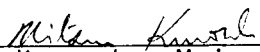
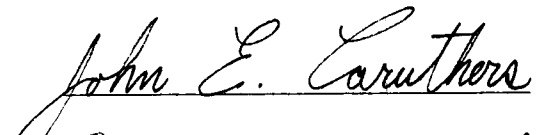


To the Graduate Council:

I am submitting herewith a thesis written by J. Michael McGee entitled "An Investigation of Unsteady Disturbances in Swirling Flows in an Annulus." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Aerospace Engineering.


M. Kurosaka, Major Professor

We have read this thesis
and recommend its acceptance:




Accepted for the Council:


Vice Chancellor
Graduate Studies and Research

AN INVESTIGATION OF UNSTEADY DISTURBANCES
IN SWIRLING FLOWS IN AN ANNULUS

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

J. Michael McGee
December 1981

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ABSTRACT

The acoustic wave propagation due to a swirling flow within a coannular duct is investigated where the swirling flow is assumed to be a combination of both a free and forced vortex. The acoustic frequency, based on the swirling velocity function, is computed as the solution of a characteristic value problem using the fourth-order Runge-Kutta method. A small swirl velocity approximation is also derived, to generate starting values for the numerical solution, and proves to be particularly accurate for estimating the effect of the free vortex on the acoustic frequency.

In general, the acoustic frequency increases as the magnitude of the swirling flow, moving opposite to the acoustic waves, is increased. As the ratio of outer to inner radius is increased the effect of the free vortex diminishes while the effect of the forced vortex remains essentially constant. Finally, the effect of the circumferential wave number is that of a multiplicative constant over the base value when that wave number equals one.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION.	1
II. MATHEMATICAL FORMULATION OF THE PROBLEM.	3
III. SMALL SWIRL VELOCITY APPROXIMATION.	14
IV. APPROXIMATE SOLUTION NEAR A SINGULAR POINT.	24
Mapping of Singular Points.	24
Method of Solution.	27
Singularities in $\frac{1}{\lambda_1}$	33
Singularities in A_1	38
V. NUMERICAL SOLUTION.	41
VI. CONCLUSIONS	48
BIBLIOGRAPHY.	79
APPENDICES.	81
A. COEFFICIENTS FOR A SOLUTION VALID NEAR A SINGULAR POINT . .	82
B. COMPUTED EIGENVALUES.	95
VITA.	144

LIST OF FIGURES

FIGURE	PAGE
1. Initial Grid.	26
2. Occurrence of Singular Points for $m = 1$	28
3. Occurrence of Singular Points for $m = 2$	30
4. Division of Integration Range	43
5. Flowchart of the Integration Process.	45
6. Order of Solution	47
7. Comparison with Small Swirl Approximation for $\Phi = \text{Constant}, m = 1, \mu = 1.01$	50
8. Comparison with Small Swirl Approximation for $\Phi = \text{Constant}, m = 1, \mu = 1.50$	51
9. Comparison with Small Swirl Approximation for $\Phi = \text{Constant}, m = 1, \mu = 3.00$	52
10. Comparison with Small Swirl Approximation for $\Phi = \text{Constant}, m = 1, \mu = 5.00$	53
11. Comparison with Small Swirl Approximation for $\Theta = \text{Constant}, m = 1, \mu = 1.01$	54
12. Comparison with Small Swirl Approximation for $\Theta = \text{Constant}, m = 1, \mu = 1.50$	55
13. Comparison with Small Swirl Approximation for $\Theta = \text{Constant}, m = 1, \mu = 3.00$	56
14. Comparison with Small Swirl Approximation for $\Theta = \text{Constant}, m = 1, \mu = 5.00$	57

FIGURE

PAGE

15. Effect of μ for $\Phi = \text{Constant}$, $\Theta = -2$, $m = 1$	58
16. Effect of μ for $\Phi = \text{Constant}$, $\Theta = 0$, $m = 1$	59
17. Effect of μ for $\Phi = \text{Constant}$, $\Theta = 2$, $m = 1$	60
18. Effect of μ for $\Theta = \text{Constant}$, $\Phi = -2$, $m = 1$	61
19. Effect of μ for $\Theta = \text{Constant}$, $\Phi = 0$, $m = 1$	62
20. Effect of μ for $\Theta = \text{Constant}$, $\Phi = 2$, $m = 1$	63
21. Effect of μ for $\Phi = \text{Constant}$, $\Theta = -2$, $m = 2$	64
22. Effect of μ for $\Phi = \text{Constant}$, $\Theta = 0$, $m = 2$	65
23. Effect of μ for $\Phi = \text{Constant}$, $\Theta = 2$, $m = 2$	66
24. Effect of μ for $\Theta = \text{Constant}$, $\Phi = -2$, $m = 2$	67
25. Effect of μ for $\Theta = \text{Constant}$, $\Phi = 0$, $m = 2$	68
26. Effect of μ for $\Theta = \text{Constant}$, $\Phi = 2$, $m = 2$	69
27. Curves of Constant Θ for $\mu = 1.01$, $m = 1$	71
28. Curves of Constant Θ for $\mu = 1.50$, $m = 1$	72
29. Curves of Constant Θ for $\mu = 3.00$, $m = 1$	73
30. Curves of Constant Θ for $\mu = 5.00$, $m = 1$	74
31. Curves of Constant Θ for $\mu = 1.01$, $m = 2$	75
32. Curves of Constant Θ for $\mu = 1.50$, $m = 2$	76
33. Curves of Constant Θ for $\mu = 3.00$, $m = 2$	77
34. Curves of Constant Θ for $\mu = 5.00$, $m = 2$	78

LIST OF SYMBOLS

$a(r)$	Local acoustic velocity
a_*	Reference acoustic velocity
C_v	Specific heat at constant volume
J_m	Bessel function of the first kind of the m^{th} order
k	Axial wave number
m	Circumferential wave number
p	Pressure of the fluid
r	Radial position
r_i	Inner radius
r_o	Outer radius
R	Gas constant
S	Enthalpy of the fluid
T	Temperature of the fluid
u_r, u_θ, u_z	Radial, circumferential, and axial velocity components
$V(r)$	Swirl velocity
$W(r)$	Axial velocity
W_r	Wronskian
X	Estimated eigenvalue
$X(\theta, \phi)$	Computed eigenvalue for value of θ and ϕ
$X_{ss}(\theta, \phi)$	Small swirl approximation of eigenvalue for value of θ and ϕ
Y_m	Bessel function of the second kind of the m^{th} order
z	vr_i

Greek Letters

α	Small swirl approximation coefficient for free vortex
β	Small swirl approximation coefficient for forced vortex
γ	Ratio of specific heats
Γ	Strength of free vortex
η	Nondimensionalized radial position
η_c	Location of singular point
Θ	Nondimensionalized strength of free vortex
μ	Ratio of outer and inner radii
ξ	Separation from singular point
ρ	Density of the fluid
ρ_0	Zero swirl solution
Φ	Nondimensionalized strength of forced vortex
ω	Frequency of acoustic wave of swirling flow
Ω	Strength of forced vortex

CHAPTER I

INTRODUCTION

It has long been recognized that swirling flow is capable of sustaining periodic motion. In 1880, Lord Kelvin [1]¹ first demonstrated this possibility theoretically for an incompressible flow contained in a cylindrical vessel. This was later confirmed in an experiment involving a rotating cylinder filled with water.

From a practical point of view, oscillatory disturbances frequently occur in the swirling flows of rotating machinery. For example, such a phenomenon is not uncommon in the draft tube of hydroelectric plants where a hydrodynamic oscillation involving spinning disturbances is found to exist in the rotational flow caused by water turbines. In aircraft gas turbines, the spinning pure tone disturbance in swirling flow, or "vortex whistle," is known to cause both aeroelastic and aeroacoustic problems. Instances have been documented where vortex whistle in the swirling flow of a cooling air cavity seriously jeopardized the structural integrity of the turbine.

In order to fix the natural frequency of various engine components away from this vortex whistle, it is of course necessary to predict the frequency of this pure tone noise. The frequency is dependent, in part, on the base, steady state, swirling flow upon which the unsteady oscillation is imposed. When the base flow is a

¹Numbers in brackets refer to similarly numbered references in the Bibliography.

free-vortex, the frequency relationship has been analyzed [2]. However, in most of the flow in turbomachinery, the steady swirl distribution is neither free-vortex nor forced-vortex but may approximately be regarded as a combination of both [3].

Thus, the objective of this study is to investigate the frequency of spinning waves superimposed upon a base flow which consists of a combination of free and forced vortex. A nondimensionalized second-order differential equation is developed to model a swirling flow in a coannular duct. The general numerical solution of this equation involves some significant difficulties, not the least of which is the occurrence of singular points within the range of numerical integration. Therefore, a numerical approximation that is valid in the neighborhood of a singular point is developed to facilitate the continuous integration across the entire range that is required. Finally, a small swirl approximation of the desired frequency is developed to provide initial values for the numerical integration.

CHAPTER II

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a swirling flow in an annulus with an inner radius, r_i , and an outer radius, r_o . Let u_r , u_θ , and u_z be the radial, angular, and axial velocities of the fluid; ρ and p the density and pressure of the fluid, and r the radial position of a fluid element. The continuity, momentum, and energy equations, in terms of cylindrical coordinates, can then be written as:

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r u_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho u_\theta) + \frac{\partial}{\partial z} (\rho u_z) = 0 . \quad (2.1)$$

Momentum

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} , \quad (2.2)$$

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = -\frac{1}{\rho} \frac{1}{r} \frac{\partial p}{\partial \theta} , \quad (2.3)$$

$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} . \quad (2.4)$$

Energy

$$\frac{\partial S}{\partial t} + u_r \frac{\partial S}{\partial r} + u_\theta \frac{\partial S}{\partial \theta} + u_z \frac{\partial S}{\partial z} = 0 , \quad (2.5)$$

where,

$$S = C_V \ln \left(\frac{p}{\rho^\gamma} \right) = C_V \ln \left(\frac{T}{\rho^{\gamma-1}} \right),$$

$$p = \rho RT,$$

C_V = specific heat at constant volume ,

γ = ratio of specific heats

(assumed = 1.4 for air) .

Each of the quantities is now expressed as the sum of a steady-state component, which is assumed to be a function of r only, and an unsteady small perturbation component. Denoting the steady-state components with a superscript "-" and the small perturbation quantities by " ' ":

$$u_r = u', \quad (2.6a)$$

$$u_\theta = \bar{V}(r) + v', \quad (2.6b)$$

$$u_z = \bar{W}(r) + w', \quad (2.6c)$$

$$p = \bar{p}(r) + p', \quad (2.6d)$$

$$\rho = \bar{\rho}(r) + \rho', \quad (2.6e)$$

$$S = \bar{S}(r) + S', \quad (2.6f)$$

$$T = \bar{T}(r) + T', \quad (2.6g)$$

where $\bar{U}(r) = 0$ by assumption and $\bar{V}(r)$ and $\bar{W}(r)$ are the swirl and axial velocities, respectively.

These quantities are substituted into Eqs. (2.1) through (2.5) and second-order terms are ignored. Collecting the results for steady and unsteady flow separately,

Steady Flow

$$\frac{1}{r} \bar{V}^2(r) = \frac{1}{\bar{\rho}(r)} \frac{\partial \bar{p}(r)}{\partial r}, \quad (2.7a)$$

$$\bar{S}(r) = C_V \ln \left[\frac{\bar{p}(r)}{(\bar{\rho}(r))^\gamma} \right], \quad (2.7b)$$

$$\frac{\bar{p}(r)}{\bar{\rho}(r)} = R\bar{T}(r). \quad (2.7c)$$

Assuming the entropy to be constant in the r -direction at the entrance to the annulus, from Eq. (2.7b),

$$\frac{\bar{p}(r)}{(\bar{\rho}(r))^\gamma}$$

is constant in the r -direction and can be assumed to be constant everywhere. Then Eq. (2.7a) becomes

$$\frac{1}{r} \bar{V}^2(r) = \frac{\partial}{\partial r} \left(\frac{\gamma}{\gamma-1} \frac{\bar{p}(r)}{\bar{\rho}(r)} + \text{constant} \right)$$

or

$$\frac{\bar{p}(r)}{\bar{\rho}(r)} = \frac{\bar{p}(r=r_{\text{ref}})}{\bar{\rho}(r=r_{\text{ref}})} + \frac{\gamma-1}{\gamma} \int_{r=r_{\text{ref}}}^r \frac{1}{\sigma} \bar{V}^2(\sigma) d\sigma \quad (2.8)$$

and substituting Eq. (2.7c),

$$\bar{T}(r) = \bar{T}(r = r_{\text{ref}}) + \frac{\gamma-1}{\gamma R} \int_{r=r_{\text{ref}}}^r \frac{1}{\sigma} \bar{V}^2(\sigma) d\sigma . \quad (2.9)$$

Unsteady Flow

$$\begin{aligned} \frac{\partial \rho'}{\partial t} + \frac{1}{r} \bar{V}(r) \frac{\partial \rho'}{\partial \theta} + \bar{W}(r) \frac{\partial \rho'}{\partial z} \\ + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{\rho}(r) u') + \frac{1}{r} \bar{\rho}(r) \frac{\partial v'}{\partial \theta} + \bar{\rho}(r) \frac{\partial w'}{\partial z} = 0 , \end{aligned} \quad (2.10a)$$

$$\begin{aligned} \frac{\partial u'}{\partial t} + \frac{1}{r} \bar{V}(r) \frac{\partial u'}{\partial \theta} + \bar{W}(r) \frac{\partial u'}{\partial z} \\ - \frac{2\bar{V}(r)v'}{r} = - \frac{1}{\bar{\rho}(r)} \frac{\partial p'}{\partial r} + \frac{\rho'}{\bar{\rho}(r)^2} \frac{\partial \bar{\rho}(r)}{\partial r} , \end{aligned} \quad (2.10b)$$

$$\begin{aligned} \frac{\partial v'}{\partial t} + \frac{1}{r} \bar{V}(r) \frac{\partial v'}{\partial \theta} + \bar{W}(r) \frac{\partial v'}{\partial z} \\ + u' \left[\frac{\partial \bar{V}(r)}{\partial r} + \frac{\bar{V}(r)}{r} \right] = - \frac{1}{r \bar{\rho}(r)} \frac{\partial p'}{\partial \theta} , \end{aligned} \quad (2.10c)$$

$$\begin{aligned} \frac{\partial w'}{\partial t} + \frac{1}{r} \bar{V}(r) \frac{\partial w'}{\partial \theta} + \bar{W}(r) \frac{\partial w'}{\partial z} \\ + u' \frac{\partial \bar{W}(r)}{\partial r} = - \frac{1}{\bar{\rho}(r)} \frac{\partial p'}{\partial z} , \end{aligned} \quad (2.10d)$$

$$\frac{\partial S'}{\partial t} + \frac{1}{r} \bar{V}(r) \frac{\partial S'}{\partial \theta} + \bar{W}(r) \frac{\partial S'}{\partial z} = 0 , \quad (2.10e)$$

$$S' = C_v \left[\frac{p'}{\bar{p}(r)} - \frac{\gamma p'}{\bar{\rho}(r)} \right] . \quad (2.10f)$$

Now set

$$u' = \tilde{u} e^{i(kz+m\theta-\omega t)}$$

$$v' = \tilde{v} e^{i(kz+m\theta-\omega t)}$$

$$w' = \tilde{w} e^{i(kz+m\theta-\omega t)}$$

$$p' = \tilde{p} e^{i(kz+m\theta-\omega t)}$$

$$\rho' = \tilde{\rho} e^{i(kz+m\theta-\omega t)}$$

$$S' = \tilde{S} e^{i(kz+m\theta-\omega t)}$$

where $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, $\tilde{p}$, $\tilde{\rho}$, and $\tilde{S}$ are the Fourier transforms of u' , v' , w' , p' , ρ' , and S' in k , m , and ω . Substituting into Eq. (2.10), taking full advantage of the result $\tilde{S} = 0$, and dropping the superscript "-" from the steady flow quantities yields

$$i\lambda(r)\tilde{\rho} + \frac{1}{r} \frac{\partial}{\partial r} (r\rho(r)\tilde{u}) + \frac{1}{r} \rho(r)im\tilde{v} + \rho(r)ik\tilde{w} = 0, \quad (2.11a)$$

$$i\lambda(r)\tilde{u} - \frac{2V(r)}{r} \tilde{v} = - \frac{a^2(r)}{\rho(r)} \frac{\partial \tilde{\rho}}{\partial r} + (2-\gamma) \frac{V^2(r)}{r} \frac{\tilde{\rho}}{\rho(r)}, \quad (2.11b)$$

$$i\lambda(r)\tilde{v} + \tilde{u} \frac{1}{r} \frac{d}{dr} (rV(r)) = - \frac{ima^2(r)}{r} \frac{\tilde{\rho}}{\rho(r)}, \quad (2.11c)$$

$$i\lambda(r)\tilde{w} + \tilde{u} \frac{dW(r)}{dr} = - ika^2(r) \frac{\tilde{\rho}}{\rho(r)}, \quad (2.11d)$$

where

$$\lambda(r) = kW(r) + \frac{m}{r} V(r) - \omega.$$

The equations in Eq. (2.11) are equivalent to those derived by Kerrebrock [3].

To this point, the derivation has paralleled the work done by Chu [2] who develops an equation in terms of the angular momentum and assumes the swirling flow to be composed of only a free vortex.

For the purpose of this investigation, $W(r)$ is assumed to be independent of r , so that

$$\tilde{w} = - \frac{ka^2(r)}{\lambda(r)\rho(r)} \tilde{p} . \quad (2.12)$$

Solving Eqs. (2.11b) and (2.11c) for $\tilde{u}$ and $\tilde{v}$,

$$\tilde{u} = iA(r) \left\{ - \lambda(r) \frac{a^2(r)}{\rho(r)} \frac{\partial \tilde{p}}{\partial r} + \frac{V(r)\tilde{p}}{r\rho(r)} \left[(2-\gamma)\lambda(r)V(r) - \frac{2ma^2(r)}{r} \right] \right\} \quad (2.13a)$$

$$\begin{aligned} \tilde{v} = A(r) \left\{ \frac{a^2(r)}{\rho(r)} \frac{1}{r} \frac{d}{dr} [rV(r)] \frac{\partial \tilde{p}}{dr} \right. \\ \left. + \frac{\tilde{p}}{\rho(r)} \left[\frac{m\lambda(r)a^2(r)}{r} - \frac{(2-\gamma)V^2(r)}{r^2} \frac{d}{dr} [rV(r)] \right] \right\}, \quad (2.13b) \end{aligned}$$

where

$$A(r) = - \frac{1}{\lambda^2(r) + \frac{2V(r)}{r^2} \frac{d}{dr} [rV(r)]} .$$

The base swirling flow is now assumed to be a combination of a free vortex and a forced vortex; that is,

$$V(r) = \frac{\Gamma}{r} + \Omega r .$$

Physically, the free-vortex component of the swirling flow in a gas turbine is attributable to the inlet guide vanes, while the forced vortex results from the motion of the compressor blades in that turbine.

Then

$$\frac{dV(r)}{dr} = -\frac{\Gamma}{r^2} + \Omega, \quad (2.14a)$$

$$\frac{d}{dr}[rV(r)] = 2\Omega r, \quad (2.14b)$$

$$\frac{d\lambda(r)}{dr} = -\frac{2\Gamma m}{r^3}, \quad (2.14c)$$

$$\frac{dA(r)}{dr} = -A^2(r) \left[\frac{4\lambda(r)\Gamma m - 8\Omega\Gamma}{r^3} \right], \quad (2.14d)$$

$$\frac{da(r)}{dr} = \frac{\gamma-1}{2} \frac{V^2(r)}{ra(r)}. \quad (2.14e)$$

Substituting Eqs. (2.12) and (2.13) into Eq. (2.11a) and employing the relationships in Eq. (2.14) eventually yields

$$\begin{aligned} & \frac{\partial^2 \tilde{\rho}}{\partial r^2} + \frac{1}{r} \left\{ 1 + (2\gamma-3) \frac{V^2(r)}{a^2(r)} - \frac{4A(r)\Gamma}{r^2} [m\lambda(r) - 2\Omega] \right\} \frac{\partial \tilde{\rho}}{\partial r} \\ & + \left\{ \frac{1}{A(r)} \left[\frac{k^2}{\lambda^2(r)} - \frac{1}{a^2(r)} \right] - \frac{m^2}{r^2} + \frac{2(2-\gamma)V(r)}{r^2 a^2(r)} [V(r) - 2\Omega r] \right. \\ & + \frac{2mV^3(r)}{r^3 \lambda(r) a^2(r)} - \frac{4m\Gamma}{r^4 \lambda(r)} \\ & \left. + \frac{4A(r)V(r)\Gamma}{r^4} \left[(2-\gamma) \frac{V(r)}{a^2(r)} - \frac{2m}{r\lambda(r)} \right] [m\lambda(r) - 2\Omega] \right\} \tilde{\rho} = 0. \quad (2.15a) \end{aligned}$$

The boundary condition is chosen as

$$\tilde{u} = 0 \text{ at } r = r_i \text{ and } r = r_o$$

or

$$\frac{\partial \tilde{p}}{\partial r} + \left\{ \frac{2mV(r)}{r^2 \lambda(r)} - \frac{(2-\gamma)V^2(r)}{r a^2(r)} \right\} \tilde{p} = 0 \quad (2.15b)$$

$$\text{at } r = r_i \text{ and } r = r_o .$$

The acoustic velocity function, $a(r)$, is now specified in terms of a selected reference velocity and the swirl velocity $V(r)$. The choice of reference point is arbitrary, but choosing either the inner or outer wall of the annulus is considered to be the most physically significant. Forcing $a(r)$ to be constant at such a point is equivalent to holding the temperature constant at that point; a condition that is most likely to be encountered in an actual environment. In addition, choosing the reference point at $r = r_i$ eliminates the singularities that arise in later formulations if the reference point is chosen as $r = r_o$. Thus defining

$$a_{\star}^2 = a^2 (r = r_i) .$$

From Eq. (2.9),

$$T(r) = T(r = r_i) + \frac{\gamma-1}{\gamma R} \int_{r_i}^r \frac{1}{\sigma} V^2(\sigma) d\sigma$$

or

$$\begin{aligned}
 a^2(r) &= a^2(r=r_i) + \gamma-1 \int_{r_i}^r \frac{1}{\sigma} V^2(\sigma) d\sigma \\
 &= a^2(r=r_i) + \frac{\gamma-1}{2} \left[\frac{\Gamma^2}{r_i^2} - \frac{\Gamma^2}{r^2} + 4\Omega\Gamma \ln\left(\frac{r}{r_i}\right) + \Omega^2 (r^2 - r_i^2) \right].
 \end{aligned}$$

To nondimensionalize Eq. (2.15), thus rendering it in a form that can be solved numerically, introduce the transformation

$$\frac{r}{r_i} = \eta, \quad (2.16a)$$

$$\frac{r_0}{r_i} = \mu, \quad (2.16b)$$

$$kr_i = \kappa, \quad (2.16c)$$

$$\frac{(\omega - kW)r_i}{a_*} = \chi, \quad (2.16d)$$

$$\frac{\Gamma}{r_i a_*} = \Theta, \quad (2.16e)$$

$$\frac{\Omega r_i}{a_*} = \Phi. \quad (2.16f)$$

Then

$$\begin{aligned}
 V(r) &= a_* \left[\frac{\Theta}{\eta} + \Phi \eta \right] \\
 &= a_* V_1, \quad (2.17a)
 \end{aligned}$$

$$\begin{aligned}
 \lambda(r) &= \frac{a_*}{r_i} \left[\frac{mV_1}{\eta} - x \right] \\
 &= \frac{a_*}{r_i} \lambda_1, \quad (2.17b)
 \end{aligned}$$

$$\begin{aligned}
 A(r) &= \frac{1}{\frac{a_*^2}{r_i^2} \left[-\lambda_1^2 + \frac{4\Phi V_1}{\eta} \right]} \\
 &= \frac{r_i^2}{a_*^2} A_1, \quad (2.17c)
 \end{aligned}$$

$$\begin{aligned}
 a^2(r) &= a_*^2 \left\{ 1 + \frac{\gamma-1}{2} \left[\Theta^2 - \frac{\Theta^2}{\eta^2} + 4\Theta\Phi\ln(\eta) + \Phi^2 (\eta^2 - 1) \right] \right\} \\
 &= a_*^2 a_1. \quad (2.17d)
 \end{aligned}$$

A final substitution into Eq. (2.15) yields the second-order homogeneous differential equation,

$$\begin{aligned}
 \frac{\partial^2 \tilde{\rho}}{\partial \eta^2} + \frac{1}{\eta} \left\{ 1 + (2\gamma-3) \frac{V_1^2}{a_1^2} - \frac{4A_1\Theta}{\eta^2} (m\lambda_1 - 2\Phi) \right\} \frac{\partial \tilde{\rho}}{\partial \eta} \\
 + \left\{ \frac{1}{A_1} \left(\frac{\kappa^2}{\lambda_1^2} - \frac{1}{a_1} \right) - \frac{m^2}{\eta^2} + \frac{2(2-\gamma)V_1}{\eta^2 a_1} (V_1 - 2\Phi\eta) + \frac{2mV_1^3}{\eta^3 \lambda_1 a_1} \right. \\
 \left. - \frac{4m\Theta}{\eta^4 \lambda_1} + \frac{4A_1 V_1 \Theta}{\eta^4} \left[(2-\gamma) \frac{V_1}{a_1} - \frac{2m}{\eta \lambda_1} \right] (m\lambda_1 - 2\Phi) \right\} \tilde{\rho} = 0, \quad (2.18a)
 \end{aligned}$$

with the homogeneous boundary condition,

$$\frac{\partial \tilde{p}}{\partial \eta} + \left\{ \frac{2mV_1}{\eta^2 \lambda_1} - \frac{(2-\gamma)V_1^2}{\eta a_1} \right\} \tilde{p} = 0$$

$$\text{at } \eta = 1.0 \text{ and } \eta = \mu . \quad (2.18b)$$

Solution of this boundary value problem will yield the eigenvalues, λ , from which the frequency, ω , of the acoustic waves of the unsteady swirling flow can be determined once the geometry of the annulus, r_o/r_i , swirl flow, $V(r)$, and wave number in the axial, κ , and circumferential, m , direction are specified.

CHAPTER III

SMALL SWIRL VELOCITY APPROXIMATION

To provide starting values for the numerical solution of the problem, a small swirl velocity approximation is determined as follows. For Γ and $\Omega \neq 0$ but small, assume

$$\omega = kW \pm a_* \sqrt{k^2 + v^2} + \alpha\Gamma + \beta\Omega ,$$

where

$$v^2 = k^2 + \frac{\lambda^2(r)}{a^2(r)} ,$$

and the zero swirl solution,

$$\omega = kW \pm a_* \sqrt{k^2 + v^2} ,$$

is easily derived from the basic equation.

Then neglecting terms of order Γ^2 , Ω^2 , and greater,

$$\lambda^2(r) = a_*^2 (k^2 + v^2) \pm 2 \left[\alpha\Gamma + \beta\Omega - \frac{mV(r)}{r} \right] a_* \sqrt{k^2 + v^2}$$

$$A(r) = - \frac{1}{\lambda^2(r)} ,$$

and Eq. (2.15) becomes

$$\frac{\partial^2 \tilde{\rho}}{\partial r^2} + \frac{1}{r} \left\{ 1 + \frac{4m\Gamma}{r^2 \lambda(r)} \right\} \frac{\partial \tilde{\rho}}{\partial r} + \left\{ \frac{\lambda^2(r)}{a^2(r)} - k^2 - \frac{m^2}{r^2} - \frac{4m\Gamma}{r^4 \lambda(r)} \right\} \tilde{\rho} = 0, \quad (3.1a)$$

B.C.

$$\frac{\partial \tilde{\rho}}{\partial r} + \frac{2mV(r)}{r^2 \lambda(r)} \tilde{\rho} = 0 \quad \text{at } r=r_i \text{ and } r=r_o. \quad (3.1b)$$

Substitution yields

$$\begin{aligned} \frac{\partial^2 \tilde{\rho}}{\partial r^2} + \frac{1}{r} \left\{ 1 + \frac{4m\Gamma}{r^2 \left[\mp a_* \sqrt{k^2 + v^2} - \alpha\Gamma - \beta\Omega + \frac{m}{r} V(r) \right]} \right\} \frac{\partial \tilde{\rho}}{\partial r} \\ + \left\{ \frac{a_*^2 (k^2 + v^2) \pm 2 \left[\alpha\Gamma + \beta\Omega - \frac{m}{r} V(r) \right] a_* \sqrt{k^2 + v^2}}{a^2(r)} \right. \\ \left. - k^2 - \frac{m^2}{r^2} - \frac{4m\Gamma}{r^4 \left[\mp a_* \sqrt{k^2 + v^2} - \alpha\Gamma - \beta\Omega + \frac{m}{r} V(r) \right]} \right\} \tilde{\rho} = 0, \quad (3.2a) \end{aligned}$$

B.C.

$$\frac{\partial \tilde{\rho}}{\partial r} + \left\{ \frac{2mV(r)}{r^2 \left[\mp a_* \sqrt{k^2 + v^2} - \alpha\Gamma - \beta\Omega + \frac{m}{r} V(r) \right]} \right\} \tilde{\rho} = 0 \quad \text{at } r=r_i \text{ and } r=r_o, \quad (3.2b)$$

and from the assumption of small Γ and Ω ,

$$\frac{da(r)}{dr} \approx v^2(r) \approx 0$$

or

$$a(r) = a_*.$$

Now using the expansion,

$$\tilde{\rho} = \rho_0 + \Gamma \rho_\Gamma + \Omega \rho_\Omega ,$$

and again ignoring terms of order r^2 , Ω^2 and greater, Eq. (3.2) can be broken down as

$$\frac{\partial^2 \rho_0}{\partial r^2} + \frac{1}{r} \frac{\partial \rho_0}{\partial r} + \left\{ v^2 - \frac{m^2}{r^2} \right\} \rho_0 = 0 , \quad (3.3a)$$

B.C.

$$\frac{\partial \rho_0}{\partial r} = 0 \text{ at } r = r_i \text{ and } r = r_o . \quad (3.3b)$$

$$\begin{aligned} \frac{\partial^2 \rho_\Gamma}{\partial r^2} + \frac{1}{r} \frac{\partial \rho_\Gamma}{\partial r} + \left\{ v^2 - \frac{m^2}{r^2} \right\} \rho_\Gamma &= \left\{ \frac{4m}{r^3 \left[\pm a_* \sqrt{k^2 + v^2} \right]} \right\} \frac{\partial \rho_0}{\partial r} \\ &- \left\{ \frac{4m}{r^4 \left[\pm a_* \sqrt{k^2 + v^2} \right]} \pm \frac{2 \left(\alpha - \frac{m^2}{r^2} \right) \sqrt{k^2 + v^2}}{a^*} \right\} \rho_0 , \end{aligned} \quad (3.4a)$$

B.C.

$$\frac{\partial \rho_\Gamma}{\partial r} = \frac{2m}{r^3 \left[\pm a_* \sqrt{k^2 + v^2} \right]} \rho_0 \text{ at } r = r_i \text{ and } r = r_o . \quad (3.4b)$$

$$\frac{\partial^2 \rho_\Omega}{\partial r^2} + \frac{1}{r} \frac{\partial \rho_\Omega}{\partial r} + \left\{ v^2 - \frac{m^2}{r^2} \right\} \rho_\Omega = \mp \left\{ \frac{2(\beta-m) \sqrt{k^2 + v^2}}{a_*} \right\} \rho_0, \quad (3.5a)$$

B.C.

$$\frac{\partial \rho_\Omega}{\partial r} = \frac{2m}{r \left[\pm a_* \sqrt{k^2 + v^2} \right]} \rho_0$$

at $r = r_i$ and $r = r_o$. (3.5b)

To simplify Eq. (3.4), introduce ψ defined by

$$\rho_\Gamma = \psi - \frac{\left(\frac{m^2}{r^2} - \alpha \right)}{\pm a_* \sqrt{k^2 + v^2}} \rho_0;$$

Eq. (3.4) can then be written as

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \left\{ v^2 - \frac{m^2}{r^2} \right\} \psi = \mp \left\{ \frac{2 \left(\alpha - \frac{m}{r^2} \right) \sqrt{k^2 + v^2}}{a_*} \right\} \rho_0, \quad (3.6a)$$

B.C.

$$\frac{\partial \psi}{\partial r} = 0 \text{ at } r = r_i \text{ and } r = r_o. \quad (3.6b)$$

Equations (3.5) and (3.6) are now in the proper form to generate solutions involving integrals of Bessel functions which can be evaluated.

For $m > 0$ the solution to Eq. (3.3a) is

$$\rho_0 = AJ_m(vr) + BY_m(vr),$$

where

$$B = - \frac{J'_m(\nu r_i)}{Y'_m(\nu r_i)} A$$

from the boundary condition, Eq. (3.3b), at $r = r_i$, and

$$\begin{vmatrix} J'_m(\nu r_i) & Y'_m(\nu r_i) \\ J'_m(\nu r_0) & Y'_m(\nu r_0) \end{vmatrix} = 0 \quad (3.7)$$

from the boundary condition at both $r = r_i$ and $r = r_0$. To obtain the inhomogeneous solution for ψ and ρ_Ω which will yield the values for α and β , evaluate the Wronskian

$$\begin{aligned} W_r &= \nu [J_m(\nu r)Y'_m(\nu r) - Y_m(\nu r)J'_m(\nu r)] \\ &= \frac{2}{\pi r} \end{aligned}$$

according to Abramowitz and Stegun [4]. The solutions for ψ and ρ_Ω are then given by

$$\begin{aligned} \psi &= -J_m(\nu r) \int_{r_i}^r \frac{\pi \sigma}{2} Y_m(\nu \sigma) \left[\mp 2 \left(\alpha - \frac{m}{\sigma^2} \right) \frac{\sqrt{k^2 + \nu^2}}{a_*} \right] \left[AJ_m(\nu \sigma) + BY_m(\nu \sigma) \right] d\sigma \\ &\quad + Y_m(\nu r) \int_{r_i}^r \frac{\pi \sigma}{2} J_m(\nu \sigma) \left[\mp 2 \left(\alpha - \frac{m}{\sigma^2} \right) \frac{\sqrt{k^2 + \nu^2}}{a_*} \right] \left[AJ_m(\nu \sigma) + BY_m(\nu \sigma) \right] d\sigma \\ &\quad + CJ_m(\nu r) + DY_m(\nu r), \end{aligned} \quad (3.8)$$

$$\begin{aligned}
\rho_{\Omega} = & -J_m(\nu r) \int_{r_i}^r \frac{\pi \sigma}{2} Y_m(\nu \sigma) \left[\mp \frac{2(\beta-m) \sqrt{k^2+\nu^2}}{a_*} \right] \left[A J_m(\nu \sigma) + B Y_m(\nu \sigma) \right] d\sigma \\
& + Y_m(\nu r) \int_{r_i}^r \frac{\pi \sigma}{2} J_m(\nu \sigma) \left[\mp \frac{2(\beta-m) \sqrt{k^2+\nu^2}}{a_*} \right] \left[A J_m(\nu \sigma) + B Y_m(\nu \sigma) \right] d\sigma \\
& + E J_m(\nu r) + F Y_m(\nu r) .
\end{aligned} \tag{3.9}$$

Solving first for α , evaluate the boundary, Eq. (3.6b), using Eq. (3.8). Then

$$C \nu J'_m(\nu r_i) + D \nu Y'_m(\nu r_i) = 0$$

$$C \nu J'_m(\nu r_0) + D \nu Y'_m(\nu r_0) = \Sigma ,$$

where

$$\begin{aligned}
\Sigma = & \nu J'_m(\nu r_0) \int_{r_i}^{r_0} \frac{\pi \sigma}{2} Y_m(\nu \sigma) \left[\mp 2 \left(\alpha - \frac{m}{\sigma^2} \right) \frac{\sqrt{k^2+\nu^2}}{a_*} \right] \left[A J_m(\nu \sigma) + B Y_m(\nu \sigma) \right] d\sigma \\
& - \nu Y'_m(\nu r_0) \int_{r_i}^{r_0} \frac{\pi \sigma}{2} J_m(\nu \sigma) \left[\mp 2 \left(\alpha - \frac{m}{\sigma^2} \right) \frac{\sqrt{k^2+\nu^2}}{a_*} \right] \left[A J_m(\nu \sigma) + B Y_m(\nu \sigma) \right] d\sigma .
\end{aligned}$$

To solve for C

$$C = \frac{\begin{vmatrix} 0 & vY'_m(vr_i) \\ \Sigma & vY'_m(vr_o) \end{vmatrix}}{\begin{vmatrix} vJ'_m(vr_i) & vY'_m(vr_i) \\ vJ'_m(vr_o) & vY'_m(vr_o) \end{vmatrix}} ;$$

The determinant in the denominator is equal to zero from Eq. (3.7).

If the value of C is to be finite, the determinant in the numerator must also equal zero. Therefore,

$$\begin{aligned} A \left\{ J'_m(vr_o) \int_{r_i}^{r_o} Y_m(v\sigma) \left(\alpha\sigma - \frac{m}{\sigma} \right) J_m(v\sigma) d\sigma \right. \\ - Y'_m(vr_o) \int_{r_i}^{r_o} J_m(v\sigma) \left(\alpha\sigma - \frac{m}{\sigma} \right) J_m(v\sigma) d\sigma \\ - \frac{J'_m(vr_i)}{Y'_m(vr_i)} J'_m(vr_o) \int_{r_i}^{r_o} Y_m(v\sigma) \left(\alpha\sigma - \frac{m}{\sigma} \right) Y_m(v\sigma) d\sigma \\ \left. + \frac{J'_m(vr_i)}{Y'_m(vr_i)} Y'_m(vr_o) \int_{r_i}^{r_o} J_m(v\sigma) \left(\alpha\sigma - \frac{m}{\sigma} \right) Y_m(v\sigma) d\sigma \right\} = 0 \end{aligned}$$

Ignoring the trivial solution of $A = 0$ and employing Eq. (3.7) in the form

$$J'_m(vr_o) Y'_m(vr_i) = J'_m(vr_i) Y'_m(vr_o) ,$$

it follows that

$$\alpha = m \frac{\tau_1}{\tau_2},$$

where

$$\begin{aligned} \tau_1 = & 2J'_m(vr_i) Y'_m(vr_o) \int_{r_i}^{r_o} \frac{1}{v\sigma} J_m(v\sigma) Y_m(v\sigma) v d\sigma \\ & - Y'_m(vr_i) Y'_m(vr_o) \int_{r_i}^{r_o} \frac{1}{v\sigma} J_m^2(v\sigma) v d\sigma \\ & - J'_m(vr_i) J'_m(vr_o) \int_{r_i}^{r_o} \frac{1}{v\sigma} Y_m^2(v\sigma) v d\sigma, \quad (3.10a) \end{aligned}$$

$$\begin{aligned} \tau_2 = & 2J'_m(vr_i) Y'_m(vr_o) \int_{r_i}^{r_o} \sigma J_m(v\sigma) Y_m(v\sigma) d\sigma \\ & - Y'_m(vr_i) Y'_m(vr_o) \int_{r_i}^{r_o} \sigma J_m^2(v\sigma) d\sigma \\ & - J'_m(vr_i) J'_m(vr_o) \int_{r_i}^{r_o} \sigma Y_m^2(v\sigma) d\sigma. \quad (3.10b) \end{aligned}$$

Using the same procedure,

$$\beta = m \left(1 - \frac{2}{\pi \sqrt{k^2 + v^2}} \frac{\tau_3}{\tau_2} \right),$$

where

$$\tau_3 = \left[J_m(vr_i) - \frac{J'_m(vr_i)}{Y'_m(vr_i)} Y_m(vr_i) \right] \frac{Y'_m(vr_o)}{vr_i} - \frac{J_m(vr_o) Y'_m(vr_i) - J'_m(vr_i) Y_m(vr_o)}{vr_o}. \quad (3.11)$$

The integrals in Eq. (3.10) can be evaluated according to Watson [5].

The values of vr_i and vr_o are derived as the eigenvalues of Eq. (3.7) in the form

$$J'_m(\mu Z) Y'_m(Z) - J'_m(Z) Y'_m(\mu Z) = 0.$$

Computed values of Z , α , and β are presented in Table 1.

Table 1. Small Swirl Approximation Coefficients

μ	Z		α		β	
	$m = 1$	$m = 2$	$m = 1$	$m = 2$	$m = 1$	$m = 2$
1.01	0.995	1.990	0.990	1.980	0.999	1.999
1.05	0.976	1.951	0.952	1.904	0.999	1.998
1.10	0.953	1.906	0.908	1.815	0.997	1.994
1.20	0.910	1.821	0.829	1.657	0.989	1.977
1.30	0.872	1.744	0.761	1.519	0.977	1.954
1.40	0.837	1.673	0.700	1.397	0.963	1.925
1.50	0.805	1.608	0.648	1.288	0.946	1.892
2.00	0.677	1.341	0.455	0.874	0.846	1.703
2.50	0.585	1.137	0.335	0.602	0.743	1.537
3.00	0.514	0.977	0.254	0.424	0.649	1.424
4.00	0.411	0.752	0.155	0.234	0.502	1.315
5.00	0.341	0.607	0.102	0.147	0.405	1.278

CHAPTER IV

APPROXIMATE SOLUTION NEAR A SINGULAR POINT

1. Mapping of Singular Points

Examination of the definitions of Eq. (2.17) reveals that, in general, the functions $\frac{1}{a_1}$, $\frac{1}{\lambda_1}$, and A_1 will have singular points, η_c , for selected values of Θ , Φ , X , m , and κ . The singular nature of the function $\frac{1}{a_1}$ has been eliminated by the choice of the acoustic reference velocity $a_*^2 = a^2(r=r_i)$ forcing $\frac{1}{a_1}$ to be uniformly increasing from $\eta = 1.0$ to $\eta = \mu$. A similar alternative is not available, however, for the terms $\frac{1}{\lambda_1}$ and A_1 .

An indication of the prevalence, over the range of interest, of singular points in these functions can be determined using the small swirl approximation previously derived. For selected values of Θ , Φ , m , and κ the small swirl approximation will yield an approximate value for X as

$$X = \sqrt{\kappa^2 + Z^2} + \alpha\Theta + \beta\Phi. \quad (4.1)$$

For the function $\frac{1}{\lambda_1}$ the position of the singular point, η_c , is then determined from

$$\lambda_1 = \frac{mV_1}{\eta} - X = 0$$

or

$$\eta_c = \eta_1 = \sqrt{\frac{m\Theta}{X - m\Phi}},$$

and for the function A_1 ,

$$-\lambda_1^2 + \frac{4V_1\Phi}{\eta} = 0$$

or

$$\eta_c = \eta_{2,3} = \sqrt{\frac{-B \pm \sqrt{B^2 - 4C}}{2}},$$

where

$$B = \frac{[4\Phi - 2m(m\Phi - X)]\Theta}{[4\Phi^2 - (m\Phi - X)^2]}$$

$$C = \frac{-m^2\Theta^2}{[4\Phi^2 - (m\Phi - X)^2]}.$$

Note that while there are, in general, two values of η_c for the function $\frac{1}{\lambda_1}$ and four values of η_c for the function A_1 , at most one of the former and two of the latter can fall within the range of interest, $1.0 < \eta < \mu$.

A grid of points is established that represents the values of Θ and Φ . The grid points are spaced at intervals of 0.2, and Θ and Φ range from -2.0 to 2.0 along the x- and y-axis, respectively. The value of each point is initialized to zero so that the grid appears as in Figure 1.

$\phi \backslash \theta$	-2.00	-1.80	-1.60	-1.40	-1.20	-1.00	-0.80	-0.60	-0.40	-0.20	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
2.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1.20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1.40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1.60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 1. Initial grid.

Position η_1 is calculated for each grid point and if η_1 falls within the range from 1.0 to μ the value of that point is increased by one. Positions η_2 and η_3 are then calculated. If one of the two falls within the range 1.0 to μ the value of the grid point is increased by two. If both fall within the specified range the value of the grid point is increased by four. Completed grids are presented in Figures 2 and 3 for μ and m within the range of interest.

2. Method of Solution

Thus, over a substantial portion of the range of interest, it is to be expected that the functions $\frac{1}{\lambda_1}$ and A_1 will contain singular points. As the numerical integration will break down in the vicinity of a singular point it is therefore necessary to employ the method developed by Fuchs and Frobenius [6] to generate a solution of the basic equation which is valid in the neighborhood of a singularity. Such a solution will yield values for $\tilde{\rho}$ and $\frac{\partial \tilde{\rho}}{\partial \eta}$ at some $\eta > \eta_c$ so that the integration may be continued.

The method of solution involves the expansion of the coefficients of the basic equation around the singular point so that the equation can be written as

$$\xi^2 \frac{d^2 \tilde{\rho}}{d\xi^2} = \xi S(\xi) \frac{d\tilde{\rho}}{d\xi} + T(\xi) \tilde{\rho} ,$$

where

$$\xi = \eta - \eta_c ,$$

$$S(\xi) = S_0 + S_1 \xi + S_2 \xi^2 + S_3 \xi^3 + \dots ,$$

$$T(\xi) = T_0 + T_1 \xi + T_2 \xi^2 + T_3 \xi^3 + \dots .$$

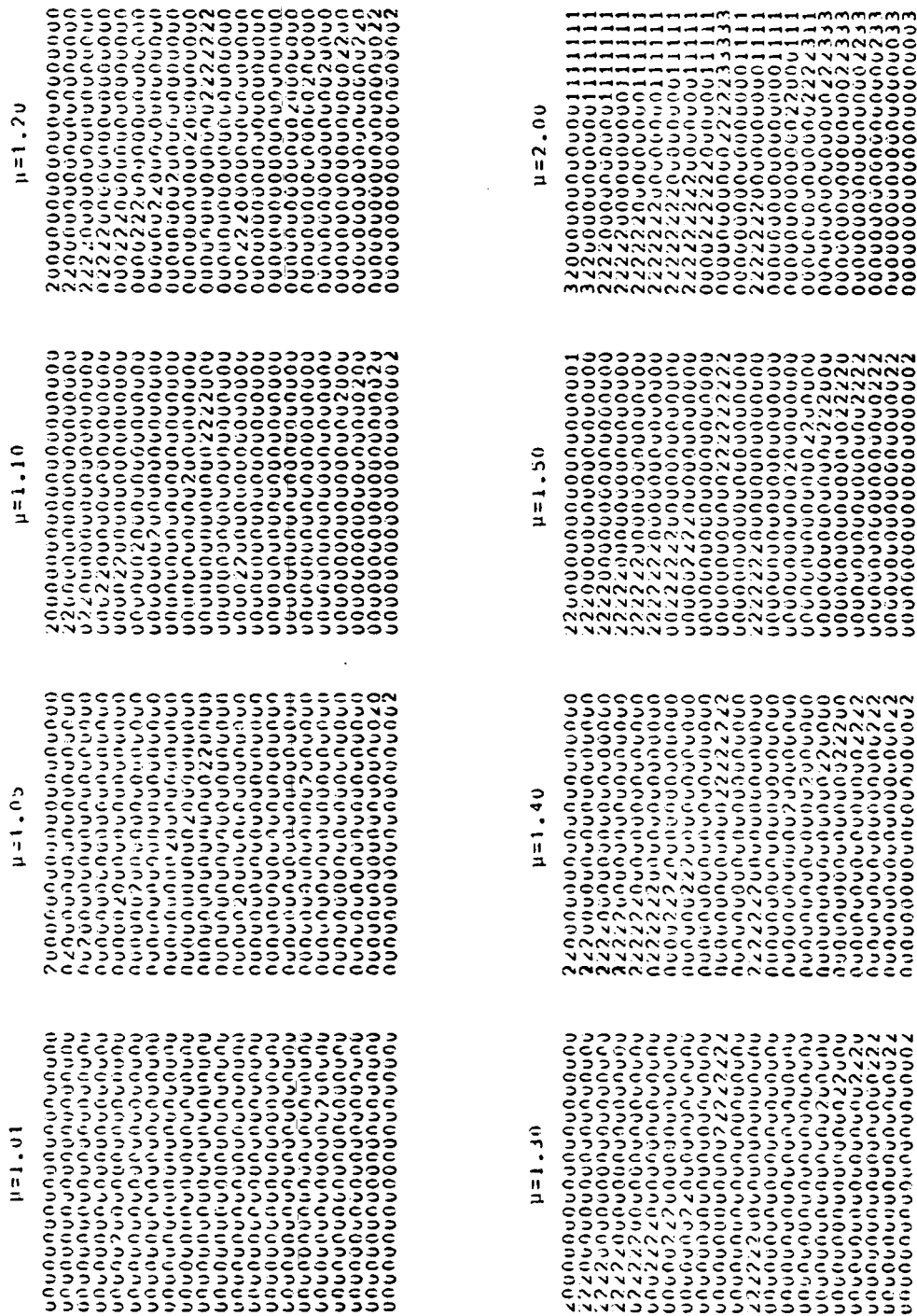


Figure 2. Occurrence of singular points for $m = 1$.

$\mu = 2.50$	$\mu = 3.00$	$\mu = 4.00$	$\mu = 5.00$
33111110001111111111	31111100000000000000	31111100000000000000	31111100000000000000
33111100001111111111	33111100000000000000	33111100000000000000	33111100000000000000
33311100000111111111	33311100000000000000	33311100000000000000	33311100000000000000
33330000000111111111	33331110000000000000	33331110000000000000	33331110000000000000
33330000000111111111	33333110000000000001	33333110000000000001	33333110000000000000
33722000000011111111	33333311100000111111	33333311100000111111	33333311100000000000
22222000000111111111	33333311101111111111	33333311101111111111	33333331110000000000
22222220000111111111	22222220000111111111	22222220000111111111	22222220001111111111
00000000002723333333	00000000002233333333	00000000001111111113	22222222200111111333
00000000000011111111	00000000000111111111	00000000000111111111	00000000000111111111
22222200000001111111	22222200000001111111	22222200000001111111	22222200000001111111
22000000000000011111	22220000000002331111	22220000000002331111	22220000000002333333
00000000000022311111	00000000000022333311	00000000000022333311	00000000000002333333
00000000000002233333	00000000000002333333	00000000000002333333	00000000000002333333
00000000000000233333	00000000000000233333	00000000000000233333	00000000000000233333
00000000000000023333	00000000000000023333	00000000000000023333	00000000000000023333
00000000000000002333	00000000000000002333	00000000000000002333	00000000000000002333
00000000000000000233	00000000000000000233	00000000000000000233	00000000000000002333
000000000000000000133	000000000000000000133	000000000000000000133	000000000000000000133
000000000000000000013	0000000000000000000013	0000000000000000000013	0000000000000000000013

Figure 2. (continued)

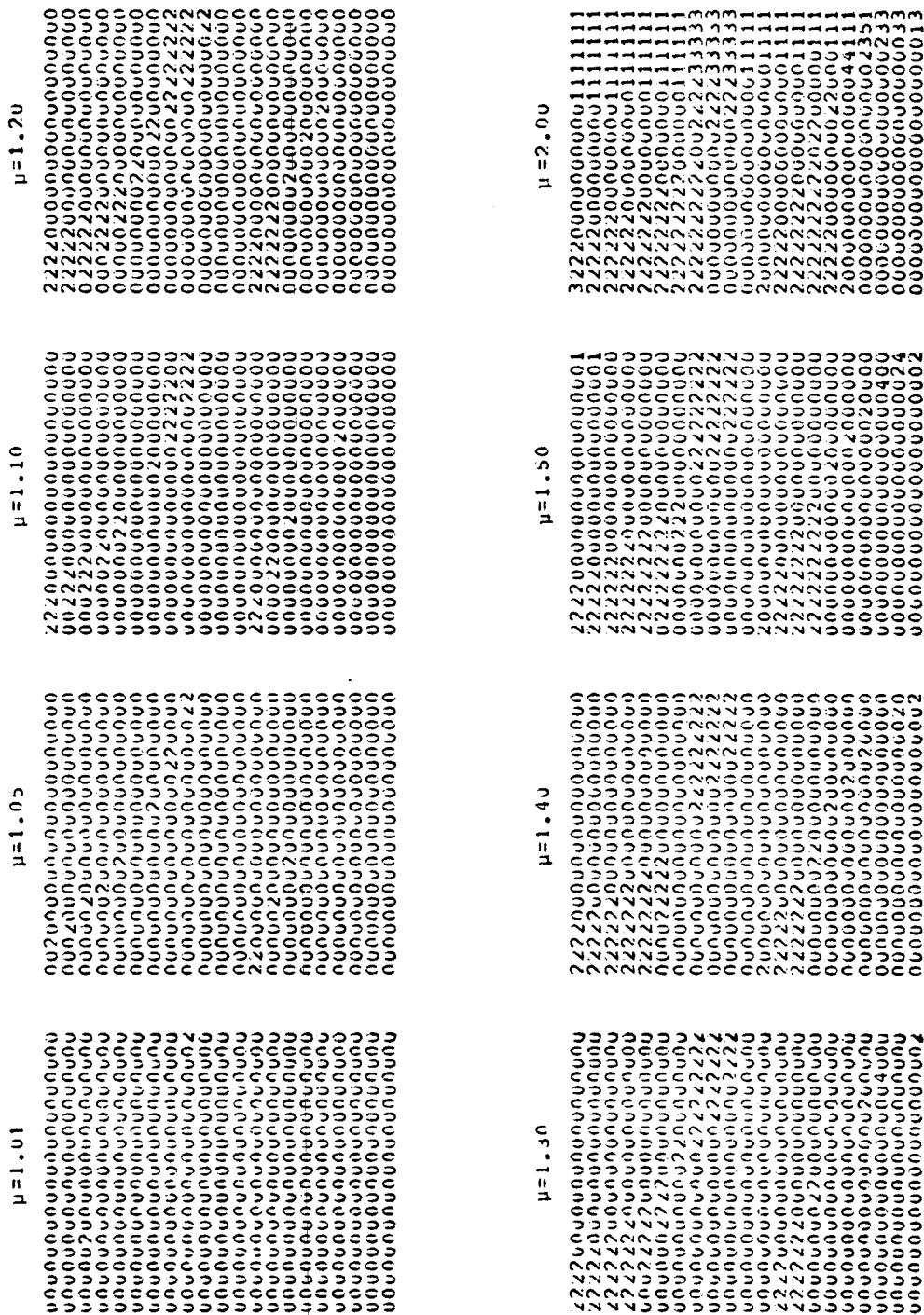


Figure 3. Occurrence of singular points for $m = 2$.

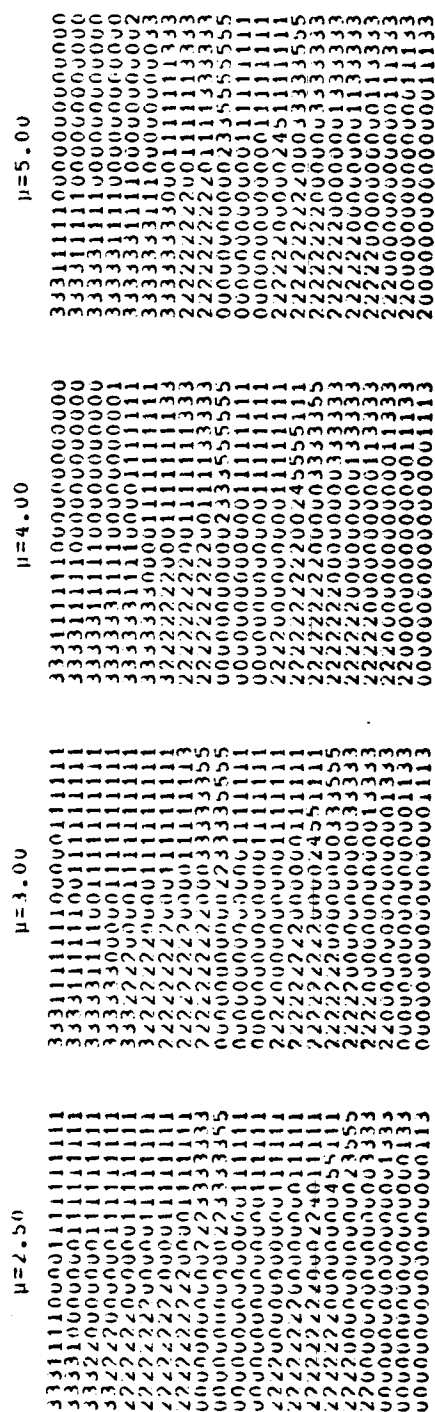


Figure 3. (continued)

Note that the functions $S(\xi)$ and $T(\xi)$ are not the entropy and temperature functions, but are chosen to be consistent with the notation in the cited reference. The resulting solutions are of the form

$$\tilde{p} = C\xi^p \left(a_0 + a_1\xi + a_2\xi^2 + a_3\xi^3 + \dots \right),$$

where the exponent p takes the value of the roots of the indicial equation

$$a_0 \left(p(p-1) - pS_0 - T_0 \right) = 0.$$

The coefficient a_0 is chosen equal to one with no loss of generality, and the remaining coefficients, a_n , are determined from the coefficients of $S(\xi)$ and $T(\xi)$.

In general then, as ξ is very small,

$$\begin{aligned} V_1 &= \frac{\Theta}{\eta} + \Phi\eta \\ &= \frac{\Theta}{\eta_c \left(1 + \frac{\xi}{\eta_c} \right)} + \Phi(\eta_c + \xi) \\ &= \frac{\Theta}{\eta_c} \left(1 - \frac{\xi}{\eta_c} + \frac{\xi^2}{\eta_c^2} - \frac{\xi^3}{\eta_c^3} \right) + \Phi(\eta_c + \xi) \\ &= \frac{\Theta}{\eta_c} + \Phi\eta_c + \left(-\frac{\Theta}{\eta_c^2} + \Phi \right) \xi + \frac{\Theta}{\eta_c^3} \xi^2 - \frac{\Theta}{\eta_c^4} \xi^3 \\ &= \gamma_1 + \gamma_2\xi + \gamma_3\xi^2 + \gamma_4\xi^3. \end{aligned}$$

Similarly,

$$\lambda_1 = Y_5 + Y_6\xi + Y_7\xi^2 + Y_8\xi^3 ,$$

$$\frac{1}{\lambda_1} = Y_{12} + Y_{13}\xi + Y_{14}\xi^2 + Y_{15}\xi^3 ,$$

$$A_1 = Y_{22} + Y_{23}\xi + Y_{24}\xi^2 + Y_{25}\xi^3 ,$$

$$\frac{1}{A_1} = Y_{26} + Y_{27}\xi + Y_{28}\xi^2 + Y_{29}\xi^3 ,$$

$$\frac{1}{a_1} = Y_{35} + Y_{36}\xi + Y_{37}\xi^2 + Y_{38}\xi^3 ,$$

where the coefficients Y_n are contained in Appendix A.

3. Singularities in $\frac{1}{\lambda_1}$

Solving for an expression that is valid near the singular point of the function $\frac{1}{\lambda_1}$, it is necessary to derive an alternate expression for $\frac{1}{\lambda_1}$ than the one contained in Section 2: one that expresses the singular nature of this function near η_c . Instead,

$$\begin{aligned} \frac{1}{\lambda_1} &= \frac{1}{\frac{mV_1}{\eta} - \chi} \\ &= \frac{1}{m\phi - \chi + \frac{m\Theta}{\eta^2}} \\ &= \frac{\eta^2}{(m\phi - \chi) \left(\eta^2 - \frac{m\Theta}{\chi - m\phi} \right)} \end{aligned}$$

$$\begin{aligned}
&= \frac{(\eta_c + \xi)^2}{(m\phi - X)\xi(\xi + 2\eta_c)} \\
&= \frac{Y_{39}}{\xi} + Y_{40} + Y_{41}\xi + Y_{42}\xi^2.
\end{aligned}$$

Substitution into the basic equation, Eq. (2.18), will yield the functions $S(\xi)$ and $T(\xi)$ from which a series solution may be generated.

The special case of $\phi = 0.0$ must be handled separately as the function A_1 reduces to $-\frac{1}{\lambda_1^2}$ in this instance. For $\phi = 0.0$, Eq. (2.18) becomes

$$\begin{aligned}
&\frac{d^2\tilde{\rho}}{d\eta^2} + \left\{ \frac{1}{\eta} + \frac{(2\gamma-3)\theta^2}{\eta^3 a_1} + \frac{4m\theta}{\eta^3 \lambda_1} \right\} \frac{d\tilde{\rho}}{d\eta} \\
&+ \left\{ \frac{\lambda_1^2}{a_1} - \kappa^2 - \frac{m^2}{\eta^2} + \frac{2(2-\gamma)\theta^2}{\eta^4 a_1} - \frac{4m\theta}{\eta^4 \lambda_1} + \frac{2(2\gamma-3)m\theta^3}{\eta^6 a_1 \lambda_1} + \frac{8m^2\theta^2}{\eta^6 \lambda_1^2} \right\} \tilde{\rho} = 0,
\end{aligned}$$

and substitution yields

$$\begin{aligned}
S_0 &= -\left(Y_{43} \right), \\
S_1 &= -\left(Y_{44} + \frac{1 - 3Y_{43}}{\eta_c} \right), \\
S_2 &= -\left(Y_{45} - \frac{3Y_{44}}{\eta_c} - \frac{1 - 6Y_{43}}{\eta_c^2} \right), \\
S_3 &= -\left(Y_{46} - \frac{3Y_{45}}{\eta_c} + \frac{6Y_{44}}{\eta_c^2} + \frac{1 - 10Y_{43}}{\eta_c^3} \right),
\end{aligned}$$

$$T_0 = - (Y_{63}) ,$$

$$T_1 = - (Y_{50} + Y_{56} + Y_{64}) ,$$

$$T_2 = - \left(\kappa^2 - \frac{m^2}{\eta_c^2} + Y_{51} + Y_{57} + Y_{65} \right) ,$$

$$T_3 = - \left(\frac{m^2}{\eta_c^2} + Y_{52} + Y_{58} + Y_{61} \right) ,$$

$$p_1 = 2 ,$$

$$p_2 = 1 .$$

A solution, ρ_1 , using $p_1 = 2$ may be generated in the form previously described. As the roots of the indicial equation differ by an integer, however, ρ_2 must be formulated as

$$\rho_2 = c\rho_1(\xi) \int \frac{d\xi}{\rho_1^2(\xi)} \exp \left[\int^\xi \frac{S(\delta)}{\delta} d\delta \right] .$$

Evaluation of the integrand yields the general expression

$$\rho_2 = c\rho_1(\xi) \int \xi^{S_0-2p_1} \left(g_0 + g_1\xi + g_2\xi^2 + g_3\xi^3 \dots \right) d\xi ,$$

where

$$g_0 = 1 ,$$

$$g_1 = S_1 - 2a_1 ,$$

$$g_2 = \frac{S_1^2 + S_2}{2} - 2a_1 S_1 + 3a_1^2 - 2a_2 ,$$

$$g_3 = \frac{S_3}{3} + \frac{S_1 S_2}{2} + \frac{S_1^3}{6} - a_1 \left(S_1^2 + S_2 \right) + \left(3a_1^2 - 2a_2 \right) S_1 \\ + 6a_1 a_2 - 4a_1^3 - 2a_3 .$$

Substituting the specific values of S_0 and p_1 yields the final expression

$$\rho_2 = C \left[h_0 + h_1 \xi + h_2 \xi^2 + h_3 \xi^3 + \left(\xi^2 + a_1 \xi^3 \right) g_1 \ln \xi \right] \\ = C \left[h(\xi) + \left(\xi^2 + a_1 \xi^3 \right) g_1 \ln \xi \right] ,$$

where

$$h_0 = 0 ,$$

$$h_1 = -1 ,$$

$$h_2 = -a_1 ,$$

$$h_3 = -a_2 + g_2 .$$

As the integration process will be terminated for some $\eta < \eta_c$, or $\xi < 0$,

$$\tilde{\rho} = \begin{pmatrix} C_1 + iC_2 & \rho_1 \\ C_3 + iC_4 & \rho_2 - i\pi g_1 (\xi^2 + a_1 \xi^3) \end{pmatrix}$$

$$\frac{d\tilde{\rho}}{d\eta} = \begin{pmatrix} C_1 + iC_2 \\ C_3 + iC_4 \end{pmatrix} \frac{d\rho_1}{d\xi} + \begin{pmatrix} C_3 + iC_4 \\ \frac{d\rho_2}{d\xi} - i\pi g_1 (2\xi + 3a_1 \xi^2) \end{pmatrix}.$$

Using the known values of $\tilde{\rho}$ and $\frac{d\tilde{\rho}}{d\eta}$, the coefficients C_n can be determined using well-established matrix methods. Once these coefficients are known, the values of $\tilde{\rho}$ and $\frac{d\tilde{\rho}}{d\eta}$ may be determined at a new point, $\xi > 0$, sufficiently past the singular point for the integration to be continued. The implementation of this procedure is discussed in greater detail in Chapter V.

For the case of $\Phi \neq 0$,

$$S_0 = 0,$$

$$S_1 = - \left(\frac{1 + Y_{67} + Y_{73}}{\eta_c} \right),$$

$$S_2 = - \left(\frac{Y_{68} + Y_{74}}{\eta_c} \right) + \left(\frac{1 + Y_{67} + Y_{73}}{\eta_c^2} \right),$$

$$S_3 = - \left(\frac{Y_{69} + Y_{75}}{\eta_c} \right) + \left(\frac{Y_{68} + Y_{74}}{\eta_c^2} \right) - \left(\frac{1 + Y_{67} + Y_{73}}{\eta_c^3} \right),$$

$$T_0 = - \left(Y_{80} \right),$$

$$T_1 = - \left(Y_{81} + Y_{90} + Y_{96} + Y_{111} \right),$$

$$T_2 = - \left(Y_{82} + \frac{m^2}{\eta_c^2} + Y_{91} + Y_{93} + Y_{97} + Y_{112} \right),$$

$$T_3 = - \left(Y_{83} + \frac{m^2}{\eta_c^3} + Y_{92} + Y_{95} + Y_{98} + Y_{113} \right) ,$$

$$p_1 = 1 ,$$

$$p_2 = 0 ,$$

and

$$\rho_2 = C \left[h(\xi) + \left(\xi + a_1 \xi^2 + a_2 \xi^3 \right) g_1 \ln \xi \right] ,$$

where

$$h_0 = -1 ,$$

$$h_1 = -a_1 ,$$

$$h_2 = g_2 - a_2 ,$$

$$h_3 = \frac{g_3}{2} + g_2 - a_3 .$$

4. Singularities in A_1

For a solution that is valid near a singular point in the function A_1 , a new expression must be determined for A_1 . From

$$\eta_c^2 = - \frac{B \pm \sqrt{B^2 - 4C}}{2} .$$

Note that for η_c to fall within the range $1.0 < \eta_c < \mu$, η_c^2 must be real and have at least one positive value. Denoting the two possible values of η_c^2 as η_a^2 and η_b^2 , let η_a^2 be positive and η_c its positive root. Then

$$\begin{aligned}
A_1 &= \frac{\eta^4}{[4\phi^2 - (m\phi - X)^2][\eta^4 + B\eta + C]} \\
&= \frac{\eta^4}{[4\phi^2 - (m\phi - X)^2](\eta - \eta_c)(\eta + \eta_c)(\eta^2 - \eta_b^2)} \\
&= \frac{1}{Y_{114}} \left(\frac{\eta_c^4}{\xi} + 4\eta_c^3 + 6\eta_c^2\xi + 4\eta_c^2\xi^2 + \xi^3 \right) \frac{1}{(\xi + 2\eta_c)(\xi^2 + 2\xi\eta_c + \eta_c^2 - \eta_b^2)} \\
&= \frac{1}{Y_{114}} \left(\frac{\eta_c^4}{\xi} + 4\eta_c^3 + 6\eta_c^2\xi + 4\eta_c^2\xi^2 + \xi^3 \right) \frac{1}{Y_{115} \left(1 + \frac{P(\xi)}{Y_{115}} \right)} \\
&= \frac{1}{Y_{114}} \left(\frac{\eta_c^4}{\xi} + 4\eta_c^3 + 6\eta_c^2\xi + 4\eta_c^2\xi^2 + \xi^3 \right) \\
&\quad \frac{1}{Y_{115} \left(1 - \frac{P(\xi)}{Y_{115}} + \frac{P^2(\xi)}{Y_{115}^2} - \frac{P^3(\xi)}{Y_{115}^3} + \dots \right)} \\
&= \frac{Y_{119}}{\xi} + Y_{120} + Y_{121}\xi + Y_{122}\xi^2.
\end{aligned}$$

Then

$$S_0 = -\frac{1}{\eta_c} (Y_{127}),$$

$$S_1 = -\frac{1}{\eta_c} \left(1 + Y_{67} + Y_{128} - \frac{Y_{127}}{\eta_c} \right),$$

$$S_2 = -\frac{1}{\eta_c} \left(Y_{68} + Y_{129} - \frac{(1 + Y_{67} + Y_{128})}{\eta_c} + \frac{Y_{127}}{\eta_c^2} \right),$$

$$S_3 = -\frac{1}{\eta_c} \left(Y_{69} + Y_{130} - \frac{(Y_{68} + Y_{129})}{\eta_c} + \frac{(1 + Y_{67} + Y_{128})}{\eta_c^2} - \frac{Y_{127}}{\eta_c^3} \right),$$

$$T_0 = 0 ,$$

$$T_1 = -(Y_{154}) ,$$

$$T_2 = - \left(Y_{131} - \frac{m^2}{\eta_c} + Y_{135} + Y_{93} + Y_{137} + Y_{155} \right) ,$$

$$T_3 = - \left(Y_{132} + \frac{m^2}{\eta_c} + Y_{136} + Y_{95} + Y_{138} + Y_{156} \right) ,$$

$$p_1 = 2 ,$$

$$p_2 = 0 ,$$

and

$$\rho_2 = c \left[h(\xi) + \left(\xi^2 + a_1 \xi^3 \right) g_2 \ln \xi \right] ,$$

where

$$h_0 = - \frac{1}{2} ,$$

$$h_1 = - g_1 - \frac{a_1}{2} ,$$

$$h_2 = - a_1 g_1 - \frac{a_2}{2} ,$$

$$h_3 = g_3 - a_2 g_1 - \frac{a_3}{2} .$$

CHAPTER V

NUMERICAL SOLUTION

The second-order differential equation, Eq. (2.18), can be reduced to a system of simultaneous first-order equations which can be solved by an application of the fourth-order Runge-Kutta methods, widely used in numerical solutions to differential equations. The Runge-Kutta method is chosen despite its requirement for four function evaluations per step, as the steps can be taken much larger than other methods for the same accuracy. In addition, the variable step size of the Runge-Kutta method lends itself well to the problems associated with integrating near a singular point.

Suppose $\bar{Y}_1(\eta)$ is the solution of Eq. (2.18) and $\bar{Y}_2(\eta)$ is its derivative

$$\bar{Y}_1(\eta) = \tilde{\rho}$$

$$\bar{Y}_2(\eta) = \frac{\partial \tilde{\rho}}{\partial \eta}.$$

Then Eq. (2.18) is transformed into the system of first-order initial value problems

$$\bar{Y}_1'(\eta) = \bar{Y}_2(\eta) \tag{5.1a}$$

$$\begin{aligned}
\bar{Y}'_2(\eta) = & -\frac{1}{\eta} \left\{ 1 + (2\gamma-3) \frac{V_1^2}{a_1} - \frac{4A_1\Theta}{\eta^2} (m\lambda_1 - 2\phi) \right\} \bar{Y}_2(\eta) \\
& - \left\{ \frac{1}{A_1} \left(\frac{\kappa^2}{\lambda_1^2} - \frac{1}{a_1} \right) - \frac{m^2}{\eta^2} + \frac{2(2-\gamma)V_1}{\eta^2 a_1} (V_1 - 2\phi\eta) + \frac{2mV_1^3}{\eta^3 \lambda_1 a_1} \right. \\
& \left. - \frac{4m\Theta}{\eta^4 \lambda_1} + \frac{4A_1 V_1 \Theta}{\eta^4} \left[(2-\gamma) \frac{V_1}{a_1} - \frac{2m}{\eta \lambda_1} \right] (m\lambda_1 - 2\phi) \right\} \bar{Y}_1(\eta) \quad (5.1b)
\end{aligned}$$

$1 < \eta < \mu$

with the initial conditions

$$\bar{Y}_1(1) = 1.0 \quad (5.2a)$$

$$\bar{Y}_2(1) = -\frac{2mV_1}{\lambda_1} + (2-\gamma)V_1^2. \quad (5.2b)$$

If there are no singular points within the range of integration, the solution is a straightforward application of the Runge-Kutta method followed by an evaluation of the boundary conditions (B.C.) (Eq. (2.18b)) at $\eta = \mu$ using the computed values of

$$\tilde{\rho} = \bar{Y}_1(\mu)$$

$$\frac{d\tilde{\rho}}{d\eta} = \bar{Y}_2(\mu).$$

If there are singular points present, the range of integration is divided into segments, as shown in Figure 4, such that there are two segments between each singular point. The initial band width from ① to ② was chosen after some trial and error to be equal to 5×10^{-4}

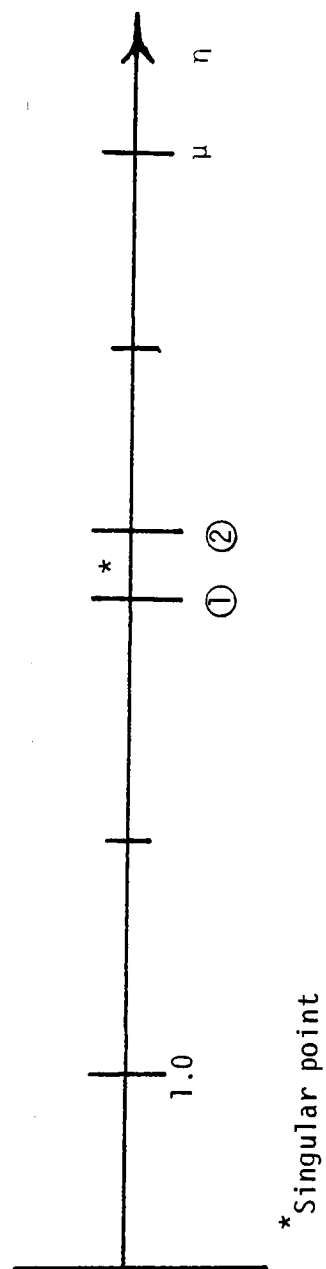
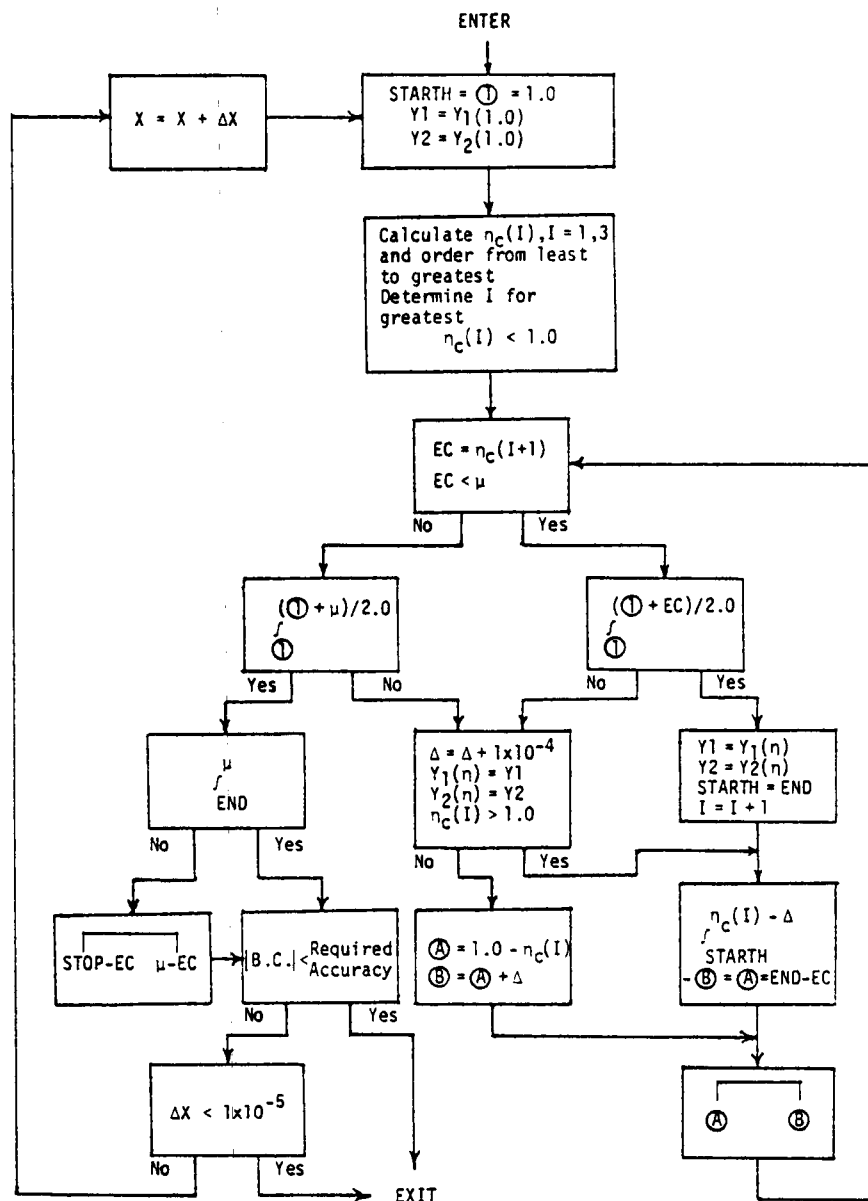


Figure 4. Division of integration range.

for singular points in $\frac{1}{\lambda_1}$ and 1×10^{-4} for singular points in A_1 . Then to transition across a singular point, as in from ① to ②, the values of $\bar{Y}_1(\eta)$ and $\bar{Y}_2(\eta)$ at ① and the approximations developed in Chapter IV are used to generate new values of $\bar{Y}_1(\eta)$ and $\bar{Y}_2(\eta)$ at ②.

The Runge-Kutta subroutine used contained a control to discontinue the integration if the stepsize was decreased, below a certain input value, to maintain the desired accuracy. In the region close to a singular point this control is exercised and the integration process either stops or cannot be started. Thus, if the integration process is discontinued for a segment followed by a singular point, it can be assumed that the bandwidth around the singular point is insufficiently large. It is, however, only necessary to set ξ equal to the value of $\eta - \eta_c$ based upon the termination point of the integration to perform a valid transition across the singularity. If the integration process cannot be initiated on a segment immediately following a singular point, it can also be assumed that the band around that singularity is insufficiently large. The solution in this case is to integrate again over the preceding segment and increase the bandwidth by an arbitrary amount that was chosen equal to 1×10^{-4} . This basic procedure is easily modified to handle the influence of singular points positioned just outside the range of integration. A flowchart of the integration process is presented as Figure 5.

Initially, it was planned to use the small swirl approximation to generate an initial value for X , modify X by a small amount, 1×10^{-2} , then use the secant method to iterate to the desired eigenvalue based upon the value of the boundary condition for X and $X - 1 \times 10^{-2}$. Some initial computations demonstrated the



Explanation of Symbols:

- ① ② Employ the appropriate approximation for the type of singularity involved using the known values of $Y_1(n)$ and $Y_2(n)$ at $n = \textcircled{A}$ to generate the new values of $Y_1(n)$ and $Y_2(n)$ at $n = \textcircled{B}$.
- ② Runge-Kutta integration from ① to ②. The value of the variable of integration at termination is denoted as END. The negative branch for the integration is for the case of $\text{END} \neq \textcircled{2}$ and the positive branch corresponds to $\text{END} = \textcircled{2}$.

Figure 5. Flowchart of the integration process.

approximation to be insufficiently accurate for large values of ϕ and μ to generate a consistent solution. Therefore, a modified approach was adapted.

Let $X(\theta, \phi)$ equal the computed eigenvalue for a grid point as depicted in Figure 1, page 26, and $X_{ss}(\theta, \phi)$ the computed small swirl approximation of the eigenvalue for that point. The initial point $X(0,0)$ was computed based solely on $X_{ss}(0,0)$ and the method described in the previous paragraph. Then the initial estimate of an eigenvalue for grid points where $\theta = 0$ was for $\phi > 0$

$$X = X_{ss}(0, \phi) + [X(0, \phi - 0.2) - X_{ss}(0, \phi - 0.2)],$$

and for grid points where $\theta \neq 0$,

$$X = X(\theta - 0.2, \phi) + 0.2\alpha \left(\frac{\theta}{|\theta|} \right).$$

These initial estimates were reduced by 1×10^{-2} for the second integration and thereafter the secant method was employed to iterate to the final solution.

The order of solution for the upper portion of the grid is shown in Figure 6. The mirror image of the process was used to evaluate the lower portion of the grid.

For this investigation, κ is specified = 0.0 and $\gamma = 1.4$ (for air). The computed eigenvalues (Tables 2 and 3) are presented in Appendix B.

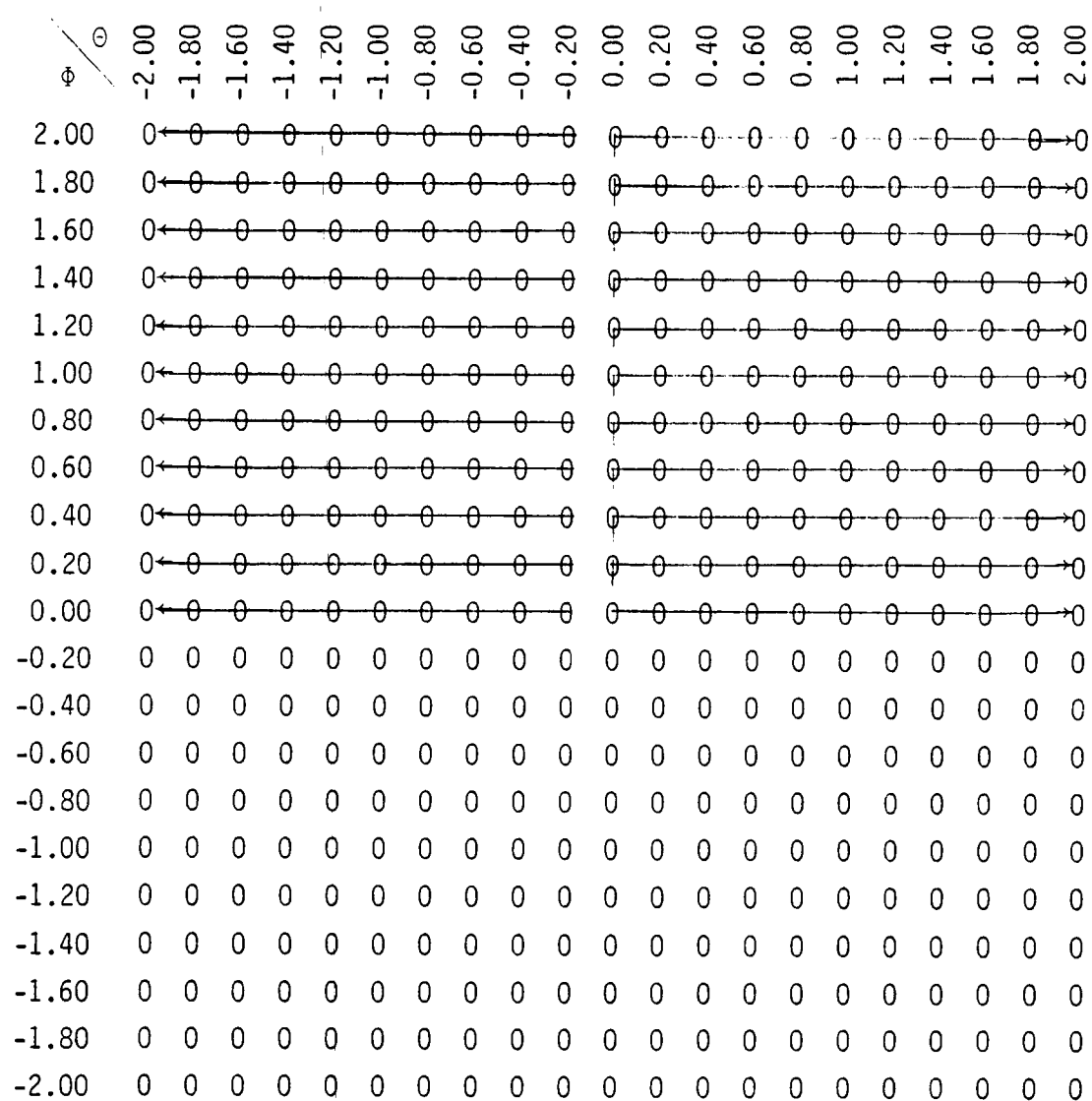


Figure 6. Order of solution.

CHAPTER VI

CONCLUSIONS

Previously, Chu [2] has conducted a similar investigation for a flow composed of only a free vortex, using an equation in terms of $r\tilde{v}$ rather than $\tilde{p}$. The acoustic reference velocity was chosen as $r = r_\infty$, but some data are available for a reference velocity at $r = r_0$. The current formulation, changed to reflect $a_* = a(r = r_0)$ was compared to Chu's results with excellent agreement over the full range of μ and Θ . As the approximations developed in Chapter IV were used for a significant portion of the comparison, they can be assumed to provide valid results over the expanded range contained in the present investigation.

As $k = 0$ for this investigation, the frequency of the acoustic waves, ω , can be assumed to be proportional to the computed eigenvalue. From the conventions established in Chapter II, the wave of the unsteady disturbance moves in the positive θ direction for $\omega > 0$. Then for Θ and $\Phi > 0$ the unsteady disturbance and the swirling flow are moving in the same direction, and for Θ and $\Phi < 0$ they are moving in the opposite direction. The reverse is, of course, true when $\omega < 0$. When Θ and Φ are not of the same sign the possibility exists for a mixed flow in which a portion of the flow moves with the unsteady disturbance. The existence of a mixed swirling flow has a definite effect on the resultant frequency of the unsteady disturbance, as will be seen.

A comparison of the computed eigenvalues, shown as symbols, and the Small Swirl (SS) Approximation, shown as solid lines, is shown in Figures 7 through 14. Although data for $m = 2$ are not shown, the curves show the same effects.

For small values of μ , the velocity is essentially constant from r_i to r_o . The SS approximation, which can be considered equivalent to an approximation for small changes in the swirl velocity, would be expected to yield good results in this instance. Figures 7, 8, 11, and 12 demonstrate that this agreement is quite satisfactory. For small values of ϕ , the SS approximation yields a good estimate of the frequency throughout the range of μ and θ . This can most probably be attributed to the nature of θ 's contribution to the swirl velocity.

In Figures 15 through 26, curves of constant θ and ϕ are plotted to show the effect of μ on the frequency. From Figure 16, it appears that the contribution due to ϕ is essentially unaffected by μ , and the frequency increases as ϕ increases. From Figure 19, it can be seen that, as μ increases, the frequency increases for $\theta < 0$, and decreases for $\theta > 0$.

For large values of μ , the effect of changes in θ , when $\phi = 0$, are minimal, as might be expected. As μ increases, the swirl velocity, except for r close to r_i , becomes very small and is not greatly affected by changes in θ . The resultant frequency would then be expected to approach asymptotically to some constant value.

The curves for $m = 1$ and $m = 2$ show the same behavior over the range of ϕ , θ , and μ , except when there is a mixed flow. The effect

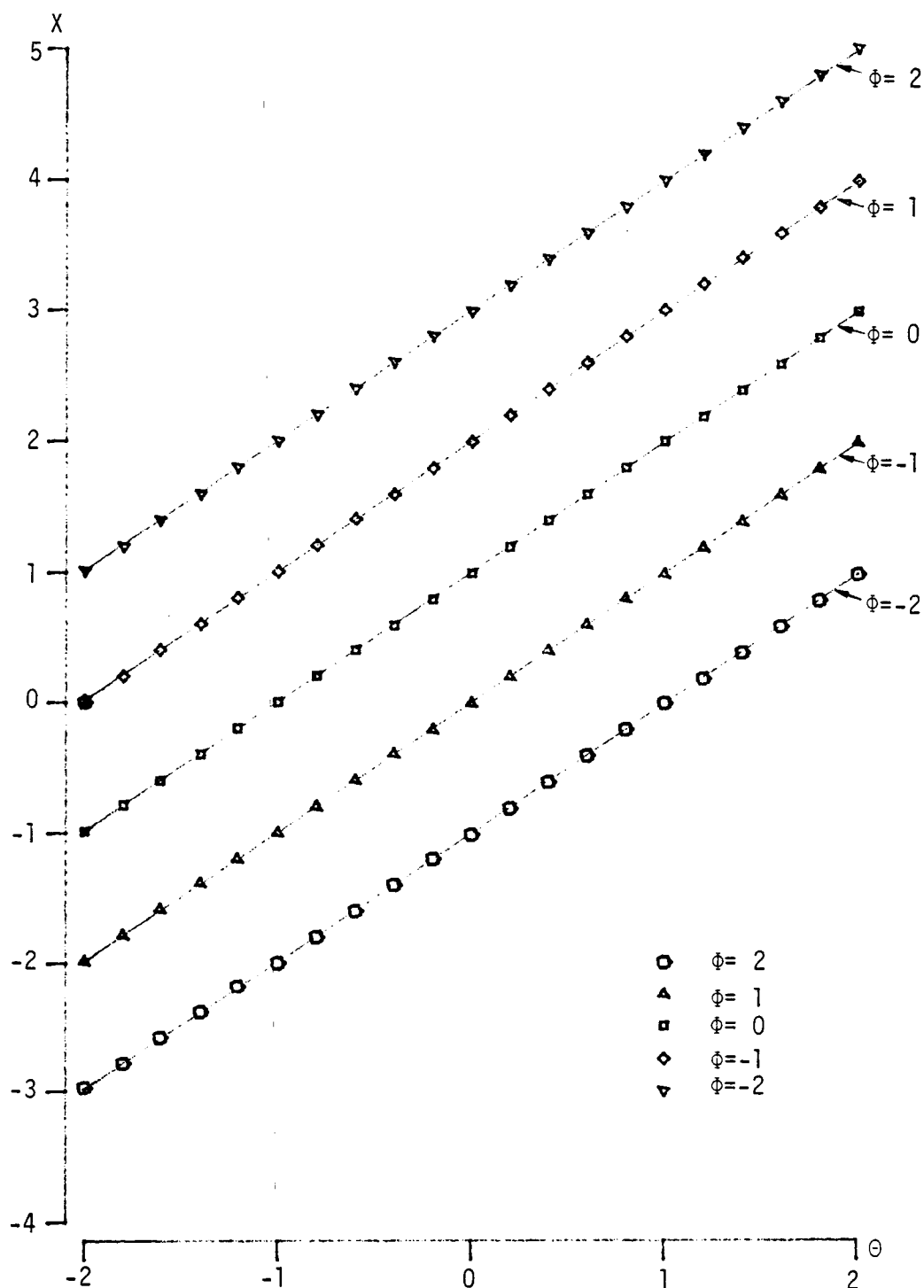


Figure 7. Comparison with Small Swirl Approximation for $\phi = \text{constant}$, $m = 1$, $\mu = 1.01$.

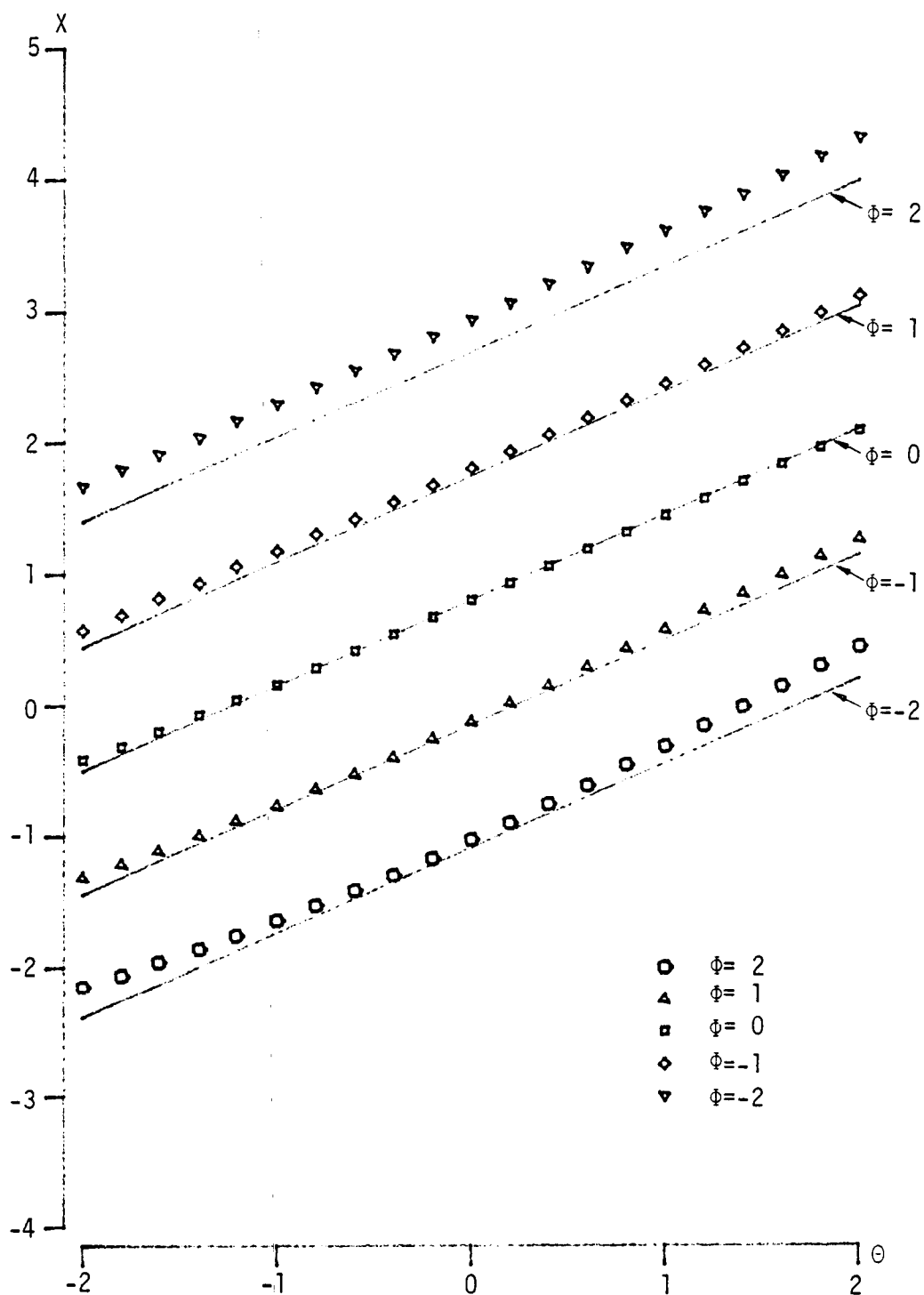


Figure 8. Comparison with Small Swirl Approximation for $\Phi = \text{constant}$, $m = 1$, $\mu = 1.50$.

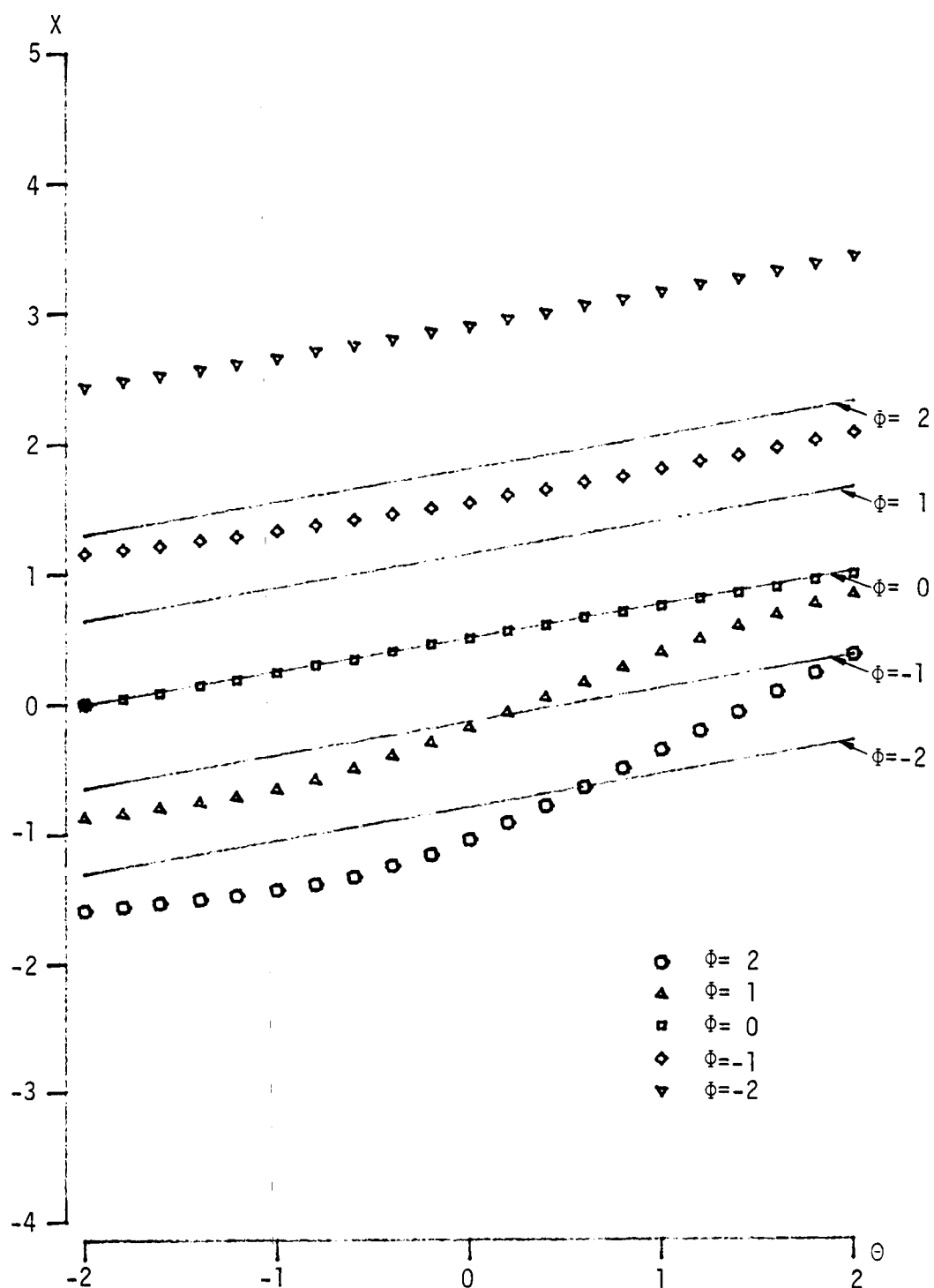


Figure 9. Comparison with Small Swirl Approximation for $\phi = \text{constant}$, $m = 1$, $\mu = 3.00$.

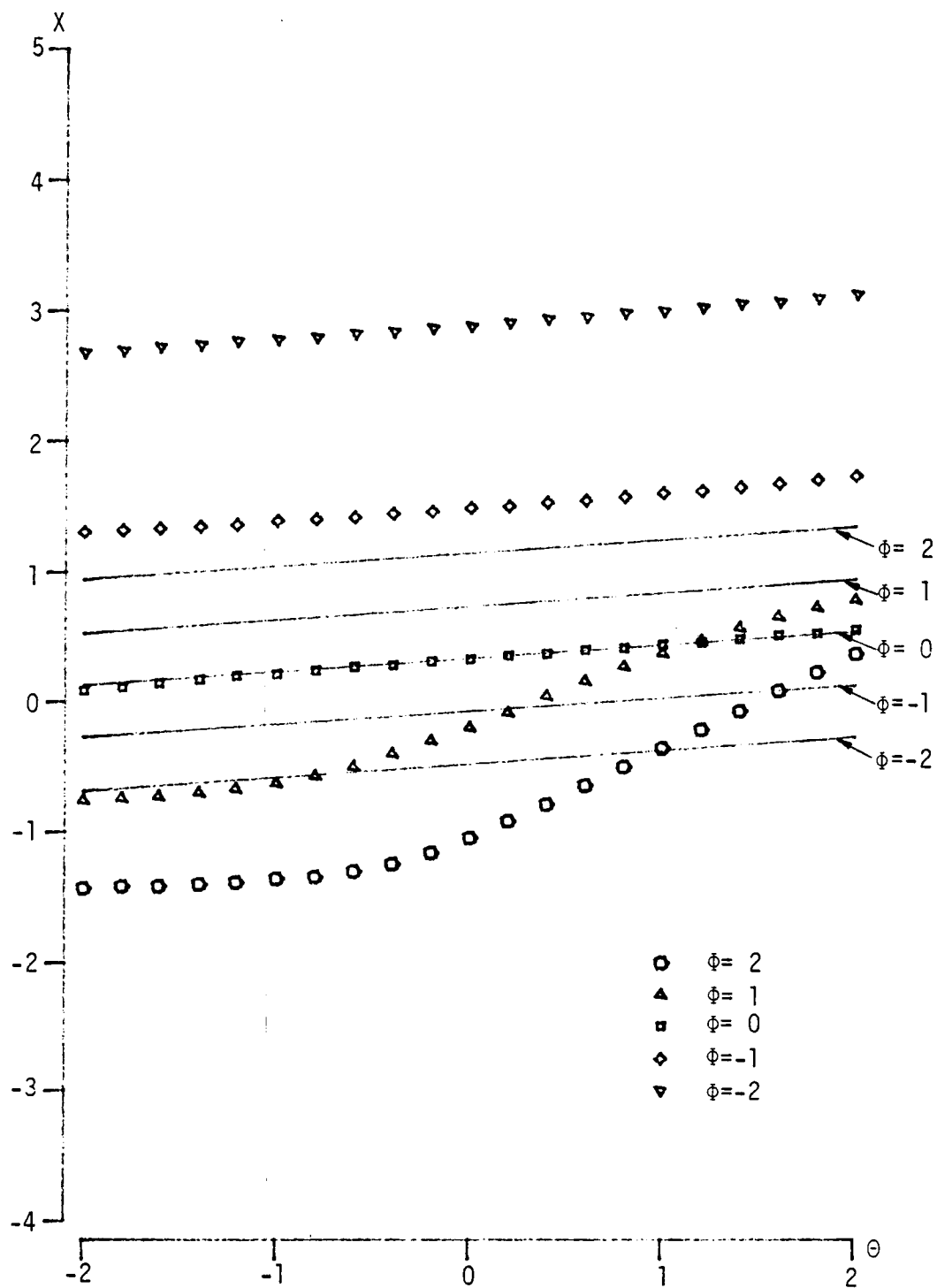


Figure 10. Comparison with Small Swirl Approximation for $\phi = \text{constant}$, $m = 1$, $\mu = 5.00$.

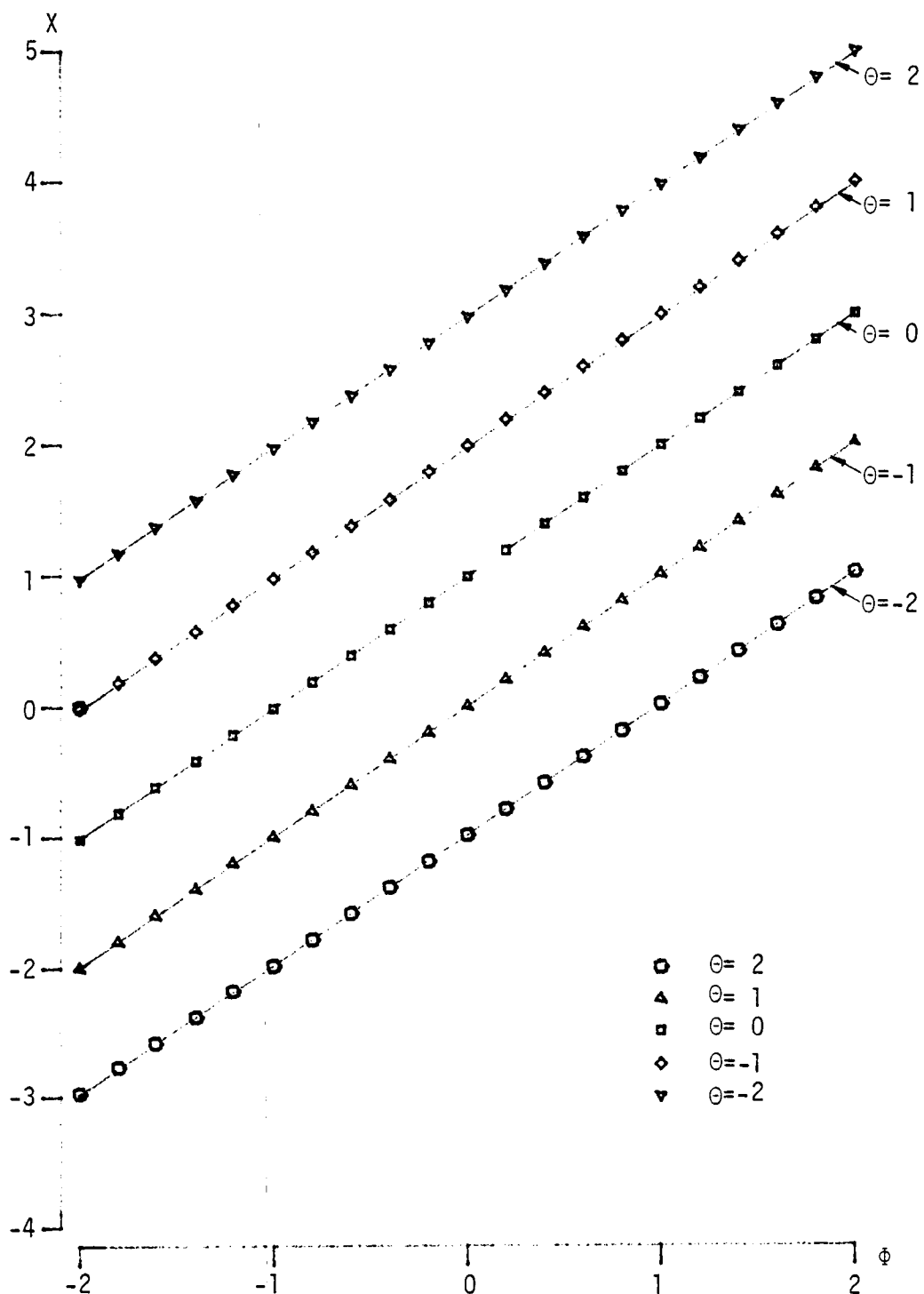


Figure 11. Comparison with Small Swirl Approximation for $\Theta = \text{constant}$, $m = 1$, $\mu = 1.01$.

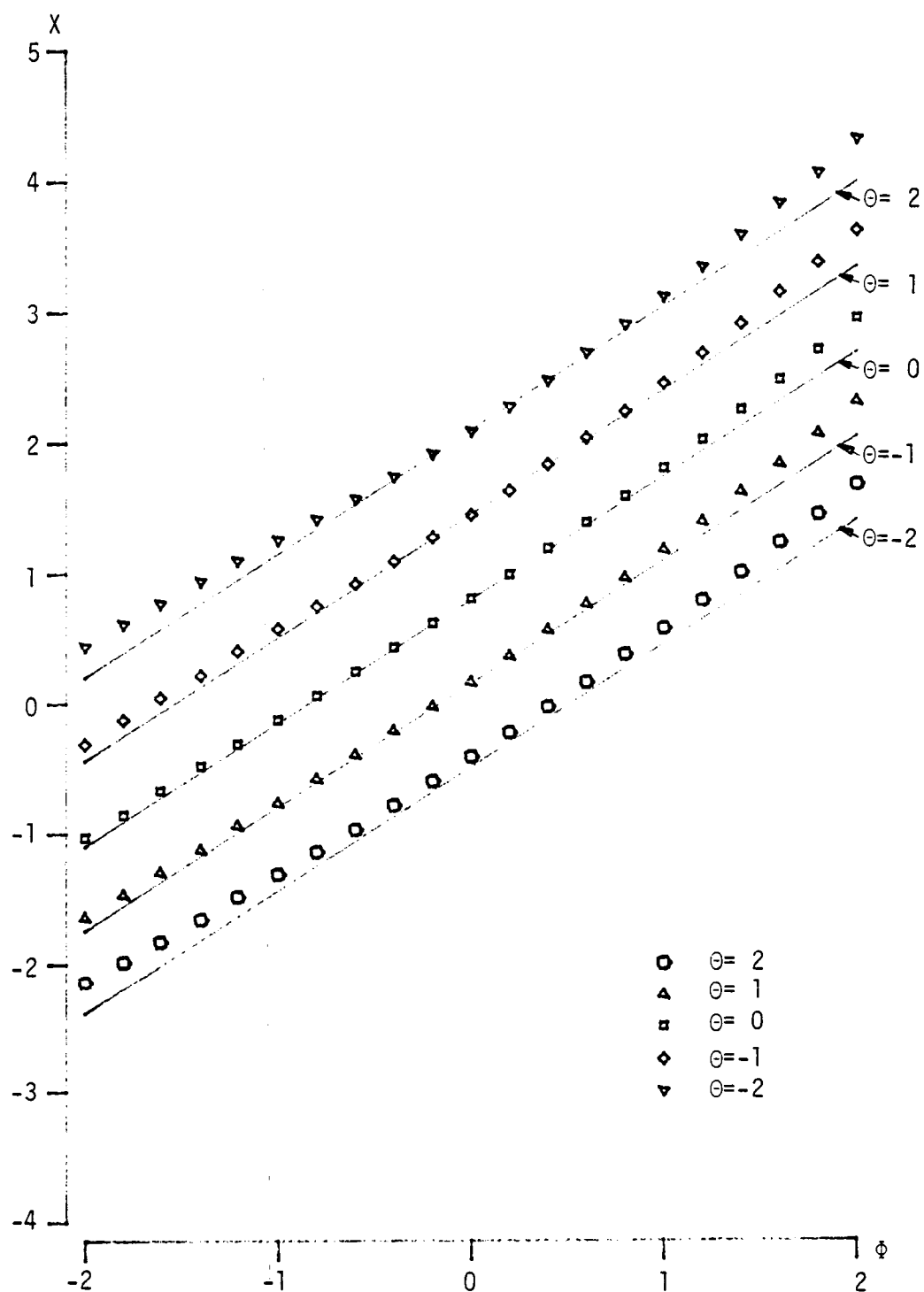


Figure 12. Comparison with Small Swirl Approximation for $\Theta = \text{constant}$, $m = 1$, $\mu = 1.50$.

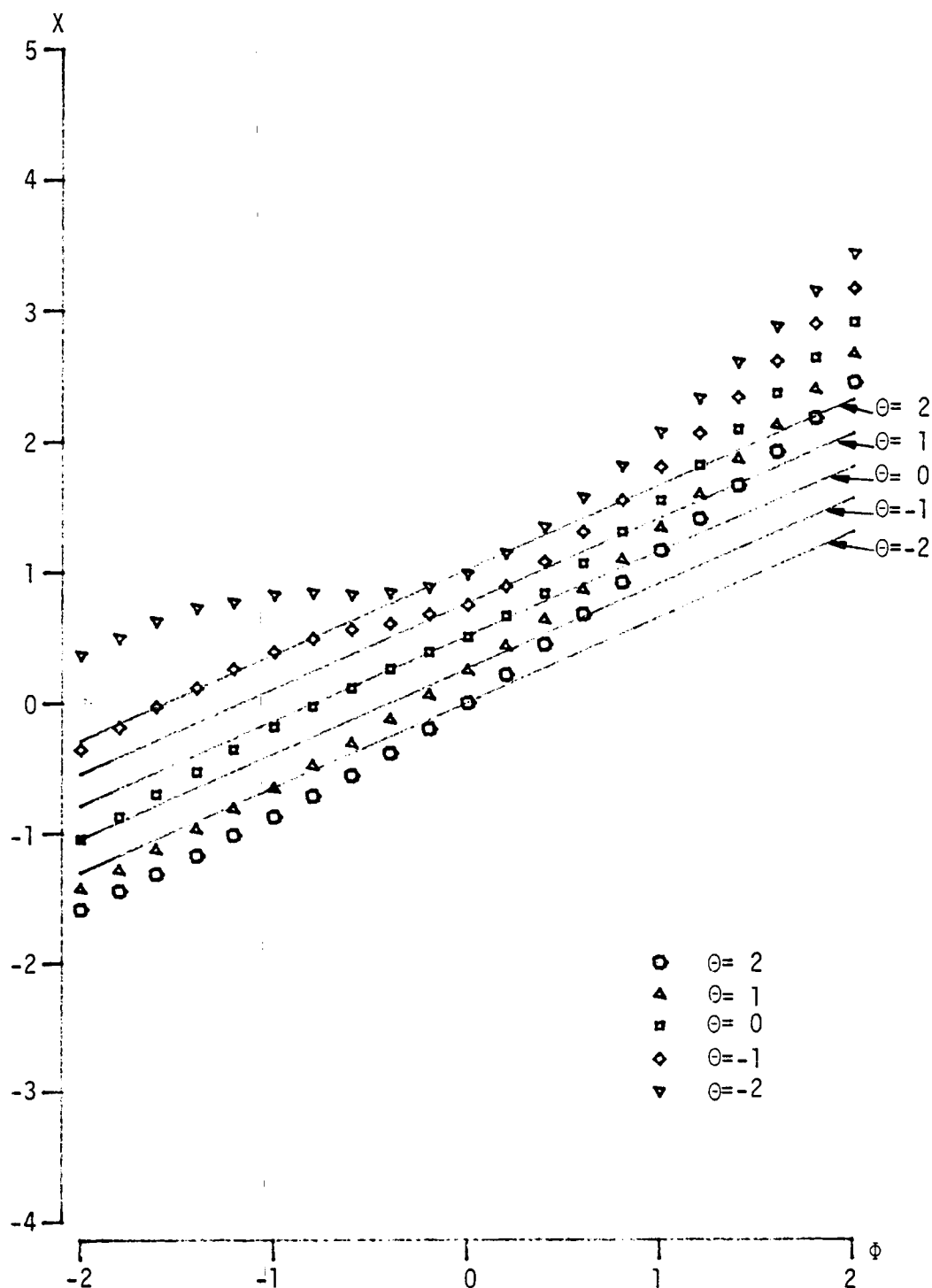


Figure 13. Comparison with Small Swirl Approximation for $\theta = \text{constant}$, $m = 1$, $\mu = 3.00$.

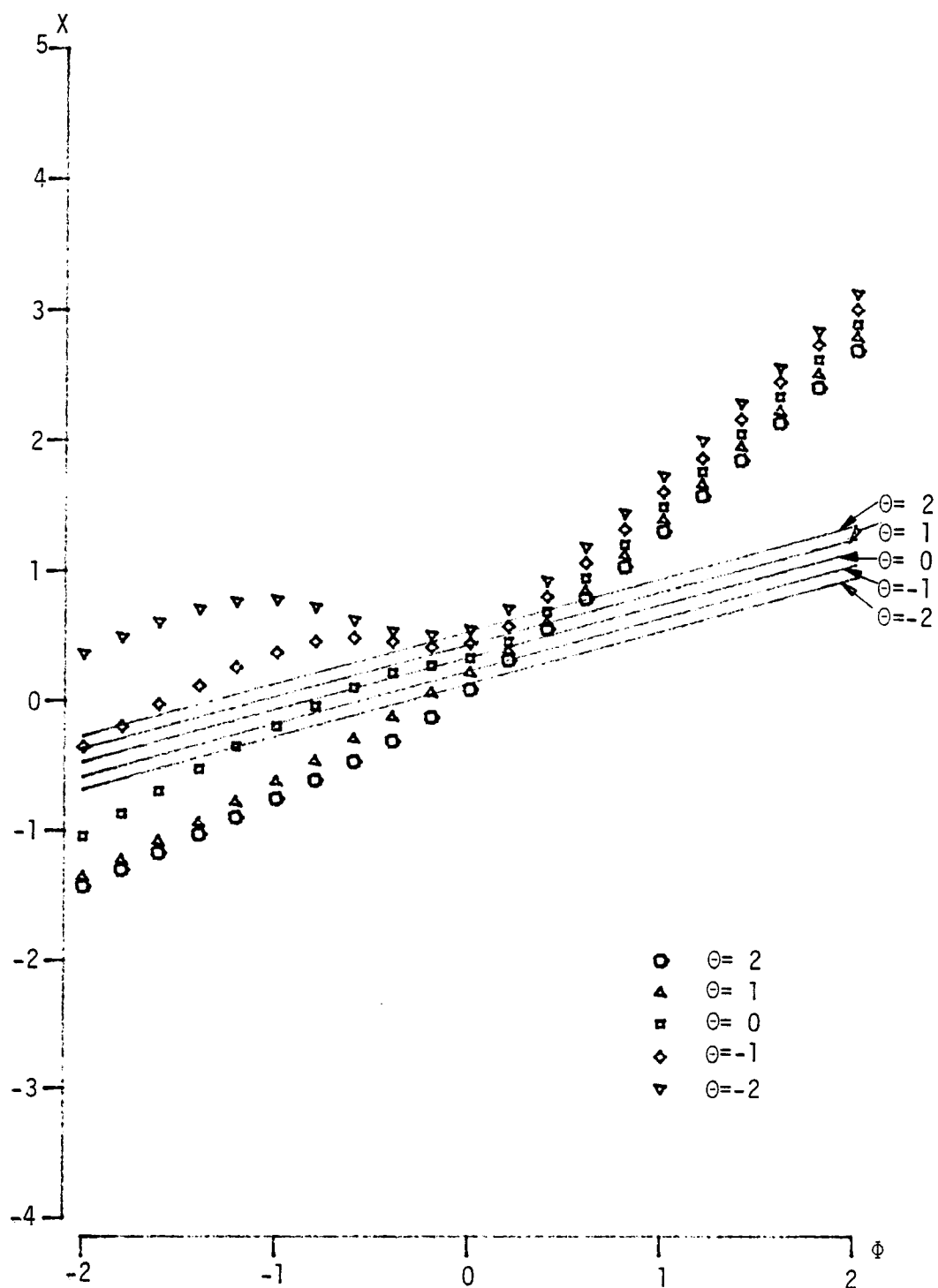


Figure 14. Comparison with Small Swirl Approximation for $\theta = \text{constant}$, $m = 1$, $\mu = 5.00$.

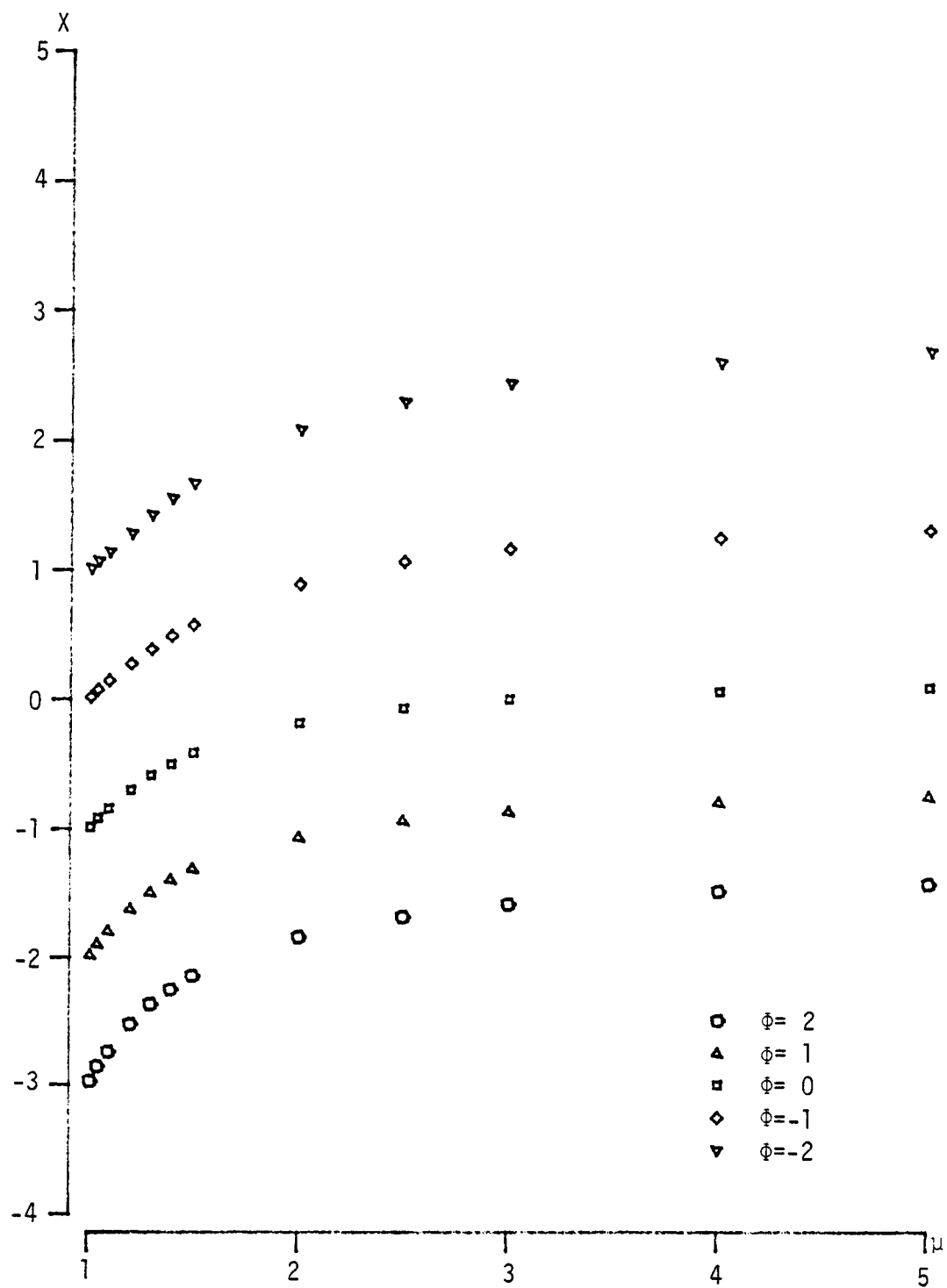


Figure 15. Effect of μ for $\phi = \text{constant}$, $\theta = -2$, $m = 1$.

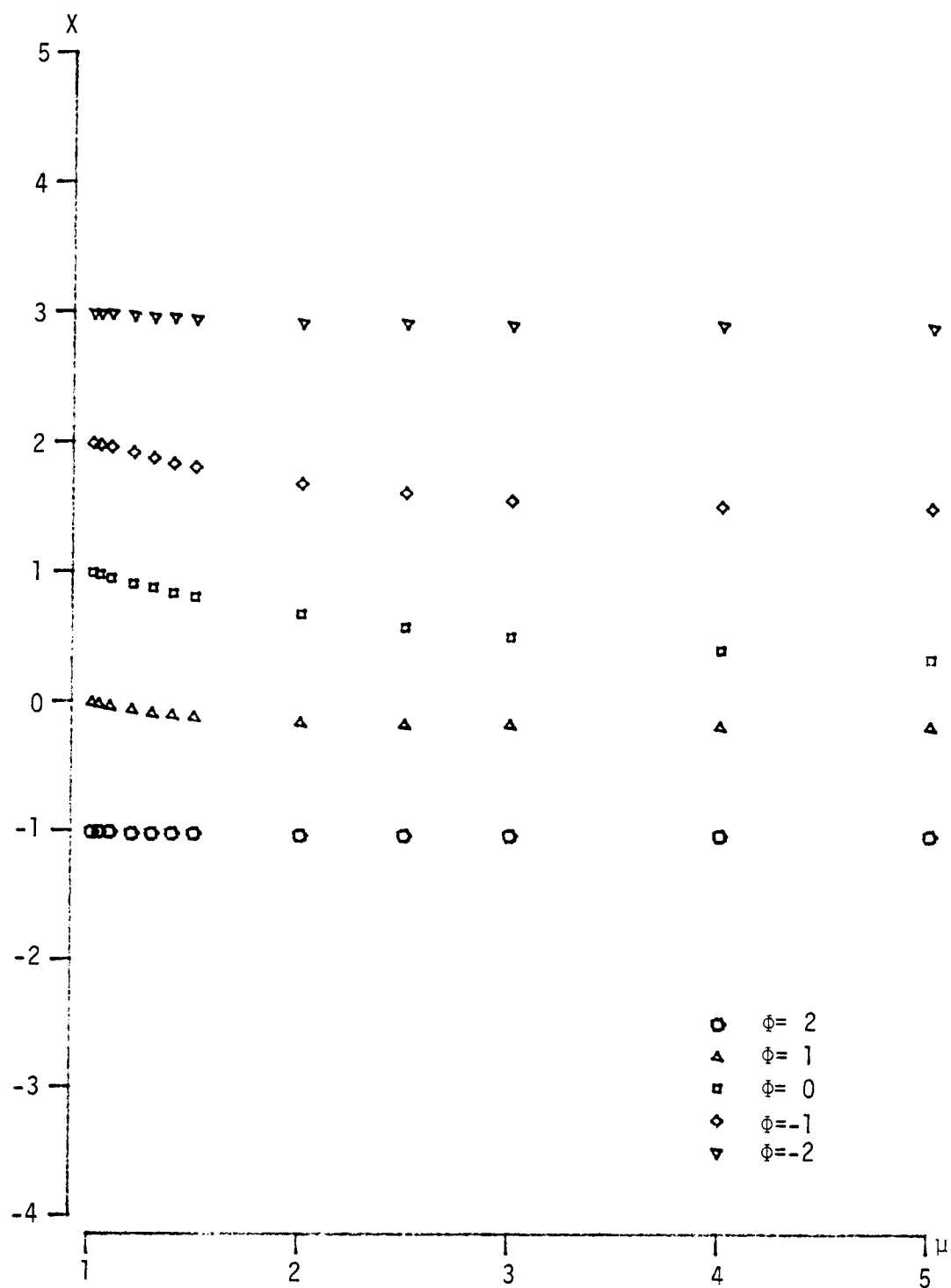


Figure 16. Effect of μ for $\Phi = \text{constant}$, $\theta = 0$, $m = 1$.

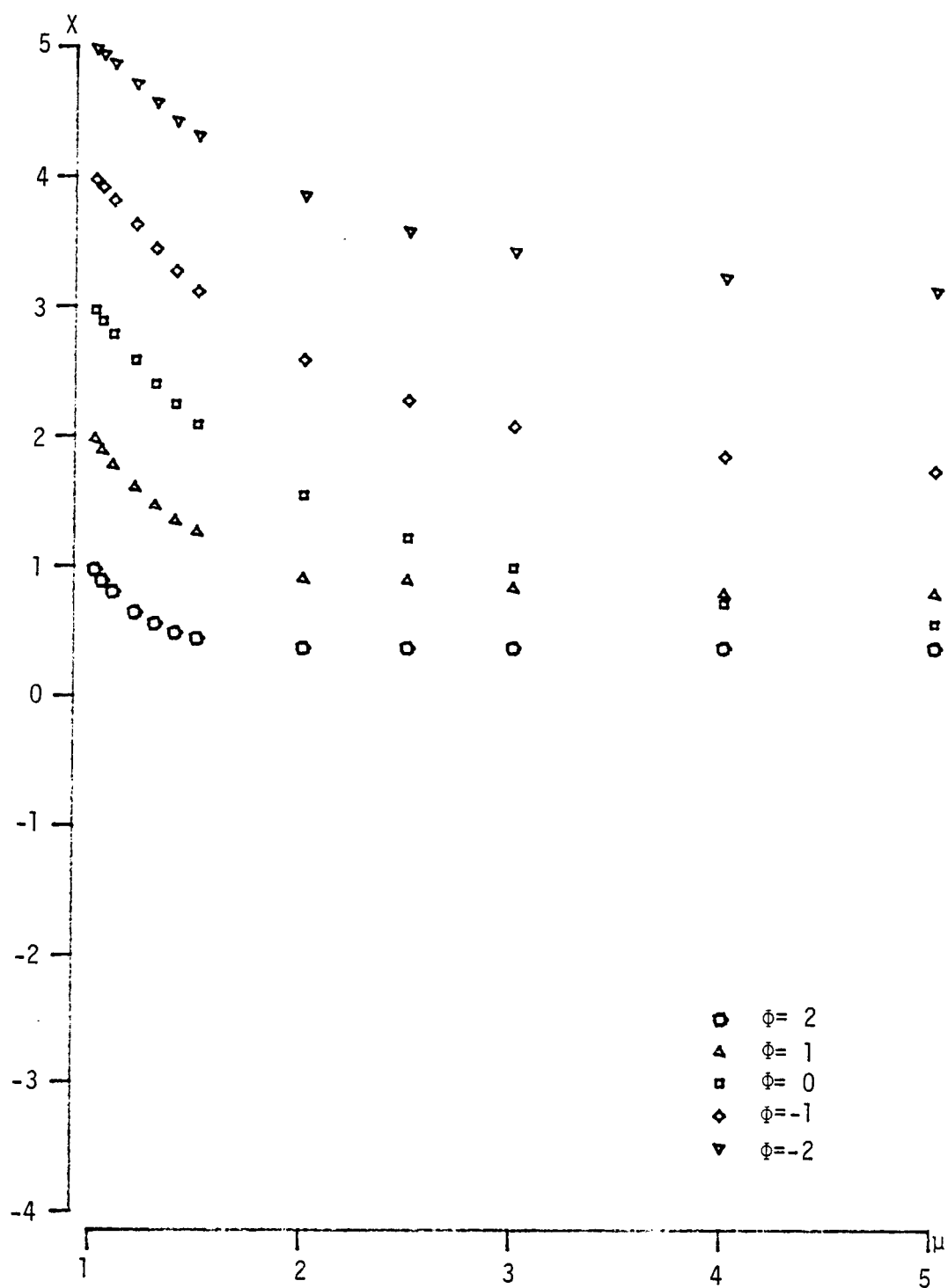


Figure 17. Effect of μ for $\Phi = \text{constant}$, $\Theta = 2$, $m = 1$.

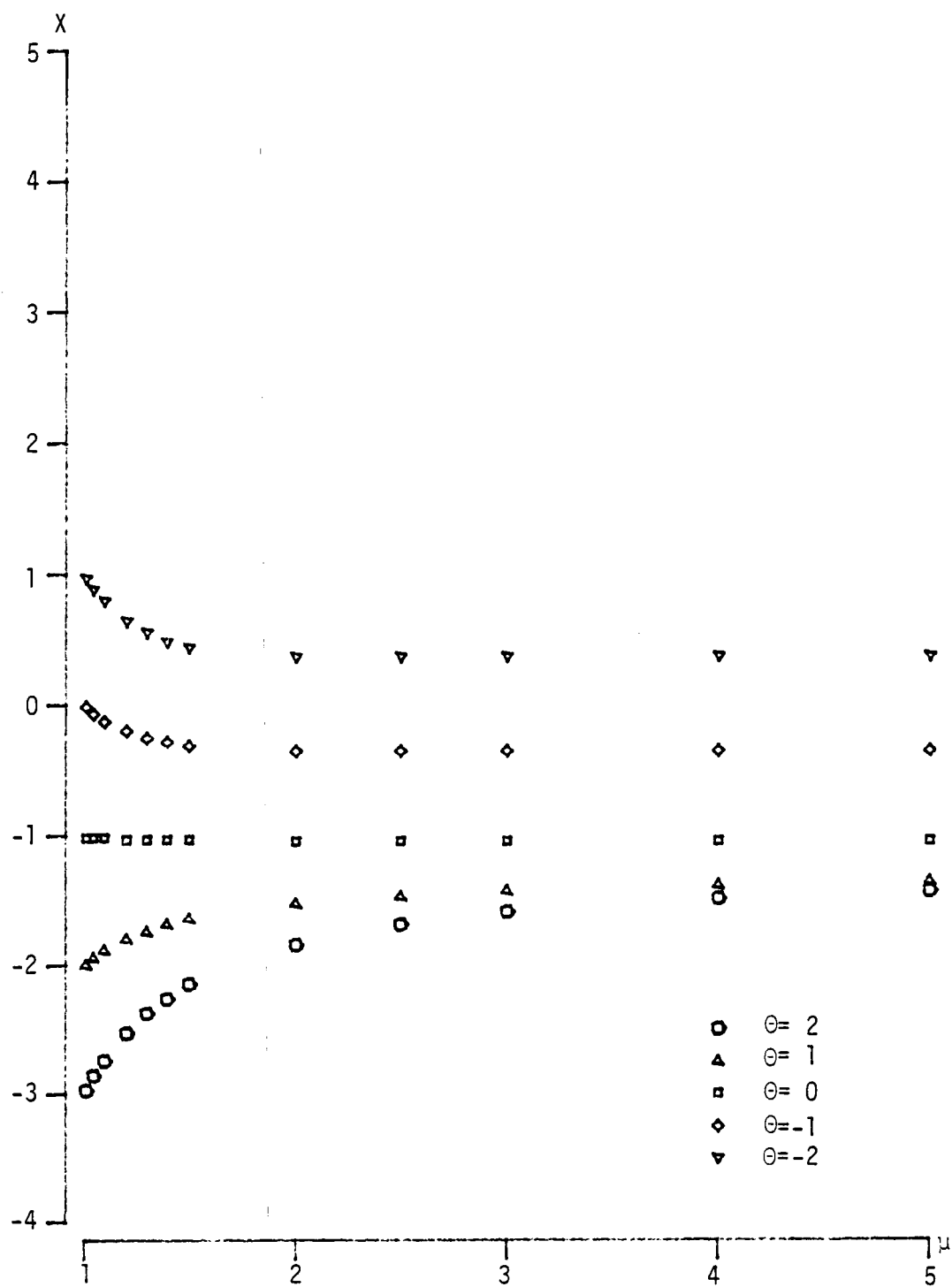


Figure 18. Effect of μ for $\theta = \text{constant}$, $\phi = -2$, $m = 1$.

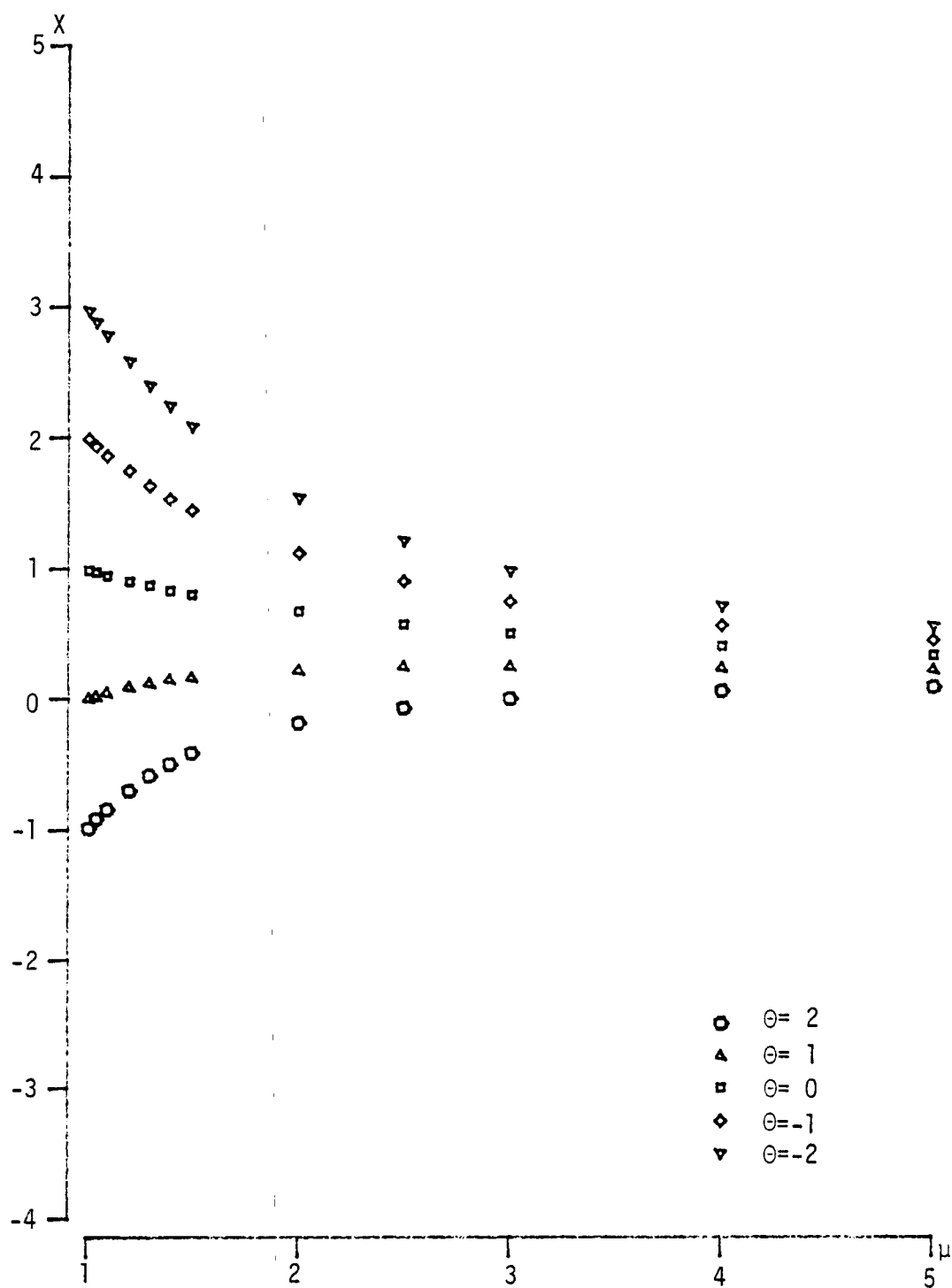


Figure 19. Effect of μ for $\Theta = \text{constant}$, $\phi = 0$, $m = 1$.

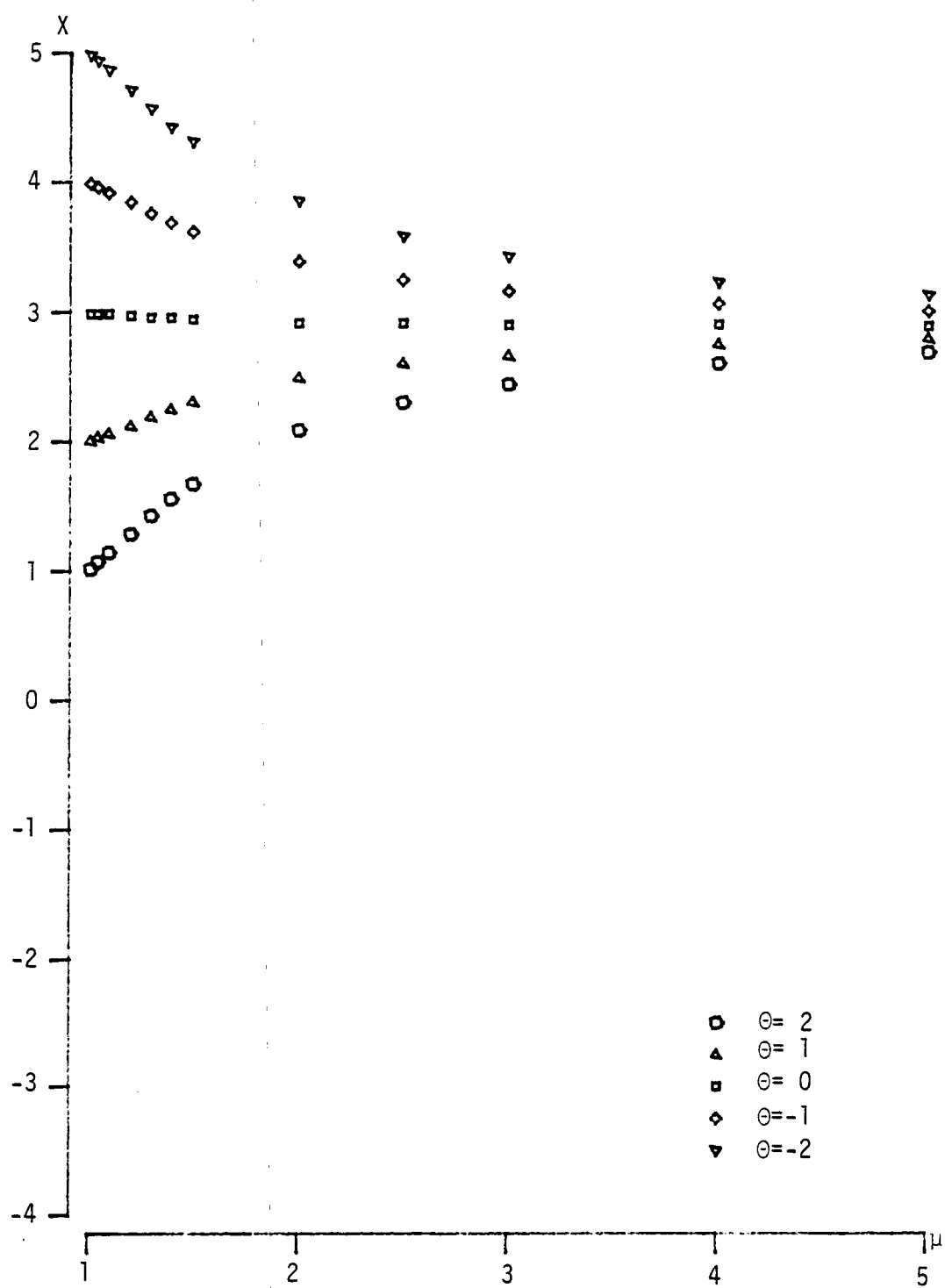


Figure 20. Effect of μ for $\theta = \text{constant}$, $\phi = 2$, $m = 1$.

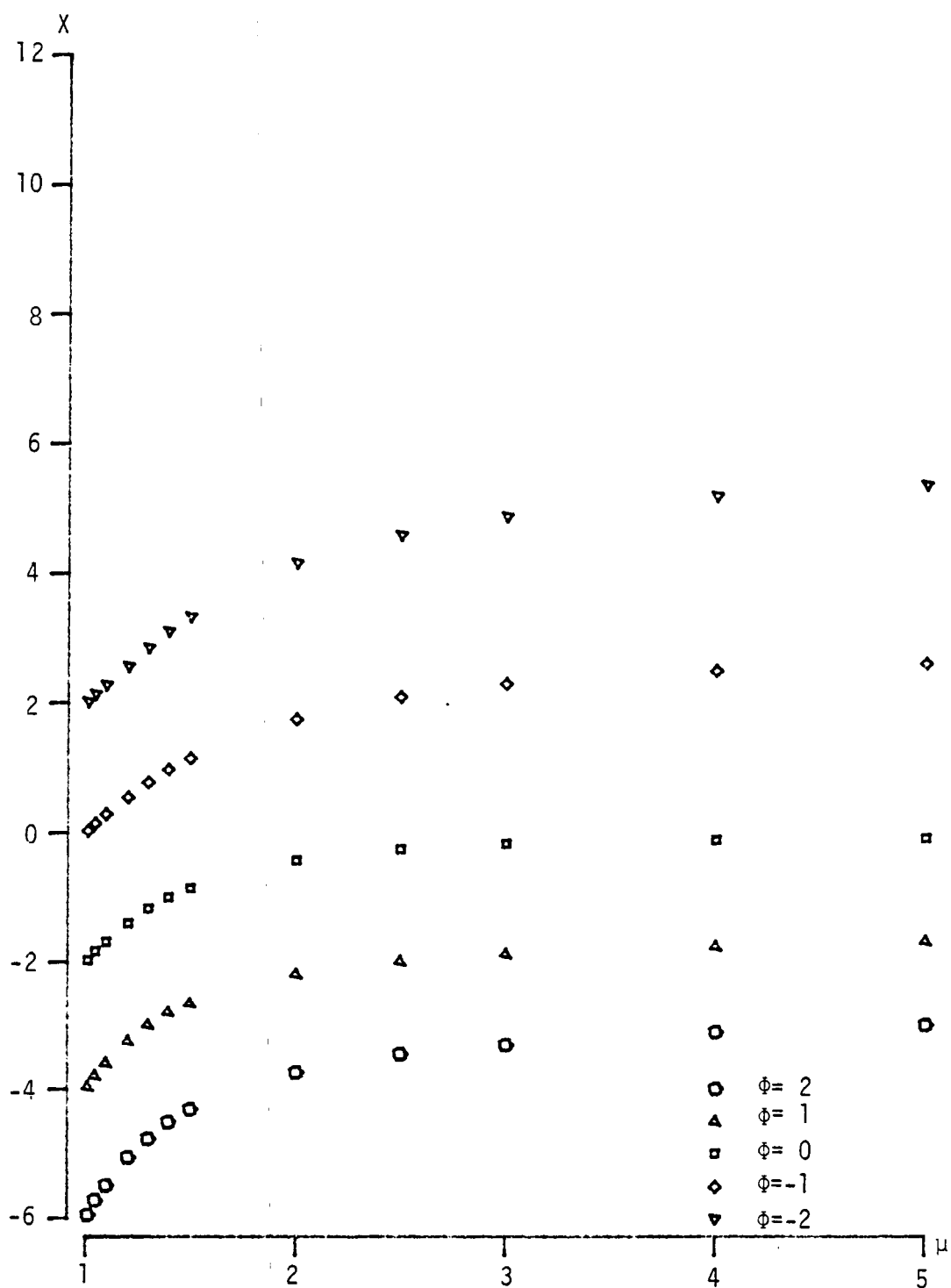


Figure 21. Effect of μ for $\phi = \text{constant}$, $\Theta = -2$, $m = 2$.

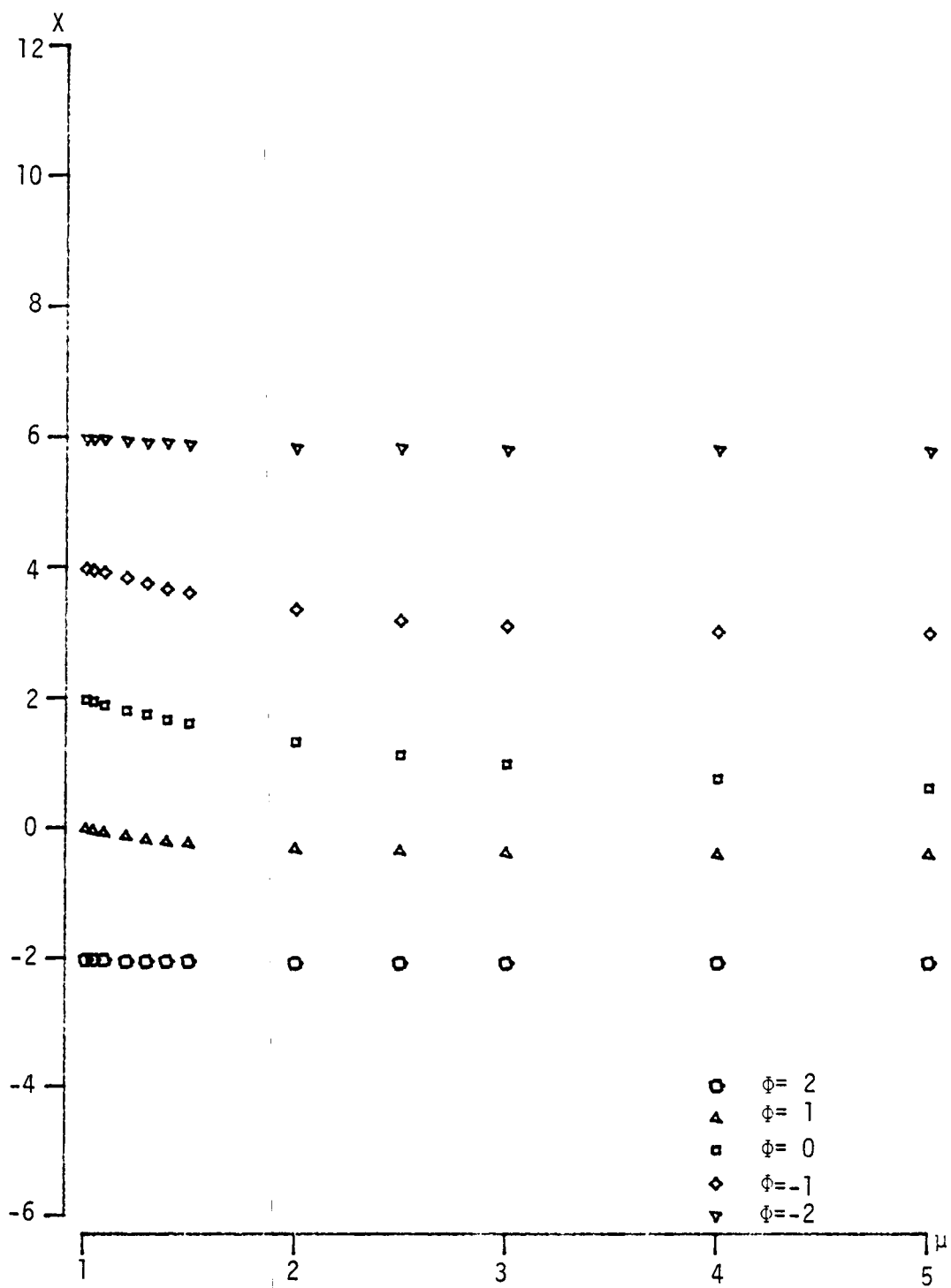


Figure 22. Effect of μ for $\Phi = \text{constant}$, $\theta = 0$, $m = 2$.

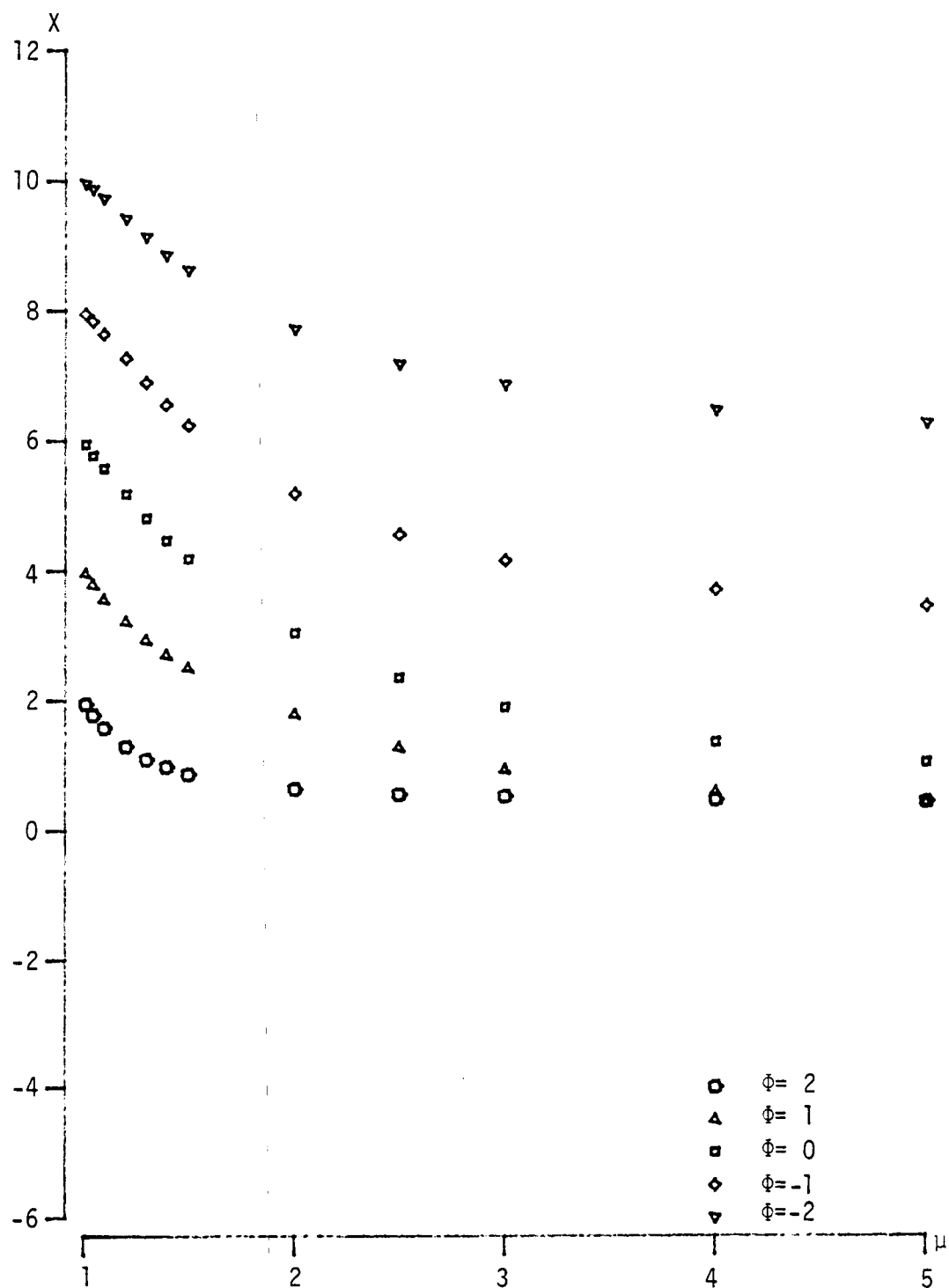


Figure 23. Effect of μ for $\Phi = \text{constant}$, $\Theta = 2$, $m = 2$.

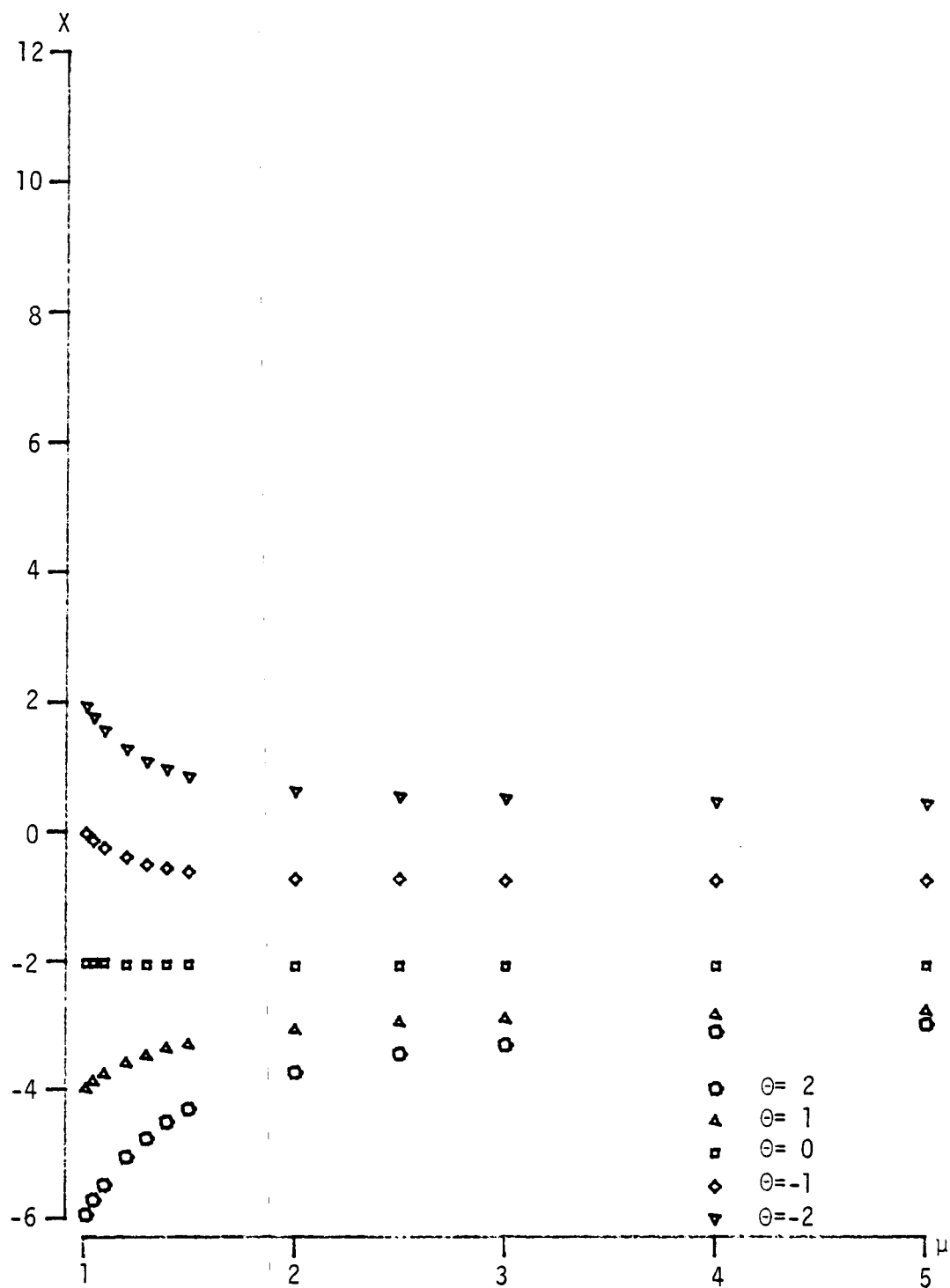


Figure 24. Effect of μ for $\theta = \text{constant}$, $\phi = -2$, $m = 2$.

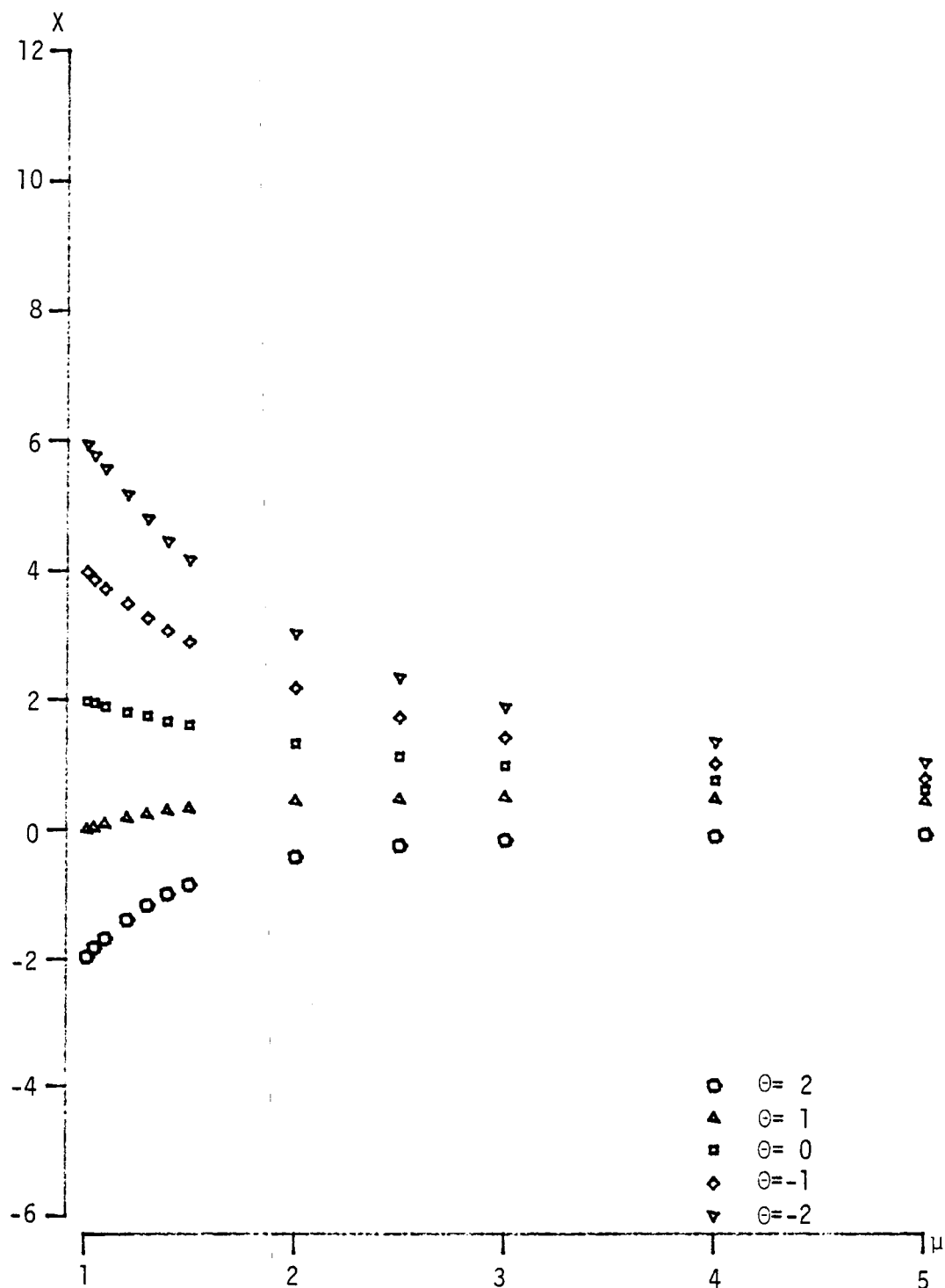
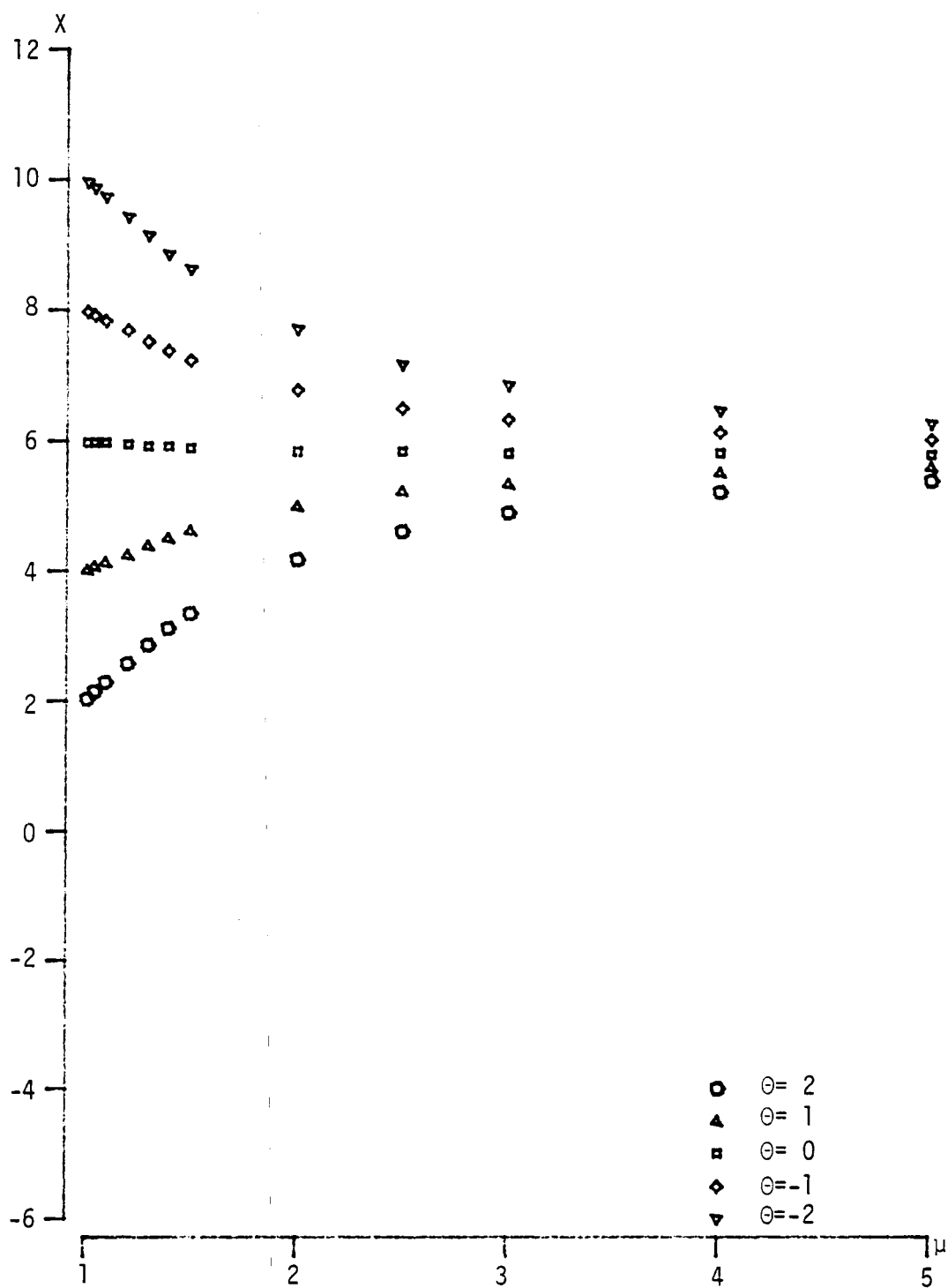


Figure 25. Effect of μ for $\theta = \text{constant}$, $\phi = 0$, $m = 2$.



of m appears to be a multiplicative constant of the basic frequency of the unsteady disturbance when $m = 1$.

Finally, in Figures 27 through 34, curves of constant Θ are plotted versus Φ to show the effects of the combination of Θ and Φ . Note that the frequency increases as Θ increases for a constant value of Φ . Also note that again, except as mentioned previously for mixed flows, the effect of increasing from $m = 1$ to $m = 2$ has been to double the resulting frequency.

In general, when Θ and Φ are of the same sign, the curves exhibit a strong linearity. When there is a mixed swirling flow, however, the relationship to the resulting frequency is more complex. It would appear that the frequency in this case is related in some way to either the average strength of the swirling flow, the result of integrating the swirling flow from r_i to r_o , or a combination of both.

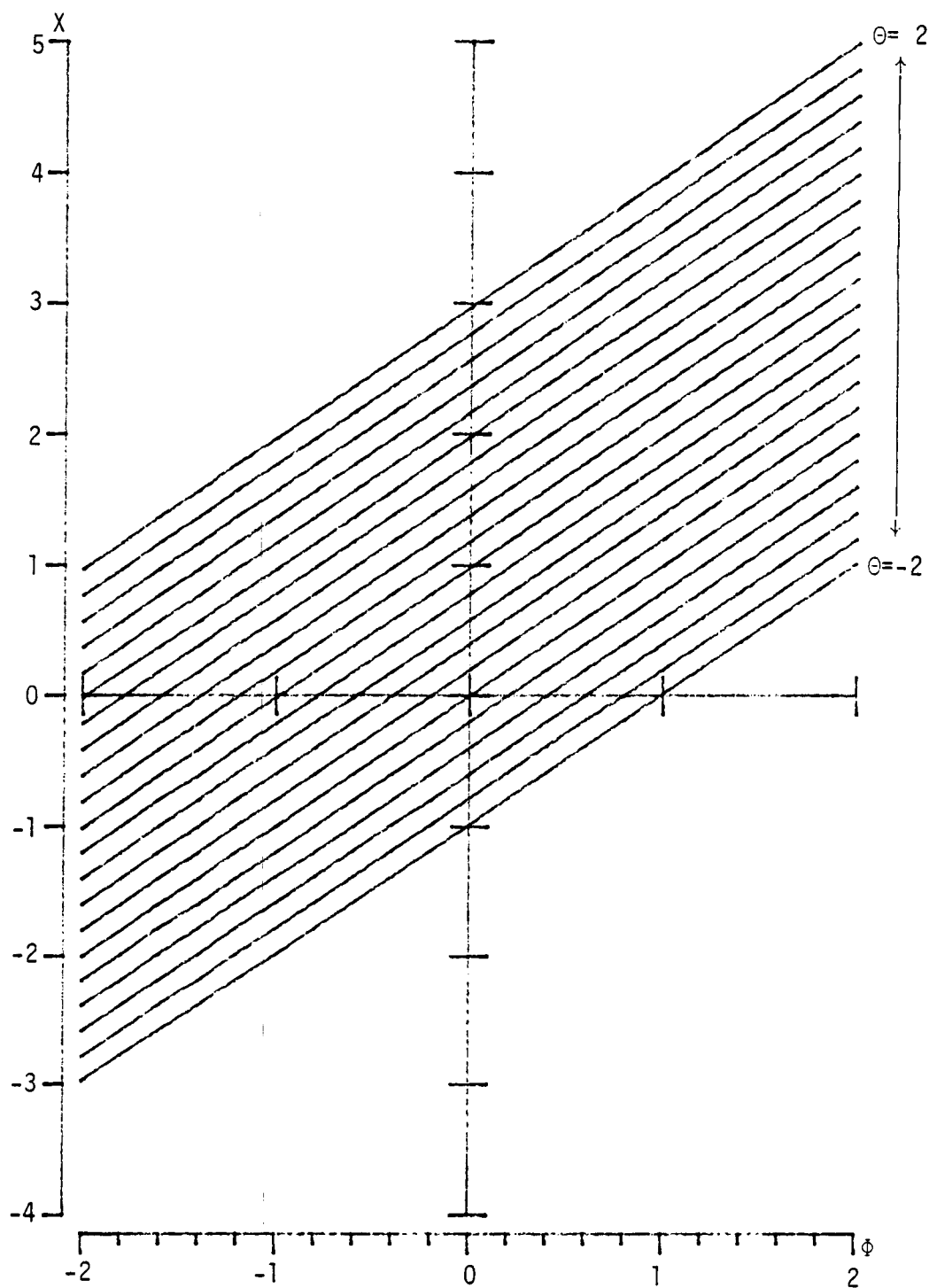


Figure 27. Curves of constant Θ for $\mu = 1.01$, $m = 1$.

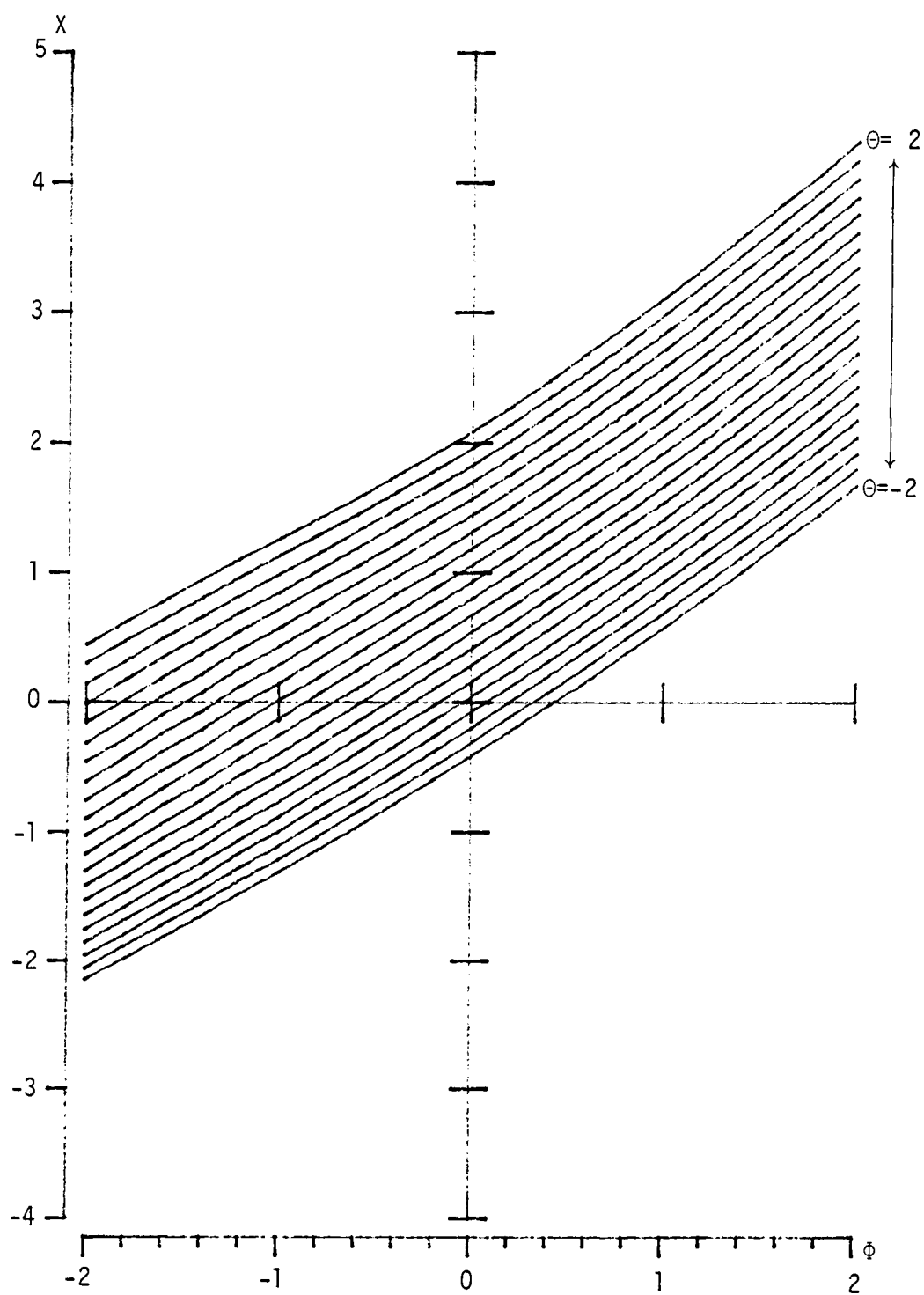


Figure 28. Curves of constant Θ for $\mu = 1.50$, $m = 1$.

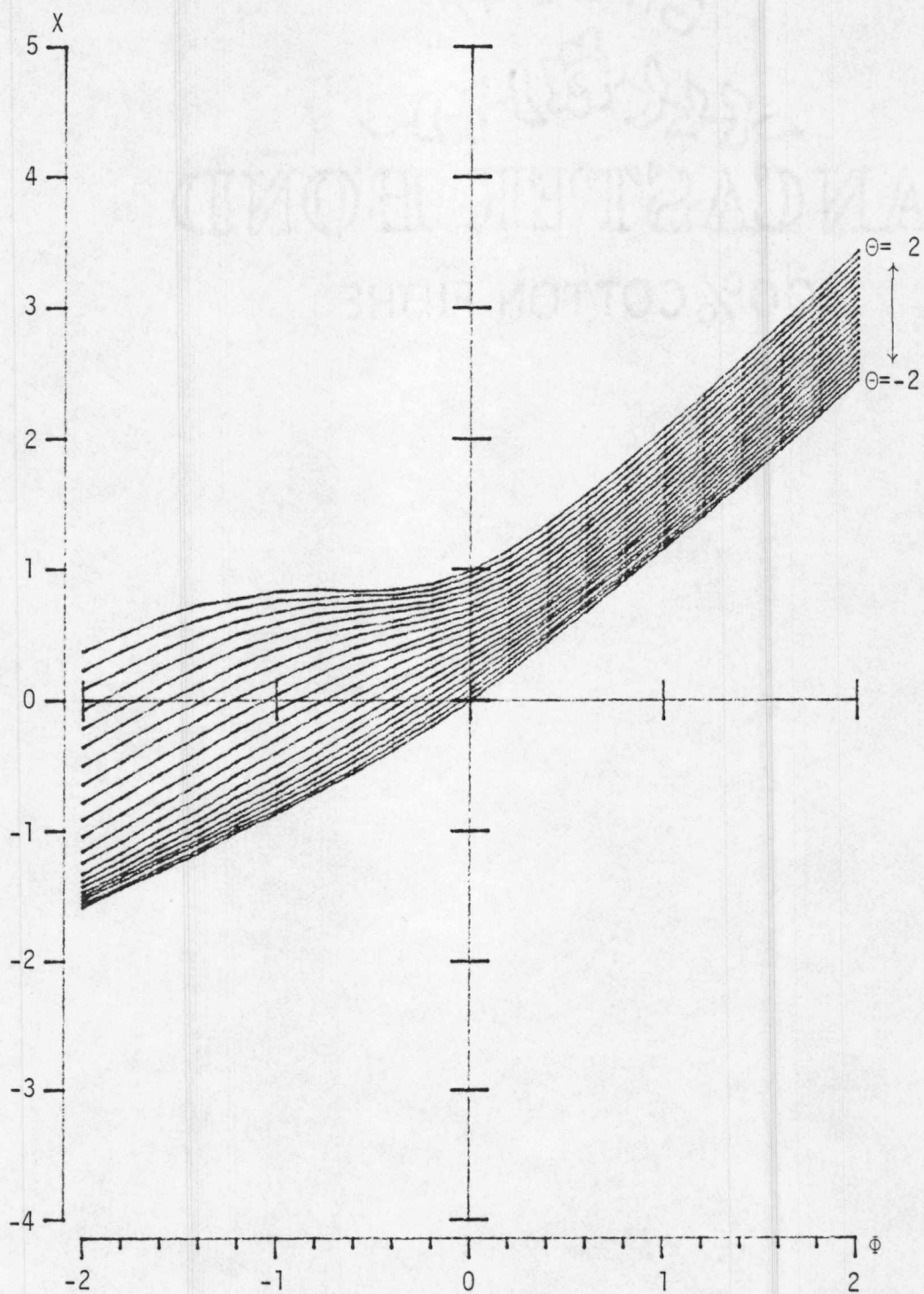


Figure 29. Curves of constant θ for $\mu = 3.00$, $m = 1$.

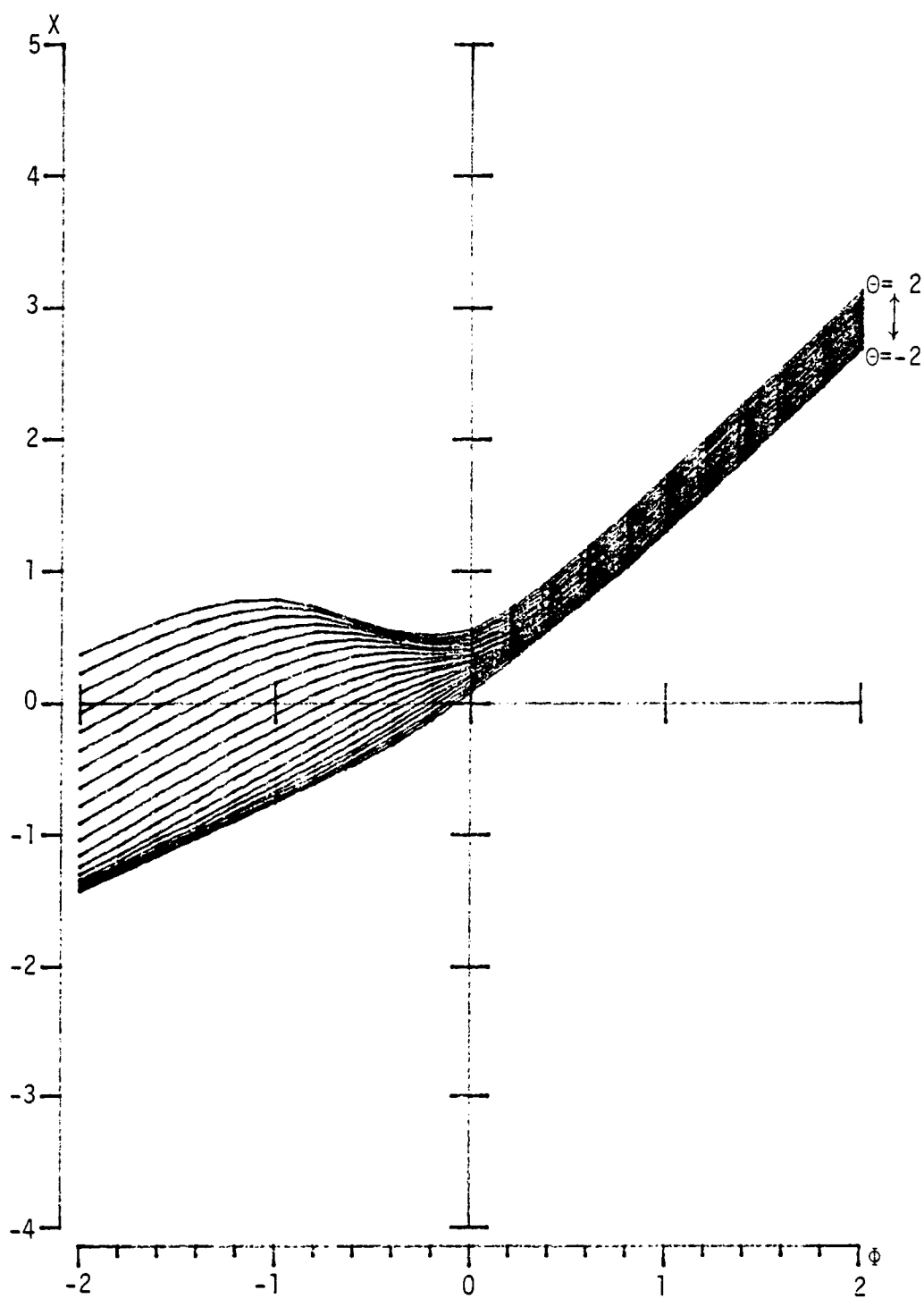


Figure 30. Curves of constant θ for $\mu = 5.00$, $m = 1$.

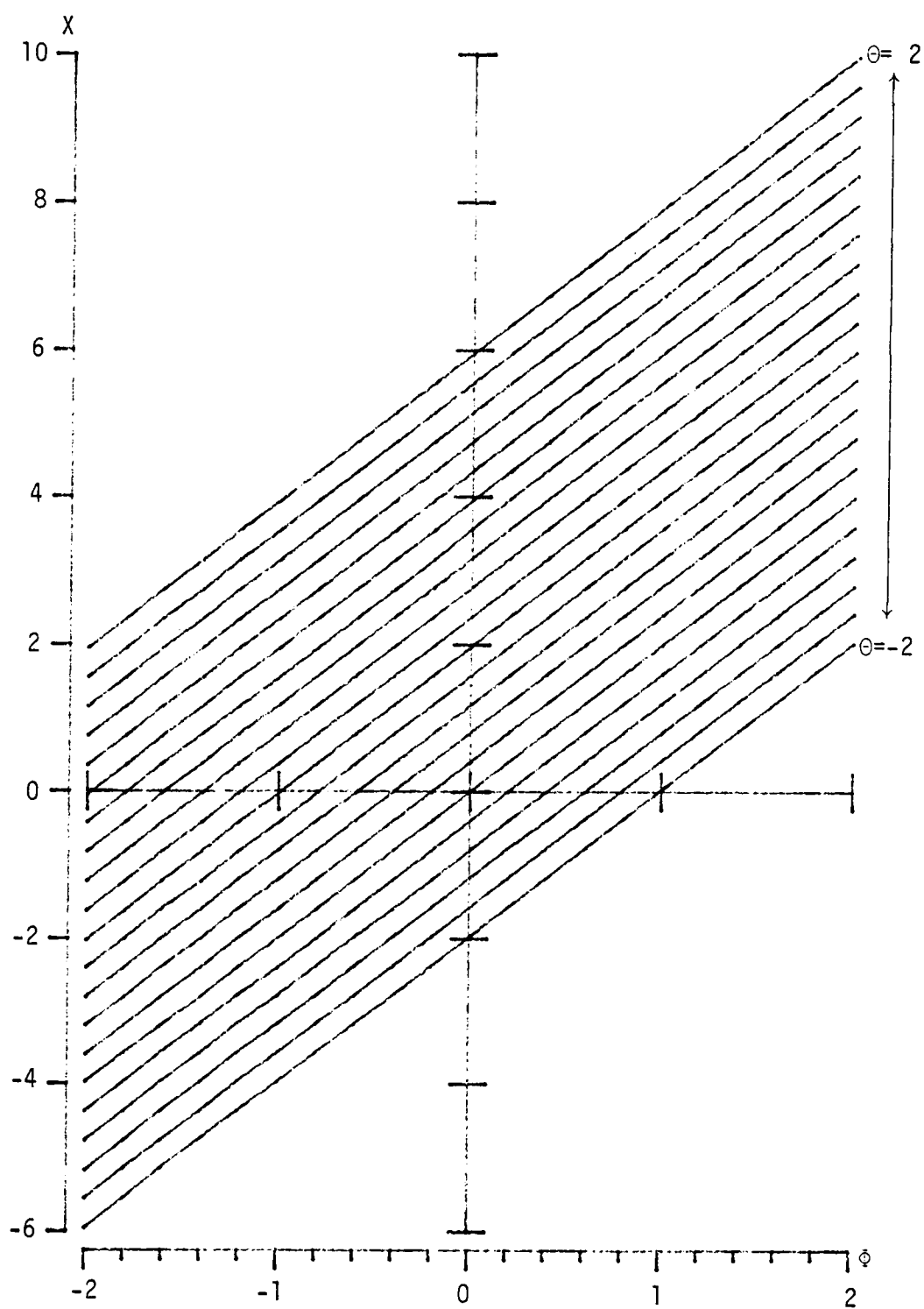


Figure 31. Curves of constant Θ for $\mu = 1.01$, $m = 2$.

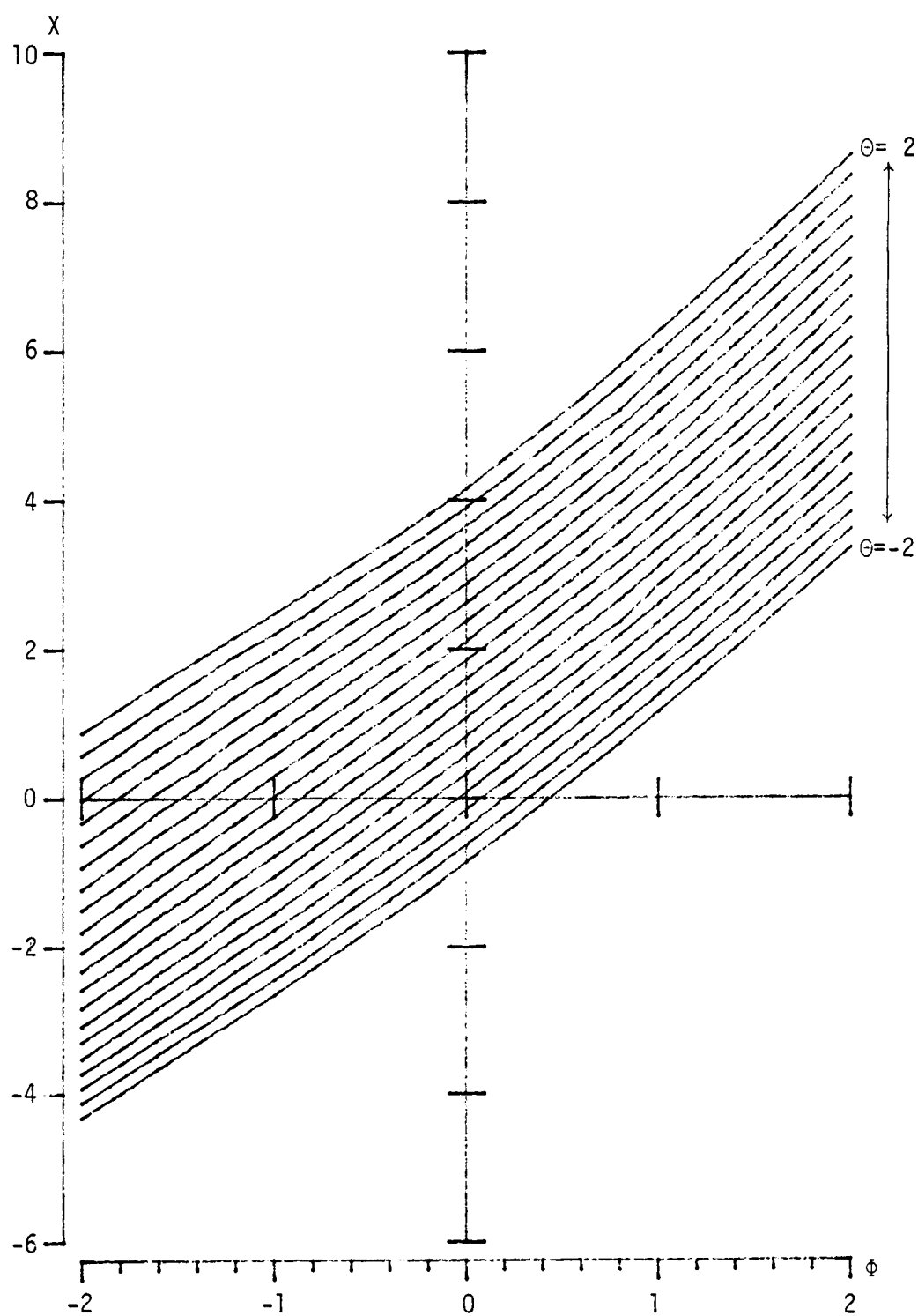


Figure 32. Curves of constant Θ for $\mu = 1.50$, $m = 2$.

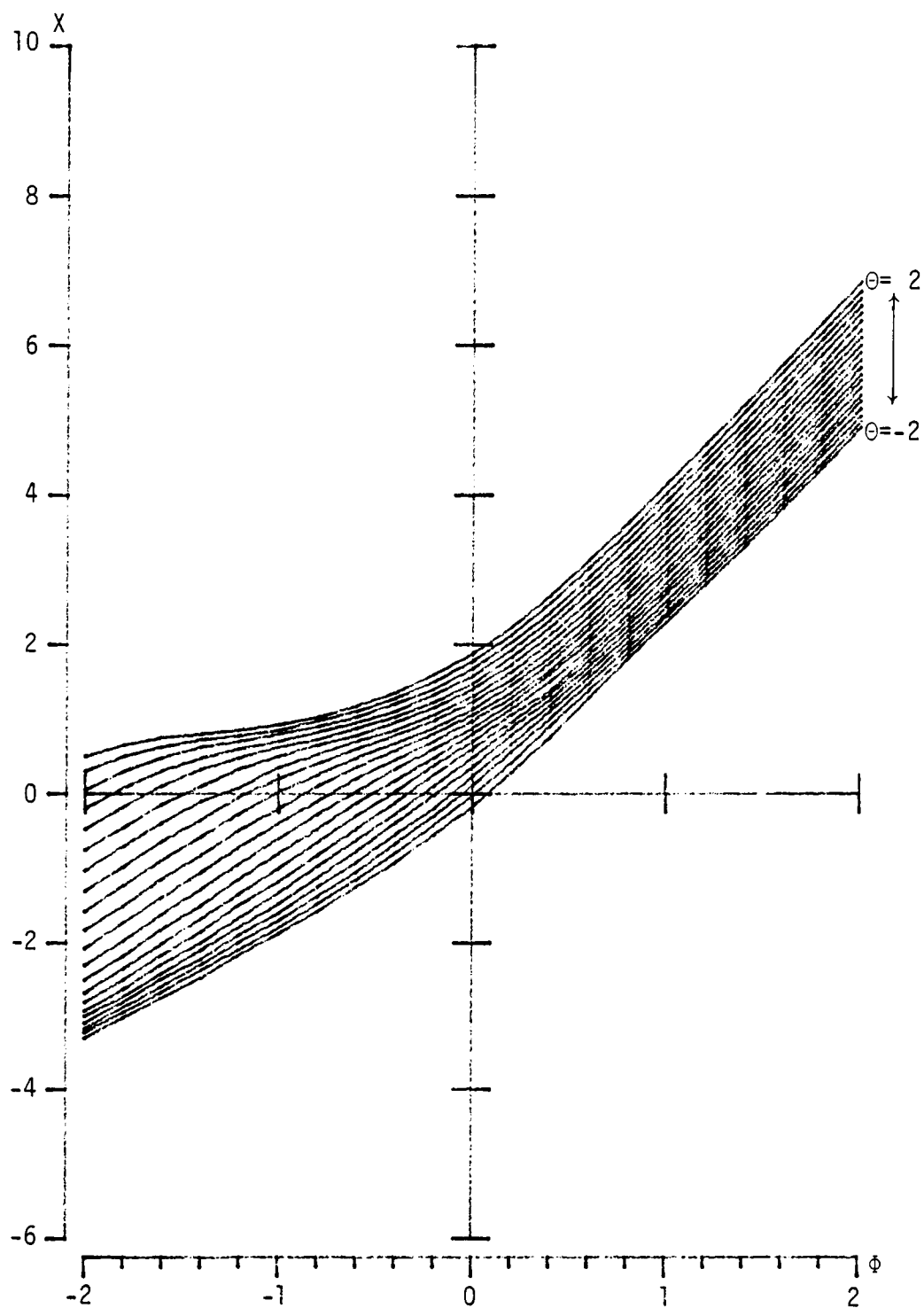


Figure 33. Curves of constant Θ for $\mu = 3.00$, $m = 2$.

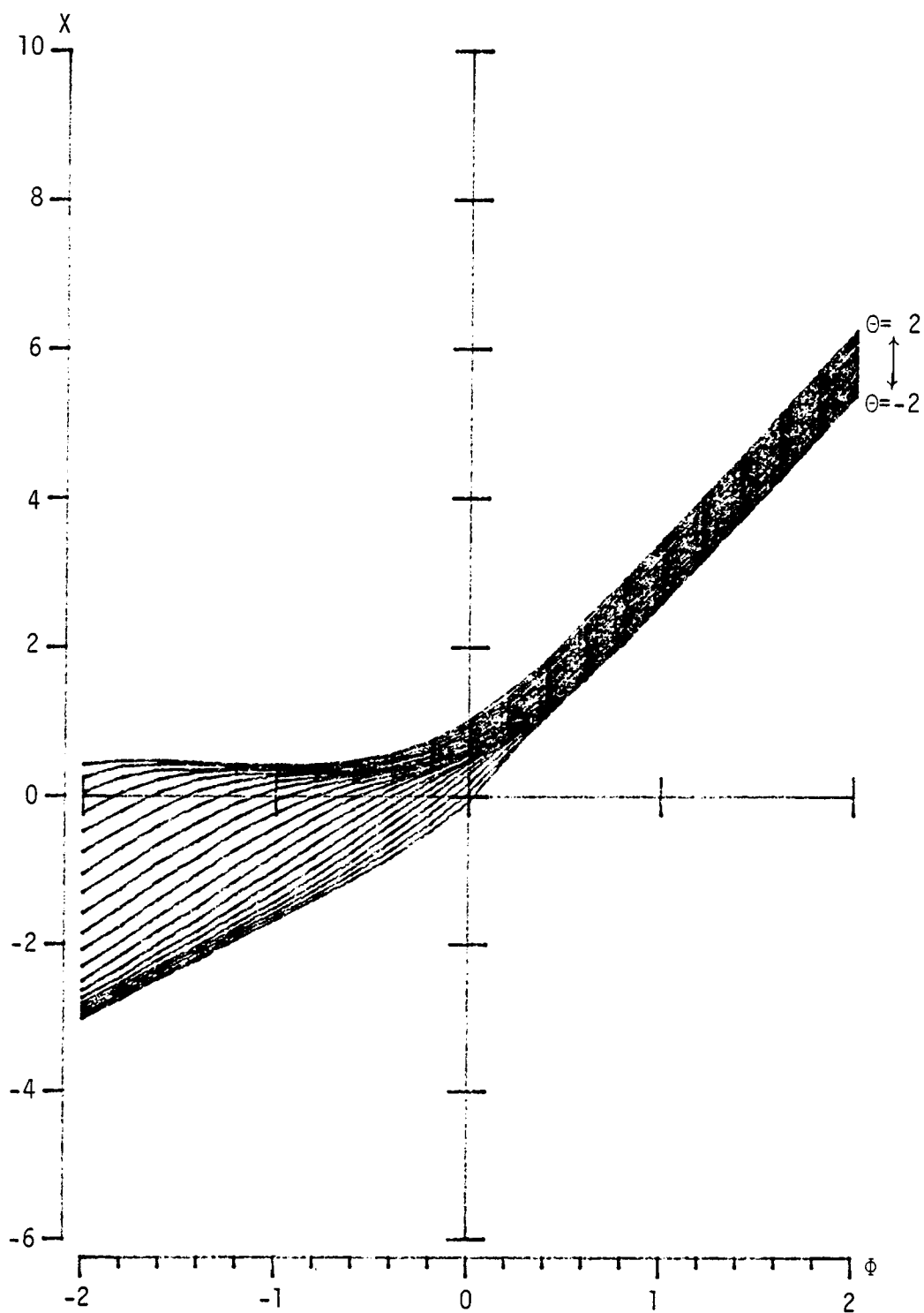


Figure 34. Curves of constant Θ for $\mu = 5.00$, $m = 2$.

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A

COEFFICIENTS FOR A SOLUTION VALID NEAR A SINGULAR POINT

$$Y_1 = \frac{\Theta}{\eta_c} + \phi \eta_c$$

$$Y_2 = \frac{-\Theta}{\eta_c^2} + \phi$$

$$Y_3 = \frac{\Theta}{\eta_c^3}$$

$$Y_4 = \frac{-\Theta}{\eta_c^4}$$

$$Y_5 = m\phi - X + \frac{m\Theta}{\eta_c^2}$$

$$Y_6 = \frac{-2m\Theta}{\eta_c^3}$$

$$Y_7 = \frac{3m\Theta}{\eta_c^4}$$

$$Y_8 = \frac{-4m\Theta}{\eta_c^5}$$

$$Y_9 = \frac{-2m\Theta}{Y_5 \eta_c^3}$$

$$Y_{10} = \frac{3m\Theta}{Y_5 \eta_c^4}$$

$$Y_{11} = \frac{-4m\Theta}{Y_5 \eta_c^5}$$

$$Y_{12} = \frac{1}{Y_5}$$

$$Y_{13} = \frac{-Y_9}{Y_5}$$

$$Y_{14} = \frac{Y_9^2 - Y_{10}}{Y_5}$$

$$Y_{15} = \frac{2Y_{10}Y_9 - Y_{11} - Y_9^3}{Y_5}$$

$$Y_{16} = 4\phi^2 - (m\phi - \chi)^2$$

$$Y_{17} = \frac{[4\phi - 2m(m\phi - \chi)] \Theta}{\eta_c^2}$$

$$Y_{18} = Y_{16} - \frac{m^2 \Theta^2}{\eta_c^4}$$

$$Y_{19} = \frac{1}{Y_{18}} \left(\frac{4m^2 \Theta^2}{\eta_c^5} - \frac{2Y_{17}}{\eta_c} \right)$$

$$Y_{20} = \frac{1}{Y_{18}} \left(\frac{-10m^2 \Theta^2}{\eta_c^6} + \frac{3Y_{17}}{\eta_c^2} \right)$$

$$Y_{21} = \frac{1}{Y_{18}} \left(\frac{20m^2 \Theta^2}{\eta_c^7} - \frac{4Y_{17}}{\eta_c^3} \right)$$

$$Y_{22} = \frac{1}{Y_{18}}$$

$$Y_{23} = \frac{-Y_{19}}{Y_{18}}$$

$$Y_{24} = \frac{Y_{19}^2 - Y_{20}}{Y_{18}}$$

$$Y_{25} = \frac{2Y_{19}Y_{20} - Y_{21} - Y_{19}^3}{Y_{18}}$$

$$Y_{26} = 4\phi^2 - Y_5^2 + \frac{4\Theta\phi}{\eta_c^2}$$

$$Y_{27} = -2Y_5Y_6 - \frac{8\Theta\phi}{\eta_c^3}$$

$$Y_{28} = - (2Y_5Y_7 + Y_6^2) + \frac{12\Theta\phi}{\eta_c^4}$$

$$Y_{29} = -2 (Y_5Y_8 + Y_6Y_7) - \frac{16\Theta\phi}{\eta_c^5}$$

$$Y_{30} = 1 + \frac{\gamma-1}{2} \left[\Theta^2 - \phi^2 + 4\Theta\phi \ln(\eta_c) \right]$$

$$Y_{31} = Y_{30} + \frac{\gamma-1}{2} \left(\phi^2 \eta_c^2 - \frac{\Theta^2}{\eta_c^2} \right)$$

$$Y_{32} = \frac{\gamma-1}{2Y_{31}} \left(\frac{2\Theta^2}{\eta_c^3} + \frac{4\Theta\phi}{\eta_c} + 2\phi\eta_c \right)$$

$$Y_{33} = \frac{\gamma-1}{2Y_{31}} \left(\frac{-3\Theta^2}{\eta_c^4} - \frac{2\Theta\phi}{\eta_c^2} + \phi^2 \right)$$

$$Y_{34} = \frac{\gamma-1}{2Y_{31}} \left(\frac{4\Theta^2}{\eta_c^5} + \frac{4\Theta\phi}{3\eta_c^3} \right)$$

$$Y_{35} = \frac{1}{Y_{31}}$$

$$Y_{36} = - \frac{Y_{32}}{Y_{31}}$$

$$Y_{37} = \frac{Y_{32}^2 - Y_{33}}{Y_{31}}$$

$$Y_{38} = \frac{2Y_{32}Y_{33} - Y_{34} - Y_{32}^3}{Y_{31}}$$

$$Y_{39} = \left(\frac{1}{m\phi - X} \right) \frac{\eta_c}{2}$$

$$Y_{40} = \left(\frac{1}{m\phi - X} \right) \frac{3}{4}$$

$$Y_{41} = \left(\frac{1}{m\phi - X} \right) \frac{1}{8\eta_c}$$

$$Y_{42} = \left(\frac{1}{m\phi - X} \right) \left(\frac{-1}{16\eta_c} \right)$$

$$Y_{43} = \frac{4m\Theta Y_{39}}{\eta_c^3}$$

$$Y_{44} = \frac{4m\Theta Y_{40} + (2\gamma - 3)\Theta^2 Y_{35}}{\eta_c^3}$$

$$Y_{45} = \frac{4m\Theta Y_{41} + (2\gamma - 3)\Theta^2 Y_{36}}{\eta_c^3}$$

$$Y_{46} = \frac{4m\Theta Y_{42} + (2\gamma - 3)\Theta^2 Y_{37}}{\eta_c^3}$$

$$Y_{47} = -4m\Theta Y_{39}$$

$$Y_{48} = 2(2 - \gamma)\Theta^2 Y_{35} - 4m\Theta Y_{40}$$

$$Y_{49} = 2(2-\gamma)\Theta^2 Y_{36} - 4m\Theta Y_{41}$$

$$Y_{50} = \frac{Y_{47}}{\eta_c^4}$$

$$Y_{51} = \frac{1}{\eta_c^4} \left(Y_{48} - \frac{4Y_{47}}{\eta_c} \right)$$

$$Y_{52} = \frac{1}{\eta_c^4} \left(Y_{49} - \frac{4Y_{48}}{\eta_c} + \frac{10Y_{47}}{\eta_c^2} \right)$$

$$Y_{53} = Y_{35}Y_{39}$$

$$Y_{54} = Y_{35}Y_{40} + Y_{36}Y_{39}$$

$$Y_{55} = Y_{35}Y_{41} + Y_{36}Y_{40} + Y_{37}Y_{39}$$

$$Y_{56} = \frac{2(2\gamma-3)m\Theta^3}{\eta_c^6} (Y_{53})$$

$$Y_{57} = \frac{2(2\gamma-3)m\Theta^3}{\eta_c^6} \left(Y_{54} - \frac{6Y_{53}}{\eta_c} \right)$$

$$Y_{58} = \frac{2(2\gamma-3)m\Theta^3}{\eta_c^6} \left(Y_{55} - \frac{6Y_{54}}{\eta_c} + \frac{21Y_{53}}{\eta_c^2} \right)$$

$$Y_{59} = Y_{39}^2$$

$$Y_{60} = 2Y_{39}Y_{40}$$

$$Y_{61} = 2Y_{39}Y_{41} + Y_{40}^2$$

$$Y_{62} = 2(Y_{39}Y_{42} + Y_{40}Y_{41})$$

$$Y_{63} = \frac{8m^2\Theta^2}{\eta_c^6} (Y_{59})$$

$$Y_{64} = \frac{8m^2\Theta^2}{\eta_c^6} \left(Y_{60} - \frac{6Y_{59}}{\eta_c} \right)$$

$$Y_{65} = \frac{8m^2\Theta^2}{\eta_c^6} \left(Y_{61} - \frac{6Y_{60}}{\eta_c} + \frac{21Y_{59}}{\eta_c^2} \right)$$

$$Y_{66} = \frac{8m^2\Theta^2}{\eta_c^6} \left(Y_{62} - \frac{6Y_{61}}{\eta_c} + \frac{21Y_{60}}{\eta_c^2} - \frac{56Y_{59}}{\eta_c^3} \right)$$

$$Y_{67} = (2Y-3)Y_1^2 Y_{35}$$

$$Y_{68} = (2Y-3) \left(Y_1^2 Y_{36} + 2Y_1 Y_2 Y_{35} \right)$$

$$Y_{69} = (2Y-3) \left(Y_1^2 Y_{37} + 2Y_1 Y_2 Y_{36} + (2Y_1 Y_3 + Y_2^2) Y_{35} \right)$$

$$Y_{70} = Y_{22} (-2\phi)$$

$$Y_{71} = Y_{22} {}^m Y_6 + Y_{23} (-2\phi)$$

$$Y_{72} = Y_{22} {}^m Y_7 + Y_{23} {}^m Y_6 + Y_{24} (-2\phi)$$

$$Y_{73} = \frac{-4\Theta}{\eta_c^2} (Y_{70})$$

$$Y_{74} = \frac{-4\Theta}{\eta_c^2} \left(Y_{71} - \frac{2Y_{70}}{\eta_c} \right)$$

$$Y_{75} = \frac{-4\Theta}{\eta_c^2} \left(Y_{72} - \frac{2Y_{71}}{\eta_c} + \frac{3Y_{70}}{\eta_c^2} \right)$$

$$Y_{76} = \kappa^2 (Y_{39}^2)$$

$$Y_{77} = \kappa^2 (2Y_{39}Y_{40})$$

$$Y_{78} = \kappa^2 (2Y_{39}Y_{41} + Y_{40}^2) - Y_{35}$$

$$Y_{79} = \kappa^2 (Y_{39}Y_{42} + Y_{40}Y_{41}) - Y_{36}$$

$$Y_{80} = Y_{26}Y_{76}$$

$$Y_{81} = Y_{26}Y_{77} + Y_{27}Y_{76}$$

$$Y_{82} = Y_{26}Y_{78} + Y_{27}Y_{77} + Y_{28}Y_{76}$$

$$Y_{83} = Y_{26}Y_{79} + Y_{27}Y_{28} + Y_{28}Y_{77} + Y_{29}Y_{76}$$

$$Y_{84} = Y_{35}Y_{39}$$

$$Y_{85} = Y_{35}Y_{40} + Y_{36}Y_{39}$$

$$Y_{86} = Y_{35}Y_{41} + Y_{36}Y_{40} + Y_{37}Y_{39}$$

$$Y_{87} = Y_1^3 Y_{84}$$

$$Y_{88} = Y_1^3 Y_{85} + 3Y_1^2 Y_2 Y_{84}$$

$$Y_{89} = Y_1^3 Y_{86} + 3Y_1^2 Y_2 Y_{85} + 3 \left(Y_1^2 Y_3 + Y_1 Y_2^2 \right) Y_{84}$$

$$Y_{90} = \frac{2m}{\eta_c^3} (Y_{87})$$

$$Y_{91} = \frac{2m}{\eta_c^3} \left(Y_{88} - \frac{3Y_{87}}{\eta_c} \right)$$

$$Y_{92} = \frac{2m}{\eta_c^3} \left(Y_{89} - \frac{3Y_{88}}{\eta_c} + \frac{6Y_{87}}{\eta_c^2} \right)$$

$$Y_{93} = \frac{2(2-\gamma)}{\eta_c^2} \left(Y_1 Y_{35} (Y_1 - 2\phi \eta_c) \right)$$

$$Y_{94} = \frac{2(2-\gamma)}{\eta_c^2} \left(Y_1 Y_{35} (Y_2 - 2\phi) + (Y_1 Y_{36} + Y_2 Y_{35}) (Y_1 - 2\phi \eta_c) \right)$$

$$Y_{95} = Y_{94} - \frac{2Y_{93}}{\eta_c}$$

$$Y_{96} = \frac{-4m\Theta}{\eta_c^4} (Y_{39})$$

$$Y_{97} = \frac{-4m\Theta}{\eta_c^4} \left(Y_{40} - \frac{4Y_{39}}{\eta_c} \right)$$

$$Y_{98} = \frac{-4m\Theta}{\eta_c^4} \left(Y_{41} - \frac{4Y_{40}}{\eta_c} + \frac{10Y_{39}}{\eta_c^2} \right)$$

$$Y_{99} = \frac{-2m}{\eta_c} (Y_{39})$$

$$Y_{100} = \frac{-2m}{\eta_c} \left(Y_{40} - \frac{Y_{39}}{\eta_c} \right) + (2-\gamma) (Y_1 Y_{35})$$

$$Y_{101} = \frac{-2m}{\eta_c} \left(Y_{41} - \frac{Y_{40}}{\eta_c} + \frac{Y_{39}}{\eta_c^2} \right) + (2-\gamma) (Y_1 Y_{36} + Y_2 Y_{35})$$

$$Y_{102} = Y_{99}(-2\phi)$$

$$Y_{103} = Y_{99} (mY_6) + Y_{100}(-2\phi)$$

$$Y_{104} = Y_{99} (mY_7) + Y_{100} (mY_6) + Y_{101}(-2\phi)$$

$$Y_{105} = Y_1 Y_{102}$$

$$Y_{106} = Y_1 Y_{103} + Y_2 Y_{102}$$

$$Y_{107} = Y_1 Y_{104} + Y_2 Y_{103} + Y_3 Y_{102}$$

$$Y_{108} = Y_{22} Y_{105}$$

$$Y_{109} = Y_{22} Y_{106} + Y_{23} Y_{105}$$

$$Y_{110} = Y_{22} Y_{107} + Y_{23} Y_{106} + Y_{24} Y_{105}$$

$$Y_{111} = \frac{4\theta}{\eta_c} (Y_{108})$$

$$Y_{112} = \frac{4\theta}{\eta_c} \left(Y_{109} - \frac{4Y_{108}}{\eta_c} \right)$$

$$Y_{113} = \frac{4\Theta}{\eta_c} \left(Y_{110} - \frac{4Y_{109}}{\eta_c} + \frac{10Y_{108}}{\eta_c^2} \right)$$

$$Y_{114} = 4\phi^2 - (m\phi - \chi)^2$$

$$Y_{115} = 2 \left(\eta_c^3 - \eta_c \eta_b^2 \right)$$

$$Y_{116} = \frac{5\eta_c^2 - \eta_b^2}{Y_{115}}$$

$$Y_{117} = \frac{4\eta_c}{Y_{115}}$$

$$Y_{118} = \frac{1}{Y_{115}}$$

$$Y_{119} = \frac{1}{Y_{114}Y_{115}} \left(\eta_c^4 \right)$$

$$Y_{120} = \frac{1}{Y_{114}Y_{115}} \left(4\eta_c^3 - \eta_c^4 Y_{116} \right)$$

$$Y_{121} = \frac{1}{Y_{114}Y_{115}} \left(6\eta_c^2 - 4\eta_c^3 Y_{116} + \eta_c^4 (Y_{116}^2 - Y_{117}) \right)$$

$$Y_{122} = \frac{1}{Y_{114}Y_{115}} \left(4\eta_c - 6\eta_c^2 Y_{116} + 4\eta_c^3 (Y_{116}^2 - Y_{117}) \right. \\ \left. + \eta_c^4 (2Y_{116}Y_{117} - Y_{118} - Y_{116}^3) \right)$$

$$Y_{123} = Y_{119} (mY_5 - 2\phi)$$

$$Y_{124} = Y_{119} mY_6 + Y_{120} (mY_5 - 2\phi)$$

$$Y_{125} = Y_{119} m Y_7 + Y_{120} m Y_6 + Y_{121} (m Y_5 - 2\phi)$$

$$Y_{126} = Y_{119} m Y_8 + Y_{120} m Y_7 + Y_{121} m Y_6 + Y_{122} (m Y_5 - 2\phi)$$

$$Y_{127} = \frac{-4\Theta}{\eta_c^2} (Y_{123})$$

$$Y_{128} = \frac{-4\Theta}{\eta_c^2} \left(Y_{124} - \frac{2Y_{123}}{\eta_c} \right)$$

$$Y_{129} = \frac{-4\Theta}{\eta_c^2} \left(Y_{125} - \frac{2Y_{124}}{\eta_c} + \frac{3Y_{123}}{\eta_c^2} \right)$$

$$Y_{130} = \frac{-4\Theta}{\eta_c^2} \left(Y_{126} - \frac{2Y_{125}}{\eta_c} + \frac{3Y_{124}}{\eta_c^2} - \frac{4Y_{123}}{\eta_c^3} \right)$$

$$Y_{131} = Y_{26} \left(\kappa^2 Y_{12}^2 - Y_{35} \right)$$

$$Y_{132} = Y_{26} \left(2\kappa^2 Y_{13} Y_{14} - Y_{37} \right) + Y_{27} \left(\kappa^2 Y_{12}^2 - Y_{35} \right)$$

$$Y_{133} = Y_1^3 Y_{35} Y_{12}$$

$$Y_{134} = Y_1^3 (Y_{35} Y_{13} + Y_{36} Y_{12}) + 3Y_1^2 Y_2 Y_{35} Y_{12}$$

$$Y_{135} = \frac{2m}{\eta_c^3} (Y_{133})$$

$$Y_{136} = \frac{2m}{\eta_c^3} \left(Y_{134} - \frac{3Y_{133}}{\eta_c} \right)$$

$$Y_{137} = \frac{-4m\Theta}{\eta_c} (Y_{12})$$

$$Y_{138} = \frac{-4m\Theta}{\eta_c} \left(Y_{13} - \frac{4Y_{12}}{\eta_c} \right)$$

$$Y_{139} = (2-\gamma) (Y_1 Y_{35})$$

$$Y_{140} = (2-\gamma) (Y_1 Y_{36} + Y_2 Y_{35})$$

$$Y_{141} = (2-\gamma) (Y_1 Y_{37} + Y_2 Y_{36} + Y_3 Y_{35})$$

$$Y_{142} = \frac{-2m}{\eta_c} (Y_{12})$$

$$Y_{143} = \frac{-2m}{\eta_c} \left(Y_{13} - \frac{Y_{12}}{\eta_c} \right)$$

$$Y_{144} = \frac{-2m}{\eta_c} \left(Y_{14} - \frac{Y_{13}}{\eta_c} + \frac{Y_{12}}{\eta_c^2} \right)$$

$$Y_{145} = (Y_{139} + Y_{142}) (mY_5 - 2\phi)$$

$$Y_{146} = (Y_{139} + Y_{142}) mY_6 + (Y_{140} + Y_{143}) (mY_5 - 2\phi)$$

$$Y_{147} = (Y_{139} + Y_{142}) mY_7 + (Y_{140} + Y_{143}) mY_6 + (Y_{141} + Y_{144}) (mY_5 - 2\phi)$$

$$Y_{148} = Y_1 Y_{145}$$

$$Y_{149} = Y_1 Y_{146} + Y_2 Y_{145}$$

$$Y_{150} = Y_1 Y_{147} + Y_2 Y_{146} + Y_3 Y_{145}$$

$$Y_{151} = Y_{119} Y_{148}$$

$$Y_{152} = Y_{119} Y_{149} + Y_{120} Y_{148}$$

$$Y_{153} = Y_{119} Y_{150} + Y_{120} Y_{149} + Y_{121} Y_{148}$$

$$Y_{154} = \frac{4\Theta}{\eta_c} (Y_{151})$$

$$Y_{155} = \frac{4\Theta}{\eta_c} \left(Y_{152} - \frac{4Y_{151}}{\eta_c} \right)$$

$$Y_{156} = \frac{4\Theta}{\eta_c} \left(Y_{153} - \frac{4Y_{152}}{\eta_c} + \frac{10Y_{151}}{\eta_c^2} \right)$$

APPENDIX B

COMPUTED EIGENVALUES

A complete list of the numerical results of this investigation is contained in this appendix. Data are presented for specified values of M , κ , and μ . The value of ϕ is varied by row and the value of θ is varied by column as indicated.

Table 2. (continued)

ϕ	$\mu = 1.01$										
	$\kappa = 0.00$					$\mu = 1.01$					
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-1.001	-0.804	-0.606	-0.409	-0.211	-0.014	0.184	0.382	0.580	0.777	0.975
-1.8	-0.802	-0.604	-0.407	-0.209	-0.012	0.186	0.384	0.582	0.779	0.977	1.175
-1.6	-0.602	-0.405	-0.207	-0.010	0.188	0.386	0.583	0.781	0.979	1.177	1.376
-1.4	-0.403	-0.205	-0.008	0.190	0.388	0.585	0.783	0.981	1.179	1.377	1.576
-1.2	-0.204	-0.006	0.192	0.390	0.587	0.775	0.983	1.181	1.379	1.578	1.776
-1.0	-0.004	0.194	0.391	0.589	0.787	0.985	1.183	1.381	1.580	1.778	1.976
-0.8	0.196	0.393	0.591	0.789	0.987	1.185	1.383	1.582	1.780	1.978	2.177
-0.6	0.395	0.593	0.791	0.989	1.187	1.385	1.584	1.782	1.980	2.179	2.377
-0.4	0.595	0.793	0.991	1.189	1.387	1.585	1.784	1.982	2.181	2.379	2.578
-0.2	0.795	0.993	1.191	1.389	1.587	1.786	1.984	2.183	2.381	2.580	2.778
0.0	0.995	1.193	1.391	1.589	1.788	1.986	2.184	2.383	2.582	2.780	2.979
0.2	1.195	1.393	1.591	1.790	1.988	2.186	2.385	2.584	2.782	2.981	3.180
0.4	1.395	1.593	1.792	1.990	2.188	2.387	2.586	2.784	2.983	3.182	3.381
0.6	1.595	1.794	1.992	2.190	2.389	2.588	2.786	2.985	3.184	3.383	3.582
0.8	1.796	1.994	2.192	2.391	2.590	2.788	2.987	3.186	3.385	3.584	3.783
1.0	1.996	2.194	2.393	2.592	2.790	2.989	3.188	3.387	3.586	3.785	3.984
1.2	2.196	2.395	2.594	2.792	2.991	3.190	3.389	3.588	3.787	3.986	4.185
1.4	2.397	2.596	2.794	2.993	3.192	3.391	3.590	3.789	3.988	4.187	4.386
1.6	2.598	2.796	2.995	3.194	3.393	3.592	3.791	3.990	4.189	4.388	4.588
1.8	2.798	2.997	3.196	3.395	3.594	3.793	3.992	4.191	4.390	4.590	4.789
2.0	2.999	3.198	3.397	3.596	3.795	3.994	4.193	4.392	4.592	4.791	4.991

Table 2. (continued)

ϕ	$M = 1$					$K = 0.00$		$\mu = 1.05$			
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-2.855	-2.672	-2.489	-2.305	-2.121	-1.936	-1.750	-1.565	-1.378	-1.192	-1.005
-1.8	-2.662	-2.479	-2.295	-2.111	-1.926	-1.741	-1.555	-1.369	-1.183	-0.996	-0.808
-1.6	-2.469	-2.286	-2.101	-1.917	-1.731	-1.546	-1.360	-1.173	-0.986	-0.799	-0.611
-1.4	-2.276	-2.092	-1.907	-1.722	-1.536	-1.350	-1.164	-0.977	-0.790	-0.602	-0.414
-1.2	-2.082	-1.897	-1.712	-1.527	-1.341	-1.155	-0.968	-0.781	-0.593	-0.405	-0.217
-1.0	-1.887	-1.702	-1.517	-1.331	-1.145	-0.958	-0.771	-0.584	-0.396	-0.208	-0.019
-0.8	-1.693	-1.507	-1.322	-1.135	-0.949	-0.762	-0.574	-0.387	-0.198	-0.010	0.179
-0.6	-1.498	-1.312	-1.126	-0.939	-0.752	-0.565	-0.377	-0.189	0.000	0.189	0.378
-0.4	-1.302	-1.116	-0.930	-0.743	-0.555	-0.368	-0.179	0.009	0.198	0.386	0.577
-0.2	-1.106	-0.920	-0.733	-0.546	-0.358	-0.170	0.018	0.207	0.397	0.586	0.776
0.0	-0.910	-0.723	-0.536	-0.348	-0.160	0.028	0.217	0.406	0.596	0.785	0.976
0.2	-0.713	-0.526	-0.339	-0.151	0.038	0.226	0.416	0.605	0.795	0.985	1.176
0.4	-0.516	-0.329	-0.141	0.047	0.236	0.425	0.615	0.805	0.995	1.185	1.376
0.6	-0.319	-0.131	0.057	0.246	0.435	0.624	0.814	1.004	1.195	1.384	1.577
0.8	-0.121	0.067	0.256	0.445	0.634	0.824	1.014	1.205	1.396	1.587	1.778
1.0	0.077	0.266	0.455	0.644	0.834	1.024	1.214	1.405	1.597	1.788	1.980
1.2	0.276	0.465	0.654	0.844	1.034	1.225	1.415	1.607	1.798	1.990	2.182
1.4	0.475	0.664	0.854	1.044	1.235	1.425	1.617	1.808	2.000	2.192	2.385
1.6	0.675	0.864	1.054	1.245	1.436	1.627	1.818	2.010	2.202	2.395	2.587
1.8	0.874	1.065	1.255	1.446	1.637	1.828	2.020	2.212	2.405	2.598	2.791
2.0	1.075	1.265	1.456	1.647	1.839	2.031	2.223	2.415	2.608	2.801	2.995

Table 2. (continued)

ϕ	$M = 1$										$\mu = 1.05$											
	$\kappa = 0.00$					$\kappa = 0.00$					$\kappa = 0.00$					$\kappa = 0.00$						
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-1.005	-0.817	-0.629	-0.441	-0.252	-0.063	0.126	0.315	0.505	0.696	0.888	-1.005	-0.817	-0.629	-0.441	-0.252	-0.063	0.126	0.315	0.505	0.696	0.888
-1.8	-0.808	-0.620	-0.432	-0.244	-0.055	0.134	0.324	0.514	0.704	0.897	1.085	-0.808	-0.620	-0.432	-0.244	-0.055	0.134	0.324	0.514	0.704	0.897	1.085
-1.6	-0.611	-0.423	-0.235	-0.046	0.143	0.333	0.523	0.713	0.903	1.094	1.285	-0.611	-0.423	-0.235	-0.046	0.143	0.333	0.523	0.713	0.903	1.094	1.285
-1.4	-0.414	-0.226	-0.037	0.152	0.342	0.531	0.722	0.912	1.103	1.294	1.485	-0.414	-0.226	-0.037	0.152	0.342	0.531	0.722	0.912	1.103	1.294	1.485
-1.2	-0.217	-0.028	0.161	0.351	0.540	0.730	0.921	1.111	1.302	1.494	1.685	-0.217	-0.028	0.161	0.351	0.540	0.730	0.921	1.111	1.302	1.494	1.685
-1.0	-0.019	0.170	0.360	0.549	0.747	0.930	1.120	1.311	1.503	1.694	1.886	-0.019	0.170	0.360	0.549	0.747	0.930	1.120	1.311	1.503	1.694	1.886
-0.8	0.179	0.369	0.558	0.748	0.939	1.129	1.320	1.512	1.703	1.895	2.087	0.179	0.369	0.558	0.748	0.939	1.129	1.320	1.512	1.703	1.895	2.087
-0.6	0.378	0.568	0.758	0.948	1.138	1.329	1.521	1.712	1.904	2.096	2.288	0.378	0.568	0.758	0.948	1.138	1.329	1.521	1.712	1.904	2.096	2.288
-0.4	0.577	0.767	0.957	1.148	1.339	1.530	1.721	1.913	2.105	2.297	2.490	0.577	0.767	0.957	1.148	1.339	1.530	1.721	1.913	2.105	2.297	2.490
-0.2	0.776	0.966	1.157	1.348	1.539	1.730	1.922	2.114	2.307	2.499	2.692	0.776	0.966	1.157	1.348	1.539	1.730	1.922	2.114	2.307	2.499	2.692
0.0	0.976	1.166	1.357	1.548	1.740	1.932	2.124	2.316	2.509	2.702	2.895	0.976	1.166	1.357	1.548	1.740	1.932	2.124	2.316	2.509	2.702	2.895
0.2	1.176	1.367	1.558	1.749	1.941	2.133	2.326	2.518	2.711	2.904	3.098	1.176	1.367	1.558	1.749	1.941	2.133	2.326	2.518	2.711	2.904	3.098
0.4	1.376	1.568	1.759	1.951	2.143	2.335	2.528	2.721	2.914	3.107	3.301	1.376	1.568	1.759	1.951	2.143	2.335	2.528	2.721	2.914	3.107	3.301
0.6	1.577	1.768	1.960	2.152	2.345	2.537	2.730	2.924	3.117	3.311	3.505	1.577	1.768	1.960	2.152	2.345	2.537	2.730	2.924	3.117	3.311	3.505
0.8	1.778	1.970	2.162	2.355	2.547	2.740	2.934	3.127	3.321	3.515	3.710	1.778	1.970	2.162	2.355	2.547	2.740	2.934	3.127	3.321	3.515	3.710
1.0	1.980	2.172	2.364	2.557	2.750	2.944	3.137	3.331	3.525	3.720	3.914	1.980	2.172	2.364	2.557	2.750	2.944	3.137	3.331	3.525	3.720	3.914
1.2	2.182	2.374	2.567	2.760	2.954	3.147	3.341	3.535	3.730	3.925	4.120	2.182	2.374	2.567	2.760	2.954	3.147	3.341	3.535	3.730	3.925	4.120
1.4	2.385	2.577	2.770	2.964	3.157	3.351	3.546	3.740	3.935	4.130	4.325	2.385	2.577	2.770	2.964	3.157	3.351	3.546	3.740	3.935	4.130	4.325
1.6	2.587	2.781	2.974	3.168	3.362	3.556	3.751	3.945	4.141	4.336	4.532	2.587	2.781	2.974	3.168	3.362	3.556	3.751	3.945	4.141	4.336	4.532
1.8	2.791	2.984	3.178	3.372	3.566	3.761	3.956	4.151	4.347	4.542	4.738	2.791	2.984	3.178	3.372	3.566	3.761	3.956	4.151	4.347	4.542	4.738
2.0	2.995	3.189	3.383	3.577	3.772	3.967	4.162	4.358	4.553	4.749	4.946	2.995	3.189	3.383	3.577	3.772	3.967	4.162	4.358	4.553	4.749	4.946

Table 2. (continued)

ϕ	$M = 1$										$\mu = 1.10$											
	$\kappa = 0.00$					$\kappa = 0.00$					$\kappa = 0.00$					$\kappa = 0.00$						
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-2.730	-2.562	-2.394	-2.224	-2.053	-1.881	-1.709	-1.535	-1.360	-1.185	-1.008	-2.730	-2.562	-2.394	-2.224	-2.053	-1.881	-1.709	-1.535	-1.360	-1.185	-1.008
-1.8	-2.543	-2.375	-2.205	-2.035	-1.863	-1.691	-1.517	-1.343	-1.168	-0.992	-0.815	-2.543	-2.375	-2.205	-2.035	-1.863	-1.691	-1.517	-1.343	-1.168	-0.992	-0.815
-1.6	-2.356	-2.186	-2.016	-1.845	-1.673	-1.500	-1.326	-1.151	-0.975	-0.798	-0.621	-2.356	-2.186	-2.016	-1.845	-1.673	-1.500	-1.326	-1.151	-0.975	-0.798	-0.621
-1.4	-2.167	-1.997	-1.826	-1.655	-1.482	-1.308	-1.133	-0.958	-0.782	-0.604	-0.427	-2.167	-1.997	-1.826	-1.655	-1.482	-1.308	-1.133	-0.958	-0.782	-0.604	-0.427
-1.2	-1.978	-1.808	-1.636	-1.463	-1.290	-1.116	-0.940	-0.764	-0.587	-0.410	-0.231	-1.978	-1.808	-1.636	-1.463	-1.290	-1.116	-0.940	-0.764	-0.587	-0.410	-0.231
-1.0	-1.788	-1.617	-1.445	-1.272	-1.098	-0.923	-0.747	-0.570	-0.393	-0.215	-0.036	-1.788	-1.617	-1.445	-1.272	-1.098	-0.923	-0.747	-0.570	-0.393	-0.215	-0.036
-0.8	-1.598	-1.426	-1.253	-1.079	-0.905	-0.729	-0.553	-0.375	-0.197	-0.019	0.161	-1.598	-1.426	-1.253	-1.079	-0.905	-0.729	-0.553	-0.375	-0.197	-0.019	0.161
-0.6	-1.407	-1.234	-1.061	-0.886	-0.711	-0.535	-0.358	-0.180	-0.001	0.178	0.358	-1.407	-1.234	-1.061	-0.886	-0.711	-0.535	-0.358	-0.180	-0.001	0.178	0.358
-0.4	-1.215	-1.042	-0.867	-0.692	-0.516	-0.340	-0.162	0.016	0.195	0.375	0.555	-1.215	-1.042	-0.867	-0.692	-0.516	-0.340	-0.162	0.016	0.195	0.375	0.555
-0.2	-1.022	-0.848	-0.674	-0.498	-0.321	-0.144	0.034	0.213	0.393	0.573	0.754	-1.022	-0.848	-0.674	-0.498	-0.321	-0.144	0.034	0.213	0.393	0.573	0.754
0.0	-0.829	-0.654	-0.479	-0.303	-0.125	0.053	0.231	0.411	0.591	0.771	0.953	-0.829	-0.654	-0.479	-0.303	-0.125	0.053	0.231	0.411	0.591	0.771	0.953
0.2	-0.635	-0.460	-0.284	-0.106	0.071	0.250	0.429	0.609	0.790	0.971	1.153	-0.635	-0.460	-0.284	-0.106	0.071	0.250	0.429	0.609	0.790	0.971	1.153
0.4	-0.440	-0.264	-0.087	0.090	0.269	0.448	0.628	0.808	0.989	1.171	1.353	-0.440	-0.264	-0.087	0.090	0.269	0.448	0.628	0.808	0.989	1.171	1.353
0.6	-0.245	-0.068	0.110	0.288	0.467	0.647	0.827	1.008	1.190	1.372	1.554	-0.245	-0.068	0.110	0.288	0.467	0.647	0.827	1.008	1.190	1.372	1.554
0.8	-0.048	0.129	0.307	0.486	0.666	0.846	1.027	1.208	1.391	1.573	1.757	-0.048	0.129	0.307	0.486	0.666	0.846	1.027	1.208	1.391	1.573	1.757
1.0	0.149	0.327	0.506	0.685	0.866	1.047	1.228	1.410	1.593	1.776	1.960	0.149	0.327	0.506	0.685	0.866	1.047	1.228	1.410	1.593	1.776	1.960
1.2	0.347	0.526	0.705	0.886	1.066	1.248	1.430	1.612	1.796	1.979	2.164	0.347	0.526	0.705	0.886	1.066	1.248	1.430	1.612	1.796	1.979	2.164
1.4	0.547	0.726	0.906	1.087	1.268	1.450	1.633	1.816	2.000	2.184	2.369	0.547	0.726	0.906	1.087	1.268	1.450	1.633	1.816	2.000	2.184	2.369
1.6	0.747	0.926	1.107	1.288	1.470	1.653	1.836	2.020	2.204	2.389	2.574	0.747	0.926	1.107	1.288	1.470	1.653	1.836	2.020	2.204	2.389	2.574
1.8	0.947	1.128	1.309	1.491	1.674	1.857	2.041	2.225	2.410	2.595	2.781	0.947	1.128	1.309	1.491	1.674	1.857	2.041	2.225	2.410	2.595	2.781
2.0	1.149	1.331	1.512	1.695	1.878	2.062	2.246	2.431	2.617	2.803	2.989	1.149	1.331	1.512	1.695	1.878	2.062	2.246	2.431	2.617	2.803	2.989

Table 2. (continued)

ϕ	$\mu = 1.10$									
	$\kappa = 0.00$					$\mu = 1.10$				
	$M = 1$	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	
-2.0	-1.008	-0.831	-0.653	-0.475	-0.295	-0.116	0.065	0.246	0.428	0.610
-1.8	-0.815	-0.637	-0.459	-0.280	-0.100	0.080	0.261	0.442	0.624	0.809
-1.6	-0.621	-0.443	-0.264	-0.085	0.096	0.276	0.458	0.639	0.823	1.004
-1.4	-0.427	-0.248	-0.069	0.111	0.292	0.473	0.655	0.844	1.019	1.202
-1.2	-0.231	-0.052	0.127	0.308	0.489	0.670	0.852	1.034	1.217	1.400
-1.0	-0.036	0.144	0.324	0.505	0.686	0.868	1.050	1.233	1.416	1.599
-0.8	0.161	0.341	0.521	0.703	0.884	1.066	1.249	1.432	1.615	1.799
-0.6	0.358	0.538	0.719	0.901	1.083	1.265	1.448	1.632	1.815	1.999
-0.4	0.555	0.736	0.918	1.100	1.282	1.465	1.648	1.832	2.016	2.200
-0.2	0.754	0.935	1.117	1.299	1.482	1.665	1.849	2.033	2.218	2.402
0.0	0.953	1.135	1.317	1.500	1.683	1.867	2.051	2.235	2.420	2.605
0.2	1.153	1.335	1.518	1.701	1.885	2.069	2.253	2.438	2.623	2.809
0.4	1.353	1.536	1.719	1.903	2.087	2.271	2.456	2.642	2.827	3.013
0.6	1.554	1.738	1.921	2.105	2.290	2.475	2.660	2.846	3.032	3.219
0.8	1.757	1.940	2.124	2.309	2.494	2.680	2.866	3.052	3.239	3.426
1.0	1.960	2.144	2.329	2.514	2.699	2.885	3.072	3.258	3.446	3.633
1.2	2.164	2.348	2.534	2.719	2.905	3.092	3.279	3.466	3.654	3.842
1.4	2.369	2.554	2.740	2.926	3.112	3.299	3.487	3.675	3.863	4.052
1.6	2.574	2.760	2.947	3.133	3.320	3.508	3.696	3.885	4.073	4.263
1.8	2.781	2.968	3.155	3.342	3.530	3.718	3.906	4.095	4.285	4.475
2.0	2.989	3.176	3.364	3.552	3.740	3.929	4.118	4.307	4.497	4.688

Table 2. (continued)

ϕ	$\mu = 1.30$									
	$M = 1$	$\kappa = 0.00$	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4
-2.0	-2.370	-2.247	-2.122	-1.994	-1.864	-1.730	-1.594	-1.455	-1.312	-1.167
-1.8	-2.198	-2.074	-1.947	-1.818	-1.686	-1.552	-1.414	-1.273	-1.130	-0.983
-1.6	-2.025	-1.899	-1.771	-1.641	-1.507	-1.371	-1.232	-1.090	-0.946	-0.798
-1.4	-1.850	-1.723	-1.594	-1.462	-1.327	-1.190	-1.049	-0.906	-0.761	-0.612
-1.2	-1.674	-1.546	-1.415	-1.281	-1.145	-1.006	-0.865	-0.721	-0.574	-0.426
-1.0	-1.496	-1.367	-1.234	-1.099	-0.962	-0.822	-0.679	-0.534	-0.387	-0.238
-0.8	-1.317	-1.186	-1.052	-0.916	-0.777	-0.636	-0.492	-0.346	-0.198	0.103
-0.6	-1.135	-1.003	-0.868	-0.730	-0.590	-0.448	-0.304	-0.157	-0.009	0.142
-0.4	-0.952	-0.816	-0.682	-0.543	-0.402	-0.259	-0.113	0.034	0.183	0.333
-0.2	-0.767	-0.632	-0.494	-0.354	-0.212	-0.068	0.061	0.226	0.375	0.526
0.0	-0.580	-0.443	-0.304	-0.163	-0.020	0.125	0.272	0.420	0.570	0.720
0.2	-0.390	-0.252	-0.112	0.030	0.174	0.320	0.467	0.616	0.766	0.917
0.4	-0.198	-0.059	0.082	0.225	0.370	0.517	0.665	0.814	0.964	1.115
0.6	-0.004	0.137	0.279	0.423	0.569	0.716	0.865	1.012	1.165	1.316
0.8	0.193	0.335	0.478	0.623	0.770	0.918	1.067	1.217	1.368	1.520
1.0	0.393	0.536	0.680	0.826	0.974	1.122	1.272	1.423	1.574	1.727
1.2	0.595	0.739	0.885	1.032	1.180	1.329	1.480	1.631	1.784	1.937
1.4	0.800	0.946	1.092	1.240	1.389	1.540	1.691	1.843	1.996	2.150
1.6	1.009	1.155	1.303	1.452	1.602	1.753	1.905	2.058	2.212	2.367
1.8	1.220	1.368	1.516	1.666	1.818	1.970	2.123	2.277	2.432	2.587
2.0	1.435	1.583	1.733	1.884	2.036	2.189	2.343	2.498	2.654	2.811

Table 2. (continued)

ϕ	$M = 1$					$K = 0.00$					$\mu = 1.40$											
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-1.022	-0.879	-0.733	-0.585	-0.433	-0.280	-0.126	0.029	0.185	0.340	0.495	-1.022	-0.879	-0.733	-0.585	-0.433	-0.280	-0.126	0.029	0.185	0.340	0.495
-1.8	-0.841	-0.697	-0.550	-0.402	-0.251	-0.099	0.055	0.209	0.363	0.516	0.669	-0.841	-0.697	-0.550	-0.402	-0.251	-0.099	0.055	0.209	0.363	0.516	0.669
-1.6	-0.658	-0.514	-0.367	-0.219	-0.068	0.083	0.235	0.387	0.539	0.691	0.842	-0.658	-0.514	-0.367	-0.219	-0.068	0.083	0.235	0.387	0.539	0.691	0.842
-1.4	-0.474	-0.330	-0.184	-0.036	0.114	0.264	0.415	0.565	0.715	0.865	1.013	-0.474	-0.330	-0.184	-0.036	0.114	0.264	0.415	0.565	0.715	0.865	1.013
-1.2	-0.290	-0.146	0.000	0.148	0.296	0.445	0.594	0.742	0.890	1.038	1.184	-0.290	-0.146	0.000	0.148	0.296	0.445	0.594	0.742	0.890	1.038	1.184
-1.0	-0.105	0.039	0.184	0.331	0.478	0.625	0.772	0.919	1.065	1.210	1.354	-0.105	0.039	0.184	0.331	0.478	0.625	0.772	0.919	1.065	1.210	1.354
-0.8	0.081	0.225	0.369	0.514	0.660	0.806	0.951	1.096	1.240	1.383	1.525	0.081	0.225	0.369	0.514	0.660	0.806	0.951	1.096	1.240	1.383	1.525
-0.6	0.268	0.411	0.555	0.698	0.843	0.987	1.131	1.274	1.416	1.558	1.699	0.268	0.411	0.555	0.698	0.843	0.987	1.131	1.274	1.416	1.558	1.699
-0.4	0.456	0.598	0.741	0.884	1.026	1.169	1.311	1.453	1.595	1.735	1.875	0.456	0.598	0.741	0.884	1.026	1.169	1.311	1.453	1.595	1.735	1.875
-0.2	0.646	0.787	0.928	1.070	1.212	1.353	1.495	1.636	1.776	1.916	2.056	0.646	0.787	0.928	1.070	1.212	1.353	1.495	1.636	1.776	1.916	2.056
0.0	0.837	0.977	1.118	1.259	1.400	1.541	1.681	1.822	1.962	2.103	2.243	0.837	0.977	1.118	1.259	1.400	1.541	1.681	1.822	1.962	2.103	2.243
0.2	1.031	1.171	1.311	1.451	1.592	1.732	1.873	2.013	2.154	2.295	2.437	1.031	1.171	1.311	1.451	1.592	1.732	1.873	2.013	2.154	2.295	2.437
0.4	1.228	1.367	1.507	1.647	1.788	1.928	2.069	2.211	2.352	2.494	2.637	1.228	1.367	1.507	1.647	1.788	1.928	2.069	2.211	2.352	2.494	2.637
0.6	1.428	1.568	1.708	1.848	1.989	2.130	2.272	2.414	2.557	2.700	2.844	1.428	1.568	1.708	1.848	1.989	2.130	2.272	2.414	2.557	2.700	2.844
0.8	1.633	1.773	1.913	2.054	2.196	2.338	2.481	2.624	2.768	2.913	3.058	1.633	1.773	1.913	2.054	2.196	2.338	2.481	2.624	2.768	2.913	3.058
1.0	1.843	1.983	2.124	2.266	2.408	2.551	2.695	2.840	2.985	3.131	3.278	1.843	1.983	2.124	2.266	2.408	2.551	2.695	2.840	2.985	3.131	3.278
1.2	2.057	2.198	2.340	2.483	2.626	2.770	2.915	3.061	3.208	3.355	3.503	2.057	2.198	2.340	2.483	2.626	2.770	2.915	3.061	3.208	3.355	3.503
1.4	2.276	2.418	2.561	2.705	2.849	2.995	3.141	3.287	3.435	3.583	3.732	2.276	2.418	2.561	2.705	2.849	2.995	3.141	3.287	3.435	3.583	3.732
1.6	2.500	2.643	2.787	2.931	3.077	3.223	3.370	3.518	3.666	3.815	3.965	2.500	2.643	2.787	2.931	3.077	3.223	3.370	3.518	3.666	3.815	3.965
1.8	2.727	2.871	3.016	3.162	3.308	3.455	3.603	3.752	3.901	4.051	4.202	2.727	2.871	3.016	3.162	3.308	3.455	3.603	3.752	3.901	4.051	4.202
2.0	2.959	3.104	3.249	3.396	3.543	3.691	3.840	3.989	4.139	4.289	4.441	2.959	3.104	3.249	3.396	3.543	3.691	3.840	3.989	4.139	4.289	4.441

Table 2. (continued)

ϕ	$M = 1$						$K = 0.00$						$\mu = 1.50$									
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-2.142	-2.049	-1.952	-1.852	-1.748	-1.640	-1.527	-1.409	-1.286	-1.158	-1.025	-2.142	-2.049	-1.952	-1.852	-1.748	-1.640	-1.527	-1.409	-1.286	-1.158	-1.025
-1.8	-1.980	-1.885	-1.786	-1.685	-1.579	-1.469	-1.354	-1.234	-1.109	-0.979	-0.845	-1.980	-1.885	-1.786	-1.685	-1.579	-1.469	-1.354	-1.234	-1.109	-0.979	-0.845
-1.6	-1.815	-1.719	-1.619	-1.515	-1.407	-1.295	-1.178	-1.056	-0.930	-0.799	-0.665	-1.815	-1.719	-1.619	-1.515	-1.407	-1.295	-1.178	-1.056	-0.930	-0.799	-0.665
-1.4	-1.649	-1.551	-1.449	-1.343	-1.233	-1.119	-1.001	-0.878	-0.750	-0.619	-0.483	-1.649	-1.551	-1.449	-1.343	-1.233	-1.119	-1.001	-0.878	-0.750	-0.619	-0.483
-1.2	-1.481	-1.381	-1.277	-1.169	-1.058	-0.942	-0.822	-0.697	-0.569	-0.437	-0.302	-1.481	-1.381	-1.277	-1.169	-1.058	-0.942	-0.822	-0.697	-0.569	-0.437	-0.302
-1.0	-1.310	-1.208	-1.102	-0.993	-0.879	-0.762	-0.641	-0.515	-0.386	-0.254	-0.119	-1.310	-1.208	-1.102	-0.993	-0.879	-0.762	-0.641	-0.515	-0.386	-0.254	-0.119
-0.8	-1.137	-1.033	-0.925	-0.814	-0.699	-0.580	-0.458	-0.332	-0.203	-0.071	0.063	-1.137	-1.033	-0.925	-0.814	-0.699	-0.580	-0.458	-0.332	-0.203	-0.071	0.063
-0.6	-0.961	-0.855	-0.745	-0.632	-0.516	-0.396	-0.273	-0.147	-0.018	0.114	0.247	-0.961	-0.855	-0.745	-0.632	-0.516	-0.396	-0.273	-0.147	-0.018	0.114	0.247
-0.4	-0.781	-0.674	-0.563	-0.448	-0.331	-0.210	-0.086	0.040	0.169	0.299	0.431	-0.781	-0.674	-0.563	-0.448	-0.331	-0.210	-0.086	0.040	0.169	0.299	0.431
-0.2	-0.599	-0.489	-0.377	-0.261	-0.142	-0.021	0.102	0.229	0.357	0.487	0.617	-0.599	-0.489	-0.377	-0.261	-0.142	-0.021	0.102	0.229	0.357	0.487	0.617
0.0	-0.412	-0.301	-0.187	-0.071	0.049	0.170	0.294	0.420	0.547	0.676	0.805	-0.412	-0.301	-0.187	-0.071	0.049	0.170	0.294	0.420	0.547	0.676	0.805
0.2	-0.222	-0.110	0.005	0.123	0.243	0.365	0.489	0.614	0.740	0.868	0.996	-0.222	-0.110	0.005	0.123	0.243	0.365	0.489	0.614	0.740	0.868	0.996
0.4	-0.028	0.086	0.202	0.320	0.440	0.562	0.686	0.811	0.937	1.063	1.191	-0.028	0.086	0.202	0.320	0.440	0.562	0.686	0.811	0.937	1.063	1.191
0.6	0.170	0.285	0.402	0.521	0.642	0.764	0.887	1.012	1.137	1.264	1.390	0.170	0.285	0.402	0.521	0.642	0.764	0.887	1.012	1.137	1.264	1.390
0.8	0.373	0.489	0.607	0.726	0.847	0.970	1.093	1.217	1.343	1.469	1.596	0.373	0.489	0.607	0.726	0.847	0.970	1.093	1.217	1.343	1.469	1.596
1.0	0.580	0.697	0.816	0.936	1.057	1.180	1.303	1.428	1.554	1.680	1.808	0.580	0.697	0.816	0.936	1.057	1.180	1.303	1.428	1.554	1.680	1.808
1.2	0.792	0.910	1.029	1.150	1.272	1.395	1.519	1.644	1.770	1.898	2.026	0.792	0.910	1.029	1.150	1.272	1.395	1.519	1.644	1.770	1.898	2.026
1.4	1.009	1.127	1.247	1.369	1.491	1.615	1.740	1.866	1.993	2.121	2.250	1.009	1.127	1.247	1.369	1.491	1.615	1.740	1.866	1.993	2.121	2.250
1.6	1.229	1.349	1.470	1.592	1.715	1.840	1.965	2.092	2.220	2.349	2.479	1.229	1.349	1.470	1.592	1.715	1.840	1.965	2.092	2.220	2.349	2.479
1.8	1.454	1.574	1.696	1.819	1.943	2.069	2.195	2.323	2.452	2.582	2.713	1.454	1.574	1.696	1.819	1.943	2.069	2.195	2.323	2.452	2.582	2.713
2.0	1.682	1.803	1.926	2.050	2.175	2.302	2.429	2.558	2.688	2.819	2.951	1.682	1.803	1.926	2.050	2.175	2.302	2.429	2.558	2.688	2.819	2.951

Table 2. (continued)

ϕ	$\mu = 1.50$										
	$\kappa = 0.00$					$\mu = 1.50$					
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-1.025	-0.887	-0.746	-0.601	-0.454	-0.304	-0.153	-0.001	0.151	0.303	0.454
-1.8	-0.845	-0.707	-0.565	-0.421	-0.274	-0.126	0.024	0.174	0.323	0.472	0.620
-1.6	-0.665	-0.526	-0.385	-0.241	-0.095	0.052	0.199	0.347	0.494	0.640	0.784
-1.4	-0.483	-0.345	-0.204	-0.061	0.083	0.228	0.373	0.518	0.662	0.804	0.945
-1.2	-0.302	-0.164	-0.024	0.118	0.260	0.403	0.546	0.687	0.828	0.967	1.105
-1.0	-0.119	0.018	0.156	0.296	0.437	0.577	0.717	0.851	0.993	1.129	1.263
-0.8	0.063	0.199	0.336	0.474	0.612	0.750	0.887	1.023	1.157	1.290	1.421
-0.6	0.247	0.381	0.517	0.652	0.788	0.923	1.057	1.190	1.322	1.452	1.581
-0.4	0.431	0.564	0.698	0.831	0.964	1.097	1.229	1.359	1.489	1.618	1.746
-0.2	0.617	0.748	0.880	1.011	1.142	1.273	1.403	1.532	1.661	1.789	1.916
0.0	0.805	0.935	1.065	1.194	1.324	1.453	1.582	1.710	1.838	1.967	2.095
0.2	0.996	1.124	1.253	1.382	1.510	1.639	1.767	1.896	2.024	2.153	2.283
0.4	1.191	1.318	1.446	1.574	1.703	1.831	1.960	2.089	2.219	2.350	2.481
0.6	1.390	1.518	1.646	1.774	1.903	2.032	2.162	2.292	2.423	2.555	2.688
0.8	1.596	1.723	1.852	1.981	2.110	2.241	2.372	2.504	2.636	2.770	2.904
1.0	1.808	1.936	2.065	2.195	2.326	2.457	2.589	2.723	2.857	2.991	3.127
1.2	2.026	2.155	2.285	2.416	2.548	2.680	2.814	2.948	3.084	3.220	3.356
1.4	2.250	2.380	2.511	2.643	2.776	2.910	3.044	3.180	3.316	3.453	3.591
1.6	2.479	2.610	2.742	2.875	3.009	3.144	3.280	3.416	3.553	3.691	3.829
1.8	2.713	2.845	2.978	3.112	3.247	3.382	3.519	3.656	3.794	3.932	4.071
2.0	2.951	3.084	3.218	3.352	3.488	3.624	3.761	3.899	4.037	4.176	4.316

Table 2. (continued)

ϕ	$M = 1$						$K = 0.00$						$\mu = 2.00$									
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	-1.0	-0.8	-0.6	-0.4	-0.2	0.0				
-2.0	-1.830	-1.775	-1.718	-1.657	-1.592	-1.521	-1.442	-1.355	-1.257	-1.149	-1.031	-1.830	-1.775	-1.718	-1.657	-1.592	-1.521	-1.442	-1.355	-1.257	-1.149	-1.031
-1.8	-1.680	-1.624	-1.565	-1.502	-1.435	-1.361	-1.279	-1.188	-1.087	-0.976	-0.857	-1.680	-1.624	-1.565	-1.502	-1.435	-1.361	-1.279	-1.188	-1.087	-0.976	-0.857
-1.6	-1.528	-1.471	-1.410	-1.345	-1.275	-1.198	-1.112	-1.018	-0.915	-0.802	-0.682	-1.528	-1.471	-1.410	-1.345	-1.275	-1.198	-1.112	-1.018	-0.915	-0.802	-0.682
-1.4	-1.375	-1.315	-1.253	-1.185	-1.112	-1.031	-0.943	-0.846	-0.741	-0.628	-0.507	-1.375	-1.315	-1.253	-1.185	-1.112	-1.031	-0.943	-0.846	-0.741	-0.628	-0.507
-1.2	-1.218	-1.157	-1.092	-1.021	-0.945	-0.862	-0.771	-0.673	-0.566	-0.453	-0.334	-1.218	-1.157	-1.092	-1.021	-0.945	-0.862	-0.771	-0.673	-0.566	-0.453	-0.334
-1.0	-1.058	-0.995	-0.927	-0.854	-0.775	-0.689	-0.597	-0.497	-0.390	-0.278	-0.161	-1.058	-0.995	-0.927	-0.854	-0.775	-0.689	-0.597	-0.497	-0.390	-0.278	-0.161
-0.8	-0.894	-0.828	-0.758	-0.682	-0.601	-0.513	-0.420	-0.320	-0.214	-0.104	0.010	-0.894	-0.828	-0.758	-0.682	-0.601	-0.513	-0.420	-0.320	-0.214	-0.104	0.010
-0.6	-0.725	-0.656	-0.584	-0.506	-0.423	-0.334	-0.240	-0.141	-0.038	0.069	0.178	-0.725	-0.656	-0.584	-0.506	-0.423	-0.334	-0.240	-0.141	-0.038	0.069	0.178
-0.4	-0.550	-0.479	-0.404	-0.324	-0.240	-0.151	-0.058	0.039	0.139	0.241	0.345	-0.550	-0.479	-0.404	-0.324	-0.240	-0.151	-0.058	0.039	0.139	0.241	0.345
-0.2	-0.368	-0.295	-0.218	-0.137	-0.053	0.035	0.126	0.220	0.316	0.413	0.510	-0.368	-0.295	-0.218	-0.137	-0.053	0.035	0.126	0.220	0.316	0.413	0.510
0.0	-0.178	-0.103	-0.025	0.056	0.139	0.226	0.314	0.404	0.495	0.586	0.677	-0.178	-0.103	-0.025	0.056	0.139	0.226	0.314	0.404	0.495	0.586	0.677
0.2	0.021	0.096	0.175	0.255	0.337	0.421	0.506	0.591	0.678	0.764	0.850	0.021	0.096	0.175	0.255	0.337	0.421	0.506	0.591	0.678	0.764	0.850
0.4	0.228	0.304	0.382	0.460	0.540	0.621	0.703	0.785	0.868	0.951	1.035	0.228	0.304	0.382	0.460	0.540	0.621	0.703	0.785	0.868	0.951	1.035
0.6	0.443	0.519	0.595	0.672	0.750	0.829	0.908	0.988	1.069	1.151	1.234	0.443	0.519	0.595	0.672	0.750	0.829	0.908	0.988	1.069	1.151	1.234
0.8	0.666	0.740	0.815	0.891	0.967	1.044	1.123	1.202	1.283	1.365	1.448	0.666	0.740	0.815	0.891	0.967	1.044	1.123	1.202	1.283	1.365	1.448
1.0	0.893	0.967	1.041	1.116	1.192	1.269	1.347	1.427	1.508	1.591	1.676	0.893	0.967	1.041	1.116	1.192	1.269	1.347	1.427	1.508	1.591	1.676
1.2	1.125	1.198	1.272	1.347	1.423	1.501	1.581	1.662	1.744	1.828	1.914	1.125	1.198	1.272	1.347	1.423	1.501	1.581	1.662	1.744	1.828	1.914
1.4	1.360	1.434	1.508	1.584	1.662	1.741	1.822	1.904	1.988	2.073	2.160	1.360	1.434	1.508	1.584	1.662	1.741	1.822	1.904	1.988	2.073	2.160
1.6	1.600	1.674	1.750	1.827	1.906	1.987	2.069	2.152	2.237	2.324	2.411	1.600	1.674	1.750	1.827	1.906	1.987	2.069	2.152	2.237	2.324	2.411
1.8	1.843	1.919	1.996	2.075	2.156	2.238	2.321	2.406	2.492	2.579	2.667	1.843	1.919	1.996	2.075	2.156	2.238	2.321	2.406	2.492	2.579	2.667
2.0	2.091	2.168	2.247	2.328	2.409	2.492	2.577	2.663	2.750	2.838	2.927	2.091	2.168	2.247	2.328	2.409	2.492	2.577	2.663	2.750	2.838	2.927

Table 2. (continued)

ϕ	$M = 1$										$\mu = 2.00$												
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	
-2.0	-1.031	-0.905	-0.771	-0.633	-0.490	-0.345	-0.199	-0.052	0.095	0.240	0.383	-0.345	-0.199	-0.052	0.095	0.240	0.383	-0.345	-0.199	-0.052	0.095	0.240	0.383
-1.8	-0.857	-0.730	-0.597	-0.459	-0.318	-0.176	-0.033	0.110	0.252	0.391	0.527	-0.176	-0.033	0.110	0.252	0.391	0.527	-0.176	-0.033	0.110	0.252	0.391	0.527
-1.6	-0.682	-0.555	-0.423	-0.287	-0.150	-0.011	0.128	0.266	0.401	0.533	0.661	-0.011	0.128	0.266	0.401	0.533	0.661	-0.011	0.128	0.266	0.401	0.533	0.661
-1.4	-0.507	-0.382	-0.252	-0.119	0.015	0.149	0.283	0.414	0.542	0.665	0.784	0.015	0.149	0.283	0.414	0.542	0.665	0.015	0.149	0.283	0.414	0.542	0.784
-1.2	-0.334	-0.210	-0.083	0.046	0.175	0.303	0.429	0.553	0.672	0.787	0.896	0.175	0.303	0.429	0.553	0.672	0.787	0.175	0.303	0.429	0.553	0.672	0.896
-1.0	-0.161	-0.040	0.082	0.205	0.328	0.449	0.568	0.682	0.792	0.898	0.914	0.328	0.449	0.568	0.682	0.792	0.898	0.328	0.449	0.568	0.682	0.792	0.898
-0.8	0.010	0.126	0.243	0.360	0.475	0.588	0.698	0.803	0.905	1.009	1.097	0.475	0.588	0.698	0.803	0.905	1.009	0.475	0.588	0.698	0.803	0.905	1.097
-0.6	0.178	0.289	0.399	0.509	0.580	0.720	0.821	0.919	1.010	1.105	1.194	0.580	0.720	0.821	0.919	1.010	1.105	0.580	0.720	0.821	0.919	1.010	1.194
-0.4	0.345	0.449	0.552	0.653	0.753	0.849	0.943	1.034	1.123	1.211	1.295	0.753	0.849	0.943	1.034	1.123	1.211	0.753	0.849	0.943	1.034	1.123	1.295
-0.2	0.510	0.607	0.703	0.797	0.890	0.980	1.069	1.156	1.242	1.327	1.410	0.890	0.980	1.069	1.156	1.242	1.327	0.890	0.980	1.069	1.156	1.242	1.410
0.0	0.677	0.768	0.858	0.947	1.035	1.121	1.207	1.293	1.378	1.463	1.549	1.035	1.121	1.207	1.293	1.378	1.463	1.035	1.121	1.207	1.293	1.378	1.549
0.2	0.850	0.937	1.023	1.109	1.194	1.280	1.365	1.452	1.539	1.627	1.717	1.194	1.280	1.365	1.452	1.539	1.627	1.194	1.280	1.365	1.452	1.539	1.717
0.4	1.035	1.119	1.203	1.288	1.374	1.461	1.548	1.636	1.726	1.817	1.909	1.374	1.461	1.548	1.636	1.726	1.817	1.374	1.461	1.548	1.636	1.726	1.909
0.6	1.234	1.317	1.402	1.488	1.575	1.663	1.752	1.843	1.935	2.028	2.122	1.575	1.663	1.752	1.843	1.935	2.028	1.575	1.663	1.752	1.843	1.935	2.122
0.8	1.448	1.533	1.619	1.706	1.794	1.884	1.975	2.067	2.160	2.255	2.350	1.794	1.884	1.975	2.067	2.160	2.255	1.794	1.884	1.975	2.067	2.160	2.350
1.0	1.676	1.761	1.849	1.937	2.027	2.118	2.209	2.303	2.398	2.493	2.589	2.027	2.118	2.209	2.303	2.398	2.493	2.027	2.118	2.209	2.303	2.398	2.589
1.2	1.914	2.001	2.089	2.179	2.269	2.361	2.454	2.548	2.643	2.739	2.835	2.269	2.361	2.454	2.548	2.643	2.739	2.269	2.361	2.454	2.548	2.643	2.835
1.4	2.160	2.248	2.337	2.427	2.519	2.611	2.705	2.796	2.894	2.990	3.087	2.519	2.611	2.705	2.796	2.894	2.990	2.519	2.611	2.705	2.796	2.894	3.087
1.6	2.411	2.500	2.590	2.681	2.773	2.866	2.960	3.055	3.150	3.246	3.343	2.773	2.866	2.960	3.055	3.150	3.246	2.773	2.866	2.960	3.055	3.150	3.343
1.8	2.667	2.757	2.847	2.939	3.031	3.125	3.219	3.314	3.403	3.505	3.602	3.031	3.125	3.219	3.314	3.403	3.505	3.031	3.125	3.219	3.314	3.403	3.602
2.0	2.927	3.017	3.108	3.200	3.292	3.386	3.480	3.575	3.671	3.767	3.864	3.292	3.386	3.480	3.575	3.671	3.767	3.292	3.386	3.480	3.575	3.671	3.864

Table 2. (continued)

ϕ	$M = 1$					$K = 0.00$					$\mu = 2.50$					-0.2	0.0																								
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8			1.0	1.2	1.4	1.6	1.8	2.0																		
-2.0	-1.674	-1.637	-1.598	-1.557	-1.512	-1.460	-1.401	-1.330	-1.246	-1.146	-1.033	-0.933	-0.860	-0.804	-0.739	-0.631	-0.515	-0.344	-0.176	-0.013	0.145	0.295	0.440	0.585	0.741	0.921	1.128	1.357	1.600	1.854	2.113	2.378	2.645	2.915							
-1.8	-1.530	-1.493	-1.453	-1.410	-1.362	-1.308	-1.244	-1.168	-1.079	-0.976	-0.860	-0.767	-0.687	-0.604	-0.515	-0.419	-0.327	-0.230	-0.129	-0.022	0.045	0.143	0.206	0.283	0.362	0.449	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851
-1.6	-1.385	-1.347	-1.305	-1.260	-1.210	-1.152	-1.084	-1.003	-0.910	-0.804	-0.687	-0.585	-0.492	-0.394	-0.288	-0.176	-0.063	0.045	0.145	0.206	0.283	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851			
-1.4	-1.238	-1.198	-1.155	-1.107	-1.054	-0.992	-0.919	-0.835	-0.739	-0.631	-0.515	-0.419	-0.327	-0.230	-0.129	-0.022	0.045	0.145	0.206	0.283	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851				
-1.2	-1.089	-1.047	-1.002	-0.951	-0.894	-0.828	-0.752	-0.665	-0.567	-0.459	-0.344	-0.230	-0.129	-0.022	0.045	0.145	0.206	0.283	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851						
-1.0	-0.936	-0.892	-0.844	-0.790	-0.729	-0.660	-0.581	-0.492	-0.394	-0.288	-0.176	-0.063	0.045	0.145	0.206	0.283	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851								
-0.8	-0.778	-0.731	-0.680	-0.623	-0.559	-0.487	-0.407	-0.318	-0.222	-0.120	-0.013	0.045	0.145	0.206	0.283	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851									
-0.6	-0.614	-0.564	-0.510	-0.450	-0.384	-0.310	-0.230	-0.143	-0.051	0.045	0.145	0.206	0.283	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851											
-0.4	-0.441	-0.389	-0.332	-0.270	-0.202	-0.130	-0.051	0.031	0.117	0.206	0.295	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851													
-0.2	-0.258	-0.203	-0.145	-0.082	-0.015	0.055	0.129	0.205	0.283	0.362	0.440	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851															
0.0	-0.063	-0.007	0.052	0.114	0.178	0.244	0.312	0.380	0.449	0.517	0.585	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851																	
0.2	0.145	0.202	0.260	0.319	0.378	0.439	0.499	0.560	0.620	0.681	0.741	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851																				
0.4	0.367	0.421	0.476	0.530	0.585	0.640	0.695	0.751	0.806	0.863	0.921	1.070	1.297	1.539	1.792	2.051	2.314	2.581	2.851																						
0.6	0.596	0.647	0.698	0.749	0.800	0.852	0.904	0.958	1.013	1.070	1.128	1.357	1.600	1.854	2.113	2.378	2.645	2.915																							
0.8	0.829	0.877	0.926	0.975	1.025	1.076	1.129	1.183	1.239	1.297	1.357	1.600	1.854	2.113	2.378	2.645	2.915																								
1.0	1.064	1.111	1.160	1.209	1.260	1.312	1.367	1.423	1.480	1.539	1.600	1.854	2.113	2.378	2.645	2.915																									
1.2	1.303	1.351	1.400	1.452	1.504	1.558	1.614	1.672	1.731	1.792	1.854	2.113	2.378	2.645	2.915																										
1.4	1.547	1.597	1.649	1.702	1.756	1.812	1.870	1.929	1.989	2.051	2.113	2.378	2.645	2.915																											
1.6	1.798	1.850	1.903	1.958	2.014	2.072	2.130	2.190	2.252	2.314	2.378	2.645	2.915																												
1.8	2.054	2.107	2.162	2.219	2.276	2.335	2.395	2.456	2.518	2.581	2.645	2.915																													
2.0	2.314	2.369	2.426	2.483	2.542	2.602	2.663	2.724	2.787	2.851	2.915																														

Table 2. (continued)

ϕ	$\mu = 1$					$\mu = 2.50$					
	$\kappa = 0.00$					$\kappa = 1$					
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-1.033	-0.909	-0.777	-0.638	-0.496	-0.351	-0.205	-0.058	0.088	0.232	0.373
-1.8	-0.860	-0.735	-0.603	-0.466	-0.326	-0.184	-0.041	0.101	0.241	0.378	0.512
-1.6	-0.687	-0.562	-0.432	-0.297	-0.160	-0.022	0.115	0.251	0.384	0.513	0.636
-1.4	-0.515	-0.392	-0.263	-0.132	0.000	0.132	0.263	0.390	0.513	0.630	0.741
-1.2	-0.344	-0.224	-0.100	0.026	0.152	0.276	0.397	0.514	0.625	0.728	0.824
-1.0	-0.176	-0.061	0.058	0.176	0.293	0.407	0.516	0.619	0.716	0.805	0.888
-0.8	-0.013	0.096	0.206	0.314	0.420	0.521	0.617	0.706	0.775	0.866	0.936
-0.6	0.145	0.244	0.343	0.440	0.532	0.588	0.701	0.778	0.851	0.917	0.977
-0.4	0.295	0.384	0.470	0.553	0.633	0.708	0.779	0.847	0.911	0.970	1.026
-0.2	0.440	0.516	0.591	0.662	0.731	0.798	0.862	0.924	0.984	1.043	1.100
0.0	0.585	0.651	0.716	0.780	0.843	0.905	0.966	1.028	1.090	1.152	1.215
0.2	0.741	0.802	0.862	0.923	0.984	1.046	1.108	1.173	1.238	1.305	1.373
0.4	0.921	0.980	1.040	1.101	1.163	1.227	1.292	1.359	1.427	1.497	1.568
0.6	1.128	1.187	1.249	1.311	1.375	1.441	1.508	1.576	1.646	1.717	1.789
0.8	1.357	1.418	1.480	1.544	1.610	1.676	1.744	1.813	1.883	1.955	2.027
1.0	1.600	1.662	1.726	1.791	1.857	1.925	1.993	2.062	2.133	2.204	2.276
1.2	1.854	1.917	1.981	2.047	2.113	2.181	2.250	2.319	2.390	2.461	2.533
1.4	2.113	2.177	2.242	2.308	2.375	2.443	2.512	2.581	2.652	2.723	2.794
1.6	2.378	2.442	2.508	2.574	2.641	2.709	2.778	2.847	2.917	2.988	3.060
1.8	2.645	2.710	2.776	2.842	2.910	2.978	3.047	3.116	3.186	3.257	3.328
2.0	2.915	2.980	3.046	3.113	3.181	3.249	3.317	3.387	3.456	3.527	3.595

Table 2. (continued)

ϕ	$\mu = 3.00$										
	$\kappa = 0.00$										
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-1.582	-1.555	-1.526	-1.496	-1.462	-1.423	-1.376	-1.317	-1.240	-1.145	-1.034
-1.8	-1.442	-1.414	-1.385	-1.354	-1.318	-1.276	-1.224	-1.158	-1.075	-0.975	-0.861
-1.6	-1.301	-1.273	-1.243	-1.209	-1.171	-1.125	-1.068	-0.996	-0.908	-0.805	-0.689
-1.4	-1.159	-1.129	-1.097	-1.062	-1.020	-0.970	-0.907	-0.830	-0.738	-0.633	-0.518
-1.2	-1.014	-0.983	-0.949	-0.910	-0.865	-0.810	-0.743	-0.662	-0.567	-0.462	-0.348
-1.0	-0.866	-0.833	-0.796	-0.754	-0.704	-0.645	-0.574	-0.491	-0.396	-0.293	-0.183
-0.8	-0.713	-0.677	-0.638	-0.592	-0.538	-0.475	-0.402	-0.319	-0.226	-0.127	-0.023
-0.6	-0.553	-0.514	-0.471	-0.421	-0.365	-0.300	-0.227	-0.147	-0.059	0.033	0.127
-0.4	-0.383	-0.341	-0.295	-0.243	-0.185	-0.121	-0.051	0.024	0.103	0.184	0.265
-0.2	-0.199	-0.155	-0.106	-0.054	0.002	0.062	0.125	0.195	0.258	0.325	0.390
0.0	0.001	0.046	0.094	0.144	0.196	0.249	0.303	0.356	0.410	0.462	0.514
0.2	0.219	0.263	0.307	0.352	0.396	0.440	0.484	0.527	0.571	0.614	0.658
0.4	0.450	0.489	0.528	0.566	0.603	0.641	0.679	0.717	0.757	0.798	0.840
0.6	0.685	0.719	0.753	0.787	0.821	0.857	0.894	0.933	0.973	1.015	1.059
0.8	0.919	0.951	0.984	1.018	1.054	1.091	1.129	1.170	1.212	1.256	1.301
1.0	1.158	1.191	1.225	1.261	1.299	1.338	1.379	1.421	1.464	1.510	1.556
1.2	1.404	1.439	1.476	1.514	1.554	1.595	1.637	1.681	1.726	1.772	1.819
1.4	1.658	1.695	1.734	1.774	1.815	1.858	1.902	1.947	1.993	2.039	2.087
1.6	1.918	1.957	1.998	2.039	2.082	2.126	2.171	2.216	2.263	2.311	2.359
1.8	2.183	2.224	2.266	2.309	2.353	2.397	2.443	2.489	2.536	2.584	2.633
2.0	2.452	2.494	2.537	2.581	2.625	2.671	2.717	2.764	2.812	2.860	2.909

Table 2. (continued)

ϕ	$M = 1$										$\mu = 3.00$												
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	
-2.0	-1.034	-0.910	-0.778	-0.640	-0.497	-0.352	-0.206	-0.059	0.087	0.231	0.372	-0.352	-0.206	-0.059	0.087	0.231	0.372	-0.352	-0.206	-0.059	0.087	0.231	0.372
-1.8	-0.861	-0.737	-0.605	-0.468	-0.328	-0.186	-0.043	0.099	0.239	0.376	0.509	-0.186	-0.043	0.099	0.239	0.376	0.509	-0.186	-0.043	0.099	0.239	0.376	0.509
-1.6	-0.689	-0.565	-0.434	-0.299	-0.162	-0.025	0.113	0.248	0.380	0.508	0.630	-0.025	0.113	0.248	0.380	0.508	0.630	-0.025	0.113	0.248	0.380	0.508	0.630
-1.4	-0.518	-0.395	-0.267	-0.136	-0.004	0.128	0.258	0.384	0.506	0.621	0.729	0.128	0.258	0.384	0.506	0.621	0.729	0.128	0.258	0.384	0.506	0.621	0.729
-1.2	-0.348	-0.228	-0.105	0.020	0.145	0.269	0.388	0.502	0.609	0.708	0.773	0.269	0.388	0.502	0.609	0.708	0.773	0.269	0.388	0.502	0.609	0.708	0.773
-1.0	-0.183	-0.068	0.049	0.166	0.281	0.392	0.497	0.595	0.684	0.764	0.836	0.392	0.497	0.595	0.684	0.764	0.836	0.392	0.497	0.595	0.684	0.764	0.836
-0.8	-0.023	0.084	0.191	0.295	0.396	0.491	0.579	0.658	0.729	0.786	0.845	0.491	0.579	0.658	0.729	0.786	0.845	0.491	0.579	0.658	0.729	0.786	0.845
-0.6	0.127	0.222	0.315	0.404	0.488	0.564	0.634	0.697	0.753	0.801	0.842	0.564	0.634	0.697	0.753	0.801	0.842	0.564	0.634	0.697	0.753	0.801	0.842
-0.4	0.265	0.344	0.399	0.492	0.558	0.608	0.675	0.726	0.773	0.814	0.852	0.608	0.675	0.726	0.773	0.814	0.852	0.608	0.675	0.726	0.773	0.814	0.852
-0.2	0.390	0.453	0.513	0.569	0.623	0.674	0.722	0.768	0.812	0.855	0.897	0.674	0.722	0.768	0.812	0.855	0.897	0.674	0.722	0.768	0.812	0.855	0.897
0.0	0.514	0.564	0.613	0.661	0.708	0.755	0.801	0.848	0.896	0.944	0.994	0.755	0.801	0.848	0.896	0.944	0.994	0.755	0.801	0.848	0.896	0.944	0.994
0.2	0.658	0.702	0.747	0.792	0.839	0.887	0.936	0.987	1.039	1.093	1.147	0.887	0.936	0.987	1.039	1.093	1.147	0.887	0.936	0.987	1.039	1.093	1.147
0.4	0.840	0.884	0.930	0.976	1.025	1.075	1.127	1.180	1.234	1.290	1.346	1.075	1.127	1.180	1.234	1.290	1.346	1.075	1.127	1.180	1.234	1.290	1.346
0.6	1.059	1.104	1.151	1.192	1.250	1.301	1.353	1.407	1.462	1.518	1.575	1.301	1.353	1.407	1.462	1.518	1.575	1.301	1.353	1.407	1.462	1.518	1.575
0.8	1.301	1.348	1.396	1.445	1.496	1.547	1.600	1.654	1.709	1.765	1.821	1.547	1.600	1.654	1.709	1.765	1.821	1.547	1.600	1.654	1.709	1.765	1.821
1.0	1.556	1.604	1.653	1.702	1.753	1.805	1.858	1.912	1.967	2.022	2.078	1.805	1.858	1.912	1.967	2.022	2.078	1.805	1.858	1.912	1.967	2.022	2.078
1.2	1.819	1.867	1.917	1.967	2.018	2.070	2.123	2.177	2.231	2.286	2.342	2.070	2.123	2.177	2.231	2.286	2.342	2.070	2.123	2.177	2.231	2.286	2.342
1.4	2.087	2.136	2.186	2.236	2.288	2.340	2.392	2.446	2.500	2.555	2.610	2.340	2.392	2.446	2.500	2.555	2.610	2.340	2.392	2.446	2.500	2.555	2.610
1.6	2.359	2.408	2.458	2.509	2.560	2.612	2.665	2.718	2.771	2.826	2.881	2.612	2.665	2.718	2.771	2.826	2.881	2.612	2.665	2.718	2.771	2.826	2.881
1.8	2.633	2.682	2.733	2.783	2.835	2.887	2.939	2.992	3.046	3.100	3.154	2.887	2.939	2.992	3.046	3.100	3.154	2.887	2.939	2.992	3.046	3.100	3.154
2.0	2.909	2.958	3.009	3.059	3.111	3.163	3.215	3.268	3.321	3.375	3.430	3.163	3.215	3.268	3.321	3.375	3.430	3.163	3.215	3.268	3.321	3.375	3.430

Table 2. (continued)

ϕ	$\mu = 4.00$										
	$\kappa = 0.00$										
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-1.034	-0.911	-0.779	-0.640	-0.498	-0.353	-0.206	-0.059	0.086	0.230	0.372
-1.8	-0.862	-0.738	-0.606	-0.468	-0.328	-0.186	-0.043	0.099	0.239	0.376	0.509
-1.6	-0.690	-0.566	-0.435	-0.300	-0.163	-0.025	0.112	0.247	0.379	0.507	0.629
-1.4	-0.519	-0.396	-0.268	-0.137	-0.005	0.127	0.256	0.382	0.503	0.618	0.724
-1.2	-0.350	-0.231	-0.107	0.018	0.143	0.265	0.384	0.497	0.602	0.698	0.784
-1.0	-0.186	-0.072	0.045	0.161	0.275	0.384	0.487	0.581	0.665	0.737	0.798
-0.8	-0.030	0.076	0.181	0.283	0.381	0.471	0.551	0.610	0.679	0.726	0.760
-0.6	0.113	0.204	0.292	0.374	0.448	0.513	0.568	0.614	0.650	0.677	0.698
-0.4	0.233	0.302	0.367	0.419	0.473	0.516	0.553	0.584	0.610	0.632	0.652
-0.2	0.324	0.370	0.412	0.450	0.485	0.518	0.547	0.575	0.601	0.627	0.653
0.0	0.411	0.442	0.472	0.502	0.532	0.562	0.593	0.624	0.657	0.690	0.724
0.2	0.543	0.569	0.591	0.627	0.658	0.691	0.725	0.761	0.790	0.812	0.835
0.4	0.742	0.770	0.791	0.831	0.863	0.897	0.932	0.969	1.006	1.044	1.083
0.6	0.982	1.012	1.042	1.074	1.107	1.141	1.176	1.212	1.249	1.287	1.325
0.8	1.241	1.272	1.303	1.335	1.368	1.402	1.437	1.473	1.509	1.546	1.584
1.0	1.510	1.541	1.573	1.602	1.638	1.672	1.706	1.742	1.777	1.814	1.851
1.2	1.784	1.815	1.847	1.879	1.913	1.946	1.964	2.015	2.051	2.086	2.122
1.4	2.061	2.092	2.124	2.157	2.190	2.224	2.258	2.292	2.327	2.363	2.399
1.6	2.340	2.372	2.404	2.436	2.469	2.503	2.529	2.571	2.606	2.641	2.676
1.8	2.621	2.652	2.685	2.717	2.750	2.783	2.817	2.851	2.886	2.921	2.956
2.0	2.903	2.934	2.966	2.999	3.032	3.065	3.099	3.133	3.167	3.202	3.237

Table 2. (continued)

ϕ	$\mu = 5.00$										
	$\kappa = 0.00$										
	$M = 1$	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-1.426	-1.414	-1.402	-1.388	-1.373	-1.355	-1.331	-1.294	-1.233	-1.144	-1.034
-1.8	-1.292	-1.280	-1.268	-1.254	-1.237	-1.217	-1.189	-1.143	-1.072	-0.975	-0.862
-1.6	-1.158	-1.146	-1.133	-1.118	-1.100	-1.077	-1.042	-0.987	-0.906	-0.805	-0.690
-1.4	-1.024	-1.011	-0.997	-0.981	-0.961	-0.933	-0.890	-0.825	-0.738	-0.635	-0.519
-1.2	-0.888	-0.875	-0.860	-0.841	-0.817	-0.783	-0.731	-0.659	-0.569	-0.464	-0.351
-1.0	-0.750	-0.736	-0.719	-0.697	-0.668	-0.626	-0.567	-0.490	-0.399	-0.297	-0.187
-0.8	-0.609	-0.592	-0.572	-0.546	-0.510	-0.462	-0.398	-0.321	-0.232	-0.134	-0.031
-0.6	-0.461	-0.441	-0.416	-0.384	-0.343	-0.291	-0.227	-0.153	-0.070	0.018	0.109
-0.4	-0.300	-0.276	-0.246	-0.210	-0.167	-0.116	-0.057	0.009	0.078	0.149	0.218
-0.2	-0.118	-0.090	-0.059	-0.023	0.016	0.060	0.105	0.153	0.200	0.244	0.284
0.0	0.093	0.170	0.147	0.175	0.202	0.228	0.253	0.277	0.299	0.320	0.341
0.2	0.328	0.345	0.360	0.373	0.386	0.398	0.411	0.424	0.439	0.454	0.471
0.4	0.557	0.566	0.575	0.585	0.596	0.608	0.621	0.636	0.652	0.670	0.689
0.6	0.791	0.801	0.813	0.825	0.839	0.853	0.869	0.886	0.904	0.923	0.943
0.8	1.042	1.055	1.069	1.084	1.100	1.116	1.134	1.152	1.172	1.192	1.212
1.0	1.306	1.321	1.337	1.353	1.370	1.388	1.407	1.426	1.446	1.467	1.488
1.2	1.577	1.594	1.611	1.628	1.646	1.665	1.684	1.704	1.725	1.746	1.767
1.4	1.853	1.870	1.888	1.907	1.925	1.945	1.965	1.985	2.006	2.027	2.049
1.6	2.132	2.150	2.168	2.187	2.207	2.226	2.247	2.267	2.288	2.310	2.331
1.8	2.413	2.431	2.450	2.470	2.489	2.509	2.530	2.551	2.572	2.593	2.615
2.0	2.695	2.714	2.733	2.753	2.773	2.793	2.814	2.835	2.856	2.878	2.900

Table 3. Computed Eigenvalues for $m = 2$

ϕ	$\mu = 1.01$									
	$\kappa = 0.00$					$\mu = 1.01$				
	$M = 2$	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	
-2.0	-5.939	-5.546	-5.153	-4.760	-4.366	-3.972	-3.579	-3.185	-2.791	-2.396
-1.8	-5.542	-5.149	-4.756	-4.362	-3.969	-3.575	-3.181	-2.787	-2.392	-1.998
-1.6	-5.145	-4.752	-4.358	-3.965	-3.571	-3.177	-2.783	-2.389	-1.994	-1.600
-1.4	-4.748	-4.354	-3.961	-3.567	-3.173	-2.779	-2.385	-1.990	-1.596	-1.201
-1.2	-4.350	-3.957	-3.563	-3.169	-2.775	-2.381	-1.986	-1.592	-1.197	-0.802
-1.0	-3.953	-3.559	-3.165	-2.771	-2.377	-1.982	-1.588	-1.193	-0.798	-0.403
-0.8	-3.555	-3.161	-2.767	-2.373	-1.978	-1.584	-1.189	-0.794	-0.399	-0.004
-0.6	-3.157	-2.763	-2.369	-1.974	-1.580	-1.185	-0.790	-0.395	0.000	0.395
-0.4	-2.759	-2.365	-1.970	-1.576	-1.181	-0.786	-0.391	0.004	0.399	0.795
-0.2	-2.361	-1.966	-1.572	-1.177	-0.782	-0.387	0.008	0.403	0.799	1.194
0.0	-1.962	-1.568	-1.173	-0.778	-0.383	0.012	0.407	0.803	1.198	1.594
0.2	-1.564	-1.169	-0.774	-0.379	0.016	0.411	0.807	1.202	1.598	1.994
0.4	-1.165	-0.770	-0.375	0.020	0.415	0.811	1.206	1.602	1.998	2.394
0.6	-0.766	-0.371	0.024	0.419	0.815	1.210	1.606	2.002	2.398	2.794
0.8	-0.367	0.028	0.423	0.819	1.214	1.610	2.006	2.402	2.798	3.195
1.0	0.032	0.427	0.823	1.218	1.614	2.010	2.406	2.802	3.199	3.595
1.2	0.431	0.827	1.222	1.618	2.014	2.410	2.806	3.203	3.599	3.996
1.4	0.831	1.226	1.622	2.018	2.414	2.810	3.207	3.603	4.000	4.397
1.6	1.230	1.626	2.022	2.418	2.814	3.208	3.607	4.004	4.401	4.798
1.8	1.630	2.026	2.422	2.818	3.215	3.611	4.008	4.405	4.802	5.199
2.0	2.030	2.426	2.822	3.219	3.615	4.012	4.409	4.806	5.203	5.600

Table 3. (continued)

ϕ	$M = 2$										$\mu = 1.01$											
	$\kappa = 0.00$					$\mu = 1.01$					$\kappa = 0.00$					$\mu = 1.01$						
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.002	-1.607	-1.213	-0.818	-0.423	-0.027	0.368	0.763	1.159	1.555	1.951	-2.002	-1.607	-1.213	-0.818	-0.423	-0.027	0.368	0.763	1.159	1.555	1.951
-1.8	-1.603	-1.209	-0.814	-0.419	-0.024	0.372	0.767	1.163	1.559	1.955	2.351	-1.603	-1.209	-0.814	-0.419	-0.024	0.372	0.767	1.163	1.559	1.955	2.351
-1.6	-1.205	-0.810	-0.415	-0.020	0.376	0.771	1.167	1.563	1.959	2.355	2.751	-1.205	-0.810	-0.415	-0.020	0.376	0.771	1.167	1.563	1.959	2.355	2.751
-1.4	-0.806	-0.411	-0.016	0.380	0.775	1.171	1.567	1.963	2.359	2.755	3.151	-0.806	-0.411	-0.016	0.380	0.775	1.171	1.567	1.963	2.359	2.755	3.151
-1.2	-0.407	-0.012	0.384	0.779	1.175	1.571	1.966	2.363	2.759	3.155	3.552	-0.407	-0.012	0.384	0.779	1.175	1.571	1.966	2.363	2.759	3.155	3.552
-1.0	-0.008	0.387	0.783	1.179	1.574	1.970	2.367	2.763	3.159	3.556	3.952	-0.008	0.387	0.783	1.179	1.574	1.970	2.367	2.763	3.159	3.556	3.952
-0.8	0.391	0.787	1.183	1.578	1.974	2.370	2.767	3.163	3.560	3.956	4.353	0.391	0.787	1.183	1.578	1.974	2.370	2.767	3.163	3.560	3.956	4.353
-0.6	0.791	1.186	1.582	1.978	2.374	2.771	3.167	3.564	3.960	4.357	4.754	0.791	1.186	1.582	1.978	2.374	2.771	3.167	3.564	3.960	4.357	4.754
-0.4	1.190	1.586	1.982	2.378	2.775	3.171	3.568	3.964	4.361	4.758	5.155	1.190	1.586	1.982	2.378	2.775	3.171	3.568	3.964	4.361	4.758	5.155
-0.2	1.590	1.986	2.382	2.778	3.175	3.571	3.968	4.365	4.762	5.159	5.557	1.590	1.986	2.382	2.778	3.175	3.571	3.968	4.365	4.762	5.159	5.557
0.0	1.990	2.386	2.782	3.179	3.575	3.972	4.369	4.766	5.163	5.560	5.958	1.990	2.386	2.782	3.179	3.575	3.972	4.369	4.766	5.163	5.560	5.958
0.2	2.390	2.786	3.183	3.579	3.976	4.373	4.770	5.167	5.564	5.962	6.360	2.390	2.786	3.183	3.579	3.976	4.373	4.770	5.167	5.564	5.962	6.360
0.4	2.790	3.187	3.583	3.980	4.377	4.774	5.171	5.568	5.966	6.364	6.761	2.790	3.187	3.583	3.980	4.377	4.774	5.171	5.568	5.966	6.364	6.761
0.6	3.191	3.587	3.984	4.381	4.778	5.175	5.572	5.970	6.367	6.765	7.163	3.191	3.587	3.984	4.381	4.778	5.175	5.572	5.970	6.367	6.765	7.163
0.8	3.591	3.988	4.385	4.782	5.179	5.576	5.974	6.371	6.769	7.167	7.565	3.591	3.988	4.385	4.782	5.179	5.576	5.974	6.371	6.769	7.167	7.565
1.0	3.992	4.389	4.786	5.183	5.580	5.978	6.375	6.773	7.171	7.569	7.967	3.992	4.389	4.786	5.183	5.580	5.978	6.375	6.773	7.171	7.569	7.967
1.2	4.393	4.790	5.187	5.584	5.982	6.379	6.777	7.175	7.573	7.972	8.370	4.393	4.790	5.187	5.584	5.982	6.379	6.777	7.175	7.573	7.972	8.370
1.4	4.794	5.191	5.588	5.986	6.384	6.781	7.179	7.577	7.976	8.374	8.772	4.794	5.191	5.588	5.986	6.384	6.781	7.179	7.577	7.976	8.374	8.772
1.6	5.195	5.592	5.990	6.388	6.785	7.183	7.581	7.980	8.378	8.777	9.175	5.195	5.592	5.990	6.388	6.785	7.183	7.581	7.980	8.378	8.777	9.175
1.8	5.596	5.994	6.392	6.789	7.187	7.585	7.984	8.382	8.781	9.179	9.578	5.596	5.994	6.392	6.789	7.187	7.585	7.984	8.382	8.781	9.179	9.578
2.0	5.998	6.396	6.793	7.191	7.589	7.988	8.386	8.785	9.183	9.582	9.981	5.998	6.396	6.793	7.191	7.589	7.988	8.386	8.785	9.183	9.582	9.981

Table 3. (continued)

ϕ	$M = 2$										$\mu = 1.05$											
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.009	-1.634	-1.259	-0.882	-0.505	-0.127	0.252	0.631	1.011	1.391	1.772	-2.009	-1.634	-1.259	-0.882	-0.505	-0.127	0.252	0.631	1.011	1.391	1.772
-1.8	-1.616	-1.241	-0.864	-0.487	-0.109	0.269	0.648	1.028	1.408	1.789	2.171	-1.616	-1.241	-0.864	-0.487	-0.109	0.269	0.648	1.028	1.408	1.789	2.171
-1.6	-1.223	-0.846	-0.469	-0.092	0.287	0.666	1.045	1.426	1.806	2.188	2.570	-1.223	-0.846	-0.469	-0.092	0.287	0.666	1.045	1.426	1.806	2.188	2.570
-1.4	-0.828	-0.452	-0.074	0.304	0.683	1.063	1.443	1.824	2.205	2.587	2.970	-0.828	-0.452	-0.074	0.304	0.683	1.063	1.443	1.824	2.205	2.587	2.970
-1.2	-0.433	-0.056	0.322	0.701	1.081	1.461	1.842	2.223	2.605	2.987	3.370	-0.433	-0.056	0.322	0.701	1.081	1.461	1.842	2.223	2.605	2.987	3.370
-1.0	-0.038	0.340	0.719	1.099	1.479	1.859	2.241	2.623	3.005	3.388	3.772	-0.038	0.340	0.719	1.099	1.479	1.859	2.241	2.623	3.005	3.388	3.772
-0.8	0.359	0.737	1.117	1.497	1.877	2.259	2.641	3.023	3.406	3.790	4.174	0.359	0.737	1.117	1.497	1.877	2.259	2.641	3.023	3.406	3.790	4.174
-0.6	0.756	1.135	1.515	1.896	2.277	2.659	3.041	3.424	3.808	4.192	4.576	0.756	1.135	1.515	1.896	2.277	2.659	3.041	3.424	3.808	4.192	4.576
-0.4	1.154	1.533	1.914	2.295	2.677	3.059	3.442	3.826	4.210	4.595	4.980	1.154	1.533	1.914	2.295	2.677	3.059	3.442	3.826	4.210	4.595	4.980
-0.2	1.552	1.933	2.314	2.696	3.078	3.461	3.845	4.229	4.613	4.999	5.384	1.552	1.933	2.314	2.696	3.078	3.461	3.845	4.229	4.613	4.999	5.384
0.0	1.951	2.333	2.714	3.097	3.480	3.863	4.247	4.632	5.017	5.403	5.789	1.951	2.333	2.714	3.097	3.480	3.863	4.247	4.632	5.017	5.403	5.789
0.2	2.351	2.733	3.116	3.499	3.882	4.266	4.651	5.036	5.422	5.809	6.196	2.351	2.733	3.116	3.499	3.882	4.266	4.651	5.036	5.422	5.809	6.196
0.4	2.752	3.135	3.518	3.901	4.285	4.670	5.056	5.442	5.828	6.215	6.603	2.752	3.135	3.518	3.901	4.285	4.670	5.056	5.442	5.828	6.215	6.603
0.6	3.154	3.537	3.921	4.305	4.690	5.076	5.462	5.847	6.235	6.622	7.010	3.154	3.537	3.921	4.305	4.690	5.076	5.462	5.847	6.235	6.622	7.010
0.8	3.556	3.940	4.326	4.709	5.095	5.481	5.867	6.254	6.642	7.030	7.419	3.556	3.940	4.326	4.709	5.095	5.481	5.867	6.254	6.642	7.030	7.419
1.0	3.960	4.344	4.729	5.114	5.500	5.887	6.274	6.662	7.050	7.439	7.829	3.960	4.344	4.729	5.114	5.500	5.887	6.274	6.662	7.050	7.439	7.829
1.2	4.364	4.749	5.134	5.520	5.907	6.294	6.682	7.071	7.460	7.849	8.239	4.364	4.749	5.134	5.520	5.907	6.294	6.682	7.071	7.460	7.849	8.239
1.4	4.769	5.155	5.541	5.927	6.315	6.703	7.091	7.480	7.870	8.260	8.651	4.769	5.155	5.541	5.927	6.315	6.703	7.091	7.480	7.870	8.260	8.651
1.6	5.175	5.561	5.948	6.335	6.723	7.112	7.501	7.891	8.281	8.672	9.063	5.175	5.561	5.948	6.335	6.723	7.112	7.501	7.891	8.281	8.672	9.063
1.8	5.582	5.969	6.356	6.744	7.133	7.522	7.912	8.302	8.693	9.085	9.477	5.582	5.969	6.356	6.744	7.133	7.522	7.912	8.302	8.693	9.085	9.477
2.0	5.990	6.377	6.765	7.154	7.543	7.933	8.324	8.715	9.107	9.499	9.892	5.990	6.377	6.765	7.154	7.543	7.933	8.324	8.715	9.107	9.499	9.892

Table 3. (continued)

ϕ	$M = 2$										$\mu = 1.10$	
	$\kappa = 0.00$	-1.2	-1.4	-1.6	-1.8	-2.0	-2.2	-2.4	-2.6	-2.8	-3.0	-3.2
-2.0	-5.461	-5.125	-4.787	-4.448	-4.106	-3.763	-3.417	-3.070	-2.721	-2.370	-2.017	-1.630
-1.8	-5.087	-4.750	-4.411	-4.070	-3.727	-3.382	-3.035	-2.686	-2.336	-1.984	-1.630	-1.242
-1.6	-4.711	-4.373	-4.032	-3.690	-3.346	-3.000	-2.652	-2.302	-1.950	-1.597	-1.242	-0.853
-1.4	-4.334	-3.995	-3.653	-3.309	-2.964	-2.616	-2.267	-1.916	-1.563	-1.209	-0.853	-0.463
-1.2	-3.956	-3.615	-3.272	-2.927	-2.580	-2.231	-1.881	-1.529	-1.175	-0.820	-0.463	-0.071
-1.0	-3.577	-3.234	-2.890	-2.543	-2.195	-1.845	-1.494	-1.140	-0.785	-0.429	-0.071	0.321
-0.8	-3.196	-2.852	-2.506	-2.159	-1.809	-1.458	-1.105	-0.751	-0.397	0.037	0.321	0.715
-0.6	-2.814	-2.469	-2.121	-1.773	-1.422	-1.070	-0.715	-0.360	0.033	0.391	0.750	1.111
-0.4	-2.430	-2.084	-1.735	-1.385	-1.033	-0.679	-0.324	0.033	0.391	0.750	1.111	1.507
-0.2	-2.045	-1.697	-1.347	-0.996	-0.642	-0.288	0.069	0.426	0.785	1.146	1.507	1.905
0.0	-1.658	-1.309	-0.958	-0.605	-0.251	0.105	0.463	0.821	1.182	1.543	1.905	2.305
0.2	-1.270	-0.919	-0.567	-0.213	0.143	0.500	0.858	1.218	1.579	1.942	2.305	2.706
0.4	-0.880	-0.528	-0.175	0.181	0.537	0.896	1.255	1.616	1.978	2.342	2.706	3.109
0.6	-0.489	-0.136	0.219	0.576	0.934	1.293	1.654	2.016	2.379	2.743	3.109	3.513
0.8	-0.096	0.259	0.615	0.973	1.332	1.692	2.054	2.417	2.781	3.147	3.513	3.919
1.0	0.299	0.654	1.012	1.371	1.731	2.093	2.456	2.820	3.185	3.552	3.919	4.327
1.2	0.695	1.052	1.411	1.771	2.133	2.496	2.860	3.224	3.591	3.959	4.327	4.737
1.4	1.093	1.452	1.812	2.173	2.536	2.900	3.265	3.632	3.999	4.368	4.737	5.149
1.6	1.493	1.853	2.214	2.570	2.941	3.306	3.672	4.040	4.409	4.778	5.149	5.563
1.8	1.895	2.256	2.618	2.982	3.348	3.714	4.082	4.450	4.820	5.191	5.563	5.979
2.0	2.298	2.661	3.025	3.390	3.756	4.124	4.493	4.863	5.234	5.606	5.979	

Table 3. (continued)

ϕ	$M = 2$										$\mu = 1.10$
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.017	-1.663	-1.307	-0.950	-0.591	-0.231	0.130	0.492	0.855	1.219	1.584
-1.8	-1.630	-1.275	-0.918	-0.560	-0.200	0.160	0.522	0.887	1.248	1.613	1.978
-1.6	-1.242	-0.886	-0.528	-0.169	0.191	0.553	0.915	1.278	1.643	2.006	2.373
-1.4	-0.853	-0.496	-0.137	0.223	0.586	0.946	1.309	1.673	2.038	2.403	2.770
-1.2	-0.463	-0.105	0.255	0.616	0.977	1.340	1.704	2.069	2.434	2.800	3.167
-1.0	-0.071	0.288	0.648	1.010	1.372	1.736	2.100	2.466	2.832	3.198	3.566
-0.8	0.321	0.682	1.043	1.405	1.768	2.133	2.498	2.864	3.230	3.598	3.966
-0.6	0.715	1.076	1.438	1.802	2.166	2.531	2.896	3.263	3.630	3.998	4.367
-0.4	1.111	1.473	1.835	2.199	2.564	2.930	3.297	3.664	4.032	4.401	4.770
-0.2	1.507	1.870	2.234	2.599	2.964	3.331	3.698	4.066	4.435	4.805	5.175
0.0	1.905	2.269	2.634	2.999	3.366	3.733	4.101	4.470	4.840	5.210	5.581
0.2	2.305	2.670	3.035	3.402	3.769	4.137	4.506	4.876	5.246	5.617	5.989
0.4	2.706	3.072	3.438	3.805	4.174	4.543	4.912	5.283	5.655	6.027	6.400
0.6	3.109	3.475	3.843	4.211	4.580	4.950	5.321	5.693	6.065	6.414	6.812
0.8	3.513	3.881	4.250	4.618	4.988	5.359	5.731	6.104	6.477	6.851	7.226
1.0	3.919	4.288	4.657	5.027	5.398	5.770	6.143	6.517	6.891	7.266	7.642
1.2	4.327	4.697	5.067	5.438	5.811	6.184	6.557	6.932	7.308	7.684	8.061
1.4	4.737	5.108	5.479	5.851	6.225	6.599	6.974	7.350	7.726	8.104	8.482
1.6	5.149	5.521	5.893	6.267	6.641	7.016	7.392	7.769	8.147	8.525	8.905
1.8	5.563	5.935	6.309	6.684	7.059	7.436	7.813	8.191	8.570	8.949	9.330
2.0	5.979	6.353	6.727	7.103	7.480	7.857	8.235	8.614	8.994	9.375	9.757

Table 3. (continued)

ϕ	$M = 2$											$\mu = 1.20$
	$\kappa = 0.00$											
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	
-2.0	-2.029	-1.706	-1.380	-1.052	-0.720	-0.387	-0.052	0.285	0.623	0.949	1.301	
-1.8	-1.652	-1.328	-1.001	-0.671	-0.339	-0.006	0.330	0.667	1.004	1.343	1.681	
-1.6	-1.274	-0.948	-0.620	-0.290	0.043	0.377	0.713	1.049	1.386	1.724	2.063	
-1.4	-0.893	-0.567	-0.238	0.093	0.426	0.760	1.096	1.432	1.770	2.107	2.445	
-1.2	-0.511	-0.184	0.146	0.479	0.811	1.145	1.481	1.817	2.154	2.491	2.828	
-1.0	-0.128	0.201	0.531	0.863	1.197	1.531	1.867	2.203	2.540	2.876	3.213	
-0.8	0.258	0.587	0.918	1.250	1.584	1.919	2.254	2.591	2.927	3.264	3.601	
-0.6	0.645	0.975	1.307	1.640	1.974	2.308	2.644	2.980	3.317	3.654	3.991	
-0.4	1.035	1.365	1.697	2.031	2.365	2.700	3.036	3.372	3.709	4.046	4.384	
-0.2	1.426	1.758	2.090	2.424	2.759	3.095	3.431	3.767	4.105	4.442	4.780	
0.0	1.821	2.153	2.486	2.820	3.156	3.492	3.829	4.166	4.504	4.842	5.181	
0.2	2.218	2.550	2.885	3.220	3.556	3.892	4.230	4.568	4.907	5.247	5.587	
0.4	2.618	2.951	3.286	3.622	3.959	4.297	4.635	4.975	5.315	5.656	5.997	
0.6	3.021	3.356	3.692	4.029	4.367	4.705	5.045	5.386	5.727	6.070	6.413	
0.8	3.428	3.764	4.101	4.440	4.778	5.118	5.459	5.801	6.144	6.488	6.833	
1.0	3.839	4.176	4.514	4.854	5.194	5.536	5.878	6.222	6.566	6.912	7.259	
1.2	4.254	4.592	4.932	5.273	5.615	5.958	6.302	6.647	6.994	7.341	7.689	
1.4	4.673	5.013	5.354	5.696	6.040	6.385	6.731	7.077	7.425	7.774	8.125	
1.6	5.096	5.438	5.781	6.125	6.470	6.816	7.164	7.512	7.862	8.213	8.564	
1.8	5.524	5.867	6.212	6.558	6.904	7.252	7.601	7.952	8.303	8.655	9.008	
2.0	5.956	6.301	6.647	6.995	7.343	7.693	8.043	8.395	8.748	9.101	9.456	

Table 3. (continued)

ϕ	-2.0	-1.8	-1.6	M = 2	-1.4	-1.2	K = 0.00	$\mu = 1.30$	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-4.745	-4.499	-4.248	-3.992	-3.730	-3.462	-3.189	-2.910	-2.625	-2.334	-2.038	-1.744	-1.448
-1.8	-4.402	-4.153	-3.899	-3.639	-3.375	-3.104	-2.828	-2.547	-2.259	-1.967	-1.669	-1.374	-1.078
-1.6	-4.055	-3.803	-3.546	-3.284	-3.017	-2.744	-2.465	-2.181	-1.891	-1.597	-1.297	-1.000	-0.704
-1.4	-3.706	-3.451	-3.191	-2.926	-2.656	-2.380	-2.099	-1.813	-1.521	-1.225	-0.924	-0.628	-0.332
-1.2	-3.353	-3.095	-2.833	-2.565	-2.292	-2.014	-1.730	-1.442	-1.149	-0.851	-0.550	-0.254	-0.000
-1.0	-2.997	-2.737	-2.471	-2.201	-1.925	-1.644	-1.359	-1.068	-0.774	-0.475	-0.173	0.129	0.433
-0.8	-2.638	-2.375	-2.106	-1.833	-1.555	-1.272	-0.984	-0.693	-0.398	-0.097	0.205	0.509	0.813
-0.6	-2.275	-2.009	-1.738	-1.462	-1.182	-0.897	-0.607	-0.314	-0.017	0.283	0.586	0.890	1.194
-0.4	-1.908	-1.639	-1.366	-1.088	-0.805	-0.518	-0.227	0.068	0.365	0.666	0.969	1.272	1.576
-0.2	-1.538	-1.266	-0.990	-0.709	-0.424	-0.136	0.157	0.452	0.751	1.052	1.355	1.658	1.961
0.0	-1.163	-0.888	-0.610	-0.327	-0.040	0.250	0.544	0.840	1.139	1.441	1.744	2.047	2.350
0.2	-0.783	-0.506	-0.226	0.059	0.348	0.640	0.934	1.232	1.532	1.833	2.136	2.439	2.742
0.4	-0.399	-0.120	0.163	0.450	0.740	1.033	1.329	1.628	1.928	2.230	2.534	2.837	3.140
0.6	-0.011	0.271	0.557	0.845	1.137	1.432	1.729	2.028	2.330	2.632	2.937	3.240	3.543
0.8	0.383	0.668	0.955	1.246	1.539	1.835	2.134	2.434	2.736	3.040	3.345	3.648	3.951
1.0	0.783	1.069	1.359	1.651	1.947	2.244	2.544	2.845	3.149	3.453	3.760	4.063	4.366
1.2	1.188	1.477	1.768	2.063	2.360	2.659	2.960	3.263	3.568	3.874	4.182	4.485	4.788
1.4	1.599	1.890	2.183	2.480	2.778	3.079	3.382	3.687	3.993	4.301	4.610	4.913	5.216
1.6	2.016	2.309	2.605	2.903	3.203	3.506	3.811	4.117	4.425	4.734	5.046	5.353	5.656
1.8	2.438	2.734	3.032	3.332	3.635	3.939	4.245	4.553	4.863	5.175	5.488	5.791	6.094
2.0	2.867	3.165	3.465	3.767	4.072	4.378	4.686	4.996	5.308	5.621	5.936	6.249	6.562

Table 3. (continued)

ϕ	$M = 2$ $\kappa = 0.00$ $\mu = 1.30$										
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.038	-1.737	-1.432	-1.122	-0.809	-0.493	-0.175	0.145	0.466	0.787	1.108
-1.8	-1.669	-1.366	-1.060	-0.750	-0.437	-0.121	0.196	0.515	0.834	1.153	1.472
-1.6	-1.297	-0.994	-0.687	-0.377	-0.064	0.251	0.567	0.884	1.202	1.519	1.835
-1.4	-0.924	-0.620	-0.313	-0.003	0.310	0.624	0.939	1.254	1.569	1.884	2.198
-1.2	-0.550	-0.245	0.063	0.373	0.684	0.997	1.310	1.624	1.937	2.250	2.561
-1.0	-0.173	0.134	0.439	0.749	1.059	1.371	1.683	1.995	2.306	2.617	2.926
-0.8	0.205	0.510	0.817	1.126	1.436	1.746	2.056	2.367	2.676	2.985	3.294
-0.6	0.586	0.891	1.197	1.505	1.814	2.123	2.432	2.741	3.049	3.357	3.665
-0.4	0.969	1.274	1.580	1.887	2.195	2.503	2.811	3.119	3.426	3.734	4.041
-0.2	1.355	1.659	1.965	2.271	2.579	2.886	3.193	3.501	3.808	4.115	4.423
0.0	1.744	2.048	2.353	2.660	2.966	3.274	3.581	3.888	4.196	4.504	4.812
0.2	2.136	2.441	2.746	3.053	3.359	3.667	3.974	4.282	4.591	4.900	5.210
0.4	2.534	2.839	3.144	3.451	3.758	4.066	4.375	4.684	4.994	5.305	5.617
0.6	2.937	3.242	3.548	3.856	4.164	4.473	4.783	5.094	5.406	5.719	6.033
0.8	3.345	3.652	3.959	4.268	4.565	4.887	5.199	5.512	5.826	6.141	6.457
1.0	3.760	4.068	4.376	4.686	4.998	5.310	5.624	5.938	6.254	6.572	6.890
1.2	4.182	4.491	4.801	5.113	5.426	5.740	6.056	6.373	6.691	7.010	7.331
1.4	4.610	4.921	5.233	5.547	5.862	6.178	6.496	6.815	7.135	7.456	7.779
1.6	5.046	5.358	5.672	5.988	6.305	6.623	6.942	7.263	7.585	7.909	8.233
1.8	5.488	5.802	6.118	6.435	6.754	7.074	7.395	7.718	8.042	8.367	8.693
2.0	5.936	6.252	6.570	6.889	7.209	7.531	7.854	8.178	8.503	8.830	9.157

Table 3. (continued)

ϕ	$\mu = 1.40$										
	$\kappa = 0.00$					$M = 2$					
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-4.501	-4.286	-4.065	-3.838	-3.605	-3.364	-3.116	-2.860	-2.596	-2.324	-2.045
-1.8	-4.167	-3.949	-3.725	-3.495	-3.258	-3.014	-2.762	-2.503	-2.236	-1.962	-1.681
-1.6	-3.830	-3.609	-3.382	-3.148	-2.907	-2.660	-2.405	-2.143	-1.874	-1.598	-1.316
-1.4	-3.490	-3.265	-3.034	-2.797	-2.554	-2.303	-2.045	-1.780	-1.509	-1.232	-0.949
-1.2	-3.146	-2.918	-2.683	-2.443	-2.196	-1.942	-1.682	-1.415	-1.142	-0.863	-0.580
-1.0	-2.797	-2.560	-2.328	-2.085	-1.835	-1.578	-1.315	-1.047	-0.773	-0.493	-0.210
-0.8	-2.445	-2.210	-1.969	-1.722	-1.470	-1.210	-0.946	-0.676	-0.402	-0.123	0.162
-0.6	-2.087	-1.849	-1.605	-1.356	-1.100	-0.839	-0.573	-0.302	-0.026	0.253	0.535
-0.4	-1.725	-1.484	-1.237	-0.984	-0.752	-0.464	-0.196	0.076	0.351	0.630	0.911
-0.2	-1.357	-1.113	-0.863	-0.608	-0.349	-0.084	0.184	0.457	0.732	1.010	1.290
0.0	-0.984	-0.737	-0.484	-0.227	0.034	0.300	0.569	0.842	1.117	1.394	1.673
0.2	-0.605	-0.354	-0.100	0.159	0.422	0.689	0.959	1.231	1.506	1.783	2.061
0.4	-0.219	0.034	0.291	0.551	0.816	1.083	1.354	1.626	1.901	2.177	2.455
0.6	0.173	0.428	0.687	0.950	1.215	1.484	1.755	2.028	2.302	2.579	2.856
0.8	0.573	0.830	1.091	1.355	1.622	1.891	2.163	2.436	2.711	2.988	3.266
1.0	0.979	1.239	1.501	1.767	2.021	2.305	2.578	2.852	3.128	3.406	3.685
1.2	1.393	1.655	1.919	2.187	2.456	2.728	3.001	3.277	3.554	3.833	4.114
1.4	1.815	2.079	2.345	2.614	2.885	3.158	3.433	3.710	3.989	4.270	4.552
1.6	2.244	2.510	2.778	3.049	3.321	3.596	3.873	4.152	4.433	4.715	4.999
1.8	2.681	2.949	3.219	3.491	3.766	4.042	4.321	4.602	4.884	5.169	5.455
2.0	3.125	3.395	3.667	3.941	4.217	4.496	4.777	5.059	5.343	5.630	5.918

Table 3. (continued)

ϕ	$M = 2$										$\mu = 1.40$											
	$\kappa = 0.00$					$\mu = 1.40$					$\kappa = 0.00$					$\mu = 1.40$						
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.045	-1.759	-1.468	-1.171	-0.870	-0.566	-0.259	0.049	0.358	0.666	0.973	-2.045	-1.759	-1.468	-1.171	-0.870	-0.566	-0.259	0.049	0.358	0.666	0.973
-1.8	-1.681	-1.394	-1.102	-0.806	-0.505	-0.202	0.102	0.407	0.694	1.017	1.320	-1.681	-1.394	-1.102	-0.806	-0.505	-0.202	0.102	0.407	0.694	1.017	1.320
-1.6	-1.316	-1.028	-0.736	-0.440	-0.141	0.160	0.462	0.764	1.065	1.366	1.664	-1.316	-1.028	-0.736	-0.440	-0.141	0.160	0.462	0.764	1.065	1.366	1.664
-1.4	-0.949	-0.661	-0.369	-0.074	0.223	0.522	0.820	1.119	1.417	1.712	2.006	-0.949	-0.661	-0.369	-0.074	0.223	0.522	0.820	1.119	1.417	1.712	2.006
-1.2	-0.580	-0.293	-0.002	0.284	0.587	0.882	1.178	1.473	1.766	2.058	2.347	-0.580	-0.293	-0.002	0.284	0.587	0.882	1.178	1.473	1.766	2.058	2.347
-1.0	-0.210	0.077	0.366	0.658	0.950	1.243	1.535	1.826	2.116	2.403	2.689	-0.210	0.077	0.366	0.658	0.950	1.243	1.535	1.826	2.116	2.403	2.689
-0.8	0.162	0.448	0.735	1.025	1.314	1.604	1.893	2.180	2.466	2.750	3.032	0.162	0.448	0.735	1.025	1.314	1.604	1.893	2.180	2.466	2.750	3.032
-0.6	0.535	0.820	1.106	1.393	1.680	1.966	2.252	2.536	2.819	3.101	3.381	0.535	0.820	1.106	1.393	1.680	1.966	2.252	2.536	2.819	3.101	3.381
-0.4	0.911	1.194	1.478	1.763	2.047	2.331	2.614	2.896	3.177	3.457	3.736	0.911	1.194	1.478	1.763	2.047	2.331	2.614	2.896	3.177	3.457	3.736
-0.2	1.290	1.572	1.854	2.136	2.418	2.700	2.981	3.262	3.541	3.821	4.100	1.290	1.572	1.854	2.136	2.418	2.700	2.981	3.262	3.541	3.821	4.100
0.0	1.673	1.953	2.233	2.514	2.795	3.075	3.356	3.636	3.915	4.196	4.476	1.673	1.953	2.233	2.514	2.795	3.075	3.356	3.636	3.915	4.196	4.476
0.2	2.061	2.340	2.619	2.899	3.179	3.459	3.739	4.020	4.301	4.582	4.865	2.061	2.340	2.619	2.899	3.179	3.459	3.739	4.020	4.301	4.582	4.865
0.4	2.455	2.733	3.012	3.292	3.572	3.852	4.134	4.416	4.698	4.982	5.267	2.455	2.733	3.012	3.292	3.572	3.852	4.134	4.416	4.698	4.982	5.267
0.6	2.856	3.134	3.414	3.694	3.975	4.257	4.540	4.824	5.109	5.396	5.684	2.856	3.134	3.414	3.694	3.975	4.257	4.540	4.824	5.109	5.396	5.684
0.8	3.266	3.546	3.826	4.094	4.383	4.668	4.950	5.230	5.507	5.782	6.054	3.266	3.546	3.826	4.094	4.383	4.668	4.950	5.230	5.507	5.782	6.054
1.0	3.685	3.965	4.247	4.530	4.815	5.101	5.388	5.677	5.968	6.259	6.553	3.685	3.965	4.247	4.530	4.815	5.101	5.388	5.677	5.968	6.259	6.553
1.2	4.114	4.396	4.679	4.965	5.251	5.540	5.829	6.121	6.413	6.708	7.003	4.114	4.396	4.679	4.965	5.251	5.540	5.829	6.121	6.413	6.708	7.003
1.4	4.552	4.836	5.122	5.409	5.698	5.988	6.280	6.574	6.869	7.165	7.463	4.552	4.836	5.122	5.409	5.698	5.988	6.280	6.574	6.869	7.165	7.463
1.6	4.999	5.285	5.573	5.862	6.153	6.445	6.739	7.035	7.332	7.630	7.929	4.999	5.285	5.573	5.862	6.153	6.445	6.739	7.035	7.332	7.630	7.929
1.8	5.455	5.743	6.032	6.323	6.616	6.910	7.206	7.503	7.802	8.101	8.402	5.455	5.743	6.032	6.323	6.616	6.910	7.206	7.503	7.802	8.101	8.402
2.0	5.918	6.207	6.499	6.791	7.086	7.382	7.679	7.978	8.277	8.578	8.880	5.918	6.207	6.499	6.791	7.086	7.382	7.679	7.978	8.277	8.578	8.880

Table 3. (continued)

	M = 2						$\kappa = 0.00$						$\mu = 1.50$									
ϕ	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-4.305	-4.115	-3.919	-3.716	-3.505	-3.286	-3.058	-2.821	-2.573	-2.316	-2.050	-2.821	-2.573	-2.316	-2.050	-1.821	-1.573	-1.316	-1.050	-0.793	-0.536	-0.279
-1.8	-3.960	-3.786	-3.586	-3.380	-3.165	-2.942	-2.710	-2.469	-2.218	-1.959	-1.691	-2.469	-2.218	-1.959	-1.691	-1.469	-1.218	-0.959	-0.691	-0.436	-0.179	0.080
-1.6	-3.650	-3.453	-3.250	-3.040	-2.821	-2.594	-2.359	-2.114	-1.861	-1.599	-1.330	-2.114	-1.861	-1.599	-1.330	-1.114	-0.861	-0.599	-0.330	-0.061	0.200	0.459
-1.4	-3.317	-3.117	-2.910	-2.695	-2.473	-2.243	-2.004	-1.756	-1.500	-1.237	-0.968	-1.756	-1.500	-1.237	-0.968	-0.756	-0.500	-0.237	0.030	0.277	0.524	0.771
-1.2	-2.980	-2.776	-2.565	-2.347	-2.121	-1.887	-1.645	-1.395	-1.138	-0.874	-0.604	-1.395	-1.138	-0.874	-0.604	-0.438	-0.181	0.076	0.323	0.570	0.817	1.064
-1.0	-2.637	-2.430	-2.215	-1.993	-1.764	-1.527	-1.282	-1.031	-0.773	-0.509	-0.240	-1.031	-0.773	-0.509	-0.240	-0.076	0.181	0.428	0.675	0.922	1.169	1.416
-0.8	-2.290	-2.078	-1.860	-1.635	-1.402	-1.163	-0.917	-0.664	-0.406	-0.143	0.125	-0.664	-0.406	-0.143	0.125	0.031	0.278	0.525	0.772	1.019	1.266	1.513
-0.6	-1.937	-1.722	-1.500	-1.271	-1.036	-0.794	-0.547	-0.294	-0.035	0.227	0.492	-0.294	-0.035	0.227	0.492	0.031	0.278	0.525	0.772	1.019	1.266	1.513
-0.4	-1.578	-1.359	-1.133	-0.902	-0.665	-0.421	-0.173	0.080	0.338	0.598	0.861	0.080	0.338	0.598	0.861	0.031	0.278	0.525	0.772	1.019	1.266	1.513
-0.2	-1.245	-0.989	-0.761	-0.527	-0.288	-0.043	0.205	0.458	0.714	0.972	1.232	0.458	0.714	0.972	1.232	0.031	0.278	0.525	0.772	1.019	1.266	1.513
0.0	-0.838	-0.613	-0.382	-0.146	0.095	0.340	0.588	0.840	1.095	1.351	1.608	0.840	1.095	1.351	1.608	0.031	0.278	0.525	0.772	1.019	1.266	1.513
0.2	-0.457	-0.229	0.004	0.242	0.483	0.729	0.977	1.226	1.481	1.735	1.990	1.226	1.481	1.735	1.990	0.031	0.278	0.525	0.772	1.019	1.266	1.513
0.4	-0.068	0.163	0.398	0.637	0.879	1.124	1.372	1.622	1.873	2.126	2.380	1.622	1.873	2.126	2.380	0.031	0.278	0.525	0.772	1.019	1.266	1.513
0.6	0.329	0.562	0.799	1.039	1.282	1.527	1.775	2.024	2.275	2.526	2.779	2.024	2.275	2.526	2.779	0.031	0.278	0.525	0.772	1.019	1.266	1.513
0.8	0.736	0.971	1.209	1.450	1.693	1.938	2.186	2.433	2.686	2.931	3.191	2.433	2.686	2.931	3.191	0.031	0.278	0.525	0.772	1.019	1.266	1.513
1.0	1.151	1.388	1.627	1.869	2.113	2.359	2.607	2.856	3.108	3.360	3.615	2.856	3.108	3.360	3.615	0.031	0.278	0.525	0.772	1.019	1.266	1.513
1.2	1.576	1.814	2.055	2.297	2.542	2.789	3.038	3.288	3.541	3.795	4.051	3.288	3.541	3.795	4.051	0.031	0.278	0.525	0.772	1.019	1.266	1.513
1.4	2.009	2.249	2.491	2.735	2.981	3.229	3.479	3.731	3.985	4.241	4.499	3.731	3.985	4.241	4.499	0.031	0.278	0.525	0.772	1.019	1.266	1.513
1.6	2.451	2.692	2.936	3.181	3.429	3.678	3.930	4.184	4.440	4.698	4.958	4.184	4.440	4.698	4.958	0.031	0.278	0.525	0.772	1.019	1.266	1.513
1.8	2.901	3.144	3.389	3.636	3.885	4.137	4.390	4.646	4.904	5.164	5.426	4.646	4.904	5.164	5.426	0.031	0.278	0.525	0.772	1.019	1.266	1.513
2.0	3.359	3.603	3.850	4.099	4.350	4.603	4.859	5.116	5.376	5.638	5.902	5.116	5.376	5.638	5.902	0.031	0.278	0.525	0.772	1.019	1.266	1.513

Table 3. (continued)

ϕ	$M = 2$										$\mu = 1.50$											
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.050	-1.775	-1.494	-1.205	-0.913	-0.616	-0.317	-0.018	0.282	0.580	0.876	-2.050	-1.775	-1.494	-1.205	-0.913	-0.616	-0.317	-0.018	0.282	0.580	0.876
-1.8	-1.691	-1.415	-1.133	-0.846	-0.555	-0.261	0.035	0.330	0.622	0.917	1.206	-1.691	-1.415	-1.133	-0.846	-0.555	-0.261	0.035	0.330	0.622	0.917	1.206
-1.6	-1.330	-1.054	-0.772	-0.487	-0.198	0.093	0.384	0.674	0.963	1.249	1.531	-1.330	-1.054	-0.772	-0.487	-0.198	0.093	0.384	0.674	0.963	1.249	1.531
-1.4	-0.968	-0.692	-0.412	-0.128	0.157	0.402	0.730	1.015	1.297	1.577	1.852	-0.968	-0.692	-0.412	-0.128	0.157	0.402	0.730	1.015	1.297	1.577	1.852
-1.2	-0.604	-0.330	-0.052	0.229	0.511	0.792	1.073	1.352	1.628	1.901	2.170	-0.604	-0.330	-0.052	0.229	0.511	0.792	1.073	1.352	1.628	1.901	2.170
-1.0	-0.240	0.033	0.309	0.585	0.862	1.139	1.414	1.687	1.957	2.224	2.488	-0.240	0.033	0.309	0.585	0.862	1.139	1.414	1.687	1.957	2.224	2.488
-0.8	0.125	0.396	0.668	0.941	1.213	1.485	1.755	2.022	2.286	2.548	2.807	0.125	0.396	0.668	0.941	1.213	1.485	1.755	2.022	2.286	2.548	2.807
-0.6	0.492	0.759	1.028	1.296	1.564	1.831	2.095	2.358	2.618	2.875	3.131	0.492	0.759	1.028	1.296	1.564	1.831	2.095	2.358	2.618	2.875	3.131
-0.4	0.861	1.125	1.389	1.654	1.917	2.179	2.440	2.698	2.955	3.210	3.464	0.861	1.125	1.389	1.654	1.917	2.179	2.440	2.698	2.955	3.210	3.464
-0.2	1.232	1.493	1.754	2.015	2.274	2.533	2.790	3.046	3.301	3.556	3.810	1.232	1.493	1.754	2.015	2.274	2.533	2.790	3.046	3.301	3.556	3.810
0.0	1.608	1.866	2.124	2.381	2.638	2.895	3.150	3.406	3.661	3.916	4.171	1.608	1.866	2.124	2.381	2.638	2.895	3.150	3.406	3.661	3.916	4.171
0.2	1.990	2.245	2.501	2.757	3.012	3.268	3.524	3.779	4.036	4.293	4.552	1.990	2.245	2.501	2.757	3.012	3.268	3.524	3.779	4.036	4.293	4.552
0.4	2.380	2.634	2.888	3.144	3.399	3.655	3.912	4.170	4.429	4.689	4.951	2.380	2.634	2.888	3.144	3.399	3.655	3.912	4.170	4.429	4.689	4.951
0.6	2.779	3.033	3.288	3.544	3.801	4.058	4.317	4.578	4.840	5.103	5.368	2.779	3.033	3.288	3.544	3.801	4.058	4.317	4.578	4.840	5.103	5.368
0.8	3.191	3.445	3.701	3.959	4.217	4.477	4.732	4.978	5.221	5.534	5.802	3.191	3.445	3.701	3.959	4.217	4.477	4.732	4.978	5.221	5.534	5.802
1.0	3.615	3.871	4.128	4.388	4.649	4.911	5.176	5.442	5.710	5.979	6.250	3.615	3.871	4.128	4.388	4.649	4.911	5.176	5.442	5.710	5.979	6.250
1.2	4.051	4.309	4.569	4.830	5.094	5.359	5.626	5.894	6.165	6.436	6.710	4.051	4.309	4.569	4.830	5.094	5.359	5.626	5.894	6.165	6.436	6.710
1.4	4.499	4.759	5.021	5.285	5.551	5.818	6.087	6.358	6.630	6.904	7.179	4.499	4.759	5.021	5.285	5.551	5.818	6.087	6.358	6.630	6.904	7.179
1.6	4.958	5.220	5.484	5.750	6.018	6.287	6.558	6.831	7.105	7.380	7.657	4.958	5.220	5.484	5.750	6.018	6.287	6.558	6.831	7.105	7.380	7.657
1.8	5.426	5.690	5.956	6.224	6.493	6.764	7.037	7.311	7.586	7.863	8.141	5.426	5.690	5.956	6.224	6.493	6.764	7.037	7.311	7.586	7.863	8.141
2.0	5.902	6.168	6.435	6.705	6.975	7.248	7.522	7.797	8.074	8.351	8.630	5.902	6.168	6.435	6.705	6.975	7.248	7.522	7.797	8.074	8.351	8.630

Table 3. (continued)

ϕ	$M = 2$										$\mu = 2.00$											
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-3.728	-3.610	-3.486	-3.355	-3.216	-3.065	-2.899	-2.717	-2.517	-2.298	-2.062	-2.517	-2.381	-2.249	-2.117	-1.985	-1.853	-1.721	-1.589	-1.457	-1.325	-1.193
-1.8	-3.427	-3.306	-3.179	-3.044	-2.899	-2.742	-2.570	-2.381	-2.175	-1.952	-1.714	-2.175	-2.041	-1.909	-1.777	-1.645	-1.513	-1.381	-1.249	-1.117	-0.985	-0.853
-1.6	-3.124	-2.999	-2.868	-2.728	-2.577	-2.413	-2.235	-2.041	-1.830	-1.605	-1.365	-1.830	-1.696	-1.564	-1.432	-1.300	-1.168	-1.036	-0.904	-0.772	-0.640	-0.508
-1.4	-2.815	-2.687	-2.550	-2.405	-2.248	-2.079	-1.895	-1.696	-1.482	-1.256	-1.018	-1.482	-1.347	-1.215	-1.083	-0.951	-0.819	-0.687	-0.555	-0.423	-0.291	-0.159
-1.2	-2.502	-2.368	-2.227	-2.075	-1.913	-1.738	-1.549	-1.347	-1.132	-0.906	-0.672	-1.132	-1.000	-0.868	-0.736	-0.604	-0.472	-0.340	-0.208	-0.076	0.056	0.188
-1.0	-2.181	-2.042	-1.895	-1.738	-1.570	-1.390	-1.198	-0.995	-0.781	-0.558	-0.329	-0.781	-0.649	-0.517	-0.385	-0.253	-0.121	0.011	0.143	0.275	0.407	0.539
-0.8	-1.852	-1.707	-1.554	-1.392	-1.220	-1.036	-0.843	-0.640	-0.429	-0.211	0.000	-0.429	-0.297	-0.165	-0.033	0.099	0.231	0.363	0.495	0.627	0.759	0.891
-0.6	-1.512	-1.362	-1.204	-1.037	-0.861	-0.676	-0.483	-0.283	-0.076	0.134	0.345	-0.076	0.078	0.210	0.342	0.474	0.606	0.738	0.870	1.002	1.134	1.266
-0.4	-1.160	-1.005	-0.842	-0.672	-0.494	-0.309	-0.118	0.078	0.276	0.476	0.675	0.276	0.440	0.604	0.768	0.932	1.096	1.260	1.424	1.588	1.752	1.916
-0.2	-0.795	-0.634	-0.468	-0.295	-0.117	0.066	0.252	0.440	0.630	0.818	1.005	0.630	0.807	0.987	1.165	1.341	1.517	1.693	1.869	2.045	2.221	2.397
0.0	-0.413	-0.249	-0.080	0.093	0.269	0.447	0.627	0.807	0.987	1.165	1.341	0.987	1.182	1.353	1.522	1.690	1.858	2.026	2.194	2.362	2.530	2.698
0.2	-0.013	0.153	0.323	0.494	0.666	0.838	1.011	1.182	1.353	1.522	1.690	1.353	1.570	1.734	1.898	2.062	2.226	2.390	2.554	2.718	2.882	3.046
0.4	0.405	0.572	0.740	0.907	1.074	1.240	1.406	1.570	1.734	1.898	2.062	1.734	1.939	2.095	2.249	2.400	2.555	2.706	2.857	3.008	3.159	3.310
0.6	0.842	1.007	1.171	1.274	1.448	1.616	1.780	1.939	2.095	2.249	2.400	2.095	2.245	2.400	2.554	2.706	2.857	3.008	3.159	3.310	3.461	3.612
0.8	1.295	1.456	1.616	1.774	1.931	2.088	2.245	2.400	2.554	2.706	2.857	2.400	2.694	2.854	3.017	3.182	3.350	3.518	3.686	3.854	4.022	4.190
1.0	1.759	1.916	2.071	2.226	2.380	2.536	2.694	2.854	3.017	3.182	3.350	2.854	3.161	3.323	3.488	3.656	3.827	4.000	4.172	4.344	4.516	4.688
1.2	2.231	2.384	2.537	2.690	2.845	3.001	3.161	3.323	3.488	3.656	3.827	3.323	3.643	3.808	3.975	4.146	4.319	4.492	4.664	4.836	5.008	5.180
1.4	2.708	2.860	3.012	3.166	3.322	3.481	3.643	3.808	3.975	4.146	4.319	3.808	4.137	4.305	4.475	4.647	4.823	5.000	5.176	5.352	5.528	5.704
1.6	3.192	3.343	3.497	3.653	3.811	3.973	4.137	4.305	4.475	4.647	4.823	4.305	4.642	4.811	4.983	5.158	5.335	5.512	5.689	5.866	6.043	6.220
1.8	3.682	3.835	3.991	4.149	4.310	4.475	4.642	4.811	4.983	5.158	5.335	4.811	5.154	5.325	5.499	5.675	5.853	6.030	6.207	6.384	6.561	6.738
2.0	4.179	4.334	4.493	4.654	4.818	4.984	5.154	5.325	5.499	5.675	5.853	5.325	5.675	6.030	6.384	6.738	7.092	7.446	7.800	8.154	8.508	8.862

Table 3. (continued)

ϕ	$M = 2$										$\mu = 2.00$
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.062	-1.812	-1.549	-1.277	-0.999	-0.717	-0.435	-0.155	0.120	0.388	0.645
-1.8	-1.714	-1.463	-1.201	-0.933	-0.660	-0.386	-0.114	0.154	0.375	0.665	0.902
-1.6	-1.365	-1.115	-0.858	-0.595	-0.330	-0.066	0.194	0.447	0.690	0.921	1.137
-1.4	-1.018	-0.771	-0.519	-0.264	-0.009	0.238	0.487	0.723	0.947	1.158	1.355
-1.2	-0.672	-0.431	-0.186	0.058	0.300	0.536	0.764	0.982	1.187	1.381	1.565
-1.0	-0.329	-0.096	0.097	0.370	0.597	0.816	1.027	1.227	1.418	1.599	1.775
-0.8	0.000	0.233	0.454	0.671	0.882	1.085	1.280	1.466	1.645	1.820	1.989
-0.6	0.345	0.555	0.762	0.964	1.159	1.348	1.530	1.707	1.879	2.049	2.214
-0.4	0.675	0.872	1.065	1.253	1.436	1.613	1.787	1.957	2.126	2.293	2.456
-0.2	1.005	1.189	1.370	1.547	1.720	1.891	2.060	2.228	2.395	2.561	2.728
0.0	1.341	1.514	1.686	1.850	2.024	2.192	2.359	2.527	2.696	2.867	3.039
0.2	1.690	1.857	2.024	2.191	2.358	2.527	2.696	2.867	3.041	3.218	3.397
0.4	2.062	2.227	2.393	2.561	2.730	2.902	3.075	3.251	3.430	3.612	3.796
0.6	2.400	2.629	2.797	2.968	3.141	3.316	3.494	3.674	3.857	4.043	4.231
0.8	2.894	3.062	3.233	3.407	3.583	3.762	3.943	4.127	4.313	4.500	4.692
1.0	3.350	3.521	3.695	3.872	4.051	4.233	4.406	4.602	4.790	4.980	5.172
1.2	3.827	4.001	4.177	4.356	4.537	4.721	4.906	5.093	5.283	5.474	5.666
1.4	4.319	4.495	4.673	4.854	5.037	5.221	5.408	5.596	5.786	5.978	6.171
1.6	4.823	5.000	5.180	5.362	5.546	5.732	5.919	6.108	6.299	6.490	6.684
1.8	5.335	5.514	5.695	5.878	6.062	6.249	6.437	6.626	6.791	7.009	7.203
2.0	5.853	6.034	6.216	6.399	6.585	6.772	6.960	7.150	7.341	7.533	7.726

Table 3. (continued)

ϕ	$N = 2 \quad \kappa = 0.00 \quad \mu = 2.50$										
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-3.451	-3.366	-3.277	-3.181	-3.077	-2.961	-2.828	-2.675	-2.496	-2.293	-2.066
-1.8	-3.165	-3.077	-2.985	-2.885	-2.775	-2.651	-2.510	-2.347	-2.161	-1.951	-1.722
-1.6	-2.876	-2.785	-2.688	-2.583	-2.466	-2.335	-2.185	-2.014	-1.821	-1.608	-1.377
-1.4	-2.563	-2.488	-2.386	-2.274	-2.150	-2.011	-1.853	-1.676	-1.478	-1.264	-1.035
-1.2	-2.285	-2.185	-2.076	-1.958	-1.826	-1.679	-1.515	-1.332	-1.133	-0.921	-0.697
-1.0	-1.980	-1.873	-1.758	-1.632	-1.492	-1.339	-1.170	-0.986	-0.788	-0.581	-0.367
-0.8	-1.664	-1.551	-1.428	-1.294	-1.149	-0.990	-0.819	-0.637	-0.445	-0.247	-0.047
-0.6	-1.336	-1.214	-1.084	-0.944	-0.794	-0.634	-0.464	-0.287	-0.066	0.078	0.259
-0.4	-0.991	-0.862	-0.725	-0.580	-0.428	-0.269	-0.105	0.062	0.229	0.394	0.553
-0.2	-0.625	-0.489	-0.308	-0.200	-0.050	0.103	0.257	0.410	0.559	0.703	0.840
0.0	-0.234	-0.092	0.052	0.197	0.341	0.484	0.624	0.759	0.889	1.015	1.137
0.2	0.187	0.331	0.473	0.611	0.745	0.874	0.998	1.118	1.234	1.348	1.462
0.4	0.640	0.781	0.915	1.042	1.163	1.278	1.390	1.500	1.609	1.720	1.832
0.6	1.119	1.251	1.373	1.487	1.596	1.703	1.809	1.916	2.025	2.137	2.252
0.8	1.612	1.730	1.839	1.944	2.048	2.152	2.258	2.367	2.478	2.593	2.712
1.0	2.105	2.211	2.313	2.415	2.518	2.624	2.733	2.845	2.960	3.078	3.200
1.2	2.596	2.696	2.798	2.901	3.008	3.117	3.229	3.344	3.462	3.583	3.707
1.4	3.090	3.192	3.296	3.402	3.512	3.624	3.739	3.857	3.978	4.101	4.227
1.6	3.593	3.698	3.805	3.915	4.027	4.143	4.261	4.381	4.504	4.628	4.755
1.8	4.106	4.214	4.324	4.437	4.552	4.670	4.790	4.912	5.036	5.162	5.290
2.0	4.627	4.738	4.851	4.966	5.084	5.203	5.325	5.449	5.574	5.701	5.830

Table 3. (continued)

ϕ	$M = 2$					$\kappa = 0.00$					$\mu = 2.50$				
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0				
-2.0	-2.066	-1.821	-1.562	-1.293	-1.017	-0.739	-0.462	-0.189	0.076	0.328	0.559				
-1.8	-1.722	-1.476	-1.219	-0.955	-0.686	-0.418	-0.155	0.101	0.342	0.562	0.755				
-1.6	-1.377	-1.134	-0.882	-0.625	-0.368	-0.115	0.130	0.360	0.569	0.752	0.907				
-1.4	-1.035	-0.797	-0.553	-0.308	-0.067	0.155	0.384	0.582	0.756	0.905	1.031				
-1.2	-0.697	-0.468	-0.237	-0.009	0.124	0.416	0.603	0.769	0.912	1.038	1.150				
-1.0	-0.367	-0.151	0.062	0.268	0.460	0.636	0.794	0.933	1.059	1.173	1.279				
-0.8	-0.047	0.151	0.341	0.520	0.685	0.835	0.971	1.096	1.213	1.323	1.429				
-0.6	0.259	0.434	0.599	0.753	0.896	1.027	1.151	1.270	1.385	1.495	1.603				
-0.4	0.553	0.704	0.847	0.981	1.109	1.231	1.350	1.466	1.581	1.694	1.806				
-0.2	0.840	0.972	1.099	1.221	1.340	1.458	1.576	1.694	1.811	1.930	2.049				
0.0	1.137	1.256	1.374	1.490	1.607	1.726	1.845	1.967	2.090	2.216	2.345				
0.2	1.462	1.576	1.691	1.808	1.927	2.049	2.173	2.301	2.431	2.565	2.700				
0.4	1.832	1.947	2.065	2.185	2.308	2.435	2.564	2.697	2.833	2.971	3.111				
0.6	2.252	2.370	2.491	2.616	2.743	2.871	3.006	3.142	3.281	3.421	3.564				
0.8	2.712	2.833	2.958	3.085	3.203	3.348	3.483	3.621	3.760	3.902	4.045				
1.0	3.200	3.324	3.451	3.581	3.713	3.847	3.983	4.121	4.261	4.403	4.547				
1.2	3.707	3.833	3.962	4.093	4.226	4.361	4.498	4.636	4.777	4.918	5.062				
1.4	4.227	4.354	4.484	4.616	4.750	4.886	5.023	5.161	5.302	5.443	5.586				
1.6	4.755	4.884	5.015	5.148	5.282	5.418	5.555	5.694	5.834	5.975	6.117				
1.8	5.290	5.420	5.552	5.685	5.819	5.955	6.093	6.231	6.371	6.512	6.654				
2.0	5.830	5.961	6.093	6.226	6.361	6.497	6.634	6.773	6.912	7.053	7.182				

Table 3. (continued)

ϕ	$M = 2$					$\kappa = 0.00$					$\mu = 3.00$											
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-3.290	-3.223	-3.153	-3.078	-2.995	-2.900	-2.789	-2.653	-2.487	-2.291	-2.068	-3.290	-3.223	-3.153	-3.078	-2.995	-2.900	-2.789	-2.653	-2.487	-2.291	-2.068
-1.8	-3.012	-2.943	-2.870	-2.791	-2.702	-2.600	-2.478	-2.331	-2.155	-1.951	-1.724	-3.012	-2.943	-2.870	-2.791	-2.702	-2.600	-2.478	-2.331	-2.155	-1.951	-1.724
-1.6	-2.732	-2.660	-2.583	-2.499	-2.404	-2.292	-2.160	-2.002	-1.818	-1.609	-1.382	-2.732	-2.660	-2.583	-2.499	-2.404	-2.292	-2.160	-2.002	-1.818	-1.609	-1.382
-1.4	-2.450	-2.374	-2.292	-2.201	-2.097	-1.976	-1.834	-1.667	-1.477	-1.267	-1.042	-2.450	-2.374	-2.292	-2.201	-2.097	-1.976	-1.834	-1.667	-1.477	-1.267	-1.042
-1.2	-2.163	-2.082	-1.993	-1.894	-1.781	-1.651	-1.499	-1.327	-1.135	-0.927	-0.708	-2.163	-2.082	-1.993	-1.894	-1.781	-1.651	-1.499	-1.327	-1.135	-0.927	-0.708
-1.0	-1.870	-1.782	-1.685	-1.577	-1.455	-1.315	-1.158	-0.983	-0.793	-0.592	-0.385	-1.870	-1.782	-1.685	-1.577	-1.455	-1.315	-1.158	-0.983	-0.793	-0.592	-0.385
-0.8	-1.567	-1.470	-1.364	-1.247	-1.116	-0.971	-0.811	-0.638	-0.455	-0.267	-0.079	-1.567	-1.470	-1.364	-1.247	-1.116	-0.971	-0.811	-0.638	-0.455	-0.267	-0.079
-0.6	-1.249	-1.143	-1.027	-0.902	-0.764	-0.616	-0.459	-0.294	-0.125	0.043	0.203	-1.249	-1.143	-1.027	-0.902	-0.764	-0.616	-0.459	-0.294	-0.125	0.043	0.203
-0.4	-0.912	-0.795	-0.671	-0.539	-0.400	-0.254	-0.104	0.046	0.194	0.333	0.462	-0.912	-0.795	-0.671	-0.539	-0.400	-0.254	-0.104	0.046	0.194	0.333	0.462
-0.2	-0.548	-0.422	-0.291	-0.176	-0.020	0.117	0.251	0.379	0.499	0.609	0.712	-0.548	-0.422	-0.291	-0.176	-0.020	0.117	0.251	0.379	0.499	0.609	0.712
0.0	-0.150	-0.017	0.116	0.247	0.374	0.494	0.605	0.707	0.802	0.892	0.977	-0.150	-0.017	0.116	0.247	0.374	0.494	0.605	0.707	0.802	0.892	0.977
0.2	0.268	0.424	0.553	0.673	0.781	0.878	0.967	1.050	1.130	1.210	1.291	0.268	0.424	0.553	0.673	0.781	0.878	0.967	1.050	1.130	1.210	1.291
0.4	0.770	0.901	1.016	1.114	1.186	1.258	1.328	1.397	1.505	1.603	1.672	0.770	0.901	1.016	1.114	1.186	1.258	1.328	1.397	1.505	1.603	1.672
0.6	1.285	1.397	1.488	1.567	1.641	1.714	1.788	1.865	1.945	2.029	2.115	1.285	1.397	1.488	1.567	1.641	1.714	1.788	1.865	1.945	2.029	2.115
0.8	1.803	1.888	1.962	2.034	2.106	2.181	2.259	2.340	2.424	2.511	2.601	1.803	1.888	1.962	2.034	2.106	2.181	2.259	2.340	2.424	2.511	2.601
1.0	2.304	2.377	2.448	2.521	2.597	2.676	2.757	2.842	2.929	3.019	3.112	2.304	2.377	2.448	2.521	2.597	2.676	2.757	2.842	2.929	3.019	3.112
1.2	2.804	2.876	2.950	3.027	3.107	3.189	3.274	3.362	3.451	3.544	3.638	2.804	2.876	2.950	3.027	3.107	3.189	3.274	3.362	3.451	3.544	3.638
1.4	3.314	3.389	3.467	3.548	3.631	3.716	3.803	3.893	3.985	4.079	4.175	3.314	3.389	3.467	3.548	3.631	3.716	3.803	3.893	3.985	4.079	4.175
1.6	3.835	3.914	3.995	4.079	4.164	4.252	4.341	4.433	4.526	4.621	4.718	3.835	3.914	3.995	4.079	4.164	4.252	4.341	4.433	4.526	4.621	4.718
1.8	4.366	4.448	4.531	4.617	4.705	4.794	4.886	4.978	5.073	5.169	5.266	4.366	4.448	4.531	4.617	4.705	4.794	4.886	4.978	5.073	5.169	5.266
2.0	4.904	4.988	5.074	5.161	5.251	5.342	5.434	5.528	5.623	5.720	5.818	4.904	4.988	5.074	5.161	5.251	5.342	5.434	5.528	5.623	5.720	5.818

Table 3. (continued)

ϕ	$M = 2$											$\mu = 3.00$
	$\kappa = 0.00$											
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	
-2.0	-2.068	-1.824	-1.566	-1.297	-1.023	-0.745	-0.470	-0.199	0.060	0.301	0.515	
-1.8	-1.724	-1.481	-1.225	-0.961	-0.694	-0.429	-0.169	0.079	0.307	0.505	0.664	
-1.6	-1.382	-1.140	-0.890	-0.635	-0.381	-0.134	0.101	0.315	0.498	0.643	0.752	
-1.4	-1.042	-0.807	-0.566	-0.326	-0.092	0.109	0.327	0.494	0.627	0.729	0.809	
-1.2	-0.708	-0.484	-0.259	-0.041	0.087	0.345	0.498	0.621	0.718	0.797	0.864	
-1.0	-0.385	-0.178	0.044	0.209	0.374	0.514	0.629	0.723	0.803	0.874	0.939	
-0.8	-0.079	0.102	0.270	0.419	0.546	0.654	0.747	0.829	0.905	0.976	1.046	
-0.6	0.203	0.352	0.485	0.601	0.703	0.794	0.879	0.960	1.037	1.114	1.191	
-0.4	0.462	0.579	0.685	0.781	0.871	0.957	1.041	1.124	1.207	1.290	1.375	
-0.2	0.712	0.807	0.897	0.984	1.070	1.156	1.243	1.331	1.420	1.511	1.603	
0.0	0.977	1.062	1.146	1.231	1.318	1.408	1.499	1.593	1.690	1.789	1.891	
0.2	1.291	1.373	1.459	1.547	1.638	1.732	1.830	1.931	2.034	2.141	2.249	
0.4	1.672	1.758	1.847	1.940	2.036	2.135	2.237	2.342	2.450	2.559	2.671	
0.6	2.115	2.205	2.299	2.395	2.494	2.596	2.700	2.807	2.916	3.027	3.139	
0.8	2.601	2.694	2.790	2.888	2.989	3.092	3.197	3.304	3.413	3.524	3.636	
1.0	3.112	3.207	3.304	3.404	3.506	3.609	3.715	3.822	3.931	4.041	4.152	
1.2	3.638	3.735	3.833	3.934	4.036	4.140	4.245	4.352	4.460	4.570	4.681	
1.4	4.175	4.272	4.372	4.472	4.575	4.679	4.784	4.891	4.999	5.108	5.218	
1.6	4.718	4.816	4.916	5.017	5.120	5.209	5.329	5.435	5.531	5.651	5.760	
1.8	5.266	5.365	5.465	5.566	5.669	5.773	5.878	5.984	6.091	6.199	6.308	
2.0	5.818	5.917	6.017	6.119	6.222	6.325	6.430	6.536	6.642	6.750	6.858	

Table 3. (continued)

ϕ	-2.0	-1.8	-1.6	-1.4	$\kappa = 0.00$	$\mu = 4.00$	-0.8	-0.6	-0.4	-0.2	0.0
-2.0	-3.105	-3.058	-3.009	-2.957	-2.899	-2.831	-2.745	-2.631	-2.479	-2.289	-2.069
-1.8	-2.836	-2.788	-2.737	-2.681	-2.618	-2.542	-2.445	-2.316	-2.150	-1.951	-1.726
-1.6	-2.567	-2.516	-2.462	-2.402	-2.332	-2.247	-2.136	-1.992	-1.815	-1.610	-1.384
-1.4	-2.296	-2.242	-2.184	-2.118	-2.039	-1.941	-1.816	-1.661	-1.477	-1.270	-1.046
-1.2	-2.024	-1.965	-1.900	-1.826	-1.736	-1.625	-1.487	-1.324	-1.137	-0.932	-0.716
-1.0	-1.747	-1.682	-1.608	-1.522	-1.420	-1.296	-1.150	-0.982	-0.798	-0.602	-0.400
-0.8	-1.463	-1.388	-1.302	-1.204	-1.089	-0.956	-0.806	-0.641	-0.465	-0.286	-0.111
-0.6	-1.165	-1.076	-0.977	-0.866	-0.742	-0.605	-0.457	-0.303	-0.148	0.001	0.134
-0.4	-0.843	-0.740	-0.628	-0.507	-0.379	-0.245	-0.110	0.022	0.143	0.248	0.334
-0.2	-0.488	-0.370	-0.248	-0.095	0.001	0.120	0.228	0.322	0.401	0.467	0.524
0.0	-0.086	0.043	0.170	0.289	0.394	0.481	0.551	0.609	0.659	0.706	0.752
0.2	0.395	0.509	0.630	0.725	0.793	0.845	0.889	0.930	0.973	1.017	1.065
0.4	0.888	1.024	1.114	1.168	1.209	1.249	1.289	1.332	1.378	1.427	1.479
0.6	1.445	1.542	1.589	1.628	1.668	1.710	1.755	1.803	1.854	1.907	1.963
0.8	1.993	2.038	2.079	2.121	2.165	2.212	2.262	2.313	2.368	2.424	2.483
1.0	2.506	2.549	2.593	2.639	2.687	2.738	2.791	2.845	2.902	2.960	3.020
1.2	3.030	3.070	3.124	3.174	3.225	3.278	3.333	3.389	3.447	3.507	3.568
1.4	3.567	3.616	3.666	3.718	3.772	3.827	3.883	3.941	4.000	4.060	4.122
1.6	4.113	4.164	4.216	4.270	4.325	4.381	4.439	4.498	4.557	4.618	4.680
1.8	4.664	4.717	4.771	4.826	4.882	4.940	4.998	5.057	5.118	5.179	5.242
2.0	5.220	5.274	5.329	5.386	5.443	5.501	5.560	5.620	5.681	5.743	5.805

Table 3. (continued)

ϕ	$M = 2$												$K = 0.00$				$\mu = 4.00$					
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.069	-1.826	-1.568	-1.299	-1.025	-0.748	-0.474	-0.206	0.099	0.279	0.468	-2.069	-1.826	-1.568	-1.299	-1.025	-0.748	-0.474	-0.206	0.099	0.279	0.468
-1.8	-1.726	-1.483	-1.227	-0.964	-0.699	-0.435	-0.178	0.086	0.275	0.443	0.557	-1.726	-1.483	-1.227	-0.964	-0.699	-0.435	-0.178	0.086	0.275	0.443	0.557
-1.6	-1.364	-1.144	-0.894	-0.641	-0.390	-0.147	0.078	0.270	0.416	0.514	0.577	-1.364	-1.144	-0.894	-0.641	-0.390	-0.147	0.078	0.270	0.416	0.514	0.577
-1.4	-1.046	-0.813	-0.574	-0.338	-0.111	0.095	0.266	0.391	0.475	0.532	0.573	-1.046	-0.813	-0.574	-0.338	-0.111	0.095	0.266	0.391	0.475	0.532	0.573
-1.2	-0.716	-0.495	-0.276	-0.068	0.116	0.265	0.372	0.445	0.497	0.537	0.570	-0.716	-0.495	-0.276	-0.068	0.116	0.265	0.372	0.445	0.497	0.537	0.570
-1.0	-0.400	-0.202	-0.016	0.145	0.271	0.362	0.427	0.476	0.516	0.551	0.583	-0.400	-0.202	-0.016	0.145	0.271	0.362	0.427	0.476	0.516	0.551	0.583
-0.8	-0.111	0.050	0.185	0.290	0.369	0.394	0.476	0.517	0.555	0.591	0.628	-0.111	0.050	0.185	0.290	0.369	0.394	0.476	0.517	0.555	0.591	0.628
-0.6	0.134	0.244	0.331	0.399	0.455	0.503	0.547	0.589	0.631	0.674	0.719	0.134	0.244	0.331	0.399	0.455	0.503	0.547	0.589	0.631	0.674	0.719
-0.4	0.334	0.405	0.464	0.517	0.566	0.614	0.661	0.710	0.760	0.812	0.865	0.334	0.405	0.464	0.517	0.566	0.614	0.661	0.710	0.760	0.812	0.865
-0.2	0.524	0.576	0.626	0.676	0.727	0.779	0.833	0.890	0.948	1.009	1.071	0.524	0.576	0.626	0.676	0.727	0.779	0.833	0.890	0.948	1.009	1.071
0.0	0.752	0.800	0.850	0.902	0.957	1.016	1.077	1.141	1.207	1.276	1.347	0.752	0.800	0.850	0.902	0.957	1.016	1.077	1.141	1.207	1.276	1.347
0.2	1.065	1.115	1.170	1.227	1.288	1.352	1.419	1.489	1.561	1.635	1.710	1.065	1.115	1.170	1.227	1.288	1.352	1.419	1.489	1.561	1.635	1.710
0.4	1.479	1.535	1.594	1.655	1.720	1.785	1.822	1.927	2.001	2.076	2.152	1.479	1.535	1.594	1.655	1.720	1.785	1.822	1.927	2.001	2.076	2.152
0.6	1.963	2.022	2.083	2.147	2.213	2.280	2.350	2.421	2.494	2.568	2.644	1.963	2.022	2.083	2.147	2.213	2.280	2.350	2.421	2.494	2.568	2.644
0.8	2.483	2.543	2.606	2.670	2.736	2.804	2.873	2.944	3.016	3.089	3.164	2.483	2.543	2.606	2.670	2.736	2.804	2.873	2.944	3.016	3.089	3.164
1.0	3.020	3.082	3.145	3.210	3.276	3.343	3.412	3.482	3.554	3.626	3.699	3.020	3.082	3.145	3.210	3.276	3.343	3.412	3.482	3.554	3.626	3.699
1.2	3.568	3.630	3.694	3.759	3.825	3.892	3.961	4.030	4.101	4.172	4.245	3.568	3.630	3.694	3.759	3.825	3.892	3.961	4.030	4.101	4.172	4.245
1.4	4.122	4.185	4.249	4.314	4.380	4.447	4.504	4.584	4.654	4.725	4.796	4.122	4.185	4.249	4.314	4.380	4.447	4.504	4.584	4.654	4.725	4.796
1.6	4.680	4.743	4.807	4.873	4.938	5.005	5.052	5.142	5.211	5.281	5.352	4.680	4.743	4.807	4.873	4.938	5.005	5.052	5.142	5.211	5.281	5.352
1.8	5.242	5.305	5.369	5.434	5.500	5.567	5.634	5.702	5.771	5.841	5.911	5.242	5.305	5.369	5.434	5.500	5.567	5.634	5.702	5.771	5.841	5.911
2.0	5.805	5.869	5.933	5.998	6.064	6.130	6.197	6.265	6.334	6.403	6.473	5.805	5.869	5.933	5.998	6.064	6.130	6.197	6.265	6.334	6.403	6.473

Table 3. (continued)

ϕ	$M = 2$					$K = 0.00$					$\mu = 5.00$												
	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	
-2.0	-3.000	-2.964	-2.927	-2.887	-2.842	-2.789	-2.720	-2.620	-2.476	-2.289	-2.069	-2.964	-2.927	-2.887	-2.842	-2.789	-2.720	-2.620	-2.476	-2.289	-2.069	-2.964	-2.927
-1.8	-2.734	-2.697	-2.659	-2.617	-2.568	-2.509	-2.427	-2.309	-2.148	-1.951	-1.727	-2.734	-2.697	-2.659	-2.617	-2.568	-2.509	-2.427	-2.309	-2.148	-1.951	-1.727	-2.734
-1.6	-2.469	-2.431	-2.390	-2.344	-2.291	-2.221	-2.124	-1.988	-1.815	-1.611	-1.385	-2.469	-2.431	-2.390	-2.344	-2.291	-2.221	-2.124	-1.988	-1.815	-1.611	-1.385	-2.469
-1.4	-2.204	-2.163	-2.119	-2.068	-2.006	-1.923	-1.809	-1.659	-1.477	-1.271	-1.048	-2.204	-2.163	-2.119	-2.068	-2.006	-1.923	-1.809	-1.659	-1.477	-1.271	-1.048	-2.204
-1.2	-1.938	-1.894	-1.845	-1.786	-1.712	-1.613	-1.483	-1.323	-1.137	-0.934	-0.718	-1.938	-1.894	-1.845	-1.786	-1.712	-1.613	-1.483	-1.323	-1.137	-0.934	-0.718	-1.938
-1.0	-1.671	-1.621	-1.564	-1.493	-1.403	-1.289	-1.147	-0.983	-0.800	-0.605	-0.405	-1.671	-1.621	-1.564	-1.493	-1.403	-1.289	-1.147	-0.983	-0.800	-0.605	-0.405	-1.671
-0.8	-1.400	-1.340	-1.270	-1.184	-1.078	-0.951	-0.805	-0.642	-0.469	-0.294	-0.125	-1.400	-1.340	-1.270	-1.184	-1.078	-0.951	-0.805	-0.642	-0.469	-0.294	-0.125	-1.400
-0.6	-1.117	-1.043	-0.955	-0.853	-0.734	-0.602	-0.458	-0.308	-0.159	-0.008	0.093	-1.117	-1.043	-0.955	-0.853	-0.734	-0.602	-0.458	-0.308	-0.159	-0.008	0.093	-1.117
-0.4	-0.812	-0.718	-0.612	-0.497	-0.373	-0.244	-0.116	0.005	0.109	0.189	0.249	-0.812	-0.718	-0.612	-0.497	-0.373	-0.244	-0.116	0.005	0.109	0.189	0.249	-0.812
-0.2	-0.467	-0.353	-0.234	-0.113	0.006	0.114	0.205	0.274	0.326	0.366	0.400	-0.467	-0.353	-0.234	-0.113	0.006	0.114	0.205	0.274	0.326	0.366	0.400	-0.467
0.0	-0.066	0.064	0.189	0.303	0.392	0.453	0.493	0.524	0.552	0.579	0.607	-0.066	0.064	0.189	0.303	0.392	0.453	0.493	0.524	0.552	0.579	0.607	-0.066
0.2	0.550	0.576	0.662	0.734	0.770	0.795	0.818	0.842	0.868	0.897	0.930	0.550	0.576	0.662	0.734	0.770	0.795	0.818	0.842	0.868	0.897	0.930	0.550
0.4	1.149	1.077	1.144	1.169	1.192	1.216	1.243	1.272	1.304	1.339	1.376	1.149	1.077	1.144	1.169	1.192	1.216	1.243	1.272	1.304	1.339	1.376	1.149
0.6	1.589	1.599	1.625	1.650	1.677	1.707	1.738	1.772	1.808	1.847	1.887	1.589	1.599	1.625	1.650	1.677	1.707	1.738	1.772	1.808	1.847	1.887	1.589
0.8	2.075	2.110	2.138	2.168	2.199	2.233	2.268	2.305	2.343	2.383	2.425	2.075	2.110	2.138	2.168	2.199	2.233	2.268	2.305	2.343	2.383	2.425	2.075
1.0	2.612	2.642	2.673	2.706	2.741	2.776	2.814	2.852	2.892	2.933	2.976	2.612	2.642	2.673	2.706	2.741	2.776	2.814	2.852	2.892	2.933	2.976	2.612
1.2	3.154	3.187	3.221	3.256	3.293	3.330	3.369	3.409	3.449	3.491	3.534	3.154	3.187	3.221	3.256	3.293	3.330	3.369	3.409	3.449	3.491	3.534	3.154
1.4	3.706	3.741	3.777	3.813	3.851	3.890	3.929	3.970	4.011	4.054	4.097	3.706	3.741	3.777	3.813	3.851	3.890	3.929	3.970	4.011	4.054	4.097	3.706
1.6	4.264	4.300	4.337	4.375	4.413	4.453	4.493	4.535	4.577	4.619	4.663	4.264	4.300	4.337	4.375	4.413	4.453	4.493	4.535	4.577	4.619	4.663	4.264
1.8	4.825	4.863	4.900	4.939	4.979	5.019	5.060	5.101	5.144	5.187	5.230	4.825	4.863	4.900	4.939	4.979	5.019	5.060	5.101	5.144	5.187	5.230	4.825
2.0	5.390	5.428	5.466	5.506	5.546	5.586	5.628	5.670	5.712	5.755	5.799	5.390	5.428	5.466	5.506	5.546	5.586	5.628	5.670	5.712	5.755	5.799	5.390

Table 3. (continued)

ϕ	$M = 2$					$\kappa = 0.00$					$\mu = 5.00$											
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
-2.0	-2.069	-1.827	-1.568	-1.300	-1.025	-0.749	-0.475	-0.208	0.063	0.269	0.441	-2.069	-1.827	-1.568	-1.300	-1.025	-0.749	-0.475	-0.208	0.063	0.269	0.441
-1.8	-1.727	-1.484	-1.228	-0.965	-0.700	-0.436	-0.181	0.057	0.260	0.407	0.495	-1.727	-1.484	-1.228	-0.965	-0.700	-0.436	-0.181	0.057	0.260	0.407	0.495
-1.6	-1.385	-1.145	-0.895	-0.643	-0.392	-0.151	0.069	0.248	0.370	0.443	0.488	-1.385	-1.145	-0.895	-0.643	-0.392	-0.151	0.069	0.248	0.370	0.443	0.488
-1.4	-1.048	-0.814	-0.576	-0.341	-0.117	0.081	0.234	0.335	0.363	0.435	0.464	-1.048	-0.814	-0.576	-0.341	-0.117	0.081	0.234	0.335	0.363	0.435	0.464
-1.2	-0.718	-0.498	-0.281	-0.079	0.094	0.221	0.303	0.354	0.390	0.417	0.440	-0.718	-0.498	-0.281	-0.079	0.094	0.221	0.303	0.354	0.390	0.417	0.440
-1.0	-0.405	-0.211	-0.033	0.111	0.213	0.279	0.324	0.356	0.383	0.406	0.426	-0.405	-0.211	-0.033	0.111	0.213	0.279	0.324	0.356	0.383	0.406	0.426
-0.8	-0.125	0.023	0.137	0.216	0.270	0.309	0.339	0.366	0.389	0.412	0.435	-0.125	0.023	0.137	0.216	0.270	0.309	0.339	0.366	0.389	0.412	0.435
-0.6	0.093	0.179	0.239	0.283	0.318	0.348	0.375	0.401	0.428	0.455	0.483	0.093	0.179	0.239	0.283	0.318	0.348	0.375	0.401	0.428	0.455	0.483
-0.4	0.249	0.294	0.331	0.363	0.394	0.424	0.454	0.486	0.519	0.555	0.592	0.249	0.294	0.331	0.363	0.394	0.424	0.454	0.486	0.519	0.555	0.592
-0.2	0.400	0.431	0.462	0.494	0.527	0.563	0.601	0.641	0.683	0.727	0.773	0.400	0.431	0.462	0.494	0.527	0.563	0.601	0.641	0.683	0.727	0.773
0.0	0.607	0.638	0.671	0.707	0.747	0.789	0.834	0.882	0.932	0.984	1.038	0.607	0.638	0.671	0.707	0.747	0.789	0.834	0.882	0.932	0.984	1.038
0.2	0.930	0.965	1.004	1.046	1.090	1.138	1.188	1.240	1.294	1.350	1.407	0.930	0.965	1.004	1.046	1.090	1.138	1.188	1.240	1.294	1.350	1.407
0.4	1.376	1.416	1.458	1.503	1.549	1.598	1.649	1.701	1.755	1.810	1.836	1.376	1.416	1.458	1.503	1.549	1.598	1.649	1.701	1.755	1.810	1.836
0.6	1.887	1.929	1.972	2.018	2.065	2.113	2.163	2.215	2.267	2.321	2.376	1.887	1.929	1.972	2.018	2.065	2.113	2.163	2.215	2.267	2.321	2.376
0.8	2.425	2.468	2.512	2.558	2.605	2.653	2.702	2.752	2.804	2.856	2.910	2.425	2.468	2.512	2.558	2.605	2.653	2.702	2.752	2.804	2.856	2.910
1.0	2.976	3.019	3.064	3.110	3.157	3.204	3.247	3.303	3.353	3.405	3.457	2.976	3.019	3.064	3.110	3.157	3.204	3.247	3.303	3.353	3.405	3.457
1.2	3.534	3.578	3.623	3.669	3.715	3.763	3.811	3.860	3.910	3.961	4.012	3.534	3.578	3.623	3.669	3.715	3.763	3.811	3.860	3.910	3.961	4.012
1.4	4.097	4.141	4.186	4.232	4.278	4.325	4.373	4.422	4.471	4.521	4.572	4.097	4.141	4.186	4.232	4.278	4.325	4.373	4.422	4.471	4.521	4.572
1.6	4.663	4.707	4.752	4.797	4.844	4.891	4.938	4.987	5.035	5.085	5.135	4.663	4.707	4.752	4.797	4.844	4.891	4.938	4.987	5.035	5.085	5.135
1.8	5.230	5.275	5.320	5.365	5.411	5.458	5.505	5.553	5.602	5.651	5.700	5.230	5.275	5.320	5.365	5.411	5.458	5.505	5.553	5.602	5.651	5.700
2.0	5.799	5.844	5.889	5.934	5.980	6.009	6.048	6.121	6.170	6.218	6.267	5.799	5.844	5.889	5.934	5.980	6.009	6.048	6.121	6.170	6.218	6.267

VITA

J. Michael McGee was born in Atlanta, Georgia, on April 1, 1953. He received a Bachelor of Science degree with a major in Aerospace Engineering in August 1975 from the Georgia Institute of Technology. He was also commissioned as a second lieutenant in the United States Air Force at that time.

He began his graduate studies in June 1977 at The University of Tennessee Space Institute upon his initial assignment to the Arnold Engineering Development Center. After a one-year delay to complete pilot training for the USAF, he subsequently received a Master of Science degree with a major in Aerospace Engineering in December 1981.