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To the Graduate Council:

I am submitting herewith a dissertation written by Terry Dean Douglass, entitled "The Application of Filters to Time Analysis of Signals from Ge(Li) Detectors." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Electrical Engineering.

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THE APPLICATION OF FILTERS TO TIME ANALYSIS
OF SIGNALS FROM Ge(Li) DETECTORS

A Dissertation
Presented to
the Graduate Council of
The University of Tennessee

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

by
Terry Dean Douglass

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ABSTRACT

The measurement of the time of occurrence of a nuclear event using semiconductor nuclear radiation detectors is increasing in importance in the field of nuclear research. Because of their large volumes and high efficiency for the detection of gamma rays, particular emphasis is being placed on the use of lithium drifted germanium detectors. The accuracy of time measurement with most Ge(Li) detectors using current time measurement methods is limited by charge collection variations in the detector and noise of the detector and preamplifier. The reduction of the effects of these limitations is accomplished by the use of filters. In this dissertation, an optimum filter which minimizes the effect of noise on time measurement was determined; and the minimum timing error associated with this optimum filter was found. For the signal and noise from a Ge(Li) detector, charge-sensitive preamplifier system, the optimum filter is physically non-realizable.

The effect of certain realizable RC filters on the reduction of the time measurement error due to noise is presented. The filters examined were time-invariant and time-variant RC high-pass, RC low-pass, and the combination of RC low-pass, RC high-pass filters. Both theoretical and experimental data were examined to determine the best filter. To reduce the errors in time measurement due to both charge collection variations and noise, the time-variant filters provided several advantages. The advantages of the time-variant filters were lower discriminator levels, allowing the reduction of the effect of

charge collection variations, and lower time measurement errors due to noise at the low discriminator levels.

The use of the filters in a Ge(Li) detector, charge-sensitive preamplifier system detecting gamma rays was determined. The detector was a 23.4 cc true coaxial Ge(Li) detector, and the energy of the gamma rays was 511 keV. The filters examined were the time-invariant and time-variant filters examined to reduce the time measurement error due to noise. For the time-invariant filters, the minimum timing error was 6.81 nanoseconds fwhm and 12.6 nanoseconds fw(.1)m. This minimum was obtained using an RC high-pass filter with 2.2 RC equal to 100 nanoseconds. The best time-variant filter was one with an RC high-pass filter before the gate and an RC low-pass filter after the gate with 2.2 RC equal to 20 nanoseconds in both filters. The fwhm and corresponding fw(.1)m were 5.63 nanoseconds and 10.58 nanoseconds.

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CHAPTER I

INTRODUCTION

I. BACKGROUND

A semiconductor radiation detector is a device that transfers the energy of some incident radiation into an electrical signal. Most of the semiconductor detectors that are now being used may be classified into one of three types. These types are (1) diffused junction detectors, (2) surface barrier detectors, and (3) lithium ion-drift detectors. The semiconductor material used in any of these three types of detectors can be silicon or germanium. Each combination of detector type and material has its particular advantages and disadvantages in the detection of nuclear radiation.¹

In the detection of gamma radiation, the lithium drifted germanium detector has received considerable attention recently. The primary reason for this attention is due to the higher atomic number of germanium as opposed to silicon. A high atomic number is desirable because the cross section for photoelectric absorption of gamma rays in a material is proportional to Z^5 where Z is the atomic number of the material. Therefore, since the Z of silicon is 14 and the Z of germanium is 32, a greater efficiency in the detection of gamma rays is expected with a germanium detector. The use of the drift process creates thicker sensitive regions in the detector; and, therefore, larger volumes are possible in this process than by other processes. Thus, the combination of the lithium ion-

drift process and germanium semiconductor material produces a high efficiency gamma ray detector. This detector is normally referred to as the lithium drifted germanium detector and is abbreviated to Ge(Li) in the literature. This abbreviation will be used throughout the dissertation.

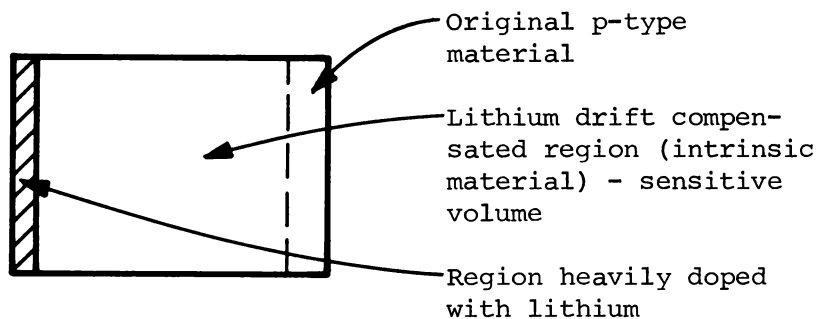
The detection of a gamma ray by a Ge(Li) detector provides two pieces of information about the gamma ray. The first piece of information is a measure of the energy absorbed in the detector. The processing of the signal from the detector to obtain energy information has been well examined and is continuing to be examined. Very good energy resolution is of utmost importance. The second piece of information is a measure of the time of occurrence of the incident radiation. The measurement of time as compared to the measurement of energy is not so well understood. Therefore, the object of this dissertation is to examine the problems of the measurement of time from the signals of Ge(Li) detectors detecting gamma radiation.

An additional stipulation must be added to the object of this dissertation. Good energy resolution, as stated previously, is necessary from a Ge(Li) detector and system. Thus, the method used for timing must not affect the energy resolution. The usual way this is done is to perform the time measurement on the signal from the charge-sensitive preamplifier. The object of this dissertation then becomes an examination of the measurement of time from the signal of the charge-sensitive preamplifier used with Ge(Li) detectors detecting gamma rays.

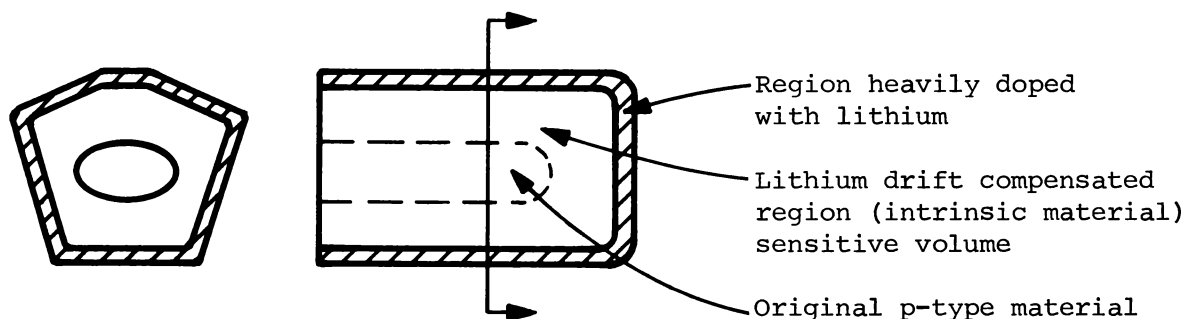
The measurement of time depends on many factors. For timing using Ge(Li) detectors the main source of timing error by current timing methods is charge collection variations in the detector. Charge collection variations are very dependent on the geometry of the detector. Presently four configurations are normally encountered, the planar configuration, the single and double open-ended five-sided configurations, and the true coaxial configuration. These configurations are represented in Figure 1.

A complete evaluation of each configuration would require a study of detection efficiency, energy resolution, charge collection properties, time resolution, cost, availability, and other factors. This study has as its purpose to examine only the time resolution of the detectors and makes no attempt to perform a complete evaluation. An examination of charge collection efficiency, energy resolution, gamma ray detection efficiency, and timing characteristics is performed by Cline² on seven planar detectors, a five-sided detector, and a true coaxial detector. Mann, et al.³ performed a study of the energy resolution of several planar detectors. Malm and Fowler⁴ examined several characteristics of five-sided detectors.

The method of performing the time measurement in most of the published information has been very similar. Discrimination on the leading edge of the signal from the charge-sensitive preamplifier is the method used in each reference cited here. For the planar configuration, Balland, et al.⁵ described the effect of both charge collection variations and noise on timing measurement. Gorni, et al.⁶ used a pulse rise time compensating network in a planar Ge(Li) detector system. A zero crossing technique was employed by Chase⁷ to reduce energy dependent

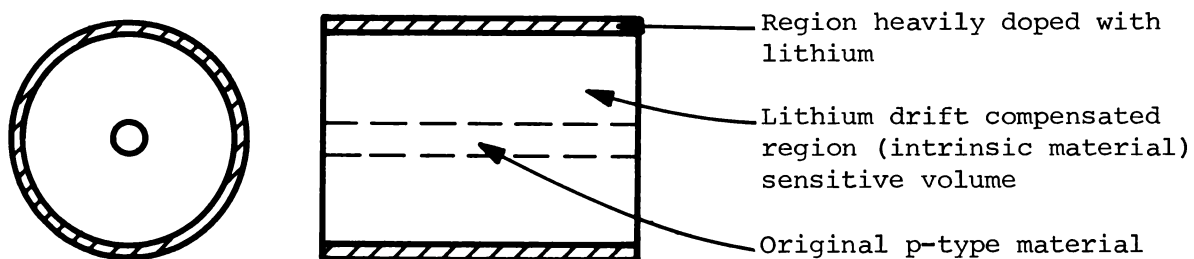


a. Planar configuration



Double open-ended obtained by removing this section

b. Single and double open-ended five-sided configuration



c. True coaxial configuration

Figure 1. Schematic configurations of Ge(Li) detectors.

and pulse shape variation timing errors in both a planar and coaxial detector system. Malm⁸ examined timing characteristics of a five-sided detector and a true coaxial detector. Graham, et al.⁹ scanned a single open-ended five-sided detector with collimated gamma rays to demonstrate the difference in charge collection times as a function of position in the detector. Time measurement from a true coaxial Ge(Li) detector was examined by Williams¹⁰ using commercially available instruments. The consensus of the information on time measurement using Ge(Li) detectors is that planar detectors are best, followed by true coaxial detectors and five-sided detectors, respectively.

II. SCOPE OF THE THESIS

The accuracy of time measurement of signals from most Ge(Li) detectors by current methods is limited by the minimum timing discriminator level possible and by charge collection variations in the detector. The minimum discriminator level is determined by the noise level into the discriminator. The noise level, and thus the discriminator level, can be decreased by filtering. However, the filtering used to obtain a low noise level may increase the effect of charge collection variations on time measurement. An examination of the effect of filtering appears to be necessary to adequately describe the problem of time measurement of signals from Ge(Li) detectors. The purpose of this dissertation is, therefore, to study the application of filters to the time analysis of signals from the charge-sensitive preamplifier used with Ge(Li) detectors detecting gamma rays.

A description of the general sources of timing error is presented in Chapter II. The sources of error in a Ge(Li) detector system are charge collection variations and noise. To demonstrate the effects of charge collection variations and noise, a determination of the signal and noise spectrum applicable in time measurement is necessary. From the signal form, time measurement as a function of charge collection variations is presented. Using both the signal and noise information, the effect of noise on time measurement is analyzed.

In Chapter III the use of filters is introduced. A calculation of the optimum filter is presented, and the minimum error in time measurement is found for the signal and noise spectrum from the preamplifier. A comparison of the time measurement of signals from several filters to the minimum value is calculated. These filters are RC low-pass, RC high-pass, and RC low-pass high-pass filters. The use of time-variant or gated filters in time measurement is presented. The gated filters are shown to have particular usefulness in a Ge(Li) detector system because of the low discriminator levels possible.

An experimental check of the results obtained theoretically is presented in Chapter IV. Also, a time measurement using a large volume true coaxial Ge(Li) detector is made. The best filter for this particular detector and system is determined, and the improvement over the normal timing procedure and the use of other filters is demonstrated.

Chapter V includes an evaluation of the results and suggestions for further study.

CHAPTER II

SOURCES OF TIMING ERROR

The accuracy of time measurement of monoenergetic signals from semiconductor radiation detectors is basically limited by two things, noise and variations in pulse shape. In many cases, either the effect of noise or the effect of variations in pulse shape is dominant. By current methods the accuracy of time measurement of signals from most Ge(Li) detector systems is dominated by pulse shape variations. However, the process of filtering creates the possibility of the effect of noise becoming noticeable. A description of both of these sources of timing error will, therefore, be made for the signal and noise from the charge-sensitive preamplifier used with a Ge(Li) detector. The signal and noise from the Ge(Li) detector-preamplifier system will be determined, and from this the effects of pulse shape variations and noise on time measurement will be described.

I. SIGNAL AND NOISE FROM A DETECTOR-PREAMPLIFIER SYSTEM

A charge-sensitive preamplifier is the term most people in nuclear instrumentation have used to describe an operational integrator. The purpose of the charge-sensitive preamplifier is to integrate the detector current signal. The integral of this detector current is a measure of the total charge released by the detector and, further, a measure of the energy of the nuclear radiation absorbed by the detector. The amplitude of the signal at the charge-sensitive preamplifier output is,

therefore, directly proportional to the energy of the radiation absorbed by the detector.

A representation of the semiconductor detector, charge-sensitive preamplifier equivalent circuit is shown in Figure 2. To facilitate the explanation, the following symbol definitions are made:

$i_d(t)$ = detector current signal (amperes),

$e_o(t)$ = output voltage signal (volts),

R_d = detector parallel resistance (ohms),

C_d = detector capacitance (farads),

R = detector series resistance (ohms),

R_g = detector bias resistance (ohms),

R_s = preamplifier series resistance (ohms),

C = input capacitance of active portion (farads),

R_i = input resistance of active portion (ohms),

R_f = feedback resistance (ohms),

C_f = feedback capacitance (farads),

$N_d(\omega)$ = detector noise current (mean square amperes/hertz),

$N_1(\omega)$ = noise voltage of R (mean square volts/hertz),

$N_g(\omega)$ = noise voltage of R_g (mean square volts/hertz),

$N_s(\omega)$ = noise voltage of R_s (mean square volts/hertz),

$N_i(\omega)$ = noise current of active portion (mean square amperes/hertz),

$N_a(\omega)$ = noise voltage of active portion (mean square volts/hertz),

$N_f(\omega)$ = noise voltage of R_f (mean square volts/hertz), and

$N_o(\omega)$ = output noise voltage (mean square volts/hertz).

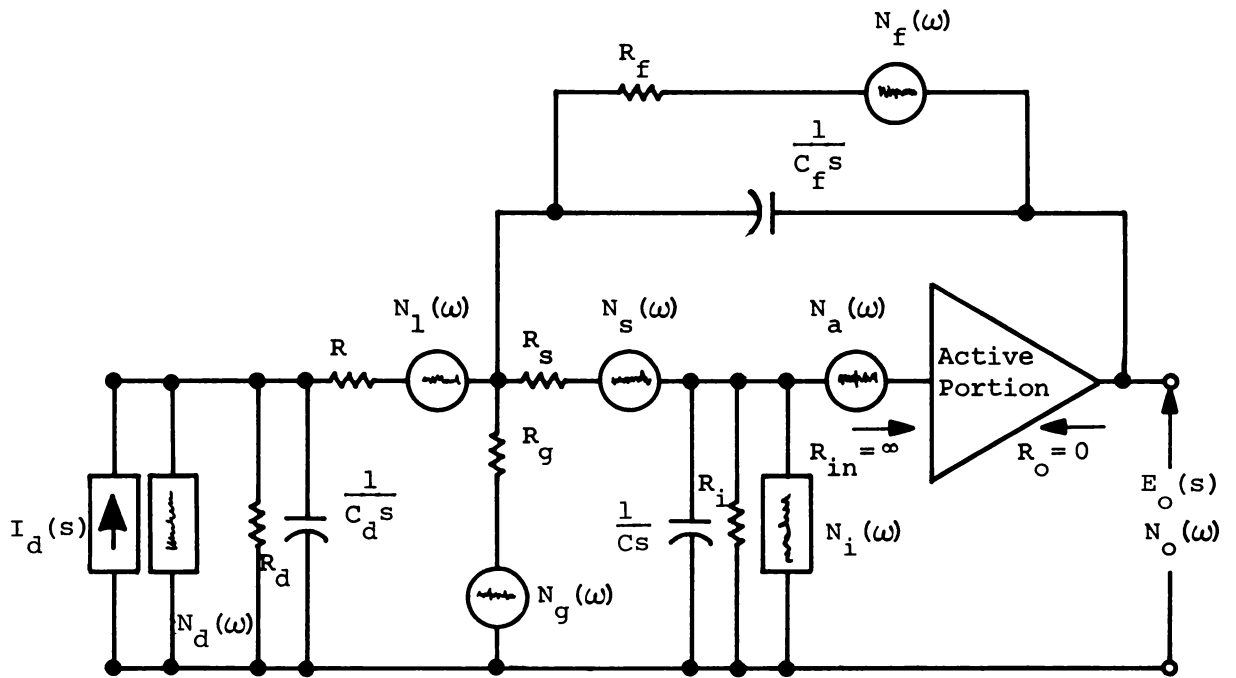


Figure 2. Equivalent circuit of the semiconductor detector, charge-sensitive preamplifier system.

The purposes of this section are to determine the output signal as a function of the input current pulse and to determine the total output noise power spectral density as a function of the noise sources.

A very detailed analysis of the semiconductor detector, charge-sensitive preamplifier system has been made by Blalock¹¹ for energy analysis using the equivalent circuit of Figure 2. Since this dissertation is concerned with the measurement of time, several simplifying assumptions can be made. An analysis of the measurement of time by discrimination on the leading edge of the preamplifier output signal requires only that the high frequency properties of the signal and noise be known. Therefore, R_d , R_g , R_i , and R_f can be assumed to have infinite values. The resistors, R and R_s , are normally small. In this analysis the values of these resistors are assumed to be zero. With these assumptions the equivalent circuit simplifies to that shown in Figure 3. In addition, the assumption of a single pole for the Laplace transform of the active portion impulse response is made. Thus, the Laplace transform of the active portion is

$$G(s) = \frac{-K}{T_a s + 1} \quad (1)$$

where K is the voltage gain, T_a is the time constant associated with the impulse response of $G(s)$, and s is the Laplace transform variable.

Transfer Function

In determining the output voltage signal as a function of the detector current signal, the noise sources in Figure 3 are neglected. Four

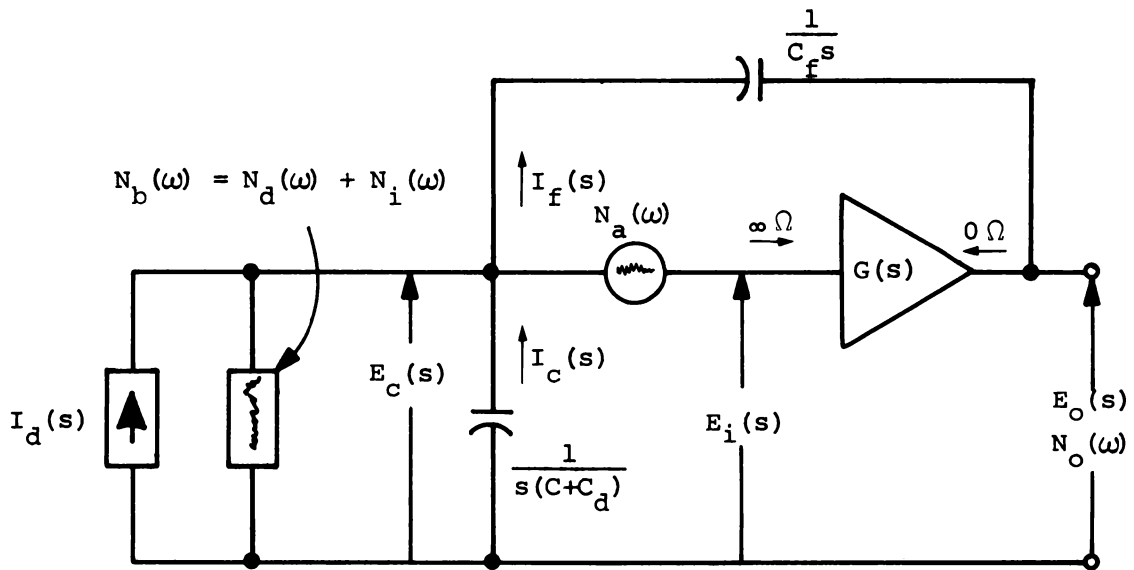


Figure 3. Simplified equivalent circuit of the semiconductor detector, charge-sensitive preamplifier system.

circuit equations can be written to find the input-to-output transfer function. These equations are:

$$I_d(s) = I_f(s) - I_c(s) \quad , \quad (2)$$

$$I_f(s) = [E_c(s) - E_o(s)]sC_f \quad , \quad (3)$$

$$I_c(s) = -E_c(s) s (C + C_d) \quad , \quad (4)$$

and

$$E_o(s) = G(s) E_c(s) \quad . \quad (5)$$

Solving Equations (1), (2), (3), (4), and (5) simultaneously, the transfer function is

$$\frac{E_o(s)}{I_d(s)} = \frac{-K}{s(C + C_d + C_f + KC_f)(T_o s + 1)} \quad , \quad (6)$$

where

$$T_o = \frac{T_a(C + C_d + C_f)}{C + C_d + C_f + KC_f} \quad . \quad (7)$$

If K is assumed to be very large, Equations (6) and (7) simplify to

$$\frac{E_o(s)}{I_d(s)} = \frac{-1}{sC_f(T_o s + 1)} \quad (8)$$

and

$$T_o = \frac{T_a(C + C_d + C_f)}{KC_f} \quad . \quad (9)$$

Detector Current Signal

To find the output voltage signal, the detector current signal must be known. The detector current signal is a function of several variables. These variables include the energy of the absorbed radiation, the location

of the charge created in the detector, the electric field distribution in the detector, the electron and hole mobility, and the detector geometry. For a particular detector and monoenergetic radiation, all of these variables are constant except for the location of the charge created in the detector. A variation in the charge location is caused by a difference in the point at which radiation enters the detector, by a variation in the range of the radiation, and by the possibility of complete absorption of the radiation from a multiple interaction process of absorption as opposed to a single interaction. A variation in the charge location results in different charge collection properties and, therefore, different detector current signal shapes. The total charge collected and the total area of the current signal, however, are equal for monoenergetic radiation.

Several attempts at describing the shape of the detector current signals have been made. Balland, et al.⁵ described the detector current signal shapes for a planar Ge(Li) detector. Both theoretical and experimental work was included in their study. Sakai¹² provided both theoretical and experimental information on the charge pulse shapes of a true coaxial Ge(Li) detector. A theoretical calculation of the detector current pulse shapes and charge pulse shapes for a single and double open-ended coaxial Ge(Li) detector was made by Poenaru.¹³ Strauss, et al.¹⁴ provided experimental data on the pulse shape distribution of the charge pulse from two Ge(Li) planar detectors.

The determination of the average detector current signal shape for a single Ge(Li) detector is very difficult. Moreover, establishing a signal shape that can be used in a theoretical analysis for a general

planar, coaxial, or five-sided detector is impossible without making some compromise. A first approximation to the average current signal shape of each of the detectors described in the references of the preceding paragraph is an exponential; that is,

$$i_d(t) = I e^{-t/T_c}, \quad (10)$$

where T_c is a measure of the charge collection time. For monoenergetic radiation the area of $i_d(t)$ is constant and a direct linear function of the energy of the radiation and the total charge, Q , released in the detector. Using this fact, the value of I can be found by determining the area of $i_d(t)$ and setting this equal to Q , as follows

$$Q = \int_0^{\infty} I e^{-t/T_c} dt. \quad (11)$$

Solving Equation (11), I is shown to be equal to Q/T_c . Thus, an expression for the current signal from a detector which is detecting monoenergetic radiation is found as a function of only one variable; i.e., the charge collection time, T_c . This expression is

$$i_d(t) = \frac{Q}{T_c} e^{-t/T_c}. \quad (12)$$

The current pulse represented by Equation (12) will be used in the theoretical analysis of charge collection effects and noise effects on time measurement.

Preamplifier Output Voltage Signal

The determination of the charge-sensitive preamplifier output signal is now quite simple. Equation (8) is the expression for the transfer function in Laplace transform notation of the input current

to the output voltage. The Laplace transform of $i_d(t)$ is

$$I_d(s) = \frac{Q}{T_c s + 1} \quad . \quad (13)$$

Substituting Equation (13) into Equation (8) and solving for $E_o(s)$, the Laplace transform of $e_o(t)$, the output voltage signal is

$$E_o(s) = \frac{-Q}{C_f s (T_o s + 1) (T_c s + 1)} \quad . \quad (14)$$

By a partial fraction expansion of Equation (14) and by finding the inverse Laplace transform of each fraction, the expression for the output voltage signal in the time domain is

$$e_o(t) = -\frac{Q}{C_f} + \frac{Q}{C_f} \frac{1}{(T_o - T_c)} \left(T_o e^{-t/T_o} - T_c e^{-t/T_c} \right) \quad . \quad (15)$$

Preamplifier Output Noise Power Spectral Density

Noise is a random process, and a description of the value of noise at a certain time can be given only in a statistical manner. For convenience in analyzing noise, the power spectral density function is introduced. A good description of random processes and the introduction of the power spectral density function is given by Brown and Nilsson.¹⁵ The power spectral density, $N(\omega)$, can be thought of as the power dissipated by noise per unit frequency and can be represented mathematically as

$$N(\omega) = 2\pi \frac{\overline{x^2}}{d\omega} \quad , \quad (16)$$

where $\overline{x^2}$ is the mean square noise voltage or current in volts squared or amperes squared depending on the noise source and ω is frequency in

radians per second. For a given power spectral density the total mean square voltage or current can be found by simply using the following equation:

$$\overline{x^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} N(\omega) d\omega \quad . \quad (17)$$

The convenience of using the power spectral density is that the relationship between the input and output power spectral densities is known if the transfer function is known. For example, if the input power spectral density is $N_i(\omega)$ in volts squared per radian per second, the output power spectral density is $N_o(\omega)$ in volts squared per radian per second, and the transfer function of voltage out to voltage in is $G(s)$, then $N_o(\omega)$ and $N_i(\omega)$ are related by

$$N_o(\omega) = \left| G(j\omega) \right|^2 N_i(\omega) \quad . \quad (18)$$

Therefore, the determination of $N_o(\omega)$ in Figure 3, page 11, requires that the proper transfer functions be found for each noise source and then applying

$$N_o(\omega) = \left| G_a(j\omega) \right|^2 N_a(\omega) + \left| G_b(j\omega) \right|^2 N_b(\omega) \quad , \quad (19)$$

where $G_b(s)$ is the transfer function of a current source at $N_b(\omega)$ and $G_a(s)$ is the transfer function of a voltage source at $N_a(\omega)$.

Effect of $N_a(\omega)$. If a voltage source replaces $N_a(\omega)$, the transfer function, $G_a(s)$, can be found using Figure 4. The following circuit equations can be written:

$$I(s) = -(C_d + C) s E_d(s) \quad , \quad (20)$$

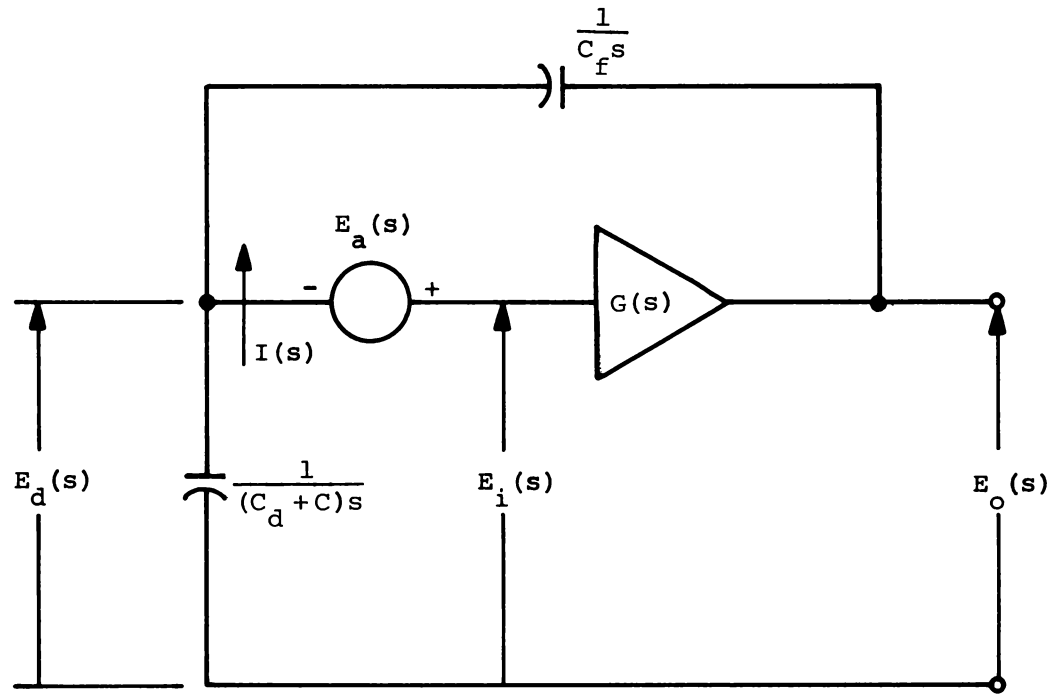


Figure 4. Equivalent circuit for the determination of the transfer function $G_a(s)$.

$$I(s) = [E_d(s) - E_o(s)] C_f s, \quad (21)$$

$$E_a(s) = E_i(s) - E_d(s), \quad (22)$$

$$E_o(s) = G(s) E_i(s), \quad (23)$$

and

$$G_a(s) = E_o(s)/E_a(s). \quad (24)$$

Solving Equations (20), (21), (22), (23), and (24) for $G_a(s)$ in terms of the circuit parameters, a useful $G_a(s)$ is obtained:

$$G_a(s) = \frac{-K (C_d + C + C_f)}{(C + C_d + C_f + KC_f)(T_o s + 1)}, \quad (25)$$

where

$$T_o = \frac{T_a (C + C_d + C_f)}{C + C_d + C_f + KC_f}. \quad (26)$$

If, once again, K is assumed to be very large Equations (25) and (26) simplify to

$$G_a(s) = \frac{-(C_d + C + C_f)}{C_f (T_o s + 1)} \quad (27)$$

and

$$T_o = \frac{T_a (C + C_d + C_f)}{KC_f}. \quad (28)$$

The effect of $N_a(\omega)$ is then found by properly substituting into Equation (18) where $|G(j\omega)|^2$ in this case is

$$|G(j\omega)|^2 = \frac{(C_d + C + C_f)^2}{C_f^2 (T_o^2 \omega^2 + 1)}. \quad (29)$$

$N_o(\omega)$ due to the $N_a(\omega)$ source is

$$N_{oa}(\omega) = \frac{(C_d + C + C_f)^2}{C_f^2 (T_o^2 \omega^2 + 1)} N_a(\omega) . \quad (30)$$

$N_a(\omega)$ is a white noise source or is constant over the entire ω range so that

$$N_{oa}(\omega) = \frac{a}{(T_o^2 \omega^2 + 1)} , \quad (31)$$

where

$$a = \frac{(C_d + C + C_f)^2}{C_f^2} N_a(\omega) . \quad (32)$$

Effect of $N_b(\omega)$. The transfer function for a current source at $N_b(\omega)$ is exactly the same as for the signal transfer function which is

$$G_b(s) = \frac{-1}{sC_f(T_o s + 1)} , \quad (33)$$

where

$$T_o = \frac{T_a (C + C_d + C_f)}{KC_f} . \quad (34)$$

Using this transfer function, $N_o(\omega)$ due to the $N_b(\omega)$ source is

$$N_{ob}(\omega) = \frac{N_b(\omega)}{C_f^2 \omega^2 (T_o^2 \omega^2 + 1)} . \quad (35)$$

$N_b(\omega)$ is also a white noise source so that $N_{ob}(\omega)$ becomes

$$N_{ob}(\omega) = \frac{b}{\omega^2 (T_o^2 \omega^2 + 1)} , \quad (36)$$

where

$$b = \frac{N_b(\omega)}{C_f^2} . \quad (37)$$

The total output power spectral density is the sum of the effects of the two uncorrelated noise sources, or

$$N_o(\omega) = \frac{a}{(T_o^2 \omega^2 + 1)} + \frac{b}{\omega^2 (T_o^2 \omega^2 + 1)} \quad . \quad (38)$$

The two terms of $N_o(\omega)$ are known to be equal in the range of $10^5 - 10^6$ radians per second for current detector-preamplifier systems. Since, for leading-edge timing, only the high frequency properties are necessary, $N_{ob}(\omega)$ is practically negligible as compared to $N_{oa}(\omega)$ in the frequency range above 10^7 radians per second. Therefore, in this study the output noise power spectral density that will be used is

$$N_o(\omega) = \frac{a}{(T_o^2 \omega^2 + 1)} \quad . \quad (39)$$

The type of noise represented by Equation (39) is commonly referred to as Markov noise.

II. EFFECT OF CHARGE COLLECTION VARIATIONS

By current time measurement methods the dominant source of error is variations in charge collection times. Charge collection variations show up as variations in the preamplifier output signal shape. The purposes of this section are to give a description of how these variations affect time measurement on the unfiltered output signal and to discuss the factors that are important in reducing the effects of charge collection variations.

The preamplifier output signal is represented by Equation (15). Since discrimination on the leading edge of the output signal is the

method used for time measurement, the representation of $e_1(t)$ can be simplified. The infinite series representation for an exponential is given by the following equation:

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^n}{n!} + \dots \quad (40)$$

For values of x less than one, an approximation to the exponential can be made by a finite series with the number of terms depending on the accuracy needed and how much x is less than one. In the exponentials of Equation (15), the arguments, $-t/T_o$ and $-t/T_c$, are both much less than one for discrimination on $e_o(t)$ at times much less than T_o and T_c . Thus, good approximations of e^{-t/T_o} and e^{-t/T_c} are

$$e^{-t/T_o} = 1 - \frac{t}{T_o} + \frac{t^2}{2T_o^2} \quad (41)$$

and

$$e^{-t/T_c} = 1 - \frac{t}{T_c} + \frac{t^2}{2T_c^2} \quad (42)$$

Substituting Equations (41) and (42) into Equation (15) and simplifying

$$e_o(t) = -\frac{Q}{C_f} \frac{t^2}{2T_o T_c} \quad (43)$$

If the discriminator level is assumed to be $-E_d$, the time of discrimination, t_d , is

$$t_d = \left(\frac{2C_f T_o T_c E_d}{Q} \right)^{1/2} \quad (44)$$

A variation in the charge collection time, T_c , results in a variation in the time of discrimination, t_d ; and, thus, an error is produced in the time measurement. The other factors in Equation (44) determine

the magnitude of the effect of variations in T_c . For a given detector and preamplifier and a given value of radiation energy, only one factor remains in the expression for t_d that will decrease the effect of variations in T_c . This factor is the discriminator level, E_d . The lower the discriminator level the lower is the error in the time measurement due to variations in T_c . The minimum discriminator level is limited by the noise level. For a given detector and preamplifier, the noise level can be reduced only by filtering. Filtering introduces the possibility of other factors being involved in determining the effect of T_c on time measurement and also the possibility of noise on the signal affecting the time measurement. The effect of noise on time measurement will be discussed in the following section.

III. EFFECT OF NOISE ON TIME MEASUREMENT

Noise has two effects on time measurement. As described in the previous section, the minimization of the effect of charge collection variations is limited by the noise level. In addition, a second effect which is a detriment to the measurement of time is caused by noise. This second effect is not currently the prime source of error in time measurement by present methods with most Ge(Li) detectors. However, by using filtering and decreasing charge collection variations, this effect may become important.

The source of error in time measurement due to noise is evident by geometrical observations. From Figure 5, this error can be mathematically described. The slope of the signal at the level of discrimination is related to e_n and σ_T in the following manner:

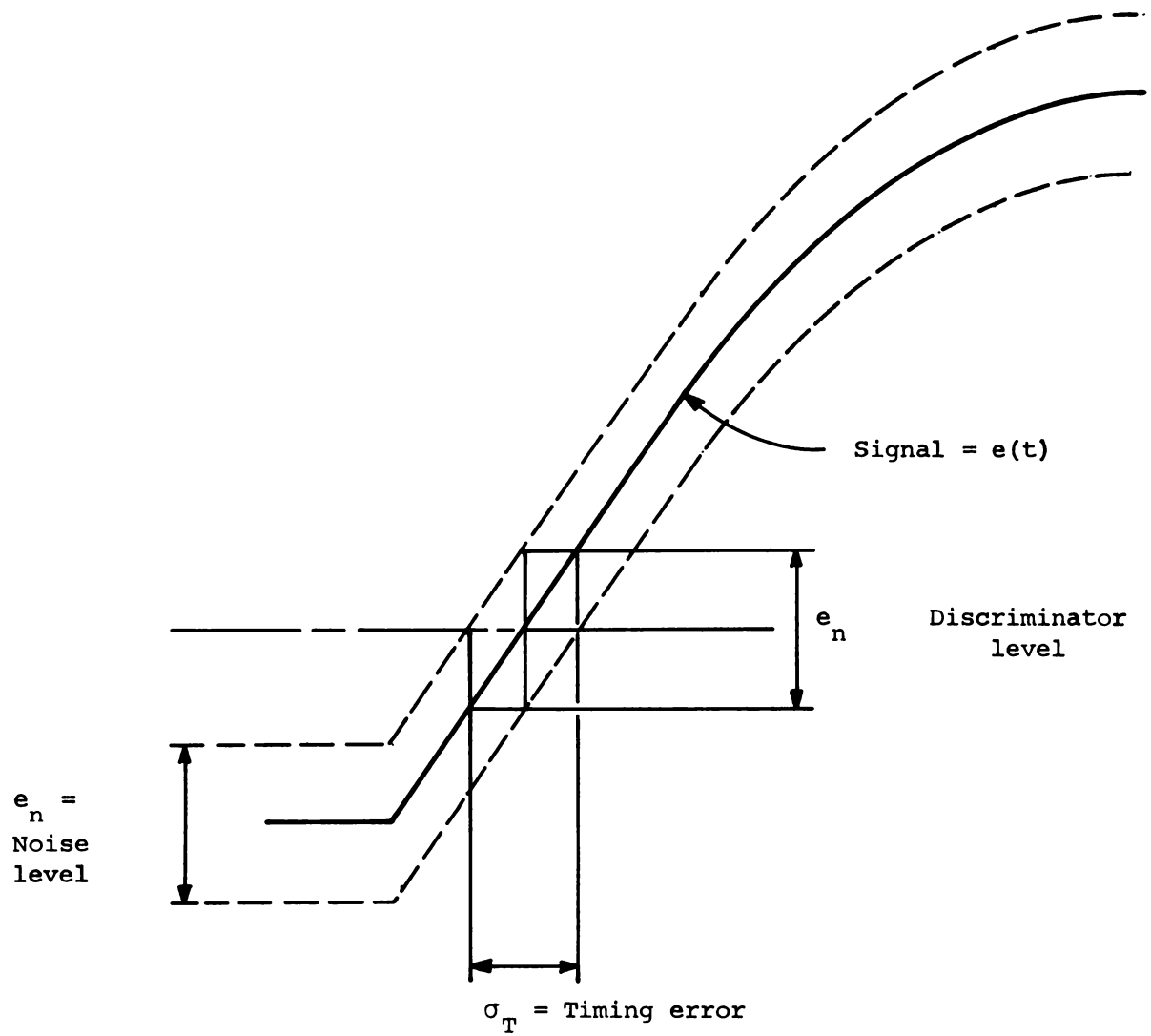


Figure 5. Signal and noise to demonstrate error in measurement of time due to noise.

$$\frac{de(t)}{dt} = \frac{e_n}{\sigma_T} . \quad (45)$$

The relationship of Equation (45) is true only if the slope does not change appreciably over the range of signal covered by the noise level. A further assumption that the noise level changes little over the interval in time of σ_T must also be made. Both of these assumptions are reasonable for the normal noise and signal from a Ge(Li) detector, charge-sensitive preamplifier system. Rearranging Equation (45),

$$\sigma_T = e_n / \left[\frac{de(t)}{dt} \right] . \quad (46)$$

Equation (46) is a representation of the timing error due to noise. This error will be referred to as noise/slope error.

CHAPTER III

EFFECT OF FILTERING

Two sources of error in the time measurement of signals from the charge-sensitive preamplifier used with a Ge(Li) detector have been described. These error sources are charge collection variations in the detector and noise/slope error. The first source of error, charge collection variations, is very dependent on the particular detector. To adequately describe the effect that filtering has on this source of error, the time spectrum for each level of the signal from the charge-sensitive preamplifier must be known. Several authors^{5,8,12,13,14} have shown that these spectra do not have a Gaussian distribution and are very irregular. Thus, an accurate description of the effect of filtering on this source of error would be very difficult to perform. In addition, since the spectra for different detectors are probably different, a general description of the effect of filtering on charge collection variations is impossible.

An analysis of the effect of filtering on the second source of error in time measurement, noise/slope error, would have general application. This source of error is presently the ultimate limitation on time measurement of signals from a charge-sensitive preamplifier. Thus, an analysis of noise/slope error could provide a standard for comparing time measurement of the signal from a charge-sensitive preamplifier used with any detector. In addition, an examination of particular filters

may lead to conclusions about the possible application of these filters to reduce errors due to charge collection variations. For example, gated filters which theoretically can have a zero noise level until the signal arrives will be examined. A zero noise level indicates that perhaps a discriminator level of near zero is possible thus reducing the effect of charge collection variations. Therefore, the purpose of this chapter is to determine the effect of filtering on the noise/slope error in time measurement. Further, the possible application of the filters described to time measurement of signals from Ge(Li) detector systems will be discussed.

I. THE OPTIMUM FILTER AND TIME MEASUREMENT

Any Signal and Noise

For an arbitrary signal and noise, an optimum filter for performing the measurement of time can be determined. Further, a minimum error in the time measurement exists with the use of the optimum filter. The purpose of this section is to determine the optimum filter and to find the optimum time measurement for any signal and noise, and to use this information to describe the optimum filter and time measurement for the signal and noise from a charge-sensitive preamplifier. For convenience the signal, noise, and filter transfer functions will be described in terms of their Fourier transform.

For a signal, with Fourier transform of $E_o(\omega)$, at the input of a filter whose Fourier transform is $G(\omega)$, the output signal is related to $E_o(\omega)$ and $G(\omega)$ by

$$E_1(\omega) = E_o(\omega)G(\omega) \quad . \quad (47)$$

The output signal as a function of time is the inverse Fourier transform of $E_1(\omega)$ or

$$e_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_o(\omega)G(\omega) e^{j\omega t} d\omega \quad . \quad (48)$$

The output noise power spectral density, $N_1(\omega)$, is related to the input noise power spectral density, $N_o(\omega)$, by

$$N_1(\omega) = |G(\omega)|^2 N_o(\omega) \quad . \quad (49)$$

By substituting Equation (49) into Equation (17), the mean square output noise voltage is

$$\overline{e_{n1}^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} N_o(\omega) |G(\omega)|^2 d\omega \quad . \quad (50)$$

To determine σ_T in Equation (46), the slope, $de_1(t)/dt$, must be known.

The Fourier transform of the slope, $F[de_1(t)/dt]$, is related to $E_1(\omega)$ by

$$F[de_1(t)/dt] = j\omega E_1(\omega) \quad . \quad (51)$$

The slope in the time domain is, therefore,

$$\frac{de_1(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_o(\omega)G(\omega)j\omega e^{j\omega t} d\omega \quad . \quad (52)$$

From Equation (46) in mean square notation because of the mean square representation for noise,

$$\overline{\sigma_T^2} = \frac{2\pi \int_{-\infty}^{\infty} N_o(\omega) |G(\omega)|^2 d\omega}{\left[\int_{-\infty}^{\infty} E_o(\omega)G(\omega)j\omega e^{j\omega t} d\omega \right]^2} \quad . \quad (53)$$

The optimum filter by definition minimizes σ_T . However, finding the maximum of $1/\sigma_T$ is more convenient and obviously gives the same result.

By definition,

$$\rho_T = \sigma_T^{-1}, \quad (54)$$

or

$$\frac{1}{\rho_T^2} = \frac{\left[\int_{-\infty}^{\infty} \frac{E_o(\omega)}{N_o(\omega)} G(\omega) j\omega e^{j\omega t} d\omega \right]^2}{2\pi \int_{-\infty}^{\infty} N_o(\omega) |G(\omega)|^2 d\omega}. \quad (55)$$

By defining

$$u(\omega) = G(\omega) N_o^{-1/2}(\omega) \quad (56)$$

and

$$v(\omega) = \frac{E_o(\omega)}{N_o^{-1/2}(\omega)} j\omega e^{j\omega t} \quad (57)$$

and by using the Schwarz Inequality,

$$\left| \int_{-\infty}^{\infty} u(\omega) v(\omega) d\omega \right|^2 \leq \int_{-\infty}^{\infty} |u(\omega)|^2 d\omega \int_{-\infty}^{\infty} |v(\omega)|^2 d\omega, \quad (58)$$

the expression

$$\begin{aligned} & \left| \int_{-\infty}^{\infty} G(\omega) E_o(\omega) j\omega e^{j\omega t} d\omega \right|^2 \\ & \leq \int_{-\infty}^{\infty} |G(\omega)|^2 N_o(\omega) d\omega \int_{-\infty}^{\infty} \frac{|E_o(\omega)|^2}{N_o(\omega)} \omega^2 d\omega \end{aligned} \quad (59)$$

is obtained. Rearranging Equation (59),

$$\frac{\left| \int_{-\infty}^{\infty} G(\omega) E_o(\omega) j\omega e^{j\omega t} d\omega \right|^2}{\int_{-\infty}^{\infty} |G(\omega)|^2 N_o(\omega) d\omega} \leq \int_{-\infty}^{\infty} \frac{|E_o(\omega)|^2}{N_o(\omega)} \omega^2 d\omega, \quad (60)$$

and substituting Equation (55) into Equation (60),

$$\overline{\rho_T^2} \leq \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|E_o(\omega)|^2}{N_o(\omega)} \omega^2 d\omega \quad . \quad (61)$$

The maximum value for $\overline{\rho_T^2}$ is, therefore,

$$\overline{\rho_T^2} \text{ (Max.)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|E_o(\omega)|^2}{N_o(\omega)} \omega^2 d\omega \quad ; \quad (62)$$

and the maximum value is obtained only when

$$u(\omega) = kv^*(\omega) \quad , \quad (63)$$

where k is some constant and $v^*(\omega)$ is the complex conjugate of $v(\omega)$. By substitution of Equations (56) and (57) into Equation (63),

$$G(\omega) N_o^{1/2}(\omega) = \frac{k E_o^*(\omega)}{[N_o^{1/2}(\omega)]^*} (-j\omega) e^{-j\omega t} \quad . \quad (64)$$

Solving for $G(\omega)$,

$$G(\omega) = \frac{k E_o^*(\omega)}{N_o(\omega)} (-j\omega) e^{-j\omega t} \quad . \quad (65)$$

Equation (65) is a representation of the optimum filter for timing.

The minimum error in the measurement of time is the inverse of Equation (62),

$$\overline{\sigma_T^2} \text{ (Opt.)} = 2\pi \left[\int_{-\infty}^{\infty} \frac{|E_o(\omega)|^2}{N_o(\omega)} \omega^2 d\omega \right]^{-1} \quad . \quad (66)$$

A similar analysis for the optimum energy filter was performed by Radeka and Karlovac.¹⁶

Equations (65) and (66) relate the optimum filter and the minimum error in time measurement for an arbitrary signal and noise, and, also, for an arbitrary measurement time. The minimum error in time measurement

is independent of the time the measurement is made; however, the optimum filter is a function of this time. Thus, if the noise and signal are known, the minimum error in time measurement is known; and if, in addition, the measurement time is chosen, the optimum filter can be described.

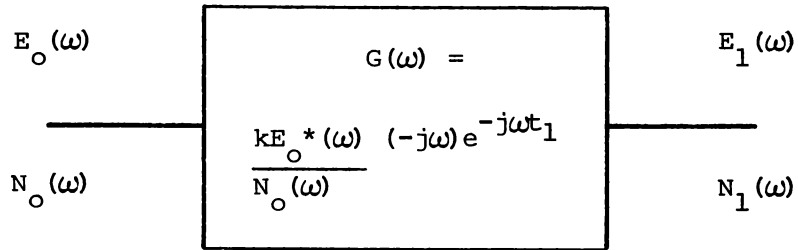
To simplify the description of the optimum filter, the filter is broken down into two parts. These parts are represented in Figure 6, where t_1 is the time the measurement is to be made. The advantage of using two filters to describe the optimum filter is that the first filter, the whitening filter, is a function of the input noise alone and the second filter, the matched filter, is a function of the signal from the whitening filter alone. The purpose of the whitening filter is to change the input noise spectrum, $N_o(\omega)$, into white noise. The filter transfer function necessary is simply

$$|G_1(\omega)| = \left[\frac{W}{N_o(\omega)} \right]^{1/2}, \quad (67)$$

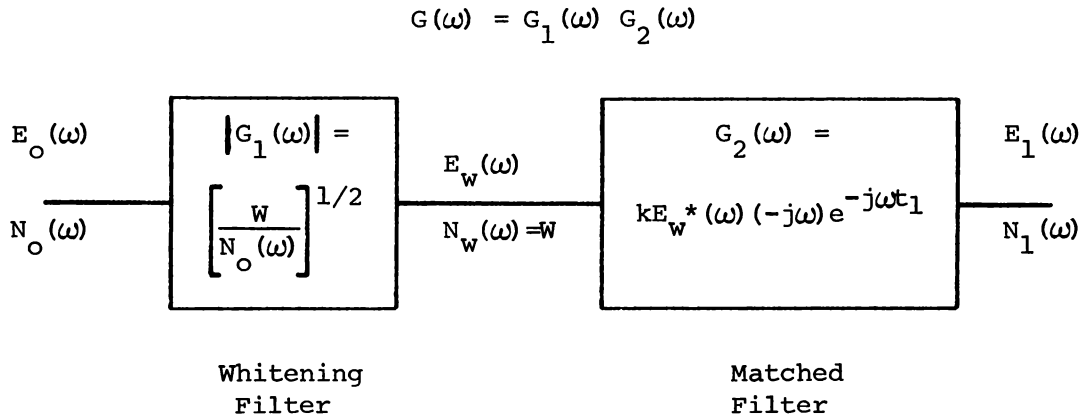
where W is some constant. The second filter then transforms the signal and white noise from the whitening filter such that the optimum time measurement at t_1 can be made. This filter transfer function is

$$G_2(\omega) = kE_w^*(\omega) (-j\omega) e^{-j\omega t_1}, \quad (68)$$

where $E_w(\omega)$ is the signal from the whitening filter. Analysis of $G_2(\omega)$ reveals that the transfer function is the conjugate of the signal slope delayed by the factor t_1 . The complex conjugate in the frequency domain implies a mirror image of the impulse response in the time domain.



a. Single transfer function



b. Noise whitening and matched filter transfer functions

Figure 6. The optimum filter for the measurement of time of a signal in the presence of noise.

Thus, the impulse response of $G_2(\omega)$ is the mirror image of the signal slope from the whitening filter delayed by the measurement time, t_1 .

The Signal and Noise from a Charge-Sensitive Preamplifier

The signal and noise from a charge-sensitive preamplifier were described in Chapter II. The noise power spectral density and the Fourier transform of the signal are, respectively,

$$N_o(\omega) = \frac{a}{T_o^2 \omega^2 + 1} \quad (69)$$

and

$$E_o(\omega) = \frac{-Q}{C_f j\omega(j\omega T_o + 1)(j\omega T_c + 1)} \quad (70)$$

Using $N_o(\omega)$ and $E_o(\omega)$ of Equations (69) and (70), an optimum filter for the measurement of time can be found. If a whitening filter with a transfer function,

$$G_1(\omega) = j\omega T_o + 1, \quad (71)$$

is applied, the noise and signal from the whitening filter are

$$N_w(\omega) = a \quad (72)$$

and

$$E_w(\omega) = \frac{-Q}{C_f j\omega(j\omega T_c + 1)} \quad (73)$$

If A is equal to $-Q/C_f$, then

$$E_w(\omega) = \frac{A}{j\omega(j\omega T_c + 1)} \quad (74)$$

With the signal from the whitening filter known, the transfer function for the matched filter portion of the optimum filter is

$$G_2(\omega) = k \left[\frac{A}{-j\omega(-j\omega T_c + 1)} \right] (-j\omega) e^{-j\omega t_1} \quad (75)$$

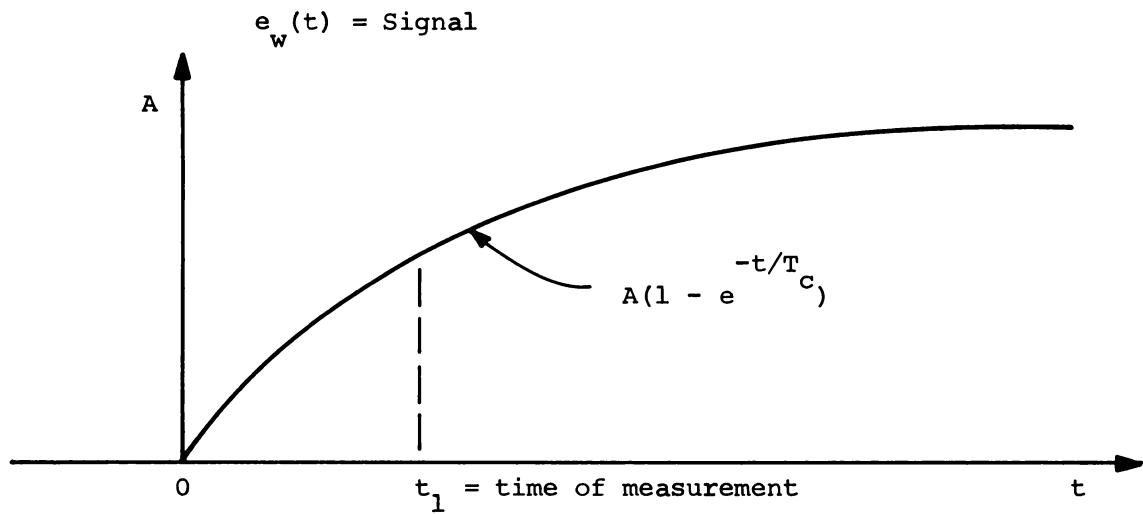
The impulse response of $G_2(\omega)$ is probably the most revealing description of the filter. As described previously, the impulse response of $G_2(\omega)$ is the mirror image of the signal slope from the whitening filter delayed by the measurement time, t_1 . In Figures 7, 8, and 9, the procedure for determining this impulse response is outlined for the signal and noise from a charge-sensitive preamplifier. A representation of the signal and slope from the whitening filter is shown in Figure 7. In Figure 8, the inverse Fourier transform of the complex conjugate of the signal slope from the whitening filter is demonstrated; and, in Figure 9, the impulse response of the optimum filter for the measurement of time at t_1 of the signal from the whitening filter is represented. For a realizable filter, the impulse response must be zero for time before zero.¹⁷ Thus, the optimum filter represented by the impulse response in Figure 9 is not physically realizable.

The optimum measurement of time is found by the substitution of Equations (69) and (70) into Equation (66). Performing these substitutions and simplifying,

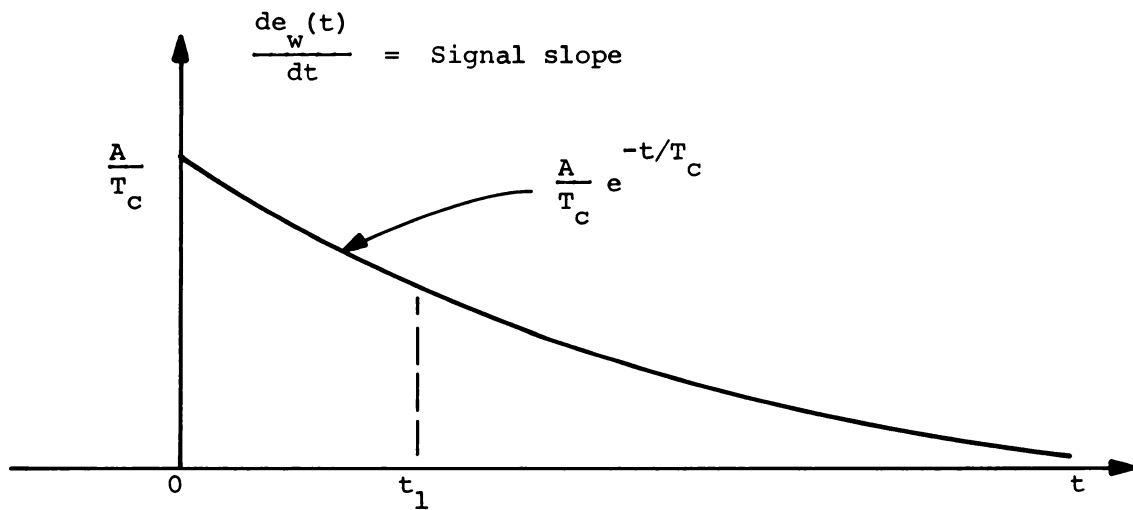
$$\overline{\sigma_T^2} \text{ (Opt.)} = \left[\frac{Q^2}{2\pi a C_f^2} \int_{-\infty}^{\infty} \left(\frac{1}{\omega^2 T_c^2 + 1} \right) d\omega \right]^{-1} \quad (76)$$

If the integral is evaluated,

$$\overline{\sigma_T^2} \text{ (Opt.)} = \frac{2a T_c C_f^2}{Q^2} \quad (77)$$



a. Signal from the noise whitening filter.



b. Signal slope from the noise whitening filter.

Figure 7. The signal and slope from the noise whitening filter used in finding the optimum filter response.

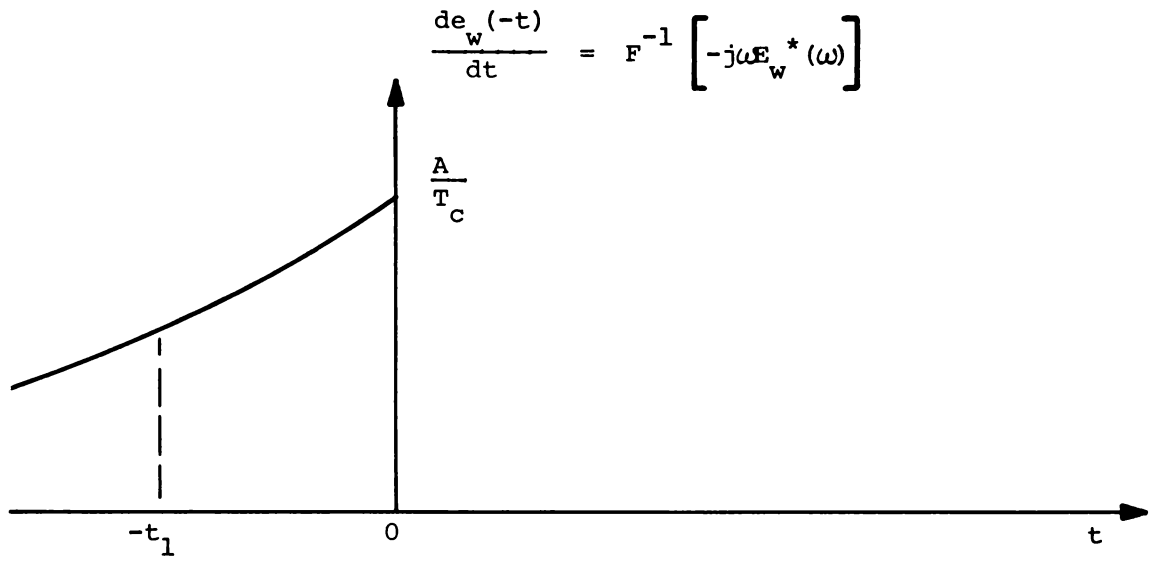


Figure 8. The mirror image of the signal slope from the noise whitening filter.

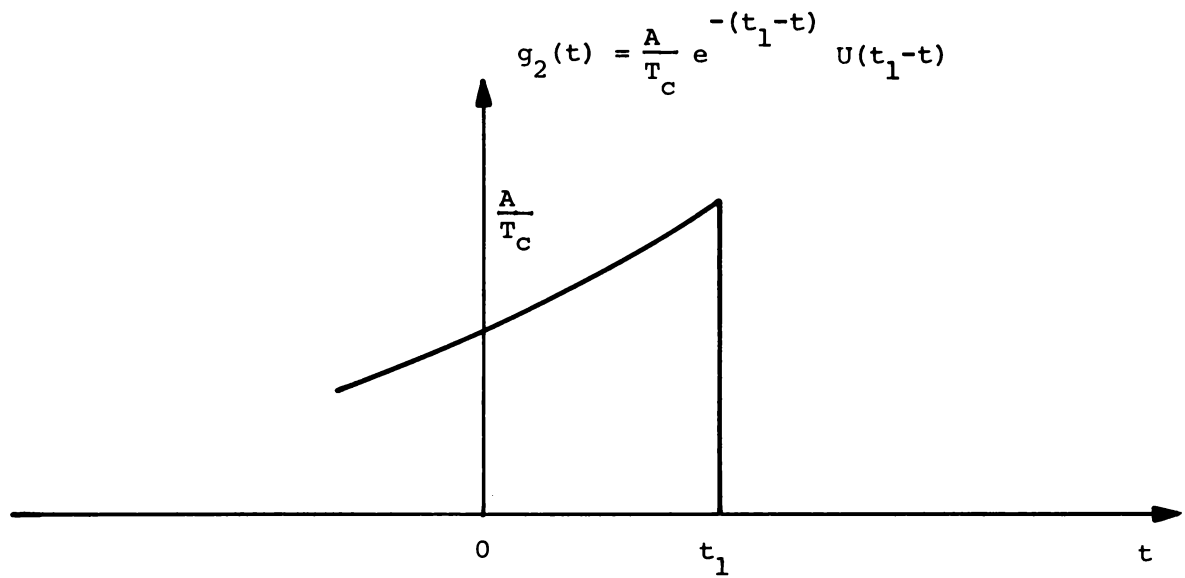


Figure 9. The impulse response of the optimum filter for the measurement of time of the signal from the noise whitening filter.

By definition A is equal to $-Q/C_f$. Thus, the root mean square value for the minimum error in the measurement of time is

$$\sigma_T(\text{Opt.}) = \frac{\sqrt{2aT_c}}{A} \quad . \quad (78)$$

Since the optimum filter is not physically realizable, the minimum error represented by Equation (78) is impossible to attain.

Although neither the optimum filter nor the minimum error in time measurement are realizable in this case, useful information is still available from a knowledge of their values. For example, the minimum error is a good standard for the comparison of realizable filters. In addition, the impulse response of the physically non-realizable optimum filter may provide insight into the design of a filter that is realizable. The remainder of this chapter will be an examination of particular filters and their comparison with the optimum.

II. TIME-INVARIANT RC FILTERS

In the previous section the optimum filter and the minimum error in the measurement of time were calculated. The optimum filter consisted of two parts. The first part, a whitening filter, had as its purpose the whitening of the noise spectrum or, in other words, making the noise power spectral density constant. The second part, a matched filter, operated on the signal from the whitening filter to obtain the best signal and noise at the output for the measurement of time. The matched filter's impulse response was found to be the mirror image of the signal slope from the whitening filter delayed by the measurement time. For the case of the signal and noise from a charge-sensitive preamplifier,

the matched filter is non-realizable. Thus, an examination of realizable filters and their comparison with the optimum filter is desirable.

A group of filters that have particular usefulness in nuclear instrumentation are the resistance-capacitance filters. These consist in the simplest forms of RC low-pass, RC high-pass, and the combination of RC low-pass, RC high-pass filters. The RC low-pass filter is sometimes referred to as an integrator, and the RC high-pass filter as a differentiator. Referring to these filters as integrators or differentiators is a misnomer except for some special cases. The simplest use of these filters is as an ungated or time-invariant, linear element, passive filter. An examination of the effect of several time-invariant, linear, passive RC filters on the measurement of time of the signal and noise from a charge-sensitive preamplifier is the object of this section.

The procedure for determining the error in time measurement will be the same for each filter. For a transfer function, $G(s)$, the filter output mean square noise voltage, the filter output signal, and the filter output signal slope for the noise and signal from a charge-sensitive preamplifier into the filter will be calculated. The mean square filter output noise voltage is

$$\overline{e_n^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{a}{T_o^2 \omega^2 + 1} \right) |G(\omega)|^2 d\omega, \quad (79)$$

where T_o and a are defined by Equations (28) and (32). The filter output signal, $e_1(t)$, is the inverse Laplace transform of

$$E_1(s) = \frac{AG(s)}{s(T_O s + 1)(T_C s + 1)} \quad , \quad (80)$$

and the filter output signal slope, $de_1(t)/dt$, is the inverse Laplace transform of

$$sE_1(s) = \frac{AG(s)}{(T_O s + 1)(T_C s + 1)} \quad . \quad (81)$$

The root mean square timing error is then

$$\sigma_T = \sqrt{e_n^2} / [de_1(t)/dt] \quad . \quad (82)$$

With the noise, signal, slope, and timing error calculated, different figures will be displayed to evaluate the particular filter's usefulness in decreasing the error in the measurement of time.

RC Low-Pass Filter

In Figure 10, the time-invariant RC low-pass filter is represented. The transfer function of this filter is

$$G(s) = \frac{1}{T_1 s + 1} \quad , \quad (83)$$

where T_1 is equal to RC. The mean square output noise voltage, output signal, and output signal slope are, respectively,

$$\overline{e_n^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{a}{(T_O^2 \omega^2 + 1)(T_1^2 \omega^2 + 1)} d\omega \quad , \quad (84)$$

$$e_1(t) = L^{-1} \left[\frac{A}{s(T_O s + 1)(T_C s + 1)(T_1 s + 1)} \right] \quad , \quad (85)$$

and

$$\frac{de_1(t)}{dt} = L^{-1} \left[\frac{A}{(T_O s + 1)(T_C s + 1)(T_1 s + 1)} \right] \quad , \quad (86)$$

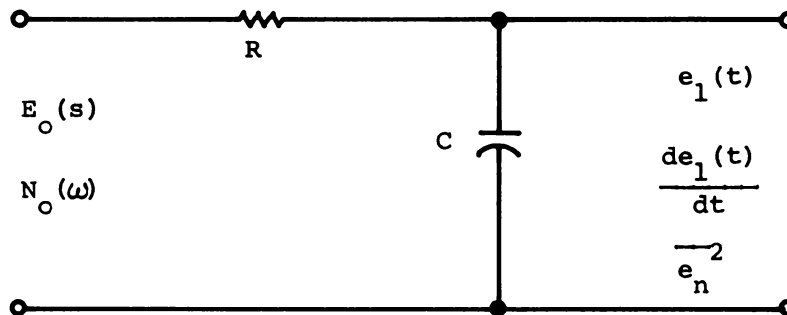


Figure 10. The time-invariant RC low-pass filter.

where L^{-1} implies the inverse Laplace transform. Evaluating the integral in Equation (84),

$$\overline{e_n^2} = \frac{a}{2(T_1 + T_o)} \quad . \quad (87)$$

The inverse Laplace transforms of Equations (85) and (86) give

$$\begin{aligned} e_1(t) = A & - \frac{AT_o^2}{(T_o - T_c)(T_o - T_1)} e^{-t/T_o} - \frac{AT_c^2}{(T_c - T_o)(T_c - T_1)} e^{-t/T_c} \\ & - \frac{AT_1^2}{(T_1 - T_o)(T_1 - T_c)} e^{-t/T_1} \end{aligned} \quad (88)$$

and

$$\begin{aligned} \frac{de_1(t)}{dt} = & \frac{AT_o}{(T_o - T_c)(T_o - T_1)} e^{-t/T_o} + \frac{AT_c}{(T_c - T_o)(T_c - T_1)} e^{-t/T_c} \\ & + \frac{AT_1}{(T_1 - T_o)(T_1 - T_c)} e^{-t/T_1} \quad . \end{aligned} \quad (89)$$

Substituting Equations (87) and (89) into Equation (82),

$$\sigma_T = \frac{\sqrt{\frac{a}{2(T_1 + T_o)}}}{\frac{AT_o e^{-t/T_o}}{(T_o - T_c)(T_o - T_1)} + \frac{AT_c e^{-t/T_c}}{(T_c - T_o)(T_c - T_1)} + \frac{AT_1 e^{-t/T_1}}{(T_1 - T_o)(T_1 - T_c)}} \quad . \quad (90)$$

Equation (90) is the general expression for the root mean square timing error using a time-invariant RC low-pass filter on the signal and noise from a charge-sensitive preamplifier.

In Equation (90), if a and A are assumed to be constants, four variables exist, T_o , T_c , T_1 , and t . With this many variables, the

evaluation of the RC low-pass filter becomes very difficult. The two variables that should be analyzed to evaluate the filter are the filter time constant, T_1 , and the time at which the measurement is made, t . If by definition T_o and T_c are made equal to T and T_1 and t are normalized with respect to T , a useful relationship for σ_T is obtained to evaluate the filter. For the case of T_o and T_c equal to T , the mean square output noise voltage, output signal, and output signal slope are

$$\overline{e_n^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{a}{(T^2 \omega^2 + 1)(T_1^2 \omega^2 + 1)} d\omega, \quad (91)$$

$$e_1(t) = L^{-1} \left[\frac{A}{s(Ts + 1)^2(T_1s + 1)} \right], \quad (92)$$

and

$$\frac{de_1(t)}{dt} = L^{-1} \left[\frac{A}{(Ts + 1)^2(T_1s + 1)} \right]. \quad (93)$$

Evaluating the integral in Equation (91),

$$\overline{e_n^2} = \frac{a}{2(T_1 + T)}. \quad (94)$$

From the inverse Laplace transforms of Equations (92) and (93) and by normalizing to T_1/T or t/T ,

$$e_1(t) = A \left[1 - \frac{(T_1/T)^2}{(1 - T_1/T)^2} e^{-t/T_1} - \frac{(t/T)}{(1 - T_1/T)} e^{t/T} - \frac{(1 - 2T_1/T)}{(1 - T_1/T)^2} e^{-t/T} \right] \quad (95)$$

and

$$\frac{de_1(t)}{dt} = \frac{A}{T} \left[\frac{(T_1/T)}{(1 - T_1/T)^2} e^{-t/T_1} - \frac{(1 - t/T)}{(1 - T_1/T)} e^{-t/T} + \frac{(1 - 2T_1/T)}{(1 - T_1/T)^2} e^{-t/T} \right] . \quad (96)$$

By defining $X = T_1/T$ and $Y = t/T$ and by substituting Equations (94) and (96) into Equation (82),

$$\sigma_T = \frac{1}{A} \sqrt{\frac{aT}{2(1+X)}} \left[\frac{1}{\frac{X}{(1-X)^2} e^{-Y/X} - \frac{(1-Y)e^{-Y}}{(1-X)} + \frac{(1-2X)}{(1-X)^2} e^{-Y}} \right] . \quad (97)$$

The timing error for the optimum filter is

$$\sigma_T(\text{Opt.}) = \frac{\sqrt{2aT}}{A} . \quad (98)$$

By normalizing Equation (97) to the optimum,

$$\sigma_L = \frac{1}{2(1+X)^{1/2}} \left[\frac{1}{\frac{X}{(1-X)^2} e^{-Y/X} - \frac{(1-Y)e^{-Y}}{(1-X)} + \frac{(1-2X)}{(1-X)^2} e^{-Y}} \right] , \quad (99)$$

where by definition

$$\sigma_L = \frac{\sigma_T}{\sigma_T(\text{Opt.})} . \quad (100)$$

Figure 11 is a graph of σ_L versus Y for seven values of X .

RC High-Pass Filter

The time-invariant RC high-pass filter is shown in Figure 12. The filter transfer function is

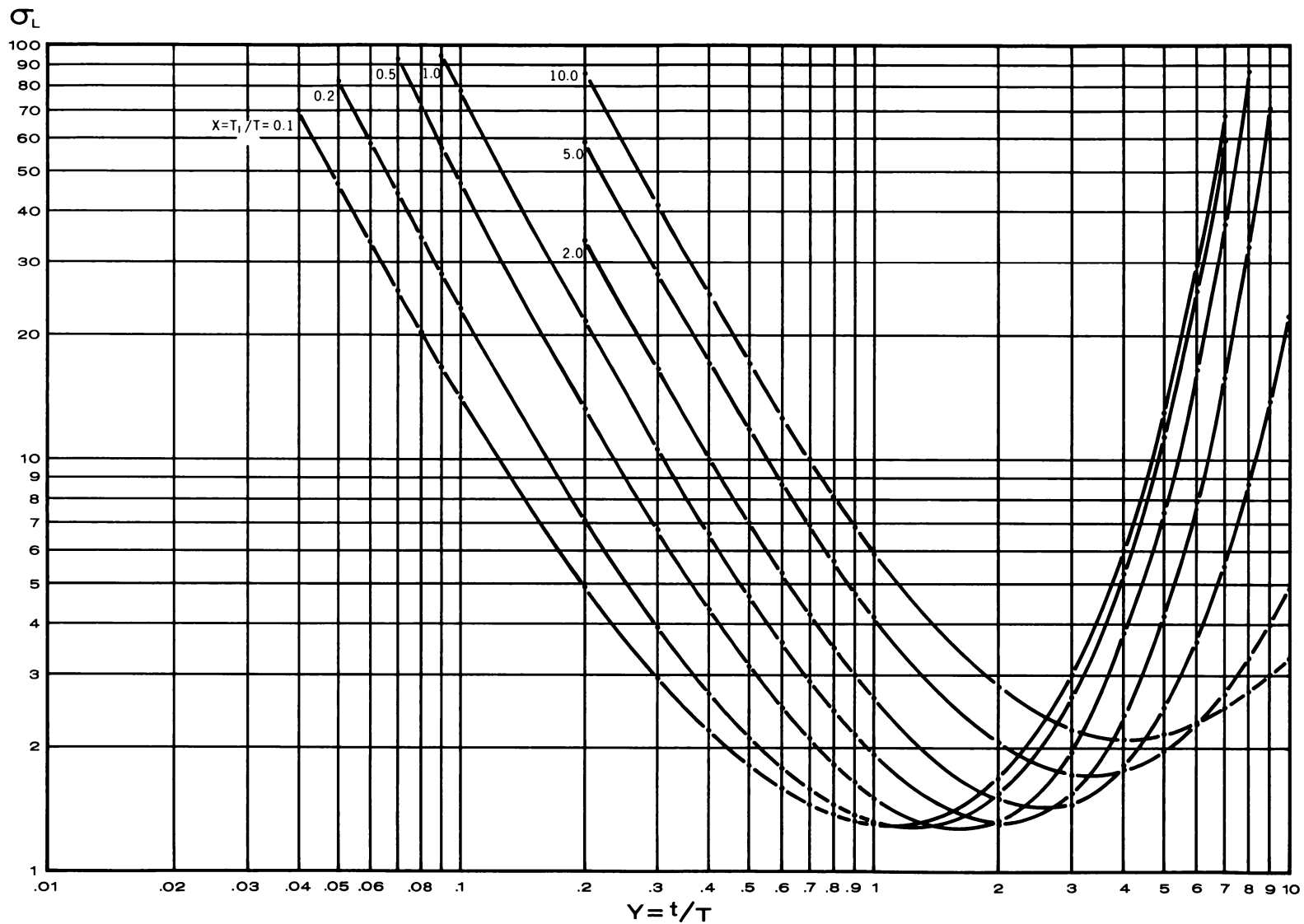


Figure 11. The effect of time-invariant RC low-pass filters on the measurement of time.

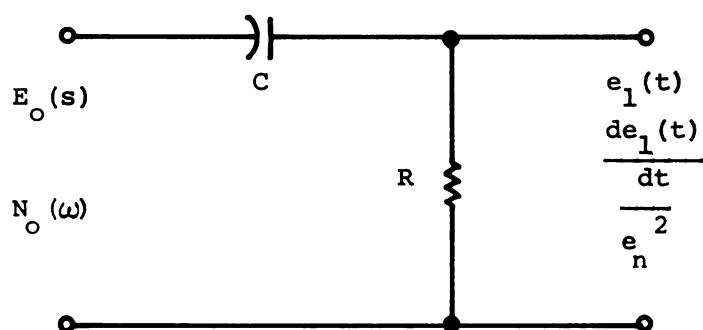


Figure 12. The time-invariant RC high-pass filter.

$$G(s) = \frac{T_1 s}{T_1 s + 1} \quad , \quad (101)$$

where T_1 is equal to RC. By proper substitution into Equations (79), (80), and (81),

$$\overline{e_n^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{aT_1^2 \omega^2}{(T_0^2 \omega^2 + 1)(T_1^2 \omega^2 + 1)} d\omega \quad , \quad (102)$$

$$e_1(t) = L^{-1} \left[\frac{AT_1}{(T_0 s + 1)(T_c s + 1)(T_1 s + 1)} \right] \quad , \quad (103)$$

and

$$\frac{de_1(t)}{dt} = L^{-1} \left[\frac{AT_1 s}{(T_0 s + 1)(T_c s + 1)(T_1 s + 1)} \right] \quad . \quad (104)$$

Evaluating Equations (102), (103), and (104),

$$\overline{e_n^2} = \frac{AT_1}{2T_0(T_1 + T_0)} \quad , \quad (105)$$

$$e_1(t) = \frac{AT_1 T_0 e^{-t/T_0}}{(T_0 - T_c)(T_0 - T_1)} + \frac{AT_1 T_c e^{-t/T_c}}{(T_c - T_0)(T_c - T_1)} + \frac{AT_1^2 e^{-t/T_1}}{(T_1 - T_0)(T_1 - T_c)} \quad , \quad (106)$$

and

$$\frac{de_1(t)}{dt} = \frac{-AT_1 e^{-t/T_0}}{(T_0 - T_c)(T_0 - T_1)} - \frac{AT_1 e^{-t/T_c}}{(T_c - T_0)(T_c - T_1)} - \frac{AT_1 e^{-t/T_1}}{(T_1 - T_0)(T_1 - T_c)} \quad . \quad (107)$$

Substituting Equations (105) and (107) into Equation (82),

$$\sigma_T = \frac{\sqrt{\frac{aT_1}{2T_0(T_1 + T_0)}}}{\frac{-AT_1 e^{-t/T_0}}{(T_0 - T_c)(T_0 - T_1)} - \frac{AT_1 e^{-t/T_c}}{(T_c - T_0)(T_c - T_1)} - \frac{AT_1 e^{-t/T_1}}{(T_1 - T_0)(T_1 - T_c)}} \quad . \quad (108)$$

Equation (108) is the general expression for the root mean square timing error using a time-invariant RC high-pass filter on the signal and noise from a charge-sensitive preamplifier.

For the case of T_c and T_o equal to T and normalizing T_1 and t to T ,

$$\overline{e_n^2} = \frac{a}{2T} \cdot \left[\frac{T_1/T}{(T_1/T + 1)} \right] , \quad (109)$$

$$e_1(t) = \frac{A}{(T/T_1 - 1)^2} \cdot \left\{ e^{-t/T_1} + [(1 - T_1/T)(t/T_1) - 1] e^{-t/T} \right\}, \quad (110)$$

and

$$\begin{aligned} \frac{de_1(t)}{dt} = \frac{A}{T} \cdot \frac{1}{(T/T_1 - 1)^2} \cdot \left\{ - (T/T_1) e^{-t/T_1} \right. \\ \left. + [T/T_1 - (1 - T_1/T)(t/T_1)] e^{-t/T} \right\} . \end{aligned} \quad (111)$$

Letting $X = T_1/T$ and $Y = t/T$ and substituting Equations (109) and (111) into Equation (82),

$$\sigma_T = \frac{1}{A} \cdot \sqrt{\frac{aTX}{2(1+X)}} \cdot \left[\frac{(1-X)^2}{X} \right] \cdot \left[\frac{1}{-e^{-Y/X} + (1-Y+XY)e^{-Y}} \right] . \quad (112)$$

Normalizing Equation (112) to the optimum,

$$\sigma_H = \frac{1}{2} \cdot \sqrt{\frac{X}{(1+X)}} \cdot \left[\frac{(1-X)^2}{X} \right] \cdot \left[\frac{1}{-e^{-Y/X} + (1-Y+XY)e^{-Y}} \right], \quad (113)$$

where

$$\sigma_H = \frac{\sigma_T}{\sigma_T(\text{Opt.})} . \quad (114)$$

Figure 13 is a graph of σ_H versus Y for six values of X .

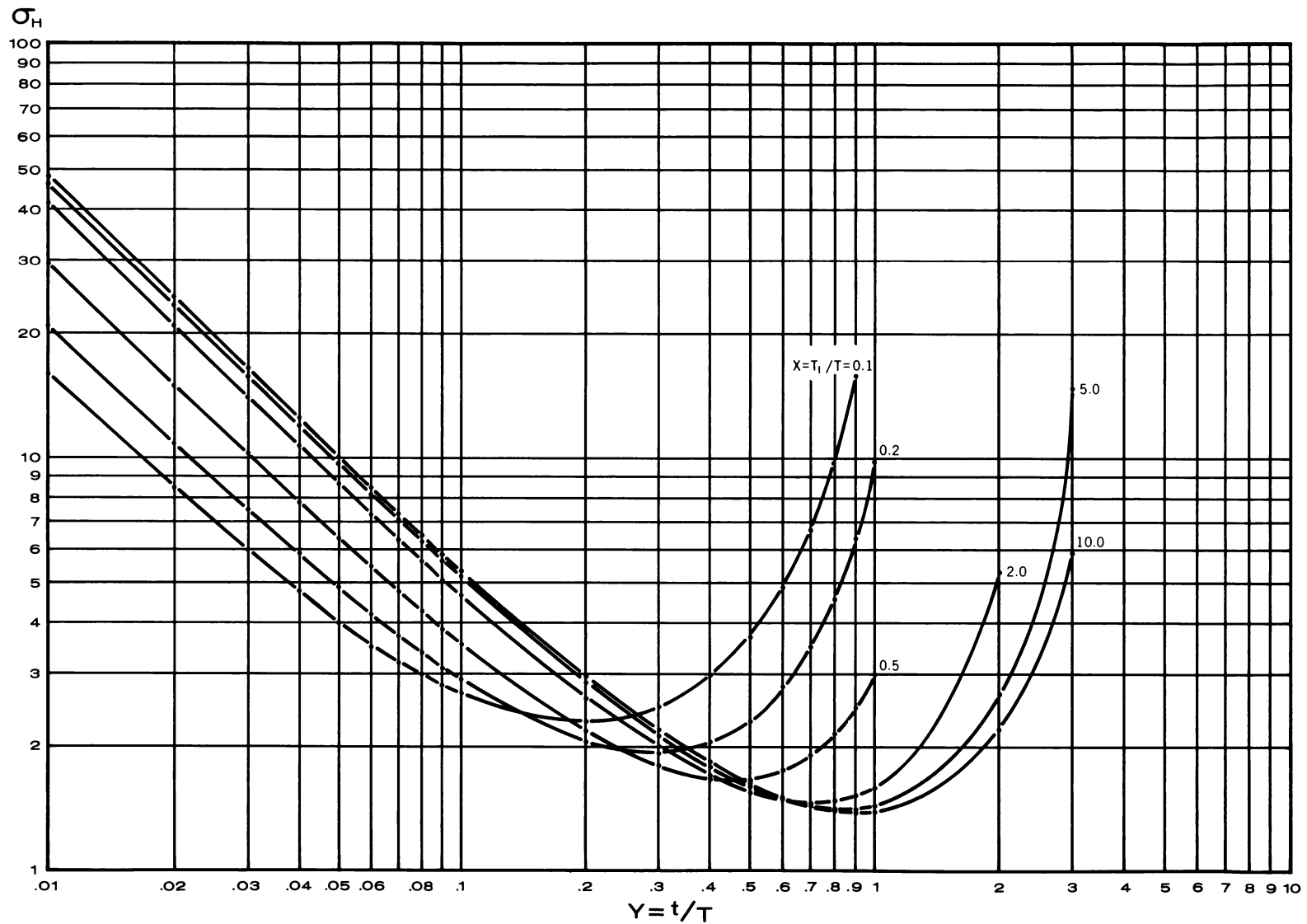


Figure 13. The effect of time-invariant RC high-pass filters on the measurement of time.

RC Low-Pass, RC High-Pass Filter

The combination of the time-invariant RC low-pass filter and RC high-pass filter is shown in Figure 14. The filter transfer function is

$$G(s) = \frac{T_2 s}{(T_1 s + 1)(T_2 s + 1)}, \quad (115)$$

where T_1 is equal to $R_1 C_1$ and T_2 is equal to $R_2 C_2$. Referring to Equations (79), (80), and (81),

$$\overline{e_n^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{a T_2^2 \omega^2}{(T_O^2 \omega^2 + 1)(T_1^2 \omega^2 + 1)(T_2^2 \omega^2 + 1)} \cdot d\omega, \quad (116)$$

$$e_1(t) = L^{-1} \left[\frac{A T_2}{(T_O s + 1)(T_C s + 1)(T_1 s + 1)(T_2 s + 1)} \right], \quad (117)$$

and

$$\frac{de_1(t)}{dt} = L^{-1} \left[\frac{A T_2 s}{(T_O s + 1)(T_C s + 1)(T_1 s + 1)(T_2 s + 1)} \right]. \quad (118)$$

Substituting Equations (116) and (118) into Equation (82),

$$\sigma_T = \frac{\left[\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{a T_2^2 \omega^2}{(T_O^2 \omega^2 + 1)(T_1^2 \omega^2 + 1)(T_2^2 \omega^2 + 1)} \cdot d\omega \right]^{1/2}}{L^{-1} \left[\frac{A T_2 s}{(T_O s + 1)(T_C s + 1)(T_1 s + 1)(T_2 s + 1)} \right]}. \quad (119)$$

The solution to Equation (119) is the general expression for the root mean square timing error using a time-invariant RC low-pass, RC high-pass filter on the signal and noise from a charge-sensitive preamplifier.

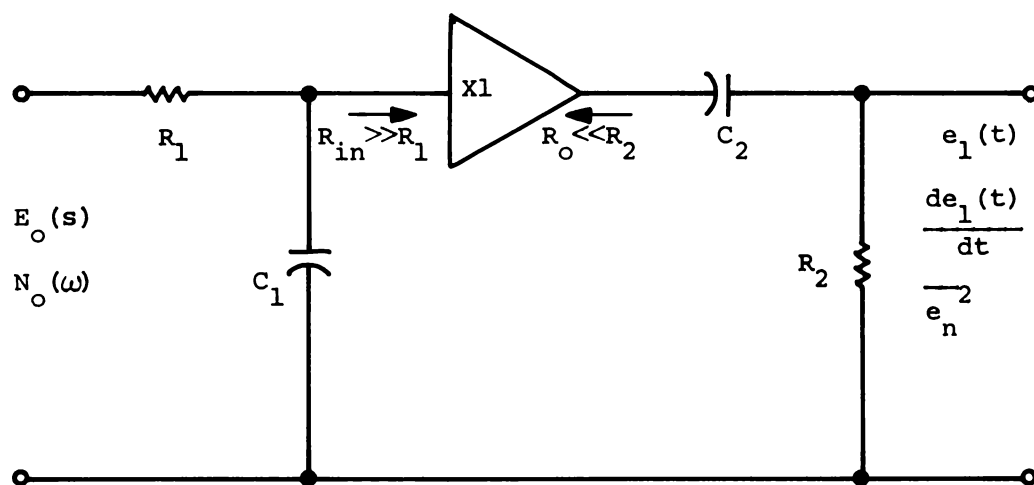


Figure 14. The time-invariant RC low-pass, RC high-pass filter.

For the case of T_c and T_o equal to T ,

$$\overline{e_n^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{aT_2^2 \omega^2}{(T^2 \omega^2 + 1) (T_1^2 \omega^2 + 1) (T_2^2 \omega^2 + 1)} d\omega, \quad (120)$$

$$e_1(t) = L^{-1} \left[\frac{AT_2}{(Ts + 1)^2 (T_1s + 1) (T_2s + 1)} \right], \quad (121)$$

and

$$\frac{de_1(t)}{dt} = L^{-1} \left[\frac{AT_2 s}{(Ts + 1)^2 (T_1s + 1) (T_2s + 1)} \right]. \quad (122)$$

Two special cases were examined to demonstrate the effect of this filter on time measurement. The first case is with equal time constants for T_1 and T_2 . Assuming T_2 is equal to T_1 and normalizing T_1 and t to T ,

$$\overline{e_n^2} = \frac{a}{T} \cdot \left[\frac{T_1/T}{4(T_1/T + 1)^2} \right], \quad (123)$$

$$e_1(t) = \frac{AT_1/T}{(1 - T_1/T)^2} \left[(t/T) \left(e^{-t/T} + e^{-t/T_1} \right) - \frac{2}{(T/T_1 - 1)} \left(e^{-t/T} - e^{-t/T_1} \right) \right], \quad (124)$$

and

$$\begin{aligned} \frac{de_1(t)}{dt} = & \frac{A}{T} \frac{T_1/T}{(1 - T_1/T)^2} \left[e^{-t/T} \left(1 - t/T + \frac{2}{T/T_1 - 1} \right) \right. \\ & \left. + e^{-t/T_1} \left(1 - t/T_1 - \frac{2}{1 - T_1/T} \right) \right]. \end{aligned} \quad (125)$$

With $X = T_1/T$ and $Y = t/T$ and substituting Equations (123) and (125) into Equation (82),

$$\sigma_T = \frac{1}{A} \cdot \frac{\sqrt{aTX}}{2(1+X)} \cdot \left[\frac{(1-X)^3}{X} \right] \cdot \left[\frac{1}{e^{-Y}(1-Y+X+XY) + e^{-Y/X}(-1-Y/X-X+Y)} \right] \quad (126)$$

Normalizing σ_T to the optimum,

$$\sigma_{LH} = \frac{\sqrt{X}}{2\sqrt{2}(1+X)} \cdot \left[\frac{(1-X)^3}{X} \right] \cdot \left[\frac{1}{e^{-Y}(1-Y+X+XY) + e^{-Y/X}(-1-Y/X-X+Y)} \right] \quad (127)$$

where

$$\sigma_{LH} = \frac{\sigma_T}{\sigma_T(\text{Opt.})} \quad (128)$$

Figure 15 is a graph of σ_{LH} , the normalized timing error for equal RC high-pass and low-pass time constants, as a function of Y for seven values of X .

The second case for the RC low-pass, RC high-pass filter is for T_2 to equal $0.1T$ and for T_1 to vary. This is approximately equivalent to differentiating the signal and noise from the charge-sensitive preamplifier and varying the time constant of an RC low-pass filter after the differentiation. For T_2 unequal to T_1 ,

$$\overline{e_n^2} = \frac{a}{T} \cdot \frac{1}{(T^2/T_2^2 - 1)} \left[\frac{1}{2(T_1/T + T_2/T)} - \frac{1}{2(T_1/T + 1)} \right] \quad (129)$$

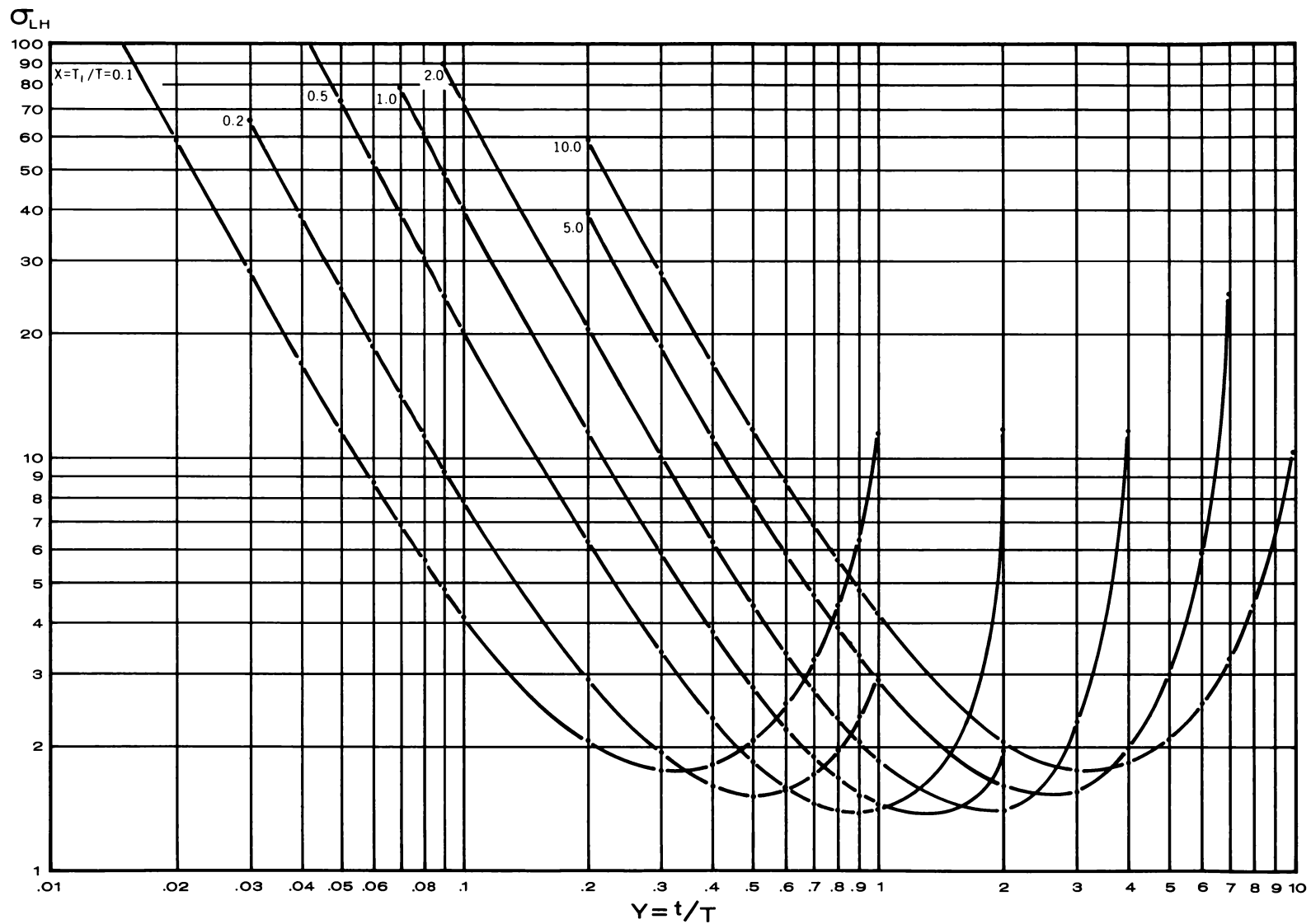


Figure 15. The effect of time-invariant RC high-pass, RC low-pass filters on the measurement of time for equal RC time constants.

$$\begin{aligned}
e_1(t) = A \left\{ \left[\frac{t/T}{(1 - T/T_2)(T_1/T - 1)} + \frac{(2T_1/T - 1 - T_1/T_2)}{(1 - T/T_2)^2 (T_1/T - 1)^2} \right] e^{-t/T} \right. \\
+ \frac{1}{(T_1/T_2 - 1)(1 - T/T_1)^2} e^{-t/T_1} \\
\left. + \frac{1}{(1 - T_1/T_2)(1 - T/T_2)^2} e^{-t/T_2} \right\}, \quad (130)
\end{aligned}$$

and

$$\begin{aligned}
\frac{de_1(t)}{dt} = \frac{A}{T} \left\{ \left[\frac{(1 - t/T)}{(1 - T/T_2)(T_1/T - 1)} - \frac{(2T_1/T - 1 - T_1/T_2)}{(1 - T/T_2)^2 (T_1/T - 1)^2} \right] e^{-t/T} \right. \\
- \frac{T_1/T}{(T_1/T_2 - 1)(T_1/T - 1)^2} e^{-t/T_1} \\
\left. - \frac{T_2/T}{(1 - T_1/T_2)(T_2/T - 1)^2} e^{-t/T_2} \right\}. \quad (131)
\end{aligned}$$

Letting $X = T_1/T$, $Y = t/T$, and $T_2 = 0.1 T$,

$$\overline{e_n^2} = \frac{a}{99T} \cdot \left[\frac{1}{2(X + 0.1)} - \frac{1}{2(X + 1)} \right], \quad (132)$$

$$\begin{aligned}
e_1(t) = A \cdot \left[\frac{Y}{-9(X - 1)} - \frac{1 + 8X}{81(X - 1)^2} \right] e^{-Y} \\
+ \frac{AX^2}{(10X - 1)(X - 1)^2} e^{-Y/X} + \frac{A}{81(1 - 10X)} e^{-10Y}, \quad (133)
\end{aligned}$$

and

$$\frac{de_1(t)}{dt} = \frac{A}{T} \cdot \left\{ \left[\frac{-(1-Y)}{9(X-1)} + \frac{1+8X}{81(X-1)^2} \right] e^{-Y} - \frac{X}{(10X-1)(X-1)^2} e^{-Y/X} - \frac{1}{8.1(1-10X)} e^{-10Y} \right\}. \quad (134)$$

Substituting Equations (132) and (134) into Equation (82) and normalizing σ_T to the optimum value, an expression for σ_{HL} is obtained. Figure 16 is a graph of σ_{HL} as a function of Y for six values of X .

III. TIME-VARIANT RC FILTERS

Recently, the use of time-variant filters has received considerable attention in the field of nuclear instrumentation. Several studies on the use of these filters in the improvement of pulse amplitude measurement have been reported.^{18, 19, 20} However, no known examination of a time-variant filter used in the measurement of time exists. For several reasons, time-variant filters appear to possibly have application in time measurement. For example, the response of a time-variant filter may more closely approximate the optimum filter response than that of the time-invariant filter. The noise/slope error in time measurement is decreased in this case. In addition, the steady-state noise level of a time-invariant filter limiting the minimum discriminator level is not a limitation using time-variant filters. The error in time measurement due to pulse shape variations can be reduced in this case. The purpose of this section is to examine the use of four time-variant filters in the measurement of time.

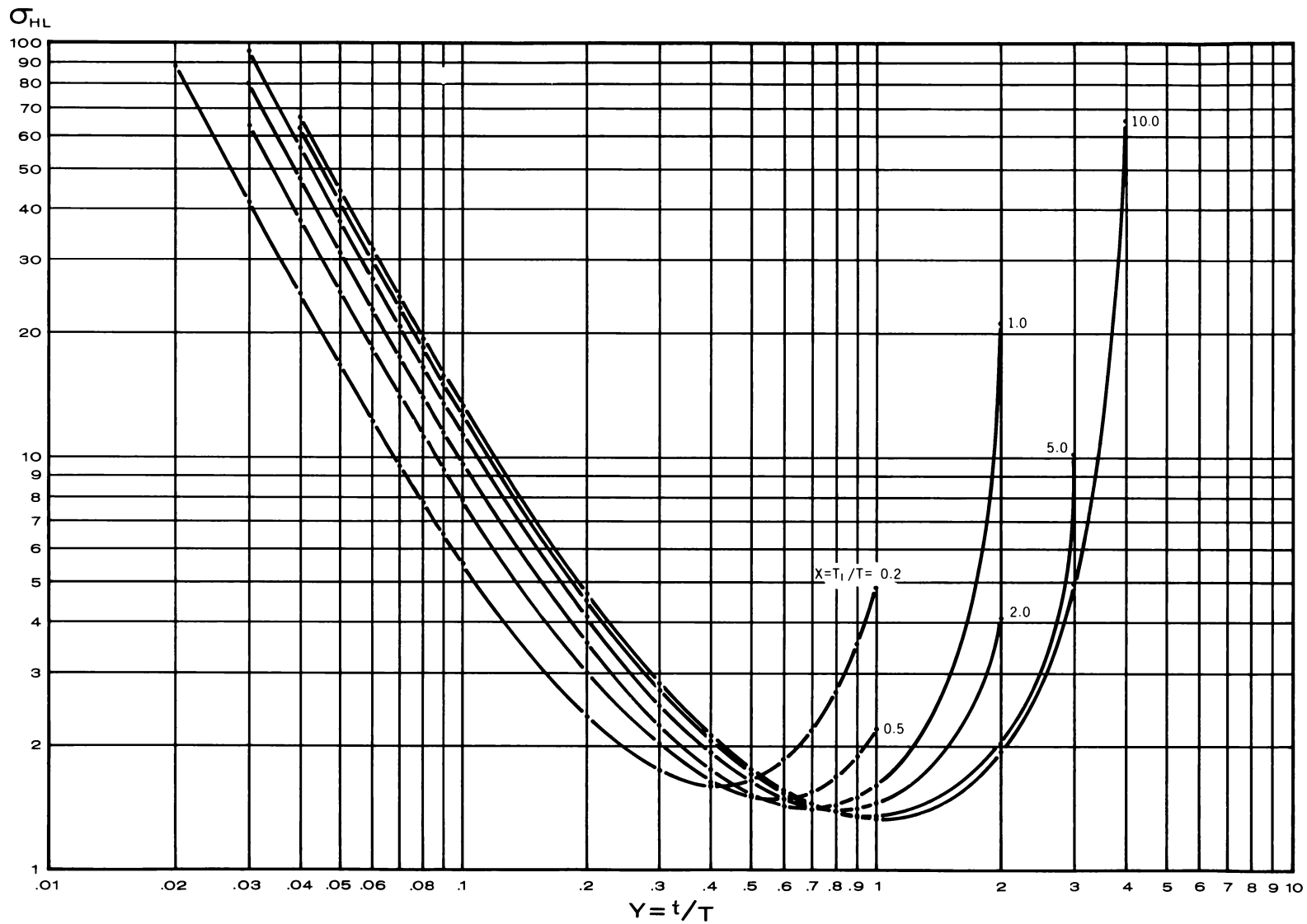


Figure 16. The effect of time-invariant RC high-pass, RC low-pass filters on the measurement of time for $T_2 = 0.1T$ and T_1 variable.

A filter whose impulse response varies as a function of the time at which the impulse is applied can be classified as a time-variant filter. The time variance may be due to a variation in the element values of the filter as a function of time, a variation in the gain of the filter as a function of time, or a combination of the two variations. A special family of time-variant filters exists in which the filter is composed of a linear gate or switch followed by a time-invariant linear filter. Nowlin and Blalock¹⁸ have examined this family of time-variant filters for the general case of the gate opening and closing at arbitrary times with respect to the time of the signal and the measurement time. A special case of this family of time-variant filters is that for which the gate opens on the arrival of the signal and closes after the measurement time. The transfer function of this filter is

$$G_g(s) = \begin{cases} G_2(s) & t_o \leq t \leq t_m + \\ 0 & t_m + < t < t_o \end{cases} \quad (135)$$

where $G_2(s)$ is the transfer function of the time-invariant filter following the gate, t_o is the time of arrival of the signal, and $t_m +$ is some time after the measurement is made.

The general representation of those filters to be examined in this section is shown in Figure 17. Since the gate is open to the signal for $t_o \leq t \leq t_m +$, the effect of filtering on the signal is time-invariant. Thus the signal, $e_1(t)$, and signal slope, $de_1(t)/dt$, at the output of the filter are the inverse Laplace transforms of

$$E_1(s) = E_o(s) G_1(s) G_2(s) \quad (136)$$

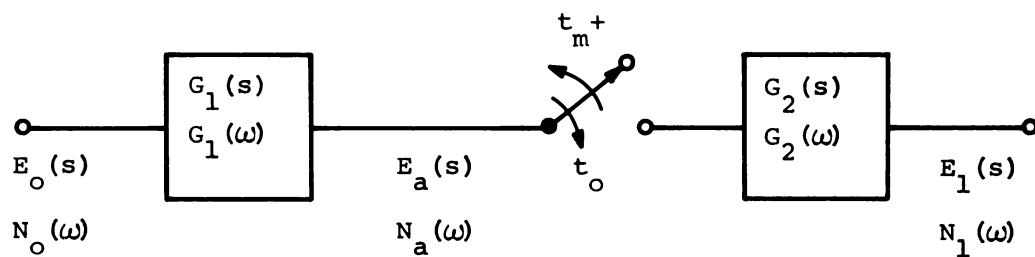


Figure 17. The time-variant filter to be examined in decreasing error in time measurement.

and

$$sE_1(s) = sE_0(s) G_1(s) G_2(s) \quad . \quad (137)$$

The effect of filtering represented by Figure 17 on noise, however, is time-variant. Lampard²¹ and Brown and Nilsson¹⁵ give equations that describe the output noise level as a function of time for the time-variant filter represented by Equation (135). The expression for the mean square value of noise at the output of the filter is

$$\tilde{e}_n^2(t) = \int_0^t \int_0^t g_2(u) g_2(v) \phi_a(u-v) du dv \quad , \quad (138)$$

where $\tilde{e}_n^2(t)$ is mean square transient noise, $g_2(t)$ is the impulse response of the time-invariant filter, $\phi_a(\tau)$ is the autocorrelation function of the gate input noise, and u and v are dummy variables. The autocorrelation function's relationship to noise power spectral density, $N_a(\omega)$, is

$$\phi_a(\tau) = F^{-1}[N_a(\omega)] \quad , \quad (139)$$

where F^{-1} is the inverse Fourier transform. $N_a(\omega)$ is related to $N_o(\omega)$ by

$$N_a(\omega) = N_o(\omega) |G_1(\omega)|^2 \quad . \quad (140)$$

Thus, for an input noise spectrum, $N_o(\omega)$, and values of $G_1(s)$ and $G_2(s)$, the output mean square transient noise, $\tilde{e}_n^2(t)$, can be found using Equations (138), (139), and (140).

The input signal and noise to the filter of Figure 17 are again the signal and noise from the charge-sensitive preamplifier, or

$$E_O(s) = \frac{A}{s(T_O s + 1)(T_C s + 1)} \quad , \quad (141)$$

and

$$N_O(\omega) = \frac{a}{(T_O^2 \omega^2 + 1)} \quad . \quad (69)$$

The special case of T_O and T_C equal to T will be examined. Therefore,

$$E_O(s) = \frac{A}{s(Ts + 1)^2} \quad , \quad (142)$$

and

$$N_O(\omega) = \frac{a}{(T^2 \omega^2 + 1)} \quad . \quad (143)$$

With the filter transfer functions known,

$$e_1(t) = L^{-1} \left[\frac{AG_1(s)G_2(s)}{s(Ts + 1)^2} \right] \quad , \quad (144)$$

and

$$\frac{de_1(t)}{dt} = L^{-1} \left[\frac{AG_1(s)G_2(s)}{(Ts + 1)^2} \right] \quad . \quad (145)$$

The output transient noise is then calculated from Equations (138), (139), (140), and (143). Knowing the noise and slope, the noise/slope error in the measurement of time using the time-variant filter of Figure 17, page 58, is

$$\tilde{\sigma}_T = \sqrt{\tilde{e}_n^2(t)} / [de_1(t)/dt] \quad . \quad (146)$$

The timing error of Equation (146) will be calculated for four RC time-variant filters, and the results will be displayed to evaluate the usefulness of the filters in the measurement of time.

RC Low-Pass Filter After the Gate

In Figure 18 the time-variant RC low-pass filter to be analyzed in this section is shown. The transfer functions of the time-invariant filter before the gate and the time-invariant filter after the gate are

$$G_1(s) = 1 \quad (147)$$

and

$$G_2(s) = \frac{1}{T_1 s + 1} \quad , \quad (148)$$

where T_1 is equal to RC. Substituting Equations (147) and (148) into Equation (144), the same result as for the ungated RC low-pass filter is obtained, or

$$e_1(t) = L^{-1} \left[\frac{A}{s(Ts + 1)^2(T_1 s + 1)} \right] \quad , \quad (92)$$

and

$$\frac{de_1(t)}{dt} = L^{-1} \left[\frac{A}{(Ts + 1)^2(T_1 s + 1)} \right] \quad . \quad (93)$$

From the inverse Laplace transform of Equations (92) and (93) and normalizing to T_1/T or t/T ,

$$e_1(t) = A \left[1 - \frac{(T_1/T)^2}{(1 - T_1/T)^2} e^{-t/T_1} - \frac{t/T}{(1 - T_1/T)} e^{-t/T} - \frac{(1 - 2T_1/T)}{(1 - T_1/T)^2} e^{-t/T} \right] \quad , \quad (95)$$

and

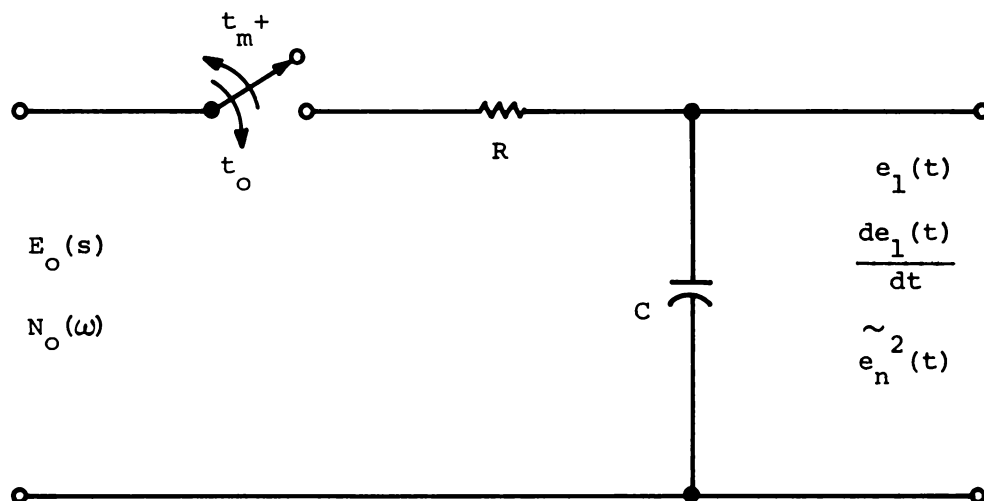


Figure 18. A time-variant RC low-pass filter.

$$\begin{aligned} \frac{de_1(t)}{dt} = \frac{A}{T} & \left[\frac{(T_1/T)}{(1 - T_1/T)^2} e^{-t/T_1} - \frac{(1 - t/T)}{(1 - T_1/T)} e^{-t/T} \right. \\ & \left. + \frac{(1 - 2T_1/T)}{(1 - T_1/T)^2} e^{-t/T} \right] . \end{aligned} \quad (96)$$

For $G_2(s) = 1$, $N_a(\omega)$ is equal to $N_o(\omega)$. Therefore,

$$\phi_a(\tau) = F^{-1} \left[\frac{a}{T^2 \omega^2 + 1} \right] , \quad (149)$$

or

$$\phi_a(\tau) = \frac{a}{2T} \cdot e^{-|\tau|/T} . \quad (150)$$

Substituting Equation (150) into Equation (138) and simplifying,

$$\tilde{e}_n^2(t) = \frac{a}{T} \int_0^t \int_0^v g_2(u) g_2(v) e^{-(v-u)/T} du dv . \quad (151)$$

Equation (151) is applicable for the case of no filter before the gate and any filter after the gate for noise from a charge-sensitive pre-amplifier.

The impulse response of the filter whose transfer function is given by Equation (148) is

$$g_2(t) = \frac{e^{-t/T_1}}{T_1} . \quad (152)$$

Substituting Equation (152) into Equation (151),

$$\tilde{e}_n^2(t) = \frac{a}{T T_1^2} \int_0^t \int_0^v e^{-u/T_1} e^{-v/T_1} e^{-v/T} e^{u/T} du dv . \quad (153)$$

Performing the integrations and normalizing to T_1/T or t/T ,

$$\begin{aligned} \tilde{e}_n^2(t) = \frac{a}{T} \frac{1}{(T_1^2/T^2 - 1)} \left[\frac{(T_1/T + 1)}{2} \left(1 - e^{-2t/T_1} \right) \right. \\ \left. + e^{-t/T} e^{-t/T_1} - 1 \right] . \end{aligned} \quad (154)$$

By defining $X = T_1/T$ and $Y = t/T$, substituting Equations (96) and (154) into Equation (146), and normalizing Equation (146) to the optimum given in Equation (78),

$$\tilde{\sigma}_L = \frac{(1-X)^2 \left[(X+1) (1 - e^{-2Y/X}) + 2e^{-Y} e^{-Y/X} - 2 \right]^{1/2}}{2(X^2 - 1)^{1/2} \left[X e^{-Y/X} - (-X + Y - XY) e^{-Y} \right]} . \quad (155)$$

Figure 19 is a graph of $\tilde{\sigma}_L$ versus Y for seven values of X .

RC High-Pass Filter After the Gate

The time-variant RC high-pass filter to be analyzed is shown in Figure 20. The transfer functions of the filter before the gate and the filter after the gate are

$$G_1(s) = 1 \quad (156)$$

and

$$G_2(s) = \frac{T_1 s}{T_1 s + 1} , \quad (157)$$

where T_1 is equal to RC . The signal and signal slope are the same as for the ungated RC high-pass filter, or

$$e_1(t) = \frac{A}{(T/T_1 - 1)^2} \cdot \left\{ e^{-t/T_1} + \left[(1 - T_1/T) (t/T_1) - 1 \right] e^{-t/T} \right\} \quad (110)$$

and

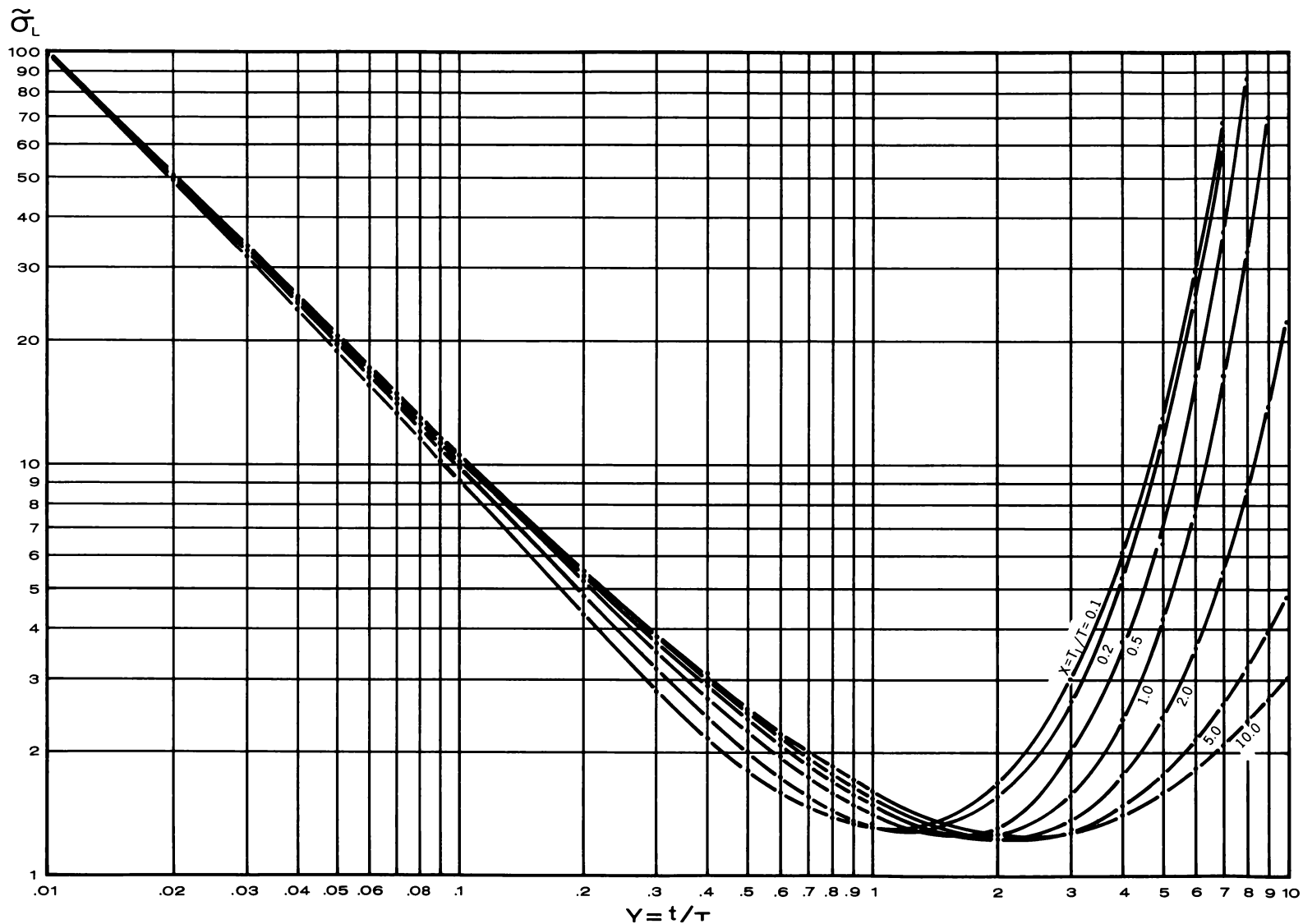


Figure 19. The effect of time-variant RC low-pass filters on the measurement of time.

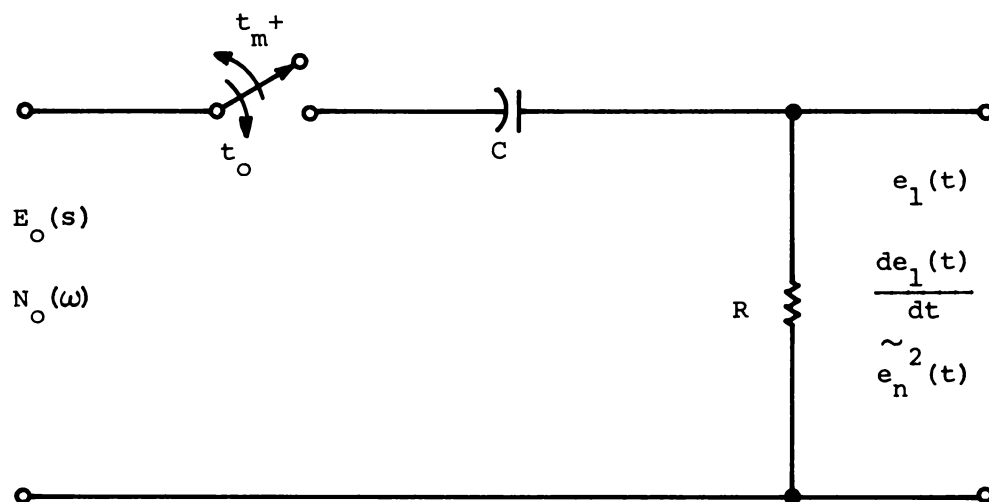


Figure 20. A time-variant RC high-pass filter.

$$\frac{de_1(t)}{dt} = \frac{A}{T} \left\{ \frac{1}{(T/T_1 - 1)^2} - (T/T_1) e^{-t/T_1} + \left[T/T_1 - (1 - T_1/T)(t/T_1) \right] e^{-t/T} \right\} . \quad (111)$$

Since $G_2(s) = 1$, Equation (151) is applicable. The impulse response of the RC high-pass filter is

$$g_2(t) = \delta(t) - \frac{e^{-t/T_1}}{T_1} , \quad (158)$$

where $\delta(t)$ is an impulse function. Substituting Equation (158) into Equation (151) and attempting to perform the integration, difficulties arise as to how to treat the impulse function. These difficulties can be overcome by using some limiting process. The method used in this analysis was to assume that

$$G_2(s) = \frac{T_1 s}{(T_1 s + 1)(T_x s + 1)} . \quad (159)$$

The procedure for determining $\tilde{e}_n^2(t)$ for an arbitrary T_x was performed, and the $\tilde{e}_n^2(t)$ for T_x equal to zero was found. T_x equal to zero in Equation (159) corresponds to the $G_2(s)$ of Equation (157). Using this procedure,

$$\begin{aligned} \tilde{e}_n^2(t) = \frac{a}{T} & \left[\frac{1}{2(1 - T_1/T)} \left(e^{-2t/T_1} - 1 \right) - \frac{1}{(1 - T_1^2/T^2)} \left(e^{-t/T_1} e^{-t/T} \right. \right. \\ & \left. \left. - 1 \right) + \frac{1}{(T_1/T + 1)} \left(e^{-t/T_1} e^{-t/T} - 1 \right) + 1/2 \right] . \quad (160) \end{aligned}$$

By defining $X = T_1/T$ and $Y = t/T$, substituting Equations (111) and (160) into Equation (146), and normalizing Equation (146) to the optimum given in Equation (78),

$$\tilde{\sigma}_H = \frac{(1-X)^2 \left[(1+X)(e^{-2Y/X} - 1) - 2X(e^{-Y/X} e^{-Y} - 1) + (1-X^2) \right]^{1/2}}{2X(1-X^2)^{1/2} \left[-e^{-Y/X} + (1-Y+XY)e^{-Y} \right]} \quad (161)$$

Figure 21 is a graph of $\tilde{\sigma}_H$ versus Y for six values of X .

RC Low-Pass, RC High-Pass Filter After the Gate

The time-variant RC low-pass, RC high-pass filter to be analyzed in this section is shown in Figure 22. The transfer functions of the filter before the gate and the filter after the gate are

$$G_1(s) = 1 \quad (162)$$

and

$$G_2(s) = \frac{T_2 s}{(T_1 s + 1)(T_2 s + 1)} \quad , \quad (163)$$

where T_1 is equal to $R_1 C_1$ and T_2 is equal to $R_2 C_2$. For T_2 equal to T_1 ,

$$G_2(s) = \frac{T_1 s}{(T_1 s + 1)^2} \quad . \quad (164)$$

The signal and signal slope are the same as for the ungated filter, or

$$e_1(t) = \frac{AT_1/T}{(1 - T_1/T)^2} \left[(t/T) \left(e^{-t/T} + e^{-t/T_1} \right) - \frac{2}{(T/T_1 - 1)} \left(e^{-t/T} - e^{-t/T_1} \right) \right] \quad , \quad (124)$$

and

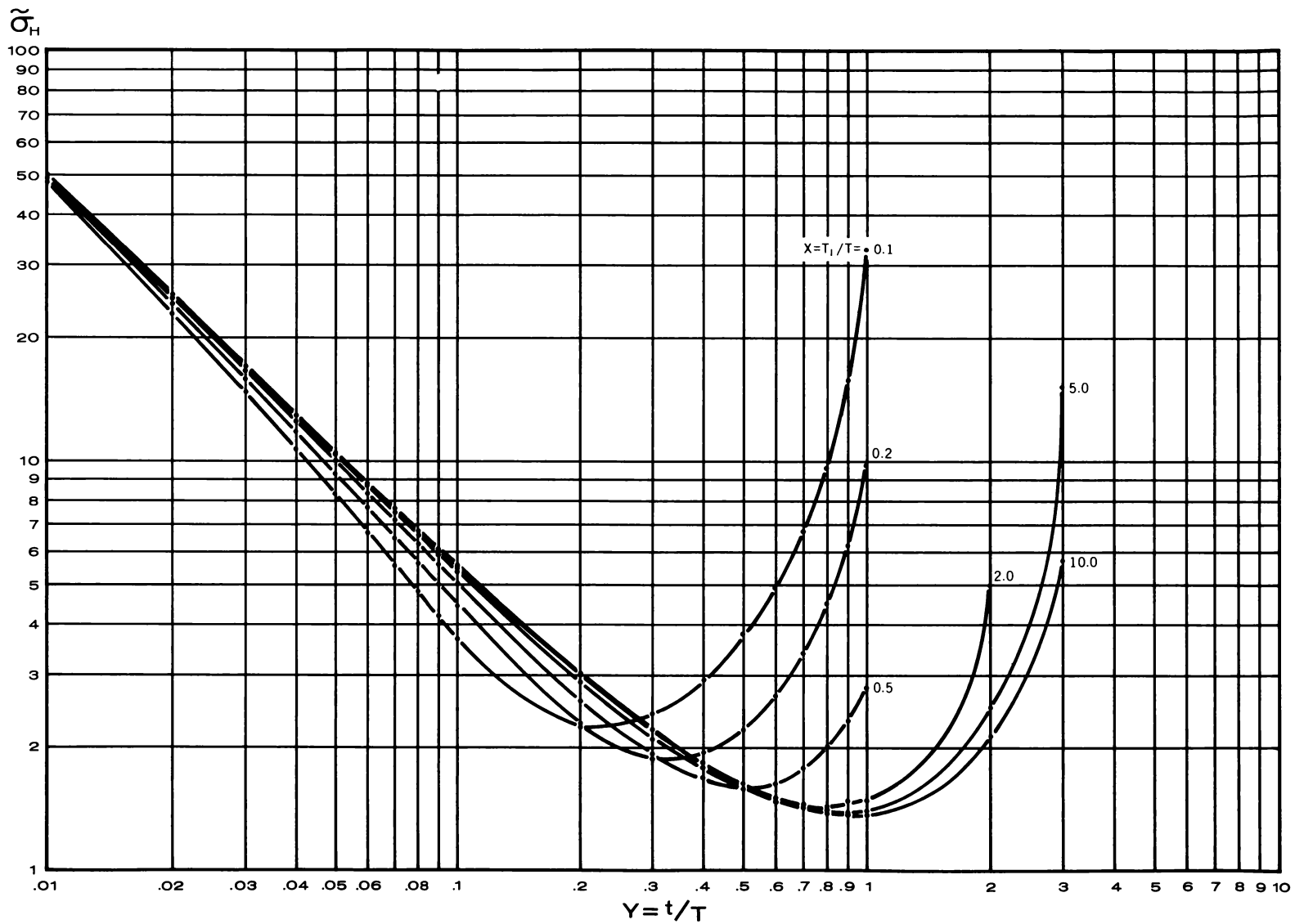


Figure 21. The effect of time-variant RC high-pass filters on the measurement of time.

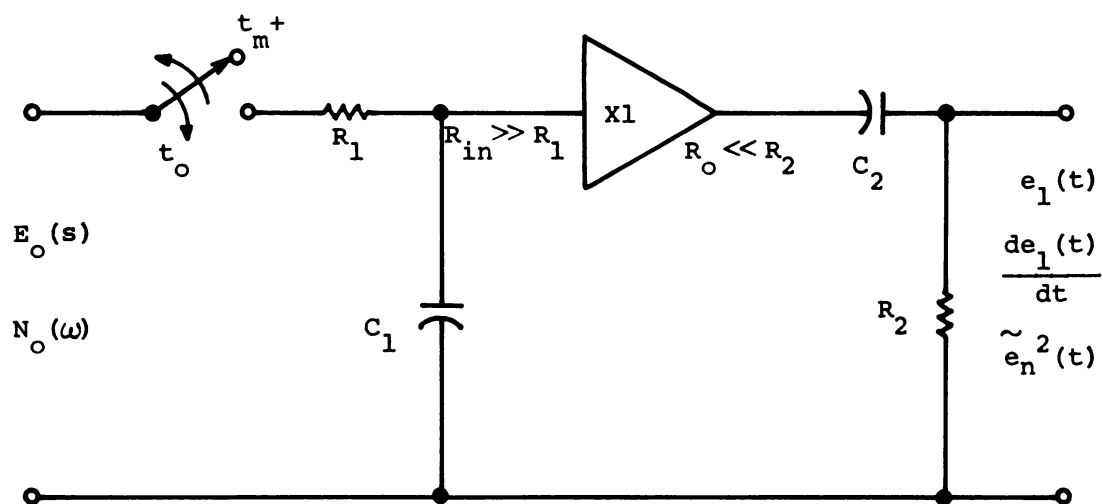


Figure 22. A time-variant RC low-pass, RC high-pass filter.

$$\begin{aligned} \frac{de_1(t)}{dt} = & \frac{A}{T} \frac{T_1/T}{(1 - T_1/T)^2} \left[e^{-t/T} \left(1 - t/T + \frac{2}{T/T_1 - 1} \right) \right. \\ & \left. + e^{-t/T_1} \left(1 - t/T_1 - \frac{2}{1 - T_1/T} \right) \right] . \end{aligned} \quad (125)$$

Equation (151) is again applicable since $G_1(s) = 1$. The impulse response of $G_2(s)$ in Equation (164) is

$$g_2(t) = \frac{e^{-t/T_1}}{T_1} (1 - t/T_1) . \quad (165)$$

Substituting Equation (165) into Equation (151) and performing the integration,

$$\begin{aligned} \tilde{e}_n^2(t) = & \frac{a}{T} \left\{ \frac{T_1/T}{4(T_1/T + 1)^2} + e^{-2t/T_1} \left[\frac{-T_1/T}{4(T_1/T - 1)^2} \right. \right. \\ & \left. + \frac{t/T}{2(T_1/T - 1)^2} - \frac{t^2/T_1^2}{2(T_1/T - 1)} \right] \\ & + e^{-t/T} e^{-t/T_1} \left[\frac{1}{(1 + T/T_1)^2 (T_1/T - 1)^2} \right. \\ & \left. \left. - \frac{t/T}{(T_1/T - 1)^2 (T_1/T + 1)} \right] \right\} . \end{aligned} \quad (166)$$

Defining $X = T_1/T$ and $Y = t/T$, substituting Equations (125) and (166) into Equation (146), and normalizing Equation (146) to the optimum given in Equation (78),

$$\begin{aligned}
\sigma_{\text{LH}}^{\sim} = (1-x)^3 \left\{ x(x-1)^2 + (x+1)^2 e^{-2Y/X} \left[-x + 2Y \right. \right. \\
\left. \left. - (2Y^2/X^2)(x-1) \right] + 4e^{-Y} e^{-Y/X} \left[x^2 - Y(x+1) \right] \right\}^{1/2} / \\
2 \sqrt{2} x(x+1)^2 \left[e^{-Y}(1-Y+X+XY) + e^{-Y/X} (-1 - Y/X \right. \\
\left. - x + Y) \right] .
\end{aligned} \tag{167}$$

Figure 23 is a graph of $\sigma_{\text{LH}}^{\sim}$ versus Y for six values of X .

RC High-Pass Before and RC Low-Pass After the Gate

In Figure 24 the time-variant RC high-pass, RC low-pass filter to be analyzed in this section is shown. The transfer functions of the filter before the gate and the filter after the gate are

$$G_1(s) = \frac{T_2 s}{(T_2 s + 1)} \tag{168}$$

and

$$G_2(s) = \frac{1}{(T_1 s + 1)} , \tag{169}$$

where T_1 is equal to $R_1 C_1$ and T_2 is equal to $R_2 C_2$. Again, the signal and slope are the same as for the ungated filter, or

$$\begin{aligned}
e_1(t) = A \left\{ \left[\frac{t/T}{(1 - T/T_2)(T_1/T - 1)} + \frac{(2T_1/T - 1 - T_1/T_2)}{(1 - T/T_2)^2 (T_1/T - 1)^2} \right] e^{-t/T} \right. \\
\left. + \frac{1}{(T_1/T_2 - 1)(1 - T/T_1)^2} e^{-t/T_1} + \frac{1}{(1 - T_1/T_2)(1 - T/T_2)^2} e^{-t/T_2} \right\} ,
\end{aligned} \tag{130}$$

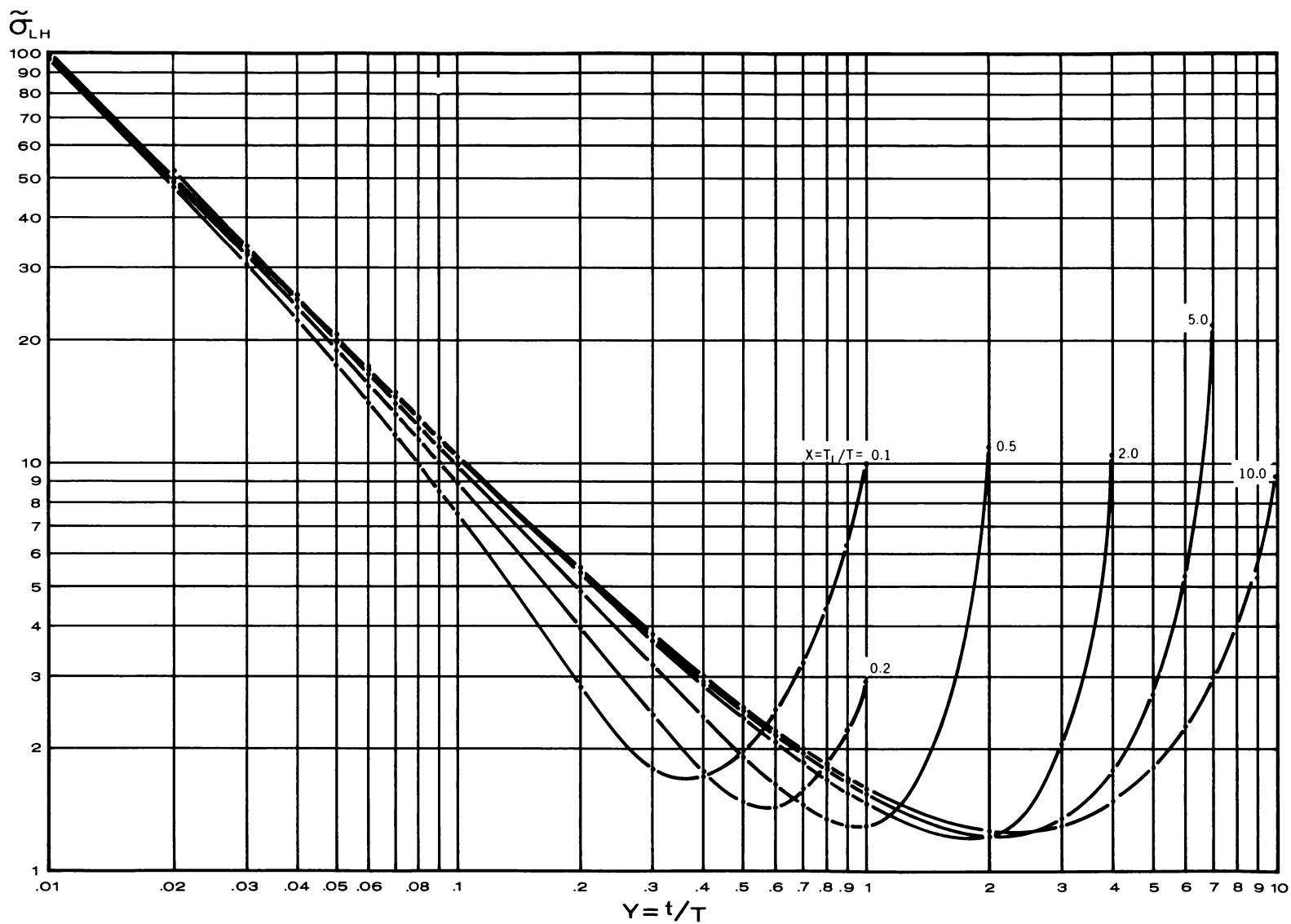


Figure 23. The effect of a time-variant RC low-pass, RC high-pass filter on the measurement of time.

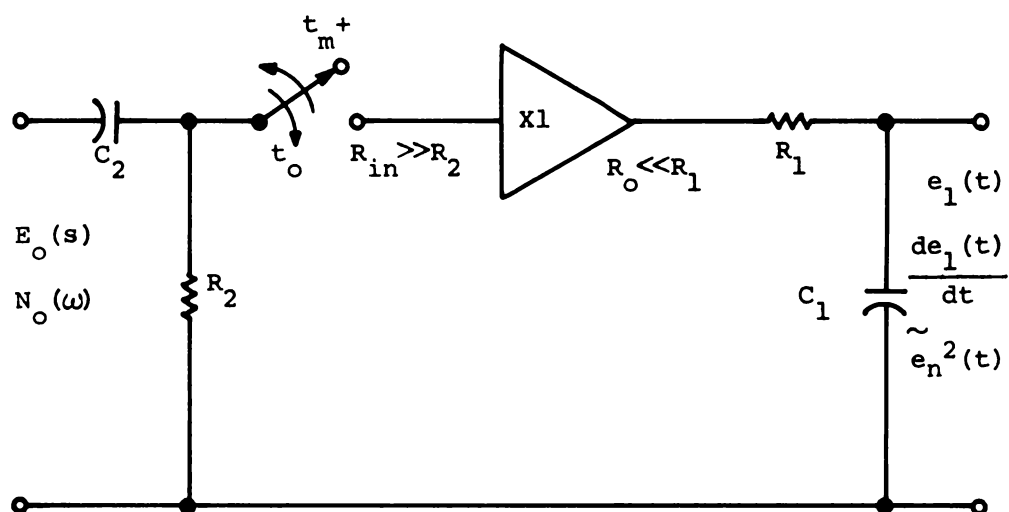


Figure 24. A time-variant RC high-pass, RC low-pass filter.

and

$$\begin{aligned} \frac{de_1(t)}{dt} = \frac{A}{T} & \left\{ \left[\frac{(1 - t/T)}{(1 - T/T_2)(T_1/T - 1)} - \frac{(2T_1/T - 1 - T_1/T_2)}{(1 - T/T_2)^2 (T_1/T - 1)^2} \right] e^{-t/T} \right. \\ & - \frac{T_1/T}{(T_1/T_2 - 1)(T_1/T - 1)^2} e^{-t/T_1} \\ & \left. - \frac{T_2/T}{(1 - T_1/T_2)(T_2/T - 1)^2} e^{-t/T_2} \right\}. \end{aligned} \quad (131)$$

Since $G_1(s)$ is unequal to 1 in this case, the general expression for the output transient noise of Equation (138) must be used. The value for $\phi_a(\tau)$ is found from Equations (139) and (140). The noise power spectral density at the gate input is

$$N_a(\omega) = \frac{a\omega^2 T_2^2}{(T^2 \omega^2 + 1)(T_2^2 \omega^2 + 1)}. \quad (170)$$

Finding the inverse Fourier transform of $N_a(\omega)$,

$$\phi_a(\tau) = \frac{aT_2^2}{2(T^2 - T_2^2)} \left[\frac{e^{-|\tau|/T_2}}{T_2} - \frac{e^{-|\tau|/T}}{T} \right]. \quad (171)$$

By substituting Equation (171) into Equation (138) and simplifying with

$$g_2(t) = \frac{e^{-t/T_1}}{T_1}, \quad (172)$$

the expression for transient noise becomes

$$\begin{aligned}
\tilde{e}_n^2(t) = & \frac{a}{T_2 (T^2/T_2^2 - 1) (T_1^2/T_2^2 - 1)} \left[\frac{(T_1/T_2 + 1)}{2} \left(1 - e^{-2t/T_1} \right) \right. \\
& + e^{-t/T_2} e^{-t/T_1} - 1 \left. \right] - \frac{a}{T(T^2/T_2^2 - 1) (T_1^2/T^2 - 1)} \left[\frac{(T_1/T + 1)}{2} \left(1 \right. \right. \\
& \left. \left. - e^{-2t/T_1} \right) + e^{-t/T} e^{-t/T_1} - 1 \right] . \quad (173)
\end{aligned}$$

To demonstrate the effect of this filter on the measurement of time, the relationships between T , T_1 , and T_2 must be simplified. The simplification chosen is for T_2 to equal $0.1T$ and for T_1 to vary. This again is approximately equal to a differentiation of the signal and noise before the gate and varying the time constant of an RC low-pass filter after the gate. Performing the substitution of $T_2 = 0.1T$, $X = T_1/T$, and $Y = t/T$ into Equations (131) and (173),

$$\begin{aligned}
\frac{de_1(t)}{dt} = & \frac{A}{T} \left\{ \left[-\frac{(1-Y)}{9(X-1)} + \frac{1+8X}{81(X-1)^2} \right] e^{-Y} \right. \\
& \left. - \frac{X}{(10X-1)(X-1)^2} e^{-Y/X} - \frac{1}{8.1(1-10X)} e^{-10Y} \right\} , \quad (174)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{e}_n^2(t) = & \frac{a}{T} \left\{ \frac{1}{9.9(100X^2 - 1)} \left[\frac{(10X+1)}{2} (1 - e^{-2Y/X}) + e^{-10Y} e^{-Y/X} - 1 \right] \right. \\
& \left. - \frac{1}{99(X^2 - 1)} \left[\frac{(X+1)}{2} (1 - e^{-2Y/X}) + e^{-Y} e^{-Y/X} - 1 \right] \right\} . \quad (175)
\end{aligned}$$

The expression for $\tilde{\sigma}_{HL}$ is then found by substituting Equations (174) and (175) into Equation (146) and normalizing to the optimum value given in Equation (78). Figure 25 is a graph of $\tilde{\sigma}_{HL}$ versus Y for six values of X .

IV. DISCUSSION

In this chapter an examination of the effect of filtering on the measurement of time has been performed. The optimum filter and the minimum noise/slope error in the measurement of time were determined for both an arbitrary signal and noise and for the signal and noise from a charge-sensitive preamplifier. The error in the measurement of time using several RC filters, both time-invariant and time-variant, was calculated; and this error was compared to the error obtained using the optimum filter. In this section, a comparison of the usefulness of the filters in the measurement of time is made. In addition, the application of these filters to the measurement of time of the signal from a Ge(Li) detector is discussed.

In situations where the noise/slope error is dominant, the minimum error, independent of the time the measurement must be made, is the important factor. In Figure 26, the minimum noise/slope error is shown as a function of the normalized filter time constant, T_1/T , for each of the filters analyzed in the previous sections. A comparison of the minimum error for each of the filters and time constants reveals that the difference in the effects of the filters is not very extreme for any filter. In general, however, the error in time measurement associated with the time-variant filters is less than that of the corresponding

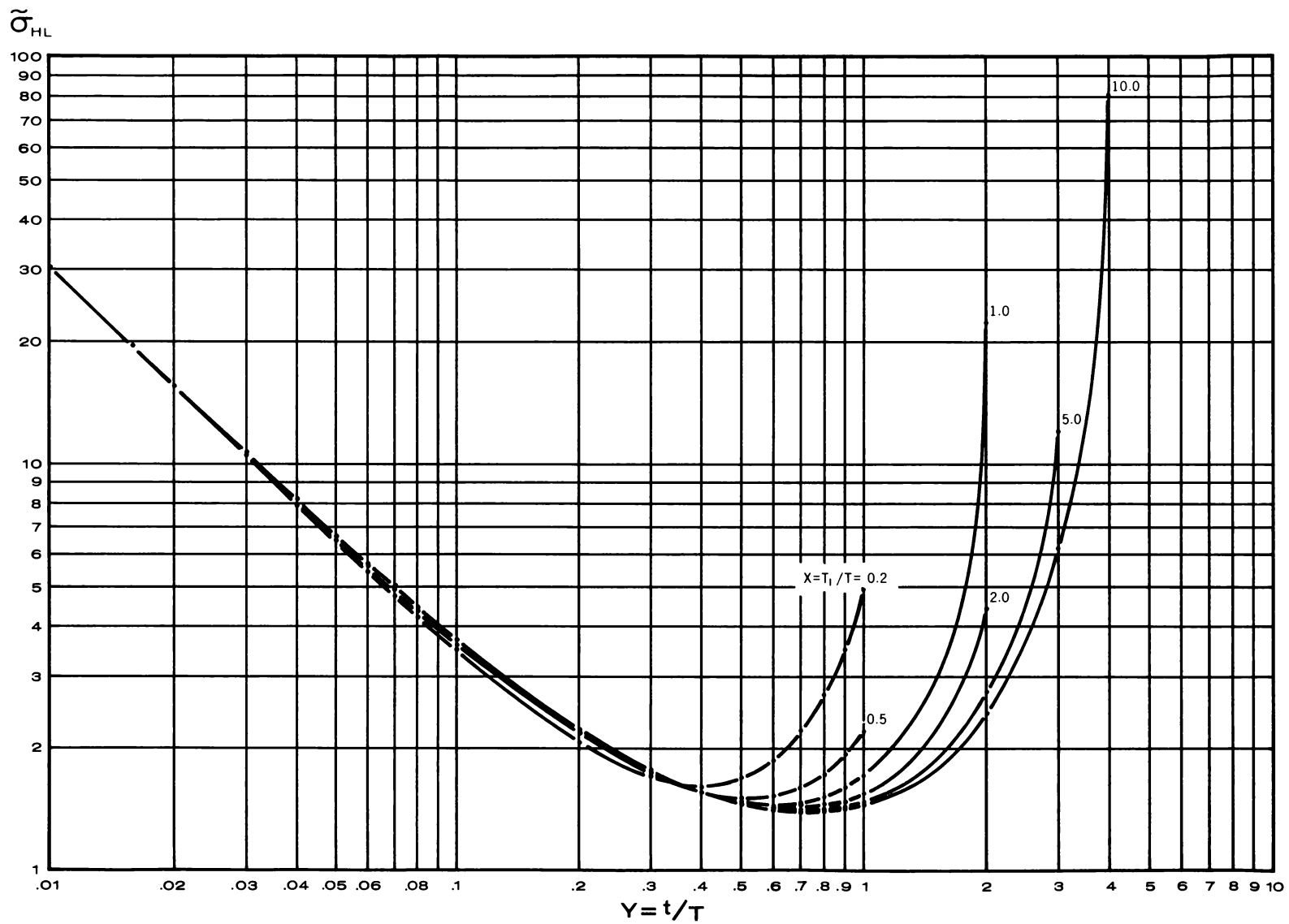


Figure 25. The effect of an RC high-pass filter before the gate and an RC low-pass filter after the gate on the measurement of time.

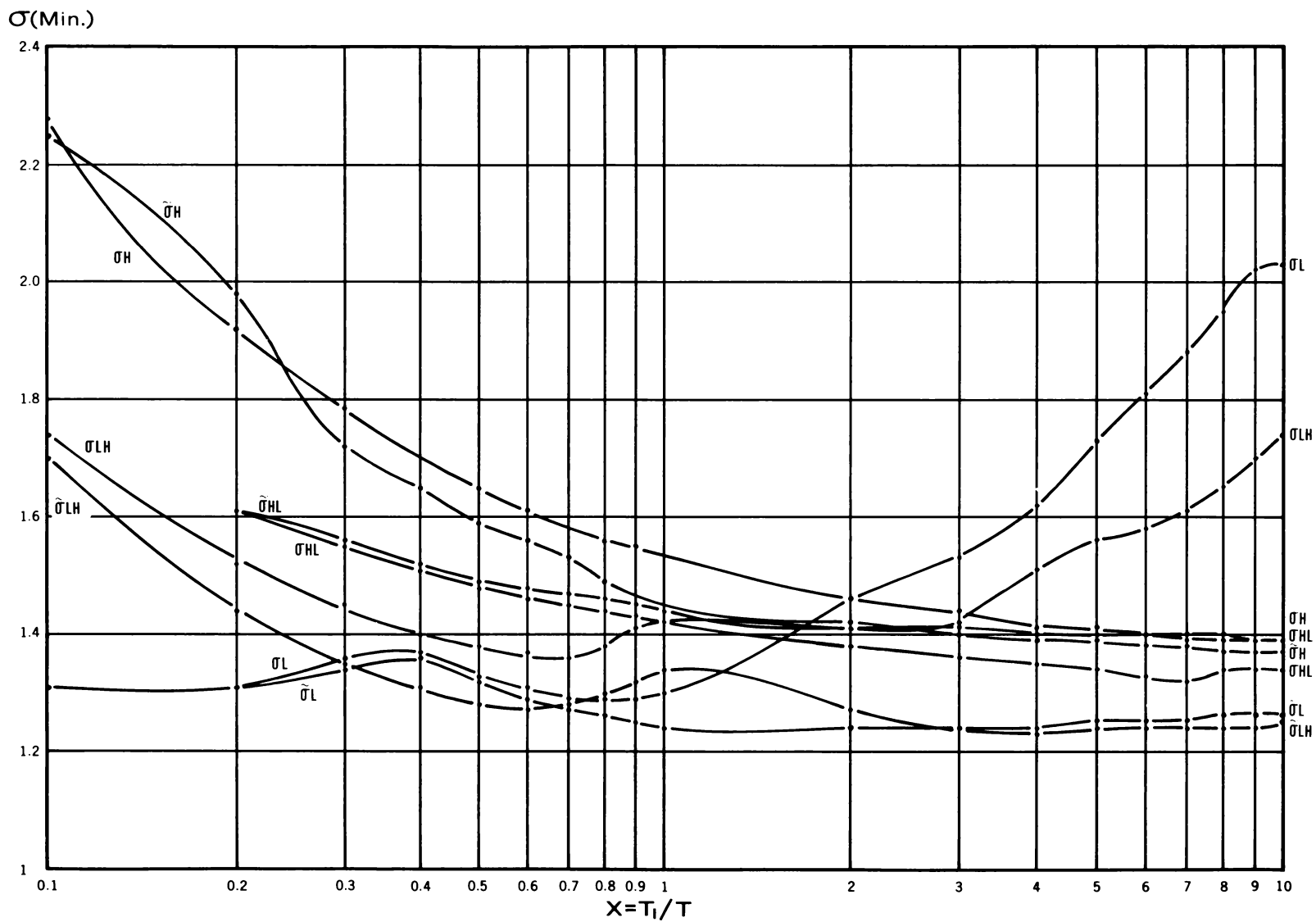


Figure 26. The minimum timing error for the time-invariant and time-variant cases.

time-invariant filter. From Figure 26, the minimum noise/slope error for the signal and noise from a charge-sensitive preamplifier for the filters analyzed is 1.23. This minimum was obtained using an RC low-pass, RC high-pass filter after the gate with the filter time constant equal to $4T$.

In the measurement of time using Ge(Li) detectors, noise/slope error is usually not dominant. The largest source of error is the charge collection variations which result in pulse shape variations at the output of any filter. To reduce the effect of charge collection variations, the lowest discriminator level possible is desired. Using time-invariant filters, the minimum discriminator level is limited by the steady-state noise from the filter. At this minimum level charge collection variations are usually still dominant; and further decreases in the level, if possible, would decrease the effect of the charge collection variations and, thus, the total error in time measurement. Using time-variant filters, the limitation of steady-state noise for the minimum discriminator level is eliminated. Theoretically, the minimum discriminator level is unlimited using a time-variant filter. A practical limit does exist, however, because of the noise produced by the gate and because of the noise that may build up during the time period between the gate opening time and the actual signal arrival time. These limitations of the time-variant filter, however, are still much less than the limitation of steady-state noise of the time-invariant filter.

In addition to decreasing the minimum discriminator level, the time-variant filter has another advantage. As the discriminator level

is lowered, charge collection variations decrease; but, the noise/slope error increases. At some level the noise/slope error becomes dominant, and further decreases in the discriminator level increase the total error in time measurement. The filter chosen must, therefore, have a low noise/slope error at low levels or early measurement times. Referring to Figure 11, page 44, Figure 13, page 48, Figure 15, page 53, Figure 16, page 56, Figure 19, page 65, Figure 21, page 69, Figure 23, page 73, and Figure 25, page 77, the noise/slope error using the time-variant filters is usually much less than the error using the time-invariant filters for measurement times near zero. An accurate evaluation to determine the best filter requires that the level or time the measurement is made be known. However, for very low levels the filter with an RC high-pass network before the gate and an RC low-pass network after the gate is, in general, the best.

An evaluation of the filters examined in this chapter reveals that for the measurement of time of signals from Ge(Li) detectors time-variant filters have two distinct advantages over time-invariant filters. Lower discriminator levels or earlier measurement times is the first advantage. This first advantage allows the reduction of charge collection variations. The second advantage is lower noise/slope error at low discriminator levels. An experimental examination to verify the evaluation of the filters presented in this chapter is the object of the following chapter.

CHAPTER IV

EXPERIMENTAL RESULTS

In the previous chapters, the sources of error in time measurement and the effect of filtering on time measurement have been examined theoretically. In this chapter, the effect on time measurement of the same filters, noise, and signal examined theoretically will be experimentally determined. In addition, those filters which are determined to have particular application to the time measurement of signals from Ge(Li) detectors will be used in a coaxial Ge(Li) detector, charge-sensitive preamplifier system. The best filter for the particular detector and group of filters will be found, and a comparison of each of the filters examined will be made.

I. EFFECT OF FILTERING

Procedure

In Chapter III, the effect of some particular filters on the noise/slope error in time measurement of the signal and noise given by Equations (14) and (39) was found theoretically. To examine this effect experimentally, the signal, noise, and filters must be fabricated by the use of circuits. The block diagram of the circuits used to obtain the signal and noise given by Equations (14) and (39) is shown in Figure 27. The circuits represented by the block diagram are shown in the Appendix. The blocks with the name ORTEC and a model number represent a commercial instrument used that is manufactured by ORTEC,

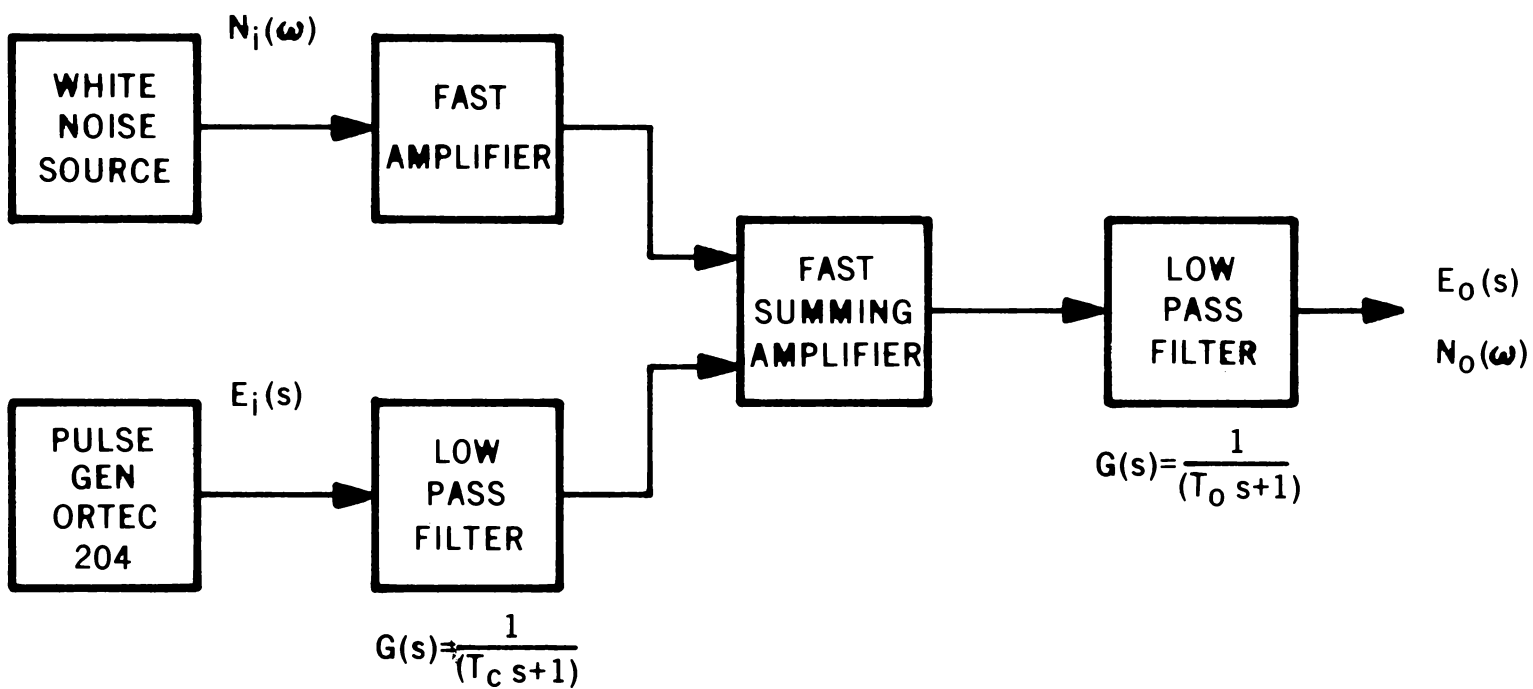


Figure 27. Block diagram of circuits used to obtain $E_o(s)$ and $N_o(\omega)$.

Incorporated. If the time constants associated with the fast amplifiers are much smaller than T_c and T_o , the transfer functions for the noise and signal are

$$\frac{N_o(\omega)}{N_i(\omega)} = \frac{K_1}{T_o^2 \omega^2 + 1} \quad (176)$$

and

$$\frac{E_o(s)}{E_i(s)} = \frac{K_2}{(T_c s + 1)(T_o s + 1)}, \quad (177)$$

where K_1 and K_2 are constants determined by the gain of the amplifiers.

For a white noise source or $N_i(\omega)$ equal to a constant, N ,

$$N_o(\omega) = \frac{NK_1}{T_o^2 \omega^2 + 1}. \quad (178)$$

If the input signal is a step function of value K_3 ,

$$E_i(s) = \frac{K_3}{s}. \quad (179)$$

Substituting Equation (179) into Equation (177),

$$E_o(s) = \frac{K_2 K_3}{s(T_c s + 1)(T_o s + 1)}. \quad (180)$$

By arbitrarily defining NK_1 equal to a and $K_2 K_3$ equal to $-Q/C_f$,

Equations (178) and (180) become

$$N_o(\omega) = \frac{a}{T_o^2 \omega^2 + 1} \quad (39)$$

and

$$E_o(s) = \frac{-Q}{C_f s(T_o s + 1)(T_c s + 1)}, \quad (14)$$

which is the noise and signal determined in Chapter II to be used in this analysis. Simplifying further, $-Q/C_f$ was defined to be equal to A ; therefore,

$$E_o(s) = \frac{A}{s(T_o s + 1)(T_c s + 1)} \quad . \quad (141)$$

The block diagram of the method of measurement of time on the signal and noise produced by the instruments and circuits of Figure 27, page 82, is shown in Figure 28. Again, the circuits represented by the blocks in the block diagram are shown in the Appendix; and the commercial instruments are represented by the name and model number. In Figure 29, the signals of the block diagram of Figure 28 are shown as a function of time. The difference in time of the start and stop signals, Δt , is converted linearly to an amplitude by the time-to-pulse-height converter; and the amplitude of the signal is stored by the multichannel analyzer. The display of the multichannel analyzer is, therefore, a measure of the variance in amplitude and thus the variance in time, Δt , due to noise/slope error in the filtered signal. The filters used are the time-invariant and time-variant RC low-pass, RC high-pass, and the RC low-pass, RC high-pass combination filters that were analyzed theoretically in Chapter III. In addition, the time constants, T_c and T_o , were set equal to T as in Chapter III.

The time-variant filters require the use of a linear gate between the two time-invariant filters represented by $G_1(s)$ and $G_2(s)$. Again, $G_1(s)$ and $G_2(s)$ are the same as in Chapter III; or $G_1(s)$ is the transfer function of the filter before the gate and $G_2(s)$ is the transfer

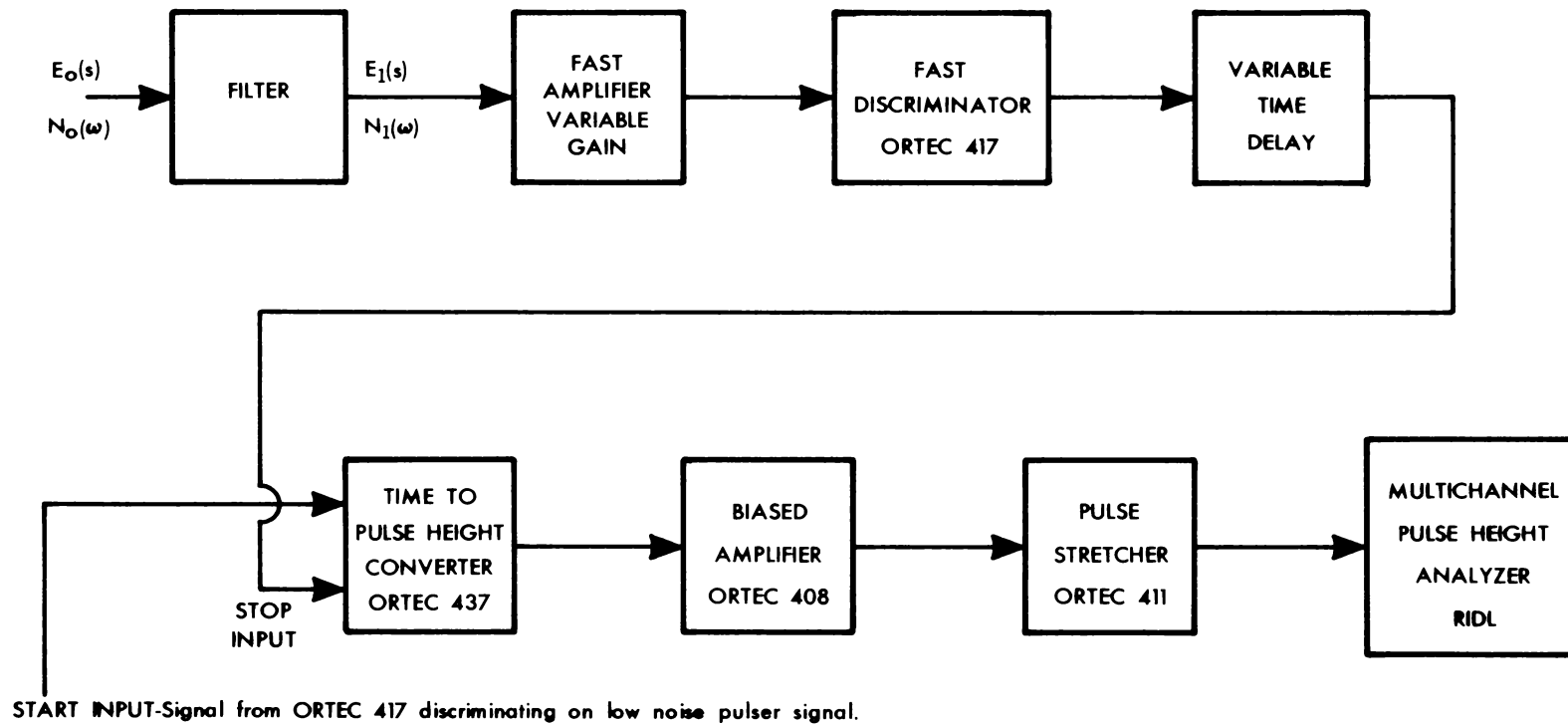


Figure 28. Block diagram to determine the effect of filtering on the measurement of time.

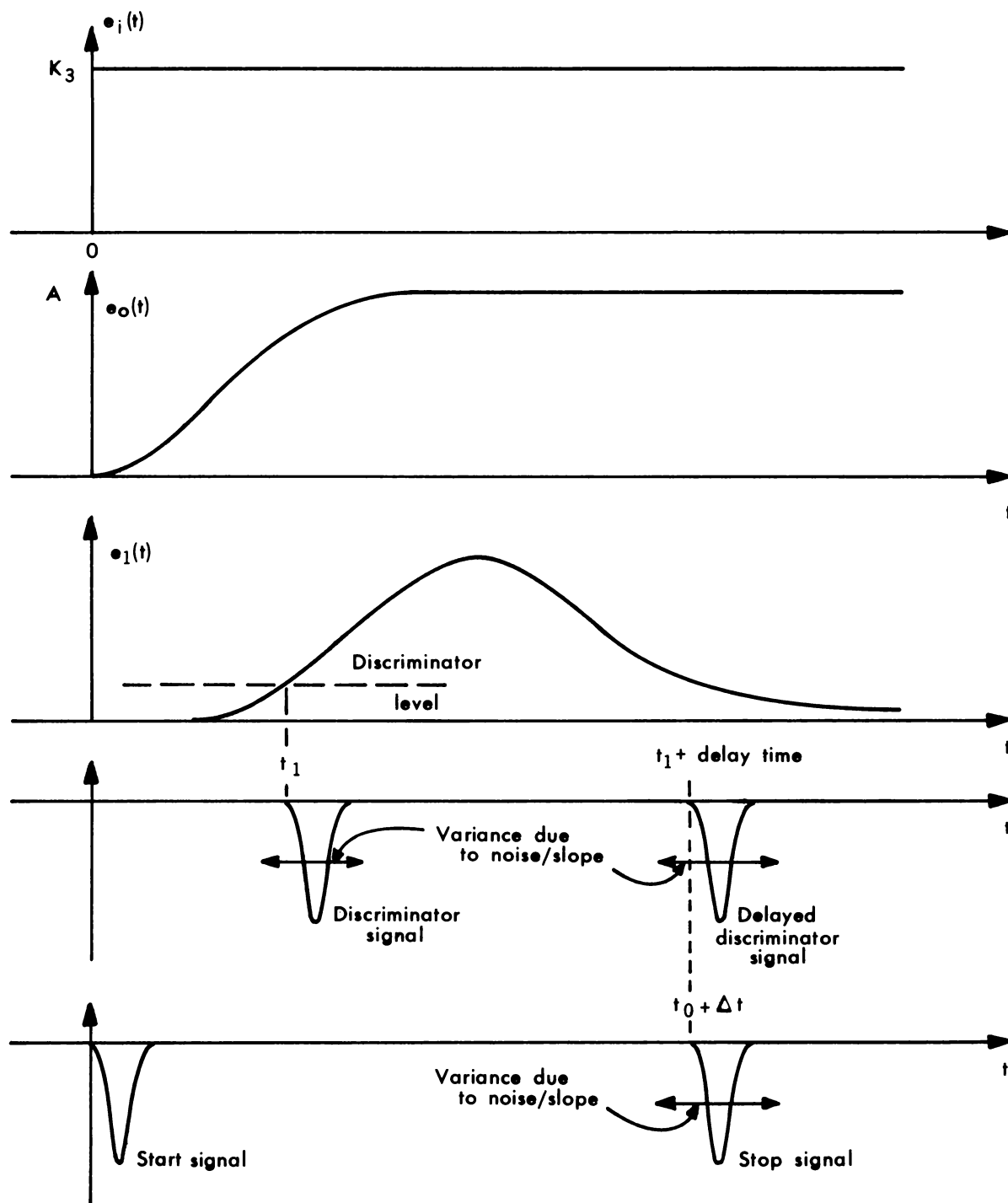


Figure 29. Signals as a function of time.

function of the filter after the gate. The block diagram of the method used in implementing the time-variant filter is shown in Figure 30.

Results

Since an amplitude-level discriminator is used on the signal from the filters, the information was taken as a function of percentage of maximum pulse height of the filtered signal rather than as a function of time as in the theoretical data. The other normalizations used in presenting the data are the same as for the theoretical information. The error in time measurement is normalized to the optimum calculated value, and the filter time constants are normalized to T . The measured value of T_c was found to be 81 nanoseconds. The calculation of the white noise power spectral density, a , from the experimental noise measurements for a signal amplitude of one volt gives a equal to 2.66 mean square millivolts per 10^6 radians/second. Substituting the values for T_c , A , and a into Equation (78), the optimum value for the error in time measurement is

$$\sigma_T(\text{Opt.}) = 0.658 \text{ nanoseconds rms} \quad . \quad (181)$$

From the information on variance displayed by the multichannel analyzer, the width of the spectrum at half the maximum value (fwhm) of the spectrum is perhaps the best measure. For a Gaussian distribution in the variance or error spectrum, the fwhm value of the variance is related to the rms value by

$$\sigma(\text{fwhm}) = 2.35 \sigma(\text{rms}) \quad . \quad (182)$$

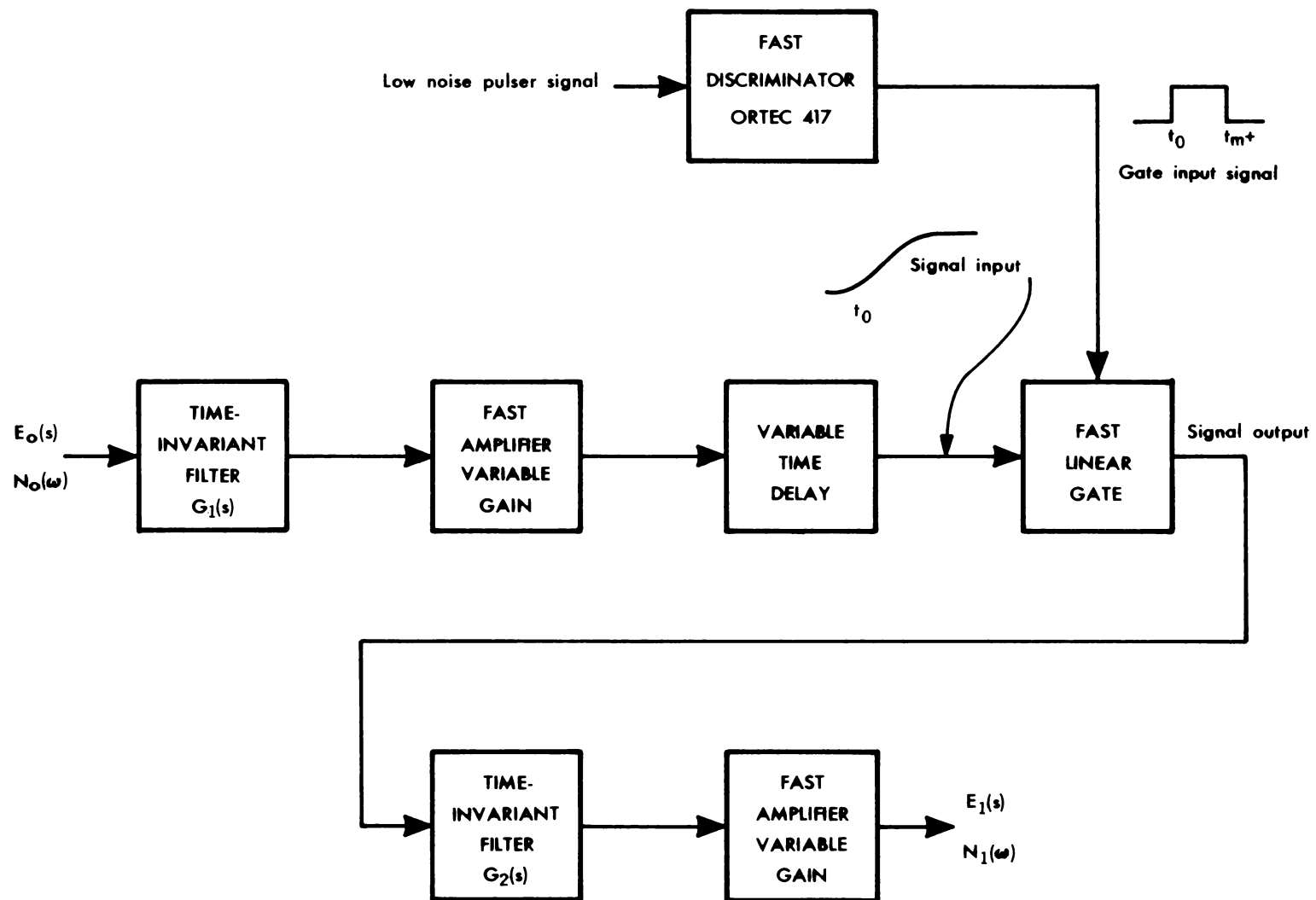


Figure 30. Block diagram of the time-variant filters used in experimental work.

The optimum value for the error in time measurement for the experimental signal and noise is, therefore,

$$\sigma_T(\text{Opt.}) = 1.545 \text{ nanoseconds fwhm} \quad . \quad (183)$$

The data taken for the error in time measurement is normalized to the optimum value of Equation (183). The experimental data taken demonstrating the effect of filtering is shown in Figures 31 through 37. The representation for each filter of the normalized error in time measurement used in the experimental data is the same as for the theoretical information. In Figure 38, the minimum experimental noise/slope error in time measurement independent of the discriminator level is shown as a function of the normalized filter time constant, T_1/T , for each of the filters analyzed.

Comparison of Experimental Results with Theoretical Results

Since the experimental data are presented as a function of discriminator level and the theoretical data as a function of discrimination time, a direct comparison of the results in each case is not possible. However, a nearly linear relationship exists between discriminator level and discrimination time; therefore, an approximate quantitative comparison can be made.

From the graphs of the theoretical and experimental data, very good correspondence exists between the two results. The greatest dissimilarity is present in the time-invariant and time-variant high-pass filters at low time constants. This difference is probably due to the signal and noise receiving additional low-pass filtering from the amplifier circuits. The conclusions concerning the filters from the theoretical

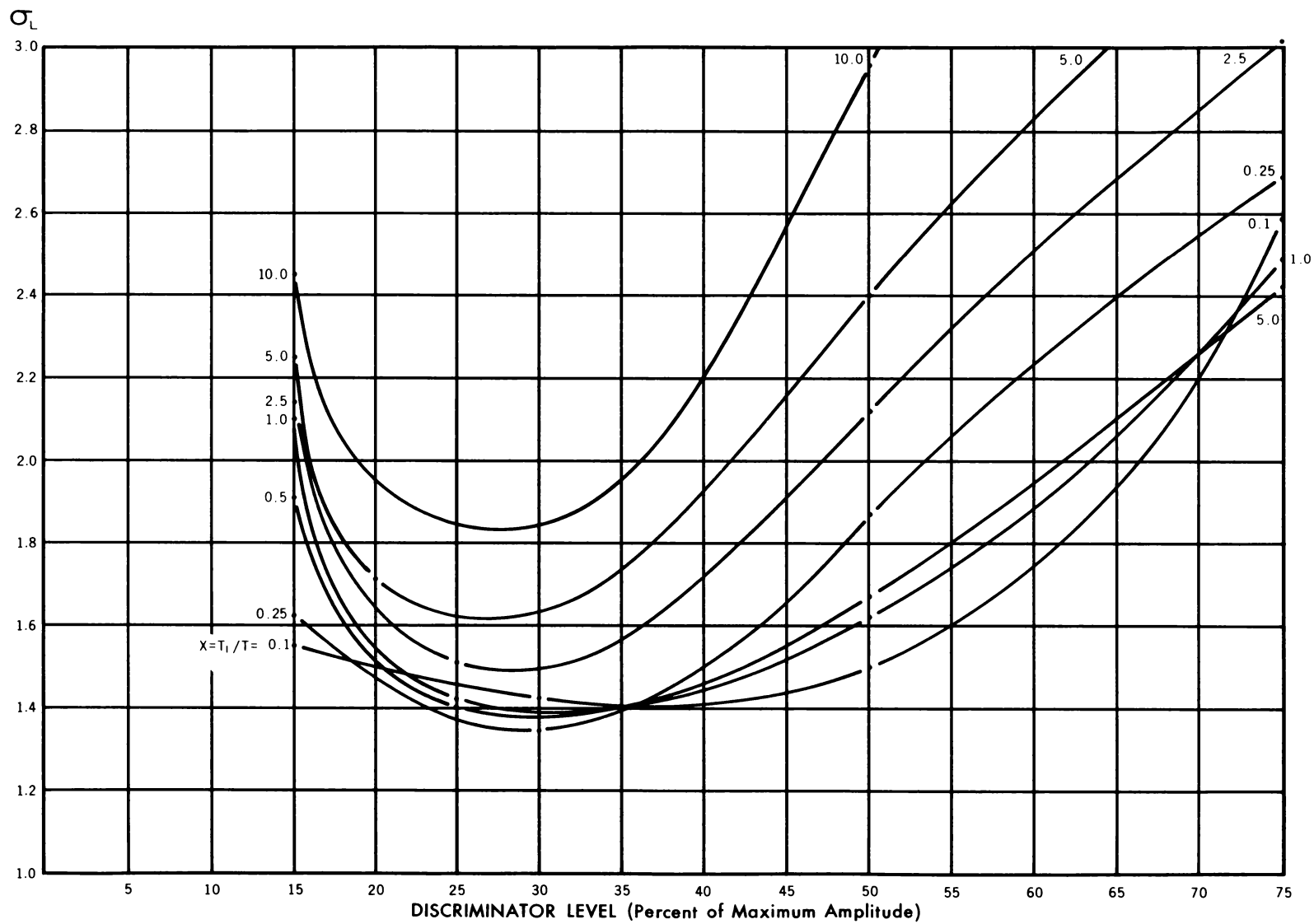


Figure 31. Experimental results using time-invariant RC low-pass filters.

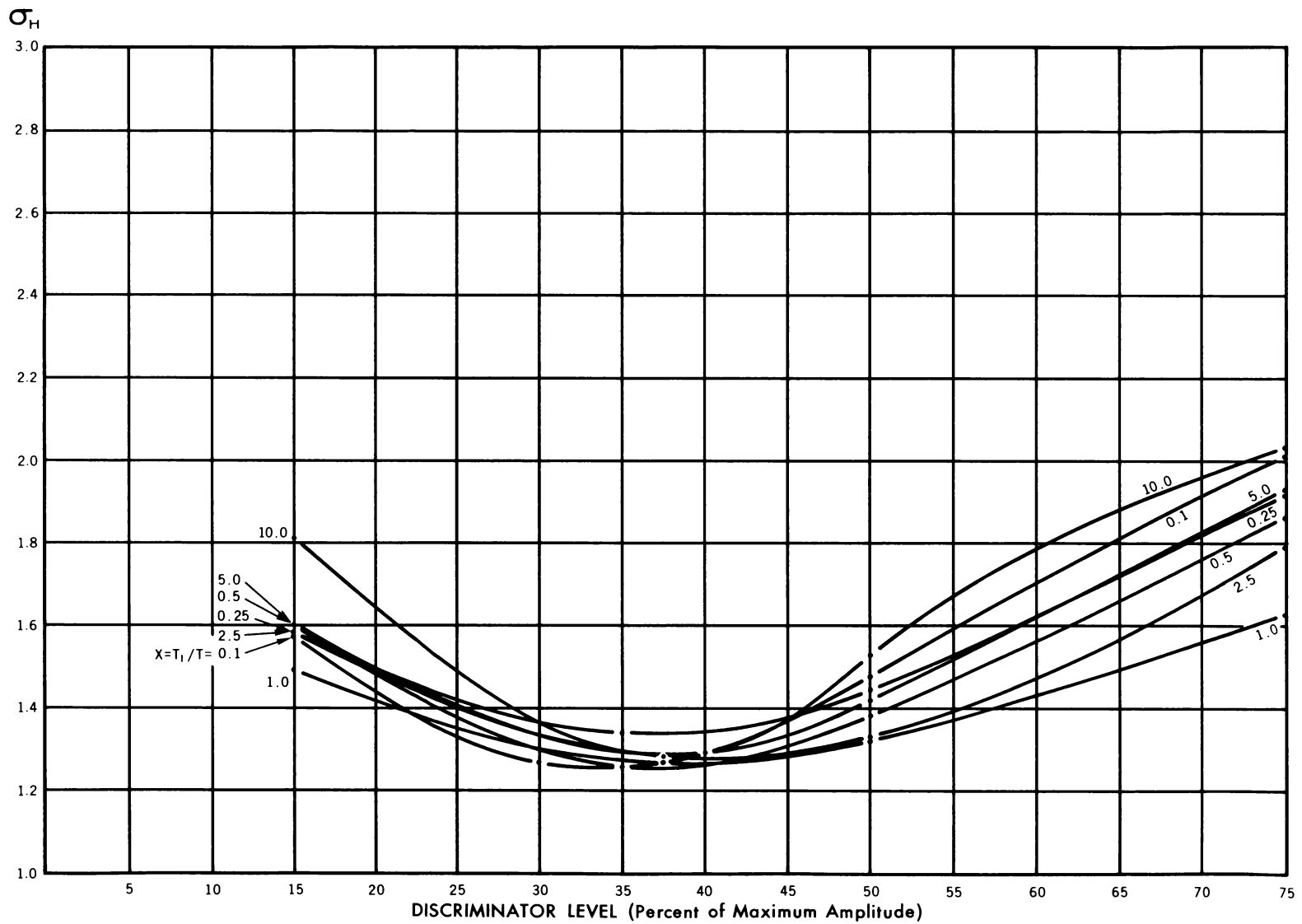


Figure 32. Experimental results using time-invariant RC high-pass filters.

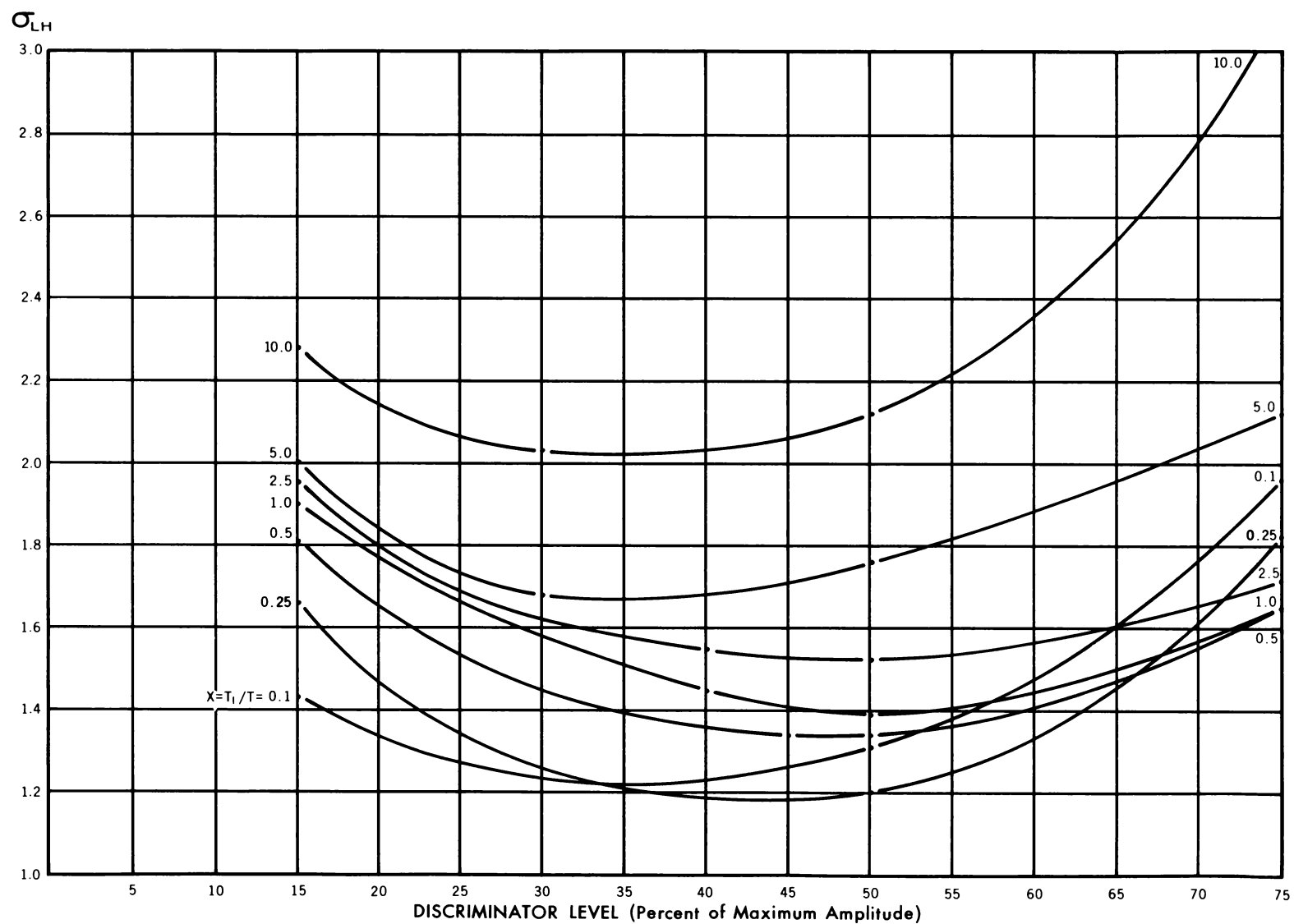


Figure 33. Experimental results using time-invariant RC high-pass, RC low-pass filters.

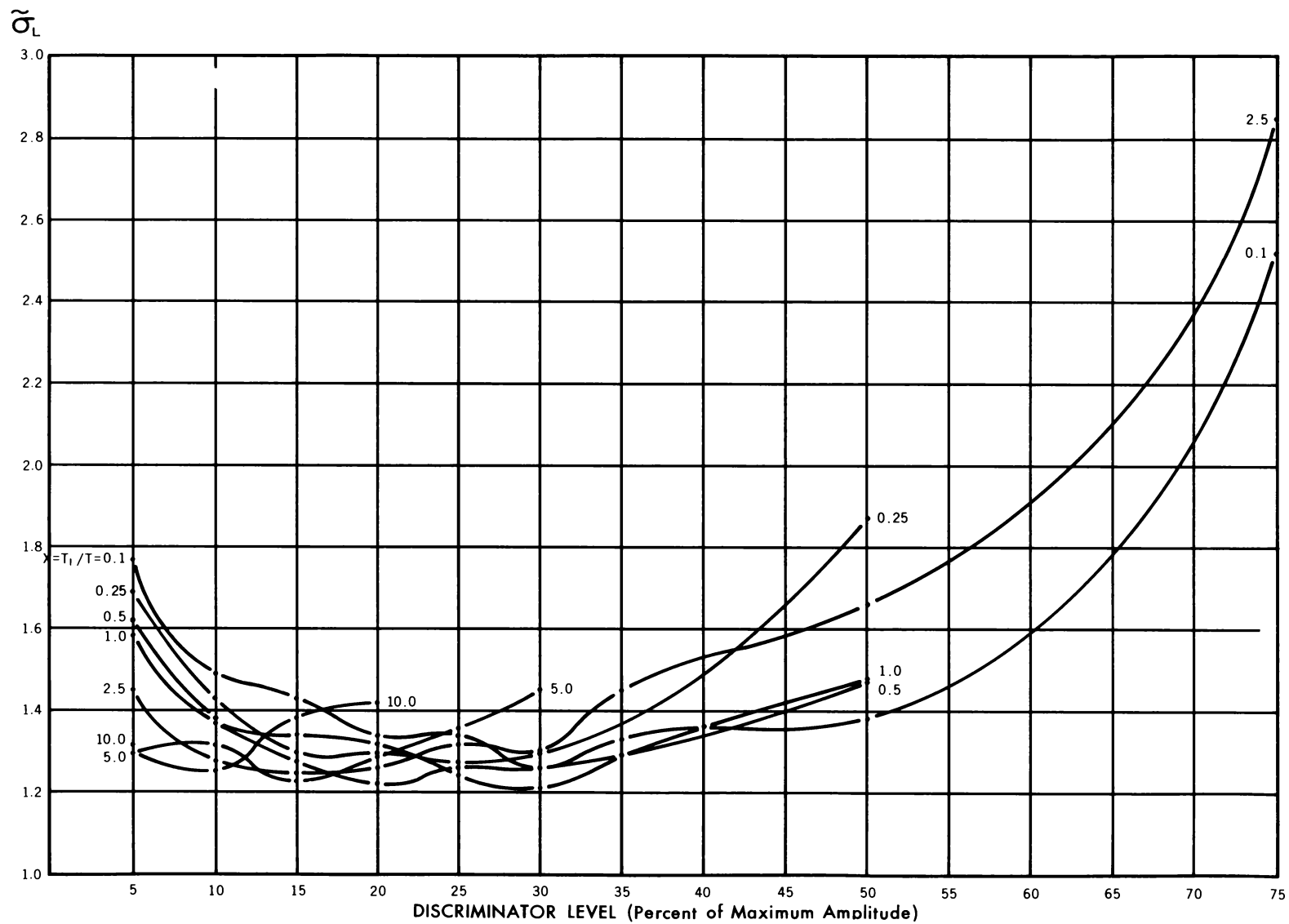


Figure 34. Experimental results using time-variant RC low-pass filters.

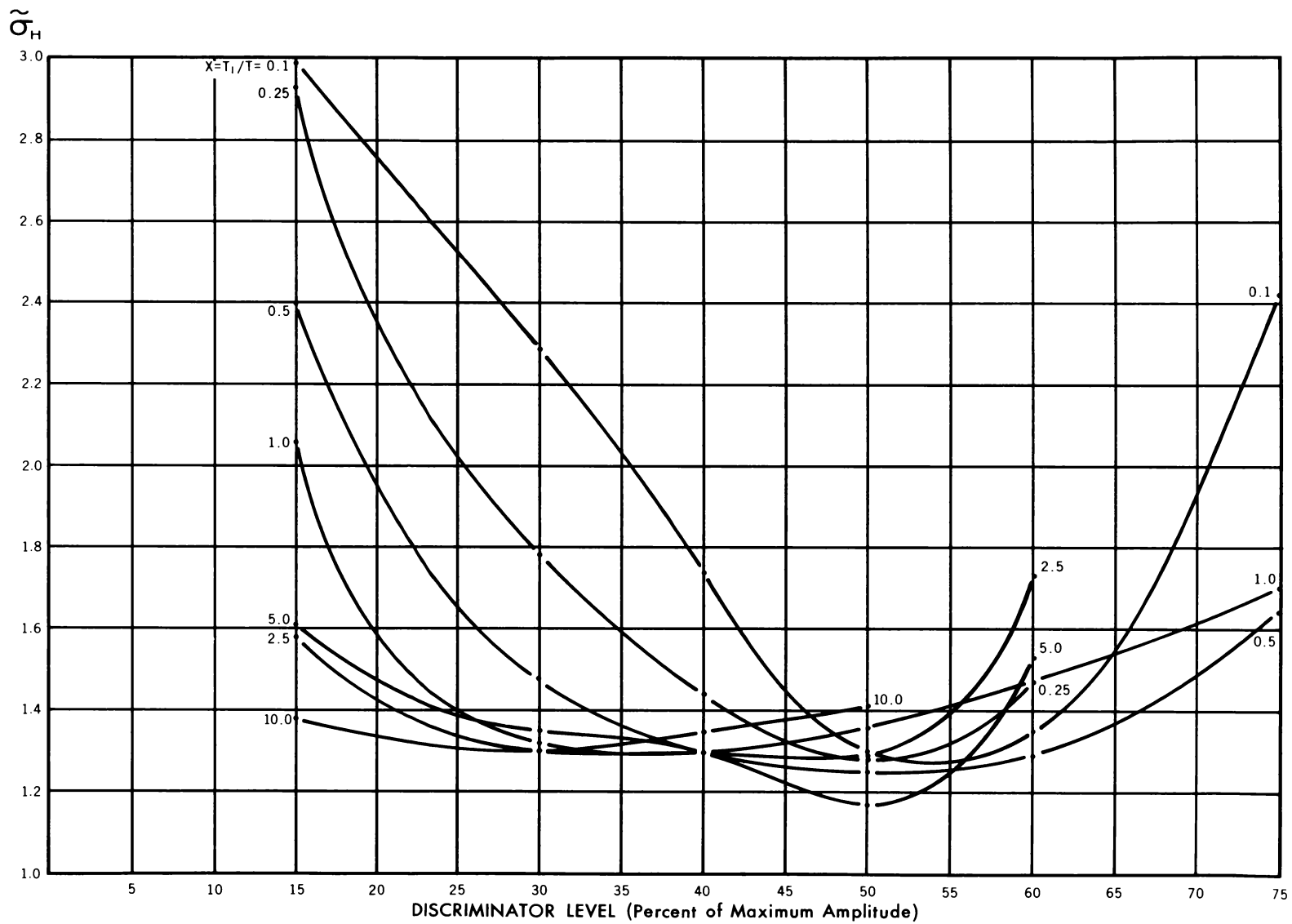


Figure 35. Experimental results using time-variant RC high-pass filters.

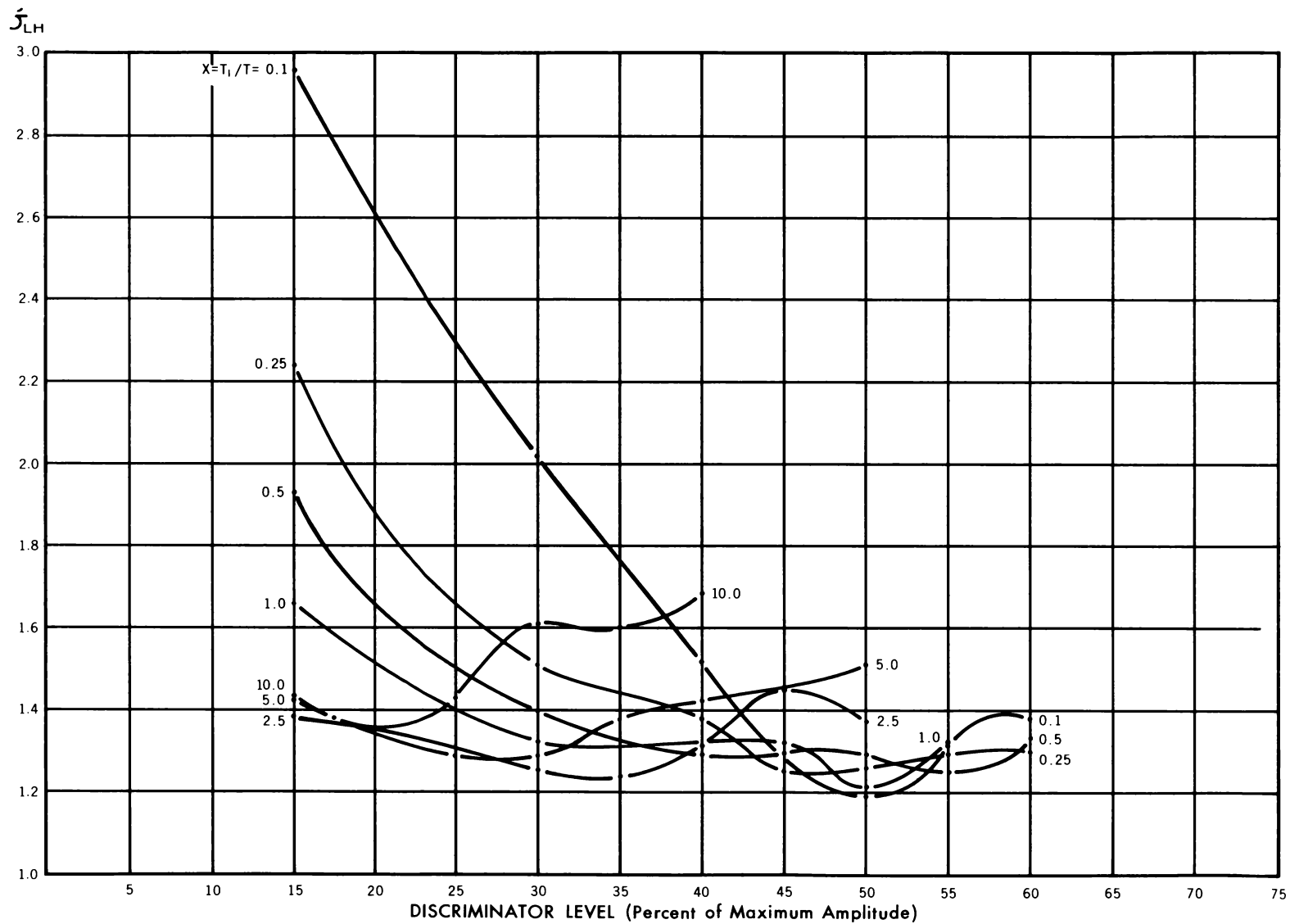


Figure 36. Experimental results using time-variant RC low-pass, RC high-pass filters.

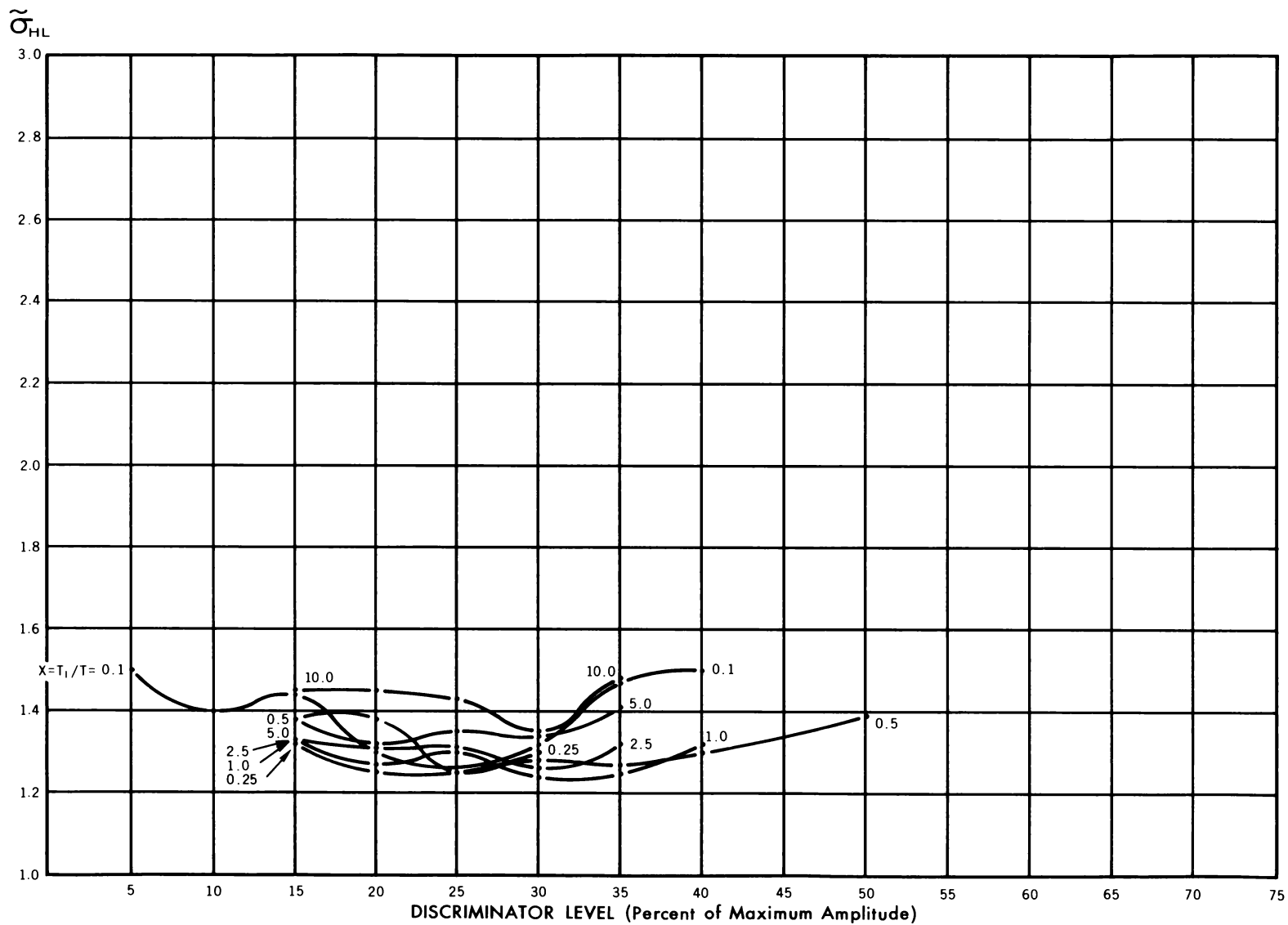


Figure 37. Experimental results using RC high-pass filters before the gate and RC low-pass filters after the gate.

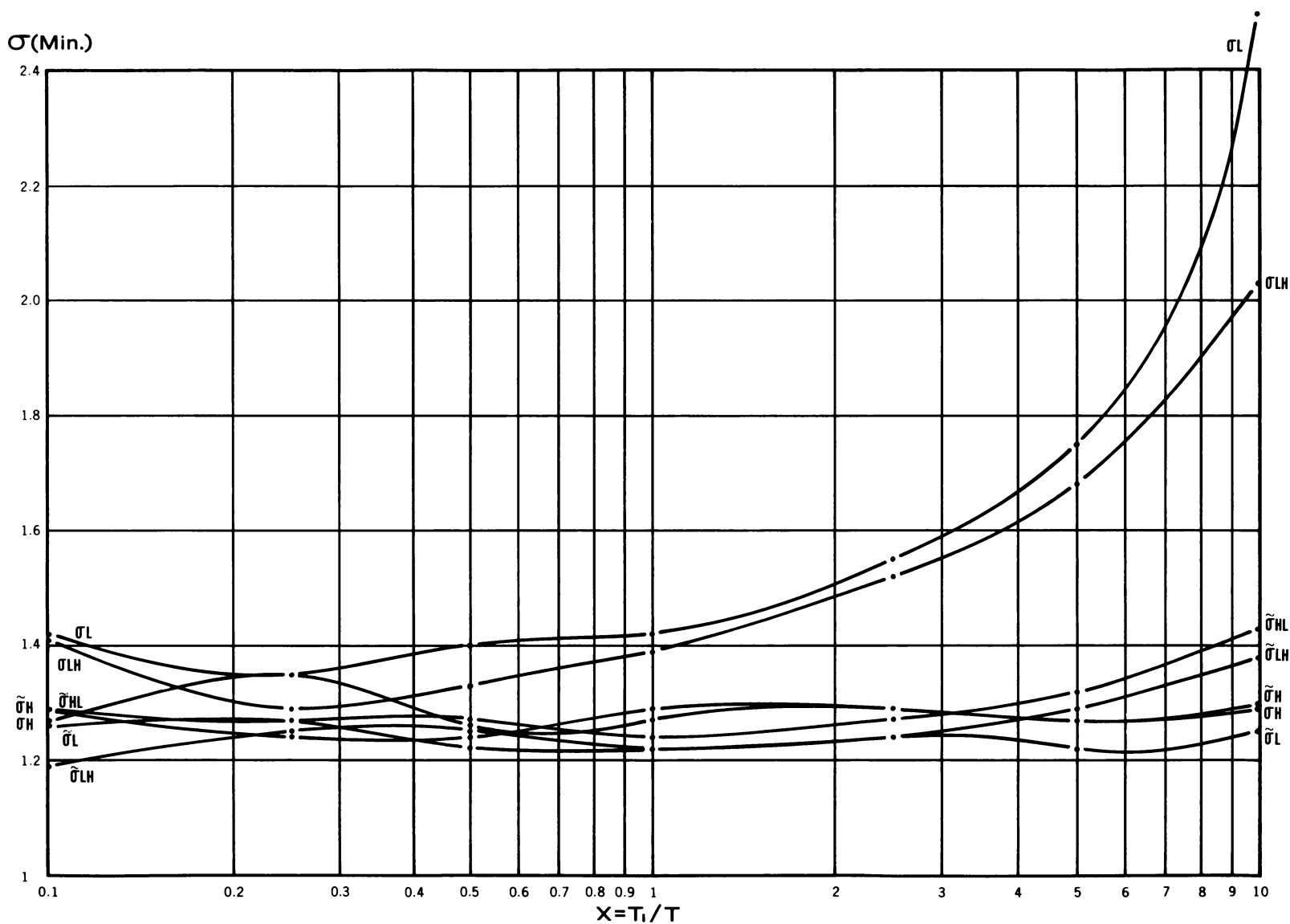


Figure 38. The minimum experimental timing error for the time-invariant and time-variant cases.

data are confirmed by the experimental results. Again, the error in time measurement associated with the time-variant filters is, in general, lower than that using the corresponding time-invariant filter. The minimum experimental noise/slope error was 1.22. This minimum error was obtained using a time-variant low-pass, high-pass filter as was the minimum theoretical noise/slope error. The data in Table I permit a comparison to be made between the minimum calculated noise/slope error and the corresponding experimental results. From this table, very good correspondence between experimental and theoretical results is evident.

The conclusions from the theoretical information concerning the use of particular filters with Ge(Li) detectors are verified by the experimental data. At low discriminator levels or early discrimination times the time-variant filter of a high-pass filter before the gate and a low-pass filter after the gate appears to be best. This same conclusion was arrived at from the theoretical information.

II. TIME MEASUREMENT USING Ge(Li) DETECTORS

Procedure

The method that is used to perform time measurement on the signals from Ge(Li) detectors is to discriminate on the leading edge of the signal from the charge-sensitive preamplifier. The block diagram of the circuit used in this analysis of the effect of filters on time measurement of Ge(Li) detector signals is shown in Figure 39. The circuit diagrams represented by the blocks are shown in the Appendix. The filter block in Figure 39 is actually composed of several circuits.

TABLE I

COMPARISON OF MINIMUM THEORETICAL NOISE/SLOPE
ERROR AND CORRESPONDING EXPERIMENTAL ERROR

Filter Noise/Slope Error Representation	Filter Time Constant $X = T_1/T$	Minimum Theoretical Error	Corresponding Experimental Error	% Difference in Theoretical and Experimental
σ_L	0.9	1.29	1.41	9.3
σ_H	10.0	1.39	1.29	7.2
σ_{LH}	0.7	1.36	1.35	0.74
$\tilde{\sigma}_L$	1.0	1.24	1.24	0.0
$\tilde{\sigma}_H$	10.0	1.37	1.30	5.1
$\tilde{\sigma}_{LH}$	4.0	1.23	1.26	2.4
$\tilde{\sigma}_{HL}$	10.0	1.39	1.43	2.9

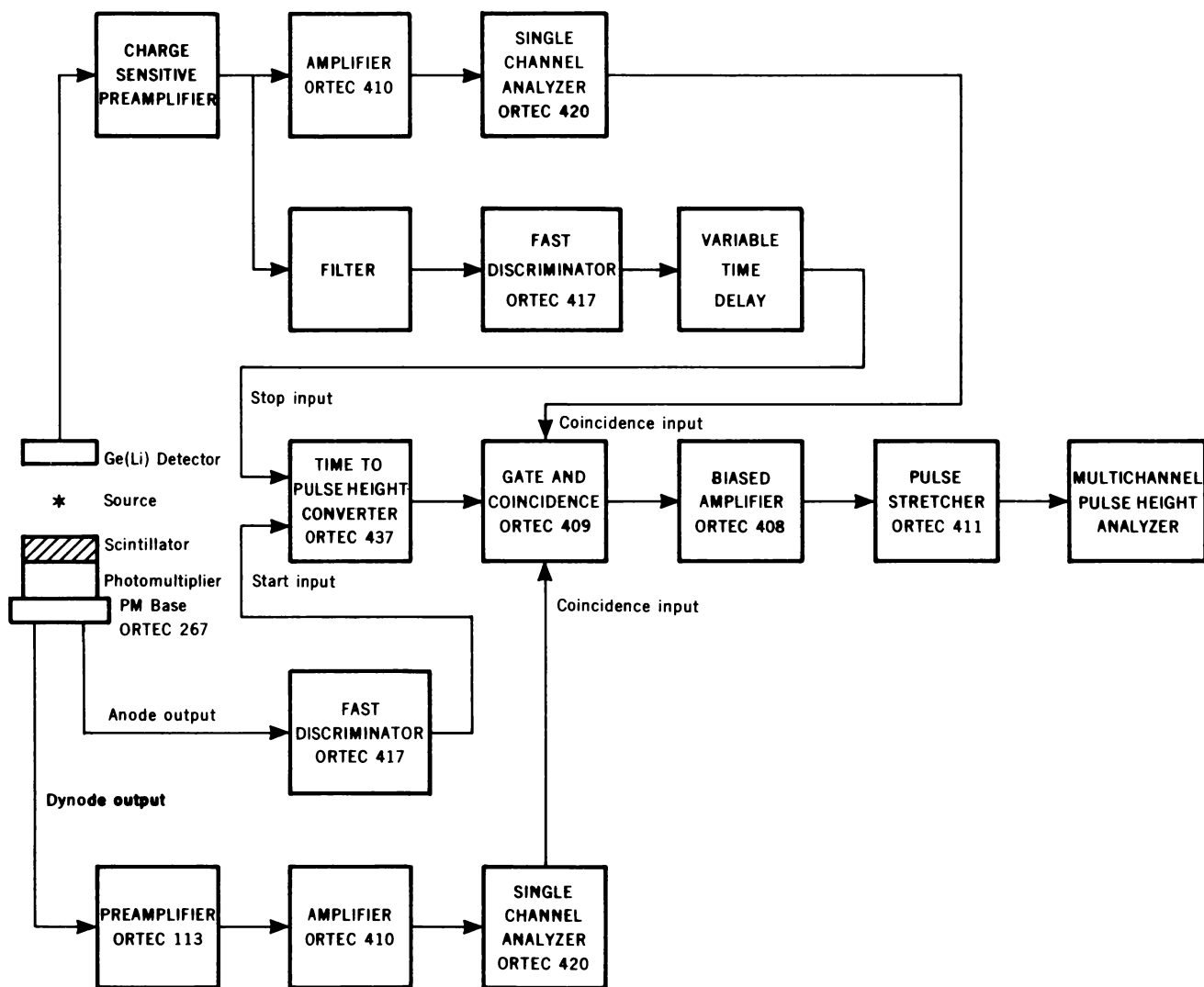


Figure 39. Block diagram of the method of time measurement using a Ge(Li) detector.

The block diagram of these circuits for the time-invariant and time-variant filters is shown in Figures 40 and 41. The circuit diagrams represented by the blocks in Figures 40 and 41 are also shown in the Appendix.

The arrangement of Figure 39 is such that the error in the measurement of coincidence of a gamma ray pair is determined. Using a Na-22 source, two gamma rays of 511 keV energy are released in coincidence by the source. One gamma ray is detected by the Ge(Li) detector, the other by the scintillator. The anode output of the photomultiplier provides a nearly invariant start input to the time-to-pulse-height converter. The stop input signal comes from discrimination on the filtered signal from the charge-sensitive preamplifier. The stop signal varies because of noise/slope error and because of variations in charge collection in the Ge(Li) detector. The difference in the start and stop times is converted into a pulse amplitude, and any variation in the stop time shows up as a pulse amplitude variation. The pulse amplitude variations are analyzed by the multichannel analyzer; and the analyzer display is, therefore, a measure of the variation in the time of the stop signal. To minimize the signals that may be obtained from non-coincident radiation, energy selection of 511 keV signals is employed. Therefore, only those time measurement signals that are coincident with 511 keV energy signals are analyzed.

Results

To properly compare the results the operating conditions of the source, Ge(Li) detector, scintillator and photomultiplier, charge-

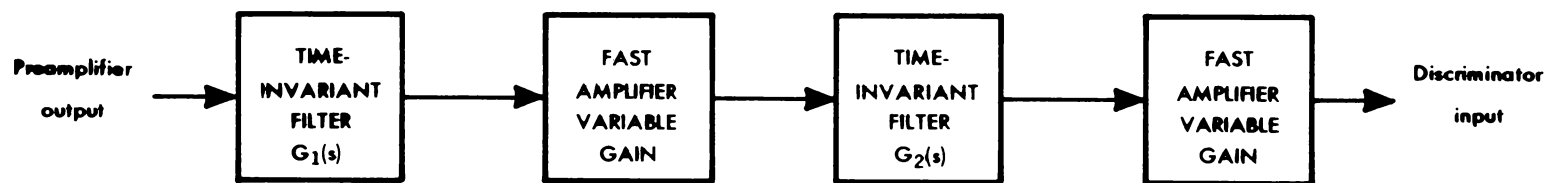


Figure 40. Block diagram of the time-invariant filters used in Ge(Li) system.

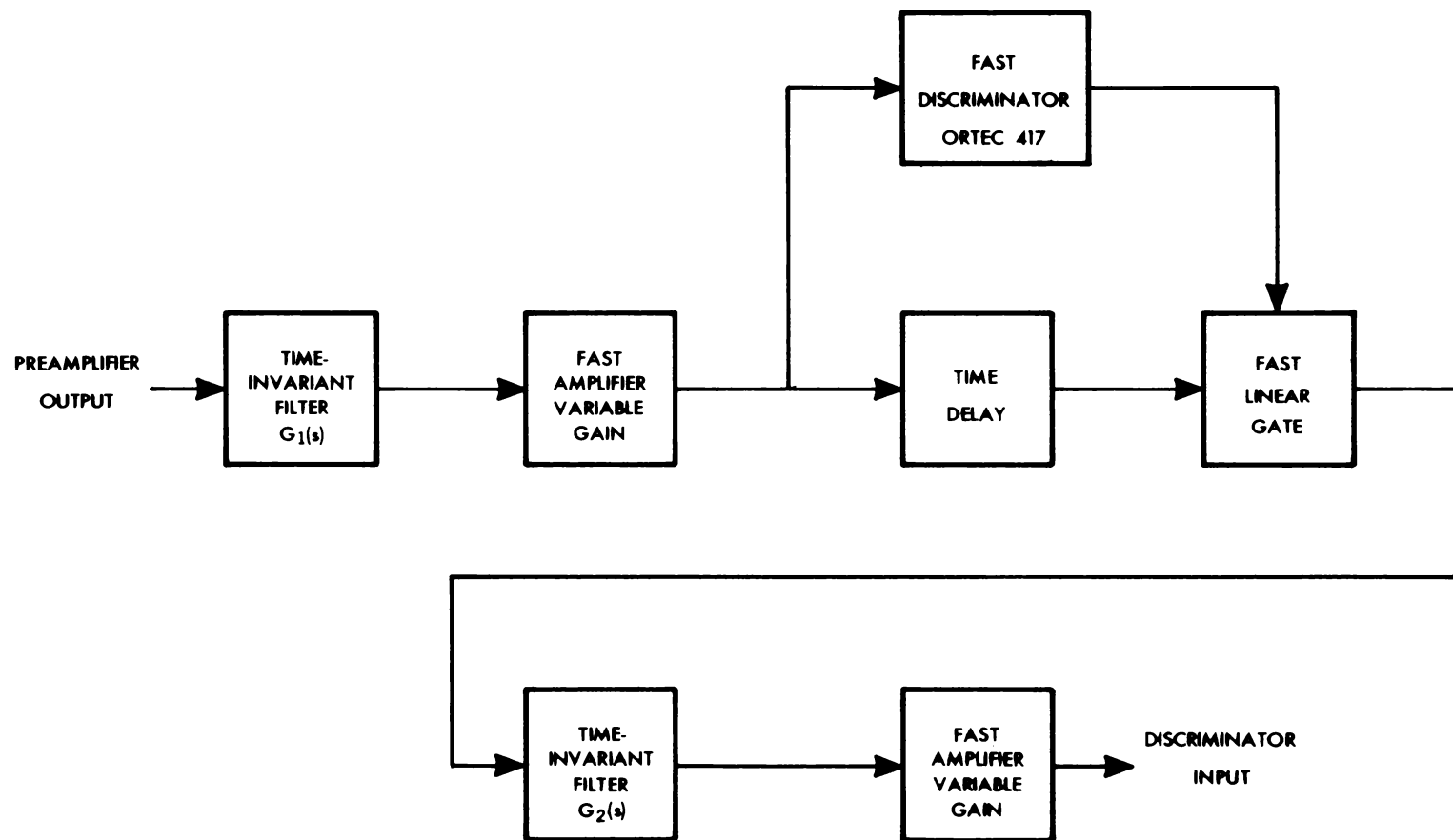


Figure 41. Block diagram of the time-variant filters used in Ge(Li) system.

sensitive preamplifier, and energy selection must be known. The source is a Na-22, 10 microcurie device emitting 511 keV gamma rays in coincidence. The Ge(Li) detector is an ORTEC true coaxial detector. The diagram of the detector with its dimensions and areas is shown in Figure 42. The active volume is 23.4 cubic centimeters. A bias of 2400 volts is applied. The scintillator is a NATON-136 plastic scintillator, and the photomultiplier tube and base are a Phillips XP1021 and an ORTEC 267, respectively. The energy selection for the photomultiplier side was 20 percent of the upper edge of the Compton distribution from the 511 keV gamma ray. From the information presented by Present, et al.,²² the contribution of the photomultiplier side (start signal) is certainly less than 0.5 nanoseconds and can be considered negligible for the time resolutions obtained from the detector side. The energy selection on the detector side was 14 keV which is much larger than the energy resolution. The information on the charge-sensitive preamplifier is presented in the Appendix with a description of the circuit.

The filters examined were the RC filters examined in the previous section. In addition, an RC high-pass filter with 2.2 RC equal to 2.0 microseconds was provided at the output of the preamplifier to ensure that the effect of low-frequency noise could be neglected. The time constants of the filters were varied from 2.2 RC equal to 20 nanoseconds to 2.0 microseconds. In Table II, the minimum timing error obtained using the various filters independent of the discriminator level or filter time constant is shown. The time constants of the filters are normalized to the average charge collection time or 100 nanoseconds.

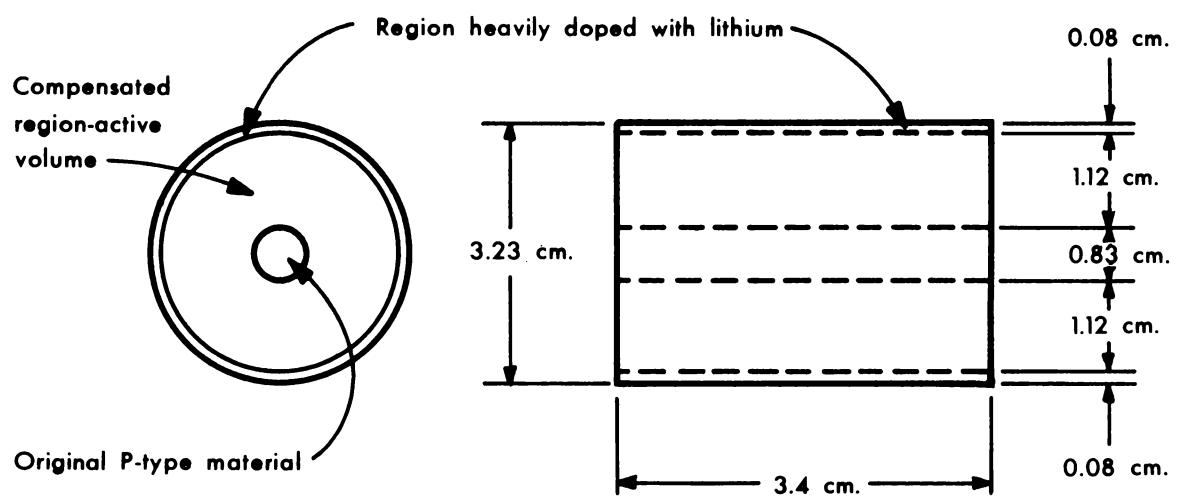


Figure 42. Ge(Li) detector used in experiment.

TABLE II

MINIMUM TIMING ERROR USING VARIOUS RC FILTERS ON THE
SIGNAL AND NOISE FROM A Ge(Li) DETECTOR

Filter Representation	Normalized Filter Time Constant T_1/T_c	Discriminator Level % of Maximum Amplitude	FWHM (ns)	FW(.1)M (ns)
σ_L	0.2	3.3	8.46	15.7
σ_H	1.0	5.0	6.81	12.6
σ_{LH}	0.2	7.5	6.82	13.7
$\tilde{\sigma}_L$	0.2	2.0	9.85	19.1
$\tilde{\sigma}_H$	0.2	15.0	9.29	17.8
$\tilde{\sigma}_{LH}$	0.2	5.0	9.9	20.6
$\tilde{\sigma}_{HL}$	0.2	3.75	5.63	10.58

For the time-invariant filters, the minimum timing error was obtained with the discriminator level set to a level just above noise. Lower timing errors are possible by setting the discriminator level where noise triggers the discriminator, but count rate problems and the problem of actually timing on the noise pulse discourages the use of low trigger levels. The time-invariant filter that produced the lowest timing error was the RC high-pass filter with $2.2 RC$ equal to 100 nanoseconds. The minimum fwhm and corresponding $fw(.1)m$ are 6.81 nanoseconds and 12.6 nanoseconds.

The minimum timing error for the time-variant filters was not limited by the discriminator level. Each minima found was at a discriminator level very much above the noise level. The difficulty involved was with those filters that had no filter before the gate. In these cases, the problem was probably the variance in the time the gate opened. From the minimum values of $\tilde{\sigma}_L$, $\tilde{\sigma}_H$, and $\tilde{\sigma}_{LH}$ compared to σ_L , σ_H , and σ_{LH} , the use of the time-variant filter increases the timing error. However, for that case where a filter was employed before and after the gate, improvement was obtained. With an RC high-pass filter before the gate and an RC low-pass filter after the gate with $2.2 RC$ equal to 20 nanoseconds in both filters, the minimum fwhm and corresponding $fw(.1)m$ are 5.63 nanoseconds and 10.58 nanoseconds. The timing spectrum for this filter is shown in Figure 43.

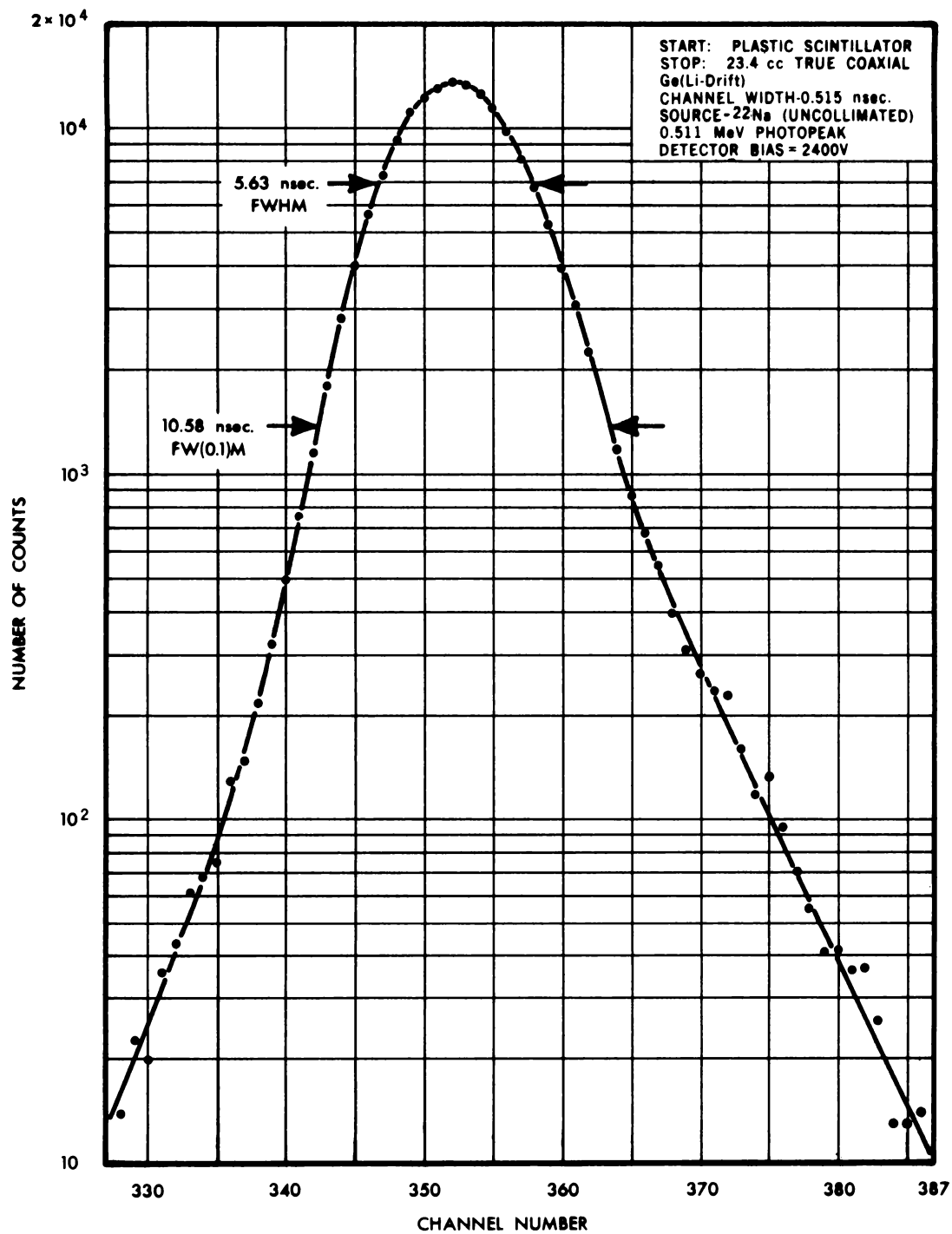


Figure 43. Timing spectrum using a 23.4 cc true coaxial Ge(Li) detector detecting 511 keV gamma rays.

CHAPTER V

CONCLUSIONS

I. SUMMARIZING DISCUSSION

The purpose of this dissertation was to study the application of filters to time analysis of signals from the charge-sensitive preamplifier used with Ge(Li) detectors. The sources of timing error were described, the optimum filter was determined, particular filters were examined, and experimental comparison was made. Particular emphasis was placed on the minimization of noise/slope error in time measurement.

In Chapter I a description of Ge(Li) detectors was presented. The uses of this detector and the advantages over other detectors in these uses were described. The problems in the measurement of time using these detectors were pointed out, and the methods of performing time measurement by other investigators were referenced. In addition, the purpose and scope of the dissertation were outlined; and the method of examination was described.

In Chapter II the sources of timing error were defined. To do this, a determination of the signal and noise from a Ge(Li) detector, charge-sensitive preamplifier system applicable to an analysis of time measurement was necessary. From this signal and noise, two sources of error in the measurement of time were analyzed. The first source of timing error examined was charge collection variations in the detector. These variations result in pulse shape variations at the preamplifier

output and, therefore, variations in the time the output signal crosses some discriminator level. To reduce the effect of charge collection variations, a low discriminator level is necessary. By present methods the minimum discriminator level is limited by the noise level. Thus, the necessity of a study of the effect of filtering was apparent. The second source of error in time measurement was due to noise inherently disturbing the time at which discrimination was made. This error source was referred to and described as noise/slope error.

The effect of filtering on the sources of error in time measurement was analyzed in Chapter III. Since charge collection variations were so dependent on a particular detector, the noise/slope error was examined quantitatively in detail; and the results were applied qualitatively to the error due to charge collection variations. In the first section, the optimum filter and corresponding minimum time measurement for the noise/slope error was found. This filter was found for any signal and noise and for the signal and noise from a charge-sensitive preamplifier. For the charge-sensitive preamplifier case, the filter was determined to be physically non-realizable.

In the next section, several time-invariant RC filters were examined; and the effect of these filters on noise/slope error was compared to the results for the optimum filter. The use of time-variant filters in time measurement was analyzed in the third section. The time-variant filters were shown to have several distinct advantages over the time-invariant filters. The first advantage is that, in general, the noise/slope error is less for the time-variant filter as

compared to the corresponding time-invariant filter. A minimum noise/slope error for those filters examined was 1.23 times the non-realizable optimum value. This minimum was obtained using an RC low-pass, RC high-pass filter after a linear gate. In reducing charge collection variations, two advantages using time-variant filters were found. The first was that the minimum discriminator level was limited by the noise of the gate or the noise that builds up during the interval between the gate opening time and the actual signal arrival time. This minimum discriminator level was much lower than the limitation of steady-state noise from a time-invariant filter. The second advantage was that at low discriminator levels the noise/slope error was less for the time-variant filter than for the time-invariant filter. The combination of these two advantages allows low-level discrimination on signals from a Ge(Li) detector, charge-sensitive preamplifier system, thereby reducing charge collection variations.

Experimental comparison of the results in Chapter III was the object of Chapter IV. In the first section the effect of filtering on noise/slope error was examined experimentally similar to the theoretical examination. The signal, noise, and filters were instrumented; and the data taken were compared to the theoretical information. In general, the theoretical and experimental data corresponded very well. In addition, the conclusions from the theoretical data were verified by the experimental work.

The second section of Chapter IV presented an examination of several time-invariant and time-variant RC filters used to perform time

measurement on the Ge(Li) detector, charge-sensitive preamplifier system. A conclusion of this section was that the best time-invariant filter examined was an RC high-pass filter of time constant ($2.2 RC$) equal to the charge collection time. An error in time measurement of 6.81 nanoseconds fwhm and 12.6 nanoseconds $fw(.1)m$ was measured for this filter. In addition, the time-variant filters composed of those with no filter before the gate produced poorer results than the corresponding time-invariant filter. This result was probably due to the variance in the gate opening time. However, the time-variant filter composed of an RC high-pass circuit before the gate and an RC low-pass circuit after the gate provided the best results. For $2.2 RC$ equal to 20 nanoseconds for both the high-pass and low-pass circuit, timing error of 5.63 nanoseconds fwhm and 10.58 nanoseconds $fw(.1)m$ was measured.

II. SUGGESTIONS FOR FURTHER STUDY

An obvious extension of this work is to examine other filters for use in time measurement. Other time-invariant filters involving networks with simple or complex poles could possibly reduce timing error. In addition, many other forms of time variability of filters exist that could be analyzed. For example, a variable-gain gate rather than a linear gate may have possible application. Also, the opening and closing of the gate at times other than those looked at here are possible.

The ultimate application of this work was to improve the time measurement of signals from Ge(Li) detectors. A very fruitful area appears to be the application of this information to silicon surface-

barrier detectors. The presentation of the theoretical noise/slope error as a function of filter and time constant would probably have to be extended to include different ratios of preamplifier rise times and charge collection times. In addition, the noise and signal spectrum as a function of frequency that is used in the examination of silicon surface-barrier detectors probably should include the entire frequency range as opposed to only the high frequency range for Ge(Li) detectors.

Another area that might be a straight-forward extension of this dissertation is an examination of the optimum filter for energy analysis where the detector current pulse is represented by some function other than an impulse function. This examination appears to be necessary because the assumption of an impulse function for the current pulse becomes invalid as the measurement time approaches the charge collection time.

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APPENDIX

APPENDIX

INSTRUMENTATION

Fast Amplifier

The circuit diagram of the fast amplifier used in the experimental work in this dissertation is shown in Figure 44. The development of this amplifier was performed by Williams and Neiler.²³ The measured rise time (10-90 percent) of the amplifier is about four nanoseconds. This circuit is used for those blocks in the block diagrams denoted by fast amplifier and variable gain fast amplifier. The variable gain is obtained by varying R_{e2} and R_{e5} . In addition, this circuit is used as the white noise source in the experimental work. The noise source of the 2.0 kilohm resistor at the input and the noise source at the input of the first transistor provide at the output white noise with a high frequency rolloff corresponding approximately to the amplifier rise time.

Fast Summing Amplifier

The amplifier used to sum the signal and noise is shown in Figure 45. The rise time of this circuit is 2.7 nanoseconds. The amplifier is a current feedback circuit. Pierce²⁴ provides an analysis of feedback amplifiers and demonstrates how to determine gain, dc properties, and high frequency properties.

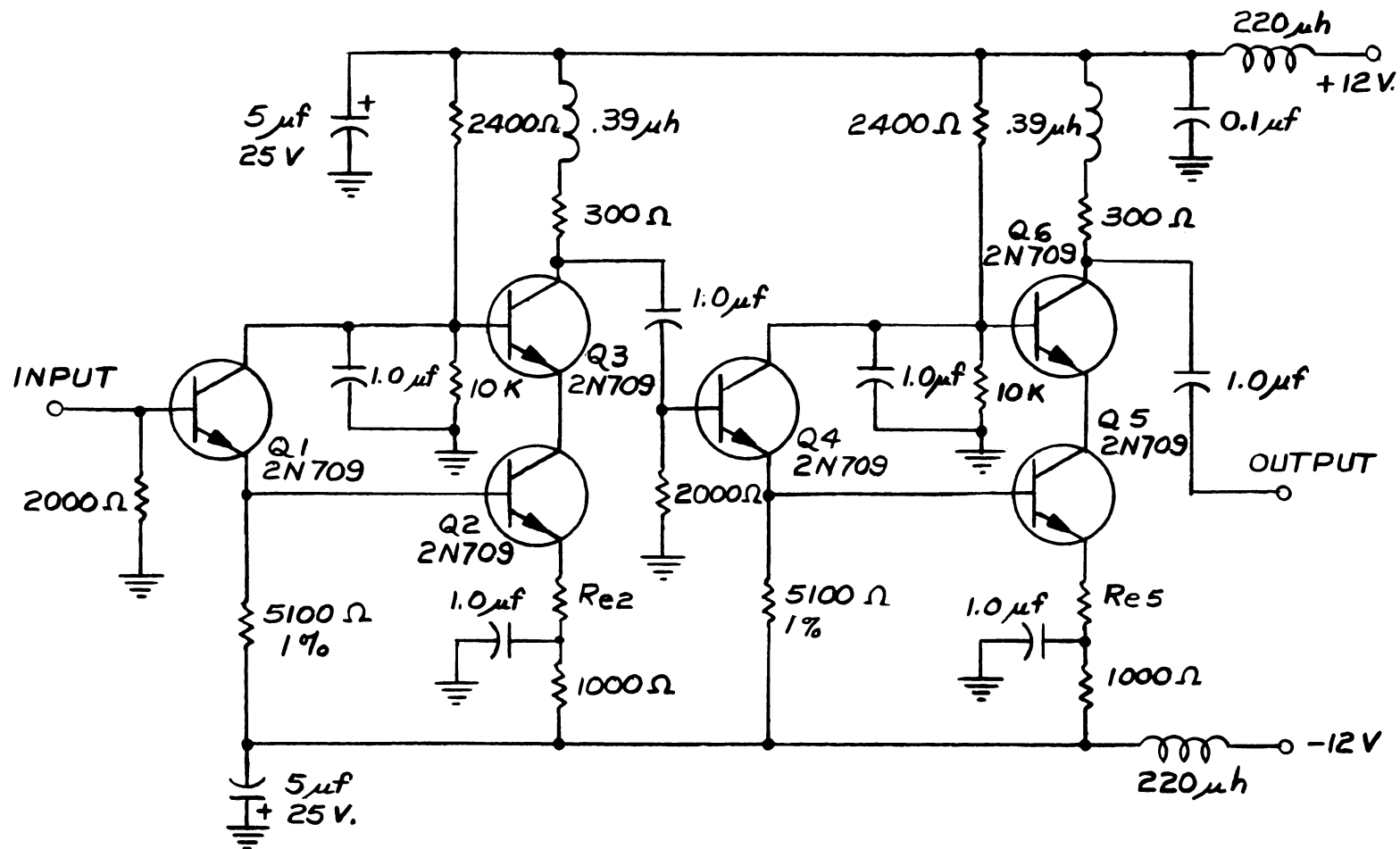
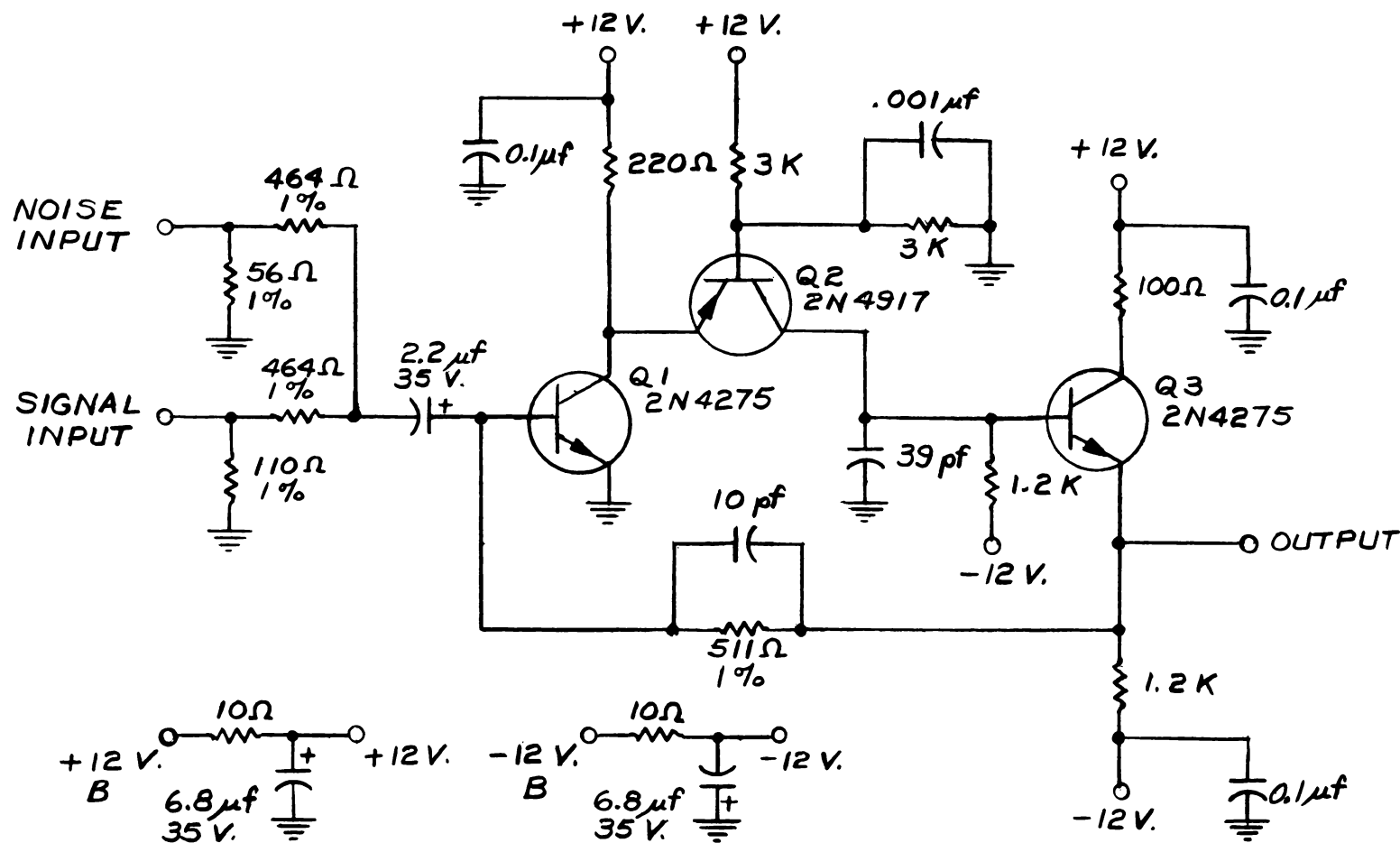


Figure 44. Circuit diagram of the fast amplifier used in experimental work.



BIAS SUPPLY FILTERS

Figure 45. Circuit diagram of the fast summing amplifier used in experimental work.

RC High-Pass, RC Low-Pass Filter

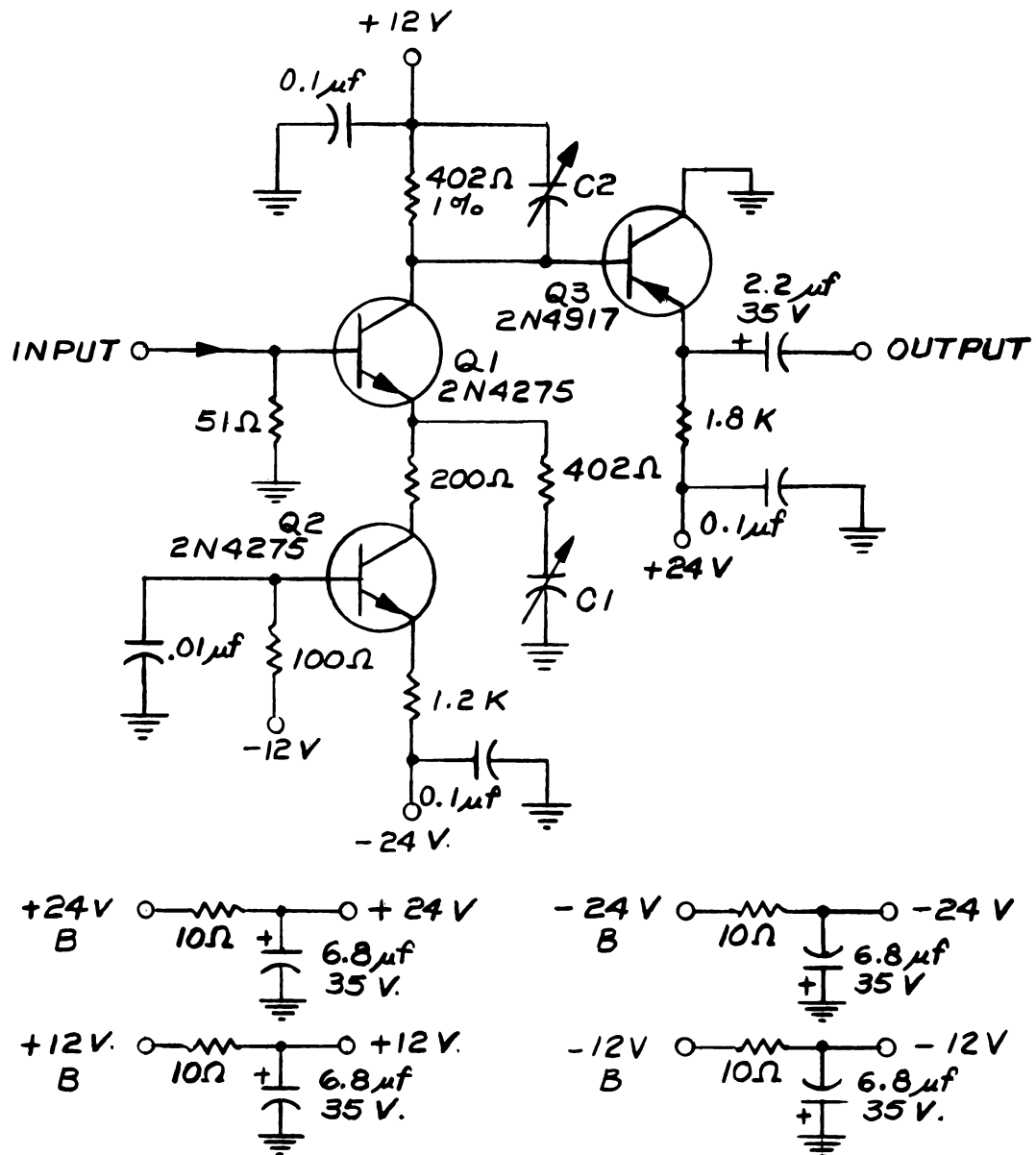
The circuit for the filter networks used in the experimental work is shown in Figure 46. The high-pass portion of the filter is determined by the RC network in the emitter of Q_1 . The low-pass portion of the filter is determined by the RC network in the collector of Q_1 . For the cases where a high-pass circuit is needed alone, C_2 is made an open circuit. Likewise, for the cases where a low-pass circuit is needed alone, C_1 is shorted out. The rise time of the circuit with C_1 equal to zero and C_2 equal to an open circuit is about 4.0 nanoseconds. The emitter follower of Q_3 can be adjusted for positive rather than negative output signals by using an NPN transistor instead of a PNP transistor and by properly biasing Q_3 .

Fast Linear Gate

The fast, linear gate used in the experimental work was a modification to the gate designed by White.²⁵ The circuit diagram of the gate is shown in Figure 47. The modification was to bias Q_4 such that a very small emitter current was present rather than Q_4 being entirely turned off. This modification made the rise time of the gate shorter and made it possible for lower level signals to be gated through. The rise time of the gate is less than 2.0 nanoseconds. The minimum signal level that can be gated through is about 10 millivolts.

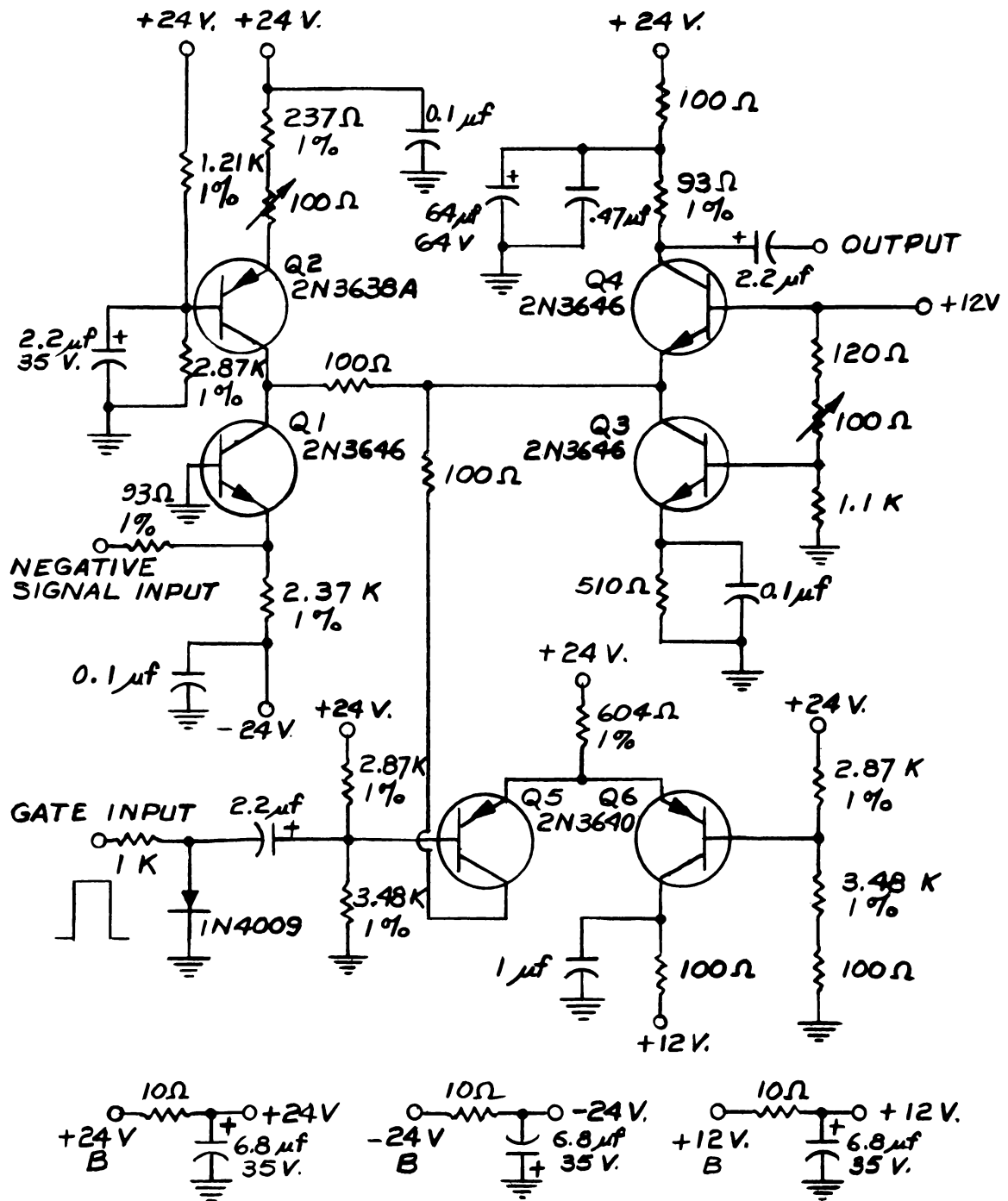
Charge-Sensitive Preamplifier

The circuit diagram of the charge-sensitive preamplifier used in this study is shown in Figure 48. The circuit is essentially the circuit of ORTEC's 118A preamplifier with the voltage amplifier taken out.



BIAS SUPPLY FILTERS

Figure 46. Circuit diagram of the RC high-pass, RC low-pass filter used in experimental work.



BIAS SUPPLY FILTERS

Figure 47. Circuit diagram of the fast linear gate used in experimental work.



Since the rise time and energy resolution depend on the detector capacitance, a value of 27 picofarads was used at the input to make the measurements. The rise time (10-90 percent) was measured to be 48 nanoseconds. The energy resolution for a 2.0 microsecond RC high-pass, low-pass filter was 2.23 keV fwhm germanium referred to the input. The measurement of charge gain gives a value of 48 millivolts per MeV germanium.

VITA

Terry Dean Douglass was born in Jackson, Tennessee, on October 26, 1942. He attended the public schools of Jackson and was graduated from Jackson High School in June 1960. In June 1965, he received the B.S. degree in electrical engineering from the University of Tennessee. While completing his work for a B.S. degree, he was a member of the Cooperative Engineering Scholarship Program doing his field work at the Jackson Utility Division, Jackson, Tennessee. During the period prior to receiving the B.S. degree, he held Schlumberger, Lockett, and Alumni Scholarships at the University of Tennessee.

In June 1966, he received the M.S. degree in electrical engineering from the University of Tennessee and is currently working toward the Ph.D. degree in electrical engineering at the University of Tennessee. While doing graduate work, he has been a recipient of a National Science Foundation Cooperative Graduate Fellowship and a National Defense Education Act Title IV Fellowship. Work on the Ph.D. dissertation and work during one summer period was performed at ORTEC, Incorporated, Oak Ridge, Tennessee.

Mr. Douglass is a member of Tau Beta Pi, Eta Kappa Nu, Phi Kappa Phi and a past student member of the Institute of Electrical and Electronic Engineers. After completion of the Ph.D. degree requirements, he plans to join ORTEC working in the life sciences products department.