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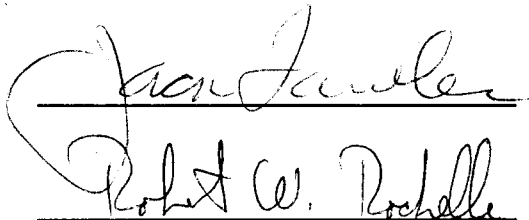
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COMPARATIVE STUDY OF VARIABLE STRUCTURE CONTROL SYSTEMS

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Gregory V. Murphy

June 1986

Dedicated
to my wife Rosalind,
my aunt and uncle Mary and John Walls,
my father Booker T. Murphy, and
my deceased mother Emma Delores Murphy
for your encouragement and help
in token of the love I have for you

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ABSTRACT

This thesis presents the results of a study on variable structure control system (VSCS) design. A simple type of VSCS using state trajectories is described and implemented. The characteristics of this simple VSCS and a linear control system design are compared. The characteristics examined are transient response and robustness of these basic design concepts. A basic analytical VSCS design method is implemented. This analytically designed VSCS is examined and seen to have some desirable characteristics. Despite the encouraging results of the basic analytically designed VSCS an undesirable characteristic exist.

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CHAPTER 1

INTRODUCTION

The theory of variable structure control has been developed and described mainly by S.V.Emel'yanov and E.A. Barbashin. U. Itkis about eight years ago presented a book which gives a concise and systematic theoretical coverage of variable structure control. This book appears to be the best source of information describing the basic concepts of variable structure control system design written in English.

Variable structure control(VSC) is a unique method of automatic control when the controller structure of the system is purposely changed during the transient process. A control system of this type is called a variable structure control system(VSCS). This change in the controller structure of the system depends on the present value of the error signal and its associated derivatives. The error and its appropriately selected order of derivatives are combined linearly to create a surface in state space at which switching of the controller structure occurs. This surface in state space will be referred to as a switching plane.

The switching of the controller structure along the switching plane in state space yields a condition called a sliding mode. A sliding mode, to be defined in a later section, is a desired characteristic of the VSCS. This VSCS concept has been found to yield certain desirable properties while operating in the sliding mode. These properties include a fast transient response, insensitivity to plant parameter variations, and simple realization .

CHAPTER 2

REVIEW OF VSC

2.1 VSC By Positive and Negative Feedback

This chapter reviews the principle operating concepts of variable structure control systems. For ease of understanding, discussion will be made using a simple system.

A VSCS approach provides a way for synthesizing a high performance stable system. This is done by switching the controller structure in a certain way. The controller structure is switched so that the system operates in stable and unstable modes alternatively during the response period, giving a stable overall response.

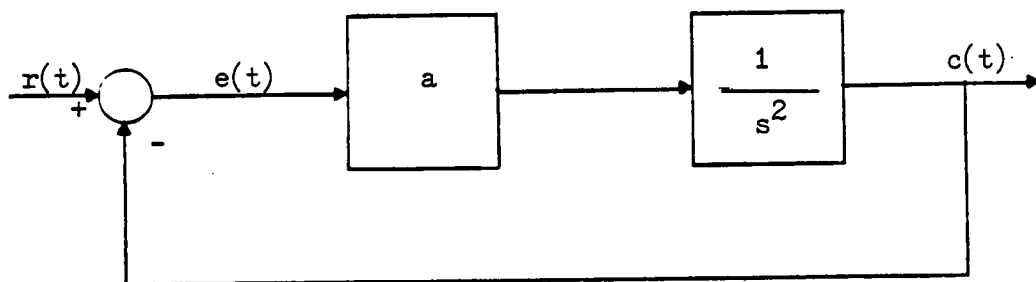
Consider the control of a plant by changing the sign of the feedback, a simple example of VSCS design. The plant is

$$G(s) = \frac{1}{s^2}.$$

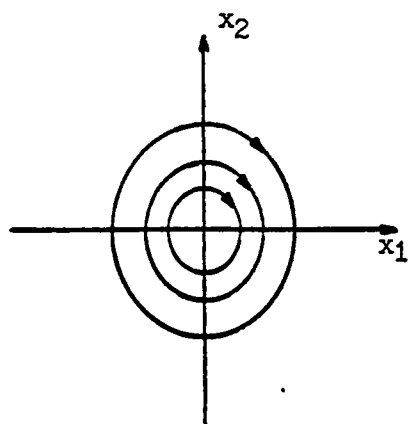
This plant is part of a stabilizing feedback loop. For negative feedback, as shown in Fig. 2.1a, the state differential equations describing the system error dynamics are

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -ax_1(t), \end{cases} \quad (2.1)$$

where $x_1 = e = r - c$ is the system error [1]. The gain a is a positive constant. The plot of the error x_1 versus x_2 provides a family of elliptic error



(a)



(b)

Figure 2.1: Negative feedback system.

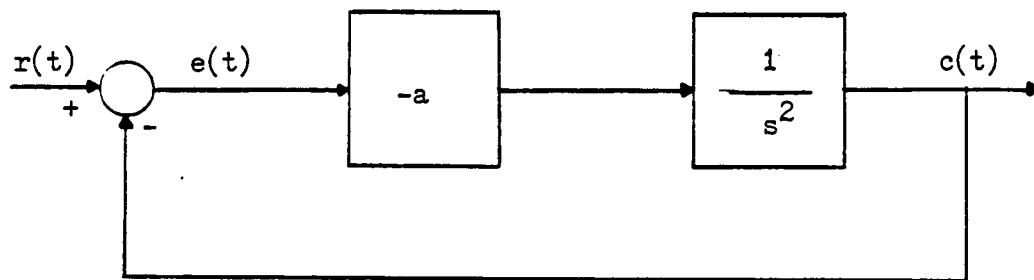
state trajectories as shown in Fig. 2.1b. With positive feedback, as shown in Fig. 2.2a, the error dynamics are described by the state differential equations

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = ax_1(t). \end{cases} \quad (2.2)$$

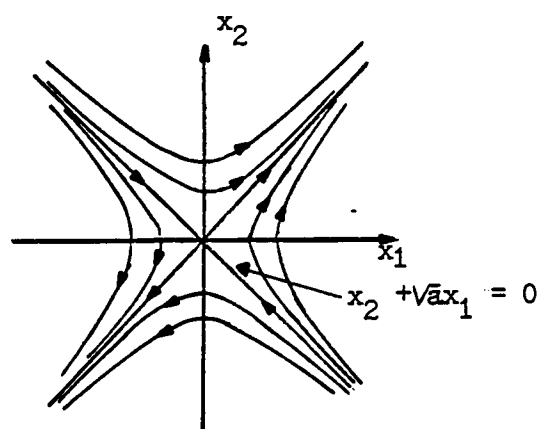
The family of hyperbolic state trajectories of this system are shown in Fig. 2.2b. Neither system described by (2.1) nor (2.2) provides a satisfactory transient response because they are respectively oscillatory and divergent. However, by an appropriate combination of the two controller structures a satisfactory error state trajectory is obtainable.

A good VSCS design has a state trajectory slope which decreases rapidly and is directed toward the origin. These characteristics of a VSCS provides a response which is fast and stable. Examining the error state trajectory of system (2.1), it is seen to decrease rapidly in the first quadrant of the state plane; region $x_1 > 0, x_2 > 0$. System (2.2) has a state trajectory in the fourth quadrant, counting counterclockwise, that has an asymptote of the family of hyperbolic trajectories directed toward the origin. Therefore by stipulating the appropriate controller structure to generate either an elliptic or hyperbolic state trajectory for various regions of the state plane, a system with a satisfactory transient response is obtainable.

To take advantage of the characteristics of both the positive and negative feedback systems, the state plane H can be divided into four pairwise symmetric regions. The regions are defined as follows:



(a)



(b)

Figure 2.2: Positive feedback system.

$$\begin{aligned}
\text{Region I} & : x_1 \geq 0, x_2 + \sqrt{a}x_1 > 0, \\
\text{Region II} & : x_1 < 0, x_2 + \sqrt{a}x_1 \geq 0, \\
\text{Region III} & : x_1 \leq 0, x_2 + \sqrt{a}x_1 < 0, \\
\text{Region IV} & : x_1 > 0, x_2 + \sqrt{a}x_1 \leq 0.
\end{aligned} \tag{2.3}$$

These regions are separated from each other by the straight lines $x_1 = 0$ and $x_2 + \sqrt{a}x_1 = 0$ as shown in Fig. 2.3a [1]. The state trajectory is elliptic in regions I and III, a characteristic of the system with negative feedback. Therefore when the state moves into region I or III, as it crosses the line $x_1 = 0$, it moves along an arc of an ellipse. The state moves along an elliptic arc until it crosses the line $x_2 + \sqrt{a}x_1 = 0$. The state trajectory is hyperbolic in regions II and IV which is characteristic of the system with positive feedback. Therefore when the state crosses the line $x_2 + \sqrt{a}x_1 = 0$, the asymptote of hyperbolic trajectories, and enters either region II or IV, it will move along an arc of a hyperbola. The state will remain on the hyperbolic trajectory until it leaves the region as it crosses the line $x_1 = 0$. Fig. 2.3b shows the overall state trajectory of the system which results from combining characteristics of both the positive and negative feedback systems. Assume the state is located in region I at some initial time $t_0 = 0$. The controller structure of the system is therefore that of negative feedback at time t_0 . The state in region I moves along an arc of an elliptic trajectory. At some time $t_1 > t_0$, the state will hit the line $x_2 + \sqrt{a}x_1 = 0$. At that time the state will enter region IV requiring the system to be switched to that of positive feedback. The state will therefore begin to move along the asymptote line of the family of hyperbolic trajectories, the line $x_2 + \sqrt{a}x_1 = 0$, at time t_1 . The state will continue to move along this line asymptotically

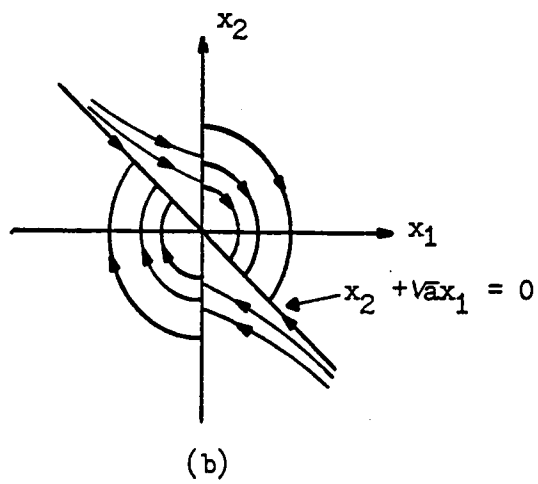
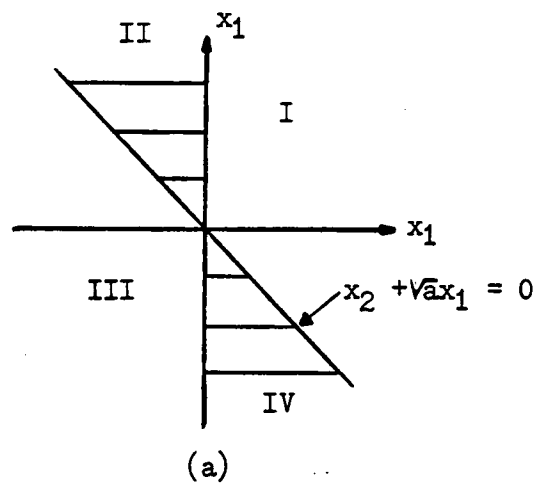


Figure 2.3: Results of combining the positive and negative feedback trajectories

approaching the origin. In the limit, as $t \rightarrow \infty$, $x_1(t) = x_2(t) = 0$.

Next consider the case where the state is in region IV at the initial time $t_0 = 0$. Region IV requires the controller structure of the system to be switched to positive feedback. Therefore the state will move along an arc of a hyperbolic trajectory crossing the ordinate axis at some time $t_2 > t_0$. At time t_2 negative feedback will be switched in, and the state will then begin to move along an arc of an elliptic trajectory. The state will move along an elliptic trajectory until it reaches the line $x_2 + \sqrt{a}x_1 = 0$. At this time the controller structure of the system instantaneously switches to that of positive feedback. The state will therefore move along the asymptotic line $x_2 + \sqrt{a}x_1 = 0$ towards the origin. Because the state crossed the ordinate axis, the sign of the error term x_1 changed, indicating that overshoot occurred in the system response. Sometimes, this is considered undesirable.

The behavior of the system when the state is in either region II or III is completely symmetric to that in region IV or I respectively.

From the previous example it is seen that by using two unsatisfactory controller structures, a globally asymptotically stable system can be obtained. In this example only one instance of overshoot can occur regardless of the initial condition of the state. The state trajectories of this VSCS example are rather idealized. In reality, the state may deviate from the asymptote because of noise or switching delays in the controller structure. The question of whether a VSCS is realizable when imperfect switching conditions exists will be discussed in the next section.

2.2 Sliding Mode in VSCS

In the previous example the VSCS was based on the premise that instantaneous switching of the controller structure is possible. The consequence of imperfect switching on a VSCS will be examined in this section. A special investigation into this question was first done by Emel'yanov in the nineteen fifties, which resulted in discovering an important advantage of a VSCS. This advantage is the synthesis of a system which is insensitive to external disturbances and to variations of the plant parameters within wide ranges. This realization is the original motivation for the further development of VSCS. In the following example the main results of Emel'yanov's work will be presented [1].

In this example t_1 is defined as the time at which the state, initially in region I, reaches the asymptote line. At this time the controller structure of the system changes from negative feedback to positive feedback. In general, a switching line can be represented by $\sigma = x_2 + cx_1 = 0$ where c is a positive constant. It should be noted that the ordinate, $x_1 = 0$, is also a switching line. The state plane is divided by the two switching lines into four regions as shown in Fig. 2.4a. The boundary of these regions are as follows:

$$\begin{aligned} \text{Region I} & : x_1 \geq 0, x_2 + cx_1 > 0, \\ \text{Region II} & : x_1 < 0, x_2 + cx_1 \geq 0, \\ \text{Region III} & : x_1 \leq 0, x_2 + cx_1 < 0, \\ \text{Region IV} & : x_1 > 0, x_2 + cx_1 \leq 0, \end{aligned} \tag{2.4}$$

where c is a positive constant. The error state trajectory of the system is elliptic in regions I and III, and hyperbolic in regions II and IV. This system

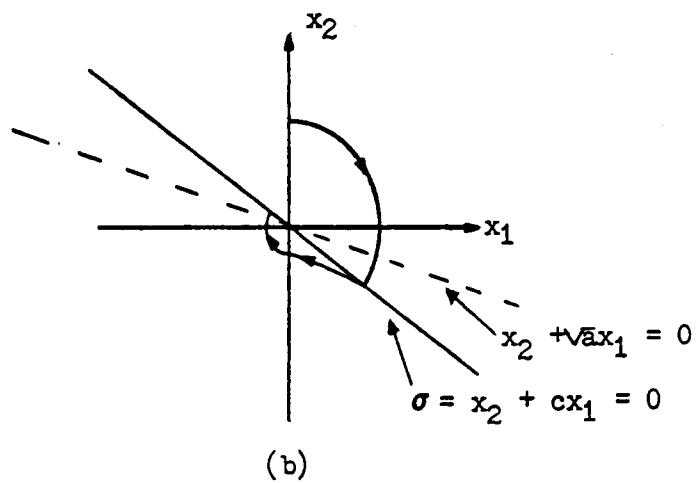
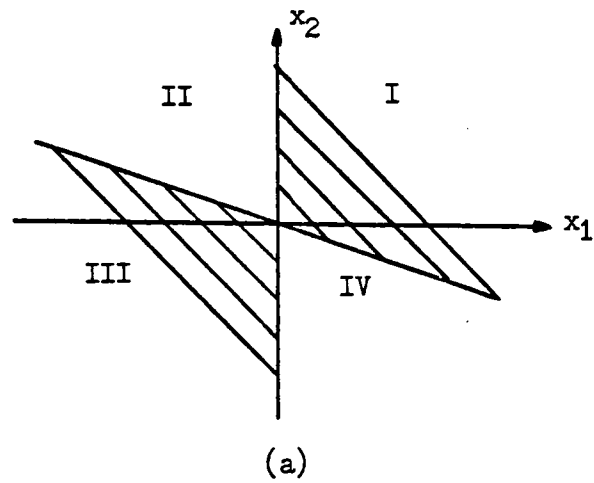


Figure 2.4: Segmented state plane and state trajectory when $c > \sqrt{a}$.

is described by equations (2.1) and (2.2)

Consider the case $c > \sqrt{a}$ which places the asymptote line $x_2 + \sqrt{a}x_1 = 0$ for the hyperbolic trajectories of the state plane completely in the regions I and III as shown in Fig. 2.4b. The state, initially in region I, moves along an elliptic trajectory. When the state hits the line $\sigma = 0$ its trajectory will change instantly to that of a hyperbolic trajectory. At this moment the state will begin to move along a hyperbolic trajectory away from the asymptote line as shown in Fig. 2.4b. Then, the state will reach the ordinate axis, when again the state trajectory is switched to the elliptic form. This process repeats itself, resulting in the switching back and forth in the controller structure of the system. This process is damped.

Suppose at time t_0 the state is located on the upper half of the ordinate axis as shown in Fig. 2.5a. If $t_1 > t_0$ is the first time at which the state crosses the line $\sigma = 0$, the absolute value of the magnitude of the ordinate, x_2 , at time t_1 is less than the value at time t_0 . Now let t_2 be the first time after t_1 at which the state crosses the lower half of the ordinate axis. The absolute value of the ordinate, x_2 , at time t_2 is less than at time t_1 . This decreasing behavior of x_2 is due to the shape of the elliptic and hyperbolic trajectories. Therefore the system is globally asymptotically stable.

Next consider the case where $0 < c < \sqrt{a}$. With this condition the motion of the trajectory of the variable structure control system will occur in a sliding mode. Motion in a sliding mode can be characterized by the switching back and forth phenomenon of the controller structure of the system at infinite frequency. As a result, the state undergoes infinitesimal oscillations about the switching line $\sigma = 0$, and "slides" toward the origin along the switching line as shown in Fig. 2.5b. Suppose the state is initially in region I at some time $t_0 > 0$. The state will then move along an elliptic trajectory and, at

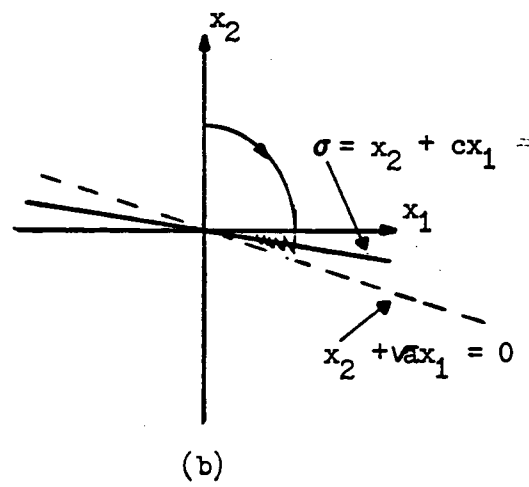
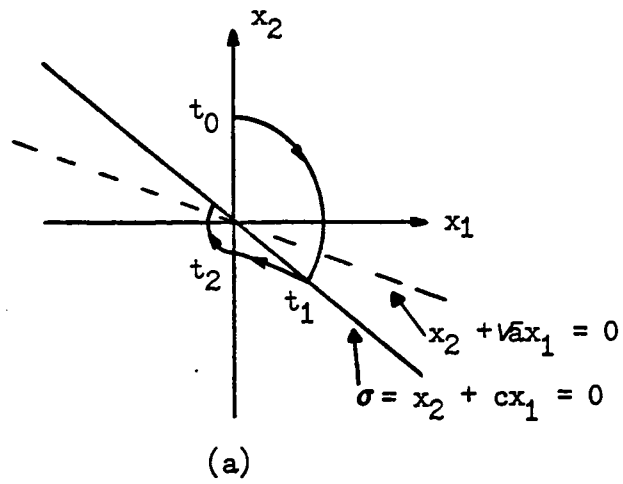


Figure 2.5: State trajectories for $c > a$ and $0 < c < \sqrt{a}$.

some time $t_1 > t_0$, hit the line $\sigma = x_2 + cx_1 = 0$. It will then begin to move along an hyperbolic trajectory which point into region I. The state thus quickly leaves region IV crossing the $\sigma = 0$ line therefore returning to region I. Since the state trajectories of region I are elliptic and directed toward the $\sigma = 0$ line, the state again returns to region IV where again it is returned to region I. This oscillation about the $\sigma = 0$ line continues as shown in Fig. 2.6a.

The state can be conjectured to oscillate with infinitesimal amplitude and infinitely high frequency. The motion of the state trajectory during the sliding mode along the switching line can be approximated by the equation

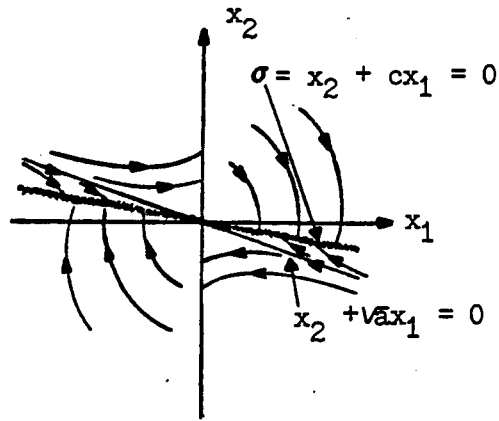
$$\dot{x}_1(t) + cx_1(t) = 0 \quad (2.5)$$

which has the solution

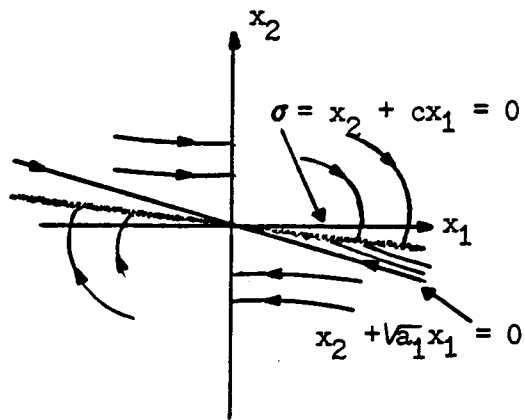
$$x_1(t) = x_1(t_0)e^{-c(t-t_1)} \quad t > t_1. \quad (2.6)$$

Thus the motion of the VSCS is described by the set of differential equations (2.1), (2.2), and (2.5).

The sliding mode is an important characteristic because it is independent of the changes in the plant parameters and of external disturbances. In the specific case just considered the VSCS is independent of the gain a . Formally this is shown by equation (2.6) which does not involve the parameter a explicitly. However since c is defined relative to the plant parameter a , the sliding mode is a consequence of the plant parameter. For example, in the case of the sliding mode the switching line $\sigma = 0$ requires $0 < c < \sqrt{a}$ which gives the state trajectory as shown in Fig. 2.6a. If the value of a is decreased



(a)



(b)

Figure 2.6: Changing the slope of the state trajectory.

to a new value a_1 the result is a change in slope of the state trajectories as shown in Fig. 2.6b. This new trajectory results in a transient response of longer duration prior to the hitting of the switching line $\sigma = 0$. Another result is the increased amount of time spent in the sliding mode once the state hits the switching line $\sigma = 0$.

CHAPTER 3

COMPARISON OF A SIMPLE VSCS TO A LINEAR CONTROL SYSTEM

3.1 Compensation of A Pure Integral Plant

In this chapter two respective control system design strategies will be implemented for both a purely integral plant, and a plant with inertial and linear friction. The two control strategies that will be used with the plants are a simple VSCS and a linear control system. In order to find out which control strategy offers the best improvement in the plant, the robustness and transient responses of the two control strategies will be compared.

The first plant to be considered is

$$G(s) = \frac{1}{s^2}$$

which is part of a negative feedback loop. The differential equations describing the system with a step input are

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) + r(t), \end{cases} \quad (3.1)$$

where $r(t)$ is a step input and $x_1(t)$ is the output of the system. It should be noted that the above state differential equations are not the error state equations given in chapter 2. The reason is because the state equations

given in chapter 2 are based on the input $r(t)$ being zero. When the input is zero, the general system state equations are identical to the error state equations. However in actual practice a non-zero input $r(t)$ is usually the case. Therefore equation (3.1) represents the general state equations of the negative feedback system. A family of error state trajectories for the negative feedback system is obtained by plotting the error, $E(t) = r(t) - x_1(t)$, versus the derivative of the error, $\dot{E}(t) = -\dot{x}_1(t)$, as shown in Fig. 3.1. With positive feedback the state differential equations are

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_1(t) - r(t). \end{cases} \quad (3.2)$$

The error state trajectory of the system (3.2) is a family of hyperbolic trajectories as shown in Fig. (3.2).

For the VSCS design a control signal must be used which allows switching between a positive and negative feedback controller structure. For this purely integral system the controller switching must be defined in order to obtain the error state trajectory characteristics as defined in section 2.2 chapter 2 for a sliding mode to occur. The controller signal therefore can be defined as

$$u = B\psi \quad (3.3)$$

where

$$\psi = \begin{cases} 1 & \sigma E(t) > 0 \\ -1 & \sigma E(t) < 0, \end{cases} \quad (3.4)$$

B is the controller gain, and σ is defined as the switching line [1]. The

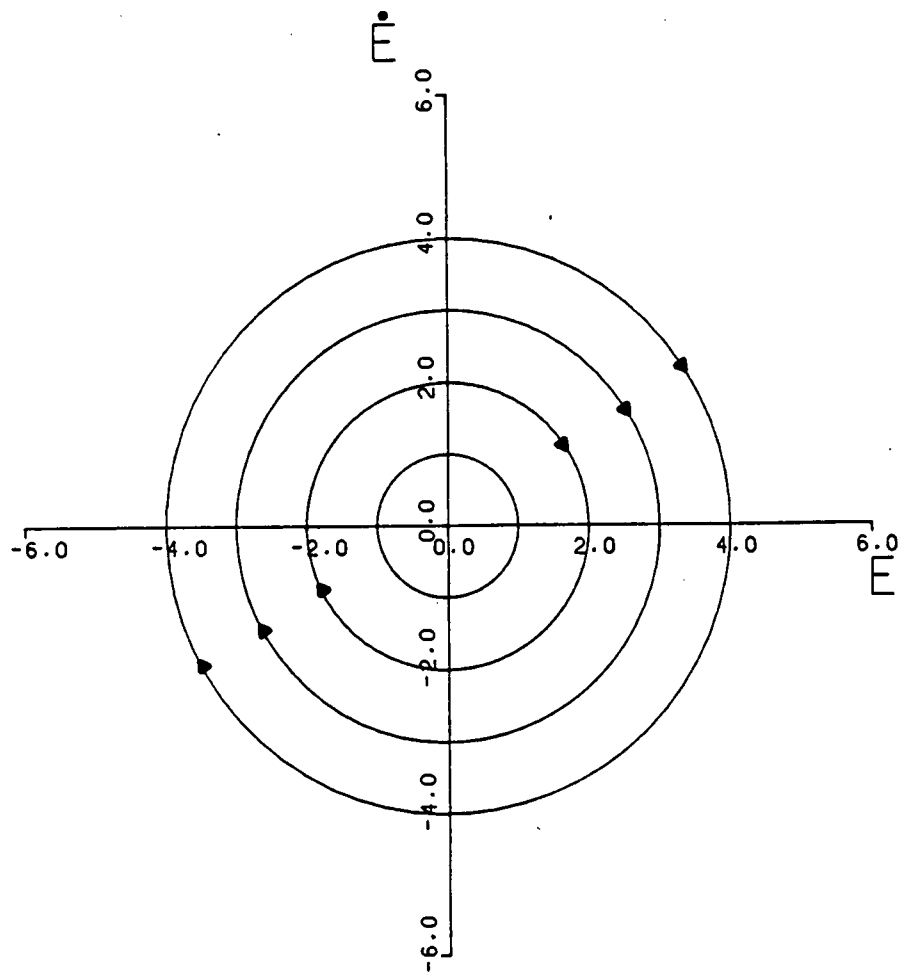


Figure 3.1: Error state trajectory of system with negative feedback.

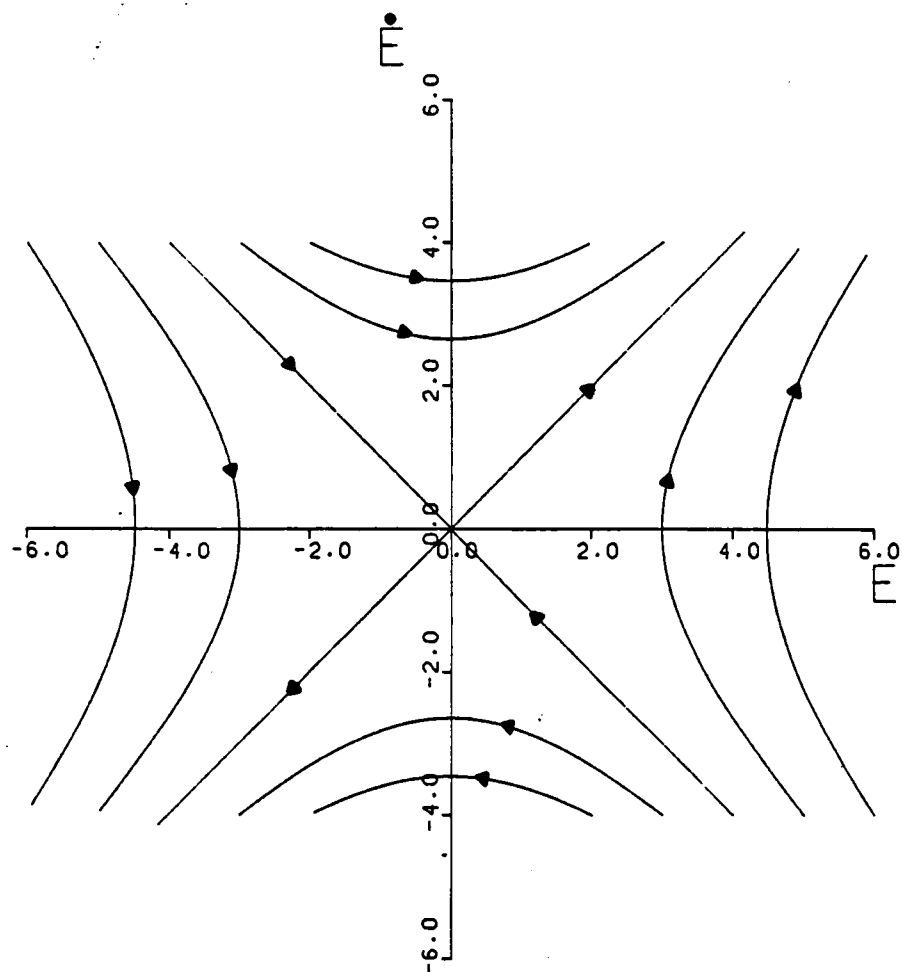


Figure 3.2: Error state trajectory of system with positive feedback.

controller gain B is included in the control signal to provide, possibly, additional improvement in the VSCS transient response. The block diagram of the required VSCS is shown in Fig. 3.3.

The switching line for this system error state trajectory must have a slope less than the asymptote line, $\dot{E}(t) + E(t)$ which is directed toward the origin, of the family of hyperbolas. The switching line along which the sliding mode will occur for this VSCS is thus

$$\sigma = \dot{E}(t) + cE(t) = 0 \quad (3.5)$$

where $0 < c < 1$. Using a digital computer simulation verification of the defined range of c for a sliding mode to exist was made. From this simulation it was found that the closer the value of c is to 1, the faster is the motion in the sliding mode and therefore the faster the overall system transient response. The VSCS transient response for different values of c is shown in Fig. 3.4. Also it was found that when $c = 1$ the error state trajectory did not move along the asymptote line. The reason for this is the lack of numerical resolution of the digital simulation of the error state trajectory. Since the digital simulation offers only a limited accuracy in obtaining the motion of the state, the state is never on the asymptote. However it was found that if the error state is initially on the asymptote line, the error state will move along the asymptote. Also it was observed that when the error state is initially on the asymptote line error state motion was found to be very slow. This slow motion along the asymptote line is contrary to what is intuitively expected.

The value of c for this VSCS is selected to be .99. The switching line for

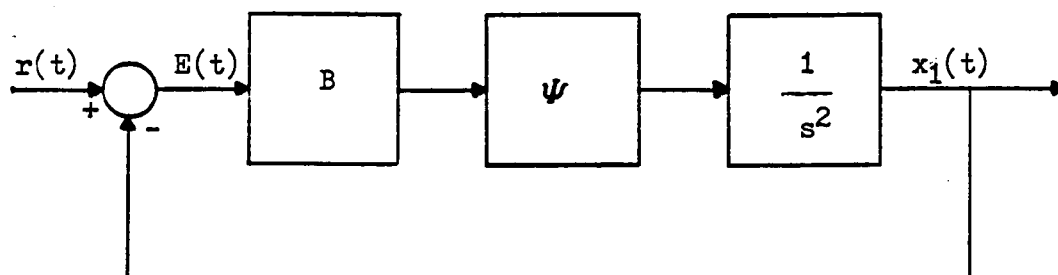


Figure 3.3: Block diagram of system compensated by variable structure.

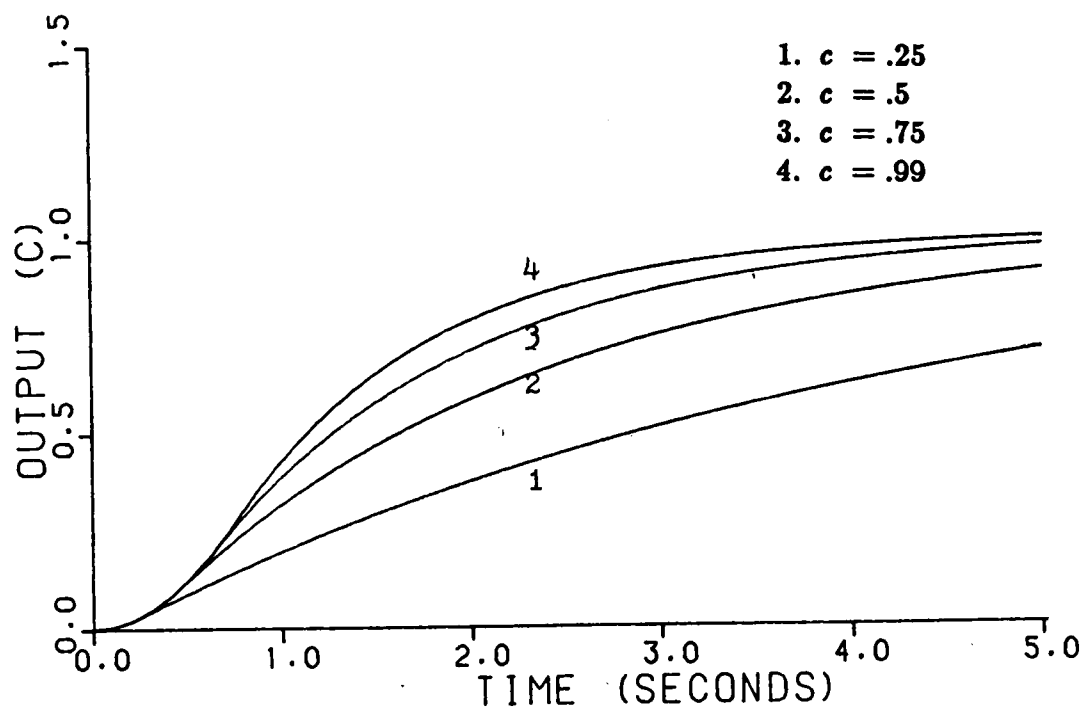


Figure 3.4: Transient response of VSCS for different values of c .

this VSCS is thus selected to be

$$\sigma = \dot{E}(t) + .99E(t) = 0$$

which ensures a sliding mode. This switching line also ensures the fastest possible transient response considering the limitation on the range of c . The error state trajectory of the VSCS for an initial condition of $\dot{E}(0) = 0$ and $E(0) = 1$ is shown in Fig. 3.5. The transient responses of the uncompensated system and the VSCS with a controller gain $B = 1$ are shown in Fig. 3.6.

In this investigation it was found that increasing the controller gain B results in even a faster VSCS transient response. The increase in controller gain however is found to be restricted because the system transient response begins to deteriorate and eventually becomes unstable after a certain value of controller gain. The improvement in transient response occurs because the increased controller gain decreases the time required for the error state to hit the switching line. The transient response for this VSCS with controller gain $B = 1000$ is shown in Fig. 3.6. The controller gain $B = 1000$ is found to be approximately the largest possible gain obtainable before the VSCS transient response starts to become unstable.

Linear Compensator Design

The linear control system design technique that will be used in this investigation is the frequency domain technique utilizing Bode plots. The design characteristics of this system will be overshoot less than 4 percent, and a steady state error less than 1 percent for a unit step input. Also in the linear control system design consideration must be given to obtaining a system which is somewhat insensitive to parameter variations and external disturbances.

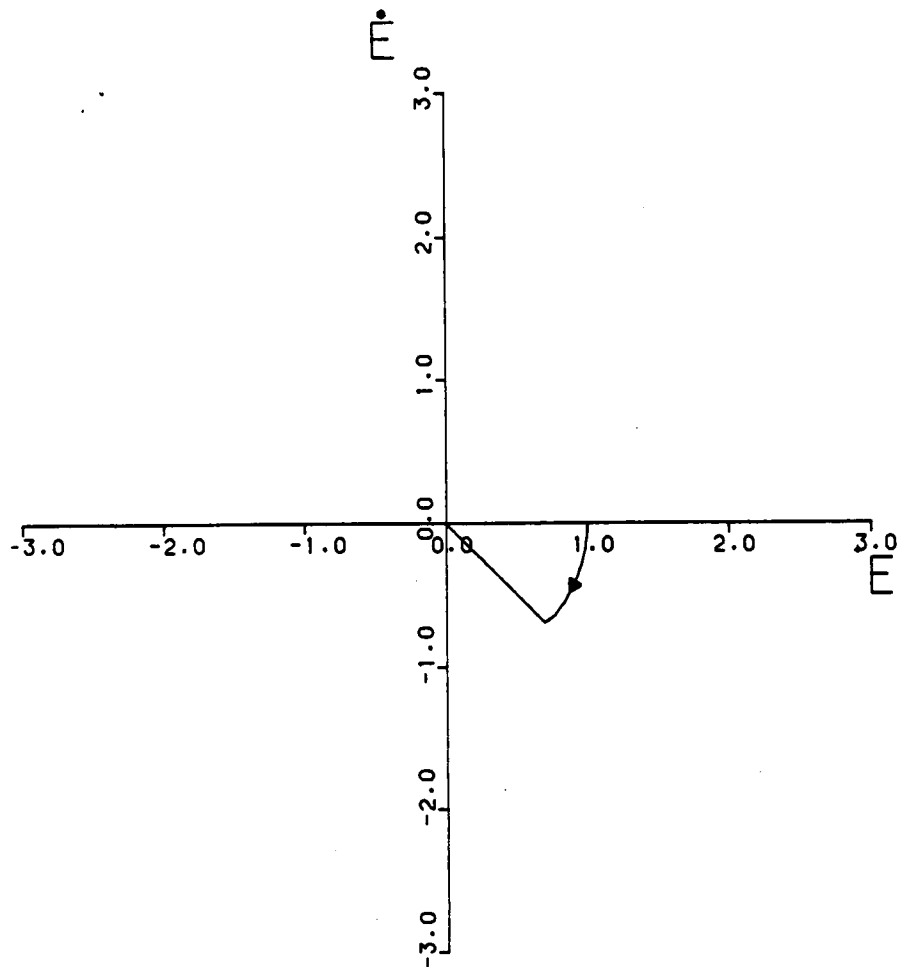


Figure 3.5: Error state trajectory of the system with a variable structure control system(VSCS) having a controller gain $B=1$.

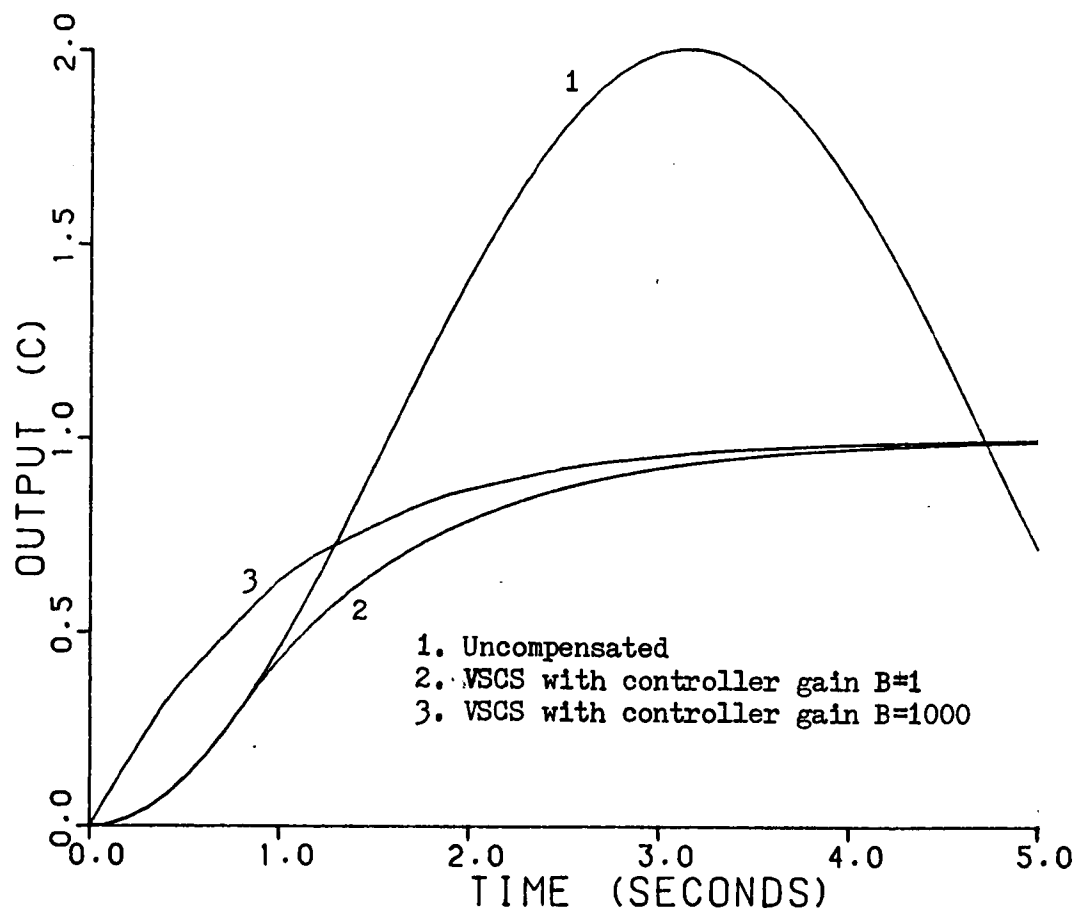


Figure 3.6: Transient response of uncompensated system and compensated system with different controller gains.

First an analytical examination of what controller specifications must be made to obtain a robust linear control system design is made[2]. The sensitivity of a closed loop system with unity feedback of the form shown in Fig. 3.7 is defined as a percentage change in

$$\frac{C(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)}$$

divided by the percentage change in the process transfer function G , or

$$\begin{aligned} S &= \frac{\frac{\partial \left(\frac{C(s)}{R(s)} \right)}{\frac{C(s)}{R(s)}}}{\frac{\partial (KG(s))}{KG(s)}} \\ &= \frac{1}{1 + KG(s)} \end{aligned} \quad (3.6)$$

Therefore from (3.6), the sensitivity to parameter variations decreases as the gain K increases when $G(s)$ is a type 0 system. Also when $G(s)$ is a type 1 or higher system and K is some finite value, the system sensitivity decreases as time increases.

A system with a significantly large gain K and a compensator which changes the forward transfer function to a type 1 or higher system must be required to obtain a system insensitive to parameter variations.

Next the requirement for a control design to obtain good disturbance rejection for a system of the form shown in Fig.3.8 will be considered. The output of the system in Fig.3.8 is

$$C(s) = \frac{KR(s)G(s)}{1 + KG(s)} + \frac{D(s)}{1 + KG(s)}. \quad (3.7)$$

Thus the disturbance $d(t)$, which is assumed constant for this analysis, has

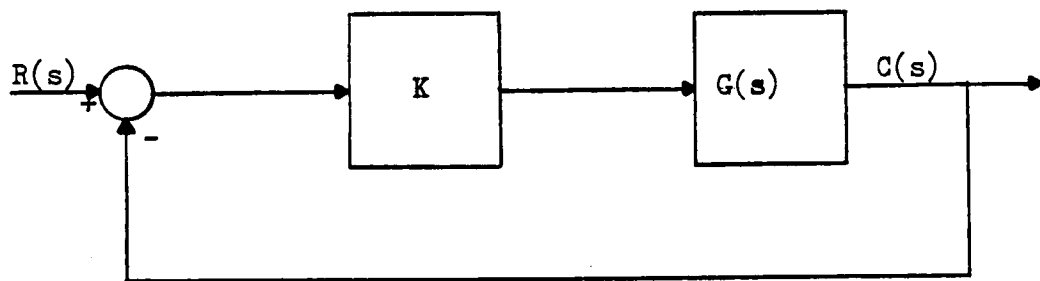


Figure 3.7: Unity feedback control system.

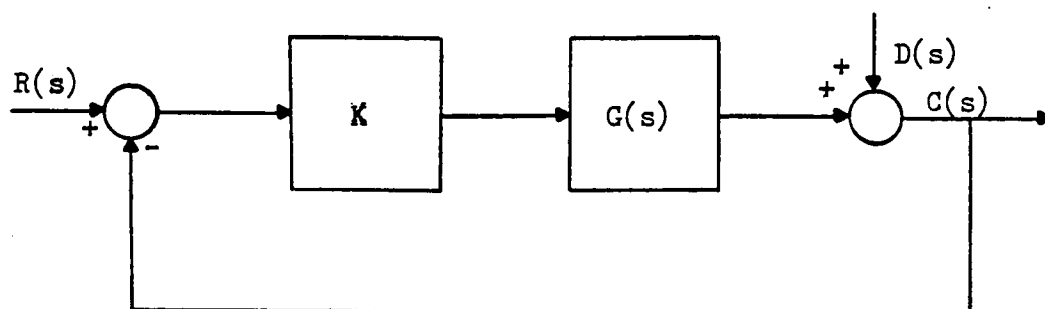


Figure 3.8: Unity feedback system with disturbance.

little effect when the gain K approaches infinity. Also as time increases when $G(s)$ is a type 1 or higher system and K is a finite value, the system disturbance rejection improves. It thus appears that the same considerations to obtain a system insensitive to parameter variations are also required to reduce the effects of disturbance in the system.

The first step in this linear control system design is to examine the phase shift of the plant's Bode plot. The phase shift of the plant is found to be -180° for all values of frequency. For this system design a system overshoot close to 0 *percent* will be attempted. To obtain this overshoot will require a lead compensator system design yielding a 90° phase margin according to a second order approximation. Obtaining a 90° phase margin theoretically requires the ratio between the zero and pole of the lead compensator to be infinity. Considering the practical limitations of the system design the compensated system will be selected to have a 85° phase margin. This phase margin will thus result in approximately a .3 *percent* overshoot. Since the phase shift of the plant is a constant -180° for all frequency values, the new gain crossover frequency can be easily selected to provide a fast compensated system response.

The new gain crossover frequency for the lead compensated system is selected to be 10 *radians/second*. The phase margin for this new gain crossover frequency will be 85° as previously stated. The resulting lead compensator using the frequency domain technique outlined in [3] is

$$4.366 \left(\frac{1 + 2.29038s}{1 + .00437s} \right)$$

where 4.366 is the gain change in the Bode plot required to obtain the desired system design. Changing the compensator into the standard form of a

transfer function for steady state analysis results in the block diagram shown in Fig. 3.9.

The steady state error of this compensated system with a step input is obviously seen to be zero as required for a type two system. The compensated open loop transfer function has a large gain and is a type two system which indicates the closed loop system should be somewhat robust.

The transient response for the linearly compensated system is shown in Fig. 3.10 along with the response for the simple variable structure compensated system (SVSCS) previously discussed. Comparing the transient response of the lead compensated system and the SVSCS shows that the compensated lead system is much faster. The overshoot of the lead compensated system is *3.56 percent* versus *0 percent* for the fastest SVSCS. The *2 percent* settling time of the fastest SVSCS is *3.8 seconds* versus *2.0 seconds* for the lead compensated network.

From this comparison it was found that the linear control system design offers more flexibility in selecting system specifications versus the SVSCS. Also it was found that certain initial conditions of the SVSCS design allow a *0 percent* overshoot system transient response to exist. However for the linear control system design, it appears practically impossible to obtain a *0 percent* overshoot transient response. The analysis of the robustness of the SVSCS and the linearly compensated system (LCS) will be examined next.

3.2 Robustness of the Compensated Integral Plant

The purpose of this section is to examine the effects of variation in plant parameters and step disturbances for the SVSCS and the linearly compen-

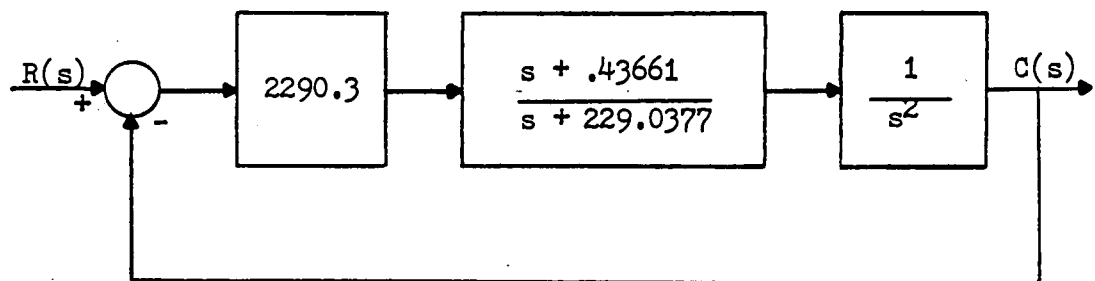


Figure 3.9: Lead compensated pure integral plant.

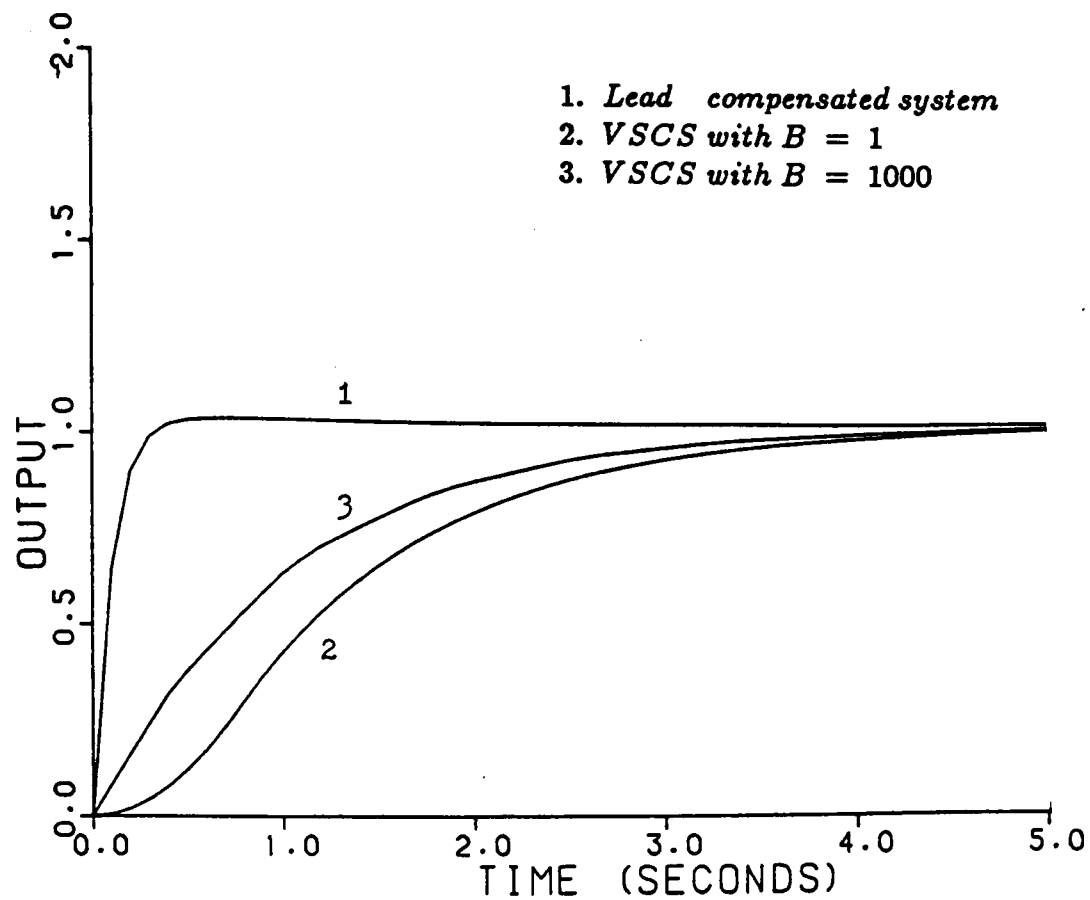


Figure 3.10: Transient response of the lead compensated system and the SVSCS.

sated system(LCS) of the previous section. From these results it will be determined whether the SVSCS or the linearly compensated system offers the most robust design.

Variations of Plant Parameter

First the effects of varying the plant parameter of the SVSCS and the linearly compensated system will be examined. According to the VSCS design concepts the sliding mode offers the desirable characteristics of insensitivity to plant parameter and external disturbances. Therefore in this analysis the change in plant parameters and application of disturbances will occur during the sliding mode of the SVSCS. The LCS parameter variations and external disturbances will be applied at times similar to those for the corresponding SVSCS.

The block diagram showing the plant parameter of the uncompensated pure integral plant is shown in Fig. 3.11. The plant parameter will be varied over the range

$$.001 \leq a_1(t) \leq 500,$$

where $a_1(t)$ is a time varying plant parameter. In this investigation the plant parameter's original value is 1.0 in accordance with the previous design considerations of section 3.1. As mentioned earlier the change in parameter will begin during the time interval in which the sliding mode exist. For this analysis the SVSCS with controller gain $B=1000$ will be used. The change in parameter will begin at .4 seconds since the sliding mode starts at .3 seconds. The transient responses for the SVSCS and the LCS with the plant parameter being varied are shown respectively in Fig. 3.12 and Fig. 3.13. Note that each of the figures show three transient responses each having a different value of the plant parameter.

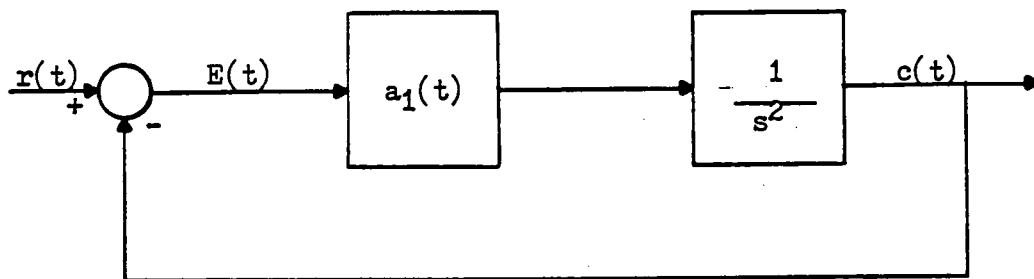


Figure 3.11: Pure integral plant with time varying plant parameter.

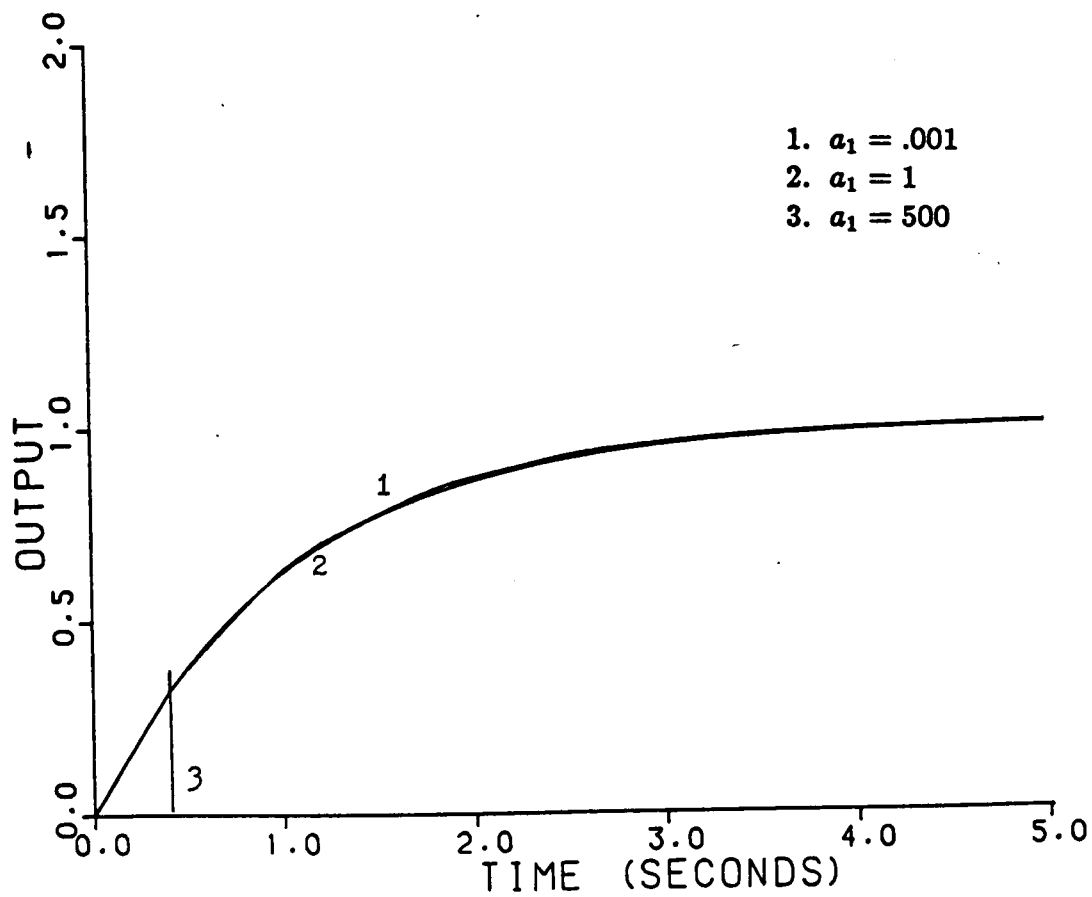


Figure 3.12: Parameter variation of a pure integral plant with a SVSCS.

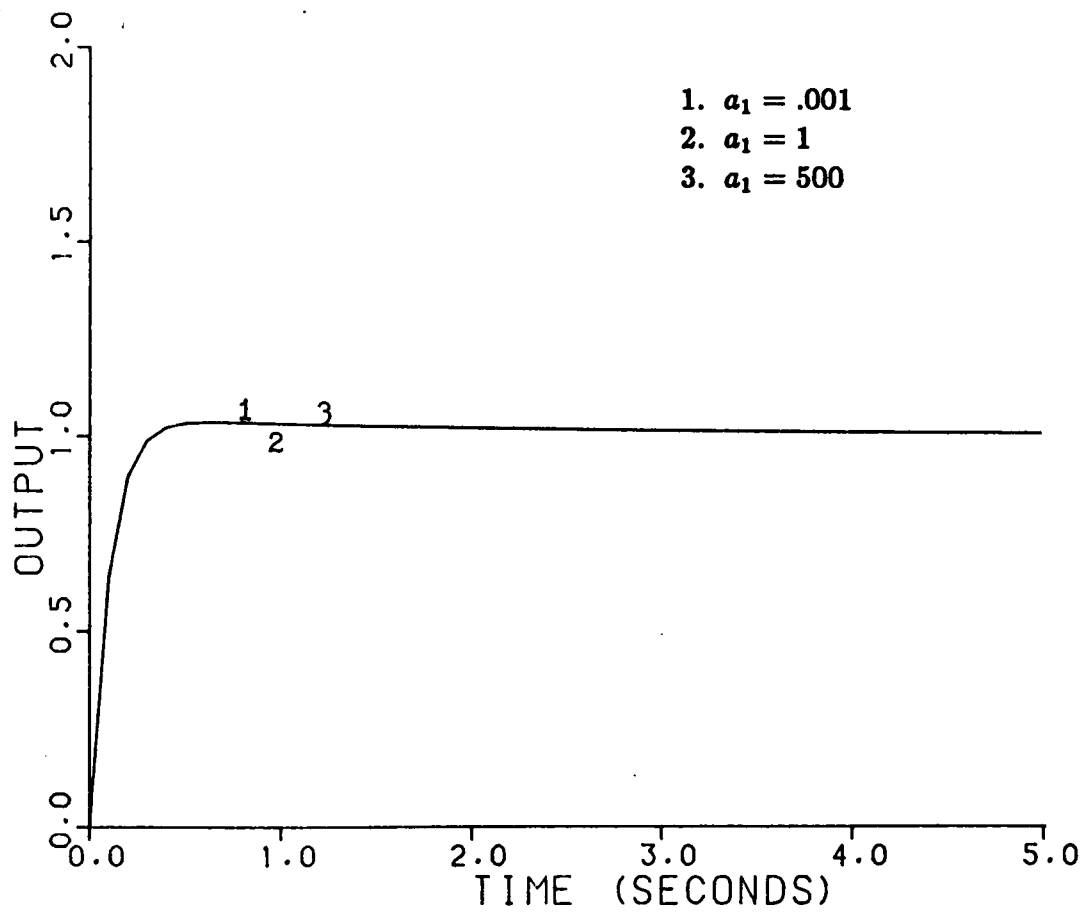


Figure 3.13: Parameter variation of a pure integral plant with a LCS.

The SVSCS transient response when the plant parameter is a value of 500 results in a system output which becomes unstable and rapidly approaches an ever increasing negative value. For the value of .001 the plant parameter causes practically no change in the transient response. The linearly compensated system appears to be completely insensitive to the changes in plant parameters. It appears that the LCS provides much greater insensitivity to parameter variations than the simplified VSCS.

Step Disturbance

Now the effects of disturbance on the SVSCS and the LCS of the pure integral plant will be examined. In this analysis it is assumed that the system will be subjected to step disturbances of two different magnitudes. The transient response showing the disturbance is shown in Fig. 3.14. The disturbance enters the system as shown previously. The transient response of the SVSCS with step disturbances is shown in Fig. 3.15. The transient response of the LCS with step disturbances is shown in Fig. 3.16.

From the transient response it appears that the SVSCS offers no immunity to disturbances. This lack of immunity to disturbances is a undesirable characteristic of the SVSCS. The LCS also is observed to be sensitive to disturbances just as the SVSCS. Even though the LCS is sensitive to disturbances a system which offer good disturbance rejection can be designed. This can be accomplished by significantly increasing the system gain and obtaining a faster system response. Therefore it appears that obtaining a LCS which has good disturbance rejection properties is a iterative design procedure.

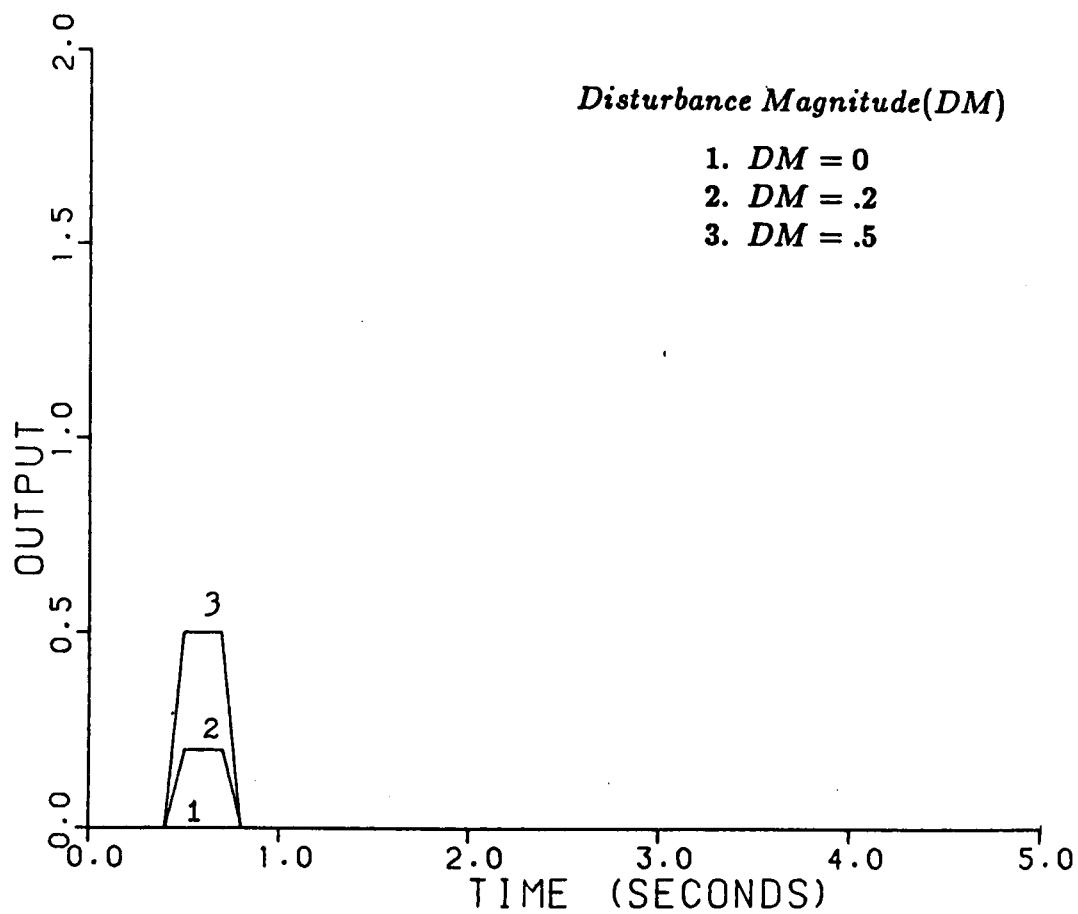


Figure 3.14: Transient response of step disturbances.

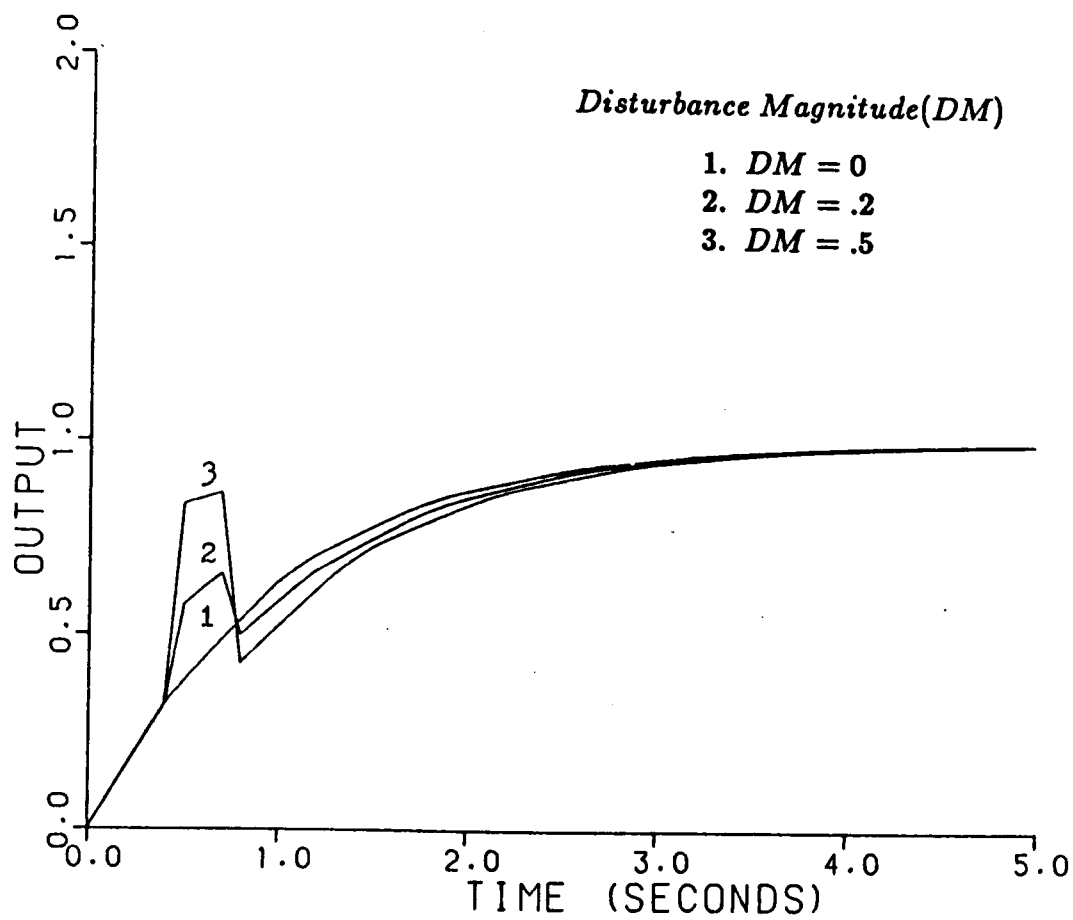


Figure 3.15: SVSCS integral plant with step disturbances.

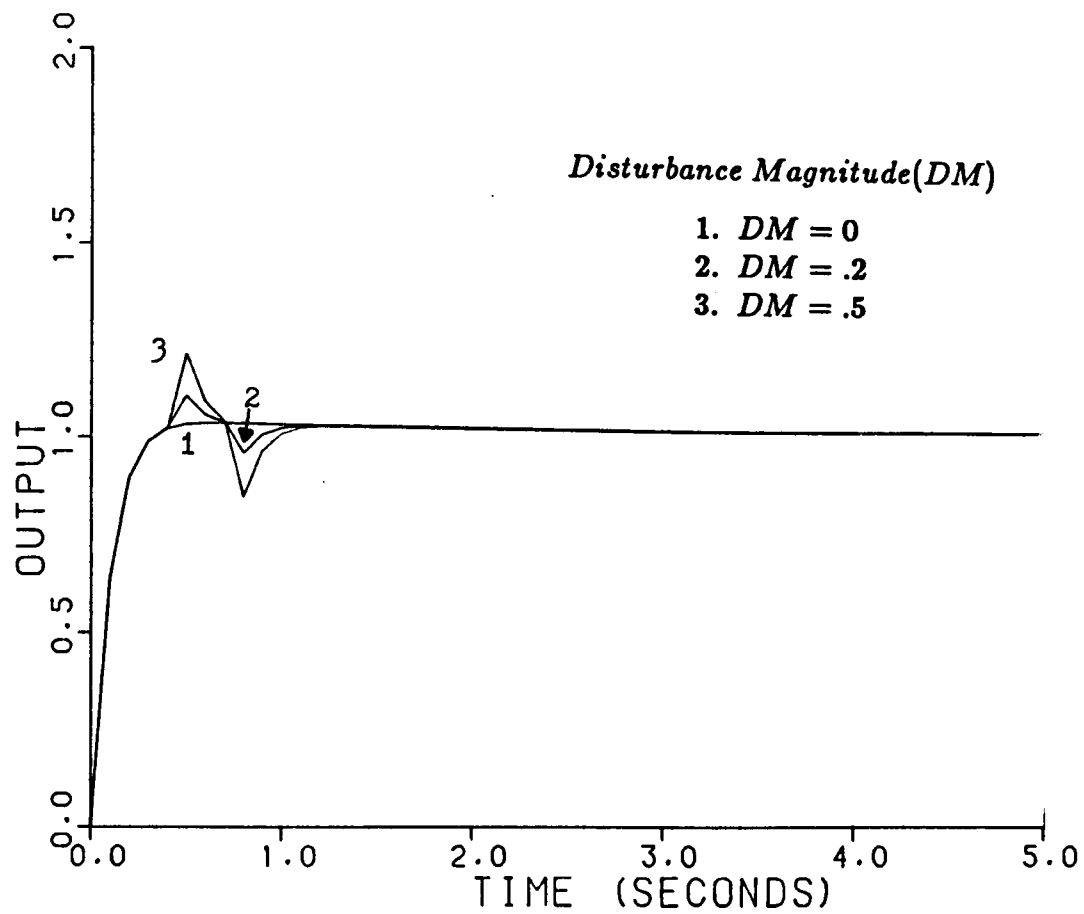


Figure 3.16: LCS integral plant with step disturbances.

3.3 Compensation of A Plant with an Integral and Linear Friction

The plant to be considered now is

$$G(s) = \frac{1}{s(s+1)}$$

which is part of a negative feedback loop. The state differential equations describing the system are

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) - x_2(t) + r(t) \end{cases} \quad (3.8)$$

where $r(t)$ is a step input and $x_1(t)$ is the output of the system. The block diagram of the system with negative feedback is shown in Fig. 3.17. The error state trajectory for this negative feedback system is a family of asymptotically stable curves as shown in Fig. 3.18. The state differential equations for the plant with positive feedback are

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) - x_2(t) - r(t). \end{cases} \quad (3.9)$$

The block diagram of the system with positive feedback is shown in Fig. 3.19. The family of trajectories describing the error state trajectories of the positive feedback system is shown in Fig. 3.20.

The control signal described by (3.3) to (3.5) is the general control strategy for a VSCS design and therefore will again be used in this system design.

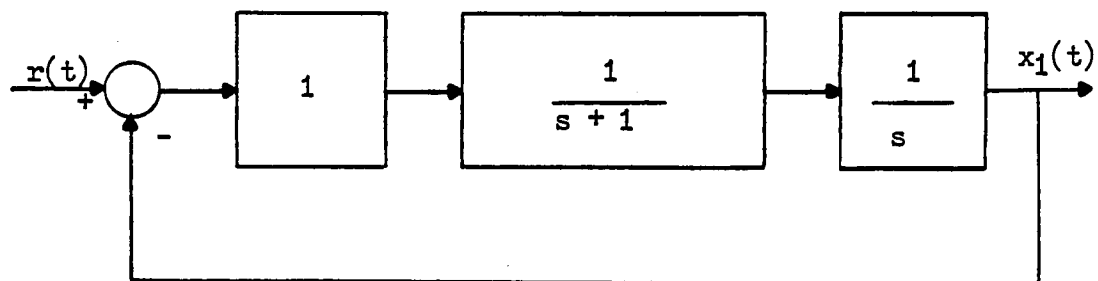


Figure 3.17: Block diagram of uncompensated system with negative feedback.

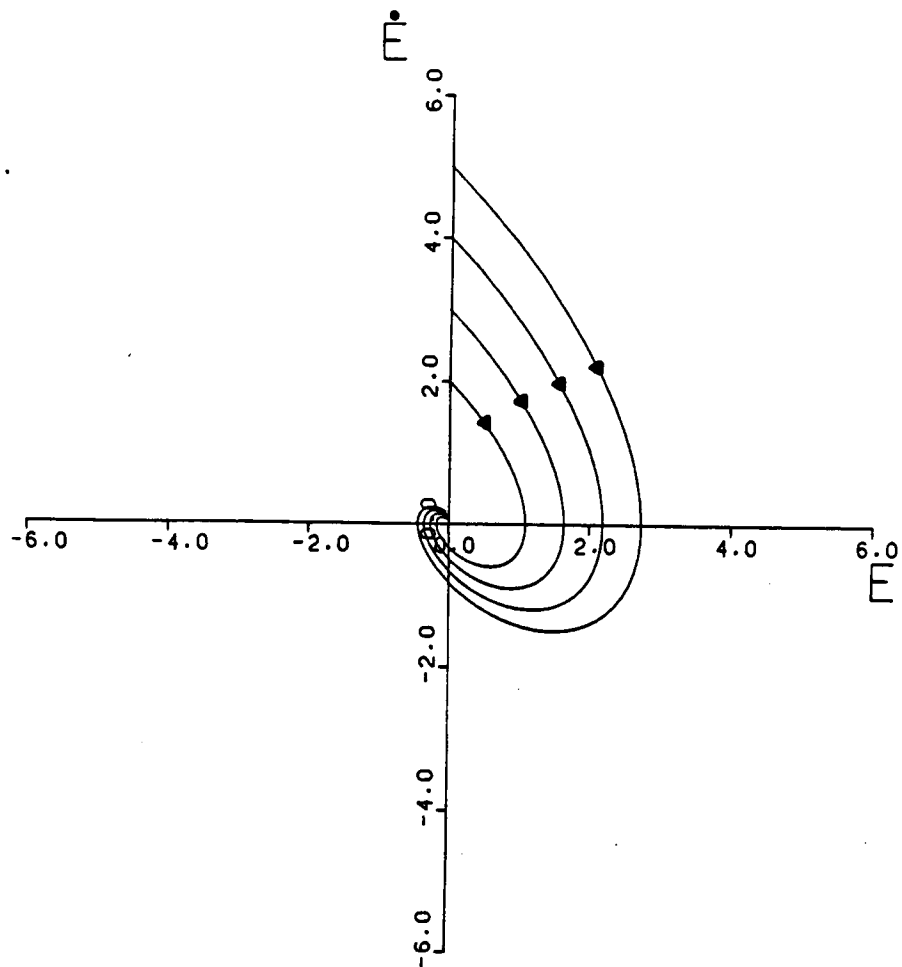


Figure 3.18: Error state trajectory of negative feedback system.

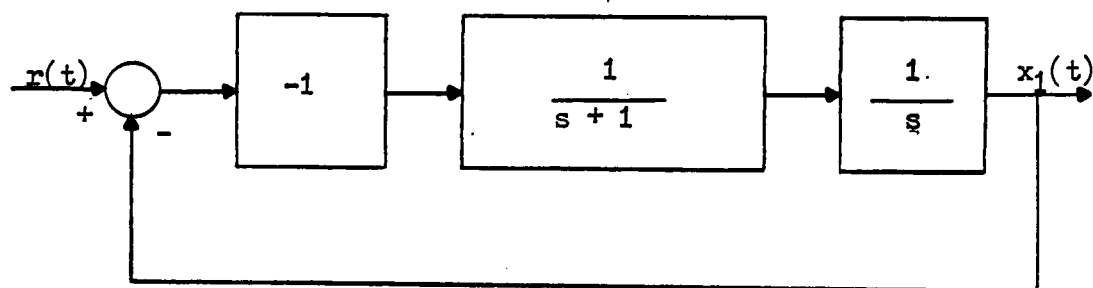


Figure 3.19: Block diagram of uncompensated system with positive feedback.

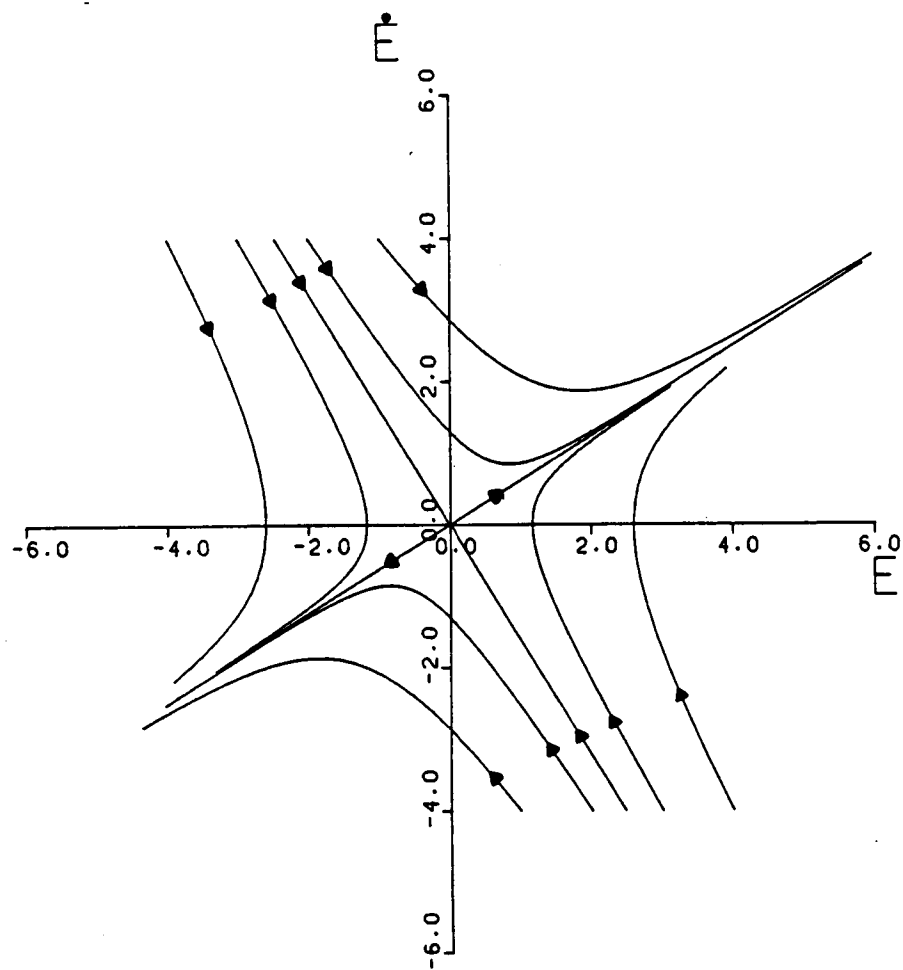


Figure 3.20: Error state trajectory of the positive feedback system.

Examining Fig. 3.20 the asymptote directed toward the origin is found approximately to be the line

$$\sigma = \dot{E}(t) + 1.61625E(t) = 0.$$

The required switching line in order for a sliding mode to exist is therefore

$$\sigma = \dot{E}(t) + cE(t) = 0$$

where $0 < c < 1.61625$. The block diagram of the simple VSCS is shown in Fig. 3.21. The transient response of the VSCS for different values of c for the switching line is shown in Fig. 3.22. The VSCS transient response is seen to have a faster response time as the value of c increases. This result is expected from examining equation (2.6).

The value of c for this VSCS design is selected to be 1.61, therefore the switching line is

$$\sigma = \dot{E}(t) + 1.61E(t) = 0.$$

The error state trajectory of the VSCS with controller parameters $c = 1.61$ and $B = 1$ is shown in Fig. 3.23.

The transient response of the uncompensated system and the VSCS is shown in Fig. 3.24. In order to further increase the speed of the VSCS transient response the controller gain is increased from $B = 1$ to $B = 1000$. The transient response of the VSCS with increased controller gain is shown in Fig. 3.24. The maximum controller gain for this simple VSCS is about 1000 before the transient response begins to deteriorate. Also it was found that with increased controller gain in the range from 25 to 1000 this simple VSCS transient responses are nearly the same.

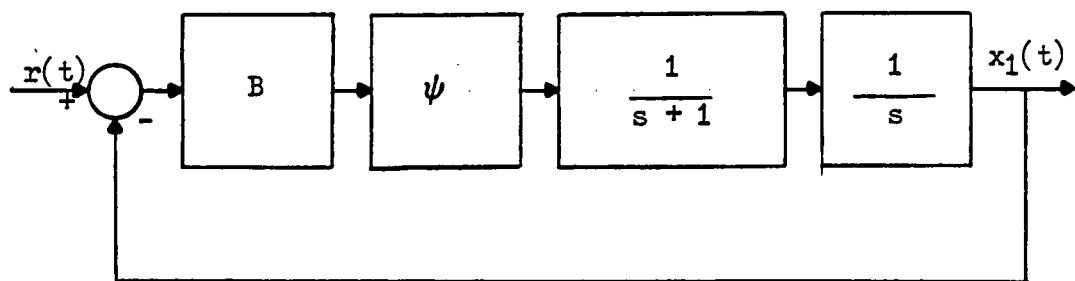


Figure 3.21: Block diagram of VSCS.

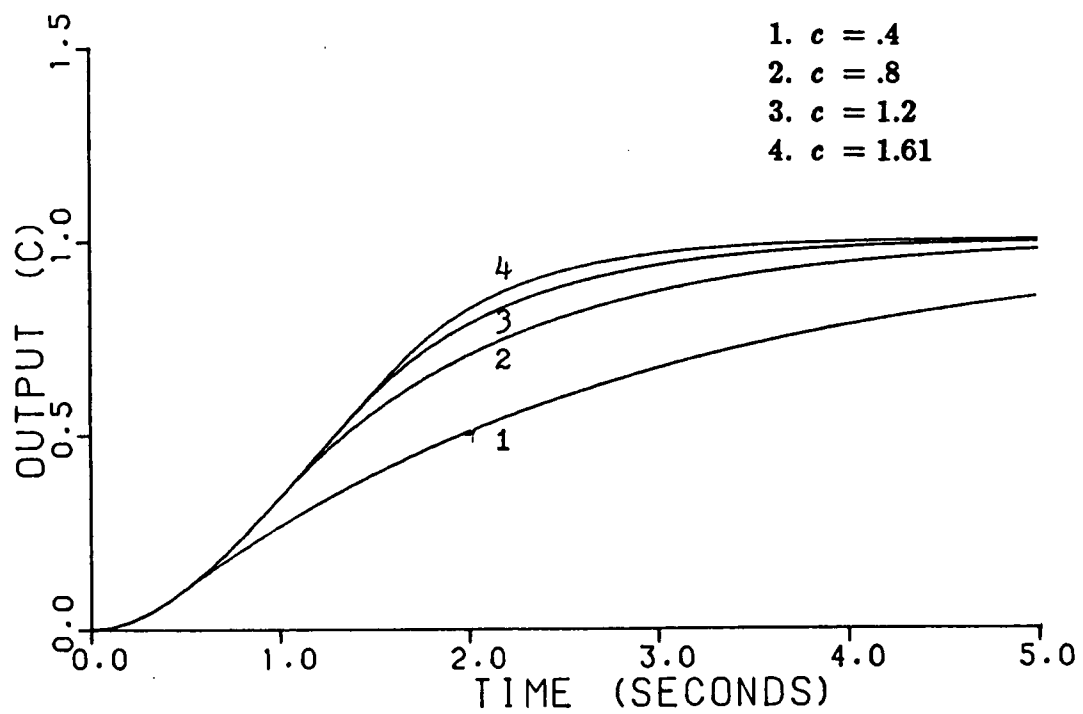


Figure 3.22: Transient response of VSCS using different values of c .

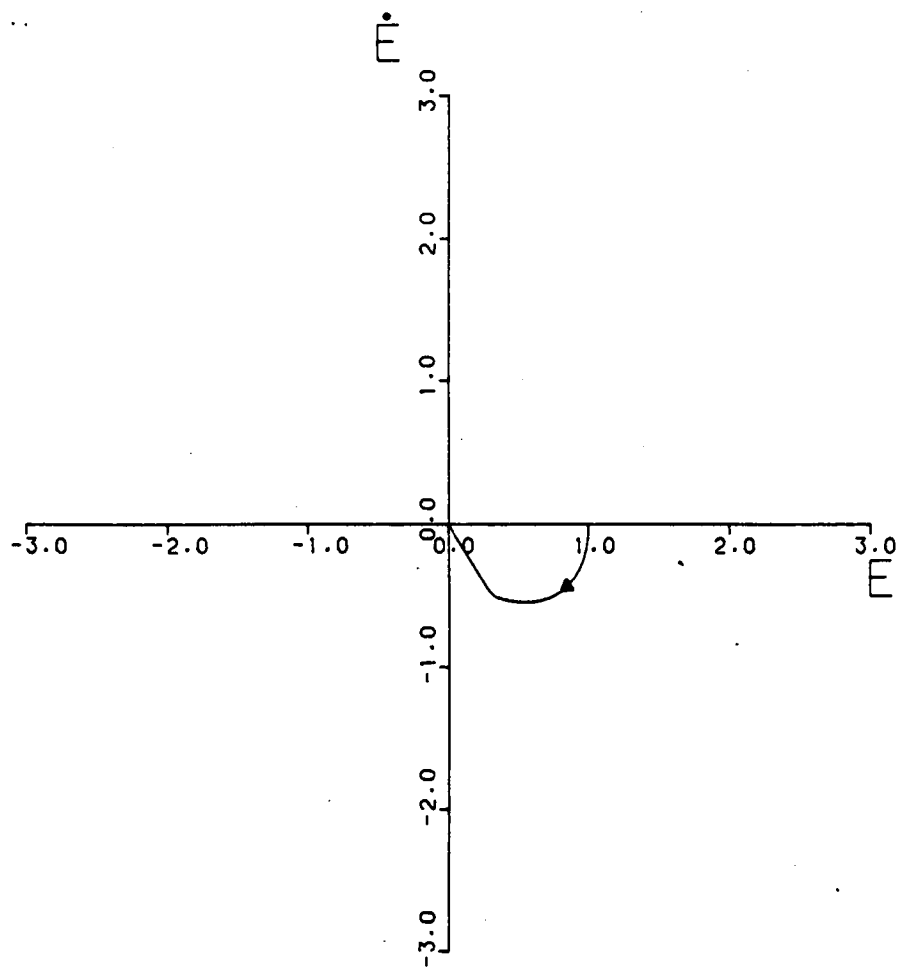


Figure 3.23: Error state trajectory of the VSCS with $c=1.61$ and $B=1$.

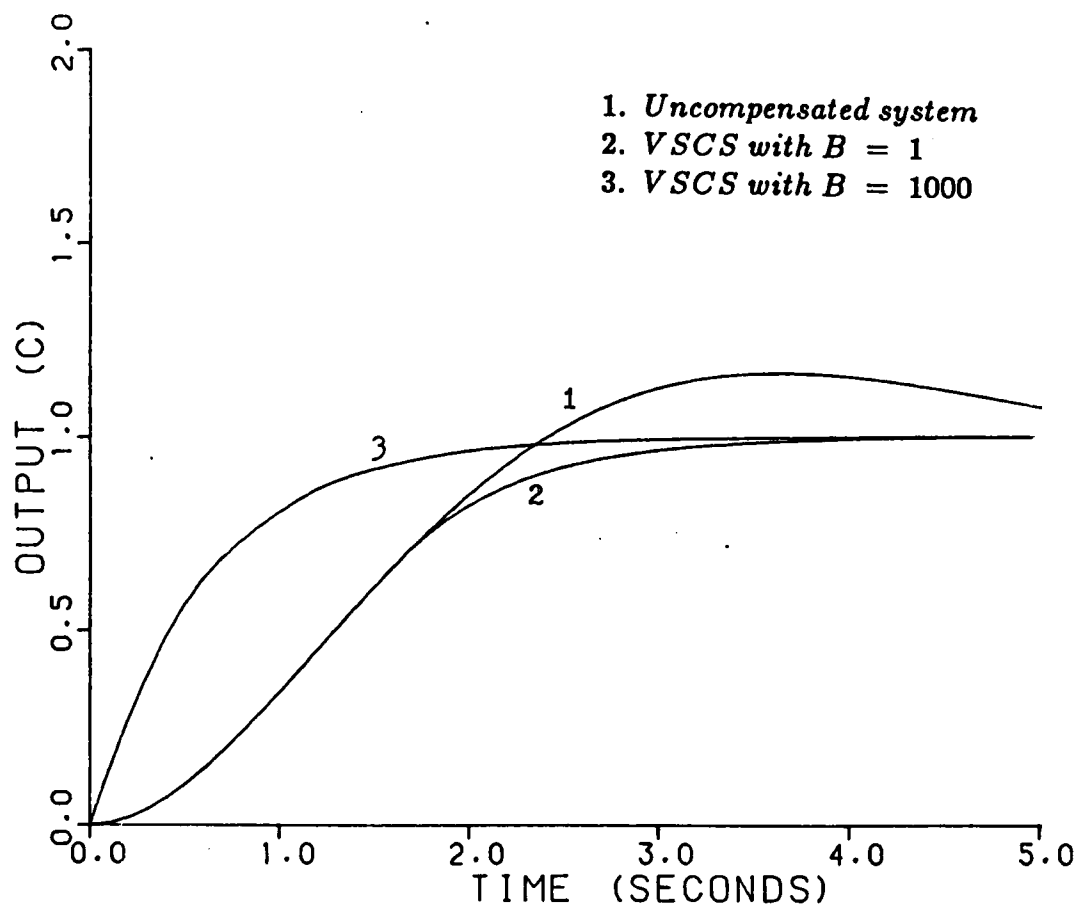


Figure 3.24: Transient response of uncompensated system and VSCS design.

Linear Control System Design

Again using the frequency domain design technique a linear compensator will be designed for the inertial and integral plant. The design characteristics will be an overshoot less than 4 percent and a steady state error less than 1 percent for a unit step input. A 2 percent settling time faster than the previously discussed simple VSCS is desired. This will require a 2 percent settling time faster than 2.3 seconds.

Examining the Bode plot characteristics of the plant, the phase margin is found to be 45° at a gain crossover frequency of 1 *radian/second*. This system phase margin provides a system overshoot of about 16 percent which does not meet the required design specification. The phase shift of the system at higher frequencies vary from -170.5° at 6 *radians/second* to -177.1° at 20 *radians/second*. Considering the behavior of the system at higher frequencies, a fast system response with minimal overshoot can be obtained by using a lead compensator at a high frequency.

The new gain crossover frequency is selected to be 20 *radians/second* with a phase margin of 85° . The phase margin at the new gain crossover frequency without the lead compensator is 3° . The lead compensator for this system design will therefore only have to provide 82° phase margin at the new gain crossover frequency. In designing this lead compensator the system gain is increased to avoid an impractical ratio between the pole and zero of the lead compensator. The new system gain is thus found to be 28.0056. The frequency domain technique outlined in [3] is used to obtain the lead compensator

$$\frac{1 + .71503s}{1 + .0034963s}$$

Changing the compensator into the standard form of a transfer function for

steady state analysis results in the block diagram shown in Fig. 3.25.

The steady state error for this system design is zero. Considering the requirements for a robust system design, this linearly compensated system LCS appears to be a robust design. The robustness of the VSCS will be examined in more detail in chapter 4.

The transient response for the linearly compensated system is shown in Fig. 3.26. The transient response of the lead compensated system has an overshoot of 1.36 *percent* versus 0 *percent* overshoot of the fastest simple VSCS. The 2 *percent* settling time of the fastest simple VSCS is 2.3 *seconds* versus .15 *seconds* for the lead compensated system.

3.4 Robustness of the Compensated Inertial and Integral Plant

Variation of Plant Parameters

The plant parameters that will be varied are shown in Fig. 3.27. The sliding mode for the VSCS starts at .3 *seconds*. Therefore the parameter variations and the input of disturbance for the VSCS and the LCS will begin at .4 *seconds*. The plant parameters for this analysis will be varied over the range

$$.001 \leq a_1(t) \leq 500$$

and

$$.001 \leq a_2(t) \leq 500$$

where $a_1(t)$ and $a_2(t)$ are plant parameters.

The transient responses of the simple VSCS and the LCS for time varying plant parameters are shown in Fig. 3.28 and Fig. 3.29 respectively. Again

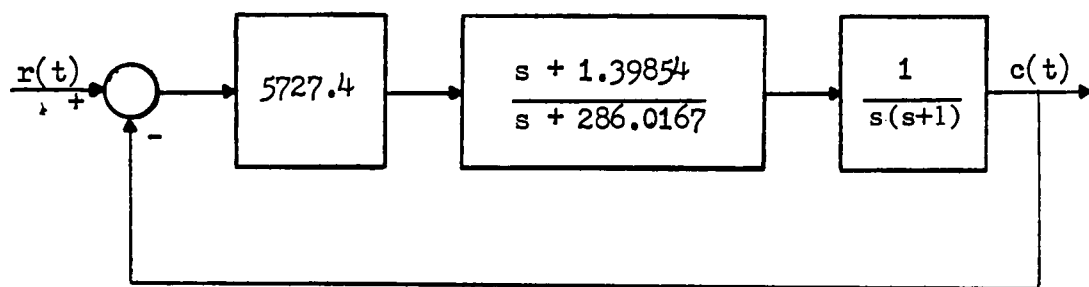


Figure 3.25: Linearly compensated system block diagram.

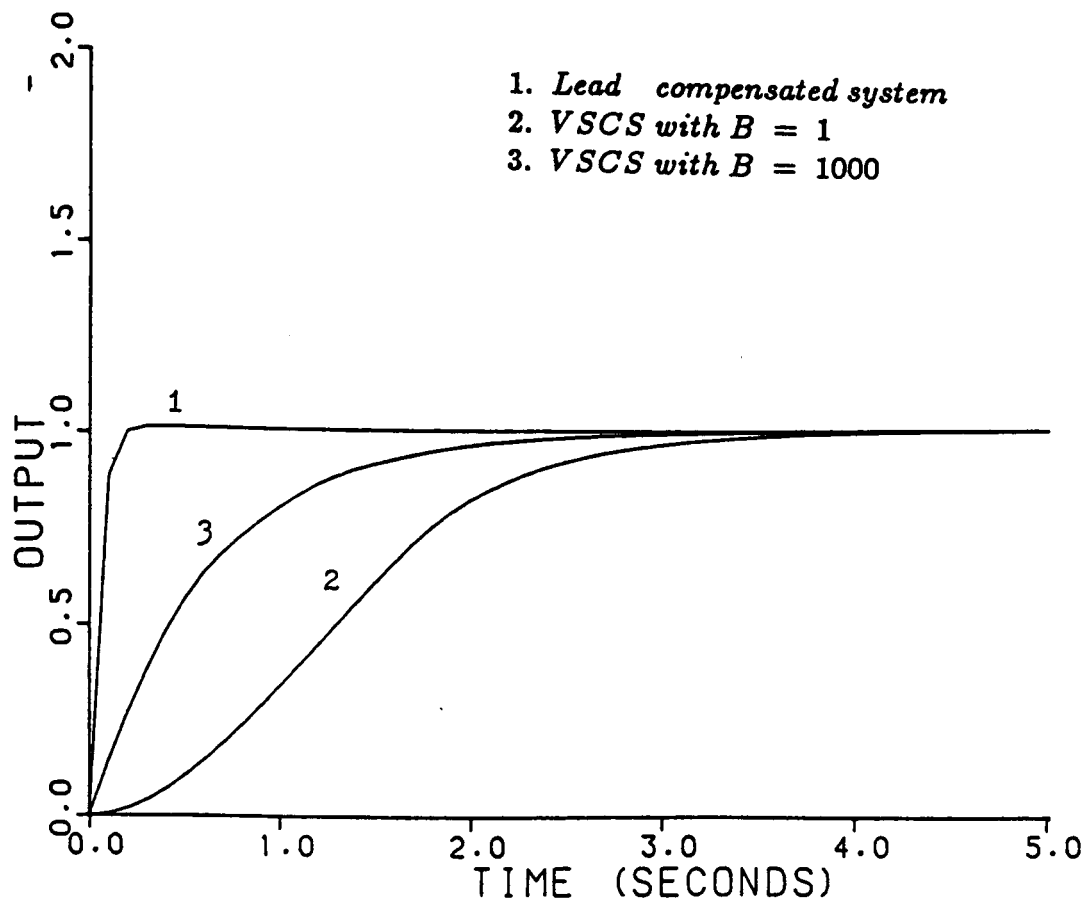


Figure 3.26: Responses of linearly the compensated system and VSCS.

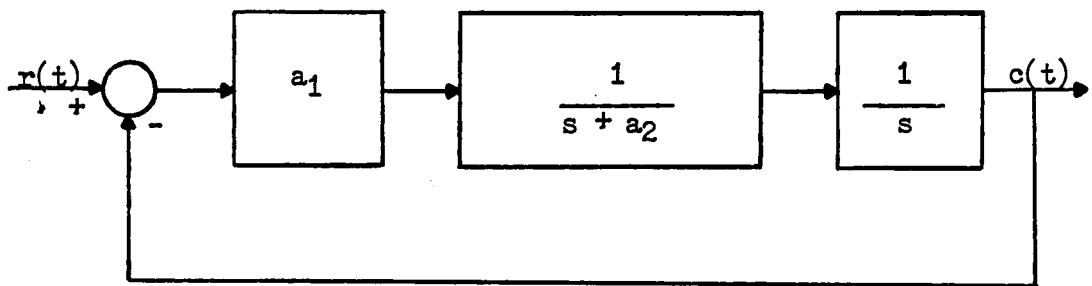


Figure 3.27: Uncompensated inertial and integral plant.

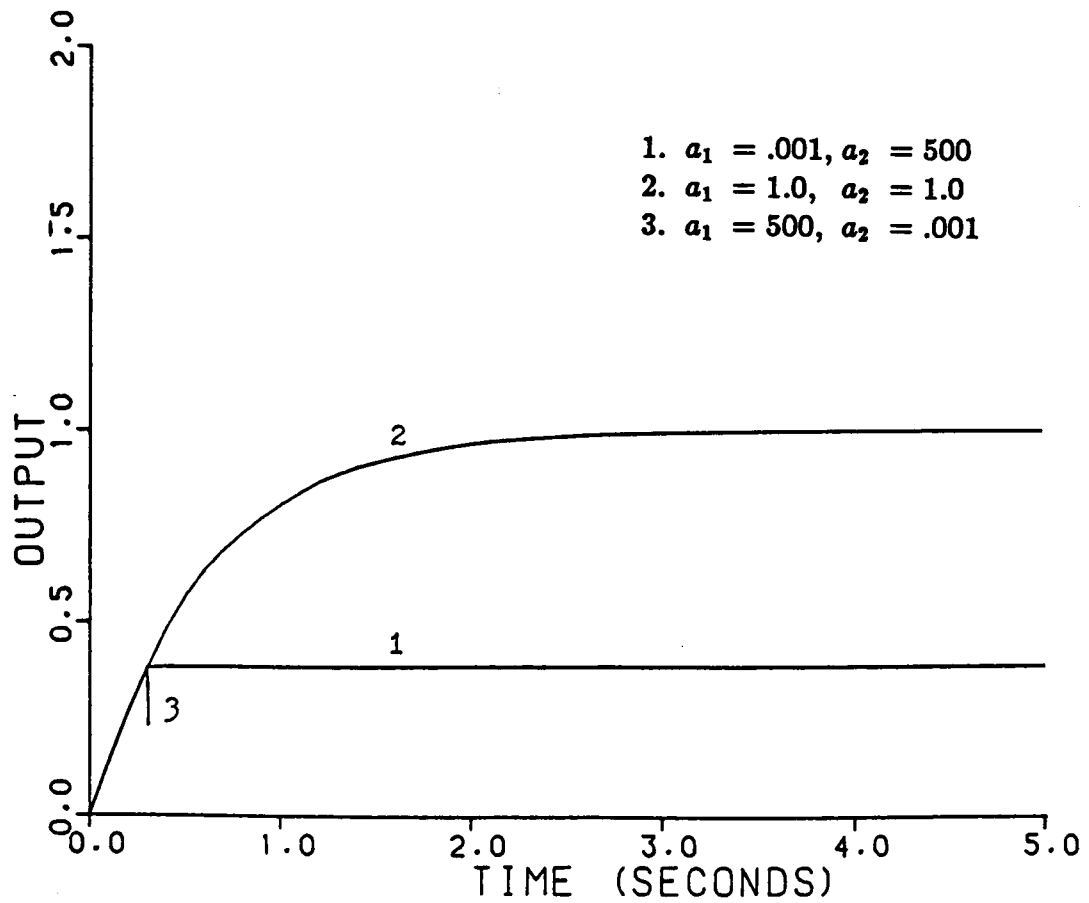


Figure 3.28: Time varying plant with VSCS.

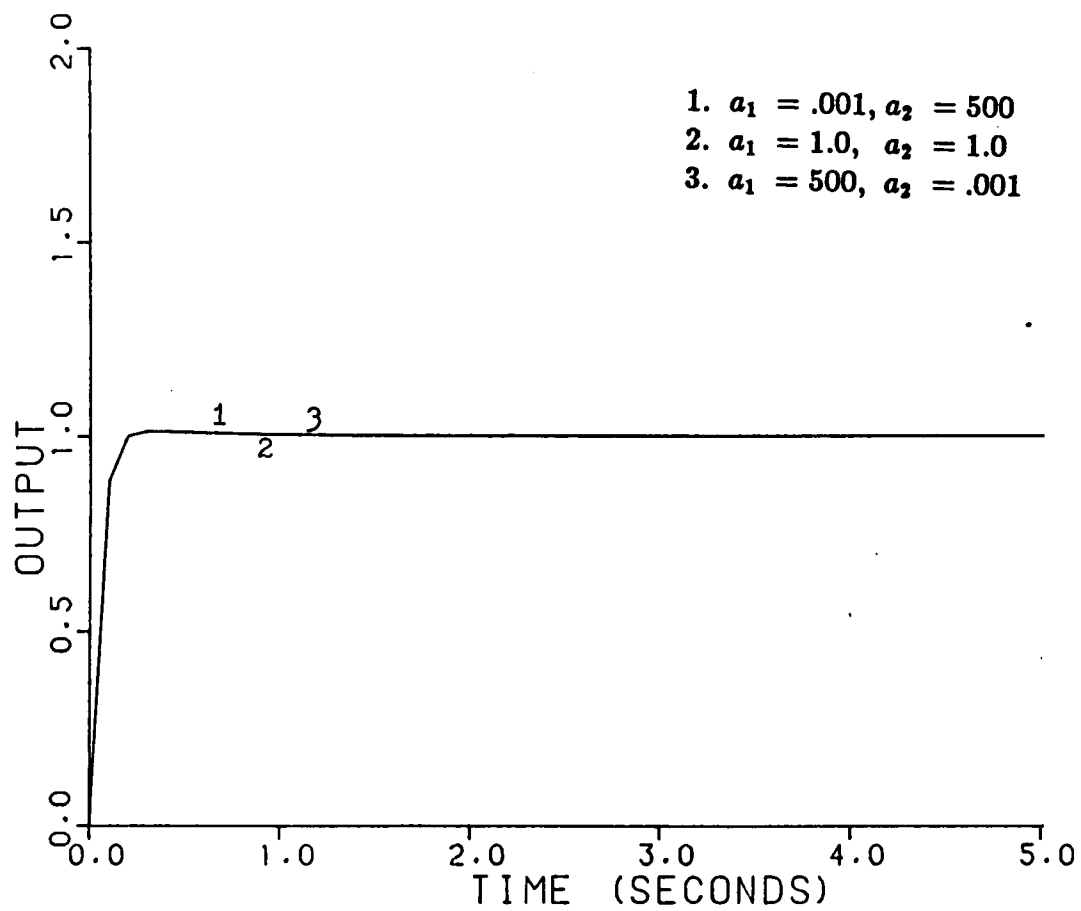


Figure 3.29: Linearly compensated time varying plant.

the transient response for the simple VSCS is not completely invariant to the complete range of parameter variations. The linearly compensated system however demonstrates extremely good insensitivity to the parameter variations.

Step Disturbance

Using the same method as presented in section 3.1 the effect of step disturbances on the simple variable structure control system and the linearly compensated system is examined. Fig. 3.30 shows that again the simple VSCS is greatly affected by step disturbances of different magnitudes. The LCS transient responses of Fig. 3.31 is only slightly affected by the disturbance. This fairly good disturbance rejection quality of this LCS easily surpasses the simple VSCS in performance.

3.5 Conclusion

This chapter presented a simplified method of obtaining a simple VSCS. This method relied on the graphical implementation of the basic concepts of variable structure control. This method proved to yield system designs which are sensitive to parameter variations and disturbances. The graphical design methods however demonstrated relationships for the controller gain and switching line slope constant that are valid and useful for the next chapter.

This chapter also examined the basic design requirements for a robust linear control system. The linear control system designs presented had very

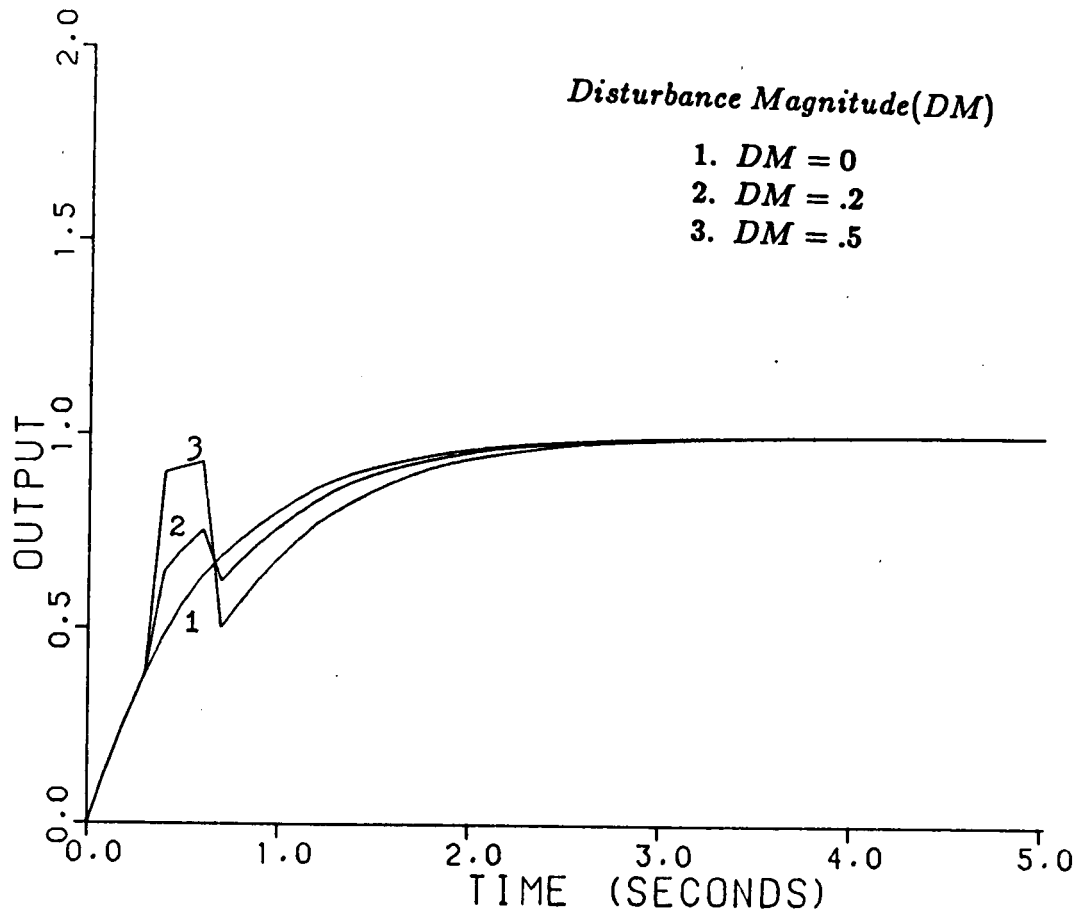


Figure 3.30: Transient responses of VSCS with disturbance.

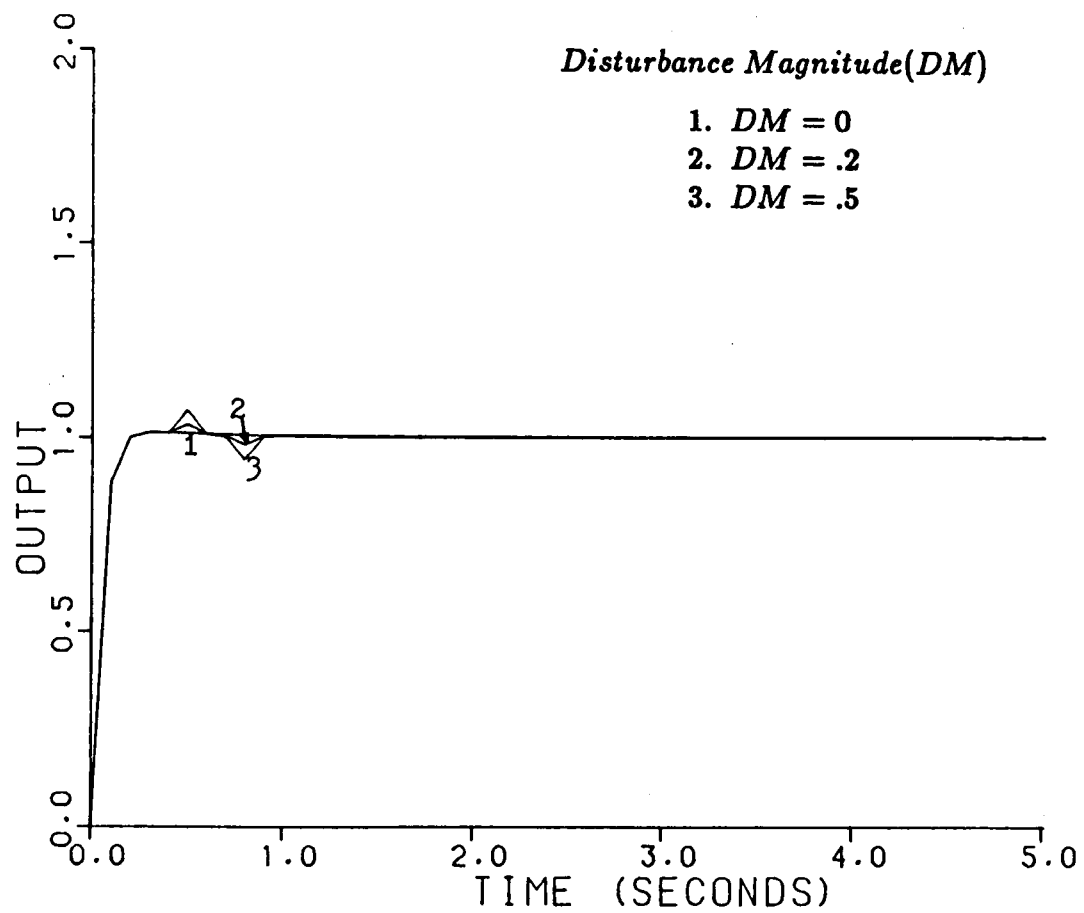


Figure 3.31: Linearly compensated transient response with disturbance.

good immunity to parameter variations but somewhat sensitive to disturbances. It was observed however that the faster the linearly compensated system response was the sensitivity to disturbances decreased.

CHAPTER 4

ANALYTICAL DESIGN OF THE VSCS

4.1 Analytical Design Concepts of the VSCS

In this chapter an analytical design method is presented to obtain a VSCS design. This method does not require plotting of the error state trajectory which reduces the time required to do a VSCS design. One important aspect of this analytical design method is that it results in a superior transient response and greater robustness than the simple VSCS of chapter 3. Even though the simple VSCS design resulted in poor performance characteristics the relationships observed for the controller gain and slope of the switching line are still valid.

The analytical method that will be discussed in this chapter is outlined by Itkis [1] for a zero reference input. A change in convention will be made in this chapter for the control law given by Itkis to consider the most practical systems which has a non-zero reference input.

Itkis presents the necessary conditions for a stable sliding mode and the hitting of the switching line for time varying plant parameters. These conditions also define a prescribed number of error derivatives which may also be switched when applying the defined control law. The switching of the error derivatives is used in order to improve the speed of the transient response while decreasing the noise immunity. Because of noise immunity problems it appears best to minimize the number of switched error derivatives and if

possible not to switch any at all. In the VSCS design in this chapter no error derivatives will be switched.

The analytical VSCS design will be applied to the second order plants of chapter 3. The design requirements for a second order plant will be generalized from Itkis [1]. The necessary conditions for a time varying plant with no switching of the error derivatives are not explicitly defined for higher order systems by Itkis. For this investigation the second order plant provides a convenient generalization to obtain no switching of the error derivatives.

To begin the generalized form of the system of differential equations and the control law are stated with a slight modification to account for a non-zero reference input. The control law is

$$u = \sum_{i=1}^{n-1} \psi_i E_i \quad (4.1)$$

where

$$\psi_i = \begin{cases} \alpha_i & \text{if } \sigma E_i > 0 \\ \beta_i & \text{if } \sigma E_i < 0, \end{cases} \quad (4.2)$$

$$i = 1, \dots, n-1 \quad (4.3)$$

$$\sigma = \sum_{i=1}^n c_i E_i = 0, \quad c_i = \text{constant} > 0, \quad c_n = 1 \quad (4.4)$$

and

$$E_1(t) = E(t) = r(t) - x_1(t) = \text{error state} \quad (4.5)$$

and

$$E_2(t), E_3(t), \dots, E_n(t)$$

are derivatives of the error state, $r(t)$ is the input to the system and $x_1(t)$ is the output of the system. The system of state differential equations describing the VSCS are

$$\begin{cases} \dot{x}_1(t) = x_{i+1}(t) & i = 1, \dots, n-1 \\ \dot{x}_n(t) = (r(t) - x_1(t))a_1(t) - \sum_{i=2}^n a_i(t)x_i + b(t) \sum_{i=1}^{n-1} \psi_i E_i(t) \end{cases} \quad (4.6)$$

where $a_i(t)$ are time varying plant parameters.

The control law and system of differential equations can be simplified for approximation to a second order system by letting $n = 2$. The control law and differential equations of the VSCS are then

$$u = \psi_1 E_1(t) = \psi_1 E(t) \quad (4.7)$$

where

$$\psi_1 = \begin{cases} \alpha_1 & \text{if } \sigma E(t) > 0 \\ \beta_1 & \text{if } \sigma E(t) < 0 \end{cases} \quad (4.8)$$

$$\sigma = cE(t) + \dot{E}(t) = 0 \quad (4.9)$$

and

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = (r(t) - x_1(t))a_1(t) - a_2(t)x_2(t) + b(t)\psi_1 E(t). \end{cases} \quad (4.10)$$

This generalization to a second order system yields the desired control law which requires no switching of the error derivatives. This particular form of the control law was desired to simplify the design procedure.

The general conditons for a sliding mode to exist along the switching line $\sigma = 0$ [1] are

$$\begin{aligned}\alpha_i &\geq \sup_t \frac{1}{b(t)} [c_{i-1} - a_i(t) + c_i(a_n(t) - c_{n-1})] \\ \beta_i &\leq \inf_t \frac{1}{b(t)} [c_{i-1} - a_i(t) + c_i(a_n(t) - c_{n-1})]\end{aligned}\tag{4.11}$$

$$\inf_t a_n(t) \geq c_{n-1}$$

The necessary conditions for hitting of the switchng line by the error state trajectory are stated as followings [1].

A sufficient condition for the error state trajectory of a system to hit the switching plane $\sigma = 0$ from any initial position in the error state space is that the characteristic equation

$$p^n + a_{n,min}p^{n-1} + \sum_{i=1}^{n-1} [a_{i,min} + \inf_t [b(t) \alpha_i]] p^{i-1} = 0 \tag{4.12}$$

has no positive real roots. Also it is required that

$$\begin{aligned}\inf_t[\alpha_i b(t)] &> -a_{i,min} \\ \sup_t[\beta_i b(t)] &< -a_{i,max}\end{aligned}\tag{4.13}$$

where

$$i = 1, \dots, n-1.$$

Note that the inf and sup notation is defined as follows [1]:

$$\begin{aligned}\inf_t[\alpha_i b(t)] &= \alpha_i b_{min} \quad \text{for } \alpha_i > 0 \\ \inf_t[\alpha_i b(t)] &= \alpha_i b_{max} \quad \text{for } \alpha_i < 0 \\ \sup_t[\beta_i b(t)] &= \beta_i b_{max} \quad \text{for } \beta_i < 0 \\ \sup_t[\beta_i b(t)] &= \beta_i b_{min} \quad \text{for } \beta_i > 0.\end{aligned}\tag{4.14}$$

Generalizing to a second order system the requirements for a sliding mode are

$$\begin{aligned}\alpha_1 &\geq \max \frac{1}{b(t)} [-a_1(t) + c_1(a_2(t) - c_1)] \\ \beta_1 &\leq \min \frac{1}{b(t)} [-a_1(t) + c_1(a_2(t) - c_1)].\end{aligned}\tag{4.15}$$

The requirements for the hitting of the switching line for the second order system are

$$p^2 + a_{2,min}p^{n-1} + a_{1,min} + \inf_t [b(t)\alpha_1] = 0\tag{4.16}$$

which has no positive real roots and

$$\begin{aligned}\inf_t [\alpha_1 b(t)] &> -a_{1,min} \\ \sup_t [\beta_1 b(t)] &< -a_{1,max}.\end{aligned}\tag{4.17}$$

4.2 Analytical VSCS Design of a Pure Integral Plant

The state equation for the pure integral plant with a variable plant parameter is given as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = (r(t) - x_1(t))a_1(t). \end{cases}\tag{4.18}$$

Therefore the state equations required for the VSCS are

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= (r(t) - x_1(t))a_1(t) - b(t)\psi_1 E(t).\end{aligned}\tag{4.19}$$

Letting $\alpha_1 = 1$ and $\beta_1 = -1$ in equations (4.7) to (4.10) is defined as

$$u = \psi_1 E\tag{4.20}$$

where

$$\psi_1 = \begin{cases} 1 & \text{if } \sigma E(t) > 0 \\ -1 & \text{if } \sigma E(t) < 0 \end{cases}\tag{4.21}$$

and

$$\sigma = c_1 \dot{E} + E, \quad c_1 = \text{constant} > 0.\tag{4.22}$$

The existence of the sliding mode for this system requires that

$$\begin{aligned} 1 &\leq \max \frac{1}{b(t)} [-a_1(t) - c_1^2] \\ -1 &\leq \min \frac{1}{b(t)} [-a_1(t) - c_1^2] . \end{aligned} \quad (4.23)$$

One of the most important aspects of the VSCS design is the selection of the constant c_1 . From observations of chapter 3 there are certain trade offs in selecting values of c_1 . When the value of c_1 is small the error state trajectory spends more time in the sliding mode. However for large values of c_1 less time is spent in the sliding mode. Since the sliding mode offers the insensitivity to varying plant parameters and disturbances, it seems advantageous to prolong time in the sliding mode. Prolonging the sliding mode however increases the system transient response time. In order to allow greater flexibility in the system design the value of c_1 will be selected to be a discrete value on the range

$$1.0 \leq c_1 \leq 10.$$

The plant parameter is selected to vary over the same range as the LCS of chapter 3 which is

$$.001 \leq a_1(t) \leq 500.$$

The controller gain for this VSCS is selected to be a constant large enough to satisfy equation (4.23) for the desired range of c_1 and the plant parameter. Depending on the size of the controller gain the hitting time of the error state trajectory may be significantly reduced. The controller gain for this VSCS is thus selected to be $b = 1000$. This value of controller gain also satisfies the requirements of equations (4.16) and (4.17).

The block diagram of the compensated system is shown in Fig. 4.1. The transient responses for different values of the constant c_1 are shown in Fig. 4.2. From these transient responses it is seen that when c_1 is 10 or 5 the system response time approaches that of the corresponding linearly compensated system of chapter 3.

The transient responses for the VSCS with parameter variations and step disturbances will be examined when $c_1 = 5.0$. The VSCS is seen to be completely insensitive to parameter variations as shown in Fig. 4.3. The VSCS however is still sensitive to disturbances as seen in Fig. 4.4.

4.3 Analytical VSCS Design of a Inertial and Integral Plant

The state equation for the inertial and integral plant with variable plant parameters is given as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = a_1(t)E(t) - a_2(t)x_2(t). \end{cases} \quad (4.24)$$

The state equations required for the VSCS is thus

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = a_1(t)E(t) - a_2(t)x_2(t) + b\psi_1 E(t). \end{cases} \quad (4.25)$$

The parameters for the control law are as previously defined in equations (4.20), (4.21), and (4.22). The existence of the sliding mode for this system requires that

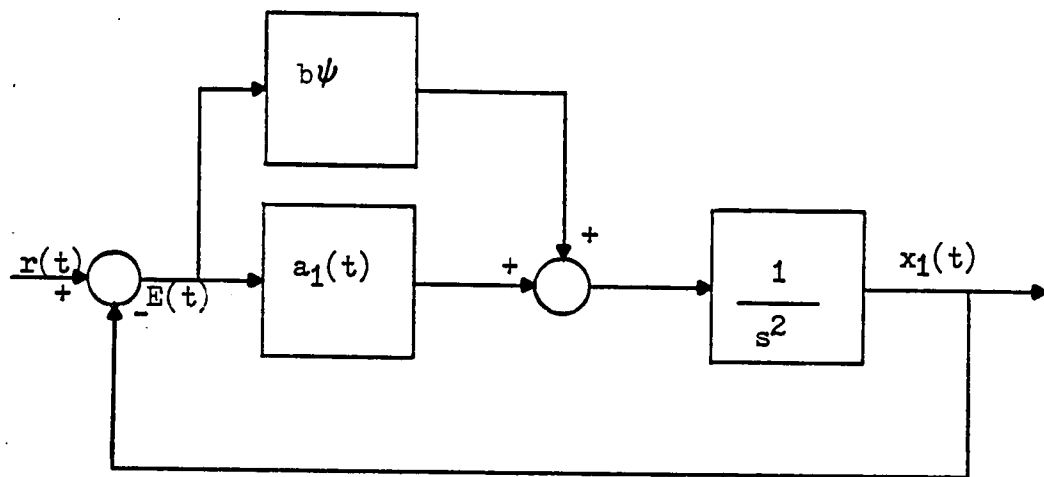


Figure 4.1: Analytical VSCS design.

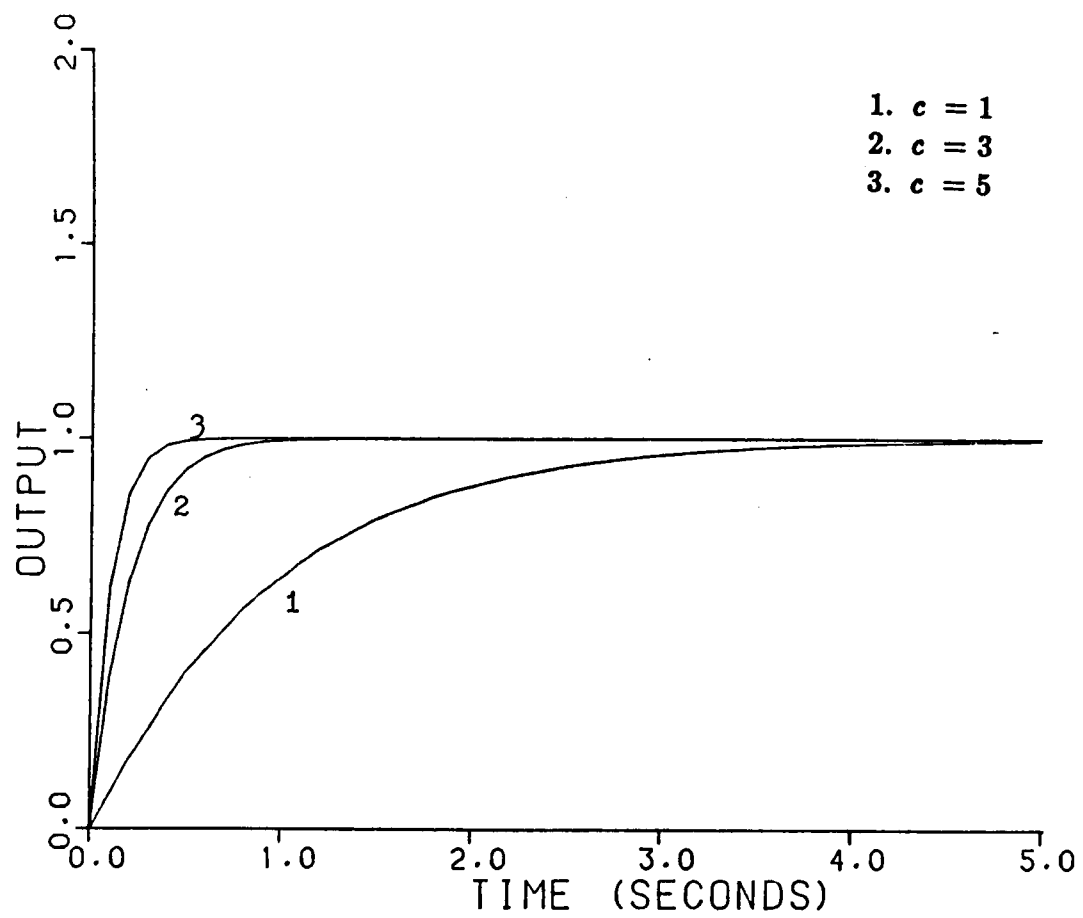


Figure 4.2: Varying the slope constant.

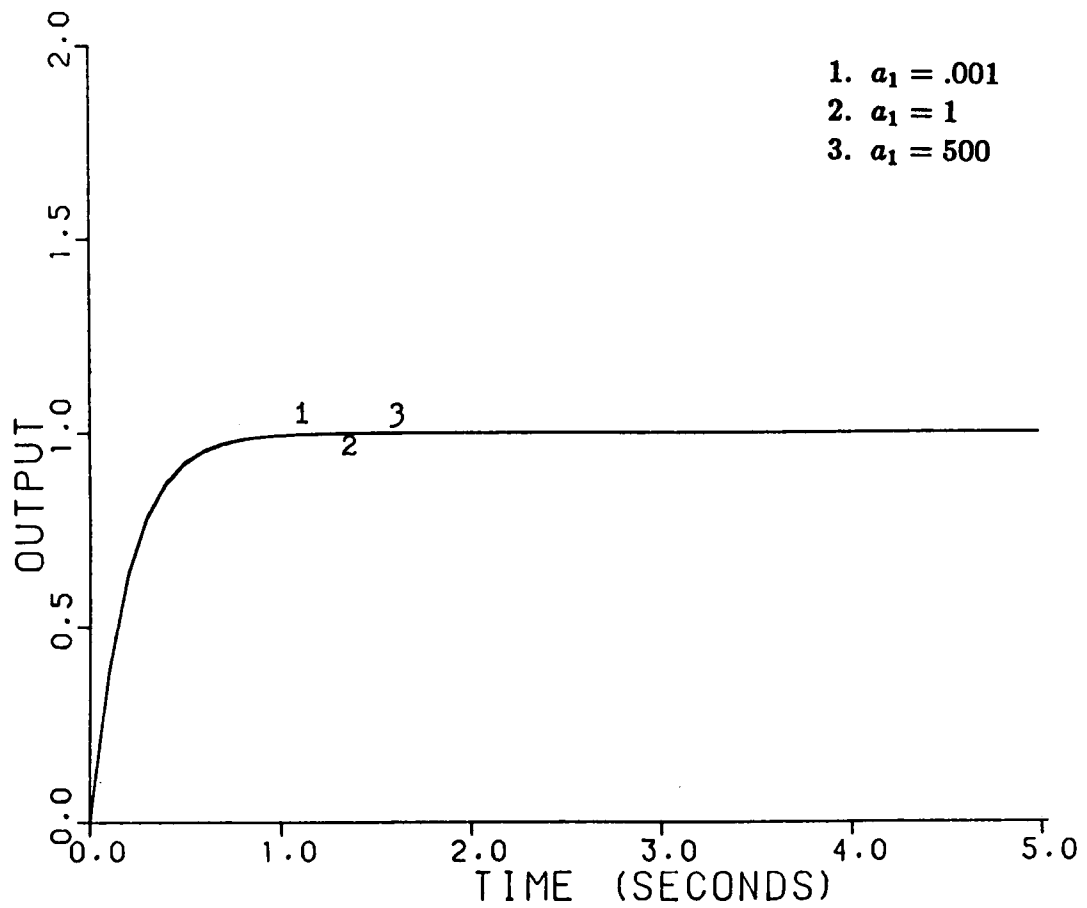


Figure 4.3: Varying the plant parameter.

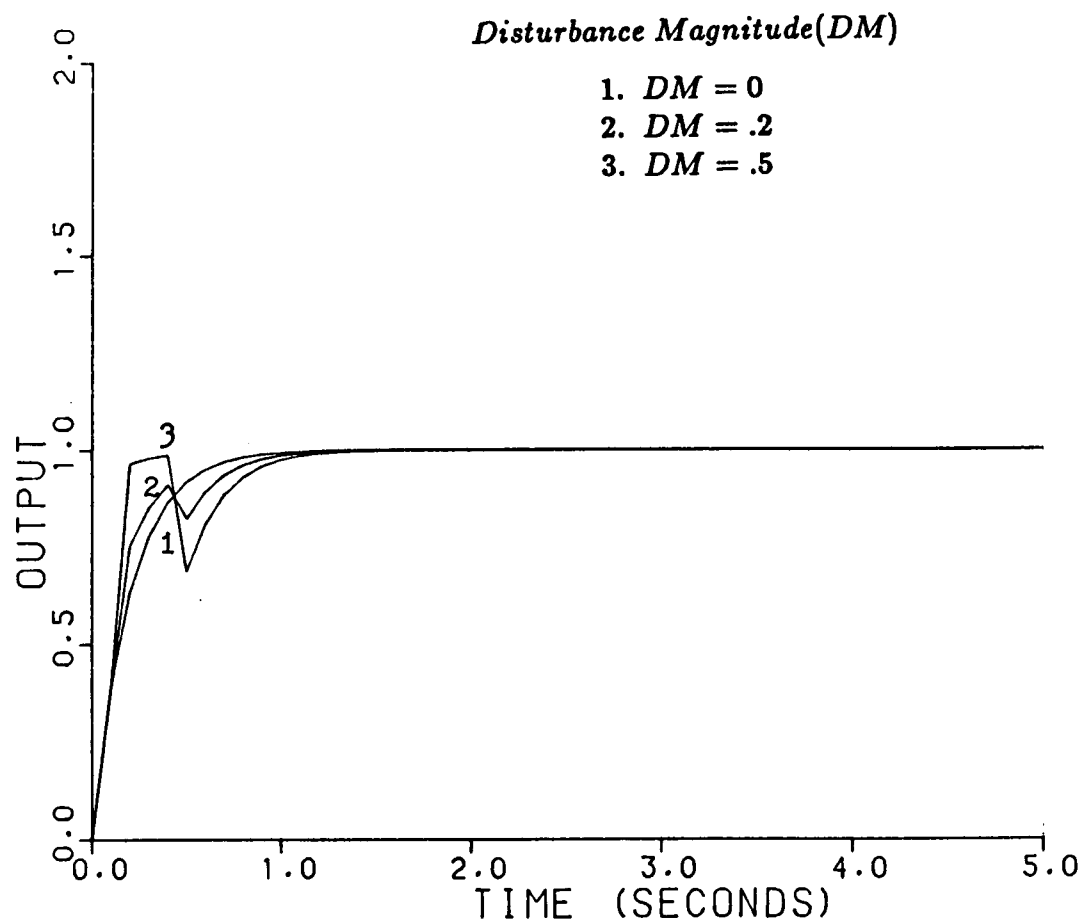


Figure 4.4: System with disturbance.

$$\begin{aligned}
1 &\geq \max \frac{1}{b(t)} [-a_1(t) + c_1 a_2(t) - c_1^2] \\
-1 &\leq \min \frac{1}{b(t)} [-a_1(t) + c_1 a_2(t) - c_1^2] .
\end{aligned}
\tag{4.26}$$

The plant parameters for this VSCS design will be required to vary over the following range

$$.001 \leq a_1(t) \leq 500$$

and

$$.001 \leq a_2(t) \leq 500.$$

The slope constant c_1 is selected to be some value on the interval

$$1.0 \leq c_1 \leq 5.0.$$

Considering the range of possible variation for the plant parameters and the slope constant, the controller gain is selected to be 3000 in order to satisfy equation (4.26). The conditions for (4.16) and (4.17) are also satisfied in this VSCS design.

The block diagram and transient response for the VSCS are shown respectively in Fig. 4.5 and Fig. 4.6. Again the analytical VSCS design method yields a good transient response characteristic especially at the higher values of slope constant. A even faster response is obtainable by increasing the controller gain to allow the selection of a higher value of slope constant. The flexibility of increasing the system rise time appears to be much easier to accomplish with the VSCS analytical design method in comparison to the linearly compensated system design.

The transient responses for the system with parameter variation and disturbance are obtained using a slope constant of $c_1 = 3.0$. The resulting

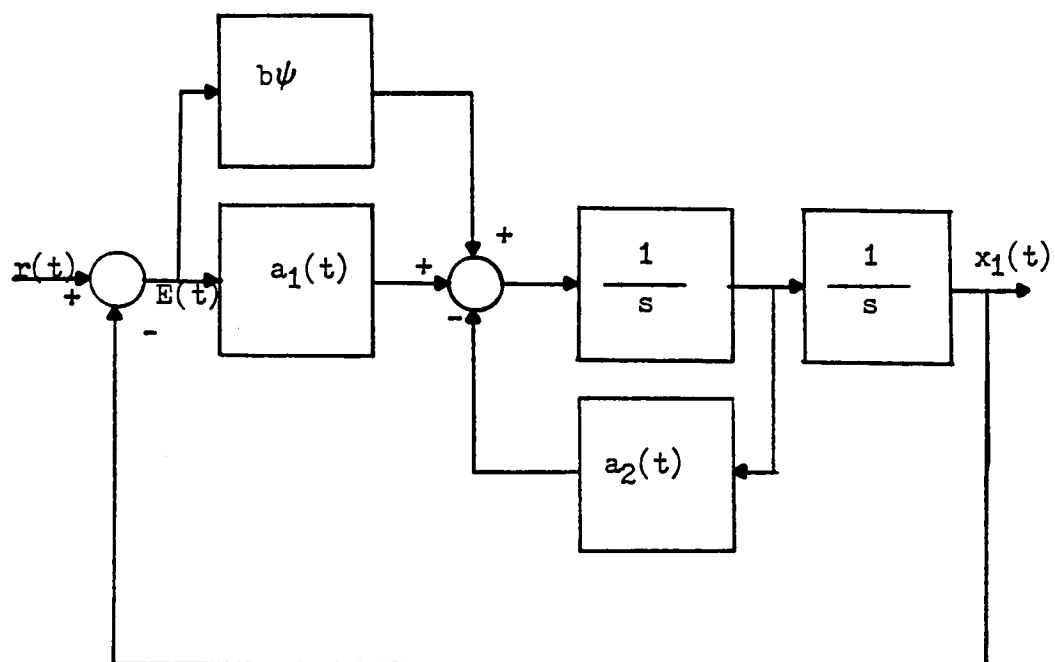


Figure 4.5: Block diagram of analytically designed VSCS.

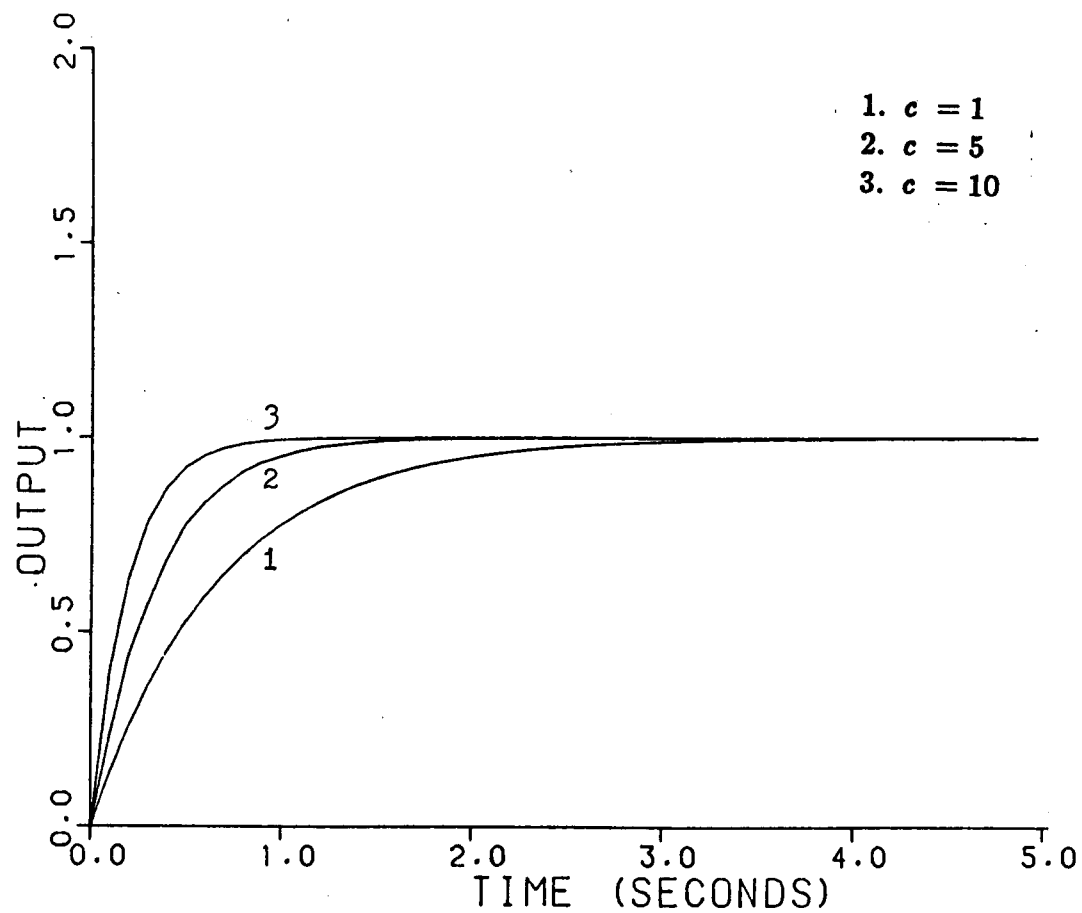


Figure 4.6: Varying values of slope constant c_1 .

transient responses are respectively shown in Fig. 4.7 and Fig. 4.8. Sensitivity to parameter variation in the system is small and may be considered acceptable depending on the application. It has been observed that sensitivity to parameter variations decreases as the rise time of the VSCS decreases. The VSCS of this section also demonstrates the extreme sensitivity to disturbance. This sensitivity is still greater than that of the linearly compensated system of chapter 3.

4.4 Conclusion

The results of this chapter show that the analytical VSCS design method results in a system with a faster response and very little sensitivity to parameter variation. These characteristics are similar to those obtained by a linear control system design method. It appears however much easier to change the transient response characteristics for the VSCS than the LCS of chapter 3. This change in response is easily obtained by varying the switching line slope constant. The variation in slope constant is obtainable only if the appropriate value of controller gain is selected which satisfy the conditions of section 4.1.

Another advantage of using this analytical VSCS design method is that it requires no plotting except for the output of the system. This is a great advantage over the linear control system design methods which requires the iterative use of approximate analytical and graphical tools to obtain a suitable design.

The only disadvantage of the basic analytical VSCS design method is the lack of inherent ability to reject disturbance.

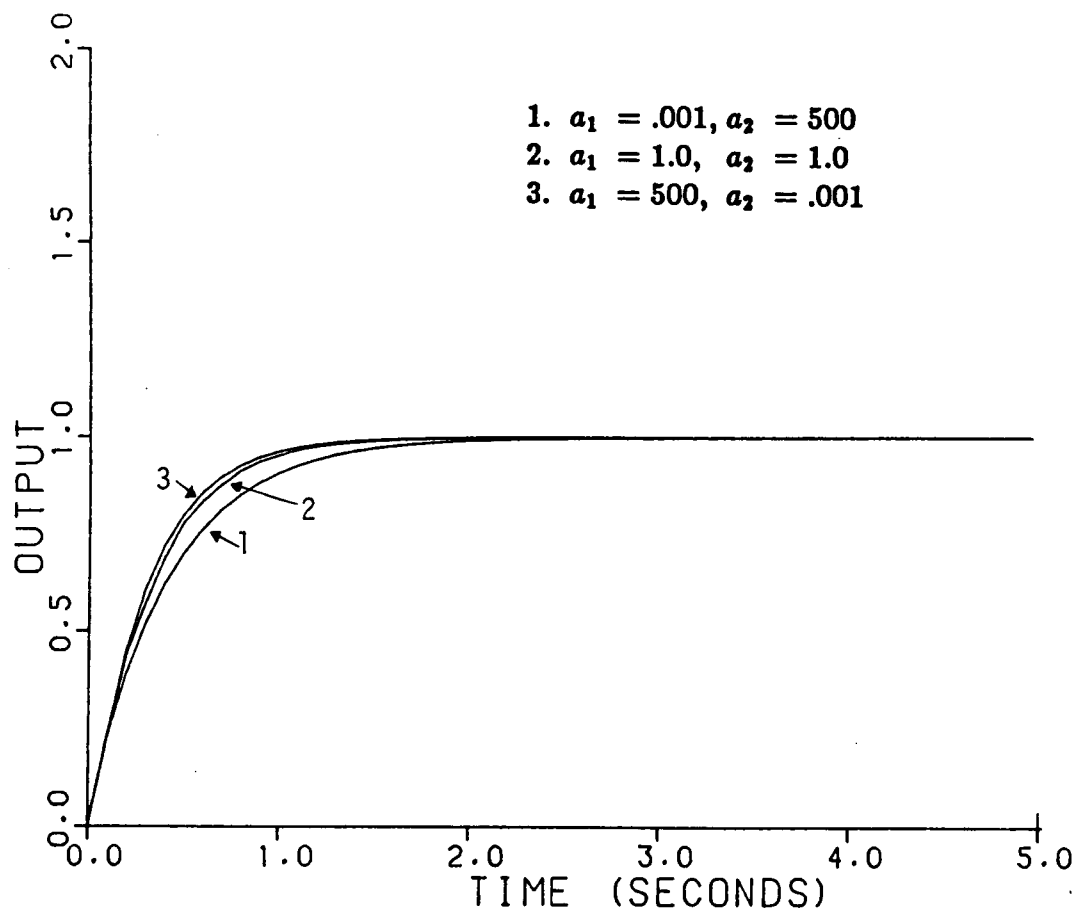


Figure 4.7: Varying the plant parameter of the VSCS.

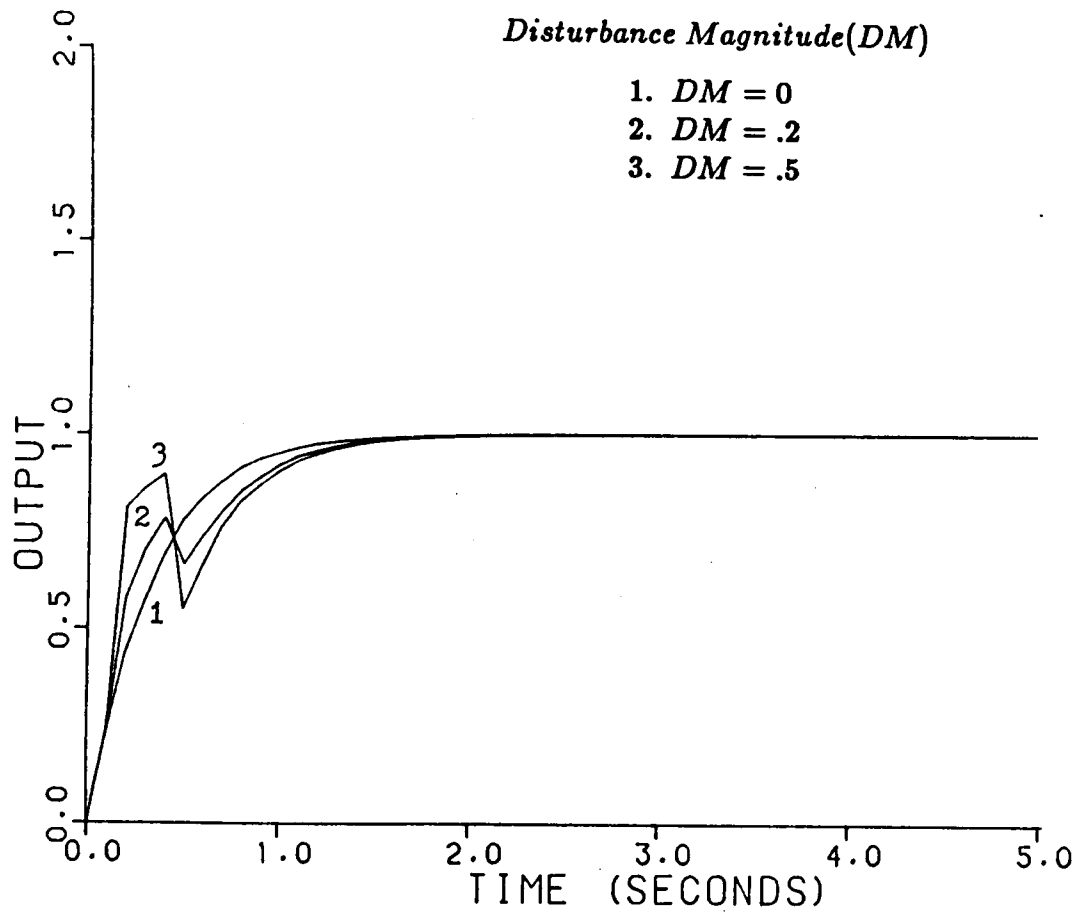


Figure 4.8: Compensated system with disturbance.

CHAPTER 5

CONCLUSION

From this investigation certain advantages and disadvantages were found to exist in the implementation of the VSCS concept. First the disadvantages will be discussed.

The basic concept of VSCS design which is introduced in this investigation can lead to an over simplification in the application of VSCS. As shown in chapter 3 a very simple VSCS can lead to an improved transient response over an already unstable or oscillatory control system. However it is found this is the only resulting improvement, which is also sensitive to parameter variations and disturbances. This is completely contrary to the improvements expected from a VSCS. Therefore from this investigation it is seen that any attempt to over simplify the design process by just putting a nonlinear switch in the forward path of the system will lead to undesirable sensitivity results. Also the disadvantage of using a simple VSCS of chapter 3 is that it lacks the flexibility in changing the system transient response characteristics over a wide range. It is seen however that a conventional Bode designed control system easily surpasses the simple VSCS in both transient and sensitivity performance.

Applying the analytical design concept of VSCS resulted in a system whose transient response rise time could be varied over a wide range by just varying one controller parameter, which is a nice advantage. Another advantage of the analytically designed VSCS is that it is rather insensitive to

changes in the plant parameters. The analytically designed VSCS however failed to be insensitive to disturbances that occurred just prior to the feedback loop of the system. This VSCS was more sensitive to this form of disturbance than the linearly compensated system.

A VSCS design procedure was found to exist in [1] to eliminate disturbance in the forward path of the system. This type of disturbance when examined is found to be of little consequence in the system transient response. After close investigation it was concluded that the basic analytical VSCS design technique does not insure disturbance rejection of the form presented in this investigation. However from initial investigation it appears that obtaining an analytically designed VSCS to reject this form of disturbance is possible. It appears that the main requirement will be to satisfy all the inequalities of the basic analytical design by rederiving the requirements with the disturbance in the system.

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VITA

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