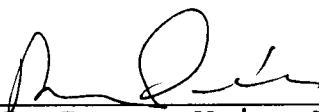


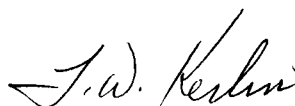
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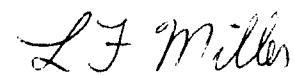
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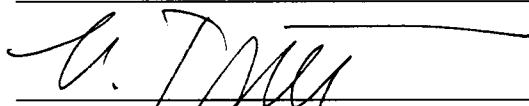


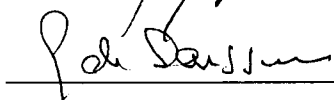
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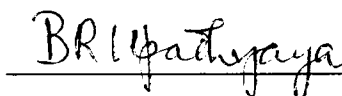
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Accepted for the Council:



Vice Provost
and Dean of the Graduate School

STUDY AND DEVELOPMENT OF
ADVANCED CONTROL TECHNIQUES FOR
NUCLEAR REACTORS AND ROBOTS

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Carlos March-Leuba

May 1989

to my wife, Nieves, and
my parents, Jose and Pilar

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ABSTRACT

This dissertation studies and develops some aspects of the Optimal Control Theory with the objective of evaluating the benefits that the nuclear industry could obtain by applying these advanced control techniques.

First, the basic relationship between Optimal Control Theory and Closed-Loop Control Design has been identified. As a result of this work, new algorithms have been developed for feedback implementations. The applicability of these new algorithms to problems such as state estimation, filtering, model update, and model decoupling has been studied. In addition, new alternatives to control design that are not based on Optimal Control Theory have been developed.

A broad range of application examples have been presented for several physical systems, including pressurized water nuclear reactors, boiling water nuclear reactors, steam generators, and robotics.

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CHAPTER 1

INTRODUCTION

The instrumentation and control hardware for nuclear reactors have been developed mostly based on analog circuits because digital instrumentation was not available when the major part of the existing nuclear reactors were built. Control algorithms were designed taking into account the analog constraints, so that an impressive amount of creative effort had to be devoted to the design of the analog controllers. Given the limited tools available to the designer, the results obtained were excellent as the performance of these control algorithms has demonstrated for a long period of time.

Analog circuits are becoming obsolete and they are not the standard hardware used by the control systems industry any more. Indeed, the major vendors of control hardware in the United States are not selling replacement analog systems or even spare parts for the existing systems. Thus, the nuclear industry is now facing the challenge of having to move to digital-based hardware systems.

Certainly, it is necessary to change the hardware now, but the question that still remains open is: do we have to change also the control algorithms?

The introduction of fast and reliable microprocessors have opened the possibility of performing sophisticated on-line calculations. With these new tools available, it will perhaps be possible to improve the dynamic behavior of nuclear reactors. Perhaps, by using more sophisticated control algorithm, nuclear reactors could be operated under more demanding conditions, or perhaps, it will be possible to decrease costly down time.

This doctoral dissertation has been devoted to study, assess, and develop these possibilities.

1.1 Motivation and Objectives

In the past three decades a significant advance in control techniques have been introduced in many of the engineering fields. In particular, aerospace engineering has been very innovative in developing algorithms emphasizing the characteristics of adaptability and robustness in the control of nonlinear systems. On the other hand, mathematicians have introduced new powerful tools to analyze and study the dynamical behavior of sophisticated systems.

A few of these advanced techniques have reached the nuclear engineering field. In the last few years, an important amount of research time has been invested in analyzing how much improvement can be achieved by applying

advanced control algorithms to nuclear reactors. This dissertation intends to be a contribution to this effort.

The objectives of this dissertation have been:

- (a) an assessment of Optimal Control Theory as a tool for the development of closed-loop control algorithms,
- (b) the development of alternative formulations of the Optimal Control Theory for nonlinear systems suitable for on-line computer implementation,
- (c) A study of the applicability of the above alternative formulations to plant-state estimation, filtering, model updating, and large model decomposition and decoupling,
- (d) an implementation of the new techniques to a broad range of typical applications, and
- (e) the development of new alternatives to control design that are not based on optimal control theory.

1.2 Background

The two most important control hardware vendors in the USA (Foxboro and Baley) have discontinued the production of analog devices. Since the industry is having difficulties to maintain the existing systems, there is an ongoing effort to replace analog by digital controllers.

The benefits of the new digital control devices over analog systems are expected to be: (a) easier

implementation, (b) better display of information for the operator, and (c) easier maintenance.

Yet control strategies have not followed this renovative process. The algorithms implemented in the digital devices remain the same as those used in the analog systems. The main reason for conserving the old algorithms is that, in general, they appear to work well and, most importantly, they are well understood.

Unfortunately, the above statement is not always true. The good safety record of nuclear industry seems to be related more to the intrinsic stability of nuclear reactors than to the performance of their control algorithms.

For instance, the level control of steam generators at low power is generally transferred to manual operation because the present automatic control system becomes unstable and causes reactor trips due to primary system overcooling. Similarly, boiling water reactors (BWR) are mostly used as base-load reactors because the control design can not handle fast load changes.

Since an increasing demand for better performing algorithms is expected in the next few years, there is a general interest in developing control algorithms that are able to perform over a wider range of operating conditions in order to avoid undesired reactor trips. Thus, it is

reasonable to predict that an investment in advanced control techniques will pay off by a reduction of expensive down time and increased availability.

Control design can be classified into two major areas: closed-loop and open-loop design. Closed-loop strategies introduce the control algorithm as a feedback to the system. Open-loop strategies are obtained off-line and, then, applied to the system. Both can be very powerful and it is important to understand their advantages and disadvantages.

Open-loop controls allow one to introduce sophisticated calculations that, in principle, can lead the system to an optimal performance. Unfortunately, these calculations have to be performed based on an approximate model of the plant. If the plant behavior is perturbed, the results are not optimal, and, in some cases can be catastrophic.

Feedback controllers are a dynamic part of the system and have the ability to adapt to unexpected perturbations. Stability can be predicted if no fundamental changes in the dynamic behavior occur. The disadvantage is that the algorithm can only use past information from the plant, and will not take into account future events or expected demands, which are known in many cases.

The research on advanced control techniques for nuclear reactors started in the sixties with the work of Rosztoczy¹. This author applied the recently published work of Pontryagin², The Maximum Principle, to optimize the control-rod movement during transients in a simplified nonlinear nuclear core model. An analog computer was developed to solve the associated two-point boundary value problem. A more global study of the optimal control techniques and their applicability to nuclear reactors was done later by Mohler and Shen³. In Europe, advanced controllers were designed and applied to nuclear reactors in the late seventies.⁴

In the past years, some isolated work has been performed in the United States. Now, a more coordinated effort is being made by the Advanced Controls Program (ACTO)⁵ at ORNL. This program represents a unique opportunity to study and analyze the benefits of advanced algorithms for the nuclear industry. This centralized and unified effort will allow to conduct cutting edge research that could not have been afforded by any particular group alone. The ACTO program can be the breakthrough point for the nuclear industry in the United States. Huge benefits could be obtained on safety and improved operative capabilities.

1.3 Preview of this Dissertation

This dissertation is organized as follows:

Chapter 2 is devoted to the study of Optimal Control based feedback controllers. The purpose of this work is to show, by stepping through a series of examples, how the basic cost function minimization problem must be reformulated to obtain a closed-loop control strategy. Some applications of Closed-Loop Optimal Control Design are proposed: state estimation, parameter tracking, uncertainty tracking, and system decomposition.

Chapter 3 presents a new formulation of a class of nonlinear optimal control problems in which the system's parameters are changing arbitrarily with time. A variational technique based on Pontryagin's Maximum Principle is used to track the time-varying parameters of the system and to calculate the optimal control action. An application to a nonlinear nuclear reactor in which feedback coefficients and sensors time constants are varying is presented.

Chapter 4 presents a new formulation of a class of nonlinear optimal control problems in which the measured signals are noisy and some system parameters are changing arbitrarily with time. The methodology is validated by an application to a nonlinear nuclear reactor model. The

reformulation of the variational technique as an initial value problem allows this microprocessor-based algorithm to perform on-line filtering, parameter-tracking, and control.

Chapter 5 shows some applications of the techniques presented in this thesis to reactor monitoring problems. First, a transient reactivity monitor is presented which was tested and validated with a dynamic model of the Advanced Neutron Source reactor (ANS). Second, a state monitor for a pressurized water reactor (PWR) is presented which is able to estimate the state variables of the system by analyzing noise-corrupted signals from the plant. Both applications use techniques presented in previous chapters.

Chapter 6 presents a comparison between the behavior of a BWR controlled with a standard proportional-integral (PI) controller and a BWR controlled with a microprocessor-based optimal control algorithm. Both control algorithms are designed to make the BWR perform as a demand-following reactor. It is shown how the PI controller fails to keep the system stable at low flow, while the optimal nonlinear control is able to keep the stability of the system at any flow; thus, improving the transient behavior.

Chapter 7 presents an adaptive optimal control algorithm for uncertain nonlinear systems (for which the model is not completely known). It represents a novel approach to the problem of decoupling a large system into a set of simplified subsystems. A variational technique based on Pontryagin's Maximum Principle is used to track the system's unknown terms, decouple the subsystem models, and calculate the optimal control. To validate the algorithm, a system representing a two-link mechanical manipulator is simulated. In the control model, the coupling and friction terms are unknown. The robot's task is to follow a prescribed trajectory and to pick up an unknown mass. The inverse kinematics problem for the trajectory prescription in generalized coordinates is also solved by applying variational techniques.

Chapter 8 presents an application of the Hamilton-Jacobi approach presented in Chapter 2 to the mechanical manipulator problem.

Chapter 9 presents a new algorithm to control steam generator water level that does not rely on feedwater flow measurements. At low power, the feedwater signal is biased; therefore, the classical three-element controller fails and forces the transfer to manual control to avoid a reactor trip due to unstable conditions in the steam generator. Manual operation has shown to be not very

effective; in fact, a large number of reactor's trip occurs under these conditions. The algorithm documented in this chapter is able to compensate for the shrink and swell phenomenon by modulating the level set point according to differential changes on the pressure signals. In this way, it is possible to keep a strong and dynamic automatic level controller from 0% to 100% power.

Finally, Chapter 10 summarizes the conclusions of this doctoral dissertation.

1.4 Original Contributions

The original contributions presented in this dissertation are:

- (a) the analysis of optimal control theory from the stability point of view and a study of the differences between open-loop and closed-loop implementation is presented,
- (b) two new algorithms that can be applied to nonlinear systems (based on optimal control theory and leading to closed-loop designs) have been developed (the adjoint technique and the adaptive linear control of nonlinear systems technique),

- (c) the application of optimal control theory to parameter tracking, uncertainty tracking, model update, subsystems decoupling and decomposition, state estimation, and filtering,
- (d) a new approach based on optimal control theory to the inverse kinematic problem of the mechanical manipulators is presented,
- (e) an alternative approach to the steam generator water level control problem has been developed,
- (f) an extensive computer validation of the proposed algorithms has been performed, including applications to PWRs, ANS, BWRs, steam generators, and mechanical manipulators.

CHAPTER 2

CLOSED-LOOP OPTIMAL CONTROL

The purpose of this Chapter is to show, by stepping through a series of examples, how the basic cost function minimization problem must be reformulated to obtain a closed-loop control strategy. Some applications of Closed-Loop Optimal Control Design are proposed for state estimation, parameter tracking, uncertainty tracking, and system decomposition.

2.1 Introduction

The earliest documented closed-loop control design was developed by Ktesibios in Alexandria in the third century B.C. He built a water clock based on a constant flow from a regulating tank to a clock tank. The level in the clock tank indicated the time. To keep the water level in the regulating tank constant, Ktesibios introduced the first known float valve to control the flow of water from the main source. The float valve acts as a feedback (closed-loop) control. If the level goes down, it opens. If the level goes up, it closes. In spite of the simplicity of this design, it is extremely efficient. It seems almost impossible to design an equivalent open-loop control.

Modern control theory is based on the mathematical analysis of dynamical systems. Closed-loop controllers are generally designed by analyzing the stability of the system via linear or nonlinear stability theory. In this chapter we study how variational techniques can help in this design.

The calculus of variations was initiated by the Bernoulli brothers in the seventeenth century with the study of the brachistochrone problem. Since then, it has become a powerful tool in almost all scientific domains. But the application of variational techniques to control design is a relatively recent development; the optimal control theory was introduced independently by Pontryagin and Bellman in the 60's.

The basic optimal control theory leads to open-loop implementations. Unfortunately, dynamical systems are subject to random perturbations; therefore, closed-loop formulations are more robust, since they can adjust to unanticipated events.

2.2 Optimal Control

This section formulates a continuous-time optimal control problem and develops the associated Pontryagin's Maximum Principle. Many books and articles have been

published on this subject since the original work of Pontryagin. This section follows the formulation presented by Luenberger⁶.

2.2.1 General Structure of an Optimal Control Problem

The basic optimal control problem is defined by:

- (a) a dynamic system (plant) for which input functions can be specified (controls), and
- (b) a cost function whose value depends on the system's dynamic behavior, and in some sense defines and measures the quality of that behavior.

For example, the dynamic system might be a nuclear reactor core with input function corresponding to the reactivity rod control, and the cost function can be defined as the error between the actual neutron density and the set point for the reactor.

The optimal control theory goal is to determine which are the input functions that minimize the value of the cost function. In our example, the goal would be to obtain a control rod reactivity strategy that minimizes the error between the set point for the reactor and the actual neutron density.

2.2.2 The Maximum Principle

The publication by Pontryagin of his Maximum Principle has been one of the most important events in the development of the control theory. Pontryagin applied the calculus of variations to the problem of finding the input functions that maximize (or minimize) the cost function.

Given a dynamical system

$$dx/dt = f(x,u) \quad , \quad (2-1)$$

where x represents the state variable and u the input function (control), and a cost function (to be minimized) defined as

$$J = \int_0^{t_f} V(x,u) dt \quad , \quad (2-2)$$

the Pontryagin's Maximum Principle can be stated as follows:

Theorem (Maximum Principle). Suppose $u(t) \in U$ and $x(t)$ represent the optimal control and state trajectory for the optimal control problem. Then there is an adjoint trajectory $w(t)$ such that $u(t)$, $x(t)$, and $w(t)$ together satisfy

$$dx/dt = f(x,u) \quad (\text{system}) \quad (2-3.a)$$

$$x(0) = x_0 \quad (\text{state condition}) \quad (2-3.b)$$

$$- dw^T/dt = H_x(w,x,u) \quad (\text{adjoint}) \quad (2-3.c)$$

$$w(t_f) = 0 \quad (\text{adjoint final condition}) \quad (2-3.d)$$

and for all t , and all $v \in U$

$$H(w, x, v) < H(w, x, u) \quad (\text{maximum condition}) \quad (2-3.e)$$

where H is the Hamiltonian

$$H(w, x, u) = w^T f(x, u) + V(x, u) \quad . \quad (2-3.f)$$

Condition (2-3.e), the maximum condition, means that the derivatives of H with respect to each component of u must vanish, and, if the objective is to minimize the cost function, then (2-3.e) must be interpreted as a minimum condition. A detailed proof of this theorem can be found in ref. 2.

2.3 Closed-Loop Optimal Control: Linear Systems

Control designers feel comfortable with closed-loop implementations, i.e., the algorithm obtains the input function (control) as a function of the state variables of the system. The reason is clear, since under these conditions the stability of the control algorithm can be studied and predicted.

2.3.1 The Regulator Problem

To simplify notation and make this section easier to understand, we will use a one-dimensional example.

Let us assume that the system we want to control can be represented by the differential equation

$$dx/dt = ax + u \quad (2-4)$$

where x is the state variable and u is the control. This system has an equilibrium at $x = 0$ and $u = 0$.

Our goal is to regulate this system around its equilibrium, i.e., to design a control algorithm that, if the system equilibrium is perturbed, will place the system back in the equilibrium position. This goal can be represented by defining the function $V(x,u)$ as

$$V(x,u) = \frac{1}{2}x^2Q + \frac{1}{2}u^2R \quad (2-5)$$

where Q and R are weights.

Now we can apply the Maximum Principle. First we construct the Hamiltonian

$$H(x,w,u) = w(ax+u) + \frac{1}{2}x^2Q + \frac{1}{2}u^2R, \quad (2-6)$$

the adjoint equation ($-dw/dt = H_x$) is given by

$$dw/dt = -aw - xQ \quad (2-7)$$

and u is obtained by demanding $H_u = 0$, which results in

$$u = -w/R. \quad (2-8)$$

Substituting equation (8) into the model we obtain the set of equations

$$dx/dt = ax - w/R \quad (2-9.a)$$

$$dw/dt = -aw - xQ. \quad (2-9.b)$$

Equations (2-9) represent the system's dynamic behavior under the action of the optimal control.

Let us now assume that the system has been perturbed from equilibrium. We want to know whether both equations

together (2-9) are stable, i.e., whether they return the system back to equilibrium following a small perturbation.

We have chosen a linear example because it is easier to study its stability. First, we realize that $x = 0$ and $w = 0$ is still an equilibrium point because it makes the derivatives zero. Then, we have to analyze whether the eigenvalues of the system have negative real parts. The characteristic equation of this system can be written as

$$s^2 - a^2 - Q/R = 0 .$$

Since Q and R are positive, there is always a positive eigenvalue; therefore, an asymptotically stable equilibrium state can not be reached in general.

This example gives a heuristic understanding of why optimal control techniques are not suitable for feedback (closed-loop) applications. We can only expect to find a good initial condition for the adjoint, $w(0)$, that will place the system in the target state at a given final time. But, because of the intrinsic instability, we can not expect to keep the system in that target state.

2.3.2 The Linear-Quadratic Algorithm

The Linear-Quadratic (LQ) algorithm was developed by Kalman⁷ and it has been widely used by the control designers mainly because it allows closed-loop implementations. LQ can be applied only to problems in

which the dynamic system is linear and the cost function is quadratic. In this section the basic formulation of the problem is studied and analyzed.

The LQ Algorithm

The LQ algorithm is based on the following idea: since the system represented by (2-9) is linear, both $x(t)$ and $w(t)$ depend linearly on x_0 ; therefore, $w(t)$ should depend linearly on $x(t)$. Accordingly, we try a solution for (2-9) of the form

$$w(t) = P(t)x(t) \quad (2-10)$$

where $P(t)$ represents the linear relation between the state and the adjoint variables. Taking the derivative of (2-10) and introducing the result in (2-9) we obtain the condition

$$0 = [dP(t)/dt + 2aP(t) - P^2(t)/R + Q]x(t) \quad (2-11)$$

which will be satisfied for any $x(t)$ if $P(t)$ is chosen as a solution of the Riccati equation

$$-dP/dt = 2aP - P^2/R + Q \quad (2-12)$$

From the boundary condition $w(t_f) = 0$ we derive $P(t_f) = 0$, a final condition. We must, however, solve this equation by integrating backwards in time from $t = t_f$.

The Feedback Solution

Now, considering t_f very large, we can assume that equation (2-12) may have reached an equilibrium near $t = 0$ (we recall that here time is incremented backwards).

Thus, $P(t)$ may be approximated by its equilibrium value, P . Under this approximation, and from (2-8) we obtain

$$u(t) = -(P/R)x \quad (2-13)$$

which gives the control input with a time-invariant feedback structure. With this approximation, the optimal system is dynamically represented by

$$dx/dt = ax - (P/R)x \quad (2-14)$$

Let us analyze the steady state solutions of the Riccati equation. From (2-12) we obtain

$$P = R[a \pm (a^2 + Q/R)^{1/2}] \quad (2-15)$$

which leads to a dynamic system

$$dx/dt = [\pm(a^2 + Q/R)^{1/2}]x \quad (2-16)$$

Equation (2-15) has two solutions. By choosing the value of P that leads to a negative eigenvalue, we obtain a stable closed-loop formulation.

We have checked, by numerical calculations, that the stable solution corresponds to the value of P obtained by integrating backwards equation (2-12) with final condition $P(t_f)=0$. But the LQ approach will only lead to a stable closed-loop implementation if we keep P constant all the

time. In fact, if we try to solve equation (2-12) forward in time, the system becomes again unstable.

The Dual LQ Algorithm

It is interesting to note that the steady-state Riccati algebraic equation not only represents the equilibrium points of equation (2-12), but also the equilibrium points of the time-reversed equation

$$dP/dt = 2aP - P^2/R + Q \quad (2-17)$$

which can be derived from the dual system

$$dx/dt = -ax + w/R \quad (2-18.a)$$

$$dw/dt = aw + xQ \quad (2-18.b)$$

The dual system is obtained by integrating backwards in time the cost function, or easier, by applying two transformations: map t into $-t$, and then map $-t$ into $-t + t_f$.

Equation (2-17) must be solved forward in time and with initial condition $P(0) = 0$, and it reaches the steady state solution that makes the system stable. The simultaneous solution of equation (2-14) and (2-17) forward in time leads to a stable system with a closed-loop control.

2.3.3 The Hamilton-Jacobi Equation

Let us now study the dynamic behavior of the cost function. First we define a dynamic cost function as

$$J(t) = \int_0^t V(x,u) \, d\tau \quad (2-19)$$

Taking the total derivative of $J(t)$ with respect to t

$$J_t = -[J_x dx/dt - V(x,u)] \quad (2-20)$$

and defining the Hamiltonian as

$$H(J_x, x, u) = J_x dx/dt - V(x, u) \quad (2-21)$$

we obtain

$$J_t = -H(J_x, x, u) \quad (2-22)$$

Equation (2-22) is known as the Hamilton-Jacobi equation in classical mechanics. In order to minimize the temporal evolution of $J(t)$ we have to select u (the control) such that it both minimizes the Hamiltonian, and keeps the value of the Hamiltonian constant and equal to zero during the evolution of the system.

Returning to the linear example used before, we first construct the Hamiltonian as

$$H = J_x(ax+u) - \frac{1}{2}x^2Q - \frac{1}{2}u^2R \quad (2-23)$$

Second, we choose u such that it minimizes H , that is,

$$H_u = 0$$

$$u = J_x/R \quad (2-24)$$

Finally, we substitute Eq. (2-24) in (2-23) and demand $H = 0$

$$0 = J_x a x + \frac{1}{2} (J_x)^2 / R - \frac{1}{2} x^2 Q . \quad (2-25)$$

Assuming now that the solution of this partial differential equation is of the form

$$J = -\frac{1}{2} P x^2 \quad (2-26)$$

it follows that

$$x^2 (-2aP + P^2/R - Q) = 0 \quad (2-27)$$

Equation (2-27) leads to a solution which is equivalent to the LQ algorithm. Given a state, x , Equation (2-24) returns the optimal control that should be applied to the system. We have not specified a final time; therefore, this optimal closed-loop formulation of the control problem will be asymptotically stable.

It is important to note that the RHS of (2-26) is the first term of the Taylor series expansion of the solution. In this linear example with a quadratic cost function, the second term of the expansion leads to the exact solution of the problem (note that second order in J means first order in the expansion of the partial of J with respect to x). In general, it will be necessary to add more terms to the expansion to solve nonlinear problems.

2.4 Closed-Loop Optimal Control: Nonlinear Systems

Models of realistic systems are seldom linear. Unfortunately, it is always more difficult to deal with nonlinear models, and the simplicity in the control design

is lost. In this section we will present some techniques to solve nonlinear control problems. Again, our goal is to be able to design a stable closed-loop control. It will be seen that there is not a general algorithm to solve the problem, and that, for each control case, the control designer will have to exercise his own creativity.

As shown in the previous section, the most clear and meaningful way of dealing with optimal control problems is the Hamilton-Jacobi equation. In many cases it will be necessary to solve this equation by expanding the solution in a Taylor series. In general, this will lead to an almost impossible analytic task. We are going to show how in some cases this work can be simplified by redefining the cost function. There will not be a general rule for doing this, and here is where the control designer will have the major role to play. Again, in order to simplify notation and make this section easier to understand we will use an example.

2.4.1 The Hamilton-Jacobi Equation

Let us suppose that we want to design a control algorithm for the following nonlinear problem

$$dx/dt = ax^2 + u \quad (2-28)$$

First we realize that the linearization technique will fail: If this model is linearized around $x = 0$, all the

information about the dynamics of the system is lost; therefore, it is necessary to deal directly with the nonlinear model.

Cost Function Transformation Approach

If we write now the cost function in the standard quadratic form (see linear example), the solution of the Hamilton-Jacobi equation will demand more than one term in the Taylor series expansion. Now, let us see if we are able to rewrite the cost function in a way that simplifies the solution.

We try

$$V(x,u) = \frac{1}{2}x^2Q + \frac{1}{2}(u + ax^2)^2R, \quad (2-29)$$

the related Hamiltonian [see eq. (2-23)] will be

$$H = w(ax^2 + u) - \frac{1}{2}x^2Q - \frac{1}{2}(u + ax^2)^2R, \quad (2-30)$$

where for simplicity we will use the notation $w = J_x$. We choose u such that this Hamiltonian is minimized, i.e., $H_u = 0$; therefore,

$$u = w/R - ax^2. \quad (2-31)$$

Introducing (2-31) into (2-30) and demanding $H = 0$, we get

$$0 = w^2/2R - \frac{1}{2}x^2Q. \quad (2-32)$$

Let us try a solution of the form

$$w = Px \quad (2-33)$$

which leads to

$$0 = (P^2/R - Q)x^2. \quad (2-34)$$

We have found a solution for P which is independent of x; therefore, we can develop a closed-loop implementation, in this case

$$u = -ax^2 - (Q/R)^{1/2}x/R , \quad (2-35)$$

and

$$dx/dt = -(Q/R)^{1/2}x/R . \quad (2-36)$$

Because Q and R are defined to be positive, this will be always a stable closed-loop implementation.

This example shows how a transformation in the cost function can simplify the analytical calculations and lead to a solution.

Direct Solution By Series Expansion

Because of the simplicity of this example, we can now try to solve it directly by applying a second order expansion. This will give us some idea as to how difficult the problem can be.

First we write again the cost function, but this time we use the standard form

$$V(x,u) = \frac{1}{2}x^2Q + \frac{1}{2}u^2R . \quad (2-37)$$

The Hamiltonian will be

$$H = w(ax^2 + u) - \frac{1}{2}x^2Q - \frac{1}{2}u^2R . \quad (2-38)$$

$H_u = 0$, and $H = 0$ leads to

$$u = w/R , \quad (2-39)$$

and

$$0 = wax + w^2/2R - \frac{1}{2}x^2Q \quad . \quad (2-40)$$

We now try a second order expansion

$$w = Px + Zx^2 \quad . \quad (2-41)$$

Introduce (2-41) into (2-40) and simplify to get

$$0 = (P^2/R - Q)x^2 \quad (2-42)$$

$$0 = (a + Z/R)Px^3 \quad . \quad (2-43)$$

The coefficient of x^4 is not analyzed because it is expected to have contributions from higher expansion terms. The solution for (2-42) and (2-43) leads to

$$u = -ax^2 - (Q/R)^{1/2}/R \quad , \quad (2-44)$$

which is equivalent to the solution that we get by minimizing the simplified cost function.

In this special case we can expect that higher order expansions will lead to the same solution; therefore, we can stop our study at the second order. But in general, it will be very difficult to decide where to stop.

2.4.2 Adaptive Linear Control of Nonlinear Systems

Sometimes, dealing directly with nonlinear systems demands too much analytic work. In these cases, control designers are tempted to linearize the system and simplify the problem. In this section we present a technique to update the linearized control.

In section 2.3.3 we have shown that the dual LQ equation (2-17) leads to the steady-state solution of the equation forward in time. We have seen that this solution is the one that gives stability in the closed-loop applications. We can use this property of the dual equation to obtain an adaptive linear control for a nonlinear system. Again, we use an example to show the methodology. It is important to know that this is a heuristic approach and it will be difficult to guarantee the stability of the final design.

Returning to our nonlinear model, we linearize around x_0

$$dx/dt = (ax_0)x + u \quad . \quad (2-45)$$

Note that this linearization is not a Taylor series expansion. It will be seen later that it works better. Again, the designer has to decide what to do.

The dual LQ equation (2-17) in this case will be

$$dP/dt = 2(ax_0)P - P^2/R + Q \quad , \quad (2-46)$$

and the optimal control is given by

$$u = -Px/R \quad . \quad (2-47)$$

The adaptive linear control algorithm can be achieved by updating the value of x_0

$$dx/dt = ax^2 - Px/R \quad , \quad (2-48)$$

and

$$dP/dt = 2axP - P^2/R + Q \quad . \quad (2-49)$$

Let us see how it works. The initial condition for equation (2-49) is the steady state value of P evaluated at the initial value of x . We can expect that if x is perturbed, equation (2-49) will evolve to the steady-state solution again (updating the value of P).

We have checked by computer simulation that this particular case works. In order to perform an analytic study of this problem, we have to simplify it. Let us assume that equation (2-49) reaches the steady state fast enough so we can assume that the value of P is always the steady state solution. Then

$$P = -axR - R[(ax)^2 + Q/R]^{\frac{1}{2}} . \quad (2-50)$$

The system evolves with time as

$$dx/dt = - [(ax)^2 + Q/R]^{\frac{1}{2}} x , \quad (2-51)$$

which is a stable solution.

2.4.3 The Dual Adjoint Technique

We have seen that by applying the dual LQ equation to the system we get a stable closed-loop control. The dual equation is obtained by integrating the cost function backwards. We can interpret this as if we were applying the backward-in-time solution for the control problem to the forward-in-time model of the plant. This interpretation leads to the dual adjoint technique. In this case, we calculate the value of the control by

integrating the adjoint backwards-in-time, and applying the result to the forward-in-time system. This is again an intuitive approach and we can not find any reason to justify it. In fact, it will be shown that this methodology will only work if the system is stable.

Let us study the dual adjoint approach by applying it to the linear example. The model and the dual adjoint will be

$$dx/dt = ax - w/R \quad (2-52.a)$$

$$dw/dt = aw + xQ \quad (2-52.b)$$

The eigenvalues of this system are

$$s = a \pm i (Q/R) \quad (2-53)$$

Therefore, the dual adjoint technique leads to a stable implementation only if the system is stable.

Even if the system is stable, we can not say that the solution will be optimal. In fact, this algorithm acts as a second order filter with the break frequencies of the original system. If this break frequencies are negative, the filter will move the system back to equilibrium.

The main advantage of this technique is that, if it works, the control algorithm is extremely easy to design. This must be taken into account when we have a multiple-input multiple-output nonlinear problem, where solving the Hamilton-Jacobi equation directly can be extremely difficult.

2.5 Applications

In this section, some applications of optimal control theory are presented. We will show what kind of cost functions need to be minimized in each case. The particular solution for the minimization problem will depend on the specific case, and any of the previously shown techniques could be used.

2.5.1 State Estimation: Filtering

In this section the optimal control methodology is applied to the problem of estimating the actual state of the plant from detector signals that are perturbed with additive noise.

We define

$$\mathbf{s} = \mathbf{y}_d + \boldsymbol{\eta} \quad , \quad (2-54)$$

as the set of signals coming from the plant detectors, where $\mathbf{y}_d$ represents the detector state variables, and $\boldsymbol{\eta}$ represents an additive noise with zero mean.

We also define $\mathbf{x}$ as the estimated plant state. The dynamic estimation of $\mathbf{x}$ can be obtained from

$$d\mathbf{x}/dt = \mathbf{G}(\mathbf{x}, \mathbf{u}) + \mathbf{v} \quad , \quad (2-55)$$

where $\mathbf{G}$ is an approximated model of the plant (predictor) and $\mathbf{v}$ is the dynamic correction (corrector) that can be obtained by minimizing the cost function

$$J_F = \int_0^{t_f} \{ [\mathbf{s} - \mathbf{x}_d]^T \mathbf{Q}_f [\mathbf{s} - \mathbf{x}_d] + \mathbf{v}^T \mathbf{R}_f \mathbf{v} \} dt \quad . \quad (2-56)$$

2.5.2 Parameter Tracking

This section shows how the optimal control methodology can be used to update the plant time-varying parameters.

We assume that there is a set of parameters a which can change arbitrarily with time. Our control model has to be updated on-line, otherwise, we can not guarantee that the calculated control will be optimal. The updating of the control model can be achieved by applying again the optimal control techniques. In this case, we assume that the magnitude of a will minimize the cost function

$$J_p = \int_0^{t_f} \{ [x-m]^T Q_p [x-m] + a^T R_p a \} dt \quad (2-57)$$

where m represent the control model state variables.

2.5.3 Uncertainty Tracking

Models of realistic systems are seldom completely known. The variational techniques that we have presented can also be used to track the unknown (uncertain) part of the model.

The formulation is similar to the one for parameter tracking. We want to find which are the optimal values for the uncertainty, p , that make the approximated model of the plant match the signals coming from the detectors, i.e., we want to minimize the cost function

$$J_u = \int_0^{t_f} \{ [x-m]^T Q_u [x-m] + p^T R_u p \} dt \quad . \quad (2-58)$$

2.5.4 System Decomposition

Often, the plant's dynamical model involves a large number of differential equation. In this cases, it may be convenient to decompose the large model into a set of decoupled subsystems. The uncertainty tracking method can be used in order to achieve an efficient decomposition without losing too much information.

Let us assume that

$$\frac{dx}{dt} = F(x,u) \quad (2-59)$$

represents the mathematical model of the plant that we wish to control, where the vector x represents the n state variables and u represents the m controls. Let

$$\frac{dx_i}{dt} = F_i(x_i, u_i) + p_i, \quad i = 1, k \quad (2-60)$$

represent a set of k subsystems formulated in such a way that they are uncoupled. To guarantee that this set of subsystems represents the whole system we introduce the unknown set of functions p_i , whose values at time t can be obtained by the optimal matching of the signals coming from the detectors with the values for the state variables given by the numerical integration of the subsystem's mathematical models. Actually, this can be formulated as an optimal control problem. We want to know what are the values of controls (unknown function p) that make the subsystems follow the demand (detector signals).

2.6 Summary

In this Chapter we reviewed the applications of optimal control theory to feedback control design. It has been seen that the Pontryagin's Maximum Principle leads to open-loop strategies. In some cases, a closed-loop implementation can be achieved by solving directly the so called Hamilton-Jacobi equation. This approach is relatively easy to implement for linear cases. Unfortunately, more analytic work is necessary in the nonlinear case. Some simplification can be obtained in these cases by choosing an "ad hoc" cost function. In general, the solution of the Hamilton-Jacobi equation will be extremely difficult. In this cases, some new alternative approaches have been presented. Finally, some of the many applications of closed-loop optimal control have been shown: filtering, model update and decomposition, and control.

CHAPTER 3

OPTIMAL PARAMETER TRACKING AND CONTROL OF NONLINEAR SYSTEMS WITH TIME-VARYING PARAMETERS

This chapter presents a new formulation of a class of nonlinear optimal control problems in which the system's parameters change arbitrarily with time. A variational technique based on Pontryagin's Maximum Principle is used to track the time-varying parameters of the system and to calculate the optimal control. The present formulation allows on-line solution for parameters and optimal control calculations by re-casting it into the form of an initial value problem. An application to a nonlinear nuclear reactor in which feedback coefficients and detector time constants are varying is presented.

3.1 Introduction

The practical applications of Pontryagin's Maximum Principle (PMP) based control algorithms has been hindered by the fact that drastic simplifications must be done to obtain on-line solutions. For example, the Linear-Quadratic (LQ) algorithm, the most popular and perhaps the only one that, to date, allows on-line solutions, requires the use of linearized models, and assumes that the relation (2-10) between system and adjoint equations is

time independent. Unfortunately, these conditions do not apply to the most common case, a nonlinear system with time varying parameters.

The ideal demand-following adaptive control should:

- (a) provide an optimal control philosophy,
- (b) provide parameter tracking so that the plant model reproduces the signals coming from the actual plant sensors at any time,
- (c) provide control flexibility, in the sense that the control limitations and possible failures are taken into account,
- (d) compensate for sensor delays and possible degradation, and
- (e) handle directly the nonlinear model, thus, making it possible to apply the algorithm under any plant conditions.

In principle, all these requirements can be satisfied by applying the PMP to a nonlinear model of the plant. The major practical problem arises in connection with the numerical solution of the resulting two-point boundary value problem (TPBVP).

The following sections present a new approach to the PMP formulation that allows on-line solutions by recasting the TPBVP into an initial value problem.

3.2 Reformulation of the Optimal Control Problem

This section shows how the free terminal time optimal control problem (FTT) can be reformulated as the solution of differential equations with a set of initial conditions, instead of the classical TPBVP.

Let

$$\frac{dy}{dt} = F(y,u) \quad (3-1)$$

be a nonlinear system, where y represents the state variables and u the controls. The FTT problem is formulated as the minimization of the extended cost function

$$J = \int_{t_0}^{t_f} [H(y,w,u) - w^T \frac{dy}{dt}] dt \quad (3-2)$$

where w is a set of Lagrange multipliers (adjoints or momenta), and H represents the Hamiltonian defined as $H = w^T \frac{dy}{dt} + V(y,u)$, where V can be interpreted as the potential that will force the system to move along the desired trajectory. We assume that H is time-invariant, $y(t_0)$ and $y(t_f)$ are given and the system is in steady-state at $t=t_0$. Because of the symmetry of the problem the minimization of the cost function can be done backwards. After a time inversion of the expression for J , and performing the equivalent steps to the PMP, the following inverted Hamilton equations are obtained

$$dy_i/dt = - dH/dw_i \quad (3-3)$$

$$dw_i/dt = dH/dy_i \quad , \quad (3-4)$$

and the conditions

$$dH/du_i = 0 \quad (3-5)$$

$$(H)_{t_0} = 0 \quad . \quad (3-6)$$

If steady-state conditions are assumed at $t=t_0$, and dV/du_i are different from zero, then, the initial conditions for w can be found. Moreover, because the state variables are time-reversal invariant, and the controls are only function of y and w , the first Hamilton equation can be solved forward instead of backwards as the result of the variations implied. Then, under these special conditions, the FTT problem can be solved forward-in-time as the solution of differential equations with initial conditions. Notice that condition (3-6) will constrain the algorithm to demand-following or regulation problems, which are the ones in which we are interested, and perhaps the only ones that need on-line solutions.

3.3 Parameter Tracking and Control

As stated before, our goal is to develop an algorithm which must be able to update the model by tracking the time-varying parameters while it is calculating the optimal control.

Let

$$dy/dt = F(y,t,b,a',u) \quad (3-7)$$

be a nonlinear system where y are the state variables and signals from the plant, t is the time, b are fixed parameters, a' are time-varying parameters, and u are the controls.

Let

$$dx/dt = F(x,t,b,a_0,u) \quad (3-8)$$

be the system and detector model, where the a_0 parameter set corresponds to a given steady state. The set of parameters a may vary with time invalidating the optimal controls calculated using this model; thus, it is necessary to track the parameters. The approach taken in this work has been to realize that the time-varying parameters can be considered as the set a that minimizes the difference between s_y and s_x (where s means the subset of y or x representing the signals). Therefore, they can be obtained by forcing the system to move through the potential V_p defined as

$$V_p = (s_y - s_x)^+ Q (s_y - s_x) + (a - a_0)^+ R (a - a_0) \quad , \quad (3-9)$$

where Q and R are weight matrices, a_0 are the steady-state parameter values, and $^+$ represents the transpose.

Following the algorithm developed in section 3.2 we can now construct the Hamiltonian, obtain the inverted Hamilton equations for the momenta (w), and the condition for the time-varying parameters

$$H_p = w^+ dx/dt + V_p \quad (3-10)$$

$$dw_i/dt = dH_p/dx_i \quad (3-11)$$

$$dH_p/da_i = 0 \quad . \quad (3-12)$$

The equations (3-11) and (3-8), in which a_0 is updated with the a obtained from (3-12), represent a mathematical model of the system that is able to update the time-varying parameters during the transient. The initial conditions for x are known, and the initial conditions for w can be obtained assuming $V_p = 0$, and $dx/dt = 0$ at $t = 0$. Notice that these assumptions support the fundamental condition (3-6).

Now, the updated model can be controlled by forcing it to move under the potential defined as

$$V_c = (d - sd_x)^+ L (d - sd_x) + (u - u_0)^+ M (u - u_0) \quad , \quad (3-13)$$

where d is a given demand for a state variables subset sd_x , and u_0 are the steady state values for the controls.

Again, following the formalism presented in section 3.2, we can obtain the momenta and conditions associated with the control problem, and again assuming $V_c = 0$ at $t = 0$, the initial conditions.

3.4 Application to a Nonlinear Nuclear Reactor

The algorithm presented in this paper has been applied to a nonlinear reactor model. The plant was simulated with a four dimensional model, and three detectors. During the transient, the feedback coefficient due to the coolant temperature, the fuel temperature detector time constant, and coolant temperature detector time constant, were time dependent. The plant was forced to follow demands in neutron density and outlet coolant temperature by adjusting two control inputs: (a) reactivity, which was constrained to a given value, and (b) inlet coolant temperature.

The time-varying parameters were changing drastically during the transient, transforming the system's dynamics; therefore, it was necessary to track them. Figure 3.1 shows how the parameters that were obtained using our algorithm match the actual parameter values.

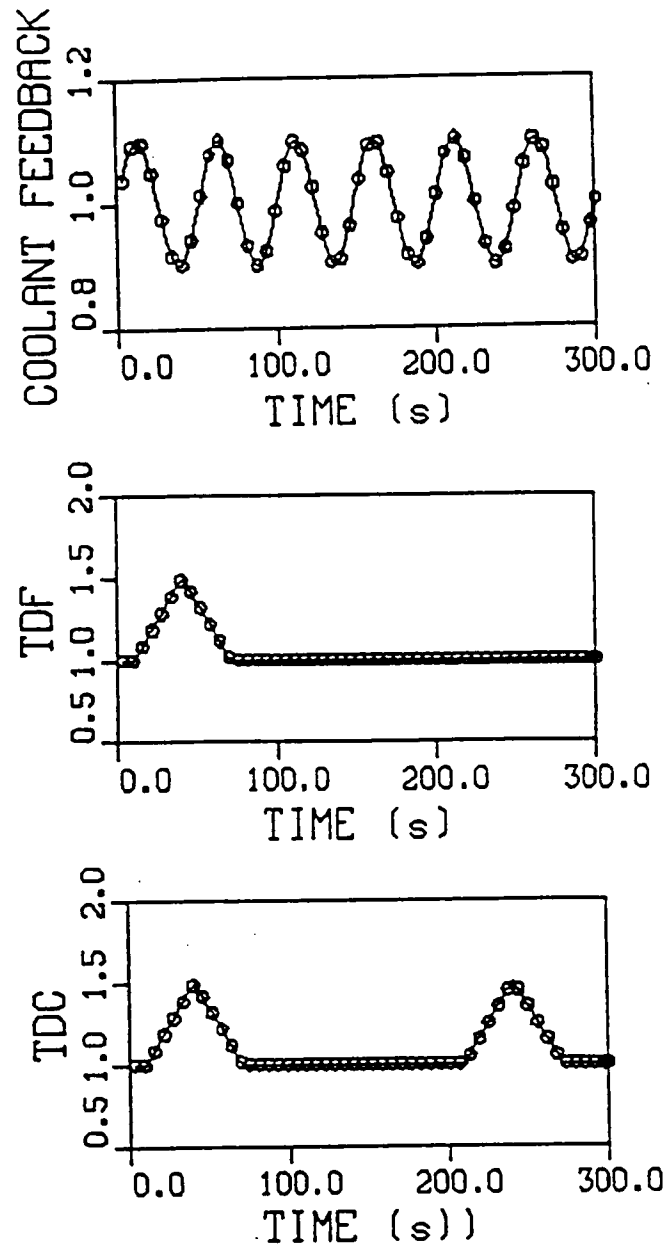


Fig. 3.1 Parameter tracking. Solid lines represent the actual values and circles the algorithm estimations: (a) coolant feedback coefficient, (b) coolant temperature detector time constant, and (c) fuel temperature time constant.

Figure 3.2 shows the controls that are necessary to follow the transient. Notice that the sinusoidal form of figure 3.2a is due to the sinusoidal change in coolant feedback coefficient. Figures 3.2a and 3.2b show how our algorithm allows both controls to interact: when u_1 reaches the maximum, u_2 carries all the work without discontinuities.

Figure 3.3 shows how both demands, in neutron density and outlet coolant temperature, were followed by the plant in spite of parameter changes and constraints in the controls.

Finally, figure 3.4 shows how the algorithm matches the actual values for (a) coolant and (b) fuel temperatures at any time, by correcting for the delays introduced by the detection process.

3.5 Summary

An optimal parameter tracking and control algorithm has been developed which allows to find on-line solutions for demand-following problems in the cases in which nonlinearities and time-varying parameters invalidated other known algorithms. A crucial result has been the recasting of the Pontryagin's Maximum Principle technique for nonlinear systems as the solution of differential equations with initial conditions. The algorithm has been

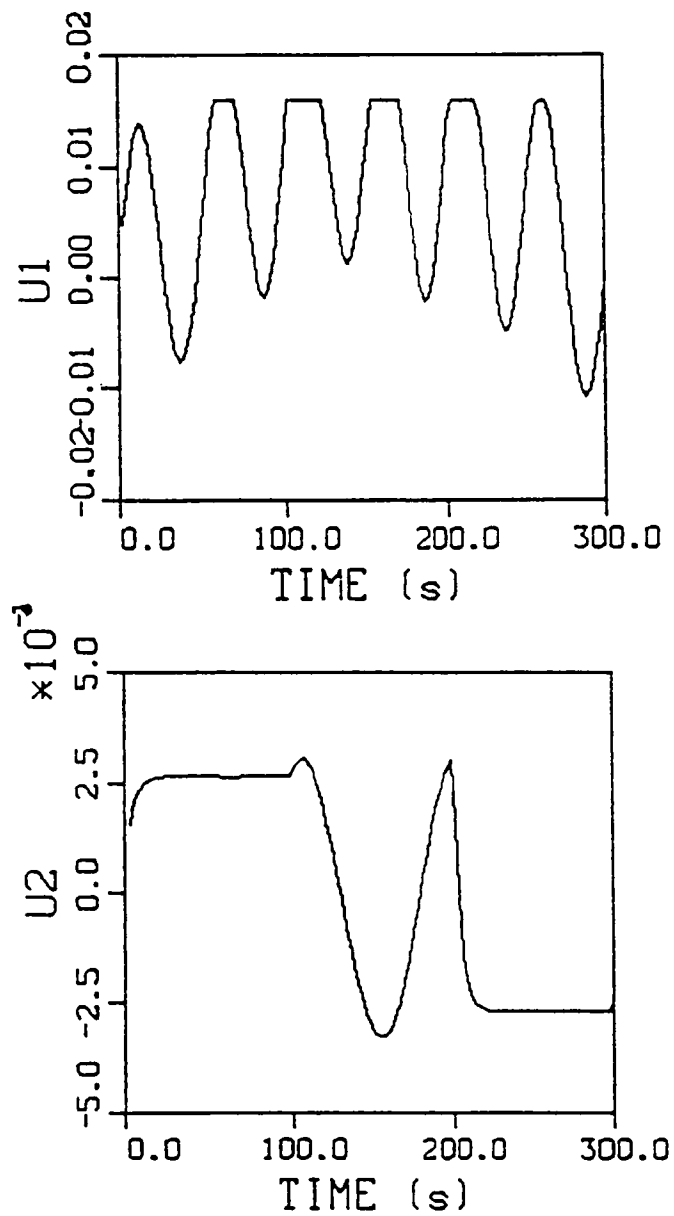


Fig. 3.2 Optimal controls: (a) u_1 , reactivity, which is constrained to $u_1 < 0.016$ (absolute units), and (b) u_2 , inlet coolant temperature (relative units).

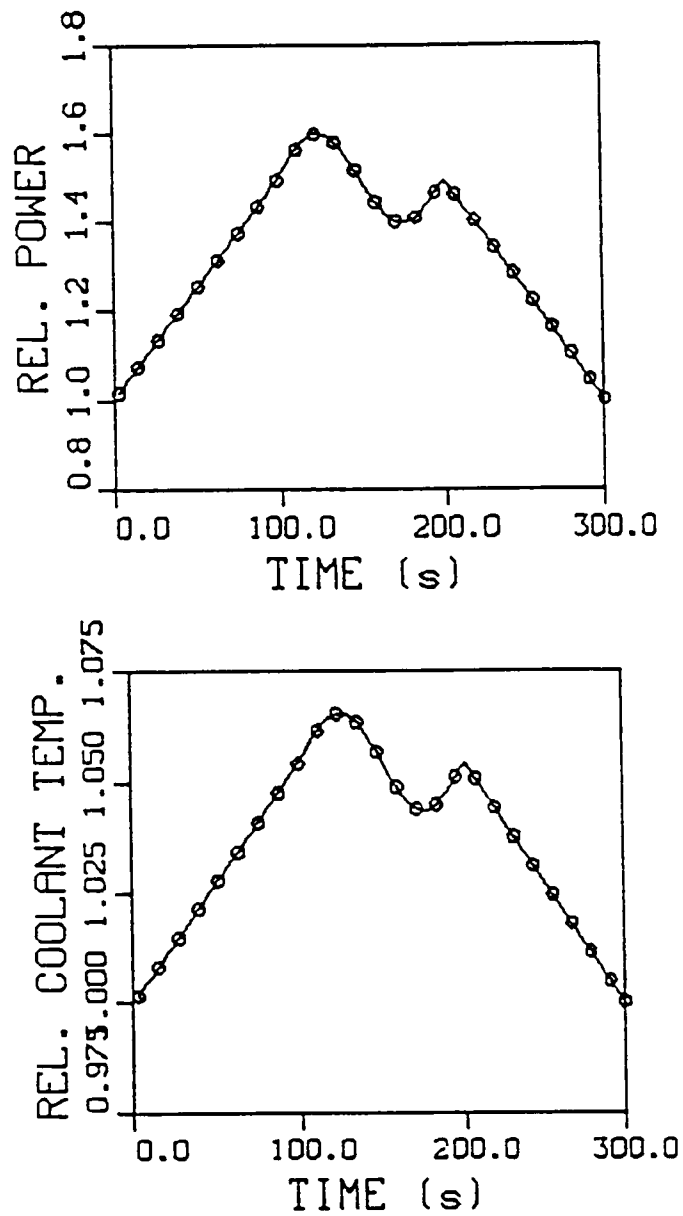


Fig. 3.3 Demand following. Solid lines represent the demand and circles the plant response: (a) relative neutron density, (b) relative outlet coolant temperature.

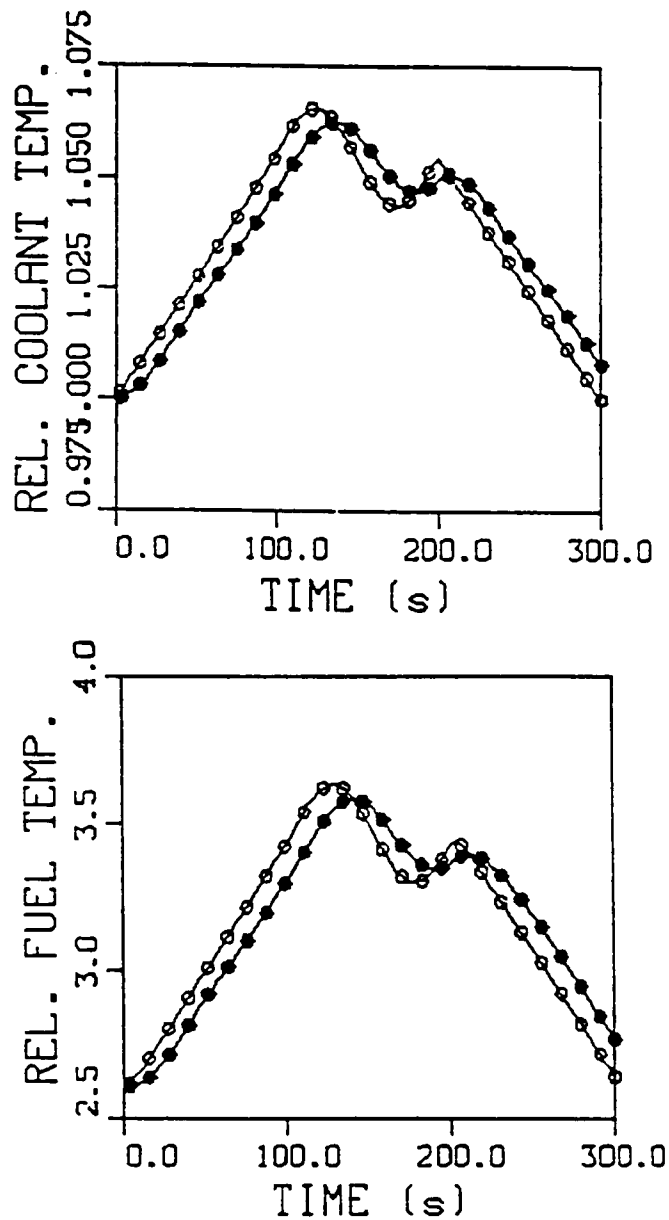


Fig. 3.4 Delay introduced by the detection process. Solid lines are the plant values, black dots and circles represent the algorithm estimations for the measured and actual temperature respectively: (a) relative coolant temperature, (b) relative fuel temperature.

successfully applied to a nonlinear nuclear reactor model. Using the plant sensor signals, the parameters were updated during the transient, allowing the calculation of the optimal control strategy.

CHAPTER 4

OPTIMAL FILTERING, PARAMETER TRACKING AND CONTROL OF NONLINEAR NUCLEAR REACTORS

This chapter presents a new formulation of a class of nonlinear optimal control problems in which the system's signals are noisy and some system's parameters are changing arbitrarily with time. The methodology is validated with an application to a nonlinear nuclear reactor model. A variational technique based on Pontryagin's Maximum Principle is used to filter the noisy signals, estimate the time-varying parameters, and calculate the optimal controls. The reformulation of the variational technique as an Initial Value Problem allows this microprocessor-based algorithm to perform on-line filtering, parameter-tracking, and control.

4.1 Introduction

The availability of fast and reliable microprocessors has opened the possibility of performing on-line numerical calculations to improve and optimize the way nuclear reactors are controlled. In particular, there is an increased interest⁸ in improving the robustness of control algorithms when the system's behavior is affected by nonlinearities, unknown time-varying parameters, and noisy

signals.

The following sections present a new microprocessor-based algorithm that is able to perform on-line filtering, parameter-tracking, and control of a nonlinear nuclear reactor. A variational technique based on Pontryagin's Maximum Principle is used to estimate the system's state and parameters, and to calculate the optimal control.

Three simultaneous optimizations are performed:

- (a) the estimate of the state variables is determined by the optimal matching of the noisy plant signals to the model,
- (b) the time varying parameter is obtained by matching the filtered estimate to the control model, and
- (c) the optimal controls are obtained by matching a set of prescribed demands.

To validate the algorithm, a nonlinear nuclear reactor was simulated in which the coolant feedback coefficient was an unknown time-varying parameter, and the signals from the plant had additive noise. The plant was forced to follow demands in neutron density and outlet coolant temperature by adjusting two controls: (a) u_1 , reactivity, which was constrained to a given set of values, and (b) u_2 , inlet coolant temperature.

4.2 The Reformulation of Optimal Control Problem

This section reviews the reformulation of the free terminal time optimal control problem (FTT).

Let

$$dy/dt = F(y,u) \quad (4-1)$$

represent the plant model, where y are the state variables and u the controls. Our goal is to find $u(t)$ such that the system will follow a given set of demands d , in the sense that the cost function defined as

$$J = \int_0^{t_f} \{ [d-H(y)]^T Q [d-H(y)] + u^T R u \} dt \quad (4-2)$$

is minimized. Q and R are weight matrices, which are allowed to change with time. The initial state $y(0)$ and final state $H y(t_f)$ are known but t_f is free.

It has been shown (see Chapter 3) that, if the system is in equilibrium at $t=0$ and d is a well-behaved function, the minimization problem is equivalent to the system:

$$dy/dt = F(y,u) \quad (4-3)$$

$$dw/dt = (dF/dy)^T w - (dH/dy) Q (d-H(y)) \quad (4-4)$$

$$u = R^{-1} w^T (dF/du) \quad (4-5)$$

$$y(0)=y_0, \quad w(0)=0, \quad (4-6)$$

where w is the set of Lagrange multipliers or adjoints.

The above reformulation of PMP can be achieved by integrating backwards the cost function and the initial conditions are known only if the initial estate is in

equilibrium. Under these special conditions the calculation of the optimal control can be easily executed on-line in a microprocessor because the classical TPBVP has been recast as an initial value problem.

4.3 State Estimation: Filtering

In this section the optimal control reformulation is applied to the problem of estimating the actual state of the plant from detector signals that are corrupted with additive noise.

Assuming that (4-1) represent a mathematical description of our system, define

$$\mathbf{s} = \mathbf{y}_d + \boldsymbol{\eta} \quad (4-7)$$

as the set of signals coming from the plant detectors, where $\mathbf{y}_d$ represents the detectors state variables, and $\boldsymbol{\eta}$ represents an additive noise with zero mean.

Let define $\mathbf{x}$ as the estimated plant state. The dynamic estimation of $\mathbf{x}$ can be obtained from

$$d\mathbf{x}/dt = \mathbf{G}(\mathbf{x}, \mathbf{u}) + \mathbf{v} \quad (4-8)$$

where $\mathbf{G}$ is an approximated model of the plant (predictor) and $\mathbf{v}$ is the dynamic correction (corrector) that can be obtained from the minimization of the cost function

$$J_F = \int_0^{t_f} \{ [\mathbf{s} - \mathbf{x}_d]^T \mathbf{Q}_f [\mathbf{s} - \mathbf{x}_d] + \mathbf{v}^T \mathbf{R}_f \mathbf{v} \} dt \quad (4-9)$$

4.4 Parameter Tracking

This section shows how the reformulation of the optimal control problem can be used to update the plant time-varying parameters.

Assume that there is a set of parameters a which can change arbitrarily with time. Our control model needs to be updated on-line, otherwise, we can not guarantee that the calculated control will be optimal. The updating of the control model can be achieved by applying again the reformulation of the optimal control problem presented in section 4.2. In this case we assume that the magnitude of a will be such as to minimize the cost function

$$J_p = \int_0^{t_f} \{ [\mathbf{x}-\mathbf{m}]^T Q_p [\mathbf{x}-\mathbf{m}] + a^T R_p a \} dt \quad , \quad (4-10)$$

where $\mathbf{m}$ represent the control model state variables.

4.5 Optimal Control

At this point, our algorithm is able to estimate the actual state variables and the time-varying parameter magnitude; therefore, we have an adaptive model to which we can apply again the reformulated optimal control problem to obtain the controls necessary to follow the demands. Now we have to minimize the cost function defined as

$$J_c = \int_0^{t_f} \{ [\mathbf{d}-\mathbf{Hm}]^T Q_c [\mathbf{d}-\mathbf{Hm}] + u^T R_c u \} dt \quad . \quad (4-11)$$

4.6 The Algorithm

As stated before, our goal is to develop an algorithm which is able to update the control model, by estimating the state variables and time-varying parameters, while calculating the optimal control. This objective is achieved by minimizing the three cost functions, J_F (4-9), J_P (4-10), and J_C (4-11), independently and simultaneously. The set of equations to solve can be summarized as follows.

$$dx/dt = G(x,a,u) + v \quad (4-12)$$

$$dm/dt = G(m,a,u) \quad (4-13)$$

$$dw_f/dt = (dG/dx)^T w_f - Q_f(s-x_d) \quad (4-14)$$

$$dw_p/dt = (dG/dm)^T w_p - Q_p(x-m) \quad (4-15)$$

$$dw_c/dt = (dG/dx)^T w_c - (dH(m)/dm) Q_c(d-H(m)) \quad (4-16)$$

$$v = (R_f)^{-1} w_f^T (dG/dv) \quad (4-17)$$

$$a = (R_p)^{-1} w_p^T (dG/da) \quad (4-18)$$

$$u = (R_c)^{-1} w_c^T (dG/dv) \quad (4-19)$$

$$m(0)=x(0)=y(0)=y_0 \quad (4-20)$$

$$w_f(0)=w_p(0)=w_c(0)=0 \quad (4-21)$$

Equations (4-12) to (4-16) can be solved simultaneously with the initial conditions given by (4-20) and (4-21) by using any standard differential equation solver. Equations (4-17) to (4-19) return us the filter corrector, v , the unknown parameters, a , and the optimal controls, u .

4.7 Application to a Nonlinear Nuclear Reactor

The algorithm presented in this section has been applied to a nonlinear reactor model. The plant was simulated with a four dimensional model, and three detectors. During the transient, the feedback coefficient due to the coolant temperature was changing sinusoidally with time. The signals coming from the simulated plant were corrupted with additive noise. The plant was forced to follow demands in neutron density and outlet coolant temperature by adjusting two control inputs: (a) reactivity, which was constrained to a given value, and (b) inlet coolant temperature.

Figure 4.1 shows how both demands in power and outlet coolant temperature were followed by the plant despite the parameter changes and constraints in the controls. Figure 4.2 shows how the algorithm was able to update the coolant temperature feedback parameter during the transient. Figure 4.3 shows the controls that were necessary to follow the transient. Finally, figure 4.4 shows how the variational filter eliminates the additive noise from the plant signals.

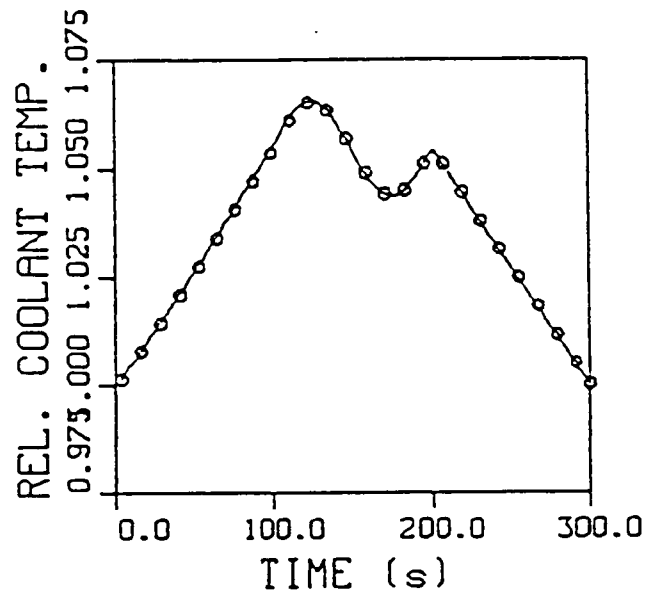
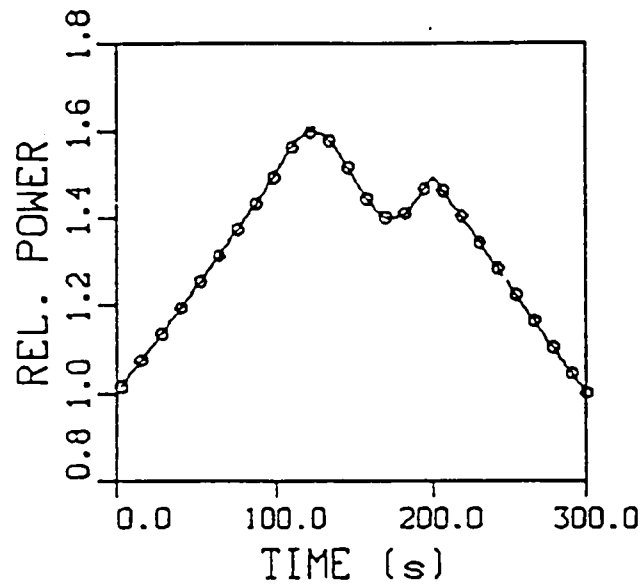


Fig. 4.1 Demand following: (a) demand on power (solid line) and plant power (circles) versus time. (b) demand on hot leg temperature (solid line) and plant hot leg temperature (circles) versus time.

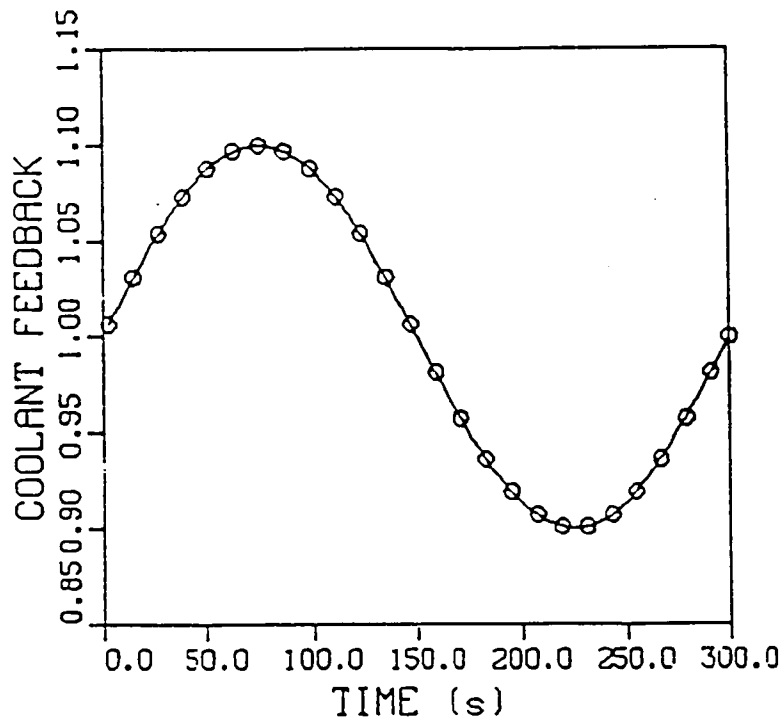


Fig. 4.2 Time varying parameter update. Plant coolant feedback coefficient (solid line) and algorithm estimate (circles) versus time.

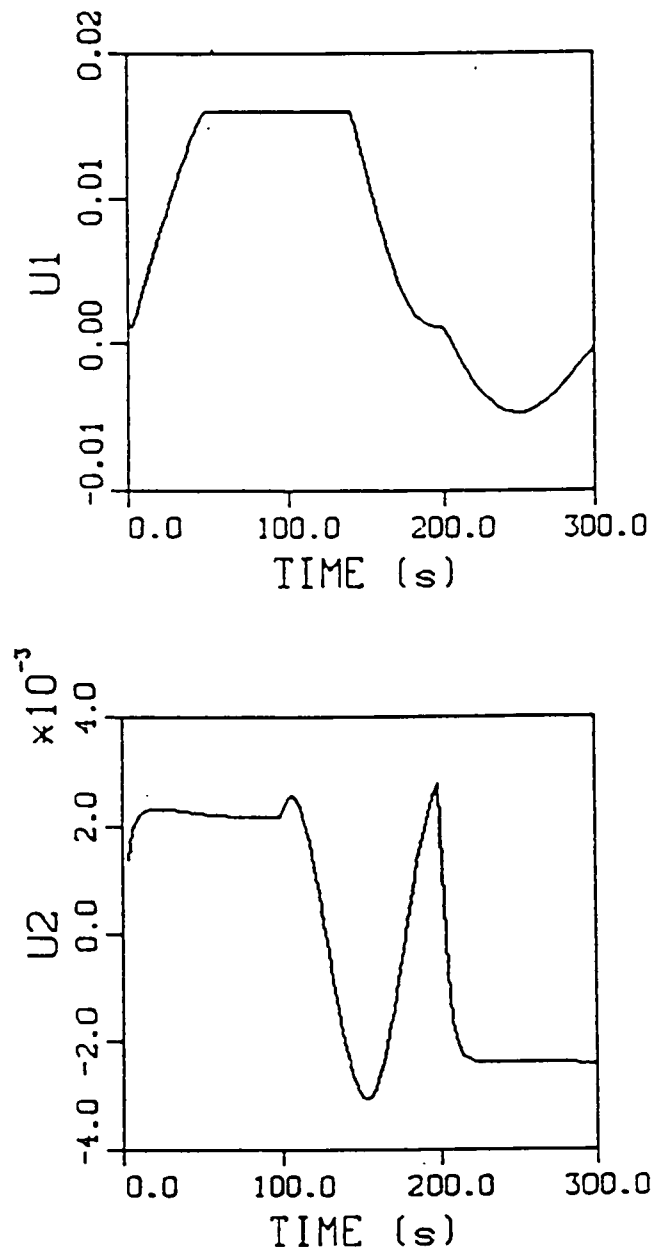


Fig. 4.3 Optimal controls: (a) u_1 , reactivity, which was constrained to be less than 0.016, and (b) u_2 , inlet coolant temperature change.

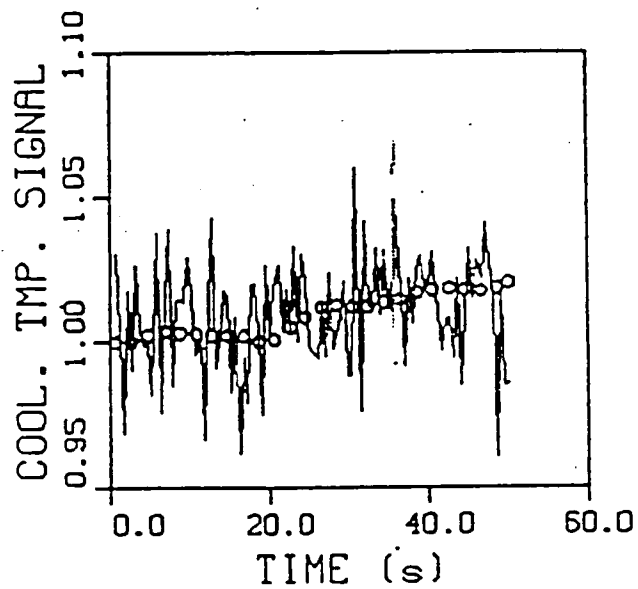
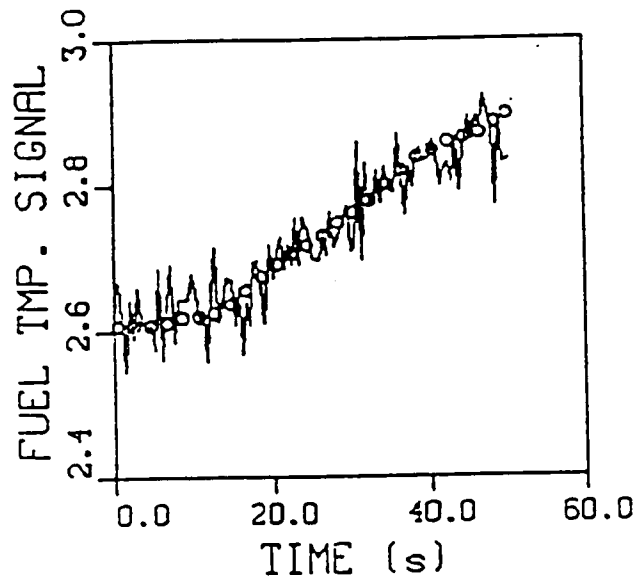


Fig. 4.4 Optimal filtering: (a) coolant temperature detector signal (solid line) and algorithm filtered estimates (circles) versus time, and (b) fuel temperature detector signal (solid line) and algorithm filtered estimates (circles) versus time.

4.8 Conclusions

An optimal filtering, parameter tracking, and control algorithm has been developed which allows one to find on-line solutions for demand-following problems in the cases in which noisy signals, nonlinearities, and time-varying parameters invalidated other known algorithms. An important result has been the recasting of the Pontryagin's Maximum Principle technique for nonlinear systems as the solution of differential equations with initial conditions. The algorithm has been applied successfully to a nonlinear nuclear reactor model. Using the plant sensor signals, all the state variables were estimated, and the time-varying parameter was updated during the transient, allowing an adaptive optimal control calculation. We can conclude that this new formulation of the optimal control problem provides a suitable methodology for on-line, microprocessor-based applications.

CHAPTER 5

REACTOR MONITORING APPLICATIONS

This Chapter shows some applications of the techniques presented in this thesis to the reactor monitoring problem.

First, a transient reactivity monitor is presented which was tested and validated with a dynamic model of the Advanced Neutron Source Reactor.

Second, a state monitor for a PWR is presented which is able to estimate the state variables of the system by analyzing the noise-corrupted signals from the plant. Both applications use the techniques presented in previous chapters.

5.1 Transient Reactivity Monitor

This section presents a technique to estimate excess reactivity values during transient conditions, which can be used to determine time-dependent reactivity coefficients. The present technique is based on optimal control theory. A control strategy is set up to minimize the difference between the measured neutron flux and a mathematical model of the reactor dynamics. The control variable is the parameter to be determined which, in this case, corresponds to the excess reactivity. The optimal

control equations have been solved and applied to several example cases. Excellent agreement is obtained between estimated and actual transient reactivity values.

5.1.1 Background

Reactivity feedback coefficients are customarily evaluated using detailed neutronic codes, and the calculated values are verified "a posteriori" against results of steady-state experiments⁹. In those experiments, the neutron flux is increased, then the reactor is allowed to stabilize, and the excess reactivity is estimated from the new control rod position. In general, though, reactivity coefficients are time dependent because of fuel expansion effects that produce geometry changes. In those cases, the steady-state coefficients might not represent accurately the prompt coefficients; indeed, they might even have different signs.

5.1.2 Theory

The approach taken in this implementation is to apply an optimal control technique to determine excess reactivity values during transients. To this end, a control strategy is set up to minimize the difference between the measured neutron flux and a mathematical model of the reactor dynamics. The control variable is the

parameter to be determined that, in this case, corresponds to p , the excess reactivity. Mathematically, we attempt to minimize the following functional:

$$J = \int_0^{t_f} [(n_m - n)^2 Q + p^2] dt \quad (5-1)$$

where Q is an adjustable weighing factor, n_m is the measured neutron flux, and n is the neutron flux predicted by a reactor dynamic model, expressed as

$$dx/dt = F(x, p) \quad (5-2)$$

where x is the vector of state variables, including n , and F is a vector function describing the reactor dynamics.

The optimal value of p that minimizes the above cost function can be readily obtained by the introduction of state variable adjoints, w , that are given by the following relations

$$dw/dt = (dF/dx)^T w - Q(n_m - n) \quad (5-3)$$

$$p = -w(dF/dp) \quad (5-4)$$

with boundary conditions

$$n(0) = n_m(0) \quad (5-5)$$

$$w(0) = 0. \quad (5-6)$$

5.1.3 Application

To illustrate this technique, we have applied it to a dynamic model of the Advance Neutron Source Reactor¹⁰. This model includes a point kinetics representation of the neutron field with thermohydraulic feedback that is

composed of fuel, core coolant, in-vessel bypass region, and the reflector region.

Using the above model as a reference reactor, transients have been performed by moving control rods under several reactivity coefficients assumptions. These transients have been analyzed using the technique outlined in the previous equations to obtain the transient reactivity. To this end, a reduced model was used that consisted only of one-delayed-group point kinetics model, without any explicit representation of the feedback terms.

That is, the reference model used to estimate the transient reactivity can be expressed as

$$dx/dt = F(x,p) \tag{5-7}$$

$$x^T = (n,c) \quad , \tag{5-8}$$

where n is the neutron flux and c is the delayed neutron precursors concentration, and $F(x,p)$ is the standard point kinetics representation with a single group of delayed neutrons.

5.1.4 Results

Figure 5.1 presents a typical result of these analyses. This figure shows the transient reactivity estimated from the present technique along with the reactivity from the reference model.

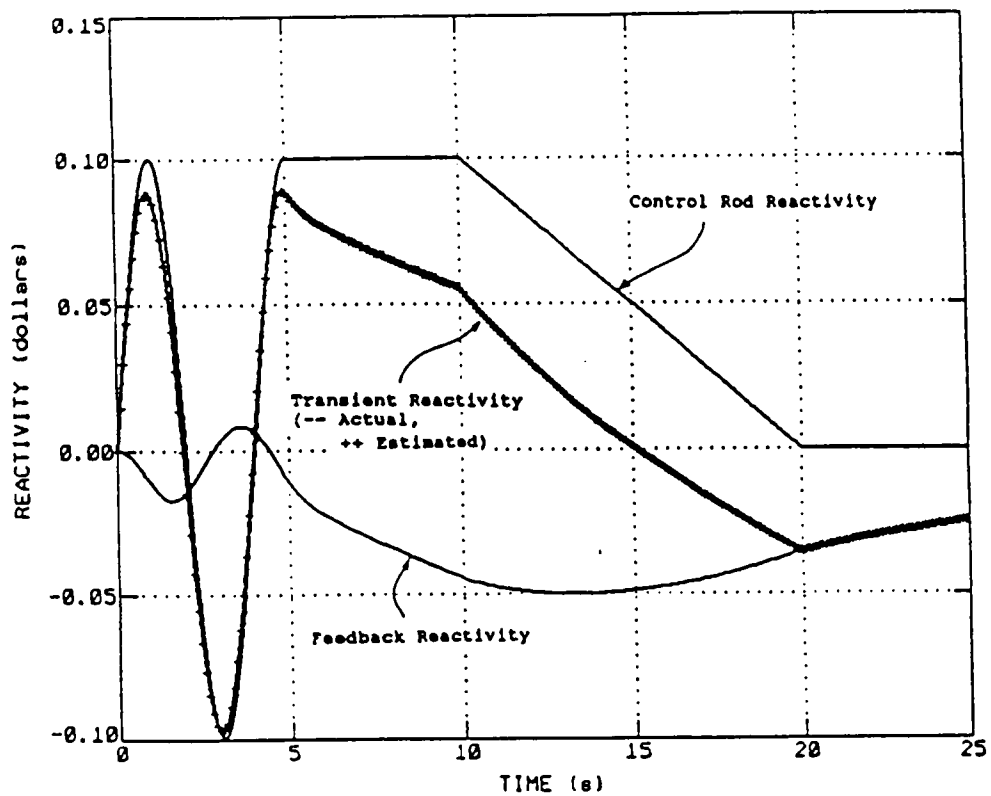


Fig. 5.1 The estimated transient reactivity (crosses) shows excellent agreement with the actual transient reactivity (solid line).

Superimposed on the same figure are the estimated feedback reactivity and the actual control rod reactivity used for the transient. This particular transient (a sinusoidal, followed by a constant, followed by a ramp in reactivity) highlights the fact that the present technique is able to extract the transient reactivity from the measured neutron flux even under extreme and unusual conditions.

5.2 State Estimator for PWR

This section documents an algorithm designed to estimate the state variables of nonlinear dynamical systems. The present algorithm relies on measurements of detector signals which are in general corrupted by additive noise, and it attempts to estimate the actual value of the state variables based on a model representation.

5.2.1 Background

Optimal control algorithms require the knowledge of many of the system state variables. In practical cases, only a subset of these variables are measured, and their values are not well known due to noise contamination. A common technique used for estimation of unmeasured state variables is known as Kalman filtering, that is based on linearized models and, thus, it may fail under special conditions when nonlinearities are dominant.

5.2.2 Theory

The goal of the present algorithm is to update an estimate, y , of the vector of state variables, x , when only a subset of signals, s , from the plant are measured. In addition, the measured signals are allowed to be contaminated by noise. The detection process can be described mathematically as $s = G(x) + \eta$, where η is a vector of additive noises, and $G(x)$ is a general vector function relating the measured signals with the actual plant variables. The state estimates are assumed to satisfy the following relation

$$dy/dt = f(y) + e, \quad (5-9)$$

where $f(y)$ is an approximate plant model that acts as a predictor of the next estimate, and e acts as the corrector. The present algorithm calculates e based on Potryagin Maximum Principle by minimizing the cost function

$$J = \int_0^{tf} [(s-G(x))^T Q (s-G(x)) + e^T R e] dt, \quad (5-9)$$

where Q and R are weight matrices.

Following the dual adjoint technique (see Chapter 2), this minimization results in the following equations for the adjoints, w , and e :

$$w = (df/dy)^T w - Q(s-G(y)) (dG/dy), \quad (5-10)$$

$$e = - R^{-1} w, \quad (5-11)$$

5.2.3 Validation

An example application of the present technique has been produced using a nonlinear model of a pressurized water nuclear reactor, which includes a point kinetics representation of the neutron field, a single-node representation of the fuel and coolant dynamics and reactivity feedback. The detection process is modeled, including detector time delays. A transient was induced in the plant model by ramping the inlet coolant temperature, and detection noise was added to the measured signals (power and fuel and coolant temperatures). Figure 5.2 shows the results of the filtering algorithm, which is a comparison between the detector responses with and without noise (solid lines) and the signal estimated by the present algorithm (dots). Satisfactory agreement is found between the estimated and actual transients despite the presence of a significant amount of measurement noise. Figure 5.3 shows how the present algorithm is able to update the unmeasured state variables.

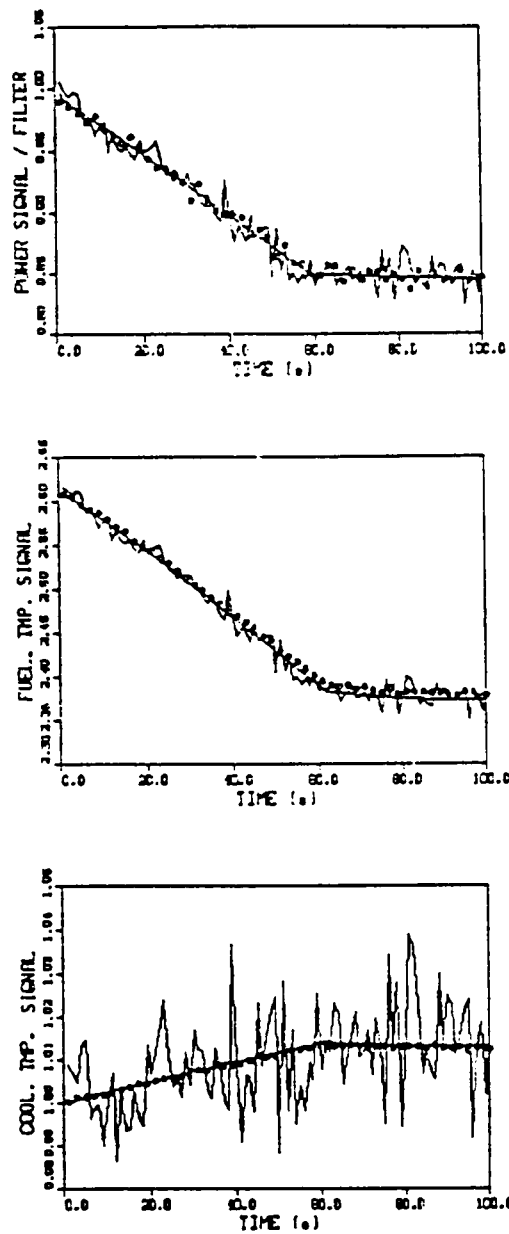


Fig. 5.2 Comparison between the detector responses with and without noise (solid lines) and the signal estimated by the present algorithm (dots).

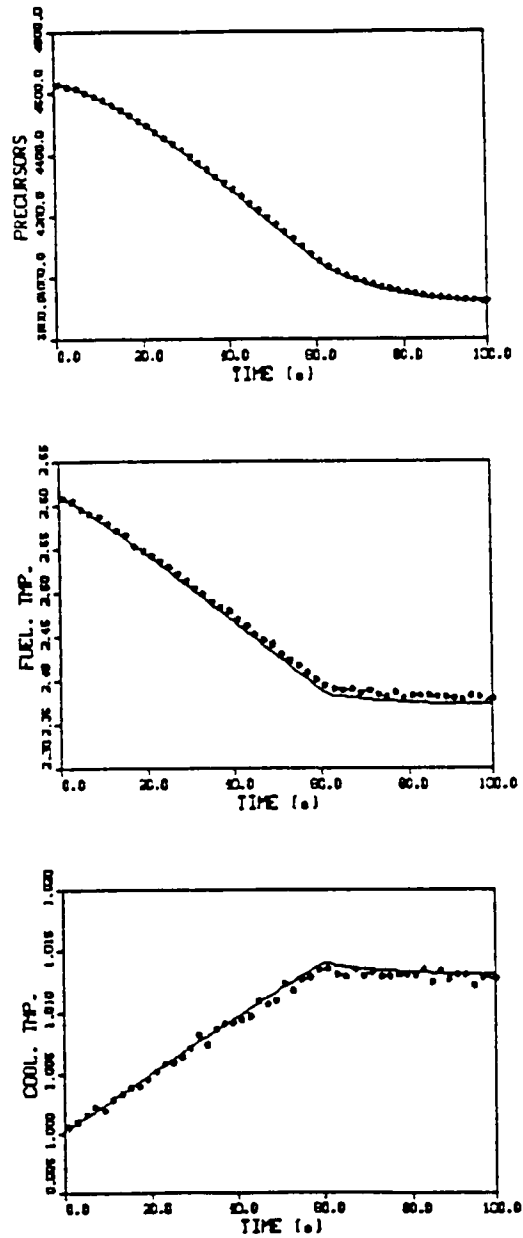


Fig. 5.3 Comparison between the unmeasured state variables estimated by the algorithm (dots) and their actual plant values (solid line).

5.2.4 Conclusions

In summary, a successful algorithm has been proposed for the estimation of unmeasured state variables based on the values of some observed variables that are contaminated by noise and detector time delays.

CHAPTER 6

A COMPARISON BETWEEN OPTIMAL CONTROL AND PI CONTROL: BWR POWER CONTROL DESIGN

This chapter presents a comparison between the behavior of a boiling water reactor (BWR) controlled with a standard proportional-integral (PI) controller and a BWR controlled with a microprocessor-based optimal control algorithm. A 5-dimensional model of BWR representing the power, precursors, fuel temperature, void fraction feedback and the first derivative of the void fraction feedback is used. The control variable is the core flow. Both control algorithms are designed to make the BWR perform as a demand-following reactor. It is shown how the PI controller fails to keep the system stable at low flow, while the optimal nonlinear control is able to keep the stability of the system at any flow, and improve its transient behavior.

6.1 The BWR Model

The dynamic behavior of a BWR can be represented by the following model:

$$dP/dt = P(\rho + \alpha T - \beta)/\Lambda + \lambda C \quad (6-1)$$

$$dC/dt = \beta P/\Lambda - \lambda C \quad (6-2)$$

$$dT/dt = a_1(P - P_0) - a_2 T \quad (6-3)$$

$$d^2\rho/dt^2 + (6/\tau)d\rho/dt + (12/\tau)\rho = (K/\tau^2)(P/U)T \quad (6-4)$$

$$P_o = .6 + (.4/.6)(U-.4) \quad (6-5)$$

$$\tau = .65/U \quad (6-7)$$

where P represents the reactor power, C the delayed neutron precursor concentration, T the fuel temperature, and ρ the void fraction feedback. The core flow is represented by U (control). It has been shown¹¹ that this simplified model is able to emulate fairly accurately the nonlinear behavior of a BWR.

6.2 PI Control Design

The PI control is designed in the standard fashion

$$U = U_p + U_i \quad (6-8)$$

where U_p is the proportional part and is given by

$$U_p = (D-P)q \quad (6-9)$$

and U_i is the integral part and is given by

$$dU_i/dt = (D-P)k \quad (6-10)$$

where D represents the demand, and q and k are the gain constants. The weights q and k are calculated such that they guarantee the stability of the system, i.e., all the real parts of the system's eigenvalues are negative. Unfortunately, it is necessary to linearize the model to perform the stability analysis, and it is not always obvious as to what are the optimal gains.

Figure 6.1 presents the reactor response to a ramp demand in power from 100% to 60% (that should correspond to a change in flow from 100% to 40%) when it is controlled using a PI technique. In this case the linearization necessary for the PI design was performed at 100% power. The values for the gains that make the system stable at this power were calculated. As Fig. 6.1 shows, these gains are able to stabilize the system at 100% power, and to make the power follow the imposed demand, but we can see that at the end of the transient the system becomes unstable. One of the system's eigenvalues has become positive. A divergent oscillatory behavior can be seen from the figure.

6.3 Optimal Control Design

In our design we are going to assume that a part of the BWR model is known. In particular we assume that we have a dynamic model for the power, precursors and temperature of the fuel. The void fraction feedback dynamic is unknown to the control system designer. We also assume that a plant power signal is available.

The knowledge of the state variables is fundamental to design and optimal control; therefore, our first goal is to obtain the unknown variables, the void fraction

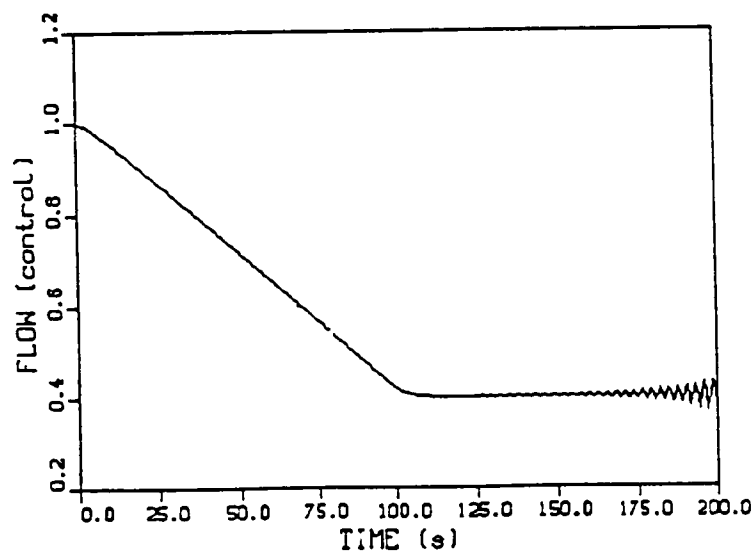
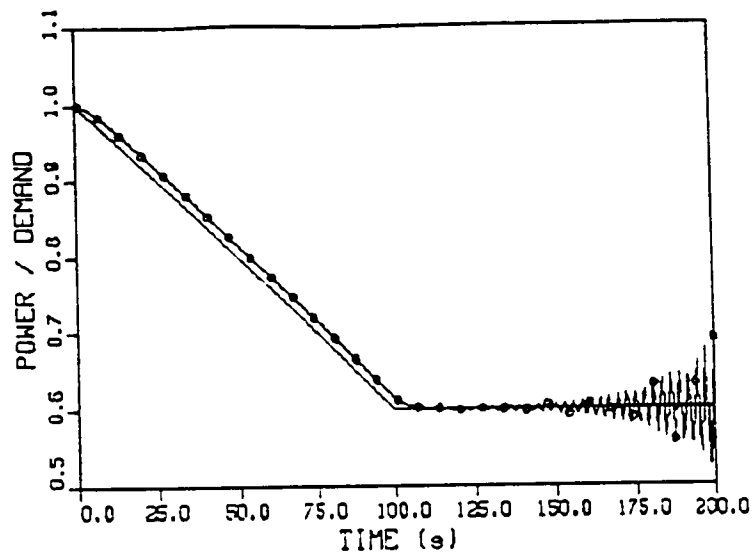


Fig. 6.1 Reactor response to a power ramp demand from 100% to 60%. Power is controlled using a PI technique with the flow being the controlled variable: (a) power, and (b) required flow control.

feedback and fuel temperature. This goal can be achieved by applying optimal control theory (see Chapter 2) to the so called state estimation problem. The idea is to find the optimal value of ρ that minimizes, in the least-squares sense, the error between the measured power signal and our control model.

Once we have obtain the unknown variable, the optimal control can be calculated following the same technique: we want to obtain the best U that minimizes the error between the demand and the control model.

To improve the stability of the control algorithm, we have also included in the minimization the error between the derivative of the demand and the derivative of the state variables.

Figure 6.2 resumes the computer simulation results:

(a) shows how the control algorithm is able to follow the demand and to stabilize the system at any power, (b) shows how the algorithm is able to track the unknown variable (void fraction feedback), and (c) shows the control action given by the algorithm.

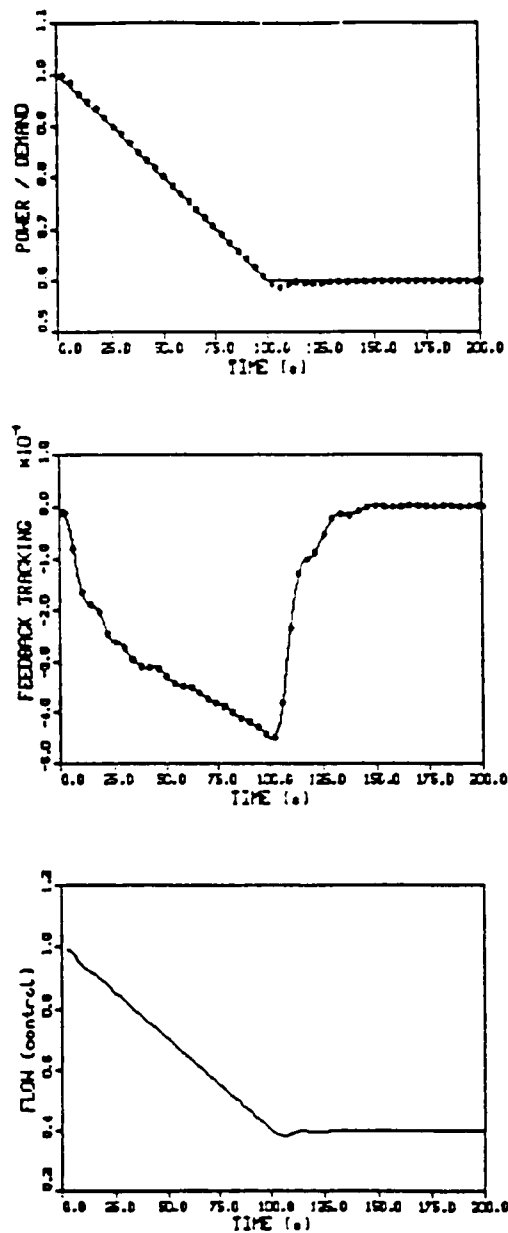


Fig. 6.2 Reactor response to a power ramp demand from 100% to 60%. Power is controlled using the optimal control technique with the flow being the controlled variable: (a) power, (b) void reactivity feedback, actual (solid line) and estimated (circles), and (c) the required flow control.

CHAPTER 7

OPTIMAL CONTROL OF UNCERTAIN NONLINEAR SYSTEMS: AN APPLICATION TO A TWO-LINK ROBOTIC ARM

This chapter presents an adaptive optimal control algorithm for uncertain nonlinear systems. A variational technique based on Pontryagin's Maximum Principle is used to track the system's unknown terms, and to calculate the optimal control. The reformulation of the variational technique as an initial value problem allows this microprocessor-based algorithm to perform as a closed-loop controller by updating the model and controlling the system on-line. To validate the algorithm a system representing a two-link mechanical manipulator is simulated. In the control model, the coupling and friction terms are unknown. The robot's task is to follow a prescribed trajectory and to pick up an unknown mass.

7.1 Introduction

The availability of fast and reliable microprocessors have opened the possibility of performing on-line numerical calculations to improve and optimize the manner in which mechanical manipulators are controlled. In particular, there is a general interest (see Ref. 8) on increasing the robustness of control algorithms when the

system's behavior is affected by nonlinearities, or when a relevant part of the system's dynamic is unknown a priori for the control designer.

This chapter presents a new microprocessor-based algorithm that is able to perform uncertainty-tracking, and control of a mechanical manipulator. A variational technique based on Pontryagin's Maximum Principle is used to update and decompose the control model, and to calculate the optimal control.

Three simultaneous optimizations are performed: (a) the trajectory in generalized coordinates is obtained by matching the kinematic model of the arm to the prescribed trajectory in cartesian coordinates, (b) the model's unknown terms are obtained by matching the signals from the arm's detectors to the control model, and (c) the optimal controls are obtained by matching the trajectory in generalized coordinates to the updated dynamic model.

To validate the algorithm a system representing a two-link robotic arm was simulated. In the control model the coupling and friction terms were unknown. The robot was forced to follow a prescribed trajectory and to pick up an unknown mass.

7.2 The Dual Adjoint Technique

This section reviews the reformulation of the free terminal time optimal control problem (FTT) presented in Chapter 2.

Let

$$dy/dt = F(y,u) \quad (7-1)$$

represent the plant model, where y are the state variables and u the controls. Our goal is to find $u(t)$ such that the system will follow a given set of demands d , in the sense that the cost function defined as

$$J = \int_0^{t_f} \{ [d-H(y)]^T Q [d-H(y)] + u^T R u \} dt \quad (7-2)$$

is minimized. Q and R are weight matrices, which are allowed to change with time. The initial state $y(0)$ and final state $H(y(t_f))$ are known but t_f is free.

If the system is in equilibrium at $t=0$ and d is a well-behaved function, the minimization problem is equivalent to the solution of the following set of equations:

$$dy/dt = F(y,u) \quad (7-3)$$

$$dw/dt = (dF/dy)^T w - (dH/dy) Q (d-H(y)) \quad (7-4)$$

$$u = R^{-1} w^T (dF/du) \quad (7-5)$$

$$y(0)=y_0 ; w(0)=0 \quad (7-6)$$

where w is the set of Lagrange multipliers or adjoints.

The above reformulation of PMP can be achieved by

integrating backwards the cost function, and the initial conditions are known only if the initial estate is in equilibrium. Under this special condition the calculation of the optimal control can be easily executed on-line, in a microprocessor, because the classical TPBVP is recast into an initial value problem.

7.3 Trajectory Prescription

This section shows how the arm angular velocities $\dot{d}$, which locates the end effector on the trajectory, can be obtained by applying the reformulated optimal control problem.

Let $\text{tra}(e)$ be a parametric representation of the trajectory in cartesian coordinates. First, we map the real time t into the parameter e by applying

$$e = 1/2[1 - \cos(\pi t/t_f)] \quad . \quad (7-7)$$

By the choice of this mapping we obtain directly the steady-state initial and final conditions. Now we can calculate the angular velocities by minimizing the cost function

$$J_T = \int_0^{t_f} \{ [\text{tra} - G(d)]^T Q [\text{tra} - G(d)] + \dot{d}^T R \dot{d} \} dt \quad (7-8)$$

where $G(d)$ represents the transformation from generalized, d , to cartesian coordinates.

7.4 Uncertainty Tracking

This section shows how the reformulation of the optimal control problem can be used to update the system's unknown terms.

Assume that there is a set of unknown terms a which can change arbitrarily with time. Our control model needs to be updated on-line, otherwise, we can not guarantee that the calculated control will be optimal. The updating of the control model can be achieved by applying the reformulation of the optimal control problem. In this case, we assume that the magnitude of the unknown terms, a , will be such that the cost function

$$J_p = \int_0^{t_f} \{ [x-m]^T Q_p [x-m] + a^T R_p a \} dt \quad (7-9)$$

is a minimum, where m represent the control model state variables and x the detector signals.

7.5 System Decoupling and Decomposition

Often, the plant's dynamical model involves a large number of differential equation. In this case, it may be convenient to decompose the large model into a set of decoupled subsystems. The uncertainty tracking method can be used to achieve an efficient decomposition without losing too much information.

Let us assume that

$$dx/dt = F(x,u) \quad (7-10)$$

represents the mathematical model of the plant that we wish to control, where the vector x represents the n state variables and u represents the m controls. Let

$$dx_i/dt = F_i(x_i, u_i) + p_i, \quad i = 1, k \quad (7-11)$$

represent a set of k subsystems formulated in such a way that they are uncoupled. To guarantee that this set of subsystems represents the whole system we introduce the unknown set of functions p_i , whose values at time t can be obtained by the optimal matching of the signals coming from the detectors with the values for the state variables given by the numerical integration of the subsystem's mathematical models. Actually, this can be formulated as an optimal control problem. We want to know which are the values of controls (unknown function p) that make the subsystems follow the demand (detector signals).

7.6 Optimal Control

At this point, our algorithm is able to update the control model; therefore, we have an adaptive model to which we can apply again the reformulated optimal control problem to obtain the controls necessary to follow the demands. Now, to implement the demand following capabilities, we have to minimize the cost function defined as

$$J_c = \int_0^{t_f} \{ [\mathbf{d}-\mathbf{m}]^T Q_c [\mathbf{d}-\mathbf{m}] + \mathbf{u}^T R_c \mathbf{u} \} dt \quad (7-12)$$

7.7 Application to a Robotic Arm

To validate the algorithm, a computer simulation of a two-link robotic arm was developed which included the friction terms. The control problem is to determine the set of torques to be supplied to the arm so that its end effector moves along a prescribed trajectory (the trajectory is given in cartesian coordinates). In the control model, the coupling and friction terms are considered unknown; therefore, it is necessary to update its values during the transient. Moreover, the arm must pick up an object whose weight is changing with time.

Figure 7.1 shows the motion of the robotic arm along the trajectory. The arm task is to follow the given trajectory (discrete line marked with the symbol +) and to pick up the unknown time-varying mass (graphically represented by a black disk).

7.8 Conclusions

This Chapter presents a novel algorithm for the control of mechanical manipulators based on optimal control theory.

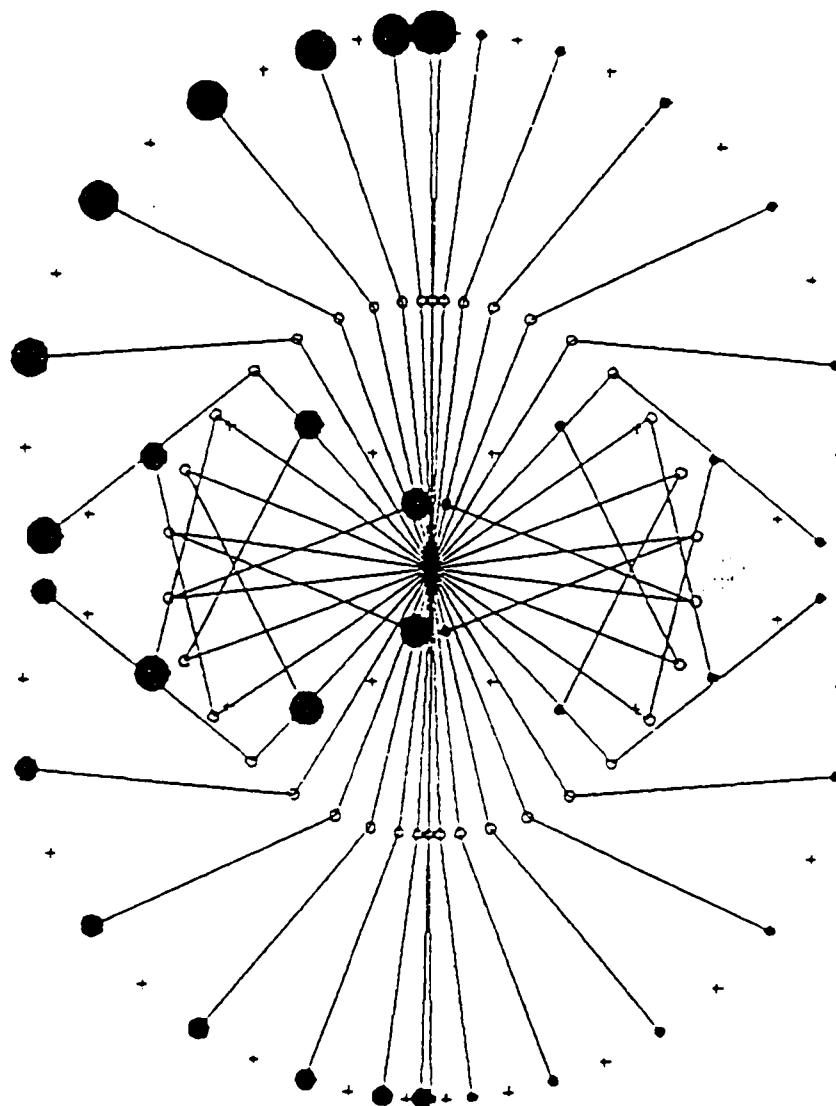


Fig. 7.1 Motion of the robotic arm along the trajectory. The arm task is to follow the given trajectory (discrete line marked with the symbol +) and to pick up the unknown time-varying mass (graphically represented by a black disk).

This new algorithm provides a suitable methodology for control problems in which only a part of the dynamical model of the robot is available to the designer. We have shown that this method can be applied to (a) transform coordinates, (b) update and decouple the control model, and (c) obtain the control inputs. The algorithm has been validated with a computer simulation of a two-link robotic arm.

CHAPTER 8

THE HAMILTON-JACOBI APPROACH

We have seen in previous chapters how easy it is to implement the so called Dual Adjoint Technique. Unfortunately, this is not an optimal solution to the problem, and, as shown in Chapter 2, it will not work for unstable systems in general. In this Chapter the more powerful Hamilton-Jacobi approach is used. To illustrate the technique a mechanical manipulator control is designed.

8.1 The Hamilton-Jacobi Equation

This section reviews the formulation of the Hamilton-Jacobi approach to optimal control presented in Chapter 2.

Let us study the dynamic behavior of the cost function. First we define a dynamic cost function as

$$J(t) = \int_0^t V(x,u) \, d\tau \quad (8-1)$$

taking the total derivative of $J(t)$ with respect to t

$$J_t = -[J_x dx/dt - V(x,u)] \quad (8-2)$$

and defining the Hamiltonian as

$$H(J_x, x, u) = J_x dx/dt - V(x, u) \quad (8-3)$$

we obtain

$$J_t = -H(J_x, x, u) \quad (8-4)$$

Equation (8-4) is known as the Hamilton-Jacobi equation in classical mechanics. In order to minimize the temporal evolution of $J(t)$ we need to select u (the control) such that it minimizes the Hamiltonian. Also we need to keep the value of the Hamiltonian constant and equal to zero during the evolution of the system.

The solution of Hamilton-Jacobi equation can be really difficult, specially for multidimensional nonlinear systems. As shown in Chapter 2, some simplification can be obtained by transforming the cost function.

8.2 Application to a Mechanical Manipulator

This section presents in detail the design process of a closed-loop control algorithm for a mechanical manipulator. First, a model of the mechanical arm is developed using the Lagrange theory. Second, the control algorithm is designed using the Hamilton-Jacobi approach. Two different formulation for the cost function are solved and compared. Because of the simplicity of this model all the calculations are made analytically.

8.2.3 The Model

The most powerful way of developing a dynamic model for a mechanical manipulator is to obtain the Lagrange-Euler equations in generalized coordinates.

Let us consider a one-link manipulator. For simplicity, we have assumed that all the mass, m , is concentrated in the end effector. The easiest approach to write the Lagrangian in generalized coordinates (in this case polar coordinates) is to write it first in Cartesian form and, then, transform to polar. In our case, the Lagrangian is

$$L = \frac{1}{2}m(v_x^2 + v_y^2) - mgy \quad (8-5)$$

The coordinates transformation is given by

$$x = r\sin(\theta) \quad (8-6)$$

$$y = -r\cos(\theta) \quad (8-7)$$

therefore,

$$v_x = -r\cos(\theta)d\theta/dt \quad (8-8)$$

$$v_y = r\sin(\theta)d\theta/dt \quad (8-9)$$

After elementary algebraic manipulation we can obtain the Lagrangian in polar coordinates as

$$L = \frac{1}{2}m(rd\theta/dt)^2 + mgr\cos(\theta) \quad (8-10)$$

The Lagrange equation is formulated as

$$T = d/dt(L_{[d\theta/dt]}) - L_{\theta} \quad (8-11)$$

where T represents the torque. Introducing (8-10) into (8-11) leads to the dynamic equation for the mechanical manipulator

$$d^2\theta/dt^2 = -(g/r)\sin(\theta) + T/mr^2 \quad (8-12)$$

defining

$$x_1 = \theta$$

$$x_2 = d\theta/dt$$

$$a_1 = (g/r)$$

$$a_2 = 1/mr^2$$

equation (8-12) can be written in state space form as

$$dx_1/dt = x_2 \quad (8-13)$$

$$dx_2/dt = -a_1 \sin(x_1) + a_2 T \quad (8-14)$$

8.2.4 The Hamilton-Jacobi Equation

Our goal is now to design a feedback control algorithm to regulate the robotic arm around a given position. Following the prescriptions from section 8.1, first we have to define a cost function, and then solve the associated Hamilton-Jacobi equation.

The efficiency of the final design depends fundamentally on the selection of the cost function. On the other hand, the analytic solution of the problem can be simplified by selecting an adequate cost function also. In this section we will show how two different cost functions lead to different results.

We first try a cost function that minimizes the position error and the acceleration. For simplicity we will assume that the desired position is $x_1=0$, then

$$J_1 = \int_0^{t_f} [\frac{1}{2}(x_1)^2 Q + \frac{1}{2}(dx_2/dt)^2 R] dt \quad (8-15)$$

the associated Hamiltonian will be

$$H_1 = w_1 x_2 + w_2 (-a_1 \sin(x_1) + a_2 T) \\ + \frac{1}{2} (x_1)^2 Q + \frac{1}{2} (-a_1 \sin(x_1) + a_2 T)^2 R \quad (8-16)$$

The optimal control must satisfy $H_u=0$; therefore,

$$w_2 a_2 + a_2 (-a_1 \sin(x_1) + a_2 T) R = 0 \quad (8-17)$$

Using (8-17) and demanding $H_1=0$ it follows

$$0 = w_1 x_2 - (w_2)^2 / 2R + \frac{1}{2} (x_1)^2 Q \quad (8-18)$$

Our goal is to obtain a solution which is independent of x_1 or x_2 , consider

$$w_1 = p_{11} x_1 + p_{12} x_2 \quad (8-19a)$$

$$w_2 = p_{12} x_1 + p_{22} x_2 \quad (8-19b)$$

Introducing (8-19) into (8-18) and performing some algebraic operations we obtain

$$0 = x_1 x_2 (p_{11} - p_{12} p_{22}) \quad (8-20a)$$

$$0 = (x_1)^2 (Q - (p_{12})^2 / R) \quad (8-20b)$$

$$0 = (x_2)^2 (p_{12} - (p_{22})^2 / 2R) \quad (8-20c)$$

We can now find a solution for equations (8-20) that is independent of the state (x_1 or x_2). Again some algebraic manipulations lead to

$$p_{12} = (QR)^{\frac{1}{2}} \quad (8-21a)$$

$$p_{22} = (2R(QR)^{\frac{1}{2}})^{\frac{1}{2}} \quad (8-21b)$$

$$p_{11} = R(2Q(QR)^{\frac{1}{2}})^{\frac{1}{2}} \quad (8-21c)$$

which is the only real solution of equations (8-20).

From equation (8-17) we can obtain the optimal control as a function of x_1 and w_2 . The quantity w_2 can be

expressed now as a function of x_1 and x_2 using equations (8-19) and (8-21). The optimal control can be, in this way, expressed on a closed-loop form as

$$T = (1/a_2)(a_1 \sin(x_1) - (Q/R)^{1/2} x_2 - (2Q(QR)^{1/2})^{1/2} x_1) \quad (8-22)$$

Introducing the optimal control in our model, we can perform stability analysis. The reader can easily check that this optimal control strategy is stable for any value of Q and R . But the system eigenvalues may have an imaginary part depending on the gains we choose. To avoid this problem, we need to come back to the original cost function and reconsider our original formulation.

Considering the cost function, we realize that the velocity has not been minimized. Now we solve again the problem by introducing this extra requirement, i.e., we try a cost function that minimizes the position error, the velocity error, and the acceleration. For simplicity we will assume that the desired position is $x_1=0$, and the desired velocity is $x_2=0$. Then

$$J_1 = \int_0^T [\frac{1}{2}(x_1)^2 Q_1 + \frac{1}{2}(x_2)^2 Q_2 + \frac{1}{2}(dx_2/dt)^2 R] dt \quad (8-23)$$

and the associated Hamiltonian will be

$$H_1 = w_1 x_2 + w_2 (-a_1 \sin(x_1) + a_2 T) + \frac{1}{2}(x_1)^2 Q_1 + \frac{1}{2}(x_2)^2 Q_2 + \frac{1}{2}(-a_1 \sin(x_1) + a_2 T)^2 R \quad (8-24)$$

The optimal control must satisfy $H_u=0$; therefore,

$$w_2 a_2 + a_2 (-a_1 \sin(x_1) + a_2 T) R = 0 \quad (8-25)$$

Using (8-25) and demanding $H_1=0$ it follows that

$$0 = w_1 x_2 - (w_2)^2 / 2R + \frac{1}{2} (x_1)^2 Q_1 + \frac{1}{2} (x_2)^2 Q_2 \quad (8-26)$$

Trying as before

$$w_1 = p_{11} x_1 + p_{12} x_2 \quad (8-27a)$$

$$w_2 = p_{12} x_1 + p_{22} x_2 \quad (8-27b)$$

and introducing (8-27) into (8-26) and after some algebraic operations we obtain

$$0 = x_1 x_2 (p_{11} - p_{12} p_{22}) \quad (8-28a)$$

$$0 = (x_1)^2 (Q_1 - (p_{12})^2 / R) \quad (8-28b)$$

$$0 = (x_2)^2 (p_{12} - (p_{22})^2 / 2R + Q_2 / 2) \quad (8-28c)$$

and

$$p_{12} = (Q_1 R)^{\frac{1}{2}} \quad (8-29a)$$

$$p_{22} = (2R(Q_1 R)^{\frac{1}{2}} + Q_2 R)^{\frac{1}{2}} \quad (8-29b)$$

$$p_{11} = R(2Q_1(Q_1 R)^{\frac{1}{2}} + Q_1 Q_2)^{\frac{1}{2}} \quad (8-29c)$$

which is the only real solution of equations (8-28). Now, we have more freedom to adjust the eigenvalues in the desired position.

Finally, we mention that if we choose to minimize the velocity and the acceleration, then, for any value of Q and R , the eigenvalues are imaginary.

8.3 Conclusions

In this chapter, we have presented a detailed example of solving the Hamilton-Jacobi equation.

The fundamental advantages of this approach are: (a) the system is stable for any selection of weights, and (b) the final implementation can be done using standard PID boxes and function generators. No differential equations are needed.

The main disadvantage of this approach is the substantial analytic work required when the systems are nonlinear. Some simplification can be obtained by redefining the cost function.

If the system is linear and our goal is to design a steady-state regulator, then the Hamilton-Jacobi approach reduces to the well-known LQ algorithm.

CHAPTER 9

A NEW APPROACH TO CONTROL THE WATER LEVEL OF U-TUBE STEAM GENERATORS

Automatic water level control in steam generators is currently achieved via the three-element controller. This algorithm is based on the measurements of level, steam flow and feedwater flow. Unfortunately, at low power the feedwater flow signal is highly unreliable, forcing the transfer to manual control. A large number of reactor trips occur under these conditions. The nuclear industry has shown a concern to this problem.

In this chapter, an alternative automatic control algorithm is proposed and validated. The new algorithm does not rely on flow signals. Instead, it uses the pressure measurement in the steam header. A level set point modulation is introduced which allows the algorithm to compensate for the shrink and swell phenomena. The standard Delta-P algorithm to control feedwater pump speed has been also modified to achieve greater performance and integration.

9.1 Steam Generator Dynamics

The U-tube steam generator (SG) dynamics is very attractive from the mathematical point of view, but it is

a nightmare for the plant operator. The SG dynamic behavior is characterized by strong nonlinearities at low power, and by the so called "non-minimum phase" problem. Both effects need to be compensated by the control designer.

Perturbations in pressure or enthalpy can make the water level and the water inventory in the SG to be out of phase for large periods (10 to 100 seconds). This means that we may be introducing more water into the system than amount of steam leaving for several seconds, and the level may be decreasing instead of increasing. This can affect dramatically the controllability of the system if it is not properly taken into account.

Out of phase perturbations can be introduced by changes in: (a) steam flow, (b) primary enthalpy, or (c) feedwater enthalpy. This phenomena is called "non-minimum phase" or "shrink and swell" problem.

On the other hand, the strong nonlinearities present in the system can also affect our design. If the system is linearized, one can get the wrong impression of having a stable model. The system can be stable, actually, only under small perturbations. A larger perturbation will make it unstable. This effect is more evident at low power, and it seems to be the most important source of failure in control designs.

9.2 Existing Controllers

This section reviews the control options available to date to the nuclear industry.

9.2.1 One-Element Controller

A minimization of the level error using a standard PID box leads to the one-element controller. Because of the out of phase period, this design can lead to instabilities.

Some naive designs have been proposed for low power operation based on this approach. These designs calculate the gains for the PID level controller based on a linearized model. The linearized model may give us the wrong idea of having a stable system. Even if this were true, the PID gains would be so small that the system would respond too slowly to the usual power ramps.

9.2.2 Three-Element Controller

Figure 9.1 shows a block diagram of a three-element controller. Now, not only the level error is minimized, but also the difference between the steam and feedwater flows. In this way, the out of phase periods are, in some way, compensated. This design is highly effective. Unfortunately, at low power the feedwater flow signal is unreliable; therefore, the three-element controller can not be used.

THREE-ELEMENT CONTROL

- Minimizes level error and flow difference

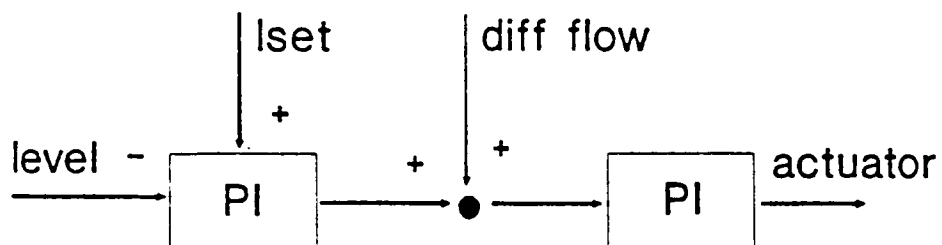


Fig. 9.1 Three-element controller diagram.

9.2.3 The Belgian Level Controller

Belgian nuclear power plants use an automatic feedwater control that is able to perform under any operating conditions¹². It is based on an adaptive three-element design but it includes a shrink and swell compensator. At low power, the feedwater flow becomes unreliable. The Belgian design solves this problem by using an one-element controller with a new shrink and swell compensator.

The compensators for low power and for full power are different. At full power the steam flow signal is used, and at low power the wide range level signal is used. We have been unable to get detailed information of how they really work, but the operational experience seems to be satisfactory.

9.3 Level Set Point Modulation

This section presents a novel algorithm to control the water level automatically under any operating conditions. The algorithm is based on the one-element controller, but it uses a level set point modulation technique to compensate for out of phase phenomena.

First, the model used to perform the steam generator simulation is presented. Then, the SG dynamics under the action of the one-element controller is studied. It is

shown how the gains needed to follow the standard power ramps are too big, and can make the system unstable at low power. Finally, a level set modulation technique able to compensate for the "non-minimum phase" phenomenon is formulated and validated.

9.3.1 Steam Generator Model

The MMS library¹³ has been used to develop the SG model. The modules used represent:

- (a) feedwater pump (includes turbine and turbine valve),
- (b) main feedwater valve and actuator,
- (c) U-tube steam generator, and
- (d) connectors and pipes.

The parameter used for this model are taken from a Combustion Engineering PWR (Arkansas Nuclear One).

Figure 9.2 gives a graphical representation of the level scales in this SG.

9.3.2 One-Element Controller Dynamics

Using the standard PID boxes of the MMS library, a one-element controller was added to the model. A power ramp of .05 per cent per second was simulated.

LEVEL SCALE

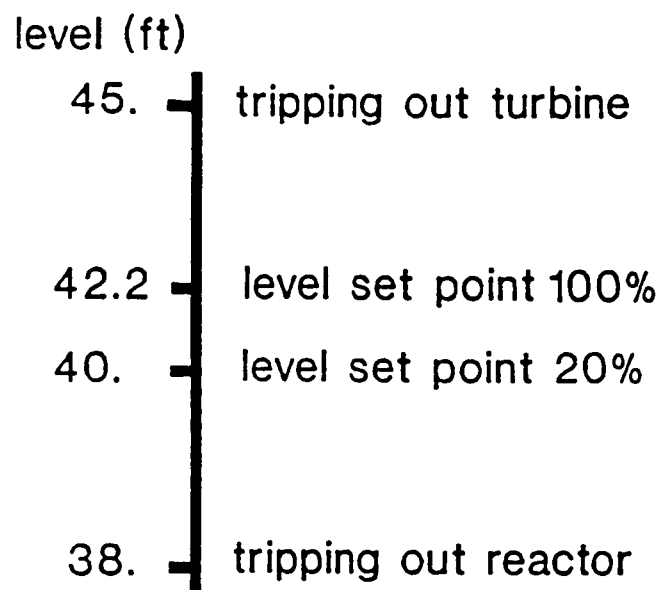


Fig. 9.2 Steam generator level scale.

The PID box was adjusted to make the level stay near the set point during the transient. Figure 9.3 presents a graphical representation of the results.

It was found that, in order to keep the level error small, it was necessary to have strong gains. The power ramp used is the one required by the nuclear industry for their level control algorithms.

A strong gain makes the level to stay near the set point, but also, can make the system unstable. Figure 9.4 shows the results of a step change on steam flow at 100% power. If the steam flow is reduced, the pressure of the steam increases, inducing a contraction of the steam bubbles in the water. The level controller is acting in the opposite direction. The feedwater flow is increasing when the steam flow has been reduced for a period of about 30 seconds. This out of phase period will produce a large overshoot.

The results are catastrophic at low power. Figure 9.5 presents a simulation of a step change in steam flow at 25% power. Although this time the step is smaller in absolute value, the system becomes unstable. Moreover, the instability has a nonlinear nature. The linearization around the set point will make us believe that the system was stable.

UT-STEAM GENERATOR power ramp

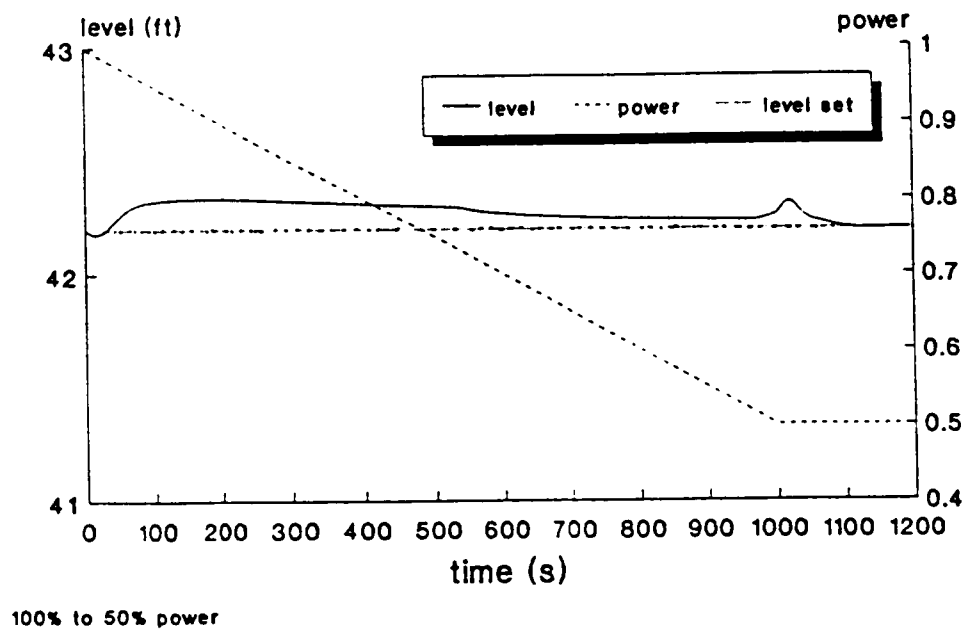


Fig. 9.3 Steam generator response to a power ramp.

UT STEAM GENERATOR

no dP/dt controller (100%)

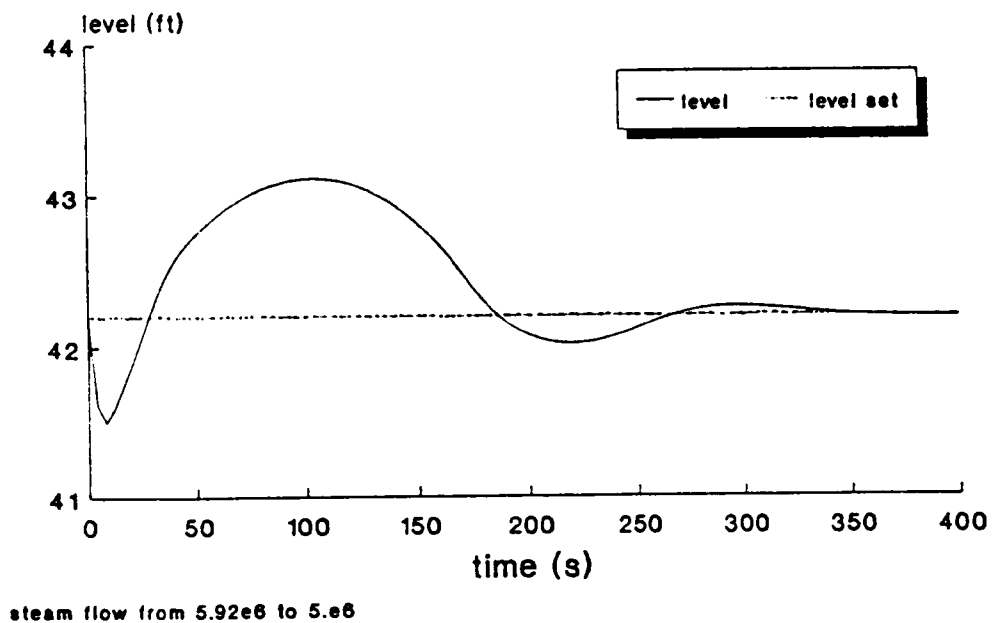


Fig. 9.4 Steam generator response to a steam flow step perturbation. 100% power.

UT STEAM GENERATOR

no dP/dt controller (25% POWER)

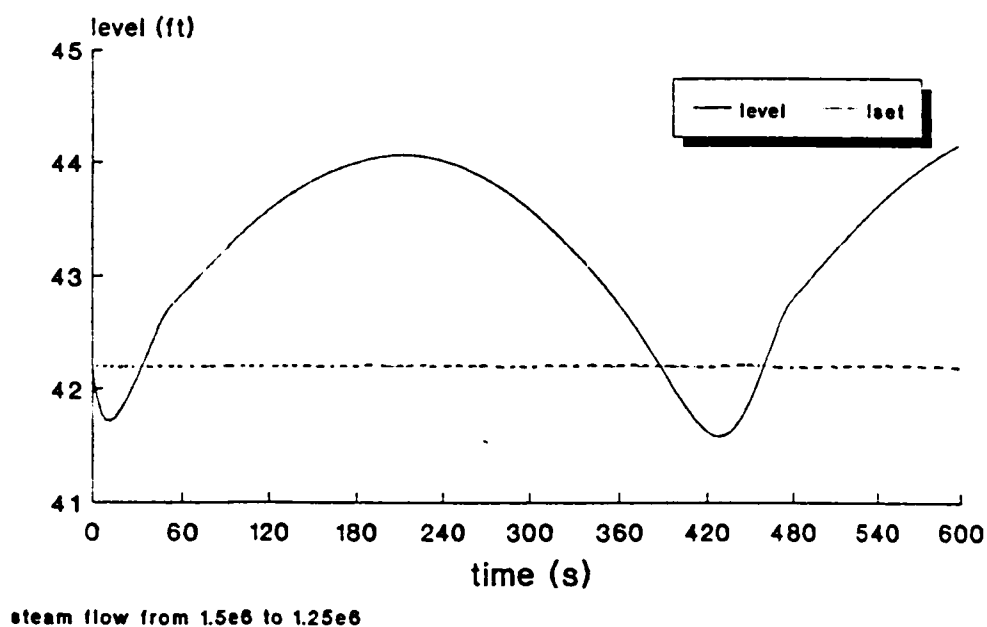


Fig. 9.5 Steam generator response to a steam flow step perturbation. 25% power.

9.3.3 Level Set Point Modulation

We have seen that the gains necessary to follow a power ramp are too large and the SG may become unstable. This instability is mainly induced by the out of phase phenomenon. The level can be decreasing while the water inventory is increasing. It is necessary to compensate in some way the level controller if a large perturbation is introduced into the system.

We have introduced a level set modulation technique to avoid these problems. We define L_{set_0} as the desired level set point. In our algorithm, the actual level set point sent to the one-element controller will be L_{set} , and define as

$$L_{set} = L_{set_0} + L_p \quad (9-1)$$

where L_p is a modulated correction factor that we obtain from

$$dL_p/dt = -L_p/\tau_p - K_p(dP/dt) \quad (9-2)$$

where P is the pressure signal, and K_p and τ_p are positive gains to be adjusted by the control designer.

9.3.4 Simulation Results

The level set modulation technique was implemented on the MMS steam generator model. A standard PID box was used for the one-element controller. The pressure signal was filtered before taking the numerical derivative. The

derivative was filtered again through a band limiter. The reason is that we are only interested in compensating for large perturbations.

Figure 9.6 shows a graphical display of the simulation results at 100% power. Figure 9.7 compares the dynamic behavior of the system: (a) without level modulation, and (b) with level modulation.

The effect of the compensation is more dramatic at low power. Figure 9.8 presents the simulation results at 25% power and figure 9.9 compares them with the ones obtained without modulation.

9.4 Pump Speed Controller

The pump speed is being controlled using the so called "Delta-P" algorithm. The idea is to operate the pump in such way that the difference of pressure between steam header and the pump exit is almost constant. The problem in this design will be that the pump speed controller does not use the level error information; therefore, in some special scenarios, it can act against the level controller.

In our design we have changed the control concept for the pumps. Two factors will contribute to the controller: (a) a valve set program, and (b) the level error.

UT STEAM GENERATOR

dP/dt controller (100% POWER)

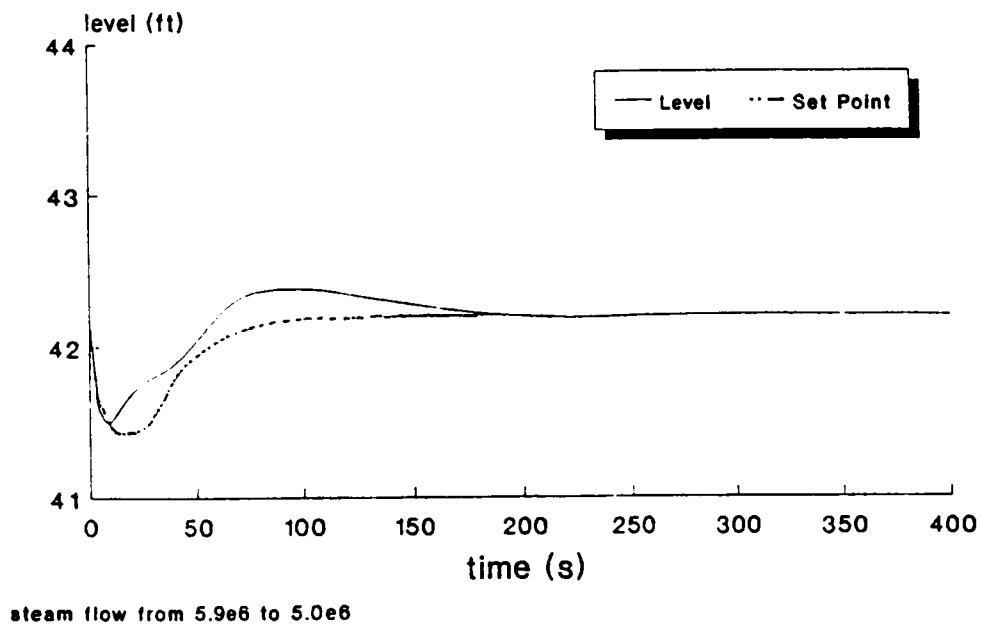
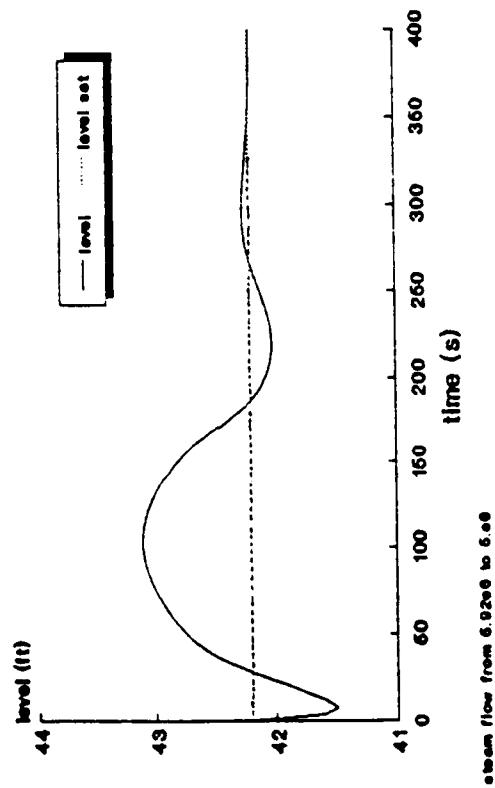


Fig. 9.6 Steam generator level response to a steam flow step perturbation. 100% power. Level modulation on.

UT STEAM GENERATOR no dP/dt controller (100%)



UT STEAM GENERATOR dP/dt controller (100% POWER)

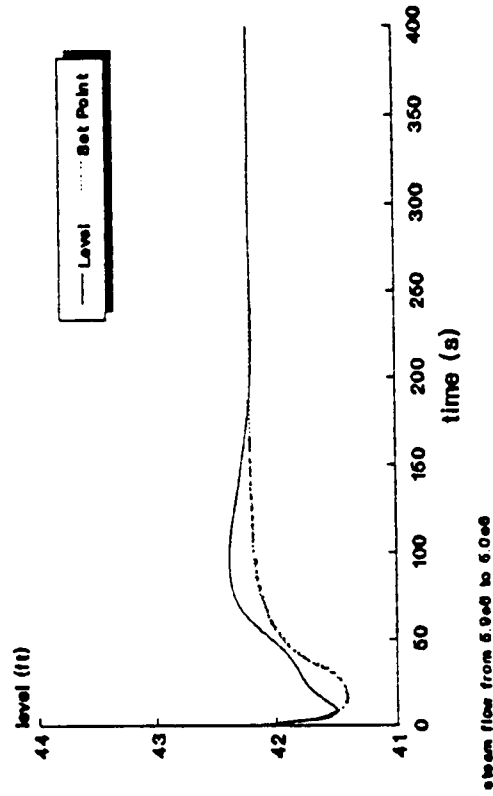


Fig. 9.7 Comparison between the steam generator response:
(a) level modulation off, and (b) level modulation on. 100% power.

UT STEAM GENERATOR dP/dt controller (25% POWER)

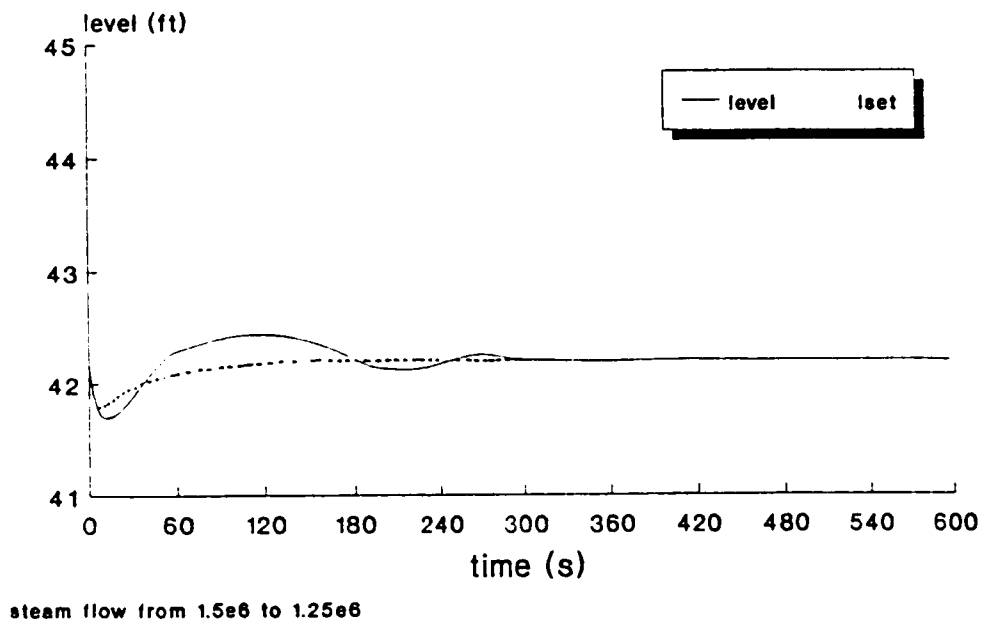
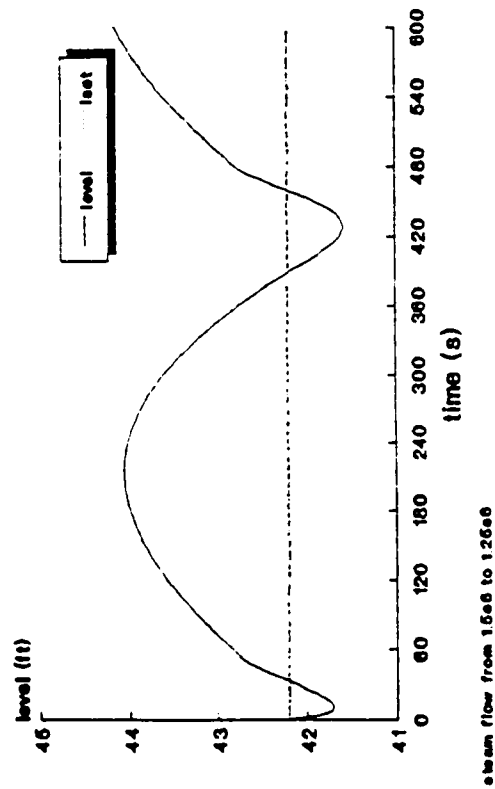


Fig. 9.8 Steam generator level response to a steam flow step perturbation. 25% power. Level modulation on.

UT STEAM GENERATOR no dP/dt controller (25% POWER)



UT STEAM GENERATOR dP/dt controller (25% POWER)

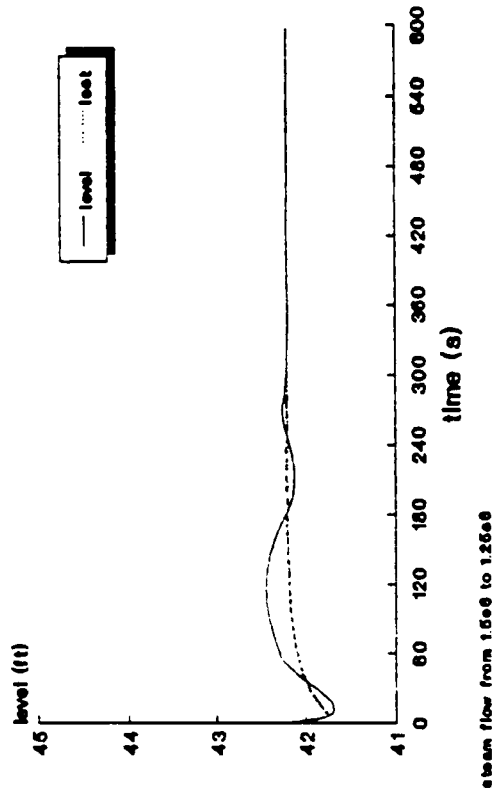


Fig. 9.9 Comparison between the steam generator response:
(a) level modulation off, and (b) level modulation on. 25% power.

Figure 9.10 is a diagram of the valve set program. Given a steam flow signal (power signal could also be used) a function generator return the optimal valve position. This function generator has been designed off-line in such way that optimizes the position of the valve to ensure a nice difference of pressure and a safe controllability margin.

The optimal valve position signal is then filter to introduce a delay (in this design the delay was of 30 seconds). This delay must be introduced to avoid abrupt reactions on the controller.

The filter signal is then added to the one-element controller signal and sent to the feedwater valve. Simultaneously, the error between the actual position of the feedwater valve and the filter desired position is added to the one element controller signal and sent to an integral box that actuates the pump valve.

Figure 9.11 gives the diagram for the integrated control algorithm: level modulator, one-element controller, and valve set program.

Figure 9.12 shows the positions of the feedwater and pump valve during the simulated 100% power transient. The behavior of the integrated controlled is clear. The fast proportional information is sent to the feedwater valve,

VALVE SET PROGRAM

- A valve set program is introduced that substitutes the delta-P controller

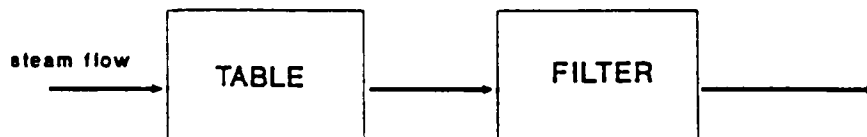


Fig. 9.10 Valve set program diagram.

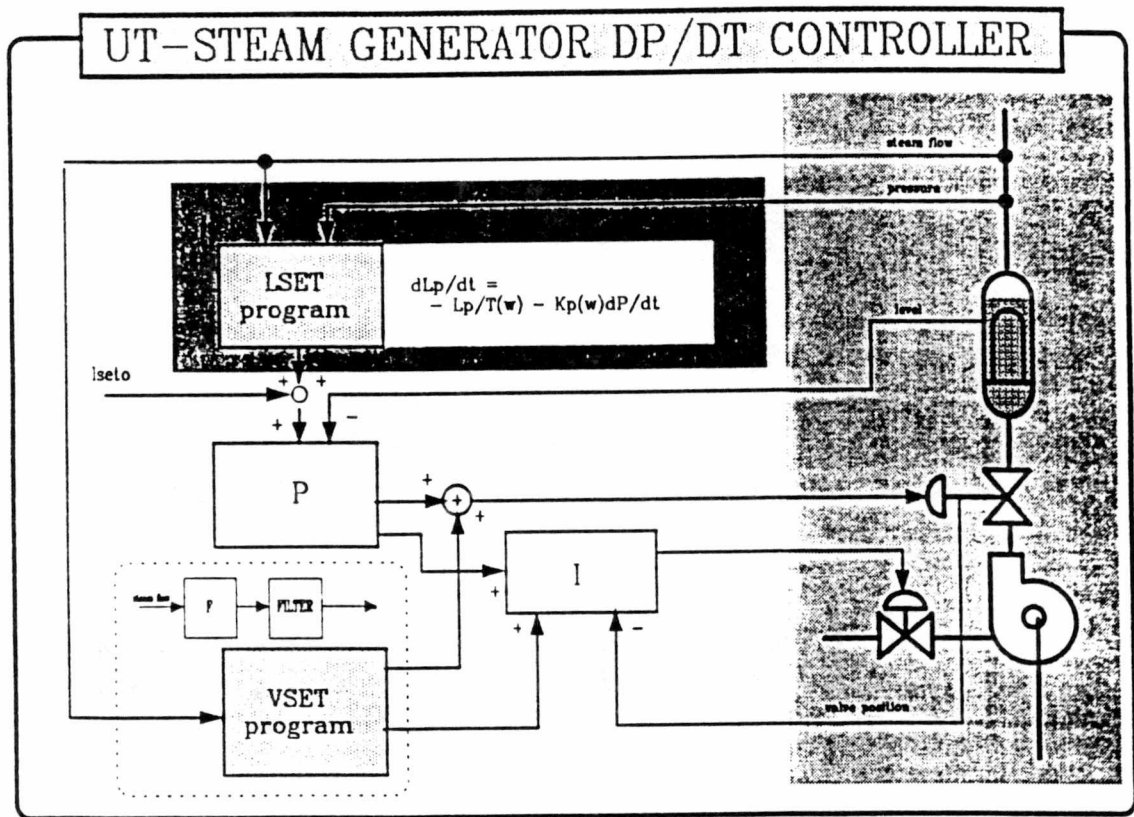


Fig. 9.11 Integrated control system diagram.

STEAM GENERATOR

dP/dt controller (100% POWER)

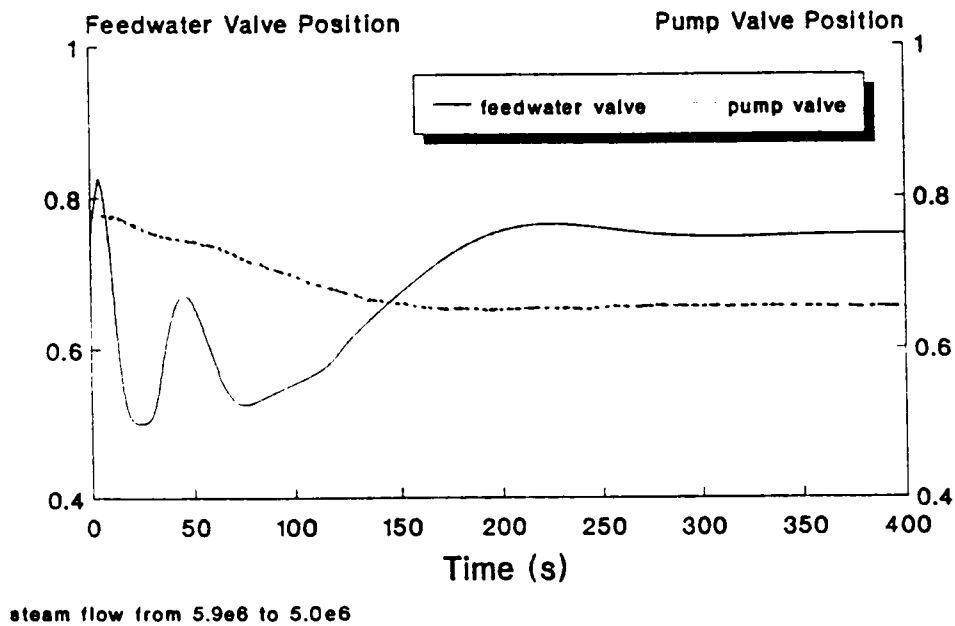


Fig. 9.12 Valves position during a transient.

while the pump valve executes a slow integral movement to correct for steady-state errors and to make the feedwater valve move to the optimal position.

9.5 Future Work

The algorithm presented in this chapter is ready to be implemented in any operating steam generator. The design was done based on the capabilities of the Foxboro hardware and, therefore, no new hardware development will be necessary.

But we think the capabilities of the algorithm could be enhanced by taking into account feedwater and primary enthalpy perturbations. Both perturbations introduce an out of phase period into the system that can be compensated by using again the level modulation technique. Let L_p , L_{tp} and L_{tf} represent the modulated corrections for pressure, primary temperature, and feedwater temperature. The modulated level set point signal sent to the one-element controller will be given by

$$L_{set} = L_{set_0} + L_p + L_{tp} + L_{tf} \quad (9-3)$$

$$dL_p/dt = -L_p/\tau_p - K_p(dP/dt) \quad (9-4)$$

$$dL_{tp}/dt = -L_{tp}/\tau_{tp} + K_{tp}(dT_p/dt) \quad (9-5)$$

$$dL_{tf}/dt = -L_{tf}/\tau_{tf} - K_{tf}(dT_f/dt) \quad (9-6)$$

where T_p , and T_f represent the primary and secondary temperature signals.

On the other hand, it will also be interesting to study the effects induced by the coupling of more than one steam generator connected to the same feedwater line. In this case, perhaps the original "delta-P" controller will perform better than the proposed valve set program. The reason is that, by keeping the pressure difference constant, it will be possible to minimize the coupling effects.

Finally, better simulation of real low power is needed. The MMS model used in this work failed at very low power. It will be necessary to use the extended range modules of the MMS library.

9.6 Conclusions

In this chapter, an alternative automatic control algorithm has been proposed. The new algorithm does not rely on flow signals. Instead, it uses the pressure measurement in the steam header. A level set point modulation is introduced which allows the algorithm to compensate for the shrink-and-swell phenomenon. The standard Delta-P algorithm to control feedwater pump speed has also been modified to achieve greater performance and integration.

The algorithm has been validated using a detailed U-tube steam generator model based on the MMS library.

CHAPTER 10

CONCLUSIONS

This doctoral dissertation has studied, developed, and validated advanced control techniques for nuclear reactors and mechanical manipulators.

Nuclear industry is now in the process of moving to digital hardware. Because of this unique opportunity to update the actual control algorithms, there is a generalized interest in knowing the benefits of applying new advanced control techniques to nuclear plants. The research presented in this thesis is a contribution to this effort.

The optimal control theory has been analyzed from the stability point of view. It has been found that the classical formulation of Pontryagin leads to open-loop strategies and that the alternative formulation which leads to stable closed-loop implementations, the so called LQ algorithm, can only be applied to linear systems.

Unfortunately, real life systems are seldom linear, and if linear, rarely an exact model of the plant is available. Even if we had the exact model, it will involve a large number of differential equations making the computation of advanced control strategies impossible.

The main effort of this dissertation has been to develop the necessary theory to simplify all these problems.

First, the basic relationship between Pontryagin's Optimal Control Theory and the alternative Kalman's LQ algorithm has been identified.

Based on this study, new algorithms have been proposed that are able to deal directly with nonlinear models, track time-varying parameters or uncertainties, decompose large model into a set of decoupled submodels, and handle noisy signals.

These new techniques have been validated with a large number of examples. Applications to PWRs, BWRs, ANS, steam generators, and mechanical manipulators have been simulated in digital computers and the results have been published in 9 peer reviewed papers.^{14,15,16,17,18,19,20,21,22}

10.1 Accomplishments

The main accomplishments of this research can be summarized as follows.

10.1.1 Closed-Loop Optimal Control

In Chapter 2, the basic relationship between Optimal Control Theory and Closed-Loop Control Design is identified. The purpose of this work has been to show how the basic cost function minimization problem must be

reformulated to obtain a closed-loop control strategy. New approaches to this problem that are able to handle directly nonlinear models have been developed. Some new applications of the Closed-Loop Optimal Control Theory have been proposed: state estimation, parameter tracking, uncertainty tracking, and system decomposition.

10.1.2 Parameter Tracking

In Chapter 3, a new formulation of a class of nonlinear optimal control problems in which the system's parameters change arbitrarily with time has been presented. The methodology has been validated with a PWR model.

10.1.3 State Estimation and Filtering

Chapter 4 presents a new formulation of a class of nonlinear optimal control problems in which the system's signals are noisy and some system's parameters are changing arbitrarily with time. The methodology has been validated with an application to a nonlinear PWR model.

10.1.4 Plant Monitoring

Chapter 5 shows some applications of the techniques presented in this thesis to the reactor monitoring problem. First, a transient reactivity monitor has been presented which was tested and validated with a dynamic

model of the Advanced Neutron Source Reactor (ANS).

Second, a state monitor for a PWR has been presented which is able to estimate the state variables of the system by analyzing the noise-corrupted signals from the plant.

Both applications use the techniques presented in previous chapters.

10.1.5 Comparison Between PI and Optimal Controllers

Chapter 6 presents a comparison between the behavior of BWR controlled with a standard PI controller and a BWR controlled with a microprocessor-based optimal control algorithm.

10.1.6 Uncertainty Tracking and Large System Decoupling

Chapter 7 presents an adaptive optimal control algorithm for uncertain nonlinear systems. It represents a novel approach to the problem of decoupling a large system into a set of simplified subsystems. A variational technique based on Pontryagin's Maximum Principle has been used to track the system's unknown terms, decouple the subsystem models, and calculate the optimal control. To validate the algorithm a system representing a two-link mechanical manipulator was simulated. The inverse kinematics problem for the trajectory prescription in generalized coordinates was also solved by applying optimal control techniques.

10.1.7 Applicability of the Hamilton-Jacobi Approach

Chapter 8 presents an application of the Hamilton-Jacobi approach to the mechanical manipulator problem.

10.1.8 Demand Modulation: SG Automatic Level Control

Chapter 9 presents a new algorithm to control steam generator water level that does not rely on feedwater flow measurements. At low power, the feedwater signal is biased; therefore, the classic three-element controller fails, forcing the transfer to manual operation. Manual operation has shown to be not very effective; in fact a substantial number of reactor trips occur in these conditions. The new algorithm presented in this thesis was able to compensate for the shrink-and-swell phenomenon by modulating the level set point according to differential changes on the pressure signals. In this manner, it was possible to keep a strong and dynamic automatic level controller from 0% to 100% power.

10.2 Comments About Some Alternative Approaches

This dissertation has been oriented to the study of the applicability of variational techniques to the advanced control design problem. Unfortunately, other powerful approaches like adaptive PID, self-tuning control, and robust control have not been analyzed.

These algorithms might be really efficient. They have the advantage of being easier to implement. It is important to understand that the main objective of advanced control algorithms is to improve the ratio between benefits and cost for the nuclear industry; therefore, the implementation cost should be taken into account when new algorithms are proposed.

On the other hand, simplicity can increase reliability. Each day more powerful computers are becoming available. There are now personal computers able to perform computations at rates faster than 8 MIPS (mega instructions per second) and 2 MFLOPS (mega floating point calculations per second). These rates can lead us to the conclusion that we can create very sophisticated algorithms working in real time.

But, by increasing the necessary computational steps, we also increase the failure rate. Given the large safety margin required for nuclear reactors, and the huge cost of down time, simplicity should have an important weight factor in our future decisions.

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