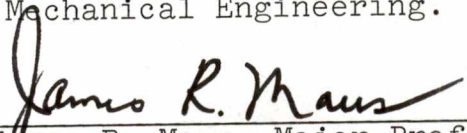
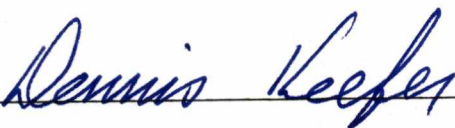


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I am submitting herewith a thesis written by Thomas Earl Hartvigsen entitled "Design of a Shock and Vibration Isolation System for a Phonograph Turntable." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

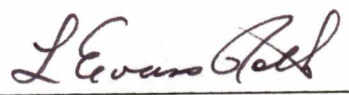

James R. Maus, Major Professor

We have read this thesis
and recommend its acceptance:





Accepted for the Council:



Vice Chancellor
Graduate Studies and Research

DESIGN OF A SHOCK AND VIBRATION ISOLATION SYSTEM
FOR A PHONOGRAPH TURNTABLE

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Thomas Earl Hartvigsen

December 1981

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ABSTRACT

Problems have been recognized for turntable operation in the presence of high sound pressure levels and floor shock disturbances in a listening room with flexible structure. These problems are due to the inadequate isolation of structure-born shock and vibrational disturbances provided by existing turntables and auxiliary isolation devices. A practical turntable isolation system design is presented in this thesis that provides low transmissibilities at frequencies of 10 Hz and above in all 6 degrees of freedom.

Both analytical and experimental results were used to select the best shock and vibration isolator. The method of eigenvalue analysis was used to obtain the natural frequencies and mode shapes for various isolation system configurations. Standing wave transmissibility equations were then derived and evaluated for various isolator configurations, assuming the isolator to be a distributed parameter rod or beam. The transmissibility of some of the more promising isolators was then measured. The shock acceleration ratio (SAR) of four isolators, selected on the basis of their transmissibility performance, was then measured with actual floor shock excitation. An isolator consisting of a 6-inch (free length) extension spring with distributed viscous damping,

having a damping factor of .041 in longitudinal vibration, was found to have the lowest transmissibility at most frequencies and a low SAR. This isolator was selected for use in the final design of the turntable isolation system.

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NOMENCLATURE

A	Area
[A]	Dynamic matrix
a	$\sqrt{\frac{E^* I}{\rho A}}$ in Equation (C-6), $\sqrt{\frac{T}{\rho}}$ in Equation (C-37), area ratio for a composite isolator
$\hat{a}$	Acceleration of $\hat{m}$ for a shock isolation system
C	Arbitrary constant
C*	Arbitrary complex constant
c	Viscous damping coefficient
c_c	c for critical damping
D	Outside diameter of bellows
D*	Arbitrary complex constant
DET	Determinant of the boundary conditions
DETZ	Determinant of the boundary conditions for impedance equation
d	Spring wire diameter
E	Young's modulus, equivalent Young's modulus
E*	Complex Young's modulus
E_ω^*	Complex Young's modulus at angular frequency ω
E_ω	Dynamic Young's modulus at angular frequency ω
$E_{\omega_{1,2}}$	E_ω for composite isolator
E_s	Young's modulus of spring wire
e	Base of natural logarithm

$\tilde{F}_0, \tilde{F}_1, \tilde{F}_2$	Sinusoidal forces applied to points 0, 1 and 2
$\{F\}$	Force matrix
$f_1 \dots f_6$	Members of force matrix
f_n	Natural frequency
f_{R_1}	Force applied at R_1
G_s	Modulus of rigidity of spring wire
h	Height of isolation system center of gravity above lower pivots
I	Moment of inertia of a plane area, equivalent moment of inertia
I_s	$\frac{\pi}{64} (d)^4$
$[I]$	Unit matrix
I_x, I_y, I_z	Mass moments of inertia about the X, Y and Z axes
j	$\sqrt{-1}$
K_1	Spring constant for Spring No. 1
$[K]$	Stiffness matrix
k_{11}	Member 1, 1 of stiffness matrix
$\hat{k}$	Stiffness of phono pickup
L	Length, active length of a bellows
L_s	Length of spring
e	Effective pendulum length
M	Mass
M_{cg}	Moment about system center of gravity
M_1, M_2	Moments about points 1 and 2

[M]	Mass matrix
$\hat{m}$	Equivalent tone-arm mass
n	Number of active coils, length of one convolution of a bellows
n^*	$\sqrt{\frac{\omega^2 \rho}{E_\omega^*}}$
P	Applied load
P_1	Static load in Spring No. 1
R	Radius
r	Mean coil radius, spring rate of one convolution of a bellows
s	Phase shift
T	Tensile force, period
$\tilde{T}$	Sinusoidal tensile force
T_m	Transmissibility
t	Time
$\tilde{V}$	Sinusoidal shear force
w	Weight of isolation system mass
X, Y, Z	Inertial coordinate axes for isolation system
X1, X2	Distance from center of gravity to pivots in plus and minus x direction
{X}	Displacement matrix
$x_1 \dots x_6$	Members of displacement matrix
x	Length coordinate for rod or beam
X_0	Displacement of $\hat{m}$ for a shock isolation system

X_1	Displacement of foundation for a shock isolation system
X_2	Displacement of m for a shock isolation system
$X_{\max}$	Maximum foundation displacement for a shock isolation system
$\{\ddot{X}\}$	Acceleration matrix
y	Transverse coordinate for a beam
$\tilde{y}$	Sinusoidal transverse displacement
y_0	Foundation displacement for a simple system
y_1	Displacement of isolated mass for a simple system
$\tilde{y}_0$	Sinusoidal transverse support motion for a beam
ZI	Distance from center of gravity to pivots in plus and minus z direction
Z_m	Driving point impedance for a rod in longitudinal vibration
Z_0	Driving point impedance for a beam or string in transverse vibration
ZN_0	$j\omega Z_0$ = The normalized driving point impedance
α	Angle
α^*	Complex variable defined by Equation (C-9)
β	Flexural rigidity of a beam
β^*	Complex variable defined by Equation (C-9)
γ	M/M_R = Mass ratio for a mass loaded rod, ω/a for a string, $\pi/\omega_0\tau$ = Rise time parameter
Δ	Deflection ($h \sin \theta$) in Appendix E
$\tilde{\Delta}$	Sinusoidal deflection

δ	Deflection
δ_1, δ_2	Pendulum displacements at points 1 and 2, deflection of Spring No. 1 and 2
δ_E	Damping factor for constant damping
δ_R	Damping factor of a shock isolation system
$\delta_{E\omega}$	Damping factor at angular frequency ω
$\delta_{E\omega_{1,2}}$	$\delta_{E\omega}$ for composite isolator
ζ	c/c_c = Viscous damping factor
η^*	Complex variable defined by Equation (A-13)
θ	Angular rotation of isolated mass, phase angle
$\tilde{\theta}$	$\partial\tilde{y}/\partial x$ = Sinusoidal angle
θ^*	Complex variable defined by Equation (A-14)
λ	An eigenvalue
$\tilde{\xi}$	Sinusoidal particle displacement
ξ^*	Complex particle displacement
ρ	Density
$\rho_{1,2}$	ρ for composite isolator
Φ^*	Complex variable defined by Equation (A-31)
τ	Rise time of a displacement step
ω	Angular frequency
ω_n	Angular natural frequency
ω_0	Angular natural frequency of a shock isolation system
$\hat{\omega}$	Angular natural frequency of $\hat{m}$ — $\hat{k}$ system

CHAPTER I

INTRODUCTION

A. The Need for Turntable Isolation

The techniques for high quality reproduction of music from phonograph records have improved in recent years. Both hardware and methods in the entire audio chain, from recording to playback, have been improved, resulting in: wider bandwidth, greater dynamic range, lower waveform distortion, and a more realistic spatial image. In order to take advantage of the available dynamic range on the finest direct-to-disc and digitally mastered records, the peak sound pressure level must be increased. The demand for higher fidelity and louder reproduction of music has exposed some problems that exist for even the finest turntables manufactured today.

In the typical music reproduction system, the turntable is operated in the presence of the sound being reproduced. The sound waves excite structural resonances in the floor, walls, and other objects in the room, which results in vibration that is structurally coupled to the turntable. This vibration of the turntable causes relative movement between the phono pickup transducer and the turntable platter, which results in spurious audio output. This phenomenon is commonly called

structure-born feedback. In severe cases, structure-born feedback will cause an audible howl or buzz; however, in most cases, it causes an annoying coloration of the bass frequencies. The problem of structure-born feedback is generally considered to occur only at frequencies below 500 Hz. There are two reasons for this: one is that most fundamental resonances of the structure occur in this frequency range, and another is that the RIAA phono equalization boosts the bass frequencies relative to the 1000 Hz level, making the system more sensitive to lower frequency disturbances. The effect of RIAA equalization on the amplifier gain is shown in Figure 1.

A phenomenon similar to structure-born feedback, which occurs when the sound waves excite structural resonances on the turntable directly, is often called acoustic feedback. The effects of acoustic feedback are normally small in comparison to structure-born feedback and can be eliminated by properly enclosing the turntable.

Another source of external disturbance for the turntable is shock excitation, which is usually caused by footsteps. These shocklike displacements of the floor are conducted through the structure to the turntable. Shocks also excite structural resonances which result in vibration being transmitted to the turntable. External disturbances, such as shock and shock-excited vibration, are capable of causing spurious audio output. Another

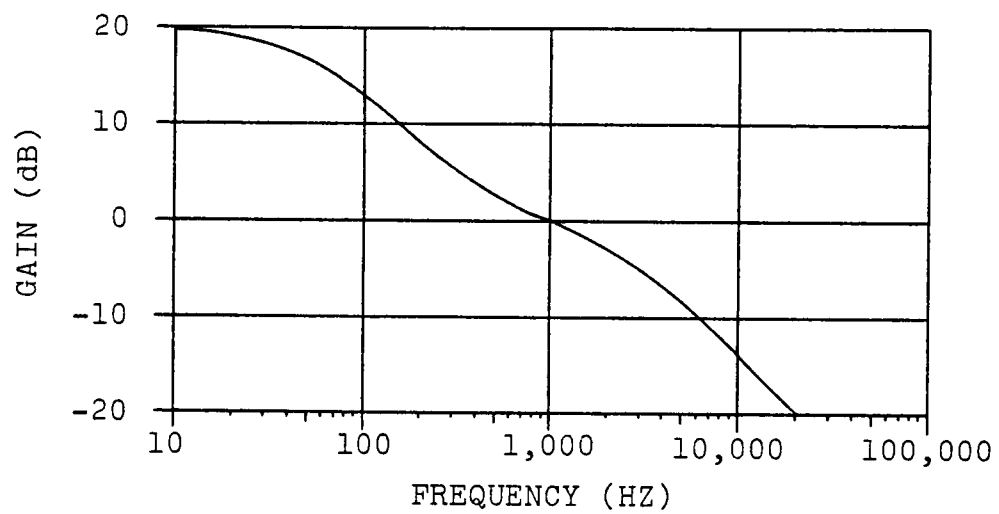


Figure 1. RIAA Playback Equalization Curve
Plotted from Data Given in [1].

problem with shock and very low frequency (infrasonic) structural resonances is that they can excite the natural frequency of the tone-arm mass/pickup compliance system. The natural frequency of this system should be placed at approximately 10 Hz for optimum performance under practical conditions, Reference [2]. This natural frequency is lightly damped with today's finest pickups and excitation of this resonance is especially harmful. The tracking ability of the pickup is momentarily reduced, which can cause audible distortion and groove damage on records containing high velocity cuts. In extreme cases, the pickup stylus may even be forced out of the groove. Therefore, when high fidelity reproduction of today's state-of-the-art records is demanded, the turntable must somehow be isolated from structure-born feedback and external shock and vibration.

B. Isolation Techniques

There are a number of valid techniques for providing turntable isolation. In rooms with a heavy concrete floor, wall, or ceiling, an effective means of reducing turntable disturbance is to rigidly attach the turntable to the concrete. The turntable is attached to a mass that has a very high mechanical impedance, which will tend to reduce the transmission of shock and vibration to acceptable levels. Rooms that are

structurally flexible can be treated in a similar manner by installing a separate concrete foundation for the turntable. A separate massive foundation is very effective; however, it can be expensive, impractical, or, in many cases, impossible.

Another isolation technique that might be considered is the active isolation system. This type of system uses electromechanical drivers with a feedback control system to keep the isolated mass motionless regardless of environmental vibration. This approach could be very effective; however, the cost of such a system is probably too high to be used for turntable isolation.

The most widely accepted method of turntable isolation is to elastically de-couple the turntable from the environment--an approach often called passive isolation. This isolation technique is incorporated in many manufactured turntables; however, the effectiveness of such systems is usually low for a number of reasons. The most common shortcoming with manufactured turntables is that the natural frequencies of the isolation system are too high to provide any isolation at the tone-arm natural frequency (10 Hz). With these systems the manufacturer will often use heavy damping to control the system resonances, which reduces the isolation at audio

frequencies. Another problem with some isolation systems is that relative movement in some directions is constrained such that practically no isolation exists in the constrained directions. Some turntables, usually of the direct-drive type, have no elastic suspension at all. It is not surprising that the performance of this type of turntable, in a structurally flexible listening room, leaves much to be desired.

Various types of accessory isolation devices have appeared on the market in the past few years, ranging from rubber isolating feet to a board with a small spring under each corner. The effectiveness of these devices is usually low and they are generally useful only for turntables that have no built-in isolation system. The one fault that all commercially-available isolating devices have in common is that they are not designed to isolate vibration in the vicinity of 10 Hz. What is needed is an elastic suspension system that is effective not only suppressing structure-born feedback, but also for isolation of shock-induced vibration at infrasonic frequencies.

C. Design Requirements

Before design work can begin, the requirements of the design must be established. The turntable isolation system design requirements are:

1. The isolators shall be of the passive, elastic type.

2. The system should provide as much isolation of vibration as possible, in the 10-500 Hz frequency range, in all 6 degrees of freedom.

3. The system should provide effective shock isolation so that dynamic changes in stylus force, due to shock, are never greater than 10 percent of the static stylus force.

4. Since the stylus force is usually applied by a slight imbalance of the tone-arm, the system must remain in a level attitude at static equilibrium.

5. The system must be simple and reliable and require little maintenance.

6. The system should not make operation of the turntable difficult.

7. The inside dimensions and maximum weight capacity of the system should be made adequate for use with virtually any existing turntable.

8. The overall dimensions of the system should be kept small enough for practical use in the home.

The objective of this thesis is to develop an optimum isolation system design, based on analysis and measurements, to satisfy the above requirements.

CHAPTER II

TRANSMISSIBILITY OF A SIMPLE SYSTEM

The design of a vibration isolation system should begin by considering the response of a simple system (Figure 2) to steady state, sinusoidal motion of the foundation. The absolute value of the amplitude ratio

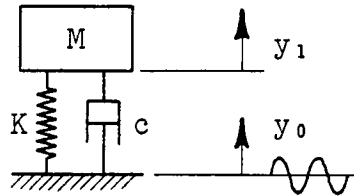


Figure 2. System Excited by Motion of the Foundation.

is called the transmissibility (T_m) and is given by the following equation from [3]:

$$T_m = \left| \frac{y_1}{y_0} \right| = \sqrt{\frac{1 + \left(2 \zeta \frac{\omega}{\omega_n} \right)^2}{\left[1 - \left(\frac{\omega}{\omega_n} \right)^2 \right]^2 + \left[2 \zeta \frac{\omega}{\omega_n} \right]^2}}, \quad (1)$$

where ζ = Damping factor = (c/c_{crit}) ,

ω = Excitation frequency,

ω_n = Natural frequency of system.

Figure 3 is a plot of Equation (1) as a function of frequency ratio for 3 values of ζ . It shows that isolation occurs only for frequencies that are above $\sqrt{2}\omega_n$ and that the transmissibility decreases with increasing frequency. It also shows that the transmissibility, for frequencies above $\sqrt{2}\omega_n$, decreases as damping is reduced. From these considerations it can be seen that, in the case of turntable isolation, the natural frequency of the system should be placed as far as possible below 10 Hz and that the damping should be minimized.

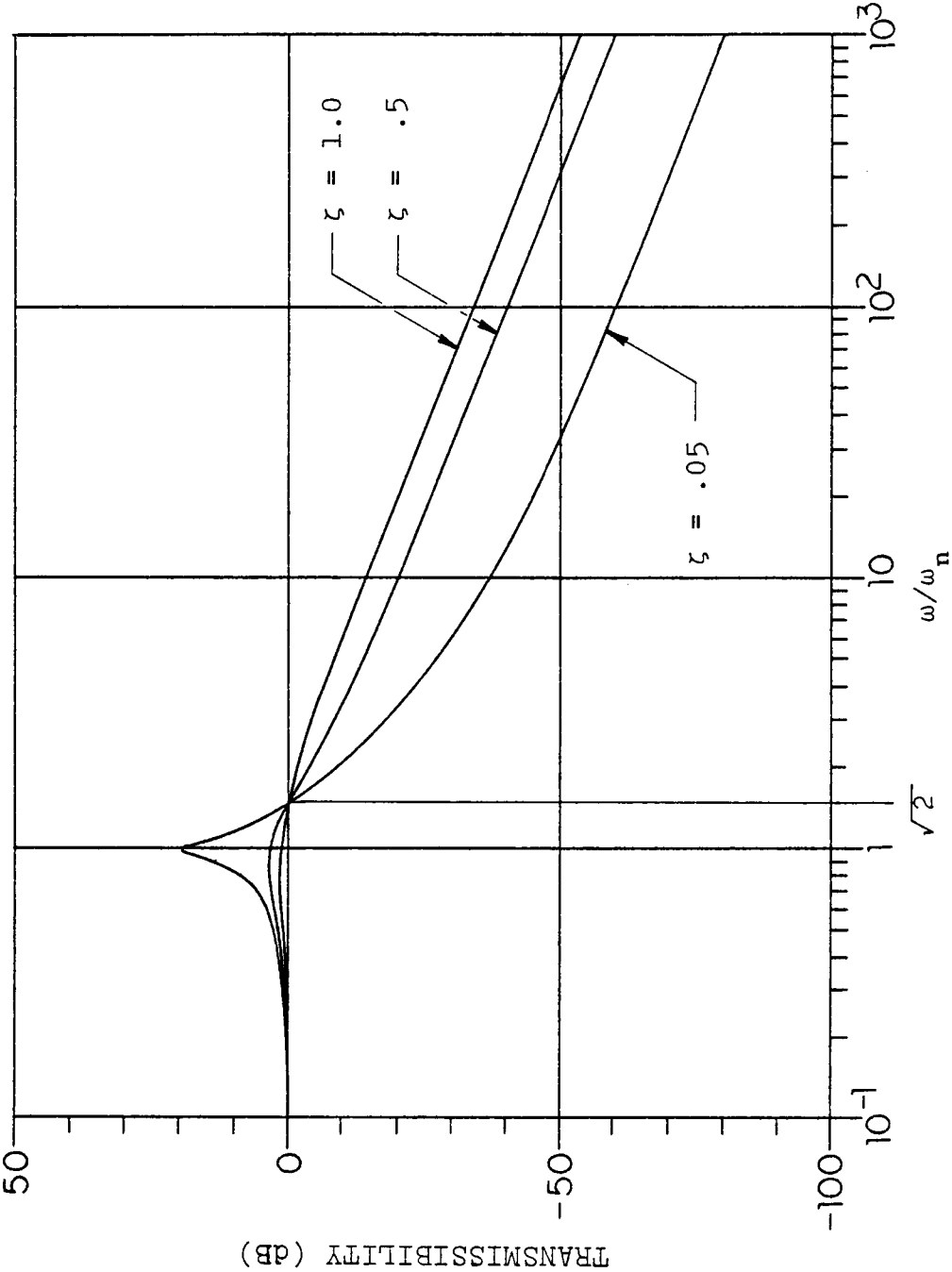


Figure 3. Plot of Equation (1).

CHAPTER III

EIGENVALUE ANALYSIS OF AN ISOLATION SYSTEM

A. Basic System Configuration

A basic configuration for a turntable isolation system is shown in Figure 4. A mass supported in this manner will have six degrees of freedom and six natural frequencies.

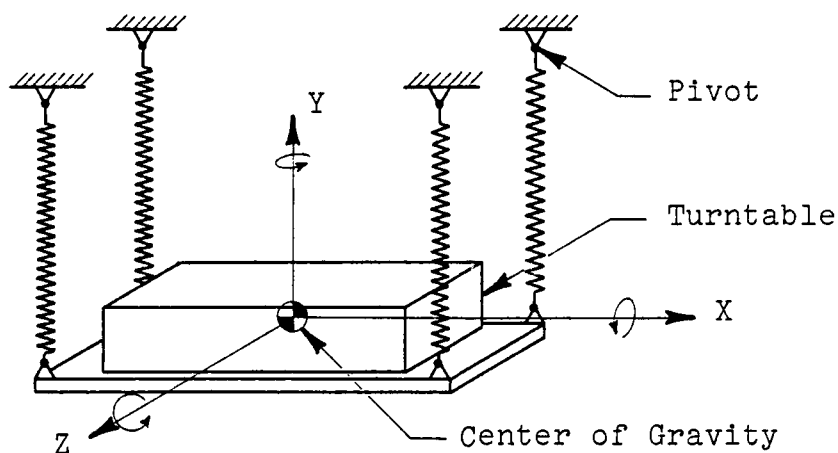


Figure 4. Basic Configuration for a Turntable Isolation System.

The degrees of freedom are translation in the X, Y, and Z directions and rotation about the X, Y, and Z axes. For the case of turntable isolation, it is desirable to keep all 6 natural frequencies of the support system as far as possible below 10 Hz and to keep damping to a minimum. Probably the best way to achieve

very low natural frequencies with low damping is to use a set of steel, helical extension springs for the elastic members. Compression springs might also be considered; however, compression springs tend to become unstable at the large static deflections required for a low natural frequency in the Y direction. The relationship between static deflection and natural frequency is shown in Figure 5.

From Figure 5 it can be seen that in order to obtain, for example, a 1 Hz natural frequency, a static deflection of approximately 10 inches is required. A 10-inch static deflection may be easily obtained by using extension springs of sufficient length. Compression springs, on the other hand, would require a very large coil diameter in order to avoid buckling problems. Because of their stability at any static deflection, extension springs will be assumed in this analysis.

The 6 natural frequencies of the suspension system will be relatively close together because they all must be placed well below 10 Hz; consequently there will tend to be some coupling between modes. In order to take coupling into account and accurately determine the natural frequencies and mode shapes, the method of eigenvalue analysis may be used. This is a matrix type solution which lends itself to machine calculation. A computerized

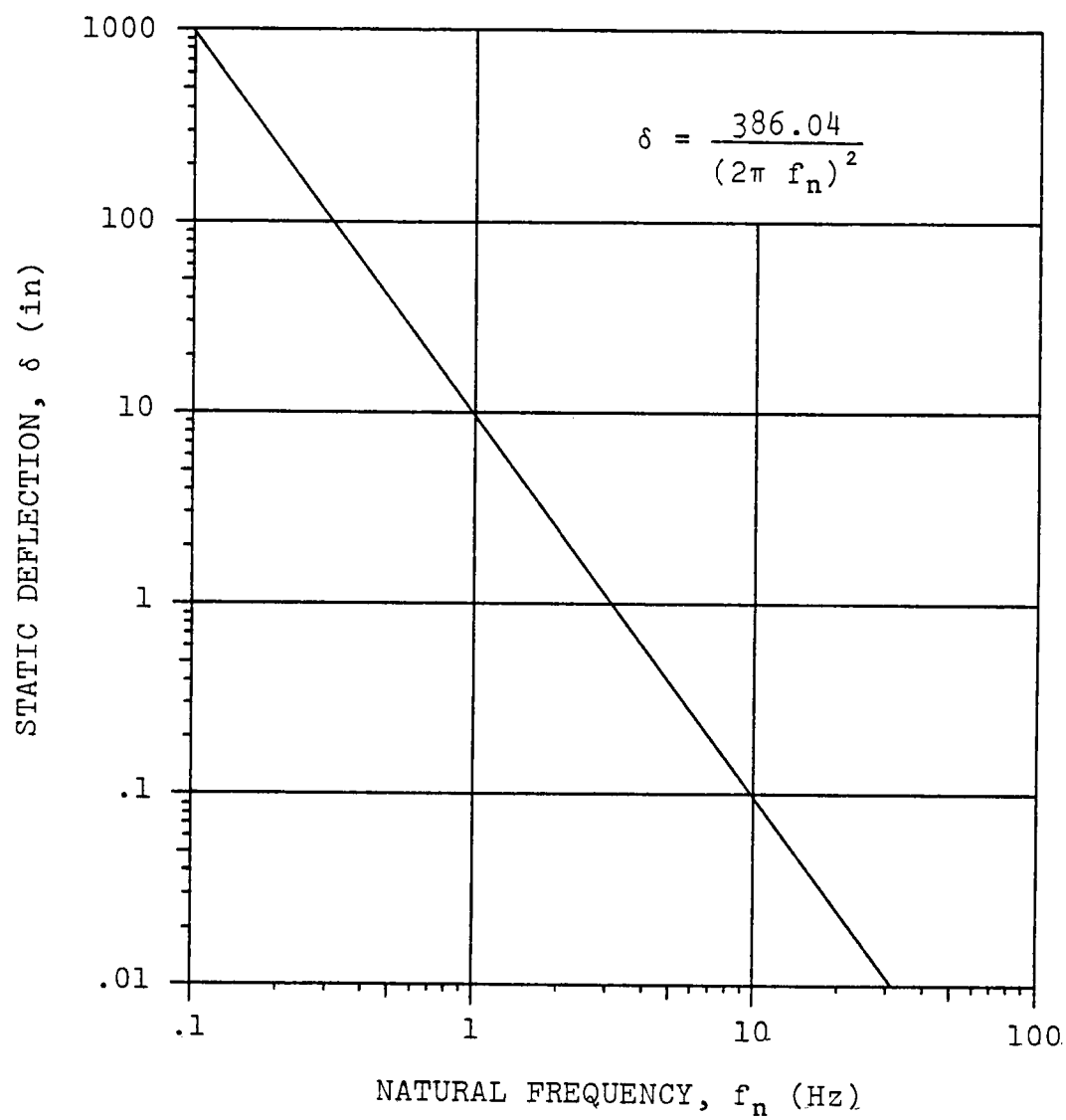


Figure 5. Static Deflection Versus Natural Frequency.

solution is handy because changes in mass distribution, spring constants, or dimensions can be made to the basic systems and easily incorporated into the solution to find the new eigenvalues (natural frequencies) and eigenvectors (mode shapes).

B. Equation Development

In this development of the eigenvalue equations, the basic theory and Equations (2) - (7) are taken from Thomson [3]. For an undamped system of several degrees of freedom, the equations of motion expressed in matrix form become

$$[M] \{\ddot{X}\} + [K] \{X\} = \{0\} \quad . \quad (2)$$

Multiplying both sides by $[M]^{-1}$,

$$[I] \{\ddot{X}\} + [M]^{-1} [K] \{X\} = \{0\} \quad (3)$$

or

$$[I] \{\ddot{X}\} + [A] \{X\} = \{0\} \quad , \quad (4)$$

where $[I]$ is the unit matrix and $[A]$ is the dynamic matrix. Assuming harmonic motion,

$$\{\ddot{X}\} = -\lambda \{X\} \quad , \quad (5)$$

where $\lambda = \omega^2$,

Equation (4) is then

$$[A - \lambda I] \{X\} = 0 \quad . \quad (6)$$

The following determinant is then formed from Equation (6):

$$|A - \lambda I| = 0 \quad . \quad (7)$$

The roots of Equation (7) are the eigenvalues which give the natural frequencies by Equation (5). To obtain the mode shape or eigenvector, the eigenvalue, λ , is substituted into Equation (6).

In order to find the eigenvalues, the dynamic matrix $[A]$ must first be found, since $[A] = [M]^{-1} [K]$, $[M]$ and $[K]$ must be obtained.

Referring to the turntable isolation system geometry shown in Figure 6, the mass matrix, $[M]$, is

$$[M] = \begin{bmatrix} M & 0 & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix} \quad \leftarrow \quad \begin{array}{l} \text{Note: No dynamic coupling} \\ \text{exists for chosen coordi-} \\ \text{nates (no off-diagonal} \\ \text{terms).} \end{array} \quad (8)$$

The $\{x\}$ is the displacement matrix, in six degrees of freedom, which are in this case:

- x_1 — Translation of cg in X direction,
- x_2 — Translation of cg in Y direction,
- x_3 — Translation of cg in Z direction,
- x_4 — Rotation of cg about X axis,

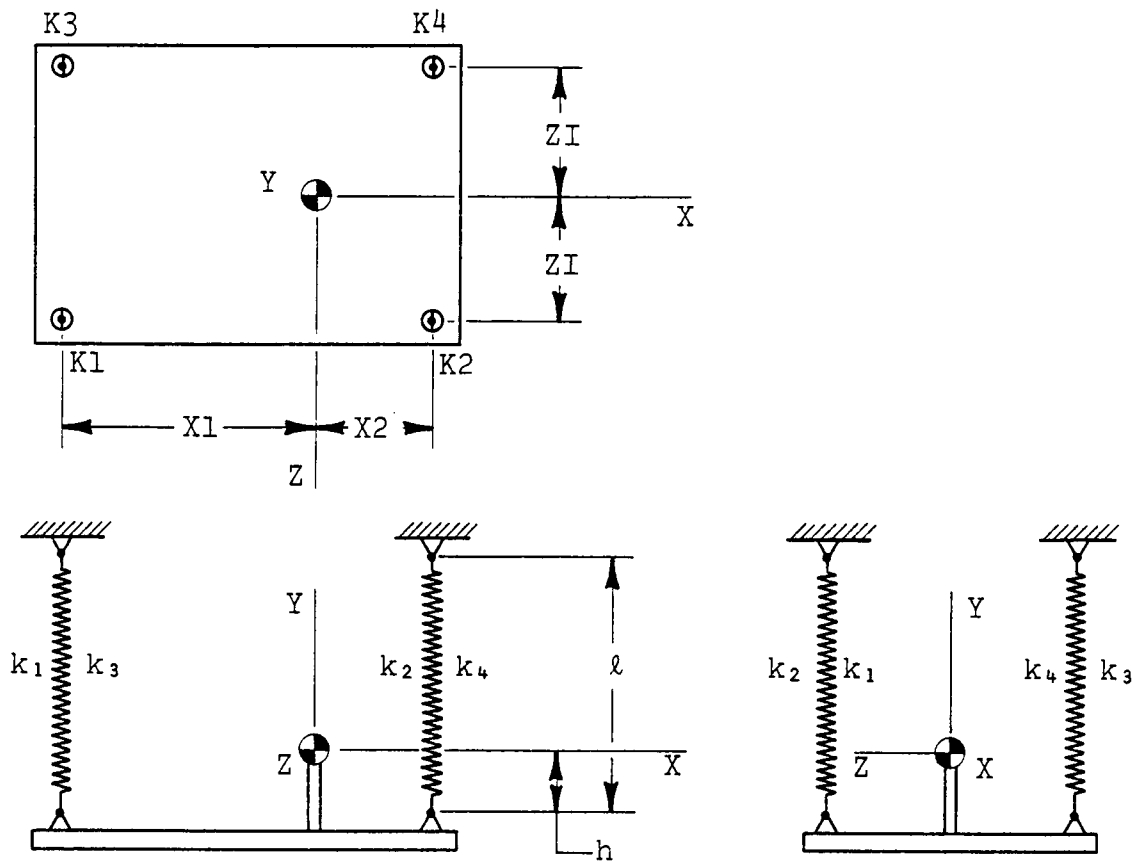


Figure 6. Basic Geometry of the Isolation System.

x_5 — Rotation of cg about Y axis,

x_6 — Rotation of cg about Z axis.

The stiffness matrix may be found by using the following relationship:

$$\{F\} = [K] \{X\} \quad , \quad (9)$$

or for the six degree of freedom problem,

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} .$$

In order to find $\begin{Bmatrix} k_{11} \\ | \\ k_{61} \end{Bmatrix}$, x_1 is set at 1, while x_2 — x_6 are held fixed at zero. The forces required to maintain this condition are then equal to the stiffness matrix

$$\begin{Bmatrix} k_{11} \\ | \\ 61 \end{Bmatrix} .$$

Each column of the stiffness matrix may be found in this manner by setting the displacement of the corresponding degree of freedom at 1 while holding all other displacements at zero. The 6 stiffness matrix columns for the isolation system geometry of Figure 6 are developed in Appendix E. The stiffness matrix, when assembled, is:

$[K] =$

$$\begin{bmatrix}
 \frac{w}{\ell} & 0 & 0 & 0 & 0 & \frac{wh}{\ell} \\
 0 & K1+K2+K3+K4 & 0 & 0 & 0 & (K2+K4)X2 + (K1+K3)X1 \\
 0 & 0 & \frac{w}{\ell} & -\frac{wh}{\ell} & 0 & 0 \\
 0 & 0 & -\frac{wh}{\ell} & (K1+K2+K3+K4)ZI^2 & 0 & 0 \\
 0 & 0 & 0 & 0 & \frac{w}{(X1+X2)\ell} [X1R_2^2 + X2R_1^2] & 0 \\
 \frac{wh}{\ell} & (K2+K4)X2 - (K1+K3)X1 & 0 & 0 & 0 & (K1+K3)X1^2 + (K2+K4)X2
 \end{bmatrix}$$

(10)

It is seen that $[K]$ is symmetrical as it should be for a linear system.

C. Results

A computer program was then written to obtain the eigenvalues and eigenvectors with the $[M]$ and $[K]$ given by Equations (8) and (9) respectively. A listing of this program with explanatory notes and a sample printout may be found in Appendix F. The program was then run with several sets of input data with the objective of finding a system with low natural frequencies and desirable mode shapes. For turntable isolation, it is desirable to have the nodes of the rotational modes located as close as possible to the tone-arm center of

gravity so that when the mode is excited, the motion amplitude and hence, acceleration, of the tone-arm would be minimized.

Input data for 5 system cases are given in Table 1. Natural frequencies, eigenvectors, and node coordinates for these 5 cases are given in Tables 2-6. The results from all 5 cases show that for systems with a 1 Hz vertical natural frequency, the highest natural frequency is approximately 1.6 Hz in a rotational mode. Cases 1, 2, and 3 show that the node locations for the rotational modes, where the maximum amplitude occurs in x_4 and x_6 , may be raised closer to the height of the tone-arm by increasing h . Cases 4 and 5 show that the node locations for these modes may be further improved by moving the system center of gravity horizontally towards the tone-arm. The highest natural frequencies are also lowered in cases 4 and 5 and are in the x_4 mode. These results indicate that certain changes may be made to the system to improve the final design. Since the highest natural frequencies are in rotational modes, it would be beneficial to lower them by increasing the moments of inertia (I_x , I_y , and I_z) by adding mass around the outside of the support plate. This additional mass should be placed as high as possible so that the system center of gravity will be raised toward the tone-arm. The lower

TABLE 1

EIGENVALUE ANALYSIS INPUT DATA FOR FIVE
VIBRATION ISOLATION SYSTEM CASES

Input Parameter (See Figure 6)	Case No.				
	1	2	3	4	5
K1 (lb/in)	1.770	1.770	1.770	1.126	1.126
K2 (lb/in)	1.770	1.770	1.770	2.414	2.414
K3 (lb/in)	1.770	1.770	1.770	1.126	1.126
K4 (lb/in)	1.770	1.770	1.770	2.414	2.414
h (in)	-2.00	0.00	2.00	2.00	3.00
ℓ (in)	17.00	17.00	17.00	17.00	17.00
ZI (in)	8.50	8.50	8.50	8.50	8.50
X1 (in)	11.00	11.00	11.00	15.00	15.00
X2 (in)	11.00	11.00	11.00	7.00	7.00
M (lb _m)	69.04	69.04	69.04	69.04	69.04
I _x (lb _m -in ²)	2039	2039	2039	2039	2039
I _y (lb _m -in ²)	5135	5135	5135	5135	5135
I _z (lb _m -in ²)	3312	3312	3312	3312	3312

TABLE 2

NATURAL FREQUENCIES, EIGENVECTORS, AND
NODE LOCATIONS FOR CASE NO. 1

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
1.5949	X ₁	-.05844	X = 0.0
	X ₂	0.0	Y = -.5854
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	0.0	
	X ₆	1.00000	
1.5739	X ₁	0.0	X = 0.0
	X ₂	0.0	Y = -.6061
	X ₃	0.06051	Z = 0.0
	X ₄	1.00000	
	X ₅	0.0	
	X ₆	0.0	
1.2225	X ₁	0.0	X = 0.0
	X ₂	0.0	Y = 0.0
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	1.00000	
	X ₆	0.0	
1.0014	X ₁	0.0	No node
	X ₂	1.00000	
	X ₃	0.0	
	X ₄	0.0	
	X ₅	0.0	
	X ₆	0.0	
0.7491	X ₁	1.00000	X = 0.0
	X ₂	0.0	Y = 82.22
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	0.0	
	X ₆	0.12183	

TABLE 2 (Continued)

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
0.7431	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = 48.89
	x ₃	1.00000	Z = 0.0
	x ₄	-.20488	
	x ₅	0.0	
	x ₆	0.0	

Note: Unit sizes on eigenvectors are as follows—x₁, x₂, x₃--1 unit = 1 inch
 x₄, x₅, x₆--1 unit = .1 radian

TABLE 3
NATURAL FREQUENCIES, EIGENVECTORS, AND
NODE LOCATIONS FOR CASE NO. 2

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
1.5905	X ₁	0.0	X = 0.0
	X ₂	0.0	Y = 0.0
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	0.0	
	X ₆	1.00000	
1.5664	X ₁	0.0	X = 0.0
	X ₂	0.0	Y = 0.0
	X ₃	0.0	Z = 0.0
	X ₄	1.00000	
	X ₅	0.0	
	X ₆	0.0	
1.2225	X ₁	0.0	X = 0.0
	X ₂	0.0	Y = 0.0
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	1.00000	
	X ₆	0.0	
1.0014	X ₁	0.0	No node
	X ₂	1.00000	
	X ₃	0.0	
	X ₄	0.0	
	X ₅	0.0	
	X ₆	0.0	
.7588	X ₁	0.0	No node
	X ₂	0.0	
	X ₃	1.00000	
	X ₄	0.0	
	X ₅	0.0	
	X ₆	0.0	

TABLE 3 (Continued)

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
.7584	x ₁	1.00000	No node
	x ₂	0.0	
	x ₃	0.0	
	x ₄	0.0	
	x ₅	0.0	
	x ₆	0.0	

Note: Unit sizes on eigenvectors are as follows—x₁, x₂, x₃--1 unit = 1 inch
 x₄, x₅, x₆--1 unit = .1 radian

TABLE 4

NATURAL FREQUENCIES, EIGENVECTORS, AND
NODE LOCATIONS FOR CASE NO. 3

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
1.5949	x ₁	0.05844	X = 0.0
	x ₂	0.0	Y = .5854
	x ₃	0.0	Z = 0.0
	x ₄	0.0	
	x ₅	0.0	
	x ₆	1.00000	
1.5739	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = 0.6061
	x ₃	-.06051	Z = 0.0
	x ₄	1.00000	
	x ₅	0.0	
	x ₆	0.0	
1.2224	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = 0.0
	x ₃	0.0	Z = 0.0
	x ₄	0.0	
	x ₅	1.00000	
	x ₆	0.0	
1.0014	x ₁	0.0	No node
	x ₂	1.00000	
	x ₃	0.0	
	x ₄	0.0	
	x ₅	0.0	
	x ₆	0.0	
0.7491	x ₁	1.00000	X = 0.0
	x ₂	0.0	Y = -82.22
	x ₃	0.0	Z = 0.0
	x ₄	0.0	
	x ₅	0.0	
	x ₆	-.12183	

TABLE 4 (Continued)

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
0.7431	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = -48.89
	x ₃	1.00000	Z = 0.0
	x ₄	0.20488	
	x ₅	0.0	
	x ₆	0.0	

Note: Unit sizes on eigenvectors are as follows—x ,x ,x --1 unit = 1 inch
 x ,x ,x --1 unit = .1 radian

TABLE 5

NATURAL FREQUENCIES, EIGENVECTORS, AND
NODE LOCATIONS FOR CASE NO. 4

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
1.5738	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = 0.6061
	x ₃	-.06051	Z = 0.0
	x ₄	1.00000	
	x ₅	0.0	
	x ₆	0.0	
1.4872	x ₁	0.07029	X = 0.0
	x ₂	0.00003	
	x ₃	0.0	
	x ₄	0.0	
	x ₅	0.0	
	x ₆	1.00000	
1.1708	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = 0.0
	x ₃	0.0	Z = 0.0
	x ₄	0.0	
	x ₅	1.00000	
	x ₆	0.0	
1.0014	x ₁	-.00002	X = 166945
	x ₂	1.00000	Y = 0.0
	x ₃	0.0	Z = 0.0
	x ₄	0.0	
	x ₅	0.0	
	x ₆	-.00006	
0.7472	x ₁	1.00000	X = 0.0
	x ₂	0.00001	Y = -68.36
	x ₃	0.0	Z = 0.0
	x ₄	0.0	
	x ₅	0.0	
	x ₆	-.14653	

TABLE 5 (Continued)

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
0.7431	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = -48.89
	x ₃	1.00000	Z = 0.0
	x ₄	0.20489	
	x ₅	0.0	
	x ₆	0.0	

Note: Unit sizes on eigenvectors are as follows—x₁, x₂, x₃--1 unit = 1 inch
 x₄, x₅, x₆--1 unit = .1 radian

TABLE 6

NATURAL FREQUENCIES, EIGENVECTORS, AND
NODE LOCATIONS FOR CASE NO. 5

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
1.5829	X ₁	0.0	X = 0.0
	X ₂	0.0	Y = 0.8957
	X ₃	-.08942	Z = 0.0
	X ₄	1.00000	
	X ₅	0.00	
	X ₆	0.00	
1.4914	X ₁	0.10413	X = 0.0
	X ₂	0.00003	Y = 1.0430
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	0.0	
	X ₆	1.00000	
1.1708	X ₁	0.0	X = 0.0
	X ₂	0.0	Y = 0.0
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	1.00000	
	X ₆	0.0	
1.0014	X ₁	-.00002	X = 0.0
	X ₂	1.00000	Y = 166945
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	0.0	
	X ₆	-.00006	
0.7333	X ₁	1.00000	X = 0.0
	X ₂	0.00002	Y = 46.15
	X ₃	0.0	Z = 0.0
	X ₄	0.0	
	X ₅	0.0	
	X ₆	-.21706	

TABLE 6 (Continued)

Natural Frequency (Hz)	Eigenvector		Node Coordinates (in)
	Degree of Freedom	Relative Amplitude	
0.7236	x ₁	0.0	X = 0.0
	x ₂	0.0	Y = -33.08
	x ₃	1.00000	Z = 0.0
	x ₄	0.30277	
	x ₅	0.0	
	x ₆	0.0	

Note: Unit sizes on eigenvectors are as follows—x₁, x₂, x₃--1 unit = 1 inch
 x₄, x₅, x₆--1 unit = .1 radian

pivot points of the springs should also be placed below the system center of gravity. The performance improvements due to asymmetrical placement of the system center of gravity (Cases 4 and 5) are interesting but they do not justify the additional expense and design complexity of using 2 different springs in the system. The final system design discussed in Chapter VIII will therefore be a symmetrical configuration.

CHAPTER IV

STANDING WAVE TRANSMISSIBILITY ANALYSIS

A. Constant Damping

In the simple system analysis, discussed in Chapter II, the spring was assumed to be massless. This is generally not a valid assumption for systems having a low natural frequency because the physical size and consequently mass of the spring is significant relative to the supported mass. With low frequency isolators, having distributed mass and elasticity, standing wave resonances will occur which account for most of the high frequency vibration that is transmitted to the isolated mass [4], [5], [6]. In order to obtain an optimum vibration isolation system, the effect of standing waves on the transmissibility must be considered. There are a number of factors which affect the transmissibility of an isolator when standing wave effects are included. Some of these factors are: the amount and type of damping, the natural frequency, the ratio of isolator mass to isolated mass, and the impedance change within an isolator. An analytical approach is required in order to understand how these factors affect the transmissibility and to obtain viable isolator designs before fabrication and testing.

In order to develop standing wave transmissibility expressions, the analytical model of the system must first be defined. The complete 6 degree of freedom system was simplified by considering the system to be 1 spring and $1/4$ of the total isolated mass. For analytical purposes, the spring was modeled as a solid elastic rod, in the case of longitudinal vibration, that simulates the mass and elastic properties of the spring. For transverse vibration, the spring was considered to be a solid elastic beam. The elastic rod and beam models are shown in Figure 7.

In order to simulate the effect of internal damping, the elastic material of the rod or beam was assumed to have a complex Young's modulus where damping is due to the imaginary part. For simplicity, the initial analysis was carried out assuming that the real and imaginary parts of the complex Young's modulus were constant. The transmissibility equations for these basic analytical models were then derived for both longitudinal and transverse vibration.

In the case of longitudinal vibration of a mass loaded rod, the rod is assumed to be a one-dimensional distributed-parameter system. This assumption is valid as long as the cross-sectional dimensions of the rod are small compared to the wavelength [4]. The vibrating

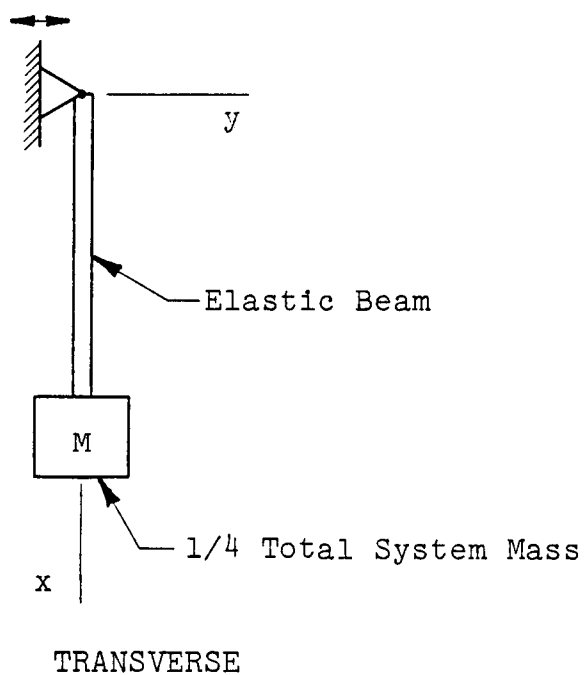
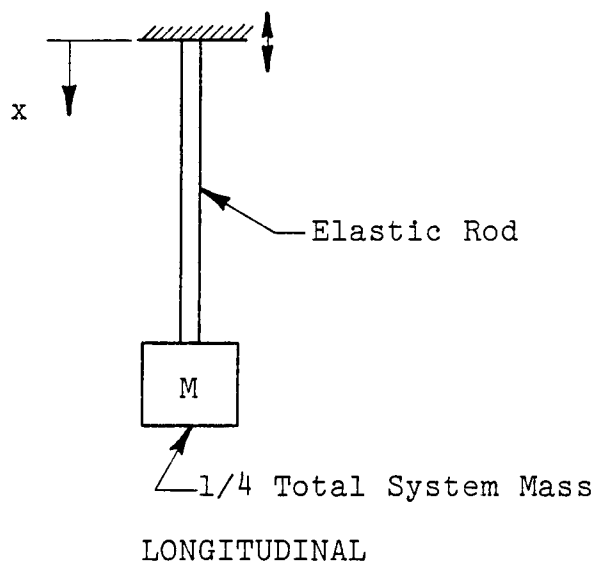


Figure 7. Analytical Models for Longitudinal and Transverse Vibration.

foundation to which the rod is attached was assumed to be infinitely rigid. The rod was also assumed to be in two segments whose properties may be different so that the effects of an impedance shift could be investigated. The analytical model for this 2-segment rod is shown in Figure 8. This figure has been rotated 90° counter-clockwise.

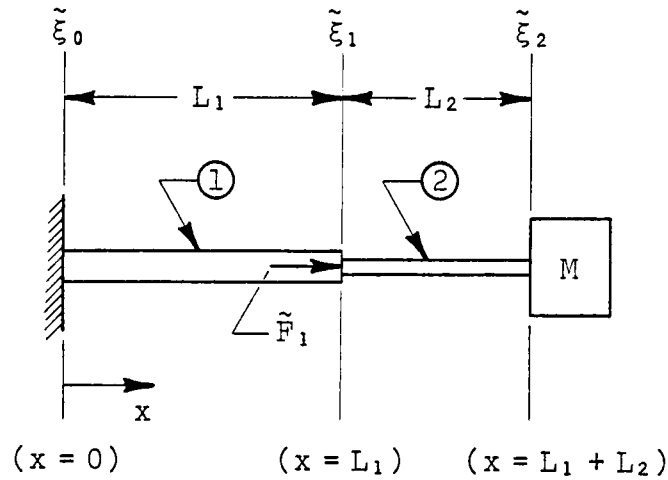


Figure 8. Mass Loaded Two-Segment Rod.

The expression for the transmissibility of this system, in longitudinal vibration, is

$$T_m = \left| \frac{1}{n_2^* \cos n_1^* L_1 - \left(\frac{n_2^* E_2^* A_2}{n_1^* E_1^* A_1} \right) \theta_2^* \sin n_1^* L_1} \right| \quad (A-24)$$

$$\text{where } \eta_2^* = \cos n_2^* L_2 - \gamma n_2^* L_2 \sin n_2^* L_2 , \quad (\text{A-13})$$

$$\theta_2^* = \sin n_2^* L_2 + \gamma n_2^* L_2 \cos n_2^* L_2 . \quad (\text{A-14})$$

The complete derivation of this equation may be found in Part 2 of Appendix A.

A computer program was then written to evaluate the transmissibility using Equation (A-24). A listing of this program with a brief explanation may be found in Appendix B.

In the case of transverse vibration of a mass loaded beam, the beam is assumed to obey the Bernoulli-Euler theory of beam vibration wherein shear deformation and rotary inertia are neglected. The Bernoulli-Euler theory is valid for relatively thin beams vibrating at low frequencies. The beam was also assumed to be in two segments with a string included between the isolated mass and the beams so that impedance effects could be investigated. This analytical model is shown in Figure 9. This figure has been rotated 90 degrees counterclockwise in order to make the axes correspond to conventional beam axes. The tension force shown in Figure 9 is due to the weight of M . The expression for the transmissibility of this system is

$$T_m = T_{m_1} T_{m_2} T_{m_3} ,$$

where T_{m_1} is the transmissibility of beam ①, T_{m_2} is the

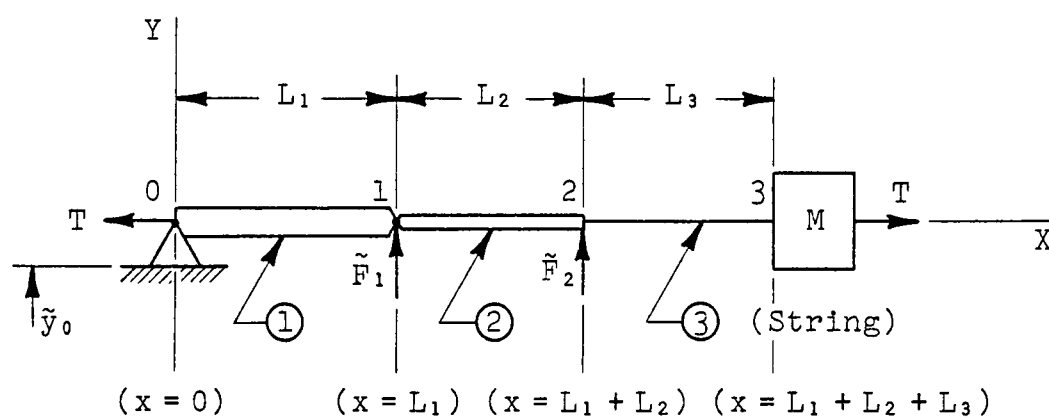


Figure 9. Mass Loaded Two-Segment Beam with a String.

transmissibility of beam ② and T_{m_3} is the transmissibility of the string. The expressions for T_{m_1} , T_{m_2} and T_{m_3} are given by Equations (C-53), (C-54) and (C-45) respectively, in Part 3 of Appendix C where they are derived.

A computer program was then written to evaluate the transmissibility using Equation (C-55). A listing of this program with a brief explanation may be found in Appendix D.

The initial input data for the constant damping analysis was obtained by assuming a load of approximately 25 lbs. per isolator. A stock extension spring with a 25.9 lb. load capacity was found in the Lee stock spring catalog [7]. The spring parameters, obtained from the catalog data, are given in Table 7. The input parameters for the computer program were then calculated using this spring data. Input parameters are given in Tables 8 and 9 for the rod cases and in Tables 10 and 11 for the beam cases.

An input parameter required for the beam cases is the equivalent moment of inertia, I . According to Wahl [8], the flexural rigidity, β , of a coil spring is given by

$$\beta = \frac{2 L_s E_s I_s G_s}{\pi r (2 G_s + E_s)} \quad , \quad (11)$$

TABLE 7
SPRING PARAMETERS

Spring No.	Outside Diameter (in)	Wire Diameter (in)	Maximum Load (lb)	Free* Length (in)	Spring Constant (lb/in)	Initial Tension (lb)	Length at Maximum Load (in)	Spring Mass (lb _m)	Number of Coils
1	.850	.085	25.9	2.75	12.188	2.4	4.678	.065	14.3
2	.850	.085	25.9	6.0	3.326	2.4	13.065	.216	52.6
3	.850	.085	25.9	12.0	1.663	2.4	26.131	.432	105.2
4	1.00	.095	30.0	4.5	4.987	2.7	9.974	.199	33.1
5	1.00	.095	30.0	6.0	3.375	2.7	14.089	.292	48.8
6	1.00	.095	30.0	12.0	1.687	2.7	28.183	.583	97.7
7**	1.460	.112	26.2	3.5	16.1	0	1.868	.100	8.2

*Measured between end loops except Spring No. 7.

**Compression Spring.

TABLE 8

INPUT PARAMETERS FOR ROD CASES 1-8

Input Parameter (See Appendix B)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
Spring No.†	2	2	2	2	3	1	*	*
REW1 (lb/in ²)	23.760	23.760	23.760	23.760	23.760	39.370	7.684	7.684
IEW1 (lb/in ²)	0	2.376	9.504	23.760	2.376	3.937	0	0
RHO1 (lb _m /in ³)	.0181	.0181	.0181	.0181	.0181	.0191	.0210	.0210
A1 (in ²)	1.828	1.828	1.828	1.828	1.828	1.448	6.143	6.143
L1 (in)	6.533	6.533	6.533	6.533	13.065	2.339	2.50	5.0
REW2 (lb/in ²)	23.760	23.760	23.760	23.760	23.760	39.370	7.684	9.349
IEW2 (lb/in ²)	0	2.376	9.504	23.760	2.376	3.937	0	0
RHO2 (lb _m /in ³)	.0181	.0181	.0181	.0181	.0181	.0191	.0210	2.46E-3
A2 (in ²)	1.828	1.828	1.828	1.828	1.828	1.448	6.143	7.068
L2 (in)	6.533	6.533	6.533	6.533	13.065	2.339	2.50	.375
MR2 (lb _m)	.1080	.1080	.1080	.1080	.2160	.0323	.3222	6.52E-3
M (lb _m)	25.90	25.90	25.90	25.90	25.90	25.90	47.2	47.2

*Theoretical compression spring.

†See Table 7 on page 39.

TABLE 9
INPUT PARAMETERS FOR ROD CASES 9-14

Input Parameter (See Appendix B)	Case 9	Case 10	Case 11	Case 12	Case 13	Case 14
Spring No.†	2	5	*	6	4	7
REW1 (lb/in ²)	23.760	60.542	1.727	60.542	63.331	6.453
IEW1 (lb/in ²)	0	0	0	0	0	0
RHO1 (lb _m /in ³)	.0181	.0263	7.77E-3	.0263	.0254	.0116
A1 (in ²)	1.828	.7854	3.227	.7854	.7854	4.599
L1 (in)	6.533	7.045	.75	14.089	4.987	.935
REW2 (lb/in ²)	23.760	60.542	1.727	60.542	63.331	6.453
IEW2 (lb/in ²)	4.752	0	0	0	0	0
RHO2 (lb _m /in ³)	.0181	.0263	7.77E-3	.0263	.0254	.0116
A2 (in ²)	1.828	.7854	3.227	.7854	.7854	4.599
L2 (in)	6.533	7.045	.75	14.089	4.987	.935
MR2 (lb _m)	.1080	.1458	.0188	.2915	.0996	.0499
M	25.90	30.00	32.98	30.00	30.00	26.2

*Stock metal bellows.

†See Table 7, page 39.

TABLE 10

INPUT PARAMETERS FOR BEAM CASES 1-6

Input Parameter (See Appendix D)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Spring No.†	2	2	2	2	2	2
REW1 (lb/in ²)	23.760	23.760	23.760	23.760	23.760	23.760
IEW1 (lb/in ²)	0	47.520	0	2.376	9.504	23.760
RHO1 (lb _m /in ³)	.0181	.0181	.0181	.0181	.0181	.0181
A1 (in ²)	1.828	1.828	1.828	1.828	1.828	1.828
I1 (in ⁴)	.2662	.2662	.2662	.2662	.2662	.2662
L1 (in)	13.065	13.065	6.533	6.533	6.533	6.533
REW2 (lb/in ²)	23.760	23.760	23.760	23.760	23.760	23.760
IEW2 (lb/in ²)	0	0	0	2.376	9.504	23.760
RHO2 (lb _m /in ³)	.0181	.0181	.0181	.0181	.0181	.0181
A2 (in ²)	1.828	1.828	1.828	1.828	1.828	1.828
I2 (in ⁴)	.2662	.2662	.2662	.2662	.2662	.2662
L2 (in)	.001	.001	6.533	6.533	6.533	6.533
LRHO3 (lb/in)	0	0	1.291E-4	1.291E-4	1.291E-4	1.291E-4
L3 (in)	0	0	3.934	3.934	3.934	3.934
T (lb)	25.90		25.90	25.90	25.90	25.90
M (lb-sec ² /in)	.0671	.0671	.0671	.0671	.0671	.0671

†See Table 7 on page 39.

TABLE 11

INPUT PARAMETERS FOR BEAM CASES 7-9

Input Parameter (See Appendix D)	Case 7	Case 8	Case 9
Spring No.†	7	*	**
REW1 (lb/in ²)	6.537	1.050	1.727
IEW1 (lb/in ²)	0	0	0
RHO1 (lb _m /in ³)	.0116	2.60×10^{-3}	$7.77E-3$
A1 (in ²)	4.603	4.089	3.227
I1 (in ⁴)	1.686	1.331	.829
L1 (in)	1.870	.60	.75
REW2 (lb/in ²)			1.727
IEW2 (lb/in ²)			0
RHO2 (lb _m /in ³)			$7.77E-3$
A2 (in ²)			3.227
I2 (in ⁴)			.829
L2 (in)			.75
LRH03(lb/in)			$1.291E-4$
L3 (in)			10.0
T (lb)	-26.23	-37.41	33.0
M (lb-sec ² /in)	.0679	.0969	.0854

†See Table 7 on page 39.

*Theoretical metal bellows.

**Stock metal bellows.

where E_s = Young's modulus of spring wire,

L_s = Length of spring,

$I_s = \pi d^4/64$

d = Spring wire diameter,

G_s = Modulus of rigidity of wire,

n = Number of active coils,

r = Mean coil radius.

The flexural rigidity, β , is also equal to the equivalent EI of a beam so that

$$I = \frac{\beta}{E} \quad (12)$$

Any convenient value of E may be chosen which will then specify I.

The effect of damping factor on the longitudinal and transverse transmissibility was investigated first. Figure 10 is a plot of the transmissibility for Rod Cases 1, 2, 3, and 4 which assume a damping factor of zero, .1, .4, and 1.0 respectively. From this plot, it can be seen that damping is very effective in reducing the transmissibility at high frequencies and especially at the standing wave resonant frequencies. For the transverse transmissibility of this isolator, it was found that damping is ineffective in reducing the transmissibility of the first standing wave resonance. This is shown in Figure 11 which is a plot of Beam Case 1 (undamped) and Beam Case 2 with a damping factor of 2.0.

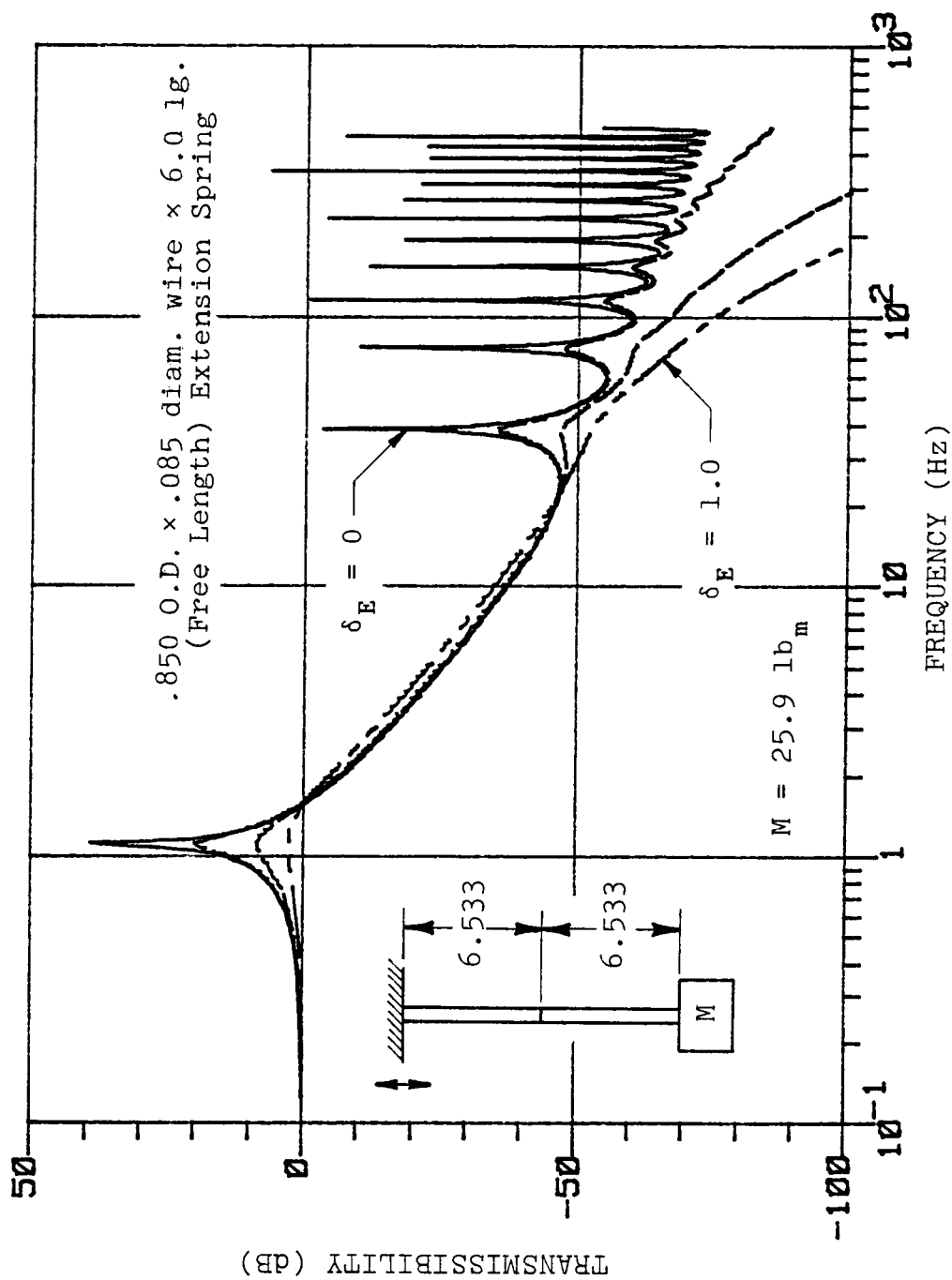


Figure 10. Transmissibility of a Mass-Loaded Rod in Longitudinal Vibration (Rod Cases 1, 2, 3, and 4). Damping factor $\delta_E = 0, 0.1, 0.4$, and 1.0 .

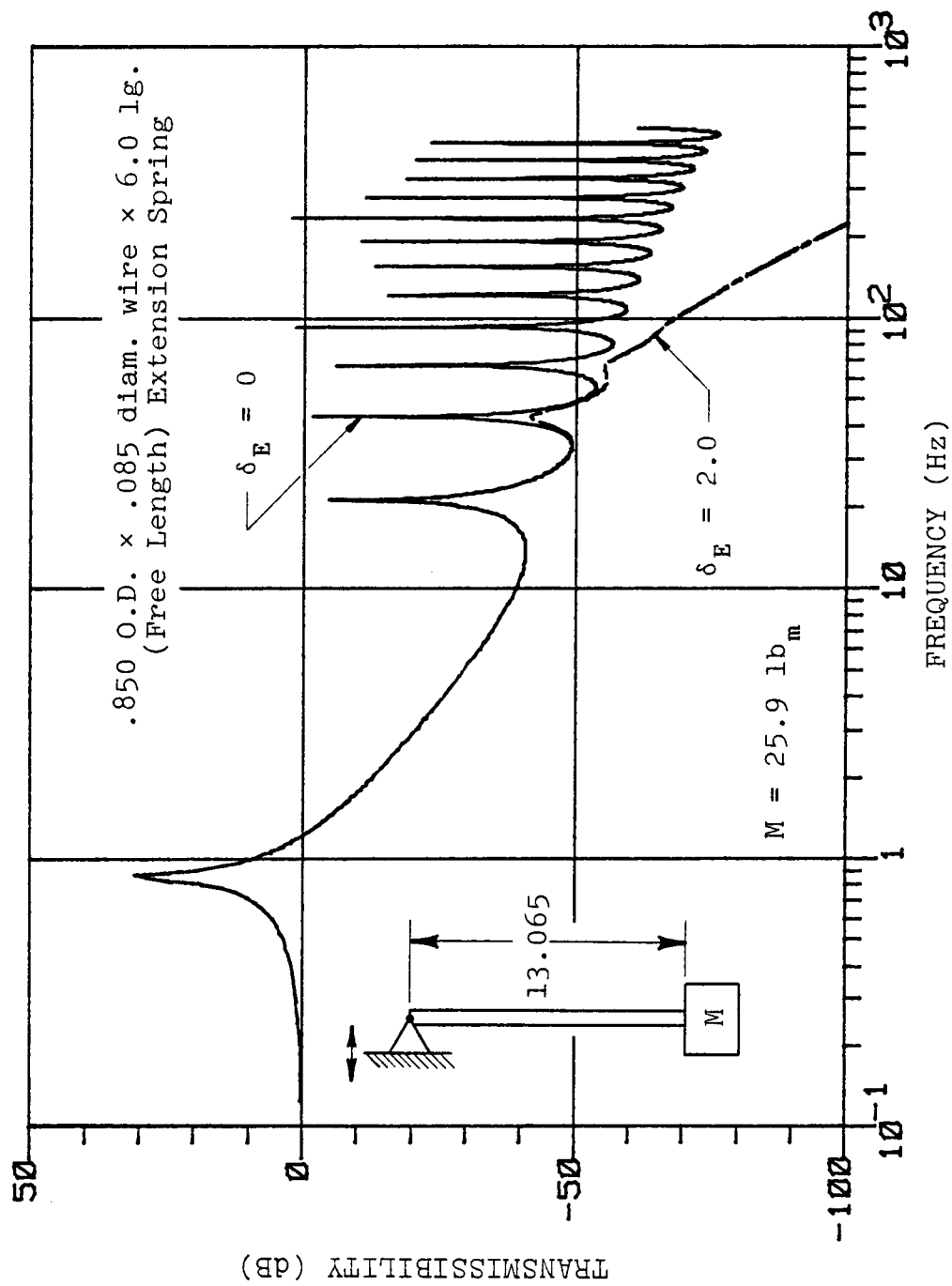


Figure 11. Transmissibility of a Mass-Loaded Beam in Transverse Vibration
(Beam Cases 1 and 2). Damping factor $\delta_E = 0$ and 2.0.

Damping is ineffective in this case probably because the bending strain is relatively small for a given deflection in the first standing wave mode.

The effect of natural frequency and mass ratio, γ , on the standing wave transmissibility was studied next. The mass ratio, γ , is the ratio of isolator mass to isolated mass. The natural frequency and mass ratio are really coupled together for coil spring isolators because the physical size and mass of the spring must increase in order to obtain a lower natural frequency. Figure 12 is a plot of the longitudinal transmissibility for Rod Cases 5, 2, and 6 which have a damping factor of .1 and a natural frequency of .792 Hz, 1.120 Hz and 2.145 Hz respectively. This plot shows that as the natural frequency and mass ratio increase, the frequency at which the first wave resonance occurs becomes higher. At the same time, the transmissibility at 10 Hz also increases. These effects are also observed for beam cases.

It is suggested in [5] that the standing wave transmissibility peaks may be reduced by placing another spring in series with the main isolating spring, thus creating an impedance change along the length of the isolator. Plotted in Figure 13 are Rod Cases 7 and 8. Rod Case 7 is for a theoretical compression spring and rod and Rod Case 8 is for the same spring with a small air

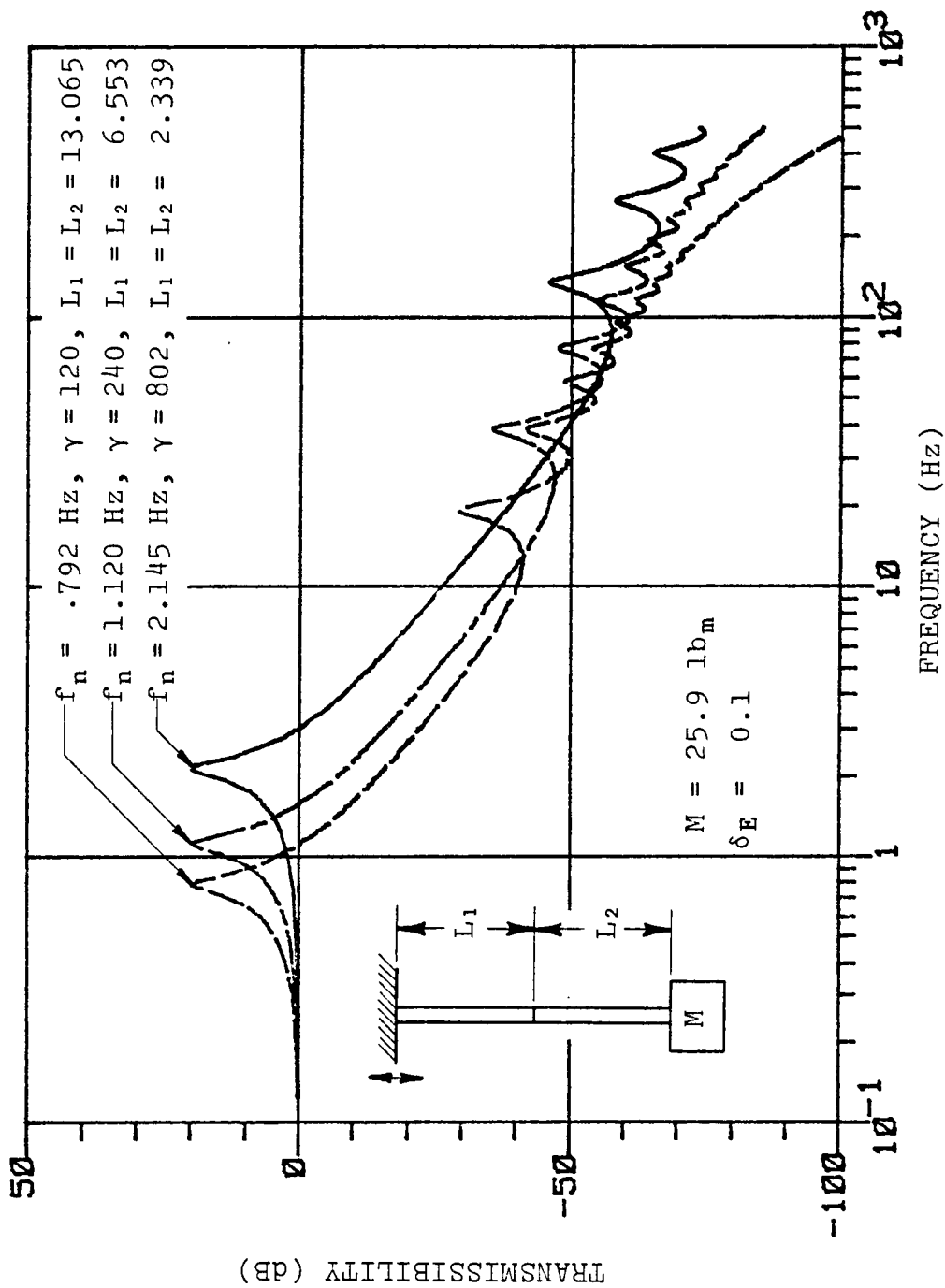


Figure 12. Transmissibility of Mass-Loaded Rods in Longitudinal Vibration (Rod Cases 5, 2, and 6). Damping factor $\delta_E = 0.1$, natural frequency $f_n = .792 \text{ Hz}$, 1.120 Hz and 2.145 Hz . Mass ratio $\gamma = 120, 240$, and 802 .

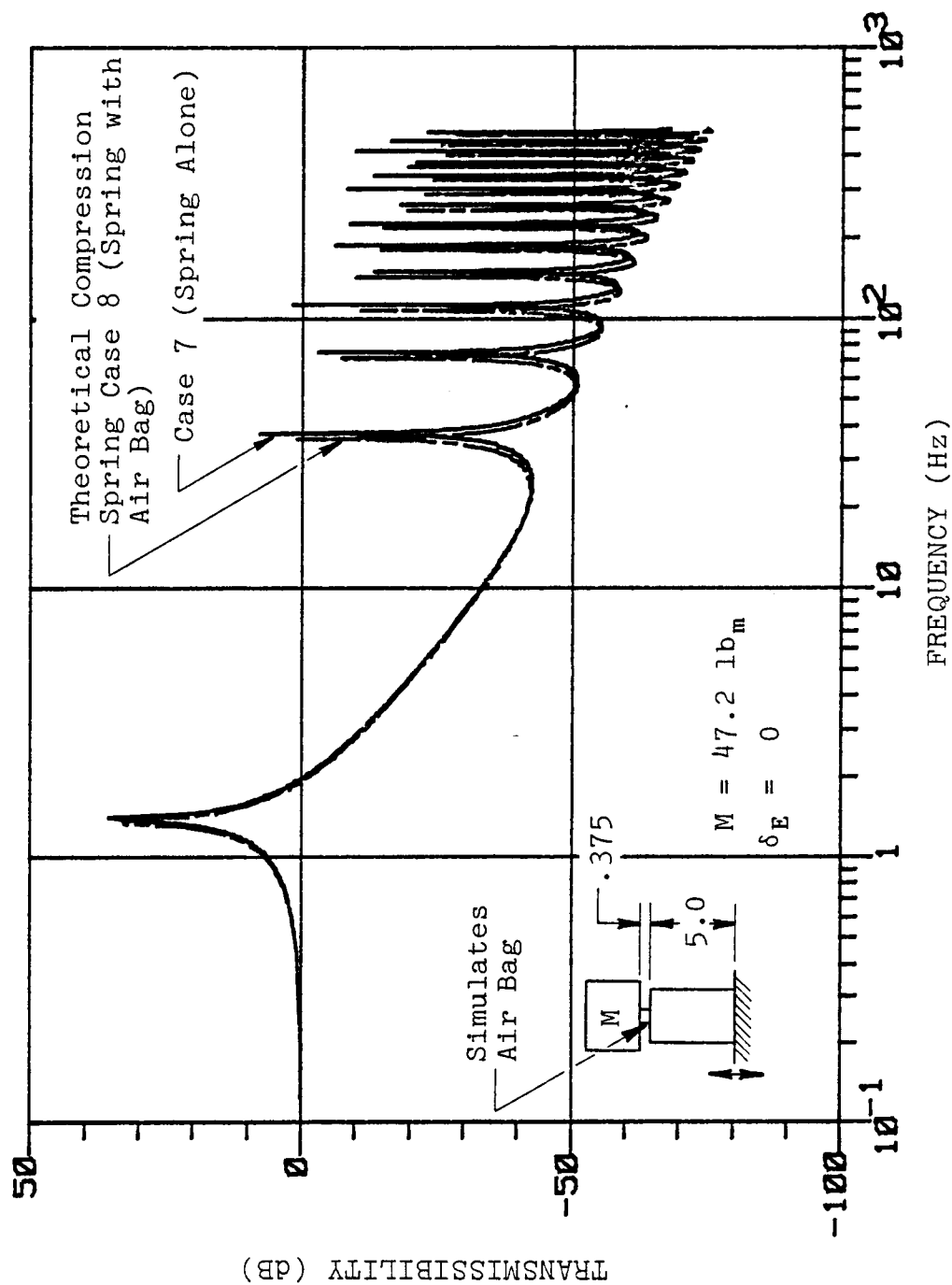


Figure 13. Transmissibility of Mass-Loaded Rods in Longitudinal Vibration (Rod Cases 7 and 8). Damping factor $\delta_E = 0$.

bag placed in series with the spring to introduce an impedance change. The small air bag is the second rod segment in this case. It can be seen from Figure 13 that the addition of the air bag causes only a slight frequency shift in the transmissibility peaks and a reduction in the transmissibility troughs of about 5 dB at 500 Hz. This plot clearly shows that the impedance shift does not provide any substantial transmissibility reduction for the first few standing wave resonances. Figure 14 is a plot of Beam Cases 3, 4, 5, and 6 which consist of 2 spring segments with damping factors of zero, .1, .4, and 1.0 in series with a 3.9-inch length of .027 diameter steel cable. It can be seen from this plot that the impedance shift, due to the addition of the cable, does not provide a noticeable transmissibility reduction for the first few standing wave resonances.

Another way to introduce an impedance shift is to apply damping only to one segment of an isolator. Figure 15 is a plot of Rod Case 2 which has a damping factor of .1 on both segments and Rod Case 9 which has a damping factor of .2 only on the second segment. This plot shows that partial damping has no particular advantage over full length damping. The transmissibility at the standing wave frequencies is lower for partial damping in this case because the total effective damping factor is higher.

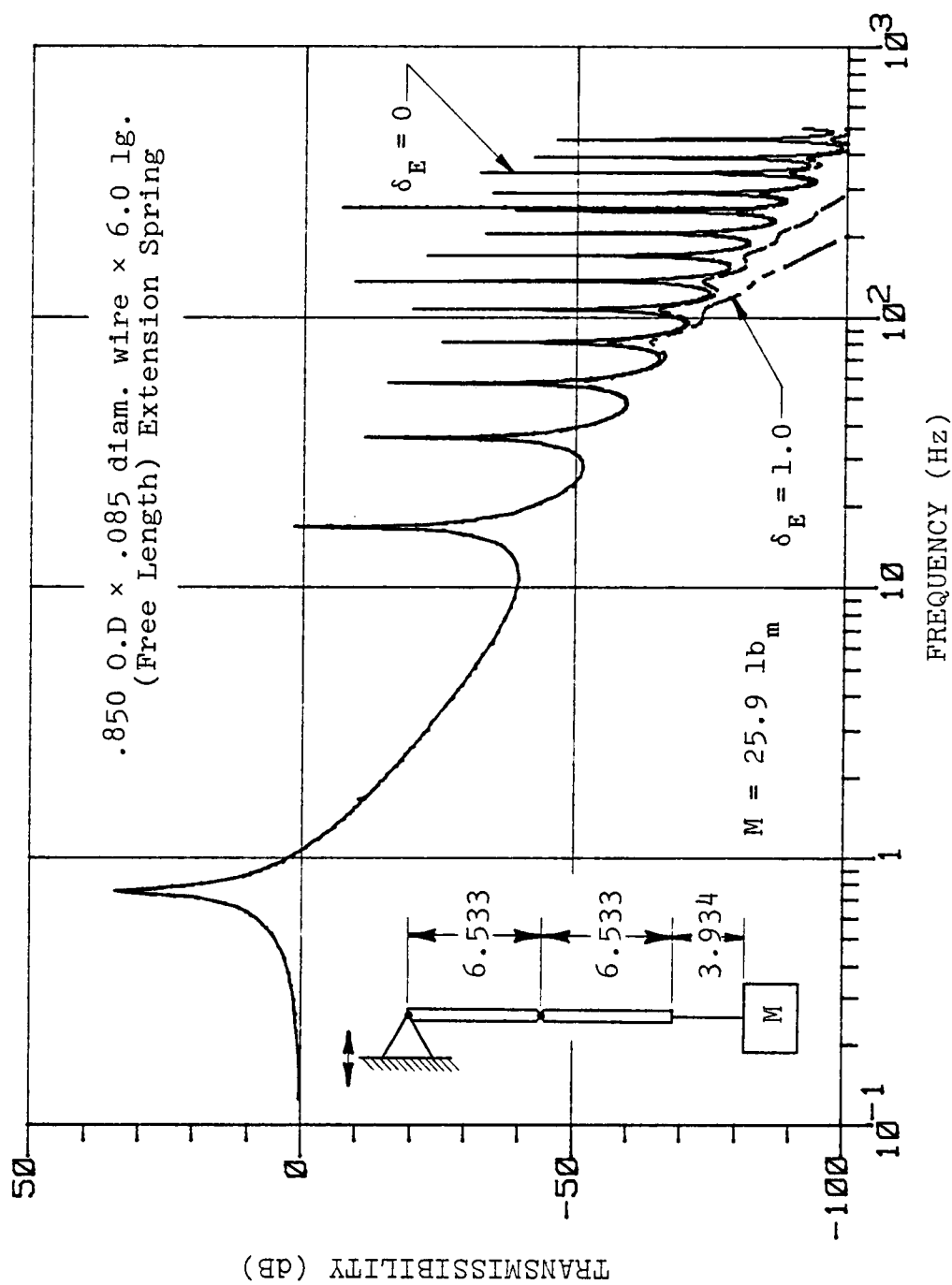


Figure 14. Transmissibility of a Mass-Loaded, 2-Segment Beam with a String, in Transverse Vibration (Beam Cases 3, 4, 5, and 6). Damping factor $\delta_E = 0, 0.1, 0.4$, and 1.0 .

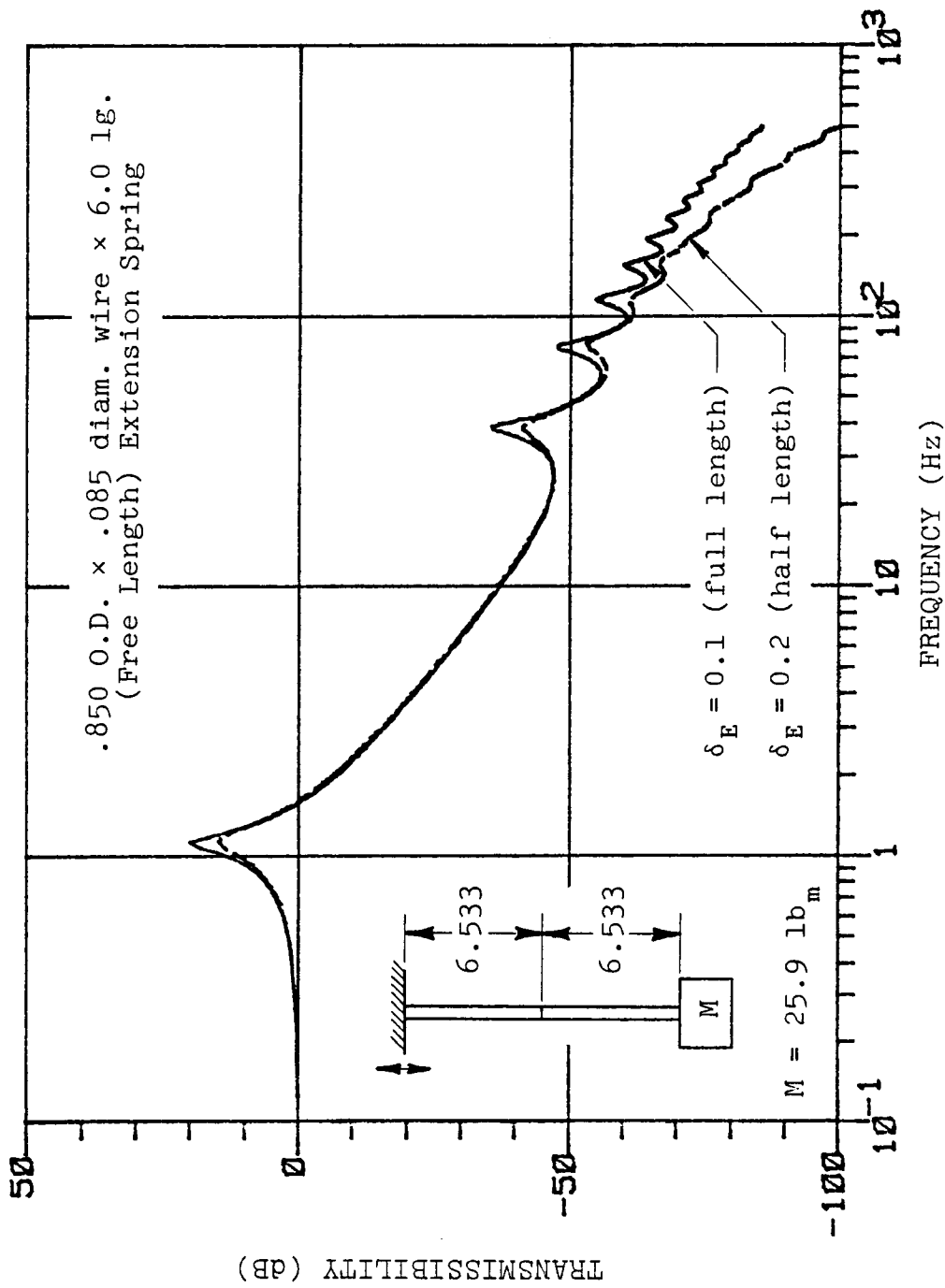


Figure 15. Transmissibility of a Mass-Loaded Rod in Longitudinal Vibration (Rod Cases 2 and 9). Damping factor $\delta_E = 0.1$ (full length) and 0.2 (half length).

From the analysis of isolators with constant internal damping it can be concluded that:

1. Damping is the only effective means of suppressing the transmissibility peaks at the standing wave frequencies for both the longitudinal vibration of a mass loaded rod and the transverse vibration of a mass loaded beam.

2. Internal damping does not provide sufficient reduction of the transmissibility peaks at the first two standing wave frequencies for a mass loaded beam.

Obviously, some means of suppressing the first two bending resonances of an extension spring isolator will be required if such an isolator is to be successfully used.

A number of concepts for suppressing the first two bending resonances were devised. The first concept (Figure 16) was to add a longitudinal flexure below the spring that is flexible in the longitudinal direction and very stiff in the transverse direction. All of the transverse isolation is then provided by the cable. The cable has a very low mass so that the first standing wave frequency will be very high, perhaps high enough so that it will not be a problem. The second concept (Figure 17) was the use of a viscous damper to perform essentially the same function as the flexure. A third

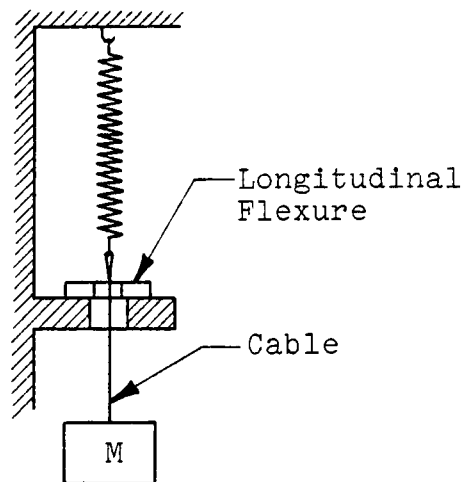


Figure 16. Extension Spring Isolator with a Longitudinal Flexure and a Cable.

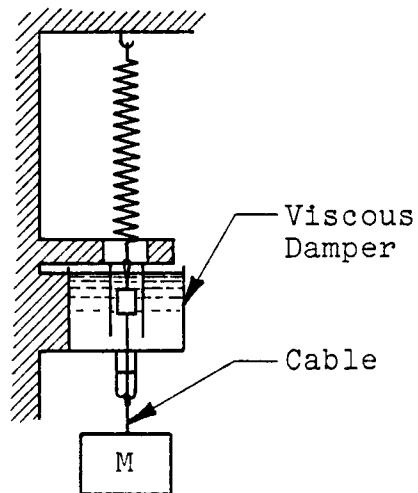


Figure 17. Extension Spring Isolator with a Viscous Damper and a Cable.

concept (Figure 18) was to use a compression spring for the isolator since it is much shorter than an extension spring and could therefore have the first standing wave resonance at a higher frequency. The fourth concept was to use an evacuated metal bellows for the isolator. The metal bellows is short, very light, and might be used in compression as shown in Figure 19 or in tension as shown in Figure 20. The fifth concept, which eventually proved to be the best approach, was to immerse the extension spring in a viscous fluid contained in a pipe as shown in Figure 21. In this case, the transverse as well as the longitudinal motion of the spring is effectively damped by the viscous fluid.

Other isolating devices which might have been considered are in the category of pressure preloaded isolators. An example of this type of isolator is the SLM mount manufactured by the Barry Corporation [9]. One problem with this type of isolator is that it leaks air through the elastometer walls and thus requires frequent re-leveling. Another problem is that the static height of this type of isolator changes with temperature due to Charles's Law, which requires a considerable linear excursion capability in order to isolate effectively at the temperature extremes. These problems have been eliminated by using a servo-leveling system with the

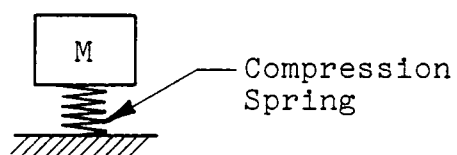


Figure 18. Compression Spring Isolator.

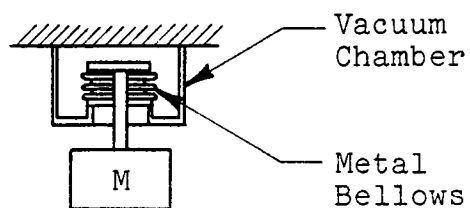


Figure 19. Evacuated Metal Bellows Isolator in Compression.

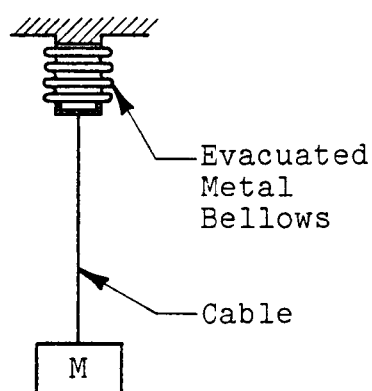


Figure 20. Evacuated Metal Bellows Isolator, in Tension, with a Cable.

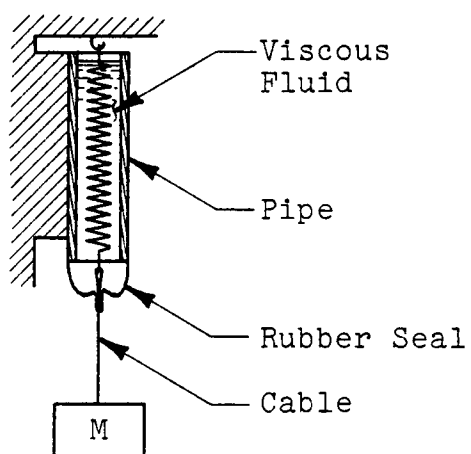


Figure 21. Extension Spring Isolator, Immersed in a Viscous Fluid, with a Cable.

mounts [9]. However, this approach was not considered to be practical for turntable isolation because it requires a continuous supply of pressurized air.

When a longitudinal flexure is added below the isolator spring as shown in Figure 16, the flexure may be considered to be a rigid constraint in the horizontal direction. The transverse isolation is then provided only by the cable. The transmissibility, in this case, may be calculated with the program for the 2-segment beam with a string by assuming very short beam lengths (say .001 inches) since a non-zero beam length is required with this program to avoid division by zero. The transverse transmissibility for a 5.0, 7.5, 10.0, and 15.0-inch length of .027 diameter steel cable is plotted in Figure 22. This plot shows that the transmissibility for all 4 cable lengths is generally very low; however, the first standing wave resonance for the 10.0 and 15.0-inch cables may still be a problem. Air damping, however, might provide a sufficient reduction of these standing wave resonances since they occur at such high frequencies.

The transverse isolation provided by the compression spring (Figure 18) and the metallic bellows (Figure 19) will now be considered. In this analysis, both the compression spring and the metallic bellows are considered to be fixed-guided beams in compression. The transmissibility equation for this case is derived in Part 4 of

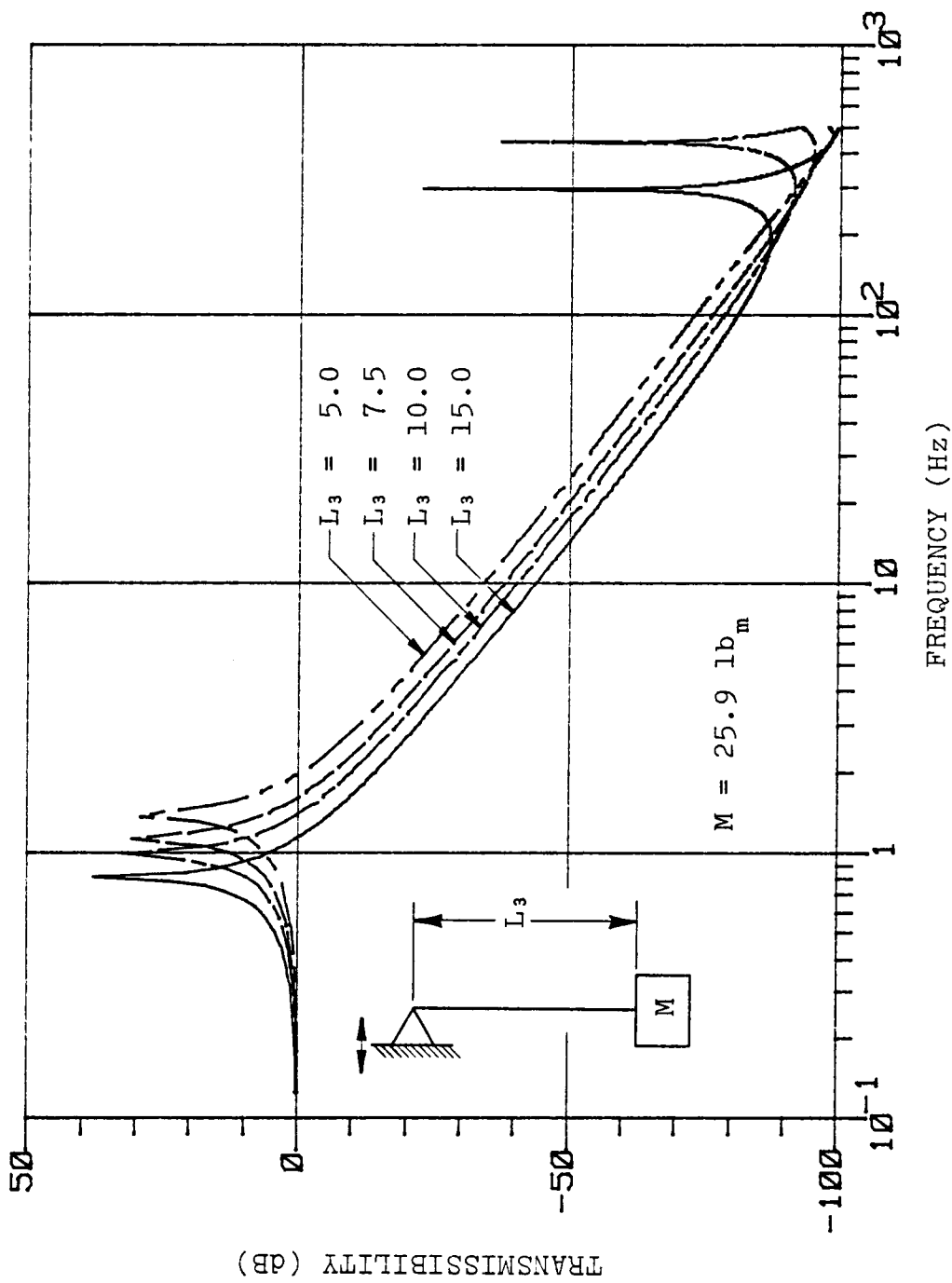


Figure 22. Transmissibility of a Mass-Loaded .027 Diameter Cable in Transverse Vibration. Length $L_3 = 5.0$, 7.5 , 10.0 , and 15.0 Inches.

Appendix C and is given by

$$T_m = \left| \frac{1}{DET} \left\{ -E^* I \alpha^* \beta^* \left[(\beta^{*3} + \alpha^{*2} \beta^*) \sinh \beta^* L + (\alpha^* \beta^{*2} + \alpha^{*3}) \sin \alpha^* L \right] \right\} \right|, \quad (C-62)$$

where DET is given by Equation (C-56) on page 244. This transmissibility equation was derived on the basis of the Bernoulli-Euler assumptions which are valid for relatively thin beams. Although neither the compression spring nor the metal bellows are relatively thin beams, their transmissibility may be estimated with reasonable accuracy at low frequencies, where the bending wavelength is several times larger than the cross-sectional dimensions of the beam, using Equation (C-62) [4]. A listing of the computer program used to evaluate Equation (C-62) is given along with a brief explanation in Appendix D.

A suitable compression spring with a 1.460 O.D. and a 26.23 lb. load capacity was found in the Associated stock spring catalog [10]. The spring parameters, obtained from the catalog data, are given in Table 7 on page 39. The metal bellows parameters were obtained from a theoretical design. The input parameters for the computer program are given in Table 11 (page 43) for both of these isolators.

An input parameter required for beam calculations is the equivalent moment of inertia, I . The I of the compression spring was obtained from Equations (11) and (12). The I of the bellows is also required. A formula for the cantilever bending deflection of a metal bellows is given in [11] as

$$\delta = \frac{5.25 P L^3}{D^2 n r}, \quad (13)$$

where δ = Deflection (in),

P = Applied load (lb),

D = Outside diameter of bellows (in),

h = Length of one convolution of the bellows (in),

L = Active length of the bellows (in),

r = Spring rate of one convolution (lb/in),

and the Young's modulus of the bellows material is 2.335×10^7 psi. The formula for the deflection of a cantilever beam according to Timoshenko [12] is

$$\delta = \frac{P L^3}{3 EI},$$

where P = Applied load (lb),

L = Beam length (in),

E = Equivalent Young's modulus (psi), and

I = Equivalent moment of inertia.

Combining Equations (13) and (14) the equivalent I for a metal bellows is given by

$$I = \frac{D^2 nr}{15.75 E} \quad .$$

Any convenient value of E may be chosen which will then specify I.

Figure 23 is a plot of the transverse transmissibility of the compression spring (Beam Case 7) and the metal bellows (Beam Case 8). This plot shows that the transmissibility of the compression spring is low, below 100 Hz, but a broad resonance occurs at about 230 Hz which probably makes this isolator unusable. The performance of the compression spring is clearly not as good as that of the 10-inch cable. The transverse transmissibility of the metal bellows is seen to be very low, between 10 Hz and 500 Hz. This is an interesting case; however, the cost of this bellows was found to be too high for use as a turntable isolator. Another problem with this bellows is that the active length is so short that changes in barometric pressure would probably place the static length of the bellows outside of its range of linear deflection.

The transverse transmissibility of an evacuated metal bellows in tension, as shown in Figure 20 on page 56, was also calculated. The bellows specifications in this case were obtained from a sample furnished by Servometer Inc. The input data for this bellows with a 10-inch cable is given as Beam Case 9 in Table 11 (on page 43). The transverse transmissibility for this case

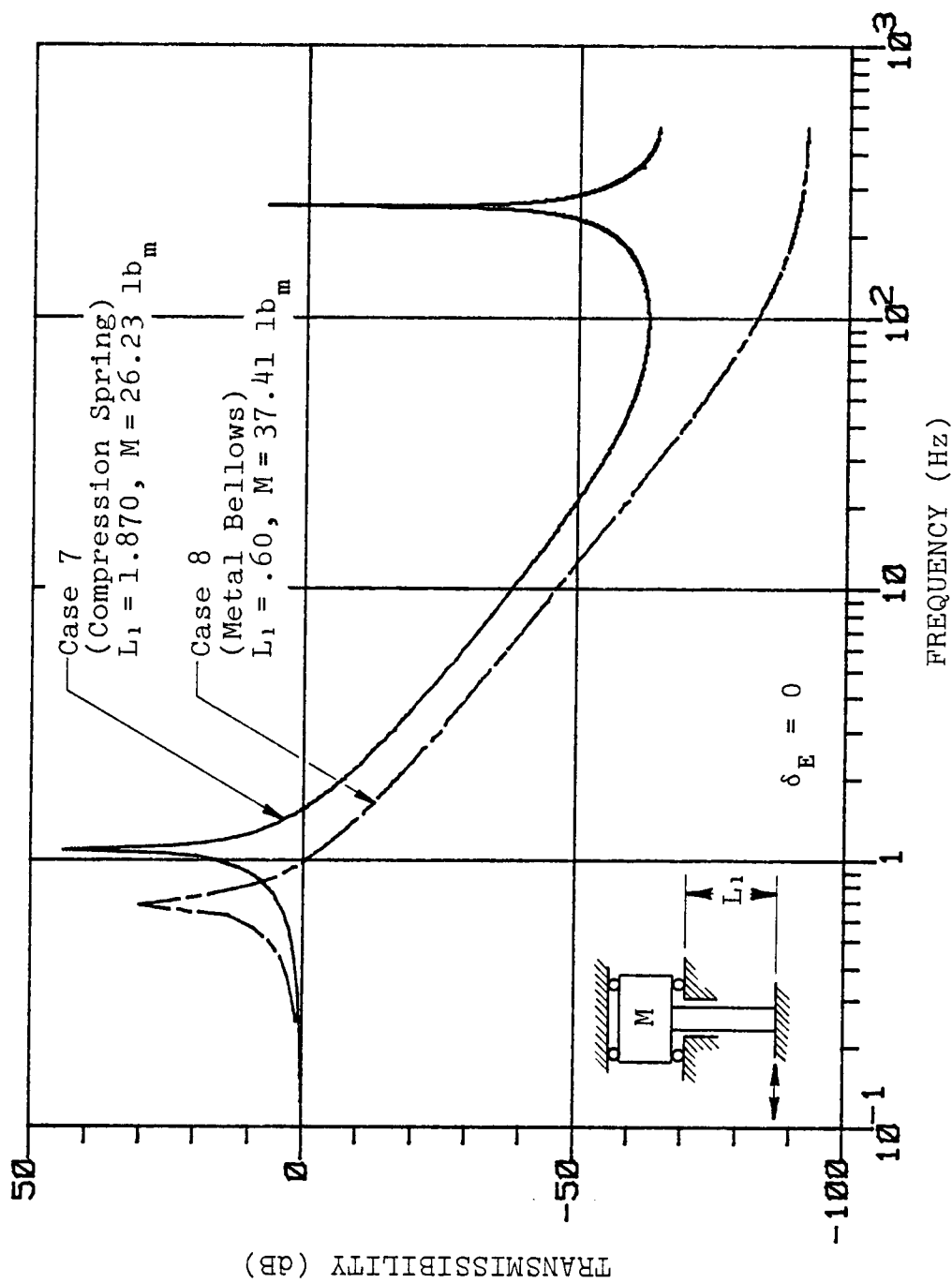


Figure 23. Transmissibility of Mass-Loaded, Fixed-Guided Beams in Transverse Vibration (Beam Cases 7 and 8). Damping factor $\delta_E = 0$.

is plotted in Figure 24. It can be seen from this plot that below 100 Hz the transmissibility is low; however, the first standing wave resonance occurs at an undesirably low frequency. Therefore, this does not appear to be a viable design for transverse isolation. The effect of an impedance change is also shown clearly in Figure 24 as an irregular spacing of the standing wave resonant peaks.

B. Solid Damping

In the previous constant damping analysis, it was shown that internal damping is very effective at reducing the transmissibility at the standing wave resonances for rods in longitudinal vibration. The constant internal damping, assumed previously, is really an idealization. In order to actually obtain internal damping with a coil spring, a high damping elastomer must be placed in parallel with the spring. This type of damping is referred to as solid damping.

The complex Young's modulus of an elastomer is given by

$$E_{\omega}^* = E_{\omega}(1 + j \delta_{E\omega}) \quad , \quad (A-1)$$

where $\delta_{E\omega}$ is the damping factor associated with the dynamic Young's modulus, E_{ω} , at any frequency ω . In the previous constant damping analysis, the values of

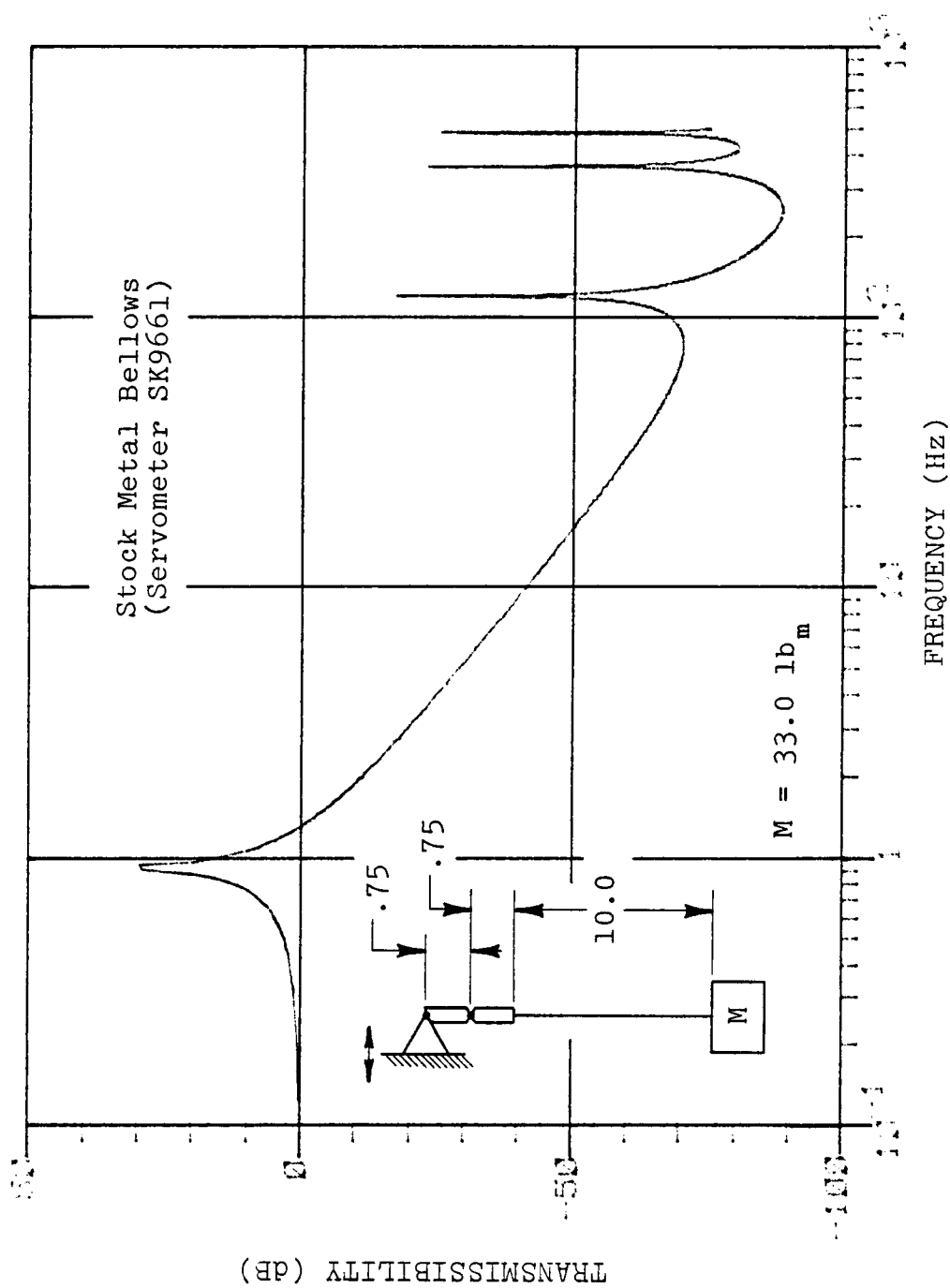
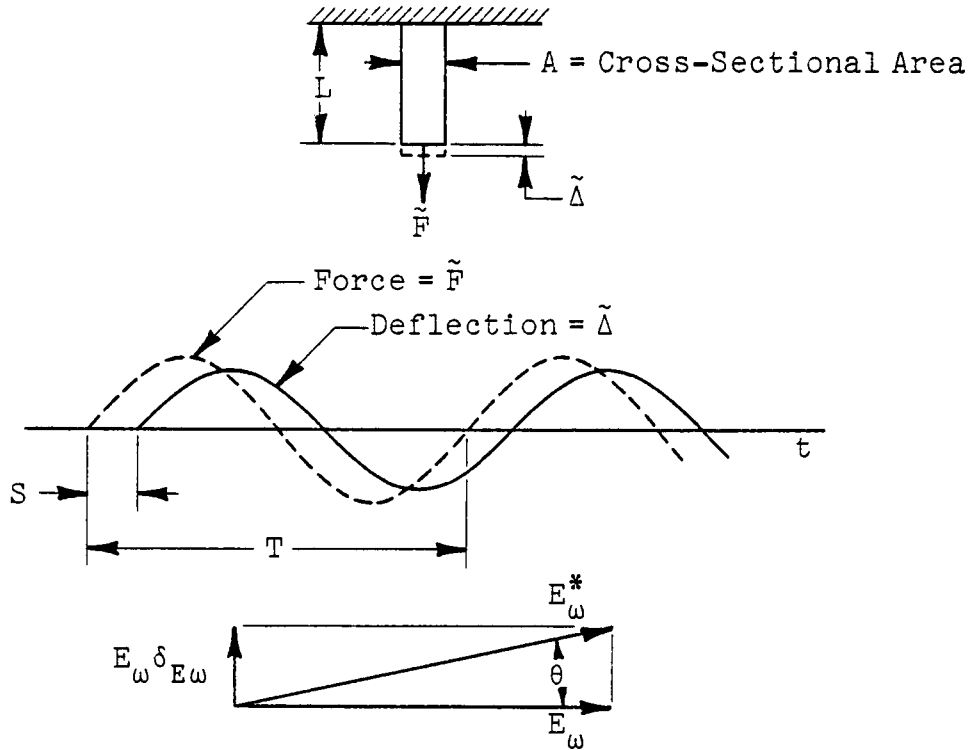


Figure 24. Transmissibility of a Mass-Loaded Beam with a String in Transverse Vibration (Beam Case 9). Damping factor $\delta_E = 0$.

E_ω (real part) and $E_\omega \delta_{E\omega}$ (imaginary part) were considered to be constant. The values of E_ω and $\delta_{E\omega}$ for high damping elastomers are shown in References [4] and [13] - [15] to have large variations with frequency at a given temperature. In order to predict the transmissibility of a spring with solid damping, over a range of frequencies, it is necessary to know the values of E_ω and $\delta_{E\omega}$ of the elastomer in this frequency range.

The values of E_ω and $\delta_{E\omega}$ for high damping elastomers are known to be very sensitive to composition, thermal treatment, and mechanical processing [14]. Because of these factors, published data from different sources for the "same" material are often in disagreement. It was also found that manufacturers and processors of elastomers generally do not have data on the frequency dependence of the dynamic Young's modulus and damping factor of their material, even when it is marketed as a damping material. Due to the lack of suitable data, it was necessary to measure E_ω and $\delta_{E\omega}$ for some available high damping materials.

The dynamic Young's modulus and damping factor of a high damping elastometer sample may be obtained by measuring the force, deflection, and phase angle between them while a sinusoidal force is applied as shown in Figure 25. From these measurements, the values of E_ω



$$|E_{\omega}^*| = \left(\frac{\text{Peak to Peak Force}}{\text{Peak to Peak Deflection}} \right) \left(\frac{L}{A} \right) \quad (16)$$

$$\theta = \frac{S(360)}{T} = \text{Phase Angle (Deg.)} \quad (17)$$

$$\delta_{E\omega} = \tan \theta = \text{Damping Factor} \quad (18)$$

$$E_{\omega} = |E_{\omega}^*| \cos \theta = \text{Dynamic Young's Modulus} \quad (19)$$

$$E_{\omega} \delta_{E\omega} = |E_{\omega}^*| \sin \theta = \text{Loss Modulus} \quad (20)$$

$$E_{\omega}^* = E_{\omega} + j E_{\omega} \delta_{E\omega} = \text{Complex Young's Modulus} \quad (21)$$

Figure 25. Measurement Technique for Obtaining the Dynamic Mechanical Properties of a High Damping Elastomer Sample.

and $\delta_{E\omega}$ may be calculated using Equations (16) - (19). If the results of such a measurement are to be valid, the dimensions of the material sample must be small relative to the wavelength. Also, the peak deflection of the sample should be kept small so that the forces and deflections are in a linear range and the sample does not experience a temperature rise due to internal energy dissipation [13]. A schematic diagram of the test apparatus is shown in Figure 26. The actual test apparatus is shown in Figure 27. Shown in Figure 28 is a material sample installed in the test apparatus.

The following components were used for the material tests:

Oscilloscope: Hewlett Packard 1220A;

Function generator: B+K Precision Model 3010;

Frequency counter: Heathkit Model Im-4100;

Power amplifier: Crown DC-300A;

Displacement transducer: Electro Corporation
Electro-Mike Model PA 12D-43;

Force transducer: Fabricated from a piezoelectric
buzzer.

Test samples were made from ten materials known to have good damping properties. A brief description of the material samples is given in Table 12. Each sample was then tested at 9 frequencies between 5 and 500 Hz. A least-squares cubic polynomial curve was then fit to

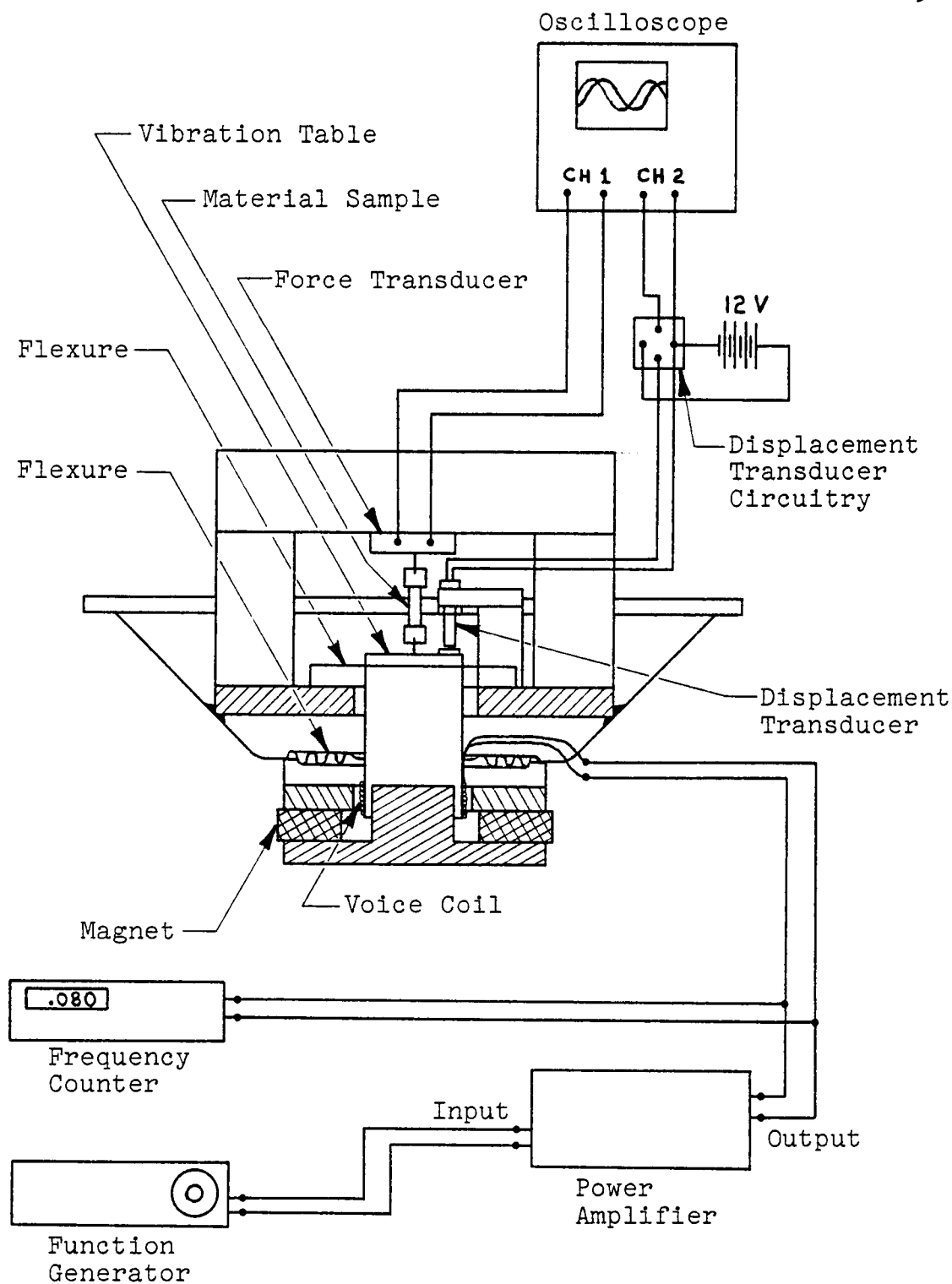


Figure 26. Schematic Diagram of Material Test Apparatus.

Figure 27. Material Test Apparatus.

Figure 28. Material Sample Installed in Test Apparatus.



Figure 27.



Figure 28.

TABLE 12

MATERIAL SAMPLE DESCRIPTIONS

Material	Sample Dimensions	Material Description	Manufacturer
1. Isodamp C-1002	.043 x .118 x .487 lg.	Blue vinyl-like material used in damping applications	EAR Corporation
2. DYAD 606	.055 x .100 x .50 lg.	Clear vinyl-like material used in damping applications	Soundcoat Company
3. EAR Foam	.540 Dia. x .406 lg.	Yellow foam material used for ear plugs	EAR Corporation
4. Poron Foam	.136 x .262 x .514 lg.	Grey urethane foam used in energy-absorbing applications	Rogers Corporation
5. Butyl Rubber	.045 x .293 x .484 lg.	Black rubber used for tire innertubes	Unknown
6. Durafoam SBR	.138 x .205 x .526 lg.	Black SBR foam	Monmouth Rubber Company
7. PVC Nitrile Foam	.139 x .179 x .50 lg.	Black PVC nitrile foam	Monmouth Rubber Company
8. Vinyl	.057 x .028 x .50 lg.	Laminated vinyl material used as a covering for drafting tables	Unknown

TABLE 12 (Continued)

Material	Sample Dimensions	Material Description	Manufacturer
9. EAR C-2003	.043 x .043 x .781 lg.	Blue, stiff, vinyl-like material used in damping applications	EAR Corporation
10. Scotch 33+ Vinyl Tape	.007 x .255 x .468 lg.	Black vinyl electrical tape	Minnesota Mining and Manufacturing

the nine measured values of $|E_{\omega}^*|$ for each sample. A least-squares parabola was fit to the nine corresponding values of $\delta_{E\omega}$. The results of the material tests, summarized in Table 13, are based on the values of $|E_{\omega}|$ and $\delta_{E\omega}$ taken from the fit curves. From these results it can be seen that the first three samples had generally higher damping factors. The Isodamp C-1002 sample has a damping factor that increases with increasing frequency. This is a desirable characteristic because the damping factor is low at low frequencies and higher at the standing wave resonant frequencies. For this reason, Isodamp C-1002 was selected for use in the solid damping analysis and also for use with the transmissibility and shock tests.

For a number of reasons, the values given in Table 13 are considered to be inaccurate at frequencies above 80 Hz. The frequencies above 150 Hz excited resonances in the test apparatus which caused waveform distortion and phase shifts which could not be compensated for. The displacement transducer could not accurately measure the very small displacements required at higher frequencies. The wavelength at 500 Hz was only about 18 times the sample length in some cases which results in a 20 degree phase angle error. The data presented, however, are considered to be useful for comparing one

TABLE 13

MEASURED VALUES OF COMPLEX YOUNG'S MODULUS AND
DAMPING FACTOR FOR 10 MATERIAL SAMPLES

Material	$ E_w^* \text{ (lb/in}^2\text{)}$					δ_{Ew}				
	5 Hz	30 Hz	80 Hz	250 Hz	500 Hz	5 Hz	30 Hz	80 Hz	250 Hz	500 Hz
1. Isodamp C-1002	1084	1456	2047	2887	2726	.411	.480	.599	.840	.733
2. DYAD 606	3169	5455	9384	16645	11854	1.04	.961	.824	.539	.617
3. EAR Foam	47.72	100.1	179.9	272.4	275.7	.742	.705	.640	.519	.619
4. Poron Foam	193.3	223.2	277.5	410.3	477.6	.328	.351	.396	.540	.724
5. Butyl Rubber	534.2	583.2	692.3	1087	1273	.454	.468	.495	.580	.687
6. Durafoam SBR	758.9	806.8	897.4	1141	1251	.406	.382	.340	.268	.359
7. PVC Nitrile Foam	457.7	493.9	583.7	927.3	807.0	.492	.482	.466	.445	.507
8. Vinyl	10211	11768	14265	17977	17451	.458	.455	.452	.465	.555
9. EAR C-2003	62068	75426	94588	106627	94950	.466	.423	.349	.206	.301
10. Scotch 33t Vinyl Tape	16576	19997	23239	17897	69337	.395	.385	.369	.370	.518

tested material to another and for rough estimating purposes.

In order to include the measured dynamic Young's moduli and damping factors with the isolator, it is necessary to obtain composite properties for the spring-material combination. For two materials in parallel, the dynamic Young's modulus, damping factor, and density for the composite isolator are given by

$$E_{\omega_1, 2} = \frac{E_{\omega_1} + a E_{\omega_2}}{(1 + a)}, \quad (22)$$

$$\delta_{E\omega_1, 2} = \frac{E_{\omega_1} \delta_{E\omega_1} + a E_{\omega_2} \delta_{E\omega_2}}{(E_{\omega_1} + a E_{\omega_2})}, \quad (23)$$

$$\rho_{1, 2} = \frac{\rho_1 A_1 + \rho_2 A_2}{A_1 + A_2}, \quad (24)$$

$$\text{where } a = \frac{A_2}{A_1}.$$

Since it was found that internal damping is ineffective for the beam cases, solid damping was considered only for rod cases. The existing computer program for the rod cases was then modified to read the composite dynamic Young's modulus and damping factor data at five frequencies. For all cases considered in this investigation, the data were read at 5, 30, 80, 250 and

500 Hz. At intermediate frequencies and frequencies below 5 Hz, the composite dynamic Young's modulus and damping factor were obtained by linear interpolation or extrapolation.

A new stock extension spring, having a 30 lb. load capacity, was chosen to allow for a higher system mass. This spring has a 1.0-inch O.D., 0.95 diam. wire, and a free length of 6 inches. The spring parameters, which were obtained from the catalog data, are given in Table 7 on page 39. Another spring, consisting of an evacuated metal bellows, was also used in this analysis. The specifications for this bellows were obtained from a data sheet furnished by Servometer Corporation. The input data for the undamped spring and undamped bellows is given in Table 9 (page 41) as Rod Cases 10 and 11 respectively. The properties of each composite isolator were calculated from Equations (22) - (24) at 3 damping factors for each isolator. The input data for these isolators are given as Solid Rod Cases 1-6 in Table 14.

When the material property tests were first run, a calibration error was made causing the values of the dynamic Young's modulus to be a factor of 10 too low. This error was not discovered until the transmissibility tests were run. The dynamic Young's modulus data given in Table 13 have been corrected; however, in this analysis

TABLE 14

INPUT PARAMETERS FOR SOLID ROD CASES 1-6

Input Parameter (See Appendix B)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Spring No.†	5	5	5	*	*	*
FQ (1) (Hz)	5.0	5.0	5.0	5.0	5.0	5.0
FQ (2) (Hz)	30.0	30.0	30.0	30.0	30.0	30.0
FQ (3) (Hz)	80.0	80.0	80.0	80.0	80.0	80.0
FQ (4) (Hz)	250.0	250.0	250.0	250.0	250.0	250.0
FQ (5) (Hz)	500.0	500.0	500.0	500.0	500.0	500.0
E1 (1) (lb/in ²)	61.547	62.586	64.733	1.923	2.169	2.916
D1 (1)	.0500	.1000	.2000	.0500	.1000	.2000
E1 (2) (lb/in ²)	63.968	67.590	79.964	1.999	2.340	3.375
D1 (2)	.0722	.1392	.2594	.0722	.1392	.2594
E1 (3) (lb/in ²)	68.102	75.918	92.368	2.128	2.631	4.158
D1 (3)	.1143	.2086	.3548	.1143	.2086	.3548
E1 (4) (lb/in ²)	76.584	93.167	128.07	2.393	3.229	5.766
D1 (4)	.2169	.3626	.5460	.2169	.3626	.5460
E1 (5) (lb/in ²)	78.224	96.504	134.98	2.444	3.345	6.077
D1 (5)	.2297	.3787	.5604	.2297	.3787	.5604
RHO 1 (lb _m /in ³)	.0281	.0299	.0337	7.87E-3	8.01E-3	8.30E-3
Al (in ²)	.8627	.9604	1.261	3.237	3.248	3.330
L1 (in)	7.045	7.045	7.045	.750	.750	.750

TABLE 14 (Continued)

Input Parameter (See Appendix B)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
E2 (1) (lb/in ²)	61.547	62.586	64.773	1.923	2.169	2.916
D2 (1)	.0500	.1000	.2000	.0500	.1000	.2000
E2 (2) (lb/in ²)	63.968	67.590	74.964	1.999	2.339	3.375
D2 (2)	.0722	.1392	.2594	.0722	.1392	.2594
E2 (3) (lb/in ²)	68.102	75.918	92.368	2.128	2.631	4.158
D2 (3)	.1143	.2086	.3548	.1143	.2086	.3548
E2 (4) (lb/in ²)	76.584	93.167	128.07	2.393	3.229	5.766
D2 (4)	.2169	.3626	.5460	.2169	.3626	.5460
E2 (5) (lb/in ²)	78.224	96.504	134.98	2.444	3.345	6.077
D2 (5)	.2297	.3787	.5604	.2297	.3787	.5604
RHO2 (lb _m /in ³)	.0281	.0299	.0337	7.87E-3	8.01E-3	8.30E-3
A2 (in ²)	.8627	.9604	1.261	3.236	3.248	3.330
L2 (in)	7.045	7.045	7.045	.750	.750	.750
MR2 (lb _m)	.1534	.1849	.2821	.0191	.0195	.0207
M (lb _m)	30.0	30.0	30.0	33.0	33.0	33.0

*Stock metal bellows.

†See Table 7 on page

the old data were used. The only effect of this error is a reduction of the mass ratio of the composite isolator. This did not change the qualitative results observed for the solid damping cases.

Plotted in Figure 29 are the transmissibility curves for Rod Case 10 and Solid Rod Cases 1, 2 and 3 which have damping factors of zero, .05, .1, and .2 respectively. This plot shows that the natural frequency of the system is higher for the higher damping cases. This is an unavoidable consequence with solid damping because stiffness is added in proportion to damping. The effect of solid damping on the resonant peaks and the transmissibility at higher frequencies is seen to be very similar to that of constant damping.

Figure 30 is a transmissibility plot for Rod Case 11 and Solid Rod Cases 4, 5, and 6, having damping factors of zero, .05, .1 and .2 respectively. This plot shows that solid damping is very effective at suppressing the first standing wave resonance when it occurs at a frequency of 100 Hz. The addition of solid damping is also seen to increase the frequency at which the first standing wave resonance occurs. From the plots of Figures 29 and 30, the optimum solid damping factor appears to be about .1 based on the transmissibility at 10 Hz versus the transmissibility at the first standing wave resonance.

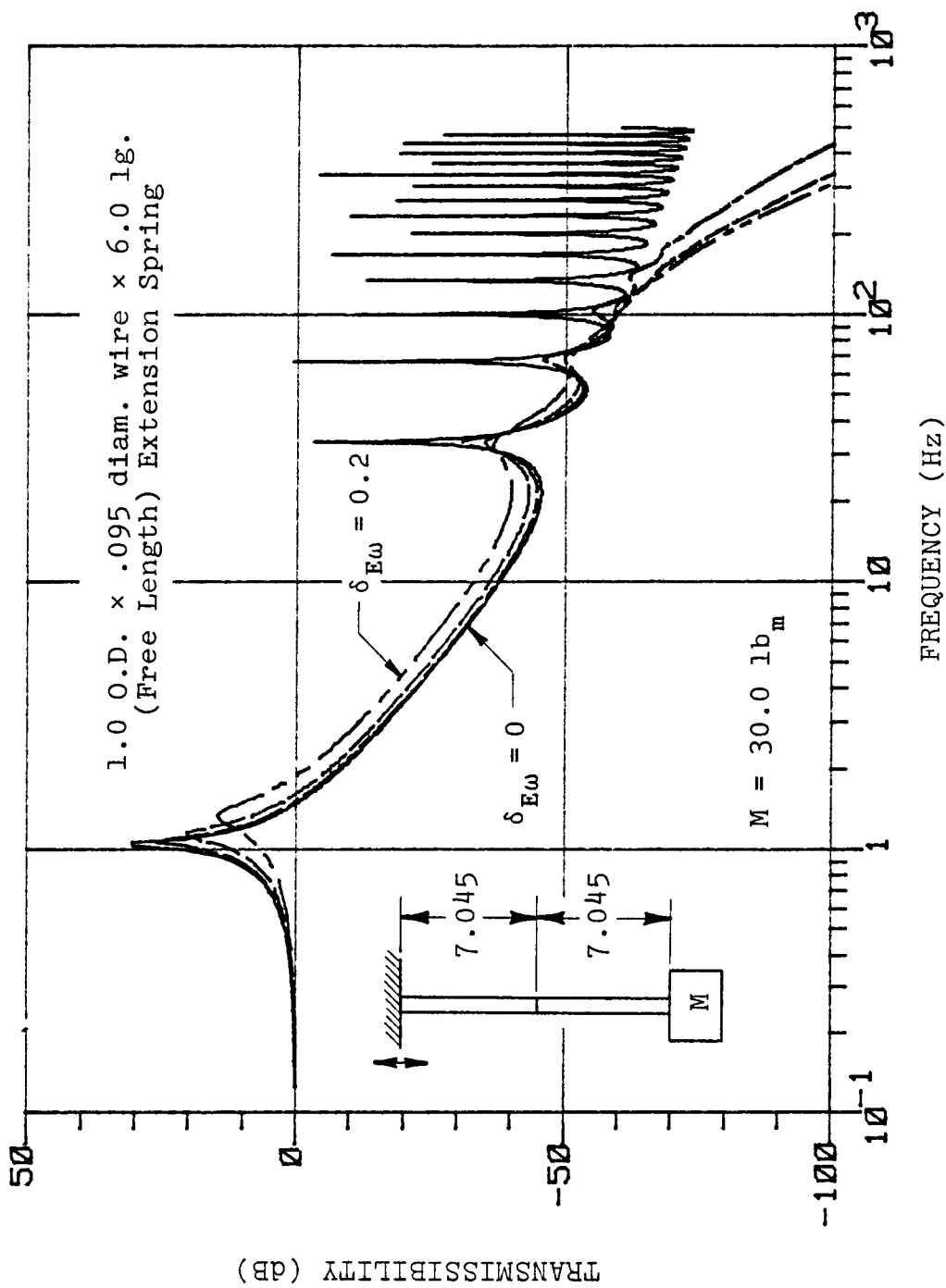


Figure 29. Transmissibility of a Mass-Loaded Rod with Solid Damping in Longitudinal Vibration (Rod Case 10 and Solid Rod Cases 1, 2, and 3). Solid damping factor $\delta_{E\omega} = 0, 0.05, 0.1, \text{ and } 0.2$

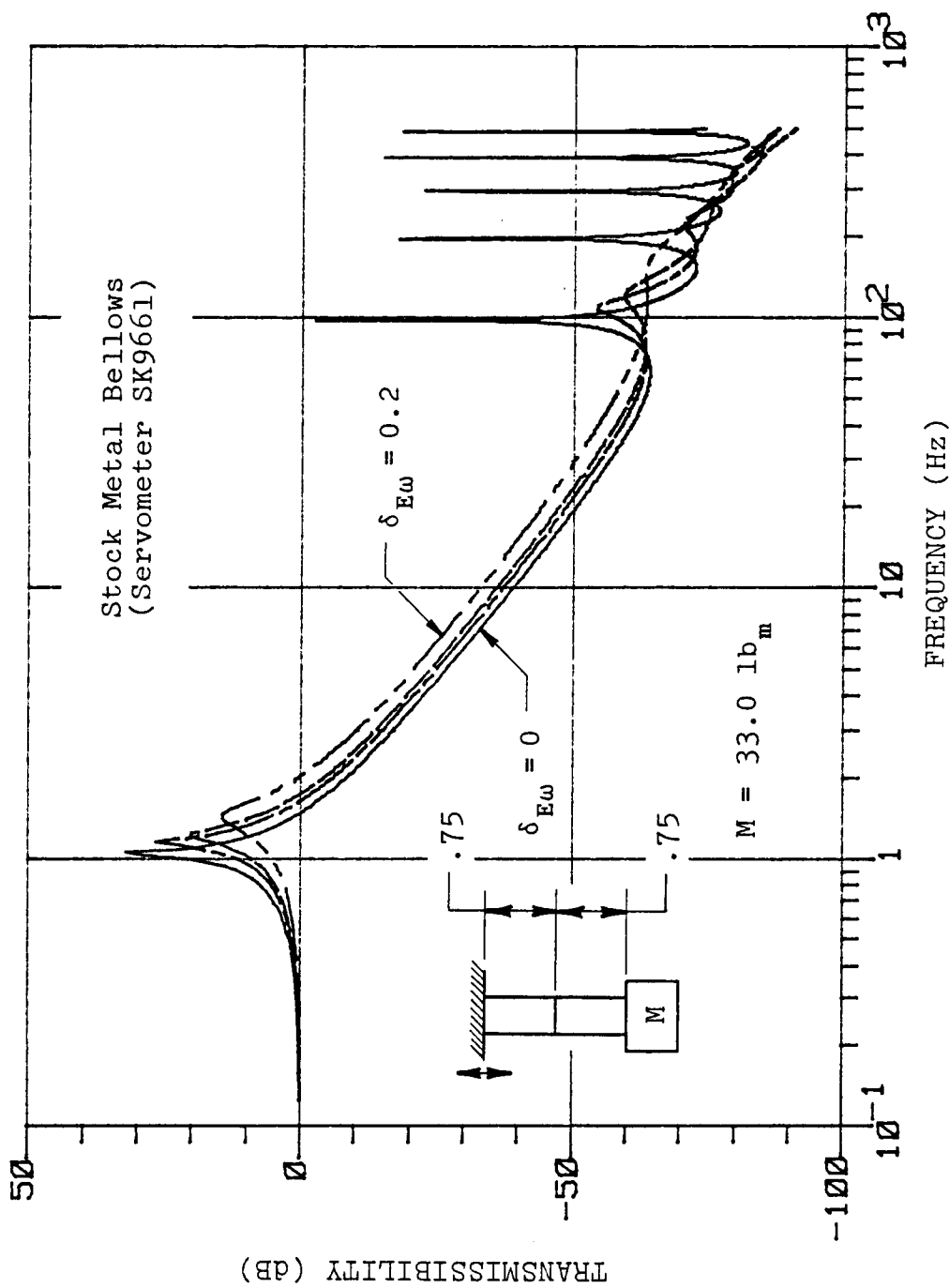


Figure 30. Transmissibility of a Mass-Loaded Rod, with Solid Damping, in Longitudinal Vibration (Rod Case 11 and Solid Rod Cases 4, 5, and 6). Solid damping factor $\delta_{E\omega} = 0, 0.05, 0.1, \text{ and } 0.2$.

C. Distributed Viscous Damping

One of the concepts for reducing the first transverse standing wave resonance was to immerse the spring in a viscous fluid as shown in Figure 21 on page 57. Due to the large effort required to develop a transverse transmissibility equation for this case, it was decided to analytically investigate only the longitudinal case. If calculations of the longitudinal transmissibility of this isolator looked promising, the transverse transmissibility could be determined from measurements.

The analytical model for an isolator with distributed viscous damping is shown in Figure 31.

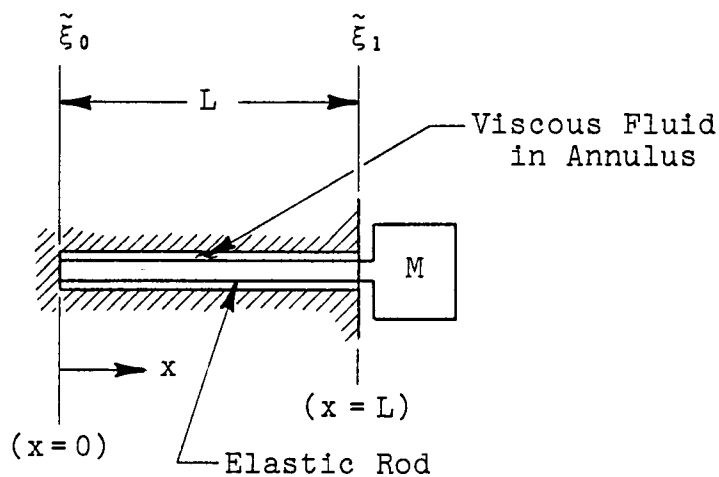


Figure 31. Mass-Loaded Elastic Rod with Distributed Viscous Damping.

The transmissibility equation for this case is derived in Part 3 of Appendix A and is given by

$$T_m = \frac{E^* n^* (1 + \Phi^* \cos n^* L)}{\left[E^* n^* \cos n^* L - \left(\frac{\omega^2 M}{A} \right) \sin n^* L \right] (1 - \Phi^*)} , \quad (A-32)$$

$$\text{where } \Phi^* = \frac{j\omega c}{j\omega c + AE(n^*)^2} , \quad (A-31)$$

and c = Damping coefficient.

The relationship between the damping coefficient, c , and the damping factor, ζ , in this case is given by

$$c = \frac{12\pi M \zeta f_n}{L} . \quad (25)$$

A branch was added to the existing computer program given in Appendix B to calculate the transmissibility using Equations (A-31) and (A-32).

The input data for 3 extension springs, a metal bellows, and a compression spring, each with damping factors of .0066, .017, and .04, are given in Tables 15-17. The five undamped cases were run previously using the constant damping part of the program. The input data for these cases (Rod Cases 12, 10, 13, 11, and 14) are given in Table 9 on page 41.

Plotted in Figures 32-36 are the transmissibility curves for Rod Cases 12, 10, 13, 11, and 14, and the

TABLE 15

INPUT PARAMETERS FOR DISTRIBUTED VISCOUS CASES 1-6

Input Parameter (See Appendix B)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Spring No.	6	6	6	5	5	5
REW2 (lb/in ²)	60.542	60.542	60.542	60.542	60.542	60.542
DAMP2 (lb-sec/in ²)	1.028E-3	2.570E-3	5.140E-3	1.473E-3	3.682E-3	8.838E-3
RHO2 (lb _m /in ³)	.0263	.0263	.0263	.0263	.0263	.0263
A2 (in ²)	.7854	.7854	.7854	.7854	.7854	.7854
L2 (in)	28.178	28.178	28.178	14.089	14.089	14.089
MR2 (lb _m)	.5830	.5830	.5830	.2915	.2915	.2915
M (lb _m)	30.0	30.0	30.0	30.0	30.0	30.0

TABLE 16

INPUT PARAMETERS FOR DISTRIBUTED VISCOUS CASES 7-12

Input Parameter (See Appendix B)	Case 7	Case 8	Case 9	Case 10	Case 11	Case 12
Spring No.†	4	4	4	*	*	*
REW2 (lb/in ²)	63.331	63.331	63.331	1.727	1.727	1.727
DAMP2 (lb-sec/in ²)	2.509E-3	6.272E-3	.0150	.0150	.0376	.0902
RHO2 (lb _m /in ³)	.0254	.0254	.0254	7.767E-3	7.767E-3	7.767E-3
A2 (in ²)	.7854	.7854	.7854	3.227	3.227	3.227
L2 (in)	9.974	9.974	9.974	1.5	1.5	1.5
MR2 (lb _m)	.1992	.1992	.1992	.0376	.0376	.0376
M (lb _m)	30.0	30.0	30.0	33.0	33.0	33.0

†See Table 7 on page 39.

*Stock metal bellows.

TABLE 17
INPUT PARAMETERS FOR DISTRIBUTED VISCOUS CASES 13-15

Input Parameter (See Appendix B)	Case 13	Case 14	Case 15
Spring No.†	7	7	7
REW2 (lb/in ²)	6.453	6.453	6.453
DAMP2 (lb-sec/in ²)	.0222	.0556	.1335
RHO2 (lb _m /in ³)	.0116	.0116	.0116
A2 (in ²)	4.599	4.599	4.599
L2 (in)	1.868	1.868	1.868
MR2 (lb _m)	.0688	.0688	.0688
M (lb _m)	26.2	26.2	26.2

†See Table 7 on page 39.

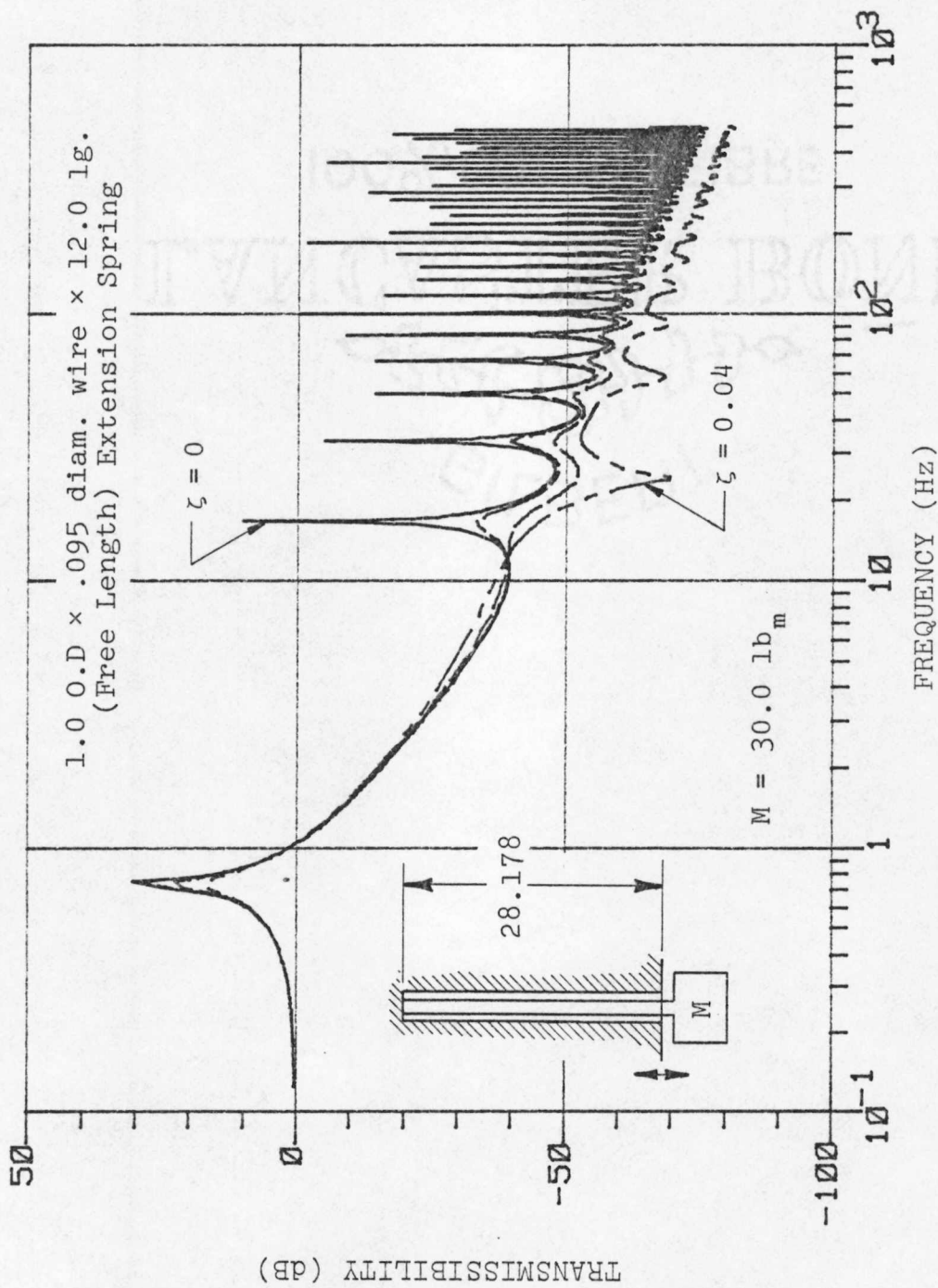


Figure 32. Transmissibility of a Mass-Loaded Rod, with Distributed Viscous Damping, in Longitudinal Vibration (Rod Case 12 and Distributed Viscous Cases 1, 2, and 3). Damping factor $\delta = 0, 0.0066, 0.017, 0.04$.

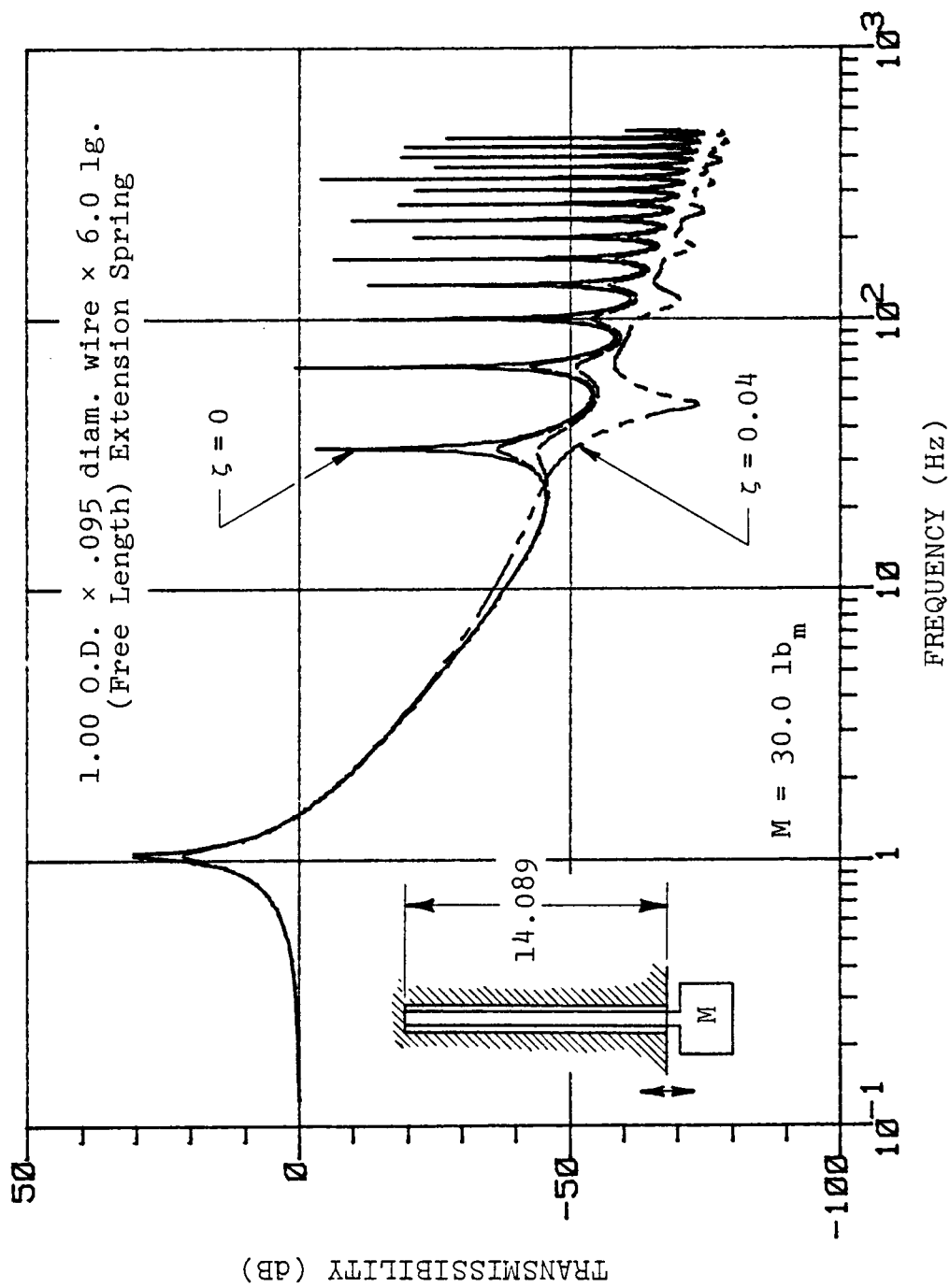


Figure 33. Transmissibility of the Mass-Loaded Rod Shown in Figure 31 (Rod Case 10 and Distributed Viscous Cases 4, 5, and 6). Damping factor $\zeta = 0$, 0.0066, 0.017, and 0.04.

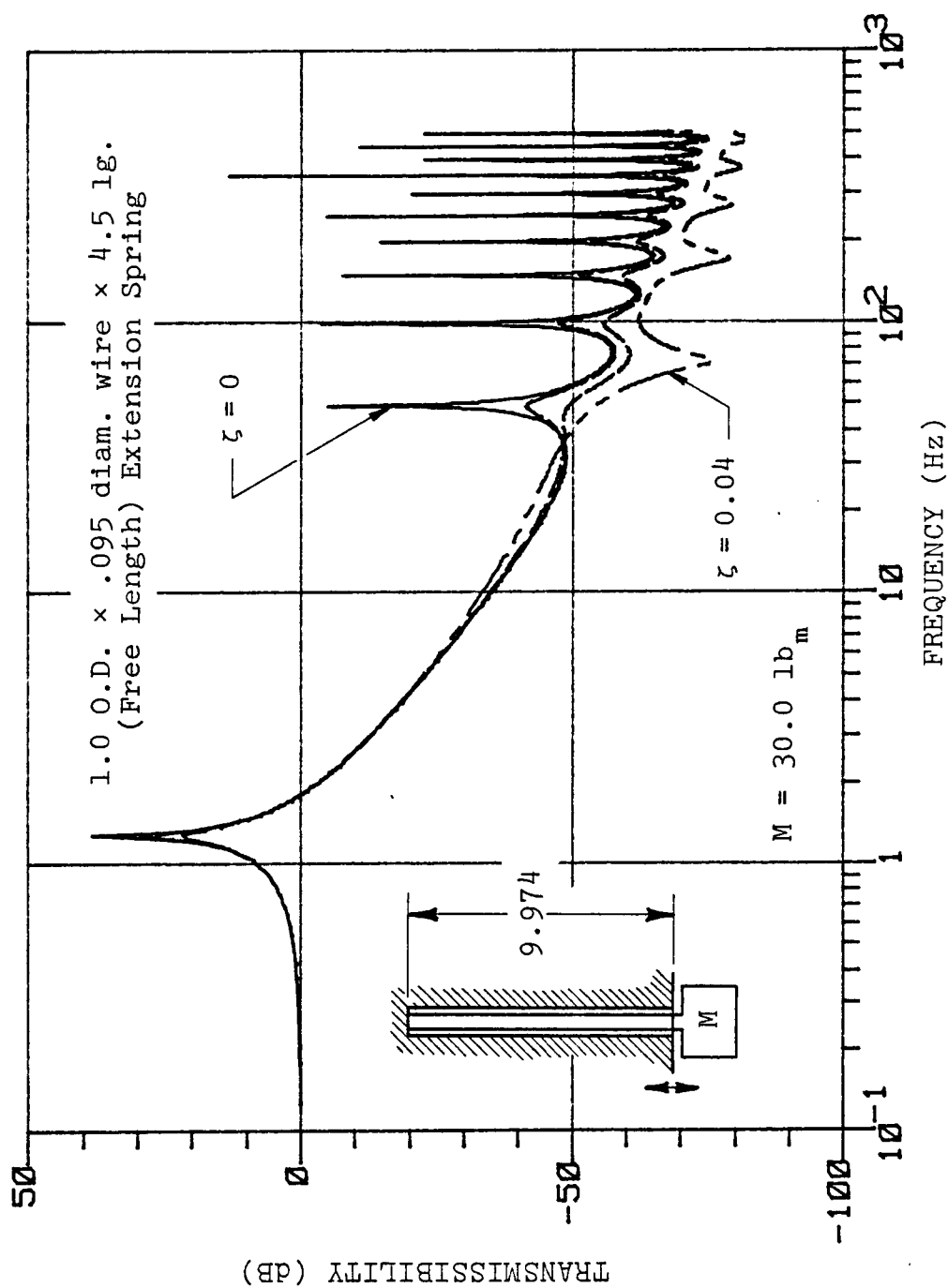


Figure 34. Transmissibility of a Mass-Loaded Rod, with Distributed Viscous Damping, in Longitudinal Vibration (Rod Case 13 and Distributed Viscous Cases 7, 8, and 9). Damping factor $\zeta = 0$, 0.0066, 0.017, and 0.04.

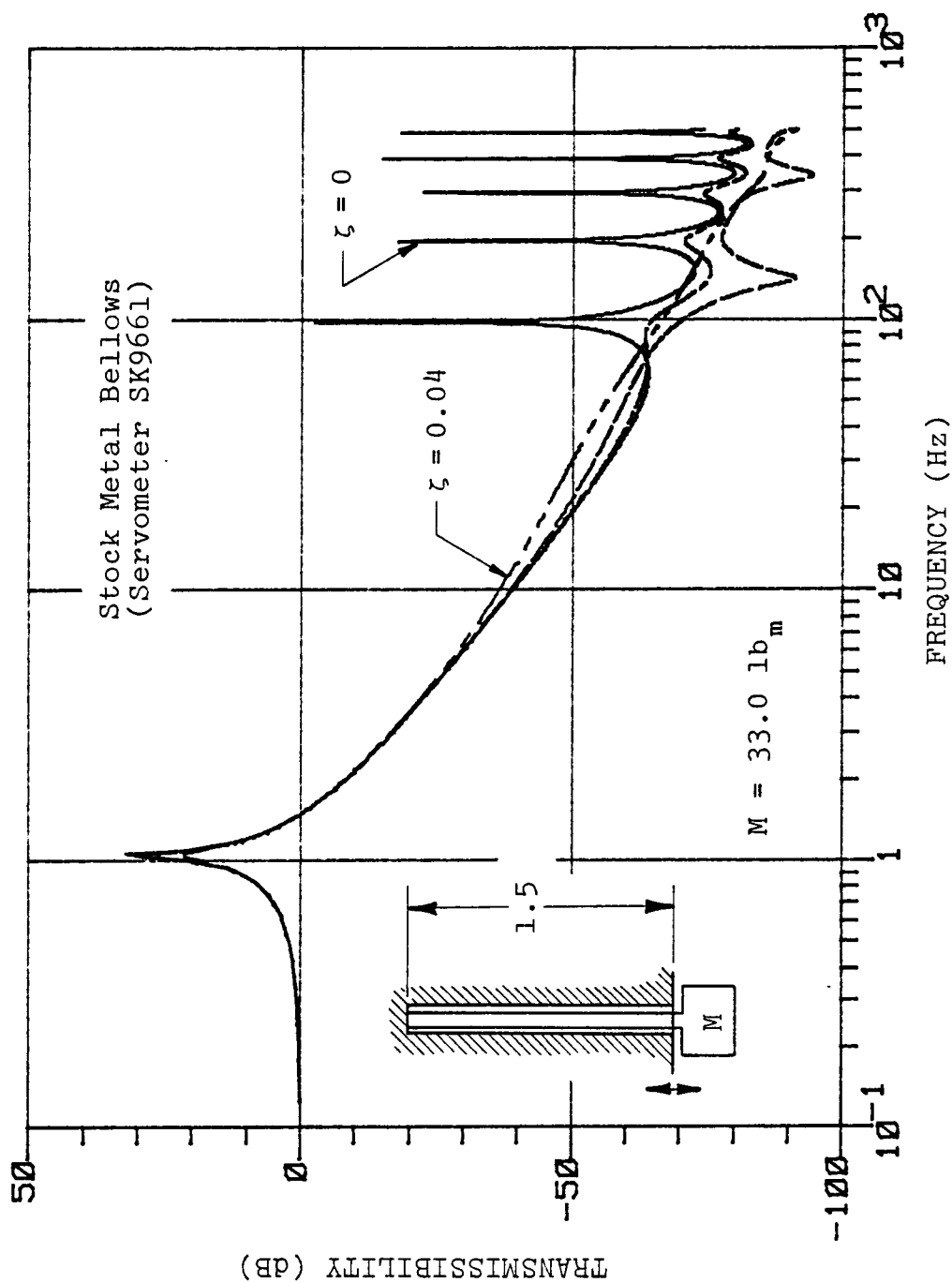


Figure 35. Transmissibility of a Mass-Loaded Rod, with Distributed Viscous Damping, in Longitudinal Vibration (Rod Case 11 and Distributed Viscous Cases 10, 11, and 12). Damping factor $\zeta = 0$, 0.0066, 0.017, and 0.04.

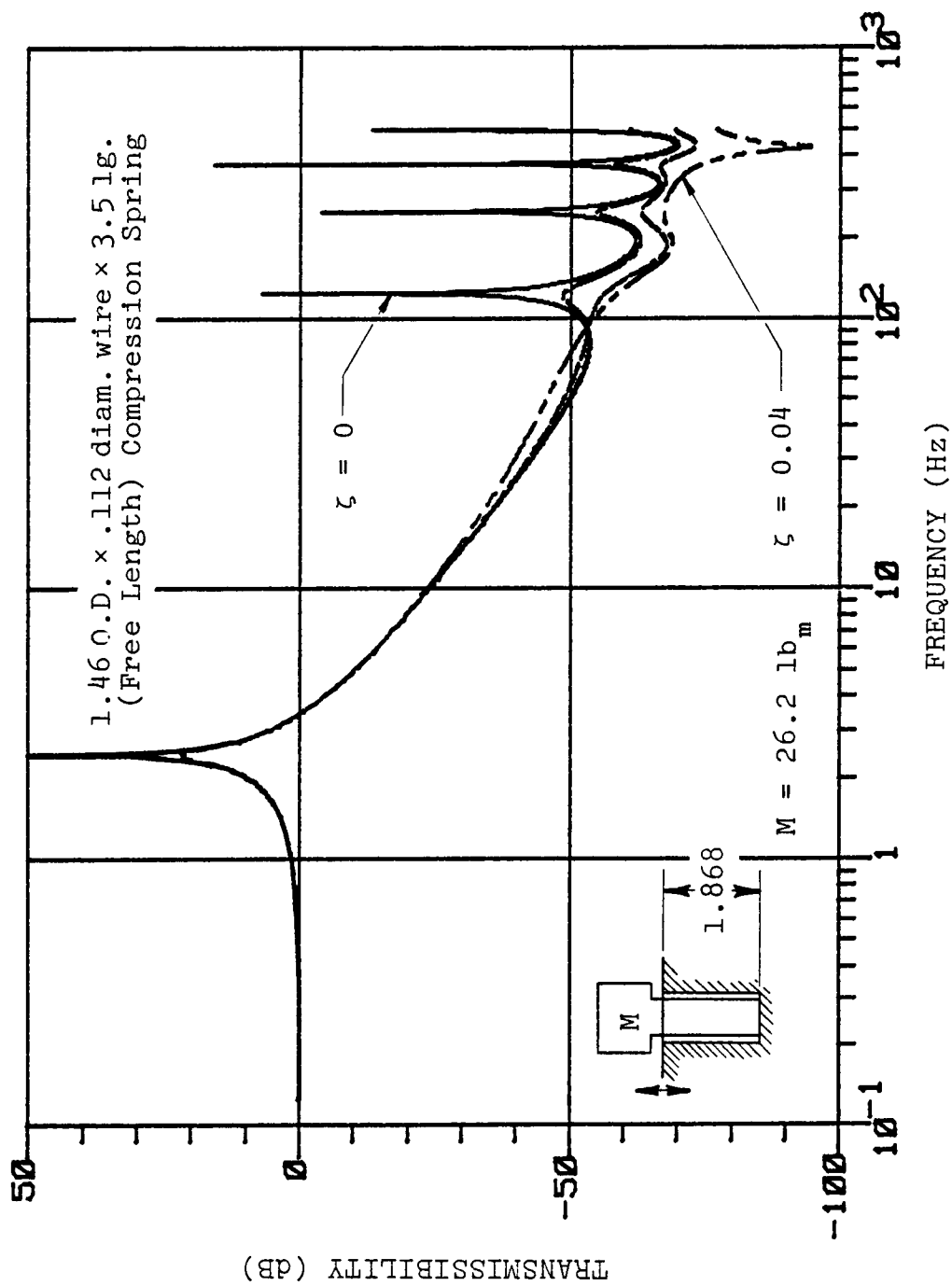


Figure 36. Transmissibility of a Mass-Loaded Rod, with Distributed Viscous Damping, in Longitudinal Vibration (Rod Case 14 and Distributed Viscous Cases 13, 14, and 15). Damping factor $\zeta = 0, 0.0066, 0.017, \text{ and } 0.04$.

corresponding Distributed Viscous Cases 1-15. The basic characteristics of distributed viscous damping can be observed on all of these plots. The plots all show that optimum damping occurs at a damping factor of .04 or less. Sharp transmissibility troughs are also observed, for damping factors near .04, at a frequency above the first, third, and so on, standing wave resonances. The best overall performances, of the distributed viscous damping cases, is seen to be provided by the metal bellows (Figure 35) with a damping factor of about .017. The low transmissibility obtained with this isolator is due to its low natural frequency and low mass.

From the plots for the extension spring cases (Figures 32-34) it can be seen that as the spring length is increased and the natural frequency reduced, the transmissibility is also reduced. The transmissibility difference between the 6-inch spring (Figure 33) and the 12-inch spring (Figure 32) is only about 2 dB at 10 Hz. This 2 dB difference probably does not justify the increased physical size of the 12-inch spring.

D. Viscous Damping

The use of a viscous damper below the isolator (Figure 17, page 54) was mentioned previously as a means of reducing the transverse transmissibility at the first

standing wave resonant frequency. The effect of a viscous damper on the longitudinal transmissibility will now be discussed.

The analytical model for this case is shown in Figure 37.

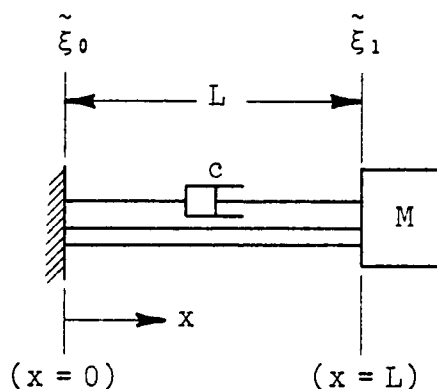


Figure 37. Mass-Loaded Elastic Rod with a Viscous Damper in Parallel.

The transmissibility equation for this case is derived in Part 4 of Appendix A and is given by

$$T = \left| \frac{AE^*n^* - j\omega c \sin n^*L}{AE^*n^* \cos n^*L - (\omega^2 M + j\omega c) \sin n^*L} \right|, \quad (A-35)$$

where \$c\$ = damping coefficient.

The relationship between the damping coefficient, \$c\$, and the damping factor, \$\zeta\$, in this case is given by

$$c = \zeta 4\pi M f_n. \quad (26)$$

Another branch was added to the existing computer program to calculate the transmissibility using Equation (A-35).

Figure 38 is a plot of the longitudinal transmissibility of a 6-inch long extension spring for viscous damping factors of zero, .1, .4 and 1.0. This plot shows that a viscous damper in parallel does not provide any noticeable reduction of the standing wave resonant peaks and generally increases the transmissibility at 10 Hz and above. Actually, the viscous damping force increases the motion of the isolated mass for transmissibilities below 0 dB and decreases the motion above 0 dB. The isolator configuration is therefore unsuitable for turntable isolation.

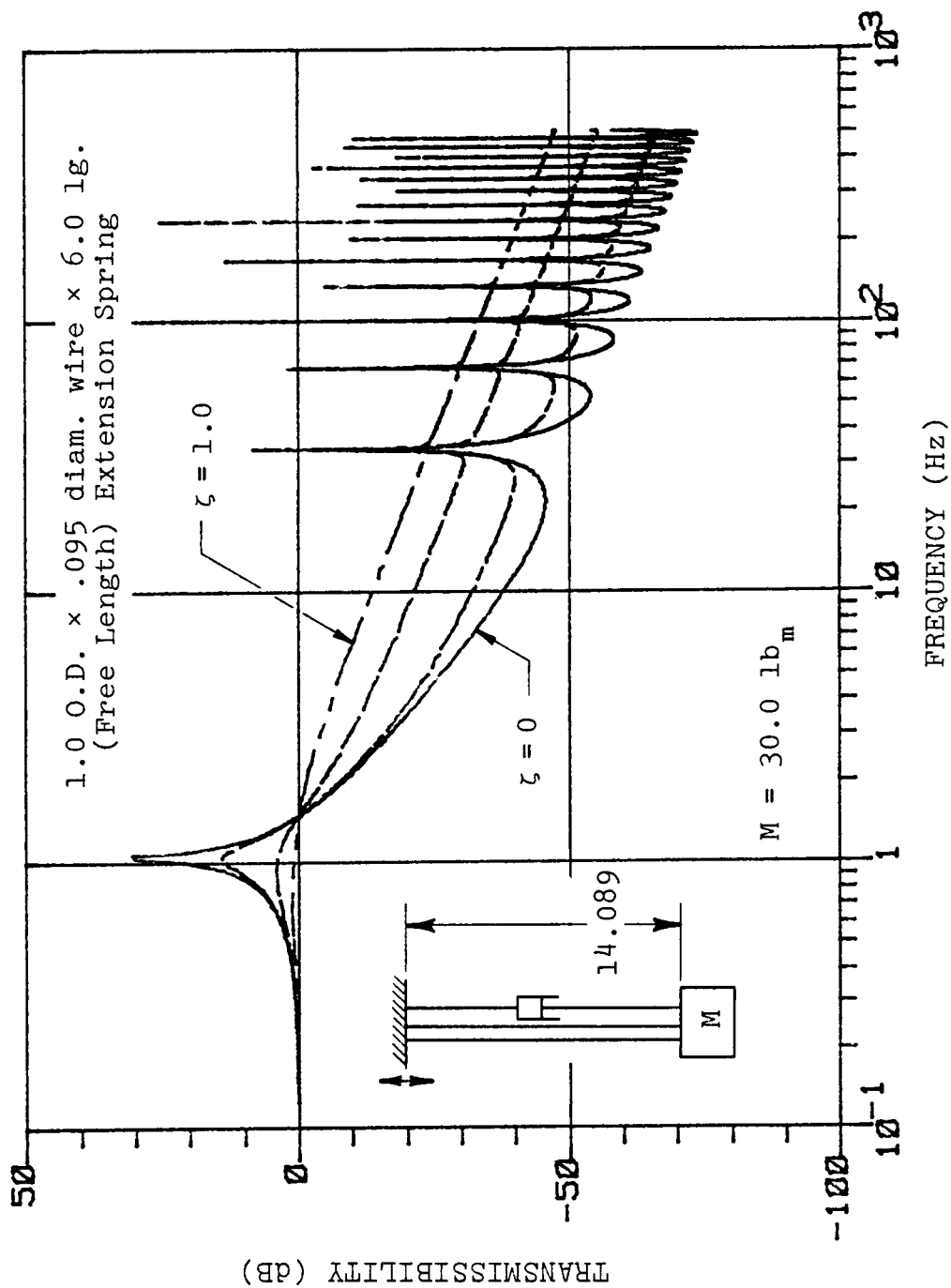


Figure 38. Transmissibility of Mass-Loaded Rod with a Viscous Damper in Longitudinal Vibration. Damping factor $\zeta = 0, 0.1, 0.4$, and 1.0 .

CHAPTER V

MEASUREMENT OF TRANSMISSIBILITY

A. Technique and Apparatus

The previous analyses of the standing wave transmissibility of various isolator configurations were based on certain simplifying assumptions which may not be valid for the actual isolator. It is now desired to check the analytical results by directly measuring the longitudinal and transverse transmissibility of the various isolator configurations. Another reason for testing is to obtain the transmissibility of isolator configurations that were not obtained analytically, such as the transverse transmissibility of beams with distributed viscous damping.

There are many ways to measure the transmissibility of an isolator over a range of frequencies. The technique used in this investigation was to vibrate a block, which simulates the environment, at a discrete frequency while measuring the acceleration of the block and isolated mass. The transmissibility is then simply the acceleration of the isolated mass divided by the acceleration of the block. Measurements must be taken at as many discrete frequencies as necessary to characterize the transmissibility in the frequency range of interest. The advantage

of this technique is that accurate results can be obtained with very simple instrumentation. The disadvantage, however, is that the measurements are very time-consuming.

A schematic diagram of the transmissibility test apparatus is shown in Figure 39. The vibrating block apparatus is shown in the longitudinal test configuration in Figures 40 and 41, and in the transverse test configuration in Figure 42. Shown in Figure 43 are the plastic and laminated metal longitudinal flexure assemblies. Some of the various isolators tested are shown installed on the vibrating block in Figures 44-52. The test control and readout instrumentation is shown in Figure 53.

The main mechanical component of the test apparatus is the vibrating block which simulates the foundation. For the test results to be valid, this block should be very massive and rigid so that there is no impedance interaction between the isolator and the block. The block should also have all of its standing wave resonant frequencies as far as possible above 500 Hz so that the block behaves as a rigid body at the test frequencies. The T-shaped, vibrating block was made of reinforced concrete with .75-inch thick steel plates epoxied to the bottom and end surfaces of the top portion, to provide

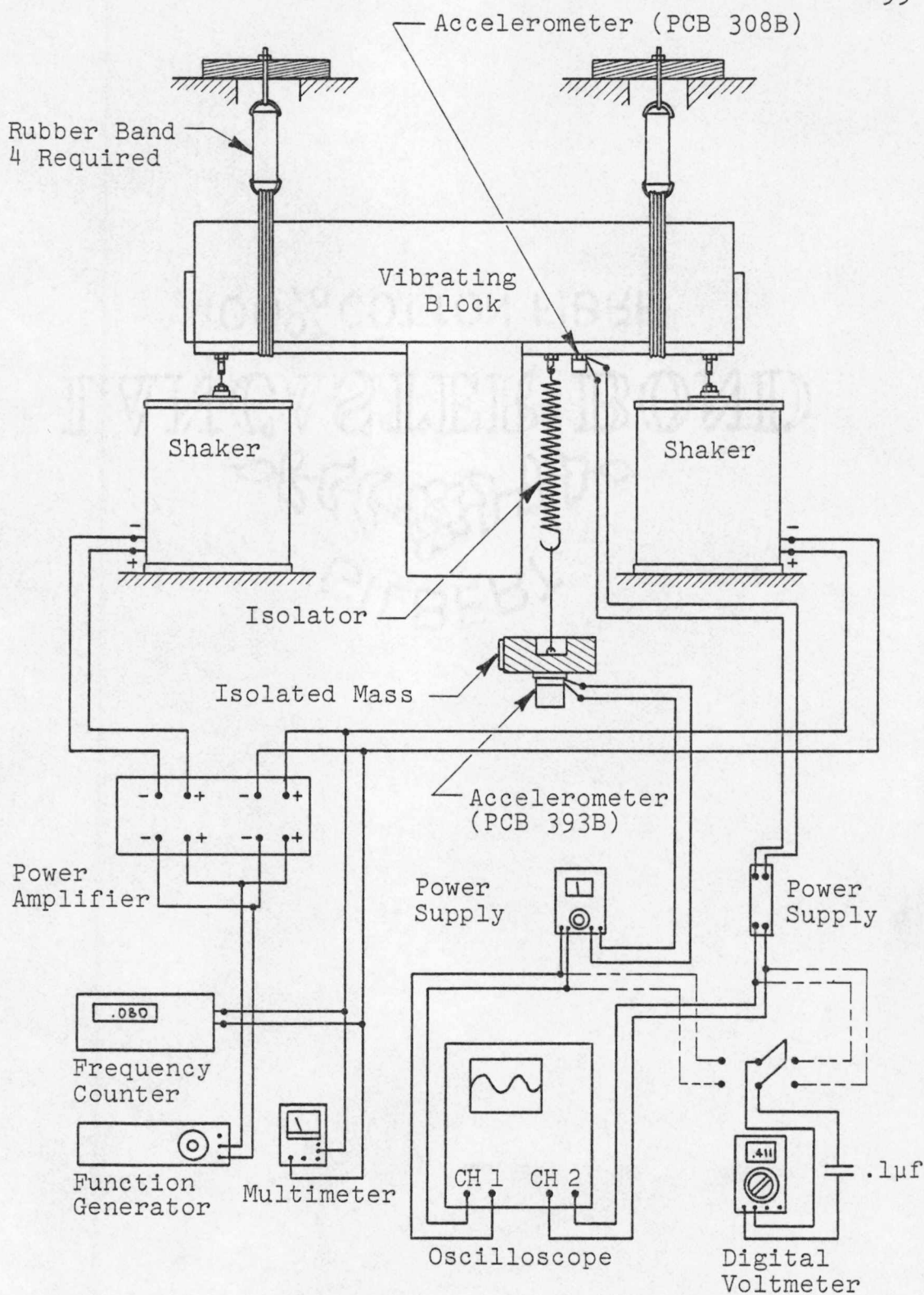


Figure 39. Schematic Diagram of Transmissibility Test Apparatus.

Figure 40. The Vibrating Block Apparatus in the Longitudinal Test Configuration.

Figure 41. End View of the Vibrating Block Apparatus Showing a Shaker and the Hardware Used to Suspend the Vibrating Block.

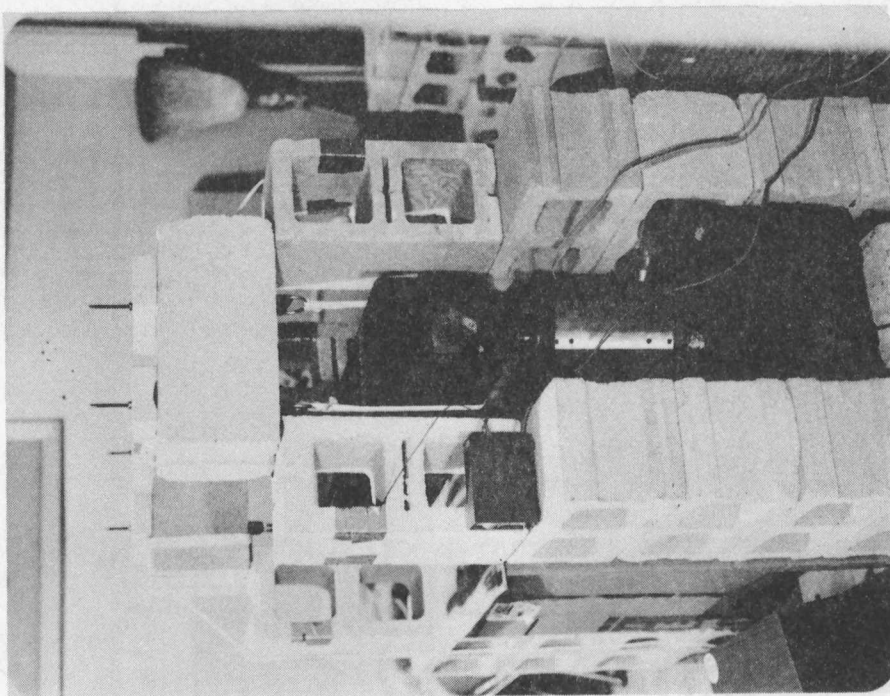


Figure 41.

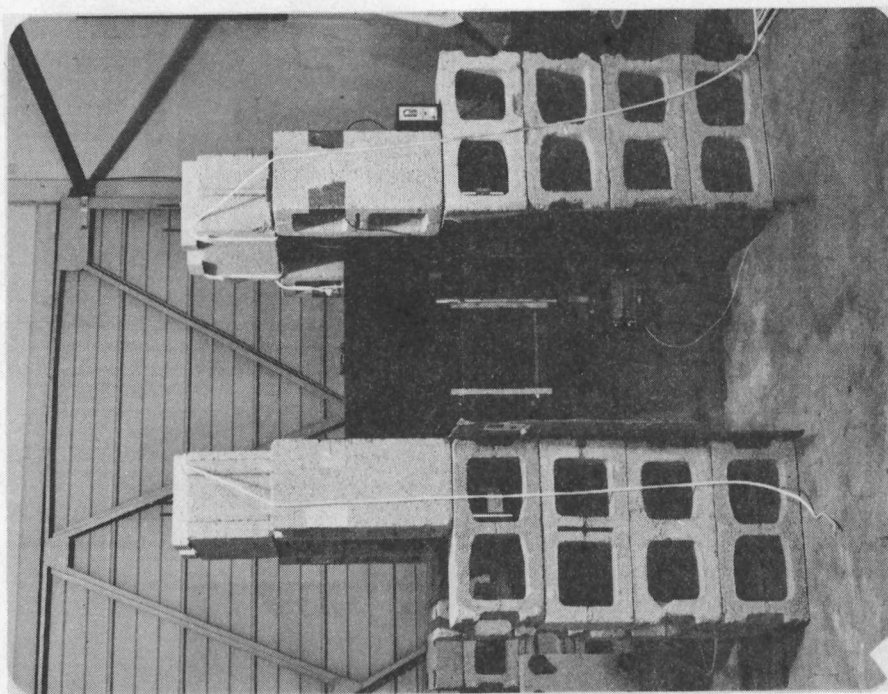


Figure 40.

Figure 42. The Vibrating Block Apparatus in the Transverse Test Configuration.

Figure 43. Plastic (Left) and Laminated Metal (Right) Longitudinal Flexure Assemblies.

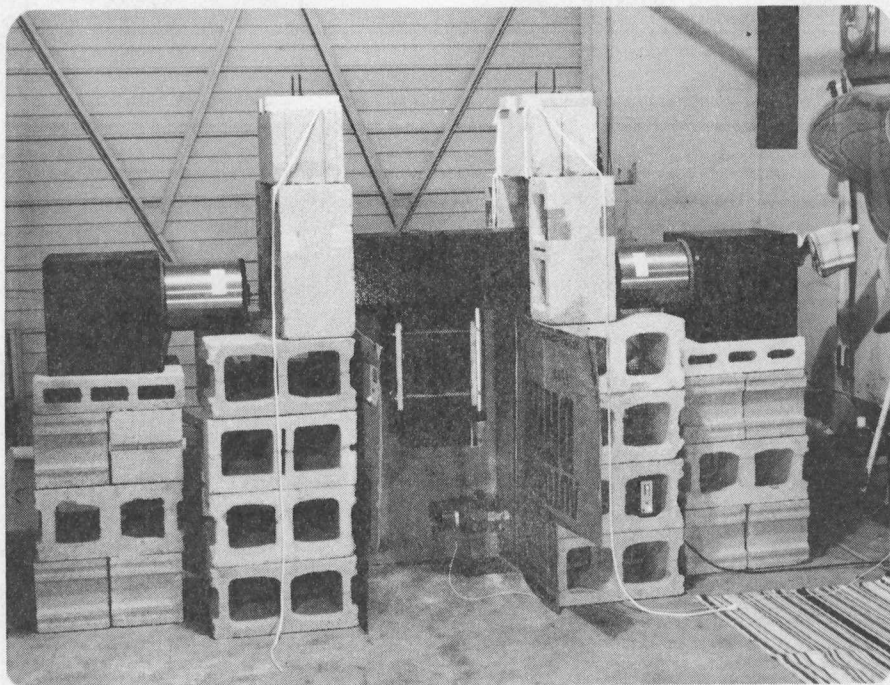


Figure 42.

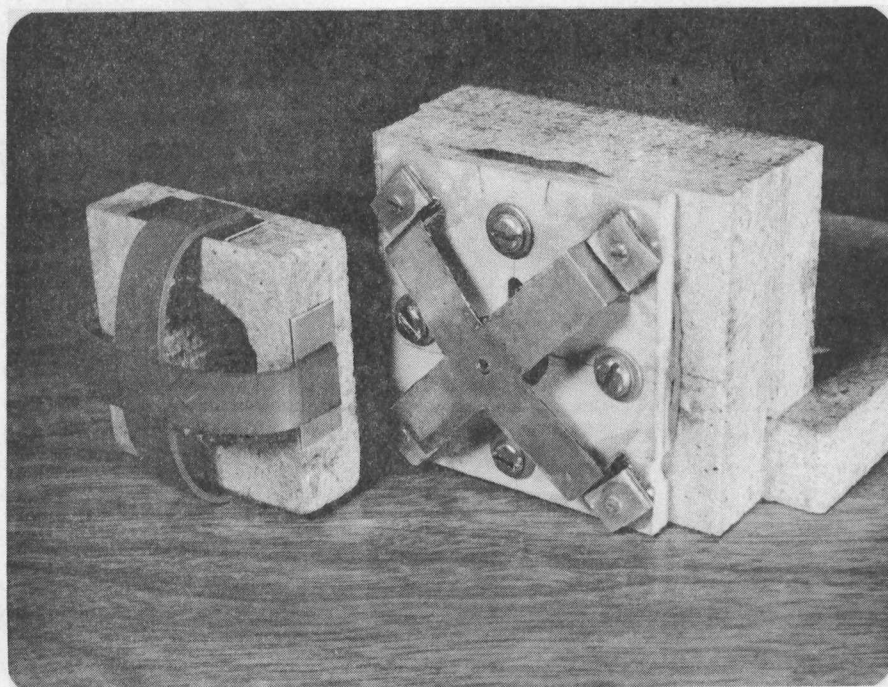


Figure 43.

Figure 44. Evacuated Metal Bellows Isolator in the Longitudinal Test Configuration. The isolated mass was 29 lb_m. The PCB 308B accelerometer, used to measure the vibrating block motion, is mounted to the right of the bellows. The PCB 393B accelerometer, used to measure the motion of the isolated mass, is mounted on the bottom of the isolated mass.

Figure 45. Evacuated Metal Bellows Isolator, with Distributed Viscous Damping, in the Longitudinal Test Configuration. The isolated mass was 29 lb_m.

Figure 46. The 6-Inch (Free Length) Extension Spring, with a Viscous Damper, in the Longitudinal Test Configuration. The isolated mass was 30.2 lb_m.

Figure 47. The 6-Inch (Free Length) Extension Spring with .175 Sq. In. (Cross-Sectional Area) of Isodamp C-1002 Bonded to It. The isolator is shown in the longitudinal test configuration. The isolated mass was 30.2 lb_m.

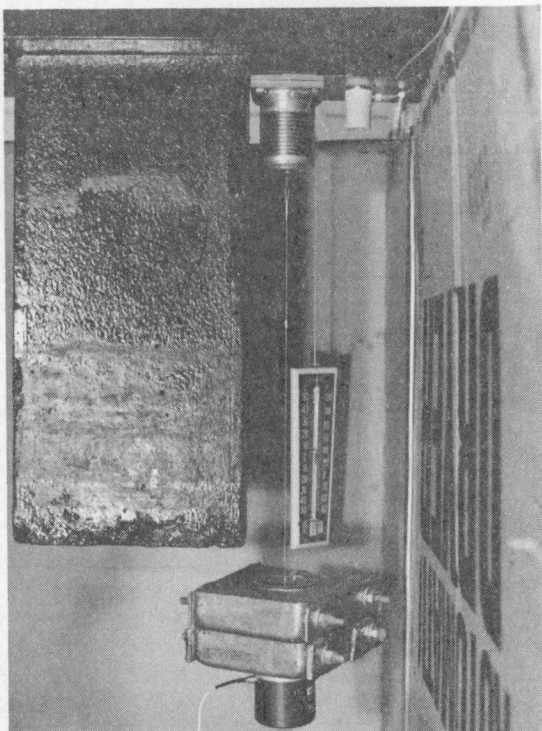


Figure 44.

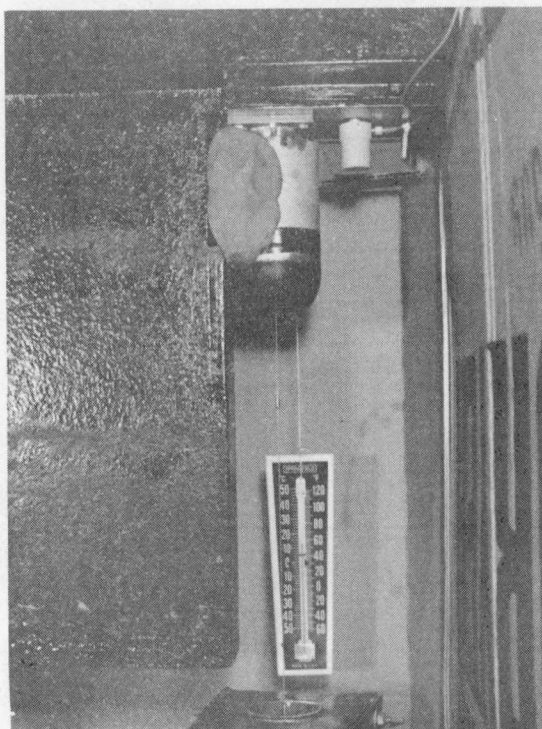


Figure 45.

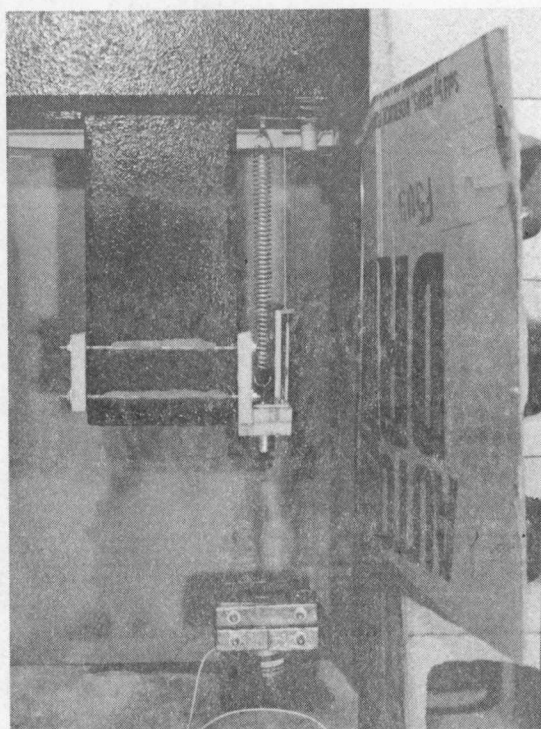


Figure 46.

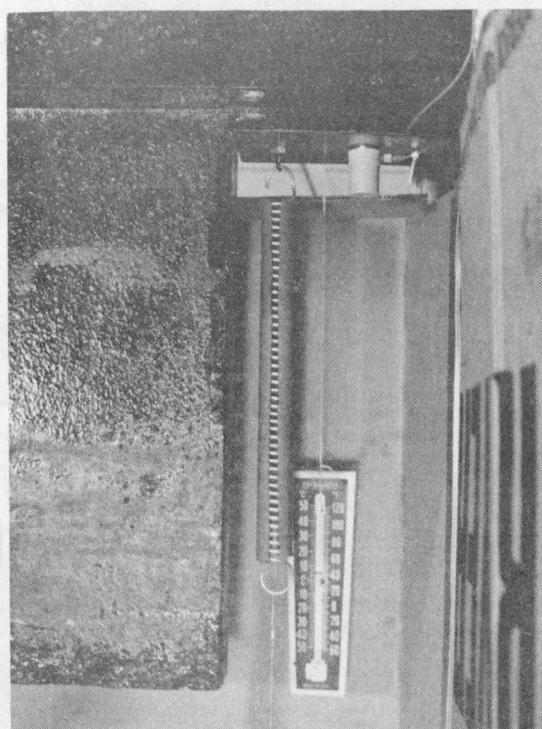


Figure 47.

Figure 48. The 6-Inch (Free Length) Extension Spring, with .044 Sq. In. (Cross-Sectional Area) of Isodamp C-1002 Bonded to It, in Series with an 11.5-Inch Length of .027 Dia. Steel Cable. The isolator is shown in the transverse test configuration. The isolated mass was 30.2 lb_m. The PCB 393B accelerometer, used to measure the motion of the isolated mass, is mounted on the side of the isolated mass.

Figure 49. The Same Isolator Shown in Figure 48 with the Laminated Metal Longitudinal Flexure (Figure 43, page 102) Installed Below the Spring. This isolator is shown in the transverse test configuration. The isolated mass was 30.2 lb_m.

Figure 50. The 6-Inch (Free Length) Extension Spring, with .0148 Sq. In. (Cross-Sectional Area) of Isodamp C-1002 Bonded to It, in Series with a 5.0-Inch Length of .027 Dia. Steel Cable, with the Plastic Longitudinal Flexure (Figure 43, page 102) Installed Below the Spring. This isolator is shown in the transverse test configuration. The isolated mass was 30.2 lb_m.

Figure 51. The 6-Inch (Free Length) Extension Spring, with Distributed Viscous Damping, in Series with a 5.0-Inch Length of .027 Dia. Steel Cable. This isolator is shown in the transverse test configuration. The isolated mass was 30.2 lb_m.



Figure 48.



Figure 49.

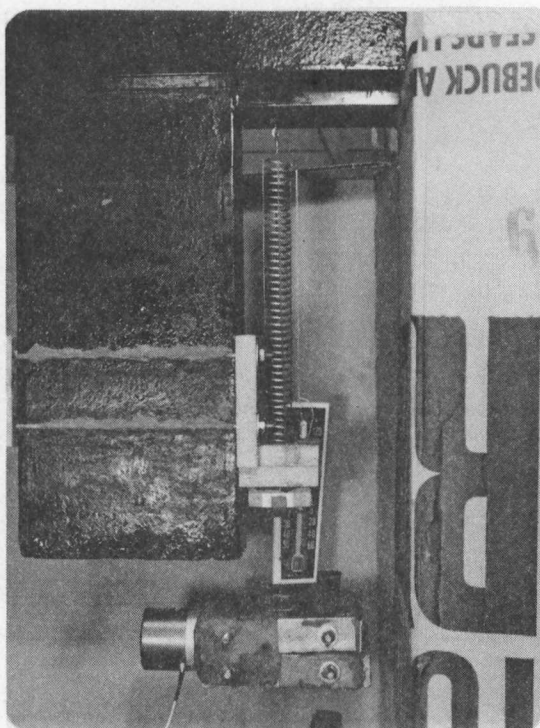


Figure 50.

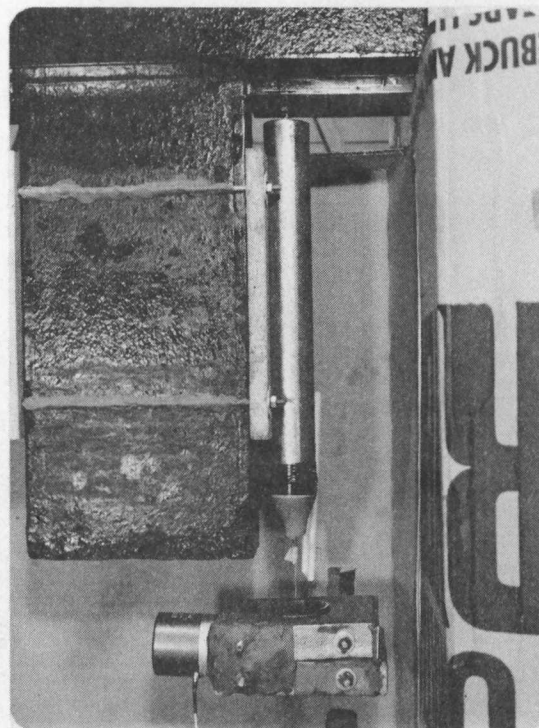


Figure 51.

Figure 52. Compression Spring Isolator in the Longitudinal Test Configuration. Cotton threads were tied to the isolated mass to provide stability. The isolated mass was 26.5 lb_m .

Figure 53. Test Control and Readout Instrumentation.

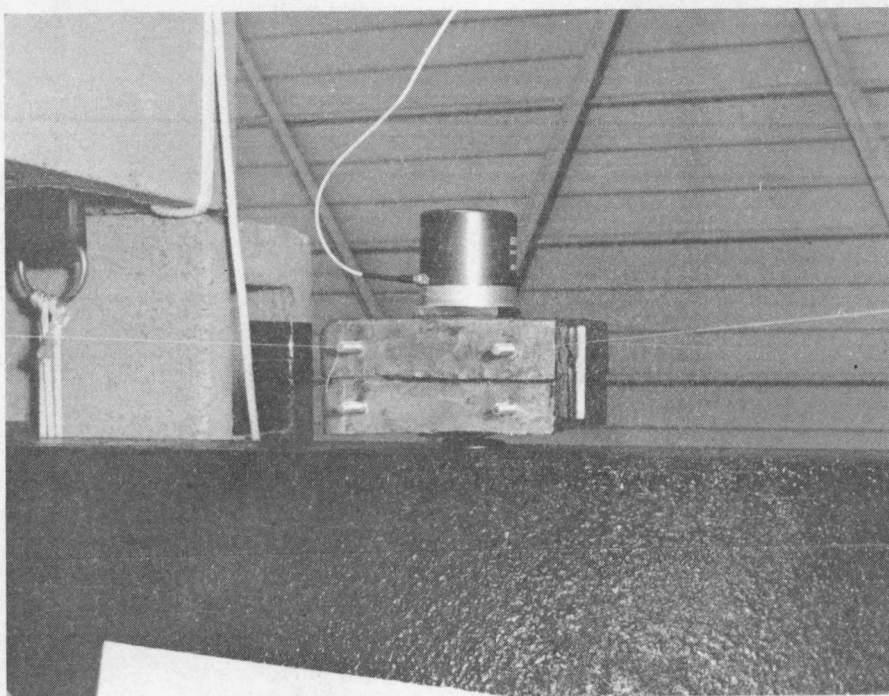


Figure 52.



Figure 53.

additional rigidity and strength where the shakers, isolator, and accelerometer are attached. The total weight of this block is approximately 240 lbs. The first bending resonance of the top portion of the block was found to occur at 750 Hz while the first bending resonance of the vertical cantilever beam was found to occur at 560 Hz. This latter resonant frequency is low enough to affect the transverse transmissibility measurements at frequencies above 300 Hz. The entire weight of the vibrating block is supported on heavy rubber bands which are attached to the concrete block structure. The concrete block structure, that supports the vibrating block and shaker blocks, was assembled using duct sealing putty as mortar to prevent any unwanted motion. This duct sealing putty which has a high damping factor was also used to prevent rattling between many other parts of the apparatus. The block used as the isolated mass was fabricated from cast lead. This block also had removable weights to allow adjustments of the block's mass and center of gravity location. The isolators, in all cases, were attached to the isolated-mass block at a point near its center of gravity. The accelerometer was rigidly fastened to the isolated mass block by epoxying a .25-inch thick steel plate to the lead block which was drilled and tapped to accept

the accelerometer mounting stud. A cardboard enclosure was built around the area of the isolator and heated to maintain the isolator damping material at the desired temperature (usually 76°F). Proper temperature control is especially important with solid damping because the dynamic Young's modulus of the Isodamp C-1002 material used is roughly doubled when the temperature is reduced from 76°F to 70°F.

The following is a list of the equipment used for the transmissibility tests:

Shakers:	Vibration Test Systems
VTS-100-8 (2 required)	
Power Amplifier:	Crown DC-300A
Function Generator:	B&K Precision Model 3010
Frequency Counter:	Heathkit Model IM-4100
Multimeter:	Micronta 1,000 Ohms/Volt
Multitester	
Accelerometers:	PCB 393B (Seismic-high
sensitivity) 1 v/g	
	PCB 308B 10 mv/g.
Power Supplies:	PCB 480C06 (used with 393 B)
	PCB 482A (used with 308B)
Oscilloscope:	Hewlett Packard 1220A
Digital Voltmeter:	Beckman Tech 330

The wiring schematic for this equipment is included in Figure 39 on page 99. The PCB 393B accelerometer, used to measure the motion of the isolated mass, was found to be capable of producing a clean output signal down to an acceleration level of .0002 g. Due to power amplifier limitations, the maximum acceleration of the vibrating block was about .3 g at 50 Hz decreasing linearly to about .15 g at 500 Hz. Because of the limited drive capability and the accelerometer signal-to-noise ratio, the lower limit for obtaining accurate transmissibility measurements was found to be about -63 dB. Although measurements were made well below -63 dB, the accuracy of these measurements is questionable. Fortunately, the area of transmissibilities below -63 dB is of less concern as this is considered to be very good isolation.

Before making measurements, it was necessary to calibrate the accelerometers. This was accomplished by comparing the output voltages of the accelerometers when they were both fastened to the vibrating block and driven at many discrete frequencies between .8 and 500 Hz.

The isolator for which the transmissibility was to be measured was first installed on the vibrating block. The test frequency was then set on the function generator by monitoring the frequency counter. The drive level was then adjusted as required to obtain a clean signal from

both accelerometers. Drive levels were set very low at frequencies near the natural frequency of the isolator in order to prevent damage to the isolator and the accelerometer attached to the isolated mass. For this reason, accurate measurements could not be made at these frequencies. At frequencies above 50 Hz, all of the available amplifier power was required to obtain a clean signal from the accelerometer attached to the isolated mass. The drive level was then set just below the amplifier clipping point. At frequencies of 15 Hz and above, the rms accelerometer output voltages were measured with a true rms averaging digital voltmeter, using a-c coupling. For most isolators, measurements were made at approximately 29 discrete frequencies between .8 and 500 Hz.

B. Longitudinal Measurements

The vibrating block apparatus was first set up in the longitudinal test configuration. The first isolator to be tested was the 1.0-inch O.D., .095 diam. wire, 6.0-inch long (free length) extension spring without damping. This isolator was analyzed previously as Rod Case 10; however, it was found that the actual spring used as the isolator had fewer active coils and was made of slightly larger diameter wire than described in the spring catalog. The actual spring specifications are:

Outside diameter	= 1.0 in
Wire diameter	= .0966 in
Maximum load	= 30.21 lb
Free length	= 6.0 in
Spring constant	= 4.028 lb/in
Initial tension	= 2.7 lb
Length at maximum load	= 12.8 in
Spring mass	= .2609 lb _m
Number of coils	= 44

This spring will hereafter be referred to as the 6-inch extension spring. When the transmissibility was calculated using the actual spring and isolated mass parameters, the resulting curve matched the measured curve very closely. The measured and calculated transmissibility curves for this isolator are plotted in Figure 54. The transmissibility of this isolator could not easily be measured above 150 Hz because of the close spacing of the resonances. It is apparent from Figure 54 that the assumption of a one-dimensional distributed parameter system is valid in this case.

The first isolator with solid damping to be tested was designed using the original material test data. The spring used was the 6-inch extension spring. The isolator was to have a composite damping factor of .10 which required a cross-sectional area of .175 sq. inches

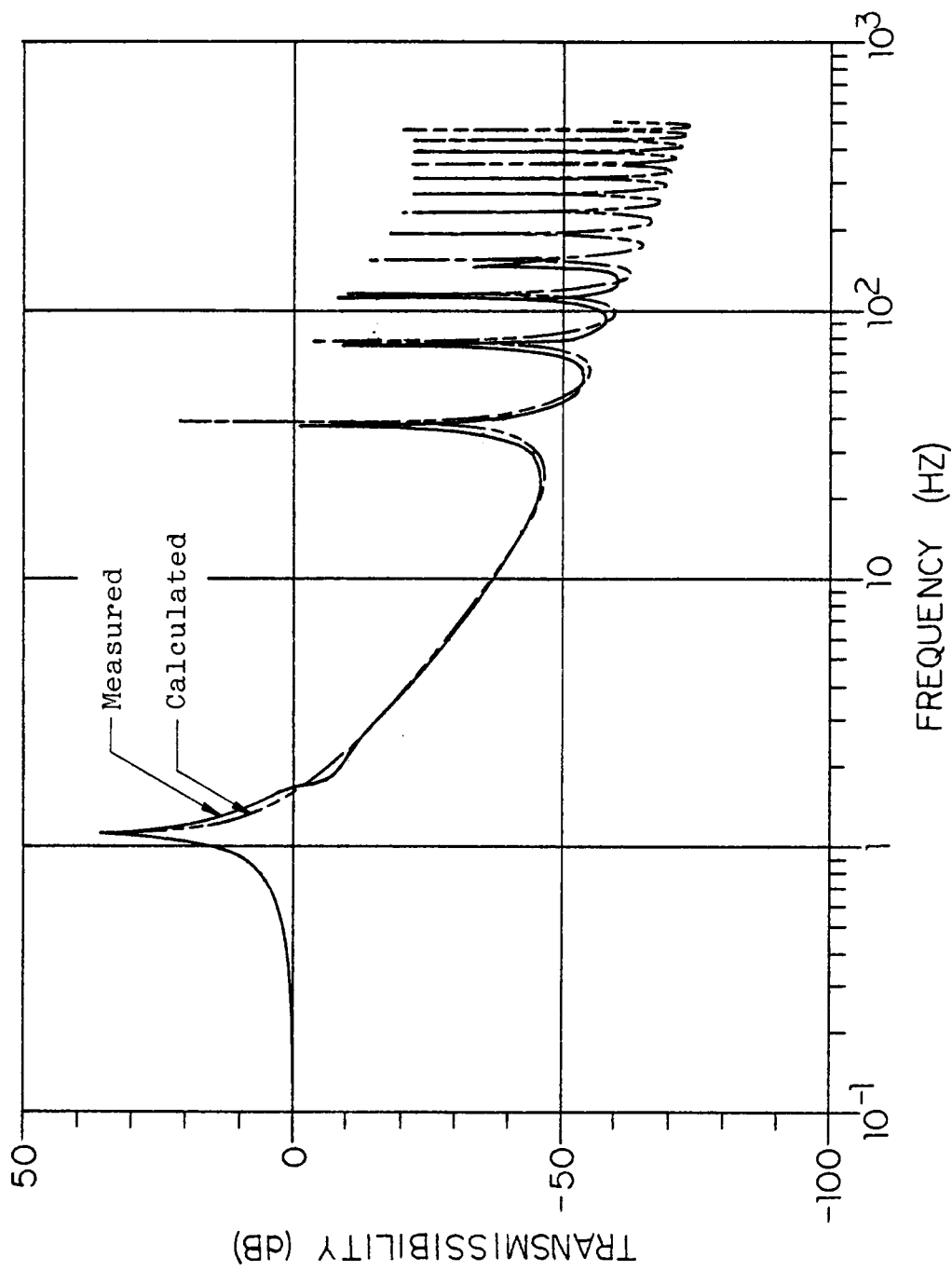


Figure 54. Measured and Calculated Transmissibility of the 6-Inch (Free Length) Extension Spring without Damping, in Longitudinal Vibration.

of Isodamp C-1002. This isolator, shown in Figure 47 on page 104, when tested was found to have a natural frequency of 2.25 Hz which was obviously too high for the calculated stiffness of the isolator. It was concluded that the measured complex Young's modulus values for the damping material were much too low. The complex Young's modulus values were found to be a factor of 10 too low due to an error in the measurement calibration. This error has been corrected in the complex Young's modulus values given in Table 13, page 75.

An isolator was then made from the 6-inch extension spring by bonding a cross-sectional area of .044 sq. inch of Isodamp C-1002 to it. The composite damping factor of this isolator at 5 Hz, if calculated using the material properties listed in Table 13, is .291. This isolator was tested alone and with the metal flexure as shown in Figures 48 and 49 on page 106. The metal flexure used, shown in Figure 43 on page 102, was of laminated construction with 2 layers of .002-inch thick brass shim stock separated by a layer of vinyl electrical tape. The measured transmissibility of these isolators in longitudinal vibration is shown in Figure 55. From these curves, it appeared that the isolator was still too heavily damped and the stiffness of the metal flexure was too high.

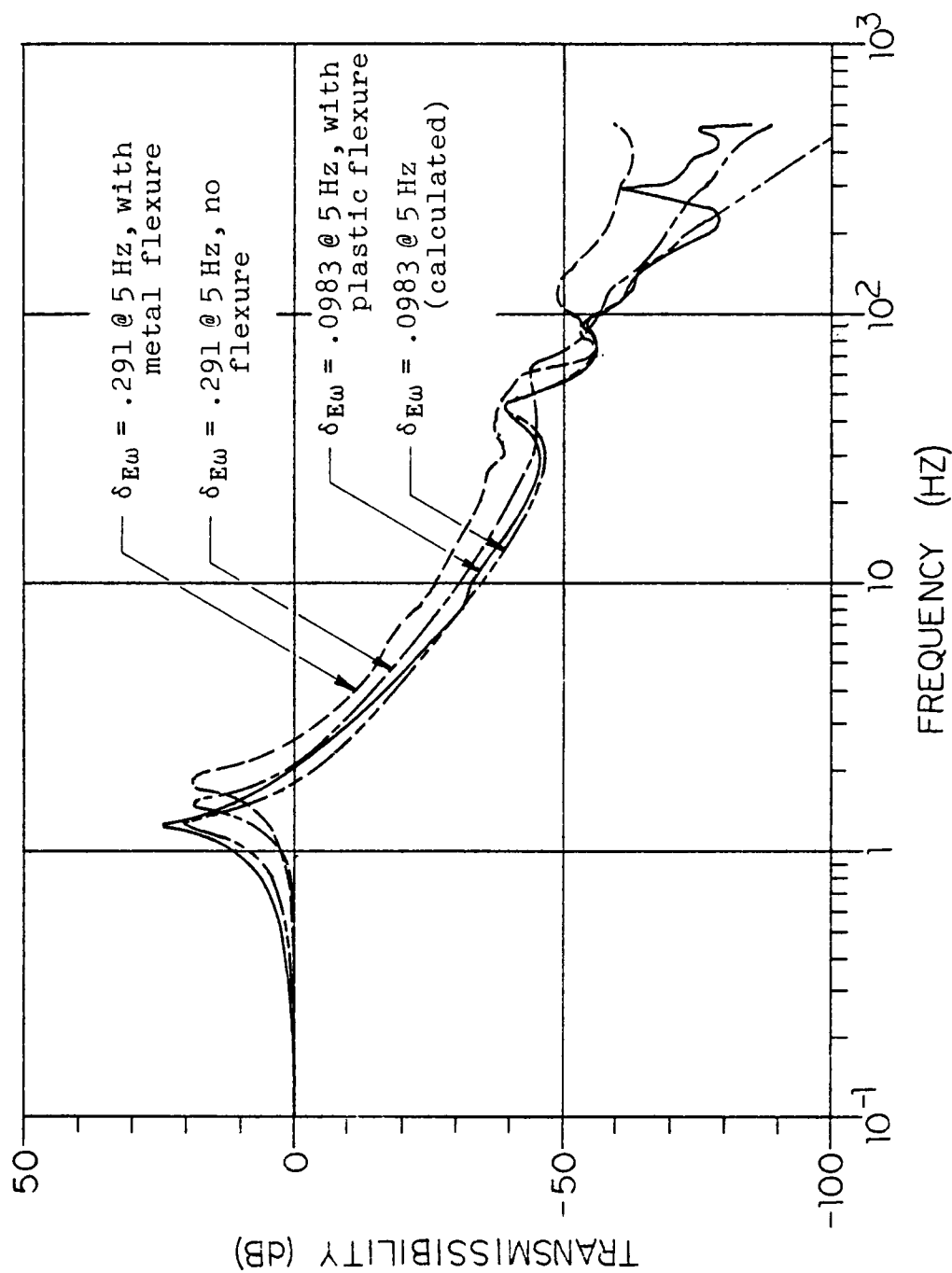


Figure 55. Measured and Calculated Transmissibility of the 6-Inch (Free Length) Extension Spring, in Longitudinal Vibration, with Solid Damping.

Another isolator was made from the 6-inch extension spring with a cross-sectional area of .0148 sq. inch of Isodamp C-1002. This isolator, shown in Figure 50 on page 106, was tested with a plastic flexure made of Isodamp C-1002. The plastic flexure assembly is shown in Figure 43 on page 102. The composite damping factor of this isolator at 5 Hz, if calculated using the correct material properties, is .0983. The measured longitudinal transmissibility and the calculated transmissibility using actual spring specifications and correct material properties are plotted in Figure 55. These curves are seen to match closely up to 200 Hz. The measured transmissibility peak at 280 Hz is probably due to a standing wave resonance in the plastic flexure. Since the measured transmissibility for this isolator is closely approximated by the calculations, and the previous analysis of Chapter IV shows the optimum damping factor to be about .1, it follows that this isolator is also nearly optimally damped.

The longitudinal transmissibility of the 6-inch extension spring with distributed viscous damping, shown in Figure 51 on page 106, was measured next. A conceptual drawing of this isolator is shown in Figure 21 on page 57. The viscous fluid container in this case consisted of a short length of 1-1/4 inch Schedule 40 pipe that was

rigidly fastened to the vibrating block and sealed at the bottom with a rubber balloon. The viscous fluid used consisted of a mixture of STP Oil Treatment and SAE 10W40 motor oil. The damping factor of the tested isolator was determined from the logarithmic decrement of the free vibration and was varied as required by adjusting the temperature of the isolator and by changing the STP/oil ratio of the viscous fluid. Plotted in Figure 56 is the measured transmissibility of this isolator with damping factors of .010, .041, and .11, and the calculated transmissibility of the isolator with a damping factor of .041 based on the actual spring specifications. From this plot it can be seen that the calculated transmissibility matches the measured transmissibility fairly well below 50 Hz. The difference between these 2 curves above 50 Hz is probably due to neglecting the mass of the viscous fluid in the calculations. It can also be seen from Figure 56 that of the 3 damping factors tested, the best overall transmissibility performance is provided by the isolator with a damping factor of .041.

A compression spring was tested in longitudinal vibration without damping as shown in Figure 52 on page 108 and with distributed viscous damping. This spring has a 1.46-inch O.D. with .112 diam. wire and a free length of 3.50 inches. The distributed viscous

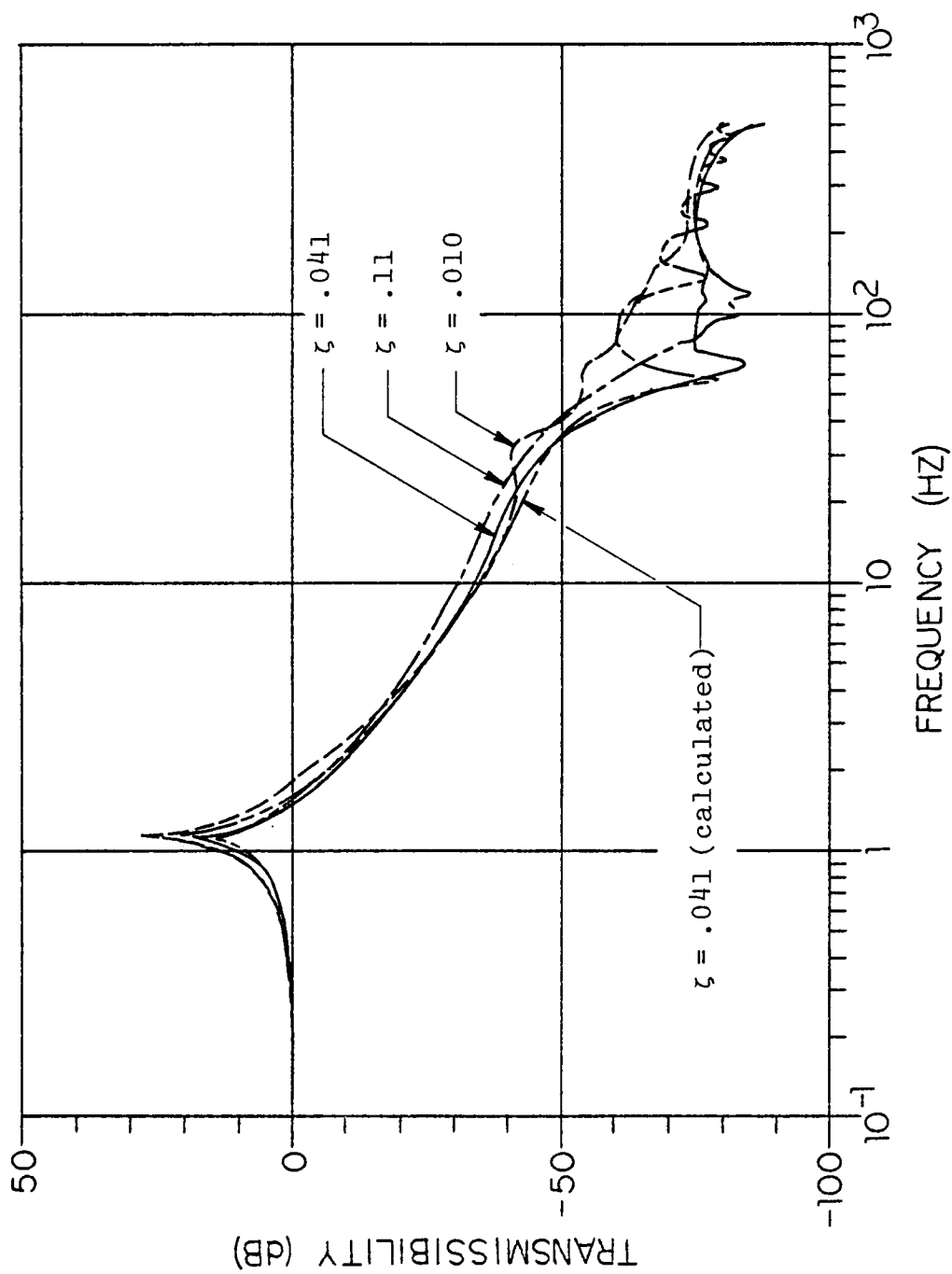


Figure 56. Measured and Calculated Transmissibility of the 6-Inch (Free Length) Extension Spring, with Distributed Viscous Damping, in Longitudinal Vibration.

damping, in this case, was obtained by placing the spring inside a small cup filled with the STP/oil mixture. Figure 57 is a plot of the measured transmissibility of this isolator with a damping factor of zero and .061 and the calculated transmissibility of the undamped spring, based on the actual spring specifications. The calculated transmissibility is seen to match the measured curve quite closely below 200 Hz. Above 200 Hz, the measured transmissibility curve is characteristic of an impedance change within the isolator which may be due to unequal coil spacing. It can also be seen from Figure 57 that distributed viscous damping is very effective in suppressing the standing wave resonances of this spring.

Another isolator tested was the 6-inch extension spring with a viscous damper in parallel as shown in Figure 46 on page 104. This isolator was tested only to verify the previous analysis of Chapter IV that showed the viscous damper to provide no suppression of the standing wave resonances. The measurements confirmed this analysis.

The evacuated metal bellows isolator, which was shown by the previous analysis to provide very low transmissibility, was tested in longitudinal vibration, undamped, and with distributed viscous damping. These

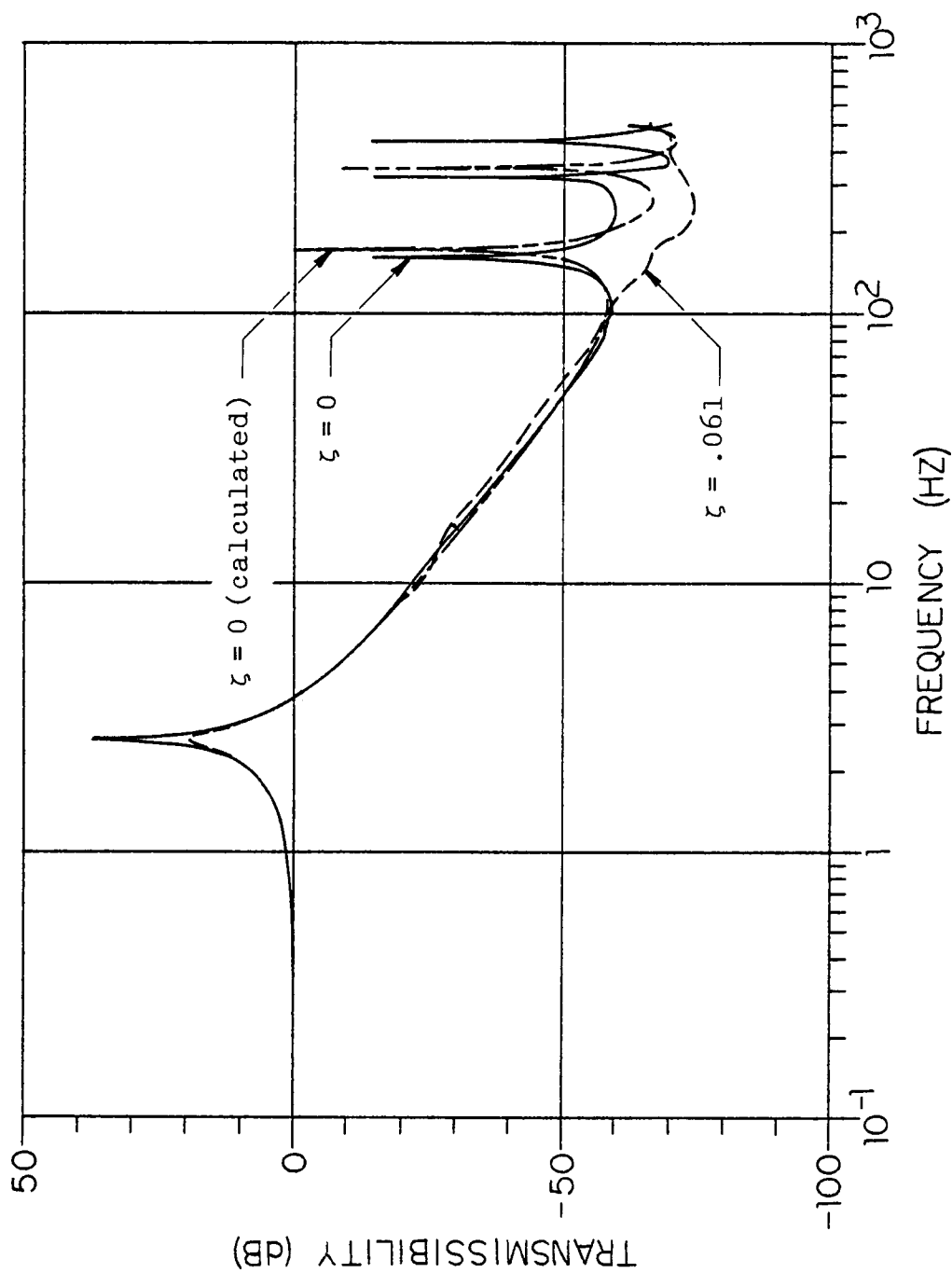


Figure 57. Measured and Calculated Transmissibility of a Compression Spring Isolator, with Distributed Viscous Damping, in Longitudinal Vibration.

isolators are shown in Figures 44 and 45 on page 104. Figure 58 is a plot of the measured transmissibility of the metal bellows with a damping factor of zero and .049 and the calculated transmissibility of the undamped bellows (Rod Case 11). The actual bellows tested was found to be much stiffer than indicated by the manufacturer's specifications which resulted in the higher natural frequency of the tested bellows compared to that of the calculations. The measured natural frequency of the bellows with distributed viscous damping was found to be higher than that of the undamped bellows. This is due to the added stiffness of the rubber seal used to contain the viscous fluid (see Figure 45 on page 104). From Figure 58 it is also seen that a damping factor of .049 is too high for this isolator. The measured transmissibility performance of this isolator is clearly not as good as that of the 6-inch extension spring with either solid or distributed viscous damping.

The basic concept of using an evacuated metal bellows as an isolator appears to be a workable one; however, there are practical problems. One problem is the relatively high cost of the bellows compared to a coil spring. The price of a bellows is roughly \$150 while the price of an extension spring is only \$2.96. Another problem with a bellows isolator with a low natural

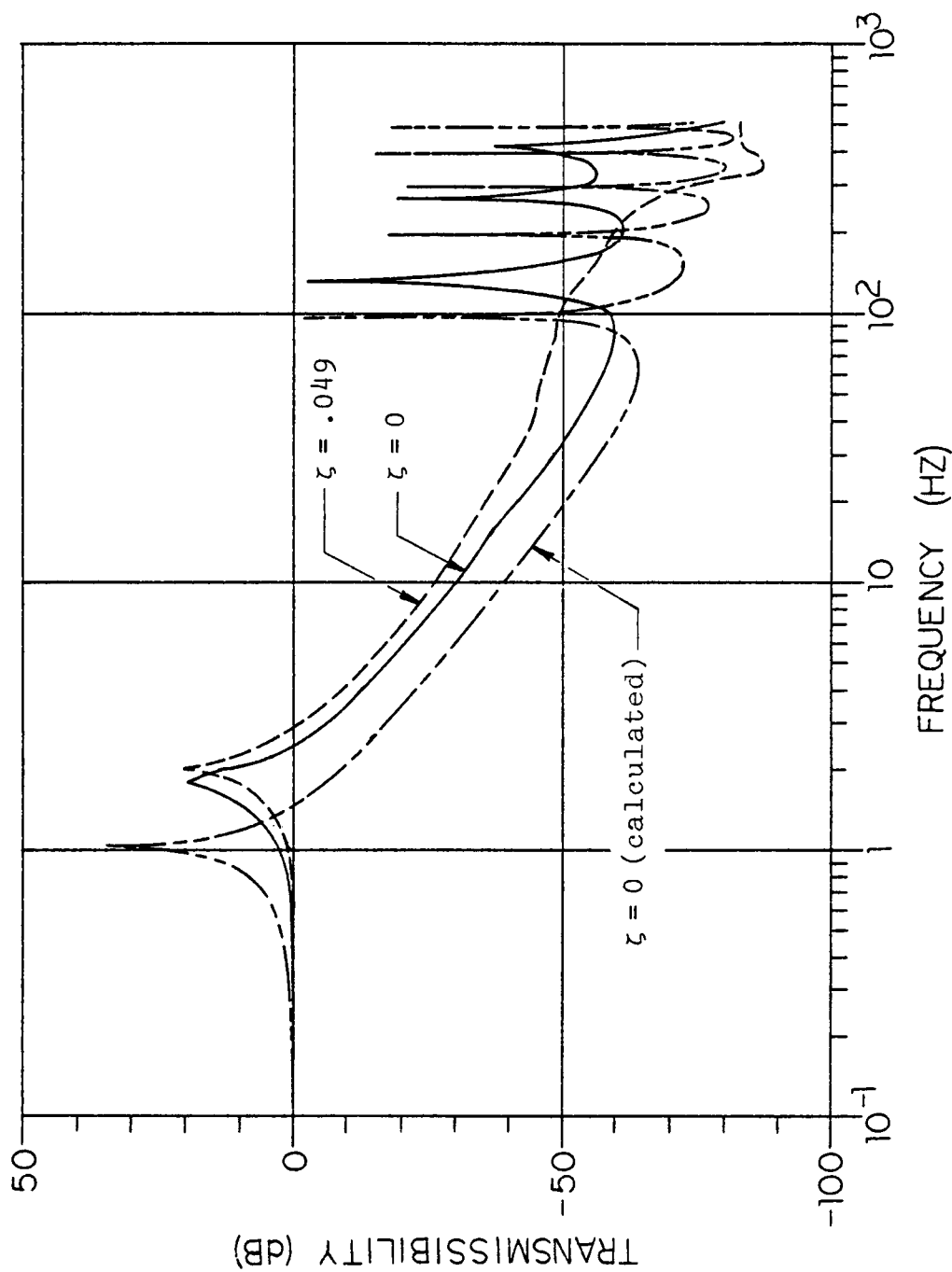


Figure 58. Measured and Calculated Transmissibility of the Evacuated Metal Bellows Isolator, with Distributed Viscous Damping, in Longitudinal Vibration.

frequency is the change in static length due to normal changes in barometric pressure. For example, an evacuated bellows isolator having a 1 Hz natural frequency would experience a change in length of .326 inch for a pressure change of 1 inch Hg. This change in length is undesirable with the present turntable isolation system design because the turntable support base would be tilted slightly with pressure changes unless the spring constants of all four bellows were closely matched. The .396-inch deflection could also place the bellows outside of its linear deflection range unless it is made sufficiently long.

C. Transverse Measurements

After completing all of the longitudinal transmissibility measurements, the vibrating block apparatus was set up in the transverse test configuration as shown in Figure 42 on page 102. The first isolator was tested to verify the previous analysis that showed the first few transverse standing wave resonances to be practically unaffected by internal damping. This isolator, shown in Figure 48 on page 106, consisted of the 6-inch extension spring with .044 sq. inch of Isodamp C-1002 bonded to it, in series with an 11.5-inch length of .027 diam. steel cable. The measured transmissibility of this isolator, and for comparison, the calculated

transmissibility based on the actual spring parameters with a constant damping factor of .38, is plotted in Figure 59. The calculated transmissibility is seen to agree very well with the measured transmissibility except at frequencies near 1 Hz. The difference there was due to the difficulty of obtaining accurate acceleration readings of the vibrating block at the low amplitudes required near the natural frequency of this isolator. It is also seen from this plot that the first two standing wave transmissibility peaks are undesirably high, as predicted.

The isolator just discussed was then tested in transverse vibration with the metal flexure installed below the spring as shown in Figure 49 on page 106. The measured and calculated transmissibility curves for this isolator are plotted in Figure 60. The calculated transmissibility for this case was based on the assumption that the upper end of the cable was rigidly fastened to the vibrating block and is seen to agree fairly well with the measured transmissibility at frequencies below 200 Hz. At frequencies above 200 Hz, the difference between the measured and calculated transmissibility is probably due to bending of the vertical beam of the vibrating block which was found to have its first bending resonance at 560 Hz. It was mentioned previously that air damping may

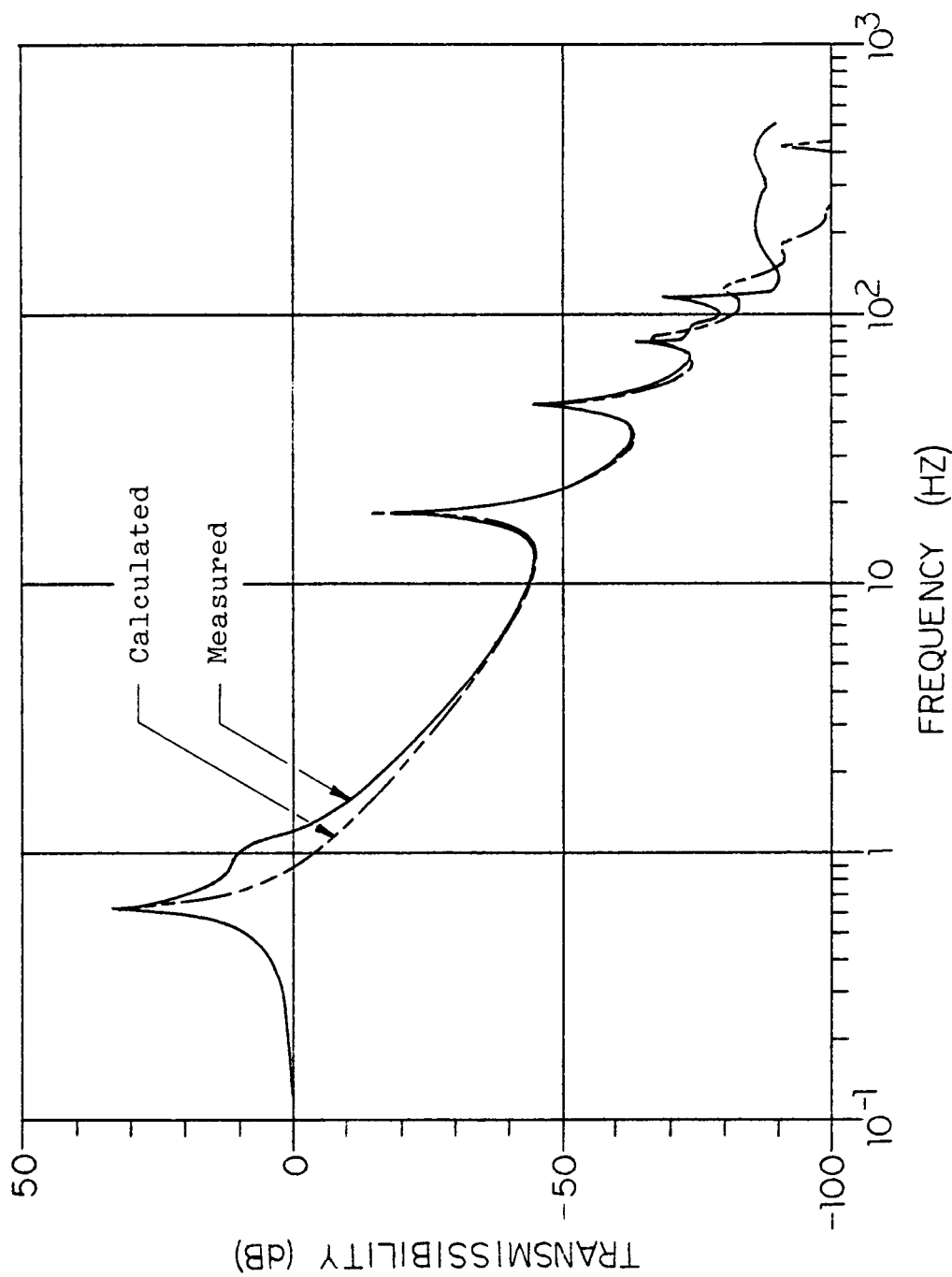


Figure 59. Measured and Calculated Transmissibility of the Undamped 6-Inch (Free Length) Extension Spring with Cable in Transverse Vibration.

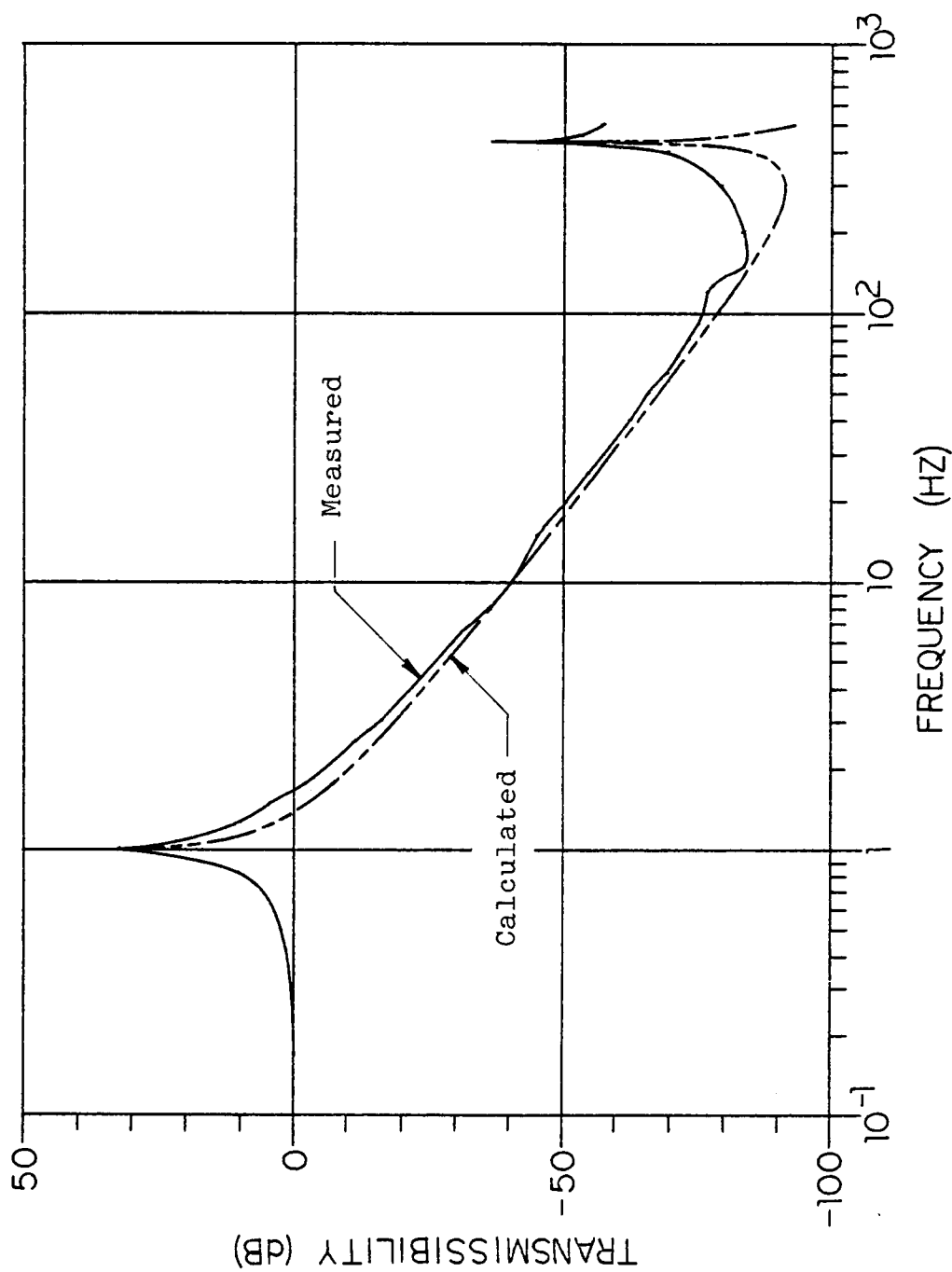


Figure 60. Measured and Calculated Transmissibility of the 6-Inch (Free Length) Extension Spring with the Metal Flexure and 10-Inch Cable in Transverse Vibration.

provide some attenuation of the standing wave resonance. Because of the flexibility of the vertical beam, it is very difficult to assess the effect of air damping from the measured transmissibility.

Another isolator tested was the 6-inch extension spring with .0148 sq. inch of Isodamp C-1002 bonded to it, in series with a 5.0-inch length of .027 diam. steel cable. The plastic flexure was installed below the spring as shown in Figure 50 on page 106. Figure 61 is a plot of the measured transmissibility of this isolator. It can be seen from Figure 61 that the standing wave resonances of the spring are apparent but are well attenuated by the plastic flexure. The plastic flexure is much more compliant in the transverse direction than the metal flexure and cannot be considered to be a rigidly attached point in the transverse direction.

The use of a plastic flexure appears to be a viable concept for controlling the transverse standing wave resonances of an extension spring isolator. Performance improvements could probably be obtained by experimenting with the geometry and transverse stiffness of the plastic flexure. Additional flexures might also be placed at various points along the length of the spring.

Also measured was the transverse transmissibility of the 6-inch extension spring with distributed viscous

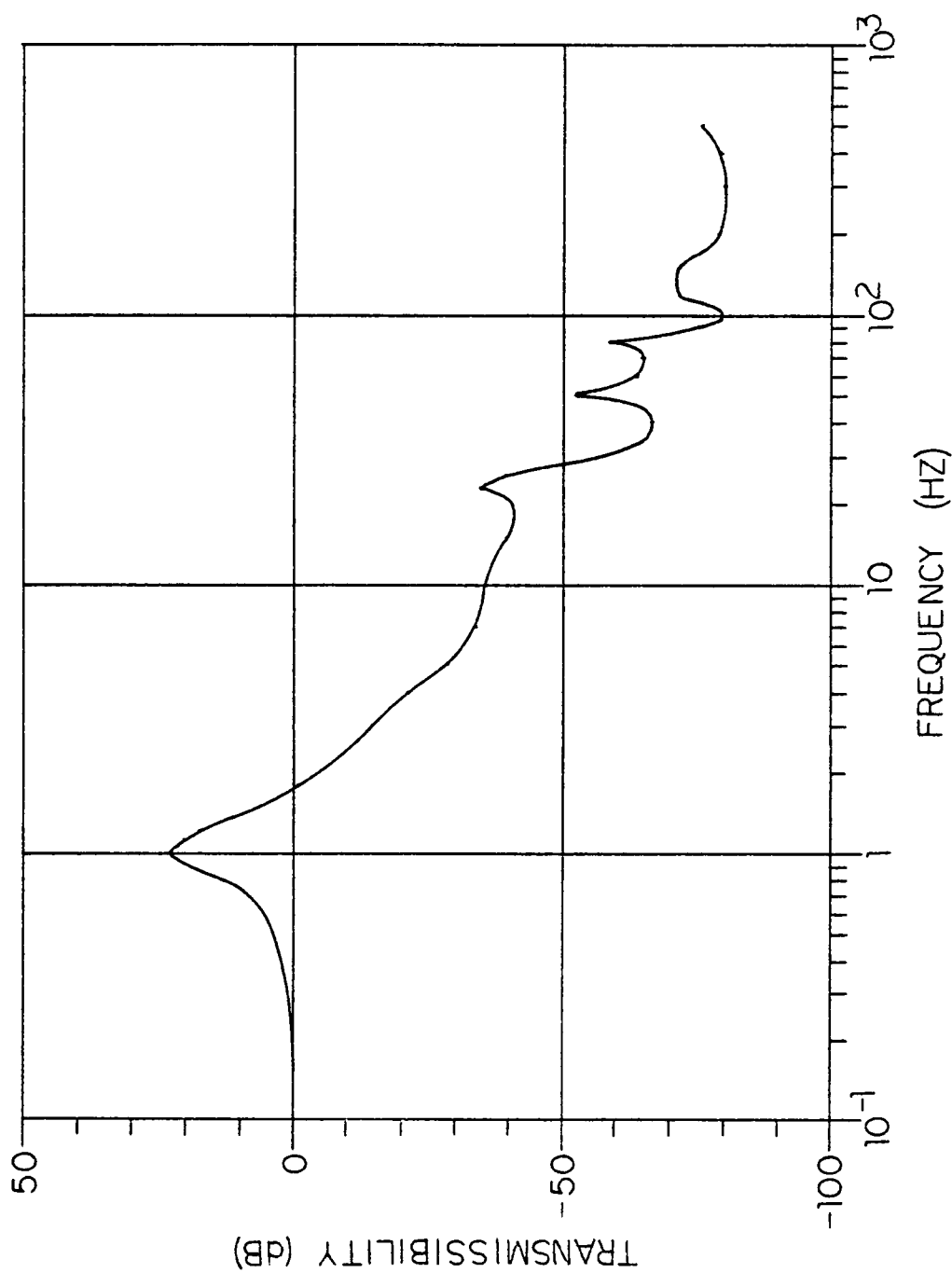


Figure 61. Measured Transmissibility of the 6-Inch (Free Length) Extension Spring, with Solid Damping and the Plastic Flexure, in Transverse Vibration.

damping in series with a 5.0-inch length of .027 diam. steel cable as shown in Figure 51 on page 106. This isolator was also tested with a 10.0-inch length of cable. The damping factor, ζ , was measured using the logarithmic decrement technique described previously. The isolators with measured damping factors of .064 (5.0-inch cable) and .01 (10-inch cable) used the same spring and viscous fluid as the longitudinal case with a measured damping factor of .041. The transverse transmissibility for the isolator with a measured damping factor of .048 was measured under the same conditions as that of the longitudinal case with a damping factor of .010. Plotted in Figure 62 is the transmissibility for these 3 cases. From this plot it can be seen that distributed viscous damping can eliminate the standing wave transmissibility peaks while providing very good overall isolation. It can also be seen that the natural frequency of this isolator is increased by increasing the fluid viscosity. This is because the damping force changes the effective upper pivot point of the isolator. For a low viscosity fluid, the effective pendulum length would be almost the total length of the cable and spring, while for a very high viscosity fluid the effective pendulum length would only be slightly more than the cable length. Because of this effect, it is seen in Figure 62 that the

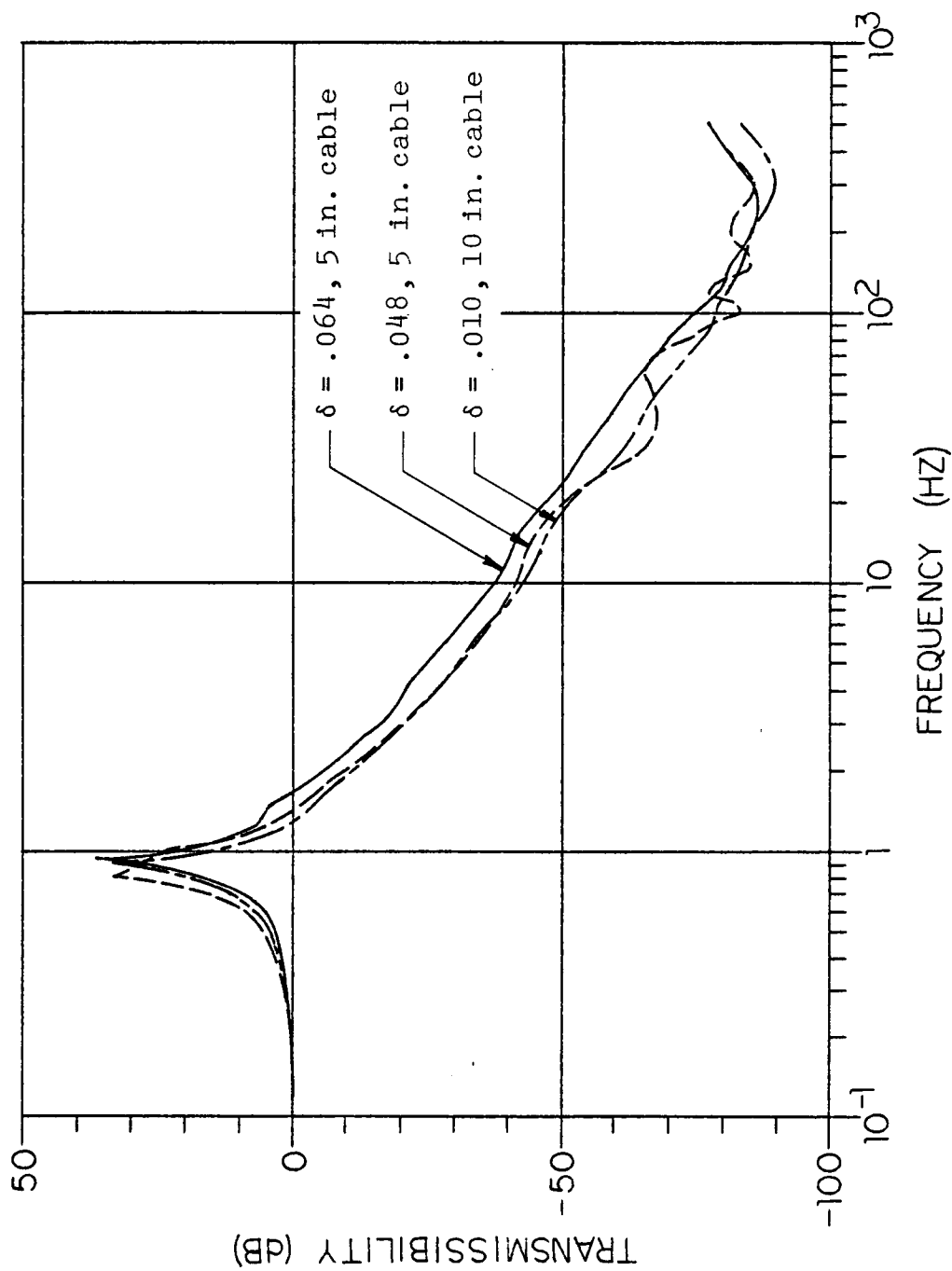


Figure 62. Measured Transmissibility of the 6-Inch (Free Length) Extension Spring, with Distributed Viscous Damping and Cable, in Transverse Vibration.

transmissibility of the isolator with the thinner fluid and a 5.0-inch cable length is roughly the same as that of the isolator with thicker fluid and a 10.0-inch length of cable. From observing the effect of fluid viscosity on the effective pivot location, it can be concluded that for a given spring and cable length, there would be a certain fluid viscosity which would give the maximum measured damping factor for the isolator.

The measured transmissibility curves for the various isolator configurations were then compared in order to select the best one. The 2 best isolators in longitudinal vibration were found to be the 6.0-inch extension spring with distributed viscous damping ($\zeta = .041$) and the same spring with solid damping ($\delta_{E\omega} = .098$ at 5 Hz) and the plastic flexure. For comparison, the longitudinal transmissibility curves for these 2 isolators are re-plotted together in Figure 63. This plot shows that lowest overall transmissibility is provided by the isolator with distributed viscous damping. The transverse transmissibility curves for the same 2 isolators are re-plotted, for comparison, in Figure 64. For transverse vibration, the lowest transmissibility, at most frequencies, is seen to be provided by the isolator with distributed viscous damping. Therefore, of the isolators tested, the best vibration isolation is

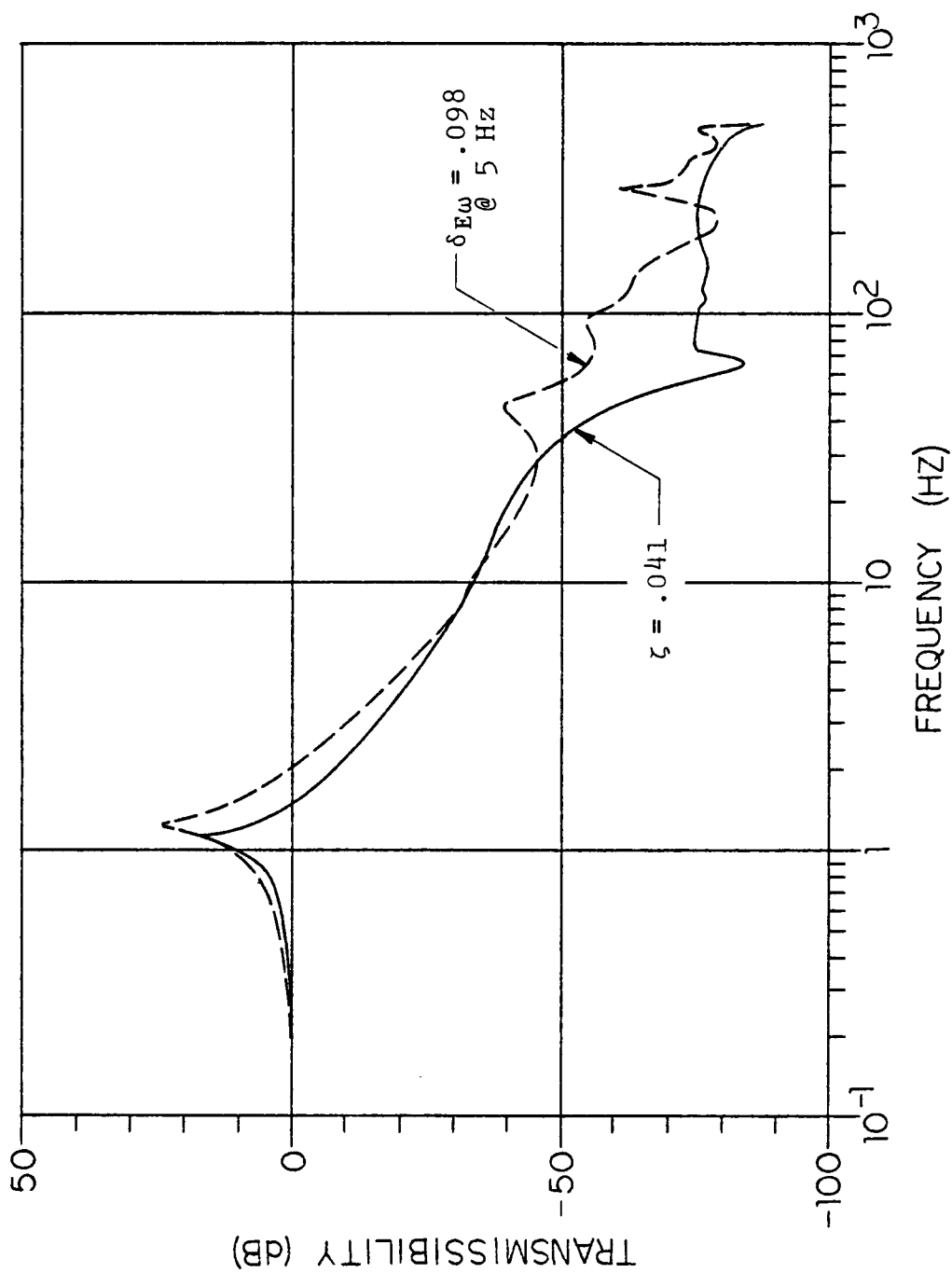


Figure 63. Measured Transmissibility of the Best Solid Damping Case and the Best Distributed Viscous Damping Case in Longitudinal Vibration.

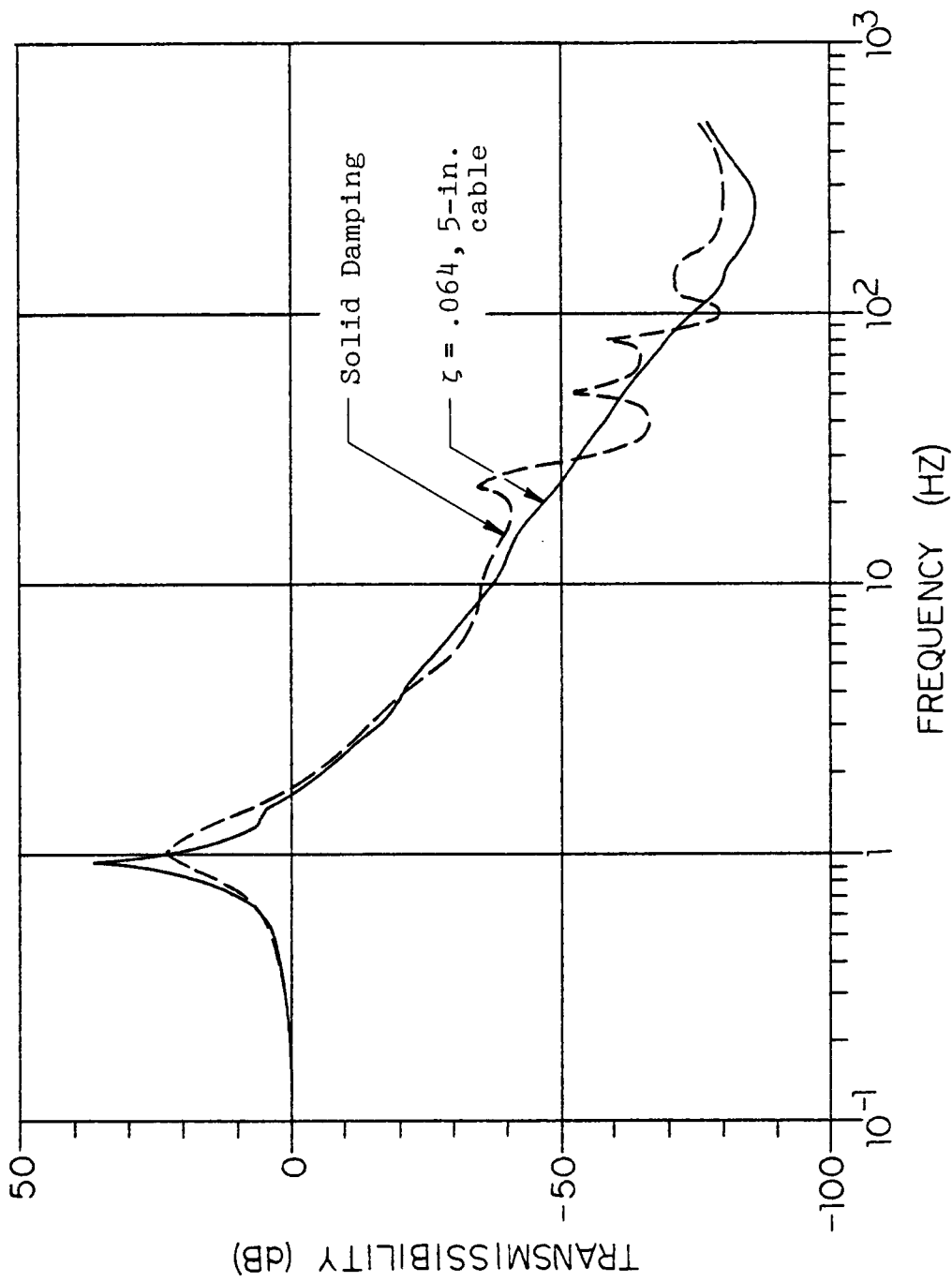


Figure 64. Measured Transverse Transmissibility of the Two Isolators Found to Have the Lowest Longitudinal Transmissibility.

provided by the 6-inch extension spring with distributed viscous damping with a longitudinal damping factor of .041.

CHAPTER VI

SHOCK ISOLATION ANALYSIS

The preceding analyses and tests were for vibration isolation only, where steady state, sinusoidal displacements of the environment were assumed. Since shocklike displacements of the environment will also occur, shock isolation must also be considered in order to achieve the optimum isolation system design.

An analysis of shock isolation that neglects wave effects appears in [4]. The input transient that is used most frequently in [4] is called the "rounded displacement step" (Figure 65) and is defined as

$$X_1(t) = 0 \quad \text{for } t < 0 \quad , \quad (27)$$

$$X_1(t) = X_{\max} \left[1 - e^{-\gamma \omega_0 t} (1 + \gamma \omega_0 t) \right] \quad , \quad (28)$$

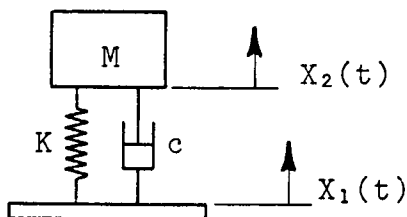


Figure 65. Simple Shock Isolator.

where ω_0 = Angular natural frequency,

t = Time,

τ = Rise time—the time required for the displacement to reach 82% of its final value, and

$$\gamma = \pi/\omega_0\tau.$$

The displacement response of the simple system according to [4] is

$$X_2 = X_{\max} \left\{ 1 - e^{-\gamma\omega_0 t} [A(\gamma\omega_0 t) + B] + e^{-\delta_R \omega_0 t} (C \cos \Delta_R \omega_0 t + D \sin \Delta_R \omega_0 t) \right\}, \quad (29)$$

where $A = 1 - \gamma^2 \Gamma$,

$$B = [1 + \gamma^2(1 - \gamma^2)\Gamma^2],$$

$$C = \gamma^2(1 - \gamma^2)\Gamma^2,$$

$$D = \gamma^2(\gamma^2 \delta_R - 2\gamma + \delta_R) (\Gamma^2/\Delta_R),$$

$$\Delta_R = (1 - \delta_R^2)^{1/2},$$

$$\Gamma = (\gamma^2 - 2\delta_R\gamma + 1)^{-1}, \text{ and}$$

δ_R = Damping factor ($\delta_R = 1$ for critical damping).

The acceleration of M , again from [4], is

$$\frac{d^2 X_2}{dt^2} = \omega_0^2 X_{\max} \left\{ -\gamma^2 e^{-\gamma\omega_0 t} [A(\gamma\omega_0 t) + (B - 2A)] + e^{-\delta_R \omega_0 t} [E \sin \Delta_R \omega_0 t + F \cos \Delta_R \omega_0 t] \right\}, \quad (30)$$

where $E = 2C \delta_R \Delta_R - D(1 - 2\delta_R^2)$, and

$$F = -C(1 - 2\delta_R^2) - 2D \delta_R \Delta_R.$$

Generally, a shock mount is used to prevent damage to some fragile element of the mounted system. In the case of a turntable shock isolation system, the "damage-prone" element is the pickup/tone-arm system as shown in Figure 66.

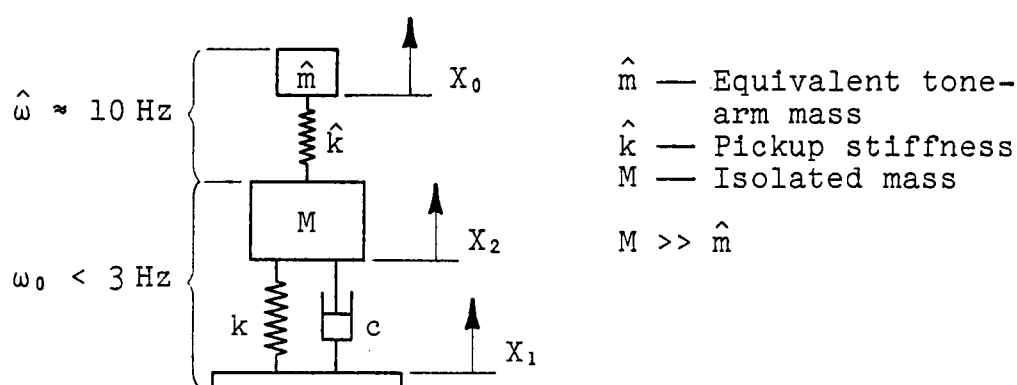


Figure 66. Simplified Model of Turntable Isolation System and Pickup/Tone-Arm System.

Since the natural frequency of the pickup-arm system is more than three times that of the mount, the "damage-prone" element is acceleration sensitive and relatively stiff so that X_0 will closely match X_2 . The acceleration of $\hat{m}$ can therefore be regarded as essentially equal to the acceleration of M . The purpose of the turntable shock isolation system is to keep the acceleration of $\hat{m}$, and consequently, M , below some acceptable level for all reasonable environmental displacement transients. For this investigation the acceptable variation of stylus

force, due to acceleration of $\hat{m}$, was assumed to be ± 10 percent of the static stylus force. The limits on acceleration by this requirement are

$$|\hat{a}| \leq .1 g \quad . \quad (31)$$

Calculations using Equation (30) show that with a damping factor of .05 and $\gamma = 50$, the allowable $X_{\max} \approx .050$ inches for a mount with a 3 Hz natural frequency.

From observing trends in Equation (30), it can be seen that the maximum acceleration of M , for a given $X_{\max}$, will be small when the damping factor is small, the rise time is long (γ small) and ω_0 is small. In order to more conveniently compare shock mount performance, some figure of merit is needed. A figure of merit, for shock isolation systems, is the "shock acceleration ratio" (SAR) which is simply the ratio of the maximum acceleration above and below the shock mount [4]. The SAR can be plotted as a function of γ which results in a design curve analogous to the transmissibility-frequency curve. Figure 67 shows that a lightly damped mount has much better performance with steplike inputs having a short rise time. Since the short rise time displacements cause the largest acceleration of the mounted item

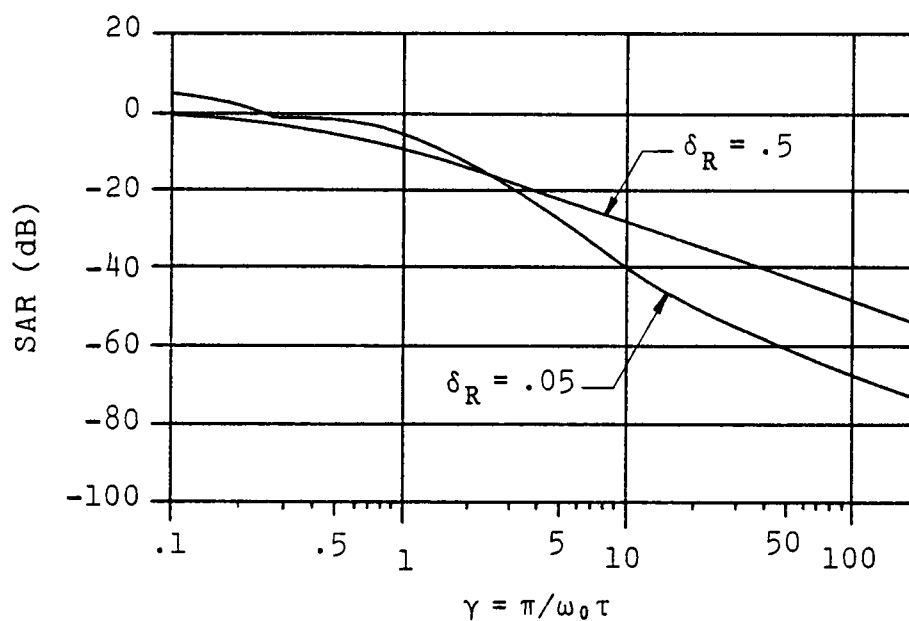


Figure 67. SAR Curve for Rounded Step Displacement.

Source: J. C. Snowden, *Vibration and Shock in Damped Mechanical Systems*, (John Wiley and Sons, New York, 1968) p. 404.

it is desirable to keep damping as low as possible. The equation for the input acceleration, obtained by differentiation of Equation (28), is

$$\frac{d^2 X_1}{dt^2} = X_{\max} \gamma^2 \omega_0^2 e^{-\gamma \omega_0 t} [1 - \gamma \omega_0 t] \quad (32)$$

and is maximum at $t = 0$ so that the maximum input acceleration is:

$$\left(\frac{d^2 X_1}{dt^2} \right)_{\max} = X_{\max} \gamma^2 \omega_0^2 \quad . \quad (33)$$

The maximum acceleration of the mounted item, for a given $X_{\max}$ and rise time, can be lowered by reducing ω_0 since the SAR becomes lower while the input acceleration, given by Equation (33), remains the same. From the above considerations it can be seen that a good shock mount would also tend to be a good vibration isolator.

CHAPTER VII

MEASUREMENT OF SHOCK ACCELERATION RATIO

A. Technique and Apparatus

In the preceding shock isolation analysis, wave effects were neglected. Neglect of wave effects in a low-frequency, low-damping shock isolation system can result in large errors in predicting the actual SAR, especially for shocks with a short rise time. In the case of shock inputs, the rapid initial displacement of the foundation is propagated through the spring as a traveling wave. When this wave reaches the isolated mass, the mass is accelerated. In order to reduce this acceleration of the isolated mass one must use the same techniques that were used to reduce the standing wave resonances in the vibration isolator. Therefore, it is expected that the best shock isolator will also be one of the best vibration isolators. Since good vibration isolators were already obtained, it was really only necessary to measure the SAR of these isolators and then use the data to choose the best shock and vibration isolator.

Since the SAR is dependent on the shape and rise time of the input displacement, the shock input should be typical of those which occur in the actual environment. For this investigation, a typical shock input was created

by a foot stomp on a carpeted, wooden floor of typical household construction. Repetitive shocks caused by normal walking were also used as the input.

Measurement of the SAR was very similar to that of transmissibility. The acceleration of the environment and the isolated mass was recorded with an oscillograph while the shock input was applied. The SAR is then obtained by dividing the peak acceleration of the isolated mass by the peak acceleration of the environment.

A schematic diagram of the test apparatus is shown in Figure 68. Photographs of the actual SAR test apparatus are shown in Figures 69 and 70. The isolators were mounted on the same T-shaped, concrete block that was used previously for the transmissibility tests. This block, due to its method of support and location near a wall, will have horizontal, vertical and angular motion components as a result of vertically applied floor shock, as illustrated in Figure 71. SAR measurements were made in the vertical (longitudinal) and horizontal (transverse) direction by changing the orientation of the accelerometers that were fastened to the T-shaped block and isolated mass as described previously for the transmissibility tests.

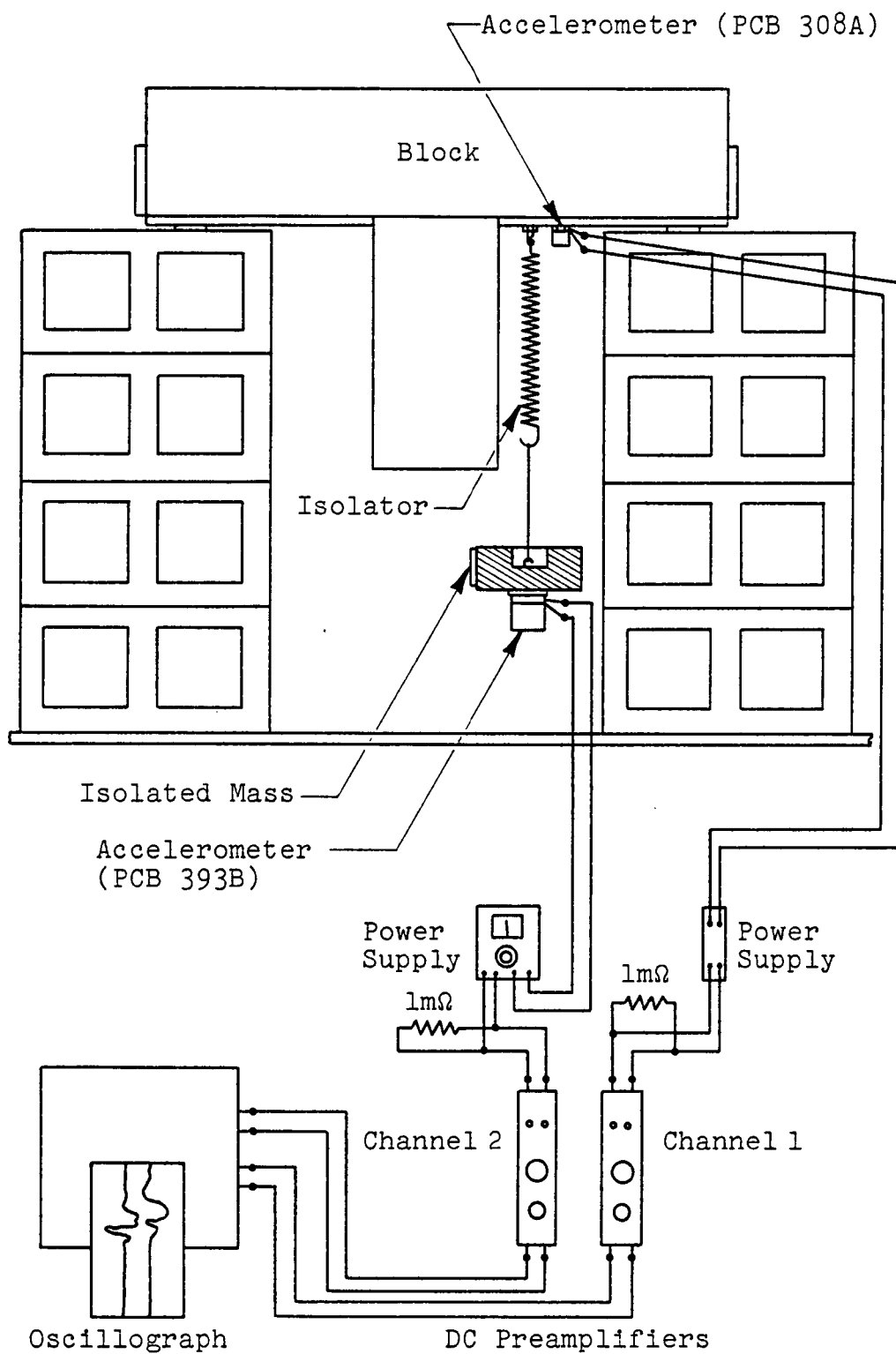


Figure 68. Schematic Diagram of SAR Test Apparatus.

Figure 69. The Shock Isolator, T-Shaped Block and Support Apparatus.

Figure 70. Shock-Recording Instrumentation.

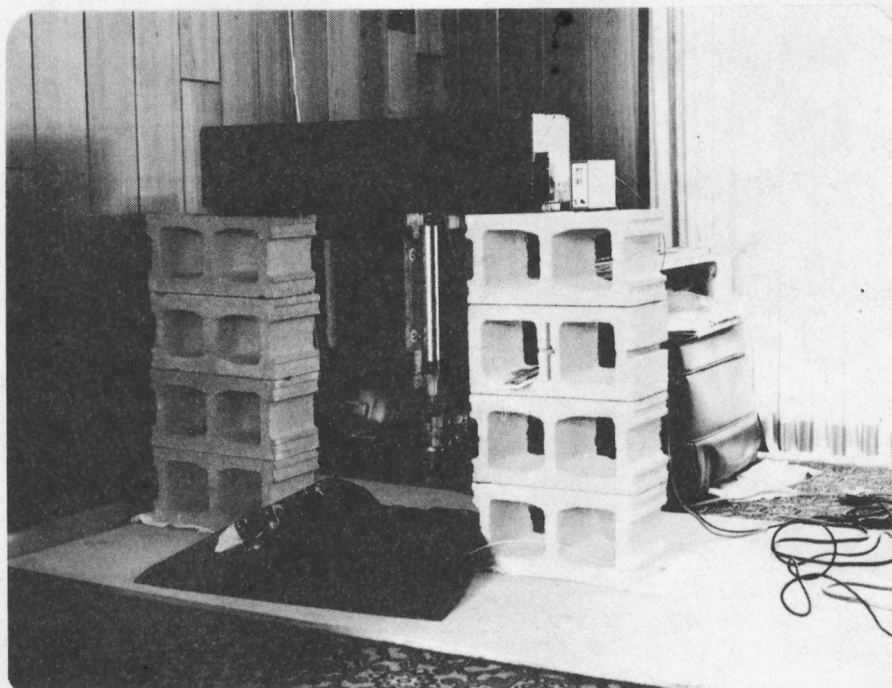


Figure 69.

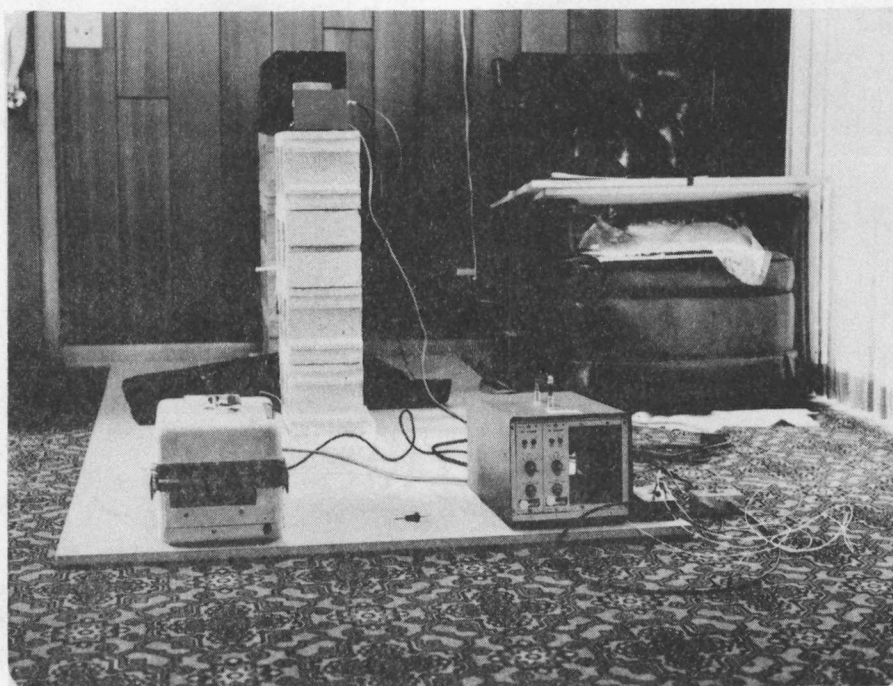


Figure 70.

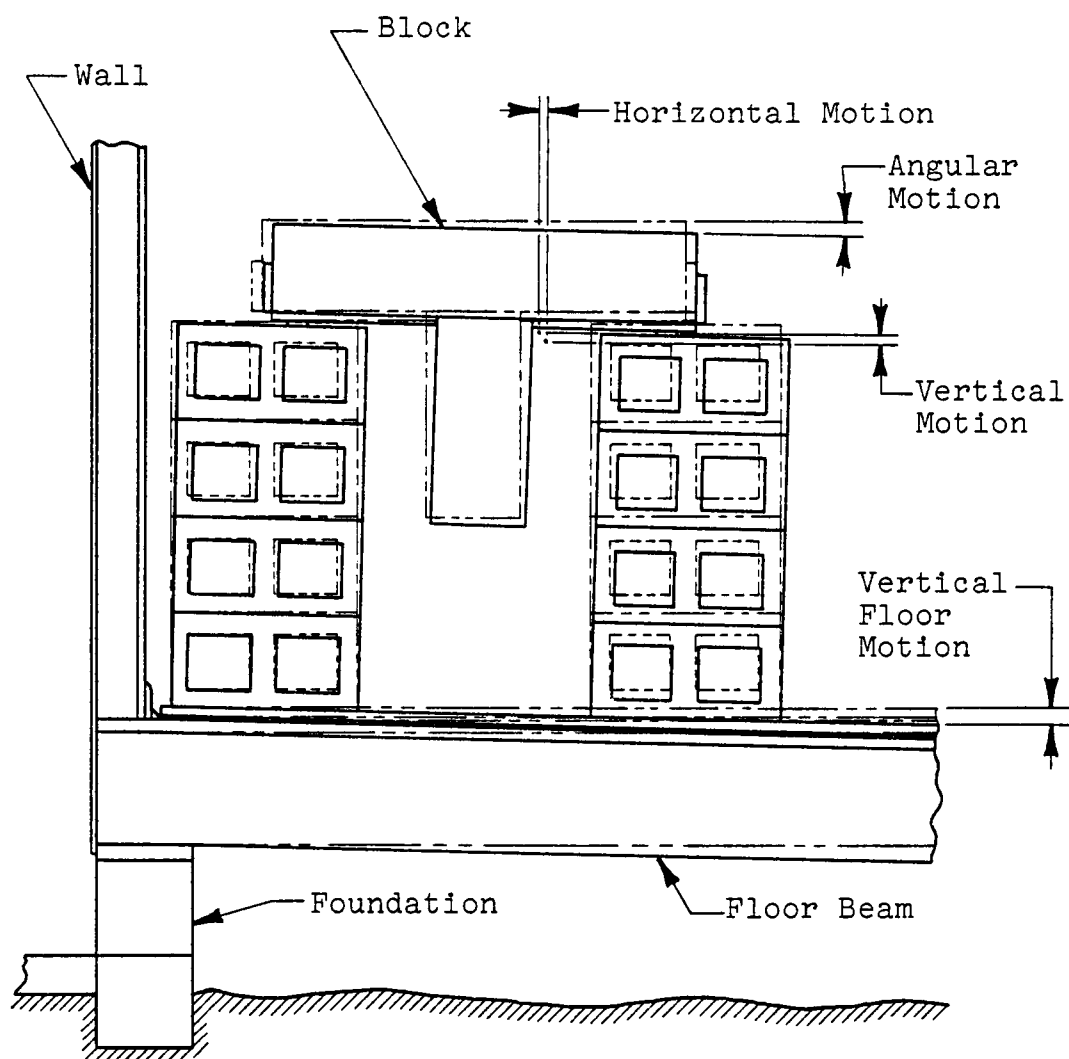


Figure 71. Motion of the Floor and SAR Test Apparatus, Shown Exaggerated for Clarity, as a Result of Vertically Applied Floor Shock.

The following instruments were used for the SAR measurements:

Accelerometers:	PCB 393B (Seismic-High Sensitivity) 1 v/g
	PCB 308B 10 mv/g
Power supplies:	PCB 480C06 (used with 393B)
	PCB 482A (used with 308B)
DC Pre-amplifiers:	Honeywell Accudata 120 (2 used)
Oscillograph:	Honeywell 906C Visicorder
	with M 1000 galvanometers

The wiring schematic for this equipment is given in Figure 68.

Before any shock tests could be made, it was necessary to obtain calibration factors for the oscillograph. This was accomplished by fastening both accelerometers to the vibrating block of the transmissibility test apparatus and driving it at 50 Hz while measuring the rms output voltage of both accelerometers with a digital voltmeter and recording the same voltages on the oscillograph. By comparing the voltages measured with the digital voltmeter and the oscillograph, the sensitivity of the oscillograph in divisions/volt was obtained. For Channel 1, which reads the output of the 308B accelerometer, the sensitivity was found to be 12.153 divisions/volt at unity gain. The sensitivity

of Channel 2, which reads the output of the 393B accelerometer, was found to be 12.184 divisions/volt at unity gain. The gain of the d-c pre-amplifiers used was adjustable in discrete steps from 10 to 1000 which must be taken into account when obtaining a voltage level. Acceleration was then obtained (approximately) by multiplying the measured voltage level by the specified voltage sensitivity of the accelerometers: .1 volt/g in the case of the 308B and 1 volt/g in the case of the 393B.

Four isolators, found previously to provide low transmissibility, were selected for shock testing in both the longitudinal and transverse directions. The first 2 isolators were the 6-inch extension spring with distributed viscous damping, as shown in Figure 53 on page 108, having damping factors of .184 and .010. The third isolator was the 6-inch extension spring with solid damping and the plastic flexure, having a damping factor, $\delta_{E\omega}$, of .098 at 5 Hz. This isolator is shown in Figure 52 on page 108. The fourth isolator was the 6-inch extension spring with solid damping and the laminated metal flexure having a damping factor, $\delta_{E\omega}$, of .291 at 5 Hz. This isolator is shown in Figure 51 on page 106. In addition, a fifth isolator, tested only in the longitudinal direction, was the 6-inch extension spring with no damping. This isolator was

tested only to observe the response of an undamped system to a shock input.

B. Measurement Results

Some sample oscillograph records of the shock tests are given in Figures 72-77. In all of these sample records, Channel 1 is the environmental acceleration while Channel 2 is the acceleration of the isolated mass. The ringing response of the undamped spring is clearly shown by the Channel 2 trace of Figure 72. The frequency of the ringing response is 37 Hz which is the frequency of the first standing wave resonance of the spring. A comparison of Figures 72 and 73 shows how a small amount of damping effectively eliminates this ringing. Figures 74 and 75 show the longitudinal and transverse response of a system to normal walking floor shock. These records clearly show how the repetitive shocks due to walking can excite the system's natural frequency. This excitation of the system natural frequency could result in a buildup of motion amplitude of the isolated mass especially when the time between footsteps closely matches the period of the natural frequency. Shown in Figure 76 is the transverse response of a system with distributed viscous damping to a shock with a short rise time, obtained from a foot stomp directly on the board beneath the T-shaped block apparatus. Figure 77 shows

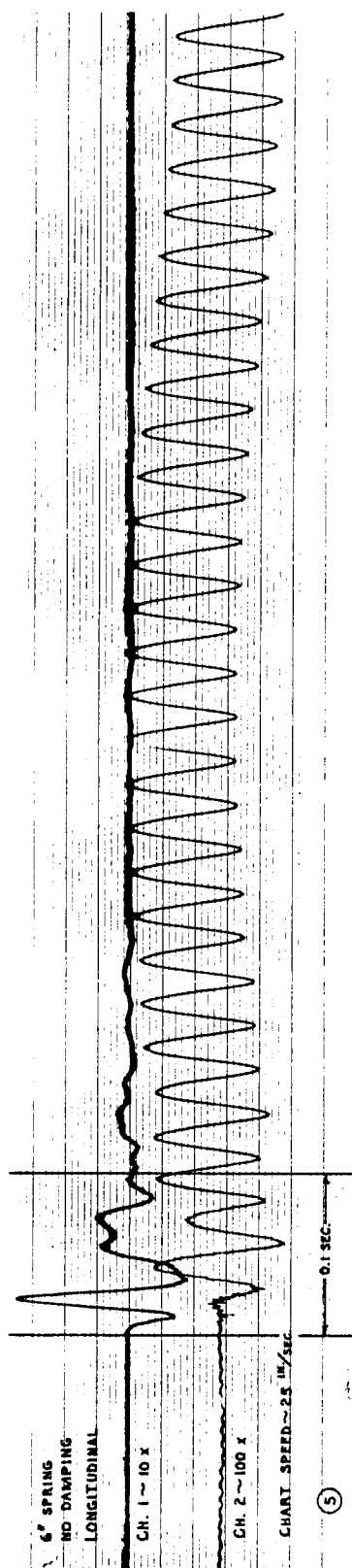


Figure 72. Oscilloscope Record of Shock Acceleration, in the Longitudinal Direction, Above and Below the 6-Inch Extension Spring Isolator without Damping.

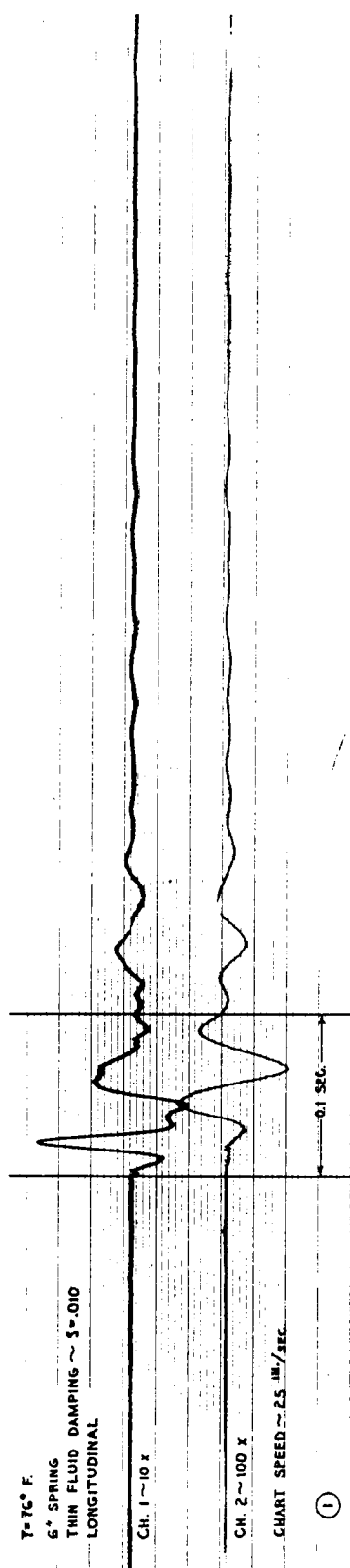


Figure 73. Oscilloscope Record of Shock Acceleration, in the Longitudinal Direction, Above and Below the 6-Inch Extension Spring Isolator with Distributed Viscous Damping.

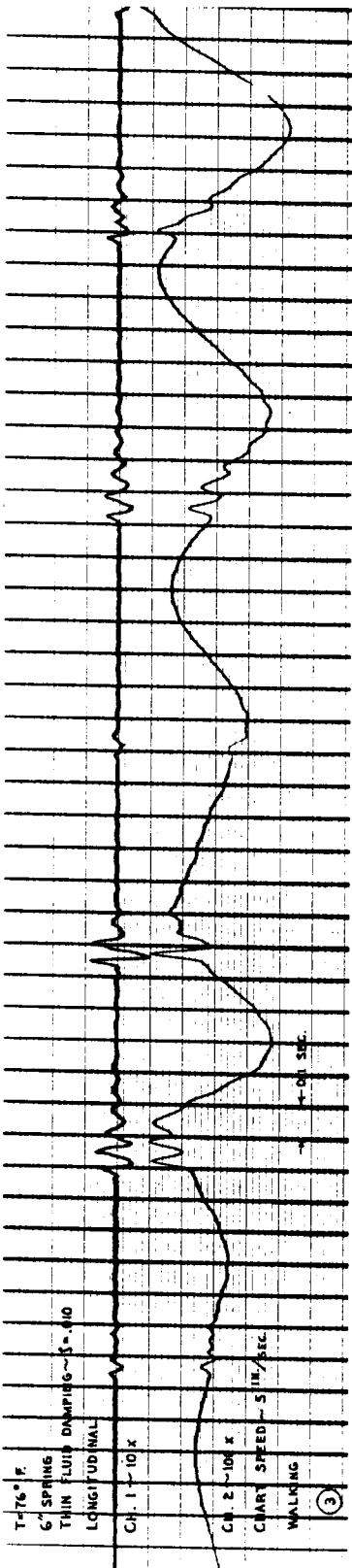


Figure 74. Oscillograph Record of Walking Shock Acceleration, in the Longitudinal Direction, Above and Below the 6-Inch Extension Spring Isolator with Distributed Viscous Damping.

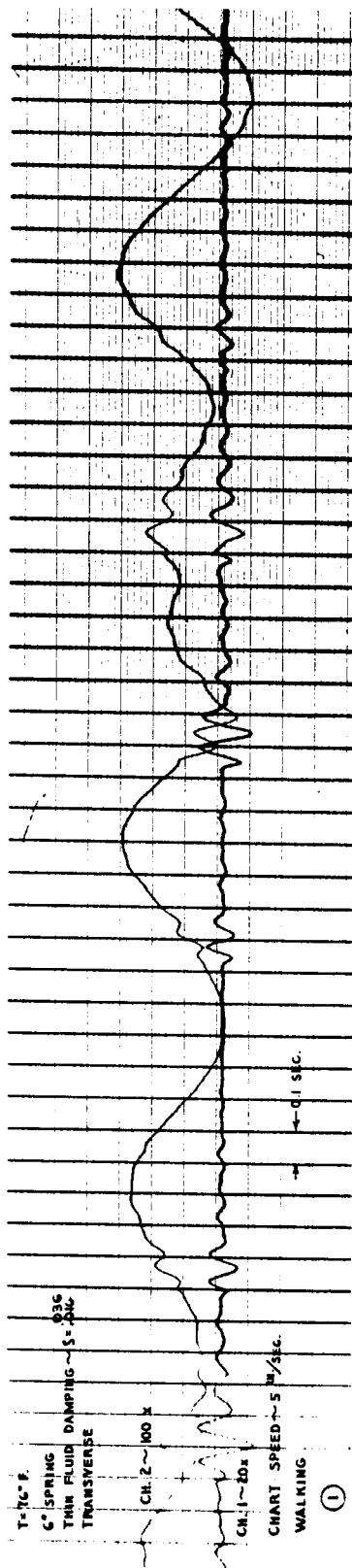


Figure 75. Oscillograph Record of Walking Shock Acceleration, in the Transverse Direction, Above and Below the 6-Inch Extension Spring Isolator with Distributed Viscous Damping. The Channel 2 trace is inverted relative to Channel 1.

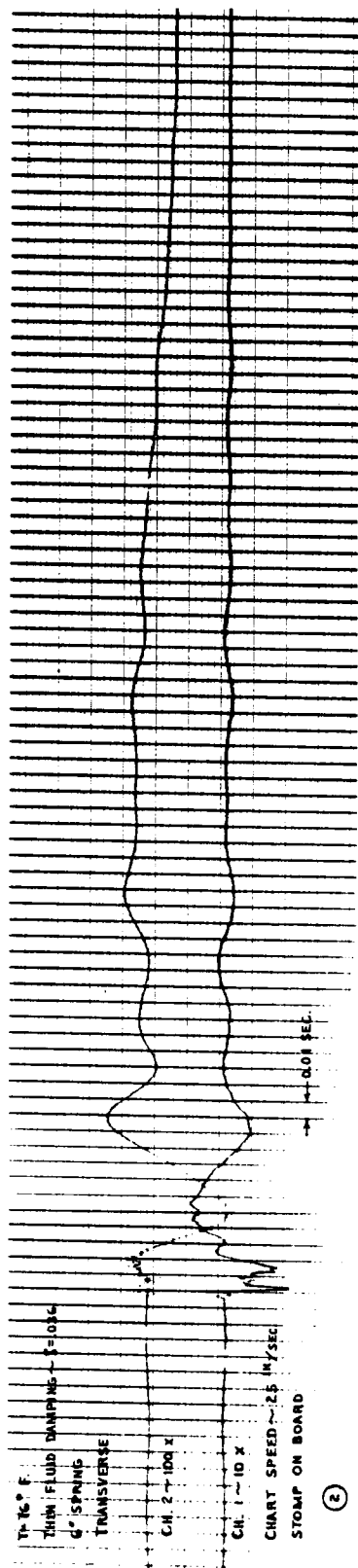


Figure 76. Oscillograph Record of Shock Acceleration, in the Transverse Direction, Above and Below the 6-Inch Extension Spring Isolator with Distributed Viscous Damping. The Channel 2 trace is inverted relative to Channel 1.

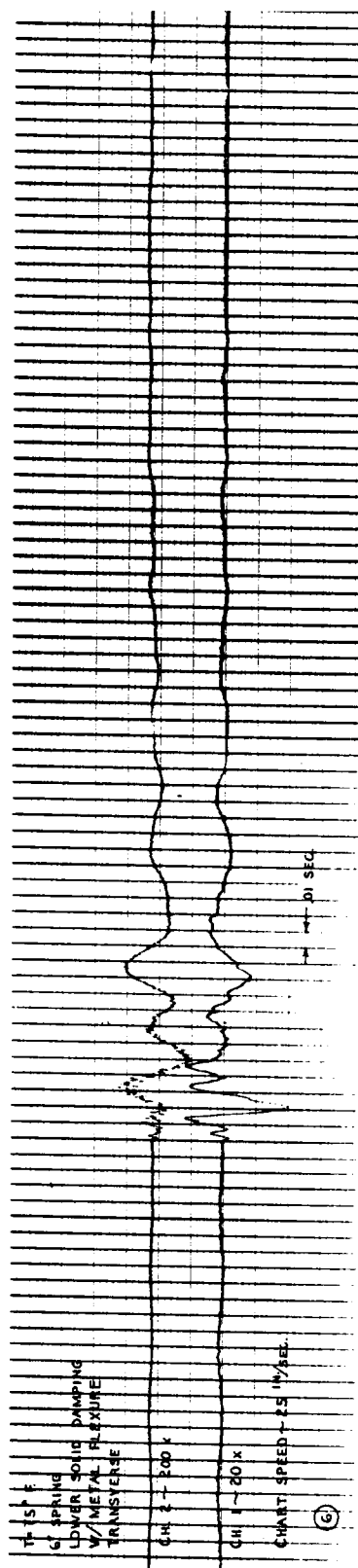


Figure 77. Oscillograph Record of Shock Acceleration, in the Transverse Direction, Above and Below the 6-Inch Extension Spring Isolator with Solid Damping and the Metal Flexure in Series with an .027 Diameter Steel Cable. The Channel 2 trace is inverted relative to Channel 1.

the transverse response of a system with the laminated metal flexure and a 10-inch length of .027 diam. steel cable to a foot stomp on the carpeted floor. Differences in the shape and rise time of the acceleration curves of the "hard" and "soft" shocks can be observed in Figures 76 and 77.

From the Channel 1 trace of Figure 72, the displacement versus time curve of a typical environmental shock was obtained by numerically integrating twice. The resulting curve is shown in Figure 78. It is seen that the shock shape is oscillatory and bears little resemblance to the rounded displacement step discussed in the preceding analysis. The oscillation frequencies of the first 2 cycles of this shock are 31 Hz and 20 Hz respectively. It might be expected then that isolators having low transmissibility in the vicinity of these frequencies should provide low SAR values with this shock.

The results of the SAR measurements in the longitudinal and transverse directions are given in Tables 18 and 19 respectively. The maximum measured peak acceleration of the environment was found to be .33 g for foot stomp shocks. This acceleration is above the .1 g criterion and could cause problems for a turntable with no isolation. The maximum measured peak acceleration of the isolated mass was found to be 3.529×10^{-3} g

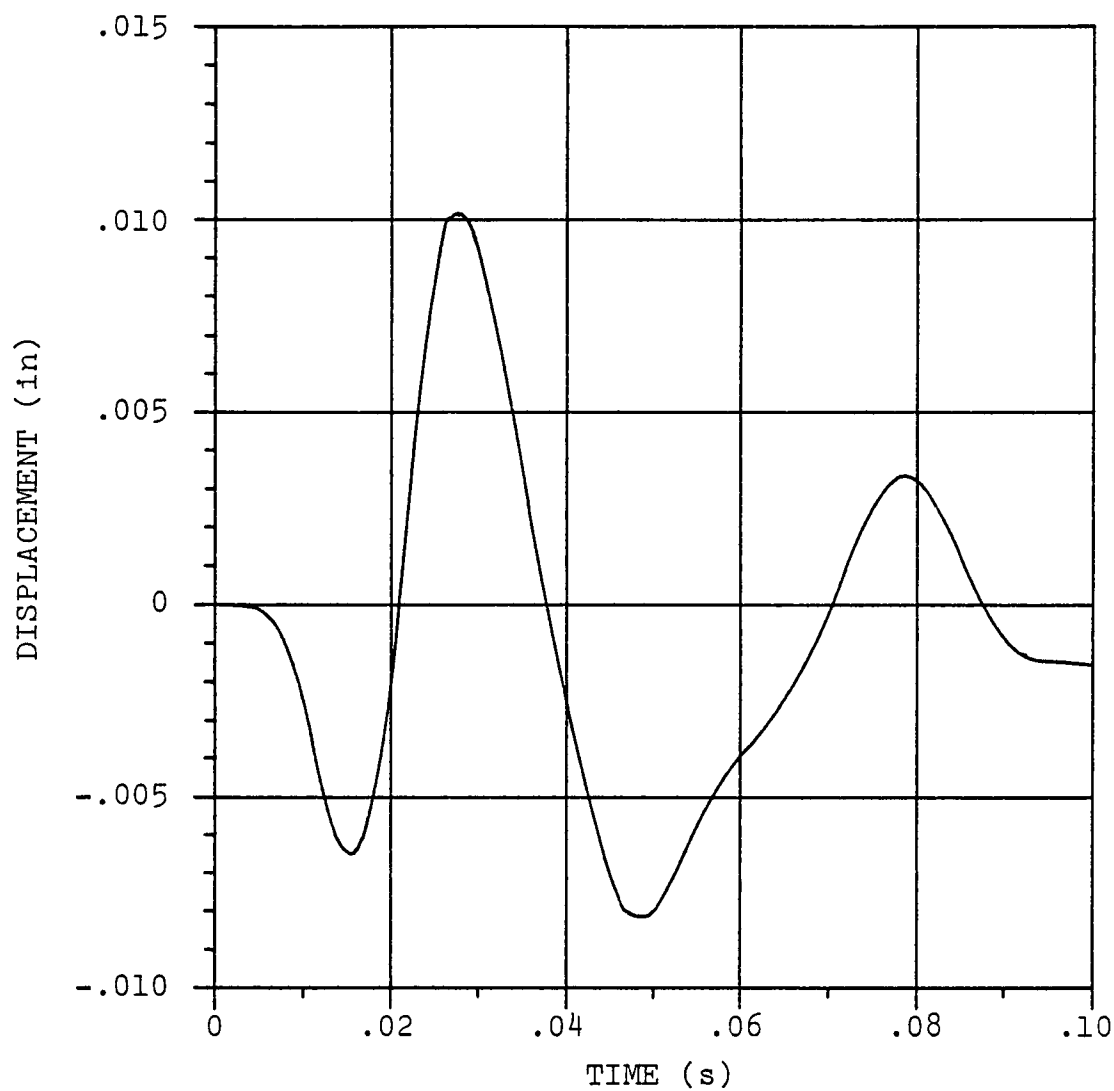


Figure 78. Displacement Versus Time Curve of the Channel 1 Trace of Figure 72. This shock, measured in the longitudinal direction, was created by a foot stomp on a carpeted, wooden floor.

TABLE 18

RESULTS OF SAR MEASUREMENTS IN THE LONGITUDINAL DIRECTION
FOR THE 6-INCH (FREE LENGTH) EXTENSION SPRING ISOLATOR

Damping	Trial No.	Shock Type	Peak		Peak Acceleration of Isolated Mass (g)	SAR (dB)	Average SAR* (dB)	Average Transmissibility 10-60 Hz (dB)
			Environmental Acceleration (g)	Shock Acceleration (g)				
Heavy Distributed Viscous $\zeta \approx .184$	1	Stomp	.1736		2.996×10^{-3}	-35.26		
	2	Walking	.100		3.152×10^{-3}	-30.03		
	3	Stomp	.0885		3.283×10^{-3}	-28.61		
	4	Stomp	.2715		3.529×10^{-3}	-37.72		
	5	Stomp	.222		3.447×10^{-3}	-36.18	-36.39	--
Thin Fluid Distributed Viscous $\zeta \approx .010$	1	Stomp	.2386		1.493×10^{-3}	-44.05		
	2	Walking	.0822		1.970×10^{-3}	-32.42		
	3	Walking	.0822		1.970×10^{-3}	-32.42		
	4	Stomp	.3374		2.257×10^{-3}	-43.49	-43.77	-46.09
Very Low Solid with Plastic Flexure $\delta E_{\omega} = .098$ @ 5 Hz	1	Stomp	.0576		5.334×10^{-4}	-40.67		
	2	Stomp	.0658		6.155×10^{-4}	-40.58		
	3	Stomp	.0921		8.617×10^{-4}	-40.57	-40.61	-43.47

TABLE 18 (Continued)

Damping	Trial No.	Shock Type	Peak Environmental Acceleration (g)	Peak Acceleration of Isolated Mass (g)	SAR (dB)	Average SAR* (dB)	Average Transmissibility 10-60 Hz (dB)
Lower Solid with Metal Flexure $\delta E_{\omega} \approx .291$ @ 5Hz	1	Stomp	.1727	1.969×10^{-3}	-38.86		
	2	Stomp	.2469	2.790×10^{-3}	-38.94		
	3	Stomp	.2798	3.037×10^{-3}	-39.29		
	4	Walking	.0699	1.806×10^{-3}	-31.76		
	5	Walking	.0494	8.617×10^{-4}	-35.16	-39.03	-36.79
Undamped	1	Stomp	.0494	7.386×10^{-4}	-36.51		
	2	Stomp	.2798	2.052×10^{-3}	-42.69		
	3	Stomp	.1049	7.797×10^{-4}	-42.58		
	4	Walk	.0675	7.797×10^{-4}	-38.75	-40.59	-43.06

*Includes only SAR values for stomp shocks.

TABLE 19

RESULTS OF SAR MEASUREMENTS IN THE TRANSVERSE DIRECTION
FOR THE 6-INCH (FREE LENGTH) EXTENSION SPRING ISOLATOR

Damping	Trial No.	Shock Type	Peak Acceleration		SAR (dB)	Average SAR* (dB)	Average Transmissibility 10-60 Hz (dB)
			Environmental Acceleration (g)	of Isolated Mass (g)			
Heavy Distributed Viscous $\zeta \approx .063$	1	Stomp	.0658	5.745×10^{-4}	-41.18		
	2	Walking	.0370	1.576×10^{-3}	-28.64		
	3	Stomp	.1317	2.134×10^{-3}	-35.81		
	4	Stomp	.0514	6.976×10^{-4}	-37.35		
	5	Stomp	.0720	7.181×10^{-4}	-40.02	-38.59	--
Thin Fluid Distributed Viscous $\zeta \approx .036$	1	Walking	.0411	1.970×10^{-3}	-26.39		
	2	Stomp†	.1563	1.970×10^{-3}	-37.99		
	3	Stomp	.0576	4.103×10^{-4}	-42.95	-40.47	-60.25
Very Low Solid with Plastic Flexure $\zeta = .038$	1	Walking	.0288	1.490×10^{-3}	-27.98		
	2	Stomp†	.1358	2.421×10^{-3}	-34.98		
	3	Walking	.0391	2.462×10^{-3}	-24.02		
	4	Stomp	.0494	7.386×10^{-4}	-36.51		
	5	Stomp	.0370	5.540×10^{-4}	-39.49	-36.99	-51.97

TABLE 19 (Continued)

Damping	Trial No.	Shock Type	Peak Environmental Acceleration (g)	Peak Acceleration of Isolated Mass (g)	SAR (dB)	Average SAR* (dB)	Average Transmissibility 10-60 Hz (dB)
Lower Solid with Metal Flexure $\zeta \approx 0$	1	Stomp	.0453	3.283×10^{-4}	-42.80		
	2	Stomp	.0617	5.745×10^{-4}	-40.62		
	3	Stomp	.0679	4.103×10^{-4}	-44.37		
	4	Stomp†	.1358	1.723×10^{-3}	-37.93		
	5	Walking	.0453	2.134×10^{-3}	-26.54		
	6	Stomp	.0823	4.514×10^{-4}	-45.22	-42.18	-58.33

*Includes only SAR values for stomp shocks.

†Stomp applied to board beneath isolator support apparatus.

which is well below the .1 g criterion. The shock isolation provided by all isolators tested should therefore be adequate for any reasonable environmental shock. Shocks generated by normal walking gave the highest SAR figures; however, the SAR is probably not meaningful in these cases. Since the input is repetitive, the SAR value obtained with walking shock depends on the motion history of the system. For this reason, the average SAR figures given in Tables 18 and 19 did not include the SAR values obtained with walking shock inputs. For measurements with a walking shock input the maximum measured peak acceleration of the isolated mass was only 3.152×10^{-3} g , which is well below the .1 g criterion. The performance ranking of the isolators based on the average SAR figures show good agreement with their ranking based on the average transmissibility between 10 and 60 Hz. This gives some support to the idea that good vibration isolation systems also make good shock isolation systems.

From the results presented in Tables 18 and 19, it is seen that the best shock isolator, based on the average SAR for the foot stomp shocks, is the 6-inch extension spring with distributed viscous damping having a damping factor of .010 in the longitudinal direction.

The best vibration isolator according to the previous transmissibility tests is the 6-inch extension spring with distributed viscous damping, having a damping factor of .041 in the longitudinal direction. Because the isolator with a damping factor of .010 was found to have generally higher transmissibility in the frequency range of interest, the isolator with a damping factor of .041 was considered to be the best shock and vibration isolator and was selected for use in the final design.

CHAPTER VIII

FINAL DESIGN

A. Design Details

So far, this thesis has been primarily concerned with choosing the best shock and vibration isolator to satisfy the first three design requirements listed in Chapter I. In order to satisfy all eight design requirements, it was necessary to consider some of the practical aspects and design details of the turntable isolation system. The final design working drawing of the complete turntable isolation system was then made. The main assemblies of this drawing, shown in Figure 79, will be referred to in the following discussion.

In addition to isolation performance, certain practical aspects of the selected isolator must be considered. Fortunately, the selected isolator with distributed viscous damping has not only a performance advantage but also has certain practical advantages over an isolator with solid damping. A problem with the high damping polymers generally used for solid damping is that their dynamic and loss moduli vary greatly with temperature. Such high damping polymers are also subject to undesirable age hardening. On the other hand, for an isolator with distributed viscous damping, only the fluid

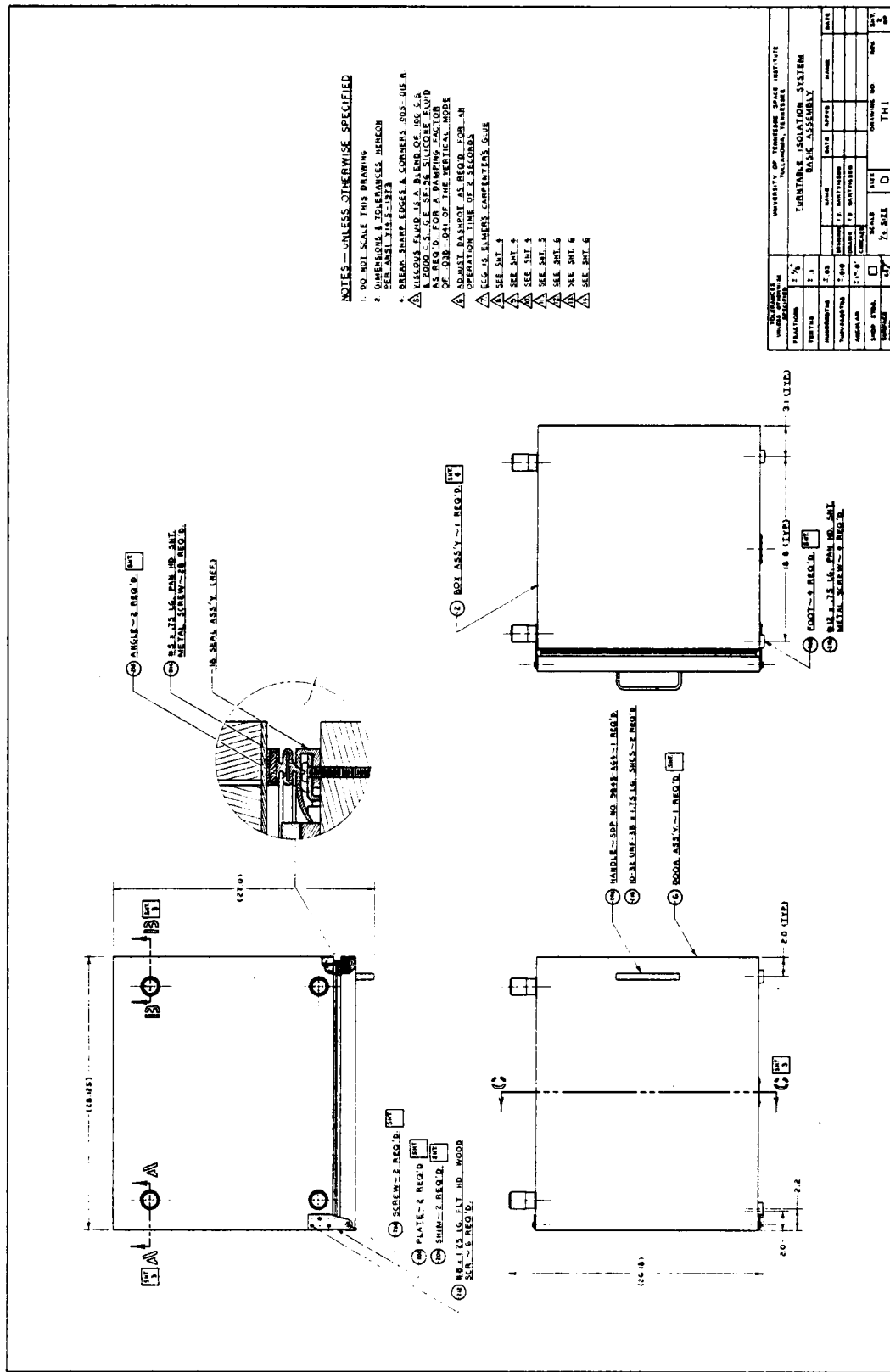
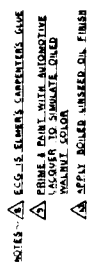
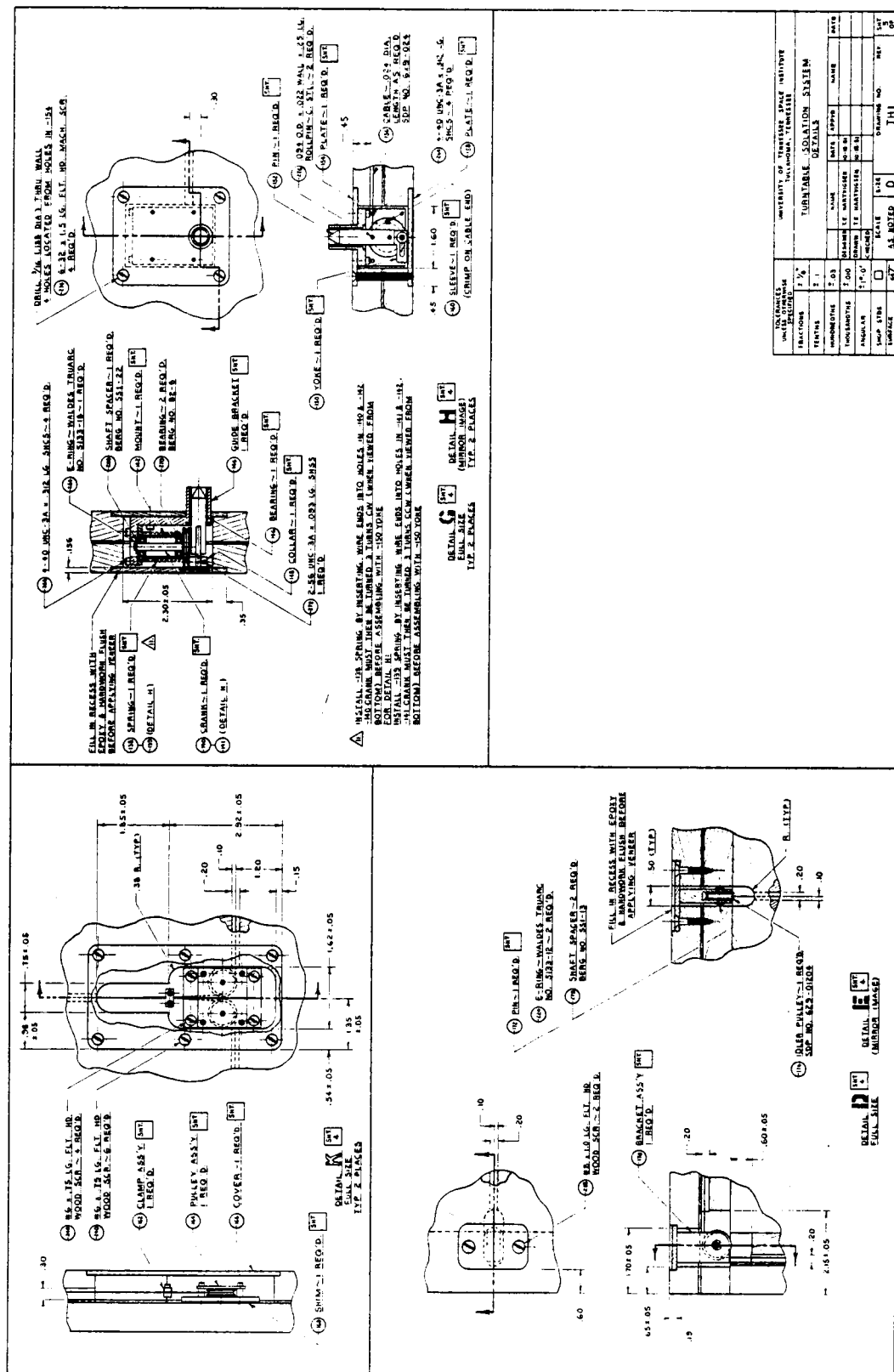
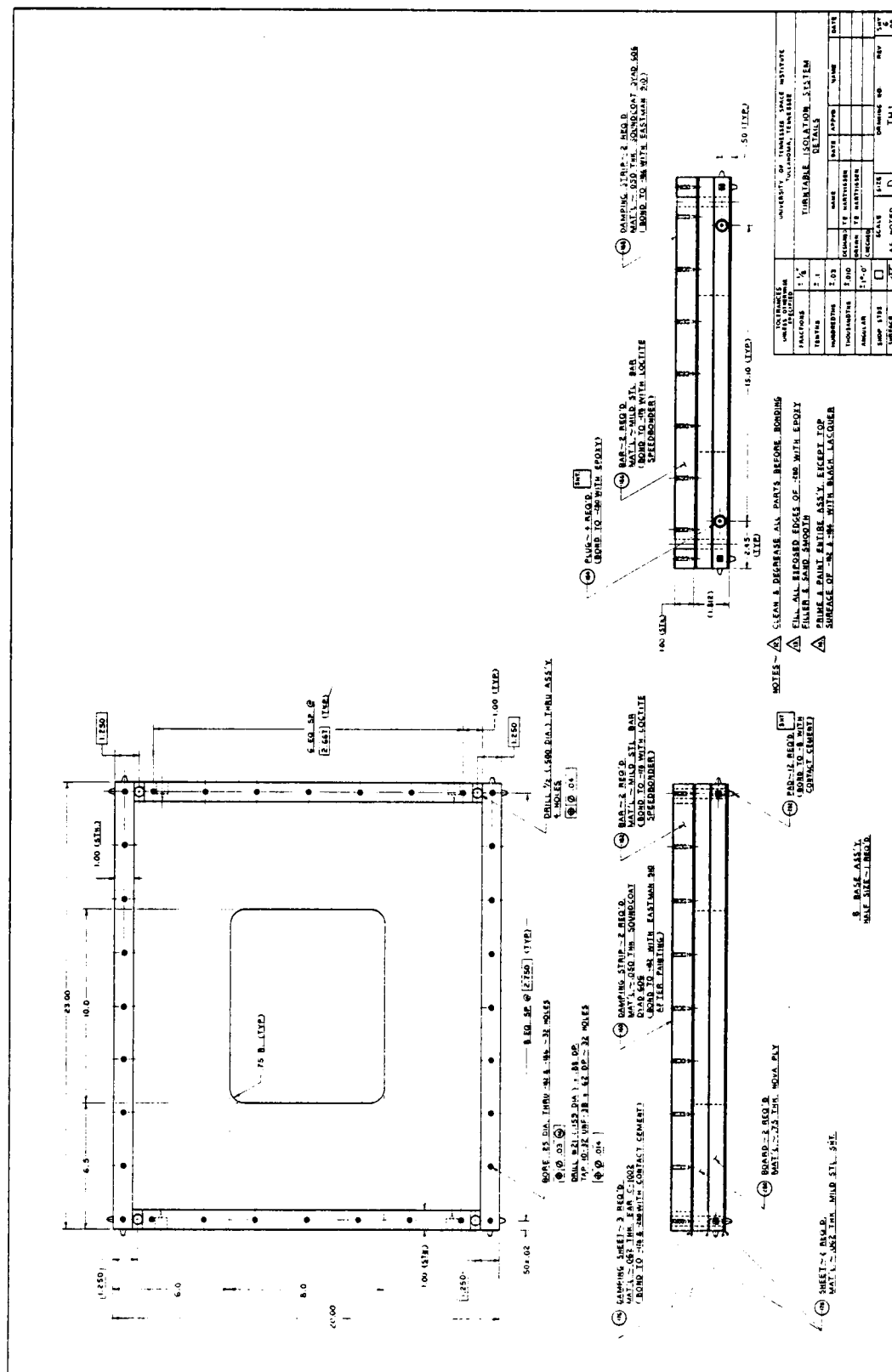


Figure 79. Main Assemblies of the Turntable Isolation System Working Drawing.









viscosity and damping change with temperature and this change is much smaller than that which occurs with solid damping, especially if a silicone based viscous fluid is used. The viscous fluid called for in Figure 79, Sections A-A and B-B, is a blend of thick and thin silicone based fluids. Other advantages with silicone based fluids are that they are unaffected by age and present no compatibility problems with most seal materials. One problem with the selected isolator was the design of the flexible fluid seal below the fluid reservoir. This seal must be very flexible and yet strong enough to withstand the static head of the viscous fluid indefinitely. The -42 bellows shown in Figure 79, Sections A-A and B-B, is a thin wall, urethane bellows which should satisfy the seal requirements.

A possible problem for a coil spring isolator with a large static deflection, such as the one selected, is deflection due to creep. The 6-inch extension spring in this design is maintained at an uncorrected shear stress of 77,063 psi. Estimating from the creep curves given by Wahl [8], a 1.08 percent creep deflection will occur from a 1-month exposure to this stress level and a 2.03 percent deflection will occur at 9 months. Beyond nine months, very little change in the creep deflection percentage is observed. Since the static deflection of

the spring is 7.5 inches, the creep deflection at 1 month is approximately .081 inch and at 9 months is .152 inch. Although the total creep deflection is large at nine months, it should be acceptable since it increases so slowly and can be compensated for by an occasional level adjustment.

A problem with isolators having low damping factors is that their natural frequency can be excited by walking shock. During the SAR tests with walking shock, the maximum peak-to-peak displacements obtained were on the order of .030; however, if the time between shocks were to closely match the period of the system's natural frequency, a large amplitude buildup could occur. Assuming sinusoidal motion, the peak displacement of the isolated mass must exceed .9 inch when vibrating at a natural frequency of 1.141 Hz in order to exceed the .1 g acceleration criterion. In this case, the displacement and consequently, acceleration, can be effectively limited by using soft, highly damped limit pads which would allow free motion of the isolated mass up to .4 inch peak-to-peak. The -282 pads shown in the -8 base assembly of Figure 79 were designed to provide the necessary soft limiting of motion of the isolated mass in all 6 degrees of freedom.

In order to allow use with virtually any existing turntable, the isolation system was designed assuming maximum turntable dimensions of 18 by 21 inches and a maximum turntable weight of 40 pounds. The assumed maximum turntable dimensions were based on specifications given in [16]. These assumed dimensions are large enough to accommodate all but 6 of the 222 turntables listed. The remaining six turntables are quite large and would require a special design.

One of the assumptions made in the design of this turntable isolation system is that the turntable and base plate behave as a single, rigid mass. If a turntable with its own built-in isolation system were used with this isolation system, the single mass assumption would be invalid and the overall transmissibility resulting from such a system could be higher, especially at 10 Hz. For this reason, the built-in isolation system on a turntable, if it has one, should somehow be disabled when used with this isolation system.

The -8 base assembly, shown in Figure 79, was designed to provide a very rigid, well damped support base for the turntable. The bottom plate of the -8 base assembly is of laminated construction using a lightweight core of Nova-Ply (cabinet grade particle board) with steel sheet facings for maximum rigidity with minimum

weight. In addition, 3 layers of EAR Isodamp C-1002 damping material are bonded between the steel and Nova-Ply laminations to provide constrained layer damping. Constrained layer damping is shown by Ungar [17] to provide high panel loss factors for a given amount of damping material. In order to keep the rotational natural frequencies as low as possible, the -8 base assembly was designed to have a high mass moment of inertia about the X, Y, and Z axes. An 8 by 10-inch hole was cut from the center of the bottom plate to keep the allowable mass away from the center of gravity. The 1-inch square steel bar was bonded around the outside edge of the -8 base assembly to increase both its rigidity and mass moment of inertia.

Some means of adjusting the mass of the turntable support base is necessary in order to keep the total isolated mass at the 120.8 lb. design point with turntables of various weights. The proper mass adjustment is obtained by fastening removable steel bars and lead weights as required to the base assembly as shown in Section C-C of Figure 79. When fastened to the base assembly, the bars and weights also increase its rigidity and mass moments of inertia. Since the removable weights are mounted above the -8 base assembly they also tend to raise the center of gravity of the assembly. This was shown to be beneficial by the eigenvalue analyses

of Chapter III. Although the mass adjustment method is somewhat inconvenient, it only needs to be made once for each turntable.

Since the turntable must remain in a level attitude at static equilibrium, some means of leveling the turntable is required. Turntable leveling is accomplished by raising or lowering the height of each isolator, as required, by turning the -190 screw shown in Sections A-A and B-B of Figure 79. After the level is adjusted, the -190 screw is locked by tightening the -192 jam nut against the -34 plug.

A problem with turntable isolation systems having low natural frequencies is that they tend to be very bouncy which makes record changing or other manual operations on the turntable difficult. To overcome this problem, a lock-down mechanism was designed to hold the turntable steady when required. The main parts of this mechanism are shown in Figure 79: Section C-C and Details G and H of the -2 box assembly. The locking action is provided by the -152 pin shown in Details G and H which, when actuated, engage a conical hole in the 4 -184 plugs of the -8 base assembly. All 4 actuators which move the -152 pin in and out are operated simultaneously by the -12 actuator assembly, shown in Section C-C, which is in turn operated manually through

the -14 lever assembly. With the -14 lever assembly in the up position, as shown, the -8 base assembly is locked down. By moving the -14 lever assembly down, the -8 base assembly is released and energy for the next locking operation is stored by the -78 spring powered reel. In order to avoid jarring of the turntable, the lock-down mechanism must operate very slowly and smoothly. The motion of this mechanism is damped by the -54 dashpot, shown in Section C-C, which is normally adjusted for an operation time of 2 seconds.

The cabinet housing the turntable isolation system was designed primarily to provide a rigid, well damped support structure for the isolators. All walls of this cabinet consist of 2 layers of .75-inch thick Nova-Ply (cabinet grade particle board) with a layer of EAR Isodamp C-1002 bonded in between to provide constrained layer damping. The cabinet can also be used to obtain beneficial acoustic isolation with relatively little additional effort. In order to limit the transmission of low frequency sound, it is necessary to have an airtight seal around the door. The hole through which the turntable line cord and audio cables exit the enclosure is also sealed airtight. In addition, the entire cabinet interior is lined with .5 inch thick Soundmat LF, a foam-lead-foam sound barrier manufactured by the Soundcoat Company.

B. Performance Estimate

Now that the turntable isolation system is designed, it is desirable to obtain an estimate of the transmissibility performance of this system in all six degrees of freedom. This estimate was obtained by first performing an eigenvalue analysis of the complete system to obtain its natural frequencies and corresponding mode shapes. The input data and calculated natural frequencies and mode shapes for this system are given in Table 24 in Appendix E. The value of the input parameter, LS , was set at 11.5 inches in this case. This 11.5-inch length is actually the effective pendulum length for the selected isolator based on the transverse transmissibility measurements. The transmissibility of the system in each degree of freedom may then be obtained by calculating the transmissibility of one of the isolators with an equivalent mass load such that the natural frequency of this system matches the natural frequency obtained from the eigenvalue analysis. This could be done with isolators for which both the longitudinal and transverse transmissibility equations were obtained, but for the selected isolator, only the longitudinal equation was obtained. By plotting the calculated longitudinal transmissibility of a rod with 3 different mass loads, as shown in Figure 80, it can be seen that the 3 curves are

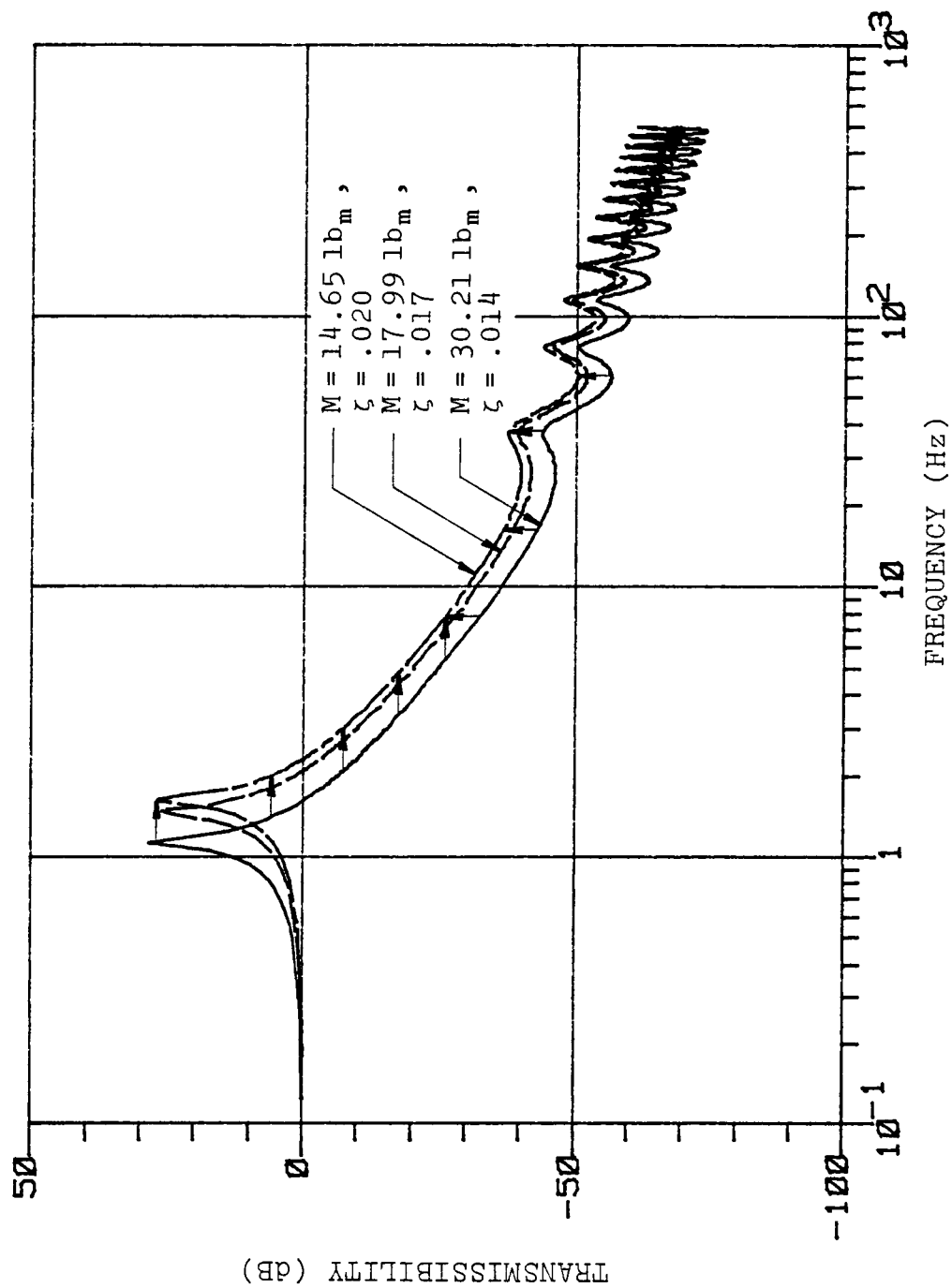


Figure 80. Calculated Longitudinal Transmissibility of a Rod with Distributed Viscous Damping for 3 Mass Loads ($M = 30.21$, 17.99 , and 14.65 lb_m). Damping factor $\zeta = .014$, $.017$, $.020$.

very similar in shape, but are displaced horizontally at low frequencies (below 7 Hz) and vertically at the higher frequencies. This suggests that a reasonably good approximation of the transmissibility curve for different equivalent masses may be obtained graphically from a single known curve by simply shifting the low frequency portion of the curve horizontally to locate the peak on the new natural frequency and then shifting the high frequency portion of the curve vertically to connect with the low frequency portion. The approximate transmissibility curves given in Figures 81 and 82 were obtained in this manner from the measured longitudinal or transverse transmissibility curve of the selected isolator. For this system, the rotational degrees of freedom are seen to have the highest transmissibility. This shows the importance of reducing the rotational natural frequencies of the system. The highest transmissibility at 10 Hz is seen to be about -27 dB in rotation about the Z-axis which is considered to be an acceptable isolation figure at 10 Hz.

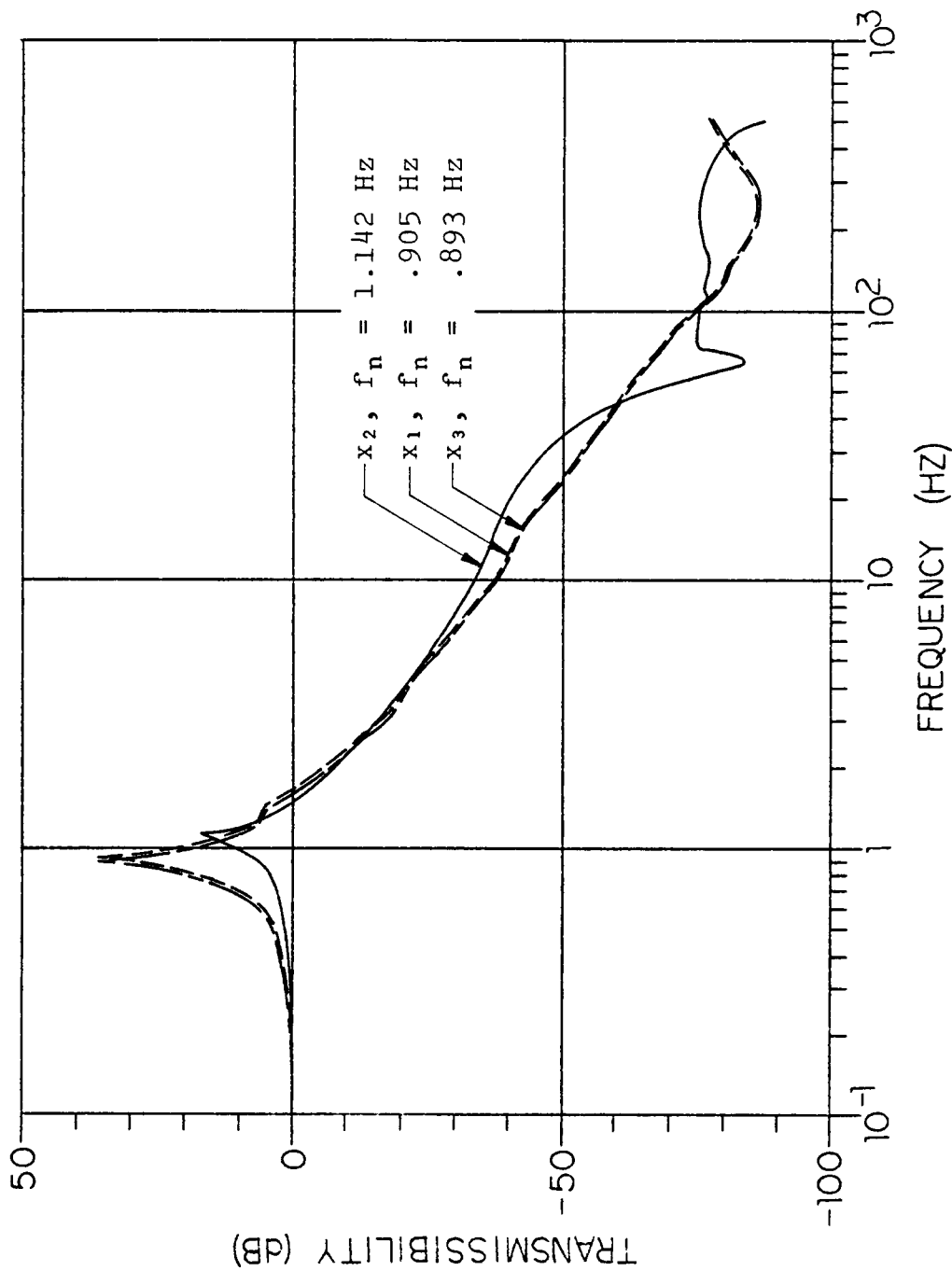


Figure 81. Approximate Transmissibility of the Complete Isolation System in the Translational Degrees of Freedom (x_1 , x_2 , and x_3).

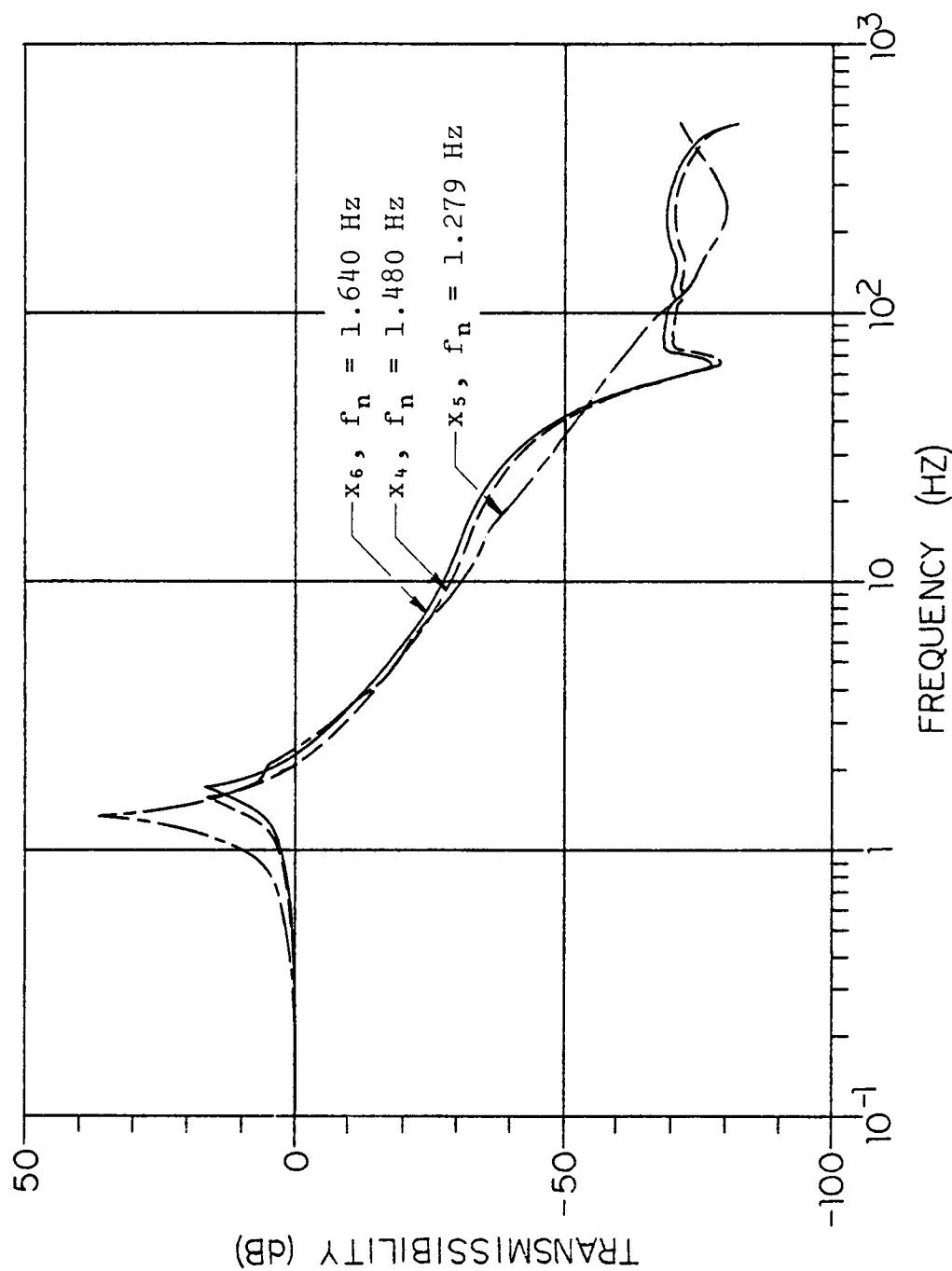


Figure 82. Approximate Transmissibility of the Complete Isolation System in the Rotational Degrees of Freedom (x_4 , x_5 , and x_6).

CHAPTER IX

CONCLUDING REMARKS

Problems have been recognized for turntable operation in the presence of high sound pressure levels and floor shock disturbances in a listening room with flexible structure. These problems are due to the inadequate isolation of external shock and vibrational disturbances provided by existing turntables and auxiliary isolation devices. The objective of this thesis was to develop an optimum isolation system design, based on analysis and measurements, to satisfy the eight design requirements listed in Chapter I. Such a design has been developed and presented. Analytical and experimental results supporting this design have also been presented.

Some of the major findings of this investigation are the following:

1. In order to reduce the transmissibility of an isolator at its standing wave resonant frequencies, damping is required. An impedance change within an isolator that does not involve damping will not reduce the standing wave transmissibility.

2. Internal damping was found to provide very little reduction in the transmissibility of the first

standing wave resonance of a mass-loaded beam in transverse vibration.

3. The optimum damping factor for minimum overall transmissibility was found to be quite low for most isolators. As a point of reference, the optimum damping factor was found to be .041, in longitudinal vibration, for a 6-inch extension spring isolator with distributed viscous damping.

4. Distributed viscous damping was found to provide the best control of standing wave resonances in both longitudinal and transverse vibration.

5. Isolators with low transmissibility were also found to provide good shock isolation.

6. Transmissibility was found to be the most useful measure of isolator performance.

7. Improvements in the performance of the isolation system can be realized by increasing the length of the extension springs and thereby reducing the natural frequencies. The optimum design in this case was found to be limited by practical height restrictions.

8. At the frequencies of greatest interest for turntable isolation (10 to 100 Hz), the analytical transmissibility equations, derived on the basis of one-dimensional rod assumptions and Bernoulli-Euler beam assumptions, appear to be sufficiently accurate

for design purposes. These equations may be used with some confidence to predict the transmissibility of new isolator designs.

9. Parallel viscous damping was found to provide no reduction of the standing wave transmissibility peaks below 0 dB. This type of damping is of no use for turntable isolation.

The following items are suggested for further study:

1. Obtain an equation for the transverse transmissibility of an extension spring isolator with distributed viscous damping. It may be possible to obtain a transverse transmissibility equation of sufficient accuracy by considering the spring to be a string rather than a beam. This would greatly simplify the derivation of this equation.

2. Consider a two-stage isolation system. The existing isolation system on some turntables might be utilized with the isolation system designed in this thesis to provide a second stage of isolation.

3. Consider the effect of resonances in the base plate supporting the turntable on the transmissibility of the complete isolation system. The base plate was assumed to be rigid in this investigation.

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APPENDIXES

APPENDIX A

DERIVATION OF TRANSMISSIBILITY EQUATIONS
FOR RODS IN LONGITUDINAL VIBRATION

In this appendix the standing wave transmissibility equations for rods in longitudinal vibration are derived. The rods are assumed to be distributed-parameter systems that are driven by sinusoidal motion on one end and loaded by a point mass on the other end. The lateral dimensions of the rods are assumed to be constant and small relative to the wavelength so that radial motion (Poisson's ratio effect) may be neglected. This assumption is valid for relatively thin rods vibrating at low frequencies. The rods are also assumed to be made of a material having a complex Young's modulus so that internal damping may be taken into account. The complex Young's modulus, E_{ω}^* , is defined by the relation

$$E_{\omega}^* = E_{\omega}(1 + j \delta_{E\omega}) \quad , \quad (A-1)$$

where $\delta_{E\omega}$ is the damping factor associated with the dynamic Young's modulus E_{ω} at any frequency ω . The * is used throughout this thesis to denote a complex quantity.

1. Single Segment Rod

The basic wave equation for the longitudinal vibration of a rod is

$$\frac{\partial^2 \tilde{\xi}}{\partial x^2} + (n^*)^2 \tilde{\xi} = 0 \quad , \quad (\text{A-2})$$

where $\tilde{\xi}$ = the sinusoidally varying particle displacement

$$\text{and } n^* = \left(\frac{\omega^2 \rho}{E_\omega^*} \right)^{1/2} . \quad (\text{A-3})$$

The $\sim$ is used throughout this thesis to denote a sinusoidally varying quantity. The assumed solution to Eq. (A-2) is

$$\tilde{\xi} = e^{j\omega t} (C^* \sin n^* x + D^* \cos n^* x) \quad , \quad (\text{A-4})$$

where C^* and D^* are constants to be evaluated from the boundary conditions. The partial derivatives of Eq. (A-4) are

$$\frac{\partial \tilde{\xi}}{\partial x} = e^{j\omega t} (-C^* n^* \cos n^* x + D^* n^* \sin n^* x) \quad , \quad (\text{A-5})$$

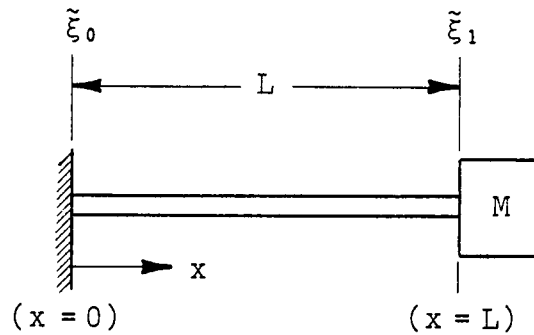
$$\frac{\partial^2 \tilde{\xi}}{\partial x^2} = e^{j\omega t} (-C^* (n^*)^2 \sin n^* x - D^* (n^*)^2 \sin n^* x) \quad , \quad (\text{A-6})$$

$$\frac{\partial \tilde{\xi}}{\partial t} = j\omega e^{j\omega t} (C^* \sin n^* x + D^* \cos n^* x) \quad , \quad (\text{A-7})$$

$$\frac{\partial^2 \tilde{\xi}}{\partial t^2} = -\omega^2 e^{j\omega t} (C^* \sin n^* x + D^* \cos n^* x) \quad . \quad (A-8)$$

The boundary conditions for the rod shown in the figure below are

1. $(\tilde{\xi})_{x=0} = \tilde{\xi}_0$ (Motion of rod end) ,
2. $AE^* \left(\frac{\partial \tilde{\xi}}{\partial x} \right)_{x=L} = M \left(\frac{\partial^2 \tilde{\xi}}{\partial t^2} \right)_{x=L}$ (Force = MA) .



After substitution of Eqs. (A-4), (A-5) and (A-8), the boundary conditions are

1. $D^* = \xi_0^*$,
2. $AE^* (-C^* n^* \cos n^* L + D^* n^* \sin n^* L)$
 $= -M\omega^2 (C^* \sin n^* L + D^* \cos n^* L)$.

Since

$$\omega^2 = \frac{(n^*)^2 E^*}{\rho} \quad , \quad (A-9)$$

$$\rho = \frac{M_R}{AL} \quad , \quad (A-10)$$

$$\gamma = \frac{M}{M_R} \quad , \quad (A-11)$$

where M_R = mass of the rod; the second boundary condition may be rewritten as

$$\begin{aligned} 2. \quad & (-C^* \cos n^*L + D^* \sin n^*L) \\ & = -\gamma n^*L(C^* \sin n^*L + D^* \cos n^*L) \quad . \end{aligned}$$

Using the boundary conditions, C^* can be expressed as

$$C^* = \xi_0^* \frac{(\sin n^*L + \gamma n^*L \cos n^*L)}{(\cos n^*L - \gamma n^*L \sin n^*L)} \quad . \quad (A-12)$$

If η^* and θ^* are defined as

$$\eta^* = \cos n^*L - \gamma n^*L \sin n^*L \quad , \quad (A-13)$$

$$\theta^* = \sin n^*L + \gamma n^*L \cos n^*L \quad , \quad (A-14)$$

then the solution at $x = L$ is

$$\tilde{\xi}_1 = e^{j\omega t} \xi_0^* \left[\left(\frac{\theta^*}{\eta^*} \right) \sin n^*L + \cos n^*L \right] \quad . \quad (A-15)$$

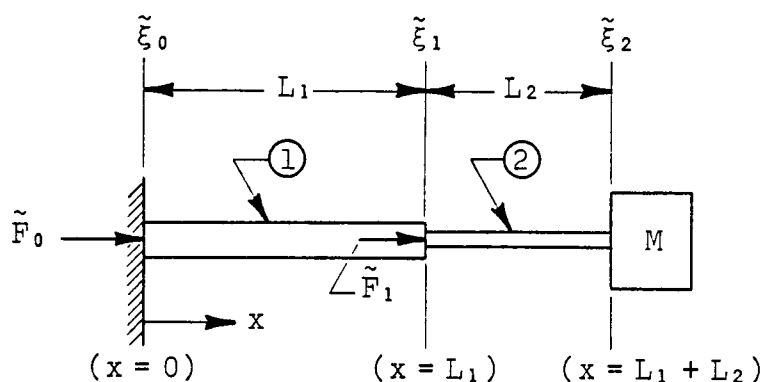
The transmissibility, T_m , of a mass loaded rod in longitudinal vibration reduces to

$$T_m = \left| \frac{\tilde{\xi}_1}{\tilde{\xi}_0} \right| = \left| \frac{1}{\eta^*} \right| \quad . \quad (A-16)$$

Eq. (A-16) is given by Snowden [4] and is in agreement with the transmissibility expressions for undamped rods found in [5] and [18] - [20].

2. Two-Segment Rod

The preceding results may now be used to obtain the transmissibility of the two-segment rod shown below.



The transmissibility of rod ② is given by Eq. (A-16). The transmissibility of rod ① must be found in order to obtain the overall transmissibility of the two-segment rod system. The boundary conditions for rod ① are

$$1. \quad (\tilde{\xi})_{x=0} = \tilde{\xi}_0 \quad (\text{Motion of rod end}) \quad ,$$

$$2. \quad A_1 E_1 * \left(\frac{\partial \tilde{\xi}_1}{\partial x} \right)_{x=L_1} = j\omega Z_m \left(\tilde{\xi}_1 \right)_{x=L_1}$$

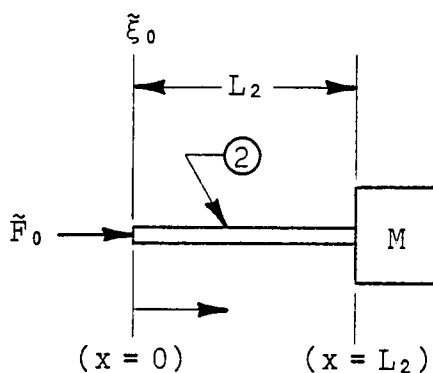
(Force = Driving point impedance $\times$ velocity) ,

where the driving point impedance, Z_m , of rod ② is defined as

$$Z_m = \frac{\tilde{F}_1}{\left(\frac{\partial \tilde{\xi}_1}{\partial t} \right)_{x=L_1}} . \quad (A-17)$$

Z_m must now be evaluated in terms of rod ② properties.

This task may be simplified by shifting the origin as shown in the figure below.



The expression for Z_m in the rod ② coordinate system is

$$Z_m = \frac{\tilde{F}_0}{j\omega \tilde{\xi}_0} . \quad (A-18)$$

$$\begin{aligned}
 \text{Since } \tilde{F}_0 &= A_2 E_2^* \left(\frac{\partial \tilde{\xi}}{\partial x} \right)_{x=0} \\
 &= A_2 E_2^* n_2^* e^{j\omega t} (-C^*) \quad (A-19)
 \end{aligned}$$

$$\text{and } \tilde{\xi}_0 = e^{j\omega t} D^* \quad , \quad (A-20)$$

the expression for the driving point impedance of rod 2 reduces to

$$Z_m = j \omega M_{R2} \left(\frac{1}{n_2^* L_2} \right) \left(\frac{\theta_2^*}{\eta_2^*} \right) \quad . \quad (A-21)$$

The boundary conditions for rod 1, after substitution of Eqs. (A-4), (A-5) and (A-21), are

$$\begin{aligned}
 1. \quad \tilde{\xi}_0^* &= D^* \quad , \\
 2. \quad A_1 E_1^* n_1^* (-C^* \cos n_1^* L_1 + D^* \sin n_1^* L_1) \\
 &= -\omega^2 M_{R2} \left(\frac{1}{n_2^* L_2} \right) \left(\frac{\theta_2^*}{\eta_2^*} \right) (C^* \sin n_1^* L_1 + D^* \cos n_1^* L_1) \quad .
 \end{aligned}$$

After substitution of Eqs. (A-9) and (A-10) the second boundary condition is

$$\begin{aligned}
 2. \quad A_1 E_1^* n_1^* (-C^* \cos n_1^* L_1 + D^* \sin n_1^* L_1) \\
 = -n_2^* E_2^* A_2 \left(\frac{\theta_2^*}{\eta_2^*} \right) (C^* \sin n_1^* L_1 + D^* \cos n_1^* L_1) \quad .
 \end{aligned}$$

Using the boundary conditions, C^* can be expressed as

$$C^* = D^* \left[\frac{-A_1 E_1^* n_1^* \sin n_1^* L_1 - n_2^* E_2^* A_2 \left(\frac{\theta_2^*}{n_2^*} \right) \cos n_1^* L_1}{-A_1 E_1^* n_1^* \cos n_1^* L_1 + n_2^* E_2^* A_2 \left(\frac{\theta_2^*}{n_2^*} \right) \sin n_1^* L_1} \right] \quad (A-22)$$

The solution at $x = L_1$ is

$$(\tilde{\xi})_{x=L_1} = e^{j\omega t} \xi_0^* \left[\frac{-A_1 E_1^* n_1^*}{-A_1 E_1^* n_1^* \cos n_1^* L_1 + n_2^* E_2^* A_2 \left(\frac{\theta_2^*}{n_2^*} \right) \sin n_1^* L_1} \right] \quad (A-23)$$

The overall transmissibility of a mass-loaded, two-segment rod in longitudinal vibration is finally given by

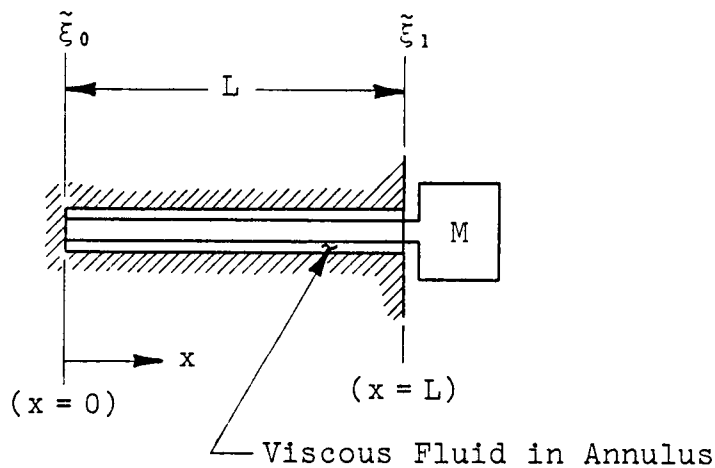
$$T_m = \left| \frac{\tilde{\xi}_2}{\tilde{\xi}_1} \right| \left| \frac{\tilde{\xi}_1}{\tilde{\xi}_0} \right|$$

$$= \left| \frac{1}{n_2} \right| \left| \frac{-A_1 E_1^* n_1^*}{-A_1 E_1^* n_1^* \cos n_1^* L_1 + n_2^* E_2^* A_2 \left(\frac{\theta_2^*}{n_2^*} \right) \sin n_1^* L_1} \right|$$

$$= \left| \frac{1}{n_2^* \cos n_1^* L_1 - \left(\frac{n_2^* E_2^* A_2}{n_1^* E_1^* A_1} \right) \theta_2^* \sin n_1^* L_1} \right|. \quad (\text{A-24})$$

3. Rod with Distributed Viscous Damping

Another useful transmissibility expression may be obtained for the case of a thin rod, in longitudinal vibration, with distributed viscous damping as shown in the figure below. The rod is again assumed to be made of a material having a complex Young's modulus. The viscous fluid is assumed to be massless.



The basic wave equation for this case is in the form of the generalized one-dimensional damped wave equation [21] and may be written as

$$\frac{\partial^2 \tilde{\xi}}{\partial x^2} - \frac{\rho}{E^*} \frac{\partial^2 \tilde{\xi}}{\partial t^2} - \frac{c}{AE^*} \left(\frac{\partial \tilde{\xi}}{\partial t} - \frac{\partial \tilde{\xi}_0}{\partial t} \right) = 0 \quad , \quad (\text{A-25})$$

where c = Viscous damping coefficient.

It is important to note that the damping force at any point on the rod is proportional to the relative velocity, $(\partial \tilde{\xi} / \partial t - \partial \tilde{\xi}_0 / \partial t)$, of that point. A transmissibility equation is derived in [4] where the damping force is assumed to be proportional to $\partial \tilde{\xi} / \partial t$, which is actually the particle velocity relative to an inertially fixed point. This assumption is clearly invalid for cases of practical significance because, if an inertially fixed point were actually available, one could simply fasten the mass to this point and obtain perfect isolation.

Eq. (A-25) may be rewritten as

$$\frac{\partial^2 \tilde{\xi}}{\partial x^2} + (n^*)^2 \tilde{\xi} + \frac{j\omega c}{AE^*} \tilde{\xi}_0 = 0 \quad , \quad (\text{A-26})$$

$$\text{where } (n^*)^2 = \left(\frac{\omega^2 \rho}{E^*} - \frac{j\omega c}{AE^*} \right) \quad . \quad (\text{A-27})$$

The assumed solution to Eq. (A-27) is

$$\tilde{\xi} = e^{j\omega t} \left(C^* \sin n^* x + D^* \cos n^* x - \frac{D^* j\omega c}{j\omega c + AE^* (n^*)^2} \right) \quad , \quad (\text{A-28})$$

where C^* and D^* are constants to be determined from the boundary conditions. The derivatives of Eq. (A-28) are given by Eqs. (A-5) - (A-8).

The boundary conditions, in this case, are

$$1. \quad (\tilde{\xi})_{x=0} = \tilde{\xi}_0 \quad (\text{Motion of rod end}) \quad ,$$

$$2. \quad AE^* \left(\frac{\partial \tilde{\xi}_1}{\partial x} \right)_{x=L} = M \left(\frac{d^2 \tilde{\xi}_1}{dt^2} \right)_{x=L} \quad (\text{Force} = MA) \quad .$$

After substitution of Eqs. (A-28), (A-5) and (A-8), the boundary conditions are

$$1. \quad \tilde{\xi}_0 = D^* \quad ,$$

$$2. \quad AE^* (-C^* n^* \cos n^* L + D^* n^* \sin n^* L) \\ = -M\omega^2 \left[C^* \sin n^* L + D^* \cos n^* L - \frac{D^* j\omega c}{j\omega c + AE^* (n^*)^2} \right] \quad .$$

From the boundary conditions, C^* can be expressed as

$$C^* = \xi_0^* \frac{AE^* n^* \sin n^* L + \omega^2 M \left[\cos n^* L - \frac{j\omega c}{j\omega c + AE^* (n^*)^2} \right]}{AE^* n^* \cos n^* L - \omega^2 M \sin n^* L} \quad .$$

(A-29)

The solution at $x = L$ is

$$\tilde{\xi}_1 = e^{j\omega t} \xi_0^* \left\{ \left[\frac{AE^* n^* \sin n^* L + \omega^2 M (\cos n^* L - \phi^*)}{AE^* n^* \cos n^* L - \omega^2 M \sin n^* L} \right] \sin n^* L + \cos n^* L - \phi^* \right\}, \quad (A-30)$$

$$\text{where } \phi^* = \frac{j\omega c}{j\omega c + AE^*(n^*)^2}. \quad (A-31)$$

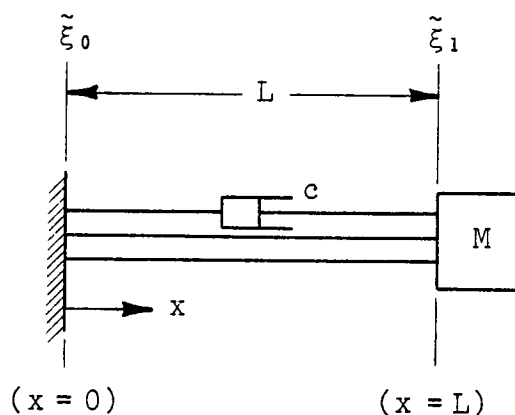
The transmissibility of a mass loaded rod, in longitudinal vibration, with distributed viscous damping, is then given by

$$T_m = \left| \frac{\tilde{\xi}_1}{\tilde{\xi}_0} \right| = \left| \frac{E^* n^* (1 - \phi^* \cos n^* L)}{\left[E^* n^* \cos n^* L - \left(\frac{\omega^2 M}{A} \right) \sin n^* L \right] (1 - \phi^*)} \right|. \quad (A-32)$$

4. Rod with a Viscous Damper in Parallel

Another transmissibility expression of interest may be obtained for the case of a thin rod, in longitudinal vibration, with a viscous damper in parallel as shown in the figure below. The rod is again assumed to be made of a material having a complex Young's modulus. The viscous

damper is idealized as being massless and frictionless. The addition of a damper in



parallel does not affect the differential equation of the rod so the basic wave equation for this case is given by Eq. (A-2). The assumed solution is given by Eq. (A-4) with partial derivatives given by Eqs. (A-5) - (A-8).

The boundary conditions for the rod shown above are

$$1. \quad (\tilde{\xi})_{x=0} = \tilde{\xi}_0 \quad (\text{Motion of rod end}) \quad ,$$

$$2. \quad AE^* \left(\frac{\partial \tilde{\xi}}{\partial x} \right)_{x=L} = M \left(\frac{\partial^2 \tilde{\xi}}{\partial t^2} \right)_{x=L} - c \left(\frac{\partial \tilde{\xi}_1}{\partial t} - \frac{\partial \tilde{\xi}_0}{\partial t} \right)$$

(Force = MA - Damping Force) ,

where c = Viscous damping coefficient.

After substitution of Eqs. (A-4), (A-5), (A-7) and (A-8), the boundary conditions are

$$\begin{aligned} 1. \quad D^* &= \xi_0^* , \\ 2. \quad AE^* (-C^* n^* \cos n^* L + D^* n^* \sin n^* L) \\ &= (-M\omega^2 - j\omega c)(C^* \sin n^* L + D^* \cos n^* L) + j\omega c D^* . \end{aligned}$$

Using the boundary conditions, C^* can be expressed as

$$C^* = \xi_0^* \frac{AE^* n^* \sin n^* L + (\omega^2 M + j\omega c) \cos n^* L - j\omega c}{AE^* n^* \cos n^* L - (\omega^2 M + j\omega c) \sin n^* L} . \quad (A-33)$$

The solution at $x = L$ is

$$\begin{aligned} \tilde{\xi}_1 = e^{j\omega t} \xi_0^* &\left\{ \left[\frac{AE^* n^* \sin n^* L + (\omega^2 M + j\omega c) \cos n^* L}{AE^* n^* \cos n^* L - (\omega^2 M + j\omega c) \sin n^* L} \right] \sin n^* L \right. \\ &\left. + \cos n^* L \right\} . \end{aligned} \quad (A-34)$$

The transmissibility of a mass loaded rod, in longitudinal vibration, with a viscous damper in parallel, is then given by

$$T_m = \left| \frac{\tilde{\xi}_1}{\tilde{\xi}_0} \right| = \left| \frac{AE^* n^* - j\omega c \sin n^* L}{AE^* n^* \cos n^* L - (\omega^2 M + j\omega c) \sin n^* L} \right| . \quad (A-35)$$

APPENDIX B

COMPUTER PROGRAM FOR CALCULATING
LONGITUDINAL TRANSMISSIBILITY

Table 20 is a listing of the computer program used to calculate the transmissibility of a mass loaded elastic rod, with damping, in longitudinal vibration. This program can be used to calculate the transmissibility of:

1. A two-segment rod with constant or solid damping.
2. A one-segment rod with distributed viscous damping.
3. A one-segment rod with a viscous damper in parallel.
4. A one-segment rod with solid and distributed viscous damping.

The program was written in FORTRAN IV to run interactively on the DEC VAX 11/780 Computer.

The main program is used for data input and controlling the frequency-step size. When the program is run, one of the five damping types must first be chosen by entering the appropriate digit in response to the prompt. The 5 damping types are: 1—constant, 2—solid, 3—viscous, 4—distributed viscous, and 5—solid + distributed viscous. Input data for the rod

TABLE 20

COMPUTER PROGRAM USED TO CALCULATE THE TRANSMISSIBILITY
OF MASS LOADED ELASTIC RODS IN
LONGITUDINAL VIBRATION

```

100 C   PROGRAM FOR OBTAINING THE TRANSMISSIBILITY OF DAMPED
200 C   ELASTIC RODS IN LONGITUDINAL VIBRATION
300 C
400   DIMENSION FQ(5),E1(5),D1(5),E2(5),D2(5)
500   COMMON REW1,IEW1,RHO1,A1,L1,REW2,IEW2,RHO2,A2,L2,MR2,M,
600   1DAMP1,DAMP2,JTYP
700   DOUBLE PRECISION RHO1,A1,L1,RHO2,A2,L2,MR2,M,PI,
800   1WSQ,REW1,IEW1,REW2,IEW2,FQ,E1,D1,E2,D2
900   TYPE*, 'ENTER DAMPING TYPE: 1-CONSTANT, 2-SOLID, 3-VISCOUS, '
1000  TYPE*, '4-DISTRIBUTED VISCOUS, 5-SOLID+DISTRIBUTED VISCOUS'
1100  ACCEPT*,JTYP
1200  IF (JTYP.EQ.1) GO TO 11
1300  IF (JTYP.EQ.2) GO TO 22
1400  IF (JTYP.EQ.3) GO TO 33
1500  IF (JTYP.EQ.4) GO TO 33
1600  IF (JTYP.EQ.5) GO TO 44
1700  11 OPEN (UNIT=2,NAME='PARAMETER.DAT',TYPE='OLD')
1800  READ (2,40) REW1,IEW1,RHO1,A1,L1,REW2,IEW2,RHO2,A2,L2,MR2,M
1900  GO TO 55
2000  22 OPEN (UNIT=2,NAME='SOLID.DAT',TYPE='OLD')
2100  READ (2,40) FQ(1),FQ(2),FQ(3),FQ(4),FQ(5),E1(1),D1(1),E1(2),D1(2),
2200  1E1(3),D1(3),E1(4),D1(4),E1(5),D1(5),RHO1,A1,L1,E2(1),D2(1),E2(2),
2300  2D2(2),E2(3),D2(3),E2(4),D2(4),E2(5),D2(5),RHO2,A2,L2,MR2,M
2400  GO TO 55
2500  33 OPEN (UNIT=2,NAME='VISCOUS.DAT',TYPE='OLD')
2600  READ (2,40) REW2,DAMP2,RHO2,A2,L2,MR2,M
2700  IEW1=0.0
2800  IEW2=0.0
2900  GO TO 55
3000  44 OPEN (UNIT=2,NAME='SOLVISC.DAT',TYPE='OLD')
3100  READ (2,40) FQ(1),FQ(2),FQ(3),FQ(4),FQ(5),E2(1),D2(1),E2(2),
3200  1D2(2),E2(3),D2(3),E2(4),D2(4),E2(5),D2(5),RHO2,A2,L2,MR2,M,
3300  2DAMP2
3400  55 CONTINUE
3500  40 FORMAT (/,'1X,E15.6')
3600  TYPE*, 'ENTER F START'
3700  ACCEPT*,FSTART
3800  TYPE*, 'ENTER F STEP'
3900  ACCEPT*,FSTEP
4000  TYPE*, 'ENTER SMALL STEP'
4100  ACCEPT*,SSTEP
4200  TYPE*, 'ENTER F STOP'
4300  ACCEPT*,FSTOP
4400  FFSSTP=FSTEP/4.0
4500  FSSSTP=SSTEP/4.0
4600  FOLD=0.0
4700  TMDBO=1.0
4800  THS=6.0/(FSTEP+0.5)
4900  F=FSTART
5000  70 CONTINUE
5100  IF (F.GT.FSTOP) GO TO 80
5200  IF (JTYP.EQ.3) GO TO 333
5300  IF (JTYP.EQ.4) GO TO 333
5400  IF (JTYP.EQ.5) GO TO 444
5500  DAMP1=0.0
5600  DAMP2=0.0
5700  IF (JTYP.EQ.1) GO TO 333
5800  222 CALL SOLID (F,FQ,E1,D1,REW1,IEW1)
5900  444 CALL SOLID (F,FQ,E2,D2,REW2,IEW2)
6000  333 CONTINUE
6100  CALL TMB(F,TM,TMDB)
6200  SLOPE=ABS((TMDB-TMDBO)/(F-FOLD))
6300  IF (SLOPE.GT.THIS) GO TO 50
6400  IF (SLO.GT.THIS) GO TO 50
6500  SLO=SLOPE
6600  TMDBO=TMDB
6700  FOLD=F
6800  IF (F.GT.3.0) GO TO 60
6900  F=F+FFSTP
7000  GO TO 70
7100  60 CONTINUE
7200  F=F+FSTEP
7300  GO TO 70
7400  50 SLO=SLOPE
7500  TMDBO=TMDB
7600  FOLD=F
7700  IF (F.GT.3.0) GO TO 90
7800  F=F+FSSTP
7900  GO TO 70
8000  90 CONTINUE
8100  F=F+SSTEP
8200  GO TO 70
8300  80 CONTINUE
8400  END

```

TABLE 20 (Continued)

```

100 C SUBROUTINE TMB FOR OBTAINING THE TRANSMISSIBILITY
200 C OF DAMPED ELASTIC RODS IN LONGITUDINAL VIBRATION
300 C
400 C F -FREQUENCY (HZ)
500 C TM -TRANSMISSIBILITY
600 C TMDB-TRANSMISSIBILITY (DB)
700 C
800 SUBROUTINE TMB(F,TM,TMDB)
900 COMMON REW1,IEW1,RHO1,A1,L1,REW2,IEW2,RHO2,A2,L2,MR2,M,
1000 1DAMP1,DAMP2,JTYP
1100 DOUBLE PRECISION RHO1,A1,L1,RHO2,A2,L2,MR2,M,PI,
1200 1WSQ,REW1,IEW1,REW2,IEW2,W
1300 COMPLEX*8 EW1,EW2,N1,N2,C1,C2,S1,S2,U,DEN,IM,V,NUM,
1400 1ETA2,THETA2,PHI2
1500 PI=3.141592654
1600 W=F*2.0*PI
1700 WSQ=(F*2.0*PI)**2
1800 IM=(0.0,1.0)
1900 EW2=CMPLX(SNGL(REW2),SNGL(IEW2))
2000 N2=CSQRT((SNGL(RHO2*WSQ)/(EW2*386.04))-(SNGL(W)*DAMP2
2100 2*IM/(SNGL(A2)*EW2)))
2200 C2=CCOS(N2*SNGL(L2))
2300 S2=CSIN(N2*SNGL(L2))
2400 IF (JTYP.EQ.3) GO TO 666
2500 IF (JTYP.EQ.4) GO TO 777
2600 IF (JTYP.EQ.5) GO TO 777
2700 EW1=CMPLX(SNGL(REW1),SNGL(IEW1))
2800 N1=CSQRT(SNGL(RHO1*WSQ)/(EW1*386.04))
2900 C1=CCOS(N1*SNGL(L1))
3000 S1=CSIN(N1*SNGL(L1))
3100 ETA2=C2-(M/MR2)*N2*L2*S2
3200 THETA2=S2+(M/MR2)*N2*L2*C2
3300 U=(N2*EW2*A2)/(N1*EW1*A1)
3400 DEN=ETA2*C1-U*THETA2*S1
3500 TM=CABS(1.0/DEN)
3600 GO TO 555
3700 666 N2=CSQRT(SNGL(RHO2*WSQ)/(EW2*386.04))
3800 C2=CCOS(N2*SNGL(L2))
3900 S2=CSIN(N2*SNGL(L2))
4000 U=IM*W*DAMP2
4100 V=A2*EW2*N2
4200 NUM=V-U*S2
4300 DEN=V*C2-((WSQ*M/386.04)+U)*S2
4400 TM=CABS(NUM/DEN)
4500 GO TO 555
4600 777 PHI2=(IM*W*DAMP2)/(IM*W*DAMP2+A2*EW2*N2*N2)
4700 NUM=EW2*N2*(1.0-C2*PHI2)
4800 DEN=(EW2*N2*C2-(WSQ*M*S2/(A2*386.04)))*(1.0-PHI2)
4900 TM=CABS(NUM/DEN)
5000 555 TMDB=20.0*LOG10(TM)
5100 30 FORMAT (5X,3H F=,F12.6,5X,6H TMDB=,F7.2,5X,4H TM=,1PE10.3)
5200 WRITE (1,30) F,TMDB,TM
5300 RETURN
5400 END

```

TABLE 20 (Continued)

```

100 C SUBROUTINE SOLID FOR OBTAINING VALUES OF THE DYNAMIC YOUNG'S
200 C MODULUS AND DAMPING FACTOR AS A FUNCTION OF FREQUENCY FOR A
300 C SYSTEM WITH SOLID DAMPING. THE VALUES ARE FOUND BY LINEAR
400 C INTERPOLATION BETWEEN FIVE DATA POINTS.
500 C
600 C F -FREQUENCY (HZ)
700 C FQ -TABULATED FREQUENCY (HZ)
800 C E -TABULATED YOUNG'S MODULUS (PSI)
900 C D -TABULATED DAMPING FACTOR
1000 C REWN-REAL PART OF COMPLEX MODULUS
1100 C IEWN-IMAGINARY PART OF COMPLEX YOUNG'S MODULUS
1200 C
1300 SUBROUTINE SOLID (F,FQ,E,D,REWN,IEWN)
1400 DOUBLE PRECISION F,FQ,E,D,REWN,IEWN
1500 DIMENSION FQ(5),E(5),D(5)
1600 J=1
1700 IF (F.LT.FQ(2)) GO TO 10
1800 J=2
1900 IF (F.LT.FQ(3)) GO TO 10
2000 J=3
2100 IF (F.LT.FQ(4)) GO TO 10
2200 J=4
2210 10 REWN=E(J)+((F-FQ(J))*(E(J+1)-E(J))/(FQ(J+1)-FQ(J)))
2220 IEWN=REWN*(D(J)+((F-FQ(J))*(D(J+1)-D(J))/(FQ(J+1)-FQ(J))))
2230 RETURN
2240 END

```

is then read from the appropriate data file for the selected damping type. The program calculates the transmissibility at discrete frequency steps between the specified frequency limits. The values in Hz of the starting frequency (F START), step size (F STEP) and stopping frequency (F STOP) must be entered in response to the prompts. In order to obtain good definition of the sharp resonant peaks, a reduced step size is used when the absolute value of the slope of the transmissibility curve exceeds a set threshold (THS). The size of this reduced step must be entered in response to the "ENTER SMALL STEP" prompt. Values typically used for undamped rods were:

F START = .125

F STEP = .5

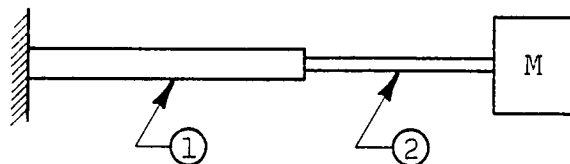
SMALL STEP = .125

F STOP = 500

Subroutine TMB is called by the main program to calculate the transmissibility at each frequency. The appropriate transmissibility equation is selected according to the chosen damping type. Equations (A-24), (A-32), and (A-35) are programmed in TMB. At each frequency, the frequency, transmissibility and transmissibility in dB are written to an output file. The data in this file were then used to obtain a computer drawn transmissibility plot.

Subroutine SOLID is called by the main program to obtain the real and imaginary parts of the complex Young's modulus, at any frequency, for a system with solid damping. The values of the real and imaginary parts of the complex Young's modulus are obtained from the five values of the composite dynamic Young's modulus and damping factor, which were read into the input data file.

The nomenclature used for the input parameters is as follows:



REW1	Real part of complex Young's modulus for Rod ① (psi)
IEW1	Imaginary part of complex Young's modulus for Rod ① (psi)
RH01	Density of Rod ① (lb_m/in^3)
A1	Cross-sectional area of Rod ① (in^2)
L1	Length of Rod ① (in)
MR2	Mass of Rod ② (lb_m)
M	Mass of m (lb_m)
FQ(1)	Lowest frequency of input data (Hz)

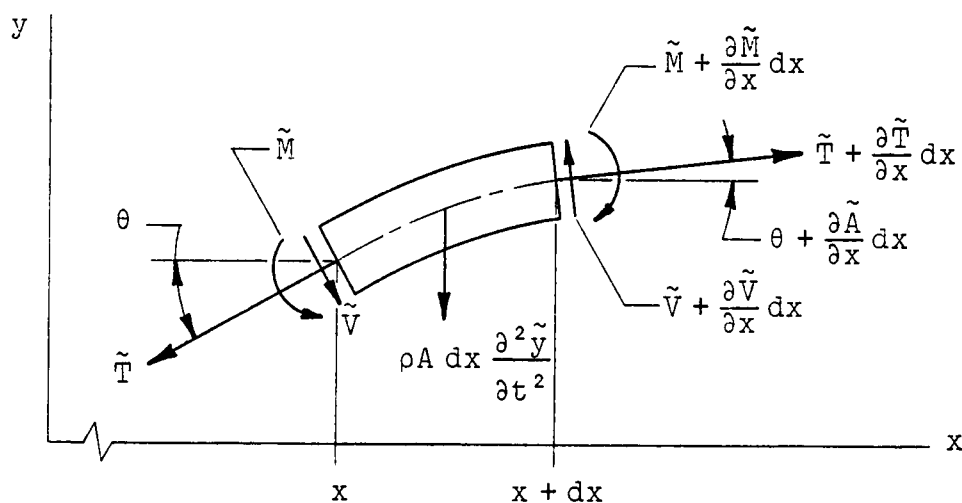
FQ(S)	Highest frequency of input data (Hz)
E2(1)	Dynamic Young's modulus of Rod (2) at lowest frequency (psi)
D2(1)	Damping factor of Rod (2) at lowest frequency
E1(S)	Dynamic Young's modulus of Rod (1) at highest frequency
D1(S)	Damping factor of Rod (1) at highest frequency
DAMP2	Distributed viscous damping coefficient defined by Equation (25) ($lb_F - S/in^2$) or viscous damping coefficient defined by Equation (26) ($lb_F - S/in$)

APPENDIX C

DERIVATION OF TRANSMISSIBILITY EQUATIONS
FOR BEAMS IN TRANSVERSE VIBRATION

In this appendix the standing wave transmissibility equations for beams in transverse vibration are derived. The beams are assumed to be distributed-parameter systems whose motion is described by the Bernoulli-Euler theory of beam vibration. This theory assumes that the bending wavelength is several times larger than the cross-sectional dimensions of the beam, and that rotary inertia and shear displacement have negligible effect on beam vibration. These assumptions are valid for relatively thin beams vibrating at low frequencies. The beams are also assumed to be made of a material having a complex Young's modulus as defined by Eq. (A-1).

The forces and moments acting on an element of a beam with a tensile load, T , are shown in the figure below. The $\sim$ denotes a sinusoidally varying quantity. The $*$ denotes a complex quantity.



The moment-curvature relationship is given by

$$\frac{\partial^2 \tilde{y}}{\partial x^2} = - \frac{M}{E^* I} \quad (C-1)$$

The equation of motion in the vertical direction for the element is given by

$$- \tilde{V} + \left(\tilde{V} + \frac{\partial \tilde{V}}{\partial x} dx \right) - \tilde{T} \tilde{\theta} + \left(\tilde{T} + \frac{\partial \tilde{T}}{\partial x} dx \right) \left(\tilde{\theta} + \frac{\partial \tilde{\theta}}{\partial x} dx \right) = \rho A dx \frac{\partial^2 \tilde{y}}{\partial t^2} \quad (C-2)$$

Neglecting the higher order term in dx^2 appearing in conjunction with the tension term, one obtains:

$$\frac{\partial \tilde{V}}{\partial x} + \frac{\partial}{\partial x} (\tilde{T} \tilde{\theta}) = \rho A \frac{\partial^2 \tilde{y}}{\partial t^2} \quad (C-3)$$

The moment equation is simply

$$\tilde{V} = \frac{\partial \tilde{M}}{\partial x} \quad . \quad (C-4)$$

Using the relationship $\tilde{\theta} = \partial \tilde{y} / \partial x$ and

Eqs. (C-1) and (C-4) in Eq. (C-3), one obtains:

$$E^* I \frac{\partial^4 \tilde{y}}{\partial x^4} - \frac{\partial}{\partial x} \left(\tilde{T} \frac{\partial \tilde{y}}{\partial x} \right) + \rho A \frac{\partial^2 \tilde{y}}{\partial t^2} = 0 \quad . \quad (C-5)$$

The assumption that axial inertia effects are negligible implies that $\partial \tilde{T} / \partial x = 0$, or that tension is constant. The final form of the transverse wave equation is then

$$\frac{\partial^4 \tilde{y}}{\partial x^4} - \frac{T}{E^* I} \left(\frac{\partial^2 \tilde{y}}{\partial x^2} \right) + \frac{1}{a^2} \frac{\partial^2 \tilde{y}}{\partial t^2} = 0 \quad , \quad (C-6)$$

where $a^2 = \frac{E^* I}{\rho A}$.

Considering the propagation of harmonic waves of the form $\exp\{j(\gamma x - \omega t)\}$, Graff [18] obtains the following wavenumber-frequency relation:

$$\gamma^4 + \frac{T}{E^* I} \gamma^2 - \frac{\omega^2}{a^2} = 0 \quad , \quad (C-7)$$

which has roots

$$\gamma = \pm \left\{ -\frac{T}{2E^* I} \pm \left[\left(\frac{T}{2E^* I} \right)^2 + \frac{\omega^2}{a^2} \right]^{1/2} \right\}^{1/2} \quad . \quad (C-8)$$

The wavenumbers will then be of the form

$$\gamma = \pm \alpha^* \quad , \quad \gamma = \pm j\beta^* \quad ,$$

where

$$\alpha^* = \left\{ -\frac{T}{2E^*I} + \left[\left(\frac{T}{2E^*I} \right)^2 + \frac{\omega^2}{a^2} \right]^{1/2} \right\}^{1/2} \quad ,$$

(C-9)

$$\beta^* = \left\{ \frac{T}{2E^*I} + \left[\left(\frac{T}{2E^*I} \right)^2 + \frac{\omega^2}{a^2} \right]^{1/2} \right\}^{1/2} \quad .$$

The general solution to Eq. (C-6) is assumed to be of the form

$$\tilde{y} = e^{j\omega t} (C_1^* \cos \alpha^* x + C_2^* \sin \alpha^* x + C_3^* \cosh \beta^* x + C_4^* \sinh \beta^* x) \quad ,$$

where $C_1^* - C_4^*$ are constants to be determined from boundary conditions.

1. Single Segment Beam

The figure below shows the configuration for which a solution at $x = L$ is desired. The left end of the beam is simply supported with sinusoidal motion $\tilde{y}_0$ of the support. The right end of the beam is loaded by a point mass M . The beam is also loaded by an axial tension force, T , which is actually due to the weight of M . The figure below

Expressions for the partial derivatives of Eq. (C-10) are:

$$\frac{\partial \tilde{y}}{\partial x} = e^{j\omega t} (-C_1^* \alpha^* \sin \alpha^* x + C_2^* \alpha^* \cos \alpha^* x + C_3^* \beta^* \sinh \beta^* x + C_4^* \beta^* \cosh \beta^* x) , \quad (C-11)$$

$$\frac{\partial^2 \tilde{y}}{\partial x^2} = e^{j\omega t} (-C_1^* \alpha^{*2} \cos \alpha^* x - C_2^* \alpha^{*2} \sin \alpha^* x + C_3^* \beta^{*2} \cosh \beta^* x + C_4^* \beta^{*2} \sinh \beta^* x) , \quad (C-12)$$

$$\frac{\partial^3 \tilde{y}}{\partial x^3} = e^{j\omega t} (C_1^* \alpha^{*3} \sin \alpha^* x - C_2^* \alpha^{*3} \cos \alpha^* x + C_3^* \beta^{*3} \sinh \beta^* x + C_4^* \beta^{*3} \cosh \beta^* x) , \quad (C-13)$$

$$\frac{\partial \tilde{y}}{\partial t} = j\omega e^{j\omega t} (C_1^* \cos \alpha^* x + C_2^* \sin \alpha^* x + C_3^* \cosh \beta^* x + C_4^* \sinh \beta^* x) , \quad (C-14)$$

$$\frac{\partial^2 \tilde{y}}{\partial t^2} = -\omega^2 e^{j\omega t} (C_1^* \cos \alpha^* x + C_2^* \sin \alpha^* x + C_3^* \cosh \beta^* x + C_4^* \sinh \beta^* x) . \quad (C-15)$$

The boundary conditions, after substitution of Eqs. (C-11) - (C-15), are:

1. $C_1^* + C_3^* = y_0$,
2. $-C_1^* \alpha^{*2} + C_3^* \beta^{*2} = 0$,
3. $-C_1^* \alpha^{*2} \cos \alpha^* L - C_2^* \alpha^{*2} \sin \alpha^* L + C_3^* \beta^{*2} \cosh \beta^* L + C_4^* \beta^{*2} \sinh \beta^* L = 0$,

$$4. \quad -E^* I (C_1^* a^{*3} \sin \alpha^* L - C_2^* a^{*3} \cos \alpha^* L + C_3^* \beta^{*3} \sinh \beta^* L + C_4^* \beta^{*3} \cosh \beta^* L) + T (-C_1^* \alpha^* \sin \alpha^* L + C_2^* \alpha^* \cos \alpha^* L + C_3^* \beta^* \sinh \beta^* L + C_4^* \beta^* \cosh \beta^* L) = M \omega^2 (C_1^* \cos \alpha^* L + C_2^* \sin \alpha^* L + C_3^* \cosh \beta^* L + C_4^* \sinh \beta^* L) \quad .$$

The constants can then be evaluated by Cramer's Rule.

The determinant of the boundary conditions is:

1	0	1
$-\alpha^{*2}$	0	β^{*2}
$-\alpha^{*2} \cos \alpha^* L$	$-\alpha^{*2} \sin \alpha^* L$	$\beta^{*2} \cosh \beta^* L$
$-E^* I \alpha^{*3} \sin \alpha^* L$ $-T \alpha^* \sin \alpha^* L$ $-M \omega^2 \cos \alpha^* L$	$+E^* I \alpha^{*3} \cos \alpha^* L$ $+T \alpha^* \cos \alpha^* L$ $-M \omega^2 \sin \alpha^* L$	$-E^* I \beta^{*3} \sinh \beta^* L$ $+T \beta^* \sinh \beta^* L$ $-M \omega^2 \cosh \beta^* L$
	0	
	0	
	$\beta^{*2} \sinh \beta^* L$	
		$-E^* I \beta^{*3} \cosh \beta^* L$ $+T \beta^* \cosh \beta^* L$ $-M \omega^2 \sinh \beta^* L$
		= DET

which reduces to

$$\begin{aligned}
 \text{DET} = & (\beta^{*2} + \alpha^{*2})\alpha^{*2} \sin \alpha^* L (-E^* I \beta^{*3} \cosh \beta^* L + T \beta^* \cosh \beta L \\
 & - M \omega^2 \sinh \beta^* L) + \beta^{*4} \sinh \beta^* L (+E^* I \alpha^{*3} \cos \alpha^* L \\
 & + T \alpha^* \cos \alpha^* L - M \omega^2 \sin \alpha^* L) + \alpha^{*2} \beta^{*2} \sinh \beta^* L \\
 & (+E^* I \alpha^{*3} \cos \alpha^* L + T \alpha^* \cos \alpha^* L - M \omega^2 \sin \alpha^* L) \quad . \quad (\text{C-16})
 \end{aligned}$$

The constants, $C_1^* - C_4^*$, are

$$C_1^* = \begin{vmatrix} y_0 & 0 & 1 & 0 \\ 0 & 0 & \beta^{*2} & 0 \\ 0 & -\alpha^{*2} \sin \alpha^* L & \beta^{*2} \cosh \beta^* L & \beta^{*2} \sinh \beta^* L \\ 0 & \boxed{\begin{matrix} +E^* I \alpha^{*3} \cos \alpha^* L \\ +T \alpha^* \cos \alpha^* L \\ -M \omega^2 \sin \alpha^* L \end{matrix}} & \boxed{\begin{matrix} -E^* I \beta^{*3} \sinh \beta^* L \\ +T \beta^* \sinh \beta^* L \\ -M \omega^2 \cosh \beta^* L \end{matrix}} & \boxed{\begin{matrix} -E^* I \beta^{*3} \cosh \beta^* L \\ +T \beta^* \cosh \beta^* L \\ -M \omega^2 \sinh \beta^* L \end{matrix}} \end{vmatrix}$$

DET

$$\begin{aligned}
 C_1^* = & \frac{-y_0 \beta^{*2}}{\text{DET}} \left[+\alpha^{*2} \beta^{*3} E^* I \sin \alpha^* L \cosh \beta^* L \right. \\
 & -\alpha^{*2} \beta^* T \sin \alpha^* L \cosh \beta^* L + \alpha^{*2} M \omega^2 \sin \alpha^* L \sinh \beta^* L \\
 & -\alpha^{*3} \beta^{*3} E^* I \sinh \beta^* L \cos \alpha^* L - \alpha^* \beta^{*2} T \sinh \beta^* L \cos \alpha^* L \\
 & \left. + \beta^{*2} M \omega^2 \sinh \beta^* L \sin \alpha^* L \right] \quad . \quad (\text{C-17})
 \end{aligned}$$

$$C_2^* = \begin{vmatrix} 1 & y_0 & 1 & 0 \\ -\alpha^{*2} & 0 & \beta^{*2} & 0 \\ -\alpha^{*2} \cos \alpha^* L & 0 & \beta^{*2} \cosh \beta^* L & \beta^{*2} \sinh \beta^* L \\ \begin{pmatrix} -E^* I \alpha^{*3} \sin \alpha^* L \\ -T \alpha^* \sin \alpha^* L \\ -M \omega^2 \cos \alpha^* L \end{pmatrix} & 0 & \begin{pmatrix} -E^* I \beta^{*3} \sinh \beta^* L \\ +T \beta^* \sinh \beta^* L \\ -M \omega^2 \cosh \beta^* L \end{pmatrix} & \begin{pmatrix} -E^* I \beta^{*3} \cosh \beta^* L \\ +T \beta^* \cosh \beta^* L \\ -M \omega^2 \sinh \beta^* L \end{pmatrix} \end{vmatrix}$$

DET

$$C_2^* = \frac{-y_0}{\text{DET}} \left[+\alpha^{*2} \beta^{*5} E^* I \cosh^2 \beta^* L - \alpha^{*2} \beta^{*3} T \cosh^2 \beta^* L \right. \\ + \alpha^{*2} \beta^{*2} M \omega^2 \cosh \beta^* L \sinh \beta^* L - \alpha^{*2} \beta^{*5} E^* I \sinh^2 \beta^* L \\ + \alpha^{*2} \beta^{*3} T \sinh^2 \beta^* L - \alpha^{*2} \beta^{*2} M \omega^2 \sinh \beta^* L \cosh \beta^* L \\ - \alpha^{*2} \beta^{*5} E^* I \cos \alpha^* L \cosh \beta^* L + \alpha^{*3} \beta^{*3} T \cos \alpha^* L \cosh \beta^* L \\ - \alpha^{*2} \beta^{*2} M \omega^2 \cos \alpha^* L \sinh \beta^* L - \alpha^{*3} \beta^{*4} E^* I \sinh \beta^* L \sin \alpha^* L \\ \left. - \alpha^* \beta^{*4} T \sinh \beta^* L \sin \alpha^* L - \beta^{*4} M \omega^2 \sinh \beta^* L \cos \alpha^* L \right]$$

(C-18)

$$C_3^* = \frac{\begin{vmatrix} 1 & 0 & y_0 & 0 \\ -\alpha^{*2} & 0 & 0 & 0 \\ -\alpha^{*2} \cos \alpha^* L & -\alpha^{*2} \sin \alpha^* L & 0 & \beta^{*2} \sinh \beta^* L \\ \boxed{\begin{matrix} -E^* I \alpha^{*3} \sin \alpha^* L \\ -T \alpha^* \sin \alpha^* L \\ -M \omega^2 \cos \alpha^* L \end{matrix}} & \boxed{\begin{matrix} +E^* I \alpha^{*3} \cos \alpha^* L \\ +T \alpha^* \cos \alpha^* L \\ -M \omega^2 \sin \alpha^* L \end{matrix}} & 0 & \boxed{\begin{matrix} -E^* I \beta^{*3} \cosh \beta^* L \\ +T \beta^* \cosh \beta^* L \\ -M \omega^2 \sinh \beta^* L \end{matrix}} \end{vmatrix}}{\text{DET}}$$

$$C_3^* = \frac{-y_0 \alpha^{*2}}{\text{DET}} \left[\alpha^{*2} \beta^{*3} E^* I \sin \alpha^* L \cosh \beta^* L \right. \\ - \alpha^{*2} \beta^* T \sin \alpha^* L \cosh \beta^* L + \alpha^{*2} M \omega^2 \sin \alpha^* L \sinh \beta^* L \\ - \alpha^{*3} \beta^{*2} E^* I \sinh \beta^* L \cos \alpha^* L - \alpha^* \beta^{*2} T \sinh \beta^* L \cos \alpha^* L \\ \left. + \beta^{*2} M \omega^2 \sinh \beta^* L \sin \alpha^* L \right] \quad (C-19)$$

$$C_4^* = \frac{\begin{vmatrix} 1 & 0 & 1 & y_0 \\ -\alpha^{*2} & 0 & \beta^{*2} & 0 \\ -\alpha^{*2} \cos \alpha^* L & -\alpha^{*2} \sin \alpha^* L & \beta^{*2} \cosh \beta^* L & 0 \\ \boxed{\begin{matrix} -E^* I \alpha^{*3} \sin \alpha^* L \\ -T \alpha^* \sin \alpha^* L \\ -M \omega^2 \cos \alpha^* L \end{matrix}} & \boxed{\begin{matrix} +E^* I \alpha^{*3} \cos \alpha^* L \\ +T \alpha^* \cos \alpha^* L \\ -M \omega^2 \sin \alpha^* L \end{matrix}} & \boxed{\begin{matrix} -E^* I \beta^{*3} \sinh \beta^* L \\ +T \beta^* \sinh \beta^* L \\ -M \omega^2 \cosh \beta^* L \end{matrix}} & 0 \end{vmatrix}}{\text{DET}}$$

$$\begin{aligned}
C_4^* = \frac{-y_0}{\text{DET}} & \left[-\alpha^{*4} \beta^{*3} E^* I \sin \alpha^* L \sinh \beta^* L \right. \\
& + \alpha^{*4} \beta^* T \sin \alpha^* L \sinh \beta^* L - \alpha^{*4} M \omega^2 \sin \alpha^* L \cosh \beta^* L \\
& + \alpha^{*5} \beta^{*2} E^* I \cosh \beta^* L \cos \alpha^* L + \alpha^{*3} \beta^{*2} T \cosh \beta^* L \cos \alpha^* L \\
& \left. - \alpha^{*2} \beta^{*2} M \omega^2 \cosh \beta^* L \sin \alpha^* L - \alpha^{*5} \beta^{*2} E^* I - \alpha^{*3} \beta^{*2} T \right] .
\end{aligned}
\tag{C-20}$$

The solution at $x = L$, obtained by substituting Eqs. (C-16) - (C-20) back into (C-10), is

$$\begin{aligned}
(\tilde{y})_{x=L} = \frac{y_0 e^{j\omega t}}{\text{DET}} & \left\{ -\beta^{*2} \left[+\alpha^{*2} \beta^{*3} E^* I \sin \alpha^* L \cosh \beta^* L \right. \right. \\
& - \alpha^{*2} \beta^* T \sin \alpha^* L \cosh \beta^* L + \alpha^{*2} M \omega^2 \sin \alpha^* L \sinh \beta^* L \\
& - \alpha^{*3} \beta^{*2} E^* I \sinh \beta^* L \cos \alpha^* L \\
& \left. - \alpha^* \beta^{*2} T \sinh \beta^* L \cos \alpha^* L + \beta^{*2} M \omega^2 \sinh \beta^* L \sin \alpha^* L \right] \\
& \cos \alpha^* L - \left[+\alpha^{*2} \beta^{*5} E^* I - \alpha^{*2} \beta^{*3} T \right. \\
& + \alpha^{*2} \beta^{*2} M \omega^2 \cosh \beta^* L \sinh \beta^* L \\
& - \alpha^{*2} \beta^{*2} M \omega^2 \sinh \beta^* L \cosh \beta^* L \\
& - \alpha^{*2} \beta^{*5} E^* I \cos \alpha^* L \cosh \beta^* L \\
& \left. + \alpha^{*2} \beta^{*3} T \cos \alpha^* L \cosh \beta^* L \right]
\end{aligned}$$

$$\begin{aligned}
& -\alpha^{*2} \beta^{*3} M\omega^2 \cos \alpha^* L \sinh \beta^* L \\
& -\alpha^{*3} \beta^{*4} E^* I \sinh \beta^* L \sin \alpha^* L \\
& -\alpha^* \beta^{*4} T \sinh \beta^* L \sin \alpha^* L \\
& -\beta^{*4} M\omega^2 \sinh \beta^* L \cos \alpha^* L \Big] \sin \alpha^* L \\
& -\alpha^{*2} \Big[+\alpha^{*2} \beta^{*3} E^* I \sin \alpha^* L \cosh \beta^* L \\
& -\alpha^{*2} \beta^* T \sin \alpha^* L \cosh \beta^* L + \alpha^{*2} M\omega^2 \sin \alpha^* L \sinh \beta^* L \\
& -\alpha^{*3} \beta^{*2} E^* I \sinh \beta^* L \cos \alpha^* L - \alpha^* \beta^{*2} T \sinh \beta^* L \cos \alpha^* L \\
& + \beta^{*2} M\omega^2 \sinh \beta^* L \sin \alpha^* L \Big] \cosh \beta^* L \\
& - \Big[-\alpha^{*4} \beta^{*3} E^* I \sin \alpha^* L \sinh \beta^* L \\
& + \alpha^{*4} \beta^* T \sin \alpha^* L \sinh \beta^* L - \alpha^{*4} M\omega^2 \sin \alpha^* L \cosh \beta^* L \\
& + \alpha^{*5} \beta^{*2} E^* I \cosh \beta^* L \cos \alpha^* L \\
& + \alpha^{*3} \beta^{*2} T \cosh \beta^* L \cos \alpha^* L \\
& - \alpha^{*2} \beta^{*2} M\omega^2 \cosh \beta^* L \sin \alpha^* L \\
& - \alpha^{*5} \beta^{*2} E^* I - \alpha^{*3} \beta^{*2} T \Big] \sinh \beta^* L \Big\} . \quad (C-21)
\end{aligned}$$

Since

$$\tilde{y}_0 = y_0 e^{j\omega t} \quad (C-22)$$

and

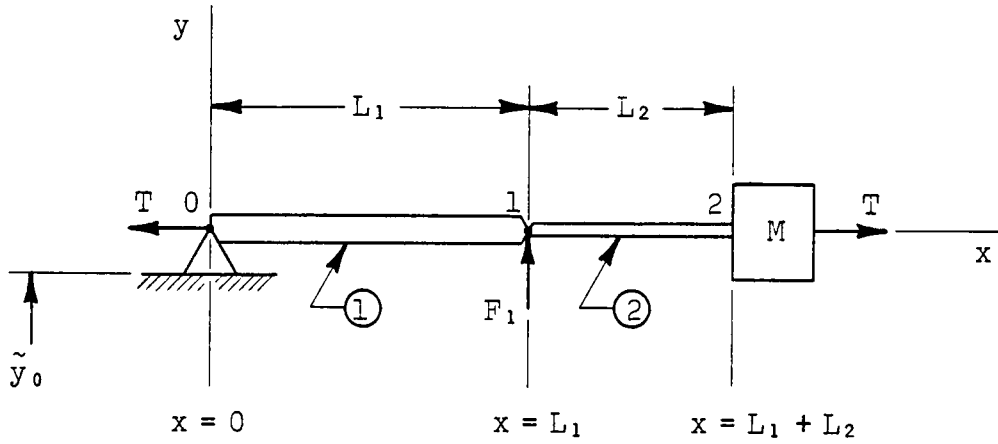
$$(\tilde{y})_{x=0} = \tilde{y}_0, \quad (C-23)$$

the equation for transmissibility, T_m , is obtained by substituting Eqs. (C-22) and (C-23) into (C-21), which reduces to

$$T_m = \left| \frac{(\tilde{y})_{x=L}}{(\tilde{y})_{x=0}} \right| = \left| \begin{array}{l} +\alpha^{*3} \beta^{*4} E^* I \sinh \beta^* L + \alpha^* \beta^{*4} T \sinh \beta^* L \\ -\alpha^{*4} \beta^{*3} E^* I \sin \alpha^* L + \alpha^{*4} \beta^* T \sin \alpha^* L \\ -\alpha^{*2} \beta^{*5} E^* I \sin \alpha^* L + \alpha^{*2} \beta^{*3} T \sin \alpha^* L \\ +\alpha^{*5} \beta^{*2} E^* I \sinh \beta^* L + \alpha^{*3} \beta^{*2} T \sinh \beta^* L \\ (\beta^{*2} + \alpha^{*2}) \alpha^{*2} \sin \alpha^* L (-E^* I \beta^{*3} \cosh \beta^* L \\ + T \beta^* \cosh \beta^* L - M \omega^2 \sinh \beta^* L) + \beta^{*4} \sinh \\ \beta^* L (+E^* I \alpha^{*3} \cos \alpha^* L + T \alpha^* \cos \alpha^* L - M \omega^2 \\ \sin \alpha^* L) + \alpha^{*2} \beta^{*2} \sinh \beta^* L (+E^* I \alpha^{*3} \\ \cos \alpha^* L + T \alpha^* \cos \alpha^* L - M \omega^2 \sin \alpha^* L) \end{array} \right| . \quad (C-24)$$

2. Two-Segment Beam

The preceding results may now be used to obtain the transmissibility of a two-segment beam, as shown in the figure below.



The transmissibility for beam ② is given by Eq. (C-24). The transmissibility for beam ① must be determined in order to obtain the overall transmissibility of the coupled system. Three of the boundary conditions for beam ① are identical to those of the previous case.

1. $(\tilde{y})_{x=0} = \tilde{y}_0$ (Motion of support) ,
2. $\left(\frac{\partial^2 \tilde{y}}{\partial x^2}\right)_{x=0} = 0$ (No bending moment) ,
3. $\left(\frac{\partial^2 \tilde{y}}{\partial x^2}\right)_{x=L_1} = 0$ (No bending moment) .

The right end of beam ① is not connected to a point mass; however, it is connected to the left end of beam ② which has a finite mechanical impedance.

The driving point impedance, Z_0 , of beam ② is defined as

$$Z_0 = \frac{\tilde{F}_1}{-\left(\frac{\partial \tilde{y}}{\partial t}\right)_{x=L_1}}, \quad (C-25)$$

where $\tilde{F}_1$ is the force exerted on beam ② by beam ①. The fourth boundary condition for beam ① is then

$$4. \quad -E_1^* I_1 \left(\frac{\partial^3 \tilde{y}}{\partial x^3} \right)_{x=L_1} + T \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=L_1} = -Z_0 \left(\frac{\partial \tilde{y}}{\partial t} \right)_{x=0}.$$

After substitution of Eqs. (C-11), (C-13) and (C-14), the fourth boundary condition is

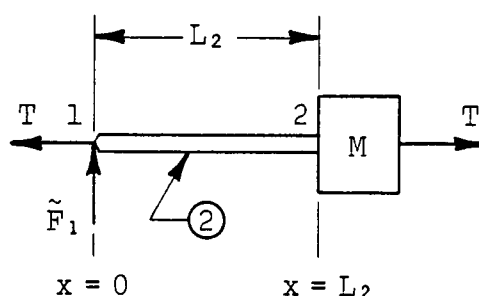
$$\begin{aligned} 4. \quad & -E_1^* I_1 (C_1^* \alpha_1^{*3} \sin \alpha_1^* L_1 - C_2^* \alpha_1^* \cos \alpha_1^* L_1 + C_3^* \beta_1^{*3} \sinh \beta_1^* L_1 \\ & + C_4^* \beta_1^{*3} \cosh \beta_1^* L_1) + T (-C_1^* \alpha_1^* \sin \alpha_1^* L_1 + C_2^* \alpha_1^* \cos \alpha_1^* L_1 \\ & + C_3^* \beta_1^* \sinh \beta_1^* L_1 + C_4^* \beta_1^* \cosh \beta_1^* L_1) \\ & = j\omega Z_0 (C_1^* \cos \alpha_1^* L_1 + C_2^* \sin \alpha_1^* L_1 + C_3^* \cosh \beta_1^* L_1 \\ & + C_4^* \sinh \beta_1^* L_1). \end{aligned}$$

If this boundary condition is compared to the fourth boundary condition of the previous case, it can be seen that they are identical except that $M\omega^2$ has been replaced by $-j\omega Z_0$. Since all boundary conditions

for beam (1) are identical to those of the previous case, with one exception, it follows that the transmissibility equation for beam (1) may be obtained by replacing $M\omega^2$ in (C-24) by $-j\omega Z_0$. The resulting equation is

$$T_{m_1} = \left| \frac{(\tilde{y})_{x=L_1}}{(\tilde{y})_{x=0}} \right| = \left| \begin{array}{l} + \alpha_1^{*3} \beta_1^{*4} E_1^* I_1 \sinh \beta_1^* L_1 \\ + \alpha_1^* \beta_1^{*4} T \sinh \beta_1^* L_1 - \alpha_1^{*4} \beta_1^{*3} E_1^* I_1 \sin \alpha_1^* L_1 \\ + \alpha_1^{*4} \beta_1^* T \sin \alpha_1^* L_1 - \alpha_1^{*2} \beta_1^{*5} E_1^* I_1 \sin \alpha_1^* L_1 \\ + \alpha_1^{*2} \beta_1^{*3} T \sin \alpha_1^* L_1 + \alpha_1^{*5} \beta_1^{*2} E_1^* I_1 \sinh \beta_1^* L_1 \\ + \alpha_1^{*3} \beta_1^{*2} T \sinh \beta_1^* L_1 \\ \hline \left(\beta_1^{*2} + \alpha_1^{*2} \right) \alpha_1^{*2} \sin \alpha_1^* L_1 \\ \left(-E_1^* I_1 \beta_1^{*3} \cosh \beta_1^* L_1 + T \beta_1^* \cosh \beta_1^* L_1 \right. \\ \left. + j\omega Z_0 \sinh \beta_1^* L_1 \right) + \beta_1^{*4} \sinh \beta_1^* L_1 \\ \left(+E_1^* I_1 \alpha_1^{*3} \cos \alpha_1^* L_1 + T \alpha_1^* \cos \alpha_1^* L_1 \right. \\ \left. + j\omega Z_0 \sin \alpha_1^* L_1 \right) + \alpha_1^{*2} \beta_1^{*2} \sinh \beta_1^* L_1 \\ \left(+E_1^* I_1 \alpha_1^{*3} \cos \alpha_1^* L_1 + T \alpha_1^* \cos \alpha_1^* L_1 \right. \\ \left. + j\omega Z_0 \sin \alpha_1^* L_1 \right) \end{array} \right|$$

Z_0 is an unknown in the above equation and must now be evaluated in terms of beam ② properties. This task may be simplified by shifting the origin, as shown in the figure below.



The equation for Z_0 in the beam ② coordinate system is

$$Z_0 = \frac{\tilde{F}_1}{-\left(\frac{\partial \tilde{y}}{\partial t}\right)_{x=0}} \quad . \quad (C-27)$$

It can be seen from this equation that Z_0 may be obtained if a solution for $(\partial \tilde{y} / \partial t)_{x=L_1}$ is found when beam ② is driven by a sinusoidal force $\tilde{F}_1$.

The boundary conditions for the beam shown above are:

$$1. \quad -E_2^* I_2 \left(\frac{\partial^3 \tilde{y}}{\partial x^3} \right)_{x=0} + T \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=0} = \tilde{F}_1$$

(Shear force + tension comp. = Force) ,

$$2. \quad \left(\frac{\partial^2 \tilde{y}}{\partial x^2} \right)_{x=L_1} = 0 \quad (\text{No bending moment}) \quad ,$$

$$3. \quad \left(\frac{\partial^2 \tilde{y}}{\partial x^2} \right)_{x=L_2} = 0 \quad (\text{No bending moment}) \quad ,$$

$$4. \quad -E_2^* I_2 \left(\frac{\partial^3 \tilde{y}}{\partial x^3} \right)_{x=L_2} + T \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=L_2} = -M \left(\frac{\partial^2 \tilde{y}}{\partial t^2} \right)_{x=L_2}$$

(Shear force + tension component = MA) .

The above boundary conditions, after substitution of Eqs. (C-11) - (C-15), are

$$1. \quad +C_2^* \alpha_2^{*3} E_2^* I_2 - C_4^* \beta_2^{*3} E_2^* I_2 + C_2^* \alpha_2^* T + C_4^* \beta_2^* T = F_1 \quad ,$$

where $\tilde{F}_1 \equiv F_1 e^{j\omega t}$;

$$2. \quad -C_1^* \alpha_2^{*2} + C_3^* \beta_2^{*2} = 0 \quad ;$$

$$3. \quad -C_1^* \alpha_2^{*2} \cos \alpha_2^* L_2 - C_2^* \alpha_2^{*2} \sin \alpha_2^* L_2 + C_3^* \beta_2^{*2} \cosh \beta_2^* L_2 + C_4^* \beta_2^{*2} \sinh \beta_2^* L_2 = 0 \quad ;$$

$$4. \quad -E_2^* I_2 (C_1^* \alpha_2^{*3} \sin \alpha_2^* L_2 - C_2^* \alpha_2^{*3} \cos \alpha_2^* L_2 + C_3^* \beta_2^{*3} \sinh \beta_2^* L_2 + C_4^* \beta_2^{*3} \cosh \beta_2^* L_2) + T (-C_1^* \alpha_2^* \sin \alpha_2^* L_2 + C_2^* \alpha_2^* \cos \alpha_2^* L_2 + C_3^* \beta_2^* \sinh \beta_2^* L_2 + C_4^* \beta_2^* \cosh \beta_2^* L_2) = +M\omega^2 (C_1^* \cos \alpha_2^* L_2 + C_2^* \sin \alpha_2^* L_2 + C_3^* \cosh \beta_2^* L_2 + C_4^* \sinh \beta_2^* L_2) \quad .$$

The determinant of the boundary conditions is:

0	$\alpha_2^3 E_2^* I_2 + \alpha_2^* T$	0
$-\alpha_2^{*2}$	0	β_2^{*2}
$-\alpha_2^{*2} \cos \alpha_2^* L_2$	$-\alpha_2^{*2} \sin \alpha_2^* L_2$	$\beta_2^{*3} \cosh \beta_2^* L_2$
$-E_2^* I_2 \alpha_2^{*3} \sin \alpha_2^* L_2$ $-T \alpha_2^* \sin \alpha_2^* L_2$ $-M \omega^2 \cos \alpha_2^* L_2$	$+E_2^* I_2 \alpha_2^3 \cos \alpha_2^* L_2$ $+T \alpha_2^* \cos \alpha_2^* L_2$ $-M \omega^2 \sin \alpha_2^* L_2$	$-E_2^* I_2 \beta_2^{*3} \sinh \beta_2^* L_2$ $+T \beta_2^* \sinh \beta_2^* L_2$ $-M \omega^2 \cosh \beta_2^* L_2$
$- \beta_2^{*3} E_2^* I_2 + \beta_2^* T$		
0		
$\beta_2^{*2} \sinh \beta_2^* L_2$		
$= \text{DETZ}$		
$-E_2^* I_2 \beta_2^{*3} \cosh \beta_2^* L_2$ $+T \beta_2^* \cosh \beta_2^* L_2$ $-M \omega^2 \sinh \beta_2^* L_2$		

which reduces to

$$\begin{aligned}
 \text{DETZ} = & E_2^{*2} I_2^2 \alpha_2^{*4} \beta_2^{*4} (-2 \alpha_2^* \beta_2^* + 2 \alpha_2^* \beta_2^* \cos \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
 & + \alpha_2^{*2} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 - \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2) \\
 & + T^2 \alpha_2^{*2} \beta_2^{*2} (2 \alpha_2^* \beta_2^* - 2 \alpha_2^* \beta_2^* \cos \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
 & + \beta_2^{*2} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 - \alpha_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2)
 \end{aligned}$$

$$\begin{aligned}
& + TE_2^* I_2 \alpha_2^{*3} \beta_2^{*3} (2 \alpha_2^{*2} - 2 \alpha_2^{*2} \cosh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
& + 4 \alpha_2^* \beta_2^* \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 + 2 \beta_2^{*2} \cosh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
& - 2 \beta_2^{*2}) \\
& + M\omega^2 E_2^* I_2 \alpha_2^{*2} \beta_2^{*2} (\alpha_2^{*3} \cos \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
& + \alpha_2^* \beta_2^{*2} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 - \alpha_2^{*2} \beta_2^* \sin \alpha_2^* L_2 \\
& - \beta_2^{*3} \cosh \beta_2^* L_2 \sin \alpha_2^* L_2) \\
& + M\omega^2 T \alpha_2^* \beta_2^* (\alpha_2^{*2} \beta_2^* \cos \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
& + \beta_2^{*3} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 + \alpha_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& + \alpha_2^* \beta_2^{*2} \cosh \beta_2^* L_2 \sin \alpha_2^* L_2) \tag{C-28}
\end{aligned}$$

The constants $C_1^* - C_4^*$ are

F_1	$\alpha_2^{*3} E_2^* I_2 + \alpha_2^* T$	0	$-\beta_2^{*3} E_2^* I_2 + \beta_2^* T$
0	0	β_2^{*2}	0
$C_1^* = 0$	$-\alpha_2^{*2} \sin \alpha_2^* L_2$	$\beta_2^{*2} \cosh \beta_2^* L_2$	$\beta_2^{*2} \sinh \beta_2^* L_2$
0	$+E_2^* I_2 \alpha_2^{*3} \cos \alpha_2^* L_2$ $+T \alpha_2^* \cos \alpha_2^* L_2$ $-M\omega^2 \sin \alpha_2^* L_2$	$-E_2^* I_2 \beta_2^{*3} \sinh \beta_2^* L_2$ $+T \beta_2^* \sinh \beta_2^* L_2$ $-M\omega^2 \cosh \beta_2^* L_2$	$-E_2^* I_2 \beta_2^{*3} \cosh \beta_2^* L_2$ $+T \beta_2^* \cosh \beta_2^* L_2$ $-M\omega^2 \sinh \beta_2^* L_2$

DETZ

$$C_1^* = \frac{F_1}{\text{DETZ}} \left[-E_2^* I_2 \alpha_2^{*2} \beta_2^{*5} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \right. \\
+ T \alpha_2^* \beta_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
- M \omega^2 \alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
+ E_2^* I_2 \alpha_2^{*3} \beta_2^{*4} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
+ T \alpha_2^* \beta_2^{*4} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
\left. - M \omega^2 \beta_2^4 \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 \right] . \quad (C-29)$$

$$C_2^* = \frac{1}{\text{DETZ}} \begin{vmatrix} 0 & F_1 & 0 & -\beta_2^{*3} E_2^* I_2 + \beta_2^* T \\ -\alpha_2^{*2} & 0 & \beta_2^{*2} & 0 \\ -\alpha_2^{*2} \cos \alpha_2^* L_2 & 0 & \beta_2^{*2} \cosh \beta_2^* L_2 & \beta_2^{*2} \sinh \beta_2^* L_2 \\ \boxed{\begin{matrix} -E_2^* I_2 \alpha_2^{*3} \sin \alpha_2^* L_2 \\ -T \alpha_2^* \sin \alpha_2^* L_2 \\ -M \omega^2 \cos \alpha_2^* L_2 \end{matrix}} & 0 & \boxed{\begin{matrix} -E_2^* I_2 \beta_2^{*3} \sinh \beta_2^* L_2 \\ +T \beta_2^* \sinh \beta_2^* L_2 \\ -M \omega^2 \cosh \beta_2^* L_2 \end{matrix}} & \boxed{\begin{matrix} -E_2^* I_2 \beta_2^{*3} \cosh \beta_2^* L_2 \\ +T \beta_2^* \cosh \beta_2^* L_2 \\ -M \omega^2 \sinh \beta_2^* L_2 \end{matrix}} \end{vmatrix}$$

$$C_2^* = \frac{F_1}{\text{DETZ}} \left[-E_2^* I_2 \alpha_2^{*2} \beta_2^{*5} + T \alpha_2^{*2} \beta_2^{*3} + E_2^* I_2 \beta_2^{*5} \cos \alpha_2^* L_2 \cosh \beta_2^* L_2 \right. \\
- T \alpha_2^{*2} \beta_2^{*3} \cos \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
\left. + M \omega^2 \alpha_2^{*2} \beta_2^{*2} \cos \alpha_2^* L_2 \sinh \beta_2^* L_2 \right]$$

$$\begin{aligned}
& + E_2^* I_2 \alpha_2^{*3} \beta_2^{*4} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 \\
& + T \alpha_2^* \beta_2^{*4} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 \\
& + M \omega^2 \beta_2^{*4} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \Big] \quad . \quad (C-30)
\end{aligned}$$

$$C_3^* = \begin{vmatrix} 0 & \alpha_2^{*3} E_2^* I_2 + \alpha_2^* T F_1 - \beta_2^{*2} E_2^* I_2 + \beta_2^* T & & \\ -\alpha_2^{*2} & 0 & 0 & 0 \\ -\alpha_2^{*2} \cos \alpha_2^* L_2 & -\alpha_2^{*2} \sin \alpha_2^* L_2 & 0 & \beta_2^{*2} \sinh \beta_2^* L_2 \\ \boxed{\begin{array}{l} -E_2^* I_2 \alpha_2^{*3} \sin \alpha_2^* L_2 \\ -T \alpha_2^* \sin \alpha_2^* L_2 \\ -M \omega^2 \cos \alpha_2^* L_2 \end{array}} & \boxed{\begin{array}{l} +E_2^* I_2 \alpha_2 \cos \alpha_2^* L_2 \\ +T \alpha_2^* \cos \alpha_2^* L_2 \\ -M \omega^2 \sin \alpha_2^* L_2 \end{array}} & 0 & \boxed{\begin{array}{l} -E_2^* I_2 \beta_2^{*3} \cosh \beta_2^* L_2 \\ +T \beta_2^* \cosh \beta_2^* L_2 \\ -M \omega^2 \sinh \beta_2^* L_2 \end{array}} \end{vmatrix}$$

DETZ

$$\begin{aligned}
C_3^* = \frac{F_1}{\text{DETZ}} & \Big[-E_2^* I_2 \alpha_2^{*4} \beta_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& + T \alpha_2^{*4} \beta_2^* \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& - M \omega^2 \alpha_2^{*4} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
& + E_2^* I_2 \alpha_2^{*5} \beta_2^{*2} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
& + T \alpha_2^{*3} \beta_2^{*2} \sinh \beta_2^* L_2 \cos \beta_2^* L_2 \\
& - M \omega^2 \alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \Big] \quad . \quad (C-31)
\end{aligned}$$

$$C_4^* = \begin{vmatrix} 0 & \alpha_2^{*3} E_2^* I_2 + \alpha_2^* T & 0 & F_1 \\ -\alpha_2^{*2} & 0 & \beta_2^{*2} & 0 \\ -\alpha_2^{*2} \cos \alpha_2^* L_2 & -\alpha_2^{*2} \sin \alpha_2^* L_2 & \beta_2^{*2} \cosh \beta_2^* L_2 & 0 \\ \boxed{\begin{matrix} -E_2^* I_2 \alpha_2^{*3} \sin \alpha_2^* L_2 \\ -T \alpha_2^* \sin \alpha_2^* L_2 \\ -M \omega^2 \cos \alpha_2^* L_2 \end{matrix}} & \boxed{\begin{matrix} +E_2^* I_2 \alpha_2^{*3} \cos \alpha_2^* L_2 \\ +T \alpha_2^* \cos \alpha_2^* L_2 \\ -M \omega^2 \sin \alpha_2^* L_2 \end{matrix}} & \boxed{\begin{matrix} -E_2^* I_2 \beta_2^{*3} \sinh \beta_2^* L_2 \\ +T \beta_2^* \sinh \beta_2^* L_2 \\ -M \omega^2 \cosh \beta_2^* L_2 \end{matrix}} & 0 \end{vmatrix}$$

DETZ

$$C_4^* = \frac{F_1}{\text{DETZ}} \left[E_2^* I_2 \alpha_2^{*4} \beta_2^{*3} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \right. \\ - T \alpha_2^{*4} \beta_2^* \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\ + M \omega^2 \alpha_2^{*4} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\ - E_2^* I_2 \alpha_2^{*5} \beta_2^{*2} \cosh \beta_2^* L_2 \cos \alpha_2^* L_2 \\ - T \alpha_2^{*3} \beta_2^{*2} \cosh \beta_2^* L_2 \cos \alpha_2^* L_2 \\ + M \omega^2 \alpha_2^{*2} \beta_2^{*2} \cosh \beta_2^* L_2 \sin \alpha_2^* L_2 \\ \left. + E_2^* I_2 \alpha_2^{*5} \beta_2^{*2} + T \alpha_2^{*3} \beta_2^{*2} \right] . \quad (C-32)$$

The driving point impedance of beam 2 is

$$Z_0 = \frac{\tilde{F}_1}{-\left(\frac{\partial \tilde{y}}{\partial t}\right)_{x=0}} \quad (C-27) ,$$

where

$$\begin{aligned}
 \left(\frac{\partial \tilde{y}}{\partial t} \right)_{x=0} &= j\omega e^{j\omega t} (C_1^* + C_3^*) \\
 &= j\omega e^{j\omega t} \frac{F_1}{\text{DET}Z} \left\{ \begin{aligned}
 & \left[-E_2^* I_2 \alpha_2^{*2} \beta_2^{*5} \sin \alpha_2^* L_2 \right. \\
 & \quad \cosh \beta_2^* L_2 + T \alpha_2^{*2} \beta_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
 & \quad - M\omega^2 \alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
 & \quad + E_2^* I_2 \alpha_2^{*3} \beta_2^{*4} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
 & \quad + T \alpha_2^* \beta_2^{*4} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
 & \quad \left. - M\omega^2 \beta_2^{*4} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 \right] \\
 & + \left[-E_2^* I_2 \alpha_2^{*4} \beta_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \right. \\
 & \quad + T \alpha_2^{*4} \beta_2^* \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
 & \quad - M\omega^2 \alpha_2^{*4} \beta_2^{*3} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
 & \quad + E_2^* I_2 \alpha_2^{*5} \beta_2^{*2} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
 & \quad + T \alpha_2^{*3} \beta_2^{*2} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
 & \quad \left. - M\omega^2 \alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \right] \Big\} . \quad (C-33)
 \end{aligned}
 \right.
 \end{aligned}$$

Recalling that $\tilde{F}_1 = F_1 e^{j\omega t}$, the driving point impedance of beam ② is given by

$$Z_0 = \frac{\tilde{F}_1}{-\left(\frac{\partial y}{\partial t}\right)_{x=0}} = \frac{DETZ}{-j\omega \{\text{Terms from (C-33)}\}} .$$

If $ZN_0 = j\omega Z_0$, ZN_0 , the normalized driving point impedance, is given in terms of beam 2 properties as

$$\begin{aligned} ZN_0 = & \frac{DETZ}{E_2^* I_2 \alpha_2^{*2} \beta_2^{*2} (+\beta_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2} \\ & - \alpha_2^* \beta_2^{*2} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 + \alpha_2^{*2} \beta_2^* \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\ & - \alpha_2^{*3} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2) - T \alpha_2^* \beta_2^* (\alpha_2^* \beta_2^{*2} \sin \alpha_2^* L_2 \\ & \cosh \beta_2^* L_2 + \beta_2^{*3} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 \\ & + \alpha_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 + \alpha_2^{*2} \beta_2^* \sinh \beta_2^* L_2 \cos \alpha_2^* L_2) \\ & + Mw^2 (\alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\ & + \beta_2^{*4} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 + \alpha_2^{*4} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\ & + \alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2) . \end{aligned} \quad (C-34)$$

The complete transmissibility equation for the two-segment beam, using Eqs. (C-24), (C-26) and (C-34), is given by

$$T_m = (T_{m_1})(T_{m_2}) .$$

$$\begin{aligned}
T_m = & \left[+\alpha_1^{*3} \beta_1^{*4} E_1^* I_1 \sinh \beta_1^* L_1 + \alpha_1^* \beta_1^{*4} T \sinh \beta_1^* L_1 \right. \\
& - \alpha_1^{*4} \beta_1^{*3} E_1^* I_1 \sin \alpha_1^* L_1 + \alpha_1^{*4} \beta_1^* T \sin \alpha_1^* L_1 \\
& - \alpha_1^{*2} \beta_1^{*5} E_1^* I_1 \sin \alpha_1^* L_1 + \alpha_1^{*2} \beta_1^{*3} T \sin \alpha_1^* L_1 \\
& \left. + \alpha_1^{*5} \beta_1^{*2} E_1^* I_1 \sinh \beta_1^* L_1 + \alpha_1^{*3} \beta_1^{*2} T \sinh \beta_1^* L_1 \right] \\
& \left[+\alpha_2^* \beta_2^{*4} E_2^* I_1 \sinh \beta_2^* L_2 + \alpha_2^* \beta_2^{*4} T \sinh \beta_2^* L_2 \right. \\
& - \alpha_2^{*4} \beta_2^{*3} E_2^* I_2 \sin \alpha_2^* L_2 + \alpha_2^{*4} \beta_2^* T \sin \alpha_2^* L_1 \\
& - \alpha_2^{*2} \beta_2^{*5} E_2^* I_2 \sin \alpha_2^* L_2 + \alpha_2^{*2} \beta_2^{*3} T \sin \alpha_2^* L_2 \\
& \left. + \alpha_2^{*5} \beta_2^{*2} E_2^* I_2 \sinh \beta_2^* L_2 + \alpha_2^{*3} \beta_2^{*2} T \sinh \beta_2^* L_2 \right] \\
& \left[(\beta_1^{*2} + \alpha_1^{*2}) \alpha_1^{*2} \sin \alpha_1^* L_1 (-E_1^* I_1 \beta_1^{*3} \cosh \beta_1^* L_1 \right. \\
& + T \beta_1^* \cosh \beta_1^* L_1 + ZN_0 \sinh \beta_1^* L_1) + \beta_1^{*4} \sinh \beta_1^* L_1 \\
& (+E_1^* I_1 \alpha_1^{*3} \cos \alpha_1^* L_1 + T \alpha_1^* \cos \alpha_1^* L_1 + ZN_0 \sin \alpha_1^* L_1) \\
& + \alpha_1^{*2} \beta_1^{*2} \sinh \beta_1^* L_1 (+E_1^* I_1 \alpha_1^{*3} \cos \alpha_1^* L_1 \\
& + T \alpha_1^* \cos \alpha_1^* L_1 + ZN_0 \sin \alpha_1^* L_1) \left. \right] \left[(\beta_2^{*2} + \alpha_2^{*2}) \alpha_2^{*2} \right. \\
& \sin \alpha_2^* L_2 (-E_2^* I_2 \beta_2^{*3} \cosh \beta_2^* L_2 + T \beta_2^* \cosh \beta_2^* L_2 \\
& - M\omega^2 \sinh \beta_2^* L_2) + \beta_2^{*4} \sinh \beta_2^* L_2 (+E_2^* I_2 \alpha_2^{*3} \cos \alpha_2^* L_2 \\
& + T \alpha_2^* \cos \alpha_2^* L_2 - M\omega^2 \sin \alpha_2^* L_2) + \alpha_2^{*2} \beta_2^* \sinh \beta_2^* L_2 \\
& \left. (+E_2^* I_2 \alpha_2^{*3} \cos \alpha_2^* L_2 + T \alpha_2^* \cos \alpha_2^* L_2 - M\omega^2 \sin \alpha_2^* L_2) \right] . (C-35)
\end{aligned}$$

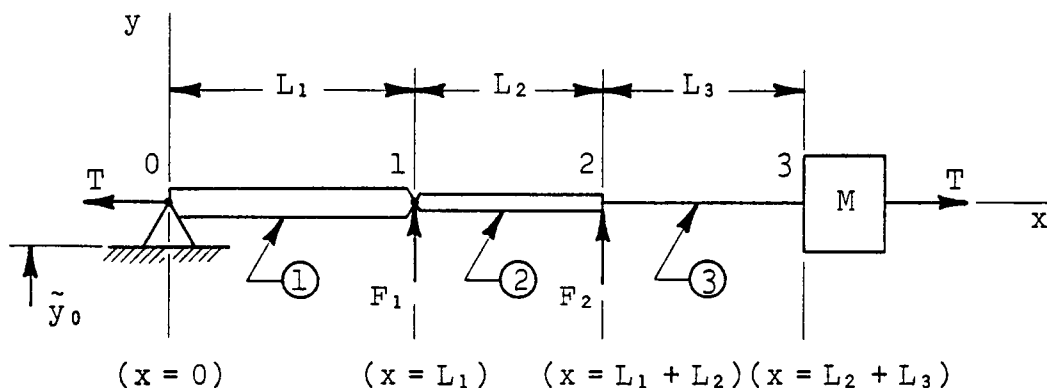
Equation (C-35) may be used to calculate the transmissibility of a two-segment beam with any given variation of dynamic Young's modulus and damping factor with frequency by using the relation

$$E_{\omega}^* = E\omega(1 + j\delta_{E\omega}) \quad (A-1)$$

and calculating α^* and β^* using E_{ω}^* .

3. Two-Segment Beam with a String

The above results for the two-segment beam can be further extended to obtain the transmissibility of a two-segment beam with a string, as shown in the figure below.



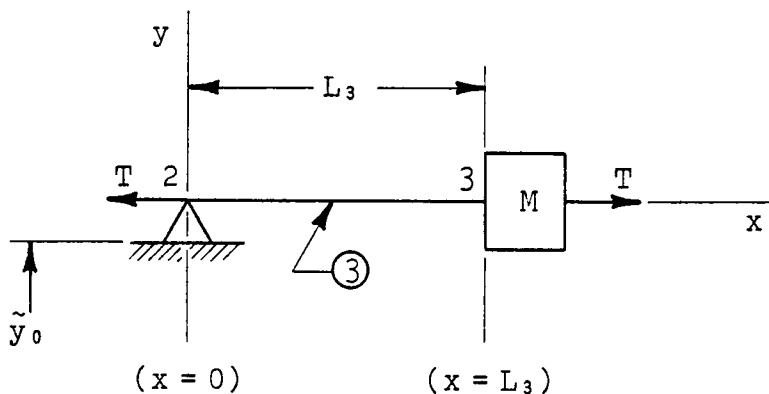
The overall transmissibility of the above system is

$$T_m = T_{m_1} T_{m_2} T_{m_3} .$$

In this case, the value of T_{m_2} is dependent on the driving point impedance of the string. T_{m_1} is dependent on the driving point impedance of beam ②, which is also affected by the addition of the string. In order to obtain T_m , one must first obtain T_{m_3} . The previous results for the two-segment beam must then be modified in order to obtain the new T_{m_1} and T_{m_2} , by replacing all $M\omega^2$ terms in $DETZ$, ZN_0 , and T_{m_2} , with $-j\omega Z_{03}$, where

$$Z_{03} = \frac{\tilde{F}_2}{-\left(\frac{\partial \tilde{y}}{\partial t}\right)_{x=L_1+L_2}} \quad (C-36)$$

T_{m_3} must now be evaluated in terms of string ③ properties. This task is simplified by shifting the origin as shown in the figure below.



The basic wave equation for transverse vibration of a string is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad , \quad (C-37)$$

where

$$a = \sqrt{\frac{T}{\rho}} \quad ,$$

ρ = Mass per unit length,

$$\gamma = \frac{\omega}{a} \quad ,$$

T = Tensile force.

The solution to Eq. (C-37) is assumed to be of the form

$$\tilde{y} = e^{j\omega t} (C_1 \cos \gamma x + C_2 \sin \gamma x) \quad , \quad (C-38)$$

where C_1 and C_2 are constants to be determined from the boundary conditions. The necessary partial derivatives of Eq. (C-38) are

$$\frac{\partial \tilde{y}}{\partial x} = e^{j\omega t} (-C_1 \gamma \sin \gamma x + C_2 \gamma \cos \gamma x) \quad , \quad (C-39)$$

$$\frac{\partial \tilde{y}}{\partial t} = j\omega e^{j\omega t} (C_1 \cos \gamma x + C_2 \sin \gamma x) \quad , \quad (C-40)$$

$$\frac{\partial^2 \tilde{y}}{\partial t^2} = -\omega^2 e^{j\omega t} (C_1 \cos \gamma x + C_2 \sin \gamma x) \quad . \quad (C-41)$$

The boundary conditions for string ③ are:

1. $(\tilde{y})_{x=0} = \tilde{y}_0$ (Motion of support) ,
2. $T \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=L_3} = -M \left(\frac{\partial^2 \tilde{y}}{\partial t^2} \right)_{x=L_3}$ (Tension component = MA) .

The constants, obtained directly by substituting Eqs. (C-39), (C-40), and (C-41) into the boundary conditions, are:

$$C_1 = y_0 , \quad (C-42)$$

$$C_2 = \frac{y_0 (T \gamma \sin \gamma L_3 + M \omega^2 \cos \gamma L_3)}{(T \gamma \cos \gamma L_3 - M \omega^2 \sin \gamma L_3)} . \quad (C-43)$$

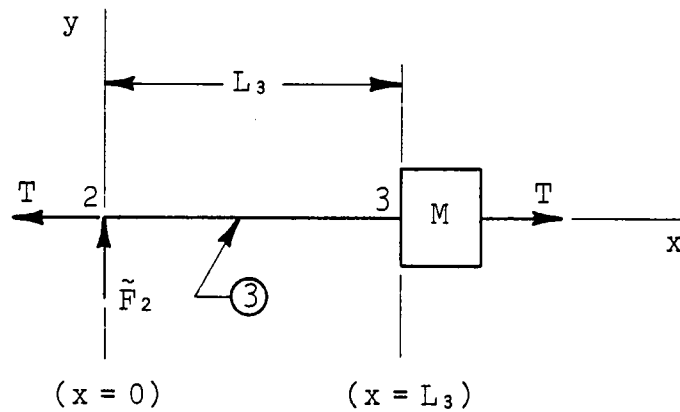
The solution at $x = L_3$, obtained by substituting Eqs. (C-42) and (C-43) back into (C-38), is:

$$(\tilde{y})_{x=L_3} = y_0 e^{j\omega t} \left[\cos \gamma L_3 + \frac{(T \gamma \sin \gamma L_3 + M \omega^2 \cos \gamma L_3)}{(T \gamma \cos \gamma L_3 - M \omega^2 \sin \gamma L_3)} \sin \gamma L_3 \right] . \quad (C-44)$$

The transmissibility, T_{m_3} , is then

$$T_{m_3} = \left| \frac{(\tilde{y})_{x=L_3}}{(\tilde{y})_{x=0}} \right| = \left[\cos \gamma L_3 + \frac{(T \gamma \sin \gamma L_3 + M \omega^2 \cos \gamma L_3)}{(T \gamma \cos \gamma L_3 - M \omega^2 \sin \gamma L_3)} \sin \gamma L_3 \right] . \quad (C-45)$$

The driving point impedance of the string, Z_{03} , must now be obtained in terms of string ③ properties. This task may be simplified, as in previous cases, by shifting the origin as shown in the figure below.



The boundary conditions, with a sinusoidal force at $x=0$, are

1. $T \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=0} = \tilde{F}$ (Tension component = Force) ,
2. $T \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=L_3} = -M \left(\frac{\partial^2 \tilde{y}}{\partial t^2} \right)_{x=L_3}$ (Tension component = MA) .

The constants, evaluated directly by substituting Eqs. (C-39), (C-40), and (C-41) into the boundary conditions, are:

$$C_2 = \frac{F}{T\gamma} \quad , \quad (C-46)$$

$$C_1 = \frac{F(T\gamma \cos \gamma L_3 - M\omega^2 \sin \gamma L_3)}{T\gamma(T\gamma \sin \gamma L_3 + M\omega^2 \cos \gamma L_3)} \quad . \quad (C-47)$$

Substituting C_1 and C_2 into Eq. (C-40) gives

$$\left(\frac{\partial \tilde{y}}{\partial t} \right)_{x=0} = j\omega e^{j\omega t} \left(\frac{F}{T\gamma} \right) \left(\frac{T\gamma \cos \gamma L_3 - M\omega^2 \sin \gamma L_3}{T\gamma \sin \gamma L_3 + M\omega^2 \cos \gamma L_3} \right) \quad . \quad (C-48)$$

The driving point impedance, Z_{03} , is then

$$Z_{03} = \frac{\tilde{F}}{- \left(\frac{\partial \tilde{y}}{\partial t} \right)} = \frac{T\gamma}{- j\omega \left(\frac{T\gamma \cos \gamma L_3 - M\omega^2 \sin \gamma L_3}{T\gamma \sin \gamma L_3 + M\omega^2 \cos \gamma L_3} \right)} \quad . \quad (C-49)$$

The normalized driving point impedance, ZN_{03} , is

$$\begin{aligned} ZN_{03} &= j\omega Z_{03} \\ &= \frac{- T\gamma(T\gamma \sin \gamma L_3 + M\omega^2 \cos \gamma L_3)}{T\gamma \cos \gamma L_3 - M\omega^2 \sin \gamma L_3} \quad . \end{aligned} \quad (C-50)$$

The expressions for DETZ and ZN_0 from the previous case, modified by replacing $M\omega^2$ terms with $- ZN_{03}$, are:

$$\begin{aligned}
\text{DET}Z_2 = & E_2^{*2} I_2^2 \alpha_2^{*4} \beta_2^{*4} (-2 \alpha_2^* \beta_2^* + 2 \alpha_2^* \beta_2^* \cos \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& + \alpha_2^{*2} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 - \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2) \\
& + T^2 \alpha_2^{*2} \beta_2^{*2} (2 \alpha_2^* \beta_2^* - 2 \alpha_2^* \beta_2^* \cos \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& + \beta_2^{*2} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 - \alpha_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2) \\
& + T E_2^* I_2 \alpha_2^{*3} \beta_2^{*2} (2 \alpha_2^{*2} - 2 \alpha_2^{*2} \cosh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
& + 4 \alpha_2^* \beta_2^* \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 + 2 \beta_2^{*2} \cosh \beta_2^* L_2 \cos \alpha_2^* L_2 \\
& - 2 \beta_2^{*2}) \\
& - ZN_{03} E_2^* I_2 \alpha_2^{*2} \beta_2^{*2} (\alpha_2^{*3} \cos \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
& + \alpha_2^* \beta_2^{*2} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 - \alpha_2^{*2} \beta_2^* \sin \alpha_2^* L_2 \\
& - \beta_2^{*3} \cosh \beta_2^* L_2 \sin \alpha_2^* L_2) \\
& - ZN_{03} T \alpha_2^* \beta_2^* (\alpha_2^{*2} \beta_2^* \cos \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
& + \beta_2^{*3} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 + \alpha_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& + \alpha_2^* \beta_2^{*2} \cosh \beta_2^* L_2 \sin \alpha_2^* L_2) \tag{C-51}
\end{aligned}$$

$$\begin{aligned}
ZN_{02} = & \frac{\text{DET}Z_2}{E_2^* I_2 \alpha_2^{*2} \beta_2^{*2} (\beta_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& - \alpha_2^* \beta_2^{*2} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 + \alpha_2^{*2} \beta_2^* \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& - \alpha_2^{*3} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2)} \tag{C-52}
\end{aligned}$$

$$\begin{aligned}
& -T\alpha_2^* \beta_2^* (\alpha_2^* \beta_2^{*2} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& + \beta_2^{*3} \sinh \beta_2^* L_2 \cos \alpha_2^* L_2 + \alpha_2^{*3} \sin \alpha_2^* L_2 \cosh \beta_2^* L_2 \\
& + \alpha_2^{*2} \beta_2^* \sinh \beta_2^* L_2 \cos \alpha_2^* L_2) \\
& -ZN_{03} (\alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
& + \beta_2^{*4} \sinh \beta_2^* L_2 \sin \alpha_2^* L_2 + \alpha_2^{*4} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2 \\
& + \alpha_2^{*2} \beta_2^{*2} \sin \alpha_2^* L_2 \sinh \beta_2^* L_2)
\end{aligned}$$

The modified expressions for T_{m_1} and T_{m_2} are

$$\begin{aligned}
T_{m_1} = & \left[\alpha_1^{*3} \beta_1^{*4} E_1^* \sinh \beta_1^* L_1 + \alpha_1^* \beta_1^{*4} T \sinh \beta_1^* L_1 \right. \\
& - \alpha_1^{*4} \beta_1^{*3} E_1^* I_1 \sin \alpha_1^* L_1 + \alpha_1^* \beta_1^* T \sin \alpha_1^* L_1 \\
& - \alpha_1^{*2} \beta_1^{*5} E_1^* I_1 \sin \alpha_1^* L_1 + \alpha_1^{*2} \beta_1^{*3} T \sin \alpha_1^* L_1 \\
& + \alpha_1^{*5} \beta_1^{*2} E_1^* I_1 \sinh \beta_1^* L_1 + \alpha_1^{*3} \beta_1^{*2} \sinh \beta_1^* L_1 \\
& \left. (\beta_1^{*2} + \alpha_1^{*2}) \alpha_1^{*2} \sin \alpha_1^* L_1 (-E_1^* I_1 \beta_1^{*3} \cosh \beta_1^* L_1 \right. \\
& + T \beta_1^* \cosh \beta_1^* L_1 + ZN_{02} \sinh \beta_1^* L_1) \\
& + \beta_1^{*4} \sinh \beta_1^* L_1 (E_1^* I_1 \alpha_1^{*3} \cos \alpha_1^* L_1 \\
& + T \alpha_1^* \cos \alpha_1^* L_1 + ZN_{02} \sin \alpha_1^* L_1) \\
& + \alpha_1^{*2} \beta_1^{*2} \sinh \beta_1^* L_1 (E_1^* I_1 \alpha_1^{*3} \cos \alpha_1^* L_1 \\
& \left. + T \alpha_1^* \cos \alpha_1^* L_1 + ZN_{02} \sin \alpha_1^* L_1) \right]
\end{aligned}$$

(C-53)

$$\begin{aligned}
T_{m_2} = & \left[\alpha_2^{*3} \beta_2^{*4} E_2^* I_2 \sinh \beta_2^* L_2 + \alpha_2^* \beta_2^{*4} T \sinh \beta_2^* L_2 \right. \\
& - \alpha_2^{*4} \beta_2^{*3} E_2^* I_2 \sin \alpha_2^* L_2 + \alpha_2^{*4} \beta_2^* T \sin \alpha_2^* L_2 \\
& - \alpha_2^{*2} \beta_2^{*5} E_2^* I_2 \sin \alpha_2^* L_2 + \alpha_2^{*2} \beta_2^{*3} T \sin \alpha_2^* L_2 \\
& \left. + \alpha_2^{*5} \beta_2^{*2} E_2^* I_2 \sinh \beta_2^* L_2 + \alpha_2^{*3} \beta_2^{*2} T \sinh \beta_2^* L_2 \right] \\
& \left[(\beta_2^{*2} + \alpha_2^{*2}) \alpha_2^{*2} \sin \alpha_2^* L_2 (-E_2^* I_2 \beta_2^{*3} \cosh \beta_2^* L_2 \right. \\
& + T \beta_2^* \cosh \beta_2^* L_2 + Z N_{03} \sinh \beta_2^* L_2) \\
& + \beta_2^{*4} \sinh \beta_2^* L_2 (E_2^* I_2 \alpha_2^{*3} \cos \alpha_2^* L_2 \\
& + T \alpha_2^* \cos \alpha_2^* L_2 + Z N_{03} \sin \alpha_2^* L_2) \\
& + \alpha_2^{*2} \beta_2^{*2} \sinh \beta_2^* L_2 (E_2^* I_2 \alpha_2^{*3} \cos \alpha_2^* L_2 \\
& \left. + T \alpha_2^* \cos \alpha_2^* L_2 + Z N_{03} \sin \alpha_2^* L_2) \right] \quad (C-54)
\end{aligned}$$

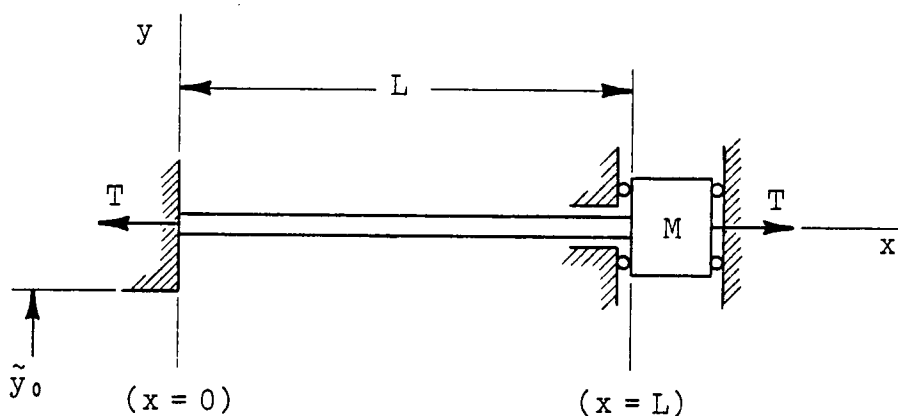
The overall transmissibility, T_m , for the case of a two-segment beam with a string is obtained by combining Eqs. (C-50) - (C-54) with (C-45).

$$T_m = T_{m_1} T_{m_2} T_{m_3} \quad (C-55)$$

4. Fixed-Guided Beam

Another useful transmissibility expression may be obtained for the case of a mass loaded beam

in tension with one end fixed and the other end guided as shown in the figure below



The boundary conditions for this beam are

$$1. \quad (\tilde{y})_{x=0} = \tilde{y}_0 \quad (\text{Motion of support})$$

$$2. \quad \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=0} = 0 \quad (\text{Slope} = 0)$$

$$3. \quad \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=L} = 0 \quad (\text{Slope} = 0)$$

$$4. \quad -E^* I \left(\frac{\partial^3 \tilde{y}}{\partial x^3} \right)_{x=L} + T \left(\frac{\partial \tilde{y}}{\partial x} \right)_{x=L} = -M \left(\frac{\partial^2 \tilde{y}}{\partial t^2} \right)_{x=L}$$

(Shear force + Tension component = MA)

After substitution of Eqs. (C-11) - (C-15), the boundary conditions are:

$$1. \quad C_1^* + C_3^* = y_0$$

$$2. \quad C_2^* \alpha^* + C_4^* \beta^*$$

$$3. \quad C_1^* \alpha^* \sin \alpha^* L + C_2^* \alpha^* \cos \alpha^* L + C_3^* \beta^* \sinh \beta^* L \\ + C_4^* \beta^* \cosh \beta^* L = 0$$

$$4. \quad -E^* I (C_1^* \alpha^{*3} \sin \alpha^* L - C_2^* \alpha^{*3} \cos \alpha^* L + C_3^* \beta^* \sinh \beta^* L \\ + C_4^* \beta^{*3} \cosh \beta^* L) + T (-C_1^* \alpha^* \sin \alpha^* L + C_2^* \alpha^* \cos \alpha^* L \\ + C_3^* \beta^* \sinh \beta^* L + C_4^* \beta^* \cosh \beta^* L) \\ = M\omega^2 (C_1^* \cos \alpha^* L + C_2^* \sin \alpha^* L + C_3^* \cosh \beta^* L + C_4^* \sinh \beta^* L)$$

The determinant of the boundary conditions is

$$\text{DET} = (\alpha^* \beta^* \sinh \beta^* L + \alpha^{*2} \sin \alpha^* L) (-E^* I \beta^{*3} \cosh \beta^* L \\ + T \beta^* \cosh \beta^* L - M\omega^2 \sinh \beta^* L) + (-\alpha^* \beta^* \cosh \beta^* L \\ + \alpha^* \beta^* \cos \alpha^* L) (-E^* I \beta^{*3} \sinh \beta^* L + T \beta^* \sinh \beta^* L \\ - M\omega^2 \cosh \beta^* L) - (\beta^{*2} \sinh \beta^* L + \alpha^* \beta^* \sin \alpha^* L) \\ (E^* I \alpha^{*3} \cos \alpha^* L + T \alpha^* \cos \alpha^* L - M\omega^2 \sin \alpha^* L) \\ + (\alpha^* \beta^* \cosh \beta^* L - \alpha^* \beta^* \cos \alpha^* L) (-E^* I \alpha^{*3} \sin \alpha^* L \\ - T \alpha^* \sin \alpha^* L - M\omega^2 \cos \alpha^* L) \quad (C-56)$$

The constants, $C_1^* - C_4^*$, are

$$\begin{aligned}
 C_1^* = \frac{Y_0}{\text{DET}} & \left[\alpha^* \beta^* \sinh \beta^* L (-E^* I \beta^{*3} \cosh \beta^* L + T \beta^* \cosh \beta^* L \right. \\
 & - M\omega^2 \sinh \beta^* L) + (-\alpha^* \beta^* \cosh \beta^* L + \alpha^* \beta^* \cos \alpha^* L) \\
 & (-E^* I \beta^{*3} \sinh \beta^* L + T \beta^* \sinh \beta^* L - M\omega^2 \cosh \beta^* L) \\
 & - \beta^{*2} \sinh \beta^* L (E^* I \alpha^{*3} \cos \alpha^* L + T \alpha^* \cos \alpha^* L \\
 & \left. - M\omega^2 \sin \alpha^* L) \right] \quad (C-57)
 \end{aligned}$$

$$\begin{aligned}
 C_2^* = \frac{-Y_0}{\text{DET}} & \left\{ \beta^* \left[-\alpha^* \sin \alpha^* L (-E^* I \beta^{*3} \sinh \beta^* L + T \beta^* \sinh \beta^* L \right. \right. \\
 & - M\omega^2 \cosh \beta^* L) - \beta^* \sinh \beta^* L (-E^* I \alpha^{*3} \sin \alpha^* L \\
 & \left. \left. - T \alpha^* \sin \alpha^* L - M\omega^2 \cos \alpha^* L) \right] \right\} \quad (C-58)
 \end{aligned}$$

$$\begin{aligned}
 C_3^* = \frac{Y_0}{\text{DET}} & \left[\alpha^{*2} \sin \alpha^* L (-E^* I \beta^{*3} \cosh \beta^* L + T \beta^* \cosh \beta^* L \right. \\
 & - M\omega^2 \sinh \beta^* L) + (\alpha^* \beta^* \cosh \beta^* L - \alpha^* \beta^* \cos \alpha^* L) \\
 & (-E^* I \alpha^{*3} \sin \alpha^* L - T \alpha^* \sin \alpha^* L - M\omega^2 \cos \alpha^* L) \\
 & - \alpha^* \beta^* \sin \alpha^* L (E^* I \alpha^{*3} \cos \alpha^* L + T \alpha^* \cos \alpha^* L \\
 & \left. - M\omega^2 \sin \alpha^* L) \right] \quad (C-59)
 \end{aligned}$$

$$C_4^* = \frac{-y_0}{\text{DET}} \left\{ -\alpha^* \left[-\alpha^* \sin \alpha^* L (-E^* I \beta^{*3} \sinh \beta^* L + T \beta^* \sinh \beta^* L \right. \right. \\ \left. \left. - M\omega^2 \cosh \beta^* L) - \beta^* \sinh \beta^* L (-E^* I \alpha^{*3} \sin \alpha^* L \right. \right. \\ \left. \left. - T \alpha^* \sin \alpha^* L - M\omega^2 \cos \alpha^* L) \right] \right\} \quad (\text{C-60})$$

The solution at $x = L$, obtained by substituting Eqs. (C-56) - (C-60) into (C-10), is

$$(\tilde{y})_{x=L} = \frac{y_0 e^{j\omega t}}{\text{DET}} \left\{ \alpha^* \beta^* \sinh \beta^* L (-E^* I \beta^{*3} \cosh \beta^* L \right. \\ \left. + T \beta^* \cosh \beta^* L - M\omega^2 \sinh \beta^* L) + (-\alpha^* \beta^* \cosh \beta^* L \right. \\ \left. + \alpha^* \beta^* \cos \alpha^* L) (-E^* I \beta^{*3} \sinh \beta^* L \right. \\ \left. + T \beta^* \sinh \beta^* L - M\omega^2 \cosh \beta^* L) - \beta^{*2} \sinh \beta^* L \right. \\ \left. (E^* I \alpha^{*3} \cos \alpha^* L + T \alpha^* \cos \alpha^* L - M\omega^2 \sin \alpha^* L) \right] \cos \alpha^* L \\ - \beta^* \left[-\alpha^* \sin \alpha^* L (-E^* I \beta^{*3} \sinh \beta^* L + T \beta^* \sinh \beta^* L \right. \\ \left. - M\omega^2 \cosh \beta^* L) - \beta^* \sinh \beta^* L (-E^* I \alpha^{*3} \sin \alpha^* L \right. \\ \left. - T \alpha^* \sin \alpha^* L - M\omega^2 \cos \alpha^* L) \right] \sin \alpha^* L \\ + \left[\alpha^{*2} \sin \alpha^* L (-E^* I \beta^{*3} \cosh \beta^* L + T \beta^* \cosh \beta^* L \right. \\ \left. - M\omega^2 \sinh \beta^* L) + (\alpha^* \beta^* \cosh \beta^* L - \alpha^* \beta^* \cos \beta^* L) \right. \\ \left. (-E^* I \alpha^{*3} \sin \alpha^* L - T \alpha^* \sin \alpha^* L - M\omega^2 \cos \alpha^* L) \right]$$

$$\begin{aligned}
& -\alpha^* \beta^* \sin \alpha^* L (E^* I \alpha^{*3} \cos \alpha^* L + T \alpha^* \cos \alpha^* L \\
& - M \omega^2 \sin \alpha^* L) \cosh \beta^* L \\
& + \alpha^* \left[-\alpha^* \sin \alpha^* L (-E^* I \beta^{*3} \sinh \beta^* L \right. \\
& + T \beta^* \sinh \beta^* L - M \omega^2 \cosh \beta^* L) - \beta^* \sinh \beta^* L \\
& \left. (-E^* I \alpha^{*3} \sin \alpha^* L - T \alpha^* \sin \alpha^* L \right. \\
& \left. - M \omega^2 \cos \alpha^* L) \right] \sinh \beta^* L \Big\} \quad (C-61)
\end{aligned}$$

The transmissibility of a mass loaded, fixed-guided beam with a tensile pre-load, in transverse vibration reduces to

$$\begin{aligned}
T_m = \left| \frac{(\tilde{y})_{x=L}}{(\tilde{y})_{x=0}} \right| = \left| \frac{1}{DET} \left\{ -E^* I \alpha^* \beta^* \left[(\beta^{*3} + \alpha^{*2} \beta^*) \sinh \beta^* L \right. \right. \right. \\
\left. \left. \left. + (\alpha^* \beta^{*2} + \alpha^{*3}) \sin \alpha^* L \right] \right\} \right| \quad (C-62)
\end{aligned}$$

Eq. (C-62) is also valid for beams with a compressive pre-load. When the beam is in compression, T is negative and the natural frequency of the system decreases as the compressive load increases. The load at which the natural frequency is zero is the Euler critical load for column buckling.

APPENDIX D

COMPUTER PROGRAMS FOR CALCULATING
TRANSVERSE TRANSMISSIBILITY

Table 21 is a listing of the computer program used to calculate the transmissibility of a mass-loaded, two-segment beam with constant damping and a string, in transverse vibration. The program was written in FORTRAN IV to run interactively on the DEC VAX 11/780 Computer. This program can be used to calculate the transverse transmissibility of:

1. A two-segment beam by assuming a string length of zero.
2. A one-segment beam by assuming very short beam and string lengths (.001 inch).
3. A string only, by assuming the beams to be very short (.001 inch).

A beam length of zero cannot be used with this program as it would result in an attempt to divide by zero.

The main program is used for data input and controlling the frequency-step size. Input data are first read from the input data file. The program calculates the transmissibility at discrete frequency steps between the specified frequency limits. The value in Hz of the

TABLE 21

COMPUTER PROGRAM USED TO CALCULATE THE TRANSMISSIBILITY
OF A MASS-LOADED, TWO-SEGMENT BEAM WITH
A STRING IN TRANSVERSE VIBRATION

```

100 C      PROGRAM FOR OBTAINING THE TRANSMISSIBILITY OF A TWO-SEGMENT
200 C      BEAM WITH A STRING IN TRANSVERSE VIBRATION
300 C
400      COMMON RHO1,REW1,IEW1,A1,I1,L1,RHO2,REW2,IEW2,A2,I2,L2,T,M,
500      1LRHU3,L3
600      DOUBLE PRECISION RHO1,A1,L1,I1,RHO2,A2,I2,L2,T,M,PI,TM,TMDB,WSQ,
700      1REW1,IEW1,REW2,IEW2,1LRHU3,L3
800      OPEN (UNIT=2,NAME='BEAM.DAT',TYPE='OLD')
900      READ (2,40) REW1,IEW1,RHO1,A1,I1,L1,REW2,IEW2,RHO2,A2,I2,L2,
1000     1LRHU3,L3,T,M
1100     40 FORMAT (/,'X',E15.6)
1200     TYPE*, 'ENTER F START'
1300     ACCEPT*, FSTART
1400     TYPE*, 'ENTER F STEP'
1500     ACCEPT*, FSTEP
1600     TYPE*, 'ENTER SMALL STEP'
1700     ACCEPT*, SSTEP
1800     TYPE*, 'ENTER F STOP'
1900     ACCEPT*, FSTOP
2000     FSTEP=FSTEP/4.0
2100     FSSSTEP=SSTEP/4.0
2200     FOLD=0.0
2300     TMDBO=1.0
2400     THS=6.0/(FSTEP+0.5)
2500     F=FSTART
2600     70 CONTINUE
2700     IF (F.GT.FSTOP) GO TO 80
2800     CALL TBEAM(F,TM,TMDB)
2900     SLOPE=ABS((TMDB-TMDBO)/(F-FOLD))
3000     IF (SLOPE.GT.THHS) GO TO 50
3100     IF (SLO.GT.THHS) GO TO 50
3200     SLO=SLOPE
3300     TMDBO=TMDB
3400     FOLD=F
3500     IF (F.GT.2.0) GO TO 60
3600     F=F+FSTEP
3700     GO TO 70
3800     60 CONTINUE
3900     F=F+FSTEP
4000     GO TO 70
4100     50 SLO=SLOPE
4200     TMDBO=TMDB
4300     FOLD=F
4400     IF (F.GT.2.0) GO TO 90
4500     F=F+FSSSTEP
4600     GO TO 70
4700     90 CONTINUE
4800     F=F+SSTEP
4900     GO TO 70
5000     40 CONTINUE
5100     END

```

TABLE 21 (Continued)

```

100 C SUBROUTINE TBEAM FOR OBTAINING THE TRANSMISSIBILITY
200 C OF A TWO-SEGMENT BEAM WITH A STRING IN TRANSVERSE
300 C VIBRATION
400 C
500 C F -FREQUENCY (HZ)
600 C TM -TRANSMISSIBILITY
700 C TMDB-TRANSMISSIBILITY (DB)
800 C
900 C SUBROUTINE TBEAM(F, TM, TMDB)
1000 COMMON RHU1, REW1, IEW1, A1, I1, L1, RHU2, REW2, IEW2, A2, I2, L2, T, M,
1100 1, LKH03, L3
1200 REAL P
1300 DOUBLE PRECISION RHU1, A1, I1, L1, RHU2, A2, I2, L2, T, M, PI, TM, TMDB, WSQ,
1400 1REW1, IEW1, REW2, IEW2, LKH03, L3, GM, C3, S3, ZN03
1500 COMPLEX*8 EW1, EW2, AP1, AP2, BT1, BT2, C1, S1, CH1, SH1, C2, S2, CH2, SH2,
1600 1DETZ, ZN02, NUM1, DEN1, NUM2, DEN2, CCOSH, CSINH, X, DNZ
1700 CCOSH(X)=CMPLX(COSH(REAL(X))*COS(AIMAG(X)), SINH(REAL(X))*
1800 1SIN(AIMAG(X)))
1900 CSINH(X)=CMPLX(SINH(REAL(X))*COS(AIMAG(X)), COSH(REAL(X))*
2000 1SIN(AIMAG(X)))
2100 PI=3.141592654
2200 WSQ=(F**2.0*PI)**2
2300 EW1=CMPLX(SNGL(REW1), SNGL(IEW1))
2400 EW2=CMPLX(SNGL(REW2), SNGL(IEW2))
2500 P=SNGL(T)
2600 AP1=CSQRT(-(P/(2.0*EW1*SNGL(I1)))+CSQRT((P/(2.0*EW1*SNGL(I1)))**2
2700 1+(SNGL(WSQ*RHO1*A1)/(EW1*SNGL(I1))*386.04))))
2800 BT1=CSQRT((P/(2.0*EW1*SNGL(I1)))+CSQRT((P/(2.0*EW1*SNGL(I1)))**2
2900 1+(SNGL(WSQ*RHO1*A1)/(EW1*SNGL(I1))*386.04))))
3000 AP2=CSQRT(-(P/(2.0*EW2*SNGL(I2)))+CSQRT((P/(2.0*EW2*SNGL(I2)))**2
3100 1+(SNGL(WSQ*RHO2*A2)/(EW2*SNGL(I2))*386.04))))
3200 BT2=CSQRT((P/(2.0*EW2*SNGL(I2)))+CSQRT((P/(2.0*EW2*SNGL(I2)))**2
3300 1+(SNGL(WSQ*RHO2*A2)/(EW2*SNGL(I2))*386.04))))
3400 GM=DSQRT(LRH03*WSQ/(T*386.04))
3500 C1=CCUS(AP1*SNGL(L1))
3600 S1=CSIN(AP1*SNGL(L1))
3700 CH1=CCOSH(BT1*SNGL(L1))
3800 SH1=CSINH(BT1*SNGL(L1))
3900 C2=CCUS(AP2*SNGL(L2))
4000 S2=CSIN(AP2*SNGL(L2))
4100 CH2=CCOSH(BT2*SNGL(L2))
4200 SH2=CSINH(BT2*SNGL(L2))
4300 C3=DCOS(GM*L3)
4400 S3=DSIN(GM*L3)
4500 ZN03=-T*GM*(T*GM*S3+M*WSQ*C3)/(T*GM*C3-M*WSQ*S3)
4600 C
4700 DETZ=(EW2**2)*(I2**2)*(AP2**4)*(BT2**4)*(-2.0*AP2*BT2+2.0*AP2
4800 1*BT2*C2*CH2+(AP2**2)*SH2*S2-(BT2**2)*S2*SH2)+T*T*(AP2**2)
4900 2*(BT2**2)*(2.0*AP2*BT2-2.0*AP2*BT2*C2*CH2+(BT2**2)*SH2*S2
5000 3-(AP2**2)*S2*SH2)+T*EW2*I2*(AP2**3)*(BT2**3)*(2.0*(AP2**2)
5100 4-2.0*(AP2**2)*CH2*C2+4.0*AP2*BT2*SH2*S2+2.0*(BT2**2)*CH2*C2
5200 5-2.0*(BT2**2)*ZN03*EW2*I2*(AP2**2)*(BT2**2)*((AP2**3)*C2
5300 6*SH2+AP2*(BT2**2)*SH2*C2-(AP2**2)*BT2*S2*CH2-(BT2**3)*CH2
5400 7*S2)-ZN03*T*AP2*BT2*((AP2**2)*BT2*C2*SH2+(BT2**3)*SH2*C2
5500 8+(AP2**3)*S2*CH2+AP2*(BT2**2)*CH2*S2)
5600 C
5700 DETZ=DETZ*1.0E-18
5800 C
5900 DNZ=((BT2**2)*(-AP2**2)*S2*(EW2*(-I2)*(BT2**3)*CH2+T*BT2*CH2
6000 1+ZN03*SH2)-(BT2**2)*SH2*(-EW2*(-I2)*(AP2**3)*C2+T*AP2*C2+ZN03
6100 2*S2))+AP2**2)*(-AP2**2)*S2*(EW2*(-I2)*(BT2**3)*CH2+T*BT2*CH2

```

TABLE 21 (Continued)

```

6200      3+ZN03*SH2)-(BT2**2)*SH2*(-EW2*(-I2)*(AP2**3)*C2+T*AP2*C2+ZN03*
6300      4S2)))
6400      C
6500      DNZ=DNZ*1.0E-18
6600      ZN02=DETZ/DNZ
6700      C
6800      NUM1=-((AP1**3)*(BT1**4)*EW1*(-I1)*SH1+AP1*(BT1**4)*T*SH1+(AP1**4)
6900      1*(BT1**3)*EW1*(-I1)*S1+(AP1**4)*BT1*T*S1+(AP1**2)*(BT1**5)*EW1*(-I
7000      21)*S1+(AP1**2)*(BT1**3)*T*S1-(AP1**5)*(BT1**2)*EW1*(-I1)*SH1
7100      3+(AP1**3)*(BT1**3)*T*SH1
7200      C
7300      DEN1=((BT1**2)+(AP1**2))*(AP1**2)*S1*(EW1*(-I1)*(BT1**3)*CH1+T*BT1
7400      1*CH1+ZN02*SH1)+(BT1**4)*SH1*(-EW1*(-I1)*(AP1**3)*C1+T*AP1*C1
7500      2+ZN02*S1)+(AP1**2)*(BT1**2)*SH1*(-EW1*(-I1)*(AP1**3)*C1+T*AP1*C1
7600      3+ZN02*S1)
7700      C
7800      NUM2=-((AP2**3)*(BT2**4)*EW2*(-I2)*SH2+AP2*(BT2**4)*T*SH2+(AP2**4)
7900      1*(BT2**3)*EW2*(-I2)*S2+(AP2**4)*BT2*T*S2+(AP2**2)*(BT2**5)*EW2*(-I
8000      22)*S2+(AP2**2)*(BT2**3)*T*S2-(AP2**5)*(BT2**2)*EW2*(-I)*SH2
8100      3+(AP2**3)*(BT2**2)*T*SH2
8200      C
8300      DEN2=((BT2**2)+(AP2**2))*(AP2**2)*S2*(EW2*(-I2)*(BT2**3)*CH2+T*BT2
8400      1*CH2+ZN03*SH2)+(BT2**4)*SH2*(-EW2*(-I2)*(AP2**3)*C2+T*AP2*C2
8500      2+ZN03*S2)+(AP2**2)*(BT2**2)*SH2*(-EW2*(-I2)*(AP2**3)*C2+T*AP2*C2
8600      3+ZN03*S2)
8700      C
8800      TM3=C3+S3*((T*GM*S3+M*WSQ*C3)/(T*GM*C3-M*WSQ*S3))
8900      NUM1=NUM1*1.0E-18
9000      DEN1=DEN1*1.0E-18
9100      NUM2=NUM2*1.0E-18
9200      DEN2=DEN2*1.0E-18
9300      TM=CABS((NUM1/DEN1)*(NUM2/DEN2)*TM3)
9400      TMDB=20.0DLOG10(TM)
9500      30  FORMAT (5X,3H F=,F12.6,5X,6H TMDB=,F7.2,5X,4H TM=,1PD10.3)
9600      WRITE (1,30) F, TMDB, TM
9700      RETURN
9800      END

```

starting frequency (F START), step size (F STEP) and stopping frequency (F STOP) must be entered in response to the prompts. In order to obtain good definition of the sharp resonant peaks, a reduced step size is used when the absolute value of the slope of the transmissibility curve exceeds a set threshold (THS). The size of this reduced step must be entered in response to the "ENTER SMALL STEP" prompt. Values typically used were:

F START = .125

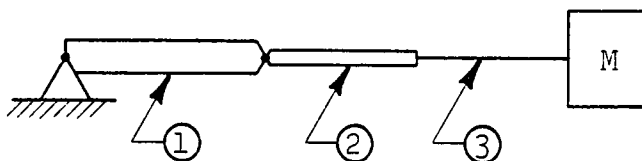
F STEP = .5

SMALL STEP = .125

F STOP = 500

Subroutine TBEAM is called by the main program to calculate the transmissibility at each frequency using Equation (C-55). At each frequency, the frequency, transmissibility, and transmissibility in dB are written to an output file. The data in this file were then used to obtain a computer drawn transmissibility plot.

The nomenclature used for the input parameters is as follows:



REW1	Real part of complex Young's modulus for Beam ① (psi)
IEW1	Imaginary part of complex Young's modulus for Beam ② (psi)
RH01	Density of Beam ① (lb_m/in^3)
A2	Cross-sectional area of Beam ② (in^2)
I2	Moment of inertia of Beam ② (in^4)
L2	Length of Beam ② (in)
LRH03	Linear density of String ③ (lb_m/in)
L3	Length of the string (in)
T	Tensile force (lb_F)
M	Mass of m (lb_m)

Table 22 is a listing of the computer program used to calculate the transmissibility of a mass-loaded, fixed-guided beam, with constant internal damping, in transverse vibration. The program was written in FORTRAN IV to run interactively on the DEC VAX 11/780 Computer. This program can be used to obtain the transverse transmissibility of a beam with either a tensile or compressive pre-load. A tensile pre-load is specified by a positive value of T and a compressive pre-load is specified by a negative value of T.

The main program is identical to that of the previous program with the exception of the input data file name which is FBM.DAT for this program. See page 248 for a description of the main program.

TABLE 22

COMPUTER PROGRAM USED TO CALCULATE THE TRANSMISSIBILITY
OF A MASS-LOADED, FIXED-GUIDED BEAM
IN TRANSVERSE VIBRATION

```

100  C      PROGRAM FOR OBTAINING THE TRANSMISSIBILITY OF A FIXED-GUIDED
200  C      BEAM IN TRANSVERSE VIBRATION
300
400  COMMON RHO1,REW1,IEW1,A1,I1,L1,T,M
500  DOUBLE PRECISION RHU1,A1,L1,I1,T,M,PI,TM,TMDB,WSQ,REW1,IEW1
700  COMPLEX*8 EW1,AP1,BT1,C1,S1,CH1,SH1,DETZ,ZNO,NUM1,DEN1,DNZ
900  OPEN (UNIT=2,NAME='FBM.DAT',TYPE='OLD')
1000 READ (2,40) REW1,IEW1,RHO1,A1,I1,L1,T,M
1100 40 FORMAT (/,'X',E15.6)
1200 TYPE*, 'ENTER F START'
1300 ACCEPT*, FSTART
1400 TYPE*, 'ENTER F STEP'
1500 ACCEPT*, FSTEP
1600 TYPE*, 'ENTER SMALL STEP'
1700 ACCEPT*, SSTEP
1800 TYPE*, 'ENTER F STOP'
1900 ACCEPT*, FSTOP
2000 FSTEP=FSTEP/4.0
2100 FSSTEP=SSTEP/4.0
2200 FOLD=0.0
2300 TMDBO=1.0
2400 THS=6.0/(FSTEP+0.5)
2500 F=FSTART
2600 70 CONTINUE
2700 IF (F.GT.FSTOP) GO TO 80
2800 CALL IFBEAM(F,TM,TMDB)
2900 SLOPE=ABS((TMDB-TMDBO)/(F-FOLD))
3000 IF (SLOPE.GT.THs) GO TO 50
3100 IF (SLO.GT.THs) GO TO 50
3200 SLO=SLOPE
3300 TMDBO=TMDB
3400 FOLD=F
3500 IF (F.GT.2.0) GO TO 60
3600 F=F+FFSTEP
3700 GO TO 70
3800 60 CONTINUE
3900 F=F+FSTEP
4000 GO TO 70
4100 50 SLO=SLOPE
4200 TMDBO=TMDB
4300 FOLD=F
4400 IF (F.GT.2.0) GO TO 90
4500 F=F+FSSTEP
4600 GO TO 70
4700 90 CONTINUE
4800 F=F+SSTEP
4900 GO TO 70
5000 80 CONTINUE
5100 END

```

TABLE 22 (Continued)

```

100 C SUBROUTINE TFBEM FOR OBTAINING THE TRANSMISSIBILITY
200 C OF A FIXED-GUIDED BEAM IN TRANSVERSE VIBRATION
300 C
400 C F -FREQUENCY (HZ)
500 C TM -TRANSMISSIBILITY
600 C TMDB-TRANSMISSIBILITY (DB)
700 C
800 C SUBROUTINE TFBEM(F, TM, TMDB)
900 C COMMON RHO1, REW1, IEW1, A1, I1, L1, T, M,
1000 C DOUBLE PRECISION RHO1, A1, I1, L1, T, M, PI, TM, TMDB, WSQ, REW1, IEW1,
1100 C 1RAP1, IAP1, A, B, DRSQRT, DISQRT, CRSQRT, CISQRT, RPW1, RQW1, IPW1, IQW1
1200 C COMPLEX*8 EW1, AP1, BT1, C1, S1, CH1, SH1
1300 C 1DETZ, ZNO, NUM1, DEN1, CCOSH, CSINH, X, UNZ
1400 C CCOSH(X)=CMPLX(COSH(REAL(X))*COS(AIMAG(X)),SINH(REAL(X)))*
1500 C 1SIN(AIMAG(X))
1600 C CSINH(X)=CMPLX(SINH(REAL(X))*COS(AIMAG(X)),COSH(REAL(X)))*
1700 C 1SIN(AIMAG(X))
1800 C DRSQRT(A,B)=DSQRT(DSQRT(A*A+B*B))*DCOS(DATAN(B/A)/2)
1900 C DISQRT(A,B)=DSQRT(DSQRT(A*A+B*B))*DSIN(DATAN(B/A)/2)
2000 C CRSQRT(A,B)=DSQRT(DSQRT(A*A+B*B))*DCOS(DATAN(B/A)/2)
2100 C CISQRT(A,B)=DSQRT(DSQRT(A*A+B*B))*DSIN(DATAN(B/A)/2)
2200 C PI=3.141592654
2300 C WSQ=(F*2.0*PI)**2
2400 C RPW1=2.0*I1*REW1
2500 C RQW1=REW1*I1*386.04
2600 C IPW1=2.0*I1*IEW1
2700 C IQW1=IEW1*I1*386.04
2800 C EW1=CMPLX(SNGL(REW1),SNGL(IEW1))
2900 C
3000 C 1RAP1=CRSQRT((-T*RPW1/((RPW1)**2+(IPW1)**2))
3100 C 1+DRSQRT(((T*RPW1/((RPW1)**2+(IPW1)**2))**2
3200 C 2+(WSQ*RHO1*A1*RQW1/((RQW1)**2+(IQW1)**2))),
3300 C 3((T*(-IPW1)/((RPW1)**2+(IPW1)**2))**2+(WSQ*
3400 C 4RHO1*A1*(-IQW1)/((RQW1)**2+(IQW1)**2))),
3500 C 5+(T*IPW1/((RPW1)**2+(IPW1)**2))+DISQRT(((T*
3600 C 6RPW1/((RPW1)**2+(IPW1)**2))**2+(WSQ*RHO1*A1*
3700 C 7RQW1/((RQW1)**2+(IQW1)**2))),((T*(-IPW1)
3800 C 8/((RPW1)**2+(IPW1)**2))**2+(WSQ*RHO1*A1*(-IQW1)
3900 C 9/((RQW1)**2+(IQW1)**2))))
4000 C
4100 C 1AP1=CISQRT((-T*RPW1/((RPW1)**2+(IPW1)**2))
4200 C 1+DRSQRT(((T*RPW1/((RPW1)**2+(IPW1)**2))**2
4300 C 2+(WSQ*RHO1*A1*RQW1/((RQW1)**2+(IQW1)**2))),
4400 C 3((T*(-IPW1)/((RPW1)**2+(IPW1)**2))**2+(WSQ*
4500 C 4RHO1*A1*(-IQW1)/((RQW1)**2+(IQW1)**2))),
4600 C 5+(T*IPW1/((RPW1)**2+(IPW1)**2))+DISQRT(((T*
4700 C 6RPW1/((RPW1)**2+(IPW1)**2))**2+(WSQ*RHO1*A1*
4800 C 7RQW1/((RQW1)**2+(IQW1)**2))),((T*(-IPW1)
4900 C 8/((RPW1)**2+(IPW1)**2))**2+(WSQ*RHO1*A1*(-IQW1)
5000 C 9/((RQW1)**2+(IQW1)**2))))
5100 C
5200 C 1RBT1=CRSQRT((+T*RPW1/((RPW1)**2+(IPW1)**2))
5300 C 1+DRSQRT(((T*RPW1/((RPW1)**2+(IPW1)**2))**2
5400 C 2+(WSQ*RHO1*A1*RQW1/((RQW1)**2+(IQW1)**2))),
5500 C 3((T*(-IPW1)/((RPW1)**2+(IPW1)**2))**2+(WSQ*
5600 C 4RHO1*A1*(-IQW1)/((RQW1)**2+(IQW1)**2))),
5700 C 5(-T*IPW1/((RPW1)**2+(IPW1)**2))+DISQRT(((T*
5800 C 6RPW1/((RPW1)**2+(IPW1)**2))**2+(WSQ*RHO1*A1*
5900 C 7RQW1/((RQW1)**2+(IQW1)**2))),((T*(-IPW1)
6000 C 8/((RPW1)**2+(IPW1)**2))**2+(WSQ*RHO1*A1*(-IQW1)
6100 C 9/((RQW1)**2+(IQW1)**2))))

```


TABLE 22 (Continued)

```

6200 C      IBT1=CISQRT((+T*RPW1/((RPW1)**2+(IPW1)**2))
6300      1+DRSQRT(((T*RPW1/((RPW1)**2+(IPW1)**2))**2
6400      2+(WSQ*RH01*A1*RQW1/((RQW1)**2+(IQW1)**2))),
6500      3((T*(-IPW1)/((RPW1)**2+(IPW1)**2))**2+(WSQ*
6600      4RH01*A1*(-IQW1)/((RQW1)**2+(IQW1)**2))),
6700      5(-T*IPW1/((RPW1)**2+(IPW1)**2))+DISQRT(((T*
6800      6RPW1/((RPW1)**2+(IPW1)**2))**2+(WSQ*RH01*A1*
6900      7RQW1/((RQW1)**2+(IQW1)**2))),((T*(-IPW1)
7000      8/((RPW1)**2+(IPW1)**2))**2+(WSQ*RH01*A1*(-IQW1)
7100      9/((RQW1)**2+(IQW1)**2))))))
7200 C
7300      AP1=CMPLX(SNGL(RAP1),SNGL(IAP1))
7400      BT1=CMPLX(SNGL(RBT1),SNGL(IBT1))
7500      C1=CCOS(AP1*SNGL(L1))
7600      S1=CSIN(AP1*SNGL(L1))
7700      CH1=CCOSH(BT1*SNGL(L1))
7800      SH1=CSINH(BT1*SNGL(L1))
7900 C
8000      NUM1=-EW1*I1*AP1*BT1*((BT1**3)+AP1*AP1*BT1)*SH1
8100      1+(AP1*BT1*BT1+(AP1**3))*S1)
8200 C
8300      DEN1=-EW1*I1*AP1*BT1*(AP1*(BT1**2)*S1*CH1+(BT1**3)*C1
8400      1*SH1+(AP1**2)*BT1*C1*SH1+(AP1**3)*S1*CH1)+M*WSQ*
8500      2(-(AP1**2)*S1*SH1+2.0*AP1*BT1-2.0*AP1*BT1*C1*CH1
8600      3+(BT1**2)*S1*SH1)
8700 C
8800      TM=CABS(NUM1/DEN1)
8900      TMDB=20.0*DLOG10(TM)
9000      30 FURMAT (5X,3H F=,F12.6,5X,6H TMDB=,F7.2,5X,4H TM=,1PD10.3)
9100      WRITE (1,30) F,TMDB,TM
9200      RETURN
9300      END
9400

```

Subroutine TFBEAM is called by the main program to calculate the transmissibility at each frequency using Equation (B-62). In order to stay within the dynamic range of the computer, it was found necessary to obtain the real and imaginary parts of α_1^* and β_1^* by directly performing the necessary complex arithmetic using only double precision (real) arguments. As in the previous program, the output data is written to a file. This file was then used to obtain a computer drawn transmissibility plot.

The nomenclature used for the input parameters is as follows:

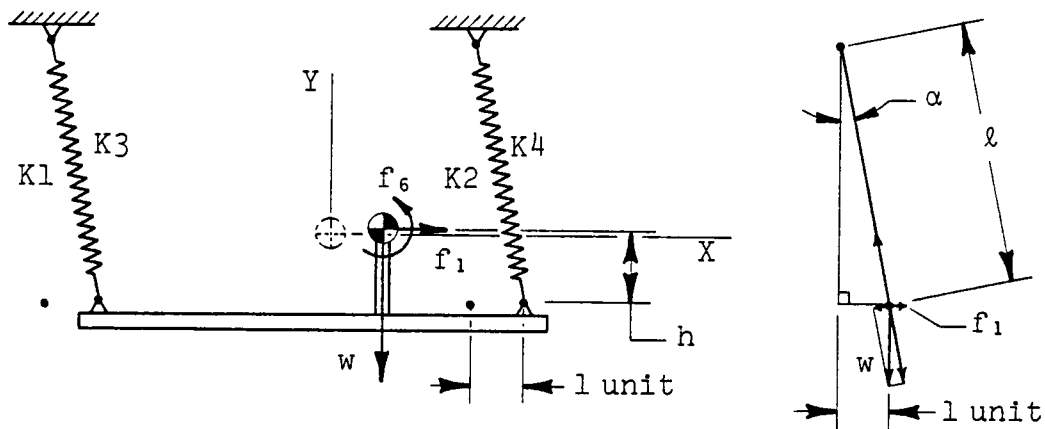
REW1	Real part of complex Young's modulus (psi)
IEW1	Imaginary part of complex Young's modulus (psi)
RHO1	Density of beam (lb_m/in^3)
A1	Cross-sectional area of beam (in^2)
I1	Moment of inertia of beam (in^4)
L1	Length of beam (in)
T	Pre-load (lb_F)
M	Mass of m (lb_m)

APPENDIX E

DEVELOPMENT OF STIFFNESS MATRIX
FOR THE ISOLATION SYSTEM

The 6 columns of the stiffness matrix for the isolation system shown in Figure 6 on page 16 are developed in this appendix.

Let $x_1 = 1$ and $x_2 \rightarrow x_6 = 0$ as shown in the figure below:

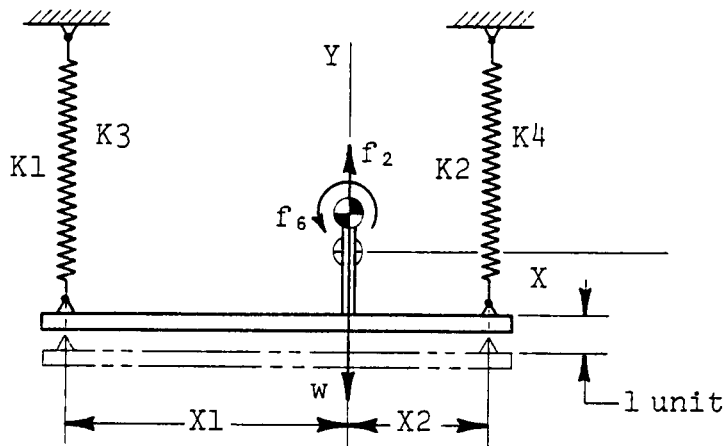


$$f_1 = w \left(\frac{l}{h} \right) = \frac{w}{h} \frac{lb}{in} = k_{11} \text{ (for small } \alpha \text{)} ,$$

$$f_6 = f_1 h = \frac{wh}{h} \frac{ln-lb}{in} = k_{61} .$$

In order to keep the system linear, the small vertical motion of the cg has been neglected.

Let $x_2 = 1$ and other $x_n = 0$ as shown below.

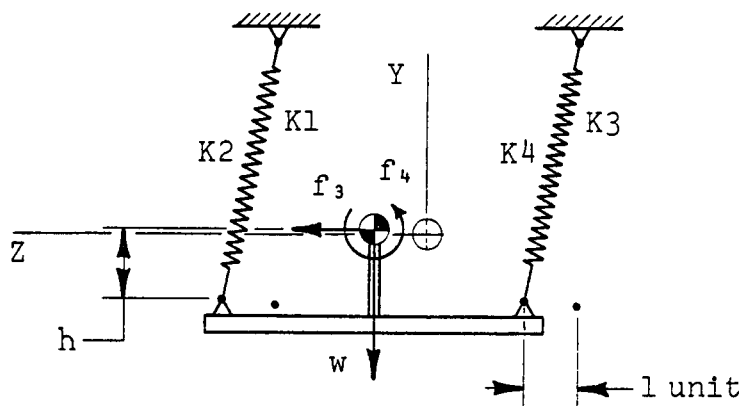


$$f_2 = K1 + K2 + K3 + K4 = k_{22} \frac{1b}{in} ,$$

$$\circlearrowleft \Sigma M_{cg} = 0 = f_6 - (K2 + K4)X2 + (K1 + K3)X1$$

$$f_6 = (K2 + K4)X2 - (K1 + K3)X1 = k_{62} \frac{in-lb}{in} .$$

Let $x_3 = 1$ and other $x_n = 0$ as shown below.

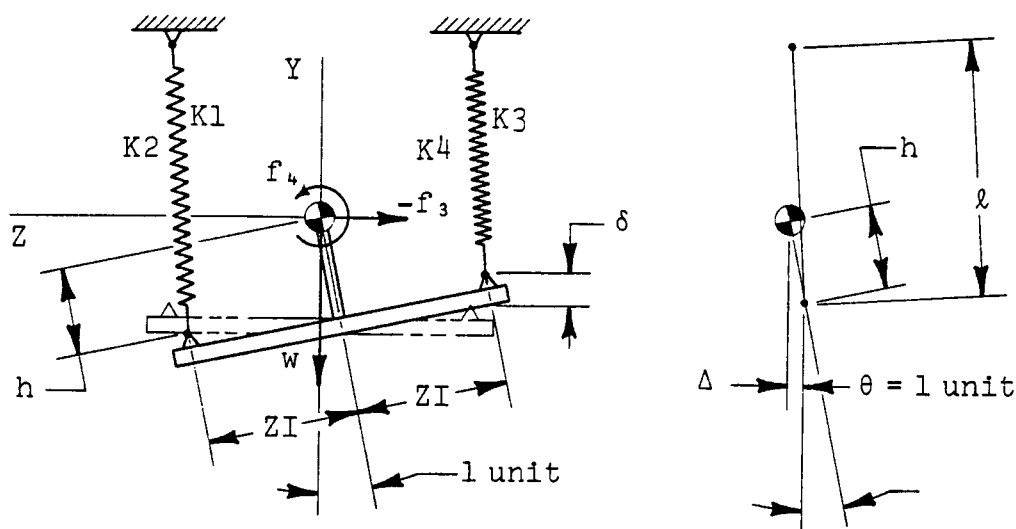


$$f_3 = \frac{w}{\ell} \frac{lb}{in} = k_{33} \text{ (same as } k_{11}) \quad ,$$

$$f_4 = -f_3 h = -\frac{wh}{\ell} \frac{in-lb}{in} = k_{43} \quad .$$

Again, as in $x_1 = 1$, the small vertical motion of the center of gravity has been neglected to maintain linearity.

Let $x_4 = 1$ and other $x_n = 0$ as shown below.



$$\delta = ZI \sin \theta \quad ,$$

$$\delta \approx ZI\theta = ZI (1 \text{ unit}) \text{ (for small } \theta) \quad ,$$

$$f_4 = (K1 + K2 + K3 + K4)(ZI)\delta$$

$$= (K1 + K2 + K3 + K4)(ZI)^2 \frac{in-lb}{rad} = k_{44} \quad ,$$

$$- f_3 = \left(\frac{W}{\ell} \right) \Delta ,$$

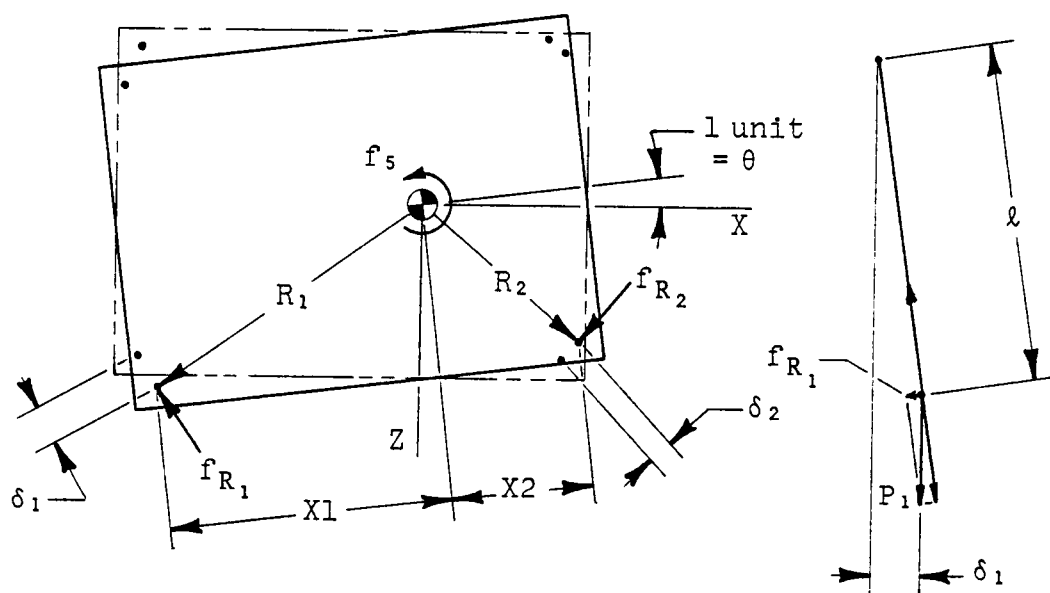
$$\Delta = h \sin \theta ,$$

$$\Delta \approx h \theta = h (1 \text{ unit}) ,$$

$$- f_3 = \frac{wh}{\ell} \frac{\text{in-lb}}{\text{in-rad}} ,$$

$$f_3 = - \frac{wh}{\ell} \frac{\text{in-lb}}{\text{in-rad}} = k_{34} .$$

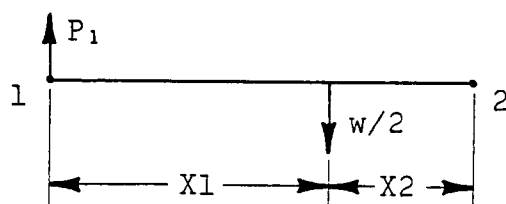
Let $x_5 = 1$ and other $x_n = 0$ as shown below.



$$\delta_1 \approx R_1 \theta = R_1 (1 \text{ unit}) \text{ (for small } \theta) ,$$

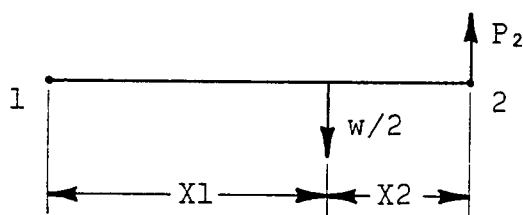
$$\delta_2 \approx R_2 (1 \text{ unit}) ,$$

$$f_{R_1} = \frac{P_1 \delta_1}{\ell} , \quad f_{R_2} = \frac{P_1 \delta_2}{\ell} .$$



$$\odot \Sigma M_2 = 0 = \frac{w}{2}(X_2) - P_1 (X_1 + X_2) \quad ,$$

$$P_1 = \frac{w(X_2)}{2(X_1 + X_2)} \quad .$$



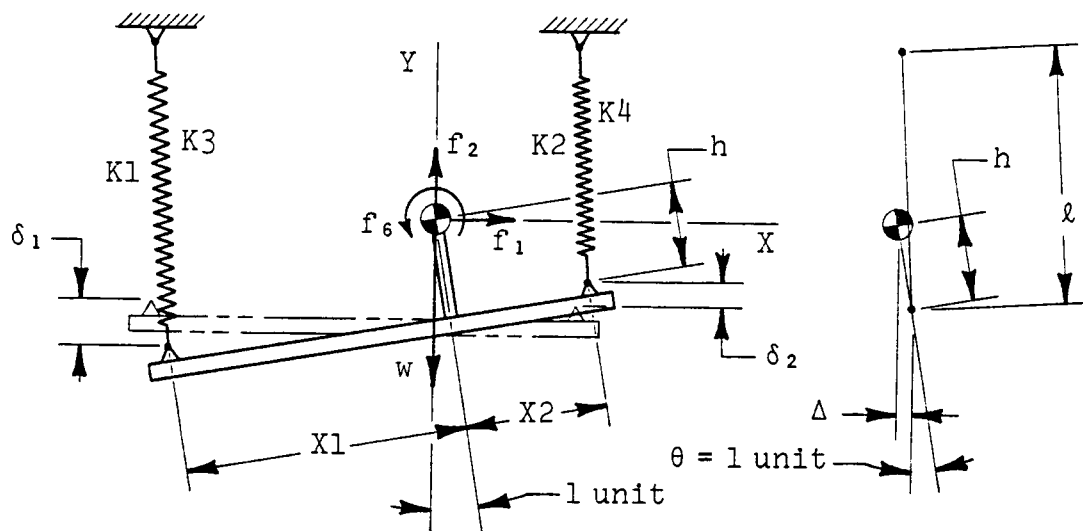
$$\odot \Sigma M_1 = 0 = -\frac{w}{2}(X_2) + P_2 (X_1 + X_2) \quad ,$$

$$P_2 = \frac{w(X_1)}{2(X_1 + X_2)} \quad .$$

$$f_5 = 2[f_{R_1} R_1 + f_{R_2} R_2] \quad ,$$

$$f_5 = \frac{w}{\ell(X_1 + X_2)} (X_2 R_1^2 + X_1 R_2^2) = k_{55} \frac{\text{in-lb}}{\text{rad}} \quad .$$

Let $x_6 = 1$ and other $x_n = 0$ as shown below.



$$\delta_1 \approx (X_1)\theta = X_1 (1 \text{ unit}) \quad ,$$

$$\delta_2 \approx (X_2)\theta = X_2 (1 \text{ unit}) \quad .$$

$$f_1 = \frac{W\Delta}{l} \quad ,$$

$$\Delta \approx h\theta = h (1 \text{ unit}) \quad ,$$

$$f_1 = \frac{Wh}{l} = k_{16} \frac{\text{in-lb}}{\text{in-rad}} \quad ,$$

$$+\uparrow \Sigma F_y = 0 = f_2 + (K_1 + K_3)\delta_1 - (K_2 + K_4)\delta_2 \quad ,$$

$$f_2 = (K_2 + K_4)X_2 - (K_1 + K_3)X_1 = k_{26} \frac{\text{in-lb}}{\text{in-rad}} \quad ,$$

$$f_6 = (K1 + K3)(X1)\delta_1 + (K2 + K4)(X2)\delta_2$$

$$= (K1 + K3)(X1)^2 + (K2 + K4)(X2)^2 = k_{66} \frac{\text{in-lb}}{\text{rad}} \quad .$$

APPENDIX F

EIGENVALUE ANALYSIS PROGRAM
WITH SAMPLE PRINTOUT

Table 23 is a listing of the computer program used to obtain the natural frequencies and mode shapes of the isolation system. This program was written in FORTRAN IV to run on the DEC VAX 11/780 Computer. The program does not require interactive inputs and can easily be modified for running on other computer systems. A sample printout obtained from this program is given in Table 24.

Comments have been included in the main program listing to explain its operation; however, the following two items require further clarification:

1. It should be noted that the members of the stiffness matrix (KVIB) are calculated according to Equation (10) and thus apply only to the basic system configuration shown in Figure 6 on page 16.
2. In order to obtain properly normalized eigenvectors for a system with both translational and rotational degrees of freedom, it is necessary to establish a proper relationship between the unit sizes. This relationship is dependent upon all of the system parameters. For the range of input parameters used in

TABLE 23
EIGENVALUE ANALYSIS PROGRAM

```

100  C      PROGRAM FOR OBTAINING NATURAL FREQUENCIES AND MODE SHAPES
200  C      OF A VIBRATION ISOLATION SYSTEM
300  C      USING THE METHOD OF EIGENVALUE ANALYSIS
400  C
500  C      NOTE: UNIT SIZES ON EIGENVECTORS ARE AS FOLLOWS
600  C
700  C          X1---1 UNIT=1 INCH
800  C          X2---1 UNIT=1 INCH
900  C          X3---1 UNIT=1 INCH
1000 C          X4---1 UNIT=0.1 RADIAN
1100 C          X5---1 UNIT=0.1 RADIAN
1200 C          X6---1 UNIT=0.1 RADIAN
1300 C
1400 C      DOUBLE PRECISION K1,K2,K3,K4,HS,LS,ZI,X1,X2,KVIB,MVIB,EGN,VECT,
1500 C      1NFRQ,K1,R2,W
1600 C      DIMENSION KVIB(6,6),MVIB(6,6),VECT(36),NFRQ(6),EGN(6)
1700 C
1800 C      READ AND WRITE SPRING STIFFNESS AND OTHER DIMENSIONAL DATA
1900 C
2000 C      5 OPEN (UNIT=2,NAME='EIGEN.DAT',TYPE='OLD')
2100 C      READ (2,9) K1,K2,K3,K4
2200 C      9 FORMAT (4F10.0)
2300 C      READ (2,10) HS,LS,ZI,X1,X2
2400 C      10 FORMAT (5F10.0)
2500 C      WRITE (1,11) K1,K2,K3,K4,HS,LS,ZI,X1,X2
2600 C      11 FORMAT (4H K1=,F10.4,12H POUNDS/INCH,27H ..... SPRING STIFFNES
2700 C      1S,/4H K2=,F10.4,12H POUNDS/INCH,/4H K3=,F10.4,12H POUNDS/INCH,/4H
2800 C      2K4=,F10.4,12H POUNDS/INCH,///4H HS=,F10.4,7H INCHES,49H .....
2900 C      3... HEIGHT OF C.G. ABOVE LOWER PIVOTS,///4H LS=,F10.4,7H INCHES,
3000 C      433H ..... LENGTH OF SPRINGS,///4H ZI=,F10.4,7H INCHES,44H
3100 C      5 ..... DISTANCE FROM C.G. TO PIVOTS,/36X,30H IN PLUS AND
3200 C      6MINUS Z DIRECTION,///4H X1=,F10.4,7H INCHES,44H ..... DIS
3300 C      7TANCE FROM C.G. TO PIVOTS,/36X,30H IN PLUS AND MINUS X DIRECTION,/
3400 C      84H X2=,F10.4,7H INCHES)
3500 C
3600 C      CLEAR KVIB AND MVIB
3700 C
3800 C      DO 12 I=1,6
3900 C      DO 12 J=1,6
4000 C      KVIB(I,J)=0.0
4100 C      MVIB(I,J)=0.0
4200 C      12 CONTINUE
4300 C
4400 C      READ MASS AND INERTIA
4500 C
4600 C      READ (2,16) MVIB(1,1),MVIB(2,2),MVIB(3,3),MVIB(4,4),MVIB(5,5),
4700 C      1 MVIB(6,6)
4800 C      16 FORMAT (6(F10.0))
4900 C
5000 C      WRITE MASS AND INERTIA
5100 C
5200 C      WRITE (1,17) MVIB(1,1),MVIB(4,4),MVIB(5,5),MVIB(6,6)
5300 C      17 FORMAT (//////4H M=,F12.5,12H POUNDS MASS,33H .....
5400 C      1 SYSTEM MASS,////4H IX=,F12.5,27H POUNDS MASS-INCHES SQUARED,27H .....
5500 C      2... INERTIA ABOUT X AXIS,////4H IY=,F12.5,27H POUNDS MASS-INCHES SQ
5600 C      3UARED,27H ..... INERTIA ABOUT Y AXIS,////4H IZ=,F12.5,27H POUNDS MA
5700 C      4SS-INCHES SQUARED,27H ..... INERTIA ABOUT Z AXIS)
5800 C
5900 C      ASSEMBLE STIFFNESS MATRIX FROM BASIC DATA
6000 C
6100 C      R1=DSQRT(X1**2+ZI**2)
6200 C      R2=DSQRT(X2**2+ZI**2)
6300 C      W=MVIB(1,1)
6400 C      KVIB(1,1)=W/LS
6500 C      KVIB(6,1)=W*HS/LS
6600 C      KVIB(2,2)=K1+K2+K3+K4
6700 C      KVIB(6,2)=(K2+K4)*X2-(K1+K3)*X1
6800 C      KVIB(3,3)=W/LS
6900 C      KVIB(4,3)=-W*HS/LS
7000 C      KVIB(3,4)=-W*HS/LS
7100 C      KVIB(4,4)=(K1+K2+K3+K4)*ZI**2
7200 C      KVIB(5,5)=(W/((X1+X2)*LS))*(X1*R2**2+X2*R1**2)
7300 C      KVIB(1,6)=W*HS/LS
7400 C      KVIB(2,6)=KVIB(6,2)
7500 C      KVIB(6,6)=(K1+K3)*X1**2+(K2+K4)*X2**2
7600 C
7700 C      WRITE MEMBERS OF KVIB
7800 C
7900 C      WRITE (1,15)
8000 C      15 FORMAT (//////6H KVIB=,/)
8100 C      DO 14 I=1,6
8200 C      WRITE (1,13) KVIB(I,1),KVIB(I,2),KVIB(I,3),KVIB(I,4),KVIB(I,5),
8300 C      1KVIB(I,6)
8400 C      13 FORMAT(6(F12.6,3X))
8500 C      14 CONTINUE

```

TABLE 23 (Continued)

```

8600 C
8700 CC CONVERT UNITS OF MVIB
8800 C
8900 DO 18 I=1,6
9000 MVIB(I,I)=MVIB(I,I)/386.04D0
9100 18 CONTINUE
9200 CC
9300 CC OBTAIN NATURAL FREQUENCIES AND EIGENVECTORS
9400 C
9500 CALL NROOT (6,KVIB,MVIB,EGN,VECT)
9600 DO 84 I=1,6
9700 NFRQ(I)=DSQRT(EGN(I))/6.2831853
9800 84 CONTINUE
9900 C
10000 CC SET 1 UNIT=0.1 RADIAN ON ANGULAR COMPONENTS
10100 CC OF EIGENVECTORS
10200 C
10300 DO 100 I=1,6
10400 IX=6*I-2
10500 VECT(IX)=10.0*VECT(IX)
10600 VECT(IX+1)=10.0*VECT(IX+1)
10700 VECT(IX+2)=10.0*VECT(IX+2)
10800 100 CONTINUE
10900 C
11000 CC NORMALIZE EIGENVECTORS
11100 C
11200 DO 94 I=1,6
11300 N=1
11400 92 CONTINUE
11500 DO 90 K=1,6
11600 IN=6*I-6+N
11700 IK=6*I-6+K
11800 IF (DABS(VECT(IK)).GT.DABS(VECT(IN))) GO TO 91
11900 90 CONTINUE
12000 GO TO 93
12100 91 N=N+1
12200 GO TO 92
12300 93 CONTINUE
12400 VECTN=VECT(IN)
12500 DO 94 J=1,6
12600 IJ=6*I-6+J
12700 VECT(IJ)=VECT(IJ)/VECTN
12800 94 CONTINUE
12900 C
13000 CC WRITE OUT NATURAL FREQUENCIES
13100 CC WITH CORRESPONDING EIGENVECTORS
13200 C
13300 85 FORMAT (1H1,44H NATURAL FREQUENCIES AND RELATIVE AMPLITUDES///4X,
13400 117H NAT. FREQ. (HZ) ,18X,5H AMP.)
13500 WRITE (1,85)
13600 DO 86 K=1,6
13700 KN=6*K-5
13800 88 FORMAT (/9X,F8.5,17H.....: X1,4X,F9.6/31X,3H X2,4X,F9.6/
13900 131X,3H X3,4X,F9.6/31X,3H X4,4X,F9.6/31X,3H X5,4X,F9.6/31X,3H X6,
14000 24X,F9.6)
14100 WRITE(1,88) NFRQ(K),VECT(KN),VECT(KN+1),VECT(KN+2),VECT(KN+3),
14200 1VECT(KN+4),VECT(KN+5)
14300 86 CONTINUE
14400 END

```

TABLE 23 (Continued)

```

100      SUBROUTINE NROOT
110      PURPOSE
120      COMPUTE EIGENVALUES AND EIGENVECTORS OF A REAL NONSYMMETRIC
130      MATRIX OF THE FORM B-INVERSE TIMES A. THIS SUBROUTINE IS
140      NORMALLY CALLED BY SUBROUTINE CANOR IN PERFORMING A
150      CANONICAL CORRELATION ANALYSIS.
160
170      USAGE
180      CALL NROOT (M,A,B,XL,X)
190
200      DESCRIPTION OF PARAMETERS
210      M - ORDER OF SQUARE MATRICES A, B, AND X.
220      A - INPUT MATRIX (M X M).
230      B - INPUT MATRIX (M X M).
240      XL - OUTPUT VECTOR OF LENGTH M CONTAINING EIGENVALUES OF
250      B-INVERSE TIMES A.
260      X - OUTPUT MATRIX (M X M) CONTAINING EIGENVECTORS COLUMN-
270      WISE.
280
290      REMARKS
300      NONE
310
320      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
330      EIGEN
340
350      METHOD
360      REFER TO W. W. COOLEY AND P. R. LOHNES, 'MULTIVARIATE PRO-
370      CEDURES FOR THE BEHAVIORAL SCIENCES', JOHN WILEY AND SONS,
380      1962, CHAPTER 3.
390
400      SUBROUTINE NROOT (M,A,B,XL,X)
410      DIMENSION A(1),B(1),XL(1),X(1)
420
430      IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
440      C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
450      STATEMENT WHICH FOLLOWS.
460
470      DOUBLE PRECISION A,B,XL,X,SUMV
480
490      THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
500      APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
510      ROUTINE.
520
530      THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
540      CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS. SQRT IN STATEMENTS
550      110 AND 175 MUST BE CHANGED TO DSQRT. ABS IN STATEMENT 110
560      MUST BE CHANGED TO DABS.
570
580      COMPUTE EIGENVALUES AND EIGENVECTORS OF B
590
600      K=1
610      DO 100 J=2,M
620      L=M*(J-1)
630      DO 100 I=1,J
640      L=L+1
650      K=K+1
660      100 B(K)=B(L)
670
680      THE MATRIX B IS A REAL SYMMETRIC MATRIX.
690
700      MV=0
710      CALL EIGEN (B,X,M,MV)
720
730      FORM RECIPROCAL OF SQUARE ROOT OF EIGENVALUES. THE RESULTS
740      ARE PREMULTIPLIED BY THE ASSOCIATED EIGENVECTORS.
750
760      L=0
770      DO 110 J=1,M
780      L=L+J
790      110 XL(J)=1.0/DSQRT(DABS(B(L)))
800      K=0
810      DO 115 J=1,M
820      DO 115 I=1,M
830      K=K+1
840      115 B(K)=X(K)*XL(J)
850

```

TABLE 23 (Continued)

8600	C		NROO 860
8700	C	FORM (B**(-1/2))PRIME * A * (B**(-1/2))	NROO 870
8800	C		NROO 880
8900		DO 120 I=1,M	NROO 890
9000		N2=0	NROO 900
9100		DO 120 J=1,M	NROO 910
9200		N1=M*(J-1)	NROO 920
9300		L=M*(J-1)+I	NROO 930
9400		X(L)=0.0	NROO 940
9500		DO 120 K=1,M	NROO 950
9600		N1=N1+1	NROO 960
9700		N2=N2+1	NROO 970
9800	120	X(L)=X(L)+B(N1)*A(N2)	NROO 980
9900		L=L+1	NROO 990
10000		DO 130 J=1,M	NROO1000
10100		DO 130 I=1,J	NROO1010
10200		N1=1-M	NROO1020
10300		N2=M*(J-1)	NROO1030
10400		L=L+1	NROO1040
10500		A(L)=0.0	NROO1050
10600		DO 130 K=1,M	NROO1060
10700		N1=N1+M	NROO1070
10800		N2=N2+1	NROO1080
10900	130	A(L)=A(L)+X(N1)*B(N2)	NROO1090
11000	C		NROO1100
11100	C	COMPUTE EIGENVALUES AND EIGENVECTORS OF A	NROO1110
11200	C		NROO1120
11300		CALL EIGEN (A,X,M,MV)	NROO1130
11400		L=0	NROO1140
11500		DO 140 I=1,M	NROO1150
11600		L=L+I	NROO1160
11700	140	XL(I)=A(L)	NROO1170
11800	C		NROO1180
11900	C	COMPUTE THE NORMALIZED EIGENVECTORS	NROO1190
12000	C		NROO1200
12100		DO 150 I=1,M	NROO1210
12200		N2=0	NROO1220
12300		DO 150 J=1,M	NROO1230
12400		N1=1-M	NROO1240
12500		L=M*(J-1)+I	NROO1250
12600		A(L)=0.0	NROO1260
12700		DO 150 K=1,M	NROO1270
12800		N1=N1+M	NROO1280
12900		N2=N2+1	NROO1290
13000	150	A(L)=A(L)+B(N1)*X(N2)	NROO1300
13100		L=L+1	NROO1310
13200		K=0	NROO1320
13300		DO 180 J=1,M	NROO1330
13400		SUMV=0.0	NROO1340
13500		DO 170 I=1,M	NROO1350
13600		L=L+1	NROO1360
13700	170	SUMV=SUMV+A(L)*A(L)	NROO1370
13800	175	SUMV=DSQRT(SUMV)	NROO1380
13900		DO 180 I=1,M	NROO1390
14000		K=K+1	NROO1400
14100	180	X(K)=A(K)/SUMV	NROO1410
14200		RETURN	NROO1420
14300		END	NROO1430

TABLE 23 (Continued)

100	C		EIGE	10
200	C		EIGE	20
300	C		EIGE	30
400	C	SUBROUTINE EIGEN	EIGE	40
500	C		EIGE	50
600	C	PURPOSE	EIGE	60
700	C	COMPUTE EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC	EIGE	70
800	C	MATRIX	EIGE	80
900	C		EIGE	90
1000	C	USAGE	EIGE	100
1100	C	CALL EIGEN(A,R,N,MV)	EIGE	110
1200	C		EIGE	120
1300	C	DESCRIPTION OF PARAMETERS	EIGE	130
1400	C	A -- ORIGINAL MATRIX (SYMMETRIC), DESTROYED IN COMPUTATION.	EIGE	140
1500	C	RESULTANT EIGENVALUES ARE DEVELOPED IN DIAGONAL OF	EIGE	150
1600	C	MATRIX A IN DESCENDING ORDER.	EIGE	160
1700	C	R - RESULTANT MATRIX OF EIGENVECTORS (STORED COLUMNWISE,	EIGE	170
1800	C	IN SAME SEQUENCE AS EIGENVALUES)	EIGE	180
1900	C	N - ORDER OF MATRICES A AND R	EIGE	190
2000	C	MV- INPUT CODE	EIGE	200
2100	C	0 COMPUTE EIGENVALUES AND EIGENVECTORS	EIGE	210
2200	C	1 COMPUTE EIGENVALUES ONLY (R NEED NOT BE	EIGE	220
2300	C	DIMENSIONED BUT MUST STILL APPEAR IN CALLING	EIGE	230
2400	C	SEQUENCE)	EIGE	240
2500	C		EIGE	250
2600	C	REMARKS	EIGE	260
2700	C	ORIGINAL MATRIX A MUST BE REAL SYMMETRIC (STORAGE MODE=1)	EIGE	270
2800	C	MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX R	EIGE	280
2900	C		EIGE	290
3000	C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	EIGE	300
3100	C	NONE	EIGE	310
3200	C		EIGE	320
3300	C	METHOD	EIGE	330
3400	C	DIAGONALIZATION METHOD ORIGINATED BY JACOBI AND ADAPTED	EIGE	340
3500	C	BY VON NEUMANN FOR LARGE COMPUTERS AS FOUND IN 'MATHEMATICAL	EIGE	350
3600	C	METHODS FOR DIGITAL COMPUTERS', EDITED BY A. RALSTON AND	EIGE	360
3700	C	H.S. WILF, JOHN WILEY AND SONS, NEW YORK, 1962, CHAPTER 7	EIGE	370
3800	C		EIGE	380
3900	C		EIGE	390
4000	C		EIGE	400
4100	C	SUBROUTINE EIGEN(A,R,N,MV)	EIGE	410
4200	C	DIMENSION A(36),R(36)	EIGE	420
4300	C		EIGE	430
4400	C		EIGE	440
4500	C		EIGE	450
4600	C	IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE	EIGE	460
4700	C	C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION	EIGE	470
4800	C	STATEMENT WHICH FOLLOWS.	EIGE	480
4900	C		EIGE	490
5000	C	DOUBLE PRECISION A,R,ANORM,ANRMX,THR,X,Y,SINX,SINX2,COSX,	EIGE	500
5100	C	1 COSX2,SINCS,RANGE	EIGE	510
5200	C		EIGE	520
5300	C	THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS	EIGE	530
5400	C	APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS	EIGE	540
5500	C	ROUTINE.	EIGE	550
5600	C		EIGE	560
5700	C	THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO	EIGE	570
5800	C	CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS. SQRT IN STATEMENTS	EIGE	580
5900	C	40, 68, 75, AND 78 MUST BE CHANGED TO DSQRT. ABS IN STATEMENT	EIGE	590
6000	C	62 MUST BE CHANGED TO DABS. THE CONSTANT IN STATEMENT 5 SHOULD	EIGE	600
6100	C	BE CHANGED TO 1.0D-12.	EIGE	610

TABLE 23 (Continued)

6200	C		EIGE	620
6300	C		EIGE	630
6400	C		EIGE	640
6500	C	GENERATE IDENTITY MATRIX	EIGE	650
6600	C		EIGE	660
6700		5 RANGE=1.0D-12	EIGE	670
6800		IF(MV-1) 10,25,10	EIGE	680
6900	10	IQ=-N	EIGE	690
7000		DO 20 J=1,N	EIGE	700
7100		IQ=IQ+N	EIGE	710
7200		DU 20 I=1,N	EIGE	720
7300		IJ=IQ+I	EIGE	730
7400		R(IJ)=0.0	EIGE	740
7500		IF(I-J) 20,15,20	EIGE	750
7600	15	R(IJ)=1.0	EIGE	760
7700	20	CONTINUE	EIGE	770
7800	C		EIGE	780
7900	C	COMPUTE INITIAL AND FINAL NORMS (ANORM AND ANORMX)	EIGE	790
8000	C		EIGE	800
8100		25 ANORM=0.0	EIGE	810
8200		DU 35 I=1,N	EIGE	820
8300		DO 35 J=1,N	EIGE	830
8400		IF(I-J) 30,35,30	EIGE	840
8500	30	IA=1+(J*J-J)/2	EIGE	850
8600		ANORM=ANORM+A(IA)*A(IA)	EIGE	860
8700	35	CONTINUE	EIGE	870
8800		IF(ANORM) 165,165,40	EIGE	880
8900	40	ANORM=1.414*DSQRT(ANORM)	EIGE	890
9000		ANRMX=ANORM*RANGE/FLOAT(N)	EIGE	900
9100	C		EIGE	910
9200	C	INITIALIZE INDICATORS AND COMPUTE THRESHOLD, THR	EIGE	920
9300	C		EIGE	930
9400		IND=0	EIGE	940
9500		THR=ANORM	EIGE	950
9600	45	THR=THR/FLOAT(N)	EIGE	960
9700	50	L=1	EIGE	970
9800	55	M=L+1	EIGE	980
9900	C		EIGE	990
10000	C	COMPUTE SIN AND COS	EIGE	1000
10100	C		EIGE	1010
10200		60 MQ=(M*M-M)/2	EIGE	1020
10300		LQ=(L*L-L)/2	EIGE	1030
10400		LM=L+MQ	EIGE	1040
10500	62	IF(DABS(A(LM))-THR) 130,65,65	EIGE	1050
10600	65	IND=1	EIGE	1060
10700		LL=L+LQ	EIGE	1070
10800		MM=M+MQ	EIGE	1080
10900		X=0.5*(A(LL)-A(MM))	EIGE	1090
11000	68	Y=-A(LM)/DSQRT(A(LM)*A(LM)+X*X)	EIGE	1100
11100		IF(X) 70,75,75	EIGE	1110
11200	70	Y=-Y	EIGE	1120
11300	75	SINX=Y/DSQRT(2.0*(1.0+(DSQRT(1.0-Y*Y))))	EIGE	1130
11400		SINX2=SINX*SINX	EIGE	1140
11500	78	COSX=DSQRT(1.0-SINX2)	EIGE	1150
11600		COSX2=COSX*COSX	EIGE	1160
11700		SINCS =SINX*COSX	EIGE	1170
11800	C		EIGE	1180
11900	C	ROTATE L AND M COLUMNS	EIGE	1190
12000	C		EIGE	1200
12100		ILQ=N*(L-1)	EIGE	1210
12200		IMQ=N*(M-1)	EIGE	1220

TABLE 23 (Continued)

12300	DO 125 I=1,N	EIGE1230
12400	IQ=(I*I-1)/2	EIGE1240
12500	IF(I-L) 80,115,80	EIGE1250
12600	80 IF(I-M) 85,115,90	EIGE1260
12700	85 IM=I+MQ	EIGE1270
12800	GO TO 95	EIGE1280
12900	90 IM=M+IQ	EIGE1290
13000	95 IF(I-L) 100,105,105	EIGE1300
13100	100 IL=I+LQ	EIGE1310
13200	GO TO 110	EIGE1320
13300	105 IL=L+IQ	EIGE1330
13400	110 X=A(IL)*COSX-A(IM)*SINX	EIGE1340
13500	A(IM)=A(IL)*SINX+A(IM)*COSX	EIGE1350
13600	A(IL)=X	EIGE1360
13700	115 IF(MV-1) 120,125,120	EIGE1370
13800	120 ILR=ILO+I	EIGE1380
13900	IMR=IMQ+I	EIGE1390
14000	X=R(ILR)*COSX-R(IMR)*SINX	EIGE1400
14100	R(IMR)=R(ILR)*SINX+R(IMR)*COSX	EIGE1410
14200	R(ILR)=X	EIGE1420
14300	125 CONTINUE	EIGE1430
14400	X=2.0*A(LM)*SINCS	EIGE1440
14500	Y=A(LL)*COSX2+A(MM)*SINX2-X	EIGE1450
14600	X=A(LL)*SINX2+A(MM)*COSX2+X	EIGE1460
14700	A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)	EIGE1470
14800	A(LL)=Y	EIGE1480
14900	A(MM)=X	EIGE1490
15000	C TESTS FOR COMPLETION	EIGE1500
15100	C	EIGE1510
15200	C TEST FOR M = LAST COLUMN	EIGE1520
15300	C	EIGE1530
15400	C	EIGE1540
15500	130 IF(M-N) 135,140,135	EIGE1550
15600	135 M=M+1	EIGE1560
15700	GO TO 60	EIGE1570
15800	C TEST FOR L = SECOND FROM LAST COLUMN	EIGE1580
15900	C	EIGE1590
16000	C	EIGE1600
16100	140 IF(L-(N-1)) 145,150,145	EIGE1610
16200	145 L=L+1	EIGE1620
16300	GO TO 55	EIGE1630
16400	150 IF(IND-1) 160,155,160	EIGE1640
16500	155 IND=0	EIGE1650
16600	GO TO 50	EIGE1660
16700	C COMPARE THRESHOLD WITH FINAL NORM	EIGE1670
16800	C	EIGE1680
16900	C	EIGE1690
17000	160 IF(THR-ANRMX) 165,165,45	EIGE1700
17100	C SORT EIGENVALUES AND EIGENVECTORS	EIGE1710
17200	C	EIGE1720
17300	C	EIGE1730
17400	165 IQ=-N	EIGE1740
17500	DO 185 I=1,N	EIGE1750
17600	IQ=IQ+N	EIGE1760
17700	LL=1+(I*I-I)/2	EIGE1770
17800	JQ=N*(I-2)	EIGE1780
17900	DO 185 J=1,N	EIGE1790
18000	JQ=JQ+N	EIGE1800
18100	MM=J+(J*J-J)/2	EIGE1810
18200	IF(A(LL)-A(MM)) 170,185,185	EIGE1820
18300	170 X=A(LL)	EIGE1830
18400	A(LL)=A(MM)	EIGE1840
18500	A(MM)=X	EIGE1850
18600	IF(MV-1) 175,185,175	EIGE1860
18700	175 DO 180 K=1,N	EIGE1870
18800	ILR=IQ+K	EIGE1880
18900	IMR=JQ+K	EIGE1890
19000	X=R(ILR)	EIGE1900
19100	R(ILR)=R(IMR)	EIGE1910
19200	180 R(IMR)=X	EIGE1920
19300	185 CONTINUE	EIGE1930
19400	RETURN	EIGE1940
19500	END	EIGE1950

TABLE 24

SAMPLE PRINTOUT FROM EIGENVALUE
ANALYSIS PROGRAM

K1= 4.0280 POUNDS/INCH SPRING STIFFNESS
K2= 4.0280 POUNDS/INCH
K3= 4.0280 POUNDS/INCH
K4= 4.0280 POUNDS/INCH

HS= 2.1430 INCHES HEIGHT OF C.G. ABOVE LOWER PIVOTS

LS= 11.5000 INCHES LENGTH OF SPRINGS

ZI= 8.7500 INCHES DISTANCE FROM C.G. TO PIVOTS
IN PLUS AND MINUS Z DIRECTION

XI= 11.0000 INCHES DISTANCE FROM C.G. TO PIVOTS
IN PLUS AND MINUS X DIRECTION

X2= 11.0000 INCHES

M= 120.84000 POUNDS MASS SYSTEM MASS

IX= 5647.00000 POUNDS MASS-INCHES SQUARED INERTIA ABOUT X AXIS

IY= 12414.00000 POUNDS MASS-INCHES SQUARED INERTIA ABOUT Y AXIS

IZ= 7170.00000 POUNDS MASS-INCHES SQUARED INERTIA ABOUT Z AXIS

KVIB=

10.507826	0.000000	0.000000	0.000000	0.000000	22.518271
0.000000	16.112000	0.000000	0.000000	0.000000	0.000000
0.000000	0.000000	10.507826	-22.518271	0.000000	0.000000
0.000000	0.000000	-22.518271	1233.575000	0.000000	0.000000
0.000000	0.000000	0.000000	0.000000	2075.952391	0.000000
22.518271	0.000000	0.000000	0.000000	0.000000	1949.552000

TABLE 24 (Continued)

NATURAL FREQUENCIES AND RELATIVE AMPLITUDES

NAT. FREQ. (HZ)		AMP.
1.63989	X1	0.099090
	X2	0.000000
	X3	0.000000
	X4	0.000000
	X5	0.000000
	X6	1.000000
1.47958	X1	0.000000
	X2	0.000000
	X3	-0.136101
	X4	1.000000
	X5	0.000000
	X6	0.000000
1.27876	X1	0.000000
	X2	0.000000
	X3	0.000000
	X4	0.000000
	X5	1.000000
	X6	0.000000
1.14184	X1	0.000000
	X2	1.000000
	X3	0.000000
	X4	0.000000
	X5	0.000000
	X6	0.000000
0.90547	X1	1.000000
	X2	0.000000
	X3	0.000000
	X4	0.000000
	X5	0.000000
	X6	-0.167002
0.89288	X1	0.000000
	X2	0.000000
	X3	1.000000
	X4	0.291243
	X5	0.000000
	X6	0.000000

this investigation, it was found that if the size of 1 unit in the translational degrees of freedom was 1 inch, the size of 1 unit in the rotational degrees of freedom could be set at .1 radian.

Subroutine NROOT is called by the main program to obtain the eigenvalues and eigenvectors of $[M]^{-1}[K]$. Subroutine EIGEN is in turn called by NROOT. Both NROOT and EIGEN are IBM subroutines found in [22].

The nomenclature used for the input parameters (refer to Figure 6 on page 16) is as follows:

K1	Spring constant of spring number one (lb _F /in)
K4	Spring constant of spring number four (lb _F /in)
HS	h—height of center of gravity above lower pivots (in)
LS	ℓ—effective pendulum length (in)
ZI	Distance from center of gravity to pivots in plus and minus z-direction (in)
X1	Distance from center of gravity to pivots in plus x-direction (in)
X2	Distance from center of gravity to pivots in minus x-direction (in)
M	System mass (lb _m)
IX	Mass moment of inertia about x-axis (lb _m - in ²)

IZ

Mass moment of inertia about z-axis

$$(lb_m - in^2)$$

VITA

Thomas Earl Hartvigsen was born on September 7, 1952, in Davenport, Iowa, where he graduated from high school. He received a Bachelor of Science degree in Aerospace Engineering in December 1973 from Parks College of Saint Louis University.

He was employed by ARO, Inc., in January 1974 and currently works at the Arnold Engineering Development Center (AEDC) as a mechanical design engineer for Arvin/Calspan, Inc. In September 1974 he began studies at The University of Tennessee Space Institute toward a Master of Science degree in Mechanical Engineering.

Mr. Hartvigsen is a member of the Audio Engineering Society. He is married to Joan Shriner of St. Louis and they live in Tullahoma.