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**An Applied Statistical Reliability Analysis of the
Modulus of Elasticity and the Modulus of Rupture
for Wood-Plastic Composites**

**Masters Thesis
Presented for the
Master of Science Degree
The University of Tennessee, Knoxville**

**Diane Goodman Perhac
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Dedication

This thesis is dedicated to my incredibly wonderful husband, Ralph, and my beautiful and precious children Angela, Alex, and Matthew. Thank you for your love, support, patience, and numerous sacrifices throughout my academic career. This thesis and the pursuit of my goals would not have been possible without you.

Angela, Alex, and Matthew: Thank you for your encouragement, for making me laugh, and for providing great joy, beauty, and truth to my life. Each of you has shown me that learning is a life-long process. I have learned much from each of you. I love each one of you more than I can say, and I am extremely fortunate and proud to be your mother.

Ralph, I cannot possibly thank you enough for all of your love, support, and encouragement over the many years we have been together. I am eternally grateful. You are everything to me. I love you, always and forever.

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Abstract

Wood-plastic composites (WPC) are materials comprised of wood fiber within a thermoplastic matrix and are a growing and important source of alternative wood products in the forest products industry. WPC is gaining market share in the building industry because of durability/maintenance advantages of WPC over traditional wood products and because of the removal of chromated copper arsenate (CCA) pressure-treated wood from the market. The reliability methods outlined in this thesis can be used to improve the quality of WPC and lower manufacturing costs by reducing raw material inputs and minimizing WPC waste. Statistical methods are described for analyzing both tensile strength and bending measures of WPC. These key measures include stiffness (tangent modulus of elasticity: MOE) and flexural strength (modulus of rupture: MOR) results from both tensile strength and bending tests. As with any real data analysis, the possibility of outliers is assessed and addressed. With this data, different WPC subsets are evaluated with and without the presence of a coupling agent. Separate subsets without outliers are also reviewed. Descriptive statistics, histograms, probability plots, and survival curves from these test data are presented and interpreted. To provide a more objective assessment of appropriate parametric modeling, Akaike's Information Criterion is used in conjunction with probability plotting. Selection of the best underlying distribution for the data is an important result that may be used to further explore and analyze the given data. In this thesis, these underlying distributional assumptions are utilized to better understand the product's lower percentiles.

These lower percentiles provide practitioners with an evaluation of the product's early failures along with providing information for specification limits, warranty, and

cost analysis. Estimation of lower percentiles is sometimes difficult, since substantive data is often sparse in the lower tails. Bootstrap techniques provide important solutions for confidence interval assessments of these percentiles. Bootstrapping is a computer intensive resampling method that may be used for both parametric and nonparametric models. This thesis briefly describes several bootstrapping methods and applies these methods to appraise MOE and MOR test results on sampled WPC. The reliability and bootstrapping methods outlined in this thesis may directly benefit WPC manufacturers through a better evaluation of strength and stiffness measures, which can lead to process improvements with enhanced reliability, thereby creating greater manufacturer and customer satisfaction.

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Chapter 1

1. Introduction

Wood-plastic composite (WPC) lumber refers to a material comprised of wood fiber within a thermoplastic matrix. Recycled plastics, such as the thermoplastics HDPE, LDPE, PP, and PVC, in addition to wood waste materials (e.g., 20 – 60 mesh wood flour from pine, oak, or maple) and various additives are normally used in WPC production. Wood-plastic composites usually contain 30 to 60 percent wood fibers and 70 to 40 percent plastic. Depending on the end use, some of the possible additives include lubricants, pigments, coupling agents, stabilizers, reinforcing agents, blowing agents, and foaming agents. The popularity of WPC by consumers and the forest products industry is continually increasing, especially for exterior end uses such as decking applications because of the many attractive qualities stemming from the combination of wood and plastic. A few of the positive attributes are the use of recycled materials, low maintenance requirements, high moisture resistance, decay and insect resistance, low splintering, and good machinability. Several negative attributes include high initial costs, lower stiffness than wood, and thermal expansion. Additionally, long-term field durability of WPC has not been studied. There is a growing need to develop realistic methods for assessing many aspects of WPC durability. As these composites evolve, product quality and reliability of WPC must be continually addressed for these composites to maintain and experience continued growth in the market place. This thesis explores the product quality of WPC by applying classical reliability and bootstrapping methods to the stiffness and breaking strength of WPC.

Product life for WPC is measured by stiffness or strength to failure, rather than time to failure. This product life is an important reliability measure of WPC. Estimation of the stiffness or strength enables practitioners and manufacturers to determine the viability, function, safety, usefulness, and quality of their product. The most important measures for WPC that will be used for decking purposes are the bending tangent modulus of elasticity (MOE) and the bending modulus of rupture (MOR). These measures will be the focus of this thesis. In addition, other important measures include the tensile strength tangent modulus of elasticity, also denoted MOE, and the tensile strength modulus of rupture, also denoted MOR. The tensile strength measures provide additional information about the quality of WPC and will also be discussed. Both MOE and MOR are measured in megapascals (MPa). For other important measures and approaches of reliability, compare Guess and Proschan (1988), Guess, Hollander, and Proschan (1986), Guess, Walker, and Gallant (1992), Young and Guess (1994), Young and Guess (2002), Guess, León, Chen, and Young (2004), and Guess, Zhang, Young, and León (2005).

Chapter 2 of this thesis is a literature review that provides brief introductions or descriptions to the main ideas contained within the thesis. The literature review begins with a brief history and description of WPC, including the many end uses, types, market share, and expected growth. Next, statistical reliability methods and studies are presented. The importance of graphical representations including histograms, scatter plots, probability plots, and survival curves are discussed. Furthermore, Akaike's Information Criterion is examined for the purposes of choosing a "best" model to

represent the data. Finally, the literature review describes various bootstrapping methods which are applied to the WPC data discussed in this thesis.

The test data used in this analysis were obtained from the University of Maine, Advanced Engineering Wood Composites Center (AEWC), <http://www.aewc.umaine.edu/>. WPC using five different polymer resins and two pine species, *Pinus resinosa* and *Pinus strobes*, were tested. The polymer resins consisted of high density polyethylene (HDPE) and four differing variations of polypropylene (PP), including PP-impact copolymer, PP-homopolymer, PP-high crystallinity polymer, and PP-random copolymer.

Product test data examined in this thesis include strength, or modulus of rupture, and stiffness, or modulus of elasticity, of WPC. The modulus of rupture (MOR) is defined as the maximum stress that can be applied to a beam in pure bending before permanent deformation occurs. The tangent modulus of elasticity (MOE) is defined as the rate of change of strain as a function of stress and is measured as the slope of the straight line portion of a stress-strain diagram taken at any point. Chapters 3 and 4 of this thesis discuss WPC's bending tangent modulus of elasticity (MOE) and bending modulus of rupture (MOR), respectively. Chapter 5 presents an analysis for both the tensile strength MOE and the tensile strength MOR for WPC. For each analysis, the data are divided into two subsets, those WPC samples that incorporated the copolymer-coupling agent—malienated polypropylene (maleic anhydride modified polypropylene or MAPP) in the production process and those that did not. If outliers are present, a third subset without the outliers is analyzed.

Chapter 3 discusses statistical methods used to explore the reliability of WPC. Additional information is provided on WPC as well as specific information regarding the data set used for the bending tangent modulus of elasticity (MOE) study. Descriptive statistics, histograms, probability plots, and survival curves (Kaplan-Meier estimates) are shown and analyzed. These methods are used as a means to provide an easy and relatively subjective way to interpret the reliability of the data. Descriptive statistics and basic histograms should always be assessed as a first step in data analysis. Location statistics, such as the mean and median, the dispersion statistics, such as the standard deviation, variance, range, coefficient of variation, and the interquartile range, and the shape of the data, expressed by the skewness and kurtosis, may be easily assessed. Histograms also provide an easy way to quickly determine whether or not outliers are present. The possibility of outliers in any given data set should always be considered early in the analysis. Outlier reasons and removal should be carefully reviewed before proceeding with the analysis. Outlying data may provide new and important information. Probability plots are used to show how a data set conforms to a specific parametric distribution, and survival curves are nonparametric plots that demonstrate the strength to failure of the data. Finally, Akaike's Information Criterion (AIC) is discussed and utilized to identify the best parametric model for the data. AIC values provide a subjective quantitative measure that, when used along with probability plots, provides reasonable reassurance that an appropriate model has been chosen.

Chapter 4 discusses the same statistical methods used in Chapter 3 for the bending MOR data set for WPC, while Chapter 5 discusses these same methods for the tensile strength MOE and the tensile strength MOR data sets. Although the bending data sets are

more useful for WPC applications such as decking, tensile strength tests provide additional information about WPC since the mechanical test measures the composites under axial stretch loading. In contrast, the bending or flexural testing for the WPC samples used in this analysis was assessed with a four-point loading system, or flexure test, where the tensile stress is applied in the convex side of the specimen and compression stress is applied in the concave side. See <http://www.instron.us/wa/applications/default.aspx#testtypes> for more information regarding test methods. Descriptions of testing methods and equations for these measures may be found in ASTM International standards, ASTM D 638-03 (2003) and ASTM D 6109-05 (2005).

Chapter 6 presents three different bootstrap methods that are used to construct confidence intervals for the percentiles of WPC. For this analysis, the bending MOE and bending MOR data sets are used. The lower percentiles are of special interest in reliability studies since these values provide valuable information pertaining to early failures during normal use, specification limits, and warranties. Bootstrapping is utilized to obtain an empirical bootstrap distribution for the desired parameter(s) by simulating the sampling process from a large population a large number of times. This bootstrap distribution is then used to characterize the chosen population parameter. The three bootstrapping methods presented in this thesis are the fully nonparametric 95% bootstrap confidence intervals, the fully parametric 95% bootstrap confidence intervals, and the 95% nonparametric bootstrap sampling method for parametric inference confidence intervals, denoted as the 95% NBSP confidence intervals by Edwards, Guess, Young, Bensmail (2007). The bootstrap techniques applied in this thesis are powerful and

effective alternatives to using the normal approximate confidence intervals and the maximum likelihood based confidence intervals.

Finally, Chapter 7 summarizes the overall approach of this thesis and discusses possibilities for future research. Although the methods and results presented in this paper are for WPC, they may be applied more generally to any data set requiring reliability analysis.

Chapter 2

2. Literature Review

This chapter provides a brief background and some uses of WPC, offers some basic information on reliability analysis, and presents some basics of bootstrapping methodology and applications. It is assumed that the reader has little prior information on wood-plastic composites and at least some knowledge of statistical methodology and various applications. Therefore, this chapter focuses primarily on WPC, leaving many of the reliability and bootstrapping methodology details to Chapters 3, 4, 5, and 6.

Wood-plastic composites (WPC) are gaining market share in the building industry as a result of chromated copper arsenate (CCA) pressure-treated wood being removed from the market, perceived durability advantages over traditional wood products, and forest conservation concerns, compare Clemmons (2002). Smith and Wolcott (2005a, 2005b, 2006) indicate that demand for WPC decking and railings, which accounted for approximately two-thirds of the United States \$1 billion extruded WPC market in 2005, has been growing since 2002 at an annual rate of 16 percent. As a percentage of all decking, WPC decking is expected to grow from 19 percent in 2002 to 42 percent in 2010, see Anon. (2006b). Other WPC applications experiencing a high growth (between 16 and 20 percent annually) include window profiles, patio furniture, shingles, and siding, see Anon. (2003). Increased WPC decking demand will be driven by increased consumer and contractor familiarity, a growing distribution network, and substantial product improvements in terms of durability, stiffness, strength, cost, light sensitivity, and appearance, see Anon. (2006a).

Wood-plastic composite lumber refers to a material comprised of wood fiber within a thermoplastic matrix. Recycled plastics, such as the thermoplastics HDPE, LDPE, PP, and PVC, in addition to wood waste materials (e.g., 20 – 60 mesh wood flour from pine, oak, or maple) are normally used in WPC production. Other wood fiber types are being investigated for use in WPC, e.g., Kim et al. (2005) discuss how ash fibers from trees infested by emerald ash borers have been successfully tested. In the production process, wood flour is dried then mixed with plastic and additives that include lubricants, pigments, coupling agents, stabilizers, reinforcing agents, blowing agents, and foaming agents. Processing technologies used to manufacture WPC include extrusion, and injection or compression molding. Wood-plastic composites typically contain 30 to 60 percent wood fibers and 40 to 70 percent plastic. They are rigid and usually pigmented to look like natural wood.

Advantages of WPC over natural wood include utilization of recycled materials, low maintenance, dimensional stability, low moisture absorption, increased rot resistance, consistent and uniform shapes, splinter resistance, as well as not requiring periodic painting. Disadvantages include initial higher costs, lower stiffness than wood, thermal expansion, creep (increase in deformation over time while subjected to a sustained load), along with sensitivity to staining and light sensitivity, which allow notable color changes and color variations within the composites.

Wood-plastic composites have an advantage over natural wood in outdoor uses such as residential decks (Figure 2.1), fences, landscape tiers, railroad ties, road noise barriers, boat docks, window and door profiles, residential furniture such as patio furniture and bathroom and kitchen cabinets, exterior and interior trim, playground

equipment, picnic tables, benches, gazebos and walkways, in addition to naval pier decking. Zawlocki and Hermanson (2004) indicate that non-residential uses of WPC, such as heavy structural applications and use in marine structures, appears to be growing. New applications include roofing materials, siding, column foundation connection elements, and sill plates.

Improvements in durability and service life of WPC are needed to enhance performance in existing uses and accommodate new outdoor applications. Clemons (2002) discusses various research efforts focusing on WPC concerns such as increasing insect and fungal attack resistance, fire performance, ultraviolet light degradation, and creep performance. In addition, new and modified processing methods are being explored to eliminate some of the undesirable characteristics of WPC. For example, new



Figure 2.1 Illustration of WPC decking.

wood-plastic composites utilizing superior compatibilizers are under development. Geng et al. (2005) discuss how these new composites could result in cost reductions, superior stiffness, and increased strength characteristics. Another example is the inclusion of a copolymer-coupling agent, maleinated polypropylene (maleic anhydride modified polypropylene or MAPP) in the production process. Harper and Wolcott (2005) established that the MAPP copolymer migrates to the wood surfaces, altering the wood-plastic matrix of the composite by increasing the compatibility between the hydrophilic wood and hydrophobic plastic components, thereby increasing the long-term durability performance of WPC, e.g., creep potential is greatly reduced. This analysis, in fact, examines the reliability and survival function differences for WPC samples with and without this MAPP coupling agent.

In order to ensure continued market share growth, WPC manufacturers need to focus on reliability, quality, and cost. The reliability methods outlined in this thesis can be used to improve the quality of WPC, as well as lower manufacturing costs by reducing raw material inputs and minimizing WPC waste. For discussions on various approaches to measuring and understanding reliability, see Guess and Proschan (1988), Guess, Hollander, and Proschan (1986), Guess, Walker, and Gallant (1992), Young and Guess (1994), Young and Guess (2002), Guess, León, Chen, and Young (2004), and Guess, Zhang, Young, and León (2005).

Improvements in product reliability hinge on the collection and interpretation of real-time destructive test data during the manufacturing process. Hence, data quality is an important issue for WPC manufacturers. In industrial settings, real-time and destructive data often will have outliers, missing values, or require sorting according to

the final product type. Improving data quality will lead to improved statistical analysis and, ultimately, improved product quality. For guidance in this crucial area, see English (1999), Huang, Lee, and Wang (1999), and Redman (1996, 2001).

Many excellent books and articles on bootstrapping methodology are available. Chernick (1999) provides a comprehensive discussion of several bootstrap methods and applications of these methods. Efron (2003), Efron and Tibshirani (1993), and Davison and Hinkley (1997) discuss bootstrap methodology, theory, and many applications. DiCiccio and Efron (1996) present several types of bootstrap confidence intervals and their applications, including standard, percentile, and bootstrap-t among others. Polansky (2000) indicates that bootstrap confidence intervals constructed by percentile methods have an upper bound on the coverage probability that can be relatively low.

Chapter 9 of Meeker and Escobar (1998) is devoted to bootstrap confidence intervals for reliability data. They discuss two methods of bootstrap sampling that include the parametric bootstrap for parametric inference and the nonparametric bootstrap for parametric inference. Both methods require knowledge of the underlying parametric distribution of the original data. Construction of confidence intervals using these methods, and the limitations involved using these methods, are discussed.

Bootstrap applications are computer-intensive, and often data analysts or practitioners must write their own code in order to utilize these applications. Martinez and Martinez (2002) provide several basic steps in the estimation of standard error, bias, and confidence intervals, including bias-corrected and accelerated intervals, using MATLAB. Lunneborg (2000) demonstrates the construction of confidence intervals using bootstrap techniques with Resampling Stats and S-PLUS software. Algorithms for

many resampling methods are provided. For this thesis, algorithms for bootstrap confidence intervals were implemented with MATLAB.

Chapter 3

3. Exploring with Graphical and Numerical Statistical Methods the Reliability of Bending Tangent Modulus of Elasticity (Moe) in Wood-Plastic Composites

Bending strength MOE descriptive statistics were generated for two subsets of data: WPC samples with and without MAPP. These descriptive statistics include the mean, median, standard deviation, coefficient of variation, interquartile range (IQR), minimum (Min), maximum (Max), skewness, and kurtosis. The box plot and histogram for the MOE data are also evaluated. In addition to these descriptive measures, probability plots, information criteria, and reliability/survival functions are utilized in order to better understand the data. See, for example, Guess, Walker, and Gallant (1992), and Walker and Guess (2003) for how different measures of reliability can be used. Compare Guess, Edwards, Pickrell, and Young (2003) for graphical and statistical analysis of medium density fiberboard.

Descriptive statistics of the bending MOE data are summarized in Table 3.1; they characterize the location, variability, and shape of these data. Location statistics include the mean and median, while variability statistics include standard deviation, coefficient of variation, and interquartile range (IQR). The shape of the data is expressed by the skewness and kurtosis values. Skewness measures the direction and degree of asymmetry and is represented graphically by a longer tail in the skewness direction. A positive numerical value indicates skewness to the right while a negative value indicates skewness to the left. Table 3.1 shows skewness values of -0.251 and -0.272 for the bending MOE

Table 3.1 Wood-plastic composites bending tangent modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP) descriptive statistics.

Statistics	Bending MOE (MPa) without MAPP	Bending MOE (MPa) with MAPP
Mean	3868.168	3901.520
Median	3971.372	4010.012
Standard Deviation	736.152	615.116
Coefficient of Variation	19.031	15.766
IQR	1045.483	977.456
Min	2340.300	2583.728
Max	5790.445	5133.649
Skewness	-0.251	-0.272
Kurtosis	-0.539	-0.882
N	150	120

without MAPP and the bending MOE with MAPP, respectively. These negative values indicate that the distributions for both subsets are slightly left skewed.

Kurtosis is a measurement of the peakedness (narrow or broad) of a distribution. It is a measure of the extent to which the probability is concentrated around the mean and in the tails rather than in the midrange relative to a normal distribution. The kurtosis value for a normal distribution is zero. A kurtosis value less than zero is obtained for a distribution with a wide midrange, on either side of the mean, and a low peak (referred to as platykurtic) while a kurtosis value greater than zero indicates a high peak, a thin midrange, and fat tails (referred to as leptokurtic). Distributions with kurtosis values of approximately zero are referred to as mesokurtic. Higher kurtosis values indicate that more of the variance is attributed to infrequent extreme deviations, in contrast to frequent modest-sized deviations. The kurtosis values of -0.539 and -0.882, shown in Table 3.1, for both the without MAPP and with MAPP subsets indicate that the distributions are

platykurtic. This is further supported by viewing the shapes of the histograms in Figure 3.1 for the MOE data.

The histogram is useful for showing both the skewness and kurtosis of the data set. The histograms in Figure 3.1 indicate that the bending MOE data, with and without MAPP, distributions are slightly left skewed and platykurtic. Box plots are shown alongside the histograms. Box plots are valuable tools for summarizing interval data. They show the shape of the distribution, the median, the lower and upper quartiles, the minimum and maximum data values, and possible outliers. The diamond shape within each box plot represented in this paper indicates the 95% confidence interval for the mean of the data. The vertical bar within each box plot represents the median of the data. The mean is less robust with respect to the distribution of the data than is the median. Thus, the box plot of a left-skewed distribution will show the 95% confidence interval for the mean to the left of the median, while a right-skewed distribution will have the mean interval to the right of the median. Outliers, if present, are represented as points outside the ends of the whiskers (lines seen extending from the sides of the box). The box plots shown in Figure 3.1 indicate the left-skewness of the distributions. No outliers are indicated for either distribution.

Probability plots are commonly used in the analysis of reliability data, because they graphically illustrate the conformity of a particular data set to a variety of distributions. The data are ordered and plotted against the theoretical order statistics for selected distributions. If the data set is consistent with a specific distribution, the data values will fall on, or close to, a straight line for the probability plot of that specific distribution. Simultaneous confidence bands with pointwise confidence intervals provide

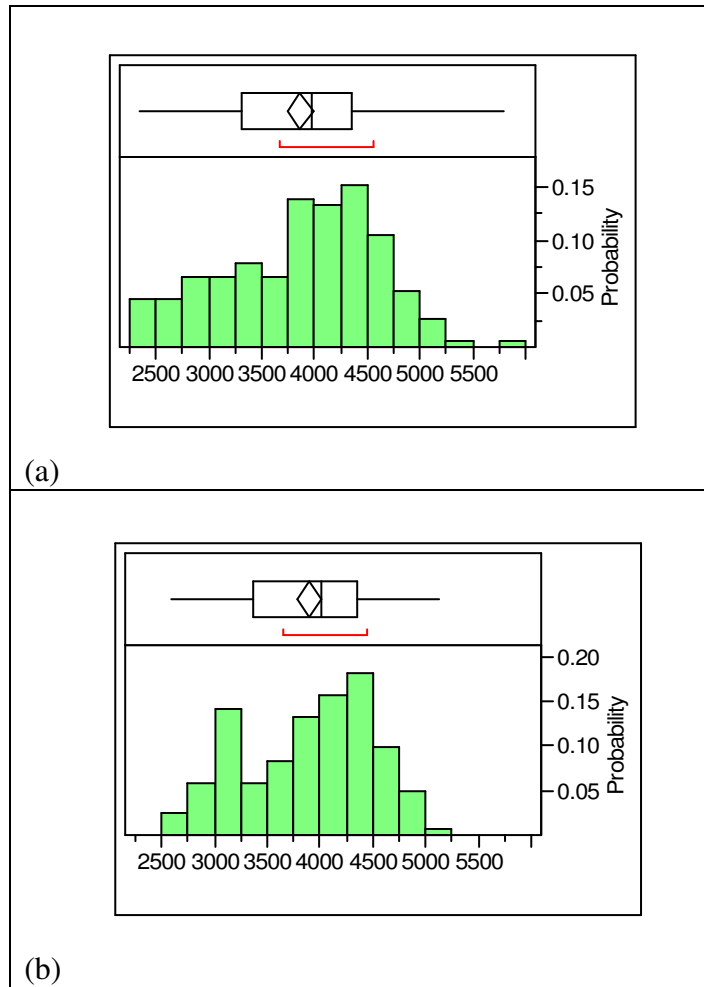


Figure 3.1 Box plots and histograms of wood-plastic composites bending tangent modulus of elasticity (MOE) (a) without coupling agent (MAPP) and (b) with coupling agent (MAPP).

objective assessments of deviations from this straight line. Data points falling outside the confidence bands indicate that the data do not adequately fit the candidate probability distribution. Refer to Chapter 6 of Meeker and Escobar (1998) for additional information. Smallest extreme value (SEV), Weibull, normal, lognormal, largest extreme value (LEV), and Frechet probability plots were produced for the bending strength MOE data using S-PLUS and SPLIDA.

Probability plots for the smallest extreme value (SEV), Weibull, normal, lognormal, largest extreme value (LEV), and Frechet distributions for the WPC MOE without MAPP data are shown in Figure 3.2. It appears that the data are best represented by the Weibull, normal, and smallest extreme value probability plots. Additional curvature is seen at either tail or both tails with the remaining three distributions. Figure 3.3 shows the probability plots for the WPC MOE with MAPP data. The best distributional representations for the MAPP data are also the Weibull, normal, and smallest extreme value distributions. These probability plot results are further supported by the results obtained from Akaike's Information Criterion (AIC).

Table 3.2 displays the log likelihood and AIC scores of select models. These scores provide a quantitative measure for choosing the best-fitting distributional model. Akaike (1973) and Bozdogan (2000) define the AIC for model selection. This criterion provides scores for each of the selected models of a particular data set, and is represented as:

$$A I C = -2 \log L(\hat{\theta}) + 2k \quad (2.1)$$

where $L(\hat{\theta})$ is the likelihood function evaluated at the maximum likelihood estimator $\hat{\theta}$ (either scalar or vector values) and k is the number of parameters evaluated in the model,

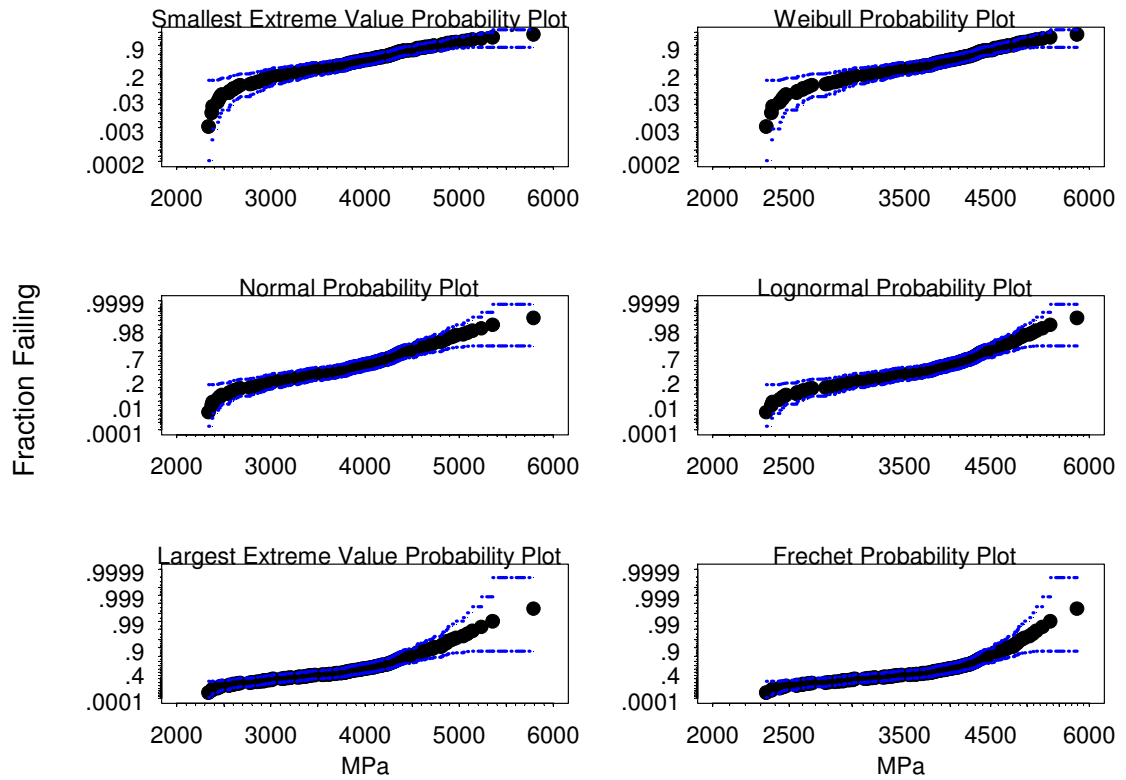


Figure 3.2 Wood-plastic composites bending tangent modulus of elasticity (MOE) with no coupling agent (MAPP) probability plots from S-PLUS and SPLIDA.

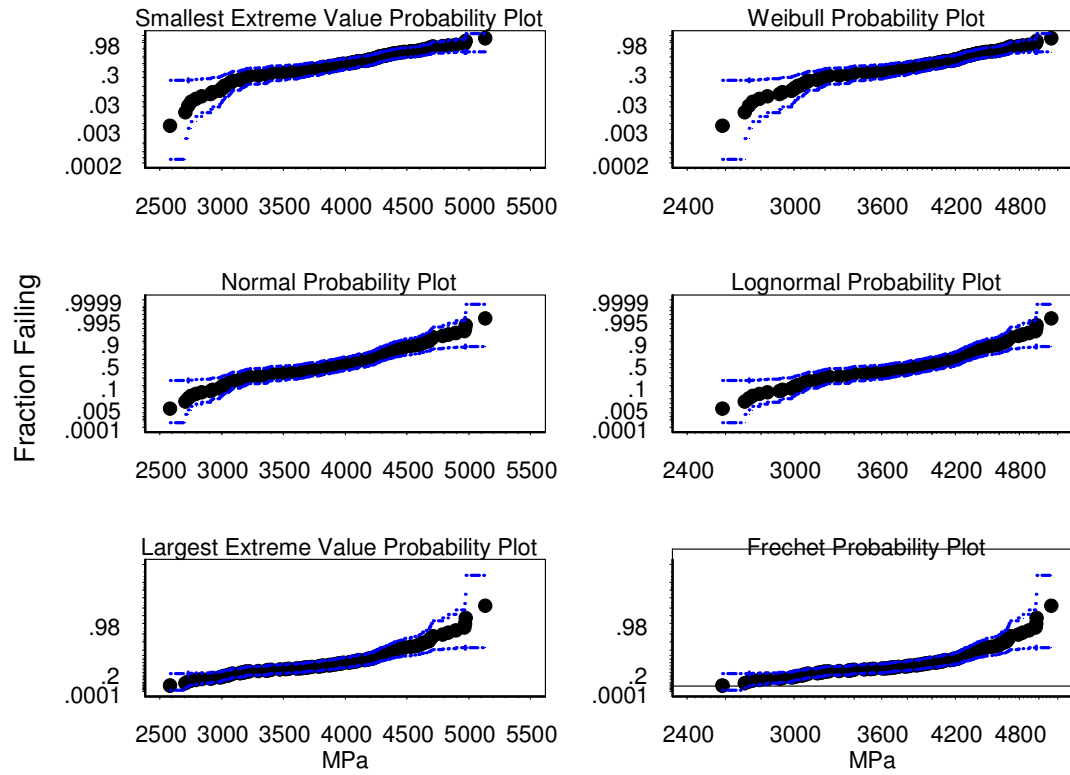


Figure 3.3 Wood-plastic composites bending tangent modulus of elasticity (MOE) with coupling agent (MAPP) probability plots from S-PLUS and SPLIDA.

Table 3.2 Selected model scores for the wood-plastic composites bending modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP).

Model Fit	Bending MOE without MAPP		Bending MOE with MAPP	
	Log Likelihood	AIC	Log Likelihood	AIC
Exponential	-1389	2782	-1112.0	2228.0
Frechet	-1230	2464	-958.1	1920.2
LEV	-1216	2436	-950.1	1904.2
Logistic	-1206	2416	-944.4	1892.8
Loglogistic	-1211	2426	-947.3	1898.6
Lognormal	-1209	2422	-943.9	1891.8
Normal	-1203	2410	-940.4	1884.8
SEV	-1206	2416	-940.4	1884.8
Weibull	-1201	2406	-938.3	1880.6

e.g., $k = 2$ for the normal model with the parameters μ and σ^2 . The model with the lowest AIC score is selected as the best model for the data. For the bending MOE without MAPP data, the lowest score is obtained for the Weibull distribution followed by the normal, SEV, and logistic distributions. This result is consistent with the probability plots shown in Figure 3.2. For the bending MOE with MAPP data, the lowest score is obtained for the Weibull distribution followed by the normal, SEV, and lognormal distributions. The Weibull distribution clearly represents the best-fitting model for both data subsets.

The reliability/survival function is used to determine the probability that the product will survive beyond a specified “time” or “pressure.” With the MOE data, pressure to failure is measured. The Kaplan-Meier estimator (origin of Product Limit Estimator) estimates the survival function from life-time (or pressure to failure) data, see Kaplan and Meier (1958). Kaplan-Meier plots, also called Product Limit graphs, are commonly used and

provide a simple means of estimating the survival curve when problematic data, such as censored data, occur. Figure 3.4 shows the combined plots for the bending MOE without MAPP and the bending MOE with MAPP. This figure shows the survival probabilities of WPC for increasing pressure. For example, the probability that the bending MOE without MAPP will be greater than 3968.63 MPa is 0.50, while the probability that the bending MOE is greater than 4728.37 MPa is 0.10. Statistically, 5% of WPC fails before reaching a pressure of 2480.15 MPa and 95% of WPC fails before attaining a pressure of 4882.73 MPa. The Kaplan-Meier plot for this WPC data set indicates that pressure to failure decreases at increasing rates between 2500 and 5250 MPa. For comparison, the probability that the bending MOE with MAPP will be greater than 3987.11 MPa is 0.50, while the probability that the bending MOE is greater than 4656.70 MPa is 0.10. Statistically, 5% of WPC fails before reaching a pressure of 2838.33 MPa and 95% of WPC fails before realizing a pressure of 4790.73 MPa. The Kaplan-Meier plot for this WPC data set indicates that pressure to failure decreases at increasing rates between 2583 and 4978 MPa. For this WPC bending MOE data, it appears that the addition of the MAPP coupling agent does not greatly alter or improve the stiffness or tangent modulus of elasticity of the product.

In order to verify that there is no difference between the bending MOE with no MAPP subset and the bending MOE with MAPP subset, a two sample t-test was conducted, assuming unequal variances. This is another way of viewing the data and is valid when the sample sizes are greater than 30 since this test is robust to the assumption of normality. For this test, a p-value of 0.6855 was obtained, thereby providing further evidence that there is, in fact, no difference between the bending MOE with no MAPP

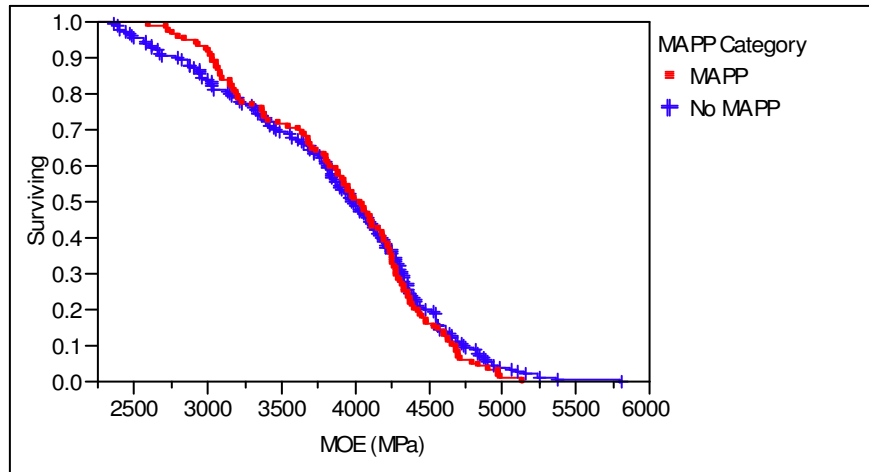


Figure 3.4 Reliability Kaplan-Meier Plot of wood-plastic composites bending tangent modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP).

and the bending MOE with MAPP subsets. Figure 3.5 provides side by side box plots showing comparisons of the two distributions. Although the dispersion for the no Mapp subset is greater, the difference in the means of the two distributions is not statistically significant. The practitioner may use the Kaplan-Meier plots to explore the effects of different wood fibers, polymer resins, and various additives as well as whether or not a coupling agent has been added, for new product development by comparing plots for the different rates. These comparisons may provide an effective means for minimizing raw material usage and reducing sources of variation, while maintaining product reliability.

3.1 SUMMARY

Graphical and numerical statistical methods used to explore the reliability of wood-plastic composites, as measured by the bending MOE, provide valuable

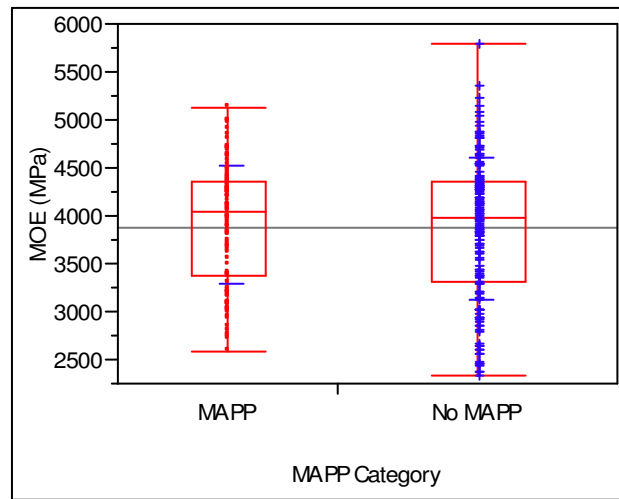


Figure 3.5 Side-by-Side Box Plots of wood-plastic composites bending tangent modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP).

information about WPC. Descriptive statistics show that the addition of the coupling agent, MAPP, to the wood-plastic composite yields approximately the same MOE values. The probability plots obtained for both the bending MOE without MAPP and the bending MOE with MAPP were almost identical. AIC results were almost identical as well. The best parametric distributional assumption for both the MOE without MAPP data and the MOE with MAPP data is the Weibull distribution. Kaplan-Meier plots and a two sample t-test for the MOE data with and without MAPP also indicate no real differences. Finding no substantial differences within a study is important information for manufacturers and practitioners. This particular finding indicates that the current process is robust with regard to the addition of the coupling agent, MAPP. Future studies could include composites with and without MAPP as well as other variables (such as lubricants, pigments, reinforcing agents, or fungicides) to assess the possibility of interactions as well as main effects.

Chapter 4

4. Exploring with Graphical and Numerical Statistical Methods the Reliability of Bending Modulus of Rupture (MOR) in Wood-Plastic Composites

Descriptive statistics for the bending modulus of rupture with and without MAPP are summarized in Table 4.1. This table also includes bending MOR with MAPP and one outlier removed since the presence of one outlier was determined in the bending MOR with MAPP subset. The skewness values of -0.619, -0.708, and -0.597 suggests mild left skewness for the bending MOR without MAPP, the bending MOR with MAPP and outlier, and the bending MOR with MAPP and outlier excluded, respectively. The kurtosis values of -0.660 and -0.123 for the bending MOR without MAPP and the bending MOR with MAPP excluding the outlier, respectively, indicate flat or platykurtic distributions. In contrast, the kurtosis value of 0.193 for the bending MOR with MAPP including the one outlier indicates a mesokurtic to mildly leptokurtic distribution. The non-peakedness or peakedness of these distributions is further supported by viewing the histogram shapes in Figure 4.1 for the MOR data.

The histograms in Figure 4.1 indicate that the bending MOR distributions are slightly left skewed and that the distributions without an outlier are relatively flat. The box plots in Figure 4.1 indicate the left skewness of the data for the three subsets and are in agreement with the skewness shown in the histograms. The WPC bending MOR with MAPP and one outlier box plot also depict the presence of the one outlier as a single point to the left of the left-sided whisker on the box plot.

Table 4.1 Wood-plastic composites bending modulus of rupture (MOR) without coupling agent (MAPP) and with coupling agent (MAPP) descriptive statistics.

Statistics	Bending MOR (MPa) without MAPP	Bending MOR (MPa) with MAPP and one Outlier	Bending MOR (MPa) with MAPP and Outlier Removed
Mean	26.970	43.717	43.856
Median	27.651	44.621	44.856
Standard Deviation	5.384	5.344	5.145
Coefficient of Variation	19.964	12.225	11.731
IQR	6.027	7.985	7.936
Min	16.187	27.193	29.942
Max	35.217	53.233	53.233
Skewness	-0.619	-0.708	-0.597
Kurtosis	-0.660	0.193	-0.123
N	150	120	119

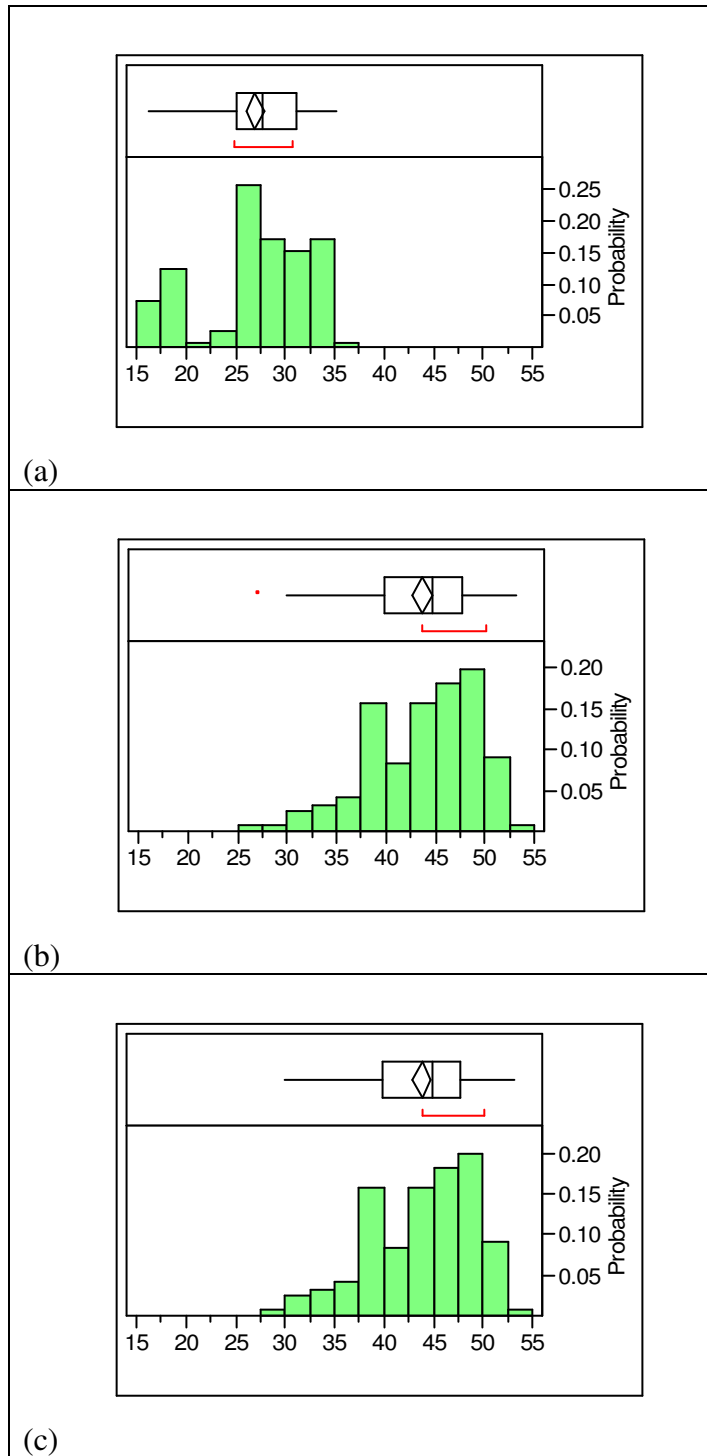


Figure 4.1 Box plots and histograms of wood-plastic composites bending modulus of rupture (MOR) (a) without coupling agent (MAPP), (b) with coupling agent (MAPP) and outlier, and (c) with coupling agent (MAPP) without outlier.

Probability plots for the smallest extreme value (SEV), Weibull, normal, lognormal, largest extreme value (LEV), and Frechet distributions for the WPC MOR data without MAPP and with MAPP including one outlier are shown in Figures 4.2 and 4.3, respectively. Figure 4.4 shows the four best, or most representative, probability plots for the WPC MOR with MAPP excluding the one outlier data. For all three subsets, it appears that the data are best represented by the smallest extreme value (SEV), Weibull, and normal probability plots. Additional curvature is seen at either tail or both tails with the remaining three distributions. These probability plot results are further supported by the results obtained from Akaike's Information Criterion.

Table 4.2 shows the log likelihood and AIC scores of select models for the bending MOR without MAPP, the bending MOR with MAPP including one outlier, and the bending MOR with MAPP excluding the one outlier. For these data, the lowest score obtained for the without MAPP and with MAPP including the outlier subsets is for the smallest extreme value (SEV) distribution. The next two lowest scores for these two subsets are for the Weibull and normal distributions, respectively. For the bending MOR excluding the outlier data, the best AIC scores obtained are for the Weibull, SEV, normal, and logistic distributions, respectively. These results are consistent with the probability plots shown in Figures 4.2, 4.3, and 4.4. It is interesting, that for the MOR with MAPP data, removal of the one outlier yielded a different choice in the underlying distribution. Although this difference is difficult to detect from viewing the probability plots, the difference is readily apparent from comparing the AIC values of 732.4 versus 720.2 for the MOR with MAPP data including the outlier and for the MOR with MAPP data excluding the outlier, respectively.

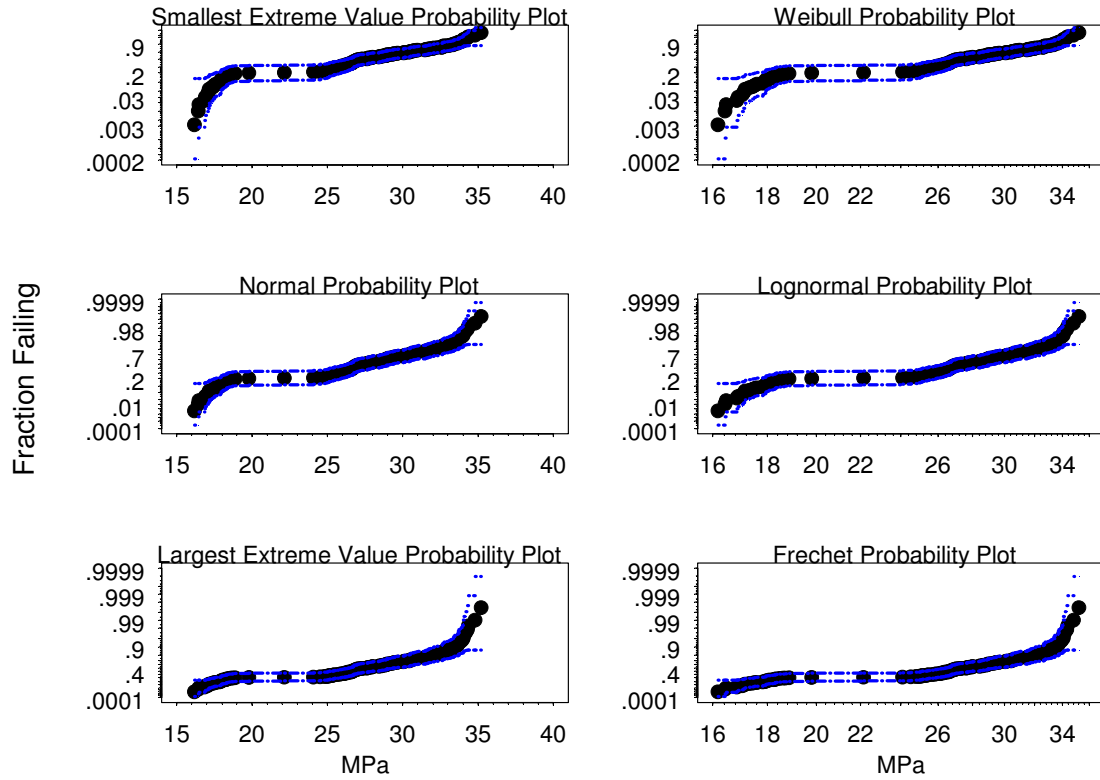


Figure 4.2 Wood-plastic composites bending modulus of rupture (MOR) without coupling agent (MAPP) probability plots from S-PLUS and SPLIDA.

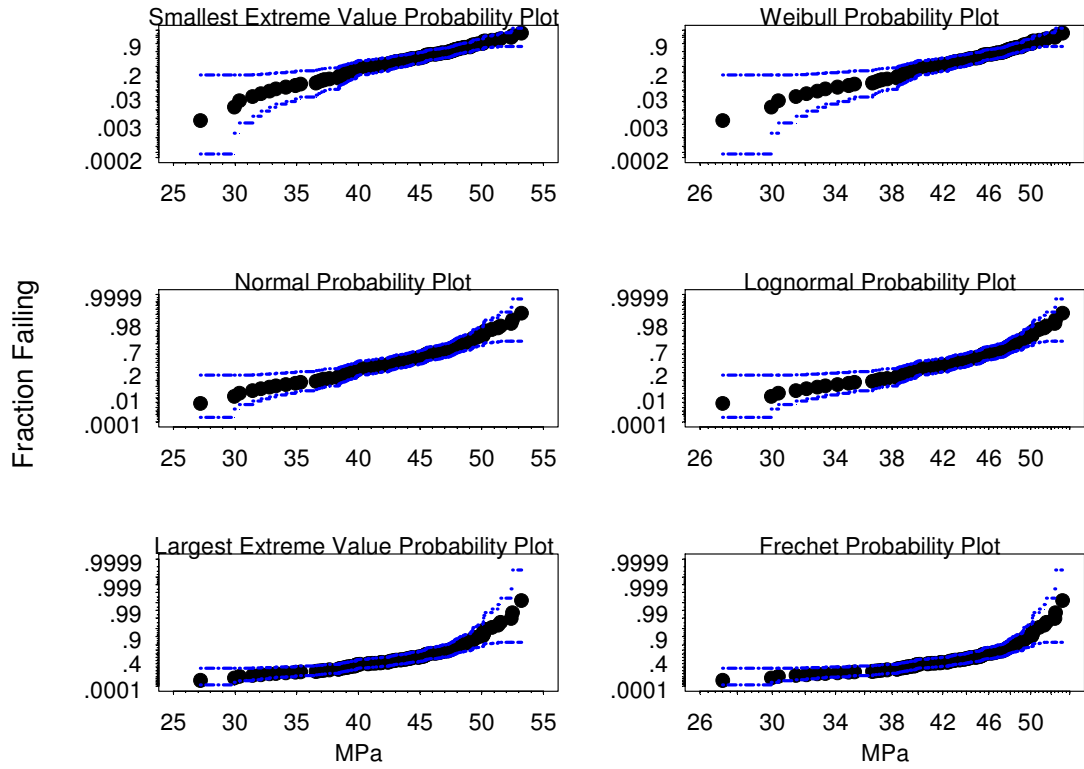


Figure 4.3 Wood-plastic composites bending modulus of rupture (MOR) with coupling agent (MAPP) probability plots from S-PLUS and SPLIDA.

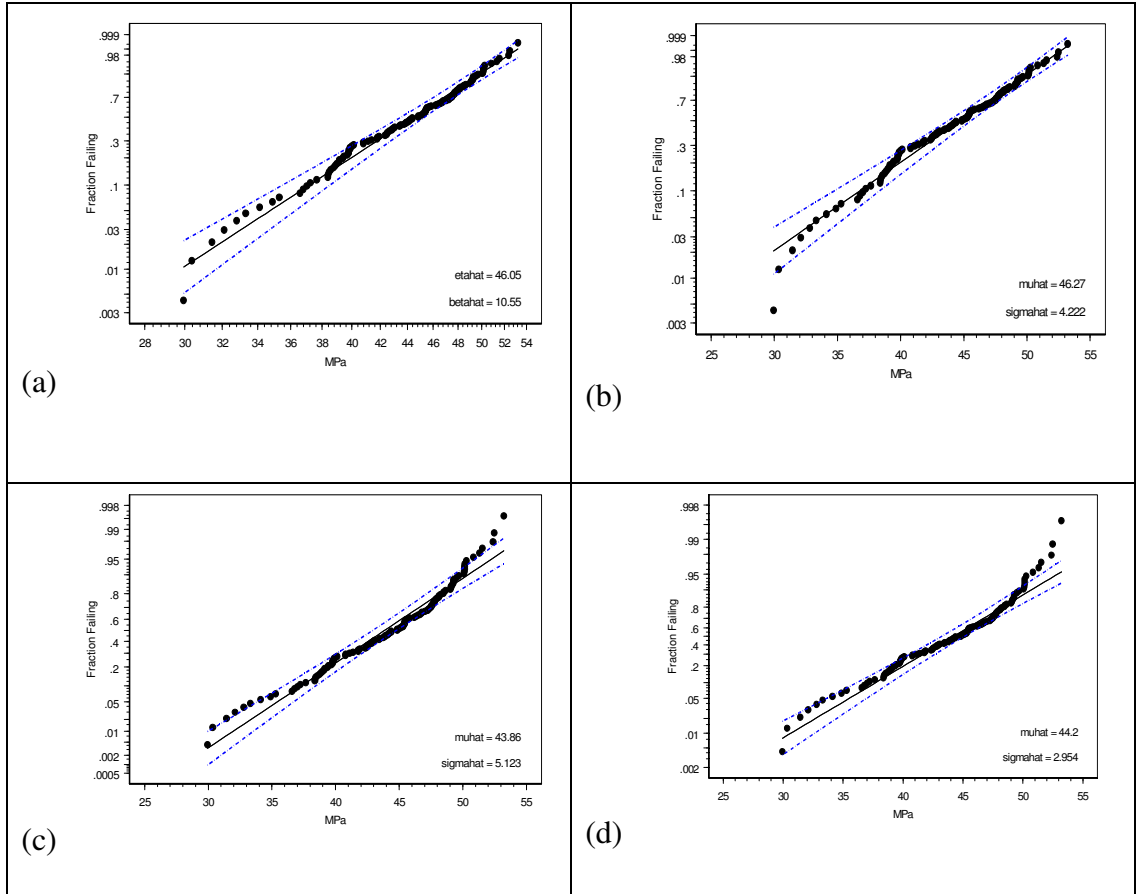


Figure 4.4 Wood-plastic composites bending modulus of rupture (MOR) probability plots with coupling agent (MAPP) and outlier removed from S-PLUS and SPLIDA (a) Weibull distribution, (b) Smallest extreme value distribution, (c) Normal distribution, and (d) Logistic distribution with horizontal axis of MPa and vertical axes of fraction failing.

Table 4.2 Selected model scores for the wood-plastic composites bending modulus of rupture (MOR) without coupling agent (MAPP), with coupling agent (MAPP) including outlier, and with coupling agent (MAPP) excluding outlier.

Model Fit	Bending MOR without MAPP		Bending MOR with MAPP Including Outlier		Bending MOR with MAPP Excluding Outlier	
	Log Likelihood	AIC	Log Likelihood	AIC	Log Likelihood	AIC
Exponential	-644.2	1292.4	-573.3	1150.6	-568.9	1141.8
Frechet	-501.1	1006.2	-404.6	813.2	-391.6	787.2
LEV	-484.1	972.2	-391.2	786.4	-380.9	765.8
Logistic	-467.8	939.6	-371.7	747.4	-364.9	733.8
Loglogistic	-477.2	958.4	-375.7	755.4	-368.2	740.4
Lognormal	-476.5	957.0	-377.6	759.2	-368.5	741.0
Normal	-464.9	933.8	-370.9	745.8	-363.3	730.6
SEV	-455.0	914.0	-364.2	732.4	-358.3	720.6
Weibull	-457.6	919.2	-364.5	733.0	-358.1	720.2

The Kaplan-Meier plots for the bending MOR data are shown in Figure 4.5. The Kaplan-Meier curves shown are for the bending MOR without MAPP and the bending MOR with MAPP excluding the one outlier. The curve for the bending MOR with MAPP including the outlier was essentially identical to the curve for the bending MOR with MAPP excluding the outlier and is not shown. For the bending MOR without MAPP subset, the probability that the bending MOR will be greater than 27.6426 MPa is 0.50, while the probability that the bending MOR is greater than 33.3801 MPa is 0.10. Statistically, 5% of WPC fails before reaching a pressure of 17.1058 MPa and 95% of WPC fails before attaining a pressure of 33.9585 MPa. The Kaplan-Meier plot for this WPC subset indicates that pressure to failure decreases at increasing rates between 16.1865 and 34.8197 MPa.

In contrast, for the bending MOR with MAPP excluding the one outlier subset, the probability that the bending MOR will be greater than 44.8556 MPa is 0.50, while the probability that the bending MOR is greater than 49.6886 MPa is 0.10. Statistically, 5% of WPC fails before realizing a pressure of 33.3224 MPa and 95% of WPC fails before attaining a pressure of 50.2882 MPa. The Kaplan-Meier plot for this WPC data set indicates that pressure to failure decreases at increasing rates between 29.9419 and 52.4952 MPa.

A two sample t-test was performed for the bending MOR with MAPP and the bending MOR without MAPP and one outlier removed. The resulting p-value is less than 0.0000, indicating a strongly significant difference in the two distributions. Figure 4.6 displays the side-by-side box plots for these two distributions. It is obvious from this comparison that the two distributions are different since almost no overlap is present.

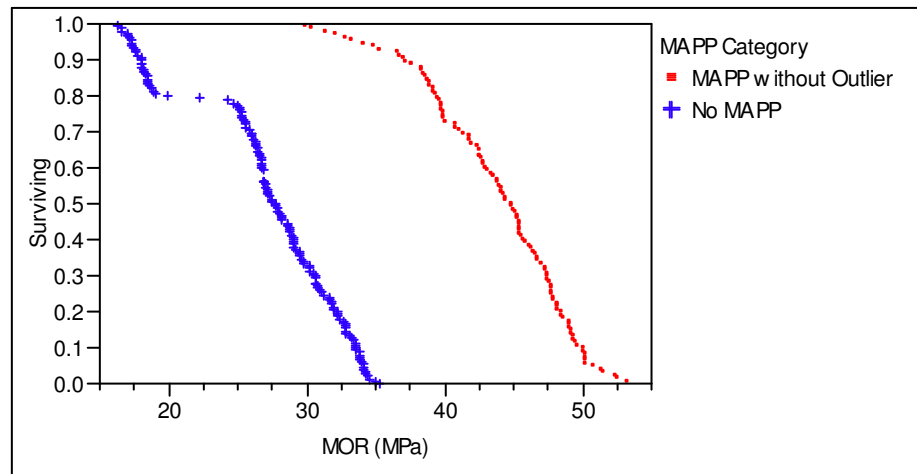


Figure 4.5 Reliability Kaplan-Meier Plot of wood-plastic composites bending modulus of rupture (MOR) without coupling agent (MAPP) and with coupling agent (MAPP) and outlier removed.

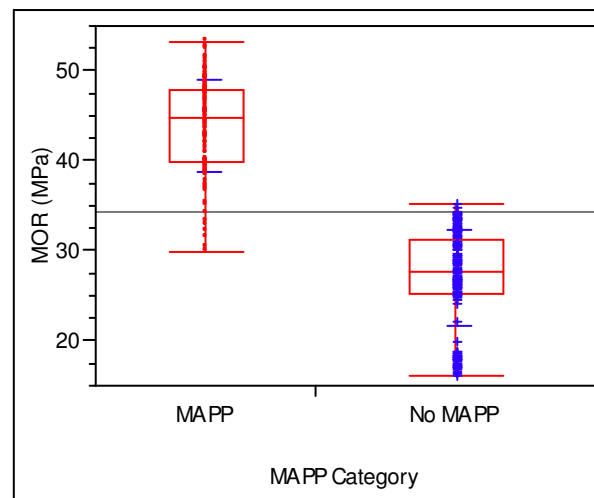


Figure 4.6 Side-by-side box plots of wood-plastic composites bending modulus of rupture (MOR) without coupling agent (MAPP) and with coupling agent (MAPP) and outlier removed.

It is interesting to see that for this WPC data, the addition of a coupling agent not only increases the breaking strength (MOR); it also helps linearize the Kaplan Meier curve. This is readily apparent from the plateau seen in the curve of the bending MOR without MAPP subset at pressures between approximately 18 and 25 MPa. Therefore, the use of MAPP increases the strength of WPC and may allow practitioners to more accurately predict failure probabilities of their manufactured WPC products. Better predictions may also help WPC manufacturers with continuous improvement of product quality and cost reductions.

4.1 SUMMARY

Exploring the reliability of wood-plastic composites by using graphical and numerical statistical methods, as measured by the bending MOR, provides valuable information about WPC. Descriptive statistics show that the addition of the coupling agent, MAPP, to the wood-plastic composite yields substantially higher MOR values. The probability plots obtained for the bending MOR without MAPP data, the bending MOR with MAPP and one outlier, and the bending MOR with MAPP excluding the outlier are almost identical. AIC comparisons did show differences for the three subsets. The best parametric distributional assumption for both the MOR without MAPP data and the MOR with MAPP and one outlier data is the smallest extreme value (SEV) distribution, while the while the best representative assumption for the MOR with MAPP excluding the outlier is the Weibull distribution, followed closely by the SEV distribution. Kaplan-Meier plots and a two sample t-test for the MOR data with and without MAPP do indicate a difference. Addition of MAPP to the composites substantially increases the

strength to failure MOR for WPC. The addition of this coupling agent also appears to linearize the Kaplan-Meier curve, thereby enabling better prediction of failure probabilities for given applied pressures. MAPP also increases the overall flexural strength, or MOR, of the WPC, which results in improved mechanical properties for many applications.

Chapter 5

5. Exploring with Graphical and Numerical Statistical Methods the Reliability of the Tensile Strength Modulus of Elasticity (MOE) and the Tensile Strength Modulus of Rupture in Wood-Plastic Composites

5.1 TENSILE STRENGTH MOE

Tensile strength test results provide additional information about WPC behavior, and thus the quality of WPC. The mechanical testing procedure for tensile strength measures the composites under axial stretch loading (one-directional) instead of the four-point flexural loading used for the bending data discussed in Chapters 3 and 4. Similarly to the bending tests described in Chapters 3 and 4, two subsets are assessed for both the tensile strength MOE and tensile strength MOR data sets; those samples where the coupling agent MAPP is a constituent and those samples that do not include MAPP. If outliers are present, a third subset without the outliers is presented and analyzed.

Descriptive statistics for the tensile strength modulus of elasticity with and without MAPP are summarized in Table 5.1. Inclusion of MAPP as a composite component appears to increase the MOE, since the mean is shown to increase from 3071.515 to 3983.097 MPa between the two distributions. Both the minimum and maximum values for the MAPP distribution have also increased substantially, from a minimum of 1738.758 to 2370.007 and from a maximum of 4421.312 to 4988.599. It is also of interest to note that the median of the MAPP distribution, 3998.583, is much closer to the mean than is the median of the distribution without MAPP, 3016.401. The skewness values of 0.174 AND -0.412 suggest very mild right skewness for the MOE

without MAPP distribution and left skewness for the tensile strength MOE with MAPP distribution, respectively. The kurtosis values of -0.426 and -0.245 for both the MOE without MAPP and the MOE with MAPP indicate flat or platykurtic distributions. The non-peakedness of these distributions is further supported by viewing the histogram shapes in Figure 5.1 for the tensile strength MOE data.

The histograms in Figure 5.1 indicate that the tensile strength MOE without MAPP subset is slightly right skewed while the tensile strength MOE with MAPP subset is slightly left skewed. Both subsets are relatively platykurtic. The box plots in Figure 5.1 indicate the right and left skewness of the data for the MOE without MAPP and the MOE with MAPP, respectively, and are in agreement with the skewness shown in the histograms.

Table 5.1 Wood-plastic composites tensile strength tangent modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP) descriptive statistics.

Statistics	Tensile Strength MOE (MPa) without MAPP	Tensile Strength MOE (MPa) with MAPP
Mean	3071.515	3983.097
Median	3016.401	3998.583
Standard Deviation	547.843	544.520
Coefficient of Variation	17.836	13.671
IQR	778.445	833.030
Min	1738.758	2370.007
Max	4421.312	4988.599
Skewness	0.174	-0.412
Kurtosis	-0.426	-0.245
N	116	115

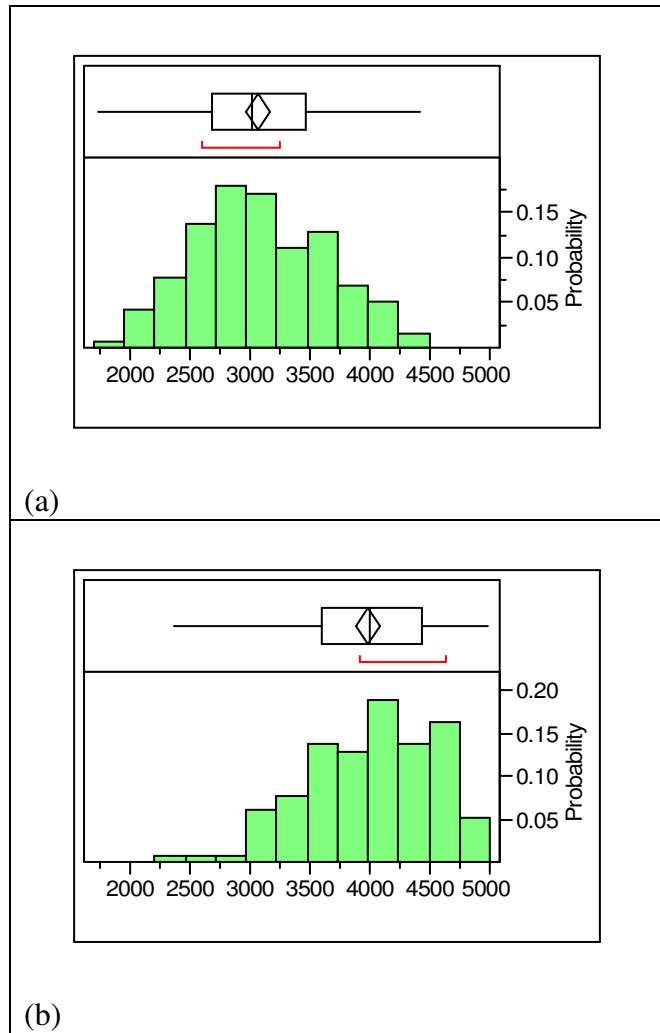


Figure 5.1 Box plots and histograms of wood-plastic composites tensile strength tangent modulus of elasticity (MOE) (a) without coupling agent (MAPP) and (b) with coupling agent (MAPP).

Probability plots for the smallest extreme value (SEV), Weibull, normal, lognormal, largest extreme value (LEV), and Frechet distributions for the tensile strength WPC MOE data without MAPP and with MAPP are shown in Figures 5.2 and 5.3, respectively. For the MOE without MAPP distribution, it appears that the data are best represented by the normal and lognormal probability plots. Additional curvature is seen at either tail or both tails with the remaining four distributions. These probability plot results are further supported by the results obtained from Akaike's Information Criterion.

Probability plots for the smallest extreme value (SEV), Weibull, normal, lognormal, largest extreme value (LEV), and Frechet distributions for the WPC MOE with MAPP data are shown in Figure 5.3. The data appear best represented by the Weibull, smallest extreme value, and normal probability plots. Again, additional curvature is seen at either tail or both tails with the remaining three distributions. These probability plot results are further supported by the results obtained from Akaike's Information Criterion (AIC).

Table 5.2 shows the log likelihood and AIC scores of select models for the tensile strength MOE without MAPP and the tensile strength MOE with MAPP distributions. For these data, the lowest score obtained for the without MAPP data is for the normal followed by the lognormal, loglogistic, and logistic distributions, respectively. For the tensile strength MOE with MAPP data, the best AIC scores obtained are for the Weibull, smallest extreme value, normal, and logistic distributions, respectively. These results are consistent with the probability plots shown in Figures 5.2 and 5.3.

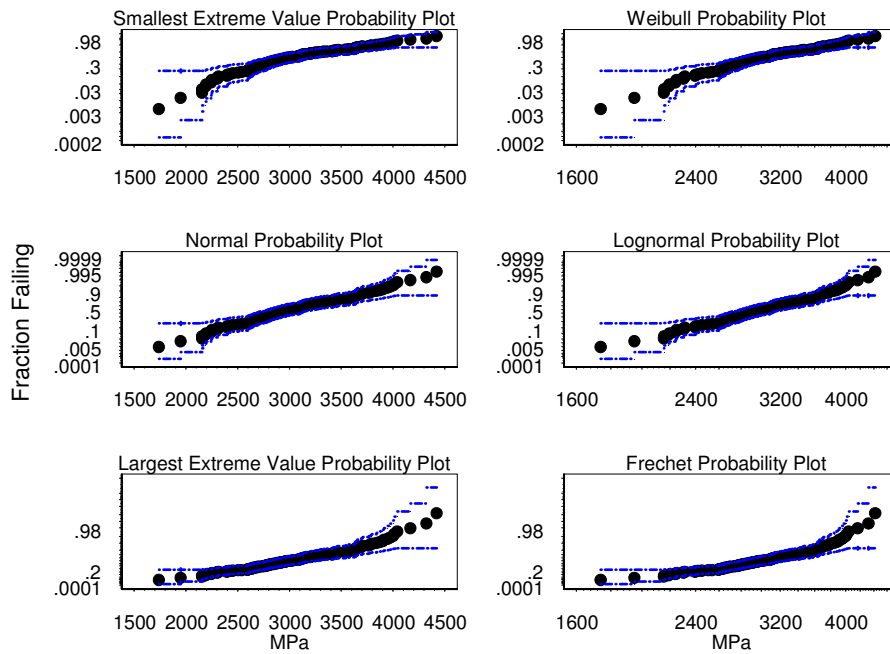


Figure 5.2 Wood-plastic composites tensile strength tangent modulus of elasticity (MOE) with no coupling agent (MAPP) probability plots from S-PLUS and SPLIDA.

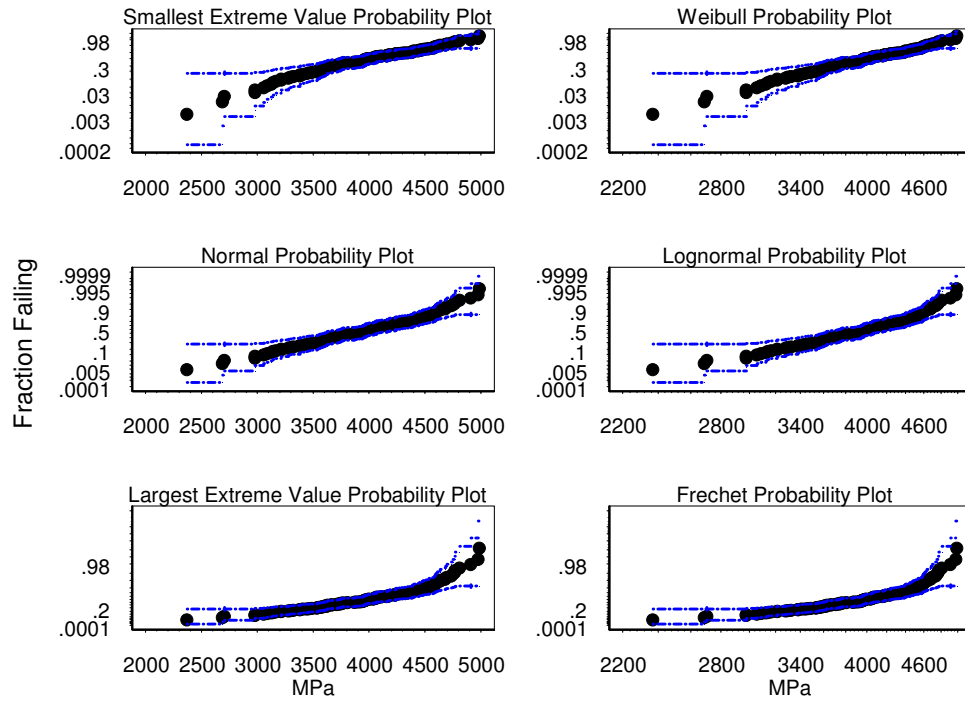


Figure 5.3 Wood-plastic composites tensile strength tangent modulus of elasticity (MOE) with coupling agent (MAPP) probability plots from S-PLUS and SPLIDA.

Table 5.2 Selected model scores for the wood-plastic composites tensile strength modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP).

Model Fit	Tensile Strength MOE without MAPP		Tensile Strength MOE with MAPP	
	Log Likelihood	AIC	Log Likelihood	AIC
Exponential	-1047.0	2098.0	-1068.0	2140.0
Frechet	-909.5	1823.0	-914.3	1832.6
LEV	-899.8	1803.6	-901.9	1807.8
Logistic	-898.0	1800.0	-889.4	1782.8
Loglogistic	-897.6	1799.2	-892.0	1788.0
Lognormal	-895.9	1795.8	-891.9	1787.8
Normal	-895.6	1795.2	-887.2	1778.4
SEV	-905.5	1815.0	-886.0	1776.0
Weibull	-898.4	1800.8	-884.6	1773.2

Figure 5.4 shows the combined plots for the tensile strength MOE without MAPP and the tensile strength MOE with MAPP. This figure shows the survival probabilities of WPC for increasing pressure. For example, the probability that the tensile strength MOE without MAPP will be greater than 3007.17 MPa is 0.50, while the probability that the tensile strength MOE is greater than 3839.98 MPa is 0.10. Statistically, 5% of WPC fails before reaching a pressure of 2198.62 MPa and 95% of WPC fails before attaining a pressure of 3974.01 MPa. The Kaplan-Meier plot for this WPC data set indicates that pressure to failure decreases at increasing rates between 1738.76 and 4323.46 MPa. For comparison, the probability that the tensile strength MOE with MAPP will be greater than 3998.58 MPa is 0.50, while the probability that the tensile strength MOE is greater than 4637.94 MPa is 0.10. Statistically, 5% of WPC fails before reaching a pressure of 3057.46 MPa and 95% of WPC fails before realizing a pressure of 4752.03 MPa. The Kaplan-Meier plot for this WPC data set indicates that pressure to failure decreases at

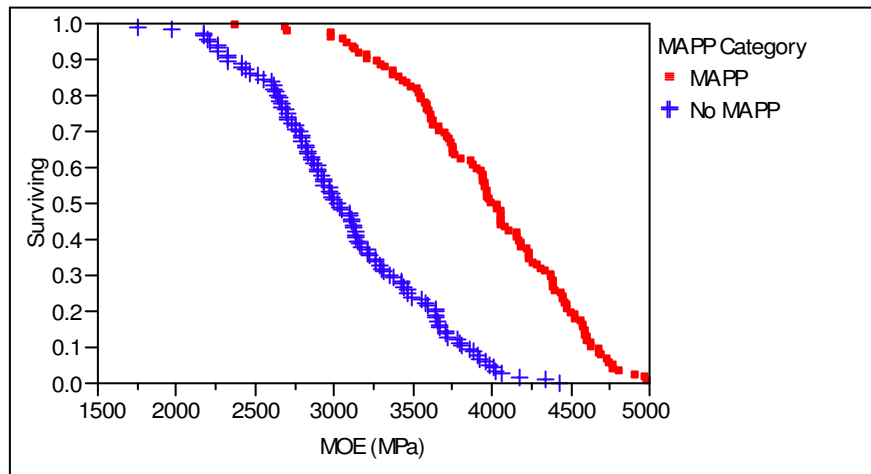


Figure 5.4 Reliability Kaplan-Meier Plot of wood-plastic composites tensile strength tangent modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP).

increasing rates between 2370.01 and 4976.20 MPa. For this WPC tensile strength MOE data, it is evident that the addition of the MAPP coupling agent improves the stiffness or tangent modulus of elasticity of the product.

To further explore this difference between the tensile strength MOE without MAPP and the tensile strength MOE with MAPP, a two sample t-test was conducted. The resulting p-value was less than 0.0000, indicating a strongly significant difference in the two distributions. A side-by-side box plot is shown in Figure 5.5, illustrating this statistical difference.

5.2 TENSILE STRENGTH MOR

Descriptive statistics for the tensile strength modulus of rupture with and without MAPP are summarized in Table 5.3. This table also includes the tensile strength MOR with MAPP and 29 outliers removed since the presence of these outliers was

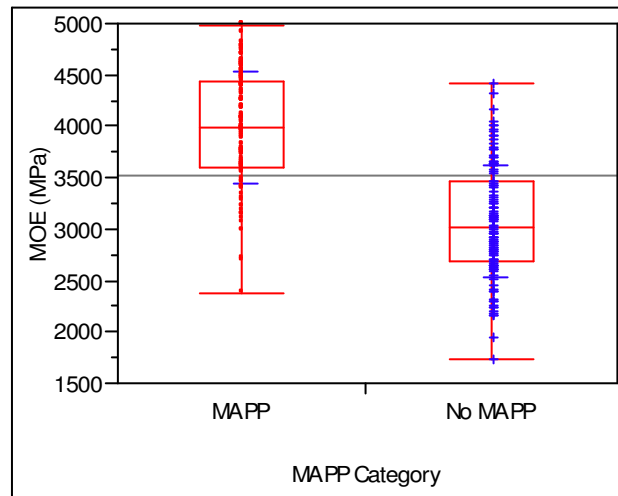


Figure 5.5 Side-by-side box plots of wood-plastic composites tensile strength tangent modulus of elasticity (MOE) without coupling agent (MAPP) and with coupling agent (MAPP).

determined in the tensile strength MOR with MAPP subset. It is interesting that all 29 outliers are in composite samples that include both MAPP and HDPE (high-density polyethylene). These samples have lower tensile strength MOR values compared with observations using plastics other than HDPE. (Note that four other plastics besides HDPE were used as the plastic constituent in this data set.) Jose, Aprem, Francis, Chandy, Werner, Alstaedt, and Thomas (2004) indicate that a natural incompatibility exists between HDPE and polypropylene, or in this case, the MAPP (maleic-anhydride-polypropylene copolymer) component of the WPC. This incompatibility is revealed as a decrease in MOR for WPC containing both HDPE and MAPP and is consistent with the similar findings of Gosselin, Rodrigue, and Riedl (2006).

Table 5.3 Wood-plastic composites tensile strength modulus of rupture (MOR) without coupling agent (MAPP) and with coupling agent (MAPP) descriptive statistics.

Statistics	Tensile Strength MOR (MPa) without MAPP	Tensile Strength MOR (MPa) with MAPP and Outliers	Tensile Strength MOR (MPa) with MAPP and Outliers Removed
Mean	13.657	22.841	26.836
Median	13.221	24.355	25.912
Standard Deviation	2.735	7.348	3.620
Coefficient of Variation	20.699	32.169	13.840
IQR	4.626	7.536	5.554
Min	8.296	1.086	18.236
Max	18.802	35.302	35.302
Skewness	0.133	-0.839	0.284
Kurtosis	-0.882	-0.227	-0.473
N	149	141	112

For WPC comprised of MAPP and any of the other four resins used in this analysis, the MOR increases substantially. The mean for the MOR data without MAPP is 13.657 while the mean for the MOR data with MAPP and no outliers is 26.836. The medians are close to the means for each data set; the median for the MOR data with no MAPP is 13.221 and the median for the MOR data with MAPP and no outliers is 25.912. Note that the standard deviations for the three subsets are 2.735, 7.348, and 3.620, respectively, for the MOR with no MAPP, the MOR with MAPP and 29 outliers, and the MOR with MAPP and no outliers. Minimum values for each of the three subsets listed previously are 8.296, 1.086, and 18.236, respectively. The skewness values 0.133 and 0.284 suggest mild right skewness for the tensile strength MOR without MAPP and the tensile strength MOR with MAPP and no outliers. The skewness value for the MOR with MAPP and outliers data is -0.839, implying that the distribution is left skewed. The kurtosis values of -0.882, -0.227, and -0.473 indicate that all three distributions are relatively flat or platykurtic. The non-peakedness of these distributions is further supported by viewing the histogram shapes in Figure 5.6 for the MOR data.

The histograms in Figure 5.6 indicate that the tensile strength MOR data without MAPP and the MOR data with MAPP and no outliers are slightly right skewed while the MOR data with MAPP and 29 outliers is slightly left skewed. All three distributions appear relatively flat. Outlier presence for the tensile strength MOR with MAPP subset is easily detected by the gaps shown on the left side of the histogram and the numerous points shown to the left of the left-sided whisker on the boxplot. The one point shown to the far left of the left whisker indicates an extreme outlier.

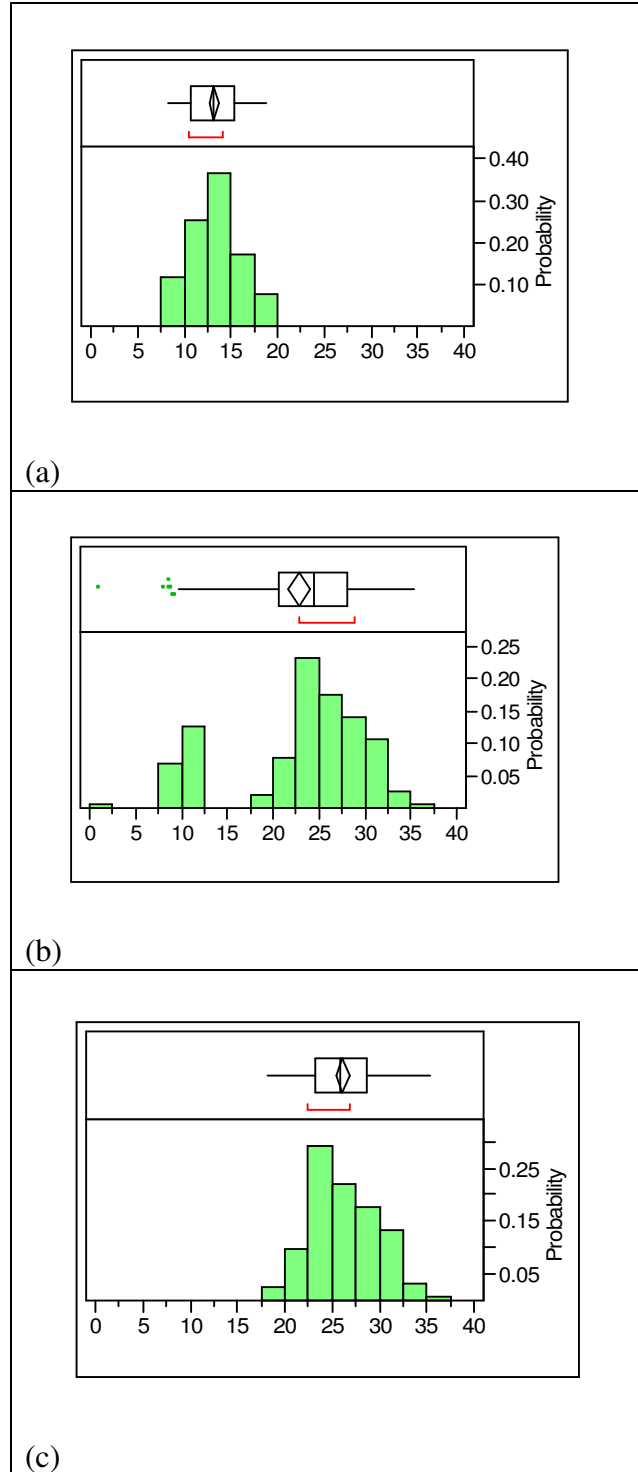


Figure 5.6 Boxplots and histograms of wood-plastic composites tensile strength modulus of rupture (MOR) (a) without coupling agent (MAPP), (b) with coupling agent (MAPP) and outliers, and (c) with coupling agent (MAPP) and excluding outliers (HDPE).

Probability plots for the smallest extreme value (SEV), Weibull, normal, lognormal, largest extreme value (LEV), and Frechet distributions for the WPC MOR data without MAPP and with MAPP including the outliers are shown in Figures 5.7 and 5.8, respectively. The tensile strength MOR data without MAPP appears to be best represented by the normal, lognormal, Weibull, and largest extreme value probability plots. For the subset with outliers, it is exceedingly difficult to determine the best distributions from the probability plots. It appears that, perhaps, the smallest extreme value, the largest extreme value or the normal may best represent the data. However, none of the plots approximate a straight line. This example graphically illustrates that some other statistical approach should be used to determine the distribution of choice for this subset. These plots clearly show that there is something unusual with the data, i.e., an extreme outlier, and that the data set should be investigated. A first step would be checking for the presence of outliers. It is interesting to see that once the outliers are removed, this graphical approach to choosing the best distribution becomes valid. Figure 5.9 depicts these same probability plots with the outliers removed. An alternative way to plot these distributions is shown in Figure 5.10, where the four best probability plots representing the MOR with MAPP and outliers removed subset are shown. This subset is best represented by the lognormal, normal, largest extreme value, and the loglogistic distributions. These probability plot results are further supported by the results obtained from Akaike's Information Criterion.

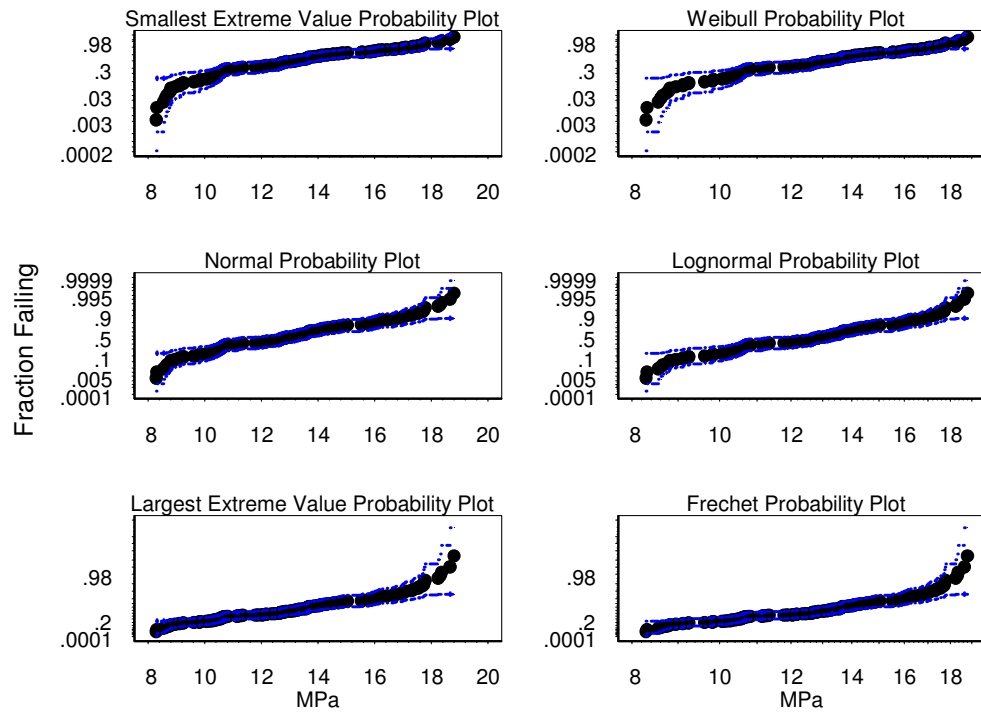


Figure 5.7 Wood-plastic composites tensile strength modulus of rupture (MOR) without coupling agent (MAPP) probability plots from S-PLUS and SPLIDA.

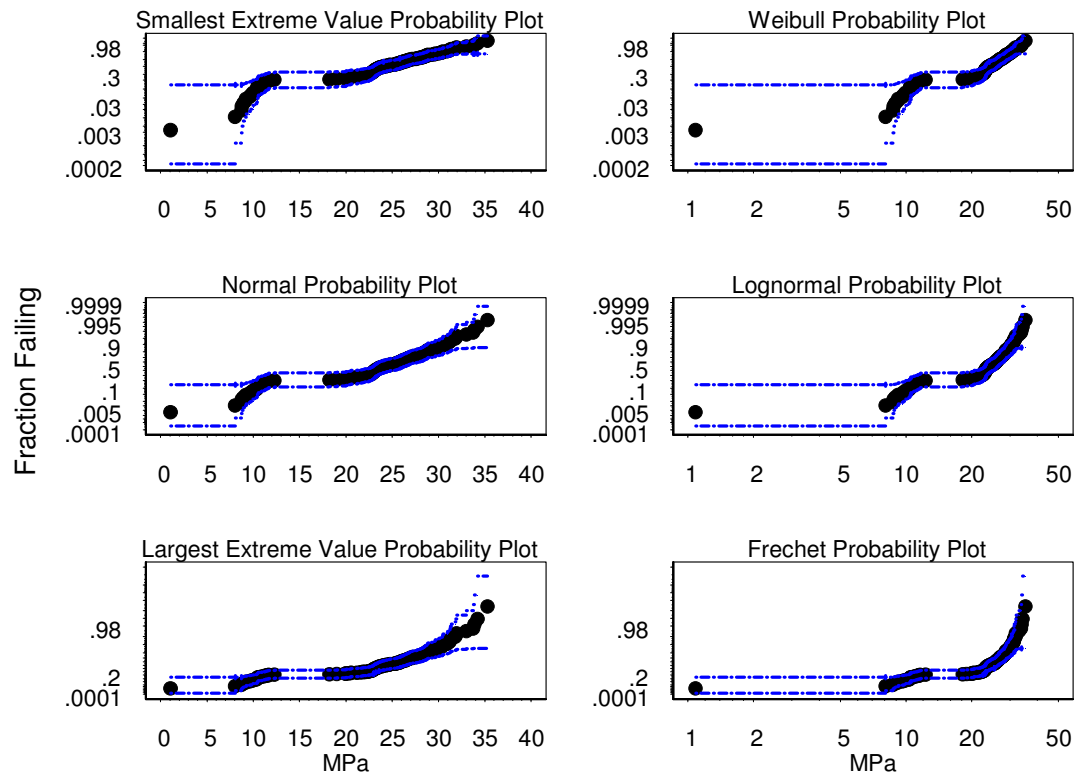


Figure 5.8 Wood-plastic composites tensile strength modulus of rupture (MOR) with coupling agent (MAPP) and outliers probability plots from S-PLUS and SPLIDA.

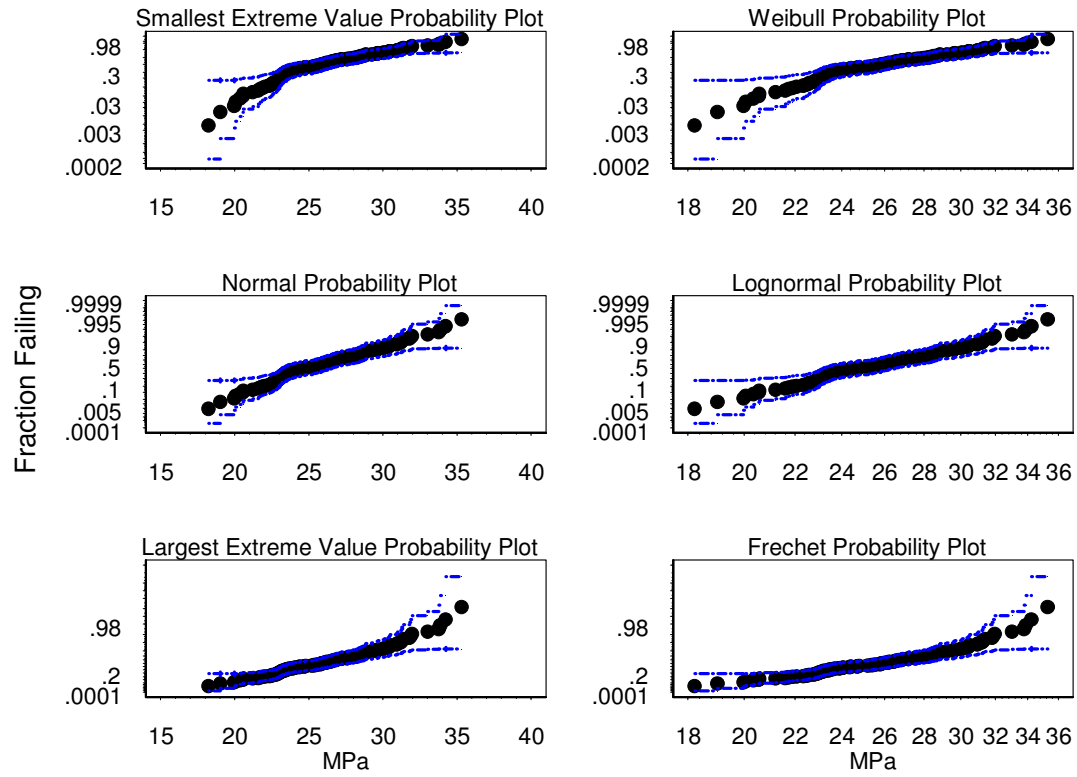


Figure 5.9 Wood-plastic composites tensile strength modulus of rupture (MOR) with coupling agent (MAPP) excluding outliers probability plots from S-PLUS and SPLIDA.

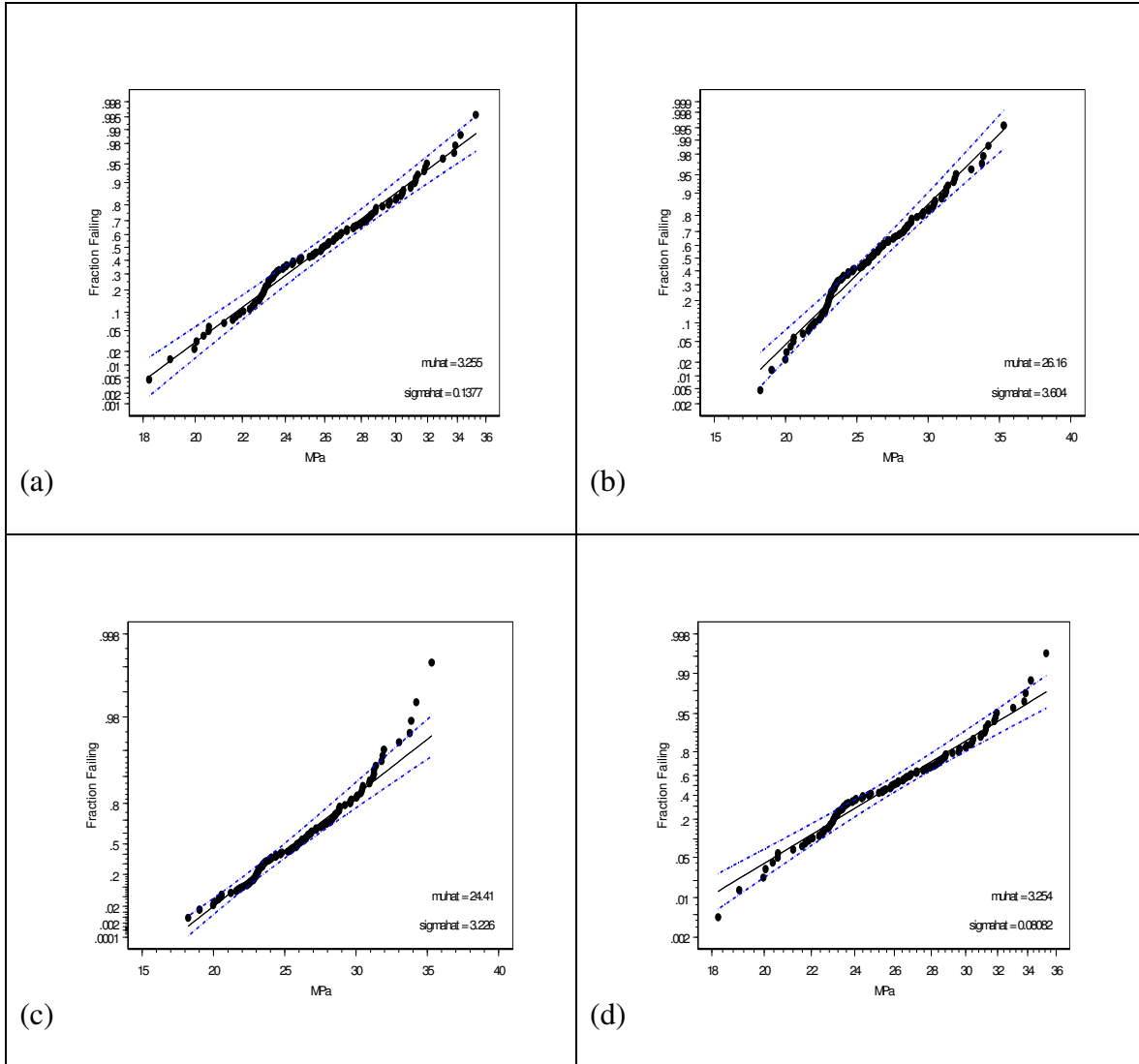


Figure 5.10 Wood-plastic composites tensile strength modulus of rupture (MOR) probability plots with coupling agent (MAPP) and outliers removed from S-PLUS and SPLIDA (a) Lognormal distribution, (b) Normal distribution, (c) Largest Extreme Value distribution, and (d) Loglogistic distribution with horizontal axis of MPa and vertical axes of fraction failing.

Table 5.4 shows the log likelihood and AIC scores of select models for the tensile strength MOR without MAPP, the tensile strength MOR with MAPP including 29 outliers, and the tensile strength MOR with MAPP excluding the outliers. The lowest scores obtained for the without MAPP subset are for the normal, lognormal, Weibull, and largest extreme value distributions. The lowest scores found for the MAPP including the outliers subsets are for the smallest extreme value, Weibull, normal, and logistic distributions. Finally, the lowest AIC values obtained for the MOR with MAPP and no outliers subset are for the lognormal, normal, largest extreme value, and the loglogistic distributions. Except for the tensile strength MOR with MAPP and 29 outliers subset, these results are consistent with the probability plots shown in Figures 5.7, 5.9, and 5.10.

The Kaplan-Meier plots for the tensile strength MOR data are shown in Figure 5.11. The Kaplan-Meier curves shown are for the tensile strength MOR without MAPP, the tensile strength MOR with MAPP and the 29 HDPE (outlier) values, and the tensile strength MOR with MAPP excluding the 29 outliers. As expected, the lowest strength to failure results are seen for the No MAPP subset. It is interesting to see that the MAPP with outliers subset starts out with an initial plateau and then a linear failure rate approximating that of the No MAPP subset followed by a plateau region of no failure and finally approximates the failure rate of the MAPP without the outliers subset. The initial or first plateau is attributed to the extreme outlier mentioned previously.

For the tensile strength MOR without MAPP subset, the probability that the tensile strength MOR will be greater than 13.2924 MPa is 0.50, while the probability that the tensile strength MOR is greater than 17.1025 MPa is 0.10. Statistically, 5% of WPC

Table 5.4 Selected model scores for the wood-plastic composites tensile strength modulus of rupture (MOR) without coupling agent (MAPP), with coupling agent (MAPP) including outliers, and with coupling agent (MAPP) excluding outliers.

Model Fit	Tensile Strength MOR without MAPP		Tensile Strength MOR with MAPP Including Outliers		Tensile Strength MOR with MAPP Excluding Outliers	
	Log Likelihood	AIC	Log Likelihood	AIC	Log Likelihood	AIC
Exponential	-533.6	1071.2	-582.1	1168.2	-477.6	959.2
Frechet	-372.6	749.2	-557.4	1118.8	-308.8	621.6
LEV	-364.0	732.0	-503.9	1011.8	-303.9	611.8
Logistic	-365.8	735.6	-481.4	966.8	-305.1	614.2
Loglogistic	-365.7	735.4	-507.4	1018.8	-304.1	612.2
Lognormal	-361.0	726.0	-521.7	1047.4	-301.4	606.8
Normal	-360.8	725.6	-480.8	965.6	-302.5	609.0
SEV	-370.4	744.8	-467.0	938.0	-313.7	631.4
Weibull	-362.2	728.4	-480.4	964.8	-307.4	618.8

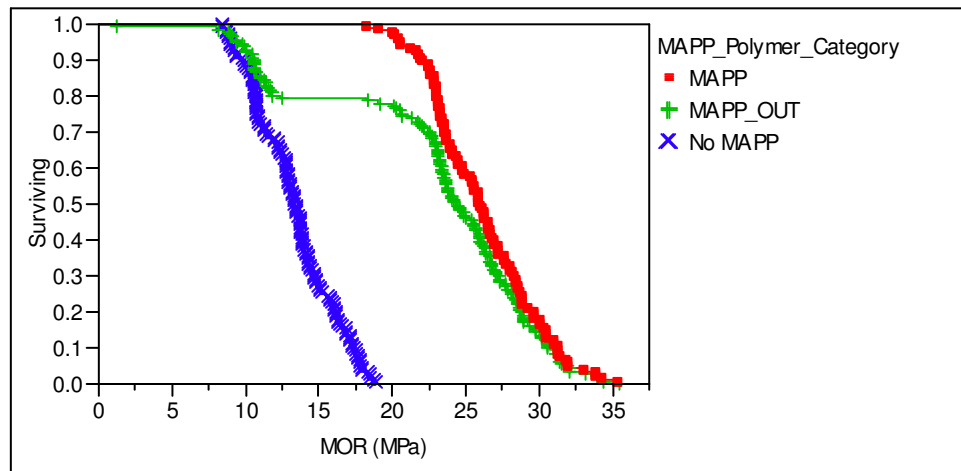


Figure 5.11 Reliability Kaplan-Meier Plot of wood-plastic composites tensile strength modulus of rupture (MOR) without coupling agent (No MAPP), with coupling agent and outliers (Outlier), and with coupling agent and outliers removed (MAPP).

fails before reaching a pressure of 8.7935 MPa and 95% of WPC fails before attaining a pressure of 17.7259 MPa. The Kaplan-Meier plot for this WPC subset indicates that pressure to failure decreases at increasing rates between 8.2962 and 18.6605 MPa.

For the outlier subset, the probability that the tensile strength MOR will be greater than 24.3552 MPa is 0.50, while the probability that the tensile strength MOR is greater than 30.4849 MPa is 0.10. Statistically, 5% of WPC fails before reaching a pressure of 9.2258 MPa and 95% of WPC fails before attaining a pressure of 31.7993 MPa. The Kaplan-Meier plot for this WPC subset indicates that pressure to failure decreases at increasing rates between 1.0858 and 34.2319 MPa.

In contrast, for the tensile strength MOR with MAPP excluding the outliers subset, the probability that the tensile strength MOR will be greater than 25.8608 MPa is 0.50, while the probability that the tensile strength MOR is greater than 31.1998 MPa is 0.10. Statistically, 5% of WPC fails before realizing a pressure of 20.5591 MPa and 95% of WPC fails before attaining a pressure of 31.8719 MPa. The Kaplan-Meier plot for this WPC data set indicates that pressure to failure decreases at increasing rates between 18.2358 and 34.2319 MPa.

In order to verify the differences between the tensile strength MOR without MAPP, with MAPP and outliers, and with MAPP and outliers removed, a Welch ANOVA was conducted as well as two sample t-tests between the MAPP and no MAPP subsets. The Welch ANOVA test resulted an F-value of 538.8348 with a p-value of <0.0001. The Tukey-Kramer test revealed that there are significant differences among the three distributions. Figure 5.12 shows the side-by-side box plots for each of the three subsets. For comparison, pairs of the three subsets were tested with two sample t-tests.

The resulting p-values for the no MAPP and the MAPP with outliers subsets is less than 0.0000, the p-value for the no MAPP and the MAPP with outliers excluded is also less than 0.0000, and the p-value for the MAPP with outliers versus the MAPP with no outliers is less than 0.0001. Thus, strongly significant differences are seen in each of the MAPP subsets. These differences are readily apparent in the side-by-side box plots shown in Figures 5.13, 5.14, and 5.15.

5.3 SUMMARY

Graphical and numerical statistical methods used to explore the reliability of wood-plastic composites, as measured by the tensile strength MOE and tensile strength MOR, provide valuable information about WPC. The axial stress loading used in tensile testing versus the flexural tests discussed in Chapters 3 and 4, provides additional information about WPC. Descriptive statistics clearly show that the addition of the

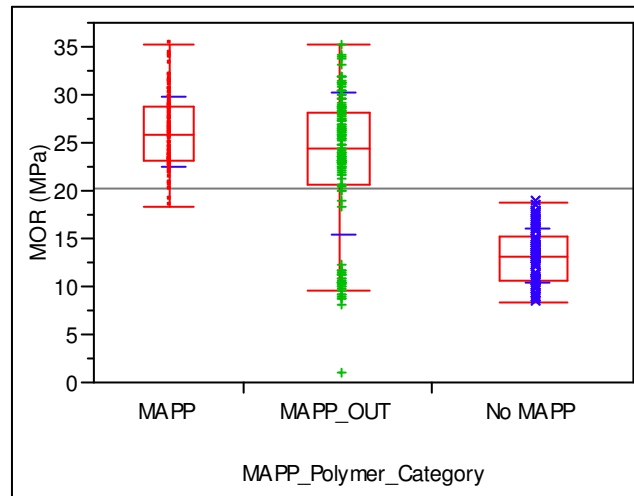


Figure 5.12 Side-by-side box plot of wood-plastic composites tensile strength modulus of rupture (MOR) without coupling agent (No MAPP), with coupling agent excluding outliers (MAPP), and with coupling agent including outliers (MAPP_OUT).

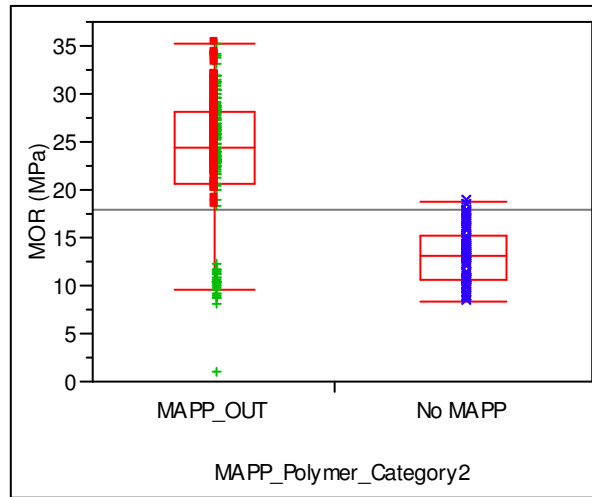


Figure 5.13 Side-by-side box plot of wood-plastic composites tensile strength modulus of rupture (MOR) without coupling agent (No MAPP) and with coupling agent and outliers (MAPP_OUT).

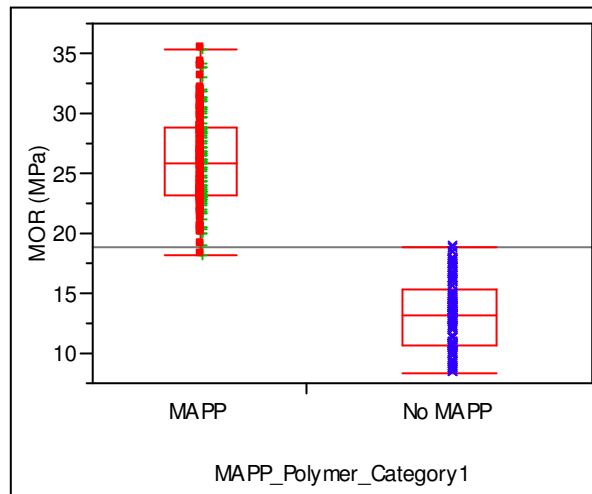


Figure 5.14 Side-by-side box plot of wood-plastic composites tensile strength modulus of rupture (MOR) without coupling agent (No MAPP) and with coupling agent and outliers removed (MAPP).

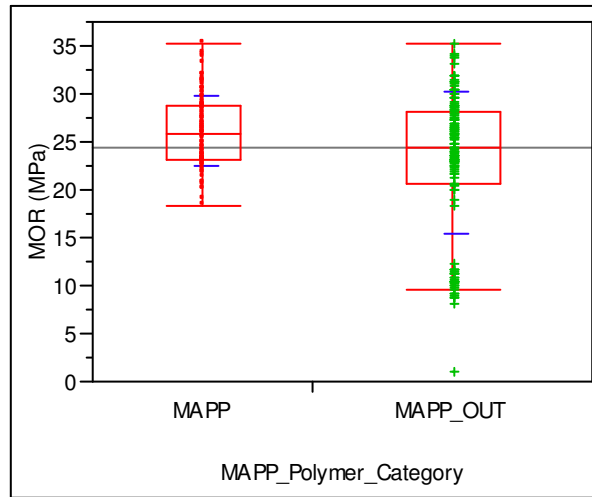


Figure 5.15 Side-by-side box plot of wood-plastic composites tensile strength modulus of rupture (MOR) with coupling agent MAPP and no outliers (MAPP) and with coupling agent MAPP and outliers removed (MAPP_OUT).

coupling agent, MAPP, to the wood-plastic composite yields greater tensile strength MOE and MOR values. Analysis of the MOR with MAPP data provides an excellent illustration of the importance of using a graphical approach to assess the reliability of WPC. Basic histograms and probability plots can provide tremendous insight for the possibility of outliers and other unusual conditions of the data, e.g., mixtures within the distribution. In this case, the combination of MAPP and HDPE resin for the tensile strength MOR test data provided interesting results, i.e., the presence of outliers.

For the tensile strength MOE and MOR data, the probability plot results for the WPC vary substantially with the addition of the MAPP coupling agent for either the MOE or MOR data. The best parametric distributional assumption for the MOE without MAPP data is the normal distribution while the best representation for the MOE with

MAPP data is the Weibull distribution. For the MOR data sets, the best representation for the data is the normal distribution for the without MAPP subset, the smallest extreme value distribution for the MAPP subset with outliers, and the lognormal distribution for the subset of MAPP with no outliers. Kaplan-Meier plots for the MOE data with and without MAPP also vary considerably. For the tensile strength MOE data, the addition of MAPP increases the strength to failure results. Except for the MOR with MAPP and HDPE samples, the addition of the MAPP coupling agent also increases the strength to failure results for the MOR of WPC, resulting in improved mechanical properties for many applications. The tensile strength MOR with MAPP including outliers provides an interesting survival curve, exhibiting strength to failure results similar to each of the other two subsets, while also revealing plateau regions of constant failure rates. This subset with outliers, therefore, makes it more difficult for the practitioner to predict failure rates for WPC. Such behavior indicates too much variation in the process and provides opportunities for process improvements.

Chapter 6

6. Bootstrap Confidence Intervals for Percentiles of Reliability of Wood-Plastic Composites

6.1 Bootstrap Introduction

As others have noted well, Meeker and Escobar (1998) remark that the “traditional parameters of a statistical model (e.g., mean and standard deviation) are not of primary interest. Instead, design engineers, reliability engineers, managers, and customers are interested in specific measures of product reliability or particular characteristics of a failure-time distribution (e.g., failure probabilities, quantiles of the life distribution, failure rates).”

This paper focuses on estimating percentiles of strengths of materials measured in megapascals (MPa). These estimation procedures apply more generally, however, to many other parameters of interest in wood science.

Reliability measurements using any parameters, e.g., percentiles, must acknowledge statistical variation so that product improvements may be realized. Thus, wood scientists, supervisors and line workers need realistic trustworthy confidence intervals to assess potential variability in their estimates. We, also, do not want to assume normality of data, since in many cases this and other assumptions will not hold in wood science settings.

Edwards, Guess, Young, and Bensmail (2007) helpfully note, “Historically, the problem of estimating percentiles was not in finding point estimators, but in finding

standard errors and thus, confidence intervals of percentiles. Serfling (1980) thoroughly examines the asymptotic distribution of the sample quantile. In particular, under mild requirements (i.e. smoothness of the distribution function), the sample quantiles are asymptotically normal.” They further comment that a normal approximate confidence interval for t_p is given

$$\hat{t}_p \pm z_{1-\alpha/2} \text{se}_{\hat{t}_p} \quad (6.1)$$

where $\text{se}_{\hat{t}_p}$ is the standard error of the estimate approximated by:

$$\text{se}_{\hat{t}_p} = \sqrt{\text{Var}(\hat{t}_p)} = \hat{t}_p \{ \text{Var}(\hat{\mu}) + 2\Phi^{-1}(p)\text{Cov}(\hat{\mu}, \hat{\sigma}) + [\Phi^{-1}(p)]^2 \text{Var}(\hat{\sigma}) \}^{1/2} \quad (6.2)$$

which is derived using the delta method and Φ^{-1} represents the inverse of the cumulative standardized location-scale distribution. $\text{Var}(\hat{\mu})$, $\text{Var}(\hat{\sigma})$, and $\text{Cov}(\hat{\mu}, \hat{\sigma})$ are obtained from the variance-covariance matrix or inverse Fisher information matrix, $\mathcal{F}^{-1}$. When the sample size is sufficiently large, these asymptotic normal intervals can provide sound approximations. In so many settings in industry and indeed labs, large enough samples to trust these approximations are too costly or too time consuming to obtain. Bootstrap methods, however, are more realistic and yield helpful methods not requiring parametric models. With various parametric models, valid bootstrap methods extend from nonparametric to these parametric models, also. Indeed, bootstrap techniques have empirical double checks on when the normal approximation is or is not appropriate for both the parametric and nonparametric models.

As Edwards, Guess, Young, Bensmail (2007) discuss on medium density fiberboard, “Bootstrapping is a computer intensive statistical method where the basic idea is to simulate the sampling process a specified (usually large) number of times and obtain an empirical bootstrap distribution for a desired population parameter. This empirical bootstrap distribution is then used to acquire characteristics (i.e. standard error, bias estimates, confidence intervals, etc.) about the population parameter. Chernick (1999) is an excellent book that provides a thorough and insightful treatment of many bootstrap methods and their applications.”

For helpful reviews on bootstrap methods, we recommend as others, Efron and Tibshirani (1993) and DiCiccio and Efron (1996). Meeker and Escobar (1998) note that the “justification for the bootstrap is based on large-sample theory”. But, even when large samples are used, there can still be problems with the tails of the sample. The sampling distribution obtained from the bootstrap may not be continuous, resulting in the calculation of inaccurate confidence intervals. Chernick (1999) discusses additional limitations.

See, also, Edwards, Guess, Young, Bensmail, and Leon (2007), where their section 3 has been adapted and shortened for this section.. Three different bootstrap methods are used in this analysis. The first and most basic bootstrap is the fully nonparametric bootstrap, described in Martinez and Martinez (2002). The steps of this procedure are described as follows. For a random sample or a given data set, $\mathbf{x} = (x_1, x_2, \dots, x_n)$ of size n , the population parameter θ is estimated by $\hat{\theta}$. Sampling is then done with replacement from the original data set to obtain a bootstrap sample of size n , denoted by $\mathbf{x}^{*b} = (x_1^{*b}, x_2^{*b}, \dots, x_n^{*b})$. Resampling with replacement is done a large number of

times, B . The same statistic as above is calculated from each bootstrap sample and is denoted as $\hat{\theta}^{*b}$, where b represents the b^{th} bootstrap sample. The empirical bootstrap distribution of $\hat{\theta}^*$, is then defined and used as an estimate for the distribution of $\hat{\theta}$. The advantage of this sampling method is that the underlying distribution does not have to be known or assumed.

The second bootstrapping method used in this paper is the fully parametric bootstrap, which requires knowledge of a parametric or underlying distribution. This method is similar to the nonparametric method described above except that with the fully parametric method, bootstrap samples are taken from the given parametric distribution using the same maximum likelihood estimates taken from the original sample. The fully parametric bootstrap is described in Efron and Tibshirani (1993), Meeker and Escobar (1998), and Chernick (1999). Meeker and Escobar (1998) state an important disadvantage of the fully parametric method, since data is simulated, is that the censoring process must be completely specified for reliability data. However, as Meeker and Escobar (1998) caution this will be, “more difficult for complicated systematic or random censoring.” This specification is simple for complete data, such as the MOE and MOR data used in this analysis.

The third and final bootstrapping method used in this analysis is described by Meeker and Escobar (1998) as the nonparametric bootstrap sampling method for parametric inference, which is conveniently denoted as NBSP by Edwards, Guess, Young, and Bensmail (2007) and Edwards (2004). Similar to the fully nonparametric method, the NBSP method samples with replacement from the original data, but for each bootstrap sample of size n , maximum likelihood estimates are realized from the specified

parametric model. These maximum likelihood estimates are then used to estimate the population parameter of interest and form the bootstrap distribution.

Different methods are available for the construction of bootstrap confidence intervals for population parameters. This paper utilizes the standard normal bootstrap confidence interval, bootstrap percentile interval, and bias-corrected bootstrap percentile interval. This paper omits the theoretical details of these methods. For a complete discussion, compare Efron and Tibshirani (1993), DiCiccio and Efron (1996), and Davison and Hinkley (1997). Edwards, Guess, Young, and Bensmail (2007) and Edwards (2004) give detailed algorithms for each of the bootstrap confidence intervals used. The actual intervals are defined as follows.

The bootstrap standard confidence interval is given by:

$$[\hat{\theta} - z^{(\alpha/2)} \mathfrak{SE}_{\hat{\theta}}, \hat{\theta} + z^{(1-\alpha/2)} \mathfrak{SE}_{\hat{\theta}}] \quad (6.3)$$

where $\mathfrak{SE}_{\hat{\theta}}$ is the standard deviation of the bootstrap estimates of θ and $z^{(\alpha/2)}$ is the $\alpha/2th$ quantile of the standard normal distribution.

The bootstrap percentile confidence interval is based upon the quantiles of the bootstrap distribution of estimates and is given by:

$$[\hat{\theta}^{*(\alpha/2)}, \hat{\theta}^{*(1-\alpha/2)}] \quad (6.4)$$

where $\hat{\theta}^{*(\alpha/2)}$ and $\hat{\theta}^{*(1-\alpha/2)}$ are actually the quantiles of the bootstrap distribution of estimates.

Finally, the bias-corrected percentile interval is defined as the amount of difference between the median of the bootstrap estimates $\hat{\theta}^{*b}$ and the estimate, $\hat{\theta}$ from

the original sample. See Efron (1981, 1987). The bias correction constant estimate, denoted by $\hat{z}_0$, is defined as:

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\#(\hat{\theta}^{*b} < \hat{\theta})}{B}\right) \quad (6.5)$$

where Φ^{-1} symbolizes the inverse cumulative normal distribution and # means “number of”

Then, a $100(1-\alpha)\%$ bias-corrected percentile confidence interval for θ is given by:

$$[\hat{\theta}^{*(\alpha_1)}, \hat{\theta}^{*(\alpha_2)}] \quad (6.6)$$

where α_1 and α_2 are the revised quantities on which to base the percentile confidence interval endpoints. These quantities are defined as:

$$\alpha_1 = \Phi(2\hat{z}_0 + z^{(\alpha/2)}) \quad (6.7)$$

and

$$\alpha_2 = \Phi(2\hat{z}_0 + z^{(1-\alpha/2)}) \quad (6.8)$$

where Φ is the cumulative standard normal distribution.

For additional, helpful comments when samples are rather small, note the insightful comments in Chernick (1999), Meeker and Escobar (1998), and Polansky (2000), .

6.2 PERCENTILE BOOTSTRAP CONFIDENCE INTERVALS FOR MOE OF WPC

For each bootstrap method of sampling, the standard normal, percentile, and bias-

corrected percentile bootstrap intervals were constructed and compared for the first, fifth, tenth, and 50th percentiles for MOE and MOR for WPC. Sample sizes are small for each of the original data sets, with the sample size for MOE equal to 120, and the sample size for MOR equal to 119. Meeker and Escobar (1998) recommend that the number of bootstrap samples be between 2000 and 4000 in order to construct confidence intervals, especially for the lower percentiles. For each sampling method for both the MOE and MOR data sets, $B = 5000$ bootstrap samples of the same size as the original sample were created. The MATLAB code used can be downloaded at <http://www.spcforwood.com>. The asymptotic normal confidence intervals will also be provided in order to compare with the bootstrap results.

Table 6.1 provides the 95% asymptotic normal confidence intervals for the MOE of WPC. The fully nonparametric MOE 95% bootstrap confidence intervals are shown in Table 6.2 while the fully parametric 95% bootstrap confidence intervals are displayed in Table 6.3. Finally, the NBSP 95% bootstrap confidence intervals are displayed in Table 6.4. In the following tables, LCL is used for the lower confidence limit, while UCL is used for the upper confidence limit. Empirical bootstrap sampling distributions for each of the four quantiles corresponding with each bootstrap interval listed above are shown in Figures 6.1, 6.2, and 6.3, respectively.

The bootstrap sampling distribution shown in Figure 6.1 indicates a constraint of the fully nonparametric bootstrap method. Because the sample size is relatively small for the MOE, the sampling distribution for the first percentile shown in Figure 6.1(a) appears discrete or “snaggle-toothed”. This discreteness would be even more apparent if the bootstrap resampling size had been smaller, i.e., less than 5000. When this pattern is

observed, practitioners should attempt other bootstrapping methods for the lower percentiles, if possible, such as the NBSP or fully parametric methods. In addition, the first percentile fully nonparametric sampling distribution is shown as heavily right skewed. Increasingly symmetrical and continuous distributions emerge with increasing percentiles, as shown in Figures 6.1(b), (c), and (d). As shown in Table 6.2, the standard interval for the first percentile is extremely wide, ranging from 2540.31 to 2800.67 MPa. The percentile interval, cuts the distribution at the 2.5 and 97.5 percentiles, yielding a confidence interval of [2583.73, 2805.97] which does not incorporate the right-skewness of the first-percentile distribution. The bias-corrected interval corrects for any skewness and gives an even smaller confidence interval of [2583.73, 2723.62]. Note you can understand the percentile interval and the bias-corrected percentile interval having the same lower bound of 2583.73, but different upper bounds result from the strong right skewness. Recall this is what can happen in the very similar case of Chi-square lower percentiles bounds versus upper percentiles bounds (e.g., see a Chi-square table with degrees of freedom being smaller).

Table 6.1 95% Asymptotic normal confidence intervals for WPC bending tangent modulus of elasticity (MOE) with coupling agent (MAPP).

P	$\hat{t}_p = \text{quantile}$	LCL	UCL
.01	2476.5	2265.5	2687.5
.05	2894.0	2725.9	3062.1
.10	3116.5	2968.6	3264.4
.50	3901.5	3791.9	4011.1

Table 6.2 Fully nonparametric 95% bootstrap confidence intervals for WPC bending tangent modulus of elasticity (MOE) with coupling agent (MAPP).

p	$\hat{t}_p = \text{quantile}$	Interval Type	LCL	UCL
.01	2676.8416	Standard	2540.3116	2800.6709
		Percentile	2583.7283	2805.9654
		Bias-Corrected	2583.7283	2723.6216
.05	2877.0572	Standard	2701.0905	3048.1973
		Percentile	2730.4557	3022.7374
		Bias-Corrected	2729.9046	3012.4229
.10	3025.8026	Standard	2923.1143	3122.3604
		Percentile	2910.9610	3121.3363
		Bias-Corrected	2910.9610	3114.3404
.50	4008.5502	Standard	3851.2728	4168.7506
		Percentile	3845.3682	4173.4349
		Bias-Corrected	3845.3682	4170.9650

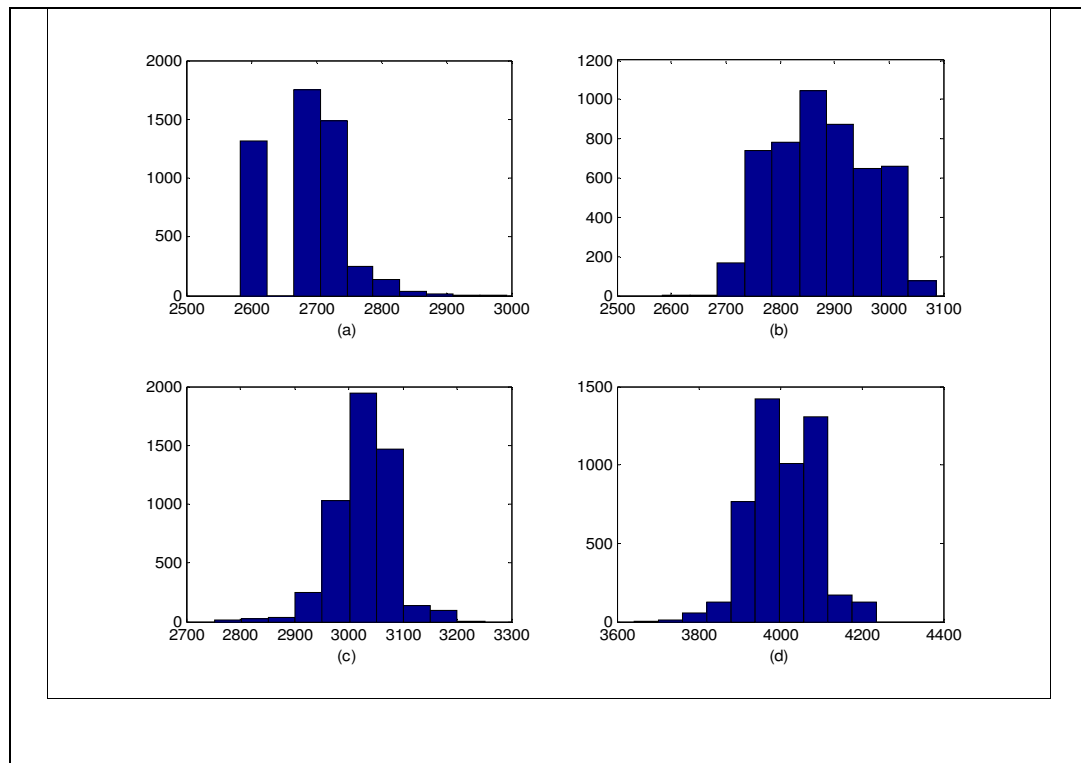


Figure 6.1 Sampling distribution of percentiles for WPC bending tangent modulus of elasticity (MOE) with coupling agent (MAPP) under the fully nonparametric bootstrap sampling method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

Table 6.3 Fully parametric 95% bootstrap confidence intervals for WPC bending tangent modulus of elasticity (MOE) with coupling agent (MAPP).

p	$\hat{t}_p = \text{quantile}$	Interval Type	LCL	UCL
.01	2266.4550	Standard	2039.0638	2472.5851
		Percentile	2050.6568	2484.7207
		Bias-Corrected	2029.9188	2466.5567
.05	2806.4363	Standard	2610.5010	2993.7206
		Percentile	2613.6001	2994.8302
		Bias-Corrected	2597.7361	2982.9236
.10	3090.5001	Standard	2915.0694	3252.4072
		Percentile	2917.0026	3254.0456
		Bias-Corrected	2902.5000	3241.5136
.50	3963.4367	Standard	3851.8431	4072.4052
		Percentile	3852.3194	4072.2753
		Bias-Corrected	3847.8997	4069.0436

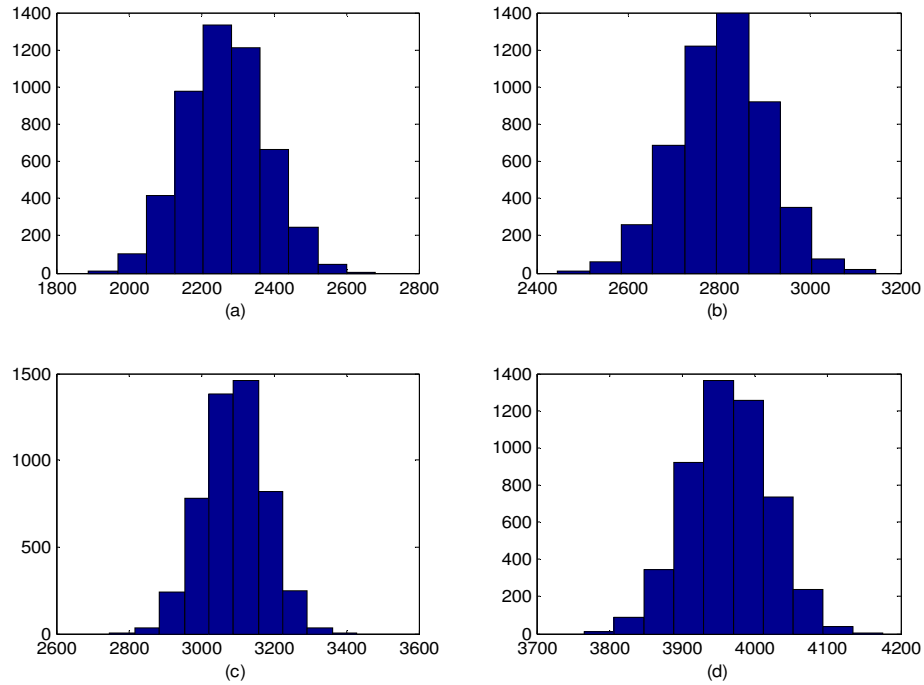


Figure 6.2 Sampling distribution of percentiles for WPC bending tangent modulus of elasticity (MOE) with coupling agent (MAPP) under the fully parametric bootstrap sampling method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

Table 6.4 NBSP 95% bootstrap confidence intervals for WPC bending tangent modulus of elasticity (MOE) with coupling agent (MAPP).

p	$\hat{t}_p = \text{quantile}$	Interval Type	LCL	UCL
.01	2265.2672	Standard	2048.5655	2463.0835
		Percentile	2058.6735	2472.5756
		Bias-Corrected	2042.4576	2458.1040
.05	2810.4823	Standard	2617.7072	2986.5144
		Percentile	2629.9603	2997.3603
		Bias-Corrected	2618.7599	2982.8038
.10	3085.9556	Standard	2918.2460	3249.2307
		Percentile	2926.7648	3257.2224
		Bias-Corrected	2922.7587	3252.3137
.50	3961.9834	Standard	3849.7168	4074.5314
		Percentile	3845.2588	4069.7234
		Bias-Corrected	3842.7912	4068.7148

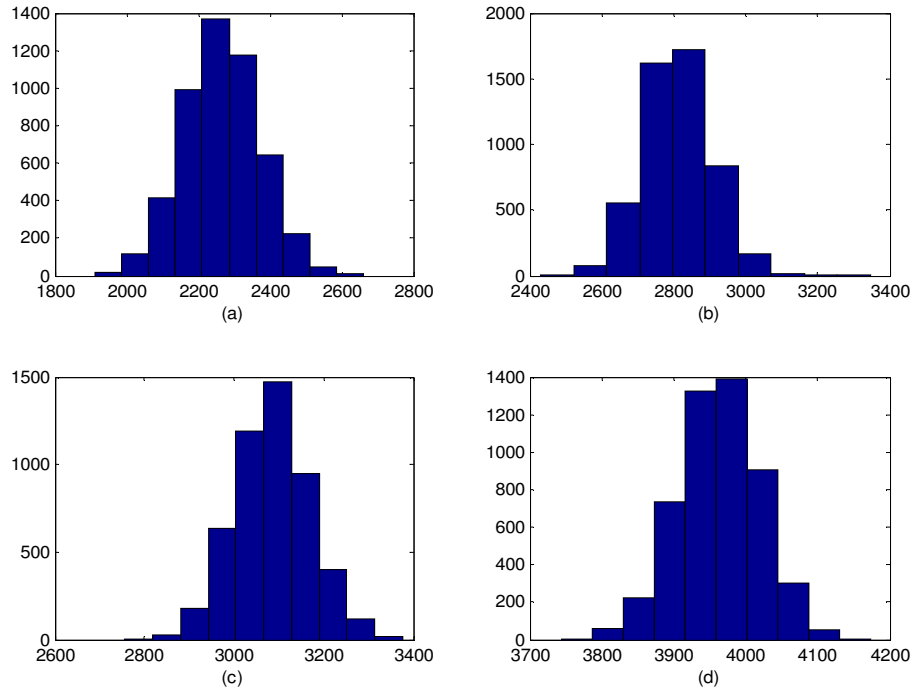


Figure 6.3 Sampling distribution of percentiles for WPC bending tangent modulus of elasticity (MOE) with coupling agent (MAPP) under the NBSP method.
(a) 1st, (b) 5th, (c) 10th, and (d) 50th.

The best underlying parametric distribution for the MOE of WPC was previously determined as Weibull followed by the normal and smallest extreme value distributions. This best model determination was used to construct the confidence intervals based on both the parametric and the NBSP methods. See Perhac, Young, Guess, and León (2007).

Fully parametric 95% bootstrap confidence intervals for the MOE are shown in Table 6.3. Unlike the fully nonparametric intervals, widths of the fully parametric intervals are agreeable for each percentile and more representative of the asymptotic normal confidence interval widths. For instance, first-percentile widths of the asymptotic normal, fully nonparametric standard, and fully parametric standard confidence intervals are 422, 260, and 434, respectively. Figure 6.2 shows that the sampling distributions for the fully parametric bootstrap sampling method are much more continuous as well as much more normally distributed than for the fully nonparametric bootstrap method.

The NBSP bootstrap method intervals are displayed in Table 6.4. These intervals are very similar to those of the fully parametric method with respect to the LCL and UCL limits. In addition, as with the fully parametric method, interval widths for each of the percentiles for the three different types of intervals are very close. Furthermore, LCL and UCL limit values and interval widths are similar to those for the asymptotic normal confidence intervals. Figure 6.3 shows the sampling distributions of the percentiles for the NBSP method. Although these sampling distributions are continuous, the normal distribution is not approximated as well as with the fully parametric method. For instance, slight right skewness is shown for the fifth percentile while minor left skewness is evident in the tenth percentile sampling distribution. However, using the NBSP

method described by Meeker and Escobar (1998) is an appropriate choice when the sample size is small and there is confidence in the underlying parametric model. This method requires a parametric assumption, but resampling is done with replacement from the original data. The main advantage of using the NBSP method over the fully parametric method is that no censoring assumptions are required; this is important for reliability data when the censoring mechanism is not known.

6.3 PERCENTILE BOOTSTRAP CONFIDENCE INTERVALS FOR MOR OF WPC

Table 6.5 provides the 95% asymptotic normal confidence intervals for the MOR of WPC. The fully nonparametric MOR 95% bootstrap confidence intervals are shown in Table 6.6 while the fully parametric 95% bootstrap confidence intervals are displayed in Table 6.7. The NBSP 95% bootstrap confidence intervals are displayed in Table 6.8. Empirical bootstrap sampling distributions for each of the four quantiles corresponding with each bootstrap interval listed above are shown in Figures 6.4, 6.5, and 6.6, respectively.

The nonparametric bootstrap sampling distributions shown in Figure 6.4 for the MOR data indicate severe right skewness for the first percentile and severe left skewness for the tenth percentile. Each of the four quantile sampling distributions appears widely dispersed, implying large sampling distribution variability and a lack of normality. As shown in Table 6.6, confidence interval widths become more consistent for three interval types with increasing percentile. Although the interval widths do not necessarily decrease with increasing percentile for the MOR data, the variation of interval width for the three interval types within each percentile does decrease with increasing percentile.

Table 6.5 95% Asymptotic normal confidence intervals for WPC bending modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p = \text{quantile}$	LCL	UCL
.01	31.938	30.166	33.710
.05	35.429	34.017	36.841
.10	37.290	36.048	38.533
.50	43.856	42.936	44.777

Table 6.6 Fully nonparametric 95% bootstrap confidence intervals for WPC bending modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p = \text{quantile}$	Interval Type	LCL	UCL
.01	30.7191	Standard	28.5386	31.9247
		Percentile	29.9419	32.8026
		Bias-Corrected	29.9419	31.1064
.05	34.0600	Standard	30.6072	36.7648
		Percentile	31.4409	37.1802
		Bias-Corrected	31.1451	36.8933
.10	37.1516	Standard	34.7542	39.4632
		Percentile	34.4382	38.8692
		Bias-Corrected	33.9534	38.7374
.50	44.6613	Standard	43.5706	46.1406
		Percentile	43.0686	45.5351
		Bias-Corrected	43.0005	45.4689

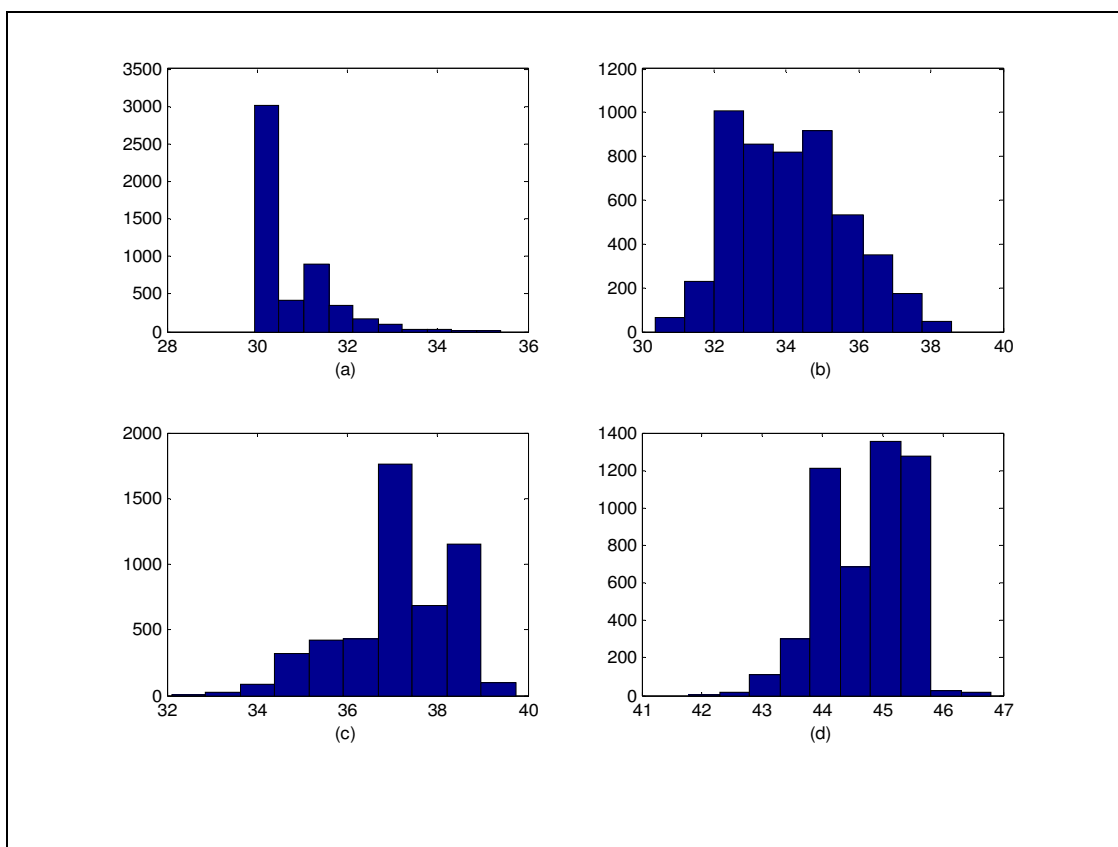


Figure 6.4 Sampling distribution of percentiles for WPC bending modulus of rupture(MOR) with coupling agent (MAPP) under the fully nonparametric bootstrap sampling method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

Table 6.7 Fully parametric 95% bootstrap confidence intervals for WPC bending modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p = \text{quantile}$	Interval Type	LCL	UCL
.01	29.8483	Standard	27.7424	31.7945
		Percentile	27.8880	31.8891
		Bias-Corrected	27.7502	31.7687
.05	34.8056	Standard	33.0784	36.4104
		Percentile	33.1728	36.4534
		Bias-Corrected	33.0446	36.3655
.10	37.2585	Standard	35.7441	38.6534
		Percentile	35.7424	38.6603
		Bias-Corrected	35.5979	38.5295
.50	44.4793	Standard	43.5809	45.3682
		Percentile	43.5567	45.3583
		Bias-Corrected	43.5338	45.3412

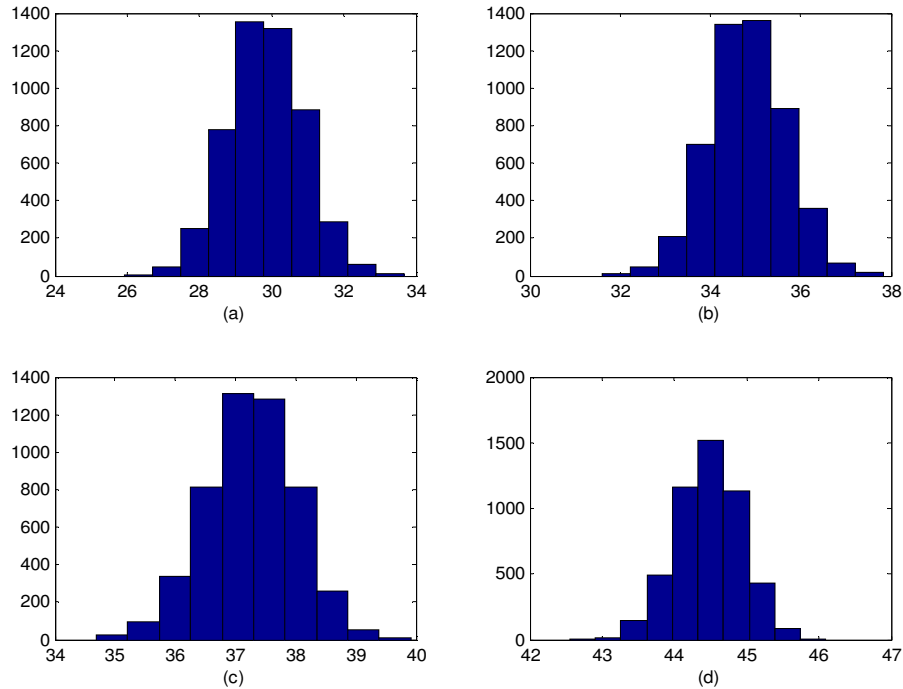


Figure 6.5 Sampling distribution of percentiles for WPC bending modulus of rupture (MOR) with coupling agent (MAPP) under the fully parametric bootstrap sampling method. (a) 1st, (b) 5th, (c) 10th, and (d) 50th.

Table 6.8 NBSP 95% bootstrap confidence intervals for WPC bending modulus of rupture (MOR) with coupling agent (MAPP).

p	$\hat{t}_p = \text{quantile}$	Interval Type	LCL	UCL
.01	29.8632	Standard	27.6952	31.8418
		Percentile	27.8165	31.9345
		Bias-Corrected	27.6759	31.7917
.05	34.7900	Standard	33.0326	36.4559
		Percentile	33.1204	36.5197
		Bias-Corrected	33.0440	36.4497
.10	37.2289	Standard	35.6665	38.7309
		Percentile	35.7217	38.7680
		Bias-Corrected	35.6899	38.7143
.50	44.4709	Standard	43.5488	45.4003
		Percentile	43.5047	45.3792
		Bias-Corrected	43.4962	45.3753

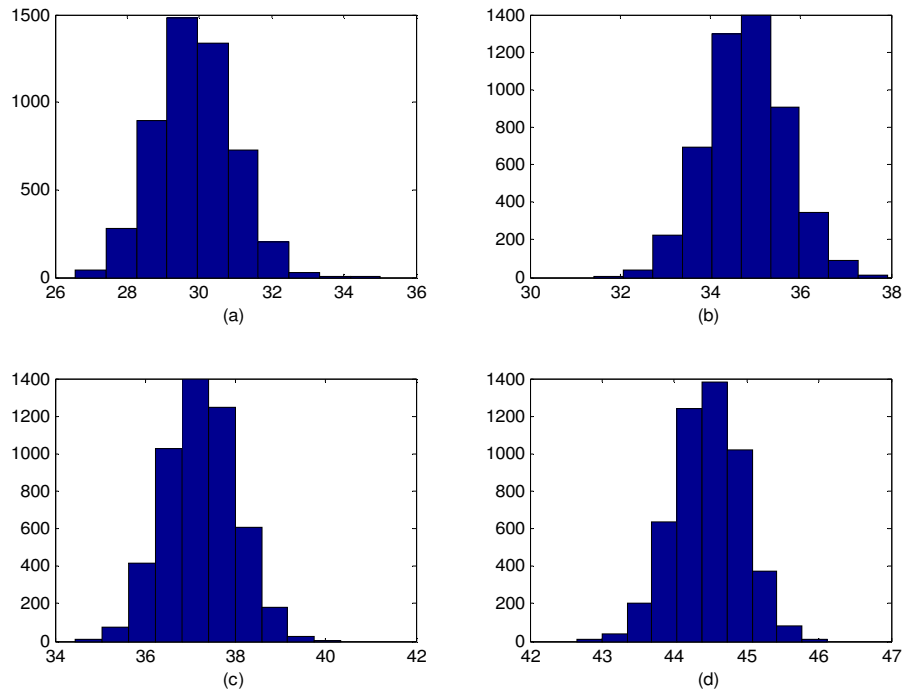


Figure 6.6 Sampling distribution of percentiles for WPC bending modulus of rupture(MOR) with coupling agent (MAPP) under the NBSP method.
(a) 1st, (b) 5th, (c) 10th, and (d) 50th.

For instance, the standard interval for the first percentile ranges from 28.54 to 31.93. The standard intervals for the fifth, tenth, and 50th percentiles are [33.08, 36.41], [35.74, 38.65], and [43.58, 45.37] MPa. Average interval differences and the standard deviations of the three interval types for the first, fifth, tenth, and 50th percentiles are 2.470 ± 1.161 , 5.882 ± 0.239 , 4.641 ± 0.186 , and 2.502 ± 0.059 , respectively.

The best underlying parametric distribution for the MOR of WPC was previously determined as Weibull followed by the smallest extreme value and normal distributions. Similarly to the MOE data, this best model determination was used to construct the confidence intervals based on both the parametric and the NBSP methods. See Perhac, Young, Guess, and León (2007).

Fully parametric 95% bootstrap confidence intervals for the MOR are shown in Table 6.7. Widths of the fully parametric intervals are more consistent within each percentile and are more representative of the asymptotic normal confidence interval widths. As the percentiles increase, the distribution widths decrease. As shown in Table 6.7, confidence interval widths are consistent for the standard, percentile, and bias-corrected intervals of each percentile. The interval widths also decrease with increasing percentile. For example, the standard interval for the first percentile ranges from 27.74 to 31.79. The standard intervals for the fifth, tenth, and 50th percentiles are [33.08, 36.41], [35.74, 38.65], and [43.58, 45.37] MPa. Average interval differences and the standard deviations of the three interval types for the first, fifth, tenth, and 50th percentiles are 4.024 ± 0.026 , 3.311 ± 0.027 , 2.920 ± 0.011 , and 1.799 ± 0.010 , respectively. As was shown with the MOE data, Figure 6.5 shows that the sampling distributions for the fully parametric bootstrap sampling method for the MOR data are more continuous and much

more normally distributed than for the fully nonparametric bootstrap method for the MOR data.

The NBSP bootstrap method intervals for the MOR data are shown in Table 6.8. These intervals are very similar to those of the fully parametric method with respect to the LCL and UCL limits. In addition, as with the fully parametric method, interval widths for each of the percentiles for the three different types of intervals are very close. As shown in Table 6.8, confidence interval widths are consistent for the standard, percentile, and bias-corrected intervals of each percentile. Similar to the fully parametric bootstrap confidence intervals, the NBSP method interval widths also decrease with increasing percentile. For example, the standard interval for the first percentile ranges from 27.70 to 31.84. The standard intervals for the fifth, tenth, and 50th percentiles are [33.03, 36.46], [35.67, 38.73], and [43.55, 45.40] MPa. Average interval differences and the standard deviations of the three interval types for the first, fifth, tenth, and 50th percentiles are 4.127 ± 0.017 , 3.409 ± 0.012 , 3.045 ± 0.020 , and 1.868 ± 0.015 , respectively. As was shown with the MOE data, Figure 6.6 shows that the sampling distributions for the NBSP sampling method for the MOR data are very similar to the sampling distributions for the fully parametric method. Both the fully parametric and the NBSP method sampling distributions appear continuous and normally distributed; the main difference between the two methods is the slight skewness shown in the lower percentiles for the NBSP method. Figure 6.6 displays the mild right skewness of the first and tenth percentiles, while mild left skewness is seen for the fifth percentile. Because the skewness is so mild and the interval results are so similar to the fully parametric method, the practitioner should certainly consider the NBSP a viable bootstrapping

method when the censoring mechanism cannot be assumed and the sample size is small. Confidence interval approximations for the NBSP method are considerably better than those obtained by using the fully nonparametric bootstrapping method.

6.4 SUMMARY

Three different bootstrapping methods have been discussed and explored. These include the fully nonparametric, fully parametric, and nonparametric bootstrap for parametric (NBSP) models. Three different types of confidence intervals were used to assess four different quantiles for each of the three bootstrapping methods. The confidence interval types included the standard, percentile, and bias-corrected intervals, which were applied to the estimation of the first, fifth, tenth, and 50th percentiles of both the MOE and MOR for WPC.

Because the sample sizes used in this analysis are not huge i.e., less than 200, using the fully nonparametric bootstrapping method is less desirable than the fully parametric or NBSP methods for smaller percentiles. For some sample sizes, sampling distributions of the lower percentiles tend to be more discrete and far from normally distributed. For the fully parametric and the NBSP methods, confidence intervals became narrower as the percentiles increased. This was not the case for the MOE and MOR of WPC for the fully nonparametric method. For this method, no consistent pattern for interval width emerged. Confidence interval widths are also more dissimilar for the standard, percentile, and bias-corrected interval types for the fully nonparametric method, especially for the MOE data. This dissimilarity and lack of normality complicates the interpretation of these intervals, but helpfully gives warning regarding our confirmation

of when to trust particular intervals more than others. Managers need warning when the numbers need to be questioned and when those numbers appear to be valid. Therefore, the fully nonparametric bootstrapping method is not recommended for small sample sizes, particularly for the lower percentile estimations, where the distributions are naturally more discrete. When the underlying distribution is not known, and the sample size is reasonable enough, utilizing the fully nonparametric method is a valid bootstrapping method. See Edwards, Guess, Young, and Bensmail (2007).

If the underlying distribution is not known and the sample size is rather small, the fully nonparametric method may be used to obtain approximate confidence intervals for the median and third quantile. For the very small quantiles, however, this approach is not recommended. One option, in the event of small sample sizes, is to use kernel smoothing to estimate the lower percentiles. See Chernick (1999), Meeker and Escobar (1998), and Polansky (2000). Meeker and Escobar (1998) indicate that another option, when prior information or previous experience is available, is to utilize a Bayesian approach to obtain reasonable estimates on the lower percentiles for small data sets.

Both the fully parametric and NBSP bootstrapping methods provide excellent results when the underlying distribution is known. While the fully parametric method provides slightly greater precision and is less susceptible to any apparent skewness, the main advantage of using the NBSP method is that no censoring mechanism knowledge is necessary. The LCL and UCL limits and the interval widths using the NBSP method are remarkably close to that of using the fully parametric method. In addition, the intervals for the three types are similar in consistency with the fully parametric method. In fact, helpfully, for all three bootstrapping methods, the type of interval does not make much

difference. This is confirming when all the intervals are rather close. The fully parametric method is valuable for validating classical textbook results, but it does not seem to provide a noteworthy advantage over the NBSP method in this data. Recall, the fully parametric method cannot be used when the censoring technique is unknown. For the reliability of WPC, it appears that the bootstrapping method of choice is the NBSP method. Normality of the bootstrap sampling distributions should always be ascertained by inspecting the histograms to prevent misuse.

Also, Meeker and Escobar (1998) recommend that between 2000 and 5000 bootstrap samples be generated in order to compute confidence intervals and that the larger samples are necessary for estimating the lower quantiles, particularly for small samples. Future study may perhaps include investigating the three bootstrapping methods on original WPC data sets with much larger sample sizes or smaller samples using a Bayesian approach.

Bootstrapping is a powerful and effective means of providing greater reliability of WPC. Estimation of the lower percentiles allows practitioners to better understand the early failures of their product. This understanding facilitates process improvements resulting in better warranties and specification limits for WPC. These process improvements ultimately lead to the production of a superior engineered wood product with enhanced reliability and better customer satisfaction.

Chapter 7

7. Concluding Remarks and Future Research

Because of an increasing interest and popularity by consumers and the forest products industry, as evidenced by a rapidly increasing market share of WPC, there is a growing need to assess the quality and reliability of WPC. In addition to the many positive attributes of WPC, e.g., use of recycled materials, low maintenance requirements, high moisture resistance, decay and insect resistance, there are several negative qualities. A few of the negative characteristics are lower stiffness than wood, thermal expansion, creep (increase in deformation over time while subjected to a sustained load), fire performance, sensitivity to staining, and light sensitivity, which allow notable color changes and color variations within the composites. To address these and other issues, WPC are continually undergoing modifications and study through increased research efforts. A lack of established performance standards as well as performance inconsistency for WPC opens the door for massive future research efforts.

One area of sustained research in recent years is assessment of the wide variety of additives that may be used in WPC. The possibility of various additives, such as stabilizers, compatibilizers, and coupling agents are continually being developed and evaluated as a way to improve the quality of WPC. The coupling agent, maleic anhydride modified polypropylene, or MAPP, is currently undergoing such assessment for improving the mechanical properties and the long-term durability of WPC. One of the purposes of this thesis has been to address the use of MAPP in WPC. For the WPC samples used in this exploratory analysis, MAPP was found to increase the bending

MOR, the tensile strength MOE, and the tensile strength MOR. The addition of MAPP had no influence on the bending MOE. Interestingly, when MAPP and HDPE were used together, the strength to failure for the tensile strength MOR was decreased over the MAPP composites with PP resins. This finding provides a future opportunity for exploring WPC with a variety of plastic resins, with and without MAPP, as well as assessing other potentially viable coupling agents and compatibilizers, e.g., maleated HDPE and PMDI-stearic acid compatibilizer system. See Geng, Li, and Simonsen (2005) for important work in this area. Inclusion of other additives, such as particular lubricants, has also been shown to adversely affect the mechanical properties of WPC. Chapter 4 of Harper (2003) provides a thorough treatment of how some additives may interact with the wood-plastic matrix, influencing the mechanical properties. Research efforts should continually assess new formulations with many of the techniques used in this thesis, i.e., descriptive statistics, graphical techniques, AIC, and study of the lower percentiles using bootstrapping methods.

Future research in this area should also focus on multivariate techniques such as principal component analysis (PCA) to determine the most important factors influencing the quality of WPC. See Clapp, Young, and Guess (2007) for important work in reliability analysis using PCA. Many experiments could be performed using a variety of additives, e.g., stabilizers, lubricants, colorants, and fungicides, and a variety of wood fibers/wood flour that may be incorporated into WPC, e.g., pine, maple, redwood, poplar. Schirp and Wolcott (2004) discuss the influence of fungal decay and moisture absorption on mechanical properties of WPC. In addition to the variety of additives, wood fiber/wood flour, and plastics, differing concentrations of each of the varieties should be

assessed. Differing mixing processes could be studied as well, e.g., varying the time of mixing dry ingredients before they are compounded and formed into a product, and varying the order or timing of when each component is added. Another option may be to assess the effects of differing processing methods, e.g., extrusion, injection molding, foam molding, etc. Besides mechanical properties, other quality measures of great concern for WPC should be studied. Some of these measures might include accelerated aging studies to assess various key responses, such as fungal decay, moisture absorption, light degradation, toxicity, and creep performance.

After PCA, a next step might include model selection and validation via various regression methods, jackknife techniques, holdout samples, and bootstrapping methods. To better understand WPC early failures and improve specification limits and warranties, Bayesian analysis, instead of confidence intervals, could be used as a predictive modeling approach to estimate the lower percentiles, especially when sample sizes are small. Finally, with the continual development and improvement of WPC, a series of designed experiments should be ongoing to assess selected quality metrics for a more thorough understanding of WPC reliability.

Descriptive statistics, histograms, box plots, probability plots, survival curves, and bootstrapping techniques are powerful and insightful tools that may help us understand the reliability of the data, indicate sources of variation, and suggest opportunities for process improvement. Compare Deming (1986, 1993). Three software packages were used to validate this analysis from a vast suite of other statistical software packages. The reliability results from this thesis may directly help WPC manufacturers better understand

the bending and tensile strength MOE and MOR measures of the WPC product and facilitate the long term reduction in variation of the product.

Practitioners, managers, and customers are interested in the lower percentiles of WPC. Knowledge of these lower percentiles provides valuable information about their product, e.g., specifications, warranty, and cost analysis. For process improvements to be realized regarding these percentiles, reliable confidence intervals are needed in order to assess potential variability in the percentile estimates. When sample sizes are sufficiently large, asymptotic normal intervals can provide reasonable approximations. However, most often, industrial and laboratory sample sizes are not sufficiently large enough to rely on these asymptotic intervals. Bootstrap methods are useful and reliable tools that may be used in cases where the underlying distribution is not known or trusted as well as in instances where a parametric assumption is valid.

The various bootstrap techniques described in this thesis provide empirical checking on whether or not the normal approximation is valid for either the parametric or nonparametric models. Engineers and managers can be confident in their lower percentile estimates if the histograms obtained from the bootstrap sampling distributions appear continuous and normally distributed, assuming that their sample sizes are reasonably large. Both the fully parametric and the NBSP bootstrapping methods provide excellent results when the underlying distribution is known. The fully parametric bootstrapping method is useful for verifying classical results using textbook formulas when the underlying distribution is known and the censoring mechanism is known in the case of censored data. The NBSP method is especially useful for censored data when the censoring mechanism is not known. Meeker and Escobar (1998) recommend using the

NBSP method first for either censored or non-censored data if the sample size is small and the underlying distribution is known. If the underlying distribution is not known and the sample size is not large, the fully nonparametric bootstrap method may be used to obtain approximate confidence intervals for the median and third quantile. But, this approach is not recommended for the lower percentiles, where the sampling distribution is often discrete. In this case, other methods are recommended, such as the bias-corrected and accelerated bootstrap percentile confidence interval discussed in Chapter 7 of Martinez and Martinez (2002) and kernel smoothing discussed in Chernick (1999), Meeker and Escobar (1998), and Polansky (2000). If prior information or previous experience is available, a Bayesian approach is recommended by Meeker and Escobar (1998) to obtain trustworthy lower percentile estimates for small data sets. Chen, León, Young, and Guess (2006) have applied forced censoring techniques to improve estimation of extremely small percentiles. Reliable percentile estimates obtained from the appropriate bootstrapping method, kernel smoothing, Bayesian modeling and forced percentile techniques help the practitioner have more precision in lower percentile estimates and provide a validation for the normal asymptotic intervals. These tools provide a means for greater reliability with improved processes and greater customer satisfaction.

Suggested improvements for this research include using both small and large sample industrial data sets to compare with the relatively small laboratory data set used for this analysis, perform more probability plotting comparisons of data subsets (comparison plotting on same graphs), use various software programs for comparisons of exploratory data analysis and bootstrapping methods, collect data over time and during

different shifts to better understand process variability and important factors for future research, and work directly with practitioners to ascertain the level of difficulty encountered in understanding and mastering the techniques presented in this thesis.

Finally, the importance of assessing outliers in real industrial data must be addressed. For the WPC data set used in this analysis, determining the reason for outliers was easy, i.e., all of the outliers in the tensile strength MOR were attributed to the combination of HDPE and MAPP. For most industrial data, determining the cause of outliers is not so simple. Practitioners and scientists must use caution before discarding any outliers when using the data for further statistical inference. In some cases, outliers, particularly extreme outliers, may be a typographical error. Often, the outliers are real and understanding the cause of these outliers can provide important and even crucial information regarding the product and the process. Allocating time and resources upfront to determine potential causes for these outliers can be an extremely helpful way to save money over time and can result in an improved product.

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Appendix

Appendix 1

MATLAB Code for Fully Nonparametric Bootstrap

```
%PrcBootCI.m is a function that creates the nonparametric standard
normal, percentile, and bias-corrected percentile bootstrap
%confidence intervals for percentiles for a given data set.

function [CINormal,CIPercentile,CIBC]=PrcBootCI_NonPara(data,alpha,B)

%DATA is one column of numerical data that is of interest for
calculating
%bootstrap CI's.
%ALPHA is the alpha level desired for a 1-ALPHA level CI. It must be
entered as a decimal.
%(Example: For a 95% CI, ALPHA=0.05)
%B is the number of desired bootstrap replications
CINormal=[];
CIPercentile=[];
CIBC=[];
data=data(:,1);
for p=[1 5 10 25 50] %Specifies a vector of percentiles to calculate
(Can be modified for other desired percentiles)
rand('seed',sum(clock*1000000)); %Set the seed by the clock
sampleX=[];
prcv=[]; %vector of percentiles
n=length(data);
pdata=prctile(data,p); %Calculate the pth quantile from the original
sample
k=0;
for i=1:B
j=ceil(n.*rand(n,1)); %Creates a random integer
sampleX=[data(j)]; %Creates a bootstrap sample
prc=prctile(sampleX,p); %Calculate the percentile for the bootstrap
sample
prcv=[prcv prc];
if prc<pdata
k=k+1;
end
end
z0=norminv(k/B); %Bias Correction Constant
BCl=2*z0-norminv(1-(alpha/2));
BCu=2*z0+norminv(1-(alpha/2));
BCalpha1=normcdf(BCl,0,1)*100; %Lower alpha level for Bias Corrected
Interval
BCalpha2=normcdf(BCu,0,1)*100; %Upper alpha level for Bias Corrected
interval
prcv=sort(prcv); %Sorts the bootstrap distribution of percentiles
prcest=mean(prcv) %Bootstrap estimate of the percentile
StErr=std(prcv); %Bootstrap standard error of the percentile
%Create one figure to show all histograms of interest
p
if p==1
subplot(2,2,1),hist(prcv);
end
```

```

if p==5
    subplot(2,2,2),hist(prcv);
end
if p==10
    subplot(2,2,3),hist(prcv);
end
if p==50
    subplot(2,2,4),hist(prcv);
end
CINormal=[CINormal; p pdata-norminv(1-(alpha/2))*StErr pdata+norminv(1-
(alpha/2))*StErr] %Creates Standard Normal Bootstrap CI
CIPercentile=[CIPercentile; p prctile(prcv,100*(alpha/2))
prctile(prcv,100*(1-(alpha/2)))] %Creates Percentile Bootstrap CI
CIBC=[CIBC; p prctile(prcv,BCalpha1) prctile(prcv,BCalpha2)] %Creates
Bias-Corrected Percentile Bootstrap CI
end

```

Appendix 2

MATLAB Code for Fully Parametric Bootstrap

`%WeibParametricBootCI.m` is a function that creates the fully parametric standard normal, percentile, and bias-corrected percentile bootstrap confidence intervals for percentiles for a given data set assumed to follow a Weibull distribution.

```
function [CINormal, CIPercentile, CIBC]=WeibParametricBootCI(data,
alpha, B)
CINormal=[];
CIPercentile=[];
CIBC=[];
data=data(:,1);
parmhat=wblfit(data) %Calculate Weibull parameter estimates from the
data (scale and shape parameters)
for p=[1 5 10 25 50] %Specifies a vector of percentiles to calculate
(Can be modified for other desired percentiles)
rand('seed',sum(clock*1000000)); %Set the seed by the clock
sampleX=[];
prcv=[]; %vector of percentiles
n=length(data);
pdata=wblinv(p/100,parmhat(1),parmhat(2)); %Calculate the pth quantile
parametrically from the original sample
k=0;
for i=1:B
    %j=ceil(n.*rand(n,1)); %Creates a random integer
    sampleX=wblrnd(parmhat(1),parmhat(2),n,1); %Creates a bootstrap sample
    from a Weibull distribution
    %with the same parameters as the data
    parmhatX=wblfit(sampleX); %Calculate Weibull parameter estimates for
    each bootstrap sample
    prc=wblinv(p/100,parmhatX(1),parmhatX(2)); %Calculate the percentile
    parametrically for the bootstrap sample
    prcv=[prcv prc];
    if prc<pdata
        k=k+1;
    end
end
z0=norminv(k/B); %Bias Correction Constant
BCl=2*z0-norminv(1-(alpha/2));
BCu=2*z0+norminv(1-(alpha/2));
BCalpha1=normcdf(BCl,0,1)*100; %Lower alpha level for Bias Corrected
Interval
BCalpha2=normcdf(BCu,0,1)*100; %Upper alpha level for Bias Corrected
interval
prcv=sort(prcv); %Sorts the bootstrap distribution of percentiles
prcest=mean(prcv) %Bootstrap estimate of the percentile
StErr=std(prcv); %Bootstrap standard error of the percentile
%Create one figure to show all histograms of interest
p
if p==1
    subplot(2,2,1),hist(prcv);
end
```

```

if p==5
    subplot(2,2,2),hist(prcv);
end
if p==10
    subplot(2,2,3),hist(prcv);
end
if p==50
    subplot(2,2,4),hist(prcv);
end
CINormal=[CINormal; p pdata-norminv(1-(alpha/2))*StErr pdata+norminv(1-
(alpha/2))*StErr] %Creates Standard Normal Bootstrap CI
CIPercentile=[CIPercentile; p prctile(prcv,100*(alpha/2))
prctile(prcv,100*(1-(alpha/2)))] %Creates Percentile Bootstrap CI
CIBC=[CIBC; p prctile(prcv,BCalpha1) prctile(prcv,BCalpha2)] %Creates
Bias-Corrected Percentile Bootstrap CI
end

```

Appendix 3

MATLAB Code for Nonparametric Bootstrap Sampling for Parametric Inference (NBSP) Bootstrap

%WeibBootCI.m is a function that creates the NBSP standard normal, percentile, and bias-corrected percentile bootstrap confidence intervals for percentiles for a given data set assumed to follow a Weibull distribution.

```
function [CINormal, CIPercentile, CIBC]=WeibBootCI_NBSP(data, alpha, B)
CINormal=[];
CIPercentile=[];
CIBC=[];
data=data(:,1);
parmhat=wblfit(data) %Calculate Weibull parameter estimates from the
data (scale and shape parameters)
for p=[1 5 10 25 50] %Specifies a vector of percentiles to calculate
(Can be modified for other desired percentiles)
rand('seed',sum(clock*1000000)); %Set the seed by the clock
sampleX=[];
prcv=[]; %vector of percentiles
n=length(data);
pdata=wblinv(p/100,parmhat(1),parmhat(2)); %Calculate the pth quantile
parametrically from the original sample
k=0;
for i=1:B
j=ceil(n.*rand(n,1)); %Creates a random integer
sampleX=[data(j)]; %Creates a bootstrap sample
parmhatX=wblfit(sampleX); %Calculate Weibull parameter estimates for
each bootstrap sample
prc=wblinv(p/100,parmhatX(1),parmhatX(2)); %Calculate the percentile
parametrically for the bootstrap sample
prcv=[prcv prc];
if prc<pdata
k=k+1;
end
end
z0=norminv(k/B); %Bias Correction Constant
BCl=2*z0-norminv(1-(alpha/2));
BCu=2*z0+norminv(1-(alpha/2));
BCalpha1=normcdf(BCl,0,1)*100; %Lower alpha level for Bias Corrected
Interval
BCalpha2=normcdf(BCu,0,1)*100; %Upper alpha level for Bias Corrected
interval
prcv=sort(prcv); %Sorts the bootstrap distribution of percentiles
prest=mean(prcv) %Bootstrap estimate of the percentile
StErr=std(prcv); %Bootstrap standard error of the percentile
%Create one figure to show all histograms of interest
p
if p==1
subplot(2,2,1),hist(prcv);
end
```

```

if p==5
    subplot(2,2,2),hist(prcv);
end
if p==10
    subplot(2,2,3),hist(prcv);
end
if p==50
    subplot(2,2,4),hist(prcv);
end
CINormal=[CINormal; p pdata-norminv(1-(alpha/2))*StErr pdata+norminv(1-
(alpha/2))*StErr] %Creates Standard Normal Bootstrap CI
CIPercentile=[CIPercentile; p prctile(prcv,100*(alpha/2))
prctile(prcv,100*(1-(alpha/2)))] %Creates Percentile Bootstrap CI
CIBC=[CIBC; p prctile(prcv,BCalpha1) prctile(prcv,BCalpha2)] %Creates
Bias-Corrected Percentile Bootstrap CI
end

```

Vita

Diane G. Perhac is a graduate research assistant for the Tennessee Forest Products Center at the University of Tennessee, Knoxville. She is currently pursuing her M.S. degree in Statistics from the University of Tennessee and plans to graduate in December 2007. Diane previously received a B.S. degree in Education with a major in Public Health and a B.S. degree in Chemical Engineering from the University of Tennessee, where she graduated with high honors and was a member of Tau Beta Pi National Honor Society. Besides various internships and research positions, her work experience includes employment as a Graduate Teaching Assistant at the University of Tennessee, a SAS programmer at the University of Michigan, and a Chemical Engineer at 3M Company. She is currently a member of the American Statistical Association (ASA), the American Institute of Chemical Engineers (AIChE), and the Forest Products Society (FPS).