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I am submitting herewith a thesis written by Nicholas Anthony Lister entitled “Determination of Thermal Diffusivity and Thermal Conductivity Using Embedded Thermocouples.” I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

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(Original signatures are on file with official student records.)

# Determination of Thermal Properties Using Embedded Thermocouples

A Thesis Presented for  
The  
Master of Science  
Degree  
The University of Tennessee Space Institute

Nicholas Anthony Lister  
May 2010

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### Acknowledgements

This thesis would not have been possible without the guidance of Trevor M. Moeller, who helped every step of the way. I also owe thanks to Roy J. Schulz, who originally came up with the idea for the thesis, for his helpfulness throughout the entire process. Thanks is also due to my mother; everything I have become is because of her. Thanks are due to Richard Joel Thompson and Denise Su A. Aktas whose moral support kept me going.

## Abstract

The Purpose of this thesis is to experimentally demonstrate an inversion analysis technique, developed by Dr. Jay Frankel (UTK), that utilizes transient temperature data from probes embedded at known locations in a material. This allows one to determine thermal properties (thermal diffusivity and thermal conductivity) of the material, surface temperature, and the surface heat flux as they change with time. Dr. Frankel's inversion method can be used to determine surface temperature and heat flux of a one-dimensional semi-infinite slab based on the transient data from one or two embedded probes, if the thermal conductivity and thermal diffusivity of the material are known. Frankel's theory suggests that the thermal properties of the material can be determined if transient data from two thermocouple (TC) probes at known locations and the heat flux at the surface are known. This thesis investigates finding the thermal properties and surface temperature of materials using a two embedded thermocouple approach. As an initial check to the inversion analysis, the theoretical temperature solution for a one-dimensional semi-infinite slab was used. This validated that the analysis could converge to the constant thermal properties for the theoretical material. An experiment was run again to provide data for the materials copper and aluminum. Using a real material is fundamentally different from using theoretical determined (analytical) data, because the thermal properties for a real material vary with temperature. Since the inversion analysis converged to a constant solution for the theoretical temperatures, it was believed that the real material will converge to a solution. However, it was seen that the thermal diffusivity for the real materials never converged to the expected value. Although, when a constant handbook value for the thermal diffusivity is used to calculate the thermal conductivities from the experimental temperature data collected from the internal probes, the inversion analysis resulted in good agreement with experiment.

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| Symbol or Abbreviation | Nomenclature                   | Meaning                         |
|------------------------|--------------------------------|---------------------------------|
| $\alpha$               | Thermal diffusivity            | $\left[ \frac{m^2}{s} \right]$  |
| $k$                    | Thermal conductivity           | $\left[ \frac{W}{mK} \right]$   |
| $C_p$                  | Specific heat                  | $\left[ \frac{J}{kgK} \right]$  |
| $q_{in}$               | Heat going into an element     | $[W]$                           |
| $q_{out}$              | Heat going out of an element   | $[W]$                           |
| $g$                    | Heat generated                 | $[W]$                           |
| $\frac{dE}{dt}$        | Heat storage                   | $[W]$                           |
| $\rho$                 | Density                        | $\left[ \frac{kg}{m^3} \right]$ |
| $V$                    | Volume                         | $[m^3]$                         |
| $t$                    | Time                           | $[s]$                           |
| $x$                    | Distance                       | $[m]$                           |
| $T$                    | Temperature                    | $[K]$                           |
| $q''$                  | Heat flux                      | $\left[ \frac{W}{m^2} \right]$  |
| $h$                    | Height of sample               | $[m]$                           |
| $r$                    | Radius of sample               | $[m]$                           |
| $A$                    | Cross-Sectional area of sample | $[m^2]$                         |
| $A_s$                  | Area of side of sample         | $[m^2]$                         |
| $f$                    | Index for space                |                                 |
| $i$                    | Index for time                 |                                 |

## **Chapter 1**

### **Introduction**

When performing any kind of thermal analysis, knowing the thermal properties of a material, along with surface temperatures and heat flux, is essential. These properties and temperatures tell the investigator how fast heat can flow through a material and provide boundary conditions for the problem. The need to know thermal properties arises from the need to design thermal systems to certain specifications. For example, an oven door must be designed to have an outside temperature cool enough to prevent severe burns when touched on the outside. The material used inside the oven door should shield people from the heat within. Knowing the thermal properties for possible insulating materials inside the oven door will help the design ensure safety standards. Other examples of important thermal analysis include rocket combustor and nozzle walls, where thermal breakdown can lead to catastrophic failure.

The objective of this thesis is to demonstrate Dr. Jay Frankel's inversion analysis technique which allows thermal properties and surface temperatures to be determined for materials with unknown thermal properties using transient temperature data [1][2] with a known heat flux on a surface; more on the theory can be found in Chapter 2. Frankel's method is attractive because "rate information, involving the time derivative, have been shown a key element for the accurate projection" [1]. Another method that is commonly used for measuring thermal conductivities is the ASTM E1225 – 04 standard test method

that solves for the thermal conductivity of solids by means of the guarded-comparative-longitudinal heat flow technique [3]. The demonstration of using Frankel's inversion technique to determine thermal properties would offer a useful alternative. The objective of this thesis is completed by using two thermocouples embedded in a material to provide transient temperature data for use in Frankel's technique. Moreover, two thermocouples on the top and bottom surfaces of the material sample are used to validate the predicted surface temperatures.

With just the transient temperature data at known locations in the materials in question, Frankel's inversion analysis can be used to determine the temperature data at the top and bottom surfaces and the thermal diffusivity of the material. Knowing the sample density and the heat flux on a surface allows one to find thermal conductivity and specific heat, as shown in Chapter 3. This analysis is important due to the capability of determining surface temperatures that can be extremely hot or hard to reach. The test apparatus (described in Chapter 2) is set up in such a way that the one-dimensional, semi-infinite slab assumption is met, a requirement for proper application of Dr. Jay Frankel's analysis. Results and discussion appear in Chapter 4, and conclusions and recommendations are in Chapter 5.

## **Chapter 2**

### **Background**

In this chapter, the experiment and data analysis used to analyze data are discussed. The experiment was designed to validate Frankel's inversion analysis and confirm that correct thermal properties for real materials could be obtained. As will be presented in detail below, the experimental setup was designed to inhibit transverse heat transfer in a finite material sample to ensure that the semi-infinite slab assumption used in Frankel's inverse analysis is not violated.

#### *Experiment*

The experiment was conducted to demonstrate that Dr. Jay Frankel's inversion analysis can be used to determine thermal properties of unknown materials. To verify that the analysis would give correct thermal properties, two materials with known thermal properties, Copper 145 and Aluminum 6061 T6, were tested in the experiment, and the results obtained using the analysis described in the next subsection were used to determine thermal properties that could be compared to accepted, handbook values. To verify the calculated surface temperatures projected from embedded thermocouples, type K thermocouples were placed on the top and bottom surfaces of the sample as a means for comparison with projected results. Besides transient temperature data from the embedded thermocouples, the thermal diffusivity and the positions of the thermocouples relative to the surface are all that is necessary to calculate the surface temperatures and heat flux data. In Chapter 3, it will be shown that one can solve for thermal diffusivity

iteratively using only the transient temperature data from two embedded probes at known positions in the sample.

### *Experimental Setup*

Thermal properties of the material test samples are shown in Table 1, and a schematic of the experimental test setup is shown in Figure 1. The setup consists of a material sample that fits inside of the cylindrical cavity of a ceramic test apparatus that enforces an axial heat flux through the sample. To ensure that the experiment did not violate the semi-infinite slab assumption, as required in Dr. Frankel's theory, the test apparatus was designed to minimize heat flow in the radial direction (so axial heat transfer dominates). Therefore, the material sample was placed in ceramic so the majority of the heat would pass through the material sample and not radially through the ceramic insulator. Moreover, only a 0.008 inch radial gap was allowed between the sample and the ceramic to accommodate for thermal expansion of the material sample.

Table 1. Thermal Properties of Material Samples

| Material Name                 | Thermal Diffusivity                | Thermal Conductivity        | Specific Heat                           | Density                          |
|-------------------------------|------------------------------------|-----------------------------|-----------------------------------------|----------------------------------|
| Aluminum<br>6061 T6<br>[4][5] | $4.9 \cdot 10^{-5} \frac{m^2}{s}$  | $167 \frac{W}{m \cdot K}$   | $1.256 \cdot 10^3 \frac{J}{kg \cdot K}$ | $2.71 \cdot 10^3 \frac{kg}{m^3}$ |
| Copper 145<br>[4][6]          | $1.03 \cdot 10^{-4} \frac{m^2}{s}$ | $354.7 \frac{W}{m \cdot K}$ | $385 \frac{J}{kg \cdot K}$              | $8937.4 \frac{kg}{m^3}$          |
| Numerical<br>data*<br>[7]     | $3.75 \cdot 10^{-6} \frac{m^2}{s}$ | $14.7 \frac{W}{m \cdot K}$  | N/A                                     | N/A                              |

\* Note that the "numerical data" represents a high temperature material that might be used as a thermal barrier.

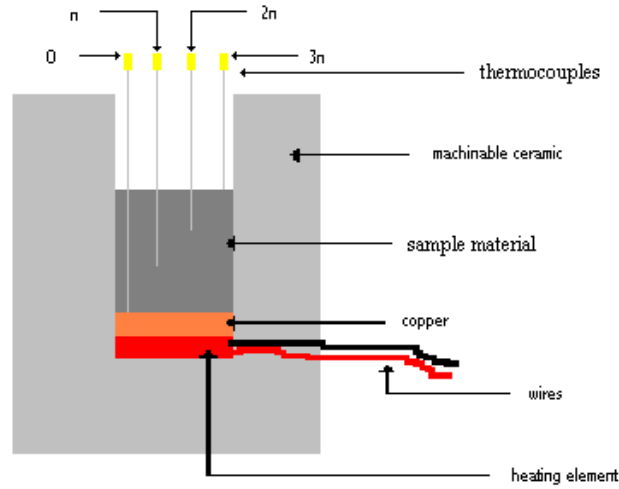


*Figure 1. Schematic of the experimental test setup*

Photographs and drawings of the test apparatus (Figures 62 through 73) can be found in the figures in Appendix A. The samples (aluminum and copper) are cylinders with a height of 1 inch and a diameter of 1 1/8 inches (see Figure 62 in the Appendix).

The thermally insulating ceramic block was fabricated out of general purpose silicate ceramic. This ceramic has a thermal conductivity of  $1.26 \frac{W}{m \cdot K}$ , which is much lower than both the copper alloy 145 thermal conductivity,  $354.7 \frac{W}{m \cdot K}$ , and the aluminum 6061 thermal conductivity,  $167 \frac{W}{m \cdot K}$  [4][5][6] (refer to Table 1). Note that “Numerical Data” in Table 1 represents a high-temperature material used to generate analytic temperature profiles used to challenge the analysis technique in a numerical



*Figure 2. Top and bottom ceramic block for insulating the samples*

experiment. To accommodate the passage of electrical leads to a heating element located in the bottom of a cylindrical hole in the ceramic, the ceramic block was comprised of two parts (see Figure 2) that mate together. The top ceramic piece is a 4 inch tall block with a 2 inch square cross-section; a 1 1/8 inch hole is drilled through to accommodate the material test sample. Note that small notches are machined on this piece to allow the passage of heating element leads (Figure 63). The bottom ceramic piece is a 2 inch square block that is 0.5 inches tall. It has a slip fit 1 1/8 inch diameter circle with 0.31 inch protrusion in the center that inserts into the top piece of ceramic (Figure 64). A photograph of both ceramic pieces is shown in Figure 2.

The notches in the bottom of the top piece of ceramic are slightly taller than the protrusion in the bottom piece. Therefore, the nicrome wires of the heating element can pass through the holes when the ceramic pieces are assembled (see Figure 65). The nicrome wire connects to a current shunt for the purpose of measuring and recording the

current. A National Instruments® data acquisition system was used to record the applied voltage and resulting current applied to the heating element. Therefore, heat flux at the bottom surface of the sample could be calculated from the product of the heating element current and voltage, divided by the cross section area of the sample material.

Each one inch tall material sample, with a  $1\frac{1}{8}$  inch round cross-section, had holes drilled in it to depths of 0, n, 2n, and 3n to accommodate thermocouple probes (refer to Figure 3). These 1/16" holes were drilled to depths 0, 0.333, 0.666, and 1 inches from the top surface (see Figure 1 and Figure 62). To minimize the effect of any radial variation in the sample, the holes were placed 90° degrees apart and radially at 0.281 inches from the center. Type K thermocouples are placed in each hole to measure the transient temperature data. The holes at 0.333 (2n) and 0.666 (n) inches from the top are used to determine the thermal properties and to project the temperature to the top (1", 3n) and bottom (0", 0) surfaces. At these locations (0 and 3n), the projected temperatures could be compared with actual thermocouple data. Precautions were taken to evenly heat the material samples. A  $\frac{1}{8}$  inch thick copper disc was placed between the electrical isolation disk and the material sample to promote a uniform heat load to the material sample.

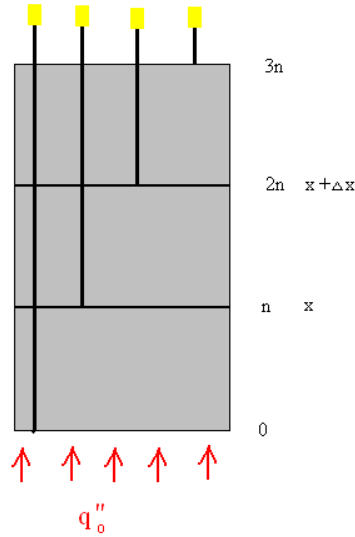


*Figure 3. Schematic of the cylindrical material sample*

The assembled apparatus includes the adjoined ceramic pieces and the nicrome heater (Figure 66) with its wires exiting through the slits in the ceramic insulator (Figure 65). A small circular piece of ceramic, less than  $\frac{1}{16}$  inch thick, was placed between the heating element and the sample to electrically isolate the sample (Figure 67). Photographs and drawings of the apparatus can be found in Appendix A.

#### *Data Analysis*

The data analysis was done in two parts, first with a preprocessor written in Matlab, then with a C++ code. Both of these were developed by the author and are provided in Appendix B. The Matlab preprocessor filtered the temperature data using a moving average, lowpass filter, with the Matlab function “smooth”. This function smooths the data by using a filter with coefficients equal to the reciprocal of the span, and in this case, the span was set to five. The lowpass filter smooths the temperature data,

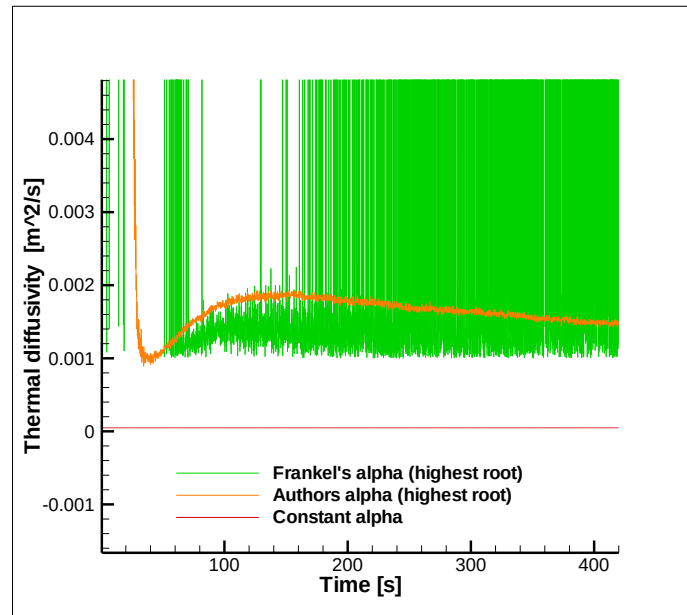


*Figure 4. Thermal diffusivity as a function of time for aluminum without moving average filter for temperature data*

so the noise obtained while taking the data has a much smaller impact on the analysis.

Plots of the thermal diffusivity data with and without smoothing the temperature data can be found in Figure 4 and Figure 5, respectively.

The purpose of the analysis coded into the C++ code is to correctly predict the thermal properties and surface temperatures of a sample material, given a known surface heat flux and transient temperature data from the embedded probes. However, Dr. Jay Frankel's theory only requires transient temperature measurements at known locations (and a surface heat flux) to determine thermal properties, assuming the material is homogenous [1][7][8]. Three approaches (detailed in Chapter 3) were taken to find the thermal properties and surface temperatures, a standard finite difference approximation

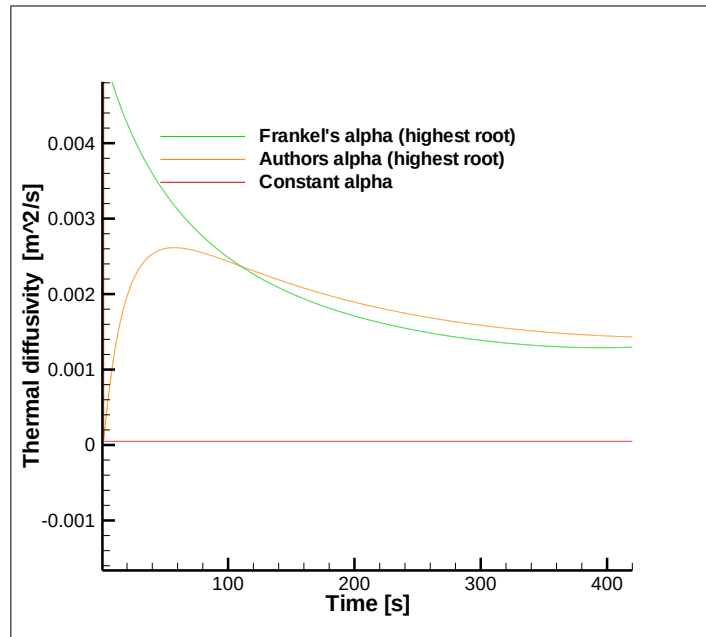


*Figure 5. Thermal diffusivity as a function of time for aluminum with moving average filter for temperature data*

(discussed on page 12), Dr. Jay Frankel's method (discussed on page 15), and finally the method developed by the author (discussed on page 20). The C++ code performs these analyses using data process by the Matlab preprocessor as input.

## Chapter 3

### Theory

#### *Knowns and Unknowns*

To solve for  $n$  unknowns, one must have  $n$  equations that depend on known parameters.

The known and unknown parameters in the analysis procedure are presented below.

#### *Knowns*

The known quantities in this problem are:

1. the positions of the thermocouples embedded in the sample material,
2. the transient temperature data for the embedded probes at locations  $n$  and  $2n$ , (see Figure 1)
3. the sample volume and mass (to give sample density), and
4. the current and voltage of the heating element from which a sample heat flux at the bottom surface can be calculated. Note that the ceramic block must have a small thermal conductivity to minimize transverse heat loss.

#### *Unknowns*

The unknown quantities of the inversion problem are:

1. the surface temperature (bottom surface in Figure 1),
2. the temperature at  $3n$  (top surface in Figure 1),
3. the thermal diffusivity,  $\alpha$ ,

4. the thermal conductivity,  $k$ , and
5. the specific heat,  $C_p$ .

Note again that the thermocouples at the bottom surface and the top surface (3n) are not needed to determine the thermal diffusivity, however they provide data for comparison with predicted results. Now, the three approaches used in this thesis to determine surface temperatures and thermal properties will be presented.

### *Finite Difference Approximation*

Finite difference approximation, FDA, is the most straight-forward way to analyze data from the experiment and is also probably the first method most would consider using. Please keep in mind that this approach is not a simulation but rather a data analysis technique. The Finite Difference Approximation, detailed below, uses actual temperature data obtained from the experimental setup.

Starting from basic principles of heat transfer, we use the energy balance equation.

$$q_{in} + g = q_{out} + \frac{dE}{dt} \quad (3-1)$$

Assuming a semi-infinite slab with no heat generation. The equation becomes

$$q_{in} \big|_x = q_{out} \big|_{x+\Delta x} + \frac{dE}{dt} \quad (3-2)$$

where  $\Delta x$  is the distance between the thermocouples, as shown in Figure 3. Fourier's law for conduction gives

$$q|_x = - \left[ kA \frac{\partial T}{\partial x} \right]_x \quad (3-3)$$

$$q|_{x+\Delta x} = - \left[ kA \frac{\partial T}{\partial x} \right]_{x+\Delta x} \quad (3-4)$$

and the change of energy in the volume is

$$\frac{dE}{dt} = \rho V c_p \frac{\partial T}{\partial t} \quad (3-5)$$

where  $\rho$  and  $C_p$  are assumed to be constant in time. Plugging these equations into (3-2) we obtain

$$- \left[ kA \frac{\partial T}{\partial x} \right]_x = - \left[ kA \frac{\partial T}{\partial x} \right]_{x+\Delta x} + \rho V c_p \frac{\partial T}{\partial t} \quad (3-6)$$

Rearranging terms and substituting  $(A\Delta x)$  for the volume,  $V$ , gives

$$\frac{\left[ kA \frac{\partial T}{\partial x} \right]_{x+\Delta x} - \left[ kA \frac{\partial T}{\partial x} \right]_x}{\Delta x} = \frac{\rho(A\Delta x)c_p}{\Delta x} \frac{\partial T}{\partial t} \quad (3-7)$$

From calculus the definition of a derivative is

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (3-8)$$

Thus,

$$\frac{\partial}{\partial x} \left( kA \frac{\partial T}{\partial x} \right) = \rho c_p A \frac{\partial T}{\partial t} \quad (3-9a)$$

Assuming constant area,  $A$ , and thermal conductivity,  $k$ , gives

$$k \frac{\partial^2 T}{\partial x^2} = \rho c_p \frac{\partial T}{\partial t} \quad (3-9b)$$

Defining the thermal diffusivity,  $\alpha$ ,

$$\alpha = \frac{k}{\rho c_p} \quad (3-10)$$

and substituting into equation (3-9b) gives

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (3-11)$$

Equation (3-11) is discretized and applied to discrete volumes in FDA. The FDA in this case is one dimensional in space because of the semi-infinite slab assumption. The FDA applied in this thesis uses nodes to find the temperature at locations away from measured values i.e. at the top and bottom surfaces. Referring to Figure 1 and Figure 3, nodes  $n$  and  $2n$  are used to calculate the temperatures at locations  $3n$  and  $0$ . Discretizing equation (3-11) using central differencing techniques about node  $n$  gives

$$\frac{T(0,t) + T(2n,t) - 2.0 \cdot T(n,t)}{(\Delta x)^2} = \frac{1}{\alpha} \frac{T(n,t+1) - T(n,t-1)}{2 \cdot \Delta t} \quad (3-12)$$

Rearranging and solving for  $T(0,t)$  gives

$$T(0,t) = 2.0 \cdot T(n,t) - T(2n,t) + \frac{(\Delta x)^2}{\alpha} \frac{T(n,t+1) - T(n,t-1)}{2 \cdot \Delta t} \quad (3-13)$$

Repeating this process for node  $2n$  and solving for  $T(3n, t)$  gives

$$T(3n, t) = 2.0 \cdot T(2n, t) - T(n, t) + \frac{(\Delta x)^2}{\alpha} \frac{T(2n, t+1) - T(2n, t-1)}{2 \cdot \Delta t} \quad (3-14)$$

Thus, the temperatures  $T(3n, t)$  and  $T(0, t)$  can be determined for a material with a known thermal diffusivity,  $\alpha$ , with thermocouples at  $n$  and  $2n$  placed a distance of  $\Delta x$  apart and  $\Delta x$  away from 0 and  $3n$  (Figure 3). The stability of the solution for the first-order central difference in time requires that  $\frac{\alpha \Delta t}{(\Delta x)^2} < \frac{1}{2}$  [8]. Using the values from Table 1 we see that

the ratio on the left hand side of this equation is well under one-half, so the numerical estimates of temperatures  $T(0, t)$  and  $T(2n, t)$  should be stable.

#### *Frankel's Method*

Frankel's method, described below, involves taking the Taylor series expansions about individual nodes, substituting Fourier's conduction law and the 1-D energy equation in the form shown below (3-17) into the Taylor series (to make it time dependent), and manipulating the equations to back out unknowns (references [1][7][8]).

The mathematical definition for the Taylor series is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x - c)^n \quad (3-15)$$

where  $f(x)$  is the function one is finding about node  $f(c)$ . Referring to Figure 3 and using equation (3-15) to write the fourth order Taylor series for  $T(n, t)$  gives

$$T(n,t) = T(2n,t) - \frac{\partial T}{\partial x}(2n,t) \cdot n + \frac{\partial^2 T}{\partial x^2}(2n,t) \cdot \frac{n^2}{2} - \frac{\partial^3 T}{\partial x^3}(2n,t) \cdot \frac{n^3}{6} + \frac{\partial^4 T}{\partial x^4}(2n,t) \cdot \frac{n^4}{24} + \dots$$

(3-16)

Substituting Fourier's Law (equation (3-3)) into the 1-D energy equation (3-11), gives

$$\frac{\partial q''}{\partial x} = -\rho c_p \frac{\partial T}{\partial t} \quad (3-17)$$

Plugging equations (3-3) and (3-17) into the Taylor series expansion (3-16), and repeating the process for nodes 0, 2n, 3n gives four relations for the temperature at each location:

$$T(n,t) = T(2n,t) + q''(2n,t) \cdot \frac{n}{k} + \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n^2}{2\alpha} + \frac{\partial q''}{\partial t}(2n,t) \cdot \frac{n^3}{6\alpha k} + \frac{\partial^2 T}{\partial t^2}(2n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-18)

$$T(2n,t) = T(n,t) - q''(n,t) \cdot \frac{n}{k} + \frac{\partial T}{\partial t}(n,t) \cdot \frac{n^2}{2\alpha} - \frac{\partial q''}{\partial t}(n,t) \cdot \frac{n^3}{6\alpha k} + \frac{\partial^2 T}{\partial t^2}(n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-19)

$$T(0,t) = T(n,t) + q''(n,t) \cdot \frac{n}{k} + \frac{\partial T}{\partial t}(n,t) \cdot \frac{n^2}{2\alpha} + \frac{\partial q''}{\partial t}(n,t) \cdot \frac{n^3}{6\alpha k} + \frac{\partial^2 T}{\partial t^2}(n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-20)

$$T(3n,t) = T(2n,t) - q''(2n,t) \cdot \frac{n}{k} + \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n^2}{2\alpha} - \frac{\partial q''}{\partial t}(2n,t) \cdot \frac{n^3}{6\alpha k} + \frac{\partial^2 T}{\partial t^2}(2n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-21)

Taking equation (3-17) and substituting it into (3-18), (3-19), (3-20), and (3-21) and manipulating gives

$$q''(n,t) = q''(2n,t) + \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n \cdot k}{\alpha} + \frac{\partial q''}{\partial t}(2n,t) \cdot \frac{n^2}{2\alpha} + \frac{\partial^2 T}{\partial t^2}(2n,t) \cdot \frac{n^3 \cdot k}{6\alpha^2} + \frac{\partial^2 q''}{\partial t^2}(2n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-22)

$$q''(3n,t) = q''(2n,t) - \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n \cdot k}{\alpha} + \frac{\partial q''}{\partial t}(2n,t) \cdot \frac{n^2}{2\alpha} - \frac{\partial^2 T}{\partial t^2}(2n,t) \cdot \frac{n^3 \cdot k}{6\alpha^2} + \frac{\partial^2 q''}{\partial t^2}(2n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-23)

$$q''(2n,t) = q''(n,t) - \frac{\partial T}{\partial t}(n,t) \cdot \frac{n \cdot k}{\alpha} + \frac{\partial q''}{\partial t}(n,t) \cdot \frac{n^2}{2\alpha} - \frac{\partial^2 T}{\partial t^2}(n,t) \cdot \frac{n^3 \cdot k}{6\alpha^2} + \frac{\partial^2 q''}{\partial t^2}(n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-24)

$$q''(0,t) = q''(n,t) + \frac{\partial T}{\partial t}(n,t) \cdot \frac{n \cdot k}{\alpha} + \frac{\partial q''}{\partial t}(n,t) \cdot \frac{n^2}{2\alpha} + \frac{\partial^2 T}{\partial t^2}(n,t) \cdot \frac{n^3 \cdot k}{6\alpha^2} + \frac{\partial^2 q''}{\partial t^2}(n,t) \cdot \frac{n^4}{24\alpha^2} + \dots$$

(3-25)

Frankel's inversion analysis [1] [7][8] provides

$$q''(f,i) = 2k \sqrt{\frac{1}{\pi \cdot \alpha}} \sum_{j=0}^{i-1} \frac{dT}{dt}(f,j) \cdot (\sqrt{t_i - t_j} - \sqrt{t_i - t_{j+1}}) \quad (3-26)$$

To simplify writing equations (3-22) through (3-25), (3-26) is written as

$$q''(f,i) = \frac{k}{\sqrt{\alpha}} \cdot q''_{ts}(f,i) \quad (3-27)$$

where

$$q''_{ts}(f,i) = \frac{2}{\sqrt{\pi}} \sum_{j=0}^{i-1} \frac{dT}{dt}(f,j) \cdot (\sqrt{t_i - t_j} - \sqrt{t_i - t_{j+1}}) \quad (3-28)$$

Now one can solve for thermal properties. Knowing that

$$\frac{q''(n,t)}{q''(2n,t)} = \frac{q''_{ts}(n,t)}{q''_{ts}(2n,t)} \quad (3-29)$$

and taking equation (3-22) (reducing its order to three to allow for comparison with Frankel's method as given in [8]), and inserting equations (3-29) and (3-28) gives

$$q''(2n,t) \cdot \frac{q''_{ts}(n,t)}{q''_{ts}(2n,t)} = q''(2n,t) + \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n \cdot k}{\alpha} + \frac{\partial q''_{ts}}{\partial t}(2n,t) \cdot \frac{n^2 \cdot k}{2\alpha^{3/2}} + \frac{\partial^2 T}{\partial t^2}(2n,t) \cdot \frac{n^3 \cdot k}{6\alpha^2} \quad (3-30)$$

Solving for  $q''(2n,t)$ , the equation is rearranged as

$$q''(2n,t) \cdot \left( \frac{q''_{ts}(n,t)}{q''_{ts}(2n,t)} - 1 \right) = \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n \cdot k}{\alpha} + \frac{\partial q''_{ts}}{\partial t}(2n,t) \cdot \frac{n^2 \cdot k}{2\alpha^{3/2}} + \frac{\partial^2 T}{\partial t^2}(2n,t) \cdot \frac{n^3 \cdot k}{6\alpha^2} \quad (3-31)$$

Taking  $\left( \frac{q''_{ts}(n,t)}{q''_{ts}(2n,t)} - 1 \right)$ , rearranging it into  $\left( \frac{q''_{ts}(n,t) - q''_{ts}(2n,t)}{q''_{ts}(2n,t)} \right)$ , and dividing into

equation (3-31) gives

$$q''(2n,t) = \frac{n \cdot k}{\alpha} \cdot \frac{\frac{\partial T}{\partial t}(2n,t) \cdot q''_{ts}(2n,t)}{(q''_{ts}(n,t) - q''_{ts}(2n,t))} + \frac{n^2 \cdot k}{2\alpha^{3/2}} \cdot \frac{\frac{\partial q''_{ts}}{\partial t}(2n,t) \cdot q''_{ts}(2n,t)}{(q''_{ts}(n,t) - q''_{ts}(2n,t))} + \frac{n^3 \cdot k}{6\alpha^2} \cdot \frac{\frac{\partial^2 T}{\partial t^2}(2n,t) \cdot q''_{ts}(2n,t)}{(q''_{ts}(n,t) - q''_{ts}(2n,t))} \quad (3-32)$$

Taking equation (3-18), with order reduced to three for comparison in Frankel's published results [8], and inserting equation (3-27) gives

$$T(n,t) = T(2n,t) + q''(2n,t) \cdot \frac{n}{k} + \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n^2}{2\alpha} + \frac{\partial q''_{ts}}{\partial t}(2n,t) \cdot \frac{n^3}{6\alpha^{3/2}} \quad (3-33)$$

Solving equation (3-33) for  $q''(2n, t)$  and inserting (3-32) gives

$$\begin{aligned} & \frac{n \cdot k}{\alpha} \frac{\frac{\partial T}{\partial t}(2n, t) \cdot q''_{ts}(2n, t)}{(q''_{ts}(n, t) - q''_{ts}(2n, t))} + \frac{n^2 \cdot k}{2\alpha^{3/2}} \frac{\frac{\partial q''_{ts}}{\partial t}(2n, t) \cdot q''_{ts}(2n, t)}{(q''_{ts}(n, t) - q''_{ts}(2n, t))} + \frac{n^3 \cdot k}{6\alpha^2} \cdot \frac{\frac{\partial^2 T}{\partial t^2}(2n, t) \cdot q''_{ts}(2n, t)}{(q''_{ts}(n, t) - q''_{ts}(2n, t))} = \\ & \frac{k}{n} (T(n, t) - T(2n, t)) - \frac{\partial T}{\partial t}(2n, t) \cdot \frac{n \cdot k}{2\alpha} - \frac{\partial q''_{ts}}{\partial t}(2n, t) \cdot \frac{n^2 \cdot k}{6\alpha^{3/2}} \end{aligned} \quad (3-34)$$

Rearranging and dividing by  $\frac{n \cdot \alpha^2}{(T(n, t) - T(2n, t))}$  gives

$$\begin{aligned} & \alpha^2 - \frac{n^2 \cdot \alpha}{(T(n, t) - T(2n, t))} \frac{\partial T}{\partial t}(2n, t) \left( \frac{1}{2} + \frac{q''_{ts}(2n, t)}{(q''_{ts}(n, t) - q''_{ts}(2n, t))} \right) \\ & - \frac{n^3 \cdot \alpha^{1/2}}{(T(n, t) - T(2n, t))} \frac{\partial q''_{ts}}{\partial t}(2n, t) \left( \frac{1}{6} + \frac{1}{2} \frac{q''_{ts}(2n, t)}{(q''_{ts}(n, t) - q''_{ts}(2n, t))} \right) \\ & - \frac{n^4}{6(T(n, t) - T(2n, t))} \left( \frac{\frac{\partial^2 T}{\partial t^2}(2n, t) \cdot q''_{ts}(2n, t)}{(q''_{ts}(n, t) - q''_{ts}(2n, t))} \right) = 0 \end{aligned} \quad (3-35)$$

This equation depends only on transient temperature data from embedded probes at known locations, and it can be used to iteratively solve for the thermal diffusivity,  $\alpha$ .

Then, thermal diffusivity can be used to find the surface temperatures at  $T(0, t)$  and

$T(3n, t)$  using equations (3-20) and (3-21), along with (3-27), to give

$$T(0, t) = T(n, t) + q''_{ts}(n, t) \cdot \frac{n}{\alpha^{1/2}} + \frac{\partial T}{\partial t}(n, t) \cdot \frac{n^2}{2\alpha} + \frac{\partial q''_{ts}}{\partial t}(n, t) \cdot \frac{n^3}{6\alpha^{3/2}} + \frac{\partial^2 T}{\partial t^2}(n, t) \cdot \frac{n^4}{24\alpha^2} + \dots \quad (3-36)$$

$$T(3n,t) = T(2n,t) - q''_{ts}(2n,t) \cdot \frac{n}{\alpha^{1/2}} + \frac{\partial T}{\partial t}(2n,t) \cdot \frac{n^2}{2\alpha} - \frac{\partial q''_{ts}}{\partial t}(2n,t) \cdot \frac{n^3}{6\alpha^{3/2}} + \frac{\partial^2 T}{\partial t^2}(2n,t) \cdot \frac{n^4}{24\alpha^2} + \dots \quad (3-37)$$

Since the thermal diffusivity is now known the thermal conductivity,  $k$ , can be found.

Using equation (3-25), again reducing the order to three and inserting into (3-27), we obtain.

$$q''(0,t) = \frac{k}{\sqrt{\alpha}} \cdot q''_{ts}(n,t) + \frac{dT}{dt}(n,t) \cdot \frac{n \cdot k}{\alpha} + \frac{\partial q''_{ts}}{\partial t}(n,t) \cdot \frac{k \cdot n^2}{2\alpha^{3/2}} \quad (3-38)$$

Recalling that  $q''(0,t)$  is a known value, (3-38) can be rearranged and solved for  $k$  to give

$$k = \frac{q''(0,t)}{\left( \frac{1}{\sqrt{\alpha}} \cdot q''_{ts}(n,t) + \frac{\partial T}{\partial t}(n,t) \cdot \frac{n}{\alpha} + \frac{\partial q''_{ts}}{\partial t}(n,t) \cdot \frac{n^2}{2\alpha^{3/2}} \right)} \quad (3-39)$$

#### *Author's Method*

The Author's method takes equation (3-19) with (3-27) and solves for the thermal diffusivity without first solving for the transient heat flux equations--equations (3-22) through (3-25). This makes it more direct to obtain a solution, as long as the transient heat flux equations are not required. Substituting equation (3-27) into equation (3-19) and rearranging gives

$$0 = T(n,t) - T(2n,t) - q''_{ts}(n,t) \cdot \frac{n}{\alpha^{1/2}} + \frac{\partial T}{\partial t}(n,t) \cdot \frac{n^2}{2\alpha} - \frac{\partial q''_{ts}}{\partial t}(n,t) \cdot \frac{n^3}{6\alpha^{3/2}} + \frac{\partial^2 T}{\partial t^2}(n,t) \cdot \frac{n^4}{24\alpha^2} \quad (3-40)$$

Now, equation (3-40) can be solved for the thermal diffusivity,  $\alpha$ . With a known  $\alpha$  the thermal conductivity,  $k$ , can easily be determined by taking equation (3-27) and rearranging to give

$$k = \frac{q''(0,t)}{\frac{1}{\sqrt{\alpha}} \cdot q''_{ts}(0,t)} \quad (3-41)$$

Finally, since the thermal diffusivity,  $\alpha$ , and thermal conductivity,  $k$ , are known, equation (3-10) can be rearranged to give a relation for the specific heat.

$$c_p = \frac{k}{\rho\alpha} \quad (3-42)$$

### *Effects of Heat Loss*

In the 1-D energy equation (3-11) used for the FDA calculation, a semi-infinite slab is assumed such that all heat leaves through the top surface, i.e. there is no heat loss. However, since the insulator in the experiment is not perfect, one would expect some heat loss though the side walls. The energy equation now will be used to examine how closely the temperature data measured in the experiment approximates the infinite plane assumption.

In the FDA analysis  $q_{out}|_{x+\Delta x}$  (heat flux out of the top surface of the sample) was the only loss mechanism included. We will now include heat loss through the side walls. Starting with equation (3-2) we split the term  $q_{out}$  into  $q_{loss}$  and  $q_{out}|_{x+\Delta x}$ , where  $q_{loss}$  is the heat loss though the sides. With this in mind, the energy equation can be written as

$$q_{in} \big|_x = q_{out} \big|_{x+\Delta x} + q_{loss} + \frac{dE}{dt} \quad (3-43)$$

From Fourier's conduction law, one can write

$$q \big|_x = - \left[ kA \frac{\partial T}{\partial x} \right]_x \quad (3-44)$$

$$q \big|_{x+\Delta x} = - \left[ kA \frac{\partial T}{\partial x} \right]_{x+\Delta x} \quad (3-45)$$

and  $\frac{dE}{dt}$  is given by

$$\frac{dE}{dt} = \rho V c_p \frac{\partial T}{\partial t} \quad (3-46)$$

Substituting equations (3-44) through (3-46) into the energy equation (3-43) gives

$$- \left[ kA \frac{\partial T}{\partial x} \right]_x = - \left[ kA \frac{\partial T}{\partial x} \right]_{x+\Delta x} + q_{loss} + \rho V c_p \frac{\partial T}{\partial t} \quad (3-47)$$

Rearranging terms and substituting  $(A\Delta x)$  for  $V$  gives

$$\frac{\left[ kA \frac{\partial T}{\partial x} \right]_{x+\Delta x} - \left[ kA \frac{\partial T}{\partial x} \right]_x}{\Delta x} = q_{loss} + \frac{\rho(A\Delta x)c_p}{\Delta x} \frac{\partial T}{\partial t} \quad (3-48)$$

From calculus the definition of a derivative is

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (3-49)$$

Thus, assuming constant  $k$  and  $A$ , equation (3-48) becomes

$$k \frac{\partial^2 T}{\partial x^2} = \frac{q_{loss}}{A\Delta x} + \rho c_p \frac{\partial T}{\partial t} \quad (3-50)$$

Note that this equation implies that  $k$  and  $C_p$  are independent of temperature. From the definition of thermal diffusivity,

$$\alpha = \frac{k}{\rho C_p} \quad (3-51)$$

and solving equation (3-50) for  $q_{loss}$  gives

$$q_{loss} = A \Delta x k \left( \frac{\partial^2 T}{\partial x^2} - \frac{1}{\alpha} \frac{\partial T}{\partial t} \right) \quad (3-52)$$

Knowing that  $A$  is the area in which heat is applied we can find  $q_{loss}$ ; however,  $q_{loss}$  is in Watts, thus dividing by the area of the sample's side (wall the through which the heat leaves) gives.

$$q_{loss}'' = \frac{q_{loss}}{A_s} \quad (3-53)$$

where the term  $A_s$  is the area of the side of the sample given by the equation  $(2\pi \cdot r \cdot h)$ .

The result is a heat flux through the side wall of the sample material. The closer this term is to zero, the more likely the semi-infinite slab assumption is met.

## Chapter 4

### Results and Discussion

In this chapter, the results of the analysis approaches presented in Chapter 3 are presented for aluminum and copper samples. In addition, the results from a numerical experiment that uses temperature profiles from exact analytical solutions for a semi-infinite plane are shown. This numerical experiment provides a mechanism for evaluating the data analysis techniques.

#### *Temperature Data used in Analysis*

The numerical data experiment uses the theoretical solution to the planar, 1-D heat diffusion equation given as

$$T(x,t) = q_0'' \sqrt{\frac{\alpha}{k^2 \pi}} \left( 2\sqrt{t} e^{-\frac{x^2}{4\alpha t}} - \sqrt{\frac{\pi x^2}{\alpha}} \operatorname{erfc} \sqrt{\frac{x^2}{4\alpha t}} \right) \quad (5-1)$$

Please see references [9][7][8] for more discussion. The values  $q_0'' = 1 \times 10^6 \text{ W/m}^2$ ,  $\alpha = 3.75 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $k = 14.7 \text{ W/(mK)}$ , and a distance of 5 mm between thermocouples, as presented in reference [8], were used as a test case. This served as a check to confirm that the analysis was working for analytical temperatures. The analytical temperature data given in Figure 6 were calculated using equation (5-1), assuming the values given above. This temperature data starts at 0K and goes up to 2500K in only 350 seconds. Notice that all the temperatures are evenly spaced. These temperature profiles will be used in the analysis of the numerical experiment revisited later in this chapter.

Experimental temperature data (Figure 7 and Figure 8) were collected for real materials (copper and aluminum) using the test apparatus in Figure 1, as discussed in Chapter 2. The temperature data for copper is given in Figure 7. The measured copper temperature data starts off at room temperature ( $\sim 293\text{K}$ ) and increases to about  $350\text{K}$ , much lower than the analytical temperature data. The measured temperature data for aluminum is given in Figure 8. The aluminum temperature is initially at room temperature and increases to about  $355\text{K}$ , also much lower than the numerical temperature data. Note that the thermocouple at the top surface ( $3n$ ) for both copper and aluminum read much lower than the rest of the thermocouples, because this particular thermocouple, which rested on the surface, was exposed to the air and likely had a large thermal contact resistance.

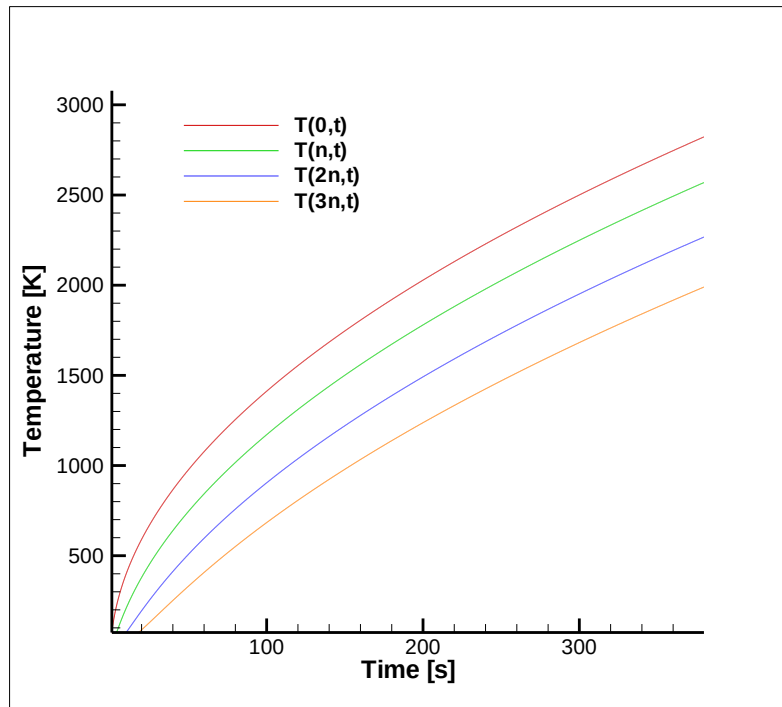


Figure 6. Analytical temperature data used in the numerical experiment.

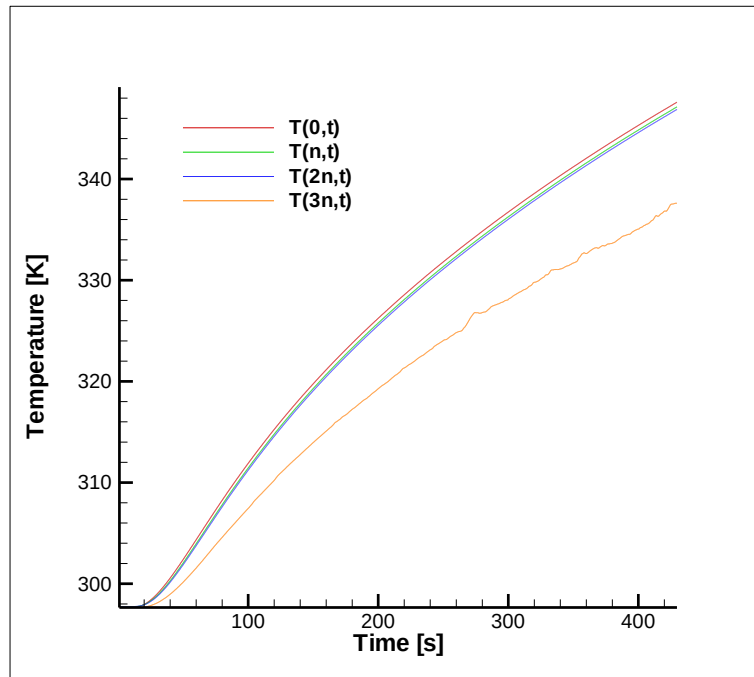


Figure 7. Copper sample temperature data measured in the experiment.

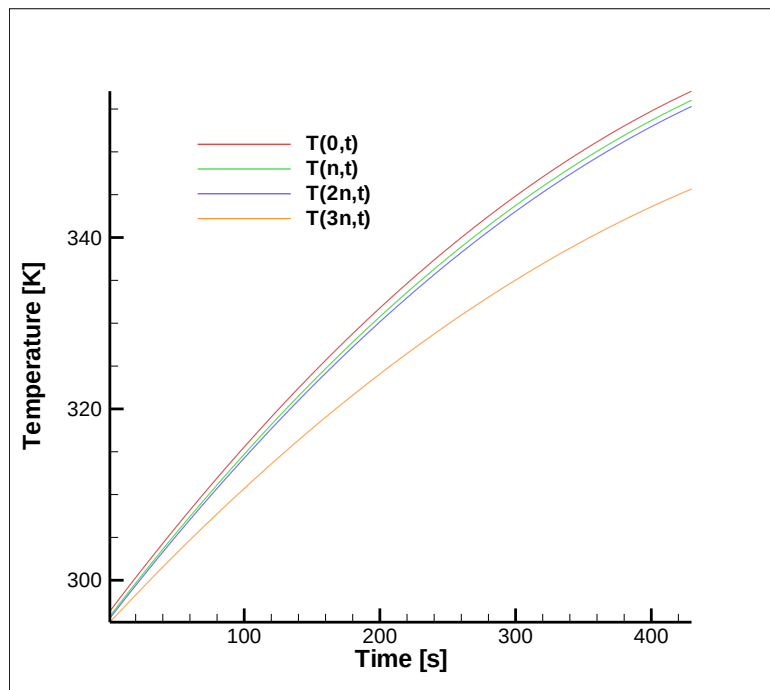


Figure 8. Aluminum temperature data measured in the experiment.

### *Thermal Diffusivity*

The thermal diffusivity was calculated using the Frankel and the Author's methods described in Chapter 3. Frankel's and the Author's methods use equations (3-35) and (3-40), respectively, to iteratively solve for thermal diffusivity. For the numerical experiment temperature data, the thermal diffusivities are shown on Figure 9 through Figure 12. Figure 9 shows that both methods converge to the accepted value of  $\alpha = 3.75 \times 10^{-6} \text{ m}^2 / \text{s}$ , with Frankel's method converging a little faster. However, because of the nature of the equations (equations 3-35 and 3-40) used to solve for the thermal diffusivity, a second root will occur. Figure 10 shows the smaller root for the numerical experiment. The larger roots were taken as the correct ones to use because they result in thermal diffusivity that are close to the accepted value used to calculate the analytic temperature profiles. The thermal diffusivities for the large and small root are also shown as functions of temperature in Figure 11 and Figure 12, respectively. Note that "const. alpha" in Figure 9 – Figure 12 refers to the thermal diffusivity used to generate the analytic temperature profiles for the numerical experiment.

The experimental thermal diffusivity for the copper (Figure 13 through Figure 16) had both analysis methods for the larger alpha, Figure 13 and Figure 15, converge to a value around  $\alpha = 0.006 \text{ m}^2 / \text{s}$ , while Frankel's method was noisier. Note that the accepted value for  $\alpha$  ( $1 \times 10^{-4} \frac{\text{m}^2}{\text{s}}$ ) is given in Table 1 and about 60 times smaller than the experimental value. The smaller roots of alpha for copper, shown in Figure 14 and Figure 16, are highly unstable.

The experimental thermal diffusivity for aluminum (Figure 17-Figure 20) for the large alpha roots analyzed using both methods smoothly converge to a value around  $\alpha = 0.0015 \text{ m}^2 / \text{s}$ . (The smaller roots for aluminum (Figure 18 and Figure 20) are on that order of  $10^{-8}$ ) Frankel's method slowly increases from time equal zero and the Authors method doesn't exist until the 220 second mark. The accepted value for  $\alpha$  ( $4.9 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$ ) is given in Table 1 and is about 50 times smaller than the experimental value.

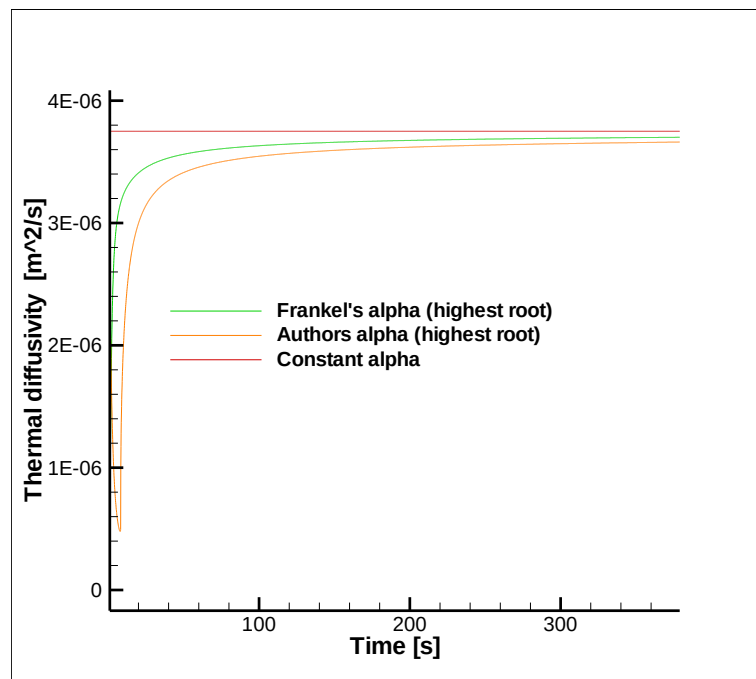


Figure 9. Large thermal diffusivity values for numerical experiment data

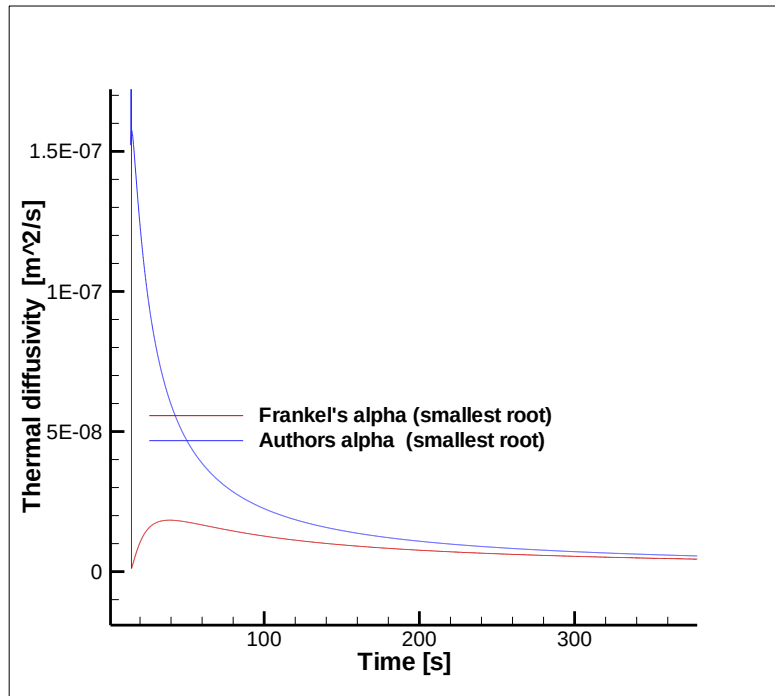


Figure 10. Small thermal diffusivity values for numerical experiment data

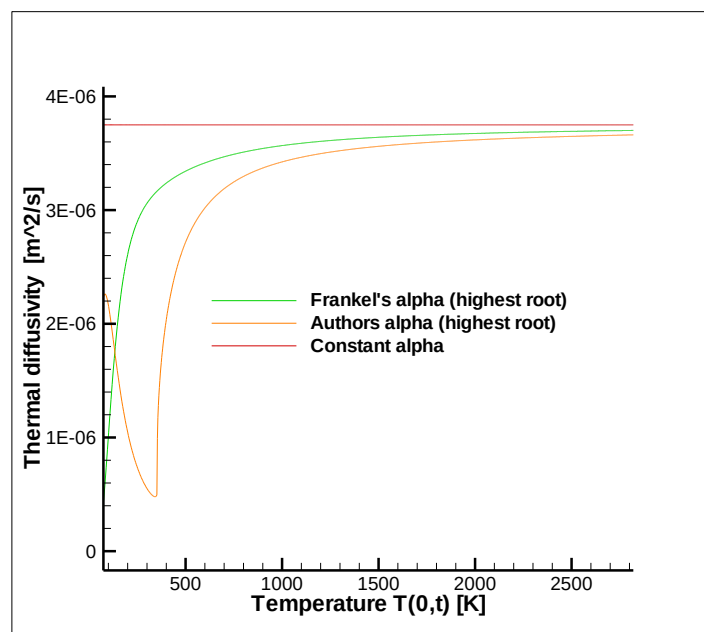


Figure 11. Large thermal diffusivity values as a function of temperature  $T(0,t)$  for numerical experiment data

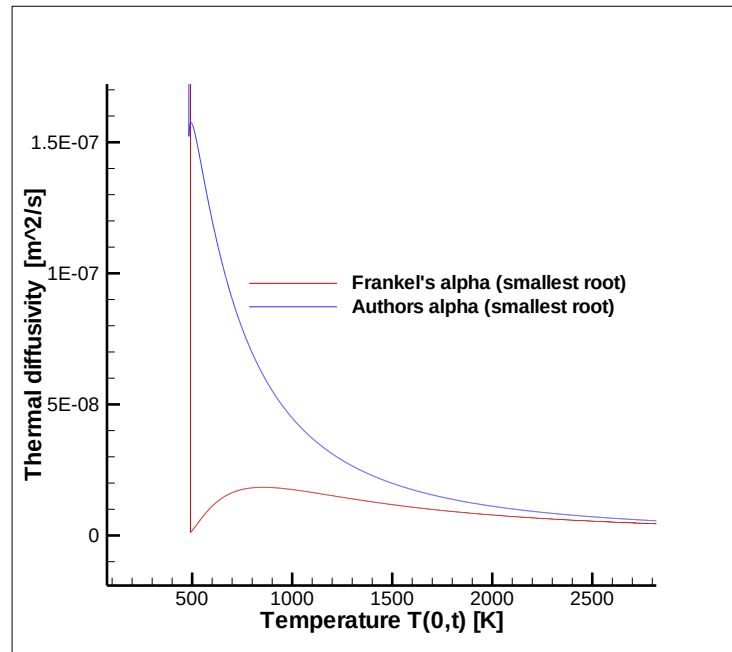


Figure 12. Small thermal diffusivity values as a function of temperature  $T(0,t)$  for numerical data

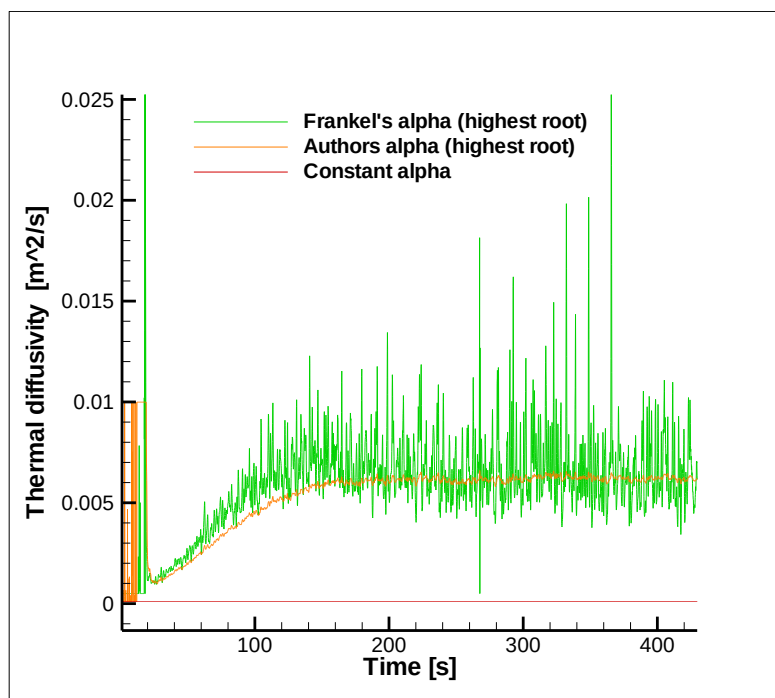


Figure 13. Copper sample large thermal diffusivity values

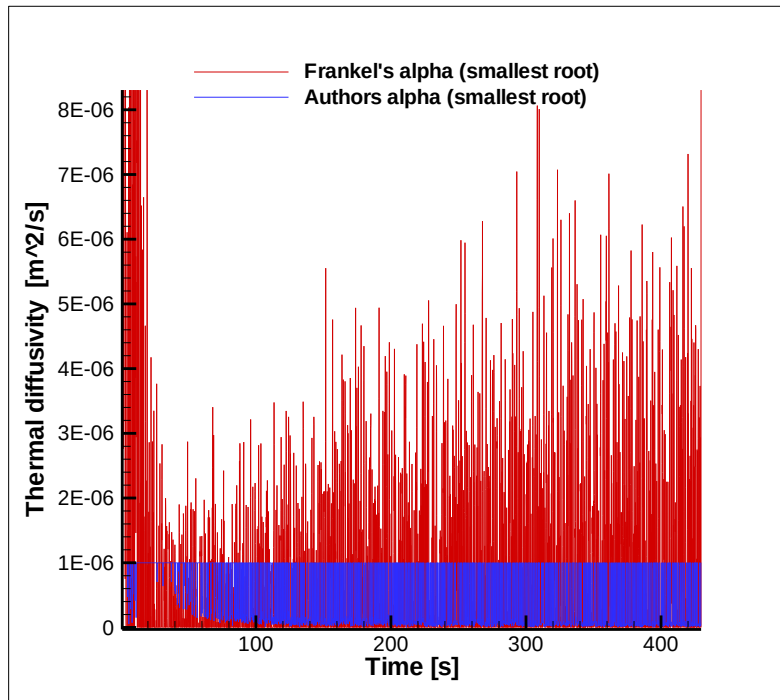


Figure 14. Copper sample small thermal diffusivity values

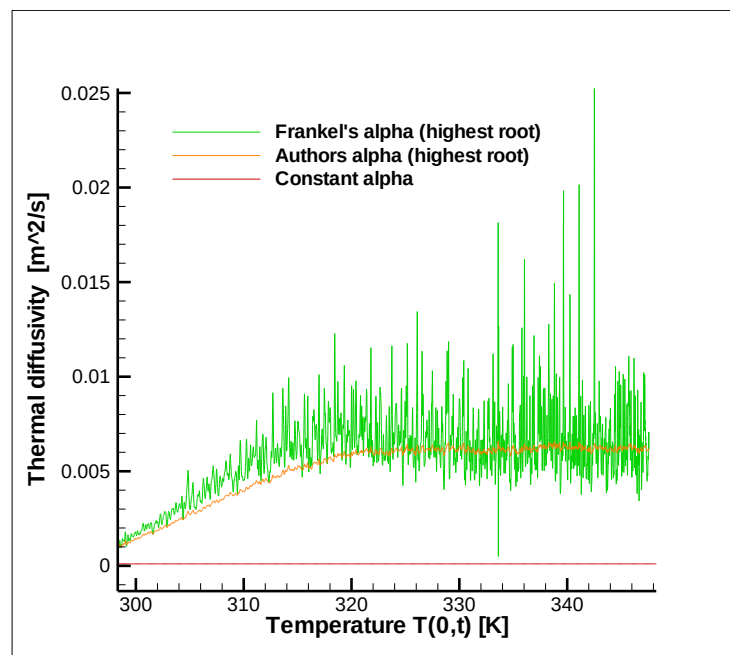


Figure 15. Copper sample large thermal diffusivity values as a function of temperature

$T(0,t)$

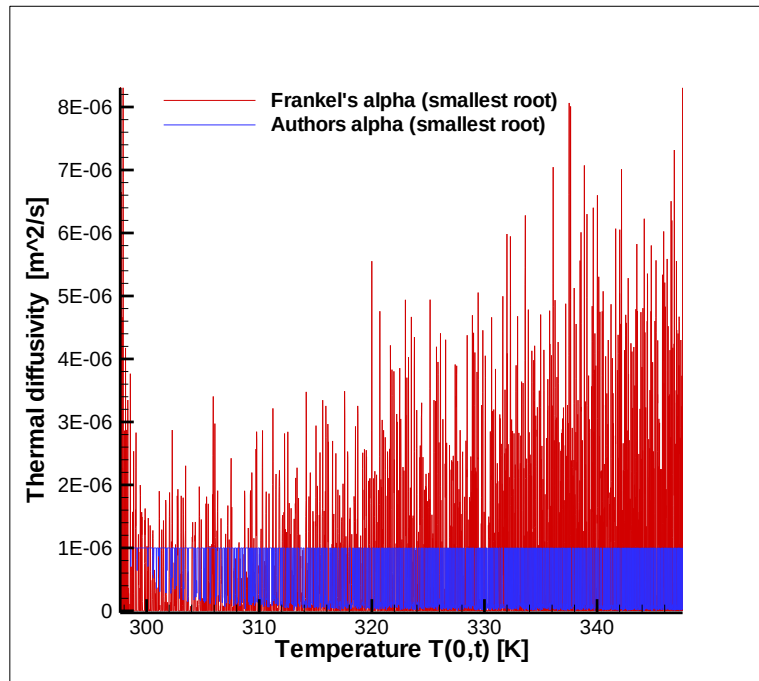


Figure 16. Copper sample small thermal diffusivity values as a function of temperature

$T(0,t)$

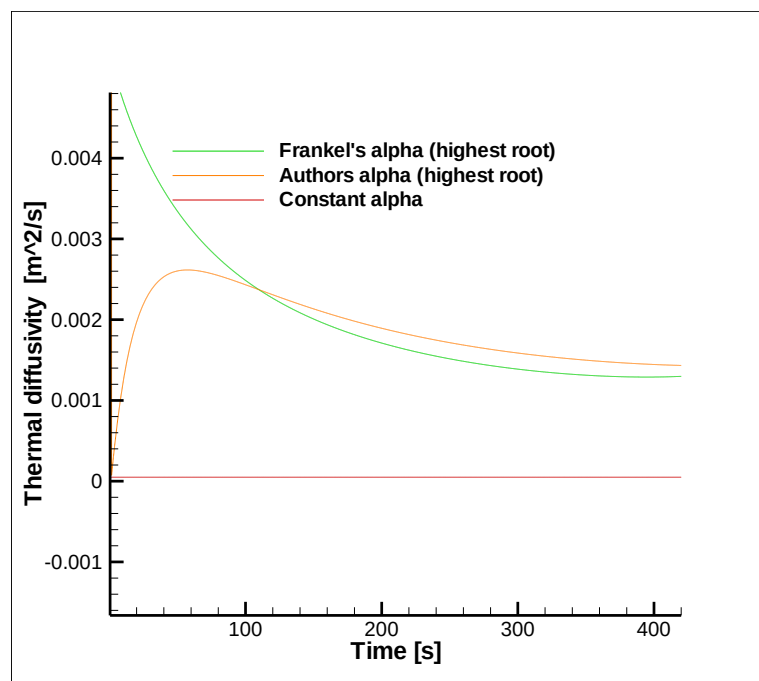


Figure 17. Large thermal diffusivity values for aluminum data

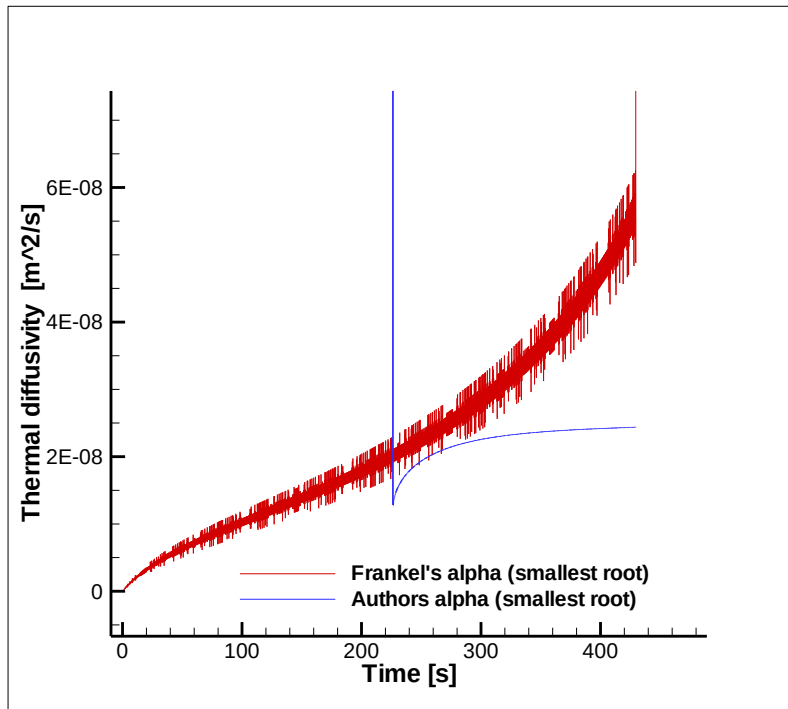


Figure 18. Small thermal diffusivity values for aluminum data

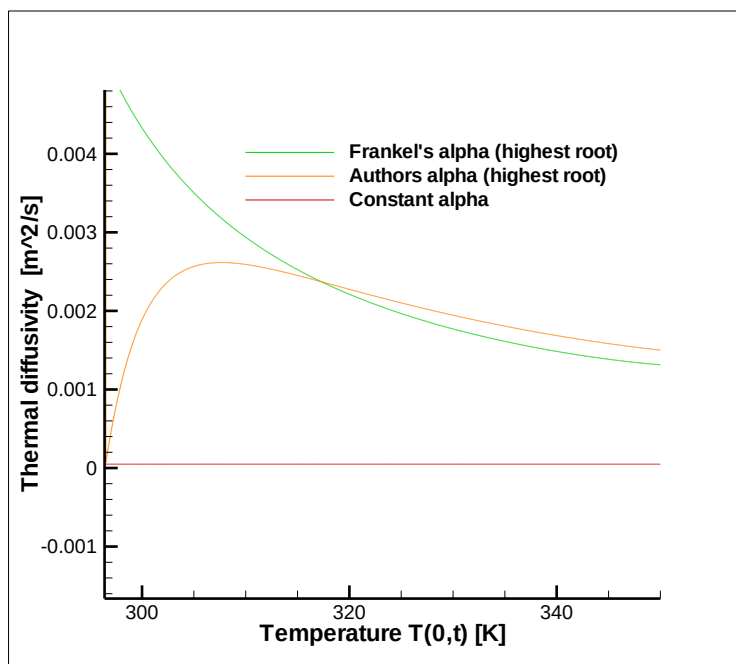


Figure 19. Large thermal diffusivity values as a function of temperature  $T(0,t)$  for aluminum data

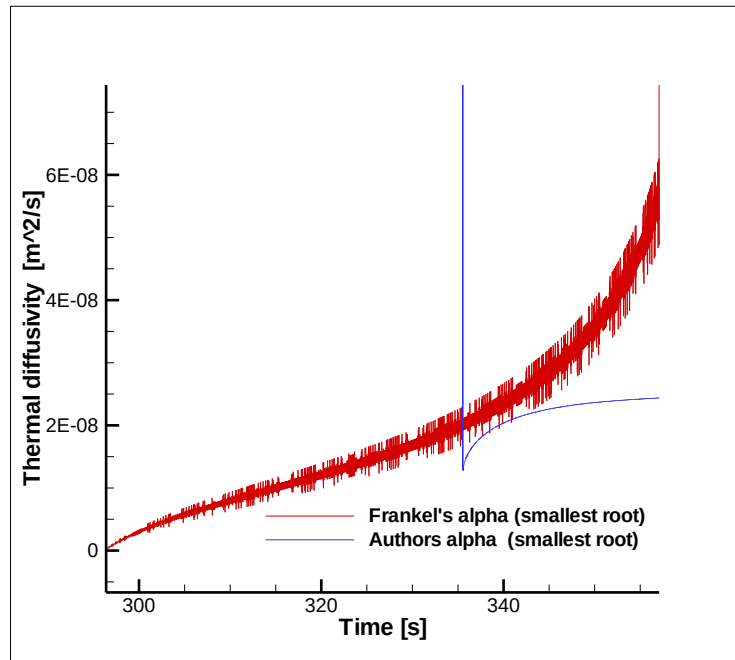


Figure 20. Small thermal diffusivity values as a function of temperature  $T(0,t)$  for aluminum data

#### Calculated Temperature Data

The temperatures  $T(0,t)$  and  $T(3n,t)$  were calculated from the experimental transient temperature data at locations  $n$  and  $2n$  and were compared to the measured temperature data at those points as a check on accuracy. This was done three different ways. First, the finite difference approximation (FDA) using equations (3-13) and (3-14), with the  $\alpha$  given in Table 1 was used. The FDA results for the numerical experiment data can be seen in Figure 21 and Figure 22. Note that the  $T(3n,t)$  line cannot be seen because the FDA calculated temperature for  $T(3n,t)$  lies directly on top of it. The FDA temperature calculated at the bottom surface ( $T(0,t)$ ) is also very close to the actual value.

The FDA analysis for the copper data experiment can be seen in Figure 23 and Figure 24. The  $T(3n,t)$  line is far from the FDA calculated  $T(3n,t)$  temperature, at the top surface of the sample, as expected. The FDA calculated  $T(0,t)$  is, however, very close to the measured data collected from the experimental data.

Finally, the FDA analysis for the aluminum data can be seen in Figure 25 and Figure 26. Again the  $T(3n,t)$  line is far from the FDA calculated  $T(3n,t)$  temperature, as expected. The FDA calculated  $T(0,t)$  is, however, very close to the actual data collected in the experiment.

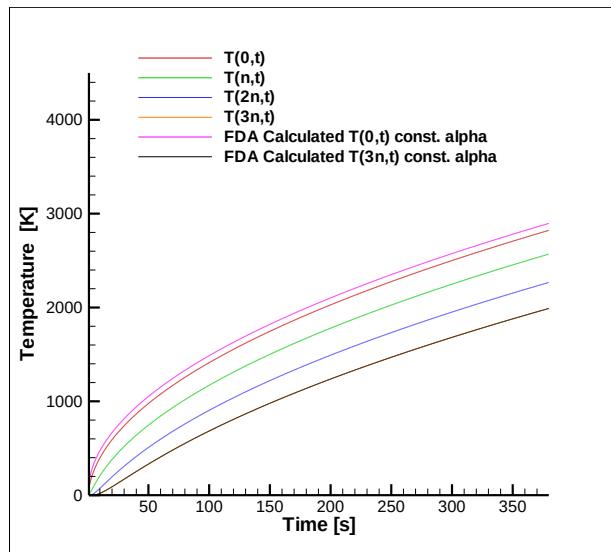


*Figure 21. Numerical experiment temperature data with FDA calculated temperatures using constant thermal diffusivity given in Table 1*

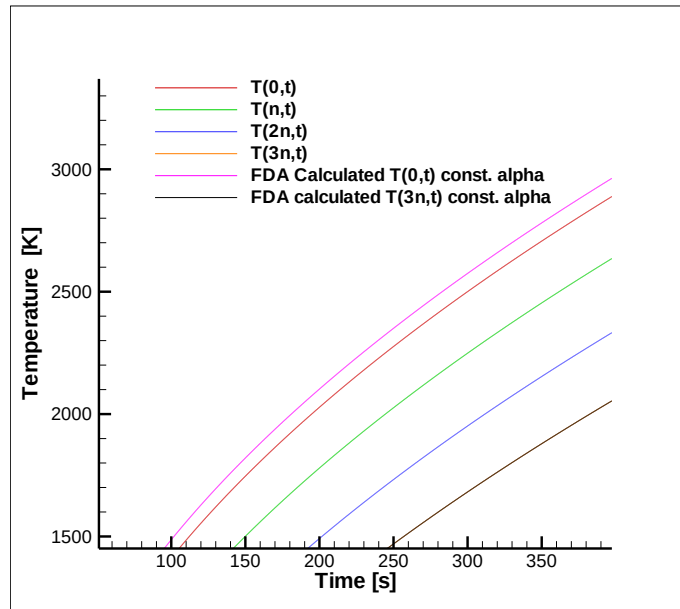


Figure 22. Numerical experiment temperature data with FDA calculated temperatures using constant thermal diffusivity given in Table 1 (expanded view of data shown in Figure 21)

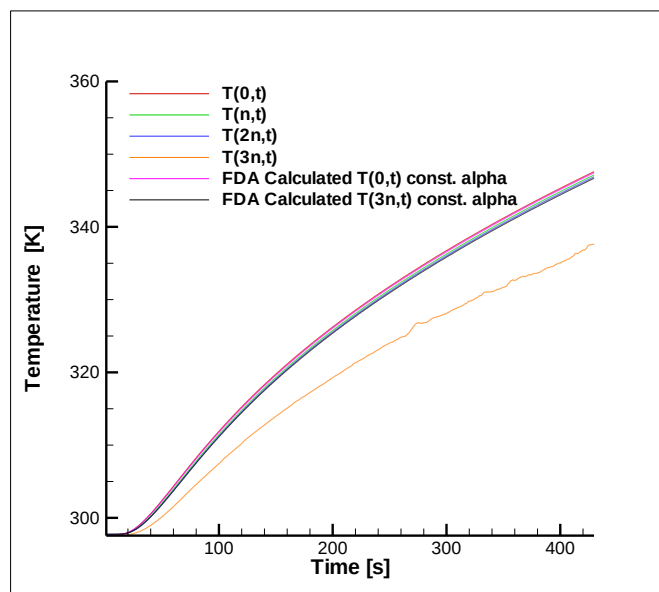


Figure 23. Copper sample temperature data with FDA calculated temperatures using constant thermal diffusivity given in Table 1

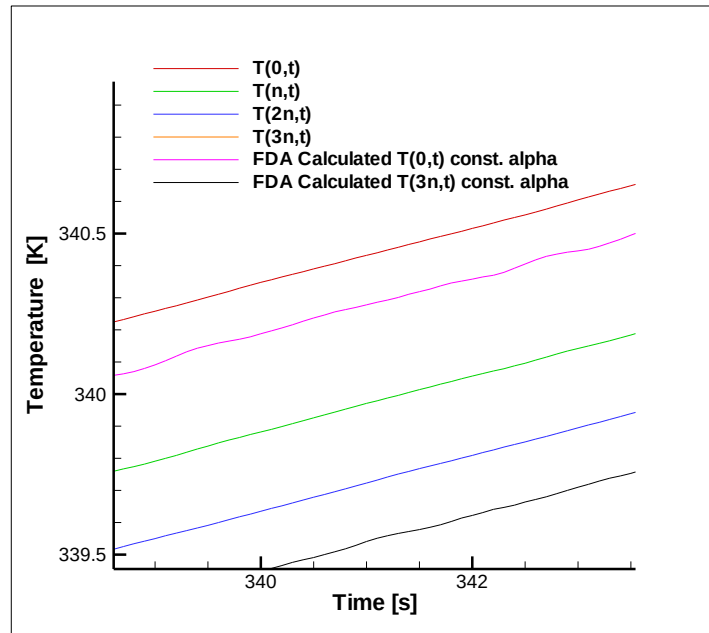


Figure 24. Copper sample temperature data with FDA calculated temperatures using constant thermal diffusivity given in Table 1 (expanded view of data in Figure 23)

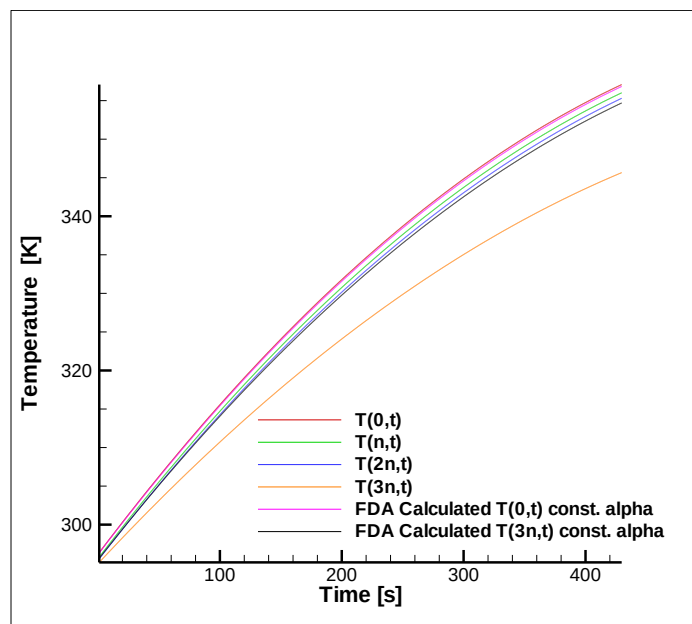


Figure 25. Aluminum temperature data with FDA calculated temperature values using constant thermal diffusivity given in Table 1

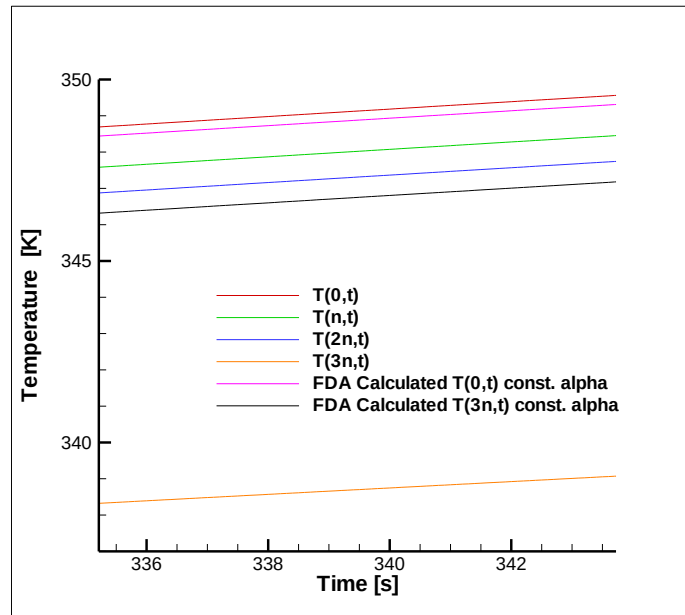


*Figure 26. Aluminum temperature data with FDA calculated temperature values using constant thermal diffusivity given in Tale 1 (expanded view of data in Figure 25)*

The numerical experiment top and bottom surface temperatures were also calculated using Frankel's equations (3-36) and (3-37), with the alpha given in Table 1. As one can see from examination of equations (3-36)(3-37) and the FDA equations (3-13)(3-14), the FDA analysis is just a lower order approximation of the former. Therefore, more accurate results are expected for Frankel's method. However, after comparing the results in Figure 27 and Figure 28 against the FDA results in Figure 21 and Figure 22 for the numerical experiment temperature data, Figure 29 and Figure 30 against the FDA Figure 23 and Figure 24 for the copper experiment temperature data, and Figure 31 and Figure 32 against the FDA Figure 25 and Figure 26 for the aluminum experiment temperature data, it is evident that there is no significant difference between the two methods of calculating the top and bottom surface temperatures  $T(0,t)$  and  $T(3n,t)$  for the data considered here.

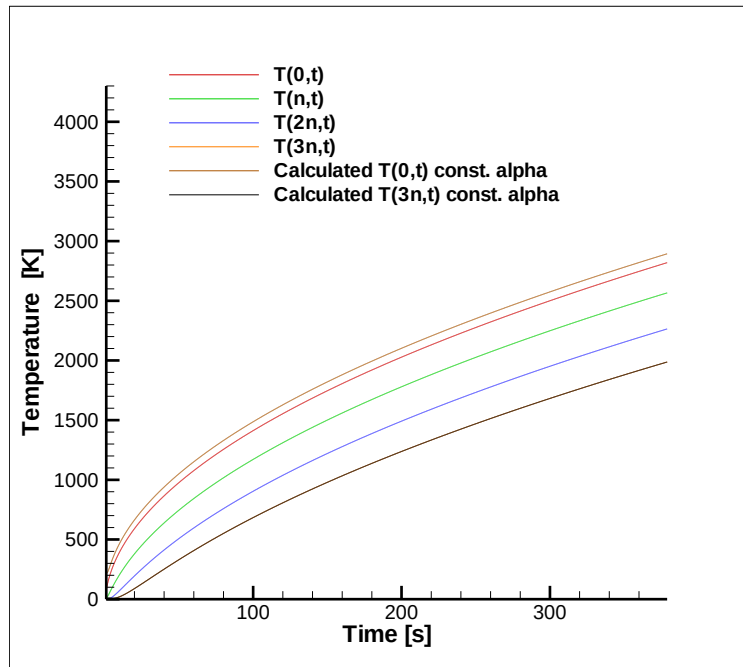


Figure 27. Numerical experiment temperature data with calculated temperatures using constant thermal diffusivity given in Table 1

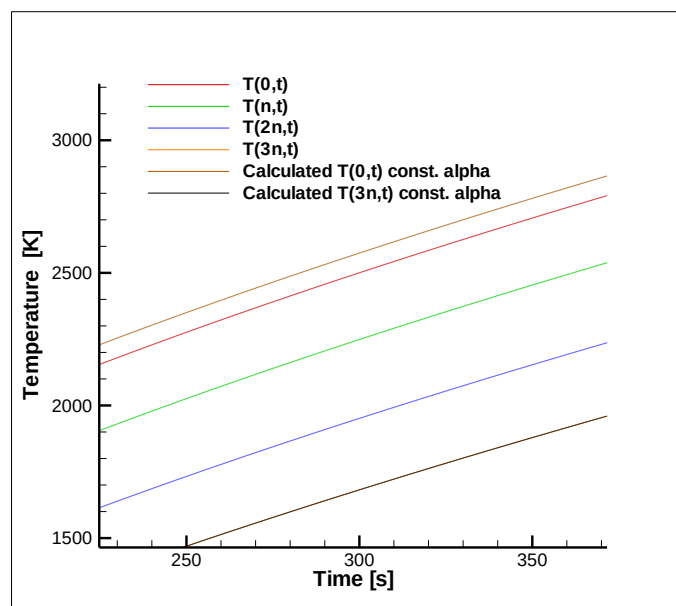


Figure 28. Numerical experiment temperature data with calculated temperatures using constant thermal diffusivity given in Table 1 (expanded view of data in Figure 27)

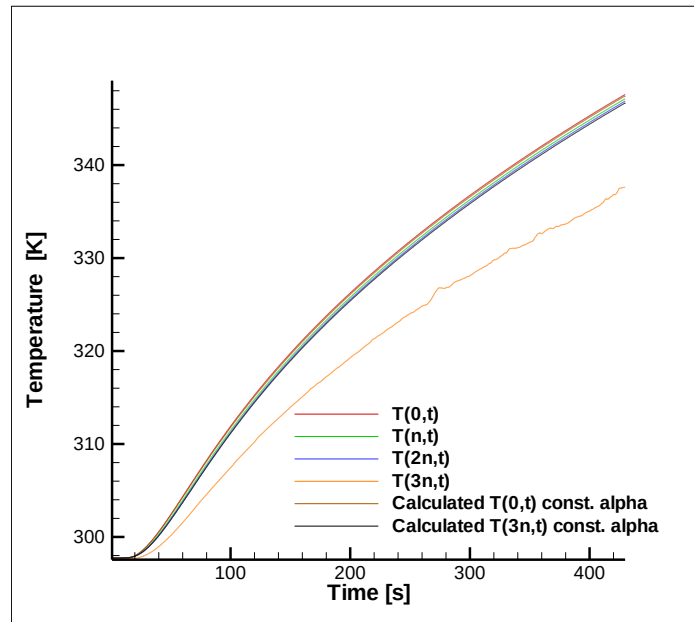


Figure 29. Copper sample temperature data with calculated temperatures using constant thermal diffusivity given in Table 1

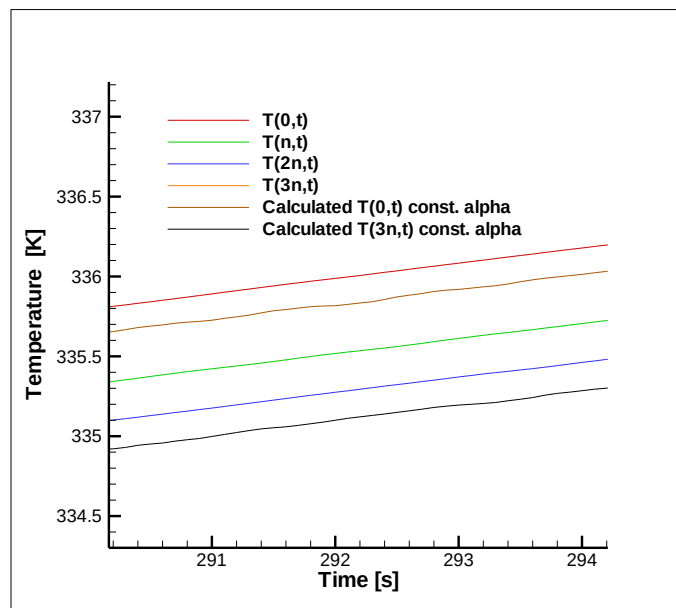


Figure 30. Copper sample temperature data with calculated temperatures using constant thermal diffusivity given in Table 1 (expanded view of data in Figure 29)

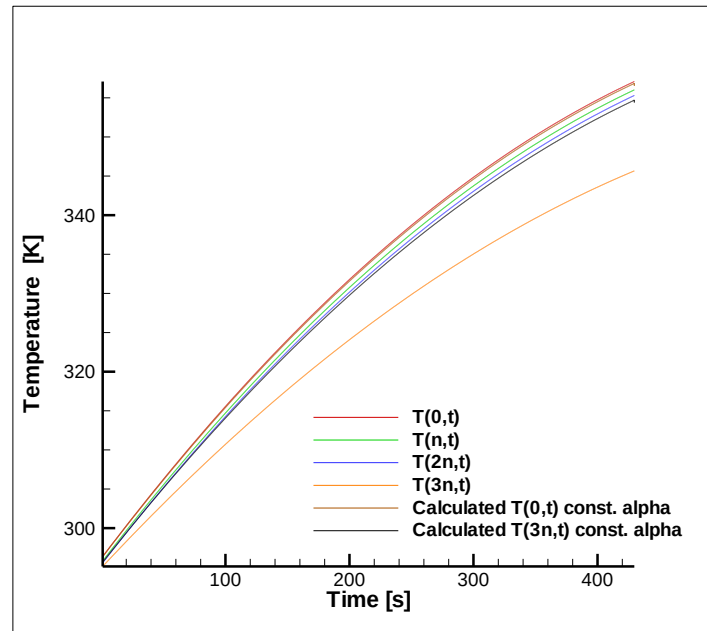


Figure 31. Aluminum temperature data with calculated temperature values using constant thermal diffusivity given in Table 1

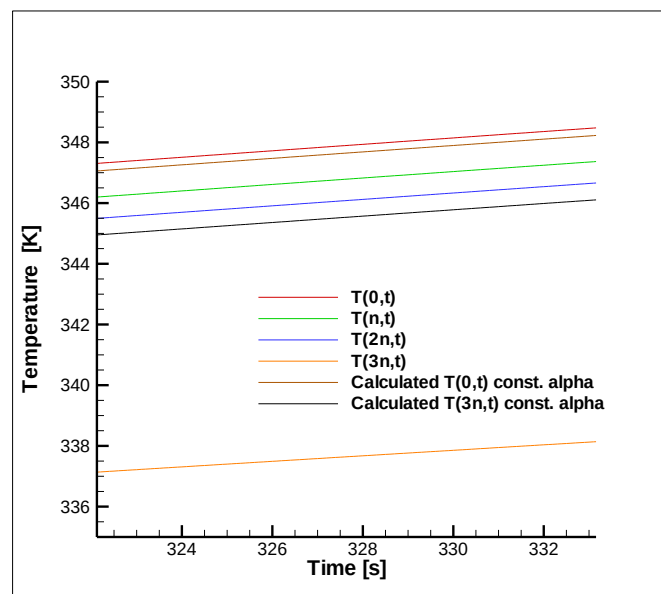


Figure 32. Aluminum temperature data with calculated temperature values using constant thermal diffusivity given in Table 1 (expanded view of data in Figure 31)

Finally,  $T(0,t)$  and  $T(3n,t)$  were calculated using the time dependent thermal diffusivity obtained from the temperature data calculated using equation (3-35) for the numerical experiment data and equation (3-40) combined with equations (3-36) and (3-37) for the experimental data. The thermal diffusivity that was used in the calculation for each case, whether numerical experiment, copper, or aluminum, was the one that converged fastest. In the numerical experiment case, the Frankel's alpha was used, while in the experimental cases the alpha from Author's method was used. The resulting numerical experiment, copper, and aluminum predicted surface temperatures (with time dependent thermal diffusivity) are shown in Figure 33- Figure 38, respectively, and have no discernable difference with the constant thermal diffusivity methods employed for the temperature profiles in Figure 21 through Figure 31.

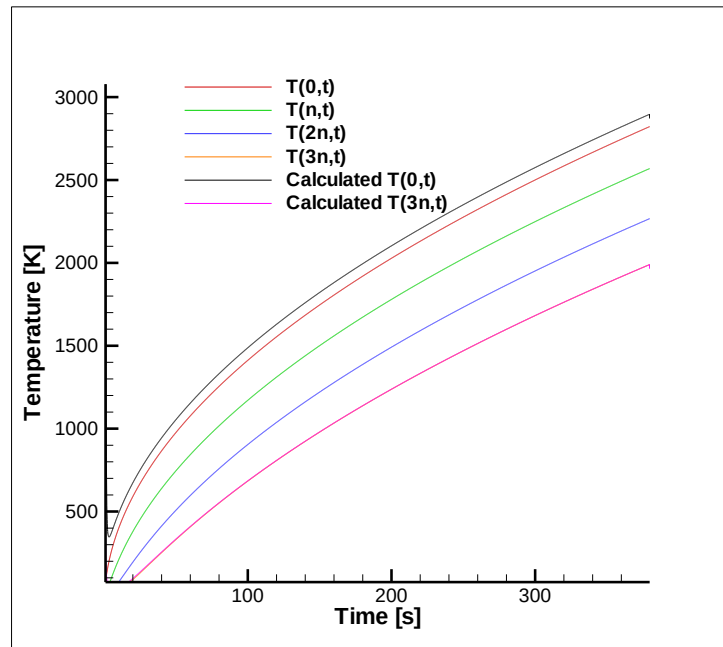


*Figure 33. Numerical experiment temperature data with calculated temperatures using time-varying alpha*

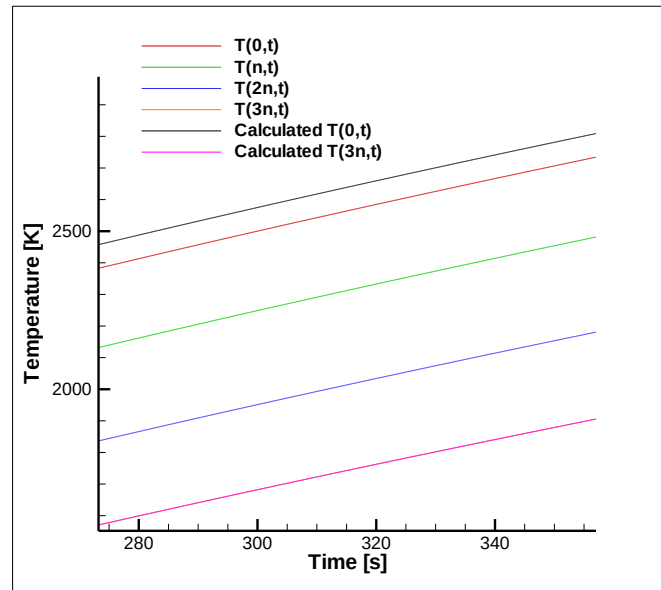


Figure 34. Numerical experiment temperature data with calculated temperatures using time-varying alpha (expanded view of data in Figure 33)

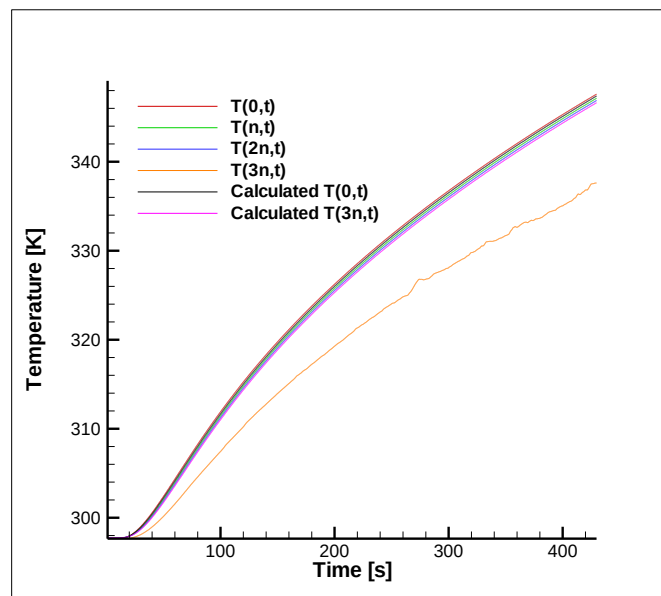


Figure 35. Copper sample temperature data with calculated temperatures using time-varying alpha

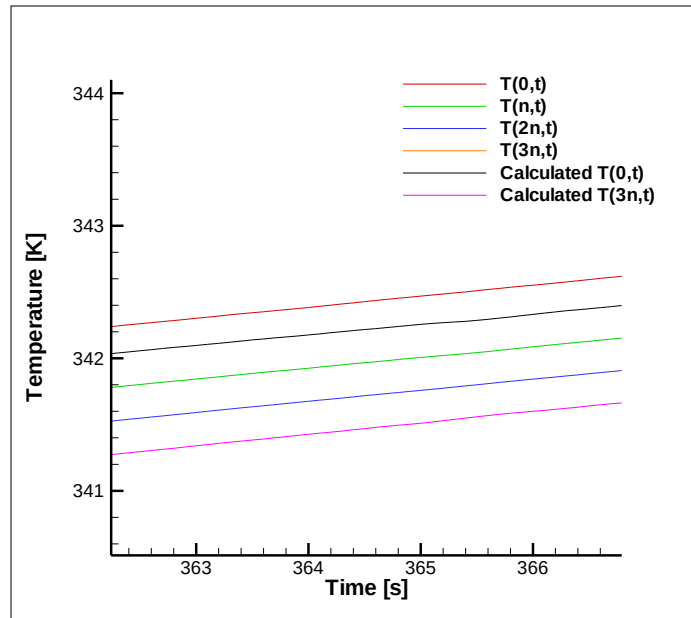


Figure 36. Copper sample temperature data with calculated temperatures using time-varying  $\alpha$  (expanded view of data in Figure 35)

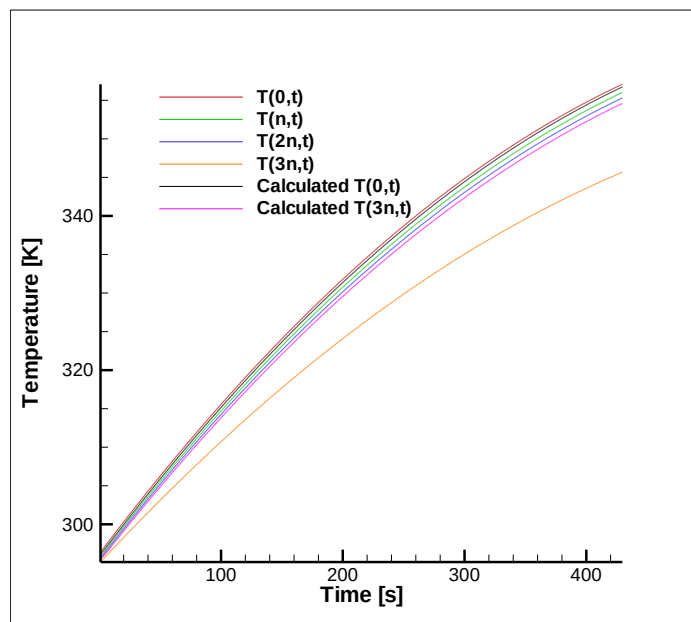


Figure 37. Aluminum temperature data with calculated temperature values using time-varying  $\alpha$

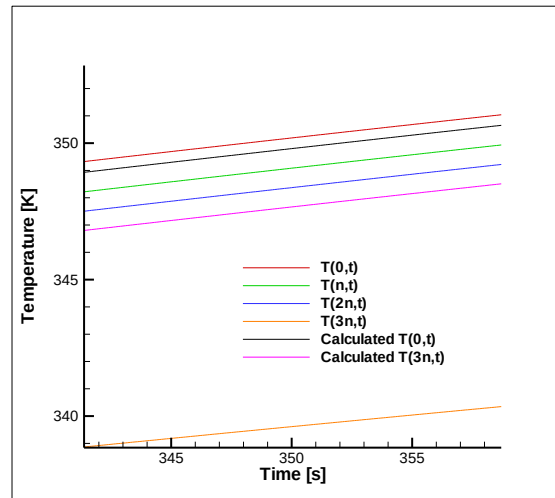


Figure 38. Aluminum temperature data with calculated temperature values using time-varying  $\alpha$  (expanded view of data in Figure 37)

### Heat Flux

Comparisons of the experimental (actual) heat flux on the bottom surface of the sample material to that calculated using Frankel's inversion analysis, equation (3-26), are shown in Figures 39-Figure 41. The thermal property values, given in Table 1, were used in equation (3-26). The heat flux plot for the numerical experiment data (Figure 39) shows the calculated heat flux converging to the actual heat flux quickly. It should be noted that the heat flux for the copper and aluminum experiment data were determined from the power supplied to the heating element, divide by the cross-sectional area of the sample, as mentioned in the experimental section. The heat flux for the copper experiment converges to the actual heat flux used in the experiment within about 3 percent, as shown in Figure 40. That of aluminum, however, under-estimates the heat flux going into the sample by approximately 25 % (Figure 41). Note that to find the heat going into the material one only has to multiply by  $A$ , the cross-sectional area of the sample material.

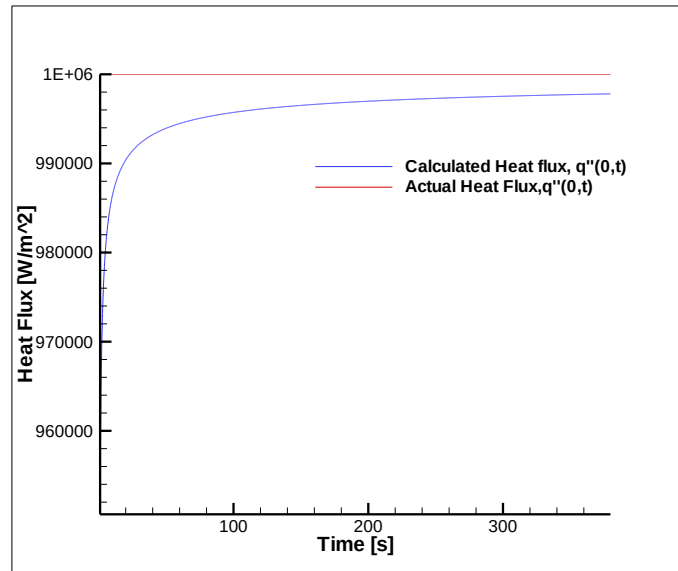


Figure 39. Applied bottom surface heat flux and calculated bottom surface heat flux using numerical experiment temperature data

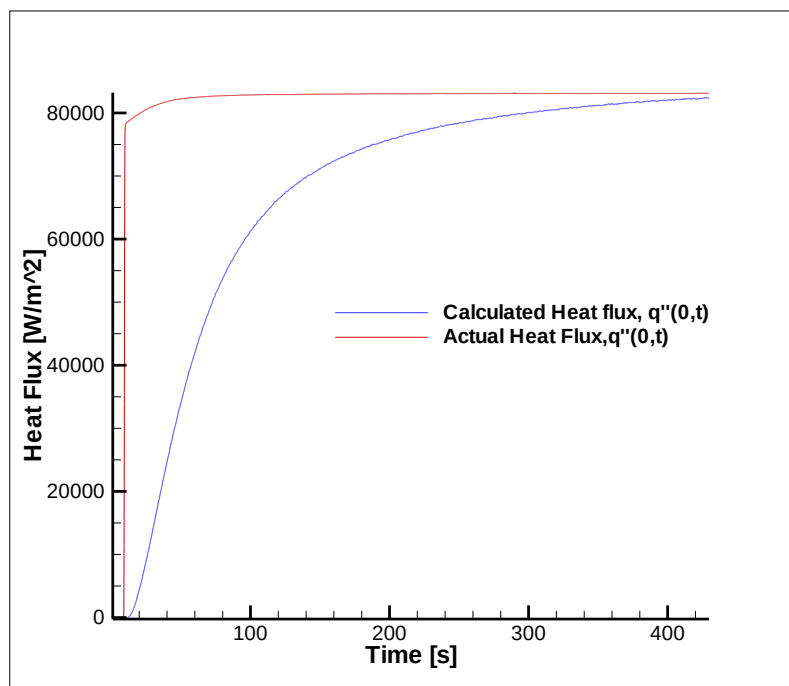


Figure 40. Copper data applied bottom surface heat flux and calculated bottom surface heat flux

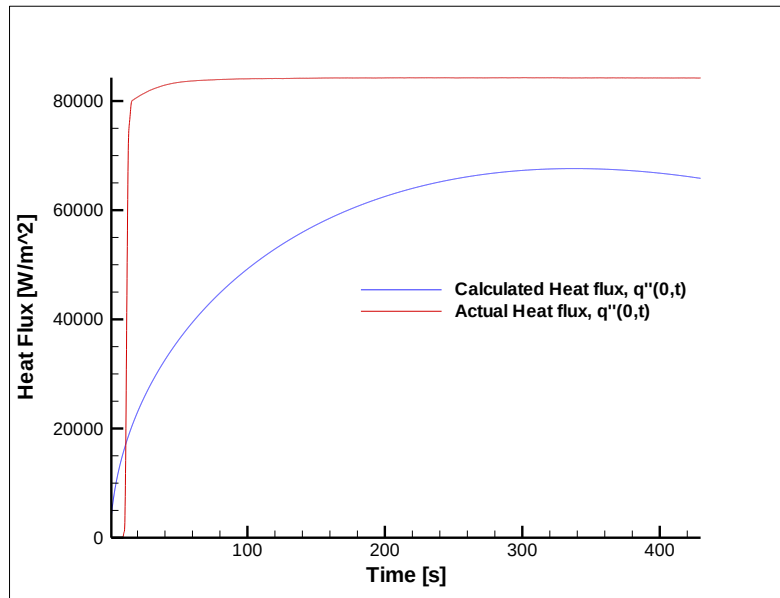


*Figure 41. Applied bottom surface heat flux and calculated bottom surface heat flux for aluminum experiment*

It is believed that radial heat loss is the dominant factor contributing to discrepancies between applied and calculated surface heat fluxes. Radial heat loss violates an assumption upon which equation (3-26) is based (semi-infinite plane). To determine how closely the experimental setup was to a semi-infinite slab. The heat loss through the side of the sample material was calculated using equations (3-52)(3-53). A heat loss for the numerical experiment data was not calculated simply because a radial heat load through an infinite area doesn't make physical sense. The heat flux loss through the side wall for the copper experiment data (Figure 42) is negative at first then steadies out to about 1500 W/m<sup>2</sup>. The percent of radial heat flux loss compared to the heat supplied by the heating element remains below 2%. Therefore, it models the semi-infinite plane pretty well. For the aluminum experiment the radial heat flux loss (Figure 43) reaches about 1400W/m<sup>2</sup>,

which again remains below about 2% of the heat flux going into the material on the bottom surface. Also to find the heat going out of the material one only has to multiply by  $A_s$  the area of the material sample sides. The ratio of heat going out of the material

compared to the heat going in is  $\frac{q_{loss}}{q_{in}} = \frac{q_{loss}'' \cdot A_s}{q_{in}'' \cdot A} = \frac{2 \cdot q_{loss}'' \cdot h}{q_{in}'' \cdot r}$ . This fraction is below

10% for both the aluminum and copper experiments.

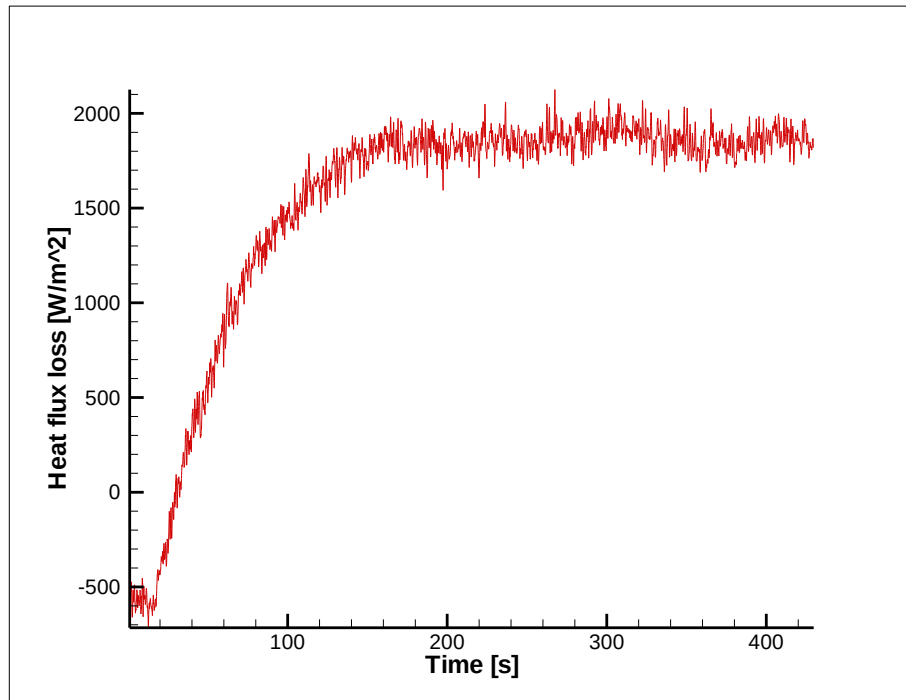


Figure 42. Calculated radial heat flux loss for copper experiment data

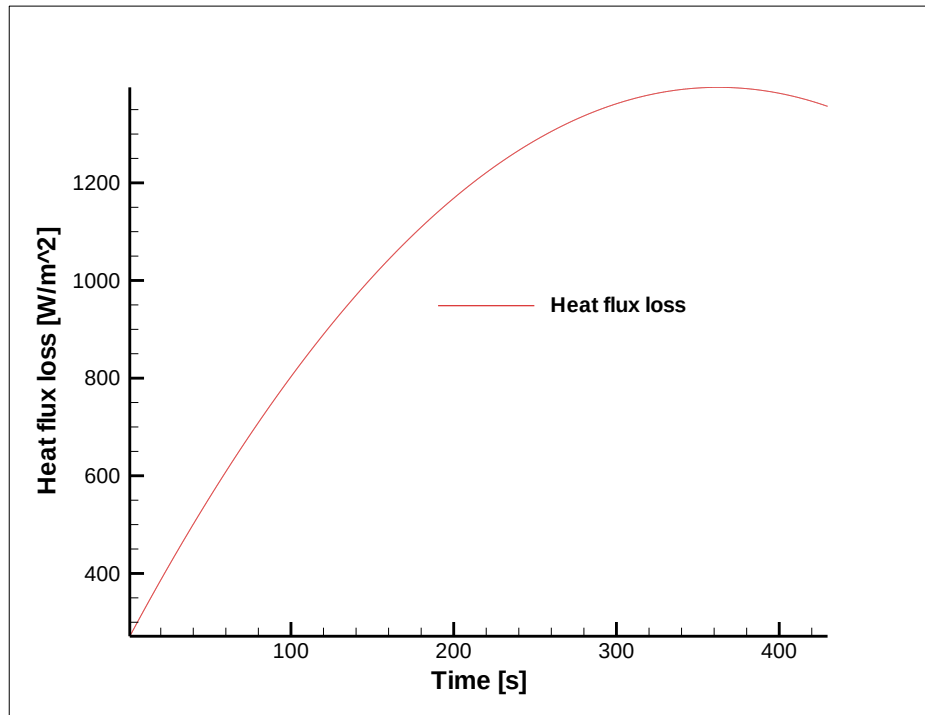


Figure 43. Calculated radial heat flux loss for aluminum experiment data

### Thermal Conductivity

Thermal conductivity,  $k$ , is a measure of the ability of a material to conduct heat. The thermal conductivity was calculated using equations (3-41) and (3-39), for the Author's and Frankel's methods, respectively. The thermal conductivity was calculated using the numerical experiment data (Figure 44 and Figure 45). The thermal conductivity in these figures is calculated in many different ways to ensure both Frankel's and the Author's methods converge to the same value using different thermal diffusivities and temperatures. The thermal conductivity for the numerical experiment data converge to the accepted value of  $k = 14.7 \text{ W/(mK)}$ , using all eight combinations of alpha and temperatures.

The thermal conductivity for the copper experiment data (Figure 46-Figure 49) show that instead of all eight methods converging to a single solution, multiple values are approached. Fortunately though, when the handbook value (constant) thermal diffusivity for copper (Table 1) and the actual (measured) bottom surface temperature ( $T(0,t)$ ) are used in the equations applied to determine  $k$ , the analysis converges to a solution of about  $k = 350W/(mK)$ , which is very close to the value  $k = 354.7W/(mK)$  in Table 1.

The aluminum experiment thermal conductivity graphs (Figure 50-Figure 53) follow suit, with only the constant thermal diffusivity and actual (measured)  $T(0,t)$  combination being close to the handbook value. This combination converges to about  $k = 205W/(mK)$  which is of the same order of magnitude as the actual thermal diffusivity,  $k = 167W/(mK)$ .

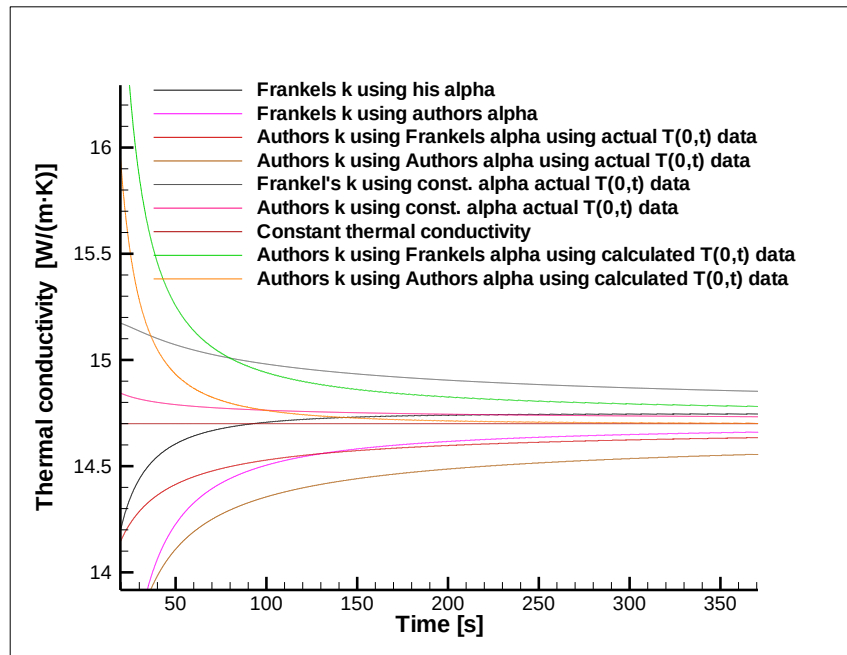


Figure 44. Thermal conductivity values calculated from numerical experiment data

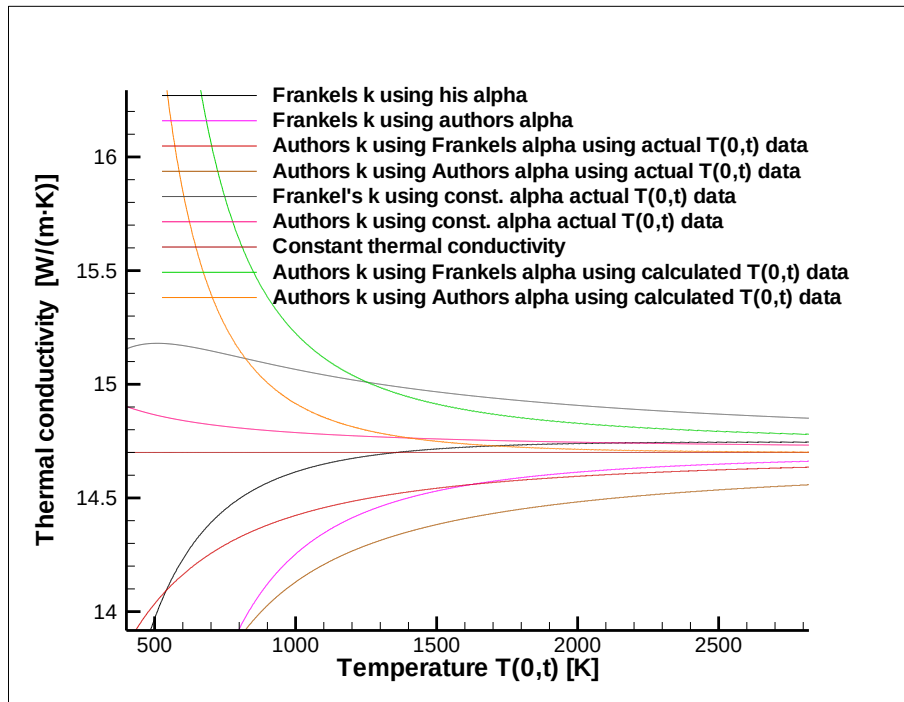


Figure 45. Thermal conductivity values as a function of temperature  $T(0,t)$  using the numerical experiment data

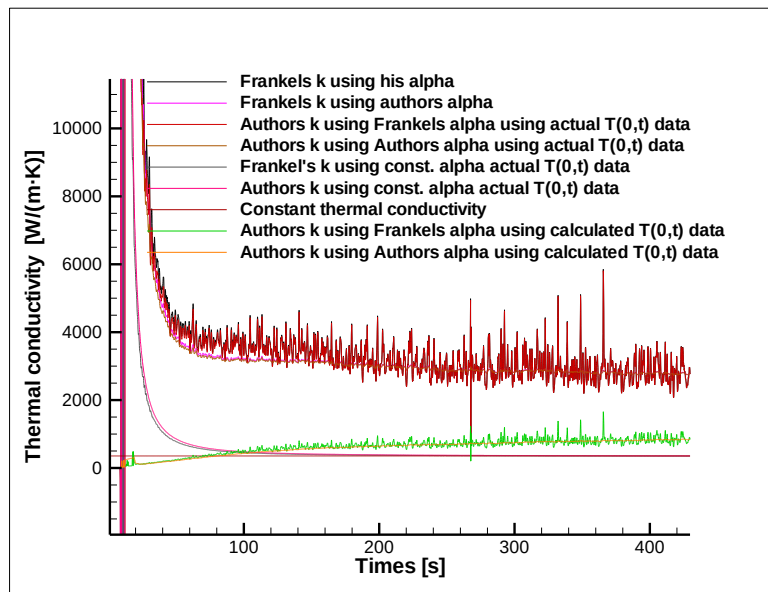


Figure 46. Copper data thermal conductivity

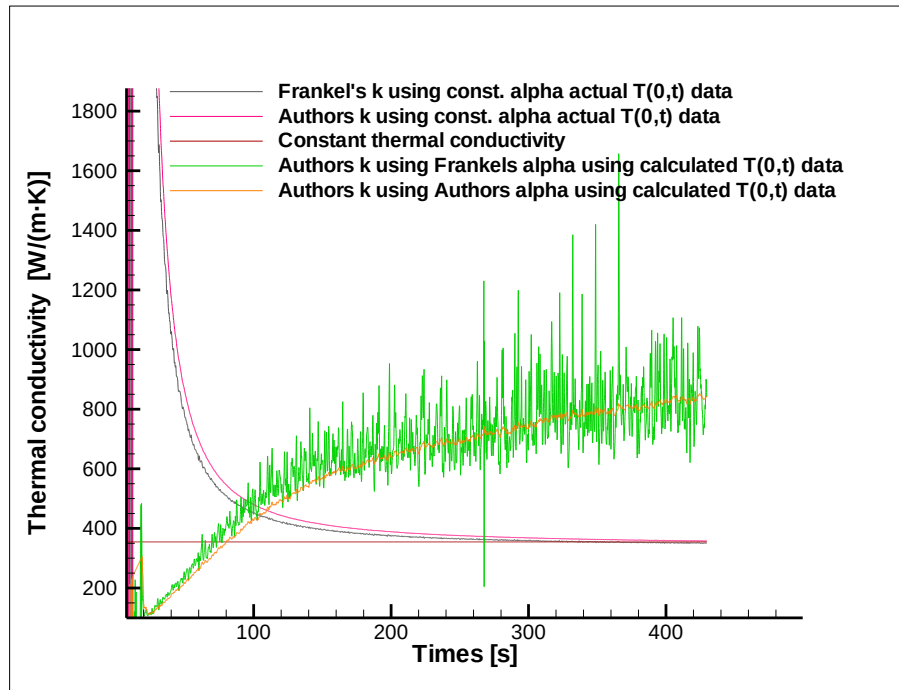


Figure 47. Copper data thermal conductivity (expanded view of data in Figure 46)

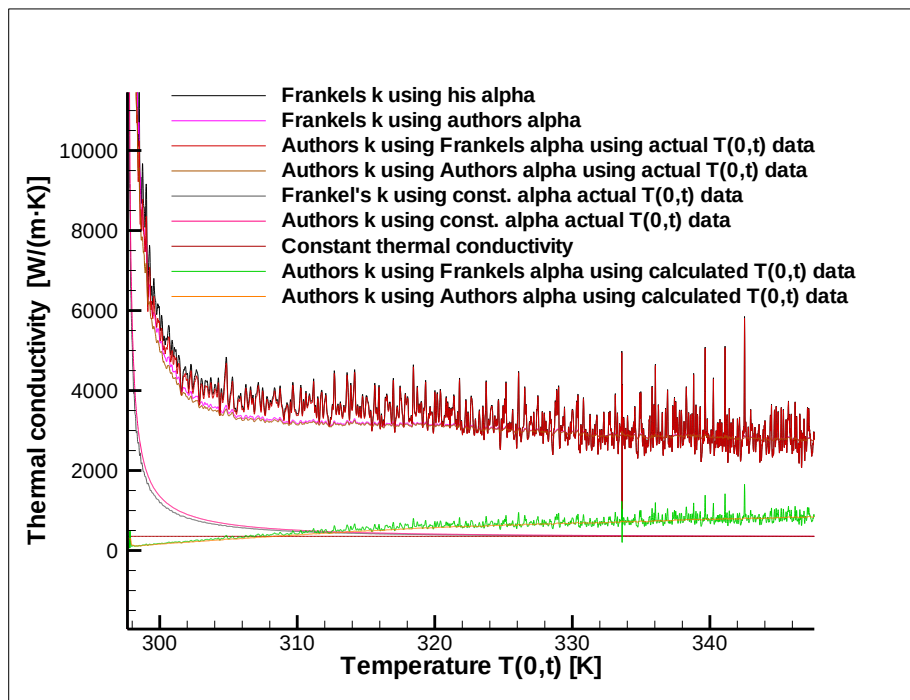


Figure 48. Copper data thermal conductivity as a function of temperature  $T(0,t)$

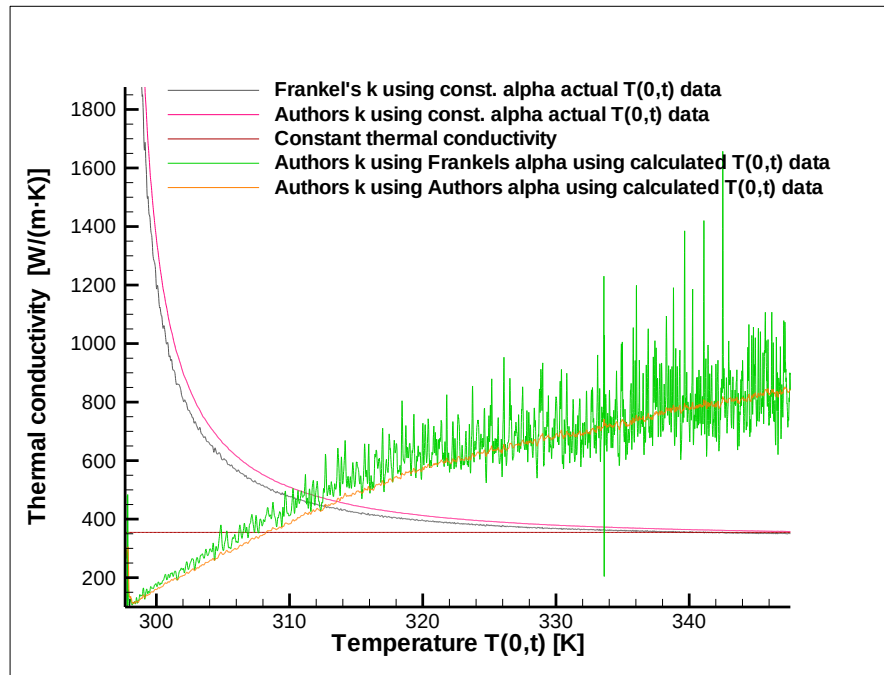


Figure 49. Copper data thermal conductivity as a function of temperature  $T(0,t)$

(expanded view of data in Figure 48)

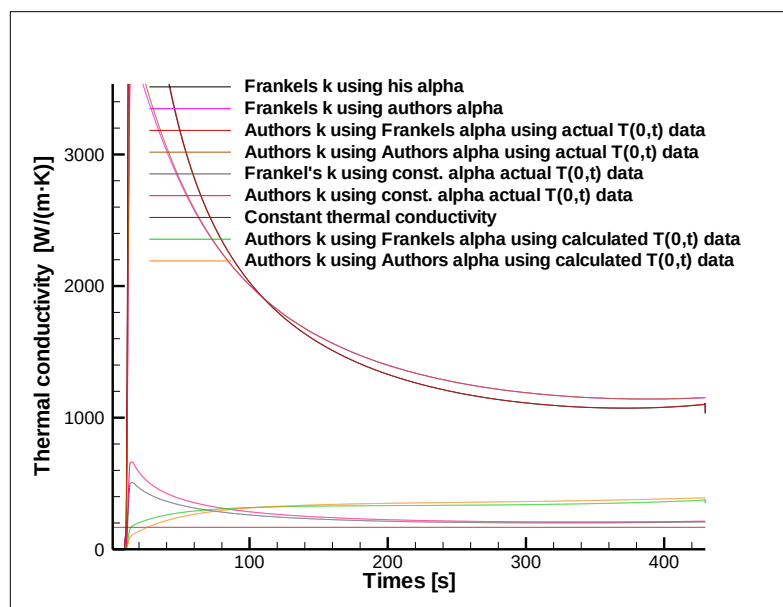


Figure 50. Thermal conductivity for aluminum data

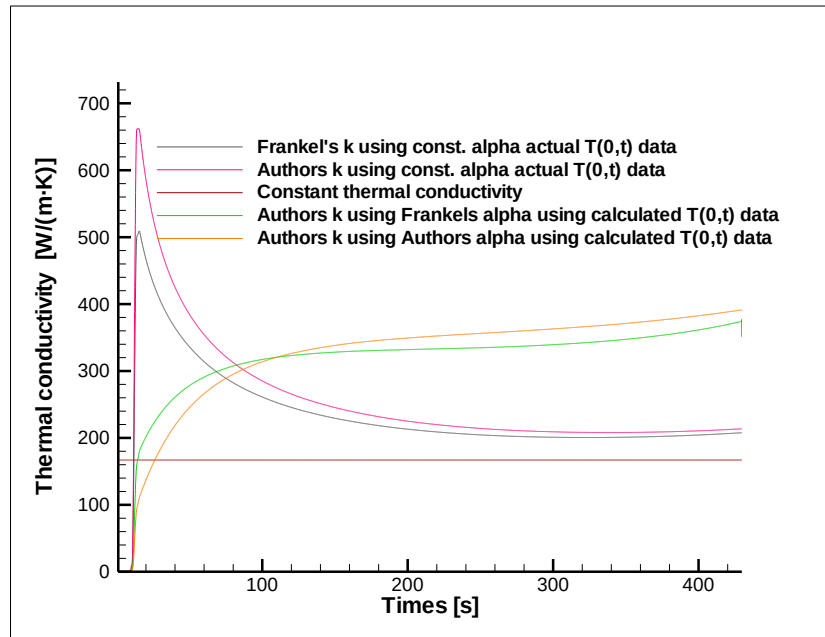


Figure 51. Thermal conductivity for aluminum data (expanded view of data in Figure 50)

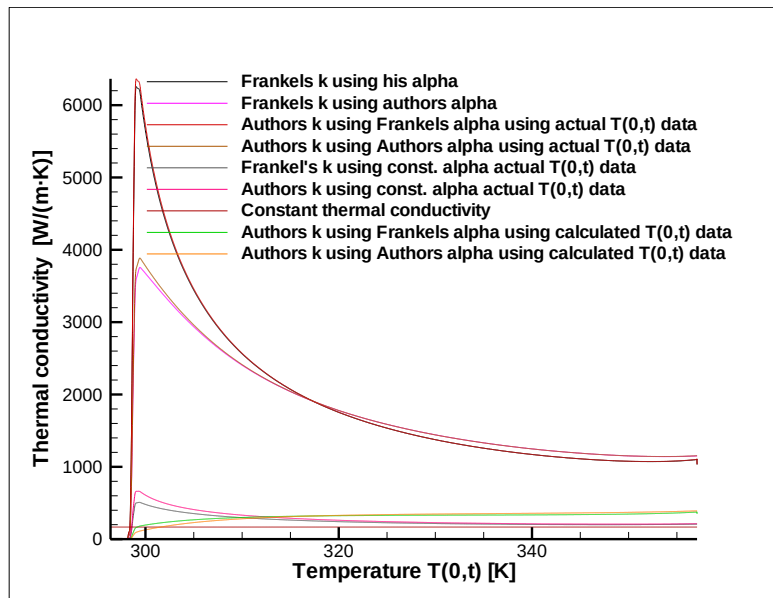


Figure 52. Thermal conductivity a function of temperature  $T(0,t)$  for aluminum data

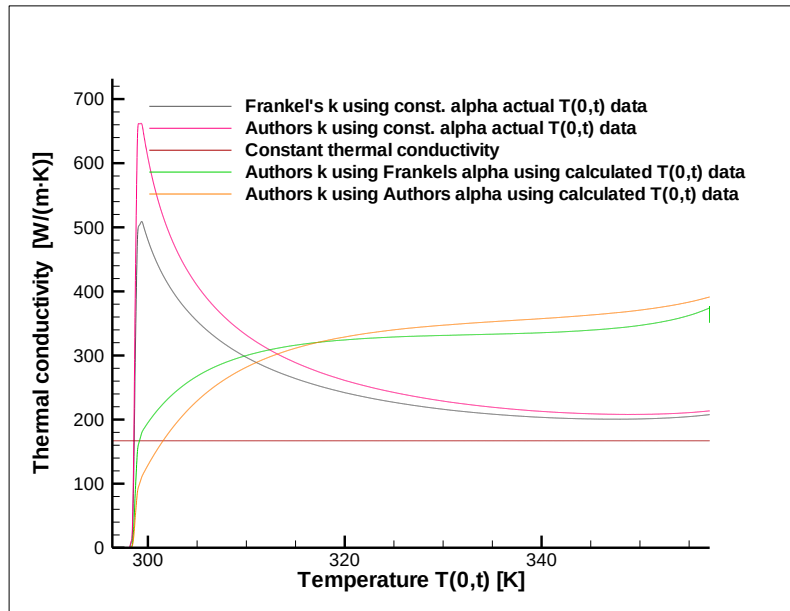


Figure 53. Thermal conductivity for aluminum as a function of temperature  $T(0,t)$  data  
(expanded view of data in Figure 52)

### Specific Heat Capacity

The specific heat capacity,  $C_p$ , is the energy required to increase the temperature per unit mass of a substance by one degree. It is calculated using equation (3-42). Since eight ways were used to calculate the thermal conductivity, there are eight solutions for specific heat that correspond to the different  $k$  values. Since all of the thermal conductivities and all of the thermal diffusivities for the numerical experiment converged, then all of the specific heats should converge to the correct solution for any material density. Knowing that the thermal conductivity only converges under one condition for both copper and aluminum, it will only converge to a correct specific heat using those conditions. Note, however, that all the ways of calculating thermal conductivity are

presented. For copper (Figure 54-Figure 57) only when the thermal diffusivity is constant and the actual  $T(0,t)$  is used does it converge to a solution of about  $C_p = 387J/(KgK)$  which agrees very well with the handbook value of  $C_p = 385J/(KgK)$ . For aluminum (Figure 58-Figure 61) again the only solution close to the handbook value,  $C_p = 1256J/(KgK)$ , was determined with the handbook value for thermal diffusivity and the actual  $T(0,t)$ . The converged value for aluminum is about  $C_p = 1550J/(KgK)$ .

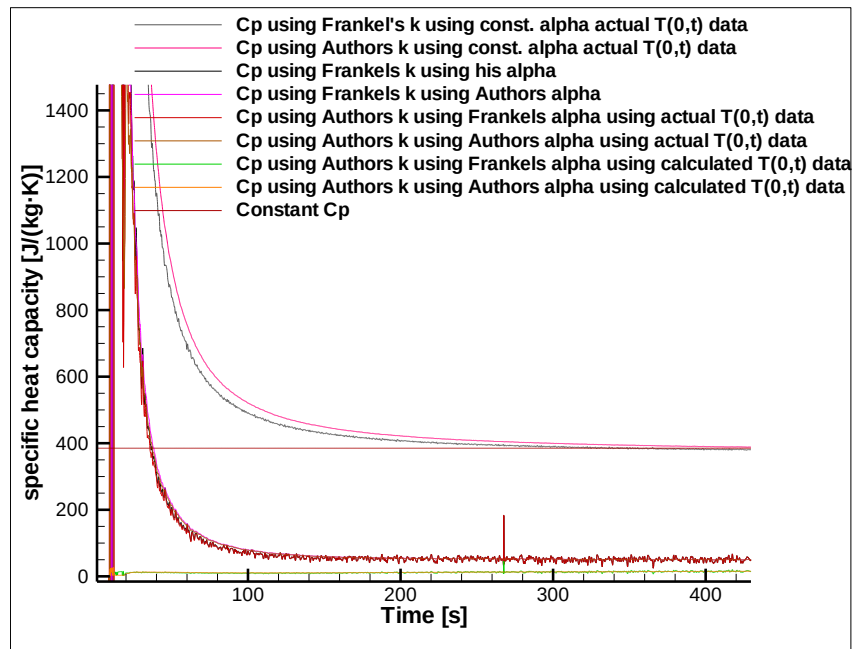


Figure 54. Copper data specific heat capacity

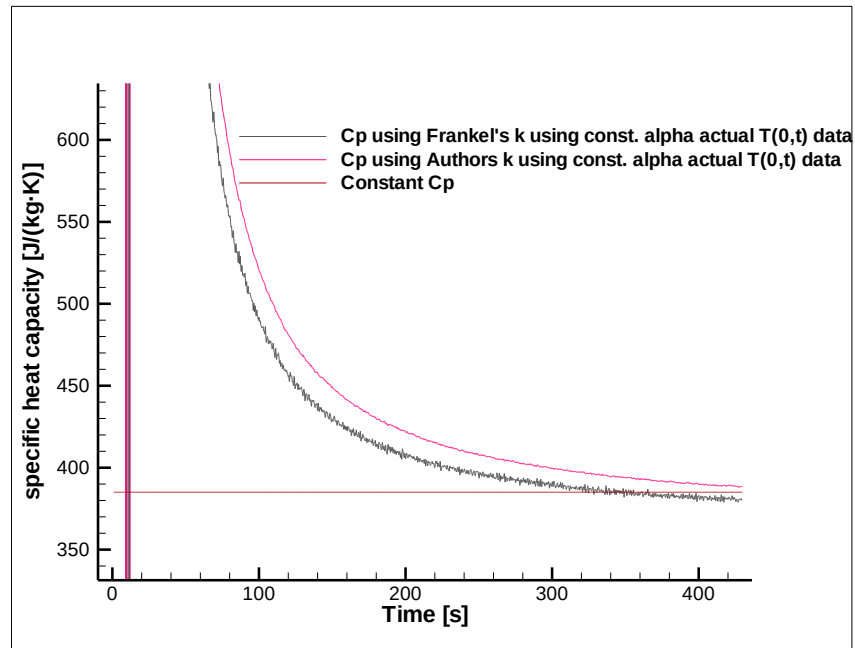


Figure 55. Copper data specific heat capacity (expanded view of data in Figure 54)

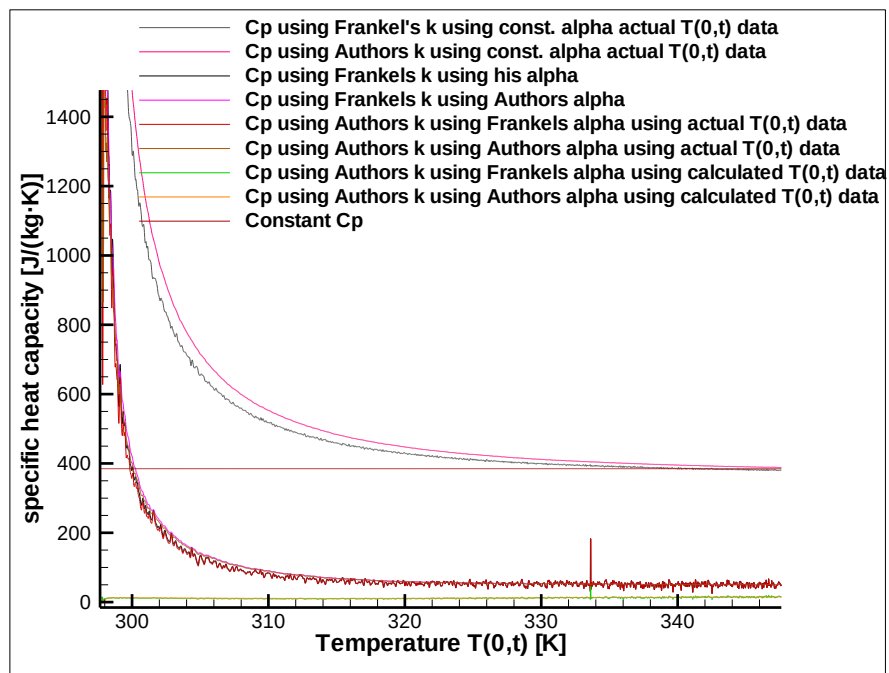


Figure 56. Copper data specific heat capacity as a function of temperature  $T(0,t)$

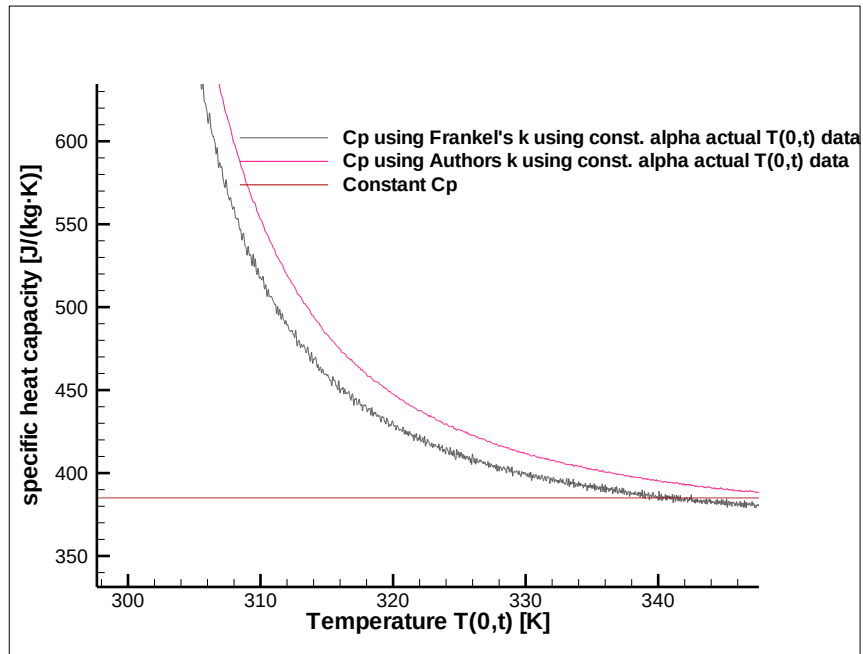


Figure 57. Copper data specific heat capacity as a function of temperature  $T(0,t)$

(expanded view of data in Figure 56)

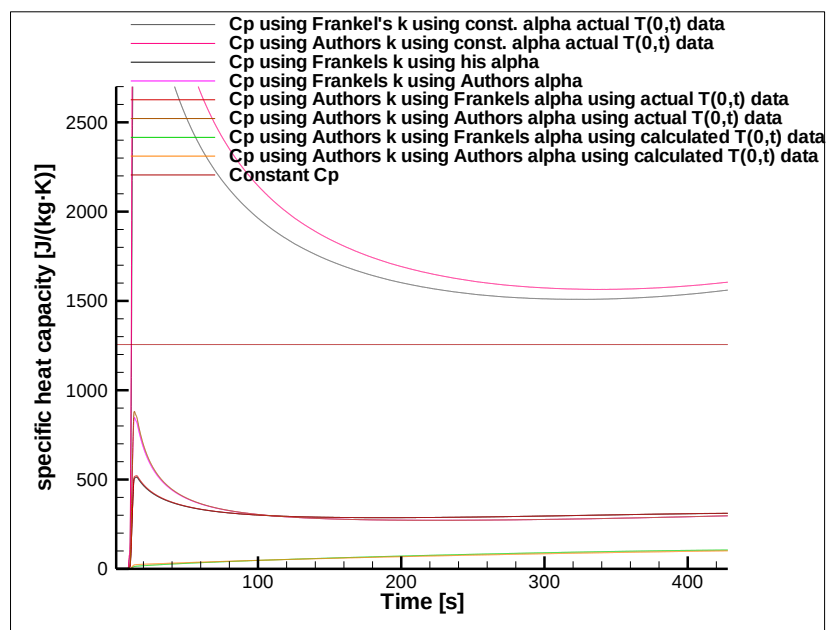


Figure 58. Specific heat capacity for aluminum data

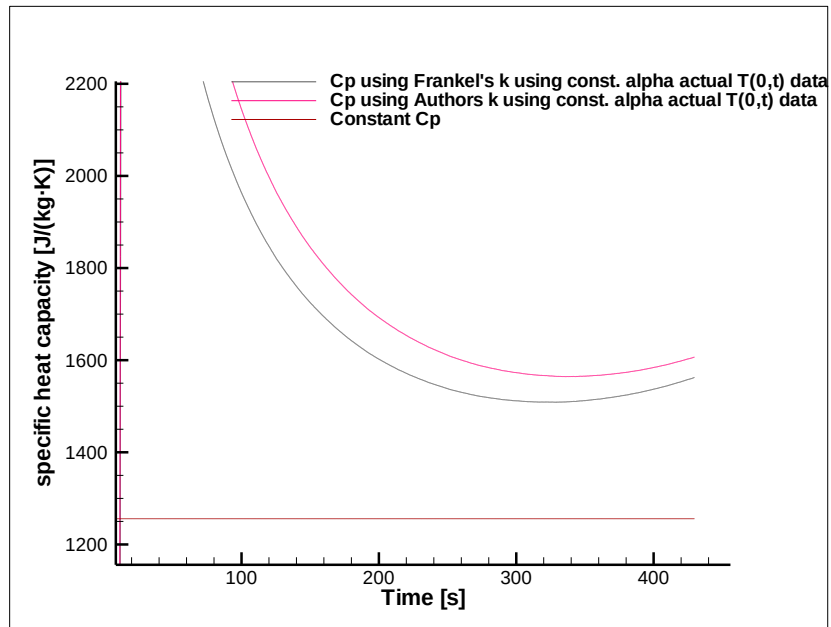


Figure 59. Specific heat capacity for aluminum data (expanded view of data in Figure 58)

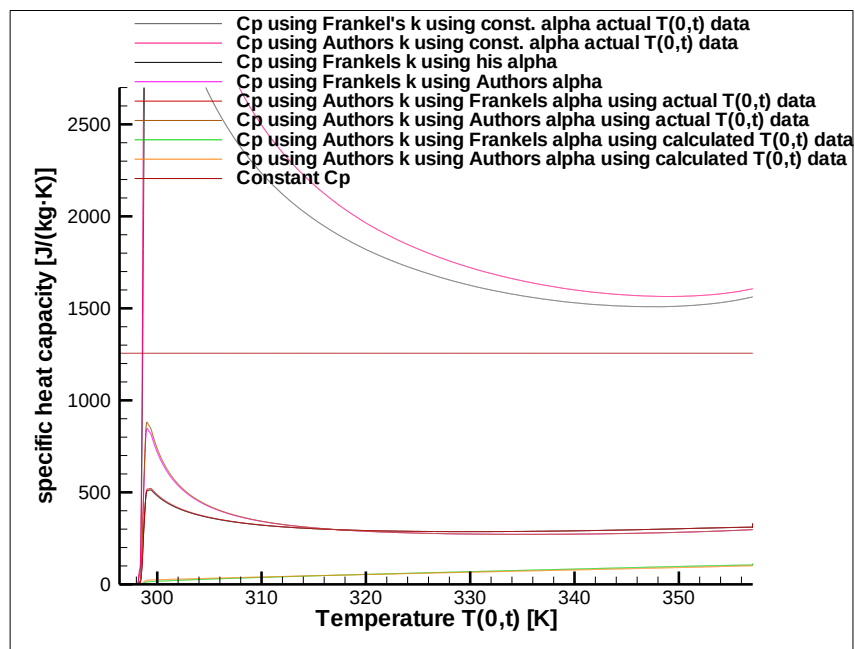


Figure 60. Specific heat capacity as a function of temperature  $T(0,t)$  for aluminum data

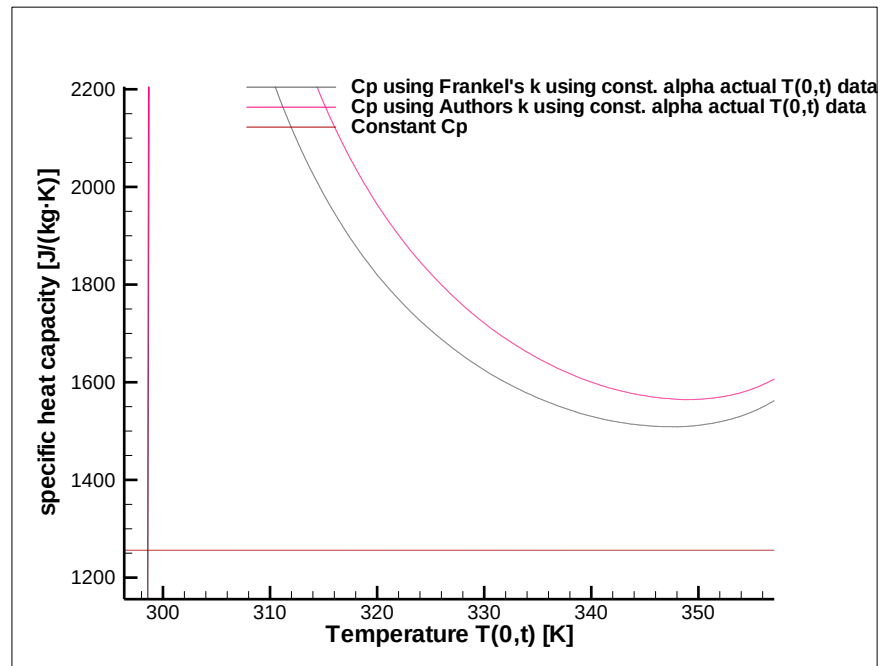


Figure 61. Specific heat capacity as a function of temperature  $T(0,t)$  for aluminum data  
(expanded view of data in Figure 60)

## Chapter 5

### Conclusions and Recommendations

#### *Conclusions*

Three methods (FDA, Frankel, Author) were used to analyze transient temperature data from embedded temperature probes in a sample with an applied axial heat flux. Both numerical experiments and physical experiments with aluminum and copper samples were performed. Analysis of these experiments resulted in values for  $\alpha$ ,  $k$ , and  $C_p$ .

The thermal diffusivity is solved correctly in the numerical experiment case (which provides a mechanism for evaluating each approach) for all analysis methods but fails to converge to the correct answer in the experimental cases. One reason for the thermal diffusivity not converging may result from the fact that the analytical temperature for the numerical experiment given from equation (5-1) uses constant thermal properties. Frankel's and the Author's methods are both based on the inversion analysis (3-26) that also assumes constant thermal properties. Perhaps these methods may only be able to converge to a solution if thermal properties are constant. This is an issue if the results are highly sensitive to varying properties because, in reality, the thermal properties change with temperature. It should be noted, however, that over the temperature range realized in the experiment,  $k$ ,  $\alpha$ , and  $C_p$  are not expected to change substantially.

Other causes for the thermal diffusivity not converging for the experimental cases (copper and aluminum) include radial heat loss and noise in the raw temperature data collected. Perhaps the spread in temperature between the embedded thermocouples is too

small to be accurately resolved by the type k thermocouples used in the experiment. In addition, the copper and aluminum material samples may be too short to prevent the top surface from influencing the results before convergence is realized, thus violating the semi-infinite plane assumption.

The bottom and top surface temperatures,  $T(0,t)$  and  $T(3n,t)$ , were calculated using many different methods; however, the surface temperatures calculated using the various techniques described in Chapter 3 do not vary significantly. The calculated temperatures at  $T(0,t)$  and  $T(3n,t)$  were also very insensitive to thermal diffusivity. This is evident from the comparison of plots that use the actual thermal diffusivity and plots that use the calculated time dependent thermal diffusivity, Figure 21-Figure 32 and Figure 33-Figure 38, respectively. From these figures we see that even though the time dependent thermal diffusivity is orders of magnitude different from the handbook value given in Table 1, temperature profiles are the same and thus insensitive to the thermal diffusivity term in equations (3-13)(3-14)(3-36)(3-37).

### *Recommendations*

To continue this research the inverse analysis developed by Dr. Jay Frankel should be expanded to address time dependent thermal properties, so that the analysis will accommodate correct thermal properties as the temperature of a material changes. Evaluation of the variable property components in the Taylor series expansion would provide clues to the sensitivity of the projection technique to variable properties. Finally, the experiment could be improved. To eliminate the effects of spatial gradients on the temperature data, the thermocouple should be placed along the isotherm of the material to

avoid error resulting from heat conduction in the thermocouple leads. Also, applying a larger heat load in the experiment would spread the temperatures recorded by the embedded probes, mitigating problems that might arise from the accuracy of the thermocouples.

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- [8] Jay I. Frankel, "*LECTURE\_thermal\_diffusivity*", 12, march, 2009
- [9] Edward B. Coy, "An Efficient Method for Calculating Surface Temperature and Heat Flux Based on Embedded Temperature Sensors" AIAA conference (2008) 26<sup>th</sup> AIAA Aerodynamic Measurement Technology and Ground Testing Conference

## **APPENDICES**

## Appendix A – Schematics and Photographs of the Test Setup

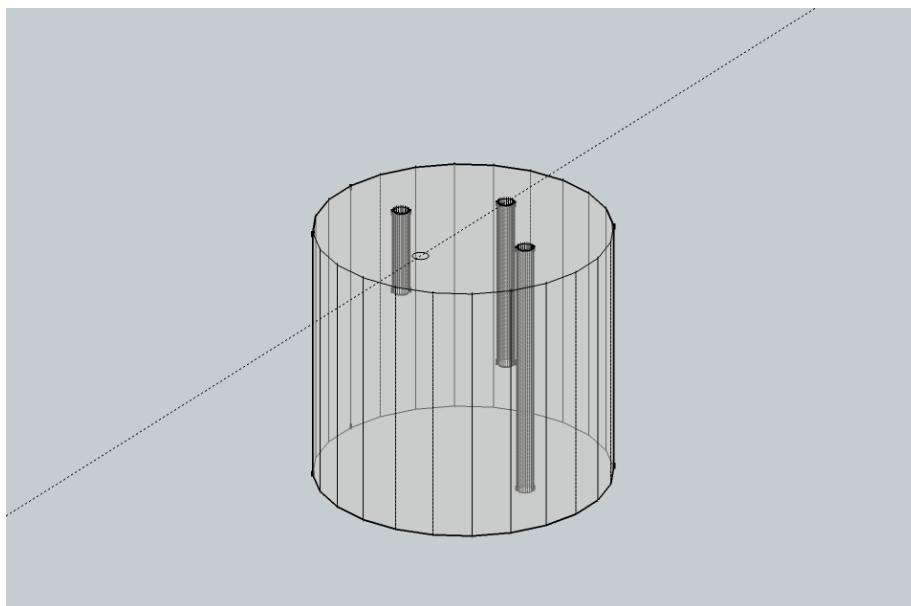


*Figure 62. Sample ISO*

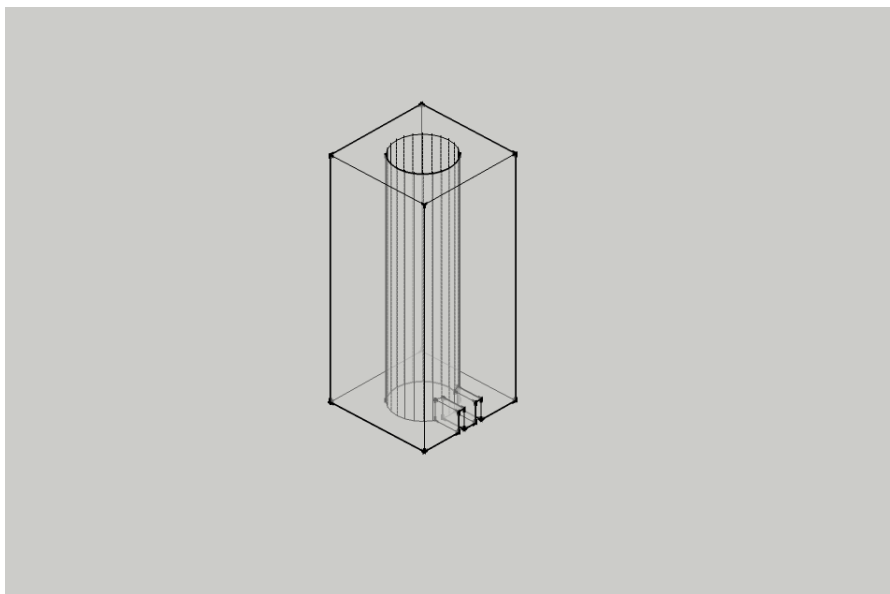


*Figure 63. ISO model of ceramic block top part*

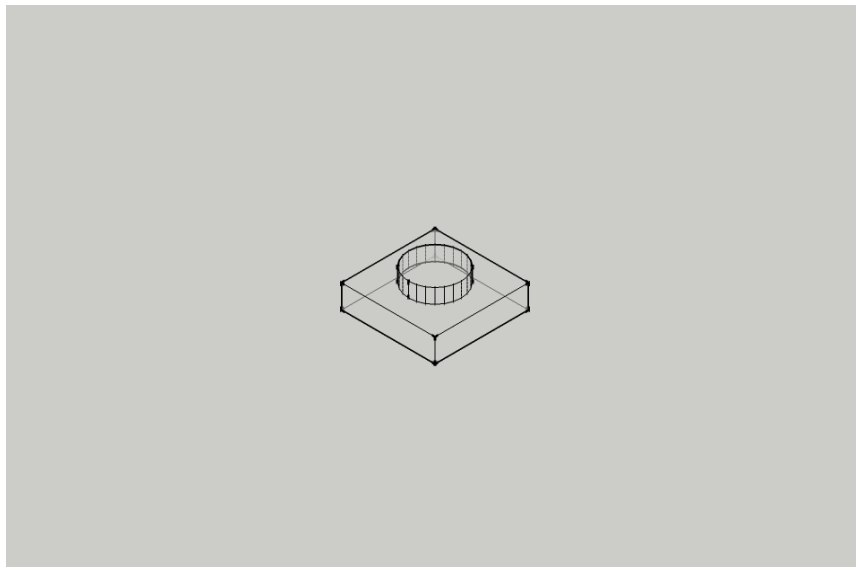


*Figure 64. ISO model of ceramic block bottom part*



*Figure 65. Test apparatus assembled with heater*



*Figure 66. Nicrome heater*



*Figure 67. Small ceramic piece used to electrically insulate the sample from the heater*

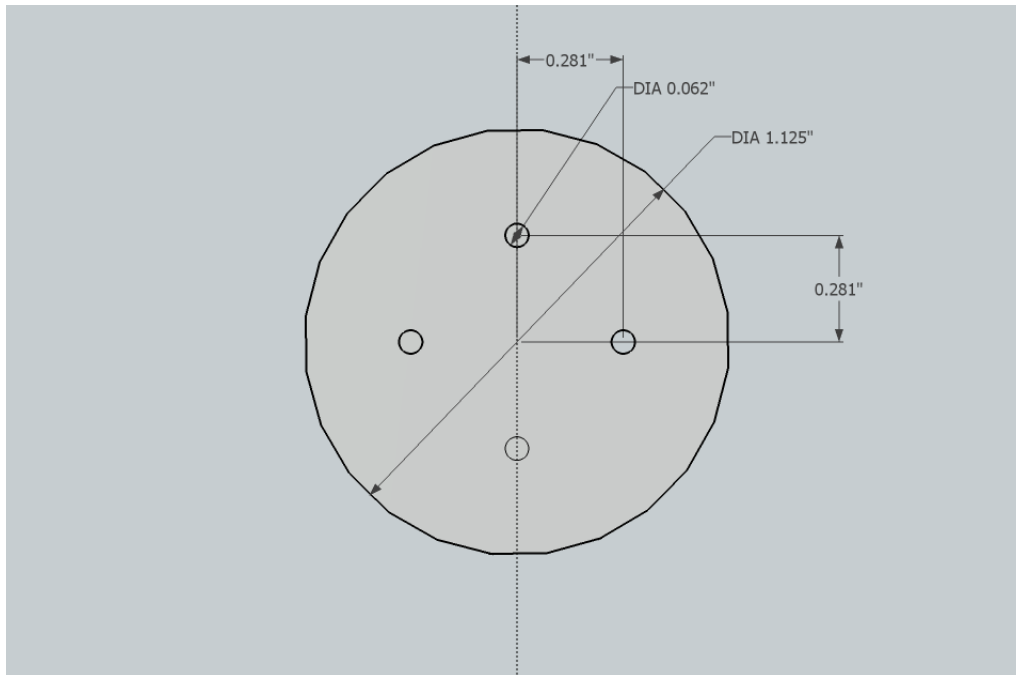


Figure 68. Diagram of top view of sample

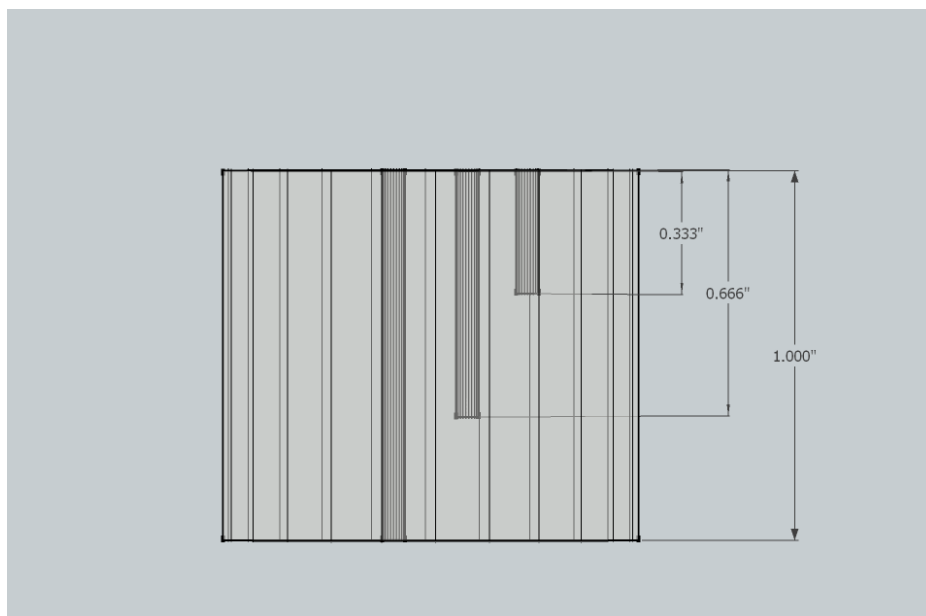


Figure 69. Diagram of side view of sample

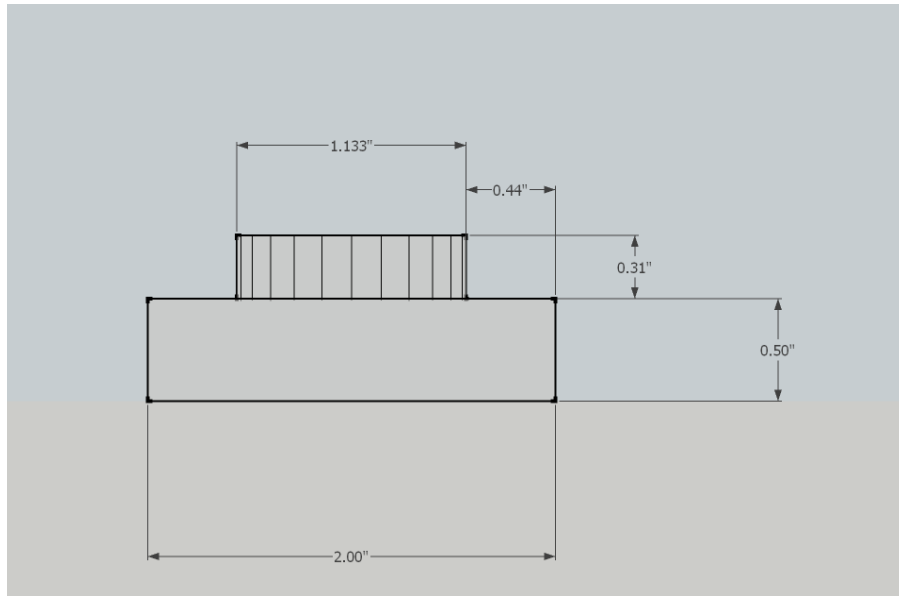


Figure 70. Diagram of side view of bottom piece of ceramic insulator

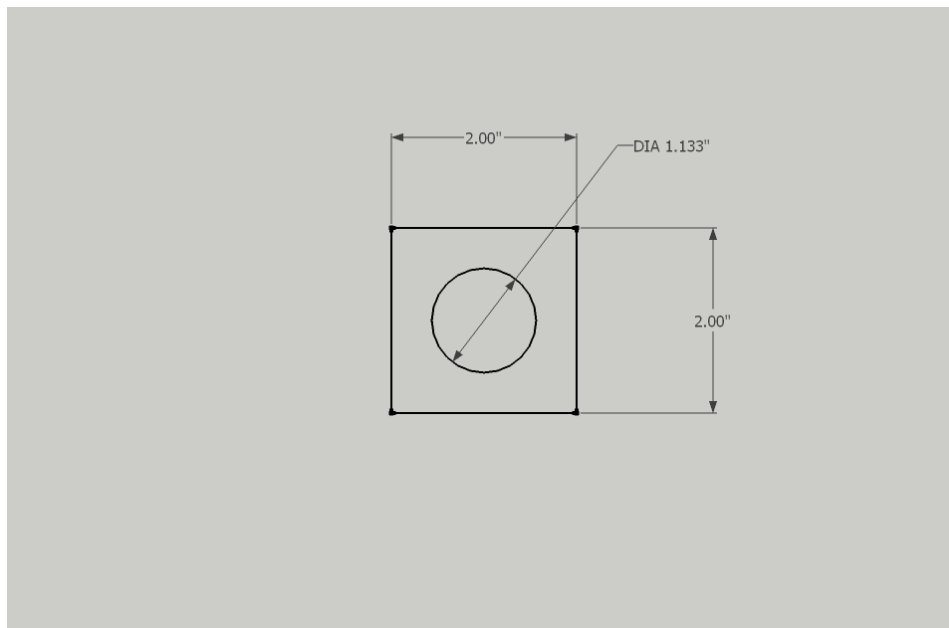


Figure 71. Diagram of top view of bottom piece of ceramic insulator

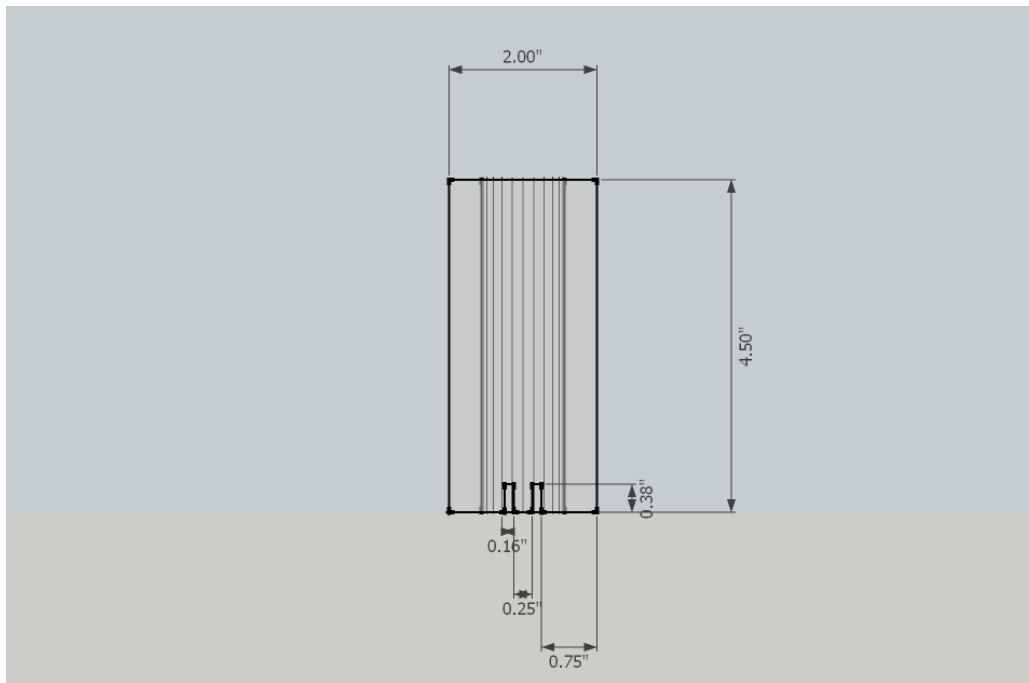


Figure 72. Diagram of side view of top piece of ceramic

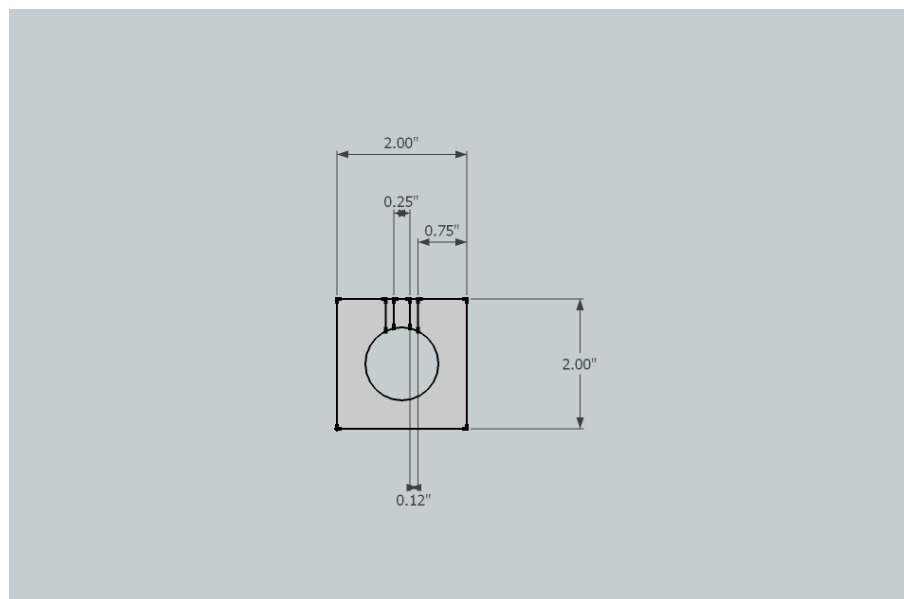


Figure 73. Diagram of top view of top piece of ceramic

## Appendix B – Code

### Program 1 Matlab filtering program

```
function smoothdata(fileToRead1)

newData1 = importdata(fileToRead1);
colheaders = genvarname(newData1.colheaders);
for i = 1:length(colheaders)
    dataByColumn1.(colheaders{i}) = newData1.data(:, i);
end
vars = fieldnames(dataByColumn1);
for i = 1:length(vars)
    assignin('base', vars{i}, dataByColumn1.(vars{i}));
end

newData1.data(:,4)=smooth(smooth((newData1.data(:,4)-32)*5.0/9.0+
273.15));
newData1.data(:,5)=smooth(smooth((newData1.data(:,5)-32)*5.0/9.0+
273.15));
newData1.data(:,6)=smooth(smooth((newData1.data(:,6)-32)*5.0/9.0+
273.15));
newData1.data(:,7)=smooth(smooth((newData1.data(:,7)-32)*5.0/9.0+
273.15));
newData1.data(:,10)=smooth(smooth(newData1.data(:,10)/.00064134));

filenameout=['F' fileToRead1];
dlmwrite(filenameout,newData1.data,'delimiter',
'\t','newline','pc','precision','%.6f')
```

**Program 2 Main Program C++ used to find Thermal Properties**

```
#include <iostream>
#include <fstream>
#include <cmath>
using namespace std;

int main()
{

    char filename[80];
    int type,typeprop,n=4000;


    double tempdata[4][n];
    double powersurf[n];
    double time[n];


    double pi=3.141592653589;

    double timestep;

    double Ac=.00064134;
    double As=.00228018;

    double Dttempdata[4][n];
    double DtDttempdata[4][n];
    double Dttempbottomsurf[n];
    double Dttemptopsurf[n];
    double Qtosolve[4][n];
    double DtQtosolve[4][n];

    double Qsolve[n];
    double Qtosolvebottom[n];
    double Qtosolvetop[n];

    double tempbottomsurf[n];
    double temptopsurf[n];
    double Ptemp2n[n];
    double Ptempn[n];
```

```
double atempbottomsurf[n];  
double atemptopsurf[n];  
double aPtemp2n[n];  
double aPtempn[n];
```

```
double featemp0[n];  
double featemp3n[n];  
double upwindfeatemp0[n];  
double upwindfeatemp3n[n];
```

```
double qloss[n];
```

```
double alpha[n];  
double alpha1[n];  
double alpha3[n];  
double alpha31[n];  
double k[n];
```

```
double k3[n];  
double km01[n];  
double km[n];
```

```
double kmm01[n];  
double kmm[n];
```

```
double ak[n];  
double ak3[n];  
double akm[n];
```

```
double cp[n];  
double cp3[n];  
double cpm[n];  
double cpmm[n];  
double cpmc[n];  
double cpmmc[n];  
double cpma[n];  
double cpa[n];
```

```
double dist;
double actk,actcp,rho,actalpha;
```

```
double fake;
```

```
cout<<"Enter the name of the file you wish to analyse \nExample
numericaltemps.dat\nRemember not to use spaces\n";
cin>>filename;
cout<<"\nIs this numerical or experimental data.\nEnter 1 for numerical data \nEnter 2
for experimental data.\n";
cin>>type;
if(type==2)
{
    cout<<"\nEnter the type of material it is if known.\nIf material is AL 6061 enter 1\nIf
material is Cu 145 enter 2\nIf the material is unknown enter 3\n";
    cin>>typeprop;
}
```

```
cout<<"Notes:\nThis program assumes a .1 second time step\nThe cp value for
numerical data will not be given\nCheck the code for more details. Enter a integer to start
program\n";
```

```
cin>>fake;
```

```
timestep=.1;// how many seconds between temperature measurements
if(typeprop==1)
{
    actk=167;
    actcp=1256;
    dist=.0084582;
    rho=2711.66;
    actalpha=actk/(rho*actcp);
}
if(typeprop==2)
{
    actk=354.7;
    actcp=385;
    dist=.0084582;
    rho=8937.41;
    actalpha=actk/(rho*actcp);
```

```

}
if(type==1)
{
    actk=14.7;
    dist=.005;
    actalpha=3.75e-6;
    actcp=1337; // no real cp given
}

```

```

ifstream in(filename);

```

```

for(int i=0;i<n;i++)
{

```

```

    double x,y,a,z,zz,p;

```

```

        in>>fake;    //1
        in>>a;        //2    time
        in>>fake;    //3
        in>>x;        //4    T15 shallow
        in>>y;        //5    T16
        in>>z;        //6    T17
        in>>zz;       //7    T18 deepest
        in>>fake;    //8
        in>>fake;    //9
        in>>p;        //10   power
        in>>fake;    //11

```

```

if(typeprop==2)
{
    time[i]=a;
    tempdata[0][i]=zz;
    tempdata[1][i]=z;
    tempdata[2][i]=y;
    tempdata[3][i]=x;

```

```

    powersurf[i]=p; // per unit area
}
if(typeprop==1)
{
    time[i]=a;
    tempdata[0][i]=zz;
    tempdata[1][i]=z;
    tempdata[2][i]=y;
    tempdata[3][i]=x;
    powersurf[i]=p; // per unit area
}

if(type==1)
{
    double timenum;
    timenum=i;
    time[i]=timenum*timestep;
    tempdata[0][i]=zz;
    tempdata[1][i]=z;
    tempdata[2][i]=y;
    tempdata[3][i]=x;
    powersurf[i]=p;
}

}

in.close();// closing file

```

```

for(int f=0;f<4;f++)
{
    Dttempdata[f][0] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempdata[f][0] + 48.0 *
tempdata[f][1] - 36.0 * tempdata[f][2] + 16.0 * tempdata[f][3] - 3.0 * tempdata[f][4] );
    Dttempdata[f][1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempdata[f][1] + 48.0 *
tempdata[f][2] - 36.0 * tempdata[f][3] + 16.0 * tempdata[f][4] - 3.0 * tempdata[f][5] );

    for (int i = 2;i<n-2;i++)
    {
        Dttempdata[f][i] = 1.0 / ( 12.0 * timestep ) * ( tempdata[f][i-2] - 8.0 * tempdata[f][i-
1] + 8.0 * tempdata[f][i+1] - tempdata[f][i+2] );
    }

    Dttempdata[f][n-2] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempdata[f][n-2] + 48.0 *
tempdata[f][n-3] - 36.0 * tempdata[f][n-4] + 16.0 * tempdata[f][n-5] - 3.0 *
tempdata[f][n-6] );
    Dttempdata[f][n-1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempdata[f][n-1] + 48.0 *
tempdata[f][n-2] - 36.0 * tempdata[f][n-3] + 16.0 * tempdata[f][n-4] - 3.0 *
tempdata[f][n-5] );
}
for(int f=0;f<4;f++)
{
    DtDttempdata[f][0] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Dttempdata[f][0] + 48.0 *
Dttempdata[f][1] - 36.0 * Dttempdata[f][2] + 16.0 * Dttempdata[f][3] - 3.0 *
Dttempdata[f][4] );
    DtDttempdata[f][1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Dttempdata[f][1] + 48.0 *
Dttempdata[f][2] - 36.0 * Dttempdata[f][3] + 16.0 * Dttempdata[f][4] - 3.0 *
Dttempdata[f][5] );

    for (int i = 2;i<n-2;i++)
    {
        DtDttempdata[f][i] = 1.0 / ( 12.0 * timestep ) * ( Dttempdata[f][i-2] - 8.0 *
Dttempdata[f][i-1] + 8.0 * Dttempdata[f][i+1] - Dttempdata[f][i+2] );
    }

    DtDttempdata[f][n-2] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Dttempdata[f][n-2] + 48.0 *
Dttempdata[f][n-3] - 36.0 * Dttempdata[f][n-4] + 16.0 * Dttempdata[f][n-5] - 3.0 *
Dttempdata[f][n-6] );
    DtDttempdata[f][n-1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Dttempdata[f][n-1] + 48.0 *
Dttempdata[f][n-2] - 36.0 * Dttempdata[f][n-3] + 16.0 * Dttempdata[f][n-4] - 3.0 *
Dttempdata[f][n-5] );
}

```

```

for(int f=0;f<4;f++)
{
    for(int i=0;i<n;i++)
    {
        double temp=0;
        for(int j=0;j<i-1;j++)
        {
            temp+=Dttempdata[f][j]*(pow((time[i]-time[j]),1.0/2.0)-pow((time[i]-
time[j+1]),1.0/2.0));
        }
        Qtosolve[f][i]=2.0*pow(pi,-1.0/2.0)*temp;
    }
}

for(int f=0;f<4;f++)
{
    DtQtosolve[f][0] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Qtosolve[f][0] + 48.0 *
Qtosolve[f][1] - 36.0 * Qtosolve[f][2] + 16.0 * Qtosolve[f][3] - 3.0 * Qtosolve[f][4] );
    DtQtosolve[f][1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Qtosolve[f][1] + 48.0 *
Qtosolve[f][2] - 36.0 * Qtosolve[f][3] + 16.0 * Qtosolve[f][4] - 3.0 * Qtosolve[f][5] );

    for (int i = 2;i<n-2;i++)
    {
        DtQtosolve[f][i] = 1.0 / ( 12.0 * timestep ) * ( Qtosolve[f][i-2] - 8.0 * Qtosolve[f][i-1]
+ 8.0 * Qtosolve[f][i+1] - Qtosolve[f][i+2] );
    }

    DtQtosolve[f][n-2] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Qtosolve[f][n-2] + 48.0 *
Qtosolve[f][n-3] - 36.0 * Qtosolve[f][n-4] + 16.0 * Qtosolve[f][n-5] - 3.0 *
Qtosolve[f][n-6] );
    DtQtosolve[f][n-1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * Qtosolve[f][n-1] + 48.0 *
Qtosolve[f][n-2] - 36.0 * Qtosolve[f][n-3] + 16.0 * Qtosolve[f][n-4] - 3.0 *
Qtosolve[f][n-5] );
}

// frankels third-order expression for alpha. smaller root
ofstream lout("dataout11.dat");
for(int i=2;i<n-2;i++)
{
    double temp,notcount,toll,noti,mini,check,temp2,max;

```

```

int count=2,flag=0;
if(type==1)
{
    toll=.0000000000001;
    check=100;
    alpha[1]=.000000001;
    max=.00000002;
}
if(typeprop==1)
{
    toll=.0000000000001;
    check=100;
    alpha[1]=.00000000001;
    max=.00000008;

}
if(typeprop==2)
{
    toll=.00000000001;
    check=100;
    alpha[1]=.000000001;
    max=.00005;

}
if(typeprop==3)
{
    toll=.000000001;
    check=100;
    alpha[1]=.0000002;
    max=.0001;

}

while(flag==0)
{

    notcount=count;

    alpha[i]=alpha[1]+toll*(notcount-1.0);

```

```

temp=alpha[i]*alpha[i]+alpha[i]*dist*dist/(2.0*(tempdata[2][i]-
tempdata[1][i]))*Dttempdata[2][i]*(1.0-2.0*Qtosolve[2][i]/(Qtosolve[2][i]-
Qtosolve[1][i]))+pow(alpha[i],1.0/2.0)*dist*dist*dist/(6.0*pow(pi,1.0/2.0)*(tempdata[2][
i]-tempdata[1][i]))*DtQtosolve[2][i]/(2.0*pow(pi,-1.0/2.0))*(1.0-
3.0*Qtosolve[2][i]/(Qtosolve[2][i]-Qtosolve[1][i]))-
dist*dist*dist*dist/(6.0*(tempdata[2][i]-
tempdata[1][i]))*DtDttempdata[2][i]*(Qtosolve[2][i]/(Qtosolve[2][i]-Qtosolve[1][i]));

```

```

mini=abs(temp);
if(type==1)
{
if(mini<.000000000000000001)
{
check=mini;
temp2=alpha[i];
flag=1;

}
}
if(typeprop==1)
{
if(mini<.000000000000000001)
{
check=mini;
temp2=alpha[i];
flag=1;

}
}
if(typeprop==2)
{
if(mini<check)
{
check=mini;
temp2=alpha[i];

}
}
if(typeprop!=2)

```



```

// frankels third-order expression for alpha. bigger root
ofstream llout("dataout12.dat");
for(int i=2;i<n-2;i++)
{
double temp,notcount,toll,noti,mini,check,temp2,max;
int count=2,flag=0;
if(type==1)
{
toll=.000000001;
check=100;
alpha1[1]=.0001;
max=.00000002;
}
if(typeprop==1)
{
toll=.00000001;
check=100;
alpha1[1]=.06;
max=.001;

}
if(typeprop==2)
{
toll=.00000001;
check=100;
alpha1[1]=.03;
max=.0005;

}
if(typeprop==3)
{
toll=.0000000001;
check=100;
alpha1[1]=.0001;
max=.00000002;

}

while(flag==0)
{

```

```
notcount=count;
```

```
alpha1[i]=alpha1[1]-toll*(notcount-1.0);
```

```
temp=alpha1[i]*alpha1[i]+alpha1[i]*dist*dist/(2.0*(tempdata[2][i]-
tempdata[1][i]))*Dttempdata[2][i]*(1.0-2.0*Qtosolve[2][i]/(Qtosolve[2][i]-
Qtosolve[1][i]))+pow(alpha1[i],1.0/2.0)*dist*dist*dist/(6.0*pow(pi,1.0/2.0)*(tempdata[2]
[i]-tempdata[1][i]))*DtQtosolve[2][i]/(2.0*pow(pi,-1.0/2.0))*(1.0-
3.0*Qtosolve[2][i]/(Qtosolve[2][i]-Qtosolve[1][i]))-
dist*dist*dist*dist/(6.0*(tempdata[2][i]-
tempdata[1][i]))*DtDttempdata[2][i]*(Qtosolve[2][i]/(Qtosolve[2][i]-Qtosolve[1][i]));
```

```
mini=abs(temp);
```

```
if(type==1)
```

```
{
```

```
if(mini<.0000000000000001)
```

```
{
```

```
check=mini;
```

```
temp2=alpha1[i];
```

```
flag=1;
```

```
}
```

```
}
```

```
if(typeprop==1)
```

```
{
```

```
if(mini<.0000000001)
```

```
{
```

```
check=mini;
```

```
temp2=alpha1[i];
```

```
flag=1;
```

```
}
```

```
}
```

```
if(typeprop==2)
```

```
{
```

```
if(mini<check)
```

```
{
```

```
check=mini;
```

```
temp2=alpha1[i];
```



```
llout.close();// closing file
```

```
ofstream pout("dataout21.dat");
for(int i=2;i<n-2;i++) // 2 TC my way
{
    double temp,notcount,toll,noti,mini,check,temp2,max;
    int count=2,flag=0;
    if(type==1)
    {
        toll=.00000000000001;
        check=100;
        alpha3[1]=.0000002;
        max=.0000000001;
    }
    if(typeprop==1)
    {
        toll=.000000000001;
        check=100;
        alpha3[1]=.0000001;
        max=.000000001;
    }

    if(typeprop==2)
    {
        toll=.000000000001;
        check=100;
        alpha3[1]=.000001;
        max=.0000000001;
    }

    if(typeprop==3)
    {
        toll=.0000000001;
        check=100;
        alpha3[1]=.0001;
        max=.0000002;
    }
}
```

```

    }
    while(flag==0)
    {

        notcount=count;

        alpha3[i]=alpha3[1]-toll*(notcount-1.0);

        temp=tempdata[1][i]-tempdata[2][i]-pow(alpha3[i],-
1.0/2.0)*Qtosolve[1][i]*dist+Dttempdata[1][i]*dist*dist/(2.0*alpha3[i])-
DtQtosolve[1][i]*dist*dist*dist/(6.0*pow(alpha3[i],3.0/2.0));

        mini=abs(temp);
        if(type==1)
        {
            if(mini<.1)
            {
                check=mini;
                temp2=alpha3[i];
                flag=1;
            }
        }
        if(typeprop==1)
        {
            if(mini<.4)
            {
                check=mini;
                temp2=alpha3[i];
                flag=1;
            }
        }
        if(typeprop==2)
        {
            if(mini<check)
            {
                check=mini;
                temp2=alpha3[i];
            }
        }
    }

```



```

{
    toll=.000000001;
    check=100;
    alpha31[1]=.0001;
    max=.0000002;
}
if(typeprop==1)
{
    toll=.0000001;
    check=100;
    alpha31[1]=.01;
    max=.00006;
}
if(typeprop==2)
{
    toll=.00001;
    check=100;
    alpha31[1]=.01;
    max=.0001;
}
if(typeprop==3)
{
    toll=.000000001;
    check=100;
    alpha31[1]=.0001;
    max=.0000002;
}
while(flag==0)
{

    notcount=count;

    alpha31[i]=alpha31[1]-toll*(notcount-1.0);

    temp=tempdata[1][i]-tempdata[2][i]-pow(alpha31[i],-
1.0/2.0)*Qtosolve[1][i]*dist+Dttempdata[1][i]*dist*dist/(2.0*alpha31[i])-
DtQtosolve[1][i]*dist*dist*dist/(6.0*pow(alpha31[i],3.0/2.0));

```

```

mini=abs(temp);
if(type==1)
{
if(mini<.1)
{
check=mini;
temp2=alpha31[i];
flag=1;

}
}

if(typeprop==1)
{
if(mini<.1)
{
check=mini;
temp2=alpha31[i];
flag=1;

}
}
if(typeprop==2)
{
if(mini<check)
{
check=mini;
temp2=alpha31[i];

}
}
if(typeprop!=2)
{
if(alpha31[i]<max)
{
flag=1;
temp2=1.0;
}
}
if(typeprop==2)
{

```



```

    }
    Ptemp2n[i]=2.0*tempdata[1][i]-
tempdata[0][i]+Dttempdata[1][i]*dist*dist/(alpha1[i]+DtDttempdata[1][i]*dist*dist*dist
*dist/(6.0*alpha1[i]*alpha1[i]));
    Ptempn[i]=2.0*tempdata[2][i]-
tempdata[3][i]+Dttempdata[2][i]*dist*dist/(alpha1[i]+DtDttempdata[2][i]*dist*dist*dist
*dist/(6.0*alpha1[i]*alpha1[i]));

}

for(int f=0;f<4;f++)
{
    Dttempbottomsurf[0] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempbottomsurf[0] + 48.0 *
tempbottomsurf[1] - 36.0 * tempbottomsurf[2] + 16.0 * tempbottomsurf[3] - 3.0 *
tempbottomsurf[4] );
    Dttempbottomsurf[1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempbottomsurf[1] + 48.0 *
tempbottomsurf[2] - 36.0 * tempbottomsurf[3] + 16.0 * tempbottomsurf[4] - 3.0 *
tempbottomsurf[5] );
    Dttemptopsurf[0] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * temptopsurf[0] + 48.0 *
temptopsurf[1] - 36.0 * temptopsurf[2] + 16.0 * temptopsurf[3] - 3.0 * temptopsurf[4] );
    Dttemptopsurf[1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * temptopsurf[1] + 48.0 *
temptopsurf[2] - 36.0 * temptopsurf[3] + 16.0 * temptopsurf[4] - 3.0 * temptopsurf[5] );

    for (int i = 2;i<n-2;i++)
    {
        Dttempbottomsurf[i] = 1.0 / ( 12.0 * timestep ) * ( tempbottomsurf[i-2] - 8.0 *
tempbottomsurf[i-1] + 8.0 * tempbottomsurf[i+1] - tempbottomsurf[i+2] );
        Dttemptopsurf[i] = 1.0 / ( 12.0 * timestep ) * ( temptopsurf[i-2] - 8.0 * temptopsurf[i-
1] + 8.0 * temptopsurf[i+1] - temptopsurf[i+2] );

    }

    Dttempbottomsurf[n-2] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempbottomsurf[n-2] +
48.0 * tempbottomsurf[n-3] - 36.0 * tempbottomsurf[n-4] + 16.0 * tempbottomsurf[n-5] -
3.0 * tempbottomsurf[n-6] );
    Dttempbottomsurf[n-1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * tempbottomsurf[n-1] +
48.0 * tempbottomsurf[n-2] - 36.0 * tempbottomsurf[n-3] + 16.0 * tempbottomsurf[n-4] -
3.0 * tempbottomsurf[n-5] );
    Dttemptopsurf[n-2] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * temptopsurf[n-2] + 48.0 *
temptopsurf[n-3] - 36.0 * temptopsurf[n-4] + 16.0 * temptopsurf[n-5] - 3.0 *
temptopsurf[n-6] );

```

```

    Dttemptopsurf[n-1] = 1.0 / ( 12.0 * timestep ) * ( -25.0 * temptopsurf[n-1] + 48.0 *
temptopsurf[n-2] - 36.0 * temptopsurf[n-3] + 16.0 * temptopsurf[n-4] - 3.0 *
temptopsurf[n-5] );

}

for(int i=0;i<n;i++)
{
    double temp=0,temp2=0;
    for(int j=0;j<i-1;j++)
    {
        temp+=Dttemptbottomsurf[j]*(pow((time[i]-time[j]),1.0/2.0)-pow((time[i]-
time[j+1]),1.0/2.0));
        temp2+=Dttemptopsurf[j]*(pow((time[i]-time[j]),1.0/2.0)-pow((time[i]-
time[j+1]),1.0/2.0));
    }
    Qtosolvebottom[i]=2.0*pow(pi,-1.0/2.0)*temp;
    Qtosolvetop[i]=2.0*pow(pi,-1.0/2.0)*temp2;
}

for(int i=4;i<n-2;i++)
{
    k[i]=powersurf[i]/(pow(alpha1[i],-1.0/2.0)*Qtosolve[1][i]+dist*pow(alpha1[i],-
1.0)*Dttempdata[1][i]+pow(alpha1[i],-3.0/2.0)*dist*dist/2.0*DtQtosolve[1][i]);
    k3[i]=powersurf[i]/(pow(alpha31[i],-1.0/2.0)*Qtosolve[1][i]+dist*pow(alpha31[i],-
1.0)*Dttempdata[1][i]+pow(alpha31[i],-3.0/2.0)*dist*dist/2.0*DtQtosolve[1][i]);
}

for(int i=4;i<n-2;i++)
{
    km01[i]=powersurf[i]/(pow(alpha1[i],-1.0/2.0)*Qtosolvebottom[i]);
    km[i]=powersurf[i]/(pow(alpha1[i],-1.0/2.0)*Qtosolve[0][i]);

    kmm01[i]=powersurf[i]/(pow(alpha31[i],-1.0/2.0)*Qtosolvebottom[i]);
    kmm[i]=powersurf[i]/(pow(alpha31[i],-1.0/2.0)*Qtosolve[0][i]);
}

```

```

// with a alpha from a book
for(int i=2;i<n-2;i++)
{
    atempbottomsurf[i]=2.0*tempdata[1][i]-
tempdata[2][i]+Dttempdata[1][i]*dist*dist/(actalpha)+DtDttempdata[1][i]*dist*dist*dist*
dist/(6.0*actalpha*actalpha);
    atemptopsurf[i]=2.0*tempdata[2][i]-
tempdata[1][i]+Dttempdata[2][i]*dist*dist/(actalpha)+DtDttempdata[2][i]*dist*dist*dist*
dist/(6.0*actalpha*actalpha);
    aPtemp2n[i]=2.0*tempdata[1][i]-
tempdata[0][i]+Dttempdata[1][i]*dist*dist/(actalpha)+DtDttempdata[1][i]*dist*dist*dist*
dist/(6.0*actalpha*actalpha);
    aPtempn[i]=2.0*tempdata[2][i]-
tempdata[3][i]+Dttempdata[2][i]*dist*dist/(actalpha)+DtDttempdata[2][i]*dist*dist*dist*
dist/(6.0*actalpha*actalpha);

}

for(int i=4;i<n-2;i++)
{
    ak[i]=powersurf[i]/(pow(actalpha,-1.0/2.0)*Qtosolve[1][i]+dist*pow(actalpha,-
1.0)*Dttempdata[1][i]+pow(actalpha,-3.0/2.0)*dist*dist/2.0*DtQtosolve[1][i]);

    akm[i]=powersurf[i]/(2.0*pow(pi*actalpha,-1.0/2.0)*Qtosolve[0][i]/(2.0*pow(pi,-
1.0/2.0)));

}
for(int i=2;i<n-2;i++)
{
    cpa[i]=ak[i]/(actalpha*rho);
    cpma[i]=akm[i]/(actalpha*rho);
    cp[i]=k[i]/(alpha1[i]*rho);
    cp3[i]=k3[i]/(alpha31[i]*rho);

    cpm[i]=km[i]/(alpha1[i]*rho);

```

```

cpmm[i]=kmm[i]/(alpha31[i]*rho);

cpmc[i]=km01[i]/(alpha1[i]*rho);
cpmmc[i]=kmm01[i]/(alpha31[i]*rho);
}

for(int i=4;i<n-2;i++)
{
    featemp0[i]=2.0*tempdata[1][i]-tempdata[2][i]+dist*dist/actalpha*Dtempdata[1][i];
    featemp3n[i]=2.0*tempdata[2][i]-tempdata[1][i]+dist*dist/actalpha*Dtempdata[2][i];
    upwindfeatemp0[i]=2.0*tempdata[1][i]-
tempdata[2][i]+dist*dist/actalpha*Dtempdata[1][i+1];
    upwindfeatemp3n[i]=2.0*tempdata[1][i]-
tempdata[2][i]+dist*dist/actalpha*Dtempdata[1][i+1];

}

for(int i=2;i<n-2;i++)
{
    qlloss[i]=1.0/As*Ac*dist*actk*((tempdata[2][i]+tempdata[0][i]-
2.0*tempdata[1][i])/(dist*dist)-1.0/actalpha*Dtempdata[1][i]);

}

for(int i=4;i<n-2;i++)
{
    Qsolve[i]=actk/pow(actalpha,1.0/2.0)*Qtosolve[0][i];
}

ofstream out("dataout.dat");

for(int i=10;i<n-2;i++)
{

    out<<time[i]<<" ";           //1
    out<<tempdata[0][i]<<" ";     //2
    out<<tempdata[1][i]<<" ";     //3

```

```

    out<<tempdata[2][i]<<" "; //4
    out<<tempdata[3][i]<<" "; //5
    out<<tempbottomsurf[i]<<" "; //6
    out<<temptopsurf[i]<<" "; //7
    out<<alpha[i]<<" "; //8
    out<<alpha1[i]<<" "; //9
    out<<alpha3[i]<<" "; //10
    out<<alpha31[i]<<" "; //11
    out<<k[i]<<" "; //12
    out<<k3[i]<<" "; //13
    out<<km[i]<<" "; //14
    out<<kmm[i]<<" "; //15
    out<<atempbottomsurf[i]<<" "; //16
    out<<atemptopsurf[i]<<" "; //17
    out<<ak[i]<<" "; //18
    out<<akm[i]<<" "; //19
    out<<actk<<" "; //20
    out<<actcp<<" "; //21
    out<<actalpha<<" "; //22
    out<<featemp0[i]<<" "; //23
    out<<featemp3n[i]<<" "; //24
    out<<Qsolve[i]<<" "; //25
    out<<km01[i]<<" "; //26
    out<<kmm01[i]<<" "; //27
    out<<powersurf[i]<<"\\n"; //28

}

out.close();// closing file

if(type!=1)
{

ofstream mnout("dataoutcp.dat");

for(int i=10;i<n-2;i++)
{

    mnout<<time[i]<<" "; //1
    mnout<<tempdata[0][i]<<" "; //2
    mnout<<cpa[i]<<" "; //40
    mnout<<cpma[i]<<" "; //41

```

```

        mnout<<cp[i]<<" "; //40
        mnout<<cp3[i]<<" "; //41
        mnout<<cpm[i]<<" "; //40
        mnout<<cpmm[i]<<" "; //41
        mnout<<cpmc[i]<<" "; //40
        mnout<<cpmmc[i]<<" "; //41
        mnout<<actcp<<"\n"; //40
    }

    mnout.close();// closing file
}

ofstream mmnout("dataoutqout.dat");

for(int i=10;i<n-2;i++)
{
    mmnout<<time[i]<<" "; //1
    mmnout<<tempdata[0][i]<<" "; //2
    mmnout<<qloss[i]<<"\n"; //3
}

mmnout.close();// closing file

ofstream mmmnout("dataoutupwind.dat");

for(int i=10;i<n-2;i++)
{
    mmmnout<<time[i]<<" "; //1
    mmmnout<<upwindfeatemp0[i]<<" "; //2
    mmmnout<<upwindfeatemp3n[i]<<" "; //3
    mmmnout<<tempdata[0][i]<<" ";
    mmmnout<<tempdata[1][i]<<" ";
    mmmnout<<tempdata[2][i]<<" ";
    mmmnout<<tempdata[3][i]<<"\n";
}

mmmnout.close();// closing file

cout<<"\b\b\b\b\b\b\b\b\b\b" << "Press any key to exit: ";
cin>>fake;

```

```
    return 0;  
}
```

**Vita**

Nick Lister was born in Illinois. He was raised in Hillsboro, IL and went to Hillsboro High School. From there, he went to Murray State University and received a degree in physics engineering. Upon graduation, he attended the University of Tennessee Space Institute in Tullahoma.