0
PDB1I:
 DB1IC_'+ ',.FM DB1I
 .GO CONT1
NDB1I:
 DB1IC_'- ',.FM (| DB1I)
CONT1:
 .GO PDB0I IF DB0I.GE 0
 .GO NDB0I IF DB0I<0
PDB0I:
 DB0IC_'+ ',.FM DB0I
 .GO CONT2
NDB0I:
 DB0IC_'- ',.FM (| DB0I)
CONT2:
 HZDI_'(Z2 ',DB1IC,' Z ',DB0IC,')'
 HZDI_ 1 80 .RO 80^HZDI
 HZD2_HZD2,[1]HZDI
 DB1_1.RV DB1
 DB0_1.RV DB0
 I_I+1
 .GO START IF I.LE N
 HZD_ 1 80 .RO ' '
 HZD_HZD,[1]1 80 .RO 80^'DENOMINATOR ='
 HZD_HZD,[1]1 80 .RO ' '
 HZD_HZD,[1]HZD2
.DL
```

```
      .DL BCPSHZ
[1]    "
[2]    "---BUTTERWORTH CHEBYSHEV BAND-PASS-STOP H(Z)---
[3]    "
[4]    .GO BP1 IF 2=(+/FT='BP')
[5]    .GO BS1 IF 2=(+/FT='BS')
[6]   BP1:
[7]    GBP_(B*N)#(H1)#((2%T)*N)
[8]    GBP_GBP%(#/DB1D)
[9]    GB_GBP
[10]   G_GB
[11]   DA2_N.RO 1
[12]   DA1_N.RO 0
[13]   DA0_N.RO .NG1
[14]   HZN2_'(Z2 - 1)',NC
[15]   HZN2_ 1 80 .RO 80^HZN2
[16]   .GO CONT1
[17]  BS1:
[18]   G5_1%(#/NB0)
[19]   GBS1_G5#G1
[20]   GBS1_GBS1%(#/DB1D)
[21]   GBS2_((4%(T*2))+(W0*2))*N
[22]   GBS_GBS1#GBS2
[23]   GB_GBS
[24]   G_GB
[25]   DA2_N.RO 1
[26]   DA1N_(2#(W0*2))-(8%(T*2))
[27]   DA1D_(4%(T*2))+(W0*2)
[28]   DA1_(DA1N%DA1D)
[29]   .GO PDA1 IF DA1.GE 0
[30]   .GO NDA1 IF DA1<0
[31]  PDA1:
[32]   DA1C_'+ ',.FM DA1
[33]   .GO CONT2
[34]  NDA1:
[35]   DA1C_'- ',.FM (| DA1)
[36]  CONT2:
[37]   DA1_N.RO DA1
[38]   DA0_N.RO 1
[39]   HZN2_'(Z2 ',DA1C,' Z + 1)',NC
[40]   HZN2_ 1 80 .RO 80^HZN2
[41]  CONT1:
[42]   HZN_ 1 80 .RO ' '
[43]   HZN_HZN,[1]1 80 .RO 80^'NUMERATOR ='
[44]   HZN_HZN,[1]1 80 .RO ' '
[45]   HZN_HZN,[1]HZN2
      .DL
```

```
.DL EPHZ
 "
 "---ELLIPTIC BAND-PASS H(Z)---
 "
 .GO ODD IF NOE='O'
 .GO EVEN IF NOE='E'
ODD:
 BA0L_1 .DA BA0
 GBP1_((2#B)%T)%(1^DB1D)
 GBP2_G1%(#/(1.DA DB1D))
 GBP2_GBP2#(#/((4%(T*2))+BA0L))
 DA2_N.RO 1
 DA1_0
 DA0_.NG1,(N-1).RO 1
 HZN2_'(Z2 - 1)'
 HZN2_ 1 80 .RO 80^HZN2
 .GO CONT
EVEN:
 BA0L_BA0
 GBP1_1
 GBP2_G1%(#/DB1D)
 GBP2_GBP2#(#/((4%(T*2))+BA0L))
 DA2_N.RO 1
 DA1_.IO 0
 DA0_N.RO 1
 HZN2_ 0 80 .RO 'INITIALIZE'
CONT:
 EPSHZ
 GBP_GBP1#GBP2
 GB_GBP
 G_GB
.DL
```

```
.DL ESHZ
 "
 "---ELLIPTIC BAND-STOP H(Z)---
 "
 .GO ODD IF NOE='O'
 .GO EVEN IF NOE='E'
ODD:
 BA0L_1 .DA BA0
 GBS1N_(1%(1^NB0))#((2%T)+(W0*2))
 GBS1D_1^DB1D
 GBS1_GBS1N%GBS1D
 GBS2_G1%(#/(1.DA DB1D))
 GBS2_GBS2#(#/((4%(T*2))+BA0L))
 DA2_N.RO 1
 DA1_((2#(W0*2))-(4%T))%((2%T)+W0)
 DA0_N.RO 1
 .GO PDA1 IF DA1.GE 0
 .GO NDA1 IF DA1<0
PDA1:
 DA1C_'+ ',.FM DA1
 .GO CONT1
NDA1:
 DA1C_'- ',.FM (| DA1)
CONT1:
 HZN2_'(Z2 ',DA1C,' Z + 1)'
 HZN2_ 1 80 .RO 80^HZN2
 .GO CONT2
EVEN:
 BA0L_BA0
 GBS1_1
 GBS2_G1%(#/DB1D)
 GBS2_GBS2#(#/((4%(T*2))+BA0L))
 DA2_N.RO 1
 DA1_.IO 0
 DA0_N.RO 1
 HZN2_ 0 80 .RO 'INITIALIZE'
CONT2:
 EPSHZ
 GBS_GBS1#GBS2
 GB_GBS
 G_GB
.DL
```

---------------------------- GROUP 4I ----------------------------

NGCT12 NGCDT

```
.DL NGCT12
"
"---NOISE GAIN COMPUTATION FOR TYPES I & II REALIZATIONS---
"
BE1G_.NG1#(A11I + A22I)
BE2G_(A11I#A22I)-(A12I#A21I)
PG_(BE2G-1)#(((BE2G+1)*2)-(BE1G*2))
QG_(A21I#C2PI)-(A22I#C1PI)
RG_(A12I#C1PI)-(A11I#C2PI)
W11NA_2#C1PI#QG#BE1G
W11NB_((C1PI*2)+(QG*2))#(BE2G+1)
W11N_W11NA-W11NB
W11_W11N%PG
W22NA_2#C2PI#RG#BE1G
W22NB_((C2PI*2)+(RG*2))#(BE2G+1)
W22N_W22NA-W22NB
W22_W22N%PG
NG_W11+W22
.DL
```

```
.DL NGCDT
"
"---NOISE GAIN COMPUTATION FOR DIRECT TYPE REALIZATION---
"
LG_(1^BE1)%((1^BE2)+1)
NGNA_((1^AL1P)*2)+((1^AL2P)*2)
NGNB_2#LG#(1^AL1P)#(1^AL2P)
NGN_NGNA-NGNB
NGDA_1-((1^B2)*2)
NGDB_(1-(LG*2))*2
NGD_NGDA#NGDB
NG_NGN%NGD
.DL
```

----------------------------- GROUP 4J -----------------------------

RSEMC

```
 .DL RSEM_RSEMC REV
  "
  "---REALIZATION SIGNED ELEMENT MATRIX CONSTRUCTION---
  "
  REM_(NR,1) .RO (| REV)
  REMC_.FM REM
  RSM_0 1 .RO 'INITIALIZE'
 START:
  REVI_1^REV
  .GO NEG IF REVI < 0
  .GO POS IF REVI .GE 0
 NEG:
  RSM_RSM,[1] '-'
  .GO CONT
 POS:
  RSM_RSM,[1] ' '
 CONT:
  REV_1 .DA REV
  .GO START IF (.RO REV) .GE 1
  RSEM_RSM,[2] REMC
 .DL
```

--------------------------- GROUP 5A ---------------------------

NCP

```
      .DL NCP
[1]    "
[2]    "---N COMPUTATION PARAMETERS---
[3]    "
[4]    N_.CE NE
[5]    GO SINGLE IF N=1
[6]    .GO EVEN IF N=2#.CE (N%2)
[7]   ODD:
[8]    NOE_'O'
[9]    N2_(N-1)%2
[10]   .GO CONT
[11]  EVEN:
[12]   NOE_'E'
[13]   N2_N%2
[14]   .GO CONT
[15]  SINGLE:
[16]   NOE_'S'
[17]   N2_0
[18]  CONT:
[19]   NEC_.FM NE
[20]   NC_.FM N
[21]   .GO N21 IF N2=1
[22]   N2C_.FM N2
[23]   .GO CONT1
[24]  N21:
[25]   N2C_' '
[26]  CONT1:
[27]   NPE_NE
[28]   NP_N
[29]   NB_2#N
[30]   NPEC_.FM NPE
[31]   NPC_.FM NP
[32]   NBC_.FM NB
      .DL
```

---------------------------- GROUP 5B ------------------------------

PSCP2 PSCP3

```
      .DL PSCP2
[1]    "
[2]    "---BAND-PASS-STOP COMPUTATION PARAMETERS #2---
[3]    "
[4]    .GO ODD IF NOE='O'
[5]    .GO EVEN IF NOE='E'
[6]   ODD:
[7]    NB1L_1.DA NB1
[8]    NB0L_1.DA NB0
[9]    .GO CONT
[10]  EVEN:
[11]   NB1L_NB1
[12]   NB0L_NB0
[13]  CONT:
[14]   SIG_.NG1#(NB1L%2)
[15]   SIG2_SIG*2
[16]   W2_NB0L-SIG2
[17]   W_W2*0.5
      .DL


      .DL PSCP3
[1]    "
[2]    "---BAND-PASS-STOP COMPUTATION PARAMETERS #3---
[3]    "
[4]    SIGAP_SIGA%2
[5]    WAP_WA%2
[6]    SIGBPN_((SIGA*2)-(WA*2))-(4#(W0*2))
[7]    SIGBPD_4
[8]    SIGBP_SIGBPN%SIGBPD
[9]    WBP_(WA#SIGA)%2
[10]   R_((SIGBP*2)+(WBP*2))*0.5
[11]   TH_.IO 0
[12]   I_1
[13]  START1:
[14]   .GO PSIGBP IF(1^SIGBP).GE 0
[15]   .GO NSIGBP IF(1^SIGBP)<0
[16]  PSIGBP:
[17]   THI_ARCTAN((1^WBP)%(1^SIGBP))
[18]   TH_TH,THI
[19]   .GO CONT1
[20]  NSIGBP:
[21]   THI_(ARCTAN((1^WBP)%(1^SIGBP)))+PI
[22]   TH_TH,THI
```

```
CONT1:
 SIGBP_1.RV SIGBP
 WBP_1.RV WBP
 I_I+1
 .GO START1 IF I.LE N2
 RP_R*0.5
 THP_TH%2
 SIGCP_RP#COS(THP)
 WCP_RP#SIN(THP)
 SIGD_SIGAP+SIGCP
 WD_WAP+WCP
 SIGE_SIGAP-SIGCP
 WE_WAP-WCP
 BB1D_.NG2#SIGD
 BB0D_(SIGD*2)+(WD*2)
 BB1E_.NG2#SIGE
 BB0E_(SIGE*2)+(WE*2)
 I_1
START2:
 BB1_BB1,1^BB1D
 BB1_BB1,1^BB1E
 BB0_BB0,1^BB0D
 BB0_BB0,1^BB0E
 BB1D_1.RV BB1D
 BB0D_1.RV BB0D
 BB1E_1.RV BB1E
 BB0E_1.RV BB0E
 I_I+1
 .GO START2 IF I.LE N2
.DL
```

------------------------------ GROUP 5C ------------------------------

EPSCP

```
      .DL EPSCP
[1]     "
[2]     "---ELLIPTIC BAND-PASS-STOP COMPUTATION PARAMETERS---
[3]     "
[4]     BA2_BA2,(2#N2).RO 1
[5]     BA1_BA1,(2#N2).RO 0
[6]     I_1
[7]    START:
[8]     BA0_BA0,((1^WG)+(1^WH))*2
[9]     BA0_BA0,((1^WG)-(1^WH))*2
[10]    .GO END IF I=N2
[11]    WG_1.RV WG
[12]    WH_1.RV WH
[13]    I_I+1
[14]    .GO START IF I.LE N2
[15]   END:
      .DL
```

---------------------------- GROUP 5D ----------------------------

EPSHZ

```
.DL EPSHZ
 "
 "---ELLIPTIC BAND-PASS-STOP H(Z)---
 "
START:
 DA1IN_(2#(1^BA0L))-(8%(T*2))
 DA1ID_(4%(T*2))+(1^BA0L)
 DA1I_DA1IN%DA1ID
 DA1_DA1,DA1I
 .GO PDA1I IF DA1I.GE 0
 .GO NDA1I IF DA1I<0
PDA1I:
 DA1IC_'+ ',.FM DA1I
 .GO CONT
NDA1I:
 DA1IC_'- ',.FM (| DA1I)
CONT:
 HZNI_'(Z2 ',DA1IC,' Z + 1)'
 HZNI_ 1 80 .RO 80^HZNI
 HZN2_HZN2,[1]HZNI
 BA0L_1.DA BA0L
 SBA0L_.RO BA0L
 .GO START IF SBA0L>0
 HZN_ 1 80 .RO ' '
 HZN_HZN,[1]1 80 .RO 80^'NUMERATOR ='
 HZN_HZN,[1]1 80 .RO ' '
 HZN_HZN,[1]HZN2
.DL
```

---------------------------- GROUP 6A ----------------------------

SIN COS TAN

```
     .DL B_SIN A
[1]   B_1.IO A
     .DL

     .DL B_COS A
[1]   B_2.IO A
     .DL

     .DL B_TAN A
[1]   B_3.IO A
     .DL
```

---------------------------- GROUP 6B ----------------------------

ARCSIN ARCCOS ARCTAN

```
     .DL B_ARCSIN A
[1]   B_.NG1.IO A
     .DL

     .DL B_ARCCOS A
[1]   B_.NG2.IO A
     .DL

     .DL B_ARCTAN A
[1]   B_.NG3.IO A
     .DL
```

------------------------------ GROUP 6C ------------------------------

SINH COSH TANH

```
     .DL B_SINH A
[1]   B_5.LO A
     .DL

     .DL B_COSH A
[1]   B_6.LO A
     .DL

     .DL B_TANH A
[1]   B_7.LO A
     .DL
```

------------------------------ GROUP 6D ------------------------------

ARCSINH ARCCOSH ARCTANH

```
     .DL B_ARCSINH A
[1]   B_.NG5.LO A
     .DL

     .DL B_ARCCOSH A
[1]   B_.NG6.LO A
     .DL

     .DL B_ARCTANH A
[1]   B_.NG7.LO A
     .DL
```

---------------------------- GROUP 6E ----------------------------

LOG LN IF

```
    .DL B_LOG A
[1]  B_10.LG A
    .DL

    .DL B_LN A
[1]  B_.LG A
    .DL

    .DL C_B IF A
[1]  C_A/B
    .DL
```

VITA

Walter James Eaton was born on June 28, 1947 in Philadelphia, Pennsylvania. He attended elementary and secondary schools in central Florida and graduated in 1965 from Leesburg High School in Leesburg, Florida. At that time he began working as an installer of telephone switching systems for contracting firms and also began periodic attendance at Auburn University in Auburn, Alabama. In 1970, he received a first class radio telephone license and shortly thereafter began working as a foreman of an installation crew for ITT in San Juan, Puerto Rico. ITT was then installing telephone multiplex systems for the local telephone company. In 1974, he received a B.S. degree in Electrical Engineering from Auburn University and was at that time supervising the reconfiguration of the San Juan telephone cable multiplex systems for ITT. In 1975, he transferred to the engineering department at ITT and began working with high-density microwave multiplex systems.

In 1979, Mr. Eaton took a position with Continental Telephone Company in Knoxville, Tennessee as a central office equipment engineer and began an M.S. program in Electrical Engineering in the evening school at the University of Tennessee. In 1982, Continental Telephone Company closed the Knoxville division office and Mr. Eaton began working with computer controlled machinery and robotic systems at the Department of Energy Y-12 Plant in nearby Oak Ridge, Tennessee. At that time the plant was operated by Union Carbide, Inc., and is now operated by Martin Marietta Energy Systems, Inc. That same year

Mr. Eaton was licensed as a Professional Engineer by the State of Tennessee. In March 1987, he received the M.S. degree in Electrical Engineering from the University of Tennessee.

Mr. Eaton is a member of the Institute of Electrical and Electronics Engineers and the National Society of Professional Engineers.