To the Graduate Council:

I am submitting herewith a thesis written by William F. Bethmann, Jr., entitled "Correlation of the Analytical Solution with Experimental Data on the Electrolysis Potential Probe." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Engineering Science.

Carl J. Remenyik

Carl J. Remenyik, Major Professor

We have read this thesis and recommend its acceptance:

William J. Snyder

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Accepted for the Council:

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Vice Chancellor
Graduate Studies and Research

CORRELATION OF THE ANALYTICAL SOLUTION WITH EXPERIMENTAL DATA ON THE ELECTROLYSIS POTENTIAL PROBE

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

William F. Bethmann, Jr.

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I wish to express my deep appreciation to the individuals who contributed to this work. First to the members who served on my advisory committee because they were partly responsible for the technical content of this thesis. Dr. Carl J. Remenyik, who served as my advisor, was responsible for the theory and the theoretical model. Dr. William T. Snyder and Dr. John H. Forrester provided suggestions and time for the success of this work. The committee served me well and it is my desire that they are rewarded for the precious time they spent for me and my work.

I give my thanks to the Instrumentation and Controls Division of Oak Ridge National Laboratory, for the financial support and equipment necessary to make this work possible. James O. Hylton, my supervisor, added valuable suggestions related to experimental measurements and content of the thesis and was very supportive of me and this work.

Because I was employed full time during the period of my thesis work, I owe sincere thanks to my family for remaining in the background as compared to the completion of this work. My wife, Ginger, gave me all the loving patience and support that was humanly possible, and I am completely indebted to her. My two sons, Cortney and Andy, do not really understand the meaning of this work but they were very patient to sacrifice some of the time that they desired of me.

Finally, I feel compelled to acknowledge the direction that God has provided to my path during the pursuit of this study (Proverbs 3:6). I

owe my thanks to the Lord for giving me physical strength, the mental capabilities, and the opportunity to accomplish this work, and I believe that all knowledge and wisdom comes from God (Colossians 2:3).

ABSTRACT

In response to a loss of coolant (LOCA) in a water cooled nuclear reactor, the reactor vessel is reflooded with water from standby storage tanks to carry off heat that would otherwise damage fuel rods, vessel and other structures. During the latter phases of the reflooding, a mixture of steam and water flows between the structures and a flowing film of water adheres to the surfaces. To gain some knowledge about the behavior of these films and to aid heat transfer calculations in designing reactors, instruments have been developed for the measurement of film thickness, flow velocities, and mass flow rate in reactors and their simulators. The Electrolysis Potential (EP) Probe is one such instrument and is designed to measure average liquid velocity inside the films. It was discovered that if the EP Probe was excited with a constant voltage of sufficient magnitude to produce electrolysis, the current through the sensor decreased in a repeatable manner with increasing average film velocity.

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A	Cross-sectional area of flow
$A(\delta)$	Function dependent upon film thickness
A_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are integer values from zero to infinity
$B(\delta)$	Function dependent upon film thickness
B_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are interger values from zero to infinity
c_j	Bulk concentration of species j at the cathode
$c_{j\infty}$	Bulk concentration of species j at infinity
C_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are integer values from zero to infinity
D_j	Diffusion coefficient of species j
D_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are integer values from zero to infinity
E_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are integer values from zero to infinity
$\vec{f}$	Body force vector
F	Faraday's constant
F_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are integer values from zero to infinity
G_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are integer values from zero to infinity
h	Experimental water film thickness
H_{lmn}	Constants of Fourier Series expansion of $\gamma(\xi,\eta,\zeta)$ where l,m,n, are integer values from zero to infinity
$\vec{i}$	Unity vector in the x-direction
I	Electrolytic current
$\vec{i}_j$	Current density vector of species j
I_j	Electric current for species j

I_o	Hyperbolic Bessel Function of order zero
I_1	Hyperbolic Bessel Function of order one
I_o'	First derivation of the Hyperbolic Bessel Function of order zero
$\vec{J}_j$	Flux vector of species j
k	Arbitrary constant in differential equations
K_m	Constants in the function for the electric potential
n	Arbitrary constant in differential equations
P	Pressure of the fluid
Q	Volumetric flow rate
Q_1	Modified dimensionless coordinate defined in equation (44)
r,y,θ	Polar coordinates
R	Universal gas constant
r_o	EP Probe radius
R(r)	Function dependent upon EP Probe radius
T	Absolute water Temperature
U_{belt}	Belt velocity
U_j	Electric mobility of species j
$\vec{v}$	Velocity vector
V	Average film velocity
x,y,z	Rectangular coordinates
Y(y)	Function dependent upon film thickness
z_j	Chemical valence for the species j
β_m	Dimensionless coefficient defined in equation (37)
γ_j	Ratio of the bulk concentration at the cathode to the bulk concentration at infinity of species j
δ	Theoretical water film thickness
ε	Dielectric constant

$\Theta(\theta)$	Function dependent upon EP Probe angular dimension
κ_m	Ratio of K_m to the electric potential at the cathode
ν	Kinematic viscosity of the fluid
ξ,η,ζ	Dimensionless coordinates from transformation of x,y,z
ρ	Density of the fluid
σ	Water Conductivity
Ψ	Electric potential
Ψ_o	Electric potential at the cathode

I. INTRODUCTION

The need to measure the velocity of de-entrained condensate films in high pressure steam vessels is an integral part of the Light Water Reactor Safety Research Program conducted by the United States Nuclear Regulatory Commission (USNRC). An electrical impedance probe measurement system was developed at Oak Ridge National Laboratory (ORNL) and designed to measure two-phase flow and condensate de-entrainment in experimental facilities. These facilities can simulate various types of loss-of-coolant accidents which might occur in commercial pressurized water reactors.

The major objectives of the loss of coolant experiments are to:

1. Ensure the adequacy of the emergency cooling system;
2. Measure the thermohydraulic phenomena in the vessel during the transient caused by the accident; and,
3. Develop and verify heat transfer and flow models used in design and licensing.

Since condensate de-entrainment is a major factor in determining local heat transfer during the transient, a great deal of effort has been directed toward the development of sensors for making film thickness and velocity measurements.

II. BRIEF EP PROBE HISTORY

Basic concepts for measurement of film thickness and velocity were developed at Lehigh University by Dr. John Chen and co-workers. They discovered that placing two wires with an electric potential of sufficient magnitude into flowing water to produce electrolysis behaved in such a manner that the measured current through the water decreased with an increased water velocity. The electrolysis current was found to be dependent on water conductivity and film thickness. A simple empirical correlation between these factors showed that the EP Probe response could be described by the equation

$$\frac{I}{\sigma} = A(\delta) \cdot \exp(-B(\delta)V) \ , \qquad (1)$$

where I is the electrolytic current in milliamps for a constant applied dc potential, σ is the water conductivity across the electrodes in the absence of electrolysis expressed in microsiemens per centimeter, V is the average film velocity in centimeters per second, and $A(\delta)$ and $B(\delta)$ are functions dependent upon film thickness.

At Lehigh, an electrode design consisting of two bare parallel wires exposed to the process fluid was tested in various configurations. The electrode wires were oriented along the direction of flow as well as perpendicular to the direction of flow with different gap dimensions and diffrent electrode lengths. These configurations are shown in Figure 1.

Several observations, from Lehigh's collected data, showed that the probe's behavior is dependent upon the direction of the liquid velocity over it. Secondly, it was observed that probes oriented parallel to the

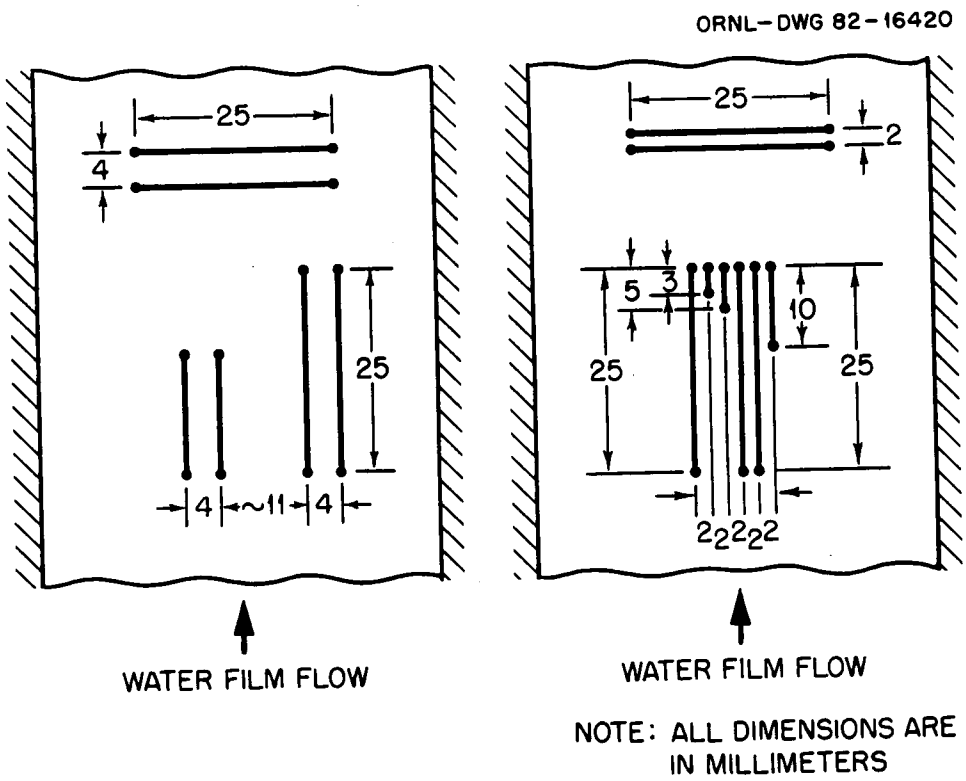

Figure 1. Lehigh University's Electrolysis Potential Probe electrode geometry configuration.

flow, having the same gap dimension, had similar current-velocity responses even though their electrode lengths differed. And thirdly, probes with the same electrode length, but greater gap dimension, showed lower sensitivity.

III. EP PROBE DESIGN AND FABRICATION

The USNRC delegated the task of further development and research on the EP Probe to ORNL. Initially, ORNL was responsible for verification of the results obtained at Lehigh. The next step was to develop an EP Probe for use in the high temperature steam-water environment encountered in reflood tests.

These instruments were required to withstand high temperatures approaching 1000 degrees Celcius and rapid temperature transients of approximately 300 degrees Celcius per second. Also, these probes must tolerate large pressure differentials and must survive the corrosive effects of superheated steam.

For ORNL to build a successful instrument, it was necessary to develop new compatible ceramic-metal materials, brazing compounds, and techniques to meet these stressful environmental conditions. Figure 2 shows the design of an EP Probe with two platinum electrodes with posts embedded flush in an electrically nonconducting material, called a cermet. This cermet is embedded in a platinum ring or collar, placed in a sensor body, and furnace brazed. After brazing, the top surface of the assembly is ground smooth removing excess brazing alloy from the cermet. This is to reduce the disturbance of the flow of the liquid film passing over it. Next, triaxial cables, end seals, an end cap, and a vent tube are assembled and furnace brazed. The vent tube provides a way to control the internal pressure during a reflood test in order to decrease the pressure differential across the sensor face. And finally, the triax leads are attached to the platinum posts and the end cap is laser welded to the sensor body. One EP Probe and three other probes of different

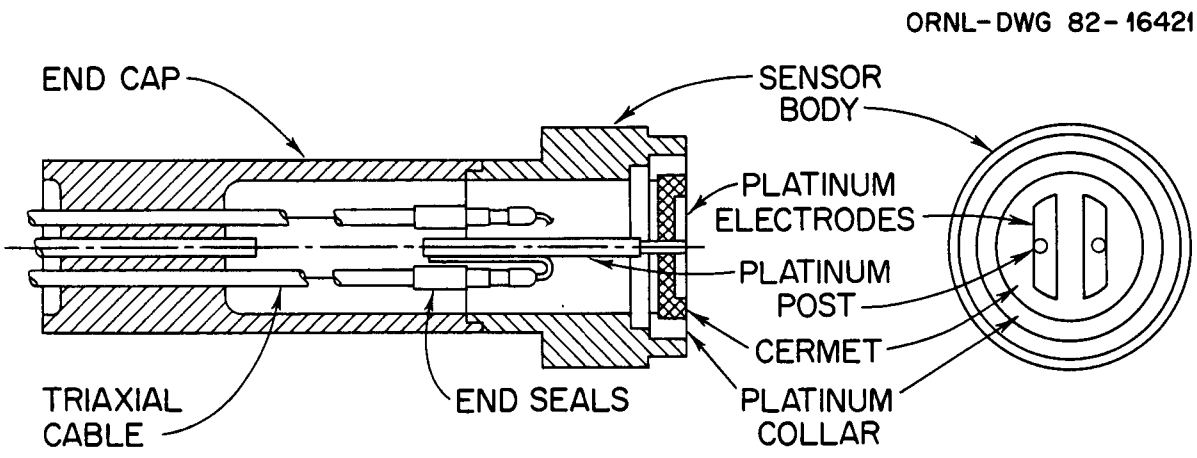

Figure 2. Oak Ridge National Laboratory's high temperature design for the Electrolysis Potential Probe.

electrode design (film thickness probes) are placed in a module housing shown in Figure 3. The housing is designed so that, if necessary, a malfunctioning probe may be removed and replaced.

One other module housing for the EP Probe is called an Incore Film Probe. The EP Probe is placed with two film thickness probes in a rod called a guide tube shown in Figure 4. The disadvantage of this design is that a malfunctioning probe cannot be replaced.

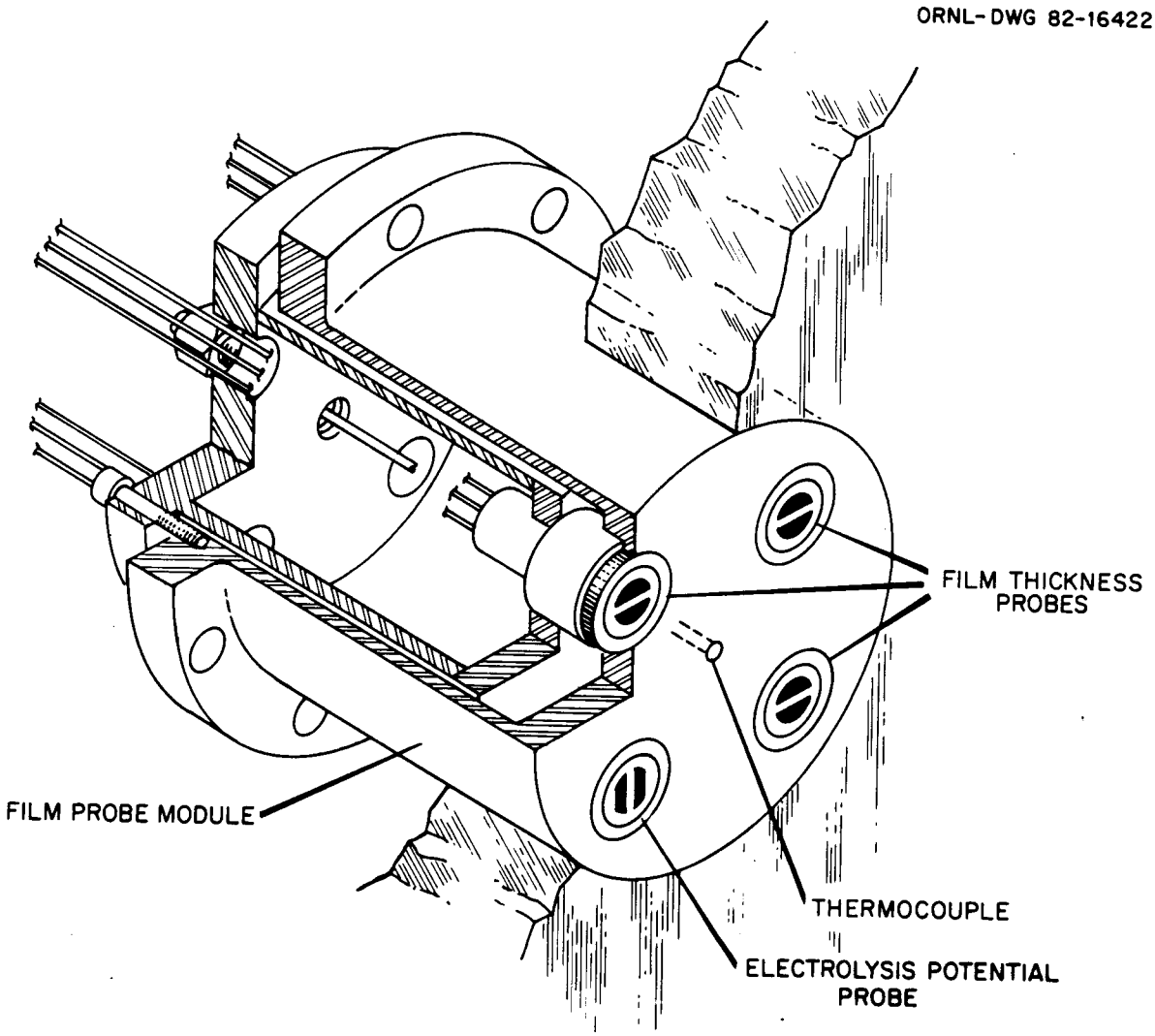

Figure 3. Film probe module housing with one EP Probe and three film thickness probes.

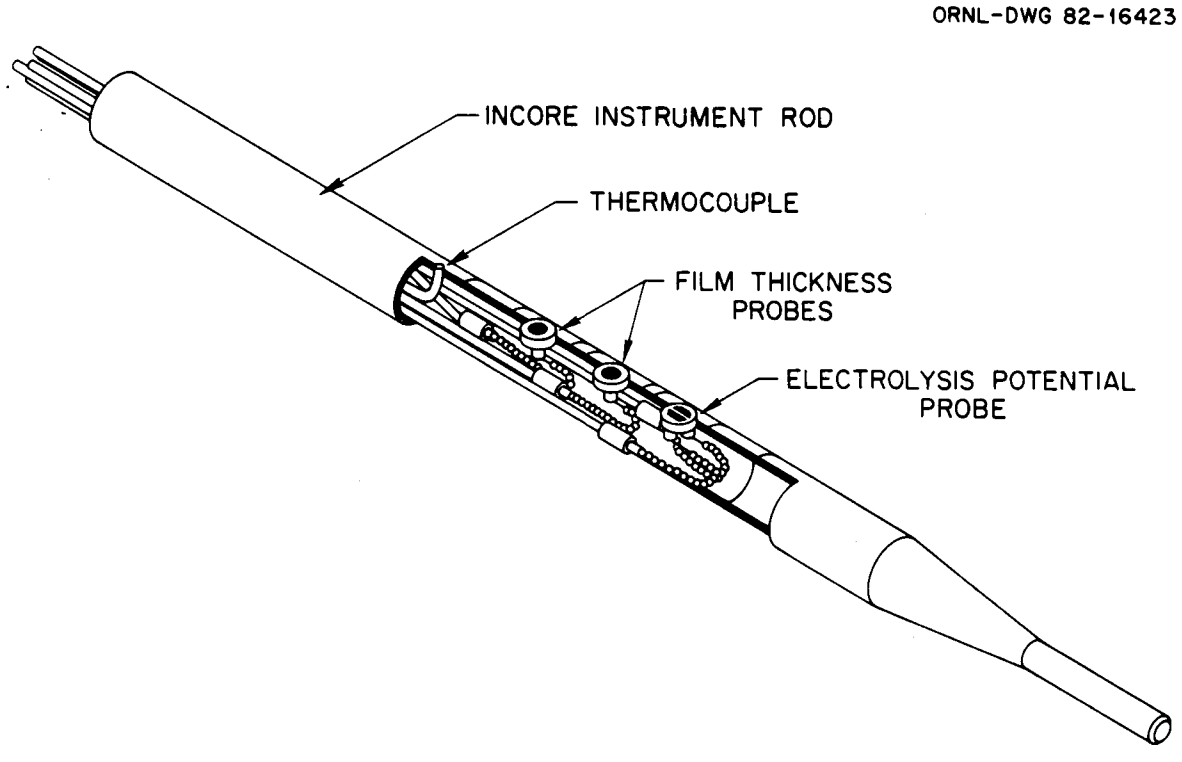

Figure 4. Incore film probe with one EP Probe and two film thickness probes.

IV. EP PROBE TESTING FACILITIES

The EP Probe has been tested in a variety of ways. One apparatus used for testing is called the EP Test Chamber shown in Figure 5. The test section, the portion housing the EP Probe, is versatile in that both a guide tube and a film probe module can be tested in an ambient temperature environment. The test loop consists of a large water reservoir where the water conductivity is monitored and held constant. Water is pumped through valves 1 and 2, used to vary the flow rate, through the rotameters obtaining a volumetric flow measurement, and then into the test section and returned to the reservoir. The test section is designed to maintain a constant film thickness for a steady state measurement. The film velocity is obtained from the flow meters and the cross-sectional area of the test section by the following relation

$$V = \frac{Q}{A} , \tag{2}$$

where V is the average film velocity in centimeters per second, Q is the volumetric flow rate in cubic centimeters per second, and A is the cross-sectional area in square centimeters. The parameters which influence the EP Probe are changed prior to a test run and then held constant during the measurement recording.

Velocity ranges for the EP Test Chamber are 5 to 250 centimeters per second while thickness measurements range between 2 and 25 millimeters. The water conductivity ranges are 5 to 100 microsiemens per centimeter.

The guide tube setup used the same geometry of pipes, valves, reservoir, etc., except for the test section. The test section shown in

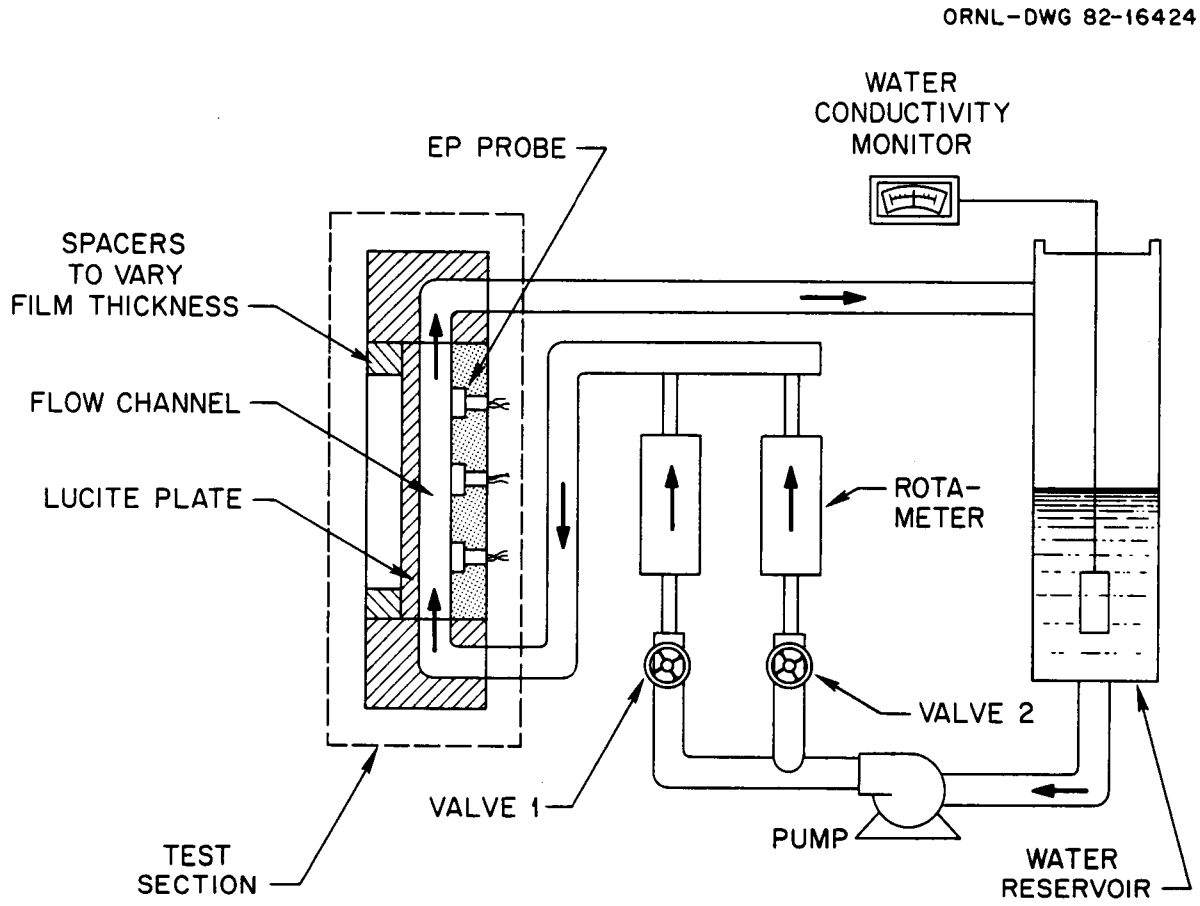

Figure 5. EP test chamber where water circulates in a loop from the reservoir to the test section.

Figure 6 was adapted to house an Incore film probe. The test section consisted of four regions in which the inner radial dimension varied such that moving the guide tube up or down produced different film thicknesses around the EP Probe. Again, steady state data is obtained by holding all measurable parameters constant.

It should be noted that data obtained from the guide tube setup was not desirable. There was great difficulty in reproducing the data because the incore rod had to be oriented perfectly in successive tests. The inner diameters of the four different regions of the tube were not completely uniform and an inaccurate placement of the sensors in these regions resulted in data that was not consistent. To complicate matters, a bending moment could be applied to the rod and the film thickness in the test region became more distorted. Therefore, this data was not used or compared to other experimental data.

Figure 7 shows another test section configuration where low velocity measurements are made in a large tube. Due to the large cross-sectional area, low flows result and data for velocities less than 20 centimeters per second were recorded.

The Film Generator in Figure 8 is the second apparatus used for testing the EP Probe. As the name implies, this device creates a liquid film to pass over the stationary module housing the EP Probe. A motor and pulley arrangement move the belt with a constant velocity. This device is placed into a water bath that sufficiently submerges the belt and sensors so the belt can pull a water film across the module face without the occurrence of air bubbles.

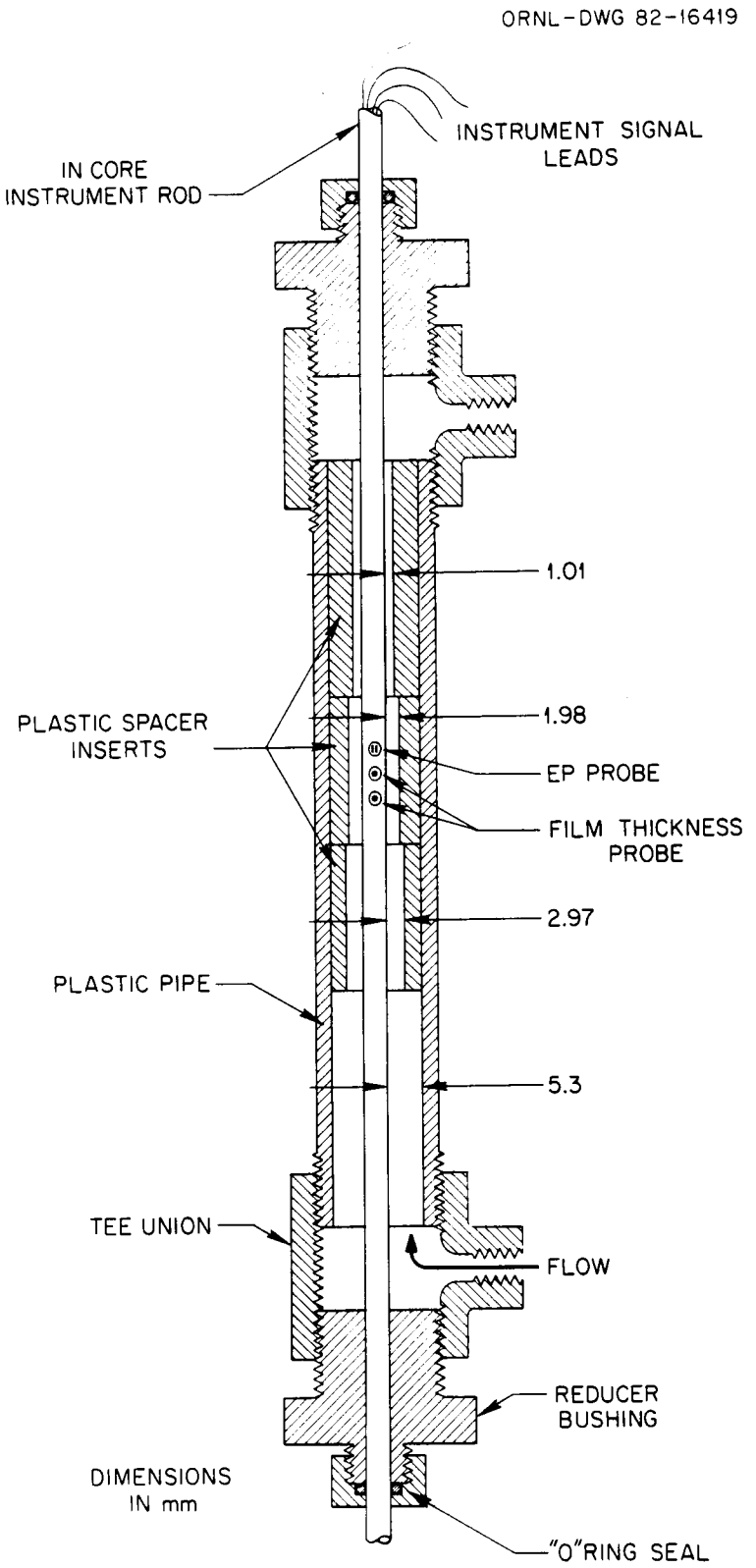

Figure 6. Guide tube test assembly with sections of different radial dimension to vary the film thickness.

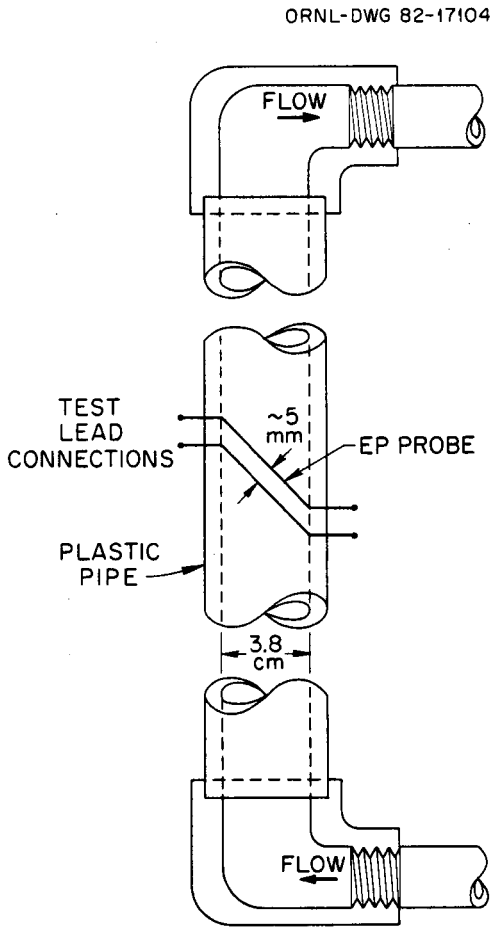

Figure 7. EP Probe low velocity configuration in a large diameter pipe.

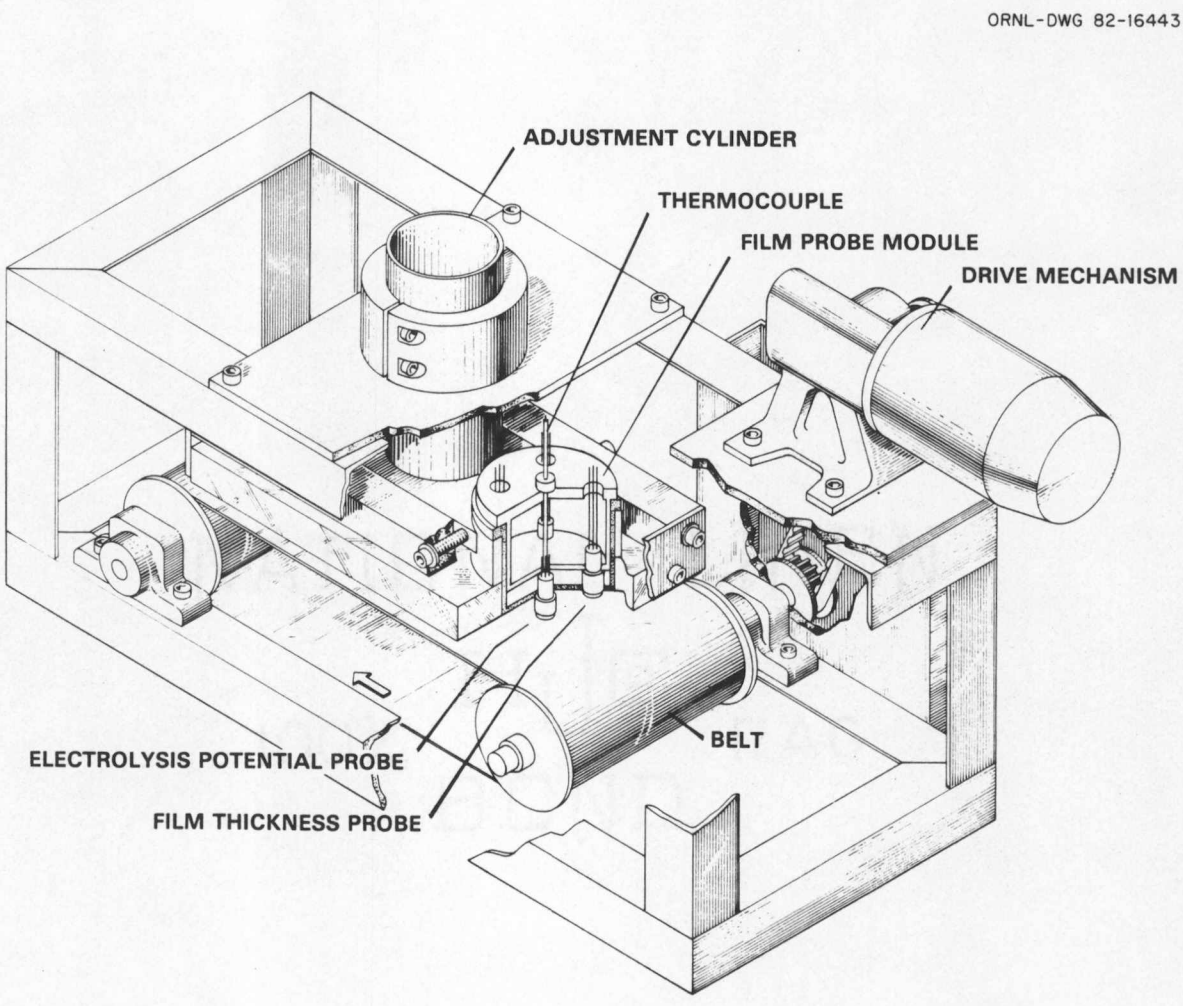

Figure 8. Film generator testing apparatus.

The conductivity of the water in the bath is constant along with the velocity of the belt during a test run. Problems with thickness measurements occur because the belt oscillates up and down and variations in belt thickness are present. Velocities in the Film Generator range from 20 to 150 centimeters per second and thickness measurements from zero to 25 millimeters are possible.

The third method for calibrating the EP Probe is the Steam-water film probe calibration chamber shown in Figure 9. Its features include capabilities of high pressures and temperatures, flow control of steam and water, adjustable nozzles for steam and water, a tilting angle to correct for accelerating and decelerating flows, viewports for visualization and photography, and micrometer measurements for film thickness. This test chamber was also used for high temperature testing to study the survivability of the ceramic to metal seal.

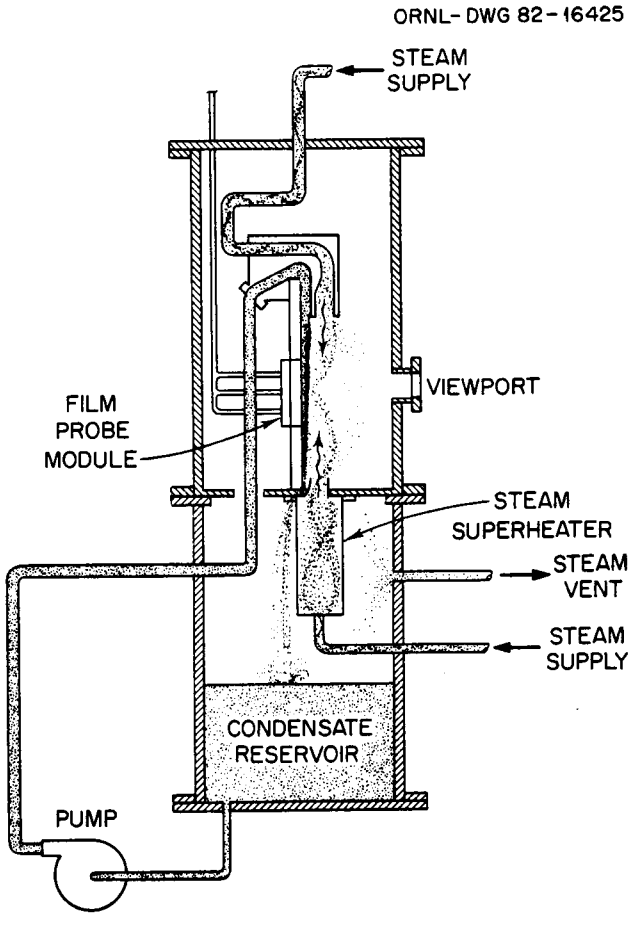

Figure 9. Steam-water film probe calibration chamber for high temperature tests.

V. PROBE EXCITATION

The simple circuit shown in Figure 10 is the excitation concept used. A voltage source of sufficient magnitude to cause electrolysis was placed across the sensor. The current flowing through the probe was measured in two different ways. First, the voltage drop measured across a 100 ohm resistor was converted to current flow. Secondly, a milliammeter was placed in the circuit to measure current directly. These two means of current measurements give a relatively accurate value for current flow through the probe.

It is noted that both measurement instruments are placed in the circuit on the negative side of the probe. It is desirable that only the electric current passing through the probe is measured and not the current supplied to the probe. Invariably, leakage current exists in the probe and a current measurement on the positive side of the probe would be invalid.

Further development on the excitation circuit resulted in a design shown in Figure 11. This circuit simply measures the voltage drop across a 100 ohm resistor and with the use of a 1 hertz relay the polarity of the sensor leads are switched. This was desirable because the sensor face would corrode with the constant polarity excitation.

The EP Probe posed another problem in which triaxial hard cables were fabricated for the sensor leads. Due to the nature of triax cables, cable capacitance plays an important role in signal conditioning and measurement. Therefore, a modification in the 1 hertz relay was made to drive the low side inner shield to the same potential as the low side conductor to cancel all cable capacitance.

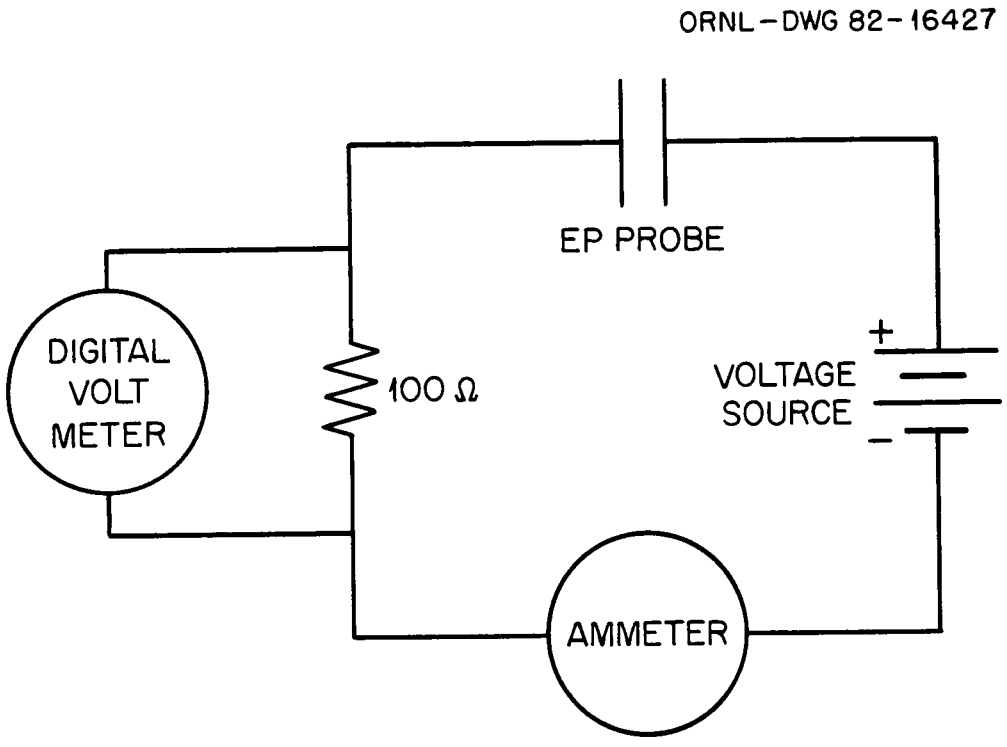

Figure 10. Simple circuit diagram for EP Probe signal measurement.

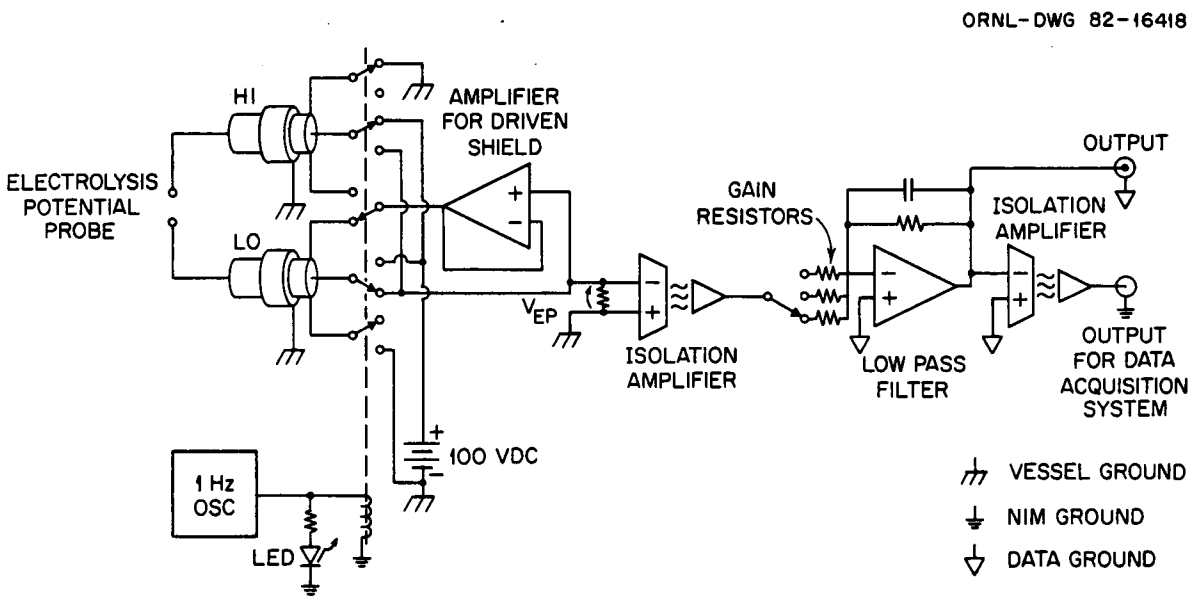

Figure 11. Developed circuit diagram for EP Probe signal measurement.

VI. EXPERIMENTAL DATA AND RESULTS

Data obtained from the low velocity testing configuration and the film generator are shown in Figure 12 and Figure 13. These figures show how different parameters cause changes in the EP Probe's current-velocity response behavior. These parameters are excitation voltage, film thickness, and water conductivity. A test run consisted of a series of data points taken through the velocity range while holding all other parameters constant.

Figure 12 shows experimental data where excitation voltage effects were seen. These curves display that the voltage amplitude must be of sufficient value to get a good current change with a velocity change. One test run used an alternating current voltage source of 100 volts with a frequency of 100 hertz. Because the sensitivity was greatly reduced, later testing was accomplished by using only a direct current voltage source.

Figure 13 shows how film thickness interacts with the current-velocity relationship. As the film thickness increases, more current is allowed to pass from one electrode to the other.

Figure 14 illustrates how the current-velocity relationship of the EP Probe varies with water conductivity. Water conductivity is a measure of the ability of water to pass current. Its value is the reciprocal of resistivity. As water becomes contaminated with more ions, the conductivity increases allowing an increase in the electric current between the electrodes.

Figure 15 shows the relationship of dividing the current by the bulk concentration of the fluid to normalize the curves shown in

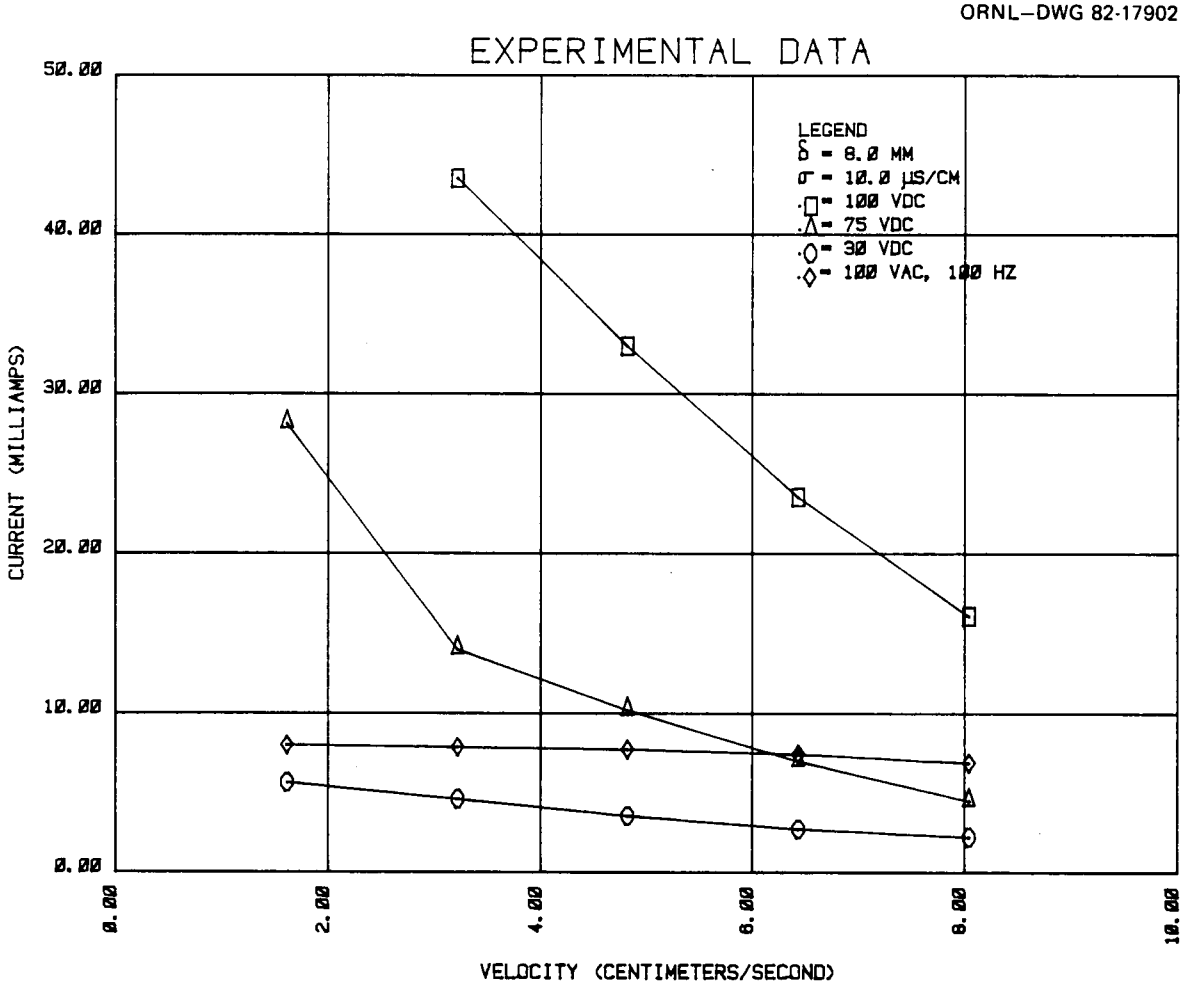

Figure 12. Current-velocity curves for different excitation voltages from experimental data.

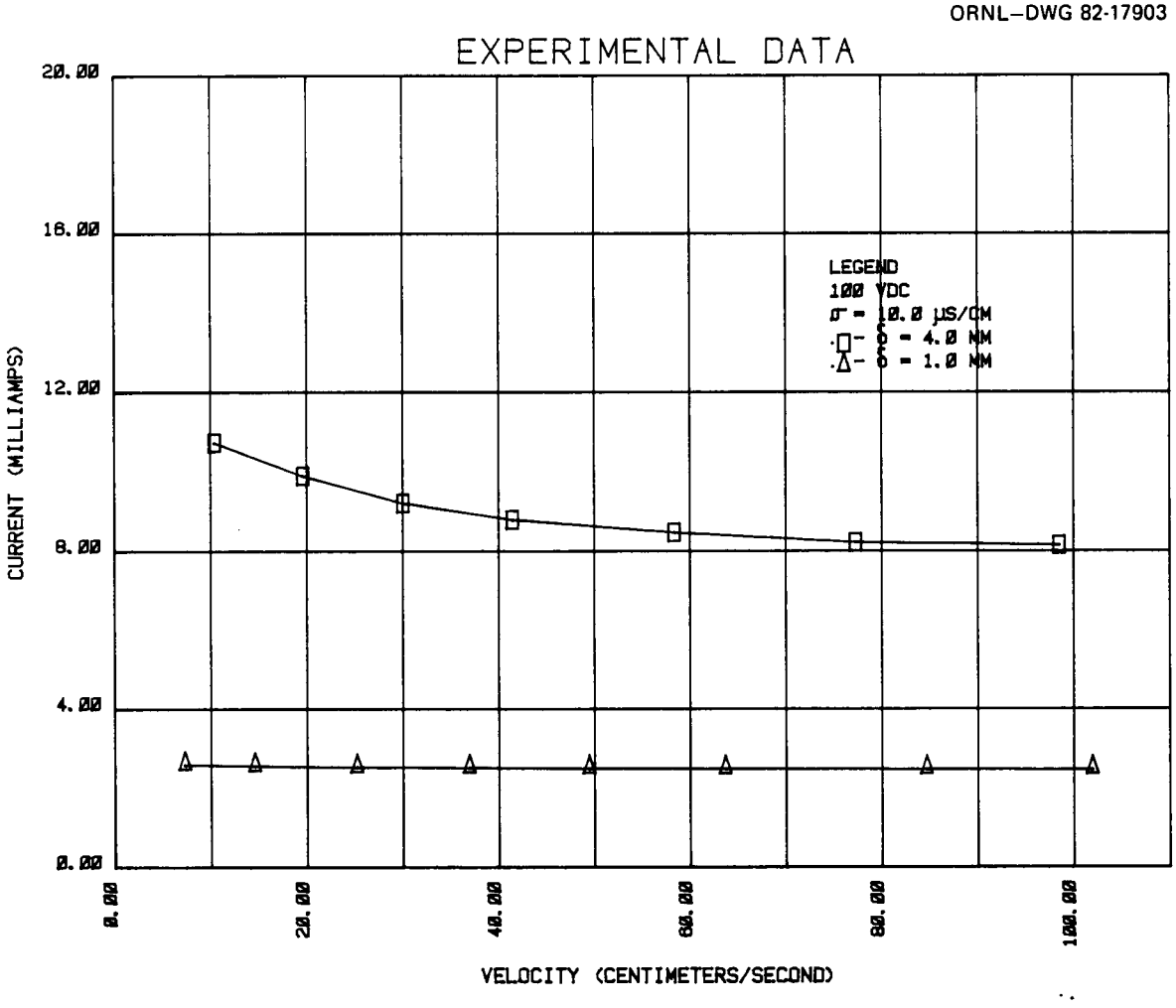

Figure 13. Current-velocity curves for different water film thicknesses at constant excitation voltage and water conductivity.

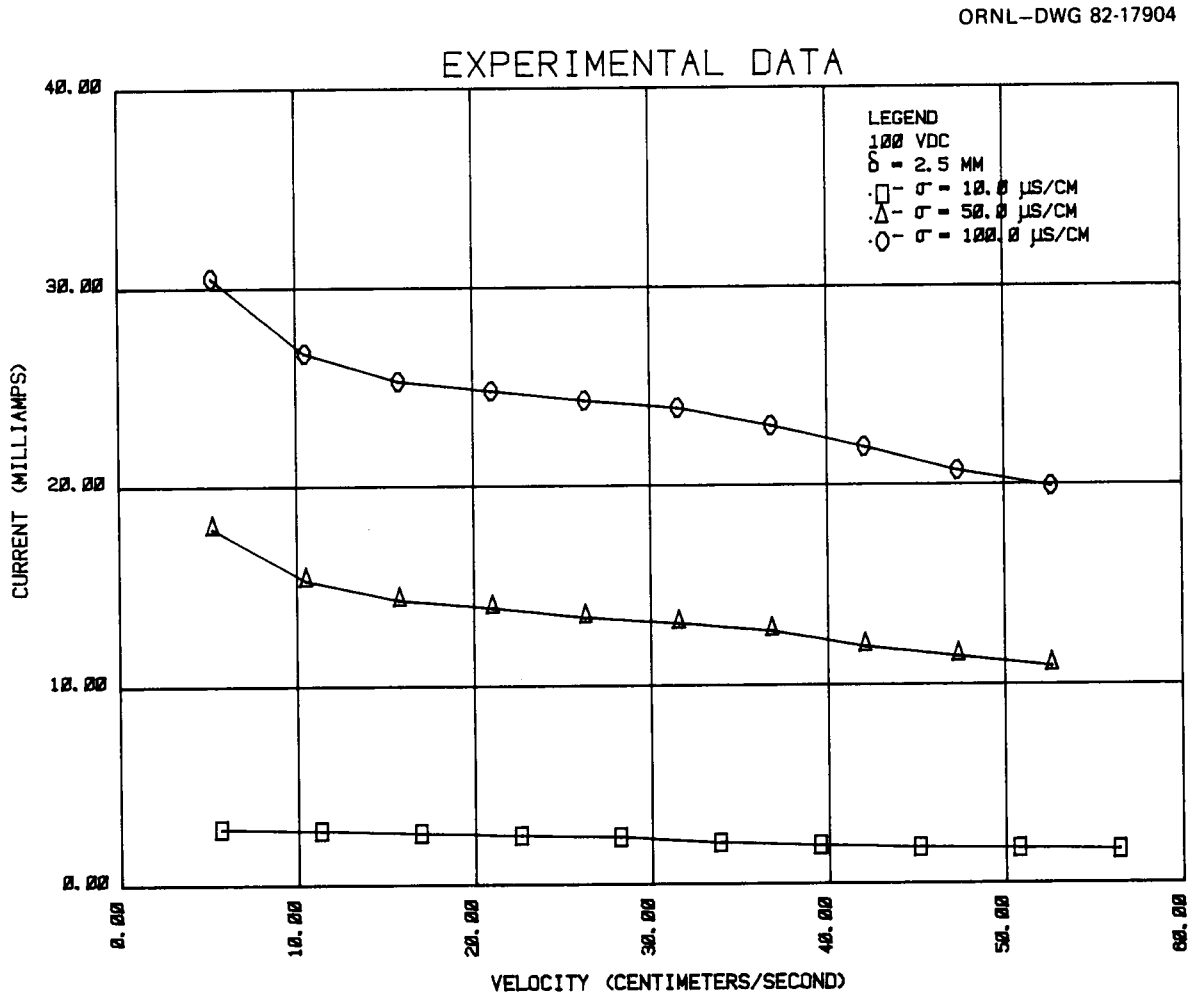

Figure 14. Current-velocity curves for different water conductivities at constant excitation voltage and water film thickness.

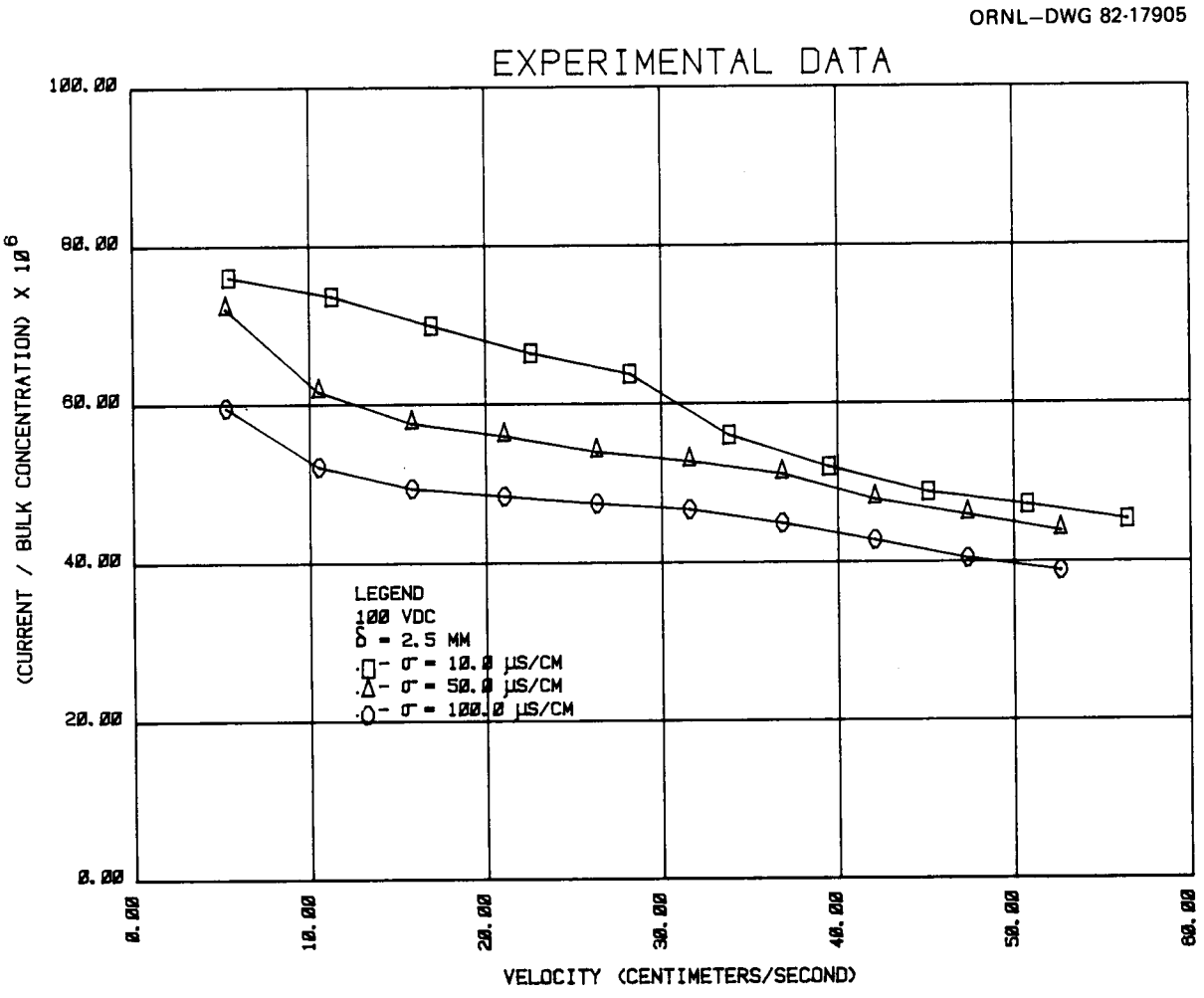

Figure 15. Normalized current-velocity curves in relation to bulk concentration and constant excitation voltage and water film thickness.

Figure 14. The values for bulk concentration were not known and assumed to be a linear function of water conductivity. The relation used to obtain the bulk concentration from water conductivity is shown in Equation (3).

$$c_{j\infty} = a + b\sigma \ , \tag{3}$$

where $c_{j\infty}$ is the bulk concentration of the fluid at infinity, σ is the water conductivity, and a and b are constants that were obtained experimentally.

VII. THEORY

It is noted that the theory for the EP Probe was not the responsibility of the author but of the personnel at ORNL. The numerical evaluation was, in part, an attempt to verify the validity of the theory and to confirm that the proposed mechanism for the current-velocity phenomenon may be valid.

To obtain an analytical solution for the EP Probe behavior in a flowing liquid, a mathematical model must be formulated. The general equations used include the continuity equation for an incompressible fluid,

$$\text{div } \vec{v} = 0 \ , \tag{4}$$

the Navier-Stokes equation of motion for an incompressible fluid,

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \text{grad } \vec{v} = -\text{grad } \frac{p}{\rho} + \nu \text{ div} \cdot (\text{grad } \vec{v}) + \frac{\vec{f}}{\rho} \ , \tag{5}$$

the continuity equation for a species j in the fluid,

$$\frac{\partial c_j}{\partial t} = \text{div} \cdot (D_j \text{ grad } c_j) - \vec{v} \cdot \text{grad } c_j - \text{div} \cdot (u_j c_j \text{ grad } \Psi) \ , \tag{6}$$

the Poisson equation,

$$\text{div} \cdot \text{grad } \Psi = \frac{-4\pi F}{\varepsilon} \sum_{j=1}^{\infty} z_j c_j \ , \tag{7}$$

and the Einstein equation, relating the electric mobility and the diffusion coefficient,

$$u_j = \frac{D_j z_j F}{RT} \ . \tag{8}$$

In applying these equations to model the EP Probe's behavior, several assumptions were made. First, the fluid flow is plane, steady and uniform Couette flow in the x-direction. Second, the liquid is electrically neutral outside a very small region at the electrodes. Third, the cathode is centered at the origin with its shape being circular and the anode located infinitely far away.

The first assumption simplifies the Navier-Stokes equation to Couette flow where the expression of flow velocity, u(y), between a stationary plane wall and a belt moving with a velocity U_{belt} at a distance, h, from the stationary wall is

$$u(y) = \frac{U_{belt}}{h} y \ . \tag{9}$$

Since the liquid is electrically neutral outside a very small region at the electrode then

$$\sum_{j=1}^{N} z_j c_j = 0 \ , \tag{10}$$

and Equation (7) is reduced to

$$\text{div} \cdot \text{grad} \ \Psi = 0 \ . \tag{11}$$

Equation (11) expressed in polar coordinates becomes

$$\nabla^2 \Psi = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0 \ , \tag{12}$$

where the solution is of the form

$$\Psi(r,\theta,y) = R(r)\Theta(\theta)Y(y) \ . \tag{13}$$

Substitution of (13) into (12) results in three ordinary differential equations with two arbitrary constants, n and k, determined by boundary conditions.

$$\frac{d^2\Theta}{d\theta^2} + n^2\Theta = 0 \ , \tag{14}$$

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(k^2 - \frac{n^2}{r^2}\right) R = 0 \ , \tag{15}$$

and

$$\frac{d^2Y}{dy^2} - k^2Y = 0 \ . \tag{16}$$

The cathode has an electric potential, Ψ_o, and so one boundary condition is

$$\Psi(r,\theta,y) = \Psi_o \text{ at } r \leq r_o \text{ and } y = 0 \ . \tag{17}$$

At the free surface or non-conducting belt, it is assumed that no electric current flows into or out of the gas phase or insulator. Therefore

$$\frac{\partial\Psi}{\partial y} = 0 \text{ at } y = h \ . \tag{18}$$

From the third assumption, since the electric field around the cathode is circularly symmetrical,

$$\frac{\partial\Psi}{\partial\theta} = 0 \ . \tag{19}$$

Solving for the arbitrary constants, one obtains n = 0 and k may be real or imaginary; its magnitude may vary so that

$$0 \leq |k| < \infty \ . \tag{20}$$

By the separation of variables technique, it can be shown that the solution to (12) becomes

$$\Psi(r,\theta,y) = \Psi(r,y) = \Psi_o + \sum_{m=1}^{\infty} K_m I_o \left[\left(m - \frac{1}{2}\right)\frac{\pi r}{h}\right] \cdot \sin\left[\left(m - \frac{1}{2}\right)\frac{\pi y}{h}\right]. \tag{21}$$

Since

$$\vec{v}(x,y,z) = u(y)\,\vec{i}\,, \tag{22}$$

from Equation (9), then (21) and (22) are substituted into (6) to yield

$$D_j\left(\frac{\partial^2 c_j}{\partial x^2} + \frac{\partial^2 c_j}{\partial y^2} + \frac{\partial^2 c_j}{\partial z^2}\right) = u(y)\frac{\partial c_j}{\partial x} + \frac{z_j F}{RT} D_j \left\{\left(x\frac{\partial c_j}{\partial x} + z\frac{\partial c_j}{\partial z}\right) \cdot \frac{1}{\sqrt{x^2+z^2}} \sum_{m=1}^{\infty} K_m \left(m - \frac{1}{2}\right)\frac{\pi}{h} I_o'\left[\left(m - \frac{1}{2}\right)\frac{\pi}{h}\sqrt{x^2+z^2}\right] \cdot \sin\left[\left(m - \frac{1}{2}\right)\frac{\pi y}{h}\right] + \frac{\partial c_j}{\partial y} \sum_{m=1}^{\infty} K_m \left(m - \frac{1}{2}\right)\frac{\pi}{h} I_o\left[\left(m - \frac{1}{2}\right)\frac{\pi}{h}\sqrt{x^2+z^2}\right]\cos\left[\left(m - \frac{1}{2}\right)\frac{\pi y}{h}\right]\right\}. \tag{23}$$

Equation (23) is transformed into a dimensionless form by substitution of the following definitions:

$$\gamma_j = \frac{c_j}{c_{j\infty}}\,, \tag{24}$$

$$\kappa_m = \frac{K_m}{\Psi_o}\,, \tag{25}$$

$$\eta = \frac{\pi}{2h}\,y\,, \tag{26}$$

$$\xi = \frac{\pi}{2r_o} x \ , \tag{27}$$

and

$$\zeta = \frac{\pi}{2r_o} z \ . \tag{28}$$

Equation (23) after reorganization becomes

$$\left(\frac{h}{r_o}\right)^2 \left(\frac{\partial^2 \gamma_j}{\partial \xi^2} + \frac{\partial^2 \gamma_j}{\partial \zeta^2}\right) + \frac{\partial^2 \gamma_j}{\partial \eta^2} = \frac{2}{\pi} \frac{h^2 u(\eta)}{r_o D_j} \frac{\partial \gamma_j}{\partial \xi}$$

$$+ \frac{z_j F \Psi_o}{RT} \left\{ \frac{h}{r_o} \frac{1}{\sqrt{\xi^2 + \zeta^2}} \left[\xi \frac{\partial \gamma_j}{\partial \xi} + \zeta \frac{\partial \gamma_j}{\partial \zeta}\right] \sum_{m=1}^{\infty} \kappa_m (2m - 1) \right.$$

$$\cdot I_o' \left[(2m - 1) \frac{r_o}{h} \sqrt{\xi^2 + \zeta^2}\right] \sin\left[(2m - 1)\eta\right] + \frac{\partial \gamma_j}{\partial \eta} \sum_{m=1}^{\infty} \kappa_m (2m - 1)$$

$$\left. \cdot I_o \left[(2m - 1) \frac{r_o}{h} \sqrt{\xi^2 + \zeta^2}\right] \cos\left[(2m - 1)\eta\right] \right\} . \tag{29}$$

The function $\gamma_j(\xi,\eta,\zeta)$ for the dimensionless concentration of species j is expanded into a Fourier Series in three variables, ξ, η, ζ.

$$\gamma_j(\xi,\eta,\zeta) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{lmn} \cos l\xi \cos m\eta \cos n\zeta$$

$$+ \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} B_{lmn} \cos l\xi \cos m\eta \sin n\zeta + \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} C_{lmn} \cos l\xi \sin m\eta \cos n\zeta$$

$$+ \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{lmn} \cos l\xi \sin m\eta \sin n\zeta + \sum_{l=1}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} E_{lmn} \sin l\xi \cos m\eta \cos n\zeta$$

$$+ \sum_{l=1}^{\infty} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} F_{lmn} \sin l\xi \cos m\eta \sin n\zeta + \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} G_{lmn} \sin l\xi \sin m\eta \cos n\zeta$$

$$+ \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} H_{lmn} \sin l\xi \sin m\eta \sin n\zeta \ . \qquad (30)$$

The boundary conditions applicable to the solution of (29) are:

$$\text{For } \sqrt{\xi^2 + \zeta^2} \leq \frac{\pi}{2} \text{ and } \eta = 0, \ \gamma_j = \gamma_{jo} \ , \qquad (31)$$

$$\eta = \frac{\pi}{2} \ , \ \frac{\partial \gamma_j}{\partial \eta} = 0 \ , \qquad (32)$$

and

$$\xi = -\frac{\pi}{2} \ , \ \zeta = 0, \ \eta = \frac{\pi}{2} \ , \ \gamma_j = 1 \ . \qquad (33)$$

To satisfy (31) all A's except $A_{000} = \gamma_{jo}$, all B's, all E's, and all F's are zero in Equation (30). Assuming a symmetrical solution with respect to $\zeta = 0$, the coefficients of terms containing sin $n\zeta$, $n = 1,2,3,\ldots$, are zero. This makes all the D's and H's equal to zero. Next, applying the boundary condition in (32), Equation (30) becomes

$$\begin{aligned}
\gamma_j = \gamma_{jo} &+ C_{010}\sin\eta + C_{011}\sin\eta\cos\zeta + C_{012}\sin\eta\cos 2\zeta \\
&+ C_{110}\cos\xi\sin\eta + C_{111}\cos\xi\sin\eta\cos\zeta + C_{112}\cos\xi\sin\eta\cos 2\zeta \\
&+ C_{210}\cos 2\xi\sin\eta + C_{211}\cos 2\xi\sin\eta\cos\zeta + C_{212}\cos 2\xi\sin\eta\cos 2\zeta \\
&+ G_{110}\sin\xi\sin\eta + G_{111}\sin\xi\sin\eta\cos\zeta + G_{112}\sin\xi\sin\eta\cos 2\zeta \\
&+ G_{210}\sin 2\xi\sin\eta + G_{211}\sin 2\xi\sin\eta\cos\zeta + G_{212}\sin 2\xi\sin\eta\cos 2\zeta \ . \qquad (34)
\end{aligned}$$

Substitute the expressions for

$$\frac{\partial \gamma_j}{\partial \xi}, \ \frac{\partial^2 \gamma_j}{\partial \xi^2}, \ \frac{\partial \gamma_j}{\partial \eta}, \ \frac{\partial^2 \gamma_j}{\partial \eta^2}, \ \frac{\partial \gamma_j}{\partial \zeta}, \text{ and } \frac{\partial^2 \gamma_j}{\partial \zeta^2}$$

obtained from (34) into Equation (29). The C's and G's must be determined so that the boundary condition in Equation (33) is satisfied. Equation (33) substituted into (34) gives

$$C_{010} + C_{011} + C_{012} - C_{210} - C_{211} - C_{212} - G_{110} - G_{111} - G_{112} = 1 - \gamma j_o \, . \tag{35}$$

Now, the K_m values in Equation (21) need to be evaluated. To be able to solve Equation (12) for the potential, Ψ, the geometry was idealized by assuming that the anode was at infinity. But in reality the two electrodes are relatively close to each other and it is assumed that almost all of the potential drop occurs in the neighborhoods of the electrodes, and the potential drop between these two regions is negligibly small. Therefore,

$$\Psi(r_o,h) - \Psi_o = \frac{\Psi_\infty - \Psi_o}{2} \, . \tag{36}$$

Defining a new variable, β_m, where

$$\beta_m = \frac{K_m}{\Psi(r_o,h) - \Psi_o}, \; m = 1, 2, 3, \ldots , \tag{37}$$

then, using (36) and (37), Equation (21) becomes

$$1 = -2 \sum_{m=1}^{\infty} \beta_m (-1)^m I_o[(m - 1/2)\frac{\pi r_o}{h}] \, . \tag{38}$$

Since little is known about the values of Ψ on the boundary of a normal circular cylinder of radius r_o, height h, and located over the circular cathode of radius r_o, all terms in Equation (38) are dropped except the first, reducing it to

$$\beta_1 = \frac{1}{2I_o(\frac{\pi r_o}{2h})} \, . \tag{39}$$

This reduces Equation (21) and converting to dimensionless coordinates gives

$$\Psi(\xi,\eta,\zeta) - \Psi_o = (\Psi_\infty - \Psi_o)\frac{I_o(\frac{r_o}{h}\sqrt{\xi^2 + \zeta^2})}{2I_o(\frac{\pi r_o}{2h})} \sin\eta \ . \qquad (40)$$

The total current I_j flowing through the cathode is obtained by integrating the current density over the surface of the electrode. The current density, $\vec{i}_j$, and the flux, $\vec{J}_j$, are related by

$$\vec{i}_j = z_j F \vec{J}_j \ , \qquad (41)$$

where

$$\vec{J}_j = c_j \vec{v} - D_j \text{grad } c_j + u_j c_j \text{grad } \Psi \ . \qquad (42)$$

From coordinate transformations, boundary conditions on the electrode surface, and Equation (41), the current density becomes

$$i_{j(\eta=0)} = \frac{\pi}{2}\frac{z_j F c_{j\infty}}{h}\left[-D_j \frac{\partial \gamma_j}{\partial \eta}_{\eta=0} + \frac{D_j z_j F}{RT}\gamma_{j(\eta=0)} \frac{\partial \Psi}{\partial \eta}_{\eta=0}\right]. \qquad (43)$$

Substitution of

$$(\frac{\partial \gamma_j}{\partial \eta})_{\eta=0}, \ \gamma_{j(\eta=0)}, \text{ and } (\frac{\partial \Psi}{\partial \eta})_{\eta=0} \ ,$$

and integrating over the surface area of the electrode the current becomes

$$I_j = \frac{\pi}{2}\frac{\pi r_o^2}{h} c_{j\infty} D_j z_j F \left[\int_{-\pi/2}^{\pi/2} (- C_{010} - C_{011}\frac{8}{\pi^3}\sin Q_1$$

$$- C_{012}\frac{4}{\pi^3}\sin 2Q_1 - C_{110}\frac{8}{\pi^3} Q_1 \cos\xi - C_{111}\frac{8}{\pi^3}\cos\xi \sin Q_1$$

$$- C_{112}\frac{4}{\pi^3}\cos\xi\sin 2Q_1 - C_{210}\frac{8}{\pi^3}Q_1\cos 2\xi - C_{211}\frac{8}{\pi^3}\cos 2\xi\sin Q_1$$

$$- C_{212}\frac{4}{\pi^3}\cos 2\xi\sin 2Q_1 - G_{110}\frac{8}{\pi^3}Q_1\sin\xi - G_{111}\frac{8}{\pi^3}\sin\xi\sin Q_1$$

$$- G_{112}\frac{4}{\pi^3}\sin\xi\sin 2Q_1 - G_{210}\frac{8}{\pi^3}Q_1\sin 2\xi - G_{211}\frac{8}{\pi^3}\sin 2\xi\sin Q_1$$

$$- G_{212}\sin 2\xi\sin 2Q_1)\,d\xi + \frac{z_jF}{RT}(\gamma_{jo})\frac{\psi_\infty-\psi_o}{I_o(\frac{\pi r_o}{2h})}\int_0^1 I_o(\frac{\pi r_o}{2h}\rho)\rho d\rho\Bigg],$$

$$\text{where } Q_1 = \sqrt{\frac{\pi^2}{4} + \xi^2}\,. \tag{44}$$

Finally, the value for the dimensionless net current through the EP Probe is defined as

$$I = -\frac{hI_j}{\pi r_o^2 c_{j\infty} D_j z_j F}\,. \tag{45}$$

This solution can be matched to the solution of the field extending from the cylinder boundary to infinity and equating coefficients. This is a lengthy process and little is known about the potential in the outer region. Therefore, this procedure is not used here but future work might produce favorable results.

There are 15 unknown coefficients in Equation (44) which requires 15 independent equations. The coefficients are determined by substituting a sufficient number of values for ξ, η, ζ and solving the system of linear equations for the coefficients. The 14 points on the domain, shown in Figure 16, were substituted into Equation (34) and its derivatives. Then, the 14 values for

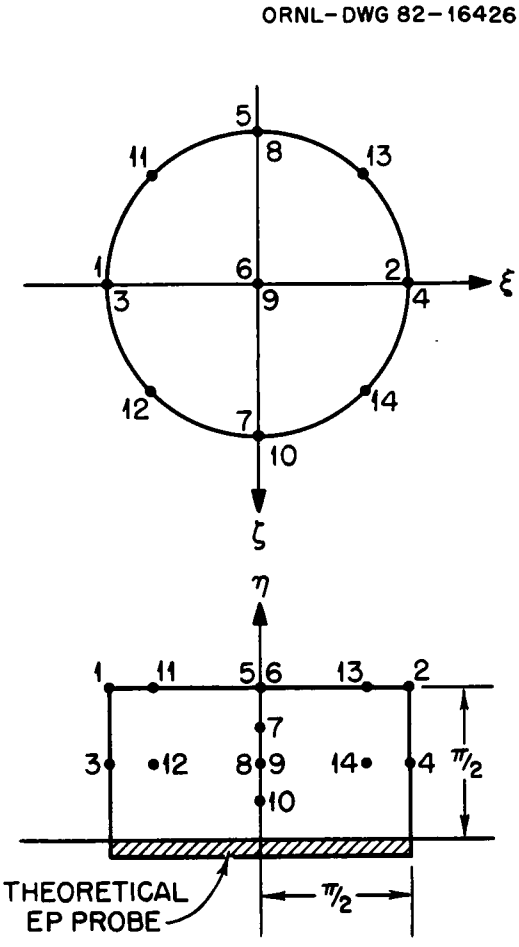

Figure 16. Fourteen points on the domain of a right circular cylinder of height $\pi/2$ and radius $\pi/2$ used in the 15 independent equations.

$$\frac{\partial \gamma_j}{\partial \xi}, \frac{\partial^2 \gamma_j}{\partial \xi^2}, \frac{\partial \gamma_j}{\partial \eta}, \frac{\partial^2 \gamma_j}{\partial \eta^2}, \frac{\partial \gamma_j}{\partial \zeta}, \text{ and } \frac{\partial^2 \gamma_j}{\partial \zeta^2}$$

were substituted into Equation (29) to obtain 14 equations. The 14 equations were used in addition to Equation (35) to create the 15 independent equations that were needed.

VIII. THEORETICAL DATA AND RESULTS

A computer model simulation program was written to input the points in the domain and solve the simultaneous equations. Appendix A shows this analysis program. It calls a matrix manipulation subroutine to solve a system of simultaneous equations using Gaussian Elimination as shown in Appendix B. Another subroutine is called which calculates the Hyperbolic Bessel Function of order zero and order one of a pre-calculated argument, shown in Appendix C, and returns the values to the main program. Appendix D gives input parameters and actual numerical values in radians of the 14 points of the domain on a right circular cylinder. Input parameters can be changed to vary the film thickness, average film velocity, and water conductivity to obtain dimensionless current values.

Dimensionless current values for the experimental data had to be calculated. Since the electrode surface configuration, Figure 2 (page 6), was not circular in shape, an r_o value had to be obtained from the actual probe to simulate an r_o value in the theory. The surface area of the cathode was calculated and set equal to the area of a circle and solved for r_o. Then dimensionless current values were calculated from Equation (45) where the experimental values for film thickness, h, and measured current values, I_j, were used.

An attempt was made to bring the theoretical results to the same order of magnitude as the experimental results by varying the parameters γ_{j_o} and z_j. Figure 17 shows the relationship of dimensionless current values as a function of average liquid film velocity. This graph portrays the data obtained from the theoretical model and experimental

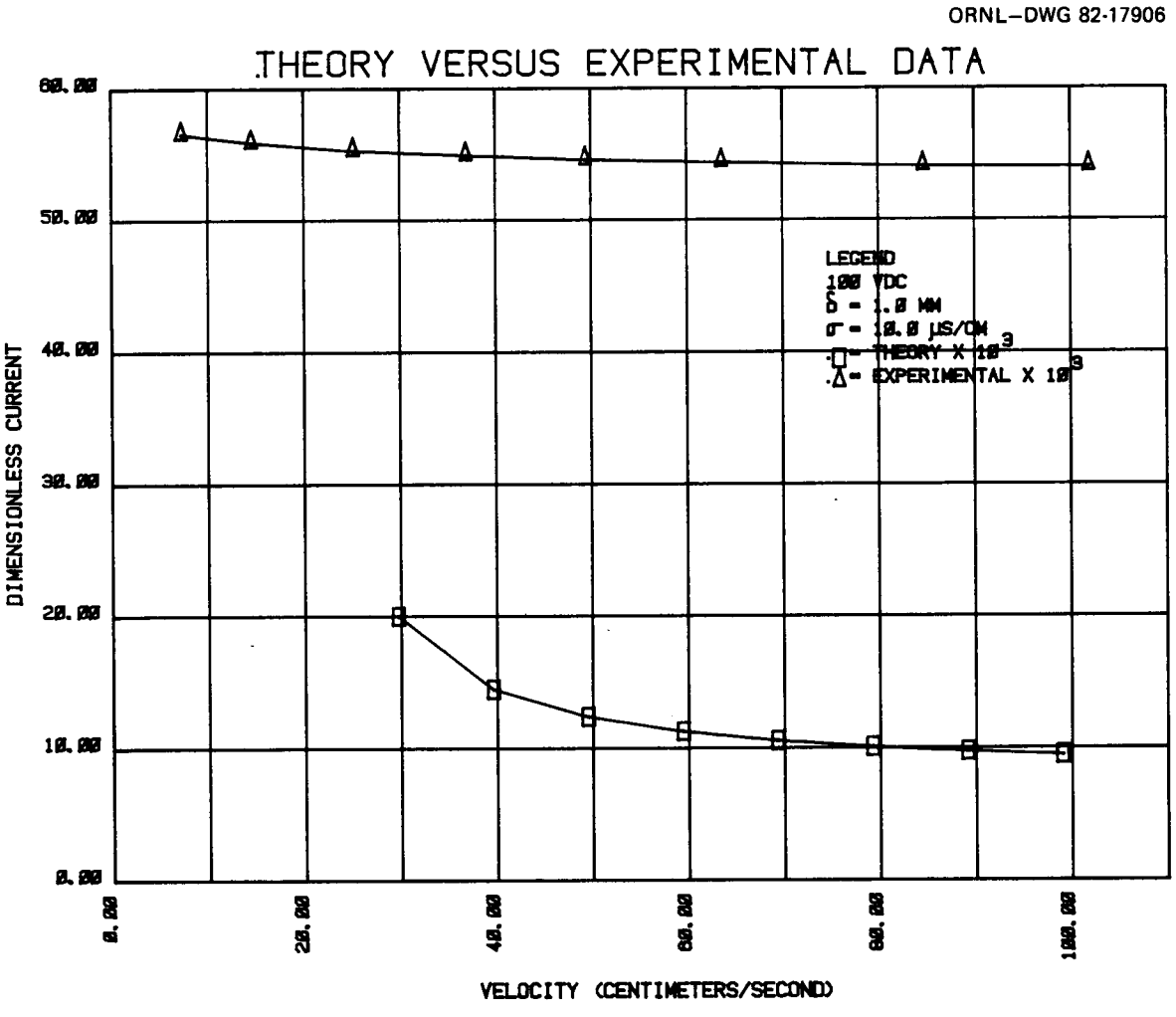

Figure 17. Dimensionless current-velocity relationship for theoretical and experimental data where the film thickness is one millimeter and the water conductivity is ten microsiemens per centimeter.

data for common parameters. These parameters included constant excitation voltage, water conductivity, and film thickness.

There may be a need here to explain the legend of the graph. The data points on the graph are multiplied by the multiplier in the legend to give the actual dimensionless current values. As will be shown, the data tend to drift apart by several orders of magnitude and it was desired to display respective data on the same graphs without having to spread the dimensionless current values far apart.

From Figure 18, theoretical data is graphed with experimental data for a different film thickness. Included in the experimental data are values where the water conductivity is changed. The correlation between the theoretical data and experimental data shows good agreement.

Figures 19 and 20 respectively show how water film thickness plays a role in the theoretical modeling. Figure 19 shows that the difference of theoretical data and experimental data at a film thickness of four millimeters is an order of magnitude. In Figure 20, with an eight millimeter thick film, the difference is about three orders of magnitude. The data in Figure 20 were obtained from the EP Probe low velocity configuration in Figure 7 (page 14). Because only velocities below ten centimeters per second were recorded, the slope is greater than the slopes seen in Figure 17, Figure 18, and Figure 19.

Collectively examining all the data obtained from the theoretical model and the experiments, it was observed that as the film thickness decreases, the correlation between the experimental data and the theoretical model becomes better.

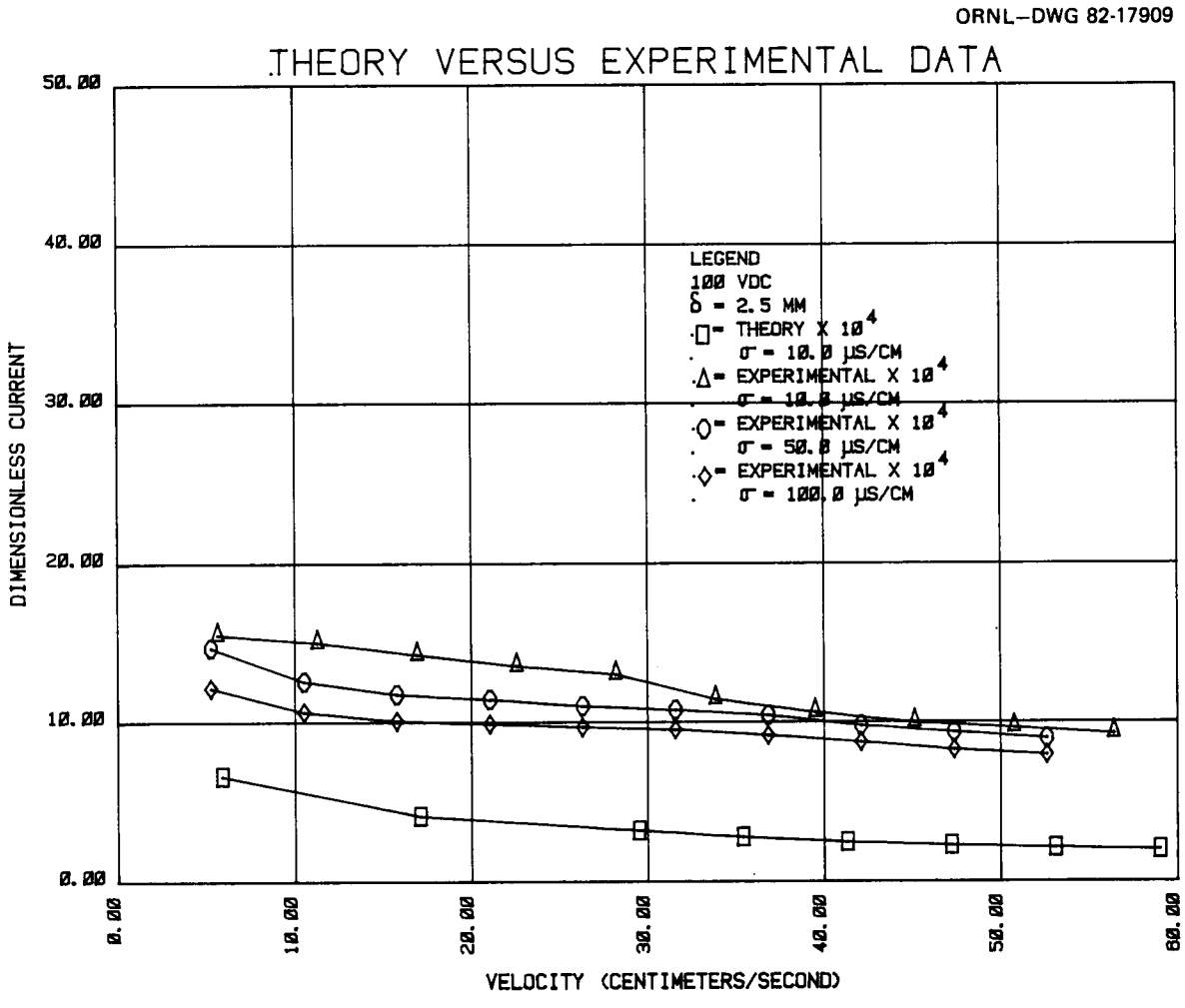

Figure 18. Dimensionless current-velocity relationship for theoretical and experimental data where the film thickness is 2.5 millimeters and the water conductivity ranged from ten to 100 microsiemens per centimeter.

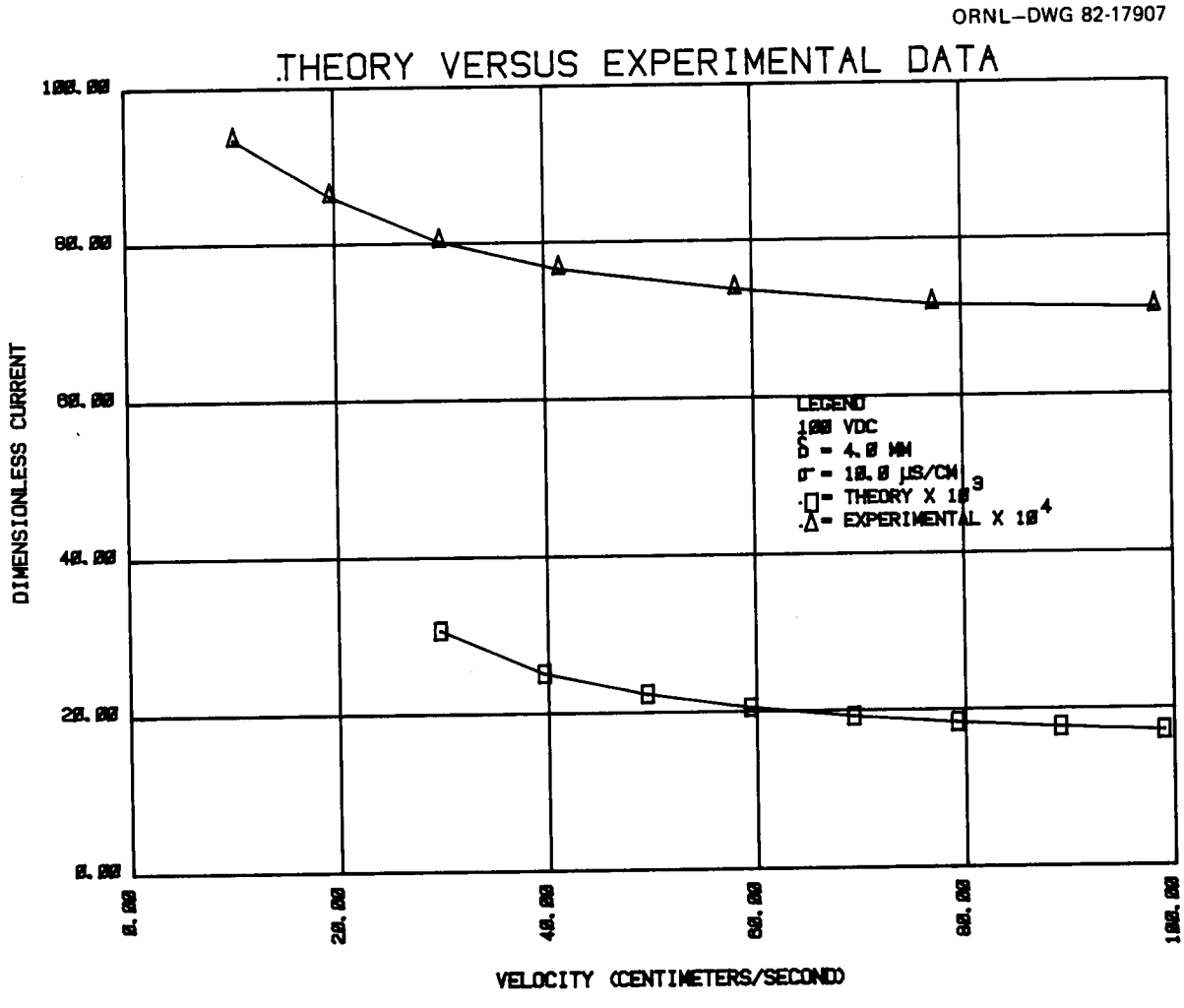

Figure 19. Dimensionless current-velocity relationship for theoretical and experimental data where the film thickness is four millimeters and the water conductivity is ten microsiemens per centimeter.

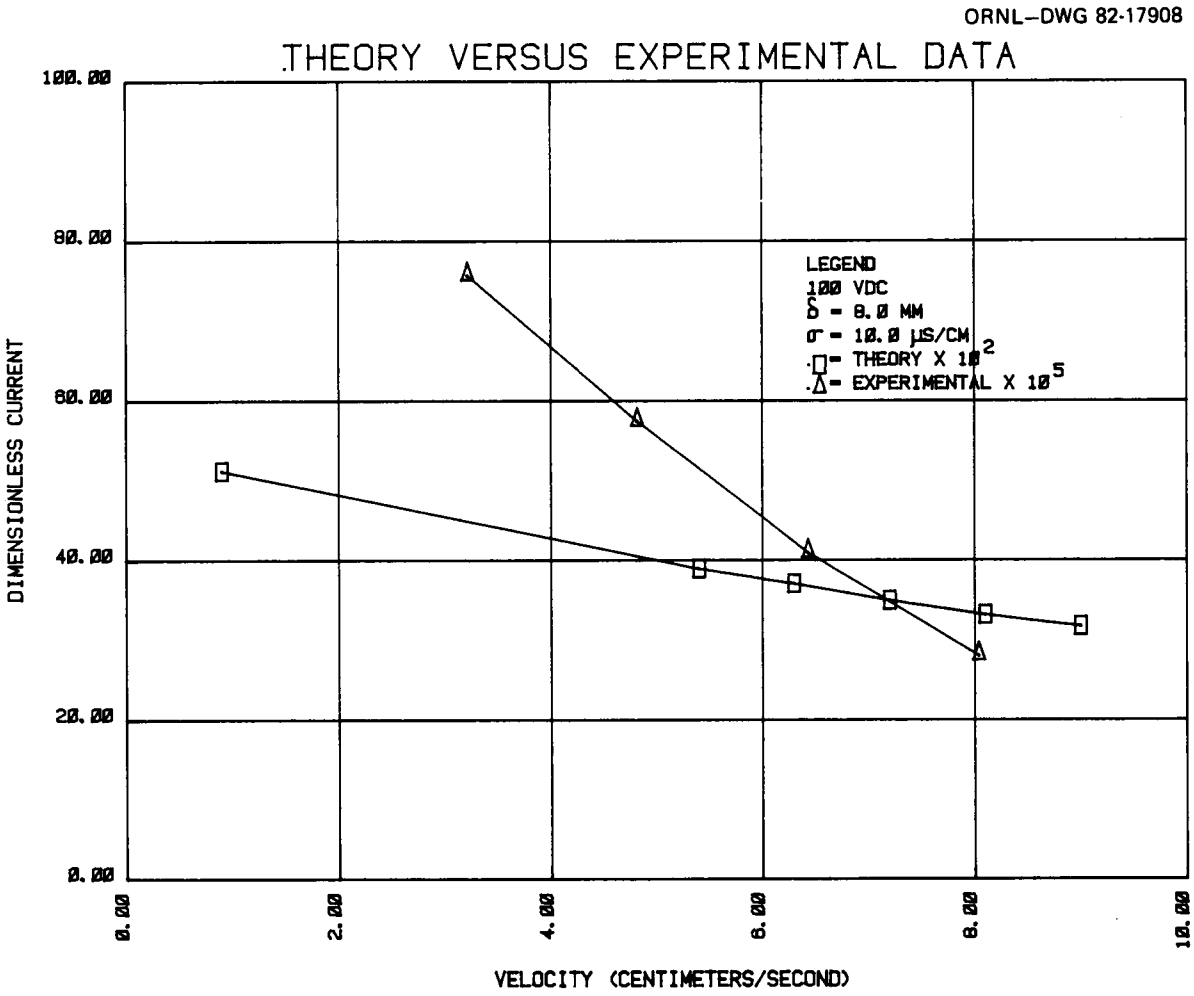

Figure 20. Dimensionless current-velocity relationship for theoretical and experimental data where the film thickness is eight millimeters and the water conductivity is ten microsiemens per centimeter.

IX. CONCLUSIONS

This research work was an effort to show that the fluid mechanical and electrochemical processes in the flowing liquid can produce the phenomenon. The current-velocity response of the EP Probe was found to be dependent upon water conductivity, film thickness, and sensitive to the ion concentration and the chemical valence of the species involved.

Many assumptions were made in the theory to simplify the mathematical complexity of the problem. First, the sensor face geometry was assumed to be circular in shape while the actual prototype EP Probe surface configuration was a modification of a rectangular shaped body. In assuming the circular geometry in the theory some mathematical simplifications were made. These simplifications could not have been applied if the prototype electrode surface geometry had been used in the theory. This is a possible source of error in the correlation of data because the electrode surface area plays a role in the occurring phenomenon. It was also assumed as a boundary condition that the distance between the cathode and anode was infinite but in reality they were a finite distance apart and this distance is significant.

It was observed during the experimentation that the parameters could not be measured accurately and held sufficiently constant. The water conductivity during an experiment might drift from ten microsiemens per centimeter to as much as 15 microsiemens per centimeter. This drift was attributed to the dissolving of ions in the water from the surfaces of the experimental apparatus during the course of the experiment. The dimensionless current measured across the electrodes changed substantially with increasing water conductivity but the dimensionless current

from the theoretical model did not. When other parameters in the theory were varied the current response began to behave in a manner similar to the experimental results. Thus, other parameters in the experiment needed to be measured, but the techniques to measure these quantities were not known. Three parameters which make the calculated current to behave similar to the measured current were the valence of the species j, z_j, the ratio of the bulk concentration of species j at the electrode to the bulk concentration of species j at infinity, γ_j, and the liquid film thickness, h. Their relationship showed that to match the experimental results as the thickness increased, z_j had to be increased and γ_j decreased. Since the values for z_j and γ_j were not known and could not be measured, they were freely adjusted but it is almost certain that the values needed for acceptable approximations are unrealistic. This leaves the question open whether this theory adequately explains the observed phenomenon, or if additional physical and chemical (possibly not yet discovered) processes have to be considered in the analysis.

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APPENDIXES

APPENDIX A

EP ANALYSIS MAIN FORTRAN PROGRAM

```
C*************************************************************
C  CREATED BY W. F. BETHMANN, JR.                             *
C                                                             *
C                                                             *
C  THIS PROGRAM WILL SOLVE AN M X N MATRIX OF                 *
C  SIMULTANEOUS EQUATIONS.  THE OPERATOR WILL NEED TO         *
C  INPUT SEVERAL PARAMETERS FOR CALCULATION OF THE            *
C  COEFFICIENTS.  THEN THESE COEFFICIENTS WILL BE USED        *
C  IN THE PROCESS OF SOLVING FOR THE UNKNOWN                  *
C  COEFFICIENTS BY GAUSS  ELIMINATION.                        *
C                                                             *
C  THE OPERATOR MAY INPUT HIS OWN COEFFICIENTS IF SO          *
C  DESIRED AND BYPASS THE COEFFICIENT CALCULATION.            *
C                                                             *
C  THIS MATRIX MANIPULATION MAY BE RUN AGAIN FOR              *
C  DIFFERENT PARAMETER VALUES.                                *
C                                                             *
C*************************************************************
C                                                             *
C  ...PARAMETERS...                                           *
C   H       :   WATER FILM THICKNESS (CM)                     *
C   RO      :   ELECTROLYSIS POTENTIAL PROBE RADIUS (CM)      *
C   UBELT   :   BELT VELOCITY (CM/SEC)                        *
C   VOLT    :   APPLIED VOLTAGE ACROSS THE ELECTRODE          *
C   DJ      :   DIFFUSION COEFFICIENT (CM**2.0/SEC)           *
C   GAMMA   :   CONCENTRATION ON SURFACE OF ELECTRODE         *
C   TEMP    :   WATER TEMPERATURE (K)                         *
C   ZJ      :   CHEMICAL VALENCE FOR THE SPECIES              *
C   F       :   FARADAY'S CONSTANT                            *
C                          (96527. COULOMBS/EQUIV. WT.)       *
C   RU      :   UNIVERSAL GAS CONSTANT                        *
C                          (8.317 JOULES/(g MOLE K))          *
C   CJ      :   BULK CONCENTRATION OF SPECIES J               *
C                                                             *
C  ...VARIABLES FOR 14 DIFFERENT POINTS IN THE DOMAIN...      *
C   X(I) :  X RADIAL DISTANCE VALUES BETWEEN + OR             *
C                                              - PI/2.0       *
C   Y(I) :  HEIGHT VALUES OF EITHER PI/2.0 OR PI/4.0          *
C   Z(I) :  Y RADIAL DISTANCE VALUES BETWEEN + OR             *
C                                              - PI/2.0       *
C   R(I) :  RATIO OF RADIAL DIMENSION DIVIDED BY PROBE        *
C           RADIUS TAKES ON VALUES OF EITHER 0.0 OR 1.0       *
C                                                             *
C  ...DEFINITIONS...                                          *
C   PI = 3.1415926                                            *
C   PR = PI * RO / (2.0 * H)                                  *
C   H1 = (H / RO)**2.0 + 1.0                                  *
C   H2 = 2.0 * ((H / RO)**2.0) + 1.0                          *
C   H4 = 4.0 * ((H / RO)**2.0) + 1.0                          *
```

```
C   H5 = 5.0 * ((H / R0)**2.0) + 1.0                                *
C   H8 = 8.0 * ((H / R0)**2.0) + 1.0                                *
C                                                                   *
C*******************************************************************
C
C  DIMENSION THE MATRIX AND XX ARRAY AND ALL VARIABLES.
C
      COMMON/EPPRAM/A(25,25), B(25,25), XX(25)
      REAL*8 X(20), Y(20), Z(20), R(20), SX(20), SY(20)
      REAL*8 SZ(20), CX(20), CY(20), CZ(20), S2X(20)
      REAL*8 S2Y(20), S2Z(20), C2X(20), C2Y(20), C2Z(20)
      REAL*8 Q010(20), Q011(20), Q012(20), Q110(20)
      REAL*8 Q111(20), Q112(20), Q210(20), Q211(20)
      REAL*8 Q212(20), T110(20), T111(20), T112(20)
      REAL*8 T210(20), T211(20), T212(20), UJ(20), IJ(20)
      REAL*8 IJP(20), HYP0(2), HYP1(2), A, B, XX, R0, H
      REAL*8 UBELT, VOLT, DJ, ZJ, GAMMA, F, RU, PI, RH
      REAL*8 PR, H1, H2, H4, H5, H8, TEMP, CJ, UBEL, INT1
      REAL*8 INT2, INT3, INT4, INT5, INT6, INT7, INT8
      REAL*8 INT9, INT10, INT11, INT12, INT13, INT14
      REAL*8 ZINTRH, FACTOR, HYP00, HYP01, HYP10, HYP11
      REAL*8 DIMCUR, CURRNT
      LOGICAL*1 YESNO
C
C*******************************************************************
C  PRINT OUT A MESSAGE DESCRIBING THE PROGRAM.
C
    5 WRITE(5,10)
   10 FORMAT(' This program solves a 25 equation system',
     1' for 25',/,' unknowns. For a test, you may input',
     2' your own coefficients. ',/,' Do you wish to',
     3' enter your coefficients? [Y/N]: ')
C
C*******************************************************************
C  TEST IF THE OPERATOR WISHES TO INPUT THE COEFFICIENTS
C  FOR THE MATRIX. FOR THE EP PROBE THE COEFFICIENTS
C  MUST BE CALCULATED BEGINNING AT LINE 170.
C
      READ(5,20)YESNO
   20 FORMAT(A1)
      IF(YESNO.NE.'Y') GO TO 170
C
C*******************************************************************
C  ASK SEVERAL QUESTIONS ABOUT THE SIZE OF THE MATRIX FOR
C  SOLUTION.
C
      WRITE(5,30)
   30 FORMAT(' How many rows are there in the matrix? ')
      READ (5, 40) M
   40 FORMAT (I2)
      WRITE (5, 50)
   50 FORMAT(' How many collumns are there in the',
     1' matrix? ')
      READ (5, 40) N
C
```

```
C*****************************************************************
C  ASK FOR THE COEFFICIENTS OF THE MATRIX.
C
      WRITE (5, 60)
   60 FORMAT (' Input the coefficients row by row, ',
     1' returning after',/,' each entry:  ')
      DO 110 I = 1, M
      DO 110 J = 1, N
   65 WRITE (5, 70) I, J
   70 FORMAT ($, ' A(', I2, ',', I2, ') = ')
      READ (5, 80) A(I, J)
   80 FORMAT (E12.5)
      WRITE (5, 90) I, J, A(I, J)
   90 FORMAT (' A(', I2, ',', I2, ') = ', G10.4)
      WRITE (5, 100)
  100 FORMAT ($, ' Is the entry correct? [Y/N]:  ')
      READ (5, 20) YESNO
      IF (YESNO .EQ. 'N') GO TO 65
  110 CONTINUE
C*****************************************************************
C  CALL THE GAUSS ELIMINATION SUBROUTINE AND TRANSFER THE
C  SIZE OF THE MATRIX AND THE COMMON A AND B AND XX ARRAY.
C
      CALL MATRIX (M, N)
C*****************************************************************
C  PRINT THE REDUCED MATRIX AND THE SOLUTION TO THE
C  SIMULTANEOUS EQUATIONS.
C
      DO 130 I = 1, M
      DO 130 J = 1, N
      WRITE (5, 120) I, J, B(I, J)
  120 FORMAT (' B(', I2, ',', I2, ') = ', G10.4)
  130 CONTINUE
      DO 150 I = 1, M
      WRITE (5, 140) I, XX(I)
  140 FORMAT (' X(', I2, ') = ', G10.4)
  150 CONTINUE
C*****************************************************************
C  TEST IF THE OPERATOR WISHES TO CONTINUE.
C
      WRITE (5, 160)
  160 FORMAT ($, ' Do you wish to continue? [Y/N]:  ')
      READ (5, 20) YESNO
      IF (YESNO .EQ. 'N') GO TO 570
      IF (YESNO .EQ. 'Y') GO TO 5
  170 CONTINUE
C*****************************************************************
C  DEFINE THE CONSTANTS USED IN THE CALCULATION OF
C  THE DIMENSIONLESS CURRENT THROUGH THE EP PROBE
C
      INT1 = 1.13366938
```

```
      INT2 = 0.28461429
      INT3 = 1.13365042
      INT4 = 0.78181136
      INT5 = 0.11821981
      INT6 = 0.28461719
      INT7 = 0.11822040
      INT8 = -0.15219291
      INT9 = 0.0
      INT10 = 0.0
      INT11 = 0.0
      INT12 = 0.0
      INT13 = 0.0
      INT14 = 0.0
C
C*****************************************************************
C  OPEN PARAMETERS FILE AND GET PARAMETERS FOR COEFFICIENT
C  CALCULATION.
C
      OPEN (UNIT=2, TYPE='OLD', NAME='EPDAT.TAB',
     1ACCESS='SEQUENTIAL', FORM='FORMATTED', ERR=998)
      READ (2, 180) RO
  180 FORMAT (16X, F12.5)
      READ (2, 180) H
      READ (2, 180) UBELT
      READ (2, 180) VOLT
      READ (2, 180) DJ
      READ (2, 180) GAMMA
      READ (2, 180) ZJ
      READ (2, 180) F
      READ (2, 180) RU
      READ (2, 180) TEMP
      READ (2, 185) CJ
  185 FORMAT (16X, E12.5)
C
C*****************************************************************
C  ENTER THE VALUES FOR THE 14 DIFFERENT POINTS IN THE
C  DOMAIN FOR X(I), Y(I), Z(I), AND R(I).
C
      DO 200 I = 1, 14
      READ (2, 190) X(I), Y(I), Z(I), R(I)
  190 FORMAT (16X, 4F12.5)
      SX(I) = SIN(X(I))
      CX(I) = COS(X(I))
      S2X(I) = SIN(2.0 * X(I))
      C2X(I) = COS(2.0 * X(I))
      SY(I) = SIN(Y(I))
      CY(I) = COS(Y(I))
      S2Y(I) = SIN(2.0 * Y(I))
      C2Y(I) = COS(2.0 * Y(I))
      SZ(I) = SIN(Z(I))
      CZ(I) = COS(Z(I))
      S2Z(I) = SIN(2.0 * Z(I))
      C2Z(I) = COS(2.0 * Z(I))
  200 CONTINUE
C
```

```
C*************************************************************
C  CLOSE THE FILE.
C
      CLOSE (UNIT=2, ERR=999)
  210 CONTINUE
C
C*************************************************************
C  OPEN OUTPUT FILE AND WRITE INPUTTED VALUES.
C
  220 WRITE (5, 230) RO, H, UBELT, VOLT, DJ, GAMMA, ZJ, F
     1RU, TEMP, CJ
  230 FORMAT (24X, 'INPUT PARAMETERS', //, 8X, 'RO    =',
     1G12.5, /, 8X, 'H     =', G12.5, /, 8X, 'UBELT =', G12.5, /,
     28X, 'VOLT  =', G12.5, /, 8X, 'DJ    =', G12.5, /, 8X,
     3'GAMMA =', G12.5, /, 8X, 'ZJ    =', G12.5, /, 8X,
     4'F     =', G12.5, /, 8X, 'RU    =', G12.5, /, 8X,
     5'TEMP  =', G12.5, /, 8X, 'CJ    =', E12.5, //)
C
C*************************************************************
C  CHECK IF ANY VALUES NEED TO BE CHANGED.
C
      WRITE (5, 240)
  240 FORMAT ($, ' Do you wish to change any values?',
     1' [Y/N]: ')
      READ (5, 20) YESNO
C
C*************************************************************
C  IF YES THEN PROMPT FOR THE NEW PARAMETER VALUES.
C  IF NO THEN GO TO LINE 184 TO CONTINUE.
C
      IF (YESNO .EQ. 'N') GO TO 410
      WRITE (5, 250)
  250 FORMAT ($, ' Input the parameter name to be',
     1' changed: ')
      READ (5, 260) SAMPLE
  260 FORMAT (A4)
      IF (SAMPLE .NE. 'RO')GO TO 290
      WRITE (5, 270) SAMPLE
  270 FORMAT ('$', A5, ' = ')
      READ (5, 280) RO
  280 FORMAT (E12.5)
  290 IF (SAMPLE .NE. 'H') GO TO 300
      WRITE (5, 270) SAMPLE
      READ (5, 280) H
  300 IF (SAMPLE .NE. 'UBEL') GO TO 310
      WRITE (5, 270) SAMPLE
      READ (5, 280) UBELT
  310 IF (SAMPLE .NE. 'VOLT') GO TO 320
      WRITE (5, 270) SAMPLE
      READ (5, 280) VOLT
  320 IF (SAMPLE .NE. 'DJ') GO TO 330
      WRITE (5, 270) SAMPLE
      READ (5, 280) DJ
  330 IF (SAMPLE .NE. 'GAMM') GO TO 340
      WRITE (5, 270) SAMPLE
```

```
      READ (5, 280) GAMMA
  340 IF (SAMPLE .NE. 'ZJ') GO TO 350
      WRITE (5, 270) SAMPLE
      READ (5, 280) ZJ
  350 IF (SAMPLE .NE. 'F') GO TO 360
      WRITE (5, 270) SAMPLE
      READ (5, 280) F
  360 IF (SAMPLE .NE. 'RU') GO TO 370
      WRITE (5, 270) SAMPLE
      READ (5, 280) RU
  370 IF (SAMPLE .NE. 'TEMP') GO TO 390
      WRITE (5, 270) SAMPLE
      READ (5, 280) TEMP
  390 IF (SAMPLE .NE. 'CJ') GO TO 400
      WRITE (5, 270) SAMPLE
      READ (5, 80) CJ
  400 GO TO 220
  410 FACTOR = UBELT / 10.
      DO 555 IJI = 1, 11
      UBEL = FACTOR * 10.
      UBELT = UBEL - ((IJI - 1) * FACTOR)
      OPEN (UNIT=1, NAME='SY:PRINT.LST', TYPE='NEW',
     1ACCESS='SEQUENTIAL', FORM='FORMATTED',
     2RECORDSIZE=132)
      WRITE (1, 230) RO, H, UBELT, VOLT, DJ, GAMMA, ZJ,
     1F, RU, TEMP, CJ
  420 WRITE (1, 430)
  430 FORMAT (' ',22X, 'X(I)', 8X, 'Y(I)', 8X, 'Z(I)',
     18X, 'R(I)',//)
      DO 450 I = 1, 14
      WRITE (1, 440) I, X(I), Y(I), Z(I), R(I)
  440 FORMAT ('0',3X,' POINT(',I2,')', 3X, 4F12.5)
      WRITE (5, 190) X(I), Y(I), Z(I), R(I)
  450 CONTINUE
C
C*************************************************************
C  ASSIGN VALUES TO THE CONSTANTS AND DECLARE DEFINITIONS.
C
      PI = 3.1415926
      RH = RO / H
      PR = PI * RO / (2.0 * H)
      H1 = (H / RO)**2.0 + 1.0
      H2 = 2.0 * ((H / RO)**2.0) + 1.0
      H4 = 4.0 * ((H / RO)**2.0) + 1.0
      H5 = 5.0 * ((H / RO)**2.0) + 1.0
      H8 = 8.0 * ((H / RO)**2.0) + 1.0
      IJK = RH * 2.0
      IF (IJK .LT. 10) IJK = 10
C
C*************************************************************
C  CALL THE HYPERBOLIC BESSEL SUBROUTINE TO OBTAIN THE
C  VALUES OF THE HYPERBOLIC BESSEL OF ORDER ZERO AND
C  ORDER ONE EVALUATED AT ZERO AND ONE FOR DIFFERENT
C  R/H ARGUMENT VALUES.
C
```

```
      CALL BESSEL (RH, IJK, HYP00, HYP01, HYP10, HYP11,
     1ZINTRH)
      HYP0(1) = HYP00
      HYP0(2) = HYP01
      HYP1(1) = HYP10
      HYP1(2) = HYP11
C
CC*************************************************************
C  BEGIN THE BIG LOOP TO CALCULATE THE COEFFICIENTS FOR
C  THE MATRIX ARRAY.
C
      DO 480 I = 1, 14
      UJ(I) = (2.0 * UBELT * H) / (PI * PR * DJ) * Y(I)
C
CC*************************************************************
C  THE VALUES OF IJ(I) AND IJP(I) DEPEND ON THE VALUE
C  OF R(I).  TEST R(I) AND THEN CALCULATE IJ(I) AND IJP(I)
C  ACCORDING TO THE R(I) VALUE.
C
      IF (R(I) .GT. 0.0) GO TO 460
      IJ(I) = (ZJ * F * VOLT * CY(I)) / (2.0 * RU * TEMP
     1* HYP0(2))
      IJP(I) = (ZJ * F * VOLT * SY(I)) / (4.0 * RU * TEMP
     1* HYP0(2))
      GO TO 470
  460 IJ(I) = (ZJ * F * VOLT * CY(I)) / (2.0 * RU * TEMP)
      IJP(I) = (ZJ * F * VOLT * SY(I) * HYP1(2)) /
     1(2.0 * RU * TEMP * PR * R(I) * HYP0(2))
C
CC*************************************************************
C  CALCULATE THE 15 COEFFICIENTS FOR EACH OF THE 14
C  POINTS IN THE DOMAIN.
C
  470 Q010(I) = -SY(I) - (IJ(I) * CY(I))
      Q011(I) = (-H1 * SY(I) * CZ(I)) - (IJ(I) * CY(I)
     1* CZ(I)) + (IJP(I) * Z(I) * SY(I) * SZ(I))
      Q012(I) = (-H4 * SY(I) * C2Z(I)) - (IJ(I) * CY(I)
     1* C2Z(I)) + (2.0 * IJP(I) * SY(I) * S2Z(I) * Z(I))
      Q110(I) = (-H1 * CX(I) * SY(I)) + ((UJ(I) + (IJP(I)
     1* X(I))) * SX(I) * SY(I)) - (IJ(I) * CX(I) * CY(I))
      Q111(I) = (-H2 * CX(I) * SY(I) * CZ(I)) + ((UJ(I)
     1+ (IJP(I) * X(I))) * SX(I) * SY(I) * CZ(I))
     2- (IJ(I) * CX(I) * CY(I) * CZ(I)) + (IJP(I) * Z(I)
     3* CX(I) * SY(I) * SZ(I))
      Q112(I) = (-H5 * CX(I) * SY(I) * C2Z(I)) + ((UJ(I)
     1+ (IJP(I) * X(I))) * SX(I) * SY(I) * C2Z(I))
     2- (IJ(I) * CX(I) * CY(I) * C2Z(I)) + (2.0 * IJP(I)
     3* Z(I) * CX(I) * SY(I) * S2Z(I))
      Q210(I) = (-H4 * C2X(I) * SY(I)) + (2.0 * ((UJ(I)
     1+ (IJP(I) * X(I))) * S2X(I) * SY(I))) - (IJ(I)
     2* C2X(I) * CY(I))
      Q211(I) = (-H5 * C2X(I) * SY(I) * CZ(I)) + (2.0
     1* ((UJ(I) + (IJP(I) * X(I))) * S2X(I) * SY(I)
     2* CZ(I))) - (IJ(I) * C2X(I) * CY(I) * CZ(I))
     3+ (IJP(I) * Z(I) * C2X(I) * SY(I) * SZ(I))
```

```
      Q212(I) = (-H8 * C2X(I) * SY(I) * C2Z(I)) + (2.0
     1* ((UJ(I) + (IJP(I) * X(I))) * S2X(I) * SY(I)
     2* C2Z(I))) - (IJ(I) * C2X(I) * CY(I) * C2Z(I))
     3+ (2.0 * IJP(I) * Z(I) * C2X(I) * SY(I) * S2Z(I))
      T110(I) = (-H1 * SX(I) * SY(I)) - ((UJ(I) + (IJP(I)
     1* X(I))) * CX(I) * SY(I)) - (IJ(I) * SX(I) * CY(I))
      T111(I) = (-H2 * SX(I) * SY(I) * CZ(I)) - ((UJ(I)
     1+ (IJP(I) * X(I))) * CX(I) * SY(I) * CZ(I))
     2- (IJ(I) * SX(I) * CY(I) * CZ(I)) + (IJP(I) * Z(I)
     3* SX(I) * SY(I) * SZ(I))
      T112(I) = (-H5 * SX(I) * SY(I) * C2Z(I)) - ((UJ(I)
     1+ (IJP(I) * X(I))) * CX(I) * SY(I) * C2Z(I))
     2- (IJ(I) * SX(I) * CY(I) * C2Z(I)) + (2.0 * IJP(I)
     3* Z(I) * SX(I) * SY(I) * S2Z(I))
      T210(I) = (-H4 * S2X(I) * SY(I)) - (2.0 * ((UJ(I)
     1+ (IJP(I) * X(I))) * C2X(I) * SY(I))) - (IJ(I)
     2* S2X(I) * CY(I))
      T211(I) = (-H5 * S2X(I) * SY(I) * CZ(I)) - (2.0
     1* ((UJ(I) + (IJP(I) * X(I))) * C2X(I) * SY(I)
     2* CZ(I))) - (IJ(I) * S2X(I) * CY(I) * CZ(I))
     3+ (IJP(I) * Z(I) * S2X(I) * SY(I) * SZ(I))
      T212(I) = (-H8 * S2X(I) * SY(I) * C2Z(I)) - (2.0
     1* ((UJ(I) + (IJP(I) * X(I))) * C2X(I) * SY(I)
     2* C2Z(I))) - (IJ(I) * S2X(I) * CY(I) * C2Z(I))
     3+ (2.0 * IJP(I) * Z(I) * S2X(I) * SY(I) * S2Z(I))
  480 CONTINUE
C
C*************************************************************
C REDEFINE ALL COEFFICIENTS INTO A MATRIX FORM FOR THE
C SUBROUTINE MATRIX.
C
      DO 490 I = 1, 14
      A(I, 1) = Q010(I)
      A(I, 2) = Q011(I)
      A(I, 3) = Q012(I)
      A(I, 4) = Q110(I)
      A(I, 5) = Q111(I)
      A(I, 6) = Q112(I)
      A(I, 7) = Q210(I)
      A(I, 8) = Q211(I)
      A(I, 9) = Q212(I)
      A(I, 10) = T110(I)
      A(I, 11) = T111(I)
      A(I, 12) = T112(I)
      A(I, 13) = T210(I)
      A(I, 14) = T211(I)
      A(I, 15) = T212(I)
      A(I, 16) = 0.0
  490 CONTINUE
      A(15, 1) = 1.0
      A(15, 2) = 1.0
      A(15, 3) = 1.0
      A(15, 4) = 0.0
      A(15, 5) = 0.0
      A(15, 6) = 0.0
```

```
      A(15, 7) = -1.0
      A(15, 8) = -1.0
      A(15, 9) = -1.0
      A(15, 10) = -1.0
      A(15, 11) = -1.0
      A(15, 12) = -1.0
      A(15, 13) = 0.0
      A(15, 14) = 0.0
      A(15, 15) = 0.0
      A(15, 16) = 1.0 - GAMMA
      M = 15
      N = 16
C
C*************************************************************
C  WRITE THE INPUTED MATRIX FOR GAUSSIAN ELIMINATION
C  TO THE OPEN FILE PRINT.LST.
C
      WRITE (1, 500)
  500 FORMAT ('1', ' INPUTED MATRIX FOR GAUSSIAN',
     1' ELIMINATION')
      WRITE (1, 520) ((A(I, J), J = 1, 8), I = 1, M)
      WRITE (1, 520) ((A(I, J), J = 9, N), I = 1, M)
C
C*************************************************************
C  CALL THE GAUSS ELIMINATION SUBROUTINE AND PASS ALL THE
C  REQUIRED ARGUMENTS.
C
      CALL MATRIX (M, N)
C
C*************************************************************
C  WRITE OUT THE RETURNED MATRIX AND THE CORRESPONDING
C  SOLUTION TO THE SYSTEM OF EQUATIONS TO THE FILE
C  PRINT.LST.
C
      WRITE (1, 510)
  510 FORMAT ('1', ' REDUCED MATRIX FROM GAUSSIAN',
     1' ELIMINATION')
      WRITE (1, 520) ((B(I, J), J = 1, 8), I = 1, M)
      WRITE (1, 520) ((B(I, J), J = 9, N), I = 1, M)
  520 FORMAT (15('0', 8(E12.6, 4X),/))
      WRITE (1, 530)
  530 FORMAT ('1', ' MATRIX SOLUTION TO THE SYSTEM OF',
     1' EQUATIONS ',/)
      DO 540 I = 1, M
      WRITE (1, 140) I, XX(I)
      WRITE (5, 140) I, XX(I)
  540 CONTINUE
C
C*************************************************************
C  SOLVE FOR THE DIMENSIONLESS CURRENT ACROSS THE EP
C  PROBE USING THE CONSTANTS INT* AND THE SOLUTION TO
C  THE MATRIX.
C
      DIMCUR = (PI / 2.0 * XX(1)) + (INT1 * XX(2))
     1+ (INT2 * XX(3)) + (INT3 * XX(4)) + (INT4 * XX(5))
```

```
     2+ (INT5 * XX(6)) + (INT6 * XX(7)) + (INT7 * XX(8))
     3+ (INT8 * XX(9)) + (INT9 * XX(10)) + (INT10
     4* XX(11)) + (INT11 * XX(12)) + (INT12 * XX(13))
     5+ (INT13 * XX(14)) + (INT14 * XX(15)) - (ZJ * F
     6/(RU * TEMP) * GAMMA * VOLT * ZINTRH)
C
C*************************************************************
C SOLVE FOR THE CURRENT THROUGH THE EP PROBE.  AND
C WRITE IT IN THE FILE PRINT.LST.
C
      CURRNT = -(DIMCUR * PI * RO**2.0 / H * DJ * ZJ * F
     1* CJ)
      WRITE (5, 550) DIMCUR, CURRNT
      WRITE (1, 550) DIMCUR, CURRNT
  550 FORMAT ('0', ' DIMENSIONLESS CURRENT = ', G10.4,/,
     1'0', ' CURRENT THROUGH EP PROBE = ', G10.4, /, '1')
C
C*************************************************************
C CLOSE THE FILE PRINT.LST.
C
      CLOSE (UNIT=1, DISP='SAVE')
  555 CONTINUE
C
C*************************************************************
C CHECK IF ANOTHER CALCULATION NEEDS TO BE PERFORMED.
C
      WRITE (5, 560)
  560 FORMAT ($, ' Do you wish to continue with other',
     1' parameter values? [Y/N]:  ')
      READ (5, 20) YESNO
      IF (YESNO .EQ. 'Y') GO TO 210
C
C*************************************************************
C IF NO THEN EXIT THE PROGRAM
C
  570 STOP
  998 STOP '...ERROR OPENING DATA FILE'
  999 STOP '...ERROR CLOSING DATA FILE'
      END
```

APPENDIX B

SUBROUTINE FOR MATRIX MANIPULATION

```
C*****************************************************************
C                                                                 *
C  CREATED BY W. F. BETHMANN, JR.                                 *
C                                                                 *
C  THIS IS A SUBROUTINE THAT SOLVES A SYSTEM OF LINEAR            *
C  EQUATIONS BY GAUSS ELIMINATION.  THE REDUCED MATRIX            *
C  IS RETURNED ALONG WITH THE SOLUTION VECTOR TO THE              *
C  MAIN PROGRAM.                                                  *
C                                                                 *
C*****************************************************************
C
      SUBROUTINE MATRIX(M, N)
C
C*****************************************************************
C  INITIALIZE AND DIMENSION THE VARIABLES AND ARRAYS.
C
      COMMON/EPPRAM/A(25,25),B(25,25),XX(25)
      REAL*8 A, B, XX, TERM, BIG, TEMP, PIVOT, YY, CONST
      LAST = M - 1.0
C
C*****************************************************************
C  START OVERALL LOOP FOR (M - 1) PIVOTS.
C
  DO 80 I = 1, LAST
C
C*****************************************************************
C  FIND THE LARGEST REMAINING TERM IN THE I-TH COLUMN
C  FOR PIVOT.
C
      BIG = 0.0
      DO 20 K = I, M
      TERM = ABS(A(K, I))
      IF (TERM - BIG) 20, 20, 10
 10   BIG = TERM
      L = K
 20   CONTINUE
C
C*****************************************************************
C  CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND.
C
      IF (BIG) 50, 30, 50
 30   WRITE (5, 40)
 40   FORMAT (' A ZERO TERM WAS FOUND. EXITING DUE TO',
     1' DIVISION',/,' BY ZERO...')
      RETURN
C
C*****************************************************************
C  L-TH ROW HAS THE BIGGEST TERM -- IS I = L?
C
```

```
 50   IF ((I - L) .EQ. 0.0) GO TO 70
C
C*************************************************************
C  I IS NOT EQUAL TO L, SWITCH ROWS I AND L.
C
      DO 60 J = 1, N
      TEMP = A(I, J)
      A(I, J) = A(L, J)
      A(L, J) = TEMP
 60   CONTINUE
C
C*************************************************************
C  NOW START PIVOTAL REDUCTION.
C
 70   PIVOT = A(I, I)
      NEXTR = I + 1.0
C
C*************************************************************
C  FOR EACH OF THE ROWS AFTER THE I-TH.
C
      DO 80 J = NEXTR, M
C
C*************************************************************
C  MULTIPYING CONSTANT FOR THE J-TH ROW IS.
C
      CONST = A(J, I) / PIVOT
C
C*************************************************************
C  NOW REDUCE EACH TERM OF THE J-TH ROW.
C
      DO 80 K = I, N
      A(J, K) = A(J, K) - (CONST * A(I, K))
 80   CONTINUE
      DO 85 I = 1, M
      DO 85 J = 1, N
      B(I, J) = A(I, J)
 85   CONTINUE
C
C*************************************************************
C  END OF PIVOTAL REDUCTION.
C  PERFORM BACK SUBSTITUTION.
C
      DO 110 I = 1, M
C
C*************************************************************
C  IREV IS THE BACKWARD INDEX, GOING FORM M BACK TO 1.
C
      IREV = M + 1.0 - I
C
C*************************************************************
C  GET Y(IREV) IN PREPARATION.
C
      YY = A(IREV, N)
      IF ((IREV - M) .EQ. 0.0) GO TO 100
C
```

```
C*****************************************************************
C  NOT WORKING ON LAST ROW, I IS 2 OR GREATER.
C
      DO 90 J = 2, I
C
C*****************************************************************
C  WORK BACKWARD FOR X(N), X(N - 1) ----, SUBSTITUTING
C  PREVIOUSLY FOUND VALUES.
C
      K = N + 1.0 - J
      YY = YY - (A(IREV, K) * XX(K))
 90   CONTINUE
C
C*****************************************************************
C  FINALLY, COMPUTE X(IREV).
C
 100  XX(IREV) = YY / A(IREV, IREV)
 110  CONTINUE
      RETURN
      END
```

APPENDIX C

SUBROUTINE FOR HYPERBOLIC BESSEL FUNCTION

```
C*****************************************************************
C                                                                *
C  CREATED BY W. F. BETHMANN, JR.                                *
C                                                                *
C  THIS IS A SUBROUTINE THAT CALCULATES THE HYPERBOLIC           *
C  BESSEL FUNCTION OF ORDER ZERO AND ORDER ONE                   *
C  EVALUATED AT INPUTTED ARGUMENT VALUES.  FOR THIS              *
C  APPLICATION THE VALUE OF THE ARGUMENT IS                      *
C  PI/2.0 * R/H WHERE R/H IS THE RATIO OF THE ELECTRODE          *
C  RADIUS TO THE THICKNESS OF THE WATER FILM.                    *
C                                                                *
C  ONLY 2 FUNCTION VALUES ARE EVALUATED;  AT X = 0.0,            *
C  AND X = 1.0.                                                  *
C                                                                *
C  THE VALUES RETURNED TO THE MAIN PROGRAM ARE:                  *
C  HYP0(1) = HYPERBOLIC BESSEL OF ORDER ZERO AT X = 0.0          *
C  HYP0(2) = HYPERBOLIC BESSEL OF ORDER ZERO AT X = 1.0          *
C  HYP1(1) = HYPERBOLIC BESSEL OF ORDER ONE AT X = 0.0           *
C  HYP1(2) = HYPERBOLIC BESSEL OF ORDER ONE AT X = 1.0           *
C  ZINTRH  = THE CONSTANT INT(r/h)                               *
C                                                                *
C*****************************************************************
C
      SUBROUTINE BESSEL(RH, N, HYP00, HYP01, HYP10, HYP11,
     1ZINTRH)
C
C*****************************************************************
C  INITIALIZE AND DIMENSION ALL VARIABLES AND ARRAYS.
C
      DIMENSION X(201), TERM(100), TM(100), HYP0(201)
      DIMENSION HYP1(2)
      REAL*8 RH, PI, X, C1, C2, TERM, TM, HYP1, HYP0
      REAL*8 HYPBES, HYP, HYP00, HYP01, HYP10, HYP11
      REAL*8 ZINTRH, AREA, ANS, DX, XVAL
      TERM(1) = 1.0
      PI = 3.1415926
C
C*****************************************************************
C  MAKE AN X(K) ARRAY FOR ALL X VALUES BETWEEN ZERO
C  AND 1.0.
C
      K = 200
      X(1) = 0.0
      DX = 1.0 / K
      DO 5 I = 2, (K + 1)
      X(I) = X(I - 1) + DX
 5    CONTINUE
C
C*****************************************************************
```

```
C CALCULATE ALL COEFFICIENTS IN THE HYPERBOLIC BESSEL
C OF ORDER ZERO AND STORE IN THE ARRAY TERM(N).
C
      DO 10 I = 2, (N + 1)
      C1 = (I - 1.0)**2.0
      C2 = PI**2.0
      TERM(I) = (TERM(I - 1) / C1) * C2 * RH * RH / 16.0
 10   CONTINUE
C
C*****************************************************************
C EVALUATE THE MODIFIED BESSEL FUNCTION OF ORDER
C ZERO USING ALL THE COEFFICIENTS CALCULATED AT THE
C X VALUES AND STORE THESE VALUES IN AN ARRAY
C HYPO(K).
C
      DO 20 M = 1, (K + 1)
      HYPBES = TERM(1)
      DO 20 J = 2, (N + 1)
      XVAL = X(M)**((J - 1) * 2.0)
      HYPO(M) = HYPBES + (TERM(J) * XVAL)
      HYPBES = HYPO(M)
 20   CONTINUE
C
C*****************************************************************
C EVALUATE THE INTEGRAL UNDER THE CURVE
C Io((PI/2.0)*(r/h)*X)*X*DX FROM 0.0 TO 1.0.
C
      AREA = 0.0
      DO 25 I = 1, K
      AREA = AREA + ((HYPO(I) * X(I)) + (HYPO(I + 1)
     1* X(I + 1))) / 2.0 * DX
 25   CONTINUE
C
C*****************************************************************
C NORMALIZE THE AREA TO THE VALUE OF HYPO(K + 1).
C
      ANS = AREA / HYPO(K + 1)
C
C*****************************************************************
C COMPUTE INT(r/h).
C
      ZINTRH = PI / 2.0 * ANS
C
C*****************************************************************
C CALCULATE ALL COEFFICIENTS IN THE HYPERBOLIC BESSEL
C OF ORDER ONE AND STORE IN THE ARRAY TM(N).
C
      TM(1) = (PI / 2.0) * (RH / 2.0)
      DO 30 I = 2, N
      TM(I) = TM(I - 1) / (I * (I - 1)) * C2 * RH / 16.0
     1* RH
 30   CONTINUE
C
C*****************************************************************
C EVALUATE THE MODIFIED BESSEL FUNCTION OF ORDER ONE
```

```
C USING ALL THE COEFFICIENTS CALCULATED AT THE TWO
C X VALUES AND STORE THESE VALUES IN AN ARRAY HYP1(K).
C
      HYP1(1) = 0.0
      HYP = 0.0
      DO 40 I = 1, N
      HYP1(2) = HYP + TM(I)
      HYP = HYP1(2)
 40   CONTINUE
C
C*************************************************************
C RETURN TO MAIN PROGRAM WITH THE VALUES HYPO(1), HYPO(2),
C HYP1(1), AND HYP1(2).
C
      HYP00 = HYPO(1)
      HYP01 = HYPO(K + 1)
      HYP10 = HYP1(1)
      HYP11 = HYP1(2)
      RETURN
      END
```

APPENDIX D

MAIN PROGRAM INPUT PARAMETERS

THIS IS A FILE THAT IS OPENED BY THE MAIN PROGRAM AND READ THE PARAMETERS MAY BE CHANGED IN THE MAIN PROGRAM PRIOR TO THE ANALYSIS IF THEY ARE NOT CORRECT.

INPUT PARAMETERS:

RO	=	0.252
H	=	0.8
UBELT	=	10.0
VOLT	=	100.0
DJ	=	0.00001
GAMMA	=	0.1E-10
ZJ	=	100.0
F	=	96517.0
RU	=	8.3170
TEMP	=	373.0
CJ	=	.374902E-7

THESE ARE 14 POINTS IN THE DOMAIN OF A RIGHT CIRCULAR CYLINDER ABOVE THE ELECTRODE SURFACE USED TO CREATE 14 INDEPENDENT EQUATIONS IN THE SOLUTION.

	X(I)	Y(I)	Z(I)	R(Y)
POINT(1)	-1.5708	1.5708	0.0	1.0
POINT(2)	1.5708	1.5708	0.0	1.0
POINT(3)	-1.5708	0.7854	0.0	1.0
POINT(4)	1.5708	0.7854	0.0	1.0
POINT(5)	0.0	1.5708	-1.5708	1.0
POINT(6)	0.0	1.5708	0.0	0.0
POINT(7)	0.0	1.17081	1.5708	1.0
POINT(8)	0.0	0.7854	-1.5708	1.0
POINT(9)	0.0	0.7854	0.0	0.0
POINT(10)	0.0	0.3927	1.5708	1.0
POINT(11)	-1.11072	1.5708	-1.11072	1.0
POINT(12)	-1.11072	0.7854	1.11072	1.0
POINT(13)	1.11072	1.5708	-1.11072	1.0
POINT(14)	1.11072	0.7854	1.11072	1.0

VITA

William F. Bethmann, Jr., was born in Franklin, Tennessee, on September 24, 1956. He attended first and second grade in Nashville, Tennessee. His family then moved to Tullahoma, Tennessee, where he completed his secondary education, graduating from Tullahoma High School in 1974. The following year he received an athletic scholarship to play basketball at Motlow State Community College in Tullahoma where he completed an Associate of Science degree in Biology in 1976. The next fall, he entered Wabash College in Crawfordsville, Indiana, with an Athletic Merit Award to continue playing basketball and complete his bachelor's degree. In 1978 he received a Bachelor of Arts degree in Biology. In the fall of 1979, he was accepted by the Graduate School at The University of Tennessee, Knoxville, and accepted employment at Union Carbide Corporation, Nuclear Division, in Oak Ridge, Tennessee, in December 1980. He received a Master of Science degree with a major in Engineering Science in December 1982.

He is married to the former Ginger Lee Gordon of Christiana, Tennessee, and has two boys, Cortney and Andrew.