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ANALYTICAL CORRECTIONS TO A NUMERICAL PROCEDURE FOR ESTIMATING GAS STAGNATION TEMPERATURE FROM THERMOCOUPLE MEASUREMENTS

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

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December 2008

ABSTRACT

An analysis study was conducted to evaluate the quantitative and qualitative predictions of a numerical (computer – coded) procedure for predicting the total or stagnation temperature of a compressible gas flow. The procedure is based on measured tip temperature provided by a radiation – shielded, self – aspirated thermocouple (TC) probe. The numerical procedure breaks down the components of the thermocouple probe into individual but thermally interacting thermal bodies. The numerical procedure assumes that the measured TC tip temperature is a true value, and carries out a heat balance for each component of the thermocouple that includes convective, conductive and radiative heat transfers where applicable. This heat balance process yields the gas recovery temperature, which can be related to the gas stagnation temperature. However, this numerical procedure has not been validated or experimentally calibrated in order to judge the accuracy of the predicted gas stagnation temperatures. Therefore, an analysis study of the numerical procedure was undertaken that compares the predicted mean temperatures of the thermocouple components (the radiation shield and the central TC sheath) to analytical or theoretical predictions of the component mean temperatures, for given measured tip and base temperatures. The analytical solutions were then used to guide a study of correction methods/schemes, applied to the numerical procedure, that minimized the total numerical error between the computer – predicted mean temperatures and the analytical mean temperatures. A "best" correction scheme was recommended, based on the minimized error.

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CHAPTER 1

INTRODUCTION

An intrusive probe is a typically a slender, cylindrical instrument that is used in fluid flows specifically for measuring certain parameters and obtaining information for diagnostic and experimental purposes. There are many types of intrusive probes such as steam thrust probes, turbine engine emission probes, mach flow angularity probes, imaging probes and thermocouple probes. Thermocouple probes have a thermocouple built inside to measure the gas temperature of the flow into which the probes are immersed.

This chapter gives the introduction, background and purpose to a specific thermocouple probe research study. Section 1.1 explains the overview, geometry, manufacturing processes and the shield designs for thermocouple probes. Section 1.2 explains the specific type of thermocouple probe that is being used in the present research study. The configuration of the thermocouple probe is explained in detail in this section. Section 1.3 explains the major steps that are required for the completion of the research study.

1.1 Introduction to Thermocouple Probes

Thermocouple probes are currently being researched and developed at AEDC, for example (ref. 1 and 2). The purpose of these thermocouple probes is to measure gas flow total temperatures and variations of total temperature in some of the project applications at AEDC: for example, finding the temperature variation in wind tunnel flows, in turbines engines exhausts, rocket motors and in exhaust plumes when they are being tested. Other types of probes have been developed for measuring static pressure, total pressure, enthalpy and chemical composition in aircraft turbine engines. Also, probes can be used for monitoring an engine during engine operation. During the testing process, several thermocouple probes are set on a vertical supporting rake structure to measure the variations in temperature at various height locations.

Most modern total temperature probes are designed as radiation – shielded, self – aspirated probes. This kind of probe design has been evaluated and developed over the last sixty years or so, for a variety of applications in high – speed flow wind tunnels and other aeronautical test facilities (ref. 1 – 6). According to ref. 2, “In the past decade, AEDC has developed and used standard rake bodies with interchangeable probes. The probes are available in diameters ranging from 0.125 to 0.5 inches, and are fabricated entirely from 304 stainless steel.” Figure 1.1, with Tables 1.1 and 1.2, shows a typical design for such thermocouple (TC) probes. In this design, a central shaft contains a TC junction at the tip of the shaft. The junction may be exposed to the flow of gas through

Table 1.1: Probe Dimensions for the Circular Thermocouple Probe in Figure 1.1 Given the Probe Internal Diameter D (Ref. 4)

D	A	B	C	E	F	G
0.15	0.04	0.25	0.24	0.156	0.007	0.35
0.144	0.04	0.187	0.177	0.156	0.007	0.35
0.056	0.04	0.08	0.076	0.06	0.007	0.35
0.02	0.01	0.038	0.034	0.024	0.0015	0.075

Note: All dimensions in inches

Table 1.2: Probe Dimensions for the Circular Thermocouple Probe in Figure 1.1 Given the Internal Probe Diameter D (Ref. 4)
L/D = Length / Diameter
Ae/Av = Area of Entrance / Area of One Vent Hole
 # Shields = Number of Shields

D	H	I	L	L/D	Ae/Av	#Shields
0.15	0.032 - 0.053	0.079	6.65 - 0.00	44.3 - 0.5	5.0 - 2.1	2, 1, or 0
0.144	0.032	0.042	1.08 - 0.00	7.5 - 0.0	5.0	2, 1, or 0
0.056	0.0125	0.021	0.28 - 0.056	5.0 – 1.0	5.0	2
0.02	0.00446	0.009	0.1 – 0.02	5.0 – 1.0	5.0	2

Note: All dimensions in inches

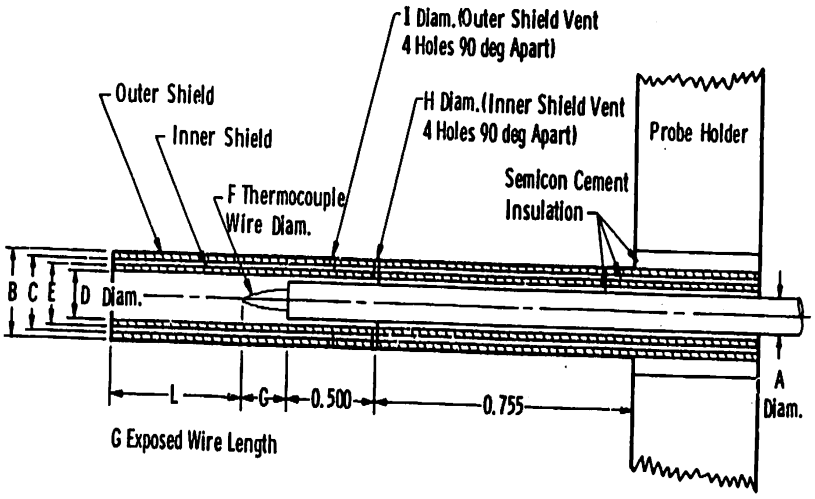


Figure 1.1: Typical Design of a Thermocouple Probe

the probe, or the junction may be protected behind a covering on the shaft tip. Surrounding this central shaft is a single radiation shield. The radiation shield has flow vent holes in the lateral sides to allow the flow of gases into the probe, around the central shaft and out of the probe. The purpose of the aspirated gas flow is to increase the convective heat transfer from the gas to the probe tip. The radiation shield minimizes radiation heat loss from the probe tip. Another source of heat loss from the probe tip is axial conduction along the length of the central shaft. These probes have been experimentally and theoretically analyzed in order to determine the best geometries and materials to minimize heat losses (ref. 1 – 6).

Because of this, thermocouple probes are manufactured in a variety of geometries and dimensions. One typical geometry for a double shielded, circular thermocouple probe used in high – speed wind tunnel applications was previously shown in Figure 1.1, with Tables 1.1 and Table 1.2. The length of a probe with an internal diameter of 0.15 inches is approximately 2.36 inches, and the height of the probe is approximately 0.25 inches. The width is the same as the height since the cross section of the probe is circular. The tolerance to these diameters is (+ or - 0.001) inches. The standard wind tunnel probe size is a probe with a 0.150 inch internal diameter. To reduce conduction heat transfer losses, the length of the exposed thermocouple wire is typically fifty times its' wire diameter. There are usually four drilled circular vent holes in the inner shield, and four drilled circular vent holes in the outer shield, if a double – shielded design is used. The cross section area for each of these holes is much smaller than the cross sectional area of the probe entrance.

When designing thermocouple probes and rake structures, damage to these probes and structures needs to be taken into account if they experience shock waves in high speed gas flows in a high temperature environment. Current probe designs for high – enthalpy flow are fabricated from 304 stainless steel and water - cooled internally by forced convective cooling using high pressure, high velocity water. These kinds of probes can withstand temperature environments up to 2,750 degrees Fahrenheit. According to Beitel, Catalano, and Edwards (ref. 1): "Conventional probe fabrication techniques limit the coolant path geometry and result in degraded cooling effectiveness and probe size restrictions." Electroforming is a newly developed fabrication technique that provides cooling channel effectiveness in thermocouple probes and rake structures that experiences shock waves resulting from high speed gas flows in a high temperature environment up to 4,500 degrees Fahrenheit and heat flux levels up to 57,000,000 Watts / meter^2. It is used in the manufacturing of certain probes and embedded, complex geometric configuration cooling channels inside probes with diameters as small as 0.1 inches. In the electroforming process, metal is electrodeposited onto a conductive substrate which then is used in process fabrication of probes and cooling channels. According to ref. 1, "Most electroforming is performed with nickel because of the wide range of mechanical properties and hardness possible." "Copper is the second most commonly electrodeposited metal because of the unique electrical and thermal properties offered." (ref 2). Another advantage of electroforming is that the electrodeposited metal

results in a higher yield strength than any kind of stainless steel. This is a very applicable to manufacturing probes that undergo a high pressure environment when a testing process is being conducted.

Radiation shield designs for thermocouple probes are usually of circular design. One benefit of having a radiation shield design is that it protects the probe against high dynamic pressure loads. The purpose of the vent holes drilled in the shield is to provide gas through – flow ventilation of the air that moves through the holes maintaining a subsonic flow inside the probe. According to Winkler (ref 3.), total stagnation temperature circular probes with a small vent – to entrance – area ratios seem to perform better than ones with larger ratios. The smaller area ratios allow the achievement of sonic flow conditions through the vent holes. With the vent holes choked, the pressure ratio across the flow and the mass flow through the probe are fixed, and depends only on the stream total conditions and vent hole total open area.

1.2 Background of the Thermocouple Probe Research Study

The thermocouple probe research study of the present investigation involves a rod shaped TC probe with a thermocouple attached inside the tip of the probe for measuring gas temperatures. The rod has a single surrounding radiation shield used for protecting the rod, thermocouple and thermocouple junction. The radiation shield protects the probe tip from heat loss due to radiation heat transfer effects. Heat losses are due to radiation from the rod to the shield surfaces and surrounding environment, and to conduction out of the base of the rod. The radiation shield has vent holes to allow some of the incoming air to escape to establish the convective heating of the rod tip.

This particular thermocouple probe design is being developed for use in high enthalpy flow streams as a gas temperature measurement device. The diagram of the thermocouple probe is shown in Figure 1.2. A temperature is measured by a thermocouple junction embedded in the probe tip of the ceramic sheath that contains and protects the thermocouple junction. Therefore, the temperature that the device records is not the gas temperature but rather, it records the tip or recovery temperature of a body (the sheath) experiencing all three modes of heat transfer: conduction (axial), convection (on its tip and lateral surfaces) and radiation heat transfer (with the gas and with any surfaces the tip and sheath can "see"). The ceramic sheath containing the thermocouple lead wires is a hollow cylindrical rod of refractory material with a spherical end cap into which the thermocouple is embedded. Surrounding the ceramic sheath is another cylindrical tube that serves as a radiation shield for the ceramic thermocouple sheath. This outer annular tube has vent holes penetrating its sides, through which the hot gas captured by the probe can be released back into the surrounding flow. Thus, the thermocouple probe is called a "self – aspirating" thermocouple probe.

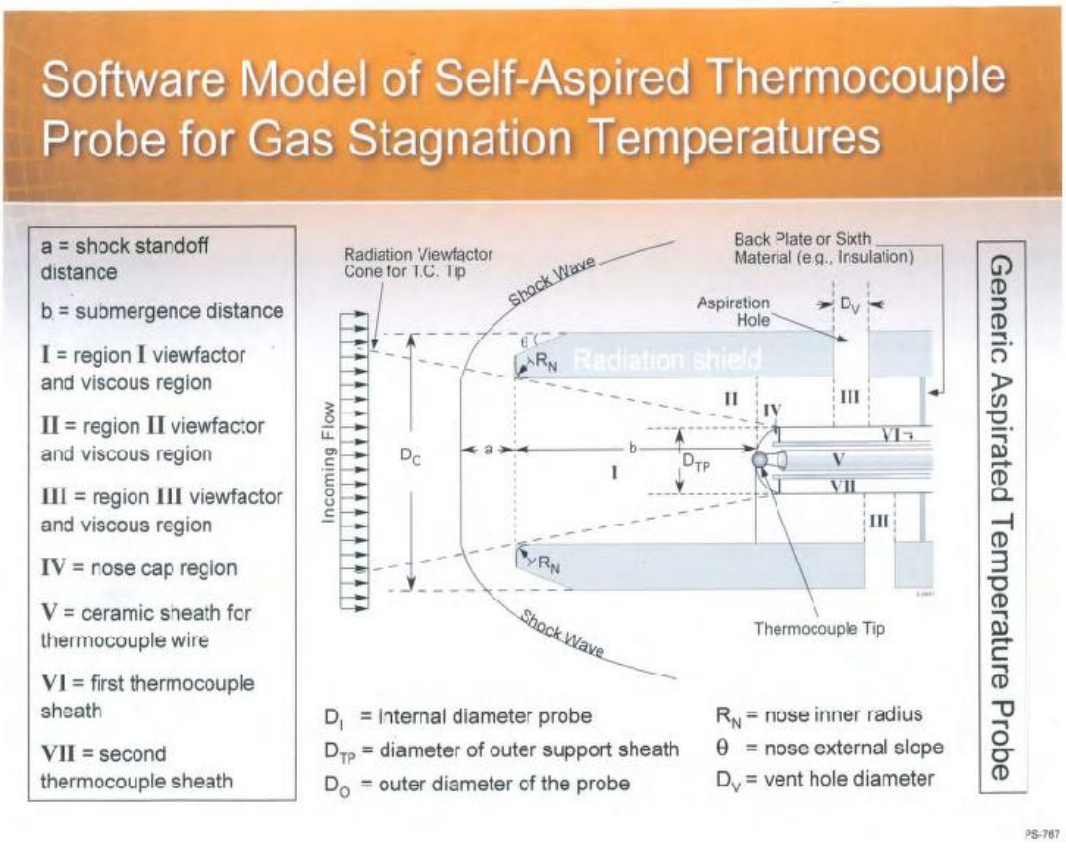


Figure 1.2: Diagram of the Thermocouple Probe used for this Research Study

1.3 Purpose of the Thermocouple Probe Research Study

The purpose of this project is to use properties of analytically calculated temperature distributions, such as the mean temperature locations and values of the sheath and radiation shield for comparing and correcting the corresponding values generated by a numerical model of the temperature probe. Heat transfers in the analytical models take into account both convection and radiation from the sheath and shield surfaces, and conduction out of the base of the sheath and shield. The purpose of the analytical corrections is to improve the capability of the computer model to predict gas stagnation temperature through a component heat balance obtained in an iterative procedure.

Based on input values for surface heat transfer along the sheath or radiation shield, the following steps to the correction process for the thermocouple probe are listed below:

Procedure for Making Analytical Corrections to Numerical Model of the Single – Shielded Thermocouple Probe.

1. Obtain the analytical predictions of the temperature distributions, mean temperature locations and mean temperature values of the thermocouple sheath.

2. Obtain the analytical temperature distributions, mean temperature locations and mean temperature values of the radiation shield surrounding the sheath.

3. Compare the analytical calculations of the temperature distributions, mean temperature locations and mean temperature values of the thermocouple sheath and radiation shield with the corresponding mean temperatures calculated numerically, for the purpose of making corrections to the numerical calculation procedure that computes the temperature losses by conduction, convection and radiation heat transfer from both the sheath and shield components.

4. Compare the predicted gas stagnation temperature values obtained in the original numerical model of the shielded thermocouple probe to the values made with the analytical - numerical corrected models.

CHAPTER 2

ANALYTICAL MODEL OF THE THERMOCOUPLE PROBE

A project at the University of Tennessee Space Institute resulted in a computer – based temperature correction procedure for a single – shielded aspirated thermocouple probe which estimates the gas temperature, based on the measured probe temperature, and heat loss estimates for the probe. The temperature estimation procedure resulted in the calculation of a single steady – state temperature for the ceramic sheath containing the thermocouple, and also a single temperature for the radiation shield surrounding the ceramic sheath. Therefore, it is necessary to estimate or model the analytical temperature distribution in the ceramic sheath in order to improve the numerical temperature correction process for the probe.

By determining the axial temperature profile in the ceramic sheath and shield, it is possible to determine the axial heat flux profile and locate on the axial temperature profile the axial position that corresponds to the single "mean" temperatures of the sheath and shield which can be compared to corresponding values calculated in the numerical heat – balance correction procedure for determining gas temperature. Section 2.1 provides the results of the temperature, heat flux and mean temperature profiles for the sheath. Section 2.2 states the results of the temperature, heat flux and mean temperature profiles for the radiation shield. Also, comparisons are made of the results of the correction on the temperature profiles for the sheath and shield.

2.1 The Sheath

The analysis of the sheath is based on an axial conduction heat transfer process of the thermocouple sheath, which accounts for convective and radiative heat transfer on its lateral sides. Two basic forms of temperature – based differential equations for conservation of energy in the thermocouple sheath were derived. These forms differ based on whether or not the "local" or x – wise temperature of the sheath is included in the term describing local surface convective and radiative heat flux. These two differential forms are given as:

A.) $T'' + K = 0$
B.) $T'' - K_1T + K_2 = 0$

These differential equations are ordinary differential equations instead of partial because the temperature distribution only depends on the axial direction along the sheath and not on the radial direction. In equation A.), K represents the convective and radiative energy input to the sides of the sheath under the assumption that it is independent of the local

surface temperature. In equation B.), K_1 and K_2 are constants that arise when the lateral convective and radiative heat transfer to the sheath are allowed to depend on the local, axial temperature of the sheath. Equation B.) is related to the standard constant – diameter /sectional area / fin heat transfer temperature equation, ref. 7. The two forms of the axial temperature equation, equation A.) and equation B.), were each solved for three different sets of boundary conditions. The boundary conditions were measured tip temperatures and known base temperatures taken from an experimental test of a self – aspirated thermocouple probe. The values of K, K_1 and K_2 were predicted by the computer program developed to estimate gas temperature, based on measured probe tip temperature and a probe overall thermal balance. One set of values of K, K_1 and K_2 were predicted for each set of known tip and base temperatures. Having the boundary conditions and the physical constants K, K_1 and K_2 corresponding to heat transfer on the lateral sides of the thermocouple sheath, a series of temperature profile calculations were made. First, the mathematical solutions to the temperature profiles were obtained subject to tip and base temperatures being the given boundary conditions. These mathematical solutions, and the axial temperature profiles that they subsequently provide, are explained below in sections 2.1.1 and 2.1.2.

2.1.1 First Model Differential Equation Analysis

The first differential equation that was derived to describe the probe was:

$$\frac{d^2 T}{dx^2} + K = 0$$

Boundary conditions were: $T_0 = T(0)$ and $T_L = T(L)$
Where:
T(0) = temperature at the tip of the sheath
T(L) = temperature at the base of the sheath

The definition of the constant K is: $\dfrac{q``(P)}{k(Ac)}$

Where the variables in K are defined as:
$q``$ = lateral heat transfer rate (Watts / meter^2)
P = perimeter (meters)
k = thermal conductivity {Watts / [meters(Kelvin)]}
Ac = cross sectional area (meters^2)

The mathematical solution to equation A.), subject to tip temperature, T(L) and base temperature, T(0) is

$$T(x) = \frac{[T(L) - T(0)](x)}{L} + \frac{(K)[(x)L - (x)^2]}{2} + T(0)$$

Again, K is a constant which came from the surface heat transfer by convection and radiation when assumed independent of T(x). The length of the sheath is 0.06063 meters. Three sets of thermocouple temperatures were used as boundary conditions for obtaining and plotting the temperature solutions. One tip temperature (T1) was obtained at a low tip temperature of 303.55 (Kelvin). The second tip temperature (T2) was obtained at a medium temperature of 589.75 (Kelvin).The third tip temperature (T3) was obtained at a high temperature of 733.15 (Kelvin).The temperatures and K values for all three supply temperatures are listed in Table 2.1. The plots of the temperature solutions for equation A.) for the three tip temperatures are shown in Figure 2.1. The temperature solution plots have a parabolic shape which is expected because the temperature solution depends on the x - squared. Based on the temperature solution plots for the three tip temperatures in Figure 2.1, the probe temperature is greatest at the middle of the probe and not at the tip. This means that the heat flux is flowing to the left before the maximum temperature is reached, and the heat flux is flowing to the right after the maximum temperature is reached. This is not physically plausible since the tip temperature is the highest temperature on the sheath. Heat flow from the tip to the gas would occur. The heat flow would be maximum at the base of the rod. Because of this predicted tip heat loss by conduction, the first model and resulting differential equation for the thermocouple probe was not consistent with expected behavior, and thus the temperature model equation, (equation A.) was discarded.

2.1.2 Second Model Differential Equation Analysis

The second model solution for the sheath axial temperature profile differential equation, equation B.), was found as:

$$\frac{d^2T}{dx^2} - K_1T + K_2 = 0$$

Boundary conditions were: $T_0 = T(0)$ and $T_L = T(L)$
Where:
T(0) = temperature at the tip of the sheath
T(L) = temperature at the base of the sheath

The physical constants in the mathematical solution were:

$$K_1 = \frac{[(hc + hr)P]}{k(Ac)}$$

- and –

$$K_2 = \frac{[(hc(T_A) + hr(Ta)P]}{[k(Ac)]}$$

Table 2.1: The Value of K for Three Given Tip Temperatures.
Where:
(k = Kelvin)

		Tip Temperature (k)	Base Temperature (k)	K (k/meter^2)
A.1	**T1**	303.55	293.82	13533.33
A.2	**T2**	589.75	293.82	404808.93
A.3	**T3**	733.15	293.82	603736.96

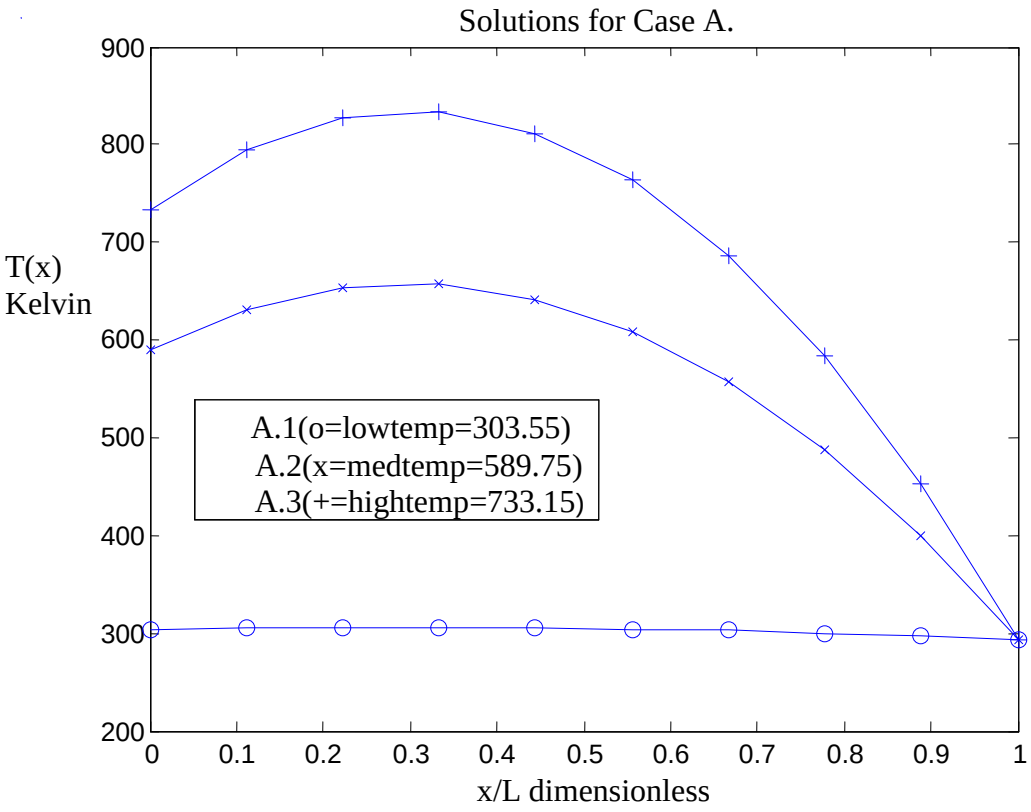


Figure 2.1: Plot for $d^2T/dx^2 + K = 0$ for the Sheath.
(Temperature versus x/L)

The parameters in K_1 and K_2 are defined as follows:
hc = surface heat transfer coefficient by convection {Watts / [(meter^2) Kelvin)]}
hr = effective surface heat transfer coefficient for radiation {Watts / [(meter^2)Kelvin)]}
P = perimeter (meters)
k = thermal conductivity {Watts / [(meters(Kelvin)]}
Ac = cross sectional area (meters^2)
T_A = gas recovery temperature (Kelvin)
Ta = radiation shield surface temperature (Kelvin)

The mathematical solution to the model equation, equation (B), was found as:

$$T(x) = \frac{K_2}{K_1} + \left[\left(T(L) - \frac{K_2}{K_1}\right) - \left(T(0) - \frac{K_2}{K_1}\right)\cosh\left(\sqrt{K_1}L\right)\right]\frac{\sinh\left(\sqrt{K_1}x\right)}{\sinh\left(\sqrt{K_1}L\right)} + \left(T(0) - \frac{K_2}{K_1}\right)\cosh\left(\sqrt{K_1}x\right)$$

As stated previously, K_1 and K_2 are constants which came from the numerically - predicted values of the lateral surface heat transfer boundary conditions, where surface heat transfer occurred by convection and radiation. The predicted values are provided by the numerical model of the thermocouple probe. The governing equation is a linear, homogeneous differential equation with constant coefficients. Two methods used in solving the equation are the Undetermined Coefficients (Superposition) approach and the Undetermined Coefficients (Annihilator) approach. It was shown that the general solution from both methods was the same. As another check, the solution was substituted back into the differential equation. After taking the first and second derivatives of the equation, the solution reduced to zero equals zero on both sides of the equation. The same sets of temperature boundary conditions were supplied to the mathematical solution for equation B.) for the low, medium and high tip temperatures. The tip temperatures, base temperatures, K_1 and K_2 as well as T_A and Ta values, were taken from the output of the computer analysis of the probe for the same three cases as before for all three tip temperature cases listed in Table 2.2, (where k = Kelvin). The plots of the temperature solutions of equation B.) are in Figure 2.2. The profiles of the three temperature solutions have hyperbolic shapes, since the general solution has hyperbolic sine and cosine terms.

The axial heat flux along the sheath was calculated using Fourier's law, which is:

$$q'' = -k\frac{dT}{dx}$$

Where: q = heat flux (Watts / meter^2)
k = thermal conductivity {Watts / [meters(Kelvin)]}
$\frac{dT}{dx}$ = derivative of temperature solution with respect to x
A_{cs} = cross sectional area in (dT/dx) equation

$$\frac{dT}{dx} = A_{cs}\sqrt{K_1}\left\{\left[\left(T(L) - \frac{K_2}{K_1}\right) - \left(T(0) - \frac{K_2}{K_1}\right)\cosh\left(\sqrt{K_1}x\right)\right]\frac{\cosh\left(\sqrt{K_1}x\right)}{\sinh\left(\sqrt{K_1}x\right)} + \left(T(0) - \frac{K_2}{K_1}\right)\sinh\left(\sqrt{K_1}x\right)\right\}$$

Table 2.2: The Values of K_1, K_2, T_A and Ta at Three Given Tip Temperatures for the Sheath
Where:
(k = Kelvin)
(Temp. = Temperature)

		Tip Temp. (k)	Base Temp. (k)	K_1 (k/meter^2)	K_2 (k/meter^2)	T_A(k)	T_a(k)
B.1	**T1**	303.55	293.82	6997	2124000	303.55	301.4
B.2	**T2**	589.75	293.82	7211	4241400	529.75	516.9
B.3	**T3**	733.15	293.82	7951	5756000	733.15	627.0

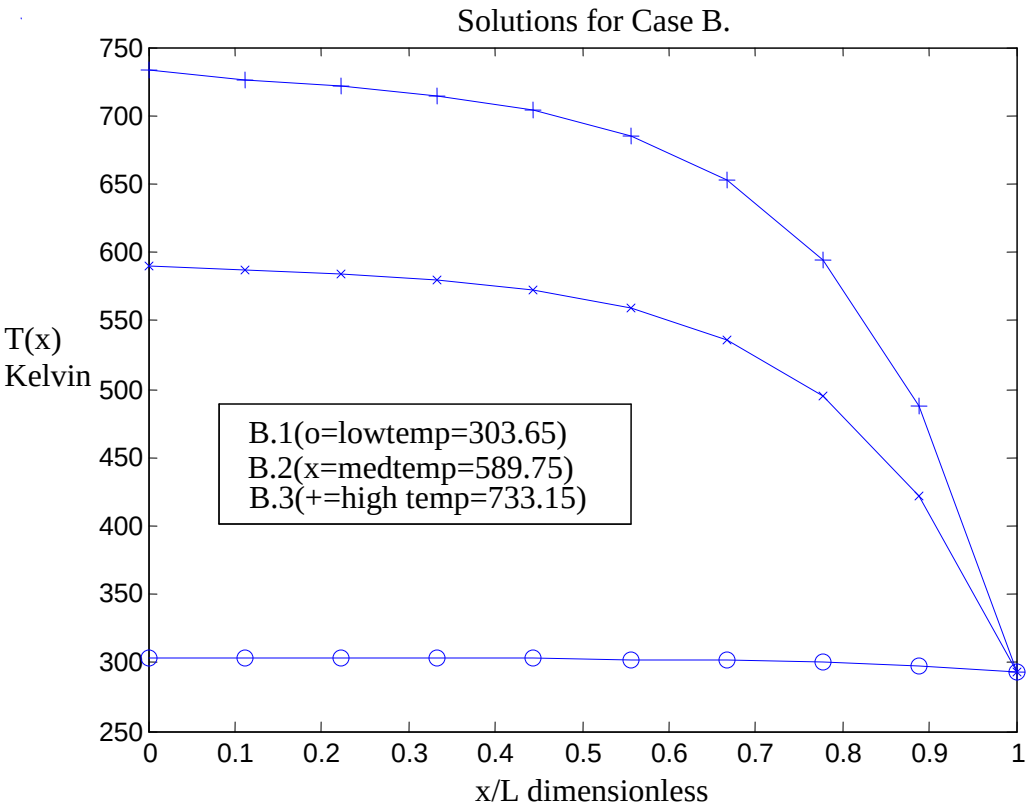


Figure 2.2: Plot of $d^2T/dx^2 - K_1T + K_2 = 0$ for the Sheath (Temperature versus x/L)

The plots of the axial heat flux in the thermocouple sheath are shown for each of the three tip temperatures in Figure 2.3, with the values of the thermal conductivities for each tip temperature in Table 2.3. Temperature is being lost inside the rod because of the heat transfer per unit lateral and cross – sectional areas, flowing to the right due to the effects of conduction, convection and radiation. The temperature that the thermocouple records is affected by these heat losses, which is why the thermocouple does not measure the gas stagnation temperature. More heat is lost when the thermocouple is operated in much higher speed flow high and enthalpy applications. According to Winkler (ref. 3), most of the temperature losses in most stagnation temperature probes are due to conduction losses in a high Mach number flow environment. However, his conclusions relate to wind tunnel The equation for the mean rod/sheath temperature was defined as:

$$\text{Tmean} = \frac{1}{L}\int_0^L T(x)dx$$

By integrating the temperature solution obtained previously, Tmean was found to be:

$$\text{Tmean} = \frac{K_2}{K_1} + \frac{1}{\sqrt{K_1 L^2}}\left\langle \left[\frac{(T(L) - K_2/K_1) - (T(0) - K_2/K_1)\cosh\left(\sqrt{K_1}L\right)}{\sinh\left(\sqrt{K_1}L\right)} \right] \left(\cosh\left(\sqrt{K_1}L\right) - 1\right) + (T(0) - K_2/K_1)\sinh\left(\sqrt{K_1}L\right) \right\rangle$$

Where the parameters of this equation are:
T(L) = base temperature (Kelvin)
T(0) = tip temperature (Kelvin)
L = sheath length (meters)

Once the mean temperatures were calculated for all three tip temperature cases, the x – coordinate on the central rod or sheath was found where the mean temperature value occurred on the temperature probe. With the x – coordinates of the mean temperatures known, the mean temperature points were plotted for all three tip temperatures. The dimensionless x – coordinate of the low tip temperature was 0.67745, for the medium tip temperature it was 0.67875, and for the high tip temperature it was 0.68283. The results are also summarized in Table 2.4 (where k = Kelvin). The plots of the mean temperature values are shown in Figure 2.4. The peak or maximum axial heat flux in all three tip temperature cases was at the base temperature, and the highest temperature was at the tip. Thus the second differential equation was taken as representing the physical thermocouple sheath temperature distribution. From the three temperature solutions the mean rod/sheath temperature was calculated for each tip temperature case, and located along the axial length of the thermocouple rod/sheath.

Table 2.3: The Values of the Thermal Conductivity at the Three Given Tip Temperatures for the Sheath
Where:
(Temp. = Temperature)
(k = Thermal Conductivity)

		Tip Temp. (Kelvin)	Base Temp. (Kelvin)	k {Watts / [meters(Kelvin)]}
B.1	**T1**	303.55	293.82	116.70
B.2	**T2**	589.75	293.82	104.99
B.3	**T3**	733.15	293.82	95.61

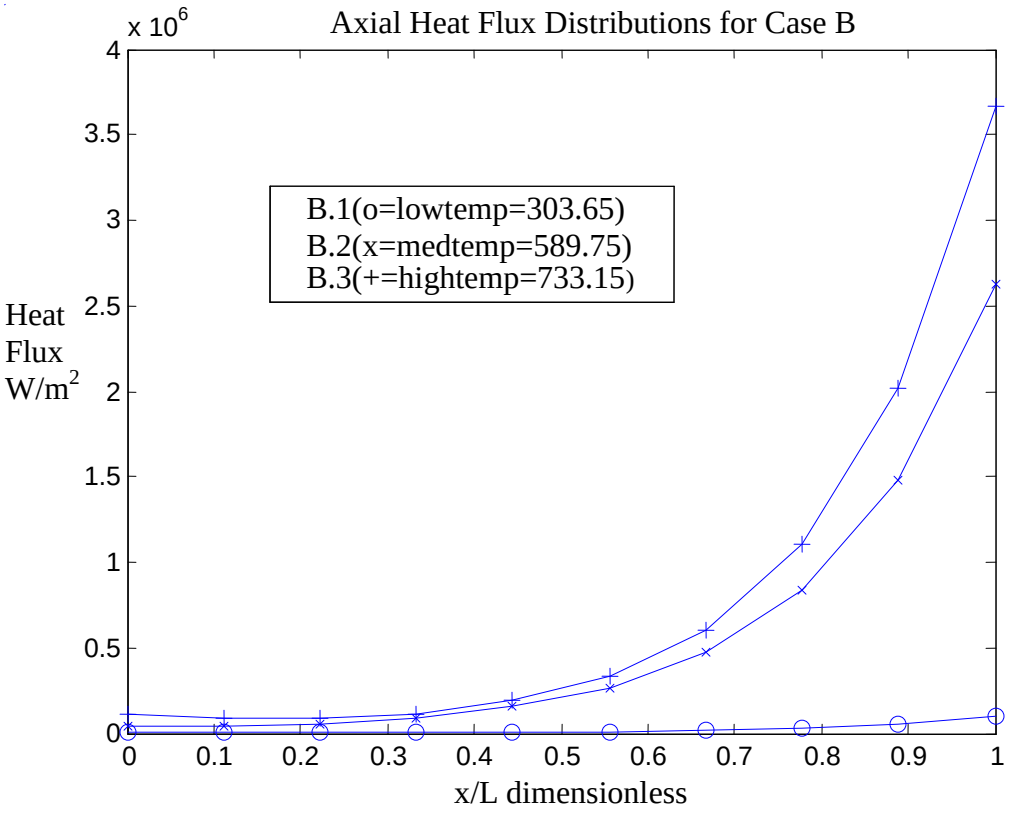


Figure 2.3: Plot of q`` = - k (dT/dx) for the Sheath.
(Heat Flux versus x/L)

Table 2.4: Values of the Mean Temperatures and Dimensionless X – Coordinates at the Three Given Tip Temperatures for the Sheath.
Where:
(k = Kelvin)
(Temp. = Temperature)

		Tip Temp. (k)	Base Temp. (k)	Mean Temp. (k)	x - coordinate
B.1	**T1**	303.55	293.82	301.65	0.6775
B.2	**T2**	589.75	293.82	531.97	0.6788
B.3	**T3**	733.15	293.82	646.77	0.6828

Note: X – coordinate is dimensionless

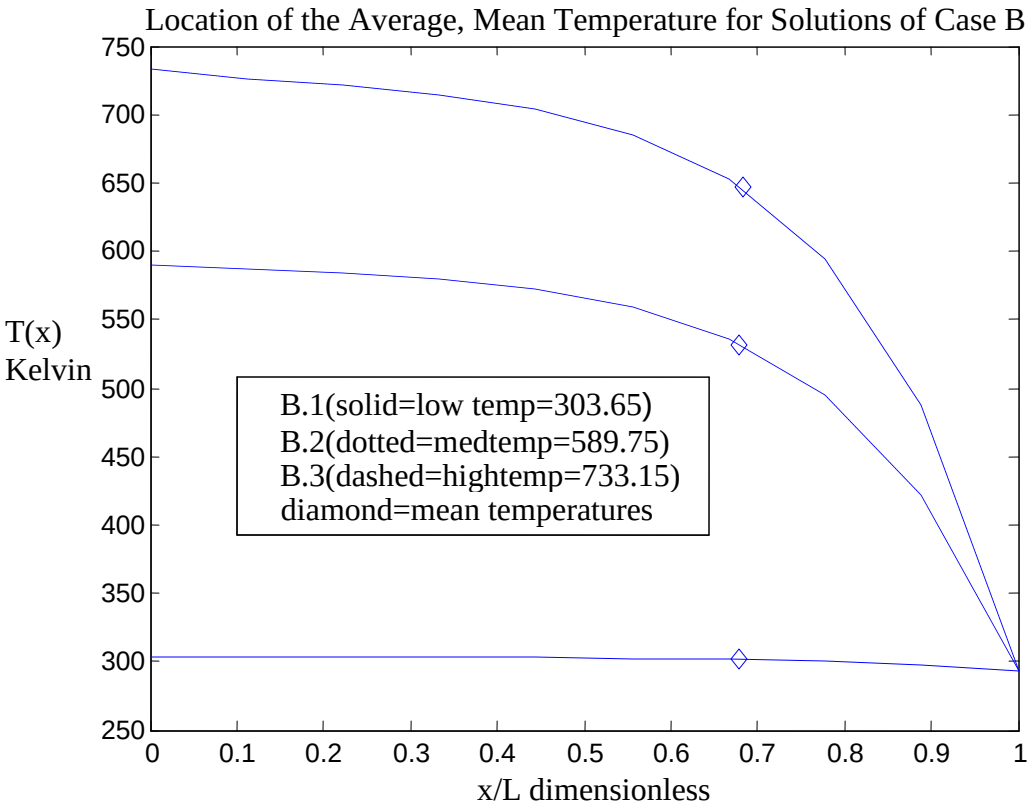


Figure 2.4: Mean Temperature Locations for each of the Three Temperature Distributions for the Sheath.
(Temperature versus x/L)

2.2 The Radiation Shield

The analysis of the radiation shield is also based on the assumption that the shield, a circular tube, has a thin wall, that in which the axial conduction heat transfer is the dominant conduction process, although it does also account for convective and radiation heat transfer on its lateral sides. The same basic form of the temperature – based differential equation for conservation of energy in the thermocouple sheath (Equation B.) is also used for the radiation shield. Equation A.) has already been discarded because the temperature solution plots for equation A, Figure 2.1 have a parabolic shape in which the temperature solution depends on the x – squared term.

2.2.1 Second Model Differential Equation Analysis

The second model solution for the radiation shield axial temperature profile differential equation, equation B.), was also found to obey the governing equation:

$$\frac{d^2T}{dx^2} - K_1T + K_2 = 0$$

Boundary conditions were: $T_0 = T(0)$ and $T_L = T(L)$
Where:
T(0) = temperature at the tip of the shield
T(L) = temperature at the base of the shield

The physical constants in the mathematical solution were

$$K_1 = \underbrace{\frac{(hc + hr)P_{21}}{k(Ac)}}_{\text{internal surface}} + \underbrace{\frac{(hc + hr)P_{22}}{k(Ac)}}_{\text{external surface}}$$

- and –

$$K_2 = \underbrace{\frac{[hc(T_A) + hr(T_1)]P_{21}}{k(Ac)}}_{\text{internal surface}} + \underbrace{\frac{[hc(T_A)+hr(To)]P_{22}}{k(Ac)}}_{\text{external surface}}$$

The parameters in K_1 and K_2 are defined as follows:
hc = surface heat transfer coefficient by convection {Watts / [(meter^2) Kelvin)]}
hr = effective surface heat transfer coefficient for radiation: includes the estimated viewfactors for the surfaces{Watts / [(meter^2)Kelvin)]}
P_{21} = perimeter of the internal surface (meters)
P_{22} = perimeter of the external surface (meters)
k = thermal conductivity of the radiation shield material{Watts / [(meters(Kelvin)]}
Ac = radiation shield cross sectional area[annulus area] (meters^2)
T_A = adiabatic wall recovery temperature of the gas flow over the shield surface. (Kelvin)

To = ambient surroundings temperature (Kelvin)
T_1 = sheath temperature (Kelvin)

The mathematical solution to the model equation, equation (B), was found to be identical to that for the central thermocouple sheath. Only the constants K_1 and K_2 are different:

$$T(x) = \frac{K_2}{K_1} + \left[\left(T(L) - \frac{K_2}{K_1}\right) - \left(T(0) - \frac{K_2}{K_1}\right)\cosh\left(\sqrt{K_1}L\right)\right]\frac{\sinh\left(\sqrt{K_1}x\right)}{\sinh\left(\sqrt{K_1}L\right)} + \left(T(0) - \frac{K_2}{K_1}\right)\cosh\left(\sqrt{K_1}x\right)$$

K_1 and K_2 account for the lateral convection and radiation on both sides of the shield. The length of the radiation shield is 0.032639 meters. K_1 and K_2 are constants which came from the lateral surface heat transfer boundary conditions, where surface heat transfer occurred by convection and radiation. K_1 and K_2 are different for the radiation shield because the perimeter, thermal convection coefficient, thermal radiation coefficient and radiative environment are different from the sheath, and there are different values for both the internal and external surfaces of the shield. The same sets of temperature boundary conditions for the radiation shield were supplied to the mathematical solution of equation B.), for the low, medium and high tip temperatures. The tip temperatures, base temperatures, K_1 and K_2 were provided by the output of the computer analysis of the probe. The same three tip temperature are used as before which are listed in Table 2.5 along with the base temperatures, K_1 and K_2 values (where k = Kelvin). The plots of the temperature solutions of equation B.) for the radiation shield are in Figure 2.5. The profiles of the three temperature solutions have hyperbolic shapes, since the general solution has hyperbolic sine and cosine terms.

The same analysis for the axial heat flux along the sheath can still be used for the radiation shield. The heat flux was calculated using Fourier's law, which is:

$$q'' = -k\frac{dT}{dx}$$

Where: q'' = heat flux (Watts / meter^2)
k = thermal conductivity {Watts / [meters(Kelvin)]}
$\frac{dT}{dx}$ = derivative of temperature solution with respect to x
A_{cs} = cross sectional area in (dT/dx) equation

$$\frac{dT}{dx} = A_{cs}\sqrt{K_1}\left\{\left[\left(T(L) - \frac{K_2}{K_1}\right) - \left(T(0) - \frac{K_2}{K_1}\right)\cosh\left(\sqrt{K_1}x\right)\right]\frac{\cosh\left(\sqrt{K_1}x\right)}{\sinh\left(\sqrt{K_1}x\right)} + \left(T(0) - \frac{K_2}{K_1}\right)\sinh\left(\sqrt{K_1}x\right)\right\}$$

Table 2.5: The Values of K_1 and K_2 at Three Given Tip Temperatures for the Radiation Shield.
Where:
(k = Kelvin)

		Tip Temperature (k)	Base Temperature (k)	K_1 (k/meter^2)	K_2 (k/meter^2)
B.1	**T1**	303.55	293.82	7382.73	2242420.84
B.2	**T2**	589.75	293.82	7031.82	4123271.53
B.3	**T3**	733.15	293.82	7430.88	5390004.41

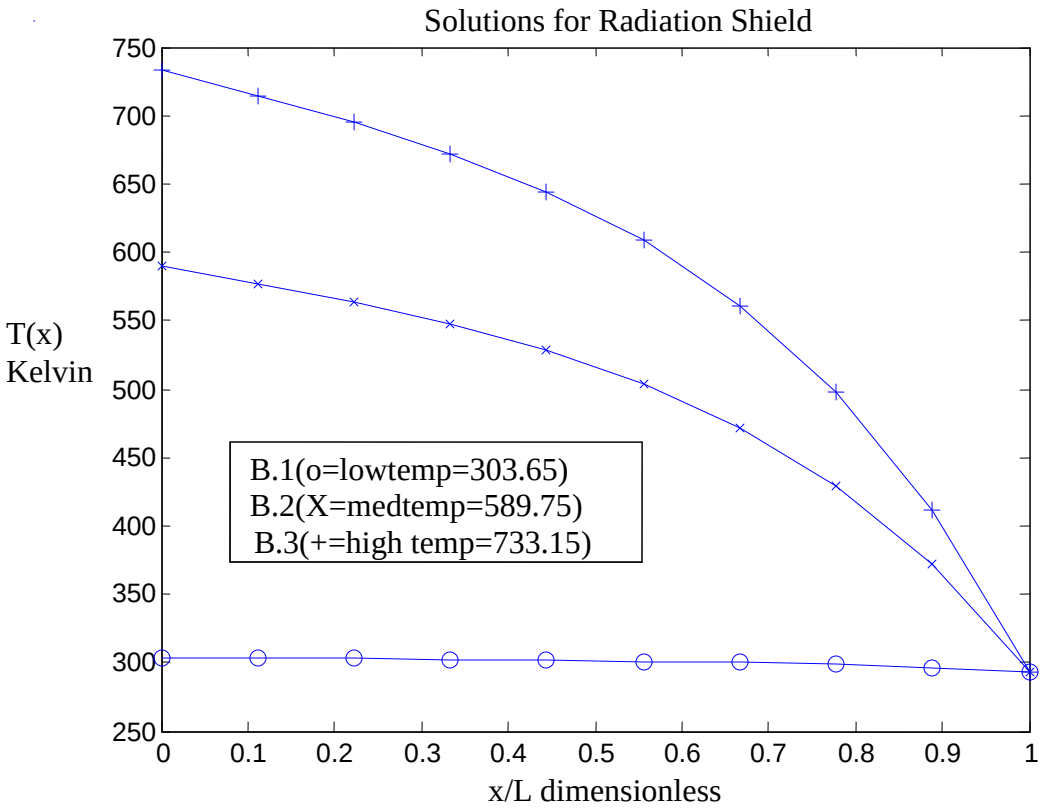


Figure 2.5: Plot of $d^2T/dx^2 - K_1T + K_2 = 0$ for the Radiation Shield. (Temperature versus x/L)

The plots of the axial heat flux in the thermocouple shield are shown for each of the three tip temperatures in Figure 2.6, with the thermal conductivity values for each tip temperature in Table 2.6. The peak or maximum axial heat flux in all three tip temperature cases was at the base temperature, thus the second differential equation can still be taken as representing the actual thermocouple shield temperature distribution as before with the thermocouple sheath distribution. The heat flux for the radiation shield will be different than the sheath because k, L, K_1 and K_2 are different for the radiation shield. From the three temperature plots, it is now possible to calculate the mean radiation shield temperature for each tip temperature case, and to locate this mean temperature at a point along the axial length of the radiation shield.

As before, the equation for the mean radiation shield temperature was defined as:

$$\text{Tmean} = \frac{1}{L}\int_0^L T(x)dx$$

By integrating, the same form for the mean temperature solution was obtained as previously. Tmean was found to be:

$$\text{Tmean} = \frac{K_2}{K_1} + \frac{1}{\sqrt{K_1 L^2}}\left\langle \left[\frac{(T(L) - K_2/K_1) - (T(0) - K_2/K_1)\cosh(\sqrt{K_1}L)}{\sinh(\sqrt{K_1 L})} \right](\cosh(\sqrt{K_1}L) - 1) + (T(0) - K_2/K_1)\sinh(\sqrt{K_1}L) \right\rangle$$

Where the parameters of this equation are:
T(L) = base temperature (Kelvin)
T(0) = tip temperature (Kelvin)
L = sheath length (meters)

Once the mean temperature was calculated for all three tip temperature cases, the dimensionless x – coordinate was found where the mean temperature value occurred on the radiation shield. With the x – coordinates of the mean temperatures known, the mean temperature points were plotted for all three tip temperatures. The x – coordinate of the low tip temperature was 0.602, for the medium tip temperature 0.5956, and for the high tip temperature 0.5978. The results are also summarized in Table 2.7 (where k = Kelvin). The plots of the mean temperature values are shown in Figure 2.7.

Table 2.6: The Values of the Thermal Conductivity at the Three Given Tip Temperatures for the Radiation Shield
Where:
(Temp. = Temperature)
(k = Thermal Conductivity)

		Tip Temp. (Kelvin)	Base Temp. (Kelvin)	k {Watts / [meters(Kelvin)]}
B.1	**T1**	303.55	293.82	151.30
B.2	**T2**	589.75	293.82	143.89
B.3	**T3**	733.15	293.82	133.32

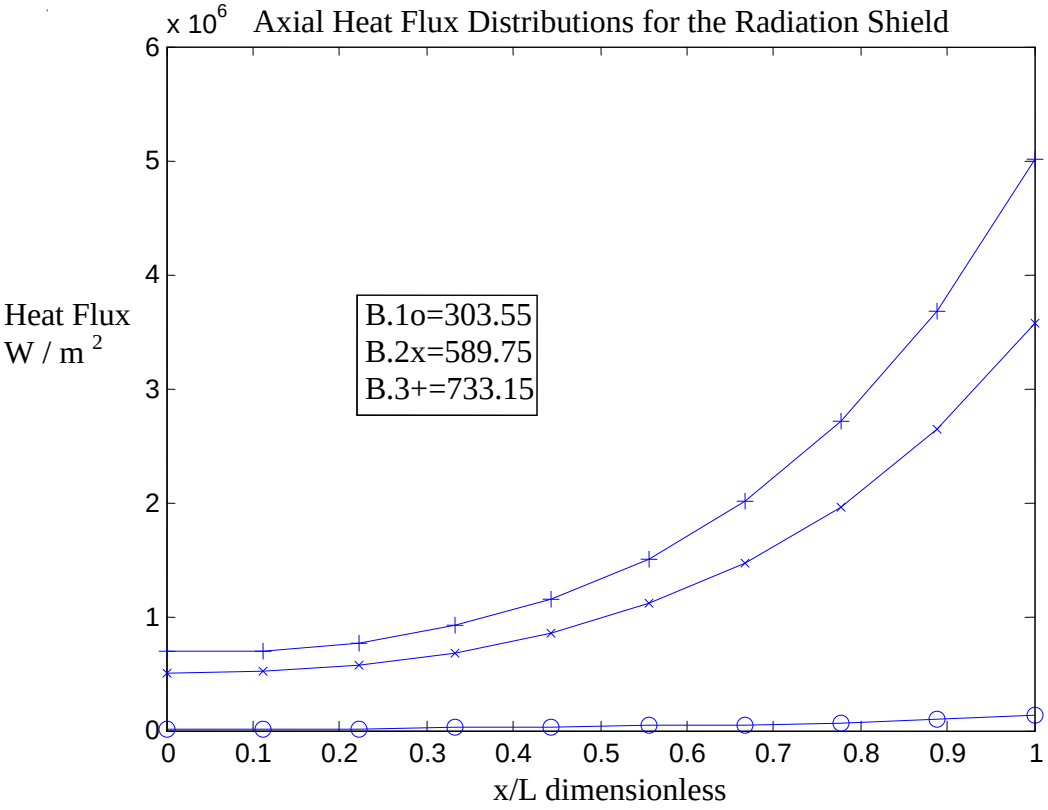


Figure 2.6: Plot of q`` = -k (dT/dx) for the Radiation Shield. (Heat Flux versus x/L)

Table 2.7: Values of the Mean Temperatures and Dimensionless X – Coordinates at the Three Given Tip Temperatures for the Radiation Shield.
Where:
(k = Kelvin)
(Temp. = Temperature)

		Tip Temp. (k)	Base Temp. (k)	Mean Temp. (k)	x - coordinate
B.1	**T1**	303.55	293.82	301.82	0.6020
B.2	**T2**	589.75	293.82	530.20	0.5956
B.3	**T3**	733.15	293.82	645.14	0.5978

Note: x – coordinate is dimensionless

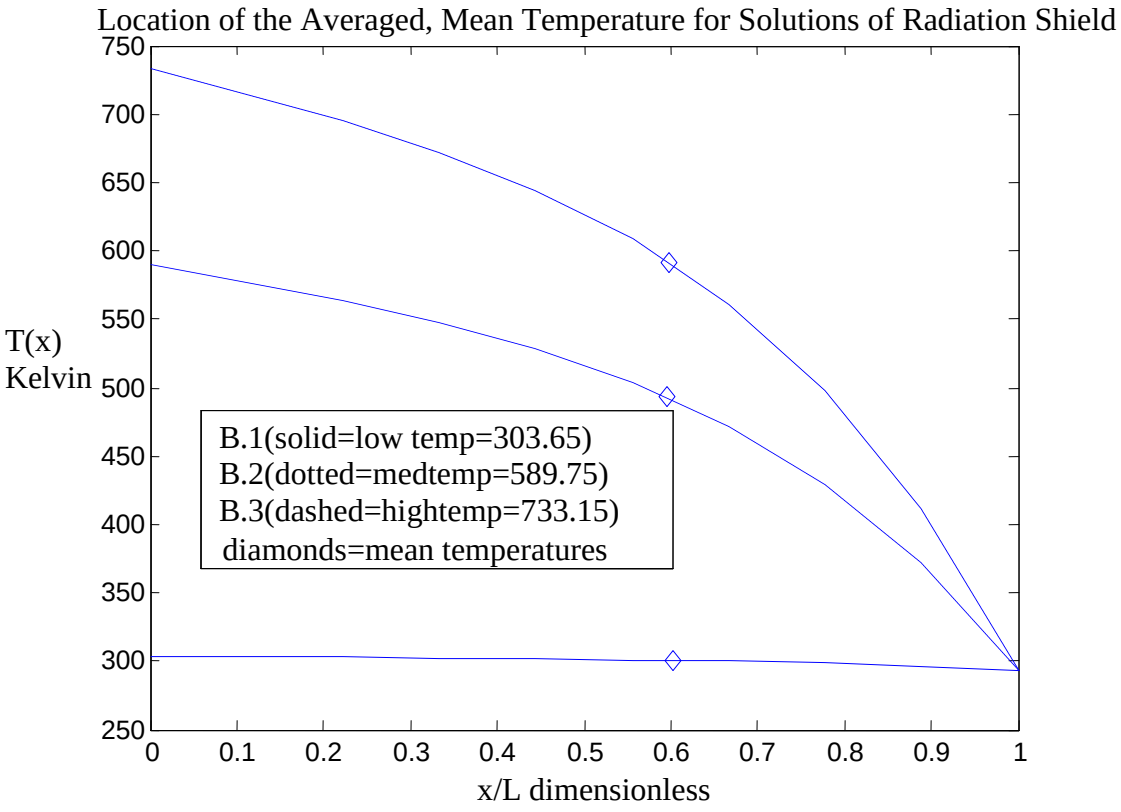


Figure 2.7: Mean Temperature Locations for each of the Three Temperature Distributions for the Radiation Shield.
(Temperature versus x/L)

2.2.2 Analysis Comparisons Between the Sheath with the Radiation Shield

The figures of the temperature distribution, axial heat flux and mean temperature locations have already been obtained for the sheath and shield. This section compares the three kinds of figures for the sheath and shield. The differences between the three kinds of figures for the sheath and shield are explained on a mathematical basis.

The temperature distributions for the sheath and shield are compared in Figure 2.8. Comparing the two distributions, the temperature distributions for the radiation shield are slanted more than the temperature distributions for the sheath. This is expected behavior for the reason that the shield is much more exposed to ambient conditions by radiation than is the sheath. Also, because the shield has a shorter thermal length than the sheath, it experiences greater conduction heat loss. Also, it has a different thermal conductivity than the sheath.

The axial heat flux distributions for the sheath and shield are in Figure 2.3 and Figure 2.6. Comparing the two figures, the axial heat flux distributions for the radiation shield is higher than the ones for the sheath at the tip temperature location, at x/L = 0. This is expected behavior for the reason that the slopes of the temperature distributions at x/L = 0 for the radiation shield in Figure 2.5 are steeper than the slopes of the temperature distributions for the sheath in Figure 2.2. According to Fourier's law, which is:

$$q'' = -k\frac{dT}{dx}$$

Where:

q'' = heat flux [varies with rod mean temperature] (Watts / meter^2)

k = thermal conductivity [Watts / [(meters(Kelvin)]

$\frac{dT}{dx}$ = derivative of temperature solution with respect to x

The (dT/dx) term is higher for the radiation shield than the sheath at the location x /L= 0. For this reason, the heat flux is higher for the radiation shield at x/L = 0.The mean temperature locations for the sheath and shield are in Figure 2.4 and Figure 2.7. Comparing the two figures, the mean temperature locations of the radiation shield occur on the average at approximately x = 0.61. , and the mean temperature locations of the sheath occur on the average at approximately x = 0.68. This is expected behavior for the reason that the mean temperature for all three tip temperature profiles for the sheath and shield seem to fall on the temperature that has the average slope between the slope of the tip temperature and the slope of the base temperature. From looking at the temperature plots, the average slope for the sheath appears at a x – location further down than the x – location of the shield.

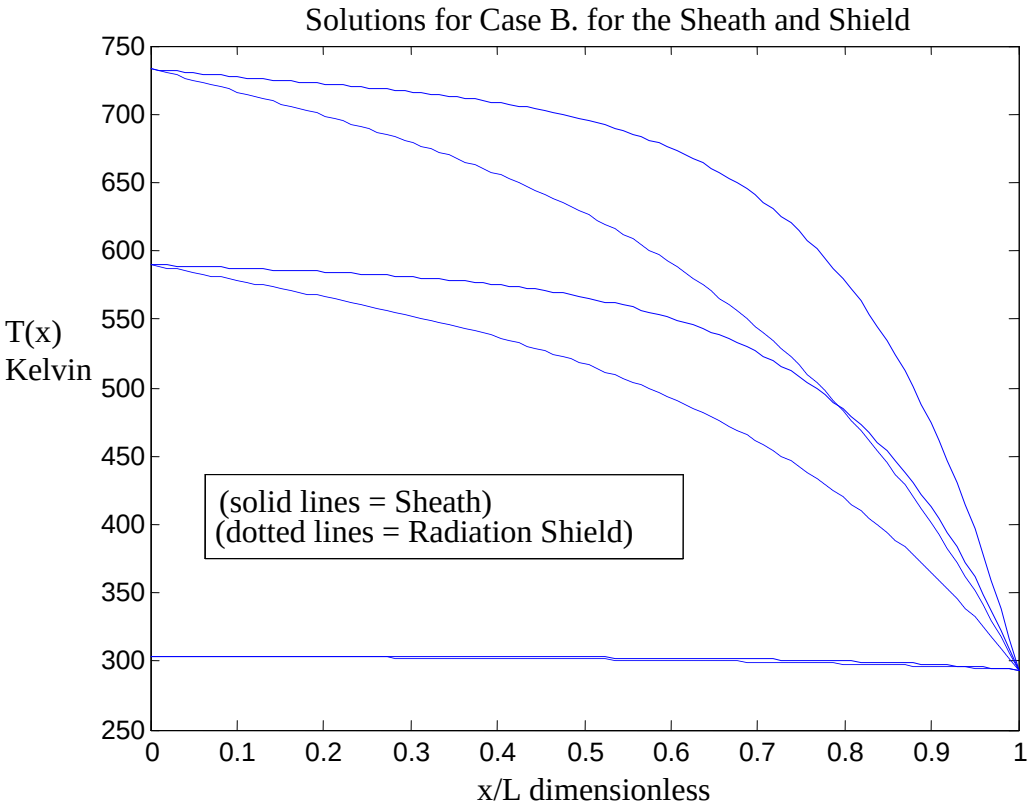


Figure 2.8: Comparisons of the Analytical Temperature Solutions Between the Sheath and Radiation Shield.
(Temperature versus x/L)

CHAPTER 3

THE NUMERICAL COMPUTER PROGRAM AND ITS MEAN TEMPERATURE LOCATIONS

The analytical mean temperature distribution model for the sheath and radiation shield has already been developed. The purpose of the analytical model is to correct the mean temperatures predicted by a numerical thermal model for the single – shielded thermocouple expression in a computer program. This chapter partially explains the program and the results of the mean temperature locations and values calculated from the program and analytical model for both the sheath and shield. Section 3.1 explains the main details of the computer program in detail. Section 3.2.1 states where the mean temperatures that are calculated from the computer program are located on the analytical temperature distribution graphs for both the sheath and shield. Section 3.2.2 explains the comparisons of the mean temperature calculations between the program and analytical model for the sheath and shield.

3.1 The Computer Program

The computer programming language used in the computer program is Fortran 77. The program name is TC_CORR created by Dr. R. J. Schulz of UTSI, ref. 10. TC_CORR stands for temperature correction. TC_CORR is actually a data reduction subroutine for use aerodynamics testing. However, to evaluate it, it has been incorporated into a program that executes its' functions. All of the variables used in the program are real numbers and are in S.I. "System International" units. Some of the main parameters calculated from the program are the mean temperatures for the sheath and shield, Prandtl number, Mach number, specific heat ratio, convection heat transfer coefficients, adiabatic wall recovery temperature and gas total stagnation temperature. Section 3.1.1 explains how the program is subdivided. Also, this section explains how the main parameters are calculated from the program. Section 3.1.2 explains the extreme case at where the computer program crashes.

3.1.1 Background Information

The block diagram for the computer program is in Figure 3.1. The computer program is divided into a main program and four subroutine programs. The main program can be subdivided into three sections. In the left section, the common statements are used to assign variable meanings to variables. The data statements are used to assign numerical values to variables, which are real numbers. There is a list of comment statements stating the definitions to some of the main variables and parameters. In the right section, the input and output files are named and opened in the executable window once the program

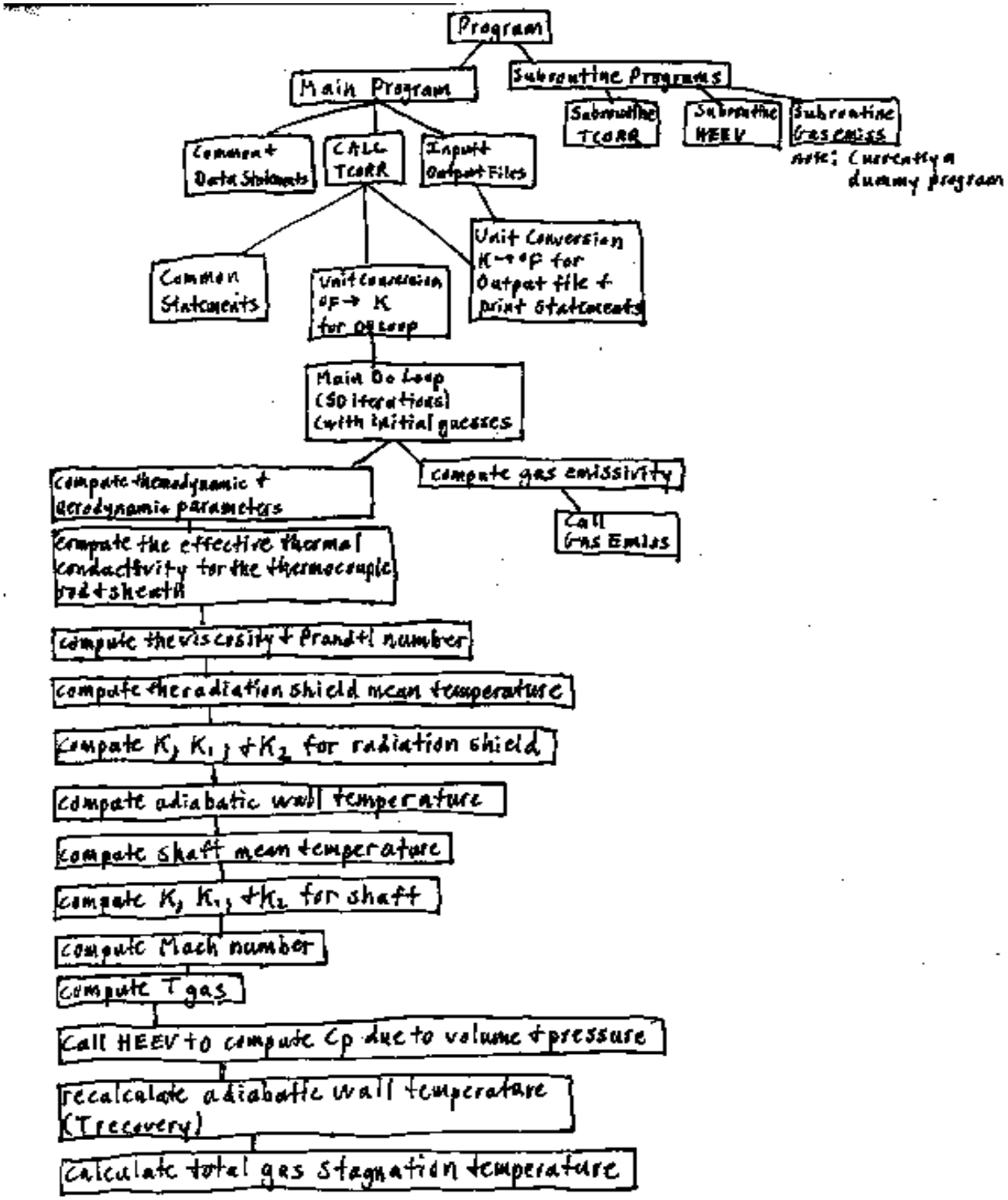


Figure 3.1: Block Diagram for the Numerical Computer Program

has been compiled and executed. Once the input file has been opened, the main program reads in a string of data representing pairs of times and tip temperatures of the thermocouple from the input file. When the main program and following subroutines are executed, the output file states the time, measured tip temperatures of the thermocouple sheath and the corresponding predicted value of the total gas stagnation temperature for each tip temperature. The tip temperatures are in degrees Fahrenheit in the input file. The units for the tip temperatures are converted to degrees Kelvin before the program executes all of the calculations. Once all of the calculations are executed, the units for the tip temperatures and calculated total stagnation gas temperatures are changed back into Fahrenheit. Then the times, tip temperatures and total gas stagnation temperatures are written into the output file.

The middle section of the main program calls the subroutine program called "TCORR" that calculates the parameters stated before for each tip temperature read from the input file. This subroutine contains the other two subroutines which are called "HEEV" and "GASEMISS", which are described below. The "TCORR" subroutine begins with listing some common statements and setting the initial guesses for the radiation shield mean temperature, rod mean temperature, adiabatic wall temperature which is the recovery temperature, gas temperature, Prandtl number and the total stagnation gas temperature. The initial guesses for these parameters are entered into the main Do loop. The Do loop calculates the parameters fifty times within the loop. By the fiftieth iteration, all of the parameters have reached steady state, in which the parameters do not change with respect to time. Also, the thermal conductivities, effective thermal conductivity, surface convection and radiation heat transfer coefficients, K, K_1 and K_2 values are calculated within the loop as well. There are three thermal conductivities calculated for the thermocouple rod and sheath which is a composite shaft. These conductivities are for the silicon carbide outer sheath, tantalum inner sheath and an aluminum oxide rod that incorporates the thermocouple lead wires. The effective rod thermal conductivity is the conductivity used in the K, K_1 and K_2 expressions for the rod or sheath. The effective conductivity is the overall conductivity of the thermocouple probe that depends on the individual material conductivities and corresponding surface areas.

For each tip temperature, the thermal heat exchange balance between the sheath, the shield and the gas has come to equilibrium once the iteration is completed and the mean temperatures have been calculated for the sheath and shield. When equilibrium is reached, The K values, K_1 values, K_2 values, Prandtl number, Mach number and gas total temperature are calculated. During the iteration sequence, the gas temperature is calculated, and the second subroutine program is called. The second subroutine program named "HEEV" calculates the specific heat of the gas at constant pressure and the specific heat at constant volume, and the rates of specific heats. After the subroutine program is executed, the total specific heat of the gas, recovery temperature (which is the adiabatic wall temperature), total stagnation gas temperature and total gas emissivity are calculated within the iteration process. The total gas emissivity and absorbtivity is calculated from the third subroutine program named "GASEMISS". After the iterative

heat balance is completed, the calculated temperature parameters are changed back into units of Fahrenheit. The calculated parameters are then printed on the execution window. After the subroutine "TCORR" has been executed, the time, measured tip temperature and predicted total gas stagnation temperatures are written in a particular format placed in the output file. Once the calculated parameters are printed on the execution window, the programmer hits the "enter" key and the next tip temperature from the input file is read into the program. The parameters stated are calculated within the main program for the next tip temperature and printed on the execution window. The programmer hits "enter" each time to run the calculations for each time and tip temperature pair in the input file until all of the tip temperatures from the input file are used. The time, tip temperature and total gas stagnation temperature for each tip temperature read from the input file is written into the output file.

3.1.2 Extreme Tip Temperature Case

The computer program was tested to see over what temperature range the program will operate. Seven tip temperatures were created in a dummy input file, each one being higher than the one before. When the programmer hits "enter" in the execution window to process a tip temperature from the input file, the program will give an error message in the execution window "TGAS IS OUT OF RANGE FOR CP CALCULATION", if a maximum allowable temperature is reached. If the program does not crash from the entered seven temperatures, the programmer needs to enter higher tip temperatures in the input file until the program crashes. After performing this task, the maximum temperature at which the program does not crash is at 3149 ^{o}F or 2004 K, which is the temperature limit for the gas specific heat calculation routine in "HEEV".

3.2 The Comparisons Between the Analytical Model and Computer Program Mean Temperature Locations

Now that the maximum tip temperature that the program can currently handle has found to be 3149 ^{o}F or 2004 K. A list of fourteen tip temperatures was chosen to the input file ranging from 400 K to 2000 K. The first seven tip temperatures will be for the low – temperature regime, and the next seven tip temperatures will be for the high - temperature regime for the sheath and shield. The input temperatures used for the low – temperature cases are 307.22 ^{o}F, 405.5 ^{o}F, 521.6 ^{o}F, 613.58 ^{o}F, 705.38 ^{o}F, 801.32 ^{o}F and 872.42 ^{o}F. The input temperatures used in the high temperature cases are 980.33 ^{o}F, 1340.33 ^{o}F, 1700.33 ^{o}F, 2060.33 ^{o}F, 2420.33 ^{o}F, 2780.33 ^{o}F and 3140.33 ^{o}F. The analytical and computer mean temperature locations and mean temperature values are calculated for the original model of the thermocouple probe. These are also calculated for each correction model in Chapter 4. The purpose of the calculations is to find the best model that has the least percent difference in location and temperature value between the analytical and computer mean temperatures in order to find the best mean temperature convergent x – location and convergent mean temperature value on the sheath and shield. The purpose of

the mean temperatures and best convergence of the mean temperature location and value is to treat the sheath and shield separately as a lumped system. Once the sheath and shield is treated separately as a lumped system, the two lumped systems are lumped together in the later part of the program to form once lumped system for the thermocouple probe in order for the program to best accurately calculate the total stagnation gas temperature for the one lumped system of the probe. Section 3.2.1 explains the comparisons of the mean temperature locations and values for the sheath. Section 3.2.2 explains the comparisons of the mean temperature locations and values for the radiation shield.

3.2.1 The Sheath

Once these fourteen temperatures were put into an input file, the computer program was executed to calculate the mean temperature for each input tip temperature for the sheath. The results are listed in Tables 3.1, 3.2 and 3.3. The plots for the regular and extreme cases are in Figure 3.2 and 3.3. From Table 3.1, the percent difference between the analytical mean temperature and computer program mean temperature increases as the tip temperature increases. The largest dispersion in the dimensionless mean temperature x – locations occur at the tip temperature of 1600 K / 2420.33 ^{o}F. The wide x – location dispersions in the extreme case is due to the flat shape of the analytical temperature graphs in the main body of the sheath. The total percent difference in x - location for the sheath comes out to be 423.98%. The total percent difference in temperature value for the sheath comes out to be 73.23 %.

3.2.2 The Radiation Shield

The same fourteen temperatures that were used for the sheath are used for the radiation shield. The computer program was executed to calculate the mean temperature for each input tip temperature for the radiation shield. The results are listed in Table 3.4, 3.5 and 3.6. The plot for the regular case is listed in Figure 3.4. The plot for the extreme case is listed in Figure 3.5.

The total percent difference of the mean temperatures between the analytical model and computer program peaks at approximately 1200 K / 2060.33 ^{o}F, and then start to go back down. Comparing the results between the sheath and radiation shield, the shield has a larger percentage difference from 400 K to 1348 K. At 1348 K / 1966.73 ^{o}F, the sheath and shield have the same percent difference. From 1348 K to 2000 K, the sheath has a larger percent difference. The total percent x – location difference for the radiation shield comes out to be 226.85%. The total percent temperature value difference for the radiation shield comes out to be 133.23%. The total percent x – location difference for both the sheath and radiation shield comes out to be 650.83%. The total percent temperature value difference for both the sheath and radiation shield comes out to be 206.46%. During a typical high speed flow wind tunnel testing process, supersonic flow of air is being blown over a thermocouple probe for which the radiation shield is a circular tube. Air has a Prandtl number of 0.7 which is near 1.0. According to Kays, “However, for the Prandtl

Table 3.1: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	258.12	259.44	0.6746	0.6686	0.89
T2	405.50	335.43	338.36	0.6746	0.6652	1.39
T3	521.60	427.21	432.45	0.6753	0.6627	1.87
T4	613.58	499.78	507.89	0.6762	0.6597	2.44
T5	705.38	572.47	584.27	0.6779	0.6566	3.14
T6	801.32	648.84	665.56	0.6797	0.6531	3.91
T7	872.42	705.79	726.96	0.6817	0.6505	4.58
T8	980.33	793.08	822.32	0.6855	0.6462	5.73
T9	1340.33	1094.22	1163.09	0.7063	0.6269	11.24
T10	1700.33	1415.21	1538.82	0.7432	0.5664	23.79
T11	2060.33	1751.63	1927.50	0.7942	0.0100	87.41
T12	2420.33	2070.77	2284.77	0.8313	0.0454	94.54
T13	2780.33	2336.27	2590.39	0.8263	0.0520	93.71
T14	3140.33	2559.11	2850.69	0.7975	0.0850	89.34

Note A: All Temperature Values are shown in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 3.2: Values of the Analytical, Computer Program Mean Temperatures and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	258.12	259.44	0.51
T2	405.50	335.43	338.36	0.87
T3	521.60	427.21	432.45	1.23
T4	613.58	499.78	507.89	1.62
T5	705.38	572.47	584.27	2.06
T6	801.32	648.84	665.56	2.58
T7	872.42	705.79	726.96	3.00
T8	980.33	793.08	822.32	3.69
T9	1340.33	1094.22	1163.09	6.29
T10	1700.33	1415.21	1538.82	8.73
T11	2060.33	1751.63	1927.50	10.04
T12	2420.33	2070.77	2284.77	10.33
T13	2780.33	2336.27	2590.39	10.89
T14	3140.33	2559.11	2850.69	11.39

Note A: All Temperature Values are shown in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 3.3: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures for the Sheath.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Gas Stagnation T
T1	307.22	258.12	259.44	308.2
T2	405.50	335.43	338.36	407.1
T3	521.60	427.21	432.45	524.4
T4	613.58	499.78	507.89	617.8
T5	705.38	572.47	584.27	711.3
T6	801.32	648.84	665.56	809.6
T7	872.42	705.79	726.96	882.8
T8	980.33	793.08	822.32	994.5
T9	1340.33	1094.22	1163.09	1373.3
T10	1700.33	1415.21	1538.82	1762.5
T11	2060.33	1751.63	1927.50	2165.8
T12	2420.33	2070.77	2284.77	2597.6
T13	2780.33	2336.27	2590.39	3077.3
T14	3140.33	2559.11	2850.69	3617.7

Note A: All Temperature Values are shown in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

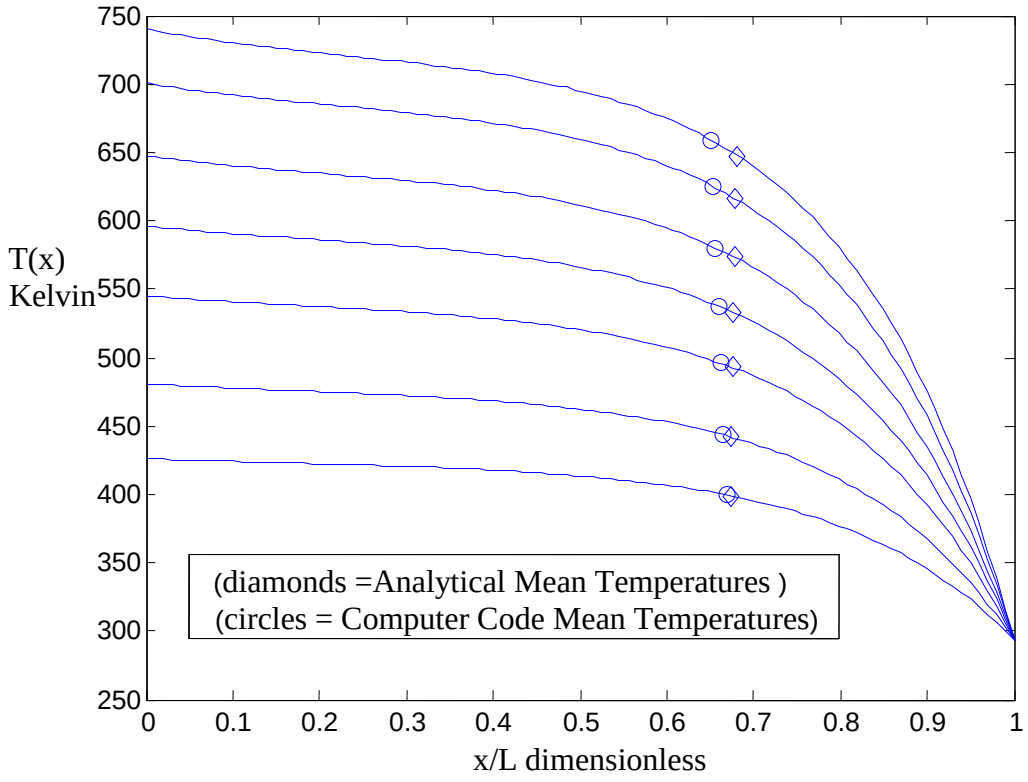


Figure 3.2: Mean Temperature Plots from the Analytical Model and Computer Program for the Sheath (Low Temperatures)

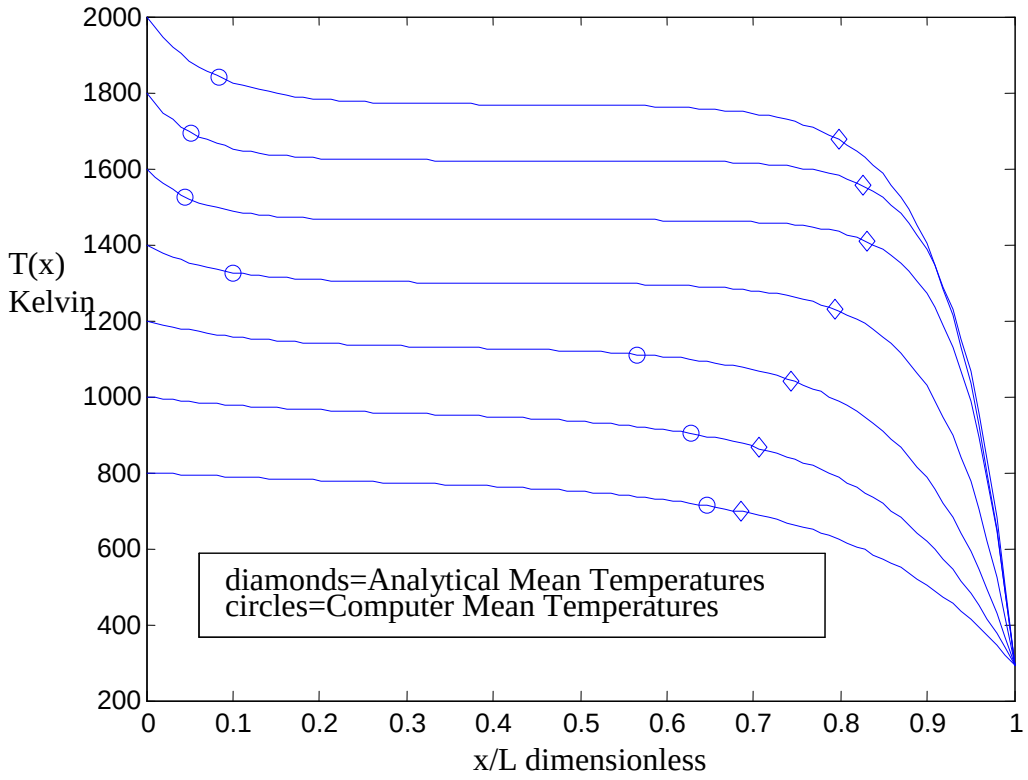


Figure 3.3: Mean Temperature Plots from the Analytical Model and Computer Program for the Sheath (High Temperatures)

Table 3.4: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	230.32	249.56	0.5965	0.499	16.35
T2	405.50	296.26	323.05	0.5959	0.501	15.93
T3	521.60	374.50	410.34	0.5956	0.501	15.88
T4	613.58	436.66	479.60	0.5956	0.502	15.72
T5	705.38	499.06	549.33	0.5961	0.502	15.79
T6	801.32	564.91	623.16	0.5970	0.502	15.91
T7	872.42	614.19	678.69	0.5981	0.501	16.23
T8	980.33	690.24	764.61	0.6000	0.501	16.50
T9	1340.33	957.49	1068.34	0.6112	0.501	18.03
T10	1700.33	1253.64	1395.77	0.6338	0.513	19.06
T11	2060.33	1574.87	1724.50	0.6679	0.548	17.95
T12	2420.33	1875.47	2017.11	0.6955	0.589	15.31
T13	2780.33	2109.83	2261.58	0.6835	0.585	14.41
T14	3140.33	2292.89	2467.93	0.6518	0.562	13.78

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 3.5: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	230.32	249.56	8.35
T2	405.50	296.26	323.05	9.04
T3	521.60	374.50	410.34	9.57
T4	613.58	436.66	479.60	9.83
T5	705.38	499.06	549.33	10.07
T6	801.32	564.91	623.16	10.31
T7	872.42	614.19	678.69	10.50
T8	980.33	690.24	764.61	10.77
T9	1340.33	957.49	1068.34	11.58
T10	1700.33	1253.64	1395.77	11.34
T11	2060.33	1574.87	1724.50	9.50
T12	2420.33	1875.47	2017.11	7.55
T13	2780.33	2109.83	2261.58	7.19
T14	3140.33	2292.89	2467.93	7.63

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 3.6: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures for the Radiation Shield.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Gas Stagnation T
T1	307.22	230.32	249.56	308.2
T2	405.50	296.26	323.05	407.1
T3	521.60	374.50	410.34	524.4
T4	613.58	436.66	479.60	617.8
T5	705.38	499.06	549.33	711.3
T6	801.32	564.91	623.16	809.6
T7	872.42	614.19	678.69	882.8
T8	980.33	690.24	764.61	994.5
T9	1340.33	957.49	1068.34	1373.3
T10	1700.33	1253.64	1395.77	1762.5
T11	2060.33	1574.87	1724.50	2165.8
T12	2420.33	1875.47	2017.11	2597.6
T13	2780.33	2109.83	2261.58	3077.3
T14	3140.33	2292.89	2467.93	3617.7

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

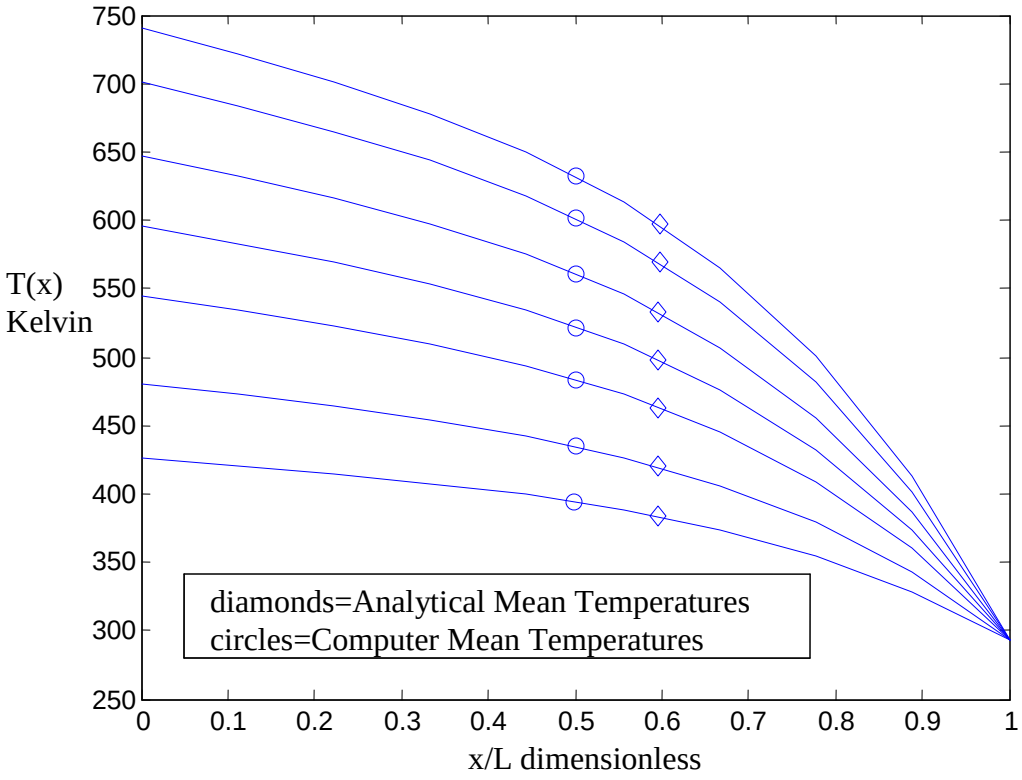


Figure 3.4: Mean Temperature Plots from the Analytical Model and Computer Program for the Radiation Shield (Low Temperatures)

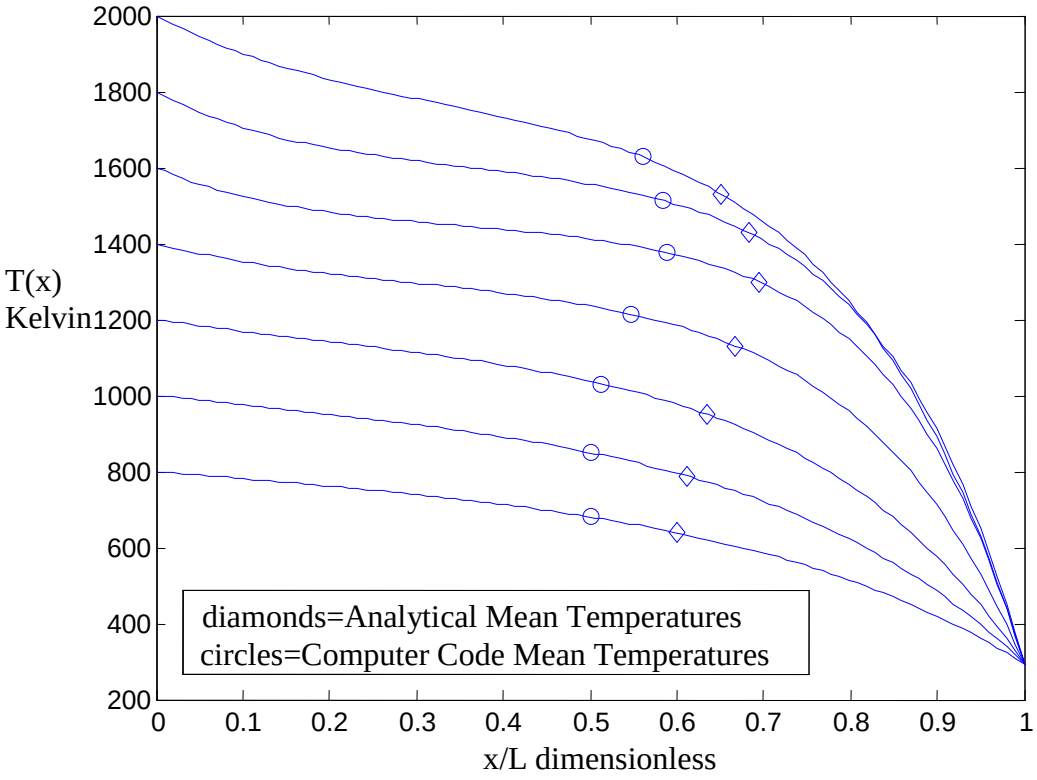


Figure 3.5: Mean Temperature Plots from the Analytical Model and Computer Program for the Radiation Shield (High Temperatures)

number range near 1.00, which includes the gas range, the velocity and temperature profiles develop at similar rates along the tube, and the assumption of a fully established velocity profile at the tube entrance can, for many applications, lead to a considerable error in predicted performance." (ref. 8). This is one reason for the error. The main reason for the "error" is that the computer program calculates only a single value for the temperature of each component treated as a uniform temperature system. So there is no temperature distribution calculated by the program in either the sheath or the radiation shield. In Chapter 4, what we are going to try to do is come up with corrections to the heat transfers in the program so that the heat balance gives a single temperature equal to the mean temperature of the analytical distributions calculated based on tip temperature, base temperature, K_1 and K_2 for the sheath and radiation shield.

CHAPTER 4

CORRECTIONS TO THE THERMOCOUPLE PROBE THERMODYNAMIC PARAMETERS USING THE ANALYTICAL MODEL AND NUMERICAL COMPUTER PROGRAM

According to the figures in Chapter 3, there is dispersion between the analytical mean temperatures and their locations for the sheath and shield. This chapter focuses on numerical experiments done to identify how these dispersions can be minimized using different types of heat transfer correction models. In Section 4.1, the different types of heat transfer rates for the thermocouple probe are explained in detail. In Section 4.2, different heat transfer correction models are explained in detail, which change the mean temperatures and their locations on the sheath and shield by making changes in the numerical computer program for each correction model. The total percent difference from the analytical and mean temperature locations and values for the sheath and shield is calculated for each correction model. The correction model with the least total percent difference from mean temperature location and value will be recommended for use as the basic TC_CORR subroutine program version for calculating the gas total or stagnation temperature.

4.1 The Heat Transfer Rates of the Thermocouple Probe

Three fundamental types of heat transfer rates are involved inside the thermocouple probe: These are heat transfer due to conduction, convection and radiation. Section 4.1.1 explains the different types of conduction, convection and radiation heat transfer rates that are involved with the sheath. Section 4.1.2 explains the different types of conduction, convection and radiation heat transfer rates that are involved with the radiation shield.

4.1.1 The Sheath

The thermocouple system heat balance in the numerical program was based on six different heat transfers due to conduction, convection and radiation that are involved with the sheath. These five different types of heat transfer rates are in Figure 4.1. The following drawings explain each heat transfer rate more in detail.

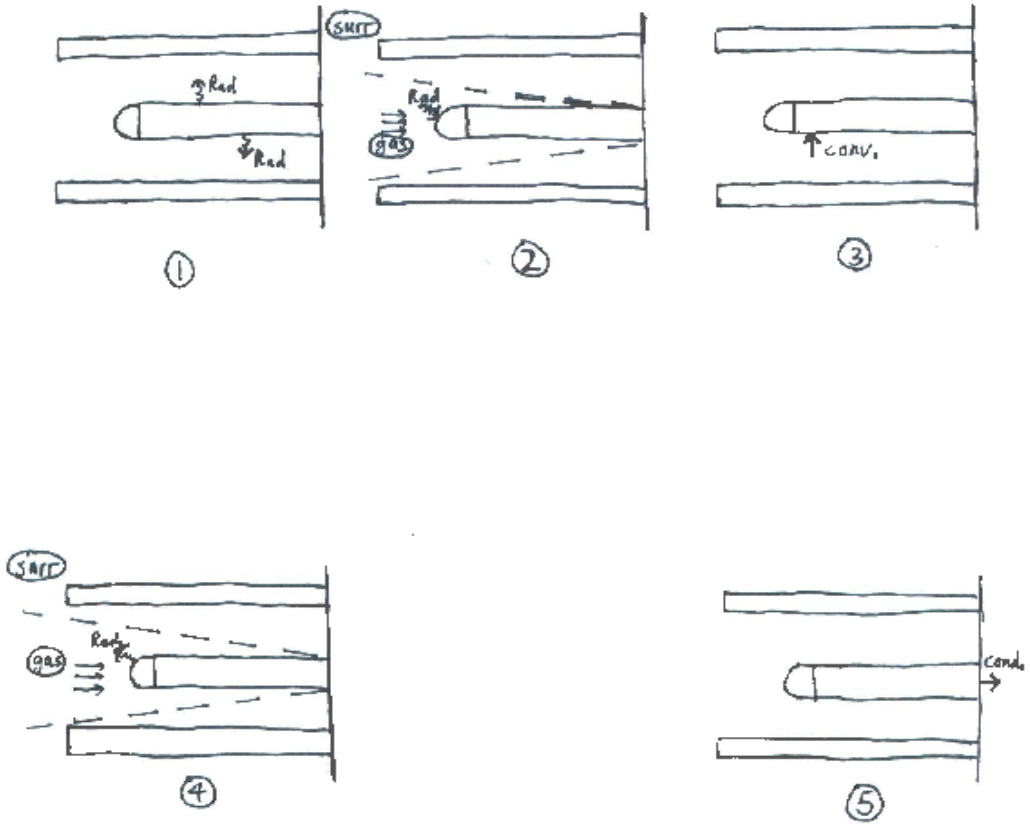


Figure 4.1: The Five Different Types of Heat Transfer Rates for the Sheath

Drawing #1 is radiation heat transfer from the sheath to the shield.
The radiation heat transfer equation from Drawing #1 is:

q'' (radiation shield into the sheath) = $h_{r1}(T_{s1} - T_2)$
h_{r1} = radiation heat transfer coefficient (HR121, FR121 in computer program)
T_{s1} = temperature of the sheath (TS1 in computer program)
T_2 = temperature of the shield (T2 in computer program)

Drawing #2 is radiation heat transfer from the gas going into the sheath. The radiation heat transfer equation from the Drawing #2 is:

q'' (radiation gas in) = $h_{r1g}(T_{s1} - T_{surr})$
h_{r1g} = radiation heat transfer coefficient (HR1GAS, FR1GAS in computer program)
T_{s1} = mean temperature of the sheath (TS1 in computer program)
T_{gas} = temperature of the surroundings (TGAS in computer program)

Drawing #3 is convection heat transfer from the gas going into the sheath. The convection heat transfer equation from the Drawing #3 is:

q'' (convection into sheath) = $h_{c1}(T_r - T_{s1})$
h_{c1} = convective heat transfer coefficient (HC1S in computer program)
T_r = adiabatic wall temperature (TAW in computer program)

Drawing #4 is radiation heat transfer between the sheath and the surroundings that passes through the gas. The radiation heat transfer equation from the Drawing #4 is:

q'' (radiation gas out) = $h_{r1a}(T_{s1} - T_{surr})$
h_{r1a} = radiation heat transfer coefficient (HR1AMB, FR1AMB in computer program)
T_{s1} = temperature of the sheath (TS1 in computer program)
T_{surr} = temperature of the surroundings (TAMB in computer program)

Drawing #5 is conduction heat transfer from the base of the sheath into the support wall.
The conduction heat transfer equation from the Drawing #5 is:

$$q'' \text{ (conduction)} = -k_1 \left.\frac{dT}{dx}\right|_L$$

k_1 = conduction heat transfer coefficient for the sheath (TKEFF in computer program)

$\left.\frac{dT}{dx}\right|_L$ = temperature gradient at the base of the rod, which is $\frac{(TS1 - TB)}{XL1/2}$ in the computer program

FR121, FR1GAS and FR1AMB are viewfactors for the respective radiation heat transfer processes. They result from either exact solutions tabulated in radiation viewfactor documents, or they are best estimates. They can contain the areas necessary to represent heat flow rates, or the appropriate areas can be included separately in the program.

4.1.2 The Radiation Shield

There are seven different types of heat transfer rates due to conduction, convection and radiation that are involved with the radiation shield. These seven different types of heat transfer rates are in Figure 4.2. Comparing the drawings in Figure 4.2, one heat transfer rate may have more of an effect on the value of the gas stagnation temperature than the other types. Depending on the value of the Peclet number, the heat transfer due to conduction may have more of an effect on the value of the gas stagnation temperature than the other two types of heat transfer inside the radiation shield, which is a cylindrical tube, and vice – versa for convection and radiation heat transfer. According to Singh (ref. 9), for laminar flow inside a cylindrical tube, the effect of axial heat conduction is negligible for the Peclet number greater than 100. The following drawings explain each heat transfer rate more in detail.

Drawing #1a is convection heat transfer in the area from the entrance of the shield to the tip of the sheath. The convection heat transfer equation from the Drawing #1a is:

q'' (convection inner entrance) = $h_{c211}\,(T_r - T_2)$
h_{c211} = convective heat transfer coefficient (HC21M1 in computer program)
T_r = adiabatic wall temperature (TAW in computer program)
T_2 = mean temperature of the shield

Drawing #1b is convection heat transfer in the area from the tip of the sheath to the wall. The convection heat transfer equation from the Drawing #1b is:

q'' (convection inner sheath) = $h_{c212}\,(T_r - T_2)$
h_{c212} = convective heat transfer coefficient (HC21M2 in computer program)
T_r = adiabatic wall temperature (TAW in computer program)
T_2 = mean temperature of the shield

Drawing #2 is convection heat transfer of the outer surface of the radiation shield. The convection heat transfer equation from the Drawing #2 is:

q'' (convection outer entrance) = $h_{c22}\,(T_r - T_2)$
h_{c22} = convective heat transfer coefficient (HC22 in computer program)
T_r = adiabatic wall temperature (TAW in computer program)
T_2 = mean temperature of the shield

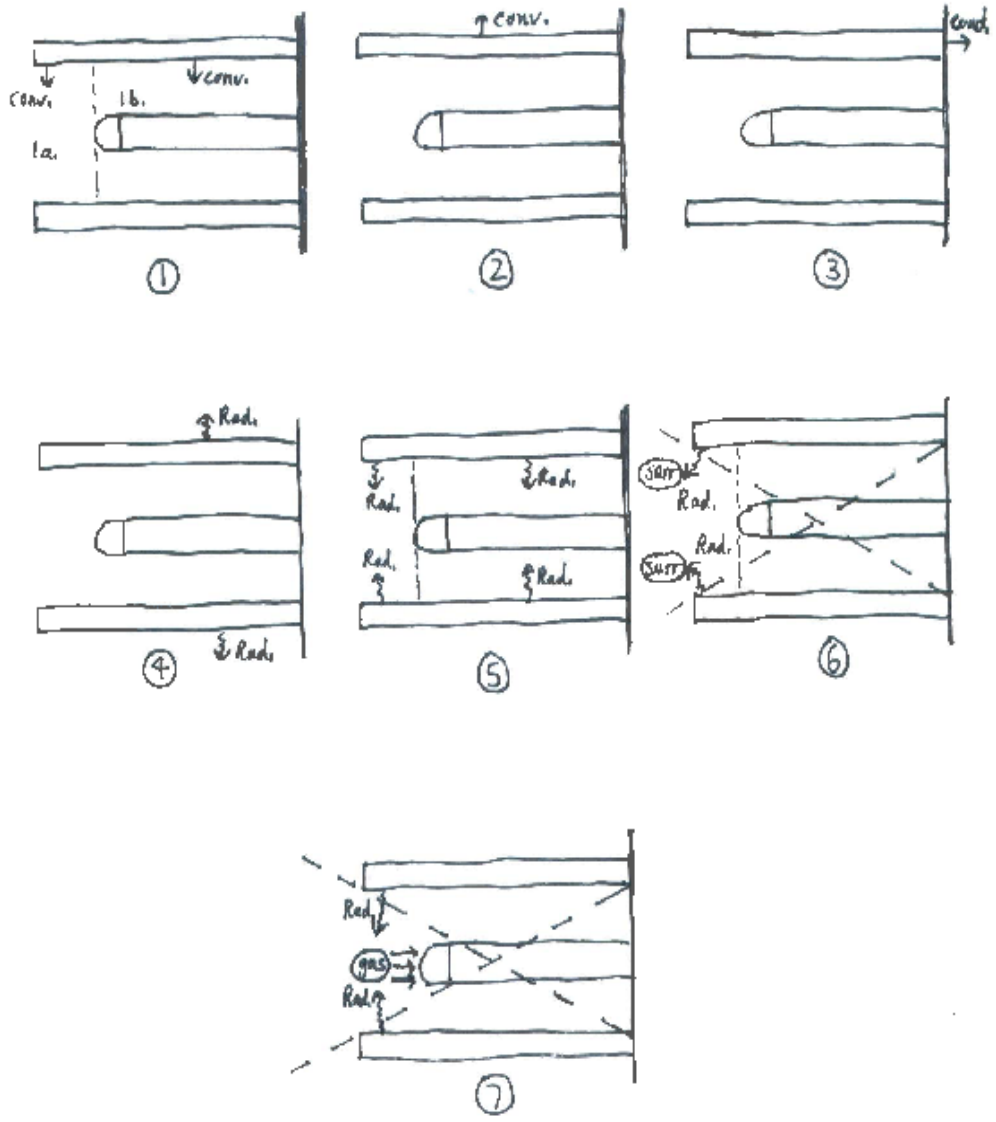


Figure 4.2: The Seven Different Types of Heat Transfer Rates for the Radiation Shield

Drawing #3 is conduction heat transfer going from the base of the shield into the support wall. The conduction heat transfer equation from the Drawing #3 is:

q'' (conduction) = $-k_2 \left.\frac{dT}{dx}\right|_L$

k_2 = conduction heat transfer coefficient for the shield (TK2 in computer program)

$\left.\frac{dT}{dx}\right|_L$ = temperature gradient at the base of the shield, which is

$$\frac{(T2 - TB)}{XL2/2}$$

in computer program

Drawing #4 is radiation heat transfer from the outer surface of the radiation shield going to the surroundings. The radiation heat transfer equation from the Drawing #4 is:

q'' (radiation shield outer surface to surroundings) = $h_{r22}\,(T_2 - T_{surr})$

h_{r22} = radiation heat transfer coefficient (HR22AMB in computer program)
T_{s2} = mean temperature of the shield (T2 in computer program)
T_{surr} = temperature of the surroundings (TAMB in computer program)

Drawing #5 is radiation heat transfer going from the radiation shield to the sheath. The radiation heat transfer equation from the Drawing #5 is:

q'' (radiation inner surface) = $h_{r211}\,(T_2 - T_{s1})$

h_{r211} = radiation heat transfer coefficient (HR211, FR211 in computer program)
T_2 = mean temperature of the shield (T2 in computer program)
T_{s1} = mean temperature of the sheath (TS1 in computer program)

Drawing #6 is radiation heat transfer going from the radiation shield inner surface to the surroundings as seen by this part of the thermocouple probe. The convection heat transfer equation from the Drawing #6 is:

q'' (radiation inner surface) = $h_{r21A}\,(T_2 - T_{surr})$

h_{r21A} = radiation heat transfer coefficient (HR21AMB,FR21AMB in computer program)
T_2 = mean temperature of the shield (T2 in computer program)
T_{surr} = temperature of the surroundings (TAMB in computer program)

Drawing #7 is radiation heat transfer from the radiation from the inner surface of the radiation shield to the gas. The radiation heat transfer equation from the Drawing #7 is:

q'' (radiation from shield into gas) = $h_{R21G}\,(T_2 - T_{GAS})$

h_{R21G} = radiative heat transfer coefficient (HR21GAS, FR21GAS in computer program)
T_2 = mean temperature of the shield (T2 in the computer program)
T_{GAS} = temperature of the gas (TGAS in computer program)

FR211, FR21AMB and FR21GAS are the radiation viewfactors for the respective radiation heat transfer processes obtained from known or estimated values. They too can include the respective areas that represent heat flow rates. In the numerical code, the areas are included separately. Drawing #1 is divided into two parts because the cross sectional mass flow area from the entrance of the probe to the tip of the sheath is different than the cross sectional mass flow area from the tip of the sheath to the wall. When the mass flow of air moves through the section enclosing the sheath, the mass flow of air has a smaller cross sectional area than the mass flow of air from the entrance to the probe to the tip of the sheath. Therefore, the gas moves faster through the inner sheath area than through the inner entrance area, changing the convective heat transfer coefficient.

4.2 Corrections to the Thermodynamic Parameters

Six different types of heat transfer correction models were identified as a way of bringing the numerical and analytical mean temperatures together, that is having the same analytical and computer mean temperature values and locations. The way these heat transfer correction models work is that certain corrections to the heat transfer equations in the numerical computer program are made. Once the corrections are made, and the program is executed, new values of K_1, K_2 and the computer mean temperature are calculated from the program and shown in the execution window. The new values for K_1 and K_2 are then put in the analytical model to calculate the new analytical mean temperature. Finally, the new values of the analytical and computer mean temperatures are plotted on the analytical temperature curves to see at what new x – location these new mean temperatures fall, hoping that the x – locations of the new mean temperatures come closer together. Convergence of the first calculation of the recovery temperature and the recalculation of the recovery temperature for each tip temperature needs to be checked for each correction model. If the two recovery temperatures do not converge for each tip temperature, then the correction model has to be discarded. The recovery temperature is taken to be the adiabatic wall temperature.

4.2.1 Conduction Correction Model

The first heat transfer correction model is a base heat transfer conduction correction model which is applied to both the sheath and the shield. The conduction heat transfer equation stated earlier for the sheath, was:

$$q''(\text{conduction}) = -k_1 \left.\frac{dT}{dx}\right|_L$$

k_1 = conduction heat transfer coefficient for the rod
(TKEFF in computer program)

$\left.\frac{dT}{dx}\right|_L$ = temperature gradient at the base of the rod, which is

$$\frac{(TS1 - TB)}{XL1/2}$$

in the computer program

The conduction heat transfer equation stated earlier for the radiation shield was:

$$q``(\text{conduction}) = -k_2 \left.\frac{dT}{dx}\right|_L$$

k_2 = conduction heat transfer coefficient for the shield
(TK2 in computer program)

$\left.\frac{dT}{dx}\right|_L$ = temperature gradient at the base of the shield, which is

$$\frac{(T2 - TB)}{XL2/2}$$

in computer program

The dT/dx terms in both equations were initially chosen as order of magnitude numerical estimates for the temperature gradients in the sheath and the shield. In the conduction correction model, the numerical dT/dx term was replaced by the analytical dT/dx term, which is on page 14, for the calculation of the analytical conduction heat transfer rate at the base of the sheath and radiation shield. The calculation of the analytical conduction heat transfer rate is used to modify the effective thermal conductivities of the sheath and shield so that the basic structure of the numerical program remains the same.

That is:

TKEFF is defined by:

$$TKEFF = \frac{-k_1 \left(dT/dx\right)_{L,\text{analytical rod}}}{\frac{(TS1 - TB)}{XL1/2}}$$

and TK2 is defined by:

$$TK2 = \frac{-k_2 \left(dT/dx\right)_{L,\text{analytical shield}}}{\left(T2 - TB\right)/XL2/2}$$

These modified thermal conductivity values were used to estimate the conduction heat loss at the base of the rod and the base of the shield, representative of the analytical predictions. The recalculated values for TKEFF and TK2 were then used in the calculations of the computer mean temperatures for the sheath and shield. These adjusted terms are only used in the calculation of the computer mean temperatures for the sheath and radiation shield. The adjusted terms are used nowhere else in the computer program.

The results for the sheath are in Tables 4.1, 4.2 and 4.3 and Figures 4.3, 4.4. The results for the radiation shield are in Tables 4.4, 4.5 and 4.6 and Figures 4.5, 4.6. The total x – location percent difference for the sheath is 530.83%. The total temperature value percent difference for the sheath is 562.41%. The total x – location percent difference for the shield is 379.83%. The total temperature value percent difference for the shield is 290.27%. The total x – location percent difference for the sheath and shield is 910.66%. The total temperature value percent difference for the sheath and shield is 852.68%. The first calculation and the recalculation of the recovery temperature converged for each tip temperature, so this correction model is valid. The reason for the greater disparity between the analytical conduction – corrected results and the original results was that the predicted heat loss from the base became larger than for the original model. In other words, the analytical gradient model was inconsistent with the order of the approximations of the numerical model. The disparity between the analytical conduction – corrected results and the original results become larger when the velocity of the gas flow in wind tunnel test applications or other engineering applications move faster. For this research analysis, the flow of gas is moving at a velocity in the transonic and supersonic region, in which the Mach number is between 0.3 and 0.8 for transonic flow and the Mach number is between 0.8 and 1.2 for supersonic flow. Careful consideration needs to be taken account when the velocity of the gas flow reaches the hypersonic region, in which the Mach number is 1.2 and above. In this very high speed region, so much more heat transfer due to conduction is lost at the base of the thermocouple probe in which the disparity between the analytical conduction – corrected results and the original results become so much larger. Future research needs to be done in correcting for the heat transfer loss at the base for gas flow velocity applications in the hypersonic region. One possible way of doing this is to better insulate the base of the wall with another material or a composite material with a very low conductivity so that less heat transfer at the base would not be lost due to conduction. For this research application, in which the gas flow is in the transonic or supersonic region, further adjustments were made to the conduction heat transfer correction model which is explained in the next section.

Table 4.1: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the First Conduction Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	228.33	138.60	0.6250	0.9070	45.12
T2	405.50	293.18	166.86	0.6243	0.9070	45.28
T3	521.60	369.61	200.68	0.6231	0.9060	45.40
T4	613.58	431.20	227.41	0.6225	0.9060	45.54
T5	705.38	490.98	254.75	0.6218	0.9050	45.55
T6	801.32	554.13	283.80	0.6220	0.9040	45.34
T7	872.42	601.20	305.89	0.6220	0.9035	45.26
T8	980.33	672.91	340.54	0.6220	0.9020	45.02
T9	1340.33	916.63	469.60	0.6258	0.8970	43.34
T10	1700.33	1171.31	631.17	0.6341	0.8910	40.51
T11	2060.33	1441.31	849.04	0.6500	0.8812	35.57
T12	2420.33	1738.31	1179.88	0.6804	0.8696	27.81
T13	2780.33	2086.97	1732.73	0.7538	0.8656	14.83
T14	3140.33	2429.33	2268.95	0.8247	0.8763	6.26

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.2: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the First Conduction Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	**Mean T [A]**	**Mean T [C]**	**% Diff.**
T1	307.22	228.33	138.60	39.30
T2	405.50	293.18	166.86	43.09
T3	521.60	369.61	200.68	45.70
T4	613.58	431.20	227.41	47.26
T5	705.38	490.98	254.75	48.11
T6	801.32	554.13	283.80	48.78
T7	872.42	601.20	305.89	49.12
T8	980.33	672.91	340.54	49.39
T9	1340.33	916.63	469.60	48.77
T10	1700.33	1171.31	631.17	46.11
T11	2060.33	1441.31	849.04	41.09
T12	2420.33	1738.31	1179.88	32.12
T13	2780.33	2086.97	1732.73	16.97
T14	3140.33	2429.33	2268.95	6.60

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.3: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Sheath for the First Conduction Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	228.33	138.60	308.6
T2	405.50	293.18	166.86	407.9
T3	521.60	369.61	200.68	525.7
T4	613.58	431.20	227.41	619.7
T5	705.38	490.98	254.75	714.1
T6	801.32	554.13	283.80	813.5
T7	872.42	601.20	305.89	887.7
T8	980.33	672.91	340.54	1001.3
T9	1340.33	916.63	469.60	1391.1
T10	1700.33	1171.31	631.17	1801.6
T11	2060.33	1441.31	849.04	2233.6
T12	2420.33	1738.31	1179.88	2679.7
T13	2780.33	2086.97	1732.73	3145.5
T14	3140.33	2429.33	2268.95	3681.1

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

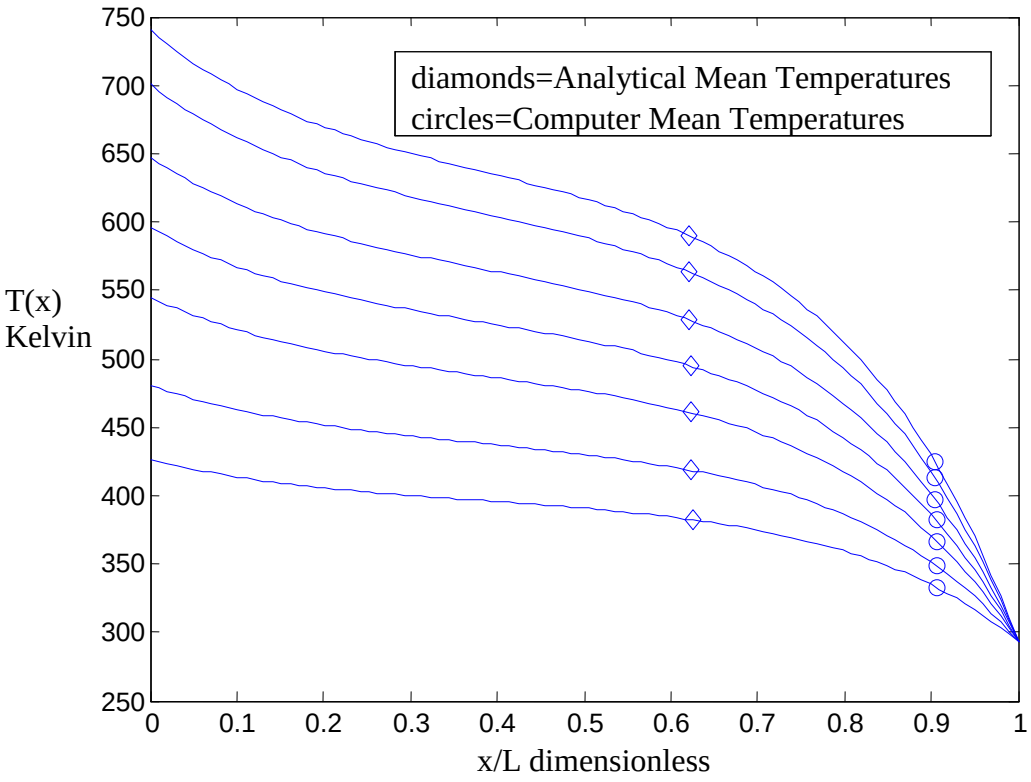


Figure 4.3: Conduction Correction for the Sheath (Low Temperatures)

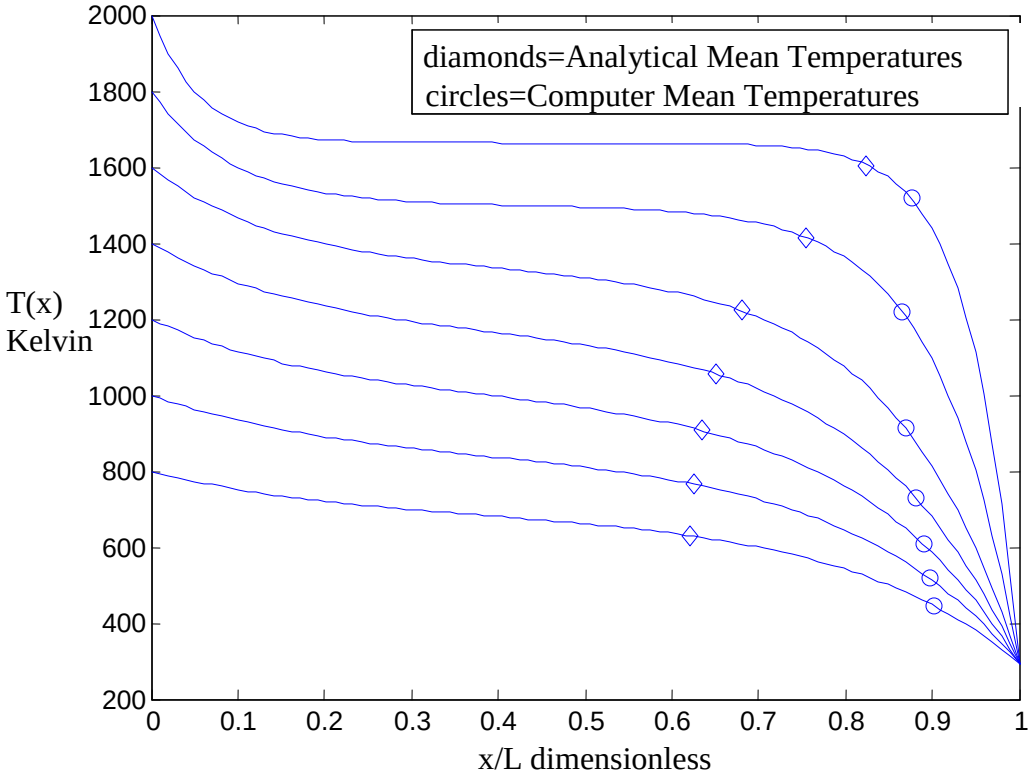


Figure 4.4: Conduction Correction for the Sheath (High Temperatures)

Table 4.4: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield for the First Conduction Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff.=Differrence)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	212.73	173.64	0.5604	0.7304	30.33
T2	405.50	271.51	215.44	0.5591	0.7316	30.85
T3	521.60	340.47	264.07	0.5588	0.7332	31.21
T4	613.58	395.51	302.70	0.5579	0.7338	31.53
T5	705.38	450.12	340.95	0.5576	0.7344	31.71
T6	801.32	503.00	380.70	0.5552	0.7310	31.66
T7	872.42	549.75	410.67	0.5573	0.7353	31.94
T8	980.33	614.48	456.24	0.5573	0.7356	31.99
T9	1340.33	834.31	617.83	0.5591	0.7334	31.18
T10	1700.33	1063.44	802.18	0.5639	0.7277	29.05
T11	2060.33	1309.69	1027.00	0.5723	0.7182	25.49
T12	2420.33	1583.51	1327.30	0.5925	0.7134	20.41
T13	2780.33	1954.67	1756.85	0.6330	0.7151	12.97
T14	3140.33	2314.31	2144.93	0.6863	0.7516	9.51

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.5: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Difference of the Mean Tempeatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield for the First Conduction Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Differrence)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	212.73	173.64	18.38
T2	405.50	271.51	215.44	20.65
T3	521.60	340.47	264.07	22.44
T4	613.58	395.51	302.70	23.47
T5	705.38	450.12	340.95	24.25
T6	801.32	503.00	380.70	24.31
T7	872.42	549.75	410.67	25.30
T8	980.33	614.48	456.24	25.75
T9	1340.33	834.31	617.83	25.96
T10	1700.33	1063.44	802.18	24.57
T11	2060.33	1309.69	1027.00	21.58
T12	2420.33	1583.51	1327.30	16.17
T13	2780.33	1954.67	1756.85	10.12
T14	3140.33	2314.31	2144.93	7.32

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.6: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Radiation Shield for the First Conduction Correction Model,
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	212.73	173.64	308.6
T2	405.50	271.51	215.44	407.9
T3	521.60	340.47	264.07	525.7
T4	613.58	395.51	302.70	619.7
T5	705.38	450.12	340.95	714.1
T6	801.32	503.00	380.70	813.5
T7	872.42	549.75	410.67	887.7
T8	980.33	614.48	456.24	1001.3
T9	1340.33	834.31	617.83	1391.1
T10	1700.33	1063.44	802.18	1801.6
T11	2060.33	1309.69	1027.00	2233.6
T12	2420.33	1583.51	1327.30	2679.7
T13	2780.33	1954.67	1756.85	3145.5
T14	3140.33	2314.31	2144.93	3681.1

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

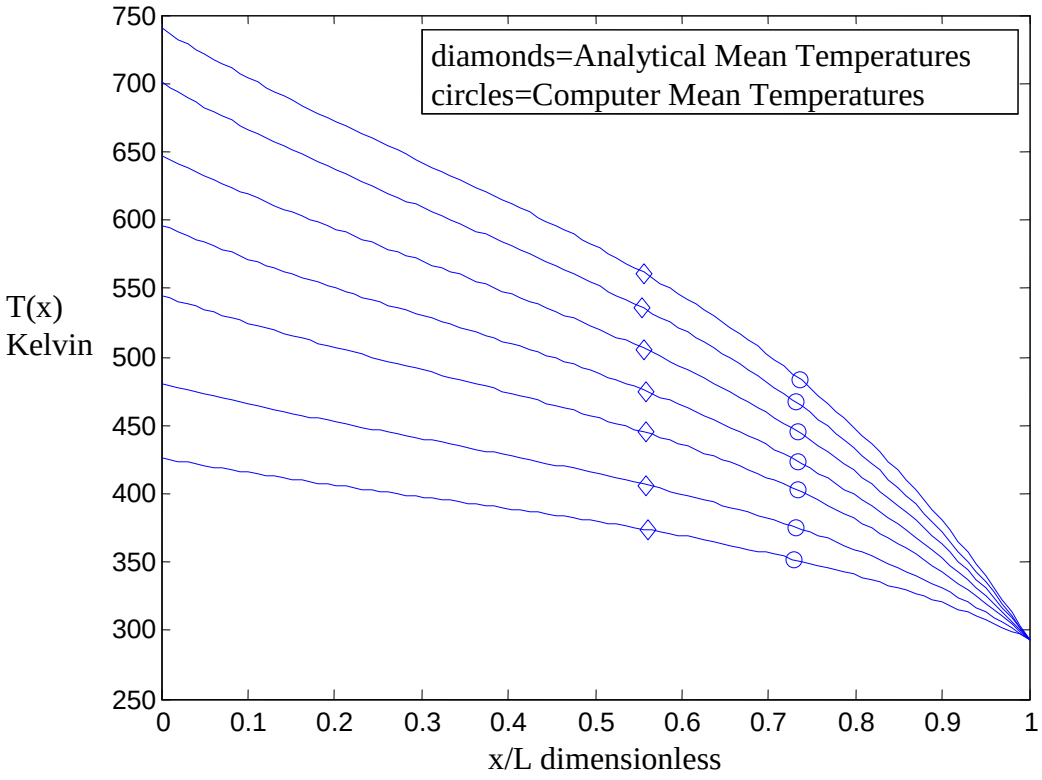


Figure 4.5: Conduction Correction for the Radiation Shield (Low Temperatures)

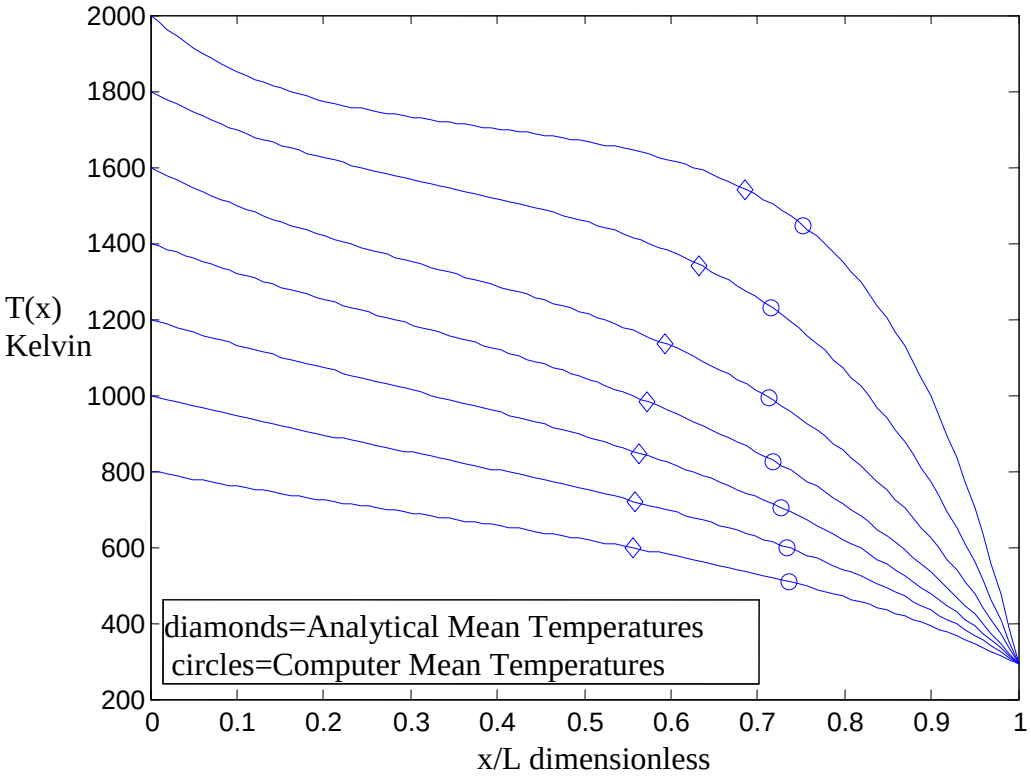


Figure 4.6: Conduction Correction for the Radiation Shield (High Temperatures)

4.2.2 Conduction, Heat Transfer Model Based On Average Heat Flux Correction

The second heat transfer correction model investigated was a conduction - averaged heat flux correction model. This model averages the base heat flux by conduction. The adjusted values of TKEFF and TK2 stated in section 4.2.1 were replaced by average values called TKEFFAVG and TK2AVG in the computer program. The value for TKEFFAVG is the average of the original value of TKEFF and the adjusted value of TKEFF. The same goes for TK2AVG. The purpose of taking the average base heat flux was to obtain base heat losses in terms of the average of numerical and analytical temperature gradients at the base. The results for the sheath are in Tables 4.7, 4.8 and 4.9 and Figures 4.7, 4.8. The results for the radiation shield are in Tables 4.10, 4.11 and 4.12 and Figures 4.9, 4.10. The total x – location percent difference for the sheath is 190.97%. The total temperature value percent difference for the sheath is 179.87%. The total x – location percent difference for the shield is 17.43%. The total temperature value percent difference for the shield is 10.17%. The total x – location percent difference for the sheath and shield is 208.40%. The total temperature value percent difference for the sheath and shield is 190.04%. The total percent difference for the sheath and shield for the heat transfer conduction - averaged heat flux correction model came out a significantly lower than the first conduction model. Therefore, a significant improvement was obtained in getting the numerically – predicted mean temperatures and the analytically – predicted mean temperatures to agree. They fall much closer together on the temperature profile curves of the previous figures. The first calculation and the recalculation of the recovery temperature converged for each tip temperature, so this correction model is valid.

4.2.3 Tip Convection, Double Nusselt Number Correction Model

The third heat transfer correction model is called herein the tip convection, double Nusselt number correction model. In this model, adjustments to the convection heat transfer rate are made at the tip of the sheath and shield. This is done by doubling the original Nusselt number for the convection heat transfer at the tip of the sheath, and the radiation shield. The results for the sheath are in Tables 4.13, 4.14 an 4.15 and Figures 4.11, 4.12. The results for the radiation shield are in Tables 4.16, 4.17 and 4.18 and Figures 4.13, 4.14. The total x – location percent difference for the sheath is 492.48%. The total temperature value percent difference for the sheath is 103.03%. The total x – location percent difference for the shield is 288.33%. The total temperature value percent difference for the shield is 159.31%. The total x – location percent difference for the sheath and shield is 780.81%. The total temperature value percent difference for the sheath and shield is 262.34%. The total percent difference for the sheath and shield for the heat transfer tip convection, double Nusselt number correction model came out higher than the original uncorrected model. The first calculation and the recalculation of the recovery temperature converged for each tip temperature, so this correction model is valid.

Table 4.7: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Differences of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath for the Second Conduction - Average Heat Flux Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	257.99	214.30	0.6739	0.8082	19.93
T2	405.50	335.17	274.64	0.6734	0.8064	19.75
T3	521.60	426.60	346.73	0.6734	0.8047	19.50
T4	613.58	498.78	404.76	0.6736	0.8031	19.23
T5	705.38	570.88	463.64	0.6741	0.8016	18.91
T6	801.32	646.47	526.62	0.6750	0.7999	18.50
T7	872.42	702.64	574.43	0.6760	0.7987	18.15
T8	980.33	788.36	649.15	0.6780	0.7968	17.52
T9	1340.33	1080.46	923.02	0.6898	0.7910	14.67
T10	1700.33	1385.69	1246.89	0.7130	0.7881	10.53
T11	2060.33	1711.31	1636.61	0.7554	0.7933	5.02
T12	2420.33	2045.57	2057.45	0.8110	0.8042	0.84
T13	2780.33	2341.31	2398.37	0.8346	0.7958	4.65
T14	3140.33	2591.87	2646.05	0.8215	0.7905	3.77

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.8: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Differences of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath for the Second Conduction - Average Heat Flux Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	257.99	214.30	16.93
T2	405.50	335.17	274.64	18.06
T3	521.60	426.60	346.73	18.72
T4	613.58	498.78	404.76	18.85
T5	705.38	570.88	463.64	18.79
T6	801.32	646.47	526.62	18.54
T7	872.42	702.64	574.43	18.25
T8	980.33	788.36	649.15	17.66
T9	1340.33	1080.46	923.02	14.57
T10	1700.33	1385.69	1246.89	10.02
T11	2060.33	1711.31	1636.61	4.37
T12	2420.33	2045.57	2057.45	0.58
T13	2780.33	2341.31	2398.37	2.44
T14	3140.33	2591.87	2646.05	2.09

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.9: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures for the Sheath for the Second Conduction- Average Heat Flux Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	257.99	214.30	308.4
T2	405.50	335.17	274.64	407.5
T3	521.60	426.60	346.73	525.1
T4	613.58	498.78	404.76	618.7
T5	705.38	570.88	463.64	712.7
T6	801.32	646.47	526.62	811.5
T7	872.42	702.64	574.43	885.2
T8	980.33	788.36	649.15	997.9
T9	1340.33	1080.46	923.02	1382.3
T10	1700.33	1385.69	1246.89	1782.1
T11	2060.33	1711.31	1636.61	2196.4
T12	2420.33	2045.57	2057.45	2629.8
T13	2780.33	2341.31	2398.37	3107.7
T14	3140.33	2591.87	2646.05	3648.2

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

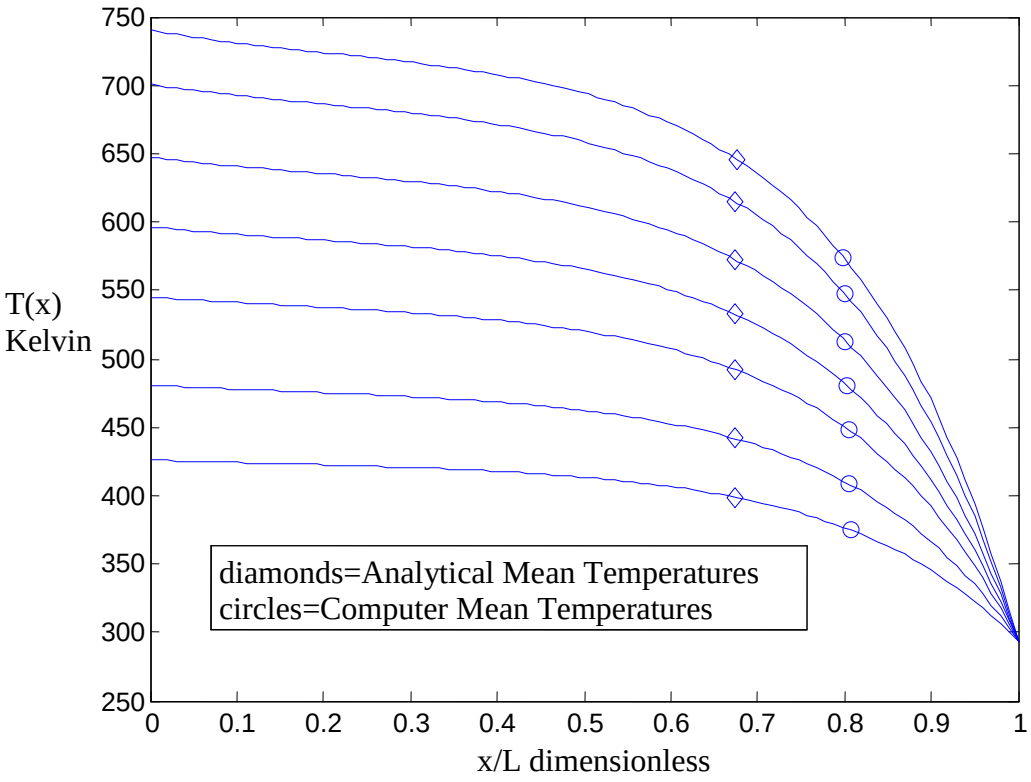


Figure 4.7: Conduction - Averaged Heat Flux Correction for the Sheath (Low Temperatures)

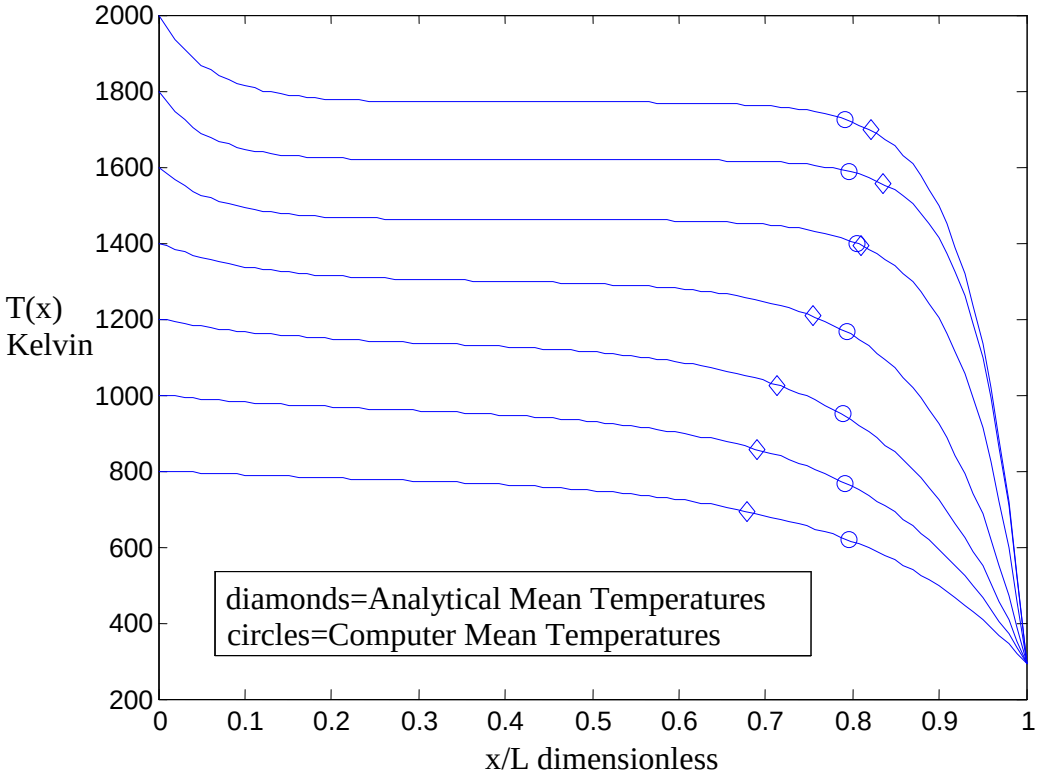


Figure 4.8: Conduction - Averaged Heat Flux Correction for the Sheath (High Temperatures)

Table 4.10: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program x – Coordinates, and Percentage Differences of the x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Second Conduction - Average Heat Flux Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	230.34	230.00	0.5955	0.5983	0.47
T2	405.50	296.17	295.20	0.5948	0.5985	0.62
T3	521.60	374.02	372.42	0.5930	0.5989	0.99
T4	613.58	435.70	433.33	0.5929	0.5990	1.03
T5	705.38	497.41	494.64	0.5934	0.5990	0.94
T6	801.32	562.24	559.17	0.5939	0.5991	0.88
T7	872.42	610.63	607.60	0.5941	0.5991	0.84
T8	980.33	684.90	682.39	0.5952	0.5991	0.65
T9	1340.33	943.50	946.92	0.5989	0.5955	0.57
T10	1700.33	1221.55	1241.83	0.6201	0.6048	2.53
T11	2060.33	1539.05	1572.35	0.6430	0.6205	3.50
T12	2420.33	1877.45	1905.53	0.6851	0.6659	2.80
T13	2780.33	2162.93	2179.31	0.6996	0.6917	1.13
T14	3140.33	2391.35	2398.91	0.6830	0.6797	0.48

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.11: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Differences of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Second Conduction - Average Heat Flux Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff.= Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	230.34	230.00	0.14
T2	405.50	296.17	295.20	0.33
T3	521.60	374.02	372.42	0.43
T4	613.58	435.70	433.33	0.54
T5	705.38	497.41	494.64	0.56
T6	801.32	562.24	559.17	0.55
T7	872.42	610.63	607.60	0.49
T8	980.33	684.90	682.39	0.37
T9	1340.33	943.50	946.92	0.36
T10	1700.33	1221.55	1241.83	1.66
T11	2060.33	1539.05	1572.35	2.16
T12	2420.33	1877.45	1905.53	1.50
T13	2780.33	2162.93	2179.31	0.76
T14	3140.33	2391.35	2398.91	0.32

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.12: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Radiation Shield for the Second Conduction - Average Heat Flux Correction Model. Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	230.34	230.00	308.4
T2	405.50	296.17	295.20	407.5
T3	521.60	374.02	372.42	525.1
T4	613.58	435.70	433.33	618.7
T5	705.38	497.41	494.64	712.7
T6	801.32	562.24	559.17	811.5
T7	872.42	610.63	607.60	885.2
T8	980.33	684.90	682.39	997.9
T9	1340.33	943.50	946.92	1382.3
T10	1700.33	1221.55	1241.83	1782.1
T11	2060.33	1539.05	1572.35	2196.4
T12	2420.33	1877.45	1905.53	2629.8
T13	2780.33	2162.93	2179.31	3107.7
T14	3140.33	2391.35	2398.91	3648.2

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

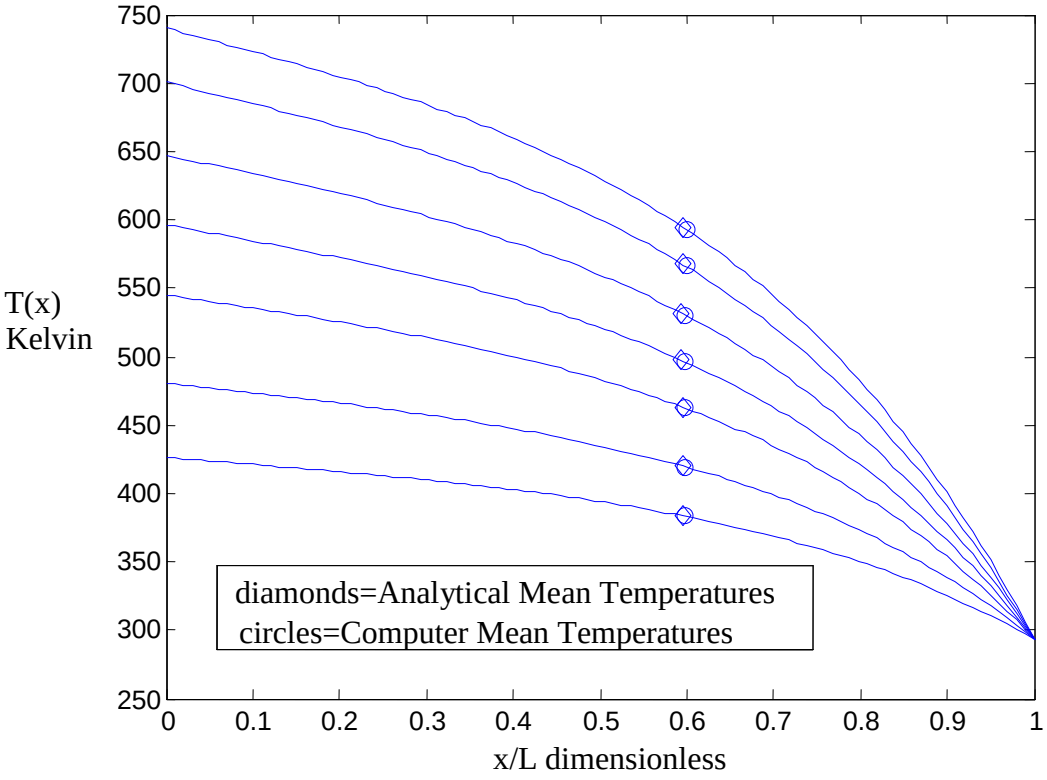


Figure 4.9: Conduction - Averaged Heat Flux Correction for the Radiation Shield (Low Temperatures)

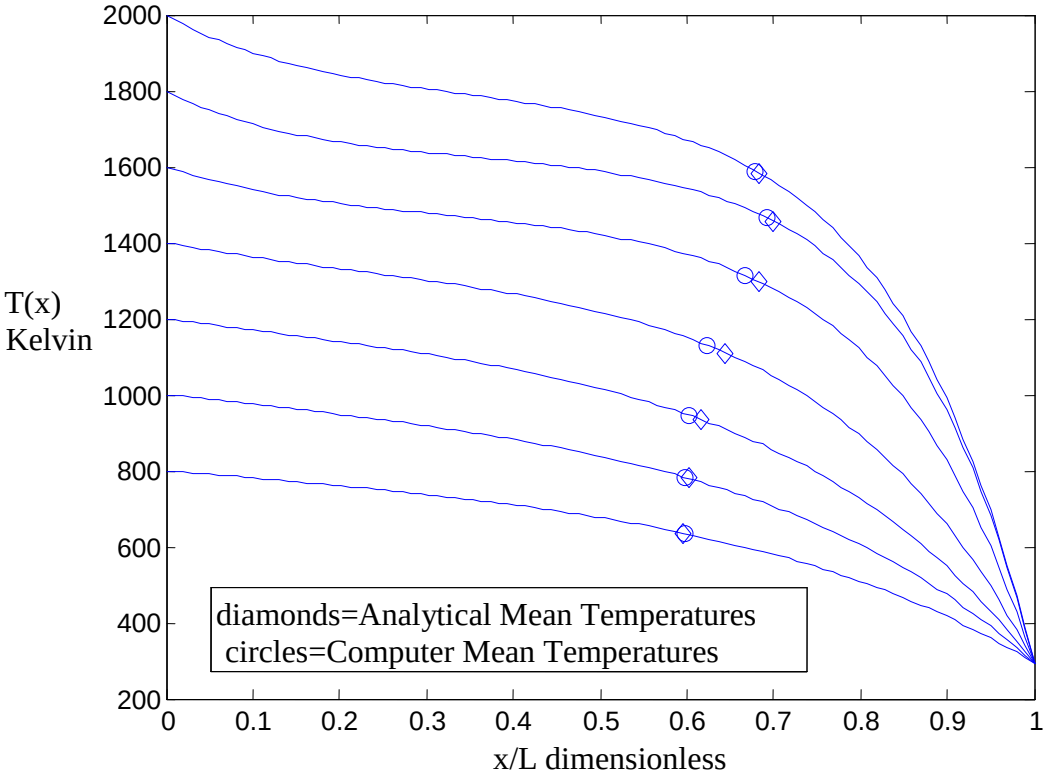


Figure 4.10: Conduction - Averaged Heat Flux Correction for the Radiation Shield (High Temperatures)

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Table 4.13: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath for the Tip Convection, Double Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	257.92	264.81	0.6746	0.6416	4.89
T2	405.50	335.08	346.06	0.6746	0.6366	5.63
T3	521.60	426.79	442.80	0.6749	0.6330	6.21
T4	613.58	498.92	520.45	0.6762	0.6284	7.07
T5	705.38	571.19	598.65	0.6777	0.6243	7.88
T6	801.32	647.28	681.71	0.6795	0.6195	8.83
T7	872.42	703.72	744.06	0.6817	0.6160	9.64
T8	980.33	790.39	840.51	0.6853	0.6103	10.94
T9	1340.33	1087.61	1180.85	0.7063	0.5797	17.92
T10	1700.33	1401.35	1546.61	0.7423	0.4123	44.46
T11	2060.33	1726.07	1919.57	0.7914	0.0846	89.31
T12	2420.33	2030.63	2263.73	0.8285	0.0443	94.65
T13	2780.33	2281.73	2559.29	0.8258	0.0482	94.16
T14	3140.33	2486.39	2810.03	0.7983	0.0727	90.89

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.14: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath for the Tip Convection, Double Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	257.92	264.81	2.67
T2	405.50	335.08	346.06	3.28
T3	521.60	426.79	442.80	3.75
T4	613.58	498.92	520.45	4.32
T5	705.38	571.19	598.65	4.81
T6	801.32	647.28	681.71	5.32
T7	872.42	703.72	744.06	5.73
T8	980.33	790.39	840.51	6.34
T9	1340.33	1087.61	1180.85	8.57
T10	1700.33	1401.35	1546.61	10.37
T11	2060.33	1726.07	1919.57	11.21
T12	2420.33	2030.63	2263.73	11.48
T13	2780.33	2281.73	2559.29	12.16
T14	3140.33	2486.39	2810.03	13.02

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.15: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Sheath for the Tip Convection, Double Nusselt Number Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	257.92	264.81	307.8
T2	405.50	335.08	346.06	406.4
T3	521.60	426.79	442.80	523.1
T4	613.58	498.92	520.45	615.8
T5	705.38	571.19	598.65	708.4
T6	801.32	647.28	681.71	805.5
T7	872.42	703.72	744.06	877.6
T8	980.33	790.39	840.51	987.4
T9	1340.33	1087.61	1180.85	1356.6
T10	1700.33	1401.35	1546.61	1731.5
T11	2060.33	1726.07	1919.57	2114.6
T12	2420.33	2030.63	2263.73	2513.3
T13	2780.33	2281.73	2559.29	2937.8
T14	3140.33	2486.39	2810.03	3396.0

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

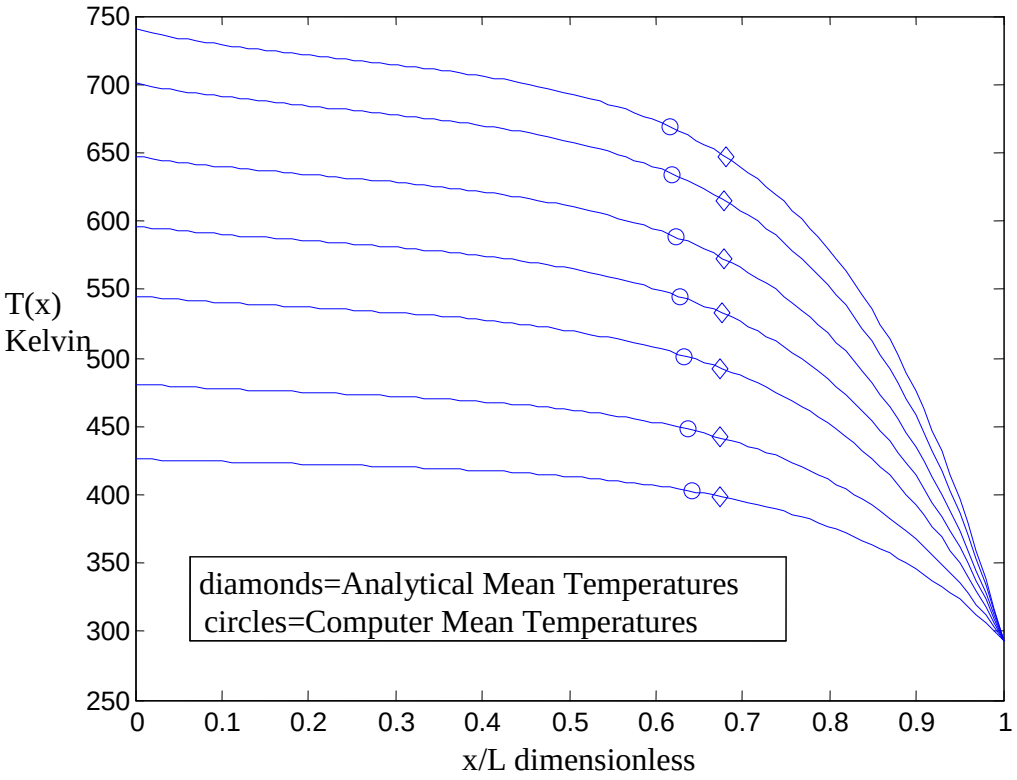


Figure 4.11: Tip Convection, Double Nusselt Number Correction for the Sheath (Low Temperatures)

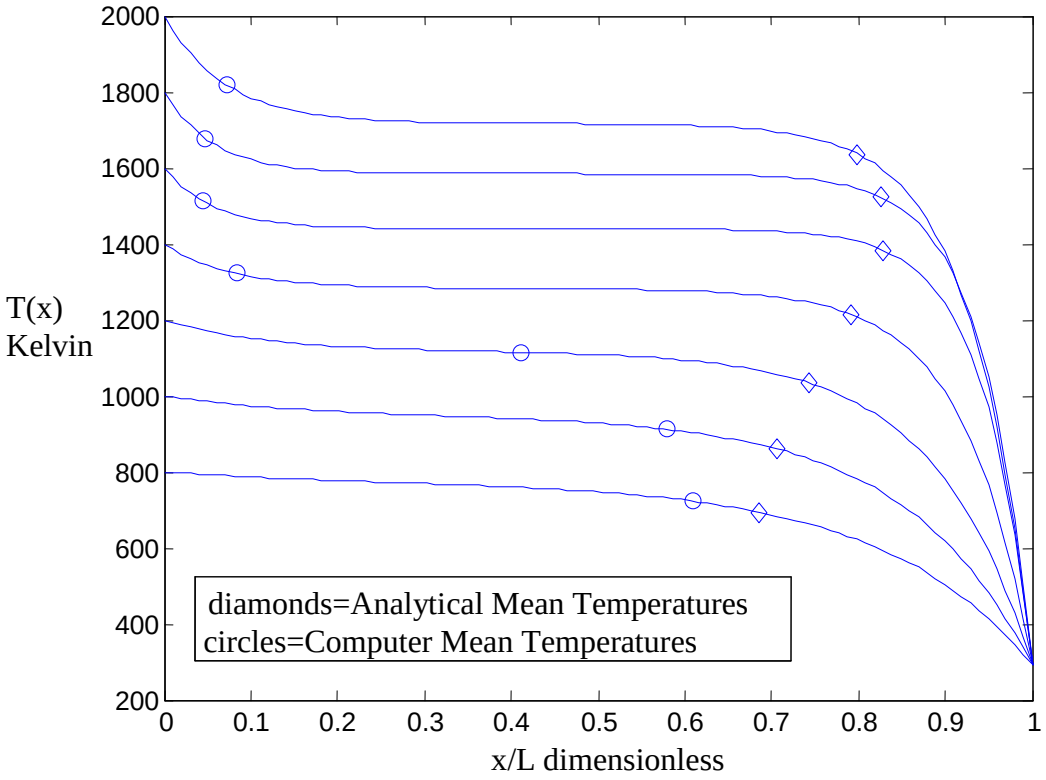


Figure 4.12: Tip Convection, Double Nusselt Number Correction for the Sheath (High Temperatures)

Table 4.16: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield for the Tip Convection, Double Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	230.31	253.74	0.5959	0.4749	20.31
T2	405.50	296.06	328.75	0.5956	0.4764	20.01
T3	521.60	374.14	418.17	0.5953	0.4758	20.07
T4	613.58	436.12	488.73	0.5953	0.4764	19.97
T5	705.38	498.38	559.74	0.5956	0.4766	19.98
T6	801.32	563.94	634.66	0.5965	0.4767	20.08
T7	872.42	612.99	690.84	0.5974	0.4767	20.20
T8	980.33	688.42	777.40	0.5994	0.4767	20.47
T9	1340.33	952.70	1080.28	0.6106	0.4776	21.78
T10	1700.33	1241.06	1400.27	0.6310	0.4878	22.69
T11	2060.33	1548.41	1717.43	0.6624	0.5135	22.48
T12	2420.33	1832.27	1998.59	0.6891	0.5408	21.52
T13	2780.33	2052.59	2230.79	0.6788	0.5405	20.37
T14	3140.33	2224.49	2422.31	0.6458	0.5270	18.40

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.17: Values of the Analytical and Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield for the Tip Convection, Double Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff.= Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	230.31	253.74	10.17
T2	405.50	296.06	328.75	11.04
T3	521.60	374.14	418.17	11.77
T4	613.58	436.12	488.73	12.06
T5	705.38	498.38	559.74	12.31
T6	801.32	563.94	634.66	12.54
T7	872.42	612.99	690.84	12.70
T8	980.33	688.42	777.40	12.93
T9	1340.33	952.70	1080.28	13.39
T10	1700.33	1241.06	1400.27	12.83
T11	2060.33	1548.41	1717.43	10.92
T12	2420.33	1832.27	1998.59	9.08
T13	2780.33	2052.59	2230.79	8.68
T14	3140.33	2224.49	2422.31	8.89

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.18: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Radiation Shield for the Tip Convection, Double Nusselt Number Correction Model. Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	230.31	253.74	307.8
T2	405.50	296.06	328.75	406.4
T3	521.60	374.14	418.17	523.1
T4	613.58	436.12	488.73	615.8
T5	705.38	498.38	559.74	708.4
T6	801.32	563.94	634.66	805.5
T7	872.42	612.99	690.84	877.6
T8	980.33	688.42	777.40	987.4
T9	1340.33	952.70	1080.28	1356.6
T10	1700.33	1241.06	1400.27	1731.5
T11	2060.33	1548.41	1717.43	2114.6
T12	2420.33	1832.27	1998.59	2513.3
T13	2780.33	2052.59	2230.79	2937.8
T14	3140.33	2224.49	2422.31	3396.0

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

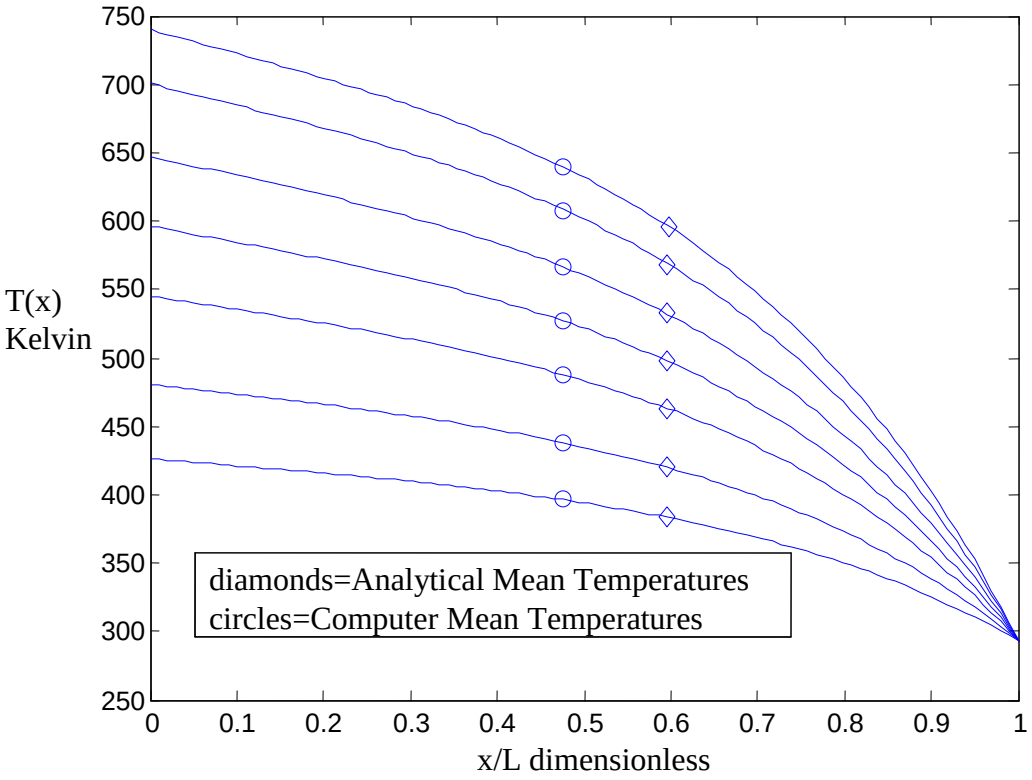


Figure 4.13: Tip Convection, Double Nusselt Number Correction for the Radiation Shield (Low Temperatures)

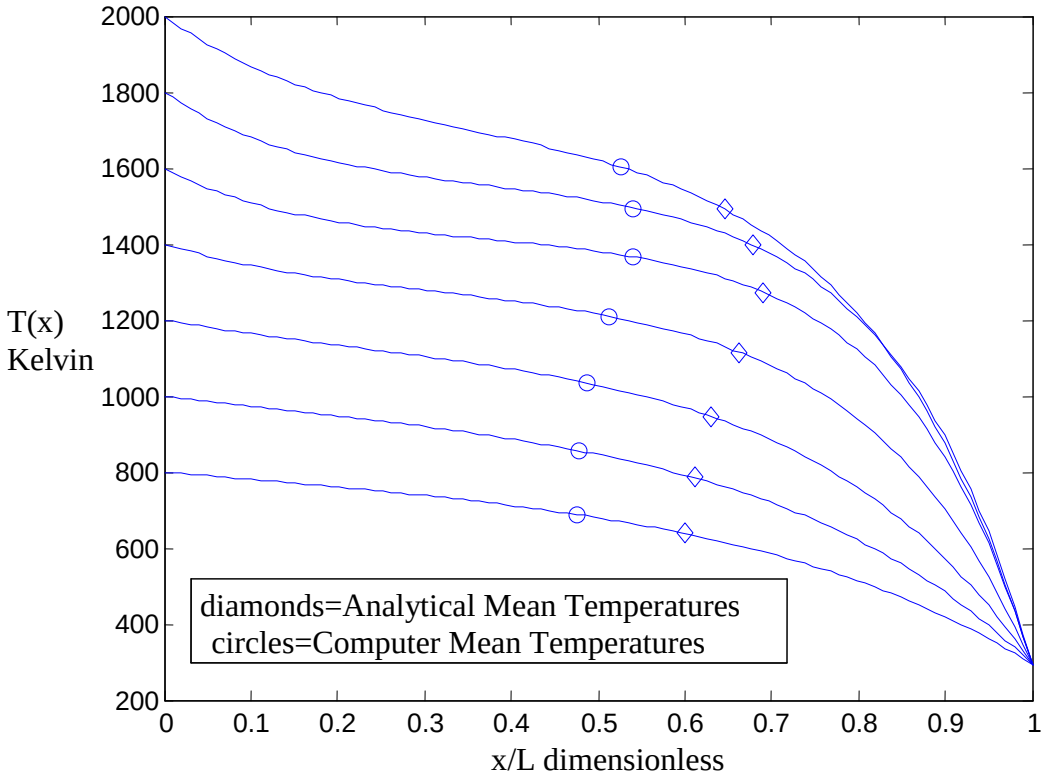


Figure 4.14: Tip Convection, Double Nusselt Number Correction for the Radiation Shield (High Temperatures)

4.2.4 Tip Convection, Half Nusselt Number Correction Model

The fourth heat transfer correction model is called the tip convection, half Nusselt number correction model. In this model, adjustments to the convection heat transfer are made at the tip of the sheath. This is done by taking 1/2 the original Nusselt number for the convection heat transfer at the tip of the sheath. The results for the sheath are in Tables 4.19, 4.20 and 4.21 and Figures 4.15, 4.16. The results for the radiation shield are in Tables 4.22, 4.23 and 4.24 and Figures 4.17, 4.18. The total x – location percent difference for the sheath is 412.02%. The total temperature value percent difference for the sheath is 59.09%. The total x – location percent difference for the shield is 204.51%. The total temperature value percent difference for the shield is 124.46%. The total x – location percent difference for the sheath and shield is 616.53%. The total temperature value percent difference for the sheath and shield is 183.55%. The total percent difference for the sheath and shield for the heat transfer tip convection, half Nusselt number correction model came out slightly lower than the original uncorrected model. In this case after executing the program, the maximum allowable temperature was 3075^{o}F / 1964 K as opposed to 3149^{o}F / 2004 K in the original case and other cases. The process for finding the maximum allowable temperature is explained in Chapter 3, Section 3.1.2. In future study the subroutine "HEEV" in the computer program, explained in Chapter 3, Section 3.1.1, needs to be adjusted to allow higher maximum temperatures. The first calculation and the recalculation of the recovery temperature did not converge for tip temperature of 3075^{o}F. The fist recovery temperature was at 3765.13^{o}F and the recalculation recovery temperature was at 3837.55^{o}F. For this reason, this correction model is not valid.

4.2.5 Double Nusselt Number Composite Correction Model

The fifth heat transfer correction model is called the double Nusselt number composite correction model. In this model, the conduction - averaged heat flux correction model and the tip convection, double Nusselt number correction model were combined in a composite model. This model applies the base heat flow conduction and convection heat transfer corrections in which the effect of the tip convection heat transfer coefficients have been doubled. The results for the sheath are in Tables 4.25, 4.26 and 4.27 and Figures 4.19, 4.20. The results for the radiation shield are in Tables 4.28, 4.29 and 4.30 and Figures 4.21, 4.22. The total x – location percent difference for the sheath is 264.54%. The total temperature value percent difference for the sheath is 145.67%. The total x – location percent difference for the shield is 28.11%. The total temperature value percent difference for the shield is 17.83%. The total x – location percent difference for the sheath and shield is 292.65%. The total temperature value percent difference for the sheath and shield is 163.50%. The total percent difference for the sheath and shield for the double Nusselt number composite correction model came out significantly lower than the original uncorrected model. The first calculation and the recalculation of the recovery temperature converged for each tip temperature, so this correction model is valid.

Table 4.19: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath for the Tip Convection, Half Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	258.46	256.42	0.6752	0.684	1.30
T2	405.50	336.29	334.24	0.6749	0.6812	0.93
T3	521.60	428.56	427.50	0.6762	0.6787	0.37
T4	613.58	501.98	502.41	0.6771	0.6762	0.13
T5	705.38	575.58	578.53	0.6787	0.6738	0.72
T6	801.32	653.12	654.35	0.6812	0.6794	0.26
T7	872.42	711.36	721.74	0.6832	0.6688	2.11
T8	980.33	800.58	818.38	0.6874	0.6652	3.23
T9	1340.33	1111.82	1169.46	0.7100	0.6497	8.49
T10	1700.33	1449.05	1565.51	0.7496	0.6061	19.14
T11	2060.33	1805.27	1975.73	0.8032	0.0790	90.16
T12	2420.33	2142.59	2350.49	0.8364	0.0300	96.41
T13	2780.33	2425.37	2671.79	0.8225	0.0436	94.70
T14	3075.01	2694.65	2976.35	0.8093	0.0480	94.07

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.20: Values of the Analytical and Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Sheath for the Tip Convection, Half Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	258.46	256.42	0.79
T2	405.50	336.29	334.24	0.61
T3	521.60	428.56	427.50	0.25
T4	613.58	501.98	502.41	0.09
T5	705.38	575.58	578.53	0.51
T6	801.32	653.12	654.35	0.19
T7	872.42	711.36	721.74	1.46
T8	980.33	800.58	818.38	2.22
T9	1340.33	1111.82	1169.46	5.18
T10	1700.33	1449.05	1565.51	8.04
T11	2060.33	1805.27	1975.73	9.44
T12	2420.33	2142.59	2350.49	9.70
T13	2780.33	2425.37	2671.79	10.16
T14	3075.01	2694.65	2976.35	10.45

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.21: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Sheath for the Tip Convection, Half Nusselt Number Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	258.46	256.42	308.9
T2	405.50	336.29	334.24	408.6
T3	521.60	428.56	427.50	527.1
T4	613.58	501.98	502.41	621.8
T5	705.38	575.58	578.53	717.1
T6	801.32	653.12	654.35	817.7
T7	872.42	711.36	721.74	892.9
T8	980.33	800.58	818.38	1008.2
T9	1340.33	1111.82	1169.46	1403.8
T10	1700.33	1449.05	1565.51	1815.5
T11	2060.33	1805.27	1975.73	2248.5
T12	2420.33	2142.59	2350.49	2731.5
T13	2780.33	2425.37	2671.79	3298.9
T14	3075.01	2694.65	2976.35	3871.0

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

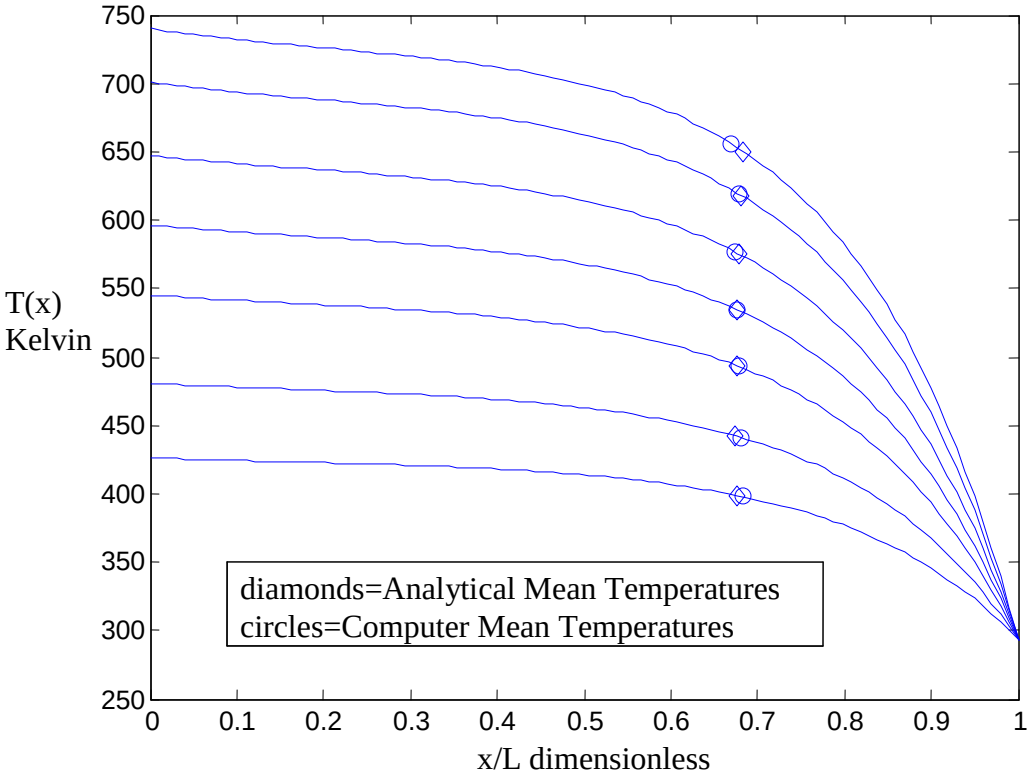


Figure 4.15: Tip Convection, Half Nusselt Number Correction for the Sheath (Low Temperatures)

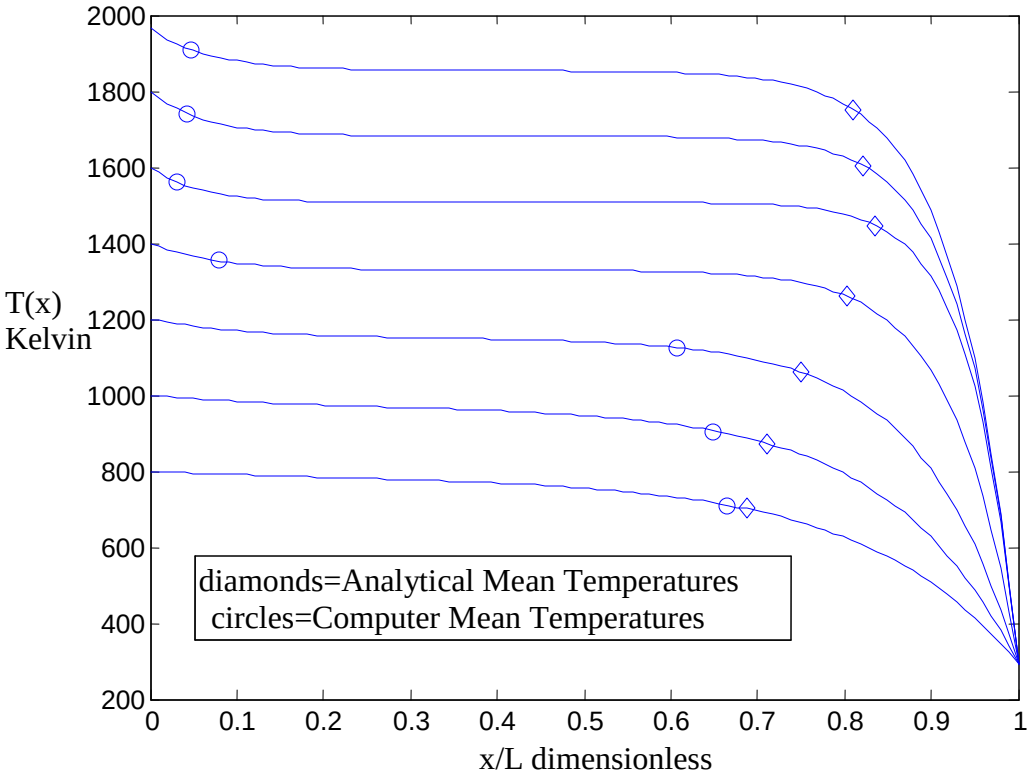


Figure 4.16: Tip Convection, Half Nusselt Number Correction for the Sheath (High Temperatures)

Table 4.22: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield for the Tip Convection, Half Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	230.67	247.59	0.5965	0.5132	13.96
T2	405.50	296.82	320.63	0.5962	0.5132	13.92
T3	521.60	375.28	407.48	0.5965	0.5132	13.96
T4	613.58	437.97	476.55	0.5965	0.5135	13.91
T5	705.38	500.99	546.46	0.5971	0.5132	14.05
T6	801.32	567.59	620.71	0.5984	0.5129	14.29
T7	872.42	617.52	677.14	0.5997	0.5120	14.62
T8	980.33	694.99	764.60	0.6020	0.5110	15.12
T9	1340.33	969.93	1077.93	0.6158	0.5104	17.12
T10	1700.33	1280.80	1421.87	0.6410	0.5251	18.08
T11	2060.33	1622.75	1767.11	0.6789	0.5705	15.97
T12	2420.33	1937.93	2074.19	0.7047	0.6134	12.96
T13	2780.33	2178.41	2335.19	0.6857	0.5988	12.67
T14	3075.01	2397.29	2596.91	0.6713	0.5781	13.88

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.23: Values of the Analytical, Computer Program Mean Temperatures, and Percentage difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures for the Radiation Shield for the Tip Convection, Half Nusselt Number Correction Model
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	230.67	247.59	7.34
T2	405.50	296.82	320.63	8.02
T3	521.60	375.28	407.48	8.58
T4	613.58	437.97	476.55	8.81
T5	705.38	500.99	546.46	9.08
T6	801.32	567.59	620.71	9.36
T7	872.42	617.52	677.14	9.65
T8	980.33	694.99	764.60	10.02
T9	1340.33	969.93	1077.93	11.13
T10	1700.33	1280.80	1421.87	11.01
T11	2060.33	1622.75	1767.11	8.90
T12	2420.33	1937.93	2074.19	7.03
T13	2780.33	2178.41	2335.19	7.20
T14	3075.01	2397.29	2596.91	8.33

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.24: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Radiation Shield for the Tip Convection, Half Nusselt Number Correction Model. Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	230.67	247.59	308.9
T2	405.50	296.82	320.63	408.6
T3	521.60	375.28	407.48	527.1
T4	613.58	437.97	476.55	621.8
T5	705.38	500.99	546.46	717.1
T6	801.32	567.59	620.71	817.7
T7	872.42	617.52	677.14	892.9
T8	980.33	694.99	764.60	1008.2
T9	1340.33	969.93	1077.93	1403.8
T10	1700.33	1280.80	1421.87	1815.5
T11	2060.33	1622.75	1767.11	2248.5
T12	2420.33	1937.93	2074.19	2731.5
T13	2780.33	2178.41	2335.19	3298.9
T14	3075.01	2397.29	2596.91	3871.0

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

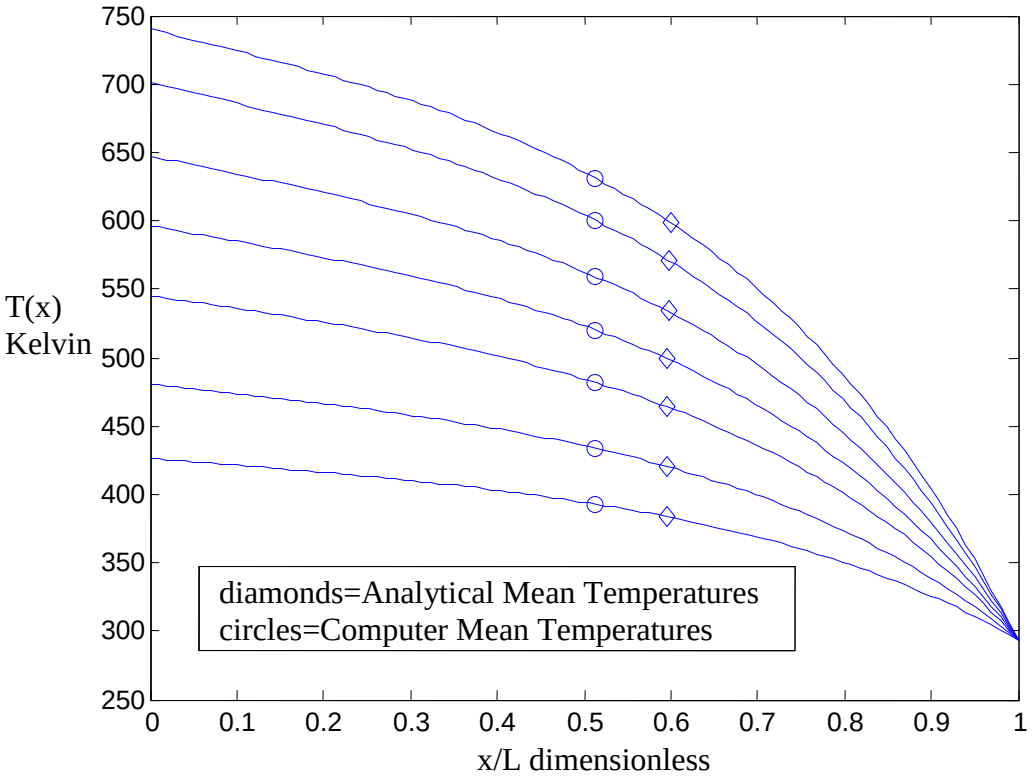


Figure 4.17: Tip Convection, Half Nusselt Number Correction for the Radiation Shield (Low Temperatures)

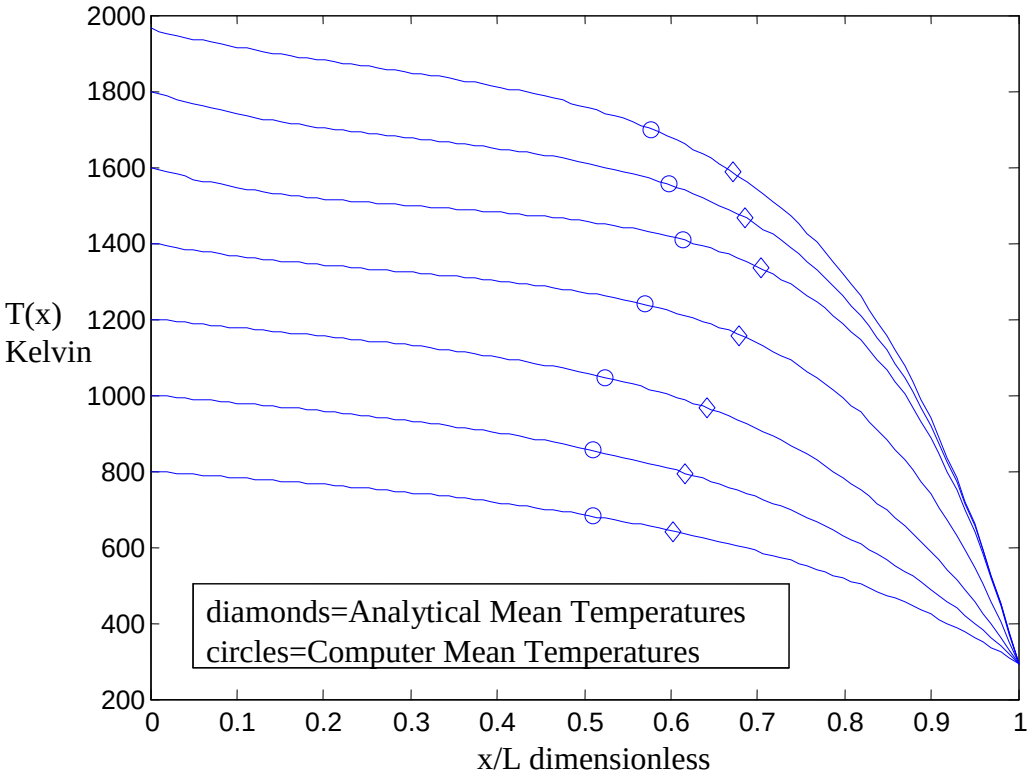


Figure 4.18: Tip Convection, Half Nusselt Number Correction for the Radiation Shield (High Temperatures)

Table 4.25: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the Double Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	245.43	212.20	0.6571	0.7813	18.90
T2	405.50	317.28	271.83	0.6568	0.7785	18.53
T3	521.60	402.42	343.00	0.6564	0.7760	18.22
T4	613.58	469.47	400.05	0.6564	0.7739	17.90
T5	705.38	536.54	457.72	0.6564	0.7719	17.60
T6	801.32	606.45	519.15	0.6573	0.7698	17.12
T7	872.42	658.49	565.47	0.6579	0.7683	16.78
T8	980.33	737.51	637.48	0.6597	0.7660	16.11
T9	1340.33	1005.91	896.22	0.6705	0.7594	13.26
T10	1700.33	1283.32	1189.98	0.6911	0.7557	9.35
T11	2060.33	1578.83	1533.83	0.7287	0.7582	4.04
T12	2420.33	1893.47	1923.17	0.7843	0.7620	2.84
T13	2780.33	2186.33	2278.13	0.8245	0.6515	20.98
T14	3140.33	2420.15	2533.37	0.8222	0.2227	72.91

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.26: Values of the Analytical and Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the Double Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	245.43	212.20	13.54
T2	405.50	317.28	271.83	14.32
T3	521.60	402.42	343.00	14.77
T4	613.58	469.47	400.05	14.79
T5	705.38	536.54	457.72	14.69
T6	801.32	606.45	519.15	14.40
T7	872.42	658.49	565.47	14.14
T8	980.33	737.51	637.48	13.56
T9	1340.33	1005.91	896.22	10.90
T10	1700.33	1283.32	1189.98	7.27
T11	2060.33	1578.83	1533.83	2.85
T12	2420.33	1893.47	1923.17	1.57
T13	2780.33	2186.33	2278.13	4.20
T14	3140.33	2420.15	2533.37	4.68

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.27: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Sheath for the Double Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	245.43	212.20	307.9
T2	405.50	317.28	271.83	406.6
T3	521.60	402.42	343.00	523.4
T4	613.58	469.47	400.05	616.2
T5	705.38	536.54	457.72	709.0
T6	801.32	606.45	519.15	806.4
T7	872.42	658.49	565.47	878.7
T8	980.33	737.51	637.48	988.9
T9	1340.33	1005.91	896.22	1360.7
T10	1700.33	1283.32	1189.98	1740.3
T11	2060.33	1578.83	1533.83	2128.5
T12	2420.33	1893.47	1923.17	2528.7
T13	2780.33	2186.33	2278.13	2952.9
T14	3140.33	2420.15	2533.37	3411.5

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

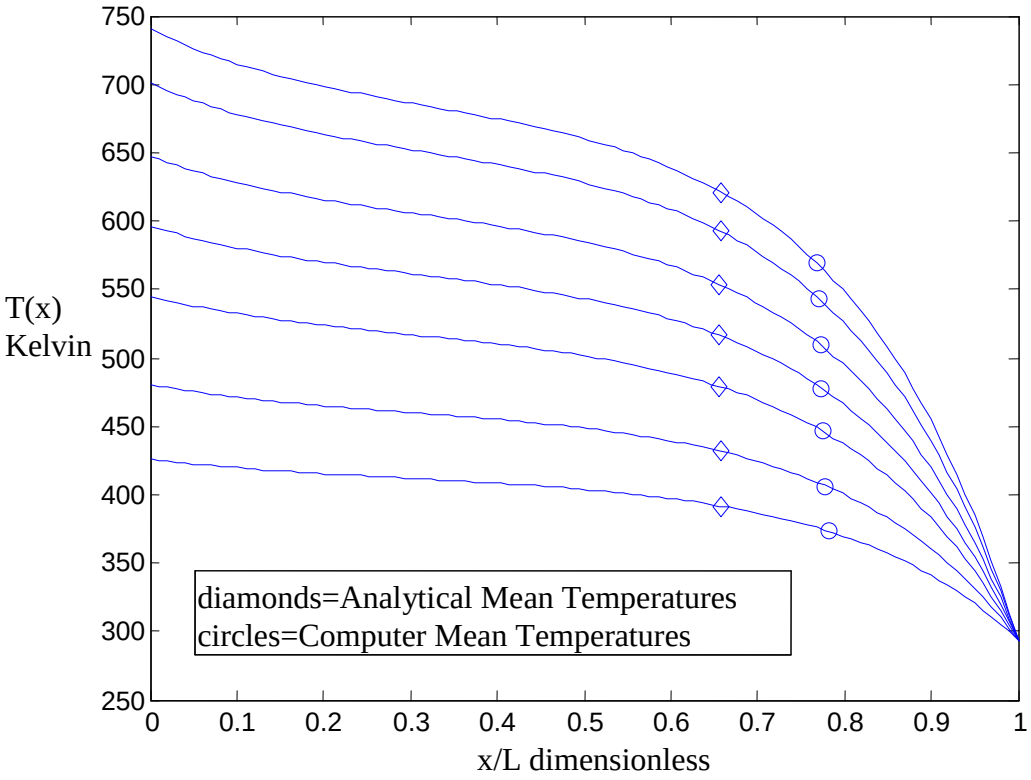


Figure 4.19: Double Nusselt Number Composite Correction for the Sheath (Low Temperatures)

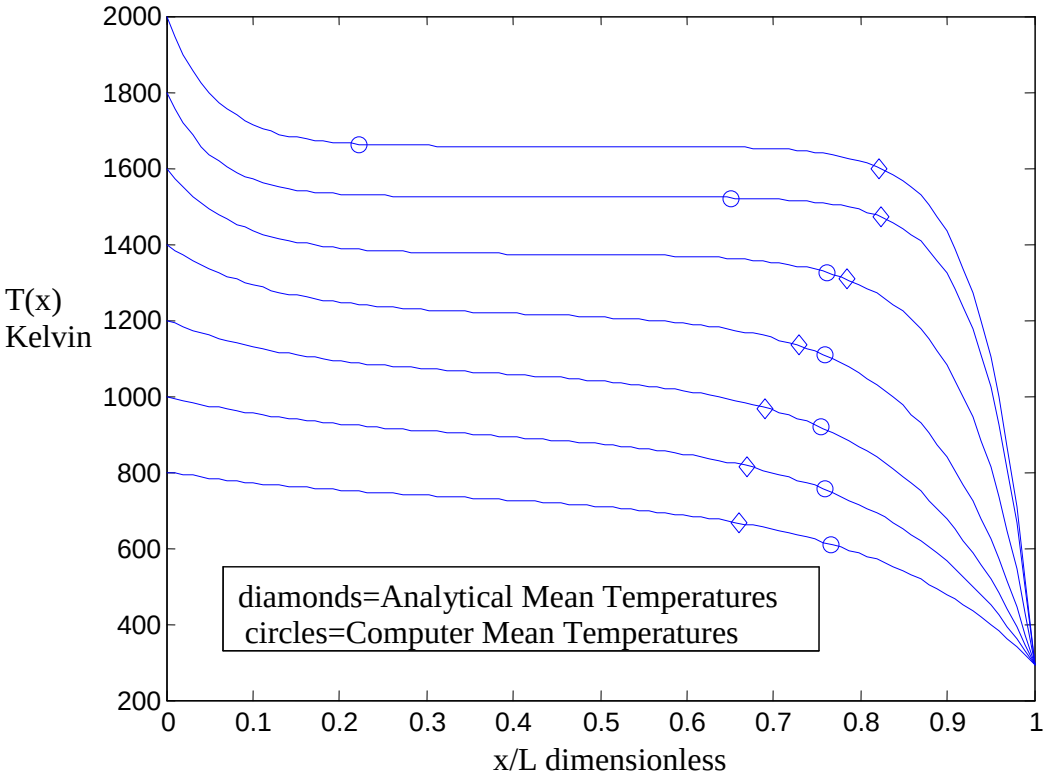


Figure 4.20: Double Nusselt Number Composite Correction for the Sheath (High Temperature)

Table 4.28: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Double Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	222.85	221.49	0.5821	0.5886	1.12
T2	405.50	285.73	283.06	0.5808	0.5898	1.55
T3	521.60	359.85	356.07	0.5804	0.5898	1.62
T4	613.58	418.55	413.58	0.5800	0.5902	1.76
T5	705.38	477.12	471.22	0.5800	0.5904	1.79
T6	801.32	538.68	531.90	0.5800	0.5904	1.79
T7	872.42	584.64	577.00	0.5800	0.5907	1.84
T8	980.33	654.44	646.79	0.5810	0.5904	1.62
T9	1340.33	894.79	891.97	0.5861	0.5887	0.44
T10	1700.33	1150.03	1160.64	0.5964	0.5886	1.31
T11	2060.33	1431.41	1460.75	0.6164	0.5973	3.01
T12	2420.33	1739.93	1780.97	0.6514	0.6253	4.01
T13	2780.33	2022.17	2061.77	0.6805	0.6563	3.56
T14	3140.33	2239.07	2271.65	0.6724	0.6543	2.69

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.29: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Double Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	222.85	221.49	0.61
T2	405.50	285.73	283.06	0.93
T3	521.60	359.85	356.07	1.05
T4	613.58	418.55	413.58	1.19
T5	705.38	477.12	471.22	1.24
T6	801.32	538.68	531.90	1.26
T7	872.42	584.64	577.00	1.31
T8	980.33	654.44	646.79	1.17
T9	1340.33	894.79	891.97	0.32
T10	1700.33	1150.03	1160.64	0.92
T11	2060.33	1431.41	1460.75	2.05
T12	2420.33	1739.93	1780.97	2.36
T13	2780.33	2022.17	2061.77	1.96
T14	3140.33	2239.07	2271.65	1.46

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.30: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Radiation Shield for the Double Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	222.85	221.49	307.9
T2	405.50	285.73	283.06	406.6
T3	521.60	359.85	356.07	523.4
T4	613.58	418.55	413.58	616.2
T5	705.38	477.12	471.22	709.0
T6	801.32	538.68	531.90	806.4
T7	872.42	584.64	577.00	878.7
T8	980.33	654.44	646.79	988.9
T9	1340.33	894.79	891.97	1360.7
T10	1700.33	1150.03	1160.64	1740.3
T11	2060.33	1431.41	1460.75	2128.5
T12	2420.33	1739.93	1780.97	2528.7
T13	2780.33	2022.17	2061.77	2952.9
T14	3140.33	2239.07	2271.65	3411.5

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

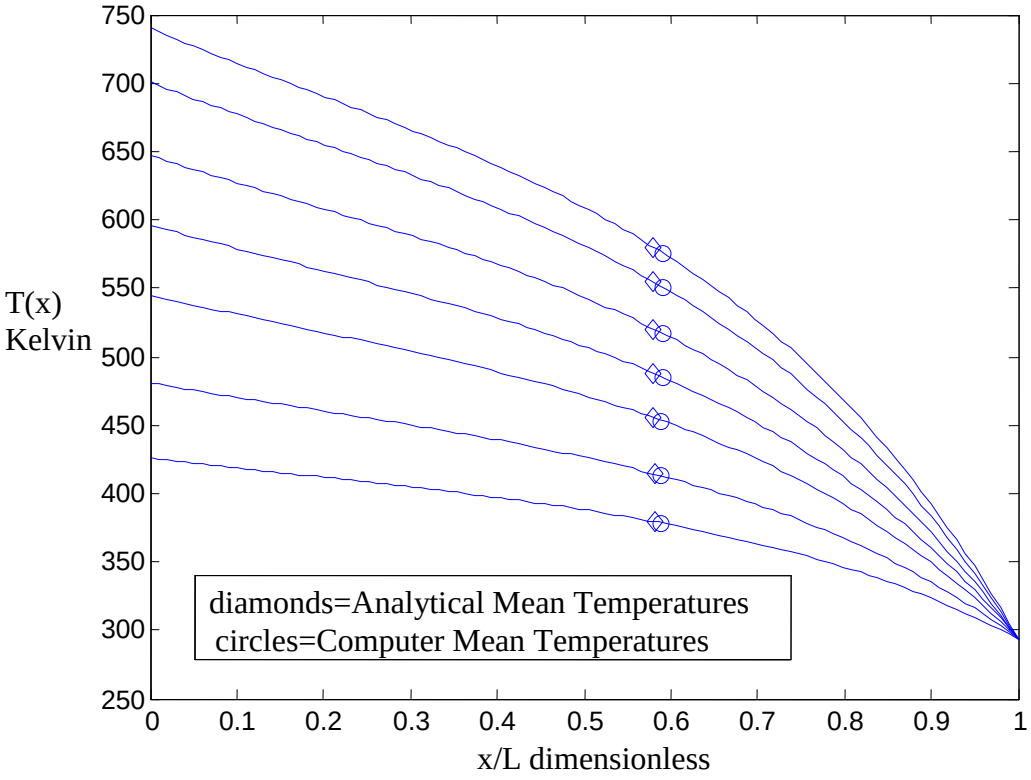


Figure 4.21: Double Nusselt Number Composite Correction for the Radiation Shield (Low Temperatures)

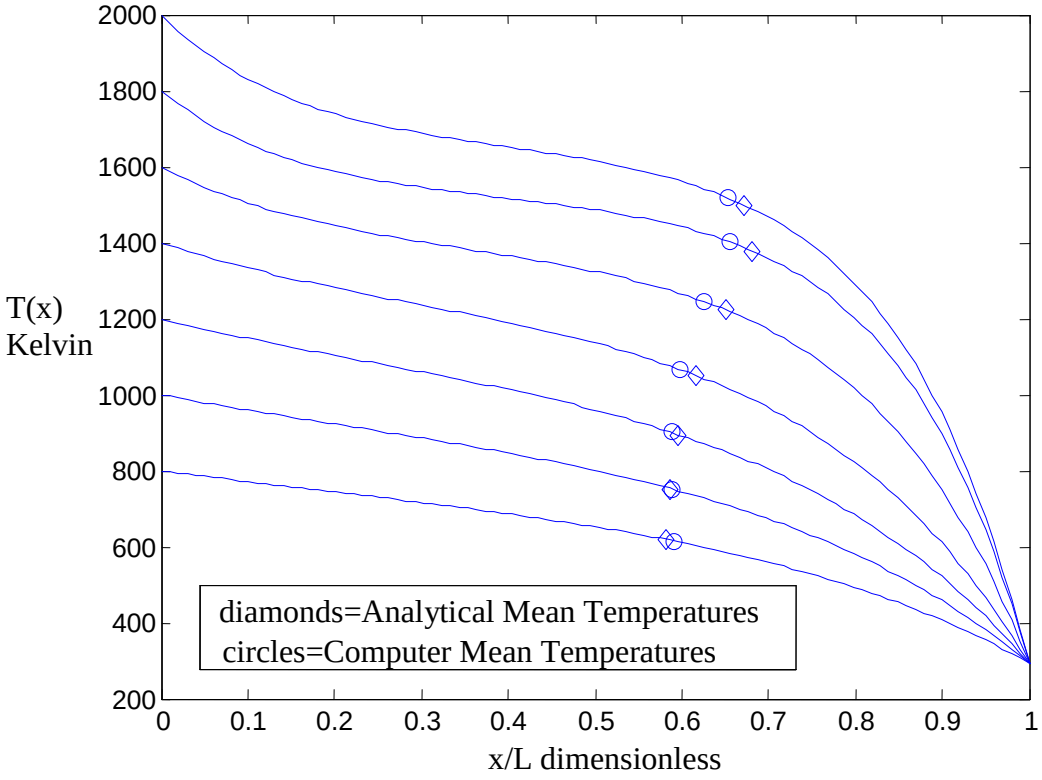


Figure 4.22: Double Nusselt Number Composite Correction for the Radiation Shield (High Temperatures)

4.2.6 Half Nusselt Number Composite Correction Model

The sixth heat transfer correction model is called the half Nusselt number composite correction model. In this model, the conduction - averaged heat flux correction model and the tip convection, half Nusselt number correction model were combined in a composite model. This model applies the base heat flow correction in conduction and the tip convection heat transfer coefficient which has been halved. The results for the sheath are in Tables 4.31, 4.32 and 4.33 and Figures 4.23, 4.24. The results for the radiation shield are in Tables 4.34, 4.35, and 4.36 and Figures 4.25, 4.26. The total x – location percent difference for the sheath is 254.10%. The total temperature value percent difference for the sheath is 237.94%. The total x – location percent difference for the shield is 88.56%. The total temperature value percent difference for the shield is 62.48%. The total x – location percent difference for the sheath and shield is 342.66%. The total temperature value percent difference for the sheath and shield is 300.42%. The total percent difference for the sheath and shield for the half Nusselt number composite correction model also came out significantly lower than the original uncorrected model. However, the total percent difference in axial positions of the mean temperatures was higher than for the double Nusselt number composite correction method of Section 4.2.5. The first calculation and the recalculation of the recovery temperature converged for each tip temperature, so this correction model is valid.

4.2.7 Correction to the Total Gas Stagnation Temperature

Six heat transfer correction models were developed for the purpose of evaluating methods of correcting the numerical computer program prediction of mean temperature locations for the sheath and shield. The results are tabulated In Table 4.37 and 4.38. The mean temperature location and mean temperature value total difference of the two best correction models, one model for the sheath and the other model for the radiation shield are tabulated in Table 4.39 and Table 4.40.

The best heat transfer correction model out of these six correction models needs to be chosen in order to correct and calculate more accurately and precisely the total gas stagnation temperature. The calculations of the total gas stagnation temperature from the original, uncorrected model needs to be compared to the calculations made from the selected models. After analyzing Tables 4.37 – 4.40, it seems that a new heat transfer correction model can be developed. A seventh heat transfer correction model composed of the Half Nusselt Composite Correction Model for the sheath will be used in conjunction with the Double Nusselt Composite Correction Model for the radiation shield. In this case after executing the program, the maximum allowable temperature was 3126^{o}F / 1992 K as opposed to 3149^{o}F / 2004 K in the original case and other cases. The first calculation and the recalculation of the recovery temperature converged for each tip temperature for the seventh model, so this correction model is valid. The results are in Tables 4.41 – 4.46 and Figures 4.27 – 4.30. After analyzing the seven correction models in Table 4.47 and 4.49, certain correction models will be selected as the best correction

model for the sheath and shield for the purpose of best accurately predicting the total gas stagnation temperature. The total gas stagnation temperature values for the original and seven correction models for the sheath and shield are listed in Table 4.52 and compared in Table 4.53. The results and conclusions from Tables 4.47 – 4.53 will be discussed in Chapter 5.

Table 4.31: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the Half Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	243.37	192.45	0.6540	0.8255	26.22
T2	405.50	314.94	243.59	0.6531	0.8243	26.21
T3	521.60	399.85	304.68	0.6533	0.8234	26.04
T4	613.58	467.02	353.89	0.6540	0.8224	25.75
T5	705.38	534.72	403.97	0.6540	0.8214	25.60
T6	801.32	605.71	457.79	0.6556	0.8202	25.11
T7	872.42	658.94	498.97	0.6566	0.8192	24.76
T8	980.33	740.46	563.52	0.6589	0.8178	24.12
T9	1340.33	1023.84	807.64	0.6714	0.8123	20.99
T10	1700.33	1329.44	1113.35	0.6944	0.8073	16.26
T11	2060.33	1668.47	1515.29	0.7378	0.8083	9.56
T12	2420.33	2039.45	1998.77	0.8039	0.8230	2.38
T13	2780.33	2373.35	2389.01	0.8357	0.8278	0.95
T14	3140.33	2647.85	2650.55	0.8237	0.8225	0.15

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.32: Values of the Analytical, Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the Half Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	243.37	192.45	20.92
T2	405.50	314.94	243.59	22.66
T3	521.60	399.85	304.68	23.80
T4	613.58	467.02	353.89	24.22
T5	705.38	534.72	403.97	24.45
T6	801.32	605.71	457.79	24.42
T7	872.42	658.94	498.97	24.28
T8	980.33	740.46	563.52	23.90
T9	1340.33	1023.84	807.64	21.11
T10	1700.33	1329.44	1113.35	16.25
T11	2060.33	1668.47	1515.29	9.18
T12	2420.33	2039.45	1998.77	1.99
T13	2780.33	2373.35	2389.01	0.66
T14	3140.33	2647.85	2650.55	0.10

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.33: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Sheath for the Half Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	243.37	192.45	309.3
T2	405.50	314.94	243.59	409.3
T3	521.60	399.85	304.68	528.3
T4	613.58	467.02	353.89	623.7
T5	705.38	534.72	403.97	719.8
T6	801.32	605.71	457.79	821.6
T7	872.42	658.94	498.97	897.9
T8	980.33	740.46	563.52	1015.3
T9	1340.33	1023.84	807.64	1423.1
T10	1700.33	1329.44	1113.35	1856.8
T11	2060.33	1668.47	1515.29	2309.3
T12	2420.33	2039.45	1998.77	2790.0
T13	2780.33	2373.35	2389.01	3354.1
T14	3140.33	2647.85	2650.55	3943.4

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

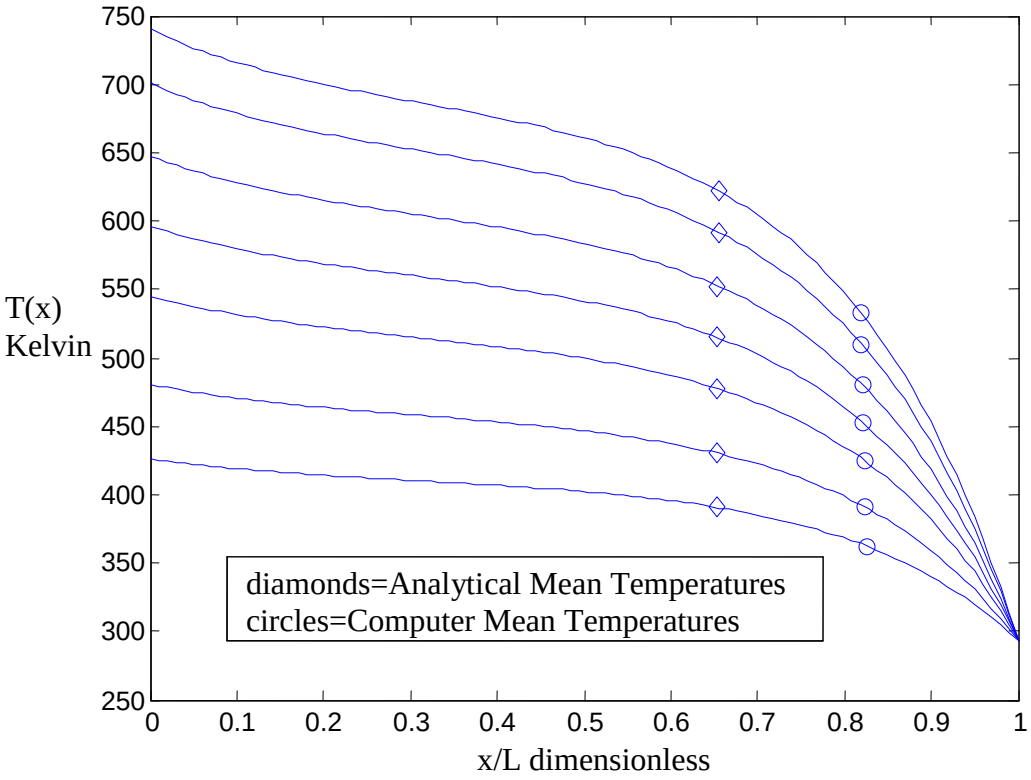


Figure 4.23: Half Nusselt Number Composite Correction for the Sheath (Low Temperatures)

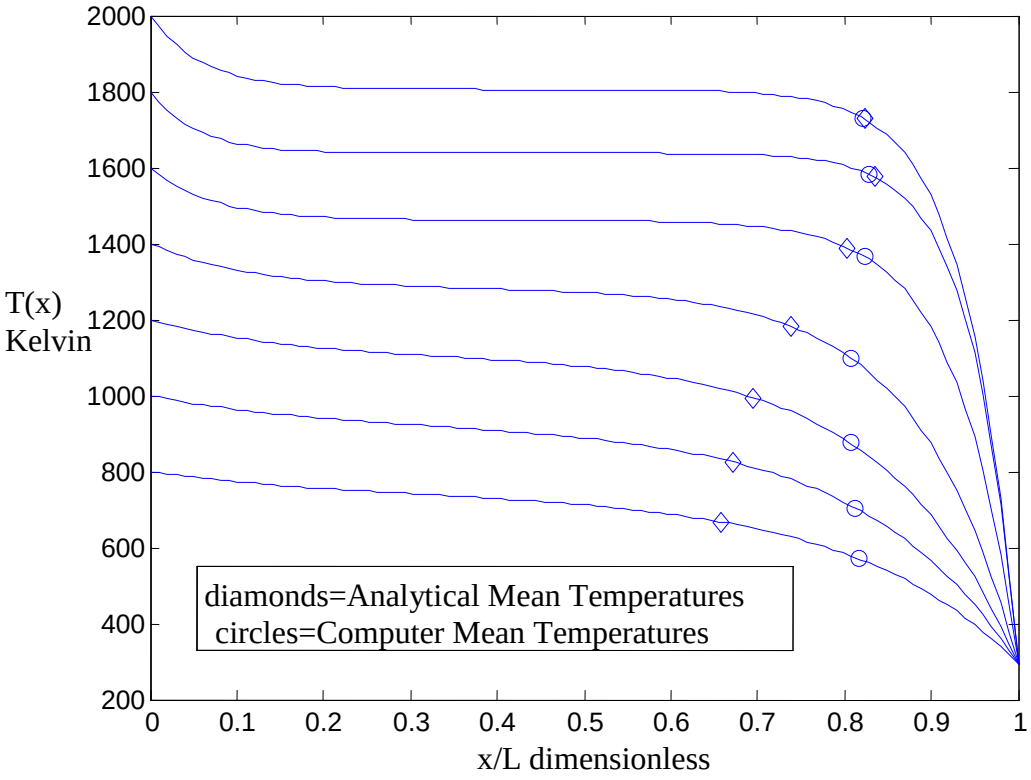


Figure 4.24: Half Nusselt Number Composite Correction for the Sheath (High Temperatures)

Table 4.34: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionles x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Half Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	221.76	209.62	0.5794	0.6342	9.46
T2	405.50	284.20	266.83	0.5791	0.6345	9.57
T3	521.60	358.23	334.56	0.5788	0.6348	9.68
T4	613.58	416.95	388.44	0.5788	0.6348	9.68
T5	705.38	475.97	442.62	0.5788	0.6348	9.68
T6	801.32	538.12	500.13	0.5791	0.6345	9.57
T7	872.42	584.47	543.70	0.5797	0.6339	9.35
T8	980.33	655.66	611.38	0.5809	0.6328	8.93
T9	1340.33	903.97	857.68	0.5886	0.6277	6.64
T10	1700.33	1177.81	1146.42	0.6028	0.6238	3.48
T11	2060.33	1499.99	1494.05	0.6302	0.6336	0.54
T12	2420.33	1874.39	1876.01	0.6783	0.6776	0.10
T13	2780.33	2199.11	2188.31	0.7036	0.7084	0.68
T14	3140.33	2449.67	2428.07	0.6889	0.6972	1.20

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.35: Values of the Analytical and Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Half Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	221.76	209.62	5.47
T2	405.50	284.20	266.83	6.11
T3	521.60	358.23	334.56	6.61
T4	613.58	416.95	388.44	6.84
T5	705.38	475.97	442.62	7.01
T6	801.32	538.12	500.13	7.06
T7	872.42	584.47	543.70	6.98
T8	980.33	655.66	611.38	6.75
T9	1340.33	903.97	857.68	5.12
T10	1700.33	1177.81	1146.42	2.67
T11	2060.33	1499.99	1494.05	0.40
T12	2420.33	1874.39	1876.01	0.09
T13	2780.33	2199.11	2188.31	0.49
T14	3140.33	2449.67	2428.07	0.88

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.36: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Radiation Shield for the Half Nusselt Number Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	221.76	209.62	309.3
T2	405.50	284.20	266.83	409.3
T3	521.60	358.23	334.56	528.3
T4	613.58	416.95	388.44	623.7
T5	705.38	475.97	442.62	719.8
T6	801.32	538.12	500.13	821.6
T7	872.42	584.47	543.70	897.9
T8	980.33	655.66	611.38	1015.3
T9	1340.33	903.97	857.68	1423.1
T10	1700.33	1177.81	1146.42	1856.8
T11	2060.33	1499.99	1494.05	2309.3
T12	2420.33	1874.39	1876.01	2790.0
T13	2780.33	2199.11	2188.31	3354.1
T14	3140.33	2449.67	2428.07	3943.4

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

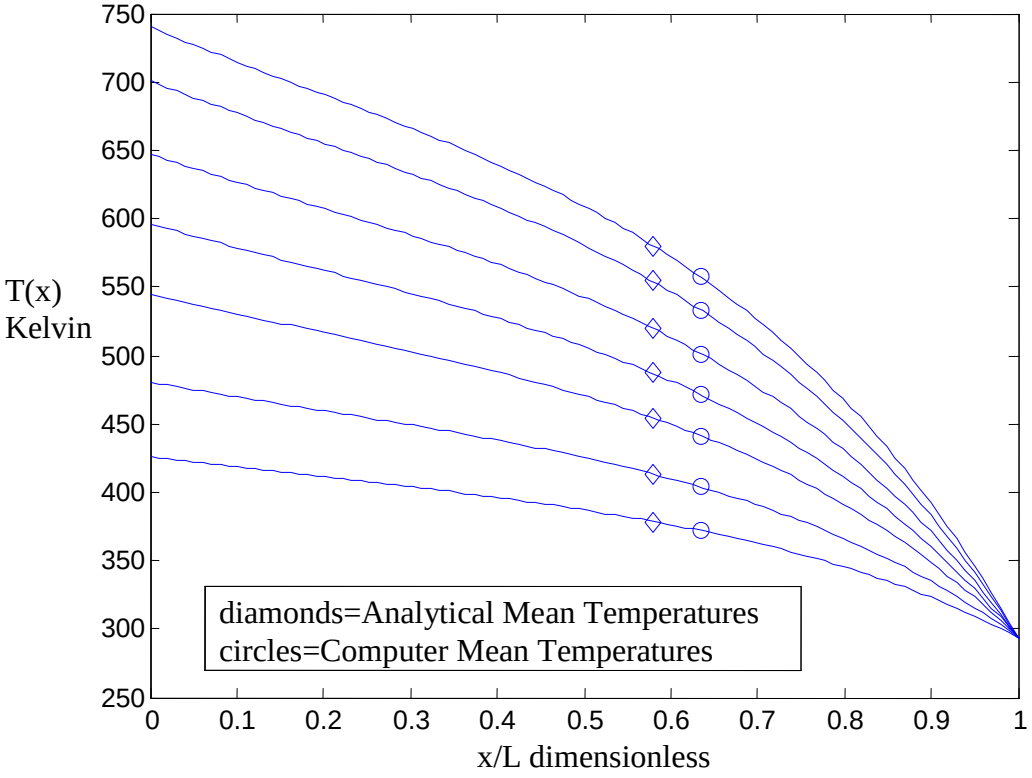


Figure 4.25: Half Nusselt Number Composite Correction for the Radiation Shield (Low Temperatures)

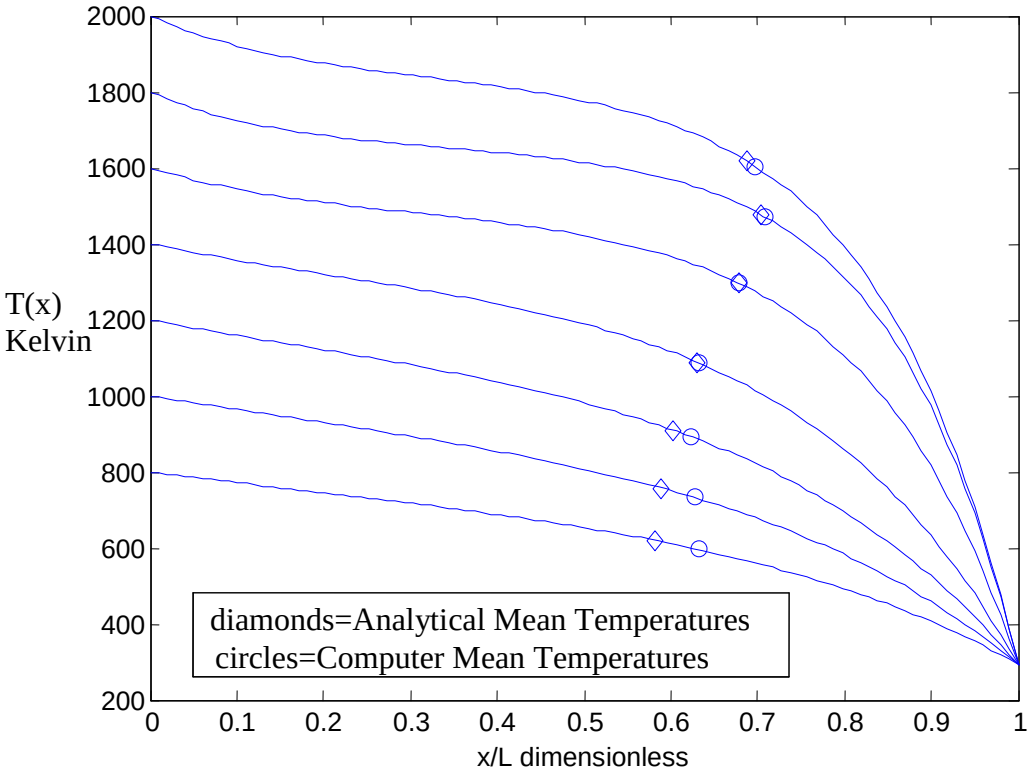


Figure 4.26: Half Nusselt Number Composite Correction for the Radiation Shield (High Temperatures)

Table 4.37: Comparisons of Percent Difference Changes in Mean Temperature Location Attributed to Correction Models
Where:
(D= Difference)

Model	Rod/Sheath %D	Shield %D	Total %D	Original/Corr. Model Change Rod/Sheath	Original/Corr. Model Change Shield
0 Original	423.98	226.85	650.83	0	0
1 Anal Base Cond.	530.83	379.83	910.66	25.20	67.44
2 Cond. Avg. Base Cond.	190.97	17.43	208.40	- 54.96	- 92.32
3 Double Tip Conv.	492.48	288.33	780.81	16.16	27.10
4 Half Tip Conv.	412.02	204.51	616.53	- 2.82	- 9.85
5 Double Comp.	264.54	28.11	292.65	- 37.61	- 87.60
6 Half Comp.	254.10	88.56	342.66	- 40.07	- 60.96

Note: Minus means percent decrease in the difference.

Table 4.38: The Minimum Percent Total Differernce in the Mean Temperature Location Attributed to Selected Correction Models for the Sheath and Shield

Min%Diff. Mean Temp. Location	Sheath	Radiation Shield
Cond Avg. Base Cond.	190.97	**----------**
Cond Avg. Base Cond.	**----------**	17.43
Total Difference		**208.40**

Table 4.39: Comparisons of Percent Difference Changes in Mean Temperature Value Attributed to Correction Models
Where:
(D= Difference)

Model	Rod/Sheath %D	Shield %D	Total %D	Original/Corr. Model Change Sheath	Original/Corr. Model Change Shield
0 Original	73.23	133.23	206.46	0	0
1 Anal Base Cond.	562.41	290.27	852.68	668.00	118.00
2Cond. Avg. Base Cond.	179.87	10.17	190.04	145.62	- 92.37
3 Double Tip Conv.	103.03	159.31	262.34	40.69	19.58
4 Half Tip Conv.	59.09	124.46	183.55	- 19.31	- 6.58
5 Double Comp.	145.67	17.83	163.50	98.92	- 86.62
6 Half Comp.	237.94	62.48	300.42	225.00	- 53.10

Note: Minus means percent decrease in the difference.

Table 4.40: The Minimum Percent Difference in the Mean Temperature Value Attributed to Selected Correction Models for the Sheath and Shield

Min%Diff. Mean Temp. Value	Sheath	Radiation Shield
Original Model	73.23	----------
Cond. Avg. Base Cond.	----------	10.17
Total Differnce		**83.40**

Note: The Half Nusselt Number Model has been discarded because of the non – convergence of the first calculation and recalculation of the recovery temperature.

Table 4.41: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Difference of the Dimensionles x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the Half Nusselt – Sheath, Double Nusselt - Shield Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff.
T1	307.22	258.44	207.36	0.6746	0.8240	22.15
T2	405.50	336.07	264.60	0.6743	0.8230	22.05
T3	521.60	428.36	333.12	0.6743	0.8220	21.90
T4	613.58	501.44	388.53	0.6746	0.8209	21.69
T5	705.38	574.65	445.01	0.6754	0.8197	21.37
T6	801.32	651.69	507.49	0.6766	0.8173	20.80
T7	872.42	709.21	581.97	0.6779	0.8176	20.61
T8	980.33	797.50	625.08	0.6802	0.8160	19.96
T9	1340.33	1102.03	899.80	0.6936	0.8107	16.88
T10	1700.33	1427.81	1241.46	0.7191	0.8077	12.32
T11	2060.33	1781.33	1671.71	0.7665	0.8141	6.21
T12	2420.33	2144.39	2130.35	0.8239	0.8304	0.79
T13	2780.33	2462.81	2477.03	0.8382	0.8318	0.76
T14	3126.00	2731.73	2720.57	0.8214	0.8255	0.50

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.42: Values of the Analytical and Computer Program Mean Temperatures, and Percentage Difference of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Sheath for the Half Nusselt – Sheath, Double Nusselt - Shield Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff.
T1	307.22	258.44	207.36	19.76
T2	405.50	336.07	264.60	21.27
T3	521.60	428.36	333.12	22.23
T4	613.58	501.44	388.53	22.52
T5	705.38	574.65	445.01	22.56
T6	801.32	651.69	507.49	22.13
T7	872.42	709.21	581.97	17.94
T8	980.33	797.50	625.08	21.62
T9	1340.33	1102.03	899.80	18.35
T10	1700.33	1427.81	1241.46	13.05
T11	2060.33	1781.33	1671.71	6.15
T12	2420.33	2144.39	2130.35	0.65
T13	2780.33	2462.81	2477.03	0.58
T14	3126.00	2731.73	2720.57	0.41

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.43: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Sheath for the Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	258.44	207.36	309.1
T2	405.50	336.07	264.60	408.9
T3	521.60	428.36	333.12	527.7
T4	613.58	501.44	388.53	622.8
T5	705.38	574.65	445.01	718.6
T6	801.32	651.69	507.49	819.8
T7	872.42	709.21	581.97	895.7
T8	980.33	797.50	625.08	1012.4
T9	1340.33	1102.03	899.80	1416.2
T10	1700.33	1427.81	1241.46	1843.8
T11	2060.33	1781.33	1671.71	2288.7
T12	2420.33	2144.39	2130.35	2760.7
T13	2780.33	2462.81	2477.03	3311.9
T14	3126.00	2731.73	2720.57	3934.9

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

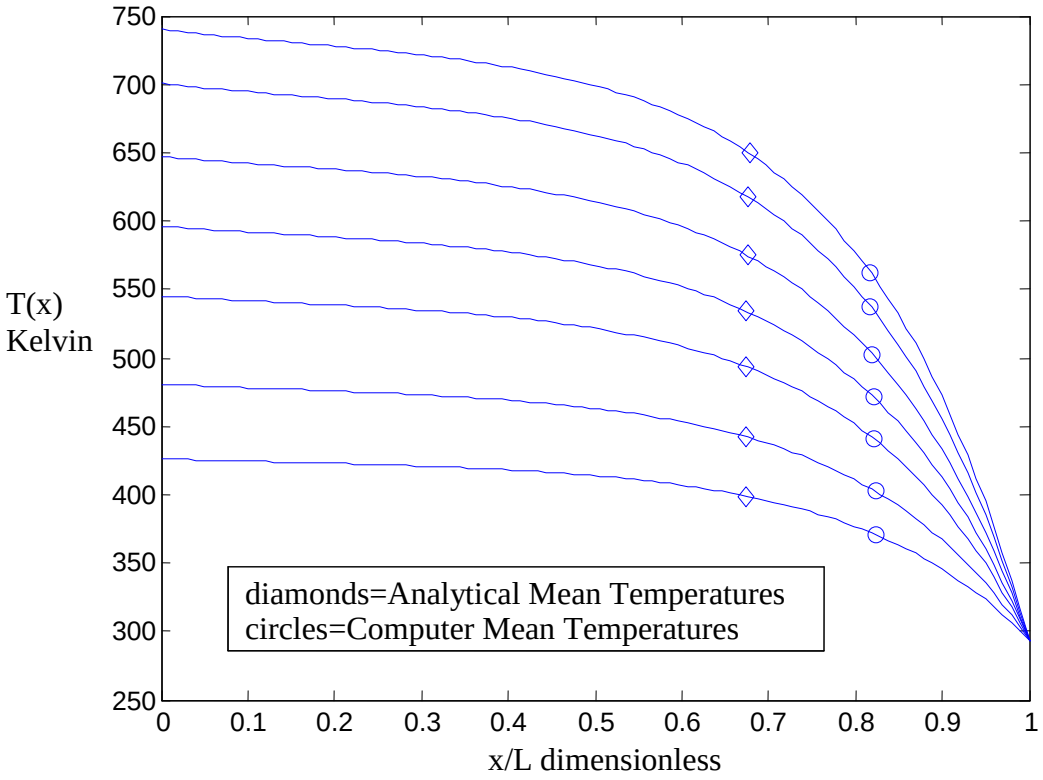


Figure 4.27: Half Nusselt – Sheath, Double Nusselt - Shield Composite Correction for the Sheath (Low Temperatures)

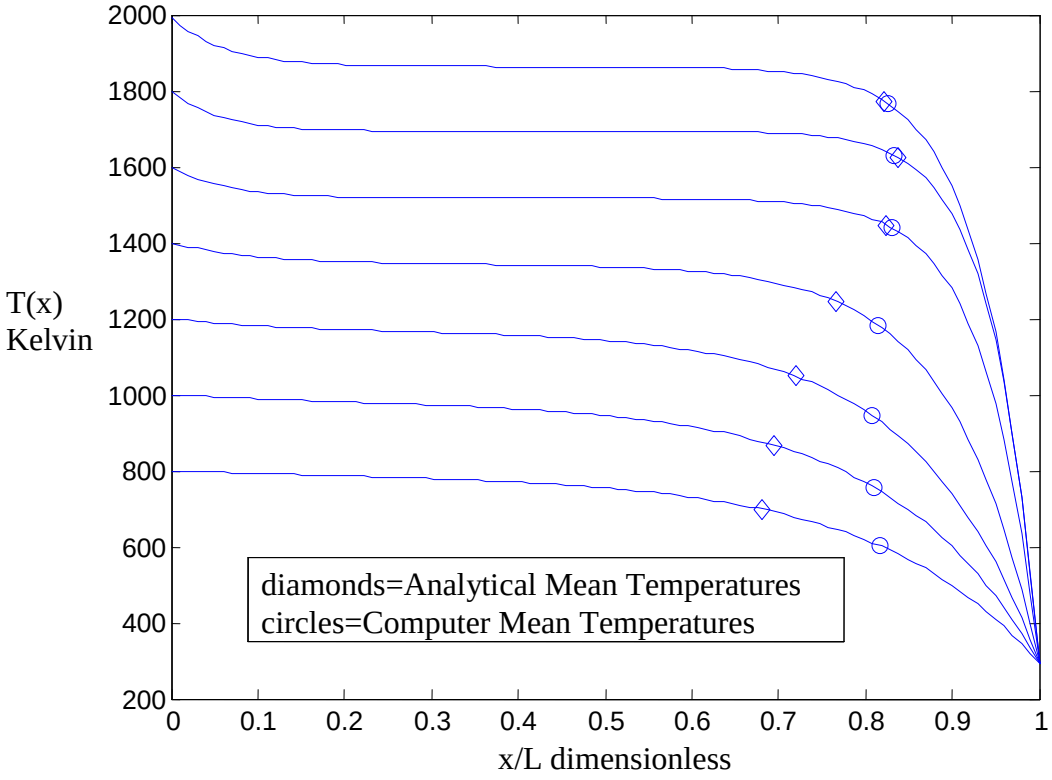


Figure 4.28: Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction for the Sheath (High Temperatures)

Table 4.44: Values of the Analytical and Computer Program Mean Temperatures, Analytical and Computer Program Dimensionless x – Coordinates, and Percentage Differences of the Dimensionless x – Coordinates between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Half Nusselt – Sheath, Double Nusselt - Shield Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	xcoord [A]	xcoord [C]	% Diff
T1	307.22	230.56	237.11	0.5967	0.5665	5.06
T2	405.50	296.58	305.67	0.5959	0.5662	4.98
T3	521.60	374.68	386.83	0.5956	0.5662	4.94
T4	613.58	436.73	451.18	0.5953	0.5662	4.89
T5	705.38	498.96	516.02	0.5953	0.5659	4.94
T6	801.32	564.28	584.62	0.5959	0.5654	5.11
T7	872.42	613.15	636.08	0.5965	0.5652	5.25
T8	980.33	688.33	715.96	0.5978	0.5644	5.59
T9	1340.33	950.25	999.41	0.6057	0.5625	7.13
T10	1700.33	1240.36	1318.08	0.6216	0.5667	8.83
T11	2060.33	1577.93	1674.59	0.6526	0.5928	9.16
T12	2420.33	1934.69	2027.57	0.6964	0.6428	7.70
T13	2780.33	2222.87	2318.99	0.7053	0.6557	7.03
T14	3126.00	2448.59	2560.19	0.6854	0.6360	7.21

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.45: Values of the Analytical and Computer Program Mean Temperatures, and Percentage Differences of the Mean Temperatures between the Analytical Model and Computer Program at the Given Tip Temperatures of the Radiation Shield for the Half Nusselt – Sheath, Double Nusselt - Shield Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)
(Diff. = Difference)

	Tip T	Mean T [A]	Mean T [C]	% Diff
T1	307.22	230.56	237.11	2.84
T2	405.50	296.58	305.67	3.06
T3	521.60	374.68	386.83	3.24
T4	613.58	436.73	451.18	3.31
T5	705.38	498.96	516.02	3.42
T6	801.32	564.28	584.62	3.60
T7	872.42	613.15	636.08	3.74
T8	980.33	688.33	715.96	4.01
T9	1340.33	950.25	999.41	5.17
T10	1700.33	1240.36	1318.08	6.27
T11	2060.33	1577.93	1674.59	6.13
T12	2420.33	1934.69	2027.57	4.80
T13	2780.33	2222.87	2318.99	4.32
T14	3126.00	2448.59	2560.19	4.56

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

Table 4.46: Values of the Analytical and Computer Program Mean Temperatures, with the Total Stagnation Gas Temperatures at the Given Tip Temperatures of the Radiation Shield for the Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction Model.
Where:
(T = Temperature)
([A] = Analytical)
([C] = Computer)

	Tip T	Mean T [A]	Mean T [C]	Stagnation T
T1	307.22	230.56	237.11	309.1
T2	405.50	296.58	305.67	408.9
T3	521.60	374.68	386.83	527.7
T4	613.58	436.73	451.18	622.8
T5	705.38	498.96	516.02	718.6
T6	801.32	564.28	584.62	819.8
T7	872.42	613.15	636.08	895.7
T8	980.33	688.33	715.96	1012.4
T9	1340.33	950.25	999.41	1416.2
T10	1700.33	1240.36	1318.08	1843.8
T11	2060.33	1577.93	1674.59	2288.7
T12	2420.33	1934.69	2027.57	2760.7
T13	2780.33	2222.87	2318.99	3311.9
T14	3140.33	2448.59	2560.19	3934.9

Note A: All Temperature Values are in degrees Fahrenheit
Note B: Temperature at the base is 69.19 degrees Fahrenheit for all Tip Temperatures

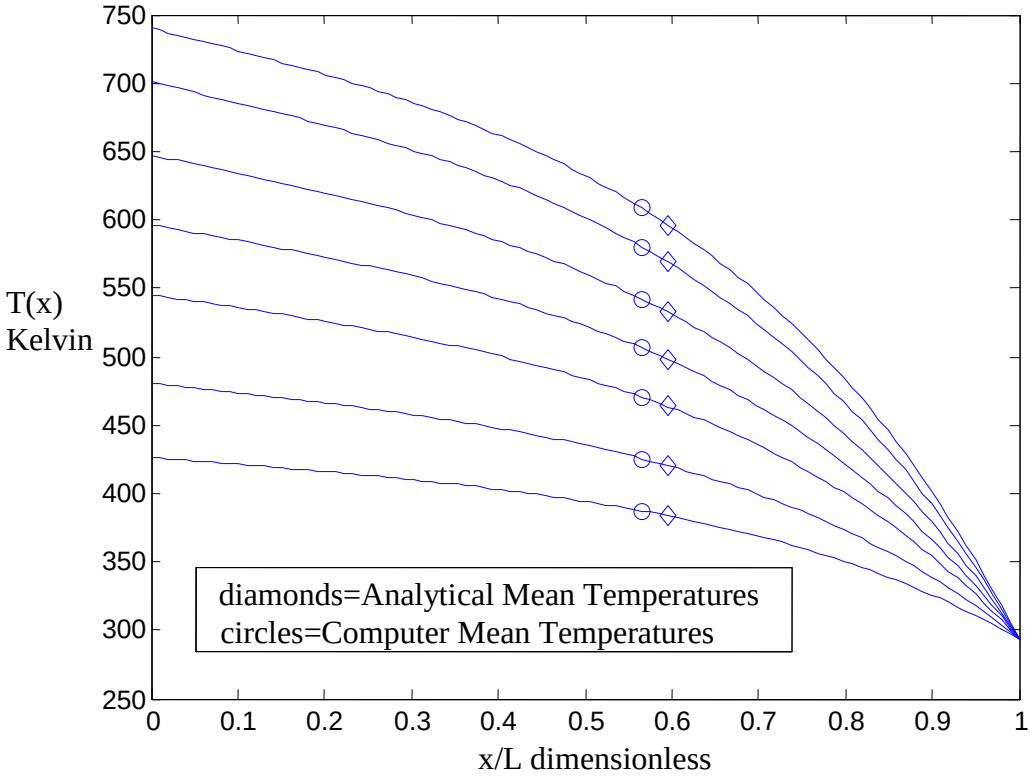


Figure 4.29: Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction for the Radiation Shield (Low Temperatures)

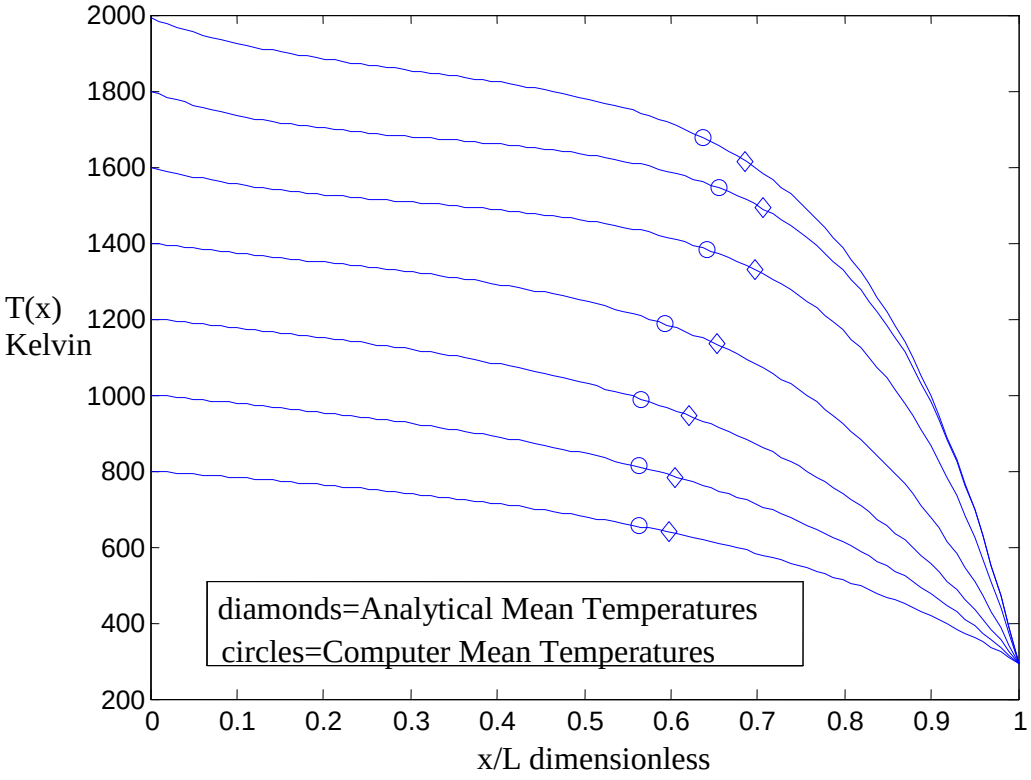


Figure 4.30: Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction for the Radiation Shield (HighTemperatures)

Table 4.47: Comparisons of Percent Difference Changes in Mean Temperature Location Attributed to Correction Models with the Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction Added
Where:
(D= Difference)

Model	Rod/Sheath %D	Shield %D	Total %D	Original/Corr. Model Change Sheath	Original/Corr. Model Change Shield
0 Original	423.98	226.85	650.83	0	0
1 Anal Base Cond.	530.83	379.83	910.66	25.20	67.44
2 Cond. Avg. Base Cond.	190.97	17.43	208.40	- 54.96	- 92.32
3 Double Tip Conv.	492.48	288.33	780.81	16.16	27.10
4 Half Tip Conv.	412.02	204.51	616.53	- 2.82	- 9.85
5 Double Comp.	264.54	28.11	292.65	- 37.61	- 87.60
6 Half Comp.	254.10	88.56	342.66	- 40.07	- 60.96
7 Half NU – Sheath Doub.NU-Shield	207.99	87.82	295.81	- 50.94	- 61.29

Note: Minus means percent decrease in the difference.

Table 4.48: The Minimum Percent Total Difference in the Mean Temperature Location Attributed to Selected Correction Models for the Sheath and Shield with the Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction Added

Min%Diff. Mean Temp. Location	Sheath	Radiation Shield
Cond Avg. Base Cond.	101.42	----------
Cond Avg. Base Cond.	----------	6.90
Total Difference		**108.32**

Table 4.49: Comparisons of Percent Difference Changes in Mean Temperature Value Attributed to Correction Models with the Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction Added
Where:
(D= Difference)

Model	Rod/Sheath %D	Shield %D	Total %D	Original/Corr. Model Change Sheath	Original/Corr. Model Change Shield
0 Original	73.23	133.23	206.46	0	0
1 Anal Base Cond.	562.41	290.27	852.68	668.00	118.00
2 Cond. Avg. Base Cond.	179.87	10.17	190.04	145.62	- 92.37
3 Double Tip Conv.	103.03	159.31	262.34	40.69	19.58
4 Half Tip Conv.	59.09	124.46	183.55	- 19.31	- 6.58
5 Double Comp.	145.67	17.83	163.50	98.92	- 86.62
6 Half Comp.	237.94	62.48	300.42	225.00	- 53.10
7 Half NU – Sheath/ Doub.NU-Shield	209.22	58.47	267.69	185.70	-56.11

Note: Minus means percent decrease in the difference.

Table 4.50: The Minimum Percent Difference in the Mean Temperature Value Attributed to Selected Correction Models for the Sheath and Shield with the Half Nusselt – Sheath, Double Nusselt – Shield Composite Correction Added

Min%Difference Mean Temp. Value	Sheath	Radiation Shield
Original Model	73.23	----------
Cond. Avg. Base Cond.	----------	10.17
Total Difference		**83.40**

Note: The Half Nusselt Number Model has been discarded because of the non – convergence of the first calculation and recalculation of the recovery temperature.

Table 4.51: Comparisons of Total Percent Difference Change in Mean Temperature Location and Mean temperature Value Attributed to Correction Models
Where:
(D= Difference)

Model	Total Sheath %D	Total Shield %D
0 Original	497.21	360.08
1 Anal Base Cond.	1093.24	670.10
2 Cond. Avg. Base Cond.	370.84	27.60
3 Double Tip Conv.	595.51	447.64
4 Half Tip Conv.	471.11	328.97
5 Double Comp.	410.21	45.94
6 Half Comp.	492.04	151.04
7 Half NU – Sheath Doub.NU-Shield	417.21	146.29

Table 4.52: Total Stagnation Gas Temperatures for the Original Model and the Seven Correction Models for the Sheath and Shield
Where:
(T=Temperature)
(OM = Original Model)
(CM(1) = Analytical Base Conduction Model)
(CM(2) = Conduction - Average Base Conduction Model)
(CM(3) = Double Tip Convection Model)
(CM(4) = Half Tip Convection Model)
(CM(5) = Double Composite Model)
(CM(6) = Half Composite Model)
(CM(7) = Half Nusselt (Sheath) – Double Nusselt(Shield) Composite Model)

Tip T	OM	CM(1)	CM(2)	CM(3)	CM(4)	CM(5)	CM(6)	CM(7)
307.22	308.2	308.6	308.4	307.8	308.9	307.9	309.3	309.1
405.50	407.1	407.9	407.5	406.4	408.6	406.6	409.3	408.9
521.60	524.4	525.7	525.1	523.1	527.1	523.4	528.3	527.7
613.58	617.8	619.7	618.7	615.8	621.8	616.2	623.7	622.8
705.38	711.3	714.1	712.7	708.4	717.1	709.0	719.8	718.6
801.32	809.6	813.5	811.5	805.5	817.7	806.4	821.6	819.8
872.42	882.8	887.7	885.2	877.6	892.9	878.7	897.9	895.7
980.33	994.5	1001.3	997.9	987.4	1008.2	988.9	1015.3	1012.4
1340.33	1373.3	1391.1	1382.3	1356.6	1403.8	1360.7	1423.1	1416.2
1700.33	1762.5	1801.6	1782.1	1731.5	1815.5	1740.3	1856.8	1843.8
2060.33	2165.8	2233.6	2196.4	2114.6	2248.5	2128.5	2309.0	2288.7
2420.33	2597.6	2679.7	2629.8	2513.3	2731.5	2528.7	2790.0	2760.7
2780.33	3077.3	3145.5	3107.7	2937.8	3298.9	2952.9	3354.1	3311.9
3140.33	3617.7	3681.1	3648.2	3396.0	3871.0	3411.5	3943.4	3934.5

Note: Temperatures are in degrees Fahrenheit

Table 4.53: Total Stagnation Gas Temperatures Percent Differences from the Original Model to each of the Seven Correction Models for the Sheath and Shield Where:
(T=Temperature)
(OM = Original Model)
(CM(1) = Analytical Base Conduction Model)
(CM(2) = Conduction - Average Base Conduction Model)
(CM(3) = Double Tip Convection Model)
(CM(4) = Half Tip Convection Model)
(CM(5) = Double Composite Model)
(CM(6) = Half Composite Model)
(CM(7) = Half Nusselt (Sheath) – Double Nusselt(Shield) Composite Model)

Tip T	OM	CM(1)	CM(2)	CM(3)	CM(4)	CM(5)	CM(6)	CM(7)
307.22	0	0.13	0.06	- 0.13	0.23	- 0.10	0.36	0.29
405.50	0	0.20	0.10	- 0.17	0.37	- 0.12	0.54	0.44
521.60	0	0.25	0.13	- 0.25	0.51	- 0.19	0.74	0.63
613.58	0	0.31	0.15	- 0.32	0.65	- 0.26	0.96	0.81
705.38	0	0.39	0.20	- 0.41	0.82	- 0.32	1.19	1.03
801.32	0	0.48	0.23	- 0.51	1.00	- 0.40	1.48	1.26
872.42	0	0.56	0.27	- 0.59	1.13	- 0.46	1.71	1.46
980.33	0	0.68	0.34	- 0.71	1.38	- 0.56	2.09	1.80
1340.33	0	1.29	0.66	- 1.22	2.22	- 0.92	3.63	3.12
1700.33	0	2.22	1.11	- 1.76	3.01	- 1.26	5.35	4.61
2060.33	0	3.13	1.41	- 2.36	3.82	- 1.72	6.61	5.67
2420.33	0	3.16	1.24	- 3.25	5.15	- 2.65	7.41	6.28
2780.33	0	2.22	0.99	- 4.53	7.20	- 4.04	8.99	7.62
3140.33	0	1.75	0.84	- 6.13	7.00	- 5.70	9.00	8.76

Note A: Temperatures are in degrees Fahrenheit
Note B: Minus means percent decrease in the difference.

CHAPTER 5

CONCLUSIONS, RECOMMENDATIONS AND FURTHER STUDY

In the previous chapter, seven heat transfer corrections models were made in order to evaluate correction procedures for the numerical prediction of mean temperature locations and values for both the sheath and shield for a self – aspirated, radiation – shielded thermocouple probe. The purpose of the study was to minimize the total difference values between the mean temperatures and where they fall on the analytical temperature distributions predicted for the TC sheath and radiation shield. Also, comparisons between the calculation of the total stagnation gas temperatures were made between the original and the seven heat transfer corrected models. This chapter states the conclusions from the results of this study. Also, recommendations and future studies are explained later in this chapter.

5.1 Conclusions of the Thermocouple Probe Research Study

Based on the results of Chapter 4, it was seen that a single correction model applied to both the sheath and the shield did not work. From comparing the analytical conduction model and the conduction, average heat flux model, it was shown that the average base heat flux model was far better than the analytical conduction model especially for the shield. From comparing the tip convection, double Nusselt number model to the tip convection, half Nusselt number model, the tip convection, half Nusselt number model came out better than the tip convection, double Nusselt number model for the sheath, but the tip convection double Nusselt number came out better for the shield. The effect of the base heat loss correction had the largest and most effect for the radiation shield at high and low temperatures, but only at very high temperatures for the sheath. It does not appear possible to decide the best single correction model for the sheath, based on minimizing the total percent reduction in difference between numerical and analytical mean temperature values and locations.

From the last work done and results presented in Chapter 4, comparisons between the calculated gas total stagnation temperature were made between the original model and each of the correction models. The stagnation temperatures predicted varied significantly with the correction model, especially in the high temperature regime. The percent changes in the gas stagnation temperature varied from a minimum of 0.19 percent (absolute value) based on CM(3) at low temperatures to a maximum value of 25.57 percent (absolute value) for CM(6) at the highest temperature. CM(3) and CM(5) resulted in reduced numerical values predicted for the gas stagnation temperatures across the entire temperature range. The base heat loss conduction – averaged correction model,

CM(2), which appeared to produce the smallest overall total differences in mean temperature and mean temperature location, resulted in a predicted gas stagnation temperature that was only about 0.8 percent higher than the original uncorrected numerical model, at worst (at the highest temperature). Thus it is felt that the values of gas stagnation temperature that the numerical program TC – CORR predicts will not change significantly, when a final composite scheme for analytical corrections is found and applied to the numerical model.

Finally, in analyzing the data in the various tables and in Table 4.51 in particular, for all of the correction models, it appears that:

1.) The conduction – averaged base heat loss model works best for the sheath and the shield in the low temperature regime.
2.) For the shield, it worked very well across the entire temperature range.
3.) Increased base heat loss or lower tip heat transfer seems to work best for the sheath in the high temperature regime.
4.) The average conduction base heat flux correction model needs more study for the sheath in the high - temperature range.

5.2 Recommendations and Further Study

Clearly, this study needs to be continued with further refinements to the sheath and shield correction models. In addition, the sensitivity of the numerical scheme for predicting gas stagnation temperature, from measured thermocouple tip temperatures to the radiation heat transfer terms needs to be investigated as well as the convection and conduction sensitivities described in the present study. Finally, experimental verification and validation must be done to the numerical scheme capability for predicting gas stagnation temperature from measured TC tip temperatures.

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