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I am submitting herewith a thesis written by Michael A. Torbett entitled "Fraunhofer Diffraction of Light and of Ultrasonic Waves by Gratings Having Multiple Periodicities." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.

M. A. Breazeale, Major Professor

We have read this thesis and recommend its acceptance:

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Graduate Studies and Research

FRAUNHOFER DIFFRACTION OF LIGHT AND OF ULTRASONIC WAVES BY GRATINGS HAVING MULTIPLE PERIODICITIES

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ABSTRACT

The theory of Fraunhofer diffraction of light by coherently illuminated amplitude transmission gratings having an arbitrary number of periodicities has been developed by Fourier transform techniques. This mathematical description has been specialized for the case of a grating having a double periodicity. The resulting expression has been tested experimentally. In addition, the case of the singly-periodic grating was also considered. In each case agreement between theory and experiment was found to be reasonable in the lower diffraction orders. In the higher diffraction orders, the agreement was not as good.

An ultrasonic schlieren apparatus made it possible to model the optical experiment with an analogous acoustical one. Photographs of the diffraction of ultrasonic waves by gratings having single and double periodicities were made. Qualitative comparison between the optical and acoustical configurations is made.

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CHAPTER I

INTRODUCTION

Diffraction has been defined by Sommerfeld (1954) to be the departure of light from a rectilinear path which cannot be attributed either to refraction or reflection. Such departures from rectilinear propagation usually occur in situations where light is required to pass close to the edge of an opaque body or through a narrow aperture.

The first reference to diffraction phenomena is attributed to the work of Leonardo da Vinci (Born and Wolf, 1964); however, such phenomena were first accurately described by Francesco Maria Grimaldi (1665) in a book published two years after his death. Grimaldi observed that the shadows cast by rods and circular apertures of light from a pinhole source were wider than predicted from the geometry of rays, and that the edges of the shadows had colored fringes associated with them. For a historical summary of such developments in diffraction theory, see Born and Wolf, 1964, and Goodman, 1968.

The phenomenon of diffraction was known both to Huygens and to Newton. Newton discusses Grimaldi's observations in his Third Book of Optics. However, even though Huygens was the proponent of the wave theory of light he did not believe in diffraction and Newton did not see in it any justification for a wave theory of light.

Christian Huygens (1690) put forth the theory that light could be treated as a wave, rather than as a stream of particles. Using Huygens' ideas of secondary emission as a basis, it is possible to produce a theory

of diffraction and, by means of it, to explain nearly all observed diffraction phenomena.

Thomas Young (1802) succeeded in experimentally putting the wave theory of light on firm ground by showing that the two beams of light can be made to interfere with each other. Young also was able to deduce from his experiments the wavelength of the light, the first such measurements of this quantity.

Augustin Jean Fresnel (1816) combined the works of Huygens and Young and showed that diffraction could be explained by the application of Huygens' principle of secondary emission together with the principle of interference. While Fresnel was forced to make some rather arbitrary assumptions concerning the effective amplitudes and phases of the secondary sources, he was able to make calculations which were in excellent agreement with experiment.

It was not until 1882 that Gustav Kirchhoff (1882, 1883) was able to put Fresnel's theory on a more sound mathematical basis by showing that the amplitudes and phases ascribed to the secondary sources by Fresnel were logical consequences of the wave nature of light. Kirchhoff's formulation of diffraction theory is based on a set of assumptions concerning the boundary conditions on the amplitude of the light disturbance at the plane of the screen containing the diffracting aperture; i.e.,

1. across the diffracting aperture, the disturbance and its normal derivative are the same as they would be in the absence of the screen, and
2. over the remaining portion of the plane of the screen, the disturbance and its normal derivative are identically zero.

These assumptions were later shown to be inconsistent with each other first by Poincaré (1892) and again by Sommerfeld (1896). In light of this internal inconsistency, Kirchhoff's formulation must be regarded as a first approximation--even though in a large number of situations its results agree quite well with experiment.

In his lectures, Sommerfeld removed the inconsistency of the Kirchhoff formulation by eliminating the necessity to simultaneously impose boundary conditions on both the disturbance and its normal derivative. The resulting treatment of diffraction has come to be known as the Rayleigh-Sommerfeld theory [cf. Rubinowicz (1917)].

In spite of Sommerfeld's refinement of Kirchhoff's theory, there is still embodied in it one major simplification--light is treated as a scalar quantity, i.e., only the scalar amplitude of one transverse component of either the electric or the magnetic field is considered. The assumption is made that any other components can be treated independently in a similar manner. This approach ignores the fact that various vector components of the electric and magnetic fields are coupled through Maxwell's equations and thus cannot be treated independently. Kottler (1923) was the first to give a satisfactory treatment of diffraction theory that accounts for this vectorial nature of light. In some situations this vector theory is necessary; however, the scalar treatment yields very good results if two conditions are met (Silver, 1962):

1. the diffracting aperture must be large compared with the wavelength of light being used, and

2. the diffraction effects must not be observed too close to the diffracting aperture.

These conditions are met in the experiments to be described. Therefore, a scalar theory will be adequate to describe the experimental results to be presented in this thesis.

The principle of the diffraction grating was discovered by Rittenhouse (1786) but attracted no particular attention. The principle was rediscovered in 1819 by Joseph Fraunhofer (1821) who extended the principles of interference and diffraction to the case of two, and later to many, openings. Fraunhofer (1823) succeeded in ruling the first glass transmission gratings using a ruling machine of his own making.

Due to the difficulty of ruling high quality gratings, considerable effort was expended to find ways of copying good gratings once they were made. Lord Rayleigh (1872, 1881, 1896) found that quite good copies could be made using a photographic process similar to contact printing. This photographic grating becomes essentially an amplitude grating (neglecting phase effects due to variations in the thickness in the film) even though the original grating could have been a phase grating. In connection with the study of these gratings, Lord Rayleigh (1881) studied Fresnel diffraction by periodic structures. Fresnel diffraction of light by phase gratings such as ultrasonic waves has also been studied (Hiedemann and Breazeale, 1959).

Associated with the development of high quality gratings was the development of a theory which would describe the effect of various imperfections of the grating on the diffraction pattern. It was observed

that any superimposed periodicity led to diffraction effects that needed to be investigated in detail if the demand for higher resolution gratings was to be met. Considerable attention was given to this problem--notable among these investigations were those of Michelson (1912, 1927), Lord Rayleigh (1874), Rowland (1893, 1902), Sparrow (1919), J. A. Anderson (1922), R. W. Wood (1924), and Stroke (1960).

Throughout most of the early theoretical work concerning the diffraction of light, it was assumed that the source of illumination was a monochromatic point source. Light from a real physical source is never strictly monochromatic, i.e., even the sharpest of spectral lines has a finite line width. In addition, a physical source must have a finite extension and therefore cannot be a true point source. Thus, an accurate comparison between theory and experiment, of necessity, must include a consideration of the effect of the partial coherence of light on the diffraction phenomenon. A general formulation of coherence theory was published by Wolf (1955) and was later applied to diffraction by a slit by Parrent and Shinner (1961). Shore, et al. (1966) considered Fraunhofer diffraction by slits and circular apertures illuminated with partially coherent light and showed that a detailed knowledge of the mutual coherence function across the diffracting aperture is required in order to fully describe the diffraction phenomena.

In this thesis, Fourier transform theory is applied to the problem of diffraction by amplitude transmission gratings having multiple periodicities. Expressions for the intensity of the Fraunhofer diffraction pattern for gratings having two, three, and N periodicities are developed. A

computer program was written to aid in the numerical evaluation of the theoretical intensity distribution.

In the experiment to be described, several amplitude transmission gratings having dual periodicities were made by means of photographic reproduction of suitable patterns. Measurements were made on the diffraction patterns produced by these gratings. The results were compared with the results calculated by using the method of Fourier transforms.

In addition, an ultrasonic schlieren system made it possible to model the optical experiment with an analogous acoustical configuration. Small, thin cylinders of graphite were mounted in such a manner that they formed a grating having a periodicity of the order of 2 mm which is comparable with the wavelength, in water, of the 2 MHz ultrasonic wave which was used. Although the assumptions of Kirchhoff's scalar diffraction theory were not satisfied in detail, a good qualitative correspondence could be seen between the optical theory and the schlieren photographs of the diffraction of ultrasonic waves by these gratings.

CHAPTER II

THEORY

The diffraction of light by a periodic grating possibly could be solved in considerable detail; however, the restriction of the theory, by making the Fraunhofer approximations, leads to an integral which is recognizable as a Fourier transform. The application of Fourier analysis to the theoretical problem can lead to an understanding of the physics involved which might otherwise be missed if a more detailed treatment were considered. The range of experimental parameters over which the theory should be valid is restricted somewhat by this procedure, but the loss of completeness is more than compensated for by the increase in intuitive understanding.

In this chapter, the Kirchhoff diffraction integral is presented, and the specialization from the Fresnel approximations to the Fraunhofer approximation is made. After showing that the resulting integral is in the form of a Fourier transform, Fourier analysis of several multiply-periodic gratings is made. Diffraction by a grating having two periodicities is considered first. [An adequate treatment of the Fourier analysis of a grating having a single period can be found in the book by Lipson and Lipson (1969).] Next, a grating having three periodicities is considered. The results of these two problems then allows one to generalize to a grating having an arbitrary number of

periodicities. The solution of this problem leads to a function which describes the diffraction pattern in such a manner that a direct comparison can be made between theory and experiment.

I. KIRCHHOFF DIFFRACTION THEORY

Consider the situation shown in Figure 1. Kirchhoff (1882) has shown that, for an aperture of area Σ illuminated by a single spherical wave of amplitude

$$U(P_1) = \frac{U_0 \exp(ikr_{21})}{r_{21}} \tag{1}$$

arising from a point source at P_2, which is at a distance r_{21} from P_1 and which is assumed to be many optical wavelengths, the amplitude of the disturbance at P_0 is given by

$$U(P_0) = \frac{U_0}{i\lambda} \iint_{\Sigma} \frac{\exp[ik(r_{21} + r_{01})]}{r_{21} r_{01}} \left[\frac{\cos(\vec{n},\vec{r}_{01}) - \cos(\vec{n},\vec{r}_{21})}{2}\right] d\xi d\eta \ . \tag{2}$$

This result is commonly known as the Fresnel-Kirchhoff diffraction integral. The method for arriving at this result is restricted to that of an aperture illuminated by a single expanding spherical wave; however, it can be shown that an arbitrary source of illumination can always be broken down into a collection of such point sources. The Fresnel-Kirchhoff diffraction integral can then be applied to each point source separately, and the results can be superposed because of the linearity of the wave equation.

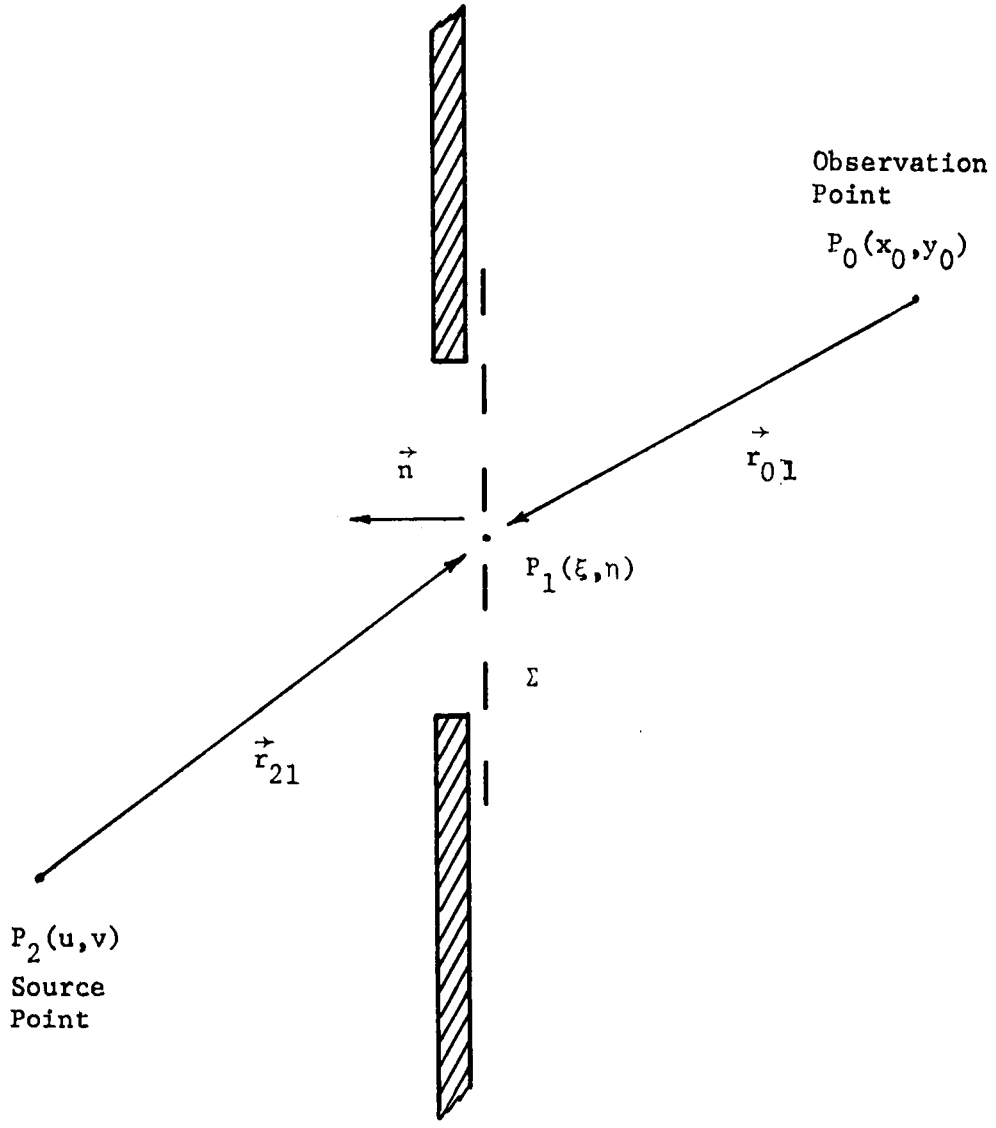


Figure 1. Application of Kirchhoff's diffraction integral to point source illumination of a plane diffracting screen.

It should be noted that a similar result was derived by Sommerfeld [cf. Rubinowicz (1917)] who eliminated, by a judicious choice of a Green's function, the necessity of imposing boundary conditions on both the disturbance and its normal derivative simultaneously as Kirchhoff had done. Sommerfeld arrived at the result

$$U(P_0) = \frac{1}{i\lambda} \iint_{\Sigma} U(P_1) \frac{\exp(ikr_{01})}{r_{01}} \cos(\vec{n},\vec{r}_{01}) ds \tag{3}$$

which when evaluated for an aperture illuminated by a spherical wave expanding from a point source at P_2, i.e.,

$$U(P_1) = U_0 \frac{\exp(ikr_{21})}{r_{21}} , \tag{4}$$

gives what is known as the Rayleigh-Sommerfeld diffraction integral

$$U(P_0) = \frac{U_0}{i\lambda} \iint_{\Sigma} \frac{\exp[ik(r_{21} + r_{01})}{r_{21} r_{01}} \cos(\vec{n},\vec{r}_{01}) ds . \tag{5}$$

Note that the only difference between this and the Fresnel-Kirchhoff formula is the obliquity factor.

Consider the general form of the Rayleigh-Sommerfeld diffraction formula given in Eq. 3. In order to facilitate the evaluation of this expression it is convenient to make certain approximations which are based on an assumption that the distance z between the diffracting aperture and the plane of observation is much, much greater than either the maximum linear dimension of the aperture or the region of interest

in the plane of observation. This will then allow the obliquity factor to take on the approximate value of unity over the area of interest; i.e.,

$$\cos(\vec{n},\vec{r}_{01}) \approx 1 \quad \text{for large } z \ . \tag{6}$$

This condition on z also allows the quantity r_{01} in the denominator of the exponential term in Eq. 5 to be replaced by z. The same r_{01} term in the exponential, however, cannot be replaced by z since the difference between r_{01} and z would be multiplied by k, a large number.

From Figure 2 it is seen that the distance r_{01} is given exactly by

$$r_{01} = \sqrt{z^2 + (x_0 - \xi)^2 + (y_0 - \eta)^2} \tag{7}$$

or

$$r_{01} = z\sqrt{1 + \left(\frac{x_0 - \xi}{z}\right)^2 + \left(\frac{y_0 - \eta}{z}\right)^2} \ . \tag{8}$$

For $z >> (x_0 - \xi)$ and $z >> (y_0 - \eta)$, Eq. 8 can be expanded, using the binomial expansion, to give

$$r_{01} \approx z\left[1 + \frac{1}{2}\left(\frac{x_0 - \xi}{z}\right)^2 + \frac{1}{2}\left(\frac{y_0 - \eta}{z}\right)^2\right] \tag{9}$$

where only the first two terms have been kept. (This is the so-called Fresnel approximation.) In terms of the approximations given by Eqs. 6 and 9, Eq. 3 can be written as

$$U(P_0) = U(x_0,y_0) = \frac{\exp ikz}{i\lambda z} \iint_{\Sigma} U(\xi,\eta) \exp\left[\frac{ik}{2z}\left[(x_0 - \xi)^2 + (y_0 - \eta)^2\right]\right] d\xi d\eta \tag{10}$$

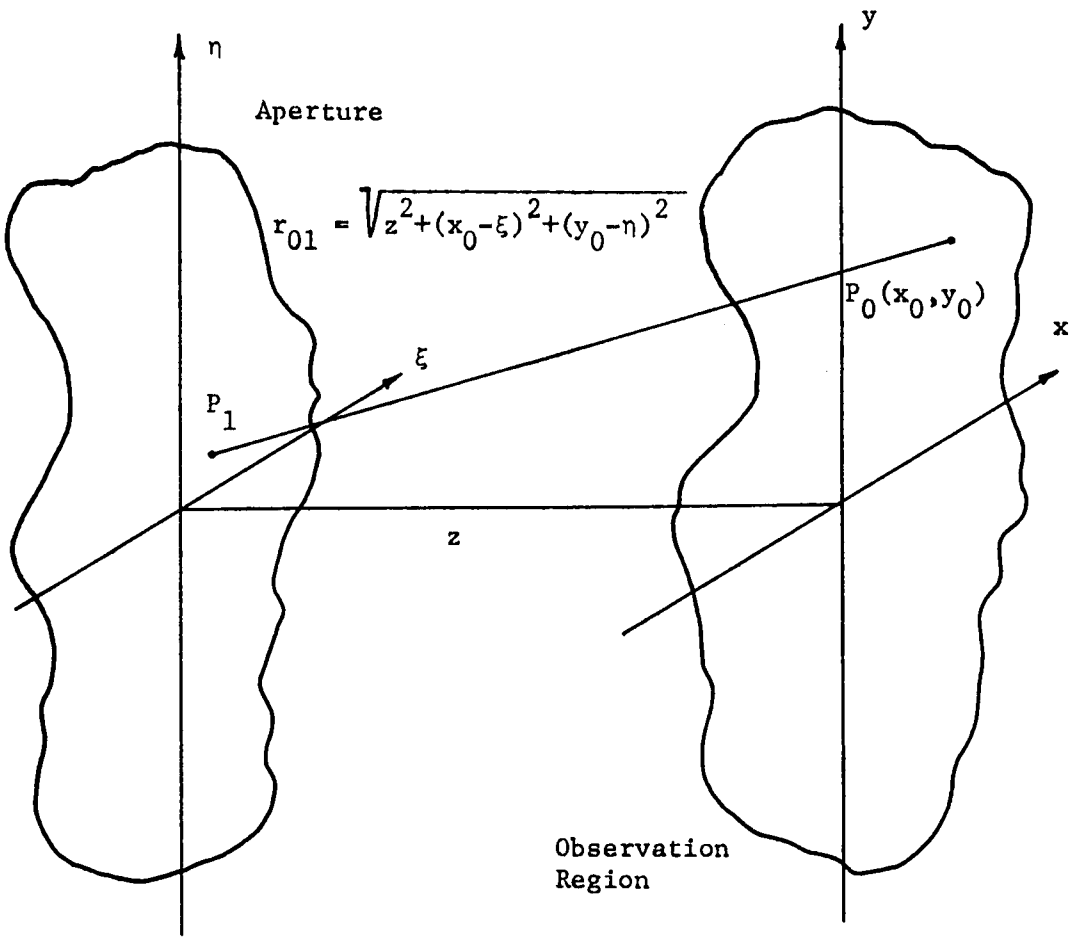


Figure 2. Diffraction geometry showing relation of the diffracting screen and the plane of observation for the application of Kirchhoff's integral to Fraunhofer diffraction.

or, by expanding the quadratic terms in the exponential function,

$$U(x,y) = \frac{\exp(ikz)}{i\lambda z} \exp[i \frac{k}{2z}(x_0^2 + y_0^2)] \iint_{\Sigma} \left[U(\xi,\eta) \exp[i \frac{k}{2z}(\xi^2 + \eta^2)]\right]$$

$$\exp[-i \frac{2\pi}{\lambda z}(x_0\xi + y_0\eta)]d\xi d\eta \ . \qquad (11)$$

The limits of integration can be taken to be infinite from this point on since, in accordance with the Kirchhoff boundary conditions, the aperture distribution $U(\xi,\eta)$ is identically zero outside the aperture Σ.

If now the stronger Fraunhofer assumption is made that

$$\frac{k}{2z}(\xi^2 + \eta^2)_{max} << 1 \ ,$$

the quadratic phase factor $\exp[i \frac{k}{2z}(\xi^2 + \eta^2)]$ in Eq. 11 is approximately unity over the entire diffracting aperture, and the disturbance in the observation plane is given by a suitable multiple of the Fourier transform of the aperture distribution itself. Thus

$$U(x_0,y_0) = K \int_{-\infty}^{\infty} U(\xi,\eta) \exp[-i \frac{2\pi}{\lambda z}(x_0\xi + y_0\eta)]d\xi d\eta \ , \qquad (12)$$

where

$$K = \frac{\exp(ikz)}{i\lambda z} \exp[i \frac{k}{2z}(x_0^2 + y_0^2)] \ . \qquad (13)$$

Note from Figure 2 that $\frac{x_0}{z} = \tan\theta_x$, but for large z, as we are assuming, $z \simeq r_{01}$ so that

$$\tan\theta_x \simeq \sin\theta_x \ . \tag{14}$$

Similarly for y_0/z we see that

$$\tan\theta_y \simeq \sin\theta_y \ . \tag{15}$$

Thus Eq. 12 can be written as

$$U(x_0,y_0) = K \iint_{\infty}^{\infty} U(\xi,\eta) \exp[-i \frac{2\pi}{\lambda}(\xi\sin\theta_x + \eta\sin\theta_y)]d\xi d\eta \tag{16}$$

which is in the form of a two-dimensional Fourier transform of $U(\xi,\eta)$, as will be shown in the next section.

II. FOURIER REPRESENTATION OF DIFFRACTION PHENOMENA

Fourier Transform Notation

In looking through the literature, one finds that several different forms of the Fourier transform exist. The form of the Fourier transform pair to be used in this thesis is given by

$$F(s) = \int_{-\infty}^{\infty} f(x)\ e^{-i2\pi sx} dx \tag{17}$$

$$f(x) = \int_{-\infty}^{\infty} F(s)\ e^{+i2\pi sx} dx \ . \tag{18}$$

It should be noted that it is not uncommon to find several other equally valid transform pairs which include

$$F(s) = \int_{-\infty}^{\infty} f(x)\ e^{-ixs} dx \tag{19}$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s)\, e^{ixs} ds \ , \tag{20}$$

and

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-ixs} dx \tag{21}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s)\, e^{ixs} ds \ . \tag{22}$$

In order to conveniently represent the amplitude transmission functions to be considered, a concise notation for a rectangular function will be chosen such that

$$\text{Rect}(x) = \begin{cases} 0 & |x| > 1/2 \\ 1/2 & |x| = 1/2 \\ 1 & |x| < 1/2 \end{cases} . \tag{23}$$

The linearity of the Fourier transform makes it an especially attractive and powerful method of solving Fraunhofer diffraction problems. In those cases in which it is possible to describe complicated diffracting objects in terms of several simpler ones, the linearity of the Fourier transform simplifies the process of obtaining the complicated diffraction pattern: it is simply the result of addition of the amplitudes of the appropriate simpler diffraction patterns. In addition, for identical apertures one can make use of the principle of convolution. An array of equally spaced similar apertures can be considered as the convolution of one such aperture with a linear array of δ-functions representing the

separation of the apertures. From the convolution theorem, then, the diffraction pattern produced by an array of similar apertures is the product of the Fourier transform of one such aperture and that of an array of δ-functions. Thus, the problem of diffraction by a grating can be divided into two parts--the evaluation of the Fourier transform of an appropriate "single" aperture--the diffraction term, $A_{diff}(\theta)$, and that of the set of δ-functions--the interference term, $A_{interf}(\theta)$.

Fourier Transform Representation of Fraunhofer Diffraction by a Grating Having Two Slit Widths

The purpose of this section is to derive an expression for the intensity of the Fraunhofer diffraction pattern produced by an amplitude transmission grating having two different slit widths.

The procedure to be followed is

1. to represent the amplitude transmission function as the convolution of an appropriate "modified single slit" (composed of two different width single slits) and a linear array of δ-functions having a spacing equal to the width of the "modified single slit";
2. to evaluate the amplitude $A_T(\theta)$ of the diffraction pattern which is given by the Fourier transform of the transmission function (using the fact that the transform of the convolution is given by the product of the transforms);
3. to calculate the intensity I given by

$$I_T(\theta) = A_T(\theta)A_T^*(\theta) \ .$$

Consider the transmission function of the "modified single slit" shown in Figure 3. The transmission function of the modified single slit in question can be considered to be composed of two rectangle or rectangular functions which will be represented by Rect(x). In particular h Rect[(x - c)/b] represents a displaced rectangle function of height h and base b, centered on x = c (see Figure 4).

The transmission function shown in Figure 3 can be represented as

$$T(x) = \text{Rect}\left[\frac{x - (b + c/2)}{c}\right] + \text{Rect}\left[\frac{x - (2b + c + a/2)}{a}\right] . \quad (24)$$

The amplitude $A_{diff}(\theta)$ of the diffraction pattern produced by the transmission function given by Eq. 24 is just the Fourier transform of Eq. 24; i.e.,

$$A_{diff}(\theta) = ce^{-i2\pi(b + c/2)s}\,\frac{\sin c\pi s}{c\pi s} + ae^{-i2\pi(2b + c + a/2)s}\,\frac{\sin a\pi s}{a\pi s} \quad (25)$$

where $s = \frac{\sin\theta}{\lambda}$ and θ is measured from the grating normal.

Let us define several new variables:

$$\gamma = c\pi s = \frac{\pi c \sin\theta}{\lambda}$$

$$\alpha = a\pi s = \frac{\pi a \sin\theta}{\lambda} \quad (26)$$

$$P = 2\pi(b + c/2)s$$

$$Q = 2\pi(2b + c + a/2)s .$$

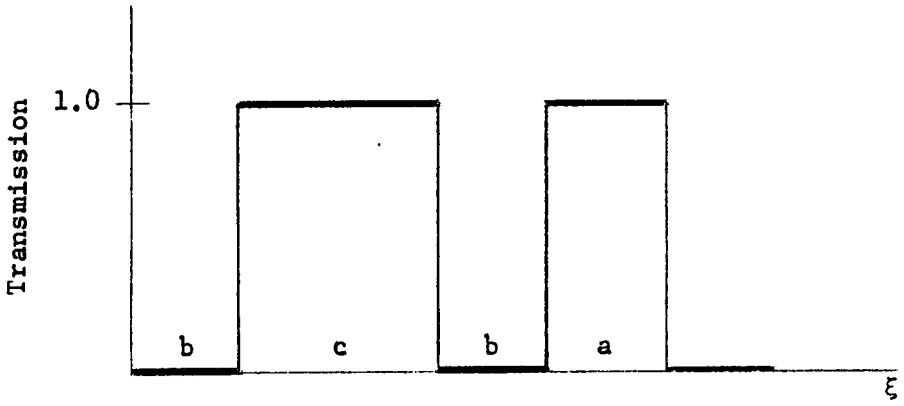


Figure 3. Representation of the amplitude transmission function of one period of a grating having two slit widths.

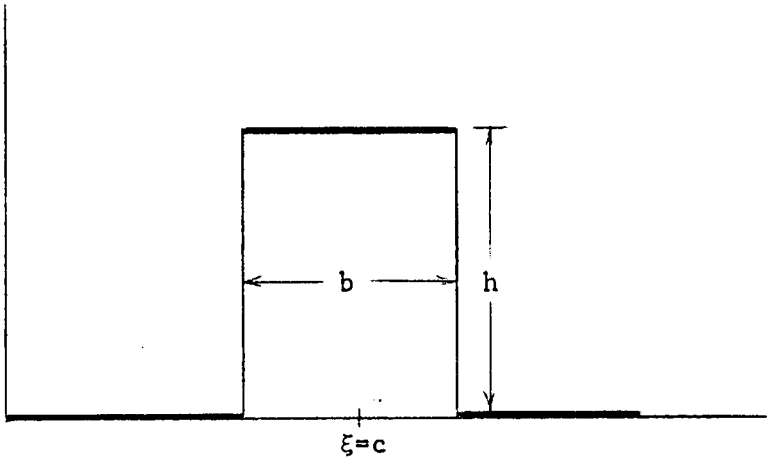


Figure 4. Representation of a general rectangle function.

Then, Eq. 25 becomes

$$A_{diff}(\theta) = ce^{-iP}\frac{\sin\gamma}{\gamma} + ae^{-iQ}\frac{\sin\alpha}{\alpha} . \tag{27}$$

The transform of a δ-function at $x = x_0$ can be calculated as follows. Let $f(x) = \delta(x - x_0) = \delta(x')$, then the Fourier transform of a δ-function located at x_0 is given by

$$F(s) = \int_{-\infty}^{\infty} \delta(x - x_0)\, \exp(i2\pi sx)dx . \tag{28}$$

Making the change of variable $x' = x - x_0$, Eq. 28 becomes

$$F(s) = \int_{-\infty}^{\infty} \delta(x')\, \exp[i2\pi s(x' - x_0)]dx' \tag{29}$$

or

$$F(s) = \exp(-i2\pi sx_0) \int_{-\infty}^{\infty} \delta(x')\, \exp(i2\pi sx')dx' . \tag{30}$$

And finally, the Fourier transform of a δ-function at x_0 is given by

$$F(s) = \exp(-i2\pi sx_0) . \tag{31}$$

Now for a collection of δ-functions at various values of x such that

$$f(x) = \sum_{m}^{M} \delta(x - x_m) \tag{32}$$

then by the linearity of Fourier transform, the Fourier transform of their array is given as

$$F(s) = \sum_{m} \exp(-i2\pi s x_m) \quad . \tag{33}$$

From this it is seen that, for a regular array of δ-functions with spacing d, the transform is

$$A_{interf}(\theta) = \sum_{m=1}^{M} \exp(-i2\pi s m d) \tag{34}$$

or

$$A_{interf}(\theta) = \exp(-i2\pi s d) \sum_{m=0}^{M-1} \exp(-i2\pi s m d) \tag{35}$$

where M is the number of δ-functions. If the geometric series is replaced by its sum, Eq. 35 becomes

$$A_{interf}(\theta) = \exp(-i2\pi s d) \left[\frac{1 - \exp(-i2\pi s M d)}{1 - \exp(-i2\pi s d)}\right] \quad . \tag{36}$$

Note that

$$1 - e^{-ix} = [e^{ix/2} - e^{-ix/2}]e^{-ix/2} \tag{37}$$

$$= [2i \sin(\tfrac{x}{2})]e^{-ix/2} \tag{38}$$

so that Eq. 36 can be written as

$$A_{interf}(\theta) = e^{-i2\pi s d} \frac{e^{\frac{-i2\pi s M d}{2}} \sin(\frac{2\pi s M d}{2})}{e^{\frac{-i2\pi s d}{2}} \sin(\frac{2\pi s d}{2})} \tag{39}$$

$$A_{interf}(\theta) = e^{-i2\pi s d}\, e^{-i\pi s M d}\, e^{+i\pi s d}\, \frac{\sin(\pi s M d)}{\sin(\pi s d)} \tag{40}$$

$$A_{interf}(\theta) = e^{-i(M+1)\pi sd} \frac{\sin M\pi sd}{\sin \pi sd} . \tag{41}$$

Let $\mu \equiv \pi sd$.

The amplitude of the diffraction pattern of a grating having two slit widths is given by the product of the transform of the "modified single slit" (Eq. 27) and the transform of the array of δ-functions (Eq. 41). Thus,

$$A_T(\theta) = A_{diff}(\theta)\, A_{interf}(\theta) \tag{42}$$

$$A_T(\theta) = \left[ce^{-iP} \frac{\sin\gamma}{\gamma} + ae^{-iQ} \frac{\sin\alpha}{\alpha}\right]\left[e^{-i(M+1)\mu} \frac{\sin M\mu}{\sin\mu}\right] . \tag{43}$$

Now the intensity is given by

$$I_T(\theta) = A_T(\theta)\, A_T^*(\theta) \tag{44}$$

$$I_T(\theta) = \left[\left[ce^{-iP} \frac{\sin\gamma}{\gamma} + ae^{-iQ} \frac{\sin\alpha}{\alpha}\right] e^{-i(M+1)\mu} \frac{\sin M\mu}{\sin\mu}\right]$$

$$\left[\left[ce^{iP} \frac{\sin\gamma}{\gamma} + ae^{iQ} \frac{\sin\alpha}{\alpha}\right] e^{i(M+1)\mu} \frac{\sin M\mu}{\sin\mu}\right] \tag{45}$$

$$I_T(\theta) = \left[c^2 \frac{\sin^2\gamma}{\gamma^2} + a^2 \frac{\sin^2\alpha}{\alpha^2} + ac \frac{\sin\alpha}{\alpha} \frac{\sin\gamma}{\gamma} \left[e^{i(Q-P)} + e^{i(P-Q)}\right]\right] \frac{\sin^2 M\mu}{\sin^2\mu} \tag{46}$$

$$I_T(\theta) = \left[c^2 \frac{\sin^2\gamma}{\gamma^2} + a^2 \frac{\sin^2\alpha}{\alpha^2} + ac \frac{\sin\alpha}{\alpha} \frac{\sin\gamma}{\gamma} [2\cos\phi]\right] \frac{\sin^2 M\mu}{\sin^2\mu} \tag{47}$$

where

$$\phi = 2\pi(b + c/2 + a/2)s \ .$$

Thus the diffraction pattern formed by an amplitude transmission grating having two slit widths is described by Eq. 47 above.

Fourier Transform Representation of Fraunhofer Diffraction by a Grating Having Three Slit Widths

The purpose of this section is to consider diffraction by an amplitude grating having three different slit widths. The procedure will be to extend the principles of the preceding section.

Consider the transmission function shown in Figure 5. Let us make the following definitions:

$$\begin{aligned}
&d = 3b + c + a + g \qquad \text{(the period)} \\
&P \equiv 2\pi(b + c/2)s \\
&Q \equiv 2\pi(2b + c + a/2)s \\
&R \equiv 2\pi(3b + c + a + g/2)s \qquad (48) \\
&\mathcal{C} \equiv c\,\frac{\sin\gamma}{\gamma} \qquad \gamma = \frac{\pi c \sin\theta}{\lambda} \\
&\mathcal{A} \equiv a\,\frac{\sin\alpha}{\alpha} \qquad \alpha = \frac{\pi a \sin\theta}{\lambda} \\
&\mathcal{G} \equiv g\,\frac{\sin\delta}{\delta} \qquad \delta = \frac{\pi g \sin\theta}{\lambda}
\end{aligned}$$

The amplitude of the diffraction pattern is given by the sum of the Fourier transforms of each slit:

$$A_{diff}(\theta) = \mathcal{C}\,e^{iP} + \mathcal{A}\,e^{iQ} + \mathcal{G}\,e^{iR} \ . \qquad (49)$$

The intensity of the diffraction term is given by:

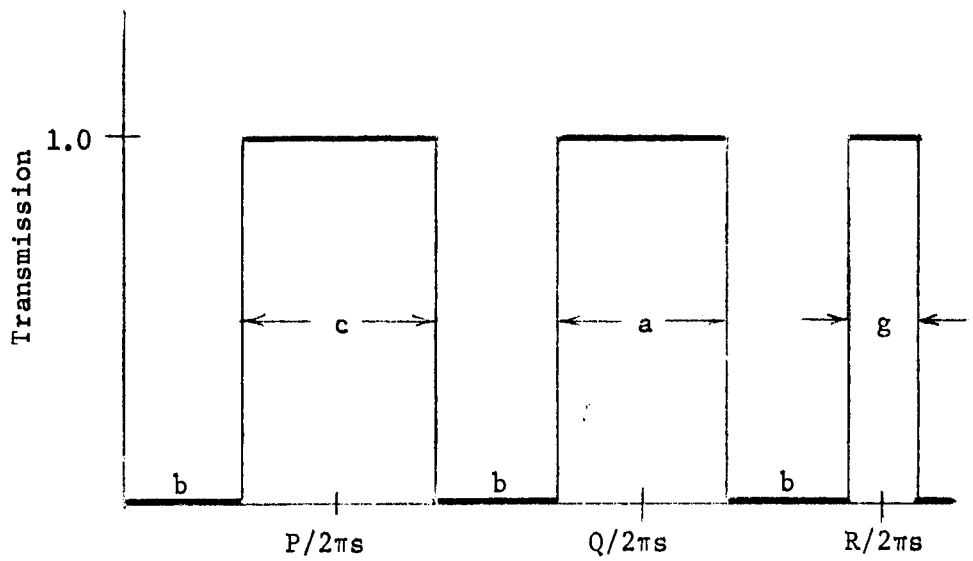


Figure 5. Representation of the amplitude transmission function of one period of a grating having three slit widths.

$$I_{diff}(\theta) = A_{diff}(\theta)\, A^{*}_{diff}(\theta) \ . \tag{50}$$

Thus

$$I_{diff}(\theta) = \mathcal{A}^2 + \mathcal{C}^2 + \mathcal{G}^2 + \mathcal{A}\mathcal{C}[e^{i(Q-P)} + e^{i(P-Q)}] + \mathcal{A}\mathcal{G}[e^{i(R-P)} + e^{i(P-R)}]$$

$$+\mathcal{C}\mathcal{G}[e^{i(R-Q)} + e^{i(Q-R)}] \ . \tag{51}$$

Now let

$$\begin{aligned} |Q-P| &= 2\pi(b + c/2 + a/2)s \equiv \phi \\ |R-P| &= 2\pi(2b + c/2 + a + g/2)s \equiv \kappa \\ |R-Q| &= 2\pi(b + a/2 + g/2)s \equiv \zeta \ . \end{aligned} \tag{52}$$

Equation 51 can now be written as

$$I_{diff}(\theta) = \mathcal{A}^2 + \mathcal{C}^2 + \mathcal{G}^2 + \mathcal{A}\mathcal{C}[e^{i\phi} + e^{-i\phi}] + \mathcal{A}\mathcal{G}[e^{i\kappa} + e^{-i\kappa}]$$

$$+\mathcal{C}\mathcal{G}[e^{i\zeta} + e^{-i\zeta}] \ , \tag{53}$$

but

$$e^{i\phi} + e^{-i\phi} = 2\cos\phi \tag{54}$$

so that Eq. 53 becomes

$$I_{diff}(\theta) = c^2 \frac{\sin^2\gamma}{\gamma^2} + a^2 \frac{\sin^2\alpha}{\alpha^2} + g^2 \frac{\sin^2\delta}{\delta^2} + 2ac \frac{\sin\alpha}{\alpha} \frac{\sin\gamma}{\gamma} \cos\phi$$

$$+ 2cg \frac{\sin\gamma}{\gamma} \frac{\sin\delta}{\delta} \cos\kappa + 2ag \frac{\sin\alpha}{\alpha} \frac{\sin\delta}{\delta} \cos\zeta \ . \tag{55}$$

The Fraunhofer diffraction pattern for a grating having three slit widths can now be formed by multiplying the expression given in Eq. 55 by the term

$$I_{interf}(\theta) = \frac{\sin^2 M\mu}{\sin^2 \mu} ,$$

where M is the number of groups of the three different slits and μ is given by

$$\mu = \frac{\pi d \sin\theta}{\lambda} \tag{56}$$

and

$$d = 3b + c + a + g .$$

Thus the diffraction pattern formed by a grating having three slit widths is described by

$$I_T(\theta) = \left\{ c^2 \frac{\sin^2\gamma}{\gamma^2} + a^2 \frac{\sin^2\alpha}{\alpha^2} + g^2 \frac{\sin^2\delta}{\delta^2} + 2ac \frac{\sin\alpha}{\alpha} \frac{\sin\gamma}{\gamma} \cos\phi \right.$$
$$\left. + 2cg \frac{\sin\gamma}{\gamma} \frac{\sin\delta}{\delta} \cos\kappa + 2ag \frac{\sin\alpha}{\alpha} \frac{\sin\delta}{\delta} \cos\zeta \right\} \frac{\sin^2 M\mu}{\sin^2 \mu} . \tag{57}$$

Fourier Transform Representation of Fraunhofer Diffraction by a Grating Having N Discrete Slit Widths

The purpose of this section is to generalize the results obtained in the preceding sections and to arrive at an expression for the intensity of the Fraunhofer diffraction pattern produced by a grating having N discrete slit widths.

Consider a "modified single slit" of the form shown in Figure 6. The "modified slit" is composed of N single slits each with a discrete slit width a_i and separated by a distance b. In order to apply the Shift Theorem, a general expression is needed for the center of each slit. The distance from the origin to the center of the nth slit is denoted by

$$\bar{a}_n = nb + a_n/2 + \sum_{i=1}^{n-1} a_i \ . \tag{58}$$

Similarly, the center of the mth slit is given by

$$\bar{a}_m = mb + a_m/2 + \sum_{i=1}^{m-1} a_i \ . \tag{59}$$

Now each single slit will interfere with every other slit so one must have a variable to express this interaction in the interference term. Let ϕ_{nm} be the argument for the interference term and be defined as

$$\phi_{nm} = 2\pi|\bar{a}_n - \bar{a}_m|\frac{\sin\theta}{\lambda} \ . \tag{60}$$

Also define the argument of the single slit diffraction term to be α_i such that

$$\alpha_i = \frac{\pi a_i \sin\theta}{\lambda} \tag{61}$$

where a_i is the width of the ith slit.

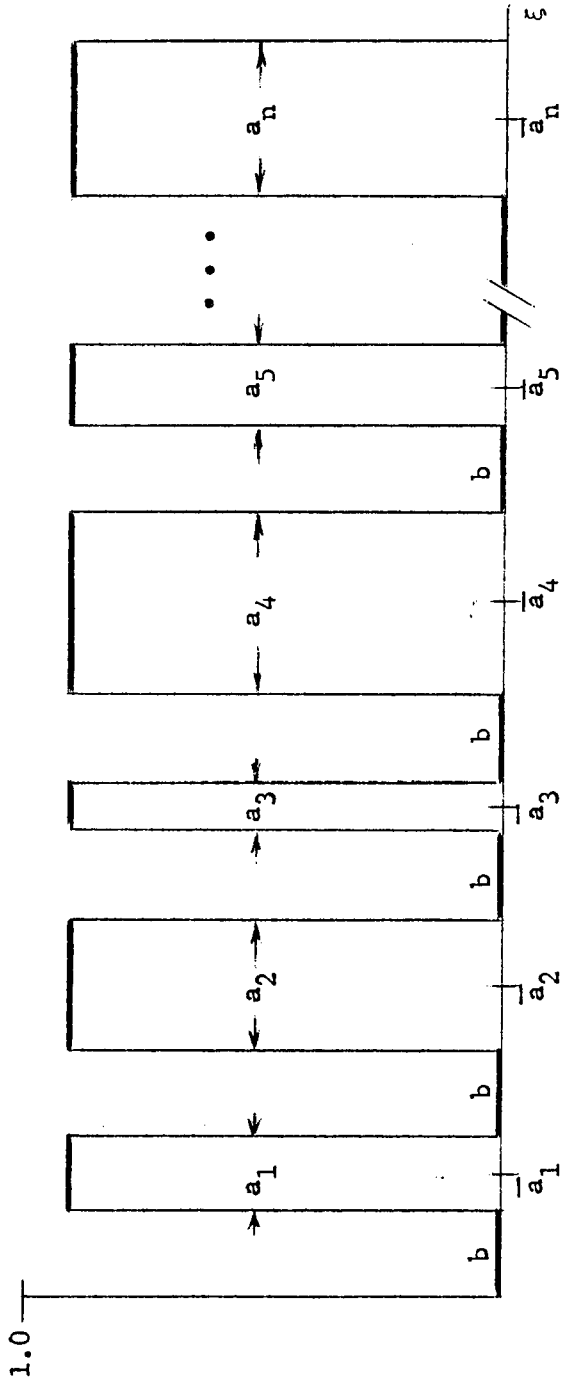


Figure 6. Representation of the amplitude transmission function of one period of a grating having N discrete slit widths.

The intensity of the diffraction pattern produced by a transmission function of the form shown in Figure 6 can now be seen to be of the form

$$I_{diff}(\theta) = \sum_{m=1}^{N} \sum_{n=1}^{N} a_n a_m \frac{\sin\alpha_n}{\alpha_n} \frac{\sin\alpha_m}{\alpha_m} \cos\phi_{nm} . \tag{62}$$

The period, d, of such a grating with N slit widths is given by

$$d = Nb + \sum_{i=1}^{N} a_i . \tag{63}$$

As was shown previously, in order to find the pattern produced by a grating composed of such "modified single slits," one has only to take the convolution of the "modified single slit" pattern and a linear array of δ-functions with a period d. The intensity of such a pattern would be given by

$$I_T(\theta) = \left(\sum_{n=1}^{N} \sum_{m=1}^{N} a_n a_m \frac{\sin\alpha_n}{\alpha_n} \frac{\sin\alpha_m}{\alpha_m} \cos\phi_{nm} \right) \frac{\sin^2 M\mu}{\sin^2 \mu}$$

where

$$\mu = \frac{\pi d \sin\theta}{\lambda}$$

and M is the number of modified single slits.

Finally, to add a last bit of generality, also let the b's vary--the b's being the dark spaces between the slit openings. Then in the above expressions for $\bar{a}_n$ and $\bar{a}_m$, one would write $\sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{m} b_i$ instead of nb and mb so that

$$\bar{a}_n = a_n/2 + \sum_{j=1}^{n} b_j + \sum_{i=1}^{n-1} a_i \tag{65}$$

and

$$\bar{a}_m = a_m/2 + \sum_{j=1}^{m} b_j + \sum_{i=1}^{m-1} . \tag{66}$$

The period would then be given by

$$d = \sum_{i=1}^{N} (b_i + a_i) , \tag{67}$$

and the intensity distribution would still be given by Eq. 64 using the definition in Eqs. 65, 66, and 67.

III. THE EFFECT OF PARTIAL COHERENCE ON DIFFRACTION PHENOMENA

In the development of diffraction theory there has been made a tacit assumption that the diffracting object was illuminated by a coherent beam of monochromatic light. It was pointed out by Shore, Thompson, and Whitney (1966) that in experiments in which the detailed structure of Fraunhofer diffraction is important, considerable divergence between theory and experiment can be observed if coherence effects are ignored. A detailed knowledge of the mutual-coherence function across the diffracting aperture is in general required to describe fully the observed diffraction phenomena, since even the qualitative characteristics of the diffraction patterns are found to vary with the form of the mutual-coherence function. It will be useful to point out the effect of partial coherence on the Fraunhofer diffraction problem being considered in this thesis.

Shore, et al. (1966) determined the Fraunhofer intensity pattern for an aperture illuminated by quasimonochromatic, partially coherent, light. Their results were based on the work of Schell (1961) who had derived an expression for the intensity distribution in the Fraunhofer diffraction pattern for such a plane, quasimonochromatic, spatially stationary aperture. His expression is proportional to the Fourier transform of the product of the aperture autocorrelation function, Ψ, and the normalized aperture mutual-intensity function, γ:

$$I(P_0) = \left(\frac{\Sigma\cos^2\theta}{\bar{\lambda}^2 Z^2}\right) \int_{\Sigma} \gamma(\vec{D},0)\ \Psi(\vec{D})\ \exp(i\bar{k}\sin\theta\hat{p}\cdot\vec{D})dD \qquad (68)$$

where

Σ = the area of the aperture

$\bar{\lambda}$ = the mean wavelength

$\bar{k} = 2\pi/\bar{\lambda}$

$\gamma(\vec{D},0)$ = the complex degree of coherence

$\Psi(\vec{D})$ = the autocorrelation amplitude transmission function

Z, θ, $\vec{D}$, and $\hat{p}$ (a unit vector) are defined as shown in Figure 7.

At this point it is necessary to define several terms in order to apply Eq. 68 to the present investigation. Consider the Young's interference experiment shown in Figure 8. Let the electromagnetic field at a point $\vec{x}$ at time t be represented by $U(\vec{x},t)$, which will in general be considered to be a scalar complex quantity.*

*Here we are restricting our attention to linearly polarized light and are disregarding the polarization phenomena in which the vector properties of light become necessary.

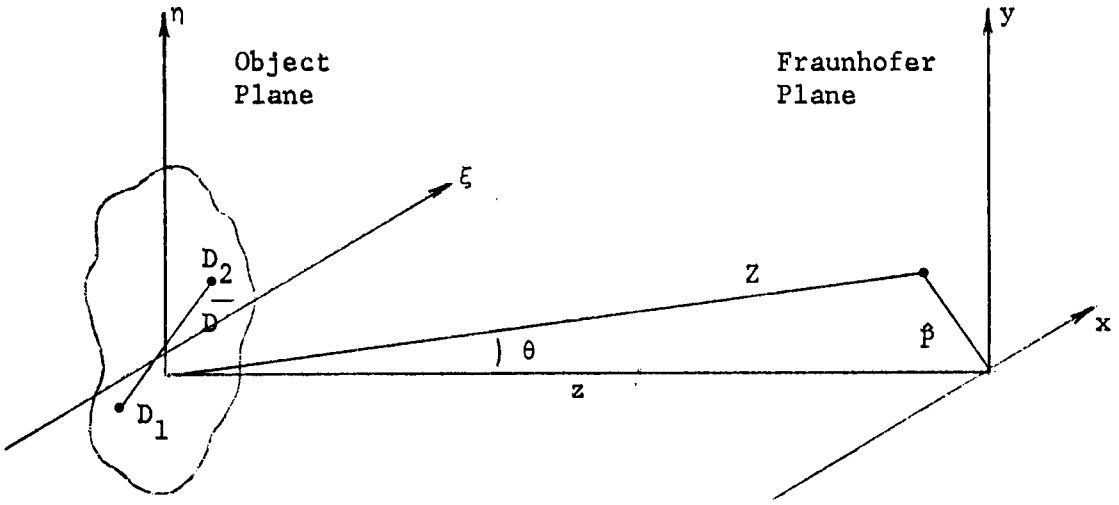


Figure 7. Coordinate system used in the expression for the intensity of the Fraunhofer diffraction pattern for a plane, quasi-monochromatic, spatially stationary aperture.

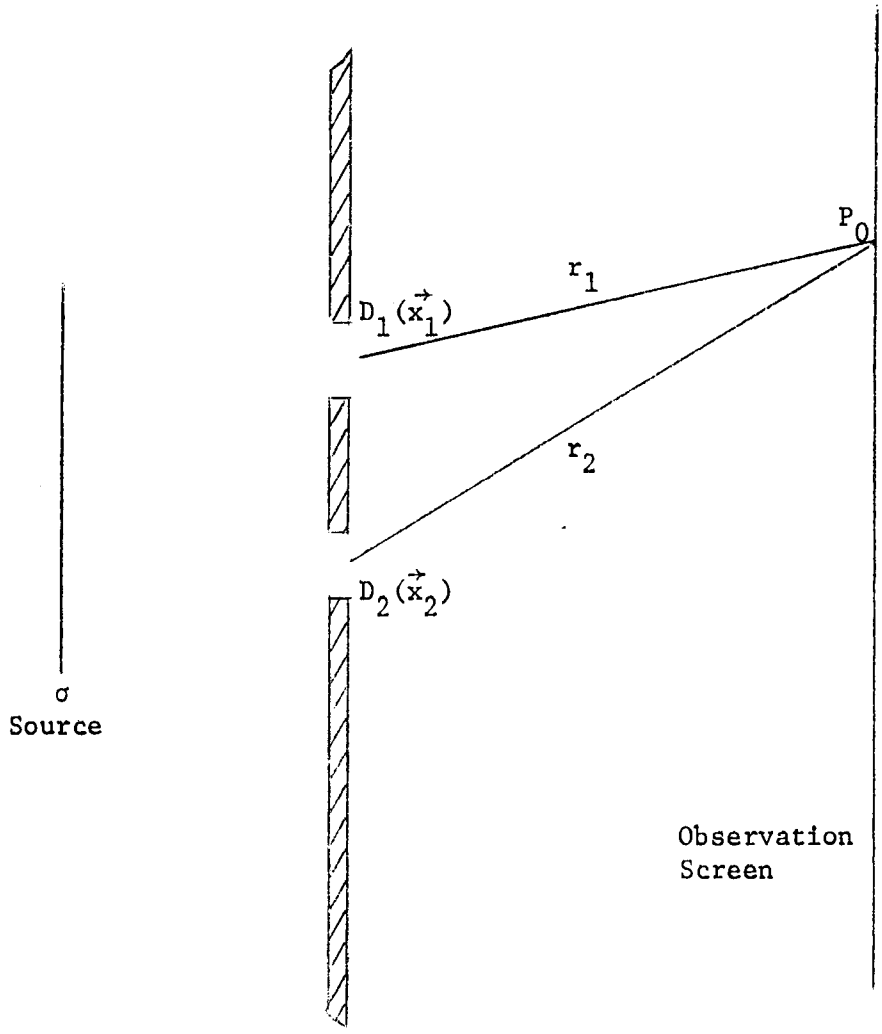


Figure 8. Young's interference experiment.

The mutual coherence function (for stationary and ergodic fields*) is then defined as:

$$\Gamma(\vec{x}_1,\vec{x}_2;\tau) \equiv \left\langle U^*(\vec{x}_1,t)\ U(\vec{x}_2,t+\tau)\right\rangle = \Gamma_{12}(\tau)$$

where the sharp brackets denote the time average

$$< \ldots > \equiv \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \ldots\ dt \tag{70}$$

and * denotes the complex conjugate.

$$\tau = t_2 - t_1 \tag{71}$$

$$t_1 = \frac{r_1}{c} \qquad t_2 = \frac{r_2}{c} \qquad c = \text{speed of light}$$

The intensities at $\vec{x}_1$ and $\vec{x}_2$ are related to the mutual coherence function by

$$I(x_j) = \Gamma(x_j,x_j;0) \qquad j = 1,2\ . \tag{72}$$

The zero ordinate of the mutual coherence function, $\Gamma(\vec{x}_1,\vec{x}_2;0)$ is called the mutual intensity function. Another useful quantity, the complex degree of coherence, denoted by $\gamma_{12}(\tau)$, is defined as the mutual coherence function normalized by the square root of the product of the intensities at $\vec{x}_1$ and $\vec{x}_2$:

---

*See Perina, Chapter 2, for a discussion of stationary and ergodic fields.

$$\gamma_{12}(\tau) = \frac{\Gamma(\vec{x}_1, \vec{x}_2; \tau)}{\sqrt{\Gamma_{11}(0)\ \Gamma_{22}(0)}} = \frac{\Gamma_{12}(\tau)}{\sqrt{I_{P_1}\ I_{P_2}}} . \tag{73}$$

The intensity at point P_0 is now given by

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1\ I_2}\ \mathrm{Re}[\gamma_{12}(\tau)] \tag{74}$$

where I_1 is the intensity at P_0 due only to the aperture at D_1 being open, and likewise for I_2. One can also write this as

$$I(P_0) = I_1 + I_2 + 2\sqrt{I_1\ I_2}\ |\gamma_{12}(\tau)|\ \cos(\beta_{12}(\tau) + \delta_{12}) \tag{75}$$

where

$$\beta_{12}(\tau) = 2\pi\nu\tau + \arg\ \gamma_{12}(\tau)$$

and

$$\delta_{12} = \frac{2\pi}{\lambda(r_2 - r_1)} .$$

We note that

$$0 \leq |\gamma_{12}(\tau)| \leq 1$$

and that the coherent limit is given by $\gamma_{12}(\tau) = 1$ and the incoherent limit as $\gamma_{12}(\tau) = 0$.

An important theorem known as the van Cittert-Zernike theorem allows us to calculate the degree of coherence in the optical field produced by an extended, incoherent, quasimonochromatic source when the medium between the source and the field is homogeneous. The van Cittert-Zernike

theorem states that, providing the dimensions of the source and the separation between D_1 and D_2 are small compared to the distance between the source and the plane containing D_1 and D_2, then the complex degree of coherence is the Fourier transform of the intensity distribution function of the incoherent source suitably normalized. Thus in terms of the coordinates shown in Figure 9:

$$\gamma_{12}(0) = \frac{\exp(ik\Phi)}{\iint_{\Sigma} I_{\Sigma}(u,v)\,dudv} \iint_{\Sigma} I_{\Sigma}(u,v)\, e^{-\frac{ik(\xi_1-\xi_2)u}{\ell}}\, e^{-\frac{ik(\eta_1-\eta_2)v}{\ell}}\, dudv \tag{76}$$

where

$$\Phi \equiv \frac{(\xi_1^2 + \eta_1^2) - (\xi_2^2 + \eta_2^2)}{2\ell}$$

and $I_{\Sigma}(u,v)$ is the intensity distribution across the source. This expression (Eq. 76) is useful in Eq. 68 as an expression for $\gamma(\vec{D},0)$, the complex degree of coherence.

In light of the above theorem, the degree of coherence arising from an extended incoherent source can easily be determined from the knowledge of the equivalent diffraction problem. For example, for a slit incoherent source of width a

$$\gamma_{12}(0) = \exp(ik\Phi)\ \operatorname{sinc}\left(\frac{k(\xi_1 - \xi_2)a}{2\pi\ell}\right) \tag{77}$$

where

$$\operatorname{sin}\, x \equiv \frac{\sin\pi x}{\pi x}\ .$$

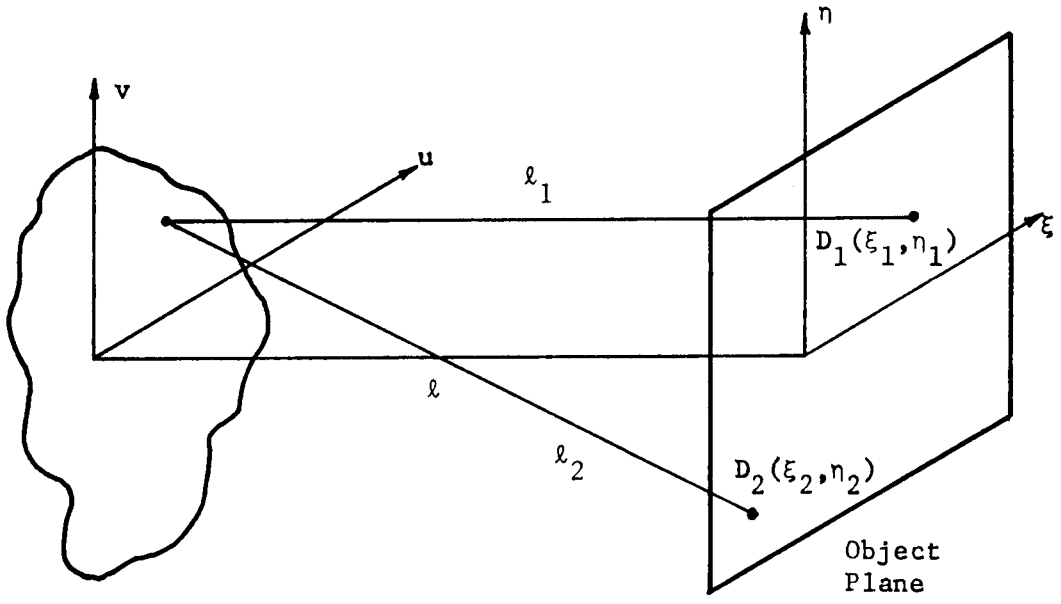


Figure 9. Coordinate system used in the van Cittert-Zernike theorem.

It is pointed out by Lipson (1972) that a negative value of $\gamma_{12}(0)$ has to be interpreted as a phase change, i.e., the interference fringes formed will have a central minimum, not a central maximum, of intensity.

Among the configurations treated by Shore, et al. (1966) is that of a slit aperture illuminated by an incoherent slit. This solution is pertinent to the present investigation. The Fraunhofer intensity distribution for this case can be solved for in closed form and is given by

$$I(\Omega,\Delta) = \frac{1}{\Delta}\left[\mathrm{Si}(\Delta+\Omega) + \mathrm{Si}(\Delta-\Omega) - \frac{1-\cos(\Delta-\Omega)}{(\Delta-\Omega)} - \frac{1-\cos(\Delta+\Omega)}{(\Delta+\Omega)}\right] \tag{78}$$

where

$\mathrm{Si}(x)$ = the sine integral $\int_0^x \frac{\sin t}{t}\,dt$

$\Omega = \bar{k}a\theta = 2\alpha$

$\Delta = a/L$

a = width of the illuminated aperture

L = correlation interval = $\frac{\lambda f_1}{\pi w}$.

In the coherent limit, $\Delta = 0$, this expression reduces to the familiar result from elementary optics:

$$I(\Omega,0) = \left[\frac{\sin \Omega/2}{\Omega/2}\right]^2 = \left[\frac{\sin\alpha}{\alpha}\right]^2 \tag{79}$$

where $\alpha = \frac{\pi a \sin\theta}{\lambda}$ as in Eq. 26 while in the incoherent limit, $V = \infty$, the intensity distribution normalized to its value at $\theta = 0$ reduces to the constant value of unity.

In order to define the coherence of a system, it is convenient to define a parameter q which is the number of correlation intervals contained in the aperture radius (or halfwidth in the case of the slit aperture). It turns out that this parameter q is a useful measure of the coherence properties of a system. (It is not to be confused with the complex degree of coherence, γ.) In terms of the experimental parameters, the correlation interval is defined as

$$L = \frac{\lambda f_1}{\pi w}$$

where f_1 and w are defined in Figure 10. Thus we have for a slit aperture of width a

$$q = \frac{a}{2L} = \frac{\pi a w}{2\lambda f_1} .$$

Figure 11 shows the results of evaluating Eq. 78 for various values of q (Shore, et al., 1966). The primary effect to be noted is the gradual filling of the minima as q increases, i.e., as the illumination becomes less coherent. A less striking effect is the gradual shifting outward of the maxima and minima which accompanies the decrease in contrast or visibility.

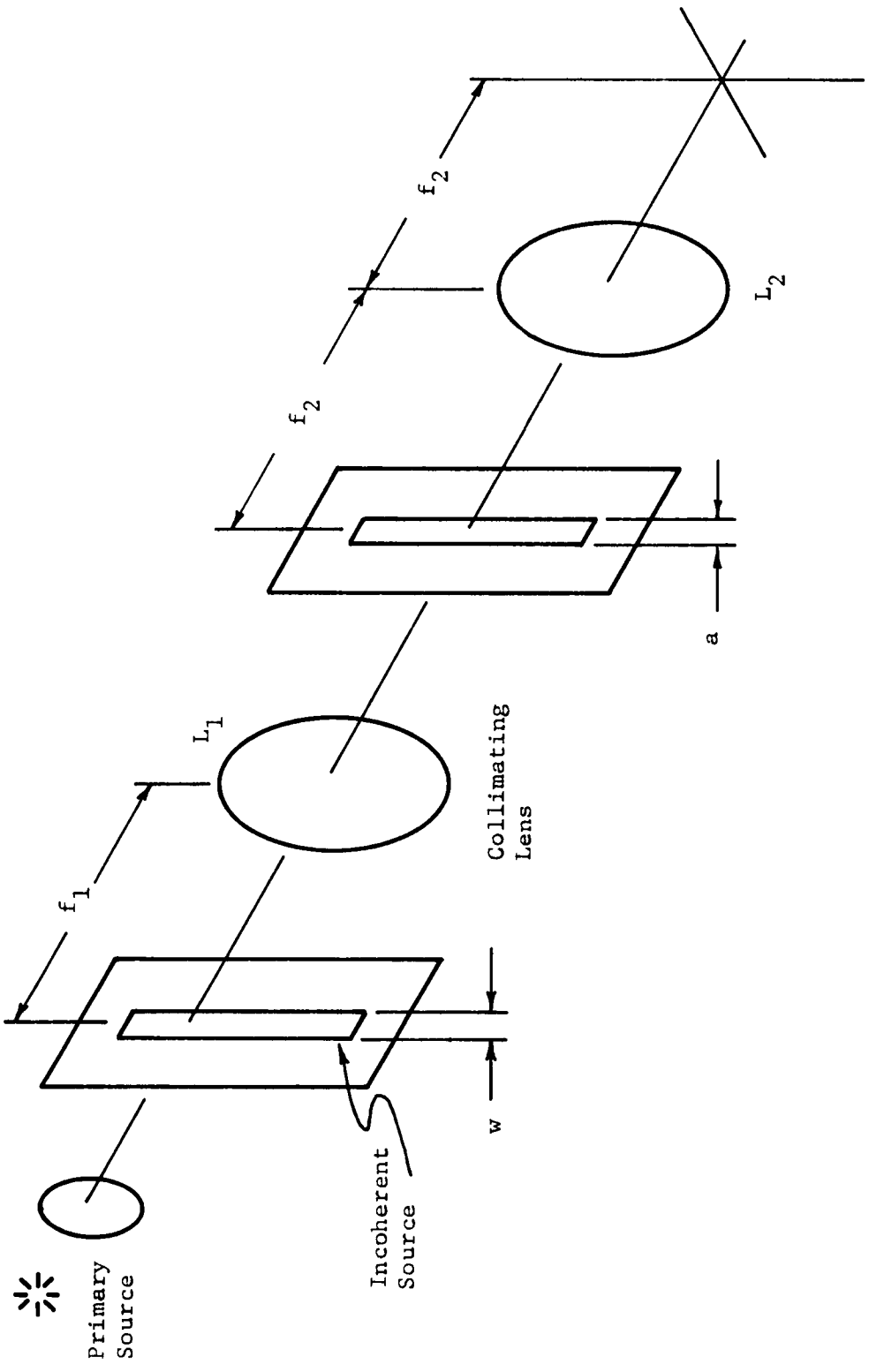


Figure 10. Experimental parameters used to define q (the number of correlation intervals contained in the slit halfwidth).

CHAPTER III

INVESTIGATION OF OPTICAL GRATINGS

I. EQUIPMENT AND PROCEDURE

The experimental apparatus used to measure the intensity of the diffraction patterns is shown in Figure 11. The elements of the optical system were mounted on an optical bench. The light source was a 100 watt mercury arc which was filtered by a combination of glass transmission filters which pass only the green line of the mercury spectrum (λ = 5460.73 Å). A condensing lens having a 5 cm focal length was used to image the arc source on the adjustable source slit. The width of the source slit was of the order of 0.5 mm. The criterion for determining this width was a compromise between maximum illumination in the diffraction pattern and good definition of the spectral orders. A collimating lens having a focal length of 122.3 mm was used to render the light from the source slit parallel. Autocollimation was used to position the collimating lens. An opaque mask, having a rectangular opening approximately 1 cm by 0.5 cm was used to pick a relatively uniform portion of the incident field for illuminating the diffraction gratings. A lens having a focal length of 80 cm was used to focus the diffraction pattern on the observation plane. An auxiliary lens, beyond this focusing lens, was used to provide variable magnification of the diffraction pattern. With this arrangement, the width of the

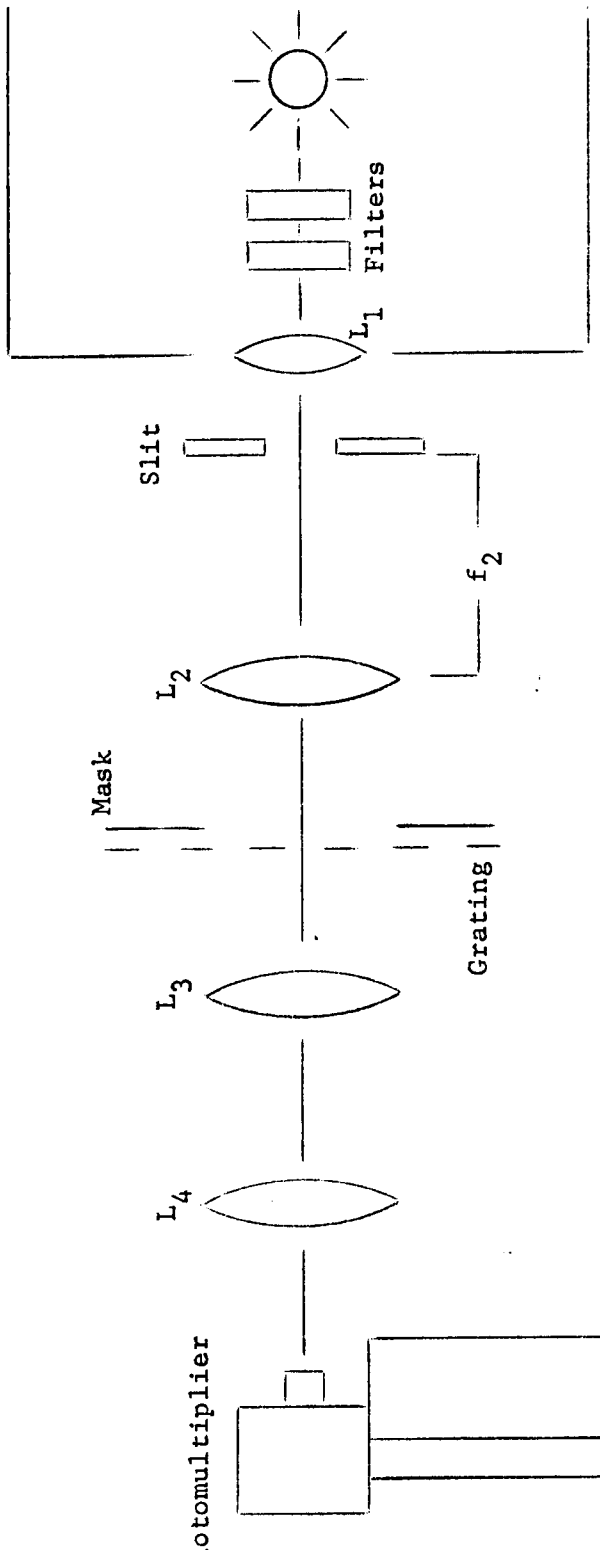


Figure 11. Experimental apparatus.

diffraction pattern could be made to coincide with the 5 cm travel of the photometer detector assembly.

The relative light intensity measurements of the diffraction pattern were made using an Eldorado Photometer Model 201 with an RCA 6199 photomultiplier tube fitted with an adjustable analyzing slit. The detector assembly was mounted on a motor-driven traveling microscope carriage which allowed accurate scanning of the diffraction pattern. The relative intensities of the various orders of the diffraction patterns were read from the meter on the photometer as relative maxima during the continuous scan of the pattern by the detector unit.

Optical Gratings

Several amplitude transmission gratings were made by photographically reducing suitable patterns of alternate black and white stripes made with matte finish, black drafting type on a white background. The first such pattern was made using tape widths of 0.25 inch and 0.125 inch which was applied to a piece of white poster board (approximately 22" x 28") with the lines of tape parallel to the narrow dimension. A combination of a metal straight edge and a steel carpenter's square was used to ensure the straightness and parallelism of the lines of tape. Spacing of the lines was accomplished by using strips of .25" and .125" drafting tape on the edges of a drafting triangle and aligning one edge of the tape with the last line produced to locate the next line.

This pattern was then photographed with a Speed Graphic (4" x 5") camera using Kodak Contrast Process Ortho Film (Estar Thick Base). The

exposure time was 1/25 sec at f8. The pattern was photographed from such a distance that the image of the grating was reduced to approximately 1.1 cm long. The developed negative was trimmed and mounted in a 35 mm slide mount for ease of handling. The area of the film around the grating image was masked to eliminate unwanted light.

A second pattern was made from the same materials using a different technique in order to eliminate the cumulative error in line position. The drafting tape was applied to tracing paper which had been laid over a large grid formed from graph paper. The lines on the graph paper were used to space and to align the drafting tape. The two widths of drafting tape used for this pattern were 1/4" and 3/16". The spaces between the tape were not equal, but rather there were two distinct spaces per period. This was due to the non-integer ratio of the tape width to the spacing of the grid lines on the graph paper. This situation can be treated by the theory which has been presented, however.

A Minolta SRT-101, 35 mm camera, using Kodak High Contrast Copy film, was used to copy the second pattern described above. Several different exposures were made from which two were chosen for the measurements.

In Figure 12 are shown enlarged negative reproductions of the doubly-periodic gratings used. The white areas on Figure 12 correspond to the opaque areas on the grating; the black areas correspond to the transparent parts. Figure 12(a) is a graph of the transmission function corresponding to the grating photograph shown in Figure 12(b). Similar transmission functions were used for the other gratings.

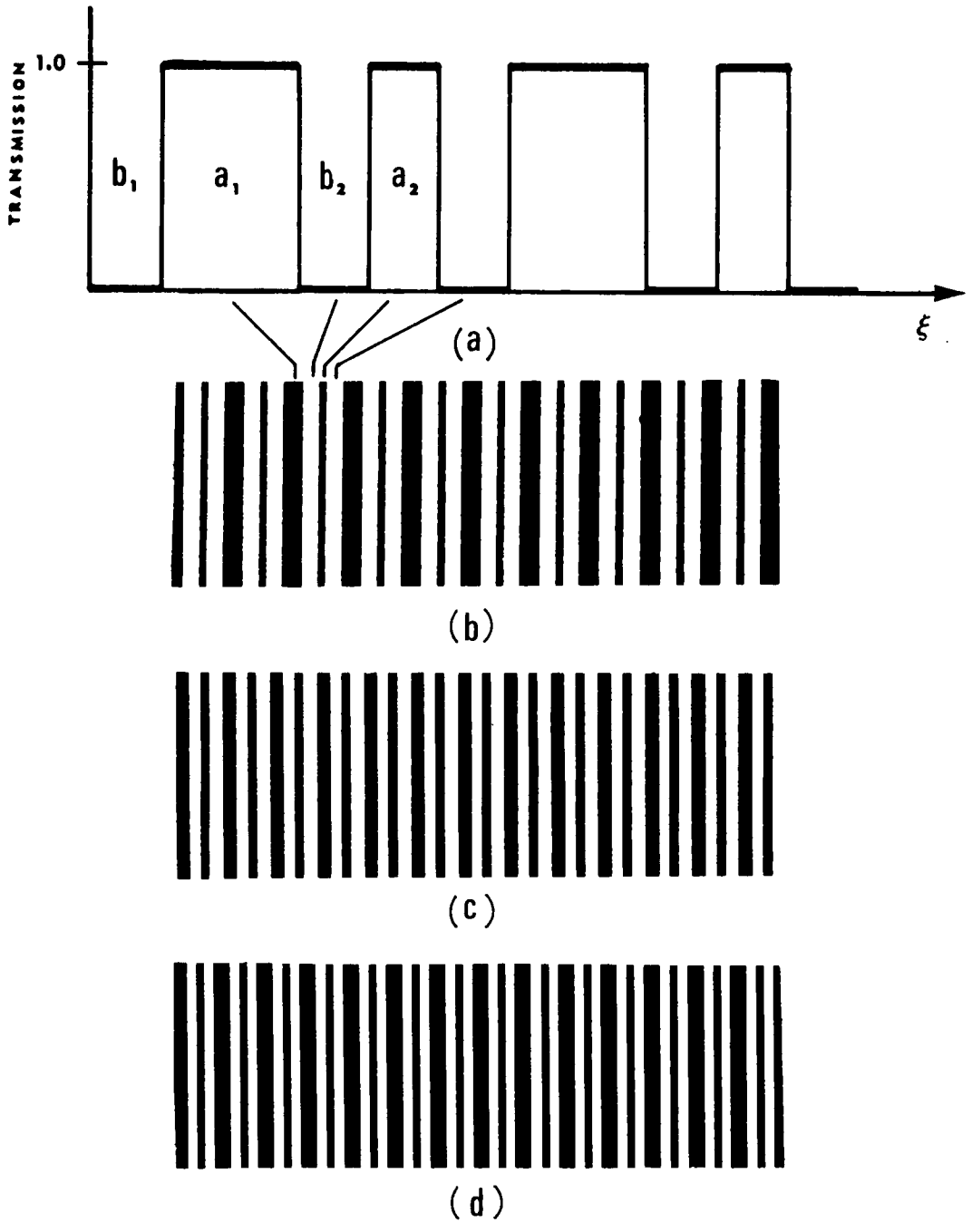


Figure 12. (a) Amplitude transmission function of doubly-periodic grating; (b) photographic negative of grating No. 2; (c) photographic negative of grating No. 3; (d) photographic negative of grating No. 4.

Four different gratings--one ordinary, singly-periodic grating and three doubly-periodic gratings--were used. Table I gives the dimensions of the gratings used according to the definitions used in Figure 12(a). The widths of the openings and dark spaces of the gratings were measured using a traveling microscope. Each opening and dark space throughout ten to twenty periods was measured. An average value for the period was obtained by first determining the average value of each subsection of the period, then adding these averages. These numbers appear in Table I.

Use of a Liquid Gate

Since photographic negatives were used as gratings, special considerations are necessary. In general, the effect of the film must be described in terms of its complex transmittance $\bar{t}$. However, the definition of $\bar{t}$ as the square root of the intensity transmittance is not acceptable. Such a definition ignores the fact that relative phase shifts can occur as light passes through the film (Goodman, 1968). These phase shifts are the result of variations in the thickness of the film base and also variations in the density of silver grains in the developed photographic emulsion. To account for these phase shifts, one describes the film's transmittance by giving both its amplitude and its phase properties; i.e.,

$$\bar{t}(\xi,\eta) = \sqrt{T(\xi,\eta)}\, \exp[i\phi(\xi,\eta)] \ .$$

In general, the phase term $\exp[i\phi(\xi,\eta)]$ is difficult to evaluate. An alternative to the description of film by its complex transmittance is to

TABLE I

DIMENSIONS OF GRATINGS

Grating	b_1 (in cm)	a_1 (in cm)	b_2 (in cm)	a_2 (in cm)	d (in cm)
Singly-Periodic #1	.00348	.00660			.01008
Doubly-Periodic #2	.00684	.0097	.0070	.00416	.02770
Doubly-Periodic #3	.0050	.0073	.0039	.0054	.0216
Doubly-Periodic #4	.0041	.00779	.00396	.00398	.01983

use a liquid gate such as that shown in Figure 13. In using the liquid gate, an attempt is made to make the phase uniform over the surface of the grating. The film, which serves as a grating, is sandwiched between two optical flats along with thin layers of oil whose refractive index is a mean value between that of the glass, the film base, and the emulsion.

By using the liquid gate, the optical path over the surface of the film is made uniform enough that relative phase shifts at different points are small. One can thus use the theory developed in Chapter II in which the transmittance has been assumed to be real.

The effect of the liquid gate can be seen in the data shown in Figure 14. A photographic grating having a double periodicity was used. Measured intensities of the various diffraction orders are plotted twice: with the liquid gate and without it. The lower orders agree to within approximately a factor of two. The fourth orders differ by more than an order of magnitude, and the seventh by almost three orders of magnitude. It is difficult to determine how closely the experimental configuration using the liquid gate is described by the theory of Chapter II--this is the purpose of the work described in this thesis--but it is clear from the data presented in Figure 14 that the liquid gate does affect the intensity pattern observed. It is to be assumed that the configuration using the liquid gate more closely approximates the theoretical assumptions. A liquid gate was used with all the photographic gratings studied.

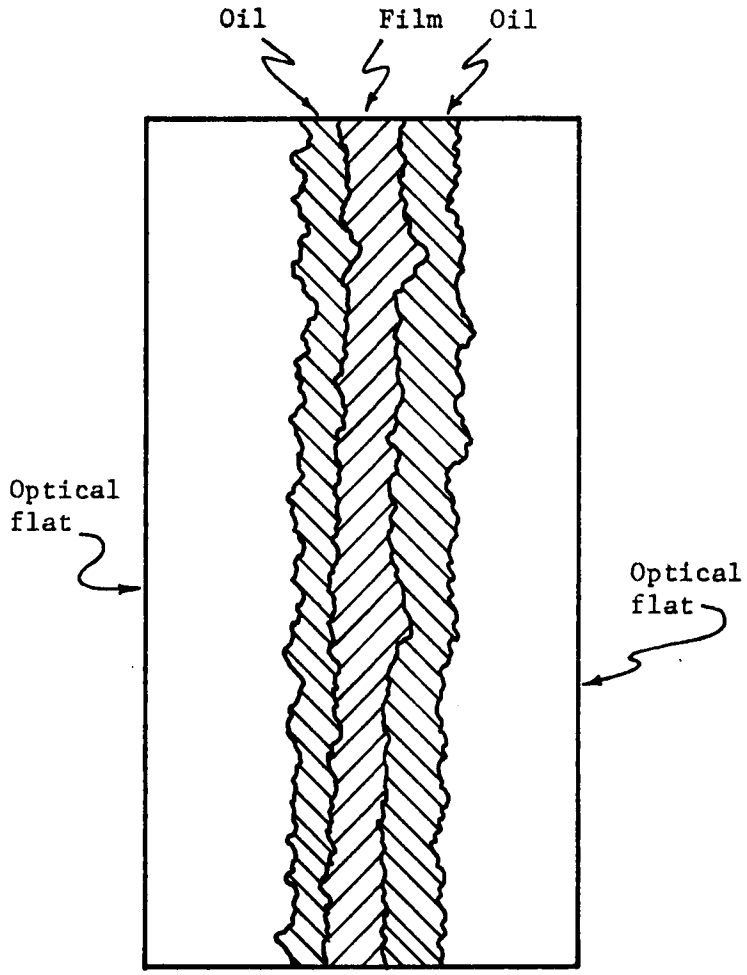


Figure 13. Liquid Gate

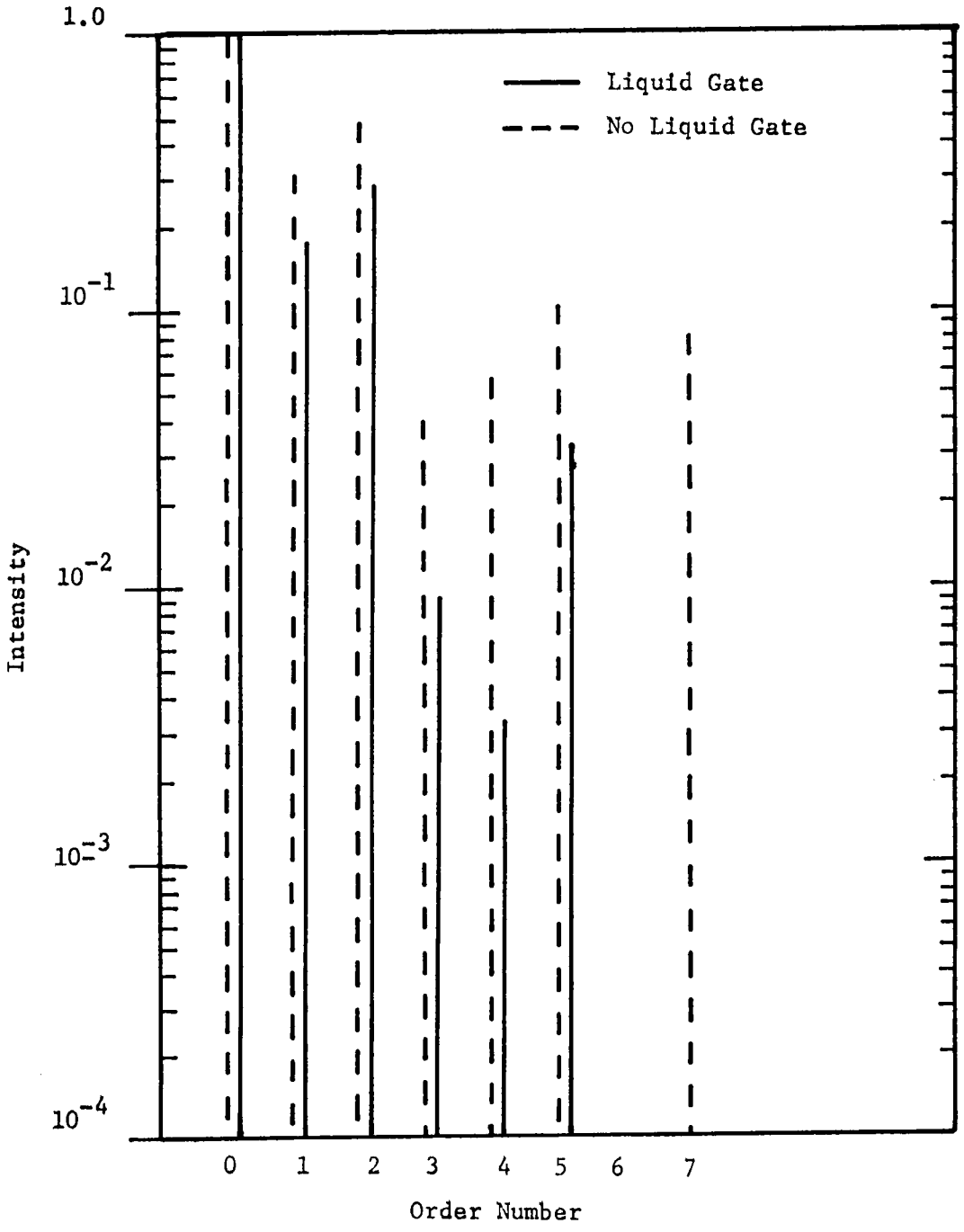


Figure 14. Experimentally measured diffraction patterns for Grating No. 2 showing the effect of the liquid gate.

II. RESULTS

The experimentally measured values for the intensity of various orders of the Fraunhofer diffraction pattern for the four gratings used in this experiment are shown in Figures 15 through 18. In order to facilitate comparison with the predicted theoretical results, the data have been normalized to unity with respect to the central maximum in each case.

The spectrum for the ordinary singly-periodic grating #1 is shown in Figure 15. Good agreement is found between theory and experiment for the first, second, and fourth orders and to a lesser extent for the fifth orders. The third order experimental value is seen to exceed the predicted value by about an order of magnitude.

Figure 16 shows the comparison of the results for grating #2. With the exception of the third order, the experimental values are found to be greater than those predicted by theory. Good agreement exists for orders one, two, three, and five. Order four is found to disagree by an order of magnitude.

Examination of Figure 17, showing the spectrum for grating #3, reveals, that, with the exception of the first order, the experimentally measured values are consistently less than those predicted by theory. Agreement among the low orders is good, but the agreement is not as good in the higher orders. It is also noted that the experimentally measured spectrum was not symmetric with respect to the orders (i.e., the intensities of the positive and negative orders do not agree within experimental

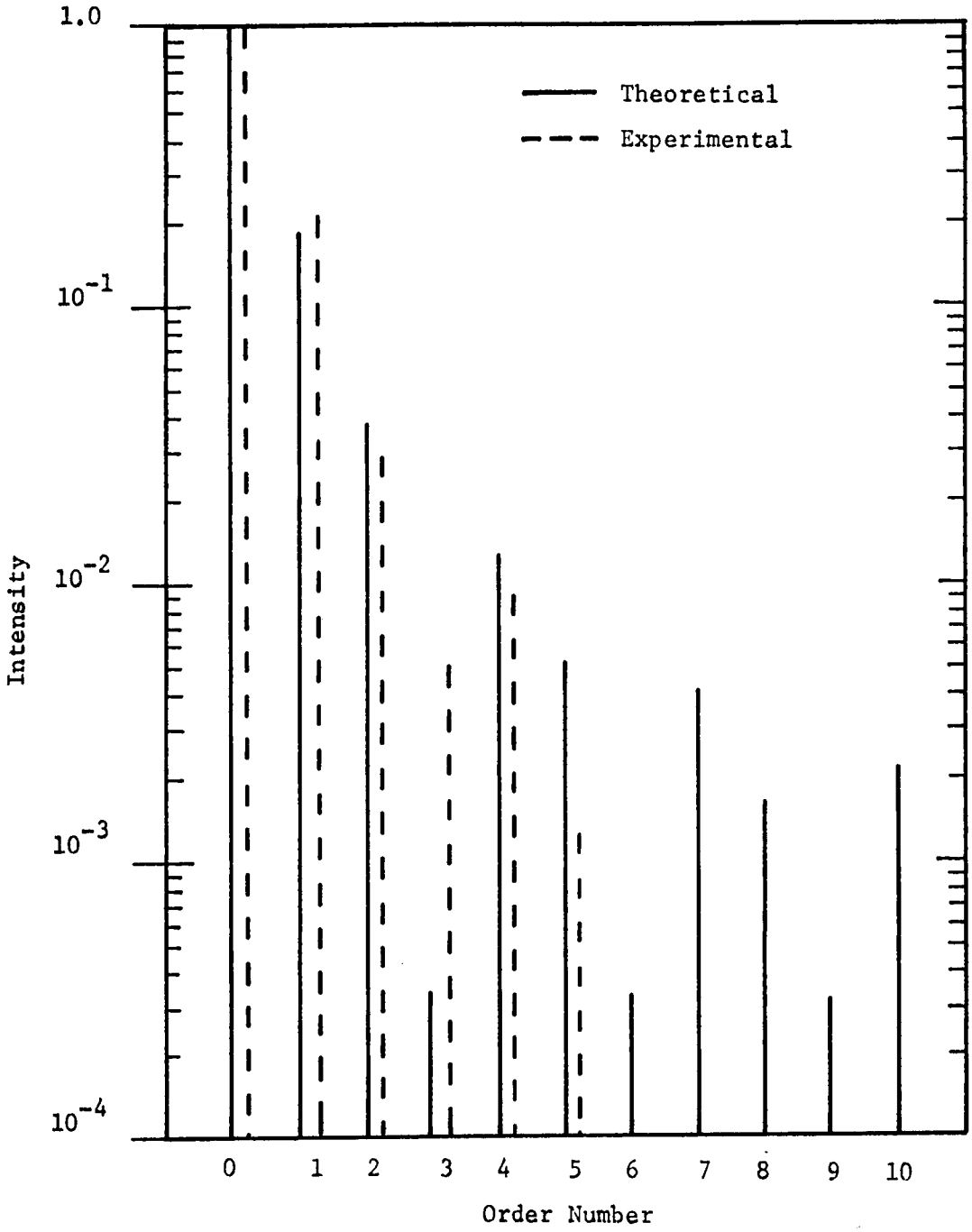


Figure 15. Comparison between theoretical and experimental diffraction patterns for singly-periodic Grating No. 1.

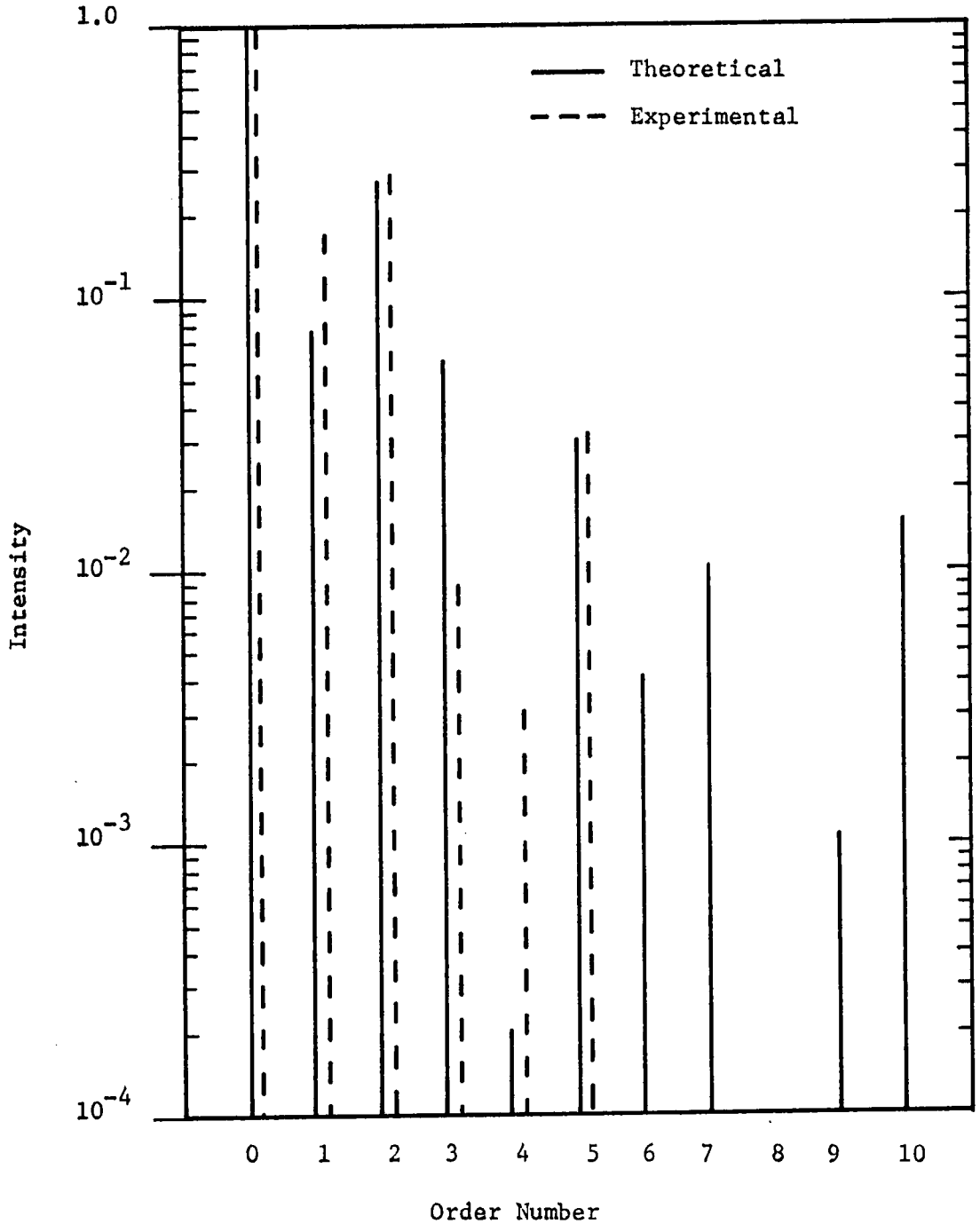


Figure 16. Comparison between theoretical and experimental diffraction patterns for doubly-periodic Grating No. 2.

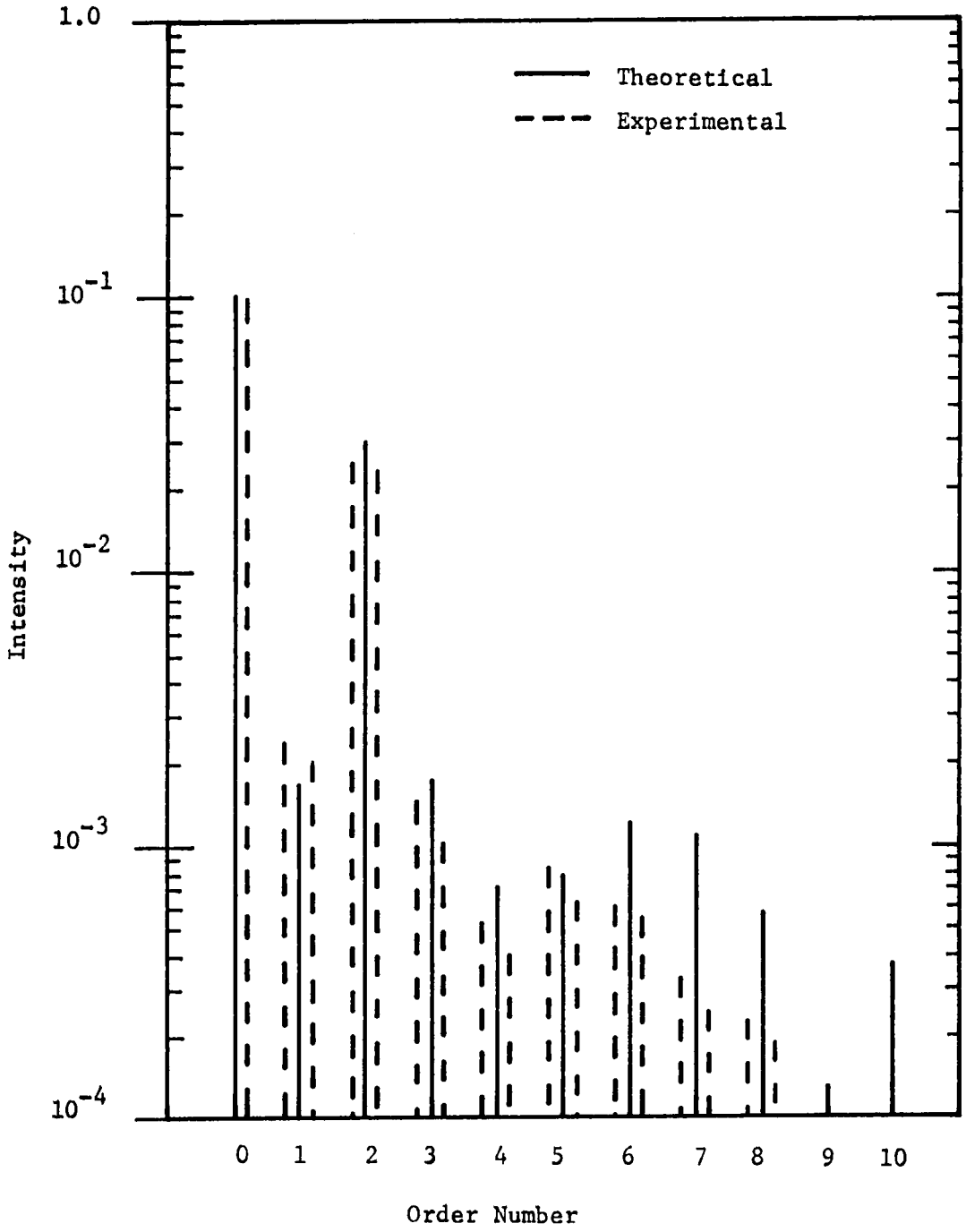


Figure 17. Comparison between theoretical and experimental diffraction patterns for doubly-periodic Grating No. 3.

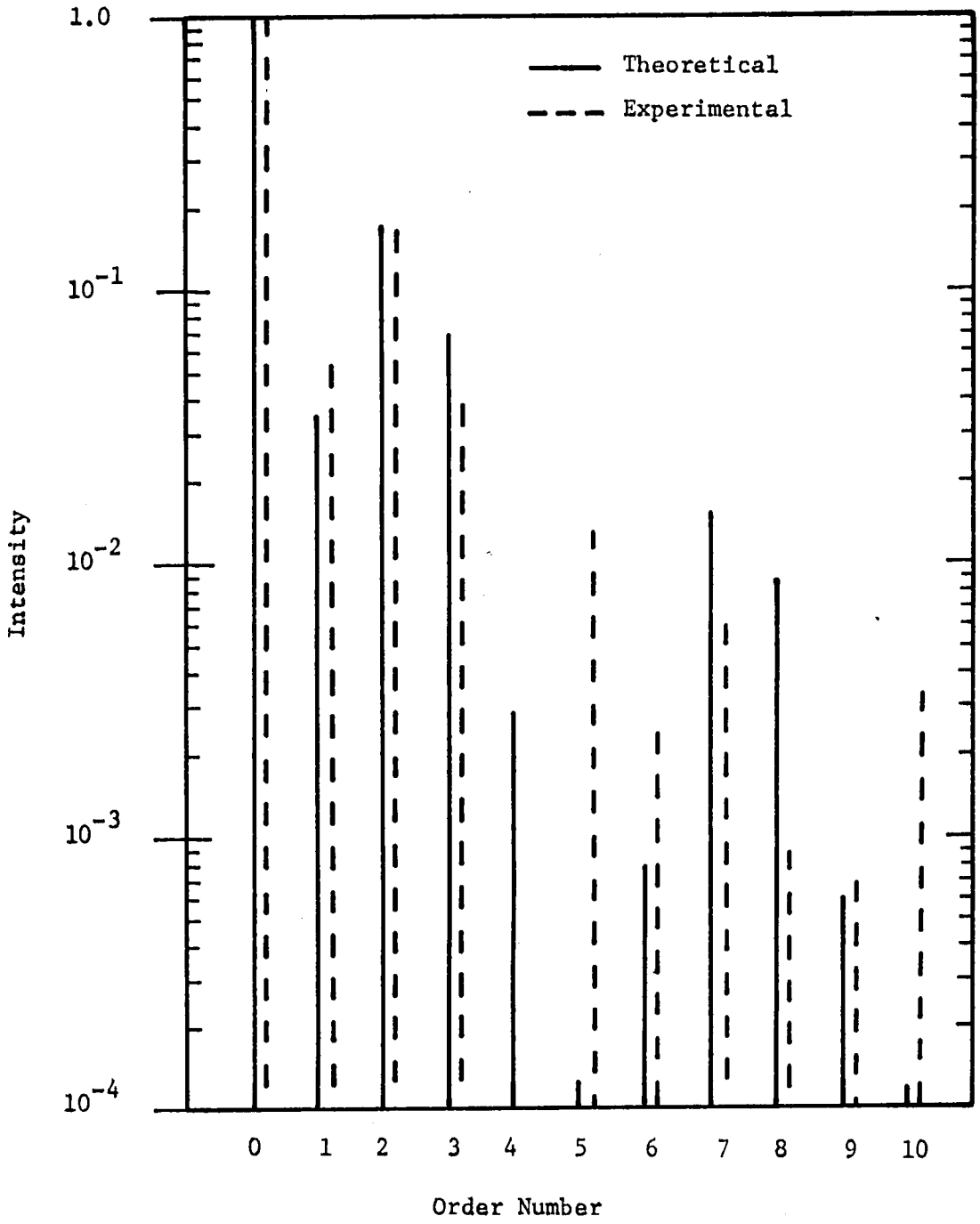


Figure 18. Comparison between theoretical and experimental diffraction patterns for doubly-periodic Grating No. 4.

error). The positive orders are consistently greater than the negative ones. It is also noted that both theory and experiment substantiate one peculiar feature of this grating. The second order is seen to be an order of magnitude greater than any other order except the zeroth order. Such behavior is reminiscent of that of the echelon or "blazed" gratings of Michelson.

The spectrum of grating #4, shown in Figure 18, exhibits good agreement in the first three orders and in the seventh and ninth orders.

Examining the data as a whole, one finds it to have the following characteristics. First, the data for the ordinary, singly-periodic grating do not exhibit significantly better agreement than do the doubly-periodic ones. Second, each set of data shows reasonable agreement in the lower orders while the agreement is not as good in the higher orders.

III. DISCUSSION

The results of these experiments and their agreement with the theory presented can best be examined by enumerating the theoretical assumptions and determining the extent to which the experimental configuration satisfies these assumptions. For mathematical convenience it was necessary to assume that a (1) monochromatic, (2) plane waves (3) uniformly, and (4) coherently illuminates (5) an error-free grating whose amplitude transmission function can be represented by a (6) periodic series of rectangular functions and whose (7) phase properties are constant across its surface.

These assumptions will be examined in turn.

1. Monochromaticity. A mercury arc source was used with a high quality absorption filter to isolate the 5460.73 Å green line. Although this line is broader than a line from a laser, the stability of the mercury source (in conjunction with a voltage regulator) was far better than that of the readily available laser sources. The diffraction orders were separated enough that no problem resulted from the width of the mercury line.

2. Plane wave. The source slit used was of finite extent; thus, some deviation from an ideal plane wave did exist. However, the source slit was made as small as possible consistent with sufficient illumination to allow measurements to be made on from 8 to 10 orders on either side of the central maximum. Lenses of good quality were used which tended to make negligible the effect of lens aberrations on the plane wave fronts.

3. Uniform illumination. Careful alignment and focusing of an image of the mercury lamp on the source slit provided illumination which did not vary more than approximately 10 percent. The effect of Fresnel diffraction resulting from the mask located between the collimator and the grating was reduced to a negligible value by careful positioning of the mask.

4. Coherent illumination. The fact that an image of the mercury source was projected onto the source slit so that each part of the slit was illuminated separately means that the illumination approaches the incoherent limit. If one ignores the effects of diffraction and lens

aberrations which tend to make complete incoherence impossible, the van Cittert-Zernike theorem can be used to relate the intensity across the incoherent source slit to the degree of coherence in the plane of the diffracting object.

The effect of the degree of coherence on the intensity of the Fraunhofer diffraction pattern can be seen by referring to the expression for the intensity of the diffraction pattern produced by a slit aperture illuminated by a slit source as given in Eq. 78. Figure 19 shows a plot of the results of evaluating Eq. 78 for various values of the correlation interval, q.

The primary effect to be noted in the gradual filling of the minima as q increases; i.e., as the illumination becomes less coherent. A less striking effect is the gradual shifting outward (increasing θ) of the maxima and minima which accompanies the decrease of contrast or visibility. It is well known from elementary optics that, for a grating, the diffraction term associated with the width of the single slit determines the envelope of the diffraction pattern and thus the intensity of each order. As can be seen in Figure 19, this envelope term is very much influenced by the coherence of the system.

The diffraction pattern of an array of identical apertures is best defined when the illumination is coherent over the whole array. (The diffraction pattern from each individual aperture is coherent with respect to every other aperture and their <u>amplitudes</u> are added, giving rise to the characteristic interference pattern.) As the coherence interval decreases and approaches the array spacing of the diffracting

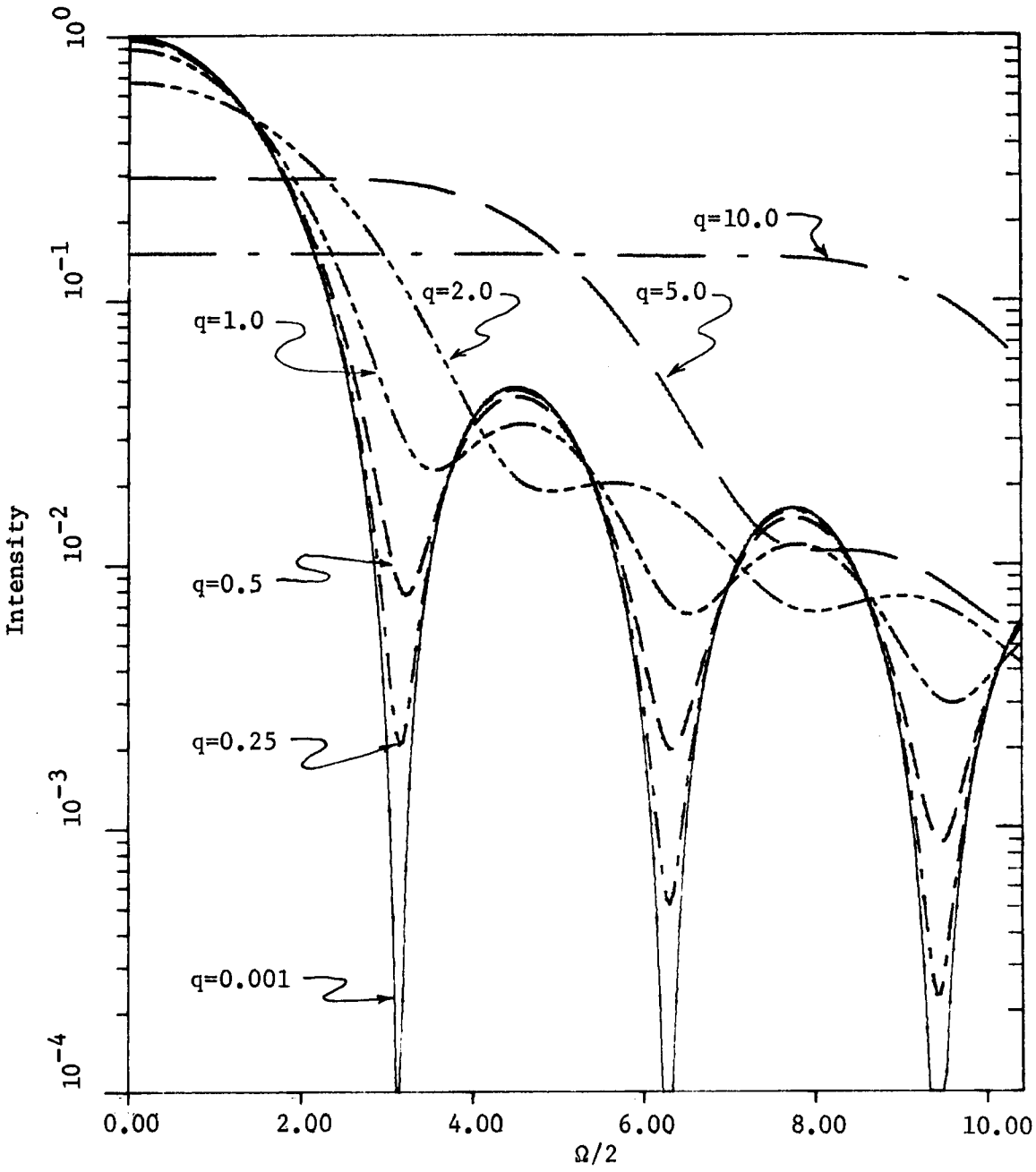


Figure 19. Theoretical Fraunhofer intensity distribution for a slit aperture illuminated by a slit source for various values of q.

object, the diffraction pattern approaches that of one member aperture of the array with the resulting incoherent addition of intensity greatly increasing the observed intensity.

For the experimental configuration used, the source slit width was of the order of 0.05 cm, the collimating lens had a focal length of 12.23 cm, and the wavelength of the light was 5460.73 Å. If these parameters are used in conjunction with the width of the aperture opening of the singly-periodic grating (.0066 cm), one arrives at a value for the correlation interval of $q = 0.776$.

Figure 20 shows the intensities of the first five orders of the diffraction pattern of a singly-periodic grating (#1) for $q = 0$ (the coherent limit), $q = .776$, and the experimentally measured values. It can be seen that for the third order the agreement between the experiment and the theory with q taken to be 0.776 is greatly improved over the value for the coherent limit. It is also to be noted that the agreement for the fifth order is not improved further by considering the effect of coherence. From this one might observe that the intensities of the low orders of the diffraction pattern are governed primarily by coherence effects while for the higher orders other factors must be considered in addition to coherence.

The basis for the sensitivity of the third order to variations in coherence can be seen by examining Figure 19 in terms of the position of the various orders. The third order of the singly-periodic grating being considered is known to occur at

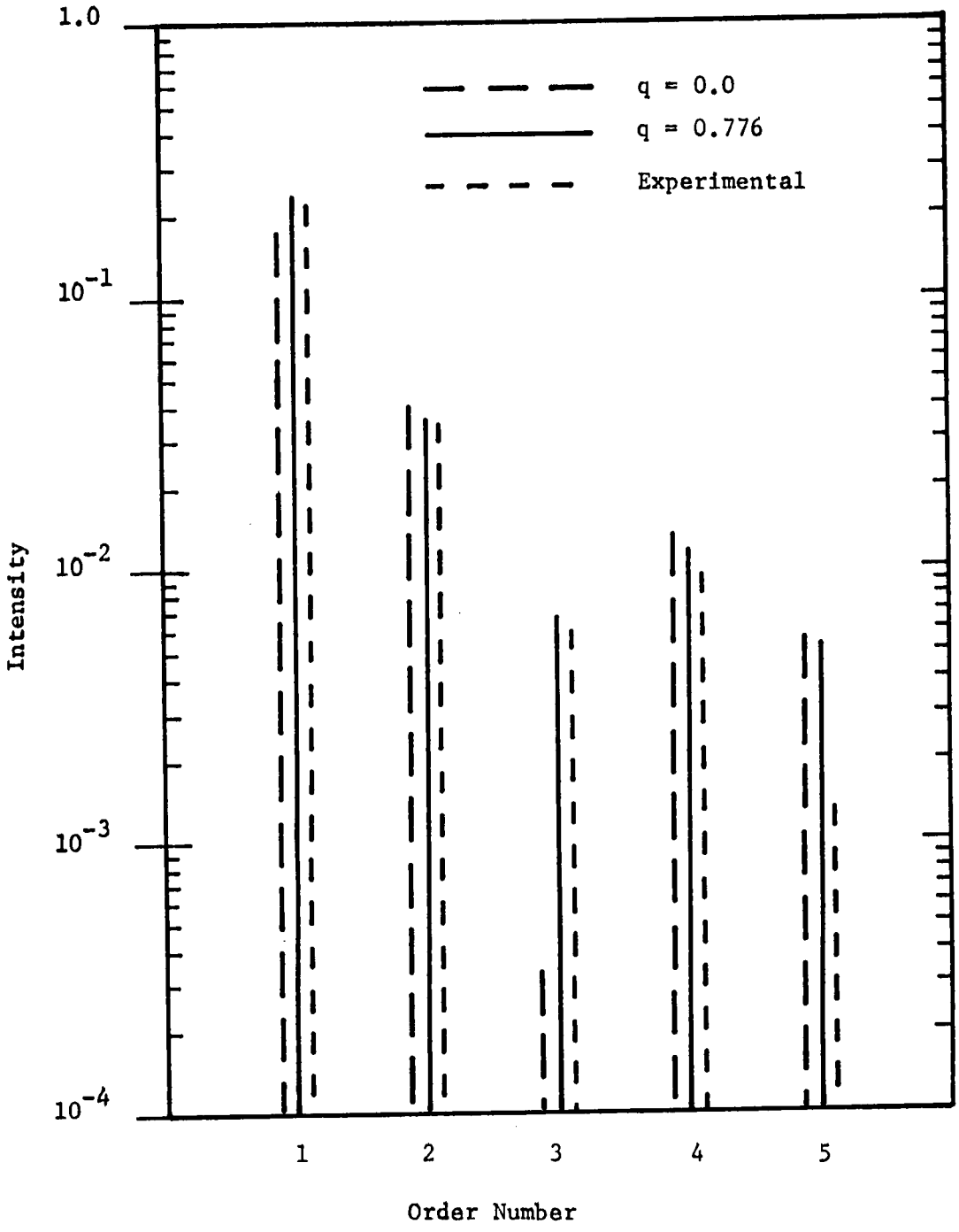


Figure 20. Comparison of theoretical and experimental diffraction patterns taking into account the coherence of the illumination.

$$\theta = 1.62 \times 10^{-2}$$

which corresponds to a value for Ω of

$$\Omega = 3.08 \ .$$

Thus the third order is seen to occur in the vicinity of a minimum of the envelope of the diffraction pattern; thus order three is highly sensitive to variations in coherence.

The fifth order, on the other hand, occurs at

$$\Omega = 5.14$$

and can be seen to show relatively small variations with coherence.

5. Error-free grating. Grating errors can be broken into two distinct categories--those which can be associated with the process of producing the grating and those associated with the measurement of the grating to determine its characteristics; i.e., its slit width(s) and period(s).

Consider first those errors associated with the production of the gratings. The fact that a large pattern of the gratings was reduced photographically made it possible to produce adequate gratings by a relatively imprecise technique. The errors in production of the lines were reduced in proportion to the reduction factor. Due to the manner in which the patterns were produced, one would expect the variations in line position on the grating to be random.

The effect of such random errors has been given by Sparrow (1919). The problem which is treated is that of a grating whose "interval

differs from the mean [interval] by a random small quantity $\frac{d\epsilon}{2\pi}$ [where d is the mean interval], the values of ϵ being distributed according to the law of error." Let p be the order of interference which is, in general, not integral. The effect of the random errors will be to produce scattered light between the spectra whose intensity is proportional to

$$2M\, p^2 \epsilon_0^{\,2}$$

where M is the number of slits composing the grating,

$\epsilon_0 = \sqrt{\Sigma\epsilon^2}$ is the mean square error of position of the lines,

and p the "order" of the spectrum which should be taken to mean the order of interference; e.g., a point midway between the first and second orders is taken to be p = 1.5.

Thus, the intensity of the scattered light is found to increase as the square of the order of interference; the higher the order, the greater the amount of scattered light. It is known that the relative intensity of the central order varies as M^2 so that the relative intensity of the scattered light varies as 1/M; i.e., as the number of slits is increased, there is a linear decrease in the intensity of the scattered light. Therefore, for a relatively small number of slits as was used for this experiment, the effect of the random grating errors would become more prominent in the higher orders.

Another class of grating errors is that which is associated with periodic errors in line position. Such errors as these result in the appearance of false orders or ghosts in addition to the normal

diffraction orders. The angular separation of the ghosts and their intensities are found to depend on the relative magnitudes of the period of the error. A series of measurements was made to determine if any of the spectral orders which were measured with the photometer were actually ghost orders. Each grating in turn was mounted in a spectrometer and the position of each observed spectral line was measured. These measurements, along with the period of the grating, were used to calculate the wavelength of the source for each individual order using the grating equation

$$p\lambda = d \sin\theta \qquad p = 0 \pm 1, \pm 2 \ldots$$

The identity of a legitimate order is signalled by the good agreement between the calculated and accepted values for the wavelength of the source. No prominent ghosts were found. This indicates that the gratings were free from any major periodic errors which would cause false spectral lines to be observed.

6. Periodic series of rectangular functions. In the development of the expression for the intensity of the Fraunhofer diffraction pattern for a grating having more than one slit width, it was assumed, as is often done, that the amplitude transmission function could be represented as a series of rectangular pulses of various widths and spacings. This assumption imposes the somewhat nonphysical condition that the material, from which the grating is made, must support a discontinuity in transparency. Due to scattering of light in the emulsion

during exposure and chemical diffusion during development of the film which was used to produce several of the gratings used, the amplitude transmission function recorded on the film must of necessity deviate from the idealized case. The corners are rounded to some extent and the leading and trailing edges of the square pulses must have finite slopes, as represented in Figure 21.

The effect of this rounding of the corners of the ATF* can be investigated as follows. Let the ATF be represented as a Fourier series

$$T(\xi) = a_0 + \sum_{m=1}^{\infty} a_m \cos mk\xi + \sum_{m=1}^{\infty} b_m \sin mk\xi$$

where ξ is the distance along the grating perpendicular to the slits. Michelson (1903) has shown that if indeed the ATF can be expressed as a Fourier series, then the intensity of the diffracted light is given by

$$I_m = a_m^2 + b_m^2$$

in directions such that

$$\sin\theta = \pm \frac{m\lambda}{d} \quad m = \pm 1, \pm 2, \pm 3 \ldots$$

where d is the grating space (or period). In other words, assuming a sufficiently large number of slits, the intensity of the mth order of the Fraunhofer diffraction pattern is completely determined by the sum of the squares of amplitudes of the mth harmonic of the ATF of the grating. From experience with Fourier series, we know that sharp corners are due to the higher frequency components. Thus, given two

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*Amplitude transmission function.

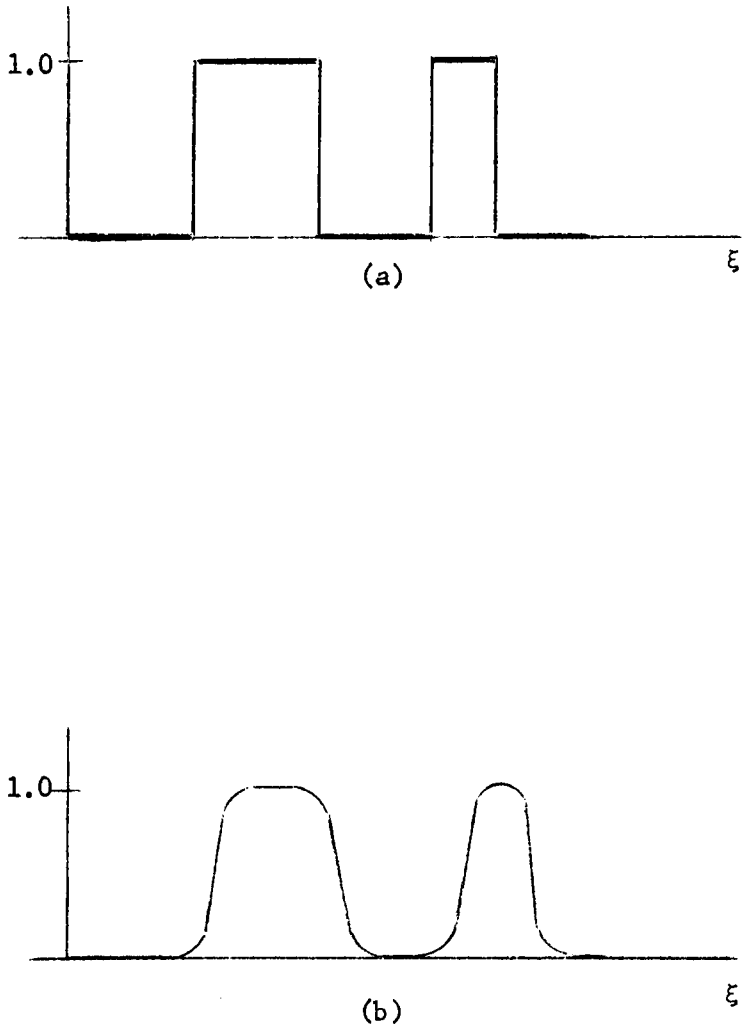


Figure 21. (a) Idealized amplitude transmission function; (b) realistic amplitude transmission function.

ATF's which are identical except that one is truncated after the kth harmonic, we would expect the first k diffraction orders to be the same for the two gratings. Furthermore, for the truncated grating, all orders k + 1 and above would be zero.

This idea then would tend, at least in part, to account for the relatively good agreement of the low orders and the relatively poor agreement of the higher orders of the measured diffraction patterns.

7. Phase properties constant across the grating surface. The purpose of the liquid gate was to make the phase properties across the surface of the grating as constant as was reasonably possible. It has been shown that a liquid gate does make a difference in the observed diffraction pattern. The agreement in the lower diffraction orders probably is improved by the liquid gate.

CHAPTER IV

ULTRASONIC ANALOGUE

In addition to the optical measurements, an ultrasonic schlieren apparatus was used to qualitatively examine the analogous problem of the diffraction of acoustic waves by a grating having two periodicities. The schlieren apparatus is shown schematically in Figure 22. The light from the mercury arc source is focused onto the entrance slit S_1 by a 5 cm condenser lens, L_1. The light is then made parallel by the collimating lens, L_2. This parallel light passes through the tank of water where it may be refracted by a gradient of refractive index set up in the water by the sound field. The lens L_3 brings the light which passes through the tank to a focus on the slit S_2 which is placed at the focus of L_3. Under usual operating conditions, S_2 is used to block the undeviated light through the system, allowing only the deviated light contained in one or more diffraction orders to pass through. This light is then imaged on the observation plane. The lenses, L_2 and L_3, have an aperture eight inches in diameter. This gives one the capability of viewing the far-field effects (Fraunhofer diffraction region) of the ultrasonic field. Such a large field of view is rarely encountered in schlieren systems which use lenses.

Two acoustical gratings were made and examined qualitatively using the schlieren apparatus. Graphite cylinders having a diameter

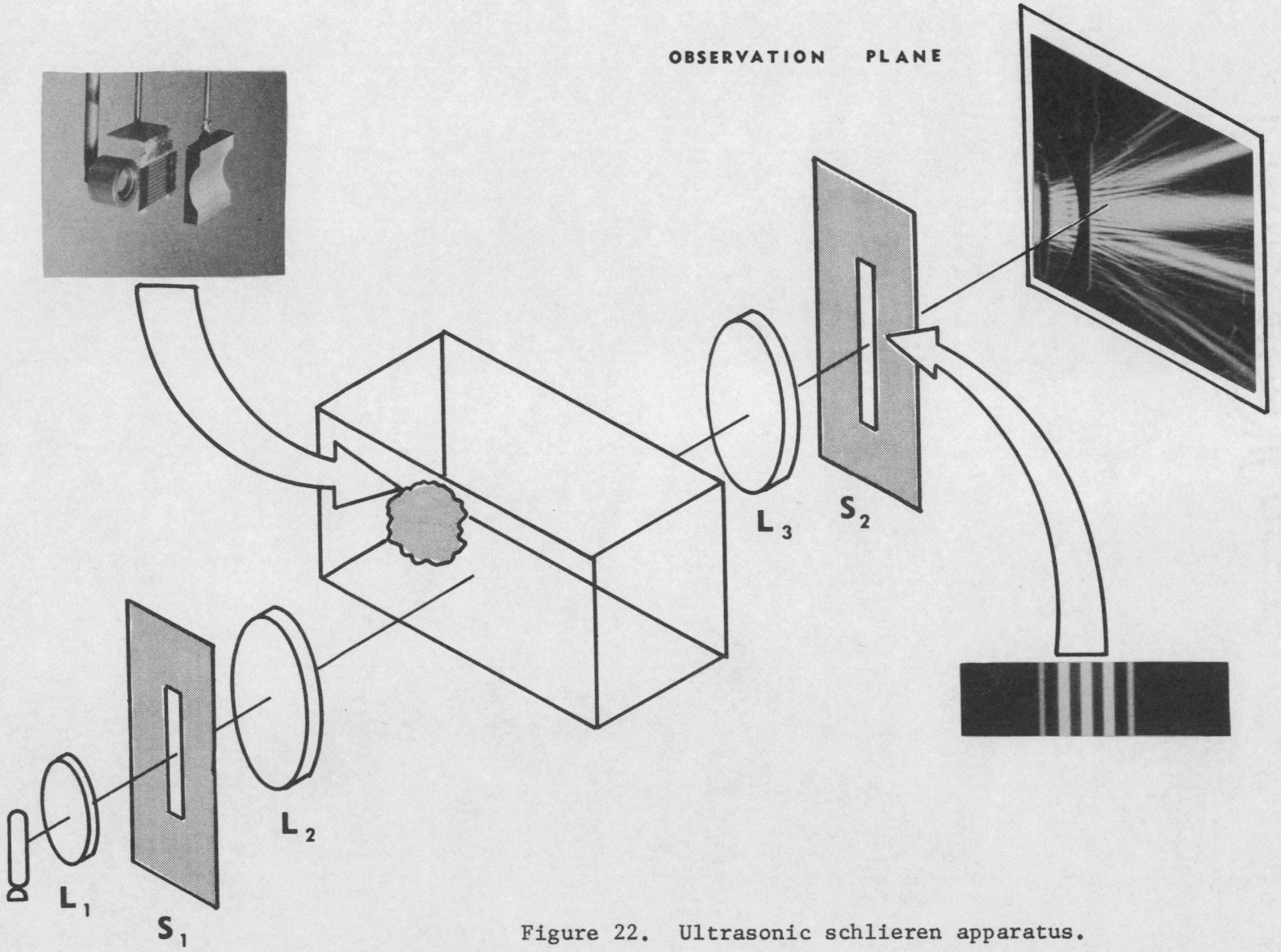


Figure 22. Ultrasonic schlieren apparatus.

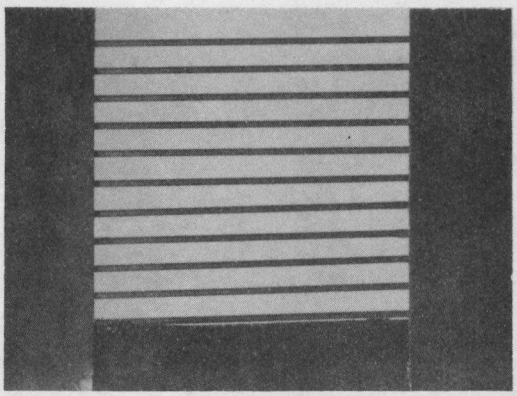
of 0.5 mm were used to construct the two gratings--one a singly-periodic grating with a period of 2.5 mm and the other a doubly-periodic grating with an overall period of 2.5 mm and secondary periods of 1.5 mm and 1.0 mm. Figure 23 shows the two acoustical gratings which were used.

The gratings were "illuminated" by a 1" diameter, 2 MHz ultrasonic beam. The wavelength in water was 0.75 mm. Initial photographs of the ultrasonic diffraction patterns for the singly- and doubly-periodic gratings are shown in Figures 24(a) and (b). The doubly-periodic grating is analogous to the optical grating shown in Figure 12(a), page 44. Although general features of the diffraction pattern could be observed, quantitative observations are not readily made.

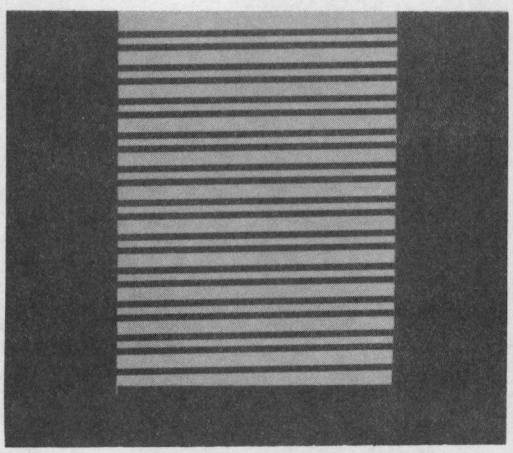
To make the analogy more complete, an acoustic lens having a focal length of 8 cm was placed behind the grating to focus the ultrasonic diffraction pattern. The results are shown in Figures 25(a) and (b).

Figure 25(a) is a photograph of the diffraction pattern produced by a singly-periodic grating. The diffraction orders are labeled. In addition to the zero order, one observes both positive and negative first and second orders. It is clear that the intensity of the second order is smaller than that of the other orders.

Figure 25(b) is a photograph of the diffraction pattern produced by a double-periodic grating. This grating has the same overall period as the singly-periodic grating. Two faint third orders are visible in addition to those appearing in Figure 24(a). In addition, both first orders are fainter than the second orders. This is completely

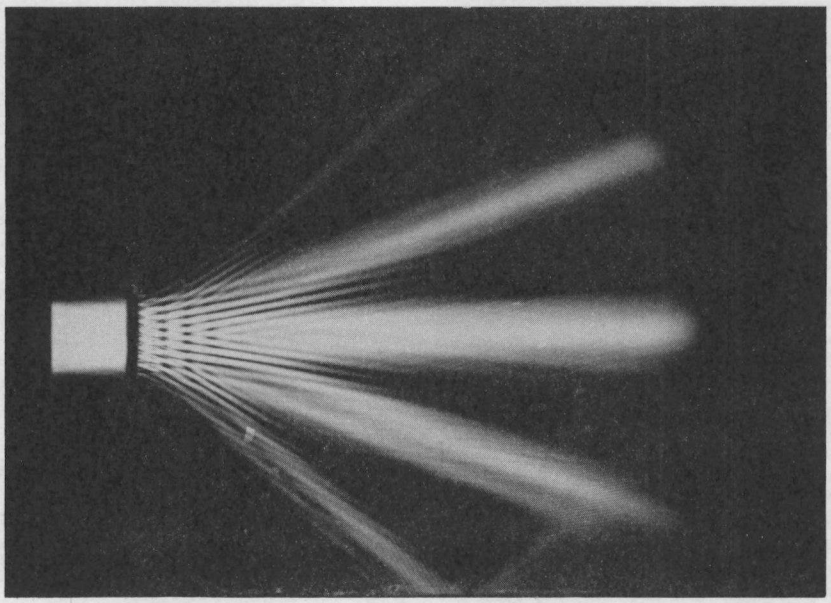


(a)

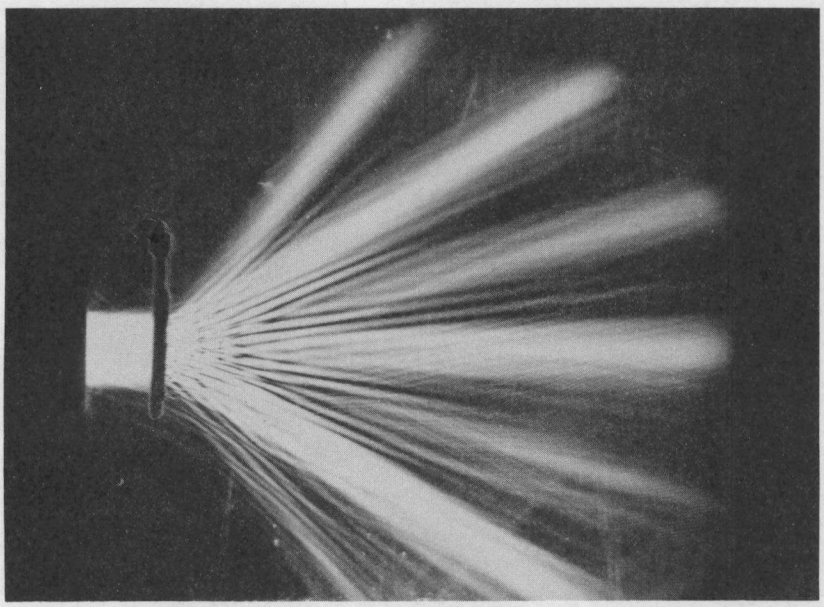


(b)

Figure 23. Ultrasonic gratings: (a) singly periodic and (b) doubly periodic.

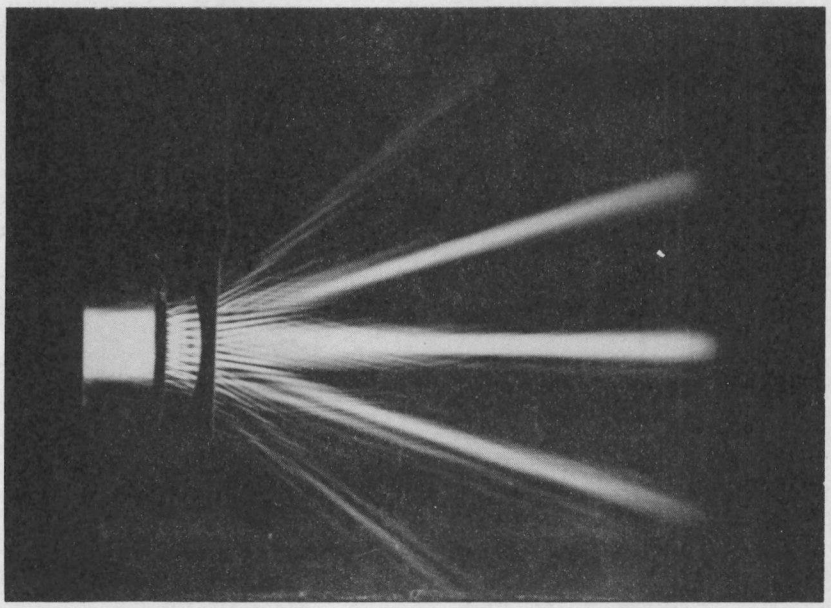


(a)

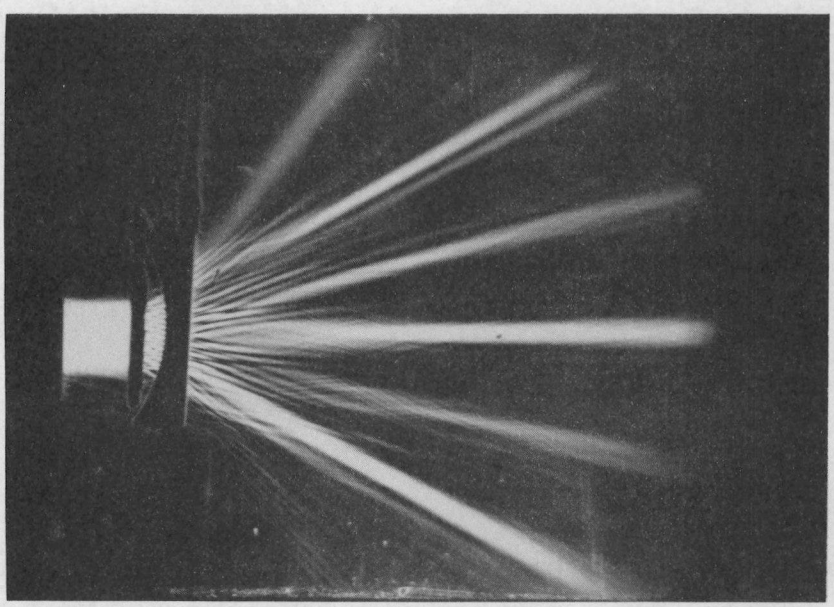


(b)

Figure 24. Schlieren photograph of diffraction of ultrasonic waves by (a) singly-periodic and (b) doubly-periodic gratings.



(a)



(b)

Figure 25. Use of a lens to focus ultrasonic waves diffracted by (a) singly-periodic and (b) doubly-periodic gratings.

consistent with the observations of the analogous optical diffraction pattern. In Figure 17, page 53, the first order was found to be an order of magnitude less intense than the second order.

In conclusion, it is seen that an ultrasonic analogue of an optical diffraction grating can be made. Such an analogue literally gives one a different point of view from which to study diffraction phenomena. In this specific example semi-quantitative observations could be made about a diffraction pattern produced by a doubly-periodic grating.

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He is a member of Tau Beta Pi, Phi Kappa Phi, and Sigma Pi Sigma.