To the Graduate Council:

I am submitting herewith a thesis written by Ricky L. Arterbury entitled "The Development of a Three-Axis Trim Program for Aerodynamic Data." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Aerospace Engineering.

William B. Baker, Jr.,
Major Professor

We have read this thesis
and recommend its acceptance:

Charles C. Limbaugh

Accepted for the Council:

Vice Provost
and Dean of The Graduate School

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a Master's degree at The University of Tennessee, Knoxville, I agree that the Library shall make it available to borrowers under rules of the Library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of the source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Head of Interlibrary Services when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my written permission.

Signature Ricky L. Arterburn

Date 4-25-90

THE DEVELOPMENT OF A THREE-AXIS TRIM PROGRAM FOR AERODYNAMIC DATA

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Ricky L. Arterbury

May 1990

ACKNOWLEDGMENTS

I wish to thank Calspan Corporation and the U. S. Air Force at AEDC for the opportunity to attend classes during business hours and for the payment of tuition and fees. Dr. W. B. Baker, Jr. has provided encouragement and advice throughout my efforts to earn the Master of Science degree. Dr. Baker also provided the "aerodynamic data simulator" program that was a vital part of the validation phase of the trim program development. Mr. T. F. Langham developed the Aerodynamic Data Analysis program and provided many helpful comments during the development of the three-axis trim option. Finally, I thank Ms. Danna Pemberton for her programing support in adding the new trim option to the Aerodynamic Data Analysis program.

ABSTRACT

This thesis presents the development of a three-axis trim program. The goal of three-axis trim is to position airplane or missile control surfaces at deflection angles which result in zero aerodynamic moments about three orthogonal axes through the vehicle center of gravity. The moment about each axis can be expressed as a single equation in terms of control effectiveness derivatives, deflection angles, and residual moments resulting from the deflections. Therefore, the essence of the problem is the solution of three equations with three unknowns (the deflections at trim). Gaussian Elimination is used to solve the equations with the control derivatives being the matrix of coefficients and the residual moments completing the augmented matrix. The solution is iterative with the control derivatives and residual moments changing for each new estimate of the trim deflections. The values of other aerodynamic parameters at the trim deflections are also available for output. The methods of updating the control effectiveness and residual moments are presented, and an example case is given.

PREFACE

The work reported herein was conducted by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), at the request of AEDC/DOFA, AEDC Project Number CL63PW. The AEDC/DOFA project manager was Mr. James McComb. The results were obtained by Calspan Corporation/AEDC Operations, operating contractor for the Aerospace Flight Dynamics testing effort at AEDC, AFSC, Arnold Air Force Base, Tennessee. The work was conducted during the period June 1988 through May 1989.

TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION	1
II.	BACKGROUND	3
	The Requirement for Trim Studies	3
	Determination of Control Effectiveness Characteristics	6
	Aerodynamic Data Analysis Program	8
III.	THREE-AXIS TRIM PROGRAM	12
	Data Requirements	12
	Computational Approach	14
	Trim Output	25
	Special Features	26
	Program Validation	32
	Program Example Case	35
IV.	CONCLUDING REMARKS	46
	BIBLIOGRAPHY	49
	APPENDIXES	51
	A. GAUSSIAN ELIMINATION WITH BACKWARD SUBSTITUTION	52
	B. EXAMPLE OUTPUT	58
	VITA	72

LIST OF FIGURES

FIGURE		PAGE
1.	Representative Flight Vehicle Forces and Moments	4
2.	Determination of Moments and Control Effectiveness from Group A Data	20
3.	Determination of Moments and Control Effectiveness from Group B Data	21
4.	Data with Variable Number of Points and Options for ALPP Limits	28
5.	Mirroring of Control Deflection Data	31
6.	Example of Aerodynamic Data Simulation	34
7.	Pitch Control Deflection Data for Example Case	36
8.	Roll Control Deflection Data for Example Case	38
9.	Yaw Control Deflection Data for Example Case	40
10.	Deflection Angles Required to Trim Example Case	43
11.	Force Coefficients at Trim for Example Case	44

NOMENCLATURE

A	Matrix of coefficients from the equations in the linear system [A][x] = [b] with elements a_{ij} where i = 1,2,3,...n and j = 1,2,3,...n
ALPP	Angle of attack, degrees
b	Vector of constants in the linear system of equations [A][x] = [b] with elements b_1, b_2, b_3, ... b_n
BETA	Angle of side slip, degrees
C_l, CLL, or CLLA	Rolling moment coefficient
C_m, CLM, or CLMA	Pitching moment coefficient
C_N, CN, or CNA	Normal force coefficient
C_n, CLN, or CLNA	Yawing moment coefficient
$C_{n_\beta)dynamic}$	Flight vehicle departure correlation function
C_X	Generalized aerodynamic coefficient
C_Y, CY, or CYA	Side force coefficient
cg	Flight vehicle center of gravity
DELTAP	Equivalent roll control deflection angle, degrees
DELTAQ	Equivalent pitch control deflection angle, degrees
DELTAR	Equivalent yaw control deflection angle, degrees
E_j	Representative equation from a system of n equations, j = 1, 2, ... n
L	Derivative of rolling moment with respect to a control deflection, per degree
M	Derivative of pitching moment with respect to a control deflection, per degree
m	Multipliers used to reduce the augmented matrix during Gaussian Elimination
N	Derivative of yawing moment with respect to a control deflection, per degree

P_n, Q_n, R_n	Specific value of roll, pitch, or yaw deflection angle, respectively, n = 1, 2, 3, ...
RMIR	Variable used in the residual moment calculation to account for the mirroring of data. The value is +1.0 when mirroring is not required and -1.0 when mirroring is required.
RUN	Sequential number used to identify independent variable sweeps.
T	Used as a suffix to indicate the value of a parameter at the trim condition
x	Vector of unknowns in the linear system of equations [A][x] = [b] with elements x_1, x_2, x_3 ....
Δ	Difference between two values
δ	Control surface deflection, degrees
Σ	Summation of a series of mathematical terms

SUBSCRIPTS

I	Current value
P	Equivalent roll deflection
Q	Equivalent pitch deflection
R	Equivalent yaw deflection
trim	Parameter value at trim condition
x	Generalized deflection subscript representing P, Q, or R

CHAPTER I

INTRODUCTION

During the 1980's Arnold Engineering Development Center (AEDC) personnel have become more directly involved in the analysis of aerodynamic data obtained in the Center's wind tunnels. As a result, a computer program has been developed for use in the analysis of aerodynamic data. The program is referred to as the Aerodynamic Data Analysis program (Hereafter called the Analysis program.) [1][1]. The Analysis program is a collection of automated techniques for performing calculations that are frequently required when using wind tunnel data, such as interpolation of pitch sweep data to even increments of pitch angle, center of gravity (cg) transfers, changes in reference length and areas, and axis transformations. In addition to these data manipulation options there are options that perform calculations to determine stability and control characteristics. For example, options are included to determine static margin and neutral point, control effectiveness and trim aerodynamics.

The Analysis program included a single-axis (longitudinal) trim option prior to the present effort. However, some flight vehicles experience significant yawing and rolling moments at the longitudinal

[1]Numbers in brackets refer to similarly numbered references in the Bibliography.

trim condition. To determine whether roll and yaw control can be maintained after trimming longitudinally, a three-axis trim analysis must be done. For this reason, the present effort was undertaken to add a three-axis trim capability to the Analysis program.

CHAPTER II

BACKGROUND

THE REQUIREMENT FOR TRIM STUDIES

An aerodynamic flight vehicle can reach its destination only if directional control is maintained throughout the flight. The control requirement means the pilot, whether human or electronic, must have the capability to generate and control turning moments (pitching, rolling, and yawing) about the vehicle center of gravity (cg), as shown in Figure 1. To initiate a turn, a moment must be created about one or more of the three orthogonal axes passing through the cg. To avoid an undesired turn, the moments about all three axes must be held to zero (trimmed). The most common method of moment control is the angular displacement of all or a part of a lifting surface. The resulting change in the forces acting on the surface changes the moment balance about the cg. A moment change from a control deflection can be used to create a desirable moment or eliminate an undesirable moment.

The amount of moment that can be produced by a control deflection is governed by several factors including the size of the deflected surface, the maximum deflection angle, the distance from the surface to the cg, and the flight dynamic pressure [2]. A common goal of trim studies, regardless of vehicle type, is to identify flight conditions where the maximum moment available from control deflections is not enough to trim the aerodynamic moments. The vehicle type determines points in the flight envelope where trim limits are most likely to

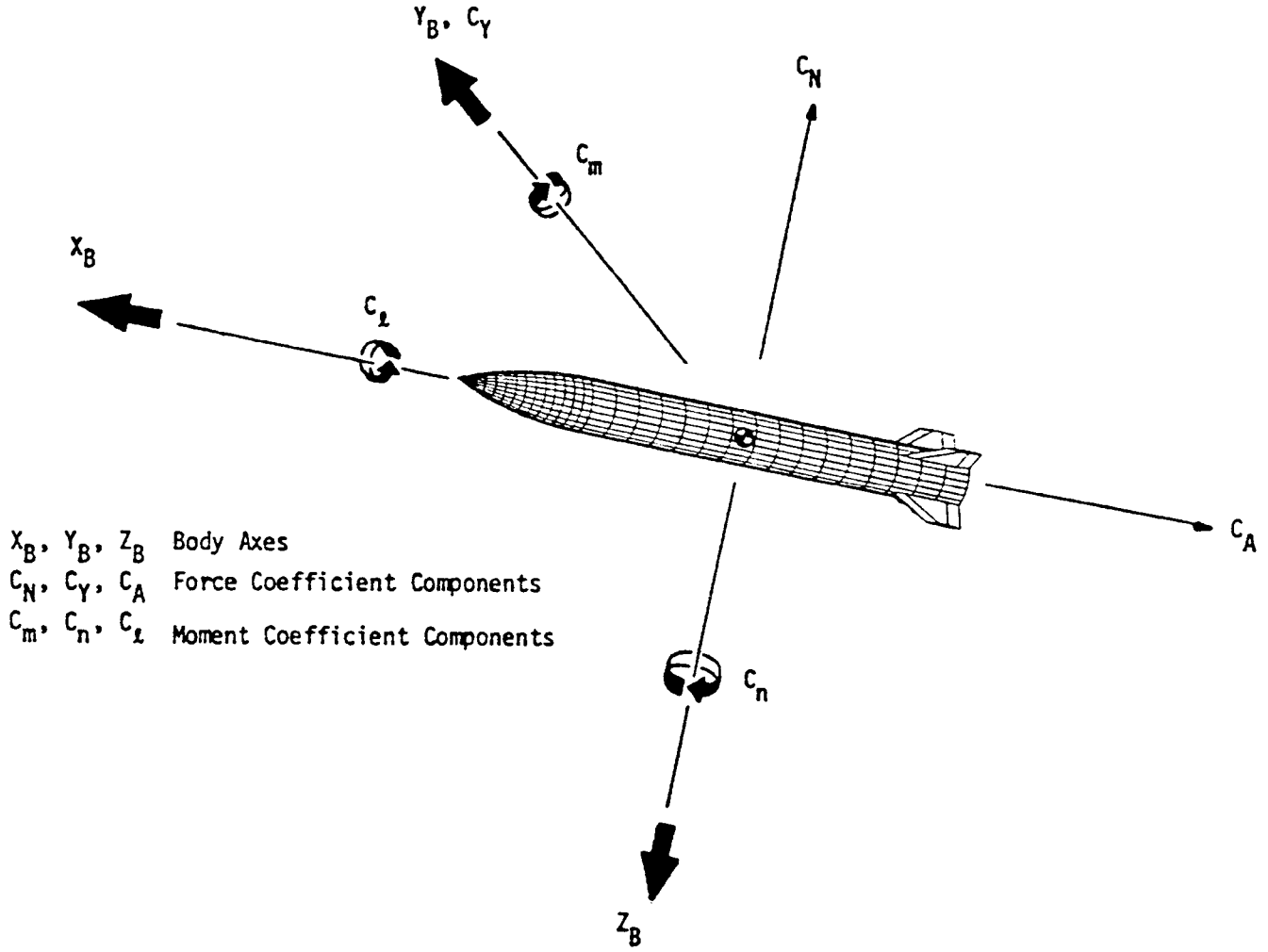


Figure 1. Representative Flight Vehicle Forces and Moments

occur. Transport aircraft are most likely to reach trim limits during takeoff and landing when the dynamic pressure is low and the angle of attack is higher than at the cruise condition. Longitudinal trim is of primary importance because transports typically make gentle turns that require small roll and yaw control deflections. Trim limits for missiles can occur near launch while the airspeed is relatively low or during a hard maneuver when high angles of attack and yaw are required. Trim about all three axes must be considered for missiles because the higher angles of attack and yaw can result in aerodynamic cross coupling [2]. (A yawing moment resulting from a pitch angle, for example.) Fighter aircraft can reach trim limits during takeoff and landing like transports or during maneuvers like missiles.

The aerodynamic forces at the trim condition are also important results of trim analysis. The values of performance parameters such as minimum sustained turn radius, maximum powered range and endurance, and maximum sustained load factor (lift/weight) depend on the forces at the trim condition [3]. The control surface hinge moments and bending moments may also be of interest when selecting actuators, hinge line location, or control surface fabrication methods. Therefore, a trim analysis capability is very important in design and performance evaluations.

Current trends in missile and aircraft performance requirements will make longitudinal-only trim analyses less applicable to the resulting designs. Missiles and fighter aircraft are being designed to operate at higher angles of attack and yaw where cross coupling in the

force and moment coefficients is very likely [4]. Unconventional control surface arrangements, such as multiple flap, rudderless aircraft, increase the likelihood of cross coupling in the control effectiveness derivatives. The use of lateral jets on missiles [5] and multiple control surfaces on aircraft for direct lift and side force generation [6] increase the complexity of trim analyses. New analysis tools must be developed to meet the challenges presented by these designs as they evolve. A three-axis trim program like the one presented here is one of the necessary tools.

DETERMINATION OF CONTROL EFFECTIVENESS CHARACTERISTICS

Control effectiveness is a measure of the moment increments about the vehicle center of gravity resulting from a known increment of control deflection angle. The control effectiveness characteristics of a vehicle must be available to perform a trim analysis. The first step in calculating control effectiveness is to define the control deflections. The actual angular deflections of the elevator, aileron, and rudder are used in the control effectiveness calculations for a conventional airplane. This simple definition is possible because each control surface is used for the control of moments about a single axis of the vehicle - the elevator for pitching moment, the ailerons for rolling moment, and the rudder for yawing moment. A more complex definition of control deflections is required for missiles that use cruciform fins for moment control.

An individual control surface on a missile may be deflected by one increment for pitch control, another increment for roll control, and a

third increment for yaw control. Therefore, each fin in a set of missile control surfaces may be deflected through a different angle. The concept of "equivalent" control deflections is used in calculating the control effectiveness of missiles. An arbitrary arrangement of the individual surfaces is equivalent to a pitch-only deflection, plus a roll-only deflection, plus a yaw-only deflection. Equivalent pitch, roll, and yaw deflections are defined as functions of the individual fin deflections. The exact equations used to define the equivalent deflections depend on the sign convention of the individual fin deflections and the axis system of the moment coefficients used in the control effectiveness calculations. In this thesis, deflections are equivalent deflections unless stated otherwise.

After the control deflections have been defined, the control effectiveness must be calculated for each vehicle configuration and flight condition (Mach number, roll angle, altitude, etc.) of interest. The data required are usually obtained by testing the model in pitch sweeps for each flight condition with each deflection type (pitch, roll, and yaw) and value. Ideally, the deflection value would vary from the negative limit to the positive limit by a small increment for each deflection type. Unfortunately, the "ideal" approach results in a prohibitively large number of test conditions. Therefore, tests are conducted at a selected subset of the conditions, and the analysis is based on assumptions about the aerodynamics at the other conditions.

A common approach to selecting test conditions is to test with only positive or only negative control deflections. The assumption

underlying this approach is that the control effectiveness is independent of the direction of the control deflection. Typically, four or five deflection values are selected varying from zero to the maximum deflection angle in the chosen direction (positive or negative).

An increasingly popular test technique for missiles involves deflecting a single control surface (as opposed to an equivalent deflection requiring multiple deflected panels) and rolling the model through 360 degrees at various pitch angles for each flight condition. The aerodynamic coefficients for the case of multiple surfaces deflected can be computed by difference function methods [7, 8].

AERODYNAMIC DATA ANALYSIS PROGRAM

At AEDC, a team of engineers often begins analysis of wind tunnel data while the test is still in progress and presents the analysis results at the same time as the final wind tunnel data package. This type of analysis depends on a method of performing the required calculations rapidly. The Aerodynamic Data Analysis program facilitates quick analysis of aerodynamic data to determine stability and control characteristics.

The Aerodynamic Data Analysis program was developed at AEDC to automate several calculations that are frequently performed during the analysis of aerodynamic data for flight vehicles. The specific data manipulation options incorporated into the program were established by a survey of AEDC personnel who frequently analyze wind tunnel data. These data manipulation options include corrections for bias in a data

variable, interpolation and extrapolation to specific values of the independent variable, cg transfer, axis transformation, reference dimension changes, skin friction and base drag corrections, and sign changes for selected variables. Additional options provided for analysis support include determination of stability characteristics, and control effectiveness characteristics and longitudinal trim. The operations available through the Analysis program are listed in Table 1. The data manipulation options are shown with a typical application. The original analysis options are listed with the output generated by the option. The remainder of this thesis documents the addition of the three-axis trim program which has been added as an analysis option.

Table 1. List of Analysis Program Options

Data Option	Typical Application
Variable Bias Corrections	Correct data for model imperfections such as bent or misaligned control surface.
Independent Variable Range	Interpolate data to even increments of the independent variable. For example, angles of attack of 0.1, 1.97, 3.95 would be interpolated/extrapolated to 0, 2, 4.
Center of Gravity Transfer	Adjust data from different sources (i.e., current test and earlier test or prediction and experiment) to the same center of gravity location for direct comparison.
Axis Transformation	Transform data from different sources to the same axis system for direct comparison.
Change in Reference Dimensions	Modify coefficients based on different reference dimensions for configuration synthesis (i.e., missile fin data based on root chord added to body data based on diameter) or for comparison of data from different sources.
Skin Friction & Base Drag Correction	Scale drag data from model to full scale flight vehicle.
Variable Sign Change	Change between sign conventions for control surface deflection.

Table 1. Concluded

Data Option	Typical Application
Determination of Stability Characteristics	Lift-to-drag ratio, static margin, neutral point, and a departure correlation function $(C_{n_\beta})_{dynamic}$
Determination of Control Effectiveness	Divided difference approximations to the partial derivative of each aerodynamic coefficient with respect to each control surface deflection.
Single-Axis Trim	The values of trim deflection for each independent variable value where trim is possible and the values of any other parameter at trim. Typically, the two untrimmed moments and the force coefficients are output.

CHAPTER III

THREE-AXIS TRIM PROGRAM

DATA REQUIREMENTS

The three-axis trim program requires data for pitching moment, rolling moment, and yawing moment coefficients at various combinations of pitch, roll, and yaw control surface deflections. If the aerodynamic data were obtained by deflecting a single panel and rolling the model, then the conversion to pitch sweeps with equivalent deflections must be made before executing the three-axis trim program. As a minimum, sweeps are required of the independent variable (usually pitch angle) with a single nonzero deflection per data run. A data matrix of this type is shown symbolically in Table 2 and is referred to hereafter as "Group A" data.

When working with Group A data there is an implicit assumption that the effectiveness of each deflection is independent of the value of the other two deflections. In general, the effectiveness of each type of deflection depends on the magnitude of the other deflections. In the case of a missile, for example, a single control surface may be positioned near the stall angle of the surface as a result of pitch trim. Any additional deflection of the same surface for roll or yaw trim could drive that surface beyond the stall angle. In this example the initial value of pitch deflection reduced the roll and yaw effectiveness of that surface.

Table 2. Generalized Group A Data Matrix

Pitch Defl.	Roll Defl.	Yaw Defl.
0.0	0.0	0.0
Q_1	0.0	0.0
Q_2	0.0	0.0
.	.	.
.	.	.
.	.	.
Q_n	0.0	0.0
0.0	P_1	0.0
0.0	P_2	0.0
.	.	.
.	.	.
.	.	.
0.0	P_m	0.0
0.0	0.0	R_1
0.0	0.0	R_2
.	.	.
.	.	.
.	.	.
0.0	0.0	R_i

Some of the dependencies among the control effectiveness derivatives can be quantified if the additional data shown in Table 3 are available ("Group B" data). The Group B data are independent variable sweeps with nonzero pitch deflection and one other nonzero deflection. The additional information can be used to estimate the roll and yaw control effectiveness at nonzero pitch deflection and the pitch control effectiveness at nonzero roll and yaw deflections. Still more data would be required to evaluate the effect of roll deflection on yaw control and vice versa.

The three-axis trim program can be used for trim calculations about one or two axes by using the available control deflection data and specifying a large tolerance for the moment coefficient that is not to be trimmed. (More will be said about tolerances in the next section.) For example, some missiles are allowed to roll during flight, therefore roll control effectiveness would not be obtained for those missiles. For the rolling missile case, pitch and yaw deflection data would be input, and a large rolling-moment coefficient tolerance would be specified.

COMPUTATIONAL APPROACH

The three-axis trim program performs calculations within user-defined limits of an independent variable (usually vehicle angle of attack). The user inputs lower and upper limits and a step size for the independent variable. Trim calculations are performed at each step proceeding from the lower limit of the independent variable to the

Table 3. Generalized Group B Data Matrix

Pitch Defl.	Roll Defl.	Yaw Defl.	Data Description
Q_1	P_1	0.0	
Q_1	P_2	0.0	
.	.	.	
.	.	.	
.	.	.	
Q_1	P_n	0.0	
Q_2	P_1	0.0	Roll Deflection at Fixed Pitch Deflection
Q_2	P_2	0.0	
.	.	.	
.	.	.	
.	.	.	
Q_2	P_n	0.0	
.	.	.	
.	.	.	
.	.	.	
Q_m	P_n	0.0	
Q_1	0.0	R_1	
Q_1	0.0	R_2	
.	.	.	
.	.	.	
.	.	.	
Q_1	0.0	R_i	
Q_2	0.0	R_1	
Q_2	0.0	R_2	
.	.	.	Yaw Deflection at Fixed Pitch Deflection
.	.	.	
.	.	.	
Q_2	0.0	R_i	
.	.	.	
.	.	.	
.	.	.	
Q_m	0.0	R_i	

upper limit. For example, a lower limit of 0, an upper limit of 20, and a step size of 5 would produce calculations at 0, 5, 10, 15, and 20 degrees angle of attack.

The aerodynamic moments about the vehicle cg (Figure 1, page 4) can be represented by the following equations:

$$\begin{aligned} C_m &= C_{m_0} + M_Q\,\delta_Q + M_p\,\delta_p + M_R\,\delta_R \\ C_\ell &= C_{\ell_0} + L_Q\,\delta_Q + L_p\,\delta_p + L_R\,\delta_R \\ C_n &= C_{n_0} + N_Q\,\delta_Q + N_p\,\delta_p + N_R\,\delta_R \end{aligned} \tag{1}$$

The first term on the right side of each equation (C_{m_0}, C_{ℓ_0}, C_{n_0}) includes moment contributions from all sources other than control surface deflections. The δ_X terms represent control surface deflection angles. The control effectiveness terms are calculated as divided differences between pairs of input data runs. For example,

$$M_Q = (C_{m_2} - C_{m_1})/(\delta_{Q_2} - \delta_{Q_1}) \tag{2}$$

where the numerical subscripts indicate values from each of the two input data runs.

The divided difference approach to control effectiveness assumes a piecewise linear variation of the moments with control deflections. That is, the control effectiveness is the slope of the straight line connecting the moment values corresponding to adjacent values of the deflection angle. The assumption of linearity is valid in the absence of significant curvature of the moment data between adjacent deflection values. If unacceptable curvature between deflection values is

suspected, then additional data must be obtained at intermediate deflections. The additional data provides two options for calculating control effectiveness. The first option is to maintain the linear assumption and allow the closer spacing of deflection values to correct the problem. This option requires no modification of the existing program. The second option is to use a higher order approximation of the variation of moments with control deflections to estimate the control effectiveness. In practice, there is rarely enough data to support the second option.

The goal of three-axis trim is to determine the control surface deflections required to make the left side of Equation (1) (the total moment coefficients) zero. The calculations begin with initial guesses for the deflections and control derivatives. (Initial guesses are discussed later.) An incremental control deflection ($\delta x_{trim} - \delta x_I$) must be added to the initial guess to obtain the trim value. The increment is calculated using the following equations:

$$
\begin{aligned}
0 &= C_{m_I} + M_Q(\delta Q_{trim} - \delta Q_I) + M_P(\delta P_{trim} - \delta P_I) + M_R(\delta R_{trim} - \delta R_I) \\
0 &= C_{l_I} + L_Q(\delta Q_{trim} - \delta Q_I) + L_P(\delta P_{trim} - \delta P_I) + L_R(\delta R_{trim} - \delta R_I) \qquad (3) \\
0 &= C_{n_I} + N_Q(\delta Q_{trim} - \delta Q_I) + N_P(\delta P_{trim} - \delta P_I) + N_R(\delta R_{trim} - \delta R_I)
\end{aligned}
$$

The moment coefficients with subscript I are residual moments associated with the current estimate of trim deflections (δx_I). The $\delta trim$ terms are the actual trim deflections. The desired value of each increment is that which, when added to the previous estimate of the trim values, makes the residual moments zero.

The moment equations (3) are nonlinear and are solved iteratively because of the variation of the control effectiveness derivatives with the deflection values. A maximum of 20 iterations is permitted at each value of the independent variable. Calculations begin at the next independent variable value after the iteration converges or reaches the 20-iteration limit.

At each value of the independent variable, starting approximations of the trim deflections are required. At the first independent variable value, the starting approximations are the input deflections which minimize the absolute value of the corresponding moment coefficient. For example, assume pitch deflections of 0, -5 degrees, and -10 degrees result in pitching-moment coefficients of 0.8, -0.4, and -1.0, respectively. The program would use -5 degrees as the initial pitch deflection because -0.4 is the moment coefficient nearest to 0.0. After the first independent variable value, the starting approximations are the trim values at the previous independent variable value if the iteration was successful there. If the iteration failed at the previous independent variable value, then the technique used at the first value of the independent variable is used to find starting approximations at the current value.

The starting deflection values determine the initial values of the control effectiveness derivatives and the untrimmed moment coefficients. At each iteration, new derivatives and moment coefficients are computed by linear interpolation within the input data. Separate interpolation procedures are required for Group A data only and for

Group A plus Group B data. The moment coefficient and control effectiveness updating procedure for Group A data only will be described first.

At each iteration, approximations (δQ_I, δP_I, and δR_I) to the trim deflection angles are known. A typical case for which δQ_I is bounded by Q_2 and Q_3, δP_I is bounded by P_1 and P_2, and δR_I is bounded by R_1 and R_2 is illustrated in Figure 2. The objective is to estimate the moment coefficient and control effectiveness at (δQ_I, δP_I, δR_I) (point D of the figure). The estimate is made by first linearly interpolating all three moment coefficients to δQ_I, δP_I, and δR_I on the respective axis. Then the coefficients at the origin of the figure are subtracted from the values at δP_I and δR_I. The two differences are then added to the coefficients at (δQ_I, 0, 0) (point A) to give moment coefficients at point D. The control derivatives are simply the divided differences obtained from the bounding deflection data on each axis.

If Group B data are available, the effects of pitch deflection on roll and yaw control effectiveness, and vice versa, can be included in the trim calculations. Typical approximations of the trim deflections and the Group B data bounding them are represented by Figure 3. As before, the objective is to obtain moment coefficients and control effectiveness estimates at point D. Each corner of the boxes enclosing points B and C represents a combination of deflections (one data run) from the Group B data. Point B indicates the combination of δQ_I and δP_I, and point C represents the combination of δQ_I and δR_I. The moment coefficients at point B are calculated by interpolation to δQ_I on the

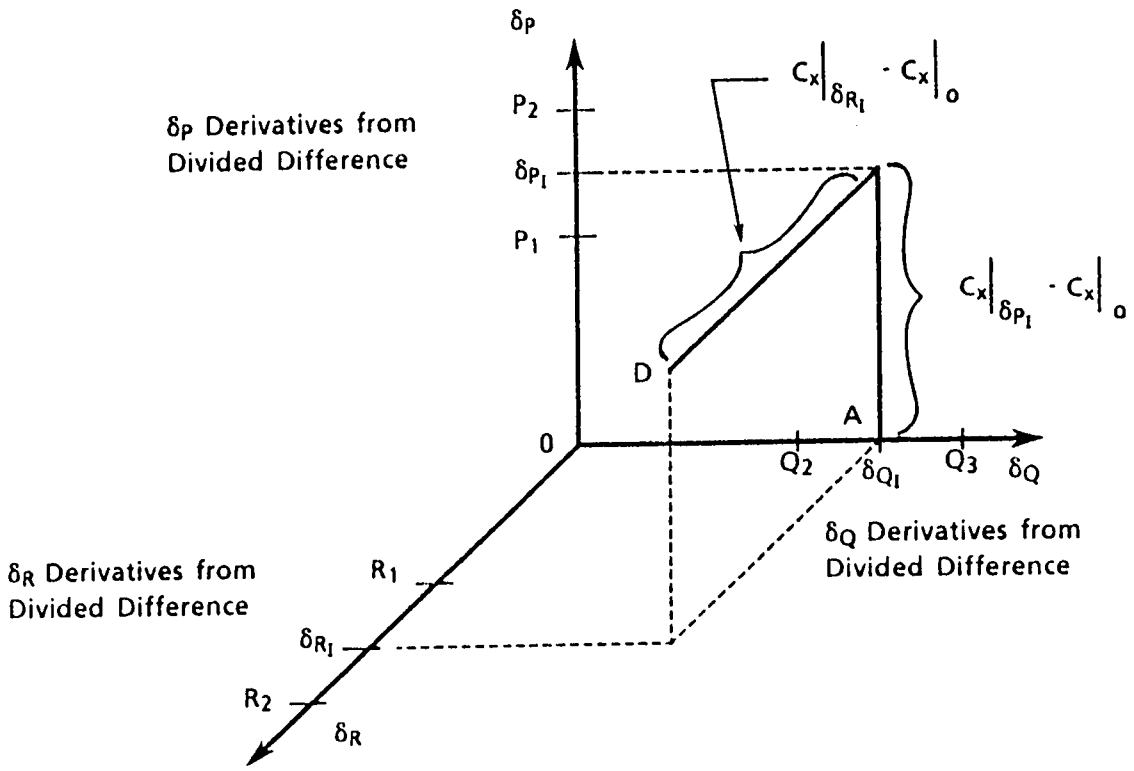


Figure 2. Determination of Moments and Control Effectiveness from Group A Data

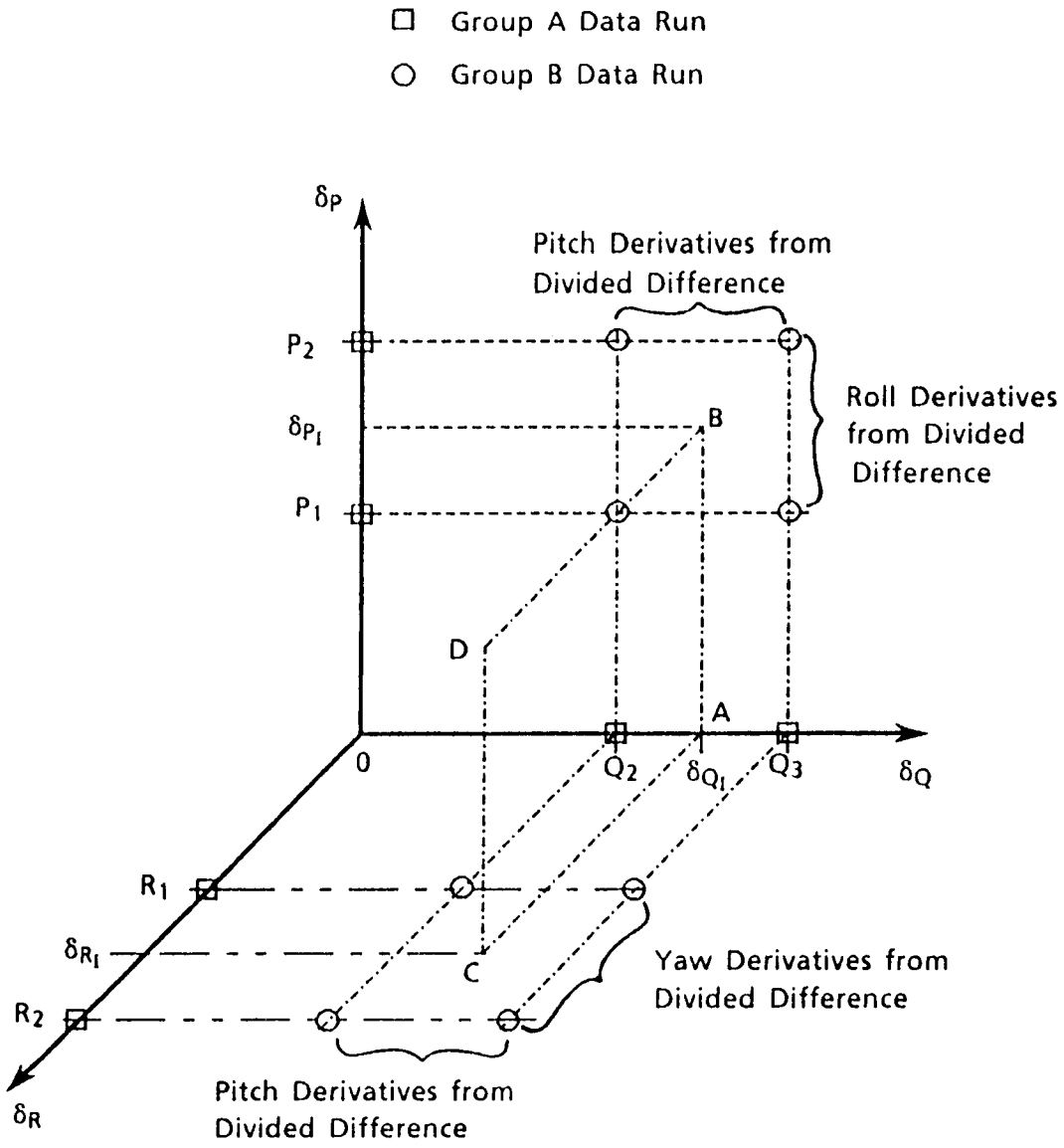


Figure 3. Determination of Moments and Control Effectiveness from Group B Data

P_1 and P_2 lines and then interpolating those results to δP_I. The same process is applied at point C. Moments at (δQ_I, 0, 0) (point A) are obtained by linear interpolation between Q_2 and Q_3. The moment coefficients at point A are subtracted from those at point B and the differences are added to the coefficients at C to give the approximation of moment coefficients at point D.

Control effectiveness derivatives at points B and C are computed as divided differences using the four bounding data runs. At point B, roll control derivatives are calculated at Q_2 and Q_3 and then interpolated to δQ_I. Pitch control derivatives are computed at P_1 and P_2 and interpolated to δP_I. The same technique is used at point C for yaw control derivatives and pitch control derivatives. The resulting roll and yaw derivatives go directly into the moment equations, Equation (1). The pitch control derivatives at (δQ_I, 0, 0) (point A) are subtracted from the values at (δQ_I, δP_I, 0) (point B) and the differences are added to the values at (δQ_I, 0, δR_I) (point C) to obtain values at (δQ_I, δP_I, δR_I) (point D). The approximations to control derivatives at D do not include the effects of roll deflection on yaw control effectiveness or vice versa.

After the control effectiveness derivatives and the untrimmed moment coefficients at the current deflections have been determined, Gaussian Elimination [9] is used to solve, Equation (3), for the next increments, $[\Delta\delta]$, to add to the current deflections. (Appendix A provides more detail on Gaussian Elimination and the application to three-axis trim.) New deflections are given by

$$[\delta]_{I+1} = [\delta]_I + [\Delta\delta]_I \tag{4}$$

where the elements of the $[\Delta\delta]$ vector are the $(\delta_{x_{trim}} - \delta_{x_I})$ terms of Equation (3). The moment coefficients and control effectiveness derivatives for the new deflections are calculated as before.

The iteration continues until the untrimmed moment coefficients are less than a specified tolerance or until 20 iterations have been completed. If the program reaches 20 iterations, then a warning message and the final moment coefficients are printed, the line of printed output for that value of the independent variable is marked with an asterisk (*), and the final deflections are used as trim values. After the iteration converges or stops, the values of other data parameters at trim are determined by the method used for moment coefficients.

The moment coefficient tolerances used to stop the iteration effect the accuracy of the results. The tolerances should be large enough to avoid unnecessary iteration, yet small enough to ensure adequate accuracy in the final results. Specific tolerance values cannot be recommended because of the wide variation of coefficient magnitudes between different vehicles. However, the following formula is offered as a guide in determining a reasonable upper limit for the tolerances.

$$C_m\ Tolerance = \left| \frac{(XHL - XMR) * S_t}{S * d} \left(\frac{dC_N}{d\delta} \right)(\varepsilon) \right| \tag{5}$$

where XHL = distance from the vehicle nose to control surface hinge line

XMR = distance from the vehicle nose to the moment reference point

S_t = planform area of the control surface

S = reference area used to nondimensionalize the input data

d = reference length used to nondimensionalize the input data

$(dC_N/d\delta)$ = change in normal force with respect to control deflection

ε = maximum allowable error in trim deflection angles (actual trim value - calculated trim value)

The same equation with appropriate substitutions will give rolling- and yawing-moment tolerances. The numbers used in the equation do not have to be precise because the order of magnitude of the result will be used to select the final tolerance values. The tolerances resulting from these calculations can be tested by running one set of data twice - once with the tolerances from the equation and again with tolerances one or more orders of magnitude smaller than the first case. If the trim deflections from the two cases differ by more than ε, then the larger tolerance is too large. The tolerances must be reduced until the calculated trim deflections change by less than ε between cases. The tolerance determination should be required only once for a given set of test data. The tolerance for any moment coefficient that is not

to be trimmed should be several orders of magnitude larger than the other tolerances.

TRIM OUTPUT

Output from the three-axis trim program may be printed, written to a plot file, or both. The format of plot files addressed by the program is AEDC-specific and will not be discussed in detail. Printed output from the program contains a summary of the Analysis program data manipulation options to be performed, the results of the data manipulation options, and the results of the analysis option executed. Appendix B contains example output from the independent variable range option and the three-axis trim option.

The list of data points for each input run number is the output from the independent variable range option. The list of points includes those generated by interpolation and extrapolation to even increments of the independent variable. If multiple data manipulation options are executed, then the data printed are the final results of all the calculations. The final data are used as input to the analysis option, if one was chosen.

The output from the three-axis trim option begins with diagnostic messages generated during the iteration. Diagnostic messages warn the user when one or more of the estimated trim deflections is not bounded by the input data deflections, when 20 iterations are completed without satisfying the moment coefficient tolerances, and when the Gaussian Elimination technique fails. The message printed to warn of unbounded deflection estimates includes the value of the independent variable

where the problem occurred, which deflection was not bounded (pitch, roll, or yaw), and the value of the unbounded deflection.

The results of the trim calculations follow the diagnostic messages. Results are printed for each value of the independent variable at which the trim deflections were bounded by input data deflections. Values of the independent variable at which trim deflections were unbounded are excluded from the output. If 20 iterations were completed, then the output for that value of the independent variable is marked with an asterisk (*), as mentioned earlier. The data parameters included in the printed output are chosen by the user. Any parameter in the plot file from which the input data are read may be included in the output. The parameter names in the printout are the original names with a "T" appended to the end to indicate a trim value of that parameter.

SPECIAL FEATURES

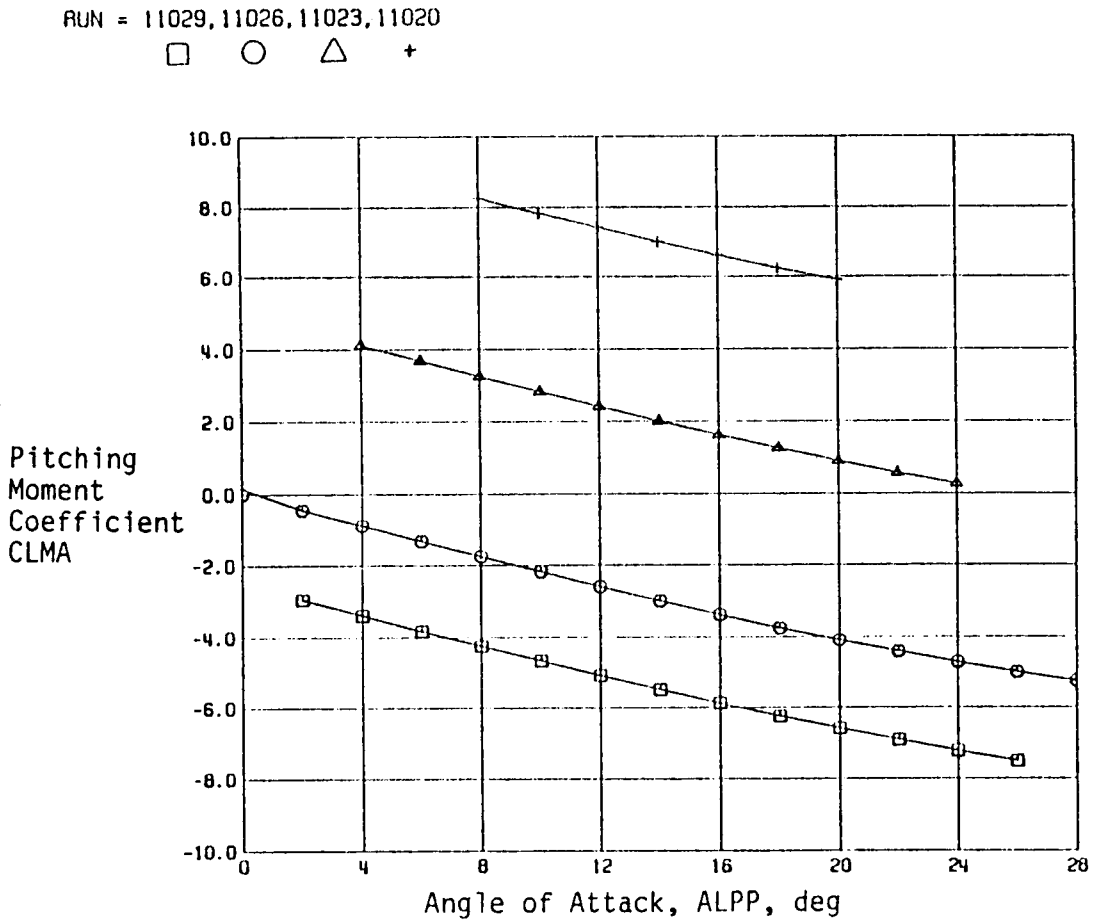
Because the primary source of input for the three-axis trim program will be wind tunnel data, the program must work with imperfect input. For example, a control deflection variable with a nominal value of 5.0 degrees may be slightly more or less than 5.0. This section describes program features designed to ensure the success of the iteration and to minimize the number of calculations performed.

Independent variable sweeps used as input must have the same number of points and the same value of the independent variable at like-numbered points. For example, if one input data run has 10 values of angle of attack, then all other runs must have 10 values, and if the

fifth value is 14 degrees, then the fifth value on all input runs must be 14 degrees. When wind tunnel data runs are used as input, all sweeps may not have the same number of points because of balance limits, model support system limits, or other test-related limitations. An example of data that would require careful selection of the independent variable limits and some options for using the data are presented in Figure 4.

The first option listed in Figure 4 requires the extrapolation of three data runs (11029, 11023, and 11010) to reach the zero and 28 degrees ALPP range limits. The Analysis program Independent Variable Range option provides the extrapolation capability required to use all four data runs and to reach the 0 and 28 degrees ALPP limits. The danger is that, in general, the extrapolation will not accurately represent the aerodynamics at the extrapolated points. The second option uses all four data runs and changes the ALPP limits such that the requirement for the same number of points on each run is met. Option 3 takes advantage of the fact that run 11020 will probably not be used in the trim calculations within the ALPP limits shown. This option excludes run 11020 and expands the ALPP limits to those allowed by the remaining runs. A combination of options 1 and 3 (excluding run 11020 and extrapolating runs 11023 and 11029) would make trim calculations possible with the ALPP limits of zero and 28 degrees.

The control effectiveness and aerodynamic moment coefficients must be known at each iteration as described in the previous section. The



OPTIONS

1. Extrapolate all runs to desired ALPP limits.
2. Use 8 deg and 20 deg as ALPP limits.
3. Exclude run 11020 and use 4 deg and 24 deg as ALPP limits without extrapolating.

Figure 4. Data with Variable Number of Points and Options for ALPP Limits

moment coefficients must be recalculated at every iteration, and the control effectiveness must be recalculated at every iteration when Group B data are used. However, the control effectiveness calculations are not required at every iteration when Group A data are used. The control effectiveness calculated using Group A data only changes when the bounding data changes. Therefore, if the current estimate of the trim deflections is bounded by the same input data as the previous estimate, then it is not necessary to search for new bounding data or to recalculate the control effectiveness. The search for bounding data and the control effectiveness calculations are required less frequently as the iteration approaches the solution because the estimated trim deflections change less. (The $[\Delta\delta]$ in Equation (4) becomes smaller.) This feature makes the Group A option much faster than the Group B option.

The determination of control effectiveness and aerodynamic coefficients requires data for deflections bounding the current estimates of the trim values. The pitch deflection must be bounded absolutely or be within 0.1 degree of the highest or lowest input data deflection to be considered bounded. The roll and yaw deflections are not required to be bounded absolutely.

Because of the cost of testing and the difficulty of determining the sign of the trim values of roll and yaw deflection a priori, data are often taken with all the deflections of a given type in the same direction. Therefore, if the roll and yaw deflections in the data do not bound the estimate, the estimate is mirrored by changing the sign

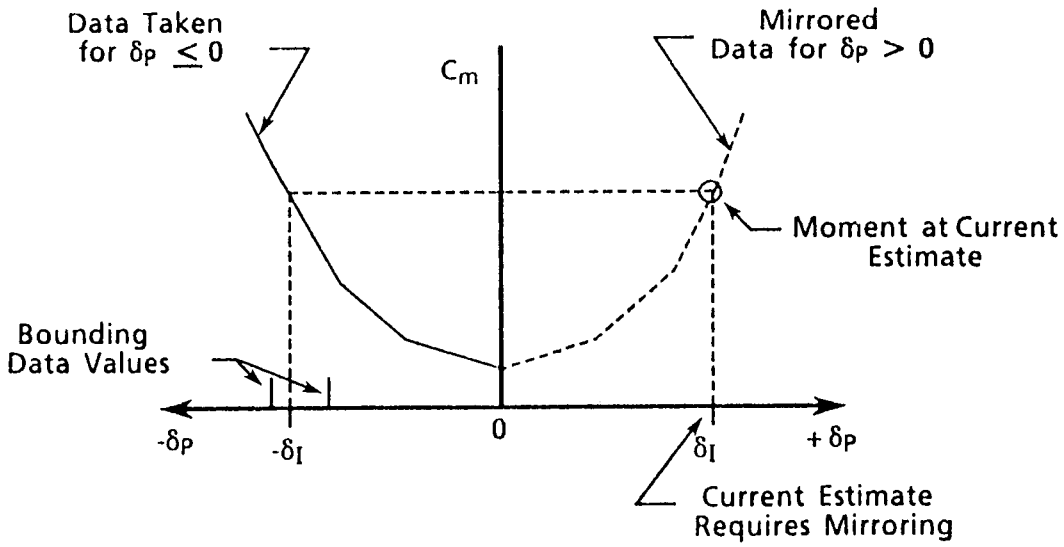
of the deflection and attempting the bounding again. If the second attempt at bounding succeeds, then the moment coefficients are interpolated to the negative of the current deflection estimate. If the mirroring is not required, then linear interpolation gives the moment coefficients directly. When the mirroring is required, the character of the variation of the moment with deflection must be considered in the calculation of moment coefficients and control effectiveness.

When mirroring is required and the moment coefficient is an even function of the deflection, as in Figure 5a, the final coefficient value is the result of interpolation only and the control effectiveness changes sign. (This is the case for lift, drag, and pitching-moment variations with roll or yaw deflections.) When the moment coefficients vary as odd functions of the deflections, as in Figure 5b, the control effectiveness does not change sign. However, for odd-function coefficients the final moment value is determined according to the following equation.

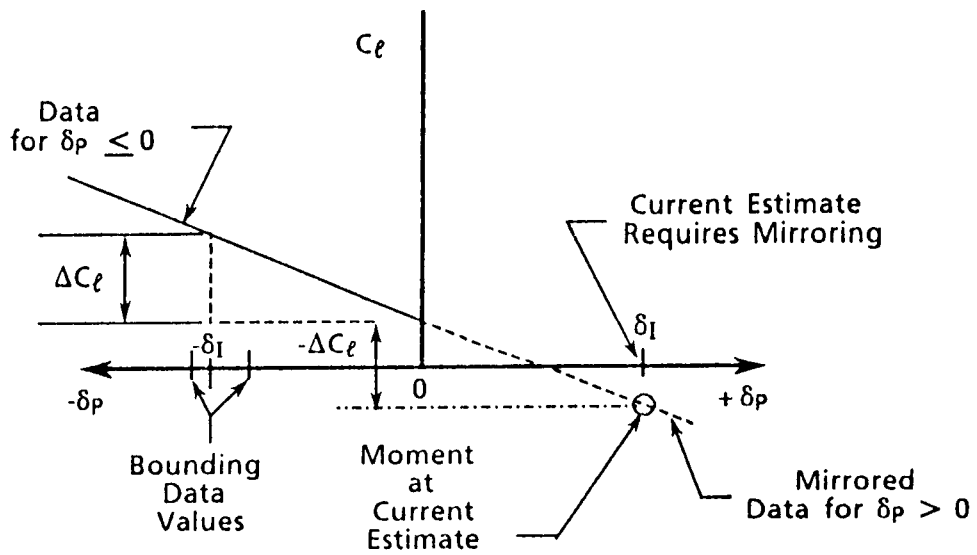
$$\begin{aligned}\text{Final Moment} = {} & \text{Moment at 0 deflection} \\ & + \text{RMIR} \times (\text{Moment at nonzero deflection} \\ & - \text{Moment at 0 deflection})\end{aligned}$$

where RMIR is 1.0 when mirroring is not required and -1.0 when mirroring is required.

The mirroring process assumes the variation of moments with control deflection is symmetric about zero deflection. Such symmetry is a favorable design characteristic and is expected to exist in most



a. Moment is Even Function of Deflection



b. Moment is Odd Function of Deflection

Figure 5. Mirroring of Control Deflection Data

cases. However, the validity of the assumption should be verified before using the three-axis trim program. If the data do not support the assumption, then data should be taken for both positive and negative deflections.

In iterating toward a trim deflection that is very near one of the extreme data values, the calculations can produce an estimate that is not bounded. For example, if the actual trim deflection is 4.93 degrees, the maximum data deflection is 5.0 degrees, and the current estimate is 5.3 degrees then the estimate is not bounded. When this happens, the program changes the current estimate to the nearest data value, 5.0 degrees in the example case, and continues the iteration. The intention is to prevent the iteration from failing because of overshoot in an intermediate result. The resetting of the estimate is allowed only once for each deflection (pitch, roll, and yaw) at each value of the independent variable. If any one of the deflections is unbounded a second time, the trim fails at that value of the independent variable and calculations begin for the next value.

PROGRAM VALIDATION

The three-axis trim program was developed in steps. Since a control effectiveness option already existed in the Analysis program, the control effectiveness calculations were programmed first. The second step was the development of the technique used to update the control effectiveness and moment coefficients at each estimate of the trim deflections. The updating technique was implemented for Group A data and then for the case with Group A plus Group B data. The method of

solving for trim deflection angles was added next. Finally, the calculation of other parameters at the trim deflections was implemented. At each step of the development, the program was checked by hand calculations and by comparison with a "pilot" program.

After the development of the approach for updating the moment coefficients and control effectiveness, a simplified computer program was written to perform the basic calculations. After the pilot program results were validated by hand calculations, the program was used as a guide by the programmer in the implementation of the method. The pilot program process was also applied to the calculation of trim deflections with control effectiveness and moment coefficient data as input.

The check cases used to validate the three-axis trim program came from actual wind tunnel data and an aerodynamic data simulator program. The experimental data included data with axisymmetric and non-axisymmetric vehicles, cruciform control surfaces and two horizontal and one vertical control surface. The data simulator is a group of equations used to produce generic data that have characteristics similar to actual aerodynamic data. The data simulator was used to produce input data with characteristics required to test particular features of the trim program. Specific characteristics simulated were: multiple trim points at a fixed deflection within the angle of attack range (see Figure 6), varying degrees of cross coupling in the control effectiveness, and varying control effectiveness with deflection angle. The data used for the example case described in the next section were generated with the data simulator to test the mirroring of yaw deflection.

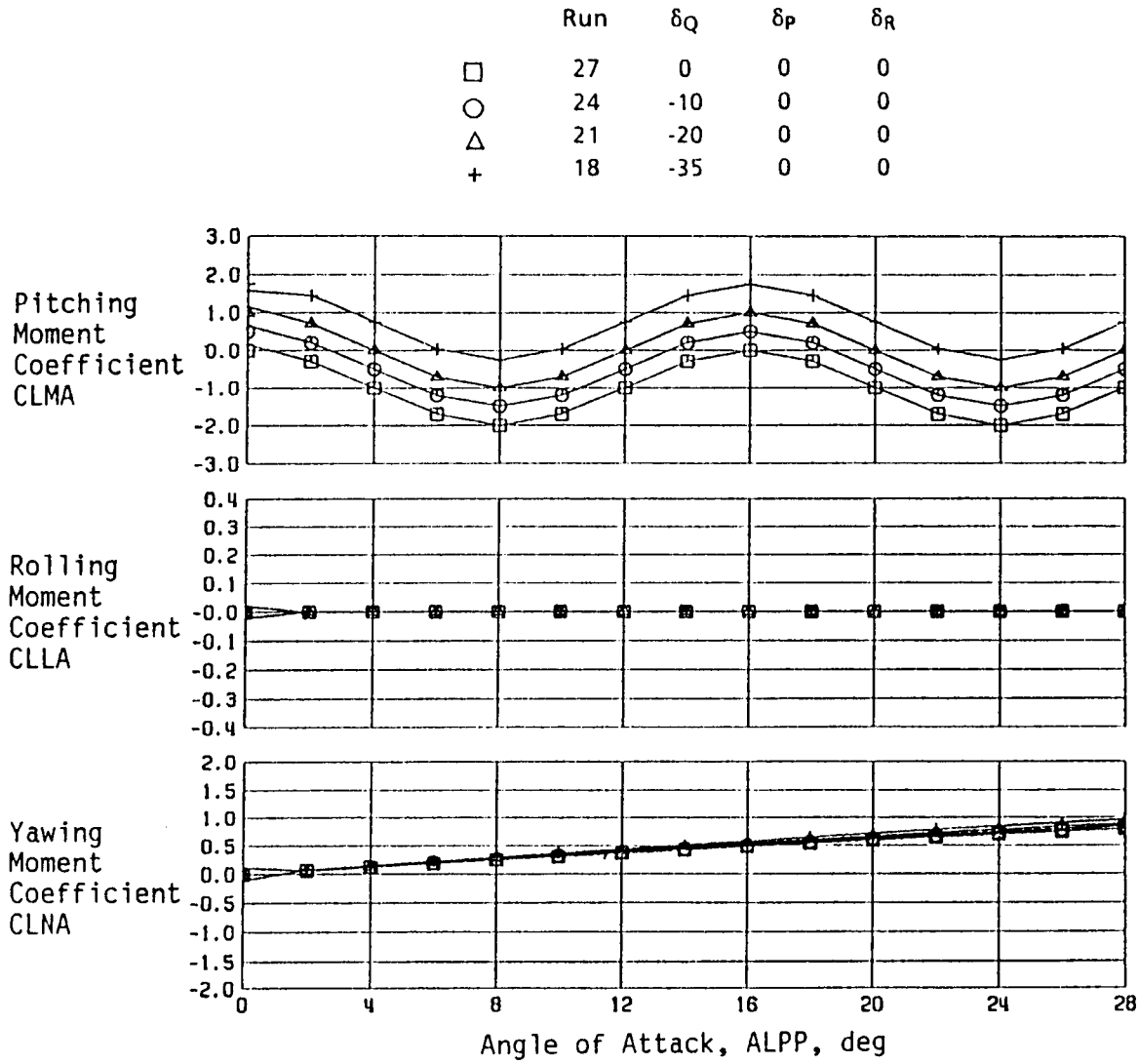


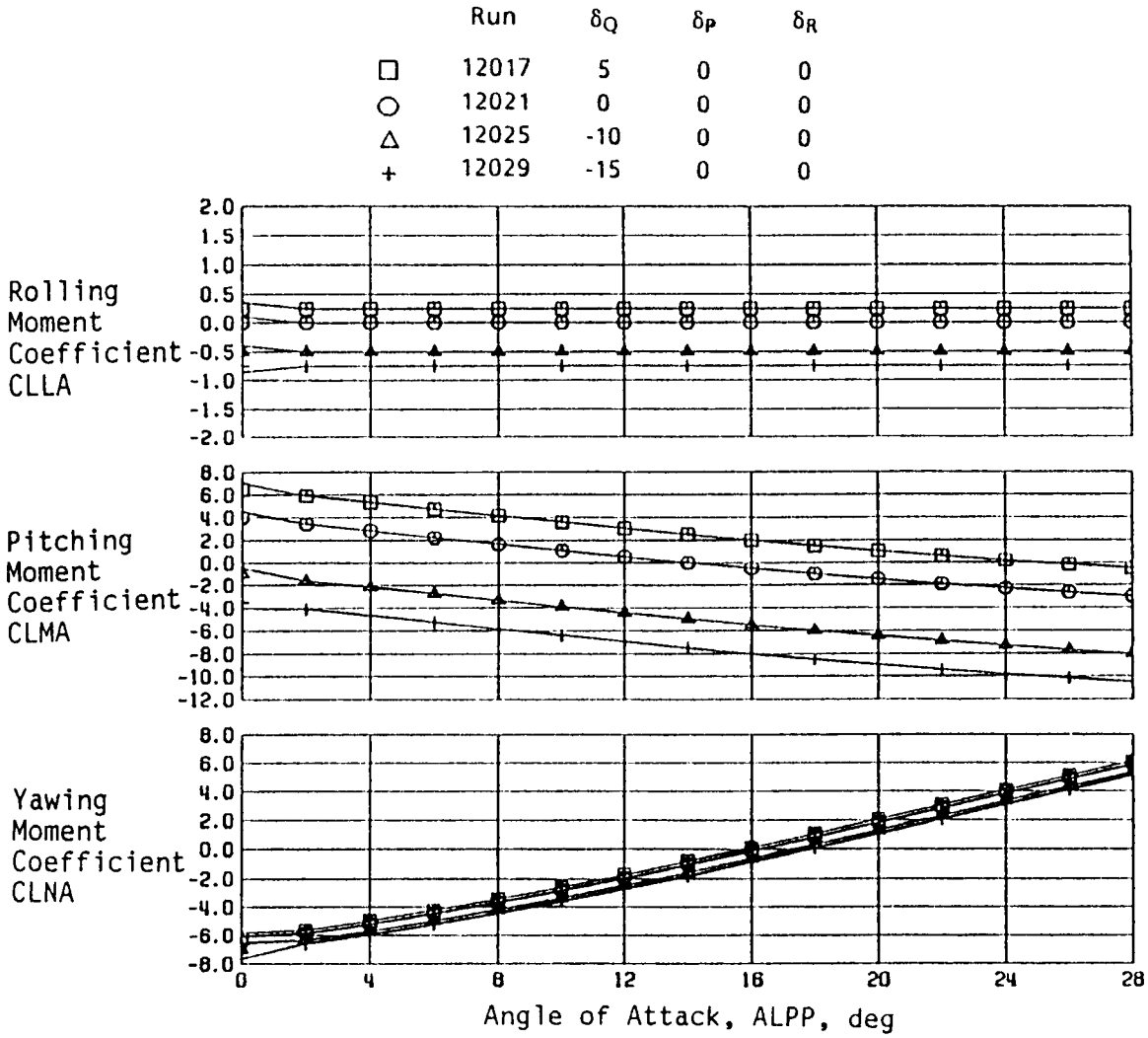
Figure 6. Example of Aerodynamic Data Simulation

PROGRAM EXAMPLE CASE

The Group A data for the example case are plotted in Figures 7 through 9. A brief analysis of the data can give some anticipated results from the trim calculations. The pitching moment data of Figure 7a indicate enough pitch deflection is available for trim from zero to about 25 degrees angle of attack. However, the yawing moment may not be trimmable at all angles of attack between those limits. The trim values of pitch deflection should be between zero and -10 degrees at angles of attack less than 14 degrees and between zero and +5 degrees at angles of attack greater than 14 degrees. Cross coupling of pitch deflection into the yaw and roll planes is also indicated in Figure 7a. The normal force at trim should vary from near -2 to just over +2 between angles of attack of zero and 14 degrees according to Figure 7b. At angles greater than 14 degrees, the normal force at trim should range from about 2 to 4.5.

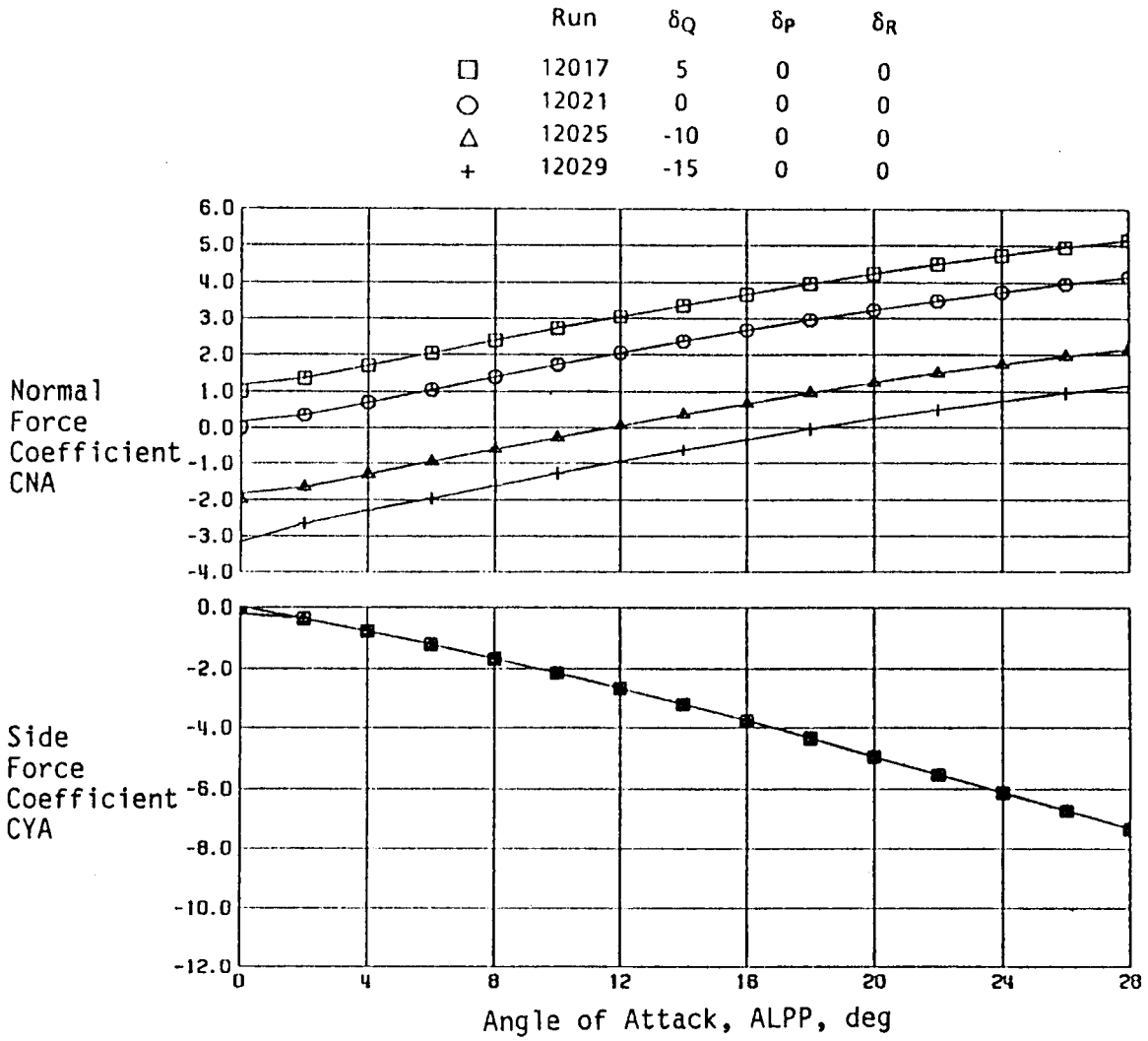
Roll control deflections, shown in Figures 8a and 8b, have very little effect on the pitching and yawing moment coefficients and no effect on the force coefficients. The rolling moment coefficients suggest that roll control will not have a limiting influence on the trim calculations.

The yaw deflection data of Figure 9 indicate trim will not be possible above 24 degrees angle of attack. The lower angle of attack limit, with data mirroring, is near eight degrees. However, the pitch deflection required for trim produces a negative increment in yawing moment coefficient and makes the eight degree angle of attack limit



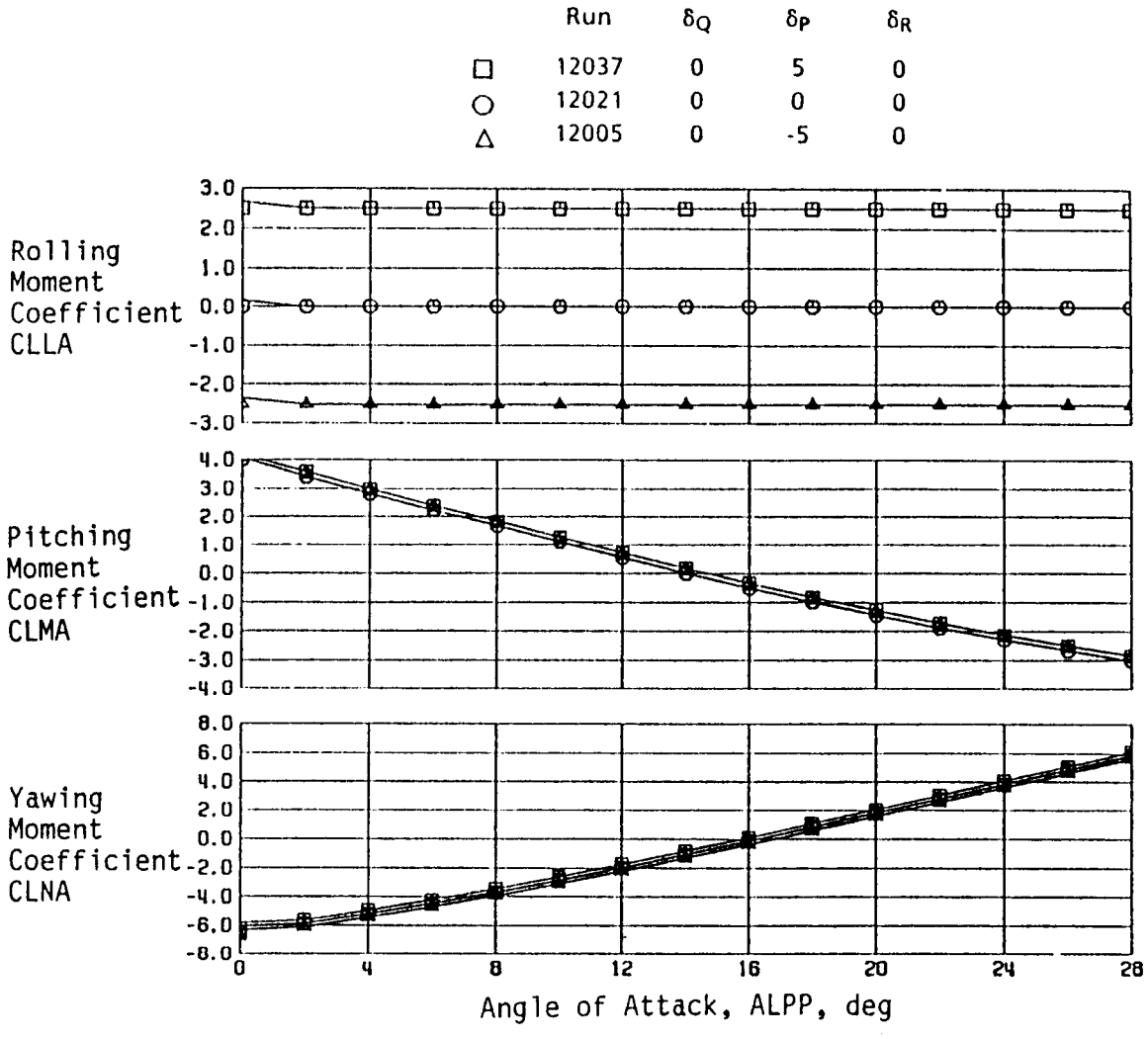
a. Moment Coefficients

Figure 7. Pitch Control Deflection Data for Example Case



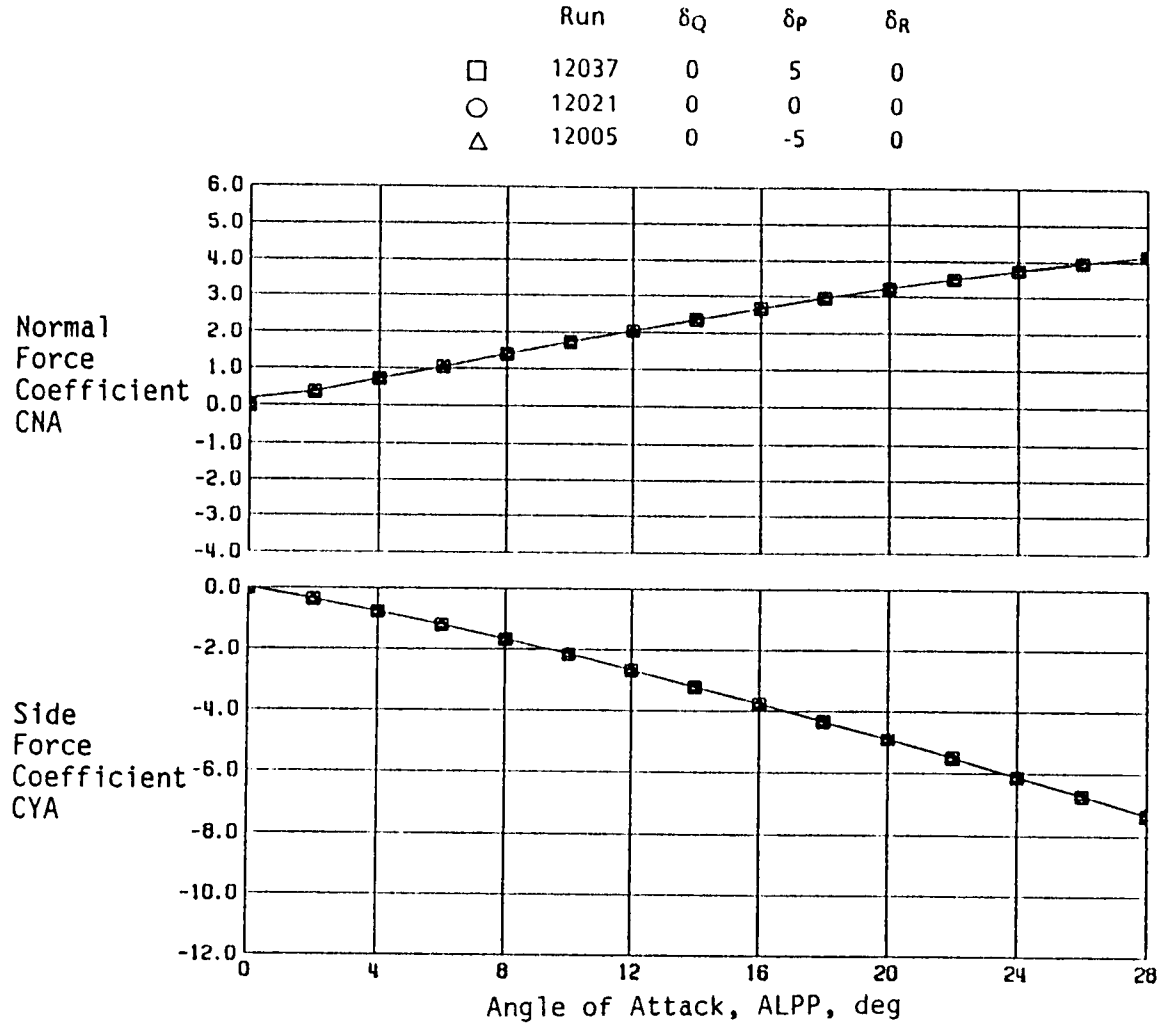
b. Force Coefficients

Figure 7. Concluded



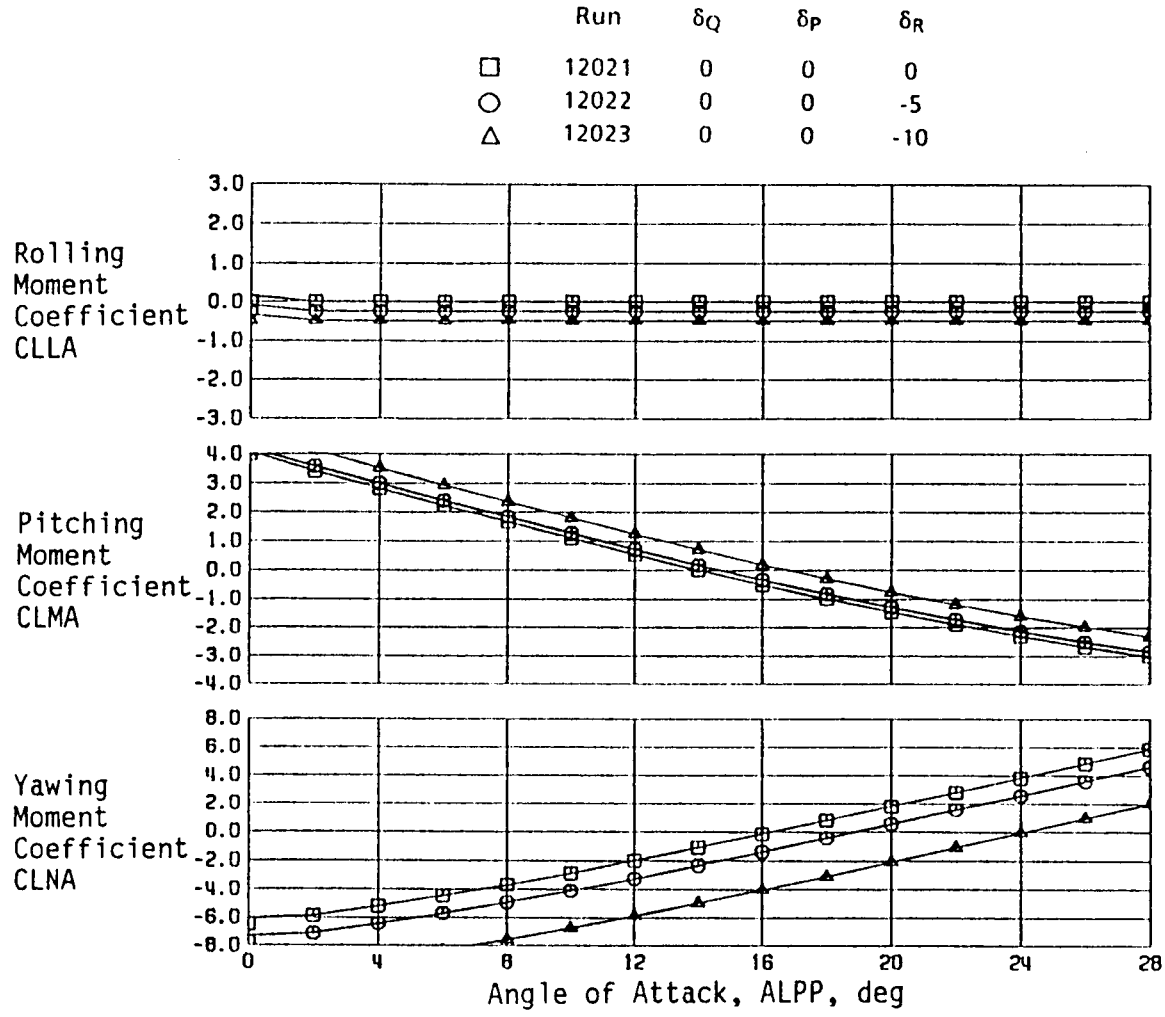
a. Moment Coefficients

Figure 8. Roll Control Deflection Data for Example Case



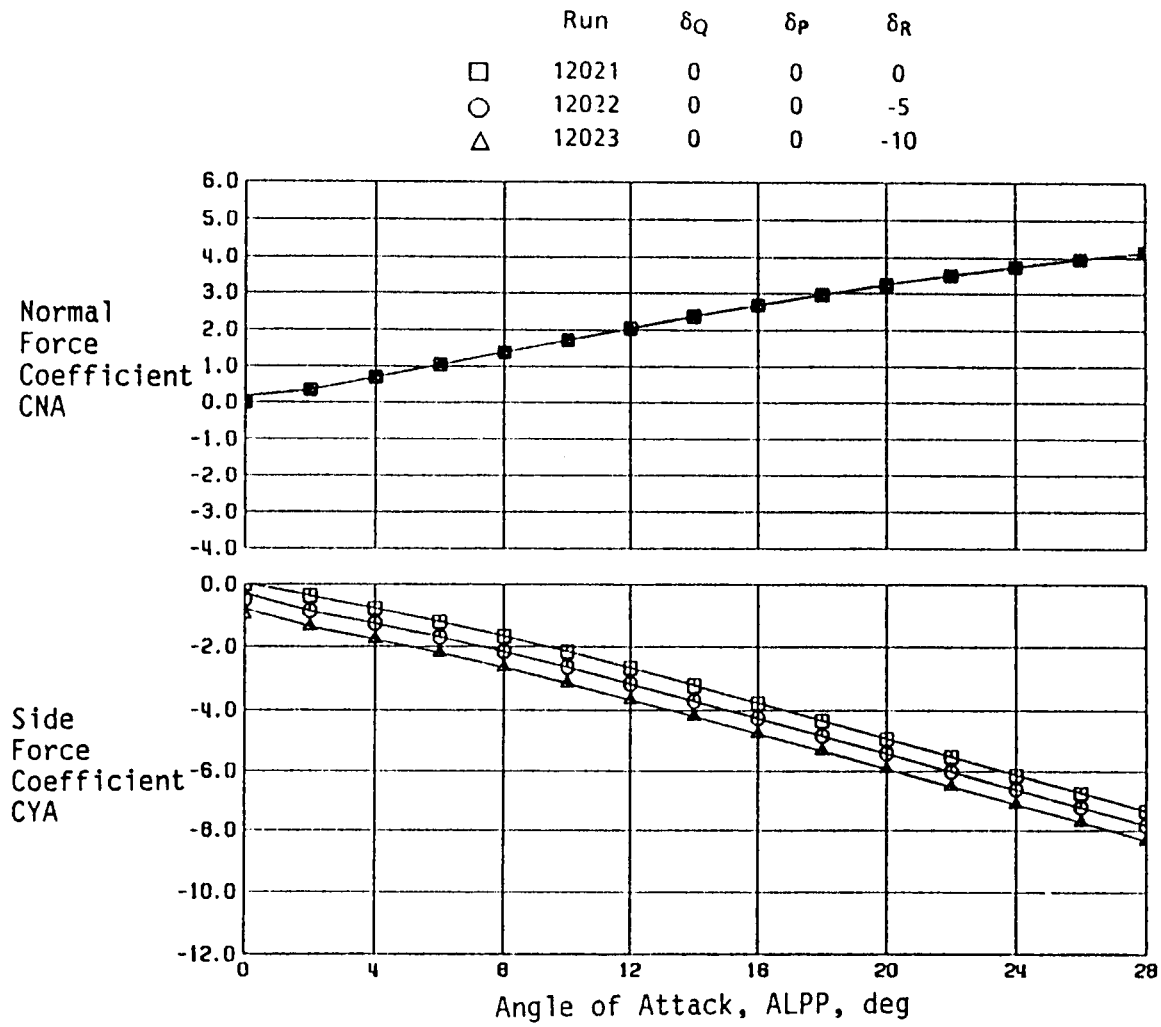
b. Force Coefficients

Figure 8. Concluded



a. Moment Coefficients

Figure 9. Yaw Control Deflection Data for Example Case



b. Force Coefficients

Figure 9. Concluded

questionable. The yaw deflection at trim should vary from +10 degrees near eight degrees angle of attack to -10 degrees near 24 degrees angle of attack. No yaw deflection should be required at some angle of attack near 16 degrees. The side force coefficient at trim should varyfrom about -0.5 at eight degrees angle of attack through -4 between 16 and 17 degrees to -7 at 24 degrees angle of attack.

With the anticipated results as a guide, the output from the new trim option can be evaluated. The printed output is listed in Appendix B, and trim results are plotted in Figures 10 and 11. At angles of attack of eight degrees and less, the yawing moment coefficient could not be trimmed. Yawing moments also prevented trim at angles of attack greater than 22 degrees. The estimated yaw deflections required are printed in the diagnostic messages shown in the example output. The control deflections at trim, shown in Figure 10, are within the ranges anticipated. Pitch deflection is zero just above 14 degrees angle of attack, and no yaw deflection is required near 16 degrees angle of attack. Roll trim required deflections of less than one degree and was not a limiting factor in the calculations.

The force coefficients at trim, plotted in Figure 11, also follow the trends predicted from the input data. The normal force at 14 degrees angle of attack is between 2.0 and 2.5 as expected. If extended to eight degrees angle of attack, the side force line would be very near -0.5, and at an angle of attack just over 16 degrees the side force is -4.

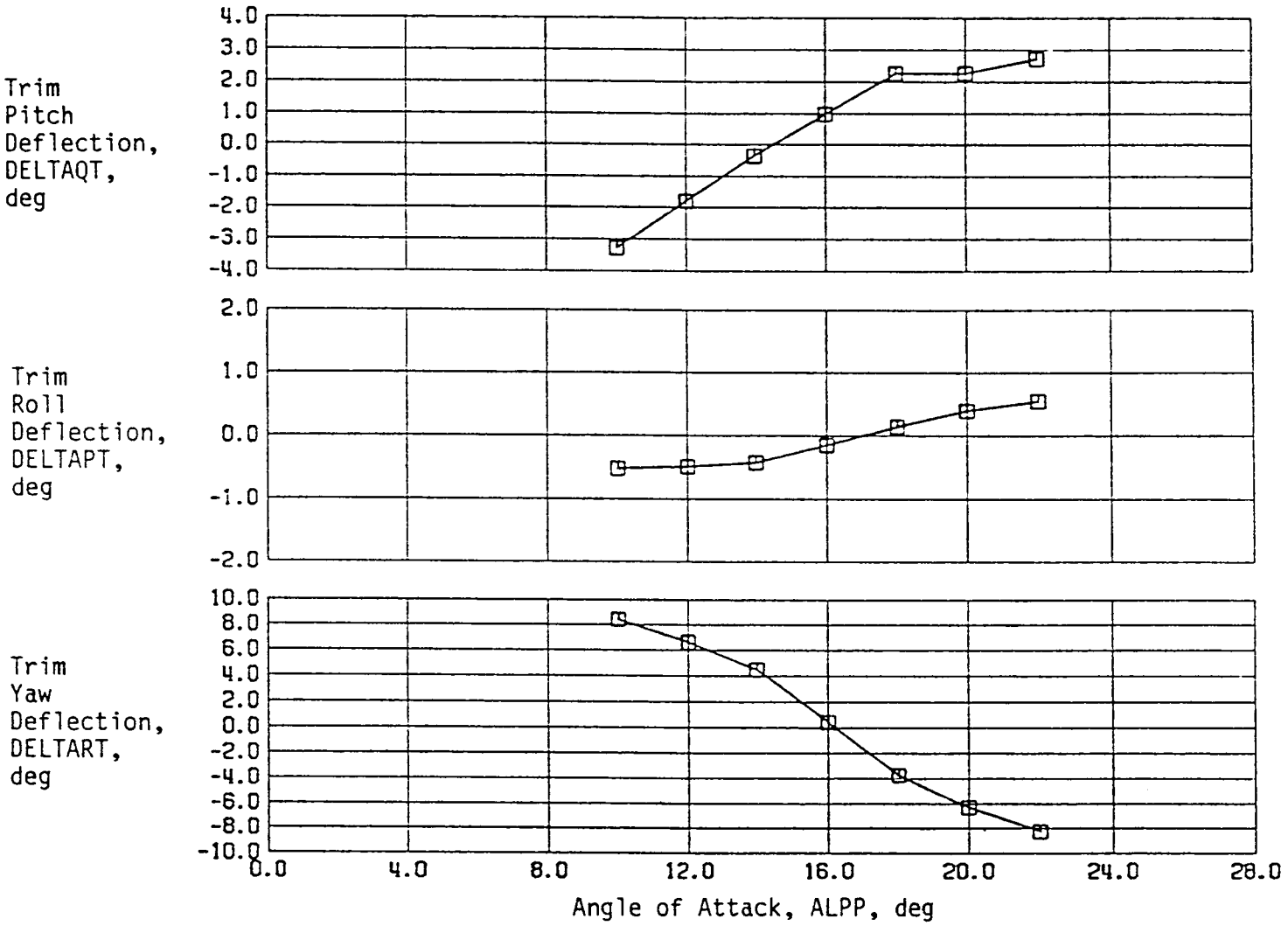


Figure 10. Deflection Angles Required to Trim Example Case

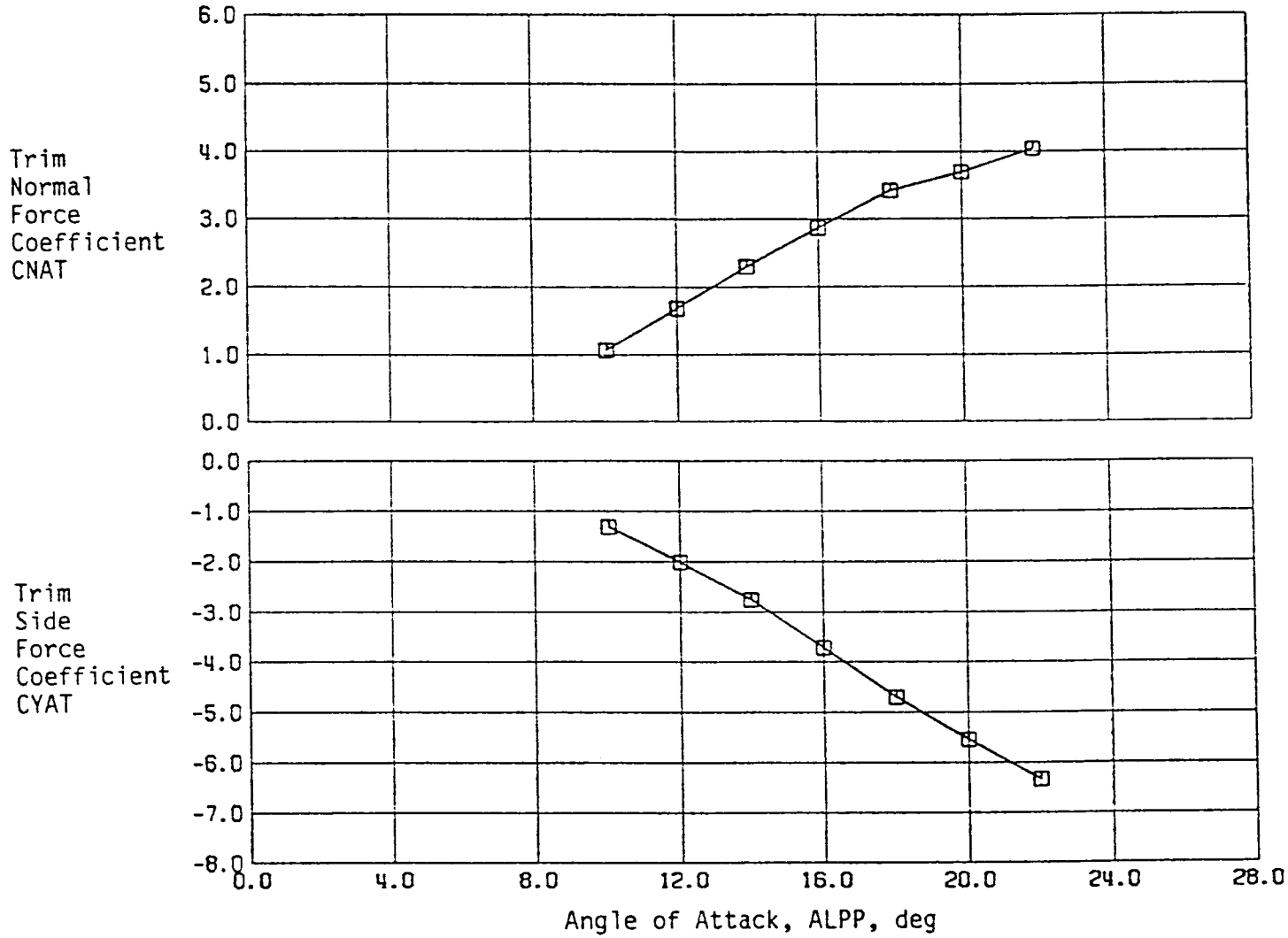


Figure 11. Force Coefficients at Trim for Example Case

The example case results match hand calculations, pilot program results, and the trends predicted from analysis of the input data very well. The same level of agreement has been demonstrated for the other check cases used during the development process.

The example case provides some insight to the computer time required for the trim calculations. That case required calculations at fifteen values of the independent variable (ALPP) based on Group A data. The calculations were completed in less than one second of computer time. While cases with Group A plus Group B data will require more time, the example case indicates the program is fast enough to be used in a test support environment.

CHAPTER IV

CONCLUDING REMARKS

The trend toward missile and fighter aircraft designs that fly at higher angles of attack increases the likelihood of cross coupling in both the aerodynamic coefficients and the control effectiveness derivatives. Because longitudinal trim analyses neglect the cross coupling, a multiple-axis trim analysis is required for the new designs. The program presented in this thesis provides a capability for trim about one, two, or three axes.

The three-axis trim program is applicable to a wide variety of flight vehicle types. If the control of a vehicle can be adequately represented by three aerodynamic moments and three control parameters, then the program should be applicable to that vehicle. Most, if not all of the vehicles presently in operation have control systems that fit this description. Vehicles with more complex control systems may require extension of the control effectiveness matrix to include additional derivatives. However, the program can be modified to apply to those vehicles.

The three-axis trim program includes features intended to ensure a high rate of success in using the program. The program is designed to accept data for the most common combinations of control deflections. The algorithm for finding input data bounding the estimated trim deflections prevents many bounding failures. The method of executing the program is very similar to that of executing the original trim

option of the Aerodynamic Data Analysis program. Therefore, engineers familiar with the original trim option can learn to use the three-axis program very easily.

The three-axis trim program has been validated by comparison with hand calculations for a variety of input data. Simulated data with the specific characteristics required to test the various features of the program were generated with a separate computer program. The simulated data facilitated testing some "worst case" input data characteristics, such as aerodynamic and control effectiveness cross coupling and nonlinear control effectiveness, before they occurred in an actual analysis effort. The program has also been successfully applied to wind tunnel data for actual flight vehicles. The results indicate accurate trim calculations can be performed under the assumptions of 1) piecewise linear control effectiveness, 2) control effectiveness symmetry about zero control deflection, 3) the validity of repetitive application of a linear solution technique (Gaussian Elimination) to the nonlinear moment equations (Equation (3)), and 4) the validity of the differencing approach (Figures 2 and 3, pages 20 and 21) to control effectiveness estimates at combinations of pitch, roll, and yaw control deflections. The first step in each application of the three-axis trim program should be an evaluation of these assumptions using the data for the specific vehicle of interest.

Analysis results must be obtained quickly if they are to be presented as a part of the final data package documenting a wind tunnel test. The ready access to the wind tunnel data and to the three-axis

trim program provided by the Aerodynamic Data Analysis program and the speed of the trim program enable analysis engineers to meet the requirement for quick results.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Langham, T. F., "An Automated System for the Analysis of Aerodynamic Stability and Control Parameters for Flight Vehicles", Master's Thesis, Unpublished.

2. Thomas, H. H. B. M., "The Aerodynamics of Aircraft Control: A General Survey in the Context of Active Control Technology", AGARD-R-711, July 1983.

3. Krieger, R. J., and Williams, J. E., "Accuracy Criteria for Evaluating Supersonic Missile Aerodynamic Coefficient Predictions", AIAA-81-1894, August 1981.

4. "U. S. Firm Claims Modified F-16 Could Match Su-27 'Cobra' Maneuver", Aviation Week & Space Technology, August 14, 1989, pp. 54-55.

5. Lacau, R. G., "A Survey of Missile Aerodynamics", presented at Nielsen Engineering and Research Missile Aerodynamics Conference, October 1988.

6. Sachs, Gottfried, "Direct Force Control", AGARD-R-711, July 1983.

7. Brebner, G. G., "The Control of Guided Weapons", Missile Aerodynamics, AGARD Lecture Series No. 98, 1979.

8. Forster, C. A., and Southgate, A. C., "The Difference Function Approach to the Overall Aerodynamics of Guided Weapons", Journal of the Royal Aeronautical Society, Vol. 64, No. 600, December 1960, pp.753-763.

9. Burden, Richard L. and Faires, J. Douglas Numerical Analysis. Prindle, Weber, and Schmidt, Boston, 1985, (Third Edition).

APPENDIXES

APPENDIX A

GAUSSIAN ELIMINATION WITH BACKWARD SUBSTITUTION

Gaussian Elimination [9] is a direct method for solving the linear system of n equations and n unknowns

$$[A][x] = [b] \tag{A1}$$

where [A] is an (n x n) matrix of coefficients, [x] is a vector of n unknowns, and [b] is a vector of known constants. The Gaussian method combines the [A] matrix and the [b] vector to form an "augmented" matrix

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots\cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} & b_2 \\ a_{31} & a_{32} & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdots\cdots & a_{nn} & b_n \end{array}\right] \tag{A2}$$

Designating each row of the augmented matrix by E_j where j = 1, 2, 3 --- n, then the replacement of row 3 by the difference between rows 1 and 2 can be expressed as $(E_1 - E_2) \rightarrow E_3$. The operation

$$(E_j - (a_{j1}/a_{11})E_1) \rightarrow E_j \tag{A3}$$

for $j = 2, 3, \cdots n$ makes the coefficient of x_1 zero in these rows. Then the operation

$$(E_j - (a_{ji}/a_{ii})E_i \rightarrow E_j \tag{A4}$$

is performed for each $i = 2, 3, \cdots n - 1$ and $j = i + 1, i + 2, \cdots n$. The result is a matrix of the form

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots\cdots & a_{1n} & a_{1,n+1} \\ 0 & a_{22} & \cdots\cdots & a_{2n} & a_{2,n+1} \\ \vdots & \searrow & & \vdots & \vdots \\ 0 & \cdots\cdots & 0 & a_{nn} & a_{n,n+1} \end{array}\right] \tag{A5}$$

The elements of rows 2 through n have different values than the corresponding elements of the original augmented matrix because of the operations performed during the reduction.

The reduced matrix represents a linear system with the same solution as the original system. The new system is in triangular form, that is

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= a_{1,n+1} \\ a_{22}x_2 + \cdots + a_{2n}x_n &= a_{2,n+1} \\ &\vdots \\ a_{nn}x_n &= a_{n,n+1} \end{aligned} \tag{A6}$$

The triangular form allows solutions of the system by backward substitution.

The backward substitution begins by solving the nth equation for x_n to give

$$x_n = a_{n,n+1}/a_{nn} \tag{A7}$$

The (n-1)st equation gives

$$x_{n-1} = [a_{n-1,n+1} - a_{n-1,n}\, x_n]/a_{n-1,n-1} \tag{A8}$$

The backward substitution continues with

$$x_i = \left[a_{i,n+1} - \sum_{j=i+1}^{n} a_{ij} x_j\right]/a_{ii} \tag{A9}$$

for each i = n-1, n-2, --- 2,1.

The a_{ii} element of the augmented matrix is referred to as the "pivot" element. The reduction of the augmented matrix will fail if the pivot element is zero because of the division in Equations (A3) and (A4). However, the solution of Equation (A1) does not depend on the order of the equations. Therefore, rows in Equation (A2) may be interchanged to avoid division by zero. As an example, consider the following matrix

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 5 \\ 0 & 0 & 4 & 2 & 1 \\ 0 & -3 & 1 & -1 & 3 \\ 0 & 1 & 2 & 5 & 6 \end{array}\right]$$

The pivot element for i = 2 in Equation (A4) (a_{22}) is zero. Either row 3 or row 4 may be switched with row 2 to obtain a nonzero pivot element. Because division by numbers with small magnitudes introduces error in machine computations, row 3 is the better choice for interchange in the example matrix. The practice of choosing the greatest magnitude below the diagonal in the ith column to be the pivot element is called maximal column pivoting.

In a computer program that must solve many systems of equations, execution time can be reduced by storing the multipliers (a_{ji}/a_{ii}) in Equation (A4). Retaining the multipliers eliminates the need to repeat the matrix reduction if the equations have the same coefficient matrix. If the coefficient condition is met, the multipliers can be applied to the [b] vector directly. The resulting vector is used as $a_{i,n+1}$ in Equation (A9) to obtain the new solution. The multipliers can be stored below the diagonal in the array used for the reduced matrix since those positions would contain zeros otherwise (see the lower left corner of Equation (A5)).

When the matrix reduction is not necessary, the values of $a_{j,n+1}$ to be used in the backward substitution are calculated from the new [b] vector using

$$a_{1,n+1} = b_1 \quad and \quad a_{j,n+1} = b_j - \sum_{i=1}^{j-1} m_{ji} b_j \tag{A10}$$

for j = n, n-1, --- 2 where m_{ji} are the multipliers calculated and stored during the matrix reduction.

The application of Gaussian Elimination to three-axis trim becomes very clear when the moment equations (3) from Chapter III are rewritten as

$$\begin{aligned} M_Q(\delta Q_{trim} - \delta Q_I) + M_P(\delta P_{trim} - \delta P_I) + M_R(\delta R_{trim} - \delta R_I) &= - C_{m_I} \\ L_Q(\delta Q_{trim} - \delta Q_I) + L_P(\delta P_{trim} - \delta P_I) + L_R(\delta R_{trim} - \delta R_I) &= - C_{\ell_I} \\ N_Q(\delta Q_{trim} - \delta Q_I) + N_P(\delta P_{trim} - \delta P_I) + N_R(\delta R_{trim} - \delta R_I) &= - C_{n_I} \end{aligned} \tag{A11}$$

The control effectiveness derivatives fill the 3 x 3 [A] matrix, the negatives of the residual moment coefficients are the [b] vector, and the unknowns are the control deflection increments required to trim.

The moment equations are not linear because the control effectiveness derivatives vary with the control deflections. The divided difference approximations of the control derivatives are constant between a given pair of input data runs. However, when the estimated trim deflection crosses the deflection value of an input data run, the control derivative value changes. If the new control derivative value is not used in subsequent calculations, then erroneous trim deflections

will result. The moment equations are solved iteratively to account for the changing control derivatives. As the deflections approach the actual trim values, the bounding deflections and the control derivatives stop changing. A second iteration with the same control derivatives will produce the trim deflections for the current value of the independent variable.

APPENDIX B

EXAMPLE OUTPUT

```
**************************************
* SUMMARY OF OPTIONS TO BE PERFORMED *
**************************************

*** ORIGINAL TEKPLOT DATA IS IN MISSILE AXIS ***


*** INDEPENDENT VARIABLE RANGE ***

ALPP:
 MINIMUM =      0.0000; MAXIMUM =     28.0000; DELTA =      2.0000

BETA:
 MINIMUM =      0.0000; MAXIMUM =      0.0000; DELTA =     10.0000

EXTRAPOLATION OPTION:
  NO EXTRAPOLATION BEYOND ACTUAL DATA (DEFAULT)
```

NONLINEAR A DATA

RUN NUMBER 12017

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	5.00000	0.00000	0.00000	6.49652	0.25000	-6.24818	1.00206	-0.00108
2.00000	5.00000	0.00000	0.00000	5.90703	0.25000	-5.62910	1.35152	-0.36808
4.00000	5.00000	0.00000	0.00000	5.31848	0.25000	-4.95194	1.70042	-0.76951
6.00000	5.00000	0.00000	0.00000	4.73427	0.25000	-4.22052	2.04675	-1.20311
8.00000	5.00000	0.00000	0.00000	4.15777	0.25000	-3.43893	2.38851	-1.66645
10.00000	5.00000	0.00000	0.00000	3.59227	0.25000	-2.61147	2.72375	-2.15698
12.00000	5.00000	0.00000	0.00000	3.04102	0.25000	-1.74265	3.05054	-2.67203
14.00000	5.00000	0.00000	0.00000	2.50713	0.25000	-0.83714	3.36704	-3.20883
16.00000	5.00000	0.00000	0.00000	1.99362	0.25000	0.10025	3.67146	-3.76453
18.00000	5.00000	0.00000	0.00000	1.50334	0.25000	1.06457	3.96210	-4.33619
20.00000	5.00000	0.00000	0.00000	1.03901	0.25000	2.05082	4.23736	-4.92086
22.00000	5.00000	0.00000	0.00000	0.60317	0.25000	3.05391	4.49574	-5.51550
24.00000	5.00000	0.00000	0.00000	0.19813	0.25000	4.06876	4.73585	-6.11712
26.00000	5.00000	0.00000	0.00000	-0.17397	0.25000	5.09026	4.95643	-6.72269
28.00000	5.00000	0.00000	0.00000	-0.51122	0.25000	6.11334	5.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12021

ALPP	DELTAQ	DELTAF	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	0.00000	0.00000	0.00000	3.99652	0.00000	-6.49818	0.00206	-0.00108
2.00000	0.00000	0.00000	0.00000	3.40703	0.00000	-5.87910	0.35152	-0.36808
4.00000	0.00000	0.00000	0.00000	2.81848	0.00000	-5.20194	0.70042	-0.76951
6.00000	0.00000	0.00000	0.00000	2.23427	0.00000	-4.47052	1.04675	-1.20311
8.00000	0.00000	0.00000	0.00000	1.65777	0.00000	-3.68893	1.38851	-1.66645
10.00000	0.00000	0.00000	0.00000	1.09227	0.00000	-2.86147	1.72375	-2.15698
12.00000	0.00000	0.00000	0.00000	0.54102	0.00000	-1.99265	2.05054	-2.67203
14.00000	0.00000	0.00000	0.00000	0.00713	0.00000	-1.08714	2.36704	-3.20883
16.00000	0.00000	0.00000	0.00000	-0.50638	0.00000	-0.14975	2.67146	-3.76453
18.00000	0.00000	0.00000	0.00000	-0.99666	0.00000	0.81457	2.96210	-4.33619
20.00000	0.00000	0.00000	0.00000	-1.46099	0.00000	1.80082	3.23736	-4.92086
22.00000	0.00000	0.00000	0.00000	-1.89683	0.00000	2.80391	3.49574	-5.51550
24.00000	0.00000	0.00000	0.00000	-2.30187	0.00000	3.81876	3.73585	-6.11712
26.00000	0.00000	0.00000	0.00000	-2.67397	0.00000	4.84026	3.95643	-6.72269
28.00000	0.00000	0.00000	0.00000	-3.01122	0.00000	5.86334	4.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12025

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	-10.00000	0.00000	0.00000	-1.00348	-0.50000	-6.99818	-1.99794	-0.00108
2.00000	-10.00000	0.00000	0.00000	-1.59297	-0.50000	-6.37910	-1.64848	-0.36808
4.00000	-10.00000	0.00000	0.00000	-2.18152	-0.50000	-5.70194	-1.29957	-0.76951
6.00000	-10.00000	0.00000	0.00000	-2.76573	-0.50000	-4.97052	-0.95325	-1.20311
8.00000	-10.00000	0.00000	0.00000	-3.34223	-0.50000	-4.18893	-0.61149	-1.66645
10.00000	-10.00000	0.00000	0.00000	-3.90773	-0.50000	-3.36147	-0.27625	-2.15698
12.00000	-10.00000	0.00000	0.00000	-4.45898	-0.50000	-2.49265	0.05054	-2.67203
14.00000	-10.00000	0.00000	0.00000	-4.99287	-0.50000	-1.58714	0.36704	-3.20883
16.00000	-10.00000	0.00000	0.00000	-5.50638	-0.50000	-0.64975	0.67146	-3.76453
18.00000	-10.00000	0.00000	0.00000	-5.99666	-0.50000	0.31457	0.96210	-4.33619
20.00000	-10.00000	0.00000	0.00000	-6.46099	-0.50000	1.30082	1.23736	-4.92086
22.00000	-10.00000	0.00000	0.00000	-6.89683	-0.50000	2.30391	1.49574	-5.51550
24.00000	-10.00000	0.00000	0.00000	-7.30187	-0.50000	3.31876	1.73585	-6.11712
26.00000	-10.00000	0.00000	0.00000	-7.67397	-0.50000	4.34026	1.95643	-6.72269
28.00000	-10.00000	0.00000	0.00000	-8.01122	-0.50000	5.36334	2.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12029

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	-15.00000	0.00000	0.00000	-3.50348	-0.75000	-7.24818	-2.99794	-0.00108
2.00000	-15.00000	0.00000	0.00000	-4.09297	-0.75000	-6.62910	-2.64848	-0.36808
4.00000	-15.00000	0.00000	0.00000	-4.68152	-0.75000	-5.95194	-2.29957	-0.76951
6.00000	-15.00000	0.00000	0.00000	-5.26573	-0.75000	-5.22052	-1.95325	-1.20311
8.00000	-15.00000	0.00000	0.00000	-5.84223	-0.75000	-4.43893	-1.61149	-1.66645
10.00000	-15.00000	0.00000	0.00000	-6.40773	-0.75000	-3.61147	-1.27625	-2.15698
12.00000	-15.00000	0.00000	0.00000	-6.95898	-0.75000	-2.74265	-0.94946	-2.67203
14.00000	-15.00000	0.00000	0.00000	-7.49287	-0.75000	-1.83714	-0.63296	-3.20883
16.00000	-15.00000	0.00000	0.00000	-8.00638	-0.75000	-0.89975	-0.32854	-3.76453
18.00000	-15.00000	0.00000	0.00000	-8.49666	-0.75000	0.06457	-0.03790	-4.33619
20.00000	-15.00000	0.00000	0.00000	-8.96099	-0.75000	1.05082	0.23736	-4.92086
22.00000	-15.00000	0.00000	0.00000	-9.39683	-0.75000	2.05391	0.49574	-5.51550
24.00000	-15.00000	0.00000	0.00000	-9.80187	-0.75000	3.06876	0.73585	-6.11712
26.00000	-15.00000	0.00000	0.00000	-10.17397	-0.75000	4.09026	0.95643	-6.72269
28.00000	-15.00000	0.00000	0.00000	-10.51122	-0.75000	5.11334	1.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12037

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	0.00000	5.00000	0.00000	4.17152	2.50000	-6.24818	0.00206	-0.00108
2.00000	0.00000	5.00000	0.00000	3.58203	2.50000	-5.62910	0.35152	-0.36808
4.00000	0.00000	5.00000	0.00000	2.99348	2.50000	-4.95194	0.70042	-0.76951
6.00000	0.00000	5.00000	0.00000	2.40927	2.50000	-4.22052	1.04675	-1.20311
8.00000	0.00000	5.00000	0.00000	1.83277	2.50000	-3.43893	1.38851	-1.66645
10.00000	0.00000	5.00000	0.00000	1.26727	2.50000	-2.61147	1.72375	-2.15698
12.00000	0.00000	5.00000	0.00000	0.71602	2.50000	-1.74265	2.05054	-2.67203
14.00000	0.00000	5.00000	0.00000	0.18213	2.50000	-0.83714	2.36704	-3.20883
16.00000	0.00000	5.00000	0.00000	-0.33138	2.50000	0.10025	2.67146	-3.76453
18.00000	0.00000	5.00000	0.00000	-0.82166	2.50000	1.06457	2.96210	-4.33619
20.00000	0.00000	5.00000	0.00000	-1.28599	2.50000	2.05082	3.23736	-4.92086
22.00000	0.00000	5.00000	0.00000	-1.72183	2.50000	3.05391	3.49574	-5.51550
24.00000	0.00000	5.00000	0.00000	-2.12687	2.50000	4.06876	3.73585	-6.11712
26.00000	0.00000	5.00000	0.00000	-2.49897	2.50000	5.09026	3.95643	-6.72269
28.00000	0.00000	5.00000	0.00000	-2.83622	2.50000	6.11334	4.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12021

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	0.00000	0.00000	0.00000	3.99652	0.00000	-6.49818	0.00206	-0.00108
2.00000	0.00000	0.00000	0.00000	3.40703	0.00000	-5.87910	0.35152	-0.36808
4.00000	0.00000	0.00000	0.00000	2.81848	0.00000	-5.20194	0.70042	-0.76951
6.00000	0.00000	0.00000	0.00000	2.23427	0.00000	-4.47052	1.04675	-1.20311
8.00000	0.00000	0.00000	0.00000	1.65777	0.00000	-3.68893	1.38851	-1.66645
10.00000	0.00000	0.00000	0.00000	1.09227	0.00000	-2.86147	1.72375	-2.15698
12.00000	0.00000	0.00000	0.00000	0.54102	0.00000	-1.99265	2.05054	-2.67203
14.00000	0.00000	0.00000	0.00000	0.00713	0.00000	-1.08714	2.36704	-3.20883
16.00000	0.00000	0.00000	0.00000	-0.50638	0.00000	-0.14975	2.67146	-3.76453
18.00000	0.00000	0.00000	0.00000	-0.99666	0.00000	0.81457	2.96210	-4.33619
20.00000	0.00000	0.00000	0.00000	-1.46099	0.00000	1.80082	3.23736	-4.92086
22.00000	0.00000	0.00000	0.00000	-1.89683	0.00000	2.80391	3.49574	-5.51550
24.00000	0.00000	0.00000	0.00000	-2.30187	0.00000	3.81876	3.73585	-6.11712
26.00000	0.00000	0.00000	0.00000	-2.67397	0.00000	4.84026	3.95643	-6.72269
28.00000	0.00000	0.00000	0.00000	-3.01122	0.00000	5.86334	4.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12005

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	0.00000	-5.00000	0.00000	4.17152	-2.50000	-6.74818	0.00206	-0.00108
2.00000	0.00000	-5.00000	0.00000	3.58203	-2.50000	-6.12910	0.35152	-0.36808
4.00000	0.00000	-5.00000	0.00000	2.99348	-2.50000	-5.45194	0.70042	-0.76951
6.00000	0.00000	-5.00000	0.00000	2.40927	-2.50000	-4.72052	1.04675	-1.20311
8.00000	0.00000	-5.00000	0.00000	1.83277	-2.50000	-3.93893	1.38851	-1.66645
10.00000	0.00000	-5.00000	0.00000	1.26727	-2.50000	-3.11147	1.72375	-2.15698
12.00000	0.00000	-5.00000	0.00000	0.71602	-2.50000	-2.24265	2.05054	-2.67203
14.00000	0.00000	-5.00000	0.00000	0.18213	-2.50000	-1.33714	2.36704	-3.20883
16.00000	0.00000	-5.00000	0.00000	-0.33138	-2.50000	-0.39975	2.67146	-3.76453
18.00000	0.00000	-5.00000	0.00000	-0.82166	-2.50000	0.56457	2.96210	-4.33619
20.00000	0.00000	-5.00000	0.00000	-1.28599	-2.50000	1.55082	3.23736	-4.92086
22.00000	0.00000	-5.00000	0.00000	-1.72183	-2.50000	2.55391	3.49574	-5.51550
24.00000	0.00000	-5.00000	0.00000	-2.12687	-2.50000	3.56876	3.73585	-6.11712
26.00000	0.00000	-5.00000	0.00000	-2.49897	-2.50000	4.59026	3.95643	-6.72269
28.00000	0.00000	-5.00000	0.00000	-2.83622	-2.50000	5.61334	4.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12021

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	0.00000	0.00000	0.00000	3.99652	0.00000	-6.49818	0.00206	-0.00108
2.00000	0.00000	0.00000	0.00000	3.40703	0.00000	-5.87910	0.35152	-0.36808
4.00000	0.00000	0.00000	0.00000	2.81848	0.00000	-5.20194	0.70042	-0.76951
6.00000	0.00000	0.00000	0.00000	2.23427	0.00000	-4.47052	1.04675	-1.20311
8.00000	0.00000	0.00000	0.00000	1.65777	0.00000	-3.68893	1.38851	-1.66645
10.00000	0.00000	0.00000	0.00000	1.09227	0.00000	-2.86147	1.72375	-2.15698
12.00000	0.00000	0.00000	0.00000	0.54102	0.00000	-1.99265	2.05054	-2.67203
14.00000	0.00000	0.00000	0.00000	0.00713	0.00000	-1.08714	2.36704	-3.20883
16.00000	0.00000	0.00000	0.00000	-0.50638	0.00000	-0.14975	2.67146	-3.76453
18.00000	0.00000	0.00000	0.00000	-0.99666	0.00000	0.81457	2.96210	-4.33619
20.00000	0.00000	0.00000	0.00000	-1.46099	0.00000	1.80082	3.23736	-4.92086
22.00000	0.00000	0.00000	0.00000	-1.89683	0.00000	2.80391	3.49574	-5.51550
24.00000	0.00000	0.00000	0.00000	-2.30187	0.00000	3.81876	3.73585	-6.11712
26.00000	0.00000	0.00000	0.00000	-2.67397	0.00000	4.84026	3.95643	-6.72269
28.00000	0.00000	0.00000	0.00000	-3.01122	0.00000	5.86334	4.15636	-7.32919

NONLINEAR A DATA

RUN NUMBER 12022

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	0.00000	0.00000	-5.00000	4.17152	-0.25000	-7.74818	0.00206	-0.50108
2.00000	0.00000	0.00000	-5.00000	3.58203	-0.25000	-7.12910	0.35152	-0.86808
4.00000	0.00000	0.00000	-5.00000	2.99348	-0.25000	-6.45194	0.70042	-1.26951
6.00000	0.00000	0.00000	-5.00000	2.40927	-0.25000	-5.72052	1.04675	-1.70311
8.00000	0.00000	0.00000	-5.00000	1.83277	-0.25000	-4.93893	1.38851	-2.16645
10.00000	0.00000	0.00000	-5.00000	1.26727	-0.25000	-4.11147	1.72375	-2.65698
12.00000	0.00000	0.00000	-5.00000	0.71602	-0.25000	-3.24265	2.05054	-3.17203
14.00000	0.00000	0.00000	-5.00000	0.18213	-0.25000	-2.33714	2.36704	-3.70883
16.00000	0.00000	0.00000	-5.00000	-0.33138	-0.25000	-1.39975	2.67146	-4.26453
18.00000	0.00000	0.00000	-5.00000	-0.82166	-0.25000	-0.43543	2.96210	-4.83619
20.00000	0.00000	0.00000	-5.00000	-1.28599	-0.25000	0.55082	3.23736	-5.42086
22.00000	0.00000	0.00000	-5.00000	-1.72183	-0.25000	1.55391	3.49574	-6.01550
24.00000	0.00000	0.00000	-5.00000	-2.12687	-0.25000	2.56876	3.73585	-6.61712
26.00000	0.00000	0.00000	-5.00000	-2.49897	-0.25000	3.59026	3.95643	-7.22269
28.00000	0.00000	0.00000	-5.00000	-2.83622	-0.25000	4.61334	4.15636	-7.82919

NONLINEAR A DATA

RUN NUMBER 12023

ALPP	DELTAQ	DELTAP	DELTAR	CLMA	CLLA	CLNA	CNA	CYA
0.00000	0.00000	0.00000	-10.00000	4.69652	-0.50000	-10.38706	0.00206	-1.00108
2.00000	0.00000	0.00000	-10.00000	4.10703	-0.50000	-9.76799	0.35152	-1.36808
4.00000	0.00000	0.00000	-10.00000	3.51848	-0.50000	-9.09083	0.70042	-1.76951
6.00000	0.00000	0.00000	-10.00000	2.93427	-0.50000	-8.35941	1.04675	-2.20311
8.00000	0.00000	0.00000	-10.00000	2.35777	-0.50000	-7.57782	1.38851	-2.66645
10.00000	0.00000	0.00000	-10.00000	1.79227	-0.50000	-6.75036	1.72375	-3.15698
12.00000	0.00000	0.00000	-10.00000	1.24102	-0.50000	-5.88154	2.05054	-3.67203
14.00000	0.00000	0.00000	-10.00000	0.70713	-0.50000	-4.97602	2.36704	-4.20883
16.00000	0.00000	0.00000	-10.00000	0.19362	-0.50000	-4.03864	2.67146	-4.76453
18.00000	0.00000	0.00000	-10.00000	-0.29666	-0.50000	-3.07432	2.96210	-5.33619
20.00000	0.00000	0.00000	-10.00000	-0.76099	-0.50000	-2.08807	3.23736	-5.92086
22.00000	0.00000	0.00000	-10.00000	-1.19683	-0.50000	-1.08498	3.49574	-6.51550
24.00000	0.00000	0.00000	-10.00000	-1.60187	-0.50000	-0.07013	3.73585	-7.11712
26.00000	0.00000	0.00000	-10.00000	-1.97397	-0.50000	0.95137	3.95643	-7.72269
28.00000	0.00000	0.00000	-10.00000	-2.31122	-0.50000	1.97445	4.15636	-8.32919

```
TRIM NOT POSSIBLE AT ALPHA OF   0.000000000E+00
 YAW DEFLECTION    16.0071564      NOT BOUNDED BY DATA
GOING TO NEXT POINT

TRIM NOT POSSIBLE AT ALPHA OF    2.00000000
 YAW DEFLECTION    14.6978970      NOT BOUNDED BY DATA
GOING TO NEXT POINT

TRIM NOT POSSIBLE AT ALPHA OF    4.00000000
 YAW DEFLECTION    13.2755823      NOT BOUNDED BY DATA
GOING TO NEXT POINT

TRIM NOT POSSIBLE AT ALPHA OF    6.00000000
 YAW DEFLECTION    11.7482834      NOT BOUNDED BY DATA
GOING TO NEXT POINT

TRIM NOT POSSIBLE AT ALPHA OF    8.00000000
 YAW DEFLECTION    10.1245289      NOT BOUNDED BY DATA
GOING TO NEXT POINT

TRIM NOT POSSIBLE AT ALPHA OF    24.0000000
 YAW DEFLECTION   -16.2074738      NOT BOUNDED BY DATA
GOING TO NEXT POINT

TRIM NOT POSSIBLE AT ALPHA OF    26.0000000
 YAW DEFLECTION   -20.4960937      NOT BOUNDED BY DATA
GOING TO NEXT POINT

TRIM NOT POSSIBLE AT ALPHA OF    28.0000000
 YAW DEFLECTION   -24.7461243      NOT BOUNDED BY DATA
GOING TO NEXT POINT
```

TRIM-3D OPTION

ALPP	DELTAQ	DELTAP	DELTAR	CLMAT	CLLAT	CLNAT	CNAT	CYAT
10.00000	-3.28722	-0.51261	8.41330	0.00000	0.00000	0.00000	1.06630	-1.31565
12.00000	-1.80678	-0.48171	6.62393	0.00000	0.00000	0.00000	1.68918	-2.00964
14.00000	-0.35849	-0.41446	4.50313	0.00000	0.00000	0.00000	2.29534	-2.75852
16.00000	0.97265	-0.14052	0.43258	0.00000	0.00000	0.00000	2.86599	-3.72127
18.00000	2.26551	0.14754	-3.74089	0.00000	0.00000	0.00000	3.41520	-4.71028
20.00000	2.27143	0.40255	-6.29698	0.00000	0.00000	0.00000	3.69164	-5.55055
22.00000	2.72149	0.55330	-8.25448	0.00000	0.00000	0.00000	4.04003	-6.34095

VITA

Ricky L. Arterbury was born in Texarkana, Arkansas on October 4, 1957. He graduated from the Arkansas Senior High School in May 1976. In September 1976 he began his undergraduate work in Aerospace Engineering at Auburn University, Auburn, Alabama. During his sophomore and junior years at Auburn, he participated in the cooperative education program and worked alternate quarters at the Arnold Engineering Development Center (AEDC) at Tullahoma, Tennessee. After receiving his Bachelor of Science degree from Auburn in June 1981, he returned to AEDC as an employee of Calspan Corporation/AEDC Operations.

In the fall of 1984 Mr. Arterbury began his graduate studies as a part-time student at the University of Tennessee Space Institute (UTSI). He expects to complete the requirements for a Masters degree in Aerospace Engineering in May 1990.