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Conditional nonlinear stochastic discount factor models as alternative explanations to stock price momentum

A Dissertation Presented for the Doctor of Philosophy Degree

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Abstract

Existing linear asset pricing models do not fully explain the abnormal profits associated with prior-return portfolios. In addition, existing nonlinear consumption-based models produce implausible risk aversion coefficient values when applied to prior-return portfolios. Measures based upon production instead of consumption reduce residual errors and drive risk aversion coefficients towards plausible values. Augmenting the existing models with a new production-based marginal utility growth proxy, supplemented by a production-based consumption proxy not previously applied to price prior-return portfolios, can explain the abnormal profits associated with prior-return portfolios and yield plausible risk aversion coefficient values.

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Dedication

This dissertation is dedicated to my parents, Robert H. Moore (who passed away during my Ph.D. pursuit) and Francelle Moore, for caring for me, sacrificing for me, instructing me, inspiring me, and encouraging me to be the best at whatever I do.

Acknowledgements

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1 Introduction

Many authors have documented the profitability of momentum investing, i.e., constructing a zero net investment portfolio long on prior winners and short on prior losers[1] and have demonstrated the momentum phenomenon through the analysis of serial correlation and post-earnings announcement drift[2]. Existing linear asset pricing models do not fully explain the abnormal profits associated with prior-return portfolios. In addition, existing nonlinear consumption-based models produce implausible risk aversion coefficient values when applied to prior-return portfolios.

Figures 1 and 2 summarize the empirical results and contribution of this thesis: augmenting the existing models with a new production-based marginal utility growth proxy, supplemented by a production-based consumption proxy not previously applied to price prior-return portfolios, can explain the abnormal profits associated with prior-return portfolios and yield plausible risk aversion coefficient values. In Figure 1A, the production-based marginal utility growth instrument produces lower residual errors, as measured by the χ^2 statistics, than the standard consumption growth instrument. Furthermore, the $p-$values associated with the χ^2 statistics indicate the models can not be rejected. In Figure 1B, the production-based aggregate consumption proxy yields significantly lower relative risk aversion coefficients than the aggregate nondurable consumption measure.

[1] Jegadeesh and Titman (1993); Fama and French (1996); Carhart (1997); Rouwenhorst (1998); Conrad and Kaul (1998); Griffin, Ji, and Martin (2003); Cooper, Gutierrez, and Hameed (2004); Bansal, Dittmar, and Lundblad (2005); Myers, Myers, and Skinner (2006); Chordia and Shivakumar (2006)

[2] Jegadeesh (1990); Chan, Jegadeesh, and Lakonishok (1996); Johnson (2002)

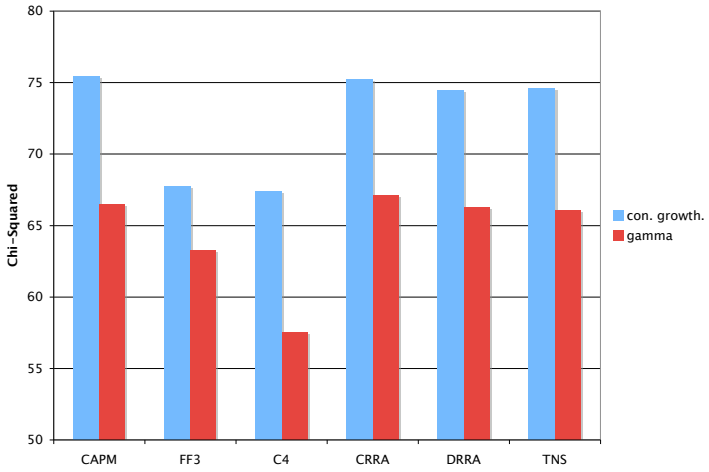


Fig. 1: Residual error reduction vs. marginal utility growth instrument by model

Residual errors χ^2 obtained from GMM estimation of linear and nonlinear stochastic discount factor models applied to 10 prior-return portfolios. Linear models include the CAPM, the Fama French 3-Factor (FF3), and the Carhart 4-factor (C4) model. The nonlinear models are based on constant relative risk aversion (CRRA), decreasing relative risk aversion (DRRA), and time non-separable (TNS) utility. Results are presented for consumption growth (con. growth) and the production-based marginal utility growth proxy (gamma) instruments.

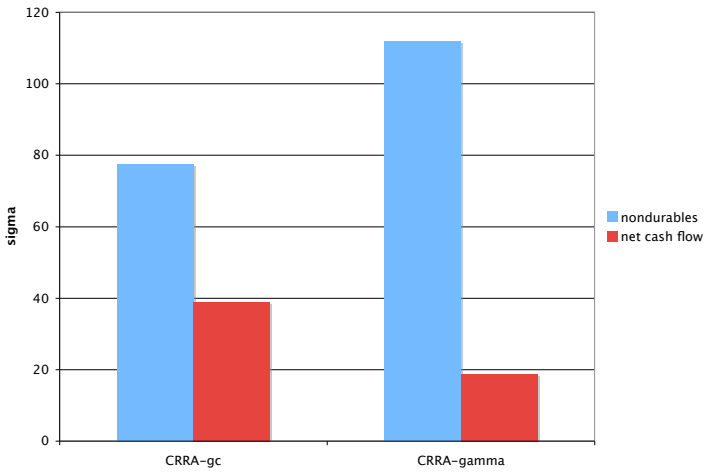


Fig. 2: Risk aversion coefficient reduction vs. consumption proxy by model
Relative risk aversion coefficients σ obtained from GMM estimation of nonlinear stochastic discount factor model based on CRRA utility with consumption growth (gc) and the marginal utility growth proxy (gamma) instruments.. Results are presented for both aggregate nondurable consumption (nondurables) and the production-based aggregate consumption proxy (net cash flows).

The results obtained in this study have implications for both researchers and practitioners. Refinements to the underlying simplifications of the model, discussed in Section 7, may lead to more robust asset pricing models capable of addressing not only the momentum phenomenon, but other asset pricing anomalies unexplained by existing models. Researchers can use models augmented with the refined consumption and marginal utility growth proxies in their examination of return patterns to potentially avoid mis-identifying those patterns as anomalies. In addition, the refined models will produce a more accurate "Jensen's alpha" measure to compare the risk-adjusted return of securities or trading strategies. Practitioners can use this refined risk-adjusted return measure to make more informed asset allocation decisions.

1.1 Existing explanations and models

Unfortunately, the usual suspects of trading-strategy profitability in the context of investor rationality have been unable to explain momentum profits. Data mining is ruled out since the extant literature demonstrates momentum profits are robust to sample period (Jegadeesh and Titman, 2001), exist internationally (Rouwenhorst, 1998), and may be robust to trading costs even when considering costs associated with shorting (Geczy, Musto, and Reed, 2002; Korajczyk and Sadka, 2004).

The CAPM and the Fama and French (1993) 3-factor model are unable explain short-term (3 to 12 month) momentum (Jegadeesh and Titman, 1993; Fama and French, 1996; Chan, Jegadeesh, and Lakonishok, 1996; Chordia and Shivakumar, 2006; Akhbari, Gressis, and Kawosa, 2006) returns. Additionally, the inclusion of a momentum-derived return factor (WML, winners

minus losers) with the Fama French 3-factor model, referred to as the Carhart (1997) four-factor model, was shown to have significant Jensen alphas (Chordia and Shivakumar, 2006).

In a recent study, Chordia and Shivakumar (2006) used standardized unexpected earnings (SUE) to form a momentum portfolio (PMN, positive minus negative unexpected earnings) long on high-SUE and short on low-SUE stocks. The authors found price momentum is explained by earnings momentum through addition of PMN to the three factor model of Fama and French (1993). However, their earnings momentum portfolio (PMN) is negatively related to GDP, and this negative relationship is posited to be a result of the "inflation illusion": investors mistaking nominal growth rates for real. Nevertheless, this "inflation illusion" explanation implies investor irrationality in that available inflation data are *not* incorporated and the apparent success of Chordia and Shivakumar's PMN factor is tainted by this implication.

Finally, even though factors based on returns may be found, such as the market risk premium of CAPM, the size and distress factors of Fama and French, the price momentum factor of Carhart, and the earnings momentum factor of Chordia and Shivakumar's, Cochrane (1996) states:

> "Though these models may successfully *describe* variation in expected returns, they will never *explain* it. To say that an asset's expected return varies over the business cycle because (say) the market expected return varies leaves unanswered the question, What real risks cause the market expected return to vary? Fur-

> thermore, fishing for asset return factors with no explicit connection to real risks can result in models that price assets by construction in a given data set..."

Therefore it is debatable whether or not priced and theoretically justifiable risk factors that can explain prior-return portfolios have been found.

1.2 Macroeconomic connection

Several authors have attempted to connect macroeconomic risk to momentum profits but results are mixed (Chordia and Shivakumar, 2002; Griffin, Ji, and Martin, 2003; Cooper, Gutierrez, and Hameed, 2004). Recently however, a stream of literature has examined the connection between cash flow sensitivity to macroeconomic variables.

Johnson (2002) noted stock price sensitivity to dividend growth rates is exponential and therefore growth rate risk increases with growth rate. In simulating his theoretical model, Johnson demonstrated growth rate risk is priced and persistent episodic growth rate shocks lead to serial correlations in returns. However, this is a theoretical model validated by simulation and not applied to available return data. Furthermore, Johnson acknowledged a systematic and persistent (cash flow) growth rate risk factor may not exist, and even so, exposure to that risk must be shown to be associated with positive expected returns.

Bansal, Dittmar, and Lundblad (2005) related aggregate consumption to the cross section of returns via a cash flow sensitivity (risk) measure. Their measure was constructed by regressing dividend growth rates onto aggregate

consumption and capturing the coefficient for momentum, size, and book to market portfolios. Such a model can be viewed as a decomposition of growth rates into systematic (aggregate consumption coefficient) and idiosyncratic (error term) risk factors. The authors found cash flow growth rates of winners are higher than that of losers and the cash flows of winners are more sensitive to aggregate consumption (i.e., they had higher cash flow betas). Furthermore, the authors found that cash flow risk is priced, consistent with Johnson (2002). However, this is inconsistent with the fact that aggregate consumption by itself can not explain the cross section of returns as suggested by Lucas (1978). In fact, the poor explanatory ability of consumption-based models are documented by the authors themselves and others (e.g., Epstein and Zin 1991 and Bansal and Yaron 2003). To summarize, a measure of cash flow *sensitivity* to aggregate consumption has explanatory power while aggregate consumption itself does not.

The authors posit two possible explanations to reconcile non-informative consumption betas with informative aggregate-consumption derived cash flow betas. First, if there are multiple risk factors in returns, then consumption beta may not capture cross-sectional variations in risk premia. Empirically, the authors identified cash-flow sensitivities to consumption as one such risk factor. The second explanation posited is the difficulty in measuring aggregate consumption, which introduces error and therefore reduces the ability for consumption beta to explain cross-sectional differences. Regarding the missing risk factor explanation, their model accounts for approximately 60% of the cross-sectional variation in returns. Therefore, the same argument regarding additional unidentified factors can be applied to

the cash-flow beta-inclusive model (to explain the remaining 40%). Regarding the problematic consumption data explanation, measurement error in aggregate consumption invariably propagates to their aggregate-consumption based measure of cash flow sensitivity. Furthermore, the authors acknowledged their cash flow risk measures are "measured with considerable error."

1.3 Specifics of existing model extensions

This research draws on both exchange and macroeconomic growth models to obtain a theoretically justifiable and rational mechanism to price momentum portfolios. The exchange economy model follows (Lucas, 1978) and Balvers, Cosimano, and McDonald (1990) to yield a production-based proxy for aggregate consumption called net cash flow. This net cash flow measure is used as a substitute for aggregate consumption in asset pricing models. The conceptual difference between aggregate net cash flow and aggregate consumption is upheld empirically. Cochrane (1991) found nondurable consumption is relatively smooth (low standard deviation) in the literature. In contrast, the data of this study over the 1947 to 2006 time period reveal the volatility of net cash flow measure is twice that of the aggregate consumption. The greater volatility of net cash flows is suggestive of greater explanatory power over aggregate consumption.

The macroeconomic growth model, based on King and Rebelo (1999), is used to obtain a productivity-based marginal utility growth expression for the purpose of connecting macroeconomic data to asset returns. The work of Madsen and Davis (2006) and Balvers and Huang (2007) supports the connection between productivity and asset returns. Madsen and Davis

(2006) demonstrated how asset prices rise then fall following a technological (productivity) shock within a partial equilibrium (firm profit maximization) framework. This thesis shows their results also hold within a general equilibrium framework (simultaneous firm and consumer maximization) and mimic the return reversals documented in many over/under-reaction studies. Thus, a rational mechanism can produce price patterns commonly considered to be irrational.

Balvers and Huang (2007) derived a productivity-based asset pricing model by including a random productivity shock in the representative agent's indirect utility function. The authors applied their model to the traditional five size by five book-to-market sorting and found the performance of their model comparable to that of consumption-based, CAPM, and Fama French 3-factor models. This thesis continues the work of Madsen and Davis (2006) and Balvers and Huang (2007) by applying the connection of productivity to market returns (Madsen and Davis) and size and book to market portfolios (Balvers and Huang) to prior-return portfolios.

The remainder of the thesis is as follows. Chapter 2 defines momentum and reviews the literature covering momentum strategies, proposed explanations of momentum profitability, neoclassical investment theory (which is later applied to the macroeconomic growth model in this study), and empirical evidence related to productivity, asset prices, and macroeconomic growth theory. Chapter 3 develops an exchange economy model to obtain an alternative measure of consumption, a macroeconomic growth model to obtain the productivity-based marginal utility growth expression, and an asset pricing framework to exploit these two results. Chapter 4 describes the data

sample, provides univariate statistics that illustrate momentum profitability persistence, establishes the process for obtaining the alternative consumption measure, and presents the methodology for testing the linear and nonlinear asset pricing models of this thesis. Chapter 5 presents the results of the empirical analysis, Chapter 6 concludes with a discussion of the results, and Chapter 7 presents avenues of future research.

2 Literature review

2.1 Momentum strategies

Momentum is a generic term used to describe the relationship between past observations and subsequent future observations. Price momentum refers to both negative and positive short-term correlation of stock prices. Earnings momentum is used to describe post-earnings announcement drift. Directional momentum considers higher moments of price patterns. Each of these classifications of momentum is discussed in turn followed by a brief discussion of other related trading strategies.

2.1.1 Serial correlation / price momentum

Very short-term (1 month or less) return reversals and short-term (3 to 12 months) price continuation have been reported by several researchers. To begin, Jegadeesh (1990) performed a cross sectional regression for each month in the sample period 1929 to 1982 using the following equation:

$$R_{it} - \overline{R}_i = a_0 + \sum_{j=1}^{12} a_{jt} R_{it-j} + a_{13} R_{it-24} + a_{14} R_{it-36} + \epsilon_{it} \tag{2.1}$$

where

R_{it} = return on security i in month t

$\overline{R}_i$ = average monthly return from $t+1$ to $t+60$ for security i

Jegadeesh found significant positive coefficients for $\widehat{a}_3$ to $\widehat{a}_{14}$ (excluding

$\widehat{a}_7$ and $\widehat{a}_8$) and a significant negative coefficient on $\widehat{a}_1$. The negative $\widehat{a}_1$ coefficient supports very short-term return reversal and the remaining positive coefficients support short-term price continuation. However, R^2 values for the regressions are less than 0.20 which may indicate model mis-specification. Jegadeesh then formulated three trading strategies to examine return predictability in the form of short-term ($\leq$ 12 months) return continuation with additional lagged returns (2 and 3 year), very short-term (1 month) return reversals, and short-term return continuation.

Strategy S1.0: This strategy was designed to exploit return predictability using lagged twelve month returns plus a two year and a three year lag. As such, returns were estimated:

$$\widehat{R}_{it} = \widehat{a}_{0t} + \sum_{j=1}^{12} \widehat{a}_{jt} R_{it-j} + \widehat{a}_{13t} R_{it-24} + \widehat{a}_{14t} R_{it-36} \tag{2.2}$$

Next, firms were sorted into deciles[3] (P1 to P10) by descending predicted contemporaneous return $\widehat{R}_{it}$. As such, decile P1 represents a portfolio of the highest predicted return stocks and decile P10 represents a portfolio of the lowest predicted return stocks.

Strategy S1.1: This strategy was designed to exploit very short-term return reversals. Securities were sorted into deciles (P1 to P10) by ascending lagged one-month return. As such, decile P1 represents a portfolio of the low-

[3] Jegadeesh states that over a half million observations were used in the cross-sectional analysis. Given the analysis window of 54 years, this works out to approximately 771 monthly returns (firms) per month. Therefore, portfolios formed by sorting these 771 firms into deciles are likely to have eliminated systematic risk since they have 77 securities in each.

est lagged one-month return securities and decile P10 represents a portfolio of the highest lagged one-month return securities.

Strategy S1.2 This strategy was designed to exploit return predictability using lagged twelve month returns. Securities were sorted into deciles (P1 to P10) by descending lagged twelve-month return. As such, decile P1 represents a portfolio of the highest lagged twelve-month return securities and P10 represents a portfolio of the lowest lagged twelve-month return securities.

For strategies S1.0 and S1.2, a zero net investment "momentum portfolio" is constructed long on decile P1 stocks (winners) and short on decile P10 stocks (losers). In the case of very short-term return reversal strategy (S1.1) the portfolio is constructed in the opposite manner: long on recent losers and short on recent winners. Jensen's alpha for each series of momentum portfolios was calculated using the market model:

$$R_{pt} - R_{Ft} = a_p + b_p MKT_t + \epsilon_{pt} \tag{2.3}$$

where $MKT = R_{mt} - R_{Ft}$ and R_{mt} is the CRSP equal-weighted return. Jegadeesh finds positive and highly significant alphas for all strategies, thereby documenting the momentum effect.

However, Fama and French (1993) found the market model of (2.3) does not capture the size and book-to-market equity common risk factors (BE/ME). Therefore, Fama and French augmented the market model and arrive at a 3-factor model:

$$R_{it} - R_{Ft} = a_i + b_i MKT_t + s_i SMB_t + h_i HML_t + e_{it} \tag{2.4}$$

where $MKT_t = R_{Mt} - R_{Ft}$ represents the excess market equity premium, SMB proxies for size effect, and HML proxies for distress. Subsequently, Fama and French (1996) investigated whether return reversals or price continuation could be captured by the size and distress factors. After applying the 3-factor model of (2.4) above to several CAPM anomaly producing portfolios including E/P, C/P, sales growth, and prior return-sorted portfolios, Fama and French (1996) found the model captured all anomalies *except* price momentum.

Several subsequent studies confirmed the results of Jegadeesh (1990). Jegadeesh and Titman (1993) adjusted strategy S1.2 by using short-term (3 to 12 months) returns as sorting criteria for momentum portfolios and confirmed short-term price continuation.

Chan, Jegadeesh, and Lakonishok (1996) showed portfolios based on six-month prior returns demonstrated the momentum effect. The authors demonstrated stocks in the higher deciles of prior 6-month returns have higher subsequent returns and the difference between returns to winners and returns to losers could not be explained by the Fama and French 3 factor model.

Several other authors have confirmed the momentum effect. Carhart (1997) found that short-term (3 to 12 month) price continuation exists. Rouwenhorst (1998) confirmed the price momentum effect is also present in 12 European markets along with internationally (European-US) diversified

portfolios. Jegadeesh and Titman (2001) repeated the analysis of Jegadeesh and Titman (1993) on an updated sample and found similar results. Korajczyk and Sadka (2004) analyzed winner portfolios to abstract from the effect of short selling and found the momentum effect to be robust to trading costs. Although Grundy and Martin (2001) found round-trip transaction costs of long/short momentum strategies in excess of 1.77% drive momentum profits to zero, Geczy, Musto, and Reed (2002) found momentum profits still existed in the presence of short selling costs. In sum, the evidence for the persistence of short-term price continuation is convincing.

2.1.2 Post earnings announcement drift (earnings momentum)

Chan, Jegadeesh, and Lakonishok (1996) also show portfolios based on earnings surprise measures demonstrate the momentum effect. The authors demonstrate past price performance is related to past earnings performance. Jegadeesh and Titman (1993) and Chordia and Shivakumar (2006) found similar results around earnings announcement dates.

2.1.3 Directional momentum

Akhbari, Gressis, and Kawosa (2006) extended momentum analysis by looking at the *direction* of momentum. The authors note that within a particular momentum decile, individual stocks will have varying degrees of momentum. In other words, price patterns of individual stocks during the formation period will vary even within a given momentum decile. This variation can provide valuable economic information that investors use to update expectations. Using data on no-load mutual funds over the 1982 to 2001 sample

period, the authors found the performance of traditional momentum strategies is mediocre in contrast to enhanced profitability when incorporating their refined "directional momentum" criteria for asset selection.

2.1.4 Other related strategies

Contrarian strategies Contrarian strategies are successful in the context of return reversals. DeBondt and Thaler (1985) find long-term return reversals by selecting stocks based on prior 3-5 year performance and holding for 3-5 years after portfolio formation. Jegadeesh (1990) and Lehmann (1990) find very-short term return reversals. The authors form portfolios based on prior *week* or *month* returns and hold those portfolios for equivalent time frames. However, Ball, Kothari, and Shanken (1995) found the profitability of contrarian strategies is significantly diminished when considering microstructure effects such as bid/ask spreads.

Relative strength strategies Relative strength strategies seek to obtain abnormal returns by selecting stocks with more favorable fundamental ratios such as price to earnings (P/E), price to cash flow (P/C), and price to sales (P/S). For instance, Fisher and Humphrey (1984), Senchack and Martin (1987), Fisher (1988), Jacobs and Levy (1988), and Aggarwal, Rao, and Hiraki (1990) apply CAPM to prove low P/S stocks outperform high P/S stocks. However, the empirical and theoretical basis of CAPM is questioned by Fama (1976), Roll (1977), Levy (1983), and Levy and Levy (1987). With this in mind, Liao and Chou (1995) instead examine the PSR effect using a stochastic dominance approach and confirm the P/S effect: low P/S

portfolios have stochastic dominance over high P/S portfolios *and* randomly selected portfolios. Finally, O'Shaughnessy (2005) documents many successful relative strength strategies.

2.2 Explanations of momentum profit persistence

This section reviews the various explanations and associated test results. Results are summarized in Table **??** at the end of this sub-Chapter. Throughout this study, the term *formation period* or *estimation period* refers to the period of time prior to forming a portfolio in which prior returns (or unexpected earnings) are used as sorting criteria. *Holding period* refers to the period of time in which a portfolio is held. Finally, *post-holding period* refers to the period of time after the holding period, typically used to asses the presence of return reversals.

2.2.1 Asset pricing review

Momentum profits may be the result of compensation for greater risk inherent in momentum strategies. That is, returns in momentum strategies are commensurate with risk of those strategies. To illustrate, the asset pricing mechanics in Cochrane (2005) are briefly discussed here.

Rational investors will maximize intertemporal utility. Lucas (1978) shows the Euler condition for a representative agent's intertemporal utility maximization is:

$$p_t u'[c_t] = \beta E_t \left[u'[c_{t+1}] (p_{t+1} + d_{t+1}) \right] \tag{2.5}$$

For details on the derivation of this condition, see Appendix 3.2.1. The equality in (2.5) reveals the cost in marginal utility ($p_t u'[c_t]$) of purchasing the asset must be equal to the discounted (β) expected gain of the future payoff ($u'[c_{t+1}](p_{t+1} + d_{t+1})$). Dividing both sides by $p_t u'[c_t]$ and defining $R_{t+1} \equiv (p_{t+1} + d_{t+1})/p_t$ yields the familiar discount factor representation:

$$E_t[m_{t+1} R_{t+1}] = 1 \tag{2.6}$$

where

$$m_{t+1} = \beta \frac{u'[c_{t+1}]}{u'[c_t]} \tag{2.7}$$

Cochrane (2005) notes the stochastic discount factor in (2.7) is unobservable and therefore empirical proxies must be used. In particular, variables that vary with consumption growth c_{t+1}/c_t should proxy for marginal utility growth. Considering K possible variables leads to the approximation:

$$m_{t+1} \approx a_0 + \sum_{k=1}^{K} a_k f_{k,t+1} \tag{2.8}$$

In addition, the approximation of (2.8) maps into the linear factor model:

$$E[R_i] = \lambda_0 + \sum_{k=1}^{K} \lambda_k b_{ik} \quad i = 1 \dots N \tag{2.9}$$

where the independent variables b_i are obtained from the time-series regression:

$$R_{i,t} = b_{i0} + \sum_{k=1}^{K} b_{ik} f_{k,t} + \epsilon_{it} \quad t = 1 \dots T \tag{2.10}$$

Excess returns The use of excess returns, $R_{ei,t} = R_{i,t} - R_{j,t}$, has two advantages. First, it simplifies the Euler condition (2.6) :

$$E_t \left[m_{t+1} R_{ei,t+1} \right] = 0 \tag{2.11}$$

since R_{ei} represents a zero net investment portfolio (i.e., one dollar is borrowed at rate $R_{j,t}$ and invested in asset i. Second, it eliminates the intercept term from the linear factor model (2.9):

$$E \left[R_{ei} \right] = \sum_{k=1}^{K} \lambda_k b_{ik} \quad i = 1 \dots N \tag{2.12}$$

Therefore, equations (2.8), (2.11), and (2.12) reflect the fact that excess returns are zero after [properly] controlling for risk.

Conditional vs. unconditional models The stochastic discount factor of equation (2.8) has a subtle implication: the coefficients on the factors are constant throughout time. The constant coefficients implication allows the *conditional* expectation of (2.11) to be written as an *unconditional* expectation: $E \left[m_{t+1} R_{ei,t+1} \right] = 0$. However, if these coefficients vary over time with respect to a macroeconomic variable z_t, the stochastic discount factor is more accurately represented by:

$$m_{t+1} = a_{0t} + a_{1t} f_{1,t+1} + a_{2t} f_{2,t+1} + \dots + a_{kt} f_{k,t+1}$$

As such, equation (2.6) can be represented by:

$$E_t \left[m_{t+1} R_{t+1} | \, z_t \right] = 0$$

2.2.2 Empirical asset pricing models

All of the explanations below are framed as specializations of the general triplet of asset pricing equations from the previous section

$$m_{t+1} = a_{0t} + \sum_{k=1}^{K} a_{kt} f_{k,t+1} \tag{2.13}$$

$$E\left[R_i\right] = \lambda_0 + \sum_{k=1}^{K} \lambda_k b_{ik} \tag{2.14}$$

$$R_{it} = b_{i0} + \sum_{k=1}^{K} b_{ik} f_{k,t} + \epsilon_{it} \tag{2.15}$$

In a correctly specified model, incorporation of the asset pricing restriction (2.14) into the time series regression (2.15) should produce error terms that are conditionally and unconditionally equal to zero.

Standard CAPM The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has a single factor, the market risk premium ($f_{1,t} = MKT_t = R_{m,t} - R_f$), and static discount factor parameters ($a_k = a_k \ \forall t$):

$$m_{t+1} = a_0 + a_1 MKT_{t+1} \tag{2.16}$$

$$E\left[R_i\right] = \lambda_0 + \lambda_m b_{im} \tag{2.17}$$

$$R_{it} = b_{i0} + b_{im} MKT_t + \epsilon_{it} \tag{2.18}$$

Carhart (1997) found a positive and highly significant b_{i0} and no explanatory power with an R^2 value of -0.002 on a portfolio long on winners and short on losers using data on mutual funds formed on 1-year lagged return from 1963 to 1993. Rouwenhorst (1998) performed a similar analysis and also found a positive and highly significant b_{i0} and no explanatory power $\left(R^2 = 0.00\right)$ when applying CAPM to momentum portfolios based on the value-weighted Morgan Stanley Capital International index. Jegadeesh and Titman (2001) found CAPM models have positive and significant b_{i0} for all performance deciles and the momentum portfolio. Bansal, Dittmar, and Lundblad (2005) found b_{i0} to be statistically insignificant (and negatively signed) along with low R^2 (0.065) when applying CAPM to a set of 30 portfolios: 10 size, 10 prior return, and 10 book to market sorted portfolios[4]. In sum, the evidence is consistent that standard CAPM is unable to explain momentum profitability.

Fama and French (1993) 3-factor model In an attempt to explain the size and value premiums, Fama and French (1993) added two factors to CAPM: $f_{2,t} = SMB_t$ which represents the returns to a portfolio long on small companies and short big companies and $f_{3,t} = HML_t$ which represents the returns to a portfolio long on high book-to-market value companies and

[4] One possible explanation is that the portfolios have too few securities therefore average return measures merely reflect compensation for idiosyncratic risk. The analysis of Bansal, Dittmar, and Lundblad (2005) was replicated and the number of securities per portfolio were computed at a point in time (June 2000) to reveal that each decile has over 200 securities and thus well diversified. For details see the Appendix.

short low book-to-market value companies:

$$m_{t+1} = a_0 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1} \tag{2.19}$$

$$E[R_i] = \lambda_0 + \lambda_m b_{im} + \lambda_s b_{is} + \lambda_h b_{ih} \tag{2.20}$$

$$R_{it} = b_{i0} + b_{im} MKT_t + b_{is} SMB_t + b_{ih} HML_t + \epsilon_{it} \tag{2.21}$$

Fama and French (1996) found the three factor model does explain size, value, and long-run return reversals, however it was unable to explain short-term (portfolios formed from $t-12$ to $t-2$) price continuation. Specifically, Fama and French found significantly negative b_{i0} for loser portfolios and significantly positive b_{i0} for winner portfolios. Jegadeesh and Titman (2001) also applied the 3-factor models and found positive and significant b_{i0} for all performance deciles and the momentum portfolio. Chordia and Shivakumar (2006) found b_{i0} to be positive and significant for momentum and SUE portfolios using the Fama and French (1993) 3-factor model. Bansal, Dittmar, and Lundblad (2005) found b_{i0} to be insignificant along with low R^2 (0.362) when applying the 3-factor model to a set of 30 portfolios: 10 size, 10 prior return, and 10 book to market sorted portfolios. In sum, the evidence suggests the 3-factor model is unable to explain momentum profits.

4-factor price momentum model Carhart (1997) analyzed price momentum but unlike Chan, Jegadeesh, and Lakonishok (1996), he extended the Fama and French 3-factor model by adding a 4th factor rather than regressing current returns on prior returns. This additional factor measures the return on a zero net investment portfolio long on prior 1 year winners and short on prior 1 year losers ($f_{4,t} = WML_t$) and is added to the 3-factor

model as follows:

$$m_{t+1} = a_0 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1} + a_4 WML_{t+1} \tag{2.22}$$

$$E\left[R_{ei}\right] = \lambda_0 + \lambda_m b_{im} + \lambda_s b_{is} + \lambda_h b_{ih} + \lambda_w b_{iw} \tag{2.23}$$

$$R_{it} = b_{i0} + b_{im} MKT_t + b_{is} SMB_t + b_{ih} HML_t + b_{iw} WML_t + \epsilon_{it} \tag{2.24}$$

Carhart found the WML factor to be a significant explanatory variable. However, a_i was significant for all but the top two deciles of prior 1-year return sorted portfolios therefore the model does not fully explain the variation in momentum portfolio returns.

4-factor earnings momentum model Chordia and Shivakumar (2006) further examined the unexplained short-term momentum by forming a factor that measures the return on a zero net investment portfolio long on high SUE firms and short on low SUE firms ($f_{4,t} = PMN_t$) and adding it to the 3-factor model as follows:

$$m_{t+1} = a_0 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1} + a_4 PMN_{t+1} \tag{2.25}$$

$$E\left[R_i\right] = \lambda_0 + \lambda_m b_{im} + \lambda_s b_{is} + \lambda_h b_{ih} + \lambda_p b_{ip} \tag{2.26}$$

$$R_{it} = b_{i0} + b_{im} MKT_t + b_{is} SMB_t + b_{ih} HML_t + b_{ip} PMN_t + \epsilon_{it} \tag{2.27}$$

Chordia and Shivakumar, who used monthly data from January 1972 to December 1999, found their four factor model described the variation in momentum portfolio returns in both time-series and cross-sectional tests. Specifically, they found the null hypothesis, $b_{i0} = 0$ for all i (where i represents deciles of prior six-month return portfolios), can *not* be rejected at the

1% level and conclude their model is well specified. Furthermore, Chordia and Shivakumar show neither the Fama and French (1993) 3-factor model (2.21) nor the Carhart (1997) four-factor model (2.24) are well-specified for explaining standardized unexpected error (SUE) portfolio returns. However, the authors acknowledge PMN's negative relationship to the macro economy is inconsistent with systematic risk and is suggestive of investor irrationality in the form of "inflation illusion," which is discussed in detail in Section.

Consumption CAPM Intuitively, marginal utility growth should be related to consumption growth. To illustrate, consider the form of the stochastic discount factor with constant relative risk aversion:

$$m_{t+1} = \beta \frac{u'\left[c_{t+1}\right]}{u'\left[c_t\right]}$$

Consumption CAPM (CCAPM) relies on the connection between consumption growth and marginal utility growth. For example, with CRRA utility the discount factor is $m_{t+1} = \beta \left(c_{t+1}/c_t\right)^{-\sigma}$ of which a simple linear approximation would be $m_{t+1} \approx a_0 + a_1(c_{t+1}/c_t)$. Let $f_{1,t} = g_c$ represents the growth rate of aggregate (and typically) nondurable consumption. The resultant asset pricing model can be expressed as:

$$m_{t+1} = a_0 + a_1 g_{c,t+1} \tag{2.28}$$

$$E\left[R_i\right] = \lambda_0 + \lambda_c b_{ic} \tag{2.29}$$

$$R_{it} = b_{i0} + b_{ic} g_{c,t} + e_{it} \tag{2.30}$$

Bansal, Dittmar, and Lundblad (2005) show the standard consumption-based CAPM (unconditional C-CAPM) is unable to explain the cross section of size, value, and momentum portfolios $\left(R^2 = 0.027\right)$ for data covering January 1967 to April 2001. Furthermore, the findings of Epstein and Zin (1991) and Bansal and Yaron (2003) also suggest consumption beta may be insufficient to measure asset risk.

Cash flow CAPM Bansal, Dittmar, and Lundblad (2005) developed what will be referred to as "Cash flow CAPM" (CFCAPM). In this model, the priced risk factor is cash flow exposure to aggregate consumption fluctuations. Note that this is distinct from *return* sensitivity to aggregate consumption (b_{ic} above). As such, their model can be expressed as:

$$E\left[R_i\right] = \lambda_0 + \lambda_{cf} b_{icf} \tag{2.31}$$

$$g_{it} = b_{icf}\left(\frac{1}{K}\sum_{j=1}^{K} g_{c,t-k}\right) + e_{it} \tag{2.32}$$

where $g_{i,t}$ represents the cash flow (dividend or dividends plus repurchases) growth rate of portfolio i and g_c represents the growth rate of aggregate nondurable consumption. The authors used demeaned growth rates to allow estimation without the intercept term, b_{i0}. The authors found that cash flow sensitivity to aggregate consumption (b_{icf}) of winner portfolios is larger than that of loser portfolios. The net effect is a dramatic improvement in cross-section R^2 values (0.027 for CCAPM, 0.6 for CFCAPM) based on quarterly data from 1967 to 2001. This means while aggregate consumption growth rates are unable to explain the cross-section of returns (including mo-

mentum), the sensitivity of dividend growth rates to aggregate consumption growth rates can.

The authors posited two possible explanations to reconcile non-informative consumption betas with informative aggregate-consumption derived cash flow betas. First, if there are multiple risk factors in returns, then consumption beta may not capture cross-sectional variations in risk premia. Empirically, the authors found cash-flow sensitivity to consumption is one such risk factor. The second explanation is the difficulty in measuring aggregate consumption introduces error and therefore reduces the ability for consumption beta to explain cross-sectional differences.

Regarding the multiple risk factor explanation, their model accounts for approximately 60% of the cross-sectional variation in returns. Therefore, the same argument regarding additional unidentified factors can be applied to CFCAPM as done with the CCAPM: there must exist additional factors that explain the remaining 40%. Secondly, measurement error in aggregate consumption invariably propagates to their aggregate-consumption based measure of cash flow sensitivity. Furthermore, the authors acknowledge their cash flow risk measures are "measured with considerable error." As such, the relatively high CFCAPM R^2 may be a spurious result[5].

In addition, CCAPM can be derived by presuming CRRA utility and utilizing a linear approximation of marginal utility growth[6], As such, the poor

[5] Another potential contributor to the unexplained variation in returns is the fact that consumption data are revised throughout the year. For instance, Runkle (1998) analyzes the impact of aggregate data revisions in the context of policy research and finds that revised data should not be used. Rather, to understand policy decisions data at the time of the policy decision must be used. Similar concerns apply in the context of asset pricing models reliant on aggregate consumption data.

[6] $m_{t+1} = (c_{t+1}/c_t)^{-\gamma} = g_c^{-\gamma} \approx a_0 + a_1 g_c$

performance of CCAPM should come at no surprise given the confounding effects of the inability of CRRA utility to account for "equity premium puzzle" and the linear approximation of marginal utility growth.

Macroeconomic risk Chordia and Shivakumar (2002) argue momentum profits are an artifact of the sensitivity of returns to macroeconomic variables. Specifically, the authors find momentum profits are related to lagged macroeconomic variables. They interpret their results as a trickle down effect: time varying economic variables impact *conditional* expectations of firms differentially and momentum profits are compensation for bearing time-varying risk (Berk, Green, and Naik, 1999). Therefore momentum profits should be higher when the macro economy is in a favorable state (expansion) than an unfavorable state (recessions).

On the surface, Cooper, Gutierrez, and Hameed (2004) agree: they find momentum profits follow favorable *market* states (lagged 1 to 3 year returns) exclusively. However, the authors also find macroeconomic variables (dividend yield, default spread, term spread, and short-term interest rates) are unrelated to momentum profits. Furthermore, Griffin, Ji, and Martin (2003) find significant momentum profits in 17 international markets in both good and bad macroeconomic states.

2.2.3 Additional asset pricing tests

In this section two models, production CAPM and conditional CAPM are reviewed even though the studies under consideration are not momentum-specific. The general lack of fit ($R^2 < 0.65$) in both cases is suggestive they

would be unsuccessful in explaining excess returns to momentum portfolios.

Conditional CAPM Jagannathan and Wang (1996) argue that the CAPM's inability to explain the cross-section of expected returns is due to the assumption of static betas. Although the authors emphasize they "do not assume that returns have a linear factor structure," their empirical specification (equation (23) in their paper) maps into a restricted linear conditional factor model as follows:

$$\begin{aligned} m_{t+1} &= a_{0t} + a_{1t} R_{vw,t+1} + a_{2t} R_{lab,t+1} \\ &= (a_{00} + a_{01} R_{prem,t}) + (a_{10} + a_{11} R_{prem,t}) R_{vw,t+1} \\ &\quad + (a_{20} + a_{21} R_{prem,t}) R_{lab,t+1} \end{aligned}$$

where $R_{prem,t}$, the yield spread between BAA- and AAA-rated bonds, serves as a proxy for the market risk premium and R_{lab} represents the rate of change in labor income. Eliminate interaction terms by setting $a_{11} = a_{21} = 0$ and let $a_0^* = a_{00}$, $a_1^* = a_{10}$, $a_2^* = a_{01}$, and $a_3^* = a_{21}$ resulting in the triplet of asset pricing equations:

$$m_{t+1} = a_0^* + a_1^* R_{vw,t+1} + a_2^* R_{prem,t} + a_3^* R_{lab,t+1} \tag{2.33}$$

$$E[R_i] = \lambda_0 + \lambda_{vw} b_{ivw} + \lambda_{prem} b_{iprem} + \lambda_{lab} b_{ilab} \tag{2.34}$$

$$R_{it} = b_{i0} + b_{ic} R_{vw,t} + b_{ip} R_{prem,t-1} + b_{ilab} R_{lab,t} + \epsilon_{it} \tag{2.35}$$

The authors analyze CRSP NYSE and AMEX stocks from 1963 to 1990. Ten size portfolios are formed and the system of moments ($E[m_{t+1} R_{i,t+1}] = 1$) is

estimated using generalized method of moments (GMM) technique of Hansen (1982) and several versions of the discount factor (2.33). Specifically, four versions of the static CAPM model were estimated: (1) plain-vanilla CAPM yielded $R^2 = 0.0135$, (2) CAPM with a size variable yielded $R^2 = 0.5756$, (3) CAPM with with a the human capital (R_{lab}) variable yielded $R^2 = 0.3046$), and finally (4) CAPM with human capital and size yielded $R^2 = 0.5855$. The respective R^2 values for the conditional CAPM model (i.e., the inclusion of $R_{prem,t-1}$, are 0.2932, 0.6166, 0.5521, and 0.6473. However, these tests were not applied to momentum portfolios.

Production CAPM Production CAPM (PCAPM) is analogous to consumption CAPM. The PCAPM partial equilibrium model derives asset prices from intertemporal profit maximization (which is based on a firm's production function) and the associated investment demand. In other words, expected returns are a function of investment growth. In this model, investment return, or the marginal rate of transformation, is the rate at which delayed or reduced production at time t (used for investment) is transformed into production at time $t + 1$. The empirical specification is:

$$R_{it} - R_{ft} = a_i + b_{i,p} PRO_t + \epsilon_{it} \tag{2.36}$$

There are two reasons for the interest in PCAPM. First, consumption measures are difficult to measure whereas production measures are related to less controversial measures of output (Balvers, Cosimano, and McDonald, 1990; Bansal, Dittmar, and Lundblad, 2005; Balvers and Huang, 2005). Sec-

ond, from a theoretical perspective, PCAPM is less sensitive to the functional form specification (Cochrane, 1991).

The PCAPM model of Cochrane (1991) predicts stock returns and investment returns should be exactly equal when (1) markets are complete in which managers can construct mimicking portfolios and (2) as arbitrage opportunities between capital markets and investment opportunities by firms are removed. Cochrane's empirical findings support his theoretical predictions. However, one might infer PCAPM models are unlikely to explain momentum returns given low R^2 values ($<$0.30) for Cochrane's regression of stock returns on investment capital ratios[7].

2.2.4 Additional risk-based explanations

Differential expected return Assuming stock prices follow a random walk with drift process, Conrad and Kaul (1998) find momentum profits are significantly related to cross-sectional differences in unconditional mean returns. However, the authors do not specifically test post-holding period returns. Their results do show longer holding periods ($>$ 24 months) are unprofitable which is suggestive of return reversals. Jegadeesh and Titman (2001) analyze holding period and post-holding period returns and find post holding period returns are in fact negative, which is contrary to the Conrad and Kaul finding of cross-sectional differences in *unconditional* mean returns.

Growth rate risk Momentum profits also result from the the combination of stock price sensitivity to expected growth rates, stochastic expected growth

[7] Similarly, after accounting for empirical issues Arroyo (1996) states "The fits of the stock return regressions are still not particularly good" in his PCAPM model.

rates, and shocks to those growth rates (Johnson, 2002). To illustrate stock price sensitivity, consider the simple dividend discount model:

$$P_0 = \frac{D_1}{k - g}$$

where P_0 is the current price, D_1 is the next period dividend, k is the discount factor given the firm's risk, and g is the dividend growth rate. Obtaining the natural log and taking the first and second derivatives with respect to g:

$$\begin{aligned} \ln[P_0] &= \ln[D_1] - \ln[k - g] \\ \frac{d \ln[P_0]}{dg} &= \frac{1}{k - g} > 0 \qquad k > g \\ \frac{d^2 \ln[P_0]}{dg^2} &= -\left(\frac{1}{k - g}\right)^2 < 0 \end{aligned}$$

Therefore, the log of stock prices is a convex function in g.

Johnson (2002) presents a theoretical argument for investor rationality in the context of momentum profits. Johnson notes stock price sensitivity to growth rates is exponential and therefore growth rate risk increases with growth rate. Johnson shows growth rate risk (sensitivity of stock price to changes in growth rate) is priced and persistent episodic growth rate shocks lead to observed momentum. Additionally, Bansal, Dittmar, and Lundblad (2005) model cash flow (dividend) growth rates as a function of aggregate consumption and firm dividend yield and find (i) cash flow growth rates of winners are higher that those of losers, (ii) cash flows of winners are more sensitive to aggregate consumption, and (iii) cash flow risk is priced, consistent with Johnson (2002).

2.2.5 Behavioral explanations

Market under-reaction Under-reaction can result from *conservatism bias* or mental accounting. Experiments by Edwards (1968) identify a "conservatism bias" in that individuals underweight new information. As a result, prices slowly adjust to new information but eventually achieve a steady state that contains all information.

Grinblatt and Han (2005) note the *disposition effect* of Shefrin and Statman (1985) that describes the tendency of investors to hold on to losing stocks too long and selling winner stocks too soon. Grinblatt and Han (2005) suggest the prospect theory of Kahneman and Tversky (1979) combined with the *mental accounting* framework of Thaler (1980) explains the disposition effect. A manifestation of the *disposition effect* is the slow but eventual incorporation of information. As such, post-holding period abnormal returns should be zero.

Empirical results are mixed. Chan, Jegadeesh, and Lakonishok (1996) find little evidence of return reversals in price and earnings momentum portfolios thereby supporting market under-reaction. In contrast, Jegadeesh and Titman (2001) observe negative post-holding period returns that are suggestive of market *over*-reaction.

Related to under-reaction and strong-form efficiency, informed traders with private information may time their purchases to conceal their identity and thereby minimize the price impacts of trade. As such, prices will slowly incorporate this private information. Hong, Lim, and Stein (2000) find holding size constant, stocks with low analyst coverage exhibit greater

momentum profits which is suggestive of slow incorporation of private information. However, the authors do not check for return reversals to confirm their results.

Market over-reaction The extant literature has put forth five over-reaction explanations: (1) the *representative heuristic* of Tversky and Kahneman (1974), (2) *self-attribution* argued by Daniel, Hirshleifer, and Subrahmanyam (1998), (3) interactions between "news watchers" and "trend watchers" discussed by Hong and Stein (1999), (4) the impact of positive feedback trading strategies discussed by DeLong, Shleifer, Summers, and Waldmann (1990), and (5) a result of earnings management suggested by Myers, Myers, and Skinner (2006).

The "representative heuristic" suggests individuals identify an uncertain event, e.g., future extraordinary earnings, with the "parent population," e.g., current earnings surprises. The "self-attribution" argument suggests informed traders attribute *ex-post* winning trades to their skill and *ex-post* losing trades to bad luck. This overconfidence leads to inflated prices of winners and perhaps also losers. Informed traders, who watch and obtain information from news, transmit this information with a delay to market participants who do not watch the news. After receiving the delayed signal, trend-watchers trading reinforces stock price inflation. Investors following positive feedback strategies purchase stocks as they rise and sell stocks as they fall irrespective of fundamental information. Finally, if abnormal returns are associated with ever increasing earnings per share (EPS) reports, managers have incentive to manipulate earnings to that effect.

In all four scenarios, the spread between actual and fundamental value widens for a finite period of time. As such, return reversals are inevitable. Initially, DeBondt and Thaler (1985) found statistically significant return reversals for both winners and losers. However, Chan, Jegadeesh, and Lakonishok (1996) find little evidence of return reversals in price and earnings momentum portfolios thereby rejecting market over-reaction. In contrast, Jegadeesh and Titman (2001) do observe return reversals. However, the authors note their results are sensitive to sample composition, sample period, and risk-adjustment of post-holding period returns. This suggests return reversals are not guaranteed and behavioral arguments provide only a partial explanation of the momentum anomaly. Cooper, Gutierrez, and Hameed (2004) find momentum profits follow market "up" states exclusively, observe return reversals, and relate their observance to the over-reaction explanations of DeBondt and Thaler (1985) and Daniel, Hirshleifer, and Subrahmanyam (1998).

Myers, Myers, and Skinner (2006) investigate the relationship between EPS reporting and abnormal returns and obtain several interesting findings. First, successive increases in EPS are not by chance, but rather, suggestive of earnings management. Second, abnormal returns on the order of 20 percent per year are associated with firms reporting successive increases in EPS. Finally, a negative stock market reaction is reported with the end of these EPS increase "strings," which is consistent with over-reaction.

Money and inflation illusion The "money illusion" hypothesis of Modigliani and Cohn (1979) suggests investors mistakenly capitalize earnings at nominal

rates as opposed to real rates. To illustrate, consider a zero growth firm with value market value $V[t]$, real earnings $X[t]$, real discount (capitalization rate) k, and inflation rate p. Under rational valuation, real earnings are capitalized at the real capitalization rate:

$$V^r[t] = \frac{X[0]e^{pt}}{k} = V^r[0]e^{pt} \tag{2.37}$$

where the superscript r is used to emphasize the rational valuation of the firm. The illusional valuation therefore is:

$$V^i[t] = \frac{X[0]e^{pt}}{k+p} \tag{2.38}$$

To see why V^r is rational and V^i is illusional, consider the expected return with assets priced according to (2.37) versus (2.38). The nominal return (capitalization rate $K = k + p$) for the rational investor is:

$$K^r = \frac{X[0] + \frac{dV^r}{dt}\big|_{t=0}}{V^r[0]} = \frac{X[0]}{V^r[0]} + \frac{pV^r[0]}{V^r[0]} = k + p$$

Therefore the firm is priced such that the realized nominal return matches the required return implied by (2.37). In contrast, for the illusional valuation:

$$K^i = \frac{X[0] + \frac{dV^i}{dt}\Big|_{t=0}}{V^i[0]} = \frac{X[0]}{V^i[0]} + \frac{pV^i[0]}{V^i[0]} = k + 2p$$

As shown, the actual nominal rate of return exceeds the required rate of return implied by (2.38).

Chordia and Shivakumar (2006), extend this particular "money illusion" manifestation by attributing the error to mistaking nominal *growth* rates for real while properly updating discount rates. [8] As a result, the authors contend the negative relationship between their risk factor (PMN) and the macroeconomy is attributable to inflation illusion. Therefore, although the risk factor (PMN) of Chordia and Shivakumar (2006) can describe momentum, the explanation of the factor's behavior relies on investor inability to evoke knowledge of inflation when it is relevant(Simon, 2000) [9].

2.2.6 Market friction

Momentum strategies are transaction intensive. Portfolios must be constructed, reconstituted, and also be engaged in short positions. Trading costs result from spreads, the price impacts of trade, trade commissions, and margin expenses for short positions. As such, the momentum profits should disappear after trading costs are considered.

Several authors have noted the majority of momentum profitability is obtained from the short position Hong, Lim, and Stein (2000); Lesmond, Schill, and Zhou (2004); Grinblatt and Moskowitz (2004). However, the re-

[8] For a full illustration see Appendix (A.1). The end result are rational and illusional values after a change in inflation from p to p^+ of:

$$V^r\left[0^+\right] = \frac{X[0]}{k-g} = V^r[0]$$

$$V^i\left[0^+\right] = \frac{X[0]}{(k+p^+)-(g+p)} = \frac{X[0]}{k-g+(p^+-p)}$$

When inflation rises $\left(p^+ > p\right)$ the illusional value is lower than the rational value therefore securities are undervalued. When inflation declines $\left(p^+ < p\right)$ the illusional value is greater than the rational value and therefore securities are overvalued.

[9] Perhaps the perceived (or actual) benefit of additional calculations an analysis (i.e., updating expected growth rates) does not outweigh the cost (time, mental effort, etc.).

sults regarding the impact of short selling costs on momentum profitability are mixed. Grundy and Martin (2001) found that once round-trip transaction costs of the long/short momentum strategy exceed 1.5% momentum profits are insignificant. In contrast, Geczy, Musto, and Reed (2002) presented evidence that momentum profitability still persists when short selling costs are incorporated.

To abstract from the impact of short selling costs, Korajczyk and Sadka (2004) analyzed the role of proportional trading costs (spread) and non-proportional costs (price impacts of trade) in the profitability of past *winner* stock portfolios. The authors found abnormal returns remained statistically significant after controlling for proportional trading costs in the form of quoted and effective spreads. Turning to non-proportional costs, the authors employed two price-impact of trade specifications: the Breen, Hodrick, and Korajczyk (2002) model and the Glosten and Harris (1988) model. The authors found that unlike proportional trading costs, the price-impact costs do drive abnormal returns to zero, but only for portfolios larger than roughly $2.0 billion. This is due to the fact that both price-impact cost specifications are increasing functions in quantity, and quantity of shares increases with portfolio size.

Korajczyk and Sadka (2004) justify the omission of direct commission costs by citing the work of Breen, Hodrick, and Korajczyk (2002) who found price impact cost estimates are larger than actual price impact costs by an amount larger than commissions. In contrast, Lesmond, Schill, and Zhou (2004) found momentum strategies require frequent trading of securities that have high trading costs and momentum profit opportunities do not exist. In

sum, the extant literature provides mixed results regarding the ability of market frictions to explain momentum strategy profitability.

2.2.7 Data mining

Momentum profits may also be the result of data mining. As such, results should be valid only in-sample. Jegadeesh and Titman (2001) repeat their earlier strategy (Jegadeesh and Titman, 1993) using a different sample (1990 to 1998 vs. the original 1965 to 1989 timeframe) and find momentum profits persist[10]. Also, Rouwenhorst (1998) finds significantly positive momentum profits in 12 European markets along with internationally (European-US) diversified portfolios. The results of these authors suggest data mining is not an explanation of the persistence of momentum profitability.

2.3 Neoclassical investment theory and *q*

2.3.1 Overview

Neoclassical investment theory (NIT), a partial equilibrium analysis of intertemporal firm profit maximization also offers potential to explain momentum profits. Work based on NIT is closely related to the production based capital asset pricing model (production-CAPM) in that both are focused on

[10] One might suspect momentum profits during both sample periods might be due to the buildup to and realization of the dot com boom. To that concern, during the January 2000 to December 2006 time period, this study found that the price momentum strategy of Jegadeesh and Titman (2001) does not produce significant mean returns (see Table 3 on page 81). However, in that same table the earnings momentum strategy of Chordia and Shivakumar (2006) does produce economically and statistically significant profits during the same time frame. Finally, Griffin, Ji, and Martin (2003) and Cooper, Gutierrez, and Hameed (2004)found that momentum profits are unrelated to macroeconomic factors.

the supply side of the economy (firm output)[11].

Numerous authors who have examined NIT models and production-CAPM have presented evidence suggestive of the applicability of NIT and the importance of supply (production) side dynamics. Lucas (1978) has shown that asset returns are related to production growth. Peng and Shawky (1997) have found that exogenous productivity shocks help explain time-varying expected asset return behavior. Kasa (1997) has found production-CAPM models perform better than consumption-CAPM. Bansal, Dittmar, and Lundblad (2001) have found that a long-run cointegrated relationship between dividends and consumption exists, consistent with the construction of the model in this thesis (see Chapter 3). Zhang (2005) has applied the neoclassical model to explain value premium. Balvers and Huang (2007) have shown that the value premium and size premium are related to productivity shocks. Madsen and Davis (2006) have demonstrated that productivity shocks have temporary effects on equity returns.

Hayashi (1982) has summarized three theories to explain the relationship between market value and capital investment: neoclassical investment theory, modified neoclassical investment theory, and the q-theory of investment. Neoclassical investment theory obtains an optimal *level* of capital stock from a firm's maximization of discounted cash flows, given technological constraints established in the production function. Unfortunately, neoclassical investment theory has two shortcomings as noted by Hayashi. First, the theory assumes exogenous output (that leads to an optimal capital

[11] Note this is in contrast to the aforementioned demand-side aggregate consumption based approach employed by Bansal, Dittmar, and Lundblad (2005).

stock *level*) which is inconsistent with perfect competition. Second, neoclassical investment theory itself does not explain the *rate* of investment. Instead it relies on an ad-hoc adjustment in the form of a distributed lag investment function to attain a rate of investment.

A rate of investment is needed given that a firm has more control over the flow variables rather than level variables due to costs of implementing new capital stock. Lucas (1967), Gould (1968), Uzawa (1969), and Treadway (1969) acknowledged the need to incorporate adjustment (installation) costs and so formulated what will be called the modified neoclassical investment theory. By incorporating adjustment costs, the neoclassical investment theory shortcomings are addressed. As a result, the *rate* of investment is determined from the firm's optimization problem given technological constraints established in the production and adjustment cost functions.

The third theory of investment summarized by Hayashi (1982) is the q-theory of investment developed by Tobin (1969) which relates the *rate* of investment to q, the ratio of market value of new investment goods to their replacement costs. In theory, q should be equal to one however in practice it is not. Hayashi attributes this to the fact that a firm can not freely adjust its capital stock *level*, which is indicative of the existence of adjustment costs. Hayashi notes the distinction between *marginal* q (market value of additional unit of capital to replacement cost) and *average* q (market value of *existing* capital to replacement cost). Hayashi also shows these two measures are equivalent if the firm is a price taker and production and adjustment costs exhibit constant returns to scale.

The equivalence of q-theory and modified neoclassical investment theory

has been shown by many authors including, but not limited to, Lucas and Prescott (1971), Abel (1977), Yoshikawa (1980), Hayashi (1982), and Abel and Blanchard (1983). Madsen and Davis (2006) utilize this equivalence to analyze the impacts of technological innovations on share prices in the "new economy," which is discussed in the next section.

2.3.2 Transitional dynamics and observed return patterns

Numerous authors have found that post-holding period returns of momentum portfolios are negative (Jegadeesh and Titman, 2001; Cooper, Gutierrez, and Hameed, 2004; Myers, Myers, and Skinner, 2006). Figure 3 is provided to illustrate the resemblance between the transitional dynamics associated with NIT and observed momentum and subsequent return reversals. Initially, the market is in a steady state, i.e., a state in which all variables grow at the *constant* growth rate m. A temporary productivity shock induces a period of transition in which the growth rate and asset prices respond accordingly (Madsen and Davis, 2006). After the transition period, the market returns to the original steady state growth rate m.

2.4 Productivity shocks and equity prices

Madsen and Davis (2006) investigate the impact of productivity shocks on equity prices. This section takes a detailed look at the authors' findings as they relate closely to the model used in this thesis.

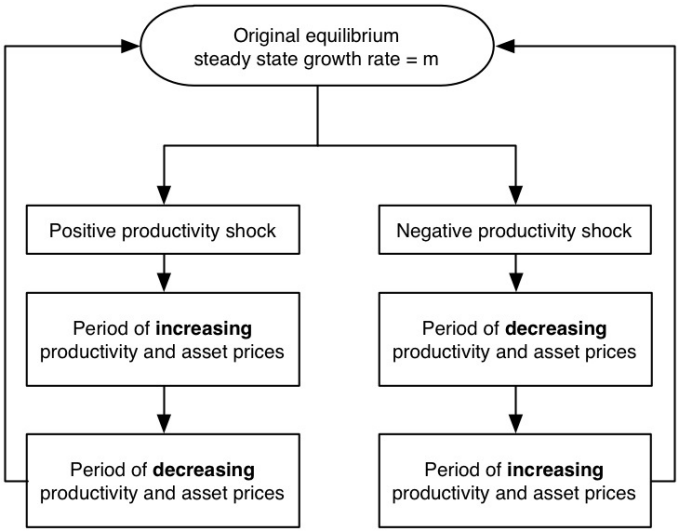


Fig. 3: Equilibrium impact of productivity shocks

2.4.1 Long-run growth rates

Begin with a Cobb-Douglas production function of the form $Y = BK^{1-\alpha}L^{\alpha}$ where B represents total factor productivity (TFP). The average and marginal productivity of labor and capital are:

$$\frac{Y}{L} = B\left(\frac{K}{L}\right)^{1-\alpha} \tag{2.39}$$

$$\frac{\partial Y}{\partial L} = \alpha B\left(\frac{K}{L}\right)^{1-\alpha} \tag{2.40}$$

$$\frac{Y}{K} = B\left(\frac{K}{L}\right)^{-\alpha} \tag{2.41}$$

$$\frac{\partial Y}{\partial K} = (1-\alpha)B\left(\frac{K}{L}\right)^{-\alpha} \tag{2.42}$$

Labor and capital marginal productivity are in constant proportion ($1/\alpha$ and $1/1-\alpha$, respectively) to average productivity; therefore average and marginal productivity growth rates are identical. Applying the time differentiation operator Δ to average labor and capital productivity yields their respective growth rates:

$$\gamma_{MPL} \equiv \Delta \ln [Y/L] = \Delta \ln[B] + (1-\alpha)\Delta \ln [K/L] \tag{2.43}$$

$$\gamma_{MPK} \equiv \Delta \ln [Y/K] = \Delta \ln[B] - \alpha\Delta \ln [K/L] \tag{2.44}$$

These equations reveal that increases in TFP increase labor and capital productivity. However, increases in K/L (capital deepening) increase labor productivity but reduce capital productivity. Empirically, the authors find K/L has grown geometrically 3.5% annually in OECD countries from 1960 to 2001 while at the same time TFP has increased by just 1.5% annually with $\alpha = 0.7$. Substituting 3.5% for $\Delta \ln [K/L]$, 1.5% for $\Delta \ln[B]$, and 0.7 for α in equations (2.43) and (2.44) yields a positive long-run growth rate in labor productivity ($\Delta \ln [Y/L] = 2.55\%$) but a decline in capital productivity ($\Delta \ln [Y/K] = -0.95\%$).

Also, the growth rate in output ($\Delta \ln[Y]$) and the growth rate in labor productivity ($\Delta \ln[Y/L]$) are biased estimates of capital productivity:

$$\begin{aligned} \Delta \ln[Y] - \Delta \ln[Y/K] &= \Delta \ln[K] \\ \Delta \ln[Y/L] - \Delta \ln[Y/K] &= \Delta \ln[K] - \Delta \ln[L] \end{aligned}$$

Therefore it would be in error to assume equality between output and productivity growth rates.

2.4.2 Theoretical model and results

The primary analytical framework employed by Madsen and Davis (2006) is a neoclassical investment theory partial equilibrium model that focuses on firm profit maximization with an exogenous discount rate. Thus, their primary model does not incorporate intertemporal utility maximization. However, the authors implicitly incorporate utility using an endogenous discount rate in a model of their associated working paper (Madsen and Davis, 2004). Both models characterize the shadow price of capital q and its associated dynamics[12].

The model is applied to aggregate investment data and aggregate equity prices (i.e., a stock price index). Technological innovations are introduced into the model via embodied technological progress (decreasing real price in investment) and disembodied technological progress (spill-over effects). The declining prices of computing and technology equipment enhances profits for positive levels of investment. Spill over effects result from increase marginal productivity of both past and current investment as a result of information and computer technology revolution. For instance, with the expansion of the Internet, both old and new computers are more productive. The authors performed comparative static analysis and also constructed phase plots to arrive at several conclusions regarding q.

[12] The shadow price of capital is the estimated price of capital in the closed economy environment which has no market for capital.

The authors found the long run shadow price of capital is unaffected by declining real prices of investment for those companies that invest in cheaper or more advanced equipment. The intuition behind this theoretical finding is that the positive effect of lower acquisition costs of new capital is offset by the negative effect of lower sales prices. However, this result applies only to firms that invest in the new technology. Firms that do not invest in the new technology will experience a shadow price drop since they must lower output prices while utilizing the relatively more expensive inputs.

The authors also found that the long-run equilibrium shadow price of capital is unaffected by spill-over effects. Similar to embodied technological progress, the positive effect of greater productivity per unit of capital is offset by the negative effect of lower output prices. Again, firms that do not, or in this case did not, undertake investment in technology that could benefit from spill-over effects will experience a decline in shadow price. In both cases, embodied and disembodied technological progress, the long-run equilibrium *level* of capital stock increases with technological innovation

The authors also investigated the transitional dynamics and results of Granger causality tests indicated share prices precede productivity gains. In other words, share prices experience an initial jump in anticipation of the expected change in future capital productivity. Productivity growth on the other hand does not jump, rather it follows a transitional path as depicted in 4. Although this figure depicts the impact of a positive productivity shock, rotating the diagram about the x-axis would produce an illustration of the general pattern for a negative shock. Examples of negative productivity shocks include Hurricane Katrina and the 9/11 terrorist attack.

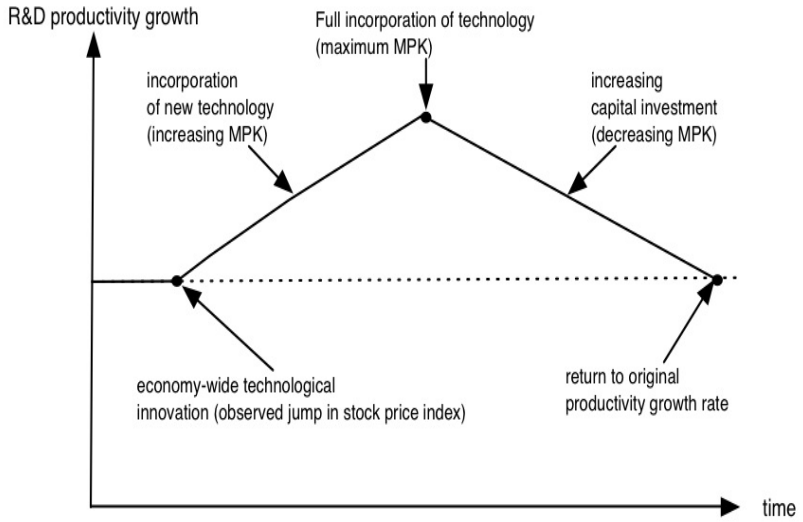


Fig. 4: Madsen and Davis (2006) theoretical implications

As shown, an initial jump in share prices reflects the economy wide technological innovation (e.g., the Internet). As time progress, firms incorporate the technological innovation into their business via investment in additional capital and perhaps adjustments to existing capital (installation of network cards in previously purchased computers). Eventually, the technology is fully incorporated and the firm arrives at the maximum productivity growth rate. From this point forward, capital investment continues but productivity growth slows due to decreasing marginal productivity of capital.

2.4.3 Empirical model and results

The theoretical implications are empirically confirmed by focusing on the causal relationship between equity prices and productivity. To begin, the authors are able to *reject* the null that equity return does *not* Granger-cause

productivity. Second, they find productivity growth does *not* precede equity returns. Therefore, equity prices reflect the future realization of technological innovation on productivity, consistent with the implications of the theoretical model (Figure 4). In addition, the authors performed panel data regressions and found no permanent effects of technology epochs (shocks) on earnings per unit of capital (capital productivity). The panel estimates are consistent with negative capital productivity growth suggested by (2.44) and declining output-capital ratios for tangible and intangible capital over the past century (Madsen and Davis (2006), Figure 1). In other words, when considering the growth rate of capital productivity (equation 2.44), the long-run growth of TFP is exceeded by the long-run growth of the capital-labor ratio thereby resulting in negative capital productivity growth.

2.5 Macroeconomic growth theory

The analytical framework utilized in investment theory has also been applied to macroeconomic growth theory. In this section three variants of macroeconomic growth models are discussed. Interestingly, one of these variants, the exogenous growth model, is shown to be consistent with the findings and model of Madsen and Davis (2006). The section concludes with the justification of the use of an exogenous growth model in this thesis.

2.5.1 Three classifications

Turnovsky (2003) summarizes three general classes of growth models: exogenous, endogenous, and non-scale growth models. In exogenous growth models (the "neo-classical" model), long run growth is determined by pop-

ulation growth, growth in labor efficiency, or growth in capital efficiency (Solow, 1956; Swan, 1956). Endogenous growth models rely on the accumulation of capital or knowledge as the source of long run growth, and this accumulation results from constant or increasing returns to scale of the accumulated factors (Romer, 1986, 1990; Rebelo, 1991; Romer, 1994). Non-scale growth models are a hybrid of exogenous and endogenous growth models in that long run growth is a function of technological parameters *and* the growth rate of labor (Jones, 1995a,b; Sergerstrom, 1998; Young, 1998).

Endogenous growth models rely on non-decreasing returns to scale in production factors such as knowledge or capital to account for long-run growth. For instance, the Research and Development model of Romer (1990) predicts long run growth rates are determined by increasing returns associated with the stock of human capital devoted to research (knowledge accumulation). However, the time series analysis of Jones (1995b) rejects the conclusions of Romer's Research and Development model. Models that rely on non-decreasing returns to capital, such as the AK model (Rebelo, 1991) are considered "fragile" due to the knife edge restriction imposed, i.e., the likelihood of returns to scale of exactly one is small (Solow, 1994). Furthermore, the empirical evidence of Jones (1995b) and Madsen and Davis (2006) rules out growth models that rely on increasing returns.

The strong rejection of Research and Development based endogenous growth models (increasing returns) by Jones (1995b) and Madsen and Davis (2006) casts doubt on the applicability of non-scale growth models since they also rely on spillover (increasing returns). To illustrate, consider the most basic non-scale model aggregate production function (Turnovsky, 2003):

$$Y = AK^{\eta+\sigma}N^{1-\sigma}$$

where Y is output, A is constant, η measures the extant of knowledge spillover, N is the number of agents (firms), and σ is a factor in determining the share of labor $(1-\sigma)$ or capital $(\eta+\sigma)$ in aggregate output. For any positive spillover, $\eta > 0$, this production function exhibits increasing returns to scale and therefore is unlikely to survive the time-series tests of Jones (1995b).

However, Arthur (1989) concludes "there may be theoretical limits, as well as practical ones, to the predictability of the economic future" in his analysis of increasing returns to scale in technology adoption. Arthur shows that increasing returns technology adoption processes have four properties. The most important property, *unpredictability*, states technology adoption is unpredictable in that ex-ante market shares can not be predicted. For example, in the recent HD-DVD vs. Blue-Ray battle, which was reminiscent of the Betamax vs. VHS battle, the eventual winner was unknown ex-ante. In each case, an ex-post perspective reveals an increasing returns to scale in investment enjoyed by the winner.

The remaining increasing returns properties identified by Arthur contribute to their unpredictability. First, the process are *non-ergodic*, meaning different sequences of historical events do not lead to the same market outcome. Increasing returns process are also *inflexible* since subsidies and tax adjustments do not always influence market outcome. They are also not

path-efficient, in hindsight investing more in the "losing" technology would not necessarily lead to higher payoffs to that technology. Therefore, empirical tests that attempt to make predictions regarding increasing returns to scale processes may be doomed from the onset. In other words, rejection of increasing returns to scale processes may merely be a result of their unpredictability.

2.5.2 Applicability of exogenous growth models

The conclusions of the previous section leave exogenous growth models as a potential choice for empirical work. However, exogenous models are not without two major criticisms. First, the model relies on labor growth and labor productivity growth as the source of long-term growth, both exogenous factors. Second, exogenous growth models suggest macroeconomic policy has no influence on long-run growth.

Regarding the first criticism, the source of long term growth, Madsen and Davis (2006) show there *has* been growth in labor efficiency in 11 OECD countries over the past 40 years (and of course, there has been population growth). This is due in part to labor productivity gains that arise from the information and communication technology revolution.

Regarding the second criticism, the influence of macroeconomic policy, it can be argued that the *recent* success of India, China, and the four Asian Tigers (Taiwan, Singapore, Hong Kong, and South Korea) are related to their respective governments' macroeconomic policy. However, macroeconomic policy influence on short-run growth is distinct from macroeconomic policy influence on long-run growth. In addition, the relationship between

macroeconomic policy and long run growth is questionable given total factor productivity has not increased over the past century Jones (1995b), the level of capital productivity has fluctuated around a constant over the past century (Mulligan, 2002), and significant momentum profits in 17 international markets in both good and bad macroeconomic states (Griffin, Ji, and Martin, 2003).

In sum, an exogenous growth model is chosen in this study for three reasons. First, exogenous growth allows for a more parsimonious model specification. Second, the focus of this study is on the influence of macroeconomic factors on equity returns. Whether or not those factors are influenced by macroeconomic policy is beyond the scope of this study. Finally, the aforementioned empirical evidence lends support to the use of exogenous growth models.

3 Theoretical foundations

Section 3.1 of this chapter discusses the attempt by Bansal, Dittmar, and Lundblad (2005) to circumvent empirical difficulties associated with aggregate consumption data and identifies theoretical inconsistencies of their model. In search of a model that is consistent with asset pricing theory and able to explain momentum profitability, an exchange economy model of the Lucas (1978) type is developed in Section 3.2 to obtain an alternative consumption measure. A macroeconomic growth model based on King and Rebelo (1999) is developed in Section 3.3 to obtain an alternative expression for marginal utility growth based on productivity. Finally, Section 3.4 combines the results of the exchange economy and macroeconomic growth models into an empirically testable asset pricing framework.

3.1 Inconsistency of cash-flow CAPM with asset pricing theory

Consider a one-factor linear asset pricing model with the linear approximation of the stochastic discount factor m_{t+1} represented by equation 3.1 and the single factor $f_{1,t+1}$. The expected return of the i-th asset $E[R_i]$ equals the zero-beta price of risk λ_0 plus the price of risk $\lambda_1$associated with the asset's sensitivity b_{i1} to the risk factor $f_{1,t}$, equation 3.2. The sensitivity to the priced risk factor $f_{1,t}$ is obtained from the time series regression of

returns $R_{i,t}$ on that factor, equation 3.3.

$$m_{t+1} = a_0 + a_1 f_{1,t+1} \tag{3.1}$$

$$E\left[R_i\right] = \lambda_0 + \lambda_1 b_{i1} \tag{3.2}$$

$$R_{i,t} = b_{i0} + b_{i1} f_t + e_{it} \tag{3.3}$$

Equation (3.2) is obtained from equation (3.1) via the innocuous asset pricing formula, $1 = E\left[mR_i\right]$, which illustrates properly discounted expected real returns should equal unity. First, note:

$$E\left[mR_i\right] = E\left[m\right] E\left[R_i\right] + cov\left[m, R_i\right] = 1$$

Next, dividing by $E[m]$ and isolating $E\left[R_i\right]$:

$$\begin{aligned} E\left[R_i\right] &= \frac{1}{E[m]} - \frac{cov\left[m, R_i\right]}{E[m]} \\ &= \frac{1}{E[m]} + \left(\frac{-var[m]}{E[m]}\right)\left(\frac{cov\left[m, R_i\right]}{var[m]}\right) \end{aligned}$$

Defining the market prices of risk $\lambda_0 = 1/E[m]$ and $\lambda_1 = -var[m]/E[m]$, and risk factor sensitivity n$b_{i1} = cov\left[m, R_i\right]/var[m]$ we arrive at equation (3.2). Therefore, b_{i1}, the sensitivity to risk factor f_1, is obtained from the time-series regression (3.3).

Now, turning to the asset pricing model of Bansal, Dittmar, and Lundblad (2005), the expected return of the i-th asset $E\left[R_i\right]$ equals the zero-beta price of risk λ_0 plus the price of risk λ_{cf} associated with the asset's sensitivity b_{icf} to aggregate consumption, equation (3.4). The sensitivity to

aggregate consumption b_{icf} is obtained by regressing the demeaned i-th asset's dividend growth rate $g_{i,t}$ on the moving average of demeaned aggregate consumption $g_{c,t}$.

$$E[R_i] = \lambda_0 + \lambda_{cf} b_{icf} \tag{3.4}$$

$$g_{i,t} = b_{icf} \left(\frac{1}{K} \sum_{j=1}^{K} g_{c,t-k} \right) + e_{it} \tag{3.5}$$

Returns, when properly discounted, should be equal to unity $E_t[m_{t+1} R_{i,t+1}] = 1$. The model of Bansal, Dittmar, and Lundblad (2005) has no discount factor m_{t+1} therefore the Euler condition can not be verified. Thus, while it is a step forward in that the CFCAPM model has greater *cross-sectional* explanatory power than consumption-CAPM, it is inconsistent with asset pricing theory, at in the sense that the *time-series* explanatory power can not be estimated.

3.2 Exchange economy model

The exchange economy model in this section follows the models of Lucas (1978) and Balvers, Cosimano, and McDonald (1990). This is a closed economy model with one representative consumer and one representative firm. A measure of consumption derived from firm net cash flows is obtained from the model and this measure is incorporated into the general equilibrium Euler condition.

3.2.1 Consumer utility maximization

The representative consumer maximizes utility by choosing a savings level s_t that maximizes the discounted value of lifetime utility. Let β represent the utility discount rate, p_t the price of one share of the firm, and d_t the net cash flow from the firm.

$$\max_{s_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left[c_t \right] \right]$$

subject to

$$c_t + p_t s_t = \left(p_t + d_t \right) s_{t-1}$$

To interpret the budget constraint, consider that at time $t-1$ the representative consumer is endowed with s_{t-1} shares of the firm. At time t those shares are worth $\left(p_t + d_t \right) s_{t-1}$ and the consumer will consume a portion (c_t) and save a portion $p_t s_t$. The solution to the maximization problem, derived in Appendix A.3.1 is the Euler condition:

$$E_t \left[\beta \frac{u' \left[c_{t+1} \right]}{u' \left[c_t \right]} R_{t+1} \right] = 1 \tag{3.6}$$

$$R_{t+1} \equiv \left(p_{t+1} + d_{t+1} \right) / p_t$$

As shown in Appendix A.3.2, for the case of N assets, the Euler condition for asset i is:

$$E_t \left[\beta \frac{u' \left[c_{t+1} \right]}{u' \left[c_t \right]} R_{t+1}^i \right] = 1 \tag{3.7}$$

It is worth emphasizing that in (3.7), the *individual* asset i return is discounted by the growth in marginal utility of *aggregate* consumption. Aggregate consumption is examined further in Section (3.3).

3.2.2 Firm value maximization

The firm maximizes the present value of all future net cash flows by choosing a level of investment (analogous to savings for a consumer). Let d_t represent future net cash flows, y_t firm output, k_t firm capital, i_t investment, δ_t the depreciation rate, and R_i the exogenous discount rate that is determined endogenously in general equilibrium (note $R_0 = 1$).

$$\max_{i_t} E_0 \left[\sum_{t=0}^{\infty} \left(\prod_{j=0}^{t} (R_{t+j})^{-1} \right) d_t \right]$$

subject to

$$d_t = y_t - i_t \tag{3.8}$$

$$y_t = f\left[k_t\right] \tag{3.9}$$

$$k_t = k_{t-1}\left(1 - \delta\right) + i_{t-1} \tag{3.10}$$

The third constraint follows the convention of Balvers, Cosimano, and McDonald (1990) that time $t-1$ investment becomes productive at time t. Incrementing (3.10) by one:

$$i_t = k_{t+1} - k_t\left(1 - \delta\right) \tag{3.11}$$

Substituting (3.11) and (3.9) into (3.8):

$$d_t = f\left[k_t\right] - \left(k_{t+1} - k_t\left(1-\delta\right)\right) \tag{3.12}$$

Note, the net cash flow constraint (3.12) differs from that of Balvers, Cosimano, and McDonald (1990) who assume 100% depreciation ($\delta = 1$). While such an assumption simplifies the model, this study follows the assumption of Poncet (2006) that capital depreciates at an annual rate of 5%.

At this point, the result of interest is the constraint (3.12) and therefore the derivation of the firm Euler condition is left for Appendix A.3.3. The constraint is of interest in that it represents aggregate net cash flows from the firm to the consumer, an important consideration for general equilibrium.

3.2.3 General equilibrium

In general equilibrium, all consumption is financed by net cash flows from the firm therefore $c_t = d_t\ \forall t$. As such, the Euler condition (3.7) becomes:

$$E_t\left[\beta\frac{u'\left[d_{t+1}\right]}{u'\left[d_t\right]}R^i_{t+1}\right] = 1 \tag{3.13}$$

with

$$d_t = f\left[k_t\right] - i_t$$

which indicates all output $f\left[k_t\right]$that is not invested i_t is consumed d_t. This result will be key to the empirical tests since empirical proxies of output $f[k]$ and capital k are less prone to measurement error than empirical proxies of consumption (Cochrane, 1991).

3.3 Macroeconomic growth model

In the previous section, the relationship between marginal utility growth and asset prices was established in equation (3.13). Given the unobservable nature of utility, obtaining a correct utility functional form is elusive. In this section, a discrete time general equilibrium macroeconomic growth model based on that of King and Rebelo (1999) is employed to arrive at an alternative expression for marginal utility growth. This productivity-based expression for marginal utility growth will be utilized in subsequent asset pricing tests as either a substitute for marginal utility growth or an instrumental variable in estimation. In the former, the difficulty in specifying utility is bypassed by using the production-based expression. In the latter, models utilizing explicit utility functions are enhanced via the addition of a theory-based proxy of marginal growth used as an instrument.

3.3.1 Choice of general equilibrium model

Madsen and Davis (2006) apply a neoclassical investment theory partial equilibrium model to aggregate stock (i.e., a price index) and investment data. In this study a general equilibrium (central planner) analysis is performed for several reasons. First, general equilibrium models allow both supply (firm) and demand (shareholder) interactions while partial equilibrium models treat one side as exogenous. For instance, macroeconomic theory and intuition suggest an increase in productivity (supply side), even if only temporary, impacts consumption (demand side). Second, Abel and Blanchard (1983), who prove the equivalence of a central planner vs. market economy

approach, state the general equilibrium (central planner) approach is

> "...very useful as it allows, when studying the effects of various shocks or policies, to use the equations of motion of the centralized economy with its unique shadow price rather than the equations of motion of the market economy with two shadow prices which themselves depend on market-determined interest rates."

Therefore the complexities associated with the inclusion of market-determined interest rates, which are beyond the scope of this study, are bypassed in the central planner approach. Again, the goal here is to establish a connection between marginal utility growth and productivity.

3.3.2 The model

As suggested in Section 2.5.2, an exogenous growth model is suited to the goals of this thesis. The model developed here follows the exogenous growth model of King and Rebelo (1999). Exogenous growth is introduced via labor augmentation consistent with Sala-i Martin (1990) who states:

> "...as Phelps showed, a necessary and sufficient condition for the existence of a steady state in an economy with exogenous technological progress is for this technological progress to be Harrod Neutral or Labor Augmenting"

As such, the production function can be specified in Cobb-Douglas form as

$$Y_t = A_t F\left[K_t, N_t X_t\right] = A_t K_t^{1-\alpha}\left(N_t X_t\right)^{\alpha} \tag{3.14}$$

where Y_t represents output, K_t capital input, N_t labor input, A_t the random productivity shock, and X_t the deterministic component of productivity which grows at a constant (and exogenous) rate $\gamma > 1$:

$$X_{t+1} = \gamma X_t$$

The Cobb-Douglas production function was chosen for several reasons. First, by construction this production function exhibits constant returns to scale, consistent with the empirical findings of Jorgenson (1972). Therefore it is not subject to the scale or non-decreasing returns effects associated with endogenous growth formulations (see above) and allows for equivalence between marginal q and average q (Hayashi, 1982). Second, the empirical evidence noted by Jorgenson (1972) suggests the estimated elasticity of substitution for the CES production function is not significantly different from unity and therefore the CES reduces to Cobb-Douglas form. Third, Arroyo (1996) suggests the Cobb-Douglas form is "probably more descriptive of aggregate technological conditions."

Maximization problem The infinitely lived central planner maximizes discounted expected utility

$$E_0\left\{\sum_{t=0}^{\infty} b^t u\left[C_t\right]\right\}$$

where $b^t < 0$ represents the rate of time preference, subject to several constraints. To begin, all output is either consumed or invested in this closed economy with no government:

$$Y_t = C_t + I_t \tag{3.15}$$

In addition, capital stock evolves according to the "perpetual inventory method":

$$K_{t+1} = I_t + (1 - \delta)K_t \tag{3.16}$$

where δ represents the rate of depreciation. All variables are expressed in per-capita (population) terms. Labor market and wages are not the focus of this study therefore the labor input is normalized to 1:

$$N_t = 1 \quad \forall t$$

Solution The first order conditions of the maximization problem, derived in Appendix A.4, combine to reveal an alternative proxy for marginal utility growth:

$$\Gamma_{t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} = \frac{u'\left[C_{t+1}\right]}{u'\left[C_t\right]} = \frac{1}{b\left(A_{t+1}F_K\left[K_{t+1}, X_{t+1}\right] + (1-\delta)\right)} \tag{3.17}$$

In the case of Cob-Douglas utility:

$$\Gamma = \frac{1}{b\left((1-\alpha)\left(Y_{t+1}/K_{t+1}\right) + (1-\delta)\right)}$$

Equation (3.17) provides a convenient and readily observable proxy for unobservable marginal utility growth. This expression shall be used both as a direct replacement for marginal utility growth and also as an instrumental variable in estimations that include explicit utility function assumptions.

3.4 Asset pricing with discount factor models

3.4.1 Linear factor models and the stochastic discount factor

All linear factor models described in Section 2.2.2 originate from the consumer's Euler condition:

$$E_t\left[\beta\frac{u'\left[c_{t+1}\right]}{u'\left[c_t\right]}R_{t+1}\right] = 1 \tag{3.18}$$

$$m_{t+1} = \beta\frac{u'\left[c_{t+1}\right]}{u'\left[c_t\right]} \tag{3.19}$$

Linear factor models are formed by making a linear approximation of the stochastic discount factor:

$$m_{t+1} = \beta\frac{u'\left[c_{t+1}\right]}{u'\left[c_t\right]} \approx a_0 + \sum_{k=1}^{K} a_1 f_{k,t+1} \tag{3.20}$$

where f_k represents the k-th factor related to marginal utility growth. This discount factor is sometimes referred to as the *pricing kernel*. Consider the case of a single factor $K = 1$. The single factor linear factor pricing model

can be summarized by three equations:

$$m_{t+1} = a_0 + a_1 f_{1,t+1} \tag{3.21}$$

$$E[R_i] = \lambda_0 + \lambda_1 b_{i1} \tag{3.22}$$

$$R_{it} = b_{i0} + b_{i1} f_{1t} + \epsilon_{it} \tag{3.23}$$

Equation (3.22) is obtained from (3.21) following the procedure of Section (3.1). The coefficient, b_{i1} is obtained from the time series regression (3.23). Following a similar procedure, the triplet of asset pricing equations for a K-factor model are:

$$m_{t+1} = a_0 + \sum_{k=1}^{K} a_k f_{k,t+1}$$

$$E[R_i] = \lambda_0 + \sum_{k=1}^{K} \lambda_k b_{ik}$$

$$R_{it} = b_{i0} + \sum_{k=1}^{K} b_{ik} f_{k,t} + \epsilon_{it}$$

3.4.2 Shortcomings of linear factor models

Cochrane (2005) points out that linear approximations to the nonlinear discount factor are not without problems. Consider the case of log utility[13]:

$$u[c_t] = \ln[c_t]$$

Let p_t^W represent the price of a claim to all future consumption:

$$p_t^W = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'[c_{t+j}]}{u'[c_t]} c_{t+j} = E_t \sum_{j=1}^{\infty} \beta^j \frac{c_t}{c_{t+j}} c_{t+j} = \frac{\beta}{1-\beta} c_t \tag{3.24}$$

From (3.24) it can be shown the wealth portfolio return is proportional to marginal utility growth:

$$R_{t+1}^W = \frac{p_{t+1}^W + c_{t+1}}{p_t^W} = \frac{\left(\frac{\beta}{1-\beta}\right) c_{t+1} + c_{t+1}}{\left(\frac{\beta}{1-\beta}\right) c_t} = \frac{c_{t+1}\left(\frac{1}{1+\beta}\right)}{\left(\frac{\beta}{1-\beta}\right) c_t} = \frac{1}{\beta}\frac{c_{t+1}}{c_t}$$

However, $u'[c] = 1/c$ therefore:

$$R_{t+1}^W = \frac{1}{\beta}\frac{u'[c_t]}{u'[c_{t+1}]} \rightarrow m_{t+1} = \frac{1}{R_{t+1}^W}$$

[13] As an aside, it is worth pausing for a moment to describe how log utility has the property that "the income effect exactly offsets the substitution effect." Consider the case of news of higher future consumption (c_{t+1}). The income effect should make the claim p_t^W more valuable. However, the substitution effect makes the claim less valuable due to the lower marginal utility in the numerator $u'[c_{t+1}]$. The net effect, i.e., the price effect, is zero as evidenced by the absence of future consumption c_{t+1} in the far right hand term of (3.24). The early work of Royama and Hamada (1967), who study the impacts of substitutability on asset choice, has interesting implications for momentum analysis. See Chapter 7 for a detailed discussion.

Therefore the linear approximation is the familiar CAPM discount factor:

$$m_{t+1} \approx a_0 + a_1 R^W_{t+1} \qquad a_1 < 0$$

However, as pointed out by Cochrane, for longer time horizons, the linear approximation loses accuracy. Therefore, in general, it is not a good idea to apply linear approximations to longer time intervals as the error of the approximation increases with the magnitude of the factor and the length of the time interval. Further complicating matters for CAPM is the imprecision with which the "wealth portfolio" is obtained. Traditionally, the Standard and Poors 500 index is used as a proxy for the wealth portfolio but this omits real estate, human capital, gold, emerging economy equity, etc.

Linearization was performed in the past due to the difficulty in estimating nonlinear models. Econometric models and computing resources have advanced such that nonlinear estimation is performed with regularity. In sum, linear factor models, due to the linear approximation of nonlinear discount factors, are best suited for short-horizon estimations. Nonlinear factor models are better suited for longer horizon models. In this thesis, level and return data are sampled at the quarterly frequency for comparison with previous studies. Therefore, nonlinear discount factor models are more appropriate.

The availability of nonlinear estimations (such as Hansen's Generalized Method of Moments or GMM) and the shortcomings of linear factor models, suggest a nonlinear asset pricing framework is in order. Several nonlinear discount factors are discussed in the following section.

3.4.3 Nonlinear discount factors

Constant relative risk aversion (CRRA) CRRA utility can be expressed in the power-utility form:

$$u\left[c\right] = \frac{c^{1-\sigma}}{1-\sigma} \tag{3.25}$$

The first and second derivatives are:

$$u'[c] = \frac{(1-\sigma)c^{-\sigma}}{1-\sigma} = c^{-\sigma}$$
$$u''[c] = -\sigma c^{-\sigma-1}$$

The degree of relative risk aversion is computed as

$$\rho_r = \frac{-cu''}{u'} = \frac{-c\sigma c^{-\sigma-1}}{c^{-\sigma}} = \sigma$$

Since σ is constant, the utility function (3.25) exhibits CRRA. The Euler condition under the CRRA utility assumption is:

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} R_{i,t+1} - 1 \middle| Z_t\right] = 0 \tag{3.26}$$

In the case of excess returns, the Euler condition simplifies to:

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} (R_{i,t+1} - R_{f,t+1}) \middle| Z_t\right] = 0 \tag{3.27}$$

Decreasing relative risk aversion (DRRA) via deterministic subsistence exclusion The smoothness (low volatility) of consumption data versus asset return data translates into a less informative (low volatility) CRRA-based

discount factor. Constantinides (1990) suggested a portion of consumption is based on habit, or in other words, required for subsistence. As such, this portion should be subtracted prior to computing utility. Meyer and Meyer (2005) constructed a deterministic trend consumption series, $x_t = \delta e^{\mu t}$, based on the growth rate of aggregate consumption in the sample[14]. Let π represent the portion of trend consumption required for subsistence. The associated utility specification is:

$$u[c_t] = \frac{(c_t - \pi x_t)^{1-\sigma}}{1-\sigma} \tag{3.28}$$

The first and second derivatives are:

$$u'[c_t] = \frac{(1-\sigma)(c_t - \pi x_t)^{-\sigma}}{1-\sigma} = (c_t - \pi x_t)^{-\sigma}$$
$$u''[c_t] = -\sigma (c_t - \pi x_t)^{-\sigma-1}$$

The degree of relative risk aversion is computed as

$$\rho_r = \frac{-cu''}{u'} = \frac{-c_t\left(-\sigma (c_t - \pi x_t)^{-\sigma-1}\right)}{(c_t - \pi x_t)^{-\gamma}} = \frac{c_t \sigma}{c_t - \pi x_t}$$

To illustrate DRRA, take the derivative with respect to c_t:

$$\frac{\partial \rho_r}{\partial c_t} = \frac{(c_t - \pi x_t)\sigma - c_t\sigma}{(c_t - \pi x_t)^2} = \frac{-\pi x_t}{(c_t - \pi x_t)^2} < 0$$

[14] Specifically, $\mu = \frac{1}{T}\log\frac{c_T}{c_0}$ and $\delta = c_0$.

Therefore the degree of relative risk aversion is decreasing with consumption. The Euler condition under the DRRA utility assumption is

$$E_t\left[\beta\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma}R_{i,t+1}-1\middle|Z_t\right]=0 \tag{3.29}$$

In the case of excess returns, the Euler condition simplifies to:

$$E_t\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma}(R_{i,t+1}-R_{f,t+1})\middle|Z_t\right]=0 \tag{3.30}$$

Time non-separable utility via stochastic subsistence exclusion Although Ferson and Harvey (1992) and Meyer and Meyer (2005) utilized a deterministic trend exclusion, the work of Constantinides (1990) calls for a *stochastic* subsistence exclusion. Specifically, the subsistence level is dependent on prior period consumption. As such the utility specification is:

$$u\left[c_t,c_{t-1}\right]=\frac{\left(c_t-\pi c_{t-1}\right)^{1-\sigma}}{1-\sigma}$$

Using similar logic from the previous section, this degree of relative risk aversion of this utility function is decreasing in c_t (current consumption). Unfortunately, the Euler condition used to obtain discount factors for the CRRA and DRRA with deterministic trend utility functions can not be applied to this multi-period (time non-separable) utility function. Appendix A.3.4 derives the Euler condition for such a utility function:

$$E_t\left[\beta\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma}-\pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right)R_{i,t+1}+\pi\beta\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma}-1\middle|Z_t\right]=0 \tag{3.31}$$

In the case of excess returns, the Euler condition simplifies to:

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma}-\pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right)(R_{i,t+1}-R_{f,t+1})\middle|Z_t\right]=0 \tag{3.32}$$

Productivity-based discount factor / instrument Equation (3.17) of Section 3.3 represents a proxy for marginal utility growth under time separable utility. Substituting (3.17) for $u'[c_{t+1}]/u'[c_t]$ in equation (3.19) yields the following Euler condition:

$$E_t\left[\beta\left(\frac{1}{b\left(A_{t+1}F_K\left[K_{t+1},X_{t+1}\right]+(1-\delta)\right)}\right)R_{i,t+1}-1\middle|Z_t\right]=0 \qquad (3.33)$$

In the case of excess returns, the Euler condition simplifies to:

$$E_t\left[\left(\frac{1}{\left(A_{t+1}F_K\left[K_{t+1},X_{t+1}\right]+(1-\delta)\right)}\right)\left(R_{i,t+1}-R_{f,t+1}\right)\middle|Z_t\right]=0 \qquad (3.34)$$

3.4.4 GMM estimation

This section provides a brief description of the GMM estimation technique of Hansen (1982). Method of moments (MM) and generalized method of moments (GMM) estimation is based on a set of population *moment conditions* that include data and unknown parameters. Estimates based on sample averages correspond to the population estimates since the sample mean is an estimate of the population mean (Cameron and Trivedi, 2005). In the over-identified case, i.e., when there are more moment conditions (equations) than parameters, GMM estimation is needed. This is quite often the case in asset pricing given the number of test assets often exceeds the number of parameters (e.g., the concavity parameter of a CRRA utility-based stochastic discount factor).

Following the discussion of Hansen's GMM estimation by Cochrane (2005), begin with the fundamental pricing equation:

$$p_t = E_t \left[m_{t+1} \left[\mathbf{a} \right] x_{t+1} \right]$$

where $\mathbf{a}$ represents a vector of parameters $[a_0 \ a_1 \ \cdots a_k]$. The equation can be rewritten in *moment condition* form:

$$E_t \left[m_{t+1}[\mathbf{a}] x_{t+1} - p_t \right] = 0 \tag{3.35}$$

The moment condition is also referred to as *orthogonality condition* The expression inside the expectations operator is the pricing error:

$$u_{t+1}[\mathbf{a}] = m_{t+1}[\mathbf{a}] x_{t+1} - p_t$$

GMM chooses parameters ($\mathbf{a}$) such that the conditional and unconditional mean of the pricing errors are zero. GMM arrives at consistent, asymptotically normal, and asymptotically efficient estimates of $\mathbf{a}$ in a two stage procedure. The first stage utilizes an arbitrary weighting matrix $\mathbf{W}$ (typically $\mathbf{W} = \mathbf{I}$)[15] to obtain a consistent and asymptotically normal parameter vector $\mathbf{a_1}$:

$$\hat{\mathbf{a}}_1 = \text{argmin}_{\{\mathbf{a}\}} g_T \left[\mathbf{a} \right]' \mathbf{W} g_T \left[\mathbf{a} \right]$$

where $g_T[\mathbf{a}]$ represents the sample mean of pricing errors ($u_t[\mathbf{a}]$). An estimate

[15] This directs GMM to price all assets equally well. The second stage weighting matrix $\mathbf{W} = \hat{\mathbf{S}}^{-1}$ takes in to account differential variance of asset returns thereby directing GMM to pay less attention to assets with high variances since their sample mean will be a less accurate measure than the population mean.

of the sample error variance-covariance matrix is obtained using $\hat{\mathbf{a}}_1$:

$$\hat{\mathbf{S}} = \sum_{j=-\infty}^{\infty} E_t \left[u_t[\hat{\mathbf{a}}_1] u_{t-j}[\hat{\mathbf{a}}_1]' \right]$$

Using $\hat{\mathbf{S}}$ as the new weighting matrix, the second stage estimate produces the consistent, asymptotically normal, *and* asymptotically efficient estimate of $\mathbf{a}$, $\hat{\mathbf{a}}_2$:

$$\hat{\mathbf{a}}_2 = \text{argmin}_{\mathbf{a}} g_T[\mathbf{a}]' \hat{\mathbf{S}}^{-1} g_T[\mathbf{a}]$$

The variance-covariance matrix of $\hat{\mathbf{a}}_2$ is:

$$\text{var}\left[\hat{\mathbf{a}}_2\right] = \frac{1}{T}\left(\mathbf{d}'\hat{\mathbf{S}}^{-1}\mathbf{d}\right)^{-1}$$

where

$$\mathbf{d} \equiv \left. \frac{\partial g_T\left[\mathbf{a}\right]}{\partial \mathbf{a}} \right|_{\cdot \mathbf{a}=\hat{\mathbf{a}}_2}$$

There are several advantages of using GMM estimation in asset pricing scenarios. Linear and nonlinear asset pricing equations (restrictions) map directly into GMM moment conditions. The estimation allows for serial correlation and non-stationarity (heteroskedasticity) in the pricing errors[16]. The inclusion of instruments, variables in investor's information set that are related to future returns or discount factors, is also straightforward. Finally, given that the system of equations for typical asset pricing tests typically exceeds the number of parameters to be estimated, GMM provides a TJ test-statistic to test if those over-identifying restrictions fit the model.

[16] For details see Hansen (1982) and Cochrane (2005)

A discussion of conditional estimation and the TJ test of over-identifying restrictions follows.

Conditional vs. unconditional estimation Begin with the fundamental asset pricing equation:

$$p_t = E_t \left[m_{t+1} x_{t+1} \right] \tag{3.36}$$

Let Ω_t represent all available time t information; therefore the asset pricing equation can be rewritten as:

$$p_t = E \left[m_{t+1} x_{t+1} | \, \Omega_t \right] \tag{3.37}$$

Let I_t represent a subset of available information at time t. Recall the law of iterated expectations

$$E \left[E \left[X | \Omega \right] | I \subset \Omega \right] = E \left[X | I \right]$$

Therefore, taking expectations of both sides of (3.36) conditional on the subset of information I_t:

$$E \left[p_t | \, I_t \right] = E \left[m_{t+1} x_{t+1} | \, I_t \right] \tag{3.38}$$

which implies

$$p_t = E \left[m_{t+1} x_{t+1} | \, I_t \right] \tag{3.39}$$

Hence, an asset can be priced using a subset of all available information. Next consider instrument z_t observed at time t. Multiplying the price and

the payoff by z_t:

$$p_t z_t = E_t \left[m_{t+1} \left(x_{t+1} z_t \right) \right] \tag{3.40}$$

Take unconditional expectations of both sides of (e) to obtain:

$$E \left[p_t z_t \right] = E \left[m_{t+1} \left(x_{t+1} z_t \right) \right] \tag{3.41}$$

An intuitive interpretation is as follows. If an investor observes that high values of z_t forecast high returns, the investor might purchase more of the asset at time t. If you consider the price $p = E \left[p_t z_t \right]$ and the payoff $x = x_{t+1} z_t$ the pricing equation can be rewritten in unconditional form

$$p = E[mx]$$

Furthermore, Cochrane (2005) shows

$$E \left[p_t z_t \right] = E \left[m_{t+1} \left(x_{t+1} z_t \right) \right] \ \forall z_t \in I_t \quad \Rightarrow \quad p_t = E \left[m_{t+1} x_{t+1} | \, I_t \right] \tag{3.42}$$

Therefore including instruments $z_t \in I_t$, where $I_t \subset \Omega_t$, and taking *unconditional* expectations is equivalent to taking *conditional* expectations.

Choice of instruments Now having established that incorporation of instruments is equivalent to taking conditional expectations, a choice of instruments must be made. Following Ferson and Harvey (1992), who also test a conditional consumption-CAPM model, I use four lags of consumption and four lags of quarterly Treasury bill rates obtained from rolling over one-month Treasury bills for three months. In addition, since the discrete

time asset pricing model relies on marginal utility growth to price assets, the productivity-based proxy for marginal utility growth (3.17) will be used as an instrument.

TJ test of over-identifying restrictions As noted in Pynnonen (2007) the reported value of the minimized objective function is the J-statistic:

$$J = g_T\left[\hat{\mathbf{a}}\right]' \hat{\mathbf{S}}^{-1} g_T\left[\hat{\mathbf{a}}\right]$$

Multiplying this by the number of time-series observations T produces the J test statistic:

$$TJ \sim \chi^2_{df}$$

where the degrees of freedom, df:

$$\begin{aligned} df &= \#\text{ of overidentifying restrictions} \\ &= \#\text{ of moments-}\#\text{ of parameters} \end{aligned}$$

The null hypothesis for the test statistic is:

$$H_0: \text{ moment conditions (pricing errors) are zero}$$

Therefore, failure to reject the null leads to acceptance of the model.

3.5 Summary

An exchange economy model was employed to obtain a more representative measure of consumption, namely, net cash flows. Not only is the net cash flow measure more representative of the dynamics associated with the model, the fact that the measure is based on production data suggests it will be more reliable that the aggregate consumption measure. The macroeconomic growth model resulted in a productivity-based expression for marginal utility growth. This expression shall be used as both a proxy for marginal utility growth as the stochastic discount factor and as an instrument when using traditional utility-based stochastic discount factors. Finally, the GMM estimation technique was presented as a means to estimate the Euler conditions for linear and nonlinear stochastic discount factor models in unconditional and conditional settings.

4 Data and Methodology

4.1 Sample construction

4.1.1 Data sources

Data for this study are obtained from a variety of sources. Table 1 summarizes the sources, frequency, and availability of the data.

Tab. 1: Data items, sources, and availability

All asset pricing tests are performed using quarterly data. Return data are converted to quarterly by continuously compounding three single month returns and then deflated by the appropriate quarterly deflator. Seasonally adjusted data are denoted by SA and not-seasonally adjusted data are denoted by NSA.

Item(s)	Source	Frequency	Availability
One-month T-Bill rate (r_f)	WRDS (via Kenneth French website)	monthly	1927.01-2007.12
10 prior-return portfolios ($M01 - M10$)	Kenneth French website	monthly	1927.01-2007.12
Individual security returns	CRSP	monthly	1925.12-2007.12
Company financial data	Compustat	quarterly	1961:1-2006:4
aggregate labor (L)	BEA NIPA tables	quarterly	1947:1-2007:4
aggregate capital (K)	BEA NIPA tables	annual	1929-2006
aggregate private nonresidential investment (I)	BEA NIPA tables	quarterly	1947:1-2007:4 (SA) 1947:1-2006:4 (NSA)
aggregate output (Y) and consumption (C)	BEA NIPA tables	quarterly	1947:1-2007:4 (SA) 1947:1-2006:4 (NSA)
SA personal consumption expenditure (SA PCE) deflator	BEA NIPA tables	quarterly	1947:1-2007:4
NSA adjusted CPI (NSA CPI) deflator	BLS LABSTAT	monthly	1947.01-2007.12

Unfortunately quarterly capital data are unavailable from the Bureau of Economic Analysis therefore the series must be estimated. The quarterly capital series is constructed from annual capital data and quarterly investment data following the procedure of Balvers and Huang (2007). Capital in quarter q is computed as:

$$K_{y,q} = \left(\frac{\sum_{i=1}^{q} I_{y,i}}{\sum_{i=1}^{4} I_{y,i}} \right) (K_y - K_{y-1}) + K_{y-1}$$

where

$$K_{y,q} = \text{capital for quarter } q \text{ in year } y$$
$$I_{y,i} = \text{investment for quarter } i \text{ in year } y$$
$$K_y = \text{capital at end of year } y$$

Since the production function is Cobb-Douglas with constant returns to scale, the quarterly marginal productivity series is computed as:

$$MPK = (1 - \alpha)\frac{Y}{K}$$

Summary statistics for MPK are provided in Table 2 Panel A. Alternatively, the quarterly marginal productivity series could be computed by first extracting the Solow residual, then estimating the stochastic (A_t) and deterministic (X_t) productivity series, and computing the marginal productivity of capital as:

$$MPK = (1 - \alpha)\, A_t K_t^{-\alpha} X_t^{\alpha}$$

Details of this method are provided in Appendix A.3.5.

4.1.2 Summary statistics

Summary statistics including means, standard deviations, and sample autocorrelation of the input data are provided in Table 2. The summary statistics for real consumption growth and the real treasury bill are comparable to those in Ferson and Harvey (1992), who study a four asset system (government bond, corporate bond, small size decile of stocks, and large size decile of stocks) in contrast to the ten asset system (prior-return deciles) of this study.

4.2 Momentum evidence and the macroeconomy

Do momentum profits still persist post Chordia and Shivakumar (2006), who look at return data up to 1999? To answer this question, price and earnings momentum portfolios are constructed over the sample period 1972 to 2006 for comparison with the results of Chordia and Shivakumar (2006). Monthly return data are obtained from CRSP and quarterly earnings data are obtained from Compustat. Following Chan, Jegadeesh, and Lakonishok (1996) and Chordia and Shivakumar (2006), the measure for earnings surprises, standardized unexpected earnings or SUE, is calculated based on a seasonal random walk model:

$$SUE_{iq} = \frac{e_{iq} - e_{iq-4}}{\sigma_{iq}} \equiv \frac{\Delta e_{iq}}{\sigma_{iq}}$$

Tab. 2: Means, standard deviations, and sample autocorrelation of input data

The variables m01-m10 represent the quarterly excess returns (nominal returns in excess of nominal one-month Treasury Bill rolled over for three months) of prior return portfolio deciles. R_f represents the one-month Treasury bill rolled over for three months and deflated by the not-seasonally adjusted CPI deflator. c_{t+1}/c_t represents the growth rate of real seasonally adjusted nondurable consumption (nominal nondurables deflated by seasonal PCE deflator), d_{t+1}/d_t represents the real "net cash flow" growth rate, MPK represents the marginal productivity of capital, and Γ represents the productivity-based marginal utility growth proxy. The sample period is 1947:1 to 2006:4.

			Sample autocorrelation					
var	mean	stddev	lag1	lag2	lag3	lag4	lag8	lag12
Panel A: Macroeconomic variables								
c	1247.71	318.368	0.9850	0.9704	0.9557	0.9406	0.8819	0.8254
d	5160.28	2122.72	0.9876	0.9749	0.9616	0.9487	0.8972	0.8451
mpk	0.2860	0.0213	0.9799	0.9549	0.9275	0.9003	0.7965	0.7085
c_{t+1}/c_t	1.011	0.011	0.3028	0.4512	0.3355	0.1356	0.0656	0.0788
d_{t+1}/d_t	1.014	0.022	0.0700	0.0466	0.0415	-0.1263	-0.0599	-0.0461
Γ_t	0.960	0.005	0.9865	0.9640	0.9354	0.9075	0.8161	0.7239
Panel B: Return variables								
m01	−0.0015	0.1336	−0.0570	0.0103	−0.0191	−0.0356	−0.0225	0.1197
m02	0.0111	0.1060	−0.0410	−0.0119	−0.0667	−0.0769	0.0073	0.0940
m03	0.0147	0.0905	−0.0186	−0.0196	−0.0176	−0.0272	−0.0035	0.0468
m04	0.0166	0.0838	0.0063	−0.0420	−0.0363	−0.0438	−0.0238	0.0436
m05	0.0163	0.0784	−0.0204	−0.0797	−0.0190	−0.0252	−0.0062	0.0437
m06	0.0186	0.0815	0.0379	−0.0721	−0.0092	−0.0570	−0.0235	−0.0281
m07	0.0200	0.0770	−0.0062	−0.0016	0.0096	−0.0069	−0.0108	−0.0072
m08	0.0254	0.0782	0.0322	0.0194	−0.0458	−0.0025	0.0381	−0.0123
m09	0.0277	0.0836	0.0739	−0.0449	−0.0600	−0.0198	0.0065	−0.0023
m10	0.0395	0.1085	0.0678	−0.0719	−0.0833	−0.0601	0.0044	−0.0445
R_f	0.0022	0.0085	0.4025	0.2935	0.3701	0.3961	0.1945	0.2802

where e_{iq} represents firm i reported earnings in quarter q and σ_{iq} represents the 8-period sample standard deviation of the change in earnings (Δe_{iq}). Stocks are sorted into 10 portfolios based on SUE and 10 portfolios based on prior six month return. In both cases the holding period is chosen to be six months for comparison with the Chordia and Shivakumar (2006) results.

Table 3 presents average monthly returns from a six-month holding period of SUE (earnings) and Jegadeesh and Titman (1993) momentum portfolios. These results are qualitatively similar to those of Chordia and Shivakumar (2006) and confirm the persistence of momentum profitability. Panel B, and to some extent Panel A, provide clues as to the cyclical nature of momentum profits. For instance, during the 1972-1999 period, earnings and price momentum strategies were profitable and statistically significant. However, in the 2000-2006 post Internet "bubble" window, the earnings momentum strategy t-value drops by over 50% and the profitability of the price momentum strategy becomes statistically insignificant.

To further illustrate the persistence of momentum profitability, monthly momentum portfolio (UMD, or up-minus-down) return data were obtained from the Kenneth French website. The UMD portfolio is constructed from all NYSE, AMEX, and Nasdaq securities by first sorting stocks into month $t-12$ to month $t-2$ returns and forming the decile portfolio in month t. From there, the UMD portfolio is long stocks with the highest prior returns and short stocks with the lowest prior returns.

The monthly series is converted to quarterly, deflated by the overall seasonally adjusted personal consumption expenditures (PCE) deflator from the

Tab. 3: Average monthly returns

Monthly returns for earnings and price momentum portfolios. Earnings portfolios are sorted into deciles (P1 through P10) based on ascending standardized unexpected earnings (SUE) from the most recent earnings announcement. Price momentum portfolios are sorted into deciles (P1 through P10) based on ascending prior six month return. In addition, mean returns are reported for a portfolio long on the highest prior-return decile (P10) and short on the lowest prior-return decile (P1). All portfolios are held for six months after formation period.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1
Panel A: Earnings (SUE) portfolios											
Jan 1972 - Dec 2006 (entire sample)											
Mean (%)	0.90	1.03	1.12	1.33	1.40	1.53	1.65	1.57	1.66	1.69	0.79
t-value	2.99	3.62	3.88	4.57	4.86	5.64	5.98	5.76	6.19	6.33	5.80
Jan 1972 - Dec 1999											
Mean (%)	0.84	0.99	1.12	1.50	1.45	1.53	1.42	1.97	1.56	1.67	0.84
t-value	2.37	3.00	3.34	3.73	4.32	4.67	4.01	3.69	4.96	5.30	5.68
Jan 2000 - Dec 2006											
Mean (%)	0.75	1.12	1.01	1.34	1.26	1.40	1.65	1.35	1.78	1.66	0.91
t-value	1.25	1.92	1.78	2.42	2.37	3.07	2.94	2.74	3.34	3.31	2.50
Panel B: Price momentum portfolios											
Jan 1972 - Dec 2006 (entire sample)											
Mean (%)	0.97	1.11	1.28	1.28	1.33	1.37	1.37	1.39	1.46	1.66	0.69
t-value	2.16	3.56	4.60	5.01	5.44	5.66	5.70	5.70	5.69	5.55	2.22
Jan 1972 - Dec 1999											
Mean (%)	0.82	1.08	1.26	1.27	1.35	1.38	1.35	1.39	1.43	1.61	0.79
t-value	1.69	3.05	3.91	4.20	4.68	4.82	4.76	4.81	4.75	4.74	2.55
Jan 2000 - Dec 2006											
Mean (%)	1.53	1.20	1.33	1.37	1.28	1.39	1.44	1.51	1.59	1.85	0.31
t-value	1.26	1.69	2.42	2.96	2.99	3.39	3.58	3.51	3.40	3.03	0.31

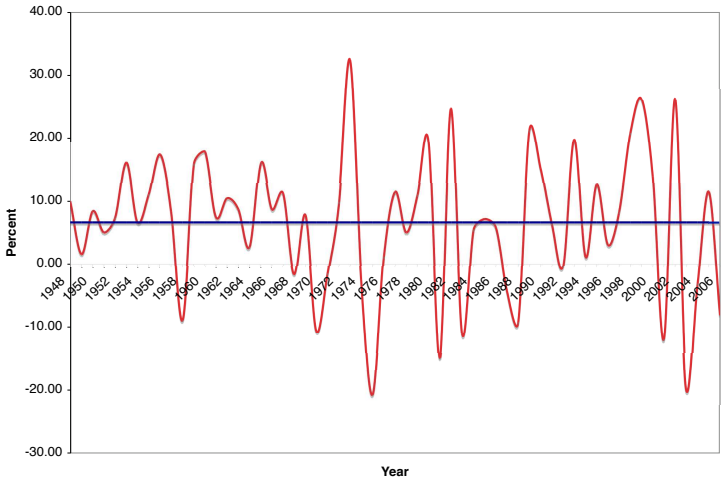


Fig. 5: Momentum portfolio (UMD) profitability vs. time

Bureau of Economic Analysis, and then continuously compounded to form a deflated annual return series. Figure 5 depicts the real returns on the price momentum portfolio (UMD), sampled annually over the 1948 to 2006 period. The graph indicates that the momentum strategy, on average, is profitable. The mean of the series, indicated by the horizontal line, is 6.6% and the standard deviation is 11.4%. This series has a roughly 34% correlation with lagged annual GDP growth suggesting momentum profitability follows periods of rising GDP. Further analysis of the GDP-momentum profitability connection is left for future research.

4.3 Aggregate consumption and firm net cash flows

The lack of success of consumption-based asset pricing models has been attributed to measurement error (Daniel and Marshall, 1997) and the relatively smooth nature of consumption series (Cochrane, 1991). Cochrane (1991) suggests the production-based model may prove more useful for linking aggregate economic activity to real returns. In particular, the aggregate production measures of output and investment not only have larger movements, but are large in magnitude therefore mitigating the impact of transactions costs. However, as described in Section 2.2.3, the production-based model is unlikely to explain momentum profitability.

In the exchange economy model presented in Section 3.2, the aggregate consumption measure is replaced by firm net cash flows to consumers, which is aggregate output less aggregate investment. Therefore consumer the marginal utility growth expression is computed with production data that is relatively more reliable than consumption data. To illustrate the differences between aggregate consumption and firm net cash flows, summary statistics for level and growth rate data are presented in Table 4.

Several observations are of note. First, regarding mean values, the level of firm net cash flows is four times larger than that of aggregate consumption and the net cash flow growth rate mean is nearly double that of aggregate consumption.. Second, regarding dispersion values, the standard deviation of firm net cash flow levels is seven times larger than that of aggregate consumption levels while the standard deviation of firm cash flow growth rates is 54% larger than that of aggregate consumption. Third, although

Tab. 4: Aggregate consumption vs. net cash flow - summary statistics

Real level aggregate consumption and firm net cash flow data are obtained by deflating nominal values by the seasonally adjusted nondurables PCE deflator. g_c represents the growth rate $(c_{t+1}/c_t)$of real seasonally adjusted nondurable consumption data whereas g_d represents the real "net cash flow" growth rate (d_{t+1}/d_t). Pearson correlation coefficients are also presented with the p-value in parentheses. The sample period is 1947:1 to 2006:4.

Variable	Mean	Std Dev	Minimum	Maximum
Panel A: Level data (dollars)				
c	1248	318	812	1972
d	5160	2123	2115	9294
$corr\,[c, d]$		0.99358 (<0.0001)		
Panel B: Growth rates				
g_c	1.011	0.0011	0.9769	1.0641
g_d	1.014	0.0022	0.8779	1.1587
$corr\,[g_c, g_d]$		0.2222 (0.0006)		

the correlation coefficient between level data is near unity, the correlation coefficient of aggregate consumption and net cash flow growth rates is less than 50%. All of these observations are consistent with the comments of Cochrane in that the production-measure-derived firm net cash flow measure may prove more useful in the upcoming asset pricing analysis.

4.4 Productivity and marginal utility growth

A key result of the general equilibrium macroeconomic growth model of Section 3.3 was that marginal utility growth is related to marginal productivity. This result is used as justification of substituting equation (3.17) for marginal utility growth and as an instrument in asset pricing tests. However, the marginal productivity series must be computed from available data.

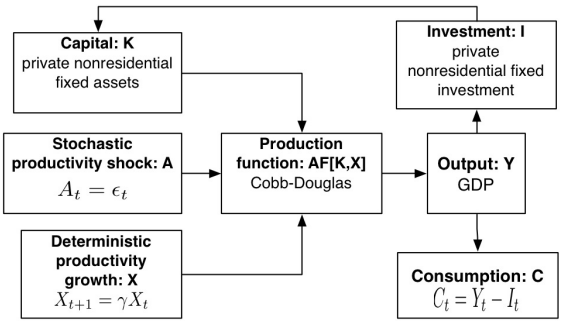


Fig. 6: Graphical representation of general equilibrium macro growth model

A crucial first step in the validation of the model is to identify the empirical proxies for the variables of interest, specifically, output, capital, and consumption. Figure 6 illustrates the relationships and definitions of variables under consideration.

Aggregate data are obtained from the Bureau of Economic Analysis National Income and Product Accounts tables (NIPA) as described in Table 1. Output (Y) is defined as gross domestic product (GDP), capital (K) is defined as private non-residential fixed assets, investment (I) is defined as private nonresidential fixed investment, and consumption (C) is defined as firm net cash flows. Note all data are converted to per-capita using population data also from the Bureau of Economic Analysis.

4.5 Asset pricing tests

All regressions are performed using generalized methods of moments (GMM) estimation with standard errors corrected for heteroskedasticity and autocorrelation via the techniques in Newey and West (1987) and Newey and West (1994). Although two-stage GMM estimation approach was described in

Section 3.4.4, iterated GMM is used in asset pricing tests. Ferson and Foerster (1994) found two-stage GMM estimation in larger (greater than 60 time series observations) complex systems (10 or more moment conditions) tends to over-reject. In such systems the authors suggest the use of an iterated approach. Given the size (over 200 time series observations) and complexity (10 assets, 8 instruments) of the model in this thesis, the iterated GMM approach is employed following Ferson and Harvey (1992) and Ferson and Foerster (1994).

If cash flows from firms are better represented by the net cash flow expression in (3.12), then standard consumption based asset pricing model performance should be improved with this refined measure. Using seasonally adjusted data and a four-asset system, Ferson and Harvey (1992) are unable to reject the simple consumption-CAPM model. Specifically, the authors obtain a p-value of 0.271 for the χ^2 test of over-identifying restrictions with consumption defined as seasonally adjusted nondurables.

In this study I follow the methodology of Ferson and Harvey (1992) for 10 prior-return portfolios described in the previous section, with two definitions of aggregate consumption: (1) aggregate non durables and (2) the firm net cash flow measure. The economical and statistical significance of these results are compared.

Also following Ferson and Harvey, I use four lags of CPI-deflated "consumption" and quarterly treasury bills. As Ferson and Harvey note, different deflators are used for the instruments (seasonally adjusted CPI) than the endogenous and exogenous variables (seasonally adjusted personal consumption expenditures or PCE) to avoid potential spurious correlations that may

arise from autocorrelated deflator measurement error.

4.5.1 Equity premium puzzle and prior return portfolios

Coefficient of risk aversion and marginal rates of substitution To illustrate the measure of risk aversion, two examples are provided. First, consider the Pratt (1964) interpretation of the risk aversion coefficient in the context of a timeless gamble as described in Ferson and Harvey (1992). An agent with power utility and degree of risk aversion σ is faced with a gamble of 1% of his wealth. Table 5 illustrates the degree of risk aversion required for this agent to be indifferent to taking a gamble with probability of winning p. As shown, the required probability of winning increases with the agent's degree of risk aversion. In other words, the more (less) risk averse agent requires a higher (lower) probability of winning.

Tab. 5: Risk aversion and winning probability for agent indifference

σ	1	10	100
p	0.50025	0.5025	0.75

To illustrate an intertemporal setting with CRRA utility, consider two period utility under certainty:

$$U\left[c_1, c_2\right] = u\left[c_1\right] + \beta u\left[c_t\right]$$

where $u[c] = \left(c^{1-\sigma}\right) / (1-\sigma)$ and β represents the rate of time preference

(patience). The marginal utilities are:

$$U_1 = c_1^{-\sigma}, \qquad U_2 = \beta c_2^{-\sigma}$$

The marginal rate of substitution (MRS), i.e, the rate at which the agent is willing to substitute period 1 consumption for period 2 is

$$MRS = \log\left[\frac{U_1}{U_2}\right] = \log\left[\frac{1}{\beta}\left(\frac{c_1}{c_2}\right)^{-\sigma}\right] = \log\left[\frac{1}{\beta}\right] - \sigma\log\left[\frac{c_1}{c_2}\right]$$

This expression can be rearranged as:

$$\log\left[\frac{c_1}{c_2}\right] = \frac{1}{\sigma}\left(\log\left[\frac{1}{\beta}\right] - \log\left[\frac{U_1}{U_2}\right]\right)$$

Therefore the intertemporal elasticity of substitution is computed as:

$$\varepsilon = \frac{d\left(\log\left[\frac{c_1}{c_2}\right]\right)}{d\left(\log\left[\frac{U1}{U_2}\right]\right)} = -\frac{1}{\sigma}$$

For example, given a 1% change in consumption, an agent with a risk aversion coefficient of 100 has a marginal rate of substitution of 1% while an agent with a coefficient of 5 has marginal rate of substitution of 20%. Therefore the rate at which an investor is willing to substitute period 1 consumption for period 2 decreases with the degree of risk aversion. In other words, the agent with the higher (lower) marginal rate of substitution has the lower (higher) coefficient of risk aversion and is more (less) willing to forgo period 1 consumption for period 2.

Negative values of σ imply risk-loving behavior, troublesome for a model

based on the assumption of investor risk aversion. However, if there are relevant omitted variables (taxes, labor, etc.), estimates of σ may be biased. The potential of omitted variables is discussed further in Chapter 7.

Equity premium puzzle and subsistence level exclusion The "equity premium puzzle," a term coined by Mehra and Prescott (1985), suggests that given CRRA utility, the degree of risk aversion required to explain the higher return on stocks than bonds is implausibly large. The practice of excluding a deterministic or stochastic subsistence level from the consumption measure has been employed by several authors (Constantinides, 1990; Ferson and Harvey, 1992; Meyer and Meyer, 2005) to address the equity premium puzzle. However, these authors did not apply such an augmentation to prior return portfolios.

Estimation of the unconditional forms of the moment conditions (3.27), (3.30), and (3.32) of Section 3.4.3 accomplish three goals. First, the existence of the equity premium puzzle in the context of prior-return portfolios is examined. Second, the impact of the use of firm cash flows in place of aggregate consumption is examined, in particular, whether or not the risk aversion coefficient can be reduced. Third, the successfulness of subsistence exclusion in reducing the risk aversion coefficient in prior return portfolios is examined.

Following Ferson and Harvey (1992), the unconditional moment restrictions of *excess* returns are estimated for a ten-asset system of prior-return portfolios for CRRA utility, DRRA utility, and time non-separable (TNS)

utility are:

$$E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}(R_{i,t+1}-R_{f,t+1})\right]=0 \quad \text{(CRRA)}$$

$$E\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma}(R_{i,t+1}-R_{f,t+1})\right]=0 \quad \text{(DRRA)}$$

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma}-\pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right)(R_{i,t+1}-R_{f,t+1})\right]=0 \quad \text{(TNS)}$$

These moment restrictions reveal the expected value of excess returns, i.e., returns from a zero investment portfolio, are zero after those returns are discounted and the discount factor varies with the utility function chosen. As suggested in Kocherlakota (1996) and Meyer and Meyer (2005) values of the risk aversion coefficient σ in the range of 0.5 to 10.0 are deemed representative of plausible investor behavior. Although σ may differ substantially from the degree of risk aversion in time non-separable models (Ferson and Harvey, 1992), it closely approximates risk aversion in models with habit persistence (Ferson and Constantinides, 1991), the scenario examined in this thesis.

4.5.2 Linear discount factor models and prior return portfolios

This thesis has suggested that nonlinear discount factor models have a theoretical advantage in explaining prior-return portfolio returns especially when using the coarser quarterly data. In order to make an empirical comparison both linear and nonlinear models must be analyzed in the same context and judged by the same criteria. To that aim, GMM is called upon to estimate discounted expected returns.

The first estimation of linear discount factor models is a test of the ability of CAPM, the Fama French 3-factor model, and the Carhart 4-factor model

to price prior-return portfolios. Following the approach of Ferson and Harvey (1992), conditional moments using excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) are estimated for CAPM, the Fama-French 3-factor model (FF3), and the Carhart 4-factor model (C4):

$$E_t\left[\left(1 + a_1 MKT_{t+1}\right) R_{ei,t+1} \middle| Z_t\right] = 0 \quad \text{(CAPM)}$$

$$E_t\left[\left(1 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1}\right) R_{ei,t+1} \middle| Z_t\right] = 0 \quad \text{(FF3)}$$

$$E_t\left[\left(1 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1} + a_4 UMD_{t+1}\right) R_{ei,t+1} \middle| Z_t\right] = 0 \quad \text{(C4)}$$

Model fit will be assessed via the χ^2 test statistic. As a further test, pricing errors, λ_i are introduced into the moment conditions be replacing $R_{ei,t}$ with $R_{ei,t} - \lambda_i$. Statistically significant pricing errors indicate model mis-specification.

4.5.3 Nonlinear discount factor models and prior return portfolios

Following the procedure of the previous section, conditional moment conditions with excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of four nonlinear factor models are evaluated: CRRA-based utility, DRRA-based utility, time non-separable (TNS) utility, and productivity (PROD) based marginal utility

growth proxy. Recalling the Euler conditions from Section 3.4.3.

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(CRRA)}$$

$$E_t\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma} R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(DRRA)}$$

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma} - \pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right) R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(TNS)}$$

$$E_t\left[\left(\frac{1}{(1-\alpha)\left(Y_{t+1}/K_{t+1}\right)+(1-\delta))}\right) R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(PROD)}$$

Model fit will be assessed via the χ^2 test statistic. As a further test, pricing errors, λ_i are introduced into the moment conditions by replacing $R_{ei,t}$ with $R_{ei,t} - \lambda_i$. Statistically significant pricing errors indicate model mis-specification.

4.6 Summary

This chapter provided evidence of momentum profitability persistence along with a test methodology for the analysis of linear and nonlinear discount factor models. The volatility of the aggregate net cash flow measure is twice that of aggregate consumption and thus should provide better explanatory power of the relatively more volatile prior return portfolio time series. The productivity-based marginal utility growth measure is employed as an alternative to consumption growth as an instrument for conditional estimations.

Several tests are performed to asses the efficacy of net cash flow and the productivity-based marginal utility growth measures in both linear and nonlinear models. The first test verifies the existence of the equity premium

puzzle when considering prior return portfolios along with the potential resolution via subsistence exclusion. The remaining tests examine the ability of linear and nonlinear factor models to explain prior-return portfolios. Results of the asset pricing tests are discussed in the following chapter.

5 Results

5.1 Equity premium puzzle and prior return portfolios

As discussed in Section 4.5.1, relative risk aversion coefficients in the range of 0.5 to 10 represent plausible investor behavior. To further illustrate, consider Figure 7, the probability of winning required for an agent with CRRA utility to be indifferent between taking a gamble of 1% of his wealth. There is an inflection point near risk aversion value of 10 and the required probability of winning is indistinguishable from 50% for values between zero and 0.5. This range of plausible values also applies to time-separable models (DRRA) with subsistence exclusion (Meyer and Meyer 2005) and time non-separable models (TNS) with habit persistence (Ferson and Constantinides, 1991) since their respective risk aversion coefficient measures closely approximate that of CRRA utility-based models.

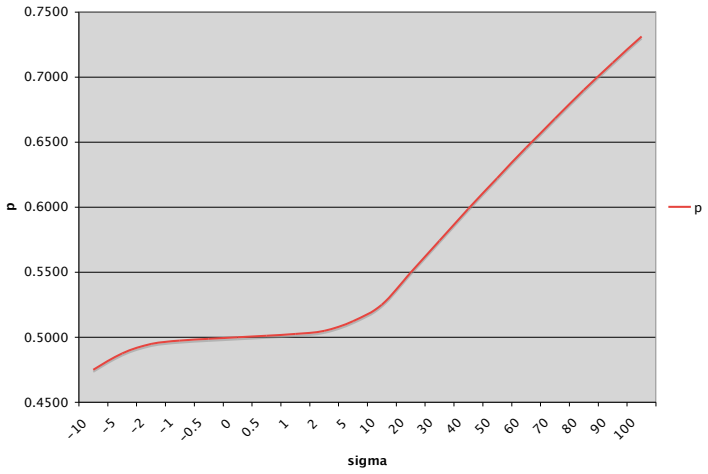


Fig. 7: Probability (p) vs. relative risk aversion coefficient (sigma)

The results presented in Table 6 provide evidence in support of the ability of the net cash flow measure to reduce the risk aversion coefficient. The relative risk aversion coefficient for aggregate consumption-based CRRA utility is 145.59, well outside the range of plausible values. Replacing aggregate consumption with net cash flows reduces the risk aversion coefficient significantly to 28.32, although still outside the range of plausible values.

Turning to DRRA utility, the magnitude of the risk aversion coefficient is significantly reduced. However, in the case of aggregate consumption, the coefficient becomes negative which implies risk-loving behavior. The risk aversion coefficient $\sigma = 0.51$for net cash flow is still within the range of plausible values. The measure is further reduced when using time non-separable utility however the parameter is negative for aggregate consumption and falls outside the range of plausible values for net cash flow.

5.2 Linear discount factors and prior return portfolios

Results of GMM estimation of the linear discount factor models of Section 4.5.2 are presented in Table 7. The coefficient signs are constant across models and instrument sets and virtually all are statistically significant. Use of the productivity-based marginal utility growth expression in place of consumption growth lowers χ^2 values for the Fama-French 3-factor (FF3) model and the Carhart 4-factor (C4) model while having a negligible effect on the CAPM model. None of the models are rejected by the χ^2 test statistics (all p-values greater than 0.5) thus warranting further investigation.

The apparent fit of the linear models to prior return portfolios is examined further by the inclusion of pricing errors λ_i by replacing $R_{i,t+1} - R_{f,t+1}$

Tab. 6: Unconditional nonlinear stochastic discount factor estimation

The degree of relative risk aversion $\sigma$is estimated using unconditional moments of CRRA, DRRA, and time non-separable (TNS) Euler conditions. The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Two measures of consumption are used: real (nominal values deflated by seasonally adjusted overall PCE deflator) quarterly aggregate nondurable consumption (c) and net cash flow (d).

$$E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(R_{i,t+1}-R_{f,t+1}\right)\right]=0 \quad \text{(CRRA)}$$

$$E\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma}\left(R_{i,t+1}-R_{f,t+1}\right)\right]=0 \quad \text{(DRRA)}$$

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma}-\pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right)\left(R_{i,t+1}-R_{f,t+1}\right)\right]=0 \quad \text{(TNS)}$$

The trend consumption level x_t is computed by geometrically detrending the respective consumption measure $\left(c=e^{\phi+\mu t}\right)$. The percent of trend consumption allotted to subsistence, π, is set to 0.9 and the time discount factor β is set to 1.0. χ^2 statistics for the test of over-identifying restrictions are provided. Standard errors are in parentheses. The sample includes quarterly data from 1948:1 to 2006:4.

Model	CRRA		DRRA		TNS	
Consumption	c	d	c	d	c	d
σ	145.59	28.32	-0.07	0.51	-1.42	0.35
	(13.21)	(4.27)	(1.22)	(0.38)	(0.14)	(0.05)

Tab. 7: Conditional linear discount factor estimation

Linear factor coefficients are estimated using CAPM, Fama French 3-factor (FF3), and Carhart 4-factor (C4) conditional moment conditions and excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of 10 prior return portfolios. The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Nominal returns are converted to real using seasonally adjusted nondurable PCE deflator.

$$E_t\left[\left(1 + a_1 MKT_{t+1}\right) R_{ei,t+1} \middle| Z_t\right] = 0 \quad \text{(CAPM)}$$

$$E_t\left[\left(1 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1}\right) R_{ei,t+1} \middle| Z_t\right] = 0 \quad \text{(FF3)}$$

$$E_t\left[\left(1 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1} + a_4 UMD_{t+1}\right) R_{ei,t+1} \middle| Z_t\right] = 0 \quad \text{(C4)}$$

Two instrument sets are used: (1) four lags of aggregate consumption growth g_c plus for lags of real risk free rate r_f and (2) four lags of the productivity-based marginal utility growth proxy Γ plus four lags of the real risk free rate. Nominal not seasonally adjusted aggregate macro data and nominal interest rates are converted to real using the not seasonally adjusted CPI deflator. χ^2 statistics for the test of over-identifying restrictions are provided with p-values in brackets $[\cdot]$. Standard errors of parameter estimates are in parentheses $(\cdot)$. Estimates in **bold** are significant at the 5% level and those in *italics* at the 10% level. The sample includes quarterly data from 1948:1 to 2006:4.

Model	CAPM		FF3		C4	
Instrument set	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$
χ^2	86.41	88.11	79.24	76.77	82.39	72.17
	[0.56]	[0.55]	[0.71]	[0.78]	[0.59]	[0.86]
a_1	**-10.24**	**-9.78**	**-9.54**	**-9.78**	**-8.71**	**-8.55**
	(0.45)	(0.43)	(0.51)	(0.48)	(0.47)	(0.42)
a_2			**9.32**	**9.43**	*1.67*	0.80
			(1.04)	(1.01)	(0.88)	(0.82)
a_3			**-5.76**	**-5.50**	**-7.92**	**-6.99**
			(0.56)	(0.75)	(0.59)	(0.66)
a_4					**-7.13**	**-7.40**
					(0.44)	(0.48)

with $R_{i,t+1} - R_{f,t+1} - \lambda_i$ as in Ferson and Harvey (1992). The pricing errors are analogous to Jensen's alpha in expected return-beta models. Results are presented in Table 8. Coefficient signs are again consistent across models and instrument sets and also with the results prior to pricing error inclusion. The χ^2 values are significantly lower after the inclusion of the pricing errors which is indicative of the significance of the added variables. For the CAPM model, 8 out of 10 pricing errors are significant when using consumption growth as the marginal utility proxy while 9 out of 10 are significant when using the productivity-based marginal utility growth proxy. Results are slightly better with the FF3 model in which 4 out of 10 pricing errors are significant when using either the consumption growth or productivity-based marginal utility growth proxy. The impact of using the productivity-based marginal utility growth proxy is more profound in the C4 model where 8 out of 10 coefficients are significant when using the consumption growth instrument while only 3 are significant when using the productivity-based instrument. As before, use of the productivity-based instrument improves the χ^2 statistics for the FF3 and C4 models, and unlike before, the CAPM model also.

In sum, three conclusions can be drawn from the results of Tables 7 and 8. First, the productivity-based marginal utility growth expression serves as a better proxy for marginal utility growth than consumption growth since it reduces the χ^2 statistics for virtually all scenarios. Second, the statistical significance of pricing errors provides evidence against the ability to explain prior return portfolios by widely used linear factor models, including the "advantaged" C4 model with its momentum factor. Third, given the consistency of discount factor coefficient estimates across models and instrument

Tab. 8: Conditional linear discount factor estimation with pricing errors

Linear factor coefficients are estimated using CAPM, Fama French 3-factor (FF3), and Carhart 4-factor (C4) conditional moment conditions and excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of 10 prior return portfolios. The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Nominal returns are converted to real using seasonally adjusted nondurable PCE deflator.

$$E_t\left[(1 + a_1 MKT_{t+1})\left(R_{ei,t+1} - R_{f,t+1}\right)\middle| Z_t\right] = 0 \quad \text{(CAPM)}$$

$$E_t\left[(1 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1})\left(R_{ei,t+1} - R_{f,t+1}\right)\middle| Z_t\right] = 0 \quad \text{(FF3)}$$

$$E_t\left[(1 + a_1 MKT_{t+1} + a_2 SMB_{t+1} + a_3 HML_{t+1} + a_4 UMD_{t+1})\left(R_{ei,t+1} - R_{f,t+1}\right)\middle| Z_t\right] = 0 \quad \text{(C4)}$$

Two instrument sets are used: (1) four lags of aggregate consumption growth g_c plus for lags of real risk free rate r_f and (2) four lags of the productivity-based marginal utility growth proxy Γ plus four lags of the real risk free rate. Nominal not seasonally adjusted aggregate macro data and nominal interest rates are converted to real using the not seasonally adjusted CPI deflator. χ^2 statistics for the test of over-identifying restrictions are provided with p-values in brackets $[\cdot]$. Standard errors of parameter estimates are in parentheses $(\cdot)$. Estimates in **bold** are significant at the 5% level and those in *italics* at the 10% level. The sample includes quarterly data from 1948:1 to 2006:4.

Model	CAPM		FF3		C4	
Instrument set	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$
χ^2	75.42	66.46	67.73	63.28	67.40	57.52
	[0.59]	[0.84]	[0.77]	[0.87]	[0.75]	[0.94]
λ_1	-0.0072	**-0.0299**	**-0.0199**	**-0.0433**	**-0.0204**	**-0.0268**
	(0.0059)	(0.0058)	(0.0069)	(0.0063)	(0.0085)	(0.0061)
λ_2	0.0045	-0.0087	**-0.0109**	**-0.0154**	0.0011	-0.0061
	(0.0048)	(0.0053)	(0.0055)	(0.0058)	(0.0067)	(0.0048)
λ_3	**0.0108**	0.0004	-0.0021	-0.0074	0.0023	0.0037
	(0.0040)	(0.0047)	(0.0046)	(0.0052)	(0.0059)	(0.0044)
λ_4	**0.0134**	0.0004	-0.0052	**-0.0118**	-0.0025	-0.0041
	(0.0036)	(0.0044)	(0.0042)	(0.0048)	(0.0054)	(0.0042)
λ_5	**0.0117**	0.0034	-0.0057	*-0.0077*	-0.0012	-0.0042
	(0.0035)	(0.0042)	(0.0041)	(0.0045)	(0.0054)	(0.0042)
λ_6	**0.0136**	0.0049	-0.0028	**-0.0095**	**-0.0107**	**-0.0128**
	(0.0036)	(0.0041)	(0.0039)	(0.0045)	(0.0052)	(0.0041)
λ_7	**0.0185**	**0.0117**	-0.0029	-0.0062	**-0.0144**	**-0.0112**
	0.0031	(0.0038)	(0.0039)	(0.0045)	(0.0051)	(0.0043)
λ_8	**0.0241**	**0.0188**	0.0054	0.0059	*-0.0092*	-0.0032
	(0.0033)	(0.0038)	(0.0039)	(0.0044)	(0.0053)	(0.0045)
λ_9	**0.0268**	**0.0164**	**0.0130**	0.0017	**-0.0117**	**-0.0116**
	(0.0035)	(0.0040)	(0.0042)	(0.0047)	(0.0058)	(0.0049)
λ_{10}	**0.0326**	**0.0256**	**0.0223**	**0.0133**	**-0.0216**	-0.0108
	(0.0048)	(0.0050)	(0.0057)	(0.0057)	(0.0082)	(0.0070)
a_1	**-6.1393**	**-6.8951**	**-8.8102**	**-8.9947**	**-7.9337**	**-7.3010**
	(0.5969)	(0.6529)	(0.7105)	(0.6459)	(0.6755)	(0.5826)
a_2			**7.7700**	-1.3349	-1.4354	-1.2713
			(1.4366)	(1.2887)	(1.1987)	(1.1149)
a_3			**-5.8901**	**-9.3797**	**-8.0833**	**-7.4577**
			(0.6857)	(0.8556)	(0.6194)	(0.6804)
a_4					**-8.5255**	**-7.0186**
					(0.6180)	(0.6519)

sets, the monotonicity of pricing errors across prior return portfolios, and the estimation technique employed produces results qualitatively similar to prior research, the methodology employed here is verified.

5.3 Nonlinear discount factors and prior return portfolios

Results of GMM estimation of the nonlinear discount factor models of Section 4.5.3 are presented in Table 9. The value of the risk aversion coefficient varies significantly across models, instrument sets, and consumption measure. In nearly all cases the value is outside the range of plausible values (0.5 to 10). However, all cases, sans the DRRA-aggregate consumption case, support the use of the productivity-based marginal utility growth proxy Γ as evidenced by lower χ^2 values than those with g_c. In addition, for the TNS-aggregate consumption and CRRA cases, replacing g_c with Γ in the instrument set produces more realistic risk aversion coefficients. In the CRRA-net cash flow case, the coefficient is reduced from 38.81 when using g_c to 18.84 when using Γ, which by at least one author (Kandel and Stambaugh, 1991) is now plausible. In the TNS-aggregate consumption cases, the coefficient is increased from an insignificant 0.03 when using g_c to a significant 0.56 when using Γ, within the range of plausible values.

The use of the net cash flow measure is also supported by several cases. In both CRRA cases, the CRRA risk aversion coefficient is reduced significantly when using net cash flow as opposed to aggregate consumption. In the DRRA-g_c case, the coefficient is increased from a risk-loving -0.09 with aggregate consumption to a risk-averse 0.18 with net cash flow. In the DRRA-Γ case, the coefficient is increased from a statistically significant -2.20 with ag-

gregate consumption to an insignificant -0.14 with net cash flow. Finally, in the TNS-g_c case, the coefficient increases from an insignificant 0.03 with aggregate consumption to a significant 0.18 with net cash flow.

The χ^2 values are higher than that of the linear discount factor models which is indicative of the greater difficulty of these nonlinear models in pricing assets. However, the models can *not* be rejected given the observed p-values in the 0.2 to 0.5 range. The lower p-values relative to the linear models warrants suspicion regarding the magnitude of pricing errors once introduced into the model.

Tables 10 and 11 present results of GMM estimation with pricing errors included when using aggregate consumption and net cash-flow measures in the Euler condition, respectively. In both consumption measure choices, the χ^2 values are reduced substantially and the majority of pricing errors are statistically significant. A larger proportion of the pricing errors are significant in the nonlinear models when compared to the linear models and they are also larger in magnitude. This result, combined with the lower χ^2 values for the linear models, suggests the linear models outperform the nonlinear models.

However, the results provide encouraging evidence regarding the use of net cash flows and the productivity based marginal utility growth proxy Γ. In the CRRA-Γ case, the risk aversion coefficient is reduced from an implausible 51.60 when using aggregate consumption to a plausible 10.28 when using net cash flows. In the TNS-g case, the coefficient increases from a statistically insignificant 0.0245 when using aggregate consumption to a significant 0.1538 when using net cash flows. Also, the use of Γ reduces the

Tab. 9: Conditional nonlinear discount factor estimation

Nonlinear factor coefficients are estimated using CRRA, DRRA, time non-separable (TNS), and productivity-based (PROD) moment conditions and excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of 10 prior return portfolios. The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Nominal returns are converted to real using seasonally adjusted nondurable PCE deflator.

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(CRRA)}$$

$$E_t\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma} R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(DRRA)}$$

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma} - \pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right) R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(TNS)}$$

$$E_t\left[\left(\frac{1}{((1-\alpha)(Y_{t+1}/K_{t+1})+(1-\delta))}\right) R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(PROD)}$$

The time discount factor β is set to 1.0, the habit persistence parameter π is set to 0.9, the labor share of output α is set to 2/3, and the depreciation rate $\delta$is set to 0.0125 per quarter (5% per year). Two instrument sets are used: (1) four lags of aggregate consumption growth g_c plus for lags of real risk free rate r_f and (2) four lags of the productivity-based marginal utility growth proxy Γ plus four lags of the real risk free rate. Nominal instrument values are based on nominal not-seasonally-adjusted data and are converted to real using not-seasonally-adjusted CPI deflator. χ^2 statistics for the test of over-identifying restrictions are provided with p-values in brackets $[\cdot]$. Standard errors of parameter estimates are in parentheses $(\cdot)$. Estimates in **bold** are significant at the 5% level and those in *italics* at the 10% level. The sample includes quarterly data from 1948:1 to 2006:4.

Model	CRRA		DRRA		TNS	
Instrument set	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$
Panel A: Aggregate consumption (c)						
χ^2	91.34	89.55	95.14	97.20	94.86	88.20
	[0.41]	[0.46]	[0.31]	[0.26]	[0.32]	[0.50]
σ	**77.54**	**111.97**	-0.09	**-2.20**	0.03	**0.56**
	(12.24)	(12.79)	(0.86)	(0.90)	(0.05)	(0.08)
Panel B: Net cash flow (d)						
χ^2	95.69	91.33	95.27	94.18	96.45	94.96
	[0.30]	[0.41]	[0.31]	[0.33]	[0.28]	[0.31]
σ	**38.81**	**18.84**	0.18	-0.14	**0.18**	**-0.13**
	(3.85)	(3.58)	(0.22)	(0.18)	(0.03)	(0.03)
Panel C: Productivity based Euler condition (PROD)						
Instrument set	$(g_c,\ r_f)$		$(g_d,\ r_f)$		$(\Gamma,\ r_f)$	
χ^2	95.15		99.44		99.04	
	[0.31]		[0.21]		[0.22]	

χ^2 values and magnitude of nearly all pricing errors.

Results for the productivity-based Euler condition (PROD) are presented in Table 12 and are similar to that of the consumption-based Euler conditions. As before, nearly all of the pricing errors are significant. However, use of the productivity-based marginal utility growth instrument Γ reduces pricing errors and χ^2 statistics.

5.4 Summary

Although the nonlinear models were unable to outperform the linear models, the results do provide evidence in support of the theoretical implications put forth in Chapter 3. First, the use of the net cash flow measure in lieu of aggregate consumption has been shown to partially address the equity premium puzzle by driving the required risk aversion coefficient towards the range of plausible values (and in some cases, within the range). Second, in all cases the use of the productivity-based marginal utility growth rate expression as an instrument lowers the χ^2 values and the magnitude of pricing errors. Potential data and model improvements are presented in Chapter 7 as a roadmap to improving nonlinear model performance.

Tab. 10: Conditional nonlinear discount factor estimation with pricing errors using aggregate consumption

Nonlinear factor coefficients are estimated using CRRA, DRRA, and time non-separable (TNS) moment conditions and excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of 10 prior return portfolios. The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Nominal returns are converted to real using seasonally adjusted nondurable PCE deflator.

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}(R_{ei,t+1}-\lambda_i)\middle|Z_t\right]=0 \quad \text{(CRRA)}$$

$$E_t\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma}(R_{ei,t+1}-\lambda_i)\middle|Z_t\right]=0 \quad \text{(DRRA)}$$

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma}-\pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right)(R_{ei,t+1}-\lambda_i)\middle|Z_t\right]=0 \quad \text{(TNS)}$$

Two instrument sets are used: (1) four lags of aggregate consumption growth g_c plus for lags of real risk free rate r_f and (2) four lags of the productivity-based marginal utility growth proxy Γ plus four lags of the real risk free rate. Nominal not seasonally adjusted aggregate macro data and nominal interest rates are converted to real using the not seasonally adjusted CPI deflator. χ^2 statistics for the test of over-identifying restrictions are provided with p-values in brackets $[\cdot]$. Standard errors of parameter estimates are in parentheses $(\cdot)$. Estimates in **bold** are significant at the 5% level and those in *italics* at the 10% level. The sample includes quarterly data from 1948:1 to 2006:4.

Model	CRRA		DRRA		TNS	
Instrument set	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$
χ^2	75.24	67.11	74.45	66.28	74.63	66.10
	[0.60]	[0.83]	[0.62]	[0.85]	[0.62]	[0.85]
λ_1	0.0080	-0.0039	0.0026	-0.0053	0.0038	-0.0040
	(0.0050)	(0.0053)	(0.0048)	(0.0052)	(0.0049)	(0.0052)
λ_2	**0.0129**	0.0062	**0.0115**	*0.0076*	**0.0113**	**0.0090**
	(0.0042)	(0.0045)	(0.0039)	(0.0043)	(0.0040)	(0.0042)
λ_3	**0.0161**	**0.0112**	**0.0135**	**0.0114**	**0.0139**	**0.0123**
	(0.0036)	(0.0050)	(0.0033)	(0.0036)	(0.0034)	(0.0035)
λ_4	**0.0183**	**0.0125**	**0.0161**	**0.0124**	**0.0163**	**0.0134**
	(0.0033)	(0.0037)	(0.0031)	(0.0035)	(0.0031)	(0.0034)
λ_5	**0.0154**	**0.0136**	**0.0149**	**0.0140**	**0.0147**	**0.0151**
	(0.0033)	(0.0033)	(0.0031)	(0.0033)	(0.0032)	(0.0032)
λ_6	**0.0210**	**0.0171**	**0.0166**	**0.0152**	**0.0174**	**0.0157**
	(0.0035)	(0.0034)	(0.0033)	(0.0033)	(0.0033)	(0.0033)
λ_7	**0.0212**	**0.0171**	**0.0196**	**0.0174**	**0.0197**	**0.0184**
	(0.0032)	(0.0033)	(0.0030)	(0.0031)	(0.0030)	(0.0031)
λ_8	**0.0271**	**0.0252**	**0.0247**	**0.0243**	**0.0251**	**0.0251**
	(0.0032)	(0.0033)	(0.0030)	(0.0031)	(0.0031)	(0.0031)
λ_9	**0.0338**	**0.0269**	**0.0284**	**0.0238**	**0.0295**	**0.0241**
	(0.0035)	(0.0034)	(0.0033)	(0.0034)	(0.0034)	(0.0034)
λ_{10}	**0.0437**	*0.0439*	**0.0396**	**0.0398**	**0.0401**	**0.0394**
	(0.0042)	(0.0040)	(0.0041)	(0.0040)	(0.0041)	(0.0040)
σ	**33.04**	**51.60**	-1.22	*-1.61*	0.02	**0.20**
	(11.08)	(11.29)	(1.01)	(0.95)	(0.06)	(0.06)

Tab. 11: Conditional nonlinear discount factor estimation with pricing errors using net cash flow

Nonlinear factor coefficients are estimated using CRRA, DRRA, and time non-separable (TNS) moment conditions and excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of 10 prior return portfolios. The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Nominal returns are converted to real using seasonally adjusted nondurable PCE deflator.

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}(R_{ei,t+1}-\lambda_i)\middle|Z_t\right]=0 \quad \text{(CRRA)}$$

$$E_t\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma}(R_{ei,t+1}-\lambda_i)\middle|Z_t\right]=0 \quad \text{(DRRA)}$$

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma}-\pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right)(R_{ei,t+1}-\lambda_i)\middle|Z_t\right]=0 \quad \text{(TNS)}$$

Two instrument sets are used: (1) four lags of net cash flow growth g_d plus for lags of real risk free rate r_f and (2) four lags of the productivity-based marginal utility growth proxy Γ plus four lags of the real risk free rate. Nominal not seasonally adjusted aggregate macro data and nominal interest rates are converted to real using the not seasonally adjusted CPI deflator. χ^2 statistics for the test of over-identifying restrictions are provided with p-values in brackets $[\cdot]$. Standard errors of parameter estimates are in parentheses $(\cdot)$. Estimates in **bold** are significant at the 5% level and those in *italics* at the 10% level. The sample includes quarterly data from 1948:1 to 2006:4.

Model	CRRA		DRRA		TNS	
Instrument set	$(g_d,\ r_f)$	$(\Gamma,\ r_f)$	$(g_d,\ r_f)$	$(\Gamma,\ r_f)$	$(g_d,\ r_f)$	$(\Gamma,\ r_f)$
χ^2	76.74	67.01	78.89	66.41	79.78	63.76
	[0.55]	[0.83]	[0.48]	[0.84]	[0.45]	[0.89]
λ_1	**0.0156**	-0.0078	0.0015	-0.0048	*0.0074*	-0.0005
	(0.0028)	(0.0055)	(0.0049)	(0.0051)	(0.0045)	(0.0051)
λ_2	**0.0277**	0.0062	**0.0138**	**0.0092**	**0.0194**	**0.0140**
	(0.0022)	(0.0046)	(0.0039)	(0.0042)	(0.0034)	(0.0042)
λ_3	**0.0269**	**0.0106**	**0.0165**	**0.0136**	**0.0195**	**0.0182**
	(0.0022)	(0.0039)	(0.0034)	(0.0035)	(0.0031)	(0.0036)
λ_4	**0.0282**	**0.0118**	**0.0174**	**0.0136**	**0.0209**	**0.0183**
	(0.0019)	(0.0037)	(0.0033)	(0.0034)	(0.0030)	(0.0036)
λ_5	**0.0253**	**0.0128**	**0.0184**	**0.0162**	**0.0242**	**0.0213**
	(0.0019)	(0.0034)	(0.0031)	(0.0032)	(0.0027)	(0.0033)
λ_6	**0.0357**	**0.0154**	**0.0211**	**0.0186**	**0.0233**	**0.0224**
	(0.0019)	(0.0035)	(0.0031)	(0.0032)	(0.0029)	(0.0032)
λ_7	**0.0346**	**0.0176**	**0.0225**	**0.0210**	**0.0252**	**0.0252**
	(0.0019)	(0.0033)	(0.0030)	(0.0032)	(0.0028)	(0.0031)
λ_8	**0.0385**	**0.0246**	**0.0280**	**0.0269**	**0.0325**	**0.0307**
	(0.0019)	(0.0033)	(0.0030)	(0.0033)	(0.0028)	(0.0032)
λ_9	**0.0453**	**0.0242**	**0.0313**	**0.0266**	**0.0352**	**0.0311**
	(0.0019)	(0.0035)	(0.0034)	(0.0036)	(0.0031)	(0.0034)
λ_{10}	**0.0566**	**0.0401**	**0.0444**	**0.0443**	**0.0499**	**0.0499**
	(0.0024)	(0.0042)	(0.0043)	(0.0044)	(0.0041)	(0.0040)
σ	**30.62**	**10.28**	-0.08	*-0.31*	**0.15**	**0.16**
	(3.90)	(3.47)	(0.1827)	(0.1844)	(0.0294)	(0.0316)

Tab. 12: Productivity-based conditional nonlinear discount factor estimation with pricing errors using net cash flow

Nonlinear factor coefficients are estimated using productivity-based (PROD) moment conditions and excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of 10 prior return portfolios using two measures of consumption: aggregate consumption (C) and net cash flow (D). The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Nominal returns are converted to real using seasonally adjusted nondurable PCE deflator.

$$E_t\left[\left(\frac{1}{(1-\alpha)\left(Y_{t+1}/K_{t+1}\right)+(1-\delta))}\right)\left(R_{ei,t+1}-\lambda_i\right)\middle| Z_t\right]=0 \quad \text{(PROD)}$$

The labor share of output α is set to 2/3 and the depreciation rate $\delta$is set to 0.0125 (5% per year).Two instrument sets are used: (1) four lags of consumption growth g_c plus for lags of real risk free rate r_f and (2) four lags of the productivity-based marginal utility growth proxy Γ plus four lags of the real risk free rate. Nominal not seasonally adjusted aggregate macro data and nominal interest rates are converted to real using the not seasonally adjusted CPI deflator. χ^2 statistics for the test of over-identifying restrictions are provided with p-values in brackets $[\cdot]$. Standard errors of parameter estimates are in parentheses $(\cdot)$. Estimates in **bold** are significant at the 5% level and those in *italics* at the 10% level. The sample includes quarterly data from 1948:1 to 2006:4.

Instrument set	$(g_c,\ r_f)$	$(g_d,\ r_f)$	$(\Gamma,\ r_f)$
χ^2	75.95	79.41	67.02
	[0.58]	[0.47]	[0.83]
λ_1	0.0018	**0.0104**	-0.0051
	(0.0050)	(0.0049)	(0.0053)
λ_2	**0.0124**	**0.0209**	*0.0074*
	(0.0040)	(0.0040)	(0.0044)
λ_3	**0.0154**	**0.0216**	**0.0114**
	(0.0035)	(0.0035)	(0.0037)
λ_4	**0.0170**	**0.0234**	**0.0128**
	(0.0032)	(0.0033)	(0.0036)
λ_5	**0.0164**	**0.0237**	**0.0140**
	(0.0032)	(0.0030)	(0.0030)
λ_6	**0.0188**	**0.0260**	**0.0158**
	(0.0034)	(0.0031)	(0.0033)
λ_7	**0.0206**	**0.0289**	**0.0176**
	(0.0031)	(0.0029)	(0.0032)
λ_8	**0.0259**	**0.0339**	**0.0250**
	(0.0032)	(0.0030)	(0.0032)
λ_9	**0.0295**	**0.0386**	**0.0248**
	(0.0034)	(0.0033)	(0.0034)
λ_{10}	**0.0410**	**0.0525**	**0.0405**
	(0.0041)	(0.0040)	(0.0040)

6 Conclusion

The persistence of momentum strategy profitability, confirmed by this thesis across a broad time series, and the underlying serial correlation in returns has generated a growing body of literature in search of a full explanation. Unfortunately the usual suspects of data mining and systematic risk compensation have been ruled out by the extant literature. Existing linear asset pricing models do not fully explain the abnormal profits associated with prior-return portfolios. In addition, existing nonlinear consumption-based models produce implausible risk aversion coefficient values when applied to prior-return portfolios.

This thesis contributes to the pursuit of momentum profitability explanation by combining the results of exchange economy and macroeconomic growth models to analyze anomalies such as momentum. The exchange economy model implicitly preserves the notion of rational investor and firm behavior through simultaneous intertemporal utility and profit maximization. The model also reveals an alternative consumption measure based on production and investment data that is better suited for asset pricing models for two reasons. First, production data are measured more reliably than consumption data. Second, the net cash flow measure exhibits twice the volatility of aggregate consumption and therefore is more likely to price the relatively high volatility asset returns.

The macroeconomic growth model yields a productivity-based proxy for marginal utility growth that bypasses two key drawbacks of traditional marginal utility growth measurements. First, the difficulty in obtaining ex-

plicit utility functions, which are unobservable, is avoided by using an observable productivity-based proxy. Second, the relatively more volatile and more reliably measured production data avoid the problems of smoothness and measurement error associated with consumption data.

The favorable characteristics of the net cash flow measure and productivity-based marginal utility growth proxy translate into improved model performance. The use of net cash flows as opposed to aggregate consumption drives risk aversion coefficients closer to the range of plausible values in the majority of scenarios examined. When using the productivity-based marginal utility growth expression in place of the commonly used consumption growth proxy, χ^2 values for both linear and nonlinear models are substantially reduced indicating greater explanatory power. However, the claim that linear approximations of marginal utility growth are at a disadvantage to their nonlinear counterparts for long-term horizons is not upheld empirically. Several data- and model-related issues are identified in Chapter 7 that, once resolved, may enhance the performance of nonlinear models.

In sum, this thesis provides evidence supporting the use of production-based consumption and marginal utility growth proxies to explain asset pricing anomalies such as momentum. Researchers can use models augmented with these measures in their examination of return patterns and potentially avoid mis-identifying those patterns as anomalies. Also, practitioners can use the improved "Jensen's alpha" measure to make more informed asset allocation decisions.

7 Areas of future research

7.1 Data issues

Several slight modifications to the input data may enhance the performance of the nonlinear models of this study. First, the use of seasonally adjusted (SA) instruments (not reported here) tends to raise the χ^2 values of the asset pricing tests relative to not seasonally adjusted (NSA) instruments indicating poorer model performance associated with the SA data. As the reader may recall, the choice of NSA instruments and deflators was meant to address the suggestion of Ferson and Harvey (1992) that "spurious correlation between endogenous variables and instruments can arise if the deflator has autocorrelated measurement error." Ferson and Harvey (1992) also note that the X-11 program used to make seasonal adjustments "removes a substantial fraction of the variability in consumption growth rates." The increased volatility associated with NSA data may be more beneficial in the context of endogenous variables. In fact, Ferson and Harvey (1992) conclude the combination of NSA data and non-separable models works better than other combinations. Therefore, using NSA endogenous variables may enhance the performance of the nonlinear models.

Balvers, Cosimano, and McDonald (1990) convert nominal values to real by using the consumption series as the numeraire. While this eliminates the ability to compare aggregate consumption vs. net cash flow, it does provide a parsimonious means to simultaneously account for growth and inflation in the aggregate data. The analysis of Table 9 was repeated following the Balvers et. al (1990) technique of nominal to real conversion and the results

are reported in Table 13. A clear advantage of employing their technique can not be gleaned from the table; however, it is worth noting the CRRA risk aversion coefficients are further reduced. Use of the relatively unreliable aggregate consumption measure as the numeraire may be contributing to the problem thus a change of numeraire (e.g., production or capital) may improve the results.

Finally, net cash flow in this thesis is constructed using output and capital measures which may introduce some look-ahead bias. Fortunately, investment data are readily available and thus repetition of the experiments with net cash flows based on output and investment may provide improved results.

7.2 Utility and production functional form modifications

7.2.1 Utility function

The nonlinear models proved unable to outperform linear factor models, at least as measured by χ^2 statistics. There are several potential avenues to explore in addition to the aforementioned data issues that may improve nonlinear factor-model performance. To begin, the utility function may be misspecified via the exclusion of another variable to which agents derive utility or improper functional form. Possible omitted variables include the labor/leisure choice (as in the original formulation of King and Rebelo, 1999), the value of the agent's wealth, or the rate of increase in the agent's wealth. Although the power utility function is widely used, it is an approximation of unobservable utility.

Tab. 13: Conditional nonlinear discount factor estimation, BCM1990

Linear factor coefficients are estimated using CAPM, Fama French 3-factor (FF3), and Carhart 4-factor (C4) conditional moment conditions and excess returns ($R_{ei,t+1} = R_{i,t+1} - R_{f,t+1}$) of 10 prior return portfolios. The quarterly Treasury bill ($R_{f,t}$) is obtained by compounding 3 one month Treasury bills. Nominal returns are converted to real using seasonally adjusted aggregate nondurable consumption as the numeraire.

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(CRRA)}$$

$$E_t\left[\left(\frac{c_{t+1}-\pi x_{t+1}}{c_t-\pi x_t}\right)^{-\sigma} R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(DRRA)}$$

$$E_t\left[\left(\left(\frac{c_{t+1}-\pi c_t}{c_t-\pi c_{t-1}}\right)^{-\sigma} - \pi\beta\left(\frac{c_{t+2}-\pi c_{t+1}}{c_t-\pi c_{t-1}}\right)^{-\sigma}\right) R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(TNS)}$$

$$E_t\left[\left(\frac{1}{(\alpha(y_{t+1}/k_{t+1})+(1-\delta))}\right) R_{ei,t+1}\middle| Z_t\right] = 0 \quad \text{(PROD)}$$

The time discount factor β is set to 1.0, the habit persistence parameter π is set to 0.9, and the capital share in output α is set to 2/3. Two instrument sets are used: (1) four lags of aggregate consumption growth g_c plus for lags of real risk free rate r_f and (2) four lags of the productivity-based marginal utility growth proxy Γ, equation (3.17) plus four lags of the real risk free rate. Nominal instrument values are converted to real using not-seasonally-adjusted aggregate nondurable consumption as the numeraire. χ^2 statistics for the test of over-identifying restrictions are provided with p-values in brackets $[\cdot]$. Standard errors of parameter estimates are in parentheses $(\cdot)$. The sample includes quarterly data from 1948:1 to 2006:4.

Model	CRRA		DRRA		TNS	
Instrument set	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$	$(g_c,\ r_f)$	$(\Gamma,\ r_f)$
Panel A: Net cash flow (d)						
χ^2	99.78	97.28	97.35	98.84	93.44	92.75
	[0.20]	[0.26]	[0.26]	[0.22]	[0.35]	[0.37]
σ	12.28	7.41	-0.12	-0.04	-0.26	-0.17
	(3.15)	(3.18)	(0.23)	(0.23)	(0.03)	(0.02)
Panel B: Productivity based Euler condition (PROD)						
Instrument set	$(g_d,\ r_f)$		$(\Gamma,\ r_f)$			
χ^2	97.95		98.87			
	[0.24]		[0.22]			

Another possibility regarding the utility function is that the risk aversion coefficient varies with time, level of wealth, or age. In fact, there are several recently published and unpublished papers that employ stochastic risk aversion (Verbrugge 2000; Bekaert, Grenadier, and Engstrom 2006; and Bekaert, Engstrom, and Xing 2006, to name a few). Further analysis (and a review of the literature) could be performed first to confirm the variability of risk aversion coefficients, which may even be stochastic, and then incorporate the variability into utility functions and empirical tests.

If labor/leisure choice is important, then there should be international and cultural differences in utility derived from consumption and leisure. For instance, leisure may be a larger factor in utility for Europeans than Americans and consumption be a larger factor for Americans than those living in China. Or, stated in a more general sense, leisure may be a larger factor of utility in developed countries than emerging countries. Once the "proper" utility functional form is found, studies on the cultural differences in determinants of utility can be examined.

7.2.2 Production function

The production function shares some of the same potential shortcomings as the utility function. The Cobb-Douglas production function is also homothetic whereas the true production function may be heterothetic (e.g., translog). Also, the capital (and labor) share of output my be time varying. These shortcomings may be addressed through incorporation of another choice variable, capacity utilization, as described in King and Rebelo (1999).

As such, the production and investment functions can be modified as follows:

$$Y_t = A_t F\left[z_t K_t, N_t X_t\right] = A_t \left(z_t K_t\right)^{1-\alpha} \left(N_t X_t\right)^{\alpha}$$
$$K_{t+1} = I_t + \left(1 - \delta\left[z_t\right]\right) K_t$$

where z_t denotes the capacity utilization rate and $\delta\left[\cdot\right]$ is convex and increasing function of the capacity utilization rate. The intuition is as follows. In periods of expansion, capacity utilization is relatively higher than that in periods of contraction, and this may be observable by simply looking at the electricity bill. An implication of the inclusion of capacity utilization is that it may increase the volatility of the productivity-based marginal utility growth rate proxy. Recall the original expression for Γ:

$$\Gamma = \frac{1}{b\left(\left(1-\alpha\right)\left(Y_{t+1}/K_{t+1}\right) + \left(1-\delta\right)\right)}$$

Note b, α, and δ are all constants in this expression and the results of Table 2 reveal this proxy has very low volatility. Thus, incorporating a time-varying capacity utilization rate is likely to increase the volatility of Γ, which, in turn, bodes well for the pricing of assets.

7.3 Substitution, complementarity, and prior return portfolios

Are individual securities within prior-return portfolios complements while individual securities in winner and loser portfolios are substitutes? In "Substitution and Complementarity in the Choice of Risky Assets," Royama and Hamada (1967) found substitution effects between asset values (demand) and

expected returns, risk, and covariance. Each of these effects are discussed in turn, as they pertain to this thesis.

The authors results are based on a one period model with beginning wealth W_0, von Neumann-Morgenstrern utility of the form $U[W] = W - \frac{1}{2}aW^2$, and an n-asset system such that $\sum_{i=1}^{n} x_i = W_0$ where x_i represents the real value of the ith asset. Royama and Hamada arrived at an asset choice analog to the consumer demand Slutsky equation:

$$S_{ij} = \left.\frac{\partial x_j}{\partial \mu_i}\right|_{W_0} - x_i \left.\frac{\partial x_j}{\partial E\,[W]}\right|_{\mu_k} \tag{7.1}$$

The left hand side of the equation represents the *total effect* of a change in expected return of asset i (μ_i) on the value for asset j. The first term on the right hand side represents the *substitution effect*, i.e., the effect on the value of asset j holding wealth constant. The final term on the right hand side represents the *income effect*, i.e., the effect on the value of asset j that results from the change in wealth due to the change in the change in the value of asset i (which is a result of the change in asset i expected return) holding all other expected returns (μ_k) constant. If the total affect is positive $S_{ij} > 0$ the assets i and j are said to be *complements* while $S_{ij} < 0$ indicates the assets are *substitutes*.

The implication for momentum analysis is that the increase in expected returns associated with prior winners will reduce the demand for prior losers, provided they are substitutes. In the context of well diversified prior return portfolios[17], it is reasonable to consider securities within a portfolio as

[17] Prior return portfolios are well diversified in that they have roughly 500 securities per portfolio. See Appendix A.2.

complements and securities in opposing portfolios (winners vs. losers) as substitutes. However, there are at least two challenges to such an analysis. First, prior return portfolios, if not reconstituted, experience return reversals. Therefore equation (7.1) requires augmentation to account for a change in expected returns. Second, the empirical measurement of S_{ij} needs to be defined.

Royama and Hamada also found several relationships between asset covariance and demand. Let $T_{ij}^k \equiv \partial x_k / \partial \sigma_{ij}$ represent the change in demand of x_k with respect to a change in the covariance of x_i and x_j. The authors examined four cases of T_{ij}^k:

Case 1 $\left(T_{ii}^i\right)$ The demand for x_i decreases with respect to risk.

$$T_{ii}^i < 0$$

Case 2 $\left(T_{ii}^k\right)$ The demand for x_k increases with the risk of x_i if x_i and x_k are substitutes. Similarly, the demand for x_k decreases with the risk of x_i if x_i and x_k are complements.

$$T_{ii}^k \begin{cases} > 0 & x_i, x_k \text{ substitutes} \\ < 0 & x_{i,} x_k \text{ complementary} \end{cases}$$

Case 3 $\left(T_{ij}^i\right)$ If x_i and x_j are complements, an increase in their covariance reduces demand for both assets.

$$T_{ij}^i < 0$$

When x_i and x_j are complements, an increase in covariance is analogous to an increase in the variance of the composite asset. If x_i and x_j are substitutes, the sign of T^i_{ij} is ambiguous.

Case 4 $\left(T^k_{ij}\right)$ If x_k is a substitute for x_i and x_j, the demand for x_k increases with σ_{ij}. Similarly, if x_k is complementary with x_i and x_j, the demand for x_k decreases with σ_{ij}.

$$T^k_{ij} \begin{cases} > 0 & x_i, x_k \text{ and } x_j, x_k \text{ substitutes} \\ < 0 & x_i, x_k \text{ and } x_j, x_k \text{ complementary} \end{cases}$$

Each of these four cases have implications for momentum analysis. Beginning with Case 1, one could posit that a change in prior return portfolio risk shall occur just prior to return reversals given the demand for the asset is decreasing in risk. For instance, in the winner portfolio, where returns are increasing (therefore demand is increasing), an increase in risk would reduce demand and this reduction in demand would translate into a reduction in returns. Regarding Case 2, since the returns of winner and loser portfolios (x_w, x_l) tend to move in opposite directions, let us assume the Slutsky equation (7.1) obtains and the assets are substitutes ($S_{wl} < 0$). Thus, an increase in the volatility of loser portfolios will increase the demand of winner portfolios and vice-versa. It would be intriguing to see if the relative volatility of winner and loser portfolios changes prior to return reversals.

Consider the final finding of Royama and Hamada, the sensitivity of the effect of a change μ_i on the demand of x_j, S_{ij}, decreases with the covariance

between x_i and x_j:

$$\frac{\partial S_{ij}}{\partial \sigma_{ij}} < 0 \tag{7.2}$$

This produces yet another intriguing implication for momentum analysis. If (7.2) is true, then intuitively one would expect the correlation of extreme portfolios (e.g, $M01$ and $M10$) to be lower than that of what will be referred to as "interior" portfolios (e.g., $M05$ and $M06$). A lower correlation between winner and loser portfolios will increase the sensitivity of demand with respect to expected returns and therefore the difference in realized returns $(M10 - M01)$. Higher correlation interior portfolios, on the other hand, will have a lower sensitivity of demand which in will in turn mitigate substitution between interior portfolios. Furthermore, momentum strategy profitability should be higher when the correlation between winner and loser portfolios is lower[18].

In sum, the early work of Royama and Hamada (1967) has several promising implications for momentum analysis. Of course, theoretical details of the incorporation of return reversals and empirical challenges of obtaining the total effect measure S_{ij} must be resolved. It is of note that microstructure theory and data could prove useful in establishing asset demands, and if necessary, very-short horizon (intraday) returns.

[18] For a truly ambitious researcher, the work of Royama and Hamada could be extended to higher order correlations.

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A Appendix

A.1 Inflation illusion derivation

Consider the rational market value of a constant growth firm

$$V^r[0] = \frac{X[0]}{k-g} \tag{A.1}$$

where g represents the real growth rate and the nominal growth rate is represented by $G = g + p$. The rational market value of a constant growth firm expressed in terms of nominal rates is equivalent to (A.1).

$$V^r[0] = \frac{X[0]}{K-G} = \frac{X[0]}{(k+p)-(g+p)} = \frac{X[0]}{k-g}$$

which verifies the rational value of a firm is unaffected by changes in inflation p.

Consider a change in inflation at time 0^+ of $p \to p^+$. Under the Fisher hypothesis, real rates (k and g) are unchanged and the rational valuation is unchanged:

$$V^r\left[0^+\right] = \frac{X[0]}{(k+p^+)-(g+p^+)} = \frac{X[0]}{k-g} = V^r[0]$$

Now consider the suggestion by Chordia and Shivakumar that the illusional investor updates the nominal discount rate but not the nominal growth rate:

$$V^i\left[0^+\right] = \frac{X[0]}{(k+p^+)-(g+p)} = \frac{X[0]}{k-g+(p^+-p)}$$

where the superscript i represents the illusional valuation. Notice $V^r \neq V^i$. When inflation rises ($p^+ > p$) the illusional value is lower than the rational

value therefore securities are undervalued. When inflation declines ($p^+ < p$) the illusional value is greater than the rational value and therefore securities are overvalued.

A.2 Diversification of size, book-to-market, and momentum deciles

One concern of strategies that yield high abnormal returns is that they are simply portfolios that are not well diversified and the abnormal returns reflect compensation for idiosyncratic risk. As a brief check of momentum strategies discussed in this study the number of securities per portfolio (decile ranking) for various authors and strategies were calculated and presented in Table 14. As shown, each decile is well diversified with a minimum of 237 firms per decile.

A.3 Discrete time dynamic programming details

A.3.1 Consumer utility maximization

Continuing from Section 3.2.1, the Bellman equation is:

$$V\left[s_{t-1}\right] = \max_{s_t} \left\{u\left[c_t\right] + \beta E_t\left[V\left[s_t\right]\right]\right\}$$

subject to

$$c_t + p_t s_t = \left(p_t + d_t\right) s_{t-1}$$

Tab. 14: Securities per size, book-to-market, and prior-return portfolios
Size and book-to-market portfolios are based on all CRSP firms with common stock data in June 2000. Momentum portfolios are based on NYSE and AMEX common stocks. Size decile rankings are based on end-of-June size breakpoints for NYSE firms. Book-to-market decile rankings are based on book equity for fiscal year end in the previous calendar year divided by market capitalization in December of previous year. Momentum decile rankings are based on prior 12 month returns.

Decile	Size	Book-to-Market	Prior return
1	2715	991	588
2	931	551	588
3	539	445	588
4	501	378	589
5	409	358	588
6	365	386	588
7	320	384	589
8	289	396	588
9	257	465	588
10	237	472	588
total	6563	4826	5882

Inserting the constraint into the Bellman equation

$$V\left[s_{t-1}\right] = \max_{s_t} \left\{ u\left[\left(p_t + d_t\right) s_{t-1} - p_t s_t\right] + \beta E_t\left[V\left[s_t\right]\right] \right\}$$

The associated first order condition is:

$$u'\left[c_t\right]\left(-p_t\right) + \beta E_t\left[V'\left[s_t\right]\right] = 0$$

$$p_t u'\left[c_t\right] = \beta E_t\left[V'\left[s_t\right]\right] \tag{A.2}$$

The indirect utility function can be eliminated by applying the envelope theorem. First, take the derivative of the Bellman equation with respect to

s_{t-1}:

$$V'[s_{t-1}] = u'[c_t](p_t + d_t)$$

then increment one time period

$$V'[s_t] = u'[c_{t+1}](p_{t+1} + d_{t+1}) \tag{A.3}$$

The Euler condition is obtained by substituting into the first order condition:

$$p_t u'[c_t] = \beta E_t \left[u'[c_{t+1}](p_{t+1} + d_{t+1})\right]$$

Dividing by p_t and noting $R_{t+1} \equiv (p_{t+1} + d_{t+1})/p_t$:

$$E_t \left[\beta \frac{u'[c_{t+1}]}{u'[c_t]} R_{t+1}\right] = 1 \tag{A.4}$$

As shown in Appendix A.3.2, for the case of N assets, the Euler condition for asset i is:

$$E_t \left[\beta \frac{u'[c_{t+1}]}{u'[c_t]} R^i_{t+1}\right] = 1 \tag{A.5}$$

A.3.2 N-asset case of discrete time consumer utility maximization

Let c_t represent consumption at time t, $\mathbf{c} = (c_0, c_{1,} \ldots)$ the vector of all consumption choices, s^i_t the quantity of the ith asset held at the beginning of period $t+1$ (or alternatively, the quantity of the ith asset purchased in period t), and $\mathbf{s}_t = \left(s^1_t, s^2_t, \ldots s^N_t\right)$ represent the vector of asset quantities.

The maximization problem therefore is:

$$\max_{\mathbf{c},\mathbf{s}_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left[c_t\right] \right] \quad \text{subject to}$$
$$c_t + \sum_{i=1}^{N} P_t^i s_t^i = \sum_{i=1}^{N} s_{t-1}^i \left(P_t^i + d_t^i \right) \tag{A.6}$$

Assume the following:

- There are N unique assets
- Each individual receives an initial endowment of one unit of each asset
- The owner of asset i receives d_t^i at the beginning of period t and the total dividend received at time t is $d_t = \sum_{i=1}^{N} d_t^i$
- Consumption is financed entirely by dividends $\therefore\ c_t = d_t\ \forall t$

The Bellman equation is:

$$V\left[\mathbf{s}_{t-1}\right] = \max_{\mathbf{s}_t} \left\{ u\left[c_t\right] + \beta E_t\left[V\left[\mathbf{s}_t\right]\right] \right\}$$

subject to the constraint (A.6). Substituting the constraint into the Bellman equation:

$$V\left[\mathbf{s}_{t-1}\right] = \max_{\mathbf{s}_t} \left\{ u\left[\sum_{i=1}^{N} s_{t-1}^i \left(P_t^i + d_t^i \right) - \sum_{i=1}^{N} P_t^i s_t^i \right] + \beta E_t\left[V\left[\mathbf{s}_t\right]\right] \right\}$$

Now compute the first order condition for the ith asset (partial derivative of expression inside braces with respect to s_t^i) :

$$\frac{\partial}{\partial s_t^i} = u'\left[c_t\right]\left(-P_t^i\right) + \beta E_t\left[\frac{\partial V\left[\mathbf{s}_t\right]}{\partial s_t^i}\right] = 0 \tag{A.7}$$

The indirect utility function can be eliminated by applying the envelope theorem. First, take the derivative of the Bellman equation with respect to s_{t-1}:

$$\frac{\partial V\left[\mathbf{s}_{t-1}\right]}{\partial s_{t-1}^i} = u'\left[c_t\right]\left(P_t^i + d_t^i\right)$$

Increment one time period

$$\frac{\partial V\left[\mathbf{s}_t\right]}{\partial s_t^i} = u'\left[c_{t+1}\right]\left(P_{t+1}^i + d_{t+1}^i\right) \tag{A.8}$$

The Euler condition is obtained by substituting (A.8) into the first order condition (A.7):

$$P_t^i u'\left[c_t\right] = \beta E_t\left[u'\left[c_{t+1}\right]\left(P_{t+1}^i + d_{t+1}^i\right)\right]$$

Dividing both sides by P_t^i, noting $R_{t+1}^i \equiv \left(P_{t+1}^i + d_{t+1}\right)/P_t^i$, and rearranging:

$$E_t\left[\beta\frac{u'\left[c_{t+1}\right]}{u'\left[c_t\right]}R_{t+1}^i\right] = 1 \tag{A.9}$$

A.3.3 Producer Euler condition derivation

Continuing from Section 3.2.2, the Bellman equation is:

$$V\left[k_t\right] = \max_{k_{t+1}} \left\{ d_t + E_t \left[\left(R_{t+1}\right)^{-1} V\left[k_{t+1}\right] \right] \right\}$$

subject to constraint (3.12). Inserting the constraint (3.12) into the Bellman equation

$$V\left[k_t\right] = \max_{k_{t+1}} \left\{ \left(f\left[k_t\right] - \left(k_{t+1} - k_t\left(1-\delta\right)\right)\right) + E_t \left[\left(R_{t+1}\right)^{-1} V\left[k_{t+1}\right] \right] \right\}$$

The associated first order condition is:

$$-1 + E_t \left[\left(R_{t+1}\right)^{-1} V'\left[k_{t+1}\right] \right] = 0$$

$$E_t \left[\left(R_{t+1}\right)^{-1} V'\left[k_{t+1}\right] \right] = 1$$

The indirect utility function can be eliminated by applying the envelope theorem. First, take the derivative of the Bellman equation with respect to k_t

$$V'\left[k_t\right] = f'\left[k_t\right] + \left(1-\delta\right)$$

then increment one time period

$$V'\left[k_{t+1}\right] = f'\left[k_{t+1}\right] + \left(1-\delta\right)$$

The Euler condition is obtained by substituting into the first order condition:

$$E_t\left[(R_{t+1})^{-1}\left(f'\left[k_{t+1}\right]+(1-\delta)\right)\right]=1 \tag{A.10}$$

A.3.4 Time non-separable Euler condition derivation

Begin with maximization problem:

$$\max_{s_t} E_0\left[\sum_{t=0}^{\infty}\beta^t u\left[c_t, c_{t-1}\right]\right] \quad \text{subject to}$$

$$c_t + p_t s_t = s_{t-1}x_t \rightarrow c_t = s_{t-1}x_t - p_t s_t \tag{A.11}$$

The Bellman equation is:

$$V\left[s_{t-1}, s_{t-2}\right] = \max_{s_t}\left\{u\left[c_t, c_{t-1}\right] + \beta E_t\left[V\left[s_t, s_{t-1}\right]\right]\right\}$$

subject to constraint (A.11). Inserting the constraint into the Bellman equation produces:

$$V\left[s_t, s_{t-1}\right] = \max_{s_t}\left\{u\left[s_{t-1}x_t - p_t s_t, s_{t-2}x_{t-1} - p_{t-1}s_{t-1}\right] + \beta E_t\left[V\left[s_t, s_{t-1}\right]\right]\right\} \tag{A.12}$$

The first order condition is:

$$u_1\left[c_t, c_{t-1}\right](-p_t) + \beta E_t\left[V_1\left[s_t, s_{t-1}\right]\right] = 0 \tag{A.13}$$

where u_1 and V_1 represents partial derivatives with respect to the first argument. Rearranging:

$$u_1 [c_t, c_{t-1}] p_t = \beta E_t [V_1 [s_t, s_{t-1}]] \tag{A.14}$$

Apply th envelope theorem by taking partial derivatives of the Bellman equation (A.12) with respect to s_{t-1} and s_{t-2}:

$$\partial/\partial_{s_{t-1}}: \quad V_1 [s_{t-1}, s_{t-2}] = u_1 [c_t, c_{t-1}] x_t + u_2 [c_t, c_{t-1}] (-p_{t-1}) + \beta E_t [V_2 [s_t, s_{t-1}]] \tag{A.15}$$

$$\partial/\partial_{s_{t-2}}: \quad V_2 [s_{t-1}, s_{t-2}] = u_2 [c_t, c_{t-1}] x_{t-1} \rightarrow V_2 [s_t, s_{t-1}] = u_2 [c_{t+1}, c_t] x_t \tag{A.16}$$

Substituting (A.16) into (A.15):

$$V_1 [s_{t-1}, s_{t-2}] = u_1 [c_t, c_{t-1}] x_t - u_2 [c_t, c_{t-1}] p_{t-1} + \beta E_t [u_2 [c_{t+1}, c_t] x_t] \tag{A.17}$$

Incrementing (A.17) by one period:

$$V_1 [s_t, s_{t-1}] = u_1 [c_{t+1}, c_t] x_{t+1} - u_2 [c_{t+1}, c_t] p_t + \beta E_t [u_2 [c_{t+2}, c_{t+1}] x_{t+1}] \tag{A.18}$$

Substituting the envelope theorem expression (A.18) into the first order condition:

$$u_1\left[c_t, c_{t-1}\right] p_t = \beta E_t\left[u_1\left[c_{t+1}, c_t\right] x_{t+1} - u_2\left[c_{t+1}, c_t\right] p_t + \beta E_t\left[u_2\left[c_{t+2}, c_{t+1}\right] x_{t+1}\right]\right] \tag{A.19}$$

Dividing both sides by p_t, defining $R_{i,t+1} \equiv x_{t+1}/p_t$, and rearranging a bit produces the time non-separable Euler condition:

$$u_1\left[c_t, c_{t-1}\right] + \beta E_t u_2\left[c_{t+1}, c_t\right] = \beta E_t\left[u_1\left[c_{t+1}, c_t\right] R_{i,t+1}\right] + \beta^2 E_t\left[u_2\left[c_{t+2}, c_{t+1}\right] R_{i,t+1}\right] \tag{A.20}$$

The left hand side represents the time t marginal utility loss from consuming one less unit. The first term on the right hand side represents the time $t+1$ marginal utility gain discounted $(\beta)$to time t and the second term represents the time $t+2$ marginal utility gain discounted $\left(\beta^2\right)$to time t.

Dividing both sides of (A.20) by $u_1\left[c_t, c_{t-1}\right]$ and rearranging again:

$$E_t\left[\beta\left(\frac{u_1\left[c_{t+1}, c_t\right]}{u_1\left[c_t, c_{t-1}\right]} + \beta\frac{u_2\left[c_{t+2}, c_{t+1}\right]}{u_1\left[c_t, c_{t-1}\right]}\right) R_{i,t+1} - \beta\frac{u_2\left[c_{t+1}, c_t\right]}{u_1\left[c_t, c_{t-1}\right]} - 1\right] = 0$$

When considering excess returns, the equation simplifies further:

$$E_t\left[\left(\frac{u_1\left[c_{t+1}, c_t\right]}{u_1\left[c_t, c_{t-1}\right]} + \beta\frac{u_2\left[c_{t+1}, c_{t+1}\right]}{u_1\left[c_t, c_{t-1}\right]}\right)\left(R_{i,t+1} - R_{f,t+1}\right)\right] = 0$$

A.3.5 Alternative quarterly productivity series construction

Begin by assuming CRTS Cobb-Douglas aggregate production function:

$$Y_t = A_t K_t^{1-\alpha} X_t^{\alpha}$$

Taking the log of both sides:

$$\log Y_t = \log A_t + (1-\alpha)\log K_t + \alpha \log X_t$$

The Solow residual, or productivity series is:

$$\log SR_t \equiv \log A_t + \alpha \log X_t = \log Y_t - (1-\alpha)\log K_t$$

which can be computed by assuming $\alpha = 2/3$:

$$\log SR_t = \log Y_t - \frac{1}{3}\log K_t$$

Recall that $X_{t+1} = \gamma X_t$. Let $X_0 = 1$ therefore:

$$X_t = \gamma^t \rightarrow \log X_t = t \log \gamma$$

Noting that A_t is the *stochastic* productivity shock, the series A_t and X_t can be derived from regressing the Solow residual on a linear trend:

$$\log SR_t = a_1 t + u_t$$

where $a_1 = \alpha \log \gamma$ and $u_t = \log A_t$

$$\gamma = \exp\left[\frac{a_1}{\alpha}\right]$$
$$A_t = \exp\left[u_t\right]$$

A.4 Macroeconomic growth model details

Continuing from Section, a central planner choice variables of consumption (C_t) and investment (via the choice of next period's capital K_{t+1}) is faced with the maximization problem:

$$\max_{C_t, K_{t+1}} \sum_{t=0}^{\infty} b^t u\left[C_t\right] \quad \text{subject to} \tag{A.21}$$

$$K_{t+1} = A_t F\left[K_t, X_t\right] + (1-\delta) K_t - C_t \tag{A.22}$$

where (A.22) is obtained by combining equations (3.14), (3.15), and (3.16). The Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} b^t u\left[C_t\right] + \sum_{t=0}^{\infty} b^t \lambda_t \left(A_t F\left[K_t, X_t\right] + (1-\delta) K_t - C_t - K_{t+1}\right)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = b^t u'\left[C_t\right] - b^t \lambda_t = 0 \Rightarrow u'\left[C_t\right] = \lambda_t \tag{A.23}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = b^{t+1} \lambda_{t+1} \left(A_{t+1} F_K\left[K_{t+1}, X_{t+1}\right] + (1-\delta)\right) - b^t \lambda_t = 0 \tag{A.24}$$

Equations (A.23) and (A.24) reveal an alternative proxy for marginal utility growth:

$$\Gamma_{t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} = \frac{u'\left[C_{t+1}\right]}{u'\left[C_t\right]} = \frac{1}{b\left(A_{t+1} F_K\left[K_{t+1}, X_{t+1}\right] + (1-\delta)\right)} \tag{A.25}$$

In the case of Cob-Douglas utility:

$$\Gamma = \frac{1}{b\left(\left(1-\alpha\right)\left(Y_{t+1}/K_{t+1}\right)+\left(1-\delta\right)\right)}$$

B Vita

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